FINITE-HORIZON ONLINE ENERGY-EFFICIENT TRANSMISSION SCHEDULING SCHEMES FOR COMMUNICATION LINKS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

BARAN TAN BACINOĞLU

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

JANUARY 2013

Approval of the thesis:

FINITE-HORIZON ONLINE ENERGY-EFFICIENT TRANSMISSION SCHEDULING SCHEMES FOR COMMUNICATION LINKS

submitted by **BARAN TAN BACINOĞLU** in partial fulfillment of the requirements for the degree of
Master of Science in Electrical and Electronics Engineering Department, Middle East Technical University by,

Prof.Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** ________________

Prof.Dr. İsmet Erkmen
Head of Department, **Electrical and Electronics Engineering** ________________

Assoc. Prof. Dr. Elif Uysal-Bıyıkoğlu
Supervisor, **Electrical and Electronics Engineering Department** ________________

Examining Committee Members:

Prof. Dr. Ömer Kırca
Department of Industrial Engineering, METU ________________

Assoc. Prof. Dr. Melek Yücel
Department of Electrical and Electronics Engineering, METU ________________

Assoc. Prof. Dr. Elif Uysal-Bıyıkoğlu
Department of Electrical and Electronics Engineering, METU ________________

Assoc. Prof. Dr. Ali Özgür Yılmaz
Department of Electrical and Electronics Engineering, METU ________________

Assist. Prof. Dr. Emre Tuna
Department of Electrical and Electronics Engineering, METU ________________

Date: ________________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name: BARAN TAN BACINOĞLU

Signature :

ABSTRACT

FINITE-HORIZON ONLINE ENERGY-EFFICIENT TRANSMISSION SCHEDULING SCHEMES FOR COMMUNICATION LINKS

Bacınoğlu, Baran Tan
M.S., Department of Electrical and Electronics Engineering
Supervisor : Assoc. Prof. Dr. Elif Uysal-Bıyıkoğlu

January 2013, 45 pages

The proliferation of embedded systems, mobile devices, wireless sensor applications and increasing global demand for energy directed research attention toward self-sustainable and environmentally friendly systems. In the field of communications, this new trend pointed out the need for study of energy constrained communication and networking. Particularly, in the literature, energy efficient transmission schemes have been well studied for various cases. However, fundamental results have been obtained mostly for offline problems which are not applicable to practical implementations. In contrast, this thesis focuses on online counterparts of offline transmission scheduling problems and provides a theoretical background for energy efficient online transmission schemes. The proposed heuristics, Expected Threshold and Expected Water Level policies, promise an adequate solution which can adapt to short-time-scale dynamics while being computationally efficient.

Keywords: Packet scheduling, energy harvesting, energy-efficient scheduling, online policy, throughput maximization

ÖZ

İLETİŞİM BAĞLANTILARI İÇİN SONLU UFUKLU ÇEVRİMİÇİ ENERJİ-VERİMLİ GÖNDERİM ÇİZELGELEME ŞEMALARI

Bacınoğlu, Baran Tan
Yüksek Lisans, Elektrik ve Elektronik Mühendisliği Bölümü
Tez Yöneticisi : Doç. Prof. Dr. Elif Uysal Bıyıkoğlu

Ocak 2013, 45 sayfa

Gömülü sistemlerin, mobil cihazların, kablosuz sensör uygulamalarının yaygınlaşması ve artan küresel enerji ihtiyacı, araştırma ilgi alanını kendi-kendine yeterli ve çevre dostu sistemlere yöneltmiştir. Telekomünikasyon alanında ise, bu yeni eğilim enerji kısıtlı iletişim ve ağ kurma araştırma çalışmalarının yapılması gerekliliğini vurgulamıştır. Özellikle, literatürde, çeşit senaryolar için enerji verimli gönderim şemaları konusu iyi bir şekilde çalışılmıştır. Ne var ki, temel sonuçlar daha çok pratik uygulamalara uygulanabilir olmayan çevrimdışı problemler için elde edilmiştir. Bunun aksine, bu tez çalışması çevrimdışı gönderim çizelgeleme problemlerinin çevrimiçi karşılıklarına odaklanıyor ve enerji verimli çevrimiçi gönderim şemaları için teorik bir altyapı sunuyor. Önerilen buluşsal yöntemler, Beklenen Eşikdeğer ve Beklenene Su Seviyesi politikaları, işlemsel verimli olarak kısa-zaman -ölçekli dinamiklere uyum sağlayabilen uygun bir çözüm vaat ediyor.

Anahtar Kelimeler: Paket çizelgeleme, enerji harmanlama, enerji-verimli çizelgeleme, çevrimiçi politıka, çıktı enyükseltme

To my beloved family

ACKNOWLEDGMENTS

First and foremost, I would like to offer my deep gratitude to my supervisor, Dr. Elif Uysal-Bıyıkoğlu, who has motivated me throughout my thesis study with her positive approach. This study would not have been possible without her continuous support and attention. In particular, I would like to thank her for her guidance and attentiveness to detail during our weekly discussions.

I am thankful to my friends and colleagues whose assistance and friendship I really appreciate, including Fatih Özçelik, Mahdi Shakiba, Neyre Tekbıyık, Ömer Gül and Sajjad Baghaee.

I am also grateful to grants form the Scientific and Technological Research Council of Turkey (TUBITAK , project no. 110E252) and Turk Telekom for providing financial support of this research and study.

Lastly, I would like to express my gratitude to my family for their love and care which has been an essential motivation to complete this study.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

It's easy to play any musical instrument: all you have to do is touch the right key at the right time and the instrument will play itself.

Johannes Sebastian Bach

With the worlwide increase in energy demand, energy renewable resources have become the center of attention and energy efficiency has gained greater priority as a design challenge in almost all technological areas. On the other hand, the developments in wireless technologies have provided mobility and freedom in many applications while eliminating wiring costs but at the same time, energy availabilities of wireless devices, have been limited as they can no longer be powered by cables. Typical examples of these applications are wireless sensor networks (WSNs) where energy consumption is a significant problem. In fact, energy consumption concerns in WSNs are a lot more important than in other wireless applications. Unlike mobile phones or some other wireless devices, wireless sensor nodes are not always under control of human users. For example, wireless sensor nodes can be spread across a vast geographical area in environment monitoring applications. In this case, if they are depleted, recharging or changing batteries of sensor nodes can become a hard task which needs to be repeated over time.

The first approach to deal with this case is to prolong battery life of sensor nodes by minimizing energy expenditures and the corresponding problem is named as *network lifetime* problem [2]. For point-to-point communication, transmission power control schemes have been studied as a solution to the network lifetime problem in wireless settings. The study in [3], can be seen as one of pioneering research efforts that formulates and solves an energy efficient packet transmission scheduling problem.

An alternative solution has come up with the advancements in energy harvesting technologies that enable the construction of various kinds of energy harvesters in small packages, which may be integrated with wireless devices in small size. The integration of energy harvesters into wireless nodes, can make them practically self-sustainable by providing almost perpetual energy. However, exploiting this energy introduces challenges for the design of transmission schemes, in particular the allocation of transmission power and rate across time, due to the unsteady or nondeterministic availability of ambient energy sources. Due to this nature of energy harvesting, transmission schedules schemes are required to be reconsidered for energy harvesting systems. In an early work [4], an optimization problem for maximizing a transmission reward on a solar energy harvesting satellite has been considered. In related work for point-to-point communication, throughput optimal scheduling policies for a single energy harvesting sensor node have been developed (e.g., [5]). The transmission completion time minimization problem on an energy harvesting communication link has been formulated and solved in [6]. A dynamic programming solution is proposed for a finite-horizon throughput maximization problem over a fading channel with an energy harvesting transmitter in [7]. In [8], a similar problem is considered, and addressed through stochastic dynamic programming, followed by the proposal of several suboptimal adaptive transmission policies. In [9], a throughput maximization problem over a Gilbert-Elliot channel with an energy harvesting source is formulated as a Markov decision problem with "transmit" and "defer" actions and it is proved that a threshold-type policy is optimal over this set of actions. For fading channels, the outage probability of an energy harvesting node is examined in [10] where the energy profile is modeled as a discrete Markov process. Some practical battery limitations such as battery size and constant battery leakage are considered and offline optimal transmission schemes under these limitations are investigated in [11].

The optimization problems in the related work can be separated into two groups depending on the information available to the transmitter before or during transmission period, offline and online transmission scheduling problems. In offline transmission scheduling problems, all external processes affecting transmission are assumed to be deterministically available prior to transmission. These external processes can be channel conditions, packet arrivals or energy harvests. The aim of studies on offline problems is to characterize the structure of the optimal solutions by disregarding the causality of information availability . On the other hand, in practical systems, side-information about external processes is reavealed causally and accordingly offline problems do not correspond to practical scenarios in general. For this reason, online problems which assume causal information can be seen as more realistic. In online transmission scheduling problems, external processes are assumed as stochastic or simply unknown before they are observed.

In the related literature, although there are valuable results for offline transmission scheduling problems, there is a serious gap for their online counterparts. With the aim to fill this gap, this thesis investigates online transmission scheduling problems and presents both optimal and heuristic solutions which can be a base for future studies.

This thesis includes 6 chapters. In the next two chapters, energy harvesting systems and some offline transmission policies are reviewed. The main contributions are presented in Chapter 4 and 5.

Chapter 2 provides information about practical energy harvesting systems and theoretical modeling and classification of harvesting dynamics. Although the information here is principally valid for any type of energy harvesting system, it is mainly focused on energy harvesting wireless sensor nodes (EH-WSNs).

Chapter 3 addresses some energy efficient offline scheduling problems and algorithms in the literature and gives background information about commonly used observations and mathematical methods.

In Chapter 4, a finite-horizon online throughput maximization problem over a point-to-point link with an energy harvesting transmitter is considered, similarly to [7] and [8]. As opposed to previous studies [5]-[8], transmission power decisions are restricted to a discrete set, motivated by practical implementation constraints. A Markovian energy harvesting process and static channel conditions are assumed. Contrary to the study in [11], a simple battery model is assumed where it behaves as an energy buffer with unlimited capacity. In this respect, this study is perhaps closest to the work in [12] which also solved a Markov decision problem (in a Gilbert-Elliot channel) with a discrete set of transmission decisions. However, while the action set [12] in was limited to “transmit” and “defer” actions and nonlinearity in power-rate relation was not taken into account, one of the contributions of this work is the structure of the optimal online policy for a general discrete transmission set, when power is a convex function of data rate. The second and main contribution is a low complexity online heuristic that exploits the optimal offline solution and approaches the performance of the optimal *online* solution. While similar dynamic programming solutions have been proposed in [5],[6]-[12], this scheduling heuristic and the approach for deriving it can be considered to be novel. Another contribution of this thesis is a comparison with the infinite-horizon optimal scheduling policy, which points to interesting future directions. Then, the problem is extended to address the time-varying case, and a policy which dynamically computes an expected water level is proposed for this case.

In Chapter 5, an online version of lazy scheduling problem is defined via dynamic programming approach which, in fact , provides an optimal solution by definition. In addition, a suboptimal policy which uses the same approach as in previous heuristic for the problem in Chapter 4, is described.

CHAPTER 2

A REVIEW ON ENERGY HARVESTING AND ENERGY HARVESTING SYSTEMS

> Give a man a fish and you feed him for a day. Teach a man to fish and you feed him for a lifetime.
>
> Chinese Proverb

The capability of energy harvesting from environmental sources can make wireless devices self-sustainable in terms of energy [13]. In today's technology, power provided by small energy harvesters can match energy expenditure of low-power wireless devices [14]. However, due to the dependence on ambient sources, energy harverters can supply power usually in a discontinuous way. As a result, to customize an energy harvesting system, it is necessary to characterize its energy availability.

Fundamentally, energy availability in an energy harvesting device relies on both its energy storage capability (i.e., battery) and the energy harvesting process. In the following two sections, theoretical treatments of these factors are described.

2.1 ENERGY STORAGE

Energy harvesting systems do not instantly consume the energy harvested from the environment and hence they need an energy storage device. This device can be a rechargeble battery or a supercapacitor. In some applications, the storage device charged by harvesting energy works as an auxiliary power source to the main battery which keeps the system operating.

Otherwise, it is required to wait until the output voltage of the storage device reaches to a certain voltage level which is sufficient for minimum operation [15] . The figure below illustrates the voltage evolution curve of the storage device for such a system.

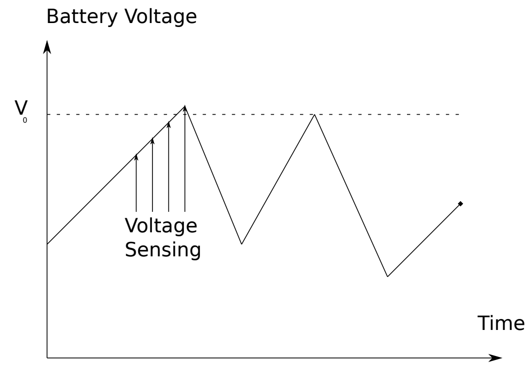


Figure 2.1: The battery voltage evolution curve of an energy harvesting system

On the other hand , in most theoretical works, energy storage device in an energy harvesting system is considered as an energy buffer. The simplest model of energy dynamics used in theoretical works for a time-slotted energy harvesting system is as follows:

$$e_{k+1} = (e_k - c_k) + h_k; c_k \leq e_k \tag{2.1}$$

where e_k denotes the available energy in energy buffer at the beginning of the kth time slot, c_k and h_k are consumed and harvested energy amounts during the same time slot.

In a practical system, energy consumption and harvesting can be done in separate time frames within the same time slot. The storage model in 2.1, can be employed to simulate such a case where the harvested energy is only available after the time slot at which it is harvested. Some other models include harvested energy during a time slot in available energy of that time slot. In this case, the difference $c_k - h_k$, if it is positive, corresponds to the energy drawn from the energy storage device or , if the harvested energy is not fully consumed, the remaining part charges the energy storage device.

$$e_{k+1} = e_k - (c_k - h_k); c_k \leq e_k + h_k \tag{2.2}$$

More realistic models take into account inefficiencies like energy leakage or harvesting inefficiency.

$$e_{k+1} = ((e_k - c_k) - \beta_l)_+ + \beta_h h_k; c_k \leq e_k \tag{2.3}$$

where β_l is energy leakage and β_h is harvesting inefficiency.

In general, the capacity of the energy storage device is assumed to be unlimited since it can be much higher than the harvested and consumed energy amounts but an even more realistic model considers the storage capacity e_{max} as in the following:

$$e_{k+1} = min\{e_{max}, ((e_k - c_k) - \beta_l)_+ + \beta_h h_k\}; c_k \leq e_k \tag{2.4}$$

Besides these, even sampling the output voltage of the storage device requires some energy and the sampling rate can be optimized accordingly [15] but this energy loss is ignored in existing theoretical models.

2.2 ENERGY HARVESTING PROCESS

Finding theoretical models for the energy harvesting process is problematic since its characteristics may vary from application to application. Furthermore, it is not always controllable or predictable, largely depending on the source of energy and the mobility of energy harvester [16]. To deal with this ambiguity, several classifications have been made in studies on energy harvesters.

In [13], energy harvesting processes are classified into following subcategories: Uncontrollable but predictable, uncontrollable and unpredictable, fully controllable and partially controllable.

The classification in [16], focuses only on the predictability considering predictable, partially predictable and stochastic energy harvesting processes. Also in [16], the effect of the mobility is underlined. For example, outdoor solar energy harvesting can be considered to be partially predictable for a static solar cell, but rather stochastic on a mobile device.

When the energy harvesting process under consideration is regarded as a stochastic process, frequenty Markovian models are preferred. In some research efforts, the validity of Markovian models has been tested. As an example, in [17], a generalized Markov (GM) model (see Figure 2.2), using a scenario paramater, is introduced to model energy harvesting processes where stationary Markov models do not suffice. Particularly, it is shown that the proposed GM model fits better for piezoelectric energy harvesting.

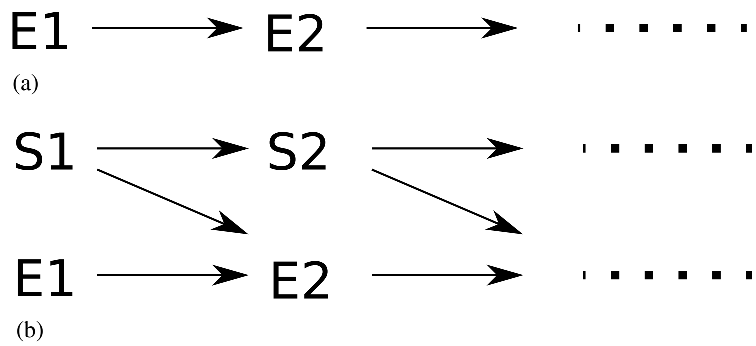


Figure 2.2: (a) The stationary Markov model where $E1, E2, ...$ are harvested energy levels (b) The generalized where $S1, S2, ...$ are additional scenario parameters.

CHAPTER 3

A REVIEW ON ENERGY EFFICIENT OFFLINE SCHEDULING ALGORITHMS

Man plans and God laughs.

Yiddish Proverb

3.1 ENERGY EFFICIENT OFFLINE SCHEDULING PROBLEMS

In offline problems, it is assumed that all the necessary information about events within the time window under consideration is known prior to the determinations of actions to be taken. In the related literature, what is meant by offline scheduling problems is meant offline optimization problems dealing with transmission schemes. Accordingly, actions in these offline problems usually correspond to transmission power or rate adaptations.

3.1.1 Offline Energy Minimization with Packet Arrival and Delay Constraints

The formulation of an energy efficient packet transmission problem dates back to the year 2001 [3]. The problem defined in [3] concerns the minimization of energy consumed to transmit packets having a common deadline but arbitrary sizes and arrival times which are known beforehand. The solution is characterized by offlines schedules that determine transmission intervals of packets. The basic observation on which this formulation relies is that a considerable amount of energy can be saved by transmitting packets over longer periods with lower power levels. It has been proved that, in optimal schedules which are named *lazy* schedules, transmission power should be constant between packet arrivals and nondecreasing as deadline approaches. An online schedule which suggets using expected values of transmission times is also proposed in [3]. In [18], the problem is generalized for broadcast and fading channels and an algorithm finding optimal offline schedule, the FlowRight algorithm, is introduced. Another version of the problem allowing packets to have individual deadlines is considered in [19].

3.1.2 Offline Throughput Maximization with Energy Arrival Constraints

The approach taken for energy efficient offline packet transmission scheduling problems, was adapted to offline transmission scheduling problems over energy harvesting links which in fact have a direct relation. In the problems with energy harvesting, harvesting events can be treated as packets arrivals and constraints related to energy consumption can be considered assuming an energy buffer similar to a data buffer that receives energy “packets” instead of data packets.

One of the early works on offline transmission scheduling for energy harvesting systems [6] considers the transmission completion time minimization problem which is a dual problem of throughput maximization problem. In [7], a discrete-time finite-horizon thoughput maximizing transmission scheduling problem is examined for fading point-to-point communication links with energy harvesters and an optimal solution making use of dynamic programming is proposed. While considering both offline and online cases, only the optimal offline solution is specified and by using KKT conditions, it is proved that the optimal offline power allocation with water filling is non-decreasing over time slots. similar optimality conditions are shown in [20], where optimal offline schedules are considered for throughput maximization and transmission completion time minimization over continuous time. Based on optimality conditions, a directional water-filling algorithm which computes the optimal offline schedule is introduced. For energy harvesting broadcast links, the transmission completion time minimization problem is solved and the structure of the optimal offline schedule is characterized in [21]. An extension of the transmission completion time minimization problem that allows packet arrivals during transmission over a point-to-point link, is considered and iterative techniques are proposed for the offline solution in [6].

3.2 OBSERVATIONS AND METHODS

3.2.1 Equalization

In related works to energy-efficient transmission scheduling, an observation named *equalization*, is frequently noted. It is based on the fact that transmission power can be assumed as a convex function of throughput and transmission rate in most high performance coding schemes. In addition, usually power-rate relation is assumed to be time-invariant for slow flow fading or static channel cases so that applying a certain power or rate has the same value over time.

Equalization characterizes the key property of the optimal solution of both related energy minimizing and throughput maximizing optimization problems. This property suggests the following: Unless causality contraints set by packet (or energy) arrivals are violated, transmission rates (or power) of different time intervals should be equalized to lower total energy consumption (or higher total thoughput). The theorem below summarizes this property:

Theorem 3.2.1 *Let us denote transmission power (or rate) during a time interval ω_i as $w(r)$ ($g(\rho)$) being a convex (or concave) function of rate (or power) r (or ρ) and let l_{ω_i} be the length of the time interval ω_i. For any pair of time intervals ω_1 and ω_2, the sum $l_{\omega_1}w(r_1) + l_{\omega_2}w(r_2)$(or $l_{\omega_1}g(\rho_1) + l_{\omega_2}g(\rho_2)$) is minimized (or maximized) when transmission rates (or powers) are equalized by transferring some amount of data (or energy) from the interval having higher rate (or power) to the one having lower rate (or power) .*

Proof. Assume $r_1 > r_2$. While conserving the total amount of data, transmission rates can be equalized at the level of $\frac{l_{\omega_1}r_1+l_{\omega_2}r_2}{l_{\omega_1}+l_{\omega_2}}$. From the convexity of the function $w(r)$, the summation of total energy consumption for equalized levels becomes less than the summation of unequalized case:

$$(l_{\omega_1} + l_{\omega_2})w\left(\frac{l_{\omega_1}r_1 + l_{\omega_2}r_2}{l_{\omega_1} + l_{\omega_2}}\right) \leq l_{\omega_1}w(r_1) + l_{\omega_2}w(r_2) \tag{3.1}$$

Since one can always find an equilibrium level for any pair of unequalized rates, equalization minimizes the sum of energy comsumption on corresponding time intervals. ■

In offline scheduling, equalization can be applied to distinct time intervals only when there is no causality constraint such as an arrival event between these intervals. Usually, in the related literature, time intervals between arrival events are called *epochs*. According to Theorem 3.2.1, transmission rate (or power) within an epoch should be constant for any optimal offline schedule. Moreover, it also shows that transmission rate (or power) should be nondecreasing otherwise more equalization can be performed by moving data from previous time intervals to next ones.

3.2.2 Stretched String Method

The optimality property in Theorem 3.2.1 and its results, can be restated by making a geometrical interpretation of the optimal offline schedule. Let us consider the departure curve $D(t)$ which is the cumulative function of transmitted data (or consumed energy). Due to causality constraints, the departure curve $D(t)$ should be always below the arrival curve $A(t)$ which represents the cumulative of data arrivals (or energy harvests). Its slope gives instantaneous transmission rate (or power) and accordingly joining any two points on the departure curve corresponds performing an equalization. On the other hand, from Theorem 3.2.1, it is known that an offline schedule can be improved by equalization and hence an equalization cannot performed on an optimal offline schedule. Also, to transmit all of the received packets (or consume all of the harvested energy), the optimal departure should reach the arrival curve at the end of transmission. Therefore, the optimal departure curve $D^*(t)$ can be defined as the departure curve that connects the starting and ending points of the arrival curve and does not contain any two points which can be joined without violating constraints. In other words, the optimal departure curve follows the shortest path from the beginning of transmission to the deadline constraint. Since it has similar mathematical description, the optimal departure curve visually resembles a stretches string attached to the end points of a staircase. The figure in 3.1.a, captures this visualization.

An algebraic expression for the derivative of the optimal departure curve $D^*(t)$ can be the following:

$$\frac{dD^*(t)}{dt} = \min_{\tau \in (t,T)} \frac{A(\tau) - D^*(t)}{\tau - t} \tag{3.2}$$

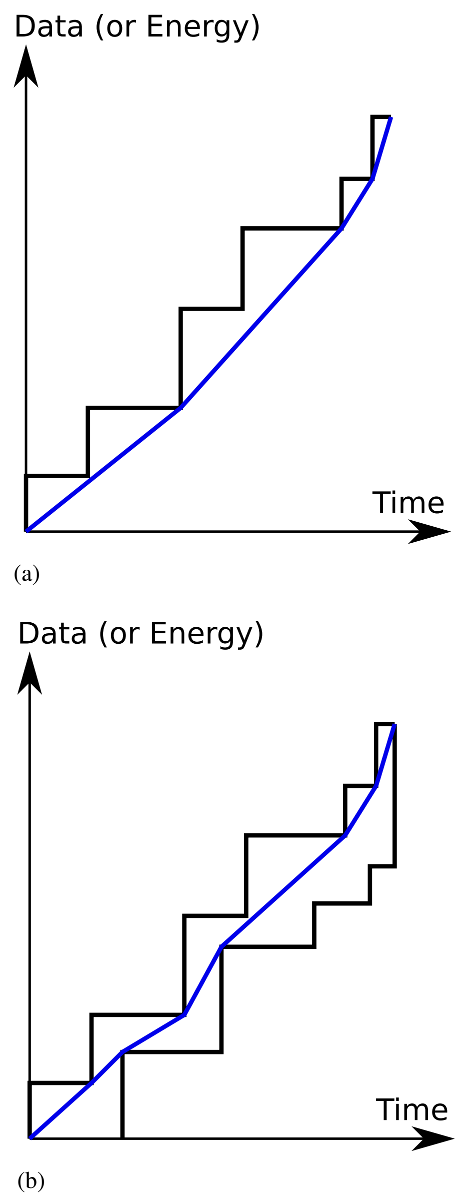


Figure 3.1: The visualization of the Stretched String Method with (a) arrival constraints, (b) arrival and departure constraints

where the time T is the end of transmission.

By using Eq. 3.2, one can compute the instantaneous optimal transmission rate (or power) which is the slope of the optimal departure curve given the present value of the optimal departure $D^*(t)$.

In addition to arrival constraints, departure constraints can be also taken into account to model individual deadline and buffer contraints. In this case, a feasible departure curve should be between arrival curve $A(t)$ and minimum departure curve $D_{min}(t)$ which determines departure constraints. The optimality property still applies but this time, the departure curve is also restricted by the minimum departure. Accordingly, the optimal offline schedule can be visualized as a stretched string between arrival and minimum departure curves in Figure 3.1.b. Despite this ease of visualization, the structure of the optimal offline schedule for this case, needs to be expressed mathematically in a more complicated way than for the previous case. Yet, a general expression for optimal offline schedules can be derived as in below:

$$\frac{dD^*(t)}{dt} = \arg\max_{s} \left(\min\left\{\tau \in (t,T) \mid (A(\tau) = D^*(t) + s(\tau - t)) \vee (D_{min}(\tau) = D^*(t) + s(\tau - t))\right\}\right) \tag{3.3}$$

Alternatively, Eq. 3.2 and Eq. 3.3 can be employed to define algorithms which give the optimal offline schedule by recursively computing the derivative of the optimal departure curve.

A detailed study of this method, namely *Stretched String* , has been presented in [19] , [22] and [23], but Eq. 3.2 and Eq. 3.3 have been obtained for this chapter.

CHAPTER 4

ONLINE THROUGHPUT MAXIMIZATION PROBLEM FOR AN ENERGY HARVESTING TRANSMITTER

> The greatest challenge to any thinker is stating the problem in a way that will allow a solution.
>
> Bertrand Russell

4.1 PROBLEM DEFINITION

Consider a point-to-point communication link with an energy harvesting transmitter. The transmitter has sufficient data to send at the beginning of the time period under consideration, and the goal is to maximize the amount of data transmitted (equivalently, throughput) over this finite horizon, by adjusting transmission rate and power in time judiciously in response to the energy harvested. The amount of data in the data buffer is assumed to be so large that data buffer cannot be emptied even the hightest transmission rate is continuously applied until the end of the transmission period. This is a reasonable assumption if the system is considered as a highly loaded queue and in the case that maximizing the throughput is a solution to stabilize the queue. Communication rate is assumed to be a concave, monotone increasing function of transmit power, hence, energy can be more efficiently spent by communicating at low rate. Time is slotted into intervals of a certain duration such that a power/rate decision will be made dynamically at the beginning of each slot. Let ρ be the energy consumption per slot when the transmission rate is chosen as $r = g(\rho)$, $g(\rho)$ is assumed to be strictly concave and increasing in ρ ([5]-[8]).

The function $g_r(e, \rho)$ below provides the number of bits delivered during a slot duration when e is the energy available for transmission at the begining of the slot. Note that this expression allows for the event that the energy e reserved for transmission is too low for transmitting at ρ for a whole slot, and in that case the transmitter will be active during part of the slot and idle in the remainder of the slot.

$$g_r(e, \rho) = g(\rho) \min\left(\frac{e}{\rho}, 1\right) \tag{4.1}$$

The slots are numbered backwards in time from the deadline such that slot 1 is the slot closest to the deadline, and slot $N < \infty$ represents the beginning of the time period. Let e_n be the stored energy at the beginning of slot n and ρ_n the transmission power level decision for this slot. The power level decision ρ_n can be picked from a finite discrete set $\mathbf{U}$. Any collection of decisions $\rho_N, ..., \rho_1$ is a *transmission trajectory*, and hence there are $|\mathbf{U}|^N$ possible trajectories.

The stored energy e_n is a function of the energy at the beginning of the previous slot, e_{n+1}, the power decision ρ_{n+1} and H_n, energy harvested *during* the previous slot:

$$e_n = (e_{n+1} - \rho_{n+1})_+ + H_n \tag{4.2}$$

Harvested energy will be modelled as a stochastic process $\{H_n\}$, $n \geq 1$, with values coming from a discrete state-space. Let H_n^m, $m \geq n$ denote the vector $[H_n, ..., H_m]$. Accordingly, stored energy at time n is a discrete random variable depending on H_n^N and the previous power decisions, ρ_{n+1} through ρ_N.

The objective function is the expectation of total throughput, over the statistics of the harvest process. An *online policy* is one that produces a decision ϱ_n at each slot n with knowledge of previous energy harvests H_n^N and the current stored energy e_n. Then, an online transmission policy ϱ, is a collection $\varrho_N, ..., \varrho_1$ and an optimal online policy ϱ^* is one that maximizes the expected throughput over N slots:

$$\varrho^* = \arg\max_{\varrho} \sum_{n=1}^{N} E\left[g_r(e_n, \varrho_n(H_n^N, e_n))\right] \tag{4.3}$$

4.2 OPTIMAL SOLUTION WITH DYNAMIC PROGRAMMING

In the rest, we shall limit attention to $\{H_n\}$, $n \geq 1$, that can be described as a first-order Markov process, with transition probabilities q_{ij} between harvest states h_i and h_j, such that: $q_{ij} = P(H_n = h_j | H_{n+1} = h_i)$. Let (e, h) be the combined state for stored energy level and harvest state, and $V_n^*(e, h)$ be the maximum expected throughput for the next n slots till the deadline. Then, the problem can be formulated using a dynamic programming equation as below.

$$V_n^*(e,h) = \max_{\rho \in \mathbf{U}} V_n(e, h_i, \rho), \; n > 1, \text{ where} \tag{4.4}$$

$$V_n(e, h_i, \rho) = g_r(e, \rho) + \sum_j q_{ij} V_{n-1}^*\left((e-\rho)_+ + h_j, h_j\right)$$

$$V_1^*(e, h_i) = \max_{\rho \in \mathbf{U}} g_r(e, \rho)$$

Note that, as the energy harvested during the last slot will not be used, $V_1^*(e, h_i)$, which represents the throughput in the last slot, does not depend on the h_i. An explicit form of the function $V_1^*(e, h_i)$ is provided in (4.5).

$$V_1^*(e, h_i) = \begin{cases} g(\rho_1)\left(\frac{e}{\rho_1}\right) & ; e < \rho_1 \\ g(\rho_1) & ; \rho_1 \leq e < \frac{g(\rho_1)}{g(\rho_2)}\rho_2 \\ g(\rho_2)\left(\frac{e}{\rho_2}\right) & ; \frac{g(\rho_1)}{g(\rho_2)}\rho_2 \leq e < \rho_2 \\ g(\rho_2) & ; \rho_2 \leq e < \frac{g(\rho_2)}{g(\rho_3)}\rho_3 \; .... \end{cases} \tag{4.5}$$

The function $V_n^*(e, h_i)$ can be evaluated by backward induction starting from $V_1^*(e, h_i)$. An optimal solution is a set of decision rules defined as:

$$\varrho_n^*(e, h_i) = \arg\max_{\rho \in \mathbf{U}} V_n(e, h_i, \rho) \tag{4.6}$$

It should be noted that the value function exhibits relatively small dependence on the energy state e_n and the harvest state h_n than time. As example, Fig. 4.1 shows plots the variation of the value function with respect to stored energy and the time (number of slots) until the end of the horizon, for two extreme harvest states (the specific state spaces will be described in Section 4.6).

Before addressing the structure of the solution, we make a final, technical assumption about the set of power levels that prevents anomalous decision regions and deems threshold results possible. It is possible to generate families of rates that do not satisfy this assumption, but it is straightforward to show the existence of sets of power levels that satisfy this assumption-such sets have been used in our numerical examples.

Assumption 1 *Let $\rho > \rho'$ where $(\rho, \rho') \in U^2$, then if $V_n(e, h_i, \rho) > V_n(e, h_i, \rho')$ for some energy level e, then $V_n(e+\delta, h_i, \rho) > V_n(e+\delta, h_i, \rho')$ for any $\delta > 0$.*

Roughly, the assumption states that for the set of rates, power levels, and harvest statistics under consideration, if a higher power level is preferred over a lower one at some energy level, it will continue to be preferred at an higher energy level. While this relationship may intuitively appear to always hold, some situations where it does not hold have been observed. The reason for this is the piecewise flatness of the value functions as seen in (4.5) for $V_1^*(e, h_i)$.

An example where the relationship does not hold is the following: Suppose that $\rho > \rho'$ and $V_2(e, h_i, \rho) - V_2(e, h_i, \rho') = \Delta > 0$ for some energy level $e > \max(\rho, \rho')$. Then, consider the difference $V_2(e+\delta, h_i, \rho) - V_2(e+\delta, h_i, \rho')$ where δ is a positive energy increment. For any given discrete set of power levels, we can find positive probability mass function values h_js for energy harvesting process such that both $e - \rho + h_j$ and $e + \delta - \rho + h_j$ are in the range $(\rho_m, \frac{g(\rho_m)}{g(\rho_{m+1})}\rho_{m+1})$ for some m and sufficiently small δ. Hence, the value function $V_2(e, h_i, \rho)$ remains constant for an energy increment δ. On the other hand, the value function $V_2(e, h_i, \rho')$ does not have to remain constant for the same setting and can increase for the same energy increment δ and this increase can be larger than $\Delta > 0$ since Δ can be infinitely small independent from δ. Therefore, the difference $V_2(e+\delta, h_i, \rho) - V_2(e+\delta, h_i, \rho')$ can be negative for some energy increment δ. However, such cases are rare and, ignoring them, we limit our attention to problems that obey Assumption 1.

Theorem 4.2.1 *Let ρ_{min} be the minimum nonzero power decision in the set* **U**. *Then, whenever the stored energy e is less than ρ_{min}, the optimal decision is ρ_{min}.*

Proof. From the concavity of the function $g(\rho)$, it can be seen that $g_r(e, \rho_{min})$ gives the largest one-slot throughput among the nonzero decisions in the set **U**. Also, when $e_n < \rho_{min}$, e_{n-1} is equal to H_{n-1} for all nonzero decisions and $e_n + H_{n-1}$ for the decision of not transmitting during that slot (being idle). But since the channel is static and transmitting with ρ_{min} is always the most efficient way to consume energy in terms of throughput per energy, idling during any slot is meaningless. Therefore, ρ_{min} is the optimal decision for every slot where $e < \rho_{min}$. ■

Now we are ready to show a set of threshold results for $\rho_n^*(e, h_i)$.

Lemma 4.2.2 *Let $\rho > \rho'$ where $(\rho, \rho') \in U^2$, then $V_n(e, h_i, \rho) > V_n(e, h_i, \rho')$ when $e > (n-1)\rho_{max} + \rho$ for any n and h_i where $\rho_{max} = \max_{\rho \in U} \rho$.*

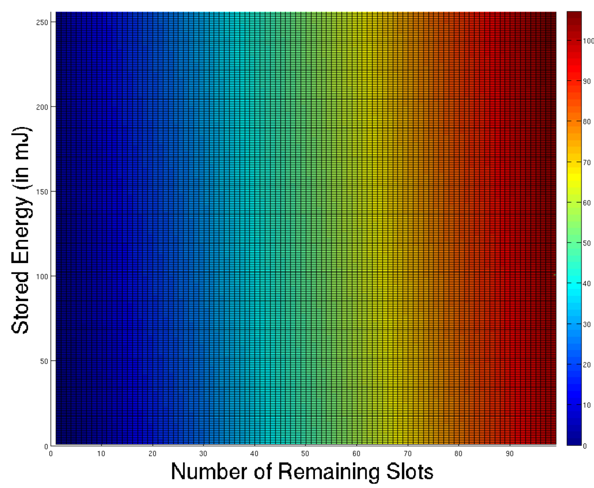
Proof. Since $g(\rho_{max})$ is the highest throughput for a slot duration, if $e > (n-1)\rho_{max} + \rho$, then $V_{n-1}^*((e-\rho)_+ + h, h)$ and $V_{n-1}^*((e-\rho')_+ + h, h)$ are both equal to $(n-1)g(\rho_{max})$ for any $h > 0$. Hence, $V_n(e, h_i, \rho) = g(\rho) + (n-1)g(\rho_{max})$ is larger than $V_n(e, h_i, \rho') = g(\rho') + (n-1)g(\rho_{max})$. ■

Lemma 4.2.3 *Let $\rho > \rho'$ where $(\rho, \rho') \in U^2$, then $V_n(e, h_i, \rho) \leq V_n(e, h_i, \rho')$ when $e \leq \frac{g(\rho')}{g(\rho)}\rho$ for any n and h_i.*

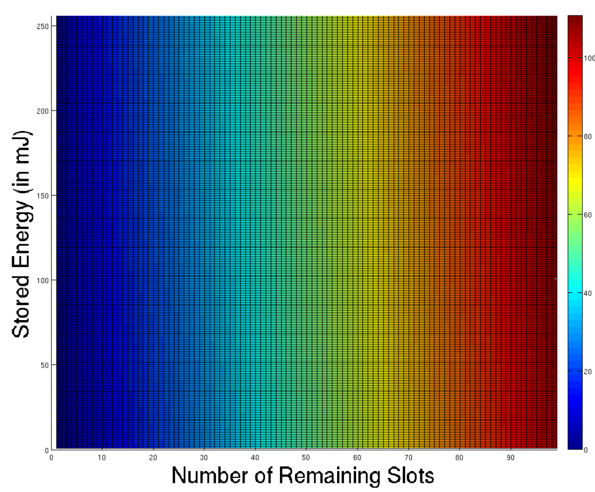
Proof. For $e \leq \frac{g(\rho')}{g(\rho)}\rho$, $V_n(e, h_i, \rho) = g(\rho)\frac{e}{\rho} + \sum_j q_{ij} V_{n-1}^*\left(h_j, h_j\right)$ and $V_n(e, h_i, \rho') = g_r(e, \rho') + \sum_j q_{ij} V_{n-1}^*\left((e-\rho')_+ + h_j, h_j\right)$. The value function is an nondecreasing function of energy, thus $V_{n-1}^*\left(h_j, h_j\right)$ is smaller than or equal to $V_{n-1}^*\left((e-\rho')_+ + h_j, h_j\right)$ for any h_j. Also, $g(\rho)\frac{e}{\rho}$ cannot be larger than $g_r(e, \rho')$ when $e \leq \frac{g(\rho')}{g(\rho)}\rho$. Therefore, $V_n(e, h_i, \rho')$ is larger than or equal to $V_n(e, h_i, \rho)$ for any n and h_i. ■

Theorem 4.2.4 *The decision rule $\rho_n^*(e, h_i)$ is an increasing (piecewise constant) function of e for any n and h_i.*

Proof. Lemma 1 shows that there is an energy level $e_{(i,j)}^{high}$ where the higher power decision $\rho_{(i)}$ is more desirable than the lower power decision $\rho_{(j)}$ for every pair $(\rho_{(i)}, \rho_{(j)}) \in U^2$. Similarly, Lemma 2 shows that there is an energy level $e_{(i,j)}^{low}$ where the lower power decision $\rho_{(j)}$ is preferable to the higher power decision $\rho_{(i)}$ for every pair of decisions $(\rho_{(i)}, \rho_{(j)}) \in U^2$. According to assumption 1, if the higher power level $\rho_{(i)}$ is preferred at a certain energy level, the higher power level will still be more desirable at a higher energy level. These imply that there is an energy level $e_{(i,j)}$ for every $(\rho_{(i)}, \rho_{(j)}) \in U^2$ such that below which the lower power decision $\rho_{(j)}$ is more desirable and above which the higher power decision $\rho_{(i)}$ is more desirable. Accordingly, there is an energy level $e_{(i)} = \max_{j, \rho_{(j)} \in U} e_{(i,j)}$ for every power decision $\rho_{(i)}$ such that the value function of $\rho_{(i)}$ is larger than the value function of any $\rho_{(j)}$ lower than $\rho_{(i)}$. Therefore, the optimal power level decisions increase in energy. ■



(a)



(b)

Figure 4.1: Value function (a) $V_n^*(e, h_0)$, (b) $V_n^*(e, h_1)$ against stored energy e_n and number of remaining slots n for burst arrival Markov model which has 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$.

Although the dynamic programming approach provides an optimal solution for the Markovian case, its computational complexity is exponential in the time horizon N. To evaluate the value functions $V_n^*(e, h_i)$, all possible transmission trajectories should be examined and since there are $|\mathbf{U}|^N$ possible transmission trajectories, a dynamic programming based algorithm has a time complexity exponential in N. This complexity will not be a problem when online computation can be substituted by a table look-up, from decision rules prepared before transmission. However, in some cases statistical information on energy harvesting process may need to be updated, which makes real-time computation a necessity. For these reasons, low complexity online policies may be prefered.

4.3 SUBOPTIMAL SOLUTIONS

In this section, two suboptimal policies will be described. The *Expected Threshold* policy is proposed as a computationally cost-effective suboptimal solution and is considered as a contribution of this work. A *Greedy* policy is proposed for performance evaluation purposes.

4.3.1 Expected Threshold Policy

The Expected Threshold Policy is defined as the following:

$$\boxed{\begin{array}{rcl} \varrho_n(H_n^N, \hat{e}_n) & = & \max\left\{\rho \in \mathbf{U} | L_n(H_n^N, \rho) \leq \hat{e}_n\right\} \\ L_n(H_n^N, \rho) & = & \max\left(\rho, \rho n - \sum_{l=1}^{n-1} E\left[H_l | H_n^N\right]\right) \\ \text{for } \rho \neq \rho_{min} & ; & L_n(H_n^N, \rho_{min}) = 0 \end{array}}$$

In the above, $L_n(H_n^N, \rho)$ is the minimum energy level at which the power level ρ is chosen. We refer to this as the "expected threshold" for the level ρ.

The computational complexity of the Expected Threshold policy is $O((|\mathbf{U}| - 1)N)$, as $(|\mathbf{U}| - 1)$ threshold calculations, each of complexity $O(N)$, are performed for each slot. It should be added that, unlike the dynamic programming solution, it does not assume a first-order Markov harvest process.

4.3.1.1 Derivation of the policy

For a moment, let us consider the offline problem where information about energy harvest amounts H_n is revealed before the start of transmission and power decision levels are picked from a continuous set such as $\mathbf{R}$. Optimal transmission power decisions can be obtained using stretched string method(see Chapter 3) [6, 22]. This optimal solution dictates that constant power transmission should be applied as long as possible with highest power levels. The optimal power level $\tilde{\rho}_n^*$ should be less than e_n and satisfy the following inequality for all $a = 1, ..., (n-1)$.

$$e_n + \sum_{l=a}^{n-1} H_l \geq (n-a+1)\tilde{\rho}_n^* \tag{4.7}$$

The optimal decision $\tilde{\rho}_n^*$ which is the highest power level satisfying this condition is given below.

$$\tilde{\rho}_n^*(e_n) = \min_{a=1,....,(n-1)} \left(e_n, \tilde{\rho}_n(e_n, a)\right), \text{ where}$$

$$\tilde{\rho}_n(e_n, a) = \frac{e_n + \sum_{l=a}^{n-1} H_l}{n-a+1}$$

Since $\tilde{\rho}_n^* \in \mathbf{R}$ and $\mathbf{U} \subset \mathbf{R}$, offline optimal throughput values achieved with real-valued power decisions $\tilde{\rho}_n^*$s dominate any online solution for every realization of the energy harvesting process.

Accordingly, an optimal online policy is one which minimizes the difference from the optimal offline throughput:

$$\varrho^* = \arg\min_{\varrho} \sum_{n=1}^{N} E\left[g(\tilde{\rho}_n^*(e_n)) - g_r(\hat{e}_n, \varrho_n)\right] \tag{4.8}$$

Note that the stored energy process of the online policy ϱ are represented with $\hat{e}_n$s since they are different than the stored energy process e_ns which depend on deterministic optimal decisions.

The online decision at slot n is informed by H_n^N and $\hat{e}_n$. Therefore, by taking the offline rule $E\left[\tilde{\rho}_n^*(\hat{e}_n)|H_n^N\right]$ as reference we define the online decision rule at slot n for $\hat{e}_n \geq \rho_{min}$ [1] as given below:

$$\varrho_n(H_n^N, \hat{e}_n) = \max\left\{\rho \in \mathbf{U} | \rho \leq E\left[\tilde{\rho}_n^*(\hat{e}_n)|H_n^N\right]\right\} \tag{4.9}$$

Applying the law of total expectation and Jensen's inequality inside the summation in (4.8), the difference between expected offline optimal throughput and the expected throughput of the online policy ϱ can be upperbounded as below.

$$\sum_{n=1}^{N} E\left[g(\tilde{\rho}_n^*(e_n))\right] - \sum_{n=1}^{N} E\left[g(\varrho_n(H_n^N, \hat{e}_n))\right] \leq$$

$$\sum_{n=1}^{N} E\left[g(E\left[\tilde{\rho}_n^*(e_n)|H_n^N\right]) - g(\varrho_n(H_n^N, \hat{e}_n))\right]$$

In the above, the LHS is positive and gets smaller as the online decision $\varrho_n(H_n^N, \hat{e}_n)$ gets close to $E\left[\tilde{\rho}_n^*(e_n)|H_n^N\right]$. The rule defined in (4.9) selects a decision close to $E\left[\tilde{\rho}_n^*(\hat{e}_n)|H_n^N\right]$ guaranteeing that $\varrho_n \leq \hat{e}_n$ but the online energy level $\hat{e}_n$ is different than the deterministic optimal energy level e_n. However, minimizing the distance between power decisions corresponds to minimizing the distance between energy levels when decisions are selected so that $\rho_n \leq e_n$. In this sense, the rule in (4.9) tracks deterministic optimal energy levels e_ns. In general, computing the expectation $E\left[\tilde{\rho}_n^*(\hat{e}_n)|H_n^N\right]$ involves a minimization over random variables. For the sake of simplicity, the online decision can be based on only the expectation of $\tilde{\rho}_n(\hat{e}_n, 1)$ which is the dominant term determining $\tilde{\rho}_n^*(\hat{e}_n)$ in most cases. Let us define such a decision rule $\varrho_n(H_n^N, \hat{e}_n)$ for $\hat{e}_n \geq \rho_{min}$ as follows:

$$\varrho_n(H_n^N, \hat{e}_n) = \max\left\{\rho \in \mathbf{U} | \rho \leq \min(\hat{e}_n, E\left[\tilde{\rho}_n(\hat{e}_n, 1)|H_n^N\right]\right\} \tag{4.10}$$

The decision rule in (4.10) is just an alternative expression of the Expected Threshold policy.

[1] For $\hat{e}_n < \rho_{min}$, $\varrho_n(H_n^N, \hat{e}_n)$ can be chosen as ρ_{min} as it is the optimal decision by Theorem 4.2.1.

4.3.2 Greedy Policy

The Greedy Policy is a simple policy that, at the beginning of any slot, sets the transmission power to the highest level that can be used for the whole slot. In other words, $\rho_n \leq e_n$. Explicitly, Greedy is defined by the following decision rule:

$$\varrho_n(H_n^N, \hat{e}_n) = \max \{\rho \in \mathbf{U} | \rho \leq \hat{e}_n\} \,; \text{for } \hat{e}_n \geq \rho_{min}$$

When harvest rate (power input) is large enough, the expected threshold $L_n(H_n^N, \rho)$ approaches ρ and Greedy makes the same choice as the Expected Threshold policy does.

4.4 EXTENSION TO A TIME VARYING CHANNEL

For completeness of the treatment, in this section we extend the problem formulation to a time-varying channel.

Provided that perfect channel state information is available at the transmitter, variation in channel state introduces just another dimension to the state space of the problem. In principle, this can be straightforwardly incorporated into the problem setup and solution method, as will be shown in the rest of this section.

However, it should be noted that the ease by which this formulation seems to handle a fading channel is because of (quite standard) assumptions that are made about the channel state process, and these assumptions may not always capture what happens in a realistic system. In wireless channels, depending on the relative movement of scatterers and transceiver units, channel fading may occur at different time scales. For example, in an indoor channel with a long coherence time (on the order of half a second), channel gain may stay relatively constant over tens of time slots (considering a slot length of about 10 ms). On the contrary, in an outdoor scenario with high mobility, channel state may significantly change from one slot to the next. Hence, the specific model for the channel state process highly depends on the choice of slot length with respect to fading dynamics. Furthermore, feedback about the channel state to the transmitter may in practice will not be perfect or timely. Acknowledging these weaknesses of the model, within the scope of this paper, we proceed with the perfect channel state information assumption.

Accordingly, let the channel gain during slot k be given by γ_k, chosen from a discrete set of values. According to our earlier definition, the communication rate r_k is $g(\gamma_k\rho)$, and the function g_r may be extended as the following.

$$g_r(e,\gamma,\rho) = g(\gamma\rho)\min\left(\frac{e}{\rho},1\right) \tag{4.11}$$

Accordingly, we can define the optimal online policy ϱ^* for a time-varying channels as:

$$\varrho^* = \arg\max_{\varrho} \sum_{n=1}^{N} E\left[g_r(e_n,\gamma_n,\varrho_n(H_n^N,\gamma_n^N,e_n))\right] \tag{4.12}$$

where γ_n^N denotes the vector $[\gamma_n, ..., \gamma_N]$.

Let us assume $\gamma_n, n \geq 1$ as a first-order Markov process with transition probability f_{uv} between channel states γ_u and γ_v, such that $f_{uv} = P(\gamma_n = \gamma_v|\gamma_{n+1} = \gamma_u)$. Then, the optimal solution for time-varying channel case can be formulated with dynamic programming as in the following:

$$\begin{gathered} V_n^*(e,h,\gamma) = \max_{\rho\in\mathbf{U}} V_n(e,h,\gamma,\rho),\ n>1, \text{ where} \\ V_n(e,h_i,\gamma_u,\rho) = g_r(e,\gamma_u,\rho)+ \\ \sum_j\sum_v q_{ij} f_{uv} V_{n-1}^*\left((e-\rho)_+ + h_j, h_j, \gamma_v\right) \\ V_1^*(e,h_i,\gamma_u) = \max_{\rho\in\mathbf{U}} g_r(e,\gamma_u,\rho) \end{gathered} \tag{4.13}$$

Similar to (4.5), an explicit form for $V_1^*(e,h_i,\gamma)$ can be written as in below:

$$V_1^*(e,h_i,\gamma) = \begin{cases} g(\gamma\rho_1)\left(\frac{e}{\rho_1}\right) & ; e<\rho_1 \\ g(\gamma\rho_1) & ; \rho_1 \leq e < \frac{g(\gamma\rho_1)}{g(\gamma\rho_2)}\rho_2 \\ g(\gamma\rho_2)\left(\frac{e}{\rho_2}\right) & ; \frac{g(\gamma\rho_1)}{g(\gamma\rho_2)}\rho_2 \leq e < \rho_2 \\ g(\gamma\rho_2) & ; \rho_2 \leq e < \frac{g(\gamma\rho_2)}{g(\gamma\rho_3)}\rho_3 \text{ ....} \end{cases} \tag{4.14}$$

Again by backward induction, the optimal online solution as a set of decision rules can be obtained:

$$\varrho_n^*(e,h_i,\gamma) = \arg\max_{\rho\in\mathbf{U}} V_n(e,h_i,\gamma,\rho) \tag{4.15}$$

4.5 EXPECTED WATER LEVEL POLICY

The approach taken for developing the Expected Threshold policy can be extended to the fading channel formulation. Given that the present state and the history of channel and energy harvesting process, the optimal offline power level $\tilde{\rho}_n^*$ may be considered as a stochastic process. For the offline solution, it is known that the optimal offline power level $\tilde{\rho}_n^*$ is always lower than the stored energy e. Thus, $g_r(e, \gamma, \rho)$ can be replaced with $g(\gamma\rho)$, arriving at the following inequality:

$$\sum_{n=1}^{N} E\left[E\left[g(\gamma_n \tilde{\rho}_n^*)|\gamma_n^N, H_n^N\right]\right] \leq \sum_{n=1}^{N} E\left[g(\gamma_n E\left[\tilde{\rho}_n^*|\gamma_n^N, H_n^N\right])\right] \tag{4.16}$$

Hence, applying the Expected Threshold policy could still provide a fairly good average throughput. However, because of the added dimensionality, the computation of the expected value of the optimal offline power level is harder in the fading case than it was in the static channel case. On the other hand, it was shown in [7] that, when power levels are continuous, the finite-horizon throughput-optimal offline policy is a *waterfilling* policy where water levels are nondecreasing as deadline approaches. Accordingly, the optimal power level for offline solution is given by $\tilde{\rho}_n^* = (\tilde{w}_n - \frac{1}{\gamma_n})_+$ where $\tilde{w}_n$ is the water level of slot n.

A lower bound for expected offline power level $\tilde{\rho}_n^*$ can be found as $E[\tilde{\rho}_n^*] \geq (E[\tilde{w}_n] - \frac{1}{\gamma_n})_+$. Then, a conservative online decision for discrete power level case can simply be the lowest power level in the set $\mathbf{U}$ that is higher than $(E[\tilde{w}_n] - \frac{1}{\gamma_n})_+$ We name this policy as "Expected Water Level Policy".

$$\begin{aligned}
\varrho_n(H_n^N, \hat{e}_n) &= \max\left\{\rho \in \mathbf{U} | L_n(H_n^N, \rho) \leq \hat{e}_n\right\} \\
L_n(H_n^N, \gamma_n^N, \rho) &= \max(\rho, e_n) \\
\text{for } \rho > 0 \quad &; \quad L_n(H_n^N, \gamma_n^N, 0) = 0 \\
\text{where } E[\tilde{w}_n(e_n)] &= \rho + \frac{1}{\gamma_n}
\end{aligned}$$

The minimum energy $L_n(\rho)$ at which ρ is the selected power level can be set so that the expected water level $E[\tilde{w}_n(e_n)]$ equals to $\rho + \frac{1}{\gamma_n}$. On the other hand, contrary to the previous case, the channel can remain idle and $L_n(0)$ can be considered as zero energy level.

The computation of $E[\tilde{w}_n(e_n)]$ is explained in Appendix A.1.

4.6 EVALUATION

The throughput performances of the optimal online solution, expected threshold policy and Greedy have been compared, along with that of a single power level policy, which is a static reference policy whose transmit power is set to the maximum power in the set U lower than the time average energy harvest rate whenever there is energy for transmission.

A first-order Markov model for the energy harvesting process is derived from an irradiance trace (measured during a car-based roadtrip) which is available in the CRAWDAD repository [1]. The time slot interval is taken as 30 s and harvested energy amounts are calculated assuming that irradiance over a 43 cm^2 area can be transformed into energy with a conversion rate of 21%. Transmission power decisions $(5, 10, 23, 26, 74, 100, 159, 256$mW) are based on single stream data rates (see Table 4.6) for 40MHz and short-guide interval (400s) in 802.11n standard. To compute power levels correspoding to standard data rates, net bitrate is assumed equal to Shannon capacity of an additive white Gaussian noise (AWGN) channel with a noise spectral density 0.83 nW/Hz. Accordingly, the capacity formula of the AWGN channel is used as the function $g(\rho)$.

Harvested and consumed energy amounts are quantized in order to discretize the state space where value functions and decision rules are evaluated for the optimal solution with dynamic programming.

Table 4.1: Single stream data rates in 802.11n standard for 40MHz channel and short-guide interval (400s)

Modulation Type	Coding Rate	Data Rate (Mbit/s)
BPSK	1/2	15.00
QPSK	1/2	30.00
QPSK	3/4	45.00
16-QAM	1/2	60.00
16-QAM	3/4	90.00
64-QAM	2/3	120.00
64-QAM	3/4	135.00
64-QAM	5/6	150.00

Achieved throughput values are averaged over 10^4 random realizations of energy harvest profiles generated with the first-order Markov model and these values are divided by the length of transmission time to find average throughput values.

In Fig. 4.3, it appears that even simple schemes such as greedy and constant suffice. However, this performance depends on the dynamics of the energy harvesting process. To illustrate such a case, policies are evaluated under another Markovian energy harvesting process assumption. This time, the time slot interval is taken as 1s, energy harvesting Markov model is assumed to have 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$ to simulate a burst arrival case. Throughput performances for this case (see Fig. 4.4) indicate that the simple schemes are limited to about half the optimal online throughput, while the Expected Threshold Policy closely follows the optimal online throughput.

Although it is proposed in [5] for stationary harvest processes and infinite horizon, throughput optimal (TO) policy is also simulated in our context. The TO policy uses the power decision equation below:

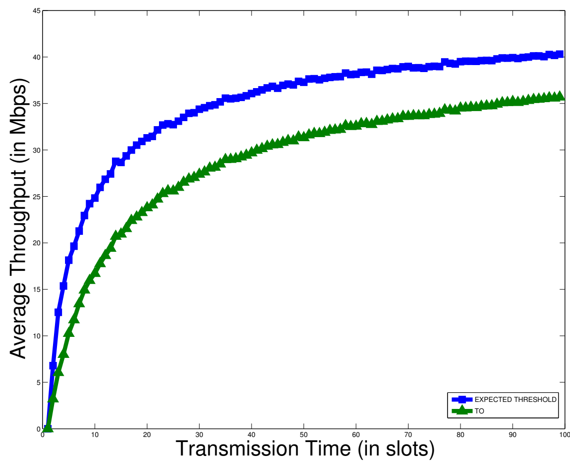
$$\varrho_n^{TO}(e) = \min(e, E[H]) \tag{4.17}$$

Note that since the energy level e and the average energy harvest rate $E[H]$ are arbitrary, usually the power decision $\varrho_n^{TO}(e)$ is not in set U.

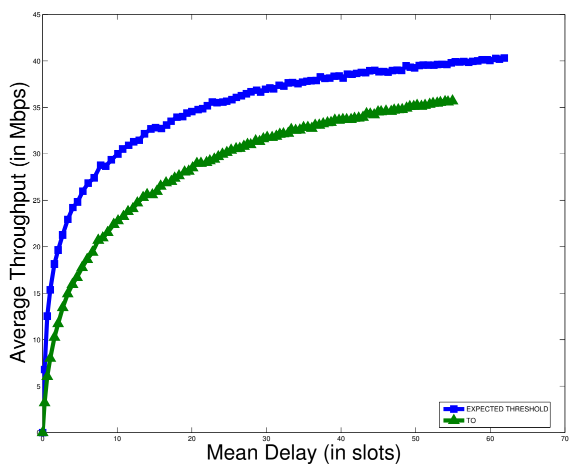
As seen in Fig. 4.2.a, the expected threshold policy performs better than TO policy in terms of average throughput. In addition, the mean delay performance of the expected threshold policy is compared against TO in Fig. 4.2.b although both policies do not consider mean delay as an optimization criterion. Mean delay is computed by time averaging over the delay values seen by each transmitted bit as in the following expression:

$$\text{Mean Delay} = \frac{\sum_{n=1}^{N}(N-n+1)g_r(e_n, \varrho_n)}{\sum_{n=1}^{N} g_r(e_n, \varrho_n)} \tag{4.18}$$

Then, using the same burst arrival Markov model for energy harvesting, the expected water level policy is tested under Rayleigh and Nakagami fading channel assumptions in Fig.4.5 and Fig. 4.6. Rayleigh and Nakagami channels are simulated as discrete channel gain processes with 7 levels ranging from 0.1 to 1.9.



(a)



(b)

Figure 4.2: Average Throughput versus (a) Transmission Time , (b) Mean Delay for Expected Threshold and TO policies assuming burst arrival Markov model which has 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$.

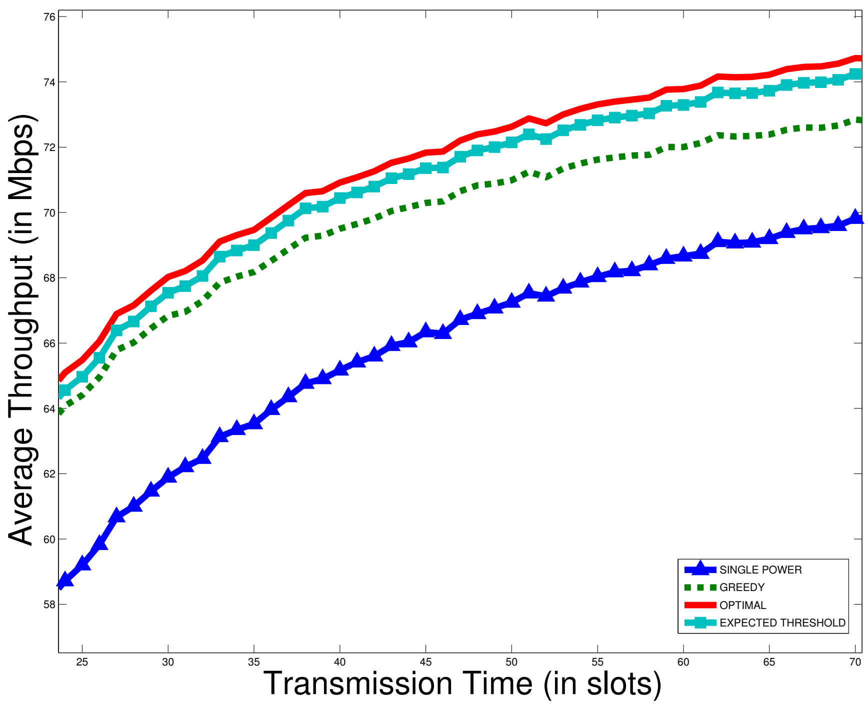


Figure 4.3: Average Throughput versus Transmission Time for single power, greedy, optimal and Expected Threshold policies assuming irradiance trace Markov model derived from an irradiance trace (measured during a car-based roadtrip) which is available in the CRAWDAD repository [1].

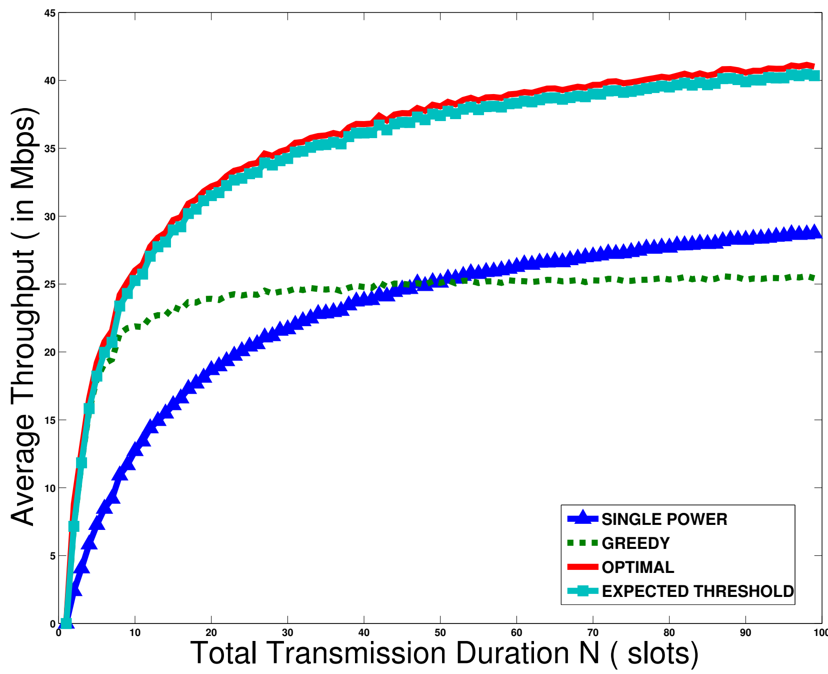


Figure 4.4: Average Throughput versus Total Transmission Duration for single power, greedy, optimal and Expected Threshold policies assuming burst arrival Markov model which has 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$ under static channel.

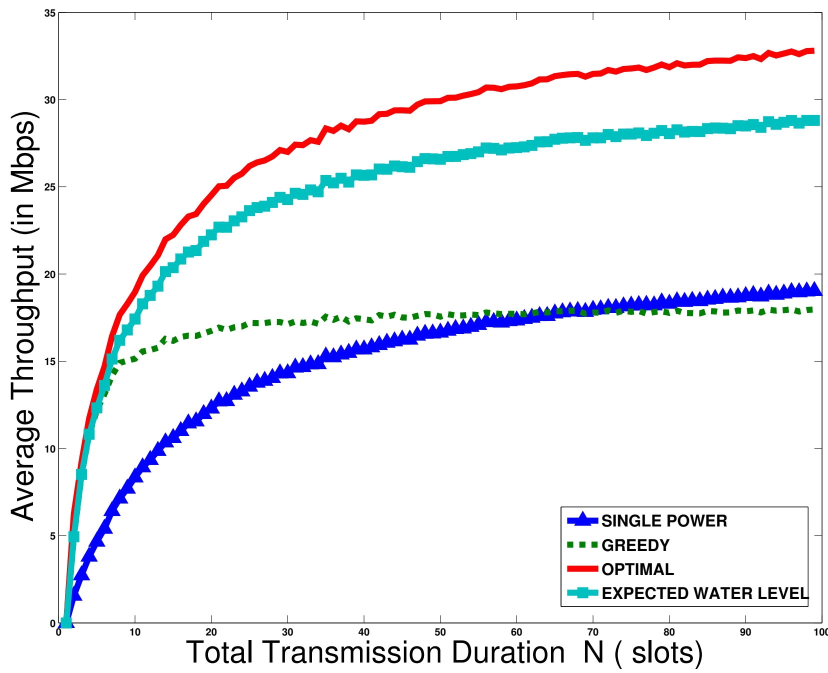


Figure 4.5: Average Throughput versus Total Transmission Duration for single power, greedy, optimal and Expected Threshold policies assuming burst arrival Markov model which has 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$ under Rayleigh fading.

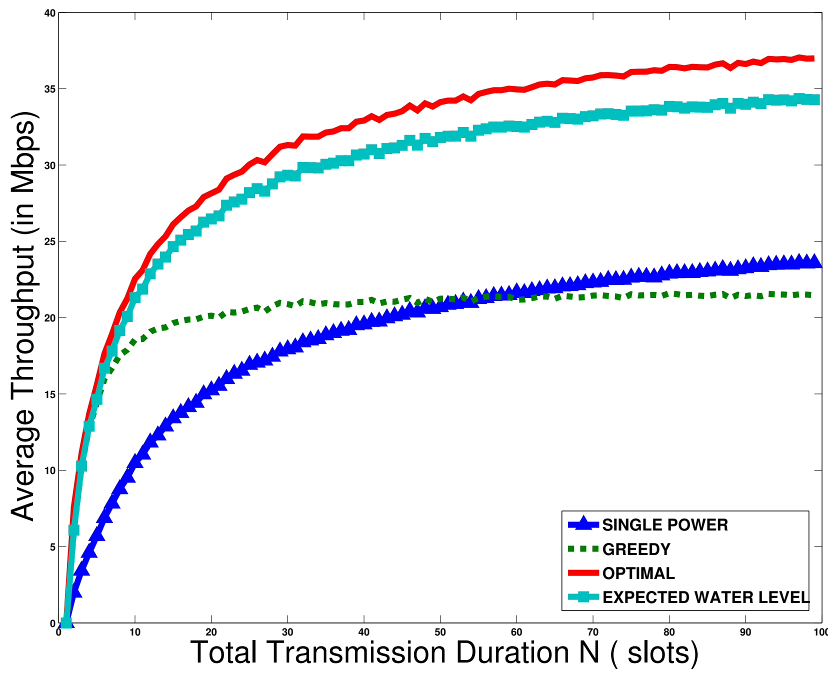


Figure 4.6: Average Throughput versus Total Transmission Duration for single power, greedy, optimal and Expected Threshold policies assuming burst arrival Markov model which has 2 states ($h_0 = 0$ and $h_1 = 256$mJ) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$ under Nakagami fading.

CHAPTER 5

ONLINE LAZY SCHEDULING

Take time for all things: great haste makes great waste.

Benjamin Franklin

5.1 PROBLEM DEFINITION

Consider a network node receiving variable-size packets that need to be sent within a finite time interval. It transmits received packets through a point-to-point link while changing its transmission rate adaptively. The problem to be considered here is to find a transmission policy that minimizes the expected total energy consumption. Let us define a slot duration as the minimum time interval that sender node can modify its transmission rate. We will consider a finite-horizon period of T slots. In other words, the slot T is the last slot that sender node can deliver its packets. However, delivering all received packets within this period may not be guaranteed by any policy. Therefore, a cost in terms of energy should be assumed for backlogging received packets at the end of the slot T.

Let n be the present number of remaining slots to the deadline and b be the present number bits stored in the sender mode's buffer. Sender node consumes $w(b, r)$ per slot time when the transmission rate r (in bits/slot) is selected.

$$w(b, r) = \varepsilon(r) \min\left(\frac{b}{r}, 1\right) \tag{5.1}$$

where $\varepsilon(r)$ is a convex function increasing in r.

Assume that the packet arrival process is a Discrete time Markov chain (DTMC) which is known beforehand. Let i be the packet arrival state and $l(i)$ be the function that returns the corresponding packet lenght when the packet arrival state is i. Then, the present state of the system can be determined by the vector $(b(n), i(n))$ when there are n slots to the deadline. It also is assumed that the sender's buffer has an infinite capacity, meaning that can store all packets received during a period of T slots.

Given that the system state is (b, i), let $J_n(b, i)$ be the minimum expected total energy constumption for the next n slots. Then, the problem can be formulated using a stochastic dynamic programming equation as below.

$$J_n(b, i) = \min_r \left[w(b, r) + \sum_j A_{ij} J_{n-1}\left((b-r)_+ + l(j), j\right) \right] \tag{5.2}$$

where A_{ij} is the transition probability between packet arrival states i and j.

The optimal solution can be identified by minimizing rates r_n^*'s :

$$r_n^* = \arg\min_r \left[w(b, r) + \sum_j A_{ij} J_{n-1}\left((b-r)_+ + l(j), j\right) \right] \tag{5.3}$$

The cost function $J_n(b, i)$ and minimizing rates r_n^*'s can be computed by backward induction starting from the last slot. The function $J_0(b, i)$ can be interpreted as a penalty function which corresponds to the cost of maintaining bits of packets not delivered until the deadline. Let $J_0(b, i)$ be defined as below.

$$J_0(b, i) = C\left(b - l(i)\right) \tag{5.4}$$

where $C(b)$ is monotone nondecreasing cost function of b and equals to zero at $b = 0$.

In practical systems, selectable transmission rates are limited and, to model this restriction, transmission rates r_n's can be assumed to be chosen from a discrete set $\mathbf{V}$.

The dynamic programming formulation of the problem in Eq. 5.2, already provides an optimal solution using backward induction. Yet, this solution has a high computational complexity which is exponential over the number of states. Moreover, practically, if the optimal solution is computed by the transmitter node, then the extra energy required for computation may exceed the energy saving of the optimal solution. Accordingly, it would be useful to focus on simpler but efficient transmission policies.

5.2 SUBOPTIMAL SOLUTIONS

This section describes some possible suboptimal solutions that require relatively less computational power compared to the optimal solution by dynamic programming.

5.2.1 Laziest Scheduling Policy

A naive transmission policy, *laziest scheduling policy*, which does not change transmission rate, is to transmit always with the lowest transmission rate independently from the sysytem state (b, i). In other words, this policy sets the transmission rate to the minimum transmission rate r_{min} in the set $\mathbf{V}$.

$$r_{min} = \min\{r \in \mathbf{V}\} \tag{5.5}$$

It can be expected that laziest scheduling policy becomes closer to the optimal policy as the cost function $C(b)$ gets closer to zero. Because, for a bounded and small cost function $C(b)$, the optimal policy tends to transmit with lower transmission rates in order to minimize total energy consumption while ignoring backlogged data.

5.2.2 Hastiest Scheduling Policy

The opposite of laziest scheduling is selecting always the highest possible transmission rate r_{max} in the set $\mathbf{V}$. Let us name this policy as *hastiest scheduling policy*.

$$r_{max} = \max\{r \in \mathbf{V}\} \tag{5.6}$$

Hastiest scheduling policy is likely to perform better if the cost function $C(b)$ is steeply increasing in b. On the other hand, the optimality of this policy is arguable since some other policies that can transmit all received packets within the transmission period may consume less energy by following optimality properties.

5.2.3 Hasty Scheduling Policy

A simple rate adapting policy can be a modification on hastiest scheduling. If the amount of backlogged data is smaller than the amount of data which can be transmitted within one slot duration with the transmission rate r_{max}, then transmitting with r_{max} is inefficient in terms of energy since a lower transmission rate requires less energy for transmitting the same amount of data. Accordingly, hastiest policy can be modified into a rate adapting policy, *hastiest scheduling policy*, which controls transmission rate $r_{hasty}(b)$ to be less than b.

$$r_{hasty}(b) = \max\{r \in \mathbf{V} | r < b\} \tag{5.7}$$

5.2.4 Expected Threshold Lazy Scheduling Policy

The heuristic approach in Chapter 4 can be applied to the online lazy scheduling problem. To do this, let us first reconsider the offline solution where transmission rates are not restricted to a discrete set.

The stretched string method described in Chapter 3 can be employed to find optimal offline transmission rates since optimal departure curve still follows the shortest path. But, depending on the cost function $C(b)$, the optimal offline solution does not have to be the one that transmits all recieved bits until the end of transmission period. Instead, the optimal offline solution may allow some part of the received information remain in exchange for minimizing total energy consumption. Thus, transmission period virtually can be assumed to be extended to the point where data buffer is emptied with the transmission rate selected at the last time slot. Accordingly, the optimal offline rate $\tilde{r}_n^*$ can be expressed as in below:

$$\tilde{r}_n^* = \min_{a=1,\ldots,(n-1)} (b_n, \tilde{r}_n(b_n, a)), \text{ where}$$

$$\tilde{r}_n(b_n, a) = \frac{b_n + \sum_{l=a}^{n-1} B_l}{n + \alpha - a + 1}$$

where B_n represents the size of the packet arriving at the slot n and α is a paramater that depends on the cost function $C(b)$ and determines the extended time.

It can be noted the parameter α goes to zero as the cost function $C(b)$ goes to infinity for any value of b.

As it is done in Chapter 4 for ET policy, online lazy scheduling transmission rates can be based on the expectation values of transmission rates obtained for offline solutions. Again, for the sake of simplicity, $E[\tilde{r}_n^*]$ can be approximated and the following expression for online decisions can be derived:

$$r_n = \max\left\{\rho \in \mathbf{V} | \rho \leq \min(b_n, E\left[\tilde{r}_n(b_n, 1) | B_n^T\right]\right\} \tag{5.8}$$

Or alternatively:

$$\boxed{\begin{aligned} r_n &= \max\left\{\rho \in \mathbf{V} | L_n(B_n^T, r) \leq b_n\right\} \\ L_n(B_n^T, r) &= \max\left(r, rn + r\alpha - \sum_{l=1}^{n-1} E\left[B_l | B_n^T\right]\right) \\ \text{for } r \neq r_{min} \quad &; \quad L_n(H_n^N, r_{min}) = 0 \end{aligned}}$$

where B_n^T is the vector of packet sizes $[B_n ... B_T]$ and $L_n(B_n^T, r)$ is the minimum data buffer level for selecting transmission rate r.

5.3 EVALUATION

A simulation experiment is performed to evaluate energy efficiencies of laziest, hastiest, hasty scheduling policies and expected threshold lazy scheduling (ETLS) policy against optimal policy using dynamic programming. For packet arrival process, a stream of 10kB packets is assumed and modeled by a Markov model having two states ($l(0) = 0$ and $l(1) = 10$kB) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$ where slot duration is 1ms. The same set of transmission rate and corresponding power levels used in the evaluation part of Chapter 4 (Section4.6) is assumed.

The cost function $C(b)$ is chosen as $w(b)$ and parameter α of ETLS policy is set to 0. For this setting, average power requirements of optimal and suboptimal scheduling policies are obtained and the comparison over varying total transmission duration T is shown in Figure 5.1. Due to the cost function, laziest scheduling policy, which backlogs large amount of data, has a much higher energy consumption compared to hastiest policy and it is excluded in Figure 5.1.

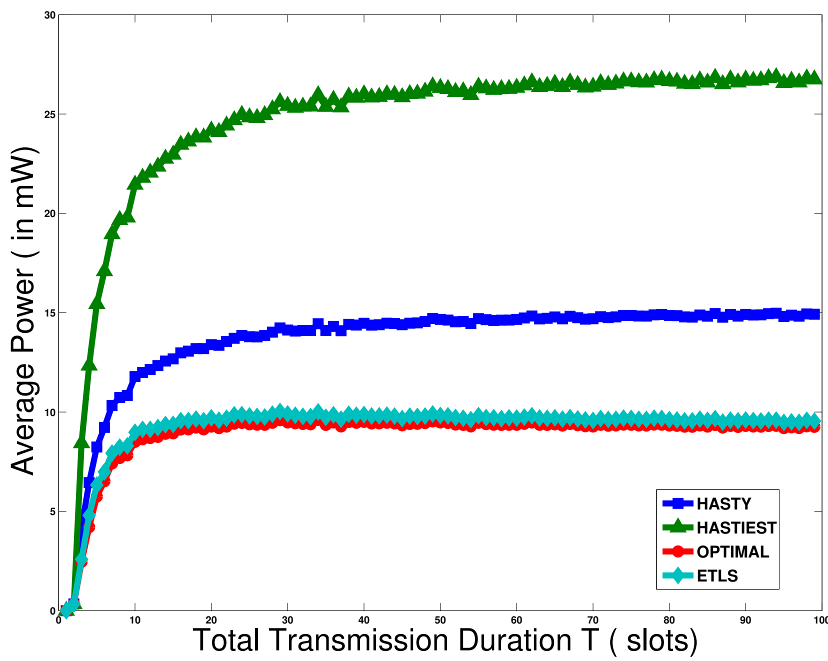


Figure 5.1: Average Power versus Total Transmission Duration for single hasty, hastiest, optimal and ETLS assuming Markov model having two states ($l(0) = 0$ and $l(1) = 10$kB) with transition probabilities $q_{00} = 0.9$, $q_{01} = 0.1$, $q_{10} = 0.5$, $q_{11} = 0.5$.

CHAPTER 6

CONCLUSIONS

> It is not the strongest of the species that survives, nor the most intelligent that survives. It is the one that is the most adaptable to change.
>
> Charles Darwin

In this thesis, a finite-horizon online throughput-maximizing scheduling problem with a discrete set of transmission actions has been formulated. The structure of the optimal solution of this problem has been studied through stochastic dynamic programming. Based on the observation of a threshold structure in the optimal policy, a low complexity heuristic solution, Expected Threshold Policy, has been proposed. The optimal and heuristic solutions are extended taking into account time-varying channels and these more general solutions are evaluated under ergodic fading. The Expected Water Level Policy, which is the proposed heuristic for the fading case, as well as the Expected Threshold Policy in the static channel case, were both observed to achieve close to optimal throughput in detailed numerical studies, significantly outperforming simple policies such as using a constant rate, or greedily spending the energy at hand. Moreover, as expected, the gap between the simple policies and the heuristic proposal widens as the energy harvesting process diverges from stationarity.

The comparison of the Expected Threshold (ET) Policy with the TO policy of [5] is particularly interesting. The TO policy is throughput-optimal in the infinite-horizon case for stationary energy harvest processes. The simulation results indicate that the Expected Threshold Policy provides higher throughput than the TO policy for any given mean delay value. The ET policy has a better average throughput performance against transmission time especially for short horizon lengths. These observations indicate that an expected threshold computation, while having a much lower complexity than computing the optimal dynamic programming solution, reaps strong benefits in terms of performance and thus such adaptation seems to be worth undertaking for dynamic energy harvesting processes in short time scales, as opposed to a time-invariant policy. Furthermore, in the considered problem, it has been seen that the performace depence on present energy level is weaker and less critical than the performance dependence on time and energy harvesting process.

This opens up an array of questions about the performance difference between stationary and time-varying policies in relation to the statistics of the energy arrival process. It is quite reasonable to believe that the design of low complexity policies that can exhibit close to optimal performance for bounded delay will be informed by and benefit from such analysis. this view of the problem should be generalized to attempt this analysis in future work.

Likewise, a finite-horizon online scheduling problem has been also formulated with this understanding. For this problem setting, an adaptation of the ET policy, the ETLS policy, has been introduced. By a simulation experiment, it has been shown that the ETLS policy can achieve close-to-optimal performance as the ET policy. One distinction from the ET policy is that the ETLS policy has an additional parameter for adjusting according to the cost of violating the transmission deadline. The relationship between this parameter and the cost function has not been investigated and hence it has been left as a future work.

The insight gained in this study can be a basis for a more generic consideration of finite-horizon online scheduling problems. Together with this, the comparison of finite-horizon scheduling solutions against infinite-horizon scheduling solutions might produce substantial results that contribute both theoretical and practical treatment of related online scheduling problems.

REFERENCES

[1] M. Gorlatova, M. Zapas, E. Xu, M. Bahlke, I. J. Kymissis, and G. Zussman, “CRAWDAD data set columbia/enhants (v. 2011-04-07).” http://crawdad.cs.dartmouth.edu/columbia/enhants, Apr. 2011.

[2] Chang, Tassiulas, J. h. Chang, and L. Tassiulas, “Routing for maximum system lifetime in wireless ad-hoc networks,” in *in 37-th Annual Allerton Conference on Communication, Control, and Computing*, 1999.

[3] B. Prabhakar, E. Uysal Biyikoglu, and A. El Gamal, “Energy-efficient transmission over a wireless link via lazy packet scheduling,” in *INFOCOM 2001. Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 1, pp. 386 –394 vol.1, Apr. 2001.

[4] A. Fu, E. Modiano, and J. Tsitsiklis, “’optimal energy allocation and admission control for communications satellites,” *Networking, IEEE/ACM Transactions on*, vol. 11, pp. 488 – 500, june 2003.

[5] V. Sharma, U. Mukherji, V. Joseph, and S. Gupta, “Optimal energy management policies for energy harvesting sensor nodes,” *Wireless Communications, IEEE Transactions on*, vol. 9, pp. 1326 –1336, April 2010.

[6] J. Yang and S. Ulukus, “Transmission completion time minimization in an energy harvesting system,” in *Information Sciences and Systems (CISS), 2010 44th Annual Conference on*, pp. 1 –6, March 2010.

[7] C. K. Ho and R. Zhang, “Optimal energy allocation for wireless communications powered by energy harvesters,” in *Proc. IEEE International Symposium on Information Theory (ISIT)*, pp. 2368 –2372, June 2010.

[8] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, “Adaptive transmission policies for energy harvesting wireless nodes in fading channels,” in *Information Sciences and Systems (CISS), 2011 45th Annual Conference on*, pp. 1–6, March 2011.

[9] M. Kashef and A. Ephremides, “Optimal scheduling for energy harvesting sources on time varying wireless channels,” in *Communication, Control, and Computing (Allerton)*, pp. 712 –718, Sept. 2011.

[10] B. Medepally, N. Mehta, and C. Murthy, “Implications of energy profile and storage on energy harvesting sensor link performance,” in *Global Telecommunications Conference, 2009. GLOBECOM 2009. IEEE*, pp. 1 –6, 30 2009-dec. 4 2009.

[11] B. Devillers and D. Gündüz, “A general framework for the optimization of energy harvesting communication systems with battery imperfections,” *Journal of Communications and Networks*, vol. 14, no. 2, pp. 130–139, 2012.

[12] M. Kashef and A. Ephremides, "Optimal packet scheduling for energy harvesting sources on time varying wireless channels," *Journal of Communications and Networks*, vol. 14, no. 2, pp. 121–129, 2012.

[13] A. Kansal, J. Hsu, S. Zahedi, and M. B. Srivastava, "Power management in energy harvesting sensor networks," *ACM Trans. Embed. Comput. Syst.*, vol. 6, Sept. 2007.

[14] J. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *Pervasive Computing, IEEE*, vol. 4, pp. 18 – 27, jan.-march 2005.

[15] P. Lee, Z. A. Eu, M. Han, and H.-P. Tan, "Empirical modeling of a solar-powered energy harvesting wireless sensor node for time-slotted operation," in *Wireless Communications and Networking Conference (WCNC), 2011 IEEE*, pp. 179–184, March 2011.

[16] M. Gorlatova, A. Wallwater, and G. Zussman, "Networking low-power energy harvesting devices: Measurements and algorithms," in *INFOCOM, 2011 Proceedings IEEE*, pp. 1602 –1610, April 2011.

[17] C. K. Ho, P. D. Khoa, and P. C. Ming, "Markovian models for harvested energy in wireless communications," in *Communication Systems (ICCS), 2010 IEEE International Conference on*, pp. 311–315, Nov. 2010.

[18] E. Uysal-Biyikoglu and A. El Gamal, "On adaptive transmission for energy efficiency in wireless data networks," *Information Theory, IEEE Transactions on*, vol. 50, pp. 3081 –3094, dec. 2004.

[19] M. A. Zafer and E. Modiano, "A calculus approach to energy-efficient data transmission with quality-of-service constraints," *IEEE/ACM Trans. Netw.*, vol. 17, pp. 898–911, June 2009.

[20] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *Selected Areas in Communications, IEEE Journal on*, vol. 29, pp. 1732 –1743, september 2011.

[21] M. Antepli, E. Uysal-Biyikoglu, and H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," *Selected Areas in Communications, IEEE Journal on*, vol. 29, pp. 1721 –1731, September 2011.

[22] M. Zafer and E. Modiano, "How to utilize a stretched string: Wireless data transmission with deadlines," in *IEEE COMSOC MMTC E-Letter*, vol. 5, pp. 11 –13, November 2010.

[23] R. Berry, E. Modiano, and M. Zafer, *Energy-Efficient Scheduling under Delay Constraints for Wireless Networks*. Synthesis Lectures on Communication Networks, Morgan & Claypool Publishers, 2012.

APPENDIX A

APPENDIX HEADING

A.1 Computing the expected water level $E[\tilde{w}_n(e_n)]$

Water levels $\tilde{w}_n$ are only constrainted by energy harvesting process and energy constraints can be written as:

$$\sum_{k=a}^{n}(\tilde{w}_k - \frac{1}{\gamma_k})_+ \leq e_n + \sum_{k=a}^{n-1} H_k \text{ where } a \in [1, n-1] \text{ and } \tilde{w}_n \leq e_n + \tfrac{1}{\gamma_n}$$

Since $\tilde{w}_n \leq \tilde{w}_{n-1}$,

$$\sum_{k=a}^{n}(\tilde{w}_n - \frac{1}{\gamma_k})_+ \leq e_n + \sum_{k=a}^{n-1} H_k \Rightarrow \sum_{k=a}^{n}(\tilde{w}_n - \frac{1}{\gamma_k}) \leq e_n + \sum_{k=a}^{n-1} H_k - \sum_{k=a}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+$$

$$\Rightarrow (n-a+1)\tilde{w}_n - \frac{1}{\gamma_n} - \sum_{k=a}^{n}\frac{1}{\gamma_k} \leq e_n + \sum_{k=a}^{n-1} H_k - \sum_{k=a}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+$$

$$\Rightarrow \tilde{w}_n \leq \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=a}^{n-1}(H_k + \frac{1}{\gamma_k}) - \sum_{k=a}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+}{n-a+1} \text{ for } a \in [1, n-1] \text{ and } \tilde{w}_n \leq e_n + \tfrac{1}{\gamma_n}$$

$$\Rightarrow \tilde{w}_n \leq \min_{a \in [1,n]} \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=a}^{n-1}(H_k + \frac{1}{\gamma_k}) - \sum_{k=a}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+}{n-a+1}$$

The inequality above is an equivalent to all constraints on $\tilde{w}_n$ and power level $\tilde{\rho}_n^*$ can be independently maximized by maximizing $\tilde{w}_n$, hence $\tilde{w}_n$ can be the maximum value which equals to the right-hand side:

$$\tilde{w}_n = \min_{a \in [1,n]} \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=a}^{n-1}(H_k + \frac{1}{\gamma_k}) - \sum_{k=a}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+}{n - a + 1}$$

The above expression can be thought as a minimization of a running average where the terms e_n and $\frac{1}{\gamma_n}$ are independent from the index a. Hence, the average is usually minimized when $a = 1$ and $\tilde{w}_n$ can be approximated as in the following equation:

$$\tilde{w}_n \simeq \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=1}^{n-1}(H_k + \frac{1}{\gamma_k}) - \sum_{k=1}^{n}(\frac{1}{\gamma_k} - \tilde{w}_n)_+}{n}$$

Then, the following approximation for the expected water level $E[\tilde{w}_n(e_n)]$ can be used:

$$E[\tilde{w}_n(e_n)] \simeq \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=1}^{n-1}(E[H_k] + E[\frac{1}{\gamma_k}]) - \sum_{k=1}^{n} E[(\frac{1}{\gamma_k} - \tilde{w}_n(e_n))_+]}{n}$$

Assuming $E[(\frac{1}{\gamma_k} - \tilde{w}_n)_+] \simeq (E[\frac{1}{\gamma_k}] - E[\tilde{w}_n])_+$, a further simplification can be made:

$$E[\tilde{w}_n(e_n)] \simeq \frac{e_n + \frac{1}{\gamma_n} + \sum_{k=1}^{n-1}(E[H_k] + E[\frac{1}{\gamma_k}]) - \sum_{k=1}^{n}(E[\frac{1}{\gamma_k}] - E[\tilde{w}_n(e_n)])_+}{n}$$