AN INVESTIGATION OF IN-SERVICE SECONDARY MATHEMATICS TEACHERS' EVOLVING KNOWLEDGE THROUGH PROFESSIONAL DEVELOPMENT ACTIVITIES BASED ON MODELING PERSPECTIVE

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ABSTRACT

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Although an increasing number of research studies in mathematics education have begun focusing their efforts on mathematical modeling as a need for change to convey mathematical ideas beyond schools, there is not enough information about the nature of the teacher knowledge for effective use of modeling in mathematics teaching and how this knowledge evolves. The goal of this study is to investigate teachers' evolving knowledge when they engage in professional development activities based on lesson study cycle from modeling perspective. Professional development program of this study included a cyclical process. Lasting a month, each cycle consisted of meeting before the implementation of the model eliciting activity, implementation of the activity and meeting after the implementation. The study took five months and was conducted in two public schools. The participants were four in-service mathematics teachers where two teachers were selected from each school by purposive sampling. The study was designed as case study. Data analyses were conducted during and after data collection and with two approaches as with-in case and cross-case analysis. As the professional development activities created learning environments for the teachers to develop their models for teaching mathematics from a modeling perspective, the results of this study showed that the professional development program used in the study had a positive effect on

teachers' evolving pedagogical content knowledge and pedagogical knowledge based on the theoretical and empirical backgrounds in the literature. Besides, implications, suggestions for professional development, for teachers and for further research are provided.

Keywords: Mathematics Education, Teacher Education, Mathematical Modeling, Teacher Knowledge, Professional Development

ÖZ

ORTAÖĞRETİM MATEMATİK ÖĞRETMENLERİNİN PEDAGOJİK ALAN VE PEDAGOJİK BİLGİLERİNDEKİ GELİŞİMİN, MODELLEME YAKLAŞIMINA GÖRE TASARLANMIŞ BİR MESLEKİ GELİŞİM VE EĞİTİM ETKİNLİĞİ SÜRECİNDE İNCELENMESİ

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Matematik eğitiminde, giderek artan sayıda araştırma çalışmaları çabalarını, matematiksel fikirlerin okul dışına taşınması gerektiği üzerine bir değişim ihtiyacından dolayı matematiksel modelleme üzerine yoğunlaştırmasına rağmen, öğretmenlerin modellemeyi matematik eğitiminde etkin bir şekilde kullanabilmeleri için nasıl bir bilgiye sahip olmaları gerektiği ve bu bilginin nasıl geliştiği üzerine yeterli bilgi bulunmamaktadır. Bu çalışmanın esas amacı, öğretmenlerin bilgilerindeki gelişimi, modelleme perspektifine uyumlu olan ders planı hazırlama üzerine dizayn edilmiş mesleki gelişim ve eğitim etkinliklerine katılımları sürecinde incelemektir. Hizmet içi öğretmen eğitimi programı döngüsel bir süreç içermektedir. Bir ay süren, her döngü modelleme aktivitelerinin uygulanmasından önce yapılan toplantıları, aktivitelerin uygulanmasını ve uygulama sonrası toplantıları içermektedir. İki devlet okulunda uygulanan çalışma beş ay sürmüştür. Çalışmanın katılımcıları her iki okuldan amaçlı örneklem yöntemiyle seçilen 4 matematik öğretmenidir. Bu çalışmada nitel araştırma desenlerinden durum çalışması kullanılmıştır. Çalışmanın veri analizi durum odaklı ve karşılaştırmalı durum analizi yaklaşımları ile veri toplarken ve veri toplandıktan sonra olmak üzere iki ana aşamada gerçekleştirilmiştir. Bu çalışmanın, mesleki gelişim ve eğitim etkinlikleri,

öğretmenlerin modelleme perspektifinden matematik öğretim modellerinin gelişmesine imkân sağlayan öğrenme ortamları sunduğu gibi, araştırmanın sonuçları hizmet içi öğretmen eğitiminin öğretmenlerin pedagojik alan ve pedagojik bilgilerindeki gelişime, alan yazındaki teorik ve ampirik köklere dayanan pozitif bir etkisinin olduğunu göstermektedir. Çalışmada, aynı zamanda mesleki gelişim, öğretmenler ve yapılacak çalışmalar için öneriler sunulmaktadır.

Anahtar Kelimeler: Matematik Eğitimi, Öğretmen Eğitimi, Matematiksel Modelleme, Öğretmen Bilgisi, Mesleki Gelişim

To My Son

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INTRODUCTION

Recent reform efforts in mathematics education in Turkey stress the importance of mathematical thinking, observing patterns and testing conjectures and estimation of results (Ministry of National Education [MoNE]-Board of Education, 2011). In addition, extraction of mathematical thinking structures are emphasized by this mathematics education vision and with the vision, establishing mathematical models, problem solving, communicating, making connections and reasoning skills are aimed to be gained by students in the mathematics curricula (MoNe, 2011, p. 13). One of the skills emphasized in the curriculum and integrated with the general vision of mathematics education is mathematical modeling. Modeling is taking important part in a reform oriented mathematics education and contributes to develop other important skills (Hodgson, 1995; Lesh & Doerr, 2003a; Ministry of National Education [MoNE], 2011; NCTM, 1989). Why is modeling taking important part in a reform oriented mathematics education? The answer to this question is begining to emerge with raising questions about appropriateness of current teaching approaches in teaching mathematics and in mathematical problem solving by many researchers (Blum & Niss, 1991; Doerr & English, 2003; Lesh & Doerr, 2003a; Sriraman & Lesh, 2006; Wyndhamm & Säljö, 1997; Yoshida, Verschaffel, & De Corte, 1997). The inadequacy of the approaches was even worse in situations when students try to work on real life problems and try to use the mathematics beyond the school (Lesh & Doerr, 2003a, 2003b; Kaiser, Blomhøj, & Sriraman, 2006). Schoenfeld (1992) and English (2003) state that a necessary distinction should be made between problem solving activity and traditional word problem solving activity and its exercises. Schoenfeld (1992) indicate that problem solving activity should enable students to use their cognitive and meta-cognitive abilities. However, in traditional word problem solving activity, givens and goals are already given and words are carefully selected to make apparent the mathematical procedure required for solving the

problem (Lesh, Yoon, & Zawojewski, 2006). Problem solving activity should be beyond a process where the givens, goals and the procedures between givens and goals to accomplish an activity are certain (Wyndhamm & Säljö, 1997). Problem solving activities that are used in schools are far from meeting the needs of this era since the individuals are needed to produce advanced and productive solutions for problem situations encountered in many areas (Greer 1997; Vershaffel, De Corte, & Lasure, 1994; Zawojewski & Lesh, 2003). Problem solving activities that are used in schools gives rise to the formation of didactic assumptions by encouraging students to think that every problem that is on book and asked by the teacher can be solved, if the problem is not understood, key words are detected in order to make apparent the mathematical procedure or to look at the similar problems' solution procedure (Greer 1997; Reusser & Stebler 1997; Verschaffel, Greer, & Corte, 2002). The results of the studies show that traditional word problem solving activities do not enable students to develop problem-solving strategies, to give meaning to the solutions since students reaching these solutions by acting according to the keywords and to relate with the real life and realistics situations (Greer, 1997; Vershaffel, De Corte, & Lasure, 1994; Yoshida, Verschaffel, & De Corte, 1997). From this point of view, many researchers focus on using mathematical modeling activities which is open-ended, not directing students by keywords or sentences, enable students to connect the problem situation with real life situations, enable students convey mathematics usage beyond schools and to develop problem solving strategies (Blum & Niss, 1991; Lesh & Doerr, 2003a; Verschaffel, Greer, & De Corte, 2002). Mathematical modeling is defined as a mathematical activity which includes many properties of mathematical thinking and learning and the properties of the activity appear in its implementation (Burkhardt & Pollak, 2006; Doerr & English, 2003; Kaiser, Blomhøj, & Sriraman, 2006; Niss, 1987). In the implementation process, learners focus on relationships and patterns beyond superficial characteristics of the problem for producing different ideas and showing mathematics usage in real-life situations (Lesh & Doerr, 2003a). Therefore, an increasing number of studies in mathematics education have begun focusing their efforts on mathematical modeling as a need for change in order to convey mathematical ideas beyond school (Lesh, Landau, & Hamilton, 1983; Lesh, Kaput, & Hamilton, 2007; Sriraman & Lesh, 2006).

On the other hand, the effective use of modeling in the classes depends on the competencies of teachers regarding mathematical modeling (Niss, Blum, & Galbraith, 2007). The extend to which the students reach the targeted objectives and the quality of the students learning experiences depends on the quality of the instruction and the quality of the instruction depends on the teachers' knowledge, skills and attitudes (Darling-Hammond & Youngs, 2002; Sowder, 2007). Yet, as Cohen and Ball (1990) stated, how teachers could teach mathematics in the same way that they have never seen and experienced. Thus, teachers should experience modeling in order to implement mathematical modeling properly in their classes.

1.1 Model and Modeling Perspective

A model is an "internal conceptual system" and "external artifact" of that system in order to use to explain other complex systems (Lesh & Doerr, 2003a, 2003b). According to modeling perspective, "internal conceptual system" and "external artifact" of that system are not different that they are interconnected in one model (Doerr & Lesh, 2003). When the internal conceptual system change, the external artifact of that system also change this means external artifacts are the auditable trails of documentation of the internal conceptual systems (Clark & Lesh 2003; Doerr & Lesh, 2003). Constructing mathematical models by students includes choosing and connecting the proper data, creating the model, testing and revising the model and expressing that model by using representational media (Lesh & Doerr, 2003a, 2003b). In addition, teachers' models for teaching mathematics not only have mathematical components but also have pedagogical components and with their model, they express components of their knowledge (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Teachers' models are constructing on their mathematical teaching and learning experiences and in teaching progress, they understand and interpret the situations with these models (Lesh & Doerr, 2003a; Schorr & Clark 2003). Accordingly, the quality of their teaching is related to the quality of their models for teaching mathematics (Schorr & Clark 2003).

This perspective provides to understand the nature and development of the teacher knowledge (Clark & Lesh 2003; Doerr & Lesh, 2003). The ways of teacher, thinking and interpreting their practices are important for teacher development from the modeling perspective (Doerr & Lesh, 2003). According to modeling perspective of teacher development, the emphasis should be on teachers' knowledge that teachers

express, test, revise and refine and extend that knowledge in order to access more effective forms for classroom teaching (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Teacher knowledge changes when they learn more about their students' thinking and their practice (Doerr & Lesh, 2003; Doerr & Lesh, 2011)

1.1.1 Teacher Development Programs from the Model and Modeling Perspective

According to model and modeling perspective, teacher development programs should give a chance to teachers to express, test, revise and refine their models (Lesh & Doerr, 2003a; Doerr & Lesh, 2011). Within this framework of teachers' professional development, teacher development takes part in when teachers express their models based on their mathematical teaching and learning experiences with the help of the external artifacts and these artifacts become more complex and sophisticated by using it in different situations and times (Doerr & Lesh, 2003; Doerr & Lesh, 2011; Schorr & Lesh, 2003). Sophisticated, powerful and comprehensible models on teaching are the outputs of the successful professional development programs (Clark & Lesh, 2003; Schorr & Lesh, 2003). Modeling activities are valuable tools in order to provide teachers an opportunity to see students' way of thinking and how this thinking develops (Doerr, 2006). From this point of view, teachers can use modeling activities in their classes in order to understand students' different way of thinking, promote these different thinking structures and develop their thinking strategies based on students' different way of thinking (Doerr, 2007). However, teachers need learning environments in order to develop their teaching models for using modeling effectively in classes (Niss, Blum, & Galbraith, 2007). These learning environments are provided with the modeling activities for teachers (Clark & Lesh, 2003; Doerr & Lesh, 2003). The most important objectives of these activities are: (i) to reveal teachers' models, (ii) to give opportunities for testing, revising and refining their models and (iii) to give opportunities for sharing models with other teachers and using models in different situations (Doerr & Lesh, 2003) . Modeling activities for teachers resemble modeling activities for students (Chamberlin, 2002; Clark & Lesh, 2003; Doerr & Lesh, 2003). Modeling activities for teachers are "*thought-revealing activities*" and are used in order to reveal teachers' internal models (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Lesson plans on modeling activities could be the teachers' model eliciting activities like

"observation sheet", "way of thinking sheet" and "concept maps" (Clark & Lesh 2003; Doerr & Lesh, 2003). Lesson plans allow for seeing the teachers' models, which are based on teaching and learning experiences of teachers, and that models describe the nature of teacher knowledge (Hiebert, Gallimore, & Stigler, 2002; Lewis & Tsuchida, 1997; Stein, Smith, & Silver, 1999). The activity of designing purposeful plans with in-service teachers served as an inquiry into curriculum, pedagogy and students thinking (Hiebert, Gallimore, & Stigler, 2002; Stein, Smith, & Silver, 1999).

Lesson study design presents an opportunity for teachers to express, test, revise, refine, and extend their model in order to access more effective forms for classroom teaching. Teachers express their models with lesson plans and test it in "research lesson" and in the post lesson revise and refine their models with the observations of the lessons and the analysis of student works (Lewis, 2002a, 2002b; Lewis & Tsuchida, 1997, 1998; Wang-Iverson & Yoshida, 2005). Schorr and Clark (2003) indicate that teacher development programs cannot be successful if the environment for testing, revising and refining is not provided. In addition, "on-the-job" teacher development program that is compatible with the modeling perspective that the student work and classroom activity becomes the parts for the "on-the-job" development and teachers are not taken away from their own teaching experience to evolve their knowledge (Doerr & Lesh, 2003). With the aim and the scope of modeling perspective, the primary goal of this study is to focus on the teachers' pedagogical content knowledge evolve and the teachers' pedagogical knowledge evolve through professional development activities based on lesson study cycle from the modeling perspective. The research questions addressed in the study are as follows:

i) How does the teachers' knowledge evolve on questioning through professional development activities based on lesson study cycle from the modeling perspective?

ii) How does the teachers' knowledge evolve on generating assessment criteria for assessing students' competency in modeling through professional development activities based on lesson study cycle from the modeling perspective?

iii) How does the teachers' knowledge evolve on classroom organization and management domain of general pedagogical knowledge through professional development activities based on lesson study cycle from the modeling perspective?

1.2 Significance of the Study

Although an increasing number of research studies in mathematics education have begun focusing their efforts on mathematical modeling as a need for change in order to convey mathematical ideas beyond schools, there is not enough information about the nature of teachers' knowledge to use modeling in mathematics teaching effectively and how this knowledge evolves (Doerr, 2007; Doerr & Lesh, 2011; Lingefjärd, 2007). The question in the literature, which is, how does teachers' knowledge evolve for proper implementation of modeling tasks? In other words, how do teachers' models develop? focuses the attention not on the static picture of what teachers know, but on the dynamics of how teachers' models or systems of interpretation develop over time (Doerr & Lesh, 2011; García & Ruiz-Higueras, 2011). These questions are automatically connected to a new fundamental questions: How should teachers be trained/supported in order to be able to implement modelling tasks effectively in their classes? And what might be the components of these professional development programs? (García & Ruiz-Higueras, 2011). The professional development programs, which take place in the teachers' naturalistic settings, "on-the-job", are offered in order to investigate the teachers' evolving knowledge by many researchers (Burkhardt, 2006; Doerr, 2007; Lingefjärd, 2007; Muller & Burkhardt, 2007; Niss, Blum, & Galbraith, 2007). Additionaly, Shulman (1987) and Doerr and Lesh (2011) emphasize the centrality of feedback and knowledge revision by constructing a cyclic model of pedagogical reasoning and action, beginning with an instance of comprehension and completing the circle with a process of reflection followed by the formation of new comprehension in the professional development programs and also added studies which combined the extensive mapping of knowledge with a close analysis of changes in that knowledge over time could yield a much clearer understanding of how the knowledge is formed and refined. The teachers' pedagogical content knowledge needed for modeling that is proposed in the LEMA project emphasize the questioning and assessment domains (Maaß & Gurlitt, 2011). Additionally, classroom organization and management domain of general pedagogical knowledge are more concerned by teachers as an

innovative pedagogical approaches and being asked for direct help by teachers in order to use modeling in their classes (García & Ruiz-Higueras, 2011; Wake, 2011).

This study tries to respond to the needs in the literature by investigating teachers' pedagogical content knowledge evolve under the questioning and assessment domain and pedagogical knowledge evolve under the classroom organization and management domain which are emphasized in order to teach via modeling tasks. Besides, teachers' evolving knowledge in order to use modeling in mathematics teaching effectively is investigated through an "on-the-job" teacher development program based on its cyclic model. In addition, with its design and the suggestions, the study proposes components of the "on-the-job" professional development programs, which are designed in teachers' naturalistics settings.

1.3 Theoretical Framework

The purpose of this study is to investigate how teachers' pedagogical content knowledge and pedagogical knowledge evolve when they engage in professional development activities based on lesson study cycle from the modeling perspective. With the scope of this aim, teachers' evolving pedagogical content knowledge was investigated under the theoretical framework suggested by Marks (1990a, 1990b) about pedagogical content knowledge. Marks (1990a, 1990b) suggests four component of pedagogical content knowledge in the context of mathematics education. Marks (1990a, 1990b) contributed curricular knowledge of Shulman's framework to PCK and offer three components: "subject matter for instructional purposes", "media for instruction" and "instructional processes" based on this contribution. One of the components of PCK which is "subject matter for instructional purposes" consist of "purposes of math instruction, justifications for learning a given topic, important ideas to teach a given topic, prerequisite ideas for a given topic, and typical school math problems" (Marks, 1990a, p.5). Other component of PCK that is offered by Marks (1990a) is "students' understanding", which is explained as "knowing students' learning process, typical understanding, common errors, things that are hard/easy for students and particular students' understanding" (Marks, 1990a, p.5). Another component of PCK that is offered by Marks (1990a) is "media for instruction"; this component is explained as "large number of references to instructional media for teaching mathematics, i.e., textbooks

and materials, usually manipulatives" (p. 100). The four components of pedagogical content knowledge and these components' subcategories are given in figure 1.1. (Marks, 1990a, p.5).

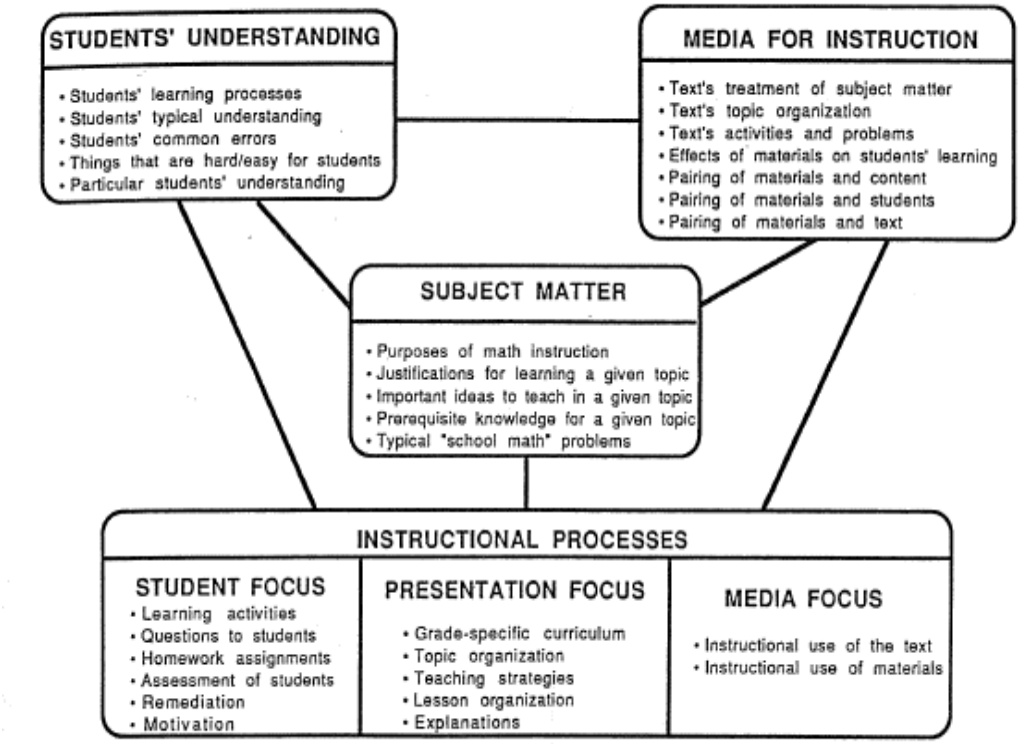


Figure 1.1 Structure of Pedagogical Content Knowledge (Marks, 1990a, p.5).

In this study, one component of Marks' (1990a, 1990b) PCK and this component's subcategories are used for investigating the teachers' evolving PCK. This component was chosen based on the PCK model needed for modeling that is proposed in the LEMA project (Maaß & Gurlitt, 2011). This model includes four main categories: modeling, tasks, lessons and assessment, and their subcategories. In task category, it is explained that teachers need to learn, how to select appropriate task for their students. In lessons category, it is explained that teachers need information about how to design lessons appropriate for modeling, how to act in the classroom, how to ask effective questions and how to reflect students' emerging thinking. In assessment category, it was emphasized "If modelling is implemented in lessons, it also has to be evaluated" (Maaß & Gurlitt, 2011, p. 632). Thus, in this study, knowledge of instructional processes-student focus component of Marks' (1990a, 1990b) conceptualization of PCK were chosen to investigate PCK evolvement because its

subcategories include the lesson and assessment categories of the PCK model needed for modeling as proposed in the LEMA project (Maaß & Gurlitt, 2011). Pedagogical content knowledge component and its subcategories utilized in investigating teachers' evolving pedagogical content knowledge are described in detail below.

Knowledge of Instructional Processes- Student Focus

"Instructional processes which emphasize the role of the student: how he or she shapes, participates in, or responds to the instruction" (Marks, 1990b, p.112).

Questions to students

Marks (1990b) stated that questions could be used for purposes of instruction or feedback. Teachers give many examples of questions they might ask to stimulate students' thinking and learning. These questions could be divided roughly into two categories, which are directive and broadening. "Directive" questions are designed to lead students along a predetermined path, by leading step by step to the desired end. Besides, "broadening" questions are aimed to move students' thinking forward by provoking students to expand their ideas (Marks, 1990b, p.116). Questioning in order to give feedback on students' difficulties and errors "can occur at any of several different levels: factual, procedural, conceptual, exploratory, evaluative and invitational" (Marks, 1990b, p.119). In this study, under this subcategory, each teacher evolving knowledge on questioning while students are working on the modeling task and questioning in order to give feedbacks on students' difficulties and errors are examined.

Assessment of students

Marks (1990b) stated that teacher knowledge about assessment requires knowledge of what and how to assess students' efforts with respect to intended goals and assessment methods. Teachers should generate effective assessment criteria based on the learning environment and the intended goals (Marks, 1990b). Teachers should understand what their students did, why they did so the teacher can generate criteria for their students' effort. In this study, under this subcategory, each teacher evolving knowledge on generating assessment criteria for assessing students' competency in modeling while students are working on the modeling task and presenting their solutions are examined.

On the other hand, teachers' evolving pedagogical knowledge was investigated under the classroom organization and management domain of general pedagogical knowledge and other domains are identified as instructional models and strategies and classroom communication and discourse (Morine-Dershimer & Kent, 1999). The varieties of sources that contribute to that domain (i.e., classroom organization and management) of the pedagogical knowledge are consistent in noting general principles of teacher behavior that promote student achievement. Student achievement descriptors are given such as "students learn more when teachers use time efficiently, implement group and instructional strategies with high levels of involvement, communicate rules and expectations clearly, and prevent problems by introducing a management system at the beginning and implement it consistently" (Morine-Dershimer & Kent, 1999, p. 25). The classroom organization and management domain of general pedagogical knowledge were chosen in order to investigate teachers' evolving pedagogical knowledge since its subcategories like class setting, introducing, warming, organizing and ending are more concerned by teachers and being asked for direct help by teachers in order to use modeling in their classes (García & Ruiz-Higueras, 2011; Wake, 2011).

The components of pedagogical content knowledge and the general pedagogical knowledge and these components' subcategories which are used in this study for investigating the teachers' evolving knowledge are given in Table 1.1.

Table 1.1 The Components of Pedagogical Content Knowledge and The General Pedagogical Knowledge Used in This Study

KNOWLEDGE OF INSTRUCTIONAL PROCESSES STUDENT FOCUS	CONSISTING OF
• Questions to students	Teachers' questions while students are working on the modeling task and questions in order to give feedbacks on students' difficulties and errors
• Assessment of students	Teachers' generating assessment criteria for assessing students' competency while students are working on the modeling task and presenting their solutions.
KNOWLEDGE OF PEDAGOGY	
• Classroom organization and management	• *Class setting: the criteria for determining the group's structure and the number of the students in one group* • *Introducing the implementation* • *providing an understanding of the MEA and to warm up the MEA* • *organizing the presentations of the solutions (groups' order, groups' presentation process)* • *ending the implementation*

1.4 Definition of Terms

Some important terms associated with this study are as the following.

Lesson Study

Lesson Study is a professional development approach. Lesson studies which has different levels and different types has three basic parts; choosing a particular topic and designing lessons on that topic, teaching the lesson to students "research lesson", discussing the lessons with the observations and the data from the conducted class and revising the lesson plan (Shimizu 2002; Yoshida 2002).

Pedagocical Content Knowledge

Shulman's (1986) description of pedagogical content knowledge (PCK) is used in this study and stated as "the particular form of content knowledge that embodies the aspect of content most germane to its teachability" (p.9).

Pedagogical Knowledge

Shulman's (1987) description of pedagogical knowledge (PK) is used in this study and stated as " general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter" (p. 8).

Modeling

Modeling definition which is a process of defining the phenomenon and the relations in it with the mathematical expressions and bringing out the mathematical patterns in this phenomenon is used in this study (Verschaffel, Greer, & De Corte, 2002).

Model-Eliciting Activities

The tools which are designing in order to promote students' and teachers' externalization of their thinking and externalization of their conceptualization steps for the problem situations are called as model eliciting activities by researchers (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Lesh & English, 2005; Lesh & Sriraman, 2005). Model eliciting activities' main characteristics are giving chance to learners in order to develop a model for a real-life situation, to describe, revise, and refine their ideas and to explain their conceptual systems by this model (Lesh& Doerr, 2003a).

CHAPTER 2

LITERATURE REVIEW

The design of this study consisted of selecting a model eliciting activity (MEA), working out to develop shared lesson plans during the meetings before the classroom implementation of the MEA, implementation of the MEA and revising lesson plans during meetings after the implementation. This sequence repeated in each month with a different MEA. Teachers were designing lesson plans for model eliciting activities. These activities were enacted in classes, and then the lesson plans were revised with feedbacks from the implementations and the data on students' thinking. Lesson study design as a professional development approach are attended in the literature review since this study was based on the lesson study cycle using modeling perspective of teacher development. In addition, revisions of the lesson plans are mostly based on students' thinking; it takes part in the review. Moreover, since the research questions are about teachers' evolving knowledge with the design that is compatible with the modeling perspective, the two other areas focused in the literature review are teacher knowledge and modeling.

2.1 Professional Development Approaches

Many studies emphasize that academic improvements and the increase in student success depend on the professional development of teachers and administrators (Ball & Cohen, 1999; Elmore & Burney, 1999; Nelson & Hammerman, 1996; Sykes, 1999). Sykes (1999) states the acts of professional development as a "significant lever for education improvement" (p. 151). Sowder (2007) categorized different types of professional development based on the teacher knowledge suggested by Cochran- Smith and Lytle (1999); namely, *knowledge-for-practice, knowledge-in-practice* and *knowledge-of-practice*. Cochran-Smith and Lytle (1999) defined *knowledge-for-practice as "* knowledge acquired by learning from formal professional development programs and university coursework"; defined

knowledge-of-practice as " teachers learn when they generate local knowledge of practice by working within the contexts of inquiry communities to theorize and construct their work and to connect it to larger social, cultural and political issues*"* and defined *knowledge-in-practice* as "it is embedded in practice and in teachers' reflections on practice" (p. 250). Professional communities, professional development schools and lesson study are settings that provide knowledge-in-practice (Sowder, 2007). Professional communities are mostly informal settings and teachers who are teaching the same grade level come together mostly outside their school district and talk about their teaching strategies, problematic issues that come from the classes and develop instruction plans based on the state standards (Hargreaves, 1995; Little, 1993). In addition, according to Grossman, Wineburg and Woolworth (2001) "a key rationale for teacher community is that it provides an ongoing venue for teacher learning" (p. 947). Other setting for learning from practice is professional development schools. Professional development schools are described as "ongoing invention and discovery; places where school and university faculty together carry on the applied study and demonstration of the good practice and policy the profession needs to improve learning for young students and prospective educators" (Lanier, 1994; p. ix). Professional development schools create an environment for teachers, students, university faculty and school faculty in order to participate in research by creating new knowledge, evaluating and revising the practices (Darling- Hammond, 1994; NRC, 2001). Another setting for learning from practice is "Lesson Study" and this professional development approach is discussed in detail in the next section since this study cycle thoughtfully adapted from the lesson study cycle.

2.1.1 Lesson Study

Teaching is considered as a *craft* and teachers obtain this knowledge in practice through "consciously reflecting on the flow of classroom action and invention of knowledge in action in order to take note of new situations, intentionally and introspectively examining those situations, and consciously enhancing and articulating what is tacit or implicit"(Cochran-Smith & Lytle, 1999, p.268). This kind of learning is the "stuff" of communities of practice, of lesson study. Japanese lesson study, a professional development approach where teachers learn about teaching by developing, examining and revising collectively, has became the focus of various

studies in recent years (e.g., see Brown, McGraw, Koc, Lynch, & Arbaugh, 2002; Chokshi & Fernandez, 2004; Fernandez, 2002; Fernandez, Canon, & Chokshi, 2003; Fernandez & Yoshida, 2004; Lewis & Tsuchida, 1997, 1998; National Research Council, 2002; Perry & Lewis, 2004; Richardson, 2004; Stepanek, 2001, 2003; Wang-Iverson & Yoshida, 2005; Watanabe, 2002; Wilms, 2003; Yoshida,1999). Lesson studies which has different levels and different types has three basic parts; choosing a particular topic and designing lessons on that topic, teaching the lesson to students "research lesson", discussing the lessons with the observations and the data from the conducted class and revising the lesson plan (Shimizu 2002; Yoshida 2002). The analysis of student work for the focus of the student thinking and the analysis of actual classroom instruction has the central role in the lesson study (Lewis, 2002b; Perry & Lewis, 2009). Hiebert and Stigler (2000) have also defined lesson study as

> The knowledge being shared through lesson study is not just collection of lesson plans that teachers can pull off the shelves and use. The goal of lesson study is not just to produce lessons that can be copied but to produce knowledge about teaching upon which colleagues can build (see Ball & Cohen, 1996). Such a knowledge base grows as a teacher reflects on and improves what others have done, working to understand the basis for the improvements (p.12).

Lesson study simply creates an environment for teachers in order to think the lessons collectively (Fernandez, 2005). What teachers learn in the lesson study, very much depend on what they bring to the table, thus actual tasks that they use to structure and organize significantly affect what teachers are apt to learn (Fernandez, 2005). Certain tasks are seen very fruitful for teachers to carry out as they try to plan, "for example, there is growing evidence that teachers can greatly benefit from doing mathematics problems that they plan to ask children to tackle and discuss. Uncovering how children tend to think about particular mathematics can be very useful in preparing teachers to teach reform-minded lessons" (Fernandez, 2005, p. 284). In lesson studies, the discussion usually conducted by the lesson teacher or the knowledgeable others (e.g. researchers, facilitators), begins with comments. The process is cyclic and the each cycle serves as preparation for the next. In the lesson studies, teachers are planning with the teams of 4 to 6 teachers, in the discussions, teachers take roles like recording and moderator or give the moderator role to the knowledgeable others. Some techniques that are used in order to make the meetings run smoothly and productively in the lesson studies are; deferring to the teachers who

will teach the lesson in cases where contentious debates arise and focusing during the post- lesson discussion on key issues targeted in the lesson with the knowledgeable others who take part in the observations and discussions (Fernandez, 2005; Fernandez & Yoshida, 2004; Lewis, 2002b; Perry, R., & Lewis, C., 2009)

Lewis, Perry, and Murata (2006) states the instructional improvements of lesson studies with two conjectures (see Figure 2.1).

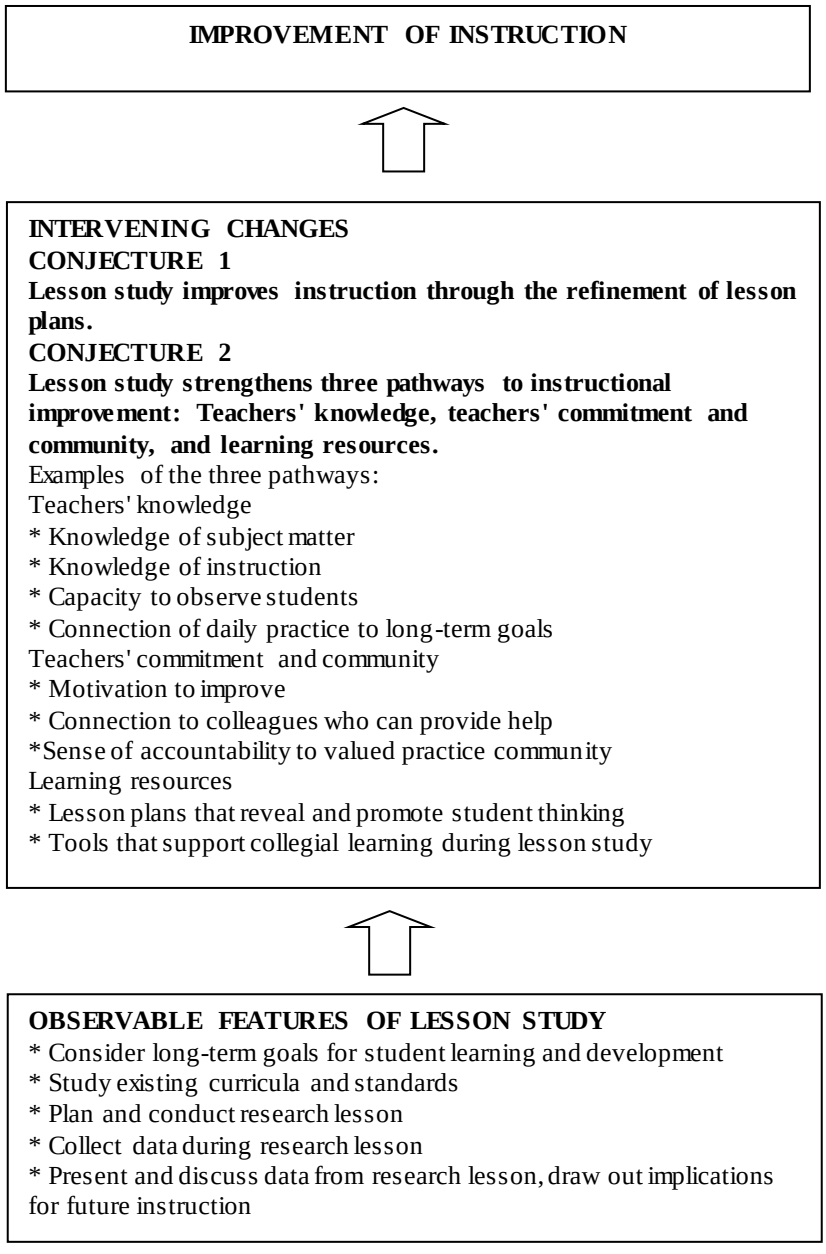


Figure 2.1 Instructional Improvements of Lesson Study: Two Conjectures (Lewis, Perry, & Murata, 2006, p.5)

The opportunities that are provided to teachers by the lesson studies are to develop pedagogical content knowledge and to learn how to reason mathematically (Fernandez, 2005). Lesson study supplies fertile ground for teacher in order to develop pedagogical content knowledge by the fact that teachers talk about topics like what problems to use in the lesson, what questions to ask students for exposing their thinking (Fernandez, 2005; Lewis 2002b). In addition, this experience gives them a chance to think collectively and in detail about the students' thinking, extend the learning and the overcome the difficulties (Cochran-Smith & Lytle, 1999; Fernandez, 2005; Lewis 2002b). Moreover, lesson study provides an opportunity to discuss the unexpected events in the research lesson and is not in their extensive planning so that teachers talk about these events in order to handle these situations and develop their knowledge about how to reason mathematically (Fernandez, 2005; Lewis 2002b). Lesson studies not only provide a meeting place for teachers but also discuss the content critically and uncover the students' thinking on a particular mathematics (Richardson, 2004). Lessons are ordinary topics for teachers to talk about daily so coming together in order to work on lessons are natural and easily sustainable units for teachers (Hiebert, Gallimore, & Stigler, 2002). Furthermore, teachers discuss the data from actual classroom after the extensive planning and it provides temporal reflection in lesson studies (Fernandez, 2005). Lesson study's contributions to instruction can be listed as (a) provide professional development (b) help teachers to gain new approaches (c) spread knowledge of content (d) connect the practices to broader goals (e) chance to discuss new visions and new pedagogical strategies (f) chance to discuss competing visions (g) create most effective role of teachers in the development of instruction (h) create a fertile ground in order to discuss for effective instruction (i) help teachers to understand students' thinking (Brown, McGraw, Koc, Lynch, & Arbaugh, 2002; Cochran-Smith, & Lytle, 1999; Fernandez, 2002; Fernandez, Cannon, & Chokshi, 2003; Lewis, 2002b; Lewis, Perry, & Murata, 2006; Richardson, 2004; Shimizu, 2002; Wang-Iverson, & Yoshida, 2005; Watanabe, 2002; Yoshida, 1999). The last item in the list but the most important contribution is to help teachers to understand students' thinking that "develop the eyes to see students" (Lewis, 2002a). In the research lessons, teachers carefully observe students' behaviors, learning and engagement and teachers have a chance to think more deeply about students' thinking than daily classroom environment and after the research lesson, the analysis of student work for the focus of the student

thinking are provided with lesson study (Lewis, 2002a). Lewis (2002a) states that "teachers see instruction through the eyes of the students" in lesson study (p.21). The importance of teachers attending to students' thinking is discussed in the next section.

2.2 Students' Thinking

In the *Principles and Standards for School Mathematics,* National Council of Teachers of Mathematics (NCTM) emphasize that "effective teaching involves observing students, listening carefully to their ideas and explanations, having mathematical goals and using the information to make instructional decisions" (NCTM, 2000, p. 19) . In addition, it was stated "knowledge of students' understandings and ways of thinking helps teachers to construct worthwhile mathematical tasks" (NCTM, 2000, p.13). "Teachers can create a learning environment that fosters the development of each student's mathematical power by respecting and valuing students' ideas, ways of thinking and mathematical dispositions" (NCTM, 1991, p. 57). Teachers can analyze the students' work on their own however analyzing with the leader who has a good pedagogical content knowledge can lead a deeper understanding of students' thinking and reasoning (Sowder, 2007). Moreover, Little (2004) indicate that student written work can be used as a source in order to strength teacher knowledge, as a basis for developing instruction and as a base for evidence of student learning. Furthermore, several strategies are offered teachers in order to understand their students' thinking and use it in their practice

> (a) pose questions that go beyond asking students to describe their solution strategies to asking them to think more deeply about the mathematics underlying those strategies; (b) understand students' mathematical thinking that differs from what might be expected based on the research-based information on students' thinking; (c) critically examine that thinking to determine whether it is mathematically valid; and (d) use what you learn about your students' thinking to create tasks that enable students to extend their understanding (Warfield, 2001, p. 137).

The students' thinking are used as a tool for developing teacher knowledge in several research projects; Cognitively Guided Instruction (Carpenter, Fennema, & Franke, 1996; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996; Franke, Carpenter, Fennema, Ansell, & Behrend, 1998), Teaching to the Big Ideas (Schifter 1998; Schifter, Russell, & Bastable, 1999), the Purdue Problem-Centered

Mathematics Project (Cobb, Wood, & Yackel, 1990; Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991), Summer Math (Simon & Schifter, 1991), Integrating Mathematics Assessment (Gearhart & Saxe, 2004; Saxe, Gearhart, & Nasir, 2001), the Kenilworth Project (Maher, Davis, & Alston, 1991; 1992; Maher & Martino, 1992), the Mathematics Case Methods Project (Barnett, 1998), the work of Gordon and MacInnis (1993), and the work of Putnam and Reineke (1993). Some of the projects were described in detail.

In a longitudinal study called *Cognitively Guided Instruction* (CGI), changes in 21 primary teachers' instruction and belief were examined (Fennema et al., 1996). In the study, teachers examined the students' thinking in order to understand the development of their students' mathematical thinking (Carpenter, Fennema, & Franke, 1996). The workshops and the classroom supports formed the teacher development program. In the workshop, videos on the solution strategies of the students for the word problems were shared and discussed by teachers. Then, teachers were encouraged in order to use the problems in their classes and take notes about their students' solution strategies and bring these notes to the next workshop. In the next workshop the notes that come from the classes and the solution strategies viewed on the videos compared and discussed. In addition teachers were encouraged to use their new knowledge of students' thinking in their instructional plans. Data were collected through observations of teachers' instruction, interviews, paper-pencil instruments and informal interactions. In addition, tests were used for students' mathematical learning. The results showed that, instructional decisions of teachers were affected by their students' thinking, the belief change from direct instruction to instruction based on students' thinking. The changes in the classroom practices from procedural skills to conceptual understanding were not decrease students' computational skills. It was indicated that "this study provides strong evidence that knowledge of children's' thinking is a powerful tool that enables teachers to transform this knowledge and use it to change instruction. These findings, when viewed in conjunction with those of other studies, provides a convincing argument that one major way to improve mathematics instruction and learning is to help teachers understand the mathematical thought processes of their students" (Fennema, et al., 1996, p.432). In addition, Franke et al.(1998) concluded that " professional development focused on children's mathematical thinking provides a basis for

teachers to engage in ongoing practical inquiry directed at understanding their own students' thinking and thus, provides a basis for teachers to engage in self-sustaining, generative growth" (p.79).

In the Teaching to the Big Ideas project, the elementary school teachers participated as a learner of mathematics especially the elementary topics which are challenging for the adults learners were addressed in the seminar, teachers also use these topics in their classes in order to see their students' thinking and teachers analyzed other teachers' students, studying videotapes on students' thinking, classroom discourses and the written materials on students' work. Teachers stated that in their practices which students' thinking took the central role, practices "become much harder to manage and much less predictable" (Schifter, 1998, p.56). Teachers also indicate that they saw the mathematics from the different perspective that aroused by the students' thinking (Schifter, Russell, & Bastable, 1999).

In *the Purdue Problem-Centered Mathematics Project*, the aim was to coordinate constructivist mathematics learning with the practice of teaching in order to look cognitive and social factors of students' mathematical learning. With this aim, a classroom teaching experiment was conducted in a second grade classroom for one school year (Cobb, Wood, & Yackel, 1990). Problem-centered mathematical activities were provided to teacher in order to implement in the class since these activities allow multiple solutions, sustain small group and class discussions and give an opportunity to see students' way of thinking. Through the project, teacher began to listen her students' thinking with the help of the activities. Many changes occurred in teacher's practice; she realized that students have more sophisticated solution strategies than she taught; she recognized that social context created by not only her authority, it was created by her and students so she used her authority in order to guide mathematical communication by listening, giving suggestions, probing students' mathematical thinking, fostering communication on math among students. To sum up, it was explained that "In the course of listening to their solutions the teacher modified her beliefs about mathematics and extended her understanding of children's learning of mathematics. By drawing on this knowledge, the teacher could better facilitate the children's construction of mathematical knowledge. In doing so, she created further opportunities to listen to creative solutions and thus further elaborated her understanding of second grade mathematics" (Cobb, Wood, & Yackel,

1990, p. 139). After the success of this teacher, similar teaching experiment was conducted with ten second grade teacher (Cobb, et al., 1991). The in-service program included one week summer workshop, in the workshop teachers analyzed videos on students' thinking, and solved activities in group which they would use in their class during the year and discussions were held about social norms and productive learning environments. At the end of the workshop, the activities and the implementation notes for use in their classes were given to teachers. During the year, four two-hour working sessions were held in order to add other activities and discussed on class instructions. At the end of the school year, performances of ten project classes were compared with the eight non project classes, it was seen that computational performances were same for both classes however, the project students had higher levels of conceptual understanding and held stronger beliefs about the importance of collaboration and understanding. Moreover, teachers' beliefs were more compatible with the constructivist perspective and teachers probed their students' thinking with the help of the activities so that their practices were changed based on the developing in students' learning (Cobb, et al., 1991).

Summer Math project are comprised by four stage intervention program based on developing teachers' constructivist view of learning in order to use it as a base for their instruction (Simon & Schifter, 1991). In the first stage of the program, teachers found a chance to learn mathematics in a constructivist environment and to compare their learning and students' learning. In the second stage, teachers' class instructions were observed and meeting with the teachers after the class sessions in order to talk about their instruction took place. In the third stage, teachers had a chance to analyze students' learning through the videotaped interview and in the last sections teachers groups with two or four and with the project staff planed the four afternoon workshops for their colleagues. Data sources were teachers' writing and the interviews with the teachers. Many changes in teachers' practices were recorded like listening more to students and designing instruction more on students' understandings and ideas (Simon & Schifter, 1991).

Another project designing on students' thinking is Integrating Mathematics Assessment (IMA) project (Gearhart & Saxe, 2004; Saxe, Gearhart, & Nasir, 2001) that aimed to help elementary school teachers to understand their students' mathematical thinking and use that knowledge in their instruction in order to guide

students to conceptual understanding of mathematics. The Professional development program had a cyclic design. During each cycle, teachers worked on curriculum-replacement unit that had been designed for California teachers for use in textbook pages on fractions, as a learner then try to understand students' mathematical thinking as a researcher and then implementation of lessons as an educator. In the implementation phase several ways were practiced in order to assess students' understanding. The results showed that teachers who participated in the professional development program focused on students' understanding of mathematics but the teachers who had not participated in the professional development program, not focused on students' understanding of mathematics, both teachers use the same curriculum-replacement unit in their classes. Moreover, IMA students had better conceptual understanding and more procedural proficiency than the students whose teachers had not participated in the professional development but use the same curriculum unit. Furthermore, researcher concluded that replacing curriculum is not sufficient for improving students' learning, teachers needed to see the activities and plan the instruction from the students' perspective.

In the Mathematics Case Methods Project, teachers engage in discussion on narratives about problematic outcomes of the classrooms and these narratives were supplemented with the copies of students' work (Barnett, 1998). In the discussions, first teachers work on the mathematics problems in the narratives individually then work in groups to identify the issues based on what might be confusing and difficult for the students and create questions based on these issues and then these questions were discussed by all the teachers. In the first discussions it was observed that teachers mostly talked about the pedagogy in the narratives based on their beliefs and external standards as seen as a good teaching. However, through the discussions it was observed that teachers began to focus more on mathematics and the students' thinking, they talked about pedagogical techniques in terms of their affect on students' learning (Barnett, 1998).

Summary of Benefits of Students' Thinking as an Approach to Teacher Learning

The results of the projects emphasize the importance of teacher attending to their students' thinking. The benefits of the teachers attending to students' thinking can be given as changing instruction from teacher centered to student centered, changing

understandings of students from operational to conceptual, teachers selecting appropriate mathematical tasks and changing beliefs of teachers and students to more positive toward mathematics. Teachers who are trained through the students' thinking approach, has integrated their knowledge in their teaching plans and contributed to the success of the students (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Franke, Carpenter, & Carey, 1993; Gearhart & Saxe, 2004; Schifter, Russell, & Bastable, 1999). In addition, Carpenter, Fennema and Franke (1996) indicated that teachers understanding of their students' thinking help to develop their pedagogical content knowledge, pedagogical knowledge and content knowledge. These categories of teachers' knowledge are discussed in the next section.

2.3 Teacher Knowledge

Three major categories of teacher knowledge are identified as subject matter, pedagogy, and general knowledge together with basic skills (Lanier & Little, 1986; Shulman, 1987). The relationship between knowledge of content and of pedagogy has a tradition extending back to ancient Athens. Aristotle asserted that the ability to teach a subject represented the highest degree of knowledge of it, so that pedagogical skill was not an object in itself but was in fact a measure of subject matter knowledge (Marks, 1990b). This tradition, with minor variation, continued through the practices of the medieval universities and on into the licensing standards for teachers in late 19th century (Shulman, 1986). While teacher education shows a broad variance, one of its few consistencies is the separation of content preparation and professional teaching studies (Lanier & Little, 1986). At the state level, standards for initial licensing typically include a test of basic content skills and ongoing assessment usually emphasizes generic teaching skills, like classroom management and cultural awareness, but the content and pedagogical skills are generally kept in isolation from one another (Shulman, 1986). The concept of pedagogical knowledge has been given shortly in most discussions of Shulman's (1987) model of teacher knowledge. Shulman (1987) described the category only as " general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter" (p. 8). The complex nature and the sources of teachers' pedagogical knowledge was investigated under the three major areas that contributing to general pedagogical knowledge;

namely, classroom organization and management, instructional models and strategies, and classroom communication and discourse by Morine-Dershimer & Kent (1999). The varieties of sources that contribute to classroom organization and management domain of the pedagogical knowledge are consistent in noting general principles of teacher behavior that promote student achievement (Brewer, Dunn, & Olszewski, 1988; Brophy & Good, 1986; Emmer, Evertson, & Anderson, 1980; Evertson, Emmer, Sanford, & Clements, 1983; Evertson & Harris , 1992). Student achievement descriptors are given such as "students learn more when teachers use time efficiently, implement group and instructional strategies with high levels of involvement, communicate rules and expectations clearly, and prevent problems by introducing a management system at the beginning and implement it consistently" (Morine-Dershimer & Kent, 1999, p. 25).

Furthermore, Shulman (1986) proposed three categories of content knowledge as subject matter knowledge, curricular knowledge and pedagogical content knowledge. Subject matter knowledge is described as knowledge about the subject, for example mathematics, and its structure (Shulman, 1986). Curricular knowledge consists of the materials used in teaching and extending and ordering the subject (Shulman, 1986). Shulman(1986) state that " the curriculum and its associated materials are the *materia medica* of pedagogy, the pharmacopeia from which the teachers draws those tools of teaching that present or exemplify particular content and remediate or evaluate the adequacy of student accomplishments." (p. 10). Lastly the pedagogical content knowledge (PCK) is described as "the particular form of content knowledge that embodies the aspect of content most germane to its teachability" (Shulman, 1986, p.9). According to Shulman (1986), pedagogical content knowledge is defined as follows.

> the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others…an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies must likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank states (p. 9-10).

In addition Shulman (1987) indicate that "the knowledge base for teaching is not fixed and final" and the knowledge base can be "discovered, invented, and refined" (p.12). From this point of view, researchers have extended and modified Shulman's framework especially through case studies of teachers working in different subject areas (An, Kulm, & Wu, 2004; Ball & Bass, 2003; Cochran, DeRuiter, & King, 1993; Even & Tirosh, 1995; Grossman,P., 1990; Fennema & Franke, 1992; Marks, 1990; Mc Ewan and Bull, 1991; McNamara, 1990; Meredith,1993;1995). Grossman (1990) integrated Shulmans' framework of curricular knowledge into pedagogical content knowledge and offer four component of pedagogical content knowledge. These four components are "knowledge of students' understanding", "knowledge of curriculum", "knowledge of instructional strategies" and "knowledge of purposes for teaching" (Grossman, 1990). Moreover, Marks (1990a, 1990b) suggested four component of pedagogical content knowledge by modifying that knowledge into mathematics education. Marks (1990a, 1990b) combined curricular knowledge of Shulman's framework to PCK and offer three components: "subject matter for instructional purposes", "media for instruction" and "instructional purposes" based on this contribution. One of the components of PCK which is "subject matter for instructional purposes" consist of "purposes of math instruction, justifications for learning a given topic, important ideas to teach a given topic, prerequisite ideas for a given topic, and typical school math problems" (Marks, 1990a, p.5). Other component of PCK that is offered by Marks (1990a) is "students' understanding", this component is explained as knowing students' learning process, common difficulties, misconceptions, and easy and difficult parts of the concepts based on the students' understanding. Furthermore, this type of knowledge "students' understanding" is raised to an important part in the Fennema and Franke's (1992) chapter in the *Handbook of Mathematics Teaching and Learning*. It is stated that teachers' decision making process and practices are influenced by the teachers' knowledge of students' understanding so thus the outcomes of the instruction is affected by this knowledge (Fennema & Franke, 1992). The specific components of "knowledge of students" are identified as "knowing that" and "knowing why" (Even & Tirosh, 1995). "Knowing that refers to research based and experienced- based knowledge about students' common conceptions and ways of thinking about the subject matter. Knowing why refers to general knowledge about possible sources of

these conceptions, and also to the understanding of the sources of the specific students' reaction in a specific case" (Graeber & Tirosh, 2008, p.121).

Ball and Bass (2003) also offer the framework of teacher knowledge from the analyses of teacher work. In the analyses they focused on the teachers' representing strategies of the mathematical concepts to students, teachers' assessing and interpreting process of the students' work and teachers' planning and managing strategies of the class (Ball & Bass, 2003). This framework has two major portions which are subject matter knowledge and pedagogical content knowledge (Ball & Bass, 2003). This framework's subject matter knowledge consists of "common content knowledge" and "specialized content knowledge" (Ball & Bass, 2003). Common content knowledge is explained as knowledge that a person who has a good mathematical background can know, "for example, what decimal is half between 1.1 and 1.11" (Ball & Bass, 2003; Graeber & Tirosh, 2008, p.122). Specialized content knowledge is described as "mathematical knowledge that is used in teaching, but not directly taught to students" (Hill, Sleep, Lewis, & Ball, 2007, p. 132). Hill, Sleep, Lewis and Ball (2007) provide an example item of a situation designed for assessing teachers' specialized content knowledge (see Figure 2.2). They state that analysis of the student work and knowledge of whole number properties are needed to conclude that all of the three of the students have employed a method valid for any two whole numbers and this knowledge can be known by a few people other than teachers because it includes both the students' work analysis and the mathematical properties.

Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x25 125 +75 875	35 x25 175 +700 875	35 x25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

Figure 2.2 An Item Designed to Measure Teachers' Specialized Content Knowledge (Hill, Sleep, Lewis, & Ball, 2007, p. 132)

In addition, in Ball and Bass' (2003) framework major portions of pedagogical content knowledge includes three categories: "knowledge of content and students", "knowledge of content and teaching" and "knowledge of curriculum". Hill et al. (2007) give a sample item (see Figure 2.3) designed for evaluating the teachers' knowledge of content and students and this example consist the knowledge that students will likely focused on the amount that is missing from each whole rather than finding common denominators (Hill, Sleep, Lewis, & Ball, 2007).

Takeem's teacher asks him to make a drawing to compare $\frac{3}{4}$ and $\frac{5}{6}$. He draws the following:

and claims that $\frac{3}{4}$ and $\frac{5}{6}$ are the same amount.

What is the most likely explanation for Takeem's answer? (Mark ONE answer.)

a) Takeem is noticing that each figure leaves one square unshaded.

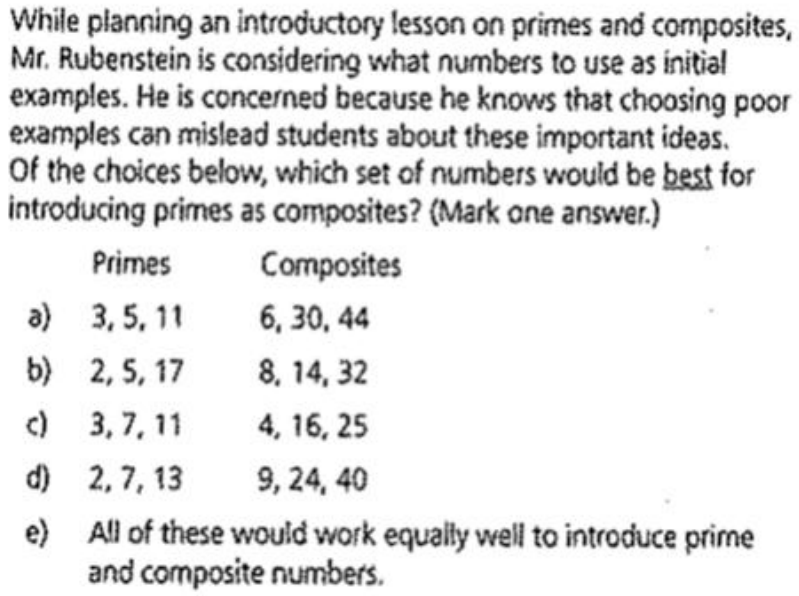
b) Takeem has not yet learned the procedure for finding common denominators.

c) Takeem is adding 2 to both the numerator and denominator of $\frac{3}{4}$, and he sees that that equals $\frac{5}{6}$.

d) All of the above are equally likely.

Figure 2.3 An Item Designed to Measure Teachers' Knowledge of Content and Students (Hill, Sleep, Lewis, & Ball, 2007, p. 132)

Furthermore, Hill et al.(2007) give a sample item (see figure 2.4) designed for evaluating the teachers' knowledge of content and teaching and this example consist the knowledge that "students have difficulty on categorizing 2 as a prime and likely to think that odd numbers cannot be composite" (Hill, Sleep, Lewis, & Ball, 2007, p. 132).



While planning an introductory lesson on primes and composites, Mr. Rubenstein is considering what numbers to use as initial examples. He is concerned because he knows that choosing poor examples can mislead students about these important ideas. Of the choices below, which set of numbers would be best for introducing primes as composites? (Mark one answer.)

	Primes	Composites
a)	3, 5, 11	6, 30, 44
b)	2, 5, 17	8, 14, 32
c)	3, 7, 11	4, 16, 25
d)	2, 7, 13	9, 24, 40
e)	All of these would work equally well to introduce prime and composite numbers.	

Figure 2.4 An Item Designed to Measure Teachers' Knowledge of Content and Teaching (Hill, Sleep, Lewis, & Ball, 2007, p. 132)

Lastly, Ball and Bass' (2003) framework major portion of pedagogical content knowledge's category "knowledge of curriculum" is described the same way with the Shulman's (1986) framework of curricular knowledge. Furthermore, this study tries to respond to the needs in the literature based on teachers' evolving knowledge in order to use modeling in mathematics teaching effectively so the modeling is discussed in the next section.

2.4 Models and Modeling

An increasing number of research studies in mathematics education have begun focusing their efforts on mathematical modeling as a need for change in order to convey mathematical idea beyond schools (Burkhardt & Pollak, 2006; Kaiser, Blomhøj, & Sriraman, 2006; Lesh, Landau, & Hamilton, 1983; Lesh, Kaput, & Hamilton, 2007; Sriraman & Lesh, 2006). However, mathematical modeling definitions and approaches cited in these studies are based on different theoretical foundations (Kaiser, 2006a; Kaiser, Blomhøj, & Sriraman, 2006). Some researchers embrace the modeling as a new approach that is beyond the constructivism and other researchers embrace modeling as expressing real life situations in mathematical language. However, in the broadest sense modeling is defined as a process of

defining the phenomenon and the relations in it with the mathematical expressions and bringing out the mathematical patterns in this phenomenon (Verschaffel, Greer, & De Corte, 2002). Most of the researchers see mathematical modeling as practical implementations of mathematics in real life situations and from this perspective these researchers have done studies on developing mathematical modeling skills in order to use it in engineering, economics, business etc.(Crouch & Haines, 2004; Haines & Crouch, 2001; Houston, 2002; Izard, Haines, Crouch, & Neill, 2003; Jensen, 2007; Lingefjard, 2000). In addition, according to Lesh and Doerr (2003b), modeling is an approach that is beyond the constructivism and they defined models as "conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s) perhaps so that the other system can be manipulated or predicted intelligently" (Lesh & Doerr, 2003a, p.10).

On the other hand, the different modeling approaches' common view is that modeling activities should be applied as a group work and the group work is the important component of the modeling process since each individual's model is discussed and evaluated in the group in order to reach the most appropriate model (Zawojewski, Lesh, & English, 2003). The difference in meaning between model and modeling terms is similar with the difference in meaning between product and process that is to say, modeling is a process of creating a model for a situation and the model is the product of this process (Lesh & Sriraman, 2005). Modeling process includes the number of iterative cycles in which the students move through (Lesh & Doerr, 2003a). Many studies are given different iterative cycles in mathematical modeling. In general, students go through the following processes: (a) Using their informal knowledge students try to understand and simplify the problem (i.e., selecting and interpreting the proper information), (b) Students develop a model where they decide the relationships among the variables, construct the hypotheses and evaluate the information, (c) Analyzing the model, students try to decide if their system has a gap or satisfy the goals, (d) Checking the model, students reflect on the solution from different perspectives with restructuring the solution in order to make their model acceptable (Lesh& Doerr, 2003a, 2003b). Lesh and Doerr (2003a) use the term "model eliciting" in order to cover the concept of the meaning of both

model and modeling terms. In addition, pedagogical purpose of the model eliciting activities is to help students to create mathematical models of the real life situations and to help students to better understand the mathematical concepts that are embedded in the activity (Lesh &Sriraman, 2005).

Model Eliciting Activities

The tools which are designing in order to promote students' and teachers' externalization of their thinking and externalization of their conceptualization steps for the problem situations are called as model eliciting activities by researchers (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Lesh & English, 2005; Lesh & Sriraman, 2005). Model eliciting activities' main characteristics are giving chance to learners in order to develop a model for a real-life situation, to describe, revise, and refine their ideas and to explain their conceptual systems by this model (Lesh & Doerr, 2003a). Model-eliciting activities create a situation that the learners can use the aspects of mathematizing by differentiating, integrating, reorganizing, transforming, and comparing (Lesh & Doerr, 2003a; Lesh & English, 2005; Lesh & Sriraman, 2005). In addition, these activities are designed for uncovering the learners' way of thinking while they are creating a model for the activity (Doerr, 2006). Generally the employer and worker relationship is emphasized in model eliciting activities in order to encourage the learners to explain their model in detail (English, 2003). Therefore, they are not only giving a one solution also their all possible solution approaches (English, 2003). In this way, model-eliciting activities create an environment for learners in order to document their own thinking and learning development sequence (Lesh & Doerr, 2003a). Moreover, model eliciting activities enable learners to develop their communication skills, mathematical language, conceptual understanding, implementation practice of math in real life and problem solving attitudes (Battye & Challis, 1997; English, 2003; English & Lesh, 2003; Lesh & Doerr, 2003a, 2003b).

Model Eliciting Activities for teachers

Model eliciting activities are valuable tools in order to provide teachers an opportunity to see students' way of thinking and how this thinking develops (Doerr, 2006). From this point of view, teachers can use modeling as a teaching approach when they understand students' different way of thinking, promote this different

thinking structures and develop their thinking strategies based on students' different way of thinking (Doerr, 2007). Teachers' models for teaching mathematics not only have mathematical components but also have pedagogical components and with their model, they express components of their knowledge (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Teachers' models are constructing on their mathematical teaching and learning experiences and in teaching progress they understand and interpret the situations with this models (Lesh & Doerr, 2003a; Schorr & Clark 2003). Accordingly, the quality of their teaching is related with the quality of their models (Schorr & Clark 2003). However, teachers need learning environments in order to develop their teaching models for using modeling effectively in classes (Niss, Blum, & Galbraith, 2007). These learning environments are provided with the modeling activities for teachers (Clark & Lesh, 2003; Doerr & Lesh, 2003). The most important objectives of these activities are: (1) to reveal teachers' models, (2) to give opportunities for testing, revising and refining their models and (3) to give opportunities for sharing models with other teachers and using models in different situations (Doerr & Lesh, 2003). Modeling activities for teachers are resembled with the modeling activities for students (Chamberlin, 2002; Clark & Lesh, 2003; Doerr & Lesh, 2003). Modeling activities for teachers are "*thought-revealing activities*" and are used in order to see teachers' internal models (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Some of the modeling activities which are designed for teachers are " observation sheets" that is used for observing the students' modeling process, "way of thinking sheet" that is created based on students' thinking structures and "library of student work" that is formed by selecting the informative student solutions (Chamberlin, 2002; Doerr & Lesh, 2003). For example, the purpose of "library of student work" is explained as "in developing a library of exemplary and illuminating responses by students, teachers reveal how they are interpreting the mathematical content, the context, and the value of the results that students produce. As the teachers select, organize and compare student work, they reveal how they are seeing the students' mathematical ideas. This may lead to mismatches between their expectations of some students based on notions and perceptions of students' abilities. It may lead to seeing students give mathematical interpretations of problem situations that the teacher had not seen. It is the resolution of such mismatches that provided the impetus for the development of teachers' knowledge." (Doerr & Lesh, 2003, p. 137).

Teachers Knowledge needed for Teaching from the Modeling Perspective

There are some theoretical and empirical studies on what knowledge a teacher should have in order to teach with the modeling perspective. LEMA (Learning and Education in and through Modeling and Implementations) was a transnational European Project (2006–2009) that attempted to tackle what is needed for using modeling in day-to-day teaching at teacher level by designing a common course of professional development in mathematical modeling. In order to design the course for professional development, the development team tried to answer the question of what knowledge a teacher needs for modeling. Based on their theoretical backgrounds, they came up with a theoretical model of the pedagogical content knowledge needed for modeling. They distinguish between four main categories, which are further divided into sub-categories.

> 1. *Modeling*: To implement modeling in lessons, teachers need background information about this concept (Sub-categories: What is modeling? Why use it?).
> 2. *Tasks*: When it comes to planning lessons, teachers need to learn how to select appropriate tasks for their students and anticipate the modeling outcomes. In line with our assumptions on how to teach modeling, a variety of tasks should be chosen. (Sub-categories: Exploring tasks, Creating tasks, Classification of tasks, e.g. according to area and context, and Variation of tasks, e.g. in order to adapt them to the specific needs of a class).
> 3. *Lessons*: Teachers need information about how to design lessons appropriate for modeling and how to act in the classroom (Sub-categories: Teaching methods, Using ICT, Supporting the development of modeling competencies, Exercising mathematical content through modeling).
> 4. *Assessment*: If modeling is implemented in lessons, it also has to be evaluated. Assessment should be used not only for grading but also for supporting learning through feedback (Sub-categories: Formative Assessment, Summative Assessment, Feedback) (Maaß & Gurlitt, 2011, p.632).

The pedagogical content knowledge needed for teaching from the modeling perspective was emphasized based on some phases by the theoretical and empirical studies in the literature. In the preparation phase, Doerr and Lesh (2011) reported that teachers think the prerequisite knowledge from the student's perspective, by probing the students' thinking while students are working on the model eliciting activities. In addition, Mousoulides et al. (2007) found a significant factor in students modeling to be the use of their informal knowledge, which is not related with the mathematic domain. Moreover, Busse (2011) stated that students did not apply all the expected mathematical knowledge and skills afforded by the tasks even upon the assumption that they had the relevant prerequisites since their real life experiences varies from person to person. Stillman (2000, p. 333-335) classify prior knowledge as (a) academic, that is, "vicarious experiences in other academic subject areas," (b) encyclopaedic – "general encyclopaedic knowledge of the world," or (c) episodic –

"truly experiential knowledge developed from personal experiences outside school or in practical school subjects" and explained which process is affected by the prior knowledge as "students' understanding of and engagement with", the task like "checking progress or the reasonableness of interim or final results, enhancing decision making, facilitating students' selection of an appropriate mathematical model or choice between two mathematical options". In addition, Gravemeijer (1994) and Venville et al. (2004) also reported that teachers have a difficulty to predict which concepts could be used by the students since students' prerequisite knowledge are differ than the teachers.

In the implementation phase, it was stated that teachers need to be able to recognize and respond to the multiplicity of ways students' models might develop and the ability to respond to students' thinking as it occurs by this way gaining ability to ask effective question in order to help the students solution process (Doerr & Lesh, 2011). Doerr and Lesh's (2011) also emphasized that "The notion that one should "let" students engage in working through their own ideas – expressing them and revising them – is a common one among the teachers we have worked with. Teachers often articulate this as "just let them work," "let them struggle," or "do not capitulate and give them the answers." Such a guideline can be a useful heuristic for making decisions as students are working on modeling tasks" (p.261). It was reported in the literature that asking questions that will move students' thinking forward needs much more than a procedural response so the teacher needs the ability to follow students' reasoning and listening to students' emerging thinking which is not an easy duty for the teachers (Davis 1997; Doerr 2006; Wallach & Even, 2005).

In the assessment phase, Maaß (2011) reported that "Teachers need to get materials and information on how modeling can actually be assessed, how a written class test can be designed, what other forms of assessment exist and how to assess students' solutions" (p.371). Furthermore, initial stages of introducing modeling approaches in mathematics lessons teachers are more concerned about general pedagogic approaches that fall outside of their usual pedagogic repertoires (Wake, 2011). Teachers are asking for direct help "*What innovative pedagogical approaches exist*?" (García & Ruiz-Higueras, 2011, p.571).

2.5 Summary of the Literature Review

Since the research questions concern teachers' evolving knowledge in a lesson study design that is compatible with modeling perspective, this literature review focuses on four areas: lesson study as a professional development approach, students' thinking, teacher knowledge and modeling. Firstly, the lesson study cycle is expressed and the opportunities that are provided by the lesson studies are given, one of the opportunities is to help teachers to understand students' thinking that "develop the eyes to see students". Then, the importance of teachers attending to students' thinking is discussed in the next section. In this section, benefits of students' thinking as an approach to teacher learning are given with the results of the projects which are designed on students' thinking. One of the benefits is that teachers understanding of their students' thinking help to develop their pedagogical content knowledge, pedagogical knowledge and content knowledge so that categories of teachers' knowledge are given in the following section. Shulman's teacher knowledge framework is explained and how this framework have been extended and modified by the researchers are given with illustrated explanations. Lastly modeling perspective is discussed with modeling approaches, modeling process, modeling activities for students and teachers and teachers knowledge that is needed for teaching from the modeling perspective since this study tries to respond to the needs in the literature based on teachers' evolving knowledge in order to use modeling in mathematics teaching effectively.

CHAPTER 3

METHODOLOGY

The primary goal of this study is to focus on the nature of the teachers' developing knowledge through lesson study design that is compatible with the modeling perspective. Specifically, the research questions addressed in the study are:

i) How does the teachers' knowledge evolve on questioning through professional development activities based on lesson study cycle from the modeling perspective?

ii) How does the teachers' knowledge evolve on generating assessment criteria for assessing students' competency in modeling through professional development activities based on lesson study cycle from the modeling perspective?

iii) How does the teachers' knowledge evolve on classroom organization and management domain of general pedagogical knowledge through professional development activities based on lesson study cycle from the modeling perspective?

3.1 Design

Qualitative research approach of case study was used in this study. As described by Creswell (2009), "case studies are a strategy of inquiry in which the researcher explores in depth a program, event, activity, process, or one or more individuals" (p. 13). The case needs not be a person "It can be whatever bounded system is of interest. An institution, a programme, a responsibility, a collection, a phenomenon or a population can be the case" (Gomm, Hammersley, & Foster, 2000, p.23). The case in this study was the phenomenon (four in-service secondary mathematics teachers' evolving knowledge through professional development activities based on lesson study cycle from the modeling perspective).

3.2 Research Context and Participants

The participants of this study were selected from among those of a larger project supported by the Scientific and Technological Research Council of Turkey (TUBITAK) (grant no 110K250). Aims of the project were "(i) to develop mathematical modeling tasks and activities that can be used with both secondary school students and pre-service and in-service teacher education programs; (ii) to develop an in-service mathematics teacher professional development program about mathematical modeling and to investigate how the program would affect teachers' beliefs, knowledge and practices; (iii) to develop an academic course for pre-service mathematics teachers and investigate how the course would affect pre-service teachers' knowledge, competencies and attitudes in terms of mathematics, mathematical modeling and using mathematical modeling in mathematics education". The participants of the professional development project were 5 secondary mathematics teachers (4 female, 1 male) from an Anatolion Teacher Training High School and 5 secondary mathematics teachers (2 female and 3 male) from an Anatolion High School in Ankara, Turkey. The schools and the teachers' participation was on volunteer basis.

In Teacher Training High Schools, students study on the same courses as other typical high schools, but also take extra education courses. The Teacher Training High School is a four-year public high school serving students in grades 9 to 10 at the time of data collection. The school creating a coeducational day and boarding school which serves Turkish students who are admitted solely on the basis of performance on a national exam. The average students rank in the top 8.8% on this exam required for admission. At the time of data collection, there were approximately 30 students in each classroom. Moreover, there were 14 mathematics teachers in the school. On the other hand, the Anatolian High School is also a four-year public high school serving students in grades 9 to 12. The school creating a coeducational day and boarding school which serves Turkish students who are admitted solely on the basis of performance on a national exam. The average students rank in the top 1% on this exam required for admission.

At the time of data collection, there are approximately 30 students in each classroom. In addition, there were 13 mathematics teachers in the school. In both schools, club activities are done by volunteers with the objective of children to work as individuals or in groups and to create a product/performance in the areas they are interested in. Other characteristics of the schools in terms of club activities and number of the students are given in Table 3.1.

Table 3.1 Schools and Their Contexts

	Teacher Training High School	**Anatolian High School**
Club Activities	football, boxing, fencing, theater	theater, volleyball, basketball, football, table tennis, tennis in courts, shooting, gymnastics
Total number of the students in the school	944	1093
Number of the students in each grade levels	Grade 9: 240	Grade 9: 299
	Grade 10: 241	Grade 10: 270
	Grade 11: 193	Grade 11: 270
	Grade 12: 270	Grade 12: 254
Number of boys and girls	Girls: 515 Boys: 429	Girls: 546 Boys: 547
Number of boarders in the schools	105	51

Consequently, this dissertation relied on a theoretical sample of informants and instruments from the parent study. Teachers from whom data are reported in this study were selected by the purposive sampling among the participants of the parent study mentioned above. Two teachers from each participating schools (i.e., Anatolion Teacher Training High School and Anatolian High School) were selected in order to represent a diversity of perspectives. Moreover, teachers teaching the same grades were preferred to hold the concepts of the model eliciting activities in that grades and the classroom implementation times same for all four teachers. Because, as it is emphasized by Marks (1990) that in order to analyze the evolving pedagogical content knowledge, the most obvious strategy is to hold grade level constant or close to each other and vary the content (subject).

None of the four teachers participated in this study had a previous experience with model eliciting activities. Participants' gender, grade levels that they were teaching, and years of teaching experience are provided in Table 3.2.

Table 3.2 Characteristics of Teacher Participants

Name*	**Gender**	**Grade Levels that teachers taught for the last 3 years**	**Teaching Experience (Year)**
Figen	Female	9-12	16
Rezzan	Female	9-10	13
Semra	Female	9-12	24
Melda	Female	9-12	23

* All names are pseudonyms

In addition, the teachers participated in this study were aware of the fact that their data will be used for a dissertation as well as reports of the parent study. They were also informed about the overarching research question: How is the nature of the teachers' developing knowledge through the professional development activities based on lesson study cycle from the modeling perspective? Moreover, All subjects were assured that any data collection from or about them would be held in confidence and the names of the individual subjects never be used in any publication that describe the research. In addition, participants were informed that participants in the study always had the right to withdraw from the study or to request that data collected about them not be used. Subjects were also informed that no one else other than the project group had access to the data.

3.2.1 Implementation Design

The professional development program followed in this study included a cyclical process. This research process consisted of meeting before the implementation of the model eliciting activity, implementation of the model eliciting activity and meeting after the implementation of the model eliciting activity in each month in each school. This cyclic design expressed in Figure 3.1. Focusing on a different model eliciting activity each month, the data collection for this study took five months. Teachers in the design of this research moved through different modeling cycles in the meetings

and the classroom practice of the model eliciting activity. However, teachers moved through comprehensive modeling cycles since the modeling process contain model eliciting activities which are based on different mathematical concepts, students' modeling behaviors and using the modeling in their classes each month.

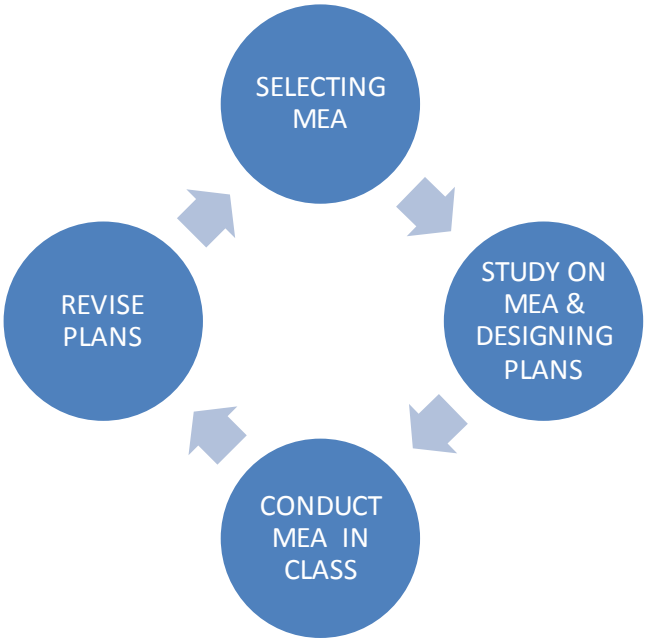


Figure 3.1 Cyclic Design Followed in the Study

Before the cyclic process of the study, teachers attended four- day workshop. The aims of the workshop were; a) sharing the objectives and the scope of the project which was going to took ten months with the teachers b) determining the preconceptions, needs and the interest of the teachers for the mathematical modeling c) creating a common view about the nature of the mathematical modeling activities and the process of modeling d) helping teachers to reconsider their mathematical knowledge and the pedagogical content knowledge via mathematical modeling activities. The workshop program that was created based on these aims was given in Appendix I. The workshop program was not included in the data analysis part since this study's theoretical framework components were not held in the workshop.

Cyclical Process of Professional Development Program in Each Month.

Teachers chose a model eliciting activity based on the curriculum and the grade level that they taught before the first week meeting. The teachers choose the MEAs that were going to be applied from the pool of MEAs, which include 60 model-

eliciting activities. Model eliciting activities' context and how they were developed were explained in Appendix D. For the first cycle, teachers chose the MEA from the pool of the MEAs; however, this process took so much time in the meetings so it was decided to select the MEAs based on the curriculum and the teachers' grade level that was going to be applied by the project group. For instance, firstly, the curriculum took into account, which subjects and which objectives were taught in that month were listed then the MEAs based on these objectives were determined, and then the MEAs were selected based on the grade level that teachers taught. At the end, the teachers determined the MEA that was going to be applied for the next month, through the selected MEAs in the third week meetings. Before the first week meetings, teachers expected to prepare lesson plans individually and create solution strategies for MEA that was going to be applied.

First Week: Meeting before the Implementation of the Model Eliciting Activity

Teachers came this meeting with their lesson plans on model eliciting activity and with their solution strategies for model eliciting activity that was going to be applied. Teachers designed these lesson plans individually and on the typical format (See Appendix A). Typical format for designing the lesson plans were distributed to the teachers before the meeting. In the meeting, firstly teachers shared their solution ways on the MEA then teachers examined and evaluated the model eliciting activity from both the student's perspective *(e.g. how the students could create models*) and the teacher's perspective (*e.g. the mathematical concepts and the relations between them that is embedded in the model eliciting activity and prerequisite knowledge required in order to supply the mathematical concepts that are embedded in the activity*). Then teachers worked out to develop a shared lesson plan for the model eliciting activity through discussion with the colleagues on each heading of the typical lesson plan format and taking into account their individual plans. In these discussions, researcher took a facilitator role, as a facilitator the headings in the lesson plan format was asked to the teachers (e.g., *what kinds of questions that the teacher can use in the solution process of the model eliciting activity and what are the aims of these questions? what kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' errors? what can be the assessment criteria while the students working on the question? what can be done in*

order to provide an understanding of the problem and to warm up the question?) and one of the teachers who was going to apply the model eliciting activity wrote the consensus decisions on the typical plan format. The individual lesson plans and video and audio recording of the meetings were collected for the analysis.

Second Week: Implementation of the Model Eliciting Activity

After the first week meeting, two teachers from each school applied the model eliciting activity that was discussed in the meeting, to their classrooms. Interviews (see interview questions in Appendix B) were conducted with teachers who were going to use the model eliciting activities before the implementation and after the implementation. The participants of this study were also interviewed before the implementation and after the implementation that they did not conducted. The interview questions include the headings of the typical format of the lesson plan which were also the theoretical framework components of this study (the components are; questions to students, assessment of students, classroom organization and management- *class setting: the criteria for determining the group's structure and the number of the students in one group- introducing the implementation- providing an understanding of the MEA and to warm up the MEA- organizing the presentations of the solutions- ending the implementation)* in order to get teachers' individual decisions other than the meetings before and after the implementations.

Implementations took two-class hours, which is approximately 100 minutes. One class hour is 45 minutes and the implementations went on during the 10 minutes break time between two-class hours. In implementations, students were organized in groups by the teachers and groups worked on the model eliciting activities in the first class hour then each group prepared poster papers and presented and defended their complete or incomplete solutions with the help of posters to the other groups in the second-class hour. In addition, in classroom practice, teacher observed the students' group working on the model eliciting activity and sometimes took informative notes or expressed their reflections based on their observations in the interviews after the implementation. Moreover, teachers who had an appropriate time also attended the implementations in order to observe the implementation-related issues (challenges, opportunities, effective strategies, and so on). After the implementation, students' solution papers were copied and distributed to the all teachers. Each schools'

students' solution papers were distributed to their own teachers and teachers examined students' solution papers based on students' solution ways and errors until the third week meetings. In addition, teachers' interviews after the implementation was transcribed and were given to the teachers who were interviewed with the aim of confirmation and a tool of recall for the teachers in the third week meetings. Furthermore, during the implementation researcher observed the teachers (Lesson Observation Form in Appendix C). The focus of the observations was the interactions between students and teacher and teacher's discourse that reveal the evolution of the teachers and how these come arrive. Researcher jot down the example conversations and evidences related to the focus of the observations (Lichtman, 2006). Additionally, video and audio recording of implementations were collected for the analysis.

Third Week: Meeting After the Implementation of the Model Eliciting Activity

Teachers brought the observation notes taken during the implementation of the model eliciting activity and the students' solution papers to the third week meetings. In these meeting, teachers had a chance to test and revise their thinking which were based on the assumptions before the implementation. Teachers tested and revised their thinking during discussion, which was enriched by the observation notes taken during the implementation of the model eliciting activity and the students' solution papers. Teachers created a shared student-thinking sheet (included students' errors and solution ways) after discussing with their colleagues. In these discussions, a second researcher took the facilitator role, teachers were asked "which solution ways that was used by the groups in the solution papers and which errors that was determined based on the solution ways" by the facilitator. Teachers discussed the different solution ways and the errors that was included in the solution ways. The aim of this activity (creating a shared student-thinking sheet) was to help teachers to reveal students' mathematical thinking ways while students were working on the model-eliciting activity. After this activity, teachers revise their shared lesson plans for the model eliciting activity that was designed before the implementation of the task. Teachers discussed each headings of the typical format of the lesson plan and revised the parts, which were stated based on the assumptions in the first week meetings with reflections on implementations, observation notes from implementations and by discussing with their colleagues. For example, under the

heading of *What kinds of students' difficulties encountered in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' difficulties;* if teachers' strategies that was decided before the implementation meeting, on one of the specific difficulty were not overcome this difficulty in the implementation, then other questions were stated and the explanations were discussed or the questions were discussed based on the errors and difficulties that was identified by the teachers in the implementations or in the solution papers, then the parts of the lesson plans were revised based on these discussions. While teachers working out to revise a shared lesson plan for the model eliciting activity, in the discussions if all suggestions in the teachers' post interviews after the implementation were not stated, researcher as a facilitator asked the teachers whether they intended to discuss these forgotten parts. For instance, if the teacher stated that gender difference was very important in the group structure for the solution process of the MEA in the post interview but not stated this suggestion in the group structure, this was reminded as "one of the teacher after the implementation, suggested that gender difference was very important in the solution process of the MEA, why this could be suggested, was it important in the group structure or not?". Besides, video and audio recording of the meeting were collected for the analysis.

The participants of the study applied three modeling activities, which were Bank Robbery, The Summer Jobs and Pack Them In! (see Appendix E) in 9-10 grades, however in other implementations, which the MEA's concept were also in 9-10 grade level, they observed their colleagues's implementations, examined students' solution papers and attended before and after meetings. Table 3.3 list the specific dates for the meetings and implementations associated with each model eliciting activity.

Table 3.3 The Dates of the Meetings and Classroom Implementations for the Modeling Activities

Model Eliciting Activity	**Dates for the Anatolian High School**	**Dates for the Anatolion Teacher Training High School**
Bank Robbery	Meeting Before Implementation:24.10.2011	Meeting Before Implementation:25.10.2011
	Implementation: 25.10.2011-04.11.2011	
	Meeting After Implementation: 14.11.2011	Meeting After Implementation: 15.11.2011
Street Parking	Meeting Before Implementation: 21.11.2011	Meeting Before Implementation: 22.11.2011
	Implementation: 22.11.2011-02.12.2011	
	Meeting After Implementation: 05.12.2011	Meeting After Implementation: 06.12.2011
The Summer Jobs	Meeting Before Implementation: 19.12.2011	Meeting Before Implementation: 20.12.2011
	Implementation: 20.12.2011-29.12.2011	
	Meeting After Implementation: 02.01.2012	Meeting After Implementation: 03.01.2012
Water Tank	Meeting Before Implementation:13.02.2012	Meeting Before Implementation: 14.02.2012
	Implementation: 14.02.2012- 24.02.2012	
	Meeting After Implementation: 27.02.2012	Meeting After Implementation: 28.02.2012
Pack Them In!	Meeting Before Implementation: 05.03.2012	Meeting Before Implementation:06.03.2012
	Implementation: 06.03.2012- 16.03.2012	
	Meeting After Implementation: 19.03.2012	Meeting After Implementation: 20.03.2012

Moreover in the implementation process, the researcher had a facilitator role in the discussions while teachers working out to develop shared lesson plans and the second researcher, had a facilitator role in the discussions while teachers filling in the

shared students' thinking sheet. The implementers were the same for all the implementation cycle and in the same way (described above) the cycle were applied and the researchers used same implementation cycle plan for all implementations. In the implementations, the meeting locations were held constant and the unplanned external events were documented. Researcher used the same instruments and collected the data as a single data collector for all the implementations in this study. Implementation cycle was conducted in two schools in teachers' naturalistic settings (in their staff room and in their classes) for five months. In addition, the missing data were controlled by arranging teachers' other appropriate times by not digressing the implementation cycle.

3.2.2 Researcher's Role

Observations were conducted while teachers applying the model eliciting activities in their classes. Researcher role in the observation was *observer as participant* (Gold, 1958). Researcher jot down the example conversations and evidences related to the focus of the observations. The focus of the observations was the interactions between students and teacher and teacher's discourse that reveal the evolution of the teachers and how these come arrive. In addition, researcher conducted semi structured interviews with teachers before and after each implementation of the model eliciting activity and took field notes in every setting that was shared with teachers. Moreover, researcher took a facilitator role when teachers working out to develop the shared lesson plan in the first week meetings and revising the plans in the third week meetings. In the first week meetings, as a facilitator the headings in the lesson plan format was asked to the teachers (e.g., *what kinds of questions that the teacher can use in the solution process of the model eliciting activity and what are the aims of these questions? what kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' errors? what can be the assessment criteria while the students working on the question? what can be done in order to provide an understanding of the problem and to warm up the question?)* and one of the teachers who was going to apply the model eliciting activity wrote the consensus decisions on the typical plan format. During the second week, the teachers' interviews after the implementation was transcribed and were given to the teachers for confirmation and as a tool of recall for the teachers in the third week

meetings. In addition, researcher took notes based on the headings that the teachers focused in their interviews about the implementation and the suggestions for the further implementations. In the third week meetings, these notes were emphasized by the researcher under the related headings of the lesson plan format. Researcher recorded all suggestions in the teachers' post interviews after the implementation and if some suggestions were forgetten, researcher asked the teachers whether they intended to discuss these forgotten parts. Furthermore, teachers in the discussion provided suggestions in different part of the components (PCK, PK) at that part; researcher revived the component and asked questions based on that component. For example, teachers in this study gave different errors and difficulties instead of discussing under the *questions to students* parts of the lesson plans; at that time, facilitator asked some questions, such as "in order to overcome these students' difficulties/errors, what kinds of questions would you ask?". To sum up, researcher controlled the components of the lesson plans in each discussion. Moreover, researcher tried to give some interventions in the discussions of the PK components of the lesson plan as a suggested strategy for the related PK component, these interventations took part in the results part and the suggestions were given based on the interventions in the suggestions for facilitator part.

3.3 Data Collection

To address the research questions, data from the five-month process were collected. Data collection procedures included videotaping and audio taping the meetings, taking field notes, collecting lesson plans, conducting semi-structured interviews with teachers before and after the each implementation of the model eliciting activity and observing the teachers in their practice while they were applying the model eliciting activity as a part of the class.

Lesson Plans

Lesson plans (see Appendix A for typical lesson plan format) on modeling activities could be the teachers' model eliciting activities like "observation sheet", "way of thinking sheet" and "concept maps" (Clark & Lesh 2003; Doerr & Lesh, 2003). Lesson plans allow for seeing the teachers' models, which are based on teaching and learning experiences of teachers, and that models describe the nature of teacher knowledge (Hiebert, Gallimore, & Stigler, 2002; Lewis & Tsuchida, 1997; Stein,

Smith, & Silver, 1999). The activity of designing purposeful plans with in-service teachers served as an inquiry into curriculum, pedagogy and students thinking (Hiebert, Gallimore, & Stigler, 2002; Stein, Smith, & Silver, 1999). Since the lesson plans are the teachers' model eliciting activities, teachers move through multiple modeling cycles while designing the lesson plans just as the students when solving the model eliciting activities (Chamberlin, 2002; Doerr & Lesh, 2003). Through these cycles, teachers' descriptions, explanations and predictions on lesson plan were tested, rejected and revised based on their discussions, reflections after the implementation of the model eliciting activity and feedbacks from discourse while they are creating a shared student thinking sheet (Schorr & Lesh, 2003). In this study, teachers designed plans on the typical format of the lesson plans (see Appendix A). Typical format of the lesson plans' headings (*Class setting (What are the criteria for determining the group's structure and the number of the students in one group?); How can the implementation be introduced; What can be done in order to provide an understanding of the problem and to warm up the question; What kinds of solution strategies that the students can use while working on the model eliciting activity; What kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' errors…)* were chosen based on the model of the pedagogical content knowledge needed for modeling that is proposed by LEMA project (Maaß & Gurlitt, 2011). This model includes four main categories which are modeling, tasks, lessons and assessment and their subcategories. In task category, it is explained that teachers need to learn, how to select an appropriate task for their students. In lessons category, it is explained that teachers need information about how to design lessons appropriate for modeling, how to act in the classroom, how to ask effective questions and how to reflect students emerged thinking. In assessment category, it was emphasized that "If modelling is implemented in lessons, it also has to be evaluated." (Maaß & Gurlitt, 2011, p. 632). In addition, headings based on classroom organization and management domain of general pedagogical knowledge were chosen since its subcategories like class setting, introducing, warming, organizing and ending are more concerned by teachers and being asked for direct help by teachers in order to use modeling in their classes (García & Ruiz-Higueras, 2011; Wake, 2011). The typical lesson plan format was examined by five mathematics education researchers based on its content, adequacy of workspace,

appropriateness of language, clarity of directions and was revised with their suggestions. The last version of the typical format of lesson plans were used from the first implementations to the last implementations and the lesson plans were collected in order to analyze how the teachers' knowledge evolves within the investigation.

Observations, Interviews and Field Notes

Observations were conducted as observer participant (Gold, 1958) while teachers applying the model eliciting activities in their classes. The focus of the observations was the interactions between students and teacher and teacher's discourse that reveal the evolution of the teachers and how these come arrive. Researcher role was to jot down the conversations and evidences related to the focus of the observations (Lichtman, 2006). The observation form includes the headings of the typical format of the lesson plan in order to observe, how the knowledge was used in the classrooms. The form was constructed by three mathematics researchers and was examined by the research team consist of ten mathematics education researchers based on its content, adequacy of workspace, appropriateness of language, clarity of directions and was revised with their suggestions. These revisons were, the observation form components' place were revised based on the process of the MEA's implementations and were gathered under these processes that were warm-up, while students are working on the MEA, listening and observing the students in the process, while students are presenting their solutions. Additionally, the directions under each process were revised in order to make it more clear and short. Besides, the revisions were made for the appropriateness of the language.

Furthermore, semi-structured interviews were conducted with teachers who applied the model eliciting activity in their classes before and after the each implementation of the model eliciting activity and interviews' audio recordings were taken in order to transcribe for the analysis. The interview questions include the headings of the typical format of the lesson plan which were also the theoretical framework components of this study (the components are; questions to students, assessment of students, classroom organization and management- *class setting: the criteria for determining the group's structure and the number of the students in one group- introducing the implementation- providing an understanding of the MEA and to warm up the MEA- organizing the presentations of the solutions- ending the*

implementation) in order to get teachers' individual decisions other than the meetings before and after the implementations. The interviews took approximately half an hour and the interviews were conducted in the rooms or the classrooms, which are suitable at that time in both schools. The questions were prepared by three mathematics researchers and its format was examined by ten-mathematics education researchers (personnel of the project the data collected within) based on its content, appropriateness of language and was revised with their suggestions. The revisions were made for the appropriateness of the language in order to make clearer and the understandable for the teachers. In addition, field notes were taken in every setting that was shared with teachers. The focus of the field notes were the teacher's discourse that reveal the evolution of the teachers and how these come arrive.

Assessment Tools

The Assessment Tool developed by teachers to assess students' modeling competency were used as a model eliciting activity for teachers. The objectives of the activity are: to reveal teachers' models on the assessment criteria ; to give opportunities for testing, revising and refining their models and to give opportunities for sharing models with their colleagues and using assessment criteria in different MEAs implementations. In the third implementation of the MEA entitled "Summer Jobs", teachers were encouraged to create an assessment tools by the researcher, and teachers revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with these tools. In the developing process of the assessment tools, researcher gave an empty tables that include only the headings which were assessment while the students working on the modeling task and assessment of students' presentation of their solutions in the third implementation to the teachers in order to help them to think the criteria under the related process more easily. Additionally, researcher questioned the assessment criteria that the teachers generated via the questions such as what was the meaning of the criterion? how did you evaluate with that criterion? in the interviews and the meetings in order to focus the teachers to think more on the criteria.

3.4 Data Analysis

Data analyses for the study were conducted in two major phases, which are during data collection and after data collection. The analysis during the data collection included all analysis from the first day of the meeting of teachers until the last day of data collection. All analyses of data that were collected from teachers were organized right after data collection. Moreover data analysis was divided into two approaches as with-in case and cross-case analysis (Merriam, 2001; Miles & Huberman, 1994). Groups of teachers were used in this study since the focus of the study was both within and across cases so with-in case and cross-case analysis approaches are suitable with the focus of the study (Gerring, 2007). With-in cases analyses were conducted on the data set for each teacher. The cross-case analyses were built on the results from with-in case analyses. Analysis for each case was also supported by the comparison analysis that used to design for generalizations across teachers (Merriam, 2001; Strauss & Corbin, 1998). Comparisons for the cross-case analyses began during the data collection since the researcher spent time with all teachers during data collection. In addition, comparisons took place as the researcher received larger sets of data and gain more experience within settings.

In addition, data were analyzed based on the paradigmatic view of knowledge. This approach views knowledge more as a guide for action, personal construction rather than as a search for objective truth. This approach rests on a normative conception of knowledge, in which "teachers would be able to describe or explain their actions in a way that makes sense. Teachers would provide reasons for their actions, thereby encouraging the growth of knowledge" (Orton, 1989, p. 15)

3.4.1 Analysis during Data Collection

The first data analysis began as the data collection was proceeding (Patton, 2002). Data analysis during the data collection mainly involved filling the form (see Appendix F) after each meeting and lesson plan design for each implementation based on the theoretical framework components of this study (the components are; questions to students, assessment of students, classroom organization and management- *class setting: the criteria for determining the group's structure and the number of the students in one group- introducing the implementation- providing an understanding of the MEA and to warm up the MEA- organizing the presentations of the solutions- ending the implementation).* The form was filled for each component

for each teacher; the sample-filled forms were given in Appendix H. The aim of filling the form was to organize, summarize the data and to create a base for further analysis. Firstly, group discussions in video and audio recordings while teachers working out to develop a shared lesson plans in meeting before the implementation of the model eliciting activity was transcribed. Each teacher transcripts, lesson plans that were designed before the meeting individually and field notes were analyzed in order to get the "descriptive codes" for summarizing the segments of data and to provide the bases for later higher order coding which is "Pattern (inferential) codes" (Miles & Huberman, 1994). After the descriptive codes, the pattern coding began in order to put together the descriptive codes to more meaningful units. Then these meaningful units, which were relating with each other, were organized together and the themes were determined (Miles & Huberman, 1994). Secondly, again group discussions in video and audio recording of discourses while teachers creating the shared student thinking sheet in meeting after the implementation, the discussions while they were revising their shared lesson plans after the implementation and the interviews before and after the implementation with the teachers were transcribed. Transcripts, observation notes from the class implementations and the field notes analyses moved through same analysis procedure just as before the implementation analysis, which were described previously: descriptive codes → pattern codes → themes. The themes which emerged from before and after implementation meetings were compared and all themes that emerged from each case (teacher) for that implementation were written in a form (see Appendix F). Moreover, case ratings for each theme for each implementation were also recorded.

In addition, "memoing" began at the start of the analysis along with the coding and determining themes since memoing points towards higher-level of pattern coding and relate different concepts to each other (Charmaz, 2006; Miles & Huberman, 1994). "A memo is the theorizing write-up of ideas about codes, themes and their relationships as they strike the analyst while coding, determining themes…it can be a sentence, a paragraph or a few pages… it exhausts the analyst's momentary ideation based on data with perhaps a little conceptual elaboration" (Miles & Huberman, 1994, p. 72). Memoing helps the analyst move from the descriptive to the conceptual level and propositions are produced at the last type of memoing (Charmaz, 2006;

Miles & Huberman, 1994). Therefore, memo recording took place in every stage of the analyses of this study.

Analysis during data collection procedure, which was described above, was followed for each implementation for each case and if new themes emerged, they were recorded in the form (see Appendix F).

3.4.2 Analysis after Data Collection

After the data collection were completed, transcripts, lesson plans, observation notes, field notes, forms (see Appendix F) that were filled for each implementations and memo recordings were organized chronologically from start to finish of the study. Firstly, each case evolving was taken account based on the theoretical framework components of this study. This step mainly involves conceptual structuring in terms of the continuum from concrete to abstract and specific to general (Miles & Huberman, 1994). More concrete levels of data were summarized and integrated in order to develop to higher order concepts (O' Leary, 2004; Richards, 2005). Secondly, what is similar and different about the cases' evolving were taken account. Comparison actually starts during the data analysis by filling the case ratings forms for each case for each implementation and memo recordings so it helps to identify more abstract concepts (Glaser, 1978). Comparison started during the data analysis since systematic comparisons is essential in order to conceptualize the development (Tesch, 1990). In addition, case ratings of themes for all implementations form (see Appendix G) were filled in order to organize and summarize the data. The form was filled for each theoretical framework components of this study based on case ratings of themes for all implementations; the sample-filled forms were given in Appendix H. Moreover, this step involved conceptual structuring of cases' evolving with its similarities and differences.

3.4.3 Summary of Data Analysis

With-in case and cross-case analysis took place both in the during data collection and after data collection phases. During data collection phase, each case's (teacher) evolving knowledge was analyzed for each implementation and also the cases' comparisons took place for each implementation. In addition, after data collection phase, one case's evolving knowledge for all implementations and case comparisons were analyzed.

3.5 Reliability and Validity

Validity stands for determining whether the findings are accurate from the point of researcher, the participant or the reader of a study in qualitative research (Creswell & Miller, 2000). In addition, the reliability expansion in qualitative research is to check for the consistent patterns theme development among several researchers. Moreover, qualitative researchers can also generalize some aspects of case analysis to other cases (Yin, 1989). Lincoln and Guba (1985) prefer to use alternative terms for the concepts of validity and reliability. They use credibility, transferability, dependability and confirmability instead of internal validity, external validity, reliability and objectivity respectively since they think that they are more suitable for the nature of the qualitative research. The criteria and the strategies in order to supply that criterion that was used in this study are given in the Table 3.4

Table 3.4 Validity and Reliability Criteria and the Strategies in order to Supply Those Criteria in the Study

Criteria	**Strategies in order to supply the criteria**
Credibility	Prolonged time in the field
	Data triangulation
	Member Checking
Transferability	Rich, thick description
Dependability	Dependability audit
Confirmability	Confirmability audit

Prolonged time in the field

The study took five months so the researcher had a chance of developing in-depth understanding of evolving knowledge and could convey the site in detail. Prolonged time obliterate the researchers' effect and the teachers' gave intimate answers and make intimate explanations in the meetings before and after the implementation, in the implementations and in the interviews. This time gave also a chance to researcher to see if the change was instant or periodic.

Data triangulation

Different data sources (Lesson plans, transcriptions of the videotapes and audiotapes of the meetings and the interviews, field notes and observation notes) were used in order to build consistent justification for themes.

Member Checking

During the study, the member check was taken by giving each member post-implementation interview transcripts in order to take their confirmation in the after implementation meetings. The observations notes and the aim of the questions were used in the implementations, and were checked by the participants in the post-implementation interviews for each implementation. In the last implementation post interview, the teachers' check were taken by asking "what do you suggest to the teacher who is going to implement the modeling task for the first time on …" on the conceptually grounded pedagogical strategies that each teacher construct during the study and the overall nature of their questioning type change were wanted to expressed by themselves and were also asked " what kinds of questions, a teacher should ask in the implementation of the modeling task?". Member check for the assessment criteria were taken on the tools that each teacher created, after transcription of each interview for the developing process of the tools. The criteria, their reasons, how it was used were checked in the interviews and in the meetings.

Rich, thick description

Researcher gave the findings in rich and thick description and mostly one to one transcriptions were used. By this way, the readers of the study can reach their conclusions and have an opportunity to interpret the raw data and dense description gave a chance to reader to understand the setting in detail.

Dependability audit

A second researcher, a Phd student in mathematics education, examined the consistency of the data collection procedure and conceptualization approach in coding process. In the conceptualization process, each researcher highlighted the themes related to the theoretical framework components of this study for each case separately, these themes were compared and contrasted. The results showed that 86%

of inter-rater dependability was reached. Then, the consensus was reached based on the discussions on non-agreed themes by each researcher.

Confirmability audit

A second researcher, a Phd student in mathematics education examined the raw data in order to confirm whether the interpretations, results and suggestions in the study could be reached from the raw data.

CHAPTER 4

RESULTS

The learning trajectories, development of pedagogical content knowledge and pedagogical knowledge that emerged in this study highlighted the development of teachers' knowledge in the following two major areas:

- Knowledge of instructional processes-student focus,
- Knowledge of classroom organization and management.

As this study is concerned specifically with knowledge used in teaching with modeling, it covers only a portion of the terrain of teachers' knowledge and the examples presented in this chapter ought to be presented as coming from the teachers. This is consistent with the paradigmatic view of knowledge in this study, which is personal construction rather than objective truth.

The data used in this dissertation were the dynamic aspects of pedagogical content knowledge and pedagogical knowledge. One of the most important dynamic aspects is how this type of knowledge is formed and modified in light of professional development activities based on lesson study cycle from the modeling perspective. The evolving knowledge was given under the two major areas based on each teacher and under each major area, the cross-case analyses were also took place. In addition, under each major, the selected representative excerpts of the teachers were given in order to reflect the evolving knowledge clearly, when the excerpt was given from the dialogue among the teachers in the meeting before and after the implementation, other teachers from both schools were coded as Teacher A, B, C, D, E and F.

4.1 Knowledge of Instructional Processes - Student Focus

4.1.1 Questions to Students

Under this subheading each teacher evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what is similar and different about the cases' evolving were taken account.

The teachers gave many examples of questions they might ask and asked to stimulate students' thinking and learning while working on the MEA, before and after the implementations. These questions were categorized as directive, broadening, procedural, invitational, exploratory and evaluative. The directive questions (How can we use the propositions? Which conjunctions will you use? Then do you establish a truth table?) were designed to lead students along a predetermined path, and broadening questions, intended to provoke students to expand their ideas. Other question types that emerged from the data was invitational (do you understand? do you have questions?), procedular (show me the truth table of the "and" conjunction), evaluative (evaluate the strategy) and exploratory questions (Is there another way to do this?) were used in order to make groups to think other approaches. Each question type, that was presented, was indicated near each excerpt.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the invitational type questions were preferred also in order to obtain feedbacks on students' difficulties and errors, the directive questions with the help of the keywords were used, during the implementation, figen did not use any question both for feedback or understanding the solution strategies, teacher did not interfere the solution processes. After the implementation, the procedural questions were denoted.

The below excerpt was taken from the pre-implementation interview

Researcher: What kinds of questions that will you use in the solution process of the model eliciting activity and what are the aims of these questions?

Figen: Firstly, I will ask them if they understand the question [bank robbery], or they have any questions [invitational]

Researcher: What are the aims of these questions?

Figen: I think, when students look at the question, they may have difficulty to begin since they are not accustomed to this kind of problem, by this way I can motivate them to begin

…

Researcher: What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' difficulties?

Figen: If they [students] will not think, then I can ask, do you read "or "and "and" statements? What do they remind you? I do this method in my classes also, for example in geometry lesson I say isosceles triangle and the height? What reminds you? and then they say that okay okay and they remember that the height is the median. I prefer to draw attention to the keywords rather than actually saying that this question is about this subject. For example in here when I draw attention to "or" and "and" keywords, I think they remember the logic and use the conjunctions [directive questions with the help of the keywords]

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Researcher: What kinds of students' difficulties encountered in the solution process of the model eliciting activity and what kinds of questions that teacher could use in order to overcome these students' difficulties?

Teacher B: they [students] made many prodecural errors; we [teachers] can show these errors to them.

Figen: we [teachers] make them [students] to realize their errors, for instance they [students] used "and" conjunction wrongly, we [teachers] can ask in which situations the "and" conjunction get the truth value, also we want them [students] to show us the truth table of the "and" conjunction then they will remember the property and they will have a chance to correct their error [procedural]

In the second implementation of the MEA entitled "Street Parking", before the implementation directive questions with the help of the keywords and procedural questions were preferred in order to obtain feedbacks on students' difficulties and errors. After the implementation, broadening questions were denoted. For example, if the MEA was not understand, the students directed to the activity with the questions and if there was misleading representations, the questions were preferred in order to realize this incorrect representation.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher A: if they [students] take wrongly parking space dimensions, the solution will go wrongly

Teacher B: I think some of the groups will not even try the angle parking, they easily say the answer that parallel park is okay

Teacher A: May be some groups use the angle parking but cannot use the trigonometry

Researcher: What kinds of questions a teacher can ask at these times?

Figen: We [teachers] can ask, are the angle important for parking the maximum numbers of cars? So with the little orientation, we can direct students to use trigonometry rather than saying use

trigonometry. I think when the angle emphasized then they will understand. [directive questions with the help of the keywords]

Teacher B: they may have also make the trigonometric calculations wrongly or while writing the ratios of the similarity

Figen: at that time we can ask them do the trigonometric operations again [procedural]

Teachers A: In here, students can make many numeric errors since it has so many operations

…

Teachers C: I think they can not manage the operations, if they make errors in the operations, the solution is not go on

Rezzan: we [teachers] can make the students do the calculations again; it can be asked can you find the similarity ratio again? Or can you show me the trigonometric ratios again?

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Figen: Most of the groups cannot understand the question completely because most of them take the hypotenuse 4.8m, so I think it is important to ask them read the question again, are you carefully examine the question, the data? [broadening- if the MEA was not understand, the students directed to the activity with the questions]

Rezzan: where is 4.8 m? What are the parking spaces dimensions? By this way they [students] read the question again and understand

…

Teacher B: some of them [students] even could not place the car on the parking space, they made wrong drawings. Then I ask do not you see your father when he is parking.

Teacher A: It is very important to make right drawing. I observed these in my class also, I ask them is your drawing right? Then they look and said yes, then I do not interfere, I could not decide what to ask what to say?

Figen: in this case we [teachers] may ask for example, when you park with angle, the car can go to the in front of the parking spaces completely? Are there any dead space? however I agree with you that we [teachers] do not know how to interfere and how much, and what kinds of questions can be asked [broadening- if there was misleading representations, the questions were preferred in order to realize this incorrect representation]

Rezzan: we [teachers] make nontraditional implementations so it is hard to ask questions? What to say? How to interfere?

In the third implementation of the MEA, entitled "Summer Jobs", before the implementation the broadening, exploratory and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. The exploratory questions were used in order to make groups to think other approaches. The broadening question were used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward. During

the implementation, the evaluative and exploratory questions were used. After the implementation, the exploratory questions and broadening questions (if the MEA is not understood, directing students to the activity with the questions were preferred) were denoted.

The below excerpt was taken from pre-implementation interview with Figen

Researcher: What kinds of students' difficulties or errors might encounter in the solution process of the model eliciting activity and what kinds of methods that you will use in order to overcome these students' difficulties?

Figen: if the group will use only the general mean, it is a missing approach, then I will ask them to look their approach and ask it is enough in order to decide… also they may be ambiguous about how to decide part time, I can ask at that time is there another way to decide? Is there another criterion? [The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward- exploratory]

Researcher: do you want to add anything?

Figen: no, it is the only way that they have difficulty

…

Researcher: What kinds of questions that you will use in the solution process of the model eliciting activity and what are the aims of these questions?

Figen: if they only use general mean and do not decide, then it can be ask can we decide based on periods or months?... [The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward]

During the implementation

Figen: what is your strategy? Why do you use this approach? (asked to all groups) [evaluative]

The question below was asked to the groups that create a table based on the money per hour and tries to decide the full time and part time worker:

Figen: Is it enough to decide based on one table? Do you create a table based on another criterion? Is there another way to do this?[evaluative-exploratory]

Figen: when we [teachers] ask them how we can do the question from another way, then they [students] think other criteria and it is good for to encourage them to think other approaches, also some of the groups cannot understand the question completely, it will be good to want them to read and examine the question again (excerpt was taken from the meeting after the implementation) [exploratory- broadening, if the MEA is not understood, directing students to the activity with the questions]

In the fourth implementation of the MEA entitled "Water Tank", before the implementation the broadening and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. The broadening question was used in order to make students compare their result if it fits to the real situation. After

the implementation, the evaluative (evaluate the strategy) questions and broadening questions (if the MEA is not understood, directing students to the activity with the questions were preferred) were denoted.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher B: I think students will have difficulty while drawing the graphs, most of them can draw the graphs linear, what can we [teachers] do at that time?

Rezzan: we [teachers] can ask do you remember the logarithmic and exponential functions' graph. So that they can relate the graphs

Teacher A: they may also confuse the concave up and concave down graphs, they may draw concave up when the water tank volume increase

Teacher B: we [teachers] can want them to draw their graph again, or ask it is true? Is the height increasing?

Figen: or we [teachers] can ask them can you examine your graphs again? or Compare the graphs and water tanks? [The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward]

…

Researher: Is there another error or difficulty? What can we ask?

Figen: I think some of the groups can draw the graph as decreasing at that time we [teachers] can ask can you imagine, how can the height be decreasing if the tank has not a hole on it [The broadening question was used in order to make students compare their result if it fits to the real situation]

Rezzan: How can the water [height as a function of the amount of water that is in the tank] decrease while the tank is filling?

Teacher A: In class time, I saw that they [students] cannot draw the graphs, they have difficulty to interpret

Rezzan: if they cannot convert their interpretations, we ask them [students] to use the models [different shapes vase and the water], by trying and observing may them convert to the graph

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher C: Most of them [students] have problem when graphing and I could not interfere

Rezzan: you [teacher C] may help by giving way like how can the height change, while its filling with water can be seen more easily if we give some values, are not?

…

Teacher B: In my class, most of the groups draw the graph by taking the time variable

Rezzan: I saw that in the papers… we [teachers] may overcome this error by asking when I want from you a bucket of water, is it important for me the time? You can fill the bucket with one tap or two tap, what is the time affect at that point?

Figen: or we [teachers] can ask what does the question ask we, which variables relation were asked? Is your graph fit with these? [the evaluative-broadening, if the MEA is not understood, directing students to the activity with the questions]

Lastly, in the fifth implementation of the MEA entitled "Pack Them In!" before the implementation the broadening, exploratory and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. In here especially the evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. The exploratory questions were used in order to make groups to think other approaches. The broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward. During the implementation, the evaluative, exploratory and broadening questions were used. The exploratory questions were used in order to make groups to think other approaches. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. The broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. In addition, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. Moreover, the broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. After the implementation, broadening questions were specified. The broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. In addition, the broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.

The below excerpt was taken from the pre-implementation interview with Figen

Researcher: What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of methods that you will use in order to overcome these students' difficulties?

Figen: may be groups find two kinds of arrangement and do not think other arrangement at that time, it can be asked, may have different installations? Can we arrange more cans in other installation? By this way students look at the question from different ways [exploratory]

…

Researcher: What kinds of students' errors might encounter in the solution process of the model eliciting activity and what kinds of methods that you will use in order to overcome these students' errors?

Figen: They may can arrange the cans but count wrongly or formulize wrongly and think that their arrangement is the appropriate one at that time, we can ask to compare their representations and their formulization, calculation…some groups may not see different arrangement, at that time we [teachers] ask them [students] to use water bottle covers [models] in order to visualize and see the blank spaces between the covers when it is arranged as regularly [the evaluative question is used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations -The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward]

During the implementation

The question below was asked to the groups that cannot think the other arrangement

Figen: When you regularly arrange, there are empty spaces, how can we decrease these empty spaces? [the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.]

The question below was asked to the groups that have a difficulty when thinking the arrangement in three dimensions:

Figen: It is a bit hard to think on three dimensions, why do not you think only the base, then other rows can be calculated based on this base, cannot? [the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective]

The question below was asked to the groups tried only the regular arrangement of the cans arrange:

Figen: Can we arrange differently? Is there another arrangement? [exploratory]

The question below was asked to the groups that arrange the cans diagonally on the base

Figen: How many cans did you arrange with this method? The empty spaces are many or not? [The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward]

The question below was asked to the groups that cannot make connection with mathematics

Figen: try to make clear sketches, how can we formulize those steps? [The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.]

Figen: it is important to think in two dimensions, because when they think in three dimensions, they cannot manage the solution, so I ask them to think only the base…most of the groups cannot think other arrangement so I ask them how we can decrease the empty spaces that are occurred when the covers [water bottle covers-models] are arranged regularly, then from this point of view they can think other arrangement (post-implementation interview with Figen) [the broadening question is used

in order to subconsciously simplify the situation by suggesting an alternative perspective, the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward]

Teacher evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors described based on before the implementation, during the implementation and after the implementation of MEAs.This evolving knowledge was summarized in the Table 4.1.

Table 4.1 Figen's Evolving Knowledge on Questioning

Implementations	**Type of Questions**
First Implementation	
Before the implementation	• invitational • directive questions with the help of the keywords
During the implementation	Did not use any question both for feedback or understanding the solution strategies, teacher did not interfere the solution processes.
After the implementation	• procedural
Second Implementation	
Before the implementation	• directive questions with the help of the keywords • procedural
After the implementation	• broadening if the MEA is not understand, the students directed to the activity with the questions if there is misleading representations, the questions are preferred in order to realize this incorrect representation.

Table 4.1 (continued)

Third Implementation	
Before the implementation	• evaluative (evaluate the strategy) The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • exploratory The exploratory questions are used in order to make groups to think other approaches. • Broadening The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.
During the implementation	• evaluative • exploratory
After the implementation	• exploratory • broadening if the MEA is not understood, directing students to the activity with the questions
Fourth Implementation	
Before the implementation	• broadening • evaluative
After the implementation	• broadening • evaluative
Fifth Implementation	
Before the implementation	• exploratory • evaluative the evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. • broadening The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward.

Table 4.1 (continued)

During the implementation	• broadening the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. • broadening the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. • exploratory • evaluative The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • broadening The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.
After the implementation	• broadening the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the procedural type questions were preferred also in order to obtain feedbacks on students' difficulties and errors, the directive questions with the help of the keywords were used. During the implementation, the procedural questions were stated. After the implementation, it was stated that, answering the students' emerging questions was not an easy task for the teachers.

Rezzan: may we [teachers] say some keywords, if they cannot establish the propositions. We may emphasize the "and" and "or" then they can remember the propositions… they [students] may make numerical errors on the truth tables of the conjunctions at that time we want them to show the truth table, to establish the truth table again so they can realize their errors while they are showing (pre-implementation interview with Rezzan) [procedural- the directive questions with the help of the keywords]

During the implementation

The question below was asked to the groups that make errors on the truth table

Rezzan: Can you show me the "and"'s truth table? Can you establish the truth table again? [procedural]

Rezzan: actually, I can only interfere the numerical errors, I do not so much interfere their [students] solutions. Actually, it is hard to answer at that time to students' questions since first we [teachers] should understand their solution process in order to understand their questions (post-implementation interview with Rezzan)

In the second implementation of the MEA entitled "Street Parking", before the implementation procedural questions were preferred. After the implementation, broadening questions were denoted. For example, if the MEA was not understood, the students directed to the activity with the questions. In addition, it was also stated that "it is hard to ask questions? What to say? How to interfere?".

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher A: if they [students] take wrongly parking space dimensions, the solution will go wrongly

Teacher B: I think some of the groups will not even try the angle parking, they easily say the answer that parallel park is okay

Teacher A: May be some groups use the angle parking but cannot use the trigonometry

Researcher: What kinds of questions a teacher can ask at this times?

Figen: We [teachers] can ask, are the angle important for parking the maximum numbers of cars? So with the little orientation, we can direct students to use trigonometry rather than saying use trigonometry. I think when the angle emphasized then they will understand.

Teacher B: they may have also make the trigonometric calculations wrongly or while writing the ratios of the similarity

Figen: at that time we can ask them do the trigonometric operations again

Teachers A: In here, students can make many numeric errors since it has so many operations

…

Teachers C: I think they can not manage the operations, if they make errors in the operations, the solution is not go on

Rezzan: we [teachers] can make the students do the calculations again; it can be asked can you find the similarity ratio again? Or can you show me the trigonometric ratios again? [procedural]

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Figen: Most of the groups cannot understand the question fully because most of them take the hypotenuse 4.8m, so I think it is important to ask them read the question again, are you carefully examine the question, the data?

Rezzan: where is 4.8 m? What are the parking spaces dimensions? By this way they [students] read the question again and understand [broadening- if the MEA was not understood, the students directed to the activity with the questions]

…

Teacher B: some of them[students] even could not place the car on the parking space, they made wrong drawings. Then I ask do not you see your father when he is parking.

Teacher A: It is very important to make right drawing. I observed these in my class also, I ask them is your drawing right? Then they look and said yes, then I do not interfere, I could not decide what to ask what to say?

Figen: in this case we [teachers] may ask for example, when you park with angle, the car can go to the in front of the parking spaces completely? Are there any dead space? however I agree with you that we [teachers] do not know how to interfere and how much, and what kinds of questions can be asked

Rezzan: we [teachers] make nontraditional implementations so it is hard to ask questions? What to say? How to interfere?

In the third implementation of the MEA entitled "Summer Jobs", before the implementation the exploratory and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. The exploratory questions were used in order to make groups to think other approaches. During the implementation, the broadening, exploratory and evaluative questions were used. The broadening question was used, for directing the students to the activity in order to be sure that the question was understood, the task was understood. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations. The exploratory questions were used in order to make groups to think other approaches. After the implementation, the exploratory questions and broadening questions (if the MEA is not understood, directing students to the activity with the questions were preferred) were denoted.

Rezzan: "first of all it is important to students understand their strategies meaning so I will ask from this point of view, what is your aim? Is your strategy fit with the boss aim?...if they tried to decide the part time workers based on the total money per hour for each worker than it can be asked is there another criterion for deciding the part time workers?" (pre-implementation interview with Rezzan) [The evaluative questions was used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward- exploratory]

During the implementation

The question below was asked to the all groups in order to see if they understand the question or not:

Rezzan: What is being asked? What does it mean the full time and part time worker?

The question below was asked to the groups whose approach and the calculations were not fit.

Rezzan: what is your approach?... okay you are using the steady, slow and busy times. Do you calculate the mean based on these periods? [The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations]

The question below was asked to most of the groups in order to make them think other approaches.

Rezzan: do you find other criteria in order to decide the part time workers? Of course based on the criteria that are given in the question [exploratory]

Rezzan: Most of the groups have a problem while they are deciding the part time workers, at that time I asked, are there another criterion in order to decide part time workers? ...I try to direct the students to the question for to be sure that the question were understood and asked questions like what is the aim? What is your mission? (post-implementation interview with Rezzan) [exploratory - broadening questions, if the MEA is not understood, directing students to the activity with the questions]

In the fourth implementation of the MEA entitled "Water Tank", before the implementation the broadening and evaluative (evaluate the strategy) questions were preferred. The broadening question was used for offering a situation in which it is clear what type of mathematics should be used and also the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. After the implementation, the broadening questions were denoted. The broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. In addition, the broadening question was used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher B: I think students will have difficulty while drawing the graphs, most of them can draw the graphs linear, what can we [teachers] do at that time?

Rezzan: we [teachers] can ask do you remember the logarithmic and exponential functions' graph. So that they can relate the graphs [The broadening question was used for offering a situation in which it is clear what type of mathematics should be used]

Teacher A: they may also confuse the concave up and concave down graphs, they may draw concave up when the water tank volume increase

Teacher B: we [teachers] can want them to draw their graph again, or ask it is true? Is the height increasing?

Figen: or we [teachers] can ask them can you examine your graphs again? or Compare the graphs and water tanks?

…

Researher: Is there another error or difficulty? What can we ask?

Figen: I think some of the groups can draw the graph as decreasing at that time we [teachers] can ask can you imagine, how can the height be decreasing if the tank has not a hole on it

Rezzan: How can the water [height as a function of the amount of water that is in the tank] decrease while the tank is filling? [The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

Teacher A: In class time, I saw that they [students] cannot draw the graphs, they have difficulty to interpret

Rezzan: if they cannot convert their interpretations, we ask them [students] to use the models [different shapes vase and the water], by trying and observing may them convert to the graph [broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations]

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher C: Most of them [students] have problem when graphing and I could not interfere

Rezzan: you [teacher C] may help by giving way like how can the height change, while its filling with water can be seen more easily if we give some values, are not? [broadening question was used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward]

…

Teacher B: In my class, most of the groups draw the graph by taking the time variable

Rezzan: I saw that in the papers… we [teachers] may overcome this error by asking when I want from you a bucket of water, is it important for me the time? You can fill the bucket with one tap or two tap, what is the time affect at that point? [broadening question was used in order to help students to imagine the real situation clearly by giving a real life example]

Figen: or we [teachers] can ask what does the question ask we, which variables relation were asked? Is your graph fit with these?

Lastly, in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation the broadening, exploratory and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. The exploratory questions were used in order to make groups to think

other approaches. The broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. In addition, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. During the implementation, broadening questions were used. The broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. Also, the broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward. In addition, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. Moreover, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. After the implementation, broadening questions were specified. The broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. In addition, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective

The below excerpt was taken from pre-implementation with Rezzan

Researcher: What kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' errors?

Rezzan: I think most of the groups can made errors based on their sketches and calculation at that time we can ask them to compare their sketches and calculations [The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

…

Researcher: What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' difficulties?

Rezzan: they [students] may have problems while thinking the different arrangement, and then we can ask them questions that make them to think the situation by connecting the real life, like how can you arrange the bottles to safes? How can arrange the cookies to tray…they [students] may make one arrangement and then do not think the other arrangement, we should ask them to think other arrangement, there is not another way for the arrangement? [broadening question was used in order to help students to imagine the real situation clearly by giving a real life example- exploratory]

Researcher: Do you want to add anyting?

Rezzan: Hmmm…Also, I think most of the students do not use the mathematics, do not formulize, I will ask at that time how we can formulize this sketch? How can we convert these operations to a mathematical formula?[broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations]

During the implementation

The questions below was asked to the groups that cannot think the different arrangement

Rezzan: Think that you are making cookies and you have a one tray but if you arrange regularly, there are cookies that cannot be arranged to the tray, how can you squeeze them?[broadening question was used in order to help students to imagine the real situation clearly by giving a real life example]

Rezzan: How can you arrange the models [water bottle covers] differently? Try it in your desk [the broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward]

The questions below was asked to the groups that cannot think in three dimension

Rezzan: Why do not think only the base; you do not need to think in tree dimension? How can you arrange the cans to the base differently? [the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective]

The question below was asked to the groups that make the different arrangement but cannot convert to math.

Rezzan: Make the sketches more clear than how can you convert your operations to a math formula? Think on it [the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations]

Rezzan: "most of the groups cannot think the other arrangement at that point I give examples and want them to think on it, for example I ask them, think that you are making cookies and you have a one tray but if you arrange regularly, there are cookies that cannot be arranged to the tray, how can you squeeze them? Then all the groups think other arrangement, I think this is important, by giving an example they can imagine easily…and some groups have a problem while they are thinking in three dimensions and I ask them why they do not think only the base? Then they think in two dimensions easily, I think they need a new perspective and they need it, then they go one step forward on their solutions" (post-implementation interview with Rezzan) [broadening question was used in order to help students to imagine the real situation clearly by giving a real life example - broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective]

Teacher evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in the table below.

Table 4.2 Rezzan's Evolving Knowledge on Questioning

Implementations	Type of Questions
First Implementation	
Before the implementation	• procedural • directive questions with the help of the keywords
During the implementation	• procedural
After the implementation	• it was stated that, answering the students' emerging questions is not an easy task for the teachers.
Second Implementation	
Before the implementation	• procedural
After the implementation	• broadening if the MEA is not understood, the students directed to the activity with the questions.
Third Implementation	
Before the implementation	• exploratory The exploratory questions are used in order to make groups to think other approaches. • evaluative (evaluate the strategy) The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.
During the implementation	• broadening The broadening question is used, for directing the students to the activity in order to be sure that the question was understood, the task was understood. • evaluative The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations. • Exploratory The exploratory questions are used in order to make groups to think other approaches.
After the implementation	• exploratory • broadening if the MEA is not understood, directing students to the activity with the questions

Table 4.2 (continued)

Fourth Implementation	
Before the implementation	• broadening The broadening question is used for offering a situation in which it is clear what type of mathematics should be used. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations. • evaluative The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.
After the implementation	• broadening The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example.
Fifth Implementation	
Before the implementation	• evaluative The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. • exploratory The exploratory questions are used in order to make groups to think other approaches. • Broadening The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.

Table 4.2 (continued)

During the implementation	• Broadening The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example. The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward. the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.
After the implementation	• broadening The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example. The broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the procedural type and invitational type questions were preferred. During the implementation, it was stated that every different approach is important and think the solution ways whatever you want and Semra did not interfere the solution processes of the groups. After the implementation, it was stated that, she let the students struggle with the question and want the students use approaches whatever they want, also did not interfere the solution process.

Semra: *they [students] may made errors while they are establishing the truth tables, in this error, I want them to look their tables again and ask them for example if there is an error in the "or" conjunction, show me the truth table based on the "or" conjunctions …in this kind of problems [MEAs], students may afraid of the problem and can not begin at that time I will plan to ask each group, do you understand? Do you have questions? So that they may begin to think on it (pre-implementation interview with Semra)* [procedural-invitational]

During the implementation

It was stated that every different approach was important and think the solution ways whatever you want. Semra did not interfere the solution processes of the groups (from observation notes)

Semra: I want them [students] struggle with the question so I did not interfere to their solutions, I said them to use every approach whatever they want (post-implementation interview with Semra)

In the second implementation of the MEA entitled "Street Parking", before the implementation procedural questions were preferred. After the implementation, procedural and broadening questions were denoted. The broadening question was used for when the MEA was not understood, the students directed to the activity with the questions.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Semra: …they [students] may use many angles but when come to the operation I think they will make many errors at that time we want them to calculate their operations again. If they write the trigonometric function wrongly, we may ask them write me the trigonometric functions again by this way they may realize their errors [procedural]

Teacher D: I think students made many calculation errors

Melda: we should ask them to calculate their operations again

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Semra: for example, most of the groups made many numerical errors, some of them get the similarity ratio wrong, and some of them have errors while they are calculating the Pythagorean Theorem, so at that time we [teachers] can ask them can you write me the Pythagorean theorem again, Can you show me the similar triangle and write the ratio again? [procedural]

Teacher D: I saw that most of the group cannot arrange the car to the parking space with the angle

Teacher F: Groups use 4.8m wrongly

Melda: Most of the groups misunderstand the 4.8m; I think they did not read the question properly, so I could ask, where is 4,8m? The length of the parking spaces or the car? What are the givens in the question? I think with these questions they could read the question again and again

…

Researcher: What kinds of students' difficulties encountered in the solution process of the model eliciting activity and what kinds of questions that could be used in order to overcome these students' difficulties?

Semra: there are problems based on not understanding the question, so we can ask where 4.8 m is? Is it belonging to parking space or the car? What is your aim? With these questions, I think groups can focus on understanding the problem [broadening, if the MEA is not understand, the students directed to the activity with the questions]

In the third implementation of the MEA entitled "Summer Jobs", before the implementation the exploratory, evaluative (evaluate the strategy) and broadening questions were preferred. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. The

exploratory questions were used in order to make groups to think other approaches. In addition, the broadening question was used for offering a situation in which it is clear what type of mathematics should be used. During the implementation, the broadening and exploratory questions were used. The exploratory questions were used in order to make groups to think other approaches. The broadening question was used for offering a situation in which it is clear what type of mathematics should be used. In addition, the broadening question was used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward. After the implementation, exploratory and evaluative (evaluate the strategy) questions were denoted. The exploratory questions were used in order to make groups to think other approaches. In addition, the evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.

The below excerpt was taken from the pre-implementation interview with Semra

Semra: I think the biggest problem will be if the groups cannot think the money per hour at that time, I can ask how you can calculate your grade. By this way, they may realize the main idea…Also they [students] can make errors as if they may want to take the busy, steady and slow periods as a criterion but calculate the money per hour based on months; at that point, it can be asked, "What is your criterion? What are your calculations are based on, which data did you use? So they may evaluate their approach [evaluative question was used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations - the broadening question was used for offering a situation in which it is clear what type of mathematics should be used]

Researcher: Other difficulties or errors?

Semra: some groups may not think the other criteria except the general mean

Researcher: What will you do?

Semra: I can ask that groups are there another criterion. Can we decide the part time workers in another way?" [exploratory]

During the implementation

The question below was asked to the groups that use ratio while comparing the workers

Semra: Is there another way in order to compare? You compare two by two is it another criterion that you can compare all the workers [exploratory]

The question below was asked to the groups that cannot establish any approach:

Semra: how can you calculate your grade at the end of the semester? Think how you can compare the workers? [broadening question was used for offering a situation in which it is clear what type of mathematics should be used]

The question below was asked to the groups that use general mean for deciding the part time workers and the full time workers

Semra: Is there another criterion? In the part-time work, months or periods [steady-busy-slow] are important or another criterion? [broadening question was used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward]

Semra: some groups use very long approaches like ratio two by two, at that time, I think it is important to make them think the other approaches so I ask is there another criterion that let the comparison between all workers…it is important for students to understand their solution ways, so I could also be ask what is the aim of your approach? Do you use the appropriate data? (post-implementation interview with Semra) [exploratory - evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward]

In the fourth implementation of the MEA entitled "Water Tank", before the implementation the broadening and evaluative (evaluate the strategy) questions were preferred. The broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. Other broadening question was used when the MEA was not understand, the students directed to the activity with the questions.The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. After the implementation, the broadening and evaluative questions were denoted. The broadening question was used for offering a situation in which it is clear what type of mathematics should be used and the evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher D: I think most of the groups cannot draw the graph or they will draw linear graph for all the tanks

Semra: In our lessons, it is also a hard work for them [students]

Researcher: What can we do?

Semra: At that point, it can be said to give height and volume of the water values and ask them to transfer these values to coordinate system. By this way, they may think how can be the graphs [broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward]

Melda: I think, they interpret truly but they may draw linearly rather than curve, after taking values as you [Semra] say, we want them [groups] control their graphs by asking, are your graphs represent your interpretations. Then they may realize that it cannot be linear

…

Semra: I think some of the groups may interpret like it is increasing but slowly increasing but they may draw decreasing functions at that time we want them to take values and control their graphs by asking, are your graphs represent your interpretations? Alternatively, we want them to read the question again and ask what are the values? What is our mission? In order to make them understand the question clearly [broadening, if the MEA is not understand, the students directed to the activity with the questions- evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

Melda: or we [teachers] can give an example like, when you filling a bottle with water, does the height decrease or increase? They may then realize their error

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher D: in my class all of them [groups] interpret truly but draw linear graphs for all tanks, they cannot draw concave up and concave down graphs

Teacher F: What do you do?

Teacher D: I asked if it is linear or not? They think on it but then did not change their graph

Semra: You [Teacher D] may ask them [students] if the linear graph represent their interpretation. When they explaining you then they may realize their incorrect drawing [evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

Teacher F: you could [Teacher D] also ask how it can be same for all the tanks?

Melda: Yes, it could be asked, do you draw a linear graph for the cylinder, Is it also same for the sphere? Or want them to take values and draw the graphs again, they may realize their inaccurate graphs

Semra: It could also be asked, do you remember the exponential numbers' graph. Then they may think the curves [broadening question was used for offering a situation in which it is clear what type of mathematics should be used]

Lastly, in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation the broadening and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. The broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. In addition, the broadening question was used when the MEA is not understood, the students directed to the activity with the questions. Moreover, the broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. During the implementation, broadening and exploratory questions were used. The exploratory questions were used in order to make groups to think other approaches. The broadening question was used when the MEA was not understood, the students

directed to the activity with the questions. Besides, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. Moreover, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. After the implementation, broadening and exploratory questions were specified. The exploratory questions was used in order to make groups to think other approaches. The broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. Additionaly, the broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward. Besides, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.

The below excerpt was taken from the pre-implementation interview with Semra

Researcher: What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' difficulties?

Semra: I think most of the groups, will not convert their operations to mathematics, so I will want them made clear sketches and then ask them how you can write your operations in an algebraic form…if the students do not think the different arrangement then I can ask what the question asks? What is the aim? Is your arrangement fit with the desired aim? I think when they understand the question clearly and compare their approaches then they will think the different arrangement [evaluative question was used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward - broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations - the broadening question was used when the MEA is not understood, the students directed to the activity with the questions]

Researcher: Another difficulty?

Semra: …groups may give up after placing regularly then I can give examples like you have many bottles and you have to arrange it to the one shelf of the refrigerator, however when you arrange regularly some of the bottles are leftover. How can we squeeze them? May with this example they can visualize more easily [the broadening question was used in order to help students to imagine the real situation clearly by giving a real life example]

During the implementation

The question below was asked to the groups that understand the question wrongly

Semra: Is your arrangement fit with the aim of the question? What the question ask us? When you turn the storage unit, is the height will be constant? [broadening question was used when the MEA was not understood, the students directed to the activity with the questions]

The question below was asked to the groups that had difficulty while thinking the arrangement in three dimensions

Semra: Can we think in two dimensions? How can we arrange the cans to the base differently?[broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective]

The question below was asked to the groups that think the regular arrangement only

Semra: Are there different arrangement of the cans? Are there another way to arrange the cans?[exploratory]

The question below was asked to the groups that cannot convert the operations to mathematics.

Semra: make clear sketches and then think how you can convert each operation to mathematics and to form a formula [broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations]

Semra: First, I think it is important to make them to think different arrangement, some of them make the regular arrangement and give up, so I said how can we arrange differently? Is there another way to arrange? Also some groups cannot imagine easily, I ask them to use models [water bottle covers], by trying, they discovered. And two groups make the arrangement in three dimensions persistently, I ask them why not you think in two dimension then they try to arrange only on base and can think the different arrangement… there are groups cannot calculate the number of the cans, I asked them how many cans in your columns?, how many cans in your rows? How can we formulize the number of the cans? Then I want them to make clear sketches and then try to formulize their steps… (post-implementation interview with Semra) [exploratory - broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective - broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward - broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations]

Teacher evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in the table below.

Table 4.3 Semra's Evolving Knowledge on Questioning

Implementations	Type of Questions
First Implementation	
Before the implementation	• procedural • invitational
During the implementation	It was stated that "every different approach is important and think the solution ways whatever you want". Semra did not interfere the solution processes of the groups.
After the implementation	it is stated that, she let the students struggle with the question and want the students use approaches whatever they want, also did not interfere the solution process.
Second Implementation	
Before the implementation	• procedural
After the implementation	• procedural • broadening if the MEA is not understand, the students directed to the activity with the questions
Third Implementation	
Before the implementation	• broadening The broadening question is used for offering a situation in which it is clear what type of mathematics should be used. • evaluative The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. • exploratory The exploratory questions are used in order to make groups to think other approaches.

Table 4.3 (continued)

During the implementation	• exploratory The exploratory questions are used in order to make groups to think other approaches. • Broadening The broadening question is used for offering a situation in which it is clear what type of mathematics should be used. The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.
After the implementation	• exploratory The exploratory questions are used in order to make groups to think other approaches. • evaluative The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.
Fourth Implementation	
Before the implementation	• broadening The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. if the MEA is not understand, the students directed to the activity with the questions • evaluative (evaluate the strategy) The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.
After the implementation	• broadening The broadening question is used for offering a situation in which it is clear what type of mathematics should be used. • evaluative (evaluate the strategy) The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.

Table 4.3 (continued)

Fifth Implementation	
Before the implementation	• broadening The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. if the MEA is not understood, the students directed to the activity with the questions The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example. • evaluative (evaluate the strategy) The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.
During the implementation	• broadening if the MEA is not understand, the students directed to the activity with the questions The broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations. • exploratory The exploratory questions are used in order to make groups to think other approaches

Table 4.3 (continued)

After the implementation	• exploratory The exploratory questions are used in order to make groups to think other approaches. • broadening The broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.

Melda's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the procedural type questions were preferred. During the implementation, directive questions were used. Melda used directive questions in order to lead the student step by step to the desired end. After the implementation, the question was not offered but it was stated that it could not be directive, it should be more general.

Melda: they [students] can made errors, especially in the truth table which include all the conjunctions, I may want them to calculate the columns that have errors or I may ask them can you establish me the truth table step by step? (pre-implementation interview with Melda) [procedural]

During the implementation

The questions below was asked to the all groups.

Melda: How can we use the propositions? Which conjunctions will you use? Then do you establish a truth table? [directive questions in order to lead the student step by step to the desired end]

The below excerpt was taken from the post-implementation interview with Melda

Melda: "All the groups use propositions and the truth table; I think as a teacher, I direct them so much. For example one of the group tried to use sets but I direct them to logic, I should not be interfere so much, I may ask more general questions"

Researcher: What kinds of questions?

Melda: I can not think know but I should not direct them so much

In the second implementation of the MEA entitled "Street Parking", before the implementation procedural question was stated. After the implementation, broadening questions was denoted. The broadening question was used for when the MEA was not understood, the students directed to the activity with the questions.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Semra: …they [students] may use many angles but when come to the operation I think they will make many errors at that time we want them to calculate their operations again. If they write the trigonometric function wrongly, we may ask them write me the trigonometric functions again by this way they may realize their errors

Teacher D: I think students made many calculation errors

Melda: we should ask them to calculate their operations again [procedural]

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Semra: for example, most of the groups made many numerical errors, some of them get the similarity ratio wrong, and some of them have errors while they are calculating the Pythagorean Theorem, so at that time we [teachers] can ask them can you write me the Pythagorean theorem again, Can you show me the similar triangle and write the ratio again?

Teacher D: I saw that most of the group cannot arrange the car to the parking space with the angle

Teacher F: Groups use 4.8m wrongly

Melda: Most of the groups misunderstand the 4.8m; I think they did not read the question properly, so I could ask, where is 4,8m? The length of the parking spaces or the car? What are the givens in the question? I think with these questions they could read the question again and again [broadening, if the MEA was not understood, the students directed to the activity with the questions]

…

Researcher: What kinds of students' difficulties encountered in the solution process of the model eliciting activity and what kinds of questions that could be used in order to overcome these students' difficulties?

Semra: there are problems based on not understanding the question, so we can ask where 4.8 m is? Is it belonging to parking space or the car? What is your aim? With these questions, I think groups can focus on understanding the problem

In the third implementation of the MEA entitled "Summer Jobs", before the implementation the exploratory and evaluative (evaluate the strategy) questions were preferred. The evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. In addition, the exploratory questions were used in order to make groups to think other approaches. During the

implementation, the broadening and evaluative (evaluate the strategy) questions were used. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations. The broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. After the implementation, broadening and evaluative (evaluate the strategy) questions were denoted. The broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. In addition, the evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations.

Melda: they [students] will have difficulty to decide the workers, because they may decide the workers that not fit with their criterion so they should know what they do, then I will ask what your criteria is? Your choices fit with your criteria?...according to me all students will try to decide all workers based on the general mean at that time I can ask how can we find the part time workers in another way? Are there other criteria? (pre-implementation interview with Melda) [exploratory - evaluative questions were used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward]

During the implementation

The question below was asked to the groups whose solutions have a mismatch between their approach and the calculations

Melda: How can you decide the part time workers? Which table did you use? Are your calculations and approach compatible? [evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations]

The question below was asked to the groups who use the general mean for both full time and part time workers

Melda: Is there another criterion in order to decide the part time workers? Can we decide based on months or busy-steady-slow times? [broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward]

Melda: in the implementation most of the time I repeat the givens in the question to the groups but when I realize that some groups cannot find starting point for the part time workers at that time I asked can we decide part time workers based on months or periods? Then they will go on their solutions, they got a new view…also most of the time I ask all the groups, what are your criteria? Which data did you use? Your approach and the calculations are compatible. I want them to look at their solutions again and again since if they made errors then they will lose many times and the using the time appropriately is important (post-implementation interview with Melda) [broadening question was used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward - evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations]

In the fourth implementation of the MEA entitled "Water Tank", before the implementation the broadening and evaluative (evaluate the strategy) questions were preferred. The broadening question was used in order to help students to imagine the real situation clearly by giving a real life example. In addition, the evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. After the implementation, evaluative questions were denoted. The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Teacher D: I think most of the groups cannot draw the graph or they will draw linear graph for all the tanks

Semra: In our lessons, it is also a hard work for them [students]

Researcher: What can we do?

Semra: At that point, it can be said to give height and volume of the water values and ask them to transfer these values to coordinate system. By this way, they may think how can be the graphs

Melda: I think, they interpret truly but they may draw linearly rather than curve, after taking values as you [Semra] say, we want them [groups] control their graphs by asking, are your graphs represent your interpretations. Then they may realize that it cannot be linear [evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

…

Semra: I think some of the groups may interpret like it is increasing but slowly increasing but they may draw decreasing functions at that time we want them to take values and control their graphs by asking, are your graphs represent your interpretations? Alternatively, we want them to read the question again and ask what are the values? What is our mission? In order to make them understand the question clearly

Melda: or we [teachers] can give an example like, when you filling a bottle with water, does the height decrease or increase? They may then realize their error [broadening question was used in order to help students to imagine the real situation clearly by giving a real life example]

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher D: in my class all of them [groups] interpret truly but draw linear graphs for all tanks, they cannot draw concave up and concave down graphs

Teacher F: What do you do?

Teacher D: I asked if it is linear or not? They think on it but then did not change their graph

Semra: You [Teacher D] may ask them [students] if the linear graph represent their interpretation. When they explaining you then they may realize their incorrect drawing

Teacher F: you could [Teacher D] also ask how it can be same for all the tanks?

Melda: Yes, it could be asked, do you draw a linear graph for the cylinder, Is it also same for the sphere? Or want them to take values and draw the graphs again, they may realize their inaccurate graphs [evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations]

Semra: It could also be asked, do you remember the exponential numbers' graph. Then they may think the curves

Lastly, in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation the broadening, evaluative and exploratory questions were preferred. The exploratory questions were used in order to make groups to think other approaches. In addition, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations. During the implementation, broadening and exploratory questions were used. The exploratory questions were used in order to make groups to think other approaches. The broadening question was used when the MEA is not understood, the students directed to the activity with the questions. Besides, the broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward. In addition, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. Moreover, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations. After the implementation, broadening and exploratory questions were specified. The exploratory questions were used in order to make groups to think other approaches. The broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward. In addition, the broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective. In addition, the broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations.

Melda: the groups can give up when they made the regular arrangement at that time we [teachers] should encourage them to think other arrangement. Your method is the most advantageous one for the maximum number of cans? Is there another arrangement? How can we arrange differently?...I think most of the student will calculate the number of cans as estimated, I will want them to use mathematics. I can ask, how can we formulize your operations? I want them try their sketches and calculations to convert to algebraic forms, formulization (pre-implementation interview with Melda)[exploratory- evaluative- broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations]

During the implementation

The question below was asked to the groups that misunderstand the question

Melda: when you convert the storage units, does the height be constant or change? What the question say us? What are the givens? What is your mission? [broadening question was used when the MEA is not understood, the students directed to the activity with the questions]

The question below was asked to the groups that made only the regular arrangement

Melda: Is there another arrangement in order to minimize the cost? How can we arrange differently in order to maximize the number of cans? [exploratory]

Melda: Why do not you use the models? How can we arrange the covers [water bottle covers] differently? [the broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward]

The question below was asked to the groups that made the arrangement in three dimensions

Melda: it is hard to think in three dimensions, is not? Why do not you think only the base think the different arrangement on base then convert it to the three dimensions?[broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective]

The question below was asked to the groups that do not use the mathematics

Melda: How many cans in each row and columns? How can we formulize this number of cans? If you make clear sketches and convert your operations to algebraic forms, this may help you [broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations]

Melda: students had difficulty when thinking the different arrangement other than regular form at that time I asked them try different arrangement by using the covers [water bottle covers] then they began to realize squeezing. Besides, one group tried to think in three dimensions, I want them to think the base only, and then they thought more easily in two dimensions. I ask every group to think different arrangement, actually one group find all the arrangement that we [teachers] found. However, most of the groups had a problem while converting to mathematics at that time I want them to make clear sketches, while some groups made clear sketches they see the special triangles and can convert to math. However some groups cannot convert, they only will estimate the number of the cans. I may direct them to the special triangles but at that time it will be my solution not them…If we direct them to the result we cannot know what they think, they only are directed to my solution. Therefore, the teacher guidance is so important at that kind of problems, if we [teachers] directed so much, what the students can gain (post-implementation interview with Melda) [exploratory - broadening question was used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward - broadening question was used in order to subconsciously simplify the situation by suggesting an alternative perspective - broadening question was used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations]

Teacher evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in the table below.

Table 4.4 Melda's Evolving Knowledge on Questioning

Implementations	**Type of Questions**
First Implementation	
Before the implementation	• procedural
During the implementation	• directive Directive questions are used to lead the student step by step to the desired end.
After the implementation	the question was not offered but it was stated that it could not be directive, it should be more general
Second Implementation	
Before the implementation	• procedural
After the implementation	• broadening if the MEA is not understood, the students directed to the activity with the questions.
Third Implementation	
Before the implementation	• evaluative The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • exploratory The exploratory questions are used in order to make groups to think other approaches.

Table 4. 4 (continued)

During the implementation	• evaluative The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations. • Broadening The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.
After the implementation	• broadening The broadening question is used in order to suggest an alternative perspective and prompt student's to give explanations that will move their thinking forward. • evaluative The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their assumptions and calculations.
Fourth Implementation	
Before the implementation	• broadening The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example. • evaluative The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.
After the implementation	• evaluative The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.

Table 4. 4 (continued)

Fifth Implementation	
Before the implementation	• exploratory The exploratory questions are used in order to make groups to think other approaches. • broadening The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations.
During the implementation	• broadening if the MEA is not understood, the students directed to the activity with the questions. • exploratory The exploratory questions are used in order to make groups to think other approaches. • broadening The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward. the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations.

Table 4. 4 (continued)

After the implementation	• broadening The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward. The broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use models, sketches and formulizations. • exploratory The exploratory questions are used in order to make groups to think other approaches.

4.1.1.1 Cross-case Analysis of Knowledge of Questions to Students

Under this subheading cross-case analysis results of knowledge of questions to students what is similar and different about the cases' evolving were taken account. Each case's evolving knowledge on questioning while students are working on the MEA and questioning in order to give feedbacks on students' difficulties and errors was compared in Table 4.5.

Table 4.5 Case Ratings of Questioning Themes for All Implementations

Case Themes	**Cases***			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
Theme 1: invitational	1		1	
Theme 2:				
• directive questions with the help of the keywords	1,2	1		
• Directive questions are used to lead the student step by step to the desired end.				1
Theme 3: procedural	1,2	1,2	1,2	1,2
Theme 4: broadening				
• if the MEA is not understood, the students directed to the activity with the questions	2,3,4	2,3	2,4,5	2,5
• if there is misleading representations, the questions are preferred in order to realize this incorrect representation.	2			
• The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.	3,5	4	3,4	3
• The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward.	5	5	5	5
• the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective.	5	5	5	5
• The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.	5	4,5	5	5
• The broadening question is used for offering a situation in which it is clear what type of mathematics should be used.		4	3,4	
• The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example.	4	4,5	5	4

Table 4. 5 (continued)

Theme 5: evaluative (evaluate the strategy)				
• The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.	3,4,5	3	3,5	3
• The evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.	5	3,4,5	3,4	3,4
Theme 6: exploratory				
• The exploratory questions are used in order to make groups to think other approaches.	3,5	3,5	3,5	3,5

* 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

The teachers gave many examples of questions they might ask and asked to stimulate students' thinking and learning while working on the MEA, before and after the implementations. These questions were categorized as directive, broadening, procedural, invitational, exploratory and evaluative. The directive questions were designed to lead students along a predetermined path, and broadening questions, intended to provoke students to expand their ideas. All the teachers questioning technique changed from directive to broadeding, from first implementations to the fifth implementation of the study. In the first implementations, all the teachers denoted invitational (do you understand? do you have questions?) and procedular (show me the truth table of the "and" conjunction) type questions. In addition, two of the teachers used directive questions with the help of the keywords (where we use the propositions?) that try to lead the students with the keywords and one of the teachers used directive questions (How can we use the propositions? Which conjunctions will you use? Then do you establish a truth table?) in order to lead the student step by step to the desired end. However, in the other implementations teachers began to state, broadening, evaluative (evaluate the strategy) and exploratory questions. The broadening questions were used for different purposes, in order to help students to imagine the real situation clearly by

giving a real life example, in order to offer a situation in which it is clear what type of mathematics should be used, in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations, in order to subconsciously simplify the situation by suggesting an alternative perspective, in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward, in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward and when the MEA is not understand, the students directed to the activity with the broadening questions. In addition, evaluative questions were specified for the puposes of prompting students to give explanations that will move their thinking forward by identifying a mismatch between their representations, assumptions and calculations and making groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. Moreover, the exploratory questions (Is there another way to do this?) were used in order to make groups to think other approaches.

It was seen that teachers questioning techniques were changed from directing students along a predetermined path to exposing their thinking with broadening, exploratory and the evaluative questions. The reason for this change might be the fact that teachers began to listen their students' way of thinking in the implementations; teachers began to try to understand students' emerging questions as time passes. This change was also occur by discussing how to ask questions on the error and difficulties that encountered or might encounter in the solution process and unexpected solutions with their colleagues. In the first implementations; the observations revealed that teachers did not listen students solution ways exactly, they immediately tried to direct them to the their solutions step by step, only use invitational types questions then let them struggle or only focused on the solution papers and asked procedural type questions based on the numerical errors. Additionally, this observation was supported by teachers' own words that it was seen as a hard work to understand students' solutions and to answer students' emerging thinking.

Rezzan: actually, I can only interfere the numerical errors, I do not so much interfere their [students] solutions. Actually, it is really hard to answer at that time to students' questions since first we [teachers] should understand their solution process in order to understand their questions (first implementation post-interview with Rezzan)

Melda: we [teachers] accustomed to help students in the class, as a teacher we want them [students] solve the question immediately so I try to give clues (first implementation post-interview with Melda)

Semra: I think, it is not important that groups reach the result or not so I let them [students] struggle and use approaches whatever they want (first implementation post- interview with Semra)

In the third, fourth and the fifth implementations, it was observed that teachers spend more time near the groups and listened their solution ways and listened students' emerging questions and asked questions in order to expose their thinking and provoke students to expand their ideas. Data analysis showed that teachers tried to produce questions based on the errors and the difficulties that was specified by their colleagues in before the implementation meetings and tried to produce questions based on the unexpected solution ways, unexpected errors and difficulties that their colleagues or they specified in after the implementation meetings.

The below excerpt was taken from the dialogue among the teachers in the meeting before the fourth implementation denoted that teachers tried to produce questions based on the errors and the difficulties that was specified by their colleagues in Anatolian High School.

Teacher B: I think students will have difficulty while drawing the graphs, most of them can draw the graphs linear, what can we [teachers] do at that time?

Rezzan: we [teachers] can ask do you remember the logarithmic and exponential functions' graph. So that they can relate the graphs

Teacher A: they may also confuse the concave up and concave down graphs, they may draw concave up when the water tank volume increase

Teacher B: we [teachers] can want them to draw their graph again, or ask it is true? Is the height increasing?

Figen: or we [teachers] can ask them can you examine your graphs again? or Compare the graphs and water tanks?

The below excerpt was taken from the dialogue among the teachers in the meeting after the fourth implementation denoted that teachers tried to produce questions based on the unexpected solution ways, unexpected errors and difficulties that their colleagues or they specified in Anatolian High School.

Teacher B: In my class, most of the groups draw the graph by taking the time variable

Rezzan: I saw that in the papers… we [teachers] may overcome this error by asking when I want from you a bucket of water, is it important for me the time? You can fill the bucket with one tap or two tap, what is the time affect at that point?

Figen: or we [teachers] can ask what does the question ask we, which variables relation were asked? Is your graph fit with these?

The below excerpt was taken from the dialogue among the teachers in the meeting before the fourth implementation denoted that teachers tried to produce questions based on the errors and the difficulties that was specified by their colleagues in Anatolian Teacher Training High School.

Teacher D: I think most of the groups cannot draw the graph or they will draw linear graph for all the tanks

Semra: In our lessons, it is also a hard work for them [students]

Researcher: What can we do?

Semra: At that point, it can be said to give height and volume of the water values and ask them to transfer these values to coordinate system. By this way, they may think how can be the graphs

Melda: I think, they interpret truly but they may draw linearly rather than curve, after taking values as you [Semra] say, we want them [groups] control their graphs by asking, are your graphs represent your interpretations. Then they may realize that it cannot be linear

…

Semra: I think some of the groups may interpret like it is increasing but slowly increasing but they may draw decreasing functions at that time we want them to take values and control their graphs by asking, are your graphs represent your interpretations? Alternatively, we want them to read the question again and ask what are the values? What is our mission? In order to make them understand the question clearly

Melda: or we [teachers] can give an example like, when you filling a bottle with water, does the height decrease or increase? They may then realize their error

The below excerpt was taken from the dialogue among the teachers in the meeting after the fourth implementation denoted that teachers tried to produce questions based on the unexpected solution ways, unexpected errors and difficulties that their colleagues or they specified in Anatolian Teacher Training High School.

Teacher D: in my class all of them [groups] interpret truly but draw linear graphs for all tanks, they cannot draw concave up and concave down graphs

Teacher F: What do you do?

Teacher D: I asked if it is linear or not? They think on it but then did not change their graph

Semra: You [Teacher D] may ask them [students] if the linear graph represent their interpretation. When they explaining you then they may realize their incorrect drawing

Teacher F: you could [Teacher D] also ask how it can be same for all the tanks?

Melda: Yes, it could be asked, do you draw a linear graph for the cylinder, Is it also same for the sphere? Or want them to take values and draw the graphs again, they may realize their inaccurate graphs

Semra: It could also be asked, do you remember the exponential numbers' graph. Then they may think the curves

In the implementations, data revealed that teachers tried to use questions which they stated and their colleagues stated while discussing how to ask questions on the error and difficulties that might encounter in the solution process, in the meetings before the implementation. The comparison of the questions which were stated in the meeting before implementation and the questions stated in the interviews and in the implementations showed that teachers took into account their colleagues suggested questions based on the specific errors and difficulties and their planned questions based on the specific errors and difficulties in the meeting before the implementation. This result was supported with the comparison of the questions which were stated in the meeting before the implementation, pre-implementation interview and the questions that were used in the implementations. Representative comparisons of the questions were given below.

Comparisons showed that Rezzan used the question, which was stated before the implementation, in her implementation.

Rezzan: first of all it is important to students understand their strategies meaning so I will ask from this point of view, what is your aim? Is your strategy fit with the boss aim?...if they tried to decide the part time workers based on the total money per hour for each worker than it can be asked is there another criterion for deciding the part time workers? (third implementation pre-interview with Rezzan)

The question below was asked to most of the groups in order to make them think other approaches during the third implementation.

Rezzan: do you find other criteria in order to decide the part time workers? Of course based on the criteria that are arrangement given in the question

Comparisons showed that Figen used the question which was stated in the meeting before the implementation by Rezzan and not stated by her, in her implementation. Additionally, Figen indicated this type in the pre-implementation interview.

Rezzan…they [students] may make one arrangement and then do not think the other arrangement, we should ask them to think other arrangement, there is not another way for the arrangement? (The excerpt was taken from the meeting before the fifth implementation)

Figen: may be groups find two kinds of arrangement and do not think other arrangement at that time, it can be asked, may have different installations? Can we arrange more cans in other installation? By this way students look at the question from different ways (Fifth implementation pre-interview)

The question below was asked to the groups tried only the regular arrangement of the cans arrange in the fifth implementation.

Figen: Can we arrange differently? Is there another arrangement?

Comparisons showed that Melda used the question which was stated in the meeting before the implementation by Semra and not stated by her, in her implementation.

Semra: …Also they [students] can make errors as if they may want to take the busy, steady and slow periods as a criterion but calculate the money per hour based on months; at that point, it can be asked, "What is your criterion? What are your calculations are based on, which data did you use? So they may evaluate their approach (The excerpt was taken from the meeting before the third implementation)

The question below was asked to the groups whose solutions have a mismatch between their approach and the calculations in the third implementation.

Melda: How can you decide the part time workers? Which table did you use? Are your calculations and approach compatible?

4.1.2 Assessment of Students

Under this subheading each teacher evolving knowledge on generating assessment criteria for assessing students' competency while students are working on MEA and presenting of their solutions described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA which entitled "Bank Robbery", before the implementation it was stated by Figen that teachers did not ask open ended questions since it could not be assessed and it was also specified that this kind of questions could be assessed based on the result. However, after the implementation, the assessment criteria began to consist like, understanding the question, the solution approaches, using the data, interpretation of the data, the result, in presentation their self-confidence, tone of voice, it was stated as presentation skills.

The below excerpt was taken from the pre-implemantation interview with Figen

Figen: we [teachers] do not ask open-ended questions since we cannot assess, for example in öss-lys [national exams], the open-ended questions are not asked since it cannot be assessed

Researcher: Do you think any possible assessment criteria

Figen: we may only look at the result like exams.

Figen: actually the important thing for the assessment are understanding the question, the solution approaches, using the data, interpretation of the data and the result…also in presentation their self-confidence, tone of voice, I think presentation skill is so important, since some groups can make monotone presentation and they cannot make other groups to listen them (post-implemantation interview with Figen)

In the second implementation of the MEA entitled "Street parking", before the implementation assessment criteria for the solution process were denoted as active participation, solution steps, mathematical conversion and the result. In addition, assessment criteria of the presentation were specified as presentation skills, tone of voice and their self-confidence. After the implementation, the result was stated especially for the assessment criterion of the solution process and for the presentation process, it was stated that teachers could ask questions about students' solution process in order to evaluate their answers if each group member dominate the solutions.

Figen: I can assess generally with their [students'] active participation, solution steps, mathematical conversion and the result (pre-implemantation interview with Figen)

Figen: especially when they present the presentation skills, tone of voice and their self-confidence are very important (pre-implemantation interview with Figen)

Figen: we [teachers] can see many different approaches but nevertheless the result is important (post-implemantation interview with Figen)

Figen: actually, in order to make every student actively participate the solution process, we [teacher] can ask some questions to the group members about their [students'] solution approaches while they are presenting. Other groups ask questions however, the teacher dominate the process mostly so teacher can ask the important points to each group member (post-implemantation interview with Figen)

In the third implementation of the MEA entitled "Summer Jobs", teachers created an assessment tools, and revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with these tools. The first version of the assessment tool was given in Table 4.6 and the last version of the assessment tool was given in Table 4.7 and its developing process was explained next.

Table 4.6 The First Version of Tool Developed by Figen to Assess Students' Modeling Competency

Assessment while the students working on the modeling task					
		Assessment Criteria			
Group	The Students in the Group	Sensing the question (individual) • Try to understand from students' conversations whether the question was understood or not Total points:18 Individually: 3 points	Establish relationships between data (individual) • Associating the given data and using it. Total points:24 Individually: 4 points	Quickly focus on the solution (group) Act immediately to solution Total points: 4	Becoming a group • Executing an idea • cooperation • group coexistence • sharing Total points: 40
Group 1					

Table 4.6 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
Transferring the steps of the solution • expressing the steps of the solution clearly Total point: 30	Self-confidence (group) • get other students to listen /pay attention Total point: 30	Result • reach the result (15 points) • not reach the result (0 points) Total point: 15	The answers to the questions (group) • The answers to questions from other groups • The answers to questions asked by the teacher Total point: 20

Table 4.7 The Last Version of Tool Developed by Figen to Assess Students' Modeling Competency

Assessment while the students working on the modeling task					
		Assessment Criteria			
Group	The Students in the Group	Sensing the question (group) • Try to understand from students' conversations whether the question was understood or not Total points:18	Establish relationships between data (group) • Associating the given data and using it. Total points:24	Quickly focus on the solution (group) Act immediately to solution Total points: 4	Becoming a group • Executing an idea • cooperation • group coexistence • sharing Total points: 40
Group 1					

Table 4.7 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
Transferring the steps of the solution • expressing the steps of the solution clearly Total point: 30	Self-confidence (group) • get other students to listen /pay attention Total point: 30	To reach conclusions • Basic Level (5 points) Demonstrate a simple approach by not thinking all the data that were given • Intermediate level (10 point) Use more data than a simple approach, but all the approaches are not be evaluated. • Advanced Level (15 points) Using all the data and thinking all possible approaches Total point: 15	The answers to the questions (group) • The answers to questions from other groups • The answers to questions asked by the teacher Total point: 20

The criteria, levels of criteria, their explanations and how they were scored, were given in the tables above. If criteria were scored as group, it was indicated as group in bracket, if the each group member scored individually based on the criteria, it was indicated as individual in bracket. In the third implementation, the result was used as a criterion and it was evaluated as the groups reach the result or not but then it was realized by the teacher that it was not appropriate since the quality of the result [model] was different so it was stated that the result could be evaluated based on the weakness and strength of it in the fourth implementation. Then the result criterion was decided to evaluate based on three level as basic, intermediate and advanced level. The result was evaluated as basic level when the group demonstrates a simple approach by not thinking all the data that were given in the question; it was evaluated as intermediate level when the group use more data than a simple approach, but not evaluating all the approaches; it was evaluated as advanced level when the group use all the data and thinking all possible approaches. In the third implementation, sensing the question and establishing relationship between data criteria were evaluated as individually, however in the fifth implementation, it was stated that scoring this criteria as group was more logical because as a group they tried to understand the question and tried to associate the data and use it. After these revisons, the assessment tool converged its last version.

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation it was stated by Rezzan that assessment was not clear and the assessment could not be as the result true or false. However, after the implementation, the assessment criteria began to consist like, active participation, cooperation, listening to each other and result while working on the questions; these criteria were mostly about communication and participation in the group rather than solution phases. In addition, the assessment criteria for the presentation were denoted as poster papers clarity, defending their approach, appeal, and answers to the questions from other groups.

Rezzan: I think we [teachers] cannot assess here like, true or false, but actually, when I see the implementation, this may be clear for me. However, I think I accept all the logical results (pre-implemantation interview with Rezzan)

Rezzan: Firstly, the active participation is so important, cooperation, listening to each other and the result (post-implemantation interview with Rezzan)

Rezzan: the poster papers should be clear, defending their [students'] ideas, they may find a result but it is also important how to defend and represent it. The group should be give convincing answer to the questions from other groups and should make other groups to listem them that their appeal is also so important (post-implemantation interview with Rezzan)

In the second implementation of the MEA entitled "Street parking", before the implementation assessment criteria for the solution process were denoted as active participation, mathematical conversion and finding the result cooperatively. In addition, assessment criteria of the presentation were specified as defending their approaches and their self-confidence. After the implementation, rather than the result solution steps, understanding the problem and the true modeling were specified for the assessment criterion of the solution process and for the presentation process, it was stated that cooperation in the presentation was also an important criterion, group member should keep their cooperation while answering the question from other groups and while defending their approach.

Rezzan: converting to math is very important, mathematization (pre-implemantation interview with Rezzan)

Rezzan: … and students' active participation, finding the result cooperatively is the good assessment criteria for me. In presentation, most important criteria are defending their [students] approaches and their self-confidence (pre-implemantation interview with Rezzan)

Rezzan: rather than the result, solution steps, understanding the problem and the true modeling are important (post-implemantation interview with Rezzan)

Rezzan: and the cooperation in the presentation is also an important criterion, group member should keep their cooperation while answering the question from other groups, while defending their approach (post-implemantation interview with Rezzan)

In the third implementation of the MEA entitled "Summer Jobs", teachers created an assessment tools, and revise these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with these tools. The first version of the assessment tool was given in Table 4.8 and the last version of the assessment tool was given in Table 4.9 and its developing process was explained next.

Table 4.8 The First Version of Tool Developed by Rezzan to Assess Students' Modeling Competency

Assessment while the students working on the modeling task					
		Assessment Criteria			
Group	The Students in the Group	Sensing the question Using the data (individual) • Try to understand from students' conversations whether the question was understood or not • Specifying how they will use the data (if they use the proper data based on their approach) Individually: 3 points	Interpretations- Trying different approaches • Using different approaches, solving the questions from different point of views Total points:20	Result • focus on the result (5 points) • reach the result partially (10 points) • reach the result completely (15 points) Total points:15	Listening to each other Cooperation Total points:20
Group 1					

Table 4.8 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
	Defend their approach • Have a command of their solutions and defend their approaches Total points:20	Appeal • get other students to listen /pay attention (used sentences, tone of voice, facial expression) Total points: 20	Cooperation • Answer to the questions from other groups cooperatively and defend solution in collaboration. Total points:20

Table 4.9 The Last Version of Tool Developed by Rezzan to Assess Students' Modeling Competency

Assessment while the students working on the modeling task						
		Assessment Criteria				
Group	The Students in the Group	Active participation in group work (Individual) Individually: 5 points	Sensing the question Using the data (group) • Try to understand from students' conversations whether the question was understood or not • Specifying how they will use the data (if they use the proper data based on their approach) Total points:15	Interpretations- Trying different approaches • Using different approaches, solving the questions from different point of views Total points:20	Result • Basic Level(5 points) Using a solution way without analyzing the all criterions. • Intermediate Level (10 points) Using alternative solutions but not using all criterions. • Advanced Level (15 points) Using all alternative solutions and all criterions. Total points:15	Listening to each other Cooperation Total points:20
Group 1						

Table 4.9 (continued)

Assessment of students' presentation of their solutions				
Assessment Criteria				
	Poster papers clarity and understandability Total points:5	Defend their approach • Have a command of their solutions and defend their approaches Total points:20	Appeal • get other students to listen /pay attention (used sentences, tone of voice, facial expression) Total points: 20	Cooperation • Answer to the questions from other groups cooperatively and defend solution in collaboration. Total points:20

The criteria, levels of criteria, their explanations and how they were scored, were given in the tables above. If criteria were scored as group, it was indicated as group in bracket, if the each group member scored individually based on the criteria, it was indicated as individual in bracket. In the third implementation, the result was used as a criterion and it was evaluated as the groups can focus on the result and reach the result completely or partially but then it was denoted that it was not sufficient, there should be more clear lines on the solution approaches since the quality of the result [model] was different based on the weakness and strength of it in the fourth implementation. Then before the fifth implementation, the result criterion was decided to evaluate based on three levels as basic, intermediate and advanced level. The result was evaluated as basic level when the group used a solution way without analyzing all criteria that was essential; it was evaluated as intermediate level when the group used alternative solutions but not using all criteria that was essential; it was evaluated as advanced level when the group used all alternative solutions and all criteria that was essential. In the third implementation, sensing the question and using the data criterion was evaluated as individually, however after the implementation, it was stated that scoring this criterion as group was more logical because as a group they tried to understand the question and tried to associate the proper data based on their approach and used it. In addition, before the fifth implementation, active participation in group work, poster papers clarity and understandability criteria were added. It was explained that each group member's participation in a group work was very important and when students made their presentation they used poster papers and explained based on that so it should be clear and understandable. After these revisons the assessment tool converged its last version.

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation it was stated by Semra that teachers could evaluate the groups but could not evaluate individually the group members. In addition, assessment criteria were denoted as group work, group participation and time organization for the solution process and make other groups to listen for the presentation process. All criteria were stated based on communication and participation in the group rather than solution phases.However, after the implementation, the assessment criteria

began to include also the solution phases, the specified criteria while students are working on the question are: groups' different solution approaches, active participation in group, time organization and the result, in presentation, presentation skills, making clear presentation and answers to the questions from teacher about group solution approach are indicated.

Semra: We [teachers] may make group assessment but we cannot evaluate individually, I think we [teachers] can evaluate based on their [students'] group work, since they are not accustomed to the group work as we and group participation and time organization is important for the group work. While presenting, it is important to make other groups listen them [the group who present] (pre-implemantation interview with Semra)

Semra: the groups' different solution approaches are important by this way they have an opportunity to compare all the approaches in one hand, the active participation in group, time organization and the result all can be criteria (post-implemantation interview with Semra)

Semra: presentation skills and making clear presentation is important, also I can ask questions to the group about their solutions in order to see that if all the group member be dominate on their approach (post-implemantation interview with Semra)

In the second implementation of the MEA entitled "Street parking", before the implementation assessment criteria for the solution process were denoted as groups' different solution approaches, active participation in group, time organization and the result. In addition, assessment criteria of the presentation were specified as presentation skills, making clear presentation and answers to the questions from teacher about group solution approach. The same criteria with the first implementation were indicated. After the implementation, to be combative and intra-group interaction and the roles with in the group were stated as the solution criteria and poster papers to be clear and understandable were added as the presentation criteria.

Semra: groups' different solution approaches, active participation in group, time organization and the result, for presentation, presentation skills, making clear presentation and answers to the questions that I asked are same for me I do not add any other criterion (pre-implemantation interview with Semra)

The below excerpt was taken from post-implemantation interview

Semra: groups being combative and dealing with the question until to the end, not giving up in a short time and not to prefer the simplest way . In addition, their roles in the group, to fulfill their responsibilities according to their role in the group and working collaboratively are also be criteria

Researcher: Do you want to add anything?

Semra: the prepared poster papers are also important; it should be systematic and clear since it shows us how they are dominate to their solutions

In the third implementation of the MEA entitled "Summer Jobs", teachers created an assessment tools, and revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with this tool. The first version of the assessment tool was given in Table 4.10 and the last version of the assessment tool was given in Table 4.11 and its developing process was explained next.

Table 4.10 The First Version of Tool Developed by Semra to Assess Students' Modeling Competency

Assessment while the students working on the modeling task						
		Assessment Criteria				
Group	The Students in the Group	Associating the data Making appropriate calculations based on their approach • Associating the data and making appropriate calculations based on their approaches and also evaluate their strategies Total points: 30	Using the time • Using the time efficiently Total points: 5	To be combative Deal with the question until the end, not give up and not to prefer the simplest way Total points:10	Intra-group interaction and the roles with in the group • To be collaborative, to fulfill their responsibilities according to their role in the group Total points: 10	Transferring their approaches to their solution papers • Writing systematically their approaches to the solution papers Total points: 5
Group 1						

TOTAL (1):

Table 4.10 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
	Making a systematic and clear presentation • Poster papers to be clear and understandable Total points: 20	Present with clear and precise language • Present confidently • Defend ideas Total points: 10	Answers to questions from the teacher • Answers given to the questions which are asked in order to clarify the presentation and make understandable to the other groups Total points: 10

TOTAL (2): **GENERAL TOTAL:**

NOTE: In here the teacher can note the individual differences

Table 4.11 The Last Version of Tool Developed by Semra to Assess Students' Modeling Competency

Assessment while the students working on the modeling task						
		Assessment Criteria				
Group	The Students in the Group	Associating the data Making appropriate calculations based on their approach • Associating the data and making appropriate calculations based on their approaches and also evaluate their strategies Total points: 30	Using the time • Using the time efficiently Total points: 5	To be combative Deal with the question until the end, not give up and not to prefer the simplest way Total points:10	Intra-group interaction and the roles with in the group • To be collaborative, to fulfill their responsibilities according to their role in the group Total points: 10	Transferring their approaches to their solution papers • Writing systematically their approaches to the solution papers Total points: 5
Group 1						

TOTAL (1):

Table 4.11 (continued)

Assessment of students' presentation of their solutions				
Assessment Criteria				
	Result: A simple approach (5 points): Using a simple approach, not using all the data that is appropriate, produce an approach with using the shortest way A systematic approach (10 points): Using a systematic approach and using all the appropriate data Total points: 10	Making a systematic and clear presentation • Poster papers to be clear and understandable Total points: 20	Present with clear and precise language • Present confidently • Defend ideas Total points: 10	Answers to questions from the teacher • Answers given to the questions which are asked in order to clarify the presentation and make understandable to the other groups Total points: 10

TOTAL (2): **GENERAL TOTAL:**

NOTE: In here the teacher can note the individual differences

The criteria, levels of criteria, their explanations and how they were scored were given in the tables above. If criteria were scored as group, it was indicated as group in bracket, if the each group member scored individually based on the criteria, it was indicated as individual in bracket. In the third implementation, the result was not used as a criterion since it was stated that it was not clear how to evaluate based on the result however it was denoted that the quality of the result [model] was different so that the result could be evaluated based on the weakness and strength of it in the fourth implementation. Then the result criterion was decided to evaluate based on two levels as simple approach and systematic approach. The result was evaluated as simple approach when the group used a simple approach, not using all the data that was appropriate, produce an approach with using the shortest way; it was evaluated as systematic approach when the group used a systematic approach and used all the appropriate data. After these revison the assessment tool converged its last version.

Melda's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation it was stated that it could be assessed like the exams not only the result but also the solution steps. However, after the implementation, the assessment criteria like active participation and collaboration in group were added, other than solution approach.

Melda: I can assess like in the exam, I cannot assess only the result but also the solution steps, for example in here I can assess if they establish the propositions, if they set up the truth table truly, if they can decide based on these truth table like that (pre-implemantation interview with Melda)

Melda: …I see that in the implementation their [students'] active participation, collaboration is also so important like the solution approach (post-implemantation interview with Melda)

In the second implementation of the MEA entitled "Street parking", before the implementation assessment criteria for the solution process were denoted as the understanding of the question, solution steps, active participation and collaboration in group and the result. These criteria were repeated after the implementation. In addition, assessment criterion of the presentation was specified as defending the solution approach.

Melda: the understanding of the question, solution steps, active participation and collaboration in-group and the result can be used as criteria (pre-implemantation interview with Melda)

Melda: …the important criteria for the presentation is defending what they do, their approaches. In addition, for the solution process, the criteria can understand of the question, solution steps, active participation and collaboration in group and the result (post-implemantation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", teachers created an assessment tools, and revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with these tools. The first version of the assessment tool was given in Table 4.12 and the last version of the assessment tool was given in Table 4.13 and its developing process was explained next.

Table 4.12 The First Version of Tool Developed by Melda to Assess Students' Modeling Competency

Assessment while the students working on the modeling task					
		Assessment Criteria			
Group	The Students in the Group	Active Participation (Individual) • To be collaborative • To take part in group (writing, calculating, checking the calculations) Total points: 10 Individually: 2.5 points	Solution Steps (Group) • Using the data (5 points) Using the data given in the question • Using the concept in the question (10 points) Using the concepts which are basis of the question • Using the old and the new concepts that is embedded under the question (5 points) Solution approach (10 points) A simple approach (5 points): Not using all the data, establish a simple approach with the shortest path A systematic approach (10 points): Establishing a systematic approach by evaluating and using all the given data Total points: 30	Result (Group) • Comparison of the data that is found at the last steps and analyzing them Total points: 10	Time (Group) • Using the time efficiently Total points: 10
GROUP 1			Using the data:		
			Using the concept in the question:		
			Solution approach:		

Table 4.12 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
The Students in the Group	Presentation Expression (Group) • The delegate of the group expressing the solution steps clearly. (expressing the approaches) (5 points) • Does the poster papers seems pretty clear that include group's approaches (5 points) Total points:10	The answers to the questions (Individual) • The group members are evaluated based on their answers to the questions from other groups. If there are group members that are not asked questions be evaluated based on their answers to the questions from teacher. Total point:10 Individually:2.5	Defend their approach (Group) • The full defense of how they have reached the result • The evaluation of the answers to the questions from other groups based on the other groups' assessments [it is asked if the answers satisfy them or not] Total point:20

Table 4.13 The Last Version of Tool Developed by Melda to Assess Students' Modeling Competency

Assessment while the students working on the modeling task					
		Assessment Criteria			
Group	The Students in the Group	Active Participation (Individual) • To be collaborative • To take part in group (writing, calculating, checking the calculations) Total points: 10 Individually: 2.5 points	Solution Steps (Group) • Using the data (5 points) Using the data given in the question • Using the concept in the question (15 points) Using the concepts which are basis of the question Solution approach (10 points) A simple approach (5 points): Not using all the data, establish a simple approach with the shortest path A systematic approach (10 points): Establishing a systematic approach by evaluating and using all the given data Total points: 30	Result (Group) • Comparison of the data that is found at the last steps and analyzing them Total points: 10	Time (Group) • Using the time efficiently Total points: 10
GROUP 1			Using the data: Using the concept in the question: Solution approach :		

Table 4.13 (continued)

Assessment of students' presentation of their solutions			
Assessment Criteria			
The Students in the Group	Presentation Expression (Group) • The delegate of the group expressing the solution steps clearly. (expressing the approaches) (5 points) • Does the poster papers seems pretty clear that include group's approaches (5 points) Total points:10	The answers to the questions (Individual) • The group members are evaluated based on their answers to the questions from other groups. If there are group members that are not asked questions be evaluated based on their answers to the questions from teacher. Total point:10 Individually:2.5	Defend their approach (Group) • The full defense of how they have reached the result • The evaluation of the answers to the questions from other groups based on the other groups' assessments [it is asked if the answers satisfy them or not] Total point:20

The criteria, levels of criteria, their explanations and how they were scored was given in the tables above. If criteria were scored as group, it was indicated as group in bracket, if the each group member scored individually based on the criteria, it was indicated as individual in bracket. In the third implementation, using the concept in the question and using the old and the new concepts that was embedded under the question criteria were specified, however in the fifth implementation, it was stated that while scoring this criteria the same things were thought so it was indicated to combine these two criterion under one criterion which was using the concept in the question. After these revisons, the assessment tool converged its last version.

4.1.2.1 Cross-case Analysis for Knowledge of Assessment of Students

Under this subheading cross-case analysis results of knowledge of assessment of students what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on generating assessment criteria for assessing students' competency while students are working on MEA and presenting of their solutions was compared in Table 4.14.

Table 4.14 Case Ratings of Assessment Criteria Themes for All Implementations

Case Themes	**Cases***			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
Theme 1: (at the beginning)				
• It was stated teachers did not ask open ended questions since it could not be assessed	X			
• assessment was not clear	X	X	X	X
assessment criteria may				
• result	X			
• solution steps				X
• communication and participation in the group			X	
Theme 2: (criteria in the assessment tools)				
Assessment while the students working on the MEA				
Understanding the question	X	X		
Associating and using the data	X	X	X	X
Quickly focusing on the solution	X			
Communication, cooperation and participation in the group	X	X	X	X

Table 4.14 (continued)

Using the concept in the question				X
Using the time efficiently			X	X
Result (simple approach, systematic approach)			X	X
Result (Basic level, Intermediate level, Advanced level)	X	X		
Result (Comparison of the data that is found at the last steps and analyzing them)				X
Trying different approaches		X		
To be combative (Deal with the question until the end, not give up and not to prefer the simplest way)			X	
Writing systematically their approaches to the solution papers			X	
Assessment of students' presentation of their solutions				
Result (simple approach, systematic approach)			X	X
Result (Basic level, Intermediate level, Advanced level)	X	X		
Present with clear ,precise language and get other students to listen /pay attention	X	X	X	X
Defending their approach	X	X	X	X
Poster papers clarity and understandability		X	X	X
The answers to the questions from other groups and teacher	X	X	X	X
The evaluation of the answers to the questions from other groups based on the other groups' assessments [it is asked if the answers satisfy them or not]				X

* X indicates that the theme was observed/presented in the case

Teacher knowledge about assessment required knowledge of what and how to assess student efforts as related to intended goals and assessment methods. Teachers should generate effective assessment criteria based on the learning environment and the intended goals. Teachers should understand what their students did, why they did so the teacher can generate criteria for their students' effort. In this study, students solved the model-eliciting activities and presented their solutions and how to assess this process, what can be the criteria were stated by the teachers based on the implementations. In the first implementations, all the teachers denoted that the assessment procedure and the criteria were not clear, although one teacher stated that it could not be assessed. Then the criteria began to appear as result, solution steps

and communication and participation in the group in general. After then, teachers created an assessment tools, and revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process and teachers used their criteria that they indicated in the first and second implementation and other criteria. In addition, teachers evaluated one group in the implementations with these tools. When the assessment criteria in the last version of the assessment tools were compared, the common criteria for the solution process were; associating and using the data, communication, cooperation and participation in the group and the result. The result criterion was evaluated both in the solution and the presentation process. Two teachers used the result criterion in solution process and the two teachers used it in the presentation process. Associating and using the data were explained as associating the data, using the proper data and making appropriate calculations based on their approaches. Other criterion which was communication, cooperation and participation in the group was explained as active participation in group, listening to each other, cooperation and to be collaborative, to fulfill responsibilities according to the roles in the group. In addition, result criterion was assessed not only true or false, two teachers evaluated result as simple approach and systematic approach, and another two teachers evaluated the result based on three level as basic level, intermediate level and advanced level. These levels were explained as the result was evaluated as simple approach when the group used a simple approach, not using all the data that was appropriate, produced an approach with using the shortest way; it was evaluated as systematic approach when the group used a systematic approach and used all the appropriate data. Also the result was evaluated as basic level when the group used a solution way without analyzing all criteria that was essential; it was evaluated as intermediate level when the group used alternative solutions but not using all criteria that was essential; it was evaluated as advanced level when the group used all alternative solutions and all criteria that was essential. Moreover, the common criteria for the presentation process were; present with clear, precise language and made other groups to listen them, defending their approach and the answers to the questions from other groups and teacher. It was explained that it was important to make clear presentation and make other groups to listen; also, the group should defend their approach and can give satisfied answers to the questions from other groups and teacher.

Intercalarily, there were also not common criteria in the assessment tool. For the solution process, two teachers were denoted understanding the question criterion and explained that if students could not understand properly then they could not produce appropriate approaches and they said that they tried to understand from students' speech that if the question be understood or not. One teacher stated quickly focusing on the solution criterion. It was stated that time was important so if they immediatly produced an approaches then they had possibility to compare more than one approach so it was important to focus on the solution quickly. In addition, one teacher specified using the concept in the question criterion and it was indicated that the basis of the concepts (big ideas of the MEAs) should be used so it was evaluated if these concepts were used or not. Besides, two teacher stated using the time efficiently criterion and it was explained that the time was limited so it was important to produce an effective approach in this limited time and this was based on how they used this time efficiently. One of the teachers added a result criterion as a solution step and stated that it was the last step of the solution that groups compared the data, the approaches and analyzed and decided the last model of their solution process so it was explained that this step was the most important step and should be evaluated. In addition, one teacher denoted trying different approaches criterion and it was stated that solving the questions from different point of views and deciding the last model based on comparison of these approaches were more efficient way so this criterion were evaluated. Moreover, one teacher added to be combative and writing systematically their approaches to the solution papers criteria, to be combative were explained as groups dealing with the question until the end, not giving up and not to prefer the simplest way was important, the group may not find the efficient model but this exertion should be evaluated and writing systematically the approaches to the solution paper was important for the teacher from this point of view that which was stated as if they did not write systematically, it was hard for the teacher to interfere efficiently, if they wrote systematically teachers could see where the groups had difficulty easily.

Moreover, there were also not common criteria for the presentation process; three teachers stated poster papers clarity and understandability criterion and explained as if this poster papers were clear and understandable, the other groups who listen the presentation could understand the group solution process more better and ask more

efficient questions to the group about their solution approach and there could be more efficient discussion occur. In addition, one teacher added the evaluation of the answers to the questions from other groups based on the other groups' assessments [it was asked if the answers satisfy them or not] criterion and it was indicated as the other groups who listen the presentation could also evaluate the presentation, they asked questions to the group who made the presentation about their solution so teacher could ask if the answer of their questions satisfied them or the group could defend their approach efficiently or not, by this way it was stated that teacher could see the assessment from the students' view.

Data revealed that the focuses of the assessment criteria were changed in the cyclic process of the study. In the first implementation, the result and solution steps were stated as assessment criteria based on teachers' assessment criteria in their exams or based on the new phenomenon (group work) in the implementations. This focus was supported by teachers' own words that it was stated the criteria could be exams criteria such as in their math exams or the important criteria in the group work.

Melda: I can assess like in the exam, I cannot assess only the result but also the solution steps, for example in here I can assess if they establish the propositions, if they set up the truth table truly, if they can decide based on these truth table like that (pre-implemantation interview with Melda)

Figen: we may only look at the result like exams.

Semra: We [teachers] may make group assessment but we cannot evaluate individually, I think we [teachers] can evaluate based on their [students'] group work, since they are not accustomed to the group work as we and group participation and time organization is important for the group work. While presenting, it is important to make other groups listen them [the group who present] (pre-implemantation interview with Semra)

In the other implementations, the focuses of the assessment criteria were changed based on the factors that affect the students' competency in modeling. These factors were determined by observing the one focus group deeply in the solutions and in the presentations and by discussing the assessment criterions with their colleagues for each implementation. The below excerpt was taken from the dialogue among the teachers in the meeting before and after the implementations denoted that teachers decided assessment criteria that is proper for assessing the students' competency in modeling by discussing with their colleagues and by observing the one focus group deeply in the solutions, such as result criteria were discussed by the colleagues and it was denoted that evaluating the groups based on reaching the result or not was not appropriate, the reason was stated as since the quality of the result (model) was

different so it was stated that the result could be evaluated based on the weakness and strength of it.

The below excerpts were taken from the dialogue among the teachers in the meeting after the implementations indicates that the result criterion level was changed based on the observations in the implementations and formed in the meeting in Anatolian High School.

Rezzan: in the implementation, I tried to assess the results as the groups focus on the result, reach the result completely or partially but it was not sufficient, there should be more clear lines on the solution approaches since the quality of the result [model] was different

Figen: some of the groups used all data and more than one approach and compare them, some of them only use one approach and reach the result, these results are not same

Teacher A: one of the results is strong, one of them weak (the above except was taken from the meeting after the third implementation).

Figen: The result can be evaluated as strong or weak but the more proper evaluation can be done based on the level like basic, intermediate and advance

Research: Can you explain these levels?

Figen: it can be evaluated as basic level when the group use a solution way without analyzing all criterions that is essential; it is evaluated as intermediate level when the group use alternative solutions but not using all criterions that is essential; it is evaluated as advanced level when the group use all alternative solutions and all criterions that is essential.

Rezzan: I think, it is so logical I can use these criterions in my tool also (the above except was taken from the meeting after the fourth implementation).

The below excerpts were taken from the dialogue among the teachers in the meeting after the fourth implementation indicates that the result criterion level was changed based on the observations in the implementations and formed in the meeting in Anatolian Teacher Training High School.

Melda: I scored the result as simple and systematic and it is suitable I think so I did not change any criteria

Semra: I may use the Semra's result criterion levels in my tool, since it can be more suitable since some groups were not use all the essential data in the question and find a basic result however some groups use all the essential data in the question and compare them and produce systematic approach

4.2 Knowledge of Classroom Organization and Management

4.2.1 Class Setting: The Criteria for Determining Group's Structure and the Number of the Students in One Group

Under this subheading each teacher evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the number of the students in the group was indicated as five or six and it was also stated that students who express themselves well (who have verbal ability-language ability) could be distributed to the groups in order to not having a problem while groups expressing their solution process. During the implementation, there were six students in each group and the teacher interfered the group structure by distributing the students who had verbal ability that expressed themselves well in their traditional class times to the each groups. After the implementation, it was denoted again that the intervention was minimum while the students constructing their groups, only the students who expressed themselves well were distributed to each group and stated that it was observed that every student could not actively participate in the group since the number of the students in group which was six was high, each student could not participate since when they expressed their ideas it was not heared, all of them tried to talk in the same time so it was specified that if the group number decreased, every member could state their ideas.

Figen: in our school [anatolian high school] generally the students' mathematical achievement are the same so while establishing the group I will distribute the students who have the verbal ability-language ability to each group by this way they have not a problem while they are expressing their solution process and I will made the groups with five or six students (pre- implementation interview with Figen)

The group structure: Six students in each group and the teacher interfered the group structure by distributing some students to the groups (During the implementation-observation notes)

Figen: I distributed the students who express themselves well to the groups but generally I do not so much interfere while they [students] are constructing their groups. They choose their friends but I said sometimes, you can be here [this group] because I afraid if I did not distribute verbal ability students to groups, the groups cannot express their solutions. Also I observed that the every student cannot actively participate in the group since the number of the students in group which is six was high, each student cannot participate since when they expressed their ideas it was not heared, all of them tried to talk in the same time but if the group number will decrease, every member can state their ideas (post- implementation interview with Figen)

In the second implementation of the MEA entitled "Street Parking", before the implementation, it was stated that teachers only interfere while distributing the students who have verbal ability and the number of the students in the group can be five in order to maximize the active participation of each group member. After the implementation it was denoted that the groups should be constructed by mixed the boys and the girls in order to minimize the speeches on other issues rather than the question, it was also specified that seating arrangement is important there are not the girls in one end and the boys in one end of the table, students should seat disorderly like one girl and one boy in order to increase the concentration to the solution.

Figen: the students should construct their groups by themselves; we [teachers] only interfere while distributing the students who have verbal ability. We can make the number of the groups five by this way each member can participate actively and we also have used the time efficiently in presentations since we will have five or six groups in the presentations (pre- implementation interview with Figen)

Figen: the groups should be constructed by mixed the boys and the girls, by this way they do not talk other subjects different than the question and also the seating arrangement is important there are not the girls in one end and the boys in one end of the table, they also seat disorderly like one girl and one boy so they will concentrate mostly on the solution (post- implementation interview with Figen)

In the third implementation of the MEA, entitled "Summer Jobs", before the implementation, it was denoted that the number of the students in the group could be five and it was stated that students should create the groups their own since the students characteristics were very different when it was compared with the traditional class times and MEAs solution process; teachers could only want students to create mixed groups with boys and girls. During the implementation, the group construction was; four groups which had five members, and one group which had four members, and the groups were mixed with boys and girls. After the implementation, the same interpretations were denoted as before the implementation. It was specified that students should create the groups their own; teachers could only want students to create mixed groups with boys and girls. In addition, it was denoted that the five members in each group worked actively in this question since it contains so much operations.

Figen: I wanted the students to construct their groups with five of them; I only stated them to construct mixed groups with boys and girls then I did not interfere the group structure since I observed that the students characteristics are very differ in the traditional class times and in solving these questions. I thought that some students would be very active and produce different ideas however; I saw that they were passive. Although, I saw that the students who are passive, not talking too much in traditional class times were more active and generate different ideas while solving these questions so I will not interfere the group structure, the ideal one is, students should create the groups their own, teachers can only want students to create mixed groups with boys and girls (pre-implementation interview with Figen)

The group structure: four groups, which had five members; one group, which had four members, and the groups were mixed with boys and girls (During the implementation-observation notes)

Figen: I think this structure is appropriate, when the groups construct their group by their own they were more active and the students who are passive in the class were also be leaders in the group this was really surprising, so it was meaningless to interfere the group structure as a teacher since we do not know their characteristics in this problems. We can only interfere groups to want them construct mixed groups [boys and girls]. In here, [Summer Jobs] five member worked actively in one group since the question contain so much operation, each member participate actively (post-implementation interview with Figen)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the number of the students in the group should change based on the question, if the question included so many operation, there could be five or four students in each group since all of them could have a chance of active participation and if the question included more interpretation rather than operation at that time, there can be three or four students in each group since all of them could have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. Besides, it was stated that mixed groups with boys and girls should be constructed since boys have practical approaches but cannot make clear sketches so they generally had problems while converting their approaches to mathematics, on the other hand girls have spend so many times on sketches and could not think as practical as boys so when they [*boys and girls*] combine in one group, there were good cooperation. In addition, it was denoted that groups should be constructed by students when students construct the groups they feel more that they were group and they paid attention mostly on communication, cooperation...e.g. Moreover, the number of the students in the groups was also specified that the number could also be change based on the grade, this was explained as when the question had so much operation the number of the students can four or three in ninth grade and tenth grade but it can be four or five in eleventh and the twelfth grade, the number could decrease when the grade increased since

eleventh and the twelfth grade's ability and knowledge more than the ninth grade and tenth grade.

Figen: I think the number of the students in the group should change based on the question, if the question includes so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question include more interpretation rather than operation at that time, there can be three or four students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group… The groups should be constructed by students and they should construct mixed groups with boys and girls (pre- implementation interview with Figen)

Figen: the mixed [boys and girls] groups are important since I observed that boys have practical approaches but cannot make clear sketches so they generally have problems while converting their approaches to mathematics, on the other hand girls have spend so many times on sketches and cannot think as practical as boys so when they [boys and girls] combine in one group, there are good cooperation. In addition, when students construct the groups they feel more that they are group and they pay attention mostly on communication, cooperation...e.g. (post- implementation interview with Figen)

Figen: the number of the students in the groups can also be change based on the grade according to me for example when the question have so much operation the number of the students can five or four in ninth grade and tenth grade but it can be four or three in eleventh and the twelfth the number can decrease when the grade increase since eleventh and the twelfth grade's ability and knowledge more than the ninth grade and tenth grade (post- implementation interview with Figen)

Teacher evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA described based on before the implementation, during the implementation and after the implementation of MEAs.This evolving knowledge was summarized in Table 4.15.

Table 4.15 Figen's Evolving Knowledge on Class Setting

Implementations	Themes*
First Implementation	
Before the implementation	• The groups with five or six members • students who express themselves well (who have verbal ability-language ability) will be distributed to the groups
During the implementation	• The groups with six members • students who express themselves well (who have verbal ability-language ability) were distributed to the groups
After the implementation	• student cannot actively participate in the group since the number of the students in group which is six was high
Second Implementation	
Before the implementation	• The groups with five members • students who express themselves well (who have verbal ability-language ability) will be distributed to the groups
After the implementation	• groups should be constructed by mixed the boys and the girls in order to minimize the speeches on other issues rather than the question • seating arrangement is important there are not the girls in one end and the boys in one end of the table, students should seat disorderly like one girl and one boy in order to increase the concentration to the solution.
Third Implementation	
Before the implementation	• The groups with five members • students should create the groups their own since the students characteristics are very different when it is compared with the traditional class times and MEAs solution process
During the implementation	• four groups, which have five members, and one group, which have four members, and the groups were mixed with boys and girls.
After the implementation	• students should create the groups their own • mixed groups with boys and girls • five members in each group worked actively in this question since it contains so much operations.

Table 4.15 (continued)

Fourth Implementation and Fifth Implementation	• the number of the students in the group should change based on the question, if the question includes so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question include more interpretation rather than operation at that time, there can be three or four students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. • mixed groups with boys and girls should be constructed since gender differences create an effective working environment • groups should be constructed by students when students construct the groups they feel more that they are group and they pay attention mostly on communication, cooperation...e.g. • the number can also be change based on the grade, when the question have so much operation the number of the students can five or four in ninth grade and tenth grade but it can be four or three in eleventh and the twelfth, the number can decrease when the grade increase since eleventh and the twelfth grade's ability and knowledge more than the ninth grade and tenth grade.

* class setting structure and its reasons were given with the teacher's sentences

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the number of the students in the group was indicated as six or seven and it was stated that the students who had good grades in her math exams were distributed to each group since they might have produced different approaches. During the implementation, there were four groups with seven members and teacher interfered as distributing some students who had good grades in her math exams to each group. After the implementation, it was denoted the number of the students in each group, which was seven, were so much since the students also construct groups in each group, and most of the students could not participate the group work. In addition, she stated that the students characteristics were very different when it was compared with the traditional class times and MEAs solution process however it was stated that students who had good grades in her exam could be distributed to each group but the reason was not that these students could produce different approaches, the reason was stated that removing groups' ideas (if the students who have good grades in her exam were in one group) like "this group member all are good in math

they will solve easily but we cannot solve" so by distributing it was thought that each group could rely on themselves.

Rezzan: the number of the students in the group can be six or seven and I want to distribute the students who have good grades in my math exams to each group. They may have produced different approaches (pre-implementation interview with Rezzan)

There were four groups with seven members and teacher interfered group structure by distributing some students who had good grades in her math exams to each group. (*During the implementation*)

Rezzan: the seven members was so much since the students also construct groups in each group, and most of the students cannot participate the group work. I also observe that students behave different in the solution process; the quiet students who also do not participate in my lessons will participate actively in the solution process, in addition, they present their group work clearly. However I prefer to distribute the students who have good grades in my exam to each group for not producing different approaches for removing groups' ideas like this group member all are good in math they will solve easily but we cannot solve so by this way all the students who have good grades in each group, each group will rely on themselves (post-implementation interview with Rezzan)

In the second implementation of the MEA entitled "Street Parking", before the implementation, it was stated that the number of the groups could be five or six and the groups should be heterogenic based on mathematical achievement in order to provide each group to rely on themselves but it was also indicated that student's characteristics were different in traditional class times and in solving these questions. After the implementation it was denoted that the groups should be constructed by mixed the boys and the girls since it was explained as, boys have different characteristics and experiences, also the girls so all of them is an advantage while solving the question. In addition, it was also specified that the seating arrangement could be one girl and one boy in order to maximize the cooperation and collaboration.

Rezzan: the number of the groups could be five or six and the groups should be heterogenic based on mathematical achievement in order to provide each group to rely on themselves but it is obvious that student's characteristics are different in traditional class times and in solving these questions (the excerpt was taken from the meeting before the implementation)

Rezzan: the groups also should be constructed based on gender differences since boys have different characteristics and experiences also the girls so all of them is an advantage while solving the question…the seating arrangement also can be one girl and one boy by this way the cooperation and collaboration will be maximum ((the excerpt was taken from the meeting after the implementation)

In the third implementation of the MEA entitled "Summer Jobs" before the implementation, it was denoted that the number of the students in the group could be five or four since the operation is so much in this question and each member could participate and it was stated that students should create the groups their own because it was observed that students' mathematical achievement and the characteristics were different between class times and MEAs solution process. During the

implementation, four group which had five students and one group that had four students. The students construct their groups. Besides, some groups had group members all girls and some groups had all boys. After the implementation, it was denoted that the students should construct groups by themselves and groups should be constructed by mixed the boys and the girls since it was explained as, boys had different characteristics and experiences the girls so all of them is an advantage while solving the question. In addition, it was denoted that the five members in each group worked actively in this question since it contains so much operations.

Rezzan: I observed that mathematical achievement and the characteristics are so different between our class times and this time [MEAs solution process] so I will let the students to construct their groups by themselves…the number of the students can be five or four since the operation is so much in this question and each member can participate (pre-implementation interview with Rezzan)

Four group which had five students and one group that had four students. The students constructed their groups. Besides, some groups had group members all girls and some groups have all boys. (*During the implementation-observation notes and field notes*)

Rezzan: I let the students to construct their groups however some groups' member all of them are girls and some groups' member all of them are boys, I have a chance to observe that the boys are think like a boss they produce different approaches however they were exited and they did not do the operations correctly and construct the tables truly, on the other hand the girls are calm, they calculate the operations correctly, fill the tables truly however did not produce different approaches so if they mixed the group work will be more effective. In other implementations, I let the students to construct the groups themselves but I also want them to construct mixed groups with girls and boys…five students is good for this questions since it has so much operation and each member actively participate (post-implementation interview with Rezzan)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the number of the students in the group should change based on the question, if the question included so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question included more interpretation rather than operation at that time, there could be four or three students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. Also, it was stated that mixed groups with boys and girls should be constructed since boys had different characteristics and experiences, also the girls so all of them was an advantage while solving the question like the boys more exited and can think more practical however they were weak on sketches and the long operations however girls were not practical as boys but they are calm and they made good and detailed sketches and could calculate the long operations with their calmness. In addition, it was denoted that groups should be

constructed by students when students constructed the groups, they fell more comfortable and work more efficiently. In addition, it was added that it was meaningless for a teacher to construct the groups because it was different process students has not the same characteristics with the traditional class times.

Rezzan: I think the group structure is clear for me since I observe that mixed groups are important since boys and girls have very different characteristics and it will be made the solution process more efficient, the boys more exited and can think more practical however they are weak on sketches and the long operations however girls are not practical as boys but they are calm and they made good and detailed sketches and can calculate the long operations with their calmness. In addition, students should construct the groups by themselves since at that time they made very effective group work ...the number of the students can be four or three when the question need more discussion and more interpretation than operation; the number of the students can be five or four when the question need more operation rather than discussions. For example the summer jobs problem need more operation and the five students collaboratively work on it, also the water tank problem need more interpretation rather than operation at that time the four student discuss actively (pre-implementation interview with Rezzan)

Rezzan: it is meaningless for a teacher to construct the groups because it is really different process students are not same with the class time, I tried to distribute the students who have good grades in my exams but I see that they could be passive, however others [who have not good grades in her exam] produced more efficient ideas, we do not know them [students] in this process, so when we [teachers] leave to their wishes, by this way, they fell more comfortable and work more efficiently (post-implementation interview with Rezzan)

Teacher evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in Table 4.16.

Table 4.16 Rezzan's Evolving Knowledge on Class Setting

Implementations	Themes*
First Implementation	
Before the implementation	• The groups with six or seven members • students who have good grades in math exams will be distributed to each group since they may have produced different approaches.
During the implementation	• The groups with seven members • students who have good grades in math exams were distributed to each group
After the implementation	• the number of the students in each group, which is seven, were so much since the students also construct groups in each group, and most of the students cannot participate the group work. • students characteristics are very different when it is compared with the traditional class times and MEAs solution process
Second Implementation	
Before the implementation	• The groups with five or six members • the groups should be heterogenic based on mathematical achievement in order to provide each group to rely on themselves • students characteristics are very different when it is compared with the traditional class times and MEAs solution process
After the implementation	• groups should be constructed by mixed the boys and the girls since boys have different characteristics and experiences, also the girls so all of them is an advantage while solving the question. • the seating arrangement could be one girl and one boy in order to maximize the cooperation and collaboration.
Third Implementation	
Before the implementation	• the students in the group could be five or four since the operation is so much in this question and each member can participate • students should create the groups their own because it was observed that students' mathematical achievement and the characteristics are different between class times and MEAs solution process

Table 4.16 (continued)

During the implementation	• four group which have five students and one group that have four students. • The students construct their groups , some groups have group members all girls and some groups have all boys
After the implementation	• students should construct groups by themselves and groups should be constructed by mixed the boys and the girls since, boys have different characteristics and experiences, also the girls so all of them is an advantage while solving the question • the five members in each group worked actively in this question since it contains so much operations.
Fourth Implementation and Fifth Implementation	• the number of the students in the group should change based on the question, if the question includes so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question include more interpretation rather than operation at that time, there can be four or three students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. • mixed groups with boys and girls should be constructed since boys have different characteristics and experiences, also the girls so all of them is an advantage while solving the question • groups should be constructed by students when students construct the groups, they fell more comfortable and work more efficiently. • it is meaningless for a teacher to construct the groups because it is different process students has not the same characteristics with the traditional class times.

*class setting structure and its reasons were given with the teacher's sentences

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the number of the students in the group was indicated as three in order to create more effective environment for discussions and it was also stated that the students who have good grades in her math exams could be distributed to each group since they might have produced different approaches. In addition, mixed groups with girls and boys were denoted and it was explained "by this way students cannot slobber over other subjects rather than the question". During the

implementation, there were nine groups with three members and teacher interfered by distributing some students who had good grades in her math exams to each group. In addition, the groups were mixed with boys and girls. After the implementation, it was denoted the number of the students in each group, which was three, were ideal since all the participants worked actively. In addition, it was stated, she observed every student whom did not had good grades participate actively and produce different ideas and it was indicated that it was meaningless to distribute the students who had good grades to groups.

Semra: I want the students to construct the groups by their wishes and I want them to construct mixed groups with girls and boys so they will not slobber over other subjects rather than the question... I want them to construct three member groups since they can listen each other and can discuss more efficient if it is more than tree each member may not explain their ideas. I will only distribute the students who are good in math [got good grades in the exam] to each group, there are six students are good in math, I will distribute them to each group but there will be ten groups and I do not know how the other groups will solve the question (pre-implementation interview with Semra)

There are nine groups with three members and teacher interfered by distributing some students who had good grades in her math exams to each group. The groups were mixed with boys and girls. (During the implementation-observation notes-field notes)

Semra: The tree member is ideal I think, all the participants work actively but I see that every student which do not have good grades participate actively and produce different ideas so it is meaningless to distribute the students who have good grades to groups (post-implementation interview with Semra)

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation, it was indicated that the tree member for the groups and the mixed groups with girls and boys were ideal, besides it was stated that distributing the students who had good grades to each groups was meaningless since the questions were up to date and from real lives so students' reasoning skill took part in the foreground. It was also denoted that the groups might be constructed by mixing the different traits like more introvert and extrovert students; by this way, it was thought that students could express their solutions more clearly.

Semra: the tree member for the groups and the mixed groups [boys-girls] are ideal, however distributing the students who have good grades to each groups is meaningless since the questions were up to date and from real lives so their [students'] reasoning skill take part in the foreground. I will may mixed the different traits for example more introvert and extrovert students so they will express their solutions more clearly (post-implementation interview with Semra)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that students should create the groups their own since it was stated that their wishes and harmony important for the group work. In addition, the number of the students in the group stated as four or three. During the

implementation, there were five groups with four members and there were two groups with three members. The students constructed their groups and the mixed groups were constructed with boys and girls. After the implementation, it was stated that students' mathematical achievement and the characteristics were different between traditional class times and MEAs solution process so the students should construct groups by them in order to work in harmony. In addition, it was indicated that the four members in each group worked actively in this question since they shared the data two by two and the collaboration was good.

Semra: I will want students to construct groups with four and three students and mixed groups with boys and girls. I will let them to construct their groups since their wishes and harmony important for the group work (pre-implementation interview with Semra)

There were five groups with four members and there were two groups with three members. The students constructed their groups and the mixed groups were constructed with boys and girls. (During the implementation-observation notes)

Semra: the groups with four member work actively, they share the data two by two and the collaboration was good. Also, I observe that the students which are extrovert can express themselves well in the groups, and one of them said she find another approach other than the group approach and explained it in the presentation, I saw that the personal traits are very different from the class times and I also see that the math grades is not important in this kinds of problems. In here, the important thing is the students should construct their group by themselves in order to work in harmony (post-implementation interview with Semra)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the number of the students in the group should be four since group could work cooperatively with four members. Besides, it was stated that "mixed groups with boys and girls should be constructed since boys had different characteristics and experiences, also the girls so all of them was an advantage while solving the question by creating effective working environment". Different characteristics were given like girls were more patient and they made clear sketches, however boys give up in a short time, they were not patient, they had practical approaches but their sketches were not clear. In addition, it was denoted that groups should be constructed by students since their wishes, harmony was important in the group work and it was added that it was meaningless for a teacher to construct the groups since students' mathematical achievement and characteristics were not same for the traditional class times and the questions' solution process so teachers' interventions were stated as meaningless.

Semra: the number of the students should be four; group will work cooperatively with four members. The groups should be mixed with boys and girls since girls are more patient and they made clear sketches, however boys are giving up in a short time, they are not patient, they have practical approaches but their sketches were not clear. If they work in one group, their different characteristics will create effective working environment… students should construct the groups since their wishes, harmony is important in the group work, and we [teachers] see that the students' mathematical achievement and their [students] characteristics are not same for the class times and the questions' solution process so our interventions are not meaningful (post-implementation interview with Semra)

Teacher evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in Table 4.17.

Table 4.17 Semra's Evolving Knowledge on Class Setting

Implementations	**Themes***
First Implementation	
Before the implementation	• The groups with three members • students who have good grades in math exams will be distributed to each group since they may have produced different approaches. • mixed groups with girls and boys wanted to have been constructed so that students cannot slobber over other subjects rather than the question.
During the implementation	• The groups with three members • students who have good grades in math exams were distributed to each group • the groups were mixed with boys and girls.
After the implementation	• the students in each group, which is three, were ideal since all the participants work actively. • every student which do not have good grades participate actively and produce different ideas and it was meaningless to distribute the students who have good grades to groups.

Table 4.17 (continued)

Second Implementation	
Before the implementation	• The groups with three members • mixed groups with girls and boys
After the implementation	• distributing the students who have good grades to each groups is meaningless since the questions were up to date and from real lives so students' reasoning skill take part in the foreground. • groups may be constructed by mixing the different traits like more introvert and extrovert students so students could express their solutions more clearly.
Third Implementation	
Before the implementation	• The groups with three or four members • students should create the groups their own since their wishes and harmony important for the group work.
During the implementation	• The groups with three or four members • students construct their groups • mixed groups were constructed with boys and girls.
After the implementation	• students' mathematical achievement and the characteristics are different between traditional class times and MEAs solution process so the students should construct groups by them in order to work in harmony. • the four members in each group worked actively
Fourth Implementation and Fifth Implementation	• number of the students in the group should be four since group will work cooperatively with four members. • mixed groups with boys and girls should be constructed since boys have different characteristics and experiences, also the girls so all of them is an advantage while solving the question by creating effective working environment. • groups should be constructed by students since their wishes, harmony is important in the group work • it is meaningless for a teacher to construct the groups since students' mathematical achievement and characteristics are not same for the traditional class times and the questions' solution process.

* class setting structure and its reasons were given with the teacher's sentences

Melda's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation the number of the students in the group was indicated as three and it was stated that the students should construct their groups since all the students math achievement are nearly same and it was also added that if the students' math achievement were not same then students who had good grades in her math exams could be distributed to each group since they might have produced different approaches. In addition, mixed groups with girls and boys wanted to have been constructed so that it was thought students could not slobber over other subjects rather than the question. During the implementation, there were nine groups with three members and there was one group with two members, students constructed their groups and the groups were mixed with boys and girls. After the implementation, it was denoted that the number of the students in each group, which was three, could be four by this way it was thought that there were fewer groups and all groups had more time in presentation while discussing their solution process. In addition, it was stated that students' construction of their mixed [*boys-girls*] groups were appropriate, since it was thought that if they were mixed they talked only about the solution approaches rather than other subjects.

Melda: I want them [students] to construct groups with three member by their own since all the students math achievement are nearly same I do not interfere the group structure, if they are not same I prefer to distribute students who have good grades in math exams to each group since they may have produced different approaches. In addition …I want them to construct mixed groups with girls and boys. If the group constructed with all girls or all boys I think they will talk on other subjects. (Pre-implementation interview with Melda)

There were nine groups with three members and there was one group with two members, students constructed their groups and the groups were mixed with boys and girls. (During the implementation-observation notes)

Melda: I think four members in each group will be more appropriate since in the presentations, we have fewer groups and we have more time to discuss on their solution approaches. However the students' construction of their mixed [boys-girls] groups are appropriate, since they are mixed they talk only about the solution approaches rather than other subjects (post-implementation interview with Melda)

In the second implementation of the MEA, entitled "Street Parking", before the implementation it was stated that the number of the students in each group should be four and mixed groups with girls and boys were ideal, and it was also added that if the students' math achievement were not same the students who had good grades in math exams could be distributed to each group since it was thought that it will be more productive. After the implementation, it was indicated, "students' math

achievement and personal characteristics were very different in MEA's solution process and in traditional class times so the students should construct their groups by their own since their harmony is more important in group work".

Melda: there are four members in each group, students will construct their mixed group with boys and girls but if the students' math achievement are not same I interfere the groups like distributing the students who have good grades in math exams, this will be more productive (pre-implementation interview with Melda)

Melda: I can observe that students are not same as we know them from our class times, the students who have low grades will produces different and efficient ideas so math achievement and personal characteristics are very different in here [MEA's solution process] so the students should construct their groups by their own since their harmony is more important (post-implementation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation and after the implementation, it was denoted that "students should create the groups their own since their wishes and harmony important for the group work". It was also added that "teachers could not interfere the group structure since teachers do not know students' characteristics and achievement in these kinds of problems". In addition, the number of the students in the group stated as four since it was thought that four members worked actively, collaboratively in each group. Moreover, it was specified that "the students should construct mixed groups with girls and boys since they do not talk on other subjects rather than the question". During the implementation, there were eight groups with four members and students constructed their groups. The groups were mixed with boys and girls.

Melda: the students' construct mixed groups with girls and boys, it was good since they do not talk on other subjects rather than the question, and the four members worked actively, collaboratively in each group…I wanted students to construct their groups since the harmony is important in group work and teachers cannot interfere since we do not know their characteristics and achievement in these kinds of problems (post-implementation interview with Melda)

There were eight groups with four members and students constructed their groups. The groups were mixed with boys and girls. (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation same things were denoted with in the third implementation that "students should create the groups their own since their wishes and harmony important for the group work". It was also added that "teachers could not interfere the group structure since teachers do not know students' characteristics and achievement in these kinds of problems". In addition, the number of the students in the group stated as four since it was thought that four members worked actively,

collaboratively in each group. Moreover, it was specified that "the students should construct mixed groups with girls and boys since they do not talk on other subjects rather than the question".

Teacher evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA described based on before the implementation, during the implementation and after the implementation of MEAs. This evolving knowledge was summarized in Table 4.18.

Table 4.18 Melda's Evolving Knowledge on Class Setting

Implementations	**Themes***
First Implementation	
Before the implementation	• The groups with three members • the students construct their groups since all the students math achievement are nearly same . If the students' math achievement are not same then students who have good grades in math exams can be distributed to each group since they may have produced different approaches. • mixed groups with girls and boys wanted to have been constructed so that students cannot slobber over other subjects rather than the question.
During the implementation	• The groups with three members • students constructed their groups • the groups were mixed with boys and girls.
After the implementation	• the students in each group, which is three, could be four by this way there are fewer groups and all groups have more time in presentation while discussing their solution process. • mixed groups with girls and boys wanted to have been constructed so that students cannot slobber over other subjects rather than the question.
Second Implementation	
Before the implementation	• The groups with four members • mixed groups with girls and boys wanted to have been constructed so that students cannot slobber over other subjects rather than the question. • if the students' math achievement are not same the students who have good grades in math exams can be distributed to each group since it will be more productive.

Table 4.18 (continued)

After the implementation	• students' math achievement and personal characteristics are very different in MEA's solution process and in traditional class times so the students should construct their groups by their own since their harmony is more important in group work.
Third Implementation	
Before the implementation *After the implementation*	• students should create the groups their own since their wishes and harmony important for the group work. • teachers could not interfere the group structure since teachers do not know students' characteristics and achievement in these kinds of problems. • the number of the students in the group should be four since four members worked actively, collaboratively in each group. • students should construct mixed groups with girls and boys since they do not talk on other subjects rather than the question.
During the implementation	• The groups with four members • students construct their groups. • groups were mixed with boys and girls.
Fourth Implementation **and** **Fifth Implementation**	• students should create the groups their own since their wishes and harmony important for the group work. • teachers could not interfere the group structure since teachers do not know students' characteristics and achievement in these kinds of problems. • the number of the students in the group stated as four since four members worked actively, collaboratively in each group. • students should construct mixed groups with girls and boys since they do not talk on other subjects rather than the question.

*class setting structure and its reasons were given with the teacher's sentences

4.2.1.1Cross-case Analysis of Knowledge of Class Setting

Under this subheading cross-case analysis results of knowledge of class setting what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on class setting that determining the group structure and the number of students in the group while students are working on the MEA was compared in Table 4.19.

Table 4.19 Case Ratings of Class Setting Themes for All Implementations

Case Themes**	**Cases***			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
students who express themselves well (who have verbal ability-language ability) will be distributed to the groups so that students could express their solutions more clearly. groups may be constructed by mixing the different traits like more introvert and extrovert students so that students could express their solutions more clearly.	1,2		2	
students who have good grades in math exams will be distributed to each group since they may have produced different approaches.		1,2	1	1
seating arrangement is important there are not the girls in one end and the boys in one end of the table, students should seat disorderly like one girl and one boy in order to increase the concentration to the solution.	2	2		
students should create the groups their own since the students characteristics are very different when it is compared with the traditional class times and MEAs solution process groups should be constructed by students when students construct the groups they feel more that they are group and they pay attention mostly on communication, cooperation...e.g. it is meaningless for a teacher to construct the groups because it is different process students has not the same characteristics with the traditional class times.	3,4,5	3,4,5	3,4,5	2,3,4,5
the number of the students in the group should change based on the question, if the question includes so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question include more interpretation rather than operation at that time, there can be three or four students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group.	4, 5	3,4,5		

Table 4.19 (continued)

the number of the students in the groups was also specified that the number can also be change based on the grade, this was explained as when the question have so much operation the number of the students can five or four in ninth grade and tenth grade but it can be four or three in eleventh and the twelfth, the number can decrease when the grade increase since eleventh and the twelfth grade's ability and knowledge more than the ninth grade and tenth grade.	5			
number of the students in the group should be four since group will work cooperatively with four members.			3,4,5	2,3,4,5
mixed groups with boys and girls should be constructed				
• since gender differences create an effective working environment	3,4,5	2,3,4,5	4,5	
• mixed groups with girls and boys so that students cannot slobber over other subjects rather than the question.			1.2,3	1,2,3,4,5

* 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

** Class setting structure and its reasons were given with the representative teachers' sentences

Teachers tried to determine the group structure and the number of students in the group while students are working on the MEA based on before the implementation, during the implementation and after the implementation of MEAs. In the first implementations, three teachers tried to interfere the group structure like distributing the students who have good grades in math exams to each group since it was thought that these students might have produced different approaches. In addition, two teachers tried to combine the introvert and extrovert students in order to help the group to express their solutions more clearly. However after the implementations, it was observed by all the teachers that the students characteristics were very different when it was compared with the traditional class times and MEAs solution process so it was decided that students should create the groups their own since it was stated that when students constructed the groups "they feel more that they are group and they pay attention mostly on communication and cooperation". Although, it was

expressed that " it is meaningless for a teacher to construct the groups because it is different process students has not the same characteristics with the traditional class times since the questions were up to date and from real lives so students' reasoning skill take part in the foreground". The number of the group member were indicated by two teachers that the number of the students in the group should change based on the question, if the question include so many operation, there can be five or four students in each group since all of them can have a chance of active participation and if the question include more interpretation rather than operation at that time, there can be three or four students in each group since all of them can have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. In addition, other two teacher were stated that number of the students in the group should be four since group could work cooperatively with four members. Moreover, one teacher also added in one of her implementation that the number of the students could also be change based on the grade, this was explained as "when the question have so much operation the number of the students can five or four in ninth grade and tenth grade but it could be four or three in eleventh and the twelfth, the number could decrease when the grade increase since eleventh and the twelfth grade's ability and knowledge more than the ninth grade and tenth grade".

All teachers added that students should construct mixed groups with boys and girls. Three teachers wanted this structure since it was thought that boys have different characteristics and experiences, the girls so, all of them was seen as an advantage while solving the question in order to create effective working environment. In addition, two teachers wanted this structure since it was thought that when the girls and boys mixed, students could not slobber over other subjects rather than the question. Intercalarily, two teachers stated that seating arrangement was important, "there should not be the girls in one end and the boys in one end of the table, students should seat disorderly like one girl and one boy in order to increase the concentration to the solution", they indicated this strategy in one implementation.

Data revealed that pedagogical strategy on class setting were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations in the implementations, and as teachers discussed strategy with colleagues. The below excerpts taken from the dialogue among the

teachers in the meeting before and after the implementations denoted that one of the component of the class setting strategy which was group structure become articulated based on their observations and the discussions with the colleagues in both schools.

The below excerpts were taken from the dialogue among the teachers in the meeting before the third implementation indicates that the teachers' group structure strategies was changed from interfering group structure such as distributing the students who had good grades in math exams and combining introvert and extrovert students; to let the students construct the groups' themselves, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Figen: in these implementation [Summer Job], I let the students construct their group, before [in the first and second implementation] I tried to combine the introvert and extrovert students in order to help the groups to express themselves well, but I saw that the characteristics of the students are very different with in this implementations and in our classes. I did not forget Salih, he did not talk in my lessons, did not participate the lessons however in the implementation, he took the leader role in the group, in the presentation he defended the group, it was really surprising

Teacher A: students' characteristics are very different in the group work, the students who you said that did not work, would participate the group work actively

Rezzan: but I am not sure that if we did not distribute the good students [students who have good grades in the math exams] to the groups, some groups may not find the result and then they feel bad

Teachers B: I did not distribute the good students and all groups reach the result, okay some of them [groups] were reach more complicated results but it does not depend on the success in the exams

Teacher A: all our students were successful so we did not need to distribute the good ones

Rezzan: yes, I observed this in my class also for example Yeşim thought very differently and explained and support her idea in the presentations, I heard this students voice for the first time and she had not got good grades in the exams…the appropriate strategy may do not interfere the group structure, let it to students"

The below excerpts were taken from the dialogue among the teachers in the meeting after the second implementation indicates that the teachers' group structure strategies was changed from interfering group structure such as distributing the students who had good grades in math exams and combining introvert and extrovert students; to let the students construct the groups' themselves, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Semra: …however distributing the students who have good grades to each group is meaningless since the questions were up to date and from real lives so their [students'] reasoning skill takes part in the foreground. I will may mixed the different traits for example more introvert and extrovert students so they will express their solutions more clearly

Teachers F: their characteristics were also different introvert students can defend their solutions very well since they pay attention more time on one question and all group member had known the steps in detail

Semra: yes, yes for example one of my students who is girl and very introvert in the class time could defend her approach in the group, I did not expect from her to defend her approach

Melda: also, the students who have low grades produced different and efficient ideas so math achievement and personal characteristics are very different in here [MEA's solution process] so the students should construct their groups by their own since their harmony is more important

4.2.2 Introducing the Implementation

Under this subheading each teacher evolving knowledge on introducing the implementation that describing the process of the implementation before the students begin to solve the MEAs described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation and during the implementation teacher did not make any explanation about the process of the implementation, after the groups were constructed, the question was distributed and wanted students to solve the question and said that "after the solution process, you will make presentation". However, after the implementation, it was indicated that the process should be explained and the time for each process should be given.

Figen: we should explain what they are going to do and we should give time for each process (post-implementation interview with Figen)

In the second implementation of the MEA, entitled "Street Parking" before the implementation, it was denoted that individual work for 5 minute, group work for 40 minutes and groups' presentations for 45 minutes could be written on board in order to show the process and its times orderly. In addition, after the implementation, it was added that this descriptions should be made before the questions were distributed "since when the questions were distributed every student concentrate on the question and cannot listen the explanations".

Figen: we [teachers] can say 5 minutes each group member read the question and work on the solution individually, and then want them to work on the solution with the group for 40 minutes, the end of the first lesson, then we can express that each group will made presentation in the second lesson [45 minutes] …we can made a table on the board that show the process and its times orderly (pre-implementation interview with Figen)

Figen: …but we [teachers] should explain the process before we distribute the question since when we distribute all the students concentrate on the questions and cannot listen our explanations (post-implementation interview with Figen)

In the third implementation of the MEA entitled "Summer Jobs", the process and its times were explained as previously stated in the second implementation. In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the process and its time should be given in detailed like "first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take arrange in the second lesson". It was indicated that this explanations was important for the groups in order to organize their work. In addition, it was indicated that how can be the group work could be explained to the students since it was thought that they were not accustomed to the group work, like "listen to each other, share, collaboratively work, discuss, analyze and establish the group solution". It was added that this explanation could be made especially to the classes where this implementations were done for the first time. Moreover, it was denoted that the concepts embedded in the MEAs should not be given in this explanations since "students try to use only given concepts so their solution strategies would be limited".

Figen: when we [teachers] explain what they do and give its times, the groups have a chance of organizing themselves. I also observe that when I explain the group work rules, like listen to each other, share, collaboratively work, discuss, analyze and establish the group solution to the classes where the implementations are done for the first time, they worked more collaboratively, so this explanations should be made since they are not accustomed to the group works… we do not say anything about the questions' concepts since if we gave then students tried to use these concepts and they would not produce many different approaches as we expected (post-implementation interview with Figen)

Rezzan's Profile

In the first implementation of the MEA which entitled "Bank Robbery", before the implementation and during the implementation teacher did not make any explanation about the process of the implementation, after the groups were constructed, the question was distributed and wanted students to solve the question

and said that "after the solution process, they will make presentation". However, after the implementation, it was indicated that the process should be explained and the time for each process should be given and this can be given in a table on board.

Rezzan: we [teachers] explain and give their times, but if we give them in a table, this will be more productive, they will look at the board and organize themselves (post-implementation interview with Rezzan)

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation, it was denoted that individual work for 5 minute, group work for 40 minutes and groups' presentations for 45 minutes could be written on board in order to show the process and its times orderly.

Rezzan: I[teachers] wanted students firstly read the question individually and try to produce solution in five minutes after then wanted them work in group and produce a group solution in nearly 40 minutes then I said that they will make presentations in the second lesson and I wrote them on board, I think by this way they knew what to do (post-implementation interview with Rezzan)

In the third implementation of the MEA entitled "Summer Jobs", the process and its times were explained as previously stated in the second implementation. In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the process and its time should be given in detailed like "first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take arrange in the second lesson". It was indicated that this explanations was important for the groups in order to organize their work. In addition, it was indicated that how can be the group work could be explained to the students since they were not accustomed to the group work, like "listen to each other, share, collaboratively work, discuss, analyze and establish the group solution". It was added that this explanation could be made especially to the classes where this implementations were done for the first time. Moreover, it was denoted that the concepts embedded in the MEAs should not be given in this explanations since it was thought that if it was emphasized, students tried to use only given concepts so their solution strategies would be limited.

Rezzan: I gave the process and the times, this made students work more organized. Also as I stated before, it is important to explained the group work to the students who we made the implementations for the first time because they do not know how to work in a group so it will be helpful for them to emphasize the collaboration, cooperation, sharing, listening to each other, discussing every member idea… I observe that when we say any title of the subject, students were directed in order to solve the question based on that subject so we should not say the concepts for not restricting their solution ways (post-implementation interview with Rezzan)

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation and during the implementation teacher did not make any explanation about the process of the implementation, after the groups were constructed, the question was distributed and wanted students to solve the question and said that "after the solution process, they will make presentation". However, after the implementation, it was indicated that the process should be explained and the time for each process should be given.

Semra: It is a good idea to give what they will do step by step and its time since they are not accustomed they thought that they could solve in five or ten minutes, so it will be good to give the process and the times so students can know what they are expected from them and in how much time (post-implementation interview with Semra)

In the second implementation of the MEA entitled "Street Parking", before the implementation, it was denoted that individual work for five or ten minute, group work for 45 minutes and groups' presentations for 30 minutes could be given in order to show the process and its times orderly. In addition, after the implementation, it was added that students accustomed to multiple-choice questions, so it could be good to express the students, whom the implementation takes arranges for the first time that "these questions solutions process is different and it may take much time" so that they can organize themselves.

Semra: we [teachers] should express what they will do with its times, for this question students will work in five or ten minutes individually, the the group work can take 45 minutes and they will be explained that they made presentations in the second lesson and this process can take 30 minutes (pre-implementation interview with Semra)

Semra: Although we should explain that these questions solutions process is different and it may take much time so that students can organize themselves. It is important since students accustomed to multiple-choice questions, so it will be good to express the students whom the implementation takes arranges for the first time (post-implementation interview with Semra)

In the third implementation of the MEA entitled "Summer Jobs", the process and its times were explained as previously stated in the second implementation. In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the

implementation it was indicated that the process and its time should be given in detailed like first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take arrange in the second lesson. It was indicated that this explanations was important for the groups in order to organize their work. In addition, it was indicated that how can be the group work could be explained to the students since they were not accustomed to the group work, like listen to each other, share, collaboratively work, discuss, analyze and establish the group solution. It was added that this explanation could be made especially to the classes where this implementations are done for the first time. Moreover, it was denoted that teachers should not be talked about the solution ways and the concepts embedded in the MEAs in this explanations since students tried to use only given concepts and solution ways so their solution strategies would be limited.

Semra: we [teachers] said what they are going to do and gives its times also we said that you will made a group work for 40 minutes and students go on to work individually because they do not know the group work, they are not accustomed to sharing, discussion and collaboration so it should be explained to the groups which the implementation is done for the first time however if we made implementation to the class, then there is no need to explain since they learned…in the introducing part we do not need to express any subject or any solution way because it will direct the students and they tried to use this solution ways, this subjects for example when we say trigonometry is important in the solution process they do not try to make from other subjects, this restrict their approaches (post-implementation interview with Semra)

Melda's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation ,during the implementation and after the implementation teacher did not make any explanation about the process of the implementation, in the implementation after the groups were constructed, the question was distributed and teacher wanted students to solve the question and said that "after the solution process, they will make presentation".

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation, it was denoted that individual work for five minute, group work for 45 minutes and groups' presentations for 30 minutes could be given in order to show the process and its times orderly.

Melda: for groups in order to organize themselves they should know what to do and their times so we [teachers] should express that we want them firstly read the question and find solution ways indivually then work in group and then make presentation and their times orderly for this question can be 5-45-30 minutes (pre-implementation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", the process and its times were explained as previously stated in the second implementation. In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", before the implementation and after the implementation it was indicated that the process and its time should be given in detailed like "first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take arrange in the second lesson". It was indicated that this explanations was important for the groups in order to organize their work. In addition, it was indicated that how can be the group work could be explained to the students since it was thought that they were not accustomed to the group work, like listen to each other, share, collaboratively work, discuss, analyze and establish the group solution. It was added that this explanation could be made especially to the classes where this implementations are done for the first time. Moreover, it was denoted that teachers should not be talked about the concepts embedded in the MEAs in this explanations since students try to use only given concepts so their solution strategies would be limited.

Melda: it is important for the groups to be expressed what they are expected to do and all steps' time in order to organize themselves. In addition, I saw that I made an explanation to one class that how they will be work in the group like you should listen each other, you should work collaboratively then I observed that the group psychology was established they rely on themselves and say our group will solve. Therefore, this explanation was important; group work should be explained to the classes where the implementation was done for the first time…we [teachers] have a chance to observe that when we say some concepts in these explanations, students tried to use them and not use other subject, their solution approaches were limited so we should not say anything about the concepts in the question (post-implementation interview with Melda)

4.2.2.1 Cross-case Analysis of Knowledge of Introducing the Implementation

Under this subheading, cross-case analysis results of knowledge of introducing the implementation what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on introducing the implementation that describing the process of the implementation before the students began to solve the MEAs was compared in Table 4.20.

Table 4.20 Case Ratings of Introducing the Implementation Themes for All Implementations

Case Themes[**]	**Cases**[*]			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
after the groups were constructed, the question was distributed and wanted students to solve the question and said that after the solution process, they were going to make presentation (during implementation)	1	1	1	1
table on the board that show the process and its times orderly	2	2		
descriptions should be made before the questions were distributed since when the questions were distributed every student concentrate on the question and cannot listen the explanations.	2			
the process and its time should be given in detailed like first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take place in the second lesson.	2,3,4,5	2,3,4,5	2,3,4,5	2,3,4,5
how can be the group work could be explained to the students since they are not accustomed to the group work, like listen to each other, share, collaboratively work, discuss, analyze and establish the group solution. It was added that this explanation could be made especially to the classes where this implementations are done for the first time.	4,5	4,5	4,5	3,4,5
the concepts embedded in the MEAs should not be given in the explanations since students try to use only given concepts so their solution strategies would be limited.	4,5	4,5	3,4,5	4,5
students accustomed to multiple-choice questions, so it will be good to express the students, which the implementation takes places for the first time that these questions solutions process is different and it may take much time so that they can organize themselves.			2	

[*] 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

[**] Introducing the implementation strategy and its reasons were given with the representative teachers' sentences

The teachers introducing the implementations that describing the process of the implementation before the students began to solve the MEAs were changed in the cyclic process of the study. In the first implementation, all teachers only distributed the questions to the groups and want them to solve the question and added after then they made presentation, however after the first implementations all teachers stated that the process and its time should be given in detailed like first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take place in the second lesson. In addition, two teachers denoted that this process and its times would be given in a table on board in order to help groups to organize their work. Moreover all teacher were indicated that how can be the group work could be explained to the students since it was thought that they were not accustomed to the group work, like "listen to each other, share, collaboratively work, discuss, analyze and establish the group solution". It was added that this explanation could be made especially to the classes where this implementations were done for the first time. All teachers were also specified that the concepts embedded in the MEAs should not be given in the explanations since it was observed by the teachers that students tried to use only given concepts so their solution strategies would be limited.

Intercalarily, one teacher specified that students accustomed to multiple-choice questions, so it could be good to express the students whom the implementation takes places for the first time that "these questions solutions process is different and it may take much time so that they can organize themselves" in one implementation. This specification was given as an intervention (i.e. suggested strategy) in the meetings before the second implementation in both schools. However, this specification was denoted by one teacher after the second implementation at once and was not going to part in the further cyclic process. In the meeting before the second implementation, the below excerpts indicated that when the researcher suggested to introduce the type of the question such as its solution process, the teachers validated however, this was not stated by the teachers in the further cyclic process of this study.

The below excerpt taken from the dialogue among the teachers in the meeting before the second implementation in Anatolian Teacher Training High School.

Teacher F: I think most of the students think that they can solve in five minutes but when process elongate, they began to think that they made wrongly, they thought wrongly

Researcher: this may be explained in the introduction part by the teachers such us this questions was not like the problems you [students] accustomed to, this questions' solution process may take much time

Semra: I think it will be good to explain by this way they can organize their solution process

Melda: or we [teachers] can say this is not like the multiple choice, this process could take much time

Teacher F: it should be explained

The below excerpt taken from the dialogue among the teachers in the meeting before the second implementation in Anatolian High School.

Researcher: In the introduction part, the teachers can introduce the type of the question such us this questions were not like the problems you [students] accustomed to, this questions' solution process may take much time, by this way they may work more organized

Rezzan: It would be good, since in my implementation [bank robbery] students began to give up when ten minutes past since they thought that they work on the wrong approach

Figen: if this was explained then students will think that I have much time and I can think more deeply

Teacher A: our students always solve multiple choice questions so it is possible to think this questions [MEA] like multiple choice and when they could not solve in short time, they might disappointed.

Besides, one teacher added that introducing should be made before the questions were distributed to students since it was thought that when the questions were distributed every student concentrates on the question and could not listen the explanations in one implementation.

Data revealed that pedagogical strategy on introducing the implementation were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations in the implementations, and as teachers discussed strategy with colleagues. The below excerpts taken from the dialogue among the teachers in the meeting before and after the implementations denoted that one of the component of the introducing the implementation strategy which was explaining the process become articulated based on their observations and the discussions with the colleagues in both schools.

The below excerpts were taken from the dialogue among the teachers in the meeting before the second implementation indicates that the teachers' explaining the process was changed from explaining roughly the process; to explaining the process in detail

and with its' times, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Rezzan: when we [teachers] explain what they [students] are going to the roughly, they are lost in the solution process and cannot organize the group work

Figen: we [teachers] should explain, what they [students] are going to do with its time

Teacher C: It can be written in a table on the board such as group work 45 minutes, poster paper preparation 10 minutes, presentation 40 minutes

Rezzan: yes, I think the same thing; they [students] can look at the board and can direct their group work

Figen: since the students are not accustomed to the group work, mostly they spend their time on the individual work

The below excerpts were taken from the dialogue among the teachers in the meeting before the second implementation indicates that the teachers' explaining the process was changed from explaining roughly the process; to explaining the process in detail and with its' times, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Semra: I think it should be explained step by step what they are going to do, since they are not accustomed they thought that they could solve in five or ten minutes, so it will be good to give the process and the times so students can know what they are expected from them and in how much time

Teacher F: it is so possible, all students accustomed to the multiple choice questions so according to me they think that the solution take 5 minutes maximum and they can solve individually

Melda: so we [teachers] should express that we want them firstly read the question and find solution ways individually then work in group and then make presentation and should give their times in order to organize themselves

4.2.3 Providing an Understanding of the MEA and to Warm up the MEA

Under this subheading each teacher evolving knowledge on providing an understanding of the MEA and to warm up the MEA before the students began to solve the MEAs described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, during the implementation and after the implementation the work was not denoted for providing an understanding of the MEA and to warm up the MEA since it was indicated that this understanding and warm-up period could direct the students' approaches and it was stated that how can this be done without directing the students.

In the second implementation of the MEA, entitled "Street Parking", before the implementation an example was given close to the question and said their actual task again based on this example question and after the implementation, it was stated that the parts which were not clear for the students in the question could be explained and focused on by teachers.

Figen: I think we [teachers] can attract their [students'] attention by another example and we can focus on their task and what they are wanted to do for instance we can say that you have an parking space and do you want to earn maximum gain, so how to place the maximum cars in your parking space (pre-implementation interview with Figen)

Figen: 3m wide and 4,8m long space including the lines is needed for a car in order to park safely part in the question is understand differently by the students and the calculations were different based on that so just like this parts which are not clear should be explained, we can say that this dimensions were the parking space dimension for a car ,not the cars' dimension at that time all of the students will understand the same thing (post-implementation interview with Figen)

In the third implementation of the MEA entitled "Summer Jobs", it was indicated that students' task and the unclear parts of the questions could be asked after the students read the question in order to provide the understanding and to warm-up. It was also added that the unclear parts would also be explained in order to eliminate the uncertainty.

After the students read the question, these questions were asked what is your task? What is full time and part time worker? and full time and part time workers were explained as "full time worker will be hired for three months and part time workers will be hired for the determined times which are based on your criteria". (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] helped students to warm-up and understand the MEA clearly. However, it was

indicated that "creating a discussion on the unclear part of the questions can direct students solutions and limit their approaches".

The below excerpt was taken from the pre-implementation interview with Figen

Figen: understanding the question is very important since if they [students] understand then there is no time-consuming, if they are not understand at that time they use irrelevant data and consume time on inappropriate approaches. Students should understand clearly, what the question asks them so it is efficient to ask what the question asks us? What is our task? What are the given? by these questions, they have a chance to look at the problem with more critic eyes…

Researcher: What can a teacher can do in order to warm up?

Figen: …it is efficient to use appropriate models, for example in here [pack them in!] with these covers [water bottle covers] they can easily see the different arrangement rather than drawing the different arrangement

Figen: when they ask me if they can rotate the storage unit, I wanted to create a discussion on it but then I gave up, I thought that if I created a discussion on these [unclear parts] then all students discuss their ideas at that time all groups heard all ideas then there will be more common approach rather than the different approaches since they are affected their ideas, according to me all groups can be done these discussions in their groups (post-implementation interview with Figen)

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, during the implementation and after the implementation the work was not denoted for providing an understanding of the MEA and to warm up the MEA.

In the second implementation of the MEA, entitled "Street Parking", before the implementation examples were given from environments close to the question and said students' actual task again based on these examples and after the implementation, it was stated that the parts which were not clear for the students in the question could not be explained by teachers, students should be directed to the question and students gave a meaning to these part since it was thought that it was also a step of the solution.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Researcher: What can a teacher do in order to provide an understanding of the activity and to warm up?

Rezzan: we [teachers] can give an example like when you see the parking spaces in front of your home and in the markets, do you think that how can we benefit from these areas efficiently and how can we place maximum number of cars, now you have a chance to think …

Teacher A: we can also want them to think our school parking spaces, it will motivate them

Rezzan: in here [street parking], I saw that most of the groups take the 4.8 m wrongly, they misunderstand the question, at that time I think, we [teachers] should ask questions about the question and make them to think again since this understanding procedure is also an important step for the solution… (the excerpt was taken from the meeting after the implementation)

In the third implementation of the MEA entitled "Summer Jobs", it was indicated that students' task and the given data could be focused on by the teacher after the students read the question in order to provide the understanding and to warm-up. In addition, when the students not clear about some parts of the question, they were directed to the question.

After the students read the question, these questions were asked: "what is your task? What are given to us?" and when the students ask full time and part time workers teacher asked "what do you understand? Can you read the question again?" with these questions students directed to the question (During the implementation-observation notes).

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] helped students to warm-up and understand the MEA clearly. In addition, it was indicated that creating a discussion on the unclear part of the questions when all groups have a problem to understand could be effective rather than explaining these parts as a teacher.

Rezzan: In here [pack them in!], they may misunderstand and can return the storage units; I can ask here Can we turn the storage units? What is our aim? with these questions they understand more clearly. if most of the groups do not understand the same part of the question I can create a discussion so that students create many ideas and try to understand all together rather than listening my explanations, my ideas (pre-implementation interview with Rezzan)

Rezzan: … models [water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question] gave students a chance to try for example in here [pack them in!] they found different arrangement with the help of the covers so it is important to want students to use models (post-implementation interview with Rezzan)

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, during the implementation and after the implementation the work was not denoted for providing an understanding of the MEA and to warm up the MEA since it was indicated that "this understanding and warm-up period can direct the students' approaches" and it was asked that how can this be done without directing the students.

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation, an example was given close to the question and said their actual task again based on this example question.

Semra: Like teacher F [colleague] said we [teachers] can give example and we can say think that you manage a parking space and your aim is to earn maximum gain, so how can you place the cars? Which different arrangement can take place? Which arrangement is the efficient way? (excerpt was taken from the meeting before the implementation)

In the third implementation of the MEA entitled "Summer Jobs", it was indicated that students' task and the given data could be focused on by the teacher after the students read the question in order to provide the understanding and to warm-up. In addition, it was stated that when the students not clear about some parts of the question, the unclear parts would also be explained in order to eliminate the uncertainty by the teacher.

After the students read the question, these questions were asked "what is your task? What are given to us? What we are going to do?" and when the students asked part time workers, teacher explained "our aim is to earn more gain as we can so we can employ the part time workers in anytime we want". (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] when appropriate, if it is not appropriate then the simulations and other real life examples could be used, helped students to warm-up and understand the MEA clearly. In addition, it was indicated that everyday language should be used in the questions in order to not direct students to some specific mathematical concepts and make the students understand clearly in one implementation.

Semra: …in my first implementations, I think that they [students] read the question and they understand that is all but I see that in order to prevent time consuming on wrong understandings, there should be the questions are asked so that we have a chance to see they understood, they know their task …when it is appropriate the model usage is very useful, students can imagine the real situation more easily like in here [Pack Them In!] most of them find the arrangement by trying [trying with models] but sometimes it cannot be possible to use models at that time simulations can be shown or we [teachers] can give other relevant real life examples in order to make the students imagine the situation more easily… (post-implementation interview with Semra)

Semra: …in the questions, the sentences and the vocabularies are important if everyday language is not used then students try to use this vocabularies like when they see the volume they tried to use the volume formulas, so it should be used amount of water that is in the tank or something like that we use in everyday language and by this way they understood the question more clearly…(post-implementation interview with Semra)

Melda's Profile

In the first implementation of the MEA, entitled "Bank Robbery", before the implementation, during the implementation and after the implementation the work was not denoted for providing an understanding of the MEA and to warm up the MEA.

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation, it was denoted that the task of the students could be focused on by the teacher with the questions.

Melda: we [teachers] can ask questions based on their [students] task in the question [MEA], for example we can ask how can we place differently? Which arrangement do you think more efficient in order to earn the maximum gain? (pre-implementation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", it was indicated that students' task and the given data could be focused on by the teacher after the students read the question in order to provide the understanding and to warm-up. In addition, it was stated that when the students not clear about some parts of the question, the teacher would also explain the unclear parts in order to eliminate the uncertainty.

After the students read the question, these questions were asked "what is your task? What are given to us? What do you understand from full time and part time worker?" after students' different explanations, teacher also explained part time worker as "our aim is to earn more gain as we can so we can employ the part time workers in anytime we want "(During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] when appropriate, if it was not appropriate then the simulations and other real life examples could be used, helped students to warm-up and understand the MEA clearly. In addition, it was indicated that everyday language should be used in the questions in order to not direct students to some specific mathematical concepts and make the students understand clearly in one implementation.

Melda: …if the question is not be understood clearly then students spend so many times on irrelevant approaches so we should ask them what are the givens? What are their tasks? so we can be sure that they understand… with the covers [models] they try and can find the situations that they cannot imagine so it is really an efficient way…like teacher D [other colleague] said if the models [concrete models] are not appropriate the simulations and real life examples can be used in order to help them imagine, for example they could imagine more easily if they see the simulation in tank problem [water tanks] which the tank is filling with water then they might graph more easily…(post-implementation interview with Melda)

Melda: …as a teacher when we read the questions we try to find the keywords and based on with the keywords we try to use related concepts and try to solve the problems. So it is important in these kinds of problems not including mathematical terms, these questions should include everyday vocabulary and should not direct the students…they [students] can understand more easily what they are going to do with these everyday language…(post-implementation interview with Melda)

4.2.3.1 Cross-case Analysis of Knowledge of Providing an Understanding of the MEA and to Warm up the MEA

Under this subheading, cross-case analysis results of knowledge of providing an understanding of the MEA and to warm up the MEA what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on providing an understanding of the MEA and to warm up the MEA before the students begin to solve the MEAs was compared in Table 4.21.

Table 4.21 Case Ratings of Providing Understand and Warm-up MEA Themes for All Implementations

Case Themes **	**Cases***			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
the work was not denoted for providing an understanding of the MEA and to warm up the MEA	1	1	1	1
• this understanding and warm-up period can direct the students' approaches and how can this be done without directing the students.	1		1	
Introduce the students to the context of the problem • an example was given close to the question and said their actual task again based on this example question • examples were given from environments close to the question and said students' actual task again based on these examples	2,3,4,5	2,3,4,5	2,3,4,5	3,4,5

Table 4.21 (continued)

Unclear parts in the question • the parts which are not clear for the students in the question could be explained and focused on by teachers.	3		3	3
• the parts which are not clear for the students in the question could not be explained by teachers, students should be directed to the question and students gave a meaning to these part since it is also a step of the solution.		2		
creating a discussion on the unclear part of the questions • it was indicated that creating a discussion on the unclear part of the questions direct students solutions and limit their approaches.	5			
• it was indicated that creating a discussion on the unclear part of the questions when all groups have a problem to understand rather than explaining these parts as a teacher.		5		
the questions about the MEA [*What is your task? What are givens?*] and the **encourage the use of concrete models** [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] help students to warm-up and understand the MEA clearly.	4,5	3,4,5	3,4,5	3,4,5
• if the concrete models is not appropriate then the simulations and other real life examples can be used,			4,5	4,5
everyday language should be used in the questions in order to not direct students to some specific mathematical concepts and make the students understand clearly.			4	4

* 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

** Providing an Understanding of the MEA and to Warm up the MEA strategy and its reasons were given with the representative teachers' sentences in some themes.

The teachers providing an understanding of the MEA and to warm up the MEA process before the students began to solve the MEAs were changed as in the first implementation, all teachers stated that it was not clear how to warm up and provide an understanding for the MEA and two teacher explained that this understanding and warm-up period could direct the students' approaches and added that "it does not known that how can this be done without directing the students". However, in other implementations, teachers introduced the context of the problem to the students by giving an example close to the MEA and said students' actual task again based on this example question. Three teachers stated that when the students not clear about some parts of the question, the teacher could also explain the unclear parts in order to eliminate the uncertainty. On the other hand one teacher denoted that the parts which were not clear for the students in the question should not be explained by teachers, students should be directed to the question and students gave a meaning to these part since it was thought that "it is also a step of the solution", also she indicated that a discussion could be created on the unclear part of the questions when all groups had a problem to understand rather than explaining these parts as a teacher. Moreover, one teacher, who denoted to explain the unclear parts as a teacher, specified that creating a discussion on the unclear part of the questions direct students solutions and limits their approaches (in one implementation).

All teachers, in the last applications indicated that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of concrete models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] helped students to warm-up and understand the MEA clearly. Intercalarily, two teachers added that if the concrete models were not appropriate then the simulations and other real life examples could be used in order to help students to warm-up and understand the MEA. In addition, these two teachers were also stated that everyday language should be used in the questions in order to not direct students to some specific mathematical concepts and make the students understand clearly in one implementation.

Data revealed that pedagogical strategy on providing an understanding of the MEA and to warm up the MEA were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations in the

implementations, and as teachers discussed strategy with colleagues. The below excerpts taken from the dialogue among the teachers in the meeting before and after the implementations denoted that teachers' providing an understanding of the MEA and to warm up the MEA strategies become articulated based on their observations and the discussions with the colleagues in both schools.

The below excerpts were taken from the dialogue among the teachers in the meeting before the second implementation indicates that one of the component of providing an understanding of the MEA and to warm up the MEA strategy which was introducing the context of the problem to the students by giving an example close to the MEA' context and said students' actual task again on this example question were changed from not warming up period; to giving an example close to the MEA' context and said students' actual task again on this example question based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Rezzan: we [teachers] can give an example like when you see the parking spaces in front of your home and in the markets, do you think that how can we benefit from these areas efficiently and how can we place maximum number of cars, now you have a chance to think, I see that students need to hear this kind of examples in order to begin with more enthusiasm

Teacher A: we can also want them to think our school parking spaces, it will motivate them

Figen: or, we [teachers] can say that you have an parking space and do you want to earn maximum gain, so how to place the maximum cars in your parking space by this way I think we [teachers] can focus on what they are wanted to do

The below excerpts were taken from the dialogue among the teachers in the meeting before the third implementation indicates that one of the component of providing an understanding of the MEA and to warm up the MEA strategy which was the questions about the MEA [*What is your task? What are givens?*], were changed from not querring the students' understanding of the MEA; to asking questions on MEA and providing understanding, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Semra: I see that in order to prevent time consuming on wrong understandings, we [teachers] should ask questions so that we have a chance to see they understood, they know their task

Teacher F: in here, students should understand that the aim is maximum gain and we will choose the workers based on that

Teachers E: we can ask what you are going to do. What are the givens?

Melda: what is your task?, we should also explain the unclear parts in order to eliminate the uncertainty, for example in here we can explain the part time and full time workers so that we can prevent the misunderstanding

One of the components of providing an understanding of the MEA and to warm up the MEA strategy which was the questions about the MEA [What *is your task? What are givens?*] was given as an intervention (i.e. suggested strategy) in the meeting after the first implementation in both schools. However, this was not stated or used by the teachers in the other implementations until they examined themselves or their colleagues examined. In the meeting after the first implementation, the below excerpts indicated that when the researcher suggested to ask questions based on the MEA such as what is the task? What are the givens? Teachers validated however, this was not stated by the teachers in the further cyclic process of this study until they examined themselves or their colleagues examined in the third implementation.

The below excerpt taken from the dialogue among the teachers in the meeting after the first implementation in Anatolian Teacher Training High School.

Semra: this understanding and warm-up period can direct the students' approaches, how can this be done without directing the students?

Melda: students can read the question several times, is this period necessary?

Researcher: teacher may ask some questions based on the activity such as what is the task? what are the givens?

Semra: it can be effective

Teacher F: by this way we [teachers] can understand if students understood or not

The below excerpt taken from the dialogue among the teachers in the meeting after the first implementation in Anatolian High School.

Teacher B: we [teachers] can ask students do you understand the question or not?

Teacher A: teacher can read the question and emphasize on some words but then we [teachers] can direct the students

Figen: how can we provide to warm-up without directing?

Researcher: teacher may ask some questions based on the activity such as what is the task? what are the givens?

Figen: yes, can be

Rezzan: it [questions that was suggested by the researcher] can be used

The below excerpts were taken from the dialogue among the teachers in the meeting before the third implementation indicates that one of the component of providing an understanding of the MEA and to warm up the MEA strategy which was the questions about the MEA [*What is your task? What are givens?*], were changed

from not querring the students' understanding of the MEA; to asking questions on MEA and providing understanding, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Semra: I see that in order to prevent time consuming on wrong understandings, we [teachers] should ask questions so that we have a chance to see they understood, they know their task

Teacher F: in here, students should understand that the aim is maximum gain and we will choose the workers based on that

Teachers E: we can ask what you are going to do. What are the givens?

Melda: what is your task?, we should also explain the unclear parts in order to eliminate the uncertainty, for example in here we can explain the part time and full time workers so that we can prevent the misunderstanding

The below excerpts were taken from the dialogue among the teachers in the meeting before the third implementation indicates that one of the component of providing an understanding of the MEA and to warm up the MEA strategy which was the questions about the MEA [*What is your task? What are givens?*], were changed from not querring the students' understanding of the MEA; to asking questions on MEA and providing understanding, based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Teacher A: in my implementation [street parking], students did not understand the givens in the activity clearly so the teacher should query the givens

Rezzan: in here we [teachers] can ask, what are given to us?and what is the aim? So we can be sure that students understand the question or not?

Teacher B: they should understand full time and part time workers

Figen: we [teachers] can ask, what is full time and part time worker? we listened students' answers if they misunderstood then we can explain this parts

4.2.4 Organizing the Presentations of the Solutions (groups' order, groups' presentation process)

Under this subheading each teacher evolving knowledge on organizing the presentations of the solutions which include groups' presentation orders and the presentation process after the students solve the MEAs described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, it was denoted that the organization of the presentation was not clear only the groups's presentation order could be based on their wishes. During the implementation each group presented their solutions, teacher only focused on the result and repeated the each groups' result after they finished their presentation when other groups tried to ask questions, they are wanted to ask their questions after the group finish their presentation by the teacher. After the implementation, it was indicated that teacher should encourage each group to share their solutions since each group deal with the solution so much and when presenting each group member should be on board and responsible to answer to other groups' questions in order to defend their groups' solution.

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation it was denoted that each group should share their solutions in order of their request and each group member should take place in the presentation and can defend their individual solutions. It was also added that after the group finish their presentation the other groups and the teacher should ask questions about the solutions in order to query its appropriateness and make it clearer. In addition, teacher also stated that the presentation time which is three or four minutes could be given to the groups, by this way giving an opportunity to groups to organize their presentation.

Figen: … each group should make presentation since all of them consume so many efforts on their solution approaches and all members should be on the board, they may also express their individual solutions…if the groups present in the order of their request, it will be more stress-free. We [teachers] should also give their presentation time; each group can organize themselves for the presentation. Each group can present in three or four minutes…our [teachers] questions and other groups' questions are very important in order to query the solution approaches and to understand clearly it, so we should encourage the groups to ask questions to the group who present…(pre-implementation interview with Figen)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was indicated that the first group could make presentation in the order of their request then the other groups, whose solution was the similar one that is presented before, could make presentation so that only the different parts could be discussed and the time was used more efficiently. It was also denoted that groups' interaction was very important so teachers should encourage groups' to ask questions

and create discussion opportunities for students to compare the ideas. In addition, it was repeated that the presentation time which was three or four minutes could be given to the groups, by this way giving an opportunity to groups to organize their presentation. During the implementation, teacher wanted one group to make presentation randomly then wanted groups who have similar solutions to make presentations consecutively. The presentation time, which was at most five minutes, was given to each group. Each group member took place in the presentations and the teacher asked" if there is any member think differently from the groups' solution". In addition, groups asked questions to each other and the teachers asked questions to the groups who present. Each groups' solution was questioned. Teacher also stated that she wanted one group to present at the end since "this group think more systematically and if this group present before the others then may be some groups do not want to present since their approaches are more basic than that group". After the implementation, it was indicated that the similar approaches should take place consecutively in order to use the time efficiently and to discuss more on the ideas, which were different. Moreover, it was denoted that it was appropriate to take the presentation, which included approach that was more systematic rather than the others at the end, since "if this group present before the others then may be some groups do not want to present since their approaches are more basic than that group".

Teacher wanted one group to make presentation randomly then wanted groups who had similar solutions to make presentations consecutively. The presentation time, which was at most five minutes, was given to each group. Each group member took place in the presentations and the teachers asked if there was any member think differently from the groups' solution. In addition, groups asked questions to each other and the teachers asked questions. Each groups' solution was questioned. Teacher also stated that she wanted one group to present at the end since "this group thinks more systematically and if this group present before the others then may be some groups do not want to present since their approaches are more basic than that group". (During the implementation - observation notes-field notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was indicated that the groups who had similar approaches could present consecutively since it was thought that the same things was not told so the different ideas could be discussed in more time. It was added that the presentations should take place from simple approaches to the systematic ones since "if the systematic approach is presented first then the other groups do not want to present". However if there was not an difference like basic and systematic or one strategy and two or more strategy then each group could make their presentation in the order of request. Moreover, it was denoted that each group

member should take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution. It was also added that group interaction was important for creating a discussion as opportunities for students to compare and select the ideas and approaches, which meet the needs of the question so teacher should encourage groups to ask questions and teacher asked questions and take a dynamic role in the discussion. Lastly, it was specified that when the groups finish their presentations, the teacher could give reflections on their errors and missing parts of the solutions.

The below excerpt was taken from the dialogue among the teachers in the meeting before the implementation.

Figen:… we [teachers] see that group discussions are very important, while discussing they compare their solutions their ideas and try to compare if these approaches meet the needs of the question so we should encourage the groups to ask questions to each other and while doing that also we should ask questions and be part of the discussions…

…

Teacher A: we can correct their error in the presentations, if we did not correct then they will learn wrongly

Teacher B: when they [groups] see errors of the group [group who present] they asked questions immediately

Figen:…when we correct their errors or show the missing parts to the groups who made presentations at that time, they do not want to continue their presentations, so I think we should make these reflections after the presentations…

The below excerpt was taken from post-implementation interview with Figen

Figen: I observed that it was an efficient way to take the presentations which are the similar ones consecutively since we [teachers] used the time more efficiently because we do not talk the same parts mostly we talk on the different parts. Also it was important to take the presentations from basic ones to the systematic ones since one group who use the basic approach, use only one strategy see the other groups' solution which includes many strategies do not want to present so we should organize the presentation from simple one to systematic ones…if there is not an difference like basic and systematic or one strategy and two or more strategy then each group can make their presentation in the order of request…

Researcher: Do you want to add anything else?

Figen: …each group member should be on board since sometimes one or two member have different ideas different than the groups and have a chance to express themselves and also each member can defend their group solution to the questions of other groups and our questions.

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, it was denoted that the organization of the presentation was not clear only the groups' presentation order could be based on their wishes. During the implementation, each groups' delegate presented the groups' solutions and teacher

asked questions to delegate about their solution process and focused on their result after they finished their presentation. After the implementation, it was indicated that "teacher should encourage each group to share their solutions since each group deal with the solution so much and when presenting each group member should be on board and responsible to answer to other groups' questions and teacher's questions in order to defend their groups' solution".

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation it was denoted that each group should share their solutions in order of their request and each group member should take place in the presentation and could defend their individual solutions. It was also added that after the group finish their presentation the other groups and the teacher should ask questions about the solutions in order to query its appropriateness, make it clearer and give reflection on the wrong or the missing parts. In addition, teacher also stated that the presentation time, which was five minutes, could be given to the groups, by this way it was thought that giving an opportunity to groups to organize their presentation.

Rezzan: …each group want to share their ideas so each group should make their presentation in the order of their request however while presenting, all group members should be on board also only the delegates present the solution, since when comes to the questions from me [teacher] or other groups. All group members should have a chance of defend their approach since we [teacher and other groups] can see that their approach inappropriateness or their missing parts, they should make clear why they think like that and how can it be improved based on our ideas and reflections. Also, sometimes group members think differently from their group so they can express their individual approaches (pre-implementation interview with Rezzan)

Rezzan:…our [teacher and other groups] questions is really important after the presentation since by these way group have an opportunity to compare their ideas with other so we should ask questions, if it's like group present then the other group at that time it cannot be effective…I join Figen's idea [colleague's idea] that it will be good to give presentation time to each group, if they know that they have five minutes then they organize their presentation and how to represent it easily with the poster papers (post-implementation interview with Rezzan)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was indicated that the first group could make presentation in the order of their request then the other groups whose solution was the similar one that was presented before, could make presentation, by this way it was thought that only the different parts could be discussed and the time was used more efficiently. It was also denoted that groups' interaction was very important so teachers should encourage groups' to ask questions and create discussion opportunities for students to compare the ideas. In addition, it was indicated that teacher could say to the

groups that while presenting the general approach was important rather than the detailed operations by this way it was thought that the general frame can be clearly expressed by the groups. During the implementation, groups' presentation order was in the order of groups' request. Each group member took place in the presentations and the teachers asked if there was any member think differently from the groups' solution. In addition, groups asked questions to each other and the teachers asked questions. Each groups' solution was queered. After the implementation, it was stated that although the groups took place based on their wishes in the implementation, it could be more effective, if the similar approaches ordered consecutively, in order to use the time efficiently and to discuss more on the ideas, which are different. Moreover, it was denoted that it was appropriate to take the presentation, which included approach that is more systematic rather than the others at the end, since it was thought that "if this group present before the others then may be some groups do not want to present since their approaches are more basic than that group". In addition, it was stated that it was seen very logical to take the systematic approaches at the end since the teachers know the groups' approaches and can organize these order.

Groups' presentation order was in the order of groups' request. Each group member took place in the presentations and the teachers asked "if there was any member think differently from the groups' solution". In addition, groups asked questions to each other and the teachers asked questions. Each groups' solution was questioned. (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was indicated that the groups who had similar approaches could present consecutively since it was thought that the same things were not told so the different ideas could be discussed in more time. In addition, it was added that the presentations should take place from simple approaches to the systematic ones since "they were completing each other and have a time to discuss the different ideas added approaches". Moreover, it was denoted that each group member should took place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution. It was also added that group interaction was important for creating a discussion as opportunities for students to compare and select the ideas and approaches, which meet the needs of the question so teacher could encourage groups to ask questions and teacher could asked questions and take a dynamic role in the discussion. Lastly it was specified that when

the groups finished their presentations, the teacher could give reflections on their errors and missing parts of the solutions also it was added that others groups absolutely to ask questions to the group who present if they had missing parts or errors in their solutions.

Rezzan:... we [teachers] have a chance to observe that it is effective to make all group members take place in presentation since they can sometimes defend their different ideas rather than the group and all of them try to defend their ideas collaboratively... Also taking the groups from simple approaches to the systematic ones is an effective way if the simple approaches and the similar ones take place firstly and when the other approaches which are more complex ones comes, they were completing each other and we have a time to discuss the different ideas added approaches, while discussing each group see their missing parts, wrong parts they have a opportunity to compare their approaches so I [teacher] always tried to ask questions and encourage groups to ask questions; by these way we correct the groups' missing part and inappropriate parts with my [teacher] questions and reflections but mostly the other groups immediately ask questions to the group who present when they see their missing part (post-implementation interview with Rezzan)

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation, it was denoted that the organization of the presentation was not clear. It was also stated that only the groups' presentation order could be based on their wishes and the delegate could present the group solution. During the implementation, each groups' delegate presented the groups' solutions, and teacher picked the groups randomly for the presentation and did not ask questions about their solution process, only focused on their result after they finished their presentation. After the implementation, it was indicated that teacher wanted each group to present since to show that all of them approaches was important for her. It was also denoted that the groups were picked randomly to the presentation since to make them not bored and make them ready for all the time just like in her class.

In the second implementation of the MEA, which was Street Parking, before the implementation and after the implementation it was denoted that each group should share their solutions in the order of randomly that the teacher choosed and each group member should take place in the presentation and could defend their individual solutions also the delegates presented the solutions. It was also added that if the time was appropriate after the group finished their presentation the other groups and the teacher should ask questions about the solutions in order to query its appropriateness, make it clearer and give reflection on the wrong or the missing parts since the group interaction was important for sharing of ideas.

Semra: I prefer to picked up the groups randomly, but I want each group make presentation since all of them wanted to express their ideas, also the each group member should be in the presentations because they may have different approaches and want to explain their ideas (post-implementation interview with Semra)

Semra: the groups' interaction was important we [teachers] should make other groups to ask questions and we should ask questions since by this way each group can see their missing and wrong part and all groups can share their ideas however if we have time, it is possible (the excerpt was taken from the meeting after the implementation)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that each group should share their solutions in the order of randomly that the teacher choosed and each group member should take place in the presentation and could defend their individual solutions also the delegates could present the solutions. In addition, it was indicated that teacher could say to the groups that while presenting the general approach was important rather than the detailed operations by this way it was thought that the general frame can be clearly expressed by the groups. Moreover, it was denoted that if the time was limited then the groups' who has different approaches could be chosen for the presentation in order to share the different ideas. Besides, it was indicated that if the groups' solution have missing or wrong parts, these parts could be corrected while ending the implementation since it was thought that if it was corrected after the presentation then the other groups who had the same missing part might not want to make presentation. During the implementation, teacher chosed the groups randomly for the presentation. Each group member took place in the presentations and the teachers asked "if there is any member think differently from the groups' solution". In addition, the teacher asked questions about the groups' approaches. However, other groups did not ask questions to the group whom present and teacher did not encourage the groups to ask questions. After the implementation, it was indicated that the groups were chosen for the presentation randomly in order to not make the students bored and made the groups ready for all the time. It was also indicated that each group member took place in the presentation and tried to answer teachers' questions collaboratively, also one delegate present the approach. It was also added that other groups' questions were not taken since the time was limited.

Teacher chosed the groups randomly for the presentation. Each group member took place in the presentations and the teacher asked "if there is any member think differently from the groups' solution". In addition, the teachers asked questions about the groups' approaches. However, other groups did not ask questions to the group whom present and teacher did not encourage the groups to ask questions. (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was indicated groups could be picked randomly by the teacher, it was added that if there was enough time, each group should share their ideas, if there was not enough time the groups who had different approaches could be picked in order to show the different approaches. In addition, it was denoted that each group member took place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution. It was also added that teacher should ask questions and took a dynamic role in the presentation. Moreover, it was stated that if the time was appropriate after the group finish their presentation the other groups should ask questions about the solutions in order to query its appropriateness, make it clearer and in order to discuss the different ideas. Lastly, it was specified that when the groups finished their presentations, the teacher could give reflections on their errors and missing parts of the solutions when ending the implementation since it was thought that if it was done at the end of the each presentation, the group who had the same missing part and the error in their approach might not want to present.It was also added that if the other groups interfered this missing part, at that time teacher could give reflection.

Semra: I chose the groups randomly to the presentation, since all of them ready and finish their solutions, it provides not to make students bored. If we have enough time I prefer every group to make their presentation and share, their approaches with us if the time is limited at that time I prefer to choose the groups who have different approaches since I want every group see the different approaches… it is effective to take every group member on board since sometimes one or two of them said that I have different approach from our group and defend their ideas, and when I ask questions all of them try to answer collaboratively so it is important to ask questions as a teacher about their solutions (post-implementation interview with Semra)

Semra:.. in my implementations, always I have limited time for the presentations so I do not let the groups' to ask questions since their discussions take more time however it is important since when they ask questions they may see the different point of views that I could not realize and there would be an effective discussion which they compare their different ideas…in my implementation [Summer Jobs] I correct one of the groups error in their table after they finish their presentation, I said that " you say that you take the months but you make your calculations on the busy-steady and slow time" then the group realized their mistake however when I picked another group they said that they did not want to present since they made the same mistake, then I said that it was not problem we also wanted to listen their ideas. Therefore, I prefer to give reflections to the groups while I ending the implementation and all the groups' finish their presentation but sometimes the other groups immediately say the missing or wrong parts of the solutions whom present at that time I do not wait the end of the lesson. I would correct it at that time; it is more appropriate (post-implementation interview with Semra)

Melda's Profile

In the first implementation of the MEA entitled " Bank Robbery", before the implementation, it was denoted that the organization of the presentation was not clear, it was also stated that only the groups who used different approaches might be picked up for the presentation in order to show the different approaches to all groups and the delegate could present the group solution. During the implementation, each group delegate presented the groups' solutions, and teacher picked the groups only who used different approaches. Teachers asked questions to the delegate of the groups about their solution process, only focused on their result after they finished their presentation. After the implementation, it was indicated that teacher wanted all groups to see the different approaches so only the groups who used different approaches were chosen for the presentation. In addition, it was denoted that it would be good to encourage groups to ask questions to the group who presented in order to discuss the different approaches.

In the second implementation of the MEA entitled "Street Parking", before the implementation and after the implementation it was denoted that groups who had different solutions should be picked up for the presentations rather they had wrong or true result since all different approaches could be shared with all groups. It was also added that after the group finished their presentation the other groups and the teacher should asked questions about the solutions in order to query its appropriateness, make it clearer and give reflection on the wrong or the missing parts since the group interaction was important for sharing of ideas.

Melda: different approaches is important for the students to see that this question can be thought from different point of views so I thought to picked up the groups who used different approaches but it is not important if it has true or wrong result. In addition, we [teachers and the other groups] should talk on the approaches, if it has missing part, wrong part, how it can be corrected with the questions from other groups and me to the groups [groups who make presentation] (pre-implementation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, and after the implementation it was denoted that if time was enough all groups should share their ideas by presenting, however if the time limited then the groups who had different approaches could be taken for the presentations in order to share different approaches and different point of views. Each group member should take place in the presentation and could defend their individual solutions and defend

their group' solutions to the questions from teacher and the other groups also the delegates presented the solutions. Moreover, it was stated that after the group finished their presentation the other groups and the teacher should ask questions about the solutions in order to query its appropriateness, make it clearer and give reflection on the wrong or the missing parts since the group interaction was important for sharing of ideas. During the implementation, first group was taken to the board for the presentation based on their wishes then the teacher picked up the other groups who had different approaches. Each group member took place in the presentations and the teachers asked if there was any member thought differently from the groups' solution. In addition, the teachers and the other groups asked questions about the groups' approaches whom present. In addition, teacher encouraged the groups to ask questions like "is there any group want to ask question".

First group was taken to the board for the presentation based on their wishes then the teacher picked up the other groups who had different approaches. Each group member took place in the presentations and the teachers asked if there was any member thought differently from the groups' solution. In addition, the teachers and the other groups asked questions about the groups' approaches whom present. In addition, teacher encouraged the groups to ask questions like "is there any group want to ask question" (During the implementation-observation notes)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that if time was enough all groups should share their ideas by presenting, however if the time limited then the groups who had different approaches could be taken for the presentations in order to share different approaches and different point of views. It was also added that the presentations should take place from simple approaches to the systematic ones since they were completing each other and have a time to discuss the different ideas and added approaches. In addition, it was denoted that each group member should take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution to the questions from teacher and the other groups also the delegates could present the solutions. Moreover, it was stated that after the group finished their presentation the other groups and the teacher should ask questions about the solutions in order to query its appropriateness, make it clearer and give reflection on the wrong or the missing parts since the group interaction was important for sharing of ideas.

Melda: every group wants to share their ideas so if we [teachers] have time we should give opportunity to all groups to present but if the time limited I prefer to pick up the groups who have different approaches also I choose them from simplest to the complex ones so that they will complete each other and we can discuss the added approaches not the same ones and I can recover all of them more easily…other groups' questions are very important in order to create effective discussions on the missing and the wrong parts of the approaches, all together we can correct or complete the approaches…each group member should be on board since I ask and other groups' ask questions and every member has responsibility to answers them by this way I can observe that all of the member can part in the solution effectively or not (post-implementation interview)

4.2.4.1 Cross-case Analysis of Knowledge of Organizing the Presentations of the Solutions (groups' order, groups' presentation process)

Under this subheading, cross-case analysis results of knowledge of organizing the presentations of the solutions which included groups' presentation orders and the presentation process what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on organizing the presentations of the solutions which included groups' presentation orders and the presentation process after the students solve the MEAs was compared in Table 4.22.

Table 4.22 Case Ratings of Organizing the Presentations Themes for All Implementations

Case Themes**	**Cases***			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
the organization of the presentation is not clear	1	1	1	1
• the groups's presentation order could be based on their wishes.	1	1	1	
• only the groups who use different approaches may be picked up for the presentation in order to show the different approaches to all groups and the delegate can present the group solution.				1
Each group should present their approaches	2,3,4,5	2,3,4,5	2,3,4,5	3,4,5
if the time is limited then the groups' who has different approaches could be chosen for the presentation in order to share the different ideas.			3,4,5	2,3,4,5
the groups who have similar approaches can present consecutively since the same things are not told so the different ideas can be discussed in more time.	3,4,5	3,4,5		

Table 4.22 (continued)

the presentations should take place from simple approaches to the systematic ones since	3,4,5	3,4,5		4,5
• if the systematic approach is presented first then the other groups do not want to present.	3,4,5			
• they were completing each other and have a time to discuss the different ideas added approaches.		3,4,5		4,5
there is not a difference in the results like basic and systematic or one strategy and two or more strategy then each group can make their presentation in the order of request.	4,5			
there is not a difference in the results like basic and systematic or one strategy and two or more strategy then the groups were picked randomly to the presentation since to make them not bored and make them ready for all the time just like in my class.			2,3,4,5	
each group member take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution.	2,3,4,5	2,3,4,5	3,4,5	3,4,5
group interaction is important for creating a discussion as opportunities for students to compare and select the ideas and approaches, which meet the needs of the question so teacher should encourage groups to ask questions and teacher ask questions and take a dynamic role in the discussion.	3,4,5	2,3,4,5	2,3,4,5	2,3,4,5
Reflections				
• when the groups finish their presentations,while ending the implementation, the teacher should not give reflections on their errors and missing parts of the solutions since if it is corrected after the group finish the presentation then the other groups who made the same mistake may not want to present	4,5		3,4,5	
• when the groups finish their presentations, the teacher can give reflections on their errors and missing parts of the solutions also it was added that others groups immediately ask questions to the group who present if they have missing parts or errors in their solutions so it is appropriate to correct at that time.		2,3,4,5	4,5	2,3,4,5

Table 4.22 (continued)

the presentation time could be given to the groups, by this way giving an opportunity to groups to organize their presentation.	2,3	2	
teacher could say to the groups that while presenting the general approach is important rather than the detailed operations by this way it was thought that the general frame can be clearly expressed by the groups.		3	3

* 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

** Organizing presentations strategy and its reasons were given with the representative teachers' sentences in some themes.

The teachers organizing the presentations of the solutions which included groups' presentation orders and the presentation process after the students solved the MEAs were changed in the cyclic process of the study. In the first implementation, the organization of the presentation was not clear for all teachers, three of them stated that the groups' presentation order could be based on their wishes and one teacher stated that only the groups who used different approaches might be picked up for the presentation in order to show the different approaches to all groups and the delegate could present the group solution. However, after the first implementations different strategies were described. All teachers denoted that each group should present their approaches but two of the teacher stated that if the time was limited then the groups who had different approaches could be chosen for the presentation in order to share the different ideas. The other presentation orders that were specified; two teachers indicated that the groups who had similar approaches could present consecutively since the same things were not told so the different ideas could be discussed in more time. In addition, three teachers denoted that the presentations should take place from simple approaches to the systematic ones and this was explained as if the systematic approach was presented first then the other groups could not want to present by one teacher and explained as they were completing each other and had a time to discuss the different ideas, added approaches by two teachers. Other presentation order that were indicated; if there was not a difference like basic and systematic or one strategy and two or more strategy then each group could make their presentation in the order of request by one teacher and another teacher denoted the groups could be picked

randomly to the presentation since to make them not bored and make them ready for all the time just like in her class. Moreover, all teachers indicated that each group member should take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution and all teachers also indicated that group interaction was important for creating a discussion in order to give opportunities for students to compare and select the ideas and approaches, which meet the needs of the question so teacher should encourage groups to ask questions and teacher ask questions and take a dynamic role in the discussion. In addition, all teachers stated that the reflections should be given on missing and wrong parts of the approaches but when this could be done is stated differently. One teacher stated that when the groups finish their presentations, while ending the implementation, the teacher could give reflections on groups' errors and missing parts of the solutions since if it was corrected after the group finish the presentation then the other groups who made the same mistake might not want to present and other two teacher denoted that when the groups finished their presentations, the teacher could give reflections on their errors and missing parts of the solutions also it was added that others groups immediately asked questions to the group who presented if they had missing parts or errors in their solutions so it was appropriate to correct at that time. In addition one teacher specified that if other groups realized the missing or wrong part and asked questions then teacher could give reflection at that time after the group finish the presentation however if there was not questions from the other groups then the teacher could give the reflection while ending the implementation in order to prevent if it was corrected after the group finish the presentation then the other groups who made the same mistake might not want to present.

Intercalarily, two teachers indicated that the presentation time could be given to the groups, by this way giving an opportunity to groups to organize their presentation in one implementation. Additionaly, two teachers stated that teacher could say to the groups that while presenting the general approach was important rather than the detailed operations by this way it was thought that the general frame could be clearly expressed by the groups in one implementation. This specification was given as an intervention (i.e. suggested strategy) in the meetings after the second implementation in both schools. However, this specification was denoted by two teachers from both schools before the third implementation at once and was not going to part in the further cyclic process. In the meetings after the second implementation, the below

excerpts indicated that when the researcher suggested that in the presentations, teacher can want groups to express the main steps of their approach rather than the operations for more effective presentations, the teachers validated this suggestion however, this was not stated by the teachers in the further cyclic process of this study.

The below excerpt taken from the dialogue among the teachers in the meeting after the second implementation in Anatolian Teacher Training High School.

Semra: in the presentations, students mostly try to express the operations in detail and this took so much time

Researcher: teacher can state this, means that teacher can emphasize groups that their approach's main steps are more important than the operations so by this way more effective presentation can take place

Teacher F: it should be emphasized since our students do not know how to present and think that they should explain every steps, every operations

Semra: yes, only the general frame can be wanted from the groups

Melda: by this way we also can gain time

The below excerpt taken from the dialogue among the teachers in the meeting after the second implementation in Anatolian High School.

Teacher B: some groups presented more effectively however some groups dwell on unnecessary parts mostly

Researcher: unnecessary parts?

Teachers B: operations

Rezzan: at that time I said pass the operations

Researcher: teacher can want groups to express the main steps of their approach rather than the operations, by this way every group can make effective presentations

Figen: if we state this at the beginning of the implementation, then students can also prepare their posters based on the main parts of the approach and they have a chance of organize their presentations

Data revealed that pedagogical strategy on organizing the presentations of the solutions were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations in the implementations, and as teachers discussed strategy with colleagues. The below excerpts taken from the dialogue among the teachers in the meeting before and after the implementations denoted that teachers' organizing the presentations of the solutions strategies become articulated based on their observations and the discussions with the colleagues in both schools.

The below excerpts were taken from the dialogue among the teachers in the meeting after the second implementation indicates that the one of the component of organizing the presentations of the solutions strategy that each group member should take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution were changed from making the presentation with the delegates of the groups to making the presentation with entire group members based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Figen: I observed that when only the delegate come to the board and present the group solution, it did not be efficient since some of the group member also indicate that they thought differently rather than the group so each group member should be in front of the board while delegate presenting the group solution.

Rezzan: yes, also when we [teacher] and other groups ask questions based on their approach at that time only the delegate try to explain but it is not proper since the entire group member should have a chance of defending the group solution

Teacher A: by this way also we [teacher] use the time more efficiently when all group member defend their approach

Figen: in order to organize time before the presentation we can also give presentation times to all groups we [teacher] can say that all groups have 3 or 5 minutes for the presentation by this way they will organize their presentations

Rezzan: it is a good idea to give presentation time

The below excerpts were taken from the dialogue among the teachers in the meeting after the third implementation indicates that the one of the component of the organizing the presentations of the solutions strategy that was group interaction was important for creating a discussion in order to give opportunities for students to compare the ideas and approaches, which meet the needs of the question were formed based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Semra: in my implementations, always I had limited time for the presentations so I did not let the groups' to ask questions since their discussions take more time however it is important since when they ask questions they may see the different point of views that I could not realize and there would be an effective discussion which they compare their different ideas…

Melda: I let the groups asked questions to each other after the group finished their presentation, groups really asked efficient questions in order to make the group' approach clearer and realized wrong or the missing parts so according to me the group interaction was important for sharing of ideas.

Teacher F: all groups spend so much time on the solutions and they can see the missing parts of the solutions more easily than the teacher since they may thought the same approach but give up or they never be thought that approach and ask questions in order to understand the approach

Melda: yes, I agree with you, their questions more efficient then us

4.2.5 Ending the Implementation

Under this subheading each teacher evolving knowledge on ending the implementation that recovering the lesson after the solution process and the presentations of the solutions described based on before the implementation, during the implementation and after the implementation of MEAs and the cross-case analysis results what was similar and different about the cases' evolving were taken account.

Figen's Profile

In the first implementation of the MEA entitled "Bank Robbery", it was denoted that the teacher could explain the result.

Teacher explained the result based on her solution approach (During the implementation - observation notes)

In the second implementation of the MEA entitled "Street Parking", before the implementation it was denoted that all solution approaches could be summarized, reflection could be given on the wrong and missing part of the solutions and teacher could explain the result. After the implementation, it was indicated that students' thoughts could be taken based on the different solution approaches and the question.

Figen: if all students find the result from different solution approaches so these solutions can be summarized and if I see missing parts and wrong parts of the solutions, these parts can be corrected and I can give reflections on them… mostly students want to see the teacher solution so I will explain the result (pre-implementation interview with Figen)

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher A: It is very different process for the students we should take their thoughts

Rezzan: they [students] see the mathematic usage in real life, and these implementations are different for them so we [teachers] should ask them what they learned, what they thought about the question and should give an opportunity to compare all the groups' different approaches

Teacher B: in my class, after presentations, they [students] began to compare their solutions

Figen: It was very normal so we [teachers] should listen them, their different ideas about the question or the solutions

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that this ending part was ambiguous since there was not one result. During the implementation, it was emphasized that "the important think using the appropriate approaches based on your criterion". After the implementation, it was indicated that the important thing was that student should

understand that there was no certain answers, these answers could be change based on their approaches that they established for their criteria so the important thing was to compare the approaches based on the criteria and choose the most appropriate one which meet the needs of the question.

Figen: in the first implementations we have one result and compare the approaches based on the result and we [teachers] focus on the result, however in here there is not a certain result how can I recover, I see in the implementation I think (pre-implementation interview with Figen)

Figen: students compare the approaches based on the criteria which the groups' choose and most of the groups criticize their approaches and try to find the most appropriate approach which is most suitable with the question want…they see that the answer can change based on the approaches and they should know that they should produce a strength approach in order to reach the needs (post-implementation interview with Figen)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the main mathematical concepts which was involved in the situation should be summarized and the models that the students produced should be asked to the students where to use in the similar situations by this way it was thought that the permanent learning would take place. In addition, it was indicated that students should compare the approaches and see the weak and strength parts of the approaches and the approaches, which could not be thought by the students, could be explained in order to give the different point of views.

Figen: the change should be explained in here [water tank] students should understand the height change based on the water in the tank, the change and its graph should be understand…they produce the change graph so we [teachers] could want them to draw the change of the based on the water on the tank while the tank ejaculating rather than filling. I think when they see that they use their result in similar situations than their permanent learning will actualize (pre-implementation interview with Figen)

Figen: students use many different approaches and they produce it by themselves rather than apply the question what we [teachers] taught so they enjoy but they should see which approach is more appropriate, which is more weak and sometimes we taught the approaches what they did not thought in these way we should share with them so they can look at the question also from the different point of view (post-implementation interview with Figen)

Rezzan's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation it was denoted that the teacher could explain the result. During the implementation, teacher explained the result based on her solution approach. After the implementation, it was denoted that students' thought could be taken since these implementations are different for them.

In the second implementation of the MEA entitled "Street Parking", it was indicated that students' thoughts could be taken based on the different solution approaches and the question. It was also added students could compare the different approaches and they could explain what they learned.

The below excerpt was taken from the dialogue among the teachers in the meeting after the implementation.

Teacher A: It is very different process for the students we should take their thoughts

Rezzan: they [students] see the mathematic usage in real life, and these implementations are different for them so we [teachers] should ask them what they learned, what they thought about the question and should give an opportunity to compare all the groups' different approaches

Teacher B: in my class, after presentations, they [students] began to compare their solutions

Figen: It was very normal so we [teachers] should listen them, their different ideas about the question or the solutions

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that this ending part was ambiguous since there was not one result, it was added that teacher might summarize the different approaches and different criteria. During the implementation, it was emphasized that the important think using the appropriate approaches based on their criterion. After the implementation, it was indicated that the important thing was that student should understand that there was no certain answers, these answers can be change based on their approaches that they established for their criteria so the important thing was to compare the approaches based on the criteria and choose the most appropriate one which meet the needs of the question.

Rezzan: …when the implementation end, students firstly ask me which result is true than they want me one result but when I explain that every group use many different criteria and many different approaches all of them is good for us in order to see the different views but we can compare the approaches and find the most suitable one which meet the needs of the question. After these explanations, I think they understand and they see the importance of the approaches (post-implementation interview with Rezzan)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the main mathematical concepts which was involved in the situation should be summarized and the models that the students produced should be asked to the students where to use in the similar situations by this way it was thought that students have more positive approaches to the math since they see math usage in real life. In addition, it was indicated that students should compare the approaches and see the weak and

strength parts of the approaches and the approaches, which could not be thought by the students, could be explained in order to give the different point of views.

Rezzan: I focused on the tangent circles and the special triangles what they [students] use in their approaches. And want them where they use their approaches in their life since I observed that they enjoy very much while solving these kind of questions since they see the math usage in their life so they look to their environment differently by the math glasses I think …they criticize their approaches while for example one group state that to the other group you use only two different arrangement but it will be placed differently and at that time the number of cans which is stored will be maximum, they see their weak and strength parts but they did not think what I thought so I explain that arrangement also since I want them to see this way also (post-implementation interview with Rezzan)

Semra's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation it was denoted that the teacher could explain the result. During the implementation, teacher explained the result based on her solution approach and took the students' thought based on the implementation. After the implementation, it was denoted that students' thought could be taken since these implementations were different for them.

In the second implementation of the MEA entitled "Street Parking", it was indicated that students' thoughts could be taken based on the different solution approaches and the question. Students could compare the different approaches and they could explain what they learned. In addition, it was indicated that teacher should explain the result even if the groups have true solutions, also it was denoted that the missing and the wrong parts of the group solutions could be corrected and reflected if it was not done in the presentations.

Semra: I think we [teachers] should ask students their thought about the implementation, different approaches, mathematic usage in real life, and what they have learned from the implementation. This implementations are first for the students and for us too so we should ask what they thought clearly…students solve from different approaches but I explain again since teacher could explain clearly so even if there were the true approaches teacher should explain and correct the missing and wrong parts of the approaches if it is not reflected and corrected in the presentations (post-implementation interview with Semra)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that this ending part was ambiguous since there was not one result, it was added that teacher might summarize the different approaches and different criteria and students' thoughts could be taken based on the different solution approaches and the question. During the implementation, it was emphasized that the important think using the appropriate approaches based on their criterion.

After the implementation, it was indicated that "students saw many different approaches from other groups however they wanted to see the teacher's approach also so the teacher should share his/her approach too". In addition, it was denoted that teacher could direct the students to think more on the question out of the school in order to produce more appropriate approaches

Semra: even if the students have seen many different approaches for the question and compare them based on the chosen criteria, they want to see the my [teacher] approach. Therefore, we should explain our approach and say that there may be more appropriate approach and want them to think more on the question out of the school (post-implementation interview with Semra)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the main mathematical concepts which was involved in the situation should be summarized and the models that the students produce should be asked to the students where to use in the similar situations by this way it was thought that students had more positive approaches to the math since they saw math usage in real life. In addition, it was indicated that students shared different approaches however sometimes they thought different approaches but did not improve it so this approaches could be given to the students as homework.

Semra: the important thing is to explain clearly the mathematical concepts which are used in order to solve the question since our [teachers'] aim is to teach these concepts... we can give home works that they can improve the approaches which are thought in the solution process but did not be improved and can apply their approach in similar situations for instance we want them also draw the graph of the height change while the water is ejaculating from the tank, by this way they see the more feasible area for the situation in the question (post-implementation interview with Semra)

Melda's Profile

In the first implementation of the MEA entitled "Bank Robbery", before the implementation and after the implementation it was denoted that the teacher could explain the result. During the implementation, teacher explained the result based on her solution approach.

In the second implementation of the MEA entitled "Street Parking", it was indicated that students' thoughts could be taken based on the different solution approaches and the question. Students could compare the different approaches and they could explain what they learned. In addition, it was indicated that teacher should explain the result even if the groups had true solutions.

Melda: students have really enjoy while solving the question, I think we [teachers] should ask them their thoughts about the question, about the approaches of the other groups by this way we have a chance to hear what they learn and how they feel, but while recovering the lesson the teacher should explain the result as a summary since to make it more clear (post-implementation interview with Melda)

In the third implementation of the MEA entitled "Summer Jobs", before the implementation, it was denoted that this ending part was ambiguous since there was not one result, it was added that teacher could explain her approach and her criterion. During the implementation, it was emphasized that the important think using the appropriate approaches based on their criterion and explained her approach and her criterion. After the implementation, it was denoted that teacher could direct the students to think more on the question out of the school in order to produce more appropriate approaches and to improve the approaches which students thought but did not improve in the implementation

Melda:.. students share many different approaches and some of them were not completed so it is appropriate to want students to complete the approaches and may produce new and more appropriate approaches, actually I want them to think on question also out of the school…for example I am waiting forward to see the groups approach whom try to use standard deviation, they said that "they will solve the question based on standard deviation and will brought me their approach (post-implementation interview with Melda)

In the fourth implementation of the MEA entitled "Water Tank" and in the fifth implementation of the MEA entitled "Pack Them In!", it was denoted that the main mathematical concepts which was involved in the situation should be summarized and the models that the students produced should be asked to the students where to use in the similar situations by this way it was thought that students had more positive approaches to the math since they saw more implementation area of math in real life. In addition, it was indicated that students shared different approaches however sometimes they thought different approaches but did not improve it so this approaches could be given to the students as homework.

Melda:…In our classes students always ask us where we use this subject in real life by this way for example they see where they can use the tangent circles and the special triangles and this is not discrete so they learn permanently… (post-implementation interview with Melda)

Melda: the graph should be explained since our aim to make the students to use this subjects and learn its usage in their life so we can increase the examples and the approaches by giving home works, we can want them to use it in different places like drawing the graph of the height change based on the water while its ejaculating from the tank or we can want them to complete their approaches or ask if there is new and more appropriate approach for what is asked in the question (post-implementation interview with Melda)

4.2.5.1 Cross-case Analysis of Knowledge of Ending the Implementation

Under this subheading, cross-case analysis results of knowledge of ending the implementation that recovering the lesson after the solution process and the presentations of the solutions what was similar and different about the cases' evolving were taken account. Each case's evolving knowledge on ending the implementation that recovering the lesson after the solution process and the presentations of the solutions was compared in Table 4.23.

Table 4.23 Case Ratings of Ending the Implementation Themes for All Implementations

Case Themes[**]	**Cases**[*]			
	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
the teacher could explain the result.	1,2	1	1,2	1,2
the main mathematical concepts which is involved in the situation should be summarized	3,4,5	3,4,5	2,3,4,5	2,3,4,5
the **mathematical models that the students produce should be asked to the students where to use in the similar situations**	4,5	4,5	3,4,5	4,5
by this way it was thought that the permanent learning would take place.	4,5			
by this way it was thought that students have more positive approaches to the math since they see math usage in real life.		4,5	4,5	4,5
students should compare the approaches and see the weak and strength parts of the approaches and the approaches, which could not be thought by the students, could be explained in order to give the different point of views.	3,4,5	2,3,4,5		
students share different approaches however sometimes they thought different approaches but did not improve it so this approaches could be given to the students as homework and teacher could direct the students to think more on the question out of the school in order to produce more appropriate approaches.			3,4,5	3,4,5

[*] 1, 2, 3, 4, 5 indicates the numbered implementation, such as 1 refers to first implementation, in which the theme corresponding in the table was observed/presented in the case at least once.

[**] Ending the implementation strategy and its reasons were given with the representative teachers' sentences in some themes.

The teachers' ending the implementation that recovering the lesson after the solution process and the presentations of the solutions were changed in the cyclic process of the study. In the first implementation, all teachers could only denote that teacher could explain the result of the MEA. However, after the first implementations different ways were described. All teachers denoted that the main mathematical concepts, which were involved in the situation, should be summarized and the mathematical models that the students produce should be asked to the students where to use in the similar situations. One teacher explained that mathematical models that the students produce should be asked to the students where to use in the similar situations since by this way it was thought that the permanent learning would take place. In addition, two teacher explained that mathematical models that the students produce should be asked to the students where to use in the similar situations since by this way it was thought that students have more positive approaches to the math since they see math usage in real life. Besides, it was stated by two teachers that students should compare the approaches and see the weak and strength parts of the approaches and it was added that the approaches, which could not be thought by the students, could be explained in order to give the different point of views to students by the teachers. Moreover, two teachers indicated that students shared different approaches however sometimes they thought different approaches but did not improve it so this approaches could be given to the students as homework and teacher could direct the students to think more on the question out of the school in order to produce more appropriate approaches.

Data revealed that pedagogical strategy on ending the implementation were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations in the implementations, and as teachers discussed strategy with colleagues. The below excerpts taken from the dialogue among the teachers in the meeting before and after the implementations denoted that teachers' ending the implementation strategies become articulated based on their observations and the discussions with the colleagues in both schools.

The below excerpts were taken from the dialogue among the teachers in the meeting before the fourth implementation indicates that the components of ending the implementation strategy which were summarizing the main mathematical

concepts and asking the students where to use their mathematical models in similar situations were formed based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian High School.

Figen: the change concept should be explained in here [water tank] students should understand the height change based on the water in the tank, the change and its graph should be understood so we can explain this concept at the end

Teacher A: they learned increasing graphs, we [teachers] can also ask the decreasing graphs as homework

Rezzan: we [teachers] can want them to draw the graphs while the tanks ejaculating rather than filling

Figen: by this way they will learn the change concept permanently

Teacher B: It is also same for us [teachers] we did not forget this question and we can give this question while we are teaching the change concept

The below excerpts were taken from the dialogue among the teachers in the meeting before the fourth implementation indicates that the one of the component of ending the implementation strategy that was giving homework the approaches which was thought but not improved in the solution process was formed based on the observations in the implementations and formed with discussing the colleagues in the meeting in Anatolian Teacher Training High School.

Melda: we [teachers] should want students to improve their approaches which are thought in the solution process but not be improved out of the school. For example, in the summer job one of the group used standard deviation but not improve this approach and give up but then I wanted them to solve this question from this approach as homework and they said that also they wondered this approach and I forward to see their approach

Teacher F: according to me when students tried to solve this question from different points of view, they began to see that math is not a stack of formulas

Semra: students began to look the math more positively because they have a chance to see the feasible areas for the situation in the question; it is good idea to give the incomplete approaches as homework

CHAPTER 5

DISCUSSIONS, CONCLUSIONS AND IMPLICATIONS

In the previous chapter, the results of data analysis were organized as profiles of the four teachers and their cross-case analyses. This chapter will begin with a review of the goal of the study and follows with a discussion of findings and conclusion of the study. Then, the implications, limitations and suggestions for professional development, for teachers and for further research are presented.

The overall goal of this study was to describe the nature of teachers' developing knowledge through lesson study design that is compatible with the modeling perspective. This study addressed three core research questions:

i) How does the teachers' knowledge evolve on questioning through professional development activities based on lesson study cycle from the modeling perspective?

ii) How does the teachers' knowledge evolve on generating assessment criteria for assessing students' competency in modeling through professional development activities based on lesson study cycle from the modeling perspective?

iii) How does the teachers' knowledge evolve on classroom organization and management domain of general pedagogical knowledge through professional development activities based on lesson study cycle from the modeling perspective?

The learning trajectories, development of pedagogical content knowledge and pedagogical knowledge that emerged in this study, highlighted teachers' knowledge in two major areas; (i) knowledge of instructional processes-student focus and (ii) knowledge of classroom organization and management. The data used in this dissertation focused on the dynamic aspects of pedagogical content knowledge and pedagogical knowledge. One of the most important dynamic aspects was how this type of knowledge is formed and modified in light of professional development

activities based on lesson study cycle from the modeling perspective. In this respect, the discussion were given firstly under the two major areas and based on the dynamic aspects how this type of knowledge was formed and modified, and then conclusions were given on the overall nature of the study.

5.1 Knowledge of Instructional Processes-Student Focus

This component of pedagogical content knowledge, proposed by Marks (1990a, 1990b), was examined under two categories, which are questions to students and assessment of students in this study.

Questions to students

The data revealed that teachers questioning techniques were changed from directing students along a predetermined path to exposing their thinking with broadening, exploratory and the evaluative questions. The result showed that in the cyclic process of the study, teachers gained ability to ask effective question in order to move students' thinking forward by listening their students' way of thinking in the implementations, by trying to understand students' emerging questions, by discussing how to ask questions on the error and difficulties that encountered or might encounter in the solution process and by discussing unexpected solutions with their colleagues. The result, the intended evolving, was stated by previous studies (Blum & Leiß, 2008; Doerr, 2006; Doerr & Lesh, 2011; García, Maaß, & Wake, 2010; Galbraith & Stillman, 2006; Wallach & Even, 2005). The intended type questions which are broadening, evaluative and exploratory questions were stated for different purposes in this study, and some of these purposes were indicated by the theoretical and the empirical studies (Blum & Leiß, 2008; Doerr, 2006; García, Maaß, & Wake, 2010; Galbraith & Stillman, 2006). The data analysis showed that evaluative questions were specified for the purposes of prompting students to give explanations that moved their thinking forward by identifying a mismatch between their representations, assumptions and calculations and for the purposes of making groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that move their thinking forward. Similar results were reported by Doerr (2006) that by asking students about the meaning of their work, by pointing to an erroneous assumption about the problem, or by suggesting an alternative perspective encourage students to give explanations that will move their thinking forward and it is also reported that asking this kind of

questions needs much more than a procedural response so the teacher needs the ability to follow students' reasoning and listening. Besides, in this study broadening questions were used by the teachers for different purposes in order to help students to imagine the real situation clearly by giving a real life example, in order to offer a situation in which it was clear what type of mathematics should be used, in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations, in order to subconsciously simplify the situation by suggesting an alternative perspective, in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward, in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. These results were consistent with the earlier studies, García, Maaß and Wake (2010) reported that it is teacher's responsibility to offer a situation in which it is clear what type of mathematics should be used when the groups stuck in the solution process, Blum and Leiß (2008) state that while teachers treat modeling in the classroom, they can give students opportunities to acquire mathematical competencies and making connections within and outside mathematics, Galbraith and Stillman (2006) indicate that in the phase of structuring and simplifying, students can draw the problem situation, and subconsciously simplify the situation and teacher can use strategic interventions like showing an alternative perspective.

Assessment of students

Data analysis of this study revealed that in the first implementation (i.e. the Bank Robbery), all the teachers denoted that the assessment procedure and the criteria in order to assess students' modeling process were not clear. In fact, one of the teachers stated that the students' modeling competency could not be assessed. Then the criteria began to appear as result, solution steps, communication, and participation in the group in general. After then, teachers created tool in order to assess students' modeling competency (see Tables 4.6, 4.7, 4.8 and 4.9) , and revised these tool during, before and after the implementations for the third, fourth and the fifth implementations. Assessment tools were created based on the solution process and the presentation process. Besides, teachers evaluated one group in the implementations with these tools. Data analysis revealed that, associating and using

the data; results based on the model quality (basic level, intermediate level and advanced level or simple approach and systematic approach); understanding the question; communication; cooperation and participation in the group were indicated as assessment criteria for the solution process and present with clear, precise language; get other students to listen /pay attention; defending their approach and the answers to the questions from other groups and teacher, were stated as assessment criteria for the presentation process. Additionally, data analysis showed that the focuses of the assessment criteria were changed in the cyclic process of the study. In the first implementation, the result and solution steps were stated as assessment criteria based on teachers' assessment criteria in their exams or based on the new phenomenon (group work) in the implementations. However, in the other implementations, the focuses of the assessment criteria were changed based on the factors that affect the students' competency in modeling. These factors were determined by observing the one focus group deeply in the solutions and in the presentations and by discussing the assessment criterions with their colleagues for each implementation. This result showed that teachers generate assessment criteria on the products and the observable actions that affect the students' modeling cycle in the solution process and in the presentation process. Similar products and observable actions of the students in the modeling cycle and in the presentations were reported as assessment criteria by the theoretic and the empirical studies (Biccard & Wessels, 2011; Ferri, 2011; Giménez & Rosich, 2011; Stillman & Galbraith, 2011; Wake, 2011)

Stillman and Galbraith (2011) reported that to evaluate the student modeling efforts, teachers used the following criteria as a basis for awarding credit: "Appropriateness of interpretation of data; Reasonableness of assumptions; Quality of mathematical model; Justification of choice of values for model parameters; Discussion of strengths and weaknesses of model; Evaluation of model" (p.691). The criteria which are appropriateness of interpretation of data; reasonableness of assumptions; quality of mathematical model and evaluation of model were also denoted as an assessment criteria in the assessment tools of the teachers in this study. In addition, Sol, Giménez and Rosich (2011) reported the Table 5.1 as an example in order to provide clues to new forms of classroom assessment and the some similar hypothetical actions which are stated in the table like identify objects and relevant relationships;

choose relevant variables, distinguishing from others; problem-solving processes involved in finding the solution; promote reflection about results; communicate the process and results when the model is valid, were used as an assessment criteria by the teachers in this study.

Table 5.1 Hypothetical Modeling Actions Related to Several Modeling Cycles (Sol, Giménez & Rosich, 2011, p.233)

Blum & Lei_ phases	Voskoglou cycle	Mason cycle	Observable hypothetical actions
1, 2	Analyse	Specify	1. Understand and recognise a mathematically manageable problem. 2. Simplify and structure. Recognise restrictions and specifications. Make decisions about a statement.
3	Mathematise	Build a model	3. Identify objects and relevant relationships. 4. Choose relevant variables, distinguishing from others. 5. State assumptions. Recognise the mathematical background that is needed. 6. Explain relationships between real objects and mathematical knowledge. 7. Check the coherence in the set of assumptions and mathematical relationships according to the real situation.
4		Formulate mathema tically	8. State the relationship among variables using mathematical language. 9. Formulate hypotheses mathematically. 10. Formulate problems and/or sub-problems in a mathematical way.
	Solve & Interpret	Find mathematic al solutions	11. Problem-solving processes involved in finding the solution.
5		Interpret	12.F ind and interpret solutions mathematically in the model used.
6	Validate	Compare with the original	13. Recognise the meaning and extent of the solutions and conclusions in the real situation. Pupils can also state the model. 14. Validate the model itself. Change the model if necessary. 15. Promote reflection about results.
7	---	Write a report	16. Communicate the process and results when the model is valid.

Besides, the criteria based on effective group work which are communication; cooperation, participation in the group were emphasized in the studies of Ferri (2011) and Wake (2011) who reported as, students have to learn how to describe their thinking processes and how to share them with others in the group work of modeling. Additionally, understanding the question criterion was also emphasized in the study

of Biccard and Wessels (2011) whom reported that since these tasks (MEAs) require a vast amount of reading, it become evident that students' understanding plays a fundamental role in modeling cycle.

5.2 Knowledge of Classroom Organization and Management

Teachers' pedagogical knowledge evolving was investigated under the classroom organization and management domain of general pedagogical knowledge (Morine-Dershimer & Kent, 1999). Classroom organization and management domain was examined under five categories, which are class setting, introducing the implementation, providing an understanding of the MEA and to warm up the MEA, organizing the presentations of the solutions (groups' order, groups' presentation process) and ending the implementation in this study.

Class setting

Data analysis of this study revealed that in the first implementations, teachers tried to interfere the group structure like distributing the students who had good grades in math exams to each group since it was thought by teachers that these students might have produced different approaches. In addition, teachers tried to combine the introvert and extrovert students and it was explained as in order to help the group to express their solutions more clearly. However, in the other implementations, it was recognized by all teachers that "the students characteristics are very different when it is compared with the traditional class times and MEAs solution process" so it was decided that students should create the groups their own since it was explained that "when students construct the groups they feel more that they are group and they pay attention mostly on communication and cooperation". Moreover, small group size were preferred by all teachers, the reason was given as in the big groups they could not observe the students and the group did not work effectively. The number of the group member were indicated by the Anatolian high school teachers that the number of the students in the group should change based on the question, if the question included so many operation, there could be four or five students in each group since all of them could have a chance of active participation and if the question included more interpretation rather than operation at that time, there could be three or four students in each group since all of them could have a chance of explaining their ideas in detail and analyzing and comparing each approach by group. In addition,

Anatolion Teacher Training high school teacher stated that number of the students in the group should be four, the reason was given as group could work cooperatively with four members. Besides, teachers stated that students should construct mixed groups with boys and girls. Teachers wanted this structure since it was realized that experiences and skill had an effective factor in the modeling cycle so it was stated that boys had different characteristics and experiences, the girls so, it was seen as an advantage for creating an effective working environment while solving the MEA by the teachers. The results showed that pedagogical strategy on class setting, which was small group size, construction of groups by the students, mixed groups with girls and boys, were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations, and as teachers discussed strategy with colleagues. These conceptually grounded pedagogical strategy components were supported by the previous studies (Bracke & Geiger, 2011; Galbraith & Clatworthy, 1990; Ikeda & Stephens, 2001; Lesh &Yoon, 2004). Lesh and Yoon (2004) stated that using small groups with three or four students in the implementation of mathematical modeling activities give chance learners in order to develop, describe, explain, manipulate the model and control important conceptual systems and also offered by Bracke and Geiger (2011) that modeling teams should consist of three to four students. Besides, a variety of studies have shown that working in small groups, support the development of modeling competencies (see e.g., Galbraith & Clatworthy, 1990; Ikeda & Stephens, 2001). Moreover, mixed group structure was emphasized by Biccard and Wessels (2001) that "students should also be exposed to a broader range of peers in their groups. This will allow for a wider scaffold for interaction, communication and reflection between the group members. This interaction and reflection will support the development of many cognitive and meta-cognitive processes and competencies" (p.382). Additionally, the rationale which was different experiences and different characteristics of girls and boys for mixed groups was consistent with the findings of Kaiser-Messmer (1993) that gender differences exist in student preferences for particular contexts.

Introducing the implementation

Data analysis showed that the teachers introducing the implementations that describing the process of the implementation before the students began to solve the MEAs were changed in the cyclic process of the study. In the first implementation, all teachers stated that they did not know how to introduce the process and in the implementations they only distributed the questions to the groups, wanted them to solve the question, and added they were going to make presentation. However after the first implementation all teachers stated that the process and its time should be given in detailed like "first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations will take place in the second lesson" since "these explanations help groups to organize their work". Moreover all teacher indicated that "how can be the group work" could be explained to the students like "listen to each other, share, collaboratively work, discuss, analyze and establish the group solution", the reason was given as students were not accustomed to the group work, by this explanation effective group work could take place. It was added that this explanation could be made especially to the classes where this implementations were take place for the first time. Furthermore, all teachers were also specified that the concepts embedded in the MEAs should not be given in the explanations since "students try to use only given concepts so their solution strategies will be limited". The results showed that pedagogical strategy on introducing the implementation, which was the process and its time should be given in detail in order to help groups to organize their work ;talk on effective group work's properties especially for students who were not accustomed to the group work in order to increase the effectiveness of the group work and the concepts embedded in the MEAs should not be given in the explanations in order to not limit the students' solution approaches, are generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations, and as teachers discussed strategy with colleagues. The similar pedagogical strategy's components were supported by the previous studies (García & Ruiz-Higueras, 2011; Ferri, 2011; Wake, 2011). García and Ruiz-Higueras (2011) reported that a possible didactic technique in order to introduce the process for the teachers are to explain briefly the

procedure like "at the end, you will produce a poster with your solution and present it" (p.573). Besides, the explanations based on effective group work were emphasized in the studies of Ferri (2011) and Wake (2011) who reported, as students have to learn how to describe their thinking processes and how to share them with others in the group in the modeling cycle.

Providing an understanding of the MEA and to warm up the MEA

Analysis of the data showed that the teachers providing an understanding of the MEA and to warm up the MEA process before the students began to solve the MEAs were changed in the cyclic process of the study. In the first implementation, all teachers state that they did not know how to warm up and provide an understanding for the MEA and it was explained that this understanding and warm-up period might direct the students' approaches and it was also expressed that "how can this be done without directing the students is not clear". However, in the other implementations, teachers offered introducing the context of the problem to the students by giving an example close to the MEA'context and said students' actual task again based on this example question in order to warm up and to help to show their task.

Teachers indicated that the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of concrete models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] help students to warm-up and understand the MEA clearly. Besides, Anatolian Teacher Training teachers added that if the concrete models were not appropriate then the simulations and other real life examples could be used in order to help students to warm-up and understand the MEA. The results show that pedagogical strategy on providing an understanding of the MEA and to warm up the MEA, which was introducing the context of the problem to the students by giving an example close to the MEA' context and said students' actual task again based on this example question; the questions about the MEA [*What is your task? What are givens?*] and the encourage the use of concrete models; if the concrete models are not appropriate then the simulations and other real life examples could be used in order to help students to warm-up and understand the MEA, are generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations, and as teachers discussed strategy with colleagues.

The similar pedagogical strategy's components were supported by the previous studies (Biccard & Wessels, 2011; Blum, 2011; Doerr & Lesh, 2011; Lesh & Doerr, 2003). Giving an example close to the MEA's context were emphasized by Lesh and Doerr (2003) who state that students can become more familiar with the situations of the case via reading the newspaper articles on MEA's context or talking on the similar situations from real life on MEA's context just like a warm-up period. Using concrete models component of the pedagogical strategy support the result of the study by Doerr and Lesh (2011) who reported, teacher's use of a physical model to demonstrate the relationship of the real life situation, as a particular "pedagogical strategy (or procedure)", for engaging the students with the task at hand. The similar components were also given in the strategic tool for teaching purposes, which was proposed in the DISUM Project, offered teachers to make the students read the text precisely, imagine the situation clearly, think about what is required from them and make a sketch for understanding the task process (Blum, 2011).

Organizing the presentations of the solutions (groups' order, groups' presentation process)

Data analyses showed that teachers organizing the presentations of the solutions which included groups' presentation orders and the presentation process after the students solve the MEAs, were changed in the cyclic process of the study. In the first implementation, the organization of the presentation was not clear for all teachers, they stated that the groups' presentation order could be based on their wishes or only the groups who used different approaches might be picked up for the presentation in order to "show the different approaches to all groups" and it was also indicated that the delegate could present the group solution by the teachers. However, in the other implementations different strategies were described. All teachers denoted that each group should present their approaches since "they focused on the solution so much time and they need to express themselves but if the time is limited then the groups who have different approaches can be chosen for the presentation in order to share the different ideas". The presentation order that were specified by all the teachers in the last implementations that presentations should take place from simple approaches to the systematic ones and this was explained as, "if the systematic approach is presented first then the other groups can not want to present" by one teacher and explained as, **"**they are completing each other and have a time to discuss the different

ideas, added approaches" by three teachers. Other presentation orders that were indicated; if there was not a difference like basic and systematic or one strategy and two or more strategy then each group could make their presentation in the order of request by one teacher and another teacher denoted the groups could be picked randomly to the presentation since to make them not bored and make them ready for all the time just like in her class. Moreover, all teachers indicated that each group member should take place in the presentations; the reason was given as, in order to give a chance to defend their individual ideas and to defend groups' solution. Additionaly, all teachers also indicated that group interaction was important for creating a discussion in order to give opportunities for students to compare and select the ideas and approaches, which meet the needs of the question so in order to supply this environment teachers stated that teacher should encourage groups to ask questions and teacher should ask questions and take a dynamic role in the discussion. Furthermore, it was denoted that the reflections should be given on missing and wrong parts of the approaches in order to make groups aware of their deficiency.

The results showed that pedagogical strategy on organizing the presentations of the solutions (groups' order, groups' presentation process), which was a) each group should present their approaches since they focused on the solution so much time and they need to express themselves but if the time is limited then the groups who have different approaches could be chosen for the presentation in order to share the different ideas b) presentations should take place from simple approaches to the systematic ones since by these way approaches completing each other and have a time to discuss the different ideas, added approaches if there was not a difference like basic and systematic or one strategy and two or more strategy then each group could make their presentation in the order of request or teacher could picked up randomly c) each group member should take place in the presentations in order to give a chance to defend their individual ideas and to defend groups' solution d) group interaction was important for creating a discussion in order to give opportunities for students to compare the ideas and approaches, which meet the needs of the question so teacher should encourage groups to ask questions and ask questions and take a dynamic role in the discussion e) reflections should be given on missing and wrong parts of the approaches in order to make groups aware of their deficiency are generated by the teachers, became more conceptually grounded as rationales for the

strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations, and as teachers discussed strategy with colleagues. The similar pedagogical strategy's components were supported by the previous studies (Blum, 2011; Doerr & Lesh, 2011; Lesh & Doerr, 2003; Mousoulides et al. 2007; Yu &Chang, 2011). Likely, Doerr and Lesh (2011) support the finding that they report

> When students work through the kinds of model development sequences, most often the process produces a diversity of ideas. From the teachers' perspective, encouraging a diversity of ideas will "make for good discussion" and discussing students' solutions to modeling tasks is a useful heuristic (or procedure). The underlying rationales for such discussion become visible as teachers use these discussions as opportunities for students to sort, select, and compare ideas (p.262).

In addition, Mousoulides et al. (2007) and Lesh and Doerr (2003) state that in the presentation, communication should take part and all groups need to describe their thinking and the teacher should be the dynamic domain of the groups' interaction. Moreover, the result is also consistent with the findings of a study (Yu &Chang, 2011) who state that in process of sharing solutions and it is the stage of presentations of solutions when "the teacher try to encourage students to not only listen to the other groups' presentations but also to try to understand the other groups' solutions and consider how well these solutions meet the needs of the client" (p.148). The reflection component of the pedagogical strategy is supported by Blum (2011) whom reports that it is important to give retrospective reflections after the students' presentations in order to make them to understand their weakness or strength.

Ending the implementation

Data analysis of this study revealed that the teachers' ending the implementation that recovering the lesson after the solution process and the presentations of the solutions were changed in the cyclic process of the study. In the first implementation, all teachers could only denote that teacher could explain the result of the MEA based on their solution approach. However, in the other implementations different ways were described. All teachers denoted that the main mathematical concepts, which were involved in the situation, should be summarized and the mathematical models that the students produced should be asked to the students where to use in the similar situations since by this way it was thought that the permanent learning would take

place and students had more positive approaches to the math since "they see their model usage in different situations in real life" by the teachers. Furthermore, it was indicated that the approaches, which could not be thought by the students, could be explained in order to give different point of views to students by the teachers and the approaches which was thought but not improved in the solution process could be given as an homework, in order to direct students to think more on the question out of the school for producing more appropriate approaches.

The results showed that pedagogical strategy on ending the implementation, which was summarizing the main mathematical concepts, that were involved in the situation; asking the students where to use their mathematical models in similar situations; explaining the approaches, which could not be thought by the students in order to give different point of views to students; giving homework the approaches which was thought but not improved in the solution process in order to direct students to think more on the question out of the school for producing more appropriate approaches, were generated by the teachers, became more conceptually grounded as rationales for the strategy become articulated, as teachers modified and adapted particular strategies across a range of contexts and problem situations, and as teachers discussed strategy with colleagues. Similar components of the pedagogical strategy were reported as a possible didactic technique that teachers can summarizes the big ideas embedded in the MEA and teacher can make the students to use their models in similar situations since the solution of the activity should be as simple as possible yet mathematical and significant and provides useful prototypes for interpreting other similar situations by García and Ruiz-Higueras (2011) and Lesh and Doerr (2003).

5.3 Conclusions on the Overall Nature of the Study

Theoretical framework of this study consists of three components of PCK and PK on theoretical and empirical backgrounds in the literature, which is needed for implementing modeling task effectively in classes by the teachers; namely, *questions to students, assessment of students* and *classroom organization and management*. In the light of the summative results of the evaluation showed that

- Teachers acquired the ability to ask effective questions in order to move students' thinking forward in the modeling cycle,
- Teachers acquired the ability to generate effective assessment criteria for assessing students' competency in modeling,
- Teachers generated effective conceptually grounded pedagogical strategies for proper implementation of modeling tasks,

Under the theoretical framework of this study, it can be concluded that the professional development activities based on lesson study cycle from the modeling perspective had a positive effect both on teachers' development of pedagogical content knowledge and pedagogical knowledge.

These conclusions revealed that teachers' models for implementing modeling task effectively in their classes can be developed through the professional development activities based on lesson study cycle from the modeling perspective.

5.4 Implications, Limitations and Suggestions

In order to develop teachers' model for implementing modeling task effectively in their classes, teachers should form and develop their model by providing a framework for planning, enacting, reflecting through integrating their knowledge and experiences; teacher should engage in multiple cycles of testing and revising those ways of thinking in particular contexts for specific goals and sharing their ideas with colleagues for replication and reuse in multiple contexts; teachers should implement broad spectrum of MEAs covering various topics, contexts and cognitive levels in their classes and in the implementation teachers should listen students' way of thinking, should try to understand students' emerging questions; teachers should discuss unexpected solutions, different way of students' thinking, how to ask questions on the error and difficulties that encountered or might encounter in the solution process with their colleagues; teachers should test the ways of assessment and should discuss the assessment criteria; teachers should test the pedagogical strategies and discuss with their colleagues.

Suggestions for Professional Development of In-service Teachers

The suggestions were given according to the overall nature of this study in order to create a learning environment that provide teachers' model development for implementing modeling task effectively in their classes.

- Professional development program should take place in the teachers' naturalistic setting by this way teachers have a chance to see different characteristics of their students and their different way of thinking.
- In the professional development program, teachers should use modeling activities in their classes in order to understand students' different way of thinking, promote these different thinking structures and develop their thinking strategies based on students' different way of thinking
- Teacher development programs should give a chance to teachers to develop, examine and revise their teaching practices collectively.
- In the professional development program, Teachers should choose MEA based on the grade level of the students and design lessons on that MEA based on the typical format of plan, which includes the components of PCK and PK needed for modeling, collectively; implement the MEA to students "research lesson" and discuss the lessons with the observations and with the solution papers of the students.
- Teacher development programs should give chance to teachers implement and assess broad spectrum of MEAs covering various topics, contexts, and cognitive levels.

Suggestions for Teachers

The suggestions were given to the teachers for implementing modeling task effectively in their classes based on the results of the study and the researcher's observation.

Teachers, who want to use modeling activities in their classes, need to be able to recognize and respond to the multiplicity of ways students' models might develop and the ability to respond to students' thinking as it occurs by this way gaining ability to ask effective question in order to help the students' solution process. Teachers should use different type of questions for different purposes. Teachers can use evaluative questions for the purposes of prompting students to give explanations

that will move their thinking forward by identifying a mismatch between their representations, assumptions and calculations and making groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. In addition, teachers can use broadening questions for different purposes in order to help students to imagine the real situation clearly by giving a real life example; in order to offer a situation in which it is clear what type of mathematics should be used; in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations; in order to subconsciously simplify the situation by suggesting an alternative perspective; in order to encourage students to use model for subconsciously simplifying the situation and prompt student's to give explanations that will move their thinking forward; in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.

Teacher role should be recast as of one of engaging students in the self-evaluation of their work and encouraging them to revise their thinking in ways that make sense to them, instead of evaluating students' work and guiding their movement along known paths as reported by Doerr (2006) and as indicated in this study.
Moreover, teacher should know how to assess students' solutions and presentations, the assessment criteria can be associating and using the data, communication, cooperation and participation in the group and the result (model quality) for the solution process and present with clear, precise language, made other groups to listen them, defending their approach and the answers to the questions from other groups and teacher for the presentation process.

In classroom organization and management some pedagogical strategies are suggested for the different parts of the implementation, while setting the class, teachers should know that the students characteristics are very different when it is compared with the traditional class times and MEAs solution process. The students who have highest grades in the math exams cannot participate effectively in the solution process of the MEA and therewithal the students who have lowest grades in the math exams can participate effectively in the solution process of the MEA and can direct the solutions. In addition, small groups with three or four students can be constructed. Students should also be exposed to a broader range of peers in their groups, especially

the mixed groups with boys and girls. This would allow for a wider scaffold for interaction, communication and reflection between the group members. While introducing the implementation, the process and its time should be explained briefly like first individually read the question for 5 minutes then work with the group to the end of the first lesson, after finishing the solution process prepare the poster papers and presentations would take place in the second lesson. In addition, how can be the group work can be explained to the students if they are not accustomed to the group work, like listen to each other, share, collaboratively work, discuss, analyze and establish the group solution. In providing an understanding of the MEA and to warm up the MEA stage, students can become more familiar with the situations of the case via reading the newspaper articles or talking on the similar situations from real life just like a warm-up period so teacher can introduce the context of the problem to the students by giving an example close to the MEA or wanted them to read the newspaper articles and said students' actual task again based on this example question or the articles. In the phase of understanding and warm-up teacher should ask questions about the MEA [*What is your task? What are givens?*] and encourage the use of concrete models [*water bottle covers in pack them in question and three dimensional figures like conics, cylinder and water for the water tank question*] in order to help students to warm-up and understand the MEA clearly. In the part of organizing the presentations of the solutions (groups' order, groups' presentation process), teachers should give a chance to all groups to present and describe their thinking clearly. In addition, teachers should encourage group interaction, should encourage groups to ask questions, since it is important for creating a discussion in order to give opportunities for students to compare and select the ideas and approaches, which meet the needs of the question and teacher should take a dynamic role in the discussion. Moreover, teacher should give retrospective reflections after the students' presentations. While ending the implementation, teacher should summarize the main mathematical concepts, which are involved in the situation and the mathematical models that the students produce should be asked to the students where to use in the similar situations since the solution of the activity should be as simple as possible yet mathematical and significant and provides useful prototypes for interpreting other similar situations.

Suggestions for facilitators

It was suggested as a facilitator who works with the teachers while teachers are working out to develop a shared lesson plan in the lesson study cycle, facilitator should conduct interview after the implementation and should record all suggestions in the teachers' post interviews after the implementation and if they forget some suggestions, facilitator should ask the teachers whether they intended to discuss these forgotten parts in the meetings after the implementations. In addition, if there are constant components that are discussed in each cyle of the lesson study, facilitator should control the component. For instance, teachers in the discussion may provide suggestions in different part of the components (e.g., components of PCK and PK) at that part; facilitator should revive the component and should ask questions based on that component. For example, if the component is questions to students on difficulties and errors, teachers can give different errors and difficulties instead of discussing under the *questions to students* parts of the lesson plans; at that time, facilitator can ask some questions, such as "in order to overcome these students' difficulties/errors, what kinds of questions would you ask?". To sum up, controlling the components of the lesson plans in each meeting would be a vital part of the lesson study, which has constant components in the lesson plans. Furthermore, facilitators should know that the intervention as a suggested strategy for the related PK component is not a proper didactic technique in the discussions. In this study, the facilitator tried to give some interventions (i.e suggesting a strategy) in the discussions of the PK components of the lesson plan, even if teachers discussed these concepts, unfortunately, these conceptions were not going to part in the further cyclic process. Teachers conceptualized the examined strategies by themselves or their colleagues in these kinds of professional development program.

Limitations and Suggestions for Further Research

The participants of the study were chosen from different school profiles however, selection of the participant was done based on teachers teaching the same grades were preferred to hold the concepts of the model eliciting activities in that grades and the classroom implementation times same for all four teachers by the way the chosen participants are not reflect the maximum variation. In the further research, the question how the knowledge evolve differ among the participants and the reasons

behind the differences can be investigated by taking the number of the cases more and choosing with maximum variation sampling.

In the meeting which the teachers collectively work, it was observed that while teachers discussing their solution approaches on the MEA, effective learning environments' occurred that gave chance to develop their subject matter knowledge, so in the further researches, teachers subject matter knowledge evolve can be investigated through the professional development activities based on lesson study cycle from the modeling perspective.

Moreover, one of the domain of the pedagogical content knowledge which is the knowledge of student's understanding (include students' learning process, students' typical understanding, students' common errors, things that are hard/easy for students, particular students' understanding) evolving can be investigated since "an understanding of the multiplicity of ways that students' thinking might develop" is emphasized as an characteristics of teachers' knowledge from the modeling perspective (Doerr & Lesh, 2011, p.257).

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APPENDIX A

TYPICAL FORMAT OF LESSON PLAN

Name-Surname:

Model Eliciting Activity:

Class:

Related Subjects:

Total Time:

Time for introducing: Time for solution process:

Time for presentations: Time for ending:

Objectives:

The skills that the students can use:

Materials:

PREPARATION

- *What are the mathematical concepts and the relations between them that is embedded in the model eliciting activity*

- *Before the implementation of the model eliciting activity, which prerequisite knowledge are required in order to supply the mathematical concepts that are embedded in the activity?*

- *Other issues that the teacher can pay attention in the preparation phase*

IMPLEMENTATION

- *Class setting (What are the criteria for determining the group's structure and the number of the students in one group?)*

- *How can the implementation be introduced*

- *What can be done in order to provide an understanding of the problem and to warm up the question.*

- *What kinds of solution strategies that the students can use while working on the model eliciting activity*

- *What kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' errors*

- *What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of questions that the teacher can use in order to overcome these students' difficulties*

- *What kinds of questions that the teacher can use in the solution process of the model eliciting activity and what are the aims of these questions*

- *What can be the assessment criteria while the students working on the question*

- *How can the teacher organize the presentations of the solutions (e.g., groups' order, groups' presentation process)*

- *What can be the assessment criteria while the students presenting the solutions*

- *How can the implementation be ended*

- *Other issues that the teacher can pay attention in the implementation phase*

APPENDIX B

INTERVIEW QUESTIONS

Pre-Implementation Interview Questions

1. Can you give some information about the class and the students that you will apply the model eliciting activity?
2. Can you briefly tell what you did in the previous lesson?
3. Did you do any work which consists of the preparations for the implementation of the model eliciting activity?
4. What are the mathematical concepts and the relations between them that is embedded in the model eliciting activity?
5. Do you think that the implementation time of the model eliciting activity is appropriate by considering the prerequisite knowledge that are required in order to supply the mathematical concepts that are embedded in the activity?
6. When and under which learning area could this question also be applied?
7. What do you expect positive and negative situations during the implementation? Can you explain?
8. Have you made any changes on shared lesson plan that you prepared with other teachers in the meeting? (If the answer yes) What changes have you made? Why did you made these changes?

In the following questions, you are expected to answer the questions by mentioning the consensus decisions while working out to develop a shared lesson plan.

9. How will you organize the time?
10. How will you set the class? What are the criteria for determining the group's structure and the number of the students in one group?
11. How will you introduce the implementation?
12. What do you plan to do in order to provide an understanding of the activity and to warm up the model eliciting activity?
13. What can be the assessment criteria while the students working on the question, if you would like to assess the students?

14. What are your expectations about students' solution approaches?

15. How will you reflect when the students explain their thinking about the solution?

16. What kinds of students' error might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' errors?

17. What kinds of students' difficulties might encounter in the solution process of the model eliciting activity and what kinds of questions that you will use in order to overcome these students' difficulties?

18. What kinds of questions that you will use in the solution process of the model eliciting activity and what are the aims of these questions?

19. How will you organize the presentations of the solutions (e.g., groups' order, groups' presentation process)?

20. What can be the assessment criteria while the students presenting the solutions, if you would like to assess the students?

21. How will you end the implementation?

22. To what extent do you feel yourself ready for the implementation? Why?

23. Is there anything else that you would like to add?

Post-Implementation Interview Questions (Teacher's answers to the this interview questions will be transcribed and will be given to the teacher before the meeting [after implementation meeting])

1. How would you rate the overall implementation process? Is it what you expected? If not, according to you what are the reasons?

2. Did you need to use of concrete materials or technology in the implementation process of the MEA?

3. There are any changes in your thought about the mathematical concepts and the relations between them that is embedded in the model eliciting activity?

4. There are any changes in your thought about the prerequisite knowledge that are required in order to supply the mathematical concepts that are embedded in the activity?

5. Do you think that the implementation time of the model eliciting activity is appropriate? When and under which learning area could this question also be applied?

6. How did you organize the time? Do you think any changes based on this organization?

7. How did you set the class? What were the criteria for determining the group's structure and the number of the students in one group?

    a. What are the positive effects of this group's structure?

    b. What are the negative effects of this group's structure?

8. What can be the assessment criteria while the students working on the question, if you would like to assess the students?

9. What are your comparisons on your expectations of the students' solution ways and the ways that you observed in the implementation?

10. In the implementation process did you observe any solution approaches that you did not expect and surprised? If yes, what were they and what did you do?

11. What kinds of students' errors that you observed in the implementation process? What had you done in order to bring out and overcome these errors?

12. What kinds of students' difficulties that you observed in the implementation process? What had you done in order to bring out and overcome these difficulties?
13. What kinds of questions that you used in the solution process of the model eliciting activity and what were the aims of these questions?

14. How did you organize the presentations of the solutions (e.g., groups' order, groups' presentation process)? Why?

15. What can be the assessment criteria while the students presenting the solutions, if you would like to assess the students?

16. Would you change anything about the implementation if you had an opportunity to make this implementation again? If yes, what would you change and why?

17. What will you share about the implementation with your colleagues in the meeting, what are the titles that you have decided. Especially which topics would you like to debate in the meeting?

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APPENDIX C

LESSON OBSERVATION FORM

Teacher		Time (Start-Finish)	
School		Subject	
Class		Observers	
Date		Number of the students	

		Observation of Frequency and Other Notes
Aim of the lesson		
Model Eliciting Activity (MEA)		
Class setting (*which stage, group type, how long*)	Small group	
	Group of two	
	Individual	
	All class	
Teacher attitudes and behaviors in the implementation process		
Phases of the lesson (*Observations in each stage will be recorded by time seconds)*	**Warm-up**	**Observation of Frequency and Other Notes**
	Description of the lesson process Warm-up activities for the MEA To provide an understanding of MEA	
	While students are working on the MEA	

	Probing the process The questions that are asked in order to bring out the students' mathematical thinking Other questions that are asked What are done in order to evaluate if the mathematical concepts and the relations between them that is embedded in the model eliciting activity are understood by students or not. What are done in order to evaluate group working	
	Listening and observing the students in the process	
	Giving feedback in the process Answers that are given to the students' questions and the ways of guidance The difficulties that are detected and the methods that are used in order to overcome these students' difficulties	
	While students are presenting their solutions	
	What are done while listening the presentations and the explanations that are given	
	Probing the solution and evaluation (*The questions that are asked,* the ways of guidance, *evaluations, questions posed to student or group*)	
	The feedbacks that are given to student or group about the solution	
	Explain the solution approaches (recovering the lesson)	
Modeling environment, and other observations regarding the implementation of the MEA		

APPENDIX D

DEVELOPMENT PROCESS OF MEAs IN THE PARENT PROJECT

The project team consist of ten mathematics researchers developed the 60 model eliciting activities. Four groups which include 2, 2, 3 and 3 researchers were constructed and then the big ideas of the MEAs were determined based on the objectives of the secondary mathematics curricula published by Turkish Ministry of National Education. The big ideas were choosen by the groups and groups began to develop the MEAs. The MEAs were constructed based on the six principles of design. These principles were described as;

1. *The Personal Meaningfulness Principle* (sometimes called *the "Reality" Principle):* Could this really happen in real life situations? Will students be encouraged to make sense of the situation based on extensions of their own personal knowledge and experiences? Will students' ideas be taken seriously, or will they be forced to conform to the teacher's (or author's) notion of the (only) correct way to think about the problem situation?
2. *The Model Construction Principle:* Does the task ensure that students clearly recognize the need for a model to be constructed, modified, extended, or refined? Does the task involve constructing, describing, explaining, manipulating, predicting, or controlling a structurally significant system? Is attention focused on underlying patterns and regularities rather than on surfacelevel information?
3. *The Self-Evaluation Principle:* Are the criteria clear to students for assessing the usefulness of alternative responses? Will students be able to judge for themselves when their responses are good enough? For what purposes are the results needed? By whom? When?
4. *The Model-Externalization Principle* (sometimes called the *Model-Documentation Principle):* Will the response require students to explicitly reveal how they are thinking about the situation (givens, goals, possible solution paths)? What kind of systems (mathematical objects, relations, operations, patterns, regularities) are they thinking about?
5. *The Simple Prototype Principle:* Is the situation as simple as possible, while still creating the need for a significant model? Will the solution provide a useful prototype for interpreting a variety of other structurally similar situations? Will the experience provide a story that will have explanatory power—or power for making sense of other structurally similar situations?
6. *The Model Generalization Principle:* Does the conceptual tool that is constructed apply to only a particular situation, or can it be modified and extended easily to apply to a broader range of situations? Students should be

> challenged to go beyond producing single-purpose ways to thinking to produce reusable, sharable, modifiable models.

by Lesh, Hole, Hoover, Kelly and Post (2000). Some MEAs were created and some of them were adapted. After the developing phase, each MEA were controlled by all the researchers in the meetings based on the six principle of instructional design, content and the clarity of language. These 60 MEA were implemented by the five mathematics teachers in their classrooms, in one of the high school in Ankara. After these implementations, the MEAs were revisied based on the teachers' suggestions and based on the analysis of the implementations. Lastly, all the MEAs were controlled by all the groups of the Project team and the last version of the MEAs were reached. These design and pilot study process were completed in one year.

APPENDIX E

MODEL-ELICITING ACTIVITIES

Bank Robbery

The night before a bank was robbed by unidentified person or persons while a power cut in İstanbul. The police soon arrived the place however failed to catch up the thieves. The suspects who are Ahmet K. (35), Burak M. (24) and Cem T. (34) were taken into the custody and taken to police station before the court yard based on the variety of evidence at the place.

The following conclusions were reached about the situation of the suspects after the interrogations:

- If Ahmet is innocent, both Burak and Cem are guilty.
- Burak or Cem is innocent.
- Ahmet is innocent or Burak is guilty.

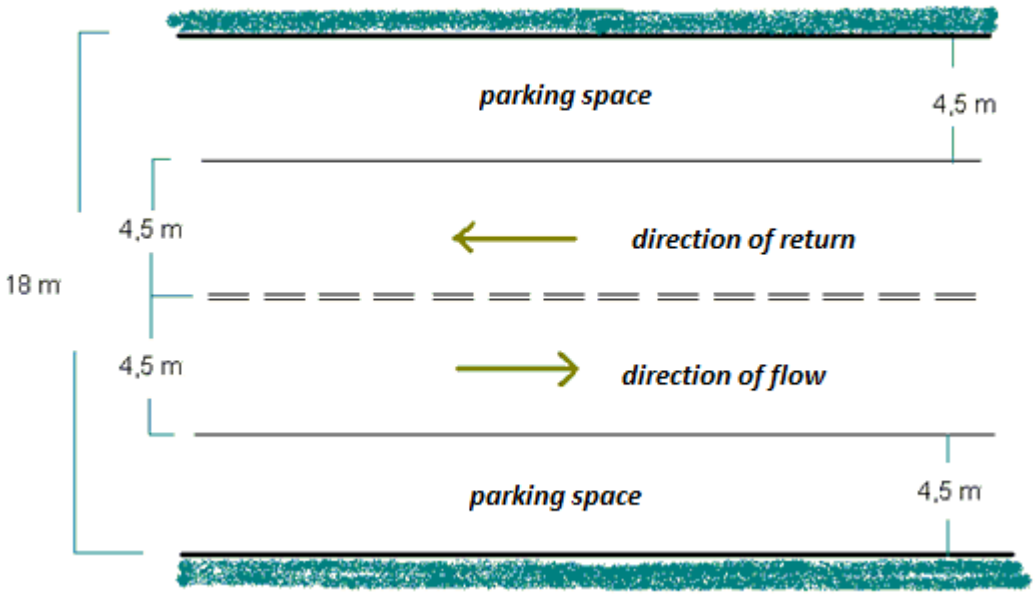
Public prosecutor will decide on the suspects to send court for arrest or to evacuate in the light of this information. You are claimed to create a method that could be used in order to decide suspects who is guilty and who is innocent by prosecutor.

"The Bank Robbery" activity was developed a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250). The activity was tested based on the validity and the realibity requirements and classroom implementations were carried during the piloting stage of the project and changes were done accordingly before further uses.

Tag: Mathematical concept under the MEA is logic that takes place in 9 th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011)

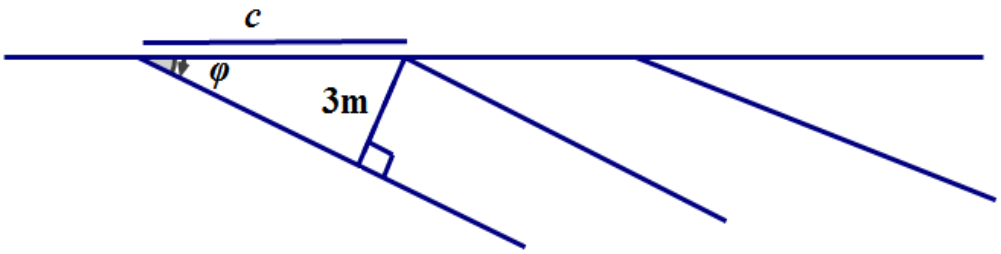
Street Parking

A city planner wants to help from you in order to design parking spaces in front of the houses. The aim of the city planner is to allow the most room for the parking of cars on both side of the roadway and still allow a two-way traffic flow. The street is 150m long and 18 m wide as shown in figure 1. 4,5 m of roadway is needed for each lane of traffic. Parking spaces are 150m long and 4,5 m wide and 3m wide and 4,8m long space including the lines is needed for a car in order to park safely. The parking spaces can be parallel to the road or can be designed to be angled as shown in figure 2 however in this case the cars should not be overflow to the roadway.



Your job is to determine which method of parking will allow the most room for parking of cars and still allow a two-way traffic flow.

a. How many cars can be parked to the street using parallel parking?
b. How many cars can be parked to the street using angle parking? You can consider the diagram below for the calculation.



Which angle allows for the greatest number of parked cars? You can try variety of angle (**φ)** values in order to calculate the number of cars that settle 150m long parking space.

c. You now have calculated the number of cars for the different method of parking. Considering this calculations, what is your recommendations to city planner on the design of the parking in order to allow the most room for parking of cars, and why?

As a part of a project supported by TUBITAK (Grant no 110K250) **"Street Parking"** activity was adapted from **"**Swetz, F. & Hartzer, J.S., (1991). *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises*. Reston, VA: NCTM. (pp. 71)"

Tag: Mathematical concepts under the MEA are the geometry of the triangle and trigonometric functions that take place in 10 th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011)

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The Summer Jobs

Last summer Arzu started a concession business at Wild Days Amusement Park. Her vendors carried candy, hot dogs, and drinks around the park, selling wherever they found customers. The business was a great success. Next summer, Arzu is expecting that all of her vendors will want to work for her again. But, the park managers told her that she won't be allowed to hire as many vendors next summer. So, she needs your help deciding which workers to rehire. If all of last year's vendors apply for a job, she'll only be able to hire about a third of them to work full time, and about a third of them to work half time.

She won't be able to hire the remaining third of them. The table below shows a sample of nine people who worked for her last summer. To try to figure out a procedure for deciding who to hire next summer. Arzu reviewed her records for the nine vendors who are shown. For each of these vendors, she totaled the number of hours they worked and the amount of money collected – when business in the park was busy (high attendance), steady (average attendance), and slow (low attendance). (See the table that follows.) She wants to rehire the vendors who will make the most money for her. But, she doesn't know how to compare them because they worked different numbers of hours; and, she isn't sure what to do about the fact that it's easier to sell more when the attendance is high.

Write a letter to Arzu describing how she can evaluate all of the vendors who worked for her last summer, and how to decide who to hire full-time and part-time. Show how your procedure works for the nine people workers who are shown in the table. Give details so Arzu can check your work, and give a clear explanation so she can decide whether your method is a good one for her to use.

HOURS WORKED LAST SUMMER

	JUNE			*JULY*			*AUGUST*		
	Busy	Steady	Slow	Busy	Steady	Slow	Busy	Steady	Slow
Melek	12.5	15	9	10	14	17.5	12.5	33.5	35
Kübra	5.5	22	15.5	53.5	40	15.5	50	14	23.5
Tülay	12	17	14.5	20	25	21.5	19.5	20.5	24.5
Jale	19.5	30.5	34	20	31	14	22	19.5	36
Çetin	19.5	26	0	36	15.5	27	30	24	4.5
Can	13	4.5	12	33.5	37.5	6.5	16	24	16.5
Remzi	26.5	43.5	27	67	26	3	41.5	58	5.5
Tekin	7.5	16	25	16	45.5	51	7.5	42	84
Veli	0	3	4.5	38	17.5	39	37	22	12

MONEY COLLECTED LAST SUMMER (IN DOLLARS)

	JUNE			*JULY*			*AUGUST*		
	Busy	Steady	Slow	Busy	Steady	Slow	Busy	Steady	Slow
Melek	690	780	452	699	758	835	788	1732	1462
Kübra	474	874	406	4612	2032	477	4500	834	712
Tülay	1047	667	284	1389	804	450	1062	806	491
Jale	1263	1188	765	1584	1668	449	1822	1276	1358
Çetin	1264	1172	0	2477	681	548	1923	1130	89
Can	1115	278	574	2972	2399	231	1322	1594	577
Remzi	2253	1702	610	4470	993	75	2754	2327	87
Tekin	550	903	928	1296	2360	2610	615	2184	2518
Veli	0	125	64	3073	767	768	3005	1253	253

Figures are given for times when park attendance was high (busy), medium (steady), and low (slow).

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250) **"The Summer Jobs"** activity adapted from **"**Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In A. E. Kelly, & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers."

Tag: Mathematical concepts under the MEA are mean and ratios and proportions that take place in 9 th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011)

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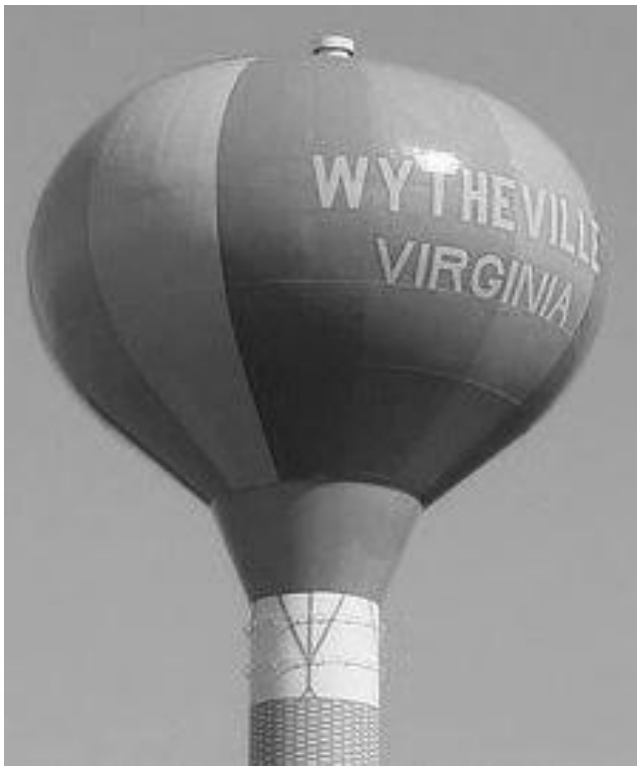
Water Tank

A company is preparing software for computer aided education. One of the team of the company job is on water tank animation that will help to develop students' graphing and interpreting the graph skills. The team needs a graph of the height as a function of the amount of water that is in the tank in order to create water tank animation.

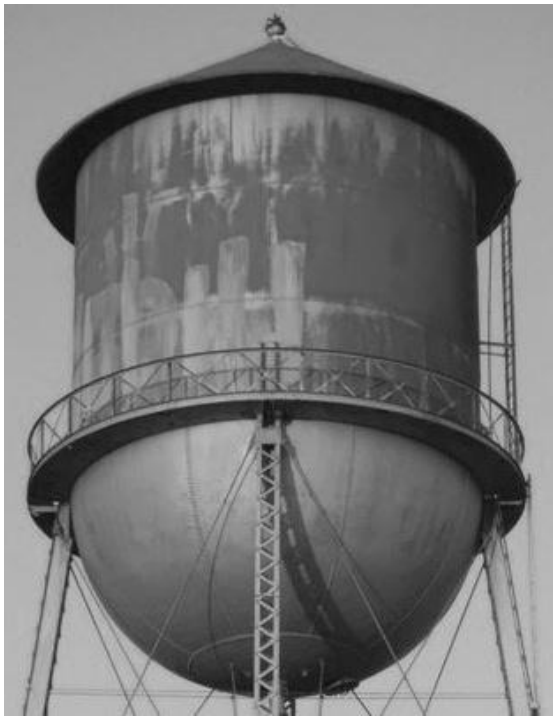
As a mathematician member of the team, drawing the graphs that are desired for the example water tanks given in the appendix are requested from you. In addition you are expected to prepare a manual that explain how to draw **a graph of the height as a function of the amount of water that is in the any shape tank**.

Water Tanks

Tank 1



Tank 2



Tank 3

Tank 4

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250), **"The Water Tank"** activity was adapted from **"**Carlson, M. P. (1998). A cross-sectional investigation of the development of the function concept. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education, III* (CBMS Issues in Mathematics Education, Vol. 7, pp. 114–162). Washington, DC: Mathematical Association of America." and "Carlson, M., Larsen, S., &Lesh, R. (2003). Integratingmodelsandmodelingperspectivewithexistingresearchandpractice. In R. Lesh&H. Doerr (Eds.), *Beyond constructivism: A modelsandmodelingperspective* (pp. 465-478). Mahwah, NJ: Lawrence ErlbaumAssociates."

Tag: Mathematical concepts under the MEA are change and concave up and concave down functions that take place in 10 th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011)

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Pack Them In!

A small business engaged in the production of organic canned food needs to find short-term storage for some cylindrical cans. The company wants to store the cans which are produced in autumn and to sell in winter and wants to do this at minimum expense. Therefore, the job is to pack them in using as little storage space as possible. The cans: the right circular cylinders with a radius of 10 cm and a height of 30cm. All 176 cans must be stored in an upright position. Storage is required for two months.

Storage units available for rents are rectangular prisms, units sizes and the rental costs are given in the table below.

Height (cm)	Width (cm)	Lenght (cm)	Rental Cost for a month (TL)
100	110	220	150

1. If you were owner of the company how do you place the cans to the storage unit in order to minimize the cost?
2. In later productions the company may need to store a large number of cans. Therefore, is it appropriate for the firm to place the cans to the storage unit all in the same way? Which way would you suggest to the firm for the most appropriate arrangement?

Note: The cans must be stored in an upright position, it is important for the security.

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250) **"The Pack Them In!"** activitiy adapted from Swetz, F. ve Hartzler, J. S. (1991) *Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises*. Reston, VA: NCTM. (pp. 12)

Tag: Mathematical concepts under the MEA are geometry and Pythagorean Theorem that take place in 9 th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011)

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APPENDIX F

THE FORM OF THE THEMES THAT EMERGED FROM EACH CASE

Table F.1 The Themes that Emerged from Each Case

Teacher Name:		
Theme 1:		
Theme 2:		
Theme 3:		
Theme 4:		
Theme 5:		
Teacher Name:		
First Implementation	**Themes**	**The Excerpt**
Before the implementation		
During the implementation		
After the implementation		
Second Implementation	**Themes**	**The Excerpt**
Before the implementation		
After the implementation		
Third Implementation	**Themes**	**The Excerpt**
Before the implementation		
During the implementation		
After the implementation		
Fourth Implementation	**Themes**	**The Excerpt**
Before the implementation		
After the implementation		
Fifth Implementation	**Themes**	**The Excerpt**
Before the implementation		
During the implementation		
After the implementation		

APPENDIX G

THE FORM OF CASE RATINGS OF THEMES FOR ALL IMPLEMENTATIONS

Table G.1 Case Ratings of Themes for All Implementations

Cases **Case Themes**	Case A:	Case B:	Case C:	Case D:

APPENDIX H

THE SAMPLE FILLED, CASE RATINGS OF THEMES FOR ALL IMPLEMENTATIONS FORM AND THE THEMES THAT EMERGED FROM EACH CASE FORM

Theoretical Framework Component: Question to Students

Table H.1 The Themes that Emerged from Figen for Questions to Students-Sample Filled Form

Teacher Name: **Figen**
Theme 1: invitational
Theme 2: directive questions with the help of the keywords
Theme 3: procedural
Theme 4: it was stated that "teachers do not know how to interfere the student groups' solution process and what kind of questions to ask."
Theme 5: broadening • if the MEA is not understand, the students directed to the activity with the questions • if there is misleading representations, the questions are preferred in order to realize this incorrect representation. • The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. • The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward. • the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. • The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.
Theme 6: evaluative (evaluate the strategy) • The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.
Theme 7: exploratory • The exploratory questions are used in order to make groups to think other approaches.

Table H.1 (continued)

Teacher Name: Figen		
First Application	**Themes**	**The Excerpt**
Before the application	• invitational • directive questions with the help of the keywords	"Can you understand the question [*bank robbery*], do you have any questions"-in order to motivate to begin "If they [students] will not think, then I can ask, do you read "or "and "and" statements? What they remind you? I do this method in my classes also, for example in geometry lesson I say isosceles triangle and the height? What reminds you? and then they say that okay okay and they remember that the height is the median. I prefer to draw attention to the keywords rather than actually saying that this question is about this subject. For example in here when I draw attention to "or" and "and" keywords, I think they remember the logic and use the conjunctions"
During the application		figen did not use any question both for feedback or understanding the solution strategies, teacher did not interfere the solution processes.
After the application	• procedural	**"***we [teachers] make them [students] to realize their errors, for instance they [students] used "and" conjunction wrongly, we [teachers] can ask in which situations the "and" conjunction get the truth value, also we want them [students] to show us the truth table of the "and" conjunction then they will remember the property and they will have a chance to correct their error"*
Second Application	**Themes**	**The Excerpt**
Before the application	• directive questions with the help of the keywords • procedural	"*We [teachers] can ask, are the angle important for parking the maximum numbers of cars? So with the little orientation, we can direct students to use trigonometry rather than saying use trigonometry. I think when the angle emphasized then they will understand*" *"Teacher B: they may have also make the trigonometric calculations wrongly or while writing the ratios of the similarity* *Figen: at that time we can ask them do the trigonometric operations again"*

Table H.1 (continued)

After the application	• broadening • if the MEA is not understand, the students directed to the activity with the questions • if there is misleading representations, the questions are preferred in order to realize this incorrect representation. • It was stated that "teachers do not know how to interfere the student groups' solution process and what kind of questions to ask."	*"Most of the groups cannot understand the question fully because most of them take the hypotenuse 4.8m, so I think it is important to ask them read the question again, are you carefully examine the question, the data? " "Teacher A: It is very important to make right drawing. I observed these in my class also, I ask them is your drawing right? Then they look and said yes, then I do not interfere, I could not decide what to ask what to say? Figen: in this case we [teachers] may ask for example, when you park with angle, the car can go to the in front of the parking spaces completely? Are there any dead space? however I agree with you that we [teachers] do not know how to interfere and how much, and what kinds of questions can be asked"*
Third Application	**Themes**	**The Excerpt**
Before the application	• evaluative (evaluate the strategy) The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • exploratory The exploratory questions are used in order to make groups to think other approaches. • Broadening The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.	**"***if the group will use only the general mean, it is a missing approach, then I will ask them to look their approach and ask it is enough in order to decide… also they may be ambiguous about how to decide part time, I can ask at that time is there another way to decide? Is there another criterion?" "if they only use general mean and do not decide, then it can be ask can we decide based on periods or months?…"*
During the application	• evaluative • exploratory	"what is your strategy? Why do you use this approach?" The question below is asked to the groups that create a table based on the money per hour and tries to decide the full time and part time worker: Figen: "Is it enough to decide based on one table? Do you create a table based on another criterion? Is there another way to do this?"

Table H.1 (continued)

After the application	• exploratory • broadening if the MEA is not understood, directing students to the activity with the questions were preferred	"when we ask them how we can do the question from another way, then they think other criteria and it is good for to encourage them to think other approaches, also some of the groups cannot understand the question completely, it will be good to want them to read and examine the question again"
Fourth Application	**Themes**	**The Excerpt**
Before the application	• evaluative • broadening The broadening question was used in order to make students compare their result if it fits to the real situation.	"*Teacher B: we [teachers] can want them to draw their graph again, or ask it is true? Is the height increasing?* *Figen: or we [teachers] can ask them can you examine your graphs again? or Compare the graphs and water tanks?"* **"***I think some of the groups can draw the graph as decreasing at that time we [teachers] can ask can you imagine, how can the height be decreasing if the tank has not a hole on it"*
After the application	• broadening • evaluative	**"***or we [teachers] can ask what does the question ask we, which variables relation were asked? Is your graph fit with these?"*
Fifth Application	**Themes**	**The Excerpt**
Before the application	• exploratory • evaluative In here especially the evaluative questions are used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations. • broadening The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward.	"*may be groups find two kinds of arrangement and do not think other arrangement at that time, it can be asked, may have different installations? Can we arrange more cans in other installation? By this way students look at the question from different ways"* "*They may can arrange the cans but count wrongly or formulize wrongly and think that their arrangement is the appropriate one at that time, we can ask to compare their representations and their formulization, calculation …some groups may not see different arrangement, at that time we [teachers] ask them [students] to use water bottle covers [models] in order to visualize and see the blank spaces between the covers when it is arranged as regularly"*

Table H.1 (continued)

During the application	• broadening the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward. • broadening the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. • exploratory • evaluative The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward. • broadening The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.	” when you regularly place, there are empty spaces, how can we decrease these empty spaces?” ” it is a bit hard to think on three dimensions, why do not you think only the base, then other rows can be calculated based on this base, cannot?” “can we place differently? Is there another arrangement?” “ How many cans did you place with this method? The empty spaces are many or not?” “try to make clear sketches, how can we formulize those steps?”
After the application	• broadening the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective. the broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.	“*it is important to think in two dimensions, because when they think in three dimensions, they cannot manage the solution, so I ask them to think only the base…most of the groups cannot think other arrangement so I ask them how we can decrease the empty spaces that are occurred when the covers [water bottle covers-models] are arranged regularly, then from this point of view they can think other arrangementsarrangement”*

Table H.2 Case Ratings of Questioning Themes for All Implementations-Sample Filled Form

Cases / **Case Themes**	Case A: Figen	Case B: Rezzan	Case C: Semra	Case D: Melda
Theme 1: invitational	X		X	
Theme 2:				
• directive questions with the help of the keywords	X	X		
• Directive questions are used to lead the student step by step to the desired end.				X
Theme 3: procedural	X	X	X	X
Theme 4: broadening				
• if the MEA is not understand, the students directed to the activity with the questions	X	X	X	X
• if there is misleading representations, the questions are preferred in order to realize this incorrect representation.	X			
• The broadening question is used in order to suggest an alternative perspective and prompt students to give explanations that will move their thinking forward.	X	X	X	X
• The broadening question is used in order to encourage students to use model for subconsciously simplifying the situation and prompt students to give explanations that will move their thinking forward.	X	X	X	X
• the broadening question is used in order to subconsciously simplify the situation by suggesting an alternative perspective.	X	X	X	X
• The broadening question is used in order to give students vast opportunities to acquire mathematical competencies and making connections within and outside mathematics by encouraging students to use sketches and formulizations.	X	X	X	X
• The broadening question is used for offering a situation in which it is clear what type of mathematics should be used.		X	X	
• The broadening question is used in order to help students to imagine the real situation clearly by giving a real life example.		X	X	X

Table H.2 (continued)

Theme 5: evaluative (evaluate the strategy)				
• The evaluative questions are used in order to make groups evaluate their strategies; understand the meaning of their approaches and to prompt students to give explanations that will move their thinking forward.	X	X	X	X
• The evaluative questions were used in order to prompt students to give explanations that will move their thinking forward by identifying a mismatch between their representations and assumptions and calculations.	X	X	X	X
Theme 6: exploratory • The exploratory questions are used in order to make groups to think other approaches.	X	X	X	X

APPENDIX I

THE PROGRAM OF THE WORKSHOP

Before the cyclic process of the study, teachers attended four- day workshop. The aims of the workshop were; a) sharing the objectives and the scope of the project which was going to take ten months with the teachers b) determining the preconceptions, needs and the interest of the teachers for the mathematical modeling c) creating a common view about the nature of the mathematical modeling activities and the process of modeling d) helping teachers to reconsider their mathematical knowledge and the pedagogical content knowledge via mathematical modeling activities.

In the first day of the workshop which took place in 13-16 September 2011 and with 14 mathematics teachers from both schools that are Anatolian High School and the Anatolian Teacher Training High School, firstly the teachers were informed about the aims of the in-service professional development program, the scope and the process of the project in detail. Additionally, teachers were wanted to create a concept map individually in order to reveal their thought and preconceptions on mathematical modeling in math teaching and learning. After the concept map activity, teachers analyzed the 5 model eliciting activities based on the mathematical concepts that are included in the activities, the nature of the activities, the differences between modeling activities and the other questions and the use of modeling activities in math teaching. Teachers also worked on the one of the MEA as a student with their groups and present their models to other groups. At the end of the day, teachers were encouraged to complete an activity that they reflected their solution process of modeling with the prepared sentences in an envelope by producing a mini-poster. The model eliciting activities' nature, place in the math education and the process of modeling were discussed with the teachers via this activity.

In the second day of the workshop, the presentation on the solution ways of the students on one of the MEA took place. Teachers had a chance of comparing their solution ways with the students' solution ways. The interpretations and the discussions on this presentation provide a better investigation on the nature of the model eliciting activities. After the presentation, the workshop continued with the presentation on the use of technology in mathematics education. In the presentation, the use of technology in the process of mathematical modeling and the technological tools and the computer software that could be used in the modeling process was explained. Then the mini workshop on Excel took place in which the teachers participated actively by using laptop computers with MS Excel spreadsheet software. Furthermore, teachers worked on one of the MEA with their groups. The groups presented their models to the other groups, after the presentation teachers evaluated the modeling process from both the teachers' perspective (e.g. the mathematical concepts embedded in the MEA and the pros and cons of using the MEA in the class) and as well as the students' perspective (e.g. how the students can create models). Besides, teachers completed an activity that they reflected their solution process of modeling with the prepared sentences in an envelope by producing a mini-poster. Moreover, teachers' were informed about how to fill the students' thinking sheet and filled the students' thinking sheet that include the students' errors and solution ways based on the students' solution papers of the same activity that they solved with their group. The consensus student thinking sheet was formed with the discussion that was moderated by one of the researcher in the project group.

Third day of the workshop began with the mini workshop on Geogebra. Firstly, the geogebra software was informed, then the workshop on Geogebra took place in which the teachers participated actively by using laptop computers with Geogebra software. After the mini workshop on Geogebra, teachers worked on one of the MEA with their groups and presented their models to the other groups. Besides, teachers completed an activity that they reflected their solution process of modeling with the prepared sentences in an envelope by producing a mini-poster. Furthermore, with the scope of the solution process of the MEA, advantages and disadvantages of the group work and the teachers' role in the modeling process were discussed.

In the last day of the workshop, the presentation took place which includes the mathematical model, the properties of the model eliciting activities, the nature of the

model eliciting activities and the modeling process. In the presentation, teachers were explained the some of the headings with the examples based on the implementations took place during the workshop. After the presentation, teachers prepared lesson plans for one of the MEA that they solved during the workshop with their groups. While teachers preparing the lesson plans, they thought the embedded mathematical concepts in the MEA, the objectives that tried to be reached in the curriculum, the students' solution ways, students' difficulties and how to overcome these difficulties and the the evaluation of the learning process of the mathematical concepts embedded in the MEA. Moreover, teachers were wanted to create a concept map individually in order to reveal their thought and conceptions on mathematical modeling in math teaching and learning through the workshop. At the end, the in-service professional development program which was going to take ten months was explained in detail and the teachers were determined by using a form of requests to participate in during the year and teachers' evaluation of the workshop were taken through reflective statements taken from forms that they filled during the last day. Consequently, through the workshop, the in-service training for teachers with the activities (e.g. preparing lesson plans, forming students' thinking sheet, focus group discussions) were tried to be given in order to provide an experience for the professional development activities that were going to place during the year.

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PUBLICATIONS

Erbaş, A.K. & Aydogan-Yenmez, A. (2011). The effect of inquiry-based explorations in a dynamic geometry environment on sixth grade students' achievements in polygons. *Computers & Education, 57* (4), 2462-2475.

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HOBBIES

Jogging, Gourmet, Movies.