OPTIMUM SEARCH STRATEGIES FOR ELECTRONIC SUPPORT MEASURES RECEIVERS

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HALİM SİNAN BALABAN

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submitted by **HALİM SİNAN BALABAN** in partial fulfillment of the requirements for the degree of **Master of Science in Electrical and Electronics Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** __________

Prof. Dr. İsmet Erkmen
Head of Department, **Electrical and Electronics Engineering** __________

Prof. Dr. Mustafa Kuzuoğlu
Supervisor, **Electrical and Electronics Engineering Dept., METU** __________

Examining Committee Members:

Prof. Dr. Tolga Çiloğlu
Electrical and Electronics Engineering Dept., METU __________________

Prof. Dr. Mustafa Kuzuoğlu
Electrical and Electronics Engineering Dept., METU __________________

Assoc. Prof. Çağatay Candan
Electrical and Electronics Engineering Dept., METU __________________

Assoc. Prof. Ali Özgür Yılmaz
Electrical and Electronics Engineering Dept., METU __________________

Aydın Bayri, M.Sc.
ASELSAN Inc. __________________

Date: 13.09.2012

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name : HALİM SİNAN BALABAN

Signature :

ABSTRACT

OPTIMUM SEARCH STRATEGIES FOR ELECTRONIC SUPPORT MEASURES RECEIVERS

Balaban, Halim Sinan

M. S., Department of Electrical and Electronics Engineering
Supervisor: Prof. Dr. Mustafa Kuzuoğlu

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Electronic Support Measures is a discipline of electronic warfare. In electronic support measures, receivers must maintain surveillance over the very wide portion of the electromagnetic spectrum in which threat emitters operate. In current receiver technology, it is not possible to have a receiver which is at once both able to discriminate multiple simultaneous emissions and highly sensitive. A common approach is to use a receiver with a relatively narrow bandwidth that sweeps its centre frequency over the threat bandwidth to search for emitters. The sequence and timing of changes in the centre frequency constitute a search strategy or sensor scheduling problem.

A good electronic support receiver should observe the threat emitters, usually radars, very soon after it first begins transmitting, so in designing search strategy we would like to ensure that the intercept time is low or the probability of intercept after a specified time is high.

In this thesis, we study the search strategies used in electronic support measures receivers. Moreover, a search strategy based on probability of intercept of the threats is proposed. The performances of the search strategies are compared at the end of the thesis.

Keywords: Electronic Support Measures receivers, search strategy, radar intercept, probability of intercept, synchronization with radar, sensor scheduling, Farey series

ÖZ

ELEKTRONİK DESTEK ALMAÇLARI İÇİN OPTİMUM ARAMA STRATEJİLERİ

Balaban, Halim Sinan

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Elektronik Destek Tedbirleri Elektronik Harbin bir disiplinidir. Elektronik Destek almaçlarının, tehditlerin çalıştığı çok geniş elektromanyetik spektrumu gözlemleyebilmesi gerekmektedir. Günümüz almaç teknolojisinde, hem birden fazla eş zamanlı tehdidin yayınını algılayabilecek hem de yeteri kadar hassasiyete sahip bir almaç mümkün değildir. Genel yaklaşım daha dar bantgenişliğine sahip almaç kullanıp, almacın merkez frekansını taranacak tehditlerin frekans spektrumu göre değiştirmektir. Merkez frekansı değişimlerinin sırası ve zamanlaması bir arama stratejisi veya sensör zamanlama problemi oluşturmaktadır.

İyi bir elektronik destek almacı; tehditleri ki bunlar genellikle radarlardır, ilk çalışmaya başladıklarından kısa bir süre sonra fark edebilmelidir, bu nedenle arama stratejisi tasarlarken yakalama zamanının kısa ya da belirli bir süre içindeki yakalama olasılığının yüksek olmasını sağlamak isteriz.

Bu tezde, elektronik destek almaçlarında kullanılan arama stratejilerini inceledik. Ayrıca tehditler için yakalama olasılığına dayalı bir arama stratejisi önerilmiştir. Tezin son kısmında arama stratejileri performansları karşılaştırılmıştır.

Anahtar Kelimeler: Elektronik Destek almaçları, arama stratejisi, radar yakalama, yakalama olasılığı, radar ile senkronizasyon, sensör zamanlama, Farey serileri

To My Family

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CHAPTER 1

INTRODUCTION

Electronic warfare, in short EW, had its true origins in World War II. It developed as the nemesis to radar. In the general usage, electronic warfare was defined [3] as: *"Military action involving the use of electromagnetic energy to determine, exploit, reduce, or prevent hostile use of the electromagnetic spectrum and action which retains friendly use of the electromagnetic spectrum."* Three divisions within EW are: ESM, ECM and ECCM. The definitions of these are given below.

ESM or electronic support measures was defined [3] as: *"Action taken under direct control of an operational commander to search for, intercept, identify, and locate sources of radiated electromagnetic energy for the purpose of immediate threat recognition. ESM provide a source of information required for immediate decisions involving ECM, ECCM, avoidance, targeting, and other tactical employment of forces."* ECM or electronic countermeasures was defined [3] as: *"Action taken to prevent or reduce an enemy's effective use of the electromagnetic spectrum.* Finally, ECCM or electronic counter-countermeasures was defined [3] as: *"Actions taken to ensure friendly effective use of the electromagnetic spectrum despite the enemy's use of ECM."*

The key task under ESM is interception of electromagnetic radiation emitted by threat systems in the shortest possible time. Nevertheless, the bandwidth in which emitters, they are usually radars, operate spans many gigahertz. In current receiver technology, it is not possible to have a receiver which is at once both able to discriminate multiple simultaneous emissions and highly sensitive. To cope with this very large bandwidth, widely preferential receiver architecture employs a receiver of more modest bandwidth with an agile centre frequency. The frequency swept super heterodyne receiver is an example of this kind of receiver [9]. In order to maintain surveillance over the entire search bandwidth, it is necessary to repeatedly re-tune the centre frequency of the receiver. That explains why they are called frequency swept super heterodyne receivers. The sequence and timing of changes to the centre frequency constitute a search strategy, which is also a sensor scheduling problem.

On the other hand, emitters also commonly employ a search strategy of their own., Radars may have a highly directional antenna, in order to gain good angular resolution. either through mechanical movement of the antenna or, in more modern and sophisticated emitters, through electronic 'beam steering', the main beam is scanned.

In an ESM system, the goal of the receiver is to try to detect radiation from the emitter in the shortest possible time, in order that the operator can be informed, the emitter be identified and the appropriate action be taken. In order for an interception to happen, the receiver must be dwelling in the 'right' band, that is, the one on which the emitter operates, whereas the emitter is pointing in the 'right' direction, that is, the one in which the receiver lies. Naturally, the performance of ESM receiver is directly related to the search strategy [10].

In this thesis, we study the search strategies used in ESM receivers. Moreover, a search strategy based on probability of intercept of the emitters is proposed. At the end of the thesis, the performances of the search strategies are compared.

1.1 PREVIOUS WORK

The literature on search strategies is sparse but not entirely absent. The only relatively substantial body of work pertains to the closely related problem of calculating intercept time between emitter and receiver or the probability of intercept when both emitter and receiver are employing periodic strategies. When both emitter and receiver are using a periodic strategy determining the intercept time requires some application of elementary number theory. The problem was first studied by Richards [4]. He examined the problem of intercept for two strictly periodic pulse trains in connection with a problem in theoretical physics. He noticed that "with certain rational ratios of the periods, the events may 'lock in step' ". Therefore, in our emitter/receiver interception problem, there is a possibility that the receiver may never intercept the emitter. This is possible if the receiver sweep period is poorly chosen, we call this situation as synchronization. Moreover, Richards admits that, despite deriving certain approximations for probability of intercept, 'the original problem is not completely solved'.

Miller and Schwarz [6] showed how intercept time for certain ratios of the periods of the pulse trains could be obtained using linear congruence when the periods were assumed to be commensurate, in other words, both were integer multiples of some common 'base' period. Friedman [5] was refined their work. However, a general formulation of the problem first appeared in the work of Kelly [7]. He describes what essentially an algorithm for determining the intercept time is. Wiley [1] also developed a formulation to calculate intercept time, by using average coincidence period and average coincidence duration. Clarkson [11] made clear the links with elementary number theory. They showed that intercept time is a problem of Diophantine approximation and can be solved either through the application of Euclid's algorithm or by the examination of adjacent fractions in a Farey series [18]. Recently, Clarkson [13, 15, 16] showed that if the sweep period of the receiver is varied, while the proportion of time spent in each band is held constant, then there is usually only a finite number of sweep periods that can cause synchronization with the emitter.

The potential for synchronization in the periodic strategy is of concern. By improper choice of receiver sweep period, the receiver might never detect the emitter. Even if the sweep period lies close to one of the 'synchronization periods', the intercept time may be arbitrarily long. Washburn [14] suggested that one way to reduce the chances of synchronization would be to introduce jitter into the receiver sweep period. Jitter can be best described as regulated noise in the sweep period of the receiver. This would have the effect of making the search pattern of the receiver ‘less’ periodic. Although a random search strategy of this type destroys any possibility of a guaranteed upper bound on the intercept time, it is possible to measure performance through, for instance, the expected intercept time. Kelly [7] carried out extensive simulation studies to evaluate the effect of varying the amount of jitter. However, rigorous analysis of the strategy appears very difficult. Another way to combat synchronization is to have good intelligence about the scan periods of the emitters that are likely to be in operation. This is the approach of Clarkson [13, 15, 16], who describes a method for selecting a sweep period for a periodic receiver search strategy that minimizes the maximum intercept time; the maximum being taken over all the emitters listed in a threat emitter list. When the intelligence is good, and the scan period parameters recorded in the threat emitter list can be relied upon, the sweep period setting calculated according to Clarkson’s algorithm gives very low intercept times.

1.2 SCOPE OF THE THESIS

This thesis has been motivated by the need to understand search strategies for an ESM receiver. The interception of radar transmission plays a key role in electronic warfare. In particular the rapid detection and identification of possible threat emitters is of vital importance. Indeed, the performance of ESM receiver is directly related to the search strategy.

The first search strategy to be examined is the ‘Simple’ Search Strategy. The Simple Search Strategy involves tuning the receiver into each frequency band that emitters operate in some sequence. In each frequency band, the receiver dwells for a certain time before re-tuning to a different band. Usually dwell time is same for all frequency bands. Once the receiver steps to the last frequency band in the sequence, it begins again from the first. On the other hand for the Simple Search Strategy there is a possibility of synchronization. In case of synchronization intercept will never occur. Clarkson’s algorithm takes into account the synchronization problem. By changing individual dwell times Clarkson’s algorithm prevents synchronization.

The second search strategy to be examined is Clarkson's algorithm. Moreover a search strategy based on probability of intercept of the threats is proposed.
Finally the performance of the search strategies is compared. Here the question is, how to determine the performance of a search strategy. At the heart of all performance comparison of a search strategy is the intercept time. That is, one search strategy is better than another if it provides smaller intercept time for the emitters. In order to compare the search strategies, for an ESM scenario given in [16] the intercept times of the emitters for each search strategy calculated. The results are given for Monte Carlo simulations.

1.3 OUTLINE OF THE THESIS

In this thesis, we are interested in search strategies for an ESM receiver.

In Chapter 2, a brief summary of radar interception problem is done. Receiver and interception models are explained. Also definitions of the parameters that are used throughout the study are given in this chapter.

Search strategies to be examined in this thesis, namely simple search and Clarkson algorithm, are given in details in Chapter 3.

In Chapter 4, the proposed algorithm is explained in details. The algorithm, which is based on probability of intercept of the threats, is explained in this part.

Chapter 5 contains the performance comparison of the algorithms described in Chapter 3 and Chapter 4. The simulation results and the performance comparisons for Monte Carlo simulations are presented.

Chapter 6 is the conclusion part of this thesis. A summary of this study and the future work are given in this part.

CHAPTER 2

INTERCEPTION OF RADAR SIGNALS

Radars and radar-intercept receivers (usually ESM) are designed to search. The search may be conducted over several parameters, including angle and center frequency. Usually the search is periodic. The same region in parameter space is periodically scanned or revisited. That is, the receiver tunes to a particular center frequency at regular intervals, similarly the radar points in a particular direction at regular intervals. The interval for the receiver and interval for the radar, i.e., the periods are usually different. In order for the receiver to intercept emissions from a radar, the scans must 'line up' at some point in time. The radar must be pointing (illuminating) towards the receiver; at the same time the receiver must be pointing towards the radar. Moreover the receiver must be tuned to the radar's frequency band. In our study, interception problem is a beam-on-frequency intercept. This is going to be explained below.

Interception is distinct from detection. Interception must have occurred before detection can take place. Detection is concerned with such factors as receiver sensitivity thresholds. [2] Interception is said to occur when any energy at all is registered at the receiver, on the other hand detection occurs only when enough energy has been received to positively identify its source. In this thesis we are interested in interception, however detection is not a subject of this study.

2.1 INTERCEPT MODEL

In this thesis we assume that we have an ESM receiver which is a frequency-swept receiver (FSR) with an omnidirectional antenna. The receiver has a number of frequency bands through which it sweeps. In fact, the sweeping process is not continuous. That is, the receiver does not continuously change its center frequency. Instead, it dwells at a particular center frequency for a certain length of time before moving on to the next center frequency. The length of this time is called dwell time, $\tau_{receiver}$. Dwell time can be same for all bands or it can be adjusted individually. A frequency-swept super heterodyne receiver is an example from the class of receivers that we call FSRs. [16]

In this study we are only interested in periodic search strategies. Periodic search strategy means that the sequence of dwells, that is the search strategy, repeats exactly after each pass through the search bandwidth. The total time of one sequence is called sweep period, $T_{receiver}$.

The receiver parameters, namely dwell time and sweep period are illustrated in Figure 2-1, for an example search strategy.

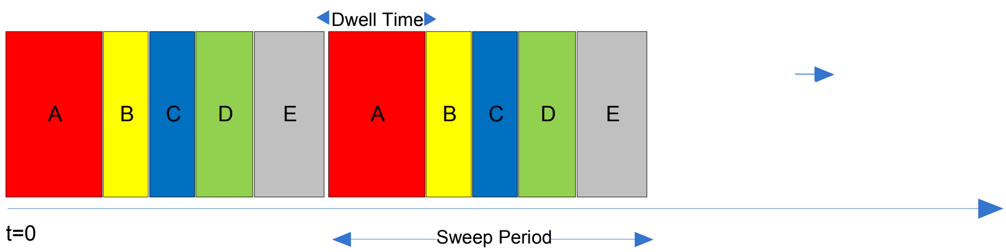

Figure 2-1: Receiver Parameters in a Search Strategy

Similarly radars also employ their own search strategy, which is commonly periodic. Radars may have a highly directional antenna in order to gain good angular resolution. Either through mechanical movement of the antenna or, in more modern and sophisticated emitters, through electronic 'beam steering', the main beam is scanned. The time at which the main beam of the emitter is pointed towards the receiver is periodic and this period is its scan period, $T_{emitter,}$ in these cases. Radar illuminates one direction for a certain length of time, which is called illumination time, $\tau_{emitter}$. Illumination time of a circularly scan radar can be calculated by using:

$$\tau_{emitter} = \frac{Beamwidth}{360} * T_{emitter} \qquad (2\text{-}1)$$

Here beamwidth defines sight of the radar when it directs its main beam to a direction, in degrees. Beamwidth is usually expressed as 3dB beamwidth. That is the angle between the half-power (3 dB) points of the main lobe, in the radio regime of an antenna pattern, when referenced to the peak effective radiated power of the main lobe. An antenna pattern example is shown in Figure 2-2.

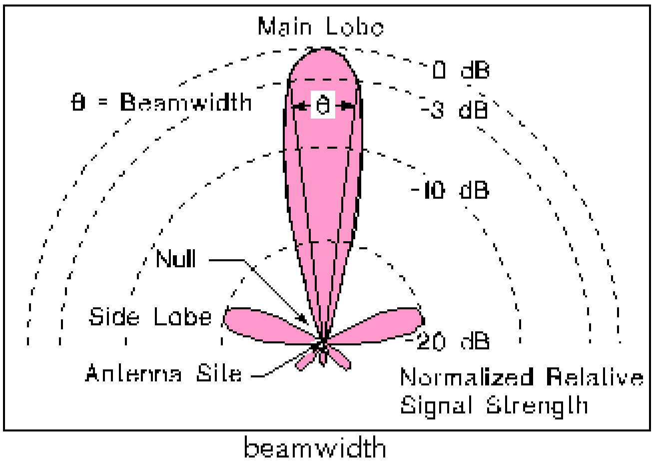

Figure 2-2: Antenna Pattern

For the antenna pattern given in Figure 2-2 the received power by the receiver is illustrated in Figure 2-3. Note that receiver can receive power only when the main beam of the radar is directed towards to the location where the receiver lies. Otherwise the received power is zero.

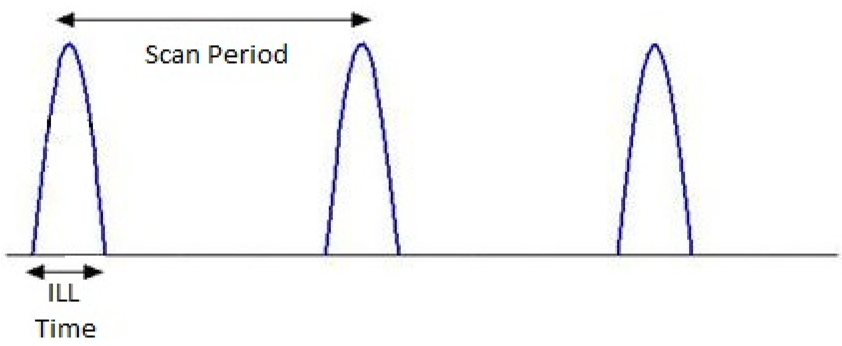

Figure 2-3: Received Power by the Receiver

Radar does not emit only one pulse during the illumination time, namely $\tau_{emitter}$. Actually radars emit many pulses one after another within an interval. The interval is determined in order to be able to detect a target in a desired range. This interval is defined as pulse repetition interval, in short PRI.

Moreover we make the following simplifying assumptions [16]:

1) We assume a search strategy which is periodic with a stable scan period, $T_{emitter}$, for the radar.
2) The frequency (RF) of the radar is also fixed.
3) The parameters of the radar namely RF, PRI, beamwidth and scan period are recorded in a threat-emitter list and are all known to good accuracy.

After these assumptions what is not known a priori by the receiver is if the radar is switched on or it is within range of the receiver.

The interception model for an EW (ESM) receiver is shown in Figure 2-4.

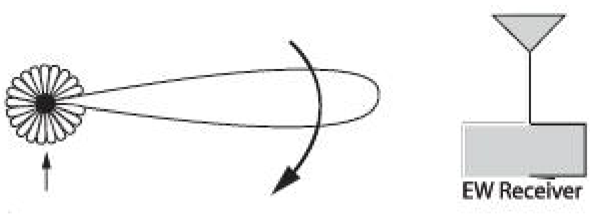

Figure 2-4: Interception Model of ESM Receiver

2.2 INTERCEPTION

Interception occurs when any energy is received by the receiver. In order to receive energy from the emitter, the receiver must be tuned to the right band, that is the operation band of the emitter, while the emitter is pointing in the right direction, that is the direction of the receiver. In other words the radar must be illuminating the receiver and the receiver must be tuned to the band of the emitter. Mathematically, the receiver being tuned to the right band can be described as an event. Whether or not the event is occurring at any particular time can be modeled by a function whose value is 1 or 0, respectively. Such a function is called a window function or pulse train. We call the time interval over which an event occurs as a window or pulse and the width of the pulses are called pulsewidth. As a result, we can describe the emitter and the receiver behaviors by pulse trains. To do this we construct one pulse train to represent whether the emitter is pointing in the right direction, and another to represent whether the receiver is tuned to the right band.

The main assumption we have done in the previous section is that both the receiver and the emitter are employing periodic search strategies, as a result the corresponding pulse trains are both periodic. Both window functions are described with the following period and pulsewidth parameters.

For the emitter pulse train,

- the period is the scan period, $T_{emitter}$,
- the pulsewidth is the illumination time, $\tau_{emitter}$,

Emitter pulse train is shown in Figure 2-5.

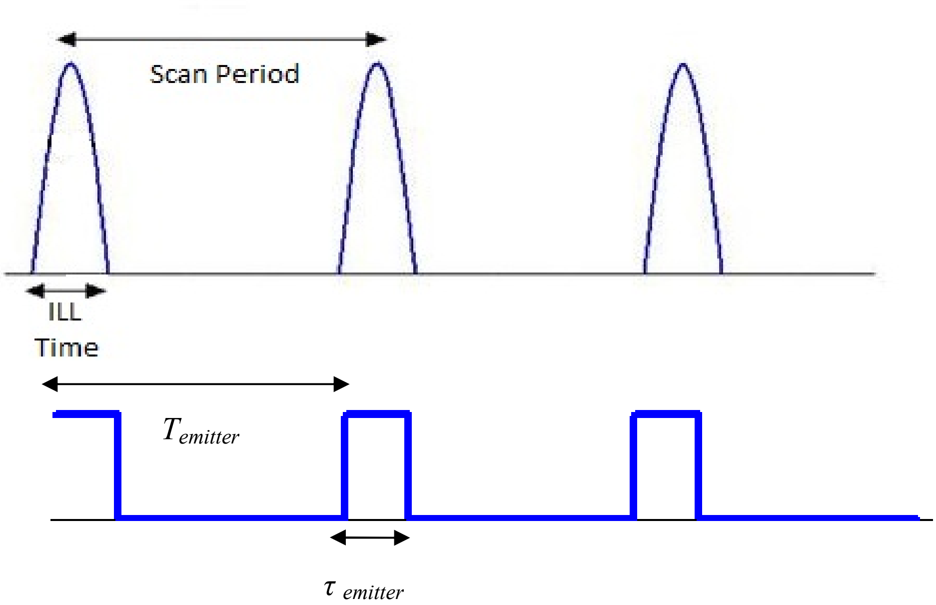

Figure 2-5: Pulse Train for the Emitter

Similarly for the receiver pulse train,

- the period is the sweep period, $T_{receiver}$,
- the pulsewidth is the dwell time on that particular band, $\tau_{receiver}$,

Receiver pulse train is shown in Figure 2-6.

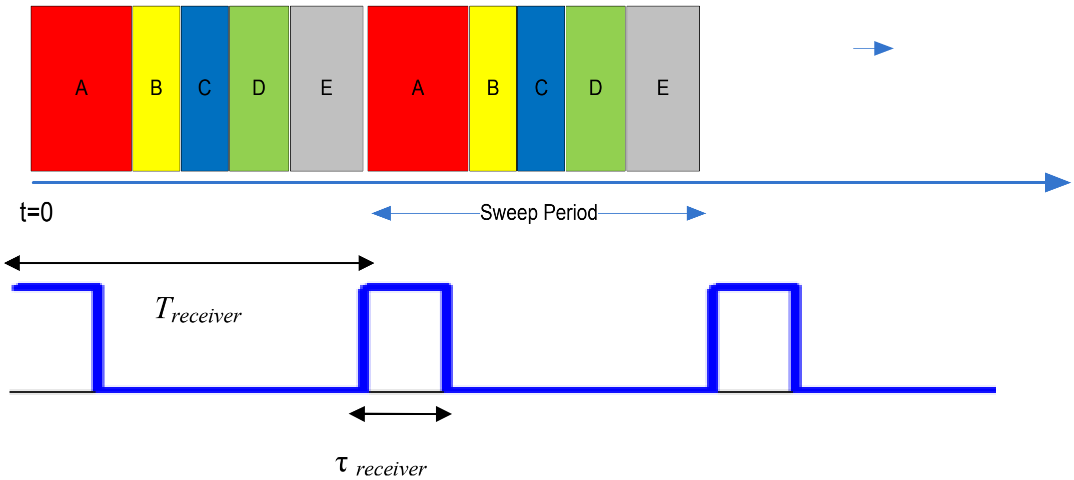

Figure 2-6: Pulse Train for the Receiver

A phase, the time offset between the center of a designated pulse and the time origin can be associated to each pulse train. Note that phase is not unique, since we can obtain another equally valid value for phase by adding the period to the phase.

After constructing pulse trains for the receiver and emitter we can define interception again by using pulse trains. For the receiver in order to intercept energy from the emitter, pulse trains must coincide, that is their value must be 1 at the same time instance. By this way the problem of interception becomes that of pulse coincidence between two pulse trains. This case is visualized in Figure 2-7.

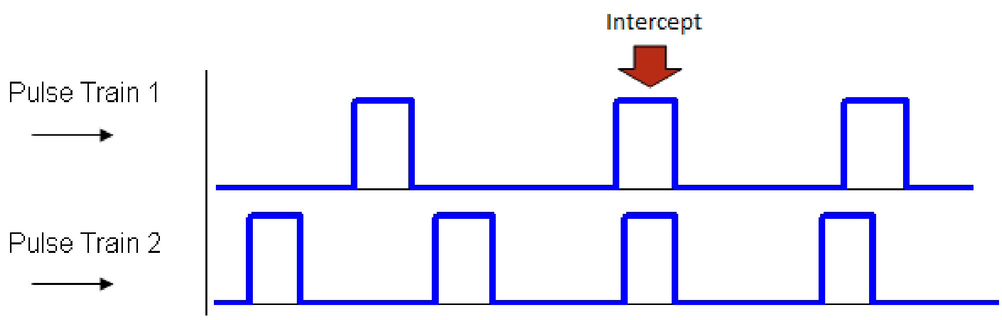

Figure 2-7: Interception (Coincidence of Periodic Pulse Trains)

CHAPTER 3

SEARCH STRATEGIES

In electronic support measures, the ability to detect or intercept users of the electromagnetic spectrum, which we call emitters (usually radars), in the shortest possible time is a key operational requirement. Nevertheless, the bandwidth in which emitters operate spans many gigahertz. In order to handle this very large bandwidth, widely favored receiver architecture employs a receiver of more modest bandwidth with an agile centre frequency. The frequency-swept superheterodyne receiver is an example of this sort of receiver [9]. To maintain surveillance over the entire search bandwidth, it is necessary to repeatedly re-tune the centre frequency of the receiver. The sequence and timing of changes to the centre frequency constitute a search strategy. An obvious and widely used strategy is the periodic strategy.

The assumptions we made in the previous chapter is that we have a priori information about the emitters that are expected to exist in the environment. A threat-emitter list contains priori information. Optimization criteria can be formulated from this a priori information. This assumption is actually realistic, since the parameters of the radars can be extracted by ELINT systems. These parameters cannot have always exact values, but their values are usually within a range. For now, we assume that the parameters of the emitters have exact values for simplicity. For each radar, RF, PRI, beamwidth and scan period are recorded. We also assume that only the radars which employ periodic search strategies are recorded in the threat-emitter list.

In this chapter at first we are going to examine calculation of (maximum) intercept time for periodic search strategies. After that, Simple Search Strategy is going to be investigated. Finally Clarkson's algorithm, in which synchronization problem is solved, is going to be given.

3.1 INTERCEPT TIME

Pulse trains for receiver and emitter are given in section 2.2. Pulse trains are shown in Figure 2-5 and Figure 2-6, respectively. Intercept time is the required time to guarantee at least one coincidence between two pulse trains, which is independent from phases of pulse trains. Here, intercept time corresponds to maximum intercept time.

Note that the k_i th pulse from pulse train i occurs when

$$\left|t - k_i T_i - \phi_i\right| \leq \frac{1}{2}\tau_i \tag{3-1}$$

where T_i is the period, τ_i is the pulsewidth and ϕ_i is the phase of the ith pulse train. Similarly k_j th pulse from pulse train j occurs when

$$\left|t - k_j T_j - \phi_j\right| \leq \frac{1}{2}\tau_j \tag{3-2}$$

Therefore, for a coincidence between two pulse trains, for an emitter and the receiver, a necessary and sufficient condition is as follows,

$$\left|k_i T_i + \phi_i - k_j T_j - \phi_j\right| \leq \frac{1}{2}(\tau_i + \tau_j) \tag{3-3}$$

In addition, if a coincidence of minimum duration of d is required then we can write (3-3) as;

$$\left|k_i T_i + \phi_i - k_j T_j - \phi_j\right| \leq \frac{1}{2}(\tau_i + \tau_j - 2d) \tag{3-4}$$

Also note that for a coincidence of duration d both pulsewidths must be greater than d. That is $\tau_{emitter} > d$ and $\tau_{receiver} > d$. Otherwise we cannot talk about a coincidence, or intercept, between pulse trains.

We can define pulse train 1 and pulse train 2as the emitter pulse train and the receiver pulse train, respectively. So the emitter scan period ($T_{emitter}$) is T_1, the illumination time ($\tau_{emitter}$) is τ_1, and its phase is φ_1. Similarly, the receiver sweep period ($T_{receiver}$) is T_2, the dwell time ($\tau_{receiver}$) is τ_2 and its phase is φ_2.
According to these definitions, let us define the following

- *p* and *q* represent pulse index from the emitter and receiver pulse train, respectively (in place of k_1 and k_2),
- the period ratio is

$$\alpha = \frac{T_{receiver}}{T_{emitter}} \tag{3-5}$$

- the (normalized) relative phase is,

$$\beta = \frac{\phi_2 - \phi_1}{T_{emitter}} \tag{3-6}$$

- the tolerance is

$$\varepsilon = \frac{\tau_{emitter} + \tau_{receiver} - 2d}{T_{emitter}} \tag{3-7}$$

By using these, equation (3-4) can be written again,

$$|q\alpha - p + \beta| \le \frac{1}{2}\varepsilon \tag{3-8}$$

Note that with $d = 0$ tolerance is the normalized sum of the pulsewidths.

3.1.1 Farey Series and Intercept Time

The Farey series of order *n*, F_n, is the sequence or series of fractions, written in lowest terms and in ascending order, with denominator less than or equal to *n*. [16] The sequence is usually defined such that it consists of only those fractions between 0 and 1. According to this definition, the first five orders are listed in Table 3-1.
Note that, we ensure that *h* and *k* have no common prime factors by writing a fraction *h/k* in lowest terms. Farey series have the following properties:

Let h/k and h'/k' be two adjacent elements of the series. Their median is defined as $(h+h')/(k+k')$. The median is itself adjacent to both h/k and h'/k' in the Farey Series of higher order, namely when the order is $k+k'$.

Moreover, for the fractions $h/k < h'/k'$, which are adjacent in a Farey Series, we have the relation: $h'k - hk' = 1$.

This property is known as unimodularity property.

Table 3-1: Farey Series up to Order 5

1st order	$\frac{0}{1}$										$\frac{1}{1}$
2nd order	$\frac{0}{1}$					$\frac{1}{2}$					$\frac{1}{1}$
3rd order	$\frac{0}{1}$			$\frac{1}{3}$		$\frac{1}{2}$		$\frac{2}{3}$			$\frac{1}{1}$
4th order	$\frac{0}{1}$		$\frac{1}{4}$	$\frac{1}{3}$		$\frac{1}{2}$		$\frac{2}{3}$	$\frac{3}{4}$		$\frac{1}{1}$
5th order	$\frac{0}{1}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{1}{1}$

ALGORITHM 1: The algorithm for producing Farey Series of order n between 0 and 1 is [18],

```
farey (h,k,h',k',n) =
    if k+k' ≤ n then
        farey (h,k,h+h',k+k',n)
        output((h+h')/(k+k'))
        farey (h+h',k+k',h',k',n)
    end
```

In [13], Farey series of appropriate order has been shown suitable to find the intercept time and to enumerate the synchronization ratio. The algorithm is given below.

ALGORITHM 2:

1. Calculate α and ε as given in (3-5) and (3-7).
2. Calculate the Farey series by using ALGORITHM 1, the order of Farey series can be calculated as,

$$n_{\text{farey}} = \text{floor}\,(\frac{1}{\varepsilon})\ - 1 \tag{3-9}$$

3. Find adjacent elements of the Farey series of order n_{farey}, h/k and h'/k', that satisfy $h/k \le \alpha \le h'/k'$. If α is equal to one of these adjacent elements and $\varepsilon < 1/k$, then this element corresponds to a synchronization ratio. In this case the intercept time is infinite and there is no further step in the algorithm.
4. Calculate x_1 as follows:

$$x_1 = \begin{cases} \dfrac{h+\varepsilon}{k}, & \text{if } k < k' \\ \dfrac{h'-\varepsilon}{k'}, & \text{otherwise} \end{cases} \tag{3-10}$$

5. Calculate the values p, q, P, Q and κ as follows:

$$p, q, P, Q = \begin{cases} h, k, h', k' & if\ \alpha < x_1 \quad or \quad (k < k'\ and\ \alpha = x_1) \\ h', k', h, k, & otherwise \end{cases} \tag{3-11}$$

$$\kappa = ceil\left(\frac{\varepsilon - |Q\alpha - P|}{|q\alpha - p|}\right) \tag{3-12}$$

6. The intercept time is

$$T_{receiver} \times (Q + q - \kappa q) \tag{3-13}$$

Note that in common search strategies a frequency band is visited only one time in a sweep period of the receiver. So, intercept time is calculated as integer multiple of $T_{receiver}$. Therefore, the intercept time can be defined as the number of consecutive "looks" required by the receiver in the radar's frequency (operating) band in order to be certain of intercepting it.

3.1.2 Geometric Construction of Intercept Time

Calculation of intercept time from Farey series is given the previous section. Farey series are calculated by using the ratio of the periods of the pulse train, α as in (3-5), and the tolerance, ε as in (3-7).

In fact, α-ε plane can be divided into these regions which show us at once where the intercept time becomes infinite, where there are jumps in intercept time as the number of looks, and where it remains constant. To get subdivided α-ε plane as in [16, 18], we use the following theorems.

THEOREM 1: *Let period ratio α and tolerance ε be as defined before. Consider two fractions h/k < h'/ k' such that h/k ≤ α ≤h'/ k'. If*

$$h - k\alpha \leq \varepsilon \ \ and \ \ k'\alpha - h' \leq \varepsilon$$

then the intercept time is not greater than k+k'

THEOREM 2: *Again* c*onsider a pair of pulse trains defined by period ratio* α *and tolerance* ε. *Suppose that h/k < h'/ k' are adjacent fractions in a Farey series such that h/k ≤ α ≤h'/ k'. If*

$$(k-k')\,\alpha - (h-h') > \varepsilon$$

then the intercept time is not less than k+k'

The proofs of THEOREM 1 and THEOREM 2 can be found in [16,18]

Consider any two adjacent Farey series elements such that $h/k \leq \alpha < h'/k'$. From THEOREM 1 we know that the intercept time is not greater than $k + k'$, when $h - k\alpha \leq \varepsilon$ and $k'\alpha - h' \leq \varepsilon$. The intersection of these regions can be found by writing the equality $h-k\alpha = k'\alpha - h'$ and using unimodularity property of Farey Series. Then, α is found to be $(h + h')/(k + k')$, and $\varepsilon = 1/(k + k')$.

Also note that at $\alpha = h/k$, $k'\alpha - h' \leq \varepsilon$ reduces to $\varepsilon \geq 1/k$, and at $\alpha = h'/k'$, $h-k\alpha \leq \varepsilon$ reduces to $\varepsilon \geq 1/k'$. Furthermore, from THEOREM 2 we know that the intercept time is not less than $k + k'$, when $(k-k')\,\alpha - (h-h') > \varepsilon$ *which* reduces to $\varepsilon < 1/k$ at $\alpha = h/k$, and $\varepsilon < 1/k'$. As a result, these boundaries form a triangle in the α-ε plane, inside which the intercept time is constant, $k + k'$ [16,18]. The vertices of the triangle are as follows:

$$\left\{ \left(\frac{h}{k},\ \frac{1}{k} \right)\ \left(\frac{h'}{k'},\ \frac{1}{k'} \right)\ \left(\frac{h+h'}{k+k'},\ \frac{1}{k+k'} \right) \right\} \tag{3-14}$$

Such a triangle is drawn in Figure 3-1 as an example. A dotted line is drawn across the top of the triangle in order to indicate that this boundary is excluded whereas the other two are included, and drawn in solid lines. *k+k'* is written in the center of the triangle, which is the intercept time everywhere within the triangle.

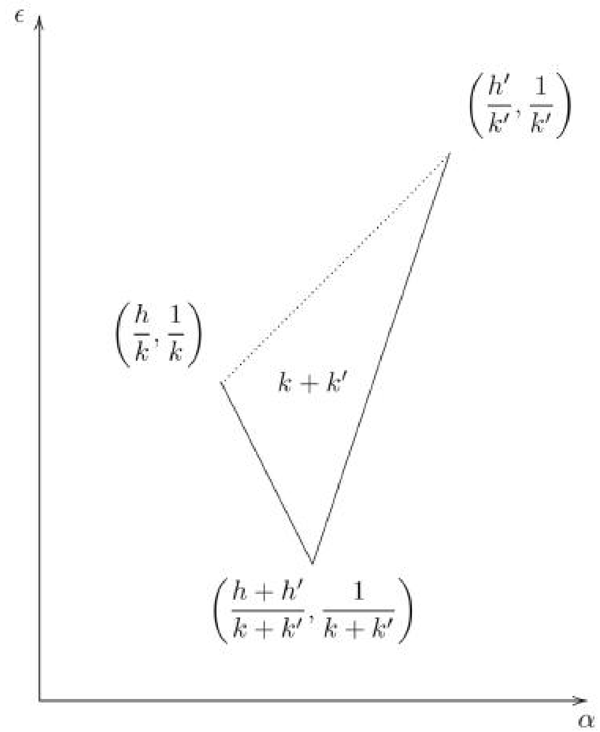

Figure 3-1: Illustration of a Triangle in the α-ε plane in which Intercept Time is Constant. [16, 18]

Note that the whole α-ε plane can be partitioned by these triangles. By this way we can see the value of intercept times depending on α and ε values. The region for which the intercept time is infinite, i.e. synchronization occurs, remains non-partitioned. These triangles are not used directly but instead just the idea is used in intercept calculations, so the details of the procedure for this partition will not be given here but can be found in [16, 18].

3.2 SIMPLE SEARCH STRATEGY

First search strategy to be examined is Simple Search Strategy. In Simple Search Strategy, the search bandwidth in which the emitters operate is divided equally by the bandwidth of the receiver. The search strategy is obtained tuning the receiver into each of the smaller frequency bands in some sequence. The receiver dwells for a certain time in each frequency band, which is called dwell time.

Usually, the receiver steps through each of the bands sequentially with (typically) equal dwell times. Once the receiver steps to the last frequency band in the sequence, it begins again from the first. The time to complete a sequence is called the sweep period. There is a relationship between individual dwell times and sweep period.

$$Dwell\ Time = \frac{Sweep\ Period}{Number\ of\ Frequency\ Bands} \tag{3-15}$$

Simple Search Strategy for five frequency bands is visualized in Figure 3-2.

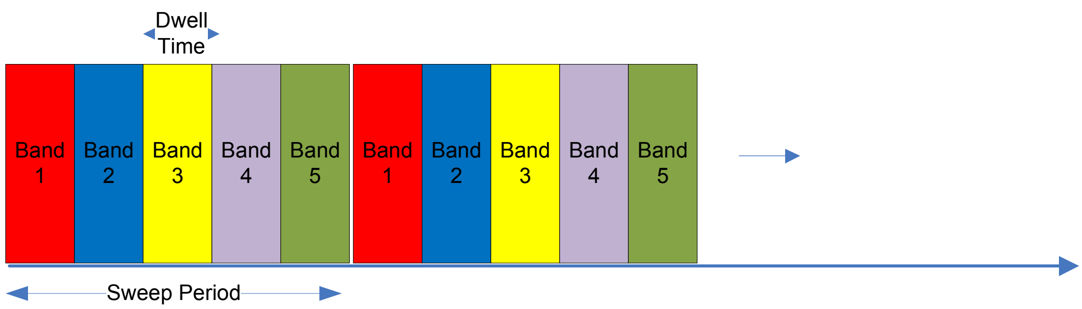

Figure 3-2: Simple Search Strategy Example

After generating search strategy, interception time for the corresponding search strategy can easily be calculated as explained in section 3.1, either using Farey Series, or Geometric Construction. Note that the order in which the frequency bands are searched is not important, as long as the order remains same on each period.

3.3 CLARKSON'S ALGORITHM

The second search strategy to be examined is Clarkson's algorithm. Simple Search Strategy is explained in the previous section. Note that the intercept time of a radar, for Simple Search Strategy can be infinite. This happens if the periods are commensurate and the sum of pulsewidths is small. These conditions are given in ALGORITHM 2 step 3. In such a case, the two pulse trains, namely receiver pulse train and emitter pulse train, are said to be synchronized. As a result the pulse trains are always out of step with each other, for certain relative phases. In an ESM system, this is a highly undesirable situation, since it means that the radar of interest may never be intercepted. On the other hand Clarkson's algorithm [16, 18] investigates synchronization problem. In this section Clarkson's algorithm is going to be given in detail. We make some improvements in Clarkson's algorithm.

In Clarkson's algorithm receiver tunes frequency bands in a sequence during sweep period as in Simple Search Strategy. However, in contrast to Simple Search Strategy, dwell times of different frequency bands do not have to be equal in Clarkson's algorithm. By adjusting dwell times of frequency bands individually synchronization problem is solved. Clarkson's algorithm is also known as min-max intercept time optimization, since the algorithm aims to minimize the maximum of intercept time at each iteration. Dwell time, $\tau_{receiver}$, for each frequency band is found for a fixed sweep period. Of course here the constraint is that sum of individual dwell times cannot exceed the sweep period. We can express this constraint as follows:

$$\sum_{i=1}^{n} \tau_{receiver}(i) \leq T_{receiver} \tag{3-16}$$

where n is the number of frequency bands, and i is the index of the frequency band. If the sum of dwell times is less than the sweep period, since there is no benefit when the receiver is idle, we allocate all the available time in "sweep period". That means increasing the dwell time of some frequency bands. As a result intercept time of the corresponding bands are expected to decrease.

The principle underlying Clarkson's algorithm is simple. If we allocate more dwell time to a particular frequency band then the intercept times with the emitters in that frequency band cannot increase. For instance if we allocate all sweep period to only one frequency band then we can intercept any emitter in that band at its first illumination. On the contrary, if we reduce dwell time then the intercept times cannot decrease. To sum up, as a function of dwell time, intercept times with emitters in that frequency band are monotonically non-increasing.

Dwell times of each frequency band cannot be less than some minimum acceptable values. The minimum dwell time value is set to be some multiple of the PRI of an emitter in that frequency band, since at least some number of pulses of the emitter should be received for interception to be possible. Note that if there is more than one emitter in the frequency band, then here PRI actually refers the maximum of all PRIs of the emitters, in order to get a minimum of acceptable dwell time for all emitters in the band. Clarkson's algorithm begins by setting the dwell times on each frequency band to their minimum allowable values.

After allocating the minimum allowable dwell times for each of the frequency bands, we compute the relative pulse train parameters, namely α and ε, as given in (3-5) and (3-7) respectively. The calculations are done for each emitter with respect to the sweep period, $T_{receiver}$, of the receiver. We can easily compute the intercept time for each emitter. For example, by using the geometric partitioning of the α-ε plane as discussed in section 3.1.2 and illustrated in Figure 3-1 or from Farey series as discussed in section 3.1.1. Just at this starting point of the algorithm, even with a minimum amount of dwell time allocated to each band, it is possible that the sum of dwell times is greater than the sweep period, $T_{receiver}$. In this case, it is not possible to find a feasible search strategy.

After calculating intercept times, it is possible that the intercept time with any one of the emitters be infinite. This occurs if, for any emitter, the ratio of periods α is rational, i.e., if $\alpha = h/k$, and the tolerance ε is less than $1/k$. This case also means synchronization. In each such case, in order to prevent synchronization extra dwell time must be allocated to the corresponding band, up to satisfy $\varepsilon = 1/k$. By this way synchronization can be avoided. Again, as a result of this allocation, the sum of the dwell times may be greater than the sweep period. In that case it is not possible to have a search strategy that satisfies finite intercept times with all emitters in the threat-emitter list.

Assuming that a feasible search strategy can be found with finite intercept times, the process continues by allocating the remaining available time in sweep period at each iteration of the algorithm. Optimization progresses iteratively by this way. In each band, we calculate the maximum intercept time over all emitters (potentially) operating in that frequency band. Since the aim is to minimize the maximum of all intercept times, optimization focuses on the frequency band which has the maximum intercept time, i.e. the one that includes the emitter with maximum of all intercept times. In the band which has the maximum of the maximums (or in any one of them if there is a tie), determine the emitter which incurs the maximum intercept time (or again choose any one if there is a tie). After that calculate how much additional dwell time must be allocated to this band in order to raise the value of ε so that the intercept time reduces. The amount of the dwell time to be added can be found by using the α-ε plane in Figure 3-1. For the emitter with maximum intercept time, α and ε are calculated by using (3-5) and (3-7) respectively. This (α, ε) point will belong to a triangle in α-ε plane, or it will be in a non-partitioned region, if the intercept time for this emitter is infinite. Then, while α is kept constant, since $T_{receiver}$ is constant, ε is increased until when (α,ε) point reaches to the upper triangle which represents a lower intercept time. This point can be found by the intersection of two lines, i.e. $\alpha = T_{receiver}$ line and the edge of upper triangle. If there is enough time left in sweep period, allocate that time to the corresponding band and repeat this procedure. If not, it is checked whether we have still significant available time in sweep period, if we have in this case we try for the emitter with next maximum intercept time. This part of Clarkson's algorithm has been changed in order to allocate all available time in sweep period. If we try for the last emitter, then the min-max intercept time has been found and a search strategy has been computed. When some additional time is allocated to a frequency band, its dwell time and as a result intercept times of the emitters of that band may be changed, so they have to be calculated again. This algorithm is given below step by step.

ALGORITHM 3:

1. Calculate the minimum dwell time for each emitter,

$$d = N \times PRI \tag{3-17}$$

   where N is the number of minimum pulses to be received.

2. Calculate the dwell time for each emitter, $\tau_{receiver}$, which equals to the minimum dwell time for this step.
3. Calculate the pulse train parameters α and ε for each emitter as given in (3-5) and (3-7) respectively.
   where $T_{receiver}$ is the sweep period of the receiver.
4. Calculate the Farey series for each emitter by using ALGORITHM 1, the order of Farey series can be calculated as given in (3-9)
5. If α equals to an element of this Farey series i.e α = *h/k* and ε is less than *1/k* then the intercept time for this emitter is infinite.
   a. So calculate how much extra dwell time must be allocated to that band in order to make ε =*1/k*.
      Let us use ε'= *1/k*.

$$dwelltime_{new} = \varepsilon' \times T_{emitter} - \tau_{emitter} + 2d \tag{3-18}$$

   b. If there is enough time left in sweep period, allocate that time to the band. Set the following parameters of the corresponding emitter

      $\varepsilon = 1/k$ and $dwelltime = dwelltime_{new}$

   c. Return to the step 4 in order to calculate the intercept time according to new pulse train parameters, according to THEOREM 1 and THEOREM 2.
6. If α does not equal to an element of this Farey series, calculate the intercept time according to THEOREM 1 and THEOREM 2 or ALGORITHM 2.

7. After calculating the intercept time for all emitters, and for all frequency bands find the emitter with maximum intercept time.
8. Calculate how much extra dwell time must be allocated to that band in order to decrease the intercept time.
    a. This calculation is done by using α-ε plane as discussed in Section 3.1.2 and illustrated in Figure 3-1. Since sweep period, $T_{receiver}$, is constant, also $T_{emitter}$, α is constant for this emitter. ε is increased up to reach dotted line in Figure 3-1. This corresponding to reaching the upper triangle. These simply calculations are done by using high school geometry. In short, calculate the formula of the dotted line by using its two points. Then find the intersection point of this line with the line $\alpha = T_{receiver}/T_{emitter}$, the value of ε at this point is, let call this as ε''can be expressed as:

$$\varepsilon'' = \left(\frac{\left\{ \left[\left(\frac{1}{k'} \right) - \left(\frac{1}{k} \right) \right] \times \left[\alpha - \left(\frac{h}{k} \right) \right] \right\}}{\left\{ \left[\left(\frac{h'}{k'} \right) - \left(\frac{h}{k} \right) \right] \right\}} \right) + \left(\frac{1}{k} \right) \qquad (3\text{-}19)$$

    b. Calculate the new dwell time by using (3-18)
    c. If there is enough time left in sweep period, allocate that time to the band.
        i. Set the following parameters of the corresponding emitter

$$\varepsilon = \varepsilon' \text{ and } \quad dwelltime = dwelltime_{new}$$

        ii. Calculate the intercept time according to new parameters, according to THEOREM 1 and THEOREM 2 or ALGORITHM 2.
    d. Otherwise return to step 8 and try other emitters with next maximum intercept time.
    e. If all emitters are tried, then the process is completed.

The dwell time of each band in the sweep period is calculated as explained above. In other words the search strategy is generated. An example search strategy for five frequency bands is visualized in Figure 3-3.

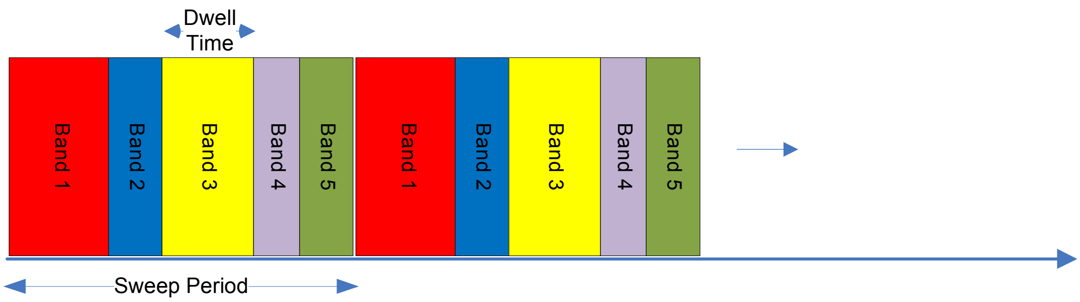

Figure 3-3: An Example Search Strategy for Clarkson's Algorithm

In contrast to Simple Search Strategy dwell times of frequency bands are not equal in this search strategy. Note that the order in which the bands are searched is not important, so long as the order remains the same on each sweep.

CHAPTER 4

A PROPOSED ALGORITHM BASED ON PROBABILITY OF INTERCEPT

Simple Search Strategy and Clarkson's algorithm are given in Chapter 3. Note that in both cases revisit period of all frequency bands are same since these algorithms aims to allocate all available time in sweep period. It is equal to sweep period of the receiver. That means receiver visits the frequency bands with a period which is equal to sweep period of the receiver. Radars also employ a periodic search. Therefore it is clear that if receiver visits a frequency band more frequently then probability of intercept increases. In other words intercept time decreases. Also we want to visit some frequency bands more frequently than others, because of the threats in that band. Moreover for these two search strategies dwell times can be longer, therefore transferring such large data from receiver to processor and processing it becomes very costly.

In this chapter an algorithm in which receiver visits frequency bands more frequently and at each visit dwells shorter, but sufficient, is proposed. Proposed algorithm is based on probability of intercept. For a radar a probability of intercept of 50% up to the time instant *t,* means that the receiver may intercept the radar with the probability of 50% at *t.*

In proposed algorithm each frequency band has its own revisit period which is calculated according to the probability of intercept (POI) of the emitters in that frequency band. Here POI refers to probability of intercepting the emitter in the first illumination of the receiver. The frequency bands are placed in "sweep period" according to their own revisit periods. We also allow for some dead time between each dwell of the receiver. Dead time is a period of time during which the receiver is re-tuning between different center frequencies. As a result, during dead times the receiver is not receiving or processing intercepts. By this way a search strategy is generated. The search strategy is repeated periodically as in previous ones. Details of the proposed algorithm are going to be given in the following sections.

4.1 PROBABILITY OF INTERCEPT

We have expressed the behaviors of emitter and receiver as pulse trains in Chapter 2. In probability of intercept problem, it is assumed that one or both of the phases of these pulse trains are uniform random variables with ranges equal to their periods. In this thesis only the phase of the emitter pulse train, φ, is taken as a random variable. We want to know the probability of intercept after N pulses (visits) from the receiver pulse train. In other words, we want to know the probability of at least one coincidence occurring with one of the first N pulses (looks) from second pulse train. Also, we assume that we know the phase of receiver pulse train. Without loss of generality, this is done by setting the time origin so that pulses from receiver pulse train occur at the times $jT_{receiver}$, where j is a non-negative integer. The relative phase, φ, is unknown and is assumed to be a random variable, which is uniformly distributed over the interval $[0, T_{emitter})$. At this point, this assumption should be justified. What we want to calculate is some measure of confidence of intercepting a pulse train within a certain number of "pulses" or "looks" from our receiver, in an ESM scenario involving a simple emitter and receiver. Note that these looks constitute the second pulse train, that is the receiver pulse train in our interception model. The time at which our receiving equipment is turned on (the first look; pulse index $j = 0$) is known to us therefore it is not a random variable. For this simple ESM scenario we can define the point $t = 0$ to be at the center of this first look. Another assumption is that the pulse train from the emitter which we wish to intercept, emitter pulse train, is present at this time. In other words the emitter is in range and at least one pulse from the emitter pulse train occurred at some time $t \leq 0$. In fact, we have no control over when the emitter begins operating. If the distribution of the illuminating time for the emitter relative to that of the receiver exists and is broad and sufficiently smooth then the distribution of the time-of-arrival of the pulse from the emitter immediately preceding the first from the receiver will be approximately uniform. In this way, the pulse index $i = 0$ is assigned to this pulse. In order to arrive at an indicative probability of intercept, we assume that the relative phase is uniform. Else, we can view the results we will describe not as a probability in the strict sense but simply as a proportion of relative phases in $[0, T_{emitter})$ that would have led to an intercept after the prescribed number of pulses from the second pulse train [18].

Let us now consider how to calculate the probability of intercept. Remember the intercept equation for two pulse trains as given in (3-8). Note that β given in (3-6) is an instance of normalized phase difference, which is a random variable. Therefore, we can say β ~ U(0, 1). If there exists some $0 \leq q < N$ such that equation (3-8) is satisfied for some integer p then an approximate coincidence with tolerance ε occurs with one of the first N pulses from receiver pulse train. In other words, for any $p,\ q \in Z$, there exists a range of normalized phase differences β for which a coincidence will occur between these two pulse trains.

Let $I_{p,q}$ be the interval on R, which we can define formally as

$$\begin{aligned} I_{p,q} &= \left\{ x \in \Re \,\middle|\, |q\alpha - p + x| \leq \frac{1}{2}\varepsilon \right\} \\ &= \left[p - q\alpha - \frac{1}{2}\varepsilon,\ \ p - q\alpha + \frac{1}{2}\varepsilon \right] \end{aligned} \tag{4-1}$$

Thus, a coincidence or intercept, with a pulse from emitter pulse train occurs with the 0^{th}, 1^{st}. . . or $(n-1)^{th}$ pulse from receiver pulse train if

$$\beta \in \bigcup_{\substack{p,q \in Z \\ 0 \leq q < n}} I_{p,q} \tag{4-2}$$

Let we define $C_N\ (\beta)$ as the characteristic function of this union. That is, $C_N\ (\beta) = 1$ if there exists some $p,\ q\ \epsilon\ Z,\ \ 0 \leq q < N$ such that $\beta\ \epsilon\ I_{p,q}$ and $C_N\ (\beta) = 0$ otherwise. Note that by this definition $C_N\ (\beta)$ is periodic with period 1.

$$C_N(\beta) = \begin{cases} C_N(\beta) = 1 & if\ \ \beta \in I_{p,q} \\ C_N(\beta) = 0 & otherwise \end{cases} \tag{4-3}$$

Finally we can define the probability of intercept after N pulses from the receiver pulse train as P_N. Then we can express P_N as

$$P_N = \int_0^1 C_N(\beta)\, d\beta \tag{4-4}$$

Equation (4-4) shows that the probability of intercept is that proportion of the range of possible relative phases (from 0 to 1) which is covered by the intervals $I_{p,q}$ with $0 \leq q < N$. Now, we could replace the interval of integration in (4-4) with any interval of length 1, since $C_N\ (\beta)$ is periodic with period 1, [18].

If $C_N\ (\beta) = 1$ over any interval of length 1, then a coincidence must have occurred with one of these N consecutive pulses, regardless of the phases of the two pulse trains. In that case, intercept time can be defined as $N{*}T_{receiver}$, where N is the least value of N such that this condition is true.

In Figure 4-1 [18] the value of the characteristic function $C_N\ (\beta)$ over the unit interval [0 1] for $N = 5$, $N = 9$ and $N = 14$ where $\alpha = 0{,}217$ and $\varepsilon = 0.1$ are shown. Note that it is not until the union with $I_{3,13}$ in $C_{14}\ (\beta)$ that this function becomes uniformly equal to 1 across the entire unit interval. Hence, in this example intercept time is $14{*}T_{receiver}$. In order words when the characteristic function becomes entirely 1 between $\beta = 0$ and $\beta = 1$, for some value of α and ε, the interception between two pulse trains becomes independent from relative phases, i.e. the interception has the probability 100%.

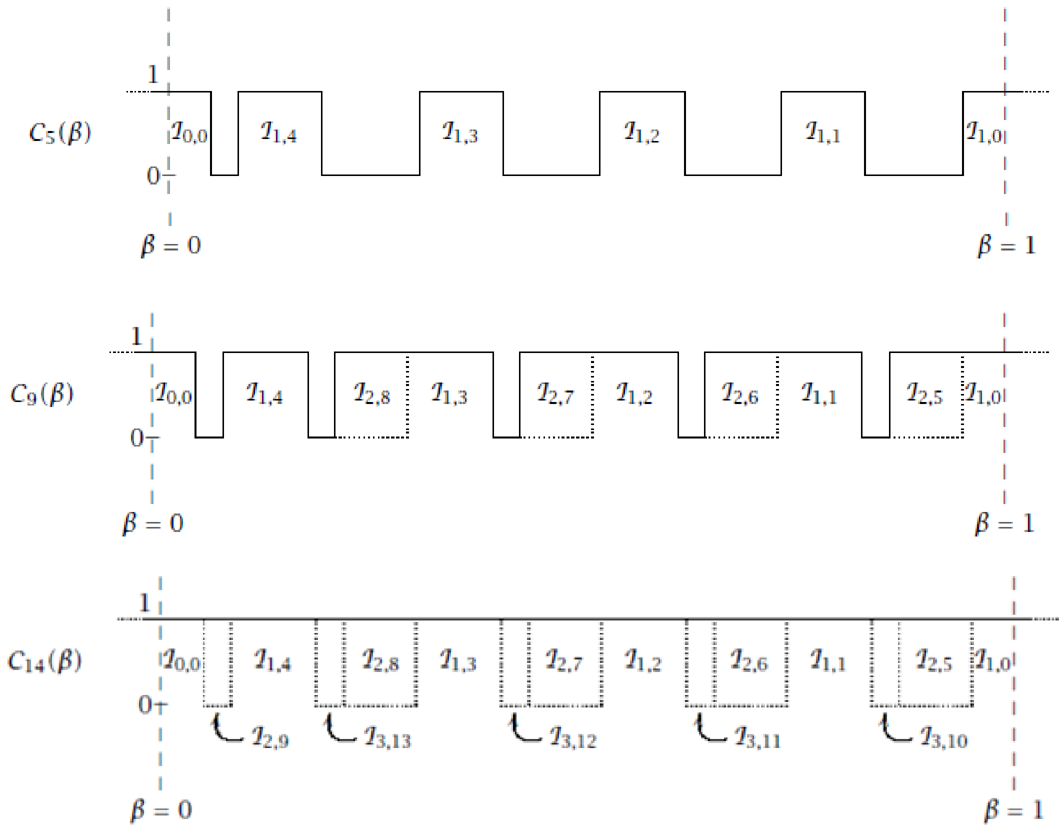

Figure 4-1: The Value of the Characteristic Function $C_N(\beta)$ for N = 5, N = 9 and N = 14

Note that, for $N \leq 5$, the intervals $I_{p,N}$ are separate and do not overlap. We can see this from the topmost illustration. As a result, the rate of growth of the probability of intercept is at its greatest. For $5 < N \leq 9$, the intervals $I_{p,N}$ overlap on one side only. Also it is illustrated in the middle illustration. Finally, for $9 < N \leq 14$, the intervals $I_{p,N}$ overlap previous intervals on both sides, filling in the last of the "gaps" in the integration interval.This is illustrated in the illustration at bottom. The value of $C_N(\beta) = 1$ everywhere and all new intervals $I_{p,N}$ are completely overlapped by previous intervals, for N > 14.

In Figure 4-2, the probability of intercept, P_N, as a function of N using the same parameters that were used in Figure 4-1, i.e., $\alpha = 0{,}217$ and $\varepsilon = 0.1$ is presented. There are four linear segments, which are clearly visible in the graph. They corresponds tostages that are explained above.

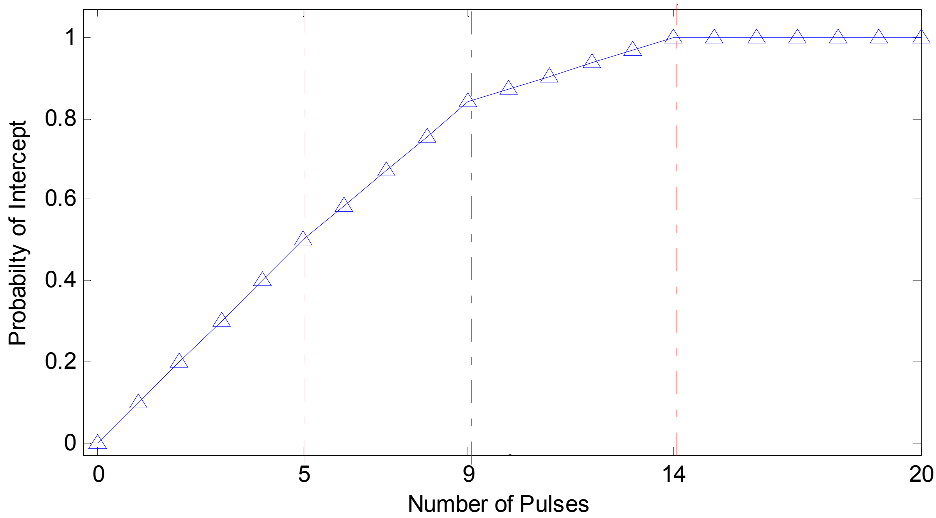

Figure 4-2: Probability of Intercept as a Function of Number of Pulses From Receiver Pulse Train

4.2 CALCULATION OF PROBABILITY OF INTERCEPT FOR AN EMITTER

Probability of intercept concept has been introduced in previous section for two pulse trains. In this section we are going to apply these derivations to our interception problem. Two kind of calculations can be done: The first one is calculating POI according to given dwell time and revisit period. On the other hand the second is calculating dwell time and revisit period for a desired POI.

In the proposed algorithm, the main aim is to find the revisit period and the dwell time to intercept radar with any desired probability or vice versa. Notice that, since the phases are assumed to be uniformly distributed, POI for a given revisit period is equal to find the instant where the characteristic function is 1.For example if the sum of intervals in $C_N\ (\beta)$ is 0.5 for a given revisit period then probability of intercept is 50% for the given revisit period. Here POI refers probability of intercepting the emitter in the first illumination of the receiver. Also the intercept time can be calculated. Here intercept time means when 100% POI is provided, in other words $C_N\ (\beta) = 1$ for all β, where $0 \leq \beta \leq 1$.

The calculation of POI for a given revisit period is given in the following algorithm.

ALGORITHM 4:

1. The calculations are done for the following receiver and emitter parameters

   $T_{emitter}$ = Scan period of the emitter

   $\tau_{emitter}$ = Illumination time of the emitter

   $T_{receiver}$ = Revisit period of the receiver for the corresponding frequency band

   $\tau_{emitter}$ = Dwell time of the receiver for the corresponding frequency band
2. Calculate α and ε as given in (3-5) and (3-7).

   where d in (3-7) is the minimum dwell time of the receiver for the corresponding frequency band calculated as given in (3-17).
3. Before calculations make synchronization control for this revisit period by using the Farey Series. Calculate the Farey series for each emitter as given ALGORITHM 1, the order of Farey series can be calculated as given in (3-9).
4. If α equals to an element of this Farey series i.e $\alpha = h/k$ and ε is less than $1/k$ then there is a probability of synchronization for this revisit period. In other words intercept time for this emitter is infinite.
   a. So in order to prevent synchronization the revisit period of the corresponding band is increased by the some amount, for example "time resolution" of the receiver.
   b. Return to the step 2 in order to calculate new α and ε parameters.
   c. Check whether or not the intercept time is infinite for new parameters.

d. This process continues until synchronization is prevented for a revisit period.

5. Calculate the number of visit (look) of the receiver to the corresponding band in one period of the emitter.

$$n = floor\,(p \times \frac{Temitter}{Treceiver}) \qquad (4\text{-}4)$$

For now calculations are done only for the first illumination of the emitter at first, so p =1.

6. Calculate $I_{p,q}$ given in (4-1) for all p,q pairs and determine covered regions of $C_N\,(\beta)$ given in (4-3) including overlaps as shown in Figure 4-3 where

   i. $0 \leq p \leq 1$

   ii. $0 \leq q \leq n-1$

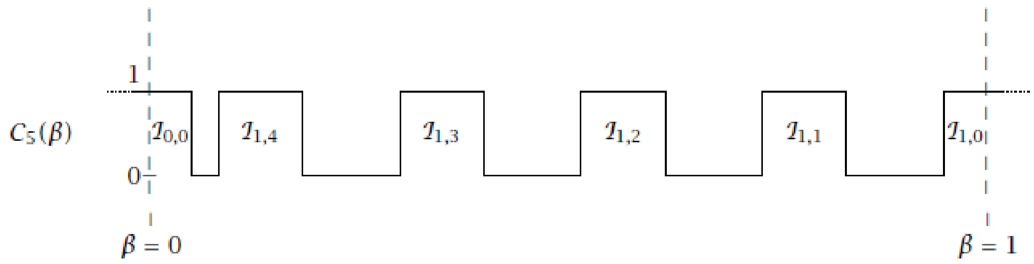

Figure 4-3: Covered Regions of $C_N\,(\beta)$

7. POI for the given revisit period is the ratio of the summation of these covered regions of $C_N\,(\beta)$ to all regions in $C_N\,(\beta)$, which is 1,

8. After calculating POI for the first illumination of the emitter, intercept time can be calculated. To do this, calculation of $I_{p,q}$ is continued from step 3, for p = 2 and so on until all the regions in regions $C_N\,(\beta)$ is covered. Then intercept time is

$$Intercept\ Time = p \times T_{emitter} \qquad (4\text{-}5)$$

In Simple Search and Clarkson's algorithm the intercept time is calculated in terms of sweep period, since the sweep period is same for all emitters. However, here, in (4-5), we have calculated intercept time in terms of emitter illumination number.
By using ALGORITHM 4 the POI corresponding to the revisit period of receiver can be calculated. Moreover the inverse, namely calculation of revisit period for a desired POI, is also possible. It is given in ALGORITHM 5

ALGORITHM 5

1. Calculate the minimum dwell time, d, as given in (3-17)
2. Calculate the dwell time of the receiver, which equals to the minimum dwell time for now.
3. Start with an initial revisit period. Let use illumination time ($\tau_{emitter}$) as the revisit period ($T_{receiver}$) of the receiver for this frequency band.
4. Calculate α and ε as given in (3-5) and (3-7).
   where d in (3-7) is the minimum dwell time of the receiver, which is calculated in step 1.
5. Make synchronization control for this revisit period by using the Farey Series. Calculate the Farey series for each emitter as given ALGORITHM 1, the order of Farey series can be calculated as given in (3-9).
6. If α equals to an element of this Farey series i.e $\alpha = h/k$ and ε is less than $1/k$ then there is a probability of synchronization for this revisit period. In other words intercept time for this emitter is infinite.
   e. So in order to prevent synchronization the revisit period of the corresponding band is increased by the some amount, for example "time resolution" of the receiver.
   f. Return to the step 4 in order to calculate new α and ε parameters.
   g. Check whether or not the intercept time is infinite for new parameters.
   h. This process continues until synchronization is prevented for a revisit period.

7. After checking the synchronization, calculate the probability of intercept as given in ALGORITM 4 in steps 5 to 7.
8. Compare the calculated POI with the desired POI
   a. If the calculated POI is equal to the desired POI, the process ends.
   b. If the calculated POI is less than the desired POI the revisit period of the frequency band is decreased by some amount, for example "time resolution" of the receiver. Go back to step 4.
   c. If the calculated POI is greater than the desired POI the revisit period of the frequency band is increased by some amount, for example "time resolution" of the receiver. Go back to step 4.
9. Finally intercept time is calculated as given in ALGORITM 4 in step 8.

4.3 CALCULATION OF RECEIVER PARAMETERS

In previous section we have seen that by using the receiver and emitter parameters, probability of intercept can be calculated for a given revisit period, or vice versa. The parameters of the emitter are given in the threat list. However the receiver parameters, namely the dwell time and revisit period, must be determined by the algorithm. For this aim the effect of dwell time and revisit period to the POI is analysed in this section.

Note that in calculation of probability of intercept in ALGORITHM 4 for each p,q pair there exists a region $I_{p,q}$ as given in (4-1) for which coincidence will occur. The region has a length of ε, where ε is given by (3-7). POI, summation of covered regions, is increased by ε for each pair when there is not any overlap in covered regions. So the dwell time of the receiver, $\tau_{receiver}$, effects POI. In order to analyse the effect of $\tau_{receiver}$ on POI, the threat list given in Table 4-1 is used. There are two emitters in the threat list all parameters, except PRI, are same for these two emitters. By this way we can also see the effect of dwell time for long and short dwell times. Minimum dwell times are calculated for N=5 consecutive RF pulses. Illumination times are calculated from beamwidth by using (2-1)

Table 4-1: Threat Emitter List

Emitter Number	Band	Scan Period (us)	PRI (us)	Beamwidth (deg)
1	A	9.9×10^6	1×10^3	2.3
2	B	9.9×10^6	0.1×10^3	2.3

For the two emitters in the threat list the dwell time is increased for a constant revisit period. POI is calculated by using ALGORITHM 4 for each case. Remember that we can not talk about an interception if the dwell time is less than minimum dwell time. So the analysis is started from minimum dwell time for each emitter. The dwell time is increased by an amount of 10% of the minimum dwell time at each iteration. POI is calculated for the corresponding receiver parameters. The results are given below in the Figure 4-4.

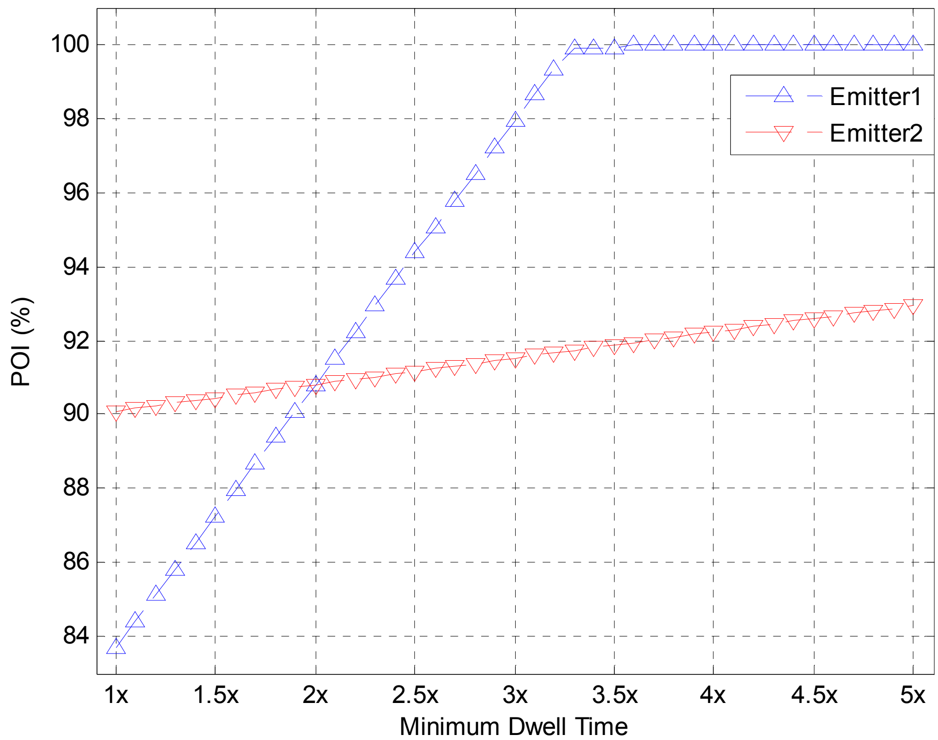

Figure 4-4: The Effect of Dwell Time on POI

As shown in Figure 4-4, for both emitters POI increases when dwell time increases, as expected. When POI reaches its maximum value (100%) increasing dwell time does not increase POI anymore, because all phase region is covered in that case. We can see this for Emitter 1. Also note that for Emitter 2, increment in POI versus dwell time is not significant. POI increases from 90.09% to 92.94%, approximalety an increment of 3%, while dwell time increases upto five times of its minimum value.

As explained above dwell time is related to ε and ε is related to POI. The formula of ε is given in (3-7). Both dwell time and its minimum value affects ε. Since minimum dwell time is constant, the effect of dwell time in ε is more for large dwell times, for Emitter 1. At the beginning ε is small for Emitter 1 so its POI is less than Emitter 2. But increasing dwell time affects ε for Emitter 1 more than Emitter 2. As a result its POI exceeds Emitter 1.

To sum up increasing dwell time increases POI but the increment is not significant for some cases. Since its sufficient to dwell as long as minimum dwell time for intercepting the emitter, the dwell time is chosen as its minimum value for proposed algorithm. However it can be increased if desired.

Note that there will be more than one emitter in a frequency band. In this case dwell time of the receiver is chosen from the minimum dwell time of the emitters in that frequency band. Maximum of the minimum dwell time is chosen, so that receiver dwells will be sufficient to intercept any of the emitters in that frequency band.

After determining the value of dwell time, we can examine the effect of revisit period. Note that in ALGORITHM 4 revisit period affects the number of looks to the frequency band of the emitter, during one scan period of the emitter. In other words, it determines how frequenty the band is visited by the receiver. When revisit period increases, the number of looks decreases so POI decreases. On the other hand when revisit period decreases POI increases as a result.

In order to analyse the effect of revisit period, $T_{receiver}$, on POI the threat list given in Table 4-1 is used again. For these two emitters in the threat list the revisit period is increased for a constant dwell time. POI is calculated by using ALGORITHM 4 for each case. In practice, if revisit period is less than illumination time then we do not miss any illumination between two concecutive visits (looks). So the analysis is started from half of the illumination time of the emitter, which is calculated by (2-1). Revisit period is increased by an amount of 10% of the illumination time at each iteration. The results are given below in the Figure 4-5.

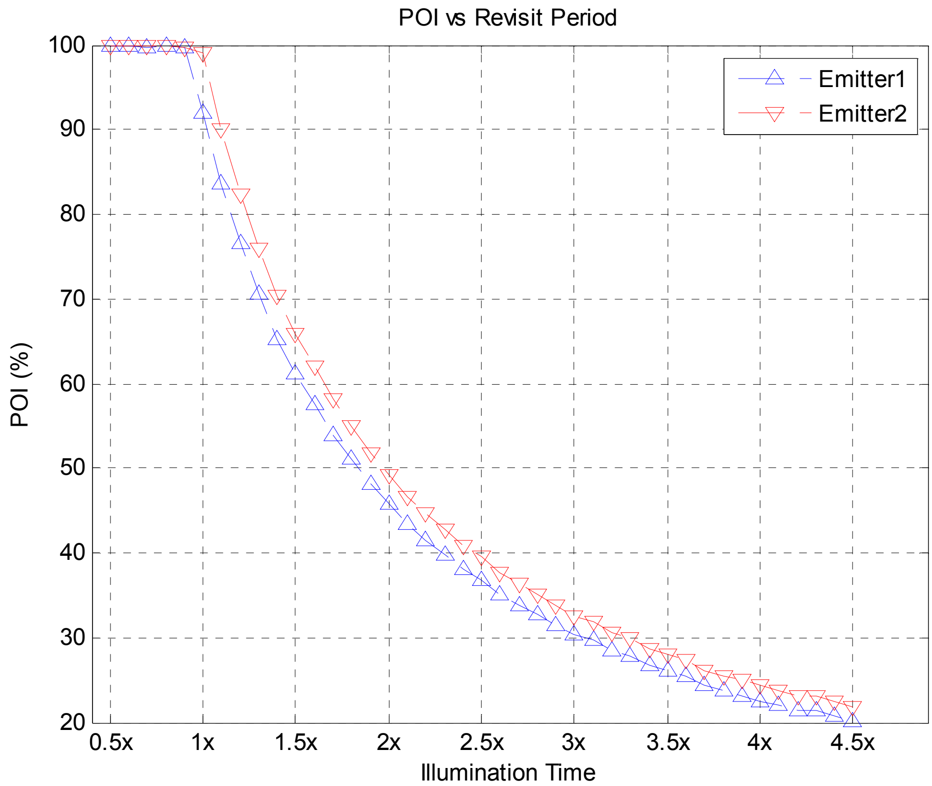

Figure 4-5: The Effect of Revisit Period on POI

As shown in Figure 4-5, for both emitters POI decreases when revisit period increases, as expected. Starting from its maximum value (100%) increasing revisit period decreases POI. Another important result that can be seen in the Figure 4-5 is that; decreasing revisit period does not affect POI after it arrives its maximum value (%100).

If we look into the Figure 4-5, for both emitters 100% POI corresponds to a revisit period of less than illumination time. In order to ensure interception at the first illumination, in other words 100% POI, revisit period should be less than illumination time. On the other hand revisit period of an illumination time usually corresponds to a POI of greater than 90%. As a result, in proposed algorithm we can choose illumination time of the emitter as a revisit period. However revisit periyot can be chosen larger or smaller if desired. Note that determining the revisit period also means determining POI. The algorithms of calculating revisit period from POI and calculating POI for desired POI as given in ALGORITHM 5 and ALGORITHM 4, respectively.

There will be more than one emitter in a frequency band. In this case revisit period of the receiver is chosen from the illumination times of the emitters in that frequency band. Minimum one is selected as revisit period, so that receiver visits frequently enough to intercept any of the emitters in that frequency band. Also note that selecting revisit period smaller, increases POI, as a result decreases intercept time, of the other emitters in that frequency band.

Finally we investigate the effect of revisit period on intercept time. In Figure 4-6 below, we see how the intercept time changes with the revisit period, $T_{receiver}$. $T_{emitter}$ is taken constant as 1, and $T_{receiver}$ is varied between 0.1 and 4, for the pulse trains parameters as in [11]. Dwell time and illumination time is constant during the anaysis, where dwell time is used as its minimum value. Intercept time is calculated in terms of receiver visit (look) number by using ALGORITHM 4.

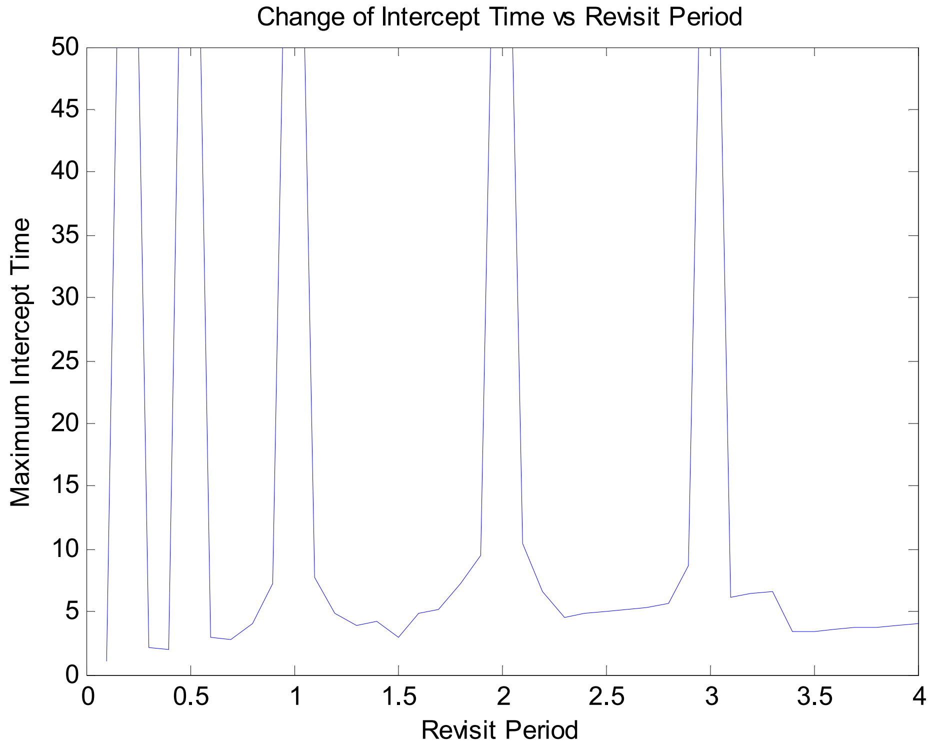

Figure 4-6: Change of Intercept Time with Respect to Revisit Period

As seen from Figure 4-6, intercept time sometimes goes to infinity which corresponds to the synchronization ratios. Also there are local minima in maximum intercept time. This property of intercept time is taken into account in revisit period calculations. The calculations are not done for only one revisit period, but for a region between desired one. The revisit period that satisfies minimum intercept time is chosen as revisit period.

4.4 PROPOSED ALGORITHM BASED ON POI

We have examined the effects of dwell time and revisit period in POI calculations in previous section. We have chosen the values of dwell time and revisit period for our proposed algorithm, as follows:

- Dwell Time = Minimum Dwell Time
- Revisit Period = Illumination Time

Now it is convenient to give all the steps of proposed algorithm. Proposed algorithm is explained in ALGORITHM 6, in detail.

ALGORITHM 6:

1. Calculate the minimum dwell time time, *d*, as given in (3-17), for each emitter.
2. Determine the dwell time for each emitter, which is equal to the minimum dwell time in proposed algorithm.
3. Determine the dwell time of the receiver, $\tau_{receiver}$, for corresponding frequency band. Choose maximum of emitter dwell time in that frequency band.
4. Determine the revisit period ($T_{receiver}$) for each emitter, as follows
    a. If any desired value is given use this desired value.
    b. Calculate by using ALGORITHM 5 if any desired POI given.
    c. Take it as illumination time ($\tau_{emitter}$) of the emitter, if it is not given.
5. Determine the revisit period of the receiver, $T_{receiver,}$ for corresponding frequency band by choosing minimum of emitter revisit period in that frequency band.
6. Find the receiver period that satisfies minimum intercept time in that frequency band. Check for a range of [$T_{receiver}$/2 2*$\mathrm{T}_{receiver}$] as shown in Figure 4-6. If a revisit period cause synchronization with any of emitters in that frequency band, synchronization control is done by using the Farey Series given in ALGORITHM 1. The order of Farey series can be calculated as in (3-9). The intercept time for this revisit can be calculated by using ALGORITHM 2. If there are more than one emitter in that frequency band choose the revisit period by summing the intercept times.
7. For each frequency band, we can calculate duty parameter, what percentage of the receiver time, namely sweep period, is used for that band as follows:

$$Duty\ (i) = 100 \times \left(\frac{Dwell\ Time(i)}{\mathrm{Re}\,visit\ Period(i)} \right) \qquad (4\text{-}6)$$

8. If sum of duties is greater than 100%, then no feasible search strategy is possible for the calculated dwell times and revisit periods.

$$\sum_{i=1}^{n} Duty\ (i) \leq 100 \qquad (4\text{-}7)$$

9. If (4-7) is not satisfied starting from highest duty, duty values are decreased, until (4-7) is satisfied. Since dwell time is set to its minimum value we can decrease duty by increasing the revisit period of the corresponding frequency band. POI and intercept time are recalculated according to step 6 for new parameters. In this case, check for a range of [$T_{receiver}$ $\mathrm{T}_{\mathrm{receiver}}$ *2] as shown in Figure 4-6.
10. After determining all frequency band parameters, for all emitters in each frequency band, calculate POI by using ALGORITHM 4.
11. After that the frequency bands are settled in “sweep period” according to their revisit periods and dwell times. We also allow for some dead time between each dwell of the receiver. Dead time is a period of time during which the receiver is re-tuning between different center frequencies. As a result, during dead times the receiver is not receiving or processing intercepts.
12. The settling starts with the highest priority frequency band. Its visits (looks) are settled according to its parameters, namely revisit period and dwell time. Its first visit is planned at $t=0$. Consecutive looks are settled according to revisit period while $t <$ *Sweep Period*.
13. For the second and next frequency bands pulse train that contains its looks is generated as in the first one. After that we try to settle this generated pulse train into the sweep period, Starting from $t_{start} = 0$ we slide the generated pulse train that is we change the starting point. If a pulse (look) of this generated pulse train overlaps with a pre-settled look in sweep period, then it is cancelled. The number of cancelled pulses is recorded.
14. If all pulses are settled successfully for any starting point, the generated pulse train is settled in sweep period with this starting point. If we cannot find such a starting point while $t_{start} \leq$ *revisit period*, then the one that satisfies minimum

number of called pulse is chosen. Cancelled pulses of the generated pulse train are not settled in the sweep period, as a result.

15. The search strategy is generated by this way. An example of search strategy is shown below in Figure 4-7.

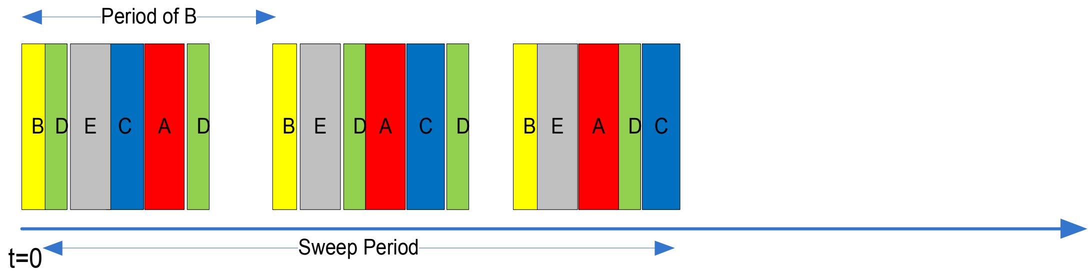

Figure 4-7: An Example Search Strategy Generated by Proposed Algorithm

Note that in proposed algorithm each frequency band has a unique revisit period which is calculated according to the Probability of Intercept (POI) of the emitters in that band. The bands are placed in a “sweep period” according to their own periods. By this way a search strategy is generated. The search strategy is repeated by the receiver periodically.

CHAPTER 5

ANALYSIS AND SIMULATIONS

This chapter includes the implementation of the algorithms, namely Simple Search, Clarkson's and proposed algorithm which are introduced in Chapters 3 and 4. The results are discussed and the performance comparison is also made in this part.

Simulations are carried out for the threat emitter list given in Clarkson's papers [15, 16]. At first, search strategies are generated by using these three algorithms separately for this threat emitter list. Search strategy generation is done in section 5.1. After that, the performance of the search strategies is investigated by means of a Monte Carlo simulation. As expressed before one search strategy is better than another if it satisfies smaller intercept time for an emitter, or for all emitters if there are more than one in threat emitter list. In order to compare the performance of generated search strategies by means of a Monte Carlo simulation, the phase, φ, of the emitter pulse train is randomly changed for these three emitters. The parameters of emitter pulse trains is shown in Figure 5-1.

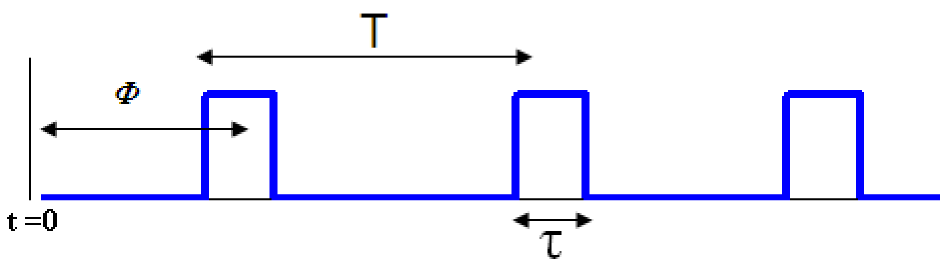

Figure 5-1: Emitter Pulse Train

For each case intercept times of these three emitters is found, for both search strategies. An intercept occurs when the receiver is tuned to the right band, that is the operation band of the emitter, and the emitter is pointing in the right direction, that is the direction of the receiver. Also a minimum dwell time overlap is necessary for an intercept. This is repeated 100 times. The results of search strategies are given and discussed in section 5.2.

5.1 GENERATING SEARCH STRATEGY

The threat emitter list used in simulations is taken from Clarkson's paper [15,16]. It is given in Table 5-1. The threat emitter list consists of three threat emitters, labeled 1-2-3. Emitters are operating in three separate frequency bands, labeled A-B-C. Suppose that the receiver is to sweep through frequency bands periodically with a sweep period of 1 second. Finally it is required to dwell long enough in each frequency band to be able to intercept five consecutive RF pulses, in order to ensure detection.

Table 5-1: Threat Emitter List

Emitter Number	Band	Scan Period (us)	PRI (us)	Beamwidth (deg)
1	A	8.4×10^6	2.38633×10^3	1.3
2	B	2.97×10^6	1.37792×10^3	2.6
3	C	10.5×10^6	9.38	2.1

The pulsewidth of emitter pulse train, that is the illumination time, can be calculated by using (2-1). Time resolution of 1us is used for dwell times and illumination times during the simulations. At first, minimum dwell times are calculated according to (3-17) with N=5. Calculated values are given in Table 5-2.

Table 5-2: Calculated Threat Emitter Parameters

Emitter Number	Band	minimum dwell time (us)	τ emitter (us)
1	A	11932	30333
2	B	6890	21450
3	C	47	61250

5.1.1 Simple Search Strategy

In Simple Search Strategy, the frequency bands are arranged one after another in sweep period. Dwell times of frequency bands are equal. Individual dwell times are calculated by using (3-15). Dwell times according to simple search strategy is given in Table 5-3.

Table 5-3: Dwell Times for Simple Search Strategy

Emitter Number	Band	Dwell Time (us)
1	A	333333
2	B	333333
3	C	333333

Search strategy is visualized in Figure 5-2.

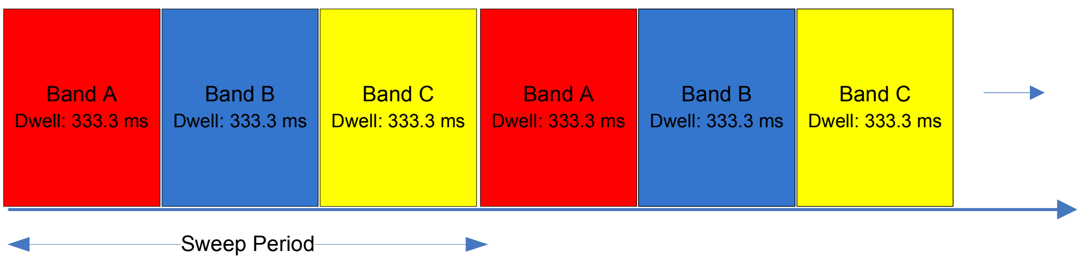

Figure 5-2: Search Strategy for Simple Search for Threat Emitter List

5.1.2 Clarkson's Search Strategy

By using the threat emitter list given in Table 5-1, ALGORITHM 3 has been run in order to generate a search strategy by using Clarkson's algorithm. At first α and ε values are calculated. Calculated values are listed in Table 5-4.

Table 5-4: α and ε Values for Threat Emitter List

Emitter Number	Band	α	ε
1	A	5/42	$2.191x10^{-3}$
2	B	100/297	$4.902x10^{-3}$
3	C	2/21	$5.892x10^{s}$

Note that, although the sum of the minimum dwells is much less than the sweep period, when intercept times are calculated the intercept time is infinite for emitters 1 and 3. This is because the tolerance with Emitter 1 needs to be at least 1/42 = $2.381x10^{-2}$ and, with Emitter 3, it needs to be at least 1/21 = $4.762x10^{-2}$. In order to satisfy these requirements, the quantity of the extra dwell time must be allocated to that band in order to make $\varepsilon = 1/k$ is calculated by using (3-18) and added to these two bands. As a result, the dwell time on band A is increased to 193531 us and, on band C, to 438844 us.

After this, the intercept times are calculated by using ALGORITHM 2, as follows

- 42 looks with Emitter 1,
- 297 looks with Emitter 2,
- 21 looks with Emitter 3.

Optimization process continues by allocating available remaining time to band with maximum intercept time, for instance it is band B for the first iteration, step by step. At the end of these iterations the intercept time with emitters are as follows.

- 42 looks with Emitter 1,
- 65 looks with Emitter 2,

- 21 looks with Emitter 3.

Note that all available time in sweep period is allocated to band B since it has the maximum intercept time for all iterations. Its intercept time is decreased to 65 looks from 297 looks. The intercept times of three emitters during the optimization process are shown in Figure 5-3.

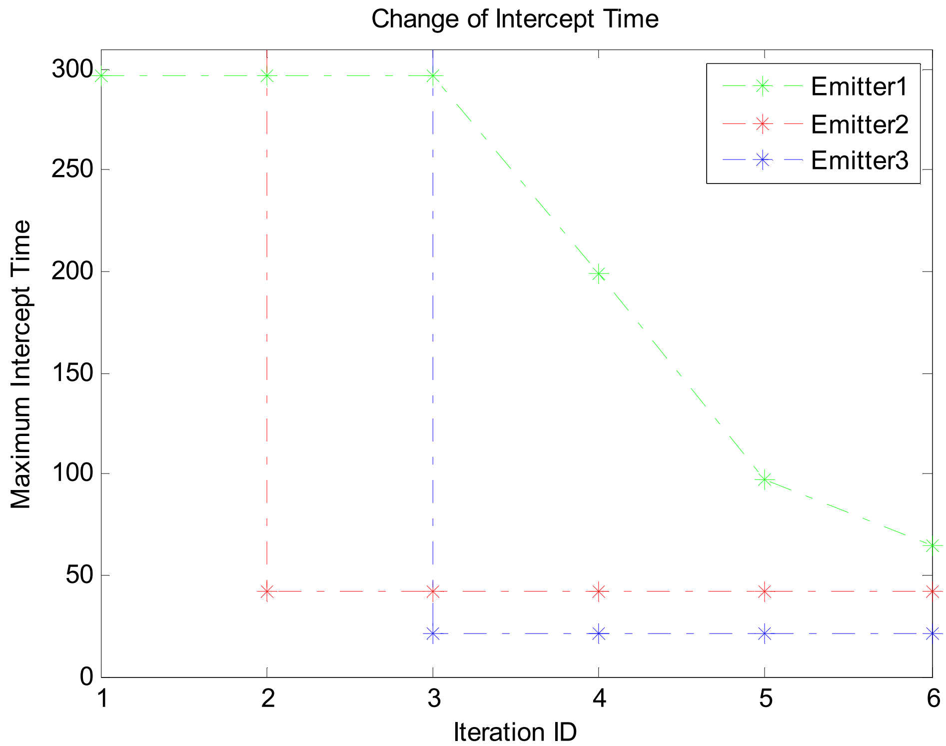

Figure 5-3: Change of Intercept Time for Threat Emitter List

Also note that we express intercept time in terms of sweep period of the receiver, namely 1 second, so for instance Emitter 1 is intercepted in 42 seconds. This therefore represents the maximum intercept time for Clarkson's Algorithm. That is also an optimal search strategy for Clarkson's Algorithm. By using Clarkson's Algorithm an optimal strategy for the emitters given in Table 5-1, with a sweep period of 1 second, have the receiver dwell times as shown in Figure 5-4 and listed in Table 5-5.

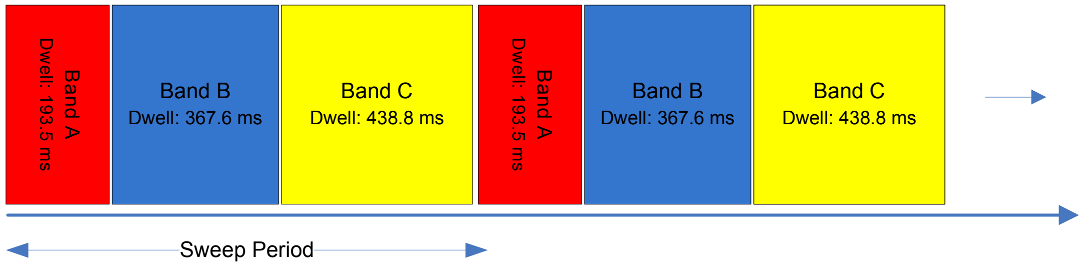

Figure 5-4: Search Strategy for Clarkson's Algorithm for Threat Emitter List

We can see in Figure 5-4 that the dwell time of Band A is decreased when we compare with Figure 5-2. In other words dwell time of Band A in Clarkson's algorithm is less than dwell time in Simple Search. On the other hand for Band B and Band C dwell times are increased with respect to Simple Search.

Table 5-5: Intercept Times and Dwell Times for Clarkson's Algorithm

Emitter Number	Band	Dwell Time (us)	Intercept time
1	A	193531	42
2	B	367625	65
3	C	438844	21

In Table 5-5, it is shown that any of the emitters in the threat-emitter list, given in Table 5-1, will be detected, in other words at least five consecutive RF pulses intercepted from the corresponding emitter, within 65 seconds of becoming operational or coming within range.

5.1.3 Proposed Search Strategy

In previous algorithms, frequency bands which are arranged in a sweep period are visited periodically. The revisit period is same for all bands. On the other hand, in proposed algorithm, receiver dwells at a particular frequency band shorter but more frequently. The revisit period and the dwell time of each frequency band is determined according to the POI.

We use a threat-emitter list as given in Table 5-1 as in previous ones. Minimum dwell times and illumination times calculated according to (3-17) and (2-1). The results are given in Table 5-2, respectively.

Revisit period ($T_{receiver}$) of each frequency band is taken as illumination time ($\tau_{emitter}$) at first. Calculations are done in the interval [$T_{receiver}/2$ $2*T_{receiver}$] as shown in Figure 4-6. Receiver parameters, namely calculated revisit periods and dwell times, of each frequency band are given in Table 5-6. Also duty parameters of frequency bands, which are calculated by (4-9) are shown in Table 5-6. Note that sum of the duties is less than %100. Therefore we do not need to decrease any duty.

Table 5-6: Receiver Parameters for Threat Emitter List

Emitter Number	Band	Dwell Time (us)	Revisit Period (us)	Duty (%)
1	A	11932	37400	39.34 %
2	B	6890	18600	32.12 %
3	C	47	61200	0.08 %

POI and intercept time for each emitter are calculated by using ALGORITHM 4 for the threat parameters given in Table 5-1 and Table 5-6. The results are given in the Table 5-7.

Table 5-7: POI and Intercept Time for Threat Emitter List

Emitter Number	Band	POI	Intercept Time
1	A	90.2%	4
2	B	92.2%	2
3	C	99.9%	1

It is seen in Table 5-7 that, Emitter 3 can be intercepted at its first illumination; on the other hand for Emitter 1 and Emitter 2 it is guaranteed to intercept at the end of forth and second illuminations, respectively. The intercept time of an emitter can be decreased by decreasing the revisit period of this band, in other words by increasing the POI for that emitter.

Finally search strategy is generated by arranging the bands in a "sweep period" of 1second as described in ALGORITHM 6. We also allow for a dead time of 5us between each dwell of the receiver, during which the receiver is re-tuning between different center frequencies. As a result the receiver is not receiving or processing intercepts during this dead time. The search strategy is shown in Figure 5-5. The first three graphs are the visit times of the each band. The upper graph is the search strategy. It is seen that revisit period of frequency bands are different. For instance, revisit period of band 2 is twice of the revisit period of Band A. Therefore Band A is visited 27 times in a sweep period of 1second, while Band B is visited 54 times and Band C is visited 17 times. Also note that there are idle times in the overall strategy, whereas all sweep periods are allocated in previous algorithms. 30.45% of the sweep period is not allocated in proposed algorithm.

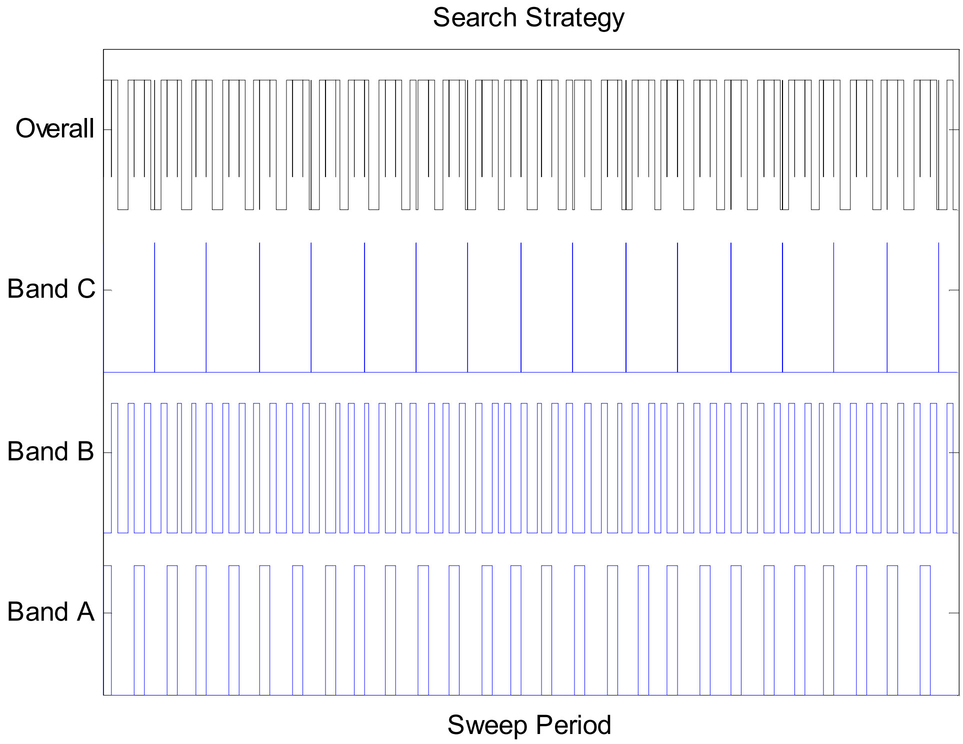

Figure 5-5: Search Strategy for Proposed Algorithm for Threat Emitter List

5.2 PERFORMANCE COMPARISON

In previous section for a given threat emitter list, search strategies are generated by using three algorithms. Each search strategy is shown in Figure 5-2 (Simple Search), Figure 5-4 (Clarkson's) and Figure 5-7 (Proposed), respectively. In this section performance comparison is done. Note that one search strategy is better than another if it provides shorter intercept time. So in order to compare the performance of search strategies their intercept times are determined for a Monte Carlo simulation. Maximum, minimum, average and standard deviation of the intercept times are also calculated for each algorithm for comparison.

Performance comparison is done for two cases . In the first case we assume that we have only one emitter in threat emitter list. However for the second case we have more than one emitter in threat emitter list.

5.2.1 Case 1: Search Strategy for one Emitter

We begin with the case where there is only one emitter in threat emitter list. Let it be Emitter 3 given in Table 5-1. For the Simple Search Strategy and Clarkson's algorithm the search strategy is simple in these cases: assign all the receiver time to Emitter 3, in other words %100 duty. As a result the intercept time is 1 for the Simple Search Strategy and Clarkson's algorithm. That means Emitter 3 is intercepted as soon as it illuminates the receiver. On the other hand for our proposed algorithm we have a search strategy as follows: dwell time is 47us and revisit period is 61200us, which are given in Table 5-8. These parameters corresponds to a duty of % 0.0768, calculated by (4-6).

Table 5-8: Receiver Parameters for Case 1

Emitter Number	Band	minimum dwell time (us)	$\tau_{emitter}$ (us)	Revisit Period, $T_{receiver}$ (us)	Duty (%)
3	C	47	61250	61200	0.0768

The ESM scenario is similar to the one shown in Figure 2-4. There is one emitter and one ESM in the environment. The phase, φ, of the emitter pulse train is randomly changed for Emitter 3. For each case, intercept time is found for the search strategy given above. This is repeated 100 times. An intercept occurs when the receiver is tuned to the right band , that is the operation band of the emitter, and the emitter is pointing in the right direction, that is the direction of the receiver, also a minimum dwell time overlap is necessary. The intercept time of each iteration is shown in the Figure 5-6. Note that intercept time of Emitter 3 for proposed algorithm is 1, for all iterations. In other words Emitter 3 is intercepted as soon as it illuminates the receiver, as in other search strategies. The same intercept time with Simple Search and Clarkson's algorithms is satisfied by proposed algorithm. On the other hand only % 0.0768 of the receiver time is used, whereas the first two algorithms uses %100 of the receiver time. That means same performance is provided by using less receiver time. This is an advantage of visiting frequency band with a revisit period, rather than dwelling continously for that particular frequency band.

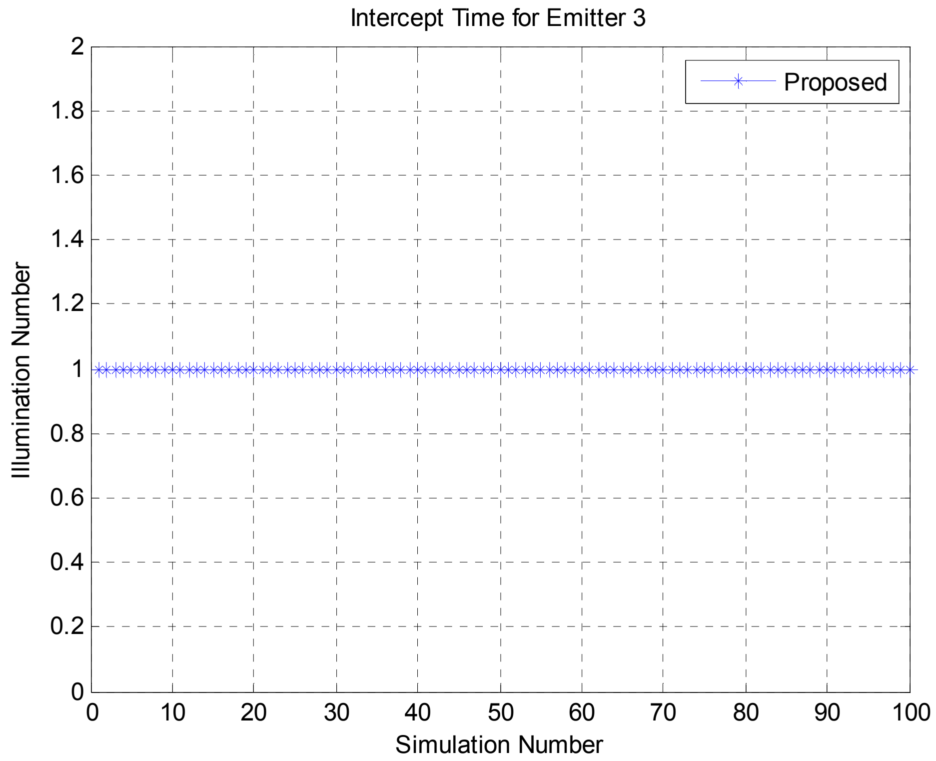

Figure 5-6: Intercept Time for Emitter 3 in Case 1

5.2.2 Case 2: Search Strategy for more than one Emitter

In this case the performance of algorithms are compared for more than one threat. Before discussing the simulation results we note that intercept times of Clarkson's Algorithm and Proposed Algorithm are given in Table 5-5 and Table 5-7, respectively. However intercept time for Simple Search Strategy has not been calculated in section 5.1.1. At first, intercept times of the emitters are calculated for Simple Search Strategy by using ALGORITHM 2. Note that the intercept time with

Emitter 3 would be infinite, since there is a possibility of synchronization. The results are given in Table 5-9.

Intercept times for Clarkson's Algorithm, given in Table 5-5, are given in terms of sweep period, $T_{receiver}$, however intercept times for Proposed Algorithm, given in Table 5-7, are given in terms of emitter illumination number, $T_{emitter}$. In order to compare intercept times, we can write the results in Table 5-5 in terms of emitter illumination number, defined as follows:

$$Illum\,ination\ Number = ceil\,(\,Pulse\ Number \times \left(\frac{T_{receiver}}{T_{emitter}}\right)) \qquad (5\text{-}1)$$

where *Pulse Number* is the intercept time calculated by using ALGORITHM 2, which refers to the number of pulses from receiver.

Theoretical results are given in Table 5-9.

Table 5-9: Intercept Time - Theoretical Results

Emitter Number	Intercept Time (Simple Search)	Intercept Time (Clarkson Algorithm)	Intercept Time (Proposed Algorithm)
1	5	5	4
2	23	22	2
3	Inf	2	1

Table 5-9 means that for instance Emitter 1 is intercepted at the end of fifth illumination if we use search strategy generated by Clarkson's Algorithm. Similarly Emitter 1 is intercepted at the end of fourth illumination if we use search strategy generated by Proposed Algorithm, or it is intercepted at the end of fifth illumination if we use search strategy generated by Simple Search Strategy algorithm. However there is a possibility of synchronization for Emitter 3 if we use search strategy generated by Simple Search Strategy algorithm. That means for some simulations Emitter 3 may not be intercepted by using a search strategy generated by Simple Search.

Monte Carlo Method is used in simulations. The ESM scenario is visualized in Figure 5-7. There are three emitters in the environment and an ESM receiver is used to intercept them. It is assumed that ESM receiver is employing one of the search strategies generated in section 5.1. On the other hand emitters are employing a search strategy according to their scan periods and illumination times. Starting point of the emitter search strategy, in other words phase of emitter pulse train, shown in Figure 5-1, is randomly changed for three emitters for each iteration. For each case, intercept times according to three search strategies derived in section 5.1. are determined. Note that in order to have an intercept, the receiver is tuned to the right band, that is the operation band of the emitter, when emitter is pointing in the right direction, that is the direction of the receiver, also a minimum dwell time overlap is necessary. This ESM scenario is repeated 100 times for different phase values of emitter pulse train.

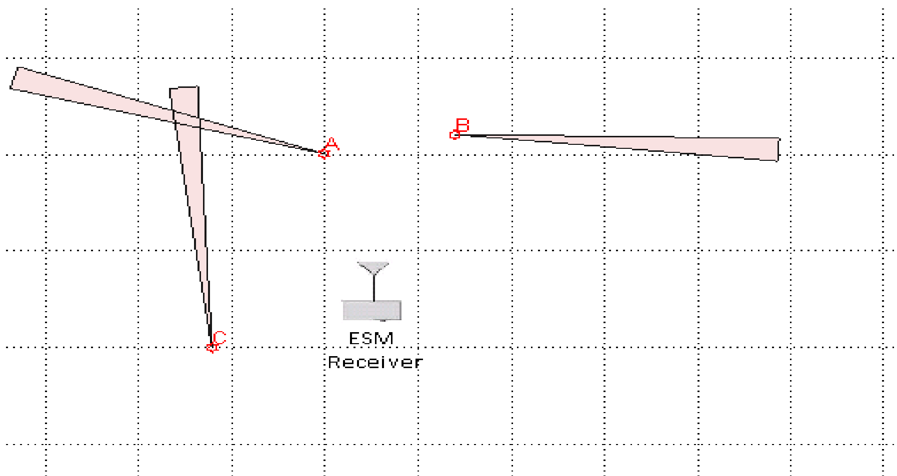

Figure 5-7: ESM Scenario for Simulations

The maximum, minimum, average values and standard deviation of intercept times of search strategies are given in tables. Results for each emitter are given below.

Results for Emitter 1: The results of Monte Carlo simulation for Emitter 1 are given in the figures below. Also some statistical results are given in Table 5-10.

Table 5-10: Intercept Time Results for Emitter 1

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Simple Search	1	5	1.97	1.0294
Clarkson's	1	5	2.73	1.4485
Proposed	1	4	1.72	0.9437

It is seen in theoretical results given in Table 5-9 that, intercept time of Emitter 1 is equal for Simple Search Strategy and Clarkson's Algorithm. However proposed algorithm provides smaller intercept times. When we examine statistical results given in Table 5-10 we see that simulation results verify theoretical results. Maximum intercept time for Emitter 1 is same, 5 illuminations, for Simple Search and Clarkson's algorithm, whereas for proposed algorithm maximum intercept time is less than them. Proposed Algorithm provides better intercept times both in average and in maximum.

Figure 5-8 demonstrates intercept times of 100 simulations for Simple Search Strategy and Clarkson's Algorithm.

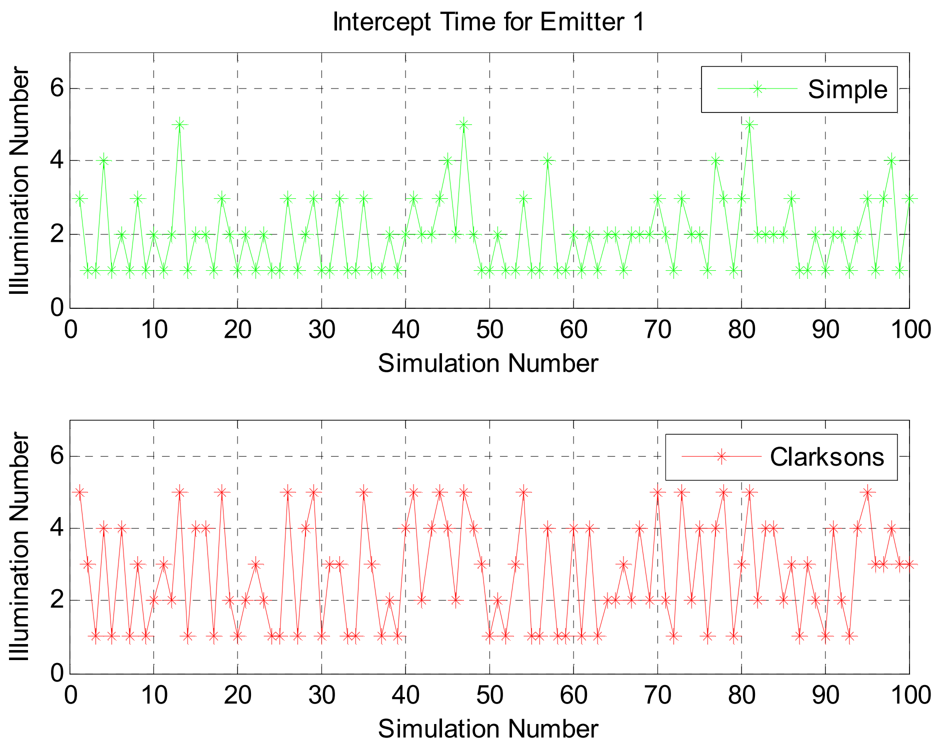

Figure 5-8: Intercept Time of Emitter 1 in Simple Search and Clarkson's Algorithm

Note that minimum and maximum values of intercept time is same for these two algorithm as given in Table 5-10. However on average Simple Search Strategy provides shorter than Clarkson's intercept time since dwell time of Emitter 1 in Simple Search is greater than the dwell time in Clarkson's Algorithm.

We see that Simple Search Strategy is better than Clarkson's Search Strategy for Emitter 1. When we compare the performances of Simple Search and Proposed Algorithm as it is seen in Figure 5-9. We can see that Proposed Algorithm provides shorter intercept time in most of the iterations. Its performance is also better than simple search on average as given in Table 5-10.

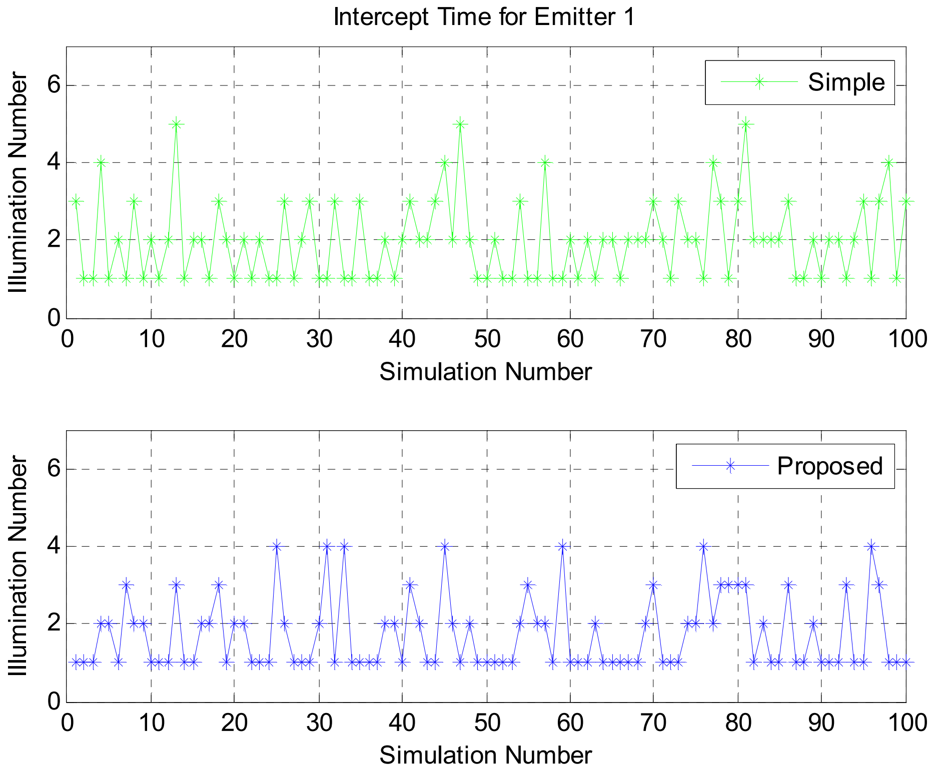

Figure 5-9: Intercept Time of Emitter 1 in Simple Search and Proposed Algorithm

Results for Emitter 2: The results of Monte Carlo simulation for Emitter 2 are given in Figure 5-10. Also some statistical results of this simulation are given in Table 5-11.

Table 5-11: Intercept Time Results for Emitter 2

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Simple Search	1	23	9.75	7.6928
Clarkson's	1	22	8.81	7.8672
Proposed	1	2	1.23	0.4230

In theoretical results given in Table 5-9, it is seen that (maximum) intercept time of Emitter 2 is approximately same for Simple Search and Clarkson's Algorithm. Since dwell time of Band B is approximately same in these two search strategies. On the other hand, proposed algorithm provides much better intercept time than both of them. The simulation results given in Table 5-11 verify theoretical results. The performance of Simple Search is similar to the performance of Clarkson's Algorithm both in terms of maximum and average intercept time. Clarkson's algorithm results are better than Simple search, since dwell time Emitter 1 in Clarkson's Algorithm is greater than the dwell time in Simple Search. However Proposed Algorithm is better than both of them, when we compare maximum and average intercept times. Intercept time of all iterations for all search strategies is shown in Figure 5-10.

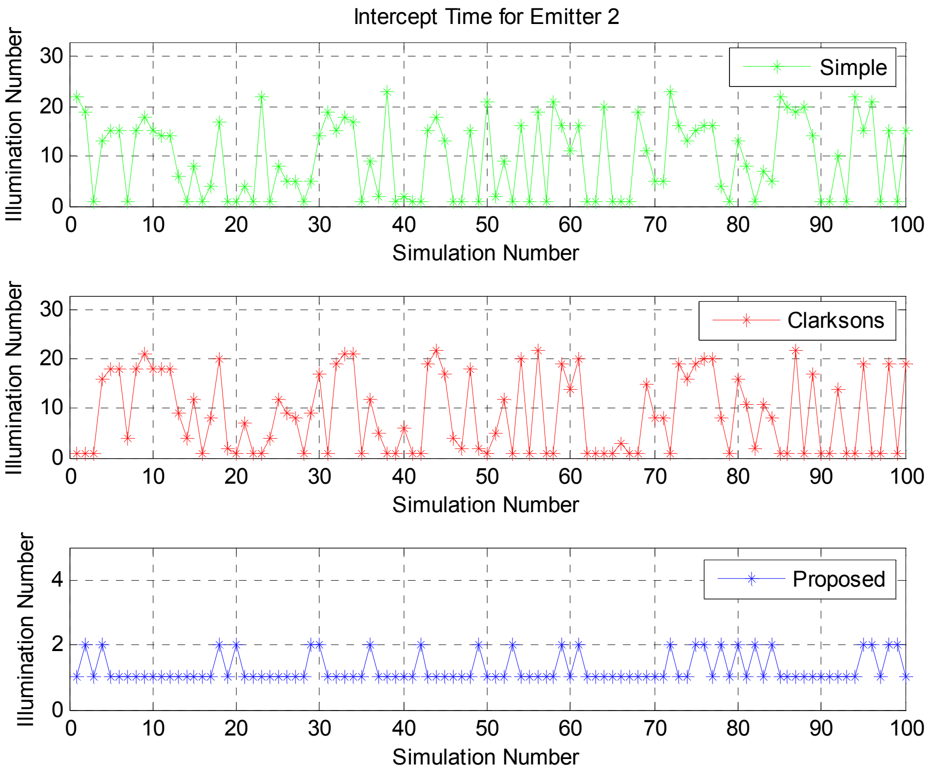

Figure 5-10: Intercept Time for Emitter 2

Results for Emitter 3: The results of Monte Carlo simulation for Emitter 3 are given in the figures below. Some statistical results of this simulation are given in Table 5-12.

Table 5-12: Intercept Time Results for Emitter 3

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Simple Search	1	Inf	-	-
Clarkson's	1	2	1.55	0.5
Proposed	1	1	1	0

In theoretical results given in Table 5-9, it is stated that there is a possibility of synchronization for Emitter 3 if we use Simple Search Strategy. That means we can not intercept Emitter 3 in some simulations by using Simple Search Strategy. In simulation results we verify this situaiton. In some iterations we have seen that we could not intercept Emitter 3 by using Simple Search Strategy, because of synchronization. However synchronization is avoided by Clarkson's Algorithm and Proposed Algorithm. So they intercept Emitter 3 in all iterations. This is shown in Figure 5-11.

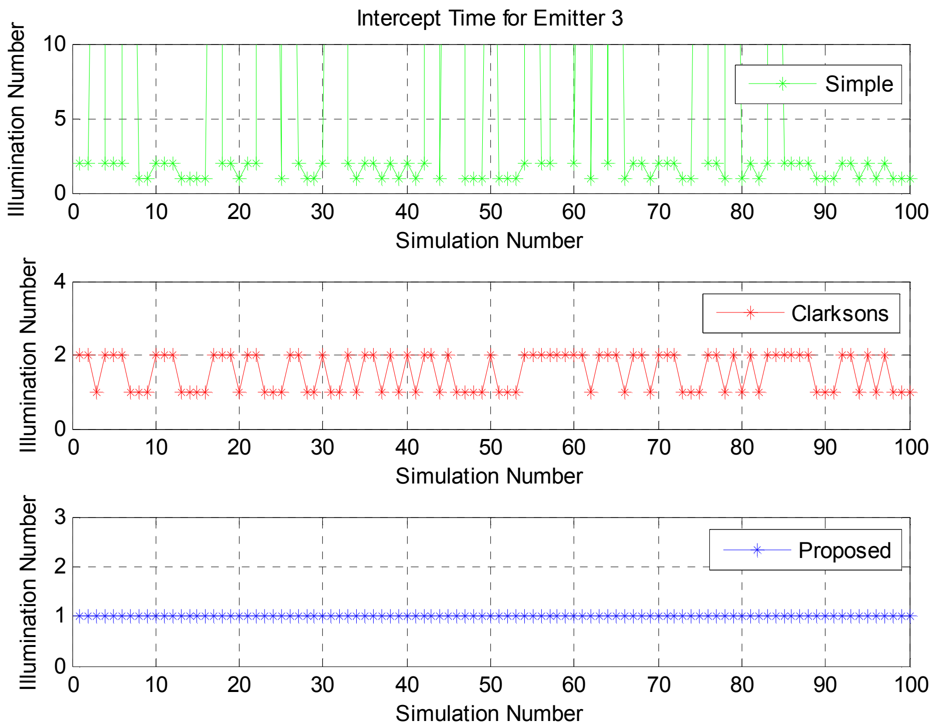

Figure 5-11: Intercept Time for Emitter 3

In theoretical results given Table 5-9 it is stated that Emitter 3 is intercepted at its first illuminaiton if we use the search strategy generated by Proposed Algorithm. Simulation results are verified this. Emitter 3 is intercepted as soon as it illuminates the receiver for all simulations, in other words the intercept time of Emitter 3 is one for all simulations. When we compare the performances of Clarkson's Algorithm and Proposed Algorithm, in which synchronization is avoided, in Figure 5-12, it is seen that Proposed Algorithm provides shorter intercept time. Also on average proposed algorithm provides shorter intercept time. Therefore its performance is better than Clarkson's Algorithm.

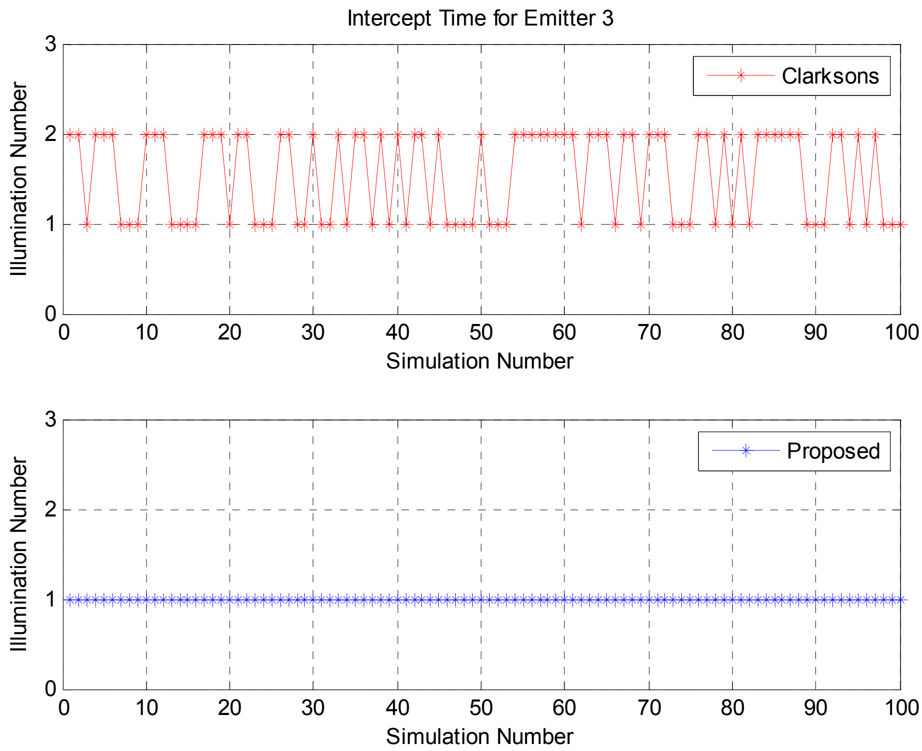

Figure 5-12: Intercept Time for Emitter 3 in Clarkson's and Proposed Algorithm

As a summary in Simple Search Strategy there is a possibility of synchronization for some cases, whereas Clarkson's algorithm is succesful in preventing synchronization. However, it is seen both in simulations and theoretical results that Clarkson's algorithm does not yield shorter intercept times for emitters. The intercept times for Emitter 1 and Emitter 2 are same for Simple Search and Clarkson's algorithm. On the other hand proposed algorithm provides shorter intercept times for all emitters besides preventing synchronization. Since in proposed algorithm we visit a frequency band more frequently. We can see that visiting frequency bands frequently but with shorter dwell times provides better intercept times.

5.2.3 Effect of Cancelled Visits

In proposed algorithm, revisit period of each frequency band is calculated according to POI, at first. After that, the pulse train for the corresponding band is generated by using revisit period and dwell time. The pulse trains of different bands are tried to settle in sweep period, search strategy is obtained by this way. There will be some overlaps during the settling process. Because of these overlaps some pulses (visits) of the corresponding frequency band will be cancelled. In fact these cancellations affect intercept time. Note that for the search strategy given in section 5.1.3 all frequency bands are settled properly, there is not any cancellation. However for some cases there will be. In order to examine the effect of cancellations, for the search strategy generated in section 5.1.3, 1/3 of the visits (pulses) are cancelled for each frequency band. They are chosen randomly. A new search strategy is obtained by this way and the same ESM scenario shown in Figure 5-7 is run only for this degenerated search strategy. Intercept time for each emitter is calculated for each simulation. It is repeated 100 times as in previous. The results are given below.

Results for Emitter 1: The results of Monte Carlo simulation for Emitter 1 is given in the Figure 5-13. Also some statistical results of for Emitter 1 is given in the Table 5-13. The results of cancellation case is compared with the properly settled case in section 5.2.2.

Table 5-13: Intercept Time Results for Emitter 1

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Proposed	1	4	1.72	0.9437
Cancellation	1	5	2.17	1.3185

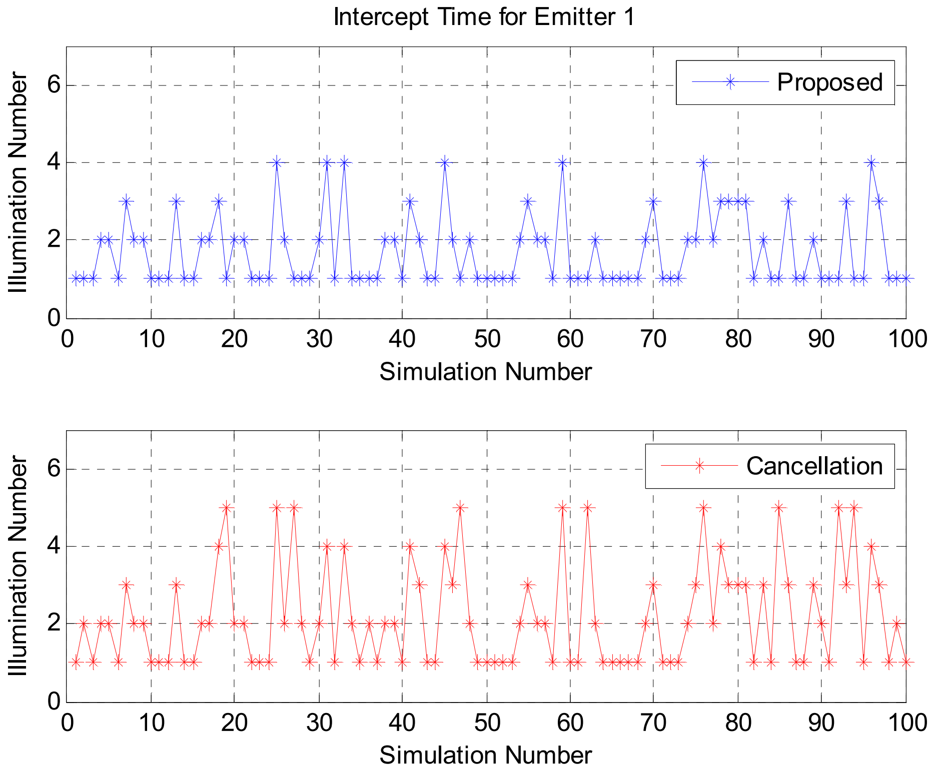

Figure 5-13: Intercept Time for Emitter 1

It is seen both in Table 5-13 and in Figure 5-13 that both maximum and average intercept time is increased for Emitter 1 in case of visit cancellation, as expected. However synchronization is still avoided.

Results for Emitter 2: The results of Monte Carlo simulation for Emitter 2 is given in the Figure 5-14. Also some statistical results of for Emitter 2 is given in Table 5-14. The results of cancellation case is compared with the properly settled case in section 5.2.2.

Table 5-14: Intercept Time Results for Emitter 2

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Proposed	1	2	1.23	0.4230
Cancellation	1	6	1.76	1.0359

It is seen both in Table 5-14 and in Figure 5-14 that both maximum and average intercept time is increased for Emitter 2 in case of visit cancellation, as expected. However synchronization is still avoided.

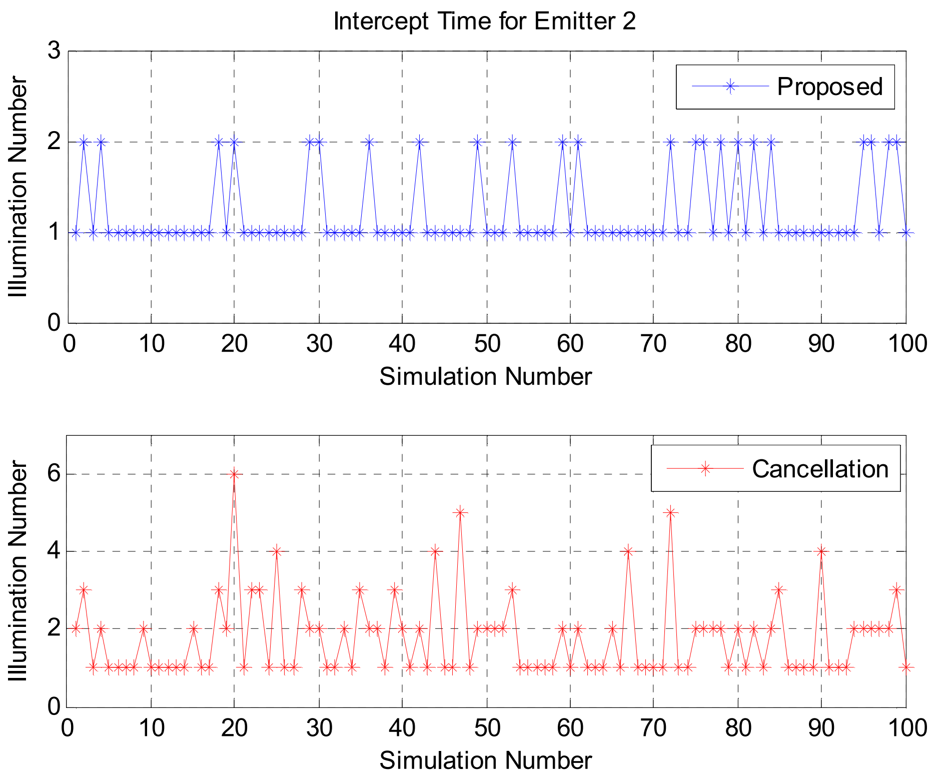

Figure 5-14: Intercept Time for Emitter 2

Results for Emitter 3: The results of Monte Carlo simulation for Emitter 3 is given in the Figure 5-15. Also some statistical results of for Emitter 3 is given in Table 5-15. The results of cancellation case is compared with the properly settled case in section 5.2.2.

Table 5-15: Intercept Time Results for Emitter 3

Method	Minimum Intercept Time	Maximum Intercept Time	Average Intercept Time	Standard Deviation of Intercept Time
Proposed	1	1	1	0
Cancellation	1	2	1.29	0.4560

It is seen both in Table 5-14 and in Figure 5-14 that both maximum and average intercept time is increased for Emitter 3 in case of visit cancellation, as expected. However synchronization is still avoided.

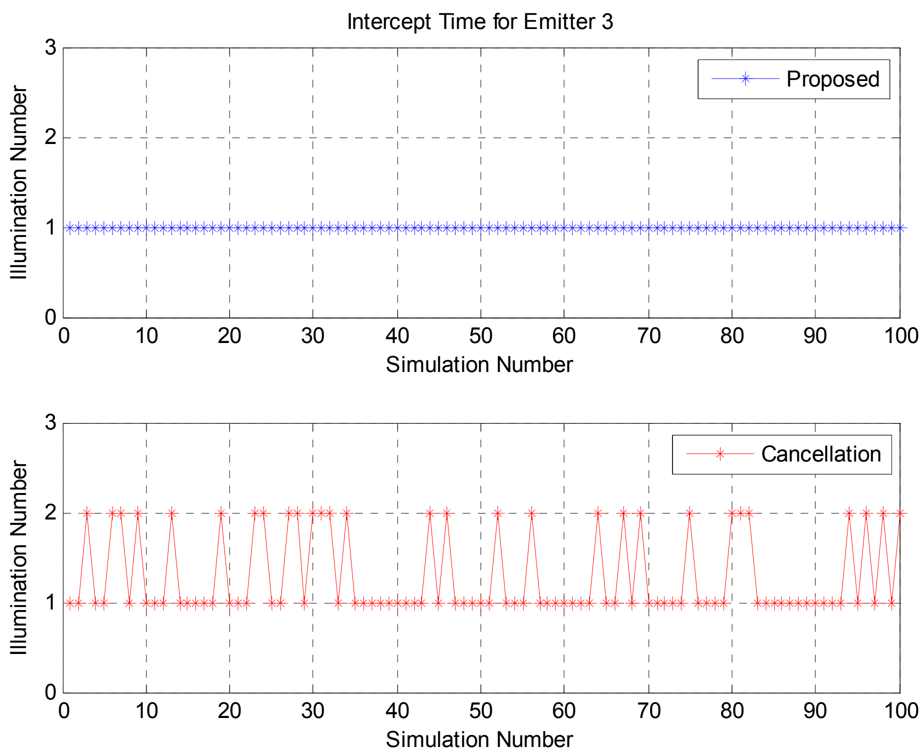

Figure 5-15: Intercept Time for Emitter 3

As a summary in both maximum and average intercept times increases in case of visits cancelations. However still there is not a possibility of synchronization. As long as cancelled visits are not periodic proposed algorithm still succesful in preventing synchronization.

5.2.4 Periodic Search

In proposed algorithm it is possible to change probability of intercept of an emitter by changing revisit period of the emitter. In this part we are going to investigate how revisit period of receiver affect probability of intercept. In other words, analysis and simulations of ALGORITHM 4 and ALGORTIHM 5 is done. Here POI refers to intercept the emitter in its first illumination. Also intercept time of the emitter, in terms of illumination number, is calculated. Calculated values are given in the Table 5-16.

Table 5-16: POI vs Revisit Period

POI (%)	Revisit Period (us)	Intercept Time
100	61200	1
90	67600	2
80	76900	2
70	87100	2
60	103500	2
50	121900	3
40	151400	3

Emitter 3 given in Table 5-1 used in simulations. For some probability of intercept values revisit period of Emitter 3 is going to be calculated bu using ALGORTIHM 5 at first. Then a search strategy is generated for the corresponding revisit period by using proposed algorithm. Note that dwell time is constant for all revisit periods. An search strategy is shown in Figure 5-16 for the first revisit period. The lower graph is the visit times of the Band C. The upper graph is the search strategy, which is same with the lower one, since there is only one frequency band to visit.

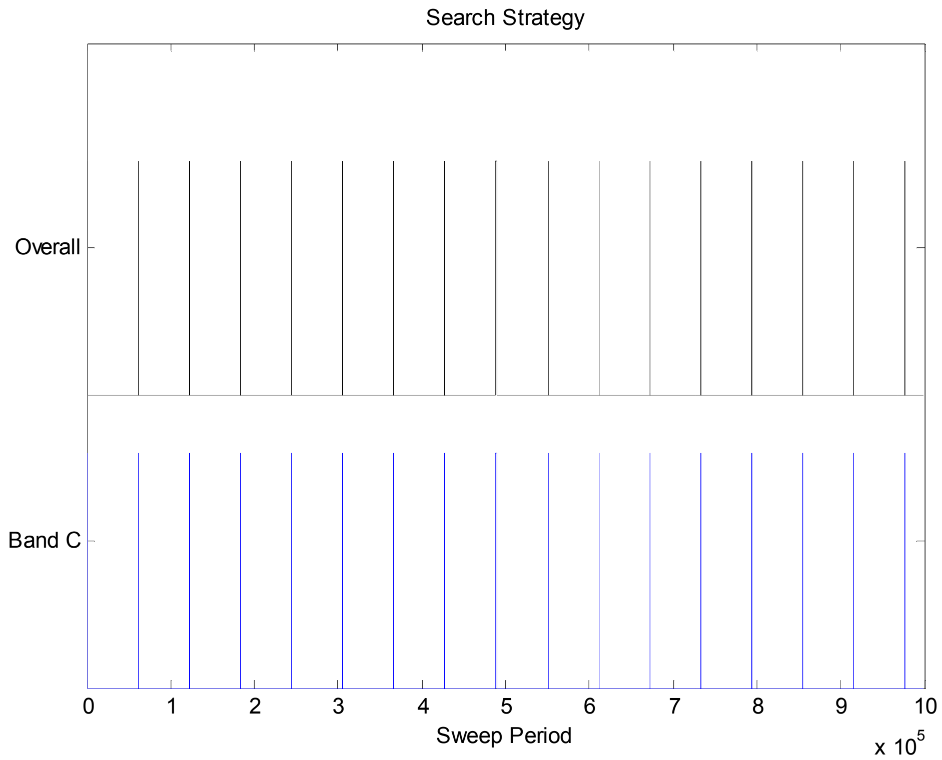

Figure 5-16: Search Strategy for Emitter 3 for revisit period of 61200us

The ESM scenario is similar to the one shown in Figure 2-4. There is one emitter and one ESM in the environment. The phase, φ, of the emitter pulse train is randomly changed for Emitter 3. For each case, intercept time is found for the search strategy generated. This is repeated 100 times. An intercept occurs when the receiver is tuned to the right band and the emitter is pointing in the right direction, also a minimum dwell time overlap is necessary.

For each search strategy intercept time is found for Emitter 3. The number of simulations in which Emitter 3 is intercepted at its first illumination is given in Table 5-17. Note that this is also corresponds to POI for that search strategy since there are 100 simulations. Also maximum intercept times are given in Table 5-17.

Table 5-17: Revisit Period vs Simulation Results

POI (%)	Revisit Period (us)	Number of simulation that Intercept Time =1	Intercept Time (max)
100	61200	100	1
90	67600	91	2
80	76900	82	2
70	87100	71	2
60	103500	63	2
50	121900	52	3
40	151400	42	3

We can see in Table 5-17 that increasing revisit period of a frequency band corresponds to decreasing probability of intercept. That measn increasing maximum intercept time. Table 5-17 shows us that, calculated revisit periods by using ALGORTIHM 5, for a given POI gives approximately same POI in practice.

CHAPTER 6

CONCLUSION AND FUTURE WORK

6.1 CONCLUSION

In this thesis, search strategies in ESM receivers are studied. Threat emitters operate over the very wide portion of the electromagnetic spectrum. ESM systems must maintain surveillance in this electromagnetic spectrum. Frequency swept super heterodyne receiver is the common used receiver in ESM systems. The center frequency of FSR can be changed and the sequence and timing of changes to the centre frequency constitute a search strategy. Success of a receiver is closely related to the search strategy. Common search strategies are periodic. Similarly radars also employ a periodic search, with their directional antenna. Therefore, mathematically, interception problem in ESM can be written as a pulse train coincidence problem. By doing so, intercept time, probability of intercept (POI) and synchronization ratios, (which means never intercepting a radar), can be calculated. In the design and analysis of ESM equipment such as radar warning receivers, calculation of these quantities is important.

Two search strategies, namely Simple Search Strategy and Clarkson's algorithm are investigated. Moreover an algorithm based on probability of intercept is proposed. In Simple Search Strategy, frequency bands are arranged one after another and search strategy is obtained by this way. Dwell times are same for this case. As a result in Simple Search synchronization is not concerned. The difference between Simple Search Strategy and Clarkson's algorithm is that in order to avoid synchronization, dwell times can be adjusted in Clarkson's algorithm. In these two algorithms a frequency band is visited only one time in a "sweep period", usually with a very large dwell time. On the other hand in Proposed Algorithm a frequency band is visited more than one time, with shorter dwell, during the sweep period. In Proposed Algorithm each frequency band has also individual revisit period. A search strategy is obtained after arranging the frequency bands into the sweep period, according to their individual revisit periods and dwell times.

It is seen both in theoretical calculations and simulations that Clarkson's algorithm does not provide shorter intercept time than Simple Search in most of the cases. But preventing synchronization is very important in ESM concept. On the other hand in theoretical calculations proposed algorithm provides better intercept time than both algorithms. This is also verified in simulation results. Therefore we can conclude that proposed algorithm provides shorter intercept times besides preventing synchronization. Another important advantage of proposed algorithm is that it does not continuously dwell on a frequency band when there is only one radar in the environment. Therefore without using all of the receiver time, proposed algorithm provides same intercept time with other algorithms, which use 100% of the receiver time in such a case. As a conclusion proposed algorithm is superior to Simple Search and Clarkson's algorithm.

In this study some assumptions are done in Chapter 3. Now we can discuss them. For instance, only circularly scanning radars are interested, their parameters are assumed to be constant. However in order to achieve better performance, modern radars are able to operate in a number of modes. During the operation radars are agile between these modes. To resolve range ambiguities PRI jittering, switching and staggering is used. Also in evading detection RF agility is useful. Moreover, the scanning strategy of the radar need not be circular, it may be concentrated in sectors. In that case scanning strategy of the radar may be spiral, raster, or lobe-switching scan strategies, or it can be non-scanning. Here the question is: How do these characteristics of a modern radar affect a receiver search strategy? In other words, how can the search strategy be adapted to take account of them?

At first, we investigate PRI agility. Note that to achieve detection, a certain number of consecutive pulses must be intercepted by the receiver. Dwell time, usually minimum dwell time is used, of a frequency band is calculated according to this. Therefore if the maximum time interval, that is maximum PRI, is known then it can be accounted for in proposed algorithm through the minimum dwell time.

The main assumption done in this study is that, radars employ periodic scanning strategy known period. This is the case in many widely used scanning strategies. However in case of non-circular scanning strategy this is also same. For instance, a radar employing a unidirectional raster or spiral scan behaves, in our emitter model given in Chapter 2, in the same way as a circularly scanning radar. That is the ESM receiver is illuminated once during each period for a certain illumination time by the emitter. As long as the minimum illumination time and period of the non-circular scanning strategy are known, the proposed algorithm still can be applied to design the search strategy for non circular scanning radars. Also for non-scanning radars, the idle times in sweep period can be used. With sufficient dwell time allocated, non-scanning radars can easily be intercepted.

Finally, RF agility can be discussed. So long as the pattern of visits to any particular RF band is periodic with known period, RF agility can be taken into account with proposed algorithm. In this case in order to devise a search strategy; RF agile emitters require multiple entries, not a single entry in the threat emitter list. Each entry identical except for the RF band. Scan period of each entry can be taken as the illumination revisit period on that RF band.

6.2 FUTURE WORK

An essential assumption done in this work is that, the receiver has omnidirectional antenna. However, in order to increase detection sensitivity, as well as to gain intelligence on the direction of arrival of radar signals, ESM receivers can employ directional antennas. In such a case they search not only in RF but also in angle. As a result the cost of this is a longer intercept and response times. The search strategy is composed of searching angular sectors in a periodic fashion. In each sector a search over RF is performed, so that the whole search strategy is also periodic at the end. Then it is also necessarry to calculate intercept time and POI in such a case. This may be a topic to work on it in the future in this area.

Proposed algorithm is based on POI for a threat and during settling in sweep period some instance of frequency bands can be cancelled, as in section 5.2.3. In this case, as a result, calculated POI values can not be satisfied and intercept times increases. There is a need to calculate the real POI values, and intercept times. This may be another topic to work on it in the future.

Moreover other search strategies such as random search strategy can be taken account. Its performance can also be calculated as a future work.

REFERENCES

[1] R. G. Wiley, “Electronic Intelligence: The Interception of Radar Signals”, Norwood, Massachusetts: Artech House, 1985.

[2] A. G. Self and B. G. Smith, “Intercept Time and its Prediction,” IEE Proc., vol. 132F, no. 4, pp. 215-222, July 1985.

[3] Joint Chiefs of Staff, Department of Defense, “Department of Defense Dictionary of Military and Associated Terms”, Joint Pub 1-02 (1 December 1989).

[4] Richards, P.I.: ‘Probability of coincidence for two periodically recurring events’, Ann. Math. Stat., 2003, 19, (1), pp. 16–29

[5] Friedman, H.D.: ‘Coincidence of pulse trains’, J. Appl. Phys., 1954, 25, (8), pp. 1001–1005

[6] Miller, K.S., and Schwarz, R.J.: ‘On the interference of pulse trains’, J. Appl. Phys., 1953, 24, (8), pp. 1032–1036

[7] Kelly, S.W., Noone, G.P., and Perkins, J.E.: ‘The effects of synchronization on the probability of pulse train interception’, IEEE Trans. Aerosp. Electron. Syst., 1996, 32, (1), pp. 213–220

[8] Wiley, Richard G., Electronic Intelligence: The Analysis of Radar Signals (Artech House, Norwood MA, 1982).

[9] Wiley, Richard G., Electronic Intelligence: The Interception of Radar Signals (Artech House, Norwood MA, 1985).

[10] B. F. Schwoerer, “Probability of Intercept in Electronic Countermeasures Receivers”, Master’s Thesis, Naval Postgraduate School, Monterey, CA, Dec. 1975.

[11] Clarkson, I.V.L., Perkins, J.E., and Mareels, I.M.Y.: ‘Number theoretic solutions to intercept time problems’, IEEE Trans. Inf. Theory, 1996, 42, (3), pp. 959–971

[12] Hardy, G.H., and Wright, E.M.: ‘An introduction to the theory of numbers’ (Oxford University Press, 1979, 5th edn.)

[13] Clarkson, I.V.L.: ‘The arithmetic of receiver scheduling for electronic support’. Proc. Aerospace Conf., 2003, vol. 5, pp. 2049–2064

[14] Washburn, A.R.: ‘Overlapping pulse trains’, IEEE Trans. Aerosp. Electron. Syst., 1981, AES-17, (3), pp. 380–385

[15] I. V. L. Clarkson, “Optimal Periodic Sensor Scheduling in Electronic Support”, in Defense Applications of Signal Processing 2005 Utah, USA, 27 Mar. – 1. Apr. 2005.

[16] Clarkson, I. V. L., “Optimization of Periodic Search Strategies for Electronic Support”, http://www.itee.uq.edu.au/~vaughan/Publications/optsearch.pdf, last visited on September 2009

[17] S. Stein and D. Johansen, “A statistical description of coincidences among random pulse trains”, Proc. IRE, vol. 46, no. 5, pp. 827-830, May 1958.

[18] Clarkson, I. V. L., “Approximation of linear forms by lattice points with applications to signal processing”, Ph.D. dissertation, The Australian National University, 1997.

[19] B. R. Hatcher, “Intercept Probability and Intercept Time”, Electronic Warfare, pp. 95-103, Mar.-Apr. 1976.

[20] Clarkson, I. V. L., El-Mahassni, E. D., and Howard, S. D. “Sensor scheduling in electronic support using Markov chains”, IEE Proceedings–Radar, Sonar & Navigation, 153, 4 (Aug. 2006), 325—332.

[21] Clarkson, I. V. L. “Optimal periodic sensor scheduling in electronic support”, Proceedings of the Defence Applications of Signal Processing Workshop, Mar. 2005, CD-Rom.