# A CRITICAL INQUIRY INTO THE DEMARCATION OF LOGICAL CONSTANTS

A THESIS SUBMITTED TO THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

TANKUT BEYGU

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILISOPHY
IN
THE DEPARTMENT OF PHILOSOPHY

JANUARY 2013

Approval of the Graduate School of Social Sciences

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## ABSTRACT

A CRITICAL INQUIRY INTO THE DEMARCATION
OF LOGICAL CONSTANTS

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January 2013, 196 pages

The dissertation aims to set out a methodological framework conducive to further research into the demarcation problem of logical constants through a critical examination of the principal proposals for the problem.

Logical constants should be characterised so as the essential values of logic, i.e., necessity, normativity and formality, are secured. Formality is central to the proposed framework in consideration of its relation to validity; necessity and normativity are established with reference to formality. Logical constanthood is analysed into logicality and constanthood to explore the conditions and constraints on logical form. On the purpose of their determination, a Wittgensteinian stance is endorsed, focusing on the view of language as a reflexive autonomous realm. The autonomy of language unfolds a specific viewpoint that indicates that logical form is existentially grounded in the possibilities presented by the autonomy.

Gentzen's natural deduction and sequent calculi are adopted as the proper perspectives to discuss the relations of logicality and constanthood to formalisation. Logical constants are required to be fixed so that the resultant logical form is sterile of content and semantically inert with respect to argument content. In addition to the conditions of harmony in logical form, mutual disharmony is introduced. Analytic and grammatical truths are specified as constraints to logical form. In particular, the conservativeness condition is found to be irrelevant to logical form.

The framework incorporates methodological pluralism as a probe into the understanding of logicality.

The dissertation suggests a bidimensional programme of research related to formal conditions and the Wittgensteinian grammatical constraints.

Keywords: Logical Constants, Autonomy of Language, Harmony, Logical Pluralism

# ÖZ

MANTIKSAL SABİTLERİN AYRIMI ÜZERİNE
ELEŞTİREL BİR ARAŞTIRMA

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Ocak 2013, 196 sayfa

Bu tez, başlıca önerişlerin eleştirisi üzerinden, mantıksal sabitlerin ayrımı sorununun daha ileri araştırılması için elverişli bir metodolojik çatı kurmayı amaçlamaktadır.

Mantıksal sabitler, mantığın özsel değerleri olan zorunluluk, normatiflik ve formalliği garanti edecek şekilde karakterize edilmelidirler. Önerilen çatı geçerliliğe olan ilişkisini göz önüne alarak, formalliği merkeze almakta, zorunluluğu ve normatifliği formalliğe başvurarak kurmaktadır. Mantıksal sabitlik, mantıksal form üzrindeki koşulları ve kısıtlamaları incelemek için mantıksallığa ve sabitliğe çözümlenmiştir. Bunların belirlenmesi için dilin yansımalı özerkliğinin odakta olduğu Wittgensteincıl dil görüşü benimsenmiştir. Dilin özekliği mantıksal formun varoluşsalolarak bu özerlik tarafından sunulan olanaklarda temellendiğini gösteren bir görüş açısı sağlar.

Gentzen'in doğal dedüksiyon ve ardışıklık (sequent) sistemleri mantıksallığın ve sabitliğin formalleştirme ile olan ilişkilerini tartışmak için uygun perspektifler olarak

alınmıştır. Mantıksal sabitler, ortaya çıkan form içerik açısından steril ve çıkarım içeriğine göre semantik olarak etkisiz olacak şekilde belirlenmelidir. Mantıksal formdaki uyum koşullarına ek olarak, karşılıklı uyumsuzluk koşulu önerilmiştir. Analitik ve gramatik doğrular mantıksal form üzerinde sınırlamalar olarak görülmektedir. Elde edilen bir sunuç korunum koşulunun mantıksal form için ilgisiz olduğudur.

Önerilen çatı metodolojik çoğulculuğu mantıksallığa bir araştırma olarak kapsamaktadır.

Tez, formal koşullara ve gramatik (Wittgensteincıl anlamda) sınırlamara ilişkin iki-boyutlu bir araştıma programı önermektedir.

Anahtar Kelimeler: Mantıksal Sabitler, Dilin Özerkliği, Uyum, Mantıksal Çoğulculuk

## ACKNOWLEDGMENTS

I would like to express my sincere appreciation to my supervisor Prof. Dr. Teo Grünberg and examining commitee members for their guidance and encouragement.

I would also like to thank the METU Department of Philosophy with my deepest gratitude for their support for the promotion of the study of philosophy.

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# CHAPTER 1

## INTRODUCTION

The notion of a strictly articulated form to represent and assess arguments is essential to logic.[1] In that form, logical consequence does subsist. The main architectural elements of the form are logical constants. Then, without a proper understanding of logical constanthood and what is distinctive of logical constants, we cannot have a satisfactory grasp of what logic is and what to expect from the study of logic. However, the plain fact is that while any textbook on logic enumerates the putative logical constants, supplementing them with some techniques to translate expressions of natural language and mathematics into and from the formal language of logic, there is hardly any considerable discussion as to what counts as a logical constant in general. Indeed, the matter unfolds as a perplexing and enduring problem beyond textbooks, once we set aside the method of enumeration and seek after principles and general characteristics.

The present inquiry brings to the fore the generic requirements of logical form that bears on a principled account of logical constants and the basis of their demarcation from nonlogical terms. Very often, the talk about logical form in the context of logical constants suggests a familiar metaphor of a non-adhesive mould getting its sturdiness from the material strength. It is presumed that the assignment of terms as logical constants *eo ipso* provides the proper form like such a mould, and then we could work out our way to the conclusion. Then, the challenge would be to find the *right* words and phrases to redefine and separate them into the logical vocabulary. We shall argue that, in some respects, this metaphor is illusionary, and in others, it is misleadingly incomplete. We think that the challenge is to specify the conditions and constraints to define a term as a logical constant so that it could take part in the logical form. This procedure involves a number of considerations about the grounds of logic and the nature of language, since logic grasps

[1] The phrase 'informal logic' is a misnomer in the philosophy of logic, though used even in some acclaimed sources –an appropriate term might be 'argumentation'.

reasoning, not in itself, but in its form in discourse. An upshot of our inquiry is that logic is, in some deep sense, an exploration into the essence of language. So, what follows can only be a part of the initial phase of a programme aimed at the characterisation of the mentioned conditions and constraints.

Therefore, our inquiry is virtually a foundational study on the methodology of the demarcation problem –an attempt to bring a methodological prudence, proposing a framework that will claim only a contribution to the basis of a programmatic elaboration.

### 1.1 PRELIMINARY REMARKS AND MAJOR SENSITIVITIES

By an overall assessment, we can agreeably identify three pillars of logic:

(1) *Formality*,

(2) *Necessity*,

(3) *Normativity*.

Our general conception of logic indoctrinates that the constructs of logic (*e.g.*, logical system,[2] logical form, logical consequence) must jointly satisfy them. Apparently, the views on logic converge about "what"; nonetheless, they diverge about "how". At any rate, these are the fundamental values whose simultaneous accomplishment logical constants should serve. Therefore, an account of logical constants should be capable of explaining the grounds of how they serve it. We regard formality, necessity and normativity as complementary notions. From our perspective, formality has the explanatory primacy. Thus, we shall account for the other two notions via formality.

Formality is presented in the syntax. Therefore, our starting point can be to get an idea of syntactic expression of logical constants. We shall refer to [Englebretsen 1989] which gives an illuminating overview of approaches to logical syntax. Englebretsen stresses that a primary task for any logical system is to demarcate those elements that determine the logical form of a sentence from those that do not. Having situated the task in the logical-

[2] We take a logical system $\mathcal{S}$ as an ordered pair $\langle \mathcal{L}, \vdash_{\mathcal{L}} \rangle$: a formal language $\mathcal{L}$ and a deductive apparatus $\vdash_{\mathcal{L}}$ defined in $\mathcal{L}$. In case that there is no formal language as in the Scholastic logic, the distinction between natural and formal language may be regarded as collapsed.

syntactic formation of sentences, he prefers the term 'formative' for logical constant. Thus, he formulates the investigative question: "What makes the terms of a sentence combine to form a single logical unit?"[3] As a response to this question, he looks into three theories of logical syntax: Aristotelian, Leibnizian and Fregean.

The Aristotelian theory is construed as a term functor logic. On this construal, every sentence consists of two terms and a term functor. The terms, *i.e.*, material parts of the sentence, can be a syntactically simple or complex expression. A term functor is a binary formative expression to combine the pairs of terms into sentences. Sentences exhibit a specific syntactic order: The terms are placed at each end (*terminus*) of the sentence and the formative element stands in between the terms connecting them.[4] For example, the sentence 'Every man is mortal' is paraphrased as 'Mortal belongs to every man' and 'Some log is white' as 'White belongs to some log' so that the terms occur at each end of the sentence. Englebretsen indicates three kinds of primitive logical functors:

(1) Unary functor is a negator –both terms and sentences can be negated (a copula is a logical constant when it is used to indicate identity; Englebretsen regards it as derivable from the primitive functors).

(2) Symmetric and associative binary term functor: such as 'belongs to some', 'and', 'both … and'.

(3) Transitive and reflexive binary term functor: such as 'belongs to every', 'only if', 'if … then'.

A logical functor is either a grammatical (undefined) functor that is a member of the mentioned three sets, or an expression that can be defined in terms of logical functors. For example, 'only', 'no', 'or', 'either … or', 'unless', 'is (identical to)' are logical functors definable in terms of grammatical functors. Hence, the Aristotelian logical constants are the logical functors.

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[3] *ibid.*, p. 382.

[4] Originally, the terms are subject and the predicate of a proposition; 'terminus' is the Latin translation of the Greek word 'horos' for limit, introduced by Aristotle, possibly to draw attention to the diagrammatic resemblance of the terms of a syllogism to those of a proportion (extremes and means).

The Leibnizian theory, which basically belongs to the Scholastic tradition of logic, construes a sentence as a concatenation of a subject and a predicate, which stand respectively as quantified terms and as qualified terms of a sentence. While subjects and predicates are syntactically simple or complex terms, a sentence itself can be a term as well. Each term consists of a formative (syncategorematic) expression and a material (categorematic) expression. The categorematic expression can occur in another sentence either as a subject-term or as a predicate-term, so subject and predicate are distinguished by the formative expression attached to them. A subject and a predicate juxtapose and yield a sentence by their natural syntactic consonance with one another. There is no essential hierarchy of complexity in a Leibnizian theory. Three kinds of logical constants (formatives) are specified in this theory:

(1) Those that apply to terms to yield new terms, such as negation.

(2) Those that apply to terms to yield subjects, such as quantifiers.

(3) Those that apply to terms to yield predicates, such as qualifiers. Qualifiers can be either affirmative ('is', 'are', etc.) or negative ('isn't', 'aren't', etc.).

Thus, Leibnizian logical constants can be thought as positive/negative signs of opposition in pairs. In this respect, they avail an algebraic representation, such that sentences are composed of expressions being added or subtracted from one another. For example, the sentence 'Some man is wise' is paraphrased as 'Some (+man) + (+wise)' and the sentence 'Every logician is unreasonable' as 'Every (+logician) + (–reasonable)'. Any expression that can be treated as a member of a positive/negative oppositional pair, or that can be defined in terms of such expressions (*e.g.*, 'none', 'only', etc.) can function as a logical constant.

In contrast to the subject-predicate model of Aristotle and Leibniz, Frege brings about function-argument model. The fundamental unit of meaningfulness in the Fregean theory is sentence, the terms acquire meaning in the context of sentence of which they are constituents. The base syntactic form of a sentence is an unsaturated expression in the sense that it contains componential holes to be filled by other expressions. Frege represents this syntactic form by a function, the holes being the argument places of the

function. The expressions that saturate the sentence are represented either by saturated arguments or by other saturated functions. Thus, the Fregean syntax has two fundamental categories of expressions, those that are incomplete and those that are completed by other complete expressions. A complete expression is of higher syntactical order than its completing (saturating) components, yielding a syntactic hierarchy of expressions. At the base, there are atomic (syntactically primitive) expressions. The semantic difference between the complete and incomplete expressions is that while the former refer to objects, the latter remains related to concepts. For example, the sentence 'Aristotle is a biologist' consists of two expressions: a predicative function '… is a biologist' and a name 'Aristotle' standing as an argument of the function.

Englebretsen favours the term functor representation of the Aristotelian system with respect to the other two in giving a rational account of logical constants, since the others do not incorporate an explicit formative element. Indeed, speaking particularly for the Fregean syntax, we observe that there is a homogeneity of functions and their arguments with the proviso that any function can be an argument of another function. Even if we suppose that the category of functions counts as logical constant, it is not determinate at all which functions are logical constants and which are variables, and furthermore, it is not what is distinctively logical about those functions in regards the notion of functions in general.

The preceding discussion reveals two points. Firstly, a specification of logical form requires a comprehensive view of language, as Hodges puts down clearly:

> The moral is that you can't hope to assess the validity of reasoning without some appreciation of the workings of the language in which the reasoning takes place. This is one reason why semantics is an essential tool for logicians.[5]

We want to distinguish those elements of an argument text that have pure inferential import; hence, the elements that transmit the warrant of the premisses to the conclusion to incorporate them into the logical form. An appealing idea is to match grammatical categories to logical categories. But as Harman warns, this idea stems from a superficial impression of the correlation of grammatical and logical categories:

[5] [Hodges 2007], p. 48.

> Logical categories are related to grammatical categories, but the relation is indirect and grammar makes many distinctions that are irrelevant to logic. For example, logic sees predicates where grammar distinguishes nouns, verbs, adjectives, and prepositions.[6]

Secondly, a logical syntax might contain no symbols for logical constants, and instead, rest on methods of concatenation and transformation of symbols. Insofar as the talk is about a logical system, this syntactic feature should not obliterate the fact there are logical constants. Suppose, from the symbols *A* and *B*, we get *AB*. If this concatenation is equivalent for '*A* and *B*', then, as we shall discuss, a coordination with truth evaluation brings out that there is an implicated logical constant. The same logical form can be expressed in different syntactic arrangements (*cf.* the standard infix notation and the Polish notation).

A word of caution is needed about the list of logical constants. The standard connectives and quantifiers, *viz.*, conjunction, disjunction, implication, biconditional, negation (designated by ∧, ∨, →, ↔, ¬, respectively), universal and existential quantifiers (designated by ∀, ∃, respectively) are on everyone's list. There is a strong tendency to be resisted which regards them as exhaustive of logical constanthood. They seem so natural that they vigorously hinder the thought of alternatives, and additional items into the list seem unnatural. An analogy can be drawn between logical constants and the number system bases. We are so deeply internalised the decimal base that we do not think an alternative to it, even though it may be more suitable to work with binary or hexadecimal numbers in some cases, or, it may be convincingly explained that, since it could to facilitate mathematical operations, the best choice for a commonly employed base would be a prime number instead of the number 10. Yet, such considerations do not interfere with the mathematical fact that any positive integer can be defined legitimately as a base, and the ground of justification for the decimal base is a plain extra-mathematical phenomenon attested by Ifrah:

> The almost universal adoption of the base 10 was undoubtedly caused by the fact that we happen to have ten fingers, since people first learned to count on

[6] [Harman 1979], p. 42. Harman offers an interesting argument claiming that *modus ponens* is a principle, but in fact, not of logic on the ground that 'if' is an operator that transforms a sentence into a name of a proposition, as opposed to the common view that it is a propositional operator.

> their fingers. If we had six fingers on each hand, our numeration would be duodecimal, that is, its base would be 12.[7]

In fact, we have already a growing list of logical constants. The modal operators of necessity and possibility (designated by □ and ◇) have been intensively studied. The unary epistemic operator $K_c$ (derived from the phrase 'c knows that …') and the doxastic operator $B_c$ (derived from the phrase 'c believes that …'), deontic operators O, P, F (derived from the phrases 'it is obligatory that …', 'it is permitted that …', 'it is forbidden that …', respectively), temporal operators G, F, H, P ('it will always be the case that …', 'it will be the case that …', 'it has always been the case that …', 'it was the case that …', respectively) are other prominent ones.[8] Beside identity and diversity ('=' and '≠'), Church's functional abstraction operator ('λ'), Hilbert's indefinite choice operator ('ϵ') and Russell's definite description operator ('ι') may be treated as logical constants.

At this point, a question suggests itself as to whether any formalisation like the above mentioned operators could be counted as genuine logical constants. We shall take up this question in connection with logical pluralism.

A verification can be done formally or informally. We can define metalogical criteria and apply them to a candidate term to see whether it complies with them or not. Alternatively, we can define informal criteria based on conceptual considerations and develop a judgement according to them. Since the notion of logical constanthood demands formal treatment, but involves a rich informal content with repercussions into a diversity of philosophical issues, we can foresee a analogical course of progress like that of the Church-Turing thesis.

The Church-Turing thesis is concerned with the notion of an effective method in the mathematical sense (involving logic and theory of computation). We may spell out the term 'effective' by setting a set of criteria Θ to be satisfied by a method M, such that:

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[7] [Ifrah 1987], p. 37.

[8] See [Gabbay and Guenthner series 2001–2010] for an overview of the variety of logical systems.

(1) M consists of a finite number of definite instructions expressed by a finite number of symbols.

(2) M produces the result in a finite number of steps.

(3) M must be such that it be executable, in principle, without any external support by human faculties.

(4) M must not require anything but operational capabilities for the human executor.

A typical method that satisfies Θ is the truth-table test for any given formula of the propositional calculus in order to determine whether the formula is a tautology or not. The method can be construed as a function that maps any formula $\varphi$ from the set of formulas of the propositional calculus to the value of 1 in case that $\varphi$ is a tautology, or else, to 0. In general, it is convenient to regard an effective method as a mathematical function. So, it can be stated for a function $f$:

(A1) $f$ is said to be effectively calculable if it accords with the set of criteria Θ.

Studies in the domain of the set Θ have conduced to devising a formal predicate that could be supplied in place of the intuitive predicate 'effectively calculable'. One construal of the formal predicate derives from Turing's model of computational machine (independent of formalism), the other one from Church's notion of λ-function. Both formal predicates due to Turing and Church are intensionally different from one another, but they define the identical class of mathematical functions. This fact is put down in what we call Church-Turing thesis, of which one version is:

(A2) $f$ is effectively calculable if and only if $f$ is Turing-computable.

The thesis asserts that the set of criteria consists of Turing-computable functions for which values can be obtained by a definite method. Kleene *et al.*, along with Turing and Church, have proposed another formal definition of a type of function whose values could be calculated by recursion. Eventually, these three methodologies (*i.e.*, based on the concepts of Turing machine, λ-function and recursively-definable function) have been proved to be extensionally equivalent in that the class of functions they define are identical to each

another. There is a prevalent presumption that the key property, effectiveness of a method (calculation, procedure, etc.), has been agreeably characterised by the thesis, as depicted by Turing back in 1948 (L.C.M. for 'logical computing machine'):

> It is found in practice that L.C.M.s can do anything that could be described as 'rule of thumb' or 'purely mechanical'. This is sufficiently well established that it is now agreed amongst logicians that 'calculable by means of an L.C.M.' is the correct accurate rendering of such phrases. There are several mathematically equivalent but superficially very different renderings.[9]

We see that the Church-Turing thesis can be considered to be composed of two clauses A1 and A2, between which a consonance should be maintained. A remarkable point is that the effectiveness property remains an informal notion, evading any further explication in formal terms, particularly, of the criterion (4) (so, a somewhat pedantic name would be 'hypothesis', rather than 'thesis'). Analogously, we may attempt to capture accurately logical constanthood for a term ξ in two similar clauses:

(B1) ξ is said to be a logical constant if it accords with the set of criteria Ξ.

(B2) ξ is a logical constant if and only if it is a formalism that satisfies Ξ.

Therefore, we can restate the demarcation problem as the challenge to make the set Ξ definite, and to formalise it properly. Looking at the matter this way uncovers that *the resultant criteria should be formalisable*, which is another point of sensitivity for our inquiry. The clauses B1 and B2 can be thought of having corresponding stages of a study. Though B1 stands as the initiating step and the criteria should be made definite to a certain extent, the efficient method is probably to have them concur with interaction. We shall focus on B1, which is clearly inchoate and philosophically more involving. In accord with these points noted, we can talk about the methodology. Before we get into that, it is worth registering several complementary notices in order to dispel confusions.

By the term 'language', our intention will be natural language augmented with other discourses such as mathematics. Thus, language is multisemiotic and, to a reasonable extent, fragmentary. We shall use the term 'proposition' as distinguished from 'sentence' when we prefer giving dominance to the truth-bearing feature of declarative sentences,

[9] [Turing 2004], p. 414.

abstracting them from the idiosyncratic features of a particular language and assuming them to be properly contextualised.[10] An ontological status independent from linguistic apprehension is not attributed to propositions.

We shall draw on the ideas and notions developed by various authors. It should be remarked that the course of an author's thought has usually followed quite differing and sometimes conflicting stages. For example, Wittgenstein's own thought had passed from an environment in which "logic must take care of itself"[11] to an environment in which "language must speak for itself".[12] While the present work is not an exegetical study of any one of them, and does not have a claim to commentary orthodoxy about those authors, we shall mostly comply with the terminological consensus on the authors' specific terms. In particular, we shall follow [Glock 1996] for the Wittgensteinian terms such as 'form of life', 'mastery of a technique'. The term 'grammar' can be an exception in that we use the term accommodating the ordinary linguistic sense in addition to the Wittgensteinian sense.

## 1.2 RIGOUR IN THE METHODOLOGY

We have stated a course of inquiry from the informal to the formal. A method oriented to this direction has already been discussed and made precise by Kreisel in [Kreisel 1967], which is later dubbed as "squeezing argument".[13]

He starts with a critique of what he calls pragmatist and positivistic philosophies of mathematics, which neglect the intuitive core with the presumption that it is quite otiose for formal studies –a neglect that has counterparts of these views in the context of logical

[10] Strictly speaking, as John L. Austin argues, not all grammatically declarative sentences are disposed to truth-conditional treatment. Such sentences as 'I apologise' and 'I bet on this' are intended to be a part of an action or an action itself. Austin makes a contrast between these sentences, the "performatives", and those that assert propositions, the "constantives". Other taxonomies of speech acts also have been proposed; see [Pagin 2012].

[11] [Wittgenstein 2001], 5.473.

[12] [Wittgenstein 1978], §27.

[13] We shall follow the clearer scheme given in [Smith 2011].

constanthood. He holds that, though rigour is mostly associated with formal methods, it is not confined to them and that rigour can be accomplished through informal methods as well. According to him, informal rigour is complementary to formal rigour to the extent that a proof can be established by means of informal and rigorous arguments. He specifies the goals of informal rigour as:

> Informal rigour wants (i) to make this analysis as precise as possible (with the means available), in particular to eliminate doubtful properties of the intuitive notions when drawing conclusions about them; and (ii) to extend this analysis, in particular not to leave undecided questions which can be decided by full use of evident properties of the intuitive notions.[14]

There is an idea shared by Kreisel and Wittgenstein: In many inconsistency cases, the fault is not in our intuitive notions, but in our improper analyses; they will dissolve when we analyse them properly. As an example of Kreisel's notion of informal rigour, we outline his argument on the extensional equivalence of the informal notion of logical validity and the model-theoretic validity for first-order languages.

Kreisel defines a primitive predicate *Val* to cover informal validity which is construed as truth in all interpretations of the language (notice that the intended notion is *not* validity *simpliciter*, which is pre-theoretically related to *any* inference). Its formal translation is the predicate *val*. Model-theoretic and proof-theoretic notions make *Val* more precise and informative, however, that should not imply that *Val* is not a well-founded notion. Kreisel's assertion is that

$$Val = val = D$$

where $D$ is the set of all sentences in a given first-order language that are theorems (provable) in (a given system of) first-order predicate logic (with identity), *Val* is the set of informally valid sentences and *val* is the set of all sentences that are model-theoretically valid. The argument runs as follows:

(1) $Val \subseteq val$, since if α is a true set-theoretic sentence, then α is logically valid (in the informal sense). *Val* signifies a sufficient condition for *val*.

[14] [Kreisel 1967], p. 138 *ff.*

(2) $val \subseteq D$ by Gödel's completeness theorem for first-order logic. $D$ signifies a necessary condition for *val*.

(3) Thus, we have $Val \subseteq val \subseteq D$, *i.e.*, the indeterminate extension of *val* is bounded by the determinate extensions of *Val* and $D$.

Also, we have another connection between *Val* and $D$, which is a proved implication, since both are formal concepts:

(4) $D \subseteq Val$ by the intuitive soundness of first-order logic.

Therefore, by (3) and (4), we establish the extensional equivalence, $Val = val = D$.

As Kreisel shows in the instance of *val*, can we make a similar case for the predicate of logical constanthood to get the exact criteria for the extension of logical constants? Such an attempt demands from us to improve informal notion of logical constanthood as well as formal conditions and constraints. We maintain that this is the proper method to track down. The informal notion is the root of a theory of logical constants. The adequacy of the formal notion is assessed against that basis. This is a relation between the informal and the formal which reminds us of Cohen saying about the course of proving independence results:

> That's primary, but it's somewhat curious that in a certain sense the continuum hypothesis and the axiom of choice are not really difficult problems – they don't involve technical complexity; nevertheless, at the time they were considered difficult. One might say in a humorous way that the attitude toward my proof was as follows. When it was first presented, some people thought it was wrong. Then it was thought to be extremely complicated. Then it was thought to be easy. But of course it is easy in the sense that there is a clear philosophical idea. There were technical points, you know, which bothered me, but *basically it was not really an enormously involved combinatorial problem; it was a philosophical idea*.[15]

So, our objective is getting clearer: to develop a conception of logical constant –the "philosophical idea" in Cohen's words– and to specify formalisable criteria to get a co-extensiveness result.

[15] [Albers *et al*. 1994], p. 53 (emphasis added).

## 1.3 A DIVISION OF PROPOSALS

It has been recognised since the era of the Stoic logic that some grammatical particles of speech should be held constant in order to evaluate arguments. There have been a wide diversity of proposals to account for logical constants and logicality –not all of them specifically directed to the demarcation problem, but dealing with it as one of the key issues in their own contexts. [MacFarlane 2009] is an excellent survey of those proposals; there is no point to duplicate it here. For us, the important issue about the proposals is that a fundamental choice needs to be made in general for the approach to be endorsed.

Some proposals consist of heuristic advices for theory virtues (like general applicability, elegance). For example, topic-neutrality of logical constants is an intuitively appealing criterion, on the presumption that logic has to be employable in any argument of whatever the subject-matter of the argument is. To be formal entails to be topic-neutral in a certain way. If we should understand something different than formality, and being topic-neutral is regarded as being "being about anything particular", then the criterion remains virtually aphoristic. The plain fact is that each logical system has a viewpoint and addresses only some kinds of arguments and leaves out others, so none of them can be said to be distinctively topic-neutral. We shall not take them into the discussion, they are more of guidelines to lay down a logical system than a theory about logical constants.

We observe that there are two main approaches that prevail in the proposals; accordingly, we can divide them into two classes, which we may call *characterisation-oriented proposals* and *adequacy-based proposals*. The former class is engaged in finding a characterisation starting with the present received conception of logical constanthood to determine the extension of logical constants. The latter class of proposals try to set out the adequacy conditions for the ascription of logical constanthood to an expression. It should be remarked that these classes should not be sharply taken mutually exclusive, because many of them connect the demarcation problem to a wide range of philosophical issues that blurs the division.

A common presupposition of characterisation-oriented proposals is that the set of standard connectives and quantifiers provides a uniquely firm ground to address the

issues concerning logicality. The method usually takes the shape of formulating a characteristic predicate of logical constanthood from a set of necessary and sufficient conditions spelled out in reference to this set of terms. Then, the objective turns out to make a definitive list of logical constants, much like in the fashion of Eratosthenes' sieve that sifts out the prime numbers from the set of natural numbers. The view that identifies logical constants with syncategorematic words and the view that distinguishes logical constants by algebraic permutation invariance are the prominent examples of this class of proposals.

We agree on that the standard connectives and quantifiers represent our base of logicality and regard them to be the elementary list by which any account of logical constants should be verified. But the question is on the basis of just what features they are conferred logical constanthood. In the light of the progress made afterwards, there is a lesson to be learned from Kant's following passage:

> Since Aristotle's time Logic has not gained much in extent, as indeed its nature forbids it should. But it may gain in accuracy, definiteness, and distinctness. There are but few sciences that can come into a permanent state, which admits of no further alteration. To these belong Logic and Metaphysics. Aristotle has omitted no essential point of the understanding; we have only become more accurate, methodical, and orderly.[16]

Even if it might seem self-evident that a contemporary system captures all there should be to logicality, a proper conception of logicality could clarify that logic is a field of substantive progress. In this respect, the characterisation-oriented views are predisposed to go astray in a manner of *post hoc, ergo propter hoc* fallacy.

The adequacy-based proposals seeks after a conception of logicality in a general philosophical setting and spell out under what conditions and constraints a term can be said to belong to logical vocabulary. The elementary logical constants serve as a test case rather than an abstraction base. The inferentialist views are examples for such an approach. We hold that this is the right approach, though we find important features about logical constanthood to learn from the characterisation-oriented works. So, we share Sher's opinion that "a characterisation of logical constants is possible"; however, reversing

[16] [Kant 1999], p. 10 *f.*

her formulation as “a mathematically precise and philosophically informative characterisation” to “a philosophically precise and mathematically informative characterisation”.

# CHAPTER 2

# AN OVERVIEW OF THE BACKGROUND

Having talked about the horizon of our objectives and methodological expectations, we shall continue with explaining against which background of proposals we proceed with our inquiry. It would not be a purposeful effort to examine comprehensively proposals with all details herein. This exposition is not because of a historical interest, nor particularly, a matter of eliminating choices –since our attempt is to present an internally coherent view providing a unified response to the demarcation problem. But going through the proposals is part of defining our viewpoint, indicating what lessons we draw from the other lines of thought.

In this chapter, we shall take into consideration the characterisation-oriented proposals that bear significance to our inquiry.

## 2.1. SYNCATEGOREMATA AS LOGICAL CONSTANTS

Scholastic philosophers divided the terms of an argument text into two basic classes as categoremata, which supply the content to the argument, and syncategoremata connecting the former into a definite form. Thus, on this view, logical constants are characterised as syncategorematic terms. Their methodology grew into a cohesive environment of investigation of with the relationship of language and logic, much in the manner of a Kuhnian paradigm.

In line with the Aristotelian tradition, the scholastic logic presumes that the primary units of logical discourse are propositions, which make up assertions and are capable of bearing a truth-value. A proposition is composed of terms, each of which has an inferential role of its own, despite that they cannot bear a truth-value by itself (hence, “term logic” in contrast to “predicate logic”). Then, an argument text is a sequence of propositions given an order in schemes constituted by syncategoremata and categoremata. Syncategorematic terms stand as derivative items with respect to categorematic terms. This point presents a

remarkable contrast to the case in logic, since syncategorematic terms are primitive items in the vocabulary of logic.

This view of logical form incorporates the theories of signification and supposition. The signification theory is concerned with the linkage of a particular sign (the term) and the object signified by it. Basically, signification is conceived as an assignment process of terms to objects at the ground level of language in which words have literal meanings. The supposition theory deals with what a significative term can be taken to stand for with respect to a particular sentential context; as such, it is actually a theory of reference. From another aspect, it can be said that signification is a precondition of the discourse relatively independent of language, whereas supposition is a linguistic act within the discourse.

The distinction between categorematic and syncategorematic terms sets it out clearly in the following passage from Albert of Saxony:

> A categorematic term is a term which, taken in its significative function, can be subject or predicate, or part of the subject or part of the distributed predicate, in a categorematical proposition. 'Man', 'animal', 'stone', for instance, are such terms. They are called categorematic terms because they have a restricted and fixed signification. A syncategorematic term, on the other hand, is a term which, when taken in its significative function, cannot be subject or predicate, or even part of the subject or part of the distributed predicate, in a categorematical proposition. Such are, for instance, the following terms: 'Every', 'none', 'some' etc., which are called signs either of universality or of particularity. So, too, the negations, as, for instance, the negation 'not', the conjunctions, as 'and', the disjunctions, as 'or', and the exclusive and exceptive prepositions, as, for instance, 'except', 'only', and the like; all these are also syncategorematic terms.
>
> To further exemplify syncategorematic terms, let us consider the following proposition: "Every man is running". 'Man' is the subject. 'Every' is neither subject nor predicate, nor is it part of either subject or predicate. Rather, it is a modification of the subject and signifies the manner of supposition in the subject itself. If 'every' were part of the subject itself, then the following propositions would not have the same subject: "Every man is running", and, "Some man is not running". Consequently, these propositions would not be contradictory, which is a gross falsity.
>
> In defining a syncategorematic term, we have designedly inserted the phrase "taken in its significative function" as applying to these terms, for if such terms as 'every', 'none', etc., are taken materially, they do function as subject or predicates of propositions. For instance, consider these propositions: 'Every' is

> a sign of universality; 'And' is a copulative conjunction; 'No' is an adverb. In these propositions the aforementioned expressions or terms are not taken in their significative function since they do not act in the capacity for which they were instituted. Thus, in the proposition, 'Every' is a sign of universality, 'Every' is no more a distributive term than 'no' is a negation in the proposition, 'No' is an adverb.[17]

This viewpoint is indeed very appealing with the almost perfect match of surface structures and has been still considered basic, even if it is in need of refinement, as we can observe in the following passage from [Gamut 1991]:

> One interesting question is this. What kinds of expressions can be treated as logical constants in a logical system? An important fact that may help us is that in logic we are interested in the structure of arguments, i.e., in argument schemata. Arguments must be valid only in virtue of their external forms and not in virtue of their content. So an expression must lend structural validity to argument schemata if it is to be treated as a logical constant. Purely descriptive terms such as *mammal*, *party*, or *airplane* are ruled out by this criterion. And expressions like *and*, *or*, *if* (… *then*), *if and only if*, the negation *not*, and the quantifying expressions *all* and *some* are clear examples of constructions which can lend structural validity to argument forms. That is indeed their only function in language. Their meaning is entirely determined by the part they play in argument, since they have no descriptive content. So the conjunctions *and*, *or*, *if* (… *then*), *if and only if*, and the negation *not* are taken as the logical constants of propositional logic; and these together with the quantifying expressions *all* and *some* form the logical constants of predicate logic.[18]

Albert of Saxony makes also a complementary and viable distinction, that of between formal and material consequences:

> A formal consequence is that which holds good for every proposition of similar form. For instance: What is B is A; therefore, what is A is B. A material consequence, however, is one which does not hold good for every proposition of similar form; or as it is commonly expressed, where the very same form is retained, such propositions are not equally valid for all terms. For instance: Man is running; therefore, an animal is running. With the following terms, however, the consequence does not hold: Man is running; therefore, wood is running. We speak here of matter and form in the sense that we understand the matter of a proposition or of a consequence to be purely categorematic

[17] Quoted from [16], p. 22 *f.*

[18] [Gamut 1991], p. 7 (emphases in the original). This is a collective pseudonym for Johan van Benthem, Jeroen Groenendijk, Dick de Jongh, Martin Stokhof and Henk Verkuyl.

> terms, that is, the subjects and predicates without the addition of the syncategorematic terms by which the former are joined or disjoined or determined to a certain mode of supposition. The rest belongs to the form. Hence, the copula of categorical and of hypothetical propositions is said to belong to the form. So, too, the negations as well as the signs (of quantification), the order of the aforementioned to each other, and the modes of signification concerning the quantity of a proposition, such as singularity or universality etc. (all these belong to the form). For instance, modal propositions are said to be a form other than that of the propositions of fact because the copula of modal propositions differs from the copula in the proposition of fact (de inesse). Because of negations and of signs (of quantification), affirmative propositions are said to be of a form different than that of a negative proposition. Likewise, the universal propositions are said to be of a form other than that of particular propositions. Because of universality, on the one hand, and distribution, discreteness (singularity) on the other, singular propositions are said to be of a form other than that of indefinite propositions. Due to the difference in order, the following propositions are of different forms: Every man is an animal, and: (An) animal is every man. The same is true of the following consequence: Every B is A; therefore, every A is B; and: Every B is A; therefore, some B is A. Furthermore, because of the relation (of a relative term) the following propositions have a different form: Man is running, and, man is not running; and: Man is running and the same is not running; for, the second is impossible because of its form while the first is not impossible.[19]

Thus, we have two kinds of arguments stated orderly by propositions. In one kind, an argument includes syncategorematic terms (*i.e.*, formal elements) and by virtue of these terms which inscribe a valid argument form, we are led to the conclusion. In the other kind, even if the argument includes syncategorematic terms, they are not capable to inscribe a valid argument form and the conclusion is reached actually by virtue of categorematic terms (*i.e.*, the material elements). It is suggested that a predication is formal if the relation of subject to a predicate is necessary; it is material if the entailment is contingent.

Categorematic and syncategorematic terms are distinguished by their difference in signification. While categorematic terms have discrete, determinate and self-contained significates and are capable of being subject or predicate of a proposition, syncategorematic terms can only co-signify with the categorematic terms or propositions. Therefore, syncategorematic terms are functional only in the presence of categorematic

[19] *ibid.*, pp. 25 *ff.* We sustain and endorse this distinction in the scope of the dissertation.

terms. Notice that the dependence involves signification and supposition, not particular meanings.

In accordance with the specifications given up to the present point, one might attempt to make a list of syncategorematic terms by undertaking a sort of corpus linguistics, browsing standard dictionaries and argument texts. A typical list would include such words as 'if', 'then', 'only', 'every', 'necessarily', 'or', 'and', 'not', 'some', 'except', which are incapable of being used for the subject or predicate of a proposition.

Syncategoremata of the scholastic logic could be classified under such topics as distribution, negation, exclusion, exception, predication, modality, conditionality, conjunction, disjunction, comparison and reduplication. So, the question suggests itself: How can we demarcate syncategoremata in the argument texts?

First, we should overcome a variety of grammatical complications (variation in phrasing, unexpressed quantifiers, etc., which are sometimes collected as the surface grammar) apart from lexical ambiguities. We take the following sentence as a simple example:

(1) Milk will be distributed to *anyone* in the room.

Modifying the underlying phrase structure, we get another reading:

(2) Milk will be distributed to *anyone in the room*.

Each version of the sentence corresponds to another logical form. Its converse is also true; different sentences can produce the same logical form:

(1) The only emeralds that are observed before time *t* are grue.

(2) All emeralds that are observed before time *t* are grue.

Such examples can be multiplied, making a definitive list syncategoremata be an elusive task. Instead, we may devise tests for the syntactic and semantic aspects to achieve a precise discrimination. We may formulate them as follows:

(a) Syntactic test: If a term can function as a subject or a predicate of a proposition, then it is categorematic, otherwise it is syncategorematic.

(b) Semantic test: If a term can singly signify something, then it is categorematic, otherwise it is syncategorematic.

The tests seem sound, however, the use of an expression may differ in a variety of cases. A conspicuous case is the use/mention distinction that may be marked by single quotes. For example, consider the word 'only' in the following sentence:

'Only' has adjectival, adverbial and conjunctive uses.

Similarly, the naming device can operate on sentences, as in:

'It is Aristotle who developed the syllogism' is true.

Search for a clear articulation of the distinction has driven the paradigm to the iteration of the method by introducing such concepts as "pure categoremata" and "pure syncategoremata", only for the deference of the difficulties. During the scholastic era, a lot of effort was made to overcome the difficulties to concord the grammatical categories and semantic contexts with the structural features of logic, trying to make two mutually exclusive lists in vain, as evinced by the volume of sophismata literature of the scholastic logic, which dealt with the resolution of logical difficulties analysing concrete instances.[20]

An objection raised against the syncategorematic approach is that it is confined to the system of term logic, which employs the subject-predicate model for the basic sentential structure (*viz*., categorical propositions) and has fallen to disuse in general. Thus, reapplying the syncategorematic idea in the different systems faces serious problems, as discussed by MacFarlane (see [MacFarlane 2009]) in the case of predicate logic which employs the function-argument model. To see the point of MacFarlane's critique, consider the sentence

(a) All protozoa are heterotrophic.

[20] In the sophismata, words take on various roles; they present different features when inspected in different contexts.

We designate the expressions 'protozoa' and 'heterotrophic' by $w$ and $x$, respectively and define a subject-functor $\varphi(.)$ and a predicate-functor $\psi(.)$. Then, we can formulate (a) as:

(b) $Q_1[\varphi(w)]\ Q_2[\psi(x)]$

where $\varphi(w)$ is the subject, $\psi(x)$ is the predicate, and $Q_1$, $Q_2$ are the quantitative and the copulative expressions, respectively. On the other side, predicate logic paraphrases (a) in one variable y:

(c) For all $y$, if $y$ is a protozoon, then it is heterotrophic.

A Fregean counterpart for (b) is a second-level function

(d) $\forall y(f(y) \rightarrow g(y))$

with first-level functions $f(y)$ designating '$y$ is a protozoon' and $g(y)$ designating '$y$ is heterotrophic'. The functional abstraction and composition enable us to go further by recursion:

(e) $y$ is heterotrophic $z$.

Thus, we have a series of functions being the argument of other functions except for the first-level functions of which arguments are objects. Then if we seek corresponding expressions for syncategoremata, the only candidates we would find are the functions – setting aside auxiliary symbols such as parentheses.

If any function is counted as syncategorematic, then the syncategorematic expressions will include such ordinary predicates as in (e) beside the standard constants. Else, if only first-level functions are specified as categoremata and second-level functions as syncategoremata, we have to find a way not to admit every second-level function logically privileged. As an illustration, we may give MacFarlane's example with minor modification to clarify the point:

> [Every $(f(x))$ is such that $g(x)$] is $h(x)$.

where the component of the sentence within the square brackets constitutes a grammatical subject to quantify over, rather than a logical composition. MacFarlane points out that the converse is a weak assertion as well. He remarks that not every first-level function is nonlogical; for example, the identity and diversity relations are usually treated as logical.

The objection has a *prima facie* appropriateness. But if we think that any logical system may have its own view of language and the standard predicate logic is not an exception, then we see that what is not general is the sense of formality. It can be claimed that there is an instability in the perception of form, which can be illustrated on an exemplary argument from Lukasiewicz:

> If all broad-leaved plants are deciduous
>
> and all vines are broad-leaved plants,
>
> then all vines are deciduous.[21]

When this argument text can be paraphrased parsing and replacing the categoremata with the metavariables *A*, *B*, *C* and an invalid form results:

> If *A* is predicated of all *B*
>
> and *B* is predicated of all *C*,
>
> then *A* is predicated of all *C*.

The scholastic conception of logical constanthood is based on grammatical categories of natural language. The justification is that the logical form is inscribed in the grammatical structure of the arguments stated in natural language, and an isomorphism between the categories of linguistic grammar and logical categories is sound and can be read off the arguments in natural language. Thus, the meanings of the logical constants have been already determined. This viewpoint does not offer prospect as Wittgenstein remarks:

> Man possesses the ability to construct languages capable of expressing every sense, without having any idea how each word has meaning or what its

[21] [Lukasiewicz 1963], p. 7.

> meaning is–just as people speak without knowing how the individual sounds are produced.
>
> Everyday language is a part of the human organism and is no less complicated than it. It is not humanly possible to gather immediately from it what the logic of language is.
>
> Language disguises thought. So much so, that from the outward form of the clothing it is impossible to infer the form of the thought beneath it, because the outward form of the clothing is not designed to reveal the form of the body, but for entirely different purposes.
>
> The tacit conventions on which the understanding of everyday language depends are enormously complicated.[22]

A correlation between natural language syntax and semantics appears to be too arbitrary in order to draw reliable conclusions about logical form and this arbitrariness invokes a naturalisation circularity: What is said is in the logical form and *vice versa*. This is in fact one of the motivations to leave the natural language as the object language of logic and set off for a formal language.

## 2.2 PERMUTATION INVARIANCE VIEW

The permutation invariance view is one of the main approaches to the demarcation problem of logical constants. This view holds that the insensitivity of logical constants to the particular identities of objects is the basis of logical constanthood, and attempts to characterise logical constants by their invariance under possible permutations of the objects in the domain they operate. It is claimed that the permutation-invariance property of logical constants is a rigorous realisation of the presumed requirement of topic-neutrality for logic.

In the following, we shall give a conceptual exposition of the view, and then argue that the ground on which it stands does not suffice to justify the approach to the problem, but on the other side, it offers prospects as its by-product to uncover various relations, primarily, for the theory of mathematical logic.

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[22] [Wittgenstein 2001], 4.002. In the passage, Wittgenstein contrasts the syntactic form of language with the form of thought. However, our interest is the strictly normative and objective realm of logic and in this respect, we preclude referring to such a realm requiring privileged access (*e.g.*, thought). For all intents and purposes, our primary object of study is the argument text.

**Automorphism as Revealing Identity.** The permutation invariance view inherits its motivation from the Erlangen Programme that Felix Klein laid out for geometry. The main idea is that each geometrical system (we may say alternatively: language of geometry) can be identified with such notions and transformations that the notions remain invariant under the algebraic groups of these transformations. Thus, each geometrical system is identified with a group of automorphisms.[23] As an illustration, consider a similarity transformation that leaves a triangle unchanged in Euclidean geometry. Hence, Euclidean geometry can characterised with similarity transformation and the basic notions that are invariant under this transformation. In the same manner, projective geometry recognises conic sections as opposed to circles and angles, since the former, not the latter are invariant under projective transformations.

In order to understand the application of Klein's idea to logic, we may go over an example. Let us consider a set of polyhedra, such as {tetrahedron, cube, octagonal prism, square antiprism as a domain of discourse and two monadic predicates in the following schemes:

(1) *x* is a polyhedron.

(2) *x* is a Platonic solid.

Substituting the term 'tetrahedron' into (1), we get true sentences:

(a) Tetrahedron is a polyhedron.

When we replace the term 'tetrahedron' with the term 'cube', we have again a true sentence:

(b) Cube is a polyhedron.

Likewise, we have true sentences replacing 'tetrahedron' with 'octagonal prism' and 'square antiprism'.

When we substitute the term 'tetrahedron' into (2), we get a true sentence:

[23] See [Klein 1892-1893] for Klein's manifesto.

(a') Tetrahedron is a Platonic solid.

Likewise, we get a true sentence for 'cube', but false sentences for 'octagonal prism' and 'square antiprism'. Thus, we may think of an "object-sensitivity" for the predicates: Being a polyhedron is not sensitive to any object in the domain, whereas being a Platonic solid exhibits a sensitivity over the domain and distinguishes the objects as to whether each one possess the particular property it designates or not. If the predicate is not object-sensitive, it retains its contribution to the truth-value of a statement to whichever term designating an object it is applied, in contradistinction to the object-sensitive predicates.

The idea of object-sensitivity can be extended to $n$-adic predicates and consider $n$-tuples of terms, $\langle x_1, \ldots, x_n \rangle$, each term $x_k$ designating an object in the domain of discourse. We may think of replacing one term with another as mapping the object it designates to another object. Hence, replacing 'tetrahedron' with 'cube' is, on this view, mapping 'tetrahedron' to 'cube' (or, one-one transformation). Thus, if $x$ is an $n$-tuple of terms, we get a mapping of $x$ as $\pi(x)$ by mapping each term as $\langle \pi(x_1), \ldots, \pi(x_n) \rangle$. Notice that such mapping is indeed a permutation of objects over the domain of discourse. The object-insensitive predicates retain invariance with respect to truth-value under this permutation. This point of view allows us to set out a mathematical expression for the idea of object-sensitivity. For this, we shall employ the notion of automorphism.

Considering that an automorphism is a homomorphic permutation, a one-one mapping of a domain of objects onto itself, preserving the structure on the domain is an automorphism. In the above example, we can regard the structures as defined by the predicates and truth-values. In the case of '… is a polyhedron', two objects $\alpha$, $\beta$ in the domain are structurally indiscernible since there is an automorphism $\rho$ on the domain such that $\beta = \rho(\alpha)$. The natural thought is that an expression should count as permutation-invariant just in case its extension on each domain of objects is invariant under all permutations of that domain. Hence, the extension of a name on a domain is the object it denotes, the extension of a monadic predicate is the set of objects in the domain to which it applies, and the extension of an $n$-adic predicate is the set of $n$ tuples of objects in the domain to which it applies. Notice that this approach does not apply to quantifiers and sentential connectives, which

do not have extensions in the usual sense, it can be translated into set-theoretical language by the following scheme due to McGee:

Given a domain of discourse and a language, the extension of a formula is the sets of assignments of values from the domain to the variables that the formula is satisfied. The extension of a connective ξ is defined as a function from sets of variable assignments of values to sets of variables assignments such that whenever formulas $\varphi_1, \ldots, \varphi_n$ are satisfied, then so is ξ ($\varphi_1, \ldots, \varphi_n$). Then, an iterative scheme of permutation of the domain is defined beginning at the level of variable assignments for formulas, proceeding through sets of variable assignments to the functions from sets of variable assignments into sets of variable assignments. For each level (*i.e.*, type), the logical notions are identified as those that remained invariant under any permutation.

Thus, automorphisms partition the domain of discourse into equivalence classes.[24] Since what we seek for by object-insensitivity is to collect all the objects of the domain into one equivalence class, we have to take into account all possible automorphisms.

This is an explication of the baseline in Tarski's proposal for the characterisation of logical constants. Indeed, its application to line dates further back than Tarski's. Beside explicit references to Erlangen Programme, it seems that there has been invariance-centric view of logical constanthood prevailing in the sphere of logic. Antonelli and May discuss in [Antonelli and May 2000] that a comparable view to Tarski's can be ascribed to Frege. As Corcoran remarks, "The history of the influence of the Erlanger Programm on the development of logic remains to be written."[25]

Clearly, the influence of Klein's idea of identification is widespread. A project that has an intimate connection Tarski's is due to Suppes. By analogy, Suppes expects that Klein?s idea could be employed to develop an extensional notion of congruence of meaning based on logical equivalence of expressions for the service of scientific and mathematical work:

[24] In an intuitive sense, Gentzen's innovative idea that proofs are structurally indiscernible if permutation of inference rules is possible and that proofs can be brought to a normal form bears an affinity to this point.

[25] [Tarski 1986], n.5, p.149.

> I characterized this view as a geometrical theory of meaning because I developed the viewpoint that different weak and strong notions of congruence are appropriate to catch different senses of 'identity' of meaning. Moreover, it seemed to me then and it still seems to me that there is much to be learned from geometry about the concept of congruence and the related concept of invariance that is applicable to the theory of meaning. We have long ago abandoned the idea of one true theory of geometry; we should do the same for meaning.[26]

Apparently, Suppes' project involves a reification of meanings into mathematical objects. Could we have objects of intended meanings, for example, of logical constants?

**Tarski's Proposal**. In connection with logical constanthood, Tarski prefers the term 'logical notions' about which he says:

> I use the term 'notion' in a rather loose and general sense, to mean, roughly speaking, objects of all possible types in some hierarchy of types like that in Principia Mathematica. Thus notions include individuals (points in the present context[27]), classes of individuals, relations of individuals, classes of classes of individuals, and so on.[28]

Underlying this terminological preference, there is a characterisation of logic implicated. Tarski conceives of logic as a field of study founded in certain notions that remain invariant under automorphisms of relevant structures, which he calls logical notions, just like geometry is founded in geometrical notions as point, line, space, etc., that remain invariant under geometrical transformations and so for other fields physics, chemistry, etc. Thus, for Tarski, demarcation of logical vocabulary from extra-logical vocabulary takes on a methodological bearing as to delineate the boundaries of epistemic fields.

Tarski starts with individuals as the most basic objects (*i.e.*, of type 0) and binary relations between them. Since any individual can be mapped to any other individual, we cannot obtain an invariant relation out of individuality, assuming that the domain has at least two individuals. Therefore, he ascends to classes of individuals (*i.e.*, objects of type 1). At this

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[26] [Suppes 1978]. For details of Suppes' view, see [Suppes 1973].

[27] *viz.*, in Klein's approach to geometry.

[28] [Tarski 1986], p. 147

level, Tarski identifies the universal relation (that holds for any two individuals) and empty relation (that holds for no individuals), identity relation and diversity relation (as the opposite of identity relation) as the only logical notions. At the level of type 2 objects (*i.e.*, classes of classes of individuals), Tarski points out that cardinalities of classes are the only properties that can be taken as logical notions. Thus, iterating the method outlined, it is suggested that we obtain the logical notions for any finite type. In regards that the permutations of values can be construed as re-interpretations, the permutation invariance view actually asserts a semantic system, albeit one that allows a quite restricted metatheory.

A precise stipulation along the same idea is offered in [Tarski and Givant 1987]. Tarski and Givant construct a series of "derivative universes" from a basic universe (of discourse) $U$ in a language equipollent to fragment of first-order language, $\bar{\mathrm{L}}^{\mathrm{x}}$, details of which are not relevant to the present context. They set out the class of n-ary relations among elements of $U$ and a chain of iterations on the class of n-ary relations among relations of elements of $U$:

> (i) Given a basic universe $U$, a member $M$ of any derivative universe $\tilde{U}$ is said to be logical, or a logical object, if it is invariant under every permutation P of $U$.
>
> (Strictly speaking, since an object $M$ can be a member of many derivative universes, we should use in (i) the phrase "*is said to be logical, or a logical object, as a member of* $\tilde{U}$.)
>
> (ii) A symbol $S$ of the formalism $\bar{\mathrm{L}}^{\mathrm{x}}$ is said to be logical, or a logical constant, if, for every given realization $U$ of this formalism with the universe $U$, $S$ denotes a logical object in some derivative universe $\tilde{U}$.

We may regard logical objects as extensions which we grasp by grasping the logical notions (*i.e.*, a definition of logical notion relative to a derivative universe), and logical constants are the corresponding expressions in the language $\mathrm{L}^{\mathrm{x}}$ (*i.e.,* an absolute definition of a logical notion). Thus, for instance, the membership relation in axiomatic set theory is not admitted as a logical constant, it does not denote a logical object in a universe.

Tarski's project is to iterate the procedure with progressively as broader groups of transformations of a domain as possible in order to obtain the set of logical constants:

> Now suppose we continue this idea, and consider the class of all one-one transformations of the space, or universe of discourse, or 'world', onto itself.

> What will be the science which deals with the notions invariant under this widest class of transformations? Here we will have very few notions, all of a very general character. I suggest that they are the logical notions, that we call a notion 'logical' if it is invariant under all possible one-one transformations of the world onto itself.[29]

Elaborating on Tarski's work in [Sher 1991], Sher stipulates the invariance not on automorphisms, but on homomorphisms between domains of equal cardinality. On any interpretation, the resultant Tarski-Sher thesis encompasses a wide range of highly controversial terms as logical notions. To borrow an example from [Bonnay 2008], the monadic quantifier $Q_{most}$ defined as $(Q_{most}x)\varphi(x)$ is true in a model if and only if most of the objects in the model satisfy $\varphi(x)$ is invariant under permutations.

The research stimulated by the permutation invariance view has yielded increasingly sophisticated formulations. [McGee 1996], [Feferman 1999], [Casanovas 2007] and [Bonnay 2008] are among those worth mentioning. The pursuit of finding the right morphism has turned to be a category-theoretical description of logical constants, rather than an exploration into logical constanthood. We shall not delve into the question of internal coherence in the proposed formulations, since they would not substantially change the philosophical claims that follow. Instead, we shall focus on the examination of its basic tenets.

In fact, it is a somewhat straightforward exercise to generate terms as logical constants that are *bona fide* with respect to permutation invariance criterion, however, in opposition to the general notion of logical constanthood. For example, an option, open at least in principle, is to define such composite terms that it is adaptive to the characteristics of each domain and will remain invariant in each domain. McGee illustrates the pitfall lying here with "wombat disjunction" $\mathcal{W}$, a connective defined by:

> $\sigma$ satisfies $(\varphi \mathcal{W} \psi)$ iff either there are wombats in the universe of discourse and $\sigma$ satisfies either $\varphi$ or $\psi$ or there are no wombats in the universe and $\sigma$ satisfies both $\varphi$ and $\psi$.[30]

[29] [Tarski 1986], p. 149.

[30] [McGee 1996], p. 575.

The connective $\mathcal{W}$ will behave either like ∨ or ∧, both of which are elementary logical constants.

A similar case is the application of the permutation-invariance idea to intensional operators, among which modal operators are the outstanding ones, via appropriate formal semantic devices like Kripke frames.[31] However, it yields results that are difficult to be conceptually sustained. For example, the necessity operator, □, is admitted as a logical constant in the modal system **S5**, while it is not in the **S4** system, since the **S5** accessibility relation is total (indeed, it might be said to be idle) over the domain, setting an equivalence relation among possible worlds, unlike that of **S4**. Hence, not only sensitivity to objects, but also sensitivity to structural features of domains should be taken into account.

The permutation invariance method claims quite reasonable results in the demarcation of logical constants. However, this does not make a distinction by itself among other proposals that secure the inclusion of the elementary set of logical constants and exclusion of those that are mostly considered dubious. As for the borderline or novel cases, it stands merely as a purely mathematical criterion of logicality.

At this point, it might be helpful to recall that the objective of type systems is basically to bind semantic categories with well-formed formulas so that the formulas are semantically well-defined as well.[32]The issue hinges on whether there is an adequate theory of language.

A usual technique employed to generate counterexamples is what we may call *modulus affixation*. We can introduce a new term as a logical constant by composing a schema with a modulus expression and indisputably logical constants, so that the logical status of the schema is also dependent on the modulus. For instance, affixing a veridical statement $\mu$ as a modulus does not result in an extensional change, but makes the constant intensionally *modulo* $\mu$. In [Peacocke 1981], Peacocke makes use of this technique in his critique of Hacking and devises an operator, ξ, similar to conjunction as follows:

[31] See [van Benthem 1989].

[32] See [Partee *et al*. 1990], p. 337.

Rockefeller is wealthy, if every element of ξ is true, then some element of ζ is true.

According to this definition, the meaning of an expression A $\underline{\wedge}$ B is dependent on by A, B and the phrase 'Rockefeller is wealthy'. Thus, if the truth-value of the phrase 'Rockefeller is wealthy' is fixed to be true as a worldly, extra-linguistic fact, $\underline{\wedge}$ will be co-extensional with $\wedge$.

Peacocke alternatively considers that $\underline{\wedge}$ might be conceived as being stipulated by the truth table of conjunction. Hence, A $\underline{\wedge}$ B is true if and only if A is true and B is true, whether or not Rockefeller is wealthy. Therefore, we have a new deducibility relation $\vdash^*$ with $\underline{\wedge}$ such that $\vdash^*$ and $\vdash$ has the same extensions in all possible worlds. For any set of propositions Γ, just as Γ $\vdash$ Γ is true with respect to any possible world, Γ $\vdash^*$ Γ is true.

Peacocke asks for considering the pair ⟨Γ, Carter is President in 1979⟩ which is true and deducible by $\vdash^*$ in the actual world, but also deducible with respect to any possible world by the given stipulation. This brings out the paradoxical consequence that 'Carter is President in 1979' is a theorem for any possible world. Hence, an affirmation of modulus could be done at the expense of degeneration in our general conception deducibility relation.

A similar counter-example is suggested by Gómez-Torrente, employing a presumably analytic truth instead of a worldly fact.[33] He constructs a first-order quantifier $\underline{\exists}$ with the meaning of 'not for all not …, if all are not male widows, and for all not …, if not all are not male widows'. We assume that $\underline{\exists}$ is primitive, not derived from other logical constants. It has the same extension as the ordinary $\exists$ does and comply with the Gentzen-style operational rules. Nevertheless, it is bound to a nonlogical index and faces similar difficulties as mentioned in Peacocke's example.

By this technique, we can compose counterexamples such that the resultant expression is logical modulo contingent facts, ontic, epistemic or some other modalities in the face which mathematical considerations remain neutral. The extension of a logical constant is the domain of indefinite extensibility (every possible domain of objects), we may talk of a

[33] [Gómez-Torrente 2002], p. 29.

necessity stemming from invariance under every permutation. Setting aside the problems of indefinite extensibility, this is not immune to modulus technique, because, at any rate, some fact about the domain can be contrived to modify the schema.

Eventually, there remains the option of appealing to the linguistic notions, as McGee resorts to:

> A connective is a logical connective if and only if it follows from the meaning of the connective that it is invariant under arbitrary bijections.[34]

He concedes that this could be only a conjecture hard to accomplish (that is, in the line of thought taken, pace proponents of permutation-invariance). Recently, Feferman has initiated an attempt, as yet inconclusive, in order to reconcile the invariance-centric view with the Gentzen-style inferential approach.[35]

It can be said that the root problem of permutation-invariance view resides in the inflationary role it attributes to logic, disregarding its role in devising arguments. We may refer to Bonnay for a succinct statement of the basic tenets of this view. Bonnay discerns two arguments justifying the permutation-invariance view. One is the generality argument:

> G.1 The distinctive feature of logic among other theories is that it is the most general theory one can think of.
>
> G.2 The bigger the group of transformations associated with a theory, the more general the theory.
>
> G.3 The biggest group of transformations is the class of all permutations.
>
> ∴ The logical notions are the notions invariant under permutations.[36]

The other one is the formality argument:

> F.1 Logic deals with formal notions, as opposed to non formal ones.

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[34] [McGee 1996], p. 578.

[35] See [Feferman 2011].

[36] [Bonnay 2008], p. 33.

F.2 Formal notions are those which are insensitive to arbitrary switchings of objects.

F.3 A notion is insensitive to arbitrary switching of objects iff it is invariant under permutation.

∴ The logical notions are the notions invariant under permutation.[37]

The first argument, lingering from the history of philosophy, is quite familiar to us. It should be clear that, ascribing such a stature to logic, this argument could not be maintained without a comprehensive metaphysical theory. As such, it might only be a part or an application of a "grand theory", but hardly a theory about logical constanthood.

The second argument is not only deficient, but furthermore, it is confusing. Logic is an endeavour to formalise arguments; along the way, it is occupied with formal languages and formal systems. However, it is not alone in dealing with "formal notions" –whatever they are– considering many fields of knowledge such as mathematics and computer science, to begin with. A reminder is in order at this point: On our view, the three constituents of the essence of logic, necessity, normativity and formality, are complementary concepts. Hence, formality should be explicated such that it is to sustain also necessity and normativity,[38] while, it should be emphasised, the proper locus of formality is the proofs.

**Concluding Remarks**. The argument of permutation invariance view fails in its presumptions and falls short of providing a clearer understanding of logical constanthood. We may surmise that there is an intrinsic connection between logical constanthood and invariance, but this does not need to be an unrestricted, definitive relation.

Evidently, the permutation-invariance method can be helpful in fostering and exploring profound connections within the progressing realm of logic; nonetheless, it remains as a theory that is rich in mathematical content, but poor as a philosophical account of logical

[37] *ibid.*, p. 34.

[38] It is noteworthy that the starting question of [McCarthy 1987], "Under what conditions will the model-theoretically valid statements of [the language] $\mathcal{L}$ be necessary?" (*op. cit.*, p. 423) is a hit to the point.

constanthood. Ultimately, conceiving of logicality as invariance under all possible transformations, it stumbles upon the question: but, of *whose* or *what* world onto itself?

## 2.3 DAVIDSON'S RECURSIVE TERM VIEW

Davidson puts forward a characterisation-oriented proposal that identifies logical constants on the ground of his theory of meaning inspired by Tarski's definition of truth as "those iterative features of the language". We can sketch a reconstruction of the Davidsonian theory as an essentially Kant-style transcendental argument in three main stages:

(a) Specification of the linguistic meaning as the object of investigation.

(b) Setting out the necessary conditions for the possibility of linguistic meaning as specified in (a).

(c) Statement of his conjecture as a consequence of (a) and (b).

We may identify several salient features of natural language that Davidson's theory aims at addressing:

(1) Reality of language. Natural language is a real system existing in the actual world with real features. How this system is embodied in the psychological reality of the actual speakers is not of primary importance for a theory of meaning.

(2) Productivity of language. There is no definite limit to the number of expressions that can be generated in the language.

(3) Finite discreteness. Language is made up of a finite set of discrete elements. The whole language consists of the systematic combination and rearrangement of those elements.

(4) Compositionality. Meaning of a compound expression is determinable by the meanings of the constituents. It should be remarked that any definition offered for this feature is a matter of disputation; however, there is a commonly agreed sense that the meaning is strongly predictable by the proper combination of the linguistic elements at levels higher than a certain level in the syntactic structural tree.

Davidson conjectures that it is strongly possible to develop a theory of meaning for a language L capitalizing on the Tarski's Convention T in his semantic conception of truth. Davison generates T-sentences using this fundamental scheme:

(M) $(\exists s)(\exists p)$('*s* means that *p*')

where 's' is the metalinguistic name of an object-language sentence and 'p' is a metalanguage sentence that gives the meaning of 's'. Suppose, for example, that the object-language is German and the metalanguage is English. Such a T-sentence of a properly developed theory of meaning for German would be one like the following:

'Schnee ist weiß' means that snow is white.

Accordingly, the theory for the language L comprises the following:

(a) A set of axioms expressing base clauses to specify the semantic primitives.

(b) A set of axioms expressing recursive clauses to specify the method to compose compound expressions.

(c) Theorems ascribing meaning to all possible sentences of L.

Thus, a theory for the object-language L will generate for every sentence of L an interpretive sentence in the metalanguage. A sentence is said to be interpretive just in case that '*p*' is synonymous to '*s*' in the schema (M). The sentence as a whole is considered as the bearer of complete meaning and the reference of a (declarative) sentence is its truth-value.

Let us consider the statement '*s* means that *p*'. We have to construct a predicate T, so that *s* is T(*p*)' is technically viable for an entailment procedure as well as explanatory for the notion of meaning without falling into vicious circularity. Davidson suggests the truth predicate is a suitable to that purpose. The claim is that there is a biconditional link between understanding meaning and understanding its truth conditions. Thus, Davidson reformulates (M) as a 'T-sentence' for L:

(T) *s* is true if and only if *p*

It is noteworthy that there is a significant difference between Davidsonian and Tarskian conventions. The '*p*' in the Tarskian Convention-T refers to a translation of '*s*' into the metalanguage, whereas in the Davidsonian version, it refers to a sentence in the metalanguage that is true if and only if the object-language sentence '*s*' is true. Davison conceives the notion of translation as intrinsically connected to the notion of meaning; he explicates this point as:

> One thing that only gradually dawned on me was that while Tarski intended to analyse the concept of truth by appealing (in Convention T) to the concept of meaning (in the guise of sameness of meaning, or translation), I have the reverse in mind. I considered truth to be the central concept, and hoped, by detailing truth's structure, to get at meaning.[39]

The theory attempts to explain meaning in extensional terms and abstain from introducing intensional contexts. Because the identity conditions of intensions, subsequently their truth conditions, are thought not clear enough due to difficulties in their individuation. In contrast, extensions offer the convenience of substitutability *salva veritate*.

The Davidsonian idea can be described mainly as the meaning of complex expressions are composed by recursion indexed by truth-conditions. The envisioned semantic structure posits an opening for intra-linguistic truth:

> A truth definition does not distinguish between analytic sentences and others, except for sentences that owe their truth to the presence alone of the constants that give the theory its grip on structure: the theory entails not only that these sentences are true but that they will remain true under all significant rewritings of their non-logical parts. A notion of logical truth is thus given limited application, related notions of logical equivalence and entailment will tag along.[40]

Then, a prospect for the theory to characterise the logical constants emerges:

> A theory of truth does not yield a definition of logical consequence or logical truth, but it will be evident from a theory of truth that certain sentences are true solely on the basis of the properties assigned to the logical constants. The

[39] [Davidson 1985], p. xiv.

[40] [Davidson 1985], p. 33.

> logical constants may be identified as those iterative features of the language that require a recursive clause in the characterisation of truth or satisfaction. Logical form, in this account, will of course be relative to the choice of a metalanguage (with its logic) and a theory of truth.[41]

So, the Davidsonian theory supposes that a meaning theory for some natural language L is achievable, if not yet.[42] Some questions concerning the theory may be accepted to be answered with that theory, but there are general problematic points that should be accounted for; for example, the question of whether a properly constructed theory of meaning can spell out generic criteria to determine conclusively that a class of the object-language sentences is logical truths is open. A logical constant does not belong to the metalanguage in which we conduct our reasoning and argumentation, even if it is translated with the same name. Let us illustrate the idea borrowing from an example from Suppes and see how the semantic behaviour of 'and' changes under the effect of the implied time indication in contrast to the object-language connective conjunction:[43]

(1) They got married and (they) had a baby.

(2) They had a baby and (they) got married.

An improved and detailed version of Davidson's view is put forward by Lepore and Ludwig focusing on explicating logical form. They give us a better understanding of what is at stake:

> We will suggest that the recursive syntactical structures of the language be treated as its logical syntax. The recursive syntax of sentences gives them structure beyond that already expressed in the number of argument places in primitive predicates. It is natural to think of arguments made valid in virtue of the presence of recursive syntax in the premises and conclusion as valid in virtue of their *structure*. This gives one clear sense to the idea that in

[41] [Davidson 1985], p. 71.

[42] . An outstanding attempt in the Davidsonian line is [Larson and Segal 1995]. Larson and Segal put forward a theoretical setting of lexical axioms, phrasal axioms and production rules, but it is hard to say that it manages to lay out satisfactory linguistic material to support the Davidsonian view of logical constanthood.

[43] See also [Klinedinst and Rothschild 2012] for non-truth functional employment of connectives.

> identifying the logical *terms* we identify those terms that we do not replace with schematic letters in identifying the structures or forms of sentences relevant to determining what other sentences similarly identified in terms of their structures they bear deductive relations to.[44]

Our critique is aimed at appealing to the realm of beliefs or concepts in accounting for logical constanthood. Since this resurface in various guises, we would like to address the issue also in Davidson's occasion. On this purpose, we turn to Quine, whose view of language and meaning shares fundamental starting points with Davidson's which are significant insofar as the demarcation of logical constants is concerned.

> The less susceptible the stimulus meaning of an occasion sentence is to the influences of collateral information, the less absurdity there is in thinking of the stimulus meaning of the sentence as the meaning of the sentence. Occasion sentences whose stimulus meanings vary none under the influence of collateral information may naturally be called *observation sentences*, and their stimulus meanings may without fear of contradiction be said to do full justice to their meanings. These are the occasion sentences that wear their meanings on their sleeves. Or, better, we may speak of degrees of observationality; for even the stimulus meaning of 'Red' can, we noted, be made to fluctuate a little from occasion to occasion by collateral information on lighting conditions. What we have is a gradation of observationality from one extreme, at 'Red' or above, to the other extreme at 'Bachelor' or below.[45]

On both Quine's and Davidson's views, meaning is individuated and determined by what belief a speaker holds true. Hence, each speaker has her own idiolect with her private dictionary. The problem is how to make a particular speaker's idiolect systematically coincide with another speaker's. This requires on the part of one speaker to set out a theory of meaning (in Davidson's case, of truth) for the other. Virtually, there is no a normative linguistic community, but there is a collection of speakers, each one trying to succeed in mapping their idiolectical meanings to others. In this picture, logical constants are, according to Quine, privileged by being the least susceptible to empirical effects, and according to Davidson, by being factors of recursivity in language. The burden of argument on Quine is how to account for the difference by degrees related to our notion of

[44] [Lepore and Ludwig 2002], p. 79 (emphases in the original).

[45] [Quine 1960], p. 42.

arguments, and on Davidson, what distinguishes the logical recursive elements from non-logical recursive elements.

We have critically examined prominent characterisation oriented views and try to see at what points they distract from an admissible characterisation. Wth what we have accrued from our criticism, we shall proceed to the adequacy-based views. Those views, in one way or another, coordinate themselves with the 'use'-theory of meaning summarised with the Wittgensteinian dictum "meaning is use". We shall expand on theory of meaning within the the scope of our discussion.

# CHAPTER 3

## IN SEARCH OF HARMONY

Concluding that the approaches seeking logical constanthood in the intrinsic features of terms are on thin grounds, we turn to the approaches that seek logical constanthood in the roles that words take on in our inferential practices. Thus, they state adequacy conditions for the terms to be endowed with logical constanthood.

We subsume those approaches under the doctrine of inferentialism. Broadly, the inferentialist views contend that the meaning of a logical constant is conferred by its distinctive inferential role in a competent speaker's discourse such that it imposes the validity of inferences in which it occurs. in terms of proof conditions, rather than truth conditions. Just what this distinctive role consists in and what requirements have to be met in order to fulfil such a role are the prime aspects in need of explanation.

The central elements of inferentialist explanation will be seen to be some harmony conditions that a logical constant should satisfy. We shall take a general look at the framework of inferentialism mainly as explicated by Brandom, one of the main exponents of inferentialism, and then, concentrate on a particular approach grounded in the concordance of Gentzen's natural deduction calculi and Wittgenstein's conception of meaning as use.

### 3.1 INFERENTIALISM

The thrust of inferentialism can be understood by reference to two main models of concepts Margolis and Laurence discuss. They identify a *container model* and an *inferential model* for concepts, in both of which a concept is a structured complex of other concepts.[46] In the container model, the concepts in the structure are proper parts of the focal concept; they are "subconcepts" of the focal concept. Occurrence of a token of the focal concept

[46] [Margolis and Laurance 1999], p. 5.

necessitates occurrence of the tokens of its subconcepts. In the inferential model, the focal concept bears a relation of inferential disposition to the other concepts in the structure. In contrast to the container model, the subconcepts may not be proper parts of the structure of the focal concept. Thus, occurrence of a token of the focal concept does not necessitate occurrence of the subconcept tokens. Inferentialism can be said to embrace a base of explanation that consists of concepts organised, as Brandom remarks, in the inferential model:

> It follows immediately that from such inferential demarcation of the conceptual that in order to master *any* concepts, one must master *many* concepts. For grasp of one concept consists in mastery of at least some of its inferential relations to other concepts. Cognitively, grasp of just one concept is the sound of one hand clapping.[47]

Granting the primacy to concepts and conceptual relations, inferentialism presents itself as a rationalist doctrine in the traditional sense. Brandom explains its main motivation as

> The idea is to understand propositional content as what can both serve as and stand in need of reasons, where the notion of a reason is understood in terms of inference. So propositional contentfulness is taken to be a matter of being able to play the both of premise and of conclusion in inferences.[48]

Brandom takes Wittgenstein's 'game' metaphor almost literally; the players are contenders in "the game of giving and asking for reasons" that exerts a normative force on them. Asking a reason is raising a challenge which involves, at least potentially, making counter-claims, and giving a reason demands making a claim which, in turn, gives rise to a need of other reasons. Thus, on this view, inferentialism envisions language as a reasoning medium like a web knitted by inferential threads. Brandom admits that "the game of giving and asking for reasons" is not the only game in language, however, he points out that it is the primary one as a locus where the rationality of human species is manifested and on which the other games rest somehow.

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[47] [Brandom 2001], p. 49 (emphases in the original).

[48] [Brandom 2007], p. 654 (emphasis in the original).

The proper size of inferential unit is a proposition, hence, inferentialism is "an essentially propositional doctrine" with a top-down approach beginning from the sentence-level, as opposed to the bottom-up approach based on the composition of sub-sentential expressions on the word-object level in the context of language-world contact. Thus, the links in the conceptual web correspond to propositional transitions. Evidently, that makes inferentialism predisposed to an arbitrary conceptual holism, and it is not clear whether there are any "nodes" in the web that delimit transitivity. A possible response might be a delimitation by "pattern-governed" linguistic behaviour, a concept worked out by Sellars, one of the founders of inferentialism. Sellars states that the meaning-constitutive linguistic behaviours are patterned-governed and systematises them in three types:

> (1) *Language Entry Transitions*: The speaker responds to objects in perceptual situations, and in certain states of himself, with appropriate linguistic activity.
>
> (2) *Intra-linguistic Moves*: The speaker's linguistic conceptual episodes tend to occur in patterns of valid inference (theoretical and practical), and tend not to occur in patterns which violate logical principles.
>
> (3) *Language Departure Transitions*: The speaker responds to such linguistic conceptual episodes as 'I will now raise my hand' with an upward motion of the hand, etc.[49]

The patterns are brought about not by the speakers' individual intentions, but through a socially developed propensity that is comparable to natural selection in biology. Those patterns are the basis of the linguistic norms that the speaker ought to accord with. The norms are related to the positions of entitlement and commitment that a speaker can assume in the practice of language. In a typical case, when one conforms to the relevant norms with respect to some assertion during the course of a conversation, she is entitled to put it forward, and by the act of her assertion, she is committed to the assertion and the associated propositions. Sellars illustrates the point comparing two speakers, Jones and Smith, each one end up with the equivalent statements 'all men are mortal' and 'no non-mortals are men':

> (A) Jones  All men are mortal
>
> So, no non-mortals are men

[49] [Sellars 1974], p. 423 *f.*

(B) Smith  If I am entitled to 'All men are mortal', I am entitled to 'No non-mortals are men'.

I am entitled to the former, I state it thus: All men are mortal

So, I am entitled to the latter, I state it thus: No nonmortals are men.[50]

Sellars points out that what Jones does is an act of inference, while Smith exhibits a process of entitlements in compliance with "instituted" patterns. Inferentialism accepts that some of the norms may take part in the chain of defining some others, but the basic norms are immanent in (or, implicitly defined by) language practice.

Inferentialism conceives of language practice as pragmatics (*cf.* Wittgenstein's conception of language practice) and contends that semantics "must be answerable to pragmatics". It rules out the possibility of semantic facts that may not be independent of the pragmatic facts. According to Brandom, conceptual content emerges through pragmatic use of concepts by expressing something in conceptual form. Any application of a concept bears on the identification of the object of the concept's application.

The task of logic as a discipline is to explicate the structure of inferential roles already implicit in language (on this view which Brandom calls this expressive theory of logic, the requirement that logical vocabulary should be a conservative extension of language is substantiated by being confined to explication):

> The content to which one is committed by using the concept or expression may be represented by the inference one implicitly endorses by such use, the inference, namely, from the circumstances of appropriate employment to the appropriate consequences of such employment … [This is] a generalization of a standard way of specifying the inferential roles of logical connectives.[51]

[50] *ibid.* p. 424.

[51] [Brandom 2001], p. 62.

Tennant summarises differentia among the inferentialist strands in five dimensions: [52]

(1) *Strong inferentialism* vs. *moderate inferentialism*. Whether inferential roles constitute the basic semantics for any expression (including such cases as singular reference) or they are constituents of the meanings of some expressions and do not exhaust the ways of an expression to be meaningful.

(2) *Holistic inferentialism* vs.*molecularist inferentialism.* Whether the size of meaning-constitutive inferential patterns are so wide to involve a cluster of expressions as ingredients or expression-specific. We may understand that this difference translates to the question of whether logical constants of a system could be learned seperately or each constant demands a grasp of others, therefore, they could only be learned jointly.

(3) *Naturalist inferentialism* vs. *hyper-rationalist inferentialism.* Whether human thought and language constitute a continuum with nature (we may understand sort of Quine's view naturalisation) or they are separated from nature by an essential gap.

(4) *Evaluative inferentialism* vs. *global inferentialism*. Whether the category of atomic facts sufficises for the evaluation of inferences or non-reductive evaluation of logically composite conclusions from logically composite premisses is also needed.

(5) *Logically* quietist inferentialism vs. *logically reformist inferentialism.* Whether classical logic is the main system or there should be a revision of system.

Tennant points out that Brandom's inferentialism is strong, holistic, hyper-rationalist, global and logically quietist, Dummett's and Prawitz's inferentialism is moderate, molecularist, global and logically reformist (advocating intuitionistic logic), while (3) is left untreated by Dummett and Prawitz.

We have discussed the severe drawbacks in giving an account of logical constanthood by appeal to the items of mental content such as concepts and beliefs. Therefore, strong inferentialism does not offer prospects for our inquiry; however, the moderate version has

[52] [Tennant 2007], p. 5. Brandom makes a similar, but coarser classification; see [Brandom 2007], p. 656 *ff.*

plausibility and this is the approach we shall proceed to.[53] First, we shall give an overview of the formal environment provided by Gentzen, which is crucial for the rest of our discussion.

**A Brief on Gentzen-Style Definition.** Indisputably, Gentzen's proof theoretic work set a paradigm on the discourse on logic and language. Gentzen introduced two proof systems with classical and intuitionistic variants: Natural deduction calculi (the classical calculus **NK** and the intuitionistic calculus **NJ**) and sequent calculi (the classical calculus **LK** and the intuitionistic calculus **LJ**).

His fundamental idea of formalism is to state separate rules for each constant, conferring meaning explicitly on each one of them. This method differs abruptly from Hilbert-style systems which comprise axioms and deductive rules. Hilbert-style systems implicitly ascribe meaning to logical constants, thus, being prone to holistic treatment in meaning-theoretical respect. In contrast, Gentzen-style systems are conducive to molecularist treatment.

In natural deduction calculi, the rules are uniformly in introduction and elimination pairs for each constant. In the sequent calculi, the architecture changes to a set of left and right rules that define logical operations performed by logical constants and structural rules that deal with the ordering of sequents. This difference implicates a change of viewpoint that can be significant as we shall discuss below.

Gentzen's influential observation is that an introduction rule confers the meaning, while the matching elimination rule ensues from this meaning:

> The introductions represent, as it were, the 'definitions' of the symbols concerned, and the eliminations are no more, in the final analysis, than the consequences of these definitions. This fact may be expressed as follows: In eliminating a symbol, we may use the formula with whose terminal symbol we are dealing only 'in the sense afforded it by the introduction of that symbol'.[54]

[53] For other problems the strong inferentialis faces see [Fodor and Lepore 2001].

[54] [Gentzen 1969], p. 80.

Subsequently, this observation has been taken as a conjecture stating that the elimination rules are to be determined *ipso facto* by the introduction rules. Caution should be taken in that Gentzen's remark on meaning is in the context of mathematical discourse, since his main objective in devising the calculi is to serve better than their Hilbert-style counterparts do to the mathematical purposes, particularly explicating the role of assumptions in mathematical proofs.

Gentzen's novelty opened up a promising area of investigation that gives weight to proof-theoretic elements (*e.g.*, subformula property, theorems for introduction and elimination rules), rather than to semantic properties (*e.g.*, bivalence, soundness, validity) in the conception of logical constanthood.

It is helpful to draw attention a significant difference between Hilbert-style axiomatic systems and Gentzen-style proof systems to see its impact on logical consequence. An appraisal in terms of the validity of a formula is given by Avron. A formula is valid if it is true under all assignments. For first order logic, Avron formulates the definitions of consequence relations (in multi-conclusion version) as follows:

Truth: $A_1 \ldots A_n \vdash_T B_1, \ldots, B_m$ iff every assignment in a first-order structure for L which makes all the $A_i$s true does the same to one of the $B_j$.

Validity: $A_1, \ldots, A_n \vdash_V B_1, \ldots, B_m$ iff if all the $A_i$s are valid, then so is at least one of the $B_i$.

The conception of the derivability relation as to whether it is based on truth, $\vdash_T$, or validity, $\vdash_V$, may make significant differences. For example, $(\forall x)A(x)$ follows from $A(x)$ according to $\vdash_V$, but not according to $\vdash_T$.

On the contrary, the classical deduction theorem holds for $\vdash_T$, but not for $\vdash_V$ –hence, the conversion of a deduction to an implication is not a theorem. Let us illustrate this with an example devised by Fitelson. With slight simplification, the example runs as follows:

Suppose that the inference from $A \rightarrow B$ *to* $C \rightarrow B$ is validity-preserving, which indicates that $C \rightarrow B$ is true on all interpretations whenever $A \rightarrow B$ is. Thus, we suppose that:

(1) $\models A \rightarrow B \Rightarrow \models C \rightarrow B$

But notice that (1) does not entail

(2) $\vDash (A \rightarrow B) \rightarrow (C \rightarrow B)$

because any interpretation on which $A \rightarrow B$ is true may not be also *the same* interpretation on which $C \rightarrow B$ is true. To see this, take A = (β → γ) and $B$ = (β → γ) for the propositions α, β and γ. Hence

(3) $A \rightarrow B \equiv (\beta \rightarrow \gamma) \rightarrow (\beta \rightarrow \alpha)$

$A \rightarrow B$ is not true on all interpretations (*i.e.* it is not valid), but is true on some interpretations. Consider, for instance, the case that α is false, β is true, and γ is true, when $A \rightarrow B$ is false, and the case that β is true, $\gamma$ is false, when $A \rightarrow B$ is true, no matter what $\alpha$ is.

In the latter case in which β is true and γ is false, $C \rightarrow B$ is false on some of the interpretations. For instance, $C \rightarrow B$ is false when α is false and $C$ is true. Therefore, the inference from $A \rightarrow B$ to $C \rightarrow B$ is not truth-preserving —though it is formula-wise validity-preserving.

Apart from the features of the Gentzen-style systems that facilitate the visibility of interpretational possibilities relative to the Hilbert-style systems, Gentzen's natural deduction approach incorporates a general notion of derivability from assumptions. Thus, each formula in a Hilbert-style derivation turns out to be virtually a theorem, but so in a Gentzen-style derivation.

In this pursuit, handling of assumptions in the derivations stands out as a critical point. Let us clarify this by a simple example from first-order logic: We can derive $\forall x$A($x$) from A($x$) by universal generalization with the restriction that A($x$) must not depend on any assumption previously imposed on $x$. Otherwise, A($x$) itself would be an assumption and it would become possible to derive $\forall x$A($x$) from A($x$), and therefore, also from $\exists x$A(x) by the rule of existential elimination.

The objective of Hilbert-style proof systems is proving theorems and there may be no assumption other than an axiom, and thus rules of inference admit axioms or (previously proved) theorems as their premisses. In that case, the distinction between in validity-

preservation and truth-preservation vanishes. Any premise guaranteed to be valid, because it is either an axiom or a theorem. Therefore, there is no question as to whether a rule of inference is truth-preserving or validity-preserving, since the validity of premisses is inherited to the conclusion. Hence, the two consequence relations are identical from the aspect of theorems (on the model-centric approach, logical truths):

$$\vdash_T A \Leftrightarrow \vdash_V A$$

There is a thriving diversity of logical systems, the stronger of which are more capable of covering a larger variety of arguments. But as Wagner points out,[55] "increasing strength yields less elementary, transparent notions of logical validity and proof" and we need more abstractive and analytic specification of the fundamental notions of logic (*cf.* Prawitz's proof-theoretic validity conception below).

One subsequence of the idea that inferential rules can be definitive of logical constants is that the discourses in which those are followed provides implicit definition. In effect, the thesis of implicit definition can be an alternative attempt in order to sustain a notion of *a priori* knowledge along with the thesis of epistemic analyticity. However, the received conception of definition is problematic, and implicit definition is *a fortiori* so.

### 3.2 THE QUESTION OF DEFINITION

To begin, a definition consists of a term, the *definiendum* and its semantic equivalent in terms that are already familiar, the *definiens*, possibly together with a matrix which an otherwise antecedently understood frame completed by the definiendum (as in the case of a recursive definition). Definitions are made on a diversity of purposes (dictionary definitions, stipulative definitions, explications, etc., see [Belnap 1993] for a discussion). Though not all of them comply with it pointedly in such as instances of abbreviation for convenience, the general motivation for definition can be said to be the Aristotelian conception that a definition should give the essence of what it is to be for the defined. We shall follow mainly [Belnap 1993] for an analysis of definition. We should be careful in that *definiendum* and *definiens* are not Leibnizian identicals; they are linguistic entities subject

[55] [Wagner 1987], p. 3.

to modification and re-interpretation.[56] We may subsume varieties of definitions a scheme of three components:

Some definitions have all of these components explicitly. In many cases, the scheme '*definiendum* is definiens' are properly employable; but an occurrence of the definiendum in the *definiens* does not violate legitimacy. Thus, the scheme of a definition for a *definiendum* '*t*' can be represented as:

$$[\ldots t \ldots] \stackrel{\text{def}}{=} \textit{definiens} \text{ for } t$$

A prevailing view is that a definition is an introduction of a new expression explaining it in terms of the expressions of which meanings are already known. Hence, a definition is merely a synonym for a certain combination of expressions, and in essence, no new knowledge is gained via definition; hence, it is not informative. This view can be said to be foundationalist, since it is always possible to trace the chain of definitions to a foundational set of terms. The definition of the exclusive disjunction, $\boxplus$, is typical in regards the claims of this view:

$$A \boxplus B \stackrel{\text{def}}{=} (A \wedge \neg B) \vee (\neg A \wedge B)$$

Then, according to Carnap, the significance of definition resides in explication:

> The task of making more exact a vague or not quite exact concept used in everyday life or in an earlier stage of scientific or logical development, or rather of replacing it by a newly constructed, more exact concept, belongs among the most important tasks of logical analysis and logical construction. We call this the task of explicating, or of giving an explication for, the earlier concept; this earlier concept, or sometimes the term used for it, is called the explicandum; and the new concept, or its term, is called an explicatum of the old one. Thus, for instance, Frege and, later, Russell took as explicandum the term 'two' in the not quite exact meaning in which it is used in everyday life and in applied mathematics; they proposed as an explicatum for it an exactly defined concept, namely, the class of pair-classes.[57]

[56] Arriving at a definition is in fact quite an intricate process, even in mathematics, involving strategies such as adjustment of definitions to exclude counterexamples; see [Lakatos 1977] for a narrative description of this process in mathematics.

[57] [Carnap 1970], p. 6 *f.*

We find a parallel remark again due to Quine:

> The definiens may be a faithful paraphrase of the definiendum into narrower notation, preserving a direct synonymy as of antecedent usage; or the definiens may, in the spirit of explication, improve upon the antecedent usage of the definiendum; or finally, the definiendum may be a newly created notation, newly endowed with meaning here and now.[58]

Thus, a definition of a term primarily serves to explicate it, or sharpen its proper usage in a pre-theoretical core meaning of the term and stipulates it into a related, but detached expression in the setting of a theory. A set of admissibility conditions can be cited for a definition in order to be a considerable definition and for us to have a coherent conception of definition:

(a) *Existence*. A definition should not nullify the existence conditions of its own definiendum, for example, containing a logical contradiction or a division by 0. Vacuous names are not excluded from the definition; what is sought for by this condition is an internal consistency.

(b) *Uniqueness*. Definition should sufficiently discriminate the *definiendum*.

(c) *Conservativeness*. The definition should be non-ampliative; it should not introduce worldly facts "true by definition".

(d) *Eliminability*. The use of a formula that contains the defined term is explained by reducing it to another formula in the basic language. Then, the definition has to be able to reduce each formula containing the defined term to a formula in the basic language. Thus, any occurrence of the definiendum can be replaced by an occurrence of the *definiens*, and vice versa.

The foundationalist view of definition is, in effect, an unjustified generalisation of the simple stipulative cases of employment of the notion. It disregards the fact that a definiendum overtakes through a definition a trajectory of its own, it is modified and re-interpreted. In simple cases, as in the example of exclusive disjunction, the definiendum is tightly dependent on the definiens. But even in this case, it can be argued that a new term

[58] [Quine 1963], p. 27.

is introduced with its own conceptual and semantic relations. This aspect bears a parallelism to Kripke's objection of the foundationalist view. Kripke argues that the purpose of some definitions is primarily to fix a certain reference, despite that they conform to the scheme of stipulation. In such definitions, the definiendum is not semantically equivalent to the definiens. The definiendum is stipulated, or more precisely, made rigid, so that it picks out or to introduces a discursive referent. However, the definiens is not rigid, since it only serves a sort of description of a choice function (thus, such a definition expresses an *a priori* truth, but a contingent one, a case we shall take up below).

A related objection is due to Wittgenstein. Wittgenstein points out that it is an illusion to think that there is a definite set of necessary and sufficient conditions common to all games and words as labels that we can apply to things, ideas, mental states, and so on, and provides us the basis whether to call something a game or not. Instead, he introduces the 'family resemblance' metaphor (which has been made the basis of the "cluster-concept view" by some theorists) according to which there are properties that are scattered to the whole of a family, but possibly none of which occurs in all the members of the family. This leads to the notion that a word must have one fixed meaning across contexts. It is not a general case that a single defining characteristic underlies all uses of a word; rather, these uses share a kind of family resemblance with one another.

**Discussion of Implicit Definition**. An implicit definition does not directly state the extension and intension of a term; instead, the definition is determined by the context it occurs. For example, a mathematical function could not be defined by an explicit mapping to values as

$$F(x) \stackrel{\text{def}}{=} y = f(x)$$

It is possible to formulate implicitly through satisfaction condition as

$$F(x) \stackrel{\text{def}}{=} f(x, y) = 0$$

For first-order languages, the foregoing interchange of formulation turns out to be a general feature, known as Beth's Definability Theorem. The theorem can be stated as follows: A term $t$ is implicitly definable by a set of propositions that we presume true if and

only if it is explicitly definable relative to the same set of propositions in terms other than *t*. This method of definition may be sometimes employed as an alternative to explicit definition in cases that explicit definition appears to be too difficult, or even impossible. For example, For example, the Euclidean primitive (undefined) terms, *e.g.*, 'point' and 'between' employed in non-Euclidean geometries are implicitly defined by satisfying the axioms, and the definition is confined to the Euclidean theory.

Implicit definition can be taken as the one with the *definiens* (for example, whatever makes rules valid, or propositions true) dispersed into the context. Boghossian's conception of implicit definition:

> Implicit definition: It is arbitrarily stipulating that certain sentences of logic are to be true, or that certain inferences are to be valid, that we attach a meaning to the logical constants. More specifically, a particular constant means that logical object, if any, which would make valid a specified set of sentences and/or inferences involving it.[59]

He quotes from Wittgenstein the following:

> It looks as if one could infer from the meaning of negation that '$\sim\sim p$' means *p*. As if the rules for the negation sign follow from the nature of negation. So that in a certain sense there is first of all negation, and then the rules of grammar.
>
> We would like to say: "Negation has the property that when it is doubled it yields an affirmation." But the rule doesn't give further description of negation, it constitutes negation.

For example, the meaning of the term 'plus' is such that the proposition 'one plus one makes two' is true and this is the way that one should grasp the meaning of 'plus'. Then, he gives the following scheme to make out the meaning of a logical constant:

> (1) If logical constant *C* is to mean what it does, then the argument-form *A* has to be valid, for *C* means whatever logical object in fact makes *A* valid.
>
> (2) *C* means what it does.
>
> Therefore,
>
> (3) *A* is valid.

[59] [Boghossian 2000], p. 348.

Thus, a logical constant is defined as what makes an argument valid.[60]

Horwich takes up the matter in a complementary manner to Boghossian's template.[61] He assumes that a matrix designated by '#—' in which 'f' is defined as '#f' is already meaningful. His proposal is based on a 'use'-theory of meaning such that "the meaning of a word is engendered by there being a certain regularity of its use":

> [I]f we suppose that the meaning conferred on 'f' by an implicit definition is that constituted by regarding a certain sentence as true, rather than that which 'f' must have to make it true, then the four difficulties are eliminated. The existence of that meaning is guaranteed by the satisfiability of the regularity – that all uses of the word stem from regarding the sentence as true; the uniqueness of the meaning is guaranteed by the uniqueness of the basic regularity governing the use of 'f'; the word's possession of that meaning is guaranteed by the fact that our use of 'f' is governed by that regularity; and the fact that 'f' does, in virtue of that usage, acquire a meaning, is explained by the very nature of meaning.[62]

Then, each occurrence in the matrices '#—' yields another sense and an appropriate context yields a definition, even if the speaker does not explicitly mention this.

We shall discuss the implicit definition proposal in general. It will be useful to keep in sight the difficulties Horwich specifies as an implicit definition should overcome:

(a) *Existence*. Given that '#—' is already meaningful, is there any meaning that 'f' could have such that the combination '#f' would be true?

(b) *Uniqueness*. Is there more than one meaning that 'f' might have that would satisfy the truth-condition of '#f'? If there is, then 'f' cannot be said to be defined unambiguously.

(c) *Possession problem*. Does 'f', in fact, come to possess the meaning, even if the meaning is unique in satisfying the truth-condition of '#f'?

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[60] *ibid.*, p. 357.

[61] See [Horwich 1998].

[62] *ibid.,* p. 137.

(d) *Explanation problem*. Granted that 'f' comes to possess it, what would explain how this happens?

We reject the characterisation of logical constants by implicit definition on two grounds. The first is that implicit definition requires a well-explicated matrix, and when possible, is highly dependent on that matrix. Mere truth or validity do not suffice to provide the required matrix to overcome existence difficulty, let alone assurance of uniqueness. We shall draw on a clear insight due to Ebert to argue for this. Ebert points out that Bohossian's template fails to transmit warrant from the premisses to the conclusion, and when the missing line is supplied, the argument fails to be expedient to establish its motivating claim.[63] The crux of his discussion is that Boghossian's template of epistemic analyticity lacks a "disquotational step" which translates the metalinguistic (3) to an object-language sentence which states that 'A' is *in fact* logically true. Then, he explicates Boghossian's template taking the logical constant *C* as conjunction and the argument form *A* as conjunction elimination:

(1) If 'and' is to mean what it does, then 'P and Q ⇒ P' has to be valid.

(2) 'and' means what it does.

(3) 'P and Q ⇒ P' is valid.

(4) P and Q ⇒ P.[64]

where '⇒' represents an inference. Ebert points out that the disquotational step presupposes that one already understands 'and'. By a suitably formulated Context Principle,[65] a grasp of 'and' requires the grasp of a sentence like (4). *Ex hypothesi*, (4) is

[63] See [Ebert 2005].

[64] [Ebert 2005], p. 210 *f.* The modified template can be made more precise at the expense of cluttering it; the present one conveys the idea, see *ibid.*, p. 511, n. 12.

[65] *I.e.*, a principle that asserts a basic size of context for semantically complete expression, which is usually set to the size of sentence. Ebert suggests Evans' Generality Constraint: "[I]f a subject can be credited with the thought that a is F, then he must have the conceptual resources for entertaining the thought that a is G, for every property of being G of which he has a conception'' ([Evans 1982], p. 104).

epistemically analytic, therefore, understanding it has to suffice for being warranted in believing it. Ebert points out that, nevertheless, the grasp of the disquotational step suffices to warrant the belief of the conclusion of the argument whatever the logical constant is. Therefore, the argument fails to transmit the warrant of the premisses to the conclusion.

Secondly, it is quite dubious that Gentzen-style definitions are really implicit definitions. We can accept that we read off the definition of a logical constant from its occurrence in a Hilbert-style axiomatisation. But do we read off the definition of a logical constant from its specific inference rules, as if there were a further linguistic entity that acts at a distance and that the inference rules would imply? To put this in another way: There is an explicit definition of *modus ponens* rule in standard Hilbert-style axiomatisations. This definition is *part* of an implicit definition of the logical constant of implication. Although schematically the same, implication elimination in Gentzen's natural deduction calculi is *part* of an explicit definition of implication.

For explicit definition, we expect a semantic equivalence in the scheme of '*definiendum* + copula + *definiens*'. But the scheme is, on the one side, related to the expressive resources of language. For example, a set $\Psi$ could be given an extensional definition as

$$\Psi = \{1, 2, 3, 4, 5\}$$

If the language has the resources for an intensional definition, the same set could be defined alternatively as:

$$\Psi = \{x \mid x \in \mathbb{N}, 1 \leq x \leq 5\}$$

On the other side, a definition may be composed of several clauses depending on the *definiendum* in question. For example, a dictionary definition of 'hammer' may offer two clauses such that hammer is "a tool with a heavy metal head mounted at right angles at the end of a handle" and" used for jobs such as breaking things and driving in nails".[66] It should be clear that there is nothing wrong, especially for abstracta, in that a definition consists of one clause that describes how to use it.

[66] Looked up in [Oxford Dictionaries].

## 3.3 THE STRANGE CONSTANT 'TONK'

Prior has ingeniously contrived a logical constant 'tonk', demonstrating that a plain scheme of Gentzen-style definitions is not adequate for the philosophy of logic.[67] Prior's counterargument is directed to the view that the meanings of logical constants consist of the inferential rules associated with each of them, and those meanings solely account for the validity of the inferences in which they occur –hence, logically valid inferences are a class of analytically valid inferences.

The inferential rules for tonk, tonk-I (for introduction) and tonk-E (for elimination) are such that their adjunction to a logical system allows A ⊢ B, for arbitrary propositions *A* and *B*. This can be seen by employing these rules consecutively:

$$\frac{A}{AtonkB}\quad (tonk\text{-}I) \qquad\qquad \frac{AtonkB}{B}\quad (tonk\text{-}E)$$

The tonk case has proved to be very conducive to the studies that provide valuable insights in the fundamentals of logic. Prior's pseudo-connective 'tonk' shows that fixing meaning of a logical constant only by its introduction and elimination rules is deficient and "a great deal of stage-setting is presupposed if a mere act of naming is to make sense". It may be said that improvement our notion of logical constanthood involves an extensive account of the stage-setting.

**Belnap's Diagnosis**. The main response in this direction is from Belnap, setting the issue independently form a particular logical system.[68] Certainly, it is possible to block tonk-triviality by altering logical consequence relation in a particular way. For example, Cook devises a substandard, but non-*ad hoc* system of logic modifying Belnap's quaternary logic, which has the independent truth-values 'true' and 'false' and the dependent truth-values 'both' (or unknown) and 'neither' (or inconsistent). The special property of his "tonk-logic"

[67] Let us an incidental digression here: There is a parallelism between Goodman's strange predicate "grue" (See [Goodman 1983]) and Prior's strange connective "tonk". They hint (the former for induction and the latter for deduction) at that there is something deficient in our conception of inferential forms.

[68] See [Belnap 1962].

is that the transitive property of the consequence relation does not hold generally, so that tonk can be added without resulting in triviality or inconsistency.

Thus, by being conservative, an extension preserves consistency of a priorly consistent system. It should be remarked that when we talk about the operations of extension and expansion, we assume that the system of logic has been already consistent, thus it is capable of discerning validity.

Surely, we do not desire indefinite extensibility for constanthood. We say that, for example, a binary relation $R$ such that $\exists y \forall x(Rxy \leftrightarrow \neg Ryy)$ is indefinitely extensible. In this case, the relation $R$ is essentially incomplete, taking each member $x$ of the domain to a non-member $y$. This point, which occurs also in semantic paradoxes, appears to have been usually overlooked in the context of logical constanthood and left to intuition.

As Belnap points out in his diagnosis of ill-defined logical constant 'tonk', the prior consistency is necessary, but not sufficient for the definition of a new constant, even if consistency would not be perturbed by the definition.

Belnap points out that the deficiency is that the transgression of the boundary of the connectives that are conservative over the base language and that are not. In effect, what Belnap defends is the traditional doctrine that deductively valid inferences have to be non-ampliative.

Belnap points out that we actually define logical constants not only in the locus of introduction and elimination rules, but in "an antecedently given context of deducibility" as well. The problem with tonk is that its definition is incompatible with our assumptions of deducibility. According to him, this context is constituted by the properties of conservativeness of extension and uniqueness. He gives an exemplary constant 'plonk' such that the system will not deduce any new statements that do not involve plonk. Thus, assuming that we have already a consistent system and have all the valid formulas, any new addition will be due to the plonk-axioms and plonk-rules.

Belnap's adjunctive requirement uniqueness is grounded in the inferential roles and their adequate definitions. Suppose we have two constants, say, plonk and plink. If we ascribe

the same inferential characteristics to both of them, then, either plonk and plink are merely naming variations, or there is a systematic fault.

### 3.4 ASSOCIATED PROPOSALS

**Hacking's Setting for the Characterisation.** In [Hacking 1979], Hacking attempts a characterisation of logical constants in a specific setting. Endorses logicism as the ground of his attempt, he takes into consideration logical constanthood not only for its own sake, but also as a key notion into a much broader context of what he calls "analytic programme". The "analytic programme" he puts forward is an exploration onto the boundaries within which mathematics could be analytically derived while admitting that mathematics does not entirely consist in analytic knowledge. His position can be regarded as a version of sub-logicism advocating that mathematics can be shown to be partially analytical in the sense of logicism, remaining noncommittal to the logicist ontology.

According to Hacking, the central notion of logic is deducibility ('⊢'). He points out that while deducibility involves transitions between propositions, logical truth is a characteristic of propositions. In this respect, he appraises Gentzen's systems of natural deduction and sequent calculus as radically important steps taken in the right direction.

The adequacy criteria to demarcate the logical from the extralogical are set down accordingly. There are three criteria –he takes the last one being essential to render the whole enterprise purposeful:

(A1) The demarcation should give the "right" class of logical constants and theorems.

(A2) The demarcation expounded through a characterisation of logical constants should provide the semantics for those constants.

(A3) The demarcation should give an account of the role of logic in the "analytic programme".

The criterion A1 is the statement of extensional adequacy. Hacking contends that the ramified theory of types (including identity relations, excluding the simplified version of the theory) covers basically the entire logic and delineates the intended extension. By

picking out this theory which he locates between first and second-order systems of logic, he comes to terms with the logicist tradition represented by Frege, Russell, early Wittgenstein and Carnap, meanwhile avoiding Quine's ontological criticism of higher-order systems in logic.

A2 is the criterion of semantic adequacy. Hacking contention is that Gentzen's method for first-order logic would provide, in his words, a "do-it-yourself semantics" if Gentzen's operational rules were framed with an abstraction of logical truth and consequence out of a language. Hacking assumes the classical conditions of bivalence for truth and truth preservation such that, for any set of propositions Γ and Θ, $\Gamma \vdash \Theta$ holds whenever all members of Γ are true, at least one member of Θ is true.

The criterion A3 relates to sub-logicism. Hacking has the idea that what makes the characterisation of logical constants an interesting issue is that it enables us to investigate how and to what extent logicist idea could be carried out.

Three relational conditions of sufficiency for (classical) deducibility are identified:

(B1) *Reflexivity*: $A \vdash A$. This condition states the deducibility of the identical propositions.

(B2) *Dilution*: If $\Gamma \vdash \Theta$ then $\Gamma, A \vdash \Theta$ and $\Gamma \vdash \Theta$, A. Multiplying premisses should not alter the deduction; thus, we rule out inductive models.

(B3) *Transitivity*: If $A \vdash$ B and $B \vdash C$ then $A \vdash C$. This is indeed a restricted form of Gentzen's cut-rule: If $\Gamma \vdash A, \Theta$ and $\Gamma, A \vdash \Theta$ then $\Gamma \vdash \Theta$. This condition is crucial to transmit formal warrant from premisses to conclusion regardless of their propositional content.

A crucial point is that these conditions do not invoke logical constanthood; therefore, they can apply also to deductions in languages that do not have logical constants such as one that contains analytic propositions, or elementary propositions in the sense of Wittgenstein's Tractatus. Hacking's strategy is to consider a language in which an antecedent conception of deducibility has been embedded and to investigate how a logical constant could be introduced by means of Gentzen's method to the language.

*Fixing Denotation of Logical Constants*. The two concerns that Hacking tackles are soundness of the resultant system after the introduction of a logical constant and conservativeness of the introduction procedure.

The soundness is alleged to obtain by means of the emergent semantics grounded in the proof-theoretic conditions imposed on introduction and elimination rules of logical constants. The claim is that assuming a semantic framework with bivalence and entailment, any introduction and elimination rules satisfying subformula property, elimination theorems for identity, dilution and cut can accurately confer a denotatum for a logical constant. Hence, supplemented by notions of truth and consequence, all the semantic properties of the constants are supposed to be determined by their syntactic properties.

Hacking states that Gentzen's structural rules correspond to the dilution (left and right) and transitivity conditions of deducibility:

$$\frac{\Gamma \vdash \Theta}{\Gamma, A \vdash \Theta} \qquad \frac{\Gamma \vdash \Theta}{\Gamma \vdash \Theta, A} \qquad \frac{\Gamma, A \vdash \Theta \; \Gamma \vdash \Theta, A}{\Gamma \vdash \Theta}$$

Thus, taking the rule A+ A as one of them in order to be concise, the structural rules specify the syntax of deducibility.

The operational rules provide the inductive steps, increasing complexity of formulas, as shown for the case of conjunction:

$$\frac{\Gamma, A \vdash \Theta}{\Gamma, A \wedge B \vdash \Theta} \qquad \frac{\Gamma, B \vdash \Theta}{\Gamma, A \wedge B \vdash \Theta, A} \qquad \frac{\Gamma \vdash A, \Theta \; \Gamma \vdash \Theta \; B, \Theta}{\Gamma \vdash A \wedge B, \Theta}$$

Returning the adequacy criteria, Hacking comments on A3; we understand that Hacking's argument yields in a way a sort of "hermeneutic circle":

(a) Analytic truth provides the grounds for logical truth.

(b) A logical truth is the one that obtains only in virtue of logical constants that occur in the proposition.

(c) Therefore, a demarcation question for logical constants is raised.

(d) A theory of deducibility reveals that the notion of logical truth is dependent on the notion of truth for a language.

(e) The notion of truth for a language essentially involves analytic truth.

In Hacking's exposition, a formal sketch goes along with the core ideas; nonetheless, it falls quite far from substantiating the defended views —[Sundholm 1981] presents a precise discussion of the technical flaws in his treatment. In [Peacoke 1981], Peacocke argues that there are shortcomings in Hacking's framework arising from the hidden assumptions about extensionality and bivalence. As for bivalence, Peacocke argues that the semantic principle of bivalence is conflated with the logical (or, logical expression of the metaphysical) law of excluded middle, a distinction crucial for intuitionistic logic. On the other side, Peacocke points out that Hacking's proof-theoretic conditions resting purely on extensionality do not suffice to assign a semantic value [69] to a logical constant. Peacocke shows that Hacking's proof-theoretic conditions fail in the face of modulo argument.

The conclusion we draw from Hacking's and his critiques' discussion is that we need more elaboration on both meaning-theoretical and proof-theoretical description of logical constants. We shall dwell on these in due course.

**Peacocke's view**. Peacocke proposes a programmatic approach similar to Dummett's, but taking a realist stance. It is understood from his discussion that a satisfactory philosophical account of logical constanthood is possible with semantical and epistemological considerations. Peacocke pursues is the idea that the conception of logical constants are not only a matter of proof-theoretical properties, but also of "cognitive phenomena involving real thinkers" that "find" obvious or "realise" ranges.

Peacocke attempts to ground the putatively substantial a priori conceptual knowledge in the frame of a general theory of concepts. He holds that there is an a priori meta-level

[69] Adapting Dever's definitions in [Denver 2006] to our context, we take semantic values as entities assigned to syntactic expressions by theories in order to account for semantic features of languages such as truth conditions of and inferential relations among sentences, ambiguity and incoherence of expressions, and to specify systematic interpretations for syntactic categories.

conceptual knowledge for logical consequence. This knowledge, underlying conceptual roles of expressions, incorporates under what conditions we possess a concept and how our concepts are connected to each other. The conceptual role of an expression is what constitutes the cognitive grounds for a speaker to assert or to accept certain sentences and to draw consequences from them. According to Peacocke, the meaning of a logical constant is specified by its conceptual role, that is, its role in inferences. He builds his framework on a general theory of concepts; he notes that:

> I have been trying to provide for the logical constants something which we have eventually every type of concept: an account on which its referential and psychological properties are fully integrated.[70]

Peacocke claims that the psychological notion of primitive obviousness functions as the justification of the inferential transitions in which concepts take part. What is primitively obvious is semantically unanalysable, whatever the cognitive causes or reasons of this state are. For example, one cannot find anything further to individuate in order to compose $\varphi \wedge \psi$ other than $\varphi$, $\psi$ and the connective '$\wedge$' in a given context and her apprehension of $\varphi \wedge \psi$ leads her to apprehend that both $\neg\varphi$ and $\neg\psi$ are incompatible with it. This kind of "obviousness" is what the speaker perceives as the end of semantic compositionality. It appears that primitive obviousness is related so much to a predeterminate cognitive capability as to language competence. As for negation, Peacocke says that

> What is primitively obvious to anyone who understands negation is just that $A$ is incompatible with $\neg A$. Unless the ordinary user of negation appreciates that $A$ and $\neg A$ cannot be both true, then he does not understand $\neg$.[71]

A concept determines the meaning of an expression only if there is a semantic value that can be attributed to it. Another way to understand this view is to consider the Fregean sense-reference pair. Concepts are similar to senses in that they constitute the ground component of meaning and take on a semantic value by corresponding to a referent.

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[70] [Peacocke 1987], p. 165 (emphasis added).

[71] *ibid.*, p. 163.

The specificity of the standard logical connectives is that we find them "obvious". They are normative in that they are determined by their conceptual role. This conceptual role determines truth conditions and justifies the introduction and elimination rules of the connective. Thus, for example, what determines the semantic value of 'and' is that function which warrants the coincidence of the semantic values of the conjunctive expressions with the instances which we find primitively obvious. It turns out that to understand logical constants is to understand these principles about them, which subsequently yields a truth-preserving form.

Peacocke gives a semantic formulation for conservativeness, making a distinction between the canonical grounds of validation for some content and the range of canonical grounds for that content:

> It is one thing to be sensitive in one's judgements to a content's possession of a certain range of canonical grounds. It is another, further, thing to come to realize by reflection that those are all the canonical grounds for that content, that there are no others.[72]

The logical constants are introduced upon the canonical grounds. If the range of canonical grounds is exhausted, then the constant is ascribed to a semantic value that is the strongest in entailment capability. This provides an equilibrium principle over all possible statements: Either a rule for a logical constant is primitively obvious, so we can introduce it, or it has the strongest semantics, so we can eliminate it.

While Peacocke deploys elements of intuitionism like Dummett, he subscribes to a realist theory of truth as opposed to his. From this aspect, Peacocke's proposal is a counter example to the view that the metaphysical issue of realism and anti-realism is intimately connected to the issue of logical constanthood.

As opposed to our framework in which truth is a property of propositions, we understand from Peacocke's discussion that truth is regarded as a property of inferential cognitions (among other types of cognitions). A concept exhibits a cognitive object that can be shared among speakers. Therefore, any two competent speakers possess the same concept of a logical constant. Assuming that such cognitions are reflected in speech with a *bona fide*

[72] *ibid.*, p. 170.

manner, it would be more appropriate in Peacocke's conception to employ the term veridicality, rather than truth, as what is preserved in a deduction.

In a more general setting, it leads to intractable series of conjectures to characterise logical constants as linguistic labels for some kind of mental content that is certified by correspondence to extra-linguistic truth *cf.* our discussion on Davidson's view resting on belief). Peacocke's view resting on conceptual roles of logical constants does not overcome this categorical problem. Concepts do not bring about normativity. For example, I may organise my conceptual content about human body as if it were a clock: It is *my* concept of human body; there is *no my* meaning of human body. You may organise your conceptual content about human body as if it were a computer network and it is your concept of human body; there is *no your* meaning of human body. *My* and *your* concepts may have overlap to a lesser and greater extent, and in an extended sense, we may talk about our (or the) concept of human body. However, in contrast to concept, meaning consists in how the expression 'human body' is woven into *our* activities; it is a linguistic entity. We may refer to Wittgenstein's private language argument in this connection. I might have something that would be replica-like in whatever ways we admit to call it a language entirely on my privacy. But it would not be a language, because, roughly speaking, I would command it, but it could not command me.

We may examine [Carrol 1895] from this respect. Alluding to Zeno's paradox, Carroll presents a dialogue between the Tortoise and Achilles on an Euclidean argument. We may schematise Carroll's narrative as follows:

The Tortoise gives a sequence of three propositions such that

(A) $\alpha$

(B) $\beta$

(Z) *Ergo*: $\gamma$

where the proposition $\gamma$ in Z is patently a logical consequence of the propositions $\alpha$ and $\beta$ in A and B. Then, it asks Achilles to take into account that someone might affirm A and B, but not the hypothetical proposition 'if A and B are true, Z must be true', because there

might be someone who affirms Z, however, neither A nor B. Thus, it compels him to insert a proposition at C:

(A) $\alpha$

(B) $\beta$

(C) If A and B are true, Z must be true.

(Z) *Ergo*: γ

Then, another proposition needs to be inserted at D such that if A and B and C are true, Z must be true. Thus, the sequence turns out to be an infinite regress in the scheme that an inference requires an inferential rule to be followed and an inferential rule involves another one in order to be obeyed.

Apparently, Carroll's idea is that intensionally binding the inference rules with the propositions involved in inferences leads immediately to profound difficulties, as we understand his explanation to the Editor of *Mind*:

> My paradox … turns on the fact that in a Hypothetical, the *truth* of the Protasis, the *truth* of the Apodosis, and the *validity of the sequence* are three distinct propositions.[73]

Smiley points out that an argument "It's Tuesday, so this must be Paris" can well be evaluated sound in connection with a traveller's timetable. In case that the scheme "*A*; so *B*" is challenged, we have two strategies: We may augment the argument supplying either a premiss *P* and obtain a scheme "*A*, *P*; so *B*" or an inference rule *R* and obtain "*A* implies *B* by *R*". The significant point for our discussion is that the first strategy meets the challenge with an augment of conceptual content, whereas the alternative strategy brings about a formalisation which cannot be taken as a variant of the former augment. That is why the distinction (or, privileged role) of logical constants should be sought in formalisation.

[73] Quoted from [Smiley 1995], p. 725.

**Harman's conception**. Harman draws our attention to the difference in meanings of logically equivalent connectives.[74] For example, consider the following (in Harman's notation with minor modification):

'P and Q' ≡ 'not((not P) or (not Q))'

Harman points out that the meaning of left-hand side cannot be said to coincide with the meaning of the right-hand side. Another example he gives is from Gentzen-style definition of disjunction ('D') and negation ('N'). The introduction rule for disjunction can be given with two clauses:

P implies D(P, Q)

Q implies D(P, Q)

Therefore, we can write:

D(P,Q) is true if P is true or Q is true.

As for the elimination rule, Harman states that the disjunction D(P, Q) by itself does not logically imply either of its disjuncts, P or Q, although it implies one of its disjuncts given also the negation of the other disjunct. Thus, an elimination rule could be:

D(P,Q), N(P) logically imply Q

D(P,Q), N(Q) logically imply P

However, this rule appeals to one logical constant (*i.e.*, negation) while defining another (*i.e.*, disjunction). It is ambiguous whether disjunction is being defined, or negation, or both. The definition also lacks generality, since it would not work for a language containing disjunction but lacking negation.

So it is customary in systems of natural deduction to adopt a more complex rule:

[74] See [Harman 1986].

IF P and certain other assumptions logically imply C,

AND Q and those other assumptions also logically imply C

THEN D(P,Q) and those other assumptions logically imply C.

However, this yields an elimination rule that is not just another way of expressing part of the truth conditions for disjunction.

Harman concludes that although different logical constants make the same contribution to the truth value of logical statements, they cannot be said to have the same meaning. Therefore, an appeal to the implicit definition of logical constants through their association with truth conditions such as it is done in the Tarskian model theory cannot be satisfactory. According to him, what is important is "which implications (and perhaps also which exclusions) are immediate". He concludes that "there is no argument here for thinking that truth conditions are more relevant to the meanings of logical constants than to the meanings of nonlogical predicates".

What are the pros and cons of Peacocke's and Harman's conceptions for our quest? Peacocke's discussion can be taken as an attempt to address Belnap's observation that "we are not defining our connectives *ab initio*, but rather in terms of an antecedently given context of deducibility, concerning which we have some definite notions" in terms of concepts. But we see that logical concepts dissolve among other concepts and there is virtually no discriminating property except for that they have inferential roles. As we have seen in our treatment of categorematic/syncategorematic division of terms, the Scholastic philosophers have noticed —and Harman's discussion partly supports— that this cannot stand as a criterion just by itself; that is why we try to be cautious employing different terms as inference and deduction. Other major problems follow: How do logical concepts veridically externalize themselves? What is the justification for the deductive transitions?

We think that a satisfactory characterisation of logical constants should involve an account of the relation of what is in logic to what is non-logic and be answerable to the question of what logic means for the non-logic. It seems that Peacocke's theory of concepts harbours too many problems to pass as a characterisation of logical constants.

The import of Harman's discussion is its clarification that an implicit definition through truth conditions of logical statements does not suffice to demarcate logical constants. But it tackles with the perplexing problem of demarcation by resorting to "immediateness" just as Peacocke does to "obviousness". A common feature of their conceptions is that the force of semantic notions such as truth and validity in the definitions of logical constants are deemphasised. But logic is a formal and normative discipline; it is neither a simulation, nor a description of reasoning, and language is not so transparent as Peacocke and Harman see it. Thus, both psychological terms remain *ad hoc* in the present quest.

It should be remarked that we do not assert that the realm of concepts is irrelevant to language, but the relation is a kind of supervenience which leaves a space to language for an opaque autonomy.

Though we will not delve into the dependency relations between language and other realms relevant to it for our present quest, we may suggest a picture of the basis of autonomy as a complement to our view: If a sentence is true or false (and a sentence may not be bivalent and there may be a sort of indeterminacy), it may be true or false in virtue of some non-linguistic realm of facts. But language is underdetermined by that realm of facts; hence, the totality of non-linguistic facts is not sufficient to determine a truth-value for each sentence. In this respect, autonomy of language provides us with a way to judge logicality without falling into a maze of indefinite regress. In the logical properties of the autonomy of language, we reach the boundary of idempotence:

**Došen's Proposal**. Došen's proposal amounts to the conception of the rules as analytic devices of a metalanguage for object language arguments and logical constants that serve as translational devices between the metalanguage and object language.[75] Došen depicts an interpretation of Gentzen's theory as a view of logical constants. He explicates of his proposal on consecutive assumptions and draws two theses from them. Since the assumptions and theses are put down quite succinctly, let us quote them in succession and then give an exposition of the underlying ideas:

(a) Logic is the science of formal deductions.

[75] See [Došen 1994].

(b) Basic formal deductions are structural deductions.

(c) Any constant of the object language on whose presence the descriptions of a non-structural formal deduction depends can be ultimately analysed in structural terms.

(d) Logic is independent of subject matter.

(e) The level of discourse of logic is higher than the level of discourse in which we treat of a particular subject matter relying on logical principles.

(Thesis I) A constant is logical if, and only if, it can be ultimately analysed in structural terms.

(Thesis II) Two logical systems are alternative if, and only if, they differ only in their assumptions on structural deductions.

Došen's prototype for deduction is the Gentzen-style proof-theory. What counts as deduction is explicated by the term 'structural' in the sense that is exemplified in Gentzen's sequent system. Thus, we have an object language which has its own items such as constants and in which premisses and conclusions of arguments are ordinarily expressed. In a metalanguage, deductions are structured into schemas expressed independently of the constants of the object language (*i.e.*, a logical form is inscribed on the expressions of the object language). Such deductions are entitled to be called formal.

A class of deductions constitutes the basic elements by combination of which compound deductions are built (from the metalinguistic aspect, the expressions of the object language are "analysed"). This analytic/constructive function pointing to the relevant structural features of object-language arguments is accomplished by the logical constants in the metalanguage. Because of this function, Došen depicts logical constants metaphorically as "punctuation marks of the object language". Hence, we take an expression $\alpha$ in an object language L as *analysandum*. There is a metalanguage M such that $\alpha$ does not belong to M and M is capable of presenting an *analysans* for $\alpha$. An analysis consists of establishing that a sentence $\sigma$ in $M \cup \{\alpha\}$ ($\alpha$ occurs once) is equivalent to a sentence $\sigma$ in M.

It is required that the language M is more "basic" than the language L, in the sense that M is simpler with fewer assumptions than L accommodates, and that M is prior to L in comprehensibility.

The properties of conservativeness and eliminability which are standard requirements for an admissible definition are not sought for an analysis in this context, but Došen's conception of analysis is subjected to other conditions.

By the analytical truth in L it is understood that its recognition requires essentially only a competent knowledge of the language L. In this sense of analyticity, M enables us to capture the completeness such that we infer every analytically true sentence of L, and the soundness such that we do not infer any sentence of L not analytically true in L. Besides, it is stipulated that if any two expressions $\alpha_1$ and $\alpha_2$ are analysed, then they have the same (unique) meaning. As an illustration, Došen gives an analysis (in his sense of the term) due to Ramsey stating the core of the redundancy theory of truth. Consider the statement

'A' is true if and only if A.

L is English (or an appropriate fragment of it), α is the predicate 'is true', and M is a fragment of English without α and with the schema A to express the sentences of L. Notice that α is assigned to an equivalent, not an interchangeable, expression.

*Prima facie*, Došen's proposal might seem to be a heuristic device to think about Gentzen-style, because what he does is to expose a way of seeing the actual formalism currently in use, rather than aiming at an explanation and justification of them. We shall present another construal of his proposal that could be more illuminative for our framework.

Došen's correlation between metalanguage and the underlying object language can be stated as a kind of supervenience; the property of being a deduction in the metalanguage supervenes on the properties of inferences in the object language, the subvenient base.

The property of being a deduction cannot be reducible particular properties of inferences in the object language. Thus, we cannot talk about a type–type identity. However, each instance of the property of being a deduction is an instance of a compound of inferential properties and relations; hence, we can say that there is token–token identity.

Inference and deduction have their own structures. But an inference has a structure out of indefinitely many variations. In contrast, as Došen emphasises, it is in the essence of deduction.

Another distinction is in the modal properties. While deduction is alleged to bear necessity, the same cannot be asserted for inference.

In this perspective, Došen's proposal does not involve any discussion of what kind of dependence metalanguage has on object language.

It appears that Hacking's and Došen's proposals expand on Gentzen's presumption that introduction rules give the meanings of logical constants and each offers an implementation of it. Nevertheless, they do not go much farther than that and leave the key issues untouched. But they serve to illuminate where the genuine problems lie, and we shall dwell on these.

There are two major features common to Hacking and Došen. The first one is that both take the sequent calculus as a prototypical metatheory to specify deducibility relations and formalise ordinary inferences, making them closer to the view that logic is more a domain of certain inferences than a domain of certain truths. In effect, the Gentzen-style schemes are the constraint set for logical constants. The second feature is that they presuppose analytic truths in their settings.

Hacking's and Došen's proposals reaffirm the view that operational rules for the logical constants are not exhaustive of their meanings, even supported by some background assumptions. Indeed, those vocabulary items are intertwined with a form of life to be comprehended and their employment requires a specific mastery of techniques. But we should draw attention to the point that we cannot attain an improved understanding of logical constanthood merely by such general considerations.

Another point to take into account is that rules are, in general, both normative and constitutive. In these proposals, the normative feature is underestimated while the other feature is overemphasized. We observe the resultant ambiguity in the instance of modal

logic: Hacking's criteria exclude the modal operators from being proper logical constants, whereas Došen's criteria include them.

It should be clear that without a framework to embed such rules, the decisions become arbitrary and reading off meanings from formalisms leads to a "symbol mysticism" analogous to the Pythagorean number mysticism. There are divergent strands as to how to frame the notion of logical constanthood by dominating rules. For instance, according to Kneale, the rules can be treated as definitions, whereas Hacking conceives of the rules "not as defining but only as characterizing the logical constants", and Došen claims that the rules unfold an "analysis", neither a definition, nor a characterisation.

Those proposals lack the semantic resources to account for logical constanthood and resort to cognitive terms. Došen's foundation is "ultimately analysed in structural terms", whereas Harman resorts to "immediateness" just as Peacocke does to "obviousness". A common feature of their conceptions is that the force of semantic notions such as truth and validity in the definitions of logical constants are deemphasized. But logic is a formal and normative discipline; it is neither a simulation, nor a description of reasoning, and language is not so transparent as Peacocke and Harman see it, leaving psychological terms remain *ad hoc* in the present quest.

We shall focus on the idea is that logical constanthood has to be characterised by the inferential rules they are subject to. This is the credo of inferentialism in general. We shall continue with Dummett's expansion on it.

## 3.5 DUMMETT ON LOGICAL CONSTANTS

Dummett's main idea is that metaphysical vision is essentially a construction of theory of meaning. He contends that a dispute on a metaphysical proposition hinges on how the issue of truth-conditions for a proposition is conceived and what to understand from the grounds to assert a proposition. In pursuit of the idea, Dummett sets out for a programme to solve the realism/anti-realism controversy based on a cogent explanation of the fundamentals concerning a theory of meaning. Logic has a significant role in this assertive programme, both as a domain to explicate Dummett's theses and as a spin-off of his theory of meaning supporting the general argument.

As many of his commentators attest, Dummett's discussion of harmony is rambling and contains conflicting statements and fine ideas side by side.

**Dual-Aspect Theory of Meaning Ascription**. A basic thesis is that "'the concepts of meaning and of truth can only be explained together". An adequate theory should explain how the composition of a sentence determines the truth-value of that sentence from the meanings of its constituents. Dummett argues that truth-conditional theories of meaning that incorporate a preconceived idea of truth, such as Davidson's theory of meaning and truth, fails in giving a satisfactory account of semantic links in compound sentences. A theory of truth assigns truth-values to the object-language sentences as dictated by the metalanguage in which semantic relations are defined. Therefore, a theory of truth cannot validate an inference by itself, a proper semantic theory, which has to be a prime component of a theory of meaning, is needed as well.

Dummett presents general considerations about language and meaning, concluding that two principles, intuitionism and anti-realism, should be supported to construe a coherent view of them. Let us briefly review them for the sake of completeness:

*Intuitionism*. Intuitionism is a stronger form of constructivism. Constructivism asserts that mathematical objects are essentially some constructions of the mind; therefore, we cannot coherently speak about them if it is assumed that they have an independent existence from the mathematical activities of the human being.

In contrast to the realist view of the foundations of mathematics (the mainstream version of which is the mathematical Platonism), constructivism claims that the mathematical objects are sensible by their constructibility through secure stages. Since the actual praxis of mathematics involves indefinitely many transcendent ideas and methods, constructivism is inevitably of highly restrictive character. Constructivist defence is that if a determinate answer to a question appears not be possible, at least in principle, then any answer is a matter of faith, rather than of fact.

The formal ground of constructivist critique is the paradoxes derived from the classical set theory. One instance is the Russell's paradox against Frege's comprehension axiom as '$\exists y \forall x (x \in y \leftrightarrow \varphi(x))$', where $\varphi$ is any property determines a set unconditionally (the set $y$

of all objects $x$ which poses the property $\varphi$). Another instance is the power set axiom which allows power sets of infinite sets to exist. Thus, it allows arbitrary infinite sets to exist, with infinite cardinalities of increasingly higher order.

Thus, a conception of mathematical truth is introduced grounded on constructive provability. Truth-value of a mathematical statement can only be specified through constructibility, since there can be no truth independent of what can be proved, at least in principle.

Consequently, the indirect method of proof (proof by contradiction) is no longer valid for existential claims, since neither $A$ nor $\neg A$ is provable constructively. Thus, the double negation elimination rule ($\neg\neg A \vdash A$) and any equivalent or any derivative of this rule turns out to be applicable only to nonexistence claims or similar negative assertions. Therefore, the Law of Excluded Middle ($\vdash A \vee \neg$A) drops out, too.

A parallelism can be perceived: It is as inferences to meaning for inferentialism as it is proofs to sensibleness for intuitionism. This may cause a tendency to put proof theory and inferential paradigm in opposition to model theory and referential paradigm. The matter is not so straightforward; we only remark that we disagree with this view and leave the issue to pursue to another text.

*Anti-realism.* Dummett associates intuitionism with the semantic anti-realism. A basic point Dummett claims is that the conditions to decide on the truth-value of a sentence conclusively may be beyond the boundaries of knowledge in some cases that Dummett calls “verification transcendent”. It is noteworthy that, with the support of his anti-realist view, Dummett-Prawitz inferentialism leads him to take a reformist stance tending to intuitionistic logic. He claims that truth-conditional theories of meaning do not meet this requirement, since there are sentences such that it is indefinite or beyond our limits to detect whether their truth-conditions obtain or not. This is the basis for his preference for anti-realism.

Such a formulation demands an understanding of the meanings of logical constants and a justification of the logical laws by means of which logical constants are defined. Dummett asserts that verification and validation of logical laws can be accomplished by a semantic

theory that successfully bridges logic and the theory of meaning. Eventually, a semantic theory itself can be justified by a theory of meaning. Thus, a chain of justification is built parallel to the thematic framework for metaphysical disputes.

Dummett argues that a workable account of meaning can be provided by verificationism and pragmatism taken jointly, corresponding to two fundamental aspects of the use of an expression in language: The grounds for the use of an expression, and the consequences of using it. Verificationism gives an account of meaning of a sentence in terms of its verification conditions, while pragmatism does in terms of its consequences:

> Our immediate concern is not with the question which, if either, of these aspects of our use of sentences should be taken as the central notion of the meaning-theory, with the mere fact that linguistic practice has these two aspects.[76]

There are two essential aspects of speaker's use of a sentence: The grounds that warrant the assertion of a sentence (the verificationist component of a meaning theory) and the consequences that follow from the sentence (the pragmatist component of a meaning theory). In order to conduct a proper practice of language, the speaker should grasp the grounds that warrant assertions and consequences drawn from it. Notice that we shift from a compositional meaning theory based on truth conditions to a compositional meaning theory based on proof conditions.

According to Dummett, the verificationist and pragmatist aspects of use find counterpart in the domain of logic by the introduction and elimination rules of logical constants. The introduction rule specifies the inferential role of the logical constant in the sentences containing it, whereas the elimination rule does in the conclusions. We justify logicality by these two aspects of language use accompanying the notion of truth. The meaning of a logical constant is stipulated by the compositional roles in specifying the truth conditions of compound sentences of language.

The verificationist and pragmatist aspects, application-conditions of a given assertoric expression and the consequences of applying it, find counterparts in the domain of logic as the Gentzen-style introduction and elimination rules of logical constants. There are two

[76] *ibid.*, p. 214 (emphasis in the original).

dominant categories for the use of statements. These categories are intended to apply generally to all kinds of expressions in language:

> The first category consists of those that have to do with the circumstances that warrant an assertion, the basis on which we may recognize as having been established. There is multiplicity within this category, according as we are concerned with when an assertion is conclusively established, or with what merely warrants its being made, though defeasibly: but principles of both these kinds fall within the same broad category. Plainly, such principles form an important part of what have to learn when we acquire language: we need to know when we are entitled to make any given assertion, and when we are required to acknowledge it as true.[77]

We see this dual-aspect theory of meaning analogues of entitlement and commitment statuses of normative inferentialism. Dummett continues:

> In acquiring language, we learn a variety of principles determining the consequences of possible utterances; these compose the second of our two categories of principles that govern our linguistic practices.[78]

Dummett's central idea about the characterisation of logical constants is harmony that is to obtain between inferential rules of a logical constants and the relevant logical system. The introduction and elimination rules must be such that no more than what is allowed to be deduced directly from the premisses can be drawn as a conclusion.

Dummett thinks that a harmony must hold between assertion (verification) conditions and the (pragmatist components of a meaning theory) consequences of the assertion; likewise, between introduction and elimination rules. We understand that being in harmony is the *modus operandi* of natural language and any disharmony distorts the proper working of linguistic practice and the two aspects of use of language are complementary to each other. Therefore, they are somehow dependent on the other.

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[77] *ibid.*, p. 210 *ff.*

[78] *ibid.*, p. 212.

The constraint of harmony is what prevents the derivation of excessive conclusions "not warranted by our methods of arriving at the premises" (another term could be 'local soundness') and discards such ill-founded constants like 'tonk':

> The notion of harmony is difficult to make precise but intuitively compelling: it is obviously not possible for the two features of the use of any expression to be determined quite independently. Given what is conventionally accepted as serving to establish the truth of a given statement, the consequences cannot be fixed arbitrarily; conversely, given what accepting a statement as true is taken to involve, it cannot be arbitrarily determined what is to count as establishing it as true.[79]

A notion at the base of harmony is local maximum (or 'local peak', a topographical metaphor Dummett deploys). A local peak for a constant ξ is any part of derivation where a ξ-introduction rule is immediately followed by a ξ-elimination rule. Dummett demands that maximal formulas in which ξ occurs as the main connective can be removed from deductions for harmony to obtain between introduction and elimination rules of a logical constant ξ. For example, the following derivation with ∧-introduction and ∨-elimination constitutes a local peak:

$$\dfrac{\dfrac{\begin{matrix}\vdots & \vdots \\ \varphi & \psi\end{matrix}}{\varphi \wedge \psi}\ \text{Elimination}}{\varphi}$$

The local peaks can be removed rewriting the derivation without the introduction-elimination pair, as in the above instance, deducing the formula $\varphi$ again. As the final outcome of successive reductions (*viz.*, the procedure of normalisation, or continuing with Dummett's topographical metaphor, levelling of local peaks), a normal (peak-free) form can be obtained.

> A conservative extension in the logicians' sense is conservative with respect to formal provability. In adapting the concept to natural language, we must take conservatism or non-conservatism as relative to whatever means exist in the language for justifying an assertion or an action consequent upon the

[79] [Dummett 1995], p. 215.

> acceptance of an assertion. The concept thus adapted offers at least a provisional method of saying more precisely what we understand by 'harmony': namely that there is harmony between the two aspects of the use of any given expression if the language as a whole is, in this adapted sense, a conservative extension of what remains of the language when that expression is subtracted from it.[80]

Harmony obtains if the grounds for asserting a proposition match the consequences of accepting it. In another perspective, it lays down a principle of encapsulation, or in Read's words, autonomy:

> But the philosophical importance of harmony is the autonomy which it confers on the logical constants. The guiding principle of a "proof-conditional theory of meaning" is that if the meaning of a logical constant is solely *i.e.*, completely) given by its introduction-rule(s), then one is entitled to infer from a formula containing it no more and no less than one can infer from the grounds for its introduction (assertion). All indirect proof reduces to direct proof. Such constants are self-justifying and autonomous. Their meaning is fully contained in the introduction-rule.[81]

Considering Belnap's discussion, Dummett holds that intrinsic harmony is not sufficient and what he calls total harmony should be another requirement:

> We may continue to treat the eliminability of local peaks as a criterion for intrinsic harmony; this is a property solely of the rules governing the logical constant in question. For total harmony, however we shall demand the addition of that logical constant produce a conservative extension of the logical theory to which it is added.[82]

In the light of these, we can state the following:[83]

*Total harmony.* We say that a logical constant ξ adjoined to a language L is in total harmony with the language L if the language is extended by the adjunction of ξ is a conservative extension of L. A method of preserving consistency is conservative extension in two ways.

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[80] [Dummett 1995], p. 218. *f.*

[81] [Read 2000], p. 131.

[82] [Dummett 1995], p. 250.

[83] For a lucid explanation of formal details of normalisation, see [van Dalen 2008], chap. 6.

First, the properties of the base-language formulas are conserved; for example, those formulas which are "synonymous" according to the base consequence relation retain this property also with respect to the consequence relation of the extended logical system. Second, the adjoined logical constant is defined to be in accordance with the existing system. Belnap puts these as

> The justification for unpacking the demand for consistency in terms of conservativeness is precisely our antecedent assumption that we already had all the universally valid deducibility-statements not involving any special connectives.[84]

*Intrinsic Harmony*. Dummett takes intrinsic harmony as "a property solely of the rules governing the logical constant in question". The idea of intrinsic harmony is based on the inversion principle, introduced by Lorenzen and developed by Prawitz.[85] Prawitz formulates the principle as follows:

> [A]n elimination rule is, in a sense, the inverse of the corresponding introduction rule: by an application of an elimination rule one essentially only restores what had already been established if the major premiss of the application was inferred by an application of an introduction rule.

Let us supplement the presentation of this line of thought with Prawitz's considerations in order to make the involved ideas more perspicuous:

Prawitz marks a characteristic that has remained untouched in Gentzen's work, that eliminations are inferences as well as introductions. He states that eliminations can be justified by showing them to be valid in reference to the meaning content of the sentences involved in the argument via the *canonical* proof of the conclusion of the elimination. Then, according to him, the task should be to lay down the considerations as to what it is for a non-canonical inference to be valid as well as a canonical inference –Prawitz claims that the canonical form of a sentence determines its meaning (explained by the verificationist idea that the meaning of a sentence is manifested by the canonical derivation of it). We shall take a look at how he sets out to develop a notion of validity applicable to showing the

[84] [Belnap 1962], p. 132.

[85] See [Moriconi and Tesconi 2008] for the evolution of the principle.

validity of certain legitimate forms of inference. First, we make a record of Prawitz's terminology:

Argument skeleton is a set of formulas in a tree form. It is to be indicated for each top sentence of the tree whether it follows from null premisses or it is inserted into the argument as an assumption. If the sentence is an assumption, then it is bound to be discharged at a certain step which is to be indicated. An inference can bind only the occurrence of an assumption or a variable in the section that is above its conclusion in the tree. If an assumption or a variable is not bound is said to be free. The following are the examples for variable and assumption:

$$\frac{\begin{array}{c}\mathcal{D}\\ A(x)\end{array}}{\forall x A(x)} \qquad\qquad \frac{\begin{array}{c}[A]\\ \mathcal{D}\\ B\end{array}}{A \to B}$$

If all occurrences of assumptions are bound and likewise all occurrences of variables that are free in the formulas are bound in an argument skeleton, then it is said to be closed, or else, it is said to be open (an open skeleton can be understood as a schema). An inference in which all the terms and assumptions are closed is said to be an appropriate instance, preserving validity. An open argument can be denoted by $\langle \mathcal{D}, \mathcal{J} \rangle$ where $\mathcal{D}$ is a derivation and $\mathcal{J}$ denotes the relevant justifying operations. An argument skeleton which ends up at an introduction is said to be in *canonical form*. There are forms of arguments for each sentence to count as canonical.

Building on these concepts, Prawitz states the principles of validity as follows:

(a) An open argument is valid iff all its appropriate instances are valid, that is, iff all those instances $\langle \mathcal{D}' \; \mathcal{J}' \rangle$ are valid where $\mathcal{J}'$ is a consistent extension of $\mathcal{J}$ and $\mathcal{D}'$ is an appropriate instance of $\mathcal{D}$.

(b) A closed argument in canonical form is valid iff its immediate subarguments are valid.

(c) A closed argument not in canonical form is valid iff it reduces to a valid argument in canonical form.

The intended interpretation is that the use of an introduction rule in the canonical (direct) proof of a sentence preserves validity by the meaning of the inferred sentence, and as for the case of a non-canonical argument, a sentence is valid (*viz.*, proved) whenever it is reducible to a valid canonical one. Prawitz notes that the three principles jointly constitute an inductive definition of the notion of validity (on logical complexity), the induction base being a set of valid canonical arguments for atomic sentences.

To be counted as a direct verification (viz., canonical proof), an argument is to satisfy not only the schematic requirement that it ends up at an introduction, but the condition of validity as well: The validity of the conclusion step is evident; iteratively, its immediate subarguments are to be valid also. Hence, the direct verification of a compound sentence consists of a chain of valid arguments in canonical form. Thus, the meaning of a compound sentence is constituted by appealing to the inductive definition of what is to be valid for it.

Prawitz stipulates a validity definition for an argument without referring to semantic notions such as truth, satisfaction in Tarski's sense, and independent of formalisation of derivability (*i.e.*, system-independent). The closed arguments consisting of introduction rules are distinguished as canonical and deemed as valid. Any other argument is evaluated to valid when there is reductive procedure to bring it to a canonical form. A particular result of Prawitz definition of validity is that it is not possible to derive $A \vdash \neg A$ either by a canonical argument or an argument reducible to a canonical argument, so the classical Law of Excluded Middle ($\vdash A \vee \neg A$) is not a valid, which imposes intuitionistic logic.

Prawitz tenuously identify the meaningful with the canonical and the canonical with the base of valid. To see that the conceptual connections that as Prawitz spells out cannot be accepted without a substantive argument, consider the collection $R$ defined as $\{x \mid x \in \mathrm{x}\}$ that leads to Russell paradox. As it stands, it has a well-formed expression in ZFC, since it complies with the intensional scheme $\{x \mid \varphi(x)\}$ where $\varphi$ denotes a property. However, it manifests a contradiction when it is forced to be a member of a collection defined on itself (*i.e.,* when a procedure is applied) and it turns out that it is not a set. Nonetheless, this does

not bar *R* still being a meaningfully definable object. Because of this fact, any collection, either in the set-theoretic realm or not, comes to be called a class. If a class is not a set, then it is called a proper class. Examples for the proper classes include the set-theoretic realm (the universe of all sets) V, the classes of all ordinal numbers and all cardinal numbers as well as the class of all algebraic groups. We see that even a contradiction renders a particular expression invalid for a theory, but may not strip a coherent meaning from it.

*Tennant's model*. Tennant aspires to demonstrate, evidently which he is conceptually convinced, that there are currently three facets of the balance notion:

(a) Conservative extension by the introduction of new operators,

(b) Reduction (normalisation) procedures,

(c) Harmony framed by the strength of conclusion and the weakness of major premiss.

Admitting that he has not accomplished his aspiration yet, he offers strength and weakness conditions for a logical constant ξ as for (c):

(S) $A \; \xi \; B$ is the strongest proposition that be inferred as a conclusion under the conditions described by ξ-introduction.

(W) $A \; \xi \; B$ is the weakest proposition that can stand as the major premiss under the conditions described by ξ-elimination.

He illustrates the idea of squeezing by strength and weakness of propositions by the case of conjunction. For this, we assume the propositions $\varphi$ and $\psi$. '$\varphi \wedge \psi$' is the strongest proposition derivable from $\varphi$ and $\psi$, since '$\varphi \wedge \psi$' has the power to deduce any proposition θ that is derivable from them:

$$\dfrac{\dfrac{\varphi \wedge \psi}{\varphi} \qquad \dfrac{\varphi \wedge \psi}{\psi}}{\theta} \; (\wedge\text{-E})$$

Simultaneously, '$\varphi \wedge \psi$' is the weakest proposition derivable from $\varphi$ and $\psi$, since any proposition $\theta$ that has the power to deduce $\varphi$ and $\psi$ can also deduce '$\varphi \wedge \psi$':

$$\dfrac{\dfrac{\theta}{\varphi} \quad \dfrac{\theta}{\psi}}{\varphi \wedge \psi} \ (\wedge\text{-I})$$

A formal treatment of Tennant's conception of harmony with a purported counterexample involving the introduction and elimination rules of existential quantification is given in [Steinberger 2009]; we shall treat it conceptually. Tennant's conception can be regarded as an instance of the "entailment model". In an entailment $A \rightarrow B$, $A$ is a sufficient condition for $B$, hence $A$ is strong enough to entail $B$. Conversely, $B$ is only a necessary condition for $A$, hence $B$ is too weak to entail $B$. Likewise, ξ-introduction is to involve sufficient conditions for ξ, while ξ-elimination is to involve necessary but not sufficient conditions for ξ. Tennant makes an analogy to Nash equilibrium in game theory and harmony —an ideal solution for the relation between introduction and elimination rules.

We tend to think that an argument form is progression from premisses to conclusion, which substantiates the understanding that there is an asymmetric relation of inference between premisses and conclusion. Since this relation is presumed to be reflexive and transitive as well, it is as if there were a partial ordering relation, like $\leq$, between premisses and conclusion over which we would proceed, like we proceed on the number line. But order of what? Even the most plausible candidate to this question, *i.e.*, the complexity of formulas, falls short of generality.

A view alternative to Prawitz's normalisability criterion is the *general-elimination (GE) harmony*.[86] GE-harmony view contends that the correct proof-theoretic correlate of harmony is signified by the inversion-principle and aims at the dependence of the specific form of an E-rule on the corresponding I-rules. Notice that on this view, it is taken into account that a logical constant may have multiple introduction rules (as it the case for ∨-I). To see its main idea, suppose that we have $\Pi_1$ and $\Pi_2$ (which can be multiplied) as the

[86] See [Read 2010] and [Francez and Dyckhoff 2012].

grounds for the assertion of some formula with the main operator ξ and a list of operands $\bar{a}$. $\Pi_1$ comprise subproofs as follows:

$$\frac{\pi_{1,1} \quad \pi_{1,2} \quad \pi_{1,3} \quad \cdots \quad \pi_{1,n}}{\xi\bar{a}}$$

Similarly, $\Pi_2$ has the following scheme:

$$\frac{\pi_{2,1} \quad \pi_{2,2} \quad \pi_{2,3} \quad \cdots \quad \pi_{2,m}}{\xi\bar{a}}$$

Then, instead of deducing from the assertion of $\xi\bar{a}$ what $\Pi_1$ and $\Pi_2$ allow, we can directly carry on with $\Pi_1$ and $\Pi_2$ and derivations of $\gamma$ from any of the grounds that establish $\xi\bar{a}$ and discharge the assumption of those grounds and get $\gamma$, since we can derive $\gamma$ from $\xi\bar{a}$. This view gives a general elimination rule:

$$\frac{\begin{array}{cccc} & \pi_{1,1} & \cdots & \cdots & (\pi_{2,m}) \\ & \vdots & \vdots & \vdots & \vdots \\ \xi\bar{a} & \gamma & \gamma & \gamma & \gamma \end{array}}{\gamma}$$

GE-harmony rejects both conservativeness and normalisability as correlates of harmony. In fact, it does not provide normal form in general, contrary to Dummett's conception of intrinsic harmony. As we shall discuss below, we agree with the GE-view on the issue of conservativeness. As for normalisability, in regards that we sustain methodological pluralism, we see that there are various systems to inspect from this aspect before a judgement on GE-harmony and normalisability can be made.

## 3.6 ON THE NOTION OF HARMONY

We should make clear at the outset that harmony is a philosophical notion, not a proof-theoretical one. There are proof-theoretic properties that are claimed to be its formal correlates; nevertheless, it cannot be said to be formalised to be a properly proof-theoretic notion.[87] Let us take a look at the significance of the notion of harmony for the inferentialistic approaches and recapitulate the main approaches to the notion of harmony

According to inferentialistic approaches, the meaning of an expression is, in principle, brought about by the inferential roles that the expression in question takes on. Therefore, whenever an expression exemplifies such a role, it is attached a meaning whatever it is. But in the case of logical constants (as for Dummett, in the general practice of language), this gives untenable results. In this respect, the notion of harmony is a response to the criticism of this untenabilitity. The counterexamples like Prior's 'tonk' to the inferentialistic meaning theory are no more destructive, what is wrong with those counterexamples are their lack of harmony.

What is intended by harmony is an agreement is sought in the circumstances of the usage of an expression so that the expression gets the proper meaning consonant with the reason forcing the expression.

We have talked about the formulations of harmony as (a) conservativeness, (b) agreement between the assertion grounds of a proposition and the consequences drawn from it, (c) an equilibrium reached by the deductive power of introduction and elimination rules. We can add to these (d) preservation of validity by reduction procedures put forward by Prawitz.

The last one brings to the fore the justificationary role of harmony. The inferentialistic approaches rely on proof-theoretic techniques, and lacking model-theoretic methods, face the difficulty of justification of the inferential rules. Prawitz version of harmony finds a base of justification of inferential rules in harmony; his conception of proof-theoretic validity we have examined above serves to this purpose.

[87] [Tennant 1996] cites harmony among the desirable features of a proof theory; but it is more appropriate to conceive of it a pre-proof-theoretical notion.

In our framework, it can be said that harmony is a description of constanthood in the context of logic.

'Harmony' may well be a typical example for a genuinely useful definition, for it appears that it names a conceptual lacuna rather than a definite content and conduces us to further investigation. Dummett, who introduces it, sees it in the generality of the proper practice of language. According to him, the conditions that count as acceptable grounds for an assertion (*viz*., an asserted proposition) and the consequences drawn from the assertion (what the speaker commits herself and is entitled to infer from her commitment) should be in harmony, and logic, *a fortiori*, should display this harmony, specifically, as a relational property of pairs of introduction and elimination rules. Dummett's envisage is to construct a recursive (if we accept to the term in a semi-formal sense) justification procedure starting from a base level harmony, subsequently, covering the whole language. Completeness itself is a kind of harmony between syntax and semantics of a system of logic; however, it is not obtainable in every logical system.

**Principles and Terminology**. We identify two principles for a logical form that are of the present interest:

(1) *Semantic inertness*: Logical form must not interfere with the applied content.

(2) *Sterility*: Logical form itself must not generate new content.

Let us define a strict ordering relation on deductive strength with respect to the capacity of the inferential rules to draw conclusions relative to particular premisses for ξ be a logical constant. We shall designate deductive strength by '⊐' and '⊏' and the equivalence of the deductive strength by '≡'. Then, we identify the following mutually exclusive conditions:

(a) I-rule ⊏ E-rule: I-rule is weaker than E-rule.

(b) I-rule ⊐ E-rule: I-rule is stronger than E-rule.

(c) I-rule ≡ E-rule: I-rule and E-rule are deductively equipotent.

Usually, the approaches to harmony attempt to seek out an Aristotelian mean between I-rules and E-rules. For instance, Tennant regards harmony as equilibrium and gives a minimax-style procedural definition to find the equilibrium. Nevertheless, as demonstrated by Steinberger, the procedure interferes with the standard restrictions for the soundness of the quantifier rules.

In the condition (a), the E-rule is strong enough to create new content within the deduction, surpassing those allowed by the I-rule and violates the principle (1). The condition (b) seems admissible from the sterility aspect, but E-rule does not allow certain inferences despite there are sufficient grounds for them and this indicates that E-rule violates the principle (2), somehow interfering with the content.

Following Gentzen, the introduction rule is usually privileged with respect to the elimination rule as the self-justified primary determinant of the meaning of a logical constant. The corresponding elimination rule is regarded as the derivative of the introduction rule, explicating the meaning. It has been argued that in some instances, it is more convenient to the determining role to the elimination rule. From our standpoint, introduction rules are no more privileged than playing White in the game of chess, having the first move; the primacy is basically an issue of technicality.

We shall employ a distinction already familiar in computer science. We shall say of a property or a condition 'local' if its conceptual scope is limited to the vicinity of a logical constant irrespective of the logical system as a whole. If the scope is the entire system, then we shall say of it 'global'. So there are global conditions related to a proof as a whole, while the scope of local conditions is limited to the individual proof steps irrespective of the specific features of a proof as a complete object. Thus, Dummett's concepts, intrinsic harmony and total harmony, are local and global, respectively.

We shall carry on with important notions:

*Uniqueness.* Logical laws should determine logical constants uniquely, otherwise they are formally indiscernible. Belnap argues that if the following holds

$$\text{plonk } B \vdash A \text{ plink } B$$

A plink B ⊢ A plonk B

a coherent understanding of logical constants requires that 'plonk' and 'plink' should be identical (see [Belnap 1962]).

*Conservativeness.* Conservativeness is a global property. There are several definitions for it. Belnap's version can be formulated as follows:

Let $\mathcal{S}$ and $\mathcal{S}^+$ be logical systems, $\langle \mathcal{L}, \vdash \rangle$ and $\langle \mathcal{L}^+, \vdash_+ \rangle$, respectively, such that $\mathcal{S}^+ \subseteq \mathcal{S}$ and $\mathcal{L} \subseteq \mathcal{L}^+$. Then $\mathcal{S}^+$ is a conservative extension of $\mathcal{L}$ if for all sentences $A \in \mathcal{L}$, $\Gamma$ being any set of $\mathcal{L}$-sentences, $\Gamma \vdash_+ A$ only if $\Gamma \vdash A$.

A new logical constant ξ adjoined to a logical system modifies the deductive apparatus of the system. In this case, conservativeness dictates that the adjunction is legitimate only if it does not allow the derivation of new propositions in which the antecedent logical constants occur, but itself does not occur. A canonical example for the violation of conservativeness is the definition of the classical negation (viz., $\neg\neg A \vdash A$ into the positive fragment of intuitionistic logic. Then, the theorem known as Peirce's law

$$((\mathrm{A} \to B) \to A) \to \mathrm{A}$$

becomes deducible, though the proposition does not contain the new definiendum. A violation of conservativeness can be interpreted as capability of the new constant to have an effect on the meanings of the antecedent vocabulary and concept of deducibility.

There is a usual intuition that an inference should not have an effect on the senses of participating terms. For example, Wittgenstein points out that the propositions in an inference should be "united in a single grammar which remains the same and after the inference":

> Whether a proposition entails another proposition must be clear from the grammar of the proposition and from that alone. It cannot be the result of any insight into a new sense: only of an insight into the old sense. It is not possible to construct a new proposition that follows from the old one which could not have been constructed (perhaps without knowing whether it was true or false) when the old one was constructed. If a new sense were discovered and

> followed from the first proposition, wouldn't that mean that that proposition had altered its sense?[88]

However, this is needed on the content side in the context of logic. What is required is the isolation of logical form and the argument content. There is no compelling reason to object the newly available proof steps by an expansion of logical vocabulary insofar as this requirement is satisfied.

For Dummett, extending a language with a rule is justified if the extended language remains conservative with respect to the basic one. If the extension were not conservative, then there would be sentences derived from other sentences expressed in the basic language, but does not belong to it. This could be possible through either a change in the meanings of the basic vocabulary, or some meanings were not determined by the use made of them. In either case, the new rule would have imposed new meanings of sentences of the basic language. He exemplifies the case in the word of 'Boche':

> It remains, nevertheless, a distinction of great importance, which is crucial to many forms of linguistic change, of the kind we should characterize as involving the rejection or revision of concepts. Such change is motivated by the desire to attain or preserve a harmony between the two aspects of an expression's meaning. A simple case would be that of a pejorative term, e.g., 'Boche'. The condition for applying the term to someone is that he is of German nationality; the consequences of its application are that he is barbarous and more prone to cruelty than other Europeans. We should envisage the connections in both directions as sufficiently tight as to be involved in the very meaning of the word: neither could be severed without altering its meaning. Someone who rejects the word does so because he does not want to permit a transition from the grounds for applying the term to the consequences of doing so. The addition of the term 'Boche' to a language which did not previously contain it would be to produce a non-conservative extension, *i.e.* one in which certain statements which did not contain the term were inferable from other statements not containing it which were not previously inferable.

Dummett's example can be laid down by the introduction and elimination rules for the word 'boche' can be given as follows:

$$\frac{\textit{German}}{\textit{boche}}\ \text{(boche-I)} \qquad\qquad \frac{\textit{boche}}{\textit{cruel}}\ \text{(boche-E)}$$

[88] [Wittgenstein 1978], p. 256.

Thus, the demand for harmony between the introduction rule governing a logical constant and the elimination rule is that the addition of the constant to a base language is to produce a conservative extension of that language.

When the eliminative companion is applied, we expect to recover the base language. Otherwise, the constant would be ill-defined infringing harmony. On this ground, Dummett states his "fundamental assumption":

> [t]hat, if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator.[89]

Thus, according to Dummett, extending a language with a rule is justified if the extended language remains conservative with respect to the basic one. If the extension were not conservative, then there would be sentences derived from other sentences expressed in the basic language, but does not belong to it. This could be possible either because a change in the meanings of the basic vocabulary had occurred, or some meanings were not determined by the use made of them. In either case, the new rule would have imposed new meanings on the sentences of the basic language.

*Separability* A logical constant is separable only if when it occurs in A and $\Gamma \vdash A$, then there is a derivation of A from $\Gamma$ only with the occurrence of the logical constants composing $\Gamma$ or A. The motivation for a logical constant to be separable is expressed by Shapiro as follows:

> The requirement of separability entails that a subject should be able to master the meaning of the logical terms one at a time, in any order, and that this meaning should suffice to determine the truth of any analytic truth involving just those terms. Just as a subject should be able to manifest her understanding of "and" without presupposing that she understands "or", she should be able to manifest her understanding of negation, identity, and first-order existential quantifiers without yet grasping "natural number", "zero", and "successor".[90]

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[89] This is, in effect, what is known in the literature as *invertibility* of a rule that gives the mutual derivability of premisses and conclusion. See [105] for a detailed exposition of its syntactic and semantic role**s** in the course of its evolution.

[90] [Shapiro 1998], p. 604.

Seen this way, separability stands as a local counterpart of conservativeness. We may extend separability to an entire logical system $\mathcal{S}$ and say of $\mathcal{S}$ that, for each logical constant $\xi_i$, $\mathcal{S}$ is separable only if $\mathcal{S}$ is a conservative extension of its restriction to $\mathcal{L}$ -{ $\xi_i$ }.

Separability provides an inductive scheme for conservativeness. According to this understanding, a system can be built up by adjunction of a logical constant $\xi_i$ at each step yielding a set of logical constants $\{\xi_1, \xi_2, \ldots, \xi_n\}$ with the guarantee of conservativeness.

Intrinsic harmony is a constraint that prevents I-rule $\sqsupset$ E-rule, but does not rule out I-rule $\sqsubseteq$ E-rule. For the prevention of the latter clause, Dummett states that the condition of stability has to obtain, in which case, whenever the consequences that E-rule draws exactly coincide with the grounds for asserting a proposition with the expression in question as stated by the corresponding I-rule.

Do we need conservativeness for harmony in the logical form? See that our concern is to guarantee that the form is semantically inert for the content of an argument, *not* that the components of the form is semantically inert among themselves. Therefore, we can directly eliminate conservativeness criterion (and for that reason, separability in the mentioned sense) from our list of desidera, and turn to normalisability as a prospective property.

Normalisability and conservativeness, though both are global, are independent, neither entails the other. For example, adding negation to the positive fragment of classical logic is a conservative addition, however, it does not normalise. On the other side, Steinberger constructs a counterexample employing Peirce's Law that satisfies normalisability, but not conservativeness.

Dummett relates intrinsic harmony alternatively to maximum formula reduction. These ideas can be illustrated by the case of conjunction with the following introduction and elimination rules:

$$\frac{\begin{array}{cc}[\Gamma_1] & [\Gamma_2] \\ A & B\end{array}}{A \wedge B}$$

$$\frac{[\Gamma_0]}{\dfrac{A \wedge B}{A}}$$

We have a maximum formula when there is an introduction step followed by an elimination step:

$$\frac{\dfrac{\Gamma_2 \qquad B}{A \wedge B}}{A}$$

To reduced form shows that conjuction satisfies intrinsic harmony:

$$\frac{\Gamma_2}{A}$$

In order to spread harmony, Dummett's proposal is to make use of normalisability. Normalisability is the natural deduction counterpart of being cut-free sequent calculus, in which use of the cut rule also possesses a cut-free proof, *i.e.*, a proof that does not make use of the cut rule, and normalised proofs have the sub-formula property as proofs without the application of cut rule. Gentzen thinks that subformula property is important for inferential purity and distinguished the cut-elimination theorem as his Hauptsatz. He says that

> No concepts enter into the proof other than those contained in its final result, and their use was therefore essential to the achievement of that result.[91]

Thus,

> The final result is, as it were, gradually built up from its constituent elements.[92]

[91] [Gentzen 1969], p. 69.

Prawitz showed that the same idea is implementable also in the natural deduction context by eliminating detours in the proofs and converting them into a normal form in which assumptions and conclusion are directly linked.

Maximum formula reduction that we have mentioned in connection with intrinsic harmony can function as the inductive step in the global procedure of normalisation. However, it is also necessary to reorder the proof steps so that any introduction step has to be followed by its associated elimination step so that they constitute a maximum formula. This reordering demands an extra procedure called permutative reduction, which may not be possible for every logical system, and it confronts logicality with normality. Notice that reduction of a maximum formula is a local property of introduction and elimination rules relative to each other, whereas admitting a permutative reduction procedure and, hence, normality is a global property.

*Stability*. Furthermore, Dummett claims that harmony is not a sufficient concept by itself and introduces a balance condition that he calls stability. If the converse of harmony also holds, then we obtain stability; hence, stability is a kind of balance condition. Dummett contends that if stability cannot be provided, the language should be revised; since we cannot be taken for granted that we use language correctly. When an eliminative rule for a constant is applied after its introductory counterpart has been employed, we expect to recover the basic language. Otherwise, the constant would be ill-defined, infringing harmony.

Stability is a stronger constraint than harmony and Dummett refers to it as a means of achieving constanthood. Rules of inference define the meanings of a logical constant they govern if and only if they are stable. Looking at the introduction rules alone we should be able to determine which elimination rules are harmonious with them, and conversely:

> If we use an upwards justification procedure, harmony validates a putative elimination rule; if we use a downwards justification procedure, it validates a putative introduction rule. In either case, harmony is guaranteed between valid rules. But, to verify that stability obtains, we have to appeal to both justification procedures. Suppose that we adopt the downwards justification

[92] *ibid.* p. 88.

> procedure, and start with a set E of elimination rules. By our procedure, we can determine which introduction rules are valid: say these form a set I. Now, with respect to this set of I of introduction rules, the upwards justification procedure is well-defined: so we can use it to determine which elimination rules are valid, according to the criteria of the upwards procedure. If we get back by this means to the set E, or to some set interderivable with E in the ordinary sense, in the presence of I, stability prevails. Otherwise not.[93]

*Normality.* Gentzen regards the cut-elimination theorem as the Hauptsatz of the sequent calculi. The theorem says that every proof can be reduced to a normal form; hence, a form clear of introduction/elimination detours. A significant corollary of the Hauptsatz is the subformula property —all formulas in the cut-free derivation are subformulas of the conclusion. Elaborating on Gentzen's work, Prawitz has shown that Gentzen's Hauptsatz for sequent calculi has a counterpart in the natural deduction calculi, *i.e.*, the normalisation of deductions.

We can make sense of the significance of normality via Dummett's intuitionistic attitude. We can rephrase his position as follows: The meaning of a statement is compositional at the level of semantic molecularism has a complexity due to compositionality. In the canonical case, the statement can be derived as a conclusion constructively without gaps in accordance with its logical structure, and the derivation is a construal of the meaning of the statement as well as is its justification. Looking at this way, normality appears as a desirable property of a system of logic. A system with normality has a deductive path to conclusion without a logical constant if that constant does not occur in the conclusion. Thus, normality is helpful in delineating the domain of logic from the non-logic. Another point is that it provides an assessment of logical equivalences between proofs and since proofs are also meaning-constituting, normality can serve also as a semantic device.

Then, we may schematically present the line of thought as follows:

(a) Introduction and elimination rules for a logical constant should be interdependently defined so that there exhibit a harmony of inferential power between them. This type of harmony is thought to be local to the logical constant and independent from the general structure of the logical system.

[93] [Prawitz 1965], p. 287.

(b) An inferential rule pair such as it is given in (a) should exhibit the property of "levelling the peak", *i.e.*, when an application of an introduction rule is immediately followed by the application of the elimination rule, it should be possible to cancel them. Notice that this is a proof-theoretical formulation along the idea expressed in (a).

(c) The second type of harmony is that a logical constant should be defined so that when it is introduced to a logical system, it should be a conservative extension of the system. This type of harmony is thought to be global in the sense that it concerns also the general structure of the logical system.

(d) A logical constant should exhibit stability. Stability is a condition such that the first type harmony holds and the introduction rule should be derivable from the elimination rule.

(e) A logical constant should be defined such that the logical system should be normalisable, *i.e.*, in a proof, if the introduction and elimination rules of the logical constant occur, then it should be possible to reduce the proof so that the detour through introduction-elimination is cancelled (hence, it should be possible to transform an indirect proof to a direct proof). Evidently, this is a global property of the logical system.

(f) The normalisability property is important, because it guarantees that there is no discrepancy of the meanings of a proposition proved directly and the same proposition proved indirectly.

(g) There is no entailment relation between intrinsic harmony and total harmony. As we have discussed in the study case, quantum logical disjunctive connective is intrinsically harmonious; nonetheless, it gives rise to a system that is not normalisable and with respect to which the classical disjunction connective does not display conservativeness. Dummett's remedy as "intrinsic harmony implies total harmony in a context where stability prevails" is obscure, since there is no satisfactory explanation of Dummett's notion of stability.

(h) It appears that it is not possible to proceed purely in terms of reduction procedures without resolving the tension between locality and globality. Let us take an overview of the relative states of local and global properties. Insofar as an encoding of global constraints

into local conditions is not found, a local conception of harmony does not suffice in the present state of the art.

### 3.7 AN EXEMPLARY CASE: QUANTUM LOGIC

An instructive case for the discussion of our view on logical constanthood is quantum logic, which was put forward originally as an interpretation of quantum mechanics by von Neumann, Birkhoff, *et al.*[94] From this respect, quantum logic is related to logicism: While logicism posits a continuity of logic and mathematics deriving from logical laws, it posits a continuity of logic and physics deriving from physical laws. Quantum logic, **QL**, also exhibits a case of a revision in logic as an alternative to classical logic, **CL**.

The starting point of **QL** is the observation of Birkhoff and von Neumann that there is a one-to-one correspondence between elementary propositions of a quantum mechanical system and the subspaces of its associated space, and quantum mechanics can be encoded into a logical system. Therefore, semantic principles of **QL** are operationally bound to quantum mechanics. Accordingly, **QL** displays deviance from **CL** in substantial aspects. We shall remark the significant ones without digressing into the details [95]

The deduction theorem does not hold in **QL** (and **QL** cannot be extended so that it holds). Then, for instance, the hypothetical syllogism of the classical propositional calculus is invalid in **QL**

$$(\alpha \rightarrow \beta), (\beta \rightarrow \gamma) \nvdash_{QL} \alpha \rightarrow \gamma$$

while the transitivity of logical consequences is valid:

$$(\vdash_{QL} \alpha \rightarrow \beta) \,\&\, (\vdash_{QL} \beta \rightarrow \gamma) \Rightarrow (\vdash_{QL} \alpha \rightarrow \gamma)$$

[94] The leading article is [Birkhoff and Von Neuman 1936].

[95] Our discussion owes to [Gibbins 2007] for **QL** and **CL** comparison.

While conjunction does not differ truth-value-wise, disjunction does and distributivity fails. So, there are true quantum propositions $\alpha, \beta, \gamma$ such that (designating the **QL** counterpart of disjunction by '$\oplus$', but **QL** conjunction by '$\wedge$' to indicate the overlapping with **CL**

$$\alpha \wedge (\beta \oplus \gamma)$$

yields truth, but the distribution of conjunction does not:

$$(\alpha \oplus \beta) \wedge (\alpha \oplus \beta)$$

The semantic theory of **QL** blocks a truth-functional interpretation to the **QL** connectives. Taking into account that **QL** formalises the negation connective as expressive of mutual inconsistency, and for that reason, designating it with '$\ominus$', we get the below truth table for it:

| A | B | $\alpha \wedge .\beta$ | $\alpha \oplus \beta$ | $\ominus\alpha$ |
|---|---|---|---|---|
| T | T | T | T | F |
| T | F | F | T | F |
| F | T | T | T | ? |
| F | F | F | ? | ? |

Notice in the table that *tertium non datur* is revoked; there are cases that cannot be interpreted by classical bivalence.

**QL** can be construed in several ways, such as a many-valued logical system, a modal logical system or a non-monotonic system of logic. But because of its tight dependence on semantics, these characteristics can be transferred to semantics as well. As an example for the transitions between the object theory and metatheory, we can cite a discussion due to Bell and Hallett. Bell and Hallett bring up a model that makes the classical conjunction and

disjunction coincide with its quantum counterparts and shift the discrepancy to the meaning of negation. They argue on the idea that classical negation is defined set-theoretical partial ordering, where as quantum negation is a vector orthogonality relation.

**QL** is not a logical system "free of existence assumptions with respect to its terms, singular and general"; nonetheless, how the existence assumptions could be properly formalised is questionable. The difficulty lies in the fact, which Teller states as, that quanta are devoid of "primitive thisness" (*viz.*, haecceity). We do not need going a metaphysical discussion on whether haecceity is a sound concept or not. At any rate, haecceity, in the present sense, is a matter of linguistic apprehension and a possible application of indexicals –-which have not been discerned in **CL**, since a member of the quantification domain has already been individuated. In the quantification domain of **CL**, that which are qualitatively identical are numerically identical as well. Teller illustrates the difference with the combination of familiar two qualitatively identical, numerically distinct pennies with fair probability for heads and tails:

On the classical view, the expected probability is 1/4 for each tossing outcomes of HH, TT, HT and TH. However, when two bosons instead of pennies are taken into a similar case, the probability is 1/3 for each one of the combinations 'both heads', 'both tails' and 'one heads and one tails', since keeping track of which object is which does not make sense in the context of quantum mechanics in contrast to the individuating space-time trajectories available in classical mechanics.

Therefore, the formula $(\exists x)\varphi(x)$ cannot be taken grammatical without restriction as in **CL** because the open sentence $\varphi(x)$ cannot be satisfied by an individual *a* such that $\varphi(a)$ could be assigned a truth value. *a* is, just in the spirit of Quine's dictum: "No entity without identity", is an indeterminate individual –it denotes neither a variable nor a constant in the classical sense. The restriction imposed on existential instantiation is that the collateral hypothesis in the minor premiss, $\Gamma_2$, has to be empty (in order to block from distributing over the existential quantifier):

$$\frac{\begin{array}{cc}[\Gamma_1] & [\Gamma_1], [\Gamma_2] \\ (\exists x)\varphi(x) & \gamma\,[a/x]\end{array}}{\gamma}$$

A view, while admitting that **QL** addresses a particular semantic domain, claims that the domain should be fundamental to our world-view. A prominent figure of this view, Putnam, claims that the relation between quantum theory and logic bears a parallelism to the relation between the general theory of relativity and geometry from the respect of non-distributivity and Euclid's parallel postulate. He thinks that just as the relativistic conception of the physical space-time continuum confirms Riemannian geometry instead of Euclidean geometry, quantum world view empirically compels us to adopt **QL** as a non-classical system of logic addressing the distinctive features of quantum mechanics. Putnam's claim is a subsequence of the view Quine brought up as:

> Any statement can be held true come what may if we make drastic enough adjustments elsewhere in the system. Even a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. Conversely, by the same token, no statement is immune from revision.
>
> Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there in principle between such a shift and the shift whereby Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?[96]

According to Putnam, the alleged paradoxes of quantum mechanics arises from adherence to classical logic and those apparent counterintuitive phenomena would vanish if the right stance were taken for logic. Otherwise, we would be forced to *ad hoc* assumptions and postulate either hidden variables or a collapse of the wave-function. Putnam's claim turns out to be to postulate **QL** and to mandate it to produce a validity relation that will facilitate the logical work of quantum mechanics. For some quantum deductions, it is essential to their correctness in quantum mechanics that they employ a non-classical consequence

[96] [Putnam 1975], p. 43.

relation $\vdash_{QL}$ (but without abandoning the principle of non-bivalence). It can be plausibly asserted that **CL** can accommodate quantum mechanics. In the realm of ordinary everyday experience, **QL** converges to **CL**.

For the sake of making the discussion plain, it should be stated that what Putnam calls paradoxes are not, indeed, incomprehensible complications. Unlike set-theoretic or semantic paradoxes, they do not have us face with genuine dilemmas. There are quite persuasive explanations that straighten out our intuitions rooted in our ordinary way of life experience and the quantum phenomena within reasonable limits. In fact, whether we have the capacity to describe the world in all its aspects is not a matter of logic.

Another point of confusion is in the comparison of the mathematical statuses of the Euclidean and non-Euclidean geometries. These systems are not competitors in the mathematical realm. A discrepancy occurs among them just when they are embedded in an interpreting physical theory, compelling us to make an exclusive choice. Likewise, there is no decisive factor that compels a choice in favour of **QL**. The simple observation is that **QL** is defined via subordination to **CL.** As a result, the grounds for **QL** cannot be said to diverge radically from those of **CL**, let alone, be superior to them.

Dummett bases his analyses for revisionism in logic on the basis of intelligibility:

> Let us assume that classical logic is at present in possession, so that a radical revision of logic will always be a revision from classical to some non-standard logic: let us call their advocates C and N. Then there are four possible cases according to which of the following two pairs of alternatives hold. (1) N rejects the classical meanings of the logical constants and proposes modified ones; or (2) N admits the classical meanings as intelligible, but proposes modified ones as more, or at least equally, interesting. And (a) C rejects N's modified meanings as illegitimate or unintelligible; or (b) he admits them as intelligible, alongside the unmodified classical meanings. If cases (2) and (b) both hold, then we are in effect in a position in which only relabeling is involved.[97]

Thus, one combination of views is that the sides are mutually unintelligible and their logical theories are incommensurable. In two others, there is a subordination of intelligibility, but there is in some way a common ground. The remaining combination is what Dummett calls relabeling, in which case "there would be no proposition that we had

[97] [Dummett 1978], p. 285.

relinquished in response to experience, only a sentence to which we found it convenient to give a different meaning". Inverting what Dummett points out makes more sense: Relabeling schematically amounts to retaining the core of an intension while projecting it onto a differentiated extension, e.g., the connective 'and' onto $\oplus$ instead of (or, beside) $\vee$ —we say this assuming the relation between two terms is not circular.

He claims that intuitionistic logic exemplifies (1), since **CL** is unintelligible from intuitionistic stance, whereas Putnam's position is an example of (2b), if backed up by realist assumptions which reduce **QL**to **CL**. Hence, what Putnam argues for amounts to a moderate variation on pragmatic purposes.

Dummett justifies the demarcation of logical constants through conceptual frameworks of speakers, while Putnam does it through the epistemic framework propounded by the best physical theory available.

The principle idea of Dummett's critique is an argument due to Quine, which is also a main line of thought followed in the comparisons of **CL** and **QL**. Quine, in his discussion of the deviant systems of logic, supposes a proposal for a heterodox system in which law of non-contradiction was rejected and a conjunction of the form $A \wedge \neg A$ could be true such that inconsistencies would be blocked in a different way. He claims that the proposal, indeed, would commit the fallacy of "changing the subject", because the classical doctrine of logic is too firmly rooted in our thought:

> They think they are talking about negation, '~', 'not'; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form '$p. \sim p$' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.[98]

A typical case is the non-distributivity in **QL**. Since truth-valuations in **CL** and **QL**, it is not possible to say that there is a valuation which affirms $A \wedge (B \vee C)$ true while negating $(A \wedge B) \vee (A \wedge C)$; hence, **CL** and **QL** talk cross each other and there is no question as to whether they are compatible or not. The $\vee$ and $\oplus$ are incompatible interpretations of disjunction. But since $\oplus$ has no occurrence in classical logic, there is no treatment of them in classical

[98] [Quine 1986], p. 181.

logic in which distributivity could fail. Non-bivalent sentences have no formalisation in classical logic. So deductions from non-bivalent premisses cannot, even if valid, be so in virtue of having valid classical forms.

Dummett wants us to consider what could be said to a child who has grasped that the squares of negative numbers are positive, but is confused by learning that $\sqrt{-1}$ is a legitimate number. He tells that the child should be informed that by $\sqrt{-1}$, we use an extended meaning for the word 'number'.

What does the Putnam-Dummett dispute make visible relevant to our quest? For both views, we can point to the plain observation that both quantum logic and intuitionistic logic do not exhibit an intelligibility problem looking from the stance of classical logic; indeed, both logical systems can be agreeably specifiable presupposing the classical meanings of the standard logical constants. The point missing on these views is that conceptual as well as epistemic frameworks are mediated by language as the common ground for logic.

In **QL** the classical laws that are dropped in intuitionistic logic, of excluded middle and double negation introduction-elimination are sustained, but the inference rules for disjunction and implication are altered. We shall deal with quantum disjunction for our discussion of harmony. The discussion may serve to two purposes:

(a) To inspect whether **QL** confirms the conjectures, specifically those that Dummett has put forward in connection to his conceptions of intrinsic harmony, total harmony and stability.

(b) To examine in the context of the considerations concerning logical constanthood whether the construction of **QL** has resulted in a robust system comparable to **CL**.

Steinberger devises deductive cases focusing on (a) and concludes that Dummett's conjectures do not hold. We shall draw on his cases, our priority being (b).

In the Gentzen-style formulation of **QL** the same restriction imposed on the existential quantification applies to $\oplus$ as well —in this instance to block the derivation of distributivity. Hence, it is required that $\Gamma_1$ and $\Gamma_2$ below must be empty:

$$\dfrac{A \oplus B \qquad \begin{matrix}[\Gamma_1] \\ A \\ \Gamma\end{matrix} \qquad \begin{matrix}[\Gamma_2] \\ \beta \\ \gamma\end{matrix}}{\Gamma}$$

How adequately $\oplus$ is defined can be tested, metaphorically speaking, by applying "a distortion" to the system and seeing how the system reacts. Let us take as base system {$\wedge$, $\oplus$}. The system is intrinsically harmonious and normalisable.

When we adjoin the classical disjunction to the base system, we observe that $\oplus$ is conflated with $\vee$, since it becomes possible to derive $\alpha \vee \beta$ from $\alpha \oplus \beta$ by $\oplus$-elimination and $\vee$-introduction:

$$\dfrac{A \oplus B \qquad \begin{matrix}[\Gamma_1] \\ \vdots \\ \alpha \vee \beta\end{matrix} \qquad \begin{matrix}[\Gamma_2] \\ \alpha \\ \alpha \vee \beta\end{matrix} \qquad \begin{matrix}[\Gamma_3] \\ \beta \\ \alpha \vee \beta\end{matrix}}{\gamma}$$

We see that the grounds of assertion and drawing consequences for $\oplus$ tangle with those of $\vee$. According to Dummett's verificationist-pragmatic dual theory of meaning, this is indicative of a weakness in the demarcation of $\oplus$ and $\vee$. We can say that the grammar of $\oplus$ is not settled so as to function separately from $\vee$. An evidence to this situation is that the system {$\wedge$, $\oplus$, $\vee$} comes out to be a non-conservative extension of the base system {$\wedge$, $\oplus$}; the missing distributivity of $\wedge$ over $\oplus$, which is the hallmark of **QL** becomes allowable in the base part of the extended system. But the genuine problem is not conservative extendibility –as we gave argued, it does not contribute to our notion of formality. The disorder of **QL** is that it does not satisfy normalisability, though it satisfies intrinsic harmony, *pace* Dummett. We shall follow Steinberger on this issue.

Steinberger applies $\vee$-introduction rule, $\vee$-elimination and $\oplus$-elimination as follows:

$$\dfrac{A \oplus B \qquad \dfrac{\dfrac{[\alpha]}{\alpha \vee \beta} \quad \dfrac{\Gamma_1, [\alpha]}{\gamma} \quad \dfrac{\Gamma_2, [\beta]}{\gamma}}{\gamma} \qquad \dfrac{\dfrac{[\beta]}{\alpha \vee \beta} \quad \dfrac{\Gamma_3, [\alpha]}{\gamma} \quad \dfrac{\Gamma_4, [\beta]}{\gamma}}{\gamma}}{\gamma}$$

The point is that the reduction of the introduction-elimination steps yields a $\oplus$-elimination which is not legitimate in this instance, since **QL** allows such a step only if $\Gamma_1$, $\Gamma_2$, $\Gamma_3$, $\Gamma_4$ are all empty. The reduction procedure has weakened to allowing illegitimate deductions. As a result, the system $\{\wedge, \oplus, \vee\}$ is not normalisable, whereas the system $\{\wedge, \oplus\}$ is.

Though **QL** seems to be indistinguishable from the definitive aspect as a logical system among other others, however, as the above cases reveal, it exhibits grammatical flaws that we do not maintain for a logical attitude. Recall that our concern is primarily methodological, not metalogical –we propose a framework for inquiries into logical constanthood. We think that this should be a part of the work on logical constanthood. Dummett's conjectures suggest us the heuristic advice that, though semi-formal arguments and quasi-proofs are not objectionable by themselves, caution should be taken on them not to draw general conclusions about logical constanthood.

To recapitulate what we have discussed, we may state our overall assessment of **QL**. We have seen that **QL** is dubious from the aspect of formality, while its necessity and normativity derive from the particular epistemic field of physics and are not connected to formality. If we may conceive of logical constants as "decision procedures" to separate the logical and non-logical domains, the view that includes **QL** into logic, albeit a deviant one, turns out to blur the boundary between a logical system and a extra-logical formal system.

# CHAPTER 4

## LANGUAGE *QUA* VIEWPOINT

The essential importance of language for the human being cannot be exaggerated. The very phrase 'natural language' is an indication of this fact. Language embodies an apprehension of the world, a medium of thought and a method of attaching meaning to what there is within the human's horizon.

It constitutes a specific realm that is fundamentally significant for logic. Our norms in accordance to which we compose argument texts cannot be understood in separation from language, in contrast to those such as cognitive or epistemic norms, which are applicable to the items that are meaningful independently from language. Language appears to be a main resource for the specification of logical constants. But we shall argue for a deeper relation between language and logical constanthood. Employing the Wittgensteinian idea that the realm of language exhibits an autonomy, we shall try to show that the basis of logical constanthood is the autonomy of language which also allows us to adopt a unique perspective we call language *qua* viewpoint.

Our framework is an attempt to look at the notion of logical constanthood not through the speaker's perspective, but through the perspective of the spoken. This is not to set up a framework of a speakerless language; rather, the language without a reference to the grasp of language of any particular speaker or group of speakers.

### 4.1 AUTONOMY OF LANGUAGE AND ITS BOUNDARIES

The idea of autonomy is familiar from various social phenomena and has been admitted as a postulate of social sciences. For example, the economic activities and behaviours involve a complexity of human and natural resources and processes. However, economics has its own laws and truths governing them, stimulating such methods of analysis as the mathematical theory of games. While the human is the originator of the economic activity, she is also subjected to the economic laws. Those laws cannot be reduced to the properties

of the resources and processes. For the integrity of the present discussion, we can envision such autonomy as a supervenience relation to other realities. By 'supervenience', we intend a determinative relation that is irreducible, but weaker than the relations of identity and definability.

We can substantiate the autonomy view of language in several aspects. Let us go through these aspects with passages from Wittgenstein:

> Suppose I am now asked "why do you choose this colour when given this order; how do you justify the choice?" In the one case I can answer "because this colour is opposite the word 'red' in my chart." In the other case there is no answer to the question and the question makes no sense. But in the first game there is no sense in this question: "why do you call 'red' the colour in the chart opposite the word 'red'?" A reason can only be given within a game. The links of the chain of reason come to an end, at the boundary of the game. (Reason and cause).
>
> If one calls to mind "that the chart does not compel us" to use it in a particular way, or even always to use it in the same way, it becomes clear to everyone that our use of the word "rule" and "game" is a fluctuating one (blurred at the edges).
>
> The connection between "language and reality" is made by definitions of words, and these belong to grammar, so that language remains self-contained and autonomous.[99]

Wittgenstein draws our attention to the constructive force of language that prevails even in the cases that, we may think, are determined beyond language. The word 'red' is not merely a label for an object to communicate its colour; we may work out a list of tasks for it: It is a measure whether to call something 'red' or not, an mutual indicator for how a colour term is to be used, a sample adjective, an exclamation to warn that the traffic light signals us to stop, etc. The case of the ostensive definition may be more illuminative for that aspect.

There is a usual tendency to regard the method of ostensive definition as a bridge between language and non-linguistic world. Those who hold this tendency compare ostension with description. We can give a description of a word as its definition to the extent of precision that we demand. We think that both the *definiendum* and the *definiens* are within the

[99] [Wittgenstein 1978], §55.

descriptive discourse, while definition by ostension is a pragmatic act of ostending the *definiendum* that stands out as an object independently from language. However, this is an illusion originating from the fact that we have mastered and been deeply immersed in the grammar of ostension. The subject to whom the definition is addressed has already been aware of what is pointed to her, and being trained in a linguistic community, knows that it is an act of definition, and that what she is to do with a definition. The whole process of ostensive definition presupposes a working knowledge of language and the configuration of the *definiendum* has been already set in language. Looking at the method in this way, ostensive definition is basically a linguistic device just as description is:

> Can an ostensive definition come into collision with the other rules for the use of a word? – It might appear so; but the rules can't collide, unless they contradict each other. That aside, it is they that determine a meaning; there isn't a meaning that they are answerable to and could contradict.
>
> Grammar is not accountable to any reality. It is grammatical rules that determine meaning (constitute it) and so they themselves are not answerable to any meaning and to that extent arbitrary.
>
> There cannot be question whether these or other rules are the correct ones for the use of "not" (that is, whether they accord with its meaning). For without these rules the word has as yet no meaning; and if we change the rules, it now has another meaning (or none), and in that case we may just as well change the word too.
>
> "The only correlate in language to an intrinsic necessity is an arbitrary rule. It is only thing which one can milk out of this intrinsic necessity into a proposition."
>
> Why don't I call cookery rules arbitrary, and why am I tempted to call the rules of grammar arbitrary? Because I think of the concept "cookery" as defined by the end of cookery, and I don't think of the concept "language" as defined by the end of language. You cook badly if you if you are guided in your cooking by rules other than the right ones; but if follow other rules than those of chess you are playing another game; and if you follow grammatical rules other than such and such ones, that does not mean you say something wrong, no you are speaking of something else.
>
> If I want to carve a block of wood into a particular shape any cut that gives it the right shape is a good one. But I don't call any argument a good argument just because it has the consequences I want (Pragmatism). I may call a calculation wrong even if the actions based on its result have led to the desired end. (Compare the joke "I've hit the jackpot and he wants to give lessons!") That shows the justifications in the two cases are different, and also that

> "justification" means something different in each case. In the one case one say "Just wait, you will soon see that it will come out right (*i.e.* as desired)". In the other case that is no justification.
>
> The connection between the rules of cookery and the grammar of the word "cook" is not the same as that between the rules of chess or that between the rules of multiplication and the grammar of the word "multiply".
>
> The rules of grammar are arbitrary in the same sense as the choice of a unit of measurement. But that means no more than that the choice is independent of the length of the objects to be measured and that the choice of one unit is not 'true' and of 'false' in the way that a statement of length is true or false. Of course that is only a remark on the grammar of the word "unit length".[100]

Wittgenstein's discussion suggests a contrast between our definition of something and the use we make of it. We may clarify the contrast taking a car as an example. In case of an urgency, for instance, under heavy rain or snow, we could use the car as a shelter. We could stay in the car for some time and run the air-conditioner and even, feel a bit at home. Nonetheless, the car is not designed and manufactured on the purpose of residing as in a house. Residing is not a definitive function of the car, unlike it is of a house. In diverse circumstances, we might employ the car in diverse ways, possibly many of which would not be related to transportation, by which the car is defined. Transportation is a construal of the essence of the car, not every imaginable use of it. It is important to note that what we talk about is not a transcendent essence. Within the present scope, it is proper to conceive of essence as an actual capability that is conducive to feasible uses. Thus, there is an "end" that the car is to achieve, and so for activities we are engaged in, the tools we employ; they are means to external ends –"external" in the sense that the ends are not part of the means.

It may be thought that language is also a device employed with expected ends, and that, above all, it is a communication device. No doubt there is an instrumental side of language practice. However, this is not what defines language. We can illustrate the difference of language with a bridge game example. During the game, the players speak for various purposes such as bidding, making a contract, warning, while they handle cards, write down points, etc. The speech is not like a soundtrack of a cinema film separately flowing; it is a part of the game, intertwined with the acts of the players, sharing the same orderliness. When a player makes a declaration, she does not utter a pre-existing "object of expression",

[100] [Wittgenstein 1978], §133.

what she does is all to it. What is to be expressed and the expression is one and the same instance, there is no regress to another instance which would be input to language practice and function as the criterion of correctness for the linguistic expression. That is to say, in language, ends are internal to means. It may be thought that players k speaks on the purpose of making a contract in the game. But this is a confusion; it is the purpose not of speech, but of a certain stage of the game to make the contract. In this connection, it can be observed that it is language that is ultimately topic-neutral (*cf.*, the topic-neutrality of logic).

Thus, we have identified two main aspects of autonomy of language: A self-contained nature and the lack of external criterion of correctness. The notion of autonomy has the following explanatory bearings onto our framework:

*Explanatory closure*. A fact about or a property of language is explained within language whatever the subvenient facts and properties are (higher-lower level actually a phenomenological ordering).

The autonomy claim does not include the claim that properties and facts explained within language are independent of properties and facts of other realities in all the aspects. It just states that the variations in other realities would not have an affect so far as the same linguist property or fact obtains.

For example, colours are supervenient upon physical properties. Colour explanations will be within the autonomy, so far as they are independent from their other realities, in particular, physics. For example, the redness of an object is realised by its emitting light at frequencies in a particular electromagnetic wave band, but that does not bear relevance to the grammar of the word 'red' – and if, indeed, it does, it is through internalisation by language.

This item contains also internal reference to linguistic phenomena.

*Internality of judgement*. The rules are within and of language, not of another reality. The properties are judged by internal criteria. Language is a realm of self-justification and self-validation. No linguistic conclusion can be reasonably inferred from entirely non-linguistic

premisses. Consequently, a grammatical rule can be neither refuted, nor falsified; it can merely be changed.

There is a view that the Wittgensteinian conception of language with autonomy and the subsequent arbitrariness of grammar can be categorised as "linguistic idealism". In consideration of Bloor's definition of linguistic idealism "the claim that some truths and realities are created by our language practice", [101]though the label obliterates, above all, the deconstructive aspect the Wittgensteinian thought, we can see that there is indeed an affinity between the Wittgensteinian and the idealist stances. The mentioned affinity is not to the Berkleian tradition summarised in the dictum "esse est percipi". There is no scepticism about a mind-independent (and devoid of meaning) reality; there exists always an openness subverting our vision of external reality. But, as it were, factuality is ours with the facts carved out from, relations set up in that reality. As Bloor points out, the Wittgensteinian thought is comparable to Hegelian tradition with its emphasis on social and cultural institutions. Language learning is a process of enculturation, a training into a language practice. That is why, as we have talked about in connection with ostensive definition, sensory stimulation of a tree does not suffice to label it as a tree as opposed to a random thing. It is a tree only in taking part in our 'rule-guided' language practice, which is not merely verbal behaviour.[102] That is what we call linguistic apprehension; we recognise things so far as they are expressed in language cast by grammar. Grammar specifies the linguistic existence conditions and specifies the possibilities of an expression to co-exist with others.

At this point, a remark is in order: It is important not to identify meanings with concepts (unless one adopts a stance that supports a sort of the Sapir-Whorf hypothesis). Concepts as cognitive contents are distinguished from meanings which are linguistic elements. For example, consider the usage of the phrase 'we can think of … as …' as in 'We have seen how to think of groups as topological objects'. How can we lay out a definitional equivalence/synonymy and *a priori* analysis for 'groups' and 'topological objects'?

[101] [Bloor 1996], p. 356.
[102] We prefer the term in distinction to `rule-governedness', as which would not admit such transgression.

Likewise, there are many instances of concepts that do not presuppose such relational properties as symmetry and transitivity, however, do involve them by inference. The following passage from Wittgenstein may be helpful to clarify the point:

> The whole idea, that a proposition has to be thought along with any proposition that entails it, rests on a false, psychologising notion. We must concern ourselves *only with what is contained in the signs and the rules*.
>
> If the criterion for p's following from q consists in "thinking of p being involved in thinking of q" then while thinking of the proposition "in the box there are 105 grains of sand", you are also thinking also of the 105 sentences "In this box there is one grain of sand" "… 2 grains of sand", etc. etc. What's the criterion here for the thought of one proposition's being involved in the thought of another?[103]

An associated property of language connected to its autonomy is its reflexive capability that language can speak of itself. A particularly interesting indication of language mirroring itself is that the discontinuity between language and the world is expressed by certain differentiations, such as property *vs.* predicate, interpretational semantics *vs.* representational semantics, substitutional *vs.* objectual quantification, *de dicto* mode *vs. de re* mode. Lucy surmises that reflexivity is one of the distinctive features of the human language, perhaps, unique to it:

> In sum, speech is permeated by reflexive activity as speakers remark on language, report utterances, index and describe aspects of the speech event, invoke conventional names, and guide listeners in the proper interpretation of their utterances. This reflexivity is so pervasive and essential that we can say that language is, by nature, fundamentally reflexive.[104]

A manifestation of the reflexive autonomy is observed in some statements that share a presumed peculiarity. Although those statements do not differ syntactically from ordinary truth-evaluable sentences, their truth-values seem amenable to nothing but a language competence. We have two items in the philosophical vocabulary for those statements, contributed, respectively, by Kant and Wittgenstein:

(1) Analytic propositions.

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[103] [Wittgenstein 1978], p. 248 *f.* (emphasis added).

[104] [Lucy 1993], p. 11.

(2) Grammatical propositions.

Analytic propositions are contrasted with synthetic ones and grammatical propositions with empirical ones. Whether either category of propositions is useful and whether the distinction either one yields is genuine have been matters of persistent controversy. Since our focus on logical constanthood, we shall not delve into these controversies. We shall be content with looking into the central points of dispute and argue that in the versions of conception we endorse, they are well-founded and significant to understand the notion of logical form, whereas the current arguments against the former one lead to an untenable to maintain coherence in a broader philosophical frame, and the latter one is a consequence of the Wittgensteinian attitude we support.

## 4.2 PROPOSITIONS VERIFIED BY LANGUAGE

Let us begin with analytic propositions:

According to the standard definition of analyticity, a proposition is said to be analytic if its truth is determined solely by virtue of its meaning. For the sake of simplicity, we may view an analytic proposition composed of two components, *analysandum* and *analysans*. If there is a proper semantic consonance between *analysandum* and *analysans*, then the proposition is true irrespective of the worldly facts. There is no substantially extra-linguistic means to refute an analytic sentence so far as such semantic consonance of the sentence is conserved. On the other side, the knowledge of meaning does not suffice to settle the truth of a synthetic proposition, some further knowledge of the facts of the world is required. It should be remarked that separability as syntactic components depends on the sentential structure and what is involved for the truth-value is not only the pair of *analysandum* and *analysans*, but the entire proposition.

While synthetic truths are usually regarded as epistemologically legitimate, analytic propositions pose a paradoxical case: Truth, which is non-linguistic, is acquired merely through our knowledge of language. We shall argue that relying on meaning as if it were an object is one of the main factors of complication, that results in the situation depicted in Katz's these vivid words:

> The true story of analyticity is surprising in many ways. Contrary to received opinion, it was the empiricist Locke rather than the rationalist Kant who had the better informal account of this type of a priori proposition. Frege and Carnap, represented as analyticity's best friends in this century, did as much to undermine it as its worst enemies. Quine and Putnam, represented as having refuted the analytic/synthetic distinction, not only did no such thing, but, in fact, contributed significantly to undoing the damage done by Frege and Carnap. Finally, the epistemological significance of the distinction is nothing like what it is commonly taken to be.[105]

Through the historical progress of the notion, we distinguish two representative approaches that underlie the common understanding of analyticity. We shall formulate them as the Kantian and the Fregean criteria of analyticity. Kant puts down the matter as follows:

> In all judgements in which the relation of a subject to the predicate is thought (if I only consider affirmative judgements, since the application to negative ones is easy) this relation is possible in two different ways. Either the predicate B belongs to the subject A as something that is (covertly) contained in this concept A; or B lies entirely outside the concept A, though to be sure it stands in connection with it. In the first case, I call the judgement analytic, in the second synthetic.

By this conception of analyticity, Kant marked an important progress in the philosophical discourse. Nevertheless, its overall internal coherence is disputable in the light of later developments, especially, in geometry and logic. In [Kant, 1999], Kant defines an analytic judgement as that which is expressed by a sentence such that the concept indicated by the sentential subject is merely explicated by the concept indicated by its predicate and it is non-ampliative in the sense that nothing new is introduced to the subject. Metaphorically speaking, an analytic truth is actually bringing out what has been already "contained" in the subject-concept through the predicate-concept. Hence, analysandum is located in the grammatical subject and analysans in the grammatical predicate. In contrast, a synthetic judgement is ampliative; it predicates of the subject something new that has not been so contained that it can be obtained merely by conceptual analysis. According to Kant, an analytic truth is *a priori* because we reach the judgement without appealing to experience and it is also necessary because we cannot think otherwise consistently. Thus, we can define the Kantian criterion as follows:

[105] [Katz 2010].

(K) A proposition is analytically true if and only if its *analysandum* implies its *analysans* by virtue of the intensional content of the *analysandum*.

*Prima facie* deficiency of the Kantian criterion is its clear grammatical limitation; it neglects propositions other than those that have the subject-predicate structure. But there is a deeper flaw that is significant for us: It is grounded in the psychology of concepts. It involves certain intensional relations between concepts that constitute a proposition. Frege gives a clear critique of this:

> On the basis of his definition, the division of judgements into analytic and synthetic is not exhaustive. What he is thinking of is the universal affirmative judgement; there, we can speak of a subject concept and ask -as his definition requires- whether the predicate concept is contained in it or not. But how can we do this, if the subject is an individual object? If the judgement is an existential one? In these cases there can simply be no question of a subject concept in Kant's sense. He seems to think of concepts as defined by giving a simple list of characteristics in no special order; but of all ways of forming concepts that is one of the least fruitful.[106]

Frege proposes a remedy in conformance with his sentential function view in logic. Fregean way of drawing the distinction is to say that a proposition is analytic if its truth depends entirely on the definition of its terms (that is, it is true by definition and eventually, by linguistic convention. Then, that is the Fregean criterion:

(F) A proposition is analytically true if and only if either it is a logical truth, or it can be converted into a logical truth by a uniform substitution *salva veritate* of definitions for non-logical terms (hence, substituting synonyms for synonyms).

Despite that the Fregean criterion, anticipating Tarski's semantic definition of truth, is an advancement over the Kantian one; nonetheless, it defers the issue to the distinction of of logical/nonlogical vocabulary. Carnap offers a well-founded conception of analyticity as part of his logical empiricist views. Carnap considers the following examples:[107]

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[106] [Frege 1960], p. 100.

[107] [Carnap 1970], p. 222.

(1) 'Fido is black or Fido is not black.'

(2) 'If Jack is a bachelor, then he is not married.'

and expounds the matter as follows:

> To ascertain the truth of (1), only meanings of the logical particles ('is', 'or', 'not') are required; the meanings of the descriptive *(i.e.*, nonlogical) words ('Fido', 'black') are irrelevant (except that they must belong to suitable types). For (2), on the other hand, the meanings of some descriptive words are involved, viz., those of 'bachelor' and 'married'.[108]

In order to account for the connections between descriptive words, Carnap introduces "meaning postulates" for a language (notice that Carnap develops his approach on formalised languages). Carnap's meaning postulates serve as non-logical axioms of a formal system comprised of language and logical apparatus. By a recursion procedure on meaning postulates, true sentences that would not be possible by the logical axioms alone are generated. Carnap identifies such derivable sentences as analytic (analyticity, definition and synonymy it turns out virtually the same in Carnap's approach). Thus, he arrives at the following convention of a language-based truth definition:

> Convention. A sentence $S_i$ is *L-true* in a semantical system *S* if and only if $S_i$ is true in *S* in such a way that its truth can be established on the basis of the semantical rules of the system *S* alone, without any reference to (extralinguistic) facts. [109]

Broadly taken, Carnap's conception of *L-true* has a similar motivation to ours. However, this approach has serious drawbacks:

(i) The recursion procedure generates a class of sentences for a particular language, but it does not provide an account of analyticity inherent in language. Analyticity defined by meaning postulates remains an arbitrary notion; it cannot help decide on whether a sentence is analytically true or true because of a worldly fact.

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[108] *ibid.*, p. 222.

[109] *ibid.*, p. 10

(ii) Enumeration of the meaning postulates for a particular language is the only way to specify meaning postulates, they do not arise from the nature of the language, as such they are unmotivated.

**Scepticism about Analyticity.** While Carnap has been bringing about a mature conception of analyticity, Quine, as the representative figure of opponents, has been setting forth counter-arguments on the issue. Pursuing the course of the dialectic between Carnap and Quine would be far digressing for us, leading into the matters of epistemology and philosophy of science for us. We shall focus on points that appear to be germane to our present discussion.

The main body of counterarguments has been given in a series of papers by Quine. Quine controverts a distinction between the analytic and the synthetic statements, and claims that the notion of analyticity in the generally received sense has no substantially explanatory value. Taking into account Quine's such other theses as confirmation holism and indeterminacy of translation, his whole discussion can be smoothly integrated to a general frame which we may call empiricist holism. We shall not dwell on Quine's discussion against logical positivism; we will concern ourselves with the points that are contributory and significant from the viewpoint of our framework.

Quine raises questions about the Fregean criterion, hence, notions of definition, meaning and synonymy. He points out that the terms 'definition', 'intension', 'possibility', and 'contradiction' stand in precisely as much need of explanation as synonymy itself. They form what seems to be a small "closed curve in space".

Our conception of '*a priori*', that is, "justifiable independently of experience", is disputably paraphrased as "unrevisable in the light of experience" by Quine. While we admit that there may be justificatory rational (not necessarily innate) principles that are revisable in the face of empirical contestation, Quine's view amounts to that if some statement is empirically revisable, it can only be empirically justifiable. It appears that Quine conducts the discussion on the hidden assumption of a picture with a Cartesian chasm, actually, turning his argument into a straw man kind.

A seeming dilemma about language confronts us: Language is embodied in the speech of its speakers and what we signify by the meaning of a statement expressed in language is actually the relevant knowledge of the competent speakers, which is, if not wholly determining, one of the main factors to recognize its truth-conditions.

Quine argues that logic could not be established by conventions. In generating all the logical truths we must eventually apply rules of inference to finitely many conventionally stipulated statements. We can conventionally stipulate that we will henceforth obey a certain inference rule. But that stipulation does not entail that we are entitled to reason in accord with the inference rule.

Mere conventional stipulation that we will henceforth obey an inference rule that ensures that the rule carries truths into truths is merely another axiom. So we require a new inference rule to draw any consequences from it, and the regress continues:

> [T]he logical truths, being infinite in number, must be given by general conventions rather than singly; and logic is needed then in the meta-theory, in order to apply the general conventions to individual cases.[110]

Therefore, employing logic to derive logic from convention generates only a vicious regress. Turning to set theory and then the rest of science, Quine goes on to argue that, although stipulative definition (what he calls "legislative postulation") "contributes truths which become integral to the corpus of truths, the artificiality of their origin does not linger as a localized quality, but suffuses the corpus".

Consider a scientist introducing a new theoretical term by definitional stipulation. The new term is embroiled in an evolving body of scientific doctrine. As this body of doctrine develops, the original legislated definition occupies no privileged status. We may reject it in light of new empirical developments. Thus, "conventionality is a passing trait, significant at the moving front of science but useless in classifying the sentences behind the lines."

A point suggested in the Quinean argument is whether there is an illusion to perceive some truths vacuous such as "truth by convention" instead of accepting them as their truth is due to their factual obviousness.

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[110] [Quine 1980], p. 132.

The question of synonymy. Quine questions whether we can explain synonymy without presupposing, either directly or indirectly, any notion of analyticity, given that we are trying to explain analyticity. Quine's adequacy constraints on any plausible account of analyticity or synonymy are as follows:

(a) Synonymy as identity of meanings. Meaning of an expression is not such a sharply specifiable entity as to employ reliably in comparisons of meaning. Thus, synonymy in the sense cannot be counted explanatory.

(b) Synonymy as mutual interchangeability *salva veritate* in simple context. It is a plain observation that the truth-value of an expression can be retained when some constituent of it is replaced by another item. This is even so when two interchangeable predicates are intended to coincide on the identical domain of objects. For example, the first one of the following propositions is trivially true, while the second one can only be a result of biological research:

(1) Every creature with a heart is a creature with a heart.

(2) Every creature with a heart is a creature with a kidney.

(c) Synonymy as mutual interchangeability *salva veritate* in an intensional context. The interchangeability criterion can be qualified by the requirement of intensional context. Again, evident counterexamples can be constructed:

(3) Necessarily, all and only brothers are brothers.

(4) Necessarily, all and only brothers are male siblings.

The proposition (3) is trivially true, while (2) can be accepted to be true if the proposition that "all and only brothers are male siblings" is settled to be true by conceptual analysis. But conceptual analysis indicates that the proposition is analytic and cannot count as an explanation of analyticity through synonymy.

(iv) Synonymy as interdefinability. Definition is an encompassing term; there are a variety of methods and conceptions of definitions. Quine's objection is not to definitions. He points

out that a factual truth cannot be introduced merely by stipulation, which would be, by definition, unrevisable.

Another case he takes into account is the dictionary definitions. He discards them as being actually lexicographers' records of expressions that are regarded as synonymous prior to the dictionary, thus cannot count as explanatory.

Thus, Quine does not find anything interesting in the analytic and synthetic distinction of propositions for the sake of epistemology. Then, how does he explain the supposedly widely observed phenomenon? His response is that it is an illusion and account for this illusion in his thesis of holism.

Thus, according to Quine, meaning is explanatorily idle and analyticity is virtually nothing but an inertial resistance to change. The paradigm examples of "high inertia" are the basic laws of logic and mathematics, but any supposedly necessary conceptual truth is, in principle, revisable.

According to Quine, beliefs every one of us holds are located in a "web of belief" in which any one belief bears evidential relations to many others. Thus, an experience does not face refutation or confirmation singly independently of its evidential relations, but together with them. In this "web", some beliefs are "central" as opposed to the "peripheral". The "central" beliefs are the ones that we hold firmer than others and are inclined to preserve in the face of contesting evidence; the "peripheral" ones are for us more easily discardable. We prefer to preserve the system with respect to its coherence, simplicity, predictive power, and elegance. Then no proposition is completely immune to revision. Every sentence can be rejected under pressure from empirical evidence plus a concern for overall coherence.

In the overall assessment, Quine's criticism of analyticity is founded on the difficulties about the reliability of intensional entities as theoretical posits, while favouring extensionality with the view that the difference between science and philosophy ought to be not of quality, but of degree of generality. Nevertheless, the plain fact is that intensional entities are essential ingredients to our thought, and expelling them results in such

untenable results that any actual practice carried out in science would not stand up to Quine's criteria of scientificity.[111]

**Disentanglement of Analyticity.** There is a usual tendency to regard the dichotomies 'analytic/synthetic', '*a priori*/*a posteriori*" and 'necessary/contingent' to be intimately correlated and to make out of them the triples 'analytic*a priori*necessary' and 'synthetic*a posteriori*contingent' and relate the former triple to logic. The argument can be outlined as follows:

(a) A proposition P of a language $\mathcal{L}$ is *a priori* if it is knowable independently of experience (*i.e.,* except for the prior experience required for the acquisition of the language $\mathcal{L}$).
(b) If experience is not determinative of truth of P, then its justification is virtually dependent on its meaning and decided on the basis of pure thought (conceptual analysis).
(c) Then truth of P is not only *a priori*, but analytic as well.
(d) If P is necessary, it must be knowable only *a priori*, since we cannot derive necessity from the experiential world.
(e) If truth of P is analytic and *a priori*, it is necessary, because there is nothing to revise or eliminate P.

Those triple patterns may appear in many situations, but there is no compelling reason to think that they are intrinsically connected, and thus, to draw stronger conclusions from analytic truth. We think that the sound framework of inquiry is to keep separately 'analytic/synthetic' distinction in the domain of semantics, *'a priori*/*a posteriori*' distinction into the domain of epistemology, and 'necessary/contingent' distinction in the domain of metaphysics. Thus, we reject conceptual implication between the other dichotomies. This does not preclude the possibility of coinciding one category with another, but this should be result of a relevant theory. Since many discussions are linked to '*a priori*/*a posteriori*' and 'necessary/contingent' distinctions, we shall examine their relations with one another and analytic/synthetic distinction.

[111] Later, Quine made concessions in his rejective attitude toward analyticity; see [Quine 1991].

***A Priori/A Posteriori Distinction.*** The terms '*a priori*' and '*a posteriori*' are employed to characterise the epistemic foundations upon which the way a proposition is known or the way a person is justified in believing the proposition, whether on the basis of experience or independently of any experience (except for the experience of learning the language in which the proposition is expressed). Traditionally, the distinction between *a priori* and *a posteriori* knowledge has been associated with the distinction between rational and empirical knowledge.

As Kant presents it, *a priori* knowledge is knowledge that one has independently of all possible sense experience. It is universal, since it is knowledge of that which constitutes the formal constraints on all possible human experiences, and it is necessity, since otherwise knowledge would not be possible. For example, knowing that every event has a cause is a case of *a priori* knowledge. By contrast, *a posteriori* knowledge is only possible through experience or introspection.

The *a priori*/*a posteriori* distinction is derivatively applied to concepts, propositions and arguments as well. So, for instance, an *a priori* concept is one that can be acquired independently of experience, which may involve its being innate. An *a priori* proposition is one that is knowable *a priori*. An *a posteriori* argument is at least one of the premisses of which is an *a posteriori* proposition.

It appears that what the category of *a priori* demands from a subject is basically to possess a non-empirical faculty of intellection. For *a priori* knowledge, experiential capacity is needed insofar as to have the required concepts, if at all. We may notice that the distinction hinges on the conception of experience. In any epistemic content that is deemed to be *a priori*, we find *a posteriori* elements originating from worldly facts and *vice versa*. Even if a proposition could be persuasively composed only by *a priori* ingredients, its truth-conditions might very well turn out to fall into the *a posteriori* domain.

We think that a proper way to understand the distinction, respecting our intuitions about the reciprocity between rational and empirical knowledge, is to regard it as modes of epistemic justification. Hence, an *a priori* proposition is the one that is justified dominantly on the grounds of reasoning and conceptual analysis, whereas an *a posteriori* proposition

requires at least indirectly grounds that can be provided only by of empirical scrutiny. Then, a synthetic proposition with *a priori* justification is any proposition constructed in the face of experience, but justification of which depend on a priorly held basis satisfy this type of statement.

In the same manner, it is possible for a statement to be reached by analytical means, but is justifiable by *a posteriori* means. Suppose that '$X \sim Y$' is an analytic truth. There may not have been possible to access to the fact that $X$ is actually $Y$. For example, a woman blind by birth may be a competent speaker of language and can assert that red is a colour solely by her grasp of language, however, until she has acquired the ability to see by a medical treatment, she may not have attested to the truth of the statement that red is a colour. Until the treatment, 'red is a colour' has been merely an *a priori* truth, after the treatment it turned into an *a posteriori* truth.

**Necessary/Contingent Distinction.** In [Kripke 1981], Kripke argues convincingly that the necessary/contingent distinction is conceptually independent from *a priori*/*a posteriori* distinction. We shall remark some highlights from his work. Let us consider the proposition:

(*) Water is $H_2O$.

How can we qualify this truth? 'Water' and '$H_2O$' are, in Kripke's terminology, rigid designators; their referents are fixed, irrespective of any possible world. 'Water' designates the substance water in all possible worlds, and '$H_2O$' designates the substance composed of $H_2O$ molecules in all possible worlds. The proposition (*) states that the two terms are co-referents, and if granted that it is true, then in every possible world, such that a counterfactual case of it cannot be made in any possible world. Therefore, the truth is necessary from the metaphysical aspect.

Though what the proposition (*) states from the metaphysical aspect is a truth in all possible worlds, it is not knowable *a priori*. That the substance ordinarily called water is composed of $H_2O$ molecules is a significant result of the progress accomplished in chemistry empirical means. Therefore, the truth is *a posteriori* from the epistemic aspect.

Alternatively, a contingent proposition can be *a priori* knowable. Thus, it is possible that a proposition comes out to be true or false, but if true, it is knowable without appealing to empirical means. Let us consider Kripke's example of the standard meter bar. Ignoring the actual history of measurement, let us grant that the following proposition about a bar B kept in a Paris museum under certain conditions is true:

The length of the bar B designates 'one meter'.

Notice that there is an incongruity exposed in the proposition between, in the Fregean terms, sense and reference. The proposition stipulates the reference of 'one meter', but does not constitute its sense. 'One meter' is a rigid designator of a certain length in all possible worlds, however, 'the length of the bar B' is not. It is possible that the length of the bar diverges from one meter long under varying physical conditions. Therefore, the proposition states a contingent truth. Since the bar B is definitive of 'one meter', the proposition is knowable *a priori*.

Kripke exemplifies the case of necessary *a posteriori* truth by the Goldbach Conjecture: Every positive even integer greater than 2 is the sum of two primes. The claim is either necessarily true or necessarily false, but discovering which one is an *a posteriori* matter.

It should be clear that, insofar as we stick to the categories of *a priori* and necessity, there is no substantial ground to admit them to be mutually inclusive. Upon these points, we dismiss the debate for and against taking analytic truth as the basis of *a priori* knowledge as impertinent.

In the overall view of the analyticity matter, we see that analyticity is indexed to language competence, which is difficult to specify. Let, for some native speaker of German, Friedrich, the following statements be true:

(a) Friedrich is a speaker of the German language.

(b) Friedrich is not a speaker of the Turkish language.

Suppose that Friedrich has some grasp of the English language as well. There must be an interpolant level of knowledge about the English language, by whose possession we can

affirm that Friedrich is a speaker of the English language, like the German language in (a) as opposed to the Turkish language in (b), so that he can discern analytic propositions from synthetic ones. Let us denote that knowledge by $K_E$. The question is where the boundary for $K_E$ is to be drawn. This intensional matter is the gist of the arguments Quine set forth. But we do not need to follow a constructive method and actually draw the line which depends on a multitude of parameters. What we need the existence of such a level, though may vary with the speakers' circumstances. Constructivist view fails to make a clear distinction between justification and truth. It indicates a procedure for constructing the object under consideration in finitely many steps, and the procedure serves as both an expression of truth and justification. A speaker's knowledge of language; it would be self-defeating for a speaker to know language and but not know the analytic relations that constitutes analytic truth. Hence, from another route, we conclude with Grice and Strawson that the analytic/synthetic distinction is a philosophically useful and widely employed one and that the blurring demarcation does not show that there is no such a distinction, rather, provides a motivation to comprehend it better. What interests us is the linguistic phenomenon of analyticity, not how many speakers share the impression that a particular proposition states an analytic truth. An expression exists in language with its grammar and its grammar specifies the possibilities for the expression to combine with other expressions. If the received meanings are changed, then they will be hardly other than mere homonyms. We can make language change –and it really undergoes changes– but we cannot speak in opposition to language, at least in the context of logic. This is like when we are speaking of the history of the chess game, it makes sense to talk of change.

The conventional aspect of language can be systematised in various ways; a particularly interesting one that incorporates the concept of truth as a fundamental factor is propounded in [Lewis 1975]. Lewis considers languages (in the formal sense of language with associated truth-functional semantics) and, the antithetical, language (in the sense of "a social phenomenon which is part of the natural history of human beings"). He offers a synthesis of these descriptions in the transcendental convention of truthfulness and trust that is present in the language practice of a population of speakers. Conventions in a population are regularities in action and belief that originate from practical and epistemic reasons for people to conform with their belief that others conform, but not requiring

agreement. A population speaks the language L (defined formally) only if there is a convention of truthfulness and trust in L. As long as someone is trusting in L, she is part of the population that conforms to the linguistic regularity. Truthfulness is truthfulness in that the speakers utter only true sentences of L; and trust in truthfulness such that they believe others (generally) to be true sentences of L, and so come to believe what they say. Since formal semantics work in terms of truth conditions, it is a straightforward matter to see what it would be for a sentence to be true. Thus, the formal is connected to the social.

**Grammatical Propositions.** As attested by the work compiled as *On Colours* from Aristotle's discourse upon it, the colour phenomena have been a subject of special interest from various aspects. With its well-understood perceptional basis and relatively standalone character, the colour vocabulary offers us a convenient investigative frame of physical and cognitive factors interacting with language. In the same vein, we shall consider Wittgenstein's treatment of the so-called "colour-exclusion problem" for the examination of grammatical propositions.

The problem has its origin in the metaphysical look of logical atomism advocated in Wittgenstein's Tractatus. See that, simultaneously taken, there is an obvious incompatibility between these two propositions:

(1) The surface of the object A is red all over.

(2) The surface of the object A is green all over.

This incompatibility is traditionally regarded as a synthetic *a priori* truth. In the Tractatus, Wittgenstein claims that, in the final analysis, supposedly, which would reveal the independent elementary propositions, the incompatibility is of logical nature:

> For example, the simultaneous presence of two colours at the same place in the visual field is impossible, in fact logically impossible, since it is ruled out by the logical structure of colour. Let us think how this contradiction appears in physics: more or less as follows—a particle cannot have two velocities at the same time; that is to say, it cannot be in two places at the same time; that is to say, particles that are in different places at the same time cannot be identical.

> (It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colours at the same time is a contradiction.).[112]

However, taking into consideration the gradable properties such as the length of a time interval, the pitch of a tone, the brightness of a colour, in [Wittgenstein], he reaches the conclusion that logical atomism is untenable, and thenceforth, progresses toward the notion of grammatical proposition. To help us clarify Wittgenstein's main idea, we may appeal to Putnam's discussion of property *vs*. conceptual synonymy. We consider 'temperature' (heat) and 'mean molecular kinetic energy'. Though there is a correspondence between them, the warmth of a surface of an object as we attribute to it grades of 'hot' and 'cold' is a different concept than the movement of the molecules making up the object. Putnam calls the correspondence a "property synonymy" and indicates that this does not necessitate a conceptual synonymy. Likewise, we may find physical correspondences to ascribe gradation to colours, such wavelength and intensity and we may think that the property synonymy we get explains our use of colour terms and the incompatibility of the propositions (1) and (2) derives from affirmation and negation of the synonymy relation. Wittgenstein points out that this is a wrongheaded view of language. There is no doubt that each item in our colour vocabulary has physical correlates. The point is that each item defines a method of comparison and samples of reference in which we have been trained during the relevant language practice, our use of this vocabulary derives from this practice. It is this grammar that makes the following statements sensible, while some others not (see [Wittgenstein 1977] for further examples and observations related to the grammar of colour terms):

(3) Black is darker than white.

(4) Nothing can be red and green all over simultaneously.

In the Wittgensteinian terminology, such statements that constitute the meanings and express the rules of use are called "grammatical proposition". By sentential features, they delude into thinking that they state something factual as any empirical proposition. However, they are informative of not factual content, but of the bounds of making sense.

[112] [Wittgenstein 2001], 6.3751

Analyticity has little, if any, bearing on what we aim with grammatical propositions. The limiting semantic feature of analytic propositions is truth, whereas that of grammatical propositions is a complex of rules of making sense. Analytic proposition expresses a relation between the meanings of its components, a grammatical proposition participates in the constitution of the meanings of its components.[113]

We claim that the basis of logical constants is grammatical propositions, as Wittgenstein remarks:

> Whoever calls "$\sim\sim p = p$" (or again "$\sim\sim p \equiv$ p") a "necessary proposition of logic" (not a stipulation about the method of presentation that we adopt) also has a tendency to say that this proposition proceeds from the meaning of negation.[114]

By the discussion so far, we may define three types of validity that do not need empirical justification with exemplary argument texts:

1. Validity justified by conceptual connections. Bealer's example[115] illustrates the point well:

(a) A function effectively calculable if and only if it is λ-definable.

(b) A function is effectively calculable if and only if it is Turing computable.

(c) A function is effectively calculable if and only if it is recursive.

The sentence (a) is Church's proposition and (b) is Turing's. It might seem that (c) follows straightforward from (a) and (b). However, it does not; it is a significant result of co-extensiveness in the computation theory due to Kleene *et al*.

2. Validity justified by grammaticality.

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[113] See [Baker and Hacker 1986], p. 268 *ff*. for an exegetical discussion on the statuses of analytic and grammatical propositions in Wittgenstein's views.

[114] [Wittgenstein 1998], p. 106.

[115] See [Bealer 1998].

(a) This thing is red.

(b) Then, it cannot be green all over.

3. Validity by analyticity.

(a) S is vixen.

(b) Then, S is a female fox.

So, how do we categorise validity by (or, "of") logical consequence?

The prevailing view on logical truth locates it as a subordinate of analytic truth, since it is claimed a sentence is logically true by virtue of the meanings of the logical constants it contains. For example, consider the sentences:

(1) This is a vixen or this is not a vixen.

(2) Every vixen is a female fox.

The sentence (1) is true is said to be true solely by virtue of the meanings of 'or' and 'not', just in the same way that the sentence (2) is true. The difference between (1) and (2) is that while (1) admits all grammatically possible interpretations for 'vixen', (2) admits effectively a single interpretation for 'vixen'. However, this account is distracting: Firstly, because (1) is not only true on the ground of the meanings of 'or' and 'not' taken together, but also on the ground of the inclusive relation between 'or' and 'not', eventually, bearing the relation to the whole logical system, and exclusive relation that they bear to other constituents. See that we want 'or' and 'not' to be as analytically *unrelated* to other constituents as possible: A logical constant makes the rest of the sentential content vacuous –this is what we understand by pure inference. Secondly, this view ascribes a mysterious capability to the meanings of some components of sentence such that they become is virtually the sole responsible of the truth value of the entire sentence.

We hold that logical constants, the building elements of logical form, are specified by relevant grammar. But grammar is the requisite context for constitution of meaning of logical constants without thereby determining what can follow from it. In view of the basis

of logical constants in the grammar and the primacy of logical consequence, which we shall take up below, we claim that a logically true proposition, despite that it can be regarded as a grammatical proposition or an analytic proposition with respect to the occasions of employment (whether they express a grammatical or an analytic relation), can be taken as an enthymematic form of a derivation. For example, the simple tautology $P \vee \neg P$ can be viewed as an enthymematic form of $P \vdash P \vee \neg P$. If this is accepted and logicality is not assimilated to grammaticality nor is it to analyticity, then we can add a fourth type:

(d) Validity justified by form.

Grammatical and analytic truths set a range of interpretations with which logical form has to accord. Just while grammatical and analytic truths constrain logical form, they also provide intelligibility of logical formulas (*cf.* the formalist view, which regards the statements of logic essentially as such statements about certain sign manipulation rules). Thus, the assertion that a proposition $P$ is a logical truth is tantamount to constituting logical form so that $P$ comes out true on the certain interpretations limited by grammatical and analytic truths. This limitation can be spelled out from various vantage points; consider, for instance, the statement of Beall and Restall:

> The argument [from *a* is red to *a* is coloured valid] is valid because in any case (that is, in any possible world) in which something is red, it is also coloured. It is impossible that something be red but that it fail to be coloured.[116]

Looking at the matter this way does not bar unintended interpretations *cf.* non-standard models in model theory). Therefore, we are required to have prior knowledge of the proposition P that what it expresses on intended interpretations is a true proposition. This is the basis of the putatively *a priori* knowledge of the logical truth of the proposition *P*.

As an upshot of our discussion, we may say that logical constanthood alludes to the logical properties of of the reflexive autonomy of language. The grammatical and the analytic, the indicators of autonomy, are among the constituent factors of logical form.

[116] [Beall and Restall 2000], p. 478 *f.*

## 4.3 LOGIC: WHICH TRUTH?

Let us begin by reviewing the fundamental notions of logic, with a tidy quote from Blanchette:

- An argument is *valid* iff (if and only if) its conclusion is a logical consequence of its premises.
- A set of claims *entails* a claim α iff α is a logical consequence of Γ.
- A set of claims Γ is *consistent* iff no contradiction is a logical consequence of it.
- A claim α is *independent* of a set of claims Γ iff α is not a logical consequence of Γ.
- A claim α is a *logical truth* iff it is a logical consequence of the empty set of claims.[117]

We see that they make up a cluster of interdefinability around the notion of logical consequence. Risking oversimplification, we can be said that logic is defined by its specific relation of consequence. Then, we can see that the form in which logical proofs are embedded comes to the fore as a central object of study. Although logic has been usually regarded akin to a calculus, that is, a system of symbolic computation to yield a resultant expression, the notion of logical form displays its aspect making it closer to an algebra, exposing a system of relations between symbols. Each proof system has its own conception of logical form. For our purposes, we prefer an approach to form that deals with logical consequence *in concreto*, so that we can get a grip on its grammar, and subsequently on its bearers, logical constants. Thus, our primary reference of proof system will be to Gentzen's natural deduction calculi which marked a key difference in the explication of logical consequence, as Feferman remarks:

> In contrast to the more usual systems of deduction where the rules of inference $A_1,\cdots, A_n$ / A may be considered to generate formulas $A$, the rules in the $N$-systems $\Gamma_1 \rightarrow A_1, \cdots, \Gamma_n \rightarrow A_n$ / $\Gamma \rightarrow A$ *generate consequence relations between formulas*, reading $\Gamma \vdash A$ as: $A$ is a consequence of Γ; when $\mathcal{D}$ is a

[117] [Blanchette 2001], p. 115.

> derivation of $\Gamma \rightarrow A$, we read this as: *A depends on the assumptions* $\Gamma$ *in* $\mathcal{D}$, or that $\mathcal{D}$ is a derivation of $A$ from $\Gamma$.[118]

Associated with this, another feature of Gentzen's approach which is very helpful for our inquiry is the definition of each logical constant separately by its inference rules. Nonetheless, these are not all for the formal part of the matter. It should be stressed that the whole proof system, for that matter, the logical form, are expressed in a formal language. Hence, the assumptions and constructive principles of a formal language are not directly inherited from the natural language; they are part of the determination of logical constants. The formal language sets limits on the range of *possible* interpretations of sentences. This is a simple, but, crucial point. An illustrative example is the the square of opposition.

The grammar of the diagram is recognized in the Aristotelian-Scholastic logic by formed by the words 'every', 'some', 'not'. On this view, affirmatives have existential import, whereas negatives do not have it, and actually, this is the formative principle of the diagram. The elements of the square and their first-order predicate calculus translations are below:

Every S is P $\forall x(Sx \rightarrow Px)$

No S is P $\forall x(Sx \rightarrow \neg Px)$

Some S is P $\exists x(Sx \wedge Px)$

Some S is not P $\exists x(Sx \wedge \neg Px)$

Notice that the universal propositions are rendered as having conditional import and the particular propositions as having existential import. Hence, conditionality and existentiality are severed in the first-order predicate logic, making the diagram vanish, though it has not lost its soundness for many speakers.

[118] [Feferman 1975], p. 232 (emphases in the original).

The traditional square of opposition with existential import can be regarded as a diagram about the grammar of certain words. If 'Every S is P' is judged to require for its truth that it has existential import (there is *S*), then it can be captured by an extra conjunct added:

$$\forall x(Sx \rightarrow Px) \wedge \exists xSx$$

While specifying the formal language, we make assumptions and stick to certain heuristics. For instance, we try to pair the syntactically simple (atomic) elements of language be also the semantically simple elements. It should also abstain from imposing structure on the domain of discourse. A view of Jakobson on natural language, which he gives credit to Franz Boas for its origin, is viable fro formal languages.[119] Jakobson states that the grammatical pattern of a language (as opposed to its lexical stock) determines those aspects of each experience that must be expressed in the given language. Different languages register different information in a particular case, for example, many languages signal gender differences in their pronouns. In order to illustrate the idea, we may adapt an example from Jakobson. German has three genders whereas Turkish has none. So, a German speaker needs supplementary information, whether agent of an action was a man or a woman, because she must make his choice between a masculine and feminine noun. On the other side, for a Turkish speaker, the question of gender may be judged immaterial or indiscreet, while it is obligatory for German. However, that should not imply a loss of information between languages, rather it indicates a careful translation. Jakobson remarks that "[l]anguages differ essentially in what they must convey and not in what they may convey". The same consideration can be made for construction of the formal language of a logical system. The grammar of such a language is designed with the constraint of what it must convey, but furthermore, *what is not to be conveyed*.

A metalogical property should not be criteria for formalisms, however desirable it is, unless it is shown for a system that it, with the metalogical property in question, provides what we expect from logic, while others do not. For example, first-order predicate calculus has a complete proof procedure, and on this feature, it is purported to be superior and furthermore, the genuine system. But consider the fact that the set of first-order validities is undecidable by Church's Theorem, while the monadic fragment is decidable. Thus, as a

[119] See [Jacobson 2000].

metalogical property, decidability is employed to mark logic off mathematics rather than completeness, then first-order predicate logic would fail and propositional logic would be entitled to be logic proper.

We borrow two questions from the lingo of software engineering that befits our discussion:

(1) Do we produce the right *system*?

(2) Do we produce the system *right*?

This bidimensional view calls for a combination of methods. We can relate the first question to logical validity, whether we have built the right system. We ensure that the principle of validity by form is satisfied with respect to our general norms about valid arguments. The second question can be related to the verification of soundness; whether we have built the system right. We answer by verification to ensure sufficient accuracy. Both validity and soundness are the two relata to ensure that the system embodying the relation of logical consequence is sufficiently accurate with reference to our norms.

We give the primary role in building logical systems to the notion of logical form. We have to give an account of the role of logical truth. This calls for reasonable grounds to designate some truths as logical and others as ordinary truths. We may conceive of logic in a Wittgensteinian way as a truth/falsity game (or, a "token passing" game in which the token is a truth value). In order to elucidate the matter further, we can appeal to a neat example from Church –while Church's own purpose is different from ours, it is not irrelevant.[120] Church states a sentence as a ground to assert further sentences, altering the sentence and the resultant sentences at each step. He obtains the following sequence of sentences:

(A1) Sir Walter Scott is the author of Waverley.

(A2) Sir Walter Scott is the man who wrote twenty-nine Waverley Novels altogether.

(A3) The number, such that Sir Walter Scott is the man who wrote that many Waverley Novels altogether, is twenty-nine.

[120] See [Church 1956], p. 23 *ff.*

(A4) The number of county in Utah is twenty-nine.

The initial sentence, A1, expresses a state of affairs and tokened with a semantic value of 'true'. In the step for A2, the singular term 'the author of Waverley' is substituted by another co-referential one. The next sentence, A3, expresses the same state of affairs, redistributing sub-sentential components —obviously, we can describe the same state of affairs in a variety of linguistic structures and it would be wrong to say that a distinct state of affairs correspond to each true sentence. Finally, we convert A3 to A4 by discharging a singular term and substituting another co-referential one for it. Notice that though the sentences are not synonymous, we appeal neither to semantic analysis, nor to import of new information, but merely to plain substitution and redistribution operations.

Church argues that what is shared by these sentences from the relevant semantic respect is nothing but what they designate, *i.e.*, their truth-values. He points out that a parallel example for false sentences can be constructed in the same manner (*i.e.*, by employing the sentence 'Sir Walter Scott is not the author of Waverley'). Church draws the conclusion from this line of reasoning (which he attributes its origin to Frege) is that, the reference of a declarative sentence is its truth-value. In our framework, meaning does not consist of reference; our interest in Church's argument is in that the propositions occurring in the logical form of an argument are not fact-stating propositions, it is a misconception of logic to regard them so, as if they would provide empirical access to certain substantive facts about the world via logic. Alternatively, we may say, for logic, there is one Eleatic fact. We can see this clearer in the equivalence resulting from replacing the truth values 'true' and 'false' by two special propositions, $\top$ and $\bot$ , incorporated into the formal language.

However, this is not to say that logic has nothing to do with the reality of the world, but to say that its epistemic justification is through language. Hence, epistemological status of logic is dependent on the epistemological status of language. A logical system is to be constructed so that logical truths will coincide with the truths that we obtain from our arguments about the factual matters. Thus, truth constrains logic in various ways. An example is the case of vacuous truth (*i.e.*, truth by default), which seems to be counterintuitive in many situations. Notice that dropping vacuous truth amounts to the

vanishing of the semantic differentiation of 'if … then … from 'and'. In the light of this, let us consider what Pap observes:

> Now, the laws of logic do not describe any contingent features of the world that can be conceived to be different. They do not even describe mental phenomena, e.g., men's habits of drawing such and such conclusions from such and such premises. For if we find a man reasoning fallaciously, *i.e.*, inferring from propositions assumed to be true a proposition that just does not follow from them, we do not say that the relevant law of law logic has been refuted. We are prepared to describe conceivable observations that would refute certain presumed laws of nature, including laws of mental association, but it would be even absurd to suppose that any observations, whether of physical or of psychological facts, might ever refute a law such as "If a thing has either property P or property Q, and it does not have P, then it has Q".
>
> According to the conventionalist's diagnosis of rationalism, the rationalist has been led to postulate a mysterious realm of necessary truths apprehended by reason because, while realizing that the valid sentences of logic do not describe empirical facts, he makes the mistaken assumption that they do describe facts of some kind. But, says the conventionalist, they are not descriptive sentences at all, they are rules. In particular, they are rules for the use of logical constants. Naturally, a rule cannot be refuted by any facts, because it does not make sense to speak of "refuting" a rule; a rule can only be violated.[121]

Therefore, logical form should not be considered in separation from truth-preservation, but we think that this semantic factor is a constraint among others on building the form. is conservation of information in the premisses, concentrating on pure inference. That suggests a kind of reversibility in the form, that is we should build a symmetry into the form, while maintaining deducibility. A grammar of logic is reversible for the consequence relation only if the relation is reversible and defined by the grammar. It may not be always possible to determine back its input, which is the case when there is a one-to-one relationship between input and output states.

A challenge raised by Novaes about logical form offers us an occasion to explain the bidimensional view.[122] Novaes criticises a general approach to the demarcation of logical

[121] [Pap 2002], p. 17.

[122] [Novaes 2012].

constants which she calls "logical hylomorphism as we know it", abbreviating as "LHAWKI". She characterises the the approach on the following main tenets:

1. In every argument, there is something that corresponds to its form and something that corresponds to its matter.

2. The form of an argument is related to a proper subset of the set of its vocabulary, in a given disposition; the matter of an argument is related to the complement set of the subset corresponding to its form.

3. The form of an argument can be rendered by means of a schema.

4. The form of a valid argument is that in virtue of which it is valid.

5. Given that logic is the systematic study of the validity of arguments, it is essentially concerned with forms of arguments.[123]

She holds that the LHAWKI approach to logical form is in the manner of mereological hylomorphism to logic, which can be defined in Aristotelian terms as:

> [A]ny account which not only views wholes as compounds of matter [hulê] and form [morphê], but which also takes both of these components (and, in particular, form) to be themselves parts, strictly and literally speaking, of the whole they compose.[124]

Hence, mereological hylomorphism regards the form as a constituent separate from the matter, allowing that the form and the matter have different ontological statuses, as opposed to the non-mereological counterpart that views the form as the principle of unity articulating the different parts of the matter, yielding the whole. Novaes claims that the LHAWKI approach gets the demarcation problem of logical constants on the wrong track. The issues she raises are centred on the three points:

(1) Ontological status of logical form is indeterminate.

[123] [Novaes 2012], p. 395.

[124] [Koslicki 2006], p. 717.

(2) Validity is understood in an absolute sense (*i.e.*, not as system-relative), implying one unique form per argument. Nevertheless, "logic is concerned with validity *simpliciter*, not just validity that holds in virtue of a limited set of 'logical forms'."[125]

(3) Focusing on logical form ignores the truth-preservation aspect (*i.e.*, the matter of arguments) of logical consequence.

Though Novaes' definition of the LHAWKI approach is too deficient to decide whether it reflects a certain position accurately, her points of criticism are the ones that our framework should be answerable, at least to explicate it better.

As for the first point, the discussion up to this point must have clarified that there is no compelling reason to differentiate the ontological status of logical form from that of language. Thus, the real horse Bucephalus and the fictitious horse Pegasus possess the same ontological status insofar as they are taken simply as genuine singular terms in language such that they are abstracted away from their referential imports. Perhaps, Novaes surmises that the metaphysical issues concerning the mathematical objects are also applicable to logic. Setting aside mathematics, logic is not, as Frege thinks, a discovery enterprise into the realm of logical objects which is, governed by laws of logic, uninfected by the vagaries imputed to natural language.

The second and third points miss how a logical form is built. The primary role of logical form does not necessitate uniqueness. It depends on how the argument is seen, metaphorically speaking, in a particular segmentation of language, like a tomographic image of an object. As we have discussed, truth-preservation is a formative constraint. We think that the view we espouse meshes smoothly with logical pluralism.

We have outlined a view of logic as an artefact (an *organon*) centred on the notion of formality. How does formality relate to normativity and satisfy rigidity and necessity? Our response to these issues shall be on the basis of Williams' interpretation of Wittgenstein's

[125] Novaes quotes [MacFarlane 2009] as a statement she endorses. The validity *simpliciter*/ validity by form distinction evokes Peirce's *logica utens*/*logica docens* based on the Medieval one (see [Haack 1978], chap. 2.). Insofar as validity by form is retained as one of the fundamental objectives of logic, the statement has no force.

view that brings into focus the import of the process of language learning on the normativity of language.[126]

**Genesis of the Essential Values of Logic.** Presumably, there is a variety of determinative relations. Consider the following counterfactual sentences Kim discusses:[127]

> By bringing about the death of Socrates, we could bring about Xantippe's widowhood.
>
> By bringing about Xantippe's widowhood, we could bring about the death of Socrates.

Under ordinary circumstances, we accept the former sentence true and the latter false and affirm the asymmetric dependence. As Kim argues, this determinative relation is not causal; it is not logical, either.

Williams supposes two agents of the initiate learning relation: a novice and a master in order to put a uniform structure on Wittgenstein's examination of drilling, correcting and learning, achieving mastery of a technique in a multitude of contexts. The novice does not have "the cognitive competence required to exercise the skill that is the object of learning" and is dependent on the support of master to make up for the shortcoming. The crucial point is that this dependence is linguistic and causal. Contrary to what is usually supposed, it is not epistemic in the respect that it is not simply a transfer of knowledge, it is an enculturation process into a certain rule-governed practice, which constitutes is the object of learning. Thus, language learning does not consist only of the acquisition of a system formulated in the epistemic field of linguistics. It is building a consensus of action and being trained into it. The novice does not interpret what the master tells him, he internalises them (*i.e.*, the relation between the novice and the rule is unmediated) as rules to accord with. Notice that this view presupposes a common "form of life" that the master and the novice are engaged in and ascribes a unitary core to their actions. The outcome of

[126] See [Williams 2011].

[127] See [Kim 1974].

this process is that the novice achieves to follow rules as he ought to do, however, the rules are not external factors his practice, they are what actually defines his practice.

Empirical regularities reflect 'what is done'; they are "mere history". Rule-following involves 'what ought to be done'. So such modelling methods as game-theoretical pay-off matrices and trade-off solutions, or statistical distributions of the observed language practices do not provide further insight into the normativity of language practice and associated behaviour, they translate them.

On this view, the mastery of a technique, in the Wittgensteinian terminology, unifies the rule of a procedure and the result of the procedure, such that a result that comes out somehow different from the mastery indicates is regarded as a mistake. We can say that the modal gap that surrounds logical form, normativity of logicality and necessity of logical consequence is bridged by language. The essence of each is bound up with another in language.[128]

Seen from the language *qua* viewpoint, normativity of meaning does not exclude alterations by prescribing a "move" and proscribing another antecedently and independently from the circumstances —these could be applicable to the speaker in a speech community in order to exhibit how she ought to display the "correct" idiom, just as how she ought to behave prudentially or morally. In contrast to such regulative norms, semantic norms emerge from the grammars that lie within language.

But the grammar is not external to the word; it is constitutive of its meaning. In this respect, the normativity of meaning is analogous to the normativity of games. So for instance, consider the game of chess: What is left over to make up the game if we set aside its rules?

A sceptical question might be raised as whether it is still legitimate to call it a rule what we have defined, since it does not implicate a normative prescription. At this point, we agree with Whiting on the understanding that the normativity of meaning consists in that

[128] See also [Schroeder 2000] for a discussion of how normative aspect of grammatical propositions derives from descriptive aspect.

"expressions are governed by distinctively semantic proprieties". But notice the statement, as such, is an exposition of the question, rather an answer to it.

It is hard to isolate the normative force in real-life examples. To illustrate the point, we may consider the beginning of the decipherment of Hittite language by the linguist Bedřich Hrozný in 1915: The text which led Hrozný to this was a particular sentence in which a single word discovered to be the word for 'bread'. Hrozný thought that a sentence in which the word bread was used might very well contain the word 'eat'. He made a prediction that the word in the same line as 'bread' and 'eat' might be related to food. With the help of his grammatical considerations based on his presumption that Hittite might be an Indo-European language, Hrozný proceeded and managed to decipher the sentence as: "Then you will eat bread and drink water". It could be asserted that what are predicted are not the meanings but how a form of life carries on. But notice that what are predicted are not the acts of eating and drinking, not even verbal behaviour, but the words of 'eat' and 'drink'. Indeed, Hrozný capitalised on the grammatical norms. These words are so strongly fused with the form of life that they appear to be inseparable; apparently, it comes out to be the same whether to predict the words or the deeds. However, they are separable, changing the view of linguistic apprehension.

Note that necessity should not be identified with a rule. The statement of a rule might be a record of a method, an instrument of teaching or training, or a manner of communication.

Looking over the history of chess, we see that the rules of the game intermittently changed. At a particular episode of the game, a rule might be competing with another resulting in a rule succeeding another, or a novel rule might be introduce and may supersede several others. These occurrences may raise several questions: Is there sufficient semantic differentiation to specify that there is another game? What bearing can this have on meaning invariance? These are questions that shed light to aspects of normativity of meaning.

We can make sense of this by referring to the contrast that Searle makes between regulative and constitutive rules:

> Regulative rules characteristically take the form of or can be paraphrased as imperatives, e.g., "When cutting food, hold the knife in the right hand", or "Officers must wear ties at dinner". Some constitutive rules take quite a different form, e.g., "A checkmate is made when the king is attacked in such a way that no move will leave it unattacked", "A touch-down is scored when a player has possession of the ball in the opponents' end zone while a play is in progress".[129]

We have seen the inferentialistic approaches translate the idea as the formulation that the rules for the use of a logical constant are constitutive of its meaning. However, seen from language *qua* viewpoint, the statement of a rule for a term collapses to just another use of the term. There is no extraneous fact of the matter about what accords with the rule and what does not. Thus, the only remaining is the inherent normativity of meaning. It is an image of the normative in the mastery of rule-following which is irreducible to non-normative element such as causal connections or mere behavioural regularities. The meaning is inherently normative in that it implies its own correct description and application, while it rules out some others. Hence, this sense of normativity should not be conceived as being prescriptive to prompt fulfilments, *e.g.*, an immediate use. Correlatively, the use is an instantiation and the rules for a term guide us, not exhaust the meaning of the term; as Goldfarb puts it:

> [T]he signpost does not determine its application, the mental picture does not determine its projection onto the world, and the rule does not supply its own interpretation. Application, projection, interpretation are practices: they are not further signposts, additional mental images, or rule-like additions to rules.[130]

We take a further step: The use in the practice is the moment of dispelling ambiguity and vagueness, if there are. Meaning of an expression is in unity with the use of the expression in the sense that there is no non-linguistic "meaning-maker" behind or beyond the use of the expression. Therefore, neither meaning nor normativity can be accounted merely by the syntactic stipulation of inferential rules for the manipulation of logical constants.

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[129] [Searl 1969], p. 34.

[130] [Goldfarb 1997], p. 81.

The matter with this kind of normativity is not whether it can be contravened or not. Contravention may actually modify the rule rather than break it.

## 4.4 LOGICALITY AND CONSTANTHOOD

A widespread metaphor is that language is a living organism. Language lives through stupendous variation from a multitude of aspects, as illustrated by the cases of divergences from "proper use" of words, the distinction of literal and figurative meaning, diversity of languages, pidgins, creoles, jargons. A dictionary containing an extensive body of etymologies and the usage records of words present numerous instances of meaning changes. It is usual to come across instantaneous meaning shifts that take place in daily life. Language is continuously subject to revision, not only in meaning, but also in grammar and pronunciation. As Hitchings states, "a language is a transcript of history, not an immutable edifice".[131] Could it be plausible to talk about a general form of an argument that supervenes on meaning while the meaning of each word was fluctuating with its different occurrences in the argument? It appears that we should draw on some methodological assumptions to bring out a form that is insusceptible to meaning variations.

**Inscrutability of Formalisation.** It is one of the Wittgensteinian themes that not only that there is no necessarily common feature to all uses of a particular word that constitutes its essence and fixes its meaning univocally, but there is no indicator external to language that commands us the proper use of a word as well. Wittgenstein's response to the question of ordinary fixity of meaning is his metaphor of family resemblance. Under these circumstances, argument forms are extracted from argument texts, and we think formal objects are precise, rigorous and unambiguous. It may be helpful to look into the denotation of quantifiers in this connection, which are *bona fide* logical constants. There is always a need to take a metatheoretical stance toward logic and take into metalogical results

An expression is said to have a referential import when its meaning depends on evidence for the existence of referents in a certain category. Then, quantifiers are expected to have a

[131] [Hitchings 2011], p. 6.

referential import. The idea that quantifiers denote second-order properties dates back to Frege, who asserted that a quantifier is a function of which argument is another function. Quantifiers do not operate directly on the individuals of a domain; they express relations among the predicate extensions that comprise individuals. The contemporary notion of quantifier is a culmination of a historical progress not without foundational difficulties, as attested by Goldfarb:

> It is certainly true that Frege and Russell saw the quantifier as a central item in their logical systems. They not only introduce symbolic notation to express "for all", but also spend a good deal of time on the question: For all what?[132]

We conceive of logical constants that they do not have referents, while they have to interoperate with the expressions that have referents. Since a definite form is the essentialising concept of logicality and logical constants are the basic elements of the form, one of the fundamental metalogical tasks for any system of logic is to specify the elements that determine the logical form.

Let us suppose the situation in which a scientist, giving an explanation, tells a layman the following:

(1) If the metal is at 280°K, then it is said to be very cold.

(2) If the metal is cold, then it is said to be at 180°K.

Suppose the layman by himself thinks in the meantime that:

(3) If the metal is very cold, then it is said to be cold.

As a competent speaker, his comprehension of language might make him take a further step and conclude that:

(C) If the metal is at 280°K, then it is said to be at 180°K.

We may consider the sentences (1), (2), (3) as premisses and (C) as conclusion, constituting together an argument text. There is obviously something wrong in the

[132] [Goldfarb 1979], p. 351.

argument; at which point? There may be several reasonable replies substantially involving semantic properties and a notion of meaning to be given.

For example, one might indicate that the argument is materially incorrect, since it is essentially a paraphrase of the sorites paradox, conflating the usual vagueness of linguistic meaning with the determinate values of scientific measurement. But this explanation makes use of extra-linguistic reasoning; we seek to confine the argumentation to the text (more precisely, the structural features of the text).

Compare with this: Another one might claim that the issue of whether the detachment of the word 'very' is a valid move or not depends on how the rules of semantic composition is set and under what conditions a meaning is ascribed to the sentences (not only to the words).

A syntactic expression is not merely a representation taking meaning over from semantic interpretation. There is a syntactic view of semantic categories and through this view, the syntactic level possesses the knowledge of semantics to some extent. Hence, syntax is not a tabula rasa for semantics to inscribe meaning. Consider an example of hyphenation, a syntactic device in order to avoid ambiguities:

(4) This is a little-used car.

(4') This is a little used car.

Notice that the word 'little' occurs as an adverb in one sentence, and as an adjective in the other through syntax. As Evans discusses in [Evans 1976], a theory of meaning should be capable of detecting the mapping between syntactic categories and semantic categories, since some syntactic units that fall under the same category can be mapped to different categories at the semantic level. This is an important issue to lay out the logical form of an argument text.

Formalisation is a part of producing logical constants. Logical constants should be thought as new vocabulary, not a translations of natural words.

An example illustrating another aspect is from Ben-Yami.[133] In the following, the sentence (2) is the translation of the sentence (1):

(1) All men are mortal.

(2) $(\forall x)$ $(Man\ x \rightarrow Mortal\ x)$

We may think of (1) as a metalanguage sentence and (2) as the object language sentence that is admissible to be written as a line in a predicate calculus proof (*cf.* our discussion of Scholastic logic).

This translation diverges in several ways from the semantics of the translated sentence: In the first sentence, 'men' used to refer to all relevant men and to them alone while 'mortal' is a predicate attributing a property to men. However, in the second one, both 'Man' and 'Mortal' are predicates erasing the distinction between reference and predication.

The other difference is that in the first sentence, the phrase 'All men' introduces a domain, whereas the second sentence presupposes one as the domain of the sentential function '$(Man\ x \rightarrow Mortal\ x)$'.

As a result of these differences, the following additional difference arises, concerning the way the quantifier functions in each.

In the first sentence, 'all' is joined to the referring expression 'men' (together they form the noun phrase 'all men'), and it determines that the predicate should apply to all the particulars that the term 'men' designates.

By contrast, in the second sentence, 'all' is joined to the variable *x*, and it determines that a complex predicate, the sentential function $(Man\ x \rightarrow Mortal\ x)$, should apply to all the particulars in a presupposed domain. Hence, the sentence does not specify any plurality of particulars, but presupposes one.

In both natural language and the predicate calculus, quantifiers determine to how many particulars from those referred to a predicate should apply. But while plural reference in

[133] [Ben-Yami 2004], p. 64.

the calculus is introduced by attaching a quantifier and a variable to a sentential function, in natural language it is made by general nouns, to which quantifiers therefore attach.

For example, suppose that we contrived the term 'mass' as a logical constant in a suitably built system of logic and it occurs in the sentences like '$\ldots\varphi_1\ldots$ mass $\ldots$ $\varphi_2\ldots$', so that it could meet the criterion of truth-preservation in the Newtonian context. However, it would not be truth-preserving since its semantic characteristics would shift radically in the relativistic context. It can be remarked that the case in the example is closely related to a meaning shift. Can we still face difficulty in preserving truth even in case of meaning fixity?

## 4.5 PROTENSION AND RIGIDITY OF FORM

Beside the generally accepted items of terminology, extension and intension, we offer a mode of meaning that we shall call *protension*. The purpose is to put emphasis on the propensity of the meaning of an expression to alter without losing its integrity, extending to novel cases, like expansion and contraction of a concept.

A contrast can be made with C. I. Lewis's term 'comprehension'. Lewis specifies comprehension of a term as the class of "all possible or consistently thinkable things to which the term would be correctly applicable".[134] In this sense, comprehension is actually the limiting case of extension to include the instances that we have not have access to, therefore cannot count as an instance, but can be subsumed into the extension in congruence with the relevant intension. To put this in the Wittgensteinian terms, there is no need to find out a connection by "family resemblance" to be included in the comprehension of a term, it has been already a member of the family.

Let us consider the discovery of black swans in Australia –-usually taken as a typical example of induction problem. If we assume that it had not been an indispensable property of swans to be of colour white, it would have been "consistently thinkable" that swans could be black as well, then the black swans would be in the comprehension of 'swan'. However, it is a discovery, not its being thinkable, that those black birds having long and slender neck were swans. It might have well be the case that they would have been given another name, for example, a local aborigine name, or they might have been supposed to be

[134] [Lewis 1946], p. 39.

a kind of goose, in which cases it might not have been in the comprehension of 'swan'. What actually happened is that the word 'swan' was projected to the novel case; certainly, with an external factor to the meaning of 'swan' which is relatively independent of one's presumptions that are held prior to its application.

It should be remarked that we do not ascribe an explanatory power to protension. It is offered as a working hypothesis to fix the externality of meaning that is unbeknownst even to the speaker of language. This spreading externality may appear specific only to figures of speech for which Chandler indicates in the context of semiotics:

> Once we employed a trope, our utterance becomes part of a larger system of associations which is *beyond our control*. For instance when we refer metaphorically to 'putting things into word' this involves a further implicit metaphor of language as a "container" – a particular view of language which has specific implications.[135]

It should be remarked that figures of speech are plain stylistic techniques; they are ways of seeing something as something else, thus making a semantic connection to it. For instance, the particular importance of metaphor for philosophy is stressed by Johnson as follows:

> The number of key concepts analysed so far, and the depth of those analyses, strongly support the prospect that our abstract concepts are defined by conceptual metaphor and metonymy. If this is so, then philosophical analysis is primarily metaphor analysis –working out the logic and inferential structure of the metaphors that ground our basic philosophical understanding of experience. Philosophical theories, like all theoretical constructions, are elaborations of conceptual metaphors. In a very strong sense, philosophy is metaphor.[136]

It may seem dubious to attribute a mode of meaning that does not refer to actuality while being on a par with intension and extension. There is a tendency to associate meaning with something that has determinable truth-conditions, something existent and verifiable. Nevertheless, it is deeply misleading to conceive of meaning as a determinate object in separation from language practice.

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[135] [Chandler 2002], p. 124 (emphasis added).

[136] [Johnson 2008], p. 44.

The protension aspect of meaning brings to the fore version of semantic externalism. Indeed, this is one of the underlying points that blocks linguistic idealism in the Wittgensteinian view of language. What we basically understand by semantic externalism is in Nuccetelli's definition:

> A property is nonlocal, external, or extrinsic if and only if it does presuppose the existence of something other than the contingent object that has it.[137]

Adapting this definition to the present discussion, we may say that protension of a term reflects an external property of a term's meaning. A term has the meaning partly in virtue of its relationships to other terms. The matter in question is neither a principle of compositionality, nor externalist accounts of epistemic justification, but interference of meanings. The idea of the dictum is that it is a mistake to separate the meaning of an expression from its use, even by some method of abstraction. Because the use is not an accompaniment of meaning; but it is an accompaniment of an actual expression. The 'is' in the dictum is neither of predicativity, nor of identity, but of unity: Meaning of an expression is in unity with its use. Meaning is attached to an expression when it actualises a use, not by corresponding to a use. The view that identifies meaning of a term with its use makes a positive criterion of meaning out of the use. But the criterion is negative; there is no meaning-maker behind or beyond the use. Dummett misconstrues this, identifying meaning with use, and mistakenly restricts his view to the observable manifestation of linguistic meaning. For Dummett, an instance of use is a moment of verification; for us, it is a moment of resolution. We may clarify our understanding of semantic externalism by referring to Wittgenstein's passage:

> It is, of course, imaginable that two people belonging to a tribe unacquainted with games should sit at a chessboard and go through the moves of a game of chess; and even with all the mental accompaniments. And if we were to see it, we'd say that they were playing chess. But now imagine a game of chess translated according to certain rules into a series of actions which we do not ordinarily associate with a game –say into yells and stamping of feet. And now suppose those two people to yell and stamp instead of playing the form of chess that we are used to; and this in such a way that what goes on is

[137] [Nuccetelli 2003], p. 3.

> translatable by suitable rules into a game of chess. Would we still be inclined to say that they were playing a game? And with what right could one say so?[138]

When two people make bodily movements, that we may devise a method of translation to a chess game does not suffice to assert that they are playing a chess game. Because a chess game does not consist of an encapsulation described by a method of translation (indeed, one may devise a method of translation from some activity to chess, thought it may not be publicly favoured). In fact, a chess game is what it is, above all, with the players following certain rules. This is so, even if all the associated behaviours are just the same as in a genuine case. Both cases that are illustrated are not interpretations of the game. We may compare the case with a chess game performed by two blindfold players telling their moves in a kind of algebraic notation, which would be an interpretation of the game and a genuine game of chess. But the matter is neither intentions of the players, nor completion of a picture by putting parts (players following rules, chess pieces, board, moves, and perhaps, spectators, etc.). The meaning is permeated (or, constituted) in the whole picture. It may be said that there is an integration to the context, rather than an aspect made dominant by the context.

As Katz defends in [Katz 2004], sense is that aspect of the grammatical structure of sentences that is responsible for their sense properties and relations (e.g., meaningfulness, ambiguity, synonymy, redundancy, and antonymy). Senses do not determine the value of either expression types or expression tokens. The theory of sense is thus independent, and consequentially, notions such as 'meaningfulness', 'ambiguity', etc., should be explained merely in terms of sense. Therefore, a theory of sense should be given in terms of the intra-linguistic properties and relations of sentences. Referential import should be conceived as a property of use.

Consider, for example, the sentence 'the hue of those boxes is blue'. Is the meaning of the word 'blue' sufficiently and determinately unequivocal? Let us consider the case of the mythological horse Pegasus and a historical one Bucephalus. Certainly, the meanings of the names are not like absolute points in space; as a matter of course, they have been modified in various ways and it is natural to expect that they will go through modifications in the

[138] [Wittgenstein 2009], §200.

future, too. There is one fundamental difference between them: Pegasus has a being in language, it is described, shaped and communicated in and through language, and whatever happens to it, in the first instance, is up to the linguistic apprehension of speakers. However, Bucephalus has a being in the world; we do not have such a freedom to talk about it as we are about Pegasus. Whatever we think of it depends on a real entity to some extent. What we might say about it could be corroborated by archaeological or historical findings, or a metaphorical usage of that name might turn out to be frivolous by means of the same findings.

The name is tied to its referent, which limits its usage contrary to the non-referential one. Metaphorically speaking, words "float free" or are "anchored" in the realm of meaning — however, this should not imply that having a referent suffices just by itself to hold meaning fixed.

It is the basic hindrance to semantic stability, thus to logical constanthood. Usually, the fixity of meaning is taken as the received view and it is attempted to demonstrate that meaning is fluctuating. We have to proceed in the reverse direction; we will take as the received view that meaning is fluctuating and attempt to show how it could be stable enough for logic. For the language *qua* viewpoint, we do have an inventory of the meanings and guidelines to look up and confer meaning on whatever items we are to speak. This feature revels that a crucial issue for logical constants is to be stable and persistent relative to language. Logical vocabulary is virtually a differentiation from the global vocabulary of language and this is where the justification of logical constants lies. In Wittgenstein's words:

> It strikes us as if something else, something over and above the use of the word 'all', must have changed if '$f$a' is no longer to follow from '(x). $f$x'; something attaching to the word itself.
>
> Isn't this like saying: "If this man were to act differently, his character would have to be different". Now this may mean something in some cases and not in others. We say "behaviour flows from character" and that is how use flows from meaning.[139]

[139] [Wittgenstein 1983], I-13.

We contend the methodological principle that logical constanthood should be treated on the basis of the conceptual componets as logicality and constanthood. It should be remarked that these components condition and qualify one another. Accordingly, we can state two corresponding clauses for a logical constant ξ to be expanded on:

(1) ξ is logical.

(2) ξ is constant.

A usual mistake is to regard a logical constant as an absolute term, for instance, like Planck constant in quantum mechanics. However, we discover Planck constant by the enforcement of natural laws. With the advancement of physics, the precision of its numerical value may be improved, it may be replaced by an analytic expression, or it may be entirely dropped. But there are not separate issues as to its "physicality" and its constanthood. In contrast to Planck constant, a logical constant, for instance, 'and', is differentiated from the vocabulary of language. It has separate clauses to satisfy showing that it serves logic as a constant of logical form.

In this connection, a remark for another mistake is in order. It is usually set as a starting point that a logical constant is a term of which interpretation is held fixed and what we can draw from its use is determinate. According to this understanding, logical constants are the building blocks of logical forms to be filled with variable content. Nevertheless, there may be other terms in a logical system of which interpretations are held fixed.

Every class of expressions can have its specific way of fixing as constant. However, if meaning is not sufficiently stabilised and its protension should be constraint, then it would not be truth-preserving. Therefore, we have to focus on preventing the logical vocabulary from fringing into non-logical vocabulary, that is, the protension of a logical constant should be confined by conditions and constraints within language, since the sole ground of logical form is argument texts. Consider Kneales' passage concerning the origin of logic as a scholarly study:

> Since logic is not simply valid argument but the reflection upon principles of validity, it will arise naturally only when there is already a considerable body of inferential or argumentative material to hand. Not every type of discourse

> provokes logical inquiry. Pure story-telling or literary discourse, for example, does not provide a sufficient amount of argumentative material. It is those types of discourse or inquiry in which proof is sought or demanded that naturally give rise to logical investigation; for to prove a proposition is to infer it validly from true premisses.[140]

Therefore, a logical constant should be univocally fixed and semantically inert to sustain an intended form. We can tackle harmony as a transcendent semantics in the sense of Evans's distinction of immanence/transcendence:

> One provides an immanent definition of some semantic term *W* if one does not define it absolutely but rather defines the notion ⌜*e* is *W* according to theory *T*⌝. One provides a transcendent definition when the definition contains no such relativity to a theory; when one says, rather, what a theory *ought* to treat as *W*.[141]

We have discussed that Dummett's notions of intrinsic and total harmony, though suggestive, are apt for the notion of logical form. Dummett's view that regards logic as a transcription of our argumentation is unsubstantiated. We conceive of the notion of harmony as the harmony of the conditions and the constraints that respect reciprocal relations between its constituents of those patterns. There are indefinitely many ways of arguing cogently for or against any propositions, and harmony can only a resultant criterion of judgement for the argument, not a pre-agreed measure in any way. To put it differently, harmony is a concern for formalisation, not pragmatics.

We can specify on what axes we can develop and formalise the kernel notion of harmony. As a result of our discussion, then, we may rephrase the types of harmony on conceptual grounds without implicating it is to be the complete list:

(a) *Local Harmony*. The pair of introduction and elimination rules is to be encapsulated by harmony. But the local harmony is not sufficient to provide conservativeness and univocally fixing of meaning. Local harmony is concerned with individual proof steps. In

[140] [Kneale and Kneale 1984], p. 1 (emphasis in the original).

[141] [Evans 1976], p. 50 (emphasis in the original).

the present context, it is to harmonise introduction and elimination rules of a logical constant.

(b) *Global Harmony*. The logical system as a whole should be stable, semantically inert and sterile. Global harmony is concerned with derivations and directed to the preservation of the given concept of validity, which has implications on the constraints on proof-theoretic decisions. One and the same argument can be judged valid in different aspects captured by different logical forms.

Local and global harmonies are the inclusive employment of the notion of harmony. We suggest that it should take on also a task of excluding the definition of other constants:

(c) *Mutual Disharmony*. It is usually thought that an excess content in the conclusion relative to the premisses is the feature that has to be abstained; however, an elimination rule that can get less that what the corresponding introduction rule licenses is also a way of interfering with content. This is an issue for logical harmony. But suppose, for example, we can have two logical constants $\xi_1$ and $\xi_2$ that have different introduction rules, but their elimination rules are stipulated such that occurrence of $\xi_1$ licenses what we will get by elimination the other (the case of quantum and classical disjunction can be interpreted as the lack of disharmony. In that case, we cannot rely on the truth-conditions to discern them. For, suppose that, as the result of a particular chess game, we say “White wins”. Though, the truth conditions of this statement just the same as “Black loses”, the meanings are not the same. The disambiguation might be devolved upon the prudence of the speaker, since there is nothing that formally binds these definitions. However, looking from language *qua* viewpoint, this is inadmissible. In regards this,it appears that a *mutual disharmony* condition should be satisfied to block such definitions. Formalisation should secure that rules would not underdetermine each other and become open to ambiguous relations to other rules and furthermore, to non-logical vocabulary.

It should be remarked that this is a different condition from separability. Separability is an inductive step to assure conservativeness at each extension of a logical system. As we have discussed, this is not what semantical inertness requires.

We shall present a logical constant '*revtonk*' in order to illuminate the condition of mutual disharmony, and further clarify the contrast between the Dummettian meaning-based approach and form-based approach we defend. We devise *revtonk* by reversing the rules of Prior's pathological connective '*tonk*'; hence, *revtonk* (designated by ⋈) is defined by the conjunction introduction rule and the disjunction elimination rule. We have pointed out that *tonk* breaches sterility requirement of logical form. We shall discuss that *revtonk* breaches semantic inertness.

The the introduction and elimination rules of *revtonk* are as follows:

$$\frac{\varphi \qquad \psi}{\varphi \bowtie \psi}\ \bowtie\text{-I}$$

$$\frac{\begin{array}{ccc} & \varphi & \psi \\ \vdots & \vdots & \vdots \\ \varphi \bowtie \psi & \sigma & \sigma \end{array}}{\sigma}\ \bowtie\text{-E}$$

Our interest is in two connectives whose elimination rules overlap – ∨ and ⋈, both of which are proof-theoretically well-behaving. Let us construct a maximum formula with a ⋈-I succeeded by ⋈-E:

$$\dfrac{\dfrac{\begin{array}{cc} \Gamma_0 & \Gamma_1 \\ \mathcal{D}_0 & \mathcal{D}_1 \\ \varphi & \psi \end{array}}{\varphi \bowtie \psi} \qquad \dfrac{\begin{array}{cc} & \\ \Gamma_{2,\varphi} & \Gamma_{3,\psi} \\ \mathcal{D}_2 & \mathcal{D}_3 \\ \sigma & \sigma \end{array}}{}}{\Sigma}$$

We can reduce the maximum formula $\varphi \bowtie \psi$ by converting the derivation into:

$$\begin{array}{c} \Delta \\ \mathcal{D}_0 \\ \varphi \\ \mathcal{D}_2 \\ \sigma \end{array}$$

where $\Delta$ is $\Gamma_0 \cup \Gamma_2$ (symmetrically, we could employ $\Gamma_1$ and $\Gamma_3$, thus $\psi$). Therefore, $\bowtie$ has the local harmony property. Also, since we can permute its introduction instances as it is done for conjunction and its elimination instances as it is done for disjunction without any restrictions, the *revtonk* connective normalises.

The proposition $\sigma$ in the above derivation could well be $\varphi \vee \psi$, that is:

$$\dfrac{\dfrac{\begin{array}{cc} \Gamma_0 & \Gamma_1 \\ \mathcal{D}_0 & \mathcal{D}_1 \\ \varphi & \Psi \end{array}}{\varphi \bowtie \psi} \quad \begin{array}{cc} \Gamma_{2,\varphi} & \Gamma_{3,\psi} \\ \mathcal{D}_2 & \mathcal{D}_3 \\ \varphi \vee \psi & \varphi \vee \psi \end{array}}{\varphi \vee \psi}$$

We observe that any grounds that warrant us for the assertion of $\varphi \bowtie \psi$ are sufficient to warrant the assertion of $\varphi \vee \psi$. According to the Dummettian stance, therefore, the rules of $\bowtie$ fail to fix a stable meaning to the connective, conflating it with disjunction.[142]. From our stance, the problem is not at this point –-consider, for instance, any grounds that warrant the assertion of $\varphi \wedge \psi$ are likewise sufficient to warrant the assertion of $\varphi \vee \psi$.

Starting with the idea that inferential rules confer meaning and then proceed to the idea that any claimed complication (*e.g.*, a detected failure of harmony) among rules leads to a defect in the adequacy of meaning is considered questionable also by Read. Read sets out a distinction as that between consistency and coherence of rules:

> Coherence and consistency are different. Coherent rules can be inconsistent, in allowing one to derive contradiction. Consistent rules can be incoherent, when

[142] See [Dummett 1995], chap. 13.

> the meaning given by one rule (e.g., tonk-I, or Curry-Prawitz ⋄I) does not cohere with that given by another (Prior's tonk-E, or Curry-Prawitz ⋄E).[143]

With respect to the distinction, he considers three cases as to the meaning of logical constants and harmony condition:[144]

1. Harmonious rules that confer coherent meaning.
2. Inharmonious rules that are compatible enough to confer a coherent meaning.
3. Inharmonious rules that are incoherent and do not make sense.

Thus, Read evaluates the pathological case of *tonk* by stating that its rules are inharmonious, but consistent and sufficiently coherent. But this way of looking at the matter iterates the question to what criteria should be employed to decide on coherence of meaning. At any rate, insofar as the rules make sense providing clear guidance to perform the operations (*cf.* 'the rules define a circular square'), it is difficult to assess incoherence.

Our main interest in whether we infer from a particular assertion of $\bowtie$ all and only what follows from the various grounds for that assertion is to detect whether logical form interferes with the content, while it should not. With this in mind, consider a segment of a derivation where $\varphi \bowtie \psi$, $\varphi \vee \psi$ and a proposition $\sigma$ that is provable by each one of assumptions $\varphi$ and $\psi$ are available. Then, we may eliminate either $\bowtie$ or $\vee$, and indifferently to what we eliminate, discharge both assumptions with the conclusion of the segment as $\sigma$. But eliminating $\bowtie$ and retaining $\vee$ is effectively weakening the assumptions which step from a stronger statement, that $\sigma$ follows from the determinate case in which both $\varphi$ and $\psi$ hold to a possibly weaker one, that $\sigma$ follows from the indeterminate case in which one from $\varphi$ and $\psi$ has to hold. Furthermore, the weakening is not an outcome of a structural rule, but of a subjective operational choice. The formal defect of *revtonk* is that it is not disharmony with disjunction. Since we can assess a particular logical form relatively within itself, the case of *revtonk* in the presence of disjunction suggests us a mutual disharmony condition should be taken into account in the characterisation of logical constants.

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[143] [Read 2010], p. 571.

[144] *ibid*.

We think that examples can be multiplied, a candidate context for clearer ones being modal systems.

## 4.6 METHODOLOGICAL PLURALISM

By *logical pluralism*, our intention is the view that there are more than one legitimate systems of logic, each with its specific consequence relation. As opposed to pluralism, *logical monism* is the view that there is one legitimate (or, in a weaker sense, orthodox) system of logic. We presume that these broad definitions will be clearer as our discussion proceeds.

Though pluralism and relativism are often regarded to imply one another in many ordinary contexts, they are independent views. We can state the distinction as follows:[145]

- An account about a particular phenomenon *X* is *relativistic* iff the correct account of *X* is a function of (relative to) some distinct set of facts *Y*.
- An account about a particular phenomenon *X* is *pluralistic* iff there is more than one correct account of *X*, even if all the accounts are functions of a fixed set of facts *Y*.

Then, relativism implies pluralism only if the accounts refer to mutually exclusive sets of facts. So, for example, being receptive about conventions varying with the cultures of different societies in different countries can be taken a kind of relativism which does not imply pluralism if each set of facts is supposed to be specified in a sharply exclusive manner by the circumstances of each corresponding country. Our interest in a pluralism that is grounded in the unity of language, therefore, one that does not devolve the questions of logic upon relativism. It is noteworthy point that the unity of language does not necessitate a meaning monism; it suffices to adopt the view that the expressions are translatable and communicable. A conception of pluralism that we can employ on this purpose has been worked out by Beall and Restall.[146] Complying with our framework, their conception respects the centrality of logical consequence and the essential values of logic.

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[145] [Cook 2010] p. 492 *f.* with slight adjustment.

[146] See [Beall and Restall 2000] and [Beall and Restall 2006].

Beall and Restall distinguishes logical systems with respect to the specific type of semantic unit, the *cases*, to which each system refers for the formalisation of logical consequence. On their view, the Leibnizian conception of necessity that underlies logical consequence takes the expression of:

> A claim is necessarily true if and only if it is true in every case.[147]

The types of cases considered are (set-theoretic) models, possible worlds, situations and constructions. Thus, classical account of logical consequence refer to models, necessary truth-preservation account to possible worlds, relevant account to situations and intuitionistic account to constructions. It is noteworthy that the cases are not specified with respect to a certain epistemic field, but arbitrarily employable (*cf.* our general assessment of quantum logic). Then, they proceed to define validity with what they call the "Generalized Tarski Thesis" (GTT) on the basis of *case* indexed by the type '*x*':

> An argument is *valid*$_x$ if and only if, in every *case*$_x$, in which the premises are true, so is the conclusion.[148]

This account of pluralism allows conflicts between logical systems, but these conflicts do not demote any one of them from being a system of logic. So, for instance, intuitionistic logic rejects the classically valid 'not not A, therefore A', while for relevant logic rejects the classically valid 'A and not A, therefore B' if B is, in a prescribed sense, unrelated to A. Admissibility of logical conflicts may seem counterintuitive, but it should be noticed that the central concern of logic is regarded as an exploration into logical consequence, rather than enunciation of logical truths.

Even though there is no consensus on the types of cases, must there be a common type of case that could be the basis of one "right" logical system? From our standpoint, a clear judgement on this question will be mere speculation, given the state-of-the-art logic. This is not to deny that the question is useless, but the response should be to investigate the points at which logical systems converge and the points at which they diverge. This is what we understand by methodological pluralism: elicitation of logic from all the wealth of

[147] [Beall and Restall 2006], p. 26.

[148] *ibid.*, p. 29.

actual argumentation, and in what ways truth is incorporated as a constraint in the definition of logical constanthood.

**Shifting the Perspective: Sequent Calculus.** We observe that the proposed correlates for harmony are usually spelt out in the context of natural deduction. Apart from the issues surrounding those proposals, it should be clear that the perspective that natural deduction systems offers is not the only one to inspect logical constanthood. Indeed, the name 'natural deduction' is given by its founder Gentzen because of the impression it gives. Gentzen says

> My starting point was this: The formalization of logical deduction, especially as it has been developed by Frege, Russell, and Hilbert, is rather far removed from the forms of deduction used in practice in mathematical proofs. Considerable formal advantages are achieved in return.
>
> In contrast, I intended first to set up a formal system which comes as close as possible to actual reasoning. The result was a 'calculus of natural deduction' ('NJ' for intuitionist, 'NK' for classical predicate logic).[149]

However, there is not an objective reason to suppose that **NK** and **NJ** stand as "natural" with respect to others. For example, a Hilbert-style system may well be suited to a certain theory as a natural choice. On the other side, a plain tautology of classical logic that can be proved straightforward by truth-tabular method may become a hard problem in the natural deduction context, demanding quite unnatural "tricks of the trade". For the present purposes, the merit of natural deduction is that it defines logical constants separately and does not imply in the axioms. However, Gentzen's alternative proof system, sequent calculus, does the same, too, right and left rules replacing introduction and elimination rules. For example, conjunction is defined as:

$$\frac{\begin{array}{c}\mathcal{D}\\ A(x)\end{array}}{\forall x A(x)} \qquad\qquad \frac{\begin{array}{c}[A]\\ \mathcal{D}\\ B\end{array}}{A \to B}$$

[149] [Gentzen 1969], p. 68.

Moreover, since there are not only operational (inferential) rules, but also another class of rules called structural rules. These are the rules that give scheme to the properties of reflexivity, weakening and transitivity, which have stated in connection to logicality. Such principles as bivalence and non-contradiction can also be cited among structural rules. Restriction (allowing absence) of structural rules yields substructural systems of logic.

Surely, we should plausibly expect that this explicit aspect about logical constants in sequent calculus is to be somehow implicit in the natural deduction system, just as the features of logical constants remain implicit in the Hilbert-style systems. Some correspondences can be identified from this respect. The right and left introduction rules in sequent calculus correspond to the introduction and elimination rules, respectively, in natural deduction, and likewise, the property of cut-free provability with normality. The structural rules are related to discharge policies.

A discharge policy specifies how many times an assumption is allowed to be discharged in a derivation (the discharge is an optional step in a derivation; an assumption may not be discharged, even if this is possible). A logical system may allow multiple discharges (*i.e.*, discharging more than one instance of a formula) or vacuous discharges (discharging zero instances of a formula in a discharge step). The vacuous discharge in natural deduction corresponds to weakening, multiple discharge corresponds to the cut-rule.

The standard policy allows both vacuous discharge and multiple discharges, whereas linear logic does neither of them. By allowing vacuous discharge and disallowing multiple discharges, we obtain affine logic; the relevant systems are its inverse, allowing multiple discharges and disallowing vacuous discharge. The structural rules yield different logical systems. The systems yielded by subtracting structural rules are called substructural systems.

This widely overlooked significance of sequent calculus has been taken up by few, Paoli and Hacking, in particular. Paoli asserts a two-aspect view of meaning. For a logical constant *c*, he identifies an "operational meaning" and a "global meaning" in the context of a sequent calculus **S**, which he explains as follows:

> The rules for *c* provide, in full accordance with the basic tenets of the proof-conditional theory, its *operational* meaning. However, Quine-type holists would rather take the meaning of *c* as encoded in the relationships between *c* itself and the other constants in the language. Such an aspect –let us call it, for the sake of definiteness, the *global* meaning of *c*– has a formal counterpart in **S**: it can be manifested by singling out the class of provable **S**-sequents containing formulae where *c* is present. In other words, within a sequent calculus framework, the 'molecular' meaning and the 'holistic' meaning of a constant coexist alongside each other. Now, it is well possible that, say, the negation connectives of two logics **L** and **L′** have the same operational meanings but different global meanings, due to the availability of different structural rules in the respective sequent calculi.[150]

Paoli thinks that this allows meet Quine's challenge of meaning variance charge against the rejection of the law of excluded middle in a deviant system of logic. The deviant logical stance points out an empirical discrepancy; nevertheless, according to Quine, all that has been done is to change the subject and to propose another conception of negation. Paoli holds that the operational meaning of negation remains the same, so no problem of "change of subject" exists in his approach. According to him, the locus of a possible disagreement is in the ascription of different properties to a logical constant by **L** and **L′**, which ascribe the same operational meanings to it.

However, structural rules are not defined individually for each logical constant as it is done with inferential rules, but for all of them collectively. Taking into account this difference, we think that the perspective offered by structural/operational distinction accords with our analytic decomposition into logicality and constanthood; thus, our view is closer to Hacking's. According to Hacking, operational rules specify the meanings of constants, the role of structural rules is to "embody basic facts about deducibility and obtain even in a language with no logical constant at all".[151] Operational rules are syntactic devices for which semantics follow from the given notion of truth and consequence:

[150] [Paoli 2007], p. 556 *f.*

[151] [Hacking 1979], p. 294.

> When we add only conservative operational rules, however, we never add any structural postulates. They hold for elementary "prelogical" formulas, and are proved to hold for arbitrary complex formulas.[152]

The import point is that the inferential rules of a logical constant do not exhaust all that is to the term, primarily because they are operative only locally. Beside Hilbert-style axiomatisation and Gentzen's natural deduction, sequent calculi provide us a third perspective to investigate logical constanthood. The sequent perspective can serve to clarify points that remain blurred in the natural deduction perspective, just as the natural deduction perspective does the same with respect to Hilbert-style axiomatisation. For an example, see the debate ([Steinberger 2009], [Tennant 2010] and [Steinberger 2011]) on Tennant's conception of harmony as an equilibrium, in which Tennant presents a sequent calculus version of harmony as equilibrium to remedy the apparent flaws of his conception in the natural deduction setting and the ensuing response from Steinberger.

## 4.7 BIDIMENSIONAL PROGRAMME

There may be miscellaneous reasons to define a logical constant, reject, or modify it. For example, while, on some views, the identity symbol, '=', counts as a logical constant, on others, it designates a binary predicate of non-logical vocabulary. Dummett gives a pro-acceptance argument specifying "quantifier conditions":

> Let us call a second-level condition which, for some domain of objects, is defined, as being satisfied or otherwise, by every predicate which in turn defined over that domain of objects. Among such second-level conditions, we may call a quantifier condition any which is invariant under each permutation of the domain of objects: *i.e.* for any predicate which applies '$F(\xi)$' and any permutation $\varphi$, it satisfies '$F(\xi)$' just in case it satisfies that predicate which applies to just those objects '$\varphi(a)$', where '$F(a)$' is true of $a$. Then we allow as also being a logical constant any expression which, with the help of the universal and existential quantifiers and sentential operators, allows us to express a quantifier condition which could not be expressed by means of those two quantifiers and the sentential operators alone. Thus, the sign of identity is recognized, on this criterion, as a logical constant, since it allows us to express

[152] *ibid.,* p. 298. By "prelogical language'', Hacking conceives of a purely descriptive language.

> the condition that a predicate applies to at most one object, which cannot be expressed without it.[153]

On the other side, Quine questions whether truths constituted by identity are indeed logical truths.[154] He points out that if a statement like '$x = x$' which expresses a truth can be generalised by quantification as $\forall(x = x)$. However, according to him, the general statement thus we obtain is against our conception of logical generality. We get logical generalities not by quantifying over variables as it is done in (empirical) sciences, but, by a semantic ascent, expressing 'Every *sentence* of the form 'p or not p' is true'.[155] Also he purports to show for a statement '$x = y$' that the identity sign is syntactically dispensable by describing the left and right hand sides of the sign by quantification and predicates such that the objects that are to be values of the variables become indistinguishable, hence being indistinguishable passes for being identical.

A logical constant may not be rejected, but exchanged with another one resulting in the demands of new conceptual explanation. For example, the negation sign, '$\neg A$', may be given an intuitionistic definition as $A \rightarrow \bot$, making *falsum* conceptually prior to negation.

But what we seek after is to have a methodology that will abstract away exigencies of logical systems on the basis of a clear understanding of logicality. Then, we think we can sensibly talk about such exigencies. Indeed, our view is akin to the Scholastic view that "conceived of logic as a science of language", but notice that we replace the linguistic grammar with what Wittgenstein would call "philosophical grammar", reading grammar into logic.

So, the study of logical form involves the study of language (including linguistic theories such as generative grammar) as well as proof conditions. We may conceive of this demand of study suggesting a *bidimensional programme*. This has to be a programmatic approach, since it should be clear that there is no straightforward answer to the demarcation

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[153] [Dummett 1981], p. 22*n*.

[154] See [Quine 1986], pp. 61 *ff*.

[155] *ibid*., p. 10 *ff*.

problem; furthermore, it appears that the preconditions for such an answer have not been worked out sufficiently. However, we can expect formally perspicuous conditions that will suffice to characterise superset and subset of the extension of logical constanthood and, like a determinate relation fixed between the metalogical properties of soundness, completeness and decidability, a relation between them is established yielding precise demarcation of logical constants.

An example may help clarify bidimensional view of logical constanthood and what we can expect from progressing with keeping it in sight. McGinn's discussion on necessity as a logical property may be insightful about the grounds of logical formalism in the grammar of relevant expressions.[156] Let us consider the expression ◇p (*mutatis mutandis*, the same considerations can be made for necessity). The standard construal of possibility operator with a proposition *p* is 'p is true at some world *w*'. Then, a question can be raised: Does the notion 'world' in this construal include or exclude *impossible* worlds? If it includes, then a necessarily false proposition holds in an impossible world and we find ourselves confused about to fix the truth conditions of modal propositions. Therefore, we may admit that the notion of 'world' excludes impossible ones. In order to abstain from the circularity implicated in the presupposition of excluding impossible worlds, following McGinn, we can explicate the construal into two clauses as 'p is true in some possible world *w*' *and* 'there are no *impossible* worlds that *p* is true'. But then, we fall into regress using modal notions. This is because we are on the boundaries of the grammar of necessity/possibility, we cannot proceed without breaking sensibility, even if we used different idioms of language corresponding to these modalities or different logical forms. One dimension has already clear about the operator, the formal conditions; see [Read 2010] for the GE-harmony version of their definitions. The other dimension is that we take into account their grammar in endowing them with logicality.

[156] See [McGinn 2000], p. *ff*

# CHAPTER 5

## CONCLUSION

We have presented a view of logic, which depicts it as basically an offshoot of language. As a substantive field of knowledge, it is responsive to the semiotic resources augmenting natural language as well as natural language itself. We have discussed that the three essential values of logic, *i.e.*, *formality*, *necessity* and *normativity* are rooted in language, and maintained that the fundamental resources for logic are argument texts that people employ in their diversity of activities intertwined with language. Logic inherits the modal status of language. Indeed, a characterisation of logical constants is an exploration into the irreducible and real logical properties and relations of language as a reflexive autonomous realm (and the associated arbitrariness of grammar).

We may return to our introductory clauses that motivate corroborative episodes of research to characterise logical constanthood for a term ξ:

(B1) ξ is said to be a logical constant if it accords with the set of criteria Ξ.

(B2) ξ is a logical constant if and only if it is a formalism that satisfies Ξ.

- The primary resources of logic are argument texts. Although logic subsists in what is sensibly spoken in language, it will be misleading to see it as confined to language. Logic refines and enhances our faculties of forming arguments and judgement, and subsequently, turns to language again, augmenting and improving it.
- The essential principles of logic are formality, necessity and normativity. We regard formality as the central one because of its direct relation to (logical) validity and treat the others with reference to formality.
- Logical constants are constitutive elements of logical form. They are the bearers of logical consequence.

- The decomposition of logical constanthood into logicality and constanthood provides a convenient method of analysis. The demarcation problem of logical constants is to specify the proper conditions and constraints along these two ingredient notions for a term to satisfy in order to be conferred logical constanthood.
- Logic deals with pure inference; it is a discipline of consequences, prior than truths. The primary function of truth from the respect of the characterisation of logical constants is to constrain logical form. The constraining truth we refer to consists in grammaticality and analyticity, and contributes to logicality of form.
- We have argued the grounds for logical constanthood are provided by the reflexive autonomy of language, a conception whose basis we borrow from the Wittgensteinian view of language. Then, language serves as the medium where the essential principles of logic are realised. The autonomy offers us a viewpoint that is proper for the characterisation of logical constanthood, which we call language *qua* viewpoint, since it is also the viewpoint language imposes upon us.
- We maintain that logical form must be such that the relation of logical consequence possesses the properties of (i) reflexivity, (ii) weakening, (iii) transitivity, and logical constants are stipulated such that a derivation possesses the properties of (iv) semantical inertness, and (v) content sterility.
- We integrate our considerations with the perspective given by Gentzen's theory of natural deduction as our framework of inquiry. Within this framework, we get the following further results:

Borrowing the concept of harmony from Dummett, we specify three positive conditions that logical form must satisfy:

(a) Local harmony for each logical constant: related to semantical inertness of a logical constant.

(b) Global harmony for each logical system: related to content sterility.

(c) Mutual disharmony for one logical constant with another: related to semantical inertness of each logical constant, since loss of content indicates an interference to content just as well as excess of content.

We draw the conclusion that conservativeness, as a condition which is stipulated by Belnap and Dummett, is not pertinent to the notion of logical form we hold. With this, the arguments favouring intuitionistic logic against classical logic fall.

Our framework supports methodological pluralism, each system insofar as it satisfies the general principles, conditions and constraints is a useful probe into logicality.

While there are informal and formal specifications to demarcate logical constants, we observe that the research on the issue is in its incipient stage. If we would say with reference to Kreisel's notion of informal rigour, the involved notions are not precise enough yet and should be worked out further. Thus, it is hardly possible to set out a squeezing argument to characterise generically the extension of logical constants. We think that our considerations suggest virtually a programmatic approach along two dimensions: First, how to stabilise logical form, mainly formalising harmony conditions, and second, how to determine the truths that constrain logical form. It appears to us that both, but especially, the second dimension, demand an interdisciplinary approach invoking resources and results of linguistics and computer science.

A concluding reflection on the demarcation problem reminds us of the words Wiles says about Fermat's Last Theorem after his proof:

> The problem with working on Fermat was that you could spend years getting nowhere. It's fine to work on any problem, so long as it generates interesting mathematics along the way –even if you don't solve it at the end of the day. The definition of a good mathematical problem is the mathematics it generates rather than the problem itself.[157]

The demarcation problem has already given rise to interesting discussions providing deep insights not only in the philosophy of logic, but also in other philosophical areas, and is yet to do further on the way.

[157] See [Wiles 2000].

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## APPENDIX A

## CURRICULUM VITAE

### PERSONAL INFORMATION

Surname, Name: Beygu, Tankut
Nationality: Turkish (TC)
Date and Place of Birth: 26 April 1963, Eskişehir
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### EDUCATION

| Degree | Institution | Year of Graduation |
|---|---|---|
| MA | METU Department of Philosophy | 2003 |
| BS | METU Electical-Electronics Engineering | 1990 |
| High School | Bornova Anadolu High School, İzmir | 1981 |

### WORK EXPERIENCE

| Year | Place | Enrollment |
|---|---|---|
| 1992-Present | Türk Telekom | Chief Engineer |
| 1992-1991 | Turkish Armed Forces | Software Engineer |
| 1991-1990 | Verap Electronics | Electronics Engineer |

### FOREIGN LANGUAGES

Advanced English, Intermediate German and Greek

### HOBBIES

Chess, Archaelogy, Trekking, Swimming

# APPENDIX B

## TURKISH SUMMARY

## MANTIKSAL SABİTLERİN AYRIMI ÜZERİNE ELEŞTİREL BİR ARAŞTIRMA

Mantıksal sabitler ayrımı mantık felsefesinin süregelen sorunlarından biridir. Argümanların değerlendirebilmesi için bazı gramatik parçacıkların sabit tutulması gerektiğinin farkındalığı Stoik mantığa dek geri gider. Söz konusu sabitlerin hangi kriterlere göre argümanların değişken olabilen bileşenlerinden ayrılacağı, sabitlerin hangi ilkelerle karakterize edileceği mantıksal sabitlerin ayrımı sorununu oluşturur.

Bu sorunun çözümü için çeşitli öneriler sunulmuştur. Önerilerin bazıları mantıksal sistemin sabitleri belirlenirken dikkat alınması gereken genel özellikler niteliğindedir. Örneğin, mantığın değişen konulara karşı nötr olması istenilen bir özellik olduğu için mantıksal sabitlerin de olabildiğince bu özelliği sağlayacak şekilde belirlenmesi uygun olacaktır. Fakat bizi ilgilendiren, böyle genel yol göstericilerden çok, mantıksal sabitlik kavramının açımlanması, bu kavramın kaplamını ortaya çıkartacak olan karakterizasyon ilkelerinin belirlenmesidir. Bu yöndeki öneriler iki ana sınıfa ayrılabilir. İlki karakterizasyon yönelimli olan önerilerdir. Bu öneriler halihazırdaki verili olan mantıksal sabitlik anlayışından hareket ederek, prototip olarak standart eklemleri ve niceleyicileri alarak bir karakterizasyon formüle etmeyi hedeflemektedirler. Öteki sınıf öneriler mantıksal sabitliği bir çatıya oturtmaya, mantıksal sabitler için yeterlilik koşullarını belirlemeye çalışırlar. İlk sınıf öneriler *post hoc, ergo propter hoc* hatasına düşme eğilimindedirler. Bu tezde ikinci sınıf yönelimin tercih edilmesi gereken olduğu savunulmakta ve daha ileri araştırmalar için temel hatlarıyla bir metodolojik bir çatı oluşturulması amaçlanmaktadır. Fakat karakterizasyon yönelimli olan önerilerden de mantıksal sabitlik kavramının geliştirilmesinde önemli katkılar sağladığı düşünülmekte ve onlara eleştiriler bir bakışla alınacak dersler bulunmaktadır.

Mantıksal sabitlik hem formal, hem de informal bileşenleri olan bir kavramdır. İnformal yönü açımlanırken, bu yöne karşılık gelen formal koşullar da tanımlanmalıdır. Bu noktada, en genel olarak göz önünde bulundurulacak sonuç, Kreisel'in sıkıştırma yöntemi uygulayabilecek bir aşamaya gelmek görünmektedir. Buna göre amacımız, mantıksal sabitlik kavramının kaplamını formal olarak tanımlayabildiğimiz bir gerek koşul olarak altküme ve yeter koşul olarak bir üstküme bulmak, bunlardan bağımsız olarak da altküme ve üstküme arasında bir gerektirme ilişkisi ortaya koymaktır. Buna göre, mantıksal sabitlik

kavramının informal bileşenini inceleyeceğiz ve formal karşılıları için izlenecek kriterleri ve daha ileri araştırma eksenlerini ortaya koymaya çalışacağız.

Mantığın temel değerleri olarak zorunluluğu, normatifliği ve formalliği belirleyebiliriz. Mantıksal sabitler mantıksal formu oluşturan öğeler olduğuna göre, bu değerleri sağlayacak nitelikte olmaları gerekir. O halde, mantıksal sabitleri nereden köken aldığına ve mantığın temel değerlerini sağlayacak hangi yeterlilik koşullara uyması gerektiğini incelemeliyiz. Bir mantıksal sabit, örneğin, fizikteki bir sabit gibi monolitik değildir; mantıksallık yönü ve sabitlik yönü olarak iki birbirini tamamlayan yönü vardır. Böylece bir terim belli koşullarla mantıksallığı sağlarken, başka koşullarla da sabitliği sağlamalıdır. Dikkat edilmesi gereken nokta bir yön için belirlenecek koşulların diğer yönden bağımsız olmadığıdır.

Mantık şu yönden yapay bir alandır: Argümanlarımızı doğal dilde ifade ediyoruz, fakat bu argümanlar belli değişmeyen öğeleri olan formlara getirildiğinde mantıksallık özelliği kazanmaktadırlar. Bu formların belirlenmesi de, matematik gibi, doğal dilden ayrı bir etkinlikle gerçekleşmektedir. Vurgulanması gereken bir nokta, mantığın dile dönerek, argümanlarımızın niteliğini ilerletmesi, kavramlarımız arasında mantıksal bağları durulaştırarak kesinleştirmesidir.

Mantıksal sabitler için sezgisel olarak yakın gelen ve Skolastik felsefede derinlemesine çalışılmış olan görüş mantıksal sabitlerin içerik sağlayan (göstergesi olan) ifadeleri (categoremata) birbirine bağlayan, geçişler sağlayan ifadeler (syncategoremata) olduğudur.

Bu görüş nesne dili/üstdil ayrımı yapmadan, doğal dili mantığın nesne dili almakta ve doğal dilin gramatik kategorileri ile mantıksal kategoriler arasında çakışma olduğunu varsaymaktadır. Fakat mantık, doğal bir dille yapıldığında bile, formal sistemlerden ve formalleştirilmiş ifadelerden oluşmaktadır. Doğal dilsel kategorilerle mantıksal kategoriler aralarında bağlantılar olsa da çakışmamaktadır. Mantıksal sabitlerin "sinkategorematik" olduğu görüşünün karşılaştığı güçlükler bize nesne dili/üst dil ayrımının ve mantıkta formal dil kullanımının önemini sergilemektedir.

Sinkategorematik anlayışı, Frege'nin tümceyi semantik birim olarak aldığı, fonksiyon-argüman yapısından mantıksal sabitliğin türetmek için uygulayabilir miyiz bakalım. Fonksiyonların bağ kurmakla görevini yerine getirdiklerini düşünürsek, argümanları kategorematik terimler, fonksiyonları da sinkategorematik terimler olarak alabiliriz. Ancak bir fonksiyon bir argümanla bütünlendikten sonra, başka bir fonksiyonun argümanı olabilmektedir. Bu durumda fonksiyonların düzeylerini ayırt ederek, belli bir düzeye kadar olanları mantıksal sabit olarak belirleyebiliriz. Fakat bu ayrım gelişigüzel kalmaktadır; belli bir düzeyi argümanların geçerliliğinde rol aldığını, ötekilerinin almadığını ileri sürmenin sağlam bir dayanağı görünmemektedir.

Formal dil kullanımına dayalı mantık anlayışına göre getirilen bir öneri permütasyon değişmezliğidir. Bu görüşün ilkesi mantıksal sabitlerin, mantıksal ifadelere konu olan söylem kümelerinin elemanlarının permütasyonu karşısında değişmez kalması kriteridir. Klein'ın geometri için ortaya attığı Erlangen Programında otomorfizm bir geometrik nesnenin kimliğini orta koyar. Tarski tarafından mantığa uyarlanan bu görüşe göre mantıksal ilişkiler, matematiksel anlamda, en genel ilişkilerdir. Bu görüş, mantık ile argüman arasındaki bağı göz ardı etmekte, mantığın en evrensel ilişkileri belirttiği metafizik savına dayanmaktadır. Bu göz ardı ediş, örneğin modal mantık sistemlerinde, gelişigüzel sonuçlar vermesiyle kendini göstermektedir. Mantık, her ne olursa olsun, öncelikle argümanlarda zorunluluk, normatiflik ve formalliktir; uslamlamanın bir modellemesi olmadığı gibi, permütasyon değişmezliği görüşünde kabul edilen metafizik rolü de tartışılmadan kabul edilebilecek bir sav değildir.

Mantık ile dil arasında formal bir ilişki kuran bir önemli bir görüş Davidson'ın yineleme yapılarına dayanan görüşüdür. Bu görüş mantıksal sabitlerin dildeki yineleme yapılarından türetmektedir ve Davidson'ın anlam teorisine dayanmaktadır. Davidson'ın anlam teorisinde bir sözcüğün anlamı doğruluk ile ayrıştırılmaktadır. Buna göre, her bir dil kullanıcısı kendi idiyolektine sahiptir. Doğruya karşılık gelme üzerinden birbirlerinin idiyolektlerini eşlerler. Bu durumda mantıksal bağlantıları kurmak için dilsel bir mekanizma gereklidir. Davidson'a göre bu mekanizma da yineleyici yapılardır. Bu görüş dil içi işleyiş ile mantık arasındaki ilişkiyi fark etmekle birlikte, mantıksal argümanlar arasındaki ilişkiyi kuramamaktadır. Mantıksal argümanların doğrulukla değerlendirilmesi ile gerçeklikteki doğruluk arasındaki ilişki Davidson'ın savunduğu gibi şeffaf değildir. Bu yanılgının sonucu mantıksallıkla ilişkilendirilemeyecek yineleme yapılarında görülmektedir.

Bu tartışmaların sonucunda çıkarımsalcılık (inferentialism) olarak adlandırılabilecek semantik görüşler ağırlık kazanmaktadır. Bunlar kavramsal içerikleri ele alışları yönünden, deneyselcilikle karşılaştırıldığında, akılcılık olarak kategorize edilebilecek görüşlerdir.

Kritik bir nokta mantıksal argümanlarda kullanılan doğruluk değerlerinin gerçeklikteki doğruluğun doğrudan bir izdüşümü olmadığıdır. Mantıksal argümanlardaki doğruluk değerleri semantik izleme amaçlıdır ve dil dolayımı ile gerçeklikteki doğrulukla bağlantılıdır. Bu durum, doğruluk değerini "verum" ve yanlışlık değerini "falsum" olarak özel önermelerle de gösterebileceğimizi göz önüne aldığımızda daha açıklık kazanır. Özünde, mantığın ontolojik ve epistemolojik statüleri dile bağımlıdır. Mantıkta kullanılan doğruluk değerleri, dolaysız olarak dil-dışı dünyaya ilişkin değil, çıkarım prosedürünü izlemek ve sınırlandırmak, doğruluğu değil, doğruluk ile ilişkiyi aktarma amaçlı dile ait semantik değerlerdir. Bu yönden bakıldığında, mantıksal çıkarımları, Wittgensteincıl anlamda, bir doğruluk-yanlışlık dil oyunu olarak da görmek mümkündür. Dolayısıyla, mantığın dil-dışı dünyadaki epistemik başarısı, bu yönden, dilin başarısına bağımlıdır.

Gentzen'in her bir mantıksal sabiti ayrı ayrı, giriş ve çıkış kuralları ile tanımladığı klasik ve sezgiselci "doğal çıkarım" calculus'ları (**NK** ve **NJ**), mantıksal sabitlik kavramına yeni bir perspektif sağlamıştır. Hilbert stili aksiyomatik sistemler, mantıksal sabitler örtük olarak tanımlandığından derinlemesine bir içsel bakışa elvermeyen bir perspektif sunmaktaydı. Bu noktada belirtilmesi gereken bir nokta şudur: Wittgenstein'ın anlam-kullanım anlayışını benimseyerek mantıksal sabitlere çıkarımsalcı yaklaşan çoğu görüş, tanıtım ve çıkış kurallarıyla bir mantıksal sabitin kullanımının belirlenmesi dolayımı ile örtük olarak tanımlandığını öne sürmekte ve tanımlamaları, sorunları ile birlikte, öyle sınıflamaktadır. Ne var ki, her bir kural ifadesi zaten bir kullanımdır. Bu yanılsamayı Hilbert ve Gentzen stili sistemleri karşılaştırarak görebiliriz. Bu bağlamda, şu olanağa dikkat çekmeliyiz: Farklı kanıtlama sistemleri bize mantıksal sabitliğin farklı yönlerini görebildiğimiz perspektifler sağlamaktadır. Bu nedenle, bir kanıt sisteminin mantıktaki önemini değerlendirmiyoruz, fakat mantıksal sabitliği araştırmamızdaki yararlılığına bakıyoruz. Örneğin, Gentzen'ın **LK** ve **LJ** sistemleri, işlemsel ve yapısal olarak iki grup kural tanımlayarak bize farklı bir perspektif sunmaktadır. İşlemsel gruptaki kurallar mantıksal sabitleri tanımlarlar, yapısal gruptaki kurallar herhangi bir terim tanımlamakta, çıkarımların yapısal yönlerini düzenlemektedir.

Paoli bu ayrımdan işlemsel anlam ve global anlam olarak iki yönlü bir anlam modeli üretir. İşlemsel anlam, mantıksal sabitin kanıtlama teorisindeki rolünü belirleyen 'moleküler' anlamdır. Global anlam, mantıksal sabitin diğer sabitlerle ilişkisinden doğan 'holistik' anlamdır. Örneğin, değillemenin işlemsel anlamı farklı formal dillerde aynıdır, fakat global anlamı ayrıdır. Bu farklı sistemlerdeki sabitlerin ortaklığını ve farklılığını açıklar. Ancak, yapısal kurallar her bir mantıksal sabit için ayrı değildir ve ayrım gözetmeksizin bütün mantıksal sabitlerin yer aldığı mantıksal çıkarım ilişkisini biçimlendirmektedir. Dolayısıyla, söz konusu ayrım, mantıksallık ve sabitlik çözümlememize denk düştüğü kanısındayız.

Örtük tanımlama görüşü mantıksal kavramların birbirleriyle olan sıkı tanımsal ilişkilerinden yararlanılarak farklı versiyonlarla karşımıza çıkmaktadır. Bu yaklaşımlara çıkış noktası, mantıksal sabitin anlamının içinde yer aldı önermeyi mantıksal olarak doğru yapan "her ne ise" o olmasıdır; özne, kavramsal olarak önermenin doğruluğunu tanıyacaktır. Bu görüşler şu yönden hatalıdır: Ya öznenin mantıksal doğruluğu tanıması için öncel olarak mantıksal sabitin anlamını biliyor olması gerekir, ya da kavramsal olarak çıkarsaması gerekir, bu seçenek de, tartıştığımız gibi, mantıksal sabitliği psikolojik alana taşımakta ve açıklayıcı bir yan ortaya koyamamaktadır.

Mantıksal sabitlerin anlamını doğruluk veya geçerlilik gibi bir referans üzerinden ortaya çıkarabileceğini ve kaplamının bulunabileceğini ileri süren görüşler için genel olarak uygulanabilecek karşı argüman "modulus iliştirme" adı verdiğimiz bir tekniktir. Bu teknikte, doğruluğu kabul edilmiş bir olgunun ifadesini modulus olarak bir mantıksal sabite iliştirerek yeni bir sabit oluştururuz. Böylece başlangıçtaki mantıksal sabitin dolayısıyla geçerli olan bütün önermeler bu değerlerini korurlar, fakat kaplam *modulo*

belirlenmiş olgu olur. Açıktır ki, mantıksal geçerliliği bir olguya bağlayamamız mantık kavramımıza ters düşmektedir.

Gentzen'in yaklaşımında bir mantıksal sabite anlamı veren asıl anlamlandırıcı kural tanıtım kuralıdır, çıkış kuralı tanıtım kuralından çıkarsanır. Şu noktaya dikkat etmek yararlı olacaktır: Gentzen, calculus'ları matematiksel kanıtlama modeli hedeflenerek ve kısıtlı bir mantıksal sabit kümesi üzerinde kurmuştur. Kuşkusuz, her halükarda, mantıksal sabitlerin ayrı kurallarla tanımlanması mantıksal sabitlik kavramında önemli bir adım olmuştur.

Mantıksal sabitler bağlamında çıkarımsalcı yaklaşım, Gentzen'ın açılımını Wittgenstein "bir dilsel ifadenin anlamı onun kullanımındadır" görüşüyle birleştirerek orta çıkmıştır. Buna göre, bir mantıksal sabitin tanıtım ve çıkış kuralları onun kullanımını tanımlayarak, tam Wittgenstein'ın görüşüne uygun olarak mantıksal sabiti belirlemektedir. Bu yaklaşımın sağladığı tartışma bağlamı mantıksal sabitlik açısından verimli sonuçlar vermiştir. Şimdi önemli sorunlarını ele alarak kendi görüşümüzü ortaya koyacağız.

Prior'un bir karşı örnek olarak tasarladığı 'tonk' eklemi düz bir çıkarımsallığın hatalı olduğunu göstermiştir. 'Tonk', giriş kuralı olarak 'veya' bağlacı şeklinde, çıkış kuralı olarak 've' bağlacı şeklinde tanımlanmış bir mantıksal sabittir. Bir mantıksal sistemde, bu kurallarla herhangi bir p önermesinden herhangi bir q önermesi türetmek mümkün olmaktadır. O halde bir mantıksal sabitin tanımının böyle patolojik durumları engelleyecek belli koşulları sağlaması gerekir. Bu yönde etkili olan Belnap'ın tanısıdır. Belnap, bir mantıksal sabitin, teklik ve mantıksal sistemin çıkarım anlayışının korunumu ilkelerine göre tanımlanması gerektiği görüşündedir. Burada, teklik ilkesi, aynı çıkarımsal rolü paylaşan terimlerin aynı mantıksal sabit olduğu (sadece adlandırma farkı olduğu) ve bir mantıksal sabitin sistemde tek olacak şekilde tanımlanması gerektiğidir. Mantıksal sistemin çıkarımsal anlayışının korunumu ilkesi, sisteme yeni tanımlanan mantıksal sabitin, kendisinin içinde bulunmadığı, önceden türetilmeyen önermeleri türetmeye yol açmamasıdır. Belnap, aksi durumu yeni mantıksal sabitin sistemin çıkarımsallık anlayışında değişikliğe yol açması olarak yorumlamaktadır. Öncel çıkarımsallık anlayışı sorunsuz kabul edildiğinde, bu değişiklik bozukluk olarak da sonuçlanabilecektir.

Dummett, Belnap'ın bu tanısı geniş bir felsefi çatıya oturtarak geliştirmiş, kendi semantik anti-realist anlayışına ve dil teorisine eklemlemiştir. Dummett bu yaklaşımlarından hareketle sezgiselci mantığın klasik mantık karşısında tercih edilmesi gereken sistem olduğunu savunur. Dummett'ın mantıksal sabitlere bakışı fazlasıyla bu teorilerle yüklüdür. Aldığı konum, yol açtığı yanlışlıklar ve bulanıklıklar ile dolaylı olarak yalın bir metodolojik çatının gerekliliğini doğrulamaktadır.

Mantıksal sabitlere çıkarımsalcı yaklaşım ile sezgiselci mantık gerekçelendirme yaklaşımı noktasında kesişmektedir. Bu noktaya ilişkin olarak, örneğin Prawitz'in atomik önermelere dayanan kanıtlama teorisinde formüle edilmiş geçerlilik kavramı, Dummett'ın dilin düzgün kullanımı gibi gerekçelendirme önerileri getirilmektedir. Öne sürdüğümüz çatı, söz konusu

düzeydeki gerekçelendirmeyi mantık ayırt edici varoluş olanaklarında bulmaktadır. Burada "ayırt edici" ile imlediğimiz bilişsel yetiler gibi ayırt edici olmayan olanakları dışarı bıraktığımızdır.

Dummett'a göre bir önerme için, onun ileri sürülebilmesine yetecek koşulların bulunması gerekir. Bunun karşılığında, ileri sürülen bir önermeden çıkarılacak sonuçlar da vardır. Bu sonuçlar da, söz konusu önermenin gerek koşullarıdır. Dummett dilin sağlıklı işleyişi için belirtilen koşulların bir uyum ("harmony") içinde olması gerektiğini savunur. Önermeden çıkarılabilecek sonuçlar ileri sürme koşullarını aşmamalıdır. Aksi durumda, örnek olarak pejoratif ifadelere dayalı önermelerde, ileri sürme koşullarının izin vermediği sonuçlara yol açılarak dilin işleyişi aksatılmaktadır. Mantıksal sabitler özelinde, Dummett iki tür uyumun sağlanması gerektiği görüşündedir: İntrinsik uyum ve toplam uyum. İntrinsik uyum, bir mantıksal sabitin giriş ve çıkış kuralları arasındaki uyumdur. Buna göre, bir mantıksal sabit bir türetmeye tanıtıldığında, eğer ardından çıkış kuralı da uygulanırsa, elenmeden çıkarılan sonucun, mantıksal sabitin tanıtılma koşullarından doğrudan elde edilebilir olması gerekir. Dummett, böylece tanıtım ve çıkış kurallarının ardı sıra uygulanması ile yeni bir içerik ortaya çıkmamasını garanti altına alındığı düşüncesindedir. Bu uyum anlayışı, özellikle Prawitz'in geliştirdiği normalizasyon teknikleriyle bir sisteme genelleştirilmiştir. Normalleştirilebilme özelliğine sahip bir mantıksal sistem, çıkış kuralı uygulaması giriş kuralının uygulamasının hemen ardından gelmese de, bir kanıtlama bu uygulamaların kanıtlamadan çıkarılabilmesine olanak veren normal forma getirilebilmektedir. Böylece her normalleştirilebilme özelliğe sahip mantıksal sistemin Dummett'in tanımladığı intrinsik uyuma sahip olmaktadır. Dummett'in tanımladığı toplam uyum ise, Belnap'ın korunum ilkesine denk düşmektedir.

Dummet'in ardından, başka uyum tanımları da yapılmıştır. Bu yöndeki araştırmaların henüz başlangıç aşamasında olduğunu söylemek yanlış olmaz.

Mantığın varoluş koşullarının dilin özerkliğinde bulunduğunu savunuyoruz. Önce Wittgensteincıl bir kavram olarak dilin özerkliğinden tartışmamız açısından ne ifade ettiğine bakalım.

Wittgenstein insanların etkinliklerini tanımlayıcı bir "son" bulunduğunu tartışır. Örneğin, yemek pişirme kuralları ve etkinliği, 'yemek pişirme' ifadesinin grameri, amaçlanan dışsal bir "son" olan ürün ile tanımlanmıştır. Dil için de bunun geçerli olduğu düşünülebilir. Dilin iletişim işlevi göz önüne alınarak dilin bu işleve göre tanımlandığı göre tanımlandığı söylenebilir. Fakat dili bir iletişim aracı olmaktan ibaret olarak görmek bir yanılsamadır. Wittgenstein dilin etkinliklerin, yaşam biçimlerinin bir bileşeni olduğunu ve onlarda içkin olduğunu tartışır. Dil kullanımının dışsal bir "sonu" yoktur; "son" kullanımın kendisindir. Dil, böylece özerk bir alan oluşturmaktadır. Bu özerklik anlayışı, aslında tanıdık ve özellikle sosyal bilimlerde yaygın olarak kullanılan bir anlayıştır. Örneğin, ekonomik etkinlikler insan ve doğal kaynakların kullanımına, fiziksel olarak ifa edilen süreçlere dayalıdır. Fakat ekonominin yasaları bu kaynaklara ve süreçlere indirgenemez; ekonomi doğası belli bir

ölçüde kendi özerk yasalarına dayalıdır. Dilin özerkliği bağlamında, belirtilmesi gereken bir nokta, dilin kendisi hakkında "konuşmasını" sağlayacak yapılara sahip olmasıdır. Dilin özerkliği, yansımalılık özelliğine sahiptir. İmdi, dilin yansımalı özerkliğinden ne bekliyoruz? Beklentimizi iki başlıkta toplayabiliriz: (1) açıklamada dışa kapalı bağlam, (2) içsel yargı. İlkinden, dışsal etkenlerin açıklayıcılığının sınırlılığını anlıyoruz. Örneğin, 'kırmızı' sözcüğünün dilde bir grameri var. Bu sözcüğün köken anlamına karşılık gelen fiziksel gerçeklik de var. Bu gerçeklikte bir nesnenin kırmızılığı belli bir dalgaboyunda ışık yansıtması ile açıklanmaktadır. Fakat 'kırmızı' sözcüğünün gramerinin açıklanması bu fiziksel gerçeklik üzerinden değil, dil içi referans iledir. Dilin yansımalı özerkliği, açıklamaların dil içinden olmasını öne çıkarmaktadır. İkincisinden, dilin grameri başka bir gerçekliğe değil, dile dayanmaktadır. Dil, öz-gerekçelendirmesi ve öz-geçerlilik sağlayan bir ortamdır. Gramatik kurallar, yanlışlanabilir değildirler, ama değiştirebilirler.

Bu tartışmadan, Wittgenstein düşüncesinin bir "dilsel idealizm" olduğu düşünülebilir. Ancak, özellikle Wittgenstein düşüncesinin dekonstrüktivist yanı göz ardı edilmemelidir. Dilsel idealizmi, Bloor'un tanımladığı gibi, bazı doğruların ve gerçekliklerin dil uygulayımı ile orta çıktığı savı olarak alırsak, gerçekten, Wittgenstein düşüncesinin idealizm geleneği ile ortak yanları vardır. İdealizm noktasından bakıldığında, Wittgenstein düşüncesi "esse est percipi" ilkesini merkez olarak alan Berkeley geleneğine değil, sosyal kurumlara olan vurgusu ile Hegel geleneğine yakındır. Dil öğrenenimi, bir yaşam biçimi edinilen bir eğitim, etkinlere alıştırma, kültür edinme eylemidir. Wittgenstein için, bir ağacın, bağımsız varlığından kuşku yoktur, fakat ağaç, dil kullanımında, ki sadece sözlü uygulama değildir, yer aldığı ölçüde bizim için bir anlam ifade eder. Biz bunu "dilsel kavrama" (linguistic apprehension) olarak adlandırıyoruz. Şeyleri, ilişkin gramerleri ile dilde varolduğu ölçüde tanıyoruz.

Bu noktada şu soruyu mantıksal sabitlik bağlamında yanıtlamak uygun olacaktır: Mantıksal sabitlerin karakterizasyonu için kavramlarımızı temel alabilir miyiz? Kavramlar ile anlamlar arasında önemli bir ayrım vardır: Kavramlar zihinsel içeriklerdir, anlamlar dilsel öğelerdir. Bireyler etkileşim içinde birbirlerinin kavramlarını biçimlendirebilirler ve bu yönden, ortak kavramlara sahip olmaktan söz edebiliriz. Fakat özünde, her bireyin kendi kavramları olduğunu unutmamalıyız. Dilsel ifadelerin anlamları, ortak yaşam biçimlerinin, dil kullanımının bir yönü olarak da görülebilir. Mantıksal sabitlerin dayanağı olarak kavramlar alanını almak, bu mantıksal öğeleri bireysel psikoloji alanına taşımak olacaktır. Halbuki, mantığın nesnelliği zorunlulukta, normatiflikte ve formallikte temellenmektedir. Dolayısıyla, biz mantıksal sabitlerin anlamları ve tanımları üzerinde duracağız ve dil alanı içinde kalacağız.

Dilin yansımalı özerkliği, mantığın varoluş koşulları yönünden önemli bir olanağı sağlamaktadır. Dil dışından gerekçelendirme gereksinmesi olmayan, uygunluğu dil içi kurallara bağlı olan doğru önermelerin olabilmesi bu temelden gelmektedir. Şu halde, (en azından doğrudan) empirik olmayan yöntemlerle gerekçelendirilen önermeleri şöyle sınıflayabiliriz: Kavramsal önermeler, gramatik önermeler, analitik önermeler. Kavramsal

önermeler, kavramlar arası ilişkilerin gösterilmesi ile gerekçelendirilen önermeleridir. Gramatik önermeler, dilin anlamlılık yapısından kaynaklanmaktadır. Bu önermeler, her ne kadar, yüzeysel olarak empirik önermelerden ayrık değil gibi görünseler de, aslında anlam oluşturan, anlamlığın sınırlarını belirleyen önermelerdir. Doğruluk değerlerini gerçekliğe empirik olarak karşılık gelmelerinden değil, dilden alırlar. Analitik önermeler anlamları dolayısıyla doğruluk değeri alan önermelerdir. Gramatik önermelerle analitik önermeler arasındaki fark şöyle belirtilebilir: Gramatik önermeler dilse sözcüklerin anlamlarını belirler ve sınırlandırırlarken, analitik önermeler belirlenmiş anlamların arasındaki ilişkiyi gösterirler. Örnek olarak, 'bekâr kişi evli olmayandır' tümcesini alalım. Bu tümce analitik bir doğruyu ifade eder, çünkü 'bekâr' ve 'evli olma' ifadelerinin gramerleri kurmaktan çok, baskın olarak, bu ifadelerin anlamlarını ilişkilendirir. Mantıksal önermeleri hangi sınıfa koyabiliriz? Mantıksal önermeler doğruluk değerlerini gösterdikleri formdan alırlar. Bu form mantıksal sabitlerle kurulmuştur. Mantıksal sabitler dilin grameri referans alınarak tanımlanırlar, fakat ortaya çıkan formun anlam oluşturucu olduğunu söylemek fazla zorlama bir belirlenim olacaktır. Mantıksal formda yer alan sabitlerin anlamlarını biliyor olacağız, fakat formda yer alan diğer bileşenlerle anlamsal bir ilişkisinin olmamasını da istiyoruz. Bu durumda, mantıksal önermelerin analitik olduğunu savunmak da güç olacaktır. Kavramsal önerme olarak sınıflandırılmamaları gerektiği de açık olduğuna göre, mantıksal önermeleri "form ile gerekçelendirilen" olarak dördünce bir sınıf olarak ele almak en uygun yol olarak görünmektedir.

Bu bağlamda şu ayrımları da belirtmek yerinde olacaktır: Kant'tan bu yana gelen yaygın bir düşünce çizgisinde, mantıksal önermeler, zorunlu, *a priori* ve analitik olarak kategorize edilmişlerdir. Özellikle Kripke'nin tartışmalarından yararlanarak, biz, zorunlu/olası dikotomisini metafizik bir ayrım, *a priori/a posteriori* ayrımını epistemolojik bir ayrım ve analitik/sentetik dikotomisini semantik bir ayrım olarak alıyoruz. İmdi, tartışmamızın merkezini analitik/sentetik dikotomisi bulunmaktadır. Diğer ayrımlar, mantıksal sabitler bağlamında, dile bağımlıdırlar. Daha genel olarak, mantığın ontolojik ve epistemolojik statülerinin dilinkinden bağımsız olmadığını söyleyebiliriz.

Analitik/sentetik önerme ayrımının hakiki bir ayrım olmadığı yönünde, Quine'ın eleştirisi ile başlayan, uzun bir tartışma vardır. Tartışmamızdan anlaşılacağı üzere, gramatik önermeleri tanımakla, analitik/sentetik ayrımını da tanıyoruz. Her ne kadar, bir kişinin bir dili biliyor kabulü, bulunulan koşullara göre değişirse de, bu sınırın esnekliği böyle bir sınırın bulunmadığını göstermez; bizim tartışmamızda önemli olan sınırın nereden çizileceği değil, var olduğu ve belli bir dil kavrayışının analitik ilişiklerin kurulmasını da birlikte getirdiğidir.

Buradan form üzerine konuşmaya başlayabiliriz. Mantıksal sabitleri formun sahip olması gereken özellikleri sağlayacak şekilde tanımlanacağı bir çatı oluşturmaya çalışıyoruz.

Mantıksal formun iki temel özelliği taşıması gerekir: Sterilite ve semantik süredurumdur. Sterilite, mantıksal formun içerik üretmeme özelliğidir. Semantik süredurum formun

argüman içeriği herhangi bir etkileşime girmemesidir. Şu halde, mantıksal sabitleri o şekilde tanımlamalıyız ki, ortaya çıkan mantık formun steril ve semantik olarak süreduran olsun. Bu özellikleri mantıksal sabitin sabitlik yönüne ilişkin alabiliriz. Dummet'ın tanımladığı intrinsik uyum bu yönde kullanılabilecek bir kavramdır. Bunu bir araştırma ekseni olarak belirliyoruz ve "lokal uyum" olarak adlandırıyoruz. "Global uyum" olarak adlandırdığımız ikinci bir araştırma ekseni bir mantıksal sistemin bir bütün olarak sterilite ve semantik süredurum niteliklerine sahip olmasıdır. Fakat bu Belnap'ın korunum ilkesi ve Dummett'in toplam uyum olarak tanımladığından farklıdır. Çünkü biz mantıksal formu, uslamlanın içeriği karşısında bir bütün olarak alıyoruz. Önemli olan mantıksal formun içerik oluşturması veya verili içerikle etkileşime girmemesidir. Bu iki araştırma eksenine ek olarak üçüncü bir eksen de "karşılıklı uyumsuzluk" (mutual disharmony) ekseni olmalıdır. Bu eksenden amaçlanan bir mantıksal sabitin giriş kuralının başka bir sabitin çıkış kuralı ile uyum içersinde olmamasıdır. Aksi durumda, ya doğru mantıksal sabiti kullanmak türetimi yapan özneye kalmaktadır, ki bu "bakış açısı olarak dil" çatısına aykırıdır, ya da form semantik olarak süreduran olma özelliğini yitirmektedir.

Uyum eksenleri formal gerekleri göstermektedir. Her bu koşulları sağlayan terimi mantıksal sabit olarak kabul edebilir miyiz? Mantıksal sistemlerin semantik tabanı dilin grameri ile uyumlu olmalıdır, aksi durumda mantıksal sistemin uslamlama ile bağını kopmuş, herhangi bir formal sistem statüsüne gelmiş olur. Örnek olarak, modal operatör olarak 'olası olmayı' alalım. Argümanlarda bir bileşeni olarak yer alan 'olası olmanın' dilsel algılanışını, anlamsal sınırlarını göz önüne alarak bu operatörün tanımını yapıyoruz. Söz konusu operatör uyumluluk koşullarını sağlamalıdır, fakat tanım aynı zamanda dilsel doğruluk ile de sınırlanmalıdır. O halde doğrulukta toplanmış olan semantik etkenler bir uyum koşullarının birer sınırlaması olarak yer almaktadır. Çıkarımsalcılığın önemli bir eksikliği bu sınırlamayı göz ardı etmesidir. Belirtilmesi gereken bir nokta, doğruluk sınırlandırması bir mantıksal sabitin mantıksallık yönüne de bir bileşen olarak girdiğidir. Bir mantıksal sabitin mantıksal yönü için nasıl koşullar getirilebilir? Dedüksiyonun (herhangi bir çıkarımı değil, mantık formunda olanı belirtmek için bu terimi kullanıyoruz) bağıntısının genel olarak kabul edilen özellikleri yansımalılık, zayıflatma ve geçişlilik olarak kabul edilmektedir. Yansımalılık, bir önermenin ileri sürülmesinin özünden gelmektedir. Zayıflatma, dedüksiyonu tümevarımsal (indüksiyon) çıkarımdan farklılaştıran bir özelliktir. Geçişlilik ise, dedüksiyonun önermelerin tekilliğinden bağımsızlığına ilişkindir. Kuşkusuz, mantıksallık ile imlediğimiz bu bağıntısal özelliklerden ibaret değildir. Mantıksallığın incelenmesi ve kapsamlı olarak ortaya konması mantıksal çoğulculukla mümkündür. Savunduğumuz çatı metodolojik çoğulculuğu, her bir sistem genel ilkelere, koşullara ve sınırlamalara uyduğunda mantıksallığa doğru bir araştırma olarak görmektedir.

Önce çoğulculuk ile görelilik arasındaki farkı belirtelim. Aynı fenomenin farklı olgu kümelerini başvurarak farklı açıklanmasını görelilik olarak, aynı fenomenin aynı olgu kümesine başvurarak farklı açıklanmasını ise çoğulculuk olarak anlıyoruz. Bu genel bakışı

mantığa uyarlarsak, aynı tabandan (dilden) yola çıkarak mantıksal çıkarım ilişkisinin birbirinden farklı (fakat yukarıda belirtildiği gibi kriterlere uygun ise, her biri geçerli) şekilde kurulmasına mantıksal çoğulculuk diyebiliriz. Bizim çatımızda, farklı mantıksal sistemlerin ıraksaması veya bir sisteme yakınsaması değil, mantıksallığın araştırılmasına yönelik bir metot olarak öne çıktığı için, savunduğumuz çatının bu bileşenini metodolojik çoğulculuk olarak adlandırıyoruz. Burada belirtilmesi geren nokta, geçerlilik kriterleri olarak sayılan genel ilkelere, koşullara ve sınırlamalara meta-mantıksal özellikleri katmadığımızdır. Bazı tekilci görüşlerde, özellikle birinci-düzey yüklem mantığı, meta-mantıksal özellikleri nedeniyle ayrıcalıklı olarak "hakiki mantık sistemi" olarak alınmaktadır. Ancak dikkat edilirse, ayrıcalığa temel olan meta-mantıksal özellikler değiştirildiğinde farklı sonuçlar alınmaktadır ki, farklı meta-mantıksal özellikler arasında önem ayrımı yapmak da gelişigüzel kalmaktadır.

Bu tezde, mantıksal sabitler ayrımı üzerine bir metodolojik altyapı geliştirirken, bağlantılı olarak temellendirilmesi dilde olan bir mantık görüşü sunduk. Bu nedenle sadece doğal dildeki argüman metinleri ile sınırlandırıldığı şeklinde anlaşılmamalıdır. Mantığın ana kaynağı argüman metinleridir. Gerçekten, mantıksal sabitlerin karakterizasyonu, yansımalı özerk bir alan olarak dilin, indirgenemez ve gerçekten var olan mantıksal özelliklerini ve bağıntılarını ortaya çıkarmak yönünde bir araştırmadır. Gözden kaçırılmaması gerek şu nokta belirtmeye değer: Bir kökleri sağlam bir bilgi alanı olarak mantık, doğal dile eklemlenen semiyotik kaynakları da, doğal dile olduğu gibi uygulanabilir, etkin olarak argümantasyonumuzu geliştirir.

Genel haliyle, mantıksal sabitlerin ayrımında iki aşama belirleyebiliriz: (1) Bir terim, belli bir Ξ kriter kümesini sağladığında mantıksal sabitlik niteliğini kazanmaktadır. (2) Bir terim Ξ kümesini sağlayan formal koşullara uygunluk gösterdiğinde mantıksal sabit olarak kabul edilecektir.

Mantığın özsel ilkelerinin zorunluluk, normatiflik ve formallik olduğunu ve mantıksal geçerlilikle doğrudan ilintili olduğundan formalliği merkeze aldığımızı belirtmiştik. Mantıksal sabitler mantıksal formu oluşturan öğeler olduklarından mantıksal çıkarımın da taşıyıcısıdırlar.

Mantık, içerikten yalıtılmış, pür (safi) çıkarım ile uğraşır; böylelikle mantık, doğrulardan önce çıkarımların disiplinidir. Mantıksal sabitlerin karakterizasyonu açısından, doğrul mantıksal formun sınırlandırılmasıdır. Sınırlandırıcı doğruluk olarak söz ettiğimiz dilin gramatik ve analitik yapılanmasından kökenini alır.

Dolayısıyla, dil mantığın özsel ilkelerinin gerçekleştirildiği bir ortam olarak hizmet etmektedir. Bunu şu akış ile açıklıyoruz: Dil ile bir form oluşturuyoruz. Dilin bir tarih ortaya koyacak şekilde değişken karakterini (örneğin, sözcüklerin etimolojilerini hatırlayabiliriz) ve dilsel ifadelerin birbirleriyle etkileşimlerini göz önüne aldığımızda, bu formun metalden bir kalıp yapar gibi, bazı sözcüklerin anlamlarını sabit tuttuğumuzu,

değişken bıraktıklarımızı yoruma tabi tutacağımızı ifade etmekle sağlayamayacağımız açıktır. Anlamın bu yanını, anlamın içlem ve kaplam gibi bir modu olarak protension terimi ile vurguluyoruz. Genel anlamıyla, semantik dışsallık protension'ın bir etkenidir.

Formu stabilize etmek için bir dizi koşullar ve sınırlamalar getireceğiz. Form stabilize olduğunda normatifliğinde olanağı ortaya çıkacaktır. Kuşkusuz, bu normatifliğin bir bileşeni de mantıksallık anlayışımız olacaktır. Formun ve normatifliğin sürdürülmesi zorunluluğu sağlar. Zorunluluk bizim dil uygulayımını içselleştirmemizden doğmaktadır. Bu içselleştirme bir usta-çırak ilişkisini andıran bir süreç sonunda ortaya çıkar. Dolayısıyla, dildeki kurallar ve düzenlilik metafizik anlamda bir zorunluluğu göstermez, bir tarihin kaydı olmaktan öte değildirler. Dilsel normlar için de benzer durum söz konusudur.

Özerklik mantıksal sabitlerin karakterizasyonu için bize tekil bir görüş açısı sunmaktadır. Bu görüş açısı aynı zamanda dilin bize empoze ettiği bir bakıştır. Metodolojik çatımızın tabanı olan bu bakış açısını "bakış açısı olarak dil" olarak adlandırıyoruz. Bunun ile mantıksal sabitliğe konuşan veya konuşulan açısından değil, özerkliğindeki dil düzleminden bakmayı amaçlıyoruz.

Lokal uyumu çözümlemekte bir mantıksal sabitin tanımlama kuralları için şöyle bir dedüktif güç sıralaması kullanabiliriz:

(a) Giriş kuralı çıkış kuralından daha zayıftır.

(b) Giriş kuralı çıkış kuralından daha güçlüdür.

(c) Giriş kuralı çıkış kuralıyla denk güçtedir.

Çözümlemelerdeki türetimlerde kullanılan öncülleri ve sonuçları karşılaştırarak kuralları birbirlerine göreli olarak değerlendirebiliriz. Buna göre, Dummett'in intrinsik uyum koşulu ele alırsak, bu koşulun tanıtım kuralının çıkış kuralından daha zayıf olmasını önlediğini görebiliriz. Ne var ki, tanıtım kuralının çıkış kuralından daha güçlü olması durumunu önlememektedir. Çünkü mantıksal çıkarımda daha baskın düşünülen yeni içerik oluşturulmamasıdır. Fakat biz daha genel olarak içerik formu yalıtmaya çabalıyoruz. Bu ise kuralların eş güçte olması ile mümkündür. Aksi durumun bir örneği olarak kuantum mantığındaki 'veya' ekleminin klasik mantıktaki 'veya' ekleminin birlikte kullanıldığın da içeriğin nasıl etkilendiğini gözleyebiliriz.

Çıkarımsalcılığın ana düşüncesinin tartışmamızı ilgilendiren yanını, Sellars'ın verdiği model üzerinden anlayabiliriz. Bu modelde, anlam oluşturma ve kavramsal içerin belirlenmesi çıkarım eylemi üzerinden tanımlanır. Üç temel çıkarım eylemi durumu vardır:

(1) Dile giriş geçişleri: Konuşmacı çevresine uygun bir dilsel etkinlikle tepki verir.

(2) Dil içi hamleler: Konuşmacının dil ile geçerli kalıplarda çıkarımlar yapar.

(3) Dilden çıkış geçişleri: Konuşmacı dilsel ortamdan geçtikten sonra bir eylemi çıktı olarak yapar.

Çıkarımsalcılığın en genel haliyle önemli sorunları vardır. Fakat, daha sınırlı versiyonları, bir terimin mantıksal sabit olarak alınmasının yeterlilik koşullarının anlaşılmasında önemli katkıları vardır.

Prawitz doğruluk gibi semantik kavramlara başvurmayan ve sistemden (belirli bir türetim kuralları kümesi varsaymayan) bir geçerlilik tanımı öne sürmüştür. Buna göre bir tanıtım kuralı ile sonlanan kapalı argümanlar kanonik argümanlar olarak adlandırılırlar ve geçerli olarak kabul edilirler. Çünkü tanıtım kuralı bir mantıksal sabitin kullanılmasının geçerli koşullarını verir. Kanonik forma indirgenebilen başka argümanlar da geçerli kabul edilir.

Prawitz böylece anlamlılığı anlamlı olanı kanonik olana ve kanonik olanı geçerliliğe bağlamıştır. Ne var ki, önerilen bu bağlar hayli zayıftır. Belirli bir forma uygunluk anlamlılık ile doğrudan bağımlı değildir. Bunun bir örneğini Russell paradoksu bağlamında görebiliriz. Russell paradoksunda kendisinin elemanı olmasına izin verilen kümenin bir çelişkiye yol açtığını biliyoruz. Ne var ki, "kendisinin elemanı olan küme" ifadesi anlamsız bir ifade de değildir. Dahası, 'küme' kavramının ötesinde de toplulukların matematiksel olarak kabul edilmesiyle, bütün kümeleri de içeren bir 'öbek' (class) kavramı da kullanılmaktadır. Öyle ise, anlamlılık, ilkesel olarak, belirli bir formalizmin ötesine geçebilir, belli bir formalizm ile sınırlandırılamayabilir.

Dummett iki yönlü bir anlam modeli ile mantıksal sabitleri temellendirmektedir. Doğrulamacı (verificationist) yön, bir önermenin anlamını, ileri sürülmesinin koşullarının tabanında açıklamaya çalışmaktadır. Pragmatik yön ise, bir önermenin anlamını, o önermeden elde edilebilecek sonuçları kendi taban almıştır. Dummett, anlamın bu iki yönün uyumlu olmasıyla dilin düzgün işleyişine uygun olarak ortaya çıktığını savunur. Dikkate değer bu nokta şudur: Dummett'ın bu görüşü çıkarımsalcılıkla uyumludur. Çıkarımsalcılıkta anlam (ve kavramsal içerik) her durumda veya baskın olarak genelde (çıkarımsalcılığın versiyonlarına göre bu vurgu değişmektedir), çıkarımsal ilişkilerden doğmaktadır. Şu halde, uyumun bulunması veya bulunmaması durumu anlamlılığı etkilemez, dilin düzgün işleyişiyle ilintilidir. Dolayısıyla, bu yönden bakıldığında uyum kavramı, dilsel ifadelerin anlamlarını çıkarımsal rollerinde arayan anlayışların eleştirisine bir karşılıktır. Çıkarımsal ilişkilere bağlanamayacak bir anlamın varlığı, artık bir karşı örnek değildir, söz konusu olan sadece gerekli uyumun bulunmamasıdır.

Dikkat edilmesi gereken bir nokta şudur: Uyum, felsefi bir kavramdır; bu kavramın formalizasyonu olduğu ileri sürülen kanıtlama teorisinde karşılıkları vardır. Bu yönde, Dummett'ın iki önerisini belirttik. Bir başka öneri Tennant'ındır. Tennant, uyumun oyun teorisindeki Nash denge noktası gibi, tanıtım ve çıkış kurallarının denge noktası olarak formüle eder. Buna göre, bir mantıksal sabitin tanıtım kuralı bir öncüllerden çıkartılabilecek dedüktif olarak en güçlü önermeyi oluşturmalı, çıkış kuralı söz konusu

mantıksal sabitin çıkartılması ile elde edilebilecek dedüktif olarak en zayıf önermeyi oluşturmalıdır. Tennant'ın önerdiği özünde bir dengeye ulaşma sürecidir. Uyum, hakikaten, belli bir dengeye gelmekle ilişkili olsa da mantıksal forma böyle bir sürecin temel oluşturması kuşkuludur. Hakikaten, standart önerme eklemlerinde Tennant'ın uyum kavramı çalışmakla birlikte, niceleyicilerde hatalı sonuç vermektedir.

Bir başka uyum anlayışı "genel-çıkış uyumu" olarak adlandırılandır. Bu görüş, bütün mantıksal sabitlere genelleştirilmiş bir çıkış kuralı şeması önermektedir. Bu görüş, korunma ilkesinin gereksizliğini savunmakla önerdiğimiz metodolojik çatı ile uyumludur. Başkaca konumuzu ilgilendirecek felsefi sonuçlar verip vermeyeceğini belirtmek için ise henüz erkendir. Fakat şu belirtilebilir: Kanıtlama teorisinin teknikleriyle formüle edilen uyumu, mantıksallık olarak hatalı olacaktır. Uyum kurallarına uygun terimler tanımlanabilir, bunlar tutarlı bir anlamsal içeriğe de sahip olabilirler, fakat mantıksal çıkarım açısından etkin bir rolleri olmayabilir. Uyum koşulları sabitliğin mantık özelindeki tarifidir denilebilir.

Diğer bir uyum anlayışı Dummett'inki ile paylaştığı yanlar olsa da, çıkarımsalcılığın mantıksal argümanların geçerliliğine genişletilmesi ana düşüncesi üzerine kuruludur. Burada, uyum kavramının gerekçelendirme yönünün vurgulandığını görüyoruz. Uyum, mantıksal sabitlerin tanımlanmasını da mantıksal olarak uygunluğunu da gerekçelendirmektedir. Dikkat edilirse, bir mantıksal sabit çıkarım kuralları ile tanımlandığında, ortaya çıkan çıkarımın geçerliliği model teorinde olduğu gibi test edilememektedir. Bu nedenle, bir yanıyla kanıtlama teorisine, diğer yanı ile çıkarımsalcılığa dayalı bu görüşlerde bir gerekçelendirme tabanına gereksinim vardır.

Argümanların mantıksal formunun tek olduğunu düşünmek bir yanılsama olacaktır. Mantıksal form bir dilin bir görünümün kurulmasını gerektirir. Örneğin, tümcenin özne-yüklem modeli ve fonksiyon-argüman modeli farklı dil görünümlerine karşılık gelmektedir. Bütün bir süreç olarak argümanların mantıksal formalizasyonu farklılıklar gösterebilir, geçerli bir argüman, farklı bir formda geçersiz duruma gelebilir. Dolayısıyla, formalizasyonun bir belirsizliği vardır. Bu belirsizliğin giderilmesinde dil görünümlerinin geliştirilmesi önemli bir etkendir. Metodolojik çoğulculuğun önemi burada da ortaya çıkmaktadır.

Dummett'ten ödünç alarak üç pozitif koşul belirledik:

(a) Lokal uyum ağırlıklı olarak mantıksal sabitlerin semantik etkisizliği ile ilintilidir.

(b) Global uyum, mantıksal sabitler vücuda gelen sistemlerin içerik yönünden steril olmasıyla ilintilidir.

(c) Bir mantıksal sabit ile bir başkası arasındaki karşılıklı uyumsuzluk semantik etkisizliğin başka bir yönüyle ilintilidir. Çünkü argüman içeriğinin bir şekilde azalması da, yeni içerik eklenmesi gibi, içerik ile formu etkileştiğini imler. Bu ekseni 'revtonk' olarak

adlandırdığımız, Prior'un 'tonk' ekleminin tersine çevrilmişi olan bir eklemle örneklendirdik.

Bu eksenler üzerinde kanıtlama teorisinin önemli katkıları olacağını öngörebiliriz. Ne var ki, halihazırdaki çalışmalar üzerinden bir değerlendirme yapmak spekülasyon olacaktır.

Tartışmamızdan Belnap ve Dummett tarafından ileri sürülen getirilen korunum ilkesinin mantıksal form anlayışı ile bağlantılı olmadığını ortaya çıkarmıştık. Böylece, tertium non datur ilkesinin sezgisel mantığa eklenerek klasik mantık elde edildiğinde Peirce kuralının klasik mantıkta kanıtlanabilir olmasını örnek göstererek, korunum ilkesinin klasik mantıkta sağlanamadığı şeklinde, Dummett'in sezgiselci mantığı tekilleştiren savı da çözülmüş olmaktadır, ki bu sonuç da metodolojik çoğulcu yaklaşımımızla uyumlu ve onu destekleyicidir.

Tartışmamızın desteklediği bir başka sonuç da şudur: İlke olarak, her hangi bir kategorideki terim için mantıksal sabitlik o terimin özsel olarak sahip olacağı bir nitelik değildir. Kuşkusuz, bazı terimler doğal dildeki konumları itibariyle mantıksal sabitlik işlevine daha yatkındırlar. Fakat bu onları, diğerlerine karşı "hakiki" veya "birincil" yapmaz. Mantıksal sabitliğin ayrımı sorunu, bu yönden bakıldığında, koşulların ve sınırlamaların belirlenmesi sorunudur.

Halen mantıksal sabitlerin ayrımı üzerine informal ve formal öneriler bulunsa da, bu sorun merkezinde geliştirilen kavramların henüz Kreisel'in "informal sağınlık" (informal rigour) olarak adlandırdığı formülasyona getirilebilecek hassaslıkta olmadığını gözlüyoruz. Dolayısıyla, bu tez bir manifesto şeklinde olmamakla birlikte, "İki-Boyutlu Program" olarak adlandırabileceğimiz programatik bir süreci öngörmektedir. Birinci boyut, mantıksal formun nasıl kararlılık düzeyine getirileceğidir. Buradan kastımız, özellikle, uyum koşullarının formal olarak ortaya konmasıdır. İkinci boyut, mantıksal formu sınırlayan doğruların nasıl belirleneceği ve mantıksal sabitlerin karakterizasyonuna katılacağıdır.

Her iki boyut için, fakat özellikle ikinci boyut için daha ağırlıklı olmak üzere, disiplinler-arası bir yaklaşımın gerektiğini düşünüyoruz. Öncelikle, dil ve bilgisayar bilimlerinin kaynaklarının ve sonuçlarının bu sürece önemli katkılar sağlayacağını öngörüyoruz.

## APPENDIX C

## TEZ FOTOKOPİSİ İZİN FORMU

**ENSTİTÜ**

| | |
|---|---|
| Fen Bilimleri Enstitüsü | |
| Sosyal Bilimler Enstitüsü | X |
| Uygulamalı Matematik Enstitüsü | |
| Enformatik Enstitüsü | |
| Deniz Bilimleri Enstitüsü | |

**YAZARIN**

Soyadı : Beygu
Adı : Tankut
Bölümü : Felsefe

**TEZİN ADI** (İngilizce) : A Critical Inquiry into the Demarcation of Logical Constants

**TEZİN TÜRÜ** : Yüksek Lisans [ ] Doktora [X]

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. [X]
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. [X]
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. [ ]

**TEZİN KÜTÜPHANEYE TESLİM TARİHİ**: