ANALYSIS OF AN OPTIONS CONTRACT IN A DUAL SOURCING SUPPLY CHAIN UNDER DISRUPTION RISK

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ABSTRACT

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In this study, value of demand information and the importance of option contracts are investigated for a supply chain consisting of a buyer and two suppliers in a single period setting. One supplier is cheap but prone to disruptions whereas the other one is perfectly reliable but expensive. At the beginning of the period, buyer orders from the unreliable supplier and reserves from the reliable supplier through a contract that gives buyer an option to use reserved units after getting disruption information of first supplier. We introduce three models which differ in terms of the level of information available when the ordering decisions are made. In the full information model, the options are exercised after getting disruption and demand information; in the partial information model, the options are exercised after getting disruption information before demand information. In the no information model, there is no options contract and units are ordered from the reliable supplier when buyer has no information about demand and disruption. Through the analysis of these models, we explore the value of advance demand and disruption information in the presence of an options contract.

Keywords: Option contract, Value of Information, Supply disruption

ÖZ

ÇİFT KAYNAKLI BİR TEDARİK ZİNCİRİNDE KESİNTİ RİSKİ ALTINDA OPSİYON SÖZLEŞMESİ ANALİZİ

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Bu çalışmada, bir üreticinin ve iki tedarikçinin olduğu bir tedarik zinciri tasarlanmış olup, talep bilgisinin değeri ve opsiyon sözleşmesinin önemi tek dönemlik bir çerçevede araştırılmıştır. Tedarikçilerden birisi ucuzdur fakat kesintilere uğrayabilir; diğer tedarikçi ise tam olarak güvenilirdir fakat pahalıdır. Dönemin başında ilk olarak, üretici güvenilir olmayan tedarikçiye ürün siparişi verir ve güvenilir tedarikçiden birinci tedarikçinin kesintiye uğrayıp uğramadığını öğrendikten sonra sipariş edebilmek üzere ürün rezerv eder. Bu rezervasyon bir opsiyon sözleşmesi ile yapılır. Ana olarak opsiyon sözleşmesi kullanıldığında, talep ve kesinti belirsizliklerinin belli olup olmadığına bağlı olarak değişen üç model tasarlanmıştır. Birincisi tam bilgi modelidir ve opsiyonlar talep ve kesinti bilgisi öğrenildikten sonra kullanılır. İkincisi kısmi bilgi modelidir ve opsiyonlar kesinti bilgisi öğrenildikten sonra, talep bilgisi öğrenilmeden önce kullanılır. Üçüncüsü sıfır bilgi modelidir ve opsiyon sözleşmesi yoktur; ürünler üçüncü tedarikçiden talep ve kesinti bilgisi öğrenilmeden sipariş edilir. Talep bilgisinin değeri ve kesintilerin tedarik zincirine verdiği etkiler araştırılmıştır.

Anahtar Kelimeler: Opsiyon sözleşmesi, Bilgi değeri, Tedarik kesintileri

To my family

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CHAPTER 1

INTRODUCTION

A company should specify and mitigate uncertainties that exist in its supply chain system in order to meet customer needs. Supply uncertainty is one of the most important difficulties to the firms on meeting customer needs. Traditional inventory models consider demand uncertainty and show how to design supply chain systems to mitigate that risk. However, the effects of supply uncertainty can have serious detriments against design and during production, possibly more than demand uncertainty can have. Recent studies in supply chain literature have begun to consider supply uncertainty as a major topic. Real life events and theoretical studies both demonstrated its impacts in supply chains when firms fail to protect or mitigate against it.

Supply uncertainty can occur in several forms. Disruptions, yield uncertainty, capacity uncertainty, lead time uncertainty, input cost uncertainties are the main forms of supply uncertainty. Disruptions are random events that cause a supplier to stop producing completely. That is, the supplier cannot produce any product during a disruption. Yield uncertainty is the event when the supplier cannot send all amount of the quantity ordered, it sends partial amount of the quantity ordered. The quantity sent is stochastic, which can be independent of or proportional to the order quantity in an additive or multiplicative fashion. So, it can be said that disruption is a special case of yield uncertainty when the quantity sent is equal to zero. This thesis considers all-or-nothing supply. Like the other analytical models, when the supplier does not face disruption, it delivers an order in full otherwise nothing can be supplied.

Real life experiences have demonstrated that supply disruptions occur during conditions such as natural disasters (earthquake, fire etc.), terrorist attacks, war and strike. For example, consider the disruption in the Toyota supply chain on Feb. 01, 1997. A fire at the Aisin Seiki Co. destroyed most of the capacity to manufacture P-valves. Because of the Aisin's ability to produce parts at low cost, Toyota had come to rely on Aisin for this product (Sheffi, 2007). According to the Wall Street Journal, Toyota officials called different part makers to obtain P-valves, including Somic (Reitman, 1997). Somic had the flexibility to free up machines and shift its production line to make P-valves. On Feb. 06, right on schedule, it delivered its first P-valves to Toyota (Reitman, 1997). In 2000, lightning caused a fire that shut down the Philips Semiconductor plant in Albuquerque, New Mexico, for six weeks, leading to a shortage of components for both Ericsson and Nokia. According to The Wall Street Journal, “company officials say they [Ericsson] lost at least $400 million in potential revenue” and “when the company revealed the damage from the fire for the first time publicly last July, its shares tumbled 14% in just hours” (Latour 2001). Hurricane Mitch caused catastrophic damage to banana production in many parts of Central America in 1998. It took many growers over a year to recover, leading to a prolonged loss of supply for Dole and Chiquita (Griffy-Brown 2003). An earthquake in Taiwan severely disrupted supply of essential components to the personal computer industry leading up to the 1999 holiday season (Burrows 1999).

There is a common and easy way for yield uncertainty solution: safety stock, that is, giving an additional order amount calculated by the cost and variability parameters beside the optimal order quantity. However, disruption requires stronger strategic decisions and models than yield uncertainty, and safety stock by itself is not sufficient. Mitigation strategies against supply disruptions are classified in Snyder et al. (2010):

1) Inventory: Extra inventory can be held so as to meet some critical customer's need during a disruption if the firm does not apply any other strategy like backup supplier option. However, as it is mentioned before only extra inventory cannot be the solution to supply disruption.

2) Diversification in Sourcing
   i) Routine Sourcing: Firms regularly source raw materials from more than one supplier. If one supplier faces disruption, the firm is not left completely without products since the other supplier(s) may still be up. Ordering process from either supplier or suppliers is done at the beginning and at the same time for all suppliers. Order quantity decision is made for each supplier and order quantities to the non-disrupted suppliers cannot be changed after a disruption occurrence in any supplier.
   ii) Contingent Sourcing: It is almost the same as routine sourcing. However, in the case of contingent sourcing, if one supplier faces disruption, the order from the non-disrupted suppliers can be changed from the pre-disruption levels.

3) Information Sharing: It is obtaining and assessing information about the disruption risk of suppliers. It can be achieved by monitoring suppliers to anticipate potential disruptions and adopt better strategies.

4) Demand Substitution: If one product of a supplier faces disruption, the firm can introduce an inferior product with decreased price or superior product with the same price.

5) Financial Mitigation: Firms may purchase some insurance to protect from disruptions.

6) Acceptance: Sometimes, the cost of mitigating disruptions is too high to do it. So, the risk can be accepted in some cases.

This thesis mainly focuses on the mitigation strategy, contingent sourcing. Being able to change the order to the reliable suppliers is the important issue of contingent sourcing. One way to do this is option contracts. In the stock and commodity markets, options contracts come in different forms. Option contract used in this thesis gives the holder of the option the choice of buying or not buying stock or commodity at a fixed price for a fixed period of time. And a payment is done for every fixed stock in the option. It is the cost of this strategy.

In this study, our primary aim is to analyze the benefits from delaying the time when the options are exercised. We consider a setting where demand uncertainty resolves after the disruption uncertainty and we introduce three models which differ in terms of the level of information available about the uncertainties when the ordering decisions are made. In the full information model, the options are exercised after getting disruption and demand information; in the partial information model, the options are exercised after getting disruption information before demand information. In the no information model, there is no options contract and units are ordered from the reliable supplier when buyer has no information about demand and disruption. That is, the option contract is utilized against both supply and demand uncertainties in the first model whereas it is used against only supply uncertainty in the second model. To perform our analysis, we consider a single-period problem with a buyer that faces random demand. The buyer has two alternative suppliers: one cheaper but prone to disruption and the other perfectly reliable but more expensive.

The remainder of the study is organized as follows. We present a review of the related studies in the literature and describe the main characteristics of the models that we analyze in Chapter 2. In Chapter 3, the no information model is introduced and analyzed. Chapter 4 includes the partial information model and Chapter 5 includes the full information model. To gain insights on the value of delaying the time when options are exercised, we perform a thorough computational analysis in Chapter 6. Finally, we conclude in Chapter 7 summarizing our major findings and offering further research directions.

CHAPTER 2

LITERATURE REVIEW AND PROBLEM ENVIRONMENT

This paper contributes to the important and growing research area of supply disruptions management. Dual sourcing mitigation strategy with options contract is used against the supply uncertainties. Supply disruption is a form of yield uncertainty so we first analyze the papers with only yield uncertainty and both of them, then the papers with only supply disruption in the literature review. We also look at how the papers that work on single sourcing mitigate supply uncertainties.

2.1. Studies with One Unreliable Supplier

Supply uncertainty was generally modeled as complete disruptions, where supply stops completely, or as yield uncertainty, where the supply quantity received varies stochastically. Early on, papers focusing on supply uncertainty usually consider single-supplier systems. (see for instance Bielecki and Kumar 1988, Parlar and Berkin 1991, Parlar and Perry 1995, Gupta 1996, Song and Zipkin 1996, Moinzadeh and Aggarwal 1997, Parlar 1997, Arreola-Risa and De Croix 1998, Schmitt, et al. 2010).Bielecki and Kumar (1988) shows that a policy similar to a zero inventory ordering policy is sometimes optimal for an unreliable supply chain system, contradicting the common belief that inventories are always valuable for buyers in uncertain environments. Parlar and Berkin (1991) is the first study that introduces disruption into the EOQ model. On and off periods of disruption have random lengths. They conclude that cost function is convex according to disruption on and off periods. Parlar and Perry (1995) extends EOQ model with disruption by allowing the reorder point to be a decision variable. Yano and Lee (1995) provide a literature review on quantitatively oriented approaches for determining lot sizes when production or supply yields are random. One of the latest papers, Schmitt, et al. (2010) considers three cases: (i) disruption, deterministic demand and deterministic supply yield, (ii) disruption, deterministic demand and stochastic supply yield and (iii) disruption, stochastic demand and deterministic supply yield. They find optimal base-stock inventory policies in a multi-period setting. There is always one unreliable supplier that is prone to disruptions. The objective is to find the parameters that make the service level maximum (minimum holding/penalty cost). Everything is deterministic in case (i), so if the firm decides to carry no safety stock, disruption would have the largest detrimental effect on the supply chain system. In the no disruption case and with equal standard deviation (either on the demand or the supply yield), cases (ii) and (iii) would stock the same safety stock quantity. Therefore they would be equally affected by disruptions. In this paper, it is concluded that the safety stock is maintained to compensate variability from demand or yield. Because, in a disruption case if the disruption lasts not short, all cases shortage as quantity of demand so disruptions should be mitigated regardless of other variability, demand or yield.

2.2. Routine Sourcing Papers

There is a growing body of literature that uses routine sourcing as a strategy to mitigate disruption risk. In routine sourcing, the buyer orders from multiple suppliers in the beginning and at the same time.

Anupindi and Akella (1993) study dual sourcing with unreliable suppliers. Different combinations of disruption and yield uncertainty result in three models. Each model has a single and a multi-period version. The first model assumes a delivery contract with each supplier that the supplier delivers a given order either in the current period with a given probability 1-p, or in the next period with probability p, when disruption occurs. It delivers nothing in the single period case when disruption occurs. The second and the third models consider yield uncertainty as well. Demand is stochastic and has a continuous distribution. The objective is to find the optimal orders from both suppliers that minimize the ordering, holding and penalty costs. They prove that the optimal ordering policy has three regions, based on the current on-hand inventory. Order nothing (if on-hand inventory is large enough), order only from the less expensive supplier (if on-hand inventory is moderate), and order from both suppliers (if on-hand inventory is small). Anupindi and Akella (1993) shows that the buyer should never order from the expensive supplier alone. Swaminathan and Shanthikumar (1999) study Anupindi and Akella's first model and show that, when the demand is deterministic, their ordering policy is no longer optimal. Swaminathan and Shanthikumar (1999) proved that ordering from only expensive supplier is a possible optimal solution. They also provide necessary conditions under which it is optimal to order at least some units from the more expensive supplier.

Lakovou et al. 2009 consider a single period supply chain system that consists of a manufacturer and two unreliable suppliers which are both prone to disruptions, such as production, transportation and security-related disruptions. They propose a single period system where a single ordering decision from both suppliers is to be made at the beginning of the period in order to maximize expected total profit. Disruption is modeled as each disruption may occur only once for each of the two suppliers, during the period with a probability. Furthermore, when a disruption occurs it is assumed that a constant percentage of the order quantity will be available in time to satisfy the demand during the period and the remaining quantity will be delivered at the end or after the end of the period.

There are also papers that use routine sourcing strategy with multiple (more than two) unreliable suppliers: See for instance Dada et al. 2007, Federgruen and Yang 2008, Federgruen and 2009, Tehrani et al. 2010. Dada et al. (2007) study a single-period model with multiple unreliable suppliers. The objective is to choose which suppliers to order from and in what quantities in order to maximize the expected profit (sales and salvage revenues minus holding and stock-out costs). Disruptions, yield uncertainty, and capacity uncertainty are all special cases in this paper. Demand is stochastic, with a continuous distribution. Two extensions about capacity uncertainty are studied in the paper. In the first extension, there exist multiple suppliers and each supplier's capacity is deterministic. In this case, the newsvendor orders as much as possible from the least expensive supplier. If the capacity of the least expensive supplier is not enough, then it orders as much as possible from the second least expensive supplier and so on. In the second extension, there exists only one supplier capacity of which is uncertain. In other words, it is not guaranteed that to receive amount ordered from the supplier. In this case, the newsvendor should order no less than the amount that would have been ordered from the supplier if the supplier’s capacity were deterministic. From the two cases, a major conclusion is that if a supplier is not chosen, then suppliers that are no more expensive than that supplier will be used whatever the reliability degree is. That is, cost trumps reliability. This result is parallel to the solution of Anupindi and Akella (1993). Another conclusion is: if a given supplier is reliable, then no more expensive suppliers than that reliable supplier will be used. They also show that the optimal order quantity is larger and the optimal service level is smaller with unreliable suppliers than it is for the classical newsboy problem. Because the variability in the demand is the same with the classical newsboy problem but the variability in the supply, capacity is extra and it is detrimental for the service level.

The model of Federgruen and Yang (2008) and Federgruen and Yang (2009) is similar to Dada et al. (2007). The suppliers are subject to yield uncertainty in the form of multiplicative yield with a general yield distribution. Disruptions are special case of yield uncertainty. Federgruen (2009) makes a modification of their 2008 model in design of cost model. The supply model is the same. In both study, they define a key quantity as “expected effective supply” that is the total expected yield from all suppliers. They find that total cost is a convex function of expected effective supply. In addition, they conclude that when the suppliers are sorted in increasing order of their per-unit costs divided by their yield factors, optimal suppliers include the first k suppliers, for some k.

Tehrani et al. (2010) consider a two-echelon inventory system with multiple unreliable suppliers. Suppliers are prone to common source disruptions so their production capacity (delivery quantities) is stochastically dependent. This is the extra uncertainty considered by Tehrani et al. (2010) in addition to Dada et al. (2007)'s uncertainties. It considers a single-period model. Two cases are studied, namely, the multi-source and assembly supply chains. In the multi-source structure, suppliers produce and send the same product to the buyer. In the assembly structure, each supplier produces a different part of the product, so the end units manufactured by the buyer equal the minimum of the supplier's order delivery quantity. For each supply chain structure, the objective is to search the impact of the dependence in capacities, induced by common-source disruptions, on the important performance measures of the buyer such as service level and to find the optimal ordering policy. They conclude that the stochastic dependence between suppliers' disruption probability has opposite impacts on the system in the two structures: total disruption risk of the assembly system increases as the level of dependence increases in the multi-source supply chain. Furthermore, as disruption risks become more dependent, the buyer should order less in the multi-source supply chain but order more in the assembly supply chain.

Berger et al. (2004), Ruiz-Torres and Mahmoodi (2007) investigate the optimal number of suppliers to use from a number of unreliable suppliers. Berger et al. (2004) assume an operating cost that is a function of number of used suppliers. Another special cost is the fixed penalty cost when all of the used suppliers disrupt simultaneously. Ruiz-Torres and Mahmoodi (2007) study a similar model with Berger et al. (2004) but one thing is different: the fixed penalty cost is incurred when only some suppliers disrupt. The main conclusion of these papers is that optimal number of unreliable suppliers used is small generally.

2.3. Studies on Contingent Rerouting

Papers that study supply disruptions that used contingent sourcing as a mitigation strategy emerged in the literature recently. The basic idea of the contingent sourcing strategy briefly is: if one supplier disrupts, the order from the non-disrupted suppliers can be changed from the pre-disruption levels. We consider such papers in more detail as our work also invokes contingent rerouting.

Tomlin (2006) discusses three strategies to overcome supply disruptions: inventory (stocking), contingent sourcing and acceptance. He studies a single-product system with two suppliers one of which is reliable that has capacity flexibility (i.e., cannot increase production levels quickly). The other one is unreliable and cheaper. The objective is to find the optimal choice among three strategies for different cases: reliable supplier has flexibility in capacity, has no flexibility in capacity, stochastic demand. Tomlin concluded that supplier's percentage uptime and the disruption behavior (frequent short or rare long) are the two main factors which determine the optimal strategy (Please see Table 2.1).

Table 2.1 Summary of Conclusions of Tomlin (2006)

	Unreliable Supplier	Reliable Supplier	Disruption behavior	**Optimal Strategy**
1	Infinite capacity/Disruption	Finite capacity	Frequent and Short	Inventory or Acceptance
2	Infinite capacity/Disruption	Finite capacity	Rare and Long	Contingent Sourcing
3	Infinite capacity/Disruption	Infinite capacity	Frequent and Short	Contingent Sourcing
4	Infinite capacity/Disruption	Infinite capacity	Rare and Long	Contingent Sourcing

Chopra et al. (2007) study a single-period model in which one supplier is subject to both yield and disruption uncertainties and the other is perfectly reliable. In contrast to Tomlin's models, both yield and disruption uncertainties are unresolved when the buyer places an order to the first supplier. This model also requires the buyer to reserve a maximum order size with the reliable supplier at a given reservation price. When order comes from the first supplier, the buyer can exercise up to the reservation quantity (maximum order size) from the reliable supplier if demand cannot be met from the first supplier's delivery. This paper assumes demand is deterministic. The objective is to find the utilization proportion of reliable and unreliable suppliers in different situations: for example decoupling risks with disruption and yield uncertainty, increasing disruption probability etc. Conclusions of this paper expand Dada's conclusions by separately considering whether the supply risk is yield uncertainty or because of disruption. When the increase in supply uncertainty is from an increase in yield uncertainty, increased use of the cheaper supplier is optimal. When the increase in supply uncertainty is from disruption, increased use of reliable supplier is optimal. In this paper, it is concluded that reliability trumps cost.

Schmitt and Snyder (2009) claim that disruptions have a significant impact on future periods, and planning for these disruptions can have a significant impact on order quantities of all periods. They also claim that one-period models are suitable for the perishable product systems or the systems that disruptions last relatively short. They extend Chopra et al. (2007) to a multi-period setting. Multi-period and single-period models' solutions are compared and it is concluded that a single-period approximation causes increases in cost, under-utilization of unreliable supplier, and spoils order quantities that is to be placed to the reliable supplier.

Qi (2009) develops a model different than the standard contingent rerouting. There are two suppliers one of which is reliable the other one is unreliable. In the standard contingent sourcing models, when unreliable supplier disrupts, the buyer's only option is to order immediately from the reliable supplier. Qi (2009) considers another option: in a disruption case the buyer can wait a while up to the unreliable supplier recovers itself. In contrast to Qi's model, in our models, there is no such a chance that in a disruption case the buyer can wait a while up to the unreliable supplier recovers itself. Demand is considered as deterministic in Qi(2009). Disruption duration is distributed exponentially. In the model, the duration that the buyer waits before ordering from the reliable supplier is a decision variable. In this multi-period setting, the buyer follows an (s, S) type review policy, with different S values depending on from which supplier order is set. Qi (2009) concluded that it is always optimal for the firm to either order from the reliable supplier immediately after the safety stock runs out or wait as long as necessary until the unreliable supplier recovers.

Hou et al. (2010) consider a two-stage supply chain with dual sourcing system. There are two suppliers one of which is main supplier that prone to disruptions, other one is back-up supplier with which the buyer can sign a buy-back contract. They consider two types of risk, namely disruption risk which results in a zero delivery and yield uncertainty which is reflected in an uncertain delivery volume.

Some papers study contingent sourcing and try to evaluate the value of advance warnings of disruptions. It is the comparison of the cases whether the exact time of disruption, disruption duration or the probability of disruption is known or not.

Snyder and Tomlin (2008) investigate how inventory systems can be designed to take advantage of advanced information of disruptions. They consider a system with an unreliable supplier that is subject to complete disruptions and a reliable supplier that is perfectly reliable. The disruption profile can change over time. Advance information of disruption means that buyer is informed about disruption characteristics continuously, which constitute what Snyder and Tomlin call "threat level", change stochastically over time, and the buyer knows the current threat level continuously. They model the system using a discrete-time Markov chain for the disruption distribution where states correspond to threat levels for disruption. The main conclusion is that advanced information is extremely beneficial and allows superb cost savings especially when the disruption probabilities are significantly different in different states. Another conclusion is that the benefit of the advanced information decreases as the capacity decreases.

2.4. Studies on Advanced Information

Saghafian and Van Oyen (2011) investigate two significant remedies to increase supply chain effectiveness. First one is contracting an option contract with a reliable supplier that can produce two products but having a shared capacity for two products. Second one is obtaining advanced disruption information as an extra mitigation strategy. There are two products and two unreliable suppliers of them. There is one reliable backup supplier that can produce both products. However, this supplier is more expensive than others. The option contract is done with this reliable backup supplier: the buyer pays a fixed reservation price to the supplier at the beginning of the contract in return for the delivery of any desired portion of the reserved shared capacity for two products at an additional purchasing price. In other words, the reliable supplier can mitigate the risk of disruption in unreliable suppliers while reducing the cost of keeping excess inventory. Demand for both products is stochastic. The problem is studied in a single period context. The objective is to find optimal values of order quantities from unreliable suppliers and capacity reserved from the reliable supplier in the first remedy. Comparing and valuing advanced information of disruption is the objective in the second remedy. It is observed that investing in a reliable backup capacity can be detrimental if the advanced information about the disruption risk of unreliable suppliers is not perfect. Monitoring unreliable suppliers to obtain better disruption estimates increases the benefit of reserving reliable backup capacity. Additionally, advanced information about disruption risk is more valuable to firms with low profit margins than those with high ones. They also showed that when suppliers are (truly) reliable enough, obtaining information is a better mitigation strategy than contracting with a reliable supplier. Another conclusion is that when unreliable suppliers are reliable enough, contracting with an expensive reliable supplier is not advantageous. However, obtaining advanced disruption risk information is still advantageous because it helps the firm to make better ordering decisions.

Our first model is closely linked to the work of Saghafian and Van Oyen (2011)'s single product special case. Our partial information model distinguishes from this work and the literature about the advanced information part. Saghafian and Van Oyen (2011) study to value advanced information of disruption risk information. However, we study to value advanced information of demand and disruption both.

2.5. Problem Environment

We consider a two-stage supply chain consisting of a single buyer and two suppliers one of which is unreliable and prone to disruption whereas the other is perfectly reliable. When disruption occurs, supply from the first supplier is zero. The buyer faces stochastic demand in a single period, which is modeled as a continuous random variable having nonnegative support (The notation is presented in Table 2.2). Contingent rerouting mitigation strategy is adopted in our paper. The buyer has a wholesale-price only contract with the unreliable supplier and options contract with the reliable supplier. The options contract provides flexibility to the supplier against uncertainties in the system.

In our models, disruption and demand uncertainties are unresolved when the buyer places an order to the unreliable supplier. In the first model, disruption and demand uncertainties are unresolved when the buyer places order from the reliable supplier similar to unreliable supplier. In other words, there is no options contract. We call this model as "No Information Model". In the second model, option-exercise takes place after supply disruption information is received but before demand uncertainty is not resolved. Hence, we call this model as "Partial Information Model". When both uncertainties are resolved, the buyer can exercise up to the reserved amount from the reliable supplier if demand cannot be met from the first supplier's delivery. We call this model as "Full Information Model". Please see Figure 2.1 for the timeline of events for each model.

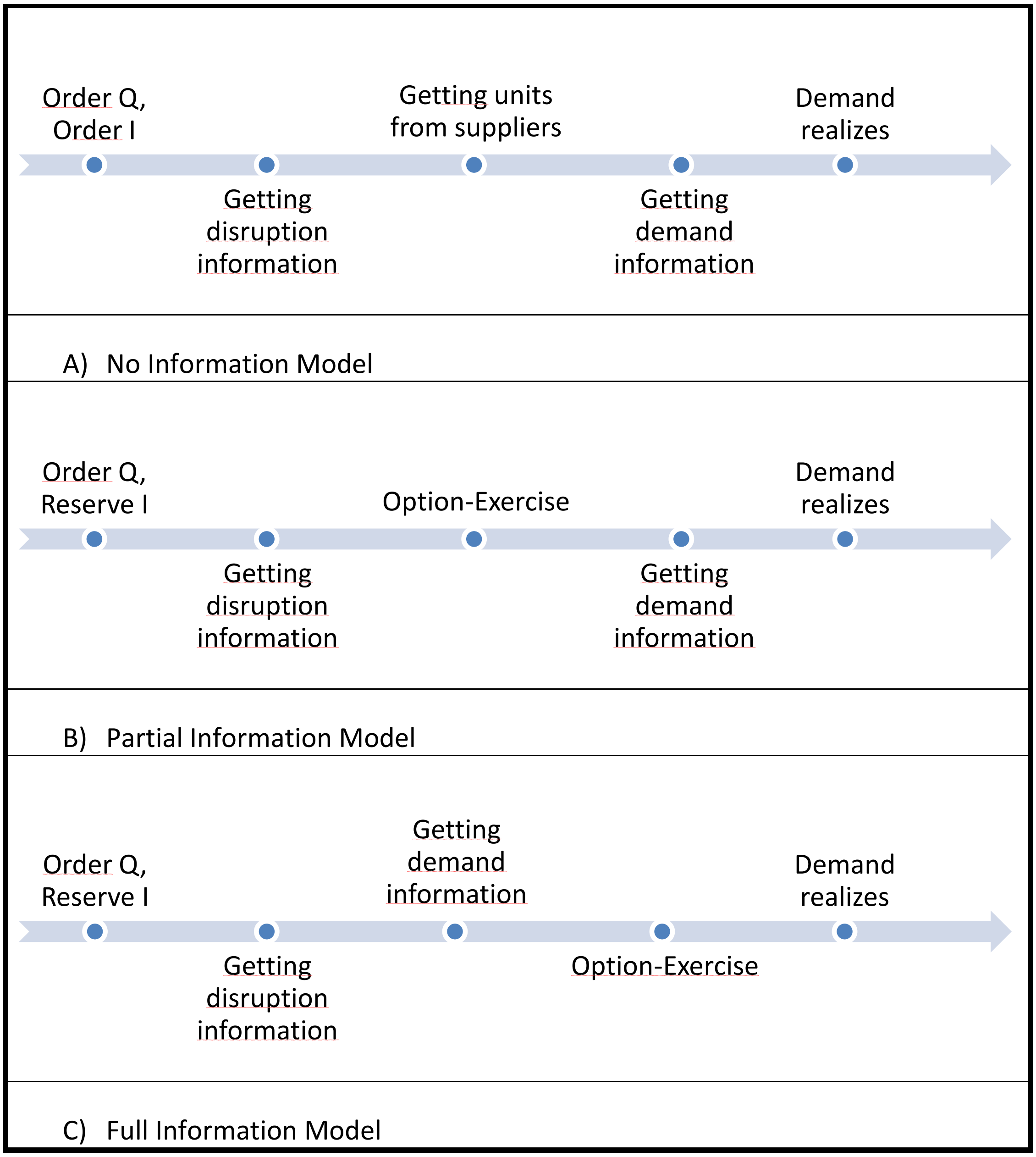

Figure 2.1 Timeline of Events

In all models, the cost of reserving plus exercising one unit of product from the reliable supplier is higher than the cost of buying one unit of product from the unreliable supplier, $c < e + h.$ Otherwise, the buyer would never use the unreliable supplier. Revenue earned from one unit is bigger than cost incurred from one unit in both supplier, $c < r$ and $e + h < r$. We assume that there is no salvage value and loss sales cost although these can be easily incorporated into our models.

Our study makes a key contribution to the literature in that option contract can be used only against disruption uncertainty. In our partial information model, demand is realized after the buyer exercises options from the second supplier. Hence, the options contract becomes an action against only the disruption uncertainty. It does not mitigate demand uncertainty because demand uncertainty has not resolved yet when options are exercised. However, both uncertainties are resolved when options are exercised in our full information model. By this way, we study the value of advanced demand information via comparison of two models. The difference between two models is whether the buyer exercises options before getting information on demand or not. So, profit difference between two models is considered as the value of deciding how many to exercise with demand information.

Table 2.2 Used Notations

X	Random variable representing customer demand
$f(x)$	Probability density function (p.d.f) of random variable X
$F(x)$	Cumulative distribution function (c.d.f) of random variable X
Y	Bernoulli random variable representing supply disruption where $Y = 1$ denotes supply disruption and $Y = 0$ denotes otherwise.
Q	Order quantity from the unreliable supplier
I	Reservation quantity from the reliable supplier
p	First supplier disruption probability
c	Cost per unit from the first supplier
h	Cost per unit reserved from the second supplier
e	Cost per unit exercised from the second supplier
r	Revenue per unit

CHAPTER 3

NO INFORMATION MODEL

In the no information model, the buyer makes order decisions from both suppliers in a situation that he knows nothing; he doesn't know disruption occurrence information and demand uncertainty has not resolved yet. This model aims to generate insights about a two supplier supply chain system; one of them is cheap but unreliable, the other is reliable but expensive. There is no option contract in other words there is no option to exercise after the uncertainties resolved. Buyer has to decide how many units to order (exercise) from the reliable supplier at the beginning. The sequence of events in this setting is as follows:

1. The buyer orders Q units from the first supplier and orders I units from the second supplier.
2. The buyer gets disruption information about the first supplier.
3. The buyer gets I units from the second supplier, Q units from the first supplier if disruption doesn't occur.
4. The buyer gets full information about demand.
5. Demand realizes.
6. The buyer satisfies the demand.

The buyer's objective is to maximize its expected profit. To characterize its expected profit, we work on order quantities from the two suppliers. As the uncertainty in supply is discrete, we examine each case (disruption and no disruption) separately. After that we take expectation with respect to random variable Y (supply disruption) to get the expected profit.

3.1. Analysis of the Disruption Case

Buyer orders Q and I from the suppliers respectively at the beginning. Then, buyer gets disruption information. When disruption occurs, supply from the first supplier is zero, from the second supplier is I. It can be thought that buyer have to exercise all of the ordered quantity from the second supplier. In disruption case, the corresponding profit realization is given by

$$\prod(Q, I|Y = 1, X = x) = -(e + h)I + r\min\{x, I\}.$$

Given that disruption has occurred, taking expectation with respect to X, we get

$$\prod(Q, I|Y = 1) = -(e + h)I + r\left(\int_{I}^{\infty} I\, f(x)dx + \int_{0}^{I} xf(x)dx\right). \qquad (3.1)$$

Lemma 3.1.1. $\prod(Q, I|Y = 1)$ is jointly concave in Q and I.

Proof: First order and second order partial derivatives with respect to Q and I are given below:

$$\frac{\partial\prod(Q,I)}{\partial Q} = 0,$$

$$\frac{\partial\prod(Q,I)}{\partial I} = [1 - \mathrm{F}(I)]r - (e + h),$$

$$\frac{\partial^2\prod(Q,I)}{\partial Q^2} = 0,$$

$$\frac{\partial^2\prod(Q,I)}{\partial I^2} = -rf(I) < 0,$$

$\frac{\partial^2 \prod (Q,I)}{\partial Q \partial I} = \frac{\partial^2 \prod (Q,I)}{\partial I \partial Q}$ = 0.

Determinant of the Hessian matrix is:

$$\left(\frac{\partial^2 \Pi}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi}{\partial I^2}\right) - \left(\frac{\partial^2 \Pi}{\partial Q \partial I}\right)^2 = 0.$$

Hence, $\prod(Q, I|Y = 1)$ is jointly concave in Q and I. ■

3.2 Analysis of the No Disruption Case

When disruption does not occur, supply from the first supplier is Q, from the second supplier is I. In the no disruption case, the corresponding profit realization is given by

$$\prod(Q, I|Y = 0, X = x) = r \min\{x, Q + I\} - cQ - (e + h)I.$$

Given that disruption has not occurred, taking expectation with respect to X, we get

$$\prod(Q, I|Y = 0, X = x) = r\left(\int_0^{Q+I} x\, f(x)dx + \int_{Q+I}^{\infty} (Q + I) f(x)dx\right) - (e + h)I - cQ \qquad (3.2)$$

Lemma 3.2.1. $\prod(Q, I|Y = 0)$ is jointly concave in Q and I.

Proof: First order and second order partial derivatives with respect to Q and $I$are shown below:

$\frac{\partial \prod (Q,I)}{\partial Q} = r[1 - \mathrm{F}(Q + I)] - c,$

$\frac{\partial \prod (Q,I)}{\partial I} = r\,[1 - \mathrm{F}(Q + I)] - e - h,$

$\frac{\partial^2 \prod (Q,I)}{\partial Q^2} = -r\, f(Q + I) < 0,$

$\frac{\partial^2 \prod (Q,I)}{\partial I^2} = -r\, f(Q + I) < 0,$

$\frac{\partial^2 \prod (Q,I)}{\partial Q \partial I} = \frac{\partial^2 \prod (Q,I)}{\partial I \partial Q} = -(r-e)f(Q+I).$

Determinant of the Hessian matrix is given as follows:

$$\left(\frac{\partial^2 \Pi}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi}{\partial I^2}\right) - \left(\frac{\partial^2 \Pi}{\partial Q \partial I}\right)^2 = (2re - e^2)\, f(Q+I) > 0.$$

Hence, $\prod(Q, I|Y=0)$ is jointly concave in Q and I. ■

3.3. Analysis of the No Information Model

In this section, we formulate and analyze the no information model by combining the results of Section 3.1 and Section 3.2. Utilizing Equation (3.1) and Equation (3.2), and taking expectation with respect to Y, the expected profit of the buyer can be characterized as

$$\prod(Q,I) = -(e+h)I - (1-p)cQ + p\left[r\left(\int_I^\infty I\, dF(X) + \int_0^I x\, dF(X)\right)\right] + (1-p)\left[r\left(\int_0^{Q+I} x\, dF(X) + \int_{Q+I}^\infty (Q+I)\, dF(X)\right)\right].$$

Then, the buyer's problem is

Max $\prod (Q,I)$

Subject to $Q, I \geq 0$.

Let Q^* (I^*) denote the optimal order (reservation) quantity from the unreliable (reliable) supplier. Then, the following theorem characterizes the optimal solution to the buyer's problem.

Theorem 3.3.1. If $-h + p(r-e) + (1-p)(c-e) < 0$, then $I^* = 0$ and $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$. Otherwise, $I^* = F^{-1}\left[\frac{-h+p(r-e)+(1-p)(c-e)}{pr}\right]$ and $Q^* = F^{-1}\left(\frac{r-c}{r}\right) - I^*$.

Proof: We have shown that $\prod(Q, I|Y = 0)$ and $\prod(Q, I|Y = 1)$ are jointly concave in Lemma 3.1.1 and Lemma 3.2.1; hence, the objective function is strictly concave. Also, constraints are linear. There exists a unique optimal solution to the unconstrained problem and it is given by the unique solution to $\frac{\partial \Pi}{\partial I} = 0$ and $\frac{\partial \Pi}{\partial Q} = 0$. From $\frac{\partial \Pi}{\partial I} = 0$, we have $I' = F^{-1}\left[\frac{-h+p(r-e)+(1-p)(c-e)}{pr}\right]$ and from $\frac{\partial \Pi}{\partial Q} = 0$, we have $Q' = F^{-1}\left(\frac{r-c}{r}\right) - I'.I^* = I'$ when $F^{-1}\left[\frac{-h+p(r-e)+(1-p)(c-e)}{pr}\right] > 0$. Otherwise, inequality constraint becomes binding and determines optimal solution as $I^* = 0$. Hence, we can argue that, when $-h + p(r-e) + (1-p)(c-e) < 0$, $I^* = 0$ and $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$, which completes the proof. ■

CHAPTER 4

PARTIAL INFORMATION MODEL

In the partial information model, buyer makes exercise decision from the second supplier in a situation that he only has disruption occurrence information and demand uncertainty has not resolved yet. Options contract is available and the buyer can reserve at the beginning and exercise after demand uncertainty is resolved. This model aims to generate insights about option contract management when it is settled only for disruption uncertainty; in other words option contract is used to mitigate only disruption uncertainty. Demand uncertainty still stays as a risk factor for the system when option contract is used. The sequence of events in this setting is as follows:

1. The buyer orders Q units from the first supplier and reserves I units from the second supplier.
2. The buyer gets disruption information about the first supplier.
3. The buyer determines the number of options to exercise, $E \leq I$.
4. The buyer gets full information about demand.
5. Demand realizes.
6. The buyer satisfies the demand.

The buyer's objective is to maximize its expected profit. To characterize its expected profit, we consider a two-stage problem setting. We work backwards and start with the second stage problem which is to determine the optimal exercise quantity, E.

Then, we continue with the first stage problem to characterize the optimal order quantity *Q,* and, reservation quantity, I, from the associated suppliers.

4.1. Second Stage Problem

The second stage problem is to determine the optimal number of options to exercise given the condition of the first supplier. It should be noted that the order and reservation quantities are fixed at this stage. Hence, only relevant costs and revenues are the unit revenue, *r*, and unit cost of exercising an option, *e*.

The second stage problem is actually the same as newsvendor problem where the number of copies of the day's paper to stock is determined in the face of uncertain demand and knowing that unsold copies will be worthless at the end of the day. In our problem, the decision of how many newspapers to stock corresponds to the decision of how many to exercise from the second supplier. Demand is uncertain and both decisions are made before demand realization. Buyer knows how many he already has (in no disruption we have Q, in disruption we have zero.) So, we can write the optimal exercise quantity for each case (disruption and no disruption) which maximizes the expected profit as the same with newsvendor problem. The buyer faces only uncertain demand and demand distribution, revenue, unit cost are the factors that influence the exercise decision.

Given that disruption has occurred ($Y = 1$), the buyer faces a capacitated newsvendor problem and the corresponding optimal number of options to exercise is given by $(E^*|Y = 1, Q, I) = \min\left\{F^{-1}\left(\frac{r-e}{r}\right), I\right\}$.

Given that disruption has not occurred ($Y = 0$); the buyer's problem is $\min_{E \leq I}\{\prod(E|Y = 0, Q, I)\}$ where $\prod(E|Y = 0, Q, I) = rE[\min\{Q + E, X\}] - \mathrm{e}E$. The problem is quite similar to the newsvendor problem and the optimal number of options to exercise is $\max\left\{0, \min\left\{F^{-1}\left(\frac{r-e}{r}\right) - Q, I\right\}\right\}$.

4.2. First Stage Problem

We start the analysis of the first stage problem by characterizing the expected profit of the buyer given the condition of the first supplier. After that, taking expectation with respect to *Y*, we characterize the expected profit in the first stage as a function of Q and I.

Let $A = F^{-1}\left(\frac{r-e}{r}\right)$. Given that disruption has occurred $(Y = 1)$ and recalling that $(E^*|Y = 1, Q, I) = \min\{A, I\}$, the expected profit is

$$\prod(Q, I|Y = 1) = -hI + \int_{0}^{min\{A,I\}} rx\, f(x)dx + \int_{min\{A,I\}}^{\infty} r\, min\{A, I\} f(x)dx - e\, min\{A, I\}$$

Given that disruption has not occurred (*Y*=0) and taking expectation with respect to *X*, the expected profit is

$$\prod(Q, I|Y = 0) = \int_{0}^{E^*+Q} rx\, f(x)dx + \int_{E^*+Q}^{\infty} r(E^* + Q)f(x)dx - eE^* - cQ - hI$$

where $E^* = \max\{0, \min\{A - Q, I\}\}$.

Let Q^* (I^*) denote the optimal order (reservation) quantity from the unreliable (reliable) supplier. Then, following Lemma simplifies the buyer's problem.

Lemma 4.2.1. The optimal reservation quantity cannot exceed $A = F^{-1}\left(\frac{r-e}{r}\right)$.

Proof: In the analysis of the second stage problem (Section 4.1), we have shown that the optimal number of options to exercise will never exceed *A*. Then, we can deduce that the options reserved in excess of A will never be utilized. Hence, it is never optimal to reserve more than A. ■

We can now characterize the expected profit of the buyer in the first stage. Due to Lemma 4.2.1, we have

$$\prod(Q,I) = -hI - (1-p)cQ + p\left(\int_0^I rx\,f(x)dx \;+\; \int_I^\infty rI\,f(x)dx \;-eI\right)$$

$$+(1-p)\begin{cases}\prod_1(Q,I) & A < Q\\ \prod_2(Q,I) & Q \le A < Q+I\\ \prod_3(Q,I) & A \ge Q+I\end{cases}$$

where

$$\prod_1(Q,I) = \int_0^Q rx\,f(x)dx \;+\; \int_Q^\infty rQ\,f(x)dx \tag{4.1}$$

$$\prod_2(Q,I) = \int_0^A rx\,f(x)dx + \int_A^\infty rA\,f(x)dx - e(A-Q) \tag{4.2}$$

$$\prod_3(Q,I) = \int_0^{Q+I} rx\,f(x)dx \;+\; \int_{Q+I}^\infty r(I+Q)f(x)dx \;\; - eI. \tag{4.3}$$

Then, the buyer's problem can be formulated as

Max $\prod(Q,I)$

subject to $\quad 0 \le I \le A$

$\qquad\qquad\qquad Q \ge 0.$

Lemma 4.3.1. $\prod(Q,I)$ is continuously differentiable.

Proof: For continuity, it is sufficient to check the breakpoints $Q = A$ and $Q + I = A$. For$Q + I = A$, we evaluate $\prod_2(Q,I)$ and $\prod_3(Q,I)$, and observe that

$$\prod\nolimits_2(Q,I) = \prod\nolimits_3(Q,I) = \int_0^A rx\,f(x)dx \;+\; \int_A^\infty rA\,f(x)dx \;\; - eI\,.$$

Hence, $\prod(Q,I)$ is continuous at the points $Q + I = A$.

For $Q = A$, we evaluate $\prod_1(Q,I)$ and $\prod_2(Q,I)$, and observe that

$$\prod\nolimits_1(Q,I) = \prod\nolimits_2(Q,I) = \int_0^A rx\,f(x)dx \;+\; \int_A^\infty rA\,f(x)dx.$$

Hence, $\prod(Q,I)$ is continuous at the points $Q = A$, which completes the proof for continuity.

For differentiability, the first derivatives of $\prod(Q,I)$ with respect to Q and I are given below:

$$\frac{\partial\prod(Q,I)}{\partial Q} = -(1-p)c + (1-p)\begin{cases} r\big(1-F(Q)\big) & ,A < Q \\ e & ,Q < A < Q + I \\ r\big(1-F(Q+I)\big) & ,A > Q + I \end{cases} \tag{4.4}$$

$$\frac{\partial\prod(Q,I)}{\partial I} = -h + p[r[1-F(I)] - e] + \begin{cases} 0 & ,A < Q \\ 0 & ,Q < A < Q + I \\ (1-p)[r[1-F(Q+I)] - e] & ,A > Q + I \end{cases} \tag{4.5}$$

It is sufficient to check the breakpoints $Q = A$ and $Q + I = A$.

At points $Q + I = A$, we evaluate $\frac{\partial \prod_2(Q,I)}{\partial Q}$, $\frac{\partial \prod_3(Q,I)}{\partial Q}$ and observe that $\frac{\partial \prod_2(Q,I)}{\partial Q} = \frac{\partial \prod_3(Q,I)}{\partial Q} = (1-p)e$ since $F(A) = \frac{r-e}{r}$ due to the definition of A.

At points $Q = A$, we evaluate $\frac{\partial \prod_1(Q,I)}{\partial Q}$, $\frac{\partial \prod_2(Q,I)}{\partial Q}$ and observe that $\frac{\partial \prod_1(Q,I)}{\partial Q} = \frac{\partial \prod_2(Q,I)}{\partial Q} = (1-p)e$ since $F(A) = \frac{r-e}{r}$ due to the definition of A. Hence, $\prod(Q,I)$ is differentiable at the points $Q = A$.

At points $Q + I = A$, we evaluate $\frac{\partial \prod_2(Q,I)}{\partial I}$, $\frac{\partial \prod_3(Q,I)}{\partial I}$ and observe that $\frac{\partial \prod_2(Q,I)}{\partial I} = \frac{\partial \prod_3(Q,I)}{\partial I} = 0$ since $F(A) = \frac{r-e}{r}$ due to the definition of A. Hence, $\prod(Q,I)$ is differentiable at the points $Q + I = A$, which completes the proof. ■

Theorem 4.3.3. For a given $I \leq A$, if $e > c$, then $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$. Otherwise, $Q^* = \max\left\{F^{-1}\left(\frac{r-c}{r}\right) - I, 0\right\}$.

Proof: We consider the case $e > c$ first. For $Q \leq A - I$, $\frac{\partial \prod(Q,I)}{\partial Q} = (1-p)[r(1-F(Q+I)) - c] > 0$ since $Q + I \leq A$ and $e > c$. For $A - I \leq Q \leq A$, $\frac{\partial \prod(Q,I)}{\partial Q} = (1-p)(e-c) > 0$. Hence, Q^* should be in the region (A,∞). For $Q > A$, $\frac{\partial \prod(Q,I)}{\partial Q} = (1-p)[r(1-F(Q)) - c] > 0$. Setting $\frac{\partial \prod(Q,I)}{\partial Q} = 0$, we get $Q^* = F^{-1}\left(\frac{r-c}{r}\right) > A$ since $e > c$.

Next we consider the case $e < c$. For $Q > A$, we have $(1-p)[r(1-F(Q)) - c] < 0$ since $e < c$ and $Q > A$. For $A - I \leq Q \leq A$, $\frac{\partial \prod(Q,I)}{\partial Q} = (1-p)(e-c) < 0$. Hence Q^* should be in the region $[0, A - I]$. For $Q \leq A - I$, $\frac{\partial \prod(Q,I)}{\partial Q} = (1-p)[r(1-F(Q+I)) - c] < 0$. Setting $\frac{\partial \prod(Q,I)}{\partial Q} = 0$, we get $Q' = F^{-1}\left(\frac{r-c}{r}\right) - I$. Then, $Q^* = \{Q', 0\}$. ■

Theorem 4.3.4. For a given $Q < A$, if $-h + p(r-e) + (1-p)[r[1-F(Q)] - e] < 0$, then $I^* = 0$. Otherwise, if $F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right] > F^{-1}\left(\frac{r-e}{r}\right) - Q$, then $I^* = F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right]$ otherwise, I^* is given by the unique solution to $-h + p[r[1-F(I)] - e] + (1-p)[r[1-F(Q+I)] - e] = 0$.

Proof: Consider the case where $-h + p(r-e) + (1-p)[r[1-F(Q)] - e] < 0$. Then, for $I \leq A - Q$, $\frac{\partial^2 \prod(Q,I)}{\partial I^2} < 0$ and $\frac{\partial \prod(Q,I)}{\partial I}(I = 0) = 0$.Hence, $\prod(Q,I)$ is decreasing in I for $I \leq A - Q$. For $I > A - Q$, $\frac{\partial^2 \prod(Q,I)}{\partial I^2} < 0$. Since the derivative is continuous, $\prod(Q,I)$ is decreasing in I for $I \leq A - Q$.

Hence, $(I^* \mid Q < A) = 0$ if $-h + p(r-e) + (1-p)[r[1-F(Q)] - e] < 0$.

Now, consider the case where $-h + p(r-e) + (1-p)[r[1-F(Q)] - e] > 0$. For $I > A - Q, \frac{\partial \prod(Q,I)}{\partial I} = -h + p[r[1-F(I)] - e]$.

Setting $\frac{\partial \prod(Q,I)}{\partial I} = 0$, we get $I' = F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right]$. If $I' > A - Q$, then it is the optimal solution. Otherwise the optimal solution is in the region $I \leq A - Q$ and can be obtained by $\frac{\partial \prod(Q,I)}{\partial I} = 0$. ■

Theorem 4.3.5 For a given $Q > A$, if $-h + p(r-e) < 0$ then $I^* = 0$. Otherwise, $I^* = F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right]$.

Proof: Given $Q > A, \frac{\partial \prod(Q,I)}{\partial I} = -h + p[r[1-F(I)] - e]$.

If $-h+p(r-e)<0$, $\frac{\partial \prod(Q,I)}{\partial I}<0 \forall I$; hence $(I^* \mid Q>A)=0$. Otherwise since $\frac{\partial^2 \prod(Q,I)}{\partial I^2}<0$, we set $\frac{\partial \prod(Q,I)}{\partial I}=0$, which completes the proof. ■

Corollary 4.3.1. The optimal order and reservation quantities under partial information are given by:

(i) If $e>c$, $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$, $I^*=\begin{cases}0, & if -h+p(r-e)<0\\ F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right], & otherwise\end{cases}$

(ii) If $e<c$,

$$(Q^*,I^*)=\begin{cases}\left(F^{-1}\left(\frac{r-c}{r}\right),0\right) & if\ -h+p(r-e)+(1-p)(c-e)<0\\ F^{-1}\left(\frac{r-c}{r}\right)-F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right], F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right] & otherwise\end{cases}$$

Proof: If $e>c$, the optimal order and reservation quantities directly follow from Theorem 4.3.3 and Theorem 4.3.5.

We next consider the case with $e<c$. If $-h+p(r-e)+(1-p)(c-e)>0$, we have $-h+p(r-e)+(1-p)[r(1-F(Q))-e]>0$ for all $Q\leq F^{-1}\left(\frac{r-c}{r}\right)$. From Theorem 4.3.3, we have $Q^*+I^*=F^{-1}\left(\frac{r-c}{r}\right)<A$. Hence, we have $I^*<A-Q$ and I^* is given by $-h+p[r[1-F(I)]-e]+(1-p)[r[1-F(Q+I)]-e]=0$. (4.10)

Since $Q^*+I^*=F^{-1}\left(\frac{r-c}{r}\right)$, Equation (4.10) reduces to $-h+p(r-e)+(1-p)(c-e)=0$, which results in $I^*=F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$.

For $-h+p(r-e)+(1-p)(c-e)<0$, consider the candidate solution $(Q^*,I^*)=\left(F^{-1}\left(\frac{r-c}{r}\right),0\right)$. From Theorem 4.3.3, we have $Q^*=F^{-1}\left(\frac{r-c}{r}\right)-I^*$ and (Q^*,I^*) satisfies it. $Q'<A$ since $e<c$. For $=Q'$, we have $-h+p(r-e)+(1-p)[r(1-F(Q))-e]<0$. Hence from Theorem 4.3.4, we have$I^*=0$ and (Q',I')

satisfies it. Since Q', I' satisfy the optimality conditions in Theorem 4.3.3 and 4.3.4, we can conclude that it is optimal. ■

This corollary demonstrates that optimal order quantity is always greater than zero in partial information as it is in full information with stochastic demand. This is because cost per unit from the first supplier is always less than the second supplier. Buyer always wants to buy from the cheaper supplier and compensate the disruption risk of the cheaper supplier with the option contract.

Expected profit per unit if the buyer uses second supplier rather than first supplier is $-h + p(r-e) + (1-p)(c-e)$ when $e < c$, otherwise it is $-h + p(r-e)$. Buyer earns $r - e$ for one exercised unit in disruption case because there is no opportunity other than using second supplier, $c - e$ in no disruption case because there is an opportunity to supply from the first supplier so opportunity cost of using second supplier rather than first supplier, $c - e$. We take expectation according to *Y* and the expected profit becomes $-h + p(r-e) + (1-p)(c-e)$. However, in no disruption case buyer always use first supplier when $e > c$ because he doesn't want to give more money when he has an opportunity to supply from the first supplier. Hence, expected profit per unit is $-h + p(r-e)$ when $e > c$.

Expected profit per unit if the buyer uses second supplier rather than first supplier

$$= \begin{cases} -h + p(r-e) + (1-p)(c-e) & , \quad if\ e < c \\ -h + p(r-e) & , \quad if\ e > c \end{cases}$$

If this profit is positive in either case, buyer always reserves from the second supplier. Otherwise, he reserves nothing.

This condition that makes buyer not reserve from the second supplier are the same as in full information with deterministic demand. We realize that buyer reserves nothing in the same condition even if buyer doesn't get demand information and demand is uncertain while exercising from the second supplier.

Total amount of units ordered or reserved, $Q^* + I^*$, converges to $F^{-1}\left(\frac{r-c}{r}\right)$ except when $e > c$ and $-h + p(r-e) > 0$. In this exception case, $Q^* + I^* > F^{-1}\left(\frac{r-c}{r}\right)$. This result is quite similar to the optimal solution of full information model with deterministic demand. Total amount of units ordered or reserved converges to demand quantity, *D*, except when $e > c$ and $-h + p(r-e) > 0$ on full information with deterministic demand. In this exception case $Q^* + I^* = 2D$.

Corollary 4.3.2. $Q^*(I^*)$ is non-increasing (non-decreasing) in the unit cost, *c*

Proof: From corollary 4.3.1, we have two different alternatives solution to Q^*; for $-h + p(r-e) + (1-p)(c-e) < 0$, $Q^* = F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$ which can be written as $F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right]$, otherwise $Q^* = \max\left\{F^{-1}\left(\frac{r-c}{r}\right), 0\right\}$. In either case, we can argue that Q^* is non-increasing in the unit cost, *c*. From corollary 4.3.1, we have two different alternative solutions to I^*, for $e > c$, $I^* = \max\left\{F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right], 0\right\}$ and for $e < c$, $I^* = \max\left\{F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right], 0\right\}$ which can be written as $\max\left\{F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right], 0\right\}$. In either case, we can argue that I^* is non-decreasing in the cost, *c*, which completes the proof. ■

Corollary 4.3.3. $Q^*(I^*)$ is non-decreasing (non-increasing) in the reservation cost, *h* and exercise price, *e*.

Proof: The proof is similar to the proof of Corollary 4.3.2 and it is presented in Appendix E. ■

Corollary 4.3.4. $Q^*(I^*)$ is non-increasing (non-decreasing) in the disruption probability, p

Proof: The proof is similar to the proof of Corollary 4.3.1 and it is presented in Appendix F. ■

CHAPTER 5

FULL INFORMATION MODEL

In the full information model, we assume the buyer makes the decision of how many products to exercise from the second supplier in an environment that he knows everything about the uncertain matters, disruption occurrence or not occurrence and demand uncertainty. The order of events is as follows:

1. The buyer orders *Q* units from the first supplier and reserves *I* units from the second supplier.
2. The buyer gets disruption information about the first supplier.
3. The buyer gets full information about demand.
4. The buyer orders $E \leq I$ units from the second supplier.
5. Demand realizes.

The buyer's objective is to maximize its expected profit. To characterize its expected profit, we work backwards and start with the optimal exercise quantity, *E*. As the uncertainty in supply is discrete, we examine each case (disruption and no disruption) separately. After that we take expectation with respect to random variable *Y* (supply disruption) to get the expected profit.

5.1. Analysis of the Disruption Case

When disruption occurs, supply from the first supplier is zero. After full demand information is received, buyer decides how many units to exercise from the second supplier and it differs according to demand and reservation quantity. Given that disruption has occurred, the optimal exercise quantity is simply given as follows:

$(E^*|Y = 1, X = x) = \min\{x, I\}$, and the corresponding profit realization is given by

$\prod(Q, I|Y = 1, X = x) = -hI + (r - e)\min\{x, I\}$.

Given that disruption has occurred, taking expectation with respect to X, we get

$$\prod(Q, I|Y = 1) = -hI + (r - e)\left(\int_{I}^{\infty} I\, f(x)dx + \int_{0}^{I} xf(x)dx\right) \tag{5.1}$$

Lemma 5.1.1. $\prod(Q, I|Y = 1)$ is jointly concave in Q and I.

Proof: First order and second order partial derivatives with respect to Q and I are given below:

$\frac{\partial \prod (Q,I)}{\partial Q} = 0,$

$\frac{\partial \prod (Q,I)}{\partial I} = [1 - \mathrm{F}(I)](r - e) - h,$

$\frac{\partial^2 \prod (Q,I)}{\partial Q^2} = 0,$

$\frac{\partial^2 \prod (Q,I)}{\partial I^2} - (r - e)f(I) < 0,$

$\frac{\partial^2 \prod (Q,I)}{\partial Q \partial I} = \frac{\partial^2 \prod (Q,I)}{\partial I \partial Q} = 0.$

Determinant of the Hessian matrix is:

$\left(\frac{\partial^2 \prod}{\partial Q^2}\right)\left(\frac{\partial^2 \prod}{\partial I^2}\right) - \left(\frac{\partial^2 \prod}{\partial Q \partial I}\right)^2 = 0.$

Hence, $\prod(Q, I|Y = 1)$ is jointly concave in Q and I. ■

5.2. Analysis of the No Disruption Case

In the no disruption case, supply from the first supplier is *Q*. After the demand is realized, buyer decides how many to exercise from the second supplier and it differs according to demand quantity, reservation quantity and order quantity. Since reliable supplier is more expensive, the options will be exercised only if demand exceeds the order quantity from the first supplier, *Q*. Hence, the optimal quantity of options exercised will be

$$(E^*|Y = 0, X = x) = \max\{0, \min\{I, x - Q\}\},$$

and the corresponding profit realization is given by,

$$\prod(Q, I|Y = 0, X = x) = r\, min\{x, Q + I\} - e \max\{0, \min\{I, x - Q\}\} - cQ - hI.$$

Given that disruption has not occurred, taking expectation with respect to X, we get

$$\prod(Q, I|Y = 0) = -hI - cQ + \int_0^Q rx\, f(x)dx + \int_Q^{I+Q} (rx - ex + eQ)f(x)dx + \int_{I+Q}^{\infty} (rI + rQ - eI)f(x)dx \quad (5.2)$$

Lemma 5.1.2. $\prod(Q, I|Y = 0)$ is jointly concave in *Q* and I.

Proof: First order and second order partial derivatives with respect to *Q* and I are shown below:

$$\frac{\partial\prod(Q,I)}{\partial Q} = e[\mathrm{F}(Q + I) - \mathrm{F}(Q)] + r[1 - \mathrm{F}(Q + I)] - c,$$

$$\frac{\partial\prod(Q,I)}{\partial I} = [1 - \mathrm{F}(Q + I)](r - e) - h,$$

$$\frac{\partial^2\prod(Q,I)}{\partial Q^2} = -(r - e)f(Q + I) - ef(Q) < 0,$$

$$\frac{\partial^2\prod(Q,I)}{\partial I^2} = -(r - e)f(Q + I) < 0,$$

$\frac{\partial^2 \prod(Q,I)}{\partial Q \partial I} = \frac{\partial^2 \prod(Q,I)}{\partial I \partial Q} = -(r-e)f(Q+I).$

Determinant of the Hessian matrix is given as follows:

$$\left(\frac{\partial^2 \Pi}{\partial Q^2}\right)\left(\frac{\partial^2 \Pi}{\partial I^2}\right) - \left(\frac{\partial^2 \Pi}{\partial Q \partial I}\right)^2 = \mathrm{e}(r-e)f(Q)\, f(Q+I) > 0.$$

Hence, $\prod(Q, I|Y=0)$ is jointly concave in Q and I. ■

5.3. Analysis of the Full Information Model

In this section, we formulate and analyze the full information model by combining the results of Section 5.1 and Section 5.2.Utilizing Equation (5.1) and Equation (5.2), and taking expectation with respect to Y, the expected profit of the buyer can be characterized as

$$\prod(Q,I) = -hI - (1-p)cQ + p\left[\int_I^\infty (r-e)\, I\, dF(X) + \int_0^I (r-e)x dF(X)\right]$$
$$+(1-p)\left[\int_0^Q rx\, dF(X) + \int_Q^{I+Q} (r-e)x + eQ\, dF(X) + \int_{I+Q}^\infty (r-e)I + eI dF(X)\right].$$

Then, the buyer's problem is

Max $\prod(Q,I)$

Subject to $\quad Q,I \geq 0$

Let Q^* (I^*) denote the optimal order (reservation) quantity from the unreliable (reliable) supplier. Then, the following theorem characterizes the optimal solution to the buyer's problem.

Theorem 5.3.1. If $< \frac{rh-c(r-e)}{(r-e)(r-c)}$, then $I^* = 0$ and $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$. Otherwise, we have $I^*, Q^* > 0$ and given by the unique solution to $\frac{\partial \Pi}{\partial I}$= 0 and $\frac{\partial \Pi}{\partial Q}$= 0.

Proof: Since the objective function is strictly concave and the constraints are linear, there exists a unique optimal solution to the buyer's problem which can be characterized by the following KKT conditions:

$$r - e - h - (r-e)[pF(I) + (1-p)F(Q+I)] + \lambda_1 = 0 \tag{5.3}$$

$$(1-p)[r - c - (r-e)F(Q+I) - eF(Q)] + \lambda_2 = 0 \tag{5.4}$$

$$\lambda_1 I = 0 \tag{5.5}$$

$$\lambda_2 Q = 0 \tag{5.6}$$

$$\lambda_1, \lambda_2, Q, I \geq 0 \tag{5.7}$$

We next analyze four possible cases (i) $\lambda_1, \lambda_2 > 0$ (ii) $\lambda_1 = 0, \lambda_2 > 0$ (iii) $\lambda_1 > 0, \lambda_2 = 0$ (iv) $\lambda_1 = 0, \lambda_2 = 0$.

(i) When $\lambda_1, \lambda_2 > 0$, we have $I = 0$ and $Q = 0$ from Equation (5.5) and Equation (5.6), respectively. Then, Equation (5.3) reduces to $\lambda_1 = -r + h + e < 0$, which violates the nonnegativity condition. Hence, $(Q, I) = (0,0)$ cannot be optimal.

(ii) When $\lambda_1 = 0, \lambda_2 > 0$ we have $I \geq 0$ and $Q = 0$ from Equation (5.5) and Equation (5.6), respectively. Then, from Equation (5.3), we get $F(I) = \frac{r-h-e}{r-e}$. Plugging it into Equation (5.4), we obtain $\lambda_2 = -(1-p)[e + h - c] < 0$, which violates the nonnegativity condition. Hence, $(Q, I) = (0, I)$ cannot be optimal.

(iii) When $\lambda_1 > 0, \lambda_2 = 0$, we have $I = 0$ and $Q \geq 0$ from Equation (5.5) and Equation (5.6), respectively. Then, Equation (5.4) reduces to $r - c -$

$rF(Q) = 0$, or $F(Q) = \frac{r-c}{r}$. Plugging this into Equation (5.3), we get $r - h - e - (r - e)(1 - p)\frac{r-c}{r} + \lambda_1 = 0$. The nonnegativity condition on λ_1 is satisfied if and only if $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$. Hence, we can conclude that $(Q, I) = (F^{-1}\left(\frac{r-c}{r}\right), 0)$ is the unique optimal if $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$ since all KKT conditions are satisfied.

(iv) Since the existence and uniqueness of the optimal solution is already established, we can deduce that when $p > \frac{rh-c(r-e)}{(r-e)(r-c)}$, KKT conditions are satisfied when $\lambda_1 = \lambda_2 = 0$. Then, Equation (5.3) and Equation (5.4) reduce to first order derivatives, which completes the proof. ■

Corollary 5.3.1. If $\frac{h}{c} < \frac{r-e}{r}$, the reliable supplier is always utilized.

Proof: If $\frac{h}{c} < \frac{r-e}{r}$, the condition $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$ can never hold since $p \geq 0$. Hence, we have $I^* > 0$, which completes the proof. ■

Contribution of one reserved product (from second supplier) and one ordered product (from first supplier) to the profit are $r - e$ and *r* respectively when demand is satisfied because, h and c are sunk costs at that time. The corollary says that the buyer always reserves from the second supplier if ratio of the contributions is greater than ratio of the sunk costs of them respectively.

Corollary 5.3.2. When both suppliers are reliable (*p* = 0), the option contract is utilized if and only if $\frac{h}{c} < \frac{r-e}{r}$.

Proof: The proof directly follows from Theorem 5.3.1 and Corollary 5.3.1. ■

When the first supplier is reliable like the second supplier, buyer uses the second supplier if and only if contribution of one reserved product from the second supplier to the profit is greater than one ordered product from the first supplier.

Since the solution is trivial when $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$, we continue our analysis for the cases with $p > \frac{rh-c(r-e)}{(r-e)(r-c)}$, where the optimal solution is given by the first order conditions.

Corollary 5.3.3. $Q^*(I^*)$ is non-increasing (non-decreasing) in the disruption probability, *p*

Proof: Let Q' and I' denote $\frac{dQ^*(p)}{\partial p}$ and $\frac{dI^*(p)}{\partial p}$ respectively. From $\frac{\partial \Pi\,(Q,I)}{\partial Q} = 0$,we have $(r-e)F(Q+I) + eF(Q) = r - c$.(5.8)

Then from the implicit derivative of $\frac{\partial \Pi\,(Q,I)}{\partial Q} = 0$ with respect to *p*, we have $(r-e)(\,I' + Q')f(I+Q) + eQ'f(Q) = 0$. Hence I' and Q'have opposite signs. Plugging (5.8) into $\frac{\partial \Pi\,(Q,I)}{\partial I} = 0$, we get $-(r-e)pF(I) - (1-p)\big(r - c - eF(Q)\big) = -r + h + e$.Taking the implicit derivative with respect to *p*, we have $-(r-e)F(I) - (r-e)pI'f(I) + r - c - eF(Q) + (1-p)eQ'f(Q) = 0$, which can be written as $-(r-e)[F(I) - pI'f(I) - F(Q+I)] + (1-p)eQ'f(Q) = 0$, noting that we have $r - c - eF(Q) = (r-e)F(Q+I)$ from Equation (5.8). In order for the last equality to hold, we need to have $Q' \leq 0$ and $I' \geq 0$, which completes the proof. ■

This corollary can be interpreted as follows: when the disruption probability increases, buyer reserves more from the second supplier which is quite intuitive because the possibility of meeting demand only from the second supplier is increased. Also, buyer should not increase its order from the first supplier because if it is increased and the first supplier does not face disruption, the buyer has to take all of the supply from the first supplier so cost of increased reserved quantity becomes a loss.

Corollary 5.3.4. $Q^*(I^*)$ is non-decreasing (non-increasing) in cost of per unit reserved from the second supplier, *h*.

Proof: The proof is similar to the proof of Corollary 5.3.3 and it is presented in Appendix A. ■

The buyer decreases its reservation quantity as the cost of per unit reserved from the second supplier increases when all other problem parameters are the same. In the case of decreased reservation quantity, buyer should increase order quantity from the first supplier in order to compensate it in a no disruption case.

Corollary 5.3.5. $Q^*(I^*)$ is non-increasing (non-decreasing) in cost of per unit from the first supplier, *c*.

Proof: The proof is similar to the proof of Corollary 5.3.3 and it is presented in Appendix B. ■

The buyer decreases its order as the cost of per unit ordered from the first supplier increases when all other problem parameters are the same. Loss due to unmet demand in a no disruption case because of the inadequacy of supply is less when cost of per unit ordered from the first supplier increases. Hence, buyer accepts this loss risk by decreasing order. In the case of decreased order quantity, buyer should increase reservation quantity from the second supplier in order to compensate it in each case, disruption and no disruption.

Corollary 5.3.6. $Q^*(I^*)$ is non-decreasing (non-increasing) in cost of per unit exercised from the second supplier, *e*.

Proof: The proof is similar to the proof of Corollary 5.3.3 and it is presented in Appendix C. ■

The buyer decreases its optimal reservation quantity as the cost of per unit exercised from the second supplier increases when all other problem parameters are the same. In this case the buyer should increase order quantity from the first supplier in order to compensate it in a no disruption case.

Corollary 5.3.7. $I^*$is non-decreasing in the revenue per unit, *r*.

Proof: The proof is similar to the proof of Corollary 5.3.3 and it is presented in Appendix D. ■

This corollary can be interpreted as follows: the buyer wants to increase reservation quantity as the revenue per unit increases. He earns more profit from one product sale if it is satisfied from reserved quantity. Although increasing reservation quantity increases the risk that more reserved quantity will be wasted when demand is satisfied from the first supplier, buyer increases reservation quantity as the revenue per unit increases. We expect Q^* is non-decreasing in the revenue but we cannot show analytically. Because when the revenue increases the buyer earns more profit from one product sale if it is satisfied from the first supplier's supply.

The summary of all analytical solution that we derived is depicted in Table 5.1.

Table 5.1 Summary of results on Q^* and I^* in all models

Optimal Quantities		Conditions			
		$p < \frac{rh-c(r-e)}{(r-e)(r-c)}$	$p > \frac{rh-c(r-e)}{(r-e)(r-c)}$	$p > \frac{rh-c(r-e)}{(r-e)(r-c)}$	
		-	$-h+p(r-e)+(1-p)(c-e)<0$	$-h+p(r-e)+(1-p)(c-e)>0$	
		-	-	$e<c$	$e>c$
No Information	Q^*	$F^{-1}\left(\frac{r-c}{r}\right)$	$F^{-1}\left(\frac{r-c}{r}\right)$	$F^{-1}\left(\frac{r-c}{r}\right)-F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$	$F^{-1}\left(\frac{r-c}{r}\right)-F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$
	I^*	0	0	$F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$	$F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$
Partial Information	Q^*	$F^{-1}\left(\frac{r-c}{r}\right)$	$F^{-1}\left(\frac{r-c}{r}\right)$	$F^{-1}\left(\frac{r-c}{r}\right)-F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$	$F^{-1}\left(\frac{r-c}{r}\right)$
	I^*	0	0	$F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$	$F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right]$
Full Information	Q^*	$F^{-1}\left(\frac{r-c}{r}\right)$	Unique solution to $\frac{\partial \Pi}{\partial Q}=0$	Unique solution to $\frac{\partial \Pi}{\partial Q}=0$	Unique solution to $\frac{\partial \Pi}{\partial Q}=0$
	I^*	0	Unique solution to $\frac{\partial \Pi}{\partial I}=0$	Unique solution to $\frac{\partial \Pi}{\partial I}=0$	Unique solution to $\frac{\partial \Pi}{\partial I}=0$

5.4. Deterministic Demand Case

Deterministic demand case is modeled in order to generate further analytical insights by reducing uncertainties to only disruption uncertainty. In such a case, the options contract with the reliable supplier serves as a means to protect against the uncertainty in supply only. We want to investigate optimal Q and Ivalues when demand is deterministic.

We can argue that the optimal order and reservation quantities from the first supplier and the second supplier, respectively, cannot exceed demand when demand is deterministic. That is, when we assume *X* = *D*, where *D* is a constant, we can say that $I \leq D$ and $Q \leq D$.

When disruption occurs, supply from the first supplier is zero and the only supply option is the units reserved from the second supplier. Demand is deterministic and buyer decides how many units to exercise from the second supplier. The optimal exercise quantity is simply: $(E^*|Y=1) = \min\{D, I\} = I$, and the corresponding profit realization is given by

$$\prod(Q, I|Y=1) = -hI + (r-e)I. \tag{5.9}$$

In the no disruption case, supply from the first supplier is *Q*. Demand is deterministic and buyer decides how many to exercise from the second supplier. Since the reliable supplier is more expensive, the options will be exercised only if demand exceeds the order quantity from the first supplier, *Q*. Hence, the optimal quantity of options exercised will be $(E^*|Y=0) = \min\{I, D-Q\}$, and the corresponding profit realization is given by

$$\prod(Q, I|Y=0) = r\min\{D, Q+I\} - e\min\{I, D-Q\} - cQ - hI. \tag{5.10}$$

Utilizing Equation (5.12) and Equation (5.13), and taking expectation with respect to *Y*, the expected profit of the buyer can be characterized as

$$\prod(Q,I) = -hI + p(r-e)I + (1-p)[r\min\{D, Q+I\} - e\min\{I, D-Q\} - cQ]$$

$$= -hI + p(r-e)I + (1-p\,)cQ$$

$$+(1-p)\begin{cases} r(Q+I) - eI & D > Q+I \\ rD - e(D-Q) & D \le Q+I \end{cases} \quad (5.11)$$

Then, the buyer's problem is

Max $\prod(Q,I)$

Subject to $D \ge Q, I \ge 0$.

Let Q^* (I^*) denote the optimal order (reservation) quantity from the unreliable (reliable) supplier.

Lemma 5.4.1. For a given *Q*, if $-h + p(r-e) > 0$, then $I^* = D$. Otherwise $I^* = D - Q$.

Proof: Taking the derivative of the profit function with respect to I, we get

$$\frac{\partial\prod(Q,I)}{\partial I} = -h + p(r-e) + (1-p)\begin{cases} r-e, & D > Q+I \\ 0, & D \le Q+I \end{cases} \quad (5.12)$$

If$-h + p(r-e) \ge 0$, from Equation (5.12) we can argue that the profit function is non-decreasing in I because the derivative is nonnegative in either region,($D > Q + I$ and $D \le Q + I$). Hence, we have $I^* = D$since D is the upper bound on I.

If $-h + p(r-e) < 0$, from Equation (5.12) we can argue that the profit function increases as the reservation quantity (I) increases until $I = D - Q$ because the derivative is $r - e - h > 0$ in this region ($I < D - Q$). For $I \ge D - Q$, the profit function decreases as the reservation quantity (I) increases because the derivative is$-h + p(r - e) < 0$. To sum up, the expected profit increases in I until $I = D - Q$ and decreases in I afterwards. Hence, the optimal reservation quantity occurs at the breakpoint, $I^* = D - Q$. ■

Lemma 5.4.2. For a given I, if $e > c$, then $Q^* = D$. Otherwise $Q^* = D - I$.

Proof: Taking the derivative of the profit function with respect to Q, we get

$$\frac{\partial \Pi\,(Q,I)}{\partial Q} = -(1-p)c + (1-p)\begin{cases} r, & D > Q + I \\ e, & D < Q + I \end{cases} . \qquad (5.13)$$

If $e > c$, from Equation (5.13) we can argue that the profit function increases as the order quantity (*Q*) increases because the derivative is always positive. Since *Q* can be at most *D*, we have $Q^* = D$.

If $e < c$, from Equation (5.13) we can argue that the profit function increases as the order quantity (*Q*) increases until $Q = D - I$because the derivative is $(1-p)(r - c) > 0$,in this region $Q < D - I$. For$Q \geq D - I$, the profit function decreases as the order quantity (*Q*) increases because the derivative, $-h + p(r - e) < 0$. To sum up, the profit function increases in Q until$Q = D - I$ and decreases in Q afterwards. Hence, the optimal order quantity occurs at the breakpoint, $Q^* = D - I$. ■

Our findings in Lemma 5.4.1 and Lemma 5.4.2 can be summarized as follows:

(i) If $-h + p(r - e) > 0$ and $e > c$, $Q^* = I^* = D$.

(ii) If $-h + p(r - e) > 0$ and $e < c$, $Q^* = 0$, $I^* = D$.

(iii) If $-h + p(r - e) < 0$ and $e > c$, $Q^* = D$, $I^* = 0$.

(iv) If $-h + p(r - e) < 0$ and $e < c$, $Q^* + I^* = D$.

The optimal solutions are already found in cases (i), (ii), (iii). We next analyze case (iv) in detail because we only know $Q^* + I^* = D$ for this case.

Lemma 5.4.3. Given that $-h + p(r-e) < 0$ and $e < c$, if $-h + p(r-e) + (1-p)(c-e) \geq 0$ then $I^* = D$, otherwise $I^* = 0$.

Proof: Plugging $Q = D - I$ into (5.11), the expected profit function reduces to

$$\prod(I) = -hI - c(1-p)(D-I) + pI(r-e) + (1-p)(rD - eI). \tag{5.14}$$

Taking the derivative of (5.14) with respect to I, we get

$$\begin{aligned}\frac{d\prod(I)}{dI} &= -h + c(1-p) + p(r-e) - e(1-p)\\ &= -h + p(r-e) + (1-p)(c-e).\end{aligned} \tag{5.15}$$

If $-h + p(r-e) + (1-p)(c-e) \geq 0$, from (5.15) we can argue that the profit function increases as the reservation quantity (I) increases because the derivative is positive. Hence, $Q^* = 0$ and $I^* = D$.

If $-h + p(r-e) + (1-p)(c-e) < 0$, from (5.15) we can argue that the profit function decreases as the reservation quantity (I) increases because the derivative is negative. Hence, $Q^* = D$ and $I^* = 0$, which completes the proof. ∎

Theorem 5.4.3. Under deterministic demand, the optimal order and reservation quantities are

Table 5.2 Optimal $Q\&I$ Values with Deterministic Demand

Conditions			$\boldsymbol{Q^*}$	$\boldsymbol{I^*}$
(i)	$-h+p(r-e)>0, e> c$	-	D	D
(ii)	$-h+p(r-e)>0, e< c$	-	0	D
(iii)	$-h+p(r-e)<0, e> c$	-	D	0
(iv)	$-h+p(r-e)<0, e< c$	$-h+p(r-e)+(1-p)(c-e)>0$	0	D
	$-h+p(r-e)<0, e< c$	$-h+p(r-e)+(1-p)(c-e)<0$	D	0

Proof: The optimal solutions and their conditions can be seen directly from Lemma 5.4.1, 5.4.2 and Lemma 5.4.3. ■

This theorem gives insights about buyer's order and reservation quantity decisions. Demand is deterministic so the buyer makes these decisions according to cost and revenue parameters and disruption uncertainty. Since the only uncertainty is disruption and uncertainty in disruption is discrete, buyer makes orders or reservation as zero or demand quantity.

Expected profit per unit if the buyer uses second supplier rather than first supplier is $-h+p(r-e)+(1-p)(c-e)$when $e<c$, otherwise it is $-h+p(r-e)$. Buyer earns $r-e$ for one exercised unit in disruption case because there is no opportunity other than using second supplier, $c-e$ in no disruption case because there is an opportunity to supply from the first supplier so opportunity cost of using

second supplier rather than first supplier, $c-e$. We take expectation according to Y and the expected profit becomes $-h+p(r-e)+(1-p)(c-e)$. However, in no disruption case buyer always uses first supplier when $e>c$ because he doesn't want to give more money when he has an opportunity to supply from the first supplier. Hence, expected profit per unit is $-h+p(r-e)$ when $e>c$.

Expected profit per unit if the buyer uses second supplier rather than first supplier

$$=\begin{cases}-h+p(r-e)+(1-p)(c-e) & , \ if\ e<c\\ -h+p(r-e) & , \ if\ e>c\end{cases}$$

If this profit is positive in either case, buyer always makes reservation as demand quantity. Otherwise, he reserves nothing.

Buyer makes order decision according to the purchasing cost difference between two suppliers and expected profit per unit if the buyer uses second supplier rather than first supplier. If the cost difference, $c-e$, is negative, buyer always makes an order as demand quantity. Buyer can also order as demand quantity when $c-e$, is positive and the expected profit per unit if the buyer uses second supplier rather than first supplier is negative, he orders as demand quantity.

Corollary 5.3.1. When $p<\frac{rh-c(r-e)}{(r-e)(r-c)}$, $I^*=0$ and $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$ whatever the information level is about demand and disruption uncertainty.

Proof: In the full information model, Theorem 5.3.1 shows that when $p<\frac{rh-c(r-e)}{(r-e)(r-c)}$, then $I^*=0$ and $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$. In the partial information model, Corollary 4.2.1 shows that when $e>c$ and $-h+p(r-e)$ then $I^*=0$ and $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$and when $e<c$ and $-h+p(r-e)+(1-p)(c-e)<0$ then $I^*=0$and $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$. In the no information model, Theorem 5.3.1 shows that when $-h+p(r-e)+(1-p)(c-e)<0$, then $I^*=0$ and $Q^*=F^{-1}\left(\frac{r-c}{r}\right)$. The condition $p<\frac{rh-c(r-e)}{(r-e)(r-c)}$ in the full information model can be written as

$\frac{c(1-p)(r-e)+r[h+p(r-e)]}{(r-e)(r-c)} < 0$ and it is equivalent to $c(1-p)(r-e)+r[-h+p(r-e)] < 0$. When this inequality holds,$-h+p(r-e) < 0$.For $e > c$, we have $-h+p(r-e)+(1-p)(c-e) < 0$ as well, and the corollary follows. For $e < c$,$[-h+p(r-e)+(1-p)(c-e)] < c(1-p)(r-e)+r[-h+p(r-e)]$. So, we conclude that if $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$, $c(1-p)(r-e)+r[-h+p(r-e)] < 0$ and $-h+p(r-e) < 0$ always hold. Hence, buyer never use second supplier whatever the information level is when $p < \frac{rh-c(r-e)}{(r-e)(r-c)}$, which completes the proof.

■

Corollary 5.3.2. When $(e < c)$ or $(e > c \text{ and } -h+p(r-e) < 0)$, disruption information is completely worthless.

Proof: From theorem 5.3.1 and corollary 4.3.1,if $e < c$, optimal Q and I values are the same in partial and no information models in the same conditions. That is, when $-h+p(r-e)+(1-p)(c-e) < 0$, we have the trivial solution we have $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$ and $I^* = 0$; otherwise we have $Q^* + I^* = F^{-1}\left(\frac{r-c}{r}\right)$ and $I^* > 0$. We next consider $e > c$ and $-h+p(r-e) < 0$. From Theorem 5.3.1 and Corollary 4.3.1, we have $Q^* = F^{-1}\left(\frac{r-c}{r}\right)$ and $I^* = 0$ in both models, which completes the proof. ■

This corollary demonstrates us disruption information is worthless not only in the trivial case $I^* = 0$, but also in some cases that buyer uses options contract.

CHAPTER 6

COMPUTATIONAL STUDY

In this chapter, we analyze the effects of disruption and demand information on the order decisions and profit levels. Since the models we presented in Chapter 3, Chapter 4 and Chapter 5 facilitate limited analytical comparisons, we perform a computational analysis to quantify decision making with the disruption and demand information.

This chapter is organized as follows: In Section 6.1, we pose the research questions that we are interested in and introduce the related performance measures. Section 6.2 analyzes the effects of problem parameters on the performance measures and profits. In section 6.3, we quantify the value of disruption and demand information considering various combinations of problem parameters.

6.1 Research Questions and Performance Measures

The major research questions that we pose and hope to provide answers for through the computational analysis can be summarized as follows:

1. What is the value of demand information for the buyer?

It is possible to quantify the effects of deciding with demand information by delaying the decision of how many to exercise after getting full information about demand onto the partial information. Eventually, delaying the decision of exercise makes the buyer have full information. The information difference between full and partial information model is deciding how many to exercise from the second supplier with demand information.

2. What is the value of disruption information for the buyer?

In partial information, disruption uncertainty is resolved prior to exercise decision by means of the options contract. If there is no chance to sign an options contract, buyer has to make the decision of order from the second supplier when all uncertainties exist. Eventually, nonexistence of options contract makes the buyer have no information. The information that the buyer knows is disruption in the partial information model.

In seeking answers to the above questions, we start with a sensitivity analysis to characterize how the performance measures mentioned below are affected by the changes in problem parameters in Section 6.2. Then we perform a full factorial design in Section 6.3 to analyze the results with a number of different parameter sets. Performance measures include the percentage improvement in expected profits due to disruption and demand information. More specifically performance measures are;

- For question 1, we compare full information with partial information and calculate % profit improvement due to full information over partial information. It is the value of demand information. We define the performance measure as follows: $FoP = \left(\frac{\Pi_{FI}-\Pi_{PI}}{\Pi_{PI}}100\right)$ for the buyer's profit.
- For question 2, we compare partial information with no information and calculate % profit improvement due to partial information over no information. It is the value of disruption information. We define the

performance measure as follows: $PoN = \left(\frac{\Pi_{PI}-\Pi_{NI}}{\Pi_{NI}}100\right)$ for the buyer's profit.

6.2 Analysis of Parameter Sensitivity

Sensitivity analysis is conducted for the parameters used in the models according to the base parameter set defined. Performance measures are evaluated while each parameter is increased and others kept as in the base parameter set. Base parameter set is selected in such a way that the condition in Corollary 5.3.1 doesn't hold. In other words, under the base parameter setting, optimal order and reservation decisions are non-zero. The base parameter set used in the sensitivity analysis is provided in Table 6.1.

Table 6.1: Base parameter set values.

r	e	c	h	p	σ	μ
150	30	24	5	0.1	10	100

6.2.1 Disruption Probability

We start with the analysis of disruption probability. Recall that uncertainty in supply disruption is discrete, first supplier disrupts according to the random variable, Y. When $Y = 1$ with the probability p, then supply from the first supplier is 0, otherwise supply is Q. Overall results for problem instances considered are provided in Table G.1 in Appendix G. p is changed between 0.01 and 0.9 with a step size of 0.05. We present our observations with respect to an increase in p:

- In the optimal solutions of all models, buyer's profit shows a decreasing behavior in a manner that in full and partial information it experiences a steep decrease until $p = 0.05$, in no information until $p = 0.1$ and after these threshold values profits decrease with lower slopes in all models.(Figure 6.1) The general decreasing behavior of profit is reasonable since buyer earns less money when probability that supply is zero from the first (cheaper) supplier increases.

- From Figure 6.1, it can be seen that the expected profits decrease in the disruption probability as expected. For all models, the decrease is more significant for lower values of p, and there is a threshold value beyond which the decrease becomes less significant, which is also reasonable since the buyer generally satisfies the demand with the supply from the first supplier in the lower values of the disruption probability. However, he doesn't rely on the first supplier further beyond a threshold value, 0.05 or 0.1. After these points, he reserves or orders from the second supplier and decreases the risks of insufficiency in satisfying demand.

- The threshold value is higher in the no information model compared to the full and partial information models which buyer decides with disruption information. In full and partial information models, the buyer has early reactions to increase the reservation quantity (Figure 6.2) because he knows whether the first supplier disrupts or not while exercising from the second supplier.

- In Figure 6.3, there is a significant decrease in the optimal order quantity for the no information model when the disruption probability increases from 0.05 to 0.1, which corresponds to the point where the reservation quantity increases significantly. The buyer does not decrease its order quantity in the partial and full information models because he wants to satisfy demand from the first supplier when disruption does not occur which can be possible due to getting disruption.

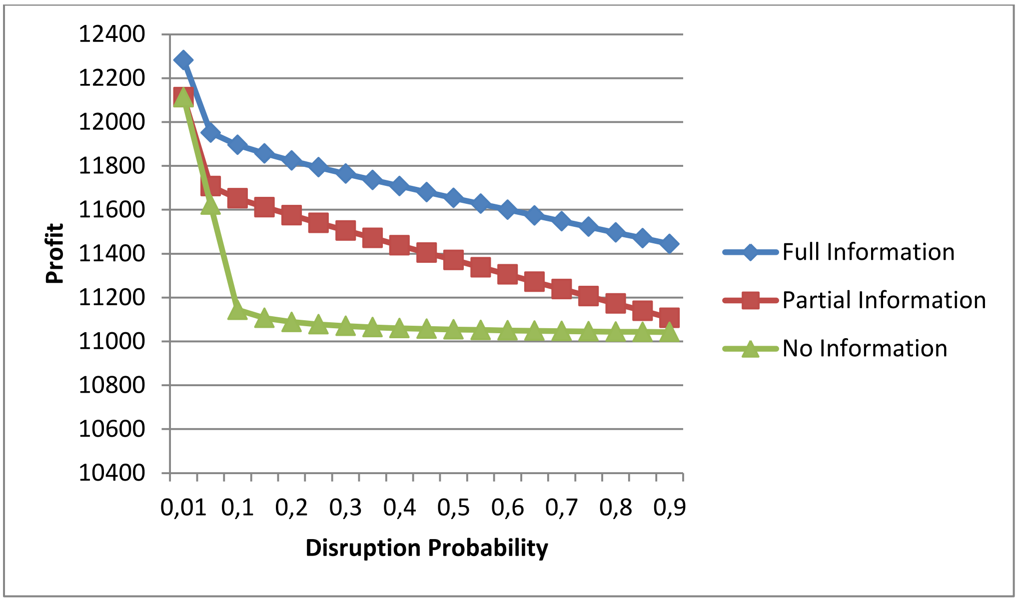

Figure 6.1 Optimal Profits when p increases

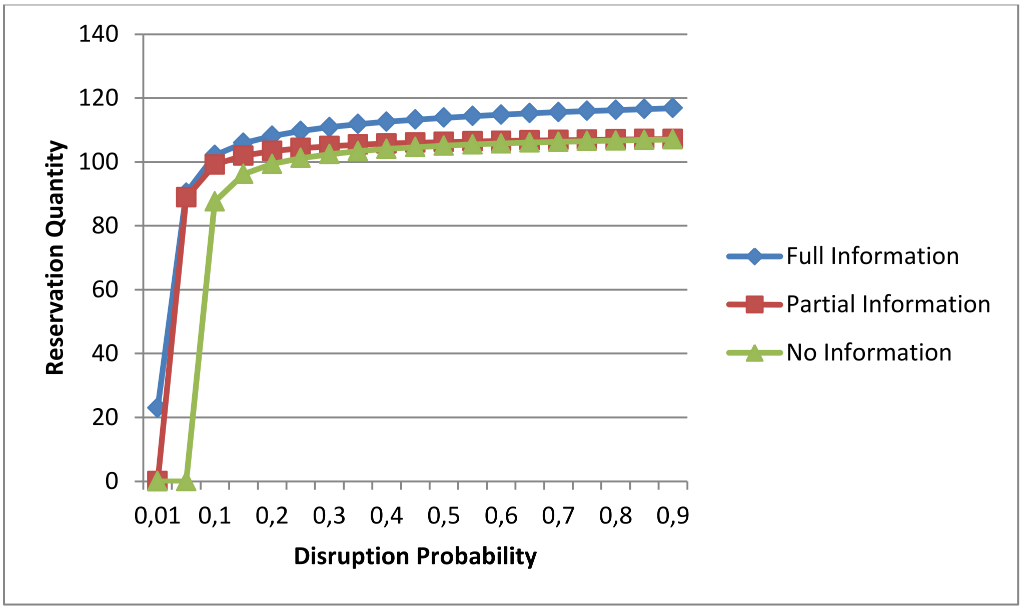

Figure 6.2 Optimal Reservation Quantity when p increases

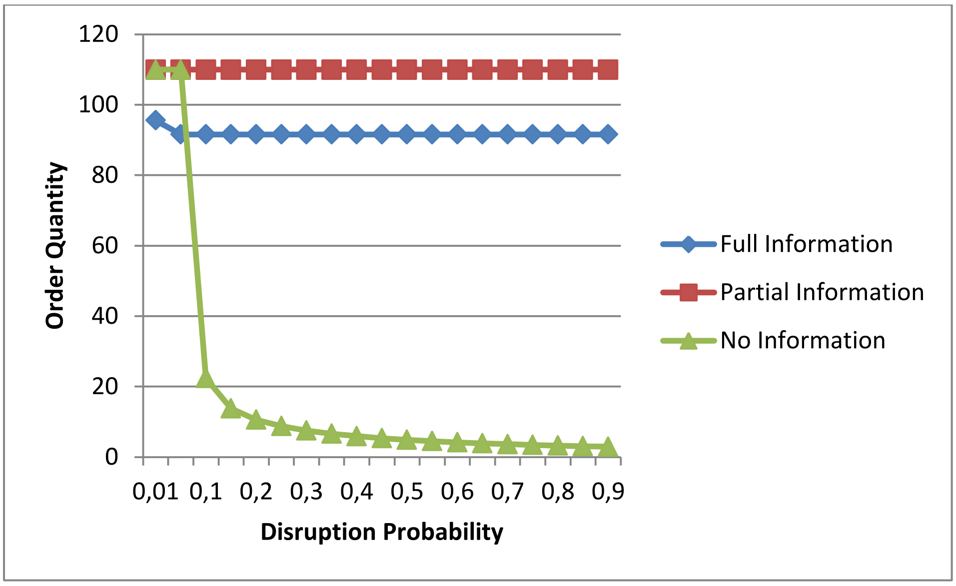

Figure 6.3 Optimal Order Quantity when p increases

- Figure 6.4 shows the percentage profit improvement due to full information over partial information (FoP) and due to partial information over no information (PoN). PoN increases until $p = 0.1$ and decreases afterwards. It is the point where the buyer begins to increase its reservation quantity also in the no information model and decrease the risk of stock-out in satisfying demand.

- FoP shows an increasing behavior since exercising after getting demand information decreases the number of units that is exercised for nothing in the full information model.

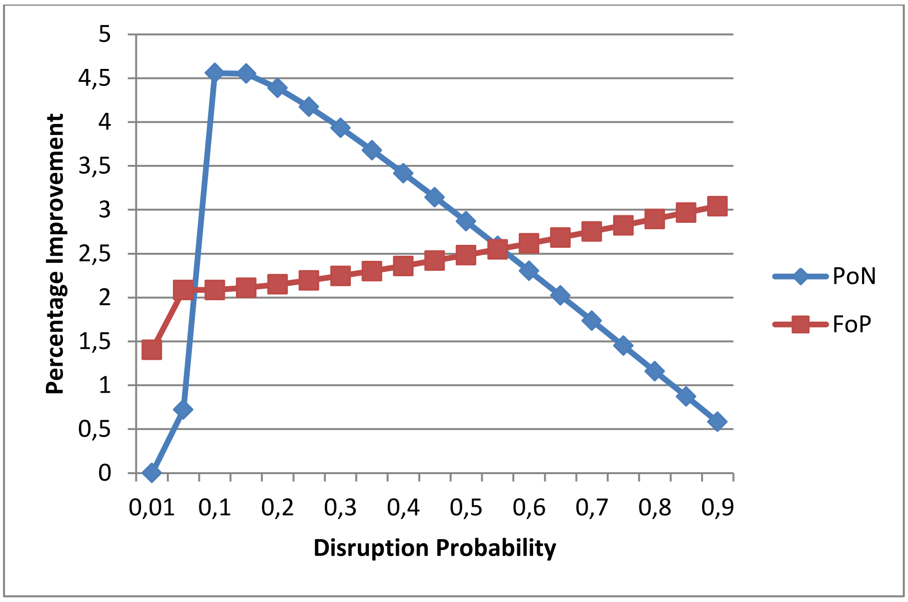

Figure 6.4 PoN and FoP when p increases

6.2.2 Standard Deviation of Demand

We analyze the sensitivity of our results to the standard deviation of the Normal random variable, σ. Overall results for the problem instances considered are provided in Table G.2 in Appendix G. σ is changed between 5 and 30 with a step size of 5. We present our observations with respect to an increase in σ:

- In the optimal solutions of all models, buyer's profit shows a decreasing behavior in standard deviation as expected. In Figure 6.5, we observe that the decrease in the partial and no information models is steeper than it is in the full information model. Since the buyer doesn't have demand information while exercising under these cases, his profit is dampened more when demand uncertainty increases.

- The value of demand information increases as the standard deviation of demand increases. As we expected, the value of demand information, that is FoP, increases in Figure 6.6. There is no significant difference in the percentage profit improvement when we compare partial and no information models (PoN). It is reasonable because in either case the demand uncertainty is not resolved.

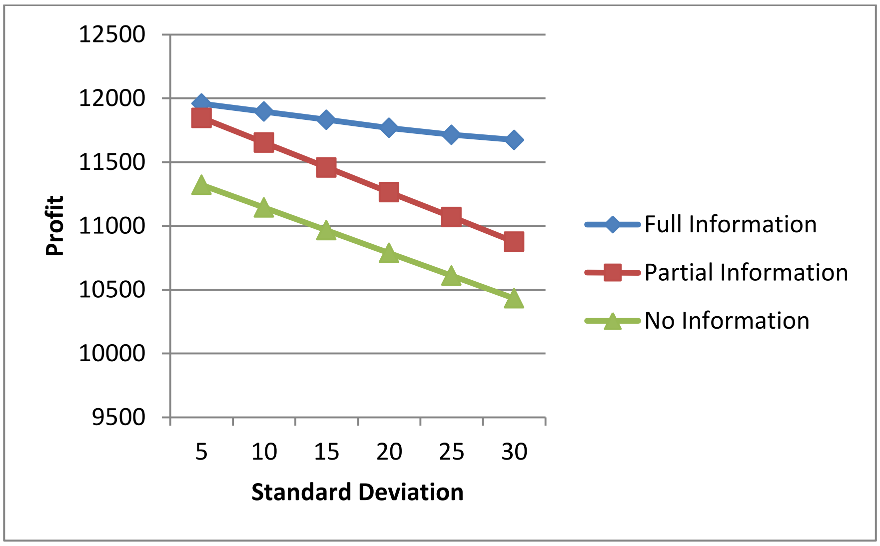

Figure 6.5 Optimal Profits when σ increases

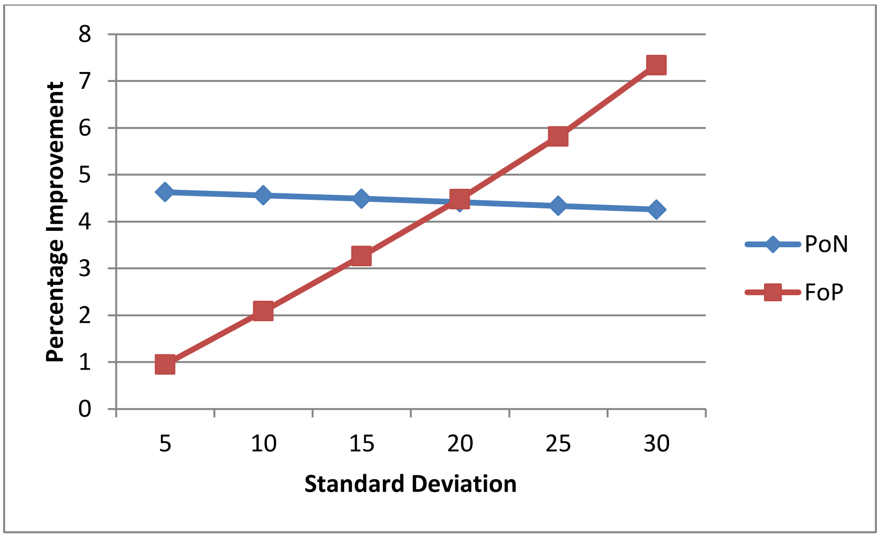

Figure 6.6 Performance Measures when σ increases

- From Figures 6.7 and 6.8, we can see that the buyer prefers to order more from the second supplier and less from the first supplier as σ increases in the full information model. However, in the partial and no information models, he prefers to order more from the first supplier.

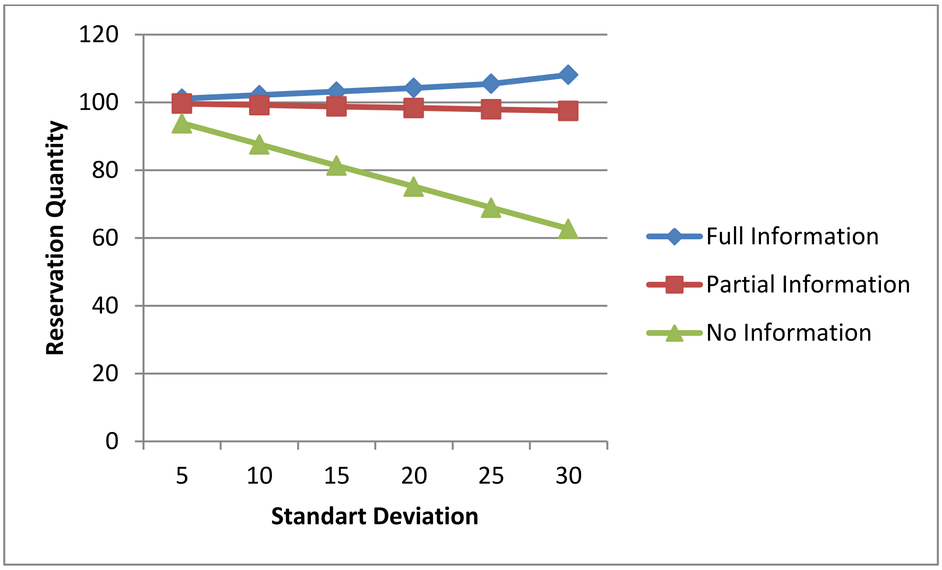

Figure 6.7 Optimal Reservation quantity when σ increases

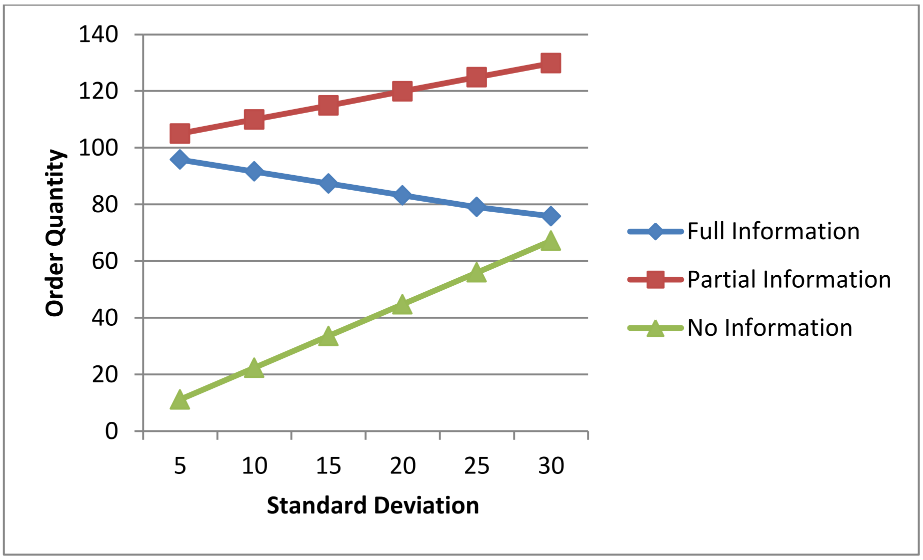

Figure 6.8 Optimal Order quantity when σ increases

6.2.3 Unit Cost of the First Supplier

We analyze the sensitivity of our results to the unit cost of the first supplier, c. Overall results for problem instances considered are provided in Table G.3 in Appendix G. c is changed between 22 and 32 with a step size of 1. We present our observations with respect to an increase in c:

- The optimal profit decreases as the unit cost increases until a threshold value after which the profits stay almost the same. We can see from Figure 6.9 that the profit under partial information decreases with the greatest slope and the profit under no information decreases with the least slope.

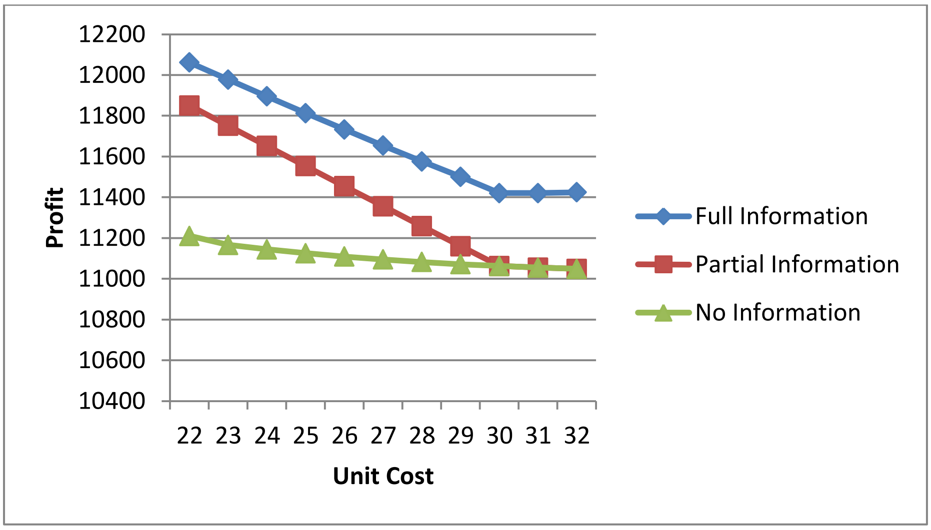

Figure 6.9 Optimal Profits when c increases

- FoP increases PoN decreases. As the profit margin when demand is satisfied from the first supplier decreases as the unit cost increases. The buyer orders less from the first supplier and orders more from the second supplier in the no information model (Figure 6.12) because the cost difference between suppliers decreases. In the same way, the buyer reserves as much as demand plus standard deviation (Figure 6.11) and having a chance to exercise after getting disruption information is now less important the in partial information model. Therefore, we can see from Figure 6.10 that the value of disruption information decreases in unit cost. After the point $c = e = 30$, the buyer orders and exercises the same amount from the suppliers and earns the same. Hence, the value of disruption information is negligible after this point on.

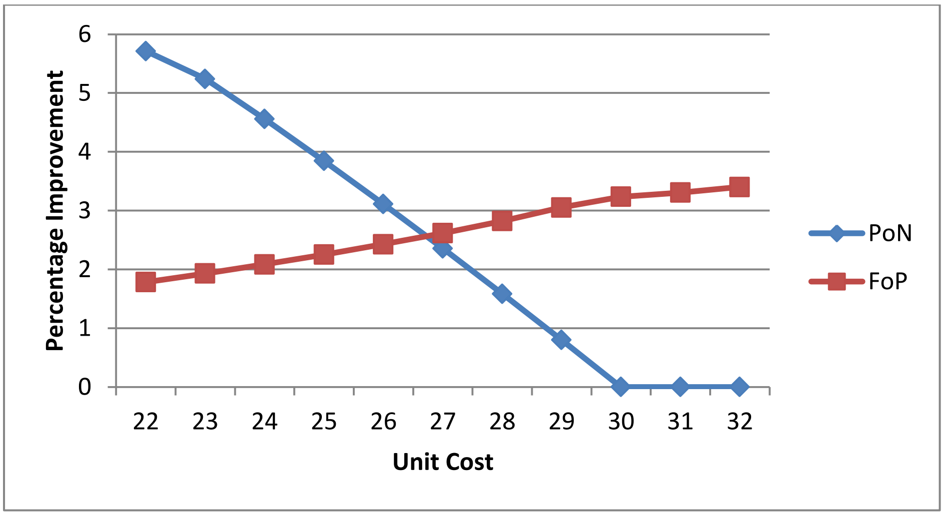

Figure 6.10 PoN and FoP when c increases

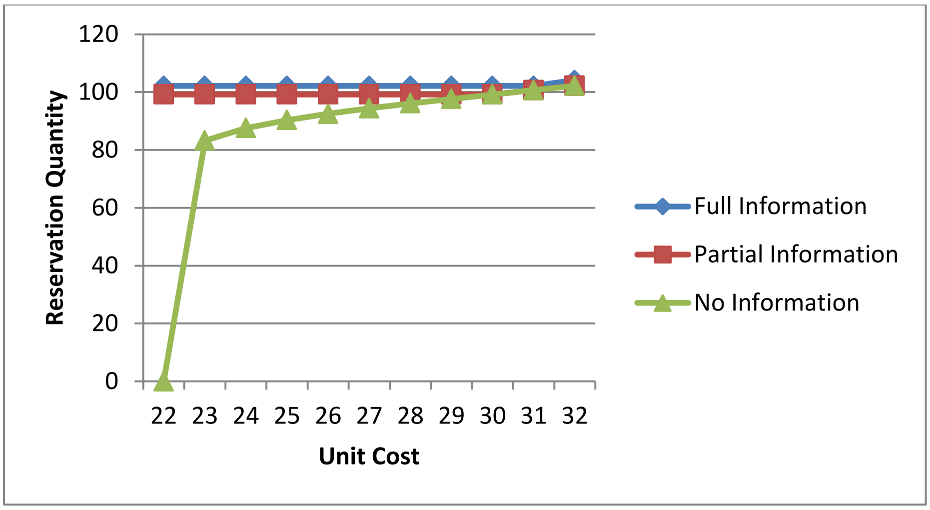

Figure 6.11 Optimal Reservation Quantity when c increases

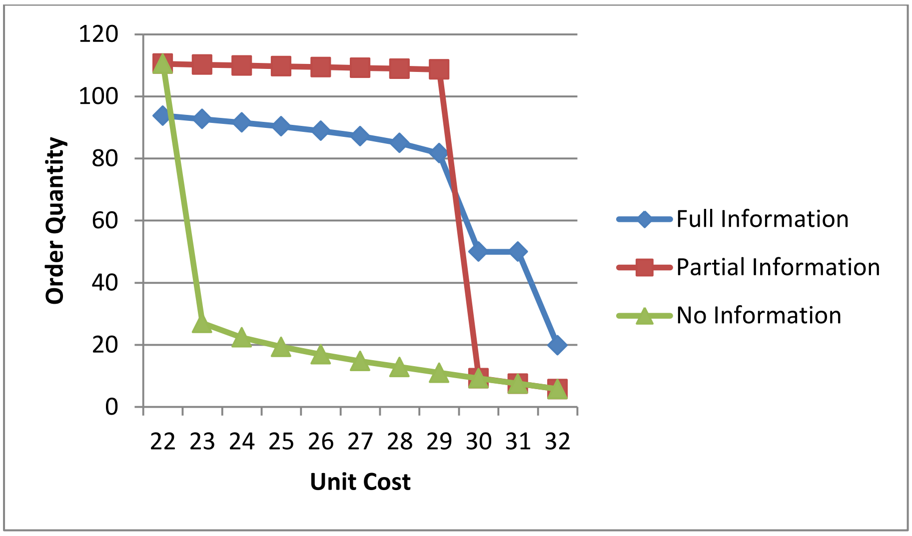

Figure 6.12 Optimal Order Quantity when c increases

6.2.4 Exercise Cost

We analyze the impact of the exercise cost of the second supplier, e on the expected profit and performance measures. Overall results for the problem instances considered are provided in Table G.4 in Appendix G. e is changed between 20 and 32 with a step size of 2. We present our observations with respect to an increase in e:

- In the optimal solutions corresponding to different e values, buyer's profit shows a decreasing behavior in a manner that in full and partial information it experiences a steep decrease until $e = 24$, after this threshold value profits decrease with lower slopes. It should be noted that the threshold value is the value of e that is equal to the unit cost $c = 24$ (Figure 6.13).

- Profit under partial information decreases almost at the same pace with profit under full information in all values of e. Figure 6.14 reflects this observation such that the value of demand information (FoP) increases very slightly until the threshold value after which it experiences a slight decrease.

- In the no information case, profit decreases almost with the same slope which is always greater than it is in the partial information case (Figure 6.13). Before the point $e = c = 24$, buyer orders and exercises the same amount from the suppliers and earns the same so disruption information is negligible before this point. The profit margin when demand is satisfied from the first supplier increases comparatively as the exercise cost increases. As observed in Figure 6.16, the buyer orders more from the first supplier and orders less from the second supplier in the no information model. In the same way, buyer orders as much as demand plus standard deviation (Figure 6.15) and having a chance to exercise after getting disruption information is more important in the partial information model. Therefore, we can see from Figure 6.14 that value of disruption information (PoN) increases in the exercise cost.

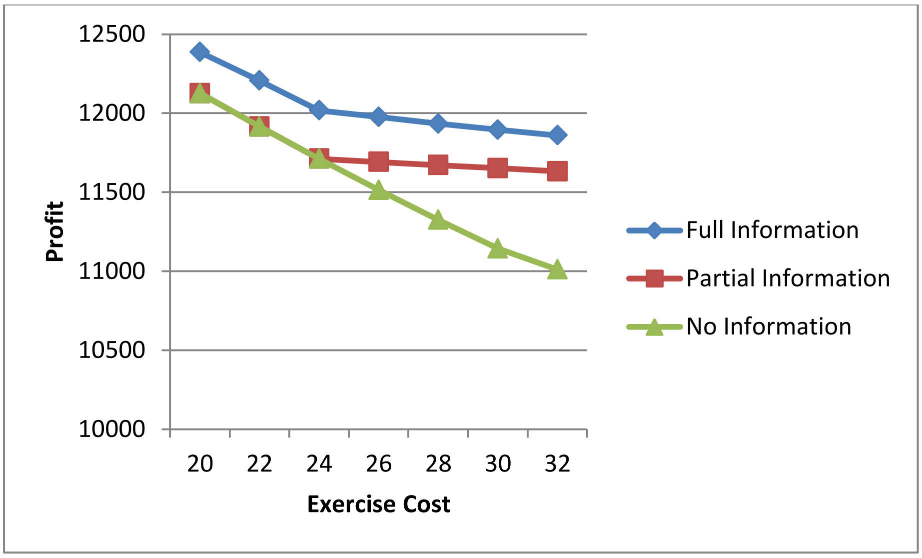

Figure 6.13 Optimal Profits when e increases

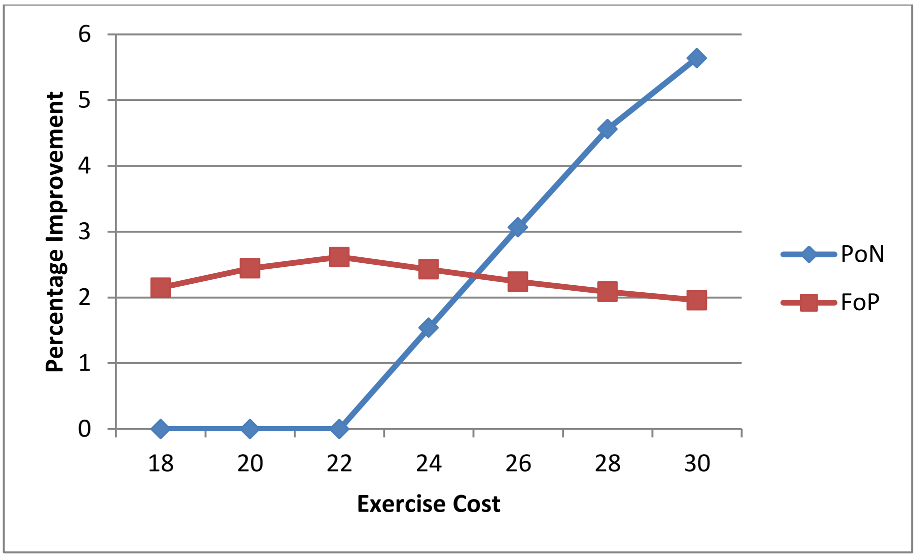

Figure 6.14 Performance Measures when e increases

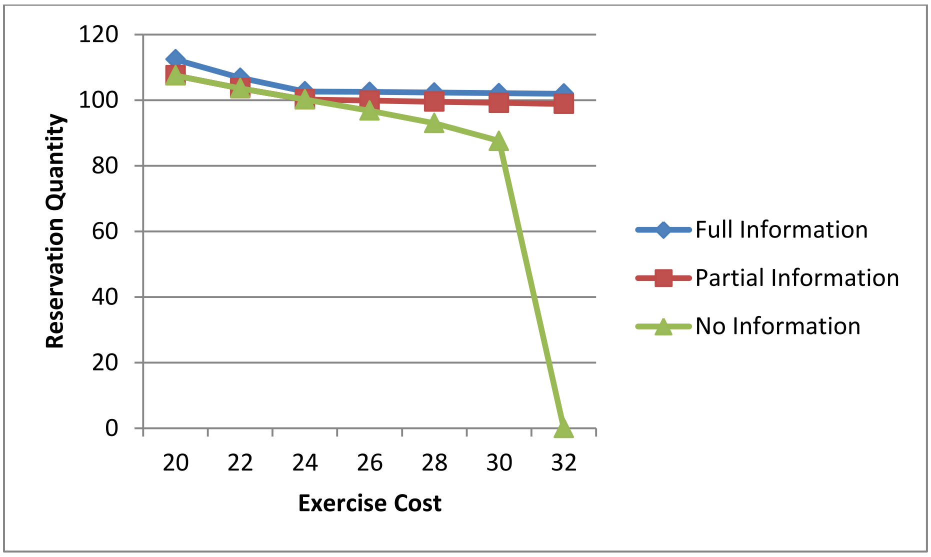

Figure 6.15 Optimal Reservation Quantity when e increases

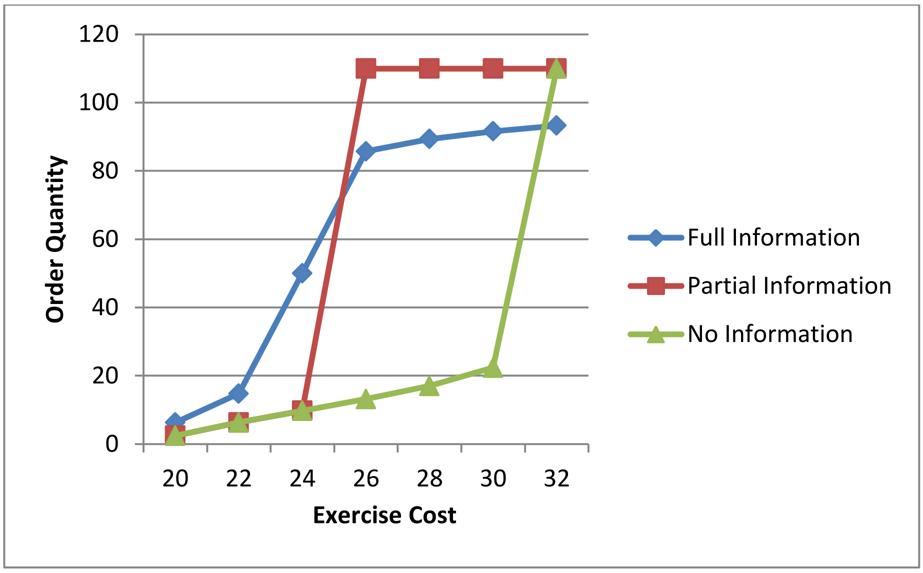

Figure 6.16 Optimal Order Quantity when e increases

6.2.5 Revenue

We analyze the impact of the revenue per unit, r on optimal profit and performance measures. Overall results for problem instances considered are provided in Appendix G. r is changed between 100 and 200 with a step size of 25. We present our observations with respect to an increase in r:

- When revenue increases, ratio of profit of one unit over cost of one unit increases exponentially. In such a case, buyer wants to reduce the risk of stock-out more since having a stock-out is now more costly. In full and partial information models, he orders and reserves about mean demand. In the no information model, he orders more from the second supplier and less from the first supplier (Figure 6.20). This makes deciding with disruption and demand information less important and in Figure 6.18, we can see the decrease in the value of disruption and demand information. There is an increase in the value of disruption information from $r = 100$ to 125 because buyer reserves nothing in these points. (Figure 6.19)

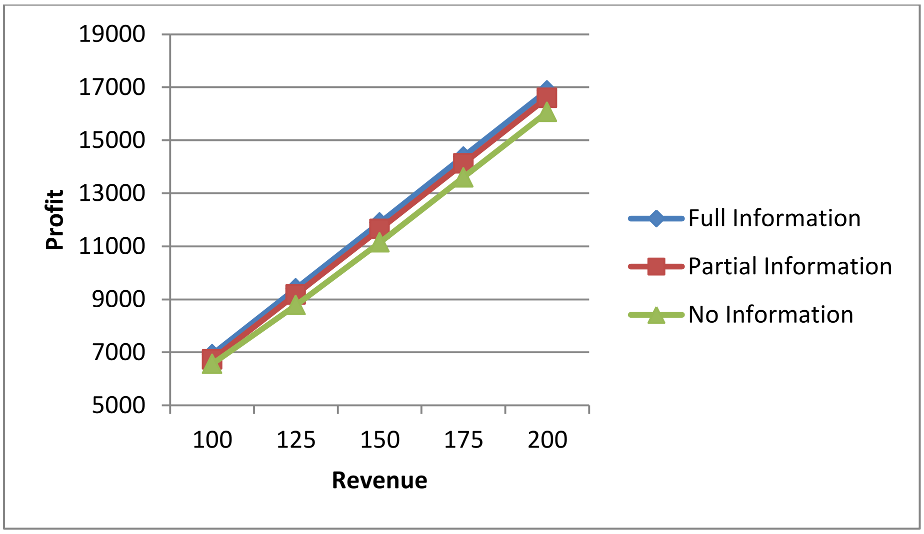

Figure 6.17 Optimal Profits when r increases

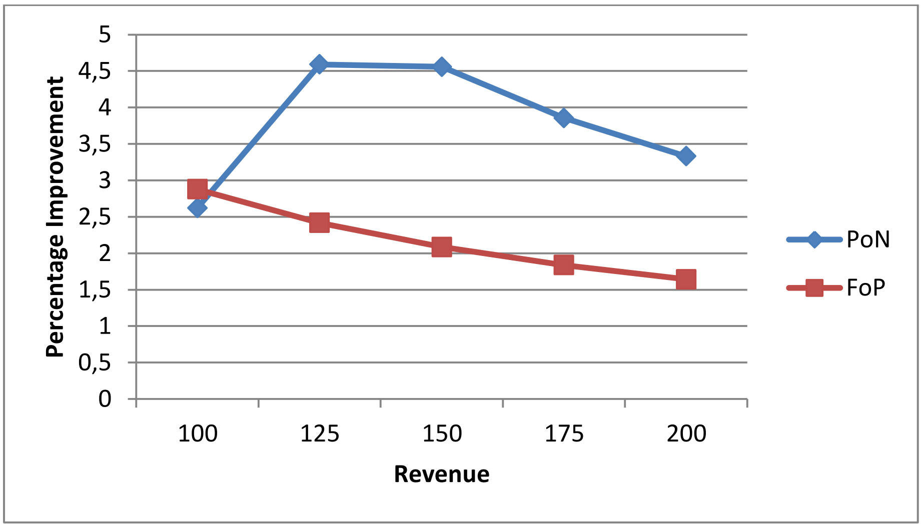

Figure 6.18 Performance Measures when r increases

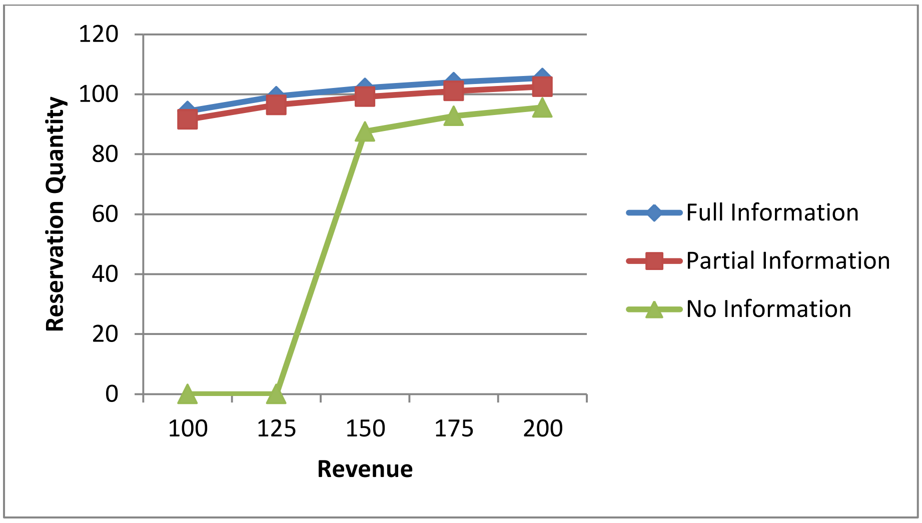

Figure 6.19 Optimal Reservation Quantity when r increases

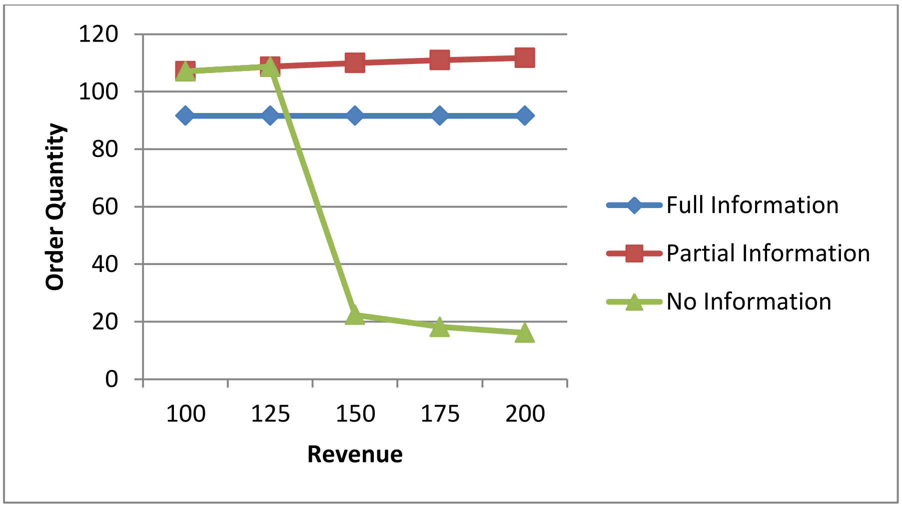

Figure 6.20 Optimal Order Quantity when r increases

6.2.6 Reservation Cost

We analyze the impact of the reservation cost per unit, h on the expected profit and performance measures. Overall results for the problem instances considered are provided in Appendix G. h is changed between 1 and 33 with a step size of 4. We present our observations with respect to an increase in h:

- In Figure 6.21, we can see that profits are getting closer to each other. After the point $h = 29$, profits are the same and the values of disruption and demand information are negligible because the buyer doesn't use options contract in any information level (Figure 6.23).

- In Figure 6.22, PoN and FoP values decrease as the reservation cost increases and after the point $h = 29$, they are the same. After this point, options contract is very costly to use, the buyer ignores the second supplier and use the first supplier in any model so profits, PoN, FoN and order quantities are the same.

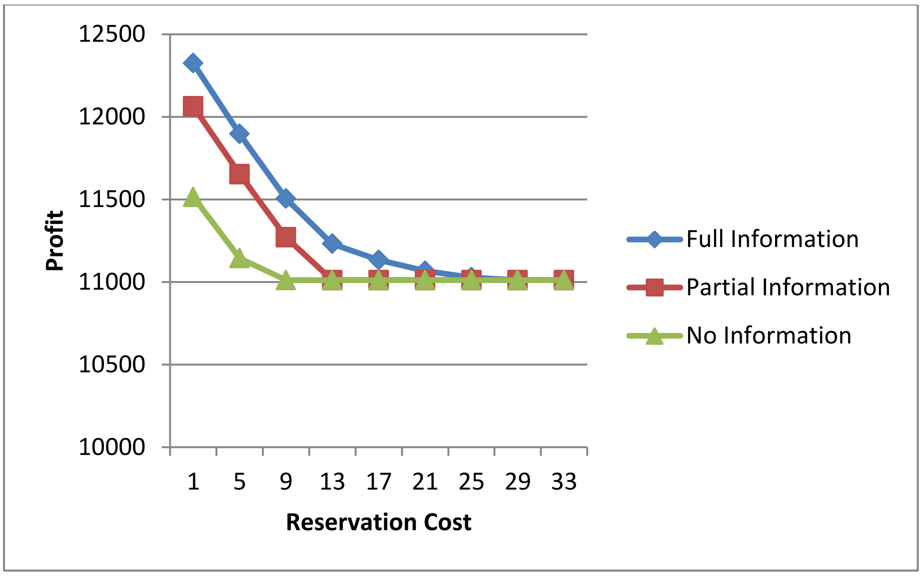

Figure 6.21 Profit when h increases

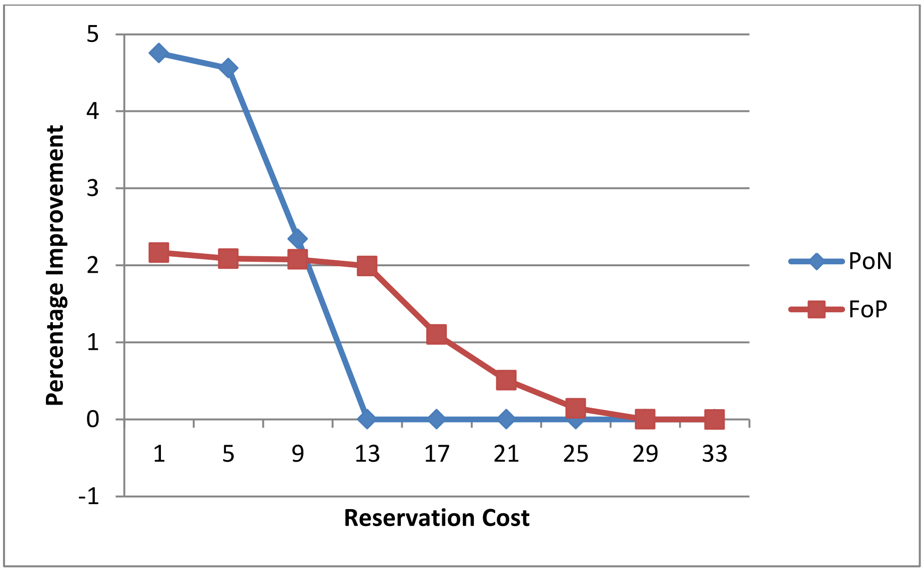

Figure 6.22 PoN and FoP when h increases

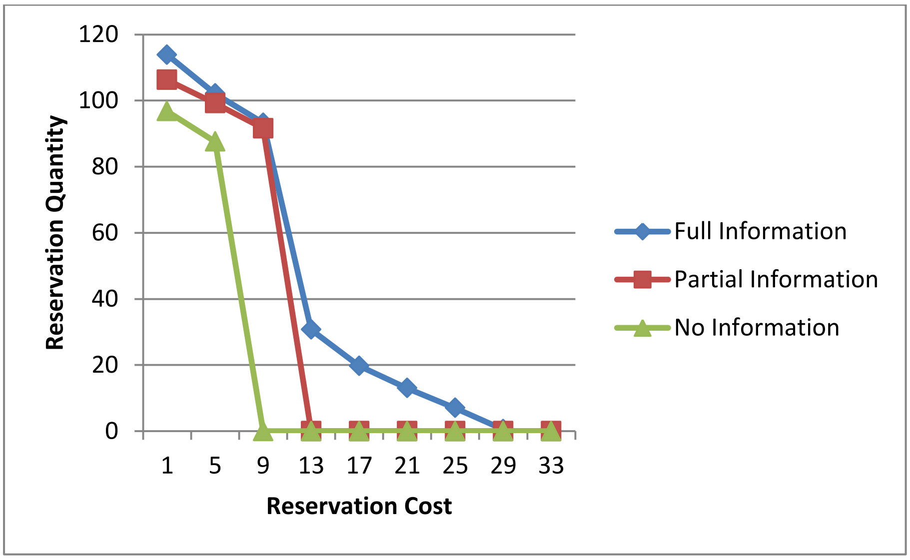

Figure 6.23 Optimal Reservation Quantity when h increases

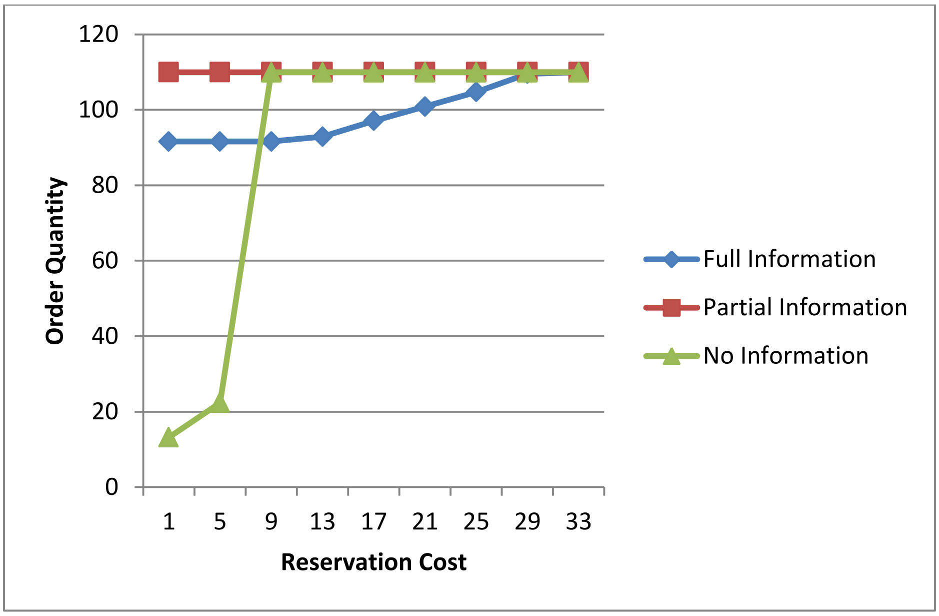

Figure 6.24 Optimal Order Quantity when h increases

6.3 Analysis of Full Factorial Design

A full factorial experiment is designed for the parameter values given in Table 6.2. The aim of the full factorial experiment is to detect the value of disruption and demand information. All combinations of these parameter values (totally 4096 problem instances) are solved for all models to analyze the results for defined research questions in Section 6.1.

Table 6.2: Parameters values used for full factorial design

r	e	c	h	p	σ	μ
100	24	10	5	0.01	10	
125	27	16	8	0.05	20	100
150	30	22	10	0.1	25	
175	33	28	12	0.25	33.3	

6.3.1 Value of Disruption Information

In partial information model, disruption uncertainty is resolved prior to the exercise decision by means of the options contract. If there is no chance to sign an options contract, buyer has to order from the second supplier when all uncertainties still exist. In partial information, buyer makes the decision of how many to exercise from the second supplier after observing the disruption or lack of it. Hence, we can quantify the value of disruption information by computing % profit improvement on partial information model's profit over no information model's profit. Average values are found over all 4096 numerical combinations.

OBSERVATION 1. Optimal profit in the partial information model is always greater than or equal to the optimal profit in the no information model. The maximum, minimum and average values of PoN over all problem instances are provided in Table 6.3. Deciding how many to exercise from the second supplier with disruption information improves buyer's profit with 18.9% maximum and 2.1% on average in our 4096 numerical combinations. The maximum value is found at a point such that disruption probability and cost difference between ordering from the first supplier and exercising from the second supplier $(e - c)$ are the highest in the parameter values of full factorial design. This coincides with the findings in sensitivity analysis part.

Table 6.3 Percentage improvements on profit under partial information over profit under no information

	MAX	MIN	AVG
% system profit improvement on PI over NI, $\left(\frac{\Pi_{PI}-\Pi_{NI}}{\Pi_{NI}}100\right)$	18.9%	0%	2.1%
r	125	100	-
c	10	28	-
e	33	27	-
h	5	5	-
p	0.25	0.25	-
σ	10	33.3	-

OBSERVATION 2. Disruption probability, p, is one of the most important parameters that change the value of disruption information. Table 6.4 shows disruption information is highly valuable for high values of disruption probability. Maximum and average profit improvements decrease as disruption probability decreases.

OBSERVATION 3. Profits of partial and no information is the same when $p = 0.01$ so disruption information is negligible.

Table 6.4 PoN values for different p values.

	$p = 0.25$	$p = 0.1$	$p = 0.05$	$p = 0.01$
max	18.9%	6.4%	1.5%	0.0%
min	0.0%	0.0%	0.0%	0.0%
avg	6.8%	1.4%	0.1%	0.0%

OBSERVATION 4. Difference between exercise cost and unit cost $e - c$, is another important parameter that changes the value of information significantly. Table 6.5 shows disruption information is highly valuable for high values of, $e - c$. Maximum and average profit improvements decrease as the cost difference decreases.

OBSERVATION 5. We can directly conclude that when $e - c < 0$, from Table 6.5, profits of partial and no information are the same so disruption information is negligible. This is also proved in Corollary 5.3.2 so that optimal order and reservation quantities are equal. We have also found the point, $e = c$ as a threshold value from the figures of sensitivity analysis. Here we prove this point is a critical value about profit improvements.

Table 6.5 PoN values for different $e - c$ values.

	$e - c = 23$	$e - c = 11$	$e - c = 2$	$e - c < 0$
max	18.9%	6.4%	2.7%	0.0%
min	0.0%	0.0%	0.0%	0.0%
avg	3.6%	2.6%	0.7%	0.0%

OBSERVATION 6. Difference between unit cost and reservation cost $c - h$, is another important parameter that changes the value of disruption information significantly. Table 6.6 shows disruption information is highly valuable for low values of, $c - h$. Maximum and average profit improvements increase as the difference decreases.

Table 6.6 PoN values for different $c - h$ values.

	$c - h = 23$	$c - h = 14$	$c - h = 6$	$c - h = -2$
max	6.7%	13.1%	14.5%	15.2%
min	0.0%	0.0%	0.0%	0.0%
avg	0.8%	1.8%	2.4%	2.5%

OBSERVATION 7. Reservation cost and standard deviation of demand do not change the value of disruption information significantly. On the other hand, increase in revenue decreases average profit improvement significantly.

Figure 6.12 shows how increase in reservation cost, revenue and standard deviation affect the average profit improvement. Parameter values are increasing from first to fourth value according to full factorial design.

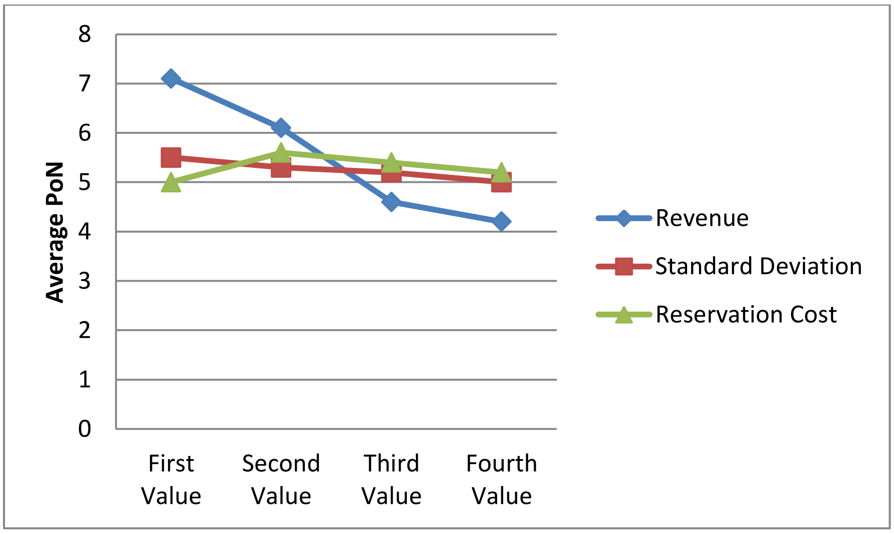

Figure 6.25 Average Profit Improvements according to Problem Parameters

OBSERVATION 8. Reservation quantity in the partial information model is always greater than or equal to the reservation quantity under no information for all parameter combinations of factorial design.

6.3.2 Value of Demand Information

In the full information model, exercise decision is delayed after getting information about demand. In partial information model, there is an uncertainty of demand while exercising from the second supplier. Hence, we can quantify the value of demand information by computing % profit improvement due to full information over partial information.

OBSERVATION 9. Optimal profit under full information is always greater than or equal to the optimal profit under partial information. The maximum, minimum and average values of FoP over all problem instances are provided in Table 6.7. Deciding how many options to exercise from the second supplier with demand information improves buyer's profit with 16.2% maximum and 3.1% on average in our 4096 numerical combinations. The maximum value is found at a point such that standard deviation is the highest and difference revenue and unit cost $(r - c)$ is the lowest in the parameter values of full factorial design. This coincides with the findings in sensitivity analysis part.

Table 6.7 Percentage improvements on profit under full information over profit under partial information

	MAX	MIN	AVG
% system profit improvement on FI over PI, $\left(\frac{\Pi_{FI}-\Pi_{PI}}{\Pi_{PI}} 100\right)$	16.2%	0%	3.1%
r	100	100	-
c	28	16	-
e	27	33	-
h	5	12	-
p	0.25	0.01	-
σ	1110	1110	-

OBSERVATION 10. Standard deviation of demand, σ, is highly significant in the value of demand information. Buyer can increase his profit by 16.2% maximum and 3.1% in average in 4096 instances if he decides the exercise quantity with demand information. Table 6.8 shows demand information is more valuable for high values of the standard deviation. Maximum and average profit improvements increase as the standard deviation increases.

Table 6.8 FoP values for different σ values.

	$\sigma = 33.3$	$\sigma = 25$	$\sigma = 20$	$\sigma = 10$
max	16.2%	11.9%	9.4%	4.4%
min	0.0%	0.0%	0.0%	0.0%
avg	4.8%	3.5%	2.8%	1.3%

OBSERVATION 11. Difference between revenue and unit cost, $r - c$, significantly impacts the value of demand information. Table 6.8 shows demand information is more valuable for low values of, $r - c$. Maximum and average profit improvements value decreases as the difference between revenue and unit cost increases.

Table 6.9 FoP values for different $r - c$ values.

	$r - c = 165$	$r - c = 128$	$r - c = 97$	$r - c = 72$
max	2.9%	8.6%	13.1%	16.2%
min	0.0%	0.2%	0.6%	0.5%
avg	1.0%	3.5%	5.9%	6.6%

OBSERVATION 12. Difference between unit cost and reservation cost $c - h$, is another important parameter that changes the value of demand information significantly. Table 6.10 shows demand information is highly valuable for high values of, $c - h$. Maximum and average profit improvements decrease as the difference decreases.

Table 6.10 FoP values for different $c - h$ values.

	$c-h=23$	$c-h=14$	$c-h=6$	$c-h=-2$
max	16.2%	12.2%	7.1%	3.7%
min	1.7%	0.5%	0.02%	0.0%
avg	6.8%	3.9%	1.8%	0.7%

OBSERVATION 13. Reservation and exercise cost do not have a significant effect on the value of demand information whereas an increase in disruption probability increases the average profit improvement significantly.

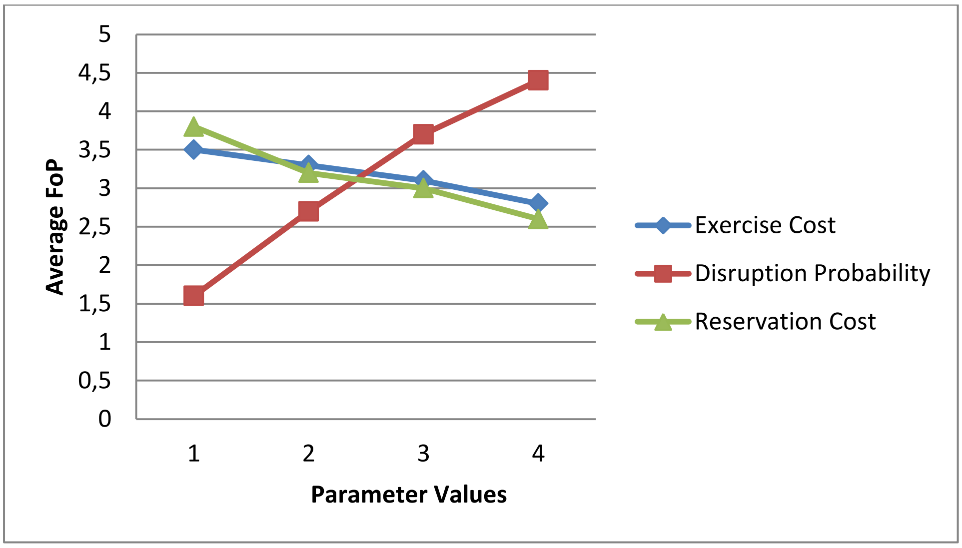

Figure 6.26 Average Profit Improvements according to Problem Parameters

OBSERVATION 14. Reservation quantity under full information is always greater than or equal to the reservation quantity under partial information for all parameter combinations.

CHAPTER 7

CONCLUSION

Supply disruption literature has been growing quickly in the past few years, motivated by increased globalization. Firms now choose suppliers from anywhere in the world because of increased price sensitivity of markets. Events like natural disasters (earthquake, fire etc.), terrorist attacks, war, strike etc. happening in another country or continent can affect your deliveries very seriously. Supply goes off totally in these circumstances so firms should mitigate the disruption risk. This is why supply disruption is an emerging interest area in the past years.

In this study, we consider a single-period problem with a buyer that faces random demand. The buyer has two alternative suppliers: one cheaper but prone to disruption and the other perfectly reliable but more expensive. The models presented here are designed to show (i) how the buyer orders from the two suppliers, (ii) in which cases options contract is used, (iii) what are the managerial benefits of higher information levels about disruption and demand risk bring.

We developed efficient approaches to quantify the value of two important supply risk mitigation strategy in a dual sourcing supply chain with options contract: (1) obtaining disruption risk information, and (2) obtaining demand risk information before using options. We defined measures for the value of obtaining disruption as well as the value of obtaining demand risk information. These measures are the upper bounds for the amount of money that a risk-neutral firm should be willing to invest to implement either of these strategies in order to mitigate uncertainties in its supply chain system.

In all information levels, we analytically characterized the buyer's behavior by explicitly identifying the optimal size of order and reservation according to all cost, revenue and uncertainty parameters. We demonstrated the circumstances that the buyer uses options contract and found closed form solutions about order and reservation quantities when options contract is used. We defined performance measures and showed analytically and theoretically how problem parameters influence these measures, profits and quantities. A computational study is conducted and our analyses reveal the following findings:

- Delaying the decision of how many to exercise from the second supplier provides greater profits due to having information about disruption and demand uncertainties.
- If the buyer doesn’t use options contract in an information level, it is also not used in the lower information levels.
- In any information level, buyer makes more reservation than he makes in lower information levels in the same values of parameters.
- When the buyer doesn’t use options contract, he orders the same amount from the first supplier in any information level.
- Increase in disruption probability increases value of disruption risk information and increase in variability of demand increases value of demand risk information.

One of the main results of Chopra (2007) says that “Growth in supply risk from increased disruption probability is best mitigated by increased use of the reliable (though more expensive) supplier and decreased use of the cheaper but less reliable supplier.” This coincides with our results such that reservation quantity is non-decreasing and order quantity is non-increasing in disruption probability in any information level. In one of the observations in Saghafian and Van Oyen (2011), it is found that disruption risk information is more valuable to firms with lower profit margins. This is parallel to our observations in computational study; we observe that changing revenue and cost parameters so that the profit margin decreases makes disruption information more attractive.

The work presented in this thesis can be extended to a multi-period setting. A different mitigation strategy, namely spot market can be suggested. Buyer can buy units immediately from a spot market however price of the product is changing overtime. In this setting, another extension can be made such that buyer has two alternatives; (1) can wait a while which is also a decision variable until the first supplier recovers itself or (2) can wait a while which is also a decision variable until the market price of the product decreases. Another extension can be making estimations about disruption and demand uncertainty and measure how estimation errors influence profit levels.

REFERENCES

[1] Anupindi, R., R. Akella. 1993. Diversification under supply uncertainty. Management Science 39(8), 944-963.

[2] Arreola-Risa, A., G. A. DeCroix. 1998. Inventory management under random supply disruption and partial backorders. Naval Research Logistics 45 687-703.

[2] Berger, P. D., A. Gerstenfeld, A. Z. Zeng. 2004. How many suppliers are best? a decision-analysis approach. Omega 32, 9-15.

[3] Bielecki, T., P. R. Kumar. 1988. Optimality of zero-inventory policies for unreliable manufacturing systems. Operations Research 36(4), 532-541.

[4] Burrows, P. 1999. The vice trapping PC makers. Business Week Online (October 25).

[5] Chopra, S., G. Reinhardt, U. Mohan. 2007. The importance of decoupling recurrent and disruption risks in a supply chain. Naval Research Logistics 54(5), 544-555.

[6] Dada, M., N. Petruzzi, L. Schwarz. 2007. A newsvendor's procurement problem when suppliers are unreliable. Manufacturing and Service Operations Management 9(1), 9-32.

[7] Lakovou, E., D. Vlachos, A. Xanthopoulos. 2009. A stochastic inventory management model for a dual sourcing supply chain with disruptions. International Journal of Systems Science Volume 41, Issue 3, 2010

[8] Federgruen, N., A.and Yang. 2009. Optimal supply diversification under general supply risks. Operations Research 57(6), 1451-1468.

[9] Federgruen, A., N. Yang. 2008. Selecting a portfolio of suppliers under demand and supply risks. Operations Research 56(4), 916-936.

[10]Griffy-Brown, C. 2003.Just-in-time to just-in-case.Graziadio Bus. Rep. 6.

[11]Gupta, D. 1996. The (Q; r) inventory system with an unreliable supplier. INFOR 34(2) 59-76.

[11]Hou, J., A. Z. Zeng, L. Zhao. 2010. Coordination with a backup supplier through buy-back contract under supply disruption. Transportation Research, 46, 881-895.

[12]Latour, A. Trial by fire: a blaze in albuquerque sets off major crisis for cell-phone giants, The Wall Street Journal; Eastern Edition, January 29, 2001.

[13] Moinzadeh, K., P. Aggarwal. 1997. Analysis of a production / inventory system subject to random disruptions. Management Science 43(11) 1577-1588.

[13]Parlar, M. 1997. Continuous-review inventory problem with random supply interruptions. European Journal of Operational Research 99, 366-385.

[14]Parlar, M., D. Berkin. 1991. Future supply uncertainty in EOQ models. Naval Research Logistics 38, 107-121.

[15]Parlar, M., D. Perry. 1995. Analysis of a (q; r; t) inventory policy with deterministic and random yields when future supply is uncertain. European Journal of Operational Research 84, 431-443.

[16]Qi, L. 2009. A continuous-review inventory model with random disruptions at the primary supplier. Working paper.

[17]Qi, L., Z.J. M. Shen, L. V. Snyder. 2010. The effect of supply disruptions on supply chain design decisions. Transportation Science 44(2), 274-289.

[18]Reitman, V. 1997. Toyota Motor shows its mettle after _re destroys parts plant. Wall Street Journal, (May,08).

[19]Ruiz-Torres, A. J., F. Mahmoodi. 2007. The optimal number of suppliers considering the costs of individual supplier failures. Omega 35, 104-115.

[20]Saghafian, S., M. P. Van Oyen. 2011. The value of flexible backup suppliers and disruption risk information: Newsvendor analysis with recourse. Submitted to manuscript.

[22]Schmitt A. J. 2007. Strategic Inventory Management for Supply Chains Subject to Supply Uncertainty. PhD Thesis

[23]Schmitt, A. J., L. V. Snyder. 2009. Infinite-horizon models for inventory control under yield uncertainty and disruptions. Forthcoming in Computers and Operations Research.

[24]Schmitt, A. J., L. V. Snyder, Z.-J. M. Shen. 2010. Inventory systems with stochastic demand and supply: Properties and approximations. European Journal of Operational Research 206(2), 313-328.

[25]Sheffi, Y. 2007. The resilient enterprise: overcoming vulnerability for competitive advantage, The MIT Press, Cambridge, Massachusetts.[25]Snyder, L. V., B. T. Tomlin. 2008. Inventory management with advanced warning of disruptions. Working Paper, P.C. Rossin College of Engineering and Applied Sciences, Lehigh University, Bethlehem, PA.

[26]Song, J.-S., P. H. Zipkin. 1996. Inventory control with information about supply conditions. Management Science 42(10) 1409-1419.

[26]Swaminathan, J. M., J. G. Shanthikumar. 1999. Supplier diversification: Effect of discrete demand. Operations Research Letters 24, 213-221.

[27]Tehrani, B. M., S. H. Xu, S. Kumara, H. Li. 2010.A single period analysis of a two-echelon inventory system with dependent supply disruptions. Working Paper, Department of Industrial and Manufacturing Engineering, Pennsylvania State University.

[28]Tomlin, B. T. 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. Management Science 52(5), 639-657.

[29]Tomlin, B. T., Y. Wang. 2005. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. Manufacturing and Service Operations Management 7(1), 37-57.

[30]Xu, N., L. Nozick. 2009. Modeling supplier selection and the use of option contracts for global supply chain design. Computers and Operations Research 36(10), 2786-2800.

APPENDIX A

PROOF OF COROLLARY 3.3.4

Let Q' and I' denote $\frac{dQ^*(h)}{\partial h}$and $\frac{dI^*(h)}{\partial h}$ respectively. From $\frac{\partial \prod (Q,I)}{\partial Q} = 0$, we have

$(r-e)F(Q+I) + eF(Q) = r - c$. (A.1)

Then, from the implicit derivative of $\frac{\partial \prod (Q,I)}{\partial Q} = 0$ with respect to *h*, we have$(r-e)(\ I' + Q')f(I+Q) + eQ'f(Q) = 0$. Hence, I' and Q' should have opposite signs. Plugging (A.1) into $\frac{\partial \prod (Q,I)}{\partial I} = 0$, we get $-(r-e)pF(I) - (1-p)\big(r - c - eF(Q)\big) = -r + h + e$.

Taking the implicit derivation with respect to *h*, we have$-(r-e)pI'f(I) + (1-p)eQ'f(Q) = 1$. Note that$(r-e)pf(I) > 0$and $(1-p)ef(Q) > 0$, so we can argue that in order for the last equality to hold, we need to have $Q' \leq 0$ and $I' \geq 0$, which completes the proof. ■

APPENDIX B

PROOF OF COROLLARY 3.3.5

Let Q' and I' denote $\frac{dQ^*(c)}{\partial c}$and $\frac{dI^*(c)}{\partial c}$ respectively. From $\frac{\partial \prod (Q,I)}{\partial I} = 0$, we get

$r - e - h = (r - e)[pF(I) + (1 - p)F(Q + I)]$.(B.1)

Then from the implicit derivative of $\frac{\partial \prod (Q,I)}{\partial I} = 0$ with respect to c, we have$(r - e)(1 - p)(I' + Q')f(I + Q) = -(r - e)pI'f(I)$. Hence, I' and Q'have opposite signs. From $\frac{\partial \prod (Q,I)}{\partial Q} = 0$, we get

$(r - e)F(Q + I) + eF(Q) = r - c$.(B.2)

Plugging (B.2) into the $\frac{\partial \prod (Q,I)}{\partial I} = 0$, we get $(r - e)oF(I) + (1 - p)[r - c - eF(Q)] = -r + h + e$.Taking implicit derivative with respect to c, we have $(r - e)pI'f - (1 - p)eQ'f(Q) = (1 - p)$. Note that $(r - e)pf(I) < 0$and $-(1 - p)ef(Q) < 0$, we can argue that in order for the last equality to hold, we need to have$Q' \geq 0$ and $I' \leq 0$, which completes the proof. ■

APPENDIX C

PROOF OF COROLLARY 3.3.6

Let Q' and I' denote $\frac{dQ^*(e)}{\partial e}$and $\frac{dI^*(e)}{\partial e}$ respectively. From $\frac{\partial \prod(Q,I)}{\partial Q} = 0$, we get $(r-e)F(Q+I) + eF(Q) = r - c$.(C.1)

Then from the implicit derivative of $\frac{\partial \prod(Q,I)}{\partial Q} = 0$ with respect to e, we have $F(Q) - F(Q+I) + (r-e)(Q'+I')f(Q+I) + eQ'f(Q) = 0$. Hence, I' and $Q'$are not negative signs together. Taking the implicit derivative of $\frac{\partial \prod(Q,I)}{\partial I} = 0$ with respect to e, we have $F(Q+I) - (r-e)(Q'+I')f(Q+I) = 1$. Hence, Q' and I' should not be positive signs together. Plugging (C.1) into $\frac{\partial \prod(Q,I)}{\partial I} = 0$, we get $-(r-e)pF(I) - (1-p)\big(r - c - eF(Q)\big) = -r + h + e$. Taking the implicit derivation with respect to e, we have $-pF(I) - (1-p)F(Q) + (r-e)pI'f(I) - (1-p)eQ'f(Q) = -1$, which can be written as $-pF(I) - (1-p)[eQ'f(Q) + F(Q)] + (r-e)pI'f(I) = -1$. In order for the last equality to hold, we need to have$Q' \geq 0$and $I' \leq 0$, which completes the proof. ■

APPENDIX D

PROOF OF COROLLARY 3.3.7

Let Q' and I' denote $\frac{dQ^*(r)}{\partial r}$and $\frac{dI^*(r)}{\partial r}$ respectively. From $\frac{\partial \prod(Q,I)}{\partial Q} = 0$, we get

$(r-e)F(Q+I) + eF(Q) = r - c$.(D.1)

Taking the implicit derivative of $\frac{\partial \prod(Q,I)}{\partial Q} = 0$ with respect to *r*, we have $(r-e)(I' + Q')f(I+Q) + F(Q+I) + eQ'f(Q) = 0$. Hence, I' and Q' should have negative signs together. Plugging (D.1) into $\frac{\partial \prod(Q,I)}{\partial I} = 0$, we get $-(r-e)pF(I) - (1-p)\big(r - c - eF(Q)\big) = -r + h + e$. Taking the implicit derivation with respect to *r*, we have $-(r-e)pI'f(I) + (1-p)eQ'f(Q) - pF(I) = -p$.In order for the last equality to hold we need to have$I' \geq 0$, which completes the proof. ■

APPENDIX E

PROOF OF COROLLARY 4.3.3

From corollary 4.3.1, we have two different alternatives solution to Q^*; for $-h + p(r-e) + (1-p)(c-e) < 0$, $Q^* = F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$ which can be written as $F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right]$, otherwise $Q^* = \max\left\{F^{-1}\left(\frac{r-c}{r}\right), 0\right\}$. In either case, we can argue that Q^* is non-decreasing in the reservation cost, *h* and exercise cost*, c*. From corollary 4.3.1, we have two different alternative solutions to I^*, for $e > c$, $I^* =$ max$\left\{F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right], 0\right\}$ and for $e < c$, $I^* =$ max$\left\{F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right], 0\right\}$ which can be written as max$\left\{F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right], 0\right\}$. In either case, we can argue that I^* is non-increasing in the reservation cost, *h*, and exercise cost*, c* which completes the proof. ■

APPENDIX F

PROOF OF COROLLARY 4.3.4

From corollary 4.3.1, we have two different alternatives solution to Q^*; for $-h + p(r-e) + (1-p)(c-e) < 0$, $Q^* = F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right]$ which can be written as $F^{-1}\left(\frac{r-c}{r}\right) - F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right]$, otherwise $Q^* = \max\left\{F^{-1}\left(\frac{r-c}{r}\right), 0\right\}$. In either case, we can argue that Q^* is non-increasing in the disruption probability, *p*. From corollary 4.3.1, we have two different alternative solutions to I^*, for $e > c$, $I^* = \max\left\{F^{-1}\left[\frac{r-e-\frac{h}{p}}{r}\right], 0\right\}$ and for $e < c$, $I^* = \max\left\{F^{-1}\left[\frac{p(r-c)-h-e+c}{pr}\right], 0\right\}$ which can be written as $\max\left\{F^{-1}\left[\frac{(1-p)c+pr-h-e}{pr}\right], 0\right\}$. In either case, we can argue that I^* is non-decreasing in the disruption probability, *p*, which completes the proof. ∎

APPENDIX G

THE OPTIMAL RESULTS FOR THE PARAMETER SENSITIVITY ANALYSIS

Table G.1: Optimal Results for the problem instances when p is increased

NO INFORMATION			
p	Profit	Q	I
0,01	12112,68	109,9414	0
0,05	11623,28	109,9414	0
0,1	11143,94	22,38281	87,55859
0,15	11106,34	13,76953	96,17676
0,2	11088,61	10,61523	99,33105
0,25	11077,7	8,769531	101,1719
0,3	11070,17	7,529297	102,417
0,35	11064,63	6,611328	103,3301
0,4	11060,36	5,908203	104,0332
0,45	11056,9	5,35	104,59
0,5	11054,1	4,89	105,053
0,55	11051,85	4,5	105,43
0,6	11049,89	4,17	105,76
0,65	11048,2	3,89	106,04
0,7	11046,74	3,65	106,28
0,75	11045,46	3,43	106,5
0,8	11044,33	3,25	106,68
0,85	11043,32	3,07	106,86
0,9	11042,4	2,92	107,02
PARTIAL INFORMATION			
p	Profit	Q	I
0,01	12112,68	109,9446	0
0,05	11707,14	109,9446	88,89229
0,1	11651,9	109,9446	99,16348
0,15	11611,73	109,9446	101,9621
0,2	11575,09	109,9446	103,4069

Table G.1: Optimal Results for the problem instances when p is increased (cont'd)

p	Profit	Q	I
0,25	11539,92	109,9446	104,3073
0,3	11505,52	109,9446	104,927
0,35	11471,56	109,9446	105,3815
0,4	11437,89	109,9446	105,7297
0,45	11404,4	109,945	106
0,5	11371,07	109,945	106,22
0,55	11337,8	109,945	106,41
0,6	11304,6	109,945	106,57
0,65	11271,5	109,945	106,7
0,7	11238,5	109,945	106,82
0,75	11205,52	109,945	106,92
0,8	11172,5	109,945	107
0,85	11139,59	109,945	107,09
0,9	11106,66	109,945	107,15
FULL INFORMATION			
p	Profit	Q	I
0,01	12282,47	95,54408	22,9796
0,05	11951,33	91,58379	90,32583
0,1	11894,95	91,58379	102,1043
0,15	11856,87	91,58379	105,8946
0,2	11823,84	91,58379	108,1222
0,25	11793,19	91,58379	109,6742
0,3	11763,91	91,58379	110,8533
0,35	11735,51	91,58379	111,7976
0,4	11707,74	91,58379	112,5816
0,45	11680,41	91,58379	113,24
0,5	11653,44	91,58379	113,82
0,55	11626,74	91,58379	114,34
0,6	11600,26	91,58379	114,79
0,65	11573,96	91,58379	115,21
0,7	11547,81	91,58379	115,58
0,75	11521,79	91,58379	115,93
0,8	11495,89	91,58379	116,24
0,85	11470,08	91,58379	116,54
0,9	11444,35	91,58379	116,81

Table G.2: Optimal Results for the problem instances when h is increased

NO INFORMATION			
h	Profit	Q	I
1	11514,73	13,17383	96,76758
5	11143,94	22,38281	87,55859
9	11011,53	109,941	0
13	11011,53	109,941	0
17	11011,53	109,941	0
21	11011,53	109,941	0
25	11011,53	109,941	0
29	11011,53	109,941	0
33	11011,53	109,941	0
PARTIAL INFORMATION			
h	Profit	Q	I
1	12062,24	109,9446	106,2292
5	11651,9	109,9446	99,16348
9	11269,53	109,9446	91,58
13	11011,53	109,9446	0
17	11011,53	109,9446	0
21	11011,53	109,9446	0
25	11011,53	109,9446	0
29	11011,53	109,9446	0
33	11011,53	109,9446	0
FULL INFORMATION			
h	Profit	Q	I
1	12323,38	91,58379	113,83
5	11894,95	91,58379	102,1043
9	11503,64	91,5838	93,25
13	11230,9	92,84	30,7
17	11132,82	97,08	19,73
21	11067,6	100,83	12,99
25	11027,4	104,71	7,01
29	11011,62	109,52	0,52
33	11011,52	109,94	0

Table G.3: Optimal Results for the problem instances when c is increased

NO INFORMATION			
c	Profit	Q	I
22	11209,93	110,5078	0
23	11165,96	27,00195	83,22266
24	11143,94	22,38281	87,55859
25	11125,22	19,35547	90,32227
26	11108,94	16,91406	92,5
27	11094,69	14,78516	94,37012
28	11082,27	12,84668	96,05469
29	11071,54	11,02051	97,63672
30	11062,41	9,24	99,16
31	11054,88	7,509766	100,6689
32	11048,9	5,761719	102,1875
PARTIAL INFORMATION			
c	Profit	Q	I
22	11850,3	110,5084	99,16348
23	11750,97	110,2224	99,16348
24	11651,9	109,9446	99,16348
25	11553,07	109,6742	99,16348
26	11454,48	109,4107	99,16348
27	11356,13	109,1537	99,16348
28	11258	108,9025	99,16348
29	11160,1	108,6568	99,16348
30	11062,41	9,25	99,1635
31	11054,88	7,511597	100,6688
32	11048,9	5,75942	102,1897
FULL INFORMATION			
c	Profit	Q	I
22	12061,75	93,77075	102,1043
23	11977,86	92,72087	102,1043
24	11894,95	91,58379	102,1043
25	11813,12	90,32578	102,1043
26	11732,5	88,89229	102,1043
27	11653,29	87,18449	102,1043
28	11575,83	84,98917	102,1043
29	11500,81	81,66084	102,1043
30	11420,37	50	102,1
31	11420,38	50	102,1043
32	11424,7	19,86061	104,0792

Table G.4: Optimal Results for the problem instances when r is increased

NO INFORMATION			
r	Profit	Q	I
100	6560,214	107,0605	0
125	8782,747	108,7061	0
150	11143,94	22,38281	87,55859
175	13600,64	18,17871	92,75391
200	16067,5	16,15234	95,60059
PARTIAL INFORMATION			
r	Profit	Q	I
100	6732,218	107,063	91,58379
125	9185,982	108,7055	96,41541
150	11651,9	109,9446	99,16348
175	14124,92	110,9325	101,0764
200	16602,66	111,7499	102,5335
FULL INFORMATION			
r	Profit	Q	I
100	6926,028	91,58379	94,34052
125	9407,981	91,58379	99,33989
150	11894,95	91,58379	102,1043
175	14384,34	91,58379	103,9932
200	16875,15	91,58379	105,414

Table G.5: Optimal Results for the problem instances when e is increased

NO INFORMATION			
e	Profit	Q	I
18	0	0	0
20	12126,35	2,441406	107,5
22	11915,41	6,357422	103,584
24	11711,69	9,77	100,16
26	11514,73	13,17383	96,76758
28	11324,9	17,00195	92,93945
30	11143,94	22,38281	87,55859
32	11011,53	109,9414	0
PARTIAL INFORMATION			
e	Profit	Q	I
18	0	0	0
20	12126,35	2,445974	107,4986
22	11915,41	6,360064	103,5845
24	11711,7	9,775	100,16
26	11691,7	109,9446	99,83288
28	11671,76	109,9446	99,49846
30	11651,9	109,9446	99,16348
32	11632,1	109,9446	98,82755
e	Profit	Q	I
FULL INFORMATION			
18	0	0	0
20	12386,62	6,307068	112,389
22	12206,17	14,79382	106,7449
24	12017,89	50	102,61
26	11975,54	85,73921	102,4501
28	11933,05	89,3243	102,2797
30	11894,95	91,58379	102,1043
32	11859,67	93,2551	101,9236

Table G.6: Optimal Results for the problem instances when σ is increased

NO INFORMATION			
σ	Profit	Q	I
5	11321,97	11,19141	93,7793
10	11143,94	22,38281	87,55859
15	10965,91	33,59	81,32
20	10787,88	44,76563	75,11719
25	10609,85	55,97	68,88
30	10431,82	67,1875	62,65625
PARTIAL INFORMATION			
σ	Profit	Q	I
5	11845,95	104,9723	99,58174
10	11651,9	109,9446	99,16348
15	11457,84	114,91	98,74
20	11263,79	119,8892	98,32695
25	11069,74	124,86	97,9
30	10875,69	129,83	97,49046
σ	Profit	Q	I
FULL INFORMATION			
5	11958,4	95,79189	101,0521
10	11894,95	91,58379	102,1043
15	11831,5	87,37	103,156
20	11768,21	83,16936	104,2115
25	11713,54	79,08	105,46
30	11673,87	75,8368	108,1039