INVESTIGATION ON COLUMN STRENGTHS
IN STEEL UNBRACED FRAMES LATERALLY RESTRAINED BY ELASTOMERIC BEARINGS
USING THE AISC CODE

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Submitted by **OMAR ISSA OMAR** in partial fulfillment of the requirements for the degree of **Master of Science in Civil Engineering Department**, **Middle East Technical University** by,

Prof. Dr. Canan Özgen
Dean, Graduate School of **Natural and Applied Sciences** _____________________

Prof. Dr. Ahmet Cevdet Yalçıner
Head of Department, **Civil Engineering** _____________________

Prof. Dr. Çetin Yılmaz
Supervisor, **Civil Engineering Dept., METU** _____________________

Examining Committee Members:

Prof. Dr. Mehmet Utku
Civil Engineering Dept., METU _____________________

Prof. Dr. Çetin Yılmaz
Civil Engineering Dept., METU _____________________

Prof. Dr. Cem Topkaya
Civil Engineering Dept., METU _____________________

Dr. Uğur Polat
Civil Engineering Dept., METU _____________________

Dr. M. Halis Günel
Department of Architecture, METU _____________________

Date: June 10, 2013

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name	: Omar Issa, OMAR
Signature	:

ABSTRACT

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Omar, Omar Issa
M.S, Department of Civil Engineering
Supervisor: Prof. Dr Çetin YILMAZ

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The restraint provided by elastomeric bearings to frames is rarely taken into account in frame stability design. If taken into account, then the consideration that the restraint they introduce might influence sway condition of the frame makes it harder to classify the frame as braced or unbraced .Conventional methods of stability analysis, e. g the Effective Length Method(ELM), which relies on such a classification, can not be readily applied in its analysis, and advanced or more general methods such as the Direct Analysis Method (DAM) will be necessary, unless the sway condition is assumed by some judgement. In this thesis, these later methods are not considered, and the objective is to study methods that are simple to use in design offices. The approach is therefore to scan codes and specifications for latest developments in frame stability design, in the context of ELM.

For this purpose, code provisions in the latest AISC specification(ANSI/AISC 360-2010) and recommendations in Structural Stability Research Council's guide(SSRC-2010) have been selected. Sway index B2, bracing design provisions and possible use of commercial software for determination of K-factors are investigated. In the investigation, a realistic set of frames, with features of frames in a building likely to be equipped with such bearings, have been selected and analysed.

This study focuses on potential methods and means that are available to a designer and not on economy of design or variations in framing configurations, and is therefore not comparative.

Keywords: Effective Length, Effective Length Method, Stability Bracing, Elastic Critical Buckling, Frame Stability.

ÖZ

YANAL ÖTELEMEYE KARŞI ELASTÖMERLERLE DONATILMIŞ ÇELİK ÇERÇEVERELERİN AISC ŞARTNAMESİNE GÖRE KOLON KAPASİTELERİN İRDELENMESİ

Omar, Omar Issa
Yüksek Lisans, İnşaatMühendisliğiBölümü
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Çerçeve stabilite analizlerinde elastomer mesnetlerin katkıları çok nadiren gözönüne alınmaktadır. Eğer tasarımcı elastomer mesnetlerin katkısını almayı tercih ederse, sözkonusu çerçevenin yanal öteleme durumunu sınıflandırmak oldukça zorlaşmaktadır. Bu yüzden geleneksel stabilite analiz yöntemleri, örneğin Etkin Boy Yöntemi (ELM), bu tür yapılara direk olarak uygulanamamaktadır. Bu nedenle eğer tasarımcı çerçevenin yanal öteleme durumu hakkında bir yorum yapamıyorsa, analizlerde direk analiz metodu kullanılması gerekliliği vardır. Bu tezde, tasarımcılar tarafından kolaylıkla kullanılabilecek metotlar üzerine bir çalışma yapılacaktır. Çalışma boyunca son dönemlerde yayımlanan standart ve/veya şartnameler incelenerek çerçeve stabilite analizlerindeki son gelişmeler mercek altına alınacaktır.

Bu amaçla, AISC'nin en son yayımlanan versiyonundaki (ANSI/AISC 360-2010) ve Stabilite Araştırma Konseyi Klavuzundaki (SSRC-2010) kod hükümleri kullanılacaktır. Bahsi geçen kaynaklar kullanılarak B2 katsayısı, çapraz tasarım hükümleri ve K faktörlerini belirlemede olası ticari yazılım kullanım olanakları incelenecektir. Bu çalışma boyunca, bir dizi elastomer mesnet barındıran örnek çerçeveler üzerinde analizler gerçekleştirilecektir. Son olarak bu tezin amacı çerçeve tiplerinin ekonomiye etkisini incelemek yerine tasarımcı mühendisler tarafından kullanılabilecek metotların karşılaştırmasını yapmaktır.

Anahtar Kelimeler: Burkulma Boyu, Etkin Boy Yöntemi, Stabilite Çapraz,Elastik Burkulma Yükü, Çerçeve Stabilitesi.

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TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

LIST OF SYMBOLS/ABBREVIATIONS

A	Cross-sectional Area or Surface Area (m^2)
A_T	Tributary Area Used in Reduction of Live Load in ASCE 7-2010
ABYBHY'07	Turkish Seismic Code, 2007
AISC	American Institute of Steel Construction
AISC'05	ANSI/AISC 360-2005, AISC specifications for Structural Steel Buildings
AISC-2005	ANSI/AISC 360-2005, AISC specifications for Structural Steel Buildings
AISC'10	ANSI/AISC 360-2010, AISC specifications for Structural Steel Buildings
AISC-2010	ANSI/AISC 360-2010, AISC specifications for Structural Steel Buildings
ASCE 7-2010	ASCE Minimum Design Loads for Buildings, 201.
ASD	Allowable Stress Design (here in the context of AISC)
B1	Moment Amplification Factor for Non-sway Condition
B2	Moment Amplification Factor for Sway Condition
C	Stability Function
Ct	A Coefficient Used in Empirical Formula for T.
DL	Dead Load
E, Et,Er	Modulus of Elasticity (MPa), Earthquake Loading, Tangent Modulus(MPa), Reduced Modulus (MPa)
ELM	Effective Length Method
Eurocode 3	European Steel Design Code for Buildings, EN 1993
Fcr	Critical Buckling Stress (MPa)
Fe	Euler Buckling Stress (MPa)
Fy	Yield Strength (MPa)
G	Shear Modulus (MPa)
G, G_A,G_B	Column- End Flexibilities.
H, H_N	Floor Height (m), Lateral Load(kN), Height of a Building (m).
I,Ix,Iy, Ir	Moment of Inertia, about x, about y axes (m^4), Moment of Inertia in Double Modulus Theory (m^4)
K,Kx,Ky	Effective Length Factor, about x, about y axes, also Structural Stiffness.

Kxy	Horizontal Stiffness of Mageba Elastomeric Bearings.
L	Length of Member or Span (m)
Lb	Unbraced Length or Length Between Nodal Supports (m).
LC	Load Combination
LL	Live Load
Lo	Initial Length(m)
L0	Unreduced Live Load (kN, kPA)
LRFD	Load and Resistance Factored Design (Here in the context of AISC)
M,Mx,My,Ma,Mb	Moment (kNm)
Mc	Available Moment Capacity of a Member (kNm)
Mn	Nominal Flexural Capacity of A Member (kNm)
Mr	Required Moment Capacity of a Member (kNm)
Ni	A Coefficient Used to Associate Ideal Stiffness with Pr/Lb in Bracing Design.
Pc	Available Axial Strength (kN)
Pcr	Critical Elastic Buckling Load (kN)
Pcr*	Critical Inelastic Buckling Load (kN)
Pe	Euler Buckling Load (kN)
Pe2	Story Elastic Sway Buckling Load (kN)
Pr	Required Axial Strength, Double Modulus Load (kN)
Pn	Nominal Axial Strength (kN)
Pt	Tangent Modulus Load (kN)
Py	Yield Strength (kN)
Pu	Ultimate Axial Strength, Factored Axial Load (kN)
R	Structural Behavior Factor Used in ABYBHY'07
R_M	Coefficient Used in Determination of Pe2.
S	Stability Function, Section Modulus (mm^3)
Sx	Section Modulus, strong axis (mm^3)
S(T)	Spectral Acceleration Coefficient used in ABYBHY'07
T	Period of vibration (seconds)
Wk	Seismic Dead Weight of a Floor used in ABYBHY'07

Z,Zx	Plastic Modulus, strong axis (mm^3)
c	Hyperbolic Functions Used to Determine Stability Functions C and S
h	Depth of section (mm), Story Height(m)
k	A variable used in stability theory ($k^2 = P/EI$)
r	Radius of Gyration (mm)
s	Hyperbolic Functions Used to Determine Stability Functions C and S.
te	Net Thickness of Rubber in Reinforced Elastomeric Bearings (mm)
v_{xyd}	Lateral Displacement of Bearings as Used by Mageba.
α	Adjustment Factor for Force Level in Relation to Inelasticity, A Parameter for Member Imperfection in Column Curves to Eurocode 3, Correction for Stiffness of Girders (Ig/Lg) in the Expression for Column Node Flexibilities G_A and G_B.
β	Spring Stiffness (kN/m)
βi	Ideal Spring Stiffness(kN/m)
βbr	Stiffness of a Bearing (kN/m)
Δ	Lateral Displacement, Floor drifts (m)
Δ0	Initial Imperfection or Displacement (m)
Δ_H	Lateral Deflection of a Story (m)
Δ_T	Total Deflection (m2)
Φ	Strength Reduction Factor Applied to Nominal Strength of A Section or A Member in LRFD
λ	Slenderness ratio KL/r, Load Factor Used in Buckling Analysis.
$\theta,\theta a,\theta b$	Member End Rotations(rads)
Ω	Factor of Safety Applied to Nominal Strength of A Section or A Member in ASD
ρ	Member End Rotations (rads), Curvature
$\tau,\tau a,\tau b$	Stiffness Reduction Factor on Account of Inelasticity.

CHAPTER 1

INTRODUCTION

1.1 Introduction

The frame in Fig.1, is the motivation of this thesis, because it presents a problem: its stability is not readily assessed if ELM is used in its design.

In the context of stability design, frames are usually classified depending on whether their lateral resistance is provided by a system of bracing or by the bending resistance of its members and connections[25], and frames with such well defined forms of lateral resistance are described respectively as braced or unbraced. Once a frame is classified as such, simplified or conventional methods are readily available for their design. One such method is ELM.

The frame in Fig. 1, on the other hand, can use the frame action alone, the lateral resistance available in the bearing element alone, if either is more dominant than the other, or combine both to some degree. The classification mentioned above can not be easily made on this frame and this presents a problem when traditional methods such as ELM, which generally requires this classification as part of the design process. Code provisions are generally very supportive of traditional methods for frames that can be classified as either braced or unbraced, but not so for frames that are not conventional, like the one shown.

DAM, adopted by the AISC since 2005, can be used for design of all kinds of frames, irrespective of how they are braced, and it is described as a method of choice for complex frames and frames with leaning columns [2]. DAM was developed following years of efforts to eliminate the ELM and its controversial K factors. In DAM, K is set to unity. However in this thesis, DAM is not considered.

But over those years and in the process of development of methods such as DAM (similar methods have been used in Europe, Canada and Australia), several studies were made on frames that can use K=1 with ELM , and in recent years, results from such studies have been incorporated in code provisions, including AISC [34] . The European and British steel specifications for buildings allow designers to use a sway index in the process of determining whether second-order effects – in relation to sensitivity to moment magnification - is significant or not [Eurocode 3, British standard BS5950-1:2000]. AISC also allows use of K=1 if a limit on sway sensitivity (measured using the moment magnification factor B2) is not exceeded. In fact, the limits of sway sensitivities by use of the European sway index and American B2 lead to the same criterion that anables K=1 [34].

Use of K=1 for braced frames and K>1 for unbraced frames is very well established, and use of K<1 for unbraced frames is viewed with suspicion in practice. This is probably because most designers are not yet familiar with these recent code provisions. With respect to braced frames too, what constitute a bracing system is not yet quantified in most codes. The practice is to consider conventional bracing systems, e.g X, V, K bracings, shear walls and connections to RC structures as bracing to a steel frame. The degree of bracing provided by any element with lateral restraint, can now be checked using the elaborate AISC bracing design criteria (introduced since 1999). With those criteria, bracing action of bearings can be verified and K=1 can be used if the criteria are met.

Surely, the designer can always choose to treat the frame of Fig. 1 as unbraced and go on to design the frame accordingly. The resulting design will surely be on the safe side and perhaps not any different from a more refined design, depending on the degree of restraint furnished by the bearings. It is not the purpose of this thesis to make a comparative study between designer's choices. The purpose is to investigate on procedures available if a designer chooses to include restraint offered by the bearings in design.

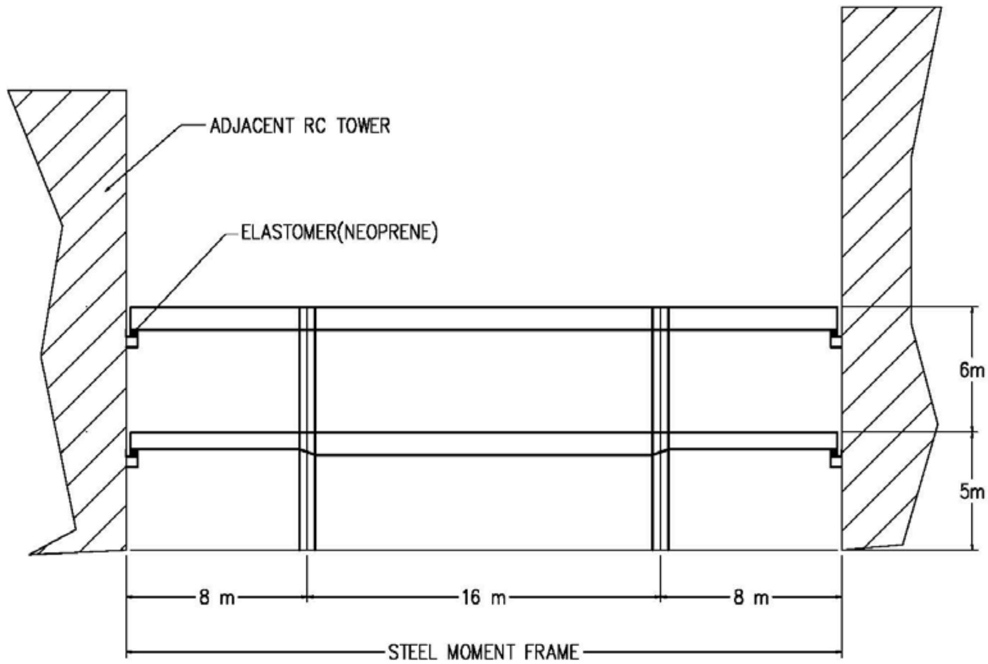


Fig.1- A steel Unbraced Frame with Bearings.

Use of bearings is often made to allow for flexibility at building joints and permit movements and rotations expected between buildings, especially movements induced by thermal forces. Bearings are rarely used for bracing purposes, and their lateral stiffness may not be worth relying upon. It is therefore entirely up to the designer to consider their contribution to stability.

A framing similar to the one above, was used in 2010 in The Zorlu Center Project, Istanbul, using ELM. Use of ELM in 2010 was widespread even in US, since DAM has only been recently popularized by the AISC code, starting 2005, but even then it was considered an alternative design method to ELM. In AISC-2010, DAM is the major method [1,2].

The Zorlu frame has strong architectural and structural characteristics that could make use of such frames to be repeated somewhere else. This thesis builds from those characteristics.

From the architecture point of view, the steel frame is desired to provide public space (e.g shopping and re-creation) in a shallow long-span building with large column-free area, squeezed in a highly prized land, between two reinforced concrete (RC) towers reserved for more private uses (e.g hotels and office floors) .

Structurally, in response to architectural needs, and the fact that the structure is located in a highly seismic zone, the frame needs to be somehow isolated from the taller and more rigid RC towers. Careful design of joints is required for both thermal and seismic movements.

In the Zorlu project, the choice was to use long span steel frames, which at floor levels were fitted with elastomeric bearings. The bearings facilitate a flexible interface with the RC towers.

Motivated by the Zorlu frames, this thesis assumes such applications can be repeated, and assumption is made that the bearings can contribute to stability.

The thesis does not address the design of the Zorlu frames, as those frames are quite irregular in plan and elevations with much higher loading at top floors. It does however use its dimensions with a slight modification, in studying frames so that a realistic analysis can be made. The Zorlu frames provide the basis for selection of frames investigated in this thesis.

In the Zorlu frames, bearings were planned at every floor level and for every frame. The distribution of bearings is uniform.

Some designers, usually for reasons of economy, may prefer to reduce number of bearings per floor with non-uniform distribution. In this thesis, such variations are not considered, since the purpose is to investigate on methods of design with bearings taken into account, and not on levels of economy. It is assumed that if bearings are not large in number, a designer can simply ignore them in stability design.

1.2 Limitations

The thesis examines a design office problem: its scope is limited by the following:

- With respect to ELM, strength of columns is reflected directly in K-values. In the thesis, K values are investigated by several code and code-permitted analysis methods.

- Provisions of AISC Specifications for Structural Steel Buildings[1,2] and guidelines of SSRC Guide to Stability Design Criteria for Metal Structures[3,4,5] .

- Commercial software as would be used in a typical design office, here SAP 2000 by Computers and Structures Inc., Berkeley California is used.

- Linear bearing behavior, i.e dynamic behavior not taken into account.

- Building plane frames of up to 4 story high, 2-3 spans between, span 8-12meters as would fit description of typical buildings in which these frames will be needed architecturally.

- Only in- plane stability is considered, since the purpose is to investigate on methods of stability design for moment frames.

- Only flexural buckling is taken into account, as this is the governing form of buckling that leads to overall instability of a framed structure.

1.3 Organization of The Thesis

Chapter 2 presents a review of structural stability and stability design concepts in relation to steel structures, believed to be important in developing methods of analysis used in the thesis. Mathematical treatments are avoided, instead emphasis is made on the nature of the solutions and important findings directly useful for stability design and for the purpose of the thesis.

Chapter 3 discusses the methods selected, namely use of sway-sensitivity checks, bracing design criteria of AISC'2005 and AISC'2010 and computer elastic buckling analysis, in great detail, presenting along with them procedures to be followed, problems that might be encountered in their application, and underlying assumptions in analysis of frames covered within the scope of this thesis.

In Chapter, 4 selected frames are analysed using procedures laid down in chapter 3.

Chapter 5 presents a summary of results and conclusion.

CHAPTER 2

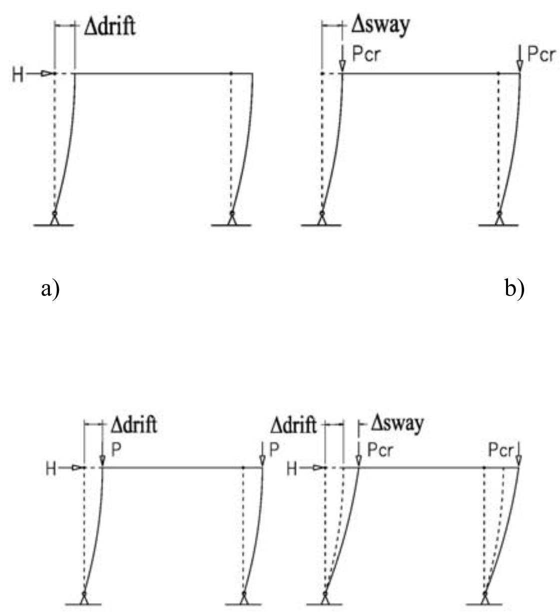
REVIEW OF CONCEPTS IN STABILITY AND STABILITY DESIGN

2.1 Purpose

This chapter presents a short review of some fundamental buckling and stability concepts that are considered important for the development of the thesis. No derivations are attempted. Equations, definitions and conclusions drawn and inferred from the sources cited are compiled for use in later chapters.

2.2 Sway(Lateral Stability) vs Drift

When used with stability concepts, sway does not refer to lateral displacement (drift) caused by lateral loads. The distinction between the two deformations is made clear by Higgins [35] and is illustrated in Fig. 2 below.



a) b)

c) d)

Fig.2: Sway v.s Drift

The term sway refers to lateral deformation caused entirely by the presence of large axial force alone as the axial force approaches the load Pcr that can cause instability of the frame as shown in Fig. 2a.

In comparison, in Fig. 2c, the lateral displacement is caused by lateral load H as vertical load P is nowhere close to Pcr. In Fig. 2d, drift caused by H is eventually magnified further by the presence of large axial load Pcr. In this figure, sway is the later part of lateral deformation associated with Pcr.

Higgins suggested that the term sway in stability be visualized with gravity loads alone in order to eliminate a source of confusion.

2.3 Buckling

Yoo and Lee[9] define buckling as a phenomenon by which a reasonably straight slender member bends laterally, usually abruptly from its longitudinal position due to compression. They describe two kinds of buckling:

- *Bifurcation type:*

  This type occurs in perfectly straight homogenous columns under pure concentric loading. It is also theoretically possible in symmetrical frames.

- *Deflection-Amplification type:*

  This type occurs in imperfect columns. It also occurs in unsymmetrical frames.

When a bifurcation buckling load is reached, the member is straight still, but a slight disturbance will cause it to deflect laterally and suddenly, without increase in loading, i.e stiffness of the member is exhausted. The straight and bent configurations are then possible at the same load P. This condition of dual shape at the same load is known as bifurcation, and the associated load is known as The Bifurcation Load.

Deflection Amplification form of buckling, is not sudden, but column starts to bend once loaded. The relationship between load and deflection however becomes indeterminate as the amplification takes place towards the buckling load. Bifurcation is not observed, but the limiting load is the same as the bifurcation load.

2.4 Euler Buckling, Critical Loads, Modeshapes and Elastic Buckling

The first engineering column formula is credited to Euler. This and further details of historical development of Euler and consequent column formulas are discussed and presented in detail by Johnston[30].

The column problem was tackled in 1744 and elaborately later in 1759. This period coincides with the first developments in small-deformation bending theory [30].

Pe is often taken to be Euler load for simple columns of all support conditions. In this section Pe is the Euler load for a pin-ended simple column.

The main interest is on assumptions in, limitations of and approach in the procedure used by Euler to derive Pe. The equations and assumptions presented, however, take the form known today- equations and form they took in Euler's time are slightly different [30].

Euler's column is perfectly straight, homogenous and is assumed to remain elastic until it fails purely by flexural buckling alone under concentric axial compression. All other forms of failure are not considered. Deformations associated with Euler buckling are flexural only (no shear and no axial deformations), and are assumed to be very small.

Equation of equilibrium is written for a slightly displaced geometry as shown in Fig. 3.

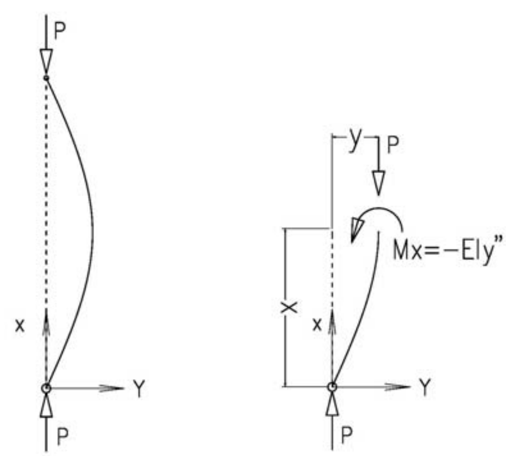


Fig.3 : Euler Column

The equation of equilibrium is a differential second-order linear differential equation of the form:

$$Mx=P.y=-EIy''$$

or

$$EIy''+Py=0$$

The characteristic equation after boundary conditions are applied is:

$$A.\sin kL=0$$

where

$$k=\sqrt{\frac{P}{EI}}$$

and the solution is

$$kL = n.\pi$$

from which for n= 1

$$P = Pe = \frac{\pi^2 EI}{L^2}$$

This is the lowest load and was indeed described by Euler as the "force necessary to bend the column even in the least degree"[30]. Higher loads correspond to $n > 1$. Such values, being larger than Pe are of no practical value. Pe is the critical buckling load for a pin-ended ideal column.

The following characteristics of the solution process are observed :

- The coefficient A remains undetermined: the magnitude of deflections are unknown, the shape however are known to be multiples of a sine curve. *Pe* is the lowest critical load and is associated with a half-sine wave. Euler solution is incapable of producing a Load-Deformation Curve. The reason for this, is the assumption of small deformations in its derivation.

- The process of determining critical loads, known as elastic critical loads, involves writing equations of equilibrium considering a deformed state, assuming small deformations and a material that remains elastic throughout loading. The result has the characteristics of a solution that takes into account geometric nonlinearity, or a linear second-order analysis with small deformations.

- Deflections are indeterminate, but equations of the deformed shapes are obtained in the solution. The problem of elastic critical buckling with small deformations has the characteristics of an eigenvalue problem.

Timoshenko [11] presented a general fourth-order equation instead of the second- order one. This equation is applicable to all cases. For each particular case relevant boundary conditions are applied on the equation. Derivations of Euler buckling load Pe for simple columns and frames are available in most standard structural stability texts, e. g [6,7,9,11].

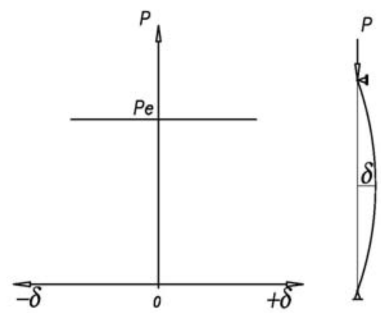


Fig.4 – Load Deformation Behavior of Euler Column

Euler loads are only useful for very slender metal columns [30].

2.5 Large Deformation Theory -Elastic Column

In this section the insignificance of large deformations with respect to buckling load of elastic columns is presented using derivations from Chajes [6] and Gaylord et al.[25].

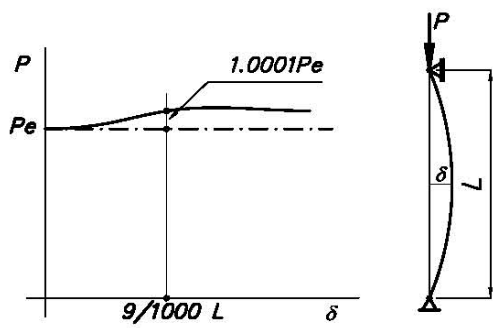


Fig.5- Influence of Large Deformation Theory on Slender Steel Columns [25]

Chajes presents equilibrium equations that include large deformations, by considering large curvature and exact equation for curvature instead of the the one used in small-deformation theory. The load-deformation curve obtained in [6] is similar to the one presented by Gaylord in Fig. 5. Gaylord's is used here since it illustrates the behavior considering a real steel column and a typical large deformation encountered in practice. The curve shows increase in capacity above Pe as deflection increases, but the increase is very small and of no significance in practice. Chajes [6] also suggests that the increased capacity is rarely realized in elastic range as large deformations force combined stresses to exceed yield strength.

The large-deformation capacity based on elastic behavior is therefore not realistic and small deformation theory is sufficient for the purpose of determination of elastic critical buckling loads.

2.6 Imperfect (Elastic) Columns

Column imperfections include all sources that would cause small initial moments once column is loaded. They can be caused by :

- Initial curvature
- Eccentricity of loading

Using the fourth-order differential equations suggested by Timoshenko, and including initial curvature or eccentricity, both of which are small, load deformation curves are obtained. The derivations can be found in [6,7].

Typical behavior is summarized in Fig. 5 and 6 [6]:

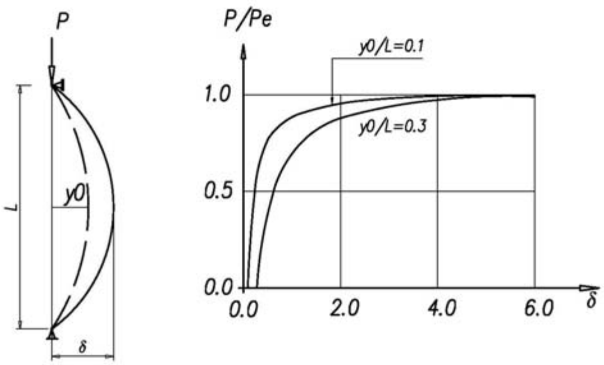


Fig.6-Effect of Initial Bow Imperfections [6]

For initial bow imperfection, the deflection curve is given by:

$$\delta=\frac{y0}{1\text{-}P/Pe}$$

In case of initial eccentricity, deflection is given by the secant formula:

$$\delta=e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{Pe}}\right)\text{-}1\right]$$

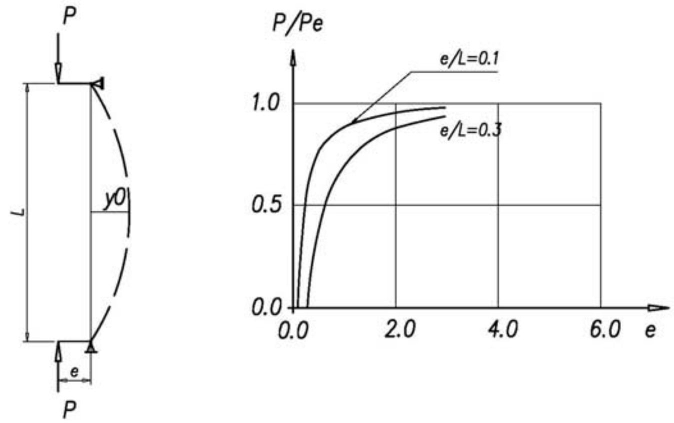


Fig.7- Effect of Initial Eccentricity [6]

The following are observed:

- Both columns start bending once loaded.

- Deflection is not sudden and not significant until load approaches Euler Load Pe, and column capacity eventually reached is Euler's Pe.

- Both models of small imperfections, behave amost the same.

- Unlike perfect columns, load-deflection curves are available

- But like perfect columns, derivations take into account only geometrical nonlinearity .

Chajes[6] makes the following remarks in relation to behavior of imperfect columns:

- The behavior of an imperfect system can be simulated by either of the two models, i.e initial curvature or eccentricity.

- Euler theory, which is based on the fictitious concept of a perfect member, provides a satisfactory design criterion for real imperfect columns, provided the imperfections are relatively minor.

2.7 Inelastic Columns

Inelasticity is due to material nonlinearity as observed in material stress-strain relationships. Inelastic behavior means stress and Elastic Modulus, vary with strain or deformations. One source of this nonlinearity , peculiar to steel shapes, especially wide-flanged shapes, is the presence of residual stresses. It is discussed in section 2.7.

Euler was aware of inelasticity and had hinted that his column theory did not account for it back in 1759. However, he did not attempt a solution. It took a 130 years after him before Engesser first presented strength of a perfect inelastic column, the Tangent Modulus Theory in 1889, which he replaced, following a criticism,with the Double Modulus Theory in the same year.

The Double Modulus Theory had a strong theoretical backing but did not agree with experimental results. The Tangent Modulus Theory, on the other hand, seemed theoretically incompatible with the very basic definition of buckling (that straight columns remains straight or bend at buckling without any increase in load) , but agreed quite well with experimental results. This column paradox lasted almost 60 years, delaying completion of the column curve, until F.R Shanley settled the matter in favor of Tangent Modulus Theory in 1947, almost 200 years since Euler's first attempt.

Shanley's Contribution is considered one single important development in column design since Euler[30].

Tangent Modulus Load is the load at which an inelastic column remains straight or starts to bend without further increase in load. Shanley, using a simple bar model, with a link of material with simple bilinear stress-strain relationship showed that Pt, is the true buckling load and further that Pr can never be attained in real columns.

This later fact is demonstrated in Fig. 8, which shows Pr as the theoretical upper-bound. In reality columns do not reach this capacity, because as the true column behavior shown shaded takes over at large deformations.

The Tangent Modulus Load, Pt*,* is given by:

$$Pt=\frac{\pi^2.Et.I}{L^2}$$

where

Et is the Tangent Modulus.

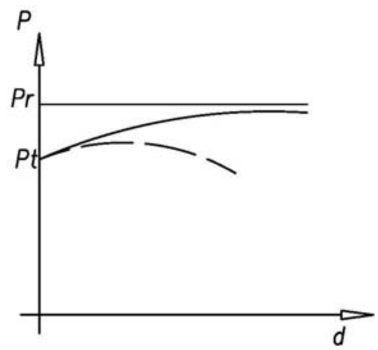


Fig.8- Relationship between Pt and Pr [6]

Pt is the smallest load corresponding to the solution of:

$$Et.I.y''+Py=0$$

The Double Modulus Load, Pr, is given by:

$$Pr=\frac{\pi^2.Er.I}{L^2}$$

Where Er is the Double Modulus, Pr is the smallest load corresponding to the solution of:

$$E.Ir.y''+Py=0$$

The Tangent modulus Theory is used widely in design of alloyed steels and aluminium columns [30]. It is not however directly applicable to steel columns, because inelastic behavior of steel columns is heavily influenced by the presence of residual stresses, and not solely by the properties of the parent material.

Inelastic column theory includes both geometrical and material nonlinearities. Analysis for Pt is a nonlinear second-order analysis.

2.8 Residual Stresses

Residual stresses are compressive stresses introduced into column sections by fabrication processes that include rolling and welding. Their magnitude can be significant in wide-flanged and welded I-sections. They are less significant in some shapes and not important at all in other materials like aluminium alloys and stainless steels. And being compressive, they are only important in members bearing mostly axial compression.

Stresses as much as 50% of yield strength reside in steel columns as residual stresses. Their consequence is to force a column to fall into inelastic range at loads that would be considered otherwise elastic loads. Their influence is more pronounced in columns of intermediate slenderness than in slender columns.

Extensive research was conducted at Lehigh University Fritz Laboratory starting 1940 under Madsen, through 1950s under Beedle and Johnston, and continued through 1970s under Beedle and Tall. Other major initial researchers include Osgood and Yang. The phenomenon started to attract attention in 1908 when J.E. Howard first reported reduction in column capacities observed from tests. This and more detailed historical account is presented by Johnston[30].

Early test results on wide-flanged sections showed a large scatter in column strengths in intermediate slenderness, compared to higher slenderness.

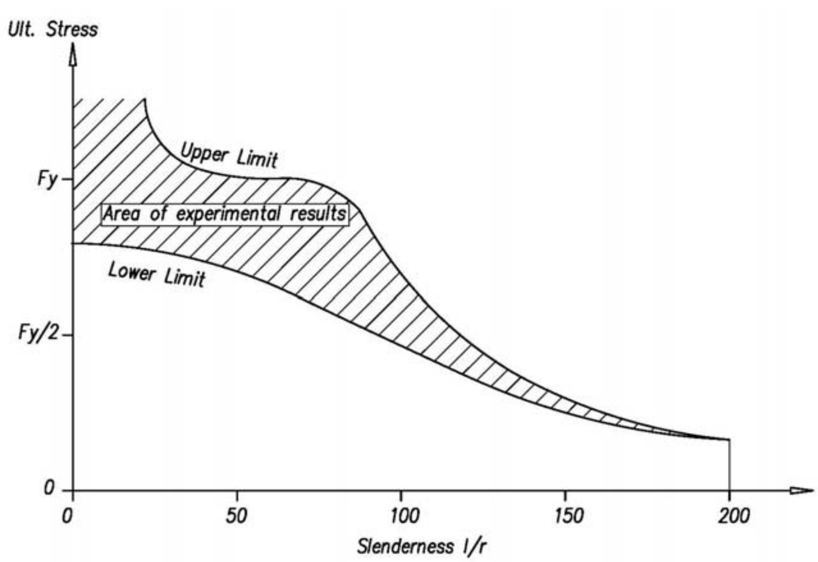


Fig.9- Results from Early Experimental Tests on Columns [24]

It is due to such a large scatter that most codes including AISC use empirically derived column curves to account for residual stresses.

2.9 Column Curves

Correct strength of a steel column is obtained from analysis or experimental work programme that takes into account all of the following.

- Geometric nonlinearity
- Material Nonlinearity
- Initial imperfections and residual stresses.

Pure analytical work is difficult to carry out and column curves are derived mostly by curve fitting experimental results. Some column curves are purely empirical, some semi-empirical [24]. But all column curves simply relate column strength to slenderness ratio, given an imperfection, material and shape.

In the US, column curves are derived from empirical work [4]. A single curve , derived from a set of 3 curves, based mostly on work in the 1972 PhD dissertation by Bjorhovde [7,21], is used. Bjorhovde showed that 3 curves can sufficiently present column strengths in all cases.

In Europe, semi-empirical relationships - such as the Perry-Ayrton equation- are used. An empirical parameter, such as the imperfection factor α in Eurocode 3, is included to take into account material and shape. There are 4 separate curves, a-d, for each class of shape (I sections, channels etc) and each assigned an α. Residual stresses, are treated simply as an imperfection.

Historical account of column curves is covered in [24].

2.10 Effective Length Factors

Effective length factors (K-factors) are required in determination of slenderness ratios entered in column curves in the determination of the available strength Pc of a column or a compression member. They are also required in design of members in frames, whenever ELM is used. ELM has been used in AISC code since 1961[13] and has only been recently (in the 2010 edition) been declared to be an alternative to DAM in stability design of frames. Effective Length factors are still required in design of pure compression members, covered in Chapter E of the code.

K-factors are factors which when multiplied by the actual length of the end-restrained column gives the equivalent pinned-pinned column whose buckling load is the same as that of the end-restrained column[10].

Using Euler Formula with length KL:

$$Pcr=\frac{\pi^2 EI}{(KL)^2}=\frac{Pe}{K^2}$$

hence,

$$K=\sqrt{\frac{Pe}{Pcr}}$$

where Pe and Pcr are elastic critical buckling loads for the equivalent pin-ended and the original column with different boundary conditions, respectively.

Euler had derived Pe as well as Pcr for other simple columns with different support conditions[30]. The assumptions and approach in his derivations are the same for all columns he studied.

From the discussion above it is observed that K is an elastic property of a perfect elastic column. K also becomes a one-to-one measure of elastic critical buckling load of a column or a frame- for every K is associated an elastic critical buckling load and vice versa for a given column or a frame.

For the case of inelastic columns, simple methods pioneered mostly by Yura [13] have been presented in the literature. In these methods inelastic K-values are generally obtained by simply modifying stiffness of a column by a correction factor τ. This factor depends on the level of factored axial load with respect to yield load.

For most stability analyses , either elastic K-values or Pcr needs to be determined first and corrections for inelasticity made later.

In this thesis, since K and Pcr are directly related, the problem of column strength is considered a problem of determining appropriate K, whether elastic or inelastic.

2.11 Analysis for Elastic Critical Buckling Loads Pcr.

Elastic Critical Loads can be determined by the following methods [6]:

- **Neutral Equillibrium**

  Equation of equilibrium are written considering a slightly deformed shape, employing the differential equations for columns and standard slope-deflection equations for beams.

  This method is usually used for isolated columns or simple frames.

For simple columns, analytical solution to the differential equations may be available, otherwise numerical methods must be used. One such method is the finite-difference method [6]. The method is, for example ,very useful in the determination of elastic buckling loads of non-prismatic columns.

- **Slope- Deflection Equations**

Slope- deflection equations for axially loaded elements are well established and found in most stability texts. Winter, Hsu and Koo demonstrated that using these equations for columns and standard slope deflection for beams reduces the complexity in analysis significantly. Most plane frames are readily analysed by this method [6].

Slope- deflection equations were employed to derive limit equations for alignment charts and were also used by Chen et al [10,12] in deriving equations to correct the alignment charts for different conditions not considered in the original charts.

The slope-deflection equations, for a prismatic member, compatible with the sign convention shown in [7] are :

$$Ma=\frac{EI}{L}\{C\theta a+S\theta b-(C+S)\rho\}-FEMa$$

$$Mb=\frac{EI}{L}\{S\theta a+C\theta b-(C+S)\rho\}-FEMb$$

where ρ is the clockwise bar rotation(sway), C and S are stability functions, that depend on P,EI and L, all combined in parameter kL:

$$kL=\sqrt{\frac{PL^2}{EI}}$$

$$C=\frac{c}{c^2-s^2}$$

and

$$S=\frac{s}{c^2-s^2}$$

c and s are functions given as:

$$c=\frac{1}{(kL)^2}\left[1-\frac{kL}{tankL}\right]$$

$$s=\frac{1}{(kL)^2}\left[\frac{kL}{sinkL}-1\right]$$

When P=0, C and S equal 4 and 2 respectively, and the slope-deflection equations reduce to the standard slope-deflection equations for flexural members. The form of equations for the case of tension, is obtained by simply reversing sign of c an s.

- Matrix Methods

Hartz demonstrated in 1965 that the matrix method can effectively be used for determination of critical loads of an entire frame [6] .Matrix methods can be force or displacement (stiffness) method.

Considering small deformations and elastic behavior, stiffness method is formulated as follows when flexural stiffness is assumed unaffected by presence of axial force:

$$[F]=[K][\Delta]$$

Where [F] is the nodal applied force vector, [K] the stiffness matrix and [Δ] the nodal displacement vector.

In the presence of axial forces, the equation is:

$$[F]=[K+P[Kg]][\Delta]$$

where Kg is a matrix which accounts for the effect of axial load P on the stiffness of the system.

This formulation takes into account geometric nonlinearity of the system through P[Kg].

At a certain level of compressive force P, effective stiffness of the system:

$$K + P\,[Kg]$$

deteriorates and approaches zero. The problem then becomes one of determining the corresponding load P=Pcr. This problem then translates to an eigenvalue problem. An elastic buckling problem is an eigenvalue problem. Solution will yield an eigenvalue (Pcr) is associated with each eigenvector (mode shapes). Euler solution too yielded similar results.

The Stiffness Method lends itself well to finite-element procedures. Solution generally improves with meshing and an optimum meshing is often required.

2.12 Considerations in Modelling For Elastic Buckling Analysis

Modelling for Elastic Critical Buckling Analysis using the same features of modeling used for strength analysis is not realistic, may lead to complexities, and most important, it may even lead to reduced buckling capacities. This is confirmed by the observations summarized in this section.

2.12.1 Insignificance of Primary Moments.

Masur in 1961, Lu in 1963 and Marcus in 1961 studied stability of frames under primary bending[6]. In each of these investigation it was concluded that in as long as stresses remain elastic, primary bending does not significantly lower the critical buckling load of a frame.

Elastic buckling analysis of a rigidly- jointed frame can be carried out by replacing the initial set of frame loads by a set which produces the same set of member axial forces without any bending[26]. In other words, for the purpose of elastic buckling analysis, loads on frames can conveniently be lumped on top of columns.

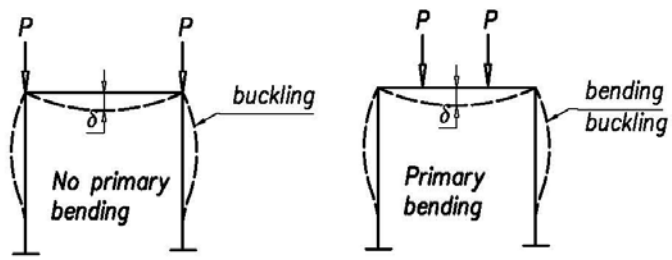


Fig.10- Effects of Primary Moments on Elastic Buckling Loads [6]

In the case of unsymmetrical frames or frames with initial imperfections, same assumption remains valid. Despite the fact that such frames start bending immediately, their capacity approaches the elastic critical buckling load(curve abc in Fig. 11).

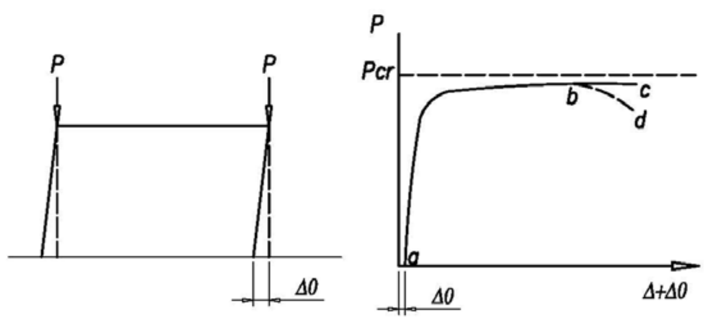


Fig.11- Behavior of Imperfect Frames Under Lumped Loads [4]

2.12.2 Insignificance of Lateral Loads

Lateral loads, unless they cause axial forces in beams, have the effect of causing no net compression in a frame, and therefore will not alter the elastic buckling loads obtained on consideration of gravity loads alone [5].

According to Timoshenko [11], the differential equations of elastic stability require that lateral loads vanish. The insignificance of lateral loads is illustrated further by Johnston [5].

2.12.3 Effects of Base Restraints

Galambos showed in 1960 that buckling load corresponding to full rigidity can be attained closely enough even with some partial fixity[7]. He illustrates that a spring stiffness ratio $\alpha Lc/EIc$ of 4.0 assumed at a base, leads to almost the same capacity as that corresponding to a fully rigid base. He also demonstrates that buckling load capacity between a flexible base with spring stiffness ratio of 0.4 is considerably larger than that corresponding to a theoretical pin base.

This observation by Galambos is the reason behind code recommended use of flexibilities, G of 10 for pinned bases instead of ∞ and 1 instead of 0 for nominally fixed bases, since 1963.

While for strength analysis, a frame may be modeled as pin-based, it needs not be modeled so flexibly in elastic buckling analysis. The available partial restraint can increase the buckling load considerably.

While no mention is made in AISC specifications or the SSRC guides on how to model bases for stability, the Eurocodes give some suggestions. They are presented in Chapter 4.

2.12.4 Effects of Joint Restraints

Influence of flexible joints on buckling loads of frames were studied by Chen in 1996[7], and a large database on these connections is now available in the *Handbook of Flexible Connections*, edited by Chen in 2011. The database includes their load –deformation curves, which can be used directly to model them in a computer analysis.

Just like bases, partially- restrained (PR) joints of sufficient stiffness can be just as good as fully rigid joints. The process of modeling PR joints is however slightly more involved in comparison to effort expended in modeling of PR bases, because, while PR bases can be modeled using code recommended simple models that account for interaction of column , concrete bases and foundations, PR joints have too many components and variables for such a similar simplification to be made.

In this thesis, PR joints are not considered- all beam-column joints are idealized as rigid.

2.13 Alignment Charts.

Theoretically, K factors can be derived from elastic critical load of a frame, and therefore from any of the methods of elastic buckling analysis outlined in 2.11.

K-factors for simple columns, the Alignment Charts, their equations and several simplified equations for the Alignment Charts, are readily available in various publications and codes. A detailed discussion of all of these is given in [12].

Alignment Charts are given in Commentary to Appendix 7 of AISC-2010.

TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

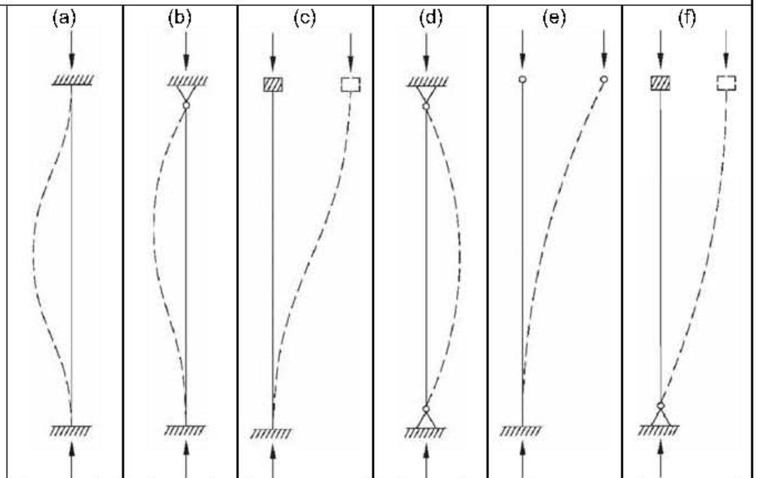
	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code	Rotation fixed and translation fixed Rotation free and translation fixed Rotation fixed and translation free Rotation free and translation free					

Fig.12- Typical K-factors for Simple Columns [2]

Both the simplified equations and the alignment charts require a frame to be first classified as either braced or unbraced, and use values of joint flexibilities G_A,G_B, evaluated beam-column joints:

$$G=\frac{\sum(Ec.Ic/Lc)}{\sum(Eg.Ig/Lg)}$$

Or in more generalized form

$$G=\frac{\sum(Ec.Ic/Lc)}{\sum\alpha(Eg.Ig/Lg)}$$

Where α is a correction factor for stiffness of the girders connected to the joint.

Chen[10,12] discusses the following simplified equations:

- ACI 318-02 equations
- Duan-King-Chen Equations
- French Equations
- Donnel Equation for Braced Frames
- Newmark Equation for Braced Frames.

Geschwindner[16] discusses other methods:

- The Modified Nomograph
- The Yura Method(∑Pe approach)
- Lim &McNamara Approach
- Le Messurier Approach
- The Commentary Equations (as given in commentary to the 1999 LRFD issue of AISC specifications).

and AISC-2005 (Commentary to chapter C) and AISC-2010(Commentary to Appendix 7), recognize:

- Le Messurier method
- The Yura Method (∑Pe approach)

The Alignment Charts, were first prepared in 1959 by Julian and Lawrence, for use in Boston building code, and have been adopted by AISC since the 1961 issue[30]. They are graphical presentations of solutions to slope-deflection equations applied to two sub-assemblages of frames, one assumed braced , the other unbraced. A sub-assemblage with definition of its constituents is shown in fig. 13. Sub-assemblages for the two cases are given along with their corresponding alignment charts in figures 14 and 15.

The alignment charts come with 9 assumptions, also listed in the commentaries. The commentaries also list corrections required for cases of different column base support conditions, different girder end-support conditions as well as corrections to account for presence of axial force in girders.

The corrections listed in the commentaries are however incomplete. These and other corrections, especially correction based on the variation of stiffness parameter $L\sqrt{(P/EI)}$ for columns and end conditions of adjacent columns, are covered in detail in Chen and Lui[10] and Chen[12].

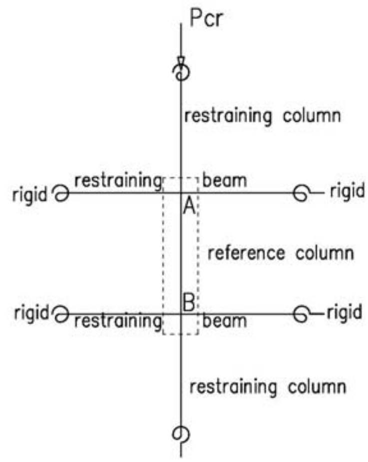
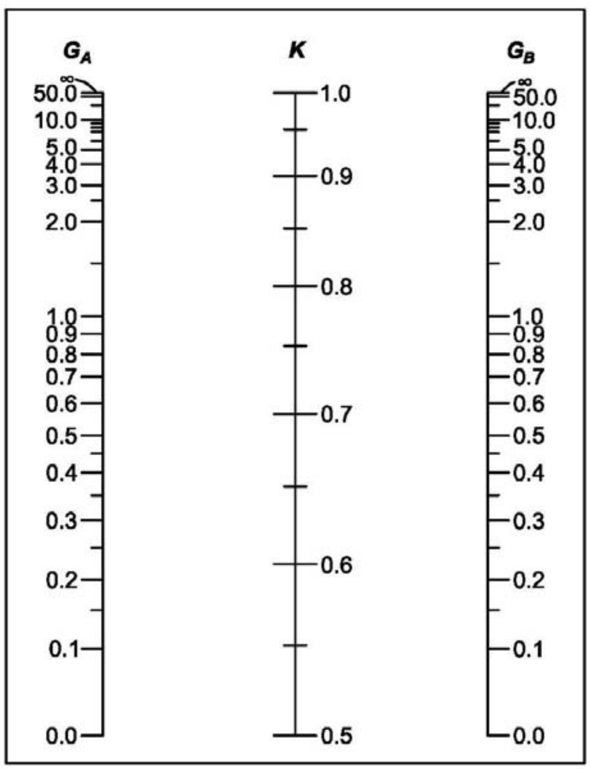


Fig.13-The Story Sub-assemblage Used in Developing Alignment Charts [10]



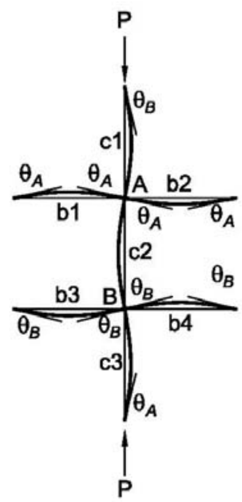


Fig.14- Alignment Chart and Assumed Shape-Braced Case [2]

For braced case:

$$\frac{G_A G_B}{4}(\pi / K)^2 + \left(\frac{G_A + G_B}{2}\right)\left(1 - \frac{\pi / K}{\tan(\pi / K)}\right) + \frac{2\tan(\pi / 2K)}{(\pi / K)} - 1 = 0$$

And for unbraced case:

$$\frac{G_A G_B(\pi/K)^2 - 36}{6(G_A + G_B)} - \frac{(\pi/K)}{\tan(\pi/K)} = 0$$

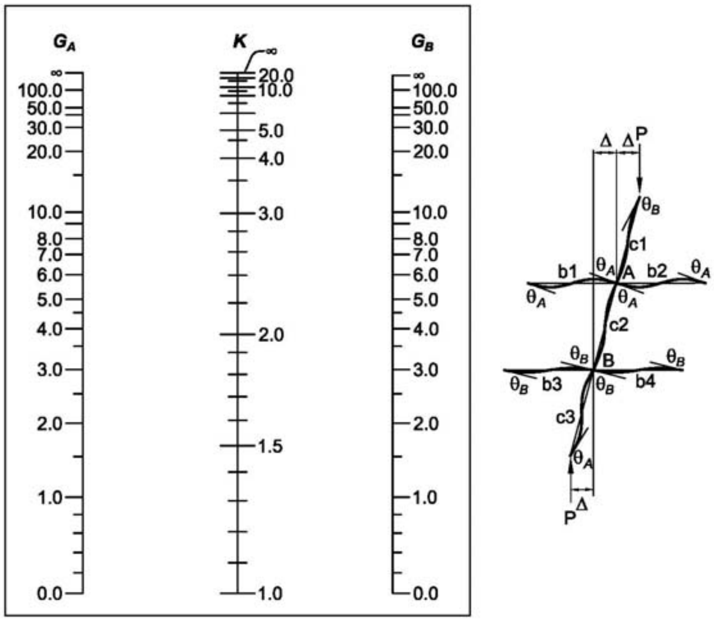


Fig.15-Alignment Chart and Assumed Shape- Unbraced Case [2]

The nine assumptions are listed in the commentary to Appendix 7 of AISC-2010 as follows:

- Behavior is purely elastic
- All members have constant cross section.
- All joints are rigid.
- For columns in frames with sidesway inhibited, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.
- For columns in frames with sidesway uninhibited, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending.
- The stiffness parameter $L\sqrt{(P/EI)}$ of all columns is equal.
- Joint restraint is distributed to the column above and below the joint in proportion to EI/L of the two columns.
- All columns buckle simultaneously.
- No significant axial compression force exists in girders.

2.14 Correction For Inelasticity In Columns

Columns are rarely elastic at buckling. Inelasticity and inelastic K- factors were first studied by Yura in 1971 and later expanded by Disque in 1973.

Inelastic K factors are derived directly from the alignment charts through a modification in G factors:

$$G^{*}=\text{Stiffness Reduction Factor}*G$$

where the Stiffness Reduction Factor(SRF) is for practical purposes taken as Et/E, and further approximated by the ratio of Inelastic Critical Stress to Elastic Critical Stress,

$$\mathrm{SRF}\cong\frac{\mathrm{Et}}{\mathrm{E}}\cong\frac{\mathrm{Fcr,inelastic}}{\mathrm{Fcr,elastic}}\cong\frac{\left(\frac{\mathrm{Pu}}{\mathrm{Ag}}\right)}{\mathrm{Fcr,elastic}}$$

Using Load and Resistance Factored expressions in AISC(1999) through AISC(2010):

$$\mathrm{Fcr(inelastic)}=\left(0.658^{\lambda^2}\right)\mathrm{Fy}$$

$$\mathrm{Fcr(elastic)}=\left(\frac{0.877}{\lambda^2}\right)\mathrm{Fy}$$

where λ=kL/r.√(E/fy) is the non-dimensional slenderness ratio.

Different versions for SRF have been developed since 1999 mostly in an effort to simplify its use, since SRF depends on the load level in a compressed member. The expression for SRF in AISC'99 is related to ratio of ultimate load Pu to squash load Py :

$$\mathrm{SRF}=\begin{cases}1.0 & \text{for } (P_u/P_y)\le\frac{1}{3}\\ -7.38\left(\frac{P_u}{P_y}\right)\log\left(\frac{(P_u/P_y)}{0.85}\right) & \text{for } (P_u/P_y)>\frac{1}{3}\end{cases}$$

The 2005 version relates it to ratio of nominal column strength Pn to squash load Py. Since Pn changes with τa, the equation must be solved iteratively.

$$\tau a=1.0 \text{ if } \frac{\mathrm{Pn}}{\mathrm{Py}}\le 0.39$$

$$\tau a=-2.724\left(\frac{\mathrm{Pn}}{\mathrm{Py}}\right)\ln\frac{\mathrm{Pn}}{\mathrm{Py}} \text{ if} \frac{\mathrm{Pn}}{\mathrm{Py}}>0.39$$

Instead of iteration, AISC'05 recommends a conservative value of Pn=α Pcr/Φc.

In the latest AISC code(2010), in which DAM is the major stability analysis method, τb, declared explicitly for use with DAM is given in Chapter C by:

$$\tau b=1.0 \qquad \text{if } \alpha\frac{\mathrm{Pr}}{\mathrm{Py}}\le 0.50$$

$$\tau b=4\left(\alpha\frac{\mathrm{Pr}}{\mathrm{Py}}\right)\left[1-\alpha\left(\frac{\mathrm{Pr}}{\mathrm{Py}}\right)\right] \text{ if } \alpha\frac{\mathrm{Pr}}{\mathrm{Py}}>0.50$$

where

α=1.0 (LRFD) and 1.6(ASD)

In this thesis, the 2010 version is used.

Effect of inelasticity can be included directly in analysis model by modifying stiffness of columns by τb. The procedure is described in detail in Chapter 3.

Yura [13] points out that columns in multistory frames (where columns are often inelastic), can often be designed on the basis of K=1.0. As columns lose stiffness, the restraint provided by girders to columns increases and K is reduced.

2.15 Analysis and Design of Frames for Instability

2.15.1 Analysis.

When frames are so well braced to eliminate local and global buckling, they will fail plastically by formation of plastic hinge mechanisms. The resulting limit state of failure is plastic instability.
However, as observed by Beedle, most building frames fail by elastic instability before plastic capacity is reached[4].

Capacity of frame furnished by an analysis depends on the type of analysis. There are several kinds of analyses, depending on how a particular analysis models aspects of frame behavior, i.e on how they take into account geometrical, material, plasticity, local buckling and their combinations, etc. The following common types of analyses exclude plasticity and local buckling, and are widely used.

- Buckling (Elastic Buckling Analysis, eigenvalue analysis)
- Elastic First-Order(Linear Analysis)
- Elastic Second- Order(Geometric Nonlinearity)
- Inelastic First- Order(Material Nonlinearity)
- Inelastic Second Order Analysis(Geometrical and Material Nonlinearity)

Capacities obtained by these analyses are illustrated in Fig. 16.

From the figure, it is clear that Inelastic Second Order Analysis is the best analysis in terms of determination of capacity of a frame [4].

An exact analysis is one that can model geometric nonlinearity, material nonlinearity, initial imperfections and effects of residual stresses correctly. Such analyses are discussed in [28]. They are not yet, however, covered in detail in codes, despite the general mention that they can be used.

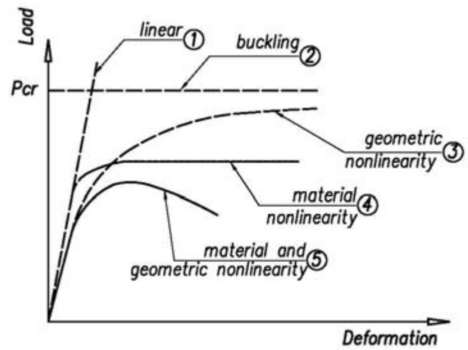


Fig.16- Strength of Frames Determined from Various Analyses [26]

Inelastic Second-Order and the more exact analyses are also difficult to carry out in practice.Within the scope of many specifications are Elastic First and Second -Order Analyses. Codes compensate for inadequacies of these popular analyses, by use of interaction curves. These curves are developed from curve-fitting of results from experimental and exact inelastic second-order analyses.

2.15.2 Design(Interaction Equations).

Methods of considering buckling behavior in analysis of frames is summarized by Galambos[4].

- Determine Axial Force and Moments using First or Second-Order Analysis.
- Determine Buckling Load Pcr, of a frame with discrete axial loads applied to the top of the columns.
- Do interaction check.

This is the outline still used in codes. While the outline applies for frame as a whole as well as members, it is for practical purposes, usually used at member level. The general assumption is that if a member in a frame can satisfy code requirements for stability, the structure will be stable.

Where first-order analyses is used (where applicable), effects of geometric-nonlinearity are taken care of at design-time by approximating second-order effects by approximate methods, e.g the moment-magnifier method outlined in Appendix 8 of AISC-2010. Material Nonlinearity is considered in code expressions for member strengths Pc and Mc [4].

Stability of a frame is considered by ensuring that each compressed frame member satisfies the interaction equation unity ratio.With ELM, it is through the K-factor that the equation accounts for interaction of the given member with the rest of the members in the frame.

The current AISC interaction equations refer to Second - Order Forces only and were developed for the first LRFD format of the specification (AISC,1986). They are based mostly on the work by

Kanchanalai in 1977[7]. Kanchanalai had curve fitted results from exact second order inelastic analyses of many sensitive benchmark frames, but did not consider geometric imperfections.

In AISC 2005 and 2010, interaction equations for a members subjected to combined axial forces and moments are given in Chapter H of the specification.

Considering only the case of axial force, major-axis bending and doubly symmetrical sections:

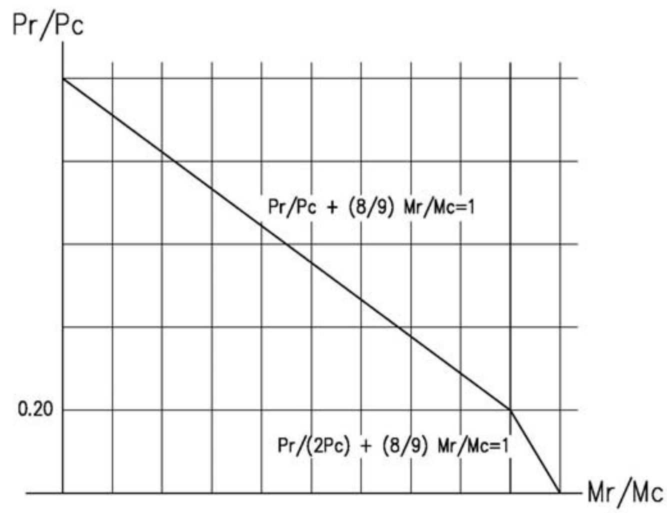


Fig.17: AISC'10 Interaction Curve For Plane Frames

For Pr/Pc ≥0.2

$$\frac{Pr}{Pc}+\frac{8}{9}\frac{Mrx}{Mcx}\leq 1.0$$

For Pr/Pc <0.2

$$\frac{Pr}{2Pc}+\frac{8}{9}\frac{Mrx}{Mcx}\leq 1.0$$

The same expressions apply for both LRFD and ASD. Their difference lies in determination of required strengths Pc and Mc. For LRFD, a safety factorΦ< 1.00 is multiplied to nominal strengths Pn and Mn to obtain capacities Pc and Mc. For ASD, Pn and Mn are divided by Ω>1.00 to obtain allowables Pa=Pc and Ma=Mc. Mn and Pn are however in both cases determined at strength level. In some cases, factors that depend on factored axial load level, e.g Pr/Pe, appear in expressions for available strengths. To bring about ASD load Pr to factored axial load level, Pr is factored by α=1.6. In both cases, the available strengths, Pc and Mc, must be determined considering provisions and method of stability analyses given in Chapter C to F of the specification.

Clause C1.1 and 1.2 (AISC- 2010) require analysis to consider the influence of second-order effects(including P-Δ and P-δ effects),flexural, shear and axial deformations, geometric imperfections and member stiffness reduction due to residual stresses on the stability of the structure and its elements is permitted.

The following methods of second-order analysis are covered in both the 2005 and 2010 editions:

- Second Order analysis by Amplified First-Order Analysis(ELM)
- First Order Analysis(A special case of DAM)
- Direct Analysis Method(DAM)

Direct Analysis Method is required when second order drifts are over 1.5 times first -order drifts. It is the major method of stability analysis and design in the current specifications (2010). It has the significant advantage that K=1 can be used in both terms of the interaction equations for beam-column elements.

Most braced frames however do not need to be designed using DAM, as they are not likely to exibit large drift magnifications. Nevertheless, DAM remains a universal method of analysis.

Frames considered in this thesis, due to large spans, are stiff enough to show drifts that are below the 1.5 limit set for validity of ELM.

CHAPTER 3

METHODS OF ANALYSIS

3.1 The B2 amplifier (Sway -Sensitivity Analysis)

According to Provision C1 of AISC-10, second-order effects must be considered in stability analysis of a structure and its elements. Structure is verified for stability using second order-forces Pr and Mr. The second-order effects result when axial compression acts on a member deformation δ (small delta) and/or frame sway deformation Δ (large delta). The difference between the two displacements is illustrated in Fig. below. The two effects are known as P-δ and P-Δ effects.

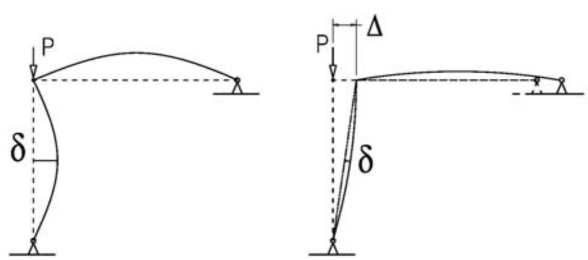


Fig.18: P-δ and P-Δ Effects

With DAM, second-order effects must be determined from a second-order analysis directly and K is set to unity in Pc and Mc. With ELM, they are approximated post first-order analysis but with an appropriate K-value that is usually greater than unity for unbraced frames. The approximate method is known as the moment-magnifier method. The method is covered in Appendix 8 of AISC'10. In this method, magnification factors B1(for P-δ, or non-sway part) and B2(for P-Δ or the sway part) are first determined, and then used to magnify forces obtained from the first-order analysis.

$$Pr = B1.\ Pnt + B2.\ Plt$$
$$Mr = B1.Mnt + B2.Mlt$$

where Pnt, Mnt are first-order axial forces and moments obtained from an analysis with sway deformations eliminated (gravity analysis). Plt, Mlt are first-order axial forces and moments obtaind from an analysis in which sway is permitted and only lateral loads act on a frame(i.e lateral analysis). In unbraced frames, B1 is close to unity and B2 does most of the magnification.

The B2 amplifier is also a measure of sway-sensitivity of a structure or a story. It is actually a ratio of second-order/first order drifts and is given as(Equation A-8-6 of the code):

$$B2 = \frac{\Delta_{2nd\text{-}order}}{\Delta_{1st\text{-}order}} = \frac{1}{1 - \frac{\alpha \Sigma Pnt}{\Sigma Pe2}}$$

where the summation applies to all columns in a story and Pe2 is the Euler load for a column using the sidesway buckling effective length K_2. α = 1.0 for LRFD and 1.6 for ASD.

B2 is determined for each story in frames in this thesis because Appendix 7 of the code allows use of $K_2 = K = 1$ if $B2 \leq 1.1$ condition is satisfied for all stories. A moment frame whose all stories satisfy this requirement is considered stiff enough and non-sway [34].

The code also provides a very simple method of evaluating $\sum$Pe2, which is based only on first-order story drifts under a set of lateral loads(Equation A-8-7 of the code).

$$\Sigma Pe2 = R_M \frac{\Sigma H.L}{\Delta_H}$$

where

R_M	= 0.85 for moment frames, 1.0 for braced frames
Δ_H	= story drift from first- order analysis.
$\sum H$	= Story force(or shear) that induces Δ_H
L	= Story Height

Commentary to Appendix 7 suggests that lateral loads H, can be any set of realistic lateral forces that can also be suitable in determination of the true lateral stiffness of a frame. The usual practice is to use actual design lateral forces, e.g load combinations that include wind or earthquake forces. For preliminary design however, story drift index $\Delta H/L$ (typically L/300) is usually assumed to correspond to story drift under design lateral forces[18]. With the use of the story drift index, lateral analysis becomes no longer necessary in determination of B2.

In this thesis, a convenient set of forces distributed to floors in inverted triangular distribution similar to that used in equivalent seismic force procedure is adopted. The magnitude of forces is such that floor displacements are of the order of a few centimeters, this being only for convenience in computation.

The use of B2 factor as a measure of degree of sway (in this thesis it will be called sway-sensitivity , to borrow from European practice) has a history and theoretical background documented in [34]. As it turns out B2 is actually similar to the sway index αcr used in Europe.

According to [34], research by Lu, et al in 1975 and Liapunov in 1973 and 1974 in the U.S had suggested within the context of AISC Allowable Stress 1989, that certain classes of rigidly-connected unbraced frames can be designed without considering P-Δ or P-δ effects, and for this class of frames, K=1 can be used in computing column strengths. One of the resulting criteria for such frames is a limit on service load story drift index $\Delta H/L$. The criteria were also published in [5] in 1976. When the limit on service drift index proposed by these earlier studies is put in a form compatible with the current LRFD format, it translates to the B2 limit of 1.10. Similar limits appear in The 1991 NEHRP Recommended Provisions for The Development of Seismic Regulations for New Buildings, Eurocode 3 and the Australian steel code AS400 [34].

In the Eurocode (EN 1993), sway-sensitivity is checked through a sway index $\alpha_{cr,}$ calculated for the whole frame. α_{cr} is the ratio of the total factored load to the elastic system or story- buckling load of a frame or structure. It is the same as the load factor λ used in elastic buckling analysis with software, presented in Section 3.3. It is used to check on whether second order analysis is required or unnecessary. When $\alpha cr \geq 10$, ie elastic critical load is over ten times the design gravity loads, a frame is considered not sway-sensitive and sway-stability analysis is not required.

Substituting this requirement on α_{cr} in equation for B2, we get almost a similar requirement on B2:

$$B2_{(\alpha cr=10)} = \frac{1}{1-\frac{1}{10}} = 1.11$$

α_{cr} is determined from elastic critical buckling analysis of a frame, or by simplified methods based on first-order drift- index, similar to the one used to simplify determination of B2. Many tables are also available for quick computation of this index for popular framed structures[27].

An extensive discussion on errors in setting K=1, for frames that satisfy the B2 criterion, is given in [34].

Frames considered in this thesis- fitted with elastomeric bearings- are likely to exhibit reduced sway-sensitivity compared to plain plane frames. With this potential, they might qualify for K=1 and B2 analysis is carried out for this purpose. B2 is calculated story-wise using provisions of AISC Chapter C, Appendix 7 and Appendix 8(Approximate Second-Order Analysis).

3.2 AISC and SSRC on Discrete Bracing

3.2.1 Winter Concept

Timoshenko [11] presented solutions of columns with intermediate flexible supports for a straight column. He showed that a certain minimum stiffness at each support , the ideal stiffness βi, would make a straight column behave as if the supports were rigid, and any further stiffness did not increase the column strength. At ideal stiffness, column strength is the Euler Load and K equals 1.

Timoshenko's solution considered effectiveness of bracing in terms of stiffness alone.

Winter ,in 1960, extended this solution to include initial crookedness. He showed that with crookedness, twice the ideal stiffness for perfectly- straight was required to reach Euler Load. If a column is imperfect, then a spring of ideal stiffness can brace the column but with an associated very large brace force. Effectiveness of bracing therefore requires both stiffness and strength considerations. His work set the beginning of dual-criteria required for bracing design in AISC code since 1999.

The discussion in this section summarize from [1, 2 & 14].

Until 1999, there were no bracing requirements in the AISC specifications. Bracings were designed mostly using the 2% rule, i.e a bracing force at 2% of the column force it braces. The 2% rule was developed for design of laces in railroad trusses, many of which had failed in the 1900s. The rule was apparently extended to all stability bracing designs eventually.

Winter considered a simple rigid bar-spring model shown in Fig. 19 to represent a relative brace. The bar is initially displaced by amount Δ0 at the top relative to the bottom, before load P is applied.

By summing moments at Point A, the equation of equilibrium is :

$$P\Delta T=\beta L(\Delta T-\Delta 0)$$

or,

$$P=\beta L(1-\Delta 0/\Delta T)$$

where

$$\Delta T = \text{total final displacement} = \Delta+\Delta 0$$

When Δ0=0, the column is initially straight and

$$P=Po=\beta i.L$$

In non-dimensional form, the equation becomes:

$$\frac{P}{Po}=\frac{\beta}{\beta i}(1-\Delta 0/\Delta T)$$

The resulting brace force is Fbr = β.L and the relationships are plotted in Fig. 20.

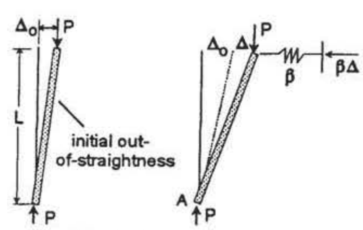


Fig.19-Winter Rigid Bar Model [2]

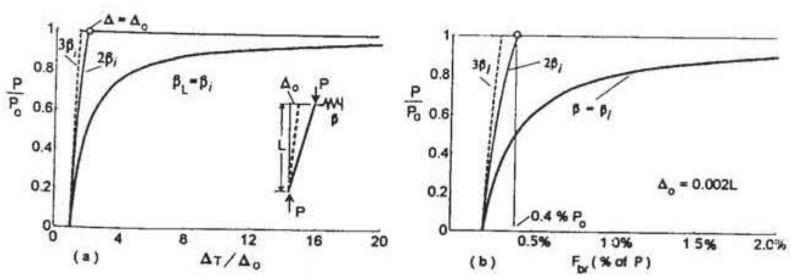


Fig.20-Relationships Between Brace Stiffness and Force in Winter Model (Taken from [2])

From Fig. 20:

- At ideal stiffness, strength Po is only possible after large deflections and brace force.

- At twice the ideal stiffness, total deformation is twice the initial deformation and the brace is at a load level of 0.4%

AISC and SSRC [3] have adopted the Winter concept in their stability design criteria. SSRC in [3] notes that many published solutions provide stiffness recommendations only for perfectly straight structural systems, and suggests that such solutions should not be used in design because very large forces may result.

In Winter's model, which assumes a rigid bar, material properties are lost. This raised a question on applicability of the model in case of inelasticity. Yura [14] settled the problem with a proof that the model is equally applicable to inelastic columns, since the original bar-model had a column element that was considered rigid, and no dependence on Elastic or Tangent modulus was included.

3.2.2 Code Requirements For Nodal Bracing

In SSRC [3], bracings are generally classified as :

- Relative Bracing
- Nodal Bracing(Discrete)
- Continuous Bracing
- Lean-On Bracing

AISC-10 provides design criteria for Relative and Nodal (Discrete) Bracing only in its Appendix 6.
A relative bracing controls the relative movement of adjacent stories or of adjacent points along the length of the column or beam. Such is a bracing offered by X-braces, shear walls and similar bracing. A nodal bracing however controls movement only at that particular brace point. An example is a connection of column to an RC shear-wall, floor or a relatively more rigid support, at some discrete points.
Only Nodal Bracing will be considered from this point forward.

Exact solution for a straight column with discrete nodal bracing was developed by Timoshenko and Gere[3, 11].

From their study it was observed that at low brace stiffness, the buckling load increase with the stiffness while a buckled shape is retained. However, as stiffness is further increased, the buckled shape changes, buckling load increases further and at a certain limiting stiffness, load increases no further. At this stiffness, additional bracing becomes ineffective.

The limiting stiffness is the ideal stiffness for a discretely or continuously braced column. For a continuously braced column , this stiffness corresponds to full bracing. Full bracing occurs at the ideal non-dimesional stiffness factor Ni=βL/Pe =3.41. For equally spaced (discrete) braces, Ni varies between 2.0 and 4.0, for one and for many intermediate braces respectively.

Using Winter model and assuming zero moment at the nodes, Yura developed discrete brace force requirement in 1993, arriving at Fbr=0.8% P at twice the ideal stiffness value, a force about twice that required for relative bracing. This value was increased to 1%P, following observation by Chen and Plaut, that Winter model was unconservative for the case of a single brace at midspan. The correction also accounts for effects of curvature, which is not included in the Winter rigid model [2].

The general form of stiffness and strength requirement is:

$$\beta = Ni.\frac{Pr}{Lb}(1+\frac{\Delta 0}{\Delta})$$

$$Fbr = 2\frac{Pr}{Lb}(\Delta+\Delta 0)$$

where

Ni =2.0~4.0,

Lb = story height between braces .

Δ0 = small displacement at the brace point from straight position, caused by sources other than gravity or compressive forces, e.g. due to initial out-of-plumbness, wind or other lateral forces.

Δ= additional displacement at the brace point due to gravity or compressive forces.

The AISC equations in Appendix 6 use:

$\Delta = \Delta 0 = 0.002L$

$Ni = 4.0$

The value of *Ni* used is the most conservative one.

The required brace strength is (eq. A-6-3):

$$Pbr = 0.01\ Pr$$

The required brace stiffness is(eq. A-6-4):

$$\beta br = \frac{1}{\Phi}\left(\frac{8Pr}{Lb}\right) \quad \text{(LRFD)} \qquad\qquad \beta br = \Omega\left(\frac{8Pr}{Lb}\right) \quad \text{(ASD)}$$

where,

Pr = required axial compressive strength using LRFD or ASD load combinations

$\Phi = 0.75$ and $\Omega = 2.00$

Definitions of displacements are as illustrated in Fig. 21:

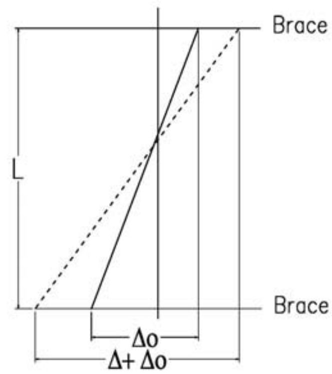


Fig.21-Definitions of Bracing Displacements [1,2,3]

Choice of the large value of Ni = 4.00 (limiting stiffness for many braces) and method of derivation mean that bracing design criteria are conservative and can not be verified by theoretical methods [3]. It is noted however that the criteria realistically use Pr instead of Pcr.

3.2.3 Notes on Code Requirements and Bracing Design Criteria

In this section, a few important points related to AISC bracing design criteria, as well as observations made by some authors are presented.

Gil and Yura[14] showed that the Winter's simplified method can also be applied to inelastic columns.

In [2], a second-order analysis that includes an initial out-of-straightness of the member to obtain brace strength and stiffness is permitted in lieu of the requirement of Appendix 6 (of AISC'10).

The bracing design criteria are a design simplification and are not fully based on an analytical method. The objective is simplicity. The stiffness of bracing is such that a value of K=1 can be used, and not that would be required for sway prevention, which is extremely large[3,4]

The Winter bar-model, and the design criteria can be extended to continuous columns and frames[1,2]. Galambos[15] in his own separate treatment of bracing, makes similar idealizations in arriving at required bracing for a frame, although he does not consider initial displacement.

If the brace stiffness provides, βact, is different from that required βbr, then the brace force can be multiplied by the factor [1,2]:

$$\frac{1}{2-\frac{\beta br}{\beta act}}$$

The unbraced length Lb, is assumed to be equal to the length Lq that enables the column to reach Pr. When the actual bracing spacing is less than Lq, the calculated stiffness can become quite conservative. In such cases Lq can be substituted for Lb in stiffness equation. Even this can still lead to conservative estimates of the stiffness requirements [2,3].

The stiffness requirements are in general conservative. This is clear from the various assumptions and corrections presented in this and previous section.

3.3 Elastic-Buckling Analysis Using SAP2000

Geschwindner [16,18] suggests use of elastic buckling analysis programs in determination of K factors and elastic buckling load for irregular or complex frames. In [16] he uses software GTSTRUDL. Use of software for similar purposes started many decades ago: Johnston[5] used software developed by Halldorson and Wang in 1968 in comparing K factors against manual methods.

SAP2000 is capable of buckling analysis and is used in this thesis. The capability of the software is verified in Appendix A, using single columns, simple frames and the frame used to develop the monographs.

For simplicity, as suggested by Galambos [4], and for the purpose of comparing with theoretical solutions, axial and shear deformations are eliminated by setting cross-sectional and shear areas to very large values, typically 1000m2. Elastic modulus E is set to 2e8 kN/m^2.

A plane-buckling analysis is invoked in SAP2000 by running the program in plane-frame mode. With this option, columns and frames are forced to buckle in modes that include only bending about their strong axes (available degrees of freedom).

Only first modes of buckling are considered since the interest is on buckling loads.

The general formula for K is:

$$K=\sqrt{\frac{Pe}{Pcr}}$$

in which Pcr is to be determined from an eigenvalue analysis by software.

Software does not directly determine Pcr, instead it determines through incremental procedures a load factor λcr, which when multiplied by the total factored column loads $\sum Pu_i$ in the model, gives the total critical elastic-buckling load Pcr of a column or a frame:

$$Pcr=\lambda cr.\ \sum Pu_i$$

Effective length Le for column i:

$$Le,i=Ki.Li=\sqrt{\frac{\pi^2 EI,i}{\lambda cr.Pu,i}}$$

Effective length factor *K* for column i:

$$Ki=\sqrt{\frac{Pe}{\lambda cr.Pu,i}}$$

For convenience, a certain reference column is selected and assigned a load Pref =1.00. This selection is best made such that the axial force in the column will be Pu=Pref=1.00. Other columns are assigned load as fractions or multiples of Pref. Then Pcr for the reference column becomes simply the factor λcrx Pu = λcr , which is obtained from the analysis of the frame. For other columns Pcr is the corresponding Pu x λcr. With this choice of loading, it is crucial that unit loads and fractions of them be distributed in the model in the same way actual loads in a given load case are distributed to columns.

3.4 The Problem of System vs Story- Buckling In Buckling-Analysis

Alignment Charts, their corresponding equations and other simplified equations, assume buckling is associated with story-buckling. Story-buckling is initiated by buckling of all columns in a single story. As Yura puts it , buckling in sway mode is a story phenomenon [13].

But buckling in complex frames may not take place in well -defined sway modes. In such frames columns in other stories may restrain columns in adjacent stories. When all columns in a frame contribute to buckling of a given frame, the frame is said to buckle in system mode.

Software can be used to model for either buckling modes. Story modes can be simulated by modeling individual stories, and system modes by all stories in a given frame.

If the results from the two models are to be compared, then equal buckling loads are not to be expected, since the resulting critical loads depend on modes captured by either model, which, might not necessarily be the same. The consequence of this potential incompatibility between the two models is that a careful decision must be made as to which model would be appropriate for analysis. In design office, making this decision might be difficult, unless a clear guidance is available.

During literature review for this thesis, it was observed that, despite agreement between most renowned authors that software analysis is the best option, the subject is not covered so well.

SSRC, through Johnston [5], and Gershwindner [16] support results from system analysis without further questioning. Both have both used computer models for two-story frames, two and one bays respectively. In the computer analysis by Johnston, very large K values-as high as 6.13-were obtained for upper story columns, which turned out to be lightly loaded. Johnston, concludes his analysis by comparing his set of computer K-values against those obtained from solution presented by Chu and Chow for corrections of Alignment Chart K-factors when column sizes and loads are not identical, i.e a case of column stiffness parameter $L\sqrt{(P/EI)}$ being not constant (Assumption 6 of Alignment Chart is violated). He does so without an additional note on the large magnitude of computer-generated K value declared as correct and in agreement with the solution by Chu and Chow.

In Gershwindner's case, upper story columns exhibited K-values that raise questions which he takes notes on, but the increase is not alarming . He explains the slight increase as a requirement for the upper columns to brace the lower ones, with no further note on potential large K-values for cases outside his particular one.

The case of lightly loaded columns bracing the more stressed columns has a sound theoretical basis, and is demonstrated in standard structural texts including Timoshenko and Gere[11] in their analysis of 2- span continuous columns, in which one span is loaded more than the other. The lightly loaded span ends up taking the larger K-factor. Chen et al.[12] makes similar demonstrations using a procedure developed by Bridge & Fraser in 1987 for columns in a continuous frame with different load and stiffness ratios.

In view of these theoretical results, it might appear that system–buckling analysis, which includes contributions of all columns to stability of frames, should be used without further questioning. However, for frames that buckle in sway modes, buckling is often considered to be a story-phenomenon, and code expressions for K-values have always been based on story-buckling approaches. Sway may be large enough to destabilize a given story before system buckles as a whole. Next, the possible anomaly in K-factors, such as that obtained in [5] raises a question as to the credibility of system-buckling analysis.

Discussion on large K-factors obtained from system-buckling analysis is made in detail in [34], White and Hajjar [31] and the Australian Steel Manual [32]. All these authors attempt to give guidelines on where system-buckling would be and would not be appropriate.

According to [31], system-buckling analysis is necessary for slender multistory frames because those frames derive their sway-flexibility from the overall cantilever action. Story by story or sub-assembly models may not capture buckling strength of slender frames properly, as most columns in most stories contribute to buckling. The same is true for innovative structural systems, such as mega-braced tall frames. In ordinary multi-story shear-building frames, story buckling behavior dominates.

The high K values for upper story columns of multi-story frames are explained as follows in [31]:

A typical top story column usually has a low axial load level, but a higher moment level, in comparison with lower story columns. This results in selection of same column size for both columns. However, it is the lower story column that is stability-critical, and the resulting governing mode of buckling is governed by lower story columns. In this particular mode, upper story columns “ride along” in rigid-mode displacement, not contributing at all to buckling resistance of the frame. Nevertheless, with large sections selected and hence large Pe , they are penalized with large K-values corresponding to critical buckling load Pcr associated mainly with the columns in lower story. One consequence of this, is that the first term in interaction equation, Pr/Pc is the same for all columns in the frame.

“If it is likely that that interaction effects between stories are negligible or can be represented by the simple idealizations of a particular story–buckling model (which is often the case for ‘shear buildings’), one can argue that column strengths should be based on a story-buckling analysis, …” [31].

In this case system buckling will tend to be a critical-story phenomenon, with the most critical story governing the buckling capacity of the whole frame.

Where anomalies in system K-values for slender and more complex frames occur, White and Hajjar go on to provide the required engineering judgement in evaluation of K from system analysis, describing the process of evaluation as an engineering, rather than scientific procedure. This required judgement is also presented in [34] and is discussed in the next paragraph. The huge effective lengths for lightly loaded columns that also do not contribute to buckling mode are also considered briefly in [32] and a few practical advices are presented. One suggestion made is to construct a special load case such that a compression member in question will have a profound effect on the buckling mode.

Perhaps the best suggestions are those made in [34], which note that the large K factor is a result of factored load being negligible compared to Euler Load Pe for the column. The case of large K will always be associated with one of the following:

i) The axial loading effects are neglible and the member is for all practical purposes be acting as a “beam”. As a beam it may or may not contribute to the buckling resistance of the frame.

In this case axial load is simply neglected, and if the member does contribute to the buckling resistance, then the results of buckling analysis is retained to keep the contribution of the member.

ii) The member is a column. However the member is not contributing significantly to the computed story or system buckling resistance. Nevertheless, if the size of the member is reduced by a large enough amount, the buckling strength of the structure (or story) might be controlled by a different buckling mode that does depend significantly on this member.

For this case, the system buckling analysis is not reliable for this member and story buckling analysis or nomographs may be more appropriate for the member.

iii) The member is a column that is contributing significantly to the computed system or story buckling resistance. In this case, if the size of the member is changed by even a small amount, the buckling resistance of the structure or a frame will be changed significantly.

In this case, the results of the analysis are correct.

For the frames in this thesis, which are not slender, the large K-value problem is not expected and although they will differ from those obtained from other analyses including the story-buckling analyses, their reliability is not expected to be poor either.

System analyses might be appropriate for use in design office in analysis of shallow long-span frames..

3.5 Considerations in Modeling For Story-Buckling Analysis: The Sub-Assemblage.

While for system-buckling analysis the whole frame is simply modeled with all available restraints and constraints, the manner in which columns are to be restrained at story levels in a story-buckling analysis is not discussed in detail in literature cited in this thesis, neither in any standard textbooks on steel design .

Sample stories used for illustration purposes in the literature, e.g [7, 13], often consist of bases either pinned or fixed, and girder restrains columns at floor levels. Close investigation of the model used to develop alignment charts, shows similar approach to story-buckling: Contribution of stiffnesses of upper stories is ignored and stories are simply treated as separate, independent sub-assemblies. Use of sub-assemblies is indeed suggested in [32].

The best story-subassemblage is probably the same one that was used in alignment charts, shown in Table 11, despite its suffering from inaccuracies related to condition 6 of alignment charts being violated (i.e when stiffness parameter is not constant). But realizing that Yura's ∑P-concept, accepted in the code, and is a truly story-based procedure, determines Pcr of non-leaning members using alignment charts, it is possible to conclude that the sub-assembly used in alignment charts can be used in modeling for story-buckling analysis.

However, that sub-assemblage, which runs vertically to include two adjacent columns, is dropped in this thesis for the following practical reasons:

- Rotational stiffnesses at ends of girders and columns in the sub-assemblies are difficult to compute. Multiple sub-assemblies might also be required to represent a story correctly if columns are sized differently.

- Vertical sub-assembly will not be attractive to designers. They would prefer to use the whole model or a complete story cut out from the complete model instead.

One possible option is to model the whole story, with girders and columns in the story, and introduce rotational restraints to represent columns above and below the story. While this is reasonably more accurate and practically possible, it assumes the adjacent columns are loaded at the same level as the columns in the story considered (case of a constant stiffness parameter) and is therefore non-conservative.

In this thesis, a conservative version developed by modifying the non-conservative story model discussed above is selected. In this model, the already compressed adjacent columns are removed from the model, both girders are retained and a single bearing is assumed to restrain a story.

The underlying assumptions are:

- That base flexibility will be reduced by the presence of bottom girders which do not seem to influence capacity of columns in a story beneath it. Base flexibility on the other hand does influence a frame for the story in question significantly and is best reduced.

- Adjacent columns have counteracting effects on stability of the reference column: while both increase nodal flexibility (as in G_A and G_B) ,the one on top, being relatively less loaded, has a bracing effect, and the one beneath, likely to be highly stressed, may have only a minor contribution to the reference column. Overall, removing both columns is likely to be mildly conservative.

Proof on validity of the sub-assembly selected is not necessary, since this sub-assemblage is a conservative simplification of the full story sub-assemblage (the one that includes all columns in neighboring floors).

3.6 K factors Accounting for Inelasticity of Columns in Sofware Analysis.

If alignment charts are used, inelastic K factors are determined by modifying G values, by procedure given by Yura in 1971 and presented in detail in [10] and [12].

In software analysis, inelastic K factors can be obtained as follows:

From White& Hajjar [31]:

$$K_{inelastic} = \sqrt{\frac{Pt}{Pcr^*}}$$

where,

$$Pt = \frac{\pi^2 \tau . EI}{L^2}$$

Pcr* is the critical load obtained from a buckling analysis that takes into account inelasticity of members, and τ is the stiffness reduction factor.

To determine Pcr* or Kinelastic, an estimate of load in each column is made, and at analysis-time, its stiffness is reduced accordingly in the same model used for Elastic-Critical Analysis. Then Pt is used instead of Pe in the equation for Kinelastic .

τ is same stiffness-reduction factor for inelasticity described in Section 2.14. In this thesis, τb as given in AISC-10 will be used.

CHAPTER 4

ANALYSIS OF SELECTED FRAMES AND RESULTS

4.1 Criteria

Geometry of selected frames is given in Section 4.2. A typical plan is given for Type A in Figure 28.

4.1.1 Loading

Loading is summarized in Table 11.All loads are assumed uniformly distributed on floors. During analysis for strength, they are applied as line loads on girders on tributary width basis. In buckling analysis, it is sufficient to lump loads on columns on basis of tributary areas of columns.

Table 1- Design Loads for Selected Frames

Dead Loads DL		kPa
Concrete in deck	150mm normal weight @24kN/m3	3.60
Finish	100mm average @20kN/m3	2.00
Joists etc		0.50
Ceiling		0.40
	Total	6.50
Live Loads		
	Floor Live Loads	5.00
	Services	0.50
	Total	5.50

Live Load Reduction(ASCE 7-2010)

Columns and beams in these frames support large areas and are not subject to overcrowding. Their live loads can be considered reducible.

Live load reduction is according to ASCE7-2010, and is summarized in Table 12, for frames in section 4.2. By inspection, and for convenience in design, a single value of 0.50 is used for all members. This choice of a single global reduction, enables a single run of analysis.

$$L=L0*\left(0.25+\frac{4.57}{\sqrt{KLL*AT}}\right)$$

Table 2-Live Load reductions to ASCE 7-10

L0	5.5	kPa					
Element	Story	Kll	At	L(kPa)	Lmin(kPa)	L,use(kPa)	Luse/L0
8m beams	All	2	64	3.60	2.2	3.60	0.65
16m beams	All	2	128	2.95	2.2	2.95	0.54
Columns(2 story)	Top Story	4	96	2.66	2.2	2.66	0.48
	First Story	4	192	2.28	2.75	2.75	0.50
Columns(4story)	Top Story	4	96	2.66	2.2	2.66	0.48
	Third Story	4	192	2.28	2.75	2.75	0.50
	Second Story	4	288	2.12	2.75	2.75	0.50
	First Story	4	384	2.02	2.75	2.75	0.50

4.1.2 Elastomers and Selection Criteria(LateralStiffness)

Elastomers can be made from natural rubber, although most elastomers are made from synthetic rubbers, also known as polyprene. Rubbers perform well only in compression-their tensile capacities are negligible-and even when they are subjected to rotations in their vertical plane, the rotation should should be such that it is accompanied only by compressive stresses in the rubber. In case design calls for uplift loading, then separate elements must be detailed for the purpose .

Strains are a critical consideration in design of elastomeric bearings. Rubbers have high poisson ratios: their strain capacity in compression is limited more by accompanying lateral tensile strains than direct strains. To increase capacity in compression, the bulge induced by lateral strains is reduced by reinforcing plates, usually thin sheets of steel 2-4mm thick, placed in between layers of rubber, as shown in Figures 22 and 23(b). The plates or laminations, increase the Elastic Modulus in compression significantly.

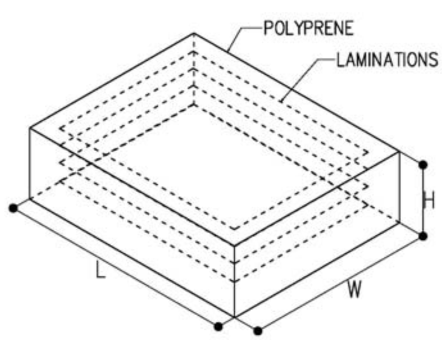


Fig.22: Elastomeric Bearing - Parts and Dimensions

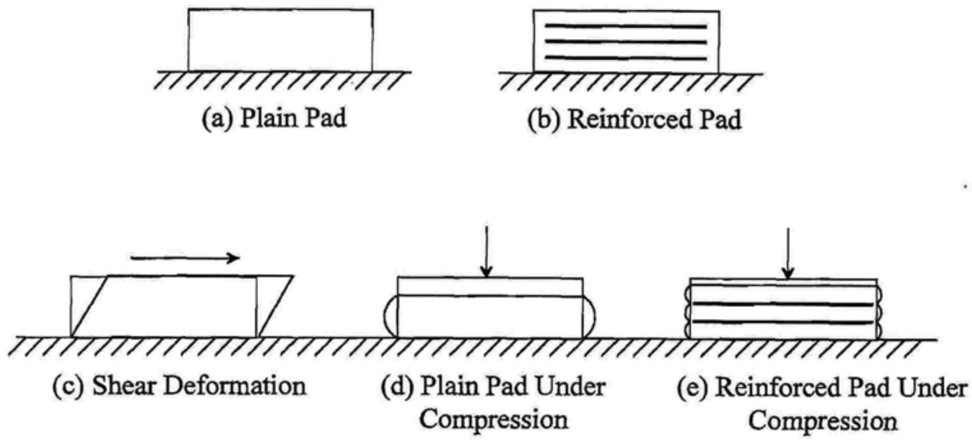


Fig.23- Elastomers - Deformations [33]

Reinforcement has no effect on shear deformation capacity.

Elastomers are nonlinear in compression with considerable hardening, but their behavior in static shear is mostly linear [33]. In this thesis in which bearings are subjected to static shear deformation only, they are modeled simply as linear springs.

Their shear strain capacity can be as high as 100%. To guard against stability failure under compressive loads, AASHTO (14thed) limits strains to 50% at service loads.

In this thesis, where stability is not a critical consideration, given small axial loads involved, large strains will be permitted.

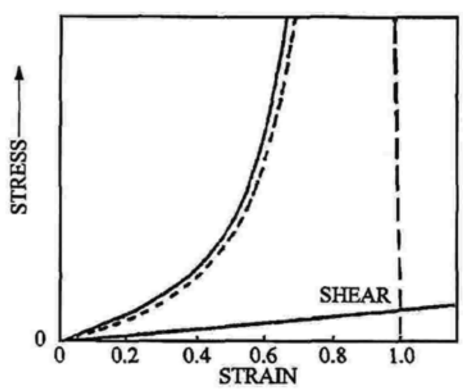


Fig.24- Stress/Strain Curves from Experiments on Bridge Bearings by [33]

Basic properties of structural elastomers and their lateral stiffness, given in Table 3, have been extracted from Long [29]. Similar properties are given in AASHTO Standard Specifications for Highway Bridges 14th edition.

Elastomers are classified by their hardness property, which is determined from Durometer tests. Common classes of hardness are International Rubber Hardness(IRHD) and Shore Hardness, both of which are essentially the same within the range of bearings used for structural applications [29].

Table 3- Basic Mechanical Properties of Elastomeric Bearings [29]

Hardness(IRHD)	Elastic Modulus E(MPa)	Shear Modulus, G(MPa)
50	2.3	0.6
60	3.7	1.0
70	6.2	1.4

In this thesis, elastomers will be modeled by linear springs, using stiffness *Kxy*, as listed in product catalogues of a typical manufacturer *Mageba*, and given in Appendix B.

The lateral stiffness, often documented in manufacturer's data, as is the case for elastomers selected in this thesis, can also be calculated by the following formula [29]:

$$Ks=G.\frac{A}{te}$$

Where A is the surface area, and te the net thickness of rubber. Lateral stiffness increases with reduced rubber thickness. Shear deformation capacity, on the contrary, increases with rubber thickness. Optimization for both stiffness, required for bracing criteria, and deformation, required for movement between buildings, is therefore a delicate task.

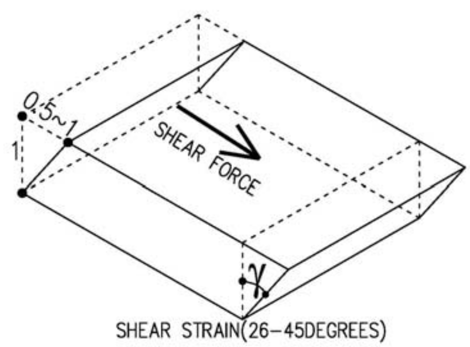


Fig.25: Shear Strains in Elastomeric Bearings

Selection of elastomers will be made to safisfy the following criteria, considered as a minimum:

- Min. deflection capacity of 50mm.
- Maximum number of elastomers is one per side of frame.

- If stiffness of the 2 items per floor selected from the largest commercial elastomer does not satisfy the bracing criteria, then the stiffness of the actual selected elastomer will be used, provided it satisfies the min. deflection capacity of 50mm.

4.1.3 Base Flexibility

For the purpose of elastic critical buckling load analysis, flexibility of a nominally pinned base is not set at its theoretical value of infinity, instead a value of 10 is assigned. Either of the two models shown in figure 26, can be used to model base flexibility of 10, depending on the capability of a given software [27]:

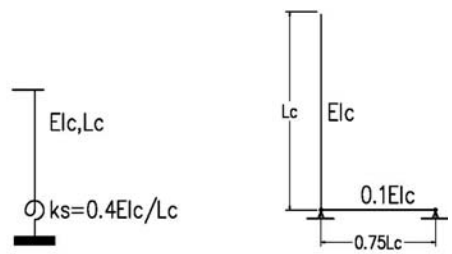


Fig.26-Models for Base Flexibility of 10 [27]

From the model on the right,

$$\text{Gbase} = 10 = \frac{\left(\frac{\text{Ic}}{\text{Lc}}\right)}{0.75 * \frac{0.1\text{Ic}}{0.75\text{Lc}}}$$

4.1.4 Story Models (Qualitative)

Story models are discussed in Section 3.5, and illustrated in Fig. 27. On the left is a typical base story, and on the right for a typical story selected from any of the stories above base.

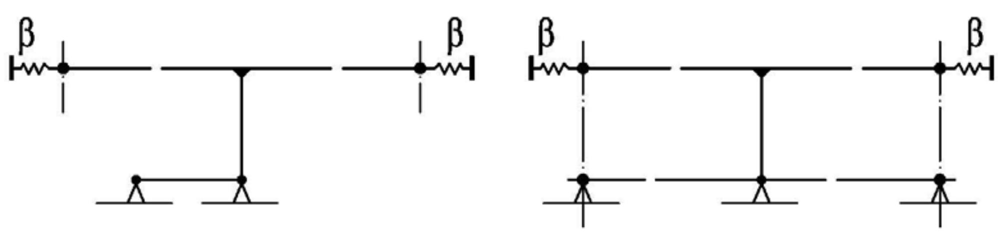


Fig.27- Basic Story-Buckling Models

4.2 Selected Frames

For the purpose of this thesis, only shallow long-span frames are considered, the reason being as explained earlier, these frames are likely to be squeezed between RC buildings(hence candidates for a potential use of bearings at joints between the two structures) for an architectural need to create a long span structure for a few public spaces. The criteria for selection are therefore a limited number of stories and free spans, and the number of frames generated are quite a few. However, since the purpose is to apply code methods and not make a comparative study, the few frames are considered sufficient, provided the analysis for each frame is detailed enough.

The number of stories is taken as 2 and 4, assumed to be typical for such applications. Span dimensions are fixed at combinations of 8m and 16m, also assumed typical. Total building dimensions or widths, from this combination of spans, are 32m (8+16+8) and 48m(8+16+16+8). The combination of number of floors and widths, gives three frames for analysis: Frame **A,B and ,C** shown in figures 29 through 31. For all frames, 8 meter bay spacing is assumed.

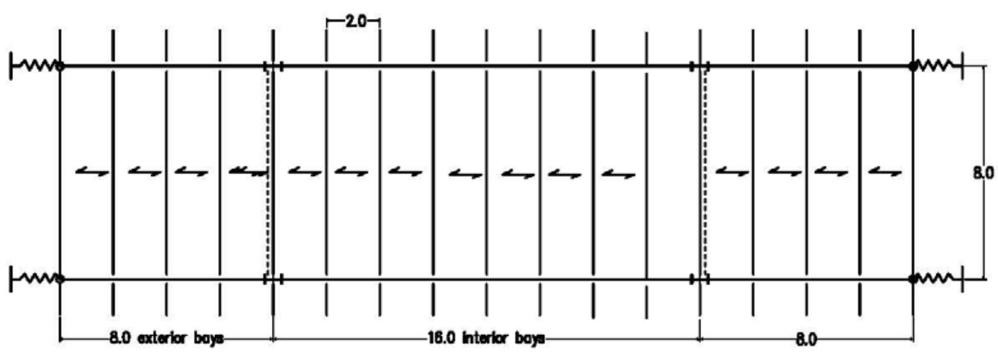


Fig.28: Typical Floor Plan

Members in these frames have been selected to satisfy design using AISC provisions under only service gravity loading DL+ LL, self weight included, using effective length method at K values corresponding to uninhibited sway(i.e unbraced), and live load deflection limit of L/360. As verified earlier, stability analysis is presented sufficiently enough when only gravity loads are taken into account. Effects of lateral loads is likely to increase the sections selected and add stiffness to the structure, making it less stability-critical. It is conservative for the purpose of stability analysis to proceed with sections selected based on loading DL + LL

The following notation is used for a given frame X:

Frame X0 : A version of frame X without bearings. It is also reffered to as a sway frame.
Frame X : A version of frame X restrained by bearings.

When not analysed for stability, both X0 and X retain hinged bases. For stability, bases are PR bases.

a)Frame A: 2 story, 3 bays

This is the basic model.

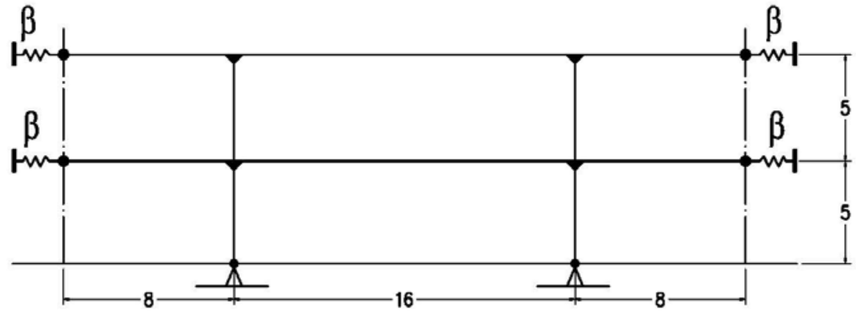


Fig.29-Frame A

b)Frame B: 4 story, 3 bays

This model adds floor bracing requirement without increasing the number of columns. It is also relatively slender compared to Frame A, hence expected to exhibit system-buckling behavior.

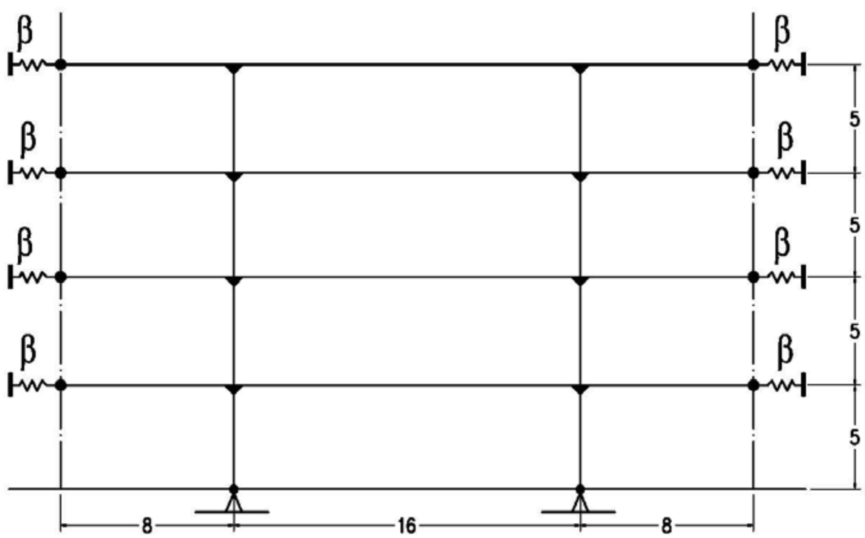


Fig.30-Frame B

c)Frame C: 2 story 4 bays

This frame adds loads to stories, putting more demand on bracing. However it does so with increased number of columns. Being the shallowest, it is expected to show shear-frame behavior.

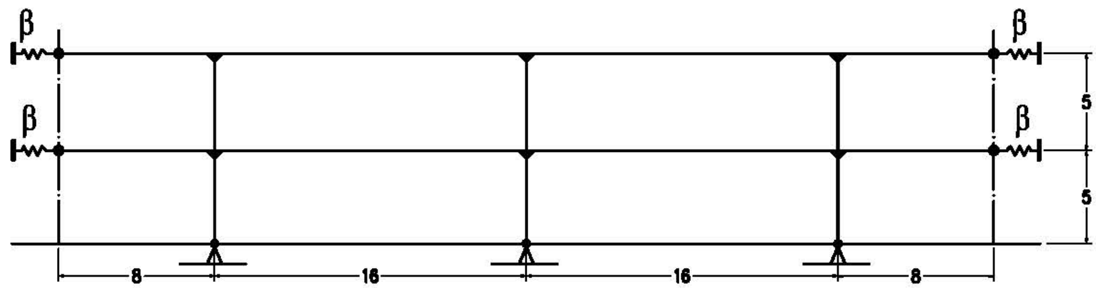


Fig.31-Frame C

4.3 Procedure

Procedure is outlined as follows and Pr is based on ASD :

- Determine gravity loading for columns in each story, hence story total, ∑Pr.

- Carry out story sway-sensitivity analysis (B2 for each story in each frame). Procedure is given in Chapter 3. If all stories satisfy the B2≤1.10 criterion, K=1.00 can be used.

- Apply AISC bracing criteria to determine required spring stiffness ,βi, for each floor.

$$\beta i, \mathrm{req} = \Omega\left(\frac{8\mathrm{Pr}}{\mathrm{Lb}}\right)$$

where Pr is the required column strength using ASD load combinations, Ω=2.00 and Lb is the braced length=floor height.

- Using the required stiffness βi, select a typical elastomer from Apendix B using selection criteria laid down in Section 4.1.2. If an elastomer is available, then in view of AISC bracing design criteria, the frame qualifies as braced, and K=1.00 can be used.

- Using the spring stiffness *Kxy* of the selected elastomers, perform buckling analysis in one or two steps:

  i) Carry out system buckling analysis to check if shear-mode governs.
  ii) If shear-mode governs, then story-buckling is an option. Carry out the analysis.

- Determine K values. Comment on the restraining effect of elastomers.

4.4 Frame A

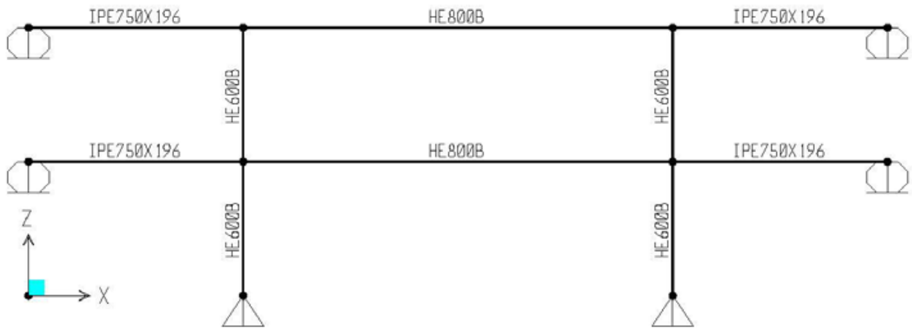


Fig.32-Basic Model and Selected Sections:Frame A

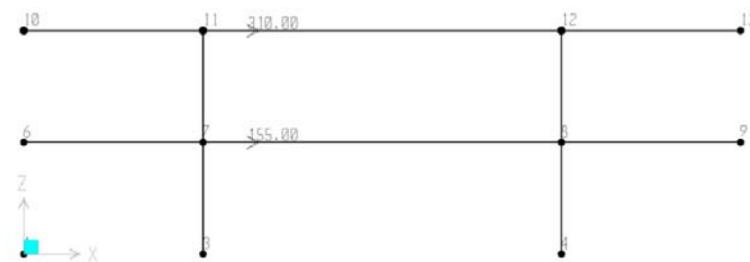


Fig.33: Loading Used to Determine Drifts- Frame A

Table 4: B2 Sway -Sensitivity : Frame A0(no bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	m	m		
Top	1776	310	0.063	0.017	1.04	Sensitive
Bottom,	3552	465	0.046	0.046	1.15	

Table 5- K factors and Pe for Sway : Frame A0(no bearings)

Story	Columns	Ix	G_A	G_B	K	Pe
		cm4				kN
Top	HE600B	171000	1.83	0.92	1.80	135016.2
Bottom,	HE600B	171000	10	1.83	2.05	135016.2

Table 6- Floor Gravity Loads and Required Bracing Stiffness/Strength-Frame A

Story	Sections	Column Load	Story Load	Story βi,req	Fi,req	Py	a.Pr/Py	Tau-b
		kN	kN	kN/m	kN	kN		
Top	HE600B	888	1776	5683.20	17.76	7425	0.19	1.00
Bottom,	HE600B	1776	3552	11366.40	35.52	7425	0.38	1.00

Table 7- Selected Bearings And Corresponding Stiffness: Frame A

Story	βreq	F,req	Bearing Description(Appendix A)	β	Δall.	Fall	Criteria	
	kN/m	kN		kN/m	mm	kN	Bracing	Δ
Top	5683.20	17.76	2 item Type B-500x600x121/89	6060	89.00	539.34	Yes	Yes
Bottom	11366.40	35.52	2 item Type B-900x900x160/125	11660	125.00	1457.5	Yes	Yes

Table 8: B2 Sway Sensitivity: Frame A(with bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	m	m		
Top	1776	310	0.024	0.007	1.02	Not Sensitive
Bottom,	3552	465	0.017	0.017	1.05	

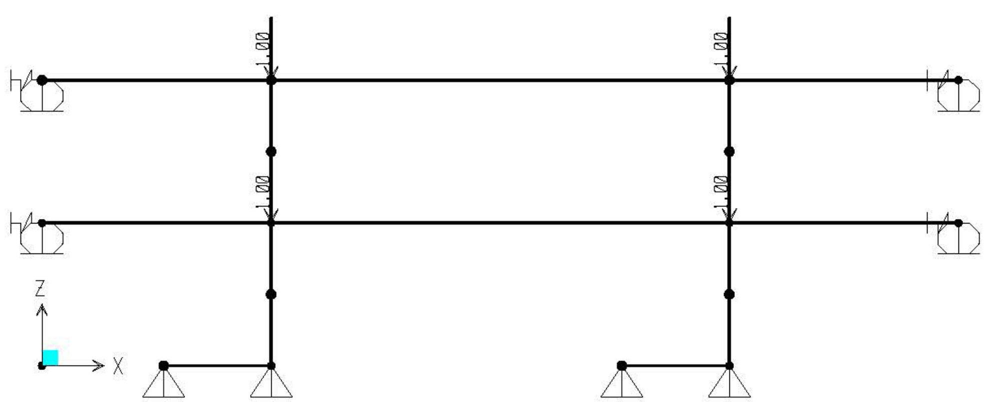


Fig.34-Model For System Buckling Analysis-Frame A(with bearings)

In Fig.34 (similar model without spring is used for comparison purposes):

- Load distribution between columns is important in buckling analysis. It is selected such that bottom story columns are loaded twice.

- Loads are lumped at column tops.

- Shear and axial deformations are eliminated.

- Base flexibility of 10 is modeled using horizontal beam elements of length 0.75 x Lc, and moment of inertia 0.1 x Ic.

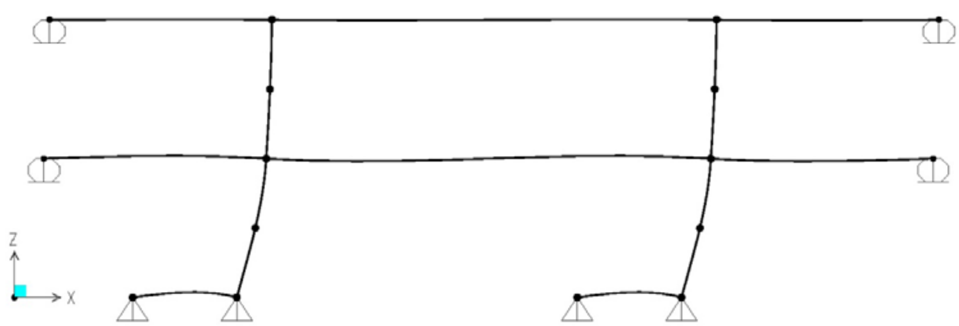


Fig.35- System Buckling: Frame A0(no bearings, λ=17261.5)

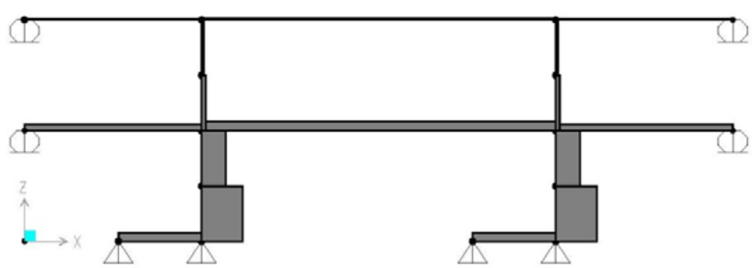


Fig.36: Shear Distribution at Buckling-Frame A0(no bearings)

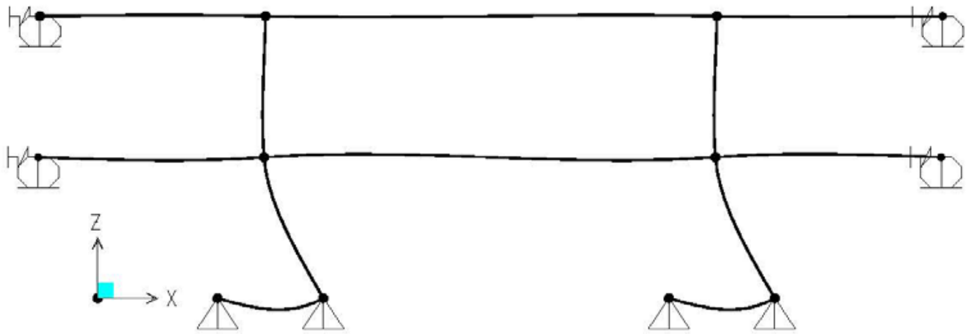


Fig.37- System Buckling Mode: Frame A with bearings (λ=35995)

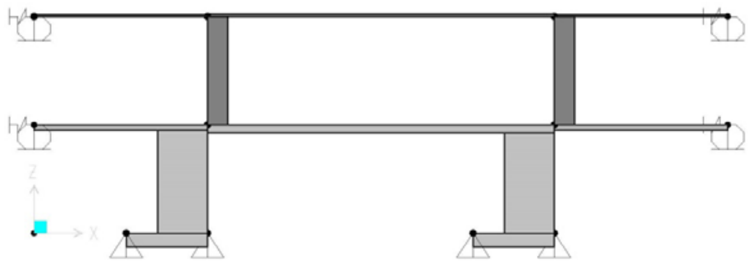


Fig.38: Shear Distribution at Buckling-Frame A with Bearings

In both Fig. 35 and 37:

- The mode shapes show shear- frame behavior.
- Top story columns sway in rigid mode: They do not actively contribute their stiffness to frame buckling in this mode. This is even more so in the case of no spring, judging from relative shear distribution between stories.
- Girders at story levels translate and deform less in bending compared to bottom story columns: They do not lend their stiffness to frame buckling in this mode, although this is less so in Fig. 35.
- The restraint at the base of the bottom columns is contributing.
- System-buckling behavior in this mode can predict strength of lower columns accurately enough but not of upper columns. For upper story columns, story-buckling can be considered instead of system buckling.

Table 9-K factors from System Buckling Analysis-Frame A0(no springs)

$$K_i = \sqrt{\frac{P_{e,i}}{\lambda . P_{ref,i}}}$$

Story	Columns	Pe	λi	Pref	Pcr	K
		kN				
Top	HE600B	135016.18	17261.5	1	17261.50	2.80
Bottom	HE600B	135016.18	17261.5	2	34523.00	1.98

Table 10- K factors from System Buckling Analysis-Frame A(with springs).

Story	Columns	Pe	λi	Pref	Pcr	K
		kN			kN	
Top	HE600B	135016.2	35995	1	35995.00	1.94
Bottom	HE600B	135016.2	35995	2	71990.00	1.37

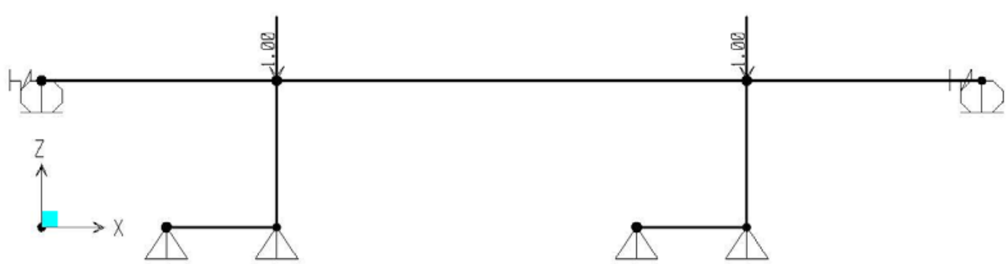


Fig.39-Model For Story Bucking Analysis of Bottom Story-Frame A

Results from story buckling analysis are summarized in Table 11 and presented in Figs. 40 and 41:

Table 11- K factors from Story Buckling Analysis-Frame A

Story	Columns	Pe	λi	Pref	Pcr	K
		kN			kN	
Top	HE600B	135016.2	94000	1	94000.00	1.20
Bottom	HE600B	135016.2	57612	2	115224.00	1.08

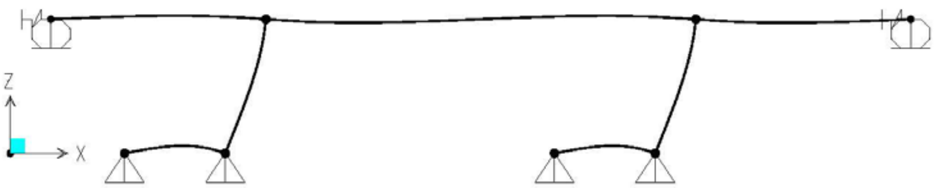


Fig.40- Story Buckling of Bottom Story-Frame A (λ=57612)

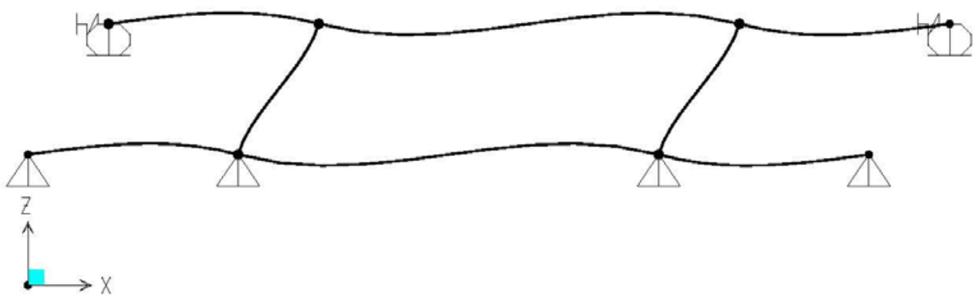


Fig.41- Story Buckling of Top Story-Frame A (λ=94000)

4.5 FRAME B

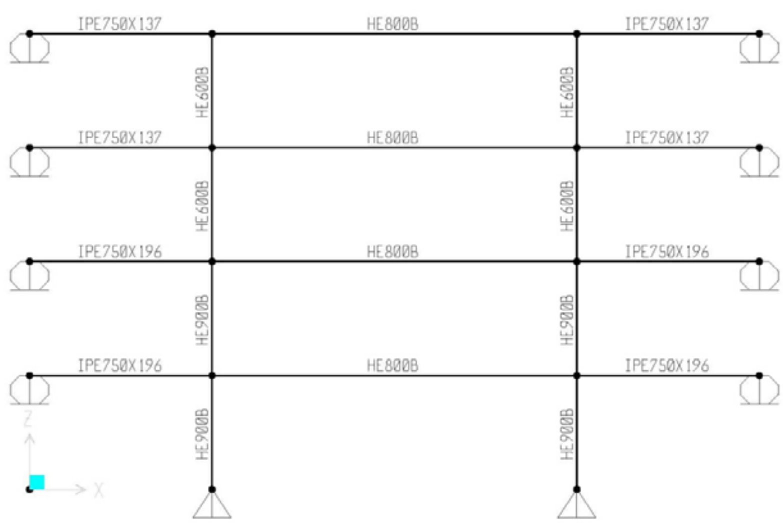


Fig.42-Basic Model and Selected Sections-Frame B.

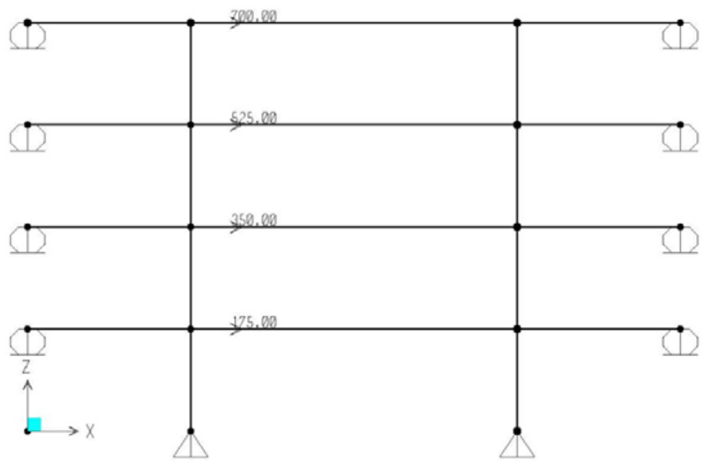


Fig.43: Loads Used to Determine Drifts-Frame B

Table 12: B2 Sway Sensitivity - Frame B0 (without bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	M	m		
Top	1776	700	0.269	0.036	1.04	Sensitive
Third	3552	1225	0.233	0.062	1.07	
Second	5328	1575	0.171	0.066	1.09	
Bottom	7104	1750	0.105	0.105	1.19	

Table 13- K factors and Pe for sway case - Frame B0

Story	Column	Ix	16m girder	Ix	8m girder	Ix	G_A	G_B	K	Pe
			kN	cm4		cm4				kN
Top	HE600B	171000	HE800B	359100	IPE750x137	159900	2.11	1.05	1.48	135016.19
3rd	HE600B	171000	HE800B	359100	IPE750x137	159900	3.55	2.11	1.78	135016.19
2nd	HE900B	494100	HE800B	359100	IPE750x396	240300	5.28	3.55	2.10	390125.72
Bottom,	HE900B	494000	HE800B	359100	IPE750x396	240300	10.00	5.28	2.60	390046.77

Table 14- Floor Gravity Loads and Required Bracing Stiffness/Strength-Frame B

Story	Section	Column Load	Story Load	Story βi,req	Story Fi,req	Py	α.Pr/Py	Tau-b
		kN	kN	kN/m	kN	kN		
Top	HE600B	888	1776	5683.20	56.832	7425	0.19	1.00
3rd	HE600B	1776	3552	11366.40	113.664	7425	0.38	1.00
2nd	HE900B	2664	5328	17049.60	170.496	10202.5	0.42	1.00
Bottom,	HE900B	3552	7104	22732.80	227.328	10202.5	0.56	0.99

Table 15–Selected Bearings and Corresponding Stiffness/Strength- Frame B

Story	β,req	F,req	Bearing (Appendix A)	β	Δ all.	Fall	Criteria	
	kN/m	kN		kN/m	mm	kN	Bracing	Δ
Top	5683.20	56.83	2 item Type B-400x600x73/53	8160	53.00	432.48	Yes	Yes
3rd	11366.40	113.66	2 item Type B-900x900x110/85	17160	85.00	1458.6	Yes	Yes
2nd	17049.60	170.50	2-item Type B-900x900x110/85	17160	85.00	1458.6	Yes	Yes
Bot.	22732.80	227.33	2-item Type B-900x900x110/85	17160	85.00	1458.6	No	Yes

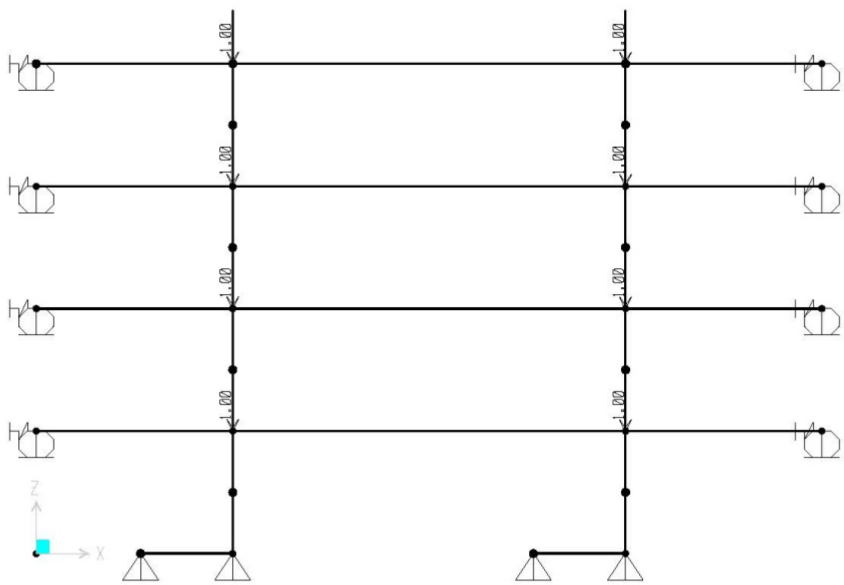


Fig.44-Model For System-Buckling Analysis : Frame B

Table 16: B2 Sway Sensitivity - Frame B (with bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	M	m		
Top	1776	700	0.047	0.013	1.01	Not Sensitive
Third	3552	1225	0.034	0.012	1.01	
Second	5328	1575	0.022	0.010	1.01	
Bottom	7104	1750	0.012	0.012	1.02	

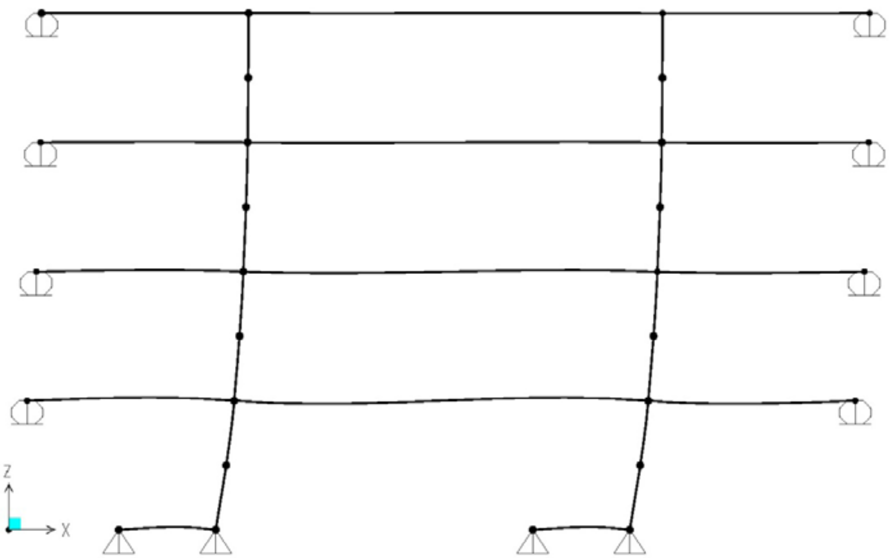


Fig.45-System Buckling: Frame B0(λ=16390.75)

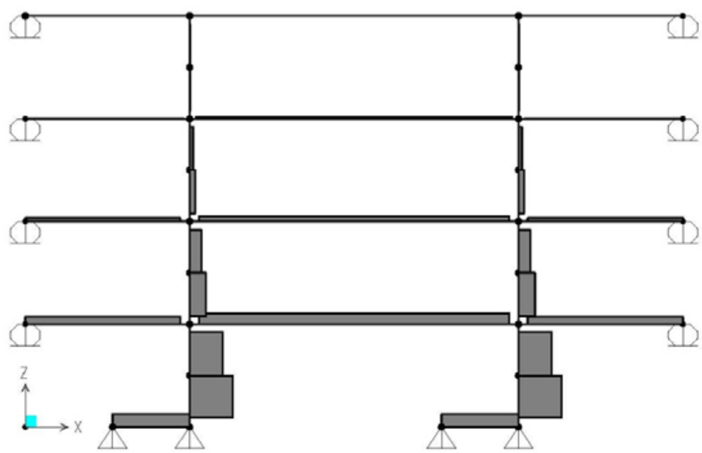


Fig.46: Shear Distribution at Buckling- Frame B0 (without bearings)

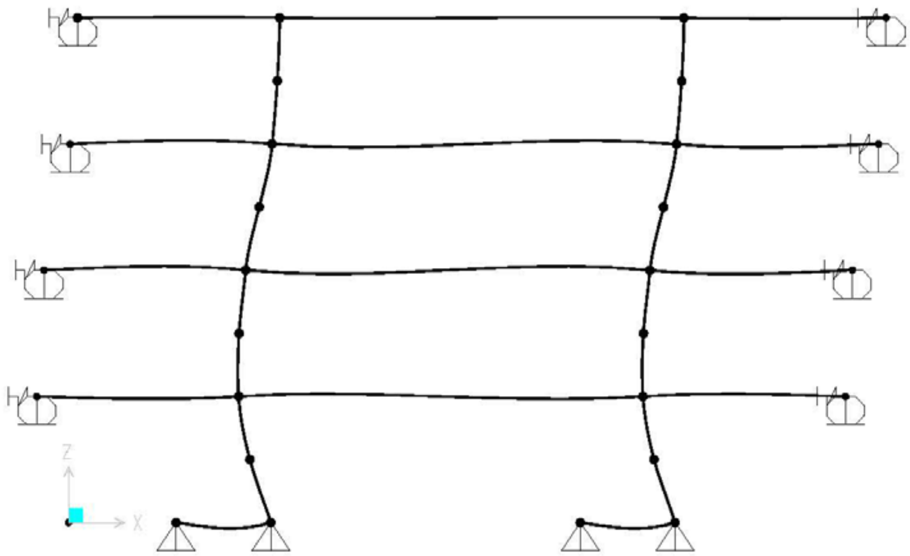


Fig.47: System Buckling : Frame B (λ =38714)

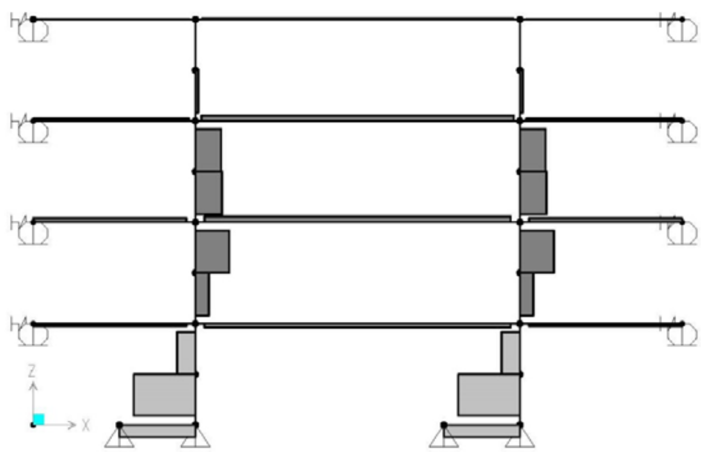


Fig.48: Shear Distribution at Buckling-Frame B (with bearings)

Fig.47 shows system-mode is dominant , but top floor show shear behavior. In Fig. 45, shear behavior fully governs.

Table 17- K factors from System-Buckling Analysis: Frame B0

Story	Column	Pe	Pref,i	λi	Pcr,i	Ki
		kN	kN		kN	
Top	HE600B	135016.19	1	16390.73	16390.73	2.87
3rd	HE600B	135016.19	2	16390.73	32781.46	2.03
2nd	HE900B	390125.72	3	16390.73	49172.19	2.82
Bottom,	HE900B	390046.77	4	16390.73	65562.92	2.44

Table 18- K factors from System- Buckling Analysis-Frame B

Story	Column	Pe	Pref,i	λi	Pcr,i	Ki
		kN	kN		kN	
Top	HE600B	135016.19	1	38714	38714	1.87
3rd	HE600B	135016.19	2	38714	77428	1.32
2nd	HE900B	390125.72	3	38714	116142	1.83
Bottom	HE900B	390046.77	4	38714	154856	1.59

Results from story buckling are summarized in Table 19 and presented in figures 49-52:

Table 19- K factors from Story-Buckling Analysis: Frame B

Story	Column	Pe	Pref,i	λi	Pcr,i	Ki
		kN	kN		kN	
Top	HE600B	135016.19	1	87667	87667	1.24
3rd	HE600B	135016.19	2	110534	221068	0.78
2nd	HE900B	390125.72	3	156633	469899	0.91
Bottom,	HE900B	390046.77	4	107037	428148	0.95

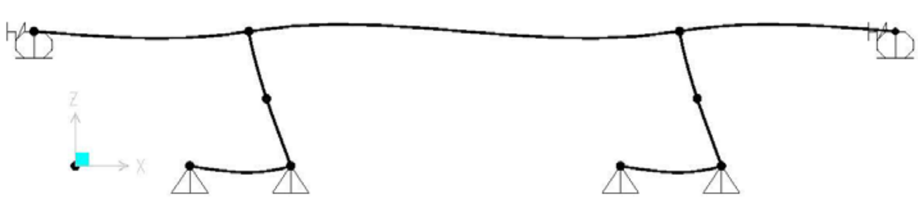


Fig.49- Story- Buckling:Frame B, Bottom Story (λ =107037)

Fig.50- Story- Buckling: Frame B, Story 2(λ =156632)

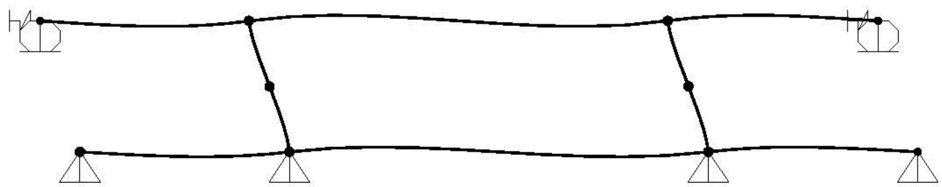


Fig.51- Story- Buckling: Frame B ,Story 3(λ =110534)

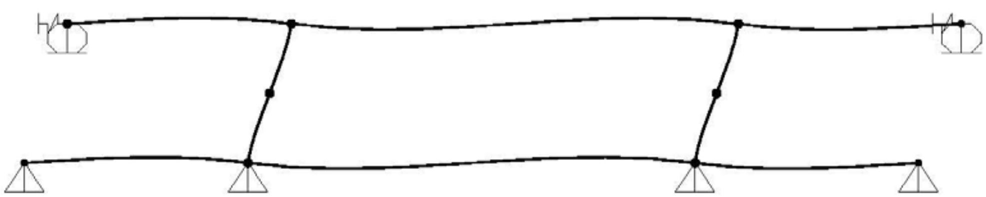


Fig.52-Story -Buckling: Frame B, Top Story(λ =87667)

4.6 FRAME C

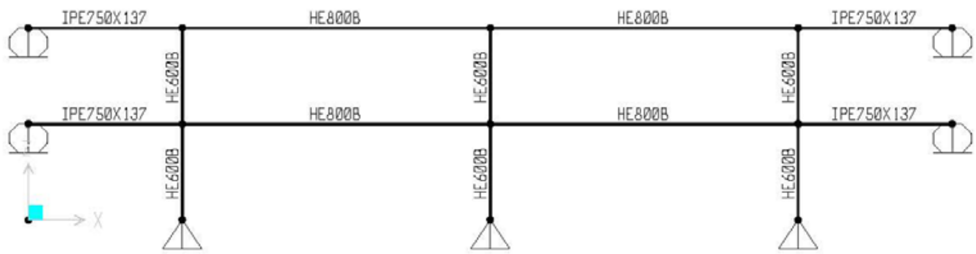


Fig.53- Basic Model and Selected Sections-Frame C

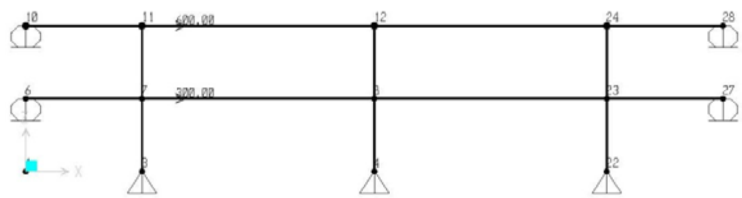


Fig.54: Loads Used to Determine Drifts-Frame C

Table 20: B2 Sway-Sensitivity - Frame C0 (without bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	M	m		
Top	2960	600	0.082	0.022	1.04	Sensitive
Bottom	5920	900	0.060	0.006	1.17	

Table 21-Floor Gravity Loads and Required Bracing Stiffness/Strength: Frame C

Floor DL	6.50			kPa	Floor LL	2.75	kPa	(reduced)	
			Tributary	End	96	m2	Mid	128	m2
	Column Sections			Column Loads			Story	Story	
Story	Left	Middle	Right	Left	Middle	Right	P	βi,req	Fi,req
				kN	kN	kN		kN/m	kN
Top	HE600B	HE600B	HE600B	888	1184	888	2960	9472.00	29.6
Bottom,	HE600B	HE600B	HE600B	1776	2368	1776	5920	18944.00	59.2

Table 22- τ Values for First Floor Columns-Frame C

Column	Section	Load	Py	α.Pr/Py	τb
			kN		
End	HE600B	1184	7425	0.26	1.00
Interior	HE600B	2368	7425	0.51	1.00

Table 23- Selected Bearings and Provided Stiffness- Frame C

Story	B,req	F,req	Bearings(Appendix A)	β	Δall.	Fall	Criteria	
	kN/m	kN		kN/m	mm	kN	Bracing	Δ
Top	9472.00	29.60	2 item Type B-500x600x73/53	10960	53.00	580.88	Yes	Yes
Bot.	18944.00	59.20	2 item Type B-900x900x110/85	17160	85.00	1458.6	No	Yes

Table 24-K factors and Pe for Sway Case: Frame C.

Story	Columns	Ix	End Columns			Interior Columns			
		cm4	G_A	G_B	K	G_A	G_B	K	Pe(kN)
Top	HE600B	171000	2.11	1.05	1.55	1.52	0.76	1.35	135016.2
Bottom,	HE600B	171000	10	2.11	2.11	10.00	1.52	2.00	135016.2

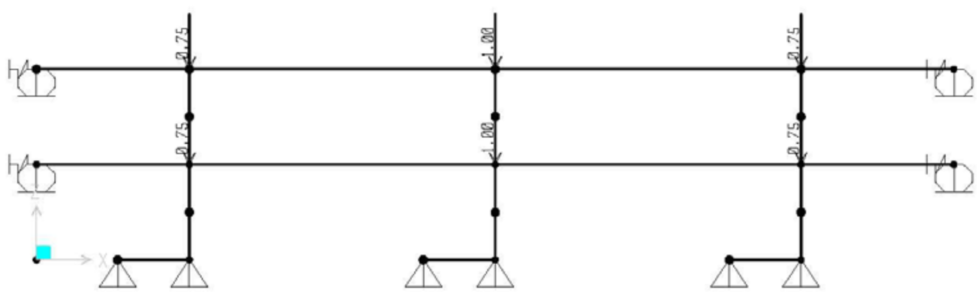


Fig.55- Model For System-Buckling Analysis: Frame C

Table 25: B2 Sway Sensitivity-Frame C (with bearings)

Story	∑P	∑H	Δtot	Δ_H	B2	Sway Sensitivity
	kN	kN	M	m		
Top	2960	600	0.027	0.009	1.02	Not Sensitive
Bottom	5920	900	0.018	0.018	1.05	

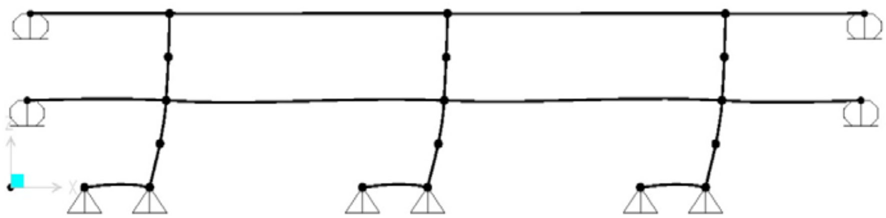


Fig.56-System Buckling: Frame C0(λ=19652.20)

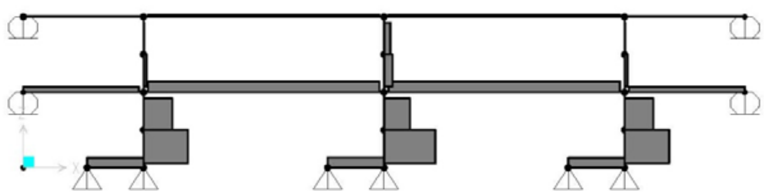


Fig.57: Shear Distribution at Buckling-Frame C0 (without bearings)

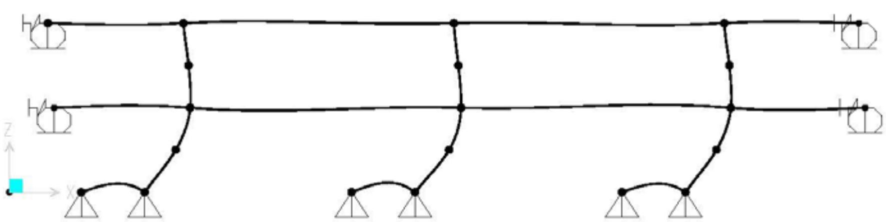


Fig.58- System- Buckling: Frame C(λ=44269)

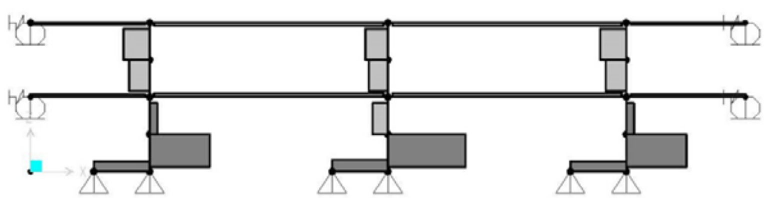


Fig.59: Shear Distribution at Buckling-Frame C (with bearings)

Figures 56 and 58 show shear – mode is dominant.

Table 26- K factors from System-Buckling Analysis: Frame C0

Story	Columns	Pe	End Columns				Interior Columns			
		kN	λi	Pref	Pcr	K	λi	Pref	Pcr	K
Top	HE600B	135016.2	19562.2	0.75	14671.65	3.03	19562	1	19562.00	2.63
Bottom	HE600B	135016.2	19562.2	1.5	29343.30	2.15	19562	2	39124.00	1.86

Table 27-K factors from System - Bucking Analysis-Frame C

Story	Columns	Pe	End Columns				Interior Columns			
		kN	λi	Pref	Pcr	K	λi	Pref	Pcr	K
Top	HE600B	135016.2	44269	0.75	33201.75	2.02	44269	1	44269.00	1.75
Bottom	HE600B	135016.2	44269	1.5	66403.50	1.43	44269	2	88538.00	1.23

Results from story buckling are summarized in Table 28, and presented in figures 60 and 61:

Table 28- K factors from Story-Buckling Analysis-Frame C

Story	Columns	Pe	End Columns				Interior Columns			
		kN	λi	Pref	Pcr	K	λi	Pref	Pcr	K
Top	HE600B	135016.2	105809	0.75	79356.75	1.30	105809	1	105809.00	1.13
Bottom	HE600B	135016.2	68872	1.5	103308.00	1.14	68872	2	137744.00	0.99

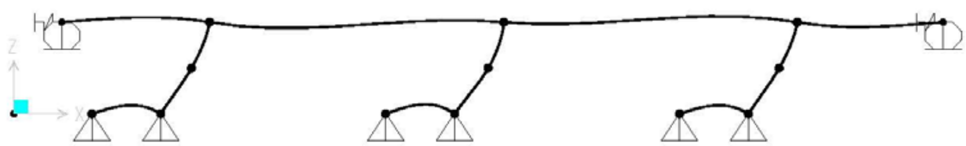


Fig.60- Story- Buckling: Frame C, Bottom Story(λ=68872)

Fig.61-Story-Buckling: Frame C, Top Story (λ=105809)

CHAPTER 5

CONCLUSION

5.1 Summary of Important Results

In the tables:

Ksway = Value of K determined using Alignment Charts on Frame X0 (i.e no bearings)
Ksystem = Value of K determined from system analysis on Frame X (i.e with bearings)
Kstory = Value of K determined from story analysis on stories of Frame X

Table 29- Summary of Results-Frame A

Column	Inelasticity	System Behavior	Bracing Criteria	Ksway	K system	Kstory	Sway Sensitivity
Top	No	Shear	Yes	1.80	1.94	**1.20**	None
Bottom	No	Shear	Yes	2.05	1.37	**1.08**	

Table 30- Summary of Results-Frame B

Column	Inelasticity	System Behavior	Bracing Criteria	Ksway	K system	Kstory	Sway Sensitivity
Top	No	Shear	Yes	1.48	1.87	**1.24**	None
3rd	No	System	Yes	1.78	**1.32**	0.78	
2nd	No	System	Yes	2.10	**1.83**	0.91	
Bottom	No	System	No	2.60	**1.59**	0.95	

Table 31- Summary of Results-Frame C

Story	Column	Inelasticity	System Behavior	Bracing Criteria	Ksway	K system	Kstory	Sway Sensitivity
Top	End	No	Shear	Yes	1.55	2.02	1.30	None
	Interior	No	Shear	Yes	1.35	1.75	1.13	
Bottom	End	No	Shear	Yes	2.11	1.43	1.14	
	Interior	No	Shear	Yes	2.00	1.23	0.99	

5.2 Discussion of Results

5.2.1 Sway-Sensitivity (B2)

For frames and bearings selected, all frames with bearings have satisfied the AISC criterion on B2 ($\leq$1.10) and can be designed for K=1. Frames without bearings have failed this criterion.

The selection of stiff bearings whose stiffness might have no real value in applications that do not consider stability, could have been criticized were it not observed that even those frames with no bearings have B2 values close to the limit of 1.10.

It can be concluded that long- span heavily- loaded frames should always be examined for stiffness using the AISC B2 criterion for potential use of K=1. This is especially useful for frames with bearings.

5.2.2 AISC Bracing Criteria

It is generally observed that due to large story loads, bracing criteria are difficult to meet unless very large bearings are used. The bearings however might at least brace the topmost floor.

Since large deflections are always associated with top floors, larger bearings are likely to be used there. For this reason, this finding from this thesis is not far from reality if bearings are considered.

If for any reason bearings are installed for seismic applications, large bearings are likely to be used in most stories, and role of bearings as a bracing element may be even more important.

In general bearings easily meet the force criterion. Satisfying the stiffness criterion appears to be difficult for these frames in which large story gravity loads must be supported.

5.2.3 Capacity (K values) with Bearings Considered As Determined using Elastic- Buckling Analysis

Simply by comparing results from system-buckling analyses between frames type X0(unrestrained by bearings) and X(restrained by bearings), significant differences in K-values(hence column strengths) are observed.

Bearings do show a considerable potential to increase strengths in view of stability limit state. Their contribution to stability of frames can be worth taking into account..

In shallow frames, story buckling modes seem to govern. The presence of bearing however, does not seem to amplify K-factors on upper floors significantly, even with these modes. The amplification effect is more pronounced in sway-permitted frames.

Story-buckling models give low K-factors, and in frames such as Frame A, where sufficient bearing stiffness is available and system- buckling mode show shear behavior, K-factors from story- buckling analysis give results that are very close to unity.

Story models could be used in such cases, finding a correct model however, is a problem not completely presented in this thesis due to limited literature on the subject. More study is required. But the fact that the model selected are highly conservative, it is possible that similar models can be used in real design.

In taller frames such as Frame B, system-buckling seems to dominate, and the few stories that show shear behavior, can still be evaluated on the basis of system-buckling, since as observed above, K-factor amplification is not so large when bearing stiffness is included in the analysis model.

In frames considered, inelasticity was not observed. But K-factors may drop further in cases of more stories where columns may be loaded into their inelastic range.

5.3 Conclusion

Alignment Charts and Story-Buckling Analysis can produce results that are comparable provided models used are approximately the same. Bracing Design and Drift Criterion (B2) can not be correlated at all even if both methods lead to use of K=1 in design. These methods can not be compared with results obtained from Story-Buckling or System-Buckling or even Alignment Charts, either.

The reason for all of the above is that most simplified methods are aimed at convenience in design and although their derivations are based on theoretical work, assumptions are made in their derivations in an effort to bring about simple procedures that can be of practical use. To cite an example, take the bracing provision. Both Winter's rigid bar and Timoshenko's discrete bracing analyses are theoretical and can be verified by any theoretical procedure. The final result from both of these analyses is the critical stiffness βi, which is tied to buckling load Pe or Pcr. This stiffness can therefore be verified from an analytical analysis. The design stiffness βbr required for bracing criteria, on the other hand, is obtained by modification on both the stiffness and the corresponding load. In ASD the stiffness is tied to a load of 16Pr instead of Pe. While this level of axial load might be close to Pe for some columns, it might not be so for every column and theoretical work might not agree with code based results.

In view of the above, it is impossible or difficult to do any comparison between K values obtained from the methods above, even though all are accepted under the same code. Judgement is required in selection between methods and evaluation of results.

In this thesis, a set of typical and realistic frames have been analysed and code provisions for sway sensitivity, bracing action and the rational computational method for determination of Pcr have been applied on those frames. The results are analysed but qualitatively.

It is possible to draw the following conclusions from this work:

- Long-span heavily loaded unbraced frames, with or without bearings, should always take advantage of possibility of K=1 using the AISC B2 criterion. These frames have large sway stiffnesses. In other word, a designer can start by assuming K=1 is possible and use drift criterion $B2 \leq 1.10$ first, and then try bracing design criteria if bearings can be considered to furnish significant lateral resistance.

- Even if bearings do not brace a floor, doubts on K-factors can be removed by simply carrying out system- buckling analyses. With this analysis, only a few top floors may have problematic K-values. For those lightly loaded floors, recommendations in [34] can be considered. Such floors may not even be stability-critical at all. If necessary, story-buckling analysis can be carried out with simple models, and for only those floors.

- Story-buckling analysis can also be used if shear- behavior is shown to be dominant in a full system analysis. Shallow buildings are likely to exhibit this behavior. Correct models are required. Alternatively, system-buckling models can be used since that where bearings are present and included in analysis model, extremely large K-factors are not likely to appear even with system analysis.

- It is possible in practice to effectively brace floors by bearings as long as total service floor loads does not exceed around 7000kN, judging from stiffnesses available in the relatively large-sized commercial bearings, as given typically in Appendix B(Mageba), but bracing criteria may not be satisfied at lower floors, due to the conflict between the fundamental need of flexible joints to deform, and the relatively less important need to brace a frame with sufficient strength and stiffness.

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APPENDIX A

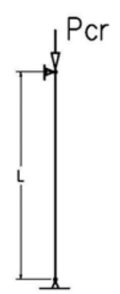
CAPABILITIES OF BUCKLING ANALYSIS: SAP2000

In all analyses below, shear and axial deformations have been eliminated. It is observed that at least three column segments (a minimum of two intermediate nodes) are required for columns not allowed to sway. For columns in frames that sway, a single segment is usually sufficient, but a minimum of two column segments (one intermediate node) generally give results even closer to theoretical ones.

Case A1: Simple Column HEA 260(Ix = 10450 e-8 m4), L=3m, Pe =22919.49kN (K=1.00)

The column is divided into 2,3 and 4 segments and analysed for each case:

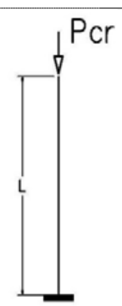
Table A1: Sap2000 Verification - Case A1



Number of segments	Pcr (kN)
2	22952.32kN
3	22927.68kN
4	22920.75kN

Case A2: Cantilever Column HEA 260(Ix = 10450 e-8 m4), L=3m, Pe =5729.85kN (K=2.00)

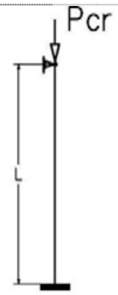
Table A2: SAP2000 verification-Case A2



Number of segments	Pcr (kN)
1	5772.77kN
2	5732.77kN
3	5730.22

Case A3: Propped Cantilever Column HEA 260(Ix = 10450 e-8 m4), L=3m, Pe =46774.31kN (K=0.70)

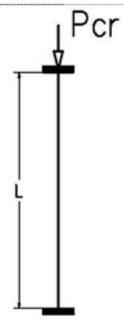
Table A3: SAP2000 verification - Case A3



Number of segments	Pcr (kN)
1	69666.67kN
2	48077.40kN
3	46968.35kN

Case A4: Fixed Ended Column HEA 260(Ix = 10450 e-8 m4), L=3m, Pe =91677.64kN (K=0.70)

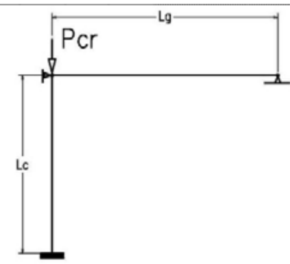
Table A4: SAP2000 verification - Case A4



Number of segments	Pcr (kN)
2	92841.05kN
3	93642.27kN
5	91919.77kN

Case A5: L-frame with no sway and fixed column base, columns HEA 260(Ix = 10450 e-8 m4), Girder IPE240 (Ix=3890e-8m4), Lc= Lg= 3m, Pcr=54247kN (K=0.65). Pcr is compared with the value obtained from alignment charts as well as the French equations for for braced frames [12].

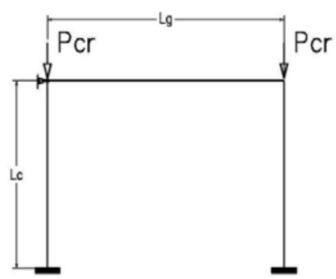
Table A5: SAP2000 verification - Case A5



Number of segments	Pcr (kN)
1	55322.75
2	54356.67
3	54111.30

Case A6 Portal frame with no sway and fixed column bases, sections HEA 260(Ix = 10450 e-8 m4), Lc= Lg= 3m, Pcr=59241kN (K=0.625). Pcr is compared with the value obtained from alignment charts as well as the French equations for braced frames.

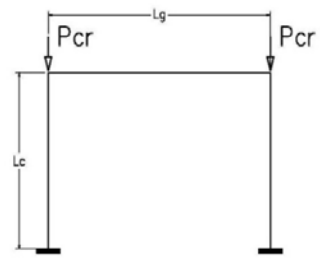
Table A6: SAP2000 Verification- Case A6



Number of segments	Pcr (kN)
1	59829.37
2	58894.81
3	58608.44

Case A7 Portal frame with sway and fixed column bases, sections HEA 260(Ix = 10450 e-8 m4), Lc= Lg= 3m, Pcr=16945.10kN (K=1.15). Pcr is compared with the value obtained from alignment charts as well as the French equation for unbraced frames [12].

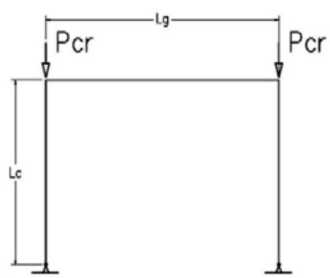
Table A7: SAP2000 Verification- Case A7



Number of segments	Pcr (kN)
1	17185.95
2	17144.51
3	17136.81

Case A8 Portal frame with sway and hinged column bases, sections HEA 260(Ix = 10450 e-8 m4), Lc= Lg= 3m, Pcr=4226.44kN (K=2.33). Pcr is compared with the value obtained from alignment charts, and exact solution given in [11] as well as the French equation(with G_B set to 100000) for unbraced frames.

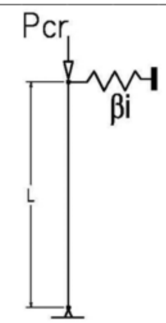
Table A8: SAP2000 verification- Case A8



Number of segments	Pcr (kN)
1	4229.92
2	4229.23
3	4229.11

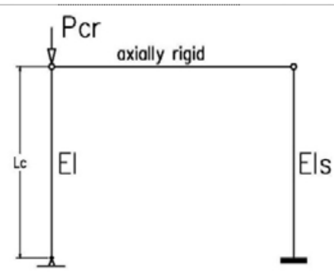
Case A9 Simple column restrained by a spring at the top, section SHS150 x 6 (Ix = 1196 e-8 m4), L= 3m, spring stiffness βi = 874kN/m = the ideal stiffness for a perfect column, Pcr= Pe = 2623kN (K=1.00). Pe is derived from Timoshenko's definition of ideal spring stiffness. A single column segment was enough to capture this value in SAP2000.

Table A9: SAP2000 Verification- Case A9

	Number of segments	**Pcr (kN)**
	1	2622.00

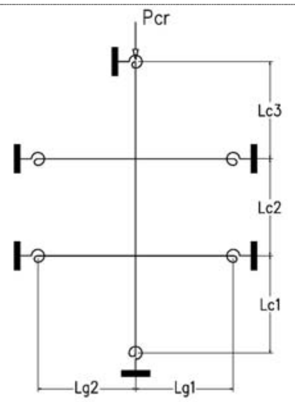
Case A10 A simple loaded column restrained by a cantilever column that provides lateral stiffness equal to ideal stiffness βi = 874kN/m . This case is the same as that of Case A9, except for the way the column is restrained by the spring. Column section SHS150 x 6 (Ix = 1196 e-8 m4), Cantilever column Is = 3933e-8 is required to achieve βi= 3EIs/Lc^3=874kN/m.   Pcr= Pe = 2623kN (K=1.00) as before.   A single column segment was enough to capture this value in SAP2000.

Table A10: SAP2000 Verification- Case A10

	Number of segments	**Pcr (kN)**
	1	2622.92

Case A11 This case verifies the subassembly used to derive the Alignment Charts for sway-permitted case. Sections HEA 260 (Ix = 10450e-8 m4) for both girders and columns. Lg=Lc and G_A=G_B=1.00. Pcr = 11682kN (K=1.34) from the charts. Pcr obtained from the software is compared to this value. The value of each rotational spring is a sum of member stiffnesses 4EI/L meeting at the joints. Unit load is applied as a reference load Pref, hence Pcr= λ. Pref = λ. λis reported directly by the software corresponding to the fundamental sway mode of buckling.

Table A11: SAP2000 Verification- Case A11

	Number of segments	Pcr (kN)
	2	11870

APPENDIX B

MAGEBA PROSPEKT LASTO-BLOCK ELASTOMERIC BEARINGS

(online catalog, mageba website,2013)

Table B 1: Mageba Lasto-Block Elastomers- Instructions and Definitions (Page 5)

mageba **Instructions and definitions 5**

Process for choosing bearing dimensions

Condition 1: $v_{xyd} = 25\% \cdot v_{xy,max}$				Condition 2: $v_{xyd} = 50\% \cdot v_{xy,max}$				Condition 3: $v_{xyd} = 100\% \cdot v_{xy,max}$				Bearing dimensions/Parameters						
N_d	N_{dmin} (concrete/ steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (concrete/steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (concrete/steel)	v_{xyd}	α_{ab}	a	b	t	Weight	T_e	K_z	K_{xy}
[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[mm]	[mm]	[mm]	[kg]	[mm]	[kN/mm]	[kN/mm]
172	(51 / 51)	4.2	2.0	159	(47 / 47)	10.5	1.7	139	(45 / 90)	21.0	1.3	100	200	30	1.8	21	55.3	0.86
391	(79 / 79)	5.8	1.8	366	(74 / 74)	14.5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	150	200	41	3.8	29	104.0	0.93
1'720	(326 / 326)	15.4	3.0	1'576	(299 / 299)	38.5	2.7	1'337	(270 / 540)	77.0	2.3	300	400	105	37.8	77	293.2	1.40

① Bearing dimensions calculated according to load condition 1

② Bearing dimensions calculated according to load condition 2

③ Bearing dimensions calculated according to load condition 3

Instructions for using the tables

Selection of the required bearing dimensions is performed in three steps (using typical load conditions) with the aid of the following tables (see pages 6-21). A suitable bearing dimension should be chosen for each load condition which can support the required input parameters (N_d, N_{dmin}, v_{xyd}, α_{ab}).

The bearing dimensions should primarily be calculated using the movement capacity to be accommodated. Vertical loads and rotations should then be checked (see calculation example).

After the three bearing dimensions have been calculated, the bearing with the smallest dimension is the size to be used (most economical solution).

Calculation example:

- Bearing type: B
- Connection: concrete both sides
- Loads: N_d = 114kN, N_{dmin} (actual) = 74kN
- Movement: v_{xy} = 13.5mm
- Rotation: α_{ab} = 1.0 %

1) Calculation of the necessary bearing dimensions using load condition 1: $v_{xyd} = 25\% \cdot v_{xy,max}$

=> Bearing dimensions: 300x400x105mm

(v_{xyd} = 15.4mm > 13.5mm, N_d = 1720kN > 114kN, α_{ab} = 3.0% > 1.0%)

Note: N_{dmin} (allowable) = 326 kN > N_{dmin} (actual)

(Because the min. load has not been met, this bearing type requires additional anchoring to prevent "shifting" e.g. shear edges. An alternative is to use an elastomeric bearing of Type C with shear lugs or bolt fastenings.)

2) Calculation of the necessary bearing dimensions using load condition 2: $v_{xyd} = 50\% \cdot v_{xy,max}$

=> Bearing dimensions: 150x200x41mm

(v_{xyd} = 14.5mm > 13.5mm, N_d = 366 kN > 114kN, α_{ab} = 1.6% > 1.0%)

Note: N_{dmin} (allowable) = 74kN = N_{dmin} (actual)

3) Calculation of the necessary bearing dimensions using load condition 3: $v_{xyd} = 100\% \cdot v_{xy,max}$

=> Bearing dimensions: 100x200x30mm

(v_{xyd} = 21.0mm > 13.5mm, N_d = 139kN > 114kN, α_{ab} = 1.3% > 1.0%)

Note: N_{dmin} (allowable) = 45kN < N_{dmin} (actual)

=> Result: the bearing with the dimensions 100x200x30 [mm] is the most economical solution for the reported loads.

Note: Please note that the following tables should only be used to calculate approximate bearing dimensions. More precise dimensions and/or optimised sizes can be requested from mageba directly.

Meaning of symbols

a	:	total bearing width (shorter measurement for rectangular bearings)
b	:	total bearing length (longer measurement for rectangular bearings)
t	:	total elastomeric bearing height
T_e	:	nominal thickness of all elastomer layers
K_z	:	vertical compressive deflection of a bearing
K_{xy}	:	horizontal compressive deflection of a bearing
N_d	:	rated value of vertical forces (design level)
N_{dmin} (concrete)	:	required min. vertical forces with concrete connection (design level)
N_{dmin} (steel)	:	required min. vertical forces with steel connection (design level)
v_{xyd}	:	resultant of the horizontal displacement (maximum for given displacement)
$v_{xy,max}$	:	resultant of the maximum horizontal displacement (under any loading)
α_{ab}	:	resultant of the rotation of a bearing

Table B 2: Mageba Lasto-Block Elastormers - Load Table Type B (Page 6)

6 Load table Type B

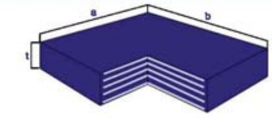
mageba

Type B elastomeric bearings are enclosed on all sides with rubber (NR/CR) and are used between concrete or steel construction components. This type of bearing can simply be positioned between the structural components.

Condition 1: $v_{xyd} = 25\% \cdot v_{xy,max}$				Condition 2: $v_{xyd} = 50\% \cdot v_{xy,max}$				Condition 3: $v_{xyd} = 100\% \cdot v_{xy,max}$				Bearing dimensions/Parameters						
N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	a	b	t	T_e	Weight	K_z	K_{xy}
[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[mm]	[mm]	[mm]	[mm]	[kg]	[kN/mm]	[kN/mm]
114	(38 / 38)	4.2	3.1	106	(35 / 35)	10.5	2.7	92	(34 / 68)	21.0	2.1	100	150	30	21	1.4	33.2	0.64
81	(37 / 37)	5.8	6.1	73	(34 / 34)	14.5	5.4	59	(34 / 68)	29.0	4.4	100	150	41	29	1.8	24.0	0.47
172	(51 / 51)	4.2	2.0	159	(47 / 47)	10.5	1.7	139	(45 / 90)	21.0	1.3	100	200	30	21	1.8	55.3	0.86
122	(50 / 50)	5.8	4.4	110	(45 / 45)	14.5	4.0	89	(45 / 90)	29.0	3.3	100	200	41	29	2.5	40.1	0.62
547	(80 / 80)	4.2	0.0	502	(76 / 76)	10.5	0.0	426	(70 / 135)	21.0	0.0	150	200	30	21	2.8	143.7	1.29
391	(79 / 79)	5.8	1.8	366	(74 / 74)	14.5	1.6	325	(68 / 135)	29.0	1.0	150	200	41	29	3.8	104.0	0.93
303	(78 / 78)	7.4	3.8	278	(72 / 72)	18.5	3.4	236	(68 / 135)	37.0	2.7	150	200	52	37	4.8	81.5	0.73
756	(101 / 101)	4.2	0.0	694	(96 / 96)	10.5	0.0	589	(88 / 169)	21.0	0.0	150	250	30	21	3.5	215.2	1.61
541	(99 / 99)	5.8	1.3	507	(93 / 93)	14.5	1.0	449	(85 / 169)	29.0	0.7	150	250	41	29	4.8	155.8	1.16
419	(98 / 98)	7.4	2.7	384	(90 / 90)	18.5	2.4	327	(85 / 169)	37.0	2.0	150	250	52	37	6.0	122.2	0.91
974	(121 / 121)	4.2	0.0	894	(116 / 116)	10.5	0.0	759	(106 / 203)	21.0	0.0	150	300	30	21	4.2	293.3	1.93
697	(120 / 120)	5.8	1.0	653	(112 / 112)	14.5	0.7	578	(102 / 203)	29.0	0.6	150	300	41	29	5.7	212.4	1.40
540	(118 / 118)	7.4	2.0	495	(109 / 109)	18.5	1.8	421	(102 / 203)	37.0	1.4	150	300	52	37	7.2	166.5	1.09
1'197	(136 / 136)	5.8	0.0	1'120	(129 / 129)	14.5	0.0	950	(119 / 225)	29.0	0.0	200	250	41	29	6.4	293.3	1.55
930	(135 / 135)	7.4	1.1	874	(126 / 126)	18.5	1.0	781	(113 / 225)	37.0	0.6	200	250	52	37	8.0	229.9	1.22
758	(133 / 133)	9.0	2.4	702	(124 / 124)	22.5	2.1	609	(113 / 225)	45.0	1.6	200	250	63	45	9.7	189.0	1.00
638	(132 / 132)	10.6	3.5	582	(121 / 121)	26.5	3.3	489	(113 / 225)	53.0	2.7	200	250	74	53	11.3	160.5	0.85
1'563	(164 / 164)	5.8	0.0	1'463	(156 / 156)	14.5	0.0	1'240	(143 / 270)	29.0	0.0	200	300	41	29	7.7	407.9	1.86
1'215	(162 / 162)	7.4	0.8	1'141	(152 / 152)	18.5	0.7	1'020	(136 / 270)	37.0	0.4	200	300	52	37	9.7	319.7	1.46
990	(161 / 161)	9.0	1.8	917	(149 / 149)	22.5	1.6	795	(135 / 270)	45.0	1.3	200	300	63	45	11.7	262.9	1.20
833	(159 / 159)	10.6	2.8	760	(145 / 145)	26.5	2.5	638	(135 / 270)	53.0	2.1	200	300	74	53	13.6	223.2	1.02
1'944	(192 / 192)	5.8	0.0	1'819	(183 / 183)	14.5	0.0	1'542	(168 / 315)	29.0	0.0	200	350	41	29	9.0	531.2	2.17
1'510	(190 / 190)	7.4	0.7	1'419	(179 / 179)	18.5	0.6	1'268	(160 / 315)	37.0	0.3	200	350	52	37	11.3	416.4	1.70
1'231	(188 / 188)	9.0	1.4	1'140	(174 / 174)	22.5	1.3	989	(158 / 315)	45.0	1.0	200	350	63	45	13.6	342.4	1.40
1'036	(187 / 187)	10.6	2.3	945	(170 / 170)	26.5	2.0	794	(158 / 315)	53.0	1.6	200	350	74	53	16.0	290.7	1.19
2'335	(219 / 219)	5.8	0.0	2'185	(209 / 209)	14.5	0.0	1'852	(192 / 360)	29.0	0.0	200	400	41	29	10.3	661.2	2.48
1'814	(218 / 218)	7.4	0.6	1'705	(205 / 205)	18.5	0.4	1'523	(183 / 360)	37.0	0.3	200	400	52	37	13.0	518.2	1.95
1'479	(216 / 216)	9.0	1.1	1'370	(200 / 200)	22.5	1.0	1'188	(180 / 360)	45.0	0.7	200	400	63	45	15.6	426.1	1.60
1'244	(214 / 214)	10.6	1.7	1'135	(195 / 195)	26.5	1.6	953	(180 / 360)	53.0	1.3	200	400	74	53	18.3	361.8	1.36
2'327	(207 / 207)	5.8	0.0	2'142	(200 / 200)	14.5	0.0	1'851	(187 / 338)	29.0	0.0	250	300	41	29	9.7	650.0	2.33
2'223	(206 / 206)	7.4	0.0	2'105	(196 / 196)	18.5	0.0	1'782	(180 / 338)	37.0	0.0	250	300	52	37	12.2	509.5	1.82
1'815	(205 / 205)	9.0	0.8	1'710	(193 / 193)	22.5	0.7	1'535	(173 / 338)	45.0	0.3	250	300	63	45	14.6	418.9	1.50
1'530	(203 / 203)	10.6	1.7	1'425	(189 / 189)	26.5	1.4	1'250	(169 / 338)	53.0	1.0	250	300	74	53	17.1	355.7	1.27
1'321	(202 / 202)	12.2	2.4	1'215	(186 / 186)	30.5	2.1	1'040	(169 / 338)	61.0	1.7	250	300	85	61	19.6	309.0	1.11
3'138	(278 / 278)	5.8	0.1	3'022	(268 / 268)	14.5	0.0	2'810	(251 / 450)	29.0	0.0	250	400	41	29	12.9	1'075.7	3.10
3'117	(276 / 276)	7.4	0.1	2'969	(263 / 263)	18.5	0.1	2'705	(242 / 450)	37.0	0.0	250	400	52	37	16.3	843.1	2.43
2'756	(275 / 275)	9.0	0.6	2'596	(259 / 259)	22.5	0.4	2'330	(232 / 450)	45.0	0.1	250	400	63	45	19.6	693.2	2.00
2'323	(273 / 273)	10.6	1.1	2'164	(254 / 254)	26.5	1.0	1'898	(225 / 450)	53.0	0.7	250	400	74	53	22.9	588.6	1.70
2'005	(271 / 271)	12.2	1.7	1'845	(249 / 249)	30.5	1.4	1'579	(225 / 450)	61.0	1.1	250	400	85	61	26.3	511.4	1.48
3'164	(334 / 334)	8.2	0.0	2'894	(320 / 320)	20.5	0.0	2'469	(296 / 540)	41.0	0.0	300	400	57	41	21.1	550.6	2.63
2'542	(331 / 331)	10.6	0.8	2'398	(313 / 313)	26.5	0.6	2'159	(282 / 540)	53.0	0.3	300	400	73	53	26.7	425.9	2.04
2'055	(329 / 329)	13.0	1.8	1'911	(306 / 306)	32.5	1.7	1'672	(270 / 540)	65.0	1.3	300	400	89	65	32.3	347.3	1.66
1'720	(326 / 326)	15.4	3.0	1'576	(299 / 299)	38.5	2.7	1'337	(270 / 540)	77.0	2.3	300	400	105	77	37.8	293.2	1.40
4'206	(419 / 419)	8.2	0.0	3'977	(401 / 401)	20.5	0.0	3'394	(371 / 675)	41.0	0.0	300	500	57	41	26.5	812.6	3.29
3'494	(416 / 416)	10.6	0.6	3'296	(392 / 392)	26.5	0.4	2'967	(353 / 675)	53.0	0.1	300	500	73	53	33.5	628.6	2.55
2'824	(412 / 412)	13.0	1.4	2'627	(384 / 384)	32.5	1.1	2'298	(338 / 675)	65.0	0.8	300	500	89	65	40.4	512.6	2.08
2'364	(409 / 409)	15.4	2.1	2'166	(375 / 375)	38.5	2.0	1'837	(338 / 675)	77.0	1.6	300	500	105	77	47.4	432.7	1.75
5'061	(505 / 505)	8.2	0.1	4'842	(483 / 483)	20.5	0.0	4'358	(446 / 810)	41.0	0.0	300	600	57	41	31.8	1'095.9	3.95
4'486	(500 / 500)	10.6	0.4	4'233	(472 / 472)	26.5	0.3	3'810	(425 / 810)	53.0	0.1	300	600	73	53	40.2	847.7	3.06
3'627	(496 / 496)	13.0	1.0	3'373	(461 / 461)	32.5	0.8	2'951	(405 / 810)	65.0	0.7	300	600	89	65	48.6	691.2	2.49
3'035	(492 / 492)	15.4	1.6	2'782	(451 / 451)	38.5	1.4	2'359	(405 / 810)	77.0	1.1	300	600	105	77	57.0	583.5	2.10
4'445	(443 / 443)	8.2	0.1	4'281	(427 / 427)	20.5	0.0	3'847	(400 / 709)	41.0	0.0	350	450	57	41	27.8	935.0	3.46
4'413	(440 / 440)	10.6	0.3	4'201	(419 / 419)	26.5	0.0	3'694	(384 / 709)	53.0	0.0	350	450	73	53	35.2	723.3	2.67
3'688	(437 / 437)	13.0	1.0	3'469	(411 / 411)	32.5	0.7	3'105	(368 / 709)	65.0	0.4	350	450	89	65	42.5	589.8	2.18
3'090	(434 / 434)	15.4	1.8	2'872	(403 / 403)	38.5	1.6	2'507	(355 / 709)	77.0	1.1	350	450	105	77	49.8	497.9	1.84
2'654	(430 / 430)	17.8	2.7	2'435	(395 / 395)	44.5	2.4	2'071	(355 / 709)	89.0	1.8	350	450	121	89	57.2	430.8	1.59
5'653	(563 / 563)	10.6	0.4	5'417	(540 / 540)	26.5	0.3	5'025	(501 / 900)	53.0	0.1	400	500	73	53	44.8	1'141.0	3.40
5'617	(560 / 560)	13.0	0.6	5'328	(531 / 531)	32.5	0.4	4'847	(483 / 900)	65.0	0.1	400	500	89	65	54.1	930.4	2.77
5'144	(556 / 556)	15.4	1.0	4'829	(522 / 522)	38.5	0.8	4'303	(465 / 900)	77.0	0.4	400	500	105	77	63.5	785.4	2.34
4'422	(553 / 553)	17.8	1.7	4'107	(513 / 513)	44.5	1.4	3'581	(450 / 900)	89.0	1.1	400	500	121	89	72.8	679.5	2.02
3'872	(549 / 549)	20.2	2.4	3'556	(505 / 505)	50.5	2.1	3'030	(450 / 900)	101.0	1.7	400	500	137	101	82.1	598.8	1.78

Table B 3: Mageba Lasto-Block Elastomers- Load Type B (Page 7)

mageba **Load table Type B 7**



Type B elastomeric bearings are enclosed on all sides with rubber (NR/CR) and are used between concrete or steel construction components. This type of bearing can simply be positioned between the structural components.

Condition 1: $v_{xyd} = 25\% \cdot v_{xy,max}$				Condition 2: $v_{xyd} = 50\% \cdot v_{xy,max}$				Condition 3: $v_{xyd} = 100\% \cdot v_{xy,max}$				Bearing dimensions/Parameters						
N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	N_d	N_{dmin} (Concrete/Steel)	v_{xyd}	α_{ab}	a	b	t	T_e	Weight	K_z	K_{xy}
[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[kN]	[kN]	[mm]	[%]	[mm]	[mm]	[mm]	[mm]	[kg]	[kN/mm]	[kN/mm]
6'802	(678 / 678)	10.6	0.4	6'519	(650 / 650)	26.5	0.4	6'046	(603 / 1'080)	53.0	0.3	400	600	73	53	53.8	1'563.0	4.08
6'759	(674 / 674)	13.0	0.6	6'412	(639 / 639)	32.5	0.4	5'832	(581 / 1'080)	65.0	0.3	400	600	89	65	65.0	1'274.5	3.32
6'691	(669 / 669)	15.4	0.7	6'281	(628 / 628)	38.5	0.6	5'597	(560 / 1'080)	77.0	0.4	400	600	105	77	76.3	1'075.8	2.81
5'752	(665 / 665)	17.8	1.3	5'342	(618 / 618)	44.5	1.1	4'658	(540 / 1'080)	89.0	0.8	400	600	121	89	87.5	930.8	2.43
5'036	(661 / 661)	20.2	1.8	4'626	(607 / 607)	50.5	1.6	3'942	(540 / 1'080)	101.0	1.3	400	600	137	101	98.7	820.2	2.14
7'694	(767 / 767)	10.6	0.6	7'410	(738 / 738)	26.5	0.4	6'938	(691 / 1'215)	53.0	0.3	450	600	73	53	60.6	1'975.8	4.58
7'651	(762 / 762)	13.0	0.7	7'303	(728 / 728)	32.5	0.6	6'724	(670 / 1'215)	65.0	0.4	450	600	89	65	73.3	1'611.0	3.74
7'608	(758 / 758)	15.4	0.8	7'196	(717 / 717)	38.5	0.7	6'510	(649 / 1'215)	77.0	0.4	450	600	105	77	85.9	1'360.0	3.16
7'565	(754 / 754)	17.8	1.0	7'089	(706 / 706)	44.5	0.8	6'296	(627 / 1'215)	89.0	0.6	450	600	121	89	98.5	1'176.6	2.73
6'913	(750 / 750)	20.2	1.4	6'416	(696 / 696)	50.5	1.1	5'589	(608 / 1'215)	101.0	0.8	450	600	137	101	111.2	1'036.8	2.41
6'144	(745 / 745)	22.6	1.8	5'647	(685 / 685)	56.5	1.7	4'819	(608 / 1'215)	113.0	1.3	450	600	153	113	123.8	926.7	2.15
8'586	(855 / 855)	10.6	0.6	8'302	(827 / 827)	26.5	0.4	7'829	(780 / 1'350)	53.0	0.3	500	600	73	53	67.4	2'417.8	5.09
8'543	(851 / 851)	13.0	0.7	8'195	(817 / 817)	32.5	0.6	7'615	(759 / 1'350)	65.0	0.4	500	600	89	65	81.5	1'971.5	4.15
8'500	(847 / 847)	15.4	0.8	8'088	(806 / 806)	38.5	0.7	7'401	(738 / 1'350)	77.0	0.6	500	600	105	77	95.5	1'664.2	3.51
8'457	(843 / 843)	17.8	1.0	7'981	(795 / 795)	44.5	0.8	7'187	(716 / 1'350)	89.0	0.7	500	600	121	89	109.6	1'439.8	3.03
8'414	(838 / 838)	20.2	1.3	7'874	(785 / 785)	50.5	1.0	6'973	(695 / 1'350)	101.0	0.7	500	600	137	101	123.6	1'268.8	2.67
8'127	(834 / 834)	22.6	1.4	7'540	(774 / 774)	56.5	1.3	6'562	(675 / 1'350)	113.0	1.0	500	600	153	113	137.7	1'134.0	2.39
7'309	(830 / 830)	25.0	1.8	6'722	(763 / 763)	62.5	1.7	5'744	(675 / 1'350)	125.0	1.4	500	600	169	125	151.7	1'025.2	2.16
9'668	(1'027 / 1'027)	13.8	0.7	9'322	(991 / 991)	34.5	0.6	8'745	(929 / 1'620)	69.0	0.3	600	600	94	69	102.6	1'639.9	4.70
9'614	(1'022 / 1'022)	17.0	0.8	9'188	(976 / 976)	42.5	0.7	8'477	(901 / 1'620)	85.0	0.4	600	600	115	85	124.2	1'331.2	3.81
9'561	(1'016 / 1'016)	20.2	1.0	9'054	(962 / 962)	50.5	0.8	8'210	(873 / 1'620)	101.0	0.4	600	600	136	101	145.8	1'120.4	3.21
9'452	(1'010 / 1'010)	23.4	1.1	8'869	(948 / 948)	58.5	1.0	7'896	(844 / 1'620)	117.0	0.6	600	600	157	117	167.4	967.1	2.77
8'268	(1'005 / 1'005)	26.6	1.8	7'685	(934 / 934)	66.5	1.6	6'712	(816 / 1'620)	133.0	1.3	600	600	178	133	189.0	850.8	2.44
7'339	(999 / 999)	29.8	2.5	6'755	(920 / 920)	74.5	2.3	5'783	(810 / 1'620)	149.0	1.8	600	600	199	149	210.6	759.4	2.17
11'301	(1'201 / 1'201)	13.8	0.7	10'896	(1'158 / 1'158)	34.5	0.6	10'222	(1'086 / 1'890)	69.0	0.4	600	700	94	69	119.9	2'170.5	5.48
11'238	(1'194 / 1'194)	17.0	0.8	10'740	(1'141 / 1'141)	42.5	0.7	9'909	(1'053 / 1'890)	85.0	0.4	600	700	115	85	145.1	1'761.9	4.45
11'176	(1'188 / 1'188)	20.2	1.0	10'583	(1'125 / 1'125)	50.5	0.8	9'596	(1'020 / 1'890)	101.0	0.6	600	700	136	101	170.3	1'482.8	3.74
11'113	(1'181 / 1'181)	23.4	1.1	10'427	(1'108 / 1'108)	58.5	1.0	9'284	(987 / 1'890)	117.0	0.7	600	700	157	117	195.5	1'280.0	3.23
10'418	(1'174 / 1'174)	26.6	1.6	9'683	(1'091 / 1'091)	66.5	1.4	8'457	(953 / 1'890)	133.0	1.0	600	700	178	133	220.8	1'126.0	2.84
9'246	(1'168 / 1'168)	29.8	2.1	8'511	(1'075 / 1'075)	74.5	2.0	7'286	(945 / 1'890)	149.0	1.6	600	700	199	149	246.0	1'005.1	2.54
13'255	(1'408 / 1'408)	13.8	0.7	12'851	(1'365 / 1'365)	34.5	0.6	12'176	(1'294 / 2'205)	69.0	0.4	700	700	94	69	140.0	2'890.7	6.39
13'193	(1'402 / 1'402)	17.0	0.8	12'694	(1'349 / 1'349)	42.5	0.7	11'864	(1'261 / 2'205)	85.0	0.6	700	700	115	85	169.5	2'346.6	5.19
13'130	(1'395 / 1'395)	20.2	1.0	12'538	(1'332 / 1'332)	50.5	0.8	11'551	(1'227 / 2'205)	101.0	0.7	700	700	136	101	198.9	1'974.9	4.37
13'068	(1'389 / 1'389)	23.4	1.3	12'382	(1'316 / 1'316)	58.5	1.1	11'238	(1'194 / 2'205)	117.0	0.8	700	700	157	117	228.4	1'704.8	3.77
13'005	(1'382 / 1'382)	26.6	1.4	12'225	(1'299 / 1'299)	66.5	1.3	10'926	(1'161 / 2'205)	133.0	1.0	700	700	178	133	257.8	1'499.7	3.32
12'943	(1'375 / 1'375)	29.8	1.6	12'069	(1'282 / 1'282)	74.5	1.4	10'613	(1'128 / 2'205)	149.0	1.0	700	700	199	149	287.3	1'338.7	2.96
12'407	(1'369 / 1'369)	33.0	1.8	11'475	(1'266 / 1'266)	82.5	1.7	9'922	(1'103 / 2'205)	165.0	1.3	700	700	220	165	316.7	1'208.9	2.67
15'171	(1'612 / 1'612)	13.8	0.7	14'708	(1'563 / 1'563)	34.5	0.6	13'936	(1'481 / 2'520)	69.0	0.4	700	800	94	69	160.1	3'663.1	7.30
15'099	(1'604 / 1'604)	17.0	0.8	14'529	(1'544 / 1'544)	42.5	0.7	13'578	(1'443 / 2'520)	85.0	0.6	700	800	115	85	193.8	2'973.6	5.93
15'028	(1'597 / 1'597)	20.2	1.0	14'350	(1'525 / 1'525)	50.5	0.8	13'220	(1'405 / 2'520)	101.0	0.7	700	800	136	101	227.5	2'502.5	4.99
14'956	(1'589 / 1'589)	23.4	1.1	14'171	(1'506 / 1'506)	58.5	1.0	12'862	(1'367 / 2'520)	117.0	0.8	700	800	157	117	261.2	2'160.3	4.31
14'885	(1'581 / 1'581)	26.6	1.4	13'992	(1'487 / 1'487)	66.5	1.1	12'504	(1'329 / 2'520)	133.0	1.0	700	800	178	133	294.9	1'900.4	3.79
14'813	(1'574 / 1'574)	29.8	1.6	13'813	(1'468 / 1'468)	74.5	1.4	12'147	(1'291 / 2'520)	149.0	1.1	700	800	199	149	328.6	1'696.3	3.38
14'741	(1'566 / 1'566)	33.0	1.7	13'634	(1'449 / 1'449)	82.5	1.6	11'789	(1'260 / 2'520)	165.0	1.1	700	800	220	165	362.3	1'531.8	3.05
13'869	(1'842 / 1'842)	17.0	1.0	13'413	(1'781 / 1'781)	42.5	0.8	12'652	(1'680 / 2'880)	85.0	0.7	800	800	110	85	197.0	2'666.7	6.78
13'797	(1'832 / 1'832)	21.0	1.3	13'234	(1'758 / 1'758)	52.5	1.1	12'294	(1'633 / 2'880)	105.0	1.0	800	800	135	105	239.0	2'158.7	5.49
13'726	(1'823 / 1'823)	25.0	1.6	13'055	(1'734 / 1'734)	62.5	1.4	11'936	(1'585 / 2'880)	125.0	1.1	800	800	160	125	280.9	1'813.3	4.61
13'654	(1'813 / 1'813)	29.0	1.8	12'876	(1'710 / 1'710)	72.5	1.7	11'578	(1'538 / 2'880)	145.0	1.3	800	800	185	145	322.9	1'563.2	3.97
13'583	(1'804 / 1'804)	33.0	2.1	12'697	(1'686 / 1'686)	82.5	1.8	11'220	(1'490 / 2'880)	165.0	1.6	800	800	210	165	364.9	1'373.7	3.49
13'511	(1'794 / 1'794)	37.0	2.4	12'518	(1'663 / 1'663)	92.5	2.1	10'862	(1'443 / 2'880)	185.0	1.7	800	800	235	185	406.9	1'225.2	3.11
13'440	(1'785 / 1'785)	41.0	2.7	12'339	(1'639 / 1'639)	102.5	2.4	10'505	(1'440 / 2'880)	205.0	2.0	800	800	260	205	448.9	1'105.7	2.81
17'636	(2'342 / 2'342)	17.0	0.8	17'122	(2'274 / 2'274)	42.5	0.8	16'265	(2'160 / 3'645)	85.0	0.7	900	900	110	85	249.6	4'092.2	8.58
17'555	(2'331 / 2'331)	21.0	1.1	16'920	(2'247 / 2'247)	52.5	1.0	15'862	(2'107 / 3'645)	105.0	0.8	900	900	135	105	302.8	3'312.7	6.94
17'475	(2'321 / 2'321)	25.0	1.4	16'719	(2'220 / 2'220)	62.5	1.3	15'459	(2'053 / 3'645)	125.0	1.0	900	900	160	125	356.0	2'782.7	5.83
17'394	(2'310 / 2'310)	29.0	1.7	16'517	(2'193 / 2'193)	72.5	1.4	15'056	(1'999 / 3'645)	145.0	1.3	900	900	185	145	409.2	2'398.9	5.03
17'313	(2'299 / 2'299)	33.0	1.8	16'316	(2'167 / 2'167)	82.5	1.7	14'653	(1'946 / 3'645)	165.0	1.4	900	900	210	165	462.4	2'108.1	4.42
17'233	(2'288 / 2'288)	37.0	2.1	16'114	(2'140 / 2'140)	92.5	2.0	14'250	(1'892 / 3'645)	185.0	1.6	900	900	235	185	515.6	1'880.2	3.94
17'152	(2'278 / 2'278)	41.0	2.3	15'912	(2'113 / 2'113)	102.5	2.1	13'847	(1'839 / 3'645)	205.0	1.8	900	900	260	205	568.8	1'696.8	3.56
17'071	(2'267 / 2'267)	45.0	2.5	15'711	(2'086 / 2'086)	112.5	2.4	13'443	(1'823 / 3'645)	225.0	2.0	900	900	285	225	622.0	1'545.9	3.24

APPENDIX C

PRELIMINARY ANALYSIS/ DESIGN OF A TYPICAL PLANE FRAME

C1. Layout

Dimensions are in metres. Forces are in kN and kNm. Stresses are in MPa.

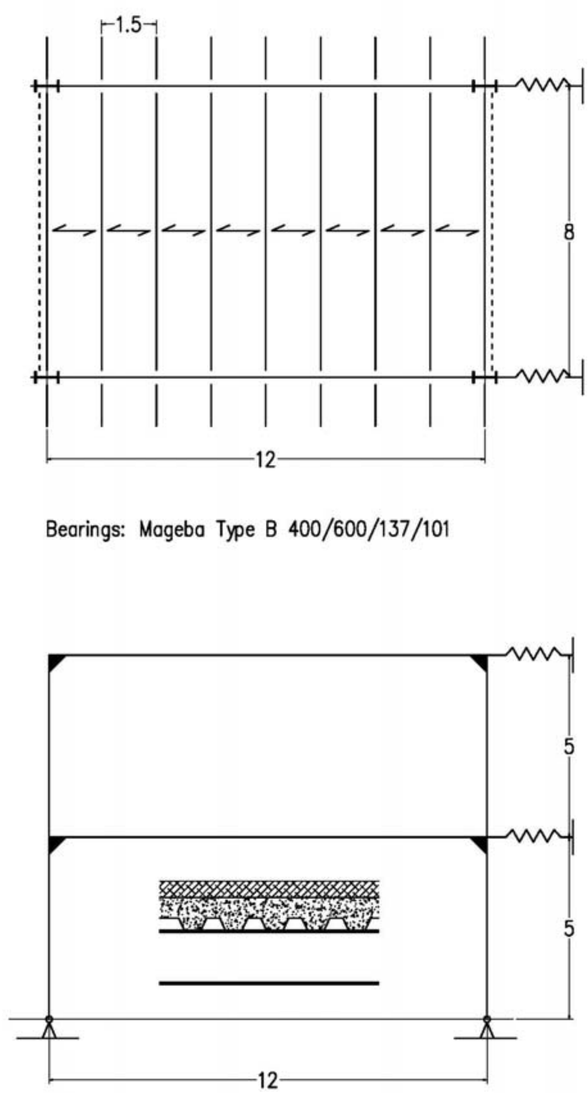


Fig. C1. Plan and Elevation of The Frame

C2. Criteria

Gravity Loads

100mm architectural finish	2.00 kPa
200mm concrete on trapezoidal deck	5.00 kPa
Floor joists @ 1500 mm centres	0.30 kPa
Self weight of frame (assumed)	0.70 kPa
Total dead load (DL)	8.00 kPa
Live Loads (LL)	5.00 kPa (reducible)
The live load is reduced since space is considered a retail space.	
For frames, reduced live load	2.50 kPa

Lateral Loads (E)

Wind load (W) is ignored. Seismic Load (E) is considered.

Seismic Zone	1
Effective Ground Acceleration Coefficient, Ao	0.40
Local Site Class	Z3 (assumed conservatively)
Site periods Ta, Tb	0.15s, 0.60s
Importance (Public Use)	1.40
Structural Behavior Factor, R	8.00
Live Load Reduction Factor, n	0.30

Note: Strength level Seismic Force, E, is calculated using the local seismic code ABYBHY'07, but load combinations are taken from ASCE 7-2010 Allowable Stress Group.

Load Combinations

ASCE7-2010 ASD, with no allowable stress increase permitted:
Gravity, LC1: 1.00 DL + 1.00 LL
Lateral, LC2 : 1.00 DL + 0.75 (0.7E) + 0.75 LL

Materials

European wide flanged columns HD, HEB profiles and HEB beams, Fy=275Mpa

C3. Analysis and Design For Gravity Loading LC1

Method:

Assume points of inflection at 0.15L from columns , apply unit distributed loads and then scale results to actual uniformly distributed load value.

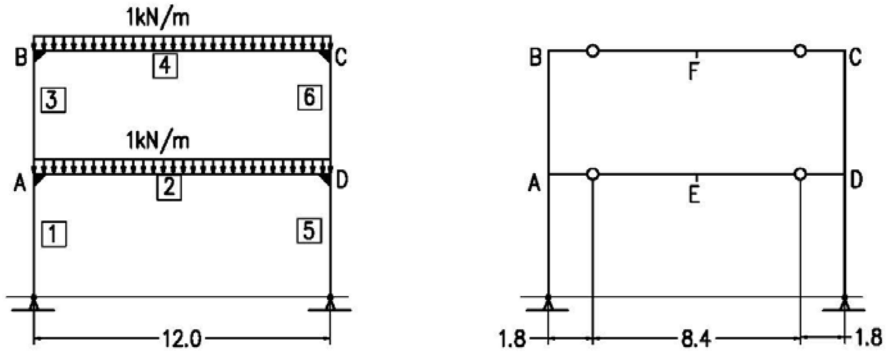


Fig. C2: Analysis Model For Gravity Loading

Moments

$M_E = M_F = 1 \times (0.70L)^2/8 = +0.06125\ L2\ (\sim qL^2/16)$
$= +8.82$ kNm

$M_{A2} = M_{B2} = (1 \times 0.70L)/2 \times 0.15L + (1 \times (0.15L)^2)/2$
$= 0.06375\ L^2\ (\sim qL^2/16)$
$= -9.18$ kNm

Girder moments M_{A2} and M_{B2} can be distributed to columns above and below girder in proportion to their stiffnesses I/L. At joint A, the ratio of bottom to top column moments is 0.75:1, since the lower column is hinged at the base.

$M_{A1} = -9.18 \times 0.75/1.75 = -3.93$kNm
$M_{A3} = -9.18 \times 1/1.75 = -5.25$ kNm

Reactions

$Rx = -M_{A1}/h = +3.93/5 = +0.786$kN
$Ry = 2\ (1 \times 12)/2 = +12$ kN

Analysis For Unit Load: 1kN/m **Analysis For LC1 : (8+2.5)8=84 kN/m**

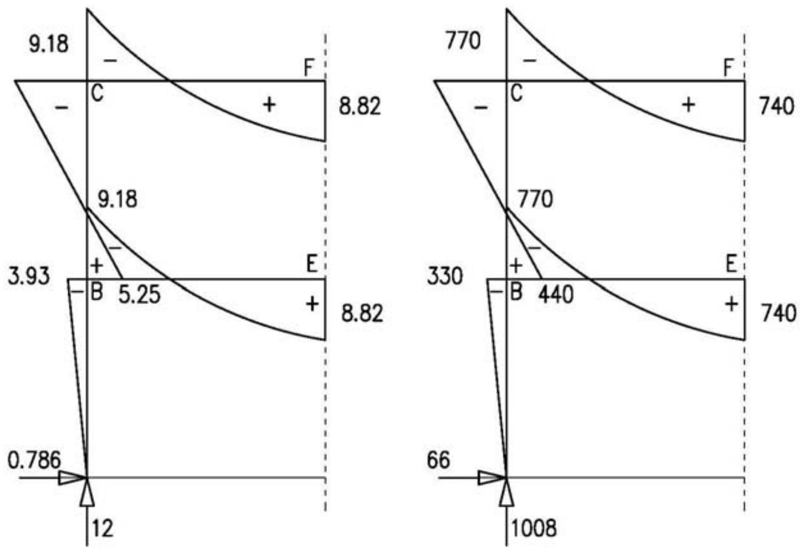


Fig. C3: Results from Analysis For Gravity Loading.

Design Loads (Required Strengths) Pr/Mr

For Columns	1008/330 or 504/770
For Girders	0/770

Design Method:

For girders, assume laterally fully braced and limit total load deflection to L/300 = 0.04m

For columns, assume the same w.r.t LTB and use method of initial sizing proposed by Gerschwindner [18]. Gerschwindner approximates the combined effects of axial compressive force N and bending moment M by an equivalent axial force Peff, and then designs the column as if it is a pure compression member.

Keep column stronger than girder for seismic considerations in final design.

a)Girder

Sx,req'd = 770 000/275 = 2800cm3

Try HEB400: Ix = 57680 cm4 ;Sx =2880cm3

Δmax ~ 5/384 x 740 x 12^4/2/57680 – 770 x 12^2/8/2/57680
= 0.076 m
> 0.04m

I, req'd = 0.076/0.04 x 57680
= 109592 cm4

Select HEB 550: Ix =136 700cm4; Sx = 4970cm3

b)Columns

Try HD400 x 382:

A	=487 cm2
Ix	= 141 300cm4 ~ Ix_{girder}
Sx	= 6794 cm3 >Sx_{girder}
Zx	= 7965 cm3
rx	= 17cm

		ry	=10.5 cm	
m	= 8/9 . A/Zx		= 8/9 x 100 x 487/7965	= 5.43/m
Peff	= N + m . Mx			

Bottom Column:	Peff	= 1008 + 543 x 330	=2800 kN
Top Column	Peff	=504 + 5.43 x 770	=4685 kN

Use the same section for both columns, Peff = 4685kN governs design.

G_A = 2 x 141300/5/[136700/12]=4.96~5.00

G_B = 5/2 =2.50

Kx = 1.80

Ky =1.00 (assumed braced)

Assuming bearings have no bracing effect on in-plane stability of the frame,

λ	= KL/r	
	= max(1.8 x 500/17, 500/10.5)	=53
λc	$=\lambda/\pi\sqrt{(Fy/E)}$	
	$=53/\pi\sqrt{(275/200000)}$	=0.63 <1.50
Fcr	$=0.658^{(\lambda c)^{\wedge}2}$. Fy	
	= $0.658^{\ 0.63^{\wedge}2}$ x 275	=233MPa
Pa	= Ag.Fcr/Ω	
	=48700 x 233/1.67/1000	=6795 kN
	>Peff	

<u>Select HD400 x 382</u>

C4. Analysis and Verification for Load LC2

Elastomers

Elastomers have been selected such that the maximum permissible drift to ASCE 7-2010 for seismic use group III (assumed appropriate for a building in Turkish Zone 1) can be achieved at the top story.

Δmax, story	= 0.01 x 5	=0.05m	
Total Δmax	= 2 x 0.01	=0.10m	= displacement at top story

Select Mageba Type B: 400/600/137/101:

v_{xyd}	= 101mm
K_{xy}	=2140 kN/m
F	= 214kN (restoring force)

Seismic Forces

Wk	= (8.00 +0.30 x 2.50) x 8 x 12 x 2	=1680kN
T	~0.085 $H_N^{0.75}$ (highly ductile or special moment frames)	
	= 0.085 x 5 $^{0.75}$	= 0.48sec
S(T)	= 2.50 since Ta<T<Tb	
E	= Ao. I. S(T)/R x Wk	
	=0.4 x 1.4 x 2.5/8 x 1680	=295 kN

Story Forces and Shears

For simplicity linear distribution of forces is assumed. Ratio of top to bottom forces is 2:1.

Ftop	= 295 x 2/3	=195kN = Vtop		
Fbot	= 295-195	= 100kN;	Vbot	=295kN

Distribution of Floor Shears Between Frames and Bearings.

Stiffness of a frame alone can be determined using the following equation taken from The Seismic Design Handbook (Naeim, 1989):

$$\Delta \quad = \sum V.\ h^2/12E\ (1/(\sum Kg)i + 1/\ (\sum Kc)i\ )$$

Where $\sum V$ is the total story shear, h story height, $(\sum Kg)i$ the total story girder stiffness $(\sum Kc)i$ the total column stiffness for the ith story. The stiffnesses are obtained by adding up I/L terms considering an assemblage for a story. Then the story stiffness is

$$\sum V/\Delta \quad = 12E/h^2\ /\ [(1/(\sum Kg)i + 1/\ (\sum Kc)i\ )]$$

This equation takes into account contribution of flexural stiffnesses of both columns and girders in resisting floor drift.

Alternatively, Muto's method can be used to determine the lateral stiffness. Muto's method is covered in details in many textbooks on structural design of framed buildings. However, at preliminary stage it may be sufficient to treat beams as rigid and assume story stiffness to be a contribution of columns alone. This on its own is definitely not a conservative assumption, but it can be argued that the whole preliminary design is already conservative in its selection of sections.

$$Kstory \quad = Kcolumns + Kspring \quad =\sum 12EI/h^3 + Kspring$$

Distribution of story shears between columns and bearings is dictated by the drift pattern, on which reasonable assumption must be made. If assumed linear, then the distribution of shears between bearings at each story by themselves will be linear compatible with the deformations.

At the top story,

Ktop = 2 x 12 x 2 x 141300/5^3 + 2140
= 54260 + 2140 = 56400kN/m
Ftop, bearing = 2140/56400 x 195 = 7.50 kN
Fbot., bearing = 0.50 x Ftop, bearing = 3.75kN

It is observed that bearing stiffness is only 3.8% of floor stiffness and resulting forces are negligibly small. For this particular frame, the whole seismic load will be resisted by the frame on its own.

Analysis

Method: The Portal Method

Muto's tables of inflection points, which are a part of Muto's method described earlier, may be used for better approximation. However, for the purpose of preliminary design, midpoints of both girders and columns can, for convenience, be taken as inflection points. Unit loads are applied at the top story and half as much at the lower story, then results from analysis with this loading is scaled by a factor 195.

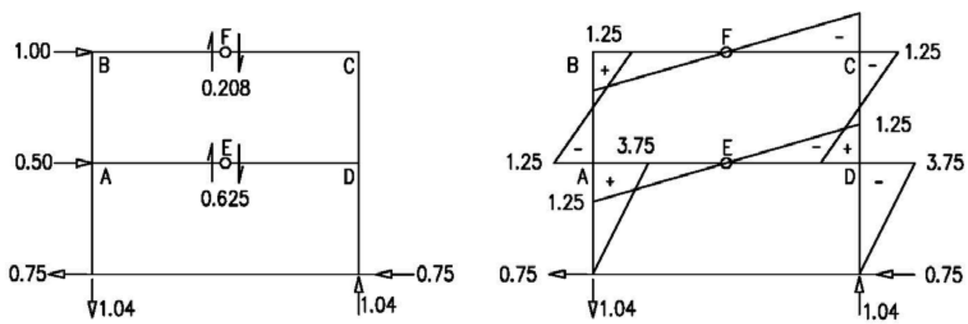


Fig. C4: Analysis for Unit Load at Top Story and Linear Distribution of Lateral Forces

DL (64 kN/m) **0.75 LL (15kN/m)** **0.525E (102kN at top,51 at bot. story)**

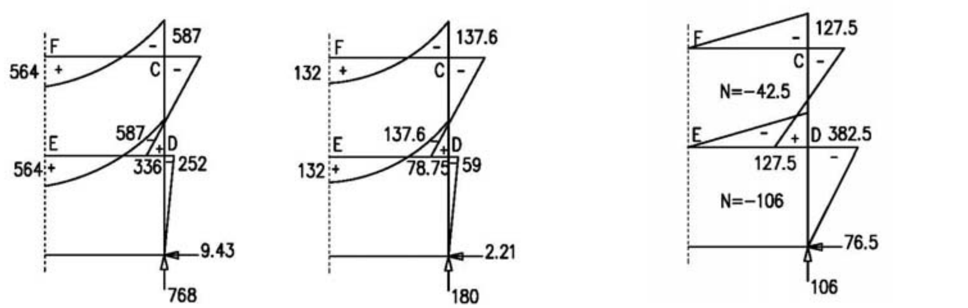


Fig. C5: Results for Individual Loads

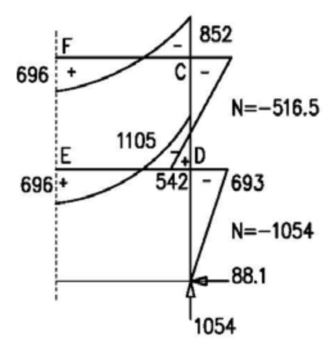


Fig. C6: Results for Load LC2

Summary of Results from Analysis

Table C1: Summary of Results from Analysis

Element		**Forces (LC2)**	**Forces (LC1)**
Top Girder	M(kNm)	852	770
Bot. Girder	M	1105	770
Top Column	M	852	770
	N(kN)	516.5	504
Bot. Column	M	693	330
	N	1054	1008

Verification of sections for LC2

Top Girder Mrx/Mcx = 852/(4970 x 0.275/1.67) = 1.04 Acceptable

Bot. Girder Mrx/Mcx =1105/(4970 x 0.275/1.67)= 1.375 Fails

In final design a haunched girder can be used. However for now assume HEB650 for bottom girder ;Sx = 6480cm3

Mrx/Mcx =1105/(6480 x 0.275/1.67)= 1.035 Acceptable.

Top Column	Peff	= 516.5 + 5.43 x 852	=5142.76 kN	< Pa = 6795kN
Bot. Column	Peff	=1054 + 5.43 x 693	= 4817kN	< Pa =6795 kN

Conclusion

Use HEB550 for top girder, HEB650 for bottom girder and HD 400 x 382 columns.

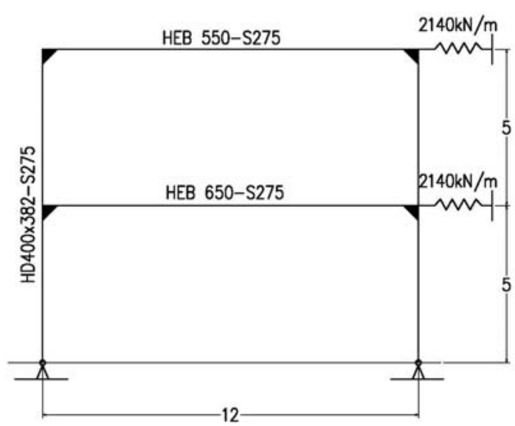


Fig. C7: Selected Sections after Preliminary Design.