AN INVESTIGATION OF PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS' PROBABILISTIC MISCONCEPTIONS AND REASONS UNDERLYING THESE MISCONCEPTIONS

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ABSTRACT

AN INVESTIGATION OF PROSPECTIVE ELEMENTARY MATHEMATICS TEACHERS' PROBABILISTIC MISCONCEPTIONS AND REASONS UNDERLYING THESE MISCONCEPTIONS

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The purpose of this study was to determine performance of prospective elementary mathematics teachers on answering the items handling the probabilistic misconceptions. The other aim was to investigate the underlying reasons behind these misconceptions of prospective elementary mathematics teachers. To address these aims, qualitative approach was performed.

The sample of this study was obtained through convenience sampling. Data were gathered during 2011-2012 spring semester by administering Probability Misconception Questionnaire to 12 senior prospective elementary mathematics teachers studying at faculty of education in Sakarya and through semi-structured interviews conducted with those prospective teachers.

None of the participants provided correct answers to items addressing misconceptions regarding time axis fallacy and compound event. Furthermore, less than half of the participants provide the correct answer to items handling misconceptions regarding conditional probability, effect of sample size, conjunction fallacy and representativeness.

Also, in this study, reasons behind those misconceptions were determined. Particularly, focusing on the first event was found to be a reason underlying time axis fallacy misconception. Also, another reason behind this misconception was misinterpretation of the problem, which also resulted in misconception regarding conditional probability. Furthermore, focusing on the ratio was found to be a reason underlying misconception regarding effect of sample size. Several participants solely focused on the narrative, which lead to misconception regarding conjunction fallacy. Moreover, seeking representativeness in samples was found to be a reason underlying misconception regarding representativeness. Lastly, in this study, it was found that ignoring order of outcomes resulted in misconception regarding compound event.

Keywords: Probabilistic misconception, prospective elementary mathematics teachers

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ OLASILIK İLE İLGİLİ KAVRAM YANILGILARI VE BU YANILGILARIN TEMELİNDE YATAN NEDENLERİN İNCELENMESİ

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Bu çalışmanın amacı, ilköğretim matematik öğretmen adaylarının, olasılık kavram yanılgılarını elen alan soruları cevaplamada performanslarının nasıl olduğunu belirlemektir. Bir diğer amaç ise, öğretmen adaylarının bu kavram yanılgılarının altında yatan nedenlerini incelemektir. Bu nedenle, bu çalışmada, nicel araştırma yöntemi kullanılmıştır.
Araştırmanın örneklemi elverişli örnekleme yoluyla elde edilmiştir. Veriler, 2011-2012 öğretim yılı bahar döneminde Sakarya ilindeki üniversitenin eğitim fakültesinden seçilen son sınıf 12 öğretmen adayına 'Olasılık Kavram Yanılgısı Testi' uygulanarak ve bu öğrenciler ile yarı yapılandırılmış görüşme yapılarak elde edilmiştir.

Adayların hiçbiri zaman çizelgesi yanılgısı ve bileşik olasılık ile ilgili kavram yanılgısını ele alan sorulara doğru yanıt verememişlerdir. Ek olarak, adayların yarısından azı koşullu olasılık, örnek uzayın etkisi, çakışma yanılgısı ve temsil kısa yolu yanılgısı ile ilgili kavram yanılgılarını ele alan sorulara doğru cevap vermişlerdir.

Ek olarak, bu çalışmada, bu kavram yanılgılarının altında yatan nedenler belirlenmiştir. İlk olaya odaklanmak zaman çizelgesi kavram yanılgısının altında yatan neden olarak bulundu. Problemin yanlış yorumlanması bu kavram yanılgısına neden olduğu gibi koşullu olasılık ile ilgili kavram yanılgısına da neden olmaktadır. Bunlara ek olarak, orana odaklanmanın, örnek uzatın etkisi ile ilgili kavram yanılgısına neden olduğu bulunmuştur. Bazı adaylar hikâyeye odaklanmış ve bu da çakışma yanılgısı ile ilgili kavram yanılgısına neden olmuştur. Ayriyeten, örneklemlerde temsil edilebilirliği aramanın temsil kısa yolu ile ilgili kavram yanılgısına neden olduğu bulunmuştur. Son olarak, bu çalışmada, çıktıların sırasını göz ardı etmenin, bileşik olasılık ile ilgili kavram yanılgısına neden olduğu bulunmuştur.

Anahtar Kelimeler: Olasılık kavram yanılgısı, ilköğretim matematik öğretmen adayı

To My Parents

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LIST OF ABBREVIATIONS

f	Frequency
HEC	Higher Education Council
MoNE	Ministry of National Education
NCTM	National Council of Teachers of Mathematics
PMQ	Probability Misconception Questionnaire

CHAPTER I

INTRODUCTION

“Probability is an unavoidable aspect of modern life. We are constantly required to make decisions in uncertain situations and-whether we are conscious of it or not-these decisions necessarily involve the concepts of randomness and probability” (Metz, 1998b, p.286).

As stated by Metz (1998b), in daily life there are many circumstances in which probabilistic thinking needs to be resorted to. Interpretation of probabilistic statements and decision-making are one of these circumstances (Gal, 2005). Shaughnessy (1992) stated that “…perhaps no other branch of mathematical sciences is as important for all students, college bound or not, as probability and statistics" (p.244). Undoubtedly, such an important topic lies at the heart of statistics and mathematics. The National Council of Teachers of Mathematics (2000) also included “Data Analysis and Probability” as one of the five content standards in elementary mathematics education program. The importance of probability topic results from the extensive usage areas of it in daily life, its connection to other disciplines and its role in the development of the critical reasoning (Batanero, Godino & Roa, 2004).

Probability is somewhat different from the other mathematical topics. It does not include just pure technical information and simple procedures which lead to solutions. New intuitions must also be created to learn it (Konold, Pollatsek, Well, Lohmeir & Lipson, 1993). Students attend class with previous experiences in beliefs regarding probability, which is not true for the other topics in mathematics (Falk, 1989; Tversky & Kahneman, 1980). If these beliefs and intuitions are in harmony with the nature of the probability theory, the students can develop their skills in probability. If not, these beliefs might result in misconceptions. In other words, students’ existing ideas about the nature of the world can conflict with the ideas behind probability. Moreover, efforts to simplify teaching probability may make it superficial. The important points in probability may be overlooked. More specifically, students might not notice the complex nature of probability, which may leads them to engage in false reasoning in probability (Tversky & Kahneman, 1980).

3.1 Statement of the Problem

The mastery of the fundamentals of probability is essential for students since probability plays an important role in the decision making skill under uncertainty, and it develops the critical thinking skill which is one of the aims of the mathematics programs. However, students seem to have difficulties in developing these skills and intuitions regarding the concepts of probability (Fischbein, Nello & Marino 1991; Garfield & Ahlgen, 1988).

Even after instruction, students can misinterpret basic ideas of probability (Shaughnessy, 1992). This may be attributed to teachers' misinterpretation since teachers transfer what they know (Even, 1990). Teachers might hold misconceptions regarding probability since this concept is found challenging by not only students but also teachers themselves. More precisely, when compared to other mathematics topic, the topic of probability is more theoretical and abstract. Thus, teachers are less comfortable with probability. This also derives from their own lack of training and experience (Jendraszek, 2008). As stated by Bulut (1994) and Stohl (2005), teachers are not sure about their abilities concerning the teaching of probability. It could be speculated that the reason for this situation is the fact that teacher preparation programs do not include how to teach probability efficiently (Shaughnessy, 1992). The need to train teachers in teaching probability is clearly articulated by Shaughnessy as follows: “We will need to develop courses which meet stochastic (probability and statistics) misconceptions and beliefs head on, and sensitize our prospective teachers to the prevalent misconceptions they can expect to encounter in their own students” (p.481).

Developing courses is not an easy task, which can be accomplished by learning about understanding of the probability and misunderstandings that prospective teachers have is required (Dollard, 2007). Hence, this study aims to investigate prospective elementary teachers' probabilistic misconceptions and thereby to provide reasonable data on this issue, which will provide insight for curriculum developers and teacher educators. To be more precise, this study aims to address the following questions:

1. How do prospective elementary mathematics teachers perform on answering items addressing probabilistic misconceptions?

    1.1 To what extent are prospective elementary mathematics teachers successful at answering items addressing *time axis fallacy, conditional probability, effect of sample size, conjunction fallacy, representativeness,* and *compound event*?

2. What are the underlying reasons of prospective elementary mathematics teachers' probabilistic misconceptions?

    2.1 What are the underlying reasons of prospective elementary mathematics teachers' probabilistic misconception of *time axis fallacy, conditional probability, effect of sample size, conjunction fallacy, representativeness, compound event* ?

3.2 Definitions of Important Terms

Probability: It refers to a measure of the likelihood of an event occurring (Van De Walle, 2007, p.479). In the current study, probability was used as a domain.

Prospective elementary mathematics teachers: They refer to teacher candidates who are students of education in university and will teach mathematics in middle school (from grade 5 to 8) after graduation. In this study, prospective teachers refer to participants who were the senior students enrolled in elementary mathematics education program in university in Sakarya.

Misconception: It refers to the erroneous concepts that students hold, leading to production of systematic pattern of errors (Smith, diSessa & Rochelle, 1993). In the present study, the probabilistic misconceptions which were encountered in the available literature were addressed to determine whether these were held by the prospective elementary mathematics teachers and what the reasons underlying these misconceptions were.

Time axis fallacy misconception: It refers to the belief that knowledge of the later event's outcome cannot be used to determine the probability of the occurrence of a previous event (Falk, 1989). In this study, the misconceptions regarding time axis fallacy were measured by means of Probability Misconception Questionnaire (PMQ).

Conditional probability misconception: It refers to the tendency to neglect the condition (Falk, 1986). In this study, the misconception regarding conditional probability was measured by means of PMQ.

Effect of the Sample Size Misconception: It refers to the belief that the magnitude of a sample cannot be used to estimate the probability of an event in this sample (Rubel, 2002). In this study, the misconceptions regarding effect of the sample size were measured by PMQ.

Conjunction Fallacy Misconception: It refers to belief that the probability of the conjunction of two events is greater than the probability of either one of its constituents, that is, erroneously assigning high probabilities to two distinct events occurring simultaneously and low probabilities to either one occurring separately (Tversky& Kahneman, 1982c; Tversky & Kahneman, 1983). In this study, this misconception of conjunction fallacy was measured by PMQ.

Representativeness Misconception: It refers to the belief that the likelihood of an event in the sample depends on how well the sample represents some aspect of its parent population (Shaughnessy, 1977; Tversky & Kahneman, 1974). In this study, misconceptions about representativeness were addressed by means of PMQ.

Compound Event Misconception: It refers to a tendency to neglect one of the outcomes of the compound event (Kustos, 2010*)*. In the present study, the misconceptions regarding compound event were measured by PMQ.

3.3 Significance of the Study

Probability has been regarded as an important part of mainstream elementary mathematics curriculum recently (Jones, 2005). The significance of the probability subject results mainly from its connections to other topics in the elementary curriculum**.** In particular, probabilistic thinking is related to proportional thinking (Lamon, 1999). Additionally, probability can be used to help elementary school students develop skills related to fraction and proportional reasoning. More precisely, since probability is the ratio of the desired outcomes to total possible outcomes, it involves part-whole relationship, which is one of the characteristics of proportional reasoning (Metz, 1998b).

On the other hand, probability is not an easy topic; has a troublesome nature. As mentioned by Konold (1991), “Probability is a particularly slippery concept...It is trying to keep one's footing in this nowhere land that is particularly disturbing. Like a frictionless surface, the conceptual landscape not only trips you up, but keeps you sliding once you're down "(p.139).

As Konold (1991) emphasized, probability could be considered as one of the difficult concepts in mathematics because of its 'slippery' nature. As a result of this, students develop a great number of misconceptions when they are confronted with this subject in school (Garfield & Ahlgen, 1998; Li, 2000). One of the reasons for these difficulties and misconceptions is that students attend class with preshaped intuitive biases which interfere with making reasonable probability judgment. Another reason why these misconceptions and mistakes might be occurring can be attributed to the teachers who are not prepared and proficient enough to teach probability (Batanero, et al., 2004) since teachers transfer what they know to their students (Even, 1990). In support of these views, Paparieonymou (2008) pointed out: “Many teachers have not studied probability in their own elementary school mathematics courses and sometimes need convincing as to why they need to learn and teach probability concept" (p.2).

It seems that there is consensus in the literature that teachers' competence in teaching probability is important. The competences of the prospective elementary mathematics teachers to teach the probability was found important as well (Watson & Moritz, 2010). To state more explicitly, the prospective elementary mathematics teachers will be in-service teachers in years. Thus, if they have such misconceptions, then they will probably continue to develop these misconceptions when they become in-service teachers. What's worse is that these misconceptions will eventually affect their students (Carnell, 1997). Hence, in order to impede this, it is important to investigate the prospective elementary mathematics teachers' misconceptions regarding probability and to understand the reasons behind those misconceptions.

Moreover, many research studies have been carried out related to misconceptions regarding probability held by students (Dereli, 2009; Kennis, 2006; Khazonov, 2005; Kustos, 2010; Mut, 2006; Rubel, 2002) and those held by teacher (Carnell, 1997; Liu, 2005); however, there is still limited research studies handling those of prospective elementary mathematics teachers (Dollard, 2007; Jendrazsek, 2008; Ozaytabak, 2004). As for studies in Turkey, studies examining probabilistic misconceptions held by prospective elementary mathematics teachers remain insufficient in the available literature. Therefore, prospective elementary mathematics teachers' probabilistic misconceptions were examined in this study.

3.4 Organization of the Study

In this chapter, the statement of the problem, research questions, definitions of important terms, and significance of the study has been explained. The second chapter, the literature review, aims to address different approaches about probability (classical, frequentist and subjective approach) and related studies on probabilistic misconceptions. The third chapter describes the method employed in the study, the participants, development of the Probability Misconception Questionnaire (PMQ), administrations and results of the pilot test, administration of PMQ, procedures of analysis, interview procedures and reliability and validity issues, assumptions and limitations. The fourth chapter reveals the findings of prospective elementary mathematics teachers' performance on the PMQ and the analysis of the reasons underlying misconceptions regarding probability. The last chapter presents the discussion and implications and provides recommendations for further research studies.

CHAPTER II

LITERATURE REVIEW

Investigating performance of prospective elementary mathematics teachers on answering the items addressing the probabilistic misconceptions was one of the purposes of this study. Exploring the underlying reasons of those misconceptions held by prospective elementary mathematics teachers was the other aim of the current study. In accordance with these aims, this chapter includes approaches on probability and probabilistic misconceptions with related studies. Finally, this chapter ends with a summary of the chapter.

4.1 Basic Approaches on Probability

It is useful to examine the different approaches about probability concept to understand its meaning. These views have been discussed by many philosophers, logicians and mathematicians. Jendraszek (2008) stated that the classical view (theoretical approach), the frequentist view (experimental approach), and the subjectivist view (belief driven approach) are the most commonly discussed views of probability concerning mathematics education.

4.1.1 Classical Approach on Probability

Classical approach is also called as theoretical approach. The theoretical probability concept has its roots in the analyses of chance games carried out by Pascal and Fermat. The analysis depends on describing likely outcomes of an event and calculating the ratio of desired outcomes to total outcomes. However, the classical approach has a shortcoming since it can only be applied in conditions where outcomes are equally likely (Batanero, Henry & Parzysz, 2005). To state specifically, classical approach is applied in classes where random chance devices like spinners and dice are used. However, Franklin (2001) and Gal (2005) claimed that these are not a good model for most cases of reasoning under uncertainty since events taking place in the real world are usually a mixture of random and non-random influences and therefore dices and spinners might not be useful for such non-random events.

4.1.2 Frequentist Approach on Probability

The frequentist interpretation of probability, also referred to as experimental probability, is based on the law of large numbers which indicates that as the number of trials increases, the probability of the event gets closer to the theoretical probability (Batenaro, et al., 2005). The frequentist interpretation defines probability as "the hypothetical number towards which the relative frequency tends when stabilizing" (Batenaro, et al., 2005. p.23). The term relative frequency can be described as the number of times that the event occurs, divided by the total number of trials. In frequentist interpretation, probability can be applied to events for which sample space cannot be described in terms of equally likely outcomes (Jendrazsek, 2008). Though frequentist interpretation expands the range of situations to which probability theory can be applied, it also has certain shortcomings. It cannot be applied to events which do not occur many times under the same conditions. In this situation, confusion is created between probability, the observed frequencies and the abstract mathematical object (Liu, 2005). The number of trials needed to estimate the probability of an event is not easily determined. Although the frequentist interpretation is not widely mentioned in research on mathematics education, it can be used in the classroom in some cases. For example, in a study conducted by Metz (1998b), students were asked to predict the outcome of a game related to spinners. If there were a time limitation, it would be suitable to make predictions depending on the small number of trials.

4.1.3 Subjective Approach on Probability

Unlike the frequentist and classical interpretation, the subjective interpretation of probability suggests that th,e probability of an event can change from one observer to another since the amount of knowledge they possess also varies. Lindley (1994) states that “subjective probability depends on two things; the event whose uncertainty is contemplated and the knowledge that you have at the time” (p.6).Therefore, according to those who favor this approach, the probability of any event would be subjective probability since it depends on the individual getting it. The subjective interpretation is the only interpretation that takes Bayes’ theorem into account and this is why it is sometimes named as the Bayesian School (Cosmides & Tooby, 1996). The main shortcoming of this approach is that it is too abstract for elementary school students to apply. However, this does not mean that it should not be used or taught since even 5-year-old children can develop a concept of likelihood of a single event.

To sum up, all three approaches have advantages and disadvantages and can be effectively applied in particular situations. Due to the fact that assuming only these approaches might not be sufficient to teach probability efficiently, all teachers should be familiar with all three types of interpretation as emphasized by Kvatinsky and Even (2002). Hence, prospective elementary mathematics teachers need to develop an understanding of these three interpretations. What’s more, an insufficient grasp of these views may lead individuals to hold misconceptions regarding probability, which is explained in the following section.

4.2 Probabilistic Misconceptions

In the literature, different definitions of the term 'misconception' exist. Smith, diSessa and Roschelle (1993) define it as " a student conception that produces a systematic pattern of error" (p.205). Different from this definition, misconceptions were referred to as alternative and naive conceptions (Hammer, 1996).Rather than defining what it means, Meyer (1993) noticed the causes of the misconception by indicating that misconceptions can be resulted from the mistakes made during the process of the interpretation of new information and from the prior misunderstanding which constitute one part of the new knowledge. To continue with the definition of the probabilistic misconception, Rubel (2002) defined it as an incorrect conception as regards probability. In this part, several studies examining probabilistic misconceptions are presented. To state specifically, misconception regarding time axis fallacy, conditional probability (Carnell, 1997; Fischbein & Schnarch,1997; Jendrazsek, 2008), effect of sample size (Dolard, 2007; Fischbein & Scnarch, 1997; Jendrazsek, 2008; Rubel, 2002; Tversky & Kahneman, 1974), conjunction fallacy misconception (Carter & Capraro, 2005; Fischbein & Scnarch, 1997; Tversky & Kahneman, 1983; Watson & Moritz, 2002), misconception regarding representativeness (Jendraszek, 2008; Kahneman & Tversky,1973; Konold et al.,1993; Rubel, 2002; Shaughnessy,1977), and misconception related to compound event (Kennis, 2006; Lecoutre & Durand, 1988; Mut, 2003; Rubel, 2002) are discussed, respectively.

4.2.1 Time Axis Fallacy Misconception

The fact that the subsequent information can be used to determine the probability of the previous event is difficult to understand for individuals. In other words, some people believe that the occurrence of the last event cannot affect the occurrence of the first event, which shows the incidence of time axis fallacy misconception (Jendrazsek, 2008). This misconception is also called as Falk Phenomenon (Falk, 1986).

When related literature is reviewed, it is noted that there are several research studies which address the time axis fallacy misconception (Carnel, 1997; Fischbein & Schnarch, 1997). In one of the studies conducted by Carnell (1997), this misconception was focused on. The following item addressing the time axis fallacy misconception was asked to 13 undergraduate prospective mathematics and science teachers, which was presented in Figure 2.1:

An urn contains two white balls and two black balls. We blindly draw two balls, one after the other, without replacement from that urn. What is the probability that the first ball is white given that the second ball is white?

Figure 4.1 Question of Carnell related to time axis fallacy misconception (1997,p.45)

Of all the participants, 31% answered it correctly. The remaining participants showed an evidence of this misconception. That is, these participants who held this misconception stated that the second ball drawn, which was white (conditioning event) could not affect the result of the first ball drawn, which was also white (target event) since the conditioning event had not occurred at the time of target event.

Rather than investigating only prospective elementary mathematics teachers' misconceptions with regard to time axis fallacy as was in the study of Carnell (1997), Fischbein and Schnarch (1997) explored the probabilistic misconceptions of 5^{th}, 7^{th}, 9^{th}, 11^{th} grade students as well as those of prospective elementary mathematics teachers. The researchers used the same problem with Carnell (1997) with some modification. Analysis of the researchers' study revealed that the frequency of the occurrence of this misconception is increasing as the participants grow except for prospective teachers. The principle of causality, which states that the antecedent event determines the consequent event, may lead to this misconception (Fischbein &Schnarch, 1997).

Similar to the previous researchers, Jendrazsek (2008) also used the same question to examine the time axis fallacy misconception held by 66 subjects who were at a graduate school of education and planned to teach mathematics at the elementary, secondary or college levels. The researcher reported that 36% of the subjects answered it correctly. More specifically, 43% of the doctoral students, 43% of the master students, and 29 % of the students who intended to teach at elementary school answered it correctly. Such low correct response rates showed the presence of this misconception among the participants. Those subjects who held this misconception focused on the sequence of the events. Furthermore, some of the participants believed that there was a causal relation between the two events. That is, they stated that the event which occurred earlier could not be caused by the event which occurred later, which reveals that these participants confused the conditioning event with the causal event (Jendrazsek, 2008).

In another study, Mut (2003) examined the time axis fallacy misconception of 885 students who were in grades 5-10 in terms of the instruction which they had previously received. The researcher used the same problem with in the previous studies to investigate this misconception. At the end of the study, Mut (2003) reported that the participants who received instruction on probability were more likely to hold this misconception when compared to the participants who did not receive instruction.

This misconception is closely related to conditional probability. In the items addressing the time axis fallacy misconception, the operations which are carried out to compute the probability of target event are the same with the operations in the items handling misconception regarding conditional probability. The difference is that the conditioning event comes after the target event in the first ones. On the other hand, such ordering does not have to occur in the items handling misconception regarding conditional probability. This misconception is explained in the next part.

4.2.2 Misconception regarding Conditional Probability

"Conditional probability refers to the probability of one event occurring given that another event occurred" (Dollard, 2007, p.27). In mathematical terms, it can be defined as, the conditional probability of event A given that event B occurred is denoted by P (A/B). It can be computed by the formula P (A/B) = $\frac{P(A \cap B)}{P(B)}$. To illustrate the conditional probability, Anton and Kolman (1978) presented the following example: "What is the probability of rolling a1 on a single toss of the die given that the number rolled is odd?" (as cited in Carnell, 1997, p.8). In this example, the target event, the event whose probability is calculated, is rolling a "1". The conditioning event is, the event which was given in the question, is rolling an odd number (Carnell, 1997). The probability of rolling an odd number is 3/6 since there are three odd numbers, such as 1, 3 and 5 and each of them has an equal chance to occur. Furthermore, the probability of the intersection of these two events (tossing 1 and an odd number) is 1/6. As a result, the ratio of the probabilities of these events is $\frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{3}$.

To continue with the misconception regarding conditional probability, those who hold misconception regarding conditional probability have a tendency to neglect prior probabilities and just consider the new sample size or ignore the conditioning event (Tversky & Kahneman, 1974). In the literature, misconceptions related to conditional probability were investigated in depth by numerous researchers (Carnell 1997; Falk, 1986; Fischbein & Schnarch, 1997; Jendrazsek, 2008; Kennis, 2006). According to Falk (1986), the reason why students experience a difficulty in understanding this concept is that students may not be able to decide which of the given events is the conditioning event. For instance, in his study, the researcher presented a problem to elementary and middle grade students in Figure 2.2:

There are three cards in a hat. One is blue on both sides, one is green on both sides, and one is blue on one side and green on the other. We draw one card blindly and put it on the table as it comes out. It shows blue face up. What is the probability that the hidden side is blue?

Figure 4.2 Question of Falk related to conditional probability misconception (1986, p.293).

The researcher indicated that almost all the participants provided an answer of "1/2". They judged the probability of the hidden side in such a way that if the one side is already blue, the card with green sides was eliminated. They thought as if the conditioning event is the card itself, not the sides. Such course of thinking way leads to incorrect identification of the conditioning event (Falk, 1986).

In another study conducted by Kennis (2006), this misconception regarding conditional probability was examined. The researcher posed two questions addressing this misconception to 427 students in grades 9, 10, 11, and 12. One of these questions was a similar version of the item used in the study of Falk (1986), which was presented above. In the item used by Kennis (2006), only the colors of the cards (i.e. red card instead of blue card and black card instead of green card) were different. A great many of the students gave "1/2" as an answer by taking into consideration only the new sample size. As opposed to these participants, some of the participants gave "1/3" an answer by ignoring the condition and focusing on the initial sample size (Kennis, 2006). The similar responses were obtained for the following question given in Figure 2.3 which was also asked in the study of Kennis (2006):

> Suppose you are on a game show and you're given a choice of three doors containing prizes; behind one of the doors is a new car, the other two doors contain pig. You pick a door, say door #1, and then, the host of the show opens a different door, say #3, to reveal a pig. The host then asks you, would you like to switch to door #2?”

Figure 4.3 Question of Kennis related to conditional probability misconception (2006, p.112).

The researcher found that more than half of the participants (57% of males and 56% of females) gave "1/2" as the probability of winning the prize whenever the door was changed. Stated that way, subjects had just considered the new sample size. Apart from these participants, several of them (25% of males and 26% of females) gave "1/3" as an answer to that question, which demonstrated that they had overlooked new information and gave answer depending on the original sample space (Kennis, 2006).

Other than this misconception, another probabilistic misconception, namely the misconception regarding effect of sample size is commonly encountered in the literature. In the following section, this misconception is explained.

4.2.3 Misconception regarding Effect of Sample Size

This misconception is mainly related to the Law of Large Numbers, which is defined simply by Pratt (2000) as follows: "the proportion of prior results for each possibility in the sample space will stabilize as an increasing number of results is considered" (p.609). In other words, according to Rubel (2002), the law of large samples means that the relative frequency of the event approaches to the theoretical probability if the number of trials increases. The individuals who hold this misconception ignore this fact. That is, they believe that it is irrelevant to use the sample size to determine the probability of an event in this sample.

This misconception has been widely addressed in several studies and researches in the literature (Dolard, 2007; Jendrazsek, 2008; Rubel, 2002). One study conducted by Jendrazsek (2008) in order to investigate students' misconceptions regarding probability. Sixty six graduate students were required to complete both a questionnaire regarding their background, views on probability and probability concepts and a 19-item questionnaire related to probabilistic misconceptions. To give an example, one of the questions was as follows in Figure 2.4:

In a certain town there are two hospitals, a small one in which there are an average of about 20 births a day and a big one in which there are an average of about 60 births a day. The likelihood of giving birth to a boy is about 50%, the same as that of giving birth to a girl. However, there are days on which more than 50% of the babies born were boys, and there are days on which more than 50% of the babies born were girls. Both hospitals like to keep track of the days when the rate significantly deviates from 50%, favoring either male or female births. (In other words, when 60% or more of the births are of either sex.) Consider, for example, the number of days in which the number of boys born exceeded 60% in the past year. In which of the two hospitals are there likely to be more such days?

a. In the big hospital there were likely more days recorded where more than 60% boys were born.

b. In the small hospital there were likely more days recorded where more boys were born.

c. The number of days for which more than 60% boys were born is likely to be equal in the two hospitals.

d. You cannot tell.

Please explain your answer and show calculations, if any

Figure 4.4 Question of Jendrazsek related to effect of sample size misconception (2008, p.249)

According to the Law of Large Numbers, the deviation from the theoretical probability, in this case "1/2", will decrease as the sample size increases. Therefore, the number of days on which the percentage of boy births would exceed 60 is more likely to be larger at the smaller hospital. Jendrazsek (2008) reported that only 24% of the subjects correctly answered this question by stating that in the small hospital, there were more likely more such days (Jendrazsek, 2008). Such a low rate of the correct response was also seen in the other research studies (Fischbein & Scnarch, 1997; Ozaytabak, 2004; Tversky & Kahneman, 1974; Watson & Moritz, 2000). Jendrazsek (2008) further added that it is necessary to acquire the understanding of the frequentist approach. In parallel to this author, Steinberg (1991) pointed out that students should get a good grasp of theoretical and experimental probability in order not to hold misconceptions regarding this issue. The results obtained from this study directly coincided with the results of Dollard (2007), who had conducted a study with 24 prospective elementary teachers. The researcher indicated that the participants did not know what theoretical, subjective and experimental probability meant. Whereas they needed to apply the law of large numbers, they ignored the magnitude of the sample, which yielded the misconception regarding effect of sample size.

In another study which was conducted by Özaytabak (2004), 248 prospective elementary mathematics teachers were asked to respond to items addressing the probabilistic misconceptions. Two of them were related to this misconception. The first item was presented in Figure 2.5:

A doctor keeps the records of newborn babies. According to his records, the probability of which of the following options is higher?

a) Out of the first 10 babies, the gender of 8 or more of them is female.

b) Out of the first 100 babies, the gender of 80 or more of them is female.

c) The probability of events (a) and (b) is the same

Figure 4.5 First question of Özaytabak related to effect of sample size misconception (2004, p.24)

Özaytabak (2004) found that more than half of the participants stated that the probabilities of these events were the same, which showed the incidence of the misconception regarding the effect of sample size. A similar incorrect response rate of the participants was obtained in the second item, which is presented as follows in Figure 2.6:

The likelihood of getting tails at least twice when tossing three coins is: a) Smaller than b) Greater than c) Equal to the likelihood of getting tails at least 200 times out of 300 times

Figure 4.6 Second question of Özaytabak related to effect of sample size misconception (2004, p.24).

Results of her study revealed that a great majority of the participants who held this misconception indicated that the probability of getting tails at least twice in three flips was the same with that of getting at least 200 tails in 300 flips. These participants ignored the sample sizes. The reason underlying this misconception might be misapplication of proportionality since the proportion of tails is the same (Jendrazsek, 2008).

Probabilistic misconceptions of individuals are not only comprised of misconceptions thus far discussed, but also the misconception regarding conjunction fallacy, which is explained in the subsequent part.

4.2.4 Misconception regarding Conjunction Fallacy

"The conjunction fallacy occurs when the assessment of the probability of an event consisting of two constituent events is viewed as more likely than one of the constituent events alone" (Jendrazsek, 2008, p.16). That is, it is believed as if the probability of two distinct events occurring simultaneously is higher than that of one of these events occurring separately (Tversky & Kahneman, 1983).

In the literature, there are several studies investigating this probabilistic misconception regarding conjunction fallacy (Carter & Capraro, 2005; Fischbein &

Scnarch, 1997; Morier & Borgida, 1984; Ozaytabak, 2004; Tversky & Kahneman, 1983; Watson & Moritz, 2002). In one part of the study conducted by Tversky and Kahneman (1983), three groups of participants, namely the naive group (not knowledgeable about statistics), the informed group (relatively knowledgeable about statistics) and the sophisticated group (experts in statistics), were selected. The researchers used the problem which they referred to in the literature as "Linda Problem" and asked the participants to rank the probability of the statements listed below the item. These are presented as follows in Figure 2.7:

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations

-Linda is active in the feminist movement

-Linda is a bank teller

-Linda is a bank teller and is an active in the feminist movement

Figure 4.7 Question of Tversky and Kahneman related to conjunction fallacy misconception(1983, p.297).

Most of the participants in these three groups stated that being both a bank teller and a feminist (the conjunctive event) was more likely to happen than being a bank teller (constituent event). Particularly, 89 %, 90%, 85 % of the participants who were in the naïve, informed and sophisticated groups, respectively, chose the conjunctive event. The effect of the subjective approach was observed in the responses of these participants (Tversky & Kahneman, 1983). That is, personal experiences affected the judgment of the probability of these outcomes. Similar to these researchers, Morier and Borgida (1984) used this question in their study where 319 undergraduate students were asked to respond to this question. According to the result of their study, the researchers reported that the mean estimate of the conjunction event (being a bank teller and active in the feminist movement) was greater than one of its constituents (being active in the feminist movement or being a bank teller), which was parallel to the result of the previous study conducted by Tversky and Kahneman (1983).

In another study, Fischbein and Schnarch (1997) conducted a study including 20 students from grade 5, 20 students from grade 7, 20 students from grade 9, 20 students from grade 11 and 18 prospective teachers specializing in mathematics. Their purpose was to investigate whether the age of the participants had a role in the judgment of the conjunctive event. In accordance with their purpose, Fischbein and Schnarch (1997) asked the following question to the participants, which was presented in Figure 2.8:

> Dan dreams of becoming a doctor. He likes to help people. When he was in high school, he volunteered for the Red Cross organization. He accomplished his studies with high performance and served in the army as a medical attendant. After ending his army service, Dan registered at the university. Which seems to you to be more likely?
>
> Dan is a student of the medical school
>
> Dan is a student

Figure 4.8 Question of Fischbein and Schnarch related to conjunction fallacy misconception (1997, p.98)

According to Fischbein and Schnarch (1997), the majority of the participants in grades 5,7, and 9 steadfastly stated that being a student of medical school was more likely to occur, which showed the incidence of the misconception. On the other hand, the percentage of the participants in grade 11 and prospective teachers thinking in that way was relatively low. In the light of these results, the researchers concluded that as the participants grow, the presence of this misconception diminishes.

Similar to these research studies investigating the effect of age on the probabilistic thinking, in a study conducted by Watson and Moritz (2002), two items were asked to provide base-line data regarding the understanding of conjunction fallacy. There were 2615 students whose grade levels ranged between 5 and11 in 20 public schools. These students were asked to respond to two questions addressing the conjunction fallacy misconception. The first one was in the frequency form. That is, the variables in this item were in the form of frequency. It was stated as follows in Figure 2.9:

> A health survey was conducted in a sample of 100 men in Australia of all ages and occupations. Please estimate:
>
> (a) How many of the 100 men have had one or more heart attacks
> (b) How many of the 100 men are over 55 years old
> (c) How many of the 100 men both are over 55 years old and have had one or more heart attacks (Watson &Moritz, 2002, p.66).

Figure 4.9 First question of Watson and Moritz related to conjunction fallacy misconception (2002, p.66)

The second one was in the form of probability. That is, the probabilities of the events were asked. In this item, the researcher aimed to decrease the percentages of the participants who held this misconception. It was as follows in Figure 2.10:

Which one of them is more probable?

(a) The probability that you will miss a whole week of school next year

(b) The probability that you will get a cold next year

(c) The probability that you will get a cold causing to miss a whole week of school next year

Figure 4.10 Second question of Watson and Moritz related to conjunction fallacy misconception (2002, p.66)

In the first problem, the means of the students' estimates for alternatives (a), (b) and (c) were found to be 29%, 39% and 29%, respectively. On the other hand, in the second problem, the means of the students' estimates for choices (a), (b) and (c) were 45%, 76% and 38%, respectively. The differences in the response of the students may have resulted from several factors according to Watson and Moritz (2002). The first one, they believe, may have been related to the context of the question. The second reason might have related to the term 'causing' in the second question since 'causing' is more restrictive than conjunction. This may have made students think that the alternative (c) (conjunctive event) could not be more probable than alteratives (a) and (b) (Watson & Moritz, 2002). Therefore, they concluded that how the conjunctive event was presented in the questions might affect the existence of this misconception among participants.

Rather than focusing on the effect of age on the presence of this misconception, Carter and Capraro (2005) conducted an online study including only 108 prospective elementary teachers. In order to investigate whether they have misconception related to conjunction fallacy, they posed the following question given in Figure 2.11:

You see a woman carrying a baby. Which of the following is more likely?

a) The woman is a doctor.

b) The woman is a doctor and a mother.

c) Both choices are equally likely (Carter &Capraro, 2005, p.21).

Figure 4.11 Question of Carter and Capraro related to conjunction fallacy misconception (2005, p.21)

Carter and Capraro (2005) reported that of all prospective teachers, 28.7% of them answered this question correctly. That is, these participants thought that being a doctor is more likely to happen when compared to the other choices. On the other hand, more than half of the subjects, namely 51.9%, judged that being a doctor and being both a doctor and a mother were equally likely. This showed that these participants could not notice that alternative (b) (conjunctive event) was less likely to occur (Carter & Capraro, 2005).

In addition to this misconception, the misconception regarding representativeness was also held by some of the individuals, which is revealed in the next section.

4.2.5 Misconception regarding Representativeness

Individuals who hold this misconception seek to impose the attributes of parent population on the small samples of that population. That is, these participants determine the probability of the sample depending on how well this sample represents its population. To explain with an example, in particular, people who have this misconception may think that among the possible birth orders in a family of four children, the sequence BGBG (boy-girl-boy-girl) is more representative than the sequence BBBB (boy-boy-boy-boy). The reason might be that the sequence BGBG includes randomness, which leads it to be perceived as more representative of its population (Jendraszek, 2008; Kennis, 2006).

When the literature is reviewed, it is seen that several research studies investigating this misconception exist (Kahneman & Tversky, 1973; Kennis (2006); Rubel, 2002; Shaughnessy, 1976). For instance, Shaughnessy (1976) conducted an experimental study including 85 undergraduate students who had already completed a mathematics course. One of the purposes of this study was to investigate the existence of this misconception. There were two groups of students, one of them had completed an activity-based course in elementary probability and the other group had completed a traditional course in elementary probability. To obtain data, a pre-test and post-test were used. In the pre-test, one of the items addressed this misconception. Subjects were asked to determine which sequence (HTTHTH or HHHHTH) was more likely to occur (Shaughnessy, 1976). The participants who held this misconception stated that the first group 'HTTHTH' was more likely to occur because of randomness. According to the pre-test results, it was found that there was no significant difference between the groups of students. That is, the percentages of the participants holding this misconception were approximately the same in both groups. In the post-test, the term ‘coin’ in the item addressing this misconception was replaced with the term the ‘gender of children’. More precisely, subjects were asked to determine which sequence of the children (BGGBGB or BBBBGB) was more likely to occur (Shaughnessy, 1976). In this item, the participants who had this misconception indicated that the first sequence 'BGGBGB' was more likely to occur. As opposed to the results of the pre-test, after formal training, it was reported that there was a significant difference between post-test scores of the two groups of students in favor of the experimental group, who had attended an activity-based lesson. That is, in the post-test, the number of those holding this misconception in the experimental group decreased. The reason underlying this misconception might be disregarding the independence and equal chances of the outcomes in these sequences (Kahneman & Tversky, 1973; Shaughnessy, 1976)

In addition to these studies, in the study of Konold et al. (1993), the misconception with regard to representativeness was addressed as well. There were 88 students who were in secondary school and college as participants of this study. In one part of their study, they used the coins version of the problem, which was also used in the study of Shaughnessy (1976) with a little modification. Firstly, participants were presented five successive flips of coins and asked to determine which one of the choices was most likely to occur among the choices: (a) HHHTT, (b) THHTH, (c) THTTT, (d) HTHTH, and (e) all four sequences are equally likely. The correct answer was "(e)" since outcomes of the sequences are independent of the each other. Konold et al. (1993) stated that most of the participants gave the correct answer. These researchers posed a second question to understand whether the participants had given this response just by chance. In this case, they were asked to determine which one of the choices was the least likely to occur among these choices. Most of the students holding this misconception chose option (d) since the sequence was in order and did not include randomness. This result coincided with the results of the study of Shaughnessy (1976) where participants tended to choose the sequence whose outcomes were in non-random ordering. The other probabilistic misconception is related to simple and compound events, which is explained in the next section.

4.2.6 Misconception regarding Compound Event

People having the misconception regarding simple and compound event cannot differentiate between a simple event and a compound event (Lecoutre & Durand, 1988). To speak more specifically, for example, rolling one 5 and one 6 is a compound event since there are two outcomes out of 36 outcomes, namely 5-5 and 6-5. On the other hand, getting two 6s out of 36 possible outcomes is a simple event since there is one possibility which is 6-6. Individuals who held this misconception thought that the chance of getting 6s was the same as getting one 5 and one 6 disregarding the other outcome, namely 6-5.

The misconception regarding simple and compound events was extensively studied in the literature (Kennis, 2006; Mut, 2003; Rubel, 2002). In one part of a study conducted by Cohen and Hansel (1958) (as cited in Kennis, 2006), students who were above 15 years of age were asked to determine the probability of the compound events. The aim of these researchers was to examine how students use the multiplicative rule while finding the probability of a compound event. The participants were presented a problem which included a game. The rules of this game to win the prize were as follows: Firstly, students had to choose the correct container which included a red disk among three identical round containers. Then, they would win the prize if they selected the correct rectangular container including the prize among three containers. The answer was (1/3)x (1/3) = 1/9. However, it was found that rather than using the multiplication rule, the subjects simply added numerators and gave "2/3" as an answer (as cited in Kennis, 2006).

In addition to these researchers, Rubel (2002) conducted a study which examined probabilistic reasoning and abilities of the middle and high school students in terms of compound event. A probability inventory was administered to the students in grades 5, 7, 9, and 11 at a private school (n= 173). After completing the inventory, 33 of the participants were interviewed to gain a deeper insight into the situations involving a compound event. There were 2 questions on compound event. One of them is called as "two event item" in the literature and was as follows: "Eminem has two quarters .What is the probability that he will get one “heads” and one “tails” if he flips them both?" (Rubel, 2002, p.74). The researcher reported that there was no statistically significant difference across the ages (χ^2 =4.524, p =.2102, df=3). Some of the participants disregarded the order of the coins although tail or head could be in the first or in the second place. Therefore, the participants incorrectly obtained the sample space. More precisely, some of the students listed sample space as "TT, HT, and "HH" rather than "TT, HT, TH, and HH". What’s more, Rubel (2002) pointed out that several participants used the 50-50 approach. That is, these participants thought that there would be always be a 50% chance of getting a head or a tail.

The second problem used in the study of Rubel (2002) is called as "four ones item" in the literature. This item reads as follows: "Suppose you roll a fair die four times. What is the probability that it lands on "ones" all four times?" (Rubel, 2002, p.90).

According to the result of his study, the percentage of the participants who correctly answered it increased as the grade level increased. Some of the students who gave the incorrect answer found the sample space by adding all 6s and thus computed the sample space as "24". All these results showed that insufficient grasp of the sample size concept may lead to this misconception (Kennis, 2006; Rubel, 2002).

In contrast to previous studies examining this issue in terms of only grade level variable, the study which was conducted by Kennis (2006) examined the existence of probabilistic misconception with respect to gender variable as well. There were 427 students in grades 9, 10, 11 and 12. Students were asked to fill out three questionnaires which were used to obtain their skills, cognition and probabilistic reasoning. The researcher posed several problems about compound event. One of them was the "two event item", which was also used in the study of Rubel (2002) and shown above. Of all the students, 74% of the females and 77% of the males gave the correct answer. That is, boys and girls did not differ from each other with respect to compound event problem. Moreover, it was indicated that the participants who gave the incorrect answer over generalized the 50-50 approach from single trail to compound event. This result was consistent with the results of the study of Rubel (2002). To speak in terms of grade level, it was reported that the correct response rates across the grades 9, 10, and 11 leveled out, while the correct response rate decreased sharply for grade 12. Students who gave incorrect responses did not notice the order of the two coins. Furthermore, it was found that the use of the 50-50 approach decreased across the ages (Kennis, 2006). Kennis (2006) posed another question, called as "four ones item", which was also used in the study of Rubel (2002) and presented above. Of all the participants, while 15% of the females gave the correct answer "$\frac{1}{6^4}$", 21% of the males responded correctly. To report the results with respect to gender, Kennis (2006) pointed out that while males could not apply the counting principle, females could not determine the sample space, which cause them to hold misconception related to compound event.

In another study which was conducted by Mut (2003), the students' probabilistic misconceptions in terms of grade level, gender, and previous instruction on probability were investigated. There were 885 students in grade levels 5, 6, 7, 8, 9, and 10. Two items were related to misconceptions of simple and compound event. According to the results of this study, it was reported that the students who had received instruction on probability were relatively successful when compared to other students. The researcher further added that in grades 5,6, and 7, the percentage of the females who had this misconception were higher than that of the males. On the other hand, this situation was vice versa for the remaining grade levels.

4.3 Summary of the Literature Review

Generally speaking, the classical approach was used in schools to teach probability wherever possible. This approach attributes a single theoretical probability on an event, which prevents students from recognizing the realistic nature of probability. Due to such a limitation of the theoretical probability, the usage of the other approaches, such as the frequentist and subjective approach, is important as well. Therefore, in the literature, there are several researches which take these approaches into consideration (Jendrazsek, 2008; Kvatinsky & Even, 2002; Liu, 2005; Metz, 1998b). However, insufficient emphasis on these approaches may lead students to have probabilistic misconceptions. Thus, there are also several studies investigating the probabilistic misconceptions. Particularly, some probabilistic misconceptions regarding time axis fallacy (Carnel,1997; Falk, 1986; Fischbein & Schnarch, 1997; Jendrazsek, 2008), conditional probability (Carnell 1997; Kennis, 2006; Jendrazsek, 2008), effect of sample size (Dolard, 2007; Jendrazsek 2008; Rubel, 2002), conjunction fallacy (Morier & Borgida,1984; Tversky & Kahneman, 1983; Watson & Moritz, 2002), representativeness (Kahneman & Tversky, 1973; Kennis, 2006; Shaughnessy, 1976) and compound event (Kennis, 2006; Mut, 2003; Rubel, 2002) are extensively studied.

Literature review indicated that some of individuals hold the misconception regarding time axis fallacy. That is, these individuals believe that the occurrence of the later event cannot affect the first event (Fischbein & Schnarch, 1997). For example, most of the elementary and middle grade students stated that it is irrelevant to use the subsequent event to determine the probability of the prior event (Fischbein & Schnarch, 1997; Mut, 2003). In addition to these students, many of the prospective and graduate students have this misconception as well (Carnel, 1997; Jendrazsek, 2008), which may result from the principle of causality, which indicates that the first event determines the occurrence of the later event (Fischbein & Schnarch, 1997)

The other misconception which is indicated in the literature review of the current study is the misconception regarding conditional probability. There are two common courses of thinking of the individuals who hold this misconception. These participants either focus on the new sample size or ignore the conditioning event and thus focus on the initial probabilities (Tversky & Kahneman, 1974). For instance, a great many of the elementary and middle grade students ignored the conditioning event (Falk, 1986). In addition, the study of Kennis (2006) revealed that the majority of the middle grade students showed an incidence of this misconception. This situation is also true for prospective teachers (Jendrazsek, 2008). Thus, this misconception may exist among the individuals regardless of their age or grade level.

Apart from these studies, several studies showed that the individuals might hold the misconception regarding effect of sample size. Those who hold this misconception cannot recognize the fact that the probability of an event in the larger sample is closer to the theoretical probability of that event (Tversky & Kahneman, 1974). According to Fischbein and Schnarch (1997) who conducted a study with students in grades 5, 7, 9, 11 and with prospective teachers, the percentage of the participants holding this misconception increases across grade levels. The incidence of this misconception is seen among some of the graduate students as well. The reason underlying this misconception might be lack of knowledge in experimental and theoretical probability (Dollard, 2007; Jendrazsek, 2008).

To continue with the next misconception, a great many of researchers have documented that many of the individuals hold the misconception regarding conjunction fallacy. People who hold this misconception believe that the conjunctive event is more likely to occur than one of its constituents. For example, to investigate whether the age of the individuals have an effect on the existence of this misconception, Fischbein and Schnarch (1997) conducted a study with students from elementary to high school students and prospective teachers. According to these researchers, even some of the prospective teachers hold this misconception, which shows that the incidence of this misconception can be seen in different age groups. Furthermore, in the literature, the effect of instruction on statistics on individuals' probabilistic misconception was investigated (Tversky & Kahneman, 1983). It was reported that instruction on statistics did not decrease the presence of the misconception regarding conjunction fallacy among the individuals. This misconception might have resulted from the subjective approach of the participants. That is, personnel experiences of individuals may have an impact on the judgment of the conjunctive event's probability (Tversky & Kahneman, 1983).

Another misconception which is mentioned in the literature review chapter of this study is the misconception regarding representativeness. Those who hold this misconception have a tendency to judge the probability of a sample by considering how well this sample represents its parent population. In the literature, there are various studies which have addressed this misconception (Kahneman & Tversky, 1973; Kennis 2006; Konold et al, 1993). For example, secondary school students, college students and prospective teachers are some individuals who hold this misconception (Konold et al, 1993; Shaughnessy, 1976). Most of them disregard the independence of the outcomes in the samples. Although these outcomes have equal chances of occurrence, these individuals might not recognize this fact.

The last but not least, the other misconception mentioned is related to compound event. Not being able to differentiate between a simple event and a compound event shows the incidence of this misconception. According to literature review, there are several studies examining this issue (Kennis,2006; Mut,2003; Rubel, 2002). All these studies indicated that most of the individuals hold this misconception whatever their grade level and gender are. The possible reason underlying this misconception might be that some of them ignore the ordering of the outcomes of the compound event and thus incorrectly assess the sample size of this event (Kennis, 2006).

All in all, as it can be understood from the literature above, studies on prospective elementary mathematics teachers' probabilistic misconceptions are limited, which shows a need for the study investigating probabilistic misconceptions held by prospective elementary mathematics teachers and the reasons underlying those misconceptions.

CHAPTER III

METHOD

One of the aims of this study was to investigate performance of prospective elementary mathematics teachers on answering the items addressing probabilistic misconceptions. This study also aimed to explore the underlying reasons behind these misconceptions that prospective elementary mathematics teachers held.

In this chapter the research design, the participants, the data collection instrument, the data collection procedure, analyses of the data, and lastly the internal and external validity of the study are dwelled on, respectively.

5.1 Research Design

The qualitative research design was used to investigate performances of prospective elementary mathematics teachers on items addressing probabilistic misconceptions and to explore reasons underlying these misconceptions. According to Denzin and Lincoln (2005), qualitative research is a field of inquiry in its own right. It helps documenting ideas of individuals in depth and detail (Patton, 2002).In a qualitative study, depth and detail ideas of individuals can be captured by interviews, observations, and documents with small number of people and cases (Patton, 2002). Therefore, in this study, qualitative research design was used to get in-depth insight about participants' probabilistic misconceptions. Particularly, Probability Misconception Questionnaire (PMQ) was administered to the participants to determine performances of prospective elementary mathematics teachers on items handling probabilistic misconceptions. In order to get in-depth understanding of reasons behind probabilistic misconceptions, semi-structured interviews were conducted.

5.2 Participants

The target population of this study was all senior prospective elementary mathematics teachers enrolled in the elementary mathematics education programs in public universities. All senior prospective elementary mathematics teachers enrolled in elementary mathematics education programs in public universities in the Marmara Region were identified as an accessible population. In order to select the sample of the study, convenience sampling method was used. In convenience sampling method, researchers collect data from the individuals who are available (Fraenkel & Wallen, 2006). The sample of this study consisted of 12 senior prospective teachers who were enrolled in elementary mathematics education program of a public university in Sakarya. Particularly, 4 female and 8 male participants voluntarily participated in this study. All the participants had attended and completed the course "Probability and Statistics" which was related to probability in their teacher education program.

5.3 Measuring Tools

One of the aims of this study was to investigate performance of prospective elementary mathematics teachers on answering the items related to probabilistic misconception. The second purpose was to investigate the underlying reasons behind these misconceptions. To address the former goal, data were collected by means of administering the Probability Misconception Questionnaire (PMQ) to the participants. As well as open ended questions of PMQ, semi-structured interviews were conducted to obtain data to address the latter goal of this study. These instruments are explained in detail in the following sections.

5.3.1 Probability Misconception Questionnaire (PMQ)

PMQ included 9 multiple choice items and 9 open-ended items. Participants were first required to answer the multiple choice items and then provided an explanation to the corresponding open ended items. The items were adapted and modified from the literature (Afantiti & Williams, 2008; Diaz & Batenero, 2009; Falk, 1986; Fischbein & Schnarch, 1997; Kennis, 2006; Rubel, 2002; Tversky & Kahneman, 1974) and a book on probability (Akdeniz, 1998). The necessary permissions were taken from these researchers separately by means of e-mail. In order to develop PMQ, in the first step, the objectives as regards to probability in the curriculum of the elementary mathematics education were examined. After examining these objectives related to the concepts of probability in the elementary mathematics curriculum, the conditional probability concept, which is not addressed in elementary mathematics curriculum, was included in the study since participants of the current study, namely prospective elementary mathematics teachers, were to be experiencing difficulty in this topic as well (Ozaytabak, 2004). In the second step, probabilistic misconceptions which were commonly encountered in the available literature were determined. From among these misconceptions drawn from the literature, those misconceptions related to concepts of probability, which were covered in the elementary school mathematics curriculum were selected to be treated in this study. These probabilistic misconceptions were time axis fallacy, conditional probability, effect of sample size, conjunction fallacy, representativeness and compound event. With respect to the concepts of probability, the following table including probabilistic misconceptions was prepared.

Table 5.1 Table of PMQ Items with corresponding Concept of Probability and Probabilistic Misconceptions

Types of Misconception	Basic concepts about probability (Equal Probability)	Basic concepts about probability (Sample Space)	Types of Events
Representativeness	Item 8		
Effect of Sample Size		Item 5, Item 6	
Simple and Compound Events			Item 9
Conjunction Fallacy			Item 7
Conditional Probability			Item 3,Item 4
Time Axis Fallacy			Item 1,Item 2

As it can be seen from Table 3.1, all topics were related to elementary mathematics curriculum except for conditional probability itself which was covered in high school curriculum. As previously mentioned, the sample of this study was comprised of prospective elementary mathematics teachers. However, analyzing the concepts of probability only in elementary mathematics curriculum would not give complete picture about the misconceptions of the prospective elementary mathematics teachers. What's more, as highlighted by Carnell (1997), the conditional probability is related to dependent and independent events. As indicated in Table 3.1, due to its connection to types of events (dependent and independent events) which were included in the elementary mathematics curriculum, the conditional probability was included in this study. The Turkish form of the PMQ was presented in Appendix A

The first and the second items of the PMQ addressed one of the probabilistic misconceptions named as time axis fallacy. They were mainly concerned about the conditional probability. In these items, the prospective elementary mathematics teachers were asked to examine whether they could see the effect of the occurrence of the latter event on the occurrence of the first event.

The first item was adapted from the study of Diaz and Batanero (2009), which was conducted with university students majoring in psychology. In this item, there were channels through which the ball could pass and exit points. The channel I was represented as the first event and the exit point R was represented as the latter event. Also, the participants were asked to explain their answers. The item is given below in Figure 5.1:

We throw a ball into the entrance E of a machine (see the figure). If the ball leaves the system through exit R, what is the probability that it passed through channel I?

a. 1/2
b. 1/3
c. 2/3
d. Cannot be computed

Explain your answer.

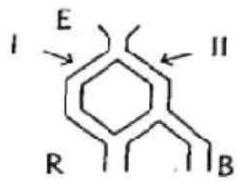

Figure 5.1 The first item of the probability misconception questionnaire

The second item was adapted from the study of Falk (1986). In this item the participants were required to compute the probability of the first event (that the first marble is white) by considering the information (that the second ball is white) which was given in advance. Moreover, they were required to provide an explanation for this item. The item is given below in Figure 5.2:

Two black and two white marbles are put in an urn. We pick a marble from the urn. Then, without putting it back into the urn, we pick a second marble at random. If the second marble is white, what is the probability that the first marble is white?

a) 1/3

b) 1/2

c) 1/6

d) Cannot be computed.

Explain your answer.

Figure 5.2 The second item of the probability misconception questionnaire

The third and forth items explored the prospective elementary mathematics teachers' misconception regarding conditional probability. The third one is a well-known problem, which is called the "Monte Hall" problem. In this item, the participants needed to consider alternative situations to find the probability of an event which was given in the item and give explanation of his/her reasoning. This item was adapted and modified depending on Turkish culture from Kennis's (2006) study which was conducted on the students at 9^{th}, 10^{th}, 11^{th}, and 12^{th} grades. The item is given below in Figure 5.3:

Suppose you are on a game show and you're given a choice of three doors containing prizes; behind one of the doors is a new car, the other two doors contain dogs. Suppose you pick the door 1 and then, the host of the show opens the door 3 to reveal a pig. The host then asks you, would you like to switch to door 2? What is the probability of getting the car which is behind the door 2?

a. 1/3

b. 1/2

c. 2/3

d.3/3

Explain your answer.

Figure 5.3 The third item of the probability misconception questionnaire

The forth items was adapted from the book "Probability and Statistic" by Akdeniz (1998). Similarly, in this item, prospective elementary mathematics teachers were asked to find the conditional probability of one event when it was known in advance that the other event had already occurred. Also, they were expected to explain their responses. The item is given below in Figure 5.4:

When the reasons of power cut were analyzed, the following results were obtained: 0.05, 0.8, and 0.01 of the power cut are resulted from failure of transformer, failure of the line, and failure of both, respectively. What is the probability of the failure of the transformer given that line is deficient?

a) 4/100

b) 1/50

c) 1/80

d) 1/100

Explain your answer.

Figure 5.4 The fourth item of the probability misconception questionnaire

The fifth and the sixth items were asked to identify prospective elementary mathematics teachers' misconception regarding effect of sample size. To speak more specifically, they were required to notice the fact that as the size of the sample increased, the probability of the event in that sample would get closer to its theoretical probability. Also, in these items, they were expected to justify their responses to these items. The fifth item is a well-known problem which is called the "Hospital problem". It was adapted from the study of Tversky and Kahneman (1974) in which undergraduate students were included. Also, it was modified by including the alternative (d). The item is given below in Figure 5.5:

In a certain town there are two hospitals, a small one in which there are an average of about 20 births a day and a big one in which there are an average of about 60 births a day. The likelihood of giving birth to a boy is about 50%, the same as that of giving birth to a girl. However, there are days on which more than 50% of the babies born were boys, and there are days on which more than 50% of the babies born were girls. Both hospitals like to keep track of the days when the rate significantly deviates from 50%, favoring either male or female births. Consider, for example, the number of days in which the number of boys born exceeded 60% in the past year. In which of the two hospitals are there likely to be more such days?

a. In the big hospital,

b. In the small hospital,

c.. The number of such days is equal for both hospitals.

d..You cannot tell anything.

Explain your answer.

Figure 5.5 The fifth item of the probability misconception questionnaire

In the sixth item, prospective elementary mathematics teachers were asked to determine the effect of the number of tosses on the probability of getting tail. This item was adapted from the study of Lamprianou and Williams (2008) which was conducted on the students whose ages ranged from 12 to 15 years old. Furthermore, it was modified by adding the alternative (d), which is given below in Figure 3.6:

Two groups of children play a game tossing a fair coin. The likelihood of getting 'Tail' when tossing the fair coin is 50%. The first group of children (group A) tosses the coin 50 times. The second group of children (group B) tosses the coin 150 times. Each time the children toss the coin, they note down the outcome. Which group of children is more likely to get 60% 'Tails' when tossing the coin? Please circle only one of the answers.

a. Group A.

b .Group B.

c. Both groups' results would be the same.

d..You cannot tell anything.

Explain your answer.

Figure 5.6 The sixth item of the probability misconception questionnaire

The seventh item was asked to explore one of the probabilistic misconceptions of the prospective elementary mathematics teachers, which is called the 'conjunction fallacy'. In this item, participants were asked to recognize the fact that the probability of the conjunctive event is less than that of its component event. Also, they were expected to provide justifications for their answers. This item was adapted from the study of Tversky and Kahneman (1983) in which undergraduate students were included. It was also modified by adding the alternative (d), which is given below in Figure 5.7:

Meltem is 32 years old, single, outspoken, and very smart. In college, she majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which of the following statements is most likely?

a. Meltem is a professor

b. Meltem is a professor who is involved with politics

c. (a) and (b) are equally likely.

d. You cannot tell anything.

Explain your answer.

Figure 5.7 The seventh item of the probability misconception questionnaire

The eighth item was asked to identify whether the prospective elementary mathematics teachers could determine the probability of a sample by considering how well this sample is representative of its population. In this item, the participants were asked to provide an explanation for their responses to this item. There were two groups of the men. In the first group, the heights of the men were in random order, while in the second group, the heights of the men were in non-random order. This item was adapted from the study of Cow and Mouw (1992) which was carried out with graduate students. In addition, it was modified by including the alternatives (c) and (d), which is presented below in Figure 3.8:

:

The mean height of the Turkish male is 175 cm. Three men were randomly selected and measured. Their heights were 178 cm, 170 cm, and 179cm, respectively. Three more men were randomly selected and measured. Their heights were 175 cm, 175 cm, and 175 cm, respectively. Which group of heights do you think is more likely to be observed if this exercise was repeated again?

a. The first group of heights is more likely to be observed

b The second group of heights is more likely to be observed

c. (a) and (b) are equally likely.

d. You cannot tell anything

Explain your answer.

Figure 5.8 The eighth item of the probability misconception questionnaire

The ninth item addressed prospective elementary mathematics teachers' misconceptions related to the compound events. In this item, the participants were expected to notice which of the events was a compound event and which of them was a simple event. Particularly, in this item, the participants were asked to judge the probability of two events, which were getting two 6s and getting one 5 and one 6. They were also required to provide an explanation on how they arrived at their answer. This item was adapted from the study of Lecoutre and Durand (1988) which was carried out with elementary school students. Also, it was modified by including the alternative (d), which is as follows in Figure 3.9:

Two fair dices are tossed. Which one of the options is more likely to occur?

a) Getting 6s

b) Getting one 5 and one 6

c) (a) and (b) are equally likely

d) You cannot tell anything.

Explain your answer.

Figure 5.9 The ninth item of the probability misconception questionnaire

To sum up, 9 items in total were included in PMQ, which was followed by a complementary interview study, which is explained in the next section.

5.3.2 Interview Procedure

The researcher performed semi-structured interviews in order to get in-depth understanding of the reasons behind the prospective elementary mathematics teachers' misconceptions after administering the PMQ. To make this aim happen, the participants who completed the PMQ were interviewed. Before each interview was conducted, the researcher explained the purpose of the study. Also, these interviews were conducted in one of the suitable classrooms in which the participants feel themselves confident. The interviews lasted between 40 and 55 minutes. In this process, the answers given by the participants were re-examined. That is, they were asked to clarify their written explanations and explain the reasons behind their solutions.

5.4 Pilot Study

A pilot study was conducted to check the validity and reliability of the PMQ which was translated into Turkish and to determine the average testing time, the possible problems that might have occurred in the actual administration. As Hambleton (2005) emphasizes, it is important to translate the expressions by considering the cultural and psychological aspects. Therefore, 3 English teachers, 2 teacher educators majored in Turkish controlled this questionnaire with respect to the clarity of the problem statements. Also, 3 teacher educators from mathematics education program checked the questionnaire according to table of specification to determine whether the items and the concepts of probability supported to each other. Depending on the feedbacks obtained from them, the questionnaire was finalized.

The pilot study of PMQ was conducted with 73 junior prospective elementary mathematics teachers who were in the same department with the participants of the main study. In the pilot study, initially, there were 15 items. The participants, namely junior prospective elementary mathematics teachers, completed the questionnaire in 60-65 minutes. During the pilot study, it was observed that the participants did not focus on all the questions and got bored because of the fact that it took a long time to complete it. In order to increase the efficiency of the questionnaire, it was decided that some of the items should be eliminated. The results of the item analysis were used as a criterion for eliminating some items. One point noticed in this process was the item discrimination value. Crocker and Algania (1986) stated that item discrimination measures "how effectively the item discriminates between examinees who are relatively high on the criterion of interest and those who are relatively low" (Crocker & Algina, 1986, p. 313). The item guideline for item discrimination used by Crocker and Algina (1986) was taken as reference in this study. This guideline is presented in Table 5.2:

Table 5.2 Item Guideline for Item-Discrimination

D =	Quality	Recommendations
> 0.39	Excellent	Retain
0.30 - 0.39	Good	Possibility for improvement
0.20 - 0.29	Mediocre	Need to check/review
0.00 - 0.20	Poor	Discard or review in depth
< -0.01	Worst	Definitely discard

According to the results of the item analysis, there were several items which had low discrimination values. These items with their discrimination values can be seen in Table 3.3:

Table 5.3 Item Discrimination Values of Items in PMQ

Item Number	Item Discrimination Value
5	0
8	.16
9	.16
14	.08
15	.027

The other criterion which was noticed in this process was the item difficulty value. Item difficulty levels used by Lord (1952) were taken as a reference in this study, which are presented as follows:

Table 5.4 Item Difficulty Levels

Format	Item Difficulty(%)
Five-response multiple-choice	70
Four-response multiple-choice	74
Three-response multiple-choice	77
True-false (two-response multiple-choice)	85

As can be seen in Table 3.4, desired item difficulty level for four-response multiple-choice items is "0.74". According to Lord (1952), an item with a higher value than "0, 74" is admitted as easy item. Seventy-eight % of the participants provided the correct answer to item 10, which showed that this item was seen as easy by these participants according to Table 3.4. Therefore, item 10 was omitted from the questionnaire. As a result, item 5, item 8, item 9, item 10, item 14, and item 15 were eliminated because of these mentioned reasons.

5.4.1 Reliability and Validity of PMQ

Validity refers to correct, appropriate useful and meaningful inferences which are made based on the collected data (Fraenkel & Wallen, 2006). To increase the content validity of the instrument, three mathematics educators examined the PMQ with the help of Table 3.1 presented before to see whether the items and the concepts of the probability in the curriculum supported each other. Also, three English teachers checked the appropriateness of the translation and two teacher educators majored in Turkish evaluated the clarity of the statements. Furthermore, a separate discussion was made with the mathematics educator in the Elementary Mathematics Education Department to judge the difficulty levels of the items.

Another concern of the study was to establish is 'reliability', which refers to "the consistency of the scored obtained from- how consistent they are for each individual from one administration of an instrument to another" (Fraenkel & Wallen, 2006, p.157). In order to establish inter-rater reliability, a rubric, which was checked by a mathematics educator, was used in this study to evaluate the explanations of the participants to open-ended items. In this study, the scoring agreement method was used in order to find an inter-rater reliability and thus the responses of 12 prospective elementary mathematics teachers were evaluated with a second coder. There was a 98% correlation between the scores of the participants.

5.5 Data Collection Procedure

Determining performance of prospective elementary mathematics teachers on answering the items addressing the probabilistic misconceptions was one of the aims of this study. Exploring the reasons underlying these misconceptions was another purpose of this study as well. In order to make these aims happen, the necessary official permissions were obtained from Middle East Technical University Human Subjects Ethics Committee and the Head of Elementary Mathematics Education program of the university to conduct the current study.

The pilot and the main studies were conducted during the second semester of the 2011-2012 academic years. In this process, in order to determine whether the prospective elementary mathematics teachers held the probabilistic misconceptions, the Probability Misconception Questionnaire (PMQ) was used after it was translated into Turkish. After the pilot study, reliability and validity of PMQ were evaluated. Then, this instrument was revised based on the results of the pilot study.

The main study was conducted with twelve 4^{th} year prospective elementary mathematics teachers who enrolled in the Elementary Mathematics Education program of a public university in Sakarya. The questionnaire was administered by the researcher in their regular class hour. It took approximately 45 minutes for participants to complete PMQ. The researcher explained the purpose of the study and how to answer each item to the participants at the beginning of the administration. Also, they were requested to answer the questions honestly. In order to establish confidentiality, it was declared at the beginning of administration that the data obtained from the questionnaire was used only in this study. Furthermore, the participants were informed that they had a right to refuse to take part in the study and to withdraw from participating.

After PMQ was completed by the participants, the interviews were conducted in order to explore the reasons behind the misconceptions of the participants, within a week of participants' completion of PMQ. A schedule representing the order of data collections is given in the Table 5.5:

Table 5.5 Time schedule for data collection

Date	Events
December 2011- April 2012	Development of the measuring tool
April 2012	Pilot study and revision of measuring tool
April-May 2012	Administration of measuring tool
April-May 2012	Conducting interviews
June-November 2012	Analysis of data

5.6 Analysis of Data

In order to answer the research questions, in-depth item based analysis was conducted. Firstly, the percentages of the participants who gave correct answers or had probabilistic misconceptions were examined for each item. Then, open-ended questions were evaluated in accordance with a holistic rubric so that there was no subjectivity in the evaluation process. More precisely, the nine open-ended items of the PMQ were scored by means of a four-level rubric. The rubric developed by the researcher was adapted for each of the items by stating the sample of the correct and incorrect answers. (See Appendix B). In the rubric, "0 point" indicated that the item was left blank."1 point" indicated an incorrect response with an insufficient explanation while "2 points" indicated a correct response with an insufficient explanation. Lastly, "3 points" meant that the participants gave the correct answer with sufficient explanation. The general form of the rubric is presented in Table 5.6:

Table 5.6 Rubric for Evaluating Open-Ended Questions

Points	Answers
0	No answer
1	Incorrect response with an insufficient explanation
2	Correct response with an insufficient explanation
3	Correct response with a sufficient explanation

Subsequently, the audiotapes of the interviews were transcribed. A second coder was recruited for data analysis in order to have agreement on findings. To code the responses of the participants, the data obtained from the interviews and open ended questions were checked simultaneously by the researcher and the coder. In cases where the disagreement arose, researcher and the second coder discussed these categories. In finally, after the discussion, the final version of categories was established.

5.7 Assumptions and Limitations

There were several assumptions of the present study as in other studies. One of these was that the participants of the study were assumed to answer the questions in the questionnaire sincerely and accurately. The other one was that there was no interaction between the subjects; otherwise, it would have affected the result of this study.

This study also limitation about generalization. The findings of this study cannot be generalized to all prospective elementary mathematic teachers since qualitative research design was used in this study. Also, this study included limited number of participants which prevented the researcher generalizing the result of this study.

5.8 Quality of the Study

The practical standards used to judge the quality of the conclusions derived from the findings of the study can be defined as the quality of the study (Miles & Huberman, 1994).

Triangulation method was used to establish credibility. According to Stake (2000), "it has been generally considered as a process of using multiple perceptions to clarify meaning, verifying the repeatability of an observation or interpretation " (p.443).To give detail, the researcher used different types of measuring tools, namely questionnaires and interviews. Results obtained from questionnaires and interviews were supported to each other and gave indepth insight about prospective elementary mathematics teachers' probabilistic misconceptions and reasons underlying these misconceptions. Also, this study included 12 prospective elementary mathematics teachers, which means that there were several participants as sources of data. Also, in the current study, a second coder evaluated the responses of the participants since the researcher bias can occur in qualitative studies. This can be possible since researcher is the main stone of the studies (Merriam, 1998). In the evaluation process, a rubric developed by the researcher of this study was used so that objectivity was established. Furthermore, the researcher of this study and second coder checked the participants' responses to open-ended items of PMQ and interviews simultaneously. During data coding, both coders tried to reach common codes to increase the quality of research study. At the time of administration of questionnaires, to speak in terms of researcher bias, the researcher did not communicate with any participants to prevent interaction.

CHAPTER IV

FINDINGS

One of the purposes of this study was to investigate performance of prospective elementary mathematics teachers on answering items related to probabilistic misconceptions. The other aim was to investigate reasons underlying these misconceptions of the prospective elementary mathematics teachers. This chapter dwells on and reports the findings of this study in six separate sections, namely, misconceptions of time axis fallacy, conditional probability, effect of sample size, conjunction fallacy, representativeness and lastly compound event. The analysis initially revealed the correct response rates and subsequently the reasons underlying those misconceptions. The chapter ends with a summary of findings.

4.1 Analysis of Items on Time Axis Fallacy Misconception

There were two items (item 1 and item 2) which addressed the time axis fallacy misconception in the Probability Misconception Questionnaire. This misconception results from the belief that the second event cannot affect the first event. In other words, while the participants think that the first event can affect the probability of the second event, the reverse is not believed to be true. More specifically, the first item was given below in Figure 4.1:

:

We throw a ball into the entrance E of a machine (see the figure). If the ball leaves the system through exit R, what is the probability that it passed through channel I?

a. 1/2
b. 1/3
c. 2/3
d. Cannot be computed

Explain your answer.

Figure 4.1 The first item of the probability misconception test

As it can be seen from the

Figure 4.1, the first event is the ball passing through channel I and the second event is the ball leaving through point R. In this item, the answer is $\frac{2}{3}$. To explain more thoroughly, the probability that the ball leaves through the point R is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. The probability that the ball passes through channel I and leaves through point R is $\frac{1}{2} .1 = \frac{1}{2}$. Thus, the answer is $\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3}$ since it was known that the ball leaves through point R. As can be seen from the equation, the probability that the ball leaves through point R (the second information) affects the probability that the ball passes through the channel I (the first information).

In addition to item 1, item 2 was also related to time axis fallacy misconception as mentioned before. It was as follows:

Two black and two white marbles are put in an urn. We pick a marble from the urn. Then, without putting it back into the urn, we pick a second marble at random. If the second marble is white, what is the probability that the first marble is white?

a) 1/3
b) 1/2
c) 1/6
d) Cannot be computed.

Explain your answer.

Figure 4.2 The second item of the probability misconception test

As it can be seen in Figure 4.2, the balls were selected without replacement. This means that the information given in the item (that the second ball is white) has changed the sample space for the first draw. In this case, two black marbles and one white marble remained for computation of the probability for the first drawing. Thus, the answer is 1/3.

The analysis of the data obtained from both questionnaire and interviews yielded the categories of the participants' responses to item 1 and item 2 as presented below in Table 4.1:

Table 4.1 Categories of Responses to Item 1 and Item 2

	Categories	
	Correct Responses (f)	**Reasons underlying Misconception (f)**
Item 1	No correct answer	• Misinterpretation of the problem (2) • Focusing on the first event (7) ➢ Focusing on the first channels • Focusing on the exit points (3)
Item 2	Correct answer with a sufficient explanation (2)	• Misinterpretation of the problem (5) • Focusing on the first event (5) ➢ Focusing on the first marble

As can clearly be seen in Table 4.1, the analysis revealed that two of the participants gave the correct answer to item 2 with correct explanation while there was no participant who gave the correct answer with a sufficient explanation to item 1. These participants were aware of the fact that the selection of the second marble changed the sample space for the first marble. To illustrate, the Participant 9 stated as follows:

> Participant 9: " Because, uh…We know that the second marble is white. Three marbles remained in the end. 1 white , 2 black marbles were remained. The probability for the other selection is 1/3."
>
> [Çünkü,ııı…2. bilyenin beyaz olduğunu biliyoruz. Sonuçta 3 tane bilye kalacak geriye. 1 beyaz, 2 tane siyah kaldı. Diğer seçim için olasılık 1/3 olur.]

When the reasons were examined for incorrect responses, there were common categories for both of the items as shown in Table 4.1. More precisely, of all participants, two of them misinterpreted item 1 and five of the participants misinterpreted item 2. For example, they interpreted item1 like the probability that the ball passes through the channel I and leaves through point R was asked. They misinterpreted the meaning of the conditional statement “it is known that the ball leaves through point R”. To give an example, the explanation of the Participant 9 for item 1 was as follows:

> Participant 9: “I think that the probability of the ball which passes through channel I and point R was asked. After the ball passes through channel I, it is not possible to pass through B”.
>
> [Bana gore topun I. kanaldan ve R den geçme olasılığı soruluyor. I. kanaldan geçtikten sonra B’ den geçmesi imkansız.]

Nearly half of the participants made the same interpretation for item 2 as in item 1.That is, these participants interpreted this item incorrectly. For example, Participant 12 provided the following response:

> Participant 12: “The probability that the first marble is white is 2/4 or 1/2. After we selected one marble, 1 white and 2 black marbles remained since we did not put it back into the urn. The probability that the second marble is white is 1/3. Thus, the answer is $\frac{1}{2}$ x $\frac{1}{3} = \frac{1}{6}$.”
>
> [1. bilyenin beyaz olma olasılığı 2/4 yani ½ dir. Bir tane bilye seçtikten sonra geriye 1 beyaz ve 2 tane siyah bilye kalıyor çünkü çektiğimizi torbanın içine geri atmıyoruz. 2. Bilyenin beyaz olma ihtimali 1/3 tür. Bu yüzden, cevap $\frac{1}{2}$ x $\frac{1}{3} = \frac{1}{6}$.]

It was obvious in the explanation of the participant 12 that he had computed the joint probability instead of the conditional probability. That is, he firstly considered the probability that the first marble is white and then the probability that the second marble is white. Finally, he just multiplied these probabilities.

The other common reason behind this misconception was that some of the participants focused on the first event in both items which was shown in Table 4.1. To speak in terms of item 1, seven of the participants focused on the first channels which were at the entrance. They ignored the details given in the remaining part of the item. Additionally, in the second item, several of them focused on the first marble. To put it in different words, they disregarded the information about the later outcome (that the second ball is white). For instance, in item 1, Participant 3 indicated as follows:

> Participant 3: " The probability that the ball passes through the channel I is ½ since there are two channels in the entrance."
>
> [Topun I. kanaldan geçme olasılığı ½ dir. Çünkü, girişte 2 tane kanal var...]

Besides, the explanation of participant 1 who focalized on the first event in item 2 was as follows:

> Participant 1: " …The second draw does not affect the first draw. If the question was as follows: 'What is the probability that the second marble is white when it is known that the first marble is white', then the first draw would affect the second draw."
>
> […2.çekim 1. yi etkilemez. Eğer soru şu şekilde olsaydı: '1.bilyenin beyaz olduğu bilindiğinde, 2. Bilyenin beyaz olma olasılığı nedir?', birinci çekim, ikinci çekimi etkilerdi.]

Apart from these reasons, the other reason was that three of the participants noticed the number of exit points in the figure given in item 1 and overlooked the rest of the figure. For example, Participant 11 concluded as follows:

> Participant 11: "There are two ways for the ball which was thrown through entrance E to leave out. It leaves out from either exit R or exit B. In this case, the probability that the ball passes through channel I is ½. That's why the probability is ½."

[E girişinden atılan bir topun çıkması için 2 seçenek var. Ya R çıkışından ya da B çıkışından çıkar. Bu durumda, I.kanaldan geçme ihtimali ½ dir. Bu yüzden olasılık 1/2 dir.]

All in all, except for two participants, nearly none of the participants were successful at answering the items related to time axis fallacy. To continue with the reasons behind this misconception, the misinterpretation of the problem and focusing on the first event were two main reasons which caused this misconception. In the following section, analyses of the items regarding conditional probability misconceptions are summarized.

4.2 Analysis of Items on Misconception of Conditional Probability

Item 3 and item 4 were asked to determine whether the prospective elementary mathematics teachers hold the misconception of the conditional probability. Also, if they had this misconception, the underlying reasons of this misconception were examined. To start with item 3, this item was a well known problem called as "Monte Hall" problem. It is given below in Figure 4.3:

Suppose you are on a game show and you're given a choice of three doors containing prizes; behind one of the doors is a new car, the other two doors contain dogs. Suppose you pick the door 1 and then, the host of the show opens the door 3 to reveal a pig. The host then asks you, would you like to switch to door 2? What is the probability of getting the car which is behind the door 2?

a. 1/3

b. 1/2

c. 2/3

d. 3/3

Explain your answer.

Figure 4.3 The third item of the probability misconception test

The correct answer of this item is “2/3”. As can be recognized that there are two hidden assumptions in this item. One of them is that the host knows where the car is located. The other one is that the host always opens the door behind which the car is not located. Therefore, as could be interpreted from these assumptions, the participants were required to think of alternative conditions to solve this item. The detailed explanation of the solution of this item is presented in

Figure 4.4:

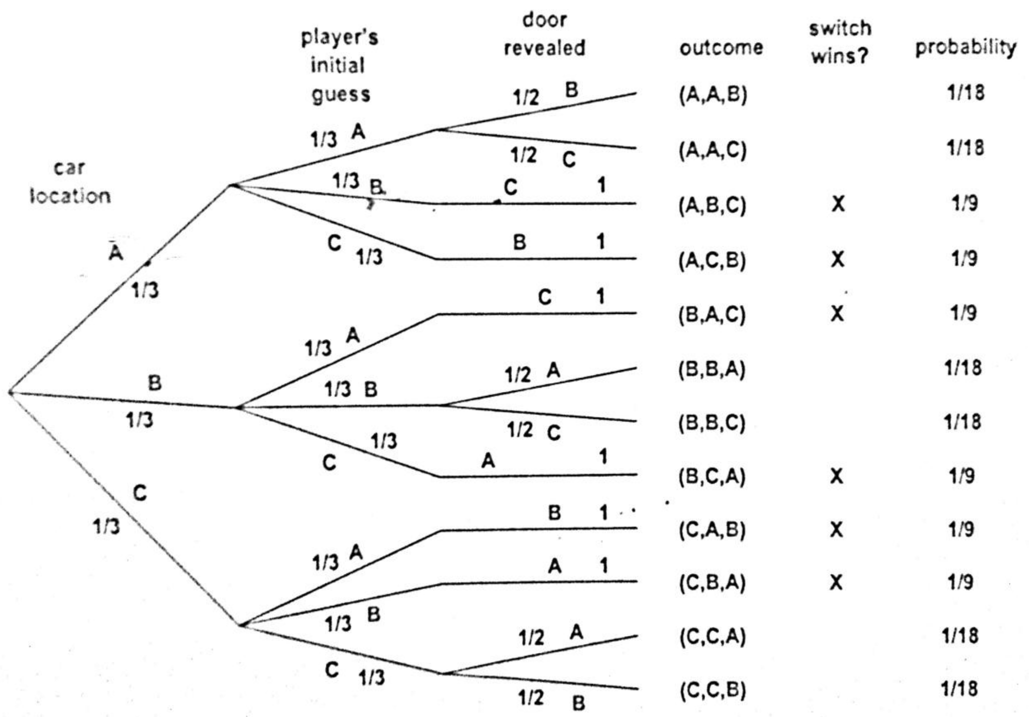

Figure 4.4 The solution of item 3

The door numbers 1, 2 and 3 are interchanged with the labels A, B, and C, respectively in Figure 4.4 in order not to confuse the door numbers with the probabilities for each event. As it can be seen in the Figure 4.5, for example, if the car is located behind door A (the probability is 1/3) and if you choose door A (the probability is 1/3), then the host opens either B or C (the probability is 1/2). As clearly seen in

Figure 4.4, the outcomes which were marked by the sign 'X' means that if you switch from your choice, you will win. There are six marked outcomes as

shown above. The probability of each marked outcome is 1/9. Thus, the probability that the player wins by switching is $6 \times \frac{1}{9} = \frac{2}{3}$. On the other hand, if one sticks to his/her initial choice, the probability that one wins the prize is 1/3. As a result, if you switch from your door to door 2, the probability will be 2/3.

The second item addressing the conception of the conditional probability was the fourth item of the PMQ, which can be seen in Figure 4.5:

When the reasons of power cut were analyzed, the following results were obtained: 5%, 80%, and 1% of the power cut are resulted from failure of transformer, failure of the line, and failure of both, respectively. What is the probability that transformer is deficient given that line is deficient? **a**) 4/100 **b)** 1/50 **c)** 1/80 **d**) 1/100 Explain your answer.

Figure 4.5 The forth item of the probability misconception test

In this item, let P (L) be the probability that the line is deficient. Let P(T) be the probability that the transformer is deficient. Then, P (L∩T) denotes the probability of the failure of both of them. Therefore, the conditional probability that the transformer is deficient when it is known that the line is deficient is P (T/L) = $\frac{P(L \cap T)}{P(L)} = \frac{0,01}{0,8} = \frac{1}{80}$.

Analyses of the data obtained from both the questionnaire and the interviews concluded that the participants' responses to item 3 and item 4 could be grouped into different categories. These categories are presented in Table 4.2 below:

Table 4.2 Categories of Responses to Item 3 and Item 4

	Categories	
	Correct Responses (f)	**Reasons underlying this Misconception (f)**
Item 3	Correct answer with an insufficient explanation (1)	• Ignorance of hidden assumptions (11)
Item 4	Correct answer with a sufficient explanation (4)	• Misinterpretation of the problem (8)

As indicated in Table 4.2, of all the participants, only one participant gave the correct answer for item 3, but did not provide a sufficient explanation to this item. The explanation was as follows:

> Participant 8: "Because when we choose the first door, the probability that the car is behind this door will be 1/3. However, when we open one of the other two doors, the dog is found. Thus, the probability that the car is behind the door which is selected is 2/3."
>
> [Çünkü ilk kapıyı seçtiğimizde araba olma ihtimali 1/3 tür. Ama diğer iki kapıdan birini açtığımızda, köpek çıkar.Bizim seçtiğimiz kapının arkasında arabamızın olma ihtimali 2/3 olur.]

The analysis revealed that when compared to item 3, the correct response rate was relatively higher for item 4. An example of a response provided by Participant 11 is as follows:

Participant 11: "Suppose that the total failure is '100x'.At this point, the failure caused by the transformer becomes '5x'. Moreover, the probability that the line is deficient is '80x'. The probability that both transformer and the line are deficient is '1x'.I did this to get rid of the percentages. The sample space will be '80x' since it was known that the line is deficient. Besides, we included the probability of both events since the probability of the failure of transformer was also asked. This becomes 'the event'. Thus, the answer is $\frac{1x}{80x}=\frac{1}{80}$ since the probability is $\frac{\text{event}}{\text{sample space}}$."

[Toplam hata "100x" olsun.Bu noktoda, trafodan kaynaklanan hata "5x" oluyor. Hattın arızalı olması olasılığı ise "80x" oluyor. Hem trafodan hem de hattan kaynaklanan arıza ise "1x" oluyor. Yani, aslında yüzdelerden kurtulmak için yaptım.Hattın arızalı olduğu bilindiğine göre , örnek uzay "80x" olacaktır. Bunun üzerine trafonun da arızalı olma ihtimalini sorduğu için her iki ihtimali de dahil etmiş oluyoruz.Yani bu da "olay" oluyor. Olasılık ise $\frac{\text{olay}}{\text{örnek uzay}}$ olduğu için, cevap $\frac{1x}{80x}=\frac{1}{80}$.]

As can be seen in the detailed explanation of the participant, she recognized the condition and used the terms 'sample space' and 'event' correctly.

Aside from these participants who gave the correct answer, there were also several participants who responded to these items incorrectly. For instance, from the given expressions of the participants for item 3, it can be seen in Table 4.2 that a vast majority of the participants could not recognize the hidden assumptions of the game which was mentioned in this item. Therefore, they failed to understand the logic behind this game. For instance, Participant 5 who gave “3/3” as an answer provided the following explanation:

> Participant 5: “If the car was behind door 1, the host would not need to open door 3 to release the dog. Thus, the dog was behind door 1. The probability that the car is behind door 2 is 100 %.”
>
> [1 nolu kapıdan araba çıkmış olsaydı, sunucunun 3 numaralı kapıyı açıp köpeği serbest bırakmasına gerek kalmayacaktır. O yüzden, 1 nolu kapıdan köpek çıkmıştır. %100 ihtimalle 2 nolu kapıda araba vardır.]

As it can be clearly seen in the above explanation, participant 5 thought that if the door which he chose included the prize, the host of the game would open his door. On the other hand, participant 11 thought that the remaining doors had the same probability to be opened by the host of the game. However, her reasoning was incorrect because the host would not open the door behind which the prize was located.

> Participant 11: “There were three doors. We know that there is a dog behind the third door. Thus, there were two doors remaining. The probability that the car is behind one of these doors is 1/2.”
>
> [3 tane kapı vardı. 3.sünde köpek olduğunu biliyoruz. O halde 2 tane kapı kaldı. Bunlardan birinde araba olma olasılığı 1/2 dir.]

To continue with item 4, it can be seen in Table 4.2 that more than half of the participants misinterpreted the question as was the case in both item 1 and item 2. More precisely, some of them interpreted the question like the probability that the line and the transformer were out of order. On the other hand, several of the participants thought that the probability of the intersections of these two events presented in the item was asked. To illustrate, the explanation of Participant 8 was as follows:

> Participant 8: "It is 1/100 since the probability of being broken down caused by both reasons was given as 1/100 in the question. Indeed, the aim of this question is to ask for this probability. Hence, the probability that both the line and the transformer were broken down was 1/100."
>
> [1/100 dür.Çünkü soruda her iki nedenden de arızalı olma ihtimali 1/100 olarak verilmiş. Aslında, sorunun amacı bu olasılığı sormak.Bu yüzden,hem hattın hem de trafonun arızalı olma ihtimali 1/100 dür.]

As can be clearly seen in the explanation of Participant 8, the intersection of these events has been mentioned. It is fairly obvious that the reason underlying this misconception was the misinterpretation of the problem. Correspondingly, participant 2 gave "1/4" as an answer, but she attempted to multiply the probabilities of these events. The explanation she provided is presented in Figure 4.6 below:

Participant 2:

Figure 4.6 Answer of Participant 2 to Item 4

As it is clear in Figure 4.6, she multiplied the probability that the line was deficient with the probability that the transformer was deficient. Then, she arrived at a wrong answer.

To sum up, less than half of the prospective elementary mathematics teachers were successful at answering the items regarding the conditional probability misconception. As mentioned above, when the reasons underlying these misconceptions were examined, it was found that more than half of the participants had misinterpreted the question and could not recognize the hidden assumptions. The following section dwells on the misconceptions of the participants regarding the effect of sample size.

4.3 Analysis of Items on the Misconception of Effect of Sample Size

There were two items, item 5 and item 6, which were asked to determine whether the prospective elementary mathematics teachers recognized the effect of sample size when estimating the likelihood of an event. The participants who have this misconception believe that the size of the sample have no effect on determining the probability of the desired event. More precisely, these participants cannot notice the fact that the probability of an event in the sample will get closer to the theoretical probability as the size of the sample increases. Also, the reasons underlying this misconception were also explored.

The first item addressing this misconception, item 5, presented in Figure 4.7, was asked to determine whether a small or big hospital was likely to have more days in which the birth rate of boys is more than or equal to 60%. It is as follows:

In a certain town there are two hospitals, a small one in which there are an average of about 20 births a day and a big one in which there are an average of about 60 births a day. The likelihood of giving birth to a boy is about 50%, the same as that of giving birth to a girl. However, there are days on which more than 50% of the babies born were boys, and there are days on which more than 50% of the babies born were girls. Both hospitals like to keep track of the days when the rate significantly deviates from 50%, favoring either male or female births. Consider, for example, the number of days in which the number of boys born exceeded 60% in the past year. In which of the two hospitals are there likely to be more such days?

a. In the big hospital,

b. In the small hospital,

c.. The number of such days is equal for both hospitals.

d..You cannot tell anything.

Explain your answer.

Figure 4.7 The fifth item of the probability misconception test

Law of Large Numbers states that the larger the sample size is, the closer the probability is to the theoretical probability. Therefore, to be more specific, in this item, as can be expected that the probability of a small hospital having more days on which births to males occurred will be higher or equal to 60%.

As for the sixth item, as can be seen in Figure 4.8, the participants were asked to determine which group of children was more likely to get tails that were 60% of all outcomes when tossing the coin.

Two groups of children play a game tossing a fair coin. The likelihood of getting 'Tail' when tossing the fair coin is 50%. The first group of children (group A) tosses the coin 50 times. The second group of children (group B) tosses the coin 150 times. Each time the children toss the coin, they note down the outcome. Which group of children is more likely to get 60% 'Tails' when tossing the coin? Please circle only one of the answers.

a. Group A.

b .Group B.

c. Both groups' results would be the same.

d..You cannot tell anything.

Explain your answer.

Figure 4.8 The sixth item of the probability misconception test

For this item, according to Law of Large Numbers, more deviation in the probability of tails is expected in the small number of tosses. That is, the probability of getting a tail will become more distant from the theoretical probability, namely 1/2, as the number of trials decreases. Therefore, the first group of children (group A) is more likely to get more tails, which are 60% of all outcomes.

The analysis of the data obtained from the responses provided to both questionnaire and interviews of item 5 and item 6 yielded the categories presented in Table 4.3 below:

Table 4.3 Categories of Responses to Item 5 and Item 6

	Categories	
	Correct Responses (f)	**Reasons Underlying this Misconception (f)**
Item 5	Correct answer with a sufficient explanation (1)	• Focusing on the ratio (2) • Focusing on the sample size (6) • Focusing on both ratio and sample size (3)
Item 6	Correct answer with a sufficient explanation (1)	• Focusing on the ratio (11)

As clearly shown in Table 4.3, the correct response rates of the participants to item 5 and item 6 were the same. There was only one participant (Participant 12) whose explanation was correct for the item 6. This participant provided a correct response to item 6 as well. Other than this participant, there was no participant who correctly answered this item correctly. His explanation for the item 5 was as follows:

> Participant 12: " I think that any increase in the number of male birth makes a big difference in terms of percentage for the small hospital. For example, assume that there are 4 births in the small hospital. If 2 male and 2 female births take place in this hospital, 50% of the births will be male births. If 3 male and 1 female births take place in this hospital, then 75% of these births will be male births. In this case, the increase in the percentage is 25%. On the other hand, assume that there are 10 births in the large hospital. If 5 male and 5 female birth take place in this hospital, 50% of the births will be boy births, again. If 6 male and 4 female births take place in this hospital, then 60% of these births will be boy births. In this circumstance, the increase in the percentage is 10%. Thus, it is more probable that small hospital has more days in which the male births will exceed or be equal to 60%."

[Bence erkek doğum sayısındaki herhangi bir artış , küçük hastanede büyük bir fark yapar. Örneğin, küçük hastanede 4 doğumun olduğunu varsayalım.Eğer bu hastanede 2 erkek ve 2 kız doğum gerçekleşiyorsa, doğumların %50si erkek doğum olacaktır.Eğer 3 erkek ve 1 kız doğum gerçekleşiyorsa, o zaman doğumların %75 i erkek doğum olacaktr.Bu durumda, yüzdedeki artış %25 tir. Öte yandan, büyük hastanede 10 doğumun olduğunu varsayalım. Eğer büyük hastanede,5 erkek ve 5 kız doğum gerçekleşiyorsa, yine doğumların %50si erkek doğum olacaktır. Eğer bu hastanede 6 erkek ve 4 kız doğum gerçekleşiyosa, o zaman doğumların %60 ı erkek doğum olacaktır. Bu durumda yüzdedeki artış %10 dur. Yani, küçük hastanede erkek doğumların %60 veya daha fazla olduğu günlerden olma ihtimali daha fazladır.]

Likewise, the reasoning of the same participant to item 6 was of the same kind. His explanation was as follows:

Participant 12: "The logic underlying this item is the same with that in the previous item. With a small number of tosses, it is more probable to make a big change in the probability of tails. Therefore, group A is more likely to get tails 60% tails of the time."

[Bu sorudaki mantık bir önceki soru ile aynı. Daha az sayıda atışla, yazı gelme olasılığında daha büyük bir değişiklik yapma ihtimali daha fazladır. Bu yüzden, grup A, daha yüksek ihtimalle %60 oranında yazı getirir.]

To continue with the reasons underlying this misconception, one can see in Table 4.3 that some of the participants had misconceptions because of several reasons. Firstly, they focused on the ratio in both items. More specifically, for example in item 5, according to two of the participants, the probability is just a ratio, regardless of the hospital. They further added that the ratio is important to determine the probability of an event. For instance, Participant 10 responded as follows:

> Participant 10: "The probability of such days is the same in both hospitals since the important thing is that 60% is the same proportion in both hospitals."
>
> [Böyle günlerin ihtimali her iki hastanede de eşittir. Çünkü önemli olan %60’ ın aynı oran olmasıdır.]

Similar to participant 10, participant 3 focused on the ratio. However, instead of mentioning that 60% is the same proportion, he focalized on the 50%. His explanation was as follows:

> Participant 3: " I think that the answer of this item is 50% or ½ because the probability of giving birth to a girl or a boy is 1/2. The number of births in the hospitals does not affect this probability. The fact that the ratio is equal is not related to the numbers..."
>
> [Bence bu sorunun cevabı %50 veya 1/2 dir. Çünkü kız veya erkek olma olasılığı 1/2 dir. Hastanelerdeki doğum sayıları bu olasılığı etkilemez. Oranların eşit olması sayılar ile alakalı değil...]

As it was the case in item 5, eleven of the participants disregarded the sample size, which was the number of the toss in this item, and just focused on the variables related to the ratio in item 6. For instance, Participant 3 stated the following:

> Participant 3 : "The probability of getting a tail is not related to the number of tosses. The ratio is 1/2 in both situations. The number of tosses is not important. Hence, the probability is the same for both groups."

[Yazı gelme olasılığı atış sayısı ile alakalı değildir. Her iki durumda da oran 1/2.Atış sayısı önemli değildir.Bu yüzden, her iki grup içinde olasılık eşittir.]

Additionally, Participant 2 reasoned in the same way. However, she selected a different choice. Her explanation was as follows:

> Participant 2: "The probability of getting a head is always 1/2. It is impossible to get a probability rate which is greater than 1/2. Therefore, we cannot say anything."

> [Yazı getirme olasılığı her zaman 1/2 dir. 1/2 'den daha fazla bir olasılık getirmek imkansız. Bu yüzden birşey söyleyemeyiz.]

Secondly, the other reason underlying this misconception was that exactly half of the participants noticed only the sample size in item 5. That is, they gave an answer depending on the size of the hospitals. For instance, Participant 7 claimed as follows:

> Participant 7: "In fact, I do not know how to find the answer by means of the formula or operation. However, I think that it is more likely that the big hospital has more days on which male births are more than or equal to 60% since more births take place in the big hospital."

> [Aslında, cevabı formülle ya da işlemlerle nasıl bulabilceğimi bilmiyorum. Ama, bence büyük hastanede %60 veya daha fazla erkek doğumun olduğu günlerin sayısı daha fazladır. Çünkü büyük hastanede daha fazla doğum gerçekleşiyor.]

As it is obvious above, the participants made a direct relation between the size of the hospital and the probability of male births. In other words, according to these participants, as the size of the hospital increases, the probability of male births increases.

Lastly, apart from these participants, three of the participants focused on both the ratio and the sample size in item 5 as indicated in Table 4.3. However, they could not decide which variable was more crucial in giving the answer to this item. Also, these participants experienced difficulty in the way to use these variables (ratio and sample size) in responding to this item. For example Participant 9 indicated as follows:

> Participant 9:" We cannot tell which hospital is more likely to have more days on which male births surpassed 60% since both birth rates are changeable and the size of the hospitals are different."
>
> [Hangi hastanede erkek doğumlarının sayısının %60 ı geçtiğini bilemeyiz çünkü hem doğum oranları değişken hem de hastenelerin büyüklüğü farklı]

In addition to the explanation of participant 9, the explanation of Participant 1 was as follows:

> Participant 1: "I could not make a connection between the birth rate and the size of a hospital. Therefore, I cannot say anything."
>
> [Doğum oranları ile hastanelerin büyüklüğü arasında bir ilişki kuramadım. O yüzden birşey söyleyemem.]

Briefly, as mentioned above, except for only one participant, nearly none of the participants were successful at answering the items addressing the misconception regarding the effect of sample size. The participants who held misconception thought that the sample size was irrelevant in estimating the probability of an event in the sample. When the reasons underlying this misconception were examined, focusing on ratio could be regarded as the major reason. Additionally, half of the participants focused on the sample size and 25% of them focused on both the ratio and the sample size.

In the following section, the prospective elementary mathematics teachers' conjunction fallacy misconception is explained.

4.4 Analysis of the Item on Conjunction Fallacy Misconception

Item 7 addressed the presence of misconception regarding conjunction fallacy in the responses of the prospective elementary mathematics teachers. The participants who hold this fallacy believe that the probability of the conjunctive event (the event that consists of two events) is less than the probability of one of its components. Furthermore, the case where participants held this misconception, the underlying reasons of this misconception were explored. The item is presented in Figure 4.9.:

Meltem is 32 years old, single, outspoken, and very smart. In college, she majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations. Which of the following statements is most likely? **a**. Meltem is a professor **b**. Meltem is a professor who is involved with politics **c**. (a) and (b) are equally likely. **d**. You cannot tell anything. Explain your answer.

Figure 4.9 The seventh item of the probability misconception test

For Meltem to be a professor and to become involved in politics, the probability of her being a professor must be multiplied by the probability of her being involved in politics to get the probability of her being both. As could be understood that a smaller fraction is obtained when they are multiplied since both probability rates are fractions. Hence, the probability of Meltem being a professor alone is higher than that of her being both. In other words, the answer is the alternative (a).

The analyses of the participants' responses are presented below in Table 4.4:

Table 4.4 Categories of Responses to Item 7

	Categories	
	Correct Responses (f)	**Reason Underlying this Misconception (f)**
Item 7	correct answer with a sufficient explanation (1)	• Focusing on the narrative (11)

It can be clearly seen in Table 4.4 that nearly all of the participants gave incorrect response to this item. To put it more precisely, there was only one participant (Participant 6) who provided the correct answer with a reasonable justification. His explanation was as below:

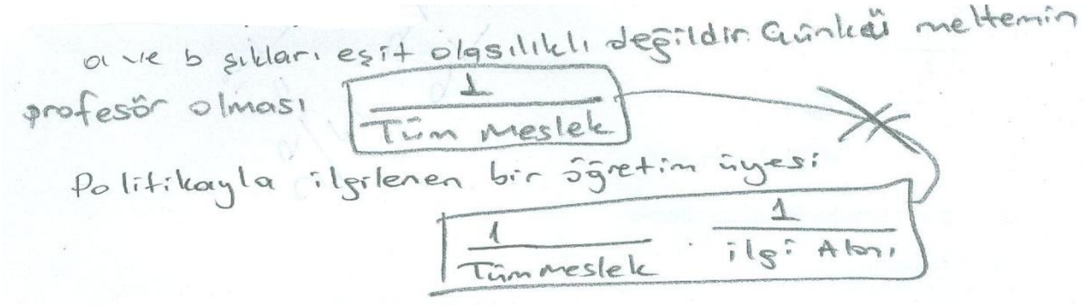

Figure 4.10 Answer of Participant 6 to Item 7

As it was obvious in Figure 4.10, he stated that the alternatives (a) and (b) were not equal to each other. He recognized that the probability of the conjunctive event (that Meltem is a professor who is involved in politics) could be found by multiplying two fractions and showed this multiplication as "$\frac{1}{\text{all professions}}$ x $\frac{1}{\text{area of interest}}$". Also, he further computed the probability of the constituent event (that Meltem is a professor) as "$\frac{1}{\text{all professions}}$".

In contrast to this participant, eleven of the participants produced incorrect responses, which showed the incidence of the conjunction fallacy misconception as indicated in Table 4.4. The reason underlying this misconception was that they focused on the narrative. In other words, they were affected by the storylike nature of the item.

These participants looked for the character's properties which fit into one of the alternatives. To illustrate, following is the explanation of Participant 10 :

> Participant 10: “The fact that Meltem was interested in discrimination and social justice can show that she can be a professor. However, there is no information which shows that she is interested in politics. Therefore, it is more likely that Meltem is a professor.”
>
> [Meltem’in ayrımcılık konuları ve sosyal adalet ile ilgilenmesi onun profesör olabileceğini gösterebilir. Fakat, onun politikayla ilgilendiğini gösteren herhangi bir bilgi yoktur. Bu yüzden, Meltem’in profesör olması daha olasıdır.]

As it can be clearly understood from the explanation above, although Participant 10 selected the correct alternative (that Meltem is a professor), her explanation was incorrect The reason was that she gave her answer by focusing on the properties of the given character rather than focusing on the fact that being a professor was more likely to occur than being a professor interested in politics.

Additionally, Participant 12 looked for Meltem’s properties which were suitable to the alternatives of the item. However, in contrast to participant 10, he produced a different explanation. His explanation was as follows:

> Participant 12: “The answer is the alternative (b) since this alternative is more suitable to the properties of Meltem. Bu iki özelliğe sahip olma ihtimali daha fazladır." Therefore, the alternative (b) is more probable.”
>
> [Cevap (b) şıkkı çünkü bu şık Meltem'in özelliklerina daha uygundur. It is more likely that she has both these aspects. (b) şıkkı daha olasıdır.]

Similar to these participants, Participant 2 was also distracted by the storylike nature of the problem and tried to determine the probability of the events in the alternatives. His explanation was as follows:

> Participant 2: "The given properties in the item could be a hobby. All these do not give information about her profession. Therefore, we cannot say anything."
>
> [Soruda verile özellikler bir hobi olabilir. Bunlar mesleği hakkında bilgi vermez. Bu yüzden, birşey söyleyemeyiz.]

To summarize, a vast majority of the prospective elementary mathematics teachers fell into the trap of the conjunction fallacy. When the reason underlying this misconception was investigated, it was found that the participants who had this misconception focused on the narrative. Therefore, they could not notice the fact that the conjunctive event was less probable when compared to the probability of its component. In the subsequent part, the findings of the participants' misconception regarding representativeness are reported.

4.5 Analysis of the Item on Representativeness Misconception

Item 8 was asked to determine whether the prospective elementary mathematics teachers held misconception regarding representativeness. Those who have this misconception believed that the probability of the sample which involves random sequence is higher than that of the sample that includes special sequence in which the outcomes are in order or the same. In case where the participants had this misconception, the underlying reasons were analyzed. This item is given below in Figure 4.11:

The mean height of the Turkish male is 175 cm. Three men were randomly selected and measured. Their heights were 178 cm, 170 cm, and 179cm, respectively. Three more men were randomly selected and measured. Their heights were 175 cm, 175 cm, and 175 cm, respectively. Which group of heights do you think is more likely to be observed if this exercise was repeated again? **a.** The first group of heights is more likely to be observed **b** The second group of heights is more likely to be observed **c**. (a) and (b) are equally likely. **d**. You cannot tell anything Explain your answer.

Figure 4.11 The eighth item of the probability misconception test

Assuming that there is a sufficient number of men in the population, the probability of each sequence is the same since the outcomes in these sequences are independent of each other. To speak more specifically, the probability of getting a man whose height is 178 cm is the same as the probability of getting a man whose height is 175 cm. Therefore, the probability of the sequence (178cm, 170cm, 179cm) is the same with the probability of the other sequence (175cm, 175cm ,175cm) as expected.

The analysis of the data obtained from both the questionnaire and the interviews are summarized below in Table 4.5:

Table 4.5 Categories of Responses to Item 8

	Categories	
	Correct Responses (f)	**Reasons Underlying this Misconception (f)**
Item 8	The correct answer with a sufficient explanation (2)	• Focusing on the mean (2) • Focusing on the number of groups (2) • Focusing on the representativeness (6)

As can be clearly seen in the above table, of all the participants, two of them answered this item correctly. These participants mentioned the equal chances of different outcomes. For example, Participant 6 stated as follows:

> Participant 6: The heights of all men were considered when determining the mean of the population. According to me, they have tried to make the mean of all three men to be 175cm. However, the probability to be selected is the same for every men selected from the population. It does not need to be 175cm."
>
> [Populasyonun ortalaması bulunurken her erkeğin boyu düşünüldü. Bana göre, her 3 erkeğin ortalamasının 175 cm olmasına çalışılmış. Fakat populasyondan seçilen her erkeğin seçilme olasılıkları eşittir. İllaki 175 cm olmak zorunda değildir.]

To continue with the reasons underlying this misconception, it could be seen that there were three different reasons, which are presented in Table 4.5. First of all, one of the reasons was that two of the participants solely focused on the mean. Specifically, they tried to find the mean of the groups. According to these participants, it was more likely that the group had a mean equal to that of the population. To give an example, below is the explanation of Participant 9 :

> Participant 9: "The second group of heights is more likely to be observed because we know that the mean is 175 cm. In the first group, the mean of the heights of the men is higher than that of the population. For a mean equaling 175cm, the group in which each of the three men had heights of 175cm must be selected."
>
> [2.grubun görülmesi daha olasıdır.Çünkü ortalamanın 175cm olduğunu biliyoruz.1. grupta erkeklerin boylarının ortalaması populasyonun ortalamasından daha yüksek.Ortalamanın 175 cm

olması için her üç erkeğin de boyunun 175 cm olduğu grup seçilmelidir.]

As can be seen above, the participant tried to establish a connection between the mean of the population and the mean of the groups.

Secondly, some of the participants noticed the number of the groups in order to determine which groups was more likely to be observed. According to them, the probabilities of these groups were the same. For instance, Participant 4 indicated as follows:

> Participant 4: "I thought the first and the second groups as a whole. That is, there are 2 groups in total. The probability of these groups is 1/2 and equal to each other."
>
> [Birinci ve ikinci grubu birer bütün olarak düşündüm. Yani, toplamda 2 grup var. Bu grupların olasılığı 1/2 ve birbirine eşittir.]

As it is clear above, the participant focused on the entire groups rather than the outcomes in each group.

In addition to these reasons, one last reason underlying their misconception was that nearly half of the participants had made a decision depending on how well the groups represent the population. These participants claimed that the probability of getting three men whose heights were the same was less probable than that of getting three men whose heights were different. For instance, participant 8 stated as follows:

> Participant 8: "The mean of the population is 175 cm. This does not show that all the men are 175 cm in height. This shows that the mean of the selected men's height is 175 cm. That is, the number of men whose heights are higher or lower than the mean is high. The second group does not represent the population well. If this exercise were repeated, the first group would be more likely to be observed."

> [Populasyonun ortalaması 175cm dir.Bu bize bütün erkeklerin boylarının 175 cm olduğunu göstermez. Seçilen erkeklerin boylarının 175 cm olduğunu gösterir. Yani, boyu ortalamanın üstünde veya altında olan erkeklerin sayısı fazladır. 2.grup populasyonu iyi temsil etmez. Eğer bu uygulama tekrarlanırsa birinci grubun görülmesi daha olasıdır.]

As it can be understood in the explanation of this participant, he was aware of the fact that the mean of the sample and population does not have to be the same; however, his selection of random sequence which included different heights showed that he was also affected by the representativeness of the group.

In conclusion, a small percentage of the participants could recognize the equal chance of the sequences and thus gave the correct answer. On the other hand, the participants who held this misconception considered that the group which was more representative was more likely to be observed and did not notice equal chances of the outcomes in these groups. Thus, the reasons underlying this misconception can be summarized as the tendencies of focusing on the mean, focusing on the number of the groups and finally focusing on the representativeness caused the participants to produce incorrect answers to this item. Finally, the participants' misconceptions regarding the compound event was analyzed and is reported in the following section.

4.6 Analysis of the Item on the Misconception regarding Compound Event

The last item (item 9), which is presented in Figure 4.12, was used to explore the presence of the misconception regarding the compound event in the prospective elementary mathematics teachers. Participants who have this misconception cannot recognize the compound event. These participants cannot notice the existence of another outcome which was obtained by changing the order of the events.

Two fair dices are tossed. Which one of the options is more likely to occur?
a) Getting 6s
b) Getting one 5 and one 6
c) (a) and (b) are equally likely
d) You cannot tell anything.
Explain your answer.

Figure 4.12 The ninth item of the probability misconception test

The correct alternative for this item is (b). To start with the first roll, it results in one of the six possible results. Moreover, there are also six possible results for the second roll. Therefore, the sample space consists of 36 outcomes of equal likelihood. Since there are two outcomes 5-6 and 6-5, the probability of getting 5 and 6, in either order is 2/36. On the other hand, the probability of getting two 6s is 1/36 since there is only one possible outcome (6-6). Thus, getting one 5 and one 6 is more likely to occur.

According to the analysis of the data obtained from both the questionnaire and the interviews, the categories of the participants' responses to this item are presented below in Table 4.6:

Table 4.6 Categories of the Responses to Item 9

	Categories	
	Correct Responses (f)	**Reasons underlying this misconception (f)**
Item 9	There is no correct answer	Ignoring the order of the outcomes (12)

As stated in Table 4.6, none of the participants correctly solved this item. All of the participants misinterpreted the question thinking that the probability of getting a 5 in the first dice and a 6 in the second dice was asked although a specific order was not mentioned in this item. They disregarded the other option, namely a 6-5. For example, the explanation of Participant 11 as follows:

> Participant 11: "The probability that the first dice lands on 5 and the second dice lands on 6 is $\frac{1}{6} x \frac{1}{6} = \frac{1}{36}$. The probability that both dices land on 6s is also $\frac{1}{6} x \frac{1}{6} = \frac{1}{36}$. Therefore, the probabilities are the same."
>
> [Birinci zarın 5, diğer zarın 6 gelmesi olasılığı $\frac{1}{6} x \frac{1}{6} = \frac{1}{36}$ dır. Her 2 zarın da 6 gelmesi olasılığı da $\frac{1}{6} x \frac{1}{6} = \frac{1}{36}$ dır.Bu yüzden, olasılıklar aynıdır.]

Similarly, Participant 1 also held this misconception. Different from participant 11, though, in his explanation, he mentioned the independence of the dices, he stated as follows:

> Participant 1: "They are equal because the dices are independent of each other. Therefore, getting 6-6 and 5-6 are equal."
>
> [Eşittir.Çünkü zarlar birbirinden bağımsızdır. Bu yüzden, 6-6 ve 5-6 gelmesi eşittir.]

To sum up, none of the participants were successful at answering this item as can clearly be seen in Table 4.6. The participants could not recognize the existence of the compound event (getting one 5 and one 6). The reason underlying this misconception was that they ignored the order of the outcomes and could not list all possible outcomes for this compound event.

4.7 Summary of the Findings

There were two aims of the current study. Determining performance of prospective elementary mathematics teachers on answering the items which addressed the probabilistic misconception was one of the aims of this study. The other aim was to examine the underlying reasons of these misconceptions in cases where they did hold these misconceptions. As mentioned before, firstly it was found that a vast majority of the participants held the time axis fallacy misconception. The reasons underlying this misconception were that they misinterpreted the items and just focused on the first event . Thus, they could not consider the fact that the second event could actually affect the probability of the first event. Secondly, the incidence of the misconception regarding the conditional probability misconception was observed in more than half the participants' responses. The main reason of this was the misinterpretation of the problem and thus ignoring the condition. Thirdly, almost all participants had the misconception regarding the effect of sample size. The major reason underlying this was that a good many of the participants just focused on the ratio regardless of the sample size. They could not notice the fact that with a large sample size, the probability of the event would get closer to its theoretical probability. Fourthly, a great majority of the participants held the conjunction fallacy misconception. The main reason of this was that they were affected by the storylike nature of the item. That is, they focused on the narrative. Fifthly, the number of the participants who held misconception regarding representativeness was significantly higher than that of the participants who provided the correct answer. The major reason behind this misconception was that those who had this misconception determined the probability of an event depending on how well the sample represents its parent population. The last but not least, all participants had the misconception regarding the compound event. They could not recognize the existence of the compound event. Therefore, they computed the probability of the compound event as if it were a simple event. The reason underlying this misconception was that they ignored the order of the events in the compound event. That is, they could not recognize that the compound event consisted of two outcomes obtained by the changing the order of these outcomes.

CHAPTER V

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The aim of this study was to determine performance of prospective elementary mathematics teachers on answering items handling the probabilistic misconceptions. The other aim was to explore the reasons underlying the misconceptions that prospective elementary mathematics teachers held. This chapter presents a discussion on the research findings by considering each probabilistic misconception and provides recommendations for the further research studies and educational implications for curriculum developers and teacher educators.

The research findings were discussed under six main sections based on the research questions. In these sections, misconceptions regarding time axis fallacy, conditional probability, effect of sample size, conjunction fallacy, representativeness, and compound events were discussed with references to previous studies, respectively.

5.1 Misconception Regarding Time Axis Fallacy

Only less than a quarter of the prospective elementary mathematics teachers correctly responded to the items regarding the time axis fallacy misconception. The participants who held this misconception could not recognize the fact that the conditioning event could come after the target event. According to these participants, there was a temporal relationship between the two events. More precisely, the participants steadfastly stated that the result of the second event did not affect the probability of the first event. According to the results of this study, the reason behind this misconception was that the participants solely focused on the first event claiming that the second event had not occurred when the first one occurred. These findings are consistent with the findings of the study carried out by Carnell (1997) with prospective middle grade students since in her study, some of the participants who had this misconception focused on the timing of the events. The other reason behind this misconception might be related to their lack of experiences in such situations where the conditioning event is followed by the target event. Additionally, this misconception might result from the lack of knowledge of the sample size concept. More precisely, several participants ignored the decrease in the sample size of the first event and thus could not correctly determine the sample size, which may lead to this misconception.

In addition to these participants who focused on the first event, nearly half of the participants of the present study computed joint probability (P (B and A)) rather than the conditional probability (P (B/A)), which resulted from the misinterpretation of the problems in the test. This result is consistent with a previous study conducted by Carnell (1997) in which the subjects confused conditional probability and joint probability. Confusing conditional probability with joint probability might be explained by lack of knowledge in the concept of the dependent event, which was also pointed out by Carnell (1997). She indicated that without a good grasp of the concept of the dependent event, it would be difficult to get in-depth insight about the concept of conditional probability. This inadequacy in dependent event concept might be attributed to insufficient instruction on this issue. More precisely, teachers may not cover all of the objectives regarding the concept of dependent event, which lead to this misconception.

5.2 Misconception Regarding Conditional Probability

The findings of the current study pointed out that more than half of the prospective elementary mathematics teachers had the misconception regarding conditional probability. Some of the participants tended to rely on the new information alone and ignored the initial probabilities. The similar results were also found in the study of Jendrazsek (2008) where graduate students focused on the new sample size. In the present study, other than the participants who had ignored initial probabilities, some of them computed the joint probability, which was the case in the items addressing the time axis fallacy misconception. Obtaining similar responses seems to be reasonable since the items handling misconceptions regarding time axis fallacy and conditional probability are, in fact, similar in that both items include the conditional probability concept. On the other hand, the difference between them is that in the items addressing the time axis fallacy misconception, the target event is the first event and the conditioning event is the larger event, which does not have to be in the items addressing the misconception regarding conditional probability. The findings of this study revealed that the reason behind this misconception was a misinterpretation of the problem.

As mentioned in the previous section, misinterpretation of the problem might be explained by the lack of knowledge in dependent event concept. Additionally, Gürbüz (2006) claimed that individuals' lack of experiences in conditional probability concept might lead to misinterpretation of problem. This claim could be reasonable since the topic of the conditional probability is a subject of instruction only at 11^{th} grade (MoNE, 2011). Therefore, the understanding of the conditional probability might not be developed sufficiently.

Misconception Regarding Effect of Sample Size

The participants' performances on the items addressing the misconception regarding the effect of sample size were poor. That is, nearly none of the students correctly responded to these items. The participants who held this misconception believed that it was relevant to use the magnitude of the sample size when estimating the likelihood of an event. According to the results of this study, the reason behind this was that the participants focused on the ratio, indicating that the probability means a ratio. In other words, these participants over generalized the meaning of ratio concept to the probability concept. The unnecessary emphasis on the ratio prevented them from noticing the sample sizes. This finding was supported by that of Fischbein and Schnarch (1997) since they claimed that the additional concepts such as ratio and proportion can impede subjects from noticing the effect of the sample size. Ignoring the sample size might have resulted from the lack of knowledge in the experimental and classical probability as emphasized by Dollard (2007) and Steinbring (1991b) who worked on the issue of insensitivity to sample size. They stated that without understanding these two concepts and the connection between them it would be difficult to understand the logic behind this law. This ignorance appeared even in the correct response of the participants of this study. More specifically, the participant who gave the correct answer justified his answer depending on the deviation, not mentioning about the experimental and theoretical probability. The lack of knowledge in experimental and theoretical probability concepts might result from its insufficient place in the curriculum. In the Turkish curriculum, the concepts theoretical and experimental probabilities are introduced only in grade 8 and the objective related to these concepts is as follows: "Students are able to explain experimental, theoretical and subjective probability "(MoNE, 2011, p.295). As could be understood, this objective represents the comprehension level of Bloom's taxonomy, which might prove to be insufficient in enabling students to apply their knowledge on these concepts to other situations and to develop a profound understanding of these concepts.

5.3 Misconception Regarding Conjunction Fallacy

The analysis of the findings revealed that the majority of the participants had misconception regarding conjunction fallacy. These participants judged the conjunctive event as being more probable when compared to its components. Stated differently, they failed to use the conjunction rule which indicates that the probability of the intersection of two events (i.e. conjunction) cannot be higher than that of one of its constituents. In this context, the findings of this study was parallel to those of a study carried out by Kennis (2006) with students in grades 9,10,11, and 12 as in his study the responses of the participants showed that they made a false reasoning in determining the probability of the conjunctive event and its component. According to the findings of the present study, it can be stated that the participants unnecessarily focused on the narrative, which led to this misconception. Therefore, they thought that the conjunction was more representative in terms of personality sketch of the given character. Focusing on narrative might be related to their experiences in daily life. More precisely, they may judge the probability of events depending on their experiences, which shows the incidence of the subjective approach. As a result, using the subjective approach might give rise to the existence of this misconception.

In addition, item addressing the misconception regarding conjunction fallacy is closely related to fraction concept. Stated clearly, the probability of conjunctive event can be computed by multiplying two fractions, which made the probability of this event be lower than that of its constituents. Hence, unnecessary emphasis on narrative might be explained by lack of knowledge in fraction concept.

Misconception Regarding Representativeness

The findings of this study indicated that a vast majority of the participants gave incorrect responses to the item addressing the misconception regarding representativeness. When compared to a sample consisting of the outcomes which are in non-random order, a sample which includes random outcomes is seen as more probable by the participants who hold this misconception. That is, these participants sought randomness in the samples. According to the findings of this study, the reason behind this misconception was that the participants evaluated the probability of the samples by considering its resemblance to their parent population. In this context, these findings were corroborated with the research done by Tversky and Kahneman (1974), where they found that the students determine the probability of getting a sample depending on how well this sample represents its population.

The outcomes in the samples were independent of each other and thus the probability of getting samples would be the same. Therefore, this misconception might be related to a lack of knowledge in independent events and equal chance of the events. In the Turkish curriculum, the concept 'independent event' has been introduced in grades 8 and 11. Particularly, the objectives related to this concept in 8 grade is “students explain the dependent and independent events” (MoNE, 2011, p.308) and in grade 11 is " students explain the dependent and independent events with examples" (MoNE, 2011, p.180). These objectives might show that the students are not expected to develop a high level of skills regarding 'independent event' based on Bloom's Taxonomy. Therefore, prospective elementary teachers who have been educated by means of the curricula implemented in elementary and high school levels will probably short fall in the concepts of independent and dependent events, which might result in the misconception regarding representativeness.

5.4 Misconception Regarding Compound Event

According to the analysis of the findings, none of the participants could provide the correct answer to the item examining the misconception regarding compound events. From the given expressions of the participants of this study, it was seen that they failed to recognize which of the events given in the item was the compound event and which of them was the simple event. Findings revealed that these participants could not recognize the order of the events, which led to this misconception. Furthermore, the participants who hold this misconception attributed the same probability to both simple and compound event. In this context, the findings of this study is supported by those of Lecoutre (1992) as in his study, the participants showed a tendency to suppose that the probabilities of the simple and compound event are the same.

The other reason underlying this misconception is the lack of focus on the sample space associated with the compound event as pointed out by Fischbein, Nello and Marino (1991) who conducted a study with students whose ages ranged from 9 to 14. These researchers indicated that a reasoning which fails to take any cognizance of the sample space of the compound event can cause individuals to hold this misconception. The other reason might be related to insufficient instruction about compound event. For example, these participants might not be asked to respond questions involving compound events. Therefore, they might not develop a deep understanding of this concept.

5.5 Implications

This study offers valuable information to mathematics teachers, textbook writers, teacher educators and curriculum developers about prospective elementary mathematics teachers' mistakes and misconceptions regarding concepts of the probability. Findings of this study revealed that there was no item for which all of the prospective elementary mathematics teachers gave correct answer with a reasonable justification. In order to overcome this circumstance, teacher and teacher educators should take probabilistic misconceptions into consideration.

To start with implications for teachers, they should ask students to respond the items treated in this study, which can be followed by experiments so that students can determine the probability of events empirically as well as theoretically. Particularly, these experiments can be in the form of actual situations or computer simulations such as “Probability Explorer” and “Tinker Plots”. By the help of these computer simulations, students can have a chance to simulate and analyze a variety of probabilistic situations which were mentioned in the items handled in this study. At this point, teachers should engage students discuss the results of these experiments so that students can get conceptual understanding of probability.

The inadequate performance on the items addressing probabilistic misconceptions might be derived from teachers' knowledge and practices during instruction on probability since teachers will probability teach what they know as stated by Even (1990). Therefore, making aware of teachers about the probabilistic misconceptions and reasons behind those misconceptions revealed in this study is important. In order to make this aim happen, seminars or in-service training programs regarding these issues can be conducted.

The implications can be extended to textbook writers. They might include the questions in the current study into these books so that teachers use them during the instruction on probability. For example, teachers can ask students to respond to these questions. By means of these questions, teachers can explore students' misconceptions during the instruction.

In addition to teachers and textbook writers, teacher educators can also benefit from the findings of the present study. For example, teacher educators can inform prospective elementary mathematics teachers about prevalence of the probabilistic misconceptions and the reasons underlying these misconceptions in mathematics teaching method course, which make prospective teachers aware of these misconceptions. Alternatively, since prospective elementary mathematics teachers education program offered by Higher Education Council (HEC) includes only a few obligatory courses related to statistics and probability, teacher educators can offer elective courses in which the issue of misconceptions regarding probability is deeply addressed so that prospective teachers may explore their own understanding of probability concept.

Analysis of findings revealed that the prospective elementary teachers have lack of knowledge about the concepts of independent and dependent events, theoretical probability, subjective probability, experimental probability, and conditional probability, which cause the prospective teachers hold several probabilistic misconceptions. Without a good grasp of the concepts of the probability, they possibly will not appropriately teach these issues to the students when they become in-service teachers. Therefore, in order to prevent this circumstance, more emphasis might be given on these concepts in teacher preparation programs. Furthermore, the lack of knowledge in these concepts might be attributed to inadequacies in the Turkish curricula since prospective teachers were educated by means of curricula implemented in elementary and high school levels. Therefore, curriculum developers should modify the objectives in relation to prevent the existence of these misconceptions.

5.6 Recommendations for Further Research

Because of the abstract nature of “probability”, individuals proceed to have difficulties in probability. Therefore, further research is always necessary to overcome these difficulties. To this end, research can be conducted on the instrument which was used in the present study; it can be improved further. Particularly, the multiple items reflecting the same misconception can be added to increase the internal consistency. Besides, more items can be included to examine other misconceptions such as negative and positive recency effect, availability heuristics, base rate fallacy, etc. since this study was merely concerned with six specific misconceptions with respect to time axis fallacy, conditional probability, and effect of sample size, conjunction fallacy, representativeness, and compound event in order to provide an in-depth insight into the prospective teachers' probabilistic misconceptions.

This study was conducted with only senior prospective elementary mathematics teachers. Therefore, the same research might be replicated with a large scale to be representative of all Turkish prospective elementary mathematics teachers. Furthermore, a longitudinal study may be conducted to get a better idea about the changes in the existence of the misconceptions regarding probability handled in this study.

Additionally, further research can be conducted by extending this study to examining the misconceptions of teachers at a large scale since this study was performed only on prospective elementary mathematics teachers in order to determine whether they hold the probabilistic misconception.

In addition, the misconceptions held by students at a large scale can be investigated as well. If these misconceptions are examined, they can be remediated during the instruction by putting more emphasis on the concepts of probability.

Several intervention studies can be carried out to investigate whether there is a difference in terms of the existence of probabilistic misconceptions before and after the instruction. For instance, the effects of technological tools can be explored to provide substantial contribution to the field of probability education.

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APPENDICES

APPENDIX A

OLASILIK KAVRAM YANILGISI ANKETİ

Sevgili öğrenciler,

Bu test sizin olasılık konusu üzerinde nasıl düşündüğünüzü ölçmek için hazırlanmıştır. **Bu test sonuçları, sadece araştırma amaçlı kullanılacak ve gizli tutulacaktır**. Test 9 tane sorudan oluşmaktadır. Her bir soru, bir tane çoktan seçmeli ve bir tane açık uçlu sorudan oluşmaktadır. Bütün soruları dikkatlice okuyup size göre doğru olduğunu düşündüğünüz seçeneği işaretleyiniz ve altına neden o seçeneği işaretlediğinizi mutlaka açıklayınız.

Katkılarınızdan dolayı teşekkür ederim.

Münevver İLGÜN

e-mail: milgun@sakarya.edu.tr

Adınız Soyadınız:.................................

Öğrenim Türü: ☐ I.Öğretim

☐ II.Öğretim

e-mail adresiniz:

SORULAR

1)Makinenin E girişine top atıyoruz. Eğer top, sistemi R çıkısından terk ederse, bu topun 1. kanaldan geçmiş olma ihtimali nedir?

a) 1/2

b) 1/3

c) **2/3**

d) Hesaplanamaz

Cevabınızı nedenleriyle açıklayınız.

2) Bir kabın içerisine 2 tane siyah ve 2 tane de beyaz bilye konuluyor. Kabın içinden 1 bilyeyi seçiyoruz. Sonra, bu bilyeyi kaba geri atmaksızın rastgele 2. bir bilye daha seçiyoruz.2. bilye beyazsa 1. bilyenin de beyaz olma olasılığı nedir?

a) **1/3**

b) 1/2

c) 1/6

d) Hesaplanamaz

Cevabınızı nedenleriyle açıklayınız

3) Bir yarışma programında olduğunuzu düşünün. Size ödüllere açılan 3 kapıdan 1 tanesini seçme şansı veriliyor. Kapılardan birisinin arkasında yeni bir araba, diğer 2 kapının arkasında ise evcilleştirilmiş köpek vardır. Diyelim ki 1 numaralı kapıyı seçtiniz. Kapınızı seçtikten sonra yarışma sunucusu başka bir kapıyı, köpeği serbest bırakmak için açıyor ve bunun da 3 numaralı kapı olduğunu varsayın. Daha sonra, yarışma sunucusu size 2. kapıyı açmak isteyip istemediğini soruyor. 2 numaralı kapının arkasında araba olma ihtimali nedir?

a) 1/3

b) 1/2

c) 2/3

d) 3/3

Cevabınızı nedenleriyle açıklayınız

4) Büyük bir şehirde yaşanan elektrik kesintilerinin nedenleri incelendiğinde, verilerden şu sonuçlar elde edilmiştir: Kesintilerin 0,05' i trafo arızasına, 0,80' i hattın arızalı olmasına ve 0,01 i ise her iki nedene de bağlıdır. Hattın arızalı olduğu bilindiğine göre, trafonun da arızalı olması olasılığı nedir?

a) 4/100

b) 1/50

c) 1/80

d) 1/100

Cevabınızı nedenleriyle açıklayınız.

5) Belirli bir kasabada 2 tane hastane vardır. Küçük olan hastanede günde ortalama 20 civarında doğum olurken büyük hastanede ise günde ortalama 60 doğum olmaktadır. Genelde doğan bebeklerin yaklaşık %50'si erkektir. Ama bazı günler vardır ki doğan bebeklerinin % 50' sinden fazlası erkektir. Her iki hastanede de, erkek doğum oranının %60 veya daha fazla olduğu günlerin kaydı tutulmuştur. Hangi hastanede böyle günlerin olma ihtimali daha fazladır?

a)Büyük hastanede

b)Küçük hastanede

c) Her iki hastane için de eşittir.

d) Bir şey söyleyemeyiz.

Cevabınızı nedenleriyle açıklayınız.

6) 2 grup çocuk yazı tura oyunu oynuyor. Hilesiz bir madeni para atıldığında yazı gelme ihtimali yüzde 50 dir. 1.grup (grup A) madeni parayı 50 kez atıyor. 2. grup (grup B) ise 150 kez atıyor. Her seferinde çocuklar gelen sonucu not alıyorlar. Hangi grubun, madeni parayı attığında % 60 oranında yazı getirme ihtimali daha fazladır?

a) Grup A
b) Grup B
c) Her iki grubun eşittir.
d) Birşey söyleyemeyiz.
Cevabınızı nedenleriyle açıklayınız

7) Meltem 32 yaşında, bekâr, açık sözlü ve çok şık bir bayandır. Üniversitede felsefe okumuştur. Öğrenci olarak ayrımcılık konuları ve sosyal adalet ile derin bir şekilde ilgilenmiştir ve ayrıca anti-nükleer gösterilere katılmıştır. Buna göre Meltem ile ilgili olan ifadelerden hangisi daha olasıdır?

a) Meltem profesördür.

b) Meltem politikayla ilgilenen bir profesördür.

c) (a) ve (b) şıkları eşit olasılıklıdır.

d) Hangisinin daha muhtemel olduğu ile ilgili bir şey söylenemez.

Cevabınızı nedenleriyle açıklayınız

8) Bir Türk erkeğinin boyu ortalama 175 cm'dir. Rastgele 3 erkek seçiliyor ve boyları ölçülüyor. Seçilen kişilerin boyları sırasıyla 178cm, 170 cm ve 179 cm'dir. Rastgele 3 erkek daha seçiliyor. Onların boyları ise sırasıyla 175, 175 ve 175 cm dir. Eğer bu uygulama devam ederse, hangi grubun görülmesi daha olasıdır?

a) 1. grubun görülmesi daha muhtemeldir.

b)2. grubun görülmesi daha muhtemeldir.

c) (a) ve (b) şıkları eşit olasılıklıdır.

d) Hangisinin daha muhtemel olduğu ile ilgili bir şey söylenemez.

Cevabınızı nedenleriyle açıklayınız

9) Hilesiz iki zar aynı anda havaya atılıyor. Aşağıdakilerden hangisinin olma olasılığı daha fazladır?

a) İki zarın da 6 gelmesi

b)Bir zarın 5 diğer zarın 6 gelmesi

c)"a" ve "b" şıklarının olma olasılıkları eşittir

d) Yukarıdaki cevapların hiçbiri doğru değildir.

Cevabınızı nedenleriyle açıklayınız.

APPENDIX B

RUBRIC FOR OPEN- ENDED ITEMS

Olasılık Kavram Yanılgısı Anketi Açık Uçlu Sorular İçin Dereceli Puanlama Anahtarı

1.Item

0. No answer

1. Incorrect response with an incorrect explanation

   For example:

   If it passes through channel I, it will be automatically get out off from exit R. Therefore, the probability is (½)x1 = ½.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

   For example:

   $P(I/R) = \frac{P(I \cap R)}{P(R)} = \frac{1/2}{1/2 + 1/4} = 2/3$

2.Item

0. No answer

1. Incorrect response with incomplete reasoning

   For example:

   It does not matter whether the second ball is white or black. The probability of getting the white marble is 2/4.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:

$$P(W_1/W_2) = \frac{P(W_1 \cap W_2)}{P(R)} = \frac{\left(\frac{2}{4}\right) x \left(\frac{1}{3}\right)}{2/4} = 1/3$$

3.Item

0. No answer

1. Incorrect response with incomplete reasoning

For instance:

The probability that one of the remaining doors includes the prize is 1/2.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:

The probability of selecting the door behind which the car is located is 1/3. If you choose the door not including the car, the probability that the host open one of the remaining doors is 1 since the host will not open the door including the prize. If you choose the door behind which the car is located, the probability that the host open one of the remaining doors is 1/2. In every case in which you have not initially selected the correct door, the door including the prize will be unopened door. Hence, the probability that the prize is behind the unopened door is 2/3.

4.Item

0. No answer

1. Incorrect response with incomplete reasoning

For example:

The probability of the desired answered is already given in the problem

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:

$P(T/H) = \frac{P(T \cap H)}{P(H)} = \frac{0,01}{0,80} = 1/80$

5.Item

0. No answer

1. Incorrect response with incomplete reasoning

For example:

The ratio is the same, namely 60%, for each of the hospital. Therefore, the probability of desired event is the same.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:

Depending on the law of large sample, with a small sample, you expect more deviation from the theoretical probability. Thus, the answer is “small hospital”.

6.Item:

0. No answer
1. Incorrect response with incomplete reasoning
2. Correct response with an insufficient explanation
3. Correct response with a sufficient explanation

For example:

Depending on the law of large sample, with a small sample, you expect more deviation from the theoretical probability. Thus, the less you toss the coin, the more the probability deviates from ½.

7.Item:

0. No answer
1. Incorrect response with incomplete reasoning

   For example:

   She interested in the social justice and discrimination. Therefore, the probability of is higher than that of the other responses.

2. Correct response with an insufficient explanation
3. Correct response with a sufficient explanation

For example:

Being a professor interested in politics is less likely when compared to being a professor since the first one can be found by multiplication of two fraction.

8.Item:

0. No answer

1. Incorrect response with incomplete reasoning

For example:

> The probability of selecting three people who are in the same height is lower than that of selecting three people who are in the different height. There must be deviation.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:

The events are independent to each other. The probability of selecting each person is the same.

9.Item:

0. No answer

1. Incorrect response with incomplete reasoning

For example: The probability of getting two 6s and one 5 and one 6 is 1/36.

2. Correct response with an insufficient explanation

3. Correct response with a sufficient explanation

For example:The probability of getting 6-6 is 1/36 while the probability of obtaining one 5 and one 6 is 1/18 since there can be 5-6 and 6-5.Thus, the probability of getting one 5 and one 6 is higher than that of two 6s.

APPENDIX C

TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü [ ]

Sosyal Bilimler Enstitüsü [X]

Uygulamalı Matematik Enstitüsü [ ]

Enformatik Enstitüsü [ ]

Deniz Bilimleri Enstitüsü [ ]

YAZARIN

Soyadı : İLGÜN
Adı : Münevver
Bölümü : İlköğretim Bölümü

TEZİN ADI (İngilizce) : An Investigation Of Prospective Elementary Mathematics Teachers' Probabilistic Misconceptions And Reasons Underlying These Misconceptions

TEZİN TÜRÜ : Yüksek Lisans [X] Doktora [ ]

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın. [ ]
2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullanıcılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) [ ]
3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) [X]

Yazarın imzası .... Tarih ....