RECTILINEAR INTERDICTION PROBLEM BY LOCATING A LINE BARRIER

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ABSTRACT

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This study is an optimization approach to the rectilinear interdiction problem by locating a line barrier. Interdiction problems study the effect of a limited disruption action on operations of a system. Network interdiction problems, where nodes and arcs of the network are susceptible to disruption actions, are extensively studied in the operations research literature. In this study, we consider a set of sink points on the plane that are being served by source points and our aim is to study the effect of locating a line barrier on the plane (as a disruption action) such that the total shortest distance between sink and source points is maximized. We compute the shortest distances after disruption using visibility concept and utilizing properties of our problem. The amount of disruption is limited by imposing constraints on the length of the barrier and also the total number of disrupted points. The suggested solution approaches are based on mixed-integer programming and a polynomial-time algorithm.

Keywords: Line barrier, Location, Interdiction

ÖZ

ÇİZGİ BARİYER YERLEŞTİRME İLE DOĞRUSAL ENGELLEME SORUNU

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Yüksek Lisans, Endüstri Mühendisliğ Bölümü
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Ortak Tez Yöneticisi : Doç. Dr. Canan Sepil

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Bu çalışma, bir çizgi bariyer yerleştirerek doğrusal engelleme sorunu için bir optimizasyon yaklaşımı geliştirir. Engelleme sorunları bir sistemin operasyonları üzerinde sınırlı bir bozulma eyleminin etkisinin araştırılmasını içerir. Bu çalışmada, hizmet vermekte olan kaynak noktaları ile hizmet alan hedef noktalari arasındaki mesafe, düzlemde bir çizgi bariyeri yerleştirerek en çoklanır. Bu bozma eyleminden dolayı yukselmiş olan uzaklık en kısa mesafe probleminin özelliklerine göre hesaplanır. Bozulma miktarı bariyerin uzunluğu ve bozulan noktaların sayısı ile sınırlıdır. Tamsayılı doğrusal programlama ve polinom zamanlı algoritmaların geliştirilimesi önerilen çözüm yaklaşımlarıdır.

Anahtar Kelimeler: Çizgi bariyer, Yer Seçimi, Engelleme

Willa, this is for you, obviously

for inspiring me to solve:

maximize $\sum_{m \in T} \heartsuit_m$

for all moments, m, in time, T

despite the occasional drab constraints of

$\heartsuit_m \leq \clock \;,\; \exists m$

$\heartsuit_m \leq \$ \;,\; \exists m$

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CHAPTER 1

INTRODUCTION AND LITERATURE REVIEW

One of the most crucial operations for organizations is logistics where a flow of resources are transported from the point of origin(source) to the point of destination(sink). Resources can be anything from food, material, equipment, troops, and artilleries to information, energy, and particles. Usually, distance between the source and sink points causes a travelling cost on transportation of resources. Besides, logistics usually has a repetitive nature which implies that the related cost and efforts are scaled by the frequency of deliveries.

To evaluate the reliability of one's logistics system, we can perform a sensitivity analysis on critical transportation components. For example, closing down some of the roads due to maintenance or traffic accidents may have an adverse effect on the performance of transportation system while shutting down a factory might lead to loss in supplying goods and services to customers. In operations research literature, studying the effect of such disruptions in a system's operations are addressed in interdiction problems. The reasons of disruption vary from technical failures and natural disasters to malicious operations.

Apart from identifying the vulnerabilities of one's system against disruption, there is another application for interdiction problems when there is a competition between two adversaries. In a competitive environment, efficiency of our logistics operations is as much important as inefficiency of our rival's. In such context, our rival's loss can be interpreted as a potential gain for us. For example, the police presumably attempts to disrupt the transportation of illegal drugs by blocking the distribution points and routes whereas drug-traffickers try to handle the operations as smooth as possible. In this case, the interdiction problem can help the police to make the most of its limited resources to optimally disrupt the rivals operations. Preventing nuclear smuggling using radiation sensors, and protecting electricity grid against terrorists attacks are another examples of interdiction problems in a competitive environment.

Interdictors actions are usually constrained by a limited budget and assumed to have a limited damage on the system. In this case, decisions are usually more complex and we examine the best decisions for protecting our own operations or disrupting the rival's. The type of disruption can be anything from removing sources, blocking routes to increasing cost or distance.

In this study, our aim is to maximize the total weighted rectilinear distance between source and sink points by locating a line barrier on the plane. Here, the barrier location is considered as the disruption action. In addition to restricting the length of the line barrier, the total number of disrupted points is also bound to a certain limit. All sink points on the plane communicate with a single source (one-to-many) or with each other (many-to-many). Sink points may have identical or different weights based on their importance. The following four problems are introduced and solved in this study:

1. One-to-many interdiction problem with a line barrier on a plane subject to a disruption constraint. In this problem, the total weight of disrupted points should not exceed a certain percentage of the total weights. For this problem, a mathematical model is proposed in Chapter 2 and solved for several test instances in Chapter 3.
2. One-to-many interdiction problem with a line barrier on a plane. This problem is special case of the above problem with no disruption constraint. For this problem, an algorithm is developed in Chapter 2 and solved for several test instances in Chapter 3.
3. Many-to-many interdiction problem with a line barrier on a plane subject to a disruption constraint. In this problem, the total weight of disrupted points should not exceed a certain percentage of the total weights. For this problem, a mathematical model is proposed and solved for several test instances in Chapter 4.
4. Many-to-many interdiction problem with a line barrier on a plane. This problem is special case of the above problem with no disruption constraint. For this problem, an algorithm is developed and solved for several instances in Chapter 4.

In this chapter, an introduction to the rectilinear interdiction, its motivation, and some possible applications are given in Section 1.1. Because of its disruptive act, the purpose of our problem becomes similar to that of interdiction problems and network interdiction problems, which are explained in Sections 1.2 and 1.3. The solution approaches to find the shortest way between facility (source) and demand (sink) points in the presence of barriers are extensively studied in facility location literature. We briefly review the facility location with barriers in Section 1.4. The difference between line (barrier) location in our context and classic line location problems in the location literature is also explained in Section 1.5.

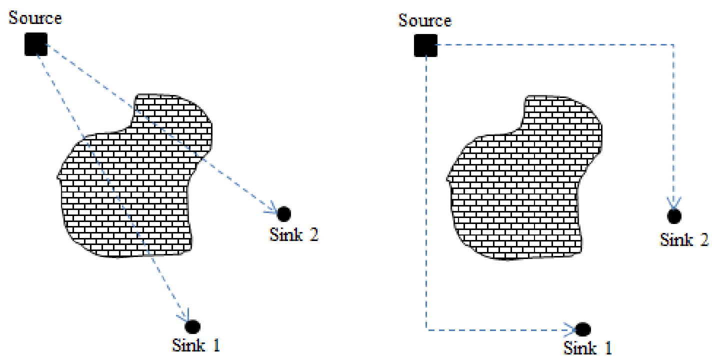
1.1 Introducing the Rectilinear Interdiction Problem

Suppose that there are n sink points located on a plane which are served by one or more source points. In planar interdiction problem, the aim is to disrupt the access of the source point(s) to sink points by increasing the distance between them.

The distance between two points on a plane is determined by the underlying metric. The most famous distance metrics for a plane are rectilinear (or Manhattan), Euclidean and Tchebychev. Increasing distances between source and sink points is a major target in planar interdiction problem and a suitable disruption action is locating a barrier on the plane.

A barrier, as the name implies, is an area on the plane through which traveling is assumed to be forbidden. Barriers can be natural entities like mountains, forests, lakes, or man-made like trenches, state borders, or no-fly zones. Since deliberate location of barriers is our interest, we only consider man-made barriers in planar interdiction problems. Once a barrier is located on a plane, it blocks some of the possible routes between points and, hence, some other ways round the barrier have to be considered. If a barrier blocks the shortest way between two points, it effectively increases the distance between these points which is the aim of this study.

The shape of the barrier and also the distance metric can affect the amount of disruption. Figure 1.1 shows an arbitrary-shaped barrier located between source and sink points. In Figure 1.1.(a), the barrier can effectively disrupt the direct way between the source and sink points when the distance metric is Euclidean. However, the barrier fails to make any disruption when the distance metric is rectilinear, as shown in Figure 1.1.(b). In planar interdiction problems, therefore, the optimal location of barrier has to be decided based on the barrier shape and the underlying distance metric.



(a) (b)

Figure 1.1: Example: an arbitrary shape barrier disrupts the Euclidean path between a source and its two sinks (a) but has no effect on their rectilinear path(b)

Some possible applications of planar interdiction are in battlefields where there is a flow of troops, arms, and logistics delivered from an arsenal of our enemy (source point) to some strongholds (sink points). The accessibility of strongholds to the enemy's arsenal is not desired and, therefore, a long barbed-wire, trench, or minefield can be located as a line barrier such that it best disrupts the undesired flow.

Another application is building a wall in a city in order to disrupt traffic flow or trespassing from one side to the other side of the wall. Once the wall is located, the disrupted flow has to go through either endpoints of the wall. In addition to increases in distances, having security checkpoints at two endpoints of the wall can provide better control and surveillance on the flow between disrupted points.

In real life, the line barrier does not have to be a physical barrier like a trench or wall all the time. For example, a line barrier can be a no-fly zone or no-drive zone which is controlled by an authority. Any traffic or transportation across the zone will be readily detected and eliminated so that it becomes impossible to trespass the zone. Therefore, the flow between points has to go round the zone and undergo the cost of increased distances. For example, drug-traffickers usually take less trodden paths through mountains and sea to enter other countries. A barrier, therefore, can be a patrolling zone that can identify and stop suspicious movements. In that case, drug-traffickers have to go through further and more difficult routes.

1.2 Interdiction Problems

Interdiction problems correspond to optimally impeding a system's operations using limited disruption actions (Cole Smith, 2010). In such problems, there are usually two opposing parties involved in a severe competition or warlike conflict. While one operates the system in order to maximize her effectiveness or efficiency, the other attempts to sabotage operations and limit the achievable objective value by the opponent. A few examples of interdiction problems among others are:

- Attacking the electricity grid on its facilities or transmission lines (Salmeron et al., 2004),
- Disrupting the supply lines (McMasters and Mustin, 1970), (Fulkerson and Harding, 1977),
- Hindering drug trafficking operations (Wood, 1993),
- Blocking the air, ground and maritime routes by creating no-fly, no-drive and no-sail zones.

Interdiction problems also help to identify vulnerabilities of a system against inauspicious attacks and perhaps can be coupled with fortification problems to protect the system (Scaparra and Church, 2006).

1.3 Network Interdiction Problems

Different disruption actions can be proposed based on possible failures and fatal vulnerabilities of different systems. In network interdiction problems, which are widely studied in operations research (Smith, 2010), disruptions usually target nodes and arcs by removing network components, decreasing arc capacities, or increasing cost of flows. The disruptions can be man-made or realize as a natural disaster on a network. Most network interdiction problems are formulated as Stackelberg games where a "leader" acts first by disrupting the system and, then, the "follower" responds to the damage by performing some recourse operations. In the literature, such subjects are sometimes referred to as "interdictor" and "operator".

The disruptions are usually partial and limited in their scale or strength such that it cannot destroy the whole system. Partial disruption reflects the limited offensive power of the leader for which has to make the best decision to attack the system. Besides, the problems with partial disruption offer more information about system's vulnerabilities than the problems with total disruption. Partial interdiction is first studied by McMasters and Mustin (1970), dealing with interdiction of military supply lines.

A possible disruptive action in network interdiction is arc removal. Wollmer (1964) and Fulkerson and Harding (1977) studied the problem of maximizing the shortest path by removing arcs in a network. Deterministic interdiction on the maximum flow through a network is studied by Wood (1993) and Cormican et al. (1998) propose a stochastic variation of this problem. Israeli and Wood (2002) analyze impact of arc removals on the shortest path between two nodes following the earlier studies by Fulkerson and Harding (1977). Lim and Smith (2007) studied a network interdiction problem on a multicommodity flow network where interdiction can be discrete (each arc must either be left alone or completely destroyed) or continuous (arc capacities are partially reduced).

Another possible interdiction is on network nodes. Scaparra and Church (2006) study the r-interdiction median problem with fortification where some of facilities can be selected to be immune (fortified) against attacks while the other facilities are left unprotected due to a budget constraint. Should an unprotected facility be attacked, its demand points will be re-allocated to further away sites. The operator's aim is to select the fortified facilities such that the total increased distance after disruption is minimized. The attacker inflicts the disruption by eliminating a set of unprotected facilities to maximize the total distance between demand points and facilities. Liberatore et al. (2011) and Losada et al. (2012) propose stochastic variations of this problem. Aksen et al. (2012) introduced partial interdiction into this problem.

Although interdiction on networks is extensively studied, to the best of our knowledge, disruption of the shortest path between source and sink points by locating a barrier on the plane has not been addressed before. As the impact of disruption on nodes or arcs is readily known in network interdiction models, the result of disruption in barrier interdiction problem has to be calculated based on the location of the barrier on the plane and the distance metric.

1.4 Facility Location with Barriers

Since the line barrier in our problem is located on a plane, calculating shortest path between points around this barrier becomes an important issue. Finding shortest path between source and sink points in the presence of barriers is mainly studied in facility location literature. A brief review of this literature is presented below.

Facility location in the presence of barriers was first studied by Katz and Cooper (1981) in terms of a circular barrier and Euclidean norm. They use heuristics approaches to solve the problem. Later on, Aneja and Parlar (1994) consider the same problem with convex and non-convex polyhedral barriers and develop a solution approach based on simulated annealing. In their approach, a facility location is generated at each iteration and subsequently a graph is constructed using all demand points, barrier vertices, and the facility point as nodes. If no barrier obstructs the direct distance between two nodes, the nodes are called ”visible” and connected to each other including a new arc. The resulting graph is called ”visibility graph”. Bischoff and Klamroth (2007) use genetic algorithm instead of simulated annealing to solve the same problem.

Decomposition of feasible region into sub-regions was first introduced by Butt and Cavalier (1996) where the shortest path between a pair of points does not change. Klamroth (2001a) introduces another decomposition approach in which the region is divided into sub-regions to keep the visibility graph as it is. A branch and bound approach is introduced by McGarvey and Cavalier (2003) where they use a variant of the big square small square method.

Larson and Sadiq (1983) study the same problem under rectilinear distance metric and they show that the problem can be discretized on a graph of edges and nodes which, in turn, can be transformed into a tree with a finite set of dominating nodes suggesting an optimal region to locate a facility. Batta et al. (1989) provide a procedure to obtain a global optimum solution in the presence of arbitrarily shaped barriers in rectilinear distance. Nandikonda et al. (2003) solve the same problem rectilinear norm with a minimax objective function. Dearing et al. (2002) propose a polynomial-time algorithm to decompose and solve the center location problem with barriers in rectilinear distance. In this paper, they introduce a set of dominating points for the optimal solution. The results are extended by Dearing et al. (2005) and rectilinear norm is replaced with block norm. Dearing and Segars(2002a,b) show that it is possible to modify non-convex barriers into convex ones without affecting the objective value. The reduced feasible region, then, is decomposed into rectangular cells which, in turn, are partitioned into convex subsets and solved optimally.

Facility location problems in the presence of line barriers are also studied in the location literature. Klamroth (2001b) considers a line barrier and a finite set of passages on the barrier that allows traveling between two sides divided by the barrier. The facility location in the presence of a probabilistic line barrier is first considered by Canbolat and Wesolowsky (2010) where the objective function is based on the expected rectilinear distances from the facility to demand points. Probabilistic line barriers are recently studied by Amiri-Aref et al. (2011a,b) for the facility center location problem and the multi-period facility location-relocation problem.

Locating a finite-size facility in the presence of barriers is studied by Savas et al. (2002) where the facility itself change the connection paths between points. Wang et al. (2002) study placement of a rectangular facility with I/O points in a layout context. Kelachankuttu et al. (2007) propose using contour lines with equal objective values to place the new facility in rectilinear distance. Sarkar et al. (2007) addressed the finite size facility placement problem with a center objective and only user-facility interactions.

Note that, in all these studies, the barrier is assumed to have a fixed size and the problem tries to minimize the total transportation costs. In our problem, the aim is to decide about the length of the barrier and locate that barrier between source and sink points with an objective so that an access from source to sink points is getting harder than it was before.

1.5 Line Location Problem

There is an extensive body of work in linear regression analysis where a line is desired to be located among points on the plane such that it minimizes the distance between the line and the points (Schöbel, 1999). Note that in our problem, the total distance between source and sink points has to be maximized and the distance from the line barrier is actually not a matter of our concern.

CHAPTER 2

THE ONE-TO-MANY RECTILINEAR INTERDICTION WITH A LINE BARRIER ON A PLANE

2.1 Problem Preliminaries and Formulation

We assume a set of points on the plane that are considered as sources and sinks where the flow is from the source towards the sink in rectilinear distance. In this chapter, we assume to have one source point communicating with many sink points on the plane, hereinafter called the one-to-many type problem. Figure 2.1.(a) shows an example of one-to-many configuration with a single source point P_0 and seven sink points.

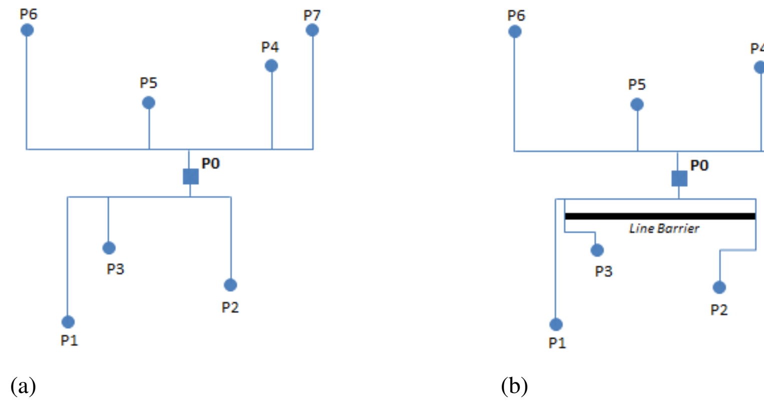


Figure 2.1: (a) A single source point P_0 communicates with 7 sink points in one-to-many configuration and (b) a horizontal line barrier disrupts the distance for sink points P_2 and P_3 (b)

The source's accessibility to sinks is not desired. Therefore, our objective is to locate a horizontal line barrier such that it maximizes the total distances between the source and sinks, as trespassing through the barrier is assumed to be impossible. Minefields, water canals, and barricades can be examples of such barriers. If a sink point is not disrupted by the barrier, its distance from the source point remains the same. Otherwise, the flow has to go round the barrier to reach the sink point and, hence, its distance changes. Therefore, identifying the disrupted points and calculating their increased distances to the source point becomes important to obtain the total distance value. Figure 2.1.(b) shows how a horizontal line barrier disrupts the distance for two sink points P_2 and P_3.

The above objective can be increased indefinitely if there is no limit or the length of the barrier. On the other hand, barriers may disrupt all the sink points on the plane if there is no limit on the number of disrupted points. Therefore, two main constraints are introduced to this generic problem:

- A constraint on barrier length
- A constraint on total weight of disrupted sink points, known as disruption constraint

In this chapter, above objective and constraints are introduced and studied in following problems:

1. The one-to-many interdiction problem with a line barrier on a plane subject to a disruption constraint. In this problem, the total weight of disrupted points should not exceed a certain percentage of the total weights.
2. The one-to-many interdiction problem with a line barrier on a plane. This problem is special case of the above problem with no disruption constraint.

In Section 2.1.1, general assumptions required for above problems are introduced. Section 2.1.2 tells us how to identify the disrupted points in presence of a barrier while Section 2.1.3 shows us how to calculate the shortest distance for disrupted points. Constraints on barrier length and amount of disruption are explained in Sections 2.1.4 and 2.1.5.

A mixed-integer programming model for the problem with disruption constraint and an algorithm for the problem with no disruption constraint are presented in Sections 2.2 and 2.3, respectively.

2.1.1 Preliminaries and Assumptions

Distance norm: All distances are computed in rectilinear (or Manhattan) metric.
Barrier Type: The barrier is a horizontal line segment with negligible width that can be located anywhere on the plane such that the total weighted distance to the source is maximized. The horizontal line is located either above or below the source point. Therefore, the plane can be partitioned into two half-spaces with respect to the source point. In Figure 2.2, the plane is partitioned into two half-spaces H_1 and H_2 with respect to the horizontal line barrier and the source point P_0. When the barrier is located below the source point, it disrupts only the sink points that are in the corresponding half-space. For example, consider three sink points P_1,P_2 and P_3 in H_2. In this example, only P_2 and P_3 are disrupted by the line barrier below the source P_0. The rectilinear path between the source and four sink points in H_1, however, cannot be disrupted given that the barrier is in H_2. Conversely, points in H_2 will not be disrupted if the barrier is located in H_1.

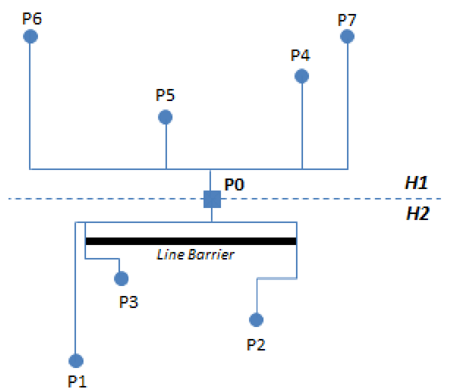


Figure 2.2: Partitioning the plane into two half-spaces H_1 and H_2

Therefore, the problem can be solved for H_1 and H_2 separately and the solution with higher weighted disruption would represent the optimal solution for the problem.

The problem with a vertical line segment can be converted into a problem with horizontal line segment by simply rotating the points on the plane as much as 90 degrees and the rectilinear distances between points remain the same. Therefore, the solution to the transformed problem can be rotated back with -90 degrees to obtain the vertical line location in the original problem. A point at (a,b) can be rotated with θ degrees using following formula:

$$\begin{bmatrix} a' \\ b' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix}$$

Hence, due to the equivalence of these horizontal and vertical problems, only location of horizontal line barrier is considered in this study as a general problem without loss of generality.

Barrier-point intersection: If a barrier falls on a sink point, that point is considered as invisible to the source. It implies that the barrier will be shifted towards the source point with a negligible amount ($\epsilon > 0$).

Distance function: Consider n sink points $P_i (i = 1, ..., n)$ on the plane with coordinates (a_i, b_i) that are being served by a source point P_0 at coordinates (a_0, b_0). The rectilinear distance between a sink point P_i and the source point P_0 is:

$$l(P_i, P_0) = |a_i - a_0| + |b_i - b_0| \tag{2.1}$$

Since the distance metric is rectilinear, there are several alternative paths between P_i and P_0 with equal distance. Note that the word ”path” used in this context simply means ”way” on the plane and it does not intend the meaning used in network terminology in which a ”path” goes through a set of nodes and arcs.

Figure 2.3 shows two alternative paths between P_i and P_0 with equal rectilinear distances. On a continuous plane, there are infinitely-many alternative paths between P_i and P_0, falling in the minimum bounding rectangle (MBR) of P_i and P_0, as depicted in Figure 2.3.

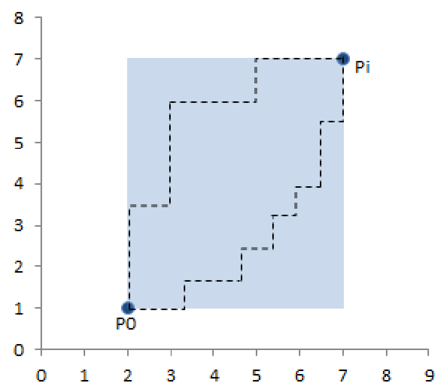


Figure 2.3: Alternative rectilinear paths between P_i and P_0

The total weighted distance from the single source P_0 to all sink points P_i's is represented by following expression:

$$\sum_{i=1}^{n} w_i l(P_i, P_0) = \sum_{i=1}^{n} w_i |a_i - a_0| + \sum_{i=1}^{n} w_i |b_i - b_0| \tag{2.2}$$

where w_i refers to the weight of the point P_i on the plane. Note that the distance expression in 2.2 does not take the presence of a line barrier into account. The source P_0 may have to find the shortest possible path to any sink P_i without passing through the barrier to minimize the total weighted distance between the source and sinks. The total distance expression in 2.1.1 can be written as:

$$\sum_{i=1}^{n} w_i l^B(P_i, P_0) \tag{2.3}$$

where $l^B(P_i, P_0)$ refers to the distance corresponding to the shortest path round the barrier. If the barrier is effectively blocking all alternative paths between P_i and P_0, the two points are said to be *invisible* to each other, and *visible* otherwise. If two points are invisible to each other, the rectilinear distance introduced in equation 2.1 is no longer valid and the shortest path that does not intersect with the barrier has to be calculated. Therefore, understanding the relationship between invisibility and the shortest path between the source and sink points in presence of a line barrier becomes essential to understand the problem nature and its complexity.

2.1.2 Identifying Disrupted Points in Presence of a Single Line Barrier Using Visibility Concept

The concept of visibility with different norms and barriers is thoroughly explained by Klamroth (2002). In this study, two points P_i and P_0 are said to be invisible to each other if all alternative rectilinear paths between them, defined by a rectangle whose edges cross at points P_i and P_0, intersects with the barrier. It means that the two points are vertically on two different sides of the line barrier and the barrier extends beyond the right and left edges of rectangle.

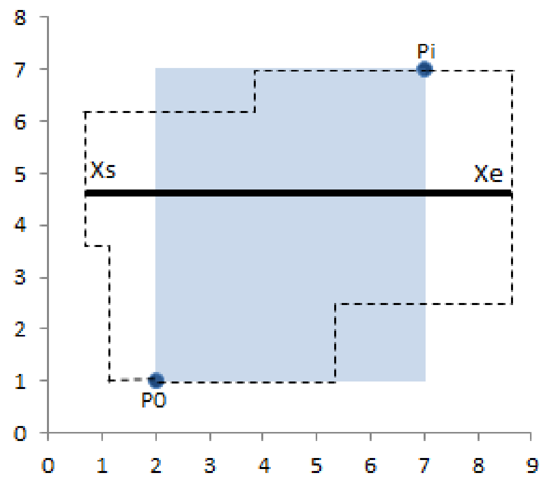
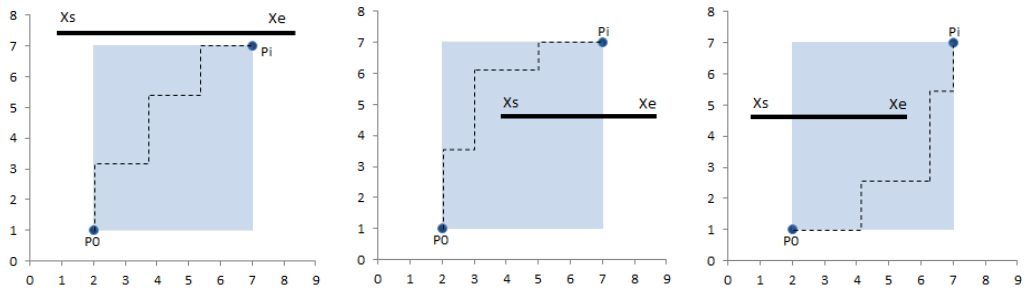


Figure 2.4: P_i and P_0 are invisible to each other due to the barrier stretched from X_s to X_e.

Figure 2.4 shows a graphical example in which two points are invisible due to a line barrier. In this example, all alternative rectilinear paths between P_i and P_0 are defined by the minimum bounding rectangle. The barrier is vertically between P_i and P_0 and it intersects with the rightmost and the leftmost alternative paths (two vertical edges of the bounding rectangle). Hence, all alternative paths between P_i and P_0 are blocked by the barrier. The shortest way, then, has to pass through either X_s or X_e.

Figure 2.5 shows three graphical examples in which points P_i and P_0 are visible to each other with an exemplary alternative path between them. The barrier in Figure 2.5.(a) is not vertically between the two points and, therefore, it cannot block any alternative way in the rectangle. The barriers in Figure 2.5.(b) and Figure 2.5.(c) are lying vertically between the points, but they fail to intersect with both vertical edges of the rectangle leaving a gap in the left or right for alternative paths.



(a) (b) (c)

Figure 2.5: Examples in which P_i and P_0 are visible to each other.

Now we formulate the invisibility conditions in Lemma 2.1.

Lemma 2.1 *Suppose that the starting and ending coordinates of the line barrier are* (x_s, y_s) *and* (x_e, y_e) *respectively and the line is horizontal, i.e.* $y_s = y_e = y$*. Two points* (a_i, b_i) *and* (a_0, b_0) *on the plane are said to be invisible to each other if:*

$$b_i < y < b_0 \vee b_0 < y < b_i \tag{2.4}$$

and

$$x_s < a_0 < x_e \wedge x_s < a_i < x_e \tag{2.5}$$

The first condition is called *y-invisibility* condition which controls whether the line barrier is vertically between the two points or they will be visible to each other regardless of the barrier. The second condition is called *x-invisibility* condition which controls if the barrier can successfully cut all vertical paths between points P_i and P_0. From *x-invisibility* conditions we infer that x_s has to be smaller than both a_i and a_0 while x_e must be greater than both a_i and a_0. In other words:

$$x_s < \min\{a_i, a_0\} \wedge \max\{a_i, a_0\} < x_e \tag{2.6}$$

P_i and P_0 are invisible to each other only if both *y-invisibility* and *x-invisibility* conditions hold. In that case, the shortest path has to go through either endpoints of the line barrier. The corresponding shortest distance calculations between such two points are explained next.

2.1.3 Calculating the Shortest Distance for Disrupted Points in the Presence of a Single Line Barrier

Property 2.1 *When a horizontal line barrier makes two points invisible, it only affects the x-distance between the points. y-distance between the points remains the same due to the rectilinear norm (Canbolat and Wesolowsky, 2010).*

Property 2.1 implies that the rectilinear distance interdiction problem with a horizontal line barrier can be reduced to a one-dimensional problem along the x-axis by taking the total distance along the y-axis as a constant value.

When two points are invisible to each other, the shortest path must go through one of the line barrier ends. Therefore, the shortest distance for a pair of invisible points P_i and P_0 is:

$$l^B(P_i, P_0) = \min\{|a_i - x_s| + |a_0 - x_s|, |x_e - a_i| + |x_e - a_0|\} + |b_i - b_0| \tag{2.7}$$

From *x-invisibility* condition we can infer that both a_i and a_0 have to be greater than x_s and less than x_e:

$$x_s < a_i, x_s < a_0, x_e > a_i, x_e > a_0 \tag{2.8}$$

Then:

$$l^B(P_i, P_0) = \min\{a_i + a_0 - 2x_s, 2x_e - a_i - a_0\} + |b_i - b_0| \tag{2.9}$$

Another way of calculating the shortest path between a pair of invisible points is to add the additional distance along the x-axis to the original distance when no barrier exists.

Figure 2.6 provides an example in which P_i and P_0 are invisible and the shortest path has to go through the endpoints of the barrier. As shown on the figure, the additional distance through X_s and X_e are $2d_s$ and $2d_e$ where $d_s = \min(a_i, a_0) - x_s$ and $d_e = x_e - \max(a_i, a_0)$.

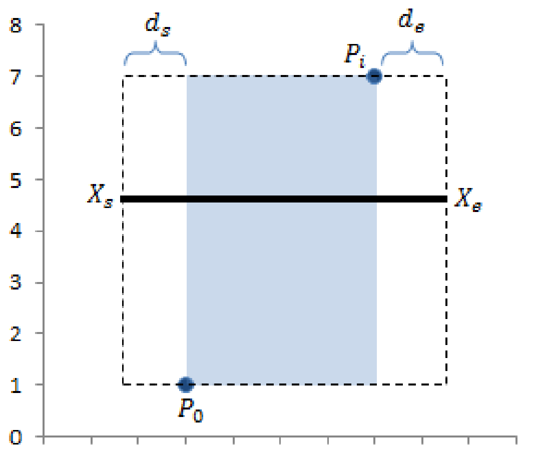


Figure 2.6: Additional distances through X_s and X_e

The minimum of these additional distances, d_s and d_e, has to be taken as the shortest added distance. Therefore, with respect to the invisibility conditions explained in (2.4) and (2.6), the additional distance can be calculated as:

$$\Delta l^B(P_i, P_0) = \begin{cases} 2\min\{\min(a_i, a_0) - x_s, x_e - \max(a_i, a_0)\} & \text{if } P_i \text{ invisible to } P_0 \\ 0 & \text{otherwise} \end{cases} \tag{2.10}$$

Therefore, the shortest distance for a pair of invisible points can be written in terms of the original distance and the added distance:

$$l^B(P_i, P_0) = l(P_i, P_0) + \Delta l^B(P_i, P_0) \tag{2.11}$$

Recall that the term $l(P_i, P_0)$ is the original distance between P_i and P_0 and can be computed beforehand. Therefore, the problem can be reduced to finding the additional distances incurred by barrier, i.e. $\Delta l^B(P_i, P_0)$.

Example 2.1 *Figure 2.7 illustrates an example of a one-to-many interdiction problem instance with the source point P_0 located at (5,5) and 3 sink points P_1, P_2 and P_3 located at (2,1), (4,3) and (6,2), respectively with unit weights. The total rectilinear distance when there is no barrier on the plane is:*

$$\sum_{i=1}^{3} l(P_0, P_i) = \sum_{i=1}^{3} (|a_0 - a_i| + |b_0 - b_i|) = (3 + 4) + (1 + 2) + (1 + 3) = 14$$

Once a line barrier is placed between (3,4) and (7,4), the shortest path between the invisible points has to be calculated.

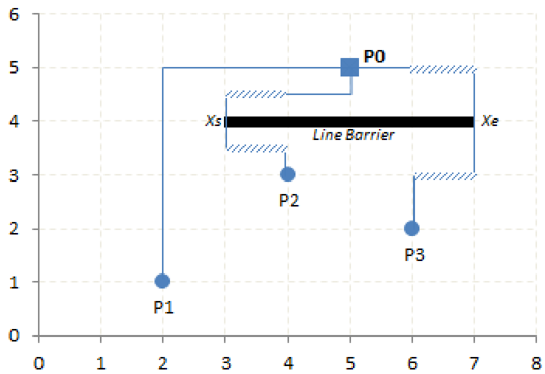


Figure 2.7: An example of a one-to-many problem instance with one source and 3 sink points

For all sink points *y-invisibility* condition hold because the barrier is vertically between the source and sink points. *x-invisibility* does not hold for P_1 ($x_s > 2$) but it does for the other sink points with $x_s < 6 < x_e$ and $x_s < 5 < x_e$ while $x_s < 5 < x_e$ holds for the source point. The additional distances for sink points (shown with hash pattern) can be calculated as:

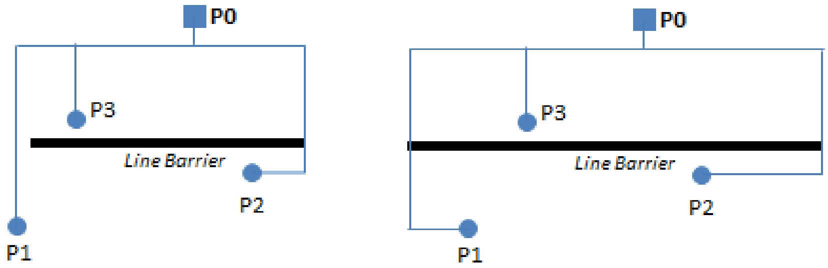
$$\begin{aligned} \Delta l^B(P_0, P_1) &= 0 \\ \Delta l^B(P_0, P_2) &= 2\min\{\min(4, 5) - 3, 7 - \max(4, 5)\} = 2\min\{1, 2\} = 2 \\ \Delta l^B(P_0, P_3) &= 2\min\{\min(6, 5) - 3, 7 - \max(6, 5)\} = 2\min\{2, 1\} = 2 \end{aligned}$$

And the total shortest distance between the source and the sink points is:

$$\sum_{i=1}^{3} l^B(P_0, P_i) = \sum_{i=1}^{3} l(P_0, P_i) + \sum_{i=1}^{3} \Delta l^B(P_0, P_i) = 14 + 4 = 18$$

2.1.4 Constraining the Barrier Length

Figure 2.8 shows the same example in Figure 2.2 but with a further or longer barrier. When the barrier is located further from the source point (a), less points become exposed to disruption. But if a longer barrier is located (b), more points can be disrupted and higher increased distance can be obtained.



(a) (b)

Figure 2.8: Further barrier (a) disrupts less. Longer barrier (b) disrupts more.

Based on above observations, a combination of barrier location and length is used in this study in which a line barrier should have a variable length proportional to its vertical distance from the source point. We think that the closer the barrier is to the source point, the shorter length it is allowed to have. Therefore, the barrier length is zero at the source location and increases gradually as it goes far from the source point. In real life problems, getting closer to the enemy's source point (stronghold, facility, etc.) may be costly and locating a long barrier near the source point will not be tolerated by the enemy. Besides, different areas on the plane may have different cost and allowance to locate a barrier.

This idea can be formulated as a constraint. Suppose that b_0 and y are the y-coordinates of the source point and the barrier respectively such that $|y - b_0|$ gives the vertical distance between them. Then, the length of the barrier, L, is determined by following constraint:

$$L \leq \alpha|y - b_0| \tag{2.12}$$

where α is a constant ratio. Above constraint can be seen as an isosceles triangle where the source point is at the vertex point and the barrier is located on the base side of the isosceles. We can define the length rate as $\alpha = 2\tan\theta$ where θ is the vertex angle between a leg and the height of the isosceles. Therefore, parameter α itself can be represented by an angle parameter θ° as $\alpha = 2\tan\theta^\circ$ where $\theta^\circ \in (0, 90)$.

As shown in Figure 2.9.(a), the closer barrier to the source point is shorter than the further barrier ($L_1 < L_2$). Besides, Figure 2.9.(b) shows that higher θ angles (or higher α ratios) allows longer barriers. When θ is increased to θ', the length of L_1 and L_2 increase to L_3 and L_4 respectively.

Since the barrier in this study is a horizontal line segment, it can only increase sink-source distances along the x-axis and y values can only affects the y-invisibility of sink points. Longer barriers can cause more interdiction which, based on above assumption, can be obtained by locating them as far as possible from the source point. Recall barrier-point intersection assumption in Section 2.1.1 in which if a barrier is located on a sink point, that point is considered as invisible to the source point.

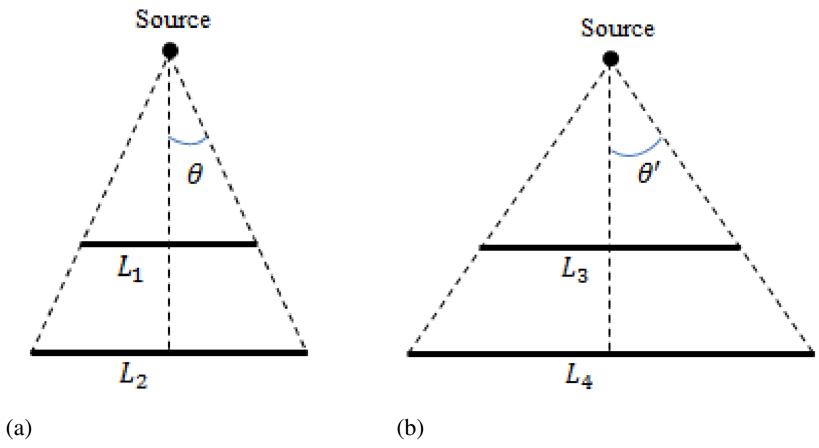


Figure 2.9: Higher angles allow longer barriers: Since $\theta' > \theta$, barriers in (b) are longer than (a).

Theorem 2.1 *There exists an optimal solution to the one-to-many rectilinear interdiction problem where the line barrier is placed on one of the sink points, i.e.* $\exists i : y^* = b_i$ *and* $a_i \in (x_s^*, x_e^*)$.

Proof. Suppose that sink points in the lower half-space are sorted descendingly according to their y-coordinate values and the barrier is not located on any of the sink points. Let the barrier be vertically located between the source point and the closest sink point, i.e. $y \in (b_0, b_1)$. For the same set of y-invisible points, we can get a longer barrier if $y = b_1$ and, hence, the shortest distance to the source definitely increases for the invisible points providing more interdiction than any other $y \in (b_0, b_1)$. When $y \in (b_1, b_2)$, P_1 becomes visible and the set of y-invisible points change. By the same token, however, $y = b_2$ would provide a better solution than any other $y \in (b_1, b_2)$. The same holds for all other y ranges and, therefore, $\exists i : y^* = b_i$.

Moreover, if $y^* = b_i$ and $a_i \notin (x_s^*, x_e^*)$ then P_i becomes visible to the source point. Let P_k be the closest point to the source in the current set of invisible points. $y = b_k$ would provide a longer barrier than $y = b_i$ for the same set of invisible points leading to a higher interdiction. Therefore, the optimal location of the barrier has to be on one the sink points, i.e. $\exists i : y^* = b_i$ and $a_i \in (x_s^*, x_e^*)$. ■

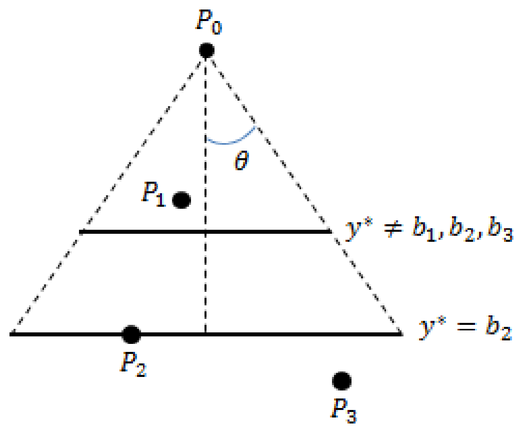


Figure 2.10: The optimal location of the line barrier must fall on one of the sink points

The example in Figure 2.10 shows how the barrier falls on one of the sink points. The shorter barrier does not intersect with any sink point whereas $y^* = b_2$ provides a longer barrier. Since the set of y-invisible points for both barrier locations are P_2 and P_3, only the longer barrier can inflict more interdiction.

The interpretation of Theorem 2.1 is that the one-to-many problem can be discretized along y-axis by locating the barrier on one of the sink points. This theorem is implemented in the algorithm in Section 2.3 where the problem is discretized along the y-axis and x-axis and the solution space is reduced to some candidate y values. This discretization on y value can be implemented in MIP models as valid inequalities.

2.1.5 Constraining the Total Weight of Disrupted Points (or Presence of a Disruption Constraint)

In interdiction literature, systems are disrupted partially and there is usually a limit on the scale of disruption. In this study, apart from limiting the length of the barrier, the total weight of disrupted points is also restricted by a disruption rate β. This rate defines the fraction of total weights allowed to be disrupted.

2.1.6 Problem Formulation

Consider n sink points $P_i (i = 1, ..., n)$ on the plane with coordinates (a_i, b_i) and weight w_i that are being served by a source point P_0 at coordinates (a_0, b_0). A barrier is going to be located between points (x_s, y) and (x_e, y) that may disrupt some of sink points. An auxiliary variable N_i would take the value 1 for disrupted point P_i:

$$N_i = \begin{cases} 1 & \text{if point } P_i \text{ is invisible to point } P_0 \\ 0 & \text{otherwise} \end{cases}$$

Therefore, a conceptual formulation of one-to-many problem can be presented as below:

$$\underset{x_s, x_e, y, L}{\text{Maximize}} \sum_{i=1}^{n} w_i z_i \tag{2.13}$$

subject to

$$L = x_e - x_s \tag{2.14}$$

$$L \leq \alpha |y - b_0| \tag{2.15}$$

$$\sum_{i=1}^{n} w_i N_i \leq \beta \sum_{i=1}^{n} w_i \tag{2.16}$$

$$z_i = |a_i - a_0| + |b_i - b_0| + N_i * 2 \min\{\min(a_i, a_0) - x_s, x_e - \max(a_i, a_0)\} \tag{2.17}$$

The objective is to maximize the total weighted distance between the source and the sink points. The length of the barrier is determined in (2.14) and limited in constraint (2.15) with respect to its distance from the source point. Constraint (2.16) is the disruption constraint where the total weight of disrupted points is limited by a constant rate β. Equation (2.17) increases the rectilinear distance between source and a sink if they are invisible to each other.

In Section 2.2, a mixed-integer programming model is developed for this problem.

2.2 Mathematical Model for the One-to-Many Rectilinear Interdiction with a Line Barrier on a Plane Subject to a Disruption Constraint

We first present preliminaries that will help to explain and understand the model. Before modelling the problem, the approach representing invisibility conditions in 2.1 and linearization of absolute terms using auxiliary variables is explained.

2.2.1 Checking an Inequality Condition

To force a linear inequality $f(x) \geq g(y)$ to hold, a binary variable, u, and a large positive value, M, can be used in following constraints:

$$f(x) - g(y) \leq Mu$$
$$g(y) - f(x) \leq M(1 - u)$$

Where u is 1 if $f(x) \geq g(y)$ holds and 0 otherwise. Later on, for inequalities in *y-invisibility* and *x-invisibility* conditions we write a pair of constraints and a binary variable as above.

2.2.2 Satisfying a Set of Conditions

In order to determine the invisibility of a pair of points, a certain number of inequalities must hold simultaneously. Since each inequality corresponds to a binary variable as explained in Section 2.2.1., sum of binary variables can give us the number of satisfied conditions. The total of m binary variables is a value between 0 and m that can be checked with the help of $m + 1$ new binary variables:

$$\sum_{i=1}^{m} u_i = \sum_{j=0}^{m} j.\mu_j$$
$$\sum_{j=0}^{m} \mu_j = 1$$

For example, μ_2 in the following constraints indicates if two binary variables u_1 and u_2 are 1 at the same time:

$$u_1 + u_2 = 0\mu_0 + 1\mu_1 + 2\mu_2$$
$$\mu_0 + \mu_1 + \mu_2 = 1$$

2.2.3 Linearization of Binary Multiplication

To have a pair of points invisible, both *x-visibility* and *y-visibility* must hold at the same time, i.e. their corresponding binary variables must take the value of 1. A binary multiplication of type $u_1 u_2$, where

u_1 and u_2 are binary variables, can be linearized using 3 inequalities and an auxiliary binary v:

$$v \leq u_1$$
$$v \leq u_2$$
$$v \geq u_1 + u_2 - 1$$

2.2.4 Linearization of Absolute Terms

The length of the barrier is defined by its vertical distance from the source point. This distance can be formulated as $d_{bj} = |y - b_0|$ which needs to be linearized. Linearization of an absolute term $A = |B - C|$ can be done using two binary variables δ_1 and δ_2 as following:

$$0 \leq A + B - C \leq 2U.\delta_1$$
$$0 \leq A + C - B \leq 2U.\delta_2$$
$$\delta_1 + \delta_2 = 1$$

Where U is an upper bound for B and C.

2.2.5 Parameters

D = Set of n points on the plane
P_i = Sink point i with coordinates (a_i, b_i) and weight w_i , $i \in D$.
P_0 = Source point with coordinates (a_0, b_0)
d_i = the rectilinear distance between P_i and P_0
$a_{\max}$ = the maximum coordinate along the x-axis in the convex hull of all points
$a_{\min}$ = the minimum coordinate in the x-axis in the convex hull of all points
$b_{\max}$ = the maximum coordinate in the y-axis in the convex hull of all points
$b_{\min}$ = the minimum coordinate in the y-axis in the convex hull of all points
β = Maximum disruption rate as a limit on total weight of disrupted points, $0 \leq \beta \leq 1$.
α = Rate of increase in the barrier length that is based on its vertical distance from the source point, P_0. As explained in Section 2.1.4, $\alpha = 2\tan\theta$ where $\theta^\circ \in (0, 90)$ is the vertex angle between a leg and the height of the isosceles. Therefore, α and θ can be used interchangeably.
M = A large positive value. The value of this parameter is important when an MIP formulation is used. If the solution method is based on the linear programming relaxation, M has to be big enough but not too big! The value of M based on the maximum vertical distance between points on the plane is:

$$M_y = b_{\max} - b_{\min}$$

We know that the maximum barrier length is $L_{\max} = \alpha(b_{\max} - b_{\min})$. Therefore, an upper bound for horizontal distance between two points on the plane can be chosen as:

$$M_x = 2 \times [L_{\max} + (a_{\max} - a_{\min})] = 2M_x$$

Finally, the maximum of these M_x and M_y can be selected as the value of parameter M:

$$M = \max\{2M_x, M_y\}$$

In order to use the M value in MIP model, all source and sink points have to be in the first quadrant, i.e. their vectors must have non-negative values. If not, all points have to be shifted into the first quadrant beforehand.

2.2.6 Variables

Recall that the problem is to find the location and length of a line barrier such that the total source-sink distance interdiction is maximized.

z_i = Shortest distance between sink P_i and source P_0 along the x-axis based on invisibility conditions with respect to the barrier
x_s = Starting point of the barrier along the x-axis
x_e = Ending point of the barrier along the x-axis
y = y-coordinate of the horizontal line barrier
L = Length of the barrier, equal to $x_e - x_s$
d_b = the vertical distance between the line barrier and the source point, equal to $|y - b_0|$

$$N_i = \begin{cases} 1 & \text{if point } P_i \text{ is invisible to point } P_0 \\ 0 & \text{otherwise} \end{cases}$$

According to Lemma 2.1, N_i would take the value of 1 if both *y-invisibility* and *x-invisibility* conditions hold between P_i and P_0. In order to have an MIP model with valid N_i values, following binary variables must be introduced.

Binary variables for linearization of $d_b = |y - b_0|$:
$\delta_1, \delta_2 \in \{0, 1\}$
Binary variable for *y-invisibility* conditions of P_i and P_0:

$$u_i = \begin{cases} 1 & \text{if } b_i \geq y \\ 0 & \text{otherwise} \end{cases}$$

$$u_0 = \begin{cases} 1 & \text{if } b_0 \geq y \\ 0 & \text{otherwise} \end{cases}$$

and $\mu_i^0, \mu_i^1, \mu_i^2 \in \{0, 1\}$.
Binary variable for *x-invisibility* conditions of P_i and P_0:

$$v_{is} = \begin{cases} 1 & \text{if } a_i > x_s \\ 0 & \text{otherwise} \end{cases}$$

$$v_{ie} = \begin{cases} 1 & \text{if } a_i < x_e \\ 0 & \text{otherwise} \end{cases}$$

$$v_{0s} = \begin{cases} 1 & \text{if } a_0 > x_s \\ 0 & \text{otherwise} \end{cases}$$

$$v_{0e} = \begin{cases} 1 & \text{if } a_0 < x_e \\ 0 & \text{otherwise} \end{cases}$$

and $\lambda_i^0, \lambda_i^1, \lambda_i^2, \lambda_i^3, \lambda_i^4 \in \{0, 1\}$.

2.2.7 Mathematical Model

In this problem, there is a single source P_0 on the plane that communicates with all points $P_i, i \in D$. The MIP formulation for this problem is as follows:

$$\underset{x_s,x_e,y,L}{\text{Maximize}} \sum_{i \in D} w_i(z_i + |b_i - b_0|) \tag{2.18}$$

subject to

$$x_e - x_s = L \tag{2.19}$$

$$0 \leq d_b + y - b_0 \leq 2y_{\max}.\delta_1 \tag{2.20}$$

$$0 \leq d_b + b_0 - y \leq 2y_{\max}.\delta_2 \tag{2.21}$$

$$\delta_1 + \delta_2 = 1 \tag{2.22}$$

$$L \leq \alpha d_b \tag{2.23}$$

$$b_0 - y \leq Mu_0 \tag{2.24}$$

$$y - b_0 \leq M(1 - u_0) \tag{2.25}$$

$$b_i - y \leq Mu_i \qquad \forall i \in D \tag{2.26}$$

$$y - b_i \leq M(1 - u_i) \qquad \forall i \in D \tag{2.27}$$

$$u_0 + u_i = 0\mu_i^0 + 1\mu_i^1 + 2\mu_i^2 \qquad \forall i \in D \tag{2.28}$$

$$\mu_i^0 + \mu_i^1 + \mu_i^2 = 1 \qquad \forall i \in D \tag{2.29}$$

$$a_0 - x_s \leq Mv_{0s} \tag{2.30}$$

$$x_s - a_0 \leq M(1 - v_{0s}) \tag{2.31}$$

$$x_e - a_0 \leq Mv_{0e} \tag{2.32}$$

$$a_0 - x_e \leq M(1 - v_{0e}) \tag{2.33}$$

$$a_i - x_s \leq Mv_{is} \qquad \forall i \in D \tag{2.34}$$

$$x_s - a_i \leq M(1 - v_{is}) \qquad \forall i \in D \tag{2.35}$$

$$x_e - a_i \leq Mv_{ie} \qquad \forall i \in D \tag{2.36}$$

$$a_i - x_e \leq M(1 - v_{ie}) \qquad \forall i \in D \tag{2.37}$$

$$v_{is} + v_{ie} + v_{0s} + v_{0e} = 0\lambda_i^0 + 1\lambda_i^1 + 2\lambda_i^2 + 3\lambda_i^3 + 4\lambda_i^4 \qquad \forall i \in D \tag{2.38}$$

$$\lambda_i^0 + \lambda_i^1 + \lambda_i^2 + \lambda_i^3 + \lambda_i^4 = 1 \qquad \forall i \in D \tag{2.39}$$

$$N_i \leq \lambda_i^4 \qquad \forall i \in D \tag{2.40}$$

$$N_i \leq \mu_i^1 \qquad \forall i \in D \tag{2.41}$$

$$N_i \geq \lambda_i^4 + \mu_i^1 - 1 \qquad \forall i \in D \tag{2.42}$$

$$\sum_{i \in D} w_i N_i \leq \beta \sum_{i \in D} w_i \tag{2.43}$$

$$z_i \leq (a_i - x_s) + (a_0 - x_s) + M(1 - N_i) \qquad \forall i \in D \tag{2.44}$$

$$z_i \leq (x_e - a_i) + (x_e - a_0) + M(1 - N_i) \qquad \forall i \in D \tag{2.45}$$

$$z_i \leq |a_i - a_0| + MN_i \qquad \forall i \in D \tag{2.46}$$

$$x_s, x_e, y \text{ are unrestricted-in-sign.} \tag{2.47}$$

$$L, d_b \geq 0 \tag{2.48}$$

$$z_i \geq 0 \quad \forall i \in D \tag{2.49}$$
$$N_i \in \{0, 1\} \quad \forall i \in D \tag{2.50}$$
$$\delta_1, \delta_2 \in \{0, 1\} \tag{2.51}$$
$$u_0, v_{0s}, v_{0e} \in \{0, 1\} \tag{2.52}$$
$$u_i, \mu_i^0, \mu_i^1, \mu_i^2, v_{is}, v_{ie}, \lambda_i^0, \lambda_i^1, \lambda_i^2, \lambda_i^3, \lambda_i^4 \in \{0, 1\} \quad \forall i \in D \tag{2.53}$$

In the objective function (2.18), we try to maximize the total weighted distance between all sink points P_i and the source P_0. Constraint (2.19) determines the relationship between the barrier length and its ending points. Constraints (2.20) to (2.22) find the absolute vertical distance between the barrier and the source point as $d_b = |y - b_0|$. This distance is crucial in determining the barrier length in constraint (2.23). The farther the barrier is from the source point, the longer barrier can be placed. Constraints (2.24) to (2.29) check the *y-invisibility* conditions between P_i and P_0 where $\mu_i^1 = 1$ if they are *y-invisible*. Constraints (2.30) to (2.33) control if $x_s \leq a_0 \leq x_e$ while constraints (2.34) to (2.37) investigate if $x_s \leq a_i \leq x_e$. Constraints (2.38) and (2.39) control if all *x-invisibility* conditions between P_i and P_0 hold to set $\lambda_i^4 = 1$. Eventually, P_i and P_0 are invisible to each other if both μ_i^1 and λ_i^4 are 1. To avoid binary multiplication in the form of $N_i = \mu_i^1 \lambda_i^4$, constraints (2.40) to (2.42) are introduced. Constraint 2.43 is the disruption constraint on total weight of interdicted points. If $N_i = 1$, the shortest path between P_i and P_0 has to go through either the starting point or the ending point of the barrier and, therefore, one of constraints (2.44) or (2.45) would be tight relatively. If $N_i = 0$, the points are visible which makes constraint (2.46) tight. Unrestricted-in-sign variables are introduced in (2.47). Constraints (2.48) and (2.49) are the non-negativity constraints while constraints (2.50) to (2.53) are integrality constraints.

For n sink points, there are $n+5$ continuous variables and $12n+5$ binary variables used across $15n+16$ constraints in this model. Hence, it is an $O(n)$ continuous variables, $O(n)$ binary variables, and $O(n)$ constraint model.

Valid inequalities for discretizing the y value:
Discretization along the y-axis explained in Theorem 2.1 can be introduced to the model via valid inequalities as follows:

$$\sum_{i \in D} \rho_i = 1 \tag{2.54}$$
$$\sum_{i \in D} b_i \rho_i = y \tag{2.55}$$

where $\rho_i, i \in D$ is a binary variable which gets the value of 1 if the barrier is decided to be on sink point P_i. Constraint 2.54 ensures that only one sink point will be selected to determine the y value. Constraint 2.55 gives the y-coordinate of the selected sink point to the barrier.

2.3 An Algorithm for the One-to-Many Rectilinear Interdiction with a Line Barrier on a Plane

The mathematical model in section 2.2.7 without the disruption constraint (2.43) applies to this problem. When there is no disruption constraint in the one-to-many problem ($\beta = 1$), we propose a polynomial-time algorithm with following outline to find the optimal solution:

1. Partition the plane along the y-axis into horizontal regions between all points
2. In each partition, maximize L by maximizing the distance from source point using Theorem 2.1 to discretize the plane along the y-axis

3. In each partition, locate the line with known y and L optimally by enumerating on a finite set of candidate locations along the x-axis
4. Repeat steps 2 and 3 for all partitions to find the optimal solution

This algorithm works mainly based on step 3 where a finite set of candidate locations that are generated with respect to the location of sink points. Since there is no disruption constraint, the line barrier can be placed on these candidate locations regardless of the number of points it disrupts.

Note that when a disruption constraint exists in the model ($\beta < 1$), placing the longest possible barrier in each partition may result in an infeasible solution due to the number of disrupted points. In that case, the feasible length of the barrier would be unknown. The infeasible solutions may get feasible by shifting the line to the right or left in combination with shortening the barrier but it comes at the expense of losing candidate locations. This repairing mechanism would also require a combinatorial selection of points to omit from right and left of the barrier which is not addressed in this algorithm but remains as a potential topic for future study.

Before developing the algorithm, we need to introduce appropriate strategies for:

- Optimally locating a fixed-length line between two *y-invisible* points
- Partitioning the plane into regions with distinct *y-invisible* point sets
- Fixing the length of the barrier in each partition

2.3.1 Optimal Location of a Line Barrier between Two *y-invisible* Points

Suppose that *y-invisibility* conditions between two points already hold and the length of the barrier is fixed as $L = x_e - x_s$. The problem is to find the optimal location for x_s and x_e such that the disruption between the two points is maximized. Since the barrier is a horizontal line and the distance norm is rectilinear, the interdiction can only happen along the x-axis and y-distance of the points cannot be increased.

Theorem 2.2 *The maximum interdiction between two y-invisible points, P_i and P_0, is obtained when $x_s = \frac{1}{2}[a_i + a_0 - L]$ and the resulting shortest distance is $l^B(P_i, P_0) = L$.*

Proof. The interdiction between P_i and P_0 happens when they are invisible. From *x-invisibility* conditions in Lemma 2.1 we know:

$$x_s < \min\{a_i, a_0\} \wedge x_e > \max\{a_i, a_0\}$$

In order to maximize the interdiction between sink and source points, the barrier line has to be stretched evenly from both sides of the rectangular hull of the points. So the optimal location of the barrier is:

$$\begin{aligned}
&\min\{a_i, a_0\} - x_s^* = x_e^* - \max\{a_i, a_0\} \\
&\Rightarrow 2x_s^* + L = \min\{a_i, a_0\} + \max\{a_i, a_0\} \\
&\Rightarrow x_s^* = \frac{1}{2}[a_i + a_0 - L] \\
&\Rightarrow x_e^* = x_s^* + L = \frac{1}{2}[a_i + a_0 + L]
\end{aligned}$$

Once the barrier is located between x_s^* and x_e^*, we can obtain the shortest distance between the two points P_i and P_0. We know that the shortest path between two invisible points passes through one of the barrier ends. Suppose that d_s and d_e represent the distances between P_i and P_0 through x_s and x_e respectively. The minimum (shortest) distance, d^*, is calculated as follows:

$$\begin{aligned} d_s &= |a_i - x_s^*| + |a_0 - x_s^*| = a_i + a_0 - 2x_s^* = a_i + a_0 - [a_i + a_0 - L] = L \\ d_e &= |x_e^* - a_i| + |x_e^* - a_0| = 2x_e^* - a_i - a_0 = [a_i + a_0 + L] - a_i - a_0 = L \\ d^* &= \min\{d_s, d_e\} = L \end{aligned}$$

Suppose that the line barrier is perturbed to the right by $\varepsilon > 0$, i.e. shifted to $(x_s^* + \varepsilon)$ and $(x_e^* + \varepsilon)$ coordinates. Then, the shortest distance between P_i and P_0 becomes:

$$\begin{aligned} d_s^{'} &= a_i + a_0 - [a_i + a_0 - L + \varepsilon] = L - \varepsilon \\ d_e^{'} &= [a_i + a_0 + L + \varepsilon] - a_i - a_0 = L + \varepsilon \\ d' &= \min\{L - \varepsilon, L + \varepsilon\} = L - \varepsilon < d^* \end{aligned}$$

The same goes if the line is perturbed to the left. ■

The interpretation of Theorem 2.2 is that projection of the midpoint of P_i and P_0 on the x-axis must be equal to that of the barrier to have the line equally stretched from both sides of P_i and P_0. Suppose that the midpoint of the points on the x-axis is $m_{i0} = \frac{1}{2}(a_i + a_0)$ and the midpoint of the line barrier on the x-axis is $m_{se} = \frac{1}{2}(x_e + x_s)$. The maximum interdiction between P_i and P_0 occurs when $m_{i0} = m_{se}$. This equation guarantees that the barrier is stretched equally from left and right.

Example 2.2 *In Figure 2.11, there is a sink point P_i at (8,7), a source point P_0 at (4,3) and a line barrier with $L = 8$ is horizontally located in between them. Using 2.2, the optimal location of the barrier is at $x_s = 0.5(8 + 4 - 8) = 2$ and $x_e = 10$ as the midpoints of P_i and P_0 overlaps with that of the barrier at 6. The projection of P_i, P_0, X_s and X_e on the x-axis along with their corresponding midpoints at 6.*

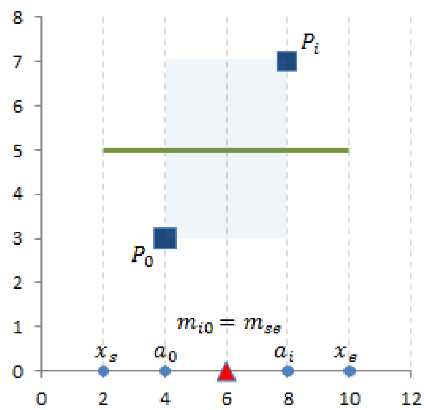


Figure 2.11: Optimal location of a barrier between two *y-invisible* points

The same rule applies when there several sink and source points on the plane.

Theorem 2.3 *Consider that weighted sink points are ascendingly sorted based on their x-coordinate values. Let a_i and a_k be two consecutive x-coordinate values with midpoints $m_{i0} = 0.5(a_i + a_0)$ and $m_{k0} = 0.5(a_k + a_0)$, respectively, and $m_{se} = 0.5(x_s + x_e)$ be that of the barrier. Then, $m_{se} = m_{i0}$ or $m_{se} = m_{k0}$ always provides a better or as good solution than any $m_{se} \in (m_{i0}, m_{k0})$.*

Proof. According to Theorem 2.2, if $m_{se} \neq m_{i0}$ then the amount of interdiction between P_i and P_0 caused by the barrier cannot be maximized. The same holds for P_k if $m_{se} \neq m_{k0}$. Therefore, $m_{se} \in (m_{i0}, m_{k0})$, cannot maximize the interdiction for neither of P_i and P_k. Shifting m_{se} towards the midpoint with higher weight would yield better interdiction because that point offers a higher "profit" for a fixed length barrier. If total disrupted weights at m_{i0} and m_{k0} are equal, then midpoints $m_{se} \in (m_{i0}, m_{k0})$ may correspond to an alternative solution but not a better one. Hence, enumerating m_{se} on all sink-source midpoints will always provide the optimal solution. ■

If *y-invisible* points and the barrier length are known, theorems 2.2 and 2.2 guarantee that optimal solution can be found by enumeration on some candidate midpoints. Therefore, the next step is to partition the plane such that *y-invisibility* remains the same within each partition.

2.3.2 Partitioning the Plane into Regions with Distinct *y-invisible* Point Sets

Locating a barrier at a particular y will divide the plane into two sides resulting in a set of *y-invisible* points.

Property 2.2 *If there are K distinct y-coordinates for all points on the plane, there are $K-1$ candidate ranges for y with $K-1$ distinct set of y-invisible points. In a particular range R_r, $r = 1, \ldots, K-1$, the value of $y_r \in R_r$ does not change the set of y-invisible points.*

In Figure 2.12, there are 8 points on the plane with P_4 being the source point. There are 7 distinctive y-coordinates which lead to 6 distinct ranges each of which having a distinct set of *y-invisible* points to the source point.

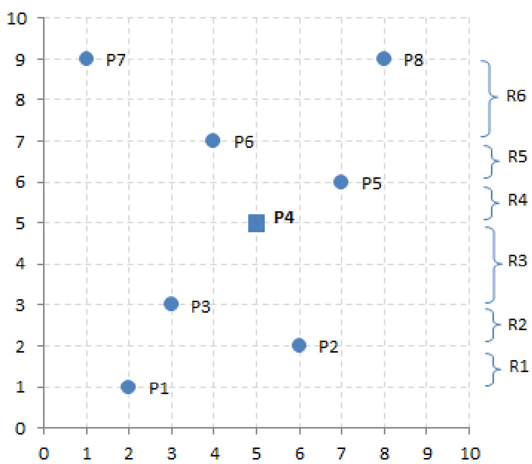


Figure 2.12: Candidate ranges for y

Table 2.1 shows the range of each candidate range in Figure 2.12 and their associated set of *y-invisible* points. For example, any horizontal line barrier in y-range [1,2) would make point P_1 *y-invisible* and it will not affect any other point on the plane. If a line barrier is placed in y-range [3,5), it would fall between the source point and P_1, P_2 and P_3 making them *y-invisible*.

Note that if a barrier falls onto a sink point, the sink point will be considered invisible. See Section 2.1.1.

Table 2.1: Candidate ranges for y and their set of *y-invisible* points

r	R_r	Set of *y-invisible* points
1	$[1, 2)$	P1
2	$[2, 3)$	P1,P2
3	$[3, 5)$	P1,P2,P3
4	$(5, 6]$	P5,P6,P7,P8
5	$(6, 7]$	P6,P7,P8
6	$(7, 9]$	P7,P8

2.3.3 Fixing the Length of the Barrier in Each Partition

Suppose that there are K distinct y-coordinates on the plane which is partitioned into ranges $R_r, r = 1, \ldots, K - 1$, according to Section 2.3.2. We know that the barrier has to be located as away as possible from the source point to get a longer length. If there is no disruption constraint in the model, the longest possible barrier within a range has to be selected in order to maximize the interdiction. Therefore, based on theorem 2.1, y_r has to be equal to the y-coordinate of the point at the border of range R_r and farthest from the source point. Therefore, the maximum barrier length in different ranges is obtained from following equation:

$$L_r = \alpha|y_r - b_0|, r = 1, \ldots, K - 1 \tag{2.56}$$

where $y_r = b_i$ and $P_i \in R_r$.

Consider the example in Figure 2.12 and suppose that $\alpha = 1$. The length of the barrier at each range can immediately be fixed at the values given in Table 2.2.

Table 2.2: Finding L_r in each range R_r

r	R_r	y_r	L_r
1	$[1, 2)$	1	4
2	$[2, 3)$	2	3
3	$[3, 5)$	3	2
4	$(5, 6]$	6	1
5	$(6, 7]$	7	2
6	$(7, 9]$	9	4

2.3.4 Algorithm for the One-to-Many Interdiction Problem with a Line Barrier on a Plane

In algorithm 1, the optimal interdiction between a single source and multiple sinks is obtained firstly by partitioning the plane into candidate ranges along the y-axis using Section 2.3.2 and assigning the maximum length using Section 2.3.3 and secondly by enumerating m_{se} on candidate midpoints in each range using Theorem 2.3:

Algorithm 1 Maximize $Z^* = \sum_{i=1}^{n} w_i l^B(P_0, P_i)$

Require: P_0 as the source point, $P_i, i = 1, ..., n$, as sink points and $\theta^\circ \in (0, 90)$
Ensure: Find x_s^*, x_e^*, L^*, Z^*

$Z_0 \leftarrow \sum_{i=1}^{n} w_i l(P_0, P_i)$ {The objective value when no barrier exists.}
$Z^* \leftarrow 0$
for $i = 1$ **to** n **do**
  $y_r \leftarrow b_i$ {find distinct y_r along the y-axis defined by P_i}
  $L_r \leftarrow 2 \tan\theta |y_r - y_0|$ {set L_r for each range along the y-axis }
  for $j = 1$ **to** n **do**
    if $(b_0 > y_r > b_j)$ or $(b_0 < y_r < b_j)$ **then**
      $m_{0j} \leftarrow 0.5(a_0 + a_j)$ {use the midpoint between P_0 and P_j if they are *y-invisible*}
      $x_{sj} \leftarrow m_{0j} - 0.5L_r$
      $x_{ej} \leftarrow m_{0j} + 0.5L_r$
      for $k = 1$ **to** n **do**
        if $b_0 > y_r > b_k$ or $b_0 < y_r < b_k$ **then**
          if $x_{sj} < a_0$ and $a_0 < x_{ej}$ and $x_{sj} < a_k$ and $a_k < x_{ej}$ **then**
            $Z_j \leftarrow Z_j + 2w_i \min\{\min(a_k, a_0) - x_s, x_e - \max(a_k, a_0)\}$
          end if
        end if
      end for
      if $Z_j + Z_0 > Z^*$ **then**
        $Z^* \leftarrow Z_j + Z_0$, $x_s^* \leftarrow x_{sj}$, $x_e^* \leftarrow x_{ej}$, $y^* \leftarrow y_r$, $L^* \leftarrow L_r$
      end if
    end if
  end for
end for
return $Z^*, x_s^*, x_e^*, y^*, L^*$

The outer loop finds the distinct range along the y-axis corresponding to the points i and calculates the y_r and L_r accordingly. In the second loop, a candidate midpoint m_{0j} is chosen from the set of *y-invisible* points with respect to y_r. Then, the barrier location is set at x_{sj} and x_{ej} using the midpoint and length information based on 2.2. The third loop calculates the shortest distance for all points with respect to the located barrier and updates the best found solutions accordingly.

In the outer loop, n sink points are used to locate the barrier along y axis. For each y value, there are, at most, n candidate places along the x-axis to locate the barrier (second loop). The third loop calculates the objective function value for each barrier location over n sink points. Therefore, the worst-case processing time needed for this algorithm is $O(n^3)$.

2.3.5 A Numerical Example for One-to-Many Interdiction Problem Using the Algorithm

Example 2.3 *Suppose that all sink point weights in Figure 2.12 are 1 and $\theta^\circ = 45$. The solution for this problem is as following:*

2: $Z_0 = (3+4) + (1+3) + (2+2) + (2+1) + (1+2) + (4+4) + (3+4) = 36$

Iteration 1:

4: Use point P_1 to partition the plane. Set $y_1 = 1$.

5: Find maximum possible length: $L_1 = 2tan45^\circ|5-1| = 8$

Locate the barrier:

8: P_1 is y-invisible to P_0. So use their midpoint: $m_1 = 0.5(5+2) = 3.5$

9: $x_{s1} = 3.5 - 0.5 \times 8 = -0.5$

10: $x_{e1} = 3.5 + 0.5 \times 8 = 7.5$

Find the disrupted distances:

14: P_1 is invisible. Therefore: $Z_1 = 2 * 2.5 = 5$

19: Since $36 + 5 > 36$ then $Z^* = 41, x_s^* = -0.5, x_e^* = 7.5, y^* = 1, L = 8$

No more y-invisible point is left in this y-range.So, go to step 4 and change y-range.

Iteration 2:

4: Use point P_2 to partition the plane. Set $y_2 = 2$.

5: Find maximum possible length: $L_2 = 2tan45^\circ|5-2| = 6$

Locate the barrier:

8: P_1 is y-invisible to P_0. So use their midpoint: $m_1 = 0.5(5+2) = 3.5$

9: $x_{s1} = 3.5 - 0.5 \times 6 = 0.5$

10: $x_{e1} = 3.5 + 0.5 \times 6 = 6.5$

Find the disrupted distances:

14: P_1 is invisible. Therefore: $Z_1 = 0 + 2 * 1.5 = 3$

14: P_2 is invisible. Therefore: $Z_1 = 3 + 2 * 0.5 = 4$

19: Since $36 + 4 \nless 41$, do not change the best solution.

Change the midpoint:

8: P_2 is y-invisible to P_0. So use their midpoint: $m_2 = 0.5(5+6) = 5.5$

9: $x_{s2} = 5.5 - 0.5 \times 6 = 2.5$

10: $x_{e2} = 5.5 + 0.5 \times 6 = 8.5$

find the disrupted distances:

14: P_2 is invisible to P_0. Therefore: $Z_2 = 0 + 2 * 2.5 = 5$

19: Since $36 + 5 \nless 41$, do not change the best solution.

After solving the problems in all iterations, the optimal solution is $Z^* = 41, x_s^* = -0.5, x_e^* = 7.5, y^* = 1, L = 8$. An alternative solution exists at $x_s^* = 2.5, x_e^* = 8.5, y^* = 2, L = 6$.

2.4 Visibility and the Shortest Path Problem in the Presence of Multiple Line Barriers and Its Complications

When several line barriers exist on a plane, two points become invisible if all alternative rectilinear paths between them are obstructed by the barriers. An algorithm is presented in Appendix A that checks if two points are invisible to each other with respect to several given line barriers. If so, the shortest way between them has to pass through a combination of barrier ends that are visible to each other.

Figure 2.13 shows how the source point find its way through 2 barriers to reach the invisible sink point

P_1 and for this, a selection of barrier endpoints have to be used. In this example, the shortest path to P_1 has to go through e_2 and s_1.

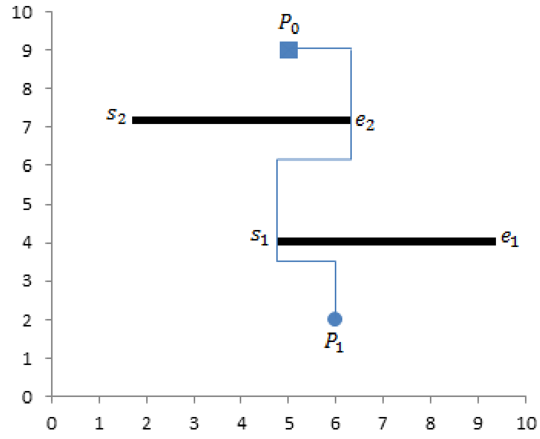


Figure 2.13: The shortest paths through multiple line barriers

If all the barriers are known, a common practice in the literature to find the shortest distances is to convert the planar problem into a graph. In this graph, the source and sink points along with barrier endpoints represent the graph nodes. If two points on the plane are visible to each other, an arc will be placed between their corresponding nodes on the graph. This graph is known as "visibility graph". There have been extensive study in the literature on shortest path in a graph and several methods are proposed for single-source, single-destination, and all-pairs shortest path problems. Among others, Dijkstra's algorithm for single-source problem and Floyd-Warshall for all-pairs problem are widely used in the literature (Cormen et al., 2009). Floyd-Warshall's algorithm is explained in Appendix A.

Figure 2.14 shows the visibility graph for above example. Since P_0 is invisible to P_1 and s_1, there is no arc between their corresponding nodes. The sink point P_1 is also invisible to e_2 for which no arc is placed between their nodes on the graph. The shortest path between P_0 and P_1 is highlighted on the graph.

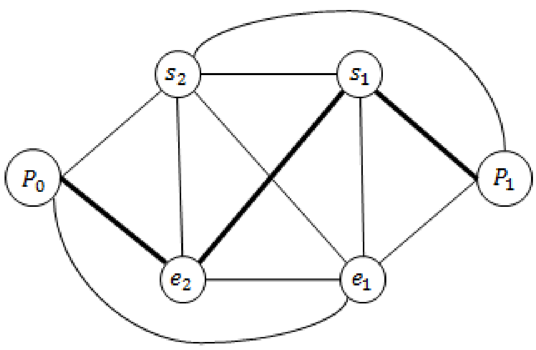


Figure 2.14: An example of visibility graph and the shortest path

However, the shortest path algorithms can only work with a static graph where all nodes and edges (visible pairs) are known. If one of the barriers is variable, as it is in this study, the visibility of any pair on the plane will be uncertain and it is not possible to assume any arc to appear on visibility graph and, therefore, shortest path calculations will not be valid.

The complexity of the problem emerges when the barriers are in ”cascade” arrangement. By ”cascade” arrangement we mean that invisibility is caused by combination of several barriers together and not by any of them alone. Figure 2.15 shows an example of cascading barriers (solid lines) and possible locations for a new barrier (dashed lines) to cause invisibility between P_0 and P_1.

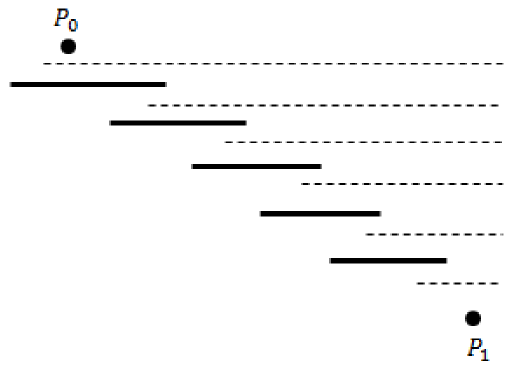


Figure 2.15: An example of visibility graph and the shortest path

The invisibility conditions, therefore, must include all the intermediate barriers, i.e. invisibility of barrier endpoints to each other and to the source and sink points. Already-invisible points will remain invisible after the location of a new barrier. But, by no means, we can assume that a pair of visible points will remain visible. Furthermore, the invisibility of source and sink points will not be discovered until visibility conditions of all barriers are checked. The visibility ”certificate” for source and sink points can only be issued when combination of all existing and new barriers fail to make the two points invisible.

Due to such complications, locating a barrier in presence of several existing barriers is not studied in this work but remains as a potential topic for future research.

CHAPTER 3

COMPUTATIONAL ANALYSIS FOR THE ONE-TO-MANY RECTILINEAR INTERDICTION PROBLEMS

In this chapter, we perform computational experimentation on a set of test problem instances for the assessment of performance of the models and solution approaches introduced in Chapter 2 for the one-to-many problem. In Section 3.1, the selected core test instances and their modifications are explained. We show that it is possible to eliminate some of the sink points based on the parameter θ in Section 3.2. The effect of parameter values, point elimination, and cuts of MIP solver on objective function and CPU time are studied in Section 3.2.1.

The one-to-many problems with disruption constraint are solved using MIP approach in Section 3.3. Objective function values and CPU time are reported for all test instances. The effect of instance specific properties and parameter values on the performance of MIP models are also analyzed. When there is no disruption constraint in the model, the algorithm is used to solve instances and the results are presented in Section 3.4. MIP model with $\beta = 1$ is also solved to validate the results for both approaches and to compare their performances.

3.1 Core Test Instances and Their Variants

In order to investigate the properties of this problem, 30 core test instances are selected mostly from the planar TSP and VRP instances available in literature. Based on the number and distribution of points on the plane, these instances can be categorized as:

- *Sparse*: Points are scattered with low density across the plane. 12 instances are in this category.
- *Clustered*: Points are concentrated in different groups (i.e. clusters) on the plane. The distance between the clusters is significant compared to the distance between points within a cluster. 11 instances are selected in this category.
- *Vertical*: Points are scattered around or along the y-axis. 5 instances are in this category.
- *Horizontal*: Points are scattered around or along the x-axis. 2 instances are in this category.

These categories may affect the performance of the MIP model. For example, having horizontal distribution of points may lead to elimination of several ”undisruptable” points considering a limit on barrier length at different y-coordinates.

For the one-to-many problems, an existing source is added to the core test instances. The location of the source point is decided to be either in the middle or at the border of the convex hull of sink points. The optimal location for the rectilinear Weber problem is chosen as the middle source while the border source is selected as a point that is vertically lower than all points in the convex hull. These variants are referred with 'M' for the middle location and 'B' for border location of the source point. Weights of the sink points are chosen to be identical (equal to 1) or randomly generated integers between 1 and 10. These variants are also addressed with '1' and 'W', respectively. Therefore, in the one-to-many problems, all core test instances are used in 4 different variants:

- **1M**: one-to-many problem with the source in the middle. All weights are equal to 1.
- **WM**: one-to-many problem with the source in the middle. Weights are randomly generated integers in [1,10].
- **1B**: one-to-many problem with the source at the border. All weights are equal to 1.
- **WB**: one-to-many problem with the source at the border. Weights are randomly generated integers in [1,10].

Considering above variations, there will be 120 instances in total for the one-to-many problems. Table 3.1 shows 30 core test instances with the location of the source in the one-to-many problems.

Instances are grouped into small and medium problems with respect to the number of their sink points. Instances with less than 100 sink points, i.e. instances 1 to 20, are considered as "small" while instances with 100 or more sink points are "medium".

3.2 Pre-processing and Point Elimination

Recall from Section 2.1.1 that the plane can be partitioned into two upper and lower half-spaces and the horizontal line barrier can only be located in one of these half-spaces at a time. Therefore, each partition can be solved separately and the best solution in both partitions would represent the solution for the instance. It is expected that solving two smaller MIP problems (one for each partition) will be faster than solving a large MIP problem. In this study, 1M and WM variants of instances can be partitioned due to the fact that the source point is somewhere in the middle of sink points. In 1B and WB variants, however, the source point is above all sink points and the lower partition would include all sink points, leaving the upper partition empty.

In addition to partitioning, it is possible to eliminate some of the sink points before constructing the MIP models. Suppose that P_0 is the source located at (a_0, b_0). To satisfy the *x-invisibility* condition, the a_i has to be between the two endpoints of the barrier, i.e. the barrier can be shifted as much as its length to the right and left side of the source. As explained in Section 2.1.4, the barrier length L at different y levels is obtained using the formula $L = \alpha|y - b_0|$. Knowing the length of the barrier, we can calculate how much the barrier can be shifted to the right and left of the source and eliminate the sink points that can never be disrupted by the barrier.

Figure 3.1 shows how the barrier with length L is shifted to the right and left side of the source point. The angle between the source point and the endpoints of the barrier is γ where: $\tan\gamma = 2\tan\theta = \alpha$. A point P_i is "disruptable" by the barrier if it falls inside the triangle with vertex angle γ, i.e. its tangent from the source point is less than $\tan\gamma$:

$$\frac{|a_i - a_0|}{|b_i - b_0|} < \alpha$$

Having P_0 and θ, it is possible to determine and eliminate "undisruptable" points before attempting to solve instances.

Table 3.1: 30 core test instances and their properties

No.	Core Instance*	n	Distribution	Middle Location		Border Location
				1M	WM	1B / WB
1	D8-Canbolat	8	sparse	(7, 4.5)	(7, 8)	(8, 1)
2	E-n22-k4	22	vertical	(146, 217)	(147, 231)	(146, 180)
3	D28	28	sparse	(156, 310)	(156, 310)	(205, -2)
4	B-n31-k5	31	clustered	(20, 27)	(21, 28)	(49.5, 5)
5	A-n32-k5	32	sparse	(50, 39)	(57, 42)	(49.5, 0)
6	D40	40	vertical	(156, 285)	(156, 285)	(155.5, 13)
7	B-n41-k6	41	clustered	(61, 64)	(61, 64)	(60, 6)
8	A-n45-k6	45	sparse	(46, 49)	(49, 53)	(49.5, 4)
9	F-n45-k4	45	horizontal	(21, 0.01)	(28, 3)	(15.5, -100)
10	att48	48	vertical	(6469, 2992)	(6344, 2874)	(6548, 1218)
11	B-n50-k7	50	clustered	(52, 35)	(52, 35)	(47.5, 0)
12	D50	50	vertical	(203, 264)	(198, 242)	(214.5, 60)
13	eil51	51	sparse	(36, 39)	(37, 40)	(34, 4)
14	berlin52	52	sparse	(700, 595)	(720, 610)	(882.5, 4)
15	A-n60-k9	60	sparse	(45, 61)	(43, 57)	(48, 4)
16	B-n68-k9	68	clustered	(36, 68)	(36, 68)	(44, 6)
17	F-n72-k4	72	vertical	(-7, 5)	(-6, 6)	(-6.5, -26)
18	rus75	75	clustered	(52, 43)	(54, 45)	(58.5, 1)
19	eil76	76	sparse	(40, 36)	(40, 35)	(38, 3)
20	A-n80-k10	80	sparse	(51, 40)	(57, 43)	(50, -1)
21	rd100	100	sparse	(486.8, 584.4)	(502.8, 612.6)	(490.4, 0.2)
22	E-n101-k14	101	sparse	(31, 35)	(31, 34)	(34.5, 2)
23	10G2	101	clustered	(50, 50)	(48, 44)	(50, 1)
24	F-n135-k7	135	horizontal	(4.8, 5.1)	(3.2, 5)	(-59.25, -36)
25	Ch150	150	sparse	(334.3, 360.3)	(343.2,370.7)	(353.7,-0.580)
26	d198	198	clustered	(1668.8, 1402.8)	(1668.8, 1402.8)	(2014.2, -1)
27	gr229	229	clustered	(30.1, 85.2)	(29.4, 83.2)	(11.8, -176.1)
28	a280	280	clustered	(132, 85)	(140, 85)	(148, 8)
29	lin318	318	clustered	(1488, 1827)	(1488, 1701)	(1575, -80)
30	fl417	417	clustered	(1114.4, 1245)	(1132.1, 1398.5)	(1203.1, 151.5)

***References for instances:**

-TSPLib (2012): Instances 10,13,14,19,21,25,26,27,28,29, and 30

-VRPH (2012): Instances 2,4,5,7,8,9,11,15,16,17,20,22,23, and 24

-Ruspini (1970): Instance 18

-Canbolat and Wesolowsky (2010): Instance 1

-Instances 3,6, and 12 are generated in this study.

Table 3.2 gives the percentage of eliminated points (%) and eliminated weights (W%) for $\theta deg = 15, 30, 45, 60$ for all variants of instances. As was expected, the percentage of eliminated points decreases as θ increases and vice versa. For example, in 1B variant of B-n31-k5 higher θ values result in less eliminated points in our computational test.

In addition to θ values, the percentage of eliminated points is highly dependent on location of the source point and distribution of the sink points on the plane as well. For example, 97% of points are eliminated when $\theta = 15$ in variant 1B of instance B-n31-k5 whereas it is 71% when the source point is in the middle (variant 1M). In variant 1B of instance d198, however, only 26% of points can be

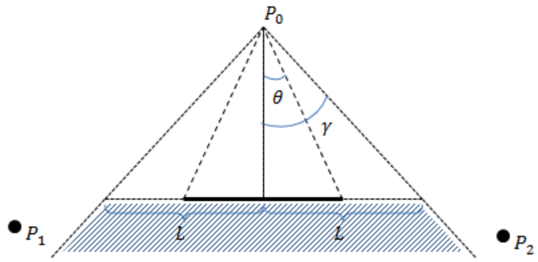


Figure 3.1: P_1 and P_2 can never be disrupted if angle θ is chosen.

Table 3.2: Percentages of eliminated points and weights based on θ levels in the one-to-many problems

Variant		1B				1M				WB								WM							
	$\theta°$	15	30	45	60	15	30	45	60	15		30		45		60		15		30		45		60	
Instance	n	%	%	%	%	%	%	%	%	%	W%	%	W%	%	W%	%	W%	%	W%	%	W%	%	W%	%	W%
D8-Canbolat	8	50	25	25	25	75	50	38	25	50	30	25	24	25	24	25	24	63	65	38	57	38	57	13	22
E-n22-k4	22	23	18	5	5	41	18	14	9	23	29	18	21	5	3	5	3	50	60	14	19	9	13	5	9
D28	28	21	7	0	0	64	43	39	21	21	18	7	6	0	0	0	0	64	64	43	49	39	43	21	19
B-n31-k5	31	97	90	23	19	71	48	42	23	97	99	90	93	23	26	19	21	65	63	52	53	45	48	29	29
A-n32-k5	32	66	31	19	19	78	53	28	19	66	65	31	32	19	17	19	17	75	77	56	56	28	29	19	19
D40	40	28	18	10	5	50	23	10	10	28	46	18	36	10	23	5	15	50	33	23	15	10	7	10	7
B-n41-k6	41	66	20	12	12	78	51	39	39	66	52	20	15	12	9	12	9	78	67	51	41	39	31	39	31
A-n45-k6	45	62	33	22	16	84	49	27	13	62	70	33	31	22	21	16	12	80	86	47	57	27	34	16	19
F-n45-k4	45	18	2	2	2	91	78	64	31	18	18	2	3	2	3	2	3	93	94	73	72	60	56	33	38
att48	48	42	29	27	19	56	27	13	8	42	43	29	27	27	25	19	17	54	49	31	33	15	15	8	6
B-n50-k7	50	44	26	18	14	76	42	14	14	44	48	26	28	18	17	14	12	76	80	42	52	14	19	14	19
D50	50	22	2	2	2	52	32	24	14	22	35	2	7	2	7	2	7	54	40	36	26	26	19	20	15
eil51	51	41	14	12	2	73	43	29	16	41	41	14	14	10	9	2	2	73	68	45	48	29	34	16	16
berlin52	52	48	25	13	10	83	63	42	25	48	55	25	29	13	15	10	10	83	85	63	67	48	48	27	23
A-n60-k9	60	42	17	8	5	72	50	27	15	42	43	17	17	8	9	5	4	68	67	45	43	22	23	10	12
B-n68-k9	68	47	29	24	15	72	43	29	25	47	45	29	24	24	19	15	14	72	70	43	46	29	35	25	29
F-n72-k4	72	8	6	3	1	54	39	28	15	8	6	6	3	3	1	1	1	51	54	40	46	32	35	21	23
rus75	75	52	25	17	9	83	61	33	13	52	52	25	27	17	18	9	12	83	82	64	63	39	38	13	10
eil76	76	39	17	11	7	71	39	22	14	39	35	17	12	11	8	7	6	72	70	42	39	24	26	14	16
A-n80-k10	80	53	26	15	6	69	46	30	18	53	51	26	22	15	12	6	5	70	70	41	43	31	35	19	19
rd100	100	47	19	13	10	76	46	27	19	47	43	19	18	13	12	10	9	76	75	43	42	29	27	15	14
E-n101-k14	101	47	16	6	4	76	42	25	14	47	46	16	13	6	5	4	3	76	77	43	46	24	27	15	20
10G2	101	51	26	14	10	80	41	41	13	51	50	26	23	14	11	10	9	77	80	42	45	31	35	19	22
F-n135-k7	135	93	72	13	3	87	76	65	46	93	93	72	69	13	13	3	2	87	85	73	73	63	63	49	49
ch150	150	45	21	13	7	79	51	28	13	45	43	21	20	13	12	7	7	77	78	50	52	31	32	16	18
d198	198	26	17	1	1	85	60	44	35	26	24	17	15	1	0	1	0	85	82	60	57	44	41	35	33
gr229	229	3	1	1	1	38	17	8	4	3	4	1	1	1	1	1	1	38	36	15	16	7	9	4	4
a280	280	70	44	24	12	83	67	48	31	70	70	44	43	24	23	12	12	85	83	68	66	48	48	31	31
lin318	318	34	18	12	7	68	33	24	13	34	37	18	20	12	13	7	7	67	67	33	33	23	22	16	16
fl417	417	39	37	37	37	41	5	3	2	39	41	37	39	37	39	37	39	43	41	6	6	3	4	2	3

eliminated with $\theta = 15$, compared to 85% elimination in its 1M variant.

3.2.1 Parameter Setting

In addition to instances, there are two parameters α and β that can affect the performance of the MIP model and the algorithm. The parameter α determines the length of the barrier based on its proximity to the source point. Since $\alpha = 2\tan\theta$, as explained in Section 2.1.4, instead of α we can use $\theta \in (0, 90°)$ as the parameter of length rate. Parameter β is the maximum disruption rate, $0 < \beta \leq 1$. In order to study the effect of these parameters on solutions, they are designed at $\theta° = \{15, 30, 45, 60\}$ and $\beta = \{0.1, 0.25, 0.5, 0.75\}$.

Following MIP cuts are also set in MIP solver of CPLEX Optimizer 10.1 (ILOG, 2006) with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

The valid inequalities introduced in Section 2.2.7 are not used in experimentations.

Table 3.3 shows the MIP solutions obtained for the test instance A-n45-k6-1B along with the required CPU time, the total number of iterations used for solving node relaxations (Niter), number of processed nodes in the active branch-and-cut search (Nodes) and the total number and percentages of MIP cuts when they are used.

Table 3.3: Parameter analysis on variant 1B of instance A-n45-k6

Parameters		**Optimal Solutions**					**Without MIP cuts**			**With MIP cuts**			**MIP Cuts**	**MIP Cuts* percentage (%)**						
θ°	β	x_s^*	x_e^*	y^*	L^*	Z^*	**CPU**	**Niter**	**Nodes**	**CPU**	**Niter**	**Nodes**		**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**IB**
15	**0.1**	32	66	68	34	3462.5	0.2	1779	216	0.7	4663	184	605	46	23	21	3	2	1	4
	0.25	38	60.5	46	22.5	3468.6	0.1	1853	233	0.9	5408	179	659	57	17	11	6	2	1	6
	0.5	38	60.5	46	22.5	3468.6	0.1	2020	227	0.8	5722	228	603	51	15	10	9	3	3	9
	0.75	38	60.5	46	22.5	3468.6	0.1	2020	227	0.9	7169	264	701	44	20	19	7	1	1	8
30	**0.1**	2.8	106.7	94	103.9	3684.2	0.4	7803	1040	1	6867	156	996	53	19	16	5	2	1	4
	0.25	12.3	86.2	68	73.9	3766.1	0.7	13290	1920	1.2	10582	522	623	43	15	17	11	3	3	8
	0.5	12.3	86.2	68	73.9	3766.1	0.7	14403	2422	1.2	10279	469	681	51	16	12	6	3	3	9
	0.75	12.3	86.2	68	73.9	3766.1	0.7	14320	2368	1.1	9733	625	902	55	12	15	9	2	1	6
45	**0.1**	-40.2	141.8	95	182	3934.5	0.6	10955	1188	1.4	11485	310	996	46	25	16	6	1	1	5
	0.25	-21.2	130.8	80	152	4316.5	1.1	20044	2815	1.1	9801	349	679	41	22	14	11	5	1	6
	0.5	-13.8	112.2	67	126	4467	1.8	34055	6515	1.2	10444	515	820	47	15	15	11	2	1	9
	0.75	-13.8	112.2	67	126	4467	1.8	34593	6029	1.2	11840	755	653	51	10	11	12	3	4	9
60	**0.1**	-102.9	212.4	95	315.2	4467.4	0.6	11378	1250	1	4782	106	713	43	21	19	6	2	0	9
	0.25	-76.9	186.4	80	263.3	5429.2	1.2	24028	3014	1.4	9824	261	794	40	21	16	11	4	1	7
	0.5	-59.9	158.4	67	218.2	5850.6	1.3	21111	3525	2.5	21843	745	874	41	23	16	9	1	2	8
	0.75	-59.9	158.4	67	218.2	5850.6	1.9	43275	5580	1.7	15364	580	770	54	19	9	6	1	3	8
											Average:		754	48	18	15	8	2	2	7

* **CQ**: Clique, **GUB**: General Upper Bound, **CV**: Cover, **FC**: Flow Cover, **MIR**: Mixed-Integer Rounding, **IB**: Implied Bound

The effect of θ and β values on objective function:
Increase in θ° angles leads to an increase in the length of the barrier which, in turn, increases the disruption and objective function value, Z^*. On the other hand, lower disruption rate β allows less disruption on points and, therefore, decreases the objective value. The results obtained in Table 3.3 approve the above-mentioned effects of θ° and β on L^* and Z^*. For example, for $\theta = 15$ and $\beta = 0.5$, the optimal barrier length is $L^* = 22.5$ yielding $Z^* = 3468.6$. This problem with $\theta = 30$ leads to longer barrier length of $L^* = 73.9$ and more disruption with $Z^* = 3766.1$ whereas the same problem with $\theta = 15$ and $\beta = 0.1$ results in reduction in the objective value down to $Z^* = 3462.5$.

The disruption constraint becomes tight with lower β rates and affects the optimal solution whereas higher β rates may result in the identical solutions. For example, the solutions obtained for $\theta^\circ = 15$ and $\beta = \{0.25, 0.5, 0.75\}$ are all identical.

The effect of MIP cuts on CPU time:
When MIP cuts are tested on this instance, the CPU time increases generally due to extra calculations required for implementing cuts, compared to the CPU time when no MIP cut is used. However, number

of processed nodes in the active branch-and-cut search and total number of iterations required for solving node relaxations (Niter) decrease if MIP cuts are used. The highest share of MIP cuts belongs to clique cuts which constitute 48% of all cuts while GUB, cover cuts and flow cover cuts are the next important cuts being 18%, 15% and 8% respectively. Although flow path, disjunctive, zero-half and multi-commodity flow cuts are enforced in CPLEX, no cut is generated in these categories.

It is expected that MIP cuts will be more effective on larger instances. Table 3.4 gives the results for 1B variant of instance 10G2 where the MIP cuts are proved to be more effective leading to lower CPU times. For example, when $\theta^\circ = 45$ and $\beta = 0.1$ and no MIP cut is used, CPLEX fails to reach the optimal solution in 1000 seconds whereas it takes only 5.7 seconds to solve when cuts are applied.

Table 3.4: Parameter analysis on variant 1B of instance 10G2

Parameters		Optimal Solutions					Without MIP cuts			With MIP cuts			MIP Cuts	MIP Cuts* percentage (%)						
θ°	β	x_s^*	x_e^*	y^*	L^*	Z^*	CPU	Niter	Nodes	CPU	Niter	Nodes		CQ	GUB	CV	FC	GF	MIR	IB
30	**0.1**	-7	105	98	112	8149.1	29	383361	56014	10.5	84585	4028	1298	35.9	25.2	12.8	13.2	4.3	1.7	6.9
	0.25	9	91	72	82	8602.6	31.7	361475	61569	8.8	86677	5502	1698	39	18	13.7	20.9	2.8	1.4	4.2
45	**0.1**	-48	146	98	194	8969	1000	10482705	1169601	5.7	54376	2582	1770	41.3	20.5	21.9	8.9	0.7	0.3	6.4
	0.25	-34	134	85	168	10061	16.8	232145	24128	4.9	41477	1724	1692	39.2	22.2	15.2	15.1	4	1.2	3.2
60	**0.1**	-34	134	85	168	10061	16.8	232145	24128	4.9	41477	1724	1692	39.2	22.2	15.2	15.1	4	1.2	3.2
	0.25	-95.5	195.5	85	291	12643.7	34	409971	56926	6	56983	1453	1284	31.9	14	19.8	21	5.4	0.4	7.6
											Average:		1577	38	20	17	16	3	1	6

* **CQ**: Clique, **GUB**: General Upper Bound, **CV**: Cover, **FC**: Flow Cover, **MIR**: Mixed-Integer Rounding, **IB**: Implied Bound

The effect of pre-processing on CPU time:
We expect that pre-processing and point elimination would improve the CPU time. Table 3.5 shows the results for A-n45-k6-1B after point elimination and applying MIP cuts. The percentage of eliminated points in instance A-n45-k6-1B with θ values at 15, 30, 45 and 60 are 62%, 33%, 22% and 16% respectively. Lower θ values result in elimination of more points and reduce the number of variables and constraints used in the MIP model which, ultimately, reduces the number of iteration (Niter) and improves the CPU time.

We expect that pre-processing and point elimination would improve the CPU time. Table 3.5 shows the results for the same problem after point elimination and applying MIP cuts. The percentage of eliminated points in instance A-n45-k6-1B with θ values at 15, 30, 45 and 60 are 62%, 33%, 22% and 16% respectively. Lower θ values result in elimination of more points and reduce the number of variables and constraints used in the MIP model which ultimately reduces the number of iteration (Niter) and improves the CPU time.

3.2.2 Selected Parameter Values

To give instances different lengths, the θ values are set to $\theta = 30, 45, 60$ and the total weight of disrupted points is constrained by $\beta = 0.1, 0.25, 1$. For $\beta < 1$ the MIP approach will be used while problems with $\beta = 1$ do not have disruption constraint and they are solved with the algorithm. The point elimination procedure will be applied to all instances in order to reduce the instance size before solving them.

3.3 Computational Results for the One-to-Many Problems with $\beta < 1$ Using MIP Model

CPLEX Optimizer 10.1 (ILOG, 2006) is used for solving MIP models while algorithms are developed in an application using VB.NET 2010 (Microsoft, 2012). All available MIP cuts in CPLEX are enabled with priority 1 and a time limit of 1000 seconds is applied for MIP problems. If a problem is

Table 3.5: Parameter analysis on instance A-n45-k6-1B after point elimination

Parameters		Optimal Solutions					With MIP cuts			MIP Cuts	MIP Cuts* percentage (%)						
$\theta°$	β	x_s^*	x_e^*	y^*	L^*	Z^*	**CPU**	**Niter**	**Nodes**		**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**IB**
15	**0.1**	32	66	68	34	3462.5	0.2	1667	129	99	33	13	12	8	13	2	18
	0.25	38	60.5	46	22.5	3468.6	0.3	2937	156	333	53	17	8	5	2	3	13
	0.5	38	60.5	46	22.5	3468.6	0.4	3900	159	440	57	18	7	7	0	1	10
	0.75	38	60.5	46	22.5	3468.6	0.4	3900	159	440	57	18	7	7	0	1	10
30	**0.1**	2.8	106.7	94	103.9	3684.2	0.6	5842	232	670	54	18	16	3	3	0	6
	0.25	12.3	86.2	68	73.9	3766.1	0.9	9094	407	661	47	17	14	11	3	2	6
	0.5	12.3	86.2	68	73.9	3766.1	0.9	8514	459	651	58	11	14	4	2	1	9
	0.75	12.3	86.2	68	73.9	3766.1	0.9	8760	472	647	51	17	14	6	2	2	9
45	**0.1**	-40.2	141.8	95	182	3934.5	0.8	5817	82	748	55	19	16	3	1	0	5
	0.25	-21.2	130.8	80	152	4316.5	0.9	7507	254	659	47	17	17	7	6	1	5
	0.5	-13.8	112.2	67	126	4467	1.3	12173	546	775	51	18	15	5	2	1	8
	0.75	-13.8	112.2	67	126	4467	1.1	9692	502	759	50	14	12	11	1	3	10
60	**0.1**	-102.9	212.4	95	315.2	4467.4	0.9	6776	128	804	40	22	18	10	2	0	8
	0.25	-76.9	186.4	80	263.3	5429.2	1.1	8368	232	852	48	18	14	9	4	1	6
	0.5	-59.9	158.4	67	218.2	5850.6	1.9	16652	456	924	54	16	14	3	1	4	8
	0.75	-59.9	158.4	67	218.2	5850.6	1.3	14348	598	786	47	17	11	11	1	4	8
								Average:		641	50	17	13	7	3	2	9

* **CQ**: Clique, **GUB**: General Upper Bound, **CV**: Cover, **FC**: Flow Cover, **MIR**: Mixed-Integer Rounding, **IB**: Implied Bound

partitioned, a time limit of 1000 seconds is applied for solving each partition. All computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

The MIP solutions are obtained for different levels of parameters θ and β. However, instead of actual objective values ($Z = \sum_{i=1}^{n} w_i l^B(P_i, P_j)$) , the amount of increase in the original objective function ($Z_0 = \sum_{i=1}^{n} w_i l(P_i, P_j)$) is used as an interdiction rate ($\Delta Z\%$):

$$\Delta Z\% = \frac{(\Delta Z = Z - Z_0)}{Z_0} \times 100$$

The original objective values (i.e. Z_0) and the interdiction rates (i.e. $\Delta Z\%$) for all instances are presented in this section while further details are available in Appendix C.

The required processing times (CPU) to solve each MIP model are reported in seconds.

A. 1B Variants

Table 3.6 shows the computational results for all instances in the one-to-many model with the source being at the border and point weights are equal to 1 (variant 1B). The number of points in the instance and the percentage of eliminated points based on θ values are given in columns n and $E\%$.

Point elimination: We know that lower θ angles provide more chance of point elimination than higher θ values. However, the effectiveness of point elimination is highly dependent on the distribution of sink points with respect to the location of source point. For example, 72% of the sink points in instance F-n135-k7 are eliminated when $\theta = 30$ whereas the elimination rate is only 3% when θ is doubled. Almost the same applies for instances B-n31-k5 and a280. In instances like D50, gr229 or F-n72-k4, on the other hand, $\theta = 30$ can eliminate only very few points due to the distribution of sink and source points on the plane. For example, F-n72-k4 has a vertical distribution of points that makes more points "disruptable" by its border source. Therefore, only 6% of its sink points can be eliminated by $\theta = 30$ and this percentage drops to 1% with $\theta = 60$.

Objective function: As the overall results suggest, the highest increase in objective value is obtained when $\theta = 60$ and $\beta = 0.25$. The reason is that higher θ values lead to longer barriers and make

Table 3.6: Computational results for the one-to-many problems with $\beta < 1$ for 1B variants

		θ	30					45					60				
		β	E%	0.1		0.25		E%	0.1		0.25		E%	0.1		0.25	
Instances	**n**	Z_0		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**
D8-Canbolat	8	65	25	0	0	18.5	0	25	0	0	37	0	25	0	0	75.2	0
E-n22-k4	22	1182	18	13.6	0.2	21.5	0.4	5	25.2	0.5	45	0.4	5	45.3	0.3	86.9	0.4
D28	28	10403	7	5.9	0.5	15.7	0.6	0	9.6	0.6	34	0.6	0	21.2	0.5	70.8	0.6
B-n31-k5	31	1694.5	90	2.9	0	2.9	0	23	6.5	0.3	6.5	0.5	19	12.6	0.5	18.2	0.5
A-n32-k5	32	2315	31	8.9	0.4	11.5	0.4	19	18.6	0.4	27	0.5	19	35.5	0.4	60.1	0.6
D40	40	11645	18	13.1	0.7	27	0.9	10	24.5	0.9	52	1.1	5	44.3	0.9	94.6	1
B-n41-k6	41	3362	20	8.1	0.7	11.5	1	12	16.9	1.2	28	1.1	12	32.7	1	59.2	0.9
A-n45-k6	45	3373.5	22	9.2	0.6	11.6	0.9	22	16.6	0.8	28	0.9	16	32.4	0.9	60.9	1.1
F-n45-k4	45	5879	2	9.4	1.4	15.5	1.7	2	17.7	1	29	1.8	2	32.1	1.1	60.4	1.7
att48	48	174125	29	14.9	0.6	23.6	0.9	27	28.6	0.8	50	0.8	19	52.5	1	95.5	1
B-n50-k7	50	3508	26	9.7	0.8	17.6	0.9	18	19.8	0.8	38	1.3	14	37.3	1.2	72.1	1
D50	50	12397	2	14.2	1.6	23.9	1.5	2	25.6	1.2	46	1.5	2	45.3	1.6	84	2.5
eil51	51	2489	14	9	1.3	14.7	1.5	12	19	1.3	35	1.4	2	36.4	1.6	71.7	2
berlin52	52	46522	25	6.8	1	11.7	1.6	13	15.5	1.2	23	1.4	10	30.5	1.2	50	1.8
A-n60-k9	60	4584	5	9	1.4	15.2	3.1	5	18.8	1.4	37	2.1	5	35.9	1.9	74.9	2.3
B-n68-k9	68	4898	29	9.8	1.6	24	2.1	24	18.3	1.4	47	2	15	33	1.5	86.5	1.9
F-n72-k4	72	2600	6	12.5	2.8	21.8	4.1	3	23.2	4.3	44	4.3	1	41.7	4	82.4	3
rus75	75	5083.5	25	4.9	2.9	9.1	2.9	17	11.1	3.3	21	4.4	9	26	4	50.5	3.8
eil76	76	3703	17	8.9	2.7	15.4	4.7	11	18.7	2.4	34	3.4	7	35.9	2.2	70.1	3.9
A-n80-k10	80	5530	26	7.7	4	14.5	4.5	15	16.8	3.4	36	2.7	6	35.2	4.6	75.5	2.9
rd100	100	83112	19	7.9	6	18.9	5	13	17.2	4.7	39	4.7	10	33.7	4.1	76	7.9
E-n101-k14	101	5029.5	16	8.5	6	13.9	13.7	6	19.1	4.7	34	16.7	4	37.5	5.2	70.4	10.4
10G2	101	7469	26	9.1	10.5	15.2	8.8	14	20.1	5.7	35	4.9	10	39.1	9.1	69.3	6
F-n135-k7	135	13761.6	72	1.5	1.1	1.5	1	13	5	7.6	5.9	13.1	3	13.1	9.8	21.9	11.5
Ch150	150	80157.7	21	7.3	12.2	11.5	25.4	13	16.2	15.1	31	16.6	7	32.7	10.4	65.1	11.8
d198	198	411628.6	17	8.9	11.4	20.4	23	1	16.5	27.9	37	61.7	1	29.5	63.6	67	76.7
gr229	229	64192.4	1	10.4	1000	25.5	73.8	1	19.6	1000	48	26.1	1	35.5	1000	86.5	1000
a280	280	43422	44	6.1	121	7.6	122	24	11.8	64.4	21	318	12	25.8	32.3	49.7	100.9
lin318	318	855394	18	9.1	1000	19	84.6	12	19.5	1000	42	123	7	37.6	1000	81.8	285.8
fl417	417	586807.7	37	9	1000	22.2	1000	37	20.2	1000	50	1000	37	39.6	1000	99.3	1000

larger areas exposed to disruption while higher β values increase the disruption capacity resulting in higher chance of increasing the objective values. For example, in instance att48, 29% of sink points are eliminated when $\theta = 30$ that leads to the least interdiction with $\Delta Z\% = 14.9$ when $\beta = 0.1$. In the same instance, $\beta = 0.25$ increases the disruption capacity and interdiction rate but at the expense of increased CPU time, which is, perhaps, the result of processing more combinations of disrupted points. When $\theta = 45$, less points are eliminated (E%=27) and higher interdiction rates (28.6% and 50%) are obtained. The same pattern is followed when $\theta = 60$.

CPU time: Processing time seems to be affected by several factors: In most instances, $\beta = 0.25$ requires more processing time than $\beta = 0.1$ regardless of θ values. For example in d198, when $\theta =$ 30 or 45, it takes almost twice as much time to solve the problem with $\beta = 0.25$ compared to the time obtained for the lower disruption rate. Even with $\theta = 60$, the MIP model is solved faster when $\beta = 0.1$. However, this is not the case for the instances like gr229 and lin318. While CPLEX cannot reach optimality for lin318 with $\theta = 30$ and $\beta = 0.1$, it takes only 84.6 seconds to hit the optimal solution when $\beta = 0.25$. The same pattern is observed for $\theta = \{45, 60\}$ in this instance.

The number of processed points and their distribution may also affect CPU time. For example, in instance a280, 44% of its points are eliminated with $\theta = 30$ compared to just 1% of gr229 that is why its CPU time is far below than that of gr229. The instance F-n135-k7 is solved faster than 10G2 with lower θ values but its CPU time gets worse when θ increases. The main reason is that the number of eliminated points for F-n135-k7 and 10G2 that are, respectively, 72% and 26% when θ=30. These figures change to 21.9% and 61.3% when θ=60.

The average interdiction rates for all instances along with CPU time for small and medium instances are given in Table 3.7. As θ values increase from 30 to 45, the average interdiction rates, $\Delta Z\%$, almost doubles for $\beta = 0.1$ and $\beta = 0.25$. The same holds when θ increases from 45 to 60. The average interdiction rate for θ=30 and β=0.25 is close to that for θ=45 and β=0.1. Therefore, a longer barrier with lower disruption rate may yield the same objective as with a shorter barrier with higher disruption rate. In general, small instances (the first 20 instances) are solved much faster than medium instances. θ=0.25 takes slightly more time to solve in small instances whereas solving θ=0.1 is more time-consuming in medium instances.

Table 3.7: Summary of results for 1B variants

θ	**30**		**45**		**60**	
β	**0.1**	**0.25**	**0.1**	**0.25**	**0.1**	**0.25**
$\Delta Z\%$	8.54	16.10	17.21	34.68	33	70
CPU (Small)	1.26	1.685	1.36	1.61	1.52	1.645
CPU (Medium)	316.82	135.73	313.01	158.48	313.45	251.1

B. 1M Variants

Table 3.8 shows the results for the same problem but with the source point being in the middle (variant 1M). Partitioning the plane is applied before solving the instances.

Table 3.8: Computational results for the one-to-many problems with $\beta < 1$ for 1M variants

		θ	**30**					**45**					**60**				
		β	**E%**	**0.1**		**0.25**		**E%**	**0.1**		**0.25**		**E%**	**0.1**		**0.25**	
Instances	**n**	Z_0		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**
D8-Canbolat	8	46	50	0	0	17.6	0.1	38	0	0	28.3	0	25	0	0	53.5	0
E-n22-k4	22	722	18	10.5	0.2	10.5	0.3	14	20.8	0.2	24.4	0.3	9	38.6	0.2	53.5	0.3
D28	28	5435	43	6.7	0.1	13.6	0.2	39	13.8	0.1	28	0.2	21	26.1	0.3	56.5	0.3
B-n31-k5	31	774	48	6.9	0.1	8	0.2	42	12.3	0.2	20.4	0.2	23	23.2	0.3	42	0.3
A-n32-k5	32	1779	53	4.9	0.1	4.9	0.1	28	11.1	0.2	11.1	0.3	19	23.4	0.3	23.6	0.4
D40	40	6058	23	13.9	0.4	21.8	0.5	10	27.3	0.5	45.8	0.8	10	50.5	0.7	87.4	0.6
B-n41-k6	41	2197	51	4.6	0.3	6.3	0.2	39	9.6	0.3	13.3	0.4	39	21.6	0.3	28.9	0.4
A-n45-k6	45	2453	27	3.9	0.3	3.9	0.3	27	10.1	0.5	10.4	0.6	13	18.9	0.6	25.9	0.8
F-n45-k4	45	2157.5	64	4.2	0	4.2	0	64	10.3	0.1	10.3	0.1	31	22.3	0.5	22.3	0.5
att48	48	141559	27	12.4	0.5	16.1	0.6	13	25.1	0.7	33.1	1	8	47.1	0.8	65	1
B-n50-k7	50	2512	42	4.6	0.4	6	0.4	14	12.8	0.7	17.9	1	14	27.1	0.8	41.6	0.8
D50	50	6211	32	9.3	0.4	10.1	0.6	24	18.1	0.6	21.9	0.9	14	33.5	0.9	43.6	1.1
eil51	51	1529	43	4.5	0.5	4.5	0.5	29	10.6	0.5	10.6	0.7	16	21.1	0.8	28.5	1.1
berlin52	52	25425	63	3.9	0.2	3.9	0.1	42	8.3	0.4	8.3	0.5	25	17.7	0.7	17.7	0.8
A-n60-k9	60	3018	15	9.9	0.5	10.7	0.6	15	18.2	0.9	21.3	1	15	30.1	1	46.6	1.4
B-n68-k9	68	3394	43	8.9	0.7	8.9	0.8	29	12.4	1.1	26.5	1.4	25	25.2	1.4	63.6	1.4
F-n72-k4	72	1142	39	9.3	0.9	9.3	1.3	28	20.7	1.2	20.8	1.6	15	40.4	1.6	42.4	2.1
rus75	75	3465	61	3.7	0.4	3.7	0.4	33	9.7	1	9.7	1.2	13	20	1.6	20	1.8
eil76	76	2353	39	4.8	1	4.8	1.2	22	10.5	1.5	12.2	1.9	14	22.7	1.7	29.6	2.1
A-n80-k10	80	3822	46	5.1	1	5.1	1.1	30	9.6	1.8	14.1	1.7	18	22.5	1.8	36.4	2.1
rd100	100	53338.2	46	4.5	1.4	4.5	1.4	27	10.8	2	11.3	2.1	19	22.6	2.6	29.3	3.5
E-n101-k14	101	3258	42	3.6	1.8	3.6	2.1	25	9.6	2.9	9.7	3.3	14	22.8	2.6	26.6	6.1
10G2	101	5084	41	4.7	1.9	4.7	1.8	41	10.2	3.6	11.2	4.3	13	24.1	5.2	31.7	5.4
F-n135-k7	135	5431.1	76	3.4	0.7	3.4	0.5	65	6.2	1.5	6.2	1.7	46	11.7	2.9	11.7	2.4
Ch150	150	51491.7	51	3.2	2.6	3.2	2.3	28	7.2	5.6	8.1	5.9	13	18.5	5.7	23.2	7.9
d198	198	158761.3	60	1.7	2.7	1.7	2.4	44	6.1	4.6	7.1	4.6	35	15.4	5.3	20.4	5.4
gr229	229	13846.8	17	10	17.3	10	22.3	8	19.4	18.2	20	76.2	4	35.9	20.3	40.9	26.1
a280	280	31918	67	1.9	17.4	1.9	9.6	48	5.4	42.2	5.4	28.4	31	12.5	65.1	15.2	1017.1
lin318	318	557400	33	4.9	72.9	4.9	77.8	24	9.6	403.2	14.4	268.7	13	20.9	51.8	37.4	107.7
fl417	417	513378.2	5	3.3	1210.4	10.5	1263.9	3	9.7	2000	24.5	2000.1	2	22.5	2000.1	56.6	2000.1

Point elimination: The percentages of eliminated points for 1M variant differ from those in 1B variant which shows how effectiveness of pre-processing is dependent on source location. For example, when θ = 30 and β = 0.1, the elimination rates for B-n31-k5 and F-n45-k4 are 48% and 78% in 1M variants compared to 90% and 2% in their 1B variants (see Table 3.6).

Objective function: Interdiction rates, $\Delta Z\%$'s, go higher as θ and the barrier length increases. At each θ level, higher β rates provide more chance of interdiction by increasing disruption capacity. For example, for $\beta = 0.1$ in D40, the interdiction rate increases as 13.9%, 27.3% and 50.5% for $\theta = 30, 45$ and 60 respectively. In addition, when $\theta = 30$, the interdiction rates rise from 13.9% to 21.8% when β increases from 0.1 to 0.25. For fl417 the optimal solution cannot be reached within the time limit of 2000 seconds for angles of 45 and 60 degrees.

In most instances, CPU time increases proportionately to θ. For example in berlin52, with $\beta = 0.1$, the required processing time is 0.2, 0.4 and 0.7 for $\theta = 30$, 45 and 60. But instances gr229 and lin318 is proved to become more time-consuming with θ=45 and β=0.25 than with θ=60 and β=0.25. Within a particular angle, CPU time usually increases as β changes from 0.1 to 0.25. For example for E-n101-k14 when θ=30, CPU time increases from 1.8 to 2.1 for β=0.1 and 0.25 respectively. The same pattern is observed for other angles. However, instances like ch150 and a280 stand as exceptions for this trend. For a280 and θ=30, CPU time drops from 17.4 to 9.6 when β is changed from 0.1 to 0.25.

Table 3.9 shows the average interdiction rates for all instances along with CPU time for small and medium instances. On average, the interdiction rate increases the angle gets wider and disruption rate increases. In small instances, CPU time slightly increases with angle and disruption rate. In medium instances, however, β=0.25 and θ=45 is, on average, more time-consuming.

Table 3.9: Summary of results for 1M variants

θ	**30**		**45**		**60**	
β	**0.1**	**0.25**	**0.1**	**0.25**	**0.1**	**0.25**
$\Delta Z\%$	5.77	7.41	12.18	16.88	24.63	37
CPU (Small)	0.4	0.475	0.625	0.79	0.815	0.965
CPU (Medium)	132.91	138.41	248.38	239.53	216.16	318.17

C. WB Variants

Table 3.10 shows the results for WB variants of instances where sink points have different weights and the source is located at the border.

Point elimination: Since the number of eliminated points is the same as in 1B variants, the percentage of eliminated weights is reported in E%.

Objective function: The highest interdiction rate ($\Delta Z\%$) is again obtained when the widest angle (θ = 60) along with highest disruption rate (β = 0.25) are chosen. For a particular β level, higher θ levels yield better objectives while for a specific angle, higher β values are more desired. For example, in eil51 with $\beta = 0.1$, the interdiction rate increases from 8.6% to 16.5% and 30.2% as θ grows from 30 to 45 and 60 degrees. Within $\theta = 30$ for the same instance, interdiction rates of 8.6% and 16.8% are obtained for $\beta = 0.1$ and $\beta = 0.25$, respectively.

CPU time: The weight of sink points seem to affect CPU time in various patterns. In B-n68-k9 CPU time increases with higher θ and β values. In A-n60-k9, with β=0.1 CPU time increases with higher θ values whereas with β=0.25 it decreases as wider θ angles are chosen. gr229 is solved very fast with highest and lowest θ and β values. The same instance cannot be solved in 1000 seconds if combination of θ=30 and β=0.25 or combination of θ=45 and β=0.1 are chosen.

CPU time in WB variants are different from those of 1B variants. For example, when $\theta = 30$ and $\beta = 0.1$, the WB variant of lin318 is solved in 434.4 seconds whereas its 1B variant cannot be solved

Table 3.10: Computational results for the one-to-many problems with $\beta < 1$ for WB variants

		θ	**30**					**45**					**60**				
		β	**E%**	**0.1**		**0.25**		**E%**	**0.1**		**0.25**		**E%**	**0.1**		**0.25**	
Instances	**n**	Z_0		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**		**ΔZ%**	**CPU**	**ΔZ%**	**CPU**
D8-Canbolat	8	336	24	0.6	0	15.7	0	24	0.6	0	28.3	0	24	0.6	0	50.1	0.1
E-n22-k4	22	6418	21	9.2	0.3	0	0.4	3	16.6	0.4	45.7	0.4	3	29.6	0.4	85.1	0.4
D28	28	59705	6	5.5	0.4	13.6	0.6	0	11.7	0.4	33.9	0.7	0	23.7	0.5	69.2	0.6
B-n31-k5	31	8595.5	93	1.2	0	1.2	0	26	2.5	0.5	5	0.6	21	8.6	0.4	18.4	0.6
A-n32-k5	32	15836	32	6.3	0.4	11.2	0.5	17	13.7	0.5	27.3	0.6	17	26.6	0.5	59	0.6
D40	40	14076.5	36	14.6	0.9	30.8	0.9	23	28	0.9	59.6	1	15	51.3	0.8	109.5	1.1
B-n41-k6	41	4324	15	6.1	0.6	18.5	0.6	9	14	0.6	35.7	1	9	27.7	0.6	69.2	0.9
A-n45-k6	45	18122.5	21	6.8	0.7	7.6	1	21	16.4	0.9	22.8	1.1	12	33.1	0.8	52.3	0.9
F-n45-k4	45	26113.5	3	6.1	1.2	12.9	2.4	3	13.4	1.3	29.3	2.2	3	26.4	2	59.2	1.8
att48	48	937901	27	10.9	1.1	22.9	0.9	25	21.1	0.9	47.3	1	17	38.8	1	89.5	0.8
B-n50-k7	50	19725.5	28	8.7	0.7	14.4	1.3	17	17.7	1.2	31.8	1.2	12	33.3	1.1	62.6	1.2
D50	50	14352	7	14.8	1	26.8	2.1	7	26.6	1.5	51.8	1.6	7	47.1	1.5	95.2	2.8
eil51	51	12302	14	8.6	1	16.8	1.4	9	16.5	1.6	38.2	1.6	2	30.2	1.9	75.5	1.8
berlin52	52	262069.5	29	5.9	1.4	12.5	1.6	15	13.6	1	21.4	1.9	10	26.9	1.5	47	1.6
A-n60-k9	60	26130	4	8.1	1.3	16.7	2.2	4	16.7	1.4	38.4	1.8	4	31.4	1.7	76.1	2
B-n68-k9	68	27920	24	10.6	1.2	22.7	1.5	19	19.8	1.3	43.6	1.5	14	35.7	1.6	79.7	2.2
F-n72-k4	72	13459	3	11.4	3.3	22.2	5.3	1	21.1	3	43.6	4.7	1	38	3.8	81.4	6
rus75	75	31343.5	27	6.4	2.2	9.4	3.1	18	12.9	3	19.2	4	12	26.7	2.9	48.2	3.3
eil76	76	19808	12	9.3	2	15.6	2.9	8	19.6	2	36	3.3	6	37.7	2.5	72.1	3.3
A-n80-k10	80	30282	22	7.4	3.4	13.9	4.9	12	17.1	3	33.7	4	5	35.4	3.9	69.2	5.3
rd100	100	513349	18	7.7	4.6	19.8	4.9	12	16.8	4.2	39.3	5.5	9	32.6	4.6	76.2	4.6
E-n101-k14	101	26667	13	8.2	6.2	13.5	8	5	16.8	4.1	32.3	10.2	3	33	7.1	67.4	6.8
10G2	101	42515	23	8.7	9.2	16.7	14.4	11	19.5	8.8	38.9	5.5	9	38.2	5.8	77.3	11
F-n135-k7	135	77720.3	69	1.3	1.2	1.3	0.9	13	5.6	7	6.2	10.8	2	13.9	11.3	22.4	11.1
Ch150	150	447334.2	20	7	18.5	12.5	24.6	12	15.4	16	31.8	15.9	7	31	11.1	66.8	11
d198	198	2353040.3	15	9	10.4	20.8	11.3	0	16.8	20.6	38.3	38.8	0	30.3	23.7	68.8	46.2
gr229	229	356116.7	1	10.3	41.5	25.9	1000	1	19.5	1000	48.2	64.9	1	35.4	1000	87	42.3
a280	280	229495	43	6.6	184.6	8.9	65.3	23	12	58.4	23	75.1	12	26.3	19.8	52.6	76.8
lin318	318	4670951	20	8.4	434.4	19.6	128.9	13	19	108.9	43.1	245	7	37.3	67.2	83.9	57.8
fl417	417	3183500.3	39	9.2	1000.1	23.5	1000	39	21	1000	52.8	1000	39	41.4	1000	103.5	1000

optimally in 1000 seconds (see). However, with $\beta = 0.25$ the same problems are solved in 128.9 and 84.6 seconds respectively.

Table 3.11 shows the average interdiction rates for all instances along with CPU time for small and medium instances. On average, the interdiction rate increases the angle gets wider and disruption rate increases. In small instances, CPU time slightly increases with angle and disruption rate. In medium instances, β=0.1, on average, has increased the CPU time except when θ=30.

Table 3.11: Summary of results for WB variants

θ	**30**		**45**		**60**	
β	**0.1**	**0.25**	**0.1**	**0.25**	**0.1**	**0.25**
ΔZ%	7.83	15.60	16.07	34.88	30.94	69
CPU (Small)	1.155	1.68	1.27	1.71	1.47	1.865
CPU (Medium)	171.07	225.83	222.8	147.17	215.06	126.76

D. WM Variants

Table 3.12 shows the results for WM variants of instances where sink points have different weights and the source is located in the middle. Partitioning the plane is applied before solving the instances.

Point elimination: Since the number of eliminated points is the same as 1M variants, the percentage of eliminated weights is reported in E%.

Objective function: As expected, the interdiction rates grow if θ or β increases. For example, in E-n22-

Table 3.12: Computational results for the one-to-many problems with $\beta < 1$ for WM variants

		θ	30					45					60				
		β	E%	0.1		0.25		E%	0.1		0.25		E%	0.1		0.25	
Instances	**n**	Z_0		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**		$\Delta Z\%$	**CPU**	$\Delta Z\%$	**CPU**
D8-Canbolat	8	187	57	2.1	0	14.7	0	57	7.5	0	33.7	0	22	16.9	0	74.9	0
E-n22-k4	22	3854	19	11.6	0.2	15	0.3	13	22.7	0.2	36.7	0.3	9	42	0.2	74.2	0.7
D28	28	27764	49	9.6	0.1	12.5	0.1	43	19.3	0.1	26	0.2	19	36.2	0.2	52.2	0.3
B-n31-k5	31	3739	53	7.7	0.2	12	0.2	48	15.3	0.2	27.3	0.2	29	28.5	0.2	54	0.2
A-n32-k5	32	12280	56	3.3	0.1	4.5	0	29	6.9	0.4	10	0.3	19	15.7	0.5	24.9	0.4
D40	40	11731	15	13.7	0.4	23.5	0.5	7	25.2	0.4	46	0.7	7	45.1	0.4	85	0.6
B-n41-k6	41	2582	41	3	0.3	8.5	0.2	31	9.3	0.3	17.3	0.3	31	22.6	0.3	31.6	0.3
A-n45-k6	45	12959	34	2.8	0.3	2.8	0.3	34	6.3	0.5	6.3	0.6	19	16.1	0.7	17.5	0.9
F-n45-k4	45	9843.5	56	4.4	0	4.4	0	56	10.5	0.1	10.5	0.1	38	21.1	0.4	21.1	0.4
att48	48	776397	33	9	0.4	15	0.6	15	18.4	0.7	31	1	6	34.8	0.8	63.2	1.2
B-n50-k7	50	13877	52	4.3	0.4	4.7	0.4	19	11.7	0.8	15.6	1	19	24.7	0.6	35.4	0.9
D50	50	9392	26	10.8	0.3	18.2	0.6	19	18.9	0.6	37.7	0.7	15	32.9	0.7	71.7	0.9
eil51	51	7288	48	5.4	0.4	6.9	0.4	34	11.3	0.6	14.9	0.6	16	24.7	0.8	33.7	1
berlin52	52	159100	67	2.7	0.2	2.8	0.2	48	5.1	0.4	6.6	0.3	23	11.4	0.7	14.1	0.8
A-n60-k9	60	16756	12	6.8	0.5	8.1	0.5	12	13.8	1	16.5	0.9	12	25.3	1.2	36.2	1.4
B-n68-k9	68	17566	46	8	0.8	8	0.6	35	15.3	1.3	24.2	1.4	29	31.9	1.3	61.4	1.3
F-n72-k4	72	5600	46	6.6	1.2	8.5	1.4	35	14.3	1.1	20.6	2	23	28.8	1.6	41.9	1.7
rus75	75	21814	63	3.9	0.4	3.9	0.4	38	9.6	0.8	9.6	0.9	10	19.6	1.5	19.7	1.5
eil76	76	11886	39	6	1	6	1.1	26	12.3	1.4	15	1.7	16	25.7	1.8	36.3	2.1
A-n80-k10	80	20073	43	3.7	1	3.7	1.1	35	9.1	1.8	11.7	1.8	19	22.7	1.9	32.2	2
rd100	100	321632.4	42	4.2	1.4	4.2	1.4	27	9.5	2.1	10.4	2.7	14	20.5	2.8	27.5	3.3
E-n101-k14	101	17024	46	4.2	1.6	4.3	1.4	27	10.7	2.7	12.3	3.4	20	21	4.1	31.1	5.1
10G2	101	28011	45	6.1	1.4	6.1	1.6	35	12.3	3.2	18.1	3	22	28.2	3.4	46.6	3.5
F-n135-k7	135	29597.7	73	4.6	0.7	4.6	0.5	63	8.5	0.6	8.5	0.9	49	15.4	2.3	15.4	1.8
Ch150	150	284698.2	52	3	2.5	3	2.6	32	7.6	7	8.6	4.1	18	18.8	5.6	25.3	7.5
d198	198	897530.6	57	2.2	2.5	2.2	1.8	41	6.7	3.9	7.1	4.1	33	16	4.3	21.3	4.8
gr229	229	77023.9	16	10.6	14.4	10.6	23.4	9	20.7	34.5	20.7	26.2	4	38.3	34.7	40.5	45.7
a280	280	169022	66	2.4	5.4	2.4	5.7	48	5.9	24.5	6.5	15	31	12.5	49.7	16.4	162.2
lin318	318	3085675	33	5.5	81.8	6	56.2	22	10.5	89	16.7	256.2	16	23.1	51.2	41.3	43
fl417	417	2899108.8	6	5.9	1053.3	13.5	1942	4	12.4	2000	31.3	2000.2	3	27.1	2000	67.7	2000

k4, $\Delta Z\%$ almost doubles moving from one θ level to another. The same pattern is observed for D40 or eil51. β values rarely affect the interdiction rates in F-n45-k4, rus75 and F-n135-k7.

CPU time: In most instances, CPU time increases as with longer barriers (higher θ values). One of the rare exceptions is lin318 for which lower CPU times are obtained at θ=60 compared to those of θ=45. When these results are compared with those of 1M variants (Table 3.8), we can observe that CPU time has improved or worsened depending on the instance. For example, in gr229-1M with θ=60 and β=0.25 the MIP model is solved in 26.1 seconds whereas its WM variant takes 45.7 seconds to be solved. However, under the same setting, 1M variant of lin318 is solved in 107.7 seconds compared to just 43 seconds obtained for WM variant.

Table 3.13 shows the average interdiction rates for all instances along with CPU time for small and medium instances. On average, the interdiction rate increases, the angle gets wider, and disruption rate increases. In both small and medium instances, CPU time increases with angle and disruption rate. On average, CPU times obtained for WM variants are slightly lower than those of 1M variants except for θ=30 and β=0.1 (see Table 3.9).

Table 3.13: Summary of results for WM variants

	30		**45**		**60**	
	0.1	**0.25**	**0.1**	**0.25**	**0.1**	**0.25**
$\Delta Z\%$	5.80	8.02	12.25	18.58	24.92	40
CPU (Small)	0.41	0.445	0.635	0.75	0.79	0.93
CPU (Medium)	116.5	203.66	216.75	231.58	215.81	227.69

3.4 Computational Results for the One-to-Many Problems with $\beta = 1$ Using Algorithm

Recall that when there is no constraint on the total weight of disrupted points ($\beta = 1$), the optimal solution can be found using the algorithm explained in Section 3.4. In addition, it is possible to solve MIP models of instances with β=1 and compare the results with those of the algorithm. Not only does this cross-checking ensure the validity of MIP model and algorithm but also provides a fair ground to compare their performance in terms of required processing time.

The application for the algorithm is programmed using visual basic .NET (Microsoft, 2012).

A. 1B Variants

Table 3.14 shows the computational results for the algorithm applied on 1B variants of instances. E% is the percentage of eliminated points based on given θ parameters. $\Delta Z\%$ shows the interdiction rate due to the barrier as a percentage of increase in the original objective function:

$$\Delta Z\% = \frac{(\Delta Z = Z - Z_0)}{Z_0} \times 100$$

where Z_0 and Z are the total weighted distance between source and sink points before and after locating the line barrier.

Table 3.14: Computational results for one-to-many problems with $\beta = 1$ for 1B variants

		θ	30				45				60			
Instances	**n**	Z_0	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	65	25	23.5	0	0.1	25	58.5	0	0	25	121.5	0	0
E-n22-k4	22	1182	18	33.8	0	0.4	5	73.9	0	0.4	5	143.4	0	0.5
D28	28	10403	7	20.3	0.02	0.8	0	57.8	0	0.5	0	123.7	0	0.5
B-n31-k5	31	1694.5	90	2.9	0	0	23	6.5	0.02	0.6	19	28.3	0.02	0.7
A-n32-k5	32	2315	31	11.5	0	0.4	19	31.5	0	0.6	19	72.8	0.02	0.6
D40	40	11645	18	48.1	0.02	0.8	10	95.5	0	0.7	5	177.5	0	0.8
B-n41-k6	41	3362	20	11.6	0.02	1.1	12	38.1	0	1.6	12	93.9	0	1.3
A-n45-k6	45	3373.5	22	11.6	0	0.9	22	32.4	0.02	1.1	16	73.4	0.02	1.5
F-n45-k4	45	5879	2	29.6	0.02	2.4	2	72.3	0.02	1.6	2	150.7	0.02	1.7
att48	48	174125	29	23.9	0.02	1	27	50.9	0.02	1.1	19	97.7	0.02	1.4
B-n50-k7	50	3508	26	17.6	0.02	1.4	18	39.1	0.02	1.4	14	93.2	0.02	1.8
D50	50	12397	2	30.4	0.02	1.9	2	65.5	0.02	2.1	2	127.3	0.02	2
eil51	51	2489	14	16.4	0.02	2.5	12	42.8	0.02	1.6	2	93.4	0.02	2
berlin52	52	46522	25	18.6	0.02	1.5	13	47.6	0.02	1.5	10	103.7	0.03	1.8
A-n60-k9	60	4584	5	16.6	0.03	2.9	5	49.2	0.03	2.2	5	107.3	0.03	2.3
B-n68-k9	68	4898	29	25.7	0.03	1.8	24	62.6	0.05	1.8	15	128.5	0.05	1.9
F-n72-k4	72	2600	6	34.2	0.06	9.3	3	76.2	0.11	6.2	1	148.8	0.06	6.3
rus75	75	5083.5	25	10.5	0.05	3.2	17	31.9	0.05	4.6	9	80.6	0.06	2.6
eil76	76	3703	17	15.4	0.06	5.3	11	41.3	0.06	4.8	7	91.7	0.08	5.3
A-n80-k10	80	5530	26	14.5	0.05	4.7	15	36.3	0.06	4.3	6	77.5	0.08	4.8
rd100	100	83112	19	19.5	0.12	4	13	50.1	0.14	6.6	10	105.2	0.17	4.3
E-n101-k14	101	5029.5	16	14.1	0.12	14	6	38.8	0.14	11.6	4	83.7	0.16	8.8
10G2	101	7469	26	15.2	0.12	14.9	14	38.2	0.14	14.4	10	89.6	0.14	39.4
F-n135-k7	135	13761.6	72	1.5	0.09	1.3	13	5.9	0.3	14.8	3	36.8	0.36	15.6
Ch150	150	80157.7	21	11.6	0.42	35.4	13	37.3	0.48	11.3	7	87.3	0.5	12.7
d198	198	411628.6	17	27.7	1.08	109.3	1	63.8	1.23	33.5	1	128.5	1.28	1000
gr229	229	64192.4	1	65.9	1.89	314.9	1	123.7	1.89	34.1	1	223.9	1.91	99.8
a280	280	43422	44	7.6	1.78	114.3	24	21	2.44	137.6	12	50.9	2.91	1000
lin318	318	855394	18	21.6	3.92	93.6	12	53.3	4.39	508.2	7	108.5	4.73	1000
fl417	417	586807.7	37	36.7	6.33	193.1	37	83.4	6.52	1000	37	164.4	6.62	1000

The Z_0 values are given in the table. The obtained objective values for all instances via algorithm and MIP model are observed to be identical which ensures that both of these approaches work correctly. For the sake of brevity, the corresponding $\Delta Z\%$ values are shown only once.

In order to compare the performance of the algorithm and MIP models two CPU times are given in tables: CPU1 is the processing time for the algorithm and CPU2 is the required time when the same problem is solved with MIP model.

Objective function: Since there is no disruption constraint for these problems, the barrier length is only restricted by θ values. Higher θ values allow longer barriers and eliminate fewer points in pre-processing, leading to higher interdiction. For example, in D28, the interdiction rate with θ=30 is just 20.3% which can be increased up to 123.7% if θ=60 is chosen. The highest interdiction rate obtained for the same instance with the same angle is 70.8% at β=0.25 (see Table 3.6), which means that disruption constraint has had a significant result on this instance. However, the same disruption constraint does not show the same effect on A-n80-k10 and a280.

As the results suggest, the algorithm solves the problems several times faster than the MIP model. Even a large instance like fl417 can be solved in a few seconds by the algorithm whereas MIP model struggles to find the optimal solution with higher θ angles. Figure 3.2 shows how CPU time grows when the MIP model solved with θ=30 for large instances while the algorithm conveniently handles all instances.

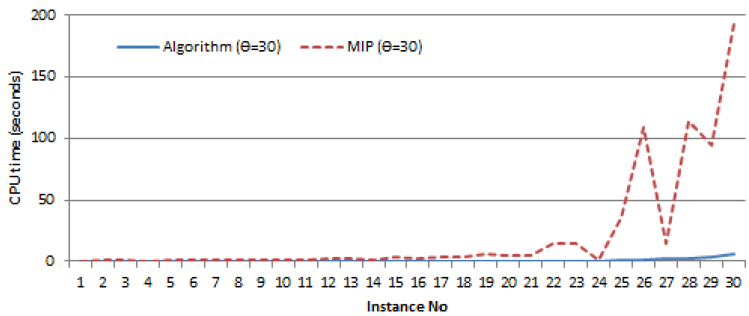


Figure 3.2: CPU time obtained for 30 instances with θ = 30 and β = 1

Table 3.15 shows the summary of results for 1B variants of instances with β=1. On average, the interdiction rate doubles by every 15 degrees increase in the angle. The algorithm's CPU time (CPU1) slightly increases as the angles gets wider. However, MIP models seem to be struggling for medium instances when θ=60. There is a significant difference between the performance of the algorithm and MIP models. In small instances (n < 100), the algorithm is more than 75 times faster than MIP models. In medium instances ($n \geq$ 100) with θ=30 the CPU1 is more than 50 times faster than MIP models which can go up to 220 times if θ=60 is chosen.

Table 3.15: Summary of results for 1B variants when β = 1

θ	**30**	**45**	**60**
ΔZ%	21.27	50.85	107.12
CPU1 (Small)	0.023	0.026	0.0275
CPU2 (Small)	2.12	1.935	1.99
CPU1 (Medium)	1.587	1.767	1.878
CPU2 (Medium)	89.48	177.21	418.06

B. 1M Variants

Table 3.16 shows the computational results for 1M variants of instances via algorithm and MIP models. Before solving MIP models, they are decomposed into two subproblems representing the upper and lower half-spaces on the plane with respect to the location of the source point. To solve each of the subproblems 1000 seconds time limit is assigned.

Table 3.16: Computational results for the one-to-many problems with $\beta = 1$ for 1M variants

		θ	**30**				**45**				**60**			
Instances	**n**	Z_0	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	46	50	17.6	0	0	38	32.6	0	0	25	70.7	0	0
E-n22-k4	22	722	18	10.5	0	0.2	14	29.1	0	0.3	9	64.4	0	0.2
D28	28	5435	43	13.6	0	0.1	39	28	0	0.2	21	56.5	0	0.3
B-n31-k5	31	774	48	8	0	0.2	42	20.4	0	0.2	23	42	0	0.3
A-n32-k5	32	1779	53	4.9	0	0.1	28	11.1	0	0.3	19	23.6	0	0.4
D40	40	6058	23	21.8	0	0.5	10	45.8	0.02	0.6	10	87.4	0.02	0.5
B-n41-k6	41	2197	51	6.3	0	0.2	39	13.3	0	0.4	39	34.8	0	0.3
A-n45-k6	45	2453	27	3.9	0	0.3	27	10.4	0	0.6	13	25.9	0.02	0.7
F-n45-k4	45	2157.5	64	4.2	0	0	64	10.3	0.02	0.2	31	22.3	0	0.4
att48	48	141559	27	16.1	0.02	0.6	13	33.1	0.02	0.8	8	65	0.02	1
B-n50-k7	50	2512	42	6	0	0.3	14	17.9	0.02	0.9	14	41.6	0.02	1
D50	50	6211	32	10.1	0	0.6	24	21.9	0.02	0.8	14	43.6	0.02	1
eil51	51	1529	43	4.5	0.02	0.5	29	10.6	0.02	0.8	16	28.5	0.02	0.9
berlin52	52	25425	63	3.9	0	0.1	42	8.3	0	0.5	25	17.7	0	0.8
A-n60-k9	60	3018	15	10.7	0.03	0.5	15	21.3	0.03	1	15	46.6	0.03	1.2
B-n68-k9	68	3394	43	8.9	0.06	0.9	29	27.2	0.05	1.3	25	69.9	0.05	1.1
F-n72-k4	72	1142	39	9.3	0.05	1.2	28	20.8	0.05	1.4	15	42.4	0.05	1.8
rus75	75	3465	61	3.7	0.03	0.4	33	9.7	0.03	1	13	20	0.05	1.9
eil76	76	2353	39	4.8	0.03	1.2	22	12.2	0.05	1.7	14	29.6	0.05	2.4
A-n80-k10	80	3822	46	5.1	0.03	1.1	30	14.1	0.06	1.5	18	36.4	0.06	2.1
rd100	100	53338.2	46	4.5	0.08	1.3	27	11.3	0.11	2.4	19	29.3	0.14	2.6
E-n101-k14	101	3258	42	3.6	0.09	1.5	25	9.7	0.12	3.2	14	26.6	0.12	7.7
10G2	101	5084	41	4.7	0.08	1.6	41	11.2	0.09	4.4	13	31.7	0.12	5
F-n135-k7	135	5431.1	76	3.4	0.11	0.4	65	6.2	0.16	1.5	46	11.7	0.19	2.2
Ch150	150	51491.7	51	3.2	0.25	2.3	28	8.1	0.36	5.6	13	23.2	0.44	8.3
d198	198	158761.3	60	1.7	0.38	2.7	44	7.1	0.52	4.4	35	20.4	0.66	4.3
gr229	229	13846.8	17	10	1.41	23.2	8	20.5	1.56	24.3	4	43.9	1.67	31.8
a280	280	31918	67	1.9	0.84	9.6	48	5.4	1.52	45.6	31	15.2	2.19	188.8
lin318	318	557400	33	4.9	3.05	58.6	24	14.4	3.61	259.5	13	38.7	4.08	153
fl417	417	513378.2	5	10.5	9.81	1141.9	3	35.5	10.38	2000.1	2	83.3	10.61	2000

Objective function: As expected, the interdiction rates ($\Delta Z\%$) increase as the angle θ gets wide. For example, in berlin52, the interdiction rate increases from 3.9% at θ=30 to 17.7% at θ=60 which are, in comparison, much lower than the rates obtained from 1B variant of this instance. The reason for that, is distribution of sink points and location of the source points that leads such huge point elimination in the 1M variant.

CPU time: The algorithm can solve the small instances in almost negligible amount of time. But when it comes to medium instances, CPU1 slightly takes off. However, algorithm outperforms MIP approach in all instances. For example, fl417, which cannot be solved by MIP model in less than 1000 seconds, is solved in about 10 seconds using the algorithm.

Table 3.17 shows the summary results for 1M variants when β=1. The average interdiction rate increases almost by two folds every time the angle is widened by 15 degrees. Algorithm is more than 30 times faster than MIP models in small instances ($n < 100$) and the gap between their CPU times even gets wider with medium instance ($n \geq 100$).

Table 3.17: Summary of results for 1M variants when $\beta = 1$

θ	**30**	**45**	**60**
$\Delta Z\%$	7.41	17.58	39.76
CPU1 (Small)	0.0135	0.0195	0.0205
CPU2 (Small)	0.45	0.725	0.915
CPU1 (Medium)	1.61	1.843	2.022
CPU2 (Medium)	124.31	235.1	240.37

C. WB Variants

Table 3.18 shows the computational results for WB variants using algorithm and MIP approaches.

Table 3.18: Computational results for one-to-many problems with $\beta = 1$ for WB variants

		θ	**30**				**45**				**60**			
Instances	**n**	Z_0	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	336	24	28.7	0	0	24	72	0	0	24	151.3	0	0
E-n22-k4	22	6418	21	30.9	0	0.3	3	72.6	0	0.4	3	144.7	0	0.5
D28	28	59705	6	25	0.02	0.6	0	65.8	0	0.6	0	137.7	0	0.5
B-n31-k5	31	8595.5	93	1.2	0	0	26	6.1	0	0.5	21	28	0	0.6
A-n32-k5	32	15836	32	11.2	0	0.5	17	31.9	0.02	0.6	17	74.5	0	0.5
D40	40	14076.5	36	39.8	0.02	0.8	23	79	0	0.9	15	146.9	0	1
B-n41-k6	41	4324	15	23.8	0.02	0.9	9	51.3	0.02	0.9	9	111.1	0	0.8
A-n45-k6	45	18122.5	21	7.6	0	1	21	25.6	0	1.2	12	68.2	0.02	1.2
F-n45-k4	45	26113.5	3	29.3	0.02	1.7	3	73.7	0.02	1.2	3	154.4	0.02	1.1
att48	48	937901	27	25.9	0.02	0.8	25	54.5	0	0.8	17	103.9	0.02	1.2
B-n50-k7	50	19725.5	28	14.4	0	1.3	17	37.8	0.02	1.4	12	94.1	0.02	1.2
D50	50	14352	7	29.5	0.02	2.3	7	62.4	0.02	2.5	7	120.2	0.03	1.8
eil51	51	12302	14	18.5	0.02	1.7	9	47.5	0.03	2	2	100.8	0.02	1.7
berlin52	52	262069.5	29	16	0.02	1.3	15	40.5	0.02	1.7	10	90.4	0.03	1.8
A-n60-k9	60	26130	4	19.5	0.02	2.1	4	53.1	0.03	2.2	4	112.1	0.03	2
B-n68-k9	68	27920	24	25.4	0.03	2	19	62.8	0.05	1.5	14	130.8	0.03	1.7
F-n72-k4	72	13459	3	33.3	0.06	8.5	1	75.9	0.06	5.6	1	150	0.06	3.1
rus75	75	31343.5	27	10.2	0.05	3.7	18	31.6	0.05	3.7	12	78.9	0.05	2.4
eil76	76	19808	12	17.6	0.06	4.9	8	46.1	0.06	6.5	6	104	0.06	4.4
A-n80-k10	80	30282	22	13.9	0.06	4.1	12	34.4	0.06	4.3	5	77.7	0.08	4.9
rd100	100	513349	18	22.7	0.12	4.4	12	53.8	0.14	6	9	110.3	0.19	5.9
E-n101-k14	101	26667	13	13.5	0.14	14	5	38	0.16	8.6	3	82.8	0.16	12.5
10G2	101	42515	23	16.7	0.12	9.8	11	40.4	0.14	12.5	9	91.9	0.16	14.6
F-n135-k7	135	77720.3	69	1.3	0.11	1	13	6.2	0.28	25.6	2	36.5	0.36	16
Ch150	150	447334.2	20	12.5	0.42	15.6	12	37.1	0.47	14.2	7	86.8	0.55	19.1
d198	198	2353040.3	15	28.5	1.11	176.7	0	65.5	1.23	43.1	0	131.5	1.23	30.4
gr229	229	356116.7	1	66.7	1.89	64.3	1	124.9	1.91	49.3	1	225.8	1.91	47.3
a280	280	229495	43	8.9	1.77	45	23	23	2.47	97.8	12	54	2.89	643.1
lin318	318	4670951	20	22.5	3.92	68.4	13	53.6	4.55	76	7	108.6	4.75	111.1
fl417	417	3183500.3	39	36.4	6.5	1000	39	82	6.52	81.1	39	160.8	6.62	332.2

Objective function: Since there is no disruption constraint in this problem, the interdiction rates are only dependent on barrier length and, therefore, increase as the angle θ increases.

CPU time: The algorithm performs almost at the same levels with 1B variants and outperforms the MIP models. However, MIP models are solved faster in compared to 1B variants in larger instances. For example, d198-1B could not be solved in 1000 seconds whereas it takes only 30.4 seconds to solve d198-WB. These problems are solved in 1.28 and 1.23 seconds with the algorithm.

Table 3.19 shows the summary of results for WB variants. The interdiction rates almost double with every 15 degrees added to the angle θ. The algorithm, on average, hits optimality at least 60 times faster than the MIP model in small instances ($n < 100$). The same ratio is observed for medium instances ($n \geq 100$) except in θ=45 where the algorithm is 23 times faster than MIP model. The reason is that 82% of points are eliminated in f417 when θ=45 leading to a huge decrease in average CPU2.

Table 3.19: Summary of results for WB variants when $\beta = 1$

θ	**30**	**45**	**60**
$\Delta Z\%$	21.71	51.64	108.96
CPU1 (Small)	0.022	0.023	0.0235
CPU2 (Small)	1.925	1.925	1.62
CPU1 (Medium)	1.61	1.787	1.882
CPU2 (Medium)	139.92	41.42	123.22

D. WM Variants

Table 3.20 shows the computational results for WM variants of instances when $\beta = 1$ where almost similar results are observed.

Table 3.20: Computational results for one-to-many problems with $\beta = 1$ for WM variants

		θ	**30**				**45**				**60**			
Instances	**n**	Z_0	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**	**E%**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	187	57	14.7	0	0	57	33.7	0	0	22	74.9	0	0
E-n22-k4	22	3854	19	15	0	0.2	13	36.7	0	0.2	9	74.2	0	0.3
D28	28	27764	49	12.5	0	0.1	43	26	0.02	0.2	19	52.2	0	0.3
B-n31-k5	31	3739	53	12	0	0.2	48	27.3	0.02	0.1	29	54	0	0.2
A-n32-k5	32	12280	56	4.5	0	0.1	29	10	0	0.3	19	24.9	0.02	0.4
D40	40	11731	15	33.7	0.02	0.5	7	70.9	0.03	0.5	7	135.3	0.02	0.5
B-n41-k6	41	2582	41	8.5	0	0.2	31	17.3	0.02	0.3	31	38.5	0	0.3
A-n45-k6	45	12959	34	2.8	0.02	0.3	34	6.3	0	0.5	19	17.5	0	0.8
F-n45-k4	45	9843.5	56	4.4	0	0	56	10.5	0	0.1	38	21.1	0.02	0.4
att48	48	776397	33	15	0.02	0.5	15	33.3	0.02	0.8	6	69.9	0.02	0.8
B-n50-k7	50	13877	52	4.7	0.02	0.4	19	15.6	0.02	0.9	19	35.4	0.02	1
D50	50	9392	26	18.2	0.02	0.5	19	37.7	0.02	0.7	15	71.7	0.02	0.8
eil51	51	7288	48	6.9	0.02	0.4	34	14.9	0.02	0.6	16	33.7	0.02	0.9
berlin52	52	159100	67	2.8	0	0.2	48	6.6	0	0.3	23	14.1	0.02	0.7
A-n60-k9	60	16756	12	8.1	0.03	0.4	12	16.5	0.03	1.2	12	36.2	0.03	1.1
B-n68-k9	68	17566	46	8	0.03	0.6	35	24.2	0.03	1.4	29	61.4	0.05	1
F-n72-k4	72	5600	46	8.5	0.03	1.2	35	20.6	0.03	1.3	23	41.9	0.05	1.4
rus75	75	21814	63	3.9	0.03	0.4	38	9.6	0.05	0.9	10	19.7	0.05	1.7
eil76	76	11886	39	6	0.05	1	26	15	0.05	1.6	16	36.3	0.05	2
A-n80-k10	80	20073	43	3.7	0.05	1.1	35	11.7	0.05	1.4	19	32.2	0.06	1.9
rd100	100	321632.4	42	4.2	0.08	1.3	27	10.4	0.11	3.3	14	27.5	0.12	2.8
E-n101-k14	101	17024	46	4.3	0.09	1.4	27	12.3	0.16	3.2	20	31.1	0.14	4.8
10G2	101	28011	45	6.1	0.09	1.5	35	18.1	0.11	3.8	22	46.6	0.12	3.4
F-n135-k7	135	29597.7	73	4.6	0.11	0.5	63	8.5	0.14	1.3	49	15.4	0.19	2.4
Ch150	150	284698.2	52	2.8	0.25	2.4	32	8.1	0.39	4	18	23.5	0.44	5.1
d198	198	897530.6	57	2.2	0.38	3.5	41	7.1	0.52	3.7	33	21.3	0.67	5
gr229	229	77023.9	16	10.6	1.36	24.8	9	20.7	1.56	20.1	4	43.4	1.67	20.7
a280	280	169022	66	2.4	0.83	5.7	48	6.5	1.53	26.7	31	16.4	2.2	124.1
lin318	318	3085675	33	6	3.06	57.6	22	16.7	3.52	365.1	16	42.7	4.02	67.8
fl417	417	2899108.8	6	14.8	9.78	1246.7	4	46.9	10.34	2000.1	3	103.9	10.58	2000

Objective function: Since there is no disruption constraint in this problem, the interdiction rates are only dependent on barrier length and, therefore, increase as the angle θ increases.

CPU time: Since algorithm is not dependent on the weight of sink points, it must presumably require almost the same amount of time for WM variants as it does for 1M variants. For example, a280-1M is solved in 2.19 seconds while a280-WM needed almost the same amount of time (2.2 seconds) to find the optimal solution.

Table 3.21 shows the summary of results for WM variants. The interdiction rates almost double with every 15 degrees added to the angle θ. The algorithm, on average, hits optimality at least 25 times faster than the MIP model in small instances ($n < 100$). This ratio even increases to at least 80 times with medium instances.

Table 3.21: Summary of results for WM variants when $\beta = 1$

θ	**30**	**45**	**60**
$\Delta Z\%$	8.40	19.99	43.90
CPU1 (Small)	0.017	0.0205	0.0225
CPU2 (Small)	0.415	0.665	0.825
CPU1 (Medium)	1.603	1.838	2.015
CPU2 (Medium)	134.54	243.13	223.61

3.5 Post-optimization Analysis on Maximum Possible Disruption Rate

Suppose that D is the set of all sink points and Q is the set of invisible sink points due to the barrier. Then, the actual interdiction rate, $\hat{\beta}$, is obtained as following:

$$\hat{\beta} = \frac{\sum_{i \in Q} w_i}{\sum_{i \in D} w_i}$$

Based on $\hat{\beta}$ values for each θ, we discover how effective the parameter β has been designed for different variants of instances. Table 3.22 shows actual interdiction rates, $\hat{\beta}$, for 10 instances after optimally locating the line barrier using β=1.

Table 3.22: Maximum interdiction rate ($\hat{\beta}$) obtained for 10 instances

Variant	**1B**			**1M**			**WB**			**WM**		
θ	**30**	**45**	**60**	**30**	**45**	**60**	**30**	**45**	**60**	**30**	**45**	**60**
E-n22-k4	0.5	0.5	0.5	0.09	0.27	0.27	0.55	0.55	0.55	0.2	0.23	0.23
B-n31-k5	0.03	0.03	0.52	0.19	0.19	0.19	0.01	0.34	0.52	0.21	0.21	0.21
D40	0.65	0.65	0.65	0.2	0.2	0.2	0.43	0.43	0.43	0.39	0.39	0.39
F-n45-k4	0.62	0.78	0.91	0.07	0.07	0.09	0.7	0.76	0.9	0.03	0.03	0.03
B-n50-k7	0.24	0.34	0.48	0.14	0.22	0.22	0.2	0.48	0.48	0.11	0.18	0.18
berlin52	0.48	0.58	0.62	0.04	0.08	0.1	0.41	0.5	0.54	0.1	0.1	0.12
F-n72-k4	0.6	0.6	0.6	0.1	0.18	0.18	0.6	0.64	0.64	0.19	0.19	0.24
F-n135-k7	0.03	0.21	0.73	0.03	0.03	0.04	0.03	0.22	0.74	0.04	0.04	0.05
d198	0.66	0.69	0.73	0.04	0.16	0.17	0.68	0.7	0.74	0.05	0.16	0.18
fl417	0.45	0.45	0.45	0.19	0.37	0.37	0.43	0.43	0.43	0.29	0.39	0.39

For example, in E-n22-k4-1B, choosing a β rate higher than 0.5 for disruption constraint does not make any difference in the final solution because at most 50% of points can be disrupted. Interestingly, if θ=30 is chosen for F-n135-k7-1B, at most 3% of points can be disrupted, thus, setting any β higher than 0.03 would result in the same solution.

The effectiveness of β and disruption constraint is highly dependent on distribution of sink points and location of source point on the plane. For example, B-n31-k5 is a clustered instance that sink points are scattered at two sides of the border source. Therefore, most of sink points are eliminated when θ=30 or 45 are chosen. θ=60, however, provides wide enough angle that can include many more sink points for disruption. In 1M variant of this instance, $\hat{\beta}$ goes up to 0.19. In a vertical instance like F-n72-k4-1B an interdiction rate up to 0.6 is effective whereas in its 1M variant only β=0.04 can make a difference.

CHAPTER 4

THE MANY-TO-MANY RECTILINEAR INTERDICTION WITH A LINE BARRIER ON A PLANE

4.1 Problem Preliminaries and Formulation

We assume a set of points on the plane that are considered as sources and sinks where the flow is from the source towards the sink in rectilinear distance. In this chapter, we assume all points on the plane have functions of both source and sink, i.e. every point communicates with every other point on the plane. This problem, hereinafter called many-to-many problem, might be seen as a complete graph in which each node on a graph with n nodes is directly "connected" to the $n-1$ nodes if it would be considered under network terminology. Figure 4.1 illustrates an example setting of many-to-many problem in which all points are connected to each other in rectilinear metric. Note that the illustrated paths are just one of many alternative rectilinear paths between points.

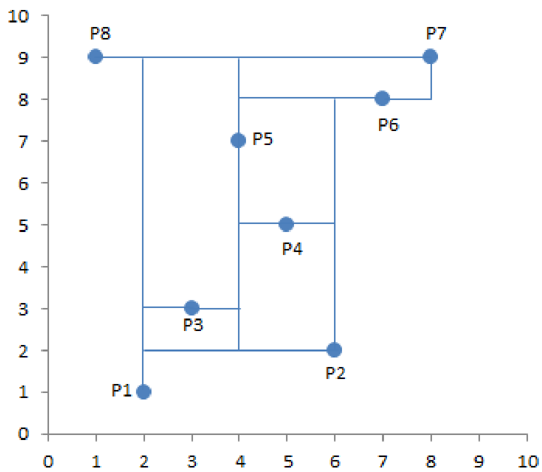


Figure 4.1: Eight source and sink points are communicating with each other in rectilinear distance

The accessibility of points to each other is not desired and, therefore, our objective is to locate a horizontal line barrier such that it maximizes the all-pair weighted rectilinear distances between points. Trespassing through the barrier is assumed to be impossible. A real life example of such problem is locating a wall in a city such that the flow on rectilinear streets are disrupted by the wall. The barrier has a finite length and the total weight of disrupted pairs can also be restricted. Based on these features, following problems are introduced and solved in this chapter:

1. The many-to-many interdiction problem with a line barrier on a plane subject to a disruption constraint. In this problem, the total weight of disrupted points should not exceed a certain percentage of the total weights.
2. The many-to-many interdiction problem with a line barrier on a plane. This problem is special case of the above problem with no disruption constraint.

Note that this problem looks similar to the one-to-many problem although the objective function is totally different, which increases the complexity of the problem. The objective function in the many-to-many problem is the total weighted rectilinear distance between all pairs of points whereas in one-to-many problem the rectilinear distance between sink points and a single source point is a matter of concern.

To define the length of barrier in the many-to-many problem, an obnoxious point is introduced such that the closer the barrier gets to this obnoxious point, the shorter length is allowed for the barrier. The obnoxious point can be seen as a difficult place to reach, a dangerous place to go or a critical place to preserve. For example, building a barrier in a mountainous area becomes more difficult as altitude increases towards the summit. A historical or strategic place that is important for us can also function as the obnoxious point. In the one-to-many problem, the source point can be seen the obnoxious point.

The required preliminaries and assumptions are briefly explained next.

4.1.1 Preliminaries and Assumptions

Distance norm: All distances are computed in rectilinear (or Manhattan) metric.

Barrier Type: The barrier is a horizontal line segment with negligible width that can be located anywhere on the plane such that the total weighted rectilinear distance between source and sink points is maximized. The problem with a vertical line segment can be converted into a problem with horizontal line segment by simply rotating the points on the plane as much as 90 degrees and the rectilinear distances between points remain the same.

Disruption: There is symmetry in disruption of points. That is, if the distance from point A to point B is disrupted by the barrier, the distance from B to A is also disrupted by the same amount. Since both points are source and sink at the same time, sum of their weights will be used in disruption constraint.

Barrier length: Line barrier has a variable length to be determined by its vertical distance from an obnoxious point. The closer the barrier is to the obnoxious point, the shorter length it is allowed to have.

Barrier-point intersection: If a barrier falls on a point, the barrier will be shifted along the y-axis towards the obnoxious point with a negligible amount $\epsilon, \epsilon > 0$.

Distance function: Consider n points on the plane. The rectilinear distance between a sink point P_i and a source point P_j is:

$$l(P_i, P_j) = |a_i - a_j| + |b_i - b_j| \tag{4.1}$$

Since the distance metric is rectilinear, there are several alternative paths between P_i and P_j with equal distances. Figure 4.2 shows two alternative paths between P_3 and P_8 with dash pattern. Note that the word ”path” used in this context simply means ”way” on the plane and it does not intend the meaning used in network terminology in which a ”path” goes through a set of nodes and arcs.

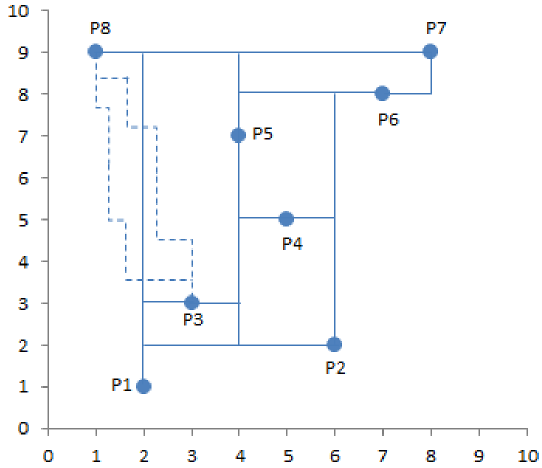


Figure 4.2: Two alternative paths between P_3 and P_8

In the many-to-many problems, the total weighted distance between all source and sink points is represented by following expression:

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i l(P_i, P_j) = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i|a_i - a_j| + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i|b_i - b_j| \tag{4.2}$$

When there is a line barrier on the plane, it may disrupt some $l(P_i, P_j)$ by blocking all alternative paths between P_i and P_j. In that case, the shortest path that does not go through the barrier will give the shortest distance between P_i and P_j. The total distance expression in 4.2 can be written as:

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i l^B(P_i, P_j) \tag{4.3}$$

where $l^B(P_i, P_j)$ refers to the distance corresponding to the shortest path round the barrier. If the barrier is effectively blocking all alternative paths between P_i and P_j, the two points are said to be *invisible* to each other, and *visible* otherwise. If two points are invisible to each other, the rectilinear distance introduced in equation 4.1 is no longer valid and the shortest path that does not intersect with the barrier has to be calculated. For this, invisible points due to the barrier have to be identified first and then the shortest distance between them has be calculated.

4.1.2 Identifying Disrupted points in Presence of a Single Line Barrier Using Visibility Concept

Lemma 2.1, introduced for visibility in Chapter 2, identifies if a pair of points are invisible to each other due to the barrier. Therefore, the conditions to call P_i and P_j invisible are as following:

$$b_i < y < b_j \vee b_j < y < b_i \tag{4.4}$$

and

$$x_s < a_i < x_e \wedge x_s < a_j < x_e \tag{4.5}$$

Throughout this chapter, conditions 4.4 are called *y-invisibility* and conditions 4.5 are called *x-invisibility*. The *x-invisibility* conditions can be written as:

$$x_s < \min\{a_i, a_j\} \wedge \max\{a_i, a_j\} < x_e \tag{4.6}$$

4.1.3 Calculating the Shortest Distance for Disrupted Points in the Presence of a Single Line Barrier

As explained in property 2.1 of Chapter 2, a horizontal line barrier can only increase the distance along x-axis and it has no effect on the distance along the y-axis whatsoever.

When two points are invisible to each other, the shortest path must go through one of the line barrier ends. Therefore, the shortest distance for a pair of invisible points P_i and P_j is:

$$l^B(P_i, P_j) = \min\{|a_i - x_s| + |a_j - x_s|, |x_e - a_i| + |x_e - a_j|\} + |b_i - b_j| \tag{4.7}$$

If P_i and P_j are invisible to each other, we know that their x-coordinates is greater than x_s and less than x_e. Therefore, the absolute terms in expression 4.7 can be written as:

$$l^B(P_i, P_j) = \min\{a_i + a_j - 2x_s, 2x_e - a_i - a_j\} + |b_i - b_j| \tag{4.8}$$

we know that a barrier can only increase the original distance between points. Therefore, the shortest distance can be obtained by appending the additional distance caused by the barrier to the original distance. The additional distance is calculated as:

$$\Delta l^B(P_i, P_j) = \begin{cases} 2\min\{\min(a_i, a_j) - x_s, x_e - \max(a_i, a_j)\} & \text{if } P_i \text{ invisible to } P_j \\ 0 & \text{otherwise} \end{cases} \tag{4.9}$$

The shortest distance for a pair of invisible points P_i and P_j can be written as:

$$l^B(P_i, P_j) = l(P_i, P_j) + \Delta l^B(P_i, P_j) \tag{4.10}$$

Example 4.1 *Figure 4.3 shows how a barrier starting at* (3.4, 6) *and ending at* (7.8, 6) *disrupts the distances* $d_{2,5}, d_{2,6}, d_{4,5}$*and* $d_{4,6}$. *The amount of interdictions (additional distances), shown with diagonal pattern on the figure, is as following:*

$$\begin{aligned} \Delta l^B(P_2, P_5) &= 2\min\{4 - 3.4, 7.8 - 6\} = 1.2 \\ \Delta l^B(P_2, P_6) &= 2\min\{6 - 3.4, 7.8 - 7\} = 1.6 \\ \Delta l^B(P_4, P_5) &= 2\min\{4 - 3.4, 7.8 - 5\} = 1.2 \\ \Delta l^B(P_4, P_6) &= 2\min\{5 - 3.4, 7.8 - 7\} = 1.6 \end{aligned}$$

The interdiction on other points is zero. Suppose that $w_4 = w_5 = 2$ *and the weights of other points are equal to 1. Then, the total amount of interdiction is:*

$$\Delta Z = \sum_{i=1}^{8} \sum_{j=1}^{8} w_i \Delta l^B(P_i, P_j) = (1 + 2)1.2 + (1 + 1) \times 1.6 + (2 + 2)1.2 + (2 + 1) \times 1.6 = 16.4$$

The total weight of all pairs of points is 70 while the total weight of disrupted pairs is:

$$(w_2 + w_5) + (w_2 + w_6) + (w_4 + w_5) + (w_4 + w_6) = 12$$

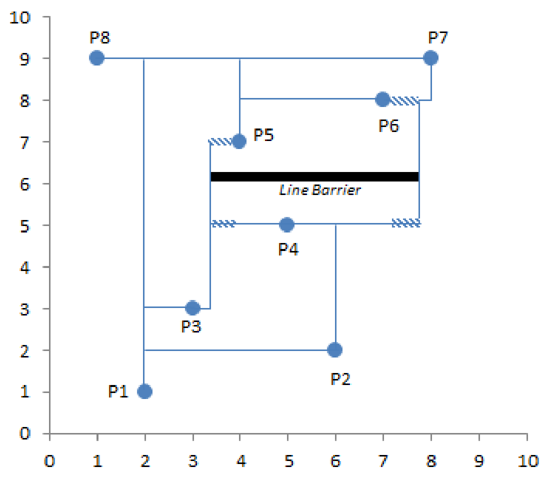


Figure 4.3: A line barrier disrupts 4 pair-distances

4.1.4 Constraining the Barrier Length

Length and location of the barrier can affect the invisibility conditions for a pair of points. In the many-to-many problem, we that assume there is an obnoxious point on the plane proximity to which is not desired. The closer does the barrier get to this obnoxious point, the shorter length it is allowed to have. For simplicity, only the distance along the y-axis between the barrier and the obnoxious point is considered.

This setting can be formulated as follows. Suppose that b_{obn} and y are the y-coordinates of the obnoxious point and the barrier, respectively, such that $|y - b_{obn}|$ gives the vertical distance between them. Then, the length of the barrier, L, is determined by following constraint:

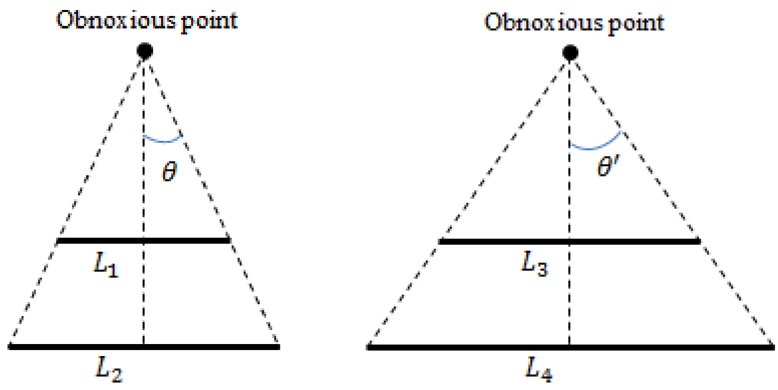
$$L \leq \alpha|y - b_{obn}|$$

where α is a constant ratio. Above constraint can be seen as an isosceles triangle where the obnoxious point is at the vertex and the barrier is located on the base side of the isosceles. We can define the length rate as $\alpha = 2\tan\theta$ where θ is the vertex angle between a leg and the height of the isosceles. Therefore, parameter α itself can be represented by an angle parameter $\theta°$ as $\alpha = 2\tan\theta°$ where $\theta° \in (0, 90)$.

As shown in Figure 4.4.(a), the closer barrier to the obnoxious point is shorter than the further barrier ($L_1 < L_2$). Besides, Figure 2.9.(b) shows that higher angles (or equivalently higher α ratios) allows longer barriers. When θ is increased to θ', the length of L_1 and L_2 increase to L_3 and L_4, respectively.

4.1.5 Constraining the Total Weight of Disrupted Points (or Presence of a Disruption Constraint)

Similar to Section 2.1.5, the total weight of disrupted points is restricted by a disruption rate β. This rate defines the fraction of total pair-weights allowed to be disrupted.



(a) (b)

Figure 4.4: Higher angles allow longer barriers: Since $\theta' > \theta$, barriers in (b) are longer than (a).

4.1.6 Problem Formulation

Consider n points $P_i (i = 1, ..., n)$ on the plane with coordinates (a_i, b_i) and weight w_i that are communicating with each other. A barrier is going to be located between points (x_s, y) and (x_e, y) so as to maximize weighted disrupted distance subject to a disruption constraint.

An auxiliary variable N_{ij} would take the value 1 if the distance between two points P_i and P_j is disrupted:

$$N_{ij} = \begin{cases} 1 & \text{if point } P_i \text{ is invisible to point } P_j \\ 0 & \text{otherwise} \end{cases}$$

Therefore, a conceptual formulation of the many-to-many problem can be presented as below:

$$\underset{x_s, x_e, y, L}{\text{Maximize}} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i z_{ij} \tag{4.11}$$

subject to

$$L = x_e - x_s \tag{4.12}$$

$$L \leq \alpha |y - b_{obn}| \tag{4.13}$$

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i N_{ij} \leq \beta \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} w_i \tag{4.14}$$

$$z_{ij} = |a_i - a_j| + |b_i - b_j| + N_{ij} * 2 \min\{\min(a_i, a_j) - x_s, x_e - \max(a_i, a_j)\} \tag{4.15}$$

Note that Z_{ij} refers to the shortest distance between points P_i and P_j in the presence of the barrier. The objective is to maximize the total weighted disrupted distance between all pairs of points. The length of the barrier is determined in (4.12) and limited in constraint (4.13) with respect to its y-distance from the obnoxious point. Constraint (4.14) is the disruption constraint where the total weight of disrupted pairs is limited by a constant rate β. Equation (4.15) increases the rectilinear distance between P_i and P_j should they be invisible to each other.

In Section 4.2, a mixed-integer programming model is developed for this problem.

4.2 Mathematical Model for the Many-to-Many Rectilinear Interdiction with a Line Barrier on a Plane Subject to a Disruption Constraint

Linearization operations required for modeling the many-to-many problems are the same as in the one-to-many problem (see Section 2.2). A complete list of parameters and variables used in the many-to-many problems are given below.

4.2.1 Parameters

D = Set of n points on the plane
P_i = Sink point i with coordinates (a_i, b_i) and weight $w_i, i \in D$.
P_j = Source point j with coordinates (a_j, b_j) and weight w_j, $j \in D$.
$a_{\max}$ = The maximum coordinate along the x-axis in the convex hull of all points
$a_{\min}$ = The minimum coordinate in the x-axis in the convex hull of all points
$b_{\max}$ = The maximum coordinate in the y-axis in the convex hull of all points
$b_{\min}$ = The minimum coordinate in the y-axis in the convex hull of all points
P_{obn} = An obnoxious point on the plane, proximity to which is not desired. In the many-to-many case, the obnoxious point is defined as:

$$\begin{aligned} a_{obn} &= \tfrac{1}{2}(a_{\max} + a_{\min}) \\ b_{obn} &= \tfrac{1}{2}(b_{\max} + b_{\min}) \end{aligned}$$

β = Maximum disruption rate as a limit on the total weight of disrupted pairs, $0 \le \beta \le 1$.
α = Rate of increase in barrier length based on its vertical distance from the source point. Suppose that $|y - b_{obn}|$ gives the vertical distance between the barrier and the source point. Then, the length of the barrier, L, is determined by following constraints:

$$L \le \alpha|y - b_{obn}|$$

As explained in Section 4.1.4, $\alpha = 2\tan\theta$ where θ is the vertex angle between a leg and the height of an isosceles which defines L.
M = A large positive value. The appropriate value for M is determined as being in Section 2.2.5.

4.2.2 Variables

z_{ij} = Shortest distance between sink P_i and source P_j along the x-axis based on invisibility conditions with respect to the barrier
x_s = Starting point of the barrier along the x-axis
x_e = Ending point of the barrier along the x-axis
y = y-coordinate of the horizontal line barrier
L = Length of the barrier, equal to $x_e - x_s$
d_b = The vertical distance between the line barrier and the obnoxious point, equal to $|y - b_{obn}|$

$$N_{ij} = \begin{cases} 1 & \text{if point } P_i \text{ is invisible to point } P_j \\ 0 & \text{otherwise} \end{cases}$$

N_{ij} would take the value of 1 if both *y-invisibility* and *x-invisibility* conditions hold for P_i and P_j. In order to have an MIP model with valid N_{ij} values, following binary variables must be introduced:

Binary variables for linearization of $d_b = |y - b_0|$:
$\delta_1, \delta_2 \in \{0, 1\}$
Binary variable for *y-invisibility* conditions of P_i and P_j:

$$u_i = \begin{cases} 1 & \text{if } b_i \geq y \\ 0 & \text{otherwise} \end{cases}$$

and $\mu^0_{ij}, \mu^1_{ij}, \mu^2_{ij} \in \{0, 1\}$.
Binary variable for *x-invisibility* conditions of P_i and P_j:

$$v_{is} = \begin{cases} 1 & \text{if } a_i > x_s \\ 0 & \text{otherwise} \end{cases}$$

$$v_{ie} = \begin{cases} 1 & \text{if } a_i < x_e \\ 0 & \text{otherwise} \end{cases}$$

and $\lambda^0_{ij}, \lambda^1_{ij}, \lambda^2_{ij}, \lambda^3_i, \lambda^4_{ij} \in \{0, 1\}$.

4.2.3 Mathematical Model

In this problem, every point on the plane communicates with all other points, thereby, all of them are considered as sources and sinks. The objective is to maximize the total weighted disrupted distances between all points by locating a line barrier. There is symmetry in interdiction which means if the path from source P_i to sink P_j is disrupted, then, the path from the source P_j to sink P_i is also disrupted with the same amount of disruption as the former. Therefore, disruption between P_i and P_j can be calculated once only by choosing $i, j \in D$ where $j > i$ that reduces the number of variables by half. The objective function and disruption constraint will be modified accordingly. Figure 4.5 shows an example of considered sink-source pairs in the model, shown as "x", for 5 points.

	P_1	P_2	P_3	P_4	P_5
P_1		×	×	×	×
P_2			×	×	×
P_3				×	×
P_4					×
P_5					

Figure 4.5: An example of reduction on the number of variables with 5 points

In that case, $w_i + w_j$ has to be considered as the weight associated to the flow between P_i and P_j. The total weight of all distances in this problem can be represented as below:

$$\sum_{i \in D} \sum_{\substack{j \in D \\ j > i}} (w_i + w_j) = (|D| - 1) \sum_{i \in D} w_i \tag{4.16}$$

The MIP formulation for the many-to-many problem is as follows:

$$\underset{x_s,x_e,y,L}{\text{Maximize}} \sum_{i\in D} \sum_{\substack{j\in D\\ j>i}} (w_i + w_j)(z_{ij} + |b_i - b_j|) \tag{4.17}$$

subject to

$$x_e - x_s = L \tag{4.18}$$

$$0 \le d_b + y - b_{obn} \le 2y_{\max}.\delta_1 \tag{4.19}$$

$$0 \le d_b + b_{obn} - y \le 2y_{\max}.\delta_2 \tag{4.20}$$

$$\delta_1 + \delta_2 = 1 \tag{4.21}$$

$$L \le \alpha d_b \tag{4.22}$$

$$b_i - y \le Mu_i \qquad \forall i \in D \tag{4.23}$$

$$y - b_i \le M(1 - u_i) \qquad \forall i \in D \tag{4.24}$$

$$u_i + u_j = 0\mu^0_{ij} + 1\mu^1_{ij} + 2\mu^2_{ij} \qquad \forall i, j \in D, j > i \tag{4.25}$$

$$\mu^0_{ij} + \mu^1_{ij} + \mu^2_{ij} = 1 \qquad \forall i, j \in D, j > i \tag{4.26}$$

$$a_i - x_s \le Mv_{is} \qquad \forall i \in D \tag{4.27}$$

$$x_s - a_i \le M(1 - v_{is}) \qquad \forall i \in D \tag{4.28}$$

$$x_e - a_i \le Mv_{ie} \qquad \forall i \in D \tag{4.29}$$

$$a_i - x_e \le M(1 - v_{ie}) \qquad \forall i \in D \tag{4.30}$$

$$v_{is} + v_{ie} + v_{js} + v_{je} = 0\lambda^0_{ij} + 1\lambda^1_{ij} + 2\lambda^2_{ij} + 3\lambda^3_{ij} + 4\lambda^4_{ij} \qquad \forall i, j \in D, j > i \tag{4.31}$$

$$\lambda^0_{ij} + \lambda^1_{ij} + \lambda^2_{ij} + \lambda^3_{ij} + \lambda^4_{ij} = 1 \qquad \forall i, j \in D, j > i \tag{4.32}$$

$$N_{ij} \le \lambda^4_{ij} \qquad \forall i, j \in D, j > i \tag{4.33}$$

$$N_{ij} \le \mu^1_{ij} \qquad \forall i, j \in D, j > i \tag{4.34}$$

$$N_{ij} \ge \lambda^4_{ij} + \mu^1_{ij} - 1 \qquad \forall i, j \in D, j > i \tag{4.35}$$

$$\sum_{i\in D} \sum_{\substack{j\in D\\ j>i}} (w_i + w_j)N_{ij} \le \beta \sum_{i\in D} \sum_{\substack{j\in D\\ j>i}} (w_i + w_j) \tag{4.36}$$

$$z_{ij} \le (a_i - x_s) + (a_j - x_s) + M(1 - N_{ij}) \qquad \forall i, j \in D, j > i \tag{4.37}$$

$$z_{ij} \le (x_e - a_i) + (x_e - a_j) + M(1 - N_{ij}) \qquad \forall i, j \in D, j > i \tag{4.38}$$

$$z_{ij} \le |a_i - a_j| + MN_{ij} \qquad \forall i, j \in D, j > i \tag{4.39}$$

$$x_s, x_e, y \text{ are unrestricted-in-sign.} \tag{4.40}$$

$$L, d_b \ge 0 \tag{4.41}$$

$$Z_{ij} \ge 0 \qquad \forall i, j \in D, j > i \tag{4.42}$$

$$N_{ij} \in \{0, 1\} \qquad \forall i, j \in D, j > i \tag{4.43}$$

$$\delta_1, \delta_2 \in \{0, 1\} \tag{4.44}$$

$$u_i, , v_{is}, v_{ie} \qquad \forall i \in D \tag{4.45}$$

$$\mu^0_{ij}, \mu^1_{ij}, \mu^2_{ij}, \lambda^0_{ij}, \lambda^1_{ij}, \lambda^2_{ij}, \lambda^3_{ij}, \lambda^4_{ij} \in \{0, 1\} \qquad \forall i, j \in D, j > i \tag{4.46}$$

In the objective function (4.17), we try to maximize the total weighted distance between all pairs of points. Constraint (4.18) determines the relationship between the barrier length and its ending points. Constraints (4.19) to (4.21) gives the vertical distance between the barrier and the obnoxious point as $d_{obn} = |y_b - y_{obn}|$. In constraint (4.22), the barrier length is determined based on its distance from the obnoxious point. The further the barrier is from the obnoxious point, the longer barrier can be placed. Constraints (4.23) to (4.26) inspect the *y-invisibility* conditions between P_i and P_j where $\mu^1_{ij} = 1$ if they are *y-invisible*. Constraints (4.27) to (4.30) control whether $x_s \leq a_i \leq x_e$ and $x_s \leq a_j \leq x_e$ hold to call P_i and P_j *x-invisible*. If so, $\lambda^4_{ij} = 1$ using constraints (4.31) and (4.32). Eventually, P_i and P_j are invisible to each other if both μ^1_{ij} and λ^4_{ij} are 1. To avoid binary multiplication in the form of $N_{ij} = \mu^1_{ij}\lambda^4_{ij}$, constraints (4.33) to (4.35) are introduced. Constraint (4.36) is the disruption constraint on the total weight of disrupted pairs of points. If $N_{ij} = 1$, the shortest path between P_i and P_j has to go through either the starting point or the ending point of the barrier. Therefore, one of constraints (4.37) and (4.38) would be tight. If $N_{ij} = 0$ then the point is visible, which makes constraint (4.39) tight. (4.40) shows the unrestricted-in-sign variables. Constraints (4.41) and (4.42) are the non-negativity constraints while constraints (4.43) to (4.46) are integrality constraints required for invisibility conditions.

For n points, there are $\frac{n(n-1)}{2} + 5$ continuous variables and $5n^2 - 2n + 2$ binary variables used across $\frac{11}{2}n^2 - \frac{1}{2}n + 10$ constraints in this model. Hence, it is an $O(n^2)$ continuous variables, $O(n^2)$ binary variables, and $O(n^2)$ constraint model.

4.3 An Algorithm for the Many-to-Many Rectilinear Interdiction with a Line Barrier on a Plane

The same model in Section 4.2.3 but without the disruption constraint (4.36) applies to this problem. All problem features introduced for the one-to-many problem in Section 2.3.4 apply for the the many-to-many problem as well. A brief review of these features are given below.

4.3.1 Optimal Location of a Line Barrier Between Two *y-invisible* Points

Due to theorem 2.2 the maximum interdiction between two points occur when the midpoint of the line barrier falls on the midpoint of the two points along the x-axis, i.e. $x_s + \frac{1}{2}L = \frac{1}{2}(a_i + a_j)$. Theorem 2.3 shows when there are multiple sink points on the plane, the midpoint of the barrier must fall on one of the sink-source midpoints. The results are also valid for the many-to-many problem. Candidate midpoints are defined by pairwise combination of all points on the plane. If the midpoint of the barrier does not fall on any of these candidate midpoints, that barrier would not incur maximum interdiction for a particular pair of points let alone optimizing on all pairwise distances. In order to use the candidate midpoints, we need to ensure that points are *y-invisible*, which is explained below.

4.3.2 Partitioning the Plane into Regions with Distinct *y-invisible* Point Sets

Before utilizing the candidate midpoints, the y-coordinate of the barrier has to be fixed to secure *y-invisibility* conditions for all points. From property 2.2 we know that if there are K distinct y-coordinates for all points on the plane (including the obnoxious point), there are $K - 1$ candidate ranges for y with $K - 1$ distinct set of *y-invisible* points. These y-ranges are mutually exclusive and collectively exhaustive. In a particular range $R_r, r = 1, \ldots, K - 1$, the value of $y_r \in R_r$ does not change the set of *y-invisible* points. In order to locate a line barrier on candidate midpoints inside a y-range, first we need to know the length of the barrier which is explained next.

Figure 4.6 shows an example in which the obnoxious point is located at (4.5, 5). There are 6 distinct ranges identified for 7 distinct y-coordinates.

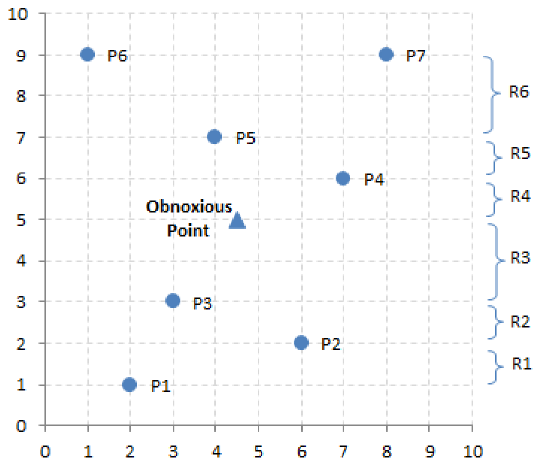


Figure 4.6: Candidate ranges for y in an 8-point system

4.3.3 Fixing the Length of the Barrier in each Partition

We know that the barrier has to be located as far as possible from the obnoxious point to get a longer length. If there is no disruption constraint, the longest possible barrier within a range has to be selected in order to maximize the interdiction. Therefore, based on theorem 2.1, y_r has to be equal to the y-coordinate of the point at the border of range R_r and farthest from the source point. Therefore, the maximum barrier length in different ranges is obtained from following equation:

$$L_r = \alpha|y_r - b_{obn}|, r = 1, \ldots, K - 1 \tag{4.47}$$

where $y_r = b_i$ and $P_i \in R_r$.

Table 4.1 shows the maximum possible length in each of these ranges with $\alpha = 1$.

Table 4.1: Maximum possible L_r in each range R_r

r	R_r	y_r	L_r
1	$[1, 2)$	1	4
2	$[2, 3)$	2	3
3	$[3, 5)$	3	2
4	$(5, 6]$	6	1
5	$(6, 7]$	7	2
6	$(7, 9]$	9	4

4.3.4 Algorithm for the Many-to-Many Rectilinear Interdiction with a Line Barrier on a Plane

In algorithm 2, the optimal interdiction between all pairs of points is obtained by partitioning the plane into candidate ranges along the y-axis, assigning the maximum length, and by enumerating on candidate midpoints in each range to find the optimal location of the barrier.

Algorithm 2 Maximize the interdiction in the many-to-many problem

Require: P_0 as the source point, $P_i, i = 1, ..., n$, as sink points and $\theta^\circ \in (0, 90)$
Ensure: x_s^*, x_e^*, L^*, Z^*
1: $Z_0 \leftarrow \sum_{i,j \in D} (w_i + w_j) l(P_i, P_j)$ {The objective value when no barrier exists.}
2: $Z^* \leftarrow 0$
3: **for** $i = 1$ **to** n **do**
4: {Find barrier's length at a y-range}
5: $y_r \leftarrow b_i$ {find distinct y_r along the y-axis defined by P_i}
6: $L_r \leftarrow 2 \tan\theta |y_r - y_{obn}|$ {set L_r for each range along the y-axis }
7: {Locate the barrier with length L using the midpoint of a pair of points P_j and P_k}
8: **for** $j = 1$ **to** n **do**
9: **for** $k = 1$ **to** n (and $k > j$) **do**
10: **if** $(b_j > y_r > b_k)$ or $(b_j < y_r < b_k)$ **then**
11: $m_{jk} \leftarrow 0.5(a_j + a_k)$ {use the midpoint if P_j and P_k are *y-invisible*}
12: $x_{sjk} \leftarrow m_{jk} - 0.5L_r$
13: $x_{ejk} \leftarrow m_{jk} + 0.5L_r$
14: {find the interdiction for all points based on this barrier.}
15: **for** $g = 1$ **to** n **do**
16: **for** $h = 1$ **to** n (and $h > g$) **do**
17: {if P_g is invisible to P_h then add their interdiction to Z_{jk}}
18: **if** $b_g > y_r > b_h$ or $b_g < y_r < b_h$ **then**
19: **if** $x_{sjk} < a_g$ and $a_g < x_{ejk}$ and $x_{sjk} < a_h$ and $a_h < x_{ejk}$ **then**
20: $Z_{jk} \leftarrow Z_{jk} + 2(w_g + w_h) * \min\{\min(a_g, a_h) - x_s, x_e - \max(a_g, a_h)\}$
21: **end if**
22: **end if**
23: **end for**
24: **end for**
25: **if** $Z_{jk} + Z_0 > Z^*$ **then**
26: $Z^* \leftarrow Z_{jk} + Z_0$, $x_s^* \leftarrow x_{sjk}$, $x_e^* \leftarrow x_{ejk}$, $y^* \leftarrow y_r$, $L^* \leftarrow L_r$
27: **end if**
28: **end if**
29: **end for** {change the midpoint}
30: **end for** {change the midpoint}
31: **end for** {change y-range}
32: **return** $Z^*, x_s^*, x_e^*, y^*, L^*$

The algorithm starts by assigning the original objective value when there is no barrier and setting the optimal interdiction amount to zero. The outer loop (steps 3-31) finds the distinct y-ranges corresponding to the points i and calculates the y_r and L_r accordingly. In the second and third loops (steps 8-30), if P_j and P_k are y-invisible due to y_r then a valid candidate midpoint m_{jk} is generated and the barrier is located at x_{sj} and x_{ej} based on theorem 2.2. In the fourth and fifth loops (steps 15-24), the invisibility conditions between every pair of points on the plane are inspected and the interdiction value corresponding to the current midpoint, i.e. Z_{jk}, is updated. The best known solution (Z^*) is, then, updated at step 26 if $Z_{jk} + Z_0$ is better than Z^*. The midpoint is changed in the same y-range level at steps 29 and 30. Once all midpoints in the range R_r are checked, the y-range is changed at step 31 and inner loops are restarted again.

The outer loop discretizes the solution space along the y-axis into n candidate levels. At each y level, the barrier can be located in n^2 different candidate locations obtained from two-way combinations of all points (the second and third loops). The fourth and the fifth loops calculate the objective function value for all pairs of points in n^2 steps. Therefore, the worst-case processing time needed for this algorithm is $O(n^5)$.

Example 4.2 *Suppose that weights of all points in Figure 4.6 are 1 and* $\theta^\circ = 45$*. The solution to this problem is as follows:*

1: $Z_0 = \sum_{i=1}^{7} \sum_{j=i+1}^{7} (w_i + w_j)(|a_i - a_j| + |b_i - b_j|) = 304$

Iteration 1:

5: Use point P_1 *to partition the plane. Set* $y_1 = 1$.

6: Find maximum possible length: $L_1 = 2tan45^\circ|5 - 1| = 8$

Locate the barrier:

11: P_2 *is y-invisible to* P_1 *. So use their midpoint:* $m_{s12} = 0.5(2 + 6) = 4$

12: $x_{s12} = 4 - 0.5 \times 8 = 0$

13: $x_{e12} = 4 + 0.5 \times 8 = 8$

Find the disrupted distances:

20: P_1 *is invisible to* P_2*. Therefore:* $Z_{12} = 0 + 2 * 2 * 2 = 6$

20: P_1 *is invisible to* P_3*. Therefore:* $Z_{12} = 6 + 2 * 2 * 2 = 12$

20: P_1 *is invisible to* P_4*. Therefore:* $Z_{12} = 12 + 2 * 1 * 2 = 16$

20: P_1 *is invisible to* P_5*. Therefore:* $Z_{12} = 16 + 2 * 2 * 2 = 22$

20: P_1 *is invisible to* P_6*. Therefore:* $Z_{12} = 22 + 2 * 1 * 2 = 26$

Update the best solution:

26: Since $304 + 26 > 304$ *then* $Z^* = 330, x_s^* = 0, x_e^* = 8, y^* = 1, L = 8$

Change the midpoint and locate a new barrier:

11: P_3 *is y-invisible to* P_1*. So use their midpoint:* $m_{s13} = 0.5(2 + 3) = 2.5$

12: $x_{s12} = 2.5 - 0.5 \times 8 = -1.5$

13: $x_{e12} = 2.5 + 0.5 \times 8 = 6.5$

Find the disrupted distances:

20: P_1 *is invisible to* P_2*. Therefore:* $Z_{13} = 0 + 2 * 0.5 * 2 = 2$

20: P_1 *is invisible to* P_3*. Therefore:* $Z_{13} = 2 + 2 * 3.5 * 2 = 16$

20: P_1 *is invisible to* P_5*. Therefore:* $Z_{13} = 16 + 2 * 2.5 * 2 = 26$

20: P_1 *is invisible to* P_6*. Therefore:* $Z_{13} = 26 + 2 * 2.5 * 2 = 36$

Update the best solution:

26: Since $304 + 36 > 340$ *then* $Z^* = 340, x_s^* = -1.5, x_e^* = 6.5, y^* = 1, L = 8$

Change the midpoint and locate a new barrier:

...

After solving the problems in all iterations, the optimal solution is:

$Z^* = 340, x_s^* = -1.5, x_e^* = 6.5, y^* = 1, L = 8.$

4.4 Computational Analysis

In this section, we perform computational experiments on different test problem instances to assess the performance of models and solution approaches introduced in this chapter. In Section 4.4.1, the selected core test problems instances and their modifications are explained. Computational settings are introduced in Section 4.4.2 and computational results are explained in sections 4.4.3 and 4.4.4.

4.4.1 Core Test Instances

In order to study the properties of the many-to-many problems and performance of the solution approaches, 24 test instances are selected from planar TSP and VRP test instances available in the literature, as mentioned in Table 4.2. Based on the number and distribution of points on the plane, these instances can be categorized as:

- *Sparse*: Points are scattered with low density across the plane. 12 instances are in this category.
- *Clustered*: Points are concentrated in different clusters on the plane. The distance between the clusters is significant compared to the distance between points in a cluster. 11 instances are selected in this category.
- *Vertical*: Points are scattered around or along the y-axis. 5 instances are in this category.
- *Horizontal*: Points are scattered around or along the x-axis. 2 instances are in this category.

An obnoxious point is also added to these instances at a specific location. In the many-to-many problem, all n points are sink and source points, simultaneously. In order to distinguish this feature from those in the one-to-many problems, the letter 'N' would be used. Weights of points are chosen to be identical (equal to 1) or randomly generated integers between 1 and 10. These variants are addressed with '1' and 'W', respectively. Therefore, for each instances, following variants will be solved:

- **1N**: All weights are equal to 1. Number of source and sink points are n.
- **WN**: Weights are random in [1,10]. Number of source and sink points are n.

All 24 instances along with their properties and obnoxious points are enlisted in Table 4.2.

4.4.2 Computational Settings

The proximity parameter values are set to $\theta = 30, 45, 60$ to give instances shorter or longer lengths and disruption rates are designed at $\beta = 0.1, 0.25, 1$. For $\beta < 1$, we use the mathematical model to solve the problems. Since the problems with $\beta = 1$ do not have disruption constraint, they are solved with the algorithm.

CPLEX Optimizer 10.1 (ILOG, 2006) is used for solving MIP models while algorithms are developed in a VB.NET 2010 application (Microsoft, 2012). A time limit of 1000 seconds is applied for running models. All computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

Following MIP cuts are set in CPLEX with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

Table 4.2: 24 core instances and their properties

No.	Core Instance*	n	Distribution	Obnoxious location
1	D8-Canbolat	8	sparse	(8, 6.5)
2	E-n22-k4	22	vertical	(146, 223)
3	D28	28	sparse	(228.5, 247)
4	B-n31-k5	31	clustered	(49.5, 41)
5	A-n32-k5	32	sparse	(49.5, 49.5)
6	D40	40	vertical	(155.5, 224)
7	B-n41-k6	41	clustered	(60, 57)
8	A-n45-k6	45	sparse	(49.5, 52.5)
9	F-n45-k4	45	horizontal	(215.5, 177)
10	att48	48	vertical	(6548, 5095)
11	B-n50-k7	50	clustered	(47.5, 46.5)
12	D50	50	vertical	(214.5, 255.5)
13	eil51	51	sparse	(34, 37.5)
14	berlin52	52	sparse	(882.5, 590)
15	A-n60-k9	60	sparse	(48, 51)
16	B-n68-k9	68	clustered	(44, 54.5)
17	F-n72-k4	72	vertical	(193.5, 200.5)
18	rus75	75	clustered	(58.5, 49)
19	eil76	76	sparse	(38, 40)
20	A-n80-k10	80	sparse	(50, 49)
21	rd100	100	sparse	(490.4232, 491.5671)
22	E-n101-k14	101	sparse	(34.5, 40)
23	10G2	101	clustered	(50, 50)
24	F-n135-k7	135	horizontal	(140.75, 212.5)

***Sources of instances:**

-TSPLib (2012): Instances 10,13,14,19 and 21

-VRPH (2012): Instances 2,4,5,7,8,9,11,15,16,17,20,22,23, and 24

-Ruspini (1970): Instance 18

-Canbolat and Wesolowsky (2010): Instance 1

-Instances 3,6, and 12 are generated in this study.

4.4.3 The Many-to-Many Problems with $\beta < 1$ Using MIP Model

The MIP solutions are obtained for different levels of parameters θ and β. Instead of actual objective values ($Z = \sum_{i=1}^{n}\sum_{j=i+1}^{n}(w_i + w_j)l^B(P_i, P_j)$), the amount of increase in the original objective function ($Z_0 = \sum_{i=1}^{n}\sum_{j=i+1}^{n}(w_i + w_j)l(P_i, P_j)$) is used as an interdiction rate ($\Delta Z\%$):

$$\Delta Z\% = \frac{(\Delta Z = Z - Z_0)}{Z_0} \times 100$$

The original objective values (Z_0) and the interdiction rates ($\Delta Z\%$) for all instances are presented in this section while further details are available in Appendix E.

The required processing times (CPU) to solve each MIP model are reported in seconds.

A. 1N Variants

Table 4.3 shows the computational results regarding solving MIP model where the weight of points are equal to 1.

Table 4.3: Computational results for the many-to-many problems with $\beta < 1$ for 1N variant of 24 instances

		θ	30				45				60			
		β	0.1		0.25		0.1		0.25		0.1		0.25	
Instances	**n**	Z_0	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**
D8-Canbolat	8	460	3.7	0.6	3.8	0.6	6.1	1.1	10.9	1	7	1.2	29.2	0.9
E-n22-k4	22	20820	5.1	81.3	7.7	1000	12.1	96	21.8	437.4	24.2	71.3	49.1	63.9
D28	28	195830	3.7	63.1	3.7	1000	7.2	111.5	8.7	157.7	11.3	132	23.7	131.5
B-n31-k5	31	37460	6.1	189.6	12.6	102.1	8.9	215	26.6	393.5	14.7	258.3	55	666.8
A-n32-k5	32	74174	2.7	188.4	2.7	157.5	6.7	212.6	7.7	1000	12.5	210.7	23.4	121.3
D40	40	329592	5.7	682.9	9.6	677.6	13.4	780.6	22.8	1000	26.8	423.8	52.2	705.6
B-n41-k6	41	117368	2.5	395.4	2.3	1000	6	580.4	6	1000	12.8	1000	17.5	1000
A-n45-k6	45	145168	1.8	1000	1.8	477.1	4.9	1000	5.2	1000.1	11.4	1000	16.8	1000.1
F-n45-k4	45	135410	4.3	1000	5.7	1000	10.5	1000	9.5	1000	24.4	1000	39.8	1000
att48	48	9336410	2.8	1000	7.3	1000.1	4.7	1000	14.5	1000	7.4	1000.1	51	1000
B-n50-k7	50	161696	2.4	1000	2.7	578.2	5.7	1000	7	1000.1	7.4	1000	20.4	1000.1
D50	50	434674	3.2	1000.1	5.7	1000.1	7.8	1000	13.6	1000	12.9	1000.1	8	1000.1
eil51	51	105980	1.8	1000.1	1.6	1000	4.2	1000	6.7	1000	5.4	1000.1	6.1	1000.1
berlin52	52	1941090	2.3	1000	2.3	622.2	4.7	1000.1	5.5	1000	8.3	1000.1	11.8	1000
A-n60-k9	60	240820	2.2	1000	2.2	1000.1	4.7	1000	8.7	1000.1	1.1	1000	5.6	1000.1
B-n68-k9	68	296504	1.6	1000.1	0.7	1000.2	0.9	1000	4.7	1000	1.6	1000.2	4.6	1000
F-n72-k4	72	114852	0.3	1000.1	0	1000	0	1000.1	0	1000	8.5	1000.1	0	1000.1
rus75	75	356392	1	1000	1.2	1000.1	0	1000.1	0	1000.1	3.4	1000.1	0	1000.1
eil76	76	242450	1	1000	0	1000	4.3	1000	0	1000.1	0.8	1000.1	0	1000
A-n80-k10	80	418736	0.9	1000.1	1.6	1000	0	1000.2	0	1000	6.4	1000.2	0	1000
rd100	100	7010846.3	0	1000.2	0	1000	0	1000.2	0	1000	0	1000.1	0	1000.2
E-n101-k14	101	439136	0	1000	0	1000.2	0	1000.1	0	1000.2	0	1000.1	0	1000.1
10G2	101	695984	0	1000.1	0	1000.1	0	1000.2	0	1000	0	1000.1	0	1000.3
F-n135-k7	135	1096068.8	0	1000.2	0	1000.4	0	1000.2	0	1000.3	0	1000.7	0	1000.1

As expected, the highest increase in objective value is obtained when $\theta = 60$ and $\beta = 0.25$. The reason is that higher θ values lead to longer barriers and make larger areas exposed to disruption while higher β values increase the disruption capacity increasing the chance of having more interdiction. For example, in instance B-n31-k5, when $\beta = 0.1$, the interdiction rate grows from 6.1% to 8.9% and 14.7% as θ increases from 30° to 45° and 60°. Moreover, with angle $\theta = 30$ for the same instance, the interdiction rate increases from 6.1% to 12.6% as disruption rate β is changed from 0.1 to 0.25.

Many instances could not be solved to optimality within the time limit. Although the modest size instance E-n22-k4 with $n = 22$ cannot be solved optimally for $\theta = 30$ and $\beta = 0.25$, the bigger instance berlin52 is solved in 622.2 seconds. A possible explanation for this observation is that the distribution of points on the plane affects the MIP performance. In the many-to-many problem, when there are more points above and below the barrier, it becomes more difficult to keep disruption constraint feasible. Placing a line barrier in a vertical instance (for example E-n22-k4) can disrupt more pairs of points than in a horizontal instance (for example A-n45-k6) or a sparse one (for example berlin52).

Since many instances are not solved to optimality, the average results are not presented for these variants.

B. 1N Variants

Table 4.4 shows the computational results regarding solving MIP models of test instances where the weight of points are randomly generated between 1 and 10.

As expected, the highest interdiction rates happen at the highest θ and β values. In A-n32-k5, 22.3% interdiction rate obtained with θ=60 and β=0.25 has not been reached by any other setting. A similar

Table 4.4: Computational results for the many-to-many problems with $\beta < 1$ for WN variant of 24 instances

		θ	30				45				60			
		β	0.1		0.25		0.1		0.25		0.1		0.25	
Instances	**n**	Z_0	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**	**ΔZ%**	**CPU**
D8-Canbolat	8	2145	3.9	0.6	4.7	0.6	5.3	1.2	12.9	0.8	7.9	0.9	27.1	1.2
E-n22-k4	22	109150	4.6	79.5	6.8	1000	11.3	55.8	20.5	479.3	23	127.1	46.9	70.5
D28	28	1061359	4.1	103	3.8	1000	7.9	118.7	9.3	105.4	12.6	139.5	23.5	1000
B-n31-k5	31	186689	7.1	205.8	13.7	65.6	11.7	179.6	27.4	150.7	23.7	220.3	56.6	247.6
A-n32-k5	32	502004	2.6	260.3	2.6	112.2	6.1	274.9	7	213.6	10.9	231.9	22.3	340.2
D40	40	552600	5.5	521.5	11.1	1000	3.1	1000	27.8	1000	20.1	1000	56.7	478.7
B-n41-k6	41	150088	2	1000	2	383.9	4.9	1000	5.5	1000	12.8	938.2	17.1	918.8
A-n45-k6	45	760509	2	372.4	2	1000	3.6	1000	4.6	1000	8.8	1000.1	15	1000
F-n45-k4	45	619773.8	4.2	1000.1	4.6	340.2	11.5	601.5	12.6	1000	27.3	755.8	33.3	1000
att48	48	49936044	4.4	1000	6.5	1000	7.2	1000	13.5	1000	3.9	1000	8.2	1000.1
B-n50-k7	50	890752	2.3	1000	2.4	661.2	2.1	1000.1	6.2	1000.1	10.9	1000.1	9.1	1000.1
D50	50	629506	1.3	1000	4.5	1000	7.6	1000	14.1	1000.1	3.2	1000	28	1000.1
eil51	51	512094	3	1000	1.7	1000	2.9	1000	6.1	1000	12.1	1000	12.1	1000
berlin52	52	11457715	1.5	1000.1	1.5	818.6	4.4	1000	3.5	1000	6	1000.1	10.5	1000
A-n60-k9	60	1352240	1.9	1000.1	2.6	1000.1	4.6	1000	7.1	1000	6.5	1000	11	1000.2
B-n68-k9	68	1601056	1.4	1000	1.4	1000	2.5	1000.1	0.7	1000.1	4	1000	2.8	1000.1
F-n72-k4	72	565058	0.5	1000.1	0	1000	1.5	1000.2	0	1000	7.7	1000	0	1000
rus75	75	2201807	0	1000	1.1	1000.1	0.3	1000.1	0	1000	0.2	1000	0	1001.4
eil76	76	1275488	0.1	1000	0.2	1000.1	0	1000	0	1000	10.4	1000.1	0	1000
A-n80-k10	80	2256298	0.1	1000	0.4	1000	0.3	1000	0	1000.1	0.1	1000.1	0	1000.2
rd100	100	42408487.6	0	1000.2	0	1000.6	0	1000.1	0	1000.3	0	1000.1	0	1000.1
E-n101-k14	101	2261022	0	1000.1	0	1000.1	0	1000.3	0	1000.2	0	1000.1	0	1000.2
10G2	101	3817569	0	1000	0	1000.2	0	1000.2	0	1000.2	0	1000.1	0	1000.2
F-n135-k7	135	6057952.9	0	1000.2	0	1000.3	0	1000.3	0	1000.4	0	1000.4	0	1000.2

result is observed for the other instances that are successfully solved to optimality. However, some instances are quite sensitive to θ and β values. For example, when θ=30, the solver cannot close the gap for B-n41-k6 within the time limit for β=0.1 whereas the optimality is reached for β=0.25. The opposite happens for A-n45-k6 whose MIP model with β=0.25 is more difficult to solve than the model with β=0.1. However, many instances could not be solved in 1000 seconds.

4.4.4 The Many-to-Many Problems with $\beta = 1$ Using Algorithm

Recall that when there is no constraint on the total weight of disrupted points ($\beta = 1$), the optimal solution can be found using the algorithm explained in Section 4.3.4. To compare the results obtained from the solutions of MIP model with those of the algorithm, we solve all the test instances with $\beta = 1$ using MIP model. Not only does this cross-checking ensure the validity of MIP model and algorithm but also provides a fair ground to compare their performances in terms of required processing time.

Since many instances are not solved to optimality, the average results are not presented for these variants.

A. 1N Variants

Table 4.5 shows the computational results of running the algorithm for 24 test instances where the weight of points are equal to 1. The solution times of the algorithm and MIP model are reported, in seconds, as CPU1 and CPU2 columns, respectively.

As expected, wider θ angles allow locating a longer barrier which, in turn, disrupts more pairs of points on the plane and increases the interdiction rate. For example, in rd100, when θ is increased from 30 to 45 and 60, the interdiction rate increases from 1.5% to 7% and 30.1%.

Table 4.5: Computational results for the many-to-many problems with $\beta = 1$ for 1N variants

		θ	**30**			**45**			**60**		
Instances	**n**	Z_0	**ΔZ%**	**CPU1**	**CPU2**	**ΔZ%**	**CPU1**	**CPU2**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	460	3.8	0	0.6	10.9	0	0.9	39.6	0	0.9
E-n22-k4	22	20820	7.7	0.06	647.8	25.9	0.06	1000	61.7	0.08	1000
D28	28	195830	3.7	0.19	85.2	8.7	0.2	347.2	25.5	0.22	1000
B-n31-k5	31	37460	12.6	0.34	86.9	30.3	0.36	1000	62.8	0.38	1000
A-n32-k5	32	74174	2.7	0.41	49.9	7.7	0.39	194	30	0.41	1000
D40	40	329592	9.6	1.2	1000	28.9	1.3	1000	65.6	1.38	1000
B-n41-k6	41	117368	2.5	1.5	188	6	1.33	1000	17.5	1.38	1000
A-n45-k6	45	145168	1.8	2.12	918.7	5.2	2.12	1000	22.3	2.16	1000
F-n45-k4	45	135410	5.9	2.05	1000	16	2.11	829.4	39.8	2.31	1000
att48	48	9336410	10.6	3	1000	38.1	3.17	1000	90.3	3.55	1000
B-n50-k7	50	161696	2.7	3.61	932.6	7.7	3.69	1000.1	28.4	3.84	1000
D50	50	434674	6.6	3.58	1000	18.4	3.73	1000	41.8	3.88	1000
eil51	51	105980	2.3	3.97	1000	8.1	4.02	1000.1	31.1	4.16	1000
berlin52	52	1941090	2.3	4.16	1000	6.2	4.14	1000	17	4.19	1000
A-n60-k9	60	240820	2.4	11	1000.1	9.6	11.2	1000	30.2	9.8	1000.1
B-n68-k9	68	296504	3.2	18.08	1000.1	10.9	17.14	1000.1	43.7	18.09	1000.1
F-n72-k4	72	114852	8.2	20.84	1000.1	26.5	22.11	1000.1	60.7	24.09	1000.1
rus75	75	356392	2.1	23.52	1000.1	7.5	24.17	1000.1	22	25.42	1000
eil76	76	242450	2.4	28.97	1000	9	29.34	1000	30.7	30.73	1000
A-n80-k10	80	418736	2.1	37.48	1000.1	7.7	37.72	1000.1	27.4	40.83	1000
rd100	100	7010846.3	1.5	114.69	1000	7	116.52	1000	30.1	121.67	1000
E-n101-k14	101	439136	1.7	114.28	1000	8	118.34	1000	28.8	126.38	1000
10G2	101	695984	1.8	115.23	1000	8.6	120.75	1000	30.1	126.98	1000
F-n135-k7	135	1096068.8	1.7	447.08	1000.7	4	523.28	1000.2	8.3	466.8	1000.4

When CPU1 and CPU2 are compared, there is a significant difference between the performance of the algorithm and the MIP model. All variants of instances are solved optimally in less than 525 seconds with the algorithm whereas the solver struggles to close the gap for more than half of MIP problems.

The average interdiction rates and CPU times for 1N variants of all instances are given in Table 4.6. On average, CPU1 does not change much with respect to θ values throughout all instances. The average time with MIP models, on the other hand, soars around 1000 seconds of time limit which proves its inefficiency when compared to the algorithm.

Table 4.6: Summary of results for 1N variants when $\beta = 1$

θ	**30**	**45**	**60**
ΔZ%	4.25	13.20	36.89
CPU1	8.304	8.415	8.845
CPU2	745.51	868.605	950.06

B. WN Variants

Table 4.7 shows the computational results of running the algorithm for 24 test instances where the weight of points are randomly generated between 1 and 10. Solution times of the algorithm and MIP model are reported, in seconds, as CPU1 and CPU2, respectively.

The interdiction rates increase as the angle θ increases and no constraint is imposed on the amount of

Table 4.7: Computational results for the many-to-many problems with $\beta = 1$ for WN variants

		θ	30			45			60		
Instances	**n**	Z_0	**ΔZ%**	**CPU1**	**CPU2**	**ΔZ%**	**CPU1**	**CPU2**	**ΔZ%**	**CPU1**	**CPU2**
D8-Canbolat	8	2145	4.7	0	0.5	12.9	0	0.9	38.9	0	1
E-n22-k4	22	109150	6.8	0.06	24.8	24	0.06	26.4	59.3	0.06	1000
D28	28	1061359	4.1	0.2	31.8	9.3	0.2	1000	28.2	0.23	1000
B-n31-k5	31	186689	13.7	0.34	59.3	33	0.36	1000	68.5	0.39	378.4
A-n32-k5	32	502004	2.6	0.41	69.3	7	0.39	200.2	29.5	0.39	1000
D40	40	552600	13.1	1.23	299.9	35.6	1.3	1000	79	1.42	163.4
B-n41-k6	41	150088	2	1.33	1000	5.5	1.33	1000	17.1	1.36	1000
A-n45-k6	45	760509	2	2.14	666.4	5.1	2.12	1000	19.3	2.17	1000
F-n45-k4	45	619773.8	4.6	2.03	530.6	13.2	2.09	799.6	33.3	2.3	1000
att48	48	49936044	10	2.97	1000	37.7	3.16	1000	89.7	3.31	1000
B-n50-k7	50	890752	2.4	3.61	744.9	6.7	3.64	1000	25.6	3.88	1000.1
D50	50	629506	10	3.55	1000	27.1	3.67	1000	59.5	3.83	1000
eil51	51	512094	3.1	3.92	1000.1	10.2	3.98	1000	32	4.17	1000
berlin52	52	11457715	1.5	4.17	1000	4.4	4.19	1000	11.8	4.19	1000
A-n60-k9	60	1352240	2.6	9.09	1000.1	10.2	9.22	1000.1	32.8	9.67	1000
B-n68-k9	68	1601056	3	16.84	1000.1	10.2	17.17	1000.1	41.6	18.12	1000
F-n72-k4	72	565058	8.5	20.83	1000.1	26.7	22.06	1000	61.7	24.09	1000.1
rus75	75	2201807	2.1	23.59	1000	7.3	24.25	1000	20.7	25.61	1000
eil76	76	1275488	2.3	29.02	1000	9.3	29.62	1000	30.6	30.89	1000
A-n80-k10	80	2256298	2	37.48	1000	7.3	37.62	1000	26.3	39	1000
rd100	100	42408488	1.7	114.72	1000	7.8	116.66	1000	31.9	121.19	1000
E-n101-k14	101	2261022	1.5	114.88	1000	6.7	118.34	1000	25.8	126.81	1000
10G2	101	3817569	2	115.5	1000	8.6	120.89	1000	29.6	127.91	1000
F-n135-k7	135	6057952.9	1.9	447.77	1008.7	4.6	454.38	1005.8	9.7	468.91	1000

disruption. For example, the interdiction rate in att48 increases from 10% to 37.7% and 89.7% as θ increases from 30 to 45 and 60. The same trend is observed for other instances as well.

When it comes to running time performance, the algorithm by far outperforms the MIP models since the algorithm is able to solve all instances to optimality in less than 500 seconds whereas CPLEX cannot close the gap within the time limit for more than half of running implementations.

The average interdiction rates and CPU times for WN variants of all instances are given in Table 4.8. On average, CPU1 does not change much with respect to θ values throughout all instances. In addition, CPU1 in WN variants are very similar to 1N variants. It follows that the algorithm is independent of the weight points. The average solution time of MIP model, on the other hand, is above 600 seconds which proves its inefficiency when compared to the algorithm.

Table 4.8: Summary of results for 1N variants when $\beta = 1$

θ	**30**	**45**	**60**
ΔZ%	4.51	13.77	37.60
CPU1	8.1405	8.3215	8.754
CPU2	671.395	851.365	877.15

CHAPTER 5

CONCLUSION AND FUTURE RESEARCH

In this study, we introduce a rectilinear interdiction problem by locating a line barrier between sink and source points on the plane. The flow from source to sink points is not desired and, therefore, we attempt to interdict this system by locating a line barrier such that the total shortest rectilinear distance between sink and source points is maximized. The related literature is dominated by the network interdiction problems where disruptive acts are defined on nodes and arcs and distances between nodes are assumed to be known. In this study, however, the distance between points on the plane after interdiction are to be determined by rectilinear distance metric.

We are interested in studying the effect of partial disruption leading to a limited damage which has to be realized with respect to a ”limited budget”. Therefore, the scale of disruption in this study is constrained by a limited barrier length and a capacity on the number of disrupted points.

Four problem types are defined with regard to the number of source points and their communication with sink points. To solve these problems, MIP models and polynomial-time algorithms are developed and tested on several test problem instances, mostly taken from the literature. It is observed that the performance of these models is highly dependent on distribution of points on the plane, barrier length, and disruption constraint. When applicable, the algorithm outperforms the MIP models by a significant margin in terms of running times, as expected.

To the best of our knowledge, this research is the first study on planar interdiction problems which seems promising to have future extensions by simply changing the assumptions introduced in this study. Some possible direction for future work are as follows:

- **A strong or ideal formulation:** Using Theorem 2.1, one might develop a strong formulation of the one-to-many problem.
- **Improved algorithm:** Again, using Theorem 2.1, one might develop a faster algorithm compared to the one in Section 2.3.4.
- **DP Algorithm:** Since solving MIP models with $\beta < 1$ are time-consuming, working on a specialized algorithm that can handle the disruption constraint becomes important. A possible approach would be based on dynamic programming.
- **Objective function:** Instead of maximizing the total interdicted distance, the number of disrupted points can be maximized.
- **Distance norm:** Current assumption for the underlying distance metric is rectilinear. A possible extension is using other well-known metrics such as Euclidean and Tchebychev.
- **Barrier shape:** Arbitrary line segments, polygons, circle and other types of barriers can be used for disruption for representing more realistic cases.
- **Number of barriers:** Instead of a single barrier, multiple barriers can be located one-by-one or simultaneously, although it would be a complicated problem as discussed in Section 2.4 even

for one-by-one barrier location.

- **Congested regions:** Trespassing through the barrier is assumed to be impossible. A possible extension to this feature is using congested regions where trespassing is possible at a certain cost.
- **Game theory:** This problem can be considered as part of a Stackelberg game in which the leader locates a barrier and the follower opens a passage through the barrier. Another possible scenario is to locate a source point on the plane such that the effect of interdiction by the opponent is minimized.
- **Source-sink assignment:** The sources on the plane can have certain capacities to serve sink points. In this study, source points are uncapacitated and serve all sink points on the plane.
- **Uncertainty:** Sink points, source points and the barrier location in this study are deterministic. A probabilistic element can be added in future studies.

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APPENDIX A

THE ONE-TO-MANY PROBLEM IN THE PRESENCE OF SEVERAL LINE BARRIERS

A.1 Identifying Invisible Points When Several Line Barriers Exist.

Suppose that there is a source point P_0 with two line barriers on the plane as shown on Figure A.1. There exists shortest rectilinear path from P_0 to sink points P_1 and P_2 that is not blocked by any the barriers and, therefore, they are " visible" to each other. P_3 is invisible to P_0 because all the possible rectilinear ways between them is blocked by the first barrier. When it comes to P_4, none the barriers can individually obstruct all possible rectilinear to P_0. However, combination of the two barriers can effectively block all possible paths between P_0 and P_4 making them invisible to each other.

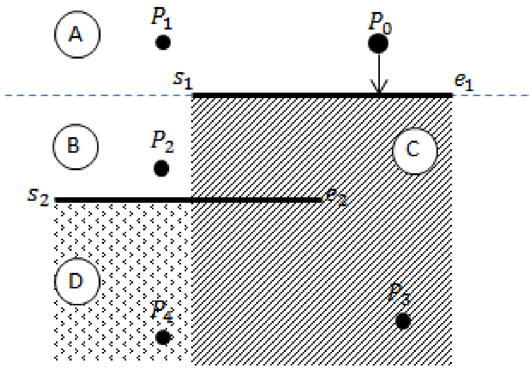

Figure A.1: Invisibility shadows of 2 barriers with cascade arrangement

To study the effect of barriers on visibility of any point on the plane in this example, we divide the plane into 4 regions of A, B, C, and D. No barrier is vertically between region A and P_0. That is, *y-invisibility* conditions do not hold for points in region A and they are immediately visible to P_0. For any point in region B, *y-invisibility* conditions hold but *x-invisibility* conditions do not hold because this region is not between s_1 and e_1. Both set of invisibility conditions hold for any point in region C making them invisible due to the first barrier. This region (marked with diagonal pattern) is called "invisibility shadow" caused by the first barrier. Any point that falls in this shadow, is invisible to P_0.

All points in region D are also invisible to P_0 because when we look at the arrangement of the barriers, we observe that some of the possible rectilinear ways are blocked by the first barrier while the rest of possible ways are obstructed by the second barrier. Therefore, we need to redefine invisibility conditions when there are more than one barrier on the plane.

Lemma A.1 *Suppose that y-invisibility conditions hold with regard to both barriers while x-invisibility conditions hold only with respect to points and barriers that are closer to each other. In that case, if any vertex of a vertically-closer barrier to a point is invisible to the other point, those two points are invisible to each other with respect to combination of barriers.*

In Figure A.1, both barrier are vertically between P_0 and P_4 but *x-invisibility* conditions do not hold with respect to individual barriers. When only the first barrier is considered, $P_0 \in (s_1, e_1)$ but $P_4 \notin (s_1, e_1)$. When only the seconds barrier is considered, $P_4 \in (s_2, e_2)$ but $P_0 \notin (s_2, e_2)$. We know that P_0 is vertically closer to the first barrier and P_4 is vertically closer to the second barrier. According to Lemma, since $s_1 \in (s_2, e_2)$ P_0 and P_4 are invisible to each other. The same is true if $s_2 \in (s_1, e_1)$ is considered. In fact, combination of barriers in this example, is extending the "invisibility shadow" from left side of the source point.

The arrangement of barriers plays an important role in identifying invisibility of points. In Figure A.2, $s_1, e_1 \notin (s_2, e_2)$ and $s_2, e_2 \notin (s_1, e_1)$. Therefore, P_0 and P_4 are visible to each other.

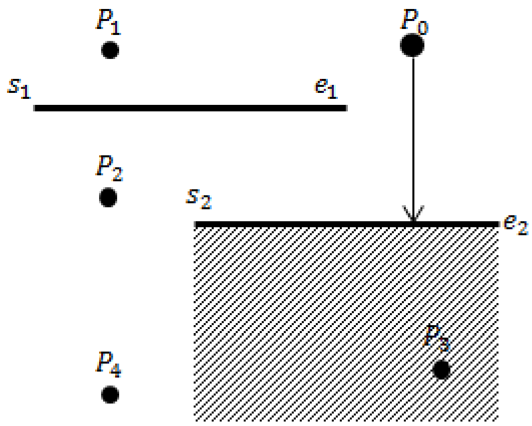

Figure A.2: The arrangement of barriers make P_0 and P_4 visible to each other.

Lemma can be extended to address invisibility issues with more than two barriers. In that case, the "invisibility shadow" for the source point will be extended from the right and left side with the help of other barriers. If a points falls in the extended "invisibility shadow" it becomes invisible to the source point.

Now, we propose an algorithm to find invisibility conditions in the presence of multiple line barriers. As the number of existing barriers increase, the computation of visibility becomes complicated and an algorithm is needed to track cascading barriers and extend "invisibility shadows" to the right and left before calling a point visible or invisible. In this section, an algorithm is introduced to determine visibility conditions when several line barriers exist. As an example, Figure A.3 shows how invisibility shadows extend to right and left through several line barriers.

Algorithm 3 A Visibility Algorithm in the Presence of Multiple Line Barriers

Require: l_k: k^{th} line barrier with vertices at (x_{s_k}, y_k) and (x_{e_k}, y_k) where $x_{s_k} < x_{e_k}$ indicating the starting and ending points, $k = 1, \ldots, K$

Require: P_0: The location of a source point at (a_0, b_0)

Require: P_i: The location of a sink point at $(a_i, b_i), i = 1, ..., n$

Require: L: The left end of the ”invisibility shadow” along x-axis

Require: R: The right end of the ”invisibility shadow” along x-axis

Require: b_1, b_2: y-coordinates defining a range along y-axis

Set all points as visible to P_0.

{Check the lower half-space (check $y_k < b_1$ in the loop)}

Set $b_1 = b_0$.

Set $L = R = a_0$.

for $k = 1$ **to** K **do**

if $(y_k < b_1)$ **then**

if $(s_k < L < e_k)$ or $(s_k < R < e_k)$ **then**

$b_2 = y_k$

for $i = 1$ **to** n **do**

if $(b_1 < b_i < b_2)$ and $(L < a_i < R)$ **then**

P_i is invisible to P_0.

end if

end for

if $s_k < L$ **then**

$L = s_k$

end if

if $e_k > R$ **then**

$R = e_k$

end if

$b_1 = b_2$

end if

end if

end for

{Repeat the above loop for the upper half-space. (this time, check $y_k > b_1$ in the loop)}

Return invisible points

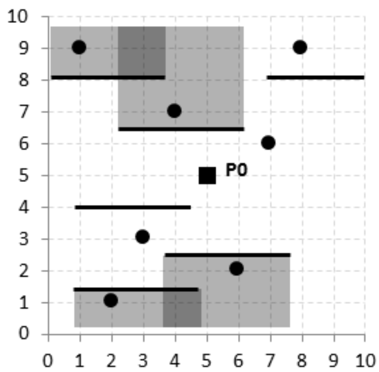

Figure A.3: Invisibility shadows with multiple line barriers

Once we know which points are invisible to the source point, we can make a graph in which nodes represent source point, sink points and barrier endpoints and edges connect nodes corresponding to a pair of visible points.

A.2 Finding the Shortest Distance between Source and Sink Points When Several Line Barriers Exist Using Floyd-Warshall's Algorithm

Floyd-Warshall algorithm is a dynamic-programming formulation to solve the all-pairs shortest-paths problem on a graph $G(V, E)$ where V and E represent the number of vertices and edges in the graph. This algorithm runs in $\Theta(V^3)$. It considers the intermediate vertices and solves the problem in a recursive fashion. Suppose that d_{ij} is the distance between any two nodes on the graph. The distance is set to infinity if the nodes are invisible to each other, which is equivalent to removing an arc from the graph. A pseudo-code for the algorithm is in below:

Algorithm 4 Floyd-Warshal's Algorithm

Require: m nodes from Source point, sink points and barrier endpoints.
Require: Direct distance between the visible pairs.

Set infinity as the distance for invisible pairs (i.e. remove the edge).
for $k = 1$ **to** m **do**
  for $i = 1$ **to** m **do**
    for $j = 1$ **to** m **do**
      $d_{ij} = \min\{d_{ij}, d_{ik} + d_{jk}\}$
    end for
  end for
end for

APPENDIX B

AN OVERVIEW ON SOLUTION PROCEDURE AND SOFTWARE PROGRAM

Following flowchart shows the solution procedure for the problems in this study. First, instances have to be created or opened in the special program developed for this thesis and they have to have a pre-defined format. Then, the parameters and instance variants are selected. The program is able to solve the problems with CPLEX 12.1 and the algorithm. However, the CPLEX optimizer has to have a valid license for above version. Therefore, sometimes it is better to extract LP models of problems using this program and solve them on a different workstation with a valid CPLEX optimizer. The algorithm results and the log files generated by CPLEX can be extracted and stored in excel files.

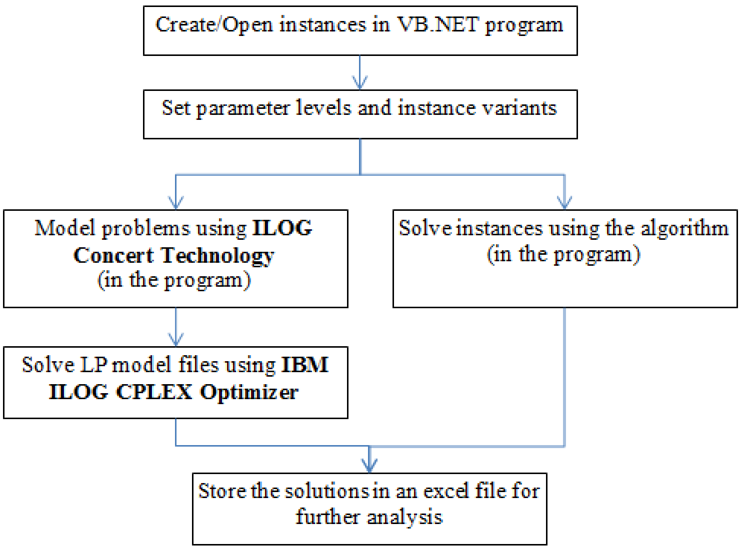

Figure B.1: The flowchart for performing computations on instances

B.1 Instance Format

All instances are stored in a tabbed text (*.txt) file. The instance data starts with sink point locations including point index (i), x-coordinate (a_i), y-coordinate(b_i) and point weight (w_i). The last two rows of instances have only 3 columns and contain the location of source points. These columns are source index (j), x-coordinate (a_j) and y-coordinate(b_j) of source points. The first source point is in the middle and the second one is at the border of the convex hull of demand points. In the one-to-many problems only one of these source points must be used and in the many-to-many problems they should be ignored.

Table B.1: The pre-defined format for instances

	index	x-coordinate	y-coordinate	Weight
	0	82	76	1
Sink points	1	96	44	1
	2	50	5	1
Middle source point	0	50	39	
Border source point	1	49.5	0	

Instances can be created or modified in an excel file and copied into a text file. Note that no excessive tab should be used in rows.

For the ease of use, a VB.NET application is developed that can show instances graphically and solve them using CPLEX Optimizer 12.1 or the algorithm. The graphical interface of this program is explained next.

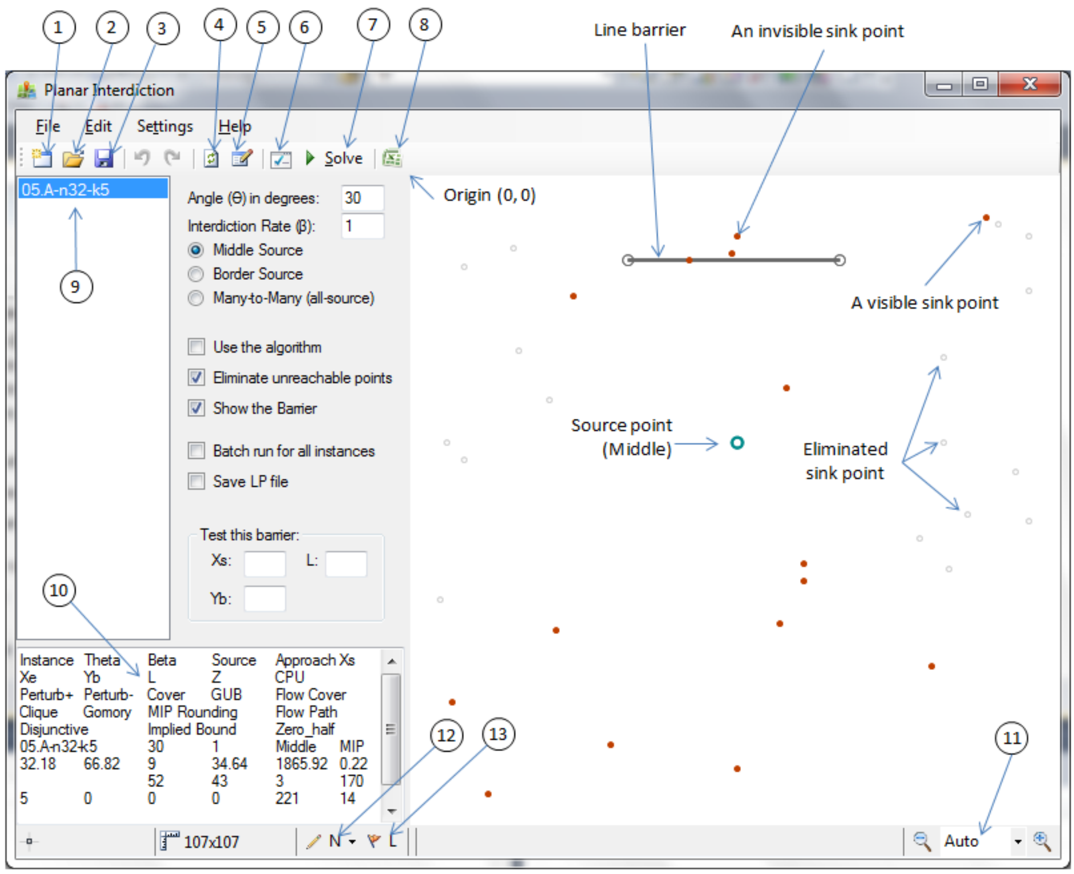

Figure B.2: The main user-interface of the barrier location program.

- Button (1) creates a blank instance. New sink or source points can be added by choosing a correct mode in (12) and then clicking on the canvas. New instances have to be saved before solving.

- Button (2) opens the instance file(s) with the proper format. The list of instances, then, appear in (9).
- Button (3) Saves the current instance in the correct instance format. Barriers on the canvas will not be saved.
- Button (4) refreshes the drawing canvas, deletes the barriers and empties the output textbox.
- Button (5) shows the details of instance data and enables us to modify values.
- Button (6) opens the settings form which contains the parameter levels and CPLEX MIP cuts. The settings form will be explained later.
- Button (7) solves the current instance based on the selected parameters. The results will be shown graphically while values will appear in the output textbox (10).
- Button (8) converts the CPLEX MIP log file into an excel file. This feature will be explained later.
- Listbox (9) contains the list of instances. An Instance will be activated by simply selecting it in this list.
- Textbox (10) contains the final solution values. It is recommended to copy the result to an excel file to view details in separate columns.
- Combobox (11) resizes the graphical shapes on the canvas. Select ”Auto” to see long barriers completely.
- Dropdown (12) enables us to graphically insert new sink/source points or move them on the plane.
- Dropdown (13) shows the labels on sink points.

B.2 Solving a Single Instance

Open the instance file you want to solve. Input your desired θ and β values in the related textboxes. For one-to-many problems, you need to select either the source point in the middle or the one at the border. If many-to-many problem is selected, all sink points will be considered as source point. The instance will be solved using CPLEX solver unless you check ”Use the algorithm”. If you want to reduce the instance size by pre-processing, check ”Eliminate unreachable points”. By clicking on ”Solve” button the solution will be computed and reported.

B.3 Solving a Batch of Instances

Select ”Batch run for all instance” if you desire to solve more than one instance. The results will be stored in a folder called ”MIP Log” in the instances' directory. In order to introduce the multiple levels for parameters, click on settings button (6). In the settings form (shown in Figure B.3), enter your desired parameter levels using comma. In the solution approach textbox, enter ”0” to use the MIP approach and ”1” if you want to use the algorithm. In source point textbox, enter ”1” for the middle source point, ”2” for the border source point and ”3” for the many-to-many variants.

When the source point is in the middle, CPU time can be reduced by partitioning the plane into upper and lower half-spaces with respect to the location of the source point. The solver will consider the half-spaces if their related checkbox is checked.

If ”Discretize Y ranges” is selected, the MIP model will consider the discretization introduced in Theorem 2.1.

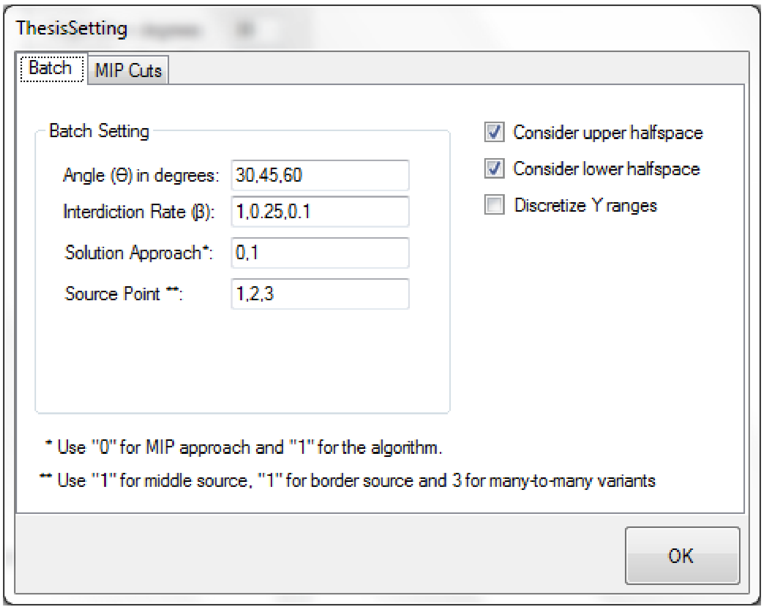

Figure B.3: The flowchart for performing computations on instances

CPLEX MIP cuts can also be enforced before solving the problems on the same workstation.

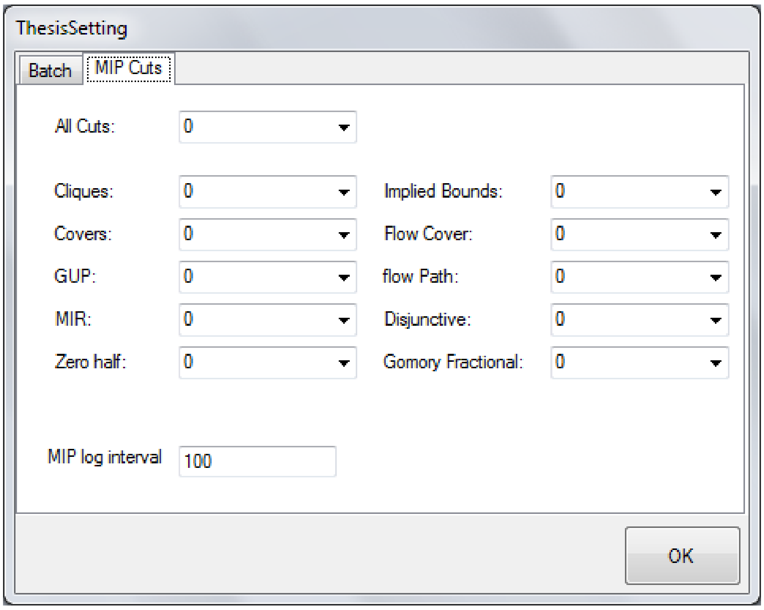

Figure B.4: The flowchart for performing computations on instances

B.4 Solving LP Files Using IBM ILOG CPLEX Optimizer

In case you do not have the license for CPLEX Optimizer 12.1, you can still solve your problems on another computer with valid CPLEX optimizer if you have the related LP files. To create only LP files without solving them, check ”Save LP file” and the LP files will be created in a folder called ”LP” in the instances’ directory. To solve LP files on a different workstation, the necessary commands can be saved in a text file and used by a batch file (*.bat) that calls CPLEX Optimizer. In this study, following CPLEX settings are saved in a file called ”cplexcommand.txt”:

```
set mip tolerances mipgap 0
set threads 1
set workmem 1000
set timelimit 1000
set mip strategy file 2
set emphasis mip 0
set output writelevel 3
Set mip cuts cliques 1
Set mip cuts covers 1
Set mip cuts disjunctive 1
Set mip cuts flowcovers 1
Set mip cuts pathcut 1
Set mip cuts gomory 1
Set mip cuts gubcovers 1
Set mip cuts implied 1
Set mip cuts mircut 1
Set mip cuts zerohalfcut 1
Set mip cuts mcfcut 1
set logfile temp.log
read mytest.lp lp
mipopt
display solution variables -
display solution objective
```

Once above command file is created, open Notepad.exe and save following script:

```
for /r ”C:\ILOG\myLP” %%f in (*.lp) do (
copy %%f mytest.lp
C:\ILOG\CPLEX101\bin\x86_win32\cplex < cplexcommand.txt
move mytest.log %%f.log
)
```

Make sure that the address of your LP folder and the directory of CPLEX optimizer are given correctly. Then change the extension of the notepad file to ”.bat” extension. Copy this windows batch file to the folder of ”cplexcommand.txt” file.

By clicking on the batch file, it will call CPLEX application and run the commands for every LP file. LP files will be copied to a temporary file (mytest) and then the related log file will be moved to the LP folder. Once log files of all LP files are collected, we can export the results into an excel file by using the button number (8) on the application.

B.5 Exporting CPLEX Log Files into an Excel File

When the export-to-excel button (number 8) is clicked on the program, following instruction will appear on the screen:

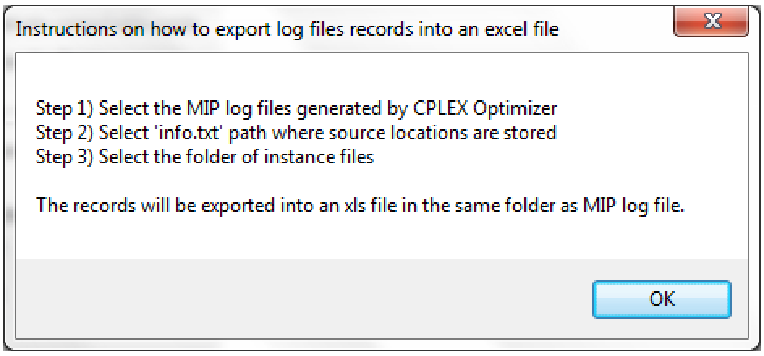

Figure B.5: The flowchart for performing computations on instances

In order to export log files into excel, 3 different files are needed:

1. CPLEX log files
2. "info.txt" file containing basic information about instances such as original objective values (Z_0), shift amount into first quadrant, Z_0 for each half-space and repeated amount in partitions, and percentages of eliminated points for each θ angle.
3. The folder of instance files is also necessary to make computations on invisible points.

For each of above steps, a dialog window will open prompting to select the related files.

APPENDIX C

COMPUTATIONAL RESULTS FOR ONE-TO-MANY INTERDICTION PROBLEM WITH A SINGLE BARRIER ON A PLANE SUBJECT TO DISRUPTION CONSTRAINT

In this appendix, computational results for one-to-many interdiction problem when $\beta < 1$ are presented. CPLEX Optimizer 10.1 is used for solving MIP models and all computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

The optimal location of the line barrier and the related objective value are reported for 1B, 1M, WB and WM variants of 30 instances and different levels of θ and β levels. x_s, x_e, y, and L represent the optimal barrier's endpoints along x-axis, its y-coordinate and length. The objective function values before and after interdiction are shown as Z_0 and ΔZ, respectively. $E\%$ shows the percentage of eliminated weights in pre-processing and $\hat{\beta}$ gives the actual disruption rate realized by the optimal solution. Since computations are terminated at the time limit of 1000 seconds, a gap percentage (Gap%) is also reported. CPU time (in seconds), number of iterations used for solving node relaxations (Niter), number of processed nodes in the active branch-and-cut search (Nodes) give the solver's performance in solving these problems. Following MIP cuts are also set in the solver with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

Number of MIP cuts (#cuts) and also the percentage of each cut used by the solver are also provided in the tables of this appendix.

Table C.1: One-to-many problem with $\beta < 1$, Core instance:D8-Canbolat

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	25	-4.5	7	11	11.5	65	0	0	0	0	45	0	36	22.2	27.8	27.8	2.8	16.7	0	0	0	2.8	0	0
		0.25	25	4	12	7.9	8	65	12	0	25	0	371	14	62	29	17.7	6.5	6.5	6.5	0	0	0	33.9	0	0
	45	0.1	25	-13	7	11	20	65	0	0	0	0	44	0	33	24.2	30.3	30.3	3	9.1	0	0	0	3	0	0
		0.25	25	1.5	18.5	9.5	17	65	24	0	25	0	468	28	80	43.8	16.2	2.5	7.5	7.5	0	0	0	22.5	0	0
	60	0.1	25	-31.1	7	12	38.1	65	0	0	0	0	40	0	39	20.5	25.6	25.6	2.6	23.1	0	0	0	2.6	0	0
		0.25	25	-7.2	22.2	9.5	29.4	65	48.9	0	25	0	338	7	59	33.9	27.1	1.7	8.5	0	1.7	0	0	27.1	0	0
1M	**30**	0.1	50	7	7	1	0	46	0	0	0	0	20	0	0	0	0	0	0	0	0	0	0	0	0	0
		0.25	50	5	9	8	4	46	8.1	0	25	0.1	167	6	30	20	13.3	3.3	6.7	3.3	10	0	0	43.3	0	0
	45	0.1	38	-3	4	1	7	46	0	0	0	0	43	0	23	21.7	26.1	30.4	4.3	13	0	0	0	4.3	0	0
		0.25	38	0.5	13.5	11	13	46	13	0	12.5	0	232	17	51	45.1	5.9	0	9.8	3.9	2	0	0	33.3	0	0
	60	0.1	25	7	7	1	0	46	0	0	0	0	45	0	24	12.5	25	29.2	4.2	25	0	0	0	4.2	0	0
		0.25	25	-1.7	15.7	9.5	17.3	46	24.6	0	25	0	269	21	71	49.3	11.3	0	12.7	5.7	1.4	0	0	19.7	0	0
WB	**30**	0.1	24	7	11	4.5	4	336	2	0	2.7	0	112	0	27	55.6	7.4	0	3.7	0	3.7	0	0	29.6	0	0
		0.25	24	1.7	13.3	11	11.5	336	52.7	0	13.5	0	258	6	52	42.3	13.5	1.9	1.9	7.7	0	0	0	32.7	0	0
	45	0.1	24	7	11	4	4	336	2	0	2.7	0	105	0	38	55.3	21.1	0	5.3	0	0	0	0	18.4	0	0
		0.25	24	-2.5	17.5	11	20	336	95	0	13.5	0	330	12	38	26.3	15.8	2.6	7.9	2.6	2.6	0	0	42.1	0	0
	60	0.1	24	7	12	4	5	336	2	0	2.7	0	129	0	32	62.5	6.2	0	9.4	0	0	0	0	21.9	0	0
		0.25	24	-9.8	24.8	11	34.6	336	168.2	0	13.5	0.1	447	7	73	43.8	19.2	5.5	8.2	0	2.7	0	0	20.5	0	0
WM	**30**	0.1	57	6	13	2	6.9	187	3.9	0	5.4	0	131	0	33	36.4	24.2	9.1	0	3	0	0	0	27.3	0	0
		0.25	57	2	9	2	6.9	187	27.5	0	18.9	0	142	0	66	31.9	18.2	10.6	0	6.1	4.5	0	0	28.8	0	0
	45	0.1	57	3.5	15.5	2	12	187	14	0	5.4	0	166	7	29	31	13.8	6.9	17.2	3.4	0	0	0	27.6	0	0
		0.25	57	-0.5	11.5	2	12	187	63	0	18.9	0	206	5	37	21.6	10.8	2.7	5.4	5.4	5.4	0	0	48.6	0	0
	60	0.1	22	-0.9	19.9	2	20.8	187	31.6	0	5.4	0	221	5	52	36.5	23.1	7.7	3.9	0	0	0	0	28.8	0	0
		0.25	22	-4.9	15.9	2	20.8	187	140.1	0	24.3	0	191	1	67	44.7	10.4	4.5	3	4.5	0	0	0	32.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.2: One-to-many problem with $\beta < 1$, Core instance:E-n22-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	18	105.7	199.3	261	93.5	1182	161.1	0	9.1	0.2	1975	75	269	46.8	21.9	8.6	6.7	1.5	0.7	0	0	13.8	0	0
		0.25	18	109.8	187.2	247	77.4	1182	253.8	0	22.7	0.4	3383	115	403	50.9	18.4	8.7	9.4	3.2	0.7	0	0	8.7	0	0
	45	0.1	5	67.5	229.5	261	162	1182	298	0	9.1	0.5	3980	69	439	41.5	19.8	15.7	9.1	2.7	0.5	0	0	10.7	0	0
		0.25	5	81.5	215.5	247	134	1182	537	0	22.7	0.4	3899	121	484	49.6	16.3	13	9.5	2.3	0.2	0	0	9.1	0	0
	60	0.1	5	8.2	288.8	261	280.6	1182	535.2	0	9.1	0.3	3498	99	369	49.9	15.2	14.9	12.2	0.5	0.3	0	0	7	0	0
		0.25	5	32.5	264.5	247	232.1	1182	1027.5	0	22.7	0.4	3535	59	438	46.6	18	12.1	11.4	2.1	0.2	0	0	9.6	0	0
1M	**30**	0.1	18	127.1	177.9	261	50.8	722	75.6	0	9.1	0.2	1196	50	291	62.2	11	6.9	3.4	3.1	1.4	0	0	12	0	0
		0.25	18	127.1	177.9	261	50.8	722	75.6	0	9.1	0.3	2081	109	272	54.4	16.2	4.4	2.6	3	1.4	0	0	18	0	0
	45	0.1	14	108.5	196.5	261	88	722	150	0	9.1	0.2	1480	52	314	54.5	16.5	8.3	3.8	2.5	0.7	0	0	13.7	0	0
		0.25	14	103	173	252	70	722	176	0	18.2	0.3	2281	103	311	58.2	16.4	2.6	2.6	1.9	1	0	0	17.4	0	0
	60	0.1	9	76.3	228.7	261	152.4	722	278.8	0	9.1	0.2	1306	20	261	56.7	19.5	3.4	2.7	1.5	1.1	0	0	15	0	0
		0.25	9	96.5	200.5	247	103.9	722	386.6	0	22.7	0.3	1765	67	251	48.6	12.8	2.8	6	7.6	2.4	0	0	19.9	0	0
WB	**30**	0.1	21	101.7	195.3	261	93.5	6418	587.7	0	6.2	0.3	2524	36	347	50.7	21	8.1	8.1	3.2	1.2	0	0	7.8	0	0
		0.25	21	96.4	179.6	252	83.1	6418	-6410.1	0	21.2	0.4	3687	203	358	55	18.7	5.6	5	5	0.8	0	0	9.8	0	0
	45	0.1	3	67.5	229.5	261	162	6418	1067	0	6.2	0.4	2997	44	383	47.8	22.2	14.1	7.6	0.8	0.3	0	0	7.3	0	0
		0.25	3	66	210	252	144	6418	2934	0	21.2	0.4	2407	74	299	36.5	19.4	12.4	11	4.7	1	0	0	15.1	0	0
	60	0.1	3	8.2	288.8	261	280.6	6418	1897.1	0	6.2	0.4	2871	53	462	51.1	20.8	11.9	6.3	3.7	0.6	0	0	5.6	0	0
		0.25	3	13.3	262.7	252	249.4	6418	5464	0	21.2	0.4	3675	97	438	47.9	16.2	11.9	9.6	3	0.7	0	0	10.7	0	0
WM	**30**	0.1	19	124.4	177.6	185	53.1	3854	448.3	0	9.7	0.2	1363	52	337	56.7	14	8	6.5	2.7	0.3	0	0	11.9	0	0
		0.25	19	129.1	172.9	193	43.9	3854	579.3	0	19.5	0.3	1694	68	300	56.7	16.3	3.7	4	3.3	2	0	0	14	0	0
	45	0.1	13	105	197	185	92	3854	876	0	9.7	0.2	1370	39	291	52.9	13.4	6.2	6.9	5.8	0	0	0	14.8	0	0
		0.25	13	113	189	193	76	3854	1414	0	23	0.3	1679	56	275	52	16.4	2.9	4.7	1.1	1.5	0	0	21.4	0	0
	60	0.1	9	71.3	230.7	185	159.3	3854	1616.8	0	9.7	0.2	1302	32	264	54.9	13.6	5.7	3	6.1	0	0	0	16.7	0	0
		0.25	9	85.2	216.8	193	131.6	3854	2860.5	0	23	0.7	1434	42	290	44.5	20.7	7.3	6.6	2.4	1.4	0	0	17.3	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.3: One-to-many problem with $\beta < 1$, Core instance:D28

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	7	-107.2	359.2	402	466.5	10403	617	0	7.1	0.5	4423	173	529	52.2	20	12.7	6.4	1.3	0	0	0	7.4	0	0
		0.25	7	-34.7	366	345	400.7	10403	1632.1	0	25	0.6	6119	337	438	43.2	16.4	13.5	14.6	6.4	1.4	0	0	4.6	0	0
	45	0.1	0	-284	536	408	820	10403	1002	0	7.1	0.6	4106	124	616	53.1	16.2	19.8	3.4	1	0	0	0	6.5	0	0
		0.25	0	-129.5	628.5	377	758	10403	3483	0	25	0.6	7109	409	419	46.1	16.2	15.8	7.4	4.1	0.7	0	0	9.8	0	0
	60	0.1	0	-584.1	836.1	408	1420.3	10403	2202.6	0	7.1	0.5	4167	130	441	36.7	29.5	17.9	5.9	0.9	0.9	0	0	8.2	0	0
		0.25	0	-406.9	905.9	377	1312.9	10403	7367.3	0	25	0.6	6724	214	597	39.5	20.4	16.9	11.9	3	1.2	0	0	7	0	0
1M	**30**	0.1	43	-15.6	247.6	82	263.3	5435	366.5	0	7.1	0.1	1782	92	261	50.5	18	6.1	8.8	3.9	0.4	0	0	12.2	0	0
		0.25	43	26.7	240.3	125	213.6	5435	741.1	0	17.9	0.2	1189	49	254	57.1	15.7	2	4.8	1.9	0.4	0	0	18.1	0	0
	45	0.1	39	-112	344	82	456	5435	752	0	7.1	0.1	1268	32	185	51.9	10.8	7.6	4.8	7	0	0	0	17.8	0	0
		0.25	39	-51.5	318.5	125	370	5435	1523	0	17.9	0.2	1275	63	187	40.1	18.2	6.4	6.4	5.9	0	0	0	23	0	0
	60	0.1	21	-278.9	510.9	82	789.8	5435	1419.6	0	7.1	0.3	1970	51	415	62.9	10.9	6.2	6.5	2.9	0	0	0	10.6	0	0
		0.25	21	-177.9	462.9	125	640.9	5435	3073.2	0	21.4	0.3	1896	89	319	60.5	10.7	1.6	4.7	5.7	0.3	0	0	16.6	0	0
WB	**30**	0.1	6	52.3	518.7	402	466.5	59705	3257	0	9	0.4	2931	61	553	59	17	15	2.7	1.3	0	0	0	5.1	0	0
		0.25	6	-53.2	366	361	419.2	59705	8144.5	0	20	0.6	6391	319	432	37.7	19.2	14.1	16.4	5.3	0.7	0	0	6.5	0	0
	45	0.1	0	-124.5	695.5	408	820	59705	6960	0	7.7	0.4	3187	93	378	47.4	16.9	22.2	4.5	1.1	0	0	0	7.9	0	0
		0.25	0	-129.5	628.5	377	758	59705	20242	0	24.5	0.7	5033	149	698	55.6	17.2	14.9	4.9	2.7	0.4	0	0	4.3	0	0
	60	0.1	0	-424.6	995.6	408	1420.3	59705	14163.4	0	7.7	0.5	4004	79	473	41.2	21.4	25.6	5.5	1.3	0	0	0	5.1	0	0
		0.25	0	-406.9	905.9	377	1312.9	59705	41328	0	24.5	0.6	5882	164	498	45.4	18.9	16.7	6.8	2	1.4	0	0	8.8	0	0
WM	**30**	0.1	49	10.9	274.1	82	263.3	27764	2671.8	0	9	0.1	844	13	202	52	17.3	5.9	2.5	5.5	0.5	0	0	16.3	0	0
		0.25	49	26.7	240.3	125	213.6	27764	3466.9	0	15.5	0.1	911	24	243	58.4	12.3	3.7	5.3	2.9	2.9	0	0	14.4	0	0
	45	0.1	43	-85.5	370.5	82	456	27764	5370	0	9	0.1	993	24	231	56.3	14.3	3.1	2.6	6.1	0.8	0	0	16.9	0	0
		0.25	43	-51.5	318.5	125	370	27764	7220	0	15.5	0.2	1072	40	170	51.2	12.9	2.4	4.7	3	1.2	0	0	24.7	0	0
	60	0.1	19	-252.4	537.4	82	789.8	27764	10043.4	0	9	0.2	1600	33	265	49.8	16.2	7.6	4.1	5.3	0	0	0	17	0	0
		0.25	19	-177.9	462.9	125	640.9	27764	14504	0	18.1	0.3	1856	81	316	53.2	10.8	2.5	7.6	7	1	0	0	18	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.4: One-to-many problem with $\beta < 1$, Core instance:B-n31-k5

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	90	-7.7	74.2	76	82	1694.5	49.5	0	3.2	0	58	0	13	0	0	0	7.7	15.4	0	0	0	76.9	0	0
		0.25	90	-7.7	74.2	76	82	1694.5	49.5	0	3.2	0	58	0	13	0	0	0	7.7	15.4	0	0	0	76.9	0	0
	45	0.1	23	-37.8	104.2	76	142	1694.5	109.5	0	3.2	0.3	1627	16	439	62.6	11.6	12.8	2.1	4.1	0	0	0	6.8	0	0
		0.25	23	-37.8	104.2	76	142	1694.5	109.5	0	3.2	0.5	4689	216	355	42	20	18.6	5.1	2.8	0.8	0	0	10.7	0	0
	60	0.1	19	-89.7	156.2	76	246	1694.5	213.5	0	3.2	0.5	4107	170	501	48.7	18	18.8	3.8	4.2	0.2	0	0	6.4	0	0
		0.25	19	-21.5	72	32	93.5	1694.5	309.2	0	22.6	0.5	4789	321	377	45.6	17.2	19.1	2.4	4	2.7	0	0	9	0	0
1M	**30**	0.1	48	-9.8	46.8	76	56.6	774	53.6	0	3.2	0.1	764	41	109	45.9	12.9	1.9	4.6	14.7	2.8	0	0	17.5	0	0
		0.25	48	14	35.9	8	21.9	774	61.6	0	19.4	0.2	1217	24	277	53.1	15.8	7.9	6.9	2.9	1.8	0	0	11.6	0	0
	45	0.1	42	-30.5	67.5	76	98	774	95	0	3.2	0.2	958	35	196	53.5	14.3	8.7	3.6	6.1	0	0	0	13.7	0	0
		0.25	42	6	44	8	38	774	158	0	19.4	0.2	1074	18	232	52.6	17.3	4.7	3.4	3.4	0	0	0	18.6	0	0
	60	0.1	23	-11.6	57.6	7	69.3	774	179.8	0	9.7	0.3	1390	31	271	42.8	16.6	8.1	8.9	3.7	0.7	0	0	19.2	0	0
		0.25	23	-7.9	57.9	8	65.8	774	324.9	0	19.4	0.3	1094	18	296	55.4	14.2	5.4	3.1	4	1.7	0	0	16.2	0	0
WB	**30**	0.1	93	-7.7	74.2	76	82	8595.5	99	0	1.2	0	75	0	34	52.9	14.7	0	5.9	0	0	0	0	26.5	0	0
		0.25	93	-7.7	74.2	76	82	8595.5	99	0	1.2	0	75	0	34	52.9	14.7	0	5.9	0	0	0	0	26.5	0	0
	45	0.1	26	-37.8	104.2	76	142	8595.5	219	0	1.2	0.5	3227	97	413	53.5	15	16.5	3.9	2.4	0.2	0	0	8.5	0	0
		0.25	26	14	56	26	42	8595.5	427	0	24.8	0.6	4987	270	513	45.4	11.3	12.7	16.4	2.5	0.8	0	0	10.9	0	0
	60	0.1	21	-30.7	80.2	37	110.9	8595.5	736.2	0	7.5	0.4	3149	183	377	49.6	12.5	21.2	5	4.2	1.3	0	0	6.1	0	0
		0.25	21	-22	71.5	32	93.5	8595.5	1585.1	0	22.4	0.6	6266	398	428	48.1	13.8	14.7	6.1	3.5	2.8	0	0	11	0	0
WM	**30**	0.1	53	11.4	35.6	7	24.2	3739	288	0	9.9	0.2	797	9	149	52.4	12.1	8.8	3.4	7.4	0	0	0	16.1	0	0
		0.25	53	13	36	8	23.1	3739	447.2	0	21.1	0.2	954	15	171	43.3	15.2	7	7.6	2.9	1.8	0	0	22.3	0	0
	45	0.1	48	2.5	44.5	7	42	3739	572	0	9.9	0.2	1265	31	174	42	16.7	5.8	9.7	3.4	3.4	0	0	18.9	0	0
		0.25	48	4.5	44.5	8	40	3739	1022	0	21.1	0.2	1111	8	247	42.1	21.5	9.7	4.4	2	2.4	0	0	17.8	0	0
	60	0.1	29	-12.9	59.9	7	72.7	3739	1063.9	0	9.9	0.2	1310	24	179	46.4	13.4	5.6	2.3	4.5	1.1	0	0	26.8	0	0
		0.25	29	-10.1	59.1	8	69.3	3739	2017.6	0	21.1	0.2	1163	3	204	29.4	20.1	9.8	2.9	6.9	1.5	0	0	29.4	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.5: One-to-many problem with $\beta < 1$, Core instance:A-n32-k5

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	30	0.1	31	-12.1	90.6	89	102.8	2315	205.8	0	9.4	0.4	2429	90	401	51.4	18.2	13.7	5.2	4	0.7	0	0	6.7	0	0
		0.25	31	21.2	89.3	59	68.1	2315	265.4	0	21.9	0.4	4238	207	467	47.8	18.8	13.1	6.2	2.6	2.8	0	0	8.8	0	0
	45	0.1	19	-49.8	128.2	89	178	2315	431.5	0	9.4	0.4	2822	55	521	40.5	21.5	24	4	3.3	0.2	0	0	6.5	0	0
		0.25	19	-29.8	108.2	69	138	2315	631.5	0	21.9	0.5	4414	184	427	49.9	17.1	12.4	5.6	5.6	1.2	0	0	8.2	0	0
	60	0.1	19	-114.9	193.4	89	308.3	2315	822.4	0	9.4	0.4	2800	56	454	41	24.2	16.5	7.3	2.4	0	0	0	8.6	0	0
		0.25	19	-77.8	147.3	65	225.2	2315	1390.3	0	25	0.6	3955	148	438	39.7	17.4	17.1	10	6.4	0.7	0	0	8.7	0	0
1M	30	0.1	53	32.2	66.8	9	34.6	1779	86.9	0	9.4	0.1	942	35	167	41.9	17.4	6.6	3.6	1.2	0.6	0	0	28.7	0	0
		0.25	53	32.2	66.8	9	34.6	1779	86.9	0	9.4	0.1	1015	34	222	57.7	9.9	5	3.2	2.3	0.5	0	0	21.6	0	0
	45	0.1	28	-10.5	89.5	89	100	1779	197	0	9.4	0.2	1750	58	385	61.3	15.1	4.7	3.4	4.1	1	0	0	10.4	0	0
		0.25	28	-10.5	89.5	89	100	1779	197	0	9.4	0.3	2611	112	469	68.8	9.6	2.3	5.4	0.4	1.7	0	0	11.7	0	0
	60	0.1	19	-47.1	126.1	89	173.2	1779	416.6	0	9.4	0.3	2659	89	552	58.9	15.2	5.5	5.4	4.5	0.9	0	0	9.6	0	0
		0.25	19	-35	114	82	149	1779	419.8	0	12.5	0.4	3381	177	476	60.3	13.9	2.3	7.3	3.6	1.2	0	0	11.4	0	0
WB	30	0.1	32	-24.4	82.9	93	107.4	15836	997.3	0	7	0.4	2227	64	378	59.3	11.1	12.4	6.1	5.6	0	0	0	5.6	0	0
		0.25	32	-5.1	74.6	69	79.7	15836	1778.4	0	22.4	0.5	4076	175	416	45.2	15.6	15.6	11.3	2.6	1.4	0	0	8.2	0	0
	45	0.1	17	-63.8	122.2	93	186	15836	2176.5	0	7	0.5	4435	114	589	49.9	18.3	17.3	5.9	3.2	0	0	0	5.3	0	0
		0.25	17	-35.2	104.8	70	140	15836	4319.5	0	22.9	0.6	4498	93	532	48.1	19.9	13.2	8.1	3.6	0.6	0	0	6.6	0	0
	60	0.1	17	-131.8	190.3	93	322.2	15836	4218.9	0	7	0.5	3755	123	512	36.1	27	20.1	5.9	3.5	0	0	0	7.4	0	0
		0.25	17	-86.5	156	70	242.5	15836	9341.4	0	22.9	0.6	4492	171	448	44.9	15	15.4	9.6	6	0.9	0	0	8.3	0	0
WM	30	0.1	56	33.9	73.1	8	39.3	12280	407.4	0	6.1	0.1	882	31	196	54.1	15.8	5.6	2.6	3.6	0	0	0	18.4	0	0
		0.25	56	33.9	72.1	9	38.1	12280	552.4	0	10.7	0	787	38	120	44.2	12.5	1.7	6.6	5	0.9	0	0	29.2	0	0
	45	0.1	29	9	103	89	94	12280	852	0	7	0.4	2371	109	354	55.1	14.7	6.8	4.8	5.6	0.3	0	0	12.7	0	0
		0.25	29	-4	90	89	94	12280	1224	0	11.2	0.3	2475	108	405	58	14.8	7.6	3.7	1.5	1	0	0	13.3	0	0
	60	0.1	19	-55.3	121.3	93	176.7	12280	1930	0	7	0.5	3624	136	480	58.1	14.8	5.5	5.7	2.7	0.6	0	0	12.7	0	0
		0.25	19	-3.7	110.7	9	114.3	12280	3053.8	0	22	0.4	2942	127	559	65.1	12.2	1.4	5.4	2.3	2.7	0	0	10.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.6: One-to-many problem with $\beta < 1$, Core instance:D40

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	18	-106.2	348.7	407	455	11645	1521.8	0	10	0.7	5212	153	764	56.3	13.7	15.4	6.7	1	0.1	0	0	6.7	0	0
		0.25	18	-60.6	333.1	354	393.8	11645	3138.5	0	25	0.9	8305	251	673	41.8	17.2	15.8	14.6	2.8	0.3	0	0	7.6	0	0
	45	0.1	10	-272.8	515.2	407	788	11645	2854	0	10	0.9	7377	173	855	43.3	21.6	18.2	7.1	2	0.6	0	0	7.1	0	0
		0.25	10	-204.8	477.2	354	682	11645	6021	0	25	1.1	9333	251	859	46.2	17.7	11.6	14.9	2.8	0.3	0	0	6.4	0	0
	60	0.1	5	-561.2	803.7	407	1364.9	11645	5161.4	0	10	0.9	5571	86	1009	50.4	18.6	17.1	5.7	1.5	0.1	0	0	6.4	0	0
		0.25	5	-457.9	723.4	354	1181.3	11645	11013.6	0	25	1	7134	138	842	42.4	17.1	12.1	19.2	1.2	0.1	0	0	7.8	0	0
1M	**30**	0.1	23	46.4	323.6	45	277.1	6058	844.5	0	10	0.4	2704	127	348	51.7	16.1	6.4	2.9	6.9	2.6	0	0	13.5	0	0
		0.25	23	60.9	309.1	70	248.3	6058	1320.1	0	20	0.5	3732	216	568	70.4	10	1.8	4	1.2	0.5	0	0	12	0	0
	45	0.1	10	-55	425	45	480	6058	1656	0	10	0.5	2762	89	587	60.6	15.3	8	3.4	4.6	0.3	0	0	7.7	0	0
		0.25	10	-32	398	70	430	6058	2774	0	20	0.8	4993	179	681	57.8	15.9	7.8	4.6	1.9	0.5	0	0	11.6	0	0
	60	0.1	10	-235.2	596.2	45	831.4	6058	3061.5	0	10	0.7	3716	101	690	55.2	15.5	11.9	3.8	5.4	0	0	0	8.2	0	0
		0.25	10	-189.4	555.4	70	744.8	6058	5292.3	0	20	0.6	4232	132	618	58.5	15.2	7.6	5	2.1	0.8	0	0	10.7	0	0
WB	**30**	0.1	36	-79.1	351.6	386	430.7	14076.5	2054.2	0	9.8	0.9	9729	249	822	43.7	17.4	16.9	14.2	1	0.2	0	0	6.6	0	0
		0.25	36	-48.5	321	333	369.5	14076.5	4331.1	0	24.6	0.9	6385	247	538	40.5	15.6	15.2	12.6	5.6	1.1	0	0	9.3	0	0
	45	0.1	23	-236.8	509.2	386	746	14076.5	3946	0	9.8	0.9	7816	190	902	45.3	18.5	17.3	11.3	1	0.3	0	0	6.2	0	0
		0.25	23	-183.8	456.2	333	640	14076.5	8388.5	0	24.6	1	7019	223	720	45	15.8	11.9	10.7	4.3	2.2	0	0	10	0	0
	60	0.1	15	-524.8	767.3	386	1292.1	14076.5	7222.7	0	9.8	0.8	5642	82	875	42.3	19.4	16.8	13.9	0.5	0.9	0	0	6.2	0	0
		0.25	15	-418	690.5	333	1108.5	14076.5	15416.2	0	24.6	1.1	7470	154	860	46.9	15.3	10.7	16.5	2.9	0.6	0	0	7.1	0	0
WM	**30**	0.1	15	21.9	329.1	19	307.2	11731	1608.9	0	9.8	0.4	3122	115	483	59.4	17	5.2	3.5	2	1.2	0	0	11.6	0	0
		0.25	15	41.9	319.1	45	277.1	11731	2761.7	0	21.3	0.5	3913	205	489	59.1	10.8	5.5	7.6	3.5	1	0	0	12.5	0	0
	45	0.1	7	-90.5	441.5	19	532	11731	2958	0	9.8	0.4	3210	100	597	65.7	11.3	8.2	2.7	4.2	0.8	0	0	7.2	0	0
		0.25	7	-59.5	420.5	45	480	11731	5399	0	21.3	0.7	4921	257	649	57.2	13.3	8.6	5.6	3.3	1.3	0	0	10.8	0	0
	60	0.1	7	-285.2	636.2	19	921.5	11731	5294.7	0	9.8	0.4	2537	69	427	52.4	13.6	10.1	1.6	8.2	1	0	0	13.1	0	0
		0.25	7	-235.2	596.2	45	831.4	11731	9967	0	21.3	0.6	4368	138	691	56.2	12.6	11.2	5.5	2.8	1.2	0	0	10.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.7: One-to-many problem with $\beta < 1$, Core instance:B-n41-k6

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	20	26	110	78.7	84	3362	272	0	9.8	0.7	6898	280	702	46.7	22.1	17.8	6	2.3	0	0	0	5.1	0	0
		0.25	20	20.9	104.1	78	83.1	3362	385.1	0	19.5	1	11866	467	609	48.6	13.6	15.6	6.7	4.3	2.8	0	0	8.4	0	0
	45	0.1	12	-49	133	97	182	3362	568	0	9.8	1.2	10421	258	938	46.6	21.7	18.4	5.8	1.3	0.2	0	0	6	0	0
		0.25	12	-8.5	133.5	77	142	3362	952	0	24.4	1.1	11833	388	634	49.5	16.6	15.3	5.7	6.6	0.9	0	0	5.4	0	0
	60	0.1	12	-115.6	199.6	97	315.2	3362	1100.9	0	9.8	1	7810	173	840	44	23.2	19.5	6.7	1.2	0	0	0	5.4	0	0
		0.25	12	-60.5	185.5	77	246	3362	1991.5	0	24.4	0.9	7074	196	687	44.1	18.5	18.9	5.1	4.7	2.3	0	0	6.4	0	0
1M	**30**	0.1	51	40	77	32	37	2197	101.8	0	9.8	0.3	1811	88	342	58.1	11.7	4.7	7.6	2.3	0.3	0	0	15.2	0	0
		0.25	51	42.3	74.7	36	32.3	2197	138	0	14.6	0.2	1381	71	294	58.2	8.8	3.8	5.1	3.7	2.4	0	0	18	0	0
	45	0.1	39	25	89	32	64	2197	210	0	9.8	0.3	2879	156	401	56.6	15.2	4	6.5	3.8	0.2	0	0	13.7	0	0
		0.25	39	30.5	86.5	36	56	2197	293	0	17.1	0.4	3784	412	354	61.3	7.1	0.8	11	2.6	0.9	0	0	16.3	0	0
	60	0.1	39	-44.1	122.1	16	166.3	2197	475.1	0	9.8	0.3	2452	118	366	56.8	14	4.7	7.3	3.9	0.2	0	0	13.1	0	0
		0.25	39	-13.2	94.2	33	107.4	2197	633.9	0	24.4	0.4	3465	274	394	64.2	8.6	1	6.9	3.3	1.8	0	0	14.2	0	0
WB	**30**	0.1	15	-3.8	89.8	87	93.5	4324	263.7	0	9.3	0.6	3156	61	548	43.6	26.5	19.3	4	2.6	0.2	0	0	3.8	0	0
		0.25	15	24	109.4	80	85.4	4324	800	0	24.1	0.6	4098	115	740	44.5	16.1	17.6	12	4.2	1.5	0	0	4.2	0	0
	45	0.1	9	-38	124	87	162	4324	606	0	9.3	0.6	4101	79	649	51.8	23	15.9	3.5	1.8	0	0	0	4	0	0
		0.25	9	-11	141	82	152	4324	1544	0	24.1	1	7287	152	755	38.7	21.5	20.1	9.8	2.9	1.5	0	0	5.6	0	0
	60	0.1	9	-97.3	183.3	87	280.6	4324	1199	0	9.3	0.6	3447	48	556	51.6	16.4	18	5.8	1.1	1.1	0	0	6.1	0	0
		0.25	9	-66.6	196.6	82	263.3	4324	2990.5	0	24.1	0.9	5012	186	720	44.4	20.8	19.4	7.1	1.8	0.7	0	0	5.7	0	0
WM	**30**	0.1	41	36.9	75.1	31	38.1	2582	78.3	0	5.6	0.3	1821	66	378	61.4	14.6	5	4.8	1.3	0.3	0	0	12.7	0	0
		0.25	41	42.3	74.7	36	32.3	2582	220	0	16.7	0.2	1309	34	293	60.1	10.2	2.1	4.1	2.7	0.3	0	0	20.5	0	0
	45	0.1	31	-6.5	87.5	17	94	2582	239	0	9.3	0.3	2876	142	376	59.8	13.3	5.1	5.6	5.1	0	0	0	11.2	0	0
		0.25	31	30.5	86.5	36	56	2582	446	0	18.5	0.3	2308	133	320	56.2	8.7	1.3	9.1	4.7	4.4	0	0	15.6	0	0
	60	0.1	31	-40.9	121.9	17	162.8	2582	583.1	0	9.3	0.3	3192	113	396	53.1	16.7	4.6	7.9	4	0.8	0	0	13.1	0	0
		0.25	31	2.5	106.5	34	103.9	2582	816	0	24.1	0.3	2846	201	339	57.2	9.1	1.2	10	1.5	1.7	0	0	19.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.8: One-to-many problem with $\beta < 1$, Core instance:A-n45-k6

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	22	2.8	106.7	94	103.9	3373.5	310.7	0	8.9	0.6	5842	232	670	54.3	18.2	15.7	3.1	2.8	0.1	0	0	5.7	0	0
		0.25	22	12.3	86.2	68	73.9	3373.5	392.6	0	20	0.9	9094	407	661	46.6	17.1	14.1	10.9	2.9	2.3	0	0	6.2	0	0
	45	0.1	22	-36.2	145.8	95	182	3373.5	561	0	8.9	0.8	5817	82	748	55.1	18.9	16.2	3.1	1.5	0.1	0	0	5.2	0	0
		0.25	22	-21.2	130.8	80	152	3373.5	943	0	22.2	0.9	7507	254	659	47.3	17.1	17.1	6.8	5.6	0.9	0	0	5	0	0
	60	0.1	16	-106.9	208.4	95	315.2	3373.5	1093.9	0	8.9	0.9	6776	128	804	39.8	21.5	18.2	10.4	2.5	0	0	0	7.6	0	0
		0.25	16	-80.9	182.4	80	263.3	3373.5	2055.7	0	22.2	1.1	8368	232	852	47.9	18.4	14	8.6	4.5	0.8	0	0	5.9	0	0
1M	**30**	0.1	27	27	79	94	52	2453	95.8	0	8.9	0.3	2196	70	358	52	14	6.4	6.1	4.7	1.4	0	0	15.4	0	0
		0.25	27	27	79	94	52	2453	95.8	0	8.9	0.3	1916	63	323	52	13.6	5.9	5.9	2.5	0	0	0	20.1	0	0
	45	0.1	27	6	96	94	90	2453	248	0	8.9	0.5	3959	219	452	51.3	14.8	5.3	5.5	8	2.4	0	0	12.6	0	0
		0.25	27	8	98	94	90	2453	254	0	11.1	0.6	5017	197	539	50.6	15.9	8.5	3.1	4.8	2.2	0	0	14.6	0	0
	60	0.1	13	-26.7	132.7	95	159.3	2453	463.4	0	8.9	0.6	6001	202	770	58.7	13.9	10.8	4.6	4.2	0.5	0	0	7.4	0	0
		0.25	13	-28.9	126.9	94	155.9	2453	635.3	0	13.3	0.8	7920	423	745	55.6	11.4	12.2	5.4	2.4	1.1	0	0	11.9	0	0
WB	**30**	0.1	21	2.8	106.7	94	103.9	18122.5	1236	0	8	0.7	6681	339	536	50.7	14.7	11.9	7.6	5.6	0.9	0	0	8.4	0	0
		0.25	21	22.7	75.8	50	53.1	18122.5	1376.5	0	18.1	1	9854	678	671	42.9	22.8	15.5	8.2	2.8	0.4	0	0	7.3	0	0
	45	0.1	21	-35.2	144.8	94	180	18122.5	2971.5	0	9.7	0.9	7551	150	769	46.6	19	18.2	7.2	2.2	0.5	0	0	6.4	0	0
		0.25	21	-25.2	126.8	80	152	18122.5	4132	0	20.3	1.1	10406	491	633	40.8	23.4	14.5	8.7	5.1	0.8	0	0	6.8	0	0
	60	0.1	12	-101.1	210.6	94	311.8	18122.5	6002.2	0	9.7	0.8	7139	154	850	45.9	22	16.5	5.6	2.9	0.6	0	0	6.5	0	0
		0.25	12	-80.9	182.4	80	263.3	18122.5	9473	0	20.3	0.9	6747	198	705	43.7	18.9	13.3	10.4	4.8	2	0	0	7	0	0
WM	**30**	0.1	34	30.3	78.7	95	48.5	12959	366	0	5.1	0.3	1653	61	292	46.9	17.1	4.8	5.8	4.8	0	0	0	20.6	0	0
		0.25	34	30.3	78.7	95	48.5	12959	366	0	5.1	0.3	1853	53	349	54.7	11.8	5.7	6.3	2.9	1.1	0	0	17.5	0	0
	45	0.1	34	13.5	95.5	94	82	12959	811	0	8	0.5	4837	223	532	42.5	18.4	10.7	7.4	6.2	1.9	0	0	13	0	0
		0.25	34	13.5	95.5	94	82	12959	811	0	8	0.6	5091	177	638	52.2	13.8	8	6.8	4.6	1.3	0	0	13.5	0	0
	60	0.1	19	-16.5	125.5	94	142	12959	2091.6	0	9.7	0.7	6803	259	660	48.6	17.9	10.8	3.9	6.3	1.5	0	0	10.9	0	0
		0.25	19	-19.9	104.9	17	124.7	12959	2264	0	20.3	0.9	7852	388	735	48	14.4	10.3	12.3	2.5	1	0	0	11.6	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.9: One-to-many problem with $\beta < 1$, Core instance:F-n45-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	30	0.1	2	-80.5	87	245	167.4	5879	549.7	0	8.9	1.4	10387	277	871	37.1	22.6	22.8	9.2	2	0.5	0	0	5.9	0	0
		0.25	2	-61.9	64	209	125.9	5879	910.5	0	24.4	1.7	20073	746	1029	46.2	20.5	12.3	12.1	4.1	0.2	0	0	4.6	0	0
	45	0.1	2	-141.8	148.2	245	290	5879	1040	0	8.9	1	7550	189	973	42.3	25.9	17.1	7	1.3	0.2	0	0	6.2	0	0
		0.25	2	-109.2	116.8	213	226	5879	1729.5	0	24.4	1.8	15653	617	887	43.7	15.8	15.1	16.3	2.9	0.1	0	0	6	0	0
	60	0.1	2	-247.6	254.6	245	502.3	5879	1889.2	0	8.9	1.1	7433	147	938	41.2	21.3	20.1	10	0.7	0.2	0	0	6.4	0	0
		0.25	2	-192	199.5	213	391.4	5879	3549.4	0	24.4	1.7	16570	441	981	35.6	21.3	15	19.4	2.2	0	0	0	6.5	0	0
1M	30	0.1	64	-23.8	36.3	252	60	2157.5	90.6	0	6.7	0	434	5	97	39.2	12.4	3.1	4.1	6.2	2.1	0	0	33	0	0
		0.25	64	-23.8	36.3	252	60	2157.5	90.6	0	6.7	0	370	6	60	35	6.7	1.7	6.7	3.3	0	0	0	46.7	0	0
	45	0.1	64	-45.7	58.2	252	104	2157.5	222.4	0	6.7	0.1	966	16	201	56.7	10	2.5	6	1	0	0	0	23.8	0	0
		0.25	64	-45.7	58.2	252	104	2157.5	222.4	0	6.7	0.1	940	27	231	57.2	10.8	3.4	3.9	3.4	0.8	0	0	20.4	0	0
	60	0.1	31	-71.9	83.9	245	155.8	2157.5	481.4	0	8.9	0.5	2954	64	432	50.5	16.9	5.1	5.1	5.5	1	0	0	16	0	0
		0.25	31	-71.7	84.2	245	155.8	2157.5	481.4	0	8.9	0.5	3481	105	601	62.4	14.8	4.3	2.2	0.5	0.2	0	0	15.6	0	0
WB	30	0.1	3	-80.5	87	245	167.4	26113.5	1605.2	0	5.8	1.2	9753	304	888	41.7	20.5	15.8	13.2	2.3	0.5	0	0	6.2	0	0
		0.25	3	-63.8	54	202	117.8	26113.5	3360	0	24.8	2.4	25308	1645	867	37.1	17	16.6	19.8	4.8	0.2	0	0	4.4	0	0
	45	0.1	3	-112.5	119.5	216	232	26113.5	3500	0	9.7	1.3	12442	488	941	34.9	27.8	20.4	8.8	1.2	0	0	0	6.9	0	0
		0.25	3	-100	118	209	218	26113.5	7640	0	23.8	2.2	19467	885	972	47.7	16.6	15.4	10.1	4.6	0.2	0	0	5.3	0	0
	60	0.1	3	-197.2	204.7	216	401.8	26113.5	6896.8	0	9.7	2	15497	412	1126	45	23.5	14.7	10.2	0.7	0.1	0	0	5.7	0	0
		0.25	3	-179.8	197.8	209	377.6	26113.5	15459.8	0	23.8	1.8	13848	385	897	40.5	21.2	16.2	11.6	3.1	0.2	0	0	7.2	0	0
WM	30	0.1	56	-2.9	114.9	101	117.8	9843.5	432.4	0	3.4	0	592	4	72	26.4	16.6	5.6	8.4	1.4	1.4	0	0	40.3	0	0
		0.25	56	-2.9	114.9	101	117.8	9843.5	432.4	0	3.4	0	522	10	68	32.4	8.8	5.9	8.9	4.4	1.5	0	0	38.2	0	0
	45	0.1	56	-46	158	101	204	9843.5	1036	0	3.4	0.1	870	9	237	53.1	16	6.4	3.4	4.2	1.3	0	0	15.6	0	0
		0.25	56	-46	158	101	204	9843.5	1036	0	3.4	0.1	810	16	222	53.6	17.1	2.2	4.5	3.2	1.4	0	0	18	0	0
	60	0.1	38	-120.7	232.7	101	353.3	9843.5	2081.3	0	3.4	0.4	2069	40	311	37.6	21.9	12.5	3.5	2.2	0	0	0	22.2	0	0
		0.25	38	-120.7	232.7	101	353.3	9843.5	2081.3	0	3.4	0.4	2735	74	380	46.6	16.5	8.4	2.7	1.8	1.1	0	0	22.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.10: One-to-many problem with $\beta < 1$, Core instance:att48

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	29	3251.2	11435.8	8306	8184.5	174125	25886.1	0	8.3	0.6	4724	93	692	54.3	16.9	14.9	5.6	2.5	0	0	0	5.8	0	0
		0.25	29	4416	9633	5736	5216.9	174125	41093.2	0	25	0.9	7743	223	715	44.5	18.5	16.8	11.2	1.1	0.7	0	0	7.3	0	0
	45	0.1	27	255.5	14431.5	8306	14176	174125	49852	0	8.3	0.8	5243	121	759	42.6	22	19.6	5.7	4.6	0.1	0	0	5.4	0	0
		0.25	27	2357.5	11393.5	5736	9036	174125	86922	0	25	0.8	5162	123	648	40.4	19	16.7	10.8	5.1	0.9	0	0	7.1	0	0
	60	0.1	19	-5538.3	19015.3	8306	24553.6	174125	91362.2	0	8.3	1	5866	128	868	44.4	20.7	19.9	5.8	2.5	0	0	0	6.7	0	0
		0.25	19	-800.9	14849.9	5736	15650.8	174125	166299.7	0	25	1	4841	122	590	37.1	18	19.7	9.3	7.1	1.7	0	0	7.1	0	0
1M	**30**	0.1	27	3631	9767	8306	6136.1	141559	17534.3	0	8.3	0.5	3819	143	634	58.5	14	10.1	3.5	5.3	0.5	0	0	8.1	0	0
		0.25	27	4431	9134	7065	4703.1	141559	22804.7	0	14.6	0.6	4505	156	688	53.4	15.5	9.6	6.1	3	1.3	0	0	11	0	0
	45	0.1	13	1990	12618	8306	10628	141559	35502	0	8.3	0.7	5104	210	750	54.1	14.3	13.4	7.2	4.1	0.4	0	0	6.4	0	0
		0.25	13	2709.5	10855.5	7065	8146	141559	46905	0	14.6	1	8530	237	950	53.4	13.7	11.3	10	1.4	1.8	0	0	8.5	0	0
	60	0.1	8	-2505.1	15903.1	8306	18408.2	141559	66622.9	0	8.3	0.8	5585	189	755	43.7	16.7	17.2	9	3.3	1	0	0	9	0	0
		0.25	8	2232.3	11737.7	5736	9505.5	141559	92081.9	0	25	1	6952	200	704	52.7	9.8	12.5	7.5	3	2.9	0	0	11.7	0	0
WB	**30**	0.1	27	3251.2	11435.8	8306	8184.5	937901	102410.3	0	6.2	1.1	15951	493	829	47	22.6	17.5	5.5	2.5	0.2	0	0	4.6	0	0
		0.25	27	4376.8	9672.2	5804	5295.5	937901	214574.9	0	22.9	0.9	5652	149	688	44.3	20.2	17.6	7.6	3.2	0	0	0	7.1	0	0
	45	0.1	25	-349.5	13826.5	8306	14176	937901	198274	0	6.2	0.9	6989	149	901	46.2	21.1	16.4	7.5	3.3	0.4	0	0	5	0	0
		0.25	25	2438.5	11610.5	5804	9172	937901	443291	0	22.9	1	7226	160	702	45.2	17.4	15.5	10	4.7	1	0	0	6.3	0	0
	60	0.1	17	-5538.3	19015.3	8306	24553.6	937901	364314.8	0	6.2	1	5361	76	873	47.4	19.6	17.4	5.5	3.4	0.1	0	0	6.5	0	0
		0.25	17	-918.7	14967.7	5804	15886.4	937901	839438.8	0	22.9	0.8	4649	65	690	37.8	19	23.3	8.4	5.4	0.6	0	0	5.5	0	0
WM	**30**	0.1	33	4105.3	10377.7	8306	6272.3	776397	69657.3	0	6.2	0.4	3599	175	512	51.6	10.9	12.9	6.5	5.3	1	0	0	11.9	0	0
		0.25	33	4300.3	9139.7	7065	4839.3	776397	116693.2	0	13.6	0.6	5365	151	631	60.2	11.6	8.1	3.5	3.3	1.7	0	0	11.5	0	0
	45	0.1	15	1809.5	12673.5	8306	10864	776397	143124	0	6.2	0.7	6148	240	720	46.8	16.1	15.8	7.5	4	1.1	0	0	8.6	0	0
		0.25	15	2529	10911	7065	8382	776397	240686	0	13.6	1	6709	205	862	54.5	13.2	12.5	5.5	2.2	2.6	0	0	9.5	0	0
	60	0.1	6	-2772	16045	8306	18817	776397	270372	0	6.2	0.8	6075	279	735	43.3	19.3	15.8	8.5	4.1	0.3	0	0	8.8	0	0
		0.25	6	1847.6	11997.4	5804	10149.8	776397	490460.2	0	22.9	1.2	7664	173	1001	55	14.8	10.5	6.5	3.2	2.8	0	0	7.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.11: One-to-many problem with $\beta < 1$, Core instance:B-n50-k7

Variant	θ	β	$E\%$	**Solution**								**Performance**			**MIP Cuts* percentage (%)**											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	26	-14.7	82.2	84	97	3508	339.5	0	10	0.8	7855	260	686	41.1	20.7	15.7	12.7	3.9	0.4	0	0	5.4	0	0
		0.25	26	-5.1	74.6	69	79.7	3508	616.1	0	24	0.9	8489	307	503	42.5	14.9	15.5	9.3	7.6	2.4	0	0	7.8	0	0
	45	0.1	18	-50.2	117.8	84	168	3508	694.5	0	10	0.8	5258	125	746	43.6	21.2	20.8	5.1	2.7	0.8	0	0	5.9	0	0
		0.25	18	-34.2	103.8	69	138	3508	1316	0	24	1.3	11474	423	727	47.2	15.8	13.9	11.8	5.4	0.6	0	0	5.4	0	0
	60	0.1	14	-111.7	179.2	84	291	3508	1309.4	0	10	1.2	7050	116	982	48.3	16.3	19.8	9.2	1.7	0.3	0	0	4.5	0	0
		0.25	14	-84.8	154.3	69	239	3508	2528.3	0	24	1	7294	202	669	36.3	17.8	16.9	15.7	5.2	1.5	0	0	6.6	0	0
1M	**30**	0.1	42	7.7	64.3	84	56.6	2512	114.9	0	10	0.4	3544	137	475	56.9	15.4	5.7	6.1	2.8	0.4	0	0	12.8	0	0
		0.25	42	8.3	63.7	83	55.4	2512	150	0	14	0.4	3082	102	382	54.2	12.3	5.2	5	3.2	1.6	0	0	18.6	0	0
	45	0.1	14	-13	85	84	98	2512	322	0	10	0.7	4431	125	572	48.1	13.8	11	10	5.7	0.3	0	0	11	0	0
		0.25	14	0	74	72	74	2512	450	0	22	1	8745	280	806	51.2	13.5	10.8	11.1	1.4	1.8	0	0	10.3	0	0
	60	0.1	14	-48.9	120.9	84	169.7	2512	680.7	0	10	0.8	5855	193	734	48.6	19.5	12.7	5.8	3.7	0.8	0	0	8.9	0	0
		0.25	14	-27.1	101.1	72	128.2	2512	1045.9	0	22	0.8	6471	283	597	49.7	14.2	8.7	7.7	4.5	0.1	0	0	14.9	0	0
WB	**30**	0.1	28	-14.7	82.2	84	97	19725.5	1717.4	0	9.1	0.7	5847	150	692	47.7	17.8	14.9	11.1	3.9	0.7	0	0	3.9	0	0
		0.25	28	-5.1	74.6	69	79.7	19725.5	2845.6	0	20	1.3	10117	291	732	47.8	15.4	14.6	9.2	5.1	1.9	0	0	6	0	0
	45	0.1	17	-50.2	117.8	84	168	19725.5	3492.5	0	9.1	1.2	10424	239	983	46.9	19	19.6	6.5	2	0.6	0	0	5.3	0	0
		0.25	17	-31.8	104.2	68	136	19725.5	6267.5	0	22.2	1.2	11125	532	672	38.4	17.7	14.3	17.6	5.8	1	0	0	5.2	0	0
	60	0.1	12	-111.7	179.2	84	291	19725.5	6567.1	0	9.1	1.1	9892	234	919	45.2	19.7	18.4	9.8	2.3	0.5	0	0	4.1	0	0
		0.25	12	-81.5	154	68	235.6	19725.5	12340.6	0	22.2	1.2	11388	285	862	53.9	14.8	13.3	7.2	3.8	0.8	0	0	6	0	0
WM	**30**	0.1	52	7.7	64.3	84	56.6	13877	594.5	0	9.1	0.4	3399	124	434	55.7	14	6.9	6	1.9	1	0	0	14.5	0	0
		0.25	52	8.3	63.7	83	55.4	13877	652.8	0	10.9	0.4	3050	92	429	57.4	12.5	2.3	8.2	2.6	1.2	0	0	15.8	0	0
	45	0.1	19	-13	85	84	98	13877	1630	0	9.1	0.8	6117	225	793	46.9	13.9	11.3	11.7	6.3	0.3	0	0	9.7	0	0
		0.25	19	-2	76	74	78	13877	2170	0	17.5	1	6584	151	781	51.5	14.3	11.9	6	2.1	2.1	0	0	12.1	0	0
	60	0.1	19	-48.9	120.9	84	169.7	13877	3423.5	0	9.1	0.6	5672	215	540	49.6	13.4	10.2	6.1	5.5	2	0	0	13.1	0	0
		0.25	19	-30.5	104.5	74	135.1	13877	4910.8	0	17.5	0.9	8176	429	778	53.3	15	7.8	8	4	1	0	0	10.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.12: One-to-many problem with $\beta < 1$, Core instance:D50

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)												
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	2	21.5	406	394	384.5	12397	1765.1	0	10	1.6	12688	187	1049	40.1	26.3	16.5	9.2	1.3	0	0	0	6.5	0	0
		0.25	2	55.4	366.1	330	310.6	12397	2964.4	0	24	1.5	12428	534	834	44.2	18.2	13.9	12.1	5.2	0.2	0	0	6.1	0	0
	45	0.1	2	-119.2	546.8	394	666	12397	3172.5	0	10	1.2	5960	113	953	43	22.1	15.7	9.2	2.5	0	0	0	7.3	0	0
		0.25	2	-58.2	479.8	330	538	12397	5693	0	24	1.5	13911	374	933	41.2	21.2	13.2	12.5	4.1	0.4	0	0	7.4	0	0
	60	0.1	2	-363	790.5	394	1153.5	12397	5610.2	0	10	1.6	10151	214	1234	47.1	19.7	15.3	9.3	1.9	0.1	0	0	6.6	0	0
		0.25	2	-255.2	676.7	330	931.8	12397	10419.1	0	24	2.5	19030	516	993	40	24.9	13.3	10.3	3.4	0.8	0	0	7.4	0	0
1M	**30**	0.1	32	132.9	283.1	394	150.1	6211	576.6	0	10	0.4	3912	119	596	56	11.9	6.5	9.9	4.2	0.3	0	0	11.1	0	0
		0.25	32	137	279	387	142	6211	626.2	0	12	0.6	4702	169	594	58.8	13.6	4	7.7	1.9	1	0	0	12.9	0	0
	45	0.1	24	78	338	394	260	6211	1126	0	10	0.6	4193	146	548	52	16	10.2	5.5	8.2	0.2	0	0	7.9	0	0
		0.25	24	101	305	162	204	6211	1362	0	18	0.9	5957	276	745	52.7	14.5	6.6	10.6	1.1	1.1	0	0	13.5	0	0
	60	0.1	14	-17.2	433.2	394	450.3	6211	2077.7	0	10	0.9	5879	265	867	53.5	13.6	15.1	5.9	3.3	0.5	0	0	8.1	0	0
		0.25	14	26.3	379.7	162	353.3	6211	2706	0	18	1.1	8668	248	970	54.9	14	12.8	6.5	1.4	1.5	0	0	8.9	0	0
WB	**30**	0.1	7	17.3	404.2	395	386.8	14352	2119.9	0	8.8	1	5360	115	723	40.2	20.5	21.6	7.9	1.9	0.1	0	0	7.7	0	0
		0.25	7	69.4	358.1	310	288.7	14352	3846	0	25	2.1	17806	396	965	41.5	19.7	13.3	13.6	3.7	1	0	0	7.3	0	0
	45	0.1	7	-124.2	545.8	395	670	14352	3819	0	8.8	1.5	8523	191	1085	39.4	23.1	18.2	11.1	2	0.1	0	0	6.1	0	0
		0.25	7	-36.2	463.8	310	500	14352	7438.5	0	25	1.6	10227	276	1065	44.1	17.8	16.8	10	4.2	0.3	0	0	6.8	0	0
	60	0.1	7	-369.5	791	395	1160.5	14352	6761.8	0	8.8	1.5	6910	110	1099	41.3	23.8	17.6	9.2	1.9	0.4	0	0	5.8	0	0
		0.25	7	-219.3	646.8	310	866	14352	13660.9	0	25	2.8	26460	1348	916	44	18.2	16.6	7	5.7	0.7	0	0	7.9	0	0
WM	**30**	0.1	26	96.6	304.4	62	207.8	9392	1014.2	0	7.4	0.3	2417	86	404	52.2	17.1	10.9	4.2	5.4	0	0	0	10.1	0	0
		0.25	26	126.6	274.4	114	147.8	9392	1705.6	0	25	0.6	3439	101	671	57.4	15.9	6.8	6.5	1.9	0.6	0	0	10.7	0	0
	45	0.1	19	20.5	380.5	62	360	9392	1775	0	7.4	0.6	3893	122	581	51.3	15.8	12.5	8.2	4.5	0	0	0	7.6	0	0
		0.25	19	72.5	328.5	114	256	9392	3545	0	25	0.7	4771	126	731	58.4	14.2	8.1	4.4	2.3	0.6	0	0	12.1	0	0
	60	0.1	15	-111.3	512.3	62	623.5	9392	3092.7	0	7.4	0.7	4592	151	769	54.3	16.7	13.7	4.8	3.1	0.4	0	0	7	0	0
		0.25	15	-21.2	422.2	114	443.4	9392	6730.9	0	25	0.9	6876	147	762	57.6	13.8	9.7	7.6	1.2	0.8	0	0	9.3	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.13: One-to-many problem with $\beta < 1$, Core instance:eil51

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	14	4.4	72.6	64	68.1	2489	223.6	0	9.8	1.3	12895	429	928	41.4	20.7	18.4	11.2	1.9	0.6	0	0	5.7	0	0
		0.25	14	8.4	62.6	52	54.3	2489	365.3	0	23.5	1.5	15003	743	747	39.2	16.5	19.3	10.4	5.9	2	0	0	6.7	0	0
	45	0.1	12	-20.5	97.5	64	118	2489	473	0	9.8	1.3	8428	183	914	43.7	17	20	10.5	0.5	0.3	0	0	8	0	0
		0.25	12	-16.5	87.5	57	104	2489	870	0	23.5	1.4	12402	351	851	34	23	16.6	14.1	4.3	0.5	0	0	7.5	0	0
	60	0.1	2	-63.7	140.7	64	204.4	2489	904.9	0	9.8	1.6	12213	238	1149	39.2	24	19.6	10.9	0.5	0.6	0	0	5.2	0	0
		0.25	2	-52.1	128.1	57	180.1	2489	1783.6	0	23.5	2	17017	379	1031	38.2	21.1	17.1	13	2.3	0.9	0	0	7.4	0	0
1M	**30**	0.1	43	24.2	50.8	16	26.6	1529	68.2	0	7.8	0.5	3919	151	520	53.1	15.6	5.2	6.2	5.4	0.9	0	0	13.7	0	0
		0.25	43	24.2	50.8	16	26.6	1529	68.2	0	7.8	0.5	3781	180	525	61.7	12	4.8	3.2	3.6	1.7	0	0	12.9	0	0
	45	0.1	29	13	59	16	46	1529	162	0	9.8	0.5	5767	314	529	49.6	16.3	8.9	7.5	6.8	0.8	0	0	10.2	0	0
		0.25	29	13	59	16	46	1529	162	0	13.7	0.7	6244	249	721	51	17.1	7.4	7.1	3.9	2.4	0	0	11.3	0	0
	60	0.1	16	-3.8	82.8	64	86.6	1529	322	0	9.8	0.8	8117	307	983	59.9	13.6	11.7	5.1	3.4	0.1	0	0	6.2	0	0
		0.25	16	-3.3	76.3	62	79.7	1529	436.1	0	17.6	1.1	9971	384	841	47.1	15.2	12.6	10.3	2.9	1.3	0	0	10.5	0	0
WB	**30**	0.1	14	-6.5	67.5	68	73.9	12302	1052.2	0	7.3	1	9048	271	810	44.1	20.7	20	7.2	2.3	0.1	0	0	5.6	0	0
		0.25	14	7.4	68.6	57	61.2	12302	2065.5	0	23.6	1.4	15375	732	680	41.8	15.9	11.8	17.4	7.1	0.6	0	0	5.6	0	0
	45	0.1	9	-33.5	94.5	68	128	12302	2026	0	7.3	1.6	12098	323	1105	45.4	22.9	18	5.7	1.4	0	0	0	6.6	0	0
		0.25	9	-15	91	57	106	12302	4704	0	24	1.6	13586	423	1000	48.3	18.7	15.4	7.5	2.8	1	0	0	6.3	0	0
	60	0.1	2	-80.4	141.4	68	221.7	12302	3712.6	0	7.3	1.9	11818	218	1185	45.8	19.2	17.3	11	0.5	0.2	0	0	6	0	0
		0.25	2	-53.8	129.8	57	183.6	12302	9282.2	0	24	1.8	15370	282	1099	38.1	22	18.6	12.1	2	1	0	0	6.2	0	0
WM	**30**	0.1	48	20.7	55.3	10	34.6	7288	392.9	0	6.9	0.4	3797	129	526	52.3	18.4	8.5	3.6	3.4	0.8	0	0	12.9	0	0
		0.25	48	24.1	51.9	16	27.7	7288	500.5	0	10.6	0.4	2652	72	431	54.5	16.2	4.4	3.9	3.3	2.5	0	0	15.1	0	0
	45	0.1	34	8	68	10	60	7288	824	0	6.9	0.6	5291	189	618	51.8	20.4	11.3	4.4	3.9	0	0	0	8.2	0	0
		0.25	34	14	62	16	48	7288	1086	0	13.4	0.6	5003	144	682	60.9	11.4	5.6	4.1	4.3	1.6	0	0	12.1	0	0
	60	0.1	16	-14	90	10	103.9	7288	1798.2	0	9.3	0.8	5561	134	730	57.7	13.4	8.8	5.8	4.6	1	0	0	8.8	0	0
		0.25	16	-5.1	78.1	16	83.1	7288	2453.8	0	17.1	1	7769	205	1047	59.5	12.4	8.9	7.1	2.5	0.8	0	0	8.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.14: One-to-many problem with $\beta < 1$, Core instance:berlin52

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	25	179.3	1283.2	960	1103.9	46522	3152	0	9.6	1	7565	347	714	41.9	21.7	18.1	7.7	4.6	0	0	0	6	0	0
		0.25	25	525	1253.6	635	728.6	46522	5434.5	0	25	1.6	14372	1119	932	58	12.6	13.7	7.2	1.4	0.4	0	0	6.7	0	0
	45	0.1	13	-224.8	1687.2	960	1912	46522	7192.5	0	9.6	1.2	8831	283	1162	50	20.5	18.8	4.9	1.4	0	0	0	4.4	0	0
		0.25	13	70.2	1392.2	665	1322	46522	10688.5	0	25	1.4	12336	420	989	42.4	15.8	13.9	20.1	4	0.2	0	0	3.6	0	0
	60	0.1	10	-924.6	2387.1	960	3311.7	46522	14190.9	0	9.6	1.2	8324	113	798	44.2	24.7	15.7	6.3	2	0.1	0	0	7	0	0
		0.25	10	-413.6	1876.1	665	2289.8	46522	23269.5	0	25	1.8	16511	612	1095	54.9	17.3	10	9.5	3.6	0.4	0	0	4.4	0	0
1M	**30**	0.1	63	331.1	948.9	1130	617.8	25425	995.5	0	3.8	0.2	1450	61	267	52.5	16.5	6	3.4	3.4	0	0	0	18.4	0	0
		0.25	63	331.1	948.9	1130	617.8	25425	995.5	0	3.8	0.1	1366	43	277	54.9	16.6	5.8	2.5	2.5	0.7	0	0	17	0	0
	45	0.1	42	247.5	977.5	960	730	25425	2120	0	7.7	0.4	3709	105	579	54.6	15.9	5.8	8.3	2.6	0.9	0	0	11.9	0	0
		0.25	42	247.5	977.5	960	730	25425	2120	0	7.7	0.5	3768	116	573	57.4	15.7	5.9	4.4	2.8	0.2	0	0	13.6	0	0
	60	0.1	25	7.8	1272.2	960	1264.4	25425	4502	0	9.6	0.7	6000	238	700	51.4	16.9	12	5.7	4	0.3	0	0	9.7	0	0
		0.25	25	7.8	1272.2	960	1264.4	25425	4502	0	9.6	0.8	6165	312	884	64.9	12	7.7	3.3	1.2	0.4	0	0	10.7	0	0
WB	**30**	0.1	29	202.4	1260.1	920	1057.7	262069.5	15475.3	0	8.8	1.4	16272	818	787	44.3	21.2	16.1	8.4	5.2	0	0	0	4.7	0	0
		0.25	29	525	1224.7	610	699.7	262069.5	32876.4	0	24.9	1.6	16177	1400	745	49.3	11.5	13.3	10.7	4.8	2.7	0	0	7.7	0	0
	45	0.1	15	-184.8	1647.2	920	1832	262069.5	35607	0	8.8	1	7357	175	759	47	19.9	20	3.6	2.4	0.7	0	0	6.5	0	0
		0.25	15	18	1340	665	1322	262069.5	55992	0	23.9	1.9	18770	901	886	39.3	20.9	17.7	13.1	3.2	0.5	0	0	5.4	0	0
	60	0.1	10	-855.3	2317.8	920	3173.1	262069.5	70476	0	8.8	1.5	13292	317	1074	45.4	22.8	16.6	7.5	1.6	0.2	0	0	5.9	0	0
		0.25	10	-439.6	1902.1	680	2341.7	262069.5	123166.3	0	23.6	1.6	14078	347	804	44.3	13.2	17.2	12.6	6.1	1.2	0	0	5.5	0	0
WM	**30**	0.1	67	441.7	938.3	180	496.5	159100	4365.2	0	3.4	0.2	1302	66	284	64.1	10.6	2.8	4.6	0.7	0.4	0	0	16.9	0	0
		0.25	67	551.4	828.6	370	277.1	159100	4403.8	0	10.1	0.2	1319	40	269	51.7	16.3	7.8	5.2	1.1	0	0	0	17.9	0	0
	45	0.1	48	412.5	902.5	365	490	159100	8045	0	7.7	0.4	2734	112	424	57.8	12.8	5.9	6.1	3.5	0.9	0	0	13	0	0
		0.25	48	450	930	370	480	159100	10490	0	10.1	0.3	2280	97	418	64.1	12.7	4.3	2.9	0.5	0.2	0	0	15.3	0	0
	60	0.1	23	233.1	1081.9	365	848.7	159100	18063.7	0	9.4	0.7	5836	224	621	48.3	16.1	13.8	8.2	5.2	0.6	0	0	7.7	0	0
		0.25	23	241.8	1073.2	370	831.4	159100	22453.5	0	11.8	0.8	6175	264	687	51.2	17.2	8.2	6.1	2.5	1.3	0	0	13.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.15: One-to-many problem with $\beta < 1$, Core instance:A-n60-k9

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	5	-10.9	91.9	93	102.8	4584	412.6	0	10	1.4	13355	460	965	40.7	23.9	19.8	6.2	3.3	0.4	0	0	5.6	0	0
		0.25	5	5.9	97.1	83	91.2	4584	697.3	0	25	3.1	26182	1570	1053	43.3	18.1	15.4	14.3	3.4	1.4	0	0	4	0	0
	45	0.1	5	-48.5	129.5	93	178	4584	864	0	10	1.4	10141	243	977	44.4	20.5	16.5	10.5	1.9	0.5	0	0	5.6	0	0
		0.25	5	-27.5	130.5	83	158	4584	1699	0	25	2.1	17294	644	995	50.7	15.9	16.6	6.1	4.8	1	0	0	4.9	0	0
	60	0.1	5	-114.7	193.7	93	308.3	4584	1645.8	0	10	1.9	8795	105	1394	47.1	19.9	18.5	7.8	0.4	0.1	0	0	6.1	0	0
		0.25	5	-85.3	188.3	83	273.7	4584	3434	0	25	2.3	13862	312	1116	44.6	21.3	15.9	7.2	2.9	1.5	0	0	6.6	0	0
1M	**30**	0.1	15	17.1	74.9	11	57.7	3018	298.4	0	10	0.5	3289	130	520	60.7	13.3	4.8	4.4	3.7	2.9	0	0	10.2	0	0
		0.25	15	20.6	71.4	17	50.8	3018	322.5	0	13.3	0.6	4915	179	626	61.2	16.9	5.8	3.5	1.5	1.7	0	0	9.4	0	0
	45	0.1	15	-5	95	11	100	3018	548	0	10	0.9	9137	315	926	46.6	15.2	15	9.3	5.7	1.9	0	0	6.2	0	0
		0.25	15	3	91	17	88	3018	644	0	16.7	1	8797	341	881	48.4	13.9	11.7	6.8	5	3	0	0	11.2	0	0
	60	0.1	15	-40.6	132.6	11	173.2	3018	907.2	0	10	1	9517	278	990	46.8	18.2	14.6	9.1	3.7	0.6	0	0	7	0	0
		0.25	15	-29.2	123.2	17	152.4	3018	1405	0	20	1.4	13700	504	1117	53.2	13.8	11.4	7.5	2.3	2.3	0	0	9.4	0	0
WB	**30**	0.1	4	-8	97	95	105.1	26130	2120.3	0	8.5	1.3	12568	316	937	42.2	16.6	17.5	14	3.8	0.2	0	0	5.7	0	0
		0.25	4	4.7	98.3	85	93.5	26130	4356.1	0	24.4	2.2	18639	785	983	37.9	18.7	14.9	16.6	5.1	0.7	0	0	6.1	0	0
	45	0.1	4	-46.5	135.5	95	182	26130	4351	0	8.5	1.4	9403	201	1053	41.2	24.6	17.9	8.3	2.1	0.1	0	0	5.8	0	0
		0.25	4	-29.5	132.5	85	162	26130	10039	0	24.4	1.8	13273	309	1072	44.5	21	14.8	9	3.5	1.3	0	0	5.8	0	0
	60	0.1	4	-113.1	202.1	95	315.2	26130	8214.8	0	8.5	1.7	7807	83	1047	40.1	20.6	20.2	9.6	0.6	0.7	0	0	8.2	0	0
		0.25	4	-88.8	191.8	85	280.6	26130	19882.2	0	24.4	2	14312	301	1058	37.6	22.6	18.4	11.2	2.9	0.9	0	0	6.3	0	0
WM	**30**	0.1	12	19.8	68.2	15	48.5	16756	1132.9	0	10	0.5	3699	158	585	53.3	15.4	6.9	6.5	6.2	0.7	0	0	11.1	0	0
		0.25	12	20.9	67.1	17	46.2	16756	1363.7	0	12.1	0.5	3702	108	606	66.5	10.4	3.3	4.6	1.2	1	0	0	13.1	0	0
	45	0.1	12	1	85	15	84	16756	2308	0	10	1	9473	291	854	46.7	17.3	18	6.2	5	0.8	0	0	5.9	0	0
		0.25	12	4	84	17	80	16756	2760	0	13.5	0.9	8631	245	796	51	16.8	10.7	4.1	2.6	0.6	0	0	14.1	0	0
	60	0.1	12	-33.7	125.7	11	159.3	16756	4237.9	0	10	1.2	11870	388	1178	48.2	16	16.5	10	2.2	0.3	0	0	6.9	0	0
		0.25	12	-25.3	113.3	17	138.6	16756	6069.8	0	17.6	1.4	11948	317	1011	47.9	15.6	11.8	7.8	3.6	1.6	0	0	11.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.16: One-to-many problem with $\beta < 1$, Core instance:B-n68-k9

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	29	-2.8	91.8	88	94.7	4898	482.1	0	8.8	1.6	9851	183	1027	38.8	23.5	21.2	8.3	1.6	0.6	0	0	6.1	0	0
		0.25	29	-8	82	84	90.1	4898	1173.1	0	25	2.1	12309	440	978	40.2	21.7	12.9	14.6	4.5	1.6	0	0	4.5	0	0
	45	0.1	24	-43	121	88	164	4898	898	0	8.8	1.4	8258	161	1007	37.3	25.2	21	8.5	1.5	1.2	0	0	5.3	0	0
		0.25	24	-41	115	84	156	4898	2294	0	25	2	14195	566	984	44.1	18.4	17.5	8.8	4.6	0.6	0	0	6	0	0
	60	0.1	15	-97.5	186.5	88	284.1	4898	1618.3	0	8.8	1.5	7336	50	1240	43.7	20.6	18.8	8.3	2.9	0.3	0	0	5.4	0	0
		0.25	15	-98.1	172.1	84	270.2	4898	4235.4	0	25	1.9	9735	134	1237	41.3	23.9	16.2	7.7	3.6	1.1	0	0	6.2	0	0
1M	**30**	0.1	43	7.3	62.7	20	55.4	3394	302.6	0	8.8	0.7	6940	391	652	47.5	13.1	13.1	8.1	4.3	2.7	0	0	11	0	0
		0.25	43	7.3	62.7	20	55.4	3394	302.6	0	8.8	0.8	8065	431	755	64.6	4.5	7	6.7	2.5	1.9	0	0	12.7	0	0
	45	0.1	29	-37	71	14	108	3394	420	0	8.8	1.1	13849	589	1099	55.9	14.5	12.4	7.9	2	0.7	0	0	6.5	0	0
		0.25	29	7.5	101.5	21	94	3394	901	0	25	1.4	15168	641	972	49.1	11.7	15	9.8	2	3.9	0	0	8.3	0	0
	60	0.1	25	-61	126	14	187.1	3394	854.4	0	8.8	1.4	15336	528	1378	48.5	17.5	16	8.6	2.5	0.4	0	0	6.5	0	0
		0.25	25	-53.1	127.1	16	180.1	3394	2158.3	0	25	1.4	10228	387	1110	54.4	12.8	12	7.9	1.4	2.6	0	0	8.9	0	0
WB	**30**	0.1	24	-4	93	90	97	27920	2965.8	0	9.7	1.2	9687	157	925	41.5	21.4	19.7	8.4	2.8	0.4	0	0	5.7	0	0
		0.25	24	-9.2	83.2	86	92.4	27920	6350.3	0	23.2	1.5	10129	251	770	32.1	17.5	16.5	20.4	6.2	1.4	0	0	5.8	0	0
	45	0.1	19	-39.5	128.5	90	168	27920	5522	0	9.7	1.3	9609	123	1055	38.3	26.8	18.5	7.3	2.9	0.5	0	0	5.7	0	0
		0.25	19	-43	117	86	160	27920	12166	0	23.2	1.5	8618	154	1065	42.5	20.4	16.1	11.6	3.6	0.1	0	0	5.7	0	0
	60	0.1	14	-103	181	88	284.1	27920	9958.1	0	10	1.6	7676	104	1063	40.1	20.1	23.7	6.2	2.4	0.1	0	0	7.4	0	0
		0.25	14	-101.6	175.6	86	277.1	27920	22239	0	23.2	2.2	12655	190	1294	47.1	17.7	15.9	9.9	2.2	0.5	0	0	6.7	0	0
WM	**30**	0.1	46	8.3	63.7	20	55.4	17566	1400.5	0	7.3	0.8	8712	423	794	47.2	16.7	16.9	5.7	3.8	1.3	0	0	8.4	0	0
		0.25	46	8.3	63.7	20	55.4	17566	1400.5	0	7.3	0.6	6943	311	728	54.4	11.4	11	5.9	2.6	1.9	0	0	12.8	0	0
	45	0.1	35	-33	75	14	108	17566	2680	0	10	1.3	15330	668	967	46.8	15.3	14.4	12.4	2.9	0.9	0	0	7.2	0	0
		0.25	35	-16	88	16	104	17566	4246	0	23.2	1.4	15427	576	1224	47.2	16.1	12.3	10.4	2.3	5.1	0	0	6.7	0	0
	60	0.1	29	-72.5	114.5	14	187.1	17566	5605.3	0	10	1.3	13751	462	1116	43.5	13.9	17.6	11.2	5	0.6	0	0	8.3	0	0
		0.25	29	-54.1	126.1	16	180.1	17566	10793.5	0	23.2	1.3	11526	464	975	50	15.4	11.4	9.7	1.3	1.7	0	0	10.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.17: One-to-many problem with $\beta < 1$, Core instance:F-n72-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	6	-29.9	24.4	221	54.3	2600	325.4	0	9.7	2.8	22333	844	1444	34.7	26	21.5	10.9	0.3	0.1	0	0	6.4	0	0
		0.25	6	-29.7	21.2	218	50.8	2600	566	0	18.1	4.1	42272	1562	1589	36	20.2	15.5	19.8	2.4	0.8	0	0	5.3	0	0
	45	0.1	3	-49.8	44.2	221	94	2600	603.5	0	9.7	4.3	43514	1000	1646	41.3	24.2	18.8	9.4	0.1	0.1	0	0	6.1	0	0
		0.25	3	-42.2	33.8	212	76	2600	1140	0	25	4.3	27141	678	1639	38.2	21.1	15.2	16.5	2.1	0.9	0	0	5.9	0	0
	60	0.1	1	-84.2	78.7	221	162.8	2600	1085.2	0	9.7	4	26028	379	1638	47.6	20.5	15	10.8	0.1	0.2	0	0	6	0	0
		0.25	1	-70.1	61.6	212	131.6	2600	2141.4	0	25	3	22799	579	1785	40.2	19.5	17	14.4	1.8	0.8	0	0	6.4	0	0
1M	**30**	0.1	39	-15.7	9.7	183	25.4	1142	105.8	0	9.7	0.9	10322	411	763	45	19.1	15.1	6.7	2.8	1	0	0	10.4	0	0
		0.25	39	-15.7	9.7	183	25.4	1142	105.8	0	9.7	1.3	15590	1069	999	55.5	14.3	12.8	5	1.5	0.9	0	0	10	0	0
	45	0.1	28	-25	19	183	44	1142	236	0	9.7	1.2	12258	406	917	43.3	16.1	17	9.8	4.5	1.3	0	0	8	0	0
		0.25	28	-17.5	8.5	218	26	1142	237	0	18.1	1.6	19108	978	1158	46.6	16.6	14.4	8.6	1.5	2.5	0	0	10	0	0
	60	0.1	15	-41.1	35.1	183	76.2	1142	461.5	0	9.7	1.6	14463	404	1312	45	17	17.1	10.7	2.4	0.9	0	0	7.1	0	0
		0.25	15	-27	18	218	45	1142	484.4	0	18.1	2.1	20321	847	1500	50.9	17.1	14.4	4.1	1.9	2.6	0	0	8.8	0	0
WB	**30**	0.1	3	-29.9	24.4	221	54.3	13459	1529.4	0	9.3	3.3	31465	987	1540	32.8	27.6	20.4	12.1	0.6	0.1	0	0	6.4	0	0
		0.25	3	-27.3	18.8	214	46.2	13459	2983.5	0	23.9	5.3	42066	1777	1338	30.6	25	15.1	16.7	2.5	1.6	0	0	8.7	0	0
	45	0.1	1	-49.8	44.2	221	94	13459	2840.5	0	9.3	3	23967	681	1434	38.1	24.5	18.8	11.3	0.1	0.3	0	0	7	0	0
		0.25	1	-43.2	34.8	213	78	13459	5873.5	0	25	4.7	29189	615	1735	30	24.5	15.7	21.8	1.3	1.2	0	0	5.5	0	0
	60	0.1	1	-84.2	78.7	221	162.8	13459	5111.3	0	9.3	3.8	35662	1156	1483	31.5	26.8	15.7	17.3	0.1	0.1	0	0	8.4	0	0
		0.25	1	-71.8	63.3	213	135.1	13459	10955.4	0	25	6	42462	563	1928	37.1	23.5	13.4	17	1.3	0.2	0	0	7.3	0	0
WM	**30**	0.1	46	-16.8	9.8	183	26.6	5600	372.3	0	6.2	1.2	13417	544	970	51.5	16.1	14	9.5	2.5	0.2	0	0	6.3	0	0
		0.25	46	-10.9	2.9	218	13.9	5600	476.4	0	18.8	1.4	14638	945	964	54.4	13.1	9.4	7.9	1.4	3.1	0	0	10.6	0	0
	45	0.1	35	-25.5	20.5	183	46	5600	800	0	6.2	1.1	10818	475	839	45.9	17.4	15.3	7.2	5.1	1.2	0	0	8	0	0
		0.25	35	-16	8	218	24	5600	1156	0	18.8	2	25982	1460	1068	50.8	12.9	11.8	8.4	1.6	3.8	0	0	10.6	0	0
	60	0.1	23	-32.9	25.9	189	58.9	5600	1610.4	0	9.3	1.6	11315	297	1334	44.3	18.9	19.5	8.2	2.7	0.9	0	0	5.4	0	0
		0.25	23	-26.6	11.6	195	38.1	5600	2349	0	24.2	1.7	14829	387	1323	51.4	15.7	12.6	7.4	1.2	2.4	0	0	9.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.18: One-to-many problem with $\beta < 1$, Core instance:rus75

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	25	14.6	121.9	94	107.4	5083.5	249.7	0	4	2.9	30821	1626	1010	30.9	26.2	16.8	14.8	4.5	0.2	0	0	6.6	0	0
		0.25	25	17	73.5	50	56.6	5083.5	463.4	0	24	2.9	27573	2249	1129	45.1	16.3	16.4	10.7	2	3.2	0	0	6.3	0	0
	45	0.1	17	-30.8	117.2	75	148	5083.5	563.5	0	9.3	3.3	24294	1081	1304	50.6	21.2	13.9	6	2.8	0.2	0	0	5.4	0	0
		0.25	17	-16.2	101.8	60	118	5083.5	1041.5	0	22.7	4.4	36648	2154	1365	41.1	19.6	16	14.6	3.1	1.1	0	0	4.5	0	0
	60	0.1	9	-84.9	171.4	75	256.3	5083.5	1321.9	0	9.3	4	29279	863	1686	40.9	24.8	17.4	10.4	1.4	0.1	0	0	5	0	0
		0.25	9	-57.7	143.2	59	200.9	5083.5	2566.5	0	24	3.8	25709	800	1379	41	20.9	12.3	16.2	4.9	0.8	0	0	4	0	0
1M	**30**	0.1	61	40.3	80.7	8	40.4	3465	127.9	0	9.3	0.4	2706	73	424	56.4	13.2	4	5	2.8	1.7	0	0	17	0	0
		0.25	61	40.3	80.7	8	40.4	3465	127.9	0	9.3	0.4	2894	112	494	62.5	10.3	3.8	5.3	3	0.6	0	0	14.4	0	0
	45	0.1	33	25.5	95.5	8	70	3465	335	0	9.3	1	7340	294	815	43.2	13.1	16.7	10.3	5.4	1	0	0	10.3	0	0
		0.25	33	25.5	95.5	8	70	3465	335	0	9.3	1.2	7971	259	1239	54.5	10.1	17	8.2	1	1.5	0	0	7.6	0	0
	60	0.1	13	-0.1	121.1	8	121.2	3465	693.7	0	9.3	1.6	15271	566	1408	55.1	16.3	15	3.8	3.2	0.5	0	0	6.2	0	0
		0.25	13	-0.1	121.1	8	121.2	3465	693.7	0	9.3	1.8	14211	703	1287	48.4	12.5	16.3	8.6	1.2	1.2	0	0	11.8	0	0
WB	**30**	0.1	27	14.6	121.9	94	107.4	31343.5	1997.3	0	5.3	2.2	18186	799	823	37.7	29.4	14.7	8	6.1	1.1	0	0	3	0	0
		0.25	27	16.3	74	51	57.7	31343.5	2957.7	0	24.3	3.1	34124	2517	1104	49.8	18.5	12.1	11.1	1.9	0.9	0	0	5.6	0	0
	45	0.1	18	-20.8	153.2	88	174	31343.5	4035	0	7.4	3	21771	829	1419	42.4	24.9	17.9	7.1	2.3	0.4	0	0	5	0	0
		0.25	18	-18.8	99.2	60	118	31343.5	6004	0	22.8	4	31457	1762	1100	52.9	18.5	10.7	8	4.7	0.7	0	0	4.5	0	0
	60	0.1	12	-63.7	196.2	76	259.8	31343.5	8374.2	0	9.4	2.9	14740	525	1568	50.1	18	15.1	8.3	2.8	0.1	0	0	5.6	0	0
		0.25	12	-60.2	140.7	59	200.9	31343.5	15109.1	0	23.6	3.3	23825	778	1147	44.6	21.9	11.2	11.2	6.9	0.2	0	0	4.1	0	0
WM	**30**	0.1	63	39.5	84.5	6	45	21814	851.3	0	8.3	0.4	2703	58	457	54.3	16.2	6.3	6.1	2.6	1.1	0	0	13.4	0	0
		0.25	63	39.5	84.5	6	45	21814	851.3	0	8.3	0.4	2126	61	361	45.7	16.6	8.3	4.4	3.6	2.8	0	0	18.5	0	0
	45	0.1	38	23	101	6	78	21814	2104	0	8.3	0.8	5631	144	855	54.7	16.1	8.5	5.2	4.1	0.9	0	0	10.7	0	0
		0.25	38	23	101	6	78	21814	2104	0	8.3	0.9	6983	225	984	54.3	9.2	14.9	8.3	1.8	1.1	0	0	10.4	0	0
	60	0.1	10	-5.5	129.5	6	135.1	21814	4273.8	0	8.3	1.5	10671	365	1361	49	13.7	19.9	6.2	3.2	0.4	0	0	7.6	0	0
		0.25	10	19.3	119.7	16	100.5	21814	4296.4	0	19.3	1.5	14964	569	1421	52.9	13.4	14.5	6.7	1.2	1	0	0	10.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.19: One-to-many problem with $\beta < 1$, Core instance:eil76

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	17	10	80.4	64	70.4	3703	328.6	0	9.2	2.7	24266	816	1408	39.5	22.2	18.7	11.9	1.9	0.9	0	0	5	0	0
		0.25	17	11.9	66.1	50	54.3	3703	569.1	0	25	4.7	39998	1982	1252	46.2	16.9	12.2	14.6	4.4	0.7	0	0	5	0	0
	45	0.1	11	-19.5	104.5	65	124	3703	693	0	9.2	2.4	14193	314	1465	45.6	19.7	18.9	8.7	1.3	0.3	0	0	5.5	0	0
		0.25	11	-14	92	56	106	3703	1274	0	22.4	3.4	35495	942	1435	43.6	20.4	14.6	12.1	4.5	0.5	0	0	4.3	0	0
	60	0.1	7	-64.9	149.9	65	214.8	3703	1328.4	0	9.2	2.2	12429	110	1544	33.2	26.7	17.7	12.6	2.1	0.3	0	0	7.4	0	0
		0.25	7	-46.6	119.6	51	166.3	3703	2594.3	0	25	3.9	26329	428	1564	38.6	21.4	18.2	13.2	3.1	0.8	0	0	4.6	0	0
1M	**30**	0.1	39	26.1	53.9	60	27.7	2353	112	0	9.2	1	11924	563	878	48.2	12.4	13.7	9	5	2.7	0	0	9	0	0
		0.25	39	26.1	53.9	60	27.7	2353	112	0	9.2	1.2	11522	456	996	53.7	16.5	10.1	6.1	2.9	0.6	0	0	10.1	0	0
	45	0.1	22	16	74	65	58	2353	246	0	7.9	1.5	18550	834	1208	47.2	17.4	14.1	8.9	5.4	1.1	0	0	6	0	0
		0.25	22	16	64	60	48	2353	288	0	13.2	1.9	19966	980	1384	51.6	16.7	12.4	6.9	0.8	1	0	0	10.6	0	0
	60	0.1	14	-6.7	93.7	65	100.5	2353	534.2	0	9.2	1.7	19195	599	1361	47	17.9	16.3	8.7	3.5	0.3	0	0	6.2	0	0
		0.25	14	0.2	79.8	59	79.7	2353	695.8	0	17.1	2.1	21988	919	1460	49.5	13.8	10.1	14.5	1.9	1.6	0	0	8.6	0	0
WB	**30**	0.1	12	3.2	74.8	65	71.6	19808	1833.3	0	8.8	2	16469	466	1009	44	19.9	16.3	10.6	2.6	0.5	0	0	6.1	0	0
		0.25	12	9	62	50	53	19808	3094	0	24.6	2.9	31943	1545	992	41	15.6	16	15.9	4.5	0.7	0	0	6.1	0	0
	45	0.1	8	-22	100	64	122	19808	3888	0	9.8	2	13209	198	1469	40	25	18.7	8.7	2	0.1	0	0	5.6	0	0
		0.25	8	-14	92	56	106	19808	7136	0	22.4	3.3	19003	382	1432	38	23.4	16.6	13.1	3.6	0.8	0	0	4.6	0	0
	60	0.1	6	-66.7	144.7	64	211.3	19808	7460.4	0	9.8	2.5	16158	285	1854	37	25.5	19.1	10.6	1.8	0.2	0	0	5.7	0	0
		0.25	6	-52.8	130.8	56	183.6	19808	14275	0	22.4	3.3	25976	898	1412	43.2	17.8	16.9	12.5	3.4	0.9	0	0	5.2	0	0
WM	**30**	0.1	39	23.1	51.9	60	28.9	11886	713	0	9.3	1	9400	374	875	44.3	14.4	12.9	13	3.4	2.2	0	0	9.7	0	0
		0.25	39	23.1	51.9	60	28.9	11886	713	0	9.3	1.1	11124	349	1075	54.3	14.8	11.2	6.6	2.3	0.6	0	0	10.2	0	0
	45	0.1	26	11	69	64	58	11886	1458	0	9.8	1.4	15908	692	1039	46.5	16	13	10.7	4.6	1.7	0	0	7.3	0	0
		0.25	26	13.5	61.5	59	48	11886	1778	0	15.1	1.7	18087	779	1433	56.6	14.1	12.3	7.2	0.9	0.5	0	0	8.4	0	0
	60	0.1	16	-10.2	90.2	64	100.5	11886	3058.4	0	9.8	1.8	17902	620	1373	49.9	17.4	14.9	7.1	4	0.5	0	0	6.3	0	0
		0.25	16	1.9	78.1	57	76.2	11886	4316.3	0	21.2	2.1	18276	614	1539	53.6	13.2	10.9	9.8	1.3	1.3	0	0	9.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.20: One-to-many problem with $\beta < 1$, Core instance:A-n80-k10

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	30	0.1	26	-17.3	82	85	99.3	5530	423.8	0	10	4	49451	1805	1323	46.1	21.8	18.3	7.1	2.9	0.5	0	0	3.3	0	0
		0.25	26	4.7	91.3	74	86.6	5530	804.2	0	21.2	4.5	35308	1620	1189	45.5	15.1	15.6	13	4.2	2	0	0	4.5	0	0
	45	0.1	15	-41.5	132.5	86	174	5530	928	0	10	3.4	23332	672	1841	47.7	19.7	18	8.9	0.5	0.5	0	0	4.7	0	0
		0.25	15	-23	125	73	148	5530	2008	0	25	2.7	24767	824	967	43.7	19.6	14.3	9.7	6.6	1.7	0	0	4.3	0	0
	60	0.1	6	-105.2	196.2	86	301.4	5530	1947	0	10	4.6	24954	345	1859	39.3	24.2	19.7	10.9	0.3	0.7	0	0	5.1	0	0
		0.25	6	-80.2	176.2	73	256.3	5530	4174.9	0	25	2.9	23009	469	1675	49.3	18.8	17.1	6.3	3.6	1.3	0	0	3.6	0	0
1M	30	0.1	46	25.4	71.6	80	46.2	3822	194.3	0	8.8	1	10640	508	819	48.1	17.1	11.7	8.3	3.6	1.2	0	0	10	0	0
		0.25	46	25.4	71.6	80	46.2	3822	194.3	0	8.8	1.1	11635	602	819	45.4	17.2	13.9	6.8	4.1	1.8	0	0	10.6	0	0
	45	0.1	30	23	107	82	84	3822	366	0	10	1.8	25699	1478	1185	45.5	17.6	16.4	8	6	1.5	0	0	5.2	0	0
		0.25	30	6	86	80	80	3822	538	0	16.2	1.7	18419	900	1279	52	14	13.1	8.3	2.1	1.6	0	0	8.9	0	0
	60	0.1	18	-44.1	136.1	92	180.1	3822	858.9	0	8.8	1.8	19457	783	1210	42.8	18.4	17.1	10.3	4.5	0.8	0	0	6	0	0
		0.25	18	-23.3	115.3	80	138.6	3822	1392.5	0	18.8	2.1	19045	846	1586	50.9	14.4	12.5	11.2	1.2	0.8	0	0	9.1	0	0
WB	30	0.1	22	-17.3	82	85	99.3	30282	2230.9	0	9.5	3.4	37525	1642	1297	46.3	21.4	17.2	7.8	2.7	0.6	0	0	3.9	0	0
		0.25	22	14.3	91.7	66	77.4	30282	4217.8	0	22.2	4.9	40026	1759	1273	53.7	15.6	13.3	7.4	2.7	2.7	0	0	4.7	0	0
	45	0.1	12	-47.5	138.5	92	186	30282	5189	0	8.8	3	16222	265	1564	43.7	21.2	16.5	10.5	0.6	1.8	0	0	5.6	0	0
		0.25	12	-19	121	69	140	30282	10204	0	23.8	4	38020	2873	1186	42.9	18.3	16.5	10.1	6	0.4	0	0	5.7	0	0
	60	0.1	5	-103.5	194.5	85	297.9	30282	10716.2	0	10	3.9	21452	316	1958	36.2	25.9	20.5	10.7	0.3	0.5	0	0	5.9	0	0
		0.25	5	-70.2	172.2	69	242.5	30282	20965.1	0	23.8	5.3	37696	702	1676	41.4	18.9	17.7	12.6	2.7	2.8	0	0	3.9	0	0
WM	30	0.1	43	33.1	75.9	80	42.7	20073	739.5	0	8.6	1	10964	578	764	37.8	18.3	13.4	8.4	4.3	2.9	0	0	14.9	0	0
		0.25	43	33.1	75.9	80	42.7	20073	739.5	0	8.6	1.1	11361	479	918	49.6	15.6	12.7	6.2	3.8	1.3	0	0	10.8	0	0
	45	0.1	35	17	115	92	98	20073	1818	0	8.2	1.8	22989	1145	1215	44.6	15.9	15	12.2	5	0.7	0	0	6.5	0	0
		0.25	35	14.5	88.5	80	74	20073	2352	0	15.4	1.8	22298	1064	1255	50.9	14.7	11.6	9.3	2.8	1.5	0	0	9.2	0	0
	60	0.1	19	-35.9	133.9	92	169.7	20073	4547.9	0	8.8	1.9	16708	611	1406	47.5	16.6	16.9	8.9	4.2	0.2	0	0	5.8	0	0
		0.25	19	-15.1	113.1	80	128.2	20073	6469.4	0	17.7	2	19066	823	1383	46	15.7	13.8	12.6	1.1	1.7	0	0	9.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.21: One-to-many problem with $\beta < 1$, Core instance:rd100

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	19	-124.4	922.6	906.9	1047	83112	6564	0	10	6	37004	1023	1928	36.5	22.9	22.3	10.9	2.1	0.2	0	0	5.1	0	0
		0.25	19	-46.2	922.6	839.2	968.8	83112	15679.5	0	25	5	34217	975	1608	42.6	13.7	15.8	18.5	3.7	0.4	0	0	5.3	0	0
	45	0.1	13	-469.5	1405.6	937.7	1875.1	83112	14279.7	0	10	4.7	24130	405	1952	37.2	22.3	22.4	9.7	3.3	0.2	0	0	4.9	0	0
		0.25	13	-383.3	1302	842.8	1685.3	83112	32332.5	0	25	4.7	24733	460	1850	39.3	20.2	17.2	14.5	3.6	0.2	0	0	4.9	0	0
	60	0.1	10	-1165.6	2082.2	937.7	3247.8	83112	28006.5	0	10	4.1	17863	321	1981	39.8	23.3	17.1	12.7	1.8	0.1	0	0	5.2	0	0
		0.25	10	-1000.1	1918.8	842.8	2918.9	83112	63174.8	0	25	7.9	48487	700	2226	35.1	22.9	18.7	15.2	1.8	0.4	0	0	5.8	0	0
1M	**30**	0.1	46	215	755.5	116.3	540.5	53338.2	2381.9	0	7	1.4	13324	519	1223	48.3	15	14.7	10.3	2.8	1.1	0	0	7.8	0	0
		0.25	46	215	755.5	116.3	540.5	53338.2	2381.9	0	7	1.4	13000	420	1285	46.1	17.1	17.1	8.6	2.8	1.2	0	0	7.2	0	0
	45	0.1	27	43	970.2	120.8	927.2	53338.2	5751.4	0	10	2	23160	928	1388	42.3	15.8	19.1	10.7	4.6	1.5	0	0	5.7	0	0
		0.25	27	-48.5	887.7	116.3	936.2	53338.2	6044.6	0	13	2.1	20520	877	1487	48.5	15.8	14.5	9	1.4	2.1	0	0	8.9	0	0
	60	0.1	19	-378.5	1376.1	77.9	1754.6	53338.2	12043.9	0	10	2.6	26096	864	1597	44.8	20.7	15.4	8.5	3.8	0.8	0	0	5.9	0	0
		0.25	19	-365.5	1204.7	131.1	1570.2	53338.2	15652.5	0	15	3.5	34675	1422	1797	48	15.4	10.5	13.6	1.1	2.4	0	0	9	0	0
WB	**30**	0.1	18	-74.7	1010.8	940.3	1085.5	513349	39384.2	0	9.7	4.6	30107	589	1668	36.9	25.5	19.7	9.5	2.6	0.1	0	0	5.8	0	0
		0.25	18	-55.7	922.6	847.4	978.3	513349	101844.1	0	24.7	4.9	32930	888	1571	43	14.8	13.9	17.2	4.3	0.6	0	0	6.2	0	0
	45	0.1	12	-472	1408.2	940.3	1880.2	513349	86269.4	0	9.7	4.2	21283	320	1920	42.4	19.7	20.9	9.7	1.6	0.5	0	0	5.2	0	0
		0.25	12	-365.8	1359.1	862.6	1724.9	513349	201926.7	0	24.7	5.5	28586	585	1910	32.8	24	16.3	18	3.3	0.6	0	0	5	0	0
	60	0.1	9	-1160.2	2096.4	940.3	3256.6	513349	167476.8	0	9.7	4.6	21264	307	2169	42	21.1	18.6	11.7	1.5	0	0	0	5.1	0	0
		0.25	9	-997.2	1990.5	862.6	2987.7	513349	391337.3	0	24.7	4.6	23137	248	1674	39.9	20.4	14.4	14.4	3.6	0.7	0	0	6.7	0	0
WM	**30**	0.1	42	206.7	779.8	116.3	573.1	321632.4	13437.9	0	5.3	1.4	11843	446	1073	43.5	18.9	14.8	9	3.7	0.9	0	0	9.1	0	0
		0.25	42	206.7	779.8	116.3	573.1	321632.4	13437.9	0	5.3	1.4	12194	368	1427	51.3	13.8	15.5	6.6	3.9	1.3	0	0	7.6	0	0
	45	0.1	27	-110.8	881.8	116.3	992.6	321632.4	30689.4	0	9.9	2.1	21155	864	1450	42.6	18.6	17.1	10.3	4.5	1.8	0	0	5.3	0	0
		0.25	27	217.3	731.2	869.5	513.9	321632.4	33486.1	0	16.6	2.7	24639	1095	1496	43.9	15.9	14.4	11.9	2.4	2.4	0	0	9.2	0	0
	60	0.1	14	-381.4	1395.1	99.8	1776.5	321632.4	65952.7	0	9	2.8	23561	817	1670	45.5	18.9	15.8	8.2	3.9	0.9	0	0	6.9	0	0
		0.25	14	-411.8	1256.1	131.1	1667.9	321632.4	88603	0	13.5	3.3	34383	1136	1811	49.6	16.7	11.2	10.8	2.2	2.3	0	0	7.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.22: One-to-many problem with $\beta < 1$, Core instance:E-n101-k14

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	16	-3.6	69.1	65	72.7	5029.5	429.5	0	9.9	6	50154	1522	2154	42.7	22.7	17.9	10.6	0.7	0.6	0	0	4.8	0	0
		0.25	16	9	61	47	52	5029.5	698.1	0	24.8	13.7	107930	5399	1638	40.7	19.5	16.6	13.9	3.6	0.7	0	0	5	0	0
	45	0.1	6	-30.2	95.8	65	126	5029.5	962	0	9.9	4.7	27522	433	1889	43.4	22.9	19.1	7.2	2.1	0.4	0	0	4.9	0	0
		0.25	6	-15.2	84.8	52	100	5029.5	1709.5	0	24.8	16.7	120234	3738	1727	34.3	20.8	21.4	13.8	4.7	0.4	0	0	4.6	0	0
	60	0.1	4	-74.4	143.9	65	218.2	5029.5	1884.4	0	9.9	5.2	25506	268	2492	45.2	20.3	17.7	9.6	2.2	0.4	0	0	4.5	0	0
		0.25	4	-51.9	121.4	52	173.2	5029.5	3539.6	0	24.8	10.4	55848	1691	2109	38	22.8	17.9	12.7	3	1.2	0	0	4.4	0	0
1M	**30**	0.1	42	16.6	45.4	60	28.9	3258	116.1	0	6.9	1.8	18818	806	1239	46.6	15.5	13.2	11.9	3.7	1.9	0	0	7.4	0	0
		0.25	42	16.6	45.4	60	28.9	3258	116.1	0	6.9	2.1	29687	1258	1426	49.6	13.5	16.1	8.4	3.3	0.9	0	0	8.1	0	0
	45	0.1	25	6	56	60	50	3258	314	0	9.9	2.9	38315	2177	1466	49.5	16.4	17.1	5.3	4.3	1.7	0	0	5.8	0	0
		0.25	25	8	54	58	46	3258	316	0	11.9	3.3	36810	2165	1620	47.6	13.6	13.7	8.8	3.9	1.9	0	0	10.4	0	0
	60	0.1	14	-19	85	65	103.9	3258	741.2	0	9.9	2.6	26180	1028	1557	46.6	16.3	16.5	8.5	4.6	0.5	0	0	6.9	0	0
		0.25	14	-10.3	76.3	60	86.6	3258	865.6	0	15.8	6.1	58209	2933	2430	48.8	16	13.1	12.3	1	1.6	0	0	7.3	0	0
WB	**30**	0.1	13	-10.7	62	65	72.7	26667	2176.4	0	9.1	6.2	42744	1307	1740	41.4	22	17.9	10.9	3.2	0.6	0	0	4	0	0
		0.25	13	1.9	59.6	52	57.7	26667	3606	0	23.2	8	84094	3493	1752	44.9	16	16.1	14.7	3.6	0.8	0	0	3.9	0	0
	45	0.1	5	-28.2	97.8	65	126	26667	4473.5	0	9.1	4.1	24695	303	2130	43.7	20.3	20.1	9.9	0.2	0.4	0	0	5.3	0	0
		0.25	5	-17.2	86.8	54	104	26667	8604.5	0	23.9	10.2	62561	1467	2033	37.4	20.5	15.8	18.7	3.5	0.3	0	0	3.7	0	0
	60	0.1	3	-74.4	143.9	65	218.2	26667	8808.7	0	9.1	7.1	37355	746	2414	39.9	25	19.1	9.3	1.8	0.6	0	0	4.3	0	0
		0.25	3	-55.3	124.8	54	180.1	26667	17968.9	0	23.9	6.8	45151	1128	1771	37.2	22.4	20	11	4.3	0.8	0	0	4.3	0	0
WM	**30**	0.1	46	15.1	42.9	58	27.7	17024	716.5	0	9.1	1.6	15720	782	1038	45.4	15.1	14.3	11	2.9	2.4	0	0	9	0	0
		0.25	46	13.6	41.4	58	27.7	17024	733.5	0	11.9	1.4	13615	602	1231	52.1	12.5	12.6	6.7	5.1	1.4	0	0	9.6	0	0
	45	0.1	27	-2	60	65	62	17024	1826	0	9.1	2.7	33651	1867	1585	43.7	17.3	17.2	10.2	4.2	1.5	0	0	6	0	0
		0.25	27	3.5	51.5	58	48	17024	2091	0	15	3.4	39245	2501	1696	50.9	16.1	12.8	6.3	2.9	1.5	0	0	9.5	0	0
	60	0.1	20	-20.7	86.7	65	107.4	17024	3567.2	0	9.1	4.1	40362	1722	1798	47	17.6	15.9	8.7	4.4	0.8	0	0	5.4	0	0
		0.25	20	-11.1	72.1	58	83.1	17024	5286	0	19.6	5.1	49336	2740	1844	49.7	15.4	14	8.6	1.5	1.9	0	0	9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.23: One-to-many problem with $\beta < 1$, Core instance:10G2

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	26	-5	107	98	112	7469	680.1	0	9.9	10.5	84585	4028	1298	35.9	25.2	12.8	13.2	4.3	1.7	0	0	6.9	0	0
		0.25	26	9	91	72	82	7469	1133.6	0	24.8	8.8	86677	5502	1698	39	18	13.7	20.9	2.8	1.4	0	0	4.2	0	0
	45	0.1	14	-48	146	98	194	7469	1500	0	9.9	5.7	54376	2582	1770	41.3	20.5	21.9	8.9	0.7	0.3	0	0	6.4	0	0
		0.25	14	-34	134	85	168	7469	2592	0	20.8	4.9	41477	1724	1692	39.2	22.2	15.2	15.1	4	1.2	0	0	3.2	0	0
	60	0.1	10	-117	219	98	336	7469	2920.2	0	9.9	9.1	61456	2649	1722	40.1	20.7	17.4	14	1.3	0	0	0	6.4	0	0
		0.25	10	-95.5	195.5	85	291	7469	5174.7	0	20.8	6	56983	1453	1284	31.9	14	19.8	21	5.4	0.4	0	0	7.6	0	0
1M	**30**	0.1	41	29.8	70.2	15	40.4	5084	238.9	0	6.9	1.9	25982	1172	1254	49.9	12.2	13.3	11.3	3.8	0.9	0	0	8.6	0	0
		0.25	41	29.8	70.2	15	40.4	5084	238.9	0	6.9	1.8	17655	878	1390	51.6	13.1	11.9	8.4	3.8	2.2	0	0	9	0	0
	45	0.1	41	3	99	2	96	5084	520	0	9.9	3.6	51479	6009	1002	42.3	17.8	15.5	9.9	5.1	2.3	0	0	7.1	0	0
		0.25	41	15	85	15	70	5084	570	0	16.8	4.3	50092	8015	1219	48.9	12.2	9	15.9	2.3	2.7	0	0	9.1	0	0
	60	0.1	13	-34.1	132.1	2	166.3	5084	1222.8	0	9.9	5.2	79111	4028	1659	48.9	13.2	14.7	10	4	1.8	0	0	7.4	0	0
		0.25	13	-10.6	110.6	15	121.2	5084	1610.1	0	20.8	5.4	49368	2777	2038	48.3	14.6	13.6	12.9	1.2	1.5	0	0	7.9	0	0
WB	**30**	0.1	23	-5	107	98	112	42515	3686.3	0	10	9.2	89728	5528	1295	40.5	20.4	16.3	11	3.6	0.1	0	0	8.2	0	0
		0.25	23	1.5	98.5	85	97	42515	7080.3	0	23.8	14.4	120549	6622	1642	40.6	19.7	9.5	20.8	3.1	2.1	0	0	4.2	0	0
	45	0.1	11	-46	148	98	194	42515	8278	0	10	8.8	63278	2364	1636	36.4	24.4	17.7	12	0.9	1	0	0	7.5	0	0
		0.25	11	-34	134	85	168	42515	16524	0	23.8	5.5	33780	1490	1577	36.1	21.9	16	16.6	3.9	1	0	0	4.4	0	0
	60	0.1	9	-117	219	98	336	42515	16231	0	10	5.8	40925	1161	1918	37.3	24.2	19.8	10.5	1.5	0.8	0	0	5.9	0	0
		0.25	9	-95.5	195.5	85	291	42515	32880.9	0	23.8	11	71167	2635	2006	40.9	20.6	16.5	12.8	2.7	1.7	0	0	4.8	0	0
WM	**30**	0.1	45	24.3	71.7	85	47.3	28011	1720.5	0	9.1	1.4	18038	1120	989	41.7	17.2	16.3	7.5	4.2	2.6	0	0	10.4	0	0
		0.25	45	24.3	71.7	85	47.3	28011	1720.5	0	9.1	1.6	18572	961	1223	43.3	16.5	18.3	7.2	3.7	1	0	0	9.8	0	0
	45	0.1	35	-4	104	98	108	28011	3458	0	10	3.2	41575	2875	1361	52.3	12.7	12.8	9.8	5	1.8	0	0	5.6	0	0
		0.25	35	8	90	85	82	28011	5074	0	22.9	3	31107	2731	1322	45.8	15	13.2	7.1	2.7	6.5	0	0	9.7	0	0
	60	0.1	22	-43.5	143.5	98	187.1	28011	7885.4	0	10	3.4	37976	1708	1664	49.2	16.1	14.2	6.7	3.8	2.1	0	0	8	0	0
		0.25	22	-22	120	85	142	28011	13047.7	0	23.8	3.5	31919	844	1943	46.1	16.5	15.9	11.8	1.8	1.3	0	0	6.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.24: One-to-many problem with $\beta < 1$, Core instance:F-n135-k7

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)												
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	30	0.1	72	-90.2	-14	230	76.2	13761.6	202.3	0	3	1.1	10555	316	778	47.3	15.9	13.9	12.1	1.5	0.9	0	0	8.4	0	0
		0.25	72	-90.2	-14	230	76.2	13761.6	202.3	0	3	1	8297	332	712	52.9	15.3	9.3	8.4	1.5	0.1	0	0	12.4	0	0
	45	0.1	13	-97.1	32.9	229	130	13761.6	683.7	0	8.1	7.6	41233	1239	1869	45.9	18.2	20.2	7.4	2.8	0.5	0	0	5	0	0
		0.25	13	-77.7	20.3	213	98	13761.6	812.9	0	20.7	13.1	61907	2595	2733	44.2	16.4	16.8	16.5	2	0.3	0	0	3.8	0	0
	60	0.1	3	-144.7	80.5	229	225.2	13761.6	1805.9	0	8.9	9.8	45222	1161	2261	44.2	20.9	19.5	7.3	0.9	0.1	0	0	7.1	0	0
		0.25	3	-113.2	61	214.3	174.2	13761.6	3013.4	0	24.4	11.5	56198	1214	2532	40.3	17.3	19.5	15	1.5	0.7	0	0	5.6	0	0
1M	30	0.1	76	-19.1	33.9	251	53	5431.1	183.6	0	3	0.7	5709	129	603	48.4	18.9	12.6	5	0.5	0.3	0	0	14.3	0	0
		0.25	76	-19.1	33.9	251	53	5431.1	183.6	0	3	0.5	3429	55	507	52.3	10.2	10.4	6.1	0.6	0.6	0	0	19.7	0	0
	45	0.1	65	-38.5	53.3	251	91.8	5431.1	338.8	0	3	1.5	12269	310	864	48.5	18.2	13.7	6.9	1.9	0	0	0	10.7	0	0
		0.25	65	-38.5	53.3	251	91.8	5431.1	338.8	0	3	1.7	18200	518	1146	54.8	10.8	12.7	6.4	4	0.6	0	0	10.8	0	0
	60	0.1	46	-68.6	83.4	249	152.1	5431.1	637.2	0	3.7	2.9	26280	798	1709	49.3	15.6	13.7	11.1	2.6	1	0	0	6.9	0	0
		0.25	46	-68.6	83.4	249	152.1	5431.1	637.2	0	3.7	2.4	17045	420	1483	51.1	12.6	12	12.1	1	0.5	0	0	10.6	0	0
WB	30	0.1	69	-87.7	-11.5	230	76.2	77720.3	1003.1	0	2.8	1.2	9673	331	930	53.4	13.2	11.2	12.4	1.7	1.7	0	0	6.3	0	0
		0.25	69	-87.7	-11.5	230	76.2	77720.3	1003.1	0	2.8	0.9	9455	320	743	49.7	17.8	11.8	7.1	2.2	0.3	0	0	11.2	0	0
	45	0.1	13	-94.1	35.9	229	130	77720.3	4353	0	9.5	7	43688	927	1656	41.9	23.6	18.2	5	2.8	1.4	0	0	7	0	0
		0.25	13	-78	20	213	98	77720.3	4814.3	0	21.5	10.8	53127	1820	2205	43.8	16	12.5	19.8	3.2	0.7	0	0	4	0	0
	60	0.1	2	-147.9	80.7	230	228.6	77720.3	10768.7	0	9.1	11.3	62589	1679	3078	37.9	24.2	25.6	6.6	0.6	0.3	0	0	4.8	0	0
		0.25	2	-111.8	62.4	214.3	174.2	77720.3	17397	0	24.9	11.1	54356	1159	2428	39.7	25	15.4	12.1	1.9	0.6	0	0	5.3	0	0
WM	30	0.1	73	-20	33.2	251	53.1	29597.7	1354.3	0	4	0.7	4118	74	570	49.5	16	12.9	6.5	0.5	0	0	0	14.7	0	0
		0.25	73	-20	33.2	251	53.1	29597.7	1354.3	0	4	0.5	3073	96	641	47.9	14.5	15.6	5.3	2.2	0.3	0	0	14.2	0	0
	45	0.1	63	-39.4	52.6	251	92	29597.7	2520.8	0	4	0.6	5622	140	582	56.2	5.3	8.1	11.3	3.8	1.5	0	0	13.7	0	0
		0.25	63	-39.4	52.6	251	92	29597.7	2520.8	0	4	0.9	7806	190	813	51.4	14	9.1	6.6	3.2	0.9	0	0	14.6	0	0
	60	0.1	49	-69.6	82.8	249	152.4	29597.7	4570.3	0	4.5	2.3	23584	754	1441	46.5	14.5	17.8	9.8	2.4	0.3	0	0	8.8	0	0
		0.25	49	-69.6	82.8	249	152.4	29597.7	4570.3	0	4.5	1.8	13924	364	1496	50.9	9.8	16.8	10.4	1.1	0.7	0	0	10.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.25: One-to-many problem with $\beta < 1$, Core instance:ch150

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	21	72.6	760.7	595.3	688.1	80157.7	5864.5	0	10	12.2	65371	1922	2344	36.3	26.5	19.9	10.7	1.1	0.4	0	0	5	0	0
		0.25	21	83.6	666.3	504.1	582.7	80157.7	9198.1	0	22.7	25.4	141400	4525	2541	31.2	21.1	17.6	22.8	2.4	0.4	0	0	4.5	0	0
	45	0.1	13	-247.4	955.8	601	1203.2	80157.7	12973.4	0	10	15.1	72894	1477	2980	34.7	27.7	20.6	11	0.6	0.1	0	0	5.3	0	0
		0.25	13	-155.7	864.1	509.3	1019.8	80157.7	24521.9	0	24.7	16.6	79147	1618	2723	35.1	21.7	17.3	17.4	2.4	0.7	0	0	5.4	0	0
	60	0.1	7	-687.8	1396.2	601	2084.1	80157.7	26185.8	0	10	10.4	39136	474	2861	35.6	24.9	21	12.3	0.6	0.3	0	0	5.2	0	0
		0.25	7	-528.9	1237.4	509.3	1766.3	80157.7	52143.6	0	24.7	11.8	44207	738	3063	34.6	22.3	18.8	17	1.5	1.1	0	0	4.7	0	0
1M	**30**	0.1	51	215.1	453.6	153.8	238.5	51491.7	1666.9	0	7.3	2.6	22703	932	1569	44	17.3	16.4	8.4	3.2	1.9	0	0	8.8	0	0
		0.25	51	215.1	453.6	153.8	238.5	51491.7	1666.9	0	7.3	2.3	22574	805	1674	50.4	14.3	15.8	7.7	2.1	1	0	0	8.7	0	0
	45	0.1	28	91.6	530.4	140.9	438.8	51491.7	3720.5	0	10	5.6	61025	2856	2217	44.4	15.6	12.9	17.8	3.9	0.9	0	0	4.4	0	0
		0.25	28	112.6	525.6	153.8	413.1	51491.7	4145.4	0	12	5.9	48062	2732	2519	45	16.9	14.5	12.6	2.2	0.8	0	0	8	0	0
	60	0.1	13	-192.1	786.2	77.9	978.3	51491.7	9503.2	0	10	5.7	50071	1762	2063	37.9	21.1	22.1	8.7	5.7	0.8	0	0	3.7	0	0
		0.25	13	-38.6	676.8	153.8	715.5	51491.7	11953.4	0	20.7	7.9	58218	2327	2864	41.6	14.2	16.7	15.7	0.4	2.2	0	0	9.3	0	0
WB	**30**	0.1	20	101.9	790	595.3	688.1	447334.2	31213.7	0	9.9	18.5	100481	2719	2993	42.4	23.7	19.6	8.4	0.8	0.4	0	0	4.6	0	0
		0.25	20	116.2	698.9	504.1	582.7	447334.2	55776.5	0	23.8	24.6	130462	3783	2980	36.3	21.9	19.1	16	2.3	0.4	0	0	4.1	0	0
	45	0.1	12	-259.6	966.3	612.4	1225.9	447334.2	68889.7	0	9.5	16	69142	2685	3055	31.4	30.3	21	11.7	0.7	0.1	0	0	4.7	0	0
		0.25	12	-137	923	529.4	1060	447334.2	142195.4	0	24.6	15.9	79732	1301	2872	32.2	23.2	16.6	19.8	1.8	0.3	0	0	6.1	0	0
	60	0.1	7	-708.4	1415	612.4	2123.4	447334.2	138890.5	0	9.5	11.1	44455	674	3004	37.9	26.7	21.1	8.7	0.8	0.1	0	0	4.6	0	0
		0.25	7	-525	1311	529.4	1836	447334.2	298941.5	0	24.6	11	36517	372	3005	36.4	21.3	16.5	18.4	0.3	1	0	0	6.1	0	0
WM	**30**	0.1	52	176.2	426.7	153.8	250.5	284698.2	8485.8	0	8.8	2.5	21595	1034	1596	43.7	15.1	16.3	12.4	2.5	2.4	0	0	7.6	0	0
		0.25	52	176.2	426.7	153.8	250.5	284698.2	8485.8	0	8.8	2.6	22309	877	1829	46	12.3	17.6	11.6	3.4	1.7	0	0	7.5	0	0
	45	0.1	32	91.6	594	119.6	502.4	284698.2	21557.8	0	9.9	7	60269	2776	2098	39.7	18.2	15.2	16.6	4	1.4	0	0	5	0	0
		0.25	32	61.1	522	140.3	460.8	284698.2	24596	0	13.9	4.1	33402	1946	2144	42.8	15.8	16.8	10.6	2.6	1.9	0	0	9.6	0	0
	60	0.1	18	-185.9	832.9	76.7	1018.7	284698.2	53404.4	0	9.7	5.6	54629	1803	2384	38.8	23.1	19.5	8.9	4.8	0.9	0	0	3.9	0	0
		0.25	18	-98.3	690.5	143	788.8	284698.2	72059.2	0	19.5	7.5	46560	1498	2896	49.5	14	11.2	14.9	0.4	1.8	0	0	8.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.26: One-to-many problem with $\beta < 1$, Core instance:d198

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	30	0.1	17	938.9	3175.8	1936.2	2236.9	411628.6	36612.5	0	9.6	11.4	83898	2154	2693	37	20.4	17.1	16.8	0.5	0	0	0	8.2	0	0
		0.25	17	1090.9	3049.2	1694.9	1958.3	411628.6	83846.4	0	24.7	23	137352	3994	3093	34.1	22.7	16.2	15.3	2.7	0.3	0	0	8.7	0	0
	45	0.1	1	120.2	3994.6	1936.2	3874.4	411628.6	67725.3	0	9.6	27.9	149528	4347	3485	29.4	26.1	25.7	9.9	0.8	0	0	0	8	0	0
		0.25	1	374.2	3766	1694.9	3391.8	411628.6	154090.1	0	24.7	61.7	242747	20262	3951	32.4	21.2	16.9	21.9	1.8	0.1	0	0	5.7	0	0
	60	0.1	1	-1298	5412.7	1936.2	6710.7	411628.6	121614.1	0	9.6	63.6	313691	10930	4276	33.9	24.2	22.7	12.1	0.6	0	0	0	6.6	0	0
		0.25	1	-867.3	5007.5	1694.9	5874.8	411628.6	275755.6	0	24.7	76.7	319999	17353	4350	27.7	20.3	20.9	20.8	1.5	0	0	0	8.7	0	0
1M	30	0.1	60	1445.8	2120.4	1987	674.6	158761.3	2742.2	0	4	2.7	25212	1349	1379	39.8	14.1	13.3	19.2	1.7	1.3	0	0	10.7	0	0
		0.25	60	1445.8	2120.4	1987	674.6	158761.3	2742.2	0	4	2.4	21256	1369	1233	34.1	15.1	12.5	19.9	0.5	2.4	0	0	15.5	0	0
	45	0.1	44	1364	2380	1910.8	1016	158761.3	9652	0	9.6	4.6	34394	1890	2233	41.1	17.9	19.1	10.5	2.2	0.8	0	0	8.4	0	0
		0.25	44	1452.9	2316.5	1834.6	863.6	158761.3	11285.2	0	15.7	4.6	40064	2326	1779	36.9	15.4	10.4	14.5	1	1	0	0	20.8	0	0
	60	0.1	35	960.8	2808.6	1936.2	1847.8	158761.3	24464.7	0	9.6	5.3	44806	2118	2054	46.5	14.7	19.4	8.3	3.6	0.6	0	0	6.9	0	0
		0.25	35	1149.5	2645.3	1834.6	1495.8	158761.3	32457.4	0	17.2	5.4	43348	3196	2145	41.6	10	14.1	12.9	2.5	1.6	0	0	17.3	0	0
WB	30	0.1	15	938.9	3175.8	1936.2	2236.9	2353040.3	212642.4	0	9.8	10.4	55556	1393	2998	40.5	19.1	18.4	10.5	0.3	0	0	0	11.1	0	0
		0.25	15	1068.9	3071.2	1733	2002.3	2353040.3	488550.3	0	24.8	11.3	66858	1550	3121	36.3	17	12.4	18.7	2.4	0.1	0	0	13	0	0
	45	0.1	0	120.2	3994.6	1936.2	3874.4	2353040.3	396044	0	9.8	20.6	116740	1204	3914	30	23	21.1	19.6	0.3	0.4	0	0	5.6	0	0
		0.25	0	336.1	3804.1	1733	3468	2353040.3	901891.6	0	24.8	38.8	196599	5568	4185	26.6	24.6	17.8	19.5	1.3	0	0	0	10.2	0	0
	60	0.1	0	-1298	5412.7	1936.2	6710.7	2353040.3	713704.8	0	9.8	23.7	121151	3632	3061	25.5	27.2	21.8	16.6	0.8	0.3	0	0	7.7	0	0
		0.25	0	-933.3	5073.5	1733	6006.8	2353040.3	1617819.7	0	24.8	46.2	195289	12828	3587	23.1	21	22.8	19	1.9	0	0	0	12.2	0	0
WM	30	0.1	57	1445.8	2120.4	1987	674.6	897530.6	20078.9	0	5.4	2.5	20719	1059	1086	42.2	15.7	9.9	15.7	2.4	0.8	0	0	13.3	0	0
		0.25	57	1445.8	2120.4	1987	674.6	897530.6	20078.9	0	5.4	1.8	13558	729	993	23.2	7.9	10.1	24.5	0.6	1.6	0	0	31.8	0	0
	45	0.1	41	1313.2	2380	1936.2	1066.8	897530.6	60082.7	0	9.5	3.9	36316	2460	1587	42.9	15.6	17	10.9	3.3	0.6	0	0	9.8	0	0
		0.25	41	1452.9	2316.5	1834.6	863.6	897530.6	64002.7	0	15.6	4.1	31499	2806	1782	45.7	13.2	10.1	11.9	0.9	0.8	0	0	17.6	0	0
	60	0.1	33	960.8	2808.6	1936.2	1847.8	897530.6	143498.9	0	9.8	4.3	37075	1566	2001	46.2	14	20.6	7.2	4.3	1.3	0	0	6.3	0	0
		0.25	33	1149.5	2645.3	1834.6	1495.8	897530.6	190783.9	0	18	4.8	29098	2013	2675	54.8	7.9	14.1	10	1.8	1.3	0	0	10	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.27: One-to-many problem with $\beta < 1$, Core instance:gr229

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	1	-186.3	180.1	341.2	366.4	64192.4	6650	22.6	9.6	1000	7188016	267981	2789	36.5	30.9	24.5	2.8	0	0	0	0	5.3	0	0
		0.25	1	-153.3	190.1	321.3	343.4	64192.4	16369.9	0	24.9	73.8	291342	10459	5528	29.7	21.1	24.1	18.8	0.7	0.8	0	0	4.8	0	0
	45	0.1	1	-320.4	314.2	341.2	634.7	64192.4	12551.2	10.6	9.6	1000	4125022	153183	4489	25.7	30.3	27.1	10.4	0.3	0.4	0	0	5.9	0	0
		0.25	1	-279	315.8	321.3	594.8	64192.4	30699.2	0	24.9	26.1	113603	2824	4926	30.9	22.1	23.4	17	0.2	1	0	0	5.4	0	0
	60	0.1	1	-552.8	546.5	341.2	1099.3	64192.4	22772.5	41.2	9.6	1000	4967969	361595	3783	37.8	19.7	31.7	5.1	0.7	0	0	0	4.9	0	0
		0.25	1	-496.7	533.5	321.3	1030.2	64192.4	55518.4	69.9	24.9	1000	5591944	268551	2690	28.1	26	19.6	14.8	2.4	0	0	0	9	0	0
1M	**30**	0.1	17	-116.3	132.2	69.9	248.5	13846.8	1388	0	3.1	17.3	106532	3852	4253	40.3	19.8	19.9	13	2.2	0.4	0	0	4.3	0	0
		0.25	17	-116.3	132.2	69.9	248.5	13846.8	1388	0	3.1	22.3	137388	4921	3893	38.8	18.5	11.8	20.1	0.2	2.1	0	0	8.3	0	0
	45	0.1	8	-186.4	202.4	90.8	388.8	13846.8	2688.7	0	3.5	18.2	98392	3051	4723	38.5	23	16.9	14.1	0.6	0.6	0	0	6.3	0	0
		0.25	8	-2.8	71.2	248.2	74.1	13846.8	2772.1	0	24	76.2	444444	30751	5591	38.6	16.9	15.3	21.9	0.5	1.6	0	0	5.2	0	0
	60	0.1	4	-328.8	344.7	90.8	673.5	13846.8	4965.7	0	3.5	20.3	116497	2325	4840	40.3	23.1	17.3	12.5	1.8	0.7	0	0	4.4	0	0
		0.25	4	-36.3	104.7	244.5	141	13846.8	5662.4	0	23.1	26.1	131609	3244	5866	36.4	16	16	24.2	0.4	2.1	0	0	5	0	0
WB	**30**	0.1	1	-181.5	184	340.4	365.5	356116.7	36841.3	0	9.5	41.5	193035	7029	3466	24.1	30.8	24.6	11.6	0.7	0.3	0	0	7.8	0	0
		0.25	1	-152.1	190	320.2	342.1	356116.7	92064	10.6	24.7	1000	4339795	122874	5620	28.2	22.4	21.3	20.6	1.1	0.1	0	0	6.3	0	0
	45	0.1	1	-315.3	317.8	340.4	633.1	356116.7	69485	18.6	9.5	1000	8015584	332147	3389	28.9	21.5	34.6	8.8	0.8	0.1	0	0	5.3	0	0
		0.25	1	-277.3	315.3	320.2	592.6	356116.7	171708.3	0	24.7	64.9	245518	5016	6196	28.7	23.7	23.1	19	0.3	0.2	0	0	5	0	0
	60	0.1	1	-547	549.5	340.4	1096.5	356116.7	126025.5	1.3	9.5	1000	5111244	241537	5336	34.8	24	23.3	10.6	0.2	0.1	0	0	7	0	0
		0.25	1	-494.2	532.2	320.2	1026.4	356116.7	309656.3	0	24.7	42.3	193076	2231	5176	27.8	22.8	21.1	20.4	0.8	0.5	0	0	6.6	0	0
WM	**30**	0.1	16	-105	117.1	90.8	222.2	77023.9	8140.4	0	3.7	14.4	87951	2539	3925	36.2	21.7	18.1	16.2	2.3	0.6	0	0	4.9	0	0
		0.25	16	-105	117.1	90.8	222.2	77023.9	8140.4	0	3.7	23.4	134737	5915	4828	42.6	15.9	13.5	18.7	0.4	1.6	0	0	7.3	0	0
	45	0.1	9	-184.8	200	90.8	384.8	77023.9	15946.9	0	3.7	34.5	218444	6145	4826	38.1	22.3	18.5	14.2	1	0.6	0	0	5.4	0	0
		0.25	9	-186.4	198.4	90.8	384.8	77023.9	15946.9	0	3.7	26.2	143461	4744	5119	33.8	19.9	15.3	21.5	0.5	3	0	0	6	0	0
	60	0.1	4	-327.2	339.3	90.8	666.5	77023.9	29468.2	0	3.7	34.7	209258	8205	5482	37.3	23.4	19.3	14	0.8	0.7	0	0	4.6	0	0
		0.25	4	-33.5	100.5	244.5	134	77023.9	31170.4	0	24.4	45.7	267759	9705	5912	36	17.2	14.9	24.4	0.6	2.4	0	0	4.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.28: One-to-many problem with $\beta < 1$, Core instance:a280

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	44	68.9	227.1	145	158.2	43422	2665.4	0	10	121.2	658992	31119	2800	36.1	23.4	15.4	15.8	1.2	0.6	0	0	7.6	0	0
		0.25	44	73.5	222.5	137	149	43422	3293.3	0	13.9	122	467202	57337	3649	32.8	19.7	13.6	23.4	0	0.7	0	0	9.8	0	0
	45	0.1	24	-9	297	161	306	43422	5138	0	9.6	64.4	270201	7543	4610	33.5	26.8	15.5	15.4	0.3	0.5	0	0	8	0	0
		0.25	24	15	273	137	258	43422	9126	0	23.2	318.4	1201004	62195	4650	24.8	21.1	15	28.9	0.5	0.6	0	0	8.9	0	0
	60	0.1	12	-118.1	398.1	157	516.2	43422	11200.2	0	10	32.3	129439	2291	4882	36.6	27.2	16.6	12.4	0.2	0.2	0	0	6.8	0	0
		0.25	12	-79.4	367.4	137	446.9	43422	21561.4	0	23.6	100.9	418043	6365	4826	32	22.3	15.2	18.7	1	0.3	0	0	10.6	0	0
1M	**30**	0.1	67	97.4	166.6	145	69.3	31918	607.1	0	6.4	17.4	130541	23965	2099	39.2	15.8	18	13.6	1.7	3.4	0	0	8.3	0	0
		0.25	67	97.4	166.6	145	69.3	31918	607.1	0	6.4	9.6	79791	13007	1969	42.1	13.7	15.6	15.3	2.3	2.4	0	0	8.7	0	0
	45	0.1	48	84	188	137	104	31918	1716	0	10	42.2	332396	32681	3313	39.3	17.1	16.1	18.1	2.1	1.8	0	0	5.6	0	0
		0.25	48	84	188	137	104	31918	1716	0	10	28.4	201405	26311	2917	35.4	16.6	15.4	19	0.8	3.6	0	0	9.2	0	0
	60	0.1	31	4.4	267.6	161	263.3	31918	3984.3	0	9.6	65.1	556284	37549	3955	36.1	19.6	17.1	17.1	3.4	0.8	0	0	6	0	0
		0.25	31	36.1	243.9	145	207.8	31918	4844.6	1.9	17.1	1017.1	3643387	753542	4136	36.1	18.2	15.2	18.4	0.7	2.9	0	0	8.6	0	0
WB	**30**	0.1	43	64.9	223.1	145	158.2	229495	15181.1	0	10	184.6	1107751	39311	2592	34.3	19.3	15.4	19	1.3	1.3	0	0	9.5	0	0
		0.25	43	73.5	222.5	137	149	229495	20423.2	0	15	65.3	331078	19380	3305	30.5	22.1	14.2	22.2	0.4	2.2	0	0	8.5	0	0
	45	0.1	23	-13	285	157	298	229495	27536	0	10	58.4	274567	6530	3846	33.2	22.8	17	17.1	0.4	0.3	0	0	9.3	0	0
		0.25	23	15	273	137	258	229495	52836	0	23.8	75.1	346172	11230	3941	26.6	22.3	15.8	23.5	0.6	1.8	0	0	9.3	0	0
	60	0.1	12	-122.1	394.1	157	516.2	229495	60258.7	0	10	19.8	84358	1212	4403	34	27.1	16.6	14.5	0.2	0.3	0	0	7.4	0	0
		0.25	12	-79.4	367.4	137	446.9	229495	120610	0	23.9	76.8	307858	7386	4898	30.3	21.9	15.2	21.9	0.7	0.3	0	0	9.7	0	0
WM	**30**	0.1	66	110	170	137	60	169022	4037.3	0	8	5.4	54197	3842	2075	41.3	16.4	18.2	11.7	2.1	1.8	0	0	8.6	0	0
		0.25	66	110	170	137	60	169022	4037.3	0	8	5.7	59492	5542	1759	40.3	16.5	15.8	10.8	3	1.7	0	0	11.9	0	0
	45	0.1	48	80	200	145	120	169022	10052	0	9.7	24.5	206603	17755	3124	42.3	19.4	14	14	2.1	1.8	0	0	6.2	0	0
		0.25	48	88	192	137	104	169022	11012	0	12.2	15	137821	11429	2891	37.9	14.4	15	14.9	1.5	2.5	0	0	13.9	0	0
	60	0.1	31	8.4	271.6	161	263.3	169022	21136.2	0	9.1	49.7	363939	16378	3879	30	19.7	14	22.9	2	0.6	0	0	10.8	0	0
		0.25	31	49.9	230.1	137	180.1	169022	27751.7	0	17.8	162.2	711056	115233	4182	41.5	15.8	13.8	16.7	1.4	2.7	0	0	8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.29: One-to-many problem with $\beta < 1$, Core instance:lin318

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1B	**30**	0.1	18	-397.7	3523.7	3316	3921.4	855394	78132.3	4.8	9.7	1000	3990990	245605	5712	32.7	22.1	21.1	16.3	0.7	0	0	0	7.2	0	0
		0.25	18	-170	3233	2867	3402.9	855394	162234.3	0	24.8	84.6	381369	8804	4818	26.4	18.3	19	25.6	2.1	0.2	0	0	8.5	0	0
	45	0.1	12	-1833	4959	3316	6792	855394	167122	15.6	9.7	1000	4312620	73609	6099	35.4	23.8	18.1	14.7	0.3	0.2	0	0	7.5	0	0
		0.25	12	-1415.5	4478.5	2867	5894	855394	359031	0	24.8	123	370441	21436	6018	26	18.9	15.1	27.9	1.7	0.1	0	0	10.3	0	0
	60	0.1	7	-4319	7445	3316	11764.1	855394	321256.8	28.1	9.7	1000	3174043	214986	5505	30.2	26.8	19.7	13.9	0.5	0.3	0	0	8.6	0	0
		0.25	7	-3572.9	6635.9	2867	10208.7	855394	699892.9	0	24.8	285.8	634020	31613	8149	27.9	19	16.1	26.8	1.1	1	0	0	8	0	0
1M	**30**	0.1	33	779.3	2133.7	654	1354.5	557400	27310.4	0	9.1	72.9	624251	48010	3317	36.4	17.8	15.6	17.7	3.2	1	0	0	8.2	0	0
		0.25	33	779.3	2133.7	654	1354.5	557400	27310.4	0	9.1	77.8	444456	57701	4623	31.6	16.1	13.3	17.4	0.7	2.5	0	0	18.5	0	0
	45	0.1	24	-143	2835	3316	2978	557400	53590	0	9.7	403.2	2283736	274166	5734	38	17.7	15.4	20.7	1.6	1.3	0	0	5.4	0	0
		0.25	24	283.5	2629.5	654	2346	557400	80431	0	21.1	268.7	901443	211659	4436	34.5	18.6	15.2	16.7	0.9	3.1	0	0	10.9	0	0
	60	0.1	13	-1330.9	4235.9	220	5566.8	557400	116566.2	0	9.7	51.8	302926	11011	5611	32.2	20.1	19	19.3	1.4	0.3	0	0	7.7	0	0
		0.25	13	-543.7	3519.7	654	4063.4	557400	208459.9	0	24.8	107.7	434027	57192	6127	34.5	14.7	14.4	24.8	0.6	2.3	0	0	8.7	0	0
WB	**30**	0.1	20	-267.7	3653.7	3316	3921.4	4670951	392616.4	0	10	434.4	1596706	62784	5506	29.5	27.7	20.5	15	0.3	0.1	0	0	7	0	0
		0.25	20	-219.6	3282.6	2953	3502.2	4670951	914669.7	0	24.9	128.9	519812	18989	5089	31.8	20.9	24.6	14.2	1.9	0.4	0	0	6.2	0	0
	45	0.1	13	-1703	5089	3316	6792	4670951	886366	0	10	108.9	359182	9628	5795	27.8	25	22.2	15.7	0.3	0.2	0	0	8.7	0	0
		0.25	13	-1501.5	4564.5	2953	6066	4670951	2014537	0	24.9	245	992065	59341	4647	26.8	19.3	17.7	23.4	2.1	0	0	0	10.6	0	0
	60	0.1	7	-4189	7575	3316	11764.1	4670951	1741565.3	0	10	67.2	241126	4006	6152	29.2	27.8	23.4	11.1	0.1	0.1	0	0	8.3	0	0
		0.25	7	-3721.8	6784.8	2953	10506.6	4670951	3919563.1	0	24.9	57.8	181164	5436	5179	26.3	24.7	19.5	16.2	1.7	0.5	0	0	11.1	0	0
WM	**30**	0.1	33	709	2346	3158	1637	3085675	168672	0	9.9	81.8	593734	45246	4050	38.9	14.8	12.6	22.2	2.3	2.8	0	0	6.4	0	0
		0.25	33	765.2	2210.8	2953	1445.7	3085675	185131.8	0	11.8	56.2	350445	31206	4433	34.2	16.5	11.4	16.6	1	3.2	0	0	17.1	0	0
	45	0.1	22	-395	2835	3316	3230	3085675	324624	0	10	89	630229	31876	5258	34.1	18.1	16.2	23.6	2.2	1.3	0	0	4.4	0	0
		0.25	22	204.5	2708.5	2953	2504	3085675	516500	0	22.6	256.2	865937	181396	5737	29.2	15.7	17.8	24.5	0.4	3	0	0	9.3	0	0
	60	0.1	16	-2100	5431	3875	7531	3085675	713236.7	0	7.1	51.2	284355	10579	5776	31.3	21.7	17.1	20.7	1.5	1.2	0	0	6.6	0	0
		0.25	16	-680.5	3656.5	2953	4337.1	3085675	1275210.7	0	24.9	43	228411	13613	5661	29.2	19.9	14.2	22.9	0.5	2.2	0	0	11.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table C.30: One-to-many problem with $\beta < 1$, Core instance:fl417

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1B	**30**	0.1	37	-57.4	2132.5	2048.1	2189.9	586807.7	52730.8	8.8	9.8	1000	2029627	347401	5028	30	21.8	19	20.3	0.9	0	0	0	8	0	0
		0.25	37	268.6	2444.9	2036.2	2176.3	586807.7	130087.3	26.3	24.9	1000	2945524	316718	4564	29.5	24.5	15.7	15.7	1	2.8	0	0	10.8	0	0
	45	0.1	37	-533.8	3259.2	2048.1	3793	586807.7	118517.9	12.7	9.8	1000	1315972	372301	6014	33.7	19.9	19.3	19.8	0.4	0.1	0	0	6.8	0	0
		0.25	37	-527.9	3241.5	2036.2	3769.4	586807.7	295773.8	6.5	24.9	1000	2328357	200148	6999	28.6	20.1	16.2	27	1.7	0.1	0	0	6.3	0	0
	60	0.1	37	-1922.2	4647.5	2048.1	6569.7	586807.7	232362.1	23	9.8	1000	3123265	330901	5854	28.3	22.4	20.6	20.9	0.4	0.1	0	0	7.4	0	0
		0.25	37	-1907.6	4621.2	2036.2	6528.8	586807.7	582751.2	11.6	24.9	1000	2462546	233414	6884	26.5	17.2	16.8	30.9	1.7	0.1	0	0	6.9	0	0
1M	**30**	0.1	5	647.3	1445.1	1935.9	797.8	513378.2	16687.8	0	9.8	1210.4	6157937	280444	6186	27.9	21.3	16.9	20.1	2.9	0.2	0	0	10.6	0	0
		0.25	5	248.6	1489.5	170.3	1241	513378.2	54030.1	6.7	18.7	1263.9	3420794	466873	7374	26.4	16.2	16.9	22.2	2.7	2.2	0	0	13.2	0	0
	45	0.1	3	211.1	2396	152.5	2184.9	513378.2	49946.6	12.1	9.8	2000	3897044	767880	7550	24.6	23.8	16.8	23.8	2	0.2	0	0	8.8	0	0
		0.25	3	225.9	2387.1	164.4	2161.2	513378.2	125982.9	8.1	24.9	2000.1	6673557	441321	9517	18.9	14.7	13.6	38.8	2.7	1.3	0	0	9.9	0	0
	60	0.1	2	-588.6	3195.7	152.5	3784.3	513378.2	115523.3	10.4	9.8	2000.1	3122755	818121	8665	31.4	22.3	20.2	16.2	1.7	0.1	0	0	8.2	0	0
		0.25	2	-562.2	3181.2	164.4	3743.4	513378.2	290431.1	11.5	24.9	2000.1	9668791	604747	8656	21.2	17.9	14.6	31.1	3.8	0.6	0	0	10.7	0	0
WB	**30**	0.1	39	261.8	2451.7	2048.1	2189.9	3183500.3	293747.3	8.1	10	1000.1	2383119	263601	6296	29.1	23.6	20.8	21	0.6	0	0	0	5	0	0
		0.25	39	271.6	2447.9	2036.2	2176.3	3183500.3	748459.8	4.1	25	1000	2657190	197694	6857	27.2	17.3	16.4	30.1	0.8	2	0	0	6.2	0	0
	45	0.1	39	-539.8	3253.3	2048.1	3793	3183500.3	668877.9	8.1	10	1000	1855607	311601	5739	29.6	21.8	20.4	21	0.6	0	0	0	6.5	0	0
		0.25	39	-525	3244.4	2036.2	3769.4	3183500.3	1680446.2	5.4	25	1000	1661849	287301	6344	25.4	15.8	14.5	35	0.4	0	0	0	8.8	0	0
	60	0.1	39	-1928.1	4641.6	2048.1	6569.7	3183500.3	1318623.2	13.5	10	1000	1755708	378161	5819	32.8	21	20.4	19.5	0.3	0	0	0	6	0	0
		0.25	39	-1904.7	4624.1	2036.2	6528.8	3183500.3	3294694.2	10.2	25	1000	2711682	191102	6803	32.7	18.7	18	21.9	1.7	0.4	0	0	6.7	0	0
WM	**30**	0.1	6	643.8	2075.7	158.5	1431.9	2899108.8	171340.4	1.9	10	1053.3	4941531	366550	5932	28.9	19.7	18.8	19.3	4	0.7	0	0	8.6	0	0
		0.25	6	169.1	1587.3	170.3	1418.3	2899108.8	392456.2	9.6	25	1942	5668411	822457	7184	26.8	13.6	18.2	20.8	2.4	1	0	0	17.3	0	0
	45	0.1	4	78.3	2558.4	158.5	2480.1	2899108.8	359067.9	7.3	10	2000	7880056	752043	8113	25.3	21	18.9	25.8	2.4	0.3	0	0	6.3	0	0
		0.25	4	87.1	2555.4	164.4	2468.3	2899108.8	906467	7.4	25	2000.2	7943322	666602	8404	27.2	17.6	15.6	26.3	3.2	2	0	0	8.1	0	0
	60	0.1	3	-823.6	3472.1	158.5	4295.7	2899108.8	785921	10.7	10	2000	4542624	786024	7872	28.7	21.8	19.8	18.6	2.7	0.1	0	0	8.3	0	0
		0.25	3	-816.3	3458.9	164.4	4275.2	2899108.8	1963518.4	11.1	25	2000	7841271	528773	9236	27	14.1	12.3	32.4	3.1	0.2	0	0	10.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

APPENDIX D

COMPUTATIONAL RESULTS FOR ONE-TO-MANY INTERDICTION PROBLEM WITH A SINGLE BARRIER ON A PLANE

In this appendix, computational results for one-to-many interdiction problem when $\beta = 1$ are presented. The algorithm is developed in a VB.NET application and all computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

The optimal location of the line barrier and the related objective value are reported for 1B, 1M, WB and WM variants of 30 instances and different levels of θ and β levels. x_s, x_e, y, and L represent the optimal barrier's endpoints along x-axis, its y-coordinate and length. The objective function values before and after interdiction are shown as Z_0 and ΔZ, respectively. $E\%$ shows the percentage of eliminated weights in pre-processing.

MIP models of all problems are also solved using CPLEX Optimizer 10.1 with a time limit of 1000 seconds. CPU time (in seconds), number of iterations used for solving node relaxations (Niter), number of processed nodes in the active branch-and-cut search (Nodes) give the solver's performance in solving these problems. $\hat{\beta}$ gives the actual disruption rate realized by the MIP solution. Following MIP cuts are also set in the solver with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

Table D.1: One-to-many problem with $\beta = 1$, Core instance:D8-Canbolat

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	25	3.5	11.5	8	8.1	65	15.3	0	0	37.5	0.1	320	15
	45	1	25	0.5	14.5	8	14	65	38	0	0	50	0	271	14
	60	1	25	-4.6	19.6	8	24.3	65	79	0	0	50	0	308	16
1M	30	1	50	5	9	8	4	46	8.1	0	0	25	0	147	8
	45	1	38	3.5	10.5	8	7	46	15	0	0	37.5	0	200	14
	60	1	25	0.9	13.1	8	12.1	46	32.5	0	0	50	0	232	16
WB	30	1	24	3.5	11.5	8	8.1	336	96.4	0	0	45.9	0	334	19
	45	1	24	0.5	14.5	8	14	336	242	0	0	70.3	0	208	7
	60	1	24	-4.6	19.6	8	24.3	336	508.5	0	0	70.3	0	218	7
WM	30	1	57	2	9	2	6.9	187	27.5	0	0	18.9	0	140	0
	45	1	57	-0.5	11.5	2	12	187	63	0	0	18.9	0	186	4
	60	1	22	-4.9	15.9	2	20.8	187	140.1	0	0	24.3	0	180	2

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.2: One-to-many problem with $\beta = 1$, Core instance:E-n22-k4

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	18	117.6	176.4	231	58.9	1182	399.8	0	0	50	0.4	4726	234
	45	1	5	96	198	231	102	1182	874	0	0	50	0.4	3331	123
	60	1	5	58.7	235.3	231	176.7	1182	1695.4	0	0	50	0.5	3698	122
1M	30	1	18	123.1	173.9	261	50.8	722	75.6	0	0	9.1	0.2	1707	119
	45	1	14	119.5	177.5	246	58	722	210	0	0	27.3	0.3	1978	100
	60	1	9	98.3	198.7	246	100.5	722	464.8	0	0	27.3	0.2	1559	96
WB	30	1	21	116.6	175.4	231	58.9	6418	1985.2	0	0	54.9	0.3	2410	92
	45	1	3	95	197	231	102	6418	4658	0	0	54.9	0.4	3065	68
	60	1	3	57.7	234.3	231	176.7	6418	9287.5	0	0	54.9	0.5	2903	74
WM	30	1	19	129.1	172.9	193	43.9	3854	579.3	0	0	19.5	0.2	1227	64
	45	1	13	113	189	193	76	3854	1414	0	0	23	0.2	1430	65
	60	1	9	85.2	216.8	193	131.6	3854	2860.5	0	0	23	0.3	1530	59

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.3: One-to-many problem with $\beta = 1$, Core instance:D28

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	7	-9.6	350.6	310	360.3	10403	2115.9	0.02	0	39.3	0.8	10005	741
	45	1	0	-88	536	310	624	10403	6018	0	0	53.6	0.5	6051	379
	60	1	0	-316.4	764.4	310	1080.8	10403	12870	0	0	53.6	0.5	3816	253
1M	30	1	43	26.7	240.3	125	213.6	5435	741.1	0	0	17.9	0.1	966	45
	45	1	39	-51.5	318.5	125	370	5435	1523	0	0	17.9	0.2	1456	101
	60	1	21	-177.9	462.9	125	640.9	5435	3073.2	0	0	21.4	0.3	1529	95
WB	30	1	6	-9.6	350.6	310	360.3	59705	14939	0.02	0	48.4	0.6	6028	295
	45	1	0	-131.5	492.5	310	624	59705	39290	0	0	60.6	0.6	3969	142
	60	1	0	-359.9	720.9	310	1080.8	59705	82229.1	0	0	60.6	0.5	3916	197
WM	30	1	49	26.7	240.3	125	213.6	27764	3466.9	0	0	15.5	0.1	919	34
	45	1	43	-51.5	318.5	125	370	27764	7220	0.02	0	15.5	0.2	1023	44
	60	1	19	-177.9	462.9	125	640.9	27764	14504	0	0	18.1	0.3	1958	84

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.4: One-to-many problem with β = 1, Core instance:B-n31-k5

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	90	-7.7	74.2	76	82	1694.5	49.5	0	0	3.2	0	58	0
	45	1	23	-37.8	104.3	76	142	1694.5	109.5	0.02	0	3.2	0.6	4605	167
	60	1	19	-3.1	69.6	26	72.8	1694.5	479.9	0.02	0	51.6	0.7	7148	447
1M	30	1	48	14	36	8	21.9	774	61.6	0	0	19.4	0.2	898	25
	45	1	42	6	44	8	38	774	158	0	0	19.4	0.2	842	19
	60	1	23	-7.9	57.9	8	65.8	774	324.9	0	0	19.4	0.3	1102	15
WB	30	1	93	-7.7	74.2	76	82	8595.5	99	0	0	1.2	0	75	0
	45	1	26	13.8	55.8	26	42	8595.5	525.5	0	0	34.2	0.5	4196	105
	60	1	21	-3.1	69.6	26	72.8	8595.5	2406.7	0	0	52.2	0.6	5049	249
WM	30	1	53	13	36.1	8	23.1	3739	447.2	0	0	21.1	0.2	1079	16
	45	1	48	4.5	44.5	8	40	3739	1022	0.02	0	21.1	0.1	827	14
	60	1	29	-10.1	59.1	8	69.3	3739	2017.6	0	0	21.1	0.2	1237	20

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.5: One-to-many problem with β = 1, Core instance:A-n32-k5

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	31	21.2	89.3	59	68.1	2315	265.4	0	0	21.9	0.4	3236	141
	45	1	19	-9.3	108.8	59	118	2315	729.5	0	0	34.4	0.6	5681	271
	60	1	19	-34.6	142.1	51	176.7	2315	1685.4	0.02	0	43.8	0.6	4347	121
1M	30	1	53	32.2	66.8	9	34.6	1779	86.9	0	0	9.4	0.1	1015	34
	45	1	28	-10.5	89.5	89	100	1779	197	0	0	9.4	0.3	2778	112
	60	1	19	-43	106	82	149	1779	419.8	0	0	12.5	0.4	3277	186
WB	30	1	32	-5.1	74.6	69	79.7	15836	1778.4	0	0	22.4	0.5	4083	138
	45	1	17	-9.3	108.8	59	118	15836	5058	0.02	0	36.4	0.6	4598	145
	60	1	17	-52.4	151.9	59	204.4	15836	11795.8	0	0	36.4	0.5	3705	114
WM	30	1	56	34	72.1	9	38.1	12280	552.4	0	0	10.7	0.1	787	38
	45	1	29	-4	90	89	94	12280	1224	0	0	11.2	0.3	2365	95
	60	1	19	-3.7	110.7	9	114.3	12280	3053.8	0.02	0	22	0.4	3413	149

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.6: One-to-many problem with β = 1, Core instance:D40

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	18	-4.7	285.2	264	289.8	11645	5600.6	0.02	0	65	0.8	8290	288
	45	1	10	-110.8	391.3	264	502	11645	11117	0	0	65	0.7	5105	117
	60	1	5	-294.5	575	264	869.5	11645	20671.7	0	0	65	0.8	5363	88
1M	30	1	23	58.9	307.1	70	248.3	6058	1320.1	0	0	20	0.5	3542	210
	45	1	10	-32	398	70	430	6058	2774	0.02	0	20	0.6	3949	240
	60	1	10	-189.4	555.4	70	744.8	6058	5292.3	0.02	0	20	0.5	3452	171
WB	30	1	36	-4.7	285.2	264	289.8	14076.5	5600.6	0.02	0	42.6	0.8	8577	327
	45	1	23	-110.8	391.3	264	502	14076.5	11117	0	0	42.6	0.9	5490	171
	60	1	15	-294.5	575	264	869.5	14076.5	20671.7	0	0	42.6	1	8358	198
WM	30	1	15	58.9	307.1	70	248.3	11731	3954.3	0.02	0	39.3	0.5	3406	188
	45	1	7	-32	398	70	430	11731	8316	0.03	0	39.3	0.5	3342	214
	60	1	7	-189.4	555.4	70	744.8	11731	15870.8	0.02	0	39.3	0.5	3534	172

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.7: One-to-many problem with $\beta = 1$, Core instance:B-n41-k6

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	20	46	113	64	67	3362	391.5	0.02	0	41.5	1.1	12513	545
	45	1	12	26.5	134.5	60	108	3362	1280	0	0	48.8	1.6	13789	624
	60	1	12	-14	173	60	187.1	3362	3155.5	0	0	58.5	1.3	9707	351
1M	30	1	51	42.3	74.7	36	32.3	2197	138	0	0	14.6	0.2	1428	74
	45	1	39	30.5	86.5	36	56	2197	293	0	0	17.1	0.4	3411	287
	60	1	39	0.5	97.5	36	97	2197	763.9	0	0	34.1	0.3	3396	259
WB	30	1	15	23.4	106.6	78	83.1	4324	1029.4	0.02	0	31.5	0.9	8027	358
	45	1	9	2	128	69	126	4324	2218	0.02	0	50	0.9	7306	392
	60	1	9	-40.7	170.7	67	211.3	4324	4806	0	0	53.7	0.8	5889	196
WM	30	1	41	42.3	74.7	36	32.3	2582	220	0	0	16.7	0.2	1125	40
	45	1	31	30.5	86.5	36	56	2582	446	0.02	0	18.5	0.3	2468	159
	60	1	31	6	103	36	97	2582	993.9	0	0	31.5	0.3	2286	156

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.8: One-to-many problem with $\beta = 1$, Core instance:A-n45-k6

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	22	12.3	86.2	68	73.9	3373.5	392.6	0	0	20	0.9	8760	472
	45	1	22	-13.8	112.3	67	126	3373.5	1093.5	0.02	0	33.3	1.1	9611	534
	60	1	16	-59.9	158.4	67	218.2	3373.5	2477.1	0.02	0	33.3	1.5	17419	626
1M	30	1	27	23	75	94	52	2453	95.9	0	0	8.9	0.3	1916	63
	45	1	27	8	98	94	90	2453	254	0	0	11.1	0.6	5127	234
	60	1	13	-28.9	126.9	94	155.9	2453	635.3	0.02	0	13.3	0.7	6486	373
WB	30	1	21	22.7	75.8	50	53.1	18122.5	1376.5	0	0	18.1	1	9211	569
	45	1	21	-13.8	112.3	67	126	18122.5	4640.5	0	0	31.6	1.2	13276	787
	60	1	12	-37.1	132.6	53	169.7	18122.5	12353.2	0.02	0	49.4	1.2	11860	399
WM	30	1	34	30.3	78.8	95	48.5	12959	366	0.02	0	5.1	0.3	1892	64
	45	1	34	13.5	95.5	94	82	12959	811	0	0	8	0.5	4307	182
	60	1	19	-11.9	112.9	17	124.7	12959	2264	0	0	20.3	0.8	7442	367

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.9: One-to-many problem with $\beta = 1$, Core instance:F-n45-k4

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	2	-43.5	61.5	-9	105.1	5879	1740.2	0.02	0	62.2	2.4	28576	1735
	45	1	2	-69.3	106.8	-12	176	5879	4249.5	0.02	0	77.8	1.6	16390	697
	60	1	2	-116.4	153.9	-22	270.2	5879	8861.7	0.02	0	91.1	1.7	14596	524
1M	30	1	64	-23.8	36.3	52	60	2157.5	90.6	0	0	6.7	0	370	6
	45	1	64	-45.7	58.2	52	104	2157.5	222.4	0.02	0	6.7	0.2	940	27
	60	1	31	-71.9	83.9	45	155.9	2157.5	481.4	0	0	8.9	0.4	3201	105
WB	30	1	3	-28.6	73.1	-12	101.6	26113.5	7657.4	0.02	0	69.9	1.7	13063	778
	45	1	3	-60.8	115.3	-12	176	26113.5	19241.5	0.02	0	76.2	1.2	10863	408
	60	1	3	-115.4	154.9	-22	270.2	26113.5	40322.2	0.02	0	90.3	1.1	7212	240
WM	30	1	56	-2.9	114.9	-99	117.8	9843.5	432.4	0	0	3.4	0	522	10
	45	1	56	-46	158	-99	204	9843.5	1036	0	0	3.4	0.1	817	16
	60	1	38	-120.7	232.7	-99	353.3	9843.5	2081.3	0.02	0	3.4	0.4	2676	58

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.10: One-to-many problem with β = 1, Core instance:att48

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	29	4405	9346	5497	4941	174125	41603.5	0.02	0	27.1	1	7669	357
	45	1	27	2596.5	11154.5	5497	8558	174125	88625	0.02	0	27.1	1.1	12600	379
	60	1	19	-536	14287	5497	14822.9	174125	170068.6	0.02	0	27.1	1.4	14541	417
1M	30	1	27	4431	9134.1	7065	4703.1	141559	22804.7	0.02	0	14.6	0.6	4278	176
	45	1	13	2709.5	10855.5	7065	8146	141559	46905	0.02	0	14.6	0.8	7842	332
	60	1	8	2232.3	11737.8	5736	9505.5	141559	92081.9	0.02	0	25	1	7781	230
WB	30	1	27	4405	9346	5497	4941	937901	243191.3	0.02	0	28.7	0.8	6179	226
	45	1	25	2596.5	11154.5	5497	8558	937901	510852	0	0	28.7	0.8	5851	248
	60	1	17	-536	14287	5497	14822.9	937901	974453.9	0.02	0	28.7	1.2	9993	181
WM	30	1	33	4300.3	9139.7	7065	4839.4	776397	116693.2	0.02	0	13.6	0.5	3657	108
	45	1	15	4150.5	9396.5	5497	5246	776397	258302	0.02	0	28.7	0.8	7984	248
	60	1	6	2230.3	11316.7	5497	9086.3	776397	542487	0.02	0	28.7	0.8	6578	212

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.11: One-to-many problem with β = 1, Core instance:B-n50-k7

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	26	-5.1	74.6	69	79.7	3508	616.1	0.02	0	24	1.4	12620	485
	45	1	18	-25.3	100.8	63	126	3508	1371.5	0.02	0	34	1.4	14422	694
	60	1	14	-50.5	147	57	197.5	3508	3270.9	0.02	0	48	1.8	15490	668
1M	30	1	42	8.3	63.7	83	55.4	2512	150	0	0	14	0.3	2583	117
	45	1	14	0	74	72	74	2512	450	0.02	0	22	0.9	8017	265
	60	1	14	-27.1	101.1	72	128.2	2512	1045.9	0.02	0	22	1	8114	336
WB	30	1	28	-5.1	74.6	69	79.7	19725.5	2845.6	0	0	20	1.3	16059	802
	45	1	17	4.8	118.8	57	114	19725.5	7458.5	0.02	0	48.4	1.4	16725	558
	60	1	12	-37	160.5	57	197.5	19725.5	18557.8	0.02	0	48.4	1.2	11512	290
WM	30	1	52	8.3	63.7	83	55.4	13877	652.8	0.02	0	10.9	0.4	2567	76
	45	1	19	-2	76	74	78	13877	2170	0.02	0	17.5	0.9	6388	180
	60	1	19	-30.6	104.6	74	135.1	13877	4910.8	0.02	0	17.5	1	9617	453

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.12: One-to-many problem with β = 1, Core instance:D50

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	2	93.7	333.8	269	240.2	12397	3772.6	0.02	0	46	1.9	15414	911
	45	1	2	5.8	421.8	269	416	12397	8114.5	0.02	0	50	2.1	19581	848
	60	1	2	-140.9	562.4	264	703.2	12397	15786.5	0.02	0	52	2	20514	683
1M	30	1	32	134	276	387	142	6211	626.2	0	0	12	0.6	5366	221
	45	1	24	101	305	162	204	6211	1362	0.02	0	18	0.8	6369	250
	60	1	14	26.3	379.7	162	353.3	6211	2706	0.02	0	18	1	8767	306
WB	30	1	7	90.1	331.4	269	241.3	14352	4230.8	0.02	0	36.8	2.3	19094	780
	45	1	7	1.8	419.8	269	418	14352	8949.5	0.02	0	39.7	2.5	19537	880
	60	1	7	-142.6	564.1	264	706.7	14352	17244.9	0.03	0	41.2	1.8	15404	503
WM	30	1	26	126.6	274.4	114	147.8	9392	1705.6	0.02	0	25	0.5	3330	130
	45	1	19	72.5	328.5	114	256	9392	3545	0.02	0	25	0.7	4845	139
	60	1	15	-21.2	422.2	114	443.4	9392	6730.9	0.02	0	25	0.8	5143	196

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.13: One-to-many problem with β = 1, Core instance:eil51

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	14	11.8	59.2	46	47.3	2489	407.5	0.02	0	31.4	2.5	23018	1463
	45	1	12	-5.5	76.5	46	82	2489	1066	0.02	0	39.2	1.6	14135	636
	60	1	2	-23.4	94.4	39	117.8	2489	2324.3	0.02	0	51	2	13850	474
1M	30	1	43	23.2	49.8	16	26.6	1529	68.2	0.02	0	7.8	0.5	3318	157
	45	1	29	13	59	16	46	1529	162	0.02	0	13.7	0.8	8614	322
	60	1	16	-3.3	76.3	62	79.7	1529	436.1	0.02	0	17.6	0.9	8035	349
WB	30	1	14	13.1	63.9	48	50.8	12302	2273	0.02	0	35.8	1.7	15764	837
	45	1	9	-6	82	48	88	12302	5844	0.03	0	40.2	2	16235	869
	60	1	2	-25.1	96.1	39	121.2	12302	12396.1	0.02	0	53.7	1.7	14775	468
WM	30	1	48	24.1	51.9	16	27.7	7288	500.5	0.02	0	10.6	0.4	2427	83
	45	1	34	14	62	16	48	7288	1086	0.02	0	13.4	0.6	5912	160
	60	1	16	-5.1	78.1	16	83.1	7288	2453.8	0.02	0	17.1	0.9	7820	240

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.14: One-to-many problem with β = 1, Core instance:berlin52

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	25	461.6	1120.9	575	659.3	46522	8670.9	0.02	0	48.1	1.5	15276	785
	45	1	13	220.3	1362.3	575	1142	46522	22130	0.02	0	57.7	1.5	13923	468
	60	1	10	-197.8	1780.3	575	1978	46522	48246.1	0.03	0	61.5	1.8	16510	774
1M	30	1	63	331.1	948.9	1130	617.8	25425	995.5	0	0	3.8	0.1	1366	43
	45	1	42	267.5	997.5	960	730	25425	2120	0	0	7.7	0.5	4864	189
	60	1	25	7.8	1272.2	960	1264.4	25425	4502	0	0	9.6	0.8	5623	264
WB	30	1	29	454.1	1113.4	575	659.3	262069.5	41836.9	0.02	0	40.7	1.3	11086	541
	45	1	15	220.3	1362.3	575	1142	262069.5	106075.5	0.02	0	50.2	1.7	11515	388
	60	1	10	-197.8	1780.3	575	1978	262069.5	236855.8	0.03	0	54.2	1.8	12142	382
WM	30	1	67	551.4	828.6	370	277.1	159100	4403.8	0	0	10.1	0.2	1319	40
	45	1	48	450	930	370	480	159100	10490	0	0	10.1	0.3	2530	93
	60	1	23	241.8	1073.2	370	831.4	159100	22453.4	0.02	0	11.8	0.7	6260	250

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.15: One-to-many problem with β = 1, Core instance:A-n60-k9

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	5	18	93	69	75.1	4584	759.3	0.03	0	38.3	2.9	24501	2471
	45	1	5	-12.5	117.5	69	130	4584	2256	0.03	0	46.7	2.2	20602	759
	60	1	5	-60.1	165.1	69	225.2	4584	4920.7	0.03	0	46.7	2.3	18608	505
1M	30	1	15	19.6	70.4	17	50.8	3018	322.5	0.03	0	13.3	0.5	4959	165
	45	1	15	3	91	17	88	3018	644	0.03	0	16.7	1	10060	401
	60	1	15	-29.2	123.2	17	152.4	3018	1405	0.03	0	20	1.2	11060	476
WB	30	1	4	18	93	69	75.1	26130	5096	0.02	0	42.4	2.1	19318	1516
	45	1	4	-10.5	119.5	69	130	26130	13878	0.03	0	47.6	2.2	22195	1095
	60	1	4	-58.1	167.1	69	225.2	26130	29294.9	0.03	0	47.6	2	13079	340
WM	30	1	12	20.9	67.1	17	46.2	16756	1363.7	0.03	0	12.1	0.4	3638	115
	45	1	12	4	84	17	80	16756	2760	0.03	0	13.5	1.2	10037	257
	60	1	12	-25.3	113.3	17	138.6	16756	6069.8	0.03	0	17.6	1.1	10676	317

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.16: One-to-many problem with $\beta = 1$, Core instance:B-n68-k9

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	29	-1	73	70	73.9	4898	1256.8	0.03	0	47.1	1.8	17910	784
	45	1	24	-26	98	68	124	4898	3068	0.05	0	51.5	1.8	16283	457
	60	1	15	-67.9	139.9	66	207.9	4898	6296.3	0.05	0	54.4	1.9	11351	283
1M	30	1	43	7.3	62.7	20	55.4	3394	302.6	0.06	0	8.8	0.9	9370	522
	45	1	29	-15	89	16	104	3394	924	0.05	0	27.9	1.3	13519	738
	60	1	25	-40.9	121.9	21	162.8	3394	2371.9	0.05	0	32.4	1.1	10002	393
WB	30	1	24	-1	73	70	73.9	27920	7091.7	0.03	0	49.7	2	20351	1123
	45	1	19	-24	96	66	120	27920	17544	0.05	0	58.4	1.5	11799	352
	60	1	14	-67.9	139.9	66	207.9	27920	36518.7	0.03	0	58.4	1.7	11508	308
WM	30	1	46	8.3	63.7	20	55.4	17566	1400.5	0.03	0	7.3	0.6	7661	374
	45	1	35	-16	88	16	104	17566	4246	0.03	0	23.2	1.4	15227	744
	60	1	29	-54.1	126.1	16	180.1	17566	10793.4	0.05	0	23.2	1	10391	343

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.17: One-to-many problem with $\beta = 1$, Core instance:F-n72-k4

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	6	-23.1	11.6	4	34.6	2600	890.1	0.06	0	59.7	9.3	87074	10311
	45	1	3	-35.8	24.3	4	60	2600	1980.5	0.11	0	59.7	6.2	44844	1704
	60	1	1	-57.7	46.2	4	103.9	2600	3869.2	0.06	0	59.7	6.3	45867	1216
1M	30	1	39	-15.7	9.7	-17	25.4	1142	105.8	0.05	0	9.7	1.2	15103	914
	45	1	28	-17.5	8.5	18	26	1142	237	0.05	0	18.1	1.4	15294	613
	60	1	15	-27	18	18	45	1142	484.4	0.05	0	18.1	1.8	15459	556
WB	30	1	3	-22.2	13.7	5	35.8	13459	4482.3	0.06	0	60.1	8.5	69766	3267
	45	1	1	-35.3	24.8	4	60	13459	10220.5	0.06	0	63.8	5.6	42826	1128
	60	1	1	-57.2	46.7	4	103.9	13459	20191	0.06	0	63.8	3.1	18271	445
WM	30	1	46	-10.9	2.9	18	13.9	5600	476.4	0.03	0	18.8	1.2	14019	748
	45	1	35	-16	8	18	24	5600	1156	0.03	0	18.8	1.3	13300	735
	60	1	23	-26.6	11.6	-5	38.1	5600	2349	0.05	0	24.2	1.4	13514	516

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.18: One-to-many problem with $\beta = 1$, Core instance:rus75

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	25	24.6	71.9	42	47.3	5083.5	534.8	0.05	0	36	3.2	29456	2482
	45	1	17	-3.8	92.3	49	96	5083.5	1620	0.05	0	45.3	4.6	41426	3095
	60	1	9	-25.8	116.3	42	142	5083.5	4099.2	0.06	0	56	2.6	21800	769
1M	30	1	61	40.3	80.7	8	40.4	3465	127.9	0.03	0	9.3	0.4	2894	112
	45	1	33	25.5	95.5	8	70	3465	335	0.03	0	9.3	1	7403	234
	60	1	13	-0.1	121.1	8	121.2	3465	693.7	0.05	0	9.3	1.9	17300	821
WB	30	1	27	19	74.5	49	55.4	31343.5	3189.8	0.05	0	30	3.7	31021	2188
	45	1	18	-2.8	93.3	49	96	31343.5	9906.5	0.05	0	46.2	3.7	29788	1140
	60	1	12	-37.9	128.4	49	166.3	31343.5	24734.9	0.05	0	46.2	2.4	19137	647
WM	30	1	63	39.5	84.5	6	45	21814	851.3	0.03	0	8.3	0.4	2126	61
	45	1	38	23	101	6	78	21814	2104	0.05	0	8.3	0.9	6513	190
	60	1	10	19.3	119.7	16	100.5	21814	4296.4	0.05	0	19.3	1.7	13114	656

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.19: One-to-many problem with $\beta = 1$, Core instance:eil76

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	17	11.9	66.1	50	54.3	3703	569.2	0.06	0	25	5.3	46728	4039
	45	1	11	2	76	40	74	3703	1528	0.06	0	44.7	4.8	34635	1944
	60	1	7	-21.6	99.6	38	121.2	3703	3394	0.08	0	48.7	5.3	35839	885
1M	30	1	39	26.1	53.9	60	27.7	2353	112	0.03	0	9.2	1.2	11213	535
	45	1	22	16	64	60	48	2353	288	0.05	0	13.2	1.7	17730	860
	60	1	14	0.2	79.8	59	79.7	2353	695.8	0.05	0	17.1	2.4	30469	1311
WB	30	1	12	19.4	58.6	37	39.3	19808	3487.4	0.06	0	41.7	4.9	42112	4233
	45	1	8	7	71	35	64	19808	9124	0.06	0	59.8	6.5	49896	1776
	60	1	6	-16.4	94.4	35	110.9	19808	20602.5	0.06	0	59.8	4.4	28863	627
WM	30	1	39	23.1	51.9	60	28.9	11886	713	0.05	0	9.3	1	10990	426
	45	1	26	13.5	61.5	59	48	11886	1778	0.05	0	15.1	1.6	16635	770
	60	1	16	1.9	78.1	57	76.2	11886	4316.3	0.05	0	21.2	2	19218	939

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.20: One-to-many problem with $\beta = 1$, Core instance:A-n80-k10

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	26	4.7	91.3	74	86.6	5530	804.2	0.05	0	21.2	4.7	36436	2068
	45	1	15	-24	124	73	148	5530	2008	0.06	0	25	4.3	29821	1339
	60	1	6	-37.3	139.3	50	176.7	5530	4288.1	0.08	0	41.2	4.8	30248	1051
1M	30	1	46	25.4	71.6	80	46.2	3822	194.3	0.03	0	8.8	1.1	11214	546
	45	1	30	6	86	80	80	3822	538	0.06	0	16.2	1.5	16926	916
	60	1	18	-23.3	115.3	80	138.6	3822	1392.5	0.06	0	18.8	2.1	24689	1144
WB	30	1	22	14.3	91.7	66	77.4	30282	4217.8	0.06	0	22.2	4.1	38938	2300
	45	1	12	-14	120	66	134	30282	10408	0.06	0	25.4	4.3	35143	2403
	60	1	5	-35.3	141.3	50	176.7	30282	23525.1	0.08	0	40.4	4.9	32058	864
WM	30	1	43	33.1	75.9	80	42.7	20073	739.5	0.05	0	8.6	1.1	11505	486
	45	1	35	14.5	88.5	80	74	20073	2352	0.05	0	15.4	1.4	14238	710
	60	1	19	-15.1	113.1	80	128.2	20073	6469.4	0.06	0	17.7	1.9	20942	920

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.21: One-to-many problem with $\beta = 1$, Core instance:rd100

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	19	35.5	880	731.5	844.4	83112	16203.1	0.12	0	34	4	27259	1023
	45	1	13	-230	1223.3	726.8	1453.3	83112	41652.9	0.14	0	43	6.6	40750	1305
	60	1	10	-762	1755.2	726.8	2517.1	83112	87399.2	0.17	0	43	4.3	17434	385
1M	30	1	46	215	755.5	116.3	540.5	53338.2	2381.8	0.08	0	7	1.3	12452	421
	45	1	27	-48.5	887.7	116.3	936.2	53338.2	6044.6	0.11	0	13	2.4	25906	1210
	60	1	19	-365.5	1204.7	131.1	1570.2	53338.2	15652.4	0.14	0	15	2.6	26191	952
WB	30	1	18	-21	911.7	807.9	932.7	513349	116767.2	0.12	0	30.9	4.4	26328	1398
	45	1	12	-263.2	1199.3	731.5	1462.6	513349	276077.6	0.14	0	44.6	6	31918	750
	60	1	9	-790.5	1726.6	726.8	2517.1	513349	566425.1	0.19	0	45.1	5.9	30449	430
WM	30	1	42	206.7	779.8	116.3	573.1	321632.4	13438	0.08	0	5.3	1.3	13799	422
	45	1	27	217.3	731.2	869.5	513.9	321632.4	33486	0.11	0	16.6	3.3	38166	1712
	60	1	14	-411.8	1256.1	131.1	1667.9	321632.4	88603	0.12	0	13.5	2.8	26159	1211

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.22: One-to-many problem with $\beta = 1$, Core instance:E-n101-k14

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	16	9.8	61.7	47	52	5029.5	711.5	0.12	0	26.7	14	92181	4983
	45	1	6	-10.3	79.8	47	90	5029.5	1953	0.14	0	33.7	11.6	95415	3865
	60	1	4	-33.5	105	42	138.6	5029.5	4210.6	0.16	0	39.6	8.8	48291	1421
1M	30	1	42	16.6	45.4	60	28.9	3258	116.1	0.09	0	6.9	1.5	16278	665
	45	1	25	6	56	60	50	3258	316	0.12	0	11.9	3.2	35708	2775
	60	1	14	-10.3	76.3	60	86.6	3258	865.6	0.12	0	15.8	7.7	106898	6679
WB	30	1	13	1.9	59.6	52	57.7	26667	3606	0.14	0	23.2	14	95161	4569
	45	1	5	-10.3	79.8	47	90	26667	10146	0.16	0	35	8.6	46771	1956
	60	1	3	-21.4	92.9	35	114.3	26667	22069.8	0.16	0	53.9	12.5	76782	1637
WM	30	1	46	13.6	41.4	58	27.7	17024	733.5	0.09	0	11.9	1.4	13239	657
	45	1	27	3.5	51.5	58	48	17024	2091	0.16	0	15	3.2	33087	2057
	60	1	20	-11.1	72.1	58	83.1	17024	5286	0.14	0	19.6	4.8	44080	2436

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.23: One-to-many problem with $\beta = 1$, Core instance:10G2

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	26	9	91	72	82	7469	1133.6	0.12	0	24.8	14.9	110326	8336
	45	1	14	-8	108	59	116	7469	2856	0.14	0	43.6	14.4	87749	9723
	60	1	10	-48.7	148.7	58	197.5	7469	6694.9	0.14	0	45.5	39.4	276276	9131
1M	30	1	41	29.8	70.2	15	40.4	5084	238.9	0.08	0	6.9	1.6	19046	966
	45	1	41	15	85	15	70	5084	570	0.09	0	16.8	4.4	47835	6829
	60	1	13	-10.6	110.6	15	121.2	5084	1610.1	0.12	0	20.8	5	60857	3774
WB	30	1	23	1.5	98.5	85	97	42515	7080.3	0.12	0	23.8	9.8	77023	3913
	45	1	11	-20	122	72	142	42515	17156	0.14	0	30	12.5	84546	4935
	60	1	9	-46.7	150.7	58	197.5	42515	39057.7	0.16	0	47.1	14.6	110352	3676
WM	30	1	45	24.3	71.7	85	47.3	28011	1720.5	0.09	0	9.1	1.5	15277	759
	45	1	35	8	90	85	82	28011	5074	0.11	0	22.9	3.8	41684	4548
	60	1	22	-22	120	85	142	28011	13047.7	0.12	0	23.8	3.4	28910	1258

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.24: One-to-many problem with $\beta = 1$, Core instance:F-n135-k7

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1B	30	1	72	-87.7	-11.5	30	76.2	13761.6	202.3	0.09	0	3	1.3	14826	491
	45	1	13	-78	20	13	98	13761.6	812.8	0.3	0	20.7	14.8	76687	3495
	60	1	3	-89.6	41.4	1.8	130.9	13761.6	5059.7	0.36	0	73.3	15.6	77005	5092
1M	30	1	76	-19.1	33.9	51	53	5431.1	183.6	0.11	0	3	0.4	3429	55
	45	1	65	-38.5	53.3	51	91.8	5431.1	338.8	0.16	0	3	1.5	12119	266
	60	1	46	-68.6	83.4	49	152.1	5431.1	637.2	0.19	0	3.7	2.2	23009	844
WB	30	1	69	-87.7	-11.5	30	76.2	77720.3	1003.1	0.11	0	2.8	1	10796	405
	45	1	13	-78	20	13	98	77720.3	4814.2	0.28	0	21.5	25.6	151941	9955
	60	1	2	-89.5	41.5	1.8	130.9	77720.3	28382.8	0.36	0	73.9	16	74829	6559
WM	30	1	73	-20	33.2	51	53.1	29597.7	1354.3	0.11	0	4	0.5	3073	96
	45	1	63	-39.4	52.6	51	92	29597.7	2520.8	0.14	0	4	1.3	10447	162
	60	1	49	-69.6	82.8	49	152.4	29597.7	4570.3	0.19	0	4.5	2.4	19345	602

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.25: One-to-many problem with $\beta = 1$, Core instance:ch150

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
	30	1	21	131.5	617.9	420.7	486.4	80157.7	9296.5	0.42	0	27.3	35.4	185368	12636
1B	45	1	13	-67.9	774.6	420.7	842.5	80157.7	29897.1	0.48	0	43.3	11.3	67042	2946
	60	1	7	-376.3	1082.9	420.7	1459.2	80157.7	69984.5	0.5	0	43.3	12.7	67228	1557
	30	1	51	215.1	453.6	153.8	238.5	51491.7	1666.9	0.25	0	7.3	2.3	21798	892
1M	45	1	28	112.6	525.6	153.8	413.1	51491.7	4145.4	0.36	0	12	5.6	51566	2846
	60	1	13	-38.6	676.8	153.8	715.5	51491.7	11953.3	0.44	0	20.7	8.3	65561	2676
	30	1	20	116.2	698.9	504.1	582.7	447334.2	55776.5	0.42	0	23.8	15.6	84956	3931
WB	45	1	12	-49.9	799.3	424	849.1	447334.2	165902.9	0.47	0	43.6	14.2	80598	2533
	60	1	7	-360.7	1110.1	424	1470.7	447334.2	388434	0.55	0	43.6	19.1	108000	3279
	30	1	52	177.8	416.3	153.8	238.5	284698.2	8111.2	0.25	0	8.8	2.4	25632	1045
WM	45	1	32	67.1	507.1	140.3	440	284698.2	22985.4	0.39	0	13.9	4	32683	1901
	60	1	18	-84.7	668.1	143	752.7	284698.2	66869.5	0.44	0	19.5	5.1	42515	1847

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.26: One-to-many problem with $\beta = 1$, Core instance:d198

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
	30	1	17	1140.5	2644.1	1301.2	1503.7	411628.6	113940	1.08	0	66.2	109.3	323149	59440
1B	45	1	1	552	3156.4	1301.2	2604.4	411628.6	262802.4	1.23	0	69.2	33.5	155039	4830
	60	1	1	-363.2	4147.8	1301.2	4511	411628.6	528768.6	1.28	1.8	73.2	1000	1979987	500944
	30	1	60	1445.8	2120.4	1987	674.6	158761.3	2742.2	0.38	0	4	2.7	24945	1586
1M	45	1	44	1452.9	2316.5	1834.6	863.6	158761.3	11285.2	0.52	0	15.7	4.4	35174	3279
	60	1	35	1149.5	2645.3	1834.6	1495.8	158761.3	32457.4	0.66	0	17.2	4.3	37280	2141
	30	1	15	1140.5	2644.1	1301.2	1503.7	2353040.3	670501.5	1.11	0	67.5	176.7	299813	120541
WB	45	1	0	577.4	3181.8	1301.2	2604.4	2353040.3	1541364.1	1.23	0	70.2	43.1	144745	13915
	60	1	0	-350.5	4160.5	1301.2	4511	2353040.3	3094622.5	1.23	0	74.4	30.4	144243	6672
	30	1	57	1445.8	2120.4	1987	674.6	897530.6	20078.9	0.38	0	5.4	3.5	38154	2338
WM	45	1	41	1452.9	2316.5	1834.6	863.6	897530.6	64002.8	0.52	0	15.6	3.7	34014	2078
	60	1	33	1149.5	2645.3	1834.6	1495.8	897530.6	190783.9	0.67	0	18	5	43082	2258

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.27: One-to-many problem with $\beta = 1$, Core instance:gr229

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
	30	1	1	-99.8	141.1	32.5	240.9	64192.4	42322.2	1.89	0	91.7	314.9	760672	128205
1B	45	1	1	-185.3	227.2	30.2	412.5	64192.4	79413	1.89	0	93	34.1	140402	2402
	60	1	1	-336.3	378.2	30.2	714.5	64192.4	143739	1.91	0	93	99.8	332733	2091
	30	1	17	-116.3	132.3	-130.1	248.6	13846.8	1388	1.41	0	3.1	23.2	182966	10158
1M	45	1	8	0	67.6	51.4	67.6	13846.8	2837.9	1.56	0	27.5	24.3	129789	4811
	60	1	4	-25	92.1	51.4	117.1	13846.8	6078.7	1.67	0	29.7	31.8	188735	8383
	30	1	1	-98.1	140.1	30.2	238.2	356116.7	237532	1.89	0	92.5	64.3	299576	13763
WB	45	1	1	-185.3	227.3	30.2	412.5	356116.7	444671.5	1.91	0	92.5	49.3	238128	2440
	60	1	1	-325.6	367.7	24	693.2	356116.7	804081.5	1.91	0	95.6	47.3	176655	1382
	30	1	16	-103.5	118.7	-109.2	222.2	77023.9	8140.4	1.36	0	3.7	24.8	192399	7125
WM	45	1	9	-184.8	200	-109.2	384.8	77023.9	15946.9	1.56	0	3.7	20.1	104806	4568
	60	1	4	-27.5	93.9	48.2	121.3	77023.9	33433.2	1.67	0	28.9	20.7	115126	3617

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.28: One-to-many problem with $\beta = 1$, Core instance:a280

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	44	73.5	222.5	137	149	43422	3293.3	1.78	0	13.9	114.3	482717	45732
	45	1	24	15	273	137	258	43422	9126	2.44	0	23.2	137.6	503486	34851
	60	1	12	-40.8	336.8	117	377.6	43422	22085.2	2.91	9.7	31.8	1000	1644586	416101
1M	30	1	67	97.4	166.6	145	69.3	31918	607.1	0.84	0	6.4	9.6	79791	13007
	45	1	48	84	188	137	104	31918	1716	1.52	0	10.4	45.6	290376	44453
	60	1	31	36.1	243.9	145	207.9	31918	4844.6	2.19	0	17.1	188.8	784491	140516
WB	30	1	43	73.5	222.5	137	149	229495	20423.2	1.77	0	15	45	240406	12397
	45	1	23	15	273	137	258	229495	52835.9	2.47	0	23.8	97.8	453495	16346
	60	1	12	-50.7	354.7	125	405.3	229495	123906.6	2.89	0	29	643.1	1209785	263763
WM	30	1	66	110	170	137	60	169022	4037.3	0.83	0	8	5.7	59492	5542
	45	1	48	88	192	137	104	169022	11012	1.53	0	12.2	26.7	264180	22651
	60	1	31	49.9	230.1	137	180.1	169022	27751.6	2.2	0	17.8	124.1	572513	99877

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.29: One-to-many problem with $\beta = 1$, Core instance:lin318

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	18	-133.6	3196.6	2804	3330.2	855394	184796.7	3.92	0	29.6	93.6	352184	12873
	45	1	12	-1092.5	4155.5	2544	5248	855394	455773	4.39	0	38.7	508.2	798446	197459
	60	1	7	-3013.4	6076.4	2544	9089.8	855394	928314.7	4.73	8.8	38.7	1000	1941583	322529
1M	30	1	33	779.3	2133.7	654	1354.5	557400	27310.4	3.05	0	9.1	58.6	362273	37492
	45	1	24	283.5	2629.5	654	2346	557400	80431	3.61	0	21.1	259.5	862030	203606
	60	1	13	-543.7	3519.7	654	4063.4	557400	215580.1	4.08	0	25.8	153	475266	111515
WB	30	1	20	-169.1	3161.1	2804	3330.2	4670951	1052389	3.92	0	30.8	68.4	229170	12318
	45	1	13	-1128	4120	2544	5248	4670951	2502443.9	4.55	0	38.8	76	216151	11333
	60	1	7	-3048.9	6040.9	2544	9089.8	4670951	5072609.7	4.75	0	38.8	111.1	128849	41947
WM	30	1	33	765.2	2210.8	2953	1445.7	3085675	185131.7	3.06	0	11.8	57.6	369621	30670
	45	1	22	204.5	2708.5	2953	2504	3085675	516499.9	3.52	0	22.6	365.1	1135191	264327
	60	1	16	-458	3363	2804	3820.9	3085675	1318150.9	4.02	0	30.8	67.8	380296	12891

$\hat{\beta}$ is the actual percentage of disrupted points.

Table D.30: One-to-many problem with $\beta = 1$, Core instance:fl417

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1B	30	1	37	156.7	2148.9	1876.8	1992.2	586807.7	215500.6	6.33	0	45.1	193.1	341299	43300
	45	1	37	-572.5	2878.1	1876.8	3450.6	586807.7	489674.7	6.52	31.8	45.1	1000	4851085	81666
	60	1	37	-1835.5	4141.1	1876.8	5976.5	586807.7	964558.2	6.62	22.3	45.1	1000	4656124	226100
1M	30	1	5	248.6	1489.5	170.3	1241	513378.2	54030.1	9.81	22.5	18.7	1141.9	3795836	482453
	45	1	3	146.1	2295.5	170.3	2149.4	513378.2	182219.2	10.38	20.5	37.4	2000.1	7999273	496474
	60	1	2	-791.4	2931.5	170.3	3722.9	513378.2	427684.6	10.61	4	37.4	2000	2876092	860087
WB	30	1	39	189.2	2181.4	1876.8	1992.2	3183500.3	1160108.7	6.5	1.7	42.5	1000	3719035	157328
	45	1	39	-540	2910.6	1876.8	3450.6	3183500.3	2609731.5	6.52	0	42.5	81.1	342416	6265
	60	1	39	-1802.9	4173.6	1876.8	5976.5	3183500.3	5120551.9	6.62	0	42.5	332.2	1150802	30916
WM	30	1	6	632.9	2051.1	170.3	1418.3	2899108.8	428266.5	9.78	11.5	29.3	1246.7	2628071	554575
	45	1	4	84.2	2540.7	170.3	2456.5	2899108.8	1360878.3	10.34	4.7	39.3	2000.1	5953544	756819
	60	1	3	-815	3439.8	170.3	4254.8	2899108.8	3013497.4	10.58	4.9	39.3	2000	4745530	665791

$\hat{\beta}$ is the actual percentage of disrupted points.

APPENDIX E

COMPUTATIONAL RESULTS FOR MANY-TO-MANY INTERDICTION PROBLEM WITH A SINGLE BARRIER ON A PLANE SUBJECT TO DISRUPTION CONSTRAINT

In this appendix, computational results for many-to-many interdiction problem when $\beta < 1$ are presented. CPLEX Optimizer 10.1 is used for solving MIP models and all computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

The optimal location of the line barrier and the related objective value are reported for 1N, WN variants of 24 instances and different levels of θ and β levels. x_s, x_e, y, and L represent the optimal barrier's endpoints along x-axis, its y-coordinate and length. The objective function values before and after interdiction are shown as Z_0 and ΔZ, respectively. Since no point elimination is possible in many-to-many problem, all $E\%$ are zero. $\hat{\beta}$ gives the actual disruption rate realized by the optimal solution. Since computations are terminated at the time limit of 1000 seconds, a gap percentage (Gap%) is also reported. CPU time (in seconds), number of iterations used for solving node relaxations (Niter), number of processed nodes in the active branch-and-cut search (Nodes) give the solver's performance in solving these problems. Following MIP cuts are also set in the solver with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

Number of MIP cuts (#cuts) and also the percentage of each cut used by the solver are also provided in the tables of this appendix.

Table E.1: Many-to-many problem with $\beta < 1$, Core instance:D8-Canbolat

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	1.8	7	2	5.2	460	16.8	0	3.6	0.6	4201	106	377	51.2	25.7	8.8	0.8	2.7	0.8	0	0	10.1	0	0
		0.25	0	2.9	8.1	11	5.2	460	17.6	0	12.5	0.6	5455	160	373	62.7	16.6	4.3	0.8	1.6	0.5	0	0	13.4	0	0
	45	0.1	0	7	16	2	9	460	28	0	7.1	1.1	8028	236	470	47	29.4	10.6	1.5	1.3	1.1	0	0	9.1	0	0
		0.25	0	1.5	10.5	11	9	460	50	0	16.1	1	12247	575	411	43.1	28	10	3.2	2.7	2.4	0	0	10.7	0	0
	60	0.1	0	7	22.6	2	15.6	460	32	0	7.1	1.2	13198	528	477	42.3	30.8	12.4	3.4	1	1.3	0	0	8.8	0	0
		0.25	0	-0.8	14.8	11	15.6	460	134.2	0	23.2	0.9	10197	324	428	51.6	22.2	7	3	3	1.4	0	0	11.7	0	0
WN	**30**	0.1	0	1.4	6.6	2	5.2	2145	82.7	0	3.9	0.6	4227	127	383	58	23.8	5	0.5	2.6	0	0	0	10.2	0	0
		0.25	0	2.9	8.1	2	5.2	2145	101	0	7.7	0.6	3796	79	454	69.8	14.8	3.3	2.2	0.9	0.4	0	0	8.6	0	0
	45	0.1	0	7.5	16.5	2	9	2145	114	0	7.3	1.2	10922	421	411	45	30.4	9.5	2.2	2.2	0.5	0	0	10.2	0	0
		0.25	0	1	10	2	9	2145	276	0	7.7	0.8	7985	305	370	44.9	25.1	8.9	2.4	2.4	2.7	0	0	13.5	0	0
	60	0.1	0	7	17.4	9.5	10.4	2145	170	0	6.2	0.9	10547	303	437	40.7	36.6	8	4.1	0.5	0.5	0	0	9.6	0	0
		0.25	0	-3.6	12	2	15.6	2145	580.9	0	10.8	1.2	12241	567	444	48.2	26.4	6.3	3.6	3.6	0.9	0	0	11	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.2: Many-to-many problem with $\beta < 1$, Core instance:E-n22-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	124.8	172.2	264	47.3	20820	1058.4	0	8.9	81.3	318724	3726	3253	32.2	38	16.8	4.8	2.2	0.1	0	0	6	0	0
		0.25	0	131.1	174.9	261	43.9	20820	1600.7	5.3	12.6	1000	3619945	410958	2668	32.2	35.8	13.7	6	5.3	0.3	0	0	6.7	0	0
	45	0.1	0	107.5	189.5	264	82	20820	2514	0	8.9	96	348220	4275	3688	27.8	41.6	15.6	7.5	1.7	0	0	0	5.7	0	0
		0.25	0	110	178	189	68	20820	4540	0	24	437.4	1266469	205418	2798	33.3	37.2	12.9	4.8	5.3	0.2	0	0	6.4	0	0
	60	0.1	0	77.5	219.5	264	142	20820	5035.2	0	8.9	71.3	248221	1944	3639	28.3	39.9	17	7.1	1.5	0	0	0	6.1	0	0
		0.25	0	85.1	202.9	189	117.8	20820	10214.9	0	24	63.9	268984	3389	3142	35.6	36.4	14.3	3.9	4.2	0.2	0	0	5.3	0	0
WN	**30**	0.1	0	125.3	172.7	264	47.3	109150	4974	0	8	79.5	301929	3667	3534	27.6	40	16.3	8.2	2	0.5	0	0	5.5	0	0
		0.25	0	129.1	172.9	185	43.9	109150	7399.9	9.3	14.1	1000	1712404	649900	2392	23.9	42.4	18.5	1.5	7	0.5	0	0	6.2	0	0
	45	0.1	0	108	190	264	82	109150	12356	0	8	55.8	223621	2086	3422	28.1	42.2	17.1	4.4	1.8	0.1	0	0	6.3	0	0
		0.25	0	112	180	189	68	109150	22360	0	28.1	479.3	972261	245574	2634	32.3	39.6	13.9	1.2	5.7	0.3	0	0	7	0	0
	60	0.1	0	78	220	264	142	109150	25142	0	8	127.1	446929	3743	4029	31.7	38.5	15.2	7.1	1.4	0.3	0	0	5.7	0	0
		0.25	0	87.1	204.9	189	117.8	109150	51182.3	0	28.1	70.5	279166	4550	3233	37.5	34.8	14.4	3.6	4.5	0.3	0	0	4.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.3: Many-to-many problem with $\beta < 1$, Core instance:D28

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	23.9	197.1	97	173.2	195830	7301.5	0	7.3	63.1	213135	1842	3931	22.9	42.5	19.1	6.3	1.3	0.3	0	0	7.6	0	0
		0.25	0	23.9	197.1	97	173.2	195830	7301.5	3.6	7.3	1000	1440277	473523	3058	23.9	41.8	20.4	1	3.6	0.4	0	0	8.9	0	0
	45	0.1	0	-45	285	82	330	195830	14088	0	6.7	111.5	337873	2295	5012	26.6	40.5	17.9	6.3	1.1	0.4	0	0	7.2	0	0
		0.25	0	-23.5	276.5	97	300	195830	17100	0	10.6	157.7	497977	10671	3984	24.6	41.3	13.5	5.6	2.9	2.3	0	0	9.8	0	0
	60	0.1	0	-216.9	320	402	536.9	195830	22208	0	9.3	132	398999	2572	5071	20	44.2	18.8	8.5	1.2	0.1	0	0	7.2	0	0
		0.25	0	-113.1	410	398	523.1	195830	46423.4	0	23.8	131.5	331771	16804	4074	22.9	44	14.1	6	3.6	0.4	0	0	9	0	0
WN	**30**	0.1	0	25.4	198.6	97	173.2	1061359	43167.2	0	6.6	103	336143	2759	4119	23.2	40.4	19.7	8.5	1.4	0.2	0	0	6.5	0	0
		0.25	0	45.6	186.4	125	140.9	1061359	40669.1	5.6	10.5	1000	1045376	464111	2876	22.8	41.3	17.1	4	3.8	0.2	0	0	10.8	0	0
	45	0.1	0	-64	266	82	330	1061359	84318	0	6.5	118.7	363441	3658	4598	21.7	45.3	20.4	3.2	1.6	0.1	0	0	7.7	0	0
		0.25	0	-26.5	273.5	97	300	1061359	98660	0	9.4	105.4	333566	8801	3885	28	41.9	14.8	2.5	3	0.5	0	0	9.3	0	0
	60	0.1	0	-185.9	351	402	536.9	1061359	133844.7	0	11.9	139.5	416127	2619	4880	26.2	44.1	16.3	4.9	1	0.1	0	0	7.4	0	0
		0.25	0	-94.8	424.8	97	519.6	1061359	249285.2	12.8	14.7	1000	1991090	250551	3999	22.6	43.1	14.3	6.3	4.3	0.6	0	0	8.9	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.4: Many-to-many problem with $\beta < 1$, Core instance:B-n31-k5

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	2.9	42.1	7	39.3	37460	2301.8	0	8.8	189.6	509058	5721	5179	23.1	41	21.2	4.1	1.3	2.4	0	0	6.9	0	0
		0.25	0	4.4	42.6	8	38.1	37460	4701.4	0	21.3	102.1	282293	3138	3750	28.4	40.3	13.5	4.7	2.8	1.2	0	0	9	0	0
	45	0.1	0	0	68	7	68	37460	3344	0	9.7	215	547041	4633	5726	25.4	40.7	20.7	5.3	1.3	0.1	0	0	6.4	0	0
		0.25	0	-10	56	8	66	37460	9968	0	23.1	393.5	906668	33245	4393	31.1	40.4	13.3	2.3	3.1	0.5	0	0	9.1	0	0
	60	0.1	0	-42.6	78.6	76	121.2	37460	5516.7	0	2.8	258.3	630232	4728	6738	25.7	38.2	21.1	7.3	0.9	0.1	0	0	6.6	0	0
		0.25	0	-34.7	79.7	8	114.3	37460	20597.4	0	23.1	666.8	1581096	68087	5130	23.1	40	16.4	7.8	2.1	1.3	0	0	9.2	0	0
WN	**30**	0.1	0	5	44.3	7	39.3	186689	13282.8	0	7.5	205.8	560808	4619	5360	23.4	41.4	22.3	3.5	1.3	1.4	0	0	6.7	0	0
		0.25	0	4.4	42.6	8	38.1	186689	25531.7	0	21.6	65.6	204016	2330	3588	22.5	39.7	17.9	3.9	3.9	2.3	0	0	9.7	0	0
	45	0.1	0	-11.5	56.5	7	68	186689	21915	0	8.4	179.6	457217	3209	5547	23.5	39.4	21.6	6.9	1	1	0	0	6.7	0	0
		0.25	0	-10.5	55.5	8	66	186689	51170	0	22.4	150.7	409784	5048	4481	31.8	38	14.3	3.5	2.3	0.7	0	0	9.5	0	0
	60	0.1	0	-36.4	81.4	7	117.8	186689	44266	0	8.4	220.3	551130	3680	6154	23.4	39.5	19.7	8.4	1.2	0.2	0	0	7.5	0	0
		0.25	0	-34.7	79.7	8	114.3	186689	105669.7	0	22.4	247.6	618388	6561	5521	27.1	41.1	18.3	2.3	2.3	0.3	0	0	8.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.5: Many-to-many problem with $\beta < 1$, Core instance:A-n32-k5

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	64.8	116.2	5	51.4	74174	2004.7	0	5.1	188.4	441575	15755	4888	26.7	41.3	20	3.2	1.3	0.3	0	0	7.2	0	0
		0.25	0	64.8	116.2	5	51.4	74174	2004.7	0	5.1	157.5	398453	15783	3981	28	37.7	20.2	2.9	1.2	0.6	0	0	9.4	0	0
	45	0.1	0	43	132	5	89	74174	4964	0	9.4	212.6	580464	4128	5262	20.8	42.6	21.6	4.6	1.3	0.2	0	0	8.9	0	0
		0.25	0	32.5	121.5	5	89	74174	5680	3.6	12.5	1000	936959	246868	4965	23.7	38.9	17.6	5.6	1.7	2.1	0	0	10.4	0	0
	60	0.1	0	9	170.1	3	161.1	74174	9266.9	0	9.5	210.7	529581	3998	6174	25.5	39.5	22.2	3.7	0.9	0.1	0	0	8.1	0	0
		0.25	0	-11.1	143.1	5	154.2	74174	17346.2	0	22.2	121.3	343511	2187	4560	21.5	39.6	16.3	8	1.7	0.5	0	0	12.3	0	0
WN	30	0.1	0	65.3	116.7	5	51.4	502004	12907.5	0	4.5	260.3	520601	33500	4587	27.6	40.4	19.8	2.9	1.3	0.4	0	0	7.6	0	0
		0.25	0	65.3	116.7	5	51.4	502004	12907.5	0	4.5	112.2	313395	5536	3979	27.9	38.5	19.1	2.8	1.5	0.6	0	0	9.7	0	0
	45	0.1	0	43	132	5	89	502004	30852	0	8.5	274.9	689911	8219	5518	25	40.1	20	5	1	0.2	0	0	8.7	0	0
		0.25	0	32.5	121.5	5	89	502004	35328	0	11.9	213.6	534322	8374	5199	26.8	37.7	15.3	7	2.2	1.3	0	0	9.7	0	0
	60	0.1	0	2	163.1	3	161.1	502004	54801.1	0	9.7	231.9	571952	3238	6009	26.7	38.3	19.2	6.4	1.3	0.1	0	0	8.1	0	0
		0.25	0	-13.6	140.6	5	154.2	502004	111858.1	0	22.1	340.2	627336	35725	5213	25.3	39	16.1	6.8	2	0.5	0	0	10.3	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.6: Many-to-many problem with $\beta < 1$, Core instance:D40

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	67.1	296.9	25	229.8	329592	18740.2	0	9.4	682.9	1286841	8402	8532	22	35.3	22.8	9.1	1.1	0.4	0	0	9.3	0	0
		0.25	0	82.5	284.5	49	202.1	329592	31531.2	0	20.2	677.6	1209105	6323	8268	22.7	33.5	17.2	13.9	1.7	0.5	0	0	10.4	0	0
	45	0.1	0	-17	381	25	398	329592	44196	0	9.6	780.6	1359869	7910	9060	22.6	33.6	21.4	12.3	1	0.3	0	0	8.7	0	0
		0.25	0	-38.5	305.5	396	344	329592	75114	20.7	22.1	1000	1796268	27307	8046	22.4	33.5	19.3	10.4	1.6	2.8	0	0	10	0	0
	60	0.1	0	-165.2	524.2	25	689.4	329592	88482.1	0	9.6	423.8	745485	2745	9035	21.9	35.8	23.7	8.6	1	0	0	0	8.9	0	0
		0.25	0	-123.6	482.6	49	606.2	329592	172174.2	0	22.1	705.6	1245241	4580	8850	24.2	34.7	18.6	11.3	0.9	0	0	0	10.3	0	0
WN	30	0.1	0	103	339.7	19	236.7	552600	30527.7	0	4.7	521.5	1017173	5646	8123	21.1	34.6	22.8	10.1	1	0.8	0	0	9.6	0	0
		0.25	0	73	286.6	39	213.6	552600	61274.3	13.1	22.6	1000	1799732	6959	8105	24.2	35.7	19	7.8	1.9	0.9	0	0	10.4	0	0
	45	0.1	0	156	554	25	398	552600	17130	27.4	8	1000	1848433	6660	8086	24.1	34.2	21.9	8.5	0.7	0.1	0	0	10.4	0	0
		0.25	0	0	370	39	370	552600	153436	22.1	23.2	1000	1716273	5223	8174	22.6	34.2	20.8	10.7	1.1	1	0	0	9.7	0	0
	60	0.1	0	-208.4	491.4	426	699.7	552600	110839.2	15.9	8	1000	1720167	8429	8731	24	36.4	23.1	5.5	0.8	1.2	0	0	9	0	0
		0.25	0	-135.4	505.4	39	640.9	552600	313513.6	0	23.2	478.7	897458	3366	8126	24.1	34.1	18.8	11.1	1.9	0.2	0	0	9.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.7: Many-to-many problem with $\beta < 1$, Core instance:B-n41-k6

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	-1.6	44.6	17	46.2	117368	2938.9	0	5.2	395.4	780074	6231	7805	27.3	34	22.9	5.4	0.4	1.6	0	0	8.5	0	0
		0.25	0	-8.7	46.7	9	55.4	117368	2734.1	3.2	3.8	1000	1052057	140800	6134	30.5	29.9	20.8	4.2	1.2	1.7	0	0	11.6	0	0
	45	0.1	0	-16	64	17	80	117368	7044	0	9.5	580.4	1030678	5713	8760	22.7	37	23.7	5.9	0.4	0.6	0	0	9.8	0	0
		0.25	0	-7	73	17	80	117368	7080	0.6	11.9	1000	1663053	26398	8475	22.5	34.6	21.3	7.2	0.8	2.1	0	0	11.5	0	0
	60	0.1	0	-67.3	99	9	166.3	117368	15049.5	10	9.3	1000	1757696	5906	8310	21	36.9	24.5	6.1	0.6	0.4	0	0	10.4	0	0
		0.25	0	-25.3	113.3	17	138.6	117368	20583.1	10.8	21.6	1000	1556024	13496	8781	25.4	32.4	17.9	11.4	0.9	1.3	0	0	10.7	0	0
WN	**30**	0.1	0	-1.6	44.6	17	46.2	150088	2938.9	0	6.1	1000	1143682	91011	7442	27.4	33.7	23.5	3.8	0.8	1.4	0	0	9.5	0	0
		0.25	0	-1.6	44.6	17	46.2	150088	2938.9	0	6.1	383.9	736360	5146	6555	31.2	32.3	21	3.4	0.7	1.3	0	0	10.1	0	0
	45	0.1	0	-15	65	17	80	150088	7366	11	10.8	1000	1625131	6469	9180	22.7	34.7	23.5	7.7	0.5	1.4	0	0	9.5	0	0
		0.25	0	-3.5	76.5	17	80	150088	8214	2	13.9	1000	1614130	21670	9806	23.6	30.9	19.3	13.7	1.1	2.2	0	0	9.2	0	0
	60	0.1	0	-46	96	16	142	150088	19168.2	0	12.4	938.2	1561470	7093	8644	26.2	33	23.1	6.8	0.2	0.1	0	0	10.6	0	0
		0.25	0	-25.8	112.8	17	138.6	150088	25690.7	0	25	918.8	1520792	11881	8779	22.7	31.8	19.7	12.5	1.5	1.1	0	0	10.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.8: Many-to-many problem with $\beta < 1$, Core instance:A-n45-k6

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	-14	27	17	41	145168	2543.3	2.7	4	1000	1539564	10111	7893	31	32.3	20.4	4.3	0.9	1.4	0	0	9.7	0	0
		0.25	0	-13.1	29.1	16	42.1	145168	2666.1	0	4.8	477.1	852510	8188	6959	30.3	30.9	22	3.3	0.4	1.3	0	0	11.7	0	0
	45	0.1	0	-22	49	17	71	145168	7144	3.3	9.3	1000	1608477	7521	9996	25.2	30.9	22.8	9.3	0.8	0.9	0	0	10.1	0	0
		0.25	0	-16.5	54.5	17	71	145168	7496	25.3	10.9	1000.1	1341954	3754	9915	28.8	30.8	19.9	7.2	0.9	1.3	0	0	11.2	0	0
	60	0.1	0	7	161.2	8	154.2	145168	16491.6	25	9.8	1000	1394662	4345	8916	27.4	33.6	22.4	4.6	0.4	0.3	0	0	11.3	0	0
		0.25	0	-20.4	116.4	13	136.8	145168	24397.4	34.3	23.3	1000.1	1361042	3458	9778	25.8	31	20.7	8.1	1.8	1.3	0	0	11.3	0	0
WN	**30**	0.1	0	-13.6	28.6	16	42.1	760509	15569.1	0	5.1	372.4	674942	5059	8268	28.3	33.9	21.2	5.2	0.4	1.4	0	0	9.6	0	0
		0.25	0	-14.1	28	16	42.1	760509	15529.2	0.5	4.5	1000	1001126	79606	7850	31.5	31.2	21.3	3.5	0.3	1.9	0	0	10.3	0	0
	45	0.1	0	1	84	94	83	760509	27520	17.1	13.5	1000	1335024	3500	9456	30.7	31.6	20.8	4.5	1.3	0.5	0	0	10.6	0	0
		0.25	0	-11	60	17	71	760509	35226	24.6	13.6	1000	1383944	3492	10145	27.8	31.6	20.1	6.2	1.5	1.6	0	0	11.1	0	0
	60	0.1	0	-22.8	141.8	5	164.5	760509	67218.3	27.3	8.7	1000.1	1434265	3111	9457	29.8	32.3	23.5	2.7	0.8	0.3	0	0	10.6	0	0
		0.25	0	-21.9	114.9	13	136.8	760509	113828.1	24	22.6	1000	1434810	10708	9743	27.3	30.7	20.6	7.3	0.8	1.7	0	0	11.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.9: Many-to-many problem with $\beta < 1$, Core instance:F-n45-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	-30	45	242	75	135410	5884	14.5	9.5	1000	1416690	3873	9913	25.5	32	23.1	7.9	0.4	1.5	0	0	9.6	0	0
		0.25	0	-46.6	40	252	86.6	135410	7709.9	6.5	7.4	1000	1356979	53240	9062	25.4	30	20.6	8.1	0.9	1.9	0	0	13.1	0	0
	45	0.1	0	-68.5	81.5	252	150	135410	14210	9.1	8.2	1000	1478578	3598	10728	28.6	32.8	21.9	6.2	0.4	0.1	0	0	10	0	0
		0.25	0	-87	63	252	150	135410	12920	69.2	7.2	1000	1367463	3107	10338	28.6	30	20.6	5.4	1.5	0.9	0	0	13	0	0
	60	0.1	0	-123.4	136.4	252	259.8	135410	33030.9	26.1	8.6	1000	1444223	3731	10532	25.6	33.6	24.2	6.3	0.2	0.1	0	0	10.1	0	0
		0.25	0	-111	124.5	245	235.6	135410	53917.3	49.1	16.4	1000	1419797	3353	9984	25.6	32.6	19.9	8.4	0.7	0.4	0	0	12.5	0	0
WN	30	0.1	0	-50.5	28	245	78.5	619773.8	25913.6	10.7	5.6	1000.1	1434092	4753	10299	28.4	32.1	21.6	6.2	1.1	1.3	0	0	9.4	0	0
		0.25	0	-43.5	35	245	78.5	619773.8	28363.7	0	6.6	340.2	655084	2946	9209	26.9	31.7	20.2	6.9	0.6	1.5	0	0	12.2	0	0
	45	0.1	0	-67.5	82.5	252	150	619773.8	71487	0	9.4	601.5	998920	3635	9723	24.5	32.2	22.4	9.3	0.4	0.2	0	0	11	0	0
		0.25	0	-54	82	245	136	619773.8	77984	46.2	12.2	1000	1416595	2023	11142	26.7	30.7	20.9	6.9	1	1.2	0	0	12.5	0	0
	60	0.1	0	-121.9	137.9	252	259.8	619773.8	169256.6	0	9.6	755.8	1198271	3518	10670	23.5	30.9	23.6	10.5	0.8	0.1	0	0	10.6	0	0
		0.25	0	-110	125.5	245	235.6	619773.8	206107.7	34	12.5	1000	1438815	3433	10744	28.3	28.9	19.5	9.8	1.1	0.2	0	0	12.2	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.10: Many-to-many problem with $\beta < 1$, Core instance:att48

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	2211.6	6469	1408	4257.4	9336410	261922.1	15.2	8	1000	1371366	3035	10294	25.4	32	25.2	5.1	0.4	0.3	0	0	11.5	0	0
		0.25	0	4911	8389	2083	3478	9336410	682451.3	30.2	22.7	1000.1	1210794	4051	10374	23.8	31	20.4	9.1	1.4	1.2	0	0	13	0	0
	45	0.1	0	5555	12525	8580	6970	9336410	443140	27.1	8.9	1000	1424313	2745	10361	25.6	29.9	22.9	9.6	0.2	0.1	0	0	11.7	0	0
		0.25	0	569	7721	1519	7152	9336410	1356796	46.3	24.4	1000	1233234	4356	10459	24.7	30.8	19.6	10.2	1.2	0.7	0	0	12.8	0	0
	60	0.1	0	-5026.8	7877	1370	12903.8	9336410	693388	45.6	9.4	1000.1	1329573	2806	11244	26.4	32.2	23.2	7.2	0.4	0	0	0	10.6	0	0
		0.25	0	-686.9	12022.9	1426	12709.8	9336410	4764705.6	48.6	22.1	1000	1239549	3867	11268	25.6	30	20	11.7	0.8	0.1	0	0	11.9	0	0
WN	30	0.1	0	5053.9	9078.1	8580	4024.1	49936044	2177169.3	8.1	8.5	1000	1403066	4862	10426	25.3	33.8	23.3	4.9	0.5	0.5	0	0	11.6	0	0
		0.25	0	5429	8907	2083	3478	49936044	3230184.6	28.8	14.5	1000	1345326	4778	9768	21.8	32.3	21.4	8.7	0.8	1.3	0	0	13.6	0	0
	45	0.1	0	427	7877	1370	7450	49936044	3583270	15.6	8.2	1000	1415857	4345	10980	25.4	28.7	21.9	12.3	0.4	0.1	0	0	11.1	0	0
		0.25	0	4246	9794	7869	5548	49936044	6759118	49.9	16.6	1000	1237502	4254	10024	24.9	30.2	21.1	9	1	0.2	0	0	13.5	0	0
	60	0.1	0	5906	17611.2	1716	11705.2	49936044	1942142	53.8	6.4	1000	1319762	2849	10312	25.9	31.5	22.7	7.7	0.6	0	0	0	11.7	0	0
		0.25	0	4422	8389	1589	3967	49936044	4089220	129.5	21.8	1000.1	1260103	4506	9518	25.3	30.8	22.3	6.3	0.9	0.2	0	0	14.3	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.11: Many-to-many problem with $\beta < 1$, Core instance:B-n50-k7

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	45.5	90	8	44.5	161696	3928	6.2	9.6	1000	1281769	3444	10991	29.4	29.9	22.8	5.8	0.6	1.5	0	0	9.9	0	0
		0.25	0	39.3	83.7	8	44.5	161696	4419.5	0	6.7	578.2	838232	6545	9804	31.3	28	18.9	6.2	1.1	1.3	0	0	13.2	0	0
	45	0.1	0	-24	49	83	73	161696	9212	12.9	8.6	1000	1285524	3300	12452	24.6	31.8	24	7.5	0.4	0.8	0	0	11	0	0
		0.25	0	-2	71	83	73	161696	11260	34.4	17.4	1000.1	1253817	2038	11344	25.9	29.6	19.6	10.8	0.8	1.7	0	0	11.7	0	0
	60	0.1	0	-70.9	59	84	129.9	161696	11904	33.3	9.6	1000	1260737	2839	11446	26.7	32.8	23.8	4	0.3	0.5	0	0	11.9	0	0
		0.25	0	3	136.4	8	133.4	161696	32982	67.2	22.6	1000.1	1150751	2187	11071	28.3	30	21.7	6	0.7	0.6	0	0	12.9	0	0
WN	30	0.1	0	41.5	86	8	44.5	890752	20713.4	8.1	5.8	1000	1292050	3725	11048	28	31.2	23.2	6.2	0.6	1.4	0	0	9.4	0	0
		0.25	0	39.3	83.7	8	44.5	890752	21077.9	0	5.2	661.2	936354	5571	10364	33.5	27.4	18.8	6.2	0.4	1.1	0	0	12.6	0	0
	45	0.1	0	28	86	6	58	890752	18448	20.4	4.4	1000.1	1324846	2092	11812	23.2	30.9	24.9	8.7	0.3	0.7	0	0	11.4	0	0
		0.25	0	18	95	8	77	890752	55362	39.9	10.2	1000.1	1160416	1423	11468	27.8	28.9	19.9	9	0.3	1.8	0	0	12.3	0	0
	60	0.1	0	-60.7	90	90	150.7	890752	97194.7	23.8	12.2	1000.1	1370483	3157	11563	24.2	30.8	24.4	7.9	0.5	0.8	0	0	11.5	0	0
		0.25	0	28	105.9	24	77.9	890752	81026	79.3	14.2	1000.1	1190384	2407	11536	25.7	30.5	22.3	7.5	0.8	1.1	0	0	12.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.12: Many-to-many problem with $\beta < 1$, Core instance:D50

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	147	286	395	139	434674	13844	18.5	7.7	1000.1	1272099	2249	12069	25.5	28.6	24.1	10.4	0.4	0.2	0	0	10.7	0	0
		0.25	0	157.2	309	387	151.8	434674	24738.9	20.2	15.9	1000.1	1363866	5312	10428	24	29	20.8	10.1	1.3	1.5	0	0	13.2	0	0
	45	0.1	0	-16	327	427	343	434674	33744	30.4	7.4	1000	1295931	2166	10573	25.3	31.4	22	8.6	0.4	0.1	0	0	12.3	0	0
		0.25	0	86	327	135	241	434674	59124	58.2	15.4	1000	1313541	2702	10320	24.3	29.8	21.3	9.4	1.2	0.5	0	0	13.4	0	0
	60	0.1	0	-130.2	540.2	62	670.3	434674	55869.8	42.3	2	1000.1	1346015	1858	11656	24	29.1	22.6	12	1.2	0.1	0	0	10.9	0	0
		0.25	0	147	292	213.6	145	434674	34880	153.8	23.5	1000.1	1180581	3695	9782	26.7	29.4	20.8	7.7	1	0.1	0	0	14.3	0	0
WN	30	0.1	0	147	222	148	75	629506	7960	19.6	6	1000	1363997	2140	12128	24.4	27.6	23.8	13.1	0.5	0.2	0	0	10.4	0	0
		0.25	0	151	259	162	108	629506	28358.4	39	21.4	1000	1200298	3039	10030	26.4	30	19.7	8.5	1.6	0.9	0	0	13	0	0
	45	0.1	0	11.5	398.5	449	387	629506	47840	23.2	4.9	1000	1395654	2700	11236	25.7	30	23.3	8.9	0.7	0.1	0	0	11.3	0	0
		0.25	0	11.5	398.5	62	387	629506	88850	50.8	2	1000.1	1243207	3368	10690	26	29.9	20.4	9.2	1.4	0.4	0	0	12.6	0	0
	60	0.1	0	164	292	108	128	629506	20132	60.5	6.8	1000	1296724	4038	11478	25	29.1	21.6	12	1	0.1	0	0	11.2	0	0
		0.25	0	-130.2	540.2	62	670.3	629506	176107.5	107.3	2	1000.1	1178519	2605	9886	29.2	31.5	21.3	3.8	0.7	0.1	0	0	13.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.13: Many-to-many problem with $\beta < 1$, Core instance:eil51

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	24.9	59	67	34.1	105980	1947.3	10.2	6.9	1000.1	1346960	4400	12121	24.6	29.6	25.3	8.3	1	1.6	0	0	9.5	0	0
		0.25	0	27	57.6	64	30.6	105980	1741.5	18.3	5.5	1000	1192470	2331	10117	31.6	30.5	20.7	2.6	0.7	0.7	0	0	13.3	0	0
	45	0.1	0	-9	52	68	61	105980	4424	20.4	6	1000	1289926	2918	10851	25.9	30.7	23.3	6.7	0.6	0.8	0	0	12	0	0
		0.25	0	12	63	63	51	105980	7100	49.2	15.1	1000	1131028	2199	10292	22.5	31.1	21.4	7.9	1.4	1.2	0	0	14.5	0	0
	60	0.1	0	25	102.9	15	77.9	105980	5728	38	7.4	1000.1	1271660	2335	11454	26.7	30	21.9	9.1	0.3	0.3	0	0	11.6	0	0
		0.25	0	-34.1	75.1	6	109.1	105980	6415.9	102.5	3.9	1000.1	1148302	2776	10986	30.5	30.5	20.5	4.2	0.8	0.5	0	0	13	0	0
WN	**30**	0.1	0	17	51.1	67	34.1	512094	15334	7.8	5.6	1000	1226237	3164	12086	29	30.2	23	6.3	0.9	1.2	0	0	9.5	0	0
		0.25	0	6.8	42	68	35.2	512094	8833.7	12.8	4.2	1000	1269394	5132	10898	30.5	30.1	20.1	4.6	0.6	1.6	0	0	12.5	0	0
	45	0.1	0	21	52	57	31	512094	14632	23.3	6.3	1000	1194940	3055	10559	27.4	32.4	23.9	2.9	0.3	0.5	0	0	12.6	0	0
		0.25	0	20	63	16	43	512094	31442	47.8	13.3	1000	1152131	2451	11322	24.9	31.2	22.5	6.2	0.9	1.3	0	0	13	0	0
	60	0.1	0	-13.6	95.6	69	109.1	512094	61873.5	33.3	8.4	1000	1214396	1795	10636	28.6	30.8	22.8	4.6	0.4	0.3	0	0	12.5	0	0
		0.25	0	-13.6	95.6	69	109.1	512094	61873.5	88	8.4	1000	1152310	2718	11199	26.3	29.5	22.9	7.3	0.4	0.6	0	0	13	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.14: Many-to-many problem with $\beta < 1$, Core instance:berlin52

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	320.7	944.3	1130	623.5	1941090	44992.9	2	4.8	1000	1294898	6118	11888	26.8	30.7	24.2	6.3	0.2	2.9	0	0	9	0	0
		0.25	0	320.7	944.3	1130	623.5	1941090	44992.9	0	4.8	622.2	883317	3490	10824	31.9	29.5	19.5	6.9	0.6	1.6	0	0	9.9	0	0
	45	0.1	0	145	885	960	740	1941090	91000	22.1	9.5	1000.1	1253878	1989	13021	25.9	27.8	22.8	12.6	0.2	0.4	0	0	10.2	0	0
		0.25	0	242.5	982.5	960	740	1941090	106960	33.2	9.8	1000	1214327	2003	12899	27.8	28.7	20.4	9.5	0.3	0.9	0	0	12.5	0	0
	60	0.1	0	25	1843.7	65	1818.7	1941090	160482.9	34.3	7.3	1000.1	1317202	2549	13225	25.3	29.6	22.5	11.5	0.1	0.2	0	0	10.8	0	0
		0.25	0	182.7	1170	875	987.3	1941090	228627.3	53.1	16.6	1000	1213736	3626	12856	27.5	28.5	21.3	8.4	1.1	0.4	0	0	12.9	0	0
WN	**30**	0.1	0	321.5	945	1130	623.5	11457715	171991.1	11.9	4	1000.1	1133735	1559	12651	31	29.2	22.5	5.5	0.3	2.8	0	0	8.7	0	0
		0.25	0	320.7	944.3	1130	623.5	11457715	172037.9	0	3.8	818.6	1132914	15772	10803	32.8	29	21.7	5.6	0.2	1.2	0	0	9.5	0	0
	45	0.1	0	235	975	960	740	11457715	506310	22.4	7.9	1000	1162366	1247	11937	26.8	29.1	24.2	8.1	0.2	0.2	0	0	11.4	0	0
		0.25	0	25	1105	1130	1080	11457715	398690	47	4.8	1000	1199510	1954	10677	27	30.7	21.7	4.5	0.5	0.9	0	0	14.6	0	0
	60	0.1	0	25	2051.5	5	2026.5	11457715	683214.4	40.6	4.5	1000.1	1256504	1777	12505	25.8	28.6	22.7	10.9	0.2	0.1	0	0	11.7	0	0
		0.25	0	6.8	1150	920	1143.2	11457715	1203462	78.5	11.6	1000	1154052	1175	12859	26.5	28.9	22.4	8.4	0.8	0.2	0	0	12.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.15: Many-to-many problem with $\beta < 1$, Core instance:A-n60-k9

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	-3.5	45	93	48.5	240820	5225.5	8.2	7.8	1000	1120105	4500	13710	26	28.4	24.1	7	0.6	2.3	0	0	11.6	0	0
		0.25	0	27	73.2	11	46.2	240820	5344.6	9.6	7.5	1000.1	1092806	2973	12961	29.7	27	21.4	6.5	0	1.1	0	0	14.3	0	0
	45	0.1	0	1	85	9	84	240820	11288	18.8	8.6	1000	1039666	1690	12105	28.4	30.6	23.1	2.1	0.3	0.4	0	0	15.2	0	0
		0.25	0	3	83	91	80	240820	20984	40.4	18.7	1000.1	1080237	540	12972	26.3	28.8	20.8	7	0.9	1	0	0	15.2	0	0
	60	0.1	0	3	37	41.2	34	240820	2544	42.3	5.6	1000	1072974	2330	13971	25.2	30.1	23.7	6.9	0.3	0.5	0	0	13.2	0	0
		0.25	0	26.6	89	69	62.4	240820	13473.2	91.9	23.2	1000.1	1001239	1452	11833	26.3	27.9	20.6	7.3	0.7	0.6	0	0	16.5	0	0
WN	30	0.1	0	-3.5	45	93	48.5	1352240	25592.8	10.4	6.4	1000.1	1091908	1573	13440	29	29.3	23.9	4.3	0.3	1.5	0	0	11.7	0	0
		0.25	0	4.8	53.2	93	48.5	1352240	34851.2	17.6	7.8	1000.1	1001390	1251	12400	29.2	28.1	21.1	5.7	0.6	0.9	0	0	14.5	0	0
	45	0.1	0	33	113	91	80	1352240	62076	19.1	7.4	1000	1020577	1489	13183	27	26.5	24.8	7	0.3	0.5	0	0	14	0	0
		0.25	0	15	95	11	80	1352240	96380	42.8	19.3	1000	1074846	1163	12207	25.6	25.9	21.6	7.8	1	1.6	0	0	16.5	0	0
	60	0.1	0	1	160.3	97	159.3	1352240	88164	36	5.5	1000	1146976	2362	12401	23.1	29.3	22.3	8.1	0.3	0.5	0	0	16.4	0	0
		0.25	0	-76.5	69	93	145.5	1352240	148416	83	12.6	1000.2	1049427	636	11783	26.4	28.3	22	4.8	0.5	0.4	0	0	17.6	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.16: Many-to-many problem with $\beta < 1$, Core instance:B-n68-k9

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	-8.5	39.5	13	47.9	296504	4728.2	12.5	5.4	1000.1	971903	498	12749	25.7	30.4	24.1	2.6	0.2	0.4	0	0	16.6	0	0
		0.25	0	24.6	79.4	102	54.8	296504	1989.4	25.9	2.1	1000.2	926362	783	7652	26.7	20.9	16.5	2.8	0.6	1.5	0	0	31	0	0
	45	0.1	0	7	51	90	44	296504	2540	25.8	4.2	1000	1032031	815	12222	20.9	31.1	19.8	5.6	0.2	0.5	0	0	21.9	0	0
		0.25	0	24	79	82	55	296504	14068	52.9	24.5	1000	1106945	255	7447	16.8	23.2	16.3	1.2	0.8	0.2	0	0	41.4	0	0
	60	0.1	0	6	52	16	46	296504	4616	45.7	5.1	1000.2	1009862	2145	11709	19.6	28.5	23	4.5	1	0.2	0	0	23.2	0	0
		0.25	0	-62.8	101.8	7	164.5	296504	13511	103.2	2.9	1000	1036892	583	7334	23.6	20.7	13.9	1.3	0.8	0.3	0	0	39.4	0	0
WN	30	0.1	0	-0.1	34	84	34.1	1601056	21822.3	13	8.5	1000	1002700	1264	10880	27.4	26.9	24.3	0.8	0.2	0.8	0	0	19.5	0	0
		0.25	0	1.5	46	16	44.5	1601056	23055.9	24.7	8.9	1000	863986	626	13702	33.7	24.5	19.7	3.6	0	0.8	0	0	17.6	0	0
	45	0.1	0	7	59	21	52	1601056	40640	23.8	6.3	1000.1	1032688	1313	10995	22.2	29.6	21.7	2.3	0.2	0.2	0	0	23.9	0	0
		0.25	0	-8	87	7	95	1601056	11282	58.9	3.3	1000.1	1132309	10	6006	19.2	18.8	11.7	0.2	0.9	0.1	0	0	49	0	0
	60	0.1	0	-63.8	100.8	7	164.5	1601056	64599.4	42.8	3.3	1000	966407	1110	10070	21.1	29.8	21.6	0.7	0.3	0	0	0	26.5	0	0
		0.25	0	-0.8	46	68	46.8	1601056	45607.5	107	13.1	1000.1	1051538	161	5163	13.4	21	10.8	0.5	1.1	0.4	0	0	52.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.17: Many-to-many problem with $\beta < 1$, Core instance:F-n72-k4

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	-6	2	221	8	114852	304	24.5	5.3	1000.1	1009285	374	7655	16.6	26.1	17.9	1.7	0.5	0.2	0	0	37	0	0
		0.25	0	-16	-16	189	0	114852	0	-	0	1000	916428	0	4430	8.4	25.1	11.8	7.5	1.6	13.8	0	0	31.8	0	0
	45	0.1	0	1	1	202	0	114852	0	-	0	1000.1	988493	1185	8760	18.7	24.6	19.5	2.6	0.4	0.1	0	0	34.1	0	0
		0.25	0	-1	-1	200.5	0	114852	0	-	0	1000	916736	0	7136	10.2	23.3	14.3	11.5	0.8	7.6	0	0	32.2	0	0
	60	0.1	0	-44.2	44.2	175	88.3	114852	9727.5	63.4	2.8	1000.1	997689	740	9072	16	29.6	20	1.8	0.3	0.4	0	0	31.9	0	0
		0.25	0	-200	-200	0	0	114852	0	-	0	1000.1	1067293	0	6774	10.4	22.5	13.6	0.8	0.9	0.6	0	0	51.3	0	0
WN	**30**	0.1	0	-20	9.4	175	29.4	565058	2919.2	24.2	2.5	1000.1	960355	400	8584	17.5	28.6	17.6	0.9	0.4	0.1	0	0	34.7	0	0
		0.25	0	-200	-200	0	0	565058	0	-	0	1000	1024341	0	6516	13.6	27.3	12.2	0.2	0.2	0.8	0	0	45.6	0	0
	45	0.1	0	-15	-4	195	11	565058	8708	42.1	9.2	1000.2	981866	480	8522	16	26.3	16.9	4.7	0.2	0.1	0	0	35.8	0	0
		0.25	0	-1	-1	200.5	0	565058	0	-	0	1000	1062722	0	7267	16.6	21.3	10.5	0.4	0.8	0.2	0	0	50.3	0	0
	60	0.1	0	-44.2	44.2	175	88.3	565058	43494.8	65.1	2.5	1000	1092476	1420	8886	15	25.4	18.8	4.8	0.4	0.6	0	0	35.1	0	0
		0.25	0	-200	-200	0	0	565058	0	-	0	1000	1052396	0	6746	12.2	22	12.9	0.3	0.8	0.8	0	0	51.1	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.18: Many-to-many problem with $\beta < 1$, Core instance:rus75

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	28	71.9	11	43.9	356392	3464.3	13.5	5.9	1000	1026252	469	10734	20.5	23.8	23	9.5	0.2	1.2	0	0	21.7	0	0
		0.25	0	47	85.1	16	38.1	356392	4383.3	24.5	5.8	1000.1	1007646	971	9247	29.9	25.1	17.2	2.7	0.3	0.6	0	0	24.2	0	0
	45	0.1	0	5	9	2	4	356392	0	27.5	0	1000.1	981278	68	6275	10.9	23.1	15.9	2.1	0.4	0.2	0	0	47.5	0	0
		0.25	0	27	27	72	0	356392	0	-	0	1000.1	919986	0	5008	6.7	20.4	12.6	17.7	0.6	7.7	0	0	34.3	0	0
	60	0.1	0	61	154.5	22	93.5	356392	12268.7	43.8	8.4	1000.1	896201	300	8182	15.1	24.8	15.9	7.3	0.2	0.2	0	0	36.4	0	0
		0.25	0	0	0	0	0	356392	0	-	0	1000.1	937164	0	7869	9.9	21.1	11.3	15.9	0.4	6.6	0	0	34.7	0	0
WN	**30**	0.1	0	34.8	38.2	52	3.5	2201807	33.9	14.6	0.2	1000	1040147	637	5858	16.7	22	17.9	0.4	0.3	1.7	0	0	41	0	0
		0.25	0	28	77	5	49	2201807	23668	24.9	3.8	1000.1	904199	430	6425	29.6	16.1	14.5	0.9	0.5	1.1	0	0	37.4	0	0
	45	0.1	0	54	77	25	23	2201807	6874	26.6	2.5	1000.1	1000417	401	6172	10.5	21.2	15	1.4	0.2	0.1	0	0	51.6	0	0
		0.25	0	0	0	0	0	2201807	0	-	0	1000	1019064	0	4489	6.4	21.1	10.4	1.9	0.5	2	0	0	57.7	0	0
	60	0.1	0	30	54.2	42	24.2	2201807	4133.2	47.7	2.3	1000	992231	597	6433	10.6	19.8	15.8	1.4	0.3	0.1	0	0	52.1	0	0
		0.25	0	0	0	0	0	2201807	0	-	0	1001.4	902442	0	5257	6	17.1	9.6	18.3	0.1	8.3	0	0	40.6	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.19: Many-to-many problem with $\beta < 1$, Core instance:eil76

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	26	53.7	64	27.7	242450	2463.5	15.9	4.6	1000	934998	800	10647	18.1	27.7	19.9	4.6	0.3	1.1	0	0	28.3	0	0
		0.25	0	36	36	26	0	242450	0	-	0	1000	1007511	234	6674	21.5	19.7	10.8	0.2	0.4	0.5	0	0	46.9	0	0
	45	0.1	0	7	70	6	63	242450	10516	26.6	9.6	1000	898120	598	7435	12.5	23.5	16	3.2	0.5	0.2	0	0	44	0	0
		0.25	0	9	9	26	0	242450	0	-	0	1000.1	873025	0	4423	6.7	17.6	8.2	10	0.3	10.5	0	0	46.8	0	0
	60	0.1	0	40	57.3	35	17.3	242450	1853.3	55.6	7.2	1000.1	900843	361	11235	16.6	24.7	20.1	9.4	0.3	0.2	0	0	28.9	0	0
		0.25	0	21	21	26	0	242450	0	-	0	1000	906413	0	5010	7	17.1	8.4	10.5	1.2	7.1	0	0	48.6	0	0
WN	**30**	0.1	0	38	52	66	14	1275488	1450	18.1	1.4	1000	1019418	649	10577	19.8	28.1	21	1.9	0.3	0.6	0	0	28.3	0	0
		0.25	0	9.9	26.1	26	16.2	1275488	2828.5	36	2.2	1000.1	988616	60	8008	23.8	21.3	14.2	0.4	0.7	0.6	0	0	39	0	0
	45	0.1	0	40	40	72	0	1275488	0	-	0	1000	991440	780	11287	16.9	28.1	18.2	7.3	0.1	0.5	0	0	29	0	0
		0.25	0	7	7	26	0	1275488	0	-	0	1000	910200	0	4603	5.1	19.4	8.3	17.4	1.3	0.6	0	0	47.9	0	0
	60	0.1	0	-14.9	109.9	4	124.7	1275488	132708.8	43.4	4	1000.1	921451	688	10701	17.5	27.8	18.7	4.2	0.3	0.1	0	0	31.5	0	0
		0.25	0	22	22	26	0	1275488	0	-	0	1000	1026416	7	5840	9	18	9.2	0.9	1	0.6	0	0	61.3	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.20: Many-to-many problem with $\beta < 1$, Core instance:A-n80-k10

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	39.2	75	80	35.8	418736	3869.6	13.8	5.1	1000.1	950962	258	6504	14.1	21.6	13.1	0	0.4	2.2	0	0	48.5	0	0
		0.25	0	20	63.9	11	43.9	418736	6492.2	25.2	6.3	1000	904718	410	10721	25.3	23.3	14.9	0.1	0.3	0.4	0	0	35.7	0	0
	45	0.1	0	39	50	22	11	418736	84	27.6	0.7	1000.2	899672	229	8097	12.9	22.5	16.3	1.8	0.4	0.1	0	0	45.9	0	0
		0.25	0	88	88	58	0	418736	0	-	0	1000	846292	0	4545	8.8	22.1	10	12.7	0.5	14.7	0	0	31.2	0	0
	60	0.1	0	3	155.4	93	152.4	418736	26748.6	40.2	6.9	1000.2	909933	177	10098	16.3	25.1	15.4	7	0.1	0.1	0	0	36	0	0
		0.25	0	88	88	58	0	418736	0	-	0	1000	916455	0	6117	10.8	21.6	12.5	0.4	1	0.6	0	0	53.2	0	0
WN	**30**	0.1	0	46	58.7	38	12.7	2256298	2002.8	15.2	1.1	1000	940715	208	6969	14.6	22.4	14.4	0.2	0.3	1.8	0	0	46.3	0	0
		0.25	0	24.3	52	73	27.7	2256298	8296.1	27.5	3.3	1000	962402	105	9039	24.5	20.2	14.6	0.1	0	0.5	0	0	40.1	0	0
	45	0.1	0	52	70	40	18	2256298	5710	28	2.7	1000	899507	170	6021	7.9	18.8	11.2	0.1	0.2	0	0	0	61.7	0	0
		0.25	0	0	0	0	0	2256298	0	-	0	1000.1	819676	0	4643	8.2	22.7	11.7	10.5	0.5	16.6	0	0	29.8	0	0
	60	0.1	0	58.7	76	54	17.3	2256298	2942	50.2	1.6	1000.1	892358	93	7761	12.2	22.5	15.7	1.5	0.2	0.1	0	0	47.8	0	0
		0.25	0	0	0	0	0	2256298	0	-	0	1000.2	817319	0	7246	10.6	19.8	11.4	16.5	0.6	6.5	0	0	34.6	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.21: Many-to-many problem with $\beta < 1$, Core instance:rd100

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	0	0	0	0	7010846.3	0	-	0	1000.2	596155	0	6721	14.2	21	18.5	18.3	0.4	5.5	0	0	22.1	0	0
		0.25	0	0	0	0	0	7010846.3	0	-	0	1000	726756	0	7067	16.1	22.5	14.8	10.8	0	0.7	0	0	35	0	0
	45	0.1	0	856.2	856.2	982	0	7010846.3	0	-	0	1000.2	569770	0	6750	13.5	22.3	13.7	17.2	0.5	0.6	0	0	32.2	0	0
		0.25	0	826.5	826.5	553.5	0	7010846.3	0	-	0	1000	587553	0	3827	11.6	22.3	11.8	23.1	0.1	1.3	0	0	29.8	0	0
	60	0.1	0	879.5	879.5	839.2	0	7010846.3	0	-	0	1000.1	601087	0	4284	8.2	19.3	13.4	13	0.8	0.1	0	0	45.1	0	0
		0.25	0	907.2	907.2	726.8	0	7010846.3	0	-	0	1000.2	577554	0	6937	11.1	20.6	11.3	30.2	0.4	1.1	0	0	25.3	0	0
WN	30	0.1	0	0	0	0	0	42408487.6	0	-	0	1000.2	617504	0	4893	12.5	21.7	17.3	13.3	0.4	4.8	0	0	30	0	0
		0.25	0	0	0	0	0	42408487.6	0	-	0	1000.6	723927	0	5147	12.2	22.3	11.3	13	0.8	1.4	0	0	38.8	0	0
	45	0.1	0	0	0	0	0	42408487.6	0	-	0	1000.1	578188	0	4452	8.4	20.8	10.5	15.7	0.7	0	0	0	43.9	0	0
		0.25	0	426.2	426.2	553.5	0	42408487.6	0	-	0	1000.3	566603	0	6211	13	21.2	11.6	25.4	0.6	1.4	0	0	26.8	0	0
	60	0.1	0	922.6	922.6	248	0	42408487.6	0	-	0	1000.1	590148	0	4343	8.3	21	12.9	15.6	0.4	0.4	0	0	41.3	0	0
		0.25	0	0	0	0	0	42408487.6	0	-	0	1000.1	589955	0	4717	9.8	20.6	8.2	28.6	1.2	0.8	0	0	30.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.22: Many-to-many problem with $\beta < 1$, Core instance:E-n101-k14

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	Gap%	$\hat{\beta}$	CPU	Niter	Nodes	#Cuts	CQ	GUB	CV	FC	GF	MIR	FP	DJ	IB	ZH	MCF
1N	30	0.1	0	23	23	40	0	439136	0	-	0	1000	531212	0	4319	11.8	21.5	17.6	10.1	0.6	1.7	0	0	36.8	0	0
		0.25	0	0	0	0	0	439136	0	-	0	1000.2	565552	0	6022	14.6	21.6	15.6	17.9	0.6	2.3	0	0	27.5	0	0
	45	0.1	0	63	63	16	0	439136	0	-	0	1000.1	504417	0	4199	12.7	21.2	13.6	14.2	0.1	1.5	0	0	36.8	0	0
		0.25	0	0	0	0	0	439136	0	-	0	1000.2	514528	0	4253	10.7	20.7	10	26.3	0.3	4	0	0	28.1	0	0
	60	0.1	0	20	20	60	0	439136	0	-	0	1000.1	520256	0	6485	10.4	23.5	14.5	17.5	0.4	0.7	0	0	33	0	0
		0.25	0	56	56	68	0	439136	0	-	0	1000.1	535761	0	4385	9.7	19.3	11.1	24.6	1.3	2.3	0	0	31.7	0	0
WN	30	0.1	0	36	36	27	0	2261022	0	-	0	1000.1	550572	0	7005	11.7	21.5	16.9	15.6	0.3	4.6	0	0	29.3	0	0
		0.25	0	0	0	0	0	2261022	0	-	0	1000.1	563558	0	6128	13.7	23.1	16.4	19.1	0.8	4.8	0	0	22.2	0	0
	45	0.1	0	35	35	48	0	2261022	0	-	0	1000.3	517594	0	6739	12.6	21.1	12.7	16.3	0.5	0.6	0	0	36.2	0	0
		0.25	0	55	55	37	0	2261022	0	-	0	1000.2	519204	0	4537	10.2	19.6	10.6	29.4	0.7	6.9	0	0	22.6	0	0
	60	0.1	0	0	0	0	0	2261022	0	-	0	1000.1	525838	0	4335	9	19.3	13.9	17.2	0.1	1	0	0	39.4	0	0
		0.25	0	28	28	35	0	2261022	0	-	0	1000.2	534420	0	6526	11.9	20.2	13	24.4	0.9	1.1	0	0	28.5	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow

$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.23: Many-to-many problem with $\beta < 1$, Core instance:10G2

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	52	52	33	0	695984	0	-	0	1000.1	475664	140	4515	17	24.6	19.1	16.6	0.7	11.2	0	0	10.9	0	0
		0.25	0	46	46	50	0	695984	0	-	0	1000.1	532687	0	4281	13.7	25.4	11.8	26.2	0.4	8.8	0	0	13.8	0	0
	45	0.1	0	0	0	0	0	695984	0	-	0	1000.2	506837	0	3667	9.9	29.6	14.6	14.9	0.9	5.9	0	0	24.1	0	0
		0.25	0	45	45	50	0	695984	0	-	0	1000	507009	0	4371	8.9	22.9	13.4	23.5	0.7	8.3	0	0	22.3	0	0
	60	0.1	0	20	20	54	0	695984	0	-	0	1000.1	510166	0	3528	10	28.7	15.2	11.9	1	6.3	0	0	26.8	0	0
		0.25	0	0	0	0	0	695984	0	-	0	1000.3	508624	0	4314	8.2	21.8	10.5	30.6	1.2	5.6	0	0	22.1	0	0
WN	**30**	0.1	0	0	0	0	0	3817569	0	-	0	1000	518806	0	3987	13.1	24.8	14.3	16.3	0.8	8.1	0	0	22.6	0	0
		0.25	0	50	50	50	0	3817569	0	-	0	1000.2	510923	0	3849	11.7	29.3	16.5	11.9	0	8.5	0	0	22	0	0
	45	0.1	0	19	19	28	0	3817569	0	-	0	1000.2	524970	0	5948	13.9	23.6	14.4	13.7	0.4	5.7	0	0	28.3	0	0
		0.25	0	45	45	50	0	3817569	0	-	0	1000.2	504087	0	3986	7.6	25.8	11.3	26.6	1.1	7.8	0	0	19.9	0	0
	60	0.1	0	31	31	56	0	3817569	0	-	0	1000.1	502786	0	3810	11.8	27.6	17.1	9.4	0.7	9.2	0	0	24.3	0	0
		0.25	0	50	50	33	0	3817569	0	-	0	1000.2	509471	0	4804	8.2	21.5	11.2	32	1.2	6.2	0	0	19.7	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

Table E.24: Many-to-many problem with $\beta < 1$, Core instance:F-n135-k7

Variant	θ	β	$E\%$	Solution								Performance			MIP Cuts* percentage (%)											
				x_s	x_e	y_b	L	Z_0	ΔZ	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**	**#Cuts**	**CQ**	**GUB**	**CV**	**FC**	**GF**	**MIR**	**FP**	**DJ**	**IB**	**ZH**	**MCF**
1N	**30**	0.1	0	15.5	15.5	260	0	1096068.8	0	-	0	1000.2	267205	0	3396	16	24.4	12	36.7	0.3	5.8	0	0	4.7	0	0
		0.25	0	3.2	3.2	215	0	1096068.8	0	-	0	1000.4	288121	0	2943	10.8	30.5	13.7	25.7	0.1	15.1	0	0	4	0	0
	45	0.1	0	-200	-200	0	0	1096068.8	0	-	0	1000.2	260239	0	6117	14.7	23.3	13.9	29.9	0.3	7.8	0	0	10.1	0	0
		0.25	0	-6	-6	208.3	0	1096068.8	0	-	0	1000.3	261336	0	3415	11.4	24.9	13.8	30.5	0.6	13.7	0	0	5	0	0
	60	0.1	0	-2	-2	260	0	1096068.8	0	-	0	1000.7	223215	20	3863	11.6	22.1	20.5	35.7	0.1	1.8	0	0	8.2	0	0
		0.25	0	-200	-200	0	0	1096068.8	0	-	0	1000.1	280894	0	3118	15.9	23	12.4	31.8	1.3	0.8	0	0	14.7	0	0
WN	**30**	0.1	0	15.5	15.5	260	0	6057952.9	0	-	0	1000.2	261554	0	5514	17.7	26.9	16.4	23.7	0.3	3.2	0	0	11.8	0	0
		0.25	0	3.2	3.2	211.7	0	6057952.9	0	-	0	1000.3	266800	0	5803	16.3	23.7	15.4	25.7	0.1	7.6	0	0	11.3	0	0
	45	0.1	0	10	10	219	0	6057952.9	0	-	0	1000.3	251818	0	4778	13.6	25.3	17.8	31.1	0.3	8.6	0	0	3.2	0	0
		0.25	0	23.3	23.3	215	0	6057952.9	0	-	0	1000.4	293023	0	5008	16.5	28	15.9	18.6	0.6	0.9	0	0	19.6	0	0
	60	0.1	0	0	0	260	0	6057952.9	0	-	0	1000.4	218857	22	3500	10.8	22.7	15.6	40.9	0.5	1.5	0	0	8.1	0	0
		0.25	0	3.2	3.2	215	0	6057952.9	0	-	0	1000.2	272939	0	2577	16.5	29.6	12.7	22.9	1.3	0.2	0	0	16.8	0	0

*CQ:Clique, GUB:General Upper Bound, CV:Cover, FC:Flow Cover, MIR:Mixed-Integer Rounding, FP:Flow Path, DJ:Disjunctive, IB:Implied Bound, ZH:Zero-half, MCF:Multi-commodity Flow
$\hat{\beta}$ is the actual percentage of disrupted points.

APPENDIX F

COMPUTATIONAL RESULTS FOR MANY-TO-MANY INTERDICTION PROBLEM WITH A SINGLE BARRIER ON A PLANE

In this appendix, computational results for many-to-many interdiction problem when $\beta = 1$ are presented. The algorithm is developed in a VB.NET application and all computations are performed on windows workstations with 3.00GHz CPU and 3.49 GB of RAM.

The optimal location of the line barrier and the related objective value are reported for 1N and WN variants of 24 instances and different levels of θ and β levels. x_s, x_e, y, and L represent the optimal barrier's endpoints along x-axis, its y-coordinate and length. The objective function values before and after interdiction are shown as Z_0 and ΔZ, respectively. $E\%$ shows the percentage of eliminated weights in pre-processing.

MIP models of all problems are also solved using CPLEX Optimizer 10.1 with a time limit of 1000 seconds. CPU time (in seconds), number of iterations used for solving node relaxations (Niter), number of processed nodes in the active branch-and-cut search (Nodes) give the solver's performance in solving these problems. $\hat{\beta}$ gives the actual disruption rate realized by the MIP solution. Following MIP cuts are also set in the solver with priority value 1:

- Clique Cuts (CQ)
- General Upper Bound Cuts (GUB)
- Cover Cuts (CV)
- Flow Cover Cuts (FC)
- Mixed-Integer Rounding Cuts (MIR)
- Implied Bound Cuts (IB)
- Flow Path Cuts (FP)
- Disjunctive Cuts (DJ)
- Zero-half Cuts (ZH)
- Multi-Commodity Flow Cuts (MCF)

Table F.1: Many-to-many problem with $\beta = 1$, Core instance:D8-Canbolat

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	1.9	7.1	2	5.2	460	17.6	0	0	7.1	0.6	4949	270
	45	1	0	1.5	10.5	11	9	460	50	0	0	16.1	0.9	13024	542
	60	1	0	0.2	15.8	2	15.6	460	182.1	0	0	32.1	0.9	9058	311
WN	30	1	0	2.9	8.1	2	5.2	2145	101	0	0	7.7	0.5	3584	126
	45	1	0	1	10	2	9	2145	276	0	0	7.7	0.9	10975	478
	60	1	0	0.2	15.8	2	15.6	2145	834.7	0	0	35.5	1	11745	415

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.2: Many-to-many problem with $\beta = 1$, Core instance:E-n22-k4

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	131.1	174.9	261	43.9	20820	1600.7	0.06	0	12.6	647.8	740682	469388
	45	1	0	116.5	176.5	193	60	20820	5386	0.06	12.2	35.7	1000	1429346	681571
	60	1	0	94.5	198.5	193	103.9	20820	12852.9	0.08	42.1	35.7	1000	1598724	633091
WN	30	1	0	136.7	171.3	193	34.6	109150	7466	0.06	0	20.1	24.8	131377	1776
	45	1	0	117.5	177.5	193	60	109150	26227	0.06	0	36.3	26.4	129569	1356
	60	1	0	95.5	199.5	193	103.9	109150	64747.5	0.06	98.4	48	1000	2583481	620451

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.3: Many-to-many problem with $\beta = 1$, Core instance:D28

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	23.9	197.1	97	173.2	195830	7301.5	0.19	0	7.3	85.2	239519	13093
	45	1	0	-23.5	276.5	97	300	195830	17100	0.2	0	10.6	347.2	602415	162415
	60	1	0	4.3	454.7	377	450.3	195830	49860	0.22	70.7	38	1000	2199830	365088
WN	30	1	0	25.4	198.6	97	173.2	1061359	43167.2	0.2	0	6.6	31.8	122258	1089
	45	1	0	-26.5	273.5	97	300	1061359	98660	0.2	5.6	9.4	1000	887781	375101
	60	1	0	2.3	452.7	377	450.3	1061359	298795.1	0.23	37.6	32.1	1000	1674659	364251

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.4: Many-to-many problem with $\beta = 1$, Core instance:B-n31-k5

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	4.5	42.6	8	38.1	37460	4701.4	0.34	0	21.3	86.9	264133	2063
	45	1	0	-10	56	8	66	37460	11340	0.36	27.5	26.5	1000	2811634	192036
	60	1	0	-34.2	80.2	8	114.3	37460	23515.5	0.38	23.1	26.5	1000	1422483	245395
WN	30	1	0	4.5	42.6	8	38.1	186689	25531.7	0.34	0	21.6	59.3	196447	1777
	45	1	0	-10.5	55.5	8	66	186689	61578	0.36	13.3	26.1	1000	1019485	304477
	60	1	0	-34.7	79.7	8	114.3	186689	127963.3	0.39	0	26.1	378.4	548528	91430

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.5: Many-to-many problem with $\beta = 1$, Core instance:A-n32-k5

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	64.8	116.2	5	51.4	74174	2004.7	0.41	0	5.1	49.9	155073	2271
	45	1	0	32.5	121.5	5	89	74174	5680	0.39	0	12.5	194	473767	19381
	60	1	0	-15.9	120.9	10	136.8	74174	22283.5	0.41	43.5	37.9	1000	2817291	103387
WN	30	1	0	65.3	116.7	5	51.4	502004	12907.5	0.41	0	4.5	69.3	210637	3076
	45	1	0	32.5	121.5	5	89	502004	35328	0.39	0	11.9	200.2	450979	22661
	60	1	0	-16.2	124.2	9	140.3	502004	147998	0.39	39.8	34.4	1000	1826917	200002

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.6: Many-to-many problem with $\beta = 1$, Core instance:D40

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	82.5	284.5	49	202.1	329592	31531.2	1.2	22.2	23.5	1000	1774139	14101
	45	1	0	-5.5	290.5	372	296	329592	95266	1.3	62	32.3	1000	1769314	53769
	60	1	0	-113.8	398.8	372	512.7	329592	216177.4	1.38	102.5	35.2	1000	2073244	38067
WN	30	1	0	89	291	49	202.1	552600	72385.4	1.23	0	28.2	299.9	672289	2655
	45	1	0	29.5	337.5	70	308	552600	196924	1.3	27.4	37.2	1000	1524909	89297
	60	1	0	-83.2	450.2	70	533.5	552600	436825.8	1.42	0	43	163.4	389685	2201

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.7: Many-to-many problem with $\beta = 1$, Core instance:B-n41-k6

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	-1.6	44.6	17	46.2	117368	2938.9	1.5	0	5.2	188	415006	3828
	45	1	0	-7	73	17	80	117368	7080	1.33	13.7	11.9	1000	1750047	34715
	60	1	0	-25.3	113.3	17	138.6	117368	20583.1	1.38	32.8	21.6	1000	1876417	20446
WN	30	1	0	-1.6	44.6	17	46.2	150088	2938.9	1.33	2.5	6.1	1000	1318602	87606
	45	1	0	-3.5	76.5	17	80	150088	8214	1.33	8.1	13.9	1000	1490290	62229
	60	1	0	-25.8	112.8	17	138.6	150088	25690.7	1.36	34.4	25	1000	1742429	44578

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.8: Many-to-many problem with $\beta = 1$, Core instance:A-n45-k6

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	-13.6	28.6	16	42.2	145168	2666.1	2.12	0	4.8	918.7	1359827	45522
	45	1	0	22	105	94	83	145168	7612	2.12	8.6	9.8	1000	1553894	6929
	60	1	0	-23.4	120.4	94	143.8	145168	32415.8	2.16	68.3	34.8	1000	1695905	12088
WN	30	1	0	-13.6	28.6	16	42.2	760509	15569.1	2.14	0	5.1	666.4	1191766	9219
	45	1	0	-17.5	53.5	17	71	760509	38663	2.12	0.3	12.5	1000	1630519	15056
	60	1	0	-22.9	120.9	94	143.8	760509	146579	2.17	52.4	30.5	1000	1665058	20571

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.9: Many-to-many problem with $\beta = 1$, Core instance:F-n45-k4

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	-43.5	35	45	78.5	135410	7959.3	2.05	6.5	7.4	1000	1503442	24900
	45	1	0	-69	67	45	136	135410	21684	2.11	0	14.3	829.4	1438272	7933
	60	1	0	-111.3	124.3	45	235.6	135410	53917.3	2.31	54	16.4	1000	1593028	13501
WN	30	1	0	-43.5	35	45	78.5	619773.8	28363.7	2.03	0	6.6	530.6	990067	7733
	45	1	0	-62	74	45	136	619773.8	81651.9	2.09	0	11.5	799.6	1466893	14670
	60	1	0	-110	125.5	45	235.6	619773.8	206107.6	2.3	32.2	9.6	1000	1655796	13391

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.10: Many-to-many problem with $\beta = 1$, Core instance:att48

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	3825.4	7827.6	1629	4002.2	9336410	988072.5	3	21.8	28.2	1000	1421062	11621
	45	1	0	2773.5	9497.5	1733	6724	9336410	3561302	3.17	51.6	43.2	1000	1335631	26346
	60	1	0	312.4	11958.7	1733	11646.3	9336410	8434388.5	3.55	77	43.2	1000	1406118	16547
WN	30	1	0	3853.9	7856.1	1629	4002.2	49936044	5005779.8	2.97	27.3	27.6	1000	1156412	47426
	45	1	0	2796	9520	1733	6724	49936044	18827771.9	3.16	52	46.3	1000	1327761	21621
	60	1	0	334.9	11981.2	1733	11646.3	49936044	44788032.8	3.31	76.4	43.1	1000	1310600	24332

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.11: Many-to-many problem with β = 1, Core instance:B-n50-k7

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	39.3	83.7	8	44.5	161696	4419.5	3.61	0	6.7	932.6	1275928	26753
	45	1	0	24	101	8	77	161696	12432	3.69	37.3	17.4	1000.1	1322673	10738
	60	1	0	-22.2	111.2	8	133.4	161696	45855.2	3.84	84.8	29.8	1000	1328833	6110
WN	30	1	0	39.3	83.7	8	44.5	890752	21077.9	3.61	0	5.2	744.9	1108169	16996
	45	1	0	25	102	8	77	890752	59974	3.64	28	18.6	1000	1359058	2766
	60	1	0	-22.2	111.2	8	133.4	890752	227755.6	3.88	80.8	28.1	1000.1	1258851	6216

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.12: Many-to-many problem with β = 1, Core instance:D50

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	132.1	283.9	387	151.8	434674	28793.6	3.58	16.1	15.4	1000	1486370	9915
	45	1	0	76.5	339.5	387	263	434674	80152	3.73	80.4	20.8	1000	1422955	8419
	60	1	0	-18.3	437.3	387	455.5	434674	181603.5	3.88	36	18.2	1000	1584606	5590
WN	30	1	0	130.8	294.2	114	163.4	629506	62778.9	3.55	13.6	17.9	1000	1537663	7392
	45	1	0	68	351	114	283	629506	170446	3.67	51.6	26.4	1000	1432678	20449
	60	1	0	-35.6	454.6	114	490.2	629506	374716	3.83	146.4	26.3	1000	1368540	11500

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.13: Many-to-many problem with β = 1, Core instance:eil51

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	19.5	53.5	67	34.1	105980	2390.7	3.97	5.2	6	1000	1420226	6002
	45	1	0	10.5	69.5	67	59	105980	8560	4.02	40.4	12.1	1000.1	1287720	2192
	60	1	0	-5.9	78.9	62	84.9	105980	32938.1	4.16	60.4	29.3	1000	1464221	7293
WN	30	1	0	19.5	53.5	67	34.1	512094	15704.2	3.92	3.1	6.2	1000.1	1448138	7781
	45	1	0	9	68	67	59	512094	52118	3.98	38.3	21.2	1000	1359107	5238
	60	1	0	-5.9	78.9	62	84.9	512094	163835.4	4.17	80.6	15.7	1000	1385395	7358

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.14: Many-to-many problem with β = 1, Core instance:berlin52

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	320.7	944.3	1130	623.5	1941090	44992.9	4.16	3.3	4.9	1000	1407822	12474
	45	1	0	187.5	1267.5	1130	1080	1941090	119510	4.14	20.9	8.3	1000	1434575	12841
	60	1	0	-200.3	1670.3	1130	1870.6	1941090	330637.1	4.19	65.6	7.5	1000	1390540	2857
WN	30	1	0	320.7	944.3	1130	623.5	11457715	172037.9	4.17	0.1	3.8	1000	1444257	9835
	45	1	0	235	975	960	740	11457715	506310	4.19	11.7	7.9	1000	1402173	4157
	60	1	0	-172.8	1697.8	1130	1870.6	11457715	1351105.7	4.19	77.8	13.1	1000	1363928	5566

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.15: Many-to-many problem with β = 1, Core instance:A-n60-k9

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	3.8	52.3	93	48.5	240820	5847.3	11	12.6	8.1	1000.1	1035344	1943
	45	1	0	-7	77	93	84	240820	23040	11.2	55.8	17.2	1000	1222149	3293
	60	1	0	-26.3	112.3	91	138.6	240820	72657.8	9.8	105.5	19.5	1000.1	1033727	1793
WN	30	1	0	4.8	53.3	93	48.5	1352240	34851.2	9.09	6.6	8.3	1000.1	1143847	2512
	45	1	0	-6	78	93	84	1352240	138245.9	9.22	62.3	18.1	1000.1	1289147	3380
	60	1	0	-24.3	114.3	91	138.6	1352240	443715.6	9.67	104.7	36.6	1000	1095614	1551

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.16: Many-to-many problem with $\beta = 1$, Core instance:B-n68-k9

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1N	30	1	0	46.3	90.7	16	44.5	296504	9462.4	18.08	25.9	6.1	1000.1	949187	1537
	45	1	0	14.5	91.5	16	77	296504	32176	17.14	69.3	37.9	1000.1	1135745	1390
	60	1	0	-24.2	109.2	16	133.4	296504	129437	18.09	102.7	43.9	1000.1	1048199	2765
WN	30	1	0	46.8	91.2	16	44.5	1601056	47865.2	16.84	26.6	5.4	1000.1	994625	2562
	45	1	0	14.5	91.5	16	77	1601056	163665.9	17.17	67.6	36	1000.1	1170252	1250
	60	1	0	-24.7	108.7	16	133.4	1601056	666563.3	18.12	104.5	44.9	1000	1033609	578

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.17: Many-to-many problem with $\beta = 1$, Core instance:F-n72-k4

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1N	30	1	0	-13.7	7.7	19	21.4	114852	9416.4	20.84	63.6	13.3	1000.1	1024124	319
	45	1	0	-23	12	18	35	114852	30456	22.11	170.8	33.2	1000.1	1116633	545
	60	1	0	-35.8	24.8	18	60.6	114852	69759.8	24.09	199.5	42.2	1000.1	1082850	2153
WN	30	1	0	-15.3	8.3	21	23.7	565058	48262.8	20.83	53.3	4.9	1000.1	1110412	772
	45	1	0	-23	12	18	35	565058	150829.9	22.06	159.8	30.2	1000	1239247	533
	60	1	0	-35.8	24.8	18	60.6	565058	348373.6	24.09	301.4	10.3	1000.1	970363	124

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.18: Many-to-many problem with $\beta = 1$, Core instance:rus75

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1N	30	1	0	41.3	88.7	8	47.3	356392	7484.8	23.52	4.1	0	1000.1	1118467	360801
	45	1	0	18	100	8	82	356392	26852	24.17	4.1	0	1000.1	1118467	360801
	60	1	0	-13.5	128.5	8	142	356392	78414.8	25.42	4.1	0	1000	1120048	361441
WN	30	1	0	41.2	90.8	6	49.7	2201807	46452.2	23.59	4.1	0	1000	1123807	363053
	45	1	0	18	104	6	86	2201807	159913.9	24.25	4.1	0	1000	1123901	363086
	60	1	0	-15	134	6	149	2201807	456587.7	25.61	4.1	0	1000	1120022	361431

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.19: Many-to-many problem with $\beta = 1$, Core instance:eil76

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1N	30	1	0	37.7	72.3	10	34.6	242450	5805.2	28.97	4.1	0	1000	1123818	363059
	45	1	0	24.5	74.5	15	50	242450	21706	29.34	4.1	0	1000	1123862	363075
	60	1	0	-0.8	85.8	15	86.6	242450	74328.9	30.73	4.1	0	1000	1119999	361416
WN	30	1	0	39.1	67.9	15	28.9	1275488	29952.5	29.02	4.1	0	1000	1123807	363053
	45	1	0	24	74	15	50	1275488	118775.9	29.62	4.1	0	1000	1120022	361431
	60	1	0	-0.3	86.3	15	86.6	1275488	390831	30.89	4.1	0	1000	1123804	363051

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.20: Many-to-many problem with $\beta = 1$, Core instance:A-n80-k10

Variant	θ	β	$E\%$	Algorithm Results							MIP Performance				
				x_s	x_e	y_b	L	Z_0	ΔZ	CPU	Gap%	$\hat{\beta}$	CPU	Niter	Nodes
1N	30	1	0	28.2	77.8	6	49.7	418736	8721.5	37.48	4.1	0	1000.1	1123930	363101
	45	1	0	18.5	104.5	6	86	418736	32372	37.72	4.1	0	1000.1	1119956	361401
	60	1	0	-13.8	117.8	11	131.6	418736	114929.6	40.83	4.1	0	1000	1123903	363087
WN	30	1	0	27.7	77.3	6	49.7	2256298	46213.2	37.48	4.1	0	1000	1120059	361444
	45	1	0	17.5	103.5	6	86	2256298	165095.9	37.62	4.1	0	1000	1123804	363051
	60	1	0	-14.3	117.3	11	131.6	2256298	594000.6	39	4.1	0	1000	1120010	361421

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.21: Many-to-many problem with $\beta = 1$, Core instance:rd100

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	287	729.1	874.5	442.2	7010846.3	108422.2	114.69	4.1	0	1000	1120005	361419
	45	1	0	4.8	760.7	869.5	756	7010846.3	492928.3	116.52	4.1	0	1000	1120010	361421
	60	1	0	-109.8	1094.4	839.2	1204.2	7010846.3	2107282.6	121.67	4.1	0	1000	1118392	360766
WN	30	1	0	283.4	725.6	874.5	442.2	42408487.6	716239.8	114.72	4.1	0	1000	1123759	363031
	45	1	0	-1.7	754.3	869.5	756	42408487.6	3303134.4	116.66	4.1	0	1000	1120010	361421
	60	1	0	-114.5	1089.7	839.2	1204.2	42408487.6	13514279.5	121.19	4.1	0	1000	1123781	363041

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.22: Many-to-many problem with $\beta = 1$, Core instance:E-n101-k14

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	7.8	40.2	12	32.3	439136	7325.8	114.28	4.1	0	1000	1118258	360703
	45	1	0	8.5	64.5	12	56	439136	35304	118.34	4.1	0	1000	1119956	361401
	60	1	0	-10.8	82.8	13	93.5	439136	126476.8	126.38	4.1	0	1000	1123908	363090
WN	30	1	0	7.3	39.7	12	32.3	2261022	34234.5	114.88	4.1	0	1000	1119798	361340
	45	1	0	8.5	62.5	13	54	2261022	152579.8	118.34	4.1	0	1000	1123804	363051
	60	1	0	-11.3	82.3	13	93.5	2261022	583724.8	126.81	4.1	0	1000	1123804	363051

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.23: Many-to-many problem with $\beta = 1$, Core instance:10G2

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	38.8	94.2	2	55.4	695984	12785.7	115.23	4.1	0	1000	1123804	363051
	45	1	0	0	96	2	96	695984	59808	120.75	4.1	0	1000	1119956	361401
	60	1	0	-33.1	133.1	98	166.3	695984	209527.3	126.98	4.1	0	1000	1120010	361421
WN	30	1	0	39.3	94.7	98	55.4	3817569	75081	115.5	4.1	0	1000	1119931	361391
	45	1	0	5.5	101.5	98	96	3817569	330084.8	120.89	4.1	0	1000	1119905	361381
	60	1	0	-31.1	135.1	98	166.3	3817569	1128682.7	127.91	4.1	0	1000	1123930	363101

$\hat{\beta}$ is the actual percentage of disrupted points.

Table F.24: Many-to-many problem with $\beta = 1$, Core instance:F-n135-k7

Variant	θ	β	$E\%$	**Algorithm Results**							**MIP Performance**				
				x_s	x_e	y_b	L	Z_0	ΔZ	**CPU**	**Gap%**	$\hat{\beta}$	**CPU**	**Niter**	**Nodes**
1N	30	1	0	-12.3	32.1	51	44.5	1096068.8	18385.4	447.08	-	0	1000.7	351276	0
	45	1	0	-29.4	47.6	51	77	1096068.8	43357.2	523.28	-	0	1000.2	319277	0
	60	1	0	-57.2	69.2	49	126.4	1096068.8	90801.7	466.8	-	0	1000.4	316455	0
WN	30	1	0	-12.3	32.1	51	44.5	6057952.9	117960.4	447.77	-	0	1008.7	334415	0
	45	1	0	-29.4	47.6	51	77	6057952.9	281641.3	454.38	-	0	1005.8	308965	0
	60	1	0	-58.2	75.2	51	133.4	6057952.9	589018.2	468.91	-	0	1000	337478	0

$\hat{\beta}$ is the actual percentage of disrupted points.