TEACHER LEARNING IN AND FROM PRACTICE: THE CASE OF A SECONDARY MATHEMATICS TEACHER

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
THE MIDDLE EAST TECHNICAL UNIVERSITY

BY

E. GÜL ÇELEBI İLHAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

IN

SECONDARY SCIENCE AND MATHEMATICS EDUCATION

JUNE 2013

Approval of the thesis:

TEACHER LEARNING IN AND FROM PRACTICE: THE CASE OF A SECONDARY MATHEMATICS TEACHER

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ABSTRACT

TEACHER LEARNING IN AND FROM PRACTICE: THE CASE OF A SECONDARY MATHEMATICS TEACHER

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June 2013,194 pages

The purpose of this research was to explore teacher learning in and from practice while inquiring their own classroom instruction/practice. Teacher professional development or learning opportunities have been receiving criticisms regarding their limitations in responding teacher's needs all around the world. Besides, with the rise of discursive research in mathematics education what teachers say and how they say it within classrooms is believed to play an important role on students' mathematical learning. The investigation was held with a high school mathematics teacher who was conducting an inquiry of her practice and focused on her learning via her discourse on and within discursive practices of her mathematics classes. Data collection utilized videotaped classroom observations and teacher interviews and teacher reflection documents. Analysis of data comprised of various methodologies consisted of narrative analysis, a combination of Systemic Functional Linguistics and Commognition theories and Michel Foucault's *toolbox* which provided theoretical tools of discourse analysis to gain new ways to look the subject under investigation. Results provided a better understanding of teacher learning as a social practice. Three themes emerged as teacher's accounts of her own practice including: discursive practices, knowing in practice and inquiry of own practice. Furthermore, all three of teacher's mathematical, social and pedagogical discourses has been found to evolve and differentiate during teacher's inquiry process. This study indicated that such a design of teacher's inquiry of her own practice can be beneficial for supporting and generating teacher learning at both in pre-service and in-service levels.

Keywords: Teacher learning, teacher discourse, systemic functional linguistics, discourse analysis, Michel Foucault's *toolbox*

ÖZ

UYGULAMADA VE UYGULAMANIN İÇİNDEN ÖĞRETMEN ÖĞRENMESİ: BİR ORTAÖĞRETİM MATEMATİK ÖĞRETMENİNİN DURUMU

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Haziran 2013,194 sayfa

Bu çalışmanın amacı, öğretmenin kendi sınıf içi uygulamalarını araştırırken/sorgularken uygulamada ve uygulamanın içinden öğrenmesini anlamaktır. Öğretmen mesleki gelişim ve öğrenme olanakları, dünyanın birçok yerinde öğretmenlerin ihtiyaçlarına cevap veremediği yönünde eleştiriler almaktadır. Bunun yanı sıra, matematik eğitiminde söylemsel araştırmaların artması ile öğretmenlerin sınıf içinde ne söylediği ve nasıl söylediğinin, öğrencilerin matematik öğrenmeleri üzerinde önemli bir rol oynadığına inanılmaktadır. Araştırma, kendi uygulamalarını/pratiğini araştıran/sorgulayan bir lise matematik öğretmeniyle gerçekleştirilmiş ve öğretmenin matematik sınıflarındaki söylemsel pratikler içerisinde ve bunlarla ile ilgili söylemi üzerinden öğrenmesine odaklanılmıştır. Veriler, öğretmen görüşmeleri, kamera kaydıyla yapılan sınıf içi gözlemler ve öğretmen yansıtma formları ile toplanmıştır. Veriler, hikâyeleme/öyküleme analizi, sistemik fonksiyonel dilbilim ve *commognition* teorilerinin karışımı ve araştırılmakta olan özneye, yeni bakış açıları edinilmesini sağlayan Michel Foucault'un *alet kutusu* kullanılarak analiz edilmiştir. Sonuçlar, öğretmen öğrenmesinin sosyal pratik olarak daha iyi anlaşılmasını sağlamıştır. Öğretmenin kendi pratiğini anlamlandırması ile ilgili 3 tema ortaya çıkmıştır: söylemsel pratik, pratikten/uygulamada öğrenme ve pratiği sorgulama/araştırma. Ayrıca öğretmenin kendi pratiğini araştırıp/sorguladığı bu süreçte öğretmenin matematiksel, sosyal ve pedagojik söylemlerinin üçü de evrilmiş ve farklılaşmıştır. Bu çalışma göstermektedir ki, öğretmenin kendi pratiğini araştırdığı/sorguladığı böyle bir model, hem hizmet öncesi hem de hizmet içi öğretmen öğrenmesini desteklemede ve üretmede faydalı olabilmektedir.

Anahtar kelimeler: Öğretmen öğrenmesi, öğretmen söylemi, sistemik fonksiyonel dilbilgisi, söylem analizi, Michel Foucault'un alet kutusu

To my babyboy Deniz Aren

ACKNOWLEDGEMENTS

I wish to thank everyone whose belief in me and support has made me to achieve this accomplishment. I would like to thank to my major professor and advisor Dr. A. Kürşat Erbaş who have provided constant constructive comments, suggestions and criticism for this doctoral research work. I also wish to thank Dr. Jeppe Skott who have encouraged me, provided valuable comments and helped me to set out a clear direction for my research during my stay as a guest researcher in Denmark .I would like to express appreciation to my dissertation committee members Dr. Ömer Faruk Özdemir and Dr. Mine Işıksal for their encouragement and feedbacks during this process.

I also wish to express my gratitude to my mother and father who have provided worthless support with their love and belief in me. Finally I would like to specially thank to my husband Murat, for being always by my side throughout this bumpy journey of my life with his loving guidance and support and to my dearest, son Deniz Aren who has recently born and been my most important motivation and moral support to get through this work.

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ABBREVIATIONS

AAMT	Australian Association of /mathematics Teachers
DA	Discourse Analysis
ICME	International Congress on Mathematical Education
MEB	Milli Eğitim Bakanlığı
NCATE	The National Council for Accreditation of Teacher Education
NCTM	National Council of Teachers of Mathematics
NSDC	National Staff Development Council
PD	Professional development
PISA	Programme for International Student Assessment
ÖSYS	Öğrenci Seçme ve Yerleştirme Sınavı
SFL	Systemic Functional Linguistics
TDA	Training and Development Agency for Schools
TUIK	Türkiye İstatistik Kurumu

CHAPTER 1

INTRODUCTION

Within the area of professional development of teachers and teacher learning, numerous research focusing on teachers, their actions and also research with teachers respectively have been carried since 1950's (Clarke & Erickson, 2004). Among these, the traditional efforts of teacher learning taking a cognitive stance to learning, offers a training model which moves from a theoretical level towards a practical level (Kelly, 2006). These kinds of professional development experiences are argued to have a limited impact on practitioners (Little, 1993). In contrast, the practice based professional development approaches engage teachers with the opportunity to participate in the production of knowing in and from their practice. These approaches are tied to the social cultural theory, assuming learning is tightly associated with the social practices in fact, it occurs within these practices. This type of professional development is also reported to be more effective because they originate from teachers' needs and difficulties coming from everyday practice and as teachers having the chance of implementing their own working plans, which have an immediate effect on their practice (Guskey, 2002).

Teacher learning is found to be a complex phenomenon including teachers' beliefs knowledge and practices, interactions between these with student's beliefs and practices and researchers (Adler, Ball, Krainer, Lin, & Novotna, 2005). That's why; studies of teacher's learning should be based on practice and position teachers as active participants in this learning process in and from practice (Matos, Powell, Stajn, Ejersbø, & Hovermill, 2009). Moreover, due to recent research about teacher learning there is a growing allowance for social dimension of teacher learning (Borko, Knuth, Peressini, Romagnano, & Willis, 2004; Jaworski, 2001; Lerman, 2001; Llinares & Krainer, 2006). Teaching and learning, should be redefined in social terms as various forms of interactions among content, teacher and students where different forms of practices of teachers and students as participants are shared and negotiated (National Research Council, 2001).

Teaching mathematics is also considered as a life-long learning process which roots back to the teacher's own learning experiences at school. Yet, without a systematic inquiry of one's own practice, teaching cannot go beyond a technical/theoretical routine that was learned before entering the profession (Jaworski, 2009). Participation to the inquiry of own practice leads teachers to own knowledge/meanings produced in and from practice and to develop an inquiry stance towards teaching which is closely related with non-traditional approaches to professional learning (Dana, 2002). Furthermore, in recent mathematics education research, there is an increased attention towards language and discourse issues pointing out the need for a discursive approach to analyze and understand learning processes (Lerman, 2009; Morgan, 2006; Sfard, 2008). As being one of the influential approaches to discourse, functionalism defines discourse as *language in use*. For instance, Lemke's (1995) definition of discourse as “a social activity of making meanings with language and other symbolic systems in some particular situation or setting” (p.15) corresponds to that approach. Due to this conceptualization of discourse, a need for addressing teacher learning/meaning-making in and from practice in terms of teachers’ use of language/discourse arises. This entails a discursive and inherently a social/participatory perspective for studying teacher learning and professional development also associating with the developing area of discursive research in mathematics education.

1.1 Problem Statement

According to the latest statistics, there are over 750 000 teachers working in primary and secondary schools in Turkey (Ministry of National Education & Turkish Statistical Institute, 2012). Needs and opportunities for professional development for in-service teachers are addressed mostly by the Department of In Service Training as a unit of National Ministry of Education and are planned and conducted as centralized activities. Yet, problems related with providing teachers with those professional development opportunities are known among the whole educational community of the country (Özoğlu, 2010). In service teachers who work in different social, cultural and physical contexts cannot meet with the PD opportunities in a way that can be addressed locally which are planned and organized due to their diversifying needs (Saban, 2000). Consequently most in service teachers can be said either to make individual efforts for their professional development or they don’t see any kind of professional development need at all. The situation for professional learning described above regarding Turkish (mathematics) teachers is also similar for most of their counterparts living in other parts of the world. In a status report on teacher development released by Darling-Hammond, Wei, Andree, Richardson and Orphanos (2009), it has been argued that although effective PD should be intensive, ongoing and connected to practice and teacher learning should be build in teacher’s work hours but most teachers in the United States do not have the opportunity to participate that kind of PD and feel that they have little influence on the decisions made about their own professional learning which is not the case for the high achieving nations on the international assessments. In the same report it has also been pointed out that there is also a significant variation in both support and the opportunity provided for professional learning between schools and districts. Moreover, in service education or professional development of practicing teachers has been reported to have many criticisms. For instance in Villages-Reimer’s (2009) report of teacher professional development based on a literature review, it has been argued that in Latin America and likewise in most parts of the world in-service teacher education programmes were not found to be responding teacher’ needs and `the educators in charge of these programmes are reported to be poorly prepared, courses are found to be theory-oriented and do not access to practical concerns and offered in difficult locations to reach particularly by those teachers who need the courses most (p.62).

On the other hand, along with ‘social turn’ in mathematics education (Lerman, 2000), perspectives viewing learners as participants within the social practices having agency in their learning; gained prominence contrasting the acquisitionist approaches that position learners as recipients of knowledge to be transferred by an expert. These new approaches to learning brought along a change in our understandings of teachers’ beliefs, knowledge and practices and in organization and conceptualization of the professional development activities for teachers as well. For instance, in their review of mathematics teachers' learning in and from practice Matos et. al. (2009) argued for a more practice based and a workplace related research is needed in contrast to time honored research and practice regarding training model of PD. Perrin-Glorian, Deblois and Robert (2008) in their review of research about professional growth of practicing teachers also put forth the need to study teaching in its context due to the changing paradigms as in Lerman (2000) above. Chapman and Da Ponte (2006) provided a substantive review of research reports in teachers' mathematics knowledge, teachers' knowledge of mathematics teaching, teachers' beliefs and conceptions and teachers' practices indicating the need for more empirical evidence regarding how teacher learning and change in practice takes place. Yet, there are few studies on how the professional development of teachers in their actual classroom settings and how associated change in their practice occur (Hoban, Butler & Lesslie, 2007; Leikin & Rota 2006). Also, in their review of the literature regarding mathematics teacher education and learning, Adler, Ball, Krainer, Lin, and Novotna (2005) identified large number of studies dealing with a particular teacher education programs' efficiency, reform issues and teacher studies in professional communities but: teachers learning outside of reform contexts and teachers learning from experience are among few researched topics. On the other hand, teachers are substantially situated in an isolated context at their schools; when they are making important instructional decisions and implementing these in their practices (Fiszer, 2003). In a study carried in Turkey, for example Dede (2006) examined mathematics teachers' interactions with each other and other teachers. He found that mathematics teachers interacted with each other and other teachers only once a month with regard to sharing their ideas about teaching, self-training, collaboration with their colleagues, and making communication with others. Other studies in Turkey also reflect teachers' views about the effects of working groups like committee of teachers' group on learning and teaching practices in schools shows that these groups are seen as superficial and impractical and has no effect on actual practice (Demirtaş, Üstüner, Özer & Cömert, 2008; Küçük, Ayvacı, & Altıntaş, 2004). Self study is proposed as an alternative perspective that supports the improvement in practice for research about practitioners who aim to learn about themselves at their workplace but it is not common in school teachers (Austin & Senese, 2004; Loughran, 2004). This approach is also argued as to remove the need for researcher intervention so the subjects could communicate with their own terms in the research process (Kieran, Forman, & Sfard, 2001). Moreover, as discursive research in the mathematics education has been increased dramatically for the last 20 years (Ryve, 2011) what teachers say and how they say it within classrooms is believed to play an important role on students' learning mathematics. Yet, relatively few studies focus on mathematics teacher's discourse together with key topics such as; mathematical classroom discourse, student discourse; the exchange of meanings between and comparison of teacher and student discourses, (e.g. Ferraira & Presmeg, 2004; Huang, Normandia, & Greer, 2005; Knott, Sriaman, & Jacob, 2008; Knuth & Peressini, 2001; Krussel, Edwards, & Springer, 2004; Springer & Dick, 2006); where almost hardly any research exists examining teacher learning with a discursive perspective (e.g. tracing teacher learning by focusing on teacher's discourse) or taking mathematics teacher discourse in its own right.

In this research, together with the issues mentioned in mathematics education practice and particularly in teacher education research above on teacher learning, I ground on these difficulties and the problems of current professional learning practices of the mathematics teachers to identify the problem situation. Addressing those issues is important for both research and practice of professional learning of mathematics teachers and teacher learning practices in general. Specifically in this research, in order to explore the professional learning process I focused on one high school mathematics teacher's learning in and from practice who was inquiring her own practice. In doing that this research addresses the following research questions:

1. How does the teacher give an account of the discursive practices of her mathematics classes while inquiring her own practice?
2. How does the teacher realize her ways of mathematizing within the discursive practice of her classes while inquiring her own practice?
3. How does teacher take part (participate) and position within the discursive practices of her mathematics classes while inquiring her own practice?
4. How does the teacher enable/organize the exchange of mathematical meanings within the discursive practices of her mathematics classes while inquiring her own practice?

In order to understand the dynamics of teacher learning in and from practice for a high school mathematics teacher inquiring own practice I followed an interpretive approach resulting in a case study by focusing on the teacher's discourse on and within discursive practices of the mathematics classes. By grounding on the combined theories approach of Tabach and Nachlieli (2011) I draw on the social semiotic perspective of systemic functional linguistics (SFL) theory (Halliday, 1978) and the theory of commogniton regarding human thinking and mathematizing (Sfard, 2008). Furthermore, since "it is in discourse where power and knowledge are joined together" (Foucault, 1990, p.110); I utilized Michel Foucault's Toolbox to discuss the teacher's perspective of the discursive practices within mathematics classroom.

1.2 Significance of the Study

The main concern of this study was to explore the professional learning of teachers who are inquiring their own classroom instruction/practice with a discursive approach to learning and communicational view of thinking. This study is significant for several reasons. Primarily, in this study discourse was used in both macro and micro meanings such as 'ways of using language' and 'how discourses create social positions and power' (Gee, 1999). Hence the methodology used and key findings of this study contributes to main topic areas of discourse research identified by Ryve (2011) By focusing on teacher talk in and about practice and grounding on the previous research on teacher learning it contributes to the area of "discourse as social interaction" and by employing Foucauldian and social semiotic perspectives it allowed for an analysis of the "production of as a social actors"(which are mainly teachers specific to this study) and of knowledge/power within the social practice(s) (Ryve, 2011, p.171). The latter perspective also enables focusing on the macro processes which have influence on teachers' and students' mathematical meaning making both inside and outside of the classroom for considering the broader cultural context of the study as well. Moreover, this study contributes to and fills a void in the literature of pre- and in-service mathematics teacher education with its aim to understand how teachers learn from their experience and about teachers' learning outside of reform contexts which were identified as under-researched areas by Adler et. al (2005). Secondly since self study was identified as rare among but recommended for teachers, with its focus on teacher learning while inquiring own practice this study attempts to fill another void in the teacher learning, development and self study literature addressed by Austin and Senese (2004) and Loughran (2004). Finally this study contributes to the discursive research in mathematics education, particularly about mathematics teacher discourse since there are few empirical research have been done about this topic as mentioned above. On the other hand, as written above professional development/learning opportunities and affordances provided to teachers at most parts of the world are limited in responding to their various needs and priorities or they are situated in an isolated context in their schools while taking important decisions and implementing them. By taking into these limitations of and criticisms that in-service professional development programmes both addressed in literature and policy documents by the practitioners the kind of professional learning of teachers who are inquiring their own practice in this study might be an alternative route for teachers, policy makers and teacher educators.

Finally, this study has several contributions to scholarly literature. First, by employing a narrative approach this study contributes to the literature regarding how teachers make sense of own practice which facilitates and extends teachers' understanding of their own practical theories as they reflect on, share and collaborate their thinking and actions as argued by Ponte (2001). Second, by building on the previous research on social semiotics and discourse research on mathematics education this study contributes to the literature about teacher learning by explaining that teachers generate meaning/knowing in and from practice while conducting an inquiry of own practice and self study as a methodology/route for teacher learning. Since literature reports that self-study is rare among classroom teachers in school settings this study contributes to the research about self-study as a professional learning experience. Third, in this study I have drawn on various disciplines/theoretical stances such as SFL and social semiotics and different traditions of discourse theory in design and data analyses as well as on mathematics education and teacher education literature. By building on the framework of Tabach and Nachlieli (2011) that was originally developed to analyze classroom discourse, I have employed above mentioned theories from various disciplines in order to explore and understand teacher learning in and from practice; utilizing rich theoretical tools and concepts available. The combination of multiple theories added richness to the results and interpretations and enabled them to be "persuasive" (Lincoln & Guba, 1985). In this context, teachers' professional learning points out a need for a shift particularly in mainstream in-service teacher education policies around the world since it can be carried on by teachers themselves. Particularly, adopting a socio-cultural perspective this study puts forth that inquiring own practice and exploring practice based opportunities for learning can lead to professional learning for teachers as knowing in practice which is collectively produced by and shared among all participants of practice (i.e. teachers, students and the physical and conceptual resources available) as argued in Kelly (2006). With its focus on teacher discourse, discursive nature of mathematics and discursive practices within mathematics classroom, this study sheds light on the need for preparation of teachers regarding discursive issues in teaching and learning mathematics at pre-service levels to be held by teacher education institutions. Finally, though acknowledging my role as a researcher and since this was not an intervention study, I strived to minimize my influence on teacher's inquiry of own practice with which I aimed easiness to replicate the inquiry process of this research for a group of teachers/researcher(s).

CHAPTER 2

REVIEW OF LITERATURE

2.1 Professional Development and Teacher Change

In the last decade, significant amount of research has been conducted on teacher professional development (PD), teacher learning and teacher change. The research literature contains different studies serving multiple purposes, varying in durations and methodologies, including intensive case studies with few teachers and large scale surveys about their professional development experiences, needs, expectations or their prior experiences of teaching and learning as students and student teachers; describing the successful practices of professional development activities (Ball, 1996; Garet, Porter, Desimone, Birman, & Yoon, 2001; Wei, Darling-Hammond, Andree, Richardson, & Orphanos, 2009).

Consequently all of these studies provide directions to and core features of designing effective professional development of teachers. Research reveals that kind of longer and focused professional development is more likely to change teachers' practices than traditional one-shot workshops that last within a few hours or days (Ball, 1996; Garet et al., 2001; Lee, 2005). By teachers' examining their own practice, reflecting in and on it, sharing their ideas, values about their practice with colleagues, the professional development activity may enhance the positive change for future practice (Suurtamn, Graves, & Vezina, 2004). In this view of professional development, teachers work collaboratively in their school setting, revise and extend their professional knowledge, skills and beliefs by being informed from their own practice.

Another element of effective professional development is the need for providing ongoing support for teachers to reach the PD objectives (Ball, 1996; Brown & Benken, 2009; Fiszer, 2003; Garet et al., 2001) Collegial support or working with an external specialist provides teachers opportunities of reflecting on their practices and learning, hence enhances the change in classroom practices and improves student learning (Guskey, 2002). In a large-scale study, Garet et al. (2001) analyzed the relationship between "best practices" of professional development activities that are reported in the PD research literature and teachers' self reported change in knowledge, skills and classroom teaching practices. They concluded that focusing on to design activities that are coherent, focusing content, promoting active learning and collective participation and emanate over longer durations tend to have more positive impact on teacher learning and change.

Aligned with these features Jin-Lee (2005) presents a professional development model that are based on teachers needs. The model was executed in a workshop form that lasted a whole year where participants were mathematics teachers who work at K-8 grade levels, attending full day meetings which lasts five days, four times a year where they have the opportunity to discuss, work collaboratively on problem solving and hands on activities, reflections and presentations by the participants. Furthermore, the evaluation of participants' assignments was done in order to address participants' needs and to prepare next workshops and by reflecting on practice of the participants individually and collaboratively to each other's videotaped lessons. These findings support Garet et.al's (2001) conclusion that participants should be considered as partners throughout the whole program as planners, implementers and evaluators of both their own professional growth and the PD program itself. In addition, the results of both above mentioned studies are coherent with NCTM's (1991) vision at professional standards for teaching mathematics, about teachers' role on their own professional development that they should systematically analyze their own teaching focusing on students' as learners of mathematics, tasks, discourse and assessment, discussing these issues related to teaching and learning with colleagues, reflecting on their teaching and learning individually and with their colleagues. This vision is in accordance with various national generic and mathematics teaching standards of various' competences related with professional development needs and responsibilities of teachers. Turkish secondary mathematics teaching competences also frames mathematics teachers' professional development in terms of self assessment, collaboration with teacher educators and other education stakeholders such as families, school administrators, colleagues etc. (MEB, 2011).

Nevertheless, though scholars seem to agree on the key components of the professional development issues, there are different approaches and models of PD in the literature. Early conceptualization of professional development was that PD is an individual activity of teachers which they were selecting one among different university courses or professional development activities of their school or government offered; which is what Little (1993) calls in a form of "training". The training models conceptualize learning as acquisition of knowledge as behaviorist theories and cognitive constructivism does; even they use different instructional techniques for an individual to be knowledgeable. Associated with change in both conceptions of knowledge and educational theories, isolated teacher in a training model leaves her/his role to a participant, designer, decision maker, and implementer of professional development programs in their natural context of teaching (Matos, Powell, Stajn, Ejersbø, & Hovermill, 2009). No matter through what perspective you look into the professional education of teachers, the common core criterion for effectiveness of the professional development activities is the quality of resulting change in teachers' professional knowledge, skills, and practices (Cohran-Smith, & Lytle, 1999; Garet et al., 2001; Wei et al., 2009). In this sense, linking teacher learning/growth with teachers' practice is essential.

As a result, the need for adapting professional development programs to the requirements of the changing educational systems and the new conceptualization of the learner, and the teacher as a learner led to a growing interest among worldwide scholars about educational practitioners who learn from in, for and from their practice (Jaworski, 2009; Matos, Powell, Stajn, Ejersbø, & Hovermill, 2009; Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007). Sfard (1998) explains that at heart of the theoretical underpinning of these emerging professional development activities, is bridging cognitive and socio cultural notions of "acquisition" and "participation" This conceptualization of learning and teacher learning is highly context dependent hence argues teachers to be active participants in designing, implementing, investigating and reflecting on their own professional developments (Matos, Powell, Stajn, Ejersbø, & Hovermill, 2009). For instance, based on a project (QUASAR) which is about middle school mathematics teachers' analyzing and reflecting on mathematical tasks in order to use in their instruction, Arbaugh and Brown (2005) conducted an initial professional development program with high school mathematics teachers to learn analyzing tasks according to the "levels of cognitive demand" criteria and to investigate the influence of their participation and reflections within the PD experience to their pedagogical content knowledge and their practice. Their work was a long term, around six months, content focused (mathematical task analysis due to the level of tasks, e.g. higher level tasks that demand procedures of connections to meaning and doing mathematics rather than rote memorization and without connections to meaning), classroom based to connect teachers' everyday work, provided appropriate professional support (researchers as facilitators of the group meetings in order to challenge teachers to analyze and reflect on their knowledge and practices) and produced positive development in their knowledge which in turn result as change in most of their practice. Another practice based professional development study was conducted by Silver and his colleagues (2007) arguing that the more teachers participate in the professional development process the more professional knowledge they would gain. Teachers' way of considering mathematical ideas in the professional development sessions that they encounter in their daily practice considering in a way of the students thinking, the process of practice, and the consequences of the decisions on these ideas enhanced their mathematical learning by reflection, knowledge sharing and building (Silver et al., 2007).

Through all mentioned above and similar studies, scholars argue that the practice based professional development programs lead to teacher learning and change in their practice nevertheless there is not an exhaustive exploration of teacher growth in relation to the wide range of knowledge, skills, decisions, judgments and their practice as a conglomerate of all of these issues of teacher competence. More empirical evidence is needed to construct such a framework. 'Which route should be followed to improve teacher professional knowledge and practice, and how it should be done' remain still as questions to be answered for the mathematics education community. In order to deal with these ambiguities, Simon and Tzur (1999) have made an effort to generate "accounts of mathematics teachers' practice" which they call as a part of "teacher development experiment" methodology. As researchers, they both play a dual role in studying and fostering teacher development by the aid of their conceptual framework and they build up a 'hypothetical learning trajectory' for teachers' learning while interacting with them and during reflection they modify these with teachers and these new trajectories leads to the new interaction phase. They argue that by these accounts of practice researchers can explore the complex relationships among different parts of teacher knowledge and their relationships to learning which is mentioned above and the results can facilitate for designing larger scaled professional development programs (Simon, & Tzur, 1999). By considering the teacher learning through the lenses of practice-based model, the context dependent manner of the effective PD is evident. This means there is a need to adapt, design and test these ideas of teacher learning in relation with individual and organizational-contextual factors of the participants and their working place and the other factors such as policies, physical environment, the local history of professional development and the structure of the community of education in Turkish National Education system.

Furthermore, much research about mathematics teacher learning is done recently with teachers who are assigned to elementary (K-5) and middle grades (6-8). There is little empirical evidence about how to identify important accounts of high school mathematics teaching, practice and how high school mathematics teacher learning and hence practice could be improved. These issues have been elaborated further in the following sections of this literature review.

2.2 Professional Knowledge in Relation to Practice

A view of specialized professional knowledge that is used while teaching mathematics emerges over past decades (Ball & Cohen, 1999). For instance, Learning Mathematics for Teaching (LMT) project aims to assess mathematics teachers' professional knowledge via examining the quality of mathematics instruction as they argue that practice is "the key indicator of an individual teacher's professional mathematics knowledge" (LMT, 2006). Researchers of the project team designed multiple choice tests to assess teachers' performance, but since there is not an agreement on or much empirical evidence about these paper and pencil tests' predicting teachers' performance on actual mathematics teaching, they developed an observation protocol; QMI (Quality of Mathematics Instruction) to understand whether their multiple choice assessment relates teachers' actual classroom based performance. They constructed 83 different codes by video observations of ten volunteered mathematics teachers which are from wide range of social, economic and cultural backgrounds who teaches at different levels of K-8 mathematics then they grouped these codes into five sections: Instructional formats and content, knowledge of mathematical terrain of enacted lesson, use of mathematics with students, mathematical features of the curriculum and the teachers' guide, use of mathematics to teach equitably. They scored each teacher's performance related to each five sections to record the quality of the mathematical instruction and the factors that might affect the quality due to "presence" of these codes and their "appropriateness". The team is still on their way of analyzing the correlation between teachers' performance on paper pencil tests related to professional mathematics knowledge and on actual observations.

Mathematics teachers' professional knowledge is also a core element of the policy documents which set the vision of the nations and societies' expectations about what teachers should "know and be able to" for recruitment and curriculum demands. Nevertheless, a debate on identifying what constitutes that knowledge is still continuing. According to Ball, Lubienski, and Mewborn (2001), one potential solution is to analyze teachers' *work of teaching*. They argue that focusing on teachers' practice will help us to better understand the characteristics of effective mathematics teaching that might serve for multiple purposes for preparation and professional learning and development of teachers. This process is also anticipated in many standards documents for teaching as, teachers should be lifelong learners and carry out their professional development by systematically analyzing their own teaching for improvement (AAMT, 2006; MEB, 2011; NCTM, 1991).

In almost all of these documents and many more, planning the process of mathematics instruction, constructing an effective learning environment and discourse that encourages learning mathematics and assessing mathematics learning constitutes main competencies of professional practice of mathematics teachers. Lesson planning is seen as an important activity for developing effective classroom instruction (Li, Chen, & Kulm, 2009). In their study Li et al. (2009) examined six Chinese mathematics teachers' daily lesson planning and associated practices in lesson plan development. They found that though all of the participant teachers were all experienced teachers they have developed useful practices by "intensive study of textbook content to be taught, considerations about their students in lesson planning and working together with their colleagues in discussing about lesson planning and classroom instruction" (p.730). By examining textbooks those teachers have developed better understandings about the important mathematics content to be covered, difficulties that students may encounter and what instructional strategies to use appropriately. In addition to the planning process of effective mathematics instruction many standards or competency frameworks for teaching documents asserts that meaningful mathematical discourse and a learning environment that discourse and the mathematical tasks embedded within are the among core standards for teaching mathematics (MEB, 2009; MEB, 2011; NCTM, 1991). Research literature about discourse embedded in a learning environment states that the meaningful mathematical discourse that the teachers should create in the learning environment should be inquiry based (Breyfogle, 2005; Casa & DeFranco, 2005; Hunter, 2008; NCTM, 1991). In order to provide an inquiry based discourse, teachers pedagogical reasoning processes and their instructional decisions regarding what mathematical ideas to pursue in the classroom discussions and in mathematical activities, when to provide information to or lead student explorations or clarifying their ideas are crucial to improve the discourse (Casa & De Franco, 2005). Both reflecting in and on practice are in conjunction with facilitating and developing these decisions that teachers make. For example, in Breyfogle's (2005) study where a secondary mathematics teacher and a researcher collaborated to create an inquiry based mathematical discourse based on teachers' reflections on the teacher's videotaped lessons within one semester, influenced teacher change in his classroom practices toward a more inquiry based discourse. Yet, teachers' practices have deep connections with not only with ones' professional knowledge for/of teaching but also with one's beliefs and attitudes related to teaching; change in these beliefs and attitudes are stated to be crucial and also a challenging issue. The researcher deals with this issue by adapting Guskey's (as cited in Breygfole, 2005) professional development model proposing change in deeper beliefs and attitudes will occur only after a change in their practices occur.

Teacher knowledge has various sources such as judgments and beliefs about teaching and learning mathematics which has a background rooted back to an individual's first years as a student. If we took both "acquisition" and "participation" metaphors of learning (Simon & Tzur, 1999) into account, we might argue that teachers' professional knowledge is a combination of somehow static knowledge of what is possessed and dynamic knowing what part of an action is (Cook & Brown, 1999). Furthermore, teacher knowledge for, in and of practice as different conceptions of the relationships among teacher knowledge and practice bounds teachers' roles as learners with being only user/consumers of knowledge, inventor of what is known or an agent of change who took different roles by "challenging their own assumptions, studying their own students, classrooms, constructing and reconstructing curriculum, and taking on roles of leadership and activism in efforts to transform schools and societies" (Cochran-Smith & Lytle, 1999, p.278).

2.3 Learning from Examining Own Study

Cochran-Smith and Lytle (1999) pose a newly emerging conception of teacher knowledge in relation to practice that they call as “knowledge of practice”. They stated that this view of professional knowledge is in accordance with perspectives where teachers examine their own experiences. This view also has common grounds with “self study”, which is common among teacher educators where they reflect on their own conceptions about teaching, learning and practice by systematically inquiring his practice (Cochran-Smith & Lytle, 1999). Though there are fewer studies related to classroom teachers’ self study in school settings, related research literature reveals teachers conducting self study as a professional learning experience can gain a deeper understanding of their practice and be able to reflect upon their teaching and learning (Aubusson & Gregson, 2006; Loughran, 2004). Austin and Senese (2004) define self study as of assigning teachers a researcher and a learner role in accordance with other kinds of practitioner research but more than that: Self study is about discovering or analyzing about self as a teacher or a person where teachers put their implicit judgments, values, and practices under examination. They believe that teachers can benefit from self-study in “practical”, “personal” and “professional” aspects (p.1256). Teachers’ potential benefits from self study are reflected also in a study of Hoban, Butler, and Lesslie (2007) who facilitated self study of two elementary teachers during their professional development programme, lasted 6 months for learning more about teaching science. They used action teaching as a framework for helping teachers to learn new ways to improve their science teaching that uses “reflection, sharing, action and feedback” process to facilitate their learning. Researchers also acted as a critical friend; a crucial part of self-studies (Loughran, 2004) to “guide teachers’ learning process, design action plans, helping them to share their ideas and to suggest ideas appropriately” (p.38). Data were collected through teacher journals, classroom observations, interviews and teachers’ individual self- developed learning models at the end of the study. Findings revealed that two teachers focused primarily on their learning and how they learned rather than the change they are going through but also they gained deeper insights about their practice, their role as professional educators and sharing ideas with colleagues and other education practitioners as professionals.

Two issues mentioned above related with self study require attention for teacher learning: Reflection and professional support. Reflection is seen crucial for sustained teacher learning as it is directly linked with teachers thinking and learning about their practice. Nevertheless, there are important differences in how and on what teachers reflect and their purposes of reflection (Clarke, 2000). Tischa and Hospesova (2006) in their study with selected elementary teachers argued that systematic self reflection and joint reflection with other teachers and/or researchers led teachers to learn professionally in their practice of preparing, implementing and analyzing several mathematics instruction experiments and is “an effective way of improving teachers’ competence” (p.150). Their results also showed that joint reflection of teachers and researchers has provided a connection between the theory and practice. It also provided a change in teachers’ role from realizing ideas of *others* towards creating their own experimental instruction. Watching their videotaped practices provided an additional insight to these roles as an educator and to reflect on their practice as an outside expert or researcher (Tischa & Hospesova, 2006). Similarly, Sherer and Steinbirg (2006) explained that a joint professional reflection between researchers and teachers, who examine student’s mathematics learning processes through their own classroom interaction, contributes to teacher learning and is also an essential component of professional knowledge. In reflective practice, teachers' investigative activities like defining a problem situation or objective which may emerge from whether theoretical or a practical problem, designing a working plan to work through, implementing and reflecting in/on/for the practice, consequently thinking and analyzing their own practice is necessary for successful professional development of teachers (Da Ponte, 2001). In line with this claim, Hall (2009) also argues that teachers autonomy on research processes result with increased engagement with research/inquiry and a transformative role that improves practice. In her study, Hall (2009) explicates aspects that contribute to teacher learning through being involved in a professional development project titled "Learning to Learn" and the way that teachers understand themselves in relation to professional inquiry and hence research processes. By particularly grounding on the Ecclestone's development of 'Learner Autonomy', she argues for a change in teachers' personal relationship with research as connected with the language used by the teacher learner (as cited in Hall, 2009) where descriptors of the increased autonomy of learner which results with learning/knowing are: "The mastery of vocabulary, the targeted use of terms in relation to the self and the engagement with definitions" (Hall, 2009, p.676).

In summary, the research literature about teacher learning, professional development and teacher change state that an effective professional development for teachers should be based on practice facilitate teachers' decision making, implementing and evaluating their own professional learning and undertaking a researcher role; promoting their reflection in, on and for their practice, provide ongoing support for the teachers for a sustainable professional learning. Furthermore practice based professional development programs assumes that teachers are learning in their practice but the mechanisms behind the interaction between teachers' learning and the change in their practice needs empirical evidence and to be validated.

2.4 Teacher Learning and Professional Development from a Socio Cultural Perspective

Lifelong learning or continuing professional development became very popular phrases for all professions today. Despite the fact that their prominence is maintained by the professional standards or competency policies; their meaning and scope still remain vague among practitioners and within academia too (Friedman & Philips, 2004). Furthermore, underlying assumptions that shape general understanding of the professional development (PD) and its focus and purposes have some limitations which are put forward by Webster-Wright (2009) in her extensive review of PD literature. Primarily according to Webster-Wright (2009), the notion of development associates with a deficient practitioner image that needs to be *developed* rather than one who takes own responsibility of learning. This image is in line with Sfard's (1998) acquisition metaphor positioning learner as a passive recipient of knowledge conveyed by an *expert* and also consistent with the *individual cognitive* perspective such as Piagetian constructivism as a subcategory of Bernstein's liberal progressive competence pedagogy restated by Lerman (2006). Secondly, separation of the professional from the learning context ends up focusing on only one particular aspect of PD whereas process, outcomes, participants and the context of learning should be inevitably interrelated. Consequently, a void in the relevant research area has been identified for addressing the real/authentic professional learning 'experience' as a whole rather than focusing solely on the development of the individual or a group of professionals.

Although educational research unfolds that effective PD should be ongoing and related to practice supporting active, engaged and social learning; most PD programs are found to reinforce the delivery of the information from a PD provider to the professional who is in a position of 'need' of development and remain separated from the everyday practice (Garet et al., 2001). These training models of PD are criticized for that they overlook the situational and contextual nature of learning (Darling-Hammond & Mclaughin, 1996). There are problematic aspects/areas which have been reported in the professional development research in and about practice. First, a majority of research about PD are about evaluating PD programs and its specific features such as the content, participation and the outcomes of the programs whilst few PD research challenge or critique the notions of those traditional training or transmitting modes. Second, the conceptualization of PD in that research is under the influence of objectivist and cognitive approaches to learning that reinforce the view of professionals who are in need of development under the supervision of an expert rather than to focus on gaining insights regarding their experiences of learning. This conceptualization promotes a view:

> …[T]hat the professional is one who is competent and develops excellence only in respect of measurable, pre-defined standards; and secondly, that professional skills can be described readily, defined meaningfully and delivered through simple transfer (with values, attitudes, knowledge and understanding being classes and subsets of general teaching skills) (Patrick, Forde, & McPhee, 2003, p.240).

The discourse of PD exemplified above highlights a passive practitioner image that is deficient of knowledge hence has to be developed through continuous relevant training delivered by an expert. Consequently, in line with cognitive approaches to learning, current prevalent PD practices view teacher knowledge as a static commodity residing in individual's mind that can be filled with up to date information and which teachers can acquire in one specific context and will be able to use in any kind of setting. There is a considerable body of research against that individualistic view of knowledge in any specific context can successfully be transferred into other contexts (Desforges, 1995). Moreover by adopting this view one would be ignoring social contexts that teachers engage with and perspectives and identities of them formed as they participate in these contexts (Wenger, 1998).

On the other hand, Billet (2001) argues that Vygotsky's sociocultural theory of learning can help us to understand how participation in to practice is viewed as a form of professional learning. Additionally, in Lave's seminal work of situated learning; working and learning are not seen separately (Lave, 1996). These widespread sociocultural theories have the common epistemological assumption that knowledge is not a possession of individual and knowing is a consequence of a constant relationship between individual and the social contexts which one takes part (Kelly, 2006). This view of knowledge is also coherent with a dynamic knowing-in and-of practice supporting the participation metaphor of Sfard (2008) in order to explain human thinking. Knowing in and of practice actively and productively shape a professional's expertise (Schön, 1987). In teaching profession, knowing in practice is distributed among all participants of the professional practice: Student's, teachers and the other physical and conceptual recourses that are at hand (Billet, 2001; Kelly, 2006, Wenger, 1998). Teacher's expertise as a professional is related to the social practices in which they engage while they are working and the discourses they produce which in turn define and shape these social practices.

However by adopting a cognitive lens, expertise of a teacher gains knowledge to acquire character and then becomes about applying these into their practice afterwards (Kelly, 2006). This is a professional expertise view supported by the traditional medical model of 'evidence based' practice and involves 'training' of the professional from theoretical to practical level which is also a very common practice in the teacher knowledge research such as UK (Hall, 2009). This view has also been critiqued due to its ignorance of the situated and the social nature of learning. Furthermore, despite Schön's (1987) knowing in practice is a challenging alternative for cognitive conception of professional knowledge; it is also problematic as being focusing on the individual practitioners mind, aligned with acquisitionist learning metaphor and as viewing practitioner separate from context. This approach is argued as "underplaying of the impact of sociocultural context in … descriptions of knowing in practice". (Webster-Wright, 2009, p.716) Thus, a need for reconceptualizing of the professional expertise that accounts for a more practice-based contextualized and employing a participationist approach in relation to professional learning becomes evident. In addressing this need Webster-Wright (2009) offers a shift from the term professional *development* of teachers to *learning* of practicing professionals.

2.5 Language and Communication in Mathematics Education

According to the extensive review titled 'language and mathematics education' Austin and Howson (1979) have given at least two reasons why a mathematics educator should take into account of linguistics or language related aspects when teaching and learning mathematics:

- First, was about the driving force of our need for communication and creation. This was given as the reason why that we are using a term called mathematical language.
- Second, the role of the language in learning and teaching in mathematics is inevitable.

Then they asked two fundamental questions. "Can linguistics help us better to understand this language [language of mathematics]?" and "How linguistics, the study of language, help increase our comprehension of the learning process and improve our techniques of teaching?" (p.162). This section provides the current situation in mathematics education regarding language and communication starting from these questions.

Learning and communication so as human thinking and language are certainly accepted as closely related to each other today just as it was many years ago. Yet we have still linguistic challenges of teaching and learning of mathematics and questions related to the nature of the language of mathematics still remains to be resolved (Schleppegrell, 2007). In her review of the collection of research papers presented at the working group on Language and Communication at the sixth International Congress of Mathematical Education (ICME), Morgan (2000) identified some of these challenges and presents an up to date vision of interaction between mathematical language and learning. She claims that though the current conceptualization of mathematical language as a formal symbol system and a technical vocabulary has shifted to a more elaborated one that embraces functional rather than a formalist approach, stemming from the idea of thought and speech/ language might not be as straightforward as it is seems; the mathematical language is still far from an agreed definition. This ground on the Vygotskian premise that there is a pure form of thought which is not related to speech at all: a speech for oneself. The striking aspect of the Morgan's argument above is related both to the disciplinarity and discursivity of mathematics. In fact mathematics is conceptualized as a form of communication but not a language in its own right (Austin & Howson, 1979).

Sociolinguist M. A. K. Halliday (1978) has pointed out that the way mathematics used in everyday language is quite different than the ways that students need to develop when learning mathematics in the schools. In that sense the students should learn to use the language that associates with certain patterns and serves new functions in mathematics. He presented his concept of *mathematical register* as "developing a language" *for* mathematics:

> A register is a set of meanings that is appropriate to a particular function of knowledge, together with the words and structures that expresses meanings. We can refer to a 'mathematics register' in the sense of the meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself) and a language must express if it is being used of mathematical purposes (p. 195).

Mohan (2011) interprets the notion of register as the system of meanings that realizes any social practice in language so that the participant of this social practice could interpret and produce new meanings. In that sense the participants of that social practice interprets and produces new meanings related to practice. Hence mathematics register helps us to understand the common patterns of mathematical language in that will help learners to interpret and constitute new mathematical meanings (Schleppegrell, 2007) and teachers to utilize these meanings and patterns within their teaching and learning practices discursively.

Similarly Lemke (2003) claims that using; learning and teaching mathematics should be understood as part of different representation systems including language and visual representations associated with different systems of meaning. This argument stems from the semiotic perspective in which different representation systems are utilized in meaning making. In her review mentioned above Morgan (2000), identifies the general tendency of functional approach to language in mathematics education and shift in researchers' interest towards the issues of discourse, communication and interaction in mathematics from more traditional approaches of mathematical symbolism, its technical vocabulary and relationship with natural language. Though she admits that this shift in our understandings about language and mathematics education has been an important progress since issues highlighted and questions addressed about the topic at Austin and Howson's review; she addresses a broader perspective or a coherent domain of research/ knowledge to interpret, discuss the research data and findings in that area. The broader domain of knowledge including the semiotic perspectives and functional views of language and more discursive approaches to both language and the teaching and learning of mathematics are in accordance with particularly to the approaches highlighted above by Lemke and Halliday. Further, in her theorization of human thinking as communication Sfard (2008) views mathematics as discourse and not as a language since these two are distinguished by their very nature. She points out that as being a discourse, mathematics constantly produces its objects of study and it is also related to human forms of doing whereas language is just one type of representation within the various semiotic systems available. Hence mathematics is given as a meta-language that includes multiple semiotic systems (graphics, geometric figures, numerical systems algebraic or other formal symbolic notations including language) but not restricted to language linguistic features.

Taken together, above discussions about the discursive nature of mathematics as a discipline or school subject and the tendency of examining the teaching and learning experiences/ practices are also reflected at curriculums and standards documents (NCTM, 2000; National Governors Association Center for Best Practices (NGA Center) & Council of Chief State School Officers (CCSSO), 2010) for teaching and learning mathematics. Furthermore, at international tests (e.g. PISA and TIMMs) designed to test quantitative literacy specifically mathematical communication as an aspect of that literacy; the highest level is stated as students' explaining or communicating the results by argumentation (Adams and Wu, cited in Kosko & Wilkins, 2011, p.4) Yet, these reflections have also some problematic aspects with respect to the discursivity of mathematics. This is argued by Onstad (2007) in his comparative study of four national curricula (England, Norway, Sweden and Romania) exploring the role of language and communication in mathematics education. Despite that all four curricula have touched upon the issues of mathematical communication and language, the emphasis of mathematics as a discipline overbalance the statements about the communicative or discursive aspects. In these statements mathematics is rather treated *as language* for learners to master its technical concepts and skills and this is also presented as a limited view for representing the interplay of language and mathematics. Adopting a semiotic perspective is proposed to provide us a better understanding about this interplay and bridging the gap between the language and mathematics while extending language to a broader semiotic sense (p.12).

Mathematical communication has also found its place among major mathematical skills that the latest Turkish national high school mathematics curriculums (MEB, 2005, 2013) aims to develop in students. The general approach of the programs is stated as:

> [E]mphasizing mathematical concepts the interrelations in between them and the basic mathematical operations and the meanings that these operations have within…aiming a conceptual approach in which the concepts are constructed through the discussions held within the classroom…With this approach it is aimed to have develop some important mathematical skills along with mathematical concepts (MEB, 2005, p.4).

- Talking, writing and listening about mathematics develop communication skills and it helps students to better understand mathematical concepts. Teacher should create classroom environments of which students will be able to explain, discuss and represent their ideas in written forms and should do appropriate examinations/inquiries for students to be able to better communicate (MEB, 2013, p.7)

The curricular framework (2005) phrased its objectives for *communication skills* to have developed in students as below:

- Being able to explain mathematical ideas with physical materials, models, pictures and diagrams
- Being able to explain and justify mathematical ideas and situations
- Being able to relate mathematical language and symbols with everyday language
- Being able to use skills of reading, listening and visualizing in order to evaluate and interpret mathematical ideas
- Being able to model verbal or written statements, concrete, picture, graphical and algebraic methods
- Being able to reach the generics by formulating the result she/he reached at the end of the mathematical discovery process
- Being able to extend and justify mathematical expressions in accordance with relevant questions
- Being able to evaluate the role and power of mathematical representations in extending mathematical ideas (MEB, 2005, p.10)

Mathematical communication is among the group of 4 mathematical skills have taken place in the 2005 curriculum. These skills are associated to the learning areas of mathematics as in the figure below:

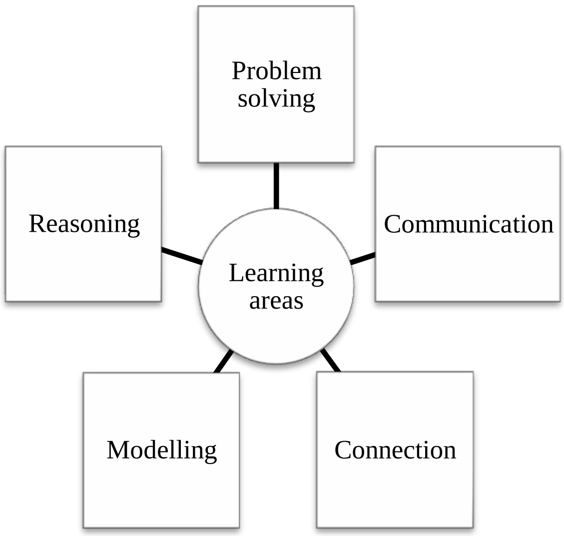


Figure 1: Learning areas and mathematical skills

The latest mathematics curriculum (MEB, 2013) while stressing more on mathematics as a language in comparison with the former curriculum at 2005, has also a similar approach for developing mathematical communication skills within students by having them to acquire following behaviors apart from those above:

- Connecting daily language with mathematical language and symbols and vice versa.
- Being aware of the mathematical language's success at expressing actual problem situations in a plain, clear and an effective manner and appreciate that.
- Being aware of the fact that mathematics is a language having distinctive symbols and terminology of that have meaningful relationships in between.
- Using mathematical language in its own right, at various disciplines and in their own lives in an appropriate and an effective manner.
- Having self confidence in using mathematical language
- Interpreting/evaluating the correctness and meaning of dialogues and ideas related with mathematics
- Having positive feelings and emotions in using mathematical language (MEB, 2013, pp.7-8)

Similar principles or standards as presented above regarding mathematical communication are also reflected in the international standards documents as well. For instance, in the Common Core State Standards document of U.S (National Governors Association Center for Best Practices (NGA Center) & Council of Chief State School Officers (CCSSO), 2010), math standards from K-12 have been grouped under two main categories as Standards for Mathematical Practice and Standards for Content where former is based on NCTM's process standards (NCTM, 2000) hence on communication as well as other strands of mathematical proficiency. Another example is Singapore mathematics curriculum which centralizes problem solving by using mathematics (Curriculum Planning and Development Division, 2012). In that curriculum mathematical communication took place together with reasoning and connections which refer to mathematical processes enable "using appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas"(p.32).

Within these models it has been stated that enabling students' expressions with a mathematical language is a crucial process for teachers of mathematics along with organizing classroom based discussions. Furthermore, explaining the mathematical solutions that they have arrived, actively participating in the classroom based discussions and group work and asking questions are stated among the roles and responsibilities of the students that relates to communication and language in learning mathematics. Nevertheless, even it seems the general approach of those curricula stresses important points about the role of language and communication in teaching and learning mathematics, the cognitive aspects and mathematical concepts dominates these frameworks. Due to the priorities of disciplinarily, aspects that should have underlined the discursive nature of the curriculum are overlooked similarly in Onstad's (2007) review about the 4 national curricula above. A particular problem with those approaches of curricula is that representing mathematics as a language with its special symbols and technical vocabulary since this view provides only a limited view making sense of mathematizing. The interpretation of mathematics as a discourse is gaining a wider acceptance and argued to be a more accurate categorization at least for two reasons. First, as I touched upon elsewhere the discourse and language belong to different ontological categories: Former is a "symbolic system" and the latter refers to "human activity that exceeds mere vocabulary and grammatical rules" (Sfard, 2008, p.130). In fact, one description for discourse is 'language in action' (Brown & Yule, as cited in Sfard, 2008). Second, language is only one form of communication mostly verbal and mathematics utilizes many other forms and semiotic systems. Through the use of multiple systems or combinations of them mathematical meanings are produced that belong to the language of mathematics or as described previously; the register of mathematics in Hallidayan terms, a broader semantic system associated with various systems of representations not just (verbal) language (Morgan, 2006).

Additionally there are also statements in both latest and the former Turkish mathematics curricula (MEB, 2005; 2013) which are found to be important in developing communicational skills in own right. Writing and talking about mathematics are among examples of these statements alongside facilitating students' mathematical meaning making. Though general communicational skills has fundamental importance in learning any school subject, these statements above don't help us to come up with a clear understanding of what is really meant with mathematical communication skills. How do we distinguish communication in mathematics, mathematically based communication skills and mathematical communication from each other? Which one helps us when and most when teaching and learning mathematics as a school subject? These aspects remain hidden in those curricula.

Language and communication related issues have also been included in the standards/ competency documents for teachers and teachers of mathematics. For instance, in professional standards of teaching mathematics (NCTM, 1991), mathematics is viewed as a discourse and frames how a teacher should organize this discourse in order to facilitate the learning of their students. Discourse is conceptualized as a main category within the standards of teaching mathematics, under which the roles of the students and teacher and the tools to enhance the discourse are presented in detail. On the other hand, standards related with the evaluation of teaching of mathematics aiming to improve teaching of mathematics requires collecting information/evidence about teaching of mathematics as communication from classroom teaching along with the processes of problem solving and reasoning (NCTM, 1991).

The Turkish standards/ documents also includes mathematical communication in the scope of competencies for both the teacher's of high school mathematics (MEB, 2011) and the teacher's of elementary mathematics teachers (MEB, 2009). Yet, when looked in detail there is not any reference to mathematics as a discourse and even hardly any explicit statement of/about mathematical communication or mathematical language or language for/ of mathematics exist in the document for high school mathematics teacher competencies. The following are the only two among the performance indicators of which are to be observed and documented within the process of classroom teaching referring to discursive aspects of the high school mathematics teaching:

- Creates a learning environment which supports the accurate and effective communication of mathematical ideas
- Creates a classroom environment that supports mathematical thinking and reasoning

The perspective provided above in Turkish standards document for teachers of mathematics can be said to put a limited emphasis on the discursivity of mathematics. Considering the trends in the ongoing research about mathematics education in order to weave communication and the mathematics together and the close relationship between language and communication; adopting more semiotic and discursive perspectives is needed and the implicit language of these documents should be revised in order to highlight the importance of the role of language and communication play in mathematics education (Onstad, 2007).

The research done about the communication within the mathematics classroom has also important implications for participants of the classroom discourse (Sierpinska, 1998; Voigt, 1995; Yackel & Cobb, 1998). The kinds of interactions and meanings produced within classroom discourse determine and also determined by the teaching and learning context (Martinho & Ponte, 2009). The teacher's role in these interactions and in the broad communicative processes is crucial when determining students' participation and agency in the mathematical meaning making process and the classroom discourse. An important means for the teacher in structuring the discourse is the nature of the questions asked by the teacher. For instance, Love and Mason (1995) identifies three main kinds of questions asked by the teacher. The first kinds are the confirmation questions that are used for checking students understanding of facts and procedures. The second kinds are the focusing questions to direct students' attention to a specific topic and the third are inquiry questions when even teacher does not know the answer.

Although it has passed over 3 decades that Austin and Howson (1979) has posed the questions about the relevance of the language and communication in mathematics we are still chasing after the answers. In the next section I elaborated on the issues around discourse, discursivity of mathematics and discursive research in mathematics education.

2.6 Discourse and Discourse Analysis

Cambridge Dictionaries Online (2013) defines discourse as "communication in speech and writing". Though this broadest definition might satisfy most of us in everyday use, due to various historical traditions and the schools of thought the definition of the discourse shows a great variety. Schiffrin et al. (2001) classify all of these into three categories. First approach views discourse as "anything beyond the sentence", second as "language use" and the last as "a broader range of social practice that includes non-linguistic and non specific instances of language" (p.1). The formalist researchers are associated with the first category and view discourse as one level up at the hierarchical sequence of words, clauses or sentences. The researchers engaging with the third type of conceptualization of discourse rejects that the discourse is always linguistic and it might cover non- linguistic elements. These elements could be gestures, signs, photographs or people's way of talking and dispositions (Juez, 2011), any material object in space, or printed or visual media (O' Halloran, 2004.) The second approach to discourse is called the functional approach. Researchers engaged with this tradition see discourse much broader than a text, they are interested in the human actions while people are using language to achieve certain goals, the way their language use operate within different socio cultural contexts as well as the particular 'context of situation' (Halliday, 1978). The significance of this latter approach lies at its emphasis on discourse as broader than the text, than the sentence or utterance and its concern with the associated *context* of language use. The meanings made through language are constructed and constrained by its environment, under certain circumstances or at particular situations. By emphasizing the *language in use* “as a goal and means of education and an instrument of social control and social change” it becomes evident why the discourse analysis has been developing a theory and practice and why it appears to be multidisciplinary (Trappes-Lomax, 2008, p.1)

The discourse analysis (DA) is a cluster of methods for studying language and its role in social life (Potter, 2008, p.217). There are multiple approaches to DA as well as to the conceptualization of the term *discourse.* According to Gee (1999) DA is analyzing the language in use to enact the activities, perspectives and identities (p.4). He argues that since any research method should be tied or based on a theory, DA as a research method associates with the domain of knowledge or the theory of *language in use.* In his approach he clearly distinguishes two forms of language in use of which he denoted with discourse and Discourse. By discourse he points out to the *language in use* while performing a communicative goal, the language on its own sense. In doing that, on the other hand, he mentions of Discourse when the non-linguistic issues cannot be explained with the *discourse* as follows:

> [In many] “ways of being in the world”, you use language and “other stuff”- ways of acting, interacting, feeling, believing, valuing together with other people and with various sorts of characteristic objects, symbols, tools and technologies to recognize yourself and others as meaning and meaningful in certain ways. In turn, you produce, reproduce sustain and transform a given “form of life” or Discourse. (p.7)

Another categorization of the approaches to the discourse is put forth by Cook (2008) in which two main approaches were mentioned. First one is ethnomethodological and the second is Foucauldian DA. In the ethnomethodological approach the primary concern is on the interactions of the human beings that produce meanings where the researchers are both interested in the structure of the language in itself and the resulted meanings. Apart from focusing on meanings embedded within the speech Foucauldian DA also pays attention to the power relations that produce social reality using language.

Another historically influential approach preceding Foucauldian DA is originated mainly from linguistics but has found a wide range of applicability in the field of education. This approach was among the first systematic approaches to DA who have been developed by Sinclair and Coulthard (Potter, 2008). In their book titled “Towards an Analysis of Discourse” published at 1975 they constructed a model of interaction takes place in the classroom by analyzing classroom discourse known as IRF, in which there is a pattern of interaction between teacher and students as “Initiation- Response and Follow up/ Feedback” widely applicable to most classrooms (Yu, 2009).

Along with Foucauldian DA, critical discourse analysis (CDA) and discursive psychology have emerged as contemporary approaches to DA at 90’s. These approaches are grounded on the social semiotics and the functional grammar of the sociolinguist M. A. K. Halliday (Huckin, 2002), and on the theories of poststructuralist thinkers such as M. Foucault and Mikhael Bakhtin. Critical discourse analysis is accepted to be the collection of approaches that utilizes text as its data to with an emphasis to social and political critique for analyzing language and discourse (Blommaert, 2005; Potter, 2008). Fairclough and Wodak sorts out the basic tenets of CDA as ‘addressing social problems, discursive power relations, discourse constituting society and culture, discourse doing ideological work, discourse as history, mediated link between text and society, interpretive and explorative approach to DA, and discourse as form of social action’(cited in Van Dijk, 2008, p.353). Although CDA shares some of its basic tenets with Foucauldian DA, both have differentiates at the level of methodology and theoretical orientations.

Discursive Psychology (DP) is also a critical approach to DA that draws mainly on the social constructionist perspective of discourse. This perspective views language use as constructions of the social reality and draws predominantly on the work of Potter and Wetherell at their book *Discourse and Social Psychology which* provides useful tools for inquiry (as cited in Jørgensen & Philips, 2002). It challenges the paradigms of social and cognitive psychology at the methodological and theoretical level and focuses on interactions at everyday or institutional settings as data and utilizes mainly conversational analysis as a methodology for analysis and interpretation (Potter, 2008). Yet, this approach differs from poststructuralist discourse traditions (e.g. Foucauldian DA) in the sense that its interest in the social interactions at the micro level of which is out of focus for the latter DA tradition.

In this section a brief summary of some major influential traditions in the study of discourse has been given. Yet more will be elaborated on the social theory and discourse especially, the Foucauldian ideas and approach to discourse and DA on the next section which will also be used to draw on to support the findings and conclusions later in the Discussion chapter.

2.7 Social Theory and Discourse: Foucauldian Perspective in DA and Foucault’s Toolbox

Post structuralism focuses on the language and production of meaning as related in the ways and relations of power and knowledge produces approved forms and knowledge and social practices (Fawcett, 2008). One of the basic tenets of this tradition is that whole reality and the social space are discursive in nature. This perspective also rejects to assume that there can be an objective reality of social spaces that is independent from human action and thought that can be studied as natural scientists do. On the other hand social theory also characterizes the social world without using the scientific method of sociology and with its skepticism about the truth; points out the differences between the nature of subjects of study of natural sciences and sociology (Juez, 2009). These two schools of thought utilizes the notion of discourse in the sense that Gee’s (1999) *Discourse*; emphasizing the ways that *Discourses* produce social positions and power which are also central notions at both perspectives. Post structuralism and social theories are often thought under the same school of thought for doing discourse analysis where Michel Foucault is a prominent figure associated with this tradition. Although it is impossible to give a brief description of that thinker’s work I will present some of his concepts of which I drew on at my research.

According to Foucault, the term discourse is comprised of a group of statements of which he conceptualized as a system of representation in a semiotic sense producing meaningful statements through its rules and practices (Potter, 2008, Jardine, 2005). Due to his genealogical/historical approach to discourse he sees discourse as a way for representing knowledge about a particular topic at a particular historical moment (Foucault, 1970). In Juez (2009, p.213) Foucault's major contributions to discourse theory are summarized as:

1. The relationship of discourse and power
2. The discursive construction of social subjects and power
3. The functioning of discourse in social change.

The first area concerns mainly on the productive nature of power and discourse, their influence on the ways of making sense of, acting on the objects of knowledge. Power and discourse produces knowledge/ statements that become meaningful for us. The second area is about the construction of objects and subjects guided by interdependent and relevant historically specific discourses and interpretation of the nature of the power as productive and being produced within social relations. The third area is about the discursive nature of the social change (Juez, 2009). Since the first area dealing with the relations of discourse and power/ knowledge well aligns with my research design and purposes of understanding a teacher's meaning making process/learning through and about herself inquiry of own practice, in this section I choose to elaborate more on the Foucauldian notion of *knowledge/ power, discourse, and discursive practices*. First, I will begin with Foucauldian notion of power. In his work Foucault (1970), characterizes power as a productive term which exists in the relations between individuals but more than that, existing only if it is put into action in the sense that certain actions modifying other actions. Furthermore, he conceptualizes knowledge and power as complementary forms of the one same unity of which knowledge spreading out the effects of, having been produced by and inducing power. These two complementary/dual notions are acting together to create and support a broader unity of knowledge called, *discursive formations* in which the rules, roles and discursive practices are formed (Jardine, 2005). Discursive practice according to Foucault (1970) is "a body of anonymous historical rules always determined in time and space that have defined a given period" (p.117). It creates knowledge and also defined by the knowledge that it forms.

Foucault did not provide a method for doing discourse analysis. By following his theoretical work yet we can have specific understandings to frame and conduct our analysis. In Foucault's words his theoretical notions can be used as a *toolbox* in gaining new ways to look into the subject under investigation. In this research, I draw on some of the methodological tools provided by Foucault of which he refers to *power relations* that I have discussed previously in order to interpret these themes as teacher's account of the discursive practices of the mathematics classroom. When discussing about his own theory he suggests his theoretical constructs should be used as a toolbox. In my interpretation of teacher's account from my perspective as a researcher I follow Simon and Tzur's (1999) approach attempting to "understand and articulate teacher's approaches to the problems of practice, how and what the teacher perceives and how she makes sense of, think of and respond to situations as they perceive them" (p.254). I utilized Foucault's theoretical notions of discourse, power/knowledge relations and discursive formations and basically in order to articulate the discursive practices within mathematics classroom from the teacher's perspective while also acknowledging my position as researcher.

2.8 Discursive Research in Mathematics Education

Recently there has been a growing interest in discourse and discourse analysis in mathematics education as a relatively new area of research (e.g. see Forman et al., 2001; Lerman, 2009; Morgan, 2000, 2006; Pimm, 2004; Ryve, 2011). According to Ryve (2011) this interest is arisen from due to several reasons: First, discourse and mathematical communication and communities of discourses were given place in reform documents like NCTM's Principles and Standards of School Mathematics. Secondly discourse and discursive research has been developed theoretical perspectives and methodologies to explore phenomena of which mathematics education research has an interest such as interaction, identity, positioning and gender. Finally, mathematics is also conceptualized as a discourse by some of the scholars within mathematics education field (e.g. O'Halloran, 2000; Sfard, 2008).

According to the functional use of discourse that I have discussed at the previous sections the *use of language* is studied in order to explore human communication. In this view discourses are seen as constructing the reality and these realities are in turn accomplishes purposes by language use and construes social action. Wetherell, Taylor, and Yates (as cited in Ryve 2011) divide the study of discourse into three main domains as: The study of social interaction, the study of minds, selves and sense making and the study of cultural and social relations. Ryve (2011) presents mathematics education researchers as conceptualizing discourse 'as social interaction' such as Cobb and Yackel (1998) with their work on sociomathematical norms and discourse and Morgan (2006) with her study based on social semiotics and interactional sociolinguistics perspective (e.g. Halliday, 1973, 1978) into the first category. In the second category researchers are interested how discourses influence individual's sense making and 'their production as social actors' (Ryve, 2011, p.172) where Atweh and Cooper (1995) focusing on the construction of learners within the mathematics classroom and Walkerdine's (1988) analysis of developing child can be categorized as exemplars of this category within the mathematics education research. Within the final category, 'macro processes of social and institutional actions' are emphasized (Ryve, 2011, p.172.). For example, McBride (1989), Hardy (1997, 2008) and Valero (2007, 2008) focus on power relations and discourses redraw on or relate to Foucaldian perspectives of discourses for instance. In addition to that Pimm (2004) divides mathematics education and discourse analysis research area into 4 categories and gives exemplars of research done in each of it as below:

> Aspects of voice, mathematical agency and addressivity (e.g. Boaler, 2002; Gerofksy, 1999; Pickering 1995, Rowland, 2000), instances of meta discourse (Chapman, 2002; Rowland, 2000), components of temporal structure (Solomon & Oneill, 1998, Pimm, 2005), elements of style (e.g. Pimm & Wagner, 2003) (p.2).

Though in this section, I make a brief introduction above instead of presenting an extensive and exhaustive review of theoretical perspectives and methodologies regarding discursive research in mathematics education I aim to review some of the prominent studies of which I chose based on their relevance to my research study and design and their influence on other studies in the related literature of discourse and mathematics education. Studies related with aspects of teacher discourse in the mathematics classroom presents that teacher discourse has distinct elements than the student discourse or the general elements of classroom discourse in terms of their purposes, functions or power relations. For instance, Knott et al.

(2008) lays out morphology of mathematics teacher discourse that features different aspects of this discourse and contrast these with student discourse. Based on their observations and analyses of classroom teacher discourse they have concluded that teacher talk falls into 5 categories. First category is called the *big ideas* related with mathematical talk, norm setting and the epistemological perspective that underlies all of the mathematical teacher discourse; second category is called *affective discourse* through which social and sociomathematical norms are established and students are motivated to learn mathematics. Third category is *discursive tools* which make mathematics happen in the classroom which comprises of content related means and resources that the teacher utilizes for students to learn in the classroom. Fourth category is, presented as *tactical tools* that teacher employs for making student learning happen and last category is the *ending discourse* by which the teacher review previous material and give students a chance to share what they have learned (p.93).

Krussel et al. (2004) also focus on teacher's discursive actions or *discourse moves* as a framework for analyzing discourse in mathematics classrooms. They argue that their framework provides an analysis of the special role that the teacher plays in the classroom discourse characterized by deliberate actions taken by a teacher *to participate in the discourse* in mathematics classrooms. Accordingly the teacher's discourse moves is argued to have a purpose, takes place in a setting (small-group or whole-class discourse), take several forms (verbal or non verbal) and may result in a variety of consequences (immediate or long-term) (p. 104).

Another framework for examining mathematics classroom discourse is based on Yuri Lotman's approach of *functional dualism* in meaning (as cited in Knuth & Peressini, 2001). Lotman himself originally drew on Bakhtin's notion of social language for this approach (Knuth & Peressini, 2001). According to Bakhtin (1986) language is inherently dialogic as every utterance actively responds to other utterances and is formed according to the possible responds of the addressee. Furthermore, besides univocality is the speaker's employment to convey meanings properly to the others or the addressee; dialogism is about generating new meanings within the discourse. The above roles of discourse that are played within the classroom have further influenced at least two prominent studies that have built on each other. First one is, Wertch and Toma's (1995) classroom discourse analysis of a 5th grade science classroom focusing on interactions between students and between students and the teacher; concluding that discourse in the classroom has either functioning to convey information from the speaker to the audience or to facilitate students and teacher's meaning making through interaction. Subsequently Knuth and Peressini (1998, 2001) by conceptualizing their work on the former, recognize the dual role of the discourse (dialogic/ univocal) at play within the mathematics classrooms. They argue that by focusing on the speaker's and listener's intent "it is possible to determine whether the transfer or generation of meaning is prevalent in discourse and in making sense of classroom discourse respectively" (p.325). In another study of Knuth and Peressini (1998) examine the nature of the discourse within a summer discrete mathematics course in which 14 high school mathematics teachers have participated and also within their actual classrooms. The central focus in analysis was how participating into the summer course as a professional development activity influenced the nature of the teachers' discourse in their classrooms. They have particularly compared and contrasted the nature of the discourses of one teacher, George, within the summer class and their actual classrooms and concluded that although

George has a dialogic mode of discourse while listening to his students his discourse switches to univocal while speaking so as general characteristics of his discourse within the classroom. On the other hand in there was also a dialogic discourse between the instructor and the in service teachers and in that sense differs from the univocality as a general tendency of George's discourse within his classes.

Another study that on the nature of the discourse within the mathematics classroom is Nathan and Knuth's (2003) examining one teacher's interpretation of reform-based ideas and how it is realized in her practice. In doing this, they have particularly examined interactions between the participants (teacher-student and student-student) and the nature of these participants' speeches to determine what it may tell about their (students' and teacher's) participations and mathematical practices. Likewise, yet based on a different theoretical perspective Greeno, Gresalfi, Hand and Martin (2008) analyzed the discourse within two middle school mathematics classrooms by focusing on the interaction between students and teacher and reconceptualize the notion of competence in terms of one's *participation* in a classroom as an activity system. In their analyses they have examined the affordances that the classroom as an activity system and how participants engage or not with these affordances. These affordances were either about the mathematical tasks or those offered by the participants themselves. Through the analyses of discourse they have concluded that the idea of competence is about participating into social setting framed by the distribution of agency and accountability between participants as a highly context dependent phenomenon.

Finally, there are also studies which focus on the use of pronouns like I/ we/ you as an account of the mathematical discourse which fit into the first category of the mathematics education and discourse analysis research area formed by Pimm (2004) as discussed above. Gerofsky presents an analysis of speech of a university lecturer showing how lecturer uses 'I' pronoun while addressing fourth year undergraduates whom she called 'junior collegues'; yet switches to using 'we' pronoun for first year students with a persuasive tone (as cited in Pimm, 2004). This use of the word 'we' by the lecturer is an exercise of power and originates from the positioning of the lecturer and the students with respect to each other.

2.9 Theoretical Perspectives

In this section I present two theoretical perspectives I have primarily ground on and their associated analytical tools relevant for my research design as a conceptual framework and analyses of data.

2.9.1 Social semiotics and systemic functional linguistics

Semiotics is known as the systematic study of signs associated with its founding father F. S Saussure. Social semiotics is a view that focuses on how people construct their meaning systems and conceptualizes productions of meaning as a process of social interaction (Chapman, 2003). The major premise of the social semiotics is that the meaning is created or constructed and it is also a functional view which focuses on how we use language as among many other semiotic systems to make particular meanings in social contexts (Lemke, 2003). Moreover, a social semiotic research utilizes a particular interpretation of signs, which is social rather than purely formal. This approach particularly matches with Systemic Functional Linguistics (SFL) theory theorizing language as a social semiotic (Halliday 1972, 1978). According to Halliday, social semiotic approach recognizes language as a social practice where meanings are exchanged in interpersonal contexts. Furthermore, adopting a social semiotic approach for language use entails considering both the immediate aspects of the situation and the broader cultural contexts in which these exchange of meaning occurs (Morgan, 2006).

Within the heart of Halliday's SFL theory lays the investigation of how people use language in social contexts. A social context can be interpreted by three components: *Field* concerning the social process or activity being carried out, *tenor* as the roles and relationships of the participants of that social activity and the *mode* refers to the role of language that plays within the situation focusing on the process of exchange of meanings (Halliday, 1978). Yet individuals not only speak or write to simply express their thoughts but they aim to effect certain changes in their environment (Morgan, 2006) or certain discursive goals (Tabach & Nachlieli, 2011). Given that language as a primary medium for meaning making/ learning for Halliday, along with many other semantic systems, meanings are realized by the choices we make through language. Moreover, meanings associated with a particular context are realized by the components of language according to the functions they serve. Halliday represents these components of which he calls *metafunctions* (of language) and their relationship with the components of the social context as in Table 1 (Halliday, 1978, p.189):

Table 1: Realization of meanings that constitute social context through language

Component of social context	Functional-semantic component through which typically realized
1 field (social process)	experiential
2 tenor (social relationship)	interpersonal
3 mode (symbolic mode)	textual

Along with this table above Halliday (1978) argues there is three level construction of meaning making in discourse that is, *ideational meaning* related to the social action, *interpersonal meaning* the roles of the participants and *the textual meaning* as organization of the text. In this research project I aimed to investigate teacher learning as a social practice during an inquiry of own practice. In doing that I have focused on the discourse of the teacher which I have further categorized into action and reflection discourses on which I have elaborated more at the method chapter. Further by grounding on the mechanism for meaning making in social contexts presented above based on three metafunctions, I have further conceptualized the second category of the teacher discourse of action into three layers, based on the work of Tabach and Nachlieli (2011).

The first layer of the teacher's discourse of action is the teacher's *mathematical discourse,* which is associated with the ideational/ experiential metafunction of the language. The ideational metafunction realizes the field of the social context. It represents the speaker's meaning potential as a sense maker of (own/ others) experiences of the social practice/action. Ideational meanings are related with the content of the situation and focuses on the conceptual elements of the teacher's discourse of action within the discursive practices of her classes. Hence in this study, teacher's mathematical discourse associated with the ideational/ experiential meanings constructed by the teacher covers teacher's ways of mathematizing and also her conceptualization of teaching and learning of mathematics.

The second layer of the teacher's discourse of action is the teacher's *social discourse;* associated with the interpersonal metafunction. The interpersonal component of the language realizes the tenor of the social context. Moreover it represents the speaker's meaning potential as a participator within a social practice. By the interpersonal meanings formed via language, speaker engages to the situation by expressing his perspective of, namely the predictions and judgments while selecting a particular role and determines the roles for the others in the speech situation (Halliday, 1973). Hence in this study the teacher's social discourse covers the teacher's discursive actions to enact the social practice of inquiry of own practice focusing on mathematical communication as a participator by which she expresses the social relations including her position within the discursive practice of the mathematics classroom.

The third layer of the teacher's discourse of action is the teacher's *pedagogical discourse*; associated with third metafunction of the language called textual. The textual component of the language realizes mode as the associated component of the social practice. The textual metafunction of the language represents the speaker's text forming potential within a social practice. By the textual meanings formed by the speaker, language becomes a text relevant to its contexts of the situation that enables the effective use of it, enabling both the ideational and interpersonal meanings come into being (Halliday, 1973). Hence the teacher's pedagogical discourse covers the teacher's discursive actions organizing the flow of communication or *exchange of meanings* through which mathematical and social meanings can be expressed within the discursive practice of the mathematics classroom through the course of inquiry of own practice focusing on mathematical communication. By these discourse moves, teacher make mathematics happen in the classroom and are meta-mathematical; they organize and enable the negotiations and exchange of mathematical meanings between participants. In that sense they are about mathematics but they are not the actual mathematical content moves: They are meta-mathematical (Knott et al. 2008).

2.9.2 Commognitive framework

The focal points of Sfard's theory of human thinking, learning and mathematical education, are communication and participation. In her theory, she begin with problematizing the acquisitionist view of learning which "makes us think of knowledge as a kind of material of human mind as a container, and of the learner as becoming an owner of the material stored in the container" (Sfard, 2008, p.49). In doing that, first she presents how time honored cognitive perspectives are not able to resolve current dilemmas of mathematics education and thinking. Then she proposes the participation metaphor to account for human thinking that could only develop from a patterned collective activity as it is for all kinds of human doing. Participationism views development of humans as the transformation in the forms of human doing rather than people themselves. Sfard (2008) argues that both individual and collective transformations are complementary to each other rather than being sharply distinct from each other. This entails the "*individualization of the collective* and *communalization of the individual*" (p.80) and a reconceptualization of "thinking as an individual version of *interpersonal communication"* (p.81). Therefore she finally comes up with a new term *commognition* as a combination of communication and cognition emphasizing that dual nature of thinking.

As a form of human doing mathematizing is also defined as participation to mathematical discourse (p.128). She conceptualizes mathematics as a discourse but not a language at least for two reasons. Mathematics is constantly producing its object of talk (Sfard, 2008). On the other hand, people participate into construction of mathematical objects and they learn about the elements of the discourse through this participation (Stahl, 2008.) She further characterizes mathematics as a discourse by its tools and its forms and outcomes of its processes as any other discourse. These tools are given as *word use* and *visual mediators* and as *endorsed narratives* and *routines* for the forms and outcomes of mathematical discursive processes.

Word use is presented as fundamentally important for any discourse since it refers to how it is used in language that is the meaning, in Witgenstenian sense (Sfard & Avigail, 2007). By the lens provided with the commognitive framework these meanings we produce represent not only what we find in but also what we are able to say in the world (Sfard, 2008). In order to examine the use of the words and visual means/ mediators Sfard (2008) argues that one should focus on how participants *realize* the words and symbols. These realizations make the words and symbols come into being, as accessible as tangible objects. The forms of the realization procedures might be *symbolic, iconic and concrete* and could be made directly or mediated by another realization procedure. Each form has its strengths and weaknesses mainly in terms of producing or substantiating mathematical narratives yet Sfard (2008) notes that "replacing student's direct realizations with discursively mediated one's is among aims of school learning" (p.226)

Like all other of discourses, mathematics is also a type of communication built on rule-regulated activities. These rules might be either at *object level* which are of the properties regarding the objects of the discourse and produces narratives about these objects or at *meta level* that is about the activities of the participants of the discourse who aims to produce object level narratives (Sfard, 2008). These meta-rules further construct the *routines* which are discursive patterns that repeat themselves in certain situations. The main aim of the mathematical routines is to produce endorsed narratives about the mathematical objects. These mathematical routines are *explorations*, but there are also other types of routines that are implementation *of rituals* and *deeds.* These are argued as 'developmental predecessors' of explorations. Exploratory routines can be conducted in three types:

- *Construction as* a discursive process resulting with new endorsed narratives
- *Substantiation* as a discursive action taken by the participants in order to decide whether to endorse the new narrative
- *Recalling* is a process of which participant conducts to remember previously endorsed narratives and to built on to construct new narratives

Due to the participation metaphor as an important tenet for commognitive theory of thinking, learning hence mathematizing, and learning occurs as the individualization of the participation to collective forms of doing. Then the initial steps of this individualization are most likely to result with rituals than of explorations and these rituals are about performing rather than knowing thus this won't lead or associate with substantiation of narratives (Sfard, 2008). This means *how* of a routine is most likely to occur than *when* and rituals are actually the most natural stage of the teaching and learning processes and learning (or becoming a participant in discourse in participationist sense) resulting from our tendency and capacity to imitate others. Final aim should be the transformation of these rituals into exploration routines. Sfard (2008) proposes that in order to grow new routines one might use these rituals associated with *deeds* well-known by the learner which would also enrich those deeds.

Finally Sfard (2008) argues that learning mathematics is a shift in one's discursive practices where meta-level learning is crucial for that. The meta-level learning occurs when one experience a *commognitive conflict* resulting from the incommensurable discourses of the interlocutors or their different ways of communicating. Hence commognitive conflict does not result from the discrepancies between individual and the world as in cognitive conflict and it is mostly resulted with a change in the interlocutors' discursive practices. Furthermore whereas the resolution of cognitive conflicts is about individual's making sense of the world, commognitive conflicts are resolved by making sense of other's thinking about the world.

2.10 Summary

The literature on teacher learning and professional development in connection to my research encompasses professional development and teacher change, professional knowledge (in relation to practice), teacher learning from examining own practice, teacher learning from a socio cultural perspective. Research has shown that an effective professional development for teachers should be connected to their practice, based on their needs, facilitate teachers' decision making, implementation and evaluation of their own professional learning and support them in undertaking a role of a researcher and their process of reflection in, on and for practice; provide an ongoing support for the teachers for a sustainable professional learning. However, in-service education programs or opportunities have been continuing to receive criticisms all over the world as not having responding to teacher's needs and having been identified by the teachers as ' not useful'; providing little professional collaboration in designing their own professional learning, being theory-oriented rather than giving access to practical concerns.

With an aim to address issues that those criticisms pointed out above; socio cultural theories of learning and the literature on teacher learning and professional development were explored and this entailed shift from the term professional *development* of teachers to a professional *learning* (Webster-Wright, 2009). On the other hand, further review of literature revealed a need to re-conceptualize professional expertise that accounts for both communication and participation metaphors as well as cognition in understanding teacher learning. Furthermore, the concept of discourse and its use in relevance to discursive research in mathematics education as an area which has been gained increased attention in mathematics education community recently has formed the fundamentals of the theoretical perspectives of this research. That strand of discursive research are connected to the socio cultural perspectives of teacher/professional learning yet, studies focusing on teacher discourse examining teacher learning with a discursive perspective are scarce. This study aimed to better understand teacher learning as a social practice during an inquiry of own practice by focusing on the discourse of the teacher which I have further categorized as action and reflection discourses.

Literature on various strands of analytical approaches to discourse has been examined in order to decide on the main theoretical perspectives upon which to base for this research. With reference to the Tabach and Nachlieli's (2011) study using Halliday's SFL theory and Sfard's commognitive theory (2008) to analyze classroom discourse I also utilized these two theories as a base to form my theoretical framework. I have made this selection because according to these theories language is seen as a social action and mathematizing as a form of human doing is seen as participating in to the mathematics discourse. In broad sense, this literature review provided crucial knowledge related to professional development and learning of teachers, and particularly in mathematics teachers, learning from examining own study, socio cultural perspectives and teacher learning/development, language, communication and discursive research in relation to mathematics education and theoretical perspectives relating to human thinking, learning and communicating.

CHAPTER 3

METHOD

The purpose of this chapter is to outline and discuss the research design, sampling, data collection procedures, and the data analysis.

3.1 Research Design

The study is a qualitative interpretive single case study employing an interpretive inquiry. Glesne (2011) puts the intent of the qualitative research as understanding how participants socially construct the world around them. Thus I worked from the view that will enable me to understand the process of teacher's construction of meanings through and about the self inquiry of her own practice. Through the interpretive inquiry, as it is for all other kinds of qualitative inquiry; the emphasis on understanding the meanings produced by people, how they make sense and construct their experiences and the social reality. In doing this, the researcher should participate into the reality of the informant to understand the social phenomenon under research and describe specific cases that will produce some forms of narratives (Bogdan & Biklen, 2007, Creswell, 1994). According to the interpretivists, people's own understandings, purposes and intentions are of strong influence on their actions and the meanings they made about themselves and others (Ezzy, 2002). An interpretivist researcher believes there is no knowledge independent from a theory and we can by no means reach the reality as pure as it is hence reality is always constructed by ourselves. As the researcher/ observer all we are able to do is presenting constructions of the world of events in a social context at particular time but this is always subject to reinterpretations based on different interests and purposes and meanings (Smith, 2008).

My initial view of teacher learning was in tune with the widespread instrumentalist view advocating a teacher expertise residing in predominantly in teacher's minds, viewing professional knowledge distinct from its associated contexts of practice and *acquisitionist* view of knowledge in the form of teacher competencies gained in a particular context transferrable to any context. During the initial stages of fieldwork my understanding about teacher learning has evolved towards a more socio cultural and an interpretivist perspective. According to socio cultural theory, learning is defined as becoming a participant in certain collective human practice/activity (Lave & Wenger, 1991; Vygotsky, 1978). Indeed, professional knowledge in socio cultural terms is understood as *knowing in practice* distributed among all participants in professional practice,as a shared collective formation including physical/conceptual resources available (Kelly, 2006). According to Packer (1999) the interpretive researcher should adopt a participationist view to understand the phenomena under research in line with socio cultural theory of learning. According to that theory, knowledge/ knowing is as a collective product of mankind where the social processes play a prominent role rather than being *received, natural or biologically determined* (Vygotksy, 1987). Hence this participatory view was also compatible with my interpretive research design focusing on the social practice of teaching and learning of the mathematics within the classroom as *a discursive practice* that indeed, is a context sensitive entity. This was also emphasized by Lerman (2000) who argued that in order to construct a more complex and true to life theory of teaching and learning, one should consider "the tension between the individualism and collectivity as combining person-in practice model with regulative features of discursive practices with the consequences of multiple practices manifested in the classroom" (p.38).

In this study, I employ an interpretive single case study methodology to investigate a mathematics teacher's professional learning process while the teacher is inquiring her own practice. For an interpretive researcher, as Myers (2009) argues the path to the reality goes through social constructions such as, language, consciousness and shared meanings. Moreover, as a sociolinguist Halliday (1994) points out human learning's as making meaning through various semiotic systems of which language is the most prevailing form. This language-based theory of learning is also in accordance with the sociocultural learning theories where knowledge/meaning is constructed through interaction in social contexts and learning emerges within specific cultural practices. Another theory of human thinking as communication and via language as a social activity is Sfard's (2008) commognitive framework. In this participationist framework the development of individual and the collective are seen as complementary processes which mean "the individualization of the collective and the communalization of the individual" (Sfard, 2008 p. 36). These perspectives are in line with that, human thinking is socioculturally situated in social practice and these social practices can be explored through the dialectic of theory and practice as in the forms of knowing and doing (Martin, Nelson, & Tobach, 1995). Within SFL theory social practice is a domain of a cultural knowledge, which is also a semiotic system carrying a *meaning potential* that enables the participants of this practice to interpret and produce texts of the social practice (Halliday, 1978). According to Mohan (2011) social practice is a frame of meaning based on the discourse of action and the discourse of reflection. While the action discourse is basically about doing of the social practice and the reflection discourse is of talking about the social practice; producing knowledge of the practice. Hence main functions of these discourses also differs yet, moving between them enables the researcher to trace the dialectic nature of learning in the form of theory and practice relationship as argued by Martin et al. (1995) above.

Based on my research design for examining the professional learning process while doing her inquiry as a social practice, I basically looked into two dimensions of one high school mathematics teacher's discourse who's inquiring her own practice: The teacher's discourse of reflection and the discourse of action. In order to explore first dimension of the discourse monthly interviews were done accompanied with few episodes from the videotapes of the teacher's mathematics/geometry classes for the teacher to reflect on her practice and reflective logs/forms right after each classroom observation were filled out by the teacher. For the second dimension, the classroom observations and the objectives or foci of inquiry/learning emerging from her inquiry process were the main methods of and sources for data collection and analysis. The main focus of inquiry was chosen by the teacher as *mathematical communication* based on the *pedagogic content knowledge* component of the High School/Secondary Mathematics Teacher Competencies in Turkey (MEB, 2011). As for the data analysis, the teacher's discourse will be analyzed based on a narrative analysis and on a combined framework fore grounded on both Halliday's SFL (1972, 1978) theory and Sfard's (2008) commognitive theory of human thinking respectively which will be discussed in the data analysis section in detail.

3.2 Single Case Study Strategy

In accordance with the interpretive constructionist research paradigm, knowledge is constructed individually or collectively through the formation of ever more sophisticated and informed formations through the process of hermeneutics/ dialectics (Lincoln & Guba, 1994; 2008). The transfer of the knowledge gained from one setting to another is often provided by case studies (Bhattacharya, 2008)

A case study is an investigation of a phenomenon within its real life context while adopting multiple perspectives rooted in that particular context (Ritchie & Lewis, 2003; Yin, 2003). According the interpretivist stance adopted in this research and the nature of the research questions the single case study methodology is chosen since it provides multiple perspectives employing multiple sources of data from the setting such as interviews, direct and participant observations, archival records thus 'thick descriptions' of the phenomenon under investigation (Yin, 2003). The case in point in this research is an experienced high school mathematics teacher teaching in a government high school. The rational for selecting the case is about its representativeness that had to be studied longitudinally due to theoretical specifications. The decision of conducting a single case study design is given based on two reasons. First, undertaking a multiple case would require extensive resources and time beyond the means of a single independent researcher (Yin, 2003). Second, conducting a single case study is found to be opting in the field of education especially when employing a particular strategy that is teacher's inquiring her own practice taken both as a process and a context for her learning (Mertens, 2005). As argued by Lincoln and Guba (1994), case studies might not be representative as their major aim in conducting the interpretive cases is to contribute to knowledge accumulation in relevant domains.

3.3 Research Site and the Participant Teacher

Based on the scope of this study the intended population is the secondary mathematics teachers. Being also a mathematics teacher myself, for the last 10 years I have a lot of contacts at school sites representing a variety of positions, experience and expertise. Starting from those contacts I have explored possible schools and teachers that might be the research site and the subject for my study. The possibilities have been narrowed as a result of my negotiations with teachers, due to their voluntariness and with some school administrators due to their opinions about some of these teachers' potential attitude and contributions to such a study. In the mean time I have employed a purposeful sampling technique in order to select participant(s) who are most eligible for the nature of my research design and the scope of my study (Koerber & McMichel, 2008). Eventually one teacher was chosen based on the following criteria:

- Mathematics teacher(s) who indicated their willingness to learn or develop professionally, had an experience regarding in service training or PD activities provided by MEB and their experience related with these training and activities was not as a full participant

- Mathematics teacher(s) who could be able to decide on a certain learning foci based on their needs/ priorities and expectations for their learning

The participant teacher was from a government high School in Ankara. She, Aylin (Pseudonym) was voluntarily participated in the study. She has been teaching for 19 years and she had been working at the same school for 8 years. At the time of data collection she was teaching both geometry and mathematics courses to the 11th grades in her school. At the first semester I had the chance to observe both her mathematics and geometry classes, but at the second semester I could only observed one of her classes in which she was teaching mathematics due to continuing changes in her schedule made by the school administration similarly to most other teacher's schedules and my work schedule back at the school.

When I first mentioned about this research to her she has voluntarily joined in and looked very confident about herself as a professional. But she was somewhat resistant towards the general educational policies and regulations of the National Ministry of Education as she put it like that in our first meeting:

> They don't care about us, our problems and requirements. They only command and want us to follow the orders. Well for me that's not the case. (25/05/2012, personal communication)

At the end of that meeting I gave her a self evaluation form that comprised of competencies and sub competency statements of core competency area; pedagogic content knowledge from the mathematics teacher competency document constructed by the Ministry of National Education. The document is expected to be helpful in planning and organization of the in service activities aimed at high school mathematics teachers and especially for the mathematics teachers in planning their own professional development (MEB, 2011). Hence the form is constituted based on that competency document to help to set a stage for identifying a foci for her inquiry of own practice. As a result of this self-evaluation form and from our personal conversation she decided to focus on mathematical communication as foci for her inquiry process. Then I have participated to an hour-long mathematics lesson of Aylin as a direct observer that day.

In Turkish education system the transition from the post secondary level to the high school is through a nationwide examination called SBS. The students who attend to high schools like the one Aylin works for, could not have been placed to any special academic high schools called Anadolu high schools which accepts registrations and admissions as a result of their exam scores and their academic achievement histories. Their achievement ranks are said to be around the mean of the national sample. While both high school types aim their students to continue to the tertiary education, regular high schools like Aylin works for are less successful than the latter academic high schools mentioned above.

3.4 Data Collection Procedures

The focus of this research is on teacher's professional meaning with and through her language in use/ discourse while inquiring her own practice. Typically a data for the analysis of discourse is taken from written texts or recordings (Brown & Yule, 1988). Several texts have been analyzed by using various qualitative data collection methods considering the purpose of this research. Indeed, using different kinds of data with different methods is recommended for the qualitative researcher namely triangulation of the data. In this research thus, I employed multiple methods for data collection that enabled deep investigation of teacher discourse to trace the teacher learning while doing a self-inquiry of practice. These methods include primarily classroom observations, teacher interviews including pre and post observation interviews and my field notes. Additionally personal communications about the school context as research site, the teacher's evaluations about her monthly objectives set according to her foci of inquiry; teacher reflection forms filled out after every observed lesson have been the other sources of data during the research.

3.4.1 Interviews

Teacher interviews are one of the primary types of data collection of this research. 6 interviews as the first and the last being pre and post study interviews have been conducted on a monthly basis starting from October 2011 to February 2012 and finally end in at May 2012. All interviews have been audio taped. Each interview has lasted approximately 45 minutes. The approach for interviewing was a general interview guide approach that stands in between the informal conversation and the standardized interviews comprising of carefully prepared open-ended question (Patton, 2002). According to this approach an outline of questions are prepared in order to cover the relevant themes or topics. This approach is advantageous in providing participant's experiences rather than researchers and minimizing personal interaction reducing the possibility of researcher's influencing the participant (Butina, 2006). Moreover, the interviews in this research are used as means to unfold teacher narratives about their practice based on the selected episodes from the videotaped lesson observations hence they are designed to be reflective in nature. This approach above is known as a narrative interviewing (Fraser, 2004). Within this approach the interview responses are treated as stories elicited by the in-depth interviewing of participants. Paget argues that distinctive feature of the in depth interviews is the answers given continually inform the evolving conversation (as cited in Mishler, 1986). Utilizing this approach researcher assumes that the "meaning is expressed in and through discourse" which is also in tune with my interpretive research design hence is selected for this study (Mishler, p.66).

Interviews serve multiple purposes in this research. In broad sense they are aimed at exploring the teacher's discourse of reflection that produce meanings or knowing about practice that is professional learning while inquiring her own practice based on her video recordings of the lessons. Specifically, first interviews aims to reveal the teacher's epistemological stance of teaching and learning of mathematics and to identify a well defined foci of inquiry. The aim of the 4 subsequent monthly interviews is to facilitate her reflection on her practice and on her ongoing inquiry of practice in based on her foci of inquiry. Finally the post study interview has the aim for exploring teacher's overall reflection on her inquiry of own practice and to gain further understanding about the meanings she had produced within the process.

3.4.2 Classroom observations

Mckehnie (2008) states that qualitative observation is, “well suited to the study of the social processes over time” as it provides “rich descriptions” and “a deeper understanding of the phenomena” (p.575). In order to complement my analysis of teacher discourse while inquiring her own practice, the teacher’s discourse of action is sought via classroom observations. I have done classroom observations for a total 12 lesson hours of videotaped sessions recorded by me. During each observation field notes were also taken to capture the additional information about the teacher discourse in practice, her enacted action discourse within the discursive practices of her mathematics classes.

As a researcher, I observed Aylin’s lessons also with the lens of a mathematics teacher with 10 years of experience. According to the constructionist-interpretivist paradigm of which this research is fore grounded there is no direct and objective observation of a reality so the position of the researcher as interpreter of this reality should be clearly asserted. So my interpretations of the classroom based both on my professional identity as a teacher and on the theoretical assumptions that are distilled from my experience, knowing and values as a researcher. Along with my writings as field notes, these repetitive classroom observations provides thick descriptions of teacher’s discourse of action which enables tracing of teacher’s professional learning within a theory-practice relationship when combined with the teacher interviews.

3.4.3 Self-evaluation form

As mentioned above, at our first meeting with Aylin she has filled out a self-evaluation (see Appendix B) form comprised of competencies and sub competencies of ‘pedagogic content knowledge’ area set by at the mathematics teacher competencies document (MEB, 2011). The form was organized to reveal the participant’s perspective in terms of the importance of these competency statements to her and identifying her priorities and needs for her inquiry of own practice. The resulting information has been used in order to identify a starting point and candidate objectives for the inquiry or reorganizing objectives stated previously before filling out the form. Since the inquiry process has planned to be ongoing and flexible these objectives or themes/foci of inquiry are tentative and have gone through various changes.

3.4.4 Teacher reflection form

Following every classroom observation the teacher had filled out a reflection document comprising of guiding questions to lead and initiate the reflection process. The questions or statements are derived from the main themes that have emerged from and monthly objectives she had set at the interviews according to the main foci of inquiry she had set at the early phases of her inquiry. Naturally, the statements are written by me thus they are also reflecting my interpretations of these themes and foci of inquiry namely the mathematical communication. Furthermore, I have made some modifications in the statements of the questions during the research process according to the emerging situations and themes relevant to my research purpose and questions (e.g. see Appendix C). Hence, the teacher reflection forms has a direct influence on teacher’s inquiry of own practice as it is discussed in the validity section later in this chapter.

3.5 Data Analysis

Starting from the transcription process I present a two level procedure for analyzing the data of this research. In the first level, the qualitative strategy of narrative research or inquiry and Foucoult's toolbox (Foucault & Deluze, 1972) will be explained then for the second level methodological framework will be presented based on the combination of the two theoretical perspectives Tabach and Nachlieli (2001) in relation to the purposes and the theoretical framework of this research.

3.5.1Transcription

The first step in data analysis was transcribing all the interviews and videotaped classroom observations. That was a very time consuming process undertaken only by myself. Edwards (2003) states that the transcripts provide invaluable opportunities for presenting events of an interaction as they have happened in real time and expressing within the form that is relevant to researcher's interest. I had to make choices about the type of information to include, the categories to use and the display of the information within a text form (Edwards, 2003). Since the interview data are analyzed by a narrative inquiry approach I have presented the teacher discourse from the interviews under the categories of narrative processes. These categories are primarily comprised of stories and additionally descriptions, theorizing, argumentations and augmentations as being other forms of narrative stories (McCormack, 2002). For the transcriptions of the videotaped observations, I have constructed transcription conventions as suggested by Tilley and Powick (2002) (see Appendix D) so that all of the transcriptions could remain consistent throughout the text. The language of all interviews and the lesson observations was Turkish. Thus first, I have transcribed all data in their original language (Turkish) and then translated them into English. In transcribing the videotapes I have both listened and watched them and listened to audio tapes of the interviews in their original language more than once in order to reduce the errors for an accurate representation of the interaction and discourse within these records. Furthermore, in order to increase the quality of transcription I had to review and compare the texts in both languages many times in terms of their linguistic, syntactic and grammatical aspects. Nevertheless it is natural even for the transcriptions where there is no translation of texts from one language to other that reviewing the texts constructed many times (Potter & Wetherel, 1987) and this process is theoretical and an analytical activity rather than technically typing out what is seen or heard (Wood & Kroger, 2000).

3.5.2 Narrative analysis

In order to explore the first research question I used a narrative inquiry approach as a qualitative methodology. In describing this methodology first I begin with definition of narrative.

3.5.2.1 Narrative

Though, there is not a consensus on the definition of the narratives among researchers, a narrative is often used synonymously with stories (Riessman & Qinney 2005). Stories are powerful ways for people to represent and make sense of their experiences (Fraser, 2004;

Ponte, 2001). Yet all forms of talk or speech are not narratives. There are many other forms of discourse including explanations, arguments, persuasions, conversations, diaries, reports to name a few. The critical feature of narrative that distinguishes it from the other forms of discourse is it's being *sequential* and *consequential* (Riessman, 2004). Namely, the events in the stories are organized and evaluated in a chronological order as to be meaningfully selected for a particular audience (Elliot, 2005; Riessman, 2004).

According to Ponte (2001) teacher's stories are also a form of narratives representing their experiences of their professional practices. He argues that these stories may provide learning goals for themselves and for other practitioners. They are also found to be closely related about their practical theories hence in order to understand teacher knowing in and of practice these stories can be an appropriate starting point (Carter 1993; Ponte, 2001).

Narrative research is often done through narrative inquiry of which it uses the stories as their data or their product (Patton, 2002; Riessman, 2008). In addition to examining participants' experiences in their context, the narrative inquiry also gives information about the cultural and social resources that they draw upon to make sense of their experiences (McCormack, 2004). Narrative inquiry is giving the researcher access to how participants interpret their reality since one does not directly access to the participants' experiences (Riessmann, 2008). Since people construct and reconstruct meanings by recreating their own stories storytelling is a means of knowledge production (Fraser, 2004; McCormack, 2002; Ponte, 2001).

Among the four main approaches to narrative inquiry (Riessman, 2008) as I use thematic analysis where the researcher identifies key themes throughout the text and organizes, presents and interprets the content by these patterns/themes. The thematic analysis is chosen due to its straightforwardness and being a systematic approach for analyzing narrative data where the main focus is the content of the discourse (Riessman, 2008).

Specifically I followed a system for analyzing the narratives offered by Fraser (2004) and McCormick (2002). First step is about listening to the stories told during the interview and reconnect with the conversations in order to reflect on feelings and thoughts during and after the interview. I have written memos in a journal during the course of interview and utilized these during analysis and the construction of the teacher narratives. Second step is about the transcription of the interviews of which I explained in detail above. Additional to these, I have erased some parts of the speech like personal comments/stories of which I found irrelevant to the teacher's focus of inquiry and my research. The third step is about locating narrative processes within the text (McCormack, 2002). After locating the stories from the interview and giving them a title according to which I interpreted as the main point of the story according to the participant teacher I have included other narrative processes such as description, argumentation, theorizing augmentations (McCormack, 2002) in the text. Then I have sent the transcribed material to the participant teacher for comments and feedbacks about this transcript. I asked her whether she thinks that stories and other narrative processes and their titles which I have identified are relevant to her experience or she thinks any aspect of her experience is omitted or overlooked and she wants to remove any of these stories from the text. She returned to me by saying that stories and my organization of them made sense to her and she did not want to correct, add or remove anything. The next step is about forming the first draft of the interpretive story/ narrative for each interview. At this step initially I have translated all the transcript from Turkish to English since all interviews was conducted in Turkish. Then I listed all of the story titles in chronological order and revised some of the titles as emergent codes from these stories as an open coding process. In doing that I reinterpreted each interview data to locate any contradictions and common themes across the stories, focusing mainly on the words used by the teacher and their meanings. This phase has also described the context of situation (i.e., the interview). On the other hand, the next step was associated with the context of culture as the broader social, historical, political conditions that in turn becomes determinant for the conditions of telling and experiencing these stories (McCormack, 2002). At that stage I have reviewed the first draft of the interpretive stories/narratives by multiple lenses and from different domains of experience (Fraser, 2004; McCormack, 2002). In doing that, I have looked into the stories to identify self talks, stories involving others than the story teller and references she made to cultural conventions and discourses. With this step I have reflected on this information provided by these multiple lenses to consider the additional understandings (McCormack, 2002). Finally, I have constructed 6 interpretive stories from 6 interviews, grouped initial story titles as codes emerged from all of the interpretive stories under broader themes or patterns and by including my own reflections I have constructed the final document.

3.5.3 Discourse analysis: A combined approach

The second level of the analysis of the teacher learning or specifically knowing in practice while doing her inquiry of own practice is analyzed through her discourse of action based on her classroom observations. By adopting an interpretive stance, I view discourses as prominent social constructs for understanding reality. In order to explore the rest of the research questions I have used a combined approach of Tabach and Nachlieli (2011) based on two theories as a methodology for discourse analysis. These theories are drawn upon the socio cultural perspectives of learning where knowledge/meanings are produced via communication and interaction, emerging within specific cultural practices. In this section I elaborate on the methodological framework in question above.

3.5.3.1 Systemic functional linguistics and the theory of commognition combined as a methodology for discourse analysis

The social practice of teacher's learning about mathematical communication during self-inquiry of her own practice is also analyzed through her discourse of action. The analysis of the teacher's discourse of action is conducted via three layers and two dimensions adapted from Tabach and Nachlieli (2011) are presented in Table 2.

The first dimension comes from SFL that categorizes teacher's discourse. These are mathematical, social and the pedagogical discourses. The teacher's ways of mathematizing/ mathematical discourse, associates with the ideational meaning adopted from Halliday's (1978) theory of SFL. It focuses on the content and the subject matter of the discourse and the (social) action taking place in the context of situation. It also indicates the intended mathematical meanings expressed by multiple semiotic systems (Lemke. 1997; Jamani, 2011, Sfard, 2008). The social discourse associates with the interpersonal meaning from the SFL theory. It focuses on the social roles, the positions of and the relationships between the participants with respect to one another (Lemke, 1997; Halliday, 1978). The pedagogical discourse associates with textual meaning from SFL theory. It focuses on the organization of the content of the discourse including strategies and tactical tools to make students participate in the mathematical discourse (Knott et al. 2008, Lemke, 1997, Tabach & Nachlieli, 2011)and it also relates the discourse to its context (Halliday. 1978).

On the other hand, according to Sfard's (2008) theory of commognition discourses are made distinct by those characteristics as *words* and *visual mediators*, *narratives endorsed* and *routines*. Despite the fact that Sfard's categories were originally developed in order to characterize the mathematical discourse, these categories were also defined to characterize teacher's social and pedagogical discourse based on the work of Tabach and Nachlieli (2011). The statements written in each cell of the Table 2 exemplifies how Sfard's discourse categories characterize each line of discourse.

In analyzing the mathematical word use and visual mediators of the teacher, first I have made a list from the words and symbols that are used mainly by the teacher and by students but approved by the teacher from the whole 12 classroom observation transcripts that are specific to mathematics, according to how are they used within the discursive practice of her classes. The extensive list is condensed in order to examine the way they are used within the context of teacher's inquiry of own practice focused on mathematical communication. With this purpose, situations instantiating the context in point are determined according to the teacher's objectives for inquiry; based on her priorities, expectations and needs she had set at monthly interviews.

A similar method is followed in construction of the list of words and visual mediators for the teacher's social and pedagogical discourse. First all observation transcripts were scanned to locate social and pedagogical word / visual mediators and then examined how they are used within the context of teacher's inquiry of own practice focused on mathematical communication. Then situations instantiate that context identified and the list of words that belong to social and pedagogical discourses are relocated within these situations.

The general characteristics of the teacher's social discourse are determined according to the *speech acts/roles* that teacher chooses and determines for herself and for the other participants and to her *expression of personal perceptions and attitudes* (Halliday, 1973). In order to determine the *speech roles* from the situations that teacher's discourse focused on mathematical communication, the situations from the 12 observation transcripts are identified according to the objectives/ foci set by the teacher then the teacher's discourse was analyzed according to her choices, made to endorse her status with respect to other participants and to express her personal perceptions or opinions (Měchura, 2005).

The general characteristics of the teacher's pedagogical discourse are determined according to the discourse moves by which teacher make mathematics happen in the classroom which are *meta-mathematical moves* identified by Knott et al (2008). Furthermore, focusing on question and answer patterns produced within the discursive practice of mathematics gives clues about the method of the production or interactivity of these texts associated with the

Table 2: Theoretical framework

		Discourse characteristics		
		Word use/visual mediators (V.M)	**Endorsed narratives**	**Routines**
Categories for teacher discourse	**Mathematical discourse**	Teacher' s mathematical word /V.M use "We have always found single value until now at problems we have solved."(O1, 29)	Mathematical narratives endorsed by the teacher "3 numbers that I make up would not always construct a triangle. Then we could only find values more than one". (O1, 39)	Mathematical routines of the teacher observed within the discursive practices of the classroom "Don't we solve questions by using triangles in general?" (O1, 30)
	Pedagogical discourse	Teacher's pedagogical word /V.M use "Can you be more explanatory?" (O8 & 9,3)	Pedagogical narratives endorsed by the teacher "You can also do by using the other triangle. You can also do it by alternative ways." (O6, 26)	Pedagogical routines of the teacher observed within the discursive practices of the classroom "We call it by seeing. What were we using in acute triangle" (O1, 49)
	Social discourse	Teacher's social word /V.M use "One by one, speak by asking your turn. You say. "(O1, 35)	Social narratives endorsed by the teacher "The faster you draw more questions we can solve." (O1, 29)	Social routines of the teacher observed within the discursive practices of the classroom "Let's wait for a while, give chance to our friends to think about it. Still we have multiple ways and use whatever you feel like to use". (O5, 17)

textual meta function of texts (Měchura, 2005). The analysis is done similarly to the social discourse presented above by focusing on these *meta-mathematical discourse moves* and *question and answer* patterns helping teacher to organize the mathematical discourse.

Over the course of observations I identified mathematical routines of and narratives endorsed by the teacher in order to describe discursive actions of the teacher in terms of their forms and consequences of her mathematizing within the discursive practices of her classes. Specifically routines were examined in terms of the narratives they produce and the narratives endorsed by the teacher.

3.6 Issues with Validity and Reliability

As my research is a qualitative case study, I have to consider specific issues to ensure *trustworthiness* for my inferences based on the evidences of the study. One main issue is known as researcher bias (Maxwell, 1996). The qualitative research paradigm assumes that researcher has somehow an effect on the participants and the context of the study. Thus the qualitative researcher understands the reality based on his/her knowledge, experiences and values. There is no objective point of view reality and the interpretations of the researchers are somehow a product of the interaction between the researcher's and the participant's perspectives. So, as a qualitative researcher I should be aware of my personal biases values and determine my role as an outsider in the research process in order to reveal and focus on the reality of participants rather than mine. As Mehra (2002) suggests, I took detailed field notes or a journal about my reactions during the fieldwork. By reviewing these notes I intended to face "… 'with my' beliefs and biases … 'which will lead me' to be more "objective" in… 'my' approach by focusing on the insiders or emic voice in research" (Mehra, 2002, Specific Questions About Bias section, ¶ 11).

Since I have established a familiarity with the participant teacher before actually starting the study I had to consider both advantages and disadvantages of that relationship. First, participant teacher might feel more comfortable to determine learning goals and share her professional development/learning process with me than a complete stranger. Since I am a teacher as well (not a complete outsider of the high school context) this might have influenced the research process positively. Yet, the situation might have been the opposite. Participants might be reluctant to share their professional practice as they percept the study as an evaluation of their teaching or might refrain from being observed by a colleague. In order take advantage of us being colleagues and to deal with these issues, my approach in this relationship was that of I was a researcher interested about the processes of teacher learning and a learner of this research process. I have explained my purposes as clear as possible to the participant teacher as, no one is the expert knowing what is "best" and she would be doing her own research as she would be conducting her own inquiry in this process of learning. This approach also helped me as a researcher "to move beyond …'my' bias and talk about the knowledge gained from the participants" (Mehra, 2002, Specific questions about bias section, ¶ 20).

I have considered four additional issues in order ensure the validity of my research design as a case study. According to Yin (2003) tests that are common to maintain the quality of most social science research and also are relevant to case studies are: Construct-internal-external validities and reliability. Moreover, Miles and Huberman (1994) identify main validity issues related to qualitative studies as: reliability/dependability, internal validity/credibility/plausibility and external validity/generalizability/transferability. Many issues were controlled to avoid the potential validity threats and establish a qualified research design as discussed below.

3.6.1 Construct validity

In order to provide correct operational measures for the teacher learning in practice and from practice, I have used triangulation (Creswell, 2003). According to Patton (2002) there are four kinds of triangulation which are triangulation of sources, methods, analysts and theories. In this research I have utilized both triangulation of sources or data and triangulation of methods. For the former, I have collected multiple sources of evidence comprised of teacher reflection notes, teacher professional development plan, teacher narratives and classroom observation transcripts of learning at different times. Additionally, in order to establish a triangulation of methods, I have combined teacher interviews and classroom observations within the research process in order to understand teacher learning as a social practice of which I have explained in detail previously in the method chapter. Further validation was established by having the participant teacher to review a draft report of my results and interpretations: a process called member checking (Maxwell, 1996). Participant teacher has only made a few changes to correct grammar and verified that findings of the research are representing herself and her ideas accurately. By doing appropriate modifications in the report I have also grounded on those comments of teacher as evidences of the objectivity of my inferences.

3.6.2 Credibility

In order to make plausible and adequate inferences from my data sources, I have provided “meaningful and context- rich descriptions” and looked for negative evidences and the rival explanations for the conclusions I have reached. Nevertheless, according to interpretive perspective, meaning is co-constructed and there is no objective truth or a reality that we can achieve for our results to compare. Lincoln and Guba (1985) considers member checking as a critical technique for increasing credibility of the data I have utilized member checking as mentioned above. Furthermore, in order to provide opportunity to readers with evaluating my interpretations and following my reasoning process I have included the transcripts of the teacher interviews and the videotaped classroom observations which comprised most of the data I have collected during this research.

3.6.3 Transferability

Whether the findings from a qualitative case study can be generalizable beyond that case is a troublesome endeavor for most case studies. As Miles and Huberman (1994) suggest, in qualitative research studies it is the researcher’s task to provide “thick descriptions for readers to assess the potential transferability and appropriateness for their own settings” (p.279) rather than the traditional generalizability notion of developing a theory from multiple cases. Thus in order to establish transferability of the study, thick descriptions of how stages of data collection, analysis and interpretation have been carried out was provided. Those included descriptions of the interviews and classroom observations, of how the transcriptions and the analysis of the discourses have been made. Furthermore in a qualitative study the key issue is as what Maxwell (1996) calls the internal generalizability rather than external generalizability which former refers to the validity of results depending "on the internal generalizability of the case as a whole" (p.97). Nonetheless as Maxwell further argues "it is the lack of external generalizability" that may give a qualitative study its value (p. 97) as the generalizability of the results often depend on a development to a theory that can be extended to other cases rather than extending the conclusions to some defined population. In fact, in this research the notion of generalizability I have tried to employ was what Simons, Kushner, Jones and James (2003) calls situated generalization grounding on practice-based evidence. In this account of generalization, an experience turns into an evidence due to its communicability, practicality and relevance to other contexts (p.359) where evidence is not a property of an individual or an environment but it is a relationship between individual and the environment as a result of collective interpretations and agreement that is tied to the situation it arose (Simons et al., 2003). In that sense in this research I have based my interpretations of teacher/practitioner learning on teacher's interpretation and reinterpretation of her experience in a particular situation which is also tied closely to that situation in her practice and our (both her and mine) collective validation of that new knowledge/learning according to contextual criteria.

3.6.4 Reliability

In order to overcome the possible threats related to the reliability of the data, my case study notes, narratives and other sources of data (audio and video recordings of the interviews and classroom observations) were stored and transcripts (of the classroom observation and interviews) are presented at appendix section for further requirements of replicating this study. Furthermore, by referring to methodological procedures in detail that allows an audit trail (Merriam, 1998), I tried to obtain a chain of evidence for an external observer to follow my steps "from conclusions back to initial research questions and from questions to conclusions" (Yin, 2003, p105).

CHAPTER 4

RESULTS

In this chapter, results of this qualitative interpretative case study are presented within 4 sections. In each section, results from the data analysis are reported in relation to the respective research question. Due to the conceptualization of professional learning as a social practice and social practice as in the form of action and reflection discourses; these results in their widest sense are related with these discourses. Accordingly, results presented under the first section should be viewed as related with the analysis of teacher discourse of reflection and the results presented under remaining sections as related with the analysis of the teacher's discourse of action.

4.1 Teacher's Accounts of Her Own Practice

In this section the results of the analysis of teacher interviews regarding the first research question is presented. The results are grouped under three themes as such: discursive practices within mathematics classroom, knowing in practice and inquiry of practice as seen in Figure 2. Furthermore while presenting the results, I also include an orientation and a reflection for each interview at the beginning or at the end of a relevant code/story title by referring to my field notes and memo's I wrote during the data collection phase of this research. This will help me to stay true to Aylin's personal experience narrative and as much as possible and offer readers a multiplicity of interpretations by including both voices; hers as participant and mine as the researcher.

4.1.1 Discursive practices within mathematics classroom

Under this theme there are 3 codes/ story titles listed: Norms of teaching and learning, routines of/for mathematizing, student participation structures.

4.1.1.1 Norms of teaching and learning

For our first interview I went to Aylin's the school at the end of September 2011. This was the third time we met since our first meeting on last may. At those previous meetings we had talked about the structure of the inquiry process for instance, on what and how she was going to focus and the objectives she will be setting for each month and when she will be filling out the reflection forms. Besides, she had already decided on 'mathematical communication' to be her general foci of inquiry after having filled out the self evaluation form. She had arranged the vice president`s room for our interview and locked the door in case of any kind of disturbance.

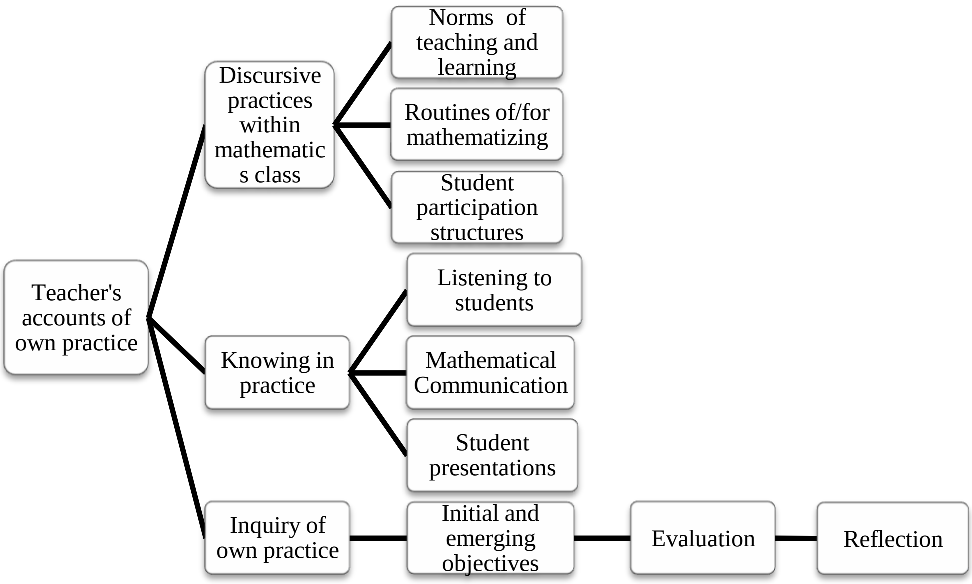


Figure 2: Teacher's accounts of her own practice

In this pre study interview, she used mainly an active voice and gave clues about her epistemological stance of teaching and learning of mathematics. There are also a lot of statements about discursive practices of teaching and learning mathematics within her classes. These statements are particularly important because they create the system of rules that are inherent in her practice and the context for her stories to have meaning. In brief, her narrative statements provide a basis for her conceptions of mathematics and clues for her practice. These fundamental statements are also important in constructionist sense (Foucault, 1970) in defining and producing the objects of her professional knowledge.

From her narratives (ISI, 1&3) the general rules or social norms that frame her discursive practices were clear and firmly determined:

> Arriving the lesson on time, note taking within a neat and a systematic manner, listening to the teacher carefully, having students to take notes on regular basis.

She used an authoritarian traditional discourse about teaching and learning positioning students as careful listeners of the teacher which was put as a necessary condition to be successful. As in the following passage that she describes her role as a teacher:

> Teaching, in my opinion, should be the responsibility of the teacher in general.
> You give essential definitions, solve sample questions. Is it possible to develop different solution techniques? Yes. In doing these, teacher should be the leader. She must lead students to alternative thinking processes. This is what I understand from saying student centered or that student is active. It is teacher`s responsibility to give the essence of the subject. Teacher usually transmits but students may generate different solutions. Transition between prior and the present knowledge should be

> left to students and the teacher should be the one who gives rise to ideas. (IS1, story2)

Here she used a special vocabulary reminding a traditional teaching and learning perspective. For instance giving essential definitions and the essence of the subject and transmitting [to students]. Solving questions and developing different solution techniques were also presented as the most important responsibilities of the teacher.

Understanding and using mathematical expressions and statements, explaining the [mathematical] procedures and showing what comes from where are featured as the prerequisites to learn mathematics. Notably, in the 2nd story there is a conflict between her role in carrying out these expectations from her students and her previous statements about "leaving the transition" between prior and present knowledge to students. Being able to *solve questions from a textbook 'very well'* is seen as the ultimate point that a student can reach. Another section from the second story well aligns with this approach:

> First of all, in my classes students should know about where mathematics or the subject being taught… should be used. Not because they are required to learn but according to where they will use them. Our classes are comprised of SM (Science-Mathematics) or TM (Turkish language-Mathematics) branch students hence they should acknowledge the priority of mathematics considering the most important exam they will take in their lives…At least what I'm trying to do is to bring them into a position of achieving their ideals considering the priority of mathematics (IS1, S2)

Here she identifies the importance of 'knowing' mathematics while prioritizing university entrance exams among reasons why students should learn mathematics and pointing out these exams as the appropriate time and place to use/extract that knowledge. She also describes her position as a mathematics teacher and the significance of her role in enabling students to *use* mathematics in accomplishing students' ideals. She emphasized knowing *how to use mathematics* or the subjects being thought [e.g. in the exams] and this determines *how it should be learned* by students.

At the third interview about three weeks later than the second, we could not find a private room in the school. Aylin looked nervous and stressed about things going on at administration department. We were at the teachers' room together with 4 or 5 other teachers. Her voice was relatively low almost like whispering compared to last two interviews.

At this interview she declared her exact preference for teaching mathematics rather than geometry and saying that she has not much opportunity for the students to reason in geometry lessons but the case was not like that in mathematics:

> In geometry I don't have much opportunity having them to reason. It is related with being able to see… This is not the case in mathematics. (IS3, S1).

…Frankly speaking, I am not very assertive in geometry. But I think that you have to develop yourself in all aspects and if you have shortfalls you complete them, thus I believe that a mathematics teacher should both give math's and geometry lessons. But I prefer mathematics more… (IS3, S1)

Nevertheless, at the previous interview she commented that students were being more likely to ask [reflective] questions to themselves like 'Can I think about it or solve it differently' and answer in geometry lessons which was not found appropriate for the mathematics lessons (IS 2, S3). This is also in tune with her comments in which she had theorized about why students are shy in geometry:

They are shier in geometry, because they don't have any backgrounds about it: *New curriculum has settled their hash.* In 11th geometry subjects are related with triangle knowledge but they don't know much about it because they overwhelm them with vectors in 10th geometry and they cannot learn thoroughly or properly and the important points are being missed (IS3, S2).

4.1.1.2 Routines of/for mathematizing

We met 5 weeks later to talk mainly about her practice of last month. In the meantime, I made two classroom observations and had all the videos with me and we have watched certain episodes that I have selected from these videos. I asked her to reflect on whenever she wanted to but she did not commented much on the videos without being prompted by me. We conducted the interview at the library where some students were working and a lot were stepping by at lesson breaks. After watching an episode from her mathematics lessons about complex numbers, she commented on that students have made progress towards her goals:

Students came to understand mathematical expressions, explain their solutions or which operation comes from where, make alternative interpretations when solving equations… Students were initially hesitating to come to the board, but now *we* have gradually overcame that. They come to board and they are gradually able to explain their solving techniques. Now they started to say: "I have done the solution I can explain it right away". Then, "I can explain the second way". I think, from now on *we* are slowly able to succeed in doing this… (IS2, S1)

… After finishing the logarithm completely, after giving all of its properties…. They must solve equations of logarithms just like they solve any other equation. That's my objective. (IS2, S1)

At that point, I asked her opinions about proofs and proving in her classes which I thought that she was overlooking in her practice. Her attitude was not positive towards considering proofs within the discursive practice of her classrooms. Furthermore, the way she was using the words 'proof' and 'proving' in her reflection discourse was reminding more of reasoning and argumentation rather than proof and proving in mathematical sense. She theorized her current position to reasoning/ argumentation [proofs and proving in original] as such:

> Although the books are directing us towards these, the total amount of time of each geometry class per week is limited, like 2 or 3 hours and the content are very intense. We put 7 or 8 properties and if we deal with all of these proofs then we don`t have enough time to make practice and solve problems, thus I prefer to pick 1-2 important one to justify in simple terms while noting the others are rooted in them. For instance we use triangles most at solving problems, "We say we are using this because of the congruency in triangles. This property comes from there". This is more practical I believe. We try to give in a fastest and a more practical manner *since we need time for solving questions* and their geometry background are not very strong sometimes I have to repeat one question three times and spent almost an hour for it. On the other hand, if I spend so many times on each question then not any chance remains to solve other important problems that I call *key questions* since I have to move on to a new topic on the next lesson. That's the reason I cannot allow so much time for the proofs in my lessons. But this is *due to the reasons beyond my control* (IS2, S2).

At the 4th story of the second interview, she explained why students at the 11th grade generally has no or little triangle or angle related knowledge. She gave the geometry curriculum in effect as a reason for this that has been implemented for few years at the time we have conducted the interview. She indicated that she was trying to make up those "knowledge deficits" (IS2, S5) by allowing time for revision within her lessons especially at the beginning of the first semester. Another way she told about for covering those deficiencies was by trying to have connected the new subject/ topics with triangles *via and on questions*. But again, timing is an important concern:

> …But making this transition is very difficult for them [students], actually after understanding it is easy but they are very slow, slow at solving questions, they think very slow, very slow. They might be aware that it is congruency but they have to look at it for 10 minutes. We have to accelerate them at that point. (IS2, S5)

Then she suggests a solution to the overall problem of making up those deficiencies:

> They have knowledge deficits, they can't see some certain things, and how can we overcome those? *With practicing, solving lots of questions.* The more we practice the more they are able to see well. What is given; which to apply in which situation? As I told before there was no note taking regularly at first couple of weeks, now they do. And one more thing as I told before, I usually have them to take notes of important points…. I make them to take notes whatever is implemented in each question…. After doing these in EA (Turkish-Mathematics branch) classes, solving problems increases a little bit. (IS2, S5)

She thought that *practicing and solving a lot of problems* will suffice to overcome the problem in point and clarified why the amount of time allowed for mathematical discussions within the classroom is limited:

> It's ok when they are at 12^{th} grade since seniors are more open to *interpretations* but for 11^{th} graders these moments of discussion are counted as spare time like a break! It might be possible in the eleventh grades' second term. Now they say, I don't care about the definitions, I think on the questions (Story 6).

At one point during watching the video recordings she has stopped the playback and commented on a technique she has learned from a mentor at her pre service years at her school practicum. It was about finding extended domain of logarithmic functions. In fact, that technique was based on the sign investigation of the roots of a function comprising of the product or division of the 2^{nd} or higher degree functions. She emphasized that the rule reduces student confusion about what to do for different degree functions and it is also advantageous for saving time.

> I give only one rule, not the products or divisions, there are lots of tables on many books. I don't give any of them. This is the most eligible one…I also get students to take notes of it as warning. What to do for instance when it is double root? etc…I make them to write down one by one. I try to proceed by filling the shortfalls in students' knowledge. But this rule is among one of my best rules (IS3, S2)

She further described how she used exercises/problems in order to have students to make comparisons between the properties of different mathematical/geometrical objects:

> Last lesson I have had them to make notes one by one again. I made the comparison myself but I asked them first. Think a while…For instance in which quadrilateral are the diagonals vertical? They can answer like rhombus and square. First, some said rectangle, then we said no their diagonals were not vertical, they were same as length, then some said parallelogram some said it is wrong, and then we ended up with …I made them to write it down…. And what else? We have properties of square discriminating it from rhombuses so that we called it a square. There were students who answered right but it was two or three, in a class of 30 and more students only a few can answer that question (IS3, S2).

4.1.1.3 Student participation structures

From the 3^{rd} story of the first interview, we have been given clues about Aylin's understands of students' collective work. In her discourse "working cooperatively" means that student helping each other to solve exercises/ questions from the tests after working on these questions individually and then explaining the solution to a peer. She also noted in S3 that student's questioning themselves about *different ways to think about* a problem happened mostly in geometry lessons that were not found as an appropriate discursive practice for the mathematics lessons.

> We solve 15 or 20 questions generally but in EA classes only 5 or 6. In SM classes they have only one or 2 or 3 questions that they could not have done. There are 10 - 12 tests after each unit, we are moving on while scanning each test. Do you have unsolved questions in the first? No. The second? We are moving like that at the SM groups but in EA's, for instance they might say I could not do 3 -10 and 13 in the

first test. We would like to solve each of them but then I say you have to work cooperatively, you have to form study groups. If you solve that you explain to your friend and if you still have questions then we will solve them at the class. (IS2, S3)

The fifth interview we have conducted was about 2 months later than the 4th interview due to the midterm holiday and changes at Aylin's lesson schedule during the first few weeks of the new semester. This was also the last interview while I was doing classroom observations that have been relatively shorter than the other interviews in the study. We could not find any free space so we had to do the interview in the teachers' room. We had only 40 minutes before the break there were 3 or 4 other teachers in the room either. So this interview was in a form of informal conversation. Under these conditions we have watched 2 episodes of her lessons. In both of the videos we have watched individual students were making their presentations.

While watching an episode from her lessons in which a student was doing a presentation for her term project she claimed that she claimed that these *presentations increased student participation*. There were two other students doing their solutions at the board in turn. One of these two students was one of the most successful students of the class and the other one was has made his presentation at the previous lesson:

> …The weaker one is the one who makes this presentation, and students who are weaker choose these projects after all, in order to get an additional mark to support their final grades… For instance this pupil had came and asked me questions many times before he did the presentation… As I say, slowly they are coming to understand the language of math or the mathematical expressions, previously they say they did not look or understand anything from the written statements from the book. After making her presentation she become more active in lessons she participates more… This was the one who made his presentation before that, his activity has increased after presentation, he presented the introduction of the sum symbol and this friend of him did the properties of the sum symbol. Look he went to the whiteboard to solve the problem you see… (IS5, S1)

Next she commented that being able to explain clearly and to be understood is the biggest concern of the students since:

> …They are very bad at verbal expression. Yet they are very ineffective at using mathematical expressions and language and also at writing them… (IS5, S3)

She told that's why she had to listen to the presenter very carefully and should always be ready for backing up the student in case students constructed mathematically wrong statements. She was very pleased with student presentations and noted that in the 11th grade mathematics lessons "*efficiency is increased and this is supported by the presentations"* (S3).

Yet she had also doubts about that method in the sense that it might not be as effective as she thought:

> Sometimes very weak students are doing presentations, for instance the girl who will be doing her presentation today is very bad, bad at mathematics. And I can guess how her performance will be more or less (IS5, S3).

In that brief interview Aylin only raised issues about student presentations and how they contributed student performance. That's how I reflected on the interview:

> *For Aylin, student participation means students' making individual presentations where they are actually responsible for teaching the topic as a whole and then solve some examples. I wanted her to consider rethinking side effects of this method on the students so with this aim I probed with some follow up questions. Yet she is very contended with that emphasizing that this works well especially in the second semester. I think that I could have asked questions more explicitly that might point out the situation. But I did not.* (Memo 5, 15/03 / 2012)

After I have finished my observations, for about 1 and a half months passed from the 5th interview, we have conducted the post study interview. In this interview I generally probed Aylin to reflect on her overall experience of the study, what changed in her practice and what might be her future objectives.

In order to provide the context of the situation for this interview I start with my memos:

> *Considering the unpleasant climate within the school for months she felt extremely under pressure. Her voice was active as it was at her other interviews, using I pronoun. No special vocabulary but lots of external dialogues between herself and her students. Within her first response she claimed that the presentations were very effective and students liked it. I wanted to dig into this.* (Memo# 6, 18/05/2012)

When I told her that whole class discussions were rarely happening under normal circumstances as far as I saw, she strongly objected*:*

> Actually it occurs a lot but there is no discussion when the camera is recording they are afraid of saying or doing something wrong. I interfere because if I don't, it turns out to be a big thing, noise increases and it becomes like you cannot understand anything, they don't listen anything, whatever you say the other continues to speak and turns out to be a quarrel. Not any consensus is made up, only a junk noise. This won't get anywhere or no result. Ok they should talk let's see if they can come up with a result like "Ok friend your idea is right". There is no such thing. Then it ends and we end up with nothing. I have to put things together again… (IS6, S6)

4.1.2 Knowing in practice

Under this theme there are 3 codes listed: *Learning about listening, a changing perspective about mathematical communication, more about student presentations: What was wrong?*

4.1.2.1 Listening to students

One month after the third interview we met for our last and the 4th interview for the first term. I invited her to reflect on the last 4 months which she spent inquiring on her own practice and to evaluate her participation in this study hence we had limited amount for watching episodes from her videos. We conducted the interview in the sports room of the school since we could not find any free room that was very noisy even though there was nobody but us. The prominent theme of the 4th interview has been the teacher's being aware of the aspects of listening to the students as allowing more time for them to talk and patiently listening to them.

> This is a science and mathematics class and they are relatively good I think mostly I reached my objectives for them. But now I realized that in the past I was not listening to what student`s say impatiently completing their answers or comments. Now I am more cautious to let them to finish their words. Yet we have the time constraint that's why I was completing student's sentences, but now I don't… (IS4, S1)

She also indicated how her listening to her students has improved:

> In the past, I was impatient I did not wait for them to finish; now I care about listening. By allowing more time to student`s opinions or ideas, focusing more on what they say or you know, I realized in myself that I am trying to activate students more in my lessons. (IS4, S1)

She stated that this was a fact that she had realized/ learnt after so many years:

> I really say that with all my heart I have discovered this year at my 19th year that how impatient I were in the past I should give more turns to speak and more time to listen to them (IS4, S1).

Yet, she distinguished the *discursive practices of geometry and mathematics classrooms i*n terms of allowing more for student talk.

> In geometry lessons we have to be in a little bit more hurry that's why I could not give so much opportunity to students… In those lessons I am the one who is more active. In mathematics this year I feel more comfortable and asking students' questions in an ease and calm manner waiting for and listening to their responses or ideas until they finish or letting them to make more comments and allowing more time for all these during my lessons. And I think this has been very effective this year. (IS4, S1)

Next she had commented on the *types of questions* can be asked and on how *solving problems all the time* might not be good for student learning when *everything is under her control*:

> …Until that I was asking questions about to what extent they can carry things [forward] they have learnt, from now on both of these types of questions can be

asked: Both aimed at *prior knowledge* and more *interpretive* questions. I take a lot from student's perspective…Students can take such various strategies or ways in order to reach solution that sometimes I could not think. But the important thing is giving chance to them. Solving problems all the time or writing a problem and asking them to solve it, it is no good. *Students might not learn very well when everything is under my control.* We can see that, if I'm not mistaken. (IS4, S4)

4.1.2.2 Mathematical communication

She had commented about her understanding about mathematical communication as below:

> When we decided to work on mathematical communication the first thing that came to my mind that teacher *leads the flow of communication when students are active.* Teacher is a resource they will consult when they feel stuck and frustrated. On the contrary, I tell and show everything then… you are so wear out. Now I feel more comfortable and contended. But you need students with good backgrounds so that they won't have problems with the operations because in that case they cannot progress that much [on their own] (IS4, S1).

Aylin perceived herself as the one who makes connections between the subjects for the students in order to view the subjects abstractly and but not distinct topics (Story 2).She also thought that when mathematizing her sentences must be comprehensible rational and clear since the ambiguity in the words or the mistakes or deficiencies of one who teaches cannot be retreated (Story 3). Due to the insufficiencies of their expressions sometimes students' explanations of what they are doing and what they are actually doing did not matched. She thinks of herself as the one who will bridge that gap. She elaborated on paying extra attention when students talked and let them finish their words but:

> …in case they can speak because in the social classes there is no one speaking and I cannot ask so many questions. You might ask why because they have a very weak or empty background that`s why students cannot generate any comments, make any connections. If not any interpretations have been made, students unfortunately can't manage the transitions from topic to topic or make connections between concepts… (IS4, S4)

She asserted that the *communication between the students* was very limited and this becomes worse when they become seniors since they see themselves as rivals or competitors. Yet her by having students to solve problem from the university exam preparation books called test- books locally, she told that she tries to have the students communicate each other. She also reflected that some topics are more suitable for having students to communicate mathematically and reason than other. She commented on the situation as below:

> For instance in the sum and product symbols we have certain rules in order to do the operations and we will give these rules. …In trigonometry I solve very few questions for instance. Talking about the concepts or the topic can help students to learn better because they are prejudiced about this topic. I try to have the class to

> reason and talk about it supporting with graphics of trigonometric functions than trying to give it very theoretically. Actually I don't go deep with graphics I usually prefer visual and brush over like "this graphic increases or decreases periodically like this". If you teach all of these slowly and piece by piece then they will understand very well. For instance, the 10th grade curriculum is very nice this year. You only teach trigonometry for the whole second semester…. Sometimes I finish lessons with solving two or one question but sometimes there remain unsolved questions, and we solve them on the next lessons. (IS4, S5)

4.1.2.3 More about student presentations: What was wrong?

She had once more stated her satisfaction about the student presentations. After all, students were doing well at the exams and the success rate was 88%. She also claimed that doing presentations were beneficial to the students in the EA classes made a progress in geometry since presentations has been fun for them but very tiring for her. She commented on the situation as below:

> Even though the students do the presentations still I have to repeat once more in order to provide student learning or they ask what they did not understand to me not to his friend. Even if they would ask to the student doing a presentation I had to step in since these students are generally weak they cannot answer properly. So I get exhausted. (IS6, S1)

Yet, she continued to express her satisfaction about that presentation as it has been beneficial for students saying that she knew because she has been utilizing this method for a long time. I have probed her further to rethink about the alternatives and the problems have arisen from student presentations that she stated above. Suddenly an idea hit her:

> Ok, let me think. It occurred to me that… We can set a better student as a mentor or teacher to them and then you can say that study with your fiend for a couple of weeks and I will… Let's say have you to make a presentation. Or like "you prepare a project and your friend might be your coordinator I will evaluate all of you"…I have not done that before but it could be a useful idea, and in order to motivate the better one also, the coordinator student might evaluate the group of students he trained. (IS6, S2)

4.1.3 Inquiry of practice

Under that theme, teacher's own accounts of her inquiry process and her comments about the inquiry process are presented. In doing that, teacher's initial stage and emerging objectives and her evaluation of previous objectives during teacher inquiry as well as her future thoughts as her reflections on the whole process are included.

4.1.3.1 Objectives set at the beginning of the inquiry

Analysis of the pre-study interview reveals two main objectives as related with Aylin's account of students' mathematical practice. These are: *understanding mathematical* *definitions and mathematical language.* Both of them mainly refer to solving problems/exercises/questions from a textbook, most likely a university entrance exam preparation textbook of which they are comprised of the tests with multiple choice items. These textbooks are locally called ‘test-banks/test books’ since they generally comprise of various multiple choice items located in different trial tests categorized under specific strands of knowledge associated with the mathematics curriculum. Generally the main concern in these questions is trying to solve them as fast as you can in order to get a higher score in the exam. She explained further that student’s understanding of mathematical definitions [of concepts] would lead them to easily understand problems from the book to recognize the differences between them by asking themselves reflective questions [like ‘what is this meant to be’].

Yet, she had formed an objective that focuses on *students’ use of mathematical expressions verbally and in written forms:*

> … We were dealing with these since last week and will deal for one week more but then I am not going to write on the board I will tell them verbally and they will write. At first some students resisted. But as they use the *mathematical expressions* both in written and verbal forms they become able to explain what they did on the on the board… (IS1, S3)
>
> I also care about mathematical expressions regarding students being able to use their oral language, in terms of self-expression…. Then I say you should do your best but you should be able to present that best…. The important thing is for students to express themselves correctly, experience the priority of both knowing mathematics and showing this to his environment. That he should both know and be able to use mathematics. (IS1, S5)

Towards the end of the interview, Aylin delimited her foci of inquiry and her initial objectives have emerged as below:

- Getting students to understand mathematical definitions and the mathematical language
- Getting students to use the mathematical language [mathematical expressions specifically] correctly and effectively,
- Having students to explain mathematical procedures they did,
- Increasing student participation

These objectives were set for the next month and she told she has already been focusing on them since two weeks. However, she has not stated any learning objectives for herself yet. She commented:

> I don’t have any expectations for myself I am at where I can reach. Can I make more students to love mathematics; can I give them more of mathematical knowledge? In that sense there must be something for my share, that`s why we are working together… (IS1, S6)

> Presently, every new system or new regulation makes the life more difficult for us [teachers]. You release your ideals after some point…. Things turn into a great mess in every 4 or 5 years. **(**IS1, S4**)**

At this stage, she looked like that she does not perceive herself as a learner, yet she seemed eager for the inquiry process. On the other hand, she was resistant towards policies of education which were being set by the Ministry of Education and changing regulations and curriculums in a very short span of time. Below are my own reflections on the interview from the memos I took on my field notes:

> *The conversation ends in a pleasant manner. She also looks pleasant and excited about talking her perspective of teaching and learning mathematics. Getting to be listened by someone… I feel that I could have expressed the purpose of this study to her beforehand so I do also feel hopeful. She was the teller and I was the listener. She mostly used "I" pronoun when she was talking about the system of rules inherent in the discursive practices of her classes, but switches to "we" when talking about the discursive practice*
> *On the other hand, I wonder whether her confidence about her practice might hinder her learning process. She seems to isolate her practice from her school, which gives an impression of a messy and unsteady workplace. Schedules are constantly changing and there is a circulation of teachers even though schools have been opened for over 3 weeks.*
> *Her critical position about the curriculums and structure of the Ministry of Education might also support her isolated position. I had the feeling that there are disagreements between her and the colleagues about some main issues so there is a limited communication between them. It is very early to decide.* (Memo #1, 26/09/2011)

4.1.3.2 Emerging objectives

By the end of the second interview, she revised her objectives for the next month as:

- Having students to *solve the problems* by working out their knowledge
- Facilitating students to make transitions between trapezoids or parallelogram and making comparisons between different quadrilaterals to *solve the problems*, to *generate different solution techniques* and to *make connections between one another.*

I reflected on the situation and generally on the interview as below:

> *There is something made me think about the interview. This should have been a process of inquiry her own practice and I am not in a position of a mentor, or an evaluator of her performance. Yet I think that I have difficulties in deciding whether to step in when she thinks that everything is ok and things are going as they are planned in her practice. But I think she is overmuch confident about what she does for actually reflecting on and think about her practice. While watching the*

videos there was nothing she told she was discontented with. This makes me rethink …about my research purposes and her goals for her inquiry of practice. We will see (Memo #2, 01/11/2011)

4.1.3.3 Evaluation of previous objectives

At the third interview she has evaluated her objectives that she had set at previous months for her inquiry of practice. She did neither revise her objectives nor set new ones. For the mathematics lessons her use of the word '*interpretation'* meant that students being "aware of the differences of the logarithm functions and the other functions" (IS3, S3).Then she told she had solved lots of graphical problems including university preparation exam questions since 'students have problems with graphs'. For the geometry lessons she reminded her aim was:

> …Getting them [students] to use prior topics rather than triangles. For instance, when solving a problem with rhombuses students can say now "I have used the property of parallelogram or rectangle (IS3, S3).

She added that she tried to make connections between these topics when teaching geometry and that she had always tried to teach like that yet she did not consider the geometry curriculum at all and that she had seen that the students became successful because of this strategy. She gave clues about the influences of her own learning experiences on her teaching.

> I try to give my students in a way that I learnt when I was at their ages… For instance in trigonometry, my teacher was using the same right corner of the board to explain the signs of the trigonometric functions and used a mnemonic code for it. I think it is very useful… *That was how we memorized the formula*. If it is like a roll even if it is short it is easier. *I have learnt like that maybe it helps*…. where does those formulas come from, ok it is nice, but one hour of two will let go for drawing out one of them (IS3, S3).

She also claimed that there is a progress in *students' use of mathematical language*. According to her statements students have gradually developed their word use and also their routines of substantiation mathematical narratives (Sfard, 2008):

> *But they started to use the mathematical language better,* they might say "since we have stated that it is an angle bisector since we have a property that comes from the parallelogram if it is an angle bisector than the other angle must be absolutely 90 degrees." When I asked why are the diagonals vertical, they started gradually saying that. (IS3, S2)

As a final word, below are my impressions about this interview from my field notes:

> *Once again I feel that she does not reflect on her practice at all. While watching the videos she focuses on a situation and from that point on she either describes her actions in the video or talks about her practice in general or gives some*

details about how and why she did it. Yet, it is difficult to keep her on a particular topic. But she looks contended with what she is doing. I did not make any comments but I expect her to be reflective at the end (Memo3, 3 /11 /2011).

4.1.3.4 Reflection on practice via own objectives

Further at the 4th and the last interview of the first term of the school, she specified what she thought she has accomplished in terms of her objectives related with mathematical communication:

> …they can now use the mathematical expressions, write them express them understand what it is written in a math book and can make interpretations. They can also connect prior topics to the present one. This was my objective… (IS4, S1)

Finally, the objective she set for the second semester was *having students to present their term projects* chosen from the topics of the second semester so that they will make individual presentations. She mounted an argument that this was a very favorable method for the students since they don't ever *miss a question* from the subject they teach ever. While preparing for the presentations they could come and ask about anything they did not understand since their classmates would be asking questions to them and they could also ask questions whenever they want to them. She described this as '*precisely like I teach*' (Story 4).

This was the first time she had been explicitly reflective on her practice after about 4 months of inquiry of her own practice. Below, there is a passage from my reflections on the interview:

> *I felt hope again with this interview. I thought she was not reflective at all during the past 4 months but it turns out that I'm wrong. She has talked about what she had learned during this time explicitly and how it had positive effects on her practice and students.* (Memo#4, 10/01/2011)

4.1.3.5Future thoughts and reflections

There was not a trustful relationship between her and her colleagues. She stated that she would repeat this kind of inquiry with a teacher she could trust since she told that she was open to anything that might develop her professionally. Yet, the candidate would be just one teacher out of 16 teachers in her school:

> ...there is only one option, one person that I can do that with but not with others… and this study can be done better with a mate from the same school because there might be some disadvantages like I might have a lesson on the time that she has lessons, I should be free in order to observe her… If we are to observe each other about one hour each month….For instance when I cannot figure out something I ask Melis and we try to find the solution together. I don't trust the others so much…. Nobody trusts each other anyway. (IS6, S6)

No matter what, she thinks that she has learned to listen to her students more:

> I learned that I have to take the position of listening students more. I really care more about students talking with more patience and much more listening, allowing them more time to talk. I wish I had more time to that more because we have to catch up with the plan. (IS6, S5)

She indicated she would have selected necessarily similar objectives each and every year due to the school policy that never let her to teach one class for two successive years since she thinks that all students are very poor at using mathematical language. She added that she would have gone one step forward, focusing on students' reasoning and proofs if she could have the chance teaching to one of her classes again next year but it was not possible due o school policy and 'unfortunately it is impossible to do these when students don't even know how to write mathematically" (IS6, S6).

4.2 Teacher's Mathematical Discourse

Under this section results of the second phase for the investigation of teacher's learning throughout an inquiry of own practice are presented. As noted above in the data analysis section; the analysis of that phase is the analysis of teacher's discourse of action conducted via three levels. First level as the teacher's ways of mathematizing or *the teacher's mathematical discourse* is seen as the construction of ideational meanings adopted from Halliday's (1978) theory of SFL. According to Sfard's categorization of properties to identify the mathematical discourse, the transcripts of the observation data are analyzed focusing on teacher's discursive actions during mathematizing.

4.2.1 Mathematical word use and the general characteristics of the teacher's mathematical discourse

According to Sfard (2008) discourses are made distinct by their tools, which are words and visual mediators. These are all important because they refer to participant's understandings of the meanings and the objects of the discourse acted upon to produce or endorse narratives of the discourse. By looking the words and symbols that are used in practice and analyzing the way teacher realizes them will give us clues about understanding her ways about knowing and understanding of mathematics.

According to the list that I have constructed for the mathematical words and symbols used by the teacher it is obvious that Aylin mainly prefers symbolic realization rather than the iconic or concrete modalities of realization within her mathematical discourse. Since symbolic realizations are basically verbal, this reflects her general attitude towards *learning/doing mathematics* in line with her emphasis on students being able to use the mathematical language correctly and effectively derived from the results of her discourse of reflection presented at the previous section.

Table 3: Teacher mathematical word use

Obs #	The word use	Modality/ies of realization	By	Turns
1	Properties of quadrilaterals, Triangle inequality	Symbolically	Teacher +Student	1-26, 36-39
1	Single value/possible values, Arbitrary	Symbolically	Teacher	29, 22
1	Middle x	Symbolically	Student	38
1	Refer to somewhere	Symbolically *	Teacher	22
2	$a+(b-1)i=5+b-ai$	Symbolically	Teacher	12
3	Extended range & three conditions	Symbolically	Teacher	5
3	Exponential function, Logarithm function	Symbolically and concretely	Teacher	9
3	"The base should be bigger than zero"	Symbolically	Teacher +Students	21
3	*x* is smaller than 5	Symbolically, iconically	Teacher +Students	22
4	Rhombus	Symbolically, iconically	Teacher +Students	1,7.8.9,10,51
4	Linear	Iconically	Teacher	8,9,47
5	$216^{\log_6 5}=6^{6\log_6 5}$	Symbolically	Teacher	4-14
6	Diagonal: Angle bisector	Symbolically,iconically	Teacher	12-26
8-9	'Polygon: conditions should a shape satisfy to be called as a polygon"	Symbolically	Teacher	1-17
10	*n*≥4 : the correctness of a proposition is demonstrable for *n*=4	Symbolically	Teacher	1-4 21-25
12	What is a_6 of the sequence whose general term is $a_n=\sum_{k=1}^{n}(\frac{1}{2})^{k-1}$	Symbolically	Teacher +students	Through out the lesson

At Table 4, there is an excerpt from the first observation giving further insight about the teacher's mathematical discourse.

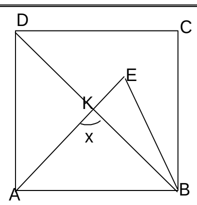
Table 4: Classroom observation excerpt 1

Turns	O1 (G) 20th October 2011 Thursday
8	T: We have called a polygon that has four sides and vertices, quadrilateral. Anything else?
9	S3: The sum of the interior angles…
10	T**:** Else? (…)
11	S4: If there are opposite angle bisectors, the difference of their angles is…
12	T: Half of the sum.. In case that our **angle bisectors** are adjacent… If our vertex angles are angle bisectors…
13	S4: What I say is this teacher. If angles are at opposite…
14	T: Yes, then it is the half of the difference**.** If our **vertex angles** that are **angle bisectors** are also **adjacent** then we told that it is the half of the sum of two angles**.** What else?
15	S5: If the **diagonals** are perpendicular then the sums of the squares of the opposite sides were equal to the sum of the squares of the other sides.
16	T: If there is a **perpendicularity** for the diagonals then this is one of the properties that we use to find the length of sides

Here the teacher is trying to have students to recall a narrative related with the quadrilaterals. In his first attempt S4 tried to explain a property related to angle bisectors of the quadrilaterals yet his utterance lacked some other important information such as the vertex angles and the teacher immediately completes S4's utterance. The S4's statement was not matching with teacher's utterance. Then S4 once more attempts to clarify what he were to say shortly but the teacher did not give him enough time and completed his words once again.

Table 5: Classroom observation excerpt 2

<table>
<tr><th>Turns</th><th>O6 (G) 6th of December 2011 Tuesday</th></tr>
<tr><td>12</td><td>We continue with doing our key questions. A new question: ABCD is a square and AKB is an equilateral triangle, what is the measure of x? Everyone should deal with it, everyone should be able to solve that. Come on S1, I expect a little bit activity from you. (Image reproduced)



</td></tr>
<tr><td>13</td><td>S2: Is it 75?</td></tr>
<tr><td>14</td><td>T: May we not tell the result immediately, what are we going to do?</td></tr>
<tr><td>15</td><td>S3: Is it 75?</td></tr>
<tr><td>16</td><td>T: Yes. What are we going to do?</td></tr>
<tr><td>17</td><td>S4: Is it 105, teacher?</td></tr>
<tr><td>18</td><td>T: The other side is 105, the alpha is 75.</td></tr>
<tr><td>19</td><td>S5:[Does the operations on the board] From the angle bisector theorem here is 1 over 5.</td></tr>
</table>

Table 5(Continued)

20	T: Is it angle bisector theorem or?
21	S5: Not that was **diagonal.**
22	T: Diagonal was the angle bisector, since DB is angle bisector it divides as 45-45 says your friend.
23	S5: Then since it is equilateral triangle here is 45 degrees and here is, since one angle of a square is 90 degrees, here remains 30 degrees. Since two interior angles gives one exterior angle 45 plus 30 equals to 75.
24	T: [*to S6 by looking to his solution*] Hmm, you can also find from there.
25	S6: I have done by using the other triangle
26	T: You can also do by using the other triangle. And our friend said that if here is 60 then 45 it remains 75 there. You can also do it by alternative ways. Another question.

At this excerpt from the 6th observation the teacher gives more time for the students to explain their mathematical ideas and allow student word use without explicitly correcting them. Instead of explicitly rejecting or endorsing the narrative constructed by S5 using a*ngle bisector theorem (19-20)* the teacher steered her to rethink that statement. S5 then corrected her expression and then the teacher substantiated S5's narrative by using deduction.

Table 6: Classroom observation excerpt 3

Turns	O12 (M)4th April 2012Wednesday
1	T What is the 6th term of the sequence whose general term is $\sum_{k=1}^{n}(\frac{1}{2})^{k-1}$ *t*hat does it mean here the 6th term fellows?
2	Ss: a_6
3	T: Ok, how do you get that?
4	S1: We may subtract those that are till a_5 from those which are a_6 ?
5	S2: It comes directly from the formula.
6	S3: Isn't it to write 6 in the place of *n*?
7	Ss: (…)
8	S1: But there is sum there
9	T: Yes but we will be finding its result.
10	S1: Just a while ago when I wrote 6 it gave me the sum of those that are from 1 to 6. If I wrote…
11	T: I don't want the sum of the terms from 1 to 6. I want the 6th term.
12	S1: I got that but I will write those which from 1 to 6 and subtract those which are till 5 so that only 6th term remains. Otherwise it doesn't.
13	S4: Ok teacher when we do from 1 to 6 we will find the sum of the terms up to 6.
14	S5: No, it says sum, then it would not have said sum.
15	S6: Can't it be like that?

Table 6 (Continued)

16	T: Fellows who thinks like that please don't confuse it with the sum of the terms. a_6 has been given as the sum symbol. We will put directly 6 to *n*'s place.
17	S1: I don't understand that teacher, should not it be like from 1 to 6 since there is a sum symbol
18	T: But what you wrote here are not the terms of the sequence it is the sum of the k's. Ok, let's do and see what it is like. I mean a_5 is from 1 to 5, a_6 is from 1 to 6. If my lower limit was a_6 then what you said would be correct but the thing that is stated here is different.
19	S7: [*Does the operations on the board and explains what she does simultaneously*] I give values on k from 1 to 6.
20	S8: and this is 63 over…
21	S7: Ok now, let me finish!
22	T: Do we have an induction formula here or am I wrong?
23	S7: Yes we have…uhmmm… *1 minus r to the power n over 1 minus r. I write ½ in the place of n and* [*completes the rest of the operations by writing and speaking about what she has done simultaneously*]
24	T: We can find the solution directly from here. Is it clear? Now, let's make that part clear. *It as* $\sum_{k=1}^{n} a_k$ *then it would be the sum from k is equal to 1 from n.*
25	S9: Does it ask only the a_6 or the sum?
26	S4: Would not it be ok if we find 6 and then subtract 5?
27	T: Your friend has said the same thing, but it asks only the 6th term not the sum. Let us articulate that when we give the sum of the first n terms at the arithmetic and the geometric sequences.

At that excerpt from the last observation contrasting with their teacher, students are not acknowledging the general term of the sequence a_n as the sum of the terms from 1 to n. This conflict stems from different realization procedures of the teacher and the students. Students directly identify what they perceive from the mathematical symbol ∑ as sum and associate this realization improperly with the a_6 as a sum of the terms of the sequence from 1 to 6, hence they think of applying the formula once for 5 and once for 6 in order to find the 6th term. On the other hand the teacher expects them to notice the inherent structure of the general term formula in a way that each term is constructed for the different values of k=1,2 3 and so on.

4.2.2 Teacher's mathematical routines and endorsed narratives

Discourses are also characterized by the forms and outcomes of their processes namely it's routines and the endorsed narratives. In order to complete the investigation for the research question about teacher's ways of realizing mathematics during inquiry of her own practice, the routines and the narratives endorsed within the teacher's mathematical discourse are analyzed in order to focus on teacher's acts of mathematizing, both discursive and practical.

The routines identified are underlined and the narratives endorsed are given by *italics* throughout the observation transcripts.

At that excerpt from the 1st lesson observation of the teacher's inquiry of her own practice, teacher is mainly emphasizing the doing process in her discursive practice which is "*how"* of a *mathematical routine*. The *how* of a routine is the set of meta level rules that determines or constrains the actions of the students and the teacher. She reminds her students of rituals that they hold to within the discursive practice: to use triangles while solving questions in general (Turn 30) and to use *Pythagoras theorem* in an acute triangle (Turn 46). The acute triangle is realized by the word *magnificent trio* which is in turn realized by direct identification of the equalities in the figure and presented as a ritualized meta rule for geometry lessons: '*We call it by seeing'* (Turn 49). Furthermore, *the magnificent trio* was a *routine* as previously

Table 7: Classroom observation excerpt 4

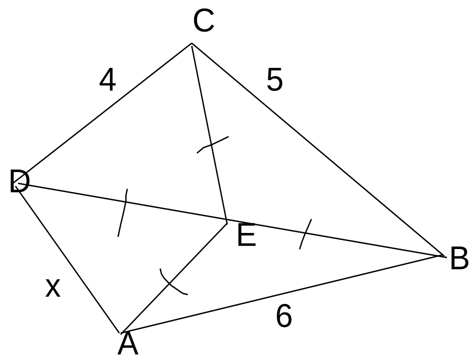
Turns	O1 (G) 20th October 2011 Thursday
28	 According to the figure what are the possible values of x? (Image reproduced)
29	29 T: Okay._We have always found a single value until now, at the problems we have solved. It says: 'possible values of *x*'. Under which condition this can be like that?
	Silence…
30	T: Don't we solve questions by using triangles in general**?** Does this ring a bell?
31	Ss: There was...an inequality.
32	T: Yes. The *triangle inequality. Remember…*
33	S4: The difference of two sides, the side that remains in between…
34	S7: It should be bigger than the sum and smaller than the difference.
35	T: One by one... Speak by asking your turn. You say.
36	S4: **If** we know two sides **then** we can find the middle *x* by taking both the sum and difference of them.
37	T: How could we? How could that middle *x* be?
38	S4: It is bigger than the sum of the two but smaller than the difference.
39	T: *We were calling that the triangle inequality.* 3 numbers that I make up would not always construct a triangle. Then *we could only find values more than one.* Then *we need to use that.*
40	Ss: Let it be 2 to 4 and hence 6 to 3.
41	T: How may it be?
42	Ss: May we equally dissect?
43	T: Ok. Everyone listens to me. Why are these equalities given?

Table 7 (Continued)

44	S1: The magnificent trio!
45	T: Yes. Where?
46	S1: At the angle C
47	T: Angle C? But why don't we say, the angle A also?
48	Ss: x is equal to 4!!!
49	T: Angle A is also…isn't it?. Now we should remember our triangle knowledge**.** *In this triangle CE is the median and is equal to the length of sides that it sets apart***.** *Since we call magnificent trio, this vertex angle is 90 degrees and A is also 90 respectively*. We call it by seeing. What were we using in acute triangle?
50	Ss: Pythagoras
51	T: Then use it**,** how were we using it?

endorsed and substantiated by the teacher. With the help of the previously endorsed triangle inequality she has realized the possible values of x (Turns 29-39). The triangle inequality also becomes a new ritual to use in the similar situations when the possible values of x are prompted. Hence, the teacher's mathematization is manifested as a *ritual* rather than an *exploration* routine and there is not actually any production of new mathematical narratives.

Table 8: Classroom observation excerpt 5

Turns	O2 (M) 20th October 2011 Thursday
1	T: What was the conjugate of *z*?
2	S1: By changing the sign of the imaginary part.
3	T: *Yes it was the symmetric of it* $\lvert z \rvert$ *to y axis*; *modulus of all is, under square root a square plus b square*. *By noting that all are equal to each other*, lets write a question. Remember we have solved questions including $a+bi$ now let it be a question including modulus *z*. $z - i = 5 - \lvert z \rvert i$ Find the complex number *z*? [first reads and then writes it on the board]
4	T: What are we going to do?
5	S2: We write $a+bi$ in substitute for *z*.
6	T: What is the difference of this question than other questions?
7	Ss: There is modulus *z*.
8	Ss: There is only absolute of *z*.
9	T: a+bi as a substitute for *z* and for modulus *z*?
10	S3: The square root of *a* squared plus *b* squared.
11	T: [*repeats S3's statement*] And what we have there? Is it a *root procedure*? Do we have problem with root procedures?
12	[*One student comes to the board hesitantly and writes* $a = bi$ *as a substitute for z. After reorganizing the equation finally writes*] $a + (b-1)i = 5 + \sqrt{a^2 + b^2}\, i$
13	T: Now here there can be seen the equality of two complex numbers.

At that episode above the teacher's ways of mathematizing are as focusing on the practical actions resulting with a physical change in the mathematical situations. Though her first discursive/mathematical action is starting with a question about what a mathematical object is [conjugate of z], the use of the *object/ word* has been customarily associated with certain situations within exercises/problems. These situations are in turn associated with the object level rules. Indeed, student S1 responds that opening question in a *routine-driven* way, expressing a routine that is applicable to certain type of exercises/problems. On the other hand, teacher's response as a re-action to S1 emphasizes the *object level rules* about the object which she has been asking at her first question (O2, 3).Then she also reminds her students about the *ritual of solving situations that contain possible objects* based on object level rules in form of questions (O2, 3). The teacher's mathematizing is a *deed* of which is aimed at performing physical action on i to solve the problem situation. Students' response to the opening question of the teacher is an example for this. By changing the sign of an imaginary part the student would only have been perform an act. The goal of this action is neither producing a new endorsed narrative nor to substantiate any. The next statement of the teacher also reminds students of a practical action of solving the given situation either: *Writing* $a+bi$ *as a substitute for z.* At the end of the procedure, the teacher herself substantiates a narrative about the complex numbers, yet for the students who are not involved in an actual *exploration* process themselves, the situation remains as a discursive or a practical deed.

Table 9: Classroom observation excerpt 6

Turns	O2 (M) 20th October 2011 Thursday
23	T::*For n is an element of N,* $\left|z\right|^n = \left|z^n\right|$. This is the property that helps us most. What does it mean here?
24	S4: The power of complex number *z* within and outside of the modulus is equal.
25	T: Now, folks, I'd like to ask you that way:_How were we taking any power of a complex number? How could we find any power with what we know so far?
26	S5: We divide it by 4
27	T: No that was for the power of *i*, the imaginary number. Now we have both real and imaginary parts, how can we find any power?
28	S5: We were multiplying the number side by side as many times as the power.
29	T: What if it is the 25th power?
30	S6: If we say 25th power of 2 we multiply 2 by itself 25 times,
31	T: Pay attention to what I say, there is also an imaginary part.
32	S3: Respectively
33	T: How come? How will you separate?
34	S3: For example if is $2+i$ then 2 to the power 25 plus to the power 25.
35	T: You cannot do that, how could we do it?
36	S8: For instance for $2+i$ to the power 25, we can write it like $2+i$ multiplied by $2+i$ to the power 24. Then we can find it.
37	T: Then I am asking how do you find $2+i$ to the power 24?
38	S8: Then we separate $2+i$ again and take $2+i$ square

Table 9 (Continued)

39	T:*[Raising her voice]* While solving power questions for complex numbers with what we know so far we have solved by using the powers of $1+i$ and $1-i$ for high powers. Now if the number we are given cannot be written in $1-i$ or $1+i$ form then how can we find that high power? There, that property helps us a lot. I find the modulus of that number regardless of whatever power it is, *The thing that I find in the modulus is a number* right? There is no imaginary part only the real part. It is easier to find that number's power right? That's why finding modulus of an arbitrary complex number or finding the power then taking the modulus, which one is easier?
40	Ss: Finding the modulus then taking the power
41	T: Yes it is much easier to find the modulus than taking the power of that number. This will help us a lot at the questions…

At this episode from the same lesson observation (O2), the teacher presents a mathematical object level rule about complex numbers for the first time. She prefers to present it directly and expects her students to make sense about the inherent regularities within. When she asked the *meaning* of the rule/property students simply read what is written in a symbolic form since the rule is identified as a new object itself directly with no references made to the *complex numbers.* Then in response to this, teacher poses questions to recall object level rules or narratives about taking the power of a complex number and evaluates student responses. As a final step the teacher verbally realizes how to use the rule, namely the procedures for applying the rule and justifies *the need for using the rule* in her last turn as: "*This (rule) will help us a lot in the questions".* The teacher's pattern of mathematizing in this episode is in the form of Initiate- response-feedback pattern where her actions are in a ritualized form since ultimate goal here is not production of an endorsed narrative. Instead, the sequence of actions is strictly defined by the teacher so that the students can perform these in an identical way [e.g *while solving power questions* (Turn 39)].

Table 10: Classroom observation excerpt 7

Turns	O5(M) 8th December 2011 Thursday
1	T: We were giving the properties of the logarithm function, we have given the base change property lastly, now let's give 1or 2 property more, fellows. The 8th property, fellows, our 8th. [*writing on the board*]. We have already given the others in the meantime and these 10 property will be enough for you.
2	8) $a^{\log_a b} = b$ 9) $\log_a b.\log_b c.\log_c k = \log_a k$ 10) $a^{\log_b c} = c^{\log_b a}$
3	S1: Is there no other property?
4	T: No, we are not going to do any, ok now, <u>how are we going to use these properties in questions,</u> let's see that. Let's do from our tests [*searching exercises from the book*]. We have solved some of these and not solved by saying that we have to give other properties, now let's start. The question is**,** What is $216^{\log_6 5} = ?$ <u>Now we try getting rid of logarithm in all questions don't we?</u>

Table 10 (Continued)

5	S2: 5
6	T: [*pretending not to hear]* How do we get rid of logarithm?
7	*S3:*By using 8. [*Referring to 8th property*]
8	T: By using which property? Which will help us? Which? We will be coming to those questions which we will be using the 10th property as well but here using only 8th serves our purposes and suffices . *Is 216 is a power of 6?*
9	Ss: Yes
10	T: *Which? Third,* <u>*then what happens if we write 216 as the cube of 6.*</u> Does the question come to that form? $6^{3\log_6 5}$ and what now? *What did we say in the properties that we give in last week's?* The expression above logarithm comes to the head as a multiplier.
11	S3: 3^5
12	S4: 5^3
13	T: Which one?
14	S5: 5^3 *[one student comes to the board]*
15	T: Ok is there anyone who has questions? Ok, another question: what does $4^{\log_2 5} + 9^{\log_3 2}$ equals to?
16	S6: 10
17	T: We shall not say the solution at once; give some chance to your friends.
18	S7: Is it 29?
19	S4: 33
20	S8: 33 yes 33.
21	T:Ok let's see when we solve it. (to the student came at board) can you explain it?
22	S9: First of all, I have separated 4 and 9 into their factors.
23	T: But why?
24	S9: In order to get rid of logarithm.
25	T: What is the base of the logarithm?
26	S9: Then I write 2 logarithm 5 squared to base 2, and here they cancel out each other.
27	T: Do they cancel out each other actually? *Due to the 5th property it transformed into 5 squared and 2 squared on the other side, right?* Is it clear? Now let's make our question some more difficult.

At this episode we see again the teacher presenting the object level rules/mathematical narratives about the logarithm function for students to perform certain discursive actions (Turn 3). Since she does not present the object level rules as a result of production or substantiation of a narrative, students identify those symbolically mediated rules as extradiscursive objects instead of mathematical narratives and use them as such (Turn 7). The teacher also explicitly reveals her practical *deed* as *to get rid of logarithm in all questions.* She displays a routine driven word use which is applicable to certain and limited situations (Turn10). What is noteworthy here S9's explanations while explaining his solution in relation to the teacher's repetitive discursive actions. He explains the reasoning behind his actions by using the same *deed* as the teacher.

Table 11: Classroom observation excerpt 8

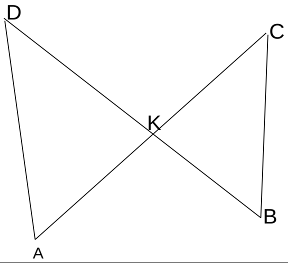
Turns	O8-9 (G) 20th January 2012 Friday
1	T: How *many sides at least should there be for a shape to be a polygon?*
2	S1: 3
3	T: *Then triangle is also a polygon*. Okay, but is there any certain property of polygons? *Which conditions should a shape satisfy to be called a polygon?* What do you say S2? Can you be more explanatory?
4	S2: For instance, a circle is not a polygon. It should consist of lines
5	S3: It should have vertices and sides.
6	T: *Okay but is every shape which has vertices and sides a polygon?* Your friend has told in detail? What did he say?
7	S4: Two consecutive points should not be linear.
8	T: [*Repeats the sentence.]* What does it mean?
9	*S4:* Means that there won't be a single line.
10	S5: There won't be three points on a single line. Not on the same line.
11	 T: [*Draws.*]As it is in that shape. A, C, D vertices are linear. On one line. Can we call it a polygon? (Image reproduced)
12	Ss: No
13	T: We have discussed it before; they are in your notebooks polygons were called in two ways?
14	Ss: Convex and concave.
15	T: Yes. [Repeats] When *does it become convex and concave?*
16	*S3:* In order to be convex you take two points then you join them.
17	T: Yes, *what was happening when you join two points from the interior of the polygon?.No matter where you take them from, stays again at the interior? Or*, we can put it like that: in order to determine it easier…*Shapes with sides that are closed inwards. But there won't be any flexure inwards. We call it closed convex polygon. Right but if the two points we took stays inside then we called it the boomerang triangle last year. A certain part of the line that it constructed falls outside. Then the polygon in that situation is called the convex polygon.* Look the vertex is flexed inwards already. And *we also told that the lengths of the sides are not equal to each other in that polygon since it is not proper*. What else? *The measure of the each interior is not equal to each other, so at the angle related knowledge that we gave since each angles can't be equal if the polygon is not a proper polygon*, since we cannot find them we can only mention the sum of the interior angles…what did we say?
18	S4: N minus 2 times 180.
19	*T: Yes* the sum of the interior angles in the polygon is *(n-2.)180* [*writes on the board*]. *The sum of the exterior angles is always 360 degrees* and we have solved one or two problems related to that I guess and what *about the number of diagonals?*
20	S6: N times n minus 3 over 2.
21	T: Yes. It was $\frac{n.(n-3)}{2}$ We had also talked about *the number of diagonals that stems from one vertex.*

Table 11 (Continued)

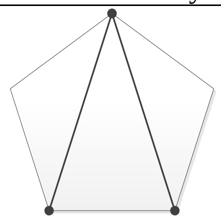
22	S6: N minus 3.
23	T: Then we can head on to proper polygon now. *A polygon whose length of its sides and the angle measures are equal is called a proper polygon n* .Ok now let's think…which of the polygons we have seen are proper or of the triangles…are proper?
24	S7: Isosceles triangle
25	S8: Square in quadrilaterals
26	T: Can't it be a rhombus?
27	S3: Can't be.
28	Ss: No. (…)
29	Ss: No their angles are not equal
30	T: *Diagonals were angle bisectors but they might not be equal to each other and they are not. In quadrilaterals we call the proper one a square.* [draws a sample isosceles triangle and square on the board] *My polygon is proper that means, equal interior angles so I can find each angle of the polygon which is n sided.*
31	Ss: $(n-2).180$ divided by n.
32	T: We already knew the sum yes, *so if there are n sides then divide it by n.One exterior angle is also* $\frac{360}{n}$. Let's do an example related to this one. *From now on let's call it proper pentagon.* We have moved towards the special, making subgroup investigations, how many sides do my polygon has?
33	Ss: 5
34	T: *Then I can find one interior and exterior angle of this polygon easily. We will put 5 in one interior angle formula right?* Let's draw the shape of it. [*drawing a pentagon*]What will be one interior angle when substituting 5 for n?
35	S6: 108
36	T: Yes, [*making the calculation on the board by using the formula*], *according to this what will be the measure of one exterior angle? If we complete to 180?* 72. <u>Then when it is called a proper pentagon we can place one interior angle as 108 and the exterior angle as 72 degrees.</u> What if we want to find the area of that pentagon? Yes, what do you say?
37	S9: From the 30-30-120?
38	S3: We can dissect it into triangles.
39	S5: Teacher shall we break them up into isosceles triangles?
40	S10: One isosceles here and one is there.
41	T: Your friend says that I break it up into triangles. [*Draws*]
42	 (Image reproduced)
43	T: Now, look *there is an isosceles.*
44	S2: They are all equal
45	S4: Right! After finding the area of one isosceles triangle.
46	T: Is there any isosceles constructed? *All of the interior angles were 108, and one is here*.
47	S7:It is 36 degrees.

Table 11 (Continued)

48	T: And there? Again another isosceles right? *Then the sum of the areas of all gives us the area of proper pentagon.* <u>We solve the area questions/problems like that.</u> Ok, we have some footnotes about the proper pentagon, let's give them at once.

In the above episode from the 8th and 9th lesson observation we can see that teacher's realizations of mathematizing have undergone some changes. The teacher begins to allow more time for substantiating and recalling previously endorsed mathematical narratives about polygons, relations between polygons and the activities related to or by using the polygons. First she starts with recalling a narrative about a polygon by asking a question then she deductively substantiate the narrative coming next and by building on these two narratives she leads an exploration routine resulting with students' construction of the *definition of a polygon.* Then she again recalls a previously endorsed narrative and a student responds with practical deed in return. After that the teacher asks questions and responds by herself of other previously endorsed narratives. Students only participate when there is a need to restate the formulas symbolically of which realized verbally by the teacher. Since then the teacher exemplifies and defines the new signifier/word *the proper polygons* and guides students' exploration process about finding the area of a polygon by working on a pentagon. She rephrases what results from this process as a new narrative about [finding] the area of a polygon [pentagon]. The amount of time the teacher allows that far was nearly half an hour, 75 % of one lesson hour.

Table 12: Classroom observation excerpt 9

Turns	O11(M) 12th March 2012 Monday
1	T:We have mentioned some properties of sequences and solved various examples. We have told that we will do this lesson with our friend, our topic is arithmetical sequences. Let's write our topic and our friend will have us to take notes of the definition, you will be solving the examples together and will be asking what you did not understand to her.
2	ST: Sequences which has a constant difference between its consecutive terms is called arithmetical sequences, take the formulas too fellows [*writing the general term for and some other formulas related with the arithmetic sequences to the board*]
3	T: Can you write a bit bigger? [*after about 2 minutes later*]. Write that on the left hand side. *It is the general term of the arithmetical sequence.*
4	ST: [*writes*] Let me may give an example right away.
5	T: Ok, ST, does everyone know a_1 among the symbols that we used here? What was it?
6	Ss: The first term
7	T: What was d? We did not say. In some textbooks it might be used r instead of d. We will say it the common difference in the arithmetic sequences but the common factor in the geometric sequences. And n-1 denotes that it is the preceding term of a_n.

Table 12 (Continued)

8	S1: r is used generally in geometric sequences
9	T:In some resources those are both called r. If you use them separately I think it will be better you will not mix them up.
10	ST: The general term of a sequence whose first term is 3 and the common difference is 4 asked**.** We can do it like that. First term is 3 and the common difference is 4, by the formula an is equal to a1 is 3 and $3+(n-1).4$ which means $3+4n-4$ is equal to $4n-1$.
11	T: *Ok do you think is there any difference between the sequences we have seen until that far and the arithmetic sequences?*
12	S2: There is
13	S3: Of course
14	S4: There is. We have said that it *increases constantly*
15	S5: The difference between the consecutive terms is constant.
16	T: What did your friend say when defining? *There is a constant difference between consecutive terms which is d.* I can therefore. ..If I know an arbitrary term and d, for example it gave the 3rd term and d how can I find the 25th term?
17	(…)
18	S4: 25 is equal to $24d+a_1$
19	T: Now since I knew the 3rd term by adding d on it, I can find the 4th term, by adding one more d I can find 5th term and by adding another d 6th term. You can go like that if the difference between the terms is small. Like 25th term or 125th term in such cases we have to use the general term formula
20	ST: [*writes a statement giving the formula for the common difference for the two arbitrary terms of an arithmetic sequence like* a_p *and* a_k *]* $d=\frac{a_p-a_k}{p-k}$. Right after she asks a question: The 18th term of an arithmetic sequence is 45 and the 8th term is 15. Then what is the 5th term? [*Solves by applying the d formula in order to find d and then she uses the general term formula to find the 5th term*].
21	T: Ok, now <u>how do I do that without using that formula?</u> I am not a good fan of too much formula since you mix them up, there will be Sn formula coming soon. You know the 18th and the 8th terms how do you find without using this formula?[*waits for a while*].Can it be by using general terms formula? Say the second way. It is ok with the formula, also practical but let's write 18th term according to general term formula. Is it a_{18} is equal to a_1+d17 . <u>Now let's also apply this to 8th term.</u> [ST applies what she says at the board, writes the formula, $=a_1+d17$]. Ok.
22	S3: She could also associate with a_8?
23	S4: But how could she without knowing d?
24	T: [*To S3 and s4*] There are two unknowns both a and d. [*To the class]* Ok. <u>Now by writing these information one under the bottom and using suppression method we can find a_1 and d.</u>
25	S5: By a_{18} being equal to a_8+10d .
26	T: a_1 plus 17 d
27	S6: By associating a_8
28	S7: Yes it also comes from that teacher!
29	T: Ok fine you show that also. Your friend has got an idea we will share it to you. Tell them.

Table 12 (Continued)

30	S5: a_{18} teacher, we have given two of them you see, a_{18} is equal to $a_8 + 10d$. From there $10d$ comes as 30 and from here it asked a_5 and I will liken that to a_8 so that it would be easy. a_8 is equal to a_5 plus $3d$. Substitute 3 for d.

The episode above is taken from the preceding lesson of the last lesson observation. This episode is different than all other episodes since it is one of the lessons in which students make presentations as student teachers [ST]. The teacher has been doing it since the beginning of the spring semester of the 2011-2012 starting from the second half of the February. The lesson presentations were done in a way that ST is mostly imitating the teacher's organization of the lessons. As expected no exploration process could have been guided by the ST and her ways of mathematizing were in a ritualized form since the focus was on performing of the certain discursive acts associated with object level rules/ narratives that were presented without substantiation and as if they were endorsed by the whole community/ participants of the discursive practice within the classroom. Students were expected to perform the mathematical procedures strictly depending on the additional instructions provided by the teacher, namely her mathematical rituals. Nevertheless, besides providing additional explanations to students and leading them towards a ritualized form of mathematizing, she has also asked more interpretive questions [Turn 16, Turn 21]. These also revealed her routines of comparing [ordinary sequences and arithmetic sequences], and of performing certain operations without using the additional formulas. Furthermore, some students have presented arguments about an alternative method to that of teachers and S5 has clearly substantiated the procedures inherent in their method.

4.3 Teacher's Social Discourse

The second layer of the teacher's discourse of action is *the teacher's social discourse*; associated with second metafunction of the language called interpersonal adopted from Halliday's SFL (1978) theory. The interpersonal metafunction of language represents the speaker's meaning potential as a participator within a social practice. By the theoretical lens of SFL I have first presented the social word use and general characteristics of the teacher's social discourse according to the *speech roles* that the teacher chooses for herself and determines for the other participants and her expression of *personal perceptions and attitudes*. Then I have moved on to the analysis of the teacher's social routines and endorsed narratives.

4.3.1 Social word use and the general characteristics of the teacher's social discourse

The analysis of the social words/ visual mediators and their use as the tools of the teacher's social discourse were done based on the combination of Sfard's (2008) theory of commognition and SFL theory. The teacher's social discourse was determined by her discursive actions that relate to how she expresses the roles and social relations in the context of inquiry of own practice within the discursive practice of mathematical classroom. Hence the teacher' words/ visual mediators that belongs to her social discourse and how she used/ realized them will enhance our understanding of her disposition towards learning, the teacher/student roles and her choices about distributing her authority and power among students.

As it can be seen from Table 13, the teacher has a limited use of words specific to social discourse whereas not any other kind of mediators used other than language to produce social meanings. When it comes to the use of these words or *meanings*, most social words are used to make meanings about students, their roles and their status within the discursive practice of the mathematics classroom and teacher's perception about learning mathematics. There are few social words used by the teacher about her role and status identifying social roles of and interactions between students and teacher.

Since most of the words used are about students most of the social meanings produced in text are *about students*. The teacher expresses her perspective of student's turn taking (O1-35, O7-17), of students turn taking in terms of equity and inclusion of all students (O7-51, O11-34), how does she evaluates students' activity (O11-43,O2-42) when does she considers student turn taking as an intervention to herself (O10-6,8), how she marks student's solution as important, hence empowering student (O11-29, O11-61, O10-16), how she attract students' attention as the most powerful participant (O1-43,O2-31) and how does she motivates a student to be more active (O6-12).

Table 13: Teacher's social word use

The word	Use/meaning	About whom	Turns
One by one, speak by asking your turn. You say. One by one, not all together!	Turn taking	Students	O1-35 O7-17
Ok. Everyone listens to me. Pay attention to what I say.	Attention attracting as the most powerful participant*	Teacher Student	O1-43 O2-31
Let's do it together	Scaffolding	Teacher Student	O3-51 O6-31 O11-53
Come on. I expect a little bit activity from you	Encouraging	Teacher / student	O6-12
Now we will say after I say it I was going to tell that	Turn taking/ Interrupting/intervening	Teacher / Student	O10-6,8
Your friend has got an idea and he will share with you. Right Very good.	Empowering	Students	O11-29 O10-16 O11-61

Table 13 (Continued)

Let's wait for a while give chance to your friends to think about it.	Turn taking, inclusion of all students Turn taking	Students	O7-51 O11-34
The silent ones. There is no activity from here folks, you're too quiet.	Evaluation of student activity	Students	O11-43 O2-42

There are also three other words that are used frequently in the text which deserves to be considered as social. These words are *give, folks/fellows* and *we.* Finally, the use of the word *give* with the *I* or mostly *we* pronoun gives remarkable clues about the teacher's disposition about teaching and learning mathematics and her role with respect to students' role in practice.

Whenever the teacher uses the word *give* combined with we/I in an active clause it only refers to teacher's transmission of a rule/ a property about a subject (e.g. logarithm, polygons) to the students as the one who owns the mathematical knowledge, or when interpreted within the constraints of social discourse being the most powerful participator

Table 14: Uses of word *give* with I/ we pronouns

O3-58	*I only give* a rule… *I* only *give* to students
O5-1	*We* have *given* the base change property We were giving the properties of logarithm
O7-2	*We* have *given* the 5th property
O7-10	*We* haven't *given* but we use them in problems
O7-21	Let's *give* 7th property
O10-10	*We* have *given* that on…

within the discursive practice of the mathematics classroom. The teacher determines her role as giving and providing and her students' role as the receiver of the information. This use of *give* is not present at last 2 lesson observations but it is not sufficient to conclude that this a deliberate action of the teacher which I articulate later on at last research question.

The general characteristics of the teacher's social discourse are determined according to the *speech roles* teacher have chosen for herself and other participants and her *expression of personal perceptions and attitudes* (Halliday, 1973).

Table 15: Classroom observation excerpt 10

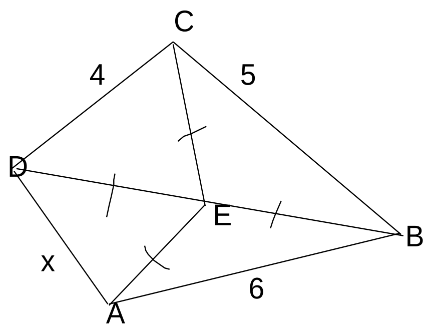
Turns	O1 (G) 20th October 2011 Thursday
11	S4: If there are opposite angle bisectors, the difference of their angles is…
12	T: [*Completes S4's statement*] Half of the sum... In case that our angle bisectors are adjacent… If our vertex angles are angle bisectors…
13	S4: What I say is this teacher. If angles are at opposite…
14	T: [*Does not wait S4 to finish her sentence*] Yes, then it is the half of the difference. If our vertex angles which are angle bisectors are also adjacent then we told that it is the half of the sum of two angles. What else?
15	S5: If the diagonals are perpendicular then the sum of the squares of the opposite sides was equal to the sum of the squares of the other sides.
16	T: If there is a perpendicularity for the diagonals then this is one of the properties that we use to find the length of sides
17	S6: Teacher shall we say "the parallelogram when having joined the middle points?"
18	T: Yes go on.
19	S6: What we do with the middle points is…if it is perpendicular is…
20	T: We have an arbitrary quadrilateral and this quadrilateral…[*pauses and demands S6 to complete the statement]*
21	S6: When we join the middle points it is a parallelogram.
22	T: Ok. When we say middle points we should refer to somewhere clearly: Middle points of what? Shouldn't we? We have an arbitrary quadrilateral at hand. When we join the middle points of the length of its sides we construct a quadrilateral once again. And this quadrilateral is a……[*pauses and waits for the students to complete the sentence]*
23	Ss: Parallelogram
24	T: If there is no other information, yes. If the first quadrilateral's diagonals are perpendicular then the quadrilateral at the middle is a rectangular.
26	 According to the figure what are the possible values of x? (Image reproduced)
27	Ss: Shall we solve on our books or on notebooks?
28	T: If you have your books ready then you may do on it. Draw the question with me. The faster you draw more questions we can solve.
29	T: Okay. We have always found single value until now at the problems we have solved. It says: 'possible values of x'. Under which condition this can be like that?
	[*Silence…]*
30	T:Don't we solve questions by using triangles in general? Does this ring a bell?
31	Ss: There was...an inequality.
32	T: Yes. The triangle inequality. Remember…
33	S4: The difference of two sides, the side which remains in between…
34	S7: It should be bigger than the sum and smaller than the difference.
35	T: One by one.. Speak by asking your turn. You say.

Table 15 (Continued)

36	S4: If we know two sides then we can find the middle x by taking both the sum and difference of them.
37	T: How could we? How could that middle x be?
38	S4: It is bigger than the sum of the two but smaller than the difference.
39	T: We were calling that the triangle inequality. 3 numbers that I make up would not always construct a triangle. Then we could only find values more than one. Then we need to use that.
40	Ss: Let it be 2 to 4 and hence 6 to 3.
41	T: How may it be?
42	Ss: May we equally dissect?
43	T: Ok. Everyone listens to me. Why are these equalities given?
44	S1: The magnificent trio!
45	T: Yes. Where?
46	S1: At the Angle C
47	T: Angle C? But why don't we say the Angle A also?
48	Ss: x is equal to 4!!!
49	T: Angle A is also…isn't it?. Now we should remember our triangle knowledge. In this triangle CE is the median and is equal to the length of sides that it sets apart. Since we call magnificent trio, this vertex angle is 90 degrees and A is also 90 respectively. We call it by seeing. What were we using in acute triangle?
50	Ss: Pythagoras
51	T: Then use it, how were we using it? (…)
52	S7: Isn't there an 6-8-10 teacher?
53	S8: It is like square root of 5
54	San: Yes
55	San: Since it is Pythagoras the square of 4 and square of 5 here. I called a there, here is $x^2 + \sqrt{41}$
56	T: Or this side is common for both, isn't it? By no need to call a we can say $x^2 + 6^2 = 544$ then we can find from here. Any questions? If not then move on.

The situation above from the geometry lesson (O1) is identified based on teacher's objectives set at the end of the pre study interview. These objectives at issue are getting students to use mathematical language correctly and effectively and increasing student participation. Throughout the above excerpt selected from the 1st observation as representative of the general characteristics of the teacher's social discourse, the teacher is only in the mood of demanding information from the students that she had *given* previously. Furthermore, there is not any *modalized* form of expression except Turn 22. There, the teacher is pointing out a rule for communicating mathematically through student word use *middle point*. The situation from above also reflects how the teacher intentionally leaves her *statements* incomplete and expects her students to complete them. Since she uses these statements as a request for information they are also used as questions. Yet, this way of using statements as prompting students might constrain the possibilities of communication in terms of exchange of meanings between students and the teacher. These statements are also used like a command: 'complete my sentence'. Additionally, between the turns 11-14 we can see that the teacher did not wait S6 to complete her statement and completed it herself on behalf of S6. At the turn 17, S6 again asks for permission and teacher's approval to express her statement. Similar commands appear on the rest of the text (O1 43, O1 35). These situations makes clear that the teacher controls the turn management that is who speaks and when, within the situation and appears as much more powerful than the students.

The teacher uses questions while solving problems (Turns 29- 51) but students respond either with a silence (Turn 29) or a humming noise (Turn 51) to these questions. There are individual responses but when these responses lead to something that is not anticipated by the teacher she suspends the dialogue with a command: everybody listens to me !(Turn 43).

In sum, based on the amounts of questions asked and the statements provided by the teacher it can be said that she determined a role of information demander as an evaluator or as the one who has the authority of possessing the [mathematical] knowledge. The prevalent modalities (e. g should/ could/ can) in her statements and questions reveals a mild level of obligation or necessity of action for the students within the discursive practice of the mathematics/ geometry classes.

4.3.2 Teacher's social discourse routines and endorsed narratives

As a final step in my analysis for the 3rd research question I analyzed routines and endorsed narratives of the social discourse by bridging the SFL theory and commogniton by Sfard (2008). The teacher's social discourse routines are the teacher's set of social meta rules defining a pattern of discursive actions of the teacher that is repeated in similar situations. Furthermore, the production or substantiation of endorsed narratives is the primary objective of the teacher's social discourse as it is the case for any level of scholarly or colloquial discourse.

Table16: Classroom observation excerpt 11

Turns	O1 (G) 20th October 2011 Thursday
18	S6: Teacher shall we say "the parallelogram when having joined the middle points?"
19	T: Yes go on.
31	T Don't we solve questions by using triangles in general? Does this ring a bell?
32	Ss: There was...an inequality.
33	T: Yes. The triangle inequality. Remember…
34	S4: The difference of two sides, the side which remains in between…
35	S7: It should be bigger than the sum and smaller than the difference.
36	T: One by one.. Speak by asking your turn. You say.

At these episodes above from the first lesson observation, the teacher's social routines are in a *ritualized* form. At the turns 19 and 35, teacher's social meta rules emerge that constrain procedures (how of a routine) of actions to be taken by *students* whereas the acceptance of these rules are dependent *on the teacher*. These routines are defining *a pattern for student participation* within the discursive practice of the mathematics classroom. Yet, these *meta rules of participation* for the students expressed tacitly by the teacher (Turn 36) and students need an associated prompt to participate in the discursive practice. Also both at the same and the next lesson another social routine of the teacher can be identified describing the forms of *social relations and the positions of the participant*s with respect to each other. At both the (O1-43) and (O2-31) the teacher tacitly set the rules about when should students stop talking about a mathematical problem or a situation and pay attention to what teacher would say as she is the only if not most authorized person to be listened to.

Following her 5th lesson observation, which after 3 months later since her inquiry of own practice teacher's ways of enacting a ritual for student participation evolved slightly. She began to express the meta rules of her social discourse more explicitly regarding the positioning of participants/students with respect to one another within the discursive practice of the classroom and that she considers the equity among participants. Examples of these descriptions of the repetitive discursive actions of the teacher are below:

> "We shall not say the solution at once; give some chance to your friends" (O11-34)

> "Let's wait for a while give chance to our friends to think about it. Still we have multiple ways and use whatever you feel like to use" (O5-17).

Also an explicit expression of her social routine for empowering students to present their ideas mathematically is lately identified in the observed lessons:

> "Your friend has got an idea. We will share it to you. Tell them." (O11- 29)

Furthermore, teacher's social narratives endorsed repetitively reflect her powerful position within the discursive practice as:

> "I will continue giving properties…"(O4-46; O10-10)

> We have given the … property (O5-1; O7-2)

Teacher's perception of self as the provider of the information/ knowledge necessary for her students and her disposition to learning as acquisition resulted from her understanding and perspective of teaching mathematics simply as a transfer of knowledge does not evolve much over time. Thus she continued exercising power on students that reveals her privileged discourse and social position among students and describing *self patterns of participation* within the discursive practices of her classroom.

Table 17: Classroom observation excerpt 12

Turns	O(10) (M) 20th February 2012 Monday
6	S1: Shall we write (k+1)! At the previous one?

Table 17 (Continued)

7	T: **Now, we will say after I say it.**
8	S2: Can I say something here? Umm... k! and (k+1)!. I think I multiply both sides with (k+1) in order to make this (k+1)!
9	T: **I was going to tell that.**
10	S2: Sorry teacher for interrupting,

At seen by the episode above the teacher' discourse helps us to understand her meta rules that constrain her relative position to students and that describe the situations of which students' participations would be considered as appropriate. She sets the boundaries for *when* the students could participate in the discourse which is after *the teacher tells important points* and these rules are also endorsed by the students. In that sense, these rules become a *norm* as enacted and widely by the whole community of the classroom endorsed by all of the participants especially the one who is in an expert position (Sfard, 2008).

As for the social narratives endorsed by the teacher, none of them are produced or substantiated as a result of a social routine. They are simply enacted by the teacher and expected to be endorsed by the student. Additionally, the teacher endorses limited amount of social narratives. Below, I present some examples of these narratives which are enacted by the teacher within the discursive practices of her classes

Table 18: Classroom observation excerpt 13

Turns	O1 (G) 20th October 2011 Thursday
28	Ss: Shall we solve on our books or on notebooks?
29	T: If you have your books ready then you may do on it. Draw the question with me. **The faster you draw more questions we can solve.**

At that excerpt above the teacher endorses a narrative about her perception of students' role within the discursive practice: They can mediate the discursive practice of the classroom by being fast so that they can practice more. There is another example below of how she evaluates students' status of participation.

Table 19: Classroom observation excerpt 14

Turns	O2(M)20th October 2011 Tuesday

Table 19 (Continued)

42	T: Now the question is if $z=(1+i)^6$ then find $\|z\|$=? Started with simple ones. Now, $1+i$ was our criterion. It feels easy to find its power. You can take the power and then find the modulus or find the modulus then take the power. **There is no activity from here folks, you're too quiet.**

After about one and a half month she shows a discontent about limited student participation into the discourse and disposition towards learning and doing mathematics. In this social narrative she trivializes finding the *correct result* and in the meantime emphasizes reasoning.

Table 20: Classroom observation excerpt 15

Turns	O4(M) 20th November 2011 Monday
18	T: Ok, let's do it on the board. **Why always the same people? Do you think that the result matters that much?** Come on let's reason. Which property of rhombus applies here? Since diagonal is given then we should obviously use diagonal related knowledge. What do we know? Let's repeat that….
52	T: **Can you do it while explaining?**
53	S2: I can do it
54	S5: By explaining…uhmmm.
55	T: I know you can, but I don't want to give you the floor all the time.
56	T: **Ok, let's do it together**, you come.

At that lesson she also shows her disposition towards knowing and learning in general. She wants her student to solve the problem by explaining her solution procedure and she tries to encourage her student by using "*let's do it together"* meaning that she is there to scaffold.

4.4 Teacher's Pedagogical Discourse

The third layer of the teacher's discourse of action is *the teacher's pedagogical discourse*; associated with third metafunction of the language called textual adopted from Halliday's SFL (1978) theory. The textual metafunction of the language represents the speaker's text forming potential within a social practice. Grounding on both SFL theory and the *meta-mathematical moves* identified by Knott et al. (2008); I first presented the pedagogical word use and the general characteristics of the teacher's pedagogical discourse by focusing on these *meta-mathematical discourse moves* and *question and answer* patterns that is used to organize mathematical discourse then I moved on to the analysis of the pedagogical routines and endorsed narratives.

4.4.1 Pedagogical word use and the general characteristics of the teacher's pedagogical discourse

The words/ visual mediators specific to teacher's pedagogical discourse are identified based on combining SFL and the *commognition theory*. The pedagogical words/visual mediators are identified based on their enabling of the social and mathematical meanings come into being which *organize* the flow of communication within the discursive practices of the classroom. According to SFL, the organization of a message given in a discursive action enables the effectiveness of this message for a given *purpose* and the context of situation (Halliday, 1978) Therefore, for each word/visual mediator a theme or purpose is determined in order to support an understanding of teacher's pedagogical purposes, aims and her dispositions towards teaching and learning mathematics.

From Table 21 teacher's use of questions/key questions draws attention as a prominent word. She uses the word *key questions* to denote the prototype questions requiring the certain knowledge of facts and solution procedures. Regardless of their modality (as iconic, symbolic) all of the *questions* selected by the teacher suits with that prototype and these different modalities are used as pedagogical visual mediators. Similar to the social use of *I/ we give*, those words have a unique use in the pedagogic discourse either. The use of *give* by the teacher with the I/ we pronoun points to the epistemological basis of the teacher discourse about teaching and learning mathematics that underlies and permeates all layers of mathematical discourse as well as pedagogical discourse.

Finally, by looking at the themes and purposes of the pedagogical words and visual mediators in the list a more detailed view about teacher's pedagogical choices and purposes and her disposition about learning mathematics can be formed. These themes are; tactical moves for students to become more active participants, leading students to communicate their mathematical ideas precisely, testing student understanding. They also point out to a pedagogical choice or purpose of the teacher; for students to evaluate the relevancy of their mathematical ideas according to the situation, about how to respond a student's question, about the organization of mathematics content and how it should be learned.

The general characteristics of the teacher's pedagogical discourse are presented by focusing on the *meta-mathematical discourse moves* and *questions and answer* patterns that are used to organize the mathematical discourse.

Table 21: Teacher's pedagogical word use

The word/ visual mediator	Theme/ purpose	Turns
Key questions	Referring to questions (problems) to be solved in the classroom	O1-3,25,59 O6-T12
We will *see on the questions*	Teacher's choice about responding student questions	07-27
What does it mean?	Interactivity Tactical move to test student understanding and to lead students to communicate their mathematical ideas precisely	02-23 O8/9-8 O12-1
Your friend says…	Tactical move for students to be more active participants	02-7 08 &9 -41
Just see that.	Pedagogical choices and/or purposes of the teacher about the organization of the mathematical content	010-3
Right, very good.	Tactical move for students to be more active participants	010-16 011-61
Please *think about it*	Tactical move for students to evaluate the relevancy of their mathematical ideas according to the situation.	011-59
Can you be more *explanatory?*	Tactical move in order to lead students to communicate their mathematical ideas precisely	08&9-3
We have discussed it before they are on your notebooks	Pedagogical choices about how mathematics should be learned	08&9-13

At the 1st and 3rd observations there was hardly a meta mathematical move identified so was any explicit pedagogical discourse of the teacher. The way questions asked by the teacher does not facilitate an exchange of meanings among students or between students and the teacher. Thus, there is not any active participation of the students since the teacher does not actively listen to them yet. The only move of the teacher in these lessons is her *summarizing* and *rephrasing* a student's answer about the properties of a logarithm function at the end of the revision section of the lesson.

Table 22: Classroom observation excerpt 16

Turns	O3(M) 14th November 2011 Monday
8	S3: Thinking of logarithm b to the based a, a should not be 1 which was the first condition. The second condition was a should be positive, and the third condition was b should be positive. If it satisfies these conditions we could find the extended range and then we can continue with the inequality solutions which we have learnt last year.

Table 22 (Continued)

9	T: Yes, now let me summarize what your friend said. In logarithm the base was the same as the exponential function. We already noted that in finding the inverse of one of it if it is exponential we should find a logarithm. The base a there and the base a in logarithm was the same we have told also when defining the exponential function, for all powers of 1 our base should be different from 1. So if the same base a is also the base for our logarithm then our base should be certainly different from 1 and bigger than zero. One thing more… As we said that the place to write is important. The expression whose logarithm is taken when it comes to logarithm the base should be written slightly below logarithm and the statement whose logarithm is taken should be written in line with the logarithm. There, the region of which those two conditions are commonly satisfied. Ok now, let's continue solving problems we have solved one or two that's not enough. We will solve more. Now let's move to questions.

By the 2nd observation there were instances of pedagogical discourse regarding teacher's objectives within the scope of mathematical communication.

Table 23: Classroom observation excerpt 17

Turns	O2 (M) 20th October 2011 Tuesday
23	T: Yes is equal to their product respectively.3rd, what is the modulus of z_1 over z_2 ?. The division of z_1 to z_2 respectively. 4th: For *n* is an element of N ,$\lvert z \rvert^n = \lvert z^n \rvert$. This is the property that helps us most. What does it mean here?
24	S4: The power of the complex number *z* within and outside of the modulus is equal.
25	T: Now, folks, I'd like to ask you that way: How were we taking any power of a complex number? How could we find any power with what we know so far?
26	S5: We divide it by 4
27	T: No that was for the power of "i" imaginary number. Now we have both real and imaginary parts, how can we find any power?
28	S5: We were multiplying the number side by side as many times as the power.
29	T: What if it is the 25th power?
30	S6: If we say 25th power of 2 we multiply 2 by itself 25 times,
31	T: Pay attention to what I say, there is also an imaginary part.
32	S3: Respectively
33	T: How come? How will you separate?
34	S3: For example if is $2+i$ then 2 to the power 25 plus i to the power 25.
35	T: You cannot do that, how could we do it?
36	S8 :For instance for $2+i$ to the power 25, we can write it like $2+i$ multiplied by $(2+i)$ to the power 24. Then we can find it.
37	T: Then I am asking how do you find the $(2+i)$ to the power 24?
38	S8: Then we separate $(2+i)$ again and take $(2+i)$ square

Table 23 (Continued)

39	T:[*Raising her voice*] While solving power questions for complex numbers with what we know so far we have solved by using the powers of $1+i$ and $1+i$ for high powers. Now if the number we are given cannot be written in the $1-i$ or $1+i$ form then how can we find that high power? There, that property helps us a lot. I find the modulus of that number regardless of whatever power it is, the thing that I find in the modulus is a number right? There is no imaginary part only the real part. It is easier to find that number's power right? That's why finding an arbitrary complex number's modulus or finding the power then taking the modulus, which one is easier?
40	Ss: Finding the modulus then taking the power

At the excerpt above the teacher enables a sequence of meta mathematical moves such as, rephrasing (Turn 25), steering and redirecting (Turns 27, 23), prompting (Turns 29, 31, 33,35, 37) that are triggered by the question "What does it mean"? (Turn 23). After the teacher asks that question an exchange of meanings occurred between the teacher and the students, though the teacher suspends the exchange finally giving the answer herself. Yet, the way teacher uses questions gets slightly different from her initial lesson. The *encounters* are defined as what are actually occurring in discourses and they are different from the interactions which assume reciprocal influence between participants (Wickmann & Östman, 2002). The pattern of the questions asked by the teacher and answers given by students and the meta mathematical moves preferred by the teacher remains more or less the same until 6th observation made at our third month of the study. By the 6th observation though, teacher's use of meta- mathematical moves and her question-answer patterns point out a change.

Table 24: Classroom observation excerpt 18

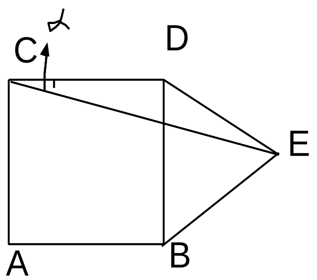
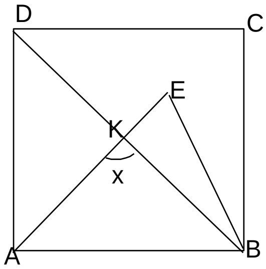
Turns	O6(G) 22th December 2011 Thursday
1	 T: The question is, ABCD is a square, BED is an equilateral triangle and here is given as α. How many degrees is alpha? (Image reproduced)
2	T: What was the property of equilateral triangle, fellows?
3	S1: Their angles being equal.
4	S2: 60 degrees
5	T: Was there any other property?
6	S3: Is it 15 degrees teacher?

Table 24 (Continued)

7	T: Yes, your friend says that all side lengths are equal. Ok, let's put these on the figure. Come on let's do it.
8	T: [*S4 is at board*] Where of angle E is 60 degrees?
9	S4: and here is 150
10	T: Ok now you have used the angles then use also the side lengths, they were all equal.
11	T: Yes, 60-60-60. Now everyone can see this is isosceles, right? Then 2 alphas is equal to…[*S4 does the rest of the operations]*
12	We continue with doing our key questions. A new question: Everyone should deal with it, everyone should be able to solve that. Come on S1, I expect a little bit activity from you. (Image reproduced) 
13	S2: Is it 75?
14	T: May we not tell the result immediately, what are we going to do?
15	S3: Is it 75?
16	T: Yes. [*repeats her question.*]
17	S4: Is it 105 teacher?
18	T: The other side is 105, alpha is 75.
19	S5:[*solves it on the board*] From the angle bisector theorem here is 1 over 5.
20	T: Is it angle bisector theorem or?
21	S5: No that was diagonal.
22	T: Diagonal was angle bisector, since DB is angle bisector it divides as 45-45 says your friend.
23	S5: Then since it is equilateral triangle here is 45 degrees and here is, since one angle of a square is 90 degrees, here remains 30 degrees. Since two interior angles gives one exterior angle 45 plus 30 equals to 75.
24	T: [*to S6 by looking to his solution*] Hmm, you can also find from there.
25	S6: I have done by using the other triangle
26	T: You can also do by using the other triangle. And our friend said that if here is 60 then 45 it remains 75 there. You can also do it by alternative ways.

At this excerpt from O6 the teacher begins to use revoicing in her pedagogical discourse for students to become more active participants of the discursive practices of the class (Turns 7, 10, 22, 26). She prompts S5 to rethink her mathematical word use (Turn 20) and validates S6's alternative solution method. Students began to participate more confidently into the discursive practice of the classroom with their ideas or explanations. At the 7^{th} observation of a mathematics lesson about two weeks later than the 6^{th}, students also start asking questions to the teacher in order to clarify their understanding of the change of bases property at the logarithm function (O7, 26, 28, 38, 39, 48, 50, 66) .Furthermore the teacher's questioning differs from her previous discursive actions as she begins to use questions in order to probe students' thinking:

"What do you say S2? *Can you be more explanatory?" (O8, 3)*

Another remarkable change regarding teacher's pedagogical discourse can be seen at the11th lesson at which a student was supposed to take on the role of a teacher and to undertake the presentation and the organization of the *arithmetic sequences.*

Table 25: Classroom observation excerpt 19

Turns	O11(M) 12th March 2012 Monday
1	T:We have mentioned some properties of sequences and solved various examples. We have told that we will do this lesson with our friend, our topic is arithmetical sequences. Let's write our topic and our friend will have us to take notes of the definition, you will be solving the examples together and will be asking what you did not understand to her.
2	ST: Sequences which has a constant difference between its consecutive terms is called arithmetical sequences, take the formulas too fellows [*writing the general term for and some other formulas related with the arithmetic sequences to the board*]
3	T: Can you write a bit bigger? [*after about 2 minutes later*]. Write that on the left hand side. It is the general term of the arithmetical sequence?
4	ST: [*writes*] Let me may give an example right away.
5	T: Ok, ST, does everyone know a_1 among the symbols that we used here? What was it?
6	Ss: The first term
7	T: What was d? We did not say. In some textbooks it might be used r instead of d. We will say it the common difference in the arithmetic sequences but the common factor in the geometric sequences. And n-1 denotes that it is the preceding term of a_n .
8	S1: r is used generally in geometric sequences
9	T: Some resources those are both called r. If you use them separately I think it will be better you will not mix them up.
10	ST: The general term of a sequence whose first term is 3 and the common difference is 4 asked. We can do it like that. First term is 3 and the common difference is 4, by the formula an is equal to a1 is 3 and $3+(n-1).4$ which means $3+4n-4$ is equal to $4n-1$.
11	T: Ok do you think is there any difference between the sequences we have seen until that far and the arithmetic sequences?
12	S2: There is
13	S3: Of course
14	S4: There is. We have said that it increases constantly
15	S5: The difference between the consecutive terms is constant.
16	T: What did your friend say when defining? There is a constant difference between consecutive terms which is d. I can therefore. If I know an arbitrary term and d, for example it gave the 3rd term and d how can I find the 25th term?
17	Ss: (…)
18	S4: 25 is equal to $22d+a_1$

Table 25 (Continued)

19	T: Now since I knew the 3rd term by adding d on it, I can find the 4th term, by adding one more d I can find 5th term and by adding another d 6th term. You can go like that if the difference between the terms is small. Like 25th term or 125th term in such cases we have to use the general term formula
20	ST: [*writes a statement giving the formula for the common difference for the two arbitrary terms of an arithmetic sequence like* a_p *and* a_k. $d = \dfrac{a_p - a_k}{p-k}$ *Right after she asks a question*]: The 18th term of an arithmetic sequence is 45 and the 8th term is 15. Then what is the 5th term? [*Solves by applying the d formula in order to find d and then she uses the general term formula to find the 5th term*].
21	T: Ok, now how do I do that without using that formula, I am not a good fan of too much formulas since you mix them up, there will be Sn formula coming soon. You know the 18th and the 8th terms how do you find without using this formula? [*waits for a while*] Can it be by using general terms formula? Say the second way. It is ok with the formula also practical but let's writes 18th term according to general term formula. Is it a_{18} is equal to $a_1 + d17$. Now let's also apply this to 8th term. [*ST applies what she says at the board, conducts the operations*]. Ok.
22	S3: She could also associate with a_8 ?
23	S4: But how could she without knowing d ?
24	T: [*To S3 and s4*] There are two unknowns both a and d. [*To the class*] Ok. Now by writing these information one under the bottom and using suppression method we can find a_1 and d .
25	S5: By a_{18} being equal to $a_8 + 10d$.
26	T: a_1 plus $17d$
27	S6: By associating a_8
28	S7: Yes it also comes from that teacher!
29	T: Ok fine you show that also. Your friend has got an idea we will share it to you. Tell them.
57	T: Ok lets think about it for a while, listen to me please I will ask you a question. For instance if you are given the sum of the first 60 term of an arithmetic sequence, by adding the 61th term, do we find the sum of the 61 terms?
58	Ss: yes
59	T: From here, when the sums are given how can we switch off to the terms how can we connect? Please think about it.
60	S5: We can subtract s_8 from s_{10} and can find $a_9 + a_{10}$
61	T: very good
62	S5: or the difference between s_9 from s_{10} gives me a_{10}
63	T: Yes, What S5 says is by taking the difference of the sums we can switch off to terms

At her first turn, the teacher reveals her ways of organization of the discursive practices of the mathematics classroom while describing what the student teacher will follow. These are note taking of expressions, solving examples/questions/problems together and asking [questions] about what is not understood to the teacher. Furthermore, the teacher uses meta mathematical moves consistently. She clarifies and revoices statements of the student teacher (Turns 3, 7, 9, 11, 16), validates alternative student solutions (Turn 29), redirecting (Turn 21), generalizing (Turns 19, 41) probing and rephrasing students' ideas. (Turns 57-63).

Here, teacher's pedagogical discourse stands out in terms of variety and the frequency among other lessons. She had reflected on that situation as below:

> …But of course I am always there for back up. Since students may skip and stutter and sometimes construct wrong sentences in terms of mathematical knowledge then I immediately step in. So I have to listen to the presenter very carefully…Necessarily in case that there is a wrong expression I have to correct it since students don't forget wrong information, they could forget the right one but not the wrong one…(IS5)
>
> I'm worrying about the others won't understand I think they won't understand when a student with a weak background what if she explains superficially and obstruct others to understand fully or if there are shortfalls this is what I worry and why I interfered a lot. (IS6)

Finally at the last lesson (O12) that I have observed, only one question was solved in one lesson hour and there was a discussion about the meaning of a symbolic mediator given in the question/example where a lot of students participated in. The teacher has *maintained her silence during the discussions and listened to her students more interpretively*. The amount of time she had allowed for discussions and listening to her students has increased considerably.

4.4.2 Teacher's pedagogical routines and endorsed narratives

The final step in the analysis for the 4th research question is the investigation of the teacher's pedagogical routines and the narratives endorsed by the teacher. In order to that meta rules underlying these routines will be identified. Secondly by grounding on Sfard (2008) the pedagogical narratives about the descriptions of the objects that the teacher uses to organize and enable the exchange of meanings in the course of inquiry of own practice, 3 main features of the teacher's pedagogical routines are identified:

- Having students to *take notes of* mathematical definitions/ properties
- Solving as much as possible key *questions* together within a lesson hour.
- *Asking students* whether there is anything unclear *about the procedure/ solution*.

Table 26: Classroom observation excerpt 20

Turns	O1 (G) 20th October Thursday
30	T: Don't we solve questions by using triangles in general**?** Does this ring a bell?
31	Ss: There was...an inequality.
32	T: Yes. The triangle inequality. Remember**…**
33	S4: The difference of two sides, the side which remains in between…
34	S7: It should be bigger than the sum and smaller than the difference.
35	T: One by one… Speak by asking your turn. You say.
36	S4: If we know two sides then we can find the middle x by taking both the sum and difference of them.
37	T: How could we? How could that middle x be?
38	S4: It is bigger than the sum of the two but smaller than the difference.
39	T: We were calling that the triangle inequality. 3 numbers that I make up would not always construct a triangle. Then we could only find values more than one. Then we need to use that.
40	Ss: Let it be 2 to 4 and hence 6 to 3.
41	T: How may it be?

At that episode from the first lesson observation the teacher and students engage in a dialogue while trying to solve the situation presented by the question at Turn 26. The teacher generalizes using triangles as a method/ strategy to *solve questions which is a pedagogical narrative endorsed by the teacher*. (Turn 30). She then has her students to recall the appropriate object level rule (Turns 32-37) where teacher endorses a pedagogical narrative at Turn 35, rephrases students' statements of that rule and clarifies how it fits into the situation represented by the question (Turns 37- 39) resulting with the substantiation of a pedagogical narrative (Turn 39) and responds with a question to the student's answer (Turn 41).

Table 27: Classroom observation excerpt 21

Turns	O1 (G) 20th October Thursday
49	T: Angle A is also…isn't it? Now we should remember our triangle knowledge. In this triangle CE is the median and is equal to the length of sides that it sets apart. Since we call magnificent trio, this vertex angle is 90 degrees and A is also 90 respectively. We call it by seeing. What were we using in acute triangle?
50	Ss: Pythagoras
51	T: Then use it, how were we using it?
	(…)

She then again prompts her students to remember more about the triangle knowledge that will lead to take an action in the solution process (identification of the vertices as acute), then concludes with a description of how she realized that action and again prompts for recalling an appropriate narrative or a factual knowledge about triangles to take another action (Turns 49 51). Finally she asks to check if everything is clear about the solution process, then moves on to another *key question.* She has her students to *write down* the object level rules (mathematical narratives) and the sequence of actions to take within the solution process (Turns 56-59).

Table 28: Classroom observation excerpt 22

Turns	O1 (G) 20th October Thursday
56	T: Or this side is common for both, isn't it? By no need to call a we can say $x^2 + 6^2 = 544$ then we can find from here. Any questions? If not then move on.
57	S9: Teacher, how did you explain?
58	T: Without using variables, without saying a to here, we can use directly Pythagoras, this side is common for both then the square of 4…
59	T: Now another key question, now I will have you written a warning. You can solve the other questions like these type since we have solved our key questions from each of them, now shall we move on to areas.

Below there is another episode representing situations that focuses on *key questions.* In this episode, the teacher sets the stage by *giving* the properties that will help students solving *a question* which is a main theme of the teacher's organization of the discursive practice of the mathematics classroom that is; of her pedagogical discourse.

Table 29: Classroom observation excerpt 23

Turns	O7 (M) 28th December 2011 Wednesday
21	T; I made a revision there. We will remember these while solving problems now, then let's remember it again. The property of change of base: $\log_a b = \dfrac{\log_c b}{\log_b a}$. And let's give the 7th property as well $\log_a b = \dfrac{1}{\log_b a}$. Now, is there any difference between them fellows?
22	Ss: Yes there is.
23	T: What kind of difference?
24	S8: One is negative and the other one is the power. I mean the 7th was according to multiplication.
25	T: This means the base and the number of which its logarithm is taken switches. And what in the other one? C has the property of being a base. What did we say at first for logarithm? The base a will be different from 1 and will be different from 0. Then I switch my base to c, look, c should have the same property as a. So it should be different from zero and bigger than 1. At the 6th property I change my base to another c base but at the 7th I switch the base and the number. How do I change that, a number at the base a to the number at the base c? The number at the base c over the old number at the base c.
26	S7: Will they give the c base to us? How it is going to be?

Table 29 (Continued)

27	T: We will see on questions, a question on the spot now.
28	S5: Can I ask something teacher? At the 6^{th} property… Is it by taking both sides' logarithm c or shall we find it at the questions by ourselves?
29	T: It will be asked at different base. If $\log_2 3 = x$ then what is $\log_{12} 18$ in terms of x. Now everyone look at the question very carefully. The logarithm of 3 to the base 2 is given and the logarithm of 18 to base 12 is asked. There were questions we have seen at our last lesson, right? Like what is equal to in terms of a. What is the difference between those questions and that?
30	Ss: The base is different

At turns 21-23, she prompts students to think about differences between two properties about logarithmic functions and then rephrases a student's response, explains the difference between the properties by describing the sequence of actions that one should take to use these in problem solving situations (Turn 25). She responds a student's question asking for prescriptions of using these rules. With this response, (Turn 27) the teacher clearly exposes her preferences of organization for the exchange of meanings within the discursive practice of the mathematics classroom. Meanings are mainly exchanged between the students and the teacher through the situations represented by *questions* where the flow of communication is predominantly from the teacher towards students. At the turn 29 the teacher repeats her way of asking the difference between the properties of the logarithmic functions this time for the *type of the question*s solved so far.

Table 30: Classroom observation excerpt 24

Turns	O6 (G) 22th December 2011 Thursday
7	T: Yes, your friend says that all side lengths are equal. Ok, let's put these on the figure. Come on let's do it.
24	T: [*to S6 by looking to his solution*] Hmm, you can also find from there.
25	S6: I have done by using the other triangle
26	T: You can also do by using the other triangle. And our friend said that if here is 60 then 45 it remains 75 there. You can also do it by alternative ways. Another question.

Above there is another utterance of the teacher from the end of the first semester. She uses revoicing and directs students' attention to their friend's words and the connection of these words with the problem situation represented by the figure. Similarly then, she validates a student's alternative solution of a problem.

On that episode below, about approximately 4 months later than the first lesson observation, the teacher organizes the lesson in a way that almost no *key questions* solved but mostly focusing on enabling students to recall and produce mathematical narratives about polygons. Below, she prompts S2 to communicate her mathematical ideas precisely and more accurately (Turn 3) and probes the mathematical meanings made by S4 (Turn 8).

Table 31: Classroom observation excerpt 25

Turns	O8-9 (G) 20th January 2012 Friday
3	T: Then triangle is also a **polygon**. Okay but is there any certain properties of polygons? Which conditions should a shape satisfy to be called a polygon? What do you say S2? Can you be more explanatory?
7	S4: Two consecutive points should not be linear.
8	T: [*Repeats the sentence.*] What does it mean?
9	*S4:* Means that there won't be a single line.

CHAPTER 5

DISCUSSION

The purpose of this research was to explore teacher professional learning process while doing an inquiry of own practice as a social practice. In analyzing the first research question teacher's accounts of the discursive practices of her classes were gathered primarily by the narrative analysis of the interview data. The rest of the research questions were analyzed to explore the teacher's discourse as teacher's mathematical, social and pedagogical discourses. Research questions in these two groups were framed due to two main discourses as *reflection* and *action discourses* of the social practice teacher's professional learning while engaging an inquiry of own practice *about knowing/talking* and *acting* respectively within discursive practices of her classes.

5.1 The Teacher's Accounts of Her Own Practice

The results of the first research question were grouped under three themes as: *the discursive practices, knowing in practice and inquiry of own practice.*

5.1.1 Discursive practices

The discursive practice is an umbrella term which is adopted from Foucault (1970) as described in the Foucauldian Discourse Analysis section. In discussing the results under the first theme I drew on some of the methodological tools provided by Michel Foucault in order to interpret them as teacher's accounts of the discursive practices of the mathematics classroom.

Norms regarding teaching and learning as being the first code regarding teacher's accounts of her discursive practice covers particular roles, responsibilities and certain preferences endorsed by the teacher. These general norms and preferences are tightly connected with her epistemological stances of teaching and learning mathematics which also denote that she acts according to particular discursive frames (Foucault, 1979). These frames are related with a traditional authoritarian discourse of teaching and learning and define her role as the norm setter whereas students to assume relatively passive positions. *Giving definitions, essence of the subject, developing different solution techniques and solving certain amounts of problems [for and on behalf of students], having students to take notes on regular basis* and *transmission of knowledge from the teacher to the students*, are sample-positions created by the teacher for the participants within her traditional discursive frames for teaching and learning of mathematics.–In this study, Aylin's expectations from her students were also manifested in a way that (e.g. arriving the lesson on time, note taking within a neat and a systematic manner, listening to the teacher carefully) defines the proper student within her classroom. This view is in tune with Walkerdine's arguments about the *production of child* by the adults who have authority over the children (Agre, 1997; Walkerdine, 1990) ideas about the *developing child* in her influential work about the mathematical activity and reasoning that child is produced within historically specific discourses as a subject of scientific investigations. The teacher's powerful position is described by her enabling students to reach their ideals on pursuing a higher level education like university. Her emphasis is on *how to use mathematics* which in turn defines *how it should be learned* by the students. Moreover, at the core of those expectations from students lies, *being able to solve exercises/ problems from a textbook.* Hence mathematics itself is also manifested as empowering students for fulfilling their ideals and this determines *why it should be learned by the students.*

The teacher's preference for teaching mathematics to geometry and her claims about the limited opportunities for having students to reason in geometry (IS3) might be associated with the teacher's knowledge of practice related with the geometry. By grounding on Kelly (2006), one might reach conclusions about that the teacher's knowledge of practice related with the geometry. This includes teacher's knowledge of geometry, of related pedagogical approaches and of the student's misconceptions when learning geometry and so on. Also, the conceptual recourses which students bring into the discursive practices of the classroom are possibly playing an important role about the teacher's disposition towards geometry. Yet, these claims are contrasting with her previous statements (IS2, S3) about students mostly questioning themselves about alternative ways to think about a problem in geometry lessons which is not appropriate for the mathematics lessons. This conflict between her statements might be attributed to her disregarding reasoning and proofs within both of her geometry and mathematics lessons as mentioned above.

Second code of the theme discursive practice was determined as routines of/for mathematizing. This code was composed of actions of the participants that are endorsed by the teacher of the discursive practice while participating in the mathematical discourse or doing mathematics. Teacher's and student's reasoning and argumentation routines which are tightly connected to and based on teacher's accounts of proof and proving, teacher's ways of covering students' 'knowledge deficits' particularly in geometry by interconnecting topics and by solving a lot exercises, giving mathematical rules in order to save time are among these routines of the teacher. Aylin's stance of proofs and proving was not positive and this did not change throughout her inquiry. In the 2nd story (IS2), she stated that doing proofs were important but through her statement of "simply giving the reasoning behind the formulas" she was actually referring to reasoning and argumentation processes rather than formal proof and proving in mathematical sense. Yet, she further contended that these were important she had still no time for them. Instead she indicated that she preferred to 'give the reasoning behind, orally' like "this is resulted from that, this comes from that". It is clear from these statements that she thought she should be giving the necessary connections within and between subjects and instead of students' actually working on or doing these proofs. There was also a covert criticism within her discourse regarding textbooks supplied by the state, from the very beginning of this research project. She was also resistant to the changes have been made especially in the geometry curriculum. This became clearer at teacher's statements of her claiming to interconnect geometry topics by solving questions/exercises as a way to cover deficits in the students' geometry knowledge base which was also due to the recently changed geometry curriculum. Moreover, underlying her critique of textbooks, there was a tension between her time related concerns for covering the intense curriculum, the prevailing phenomenon of the university entrance examinations and her focus on mathematical communication during her inquiry. Due to the criteria of selection and entrance to universities; timing is considered as a very important issue especially for and within examinations, hence solving a certain amount of test questions within a mathematics lesson becomes crucial in order to improve one's future exam score. On the contrary, the above mentioned textbooks were emphasizing proofs and proving rather than practicing exercises and test questions for the exam. Here it can be inferred that, though the curriculums and textbooks lead the discursive practices of the mathematics classrooms, the assessment and evaluation practices are dominating these practices as being the most prevalent discourse influencing mathematics education practices until now in Turkey.

Final code under the category of discursive practices was student participation structures. This code was identified due to teacher's statements of student interaction, presentations and mathematical communication and whole class discussions. The discourse of university entrance exams once again, influences teacher understands of cooperative work of students. Teacher's concern of preparing students for the university entrance exams comes to fore reflecting the broadest mission of her mathematics lessons. In order to make up for her concerns of timing and maximizing the amount of exam like questions solved in a lesson, constructing study groups in order to solve test questions together apart from the lesson is seen as a viable solution for her. She also thought that *practicing and solving a lot of exercises* would suffice to cover deficits in students' knowledge and due to the time concerns the amount of time that is allowed for mathematical discussions is limited. Teacher's claim about students' hesitation of saying something wrong when the camera is on could be a realistic argument. On the other hand, considering the prevalent themes that have arisen from her stories from the beginning, her authoritarian discourse as a teacher is beyond question. Classroom observations and the episodes we have watched during the interviews also supports that view since students rarely talk with one another and the one who talks most is the teacher, as Aylin stated once: "teacher leads the flow of communication" (IS4, S1). Furthermore due to her comments only 4 or 5 students generally participate in the discussions (IS4, S3). Although accepting that individual student presentations might not be actually as effective as she thought and admitting that she has to take more of position of a listener (IS4, S5) her stance about these discussions remains as they "*end up with nothing*". She had a tendency to think that individual presentations inherently implicate participation into the classroom discourse.

Students not appreciating the value of mathematical meanings and thinking and their perception of exercises/ problems as the only means for mathematical thinking can partly be based on her emphasis on *practicing by solving exercises* instead of overlooked whole class discussions that would lead students to produce and utilize different and multiple means for mathematical thinking. This can also be interpreted as a result of the discursive practices within the mathematics/geometry classes that are mainly controlled and regulated by the teacher's discursive actions and by the broader sites of practices such as national policies about mathematics education such as examination mechanism, textbook production, teacher proficiency standards which control and exercise power on students, teachers and educational institutions as constrained by the broader cultural context (Foucault, 1979, Morgan, 2006).

Overall, Aylin's accounts of discursive practices of her mathematics classrooms have teacher and students the resources provided by the classroom as participants which are further determined and shaped by a broader and a prevalent discursive practice: The university entrance system in Turkey and related exams. According to Foucault (1979), examinations are the *techniques of power* that enables the operation of power within groups and institutions. The discursive practices of Aylin's classrooms are hence shaped and regulated within what Valero (2007) calls these *a network of mathematics education practices* which also shape Aylin's practice and her accounts of the discursive practices of her classroom

5.1.2 Knowing in practice

From the narrative analyses one can trace how the teacher starts to reflect on her practice while she talks about her discursive practices within her classes. At the end of the first term, at our 4th interview Aylin stated that she had realized that she did neither listen to her students nor give enough time to finish their sentences but completing their sentences by her. Yet she had indicated that this was so in mathematics but not in geometry lessons in order to cover the intense (geometry) curriculum. This learning/ knowing in practice is also in line with the change in her general characteristics of her pedagogical discourse and in her ways of enacting social routines about student participation. As presented earlier in the results, a social routine of Aylin is identified from her transcript explicitly encouraging students to share their ideas with the class (O11- 29) and her statements on students to wait and listen to each other's solutions (e.g. O11-34; O5-7). There was also a change in the teacher's question-answer patterns identified at O6 and both of these results have been identified between third into fourth months since I have been doing observations and Aylin had been inquiring her own practice. During that interview she had also commented about possible types of '*questions*' that can be asked within a lesson. She explained that from that point on questions could be either interpretive or aimed at students' prior knowledge. In general, she uses the word "question" mainly instead of mathematical exercises or problems. But results from the observation transcripts also show the use of the word in both literal and mathematical meanings. According to Love and Mason (1995) classification of the types of questions that can be used by the teacher, Aylin mainly uses *confirmatory* questions in order to test students' factual knowledge of mathematical procedure and facts, following a sequence of statements at the initiation or revision sections of lessons (e.g. O1-(1-25); O2-(1-9); O7-(1-18)), *focusing* questions to direct students' attention to a specific issue or a combination of two during solving exercises/questions from the test books except O10,11 and 12. Nevertheless, there is third type of questions called *inquiry* questions which could not be identified within the lessons. These questions are asked by the teacher in order to gain information that she/he does not really know (Love & Mason, 1995). Yet she indicated that in order to use questions what she calls *interpretive* there is a need for students with good backgrounds so that they won't have problems with basic mathematical skills that would hinder their progress on their own.

Finally she had reshaped her ideas about making presentations are beneficial for all students and also increasing student participation by nature. Instead of making individual presentations an idea has emerged about a collaborative student work such as working together in a project with a better student assigned as a mentor to the others. Thus, it might be inferred that she had reconstructed at least extended her sense of mathematical communication on *listening to students, student participation and teacher participation, interaction between the teacher and students.*

5.1.3 Inquiry of own practice

Her initial objectives were focused on students' use of mathematical language and understanding mathematical definitions. By mathematical language she was mainly referring to mathematical expressions. It is important to note that she used the phrase *mathematical expressions* in a dual meaning. First, she refers to mathematics as a discourse with its unique tools and processes such as the words, symbols and other visual mediators (Sfard, 2008) signifying its objects, constructing intra discursive [mathematical] narratives and also to the natural language that enables students *expressing the mathematical procedures they did.* The latter meaning was about natural language. She was aiming to use mathematical expressions for students to improve their verbal expressions which she was thinking that students are very bad at.

Despite all, she did not indicate any learning objectives for herself. This was again in tune with my problem statement in relation with the inadequacy of professional development opportunities that are held locally and according to the teachers' diversifying needs in Turkey as well as in many parts of the world having teachers seeing no need for PD at all among its consequences. Nevertheless, it is argued (e.g Billet 2001, Kelly 2006) that working practices of workplaces can encourage or render learning/ knowing in practice which is not less important than the issues addressed about teacher learning above. Affordances as expectations of all participants of the school practices they determined about the kinds of things that can be said and done while engaging with a particular social practice (Kelly, 2006) provided by the schools for the teachers to think about, reflect on their and others' practices and share their experiences with one another also determines what can be learned or how should one act in that school setting. According to Lave and Wenger's (1991) socio cultural theory based on legitimate peripheral participation as a viewpoint for learning, it is this participation into particular social practices enables us to learn, which is not just simply derived from these social settings. Thus teacher's quality of learning is inevitably related to 'the quality of their schools as learning organizations' and conditions of schools in enabling teacher learning during their careers (Jurasaite- Harbison & Lex, 2010; Knight, 2002, p.293). In Aylin's case the conditions in the school was far from being supportive. In fact as I presented briefly by the orientations for each interview situation at the results section and teacher's statements (e. g. IS6, S3) the school conditions was explicitly hindering teacher's practice and learning especially for the last three months during my observation. Aylin was feeling suppressed and like in a "psychological war". She also thinks that their needs, requests and feedbacks as a teacher are not valued or taken into account both at administrative level in her school and at a broader level in the ministry of education. These are all constraining and influencing her stance about professional learning and the quality of her learning.

At one point in her inquiry of practice Aylin set a new set of objectives which are aimed at solving exercises by “working out their knowledge”, and facilitating students in connecting and compare topics in order to solve problems, generate different solution techniques (IS2). Here the word knowledge was used as factual or procedural knowledge that is essential mathematical facts and procedures or algorithms and rules that form the basis for doing mathematics. Her objectives were drawn away from *mathematical communication* as her main foci of inquiry since she thought that she had reached her initial objectives since students have started to “*understand mathematical expressions, explain their solutions make alternative interpretations when solving equations”*(IS2). Those were in line with teacher’s initial understanding of mathematical communication as prioritizing teacher’s control of discursive practices within the mathematics classroom derived from the narrative analysis. In that understanding teacher was characterized as the primary authority in *leading the flow of communication* (IS4, S1) and this discourse of teacher on *mathematical communication* did not change until the 4th interview; that is after about 4 months of inquiry of own practice. In relation to these, Aylin had told that students solving problems all the time and everything being under her control might not be very good (IS4, S4). She also added in the same interview that she had realized she was telling and showing everything hence she had been so exhausted and felt more comfortable and contended then, which can be interpreted as a reorganization of the distribution of authority and power among students on which I will discuss it further in next section.

Teacher’s opinions about repeating an inquiry of own practice in the future were somewhat ambiguous which was due to the insecure relationships between her and her colleagues. She told that there was only one teacher she can trust out of 16 colleagues at her school and that nobody trusts each other (IS6, S6). Here confidence and trust among the participants comes to the fore as important factors enabling the *situated generalization* of teacher inquiry as argued in Simons et. al. (2003), from a tool for learning in and from practice towards “the inquiry as a way of being” as in Jaworski (2006). In contrast to traditional accounts of transfer recommending the generalization of the results to all similar contexts, what Simons et al. (2003) proposes in situated generalization is a process of adaptation and recognition of the relationships within one situation, as common issues and problems by others to their contexts, which is only validated among colleagues/peers by collective interpretation and analysis of these relationships. Hence, despite the affordances within the school does not seem to support the confidence and trust among the colleague teachers, Aylin’s willingness of reproducing that inquiry process with an aforementioned peer may suggest that she had found this process as worthwhile which is a persuasive and as a valid social practice for teacher knowing/learning.

Final theme of the teacher’s accounts of own practice was about teacher’s inquiry process in its own sense. At the beginning teacher set initial objectives to start her inquiry process and additional objectives emerged throughout the process. As soon as she began to evaluate her previous objectives, at one point around her 4th interview, she has actually started to reflect on her practice. Finally when she completed her inquiry she had future thoughts and contributed to her professional learning with many more reflections. Hence, as Hall argued (2009) by the process of inquiry teacher’s having a complete autonomy to choose her focus of learning has been potentially developmental and enables teacher's gradual engagement in research towards engagement with research process.

5.2 Teacher's Discourse

Aylin's mathematical word or symbol use in her lessons is exclusively for the sake of mathematical discourse. The way that Aylin uses mathematical words and symbols might provide further insights about her ways of mathematizing. Even though using different types for realizations of the mathematical words or symbols is argued to extend possibilities for communication (Schleppergel, 2007) the symbolic mediation is still has a privilege among the participants of the mathematical discourse of all contexts from school mathematics to a more rigid scholar mathematical discourse (Sfard, 2008). Accordingly, besides iconic and concrete realizations symbolic realization was the most preferred mediation type for the teacher.

Additionally, how and by who of these mathematical words or symbolic artifacts were realized in that list may tell us about the nature of teacher's mathematical discourse. The teacher was the main actor in these realization processes and the teacher's use of these words was aimed at transmitting knowledge she possessed to her students which she also made explicit many times in her discourse of reflection. In her use of mathematical words and symbols her general aim was calling for students' factual knowledge instead of actively listening to them and trying to make sense of what they say (e.g. O1, 8-16).By looking at the teacher's intent as being the one who holds power to determine the discursive practices within the classroom, it can be inferred that discourse of the teacher remains *univocal* so does the nature of the discourse of classroom. Univocality is about 'conveying meanings adequately' yet it does not aim to 'generate new meanings' (Knuth & Peressini, 1998, p.108). This is also in line with teacher's being the primary participant in the discourse that defines students by her discursive practices in Foucauldian perspective of discourse. In that case, students rarely question or reject teacher's statements and the teacher is 'receiving' student statements in response rather than focusing to understand what their students think and say. On the contrary, new meanings are generated when one uses different statements of self and others to inform own understanding about other's thinking, which is the essence of *dialogic* thinking (Bakhtin, 1986). As for the difference in the realization of mathematical words by the teacher and student; resulting from the incommensurable discourses of the participants is argued to be crucial for learning. Also called *communicational breach* as in Sfard (2008), this type of difference is created primarily by the teacher towards the end of the observation period (i.e. O12, 1-27). . Through the interaction with the teacher as being more proficient participant within the discourse, learning occurs as students individualize the new realization procedures by following her steps. Furthermore a transition occurs from student's direct realization procedures to a more elaborated discursively mediated procedure which is an important task for the school mathematics discourse to accomplish (Sfard, 2008).

Similarly at her first lessons Aylin's mathematical routines were mainly focused on the sequence of actions rather to solve a mathematical exercise or a problem rather than thinking or discussing the conditions of applicability for these actions (e.g. O1, 28-51). This was also reflected in her comments on mathematizing:

> "First of all, in my classes students should know about where mathematics or the subject being taught…should be used. Not because they are required to learn but according to where they will use them.(….)hence they should acknowledge the priority of mathematics considering the most important exam they will take in their lives…" (IS1, S3)

In line with this comment she prompted her students to *use* mathematical narratives and objects in order to reach the solution of a problem situation. She was also the ultimate substantiator of these narratives (Turns 39, 49) whereas students never seem to substantiate any. Despite the word knowing signifies a mental rather than behavioral process the way that the teacher enacts her understanding above within the discursive practice of the classroom, relates to procedures to be carried out in a mathematical routine (e.g. O1, turns 30,37, 43, 49, 51) rather than knowing the applicability conditions or consequences of the discursive action.

The teacher's ways of mathematizing were predominantly in forms of rituals or practical deeds. Teacher's enacting rituals and deeds as primary mathematical routines within the discursive practices of the mathematics classroom have important implications. First, it might be inferred that both students and the teacher are focused on the courses of action instead of deciding *when* a mathematical/ discursive action is applicable, appropriate and accomplished. Hence an important feature of doing mathematics; under which conditions and according to which goals a mathematical performance would be considered as appropriate would be neglected. This is particularly important when students are faced with non routine problems required to be solved without the particular clues provided by those sequences of actions in the school context (Sfard, 2008). Secondly, teacher's focusing only on practical deeds and rituals in her ways of doing mathematizing hinders the construction of new or substantiation of previously endorsed mathematical narratives by the students. This is of crucial importance in school mathematics since "the overall goal of mathematizing is to produce narratives that can be endorsed, labeled as true, and become known as mathematical facts" (Sfard, 2008, p. 223). Teacher's presenting object level rules of mathematics directly as mathematical facts but not as a result of an exploratory process enabling the production or substantiation of a mathematical narrative leads student's to use these rules as extradiscursive objects. Hence students see these object level rules mediated symbolically as if they were non mathematical facts (e.g. O5, 7). In a way by imitating teacher's ways, students' mathematizing also takes a ritualized form which is predominantly about performing instead of knowing a mathematical/ discursive action.

Nevertheless the teacher began to realize her ways of mathematizing slightly different towards the end of her inquiry of practice. At those moments teacher's guiding exploration routines led students to construct mathematical narratives (e.g. O8 & 9). Furthermore, despite not completely abandoning a ritualized form of mathematizing and expecting that students to follow her additional instructions to perform mathematical procedures she has found an opportunity to use those "interpretive" questions (e.g. O11) that she had aimed to use (IS4). Accordingly, in O11, students have made explicit arguments for an alternative solution for the first time to that of teacher's and despite teacher's initial disapproval, they have insisted on their way and finally proposed that to the rest of the class. Hence, these could be taken as traces of a *dialogic* discourse (Knuth & Peressini, 2009) which led production of new meanings within the mathematics classroom.

In teacher's social discourse there was only a limited amount of social word use or produced meanings regarding social relations, her and students' positions and roles as participants of the discursive practices of the classroom. Mainly her social words used in practice were about students and their positions as general social norms for being a participant within the discursive practices of the classroom. Yet, in her social discourse teacher did not articulate much on her position as a participant which she took it as almost stable, and determined inherently by nature. These were compatible with teacher's statements at the pre study interview, which contained hardly any social meanings. Her discourse of reflection or her ways of talking about her practices and the process of her inquiry of practice were producing meanings predominantly about mathematical and pedagogical discursive actions within the discursive practice of her classrooms. Hence the reason why the teacher's not using a mass amount of social words in her discourse was comprehensible. Although some of her objectives for inquiry included statements regarding her dispositions of teaching and learning mathematics there was not any explicit statement focusing on her role and status within discursive practices. So this might explain the minimized social word use of the teacher about herself. Specifically teacher's frequent use of *we* might denote a conscious or unconscious attempt of the teacher to make the students feel more involved. The use of *we* also might be an attempt of the teacher's to strengthen her authority by tacitly pointing out a group of experts which are not present (Pimm, 1987) or to manipulate the conversations by taking the advantage of its vagueness in terms of its referent (Wills, 1977). Similarly the use of the *folks/ fellows* when explaining or questioning an issue can be understood as a teacher attempt to reduce the degree of social distance between her and her students.

The teacher's social routines were mainly in a ritualized form which remained the same until 5th lesson observation for about 3 months later since she has been inquiring her practice. The goal of the routines which are in the form of rituals is bridging a social bond or relationship with others and usually done with others activated by the prompts of one who has a privileged social position or power than others within a discursive practice. Since, there is a need for a prompt that associates with the ritualized actions to be performed, almost no variations of this performance is accepted. By the 5th lesson observation teacher's social routines enacted in the form of rituals has changed slightly. Before that there was not an explicit social routine enacted by the teacher be identified *regarding empowering of the students* to present their ideas mathematically. Hence only one explicit expression might not be sufficient to point out a changing meta rule. Yet, the teacher's social routines regarding empowering students can be said to evolve over the course of her inquiry or might have transformed from being tacit to a more explicit form. On the other hand this might also be due to teacher's reluctance of distributing the circulation of power among the participants within discursive practice of the classroom over the course of her inquiry. This consistent powerful position of teacher can be exemplified by the social narratives endorsed by the teacher repetitively in many situations. These narratives are about teacher's position as an expert or the most knowledgeable one among the participants of the discursive practice and are consistent with Foucault's notion of power/knowledge duality. According to Foucault (1970) the power is in the relations which exist in every social interaction which does not act directly on people. His single category of knowledge and power conceptualizes knowledge

as a form or expression of power thus of relations between individuals (McBride, 1989). Hence teacher is positioning herself with her social narratives as the most knowledgeable/ powerful participant within the discursive practices of her classes.
The use of the words/ visual mediators specific to teacher's pedagogic discourse has a complementary purpose enabling the mathematical and social meaning components come into being within her discourse. These pedagogical words and visual mediators by the teacher includes a specific theme and a purpose in order to make her mathematical meanings relevant for both the other participants/students and for the context of situation within the discursive practice of the mathematics classroom. They also enable her to convey messages/ meanings inherent in her social discourse about herself and students relevant to the teaching and learning of mathematics within the discursive rules and practices embedded in the classroom context. Viewed from this point of view, the use of *question* comes forth as a prominent theme in teacher's pedagogical discourse. Teacher uses the word question for denoting all of the problems and tasks as chosen and added by her within the discursive practice of the classroom. The nature of these *questions* is mostly factual as to test students' knowledge about facts in line with the university entrance examinations being administered nationwide. Furthermore, these questions do not often require students to work out for problem solving. In that sense, these questions were not problems for students to deal with to make sense of their mathematizing within the discourse of the school mathematics. But the teacher accepted them as the only legitimate way to have her students to meet the requirements of these exams.

As I touched upon above, teacher also uses *we/ I give* phrase as a pedagogical phrase as well. This use associates with an objectivist/ instrumentalist view of mathematical knowledge which can be transferred by a more knowledgeable one to others. Learner receives knowledge by an expert and practices it in order to gain expertise on it. Based on the results it can be inferred that teacher's use of these words did not change until the last two lessons. Nevertheless this might not indicate a real shift or change in her pedagogical discourse. At these two lessons the discursive context is slightly different than the other 10 lessons. At the 11th lesson, a student taking the role of the teacher was there and the teacher perceives herself as backing her up. Hence in teacher's eyes that student might not be in a position of *giving any knowledge* to the others. That could be the reason why she did not call on the word *give*. For the last lesson (i.e. O12) which was allowed for problem solving regarding sum and product symbols, students were allowed to discuss their ideas about possible ways to solution and the teacher had stayed behind during these discussions. Hence the roles and relationships of the students and the teachers determined different than the other lessons might have caused the word give to be out of use from the teacher's discourse.

General characteristics of teacher's pedagogical discourse have not undergone a change until 6th lesson observation based on teacher's meta mathematical moves as tactical tools to promote student learning (Knott et al., 2008). The teacher's meta mathematical moves has been diversified from summarizing and rephrasing student's answers to moves including steering, redirecting and prompting. Moreover in the first three lesson observations, questioning as a teacher pedagogical/discursive action did not seem to promote students' participation into the lesson. Though there were signs that the teacher questioning were facilitating an exchange of meanings between students and the teacher (e.g. O2, 23) they were remained as an *encounter* which does not necessitate an interaction or exchange (e.g.

meanings) between the participants (Wickmann & Ostmann, 2002). Beginning from the 6^{th} lesson observation teacher also began to use revoicing move for students to become more active participants which in turn seems to work since students also began to ask questions in order to clarify their understanding at the 7^{th} lesson. At the end of the lesson observation period Aylin has increased the amount of and diversified her meta mathematical moves and began to use questions as means for probing student thinking. Moreover she has maintained her silence during a classroom discussion where she has also listened to her students more *interpretively* which means listening for student understanding (Davis, 1997).

Finally, questions/key questions selected by the teacher in both mathematics and geometry lessons comes forth as a general theme of her pedagogical discourse and the main routine of this discourse is to have these questions solved mostly with a few minutes allowed for students to work individually. Hence most pedagogical routines of the teacher were identified from situations containing questions/ key questions where students are engaged in the solution procedures. By the end of the first semester (i.e. O6) teacher began to use revoicing as a pedagogical routine in order to make students being more active participants of the discursive practice. Next month at the 8th and 9th observation, she started to use a more *dialogic* pedagogical discourse (Knuth & Peressini, 1998) that allows and leads the generation of new meanings within the discursive practice of geometry classroom. Those were the final changes that could be identified recursively a next lessons hence as a change in teacher's pedagogical routines.

CHAPTER 6

RECOMMENDATIONS AND IMPLICATIONS

In the view of key issues raised in this research and recognizing its limitations, I will address future research possibilities and the theoretical and practical implications of this study.

6.1 Recommendations

The data in this study has been collected during eight months covering two academic semesters within certain days of the week depending on my position as a full time teacher. Due to my own work schedule I had to follow certain classes of the teacher which have comprised of geometry and mathematics classes of different 11^{th} graders in the first semester yet I could follow just one class's mathematics lesson at the second semester. Hence my interpretations and inferences regarding teacher's professional learning and the teacher's action discourse as mathematical, social and pedagogical discourses are based on relatively limited to particular classes to which Aylin teaches comprising certain cohort of students with particular (mathematical, social and pedagogical) backgrounds and motives. I can only assume that Aylin's professional learning as a social practice has similar discourse features in terms of action and reflection, but predominantly her *discourse of action* within the discursive practice of all her classes.

There are several reasons that I have chosen to work with Aylin in this research project. First she was an experienced mathematics teacher (with 19 years of experience) who was working at a standard high school representing an average achievement rank based on their scores at the SBS exam (Turkish high school placement test). She had participated in various in service professional development activities/training provided by the National Ministry of Education and indicated that those were neither beneficial nor effective for her professional development. Yet she also indicated her voluntariness as she was open for everything that might improve herself professionally. However, there is a need to conduct a similar research with teachers representing different experience levels, teaching at different school types to various student profiles.

As I have mentioned before, I have conceptualized teacher learning-while inquiring own practice- as a social practice including two main discourses which enabled the exploration of these action and reflection discourses of the teacher. A future research can be done that will focus also on student discourses and learning, track changes at these discourses and compare and contrast them with teacher discourse. In order to do that, the narrative analysis can be expanded by interviewing several students as well. Another possibility for research might be to focus on whole class mathematics discourse while taking multiple semiotic resources as linguistic, visual and symbolic systems into account in meaning making processes of the participants of discourse.

Another limitation of this study is the ignorance of other members/stakeholders of the school community. These are the other mathematics teachers or the teachers of other subjects in the same school, supervisors, school leaders and even parents which a teacher negotiates and interacts with while as part of one's profession. There are several discursive practices manifested within classroom practice each representing distinct cultural, historical, social and contextual variants in which these others participate mentioned above. Moreover, in line with the socio cultural theory and participation metaphor of learning this research entails a view of professional learning which is social and collective in nature. Hence, focusing only on only one teacher's learning in and from practice via her discourse is providing a limited view for understanding learning of teachers as a whole. Specifically a similar research could be done with a group of teachers as members of a learning/ inquiry group or co-learners which would enable the exploration of teacher learning in the course of such a collaborative inquiry process.

Another limitation of this study is somewhat inherent in itself. On one hand as a researcher I try to interpret the actions of my objects of inquiry, on the other hand participant herself also tries to understand the reasons and the consequences of he own actions. Hence, the design and resulting interpretation of both types of inquiry requires attention and diligence so that both sides are to be taken into account. Furthermore, a possible source of bias is my own professional background. As I have been working as mathematics teacher I may have underestimated inconsistencies at Aylin's accounts of practice, her stance of knowing/learning in practice and inquiry and her statements about obstacles of changing her practice in accordance with certain issues (e.g. incorporating proofs and group work into her practice) and about the affordances of the school environment. Nevertheless, by presenting teacher's accounts of own practice as interpretive stories associated with narrative inquiry methodology I aimed to open readers to multiple possible interpretations in the interview texts if the same methodology is used. I also included my voice as a researcher; in terms of my own reflections and thoughts and emotions that I have written in my field notes during my field work. By doing these I hoped to increase the transferability and the trustworthiness of my research project. Having been aware of the qualitative and interpretive nature of my research design I kept in mind, as Kellehear says that '… "research is "reading" of the world, and the task is always on persuasion rather than proving' (as cited in Fraser, 2004).

Finally, according to the results of this research project, a number of research areas/questions emerged can be looked in a future research.

- How does the network of mathematics education practices shape pre service and in service teacher's practices and their own accounts of practice?
- How does proofs and proving can be better incorporated in the Turkish (high) school mathematics practice in a way that balances both the discursive (mathematical as a discipline), social and pedagogical requirements of teaching and learning mathematics, helping to develop mathematical meaning making and caters to the demands of the university examination system which regulates and determines that practice as a disciplinary power?

- What changes (if any) in teachers' learning if the inquiry is carried with a couple of teachers who trust and respect each other from the same school?
- What changes (if any) in the teachers' knowing in practice would be found if the study is carried with a group of mathematics teachers and a researcher working as research partners conducting a collaborative inquiry?
- How do different modes/ types of realizations of signifiers (words and symbols) occur in mathematical discourse?
- What is the process that connects teachers' and students' realization processes during mathematizing?
- How does teacher's ways of mathematizing plays a role causing students' mathematical meaning making?
- How does social routines as being part of teacher's discourse determines power relations and the production of mathematical knowledge within the discursive practice of the mathematics class?
- How a mathematics teacher can be encouraged and supported to use more *inquiry* type questions within the discursive practices of her classes?

6.2 Implications

The outcomes of this study have implications on both theoretical and practical basis. First I present the theoretical implications for the researchers of mathematics education then I continue with the practical implications for the teacher educators and the practitioners of the mathematics education area.

6.2.1 For the researchers of mathematics education

One of the aims of this study was to contribute to a better understanding of teacher learning while conducting an inquiry of own practice as a social practice. In doing that, the social practice of teacher learning was identified with its discourse which was further categorized as action and reflection discourses. Framing teacher learning as a social practice enabled to explore *learning as meaning making* and this enriches the theoretical/ methodological perspectives available in terms of research design and interpretation. Whether accepting mathematics as a language or not, this enables to study how meanings created by the students and the teachers within the discursive practices of mathematics classes. This was done in this study by focusing on the three main functions of the language derived from the SFL theory and social semiotics. Further these meanings construct mathematical, social and pedagogical discourses as layers of student or teacher discourse that construct and are produced by a set of anonymous and historical rules that are, the discursive practices of the classroom.

Another implication of this study for the researchers is related with the potential of combining multiple traditions and disciplines as for method and analysis of data. Potential of using discourse analysis and social semiotic approaches in conjunction with qualitative methods in mathematics education research is promising for understanding the teacher learning process at least for two reasons. First, that methodology provides a deeper insight of how and when does meaning making occurs this might lead to more other valuable research that will help to understand about teacher knowing/learning in practice. Second, tracing teacher/student discourse both in action and reflection modes within the discursive practices of mathematics classes might help to determine and better understand the events, conditions and actions that lead to (mathematical) meaning making.

6.2.2 For teacher educators and practitioners

First of all, this study has shown that the language and mathematics are very two important areas that has to be considered together at all levels of teacher education. Nowadays, social and communicational theories of teacher learning/development and discursive approach to teaching and learning of mathematics gain prominence. Hence course units covering language/discourse of mathematics or mathematics as a discourse etc. that aims to develop appreciation and understanding of these issues in student teachers might cater for the challenges that arise in mathematics classrooms. Thus, student teachers might have the opportunity to explore different features and aspects of teacher discourses (as in mathematical, social and pedagogical discourse) that lead to mathematical meaning making.

Another implication for teacher educators is about encouraging student teachers to conduct a self inquiry process throughout their education and their school practicum. By this means student teachers might have the chance to experience workplace related learning and an inquiry before actually becoming a teacher. Accordingly, in service teachers (as it is in Aylin's case) perceive learning associates primarily with being a student hence they don't see themselves as learners and quit determining professional development or learning objectives for themselves. This is also a serious issue; I believe that must be handled at the pre service phase of teacher education, hence at the teacher education colleges or institutions. Conducting such a study or participating one might enrich student teacher's perception of being a teacher, learner and a researcher which will support to blend these in forming a professional identity. Teacher educators are also recommended to conduct such an inquiry of own practice that involve systematic and planned observation and reflection in order to gain further insight and knowing about their own practice hence cater for their own professional development.

Third, teachers generally ignore mathematical meaning making and allow for a lot of exercises resulting students to mathematize in a ritualized form. Even though rituals are the first steps to individualize other people's discourse an excessive focus on how should a mathematical routine should be performed is likely to hinder exploration processes (Sfard, 2008). In relation to that, teacher's using *confirmatory* or *focusing* type questions rather that *inquiry* questions hinders students to construct mathematical narratives themselves. Furthermore, teachers' habits of listening to their students are also problematic. This argument is supported by the participant teacher of this research, Aylin stating that this was a fact that she had discovered after 19 years of teaching. Teachers should learn about changing their listening to their students in the mathematics classes from being evaluative towards being at least interpretive which is among one of the most important skills of a teacher to develop, as Davis (1997) argues. This study showed that such a knowing can be generated from teachers' practice via an inquiry of own practice as conducted in this study.

Another implication for practitioners or teachers is about how a simple and effortless design of inquiry of practice can be beneficial for generating knowing from, in and about their own practice. Without training or attending to any external PD activities and not even allowing extra time for an academic/ professional reading or study regarding his/ her professional development foci teachers could also learn in and from their practice. Besides, the simplicity of the design which involves identifying a learning foci, teach and reflect on one's lesson with the help of videotapes and reflection logs/forms allows replication of it with a group of teachers who trust and respect each other professionally. As Aylin also indicates that she had started to learn soon after her inquiry of practice began. For instance, she told once she had realized that she did not listened to what her students say in detail and she was focusing on that for 2 months which was after 2 months since her inquiry of own practice began. Yet, it took far more time (approximately 4 months more) for her to reflect this awareness to her discursive practice. This process could again be speeded up by participating in a teacher group who are also focusing on the same or similar learning foci or objectives and conducting both individual and a collaborative inquiry of their practices.

At that point a word of caution for teachers/ researcher inclined to use this design should be made. Reflection logs/forms were the essential parts of this design that teacher's further rethinking of the events and actions take place at particular lesson and about how one handled them; how could one possibly do it differently. In this study I have already prepared these forms as a list of questions. Yet, following questions by using critical incidents from one's own practice that are times of which a teacher feels that he/she is the most and least effective as a teacher can help developing almost all reflection forms regarding any learning foci:

- What was the situation?
- What events led to this situation?
- Who was involved?
- What did I think, feel or want to do in the situation?
- What did I actually do and say?
- What was the outcome?(Cobb, 2011, p.97)

Finally, one last important thing to note for teacher educators and the practitioners is about teacher attitude towards professional development activities. Teachers like Aylin, who indicates her resistance mainly towards practices of Ministry of Education which is being criticized as being much centralized and performing redundant control over individual practices of teachers are cautious about any professional development opportunity and they might not set any professional learning goal for themselves at the initial phases. In that case they should have been given opportunities to rethink about their practice as in this study like writing up reflection logs/forms, watching short episodes of videotaped sessions from their lessons or by doing observations of other teachers' practices. Nevertheless, as I told above at initial stages of inquiry teachers might be prone to report that what they were doing were best for their students and perfectly practiced. Hence, they would not be willing to alter or simply think about their practice. Whether being a researcher from academia or a teacher/colleague who replicate a kind of inquiry of practice as in this study should be a trusted and professionally respected person for another or at least be patient enough to gain confidence of the other party. In this research process I had to wait 4 months with almost complete silence for this and also to make sure that I was not there for judging or evaluating her or her practice. Then it took almost another 2 months for Aylin to try new routines that is, new ways of doing and telling things within the discursive practice of her mathematics classes. Yet she did not represent all of those new knowing into her practice that she had identified at her discourse of reflection. Hence presumably it might take much time and effort for teachers to accomplish this goal and again communication with and participation in a teacher group forming a community of learners is a highly likely candidate for such a mission.

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APPENDIX A

DEFINITIONS OF TERMS

Discourse:

Among many definitions of the discourse from the perspective of various schools of thoughts, discourse in this study mostly refers to *language in use* where language is socially situated. As Lemke (1995, p.6) argues discourse is “a social activity of making meaning with language and other symbolic systems in some particular kind of situation and setting”. Yet according to this definition, while enacting specific activities or realizing specific ways of being in social and cultural settings we use other tools as well as language (Gee, 1999). Hence in order to be complementary “broader range of social practice including non linguistic…instances of language” (Schiffrin, Tannen, & Hamilton, 2001, p. 1) is also considered.

Discursive action:

Activities performed as part of a communicative event/ situation (Sfard, 2008). When people participate in an activity of communication their actions customarily produce re-actions. These actions generally form a well defined repertoire of actions or systems which are called communicational. Communicational (discursive) actions are *about* objects which may lead to an action on, or another communicational action about an object, whereas practical actions are direct actions *on* objects (Sfard, 2008, p. 89). An example of a teacher’s discursive actions while she engages in a conversation with student S9 from the 5th lesson observation is below:

> S9: First of all, I have separated 4 and 9 into their factors
> T: But why?
> S9: In order to get rid of logarithm.
> T: What is the base of the logarithm?
> S9: Then I write 2 logarithm 5 squared to base 2, and here they cancel out each other

Deeds:

A set of meta rules of practical actions that aim a change in the physical objects or the environment (Sfard, 2008). Implementation of a deed is one among three types of mathematical routines according to Sfard’s (2008) communicational theory of human thinking.
An example of an implementation of a deed by the teacher from the 5th lesson observation is below:

> T: Ok now, how are we going to use these properties in questions, let’s see that.
> The question is**,** What is $216^{\log_6 5} = ?$ Now **<u>we try getting rid of logarithm in all</u>**

<u>questions don't we?</u>
S2: 5
T: [*pretending not to hear]* How do **we get rid of logarithm?**
S3*:* By using 8. [*Referring to 8^{th} property*]

Discursive practice:

Discursive practices are "a body of anonymous, historical rules which produces knowledge always determined in the time and space that have defined in a given period" (Foucault, 1970, p.117). By adapting Potter's (2008) example of medicine to education, discursive practices of education might include: historically evolving discourse that shaped the educational practices, teaching and learning processes, the practices of distinguishing success from fail, examination and the normalizing procedures of the individual and also the architecture of the schools, classrooms or the teacher's room etc.

Discursive practices in the mathematics classroom:

The discursive practices of the mathematics classroom might include the ways of doing, knowing, presenting, believing and understanding of mathematics constituted by the actions of the participants their interactions with each other and; mathematical meanings and objects produced within the classroom. Textbooks and their use, teaching and learning practices, teacher and student roles; rules, norms and routines of the mathematics classroom are among examples of the discursive practices of the mathematics classroom. Hence it involves a conglomerate of the practices of the participants, of their interactions, of production and utilizing the conceptual and physical resources besides the influence of more powerful discursive practices such as curriculums, standards and examination regulations etc.

Endorsed narratives:

When narratives are defined as "any spoken or written text which describes the objects, relations between objects or activities with or by objects" (Sfard & Avigail, 2007, p.8) these narratives are inherently open to endorsement that is to be labeled as true. For example, in mathematical discourse these endorsed narratives include theorems, proofs and definitions.

Explorations:

Explorations are routines of which their performance counts as completed when they end up with producing or substantiating endorsed narratives (Sfard, 2008).

Interpretive stories:

An interpretive story is an alternative way to represent interview transcripts that preserve its situated nature and offers its readers a multiple possibility of interpretations (McCormack, 2004).

Knowing in practice:

Knowing in practice is a process that is distributed among all participants of the practice (e.g., teachers, students and the resources available in the lesson) (Kelly, 2006), rather than a commodity being possessed by an individual.

Language:

From the social semiotic perspective, language is a social practice where meanings are exchanged in interpersonal contexts (Halliday, 2003).

Mathematical communication:

Mathematical communication is a process of which enables sharing and understanding mathematical ideas, also closely related to reasoning and problem solving processes of teaching and learning mathematics (NCTM, 1991).

Mathematical discourse:

Mathematical discourse focuses on participant's ways of mathematizing. It also involves the mathematical meanings expressed by multiple semiotic systems (Jamani, 2011; Lemke, 2003; Sfard, 2008).

Mathematizing:

Mathematizing is defined as "participation in the mathematical discourse, and doing mathematics" (Sfard, 2008, p.299).

Meta level learning:

Meta level learning occurs with change(s) in meta rules of the discourse. This entails identifying objects or defining terms will be done in a different way in fact, some words will change their use (Sfard, 2008).

Meta (discursive) rules:

These are rules that define patterns in the activities of participants of discourse (Sfard, 2008).

Pedagogical discourse

This discourse focuses on the organization of the content of the discourse. It includes strategies and tactical tools for students to participate in the mathematical discourse. Pedagogical discourse is characterized by the textual meta-function of language within any instance of language use that attaches discourse to its context, as adopted from Halliday (1978).

Rituals:

Rituals are (mathematical) routines as a set of discursive actions of which their primary goal is to create and sustain a social bond with other people (Sfard, 2008).The most important thing in the ritual action is that it is strictly defined and followed with accuracy and precision so that different people can perform it in an identical way, possibly together. In this case, the precise, accurate performance of the routine procedure is the only requirement. In implementing rituals the issue is about performing not knowing; hence rituals are highly situated and associated with prompts.

Routines:

Routines are a set of meta rules that define repetitive discursive actions (Sfard, 2008). They are further categorized into two subsets: First is *how of a routine* which consists of meta rules determining the course of actions to take or the procedures to be followed throughout a discursive performance. Second is *when of a routine,* consisting of meta rules which are determining the conditions or situations of which the performance defined above would be considered as appropriate by the participants (p.208).

Social Discourse:

It focuses on the positions of and the ways of taking part of the participants in relation to each other and to the content of the discourse. Social discourse is characterized by the interpersonal meta-function of language within any instance of language use that is a participatory function of language, as adopted from Halliday (1978).

Teacher inquiry:

Teacher inquiry is a teacher's systematic and intentional study of own practices (Dana, 2002 p.5).

Teacher learning:

Teacher learning is a process which is conceptualized as a learning of practicing professionals and knowing in practice, where knowing is generated among all participants of the lesson -teacher and students-and is "socially shared and distributed across participants and resources" (Kelly, 2006, p. 510).

Teacher's practice:

Teachers practices involve teacher's *actions* that relates to their teaching as well as what they think, know and believe about which is, their *accounts of* what they do (Simon & Tzur, 1999).

Visual mediators:

Visual mediators are "the image providers of our talk while we communicate" (Sfard, 2008, p.147) and like many other scientific discourses mathematics has also got its own symbolic artifacts enabling its communicative processes.

Word use:

All discourses are made distinct primarily by the *keywords* they use. The use of these words is particularly important since "the meaning of a word is its use in the language" (Wittgenstein, 1953, p.20)

APPENDIX B

SELF EVALUATION FORM

Table A.1: Self evaluation form

	Önemi				Benim Düzeyim				Gelişimim için önceliği			
	1	2	3	4	1	2	3	4	1	2	3	4
MATEMATİK ÖĞRETİM SÜRECİNİ PLANLAMA VE UYGULAYABİLME												
Ortaöğretim matematik öğretim programlarının vizyon, felsefe ve kuramsal dayanaklarına uygun öğrenme-öğretme ortamları hazırlarım.												
Ders planlarımı matematik dersi öğretim programlarında yer alan kazanımları ve öğrencilerin hazır bulunuşluk düzeylerini göz önüne alarak hazırlarım.												
Matematik dersinin amaçlarını gerçekleştirmeye yönelik çeşitli ve etkin öğretim strateji, yöntem ve teknikleri kullanırım.												
Öğrenme ve öğretme sürecini gerçek hayatla ilişkilendiren örnek ve etkinlikler hazırlarım												
Öğrenme ve öğretme sürecini basitten karmaşığa doğru tasarlarım.												
Matematik öğretiminde matematiğin tarihsel, kültürel ve bilimsel gelişiminden örnekler veririm												
Öğrencilerin matematikteki öğrenme zorluklarını ve kavram yanılgılarını gidermeye yönelik uygulamalar yaparım.												
MATEMATİK ÖĞRENMEYİ TEŞVİK EDİCİ SINIF ORTAMI OLUŞTURABİLME												
Derste hem kavramsal anlamayı ve hem de işlemsel beceriyi geliştirmeye yönelik uygulamalar yaparım.												
Matematiksel formül ve algoritmaların nereden geldiğini gösteririm.												
Derste matematiksel fikir yürütmeyi ve muhakemeyi destekleyen bir sınıf ortamı oluştururum.												
Derste matematiksel düşüncelerin doğru ve etkili iletişimini destekleyen sınıf ortamı oluşturur.												
Öğrencilerin derse aktif katılımını sağlarım												

Table A.1 (Continued)

Öğrencilere derse katılım için eşit fırsatlar veririm.												
Her öğrencinin ulaşılabileceği yüksek öğrenme hedef ve beklentileri oluştururum.												
MATEMATİK ÖĞRETİMİNDE UYGUN KAYNAK MATERYAL VE TEKNOLOJİLERİ UYGULAYABİLME VE GELİŞTİREBİLME												
Matematik öğretimi için gerekli özel araç gereçleri kullanırım. (cetvel, pergel, iletki vb.).												
Matematik öğretimiyle ilgili ulaşılabilir kaynak ve materyalleri (kitap, video, somut araç-gereç vb.) kullanırım.												
Matematik öğretimiyle ilgili ihtiyaç duyulan materyalleri geliştiririm.												
Matematik öğretimi için geliştirilen uygun internet tabanlı kaynaklardan (web sitesi, animasyon, küçük uygulama [applet] vb.) faydalanırım.												
Matematik öğretiminde kullanılabilecek ulaşılabilir teknolojilerden (avuç içi bilgisayar, veri toplama cihaz ve algılayıcıları, grafik hesap makineleri, akıllı tahta, projeksiyon cihazı vb.) faydalanır.												
Matematik öğretimi için geliştirilen bilgisayar yazılımlarını (dinamik geometri ve istatistik yazılımları, elektronik tablo, bilgisayar cebir sistemleri, grafik çizdirme yazılımları vb.) kullanır.												
MATEMATİK ÖĞRENİM SÜRECİNİ İZLEYEBİLME VE DEĞERLENDİREBİLME												
Öğrencilerin matematikte öğrenmekte zorluk çektikleri konuları, nedenlerini ve kavram yanılgılarını belirlerim.												
Öğrencilerin öğrenme hızlarını, matematiğe yönelik ilgi, istek ve hazır bulunuşluk düzeylerini belirlerim												
Öğrencilerin kariyer planları ve akademik hedefleri doğrultusunda matematik öğrenim süreçlerini düzenlerim												
Matematik öğretim programındaki kazanımlara ve öğretim yöntemlerine uygun çeşitli ölçme-değerlendirme tekniklerini (performans görevi, proje, ürün dosyası, sunu, öz değerlendirme, kısa sınav vb.) kullanırım.												
Ölçme ve değerlendirmede hem sonucu hem de süreci göz önüne alırım.												
Ölçme ve değerlendirme süreçlerini kullanarak öğrencilerin öğrenmelerindeki eksiklikleri belirlerim.												
Ölçme ve değerlendirme sonuçlarını kullanarak öğretimde gereken düzenlemeleri yaparım.												

APPENDIX C

SAMPLE TEACHER REFLECTION FORM

Öğretmen: ____________________ Gözlemin yapıldığı tarih:

.

Sınıf: ____________________ Düşüncelerin yazıldığı tarih:

.

Öğrencilerin kavramları anlamalarına yani kavramsal anlayışlarının gelişmesine yardımcı oldu mu?

Matematiksel düşüncelerin doğru ve etkili iletişimini destekleyen bir ortam yarattı mı?

Matematiksel çıkarımlar ya da tahminler yapmaları için fırsatlar tanıdı mı?

Öğrencilerin derse katilimi için eşit fırsatlar verdi mi?

Öğrenciler verdikleri cevapları/ çözüm stratejilerini açıklayabildiler mi?

Çoklu bakış açıları veya çözüm stratejileri değerli bulunup desteklendi mi?

Öğrencilerin matematik ile ilgili ifadeleri değerli bulunup bunlar üzerine bir tartışma yapıldı mı veya bu ifadeler sınıfta ortak bir anlayış/akıl oluşturmak için kullanıldı mı?

Matematiksel formül ve işlemlerin nereden geldiğini gösterildi mi?

Bir konunun anlaşılması için öğrencilerin akıl yürütmelerine (ve muhakeme yapmalarina)ve tartışmalarına yeteri kadar izin verdim mi? Öğrencilerimi bu konuda destekleyecek neler yaptım/yapabilirdim?

APPENDIX D

CONVENTIONS FOR THE TRANSCRIPTIONS

Table A.2.: Conventions for the transcriptions

Symbol	The explanation of this symbol
Obs#	The number of observation
O1-12	Number of the observation-number of turn
O1, 12-16	Number of observation, between turns
IS1	Interpretive story number 1
IS1, S5	Interpretive story number, story number
M	Mathematics
G	Geometry
Ss	Some students
S1	Denoting a particular student
San	Denoting an anonymous student, could not be identified
[]	My explanations, or interpretations
(…)	Inaudible due to the humming noise
ST	Student who undertakes a teaching role

APPENDIX E

TRANSCRIPTS OF CLASSROOM OBSERVATIONS

Table A.3: Transcripts of classroom observations

Turns	Obs#1 October 20th 2011 Geometry lesson
1	T: We have seen the properties of quadrilaterals lately, am I wrong?
2	Ss: yes
3	T: Now, if we could make a revision from the beginning to the end, we had key questions and if we have time in the second lesson we will move on to the area of the quadrilaterals. Next week, we will continue with multiple choice problem solving and hope that we will move on to the special quadrilaterals the week after next week. We have written 8-9 properties related with quadrilaterals which we have been dealing for 2 weeks. What were they? Let's remember. No need to sort them out. Then we may begin solving our questions. What did we said in the first place?
4	S1: The sum of the interior angles is 360 degrees
5	S2: Four sides and angles are equal to each other.
6	T: Are the length of the sides equal?
7	Ss: No
8	T: We have called a polygon that has four sides and vertices, quadrilateral. Anything else?
9	S3: The sum of the interior angles…
10	T: Else?
	(…)
11	S4: If there are opposite angle bisectors, the difference of their angles is…
12	T: Half of the sum.. in case that our angle bisectors are adjacent… If our vertex angles are angle bisectors…
13	S4: What I say is this teacher. If angles are at opposite…
14	T: Yes, then it is the half of the difference. If our vertex angles which are angle bisectors are also adjacent then we told that it is the half of the sum of two angles. What else?
15	S5: If the diagonals are perpendicular then the sum of the squares of the opposite sides was equal to the sum of the squares of the other sides.
16	T: If there is a perpendicularity for the diagonals then this is one of the properties that we use to find the length of sides
17	S6: Teacher shall we say "the parallelogram when having joined the middle points?"
18	T: Yes go on.
19	S6: What we do with the middle points is…if it is perpendicular is…
20	T: We have an arbitrary quadrilateral and this quadrilateral…?
21	S6: When we join the middle points it is a parallelogram.
22	T: Ok. When we say middle points we should refer to somewhere clearly: Middle points of What? Shouldn't we? We have an arbitrary quadrilateral at hand. When we join the middle points of the length of it's sides we construct a quadrilateral once again. And this quadrilateral is a……[pauses and waits for the student to complete the sentence]

Table A. 3: Continued

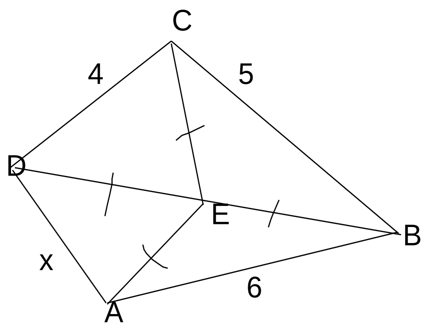
23	Ss: Parallelogram
24	T: If there is no other information, yes. If the first quadrilateral's diagonals are perpendicular then the quadrilateral at the middle is a rectangular.
25	T: The perimeter can also be found by the two sides or the length of the diagonals of the quadrilateral. We will also be emphasizing when we head on to areas in a minute. The area of that quadrilateral is the half of the bigger quadrilaterals. Hence if we can find that area we can find the other one. Now we have a few key questions, let's solve them. Let's write those. (draws the figure on the board)
26	 According to the figure what are the possible values of x? (Image reproduced)
27	Ss: Shall we solve on our books or on notebooks?
28	T: If you have your books ready then you may do on it. Draw the question with me. The faster you draw more questions we can solve.
29	T: Okay. We have always found single value until now at the problems we have solved. It says: 'possible values of x'. Under which condition this can be like that?
	Silence…
30	T: Don't we solve questions by using triangles in general? Does this ring a bell?
31	Ss: There was...an inequality.
32	32 T: Yes. The triangle inequality. Remember…
	33 S4: The difference of two sides, the side which remains in between…
	34 S7: It should be bigger than the sum and smaller than the difference.
	35 T: One by one.. Speak by asking your turn. You say.
	36 S4: If we know two sides then we can find the middle x by taking both the sum and difference of them.
	37 T: How could we? How could that middle x be?
	38 S4: It is bigger than the sum of the two but smaller than the difference.
	39 T: We were calling that the triangle inequality. 3 numbers that I make up would not always construct a triangle. Then we could only find values more than one. Then we need to use that.
	40 Ss: Let it be 2 to 4 and hence 6 to 3.
	41 T: How may it be?
	42 Ss: May we equally dissect?
	43 T: Ok. Everyone listens to me. Why are these equalities given?
	44 S1: The magnificent trio!
	45 T: Yes. Where?
	46 S1: At the Angle C
	47 T: Angle C? But why don't we say the Angle A also?
	48 Ss: x is equal to 4!!!
	49 T: Angle A is also…isn't it?. Now we should remember our triangle knowledge. In this triangle CE is the median and is equal to the length of sides that it sets apart. Since we call magnificent trio, this vertex angle is 90 degrees and A is also 90 respectively. We call it by seeing. What were we using in acute triangle?

Table A. 3: Continued

	50 Ss: Pythagoras
	51 T: Then use it, how were we using it?
	(…)
	52 S7: Isn't there an 6-8-10 teacher?
	53 S8: It is like square root of 5
	54 SAn: Yes
	55 San: Since it is Pythagoras the square of 4 and square of 5 here. I called a there, here is $x^2 + \sqrt{41}$
	56 T: Or this side is common for both, isn't it? By no need to call a we can say then $x^2 + 6^2 = 544$ we can find from here. Any questions? If not then move on.
	57 S9: Teacher, how did you explain?
	58 T: Without using variables, without saying a to here, we can use directly Pythagoras, this side is common for both then the square of 4…
	59 T: Now another key question, now I will have you written a warning. You can solve the other questions like these type since we have solved our key questions from each of them, now shall we move on to areas.
	60 T:The area of a quadrilateral is $\frac{e.f}{2}\sin\propto$. $\propto$ is the angle between the diagonals. And e and f are diagonals. Now let's solve some problems. (…)
	Observation 2: 20th October 2011 Mathematics Lesson.
1	T: What was the conjugate of z?
2	S1: By changing the sign of the imaginary part
3	T: Yes it was the symmetric of it to *y*-axis; modulus of all is under square root a square plus b square. By noting that all are equal to each other, lets write a question. Remember we have solved questions including $a+bi$, now let it be a question including modulus z. $z - i = 5 - \|z\|.i$. Find the z complex number?[first reads and then writes it on the board]
4	T: What are we going to do?
5	S2: We write $a+bi$ in substitute for z.
6	T: What is the difference of this question than other questions?
7	Ss: There is modulus z.
8	Ss: There is only absolute value of z.
9	T: $a+bi$ as a substitute for z and for modulus z ?
10	S3: The square root of a square plus b square.
11	T: [*repeats*] And what we have there? Is it a root procedure? Do we have problem with root procedures?
12	[*One student comes to the board hesitantly and writes* $a+bi$ *as a substitute for z. After reorganizing the equality finally writes* $a+(b-1)i = 5+\sqrt{a^2+b^2 i}$]
13	T: Now here there can be seen the equality of two complex numbers.
14	Ss: Yes
15	T: The real part equals to real, imaginary equals to imaginary part. Let's note it as a warning again: the properties of the absolute value. The property that I gave there is actually the first property which I will not write again. What was it? While we were doing conjugates?
16	S3: Multiplying the conjugate with it gives itself.

Table A. 3: Continued

17	T: Multiplying the conjugate with it? [repeats with no tone] What was it? A complex number….
18	S4: Was it z square?
19	T: It was a square plus b square. Now there is the square root of a square plus b square, then the product of one complex number with its conjugate is the square of its modulus? It is obvious, isn't it?
20	Ss: Yes
21	T: 2nd property: z_1 multiplied by z_2 is equal to..
22	Ss: Separately
23	T: Yes is equal to their product respectively.3rd, what is the modulus of z1 over z_2?. The division of z_1 to z_2 respectively. 4th: For n is an element of N ,$\|z\|^n = \|z^n\|$ This is the property that helps us most. What does it mean here?
24	S4: The power of z complex number within and outside of the modulus is equal.
25	T: Now, folks, I'd like to ask you that way: How were we taking any power of a complex number? How could we find any power with what we know so far?
26	S5: We divide it by 4
27	T: No that was for the power of "i" imaginary number. Now we have both real and imaginary parts, how can we find any power?
28	S5: We were multiplying the number side by side as many times as the power.
29	T: What if it is the 25th power?
30	S6: If we say 25th power of 2 we multiply 2 by itself 25 times,
31	T: Pay attention to what I say, there is also an imaginary part.
32	S3: Respectively
33	T: How come? How will you separate?
34	S3: For example if is $2+i$ then 2 to the power 25 plus i to the power 25.
35	T: You cannot do that, how could we do it?
36	S8 :For instance for $2+i$ to the power 25, we can write it like $2+i$ multiplied by $(2+i)$ to the power 24. Then we can find it.
37	T: Then I am asking how do you find the $(2+i)$ to the power 24?
38	S8: Then we separate $2+i$ again and take $(2+i)$ square
39	T:[*Raising her voice*] While solving power questions for complex numbers with what we know so far we have solved by using the powers of $1+i$ and $(1-i)$ for high powers. Now if the number we are given cannot be written in the $1-i$ or $1+i$ form then how can we find that high power? There, that property helps us a lot. I find the modulus of that number regardless of whatever power it is, the thing that I find in the modulus is a number right? There is no imaginary part only the real part. It is easier to find that number's power right? That's why finding an arbitrary complex number's modulus or finding the power then taking the modulus, which one is easier?
40	Ss: Finding the modulus then taking the power
41	T: Yes it is much easier to find the modulus than taking the power of that number. This will help us a lot at the questions, ok then, there is one more the triangle inequality in case we come across to. $\|z_1 - z_2\| \le \|z_1 + z_2\| \le \|z_1\| + \|z_2\|$

Table A. 3: Continued

42	T:Now the question is if $z=(1+i)^6$ then find $\|z\|$=? Started with simple ones. Now, $1+i$ were our criteria. It feels easy to find it's power, you can take the power and then find the modulus or find the modulus then take the power. There is no activity from here folks, you're too quiet.
43	S2: square root 65
44	S3: it comes 8…
45	S4: S2 there is no real part.
46	T: [*after waiting 40-45 seconds*]:Let's do it, now can I write it that way? Can I write like i to the power 6 and 1 to the power 6? I see that some of you writing that way. Those who write that way please think that again…I take the power of $1+i$ then the modulus of it. [*one student comes to the board to do the operations]*. Now we are at our third week, we have gradually become able to explain the operations we did. *[(to the student) teacher corrects the notation errors whisperingly that the student did while bringing his solution to terms*] Look how important is the act of writing!
	[There are two – more similar questions solved at the rest of the lesson]
47	At Tomorrow's lesson we will solve previous year's University Entrance Exam questions from 80's to 2011 related with thus far, I mean until modulus. Everyone should attend the class.
	Observation #3: 14th November 2011 Mathematics Lesson
1	We have started with the exponential function. This function constituted a first step for logarithm function. What was that step? Where did it help?
2	S1: The inverse was giving the logarithm function
3	T: Yes we have told that the inverse of the exponential function was logarithm function and we have also told that for in order that there is an inverse of a function it has to be 1-1 and onto. Since the exponential function both 1-1 and onto there is an inverse of it and it is called the logarithm function. We have investigated the graphics also; we have looked at the lines in our palms. Do we remember? Which one was the logarithm line?
4	S1: This one.
5	T: If we lay our hands parallel to our thumb the one above was exponential, the line in between was $y=x$ line, the other one was the symmetric of it which was the logarithm function. We have told that they are the inverse of each other, and then we have done some operations regarding finding inverse function. We have told that If the function which is given to us is exponential we must definitely find the inverse of it as logarithm, and vice versa. We have solved a lot of problems related to those and I give also home works, ones that haven't completed yet, please do it. After that we have started writing the extended range of the logarithm function. What did we tell and write about the extended range?
6	S2: we have drawn the increasing and decreasing graphics
7	T: No, about the extended range?
8	S3: Thinking of logarithm *b* to the base *a*, where *a* should not be 1 which was the first condition. The second condition was *a* should be positive, and the third condition was *b* should be positive. If it satisfies these conditions we could find the extended range and then we can continue with the inequality solutions which we have learnt last year.

Table A. 3: Continued

9	T: Yes, now let me summarize what your friend said. In logarithm the base was the same as the exponential function. We already noted that in finding the inverse of one of it if it is exponential we should find a logarithm. The base a there and the base a in logarithm was the same we have told also when defining the exponential function, for all powers of 1 our base should be different from 1. So if the same base a is also the base for our logarithm then our base should be certainly different from 1 and bigger than zero. One thing more… As we said that the place to write is important. The expression whose logarithm is taken when it comes to logarithm the base should be written slightly below logarithm and the statement whose logarithm is taken should be written in line with the logarithm. There, the region of which those two conditions are commonly satisfied. Ok now, let's continue solving problems we have solved one or two that's not enough. We will solve more. Now let's move to questions.
10	Find the extended range of the $\log_{5-x}(x-2)$
11	T: Now, how can we implement those three rules we told to that question?
12	S4: We make both sides bigger than zero and the base different than 1.
13	T: Two for the base and one for the statement whose logarithm is taken. What is our base here, fellows?
14	S: $5-x$
15	T: What should $5-x$ not be?
16	S3: It should not be 4
17	SAN: should not be negative
18	Ss: Should not be 1.
19	T: Yes, should not be 1. Let's write that right away? Everyone look at the board.
20	This means x is different than 1 so if we take it to the other side x is different than 4.(Also writes symbolically what she says). What was the second condition?
21	Ss: the base should be bigger than zero.
22	T: Repeats, (writes symbolically) means $5-x$ is bigger than zero. The meaning of 5 is bigger than x has the same meaning with x is smaller than 5. Right fellows? Hence (writes and reads it) $x<5$. and our third rule. The statement of which logarithm is taken is bigger than zero. Hence $x-2$ is bigger than zero. Then x comes bigger than zero?
23	Ss: Yes.
24	T: Now, let's see them at the number line. Since these are first expressions are of 1st degree, we can directly illustrate them on number line, but if we come across with 2nd or bigger degree inequalities then we have to make table investigations that we will do soon. Ok, let's put numbers one by one. 2, 4 and 5. Now, I exclude 4 you know, x must be different than 4 and smaller than 5. How can I say that as an interval?
25	Ss: the interval of 2 and 5.
26	S5: teacher the open interval of 2 and 5 different than 4.
27	T: I just say, x is smaller than 5, do not take 2 into account, we did not come to that. x is smaller than 5 means, if we say it as an interval. How can we say it?
28	S6: The open interval of minus infinity and 5
29	T: [*Repeats*] hence round parenthesis. Therefore, if I say minus infinity here and plus infinity here (writes on the number line). From minus infinity to 5, 5 is not included, we left blank inside.[*Draws the interval as an additional line segment below the number line with a blank circle at the end representing 5*] The other one is x is bigger than 2. 2 is not included,
30	Ss: Positive infinity interval

Table A. 3: Continued

31	T: The interval of 2 and, 2 is not included, plus infinity. [*Draws that in a way similar to the previous interval* $(-\infty,5)$] Look now, where is the common region? I mean where I draw the double line. Here, isn't it? Then I write that as an interval, the extended range is… 2,5 open interval, but we have noted that 4 should not be there…So minus 4 [*Writes (2,5)-4*]. Is it clear? What is the region that satisfies all three? The Extended range. Ok, let this be an inequality of a higher degree, so we can also remember 10th grade inequality solutions.
32	T: Is there anywhere which isn't understood at the question?
33	2/3 S: No
34	S2: I have a question teacher you can ask this if you want.
35	T: Ok, first let us have this written, and then have yours written.
36	S2: Fine [*seats to her seat back with her book*]
37	T: What is the extended range of the function f(x) is equal to, the question is, logarithm of x squared minus 8x minus 9 to the base x. [*First reads the statement and students writes on their notebooks, then writes* $\log_x(x^2-8x-9)$]. Here you go. What are we going to do? The region that satisfies all three. Come to the board please.
38	S7: I have done but I'm not sure.
39	T: [*Goes beside him and checks what he did*] x must be different than this and bigger than this. You should draw an inequality table…No not a number line, an inequality table.
40	[*In the meantime*]
41	S2: Can it be 9 and plus infinity interval?
42	S8: Wouldn't it be -8?
43	(…)[*students are discussing the different results, teacher is still beside the same student. She started to explain the solution by writing on that student's notebook*].
44	T: [*An additional 1 minute passes*] Yes.
45	S2: [*Explains the solution to another friend*] You start writing according to the coefficient of x squared.
46	T: Now, does it necessary to make a table fellows? This is valid only for the function [*to S8 who shows her solution*] there are also the other two conditions, you will look for the place where satisfies all three, separately. Is 1 included in 9 and plus infinity? Do you need to exclude it? Just before, 4 were in the region, that's why we excluded it. Does it necessary to exclude 1 here?
47	Ss: No, it should not be.
48	T: If it is included in that region we exclude it, fellows. Is it clear S8?
49	S8: Ok, teacher.
50	T: And the other ones…[*hangs around the class*] Why negative? Does it start with negative? [*to S9*] Now fellows if the inequalities are of bigger degree than 2nd we have to make sign investigation at the table. Now let's remember with which these signs begins and continues.
51	S9: For which do we draw table?
52	T: Can you do it while explaining?
53	S2: I can do it
54	S5: By explaining…uhmmm.
55	S2: I know you can, but I don't want to give you the floor all the time.
56	T: Ok, let's do it together, you come .
57	S5: [*Explains while writing the above mentioned conditions for the one by one according to the question*]

Table A. 3: Continued

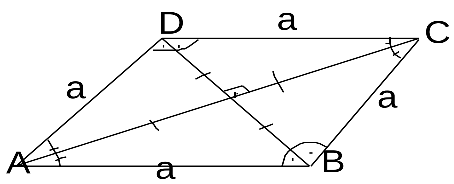
58	T: [*interrupts when she found all the roots and was about to draw the inequality*]. Now we have to make sign investigation if we have a 2^{nd} degree inequality. Let's make this investigation for that only and then look on the number line for all three. This will be more accurate. Let me draw the table again, our roots are -1 and 9. I have seen the sign errors that you made please pay attention very carefully, our 10^{th} grade knowledge. At the sign investigation for the 2^{nd} degree functions there are rules given at the textbooks, like between the roots is the inverse sign of a, which refers to the coefficient of x squared when saying a, the others are the same sign as a. But you confuse here, Was it the same or different sign of a?.I only give that rule when I am teaching that at 1oth grades: In the expression that we factored when we are making sign investigation, what is the sign of that parenthesis if you substitute the biggest number that you may able to think for x? If I subtract 9 from a very big number the sign will be positive, if I add 1 it will also be positive. The product of those is positive then it begins with positive, continues as negative and positive. This holds for all inequlaties which are of 2^{nd} or bigger degrees. I only give that to the students. Now, if we located the signs according to these, where is the place bigger than zero? Where the signs are positive, right? Here. Now we will show these two together on the number line. Minus infinity, starting with -1,0,1, 9 and plus infinity.Now x will not be 1, we excluded that, x will be bigger than zero, it is this region [*draws the interval as a line] and I should also show the other two here, the interval (-∞,-1) [draws that also] and the 9 and plus infinity interval [draws that also*]. Where do I double lined? Here. Then I write my extended range as $(9,\infty)$.
	Observation #4 20th November 2011 Geometry lesson
1.	T: Yes now say rhombus. We define it like that: It is a parallelogram which has equal length of sides. Since it is a parallelogram it has all properties of the parallelogram. Now we told that this angle and that angle were equal.  (Image reproduced)
2.	S1: Is it since its angles are different that it is not a parallelogram?
3	T: Why was it like that fellas?
4	S2: Because of the parallelogram
5	T: Among the most important properties of parallelogram is this: the opposite vertex angles were equal. And more related with angles, that we also mentioned in trapezoids, base angles which are adjacent were complementary. Angle A and D are complementary (pointing the figure). Ok what was the property related with diagonals, in parallelogram?
6	S#: They center each other in the middle

Table A. 3: Continued

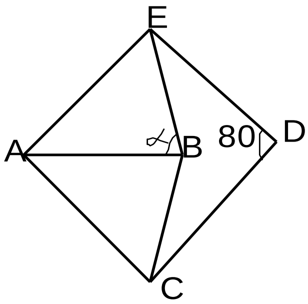
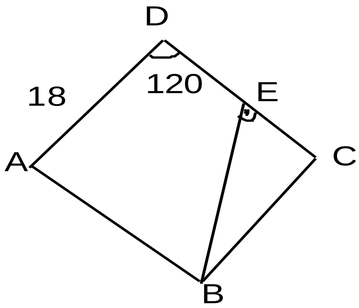
7	T: Yes, (repeats s#'s statement). If we are to draw the diagonal, and let's say e here. What did we say in parallelogram? This length is equal to this and this is equal to that length. The diagonals centered each other in the middle. Yet we have two additional information fellows? Diagonals are also angle bisectors and they are always perpendicular to each other. Do write it with in words. Did we say that at others? We will say also in the square now. Now therefore, in rhombus diagonals are perpendicular. Ok let's remember now. Under which circumstances did we tell that these diagonals were perpendicular? We have actually told that diagonals are perpendicular at parallelogram and rectangle also. Under which conditions was that angle become acute? Does it become acute? If the adjacent angles were angle bisectors, we have actually told that diagonals are always angle bisectors in rhombus. That's the reason why. Diagonals are perpendicular to each other. According to that, angles between the diagonals in rhombus are always 90 degrees. Ok then let's write a question:
8	ABCD is a rhombus, ABC is an equilateral triangle, [EC] is not linear, α is asked: (Image reproduced) 
9	(There has been many discussions, students insists assuming AC linear and try to solve the question accordingly, namely finding α is 120. Finally the student who was trying to solve the question at the board realizes equality between of sides of the triangles and the side of the rhombus and reaches the solution. Then the Teacher chooses another question.)
10	 T: Ok, let's deal with that. ABCD is a rhombus. What do you say? (Image reproduced)
11	S3: Here goes 60 and the opposite will be 30.
12	T: Why?
13	S3: It will complement to 180. And the opposite C angle will be. ..If the opposite of 90 is 18 then the opposite of 30, [AC] will be 9.
14	T: Ok most of you found I suppose. Let me see. (Walks around the class and asks one student to solve at the board).

Table A. 3: Continued

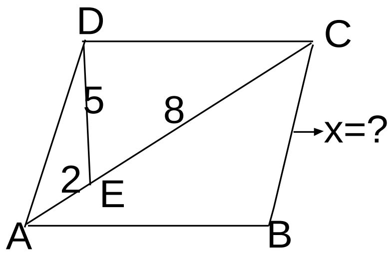
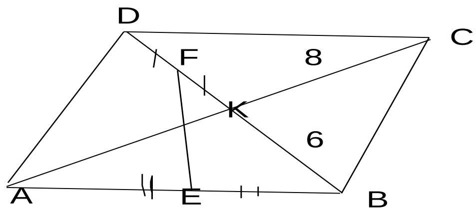
15	T: Now fellows, we know from our previous years' knowledge that the opposite of 90 is called the hypotenuse and the opposite of 30 is the half of hypotenuse. The opposite of 60 is the half of hypotenuse multiplied by square root of 3. We have used this knowledge/ (information) here. We have noted that the adjacent angles are complementary and the opposite angles were equal to each other. If the whole is 18 then E was in other words the middle point. Ok now, let's write some questions about diagonals: (Image reproduced) 
16	S#: Is AC linear now?
17	T: Yes it is, diagonal. Let your friends deal with this little bit. [*walks around the class for a while*]
18	T: Ok, let's do it on the board. Why always the same people? Do you think that the result matters that much? Come on let's reason. Which property of rhombus applies here? Since diagonal is given then we should obviously use diagonal related knowledge. What do we know? Let's repeat that. Diagonals center each other in the middle and are perpendicular. If a question was asked related with angles then we would say that the diagonals were angle bisectors but since length is asked here what do we do?, We would use diagonal lengths and their perpendicularity.
19	 [*A student solves the question on the board, teacher writes a new question right after that. She waits for a few seconds before asking*] (Image reproduced)
20	T: What is the length of [EF]=?Yes what can we do? Let's review our knowledge related with diagonals.
21	Ss: It is 3-3.
22	T: We told diagonals are perpendicular and centers each other, right?
23	S4: Is it 10?
24	S5: Is it square root 56? It comes square root 56.
25	T: If here are 6 then those will share 3-3, fellows right?
26	Ss: Yes
27	S3: These places will be 5-5.
28	S5: If it is 6-8 then it gets 10, and that will dissect as 5-5.
29	S3: AE and EB will be separated as 5-5.
30	S5:Then take our 15 from 81
31	T: Now if KC is 8 then these places will be 5-5.
32	S5: There is no need to do that teacher, here it becomes 6-8-10 triangle. The diagonals EC and AB will be 10. Therefore it will be 5-5.

Table A. 3: Continued

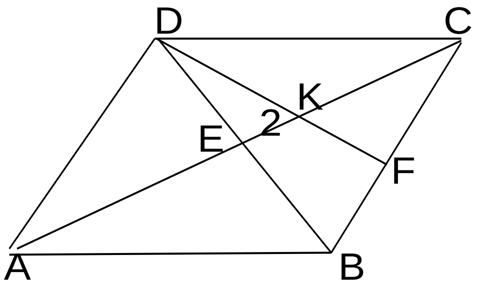
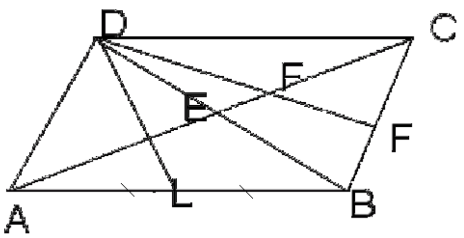
33	T: Ok now, we know that diagonals are perpendicular to each other, and I also fixate a middle point , we can say L to that, E was the middle point I conjoined them.
34	S4: Square root 56
35	T: Now does this share as 3-3?
36	S4: Square root 56
37	T: Ok now everyone looks here, does the length of EL become the middle base for the triangle KAB? Anyone could not see? E was the middle point and I fixated another middle point L by myself. I conjoined and it becomes middle base right? If it is 8 then this is also 8, and if it is middle base then it becomes 4.
38	S7: Then, there must be 90 degrees.
39	T: Yes, then there must be also 90 degrees, 6 and 4 and let's call here x. Can we use Pythagoras now? Let's use it. 6^2+4^2.
40	[*One student shows his result that insistently he was telling for a while*]
41	T: That place is not perpendicular.
42	S4: But it divides into two.
43	T: It is not acute, it might not be, it is not acute.
44	S: Is it?
45	T: Yes let's continue at the board.[*one student completes the rest of the operations*]
46	T: Is it understood? I will continue giving properties on tomorrow's lesson (writes another question)
47	 D,K, F are linear, [AE]= x =? (Image reproduced)
48	T: We have done much of these kinds of questions in parallelograms
49	S8: 2
50	S9:4
51	T: Is it 2 or 4? Where is x? [AE]. We have thought of K as the centroid of ABC, right? If F is also a middle point, what did we say? If the distance to the vertex is 1 then the distance to the side, should be 2. Did not we? If the rhombus has all of features of parallelogram then what will be the length of KC?
52	Ss: 4 units
53	 T: Since the medians centers each other the length of EC=AE that is to say 6. How could we say that here? Because it is given that the point F is a middle point, that's how we say that. What if it was like that? Just say it. Let's call here point L. (Image reproduced) How would it be then?
54	S7: Then these three would be equal
55	S9: Here, there and that would be equal to each other

Table A.3 (Continued)

56	T: We have divided as 2L, 2L do you remember? What was the whole ? 12, if you divide 12 by three each section shares as 4, 4, 4.
	Observation #5 8th December 2011 Mathematics lesson
1	T: We were giving the properties of the logarithm function, we have given the base change property lastly, now let's give 1or 2 property more, fellows. The 8th property, fellows, our 8th. (writing on the board). We have already given the others in the meantime and these 10 property will be enough for you.
	8) $a^{\log_a b} = b$ 9) $\log_a b.\log_b c.\log_c k = \log_a k$ 10) $a^{\log_b c} = c^{\log_b a}$
2	S1: Is there no other property?
3	T: No, we are not going to do, ok now, how are we going to use these properties in questions, let's see that. Let's do from our tests [*searching exercises from the book*]. We have solved some of these and not solved by saying that we have to give other properties, now let's start.
4	The question is What is $216^{\log_6 5}$? Now we try getting rid of logarithm in all questions don't we?
5	S2: 5
6	T: [*pretending not to hear*] how do we get rid of logarithm?
7	S3:By using 8. [*Referring to 8th property*]
8	T: By using which property? Which will help us? Which? We will be coming to those questions which we will be using the 10th property as well but here using only 8th serves our purposes and suffices. Is 216 is a power of 6?
9	Ss: Yes
10	T: Which? Third, then what happens if we write 216 as the cube of 6. Does the question come to that form? $6^{3.\log_6 5}$ and what now? What did we say in the properties that we give in last weeks? The expression above logarithm comes to the head as a multiplier.
11	S3: 3^5
12	S4: 5^3
13	T: Which one?
14	S5: 5^3 (one student comes to the board)
15	T: Ok is there anyone who has questions? Ok, another question: what does $4^{\log_2 5} + 9^{\log_8 2}$ equals to?
16	S6: 10
17	T: We shall not say the solution at once; give some chance to your friends.
18	S7: Is it 29?
19	S4: 33
20	S8: 33 yes 33.
21	T:Ok let's see when we solve it. (to the student came to the board) Can you explain it?
22	S9: First of all, I have separated 4 and 9 into their factors.
23	T: But why?
24	S9: In order to get rid of logarithm,
25	T: What is the base of the logarithm?
26	S9: Then I write 2 logartihm 5 squared to base 2, and here they cancel out each other.

Table A.3 (Continued)

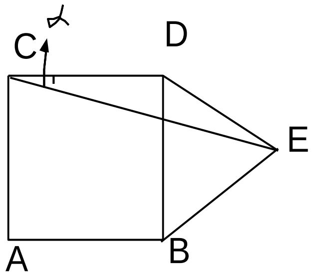
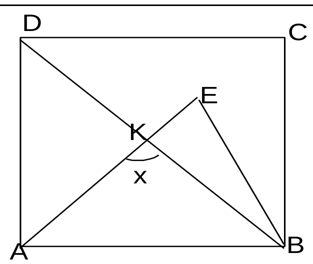
27	T: Do they cancel out each other actually? Due to the 5th property it transformed into 5 squared and 2 squared on the other side, right? Is it clear? Now let's make our question some more difficult.
28	$5^{1+\frac{1}{5}\log_{\sqrt{5}}32}=?$
29	T: What is the difference of that question from the others? What do we do?
30	S3: Is it 37 over 2?
31	T: Now we have more than one expression in sum above 5….
32	(later on question solving continues till the end of the lesson, nothing different noted)
	Observation # 6 22 December 2011 Geometry Lesson
1	T: The question is, ABCD is a square, BED is an equilateral triangle and here is given as α. How many degrees is alpha?  (Image reproduced)
2	T: What was the property of equilateral triangle, fellows?
3	S1: Their angles being equal.
4	S2: 60 degrees
5	T: Was there any other property?
6	S3: Is it 15 degrees teacher?
7	T: Yes, your friend says that all side lengths are equal. Ok, let's put these on the figure. Come on let's do it.
8	T: [*S4 is at board*] Where of angle E is 60 degrees?
9	S4: and here is 150
10	T: Ok now you have used the angles then use also the side lengths, they were all equal.
11	T: Yes, 60-60-60. Now everyone can see this is isosceles, right? Then 2 alphas is equal to…[*S4 does the rest of the operations*]
12	We continue with doing our key questions. A new question: Everyone should deal with it, everyone should be able to solve that. Come on S1, I expect a little bit activity from you. (Image reproduced) 
13	S2: Is it 75?
14	T: May we not tell the result immeadiately, what are we going to do?
15	S3: Is it 75?
16	T: Yes. [*Repeats her question.*]
17	S4: Is it 105 teachers?
18	T: The other side is 105, alpha is 75.
19	S5:[*solves it on the board*] From the angle bisector theorem here is 1 over 5.

Table A.3 (Continued)

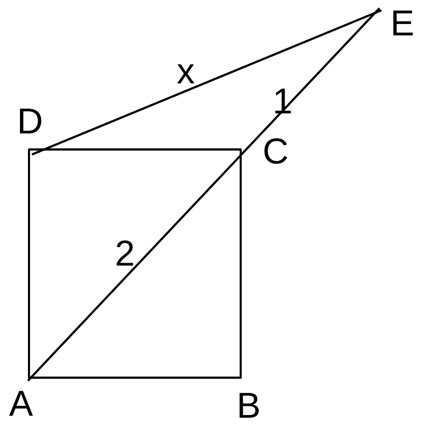
20	T: Is it angle bisector theorem or?
21	S5: Not that was diagonal.
22	T: Diagonal was angle bisector, since DB is angle bisector it divides as 45-45 says your friend.
23	S5: Then since it is equilateral triangle here is 45 degrees and here is, since one angle of a square is 90 degrees, here remains 30 degrees. Since two interior angles gives one exterior angle 45 plus 30 equals to 75.
24	T: [*to S6 by looking to his solution*] Hmm, you can also find from there.
25	S6: I have done by using the other triangle
26	 T: You can also do by using the other triangle. And our friend said that if here is 60 then 45 it remains 75 there. You can also do it by alternative ways. Another question. A, C and E is linear, AC is diagonal x=? (image reproduced)
	(Teacher walks around the class by looking to a student's solution)
27	T: Did you use that? [*to S5*]
28	S5:[*Tries to explain*] (…)
29	T: Yes it divides into equal halves.
30	S7: Square root 3?
31	T: AC is diagonal fellows. We can do that after seeing that in any case. Ok, together come on S8. Draw the other diagonal. If there is one diagonal we generally draw also the other diagonal' right? What do we know S8? We know the diagonals center each other perpendicularly. Now if here is 2 then there is 1 and here is also 1. Then their lengths are equal too then there is also 1. What does it say? It says Pythagoras. Right?
32	Ss: Yes
	S8 Compeletes the rest of the procedure.
1	**Observation # 7 28th December 2011 Mathematics Lessson.**
2	T: We have given the 5 th property finally I suppose. We have talked about the logarithm of the division. Let's review the properties of the logarithm function from last week. What did we said first initially when giving logarithm function's property?
3	S1: Logarithm a to the base a is equal to 1.
4	T: Means if the number of which the logarithm functions is taken and the base is equal the result is 1. [*Lists the properties at the right upper corner of the board, first writes* $\log_a a = 1$]. Secondly we have said logarithm 1 to the base a is zero. [*Writes* $\log_a 1 = 0$]. According to what we have told that?
5	S2: A to the power zero is equal to 1.
6	T: Ok. What was our third property?
7	S3: Logarithm x to the power n to the base a …
8	T: What was that equal to?

Table A.3 (Continued)

9	S3: N times $\log x$ to the power a .
10	T: Yes the power was going to the lead position as a factor. Then it became n times $\log x$ to the base x . [*writes* $\log_a x^n = n.\log_a x$] Ok fellows lets add this to here. We haven't given but we use this in questions. If it is log x to the power m to the base a to the power m. like base, the power of the base number the power of the number.
11	S4: M over x and log x to the power a
12	Y: Then this goes to the lead. M over x and $\log x$ to the power a. [*Writes* $\log_{a^n} x^n$] Ok? If there is only one power then it goes to the lead as a factor but if there are the power of the base and the number then these goes to the lead as division. We will do examples. Now remember our 4th property. The logarithm of the multiplication isn't it? Logarithm x multiplied by y to the base a is equal to log x to the base a plus log y to the base a, or you may use the other side of the equality. [*Writes* $\log_a xy = \log_a x + \log_a y$].You can use both sides don't you? What did we say if we find the sum of the logarithms with same bases in questions what should we recall?
13	S5: Multiplication
14	T: The logarithm of the multiplication. And the 5th property? Logarithm x over y to the base a. What did we say?
15	S6: Logarithm x minus logarithm y both to the base a.
16	T: Yes [*writes* $\log_a \frac{x}{y} = \log_a x - \log_a y$] Similarly how could we use the other side of the equality?
17	Ss: If there is minus we will divide.
18	T: One by one not altogether! You tell.
19	S7: If there is minus then it will be in a division form within questions.
20	T: We will look at the bases, if they are the same and there is a subtraction operation between them then I should recall the logarithm of the division. Now let`s move on to the 6th property. S4: We have written 5, is it the 6th property or are we going to write the follow up?
21	T; I made a revision there. We will remember these while solving problems now, then let's remember it again. The property of change of base: $\log_a b = \frac{\log_c b}{\log_c a}$. And let's give the 7th property as well $\log_a b = \frac{1}{\log_b a}$ Now, is there any difference between them fellows?
22	Ss: Yes there is.
23	T: What kind of difference?
24	S8: One is negative and the other one is the power. I mean the 7th was according to multiplication.
25	T: This means the base and the number of which its logarithm is taken switches. And what in the other one? C has the property of being a base. What did we say at first for logarithm? The base a will be different from 1 and will be different from 0. Then I switch my base to c look, c should have the same property as a. So it should be different from zero and bigger than 1. At the 6th property I change my base to another c base but at the 7th I switch the base and the number. Ho do I change that, a number at the base a to the number at the base c? The number at the base c over the old number at the base c.
26	S7: Will they give the c base to us? How it is going to be?

Table A.3 (Continued)

27	T: We will see on questions, a question on the spot now.
28	S5: Can I ask something teacher? At the 6^{th} property… Is it by taking both sides' logarithm c or shall we find it at the questions by ourselves?
29	T: It will be asked at different base. If $\log_2 3 = x$ then what is $\log_{12} 18$ in terms of x Now everyone look at the question very carefully. The logarithm of 3 to the base 2 is given and the logarithm of 18 to base 12 is asked. There were questions we have seen at our last lesson, right? Like what is equal to in terms of a. What is the difference between those questions and that?
30	Ss: The base is different
31	T: The bases of the expressions that we are given and asked were the same there. We were finding the solution by using the multiplication or division rules. Now here, the information we are given is at the base 2 and we are asked is at the base 12. Our c number is 12 here, we will change to the base 12.
32	S4: Ok now change it to 12.
33	T: 18 at the base 12 shall we factorize 18?
34	Ss: It is 2 times 3 squared
35	T: Then it is $\log_{12} 9.2$ that is $3^2.2$ and how do I write that?
36	S2: With plus sign
37	T: As the property requires, 3^{rd} property, 2 comes to the lead position $2.\log_{12} 3 + 1$ .According to 4^{th} property, which is the logarithm of the multiplication, is it $\log_{12} 2$? Then I have to use the $\log_2 3$ right? $\log_2 3 = \frac{\log_{12} 3}{\log_{12} 2}$ which is equal to x .Ok then what are we going to do now?
38	S3: Don't we subtract since it is division?
39	S7: Shall we draw the x out?
40	T: We have said $\frac{\log_{12} 3}{\log_{12} 2}$ Then let's say that we made cross multiplication, $\log_{12} 3$ is equal to x…[*Pause*]
41	T: multiplied by $\log_{12} 2$ and what now? [*going away from the board and comes closer*] Shall we
	change this to the base 2?
42	S5: As I said 2 squared plus 3.
43	T: When did you say?
44	S6: When you were there.
45	T: I haven't heard.
46	T: Did I reach a solution? In order to use the information given let's change the expression at the base 12 to the base 2. Now let's try that. That is to say, I change $\log_{12} 18$ to $\frac{\log_2 18}{\log_2 12}$. I erase these let's write it again. I have to change the number at the base 12 to base 2.Now I will think the factors of 12 and 18 again. $\frac{2\log_2 3 + \log_2 2}{2\log_2 2 + \log_2 3^2}$ [*she reads the statement while writing*].Now I can use all of the properties. $2\log_2 3$ is equal to x in any case here is 1 according to 1^{st} property and here is 1 and here is x . Then our answer is $\frac{2x+1}{x+1}$. Is it clear?

Table A.3 (Continued)

47	S9: No, teacher I did not get that
48	S1: How did we use that 2?
49	T: It is given at the question. It is given 3 at the base 2 as x. I did not choose it randomly. If the base has been given as 3 not 2 then fellows then we would have changed the base 12 to the base 3. If changing the base 2 to base 12 gives any result like we first did then it could also be used but for that question it did not turn out to something.
50	S7: According to what did we write here?
51	T: Isn't it $\log_c d$ according to the rule? Ok then let's write another question. Now did you see what c is? This is a LYS exam question of 2010: If logarithm 5 to the base 3 is equal to a and then what is logarithm 15 to the base 5 in terms of a? Don't say the result immediately give chance to your friends.
52	S5: Is it $1+a$? [*About 1 minute later one pupil asks to her friend S8*].
53	S8: $\frac{1+a}{a}$
54	T: Yes [makes one student to come to the board]. Now 5 at the base 3 is given as a at this question. Then in which base it is asked?
55	S9: At the base 3
56	T: No, it is given at the base 3 but asked at the base 5. Then what do we do?
57	S9: We will write it at the base 3.The factors of 15 are 3 and 5.
58	T: We have changed bases in order to use the information.
59	Ss: You have missed some digits
60	S9: It should be 5 at the base 3 over 5 at the base 3. It was the 4^{th} property I suppose.
61	T: It is the logarithm of the multiplication. The number of the property isn't that important. You will learn as you do the sum of the logarithms.
62	S9: It makes logarithm 3 at the base 3
63	T: Yes log 3 to the base 3. We have to be careful while writing it seems as if it is the cube of 3. We should write 3 at the base slightly downwards and the other 3 in line with the logarithm.
64	S10: The log 5 at the base 3 was a and it was $\frac{1+a}{a}$.
65	T: You see it is that simple!! It is an OSS exam question.
66	S10: Can I ask something? Why did we write log 3 to the base 3?
67	T: It is 3times 5 you see. When it is factorized as 3.5 then from the logarithm of the multiplication….Ok let's write this question fellows. It is much nicer question. A nice verbal grade for whom that solves that fellows….Another exam question from the 90's.
T	**Observation #8 and 9: 20th January 2012 Geometry Lesson**
1	T: How many sides at least should there be for a shape to be a polygon?
2	S1: 3
3	T: Then triangle is also a polygon. Okay but is there any certain properties of polygons? Which conditions should a shape satisfy to be called a polygon? What do you say S2? Can you be more explanatory?
4	S2: For instance, a circle is not a polygon. It should consist of lines
5	S3: It should have vertices and sides.
6	T: Okay but is every shape which has vertices and sides a polygon? Your friend has told in detail? What did he say?
7	S4: Two consecutive points should not be linear.
8	T: Repeats the sentence. What does it mean?

Table A.3 (Continued)

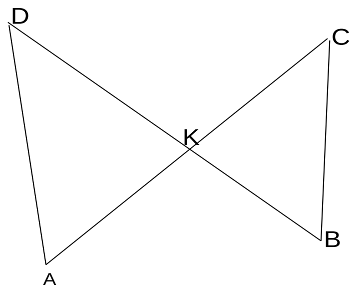
9	S4: Means that there won't be a single line.
10	S5: There won't be three points on a single line. Not on the same line.
11	 T: Draws. As it is in that shape. A, C, D vertices are linear. On one line. Can we call it a polygon? (image reproduced)
12	Ss: No
13	T: We have discussed it before, they are in your notebooks. Polygons were called in two ways?
14	Ss: Convex and concave.
15	T: Yes. Repeats. When does it become convex and concave?
16	S3: In order to be convex you take two points then you join them.
17	T: Yes, what was happening when you join two points from the interior of the polygon, no matter where you take them from, stays again at the interior? Or, we can put it like that: in order to determine it easier…Shapes with sides that are closed inwards. But there won't be any flexure inwards. We call it closed convex polygon. Right but if the two points we took stays inside then we called it the boomerang triangle last year. A certain part of the line that it constructed falls outside. Then the polygon in that situation is called the convex polygon. Look the vertex is flexed inwards already. And we also told that the lengths of the sides are not equal to each other in that polygon since it is not proper. What else? The measure of the each interior is not equal to each other, so at the angle related knowledge that we gave since each angles can't be equal if the polygon is not a proper polygon, since we cannot find them we can only mention the sum of the interior angles…what did we say?
18	S4: N minus 2 times 180.
19	T: Yes the sum of the interior angles in the polygon is $(n-2).180$ [*writes on the board*]. The sum of the exterior angles is always 360 degrees and we have solved on or two problems related to that I guess and what about the number of diagonals?
20	S6: N times n minus 3 over 2.
21	T: Yes. It was $\frac{n.(n-3)}{2}$. We had also talked about the number of diagonals that stems from one vertex.
22	S6: N minus 3.
23	T: Then we can head on to proper polygon now. A polygon whose length of its sides and the angle measures are equal is called a proper polygon.Ok now let's think…which of the polygons we have seen are proper or of the triangles…are proper?
24	S7: Isosceles triangle
25	S8: Square in quadrilaterals
26	T: Can't it be a rhombus?
27	S3: Can't be.
28	Ss: No. (…)
29	Ss: No their angles are not equal

Table A.3 (Continued)

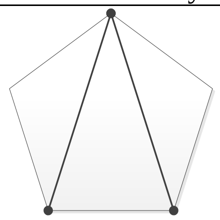
30	T: Diagonals were angle bisectors but they might not be equal to each other and they are not. In quadrilaterals we call the proper one a square. [*draws a sample isosceles triangle and square on the board*] My polygon is proper that means, equal interior angles so I can find each angle of the polygon which is n sided.
31	Ss: (n-2). 180 divided by n .
32	T: We already knew the sum yes, so if there are n sides then divide it by n .One exterior angle is also $\frac{360}{n}$. Let's do an example related to this one. From now on let's call it proper pentagon. We have moved towards the special, making subgroup investigations, how many sides do my polygon has?
33	Ss: 5
34	T: Then I can find one interior and exterior angle of this polygon easily. We will put 5 in one interior angle formula right? Let's draw the shape of it. [*drawing a pentagon*]What will be one interior angle when substituting 5 for n?
35	S6: 108
36	T: Yes, [*making the calculation on the board by using the formula*], according to this what will be the measure of one exterior angle? If we complete to 180? 72. Then when it is called a proper pentagon we can place one interior angle as 108 and the exterior angle as 72 degrees. What if we want to find the area of that pentagon? Yes, what do you say?
37	S9: From the 30-30-120?
38	S3: We can dissect it into triangles.
39	S5: Teacher shall we break them up into isosceles triangles?
40	S10: One isosceles here and one is there.
41	T: Your friend says that I break it up into triangles. [*Draws*]
42	 (image reproduced)
43	T: Now, look there is an isosceles.
44	S2: They are all equal
45	S4: Right! After finding the area of one isosceles triangle.
46	T: Is there any isosceles constructed? All of the interior angles were 108, and one is here.
47	S7:It is 36 degrees.
48	T: And there? Again another isosceles right? Then the sum of the areas of all gives us the area of proper pentagon. We solve the area questions/problems like that. Ok, we have some footnotes about the proper pentagon, let's give them at once.
	Observation #10: 20th February 2012 Mathematics lesson
1	T: For $n \geq 4$, show that. $n! \geq 2^n$ let's try a while. There is no finite sum, there is a factorial right? Why is $n \geq 4$ folks?
2	S1: Because it doesn't go, the others don't fit.

Table A.3 (Continued)

3	T: Just see that. We have told positive integers at the other questions so we have started from 1 the first element. If it says $n \geq 4$ then the first is 4. Which means the correctness of this proposition is demonstrable for $n = 4$. Then it is false with 1, 2, 3. [*Waits for about 1 minute and a half, then she hangs around in the class, talks to some students, at the end one stand up, to the board, shows the correctness for $n = 4$ and that we assume the correctness for $n = k$ and the correctness for $n = k+1$]*
4	T: Now go upwards, everybody look at the board. We will show that $2^{k+1} \leq (k+1)!$. I look at the left hand side of the statement *. What is there?
5	S1: Shall we write $(k+1)!$At the previous one?
6	T: Now we will say after I say it.
7	S2: Can I say something here? Umm... $k!$ and $(k+1)!$. I think I multiply both sides with $(k+1)$ in order to make this $(k+1)!$
8	T: I was going to tell that.
9	S2: Sorry teacher for interrupting,
10	T: $(k+1)!$has more than k!, there is no finite sum at this question, the difference of that question from the others is this. In the finite sum we were showing the more or less by writing the preceding element of $k+1$but here we can't say that since there is only factorial, but by the property of the factorial we can write $k+1!$ in terms of $k!$We have given that on permutation, combination property, for example how can we write $9!$ in terms of $8!$?
11	Ss: 8!.9
12	T: Now we can write $k+1!$ in terms of$k!.(k+1)$. What did you do there now?
13	S2: Umm, I multiplied both sides with $k+1$ in order to write in terms of that.
14	T: Now, let's write that above in terms of $k!$ so that we can see what is more or less. Now look at the* statement and the left hand side what your friend has just written. What is more?
15	Ss: $k+1$
16	T: Right , then we are not adding with $k+1$, since it is at the product form then we multiply both sides of the * with $k+1$. In short we are doing mathematical games, like thinking how can I reach that? I think of these.
17	T: Now our induction formulas (one student is writing the formulas on the board), we will be on induction on Wednesday so have them memorize until then,
18	S3: Are these sum formulas teacher?
19	T: There are also the product formulas, they are the ones that we will use most we will know them from the memory. We will use sum forms in sum symbol and the product forms in product symbol. We have proven most of them haven't we?
20	S1: We have proven 5-6 until now.
21	S2: We are using three things after all$n = k$.
22	T: This is the rule of the induction method
23	S2: Yes teacher but won't we use the same to prove these things? $n = 1$, $n = k$ and then for$k+1$.
24	T: Not all of them might be equal to 1
25	S2: Yes they may change
	Observation #11 12th March 2012 Mathematics lesson

Table A.3 (Continued)

1	T: We have mentioned some properties of sequences and solved various examples. We have told that we will do this lesson with our friend, our topic is arithmetical sequences. Let's write our topic and our friend will have us to take notes of the definition, you will be solving the examples together and will be asking what you did not understand to her.
2	ST: Sequences which has a constant difference between its consecutive terms is called arithmetical sequences, take the formulas too fellows [*writing the general term for and some other formulas related with the arithmetic sequences to the board*]
3	T4: Can you write a bit bigger? [*After about 2 minutes later*]. Write that on the left hand side. It is the general term of the arithmetical sequence.
4	ST: [*writes*] Let me may give an example right away.
5	T: Ok, ST, does everyone know a_1 among the symbols that we used here? What was it?
6	Ss: The first term
7	T: What was d ? We did not say. In some textbooks it might be used r instead of d . We will say it the common difference in the arithmetic sequences but the common factor in the geometric sequences. And $n-1$ denotes that it is the preceding term of. a_n
8	S1: r is used generally in geometric sequences
9	T: Some resources those are both called r. If you use them separately I think it will be better you will not mix them up.
10	ST: The general term of a sequence whose first term is 3 and the common difference is 4 asked. We can do it like that. First term is 3 and the common difference is 4, by the formula an is equal to a1 is 3 and $3+(n-1).4$ which means $3+4n-4$ is equal to $4n-1$.
11	T: Ok do you think is there any difference between the sequences we have seen until that far and the arithmetic sequences?
12	S2: There is
13	S3: Of course
14	S4: There is. We have said that it increases constantly
15	S5: The difference between the consecutive terms is constant.
16	T: What did your friend say when defining? There is a constant difference between consecutive terms which is d. I can therefore. If I know an arbitrary term and d , for example it gave the 3rd term and d how can I find the 25th term?
17	Ss (…)
18	S4: 25 is equal to $22d+a_1$
19	T: Now since I knew the 3rd term by adding d on it, I can find the 4th term, by adding one more d I can find 5th term and by adding another d 6th term. You can go like that if the difference between the terms is small. Like 25th term or 125th term in such cases we have to use the general term formula
20	ST: [*writes a statement giving the formula for the common difference for the two arbitrary terms of an arithmetic sequence like* a_p *and* a_k $d=\frac{a_{p-}a_k}{p-k}$. *Right after she asks a question*]: The 18th term of an arithmetic sequence is 45and the 8th term is 15. Then what is the 5th term? [*Solves by applying the d formula in order to find d and then she uses the general term formula to find the 5th term*].

Table A.3 (Continued)

21	T: Ok, now how do I do that without using that formula, I am not a good fan of too much formulas since you mix them up, there will be Sn formula coming soon. You know the 18th and the 8th terms how do you find without using this formula? [*waits for a while*] Can it be by using general terms formula? Say the second way. It is ok with the formula also practical but let's writes 18th term according to general term formula. Is it a_{18} is equal to $a_1 + d17$. Now let's also apply this to 8th term. [*ST applies what she says at the board conducts the operations*]. Ok.
22	S3: She could also associate with a_8 ?
23	S4: But how could she without knowing d ?
24	T: [*To S3 and S4*] There are two unknowns both a and d. [*To the class*] Ok. Now by writing these information one under the bottom and using suppression method we can find a_1 and d .
25	S5: By a_{18} being equal to $a_8 + 10d$.
26	T: a_1 plus $17d$
27	S6: By associating a_8
28	S7: Yes it also comes from that teacher!
29	T: Ok fine you show that also. Your friend has got an idea we will share it to you. Tell them.
30	S5: a_{18} teacher, we have given two of them you see, a_{18} is equal to $a_8 + 10d$. From there $10d$ comes as 30 and from here it asked a_5 and I will liken that to a_8 so that it would be easy. a_8 Is equal to a_5 plus $3d$. Substitute 3 for d .
31	T: There is not just one way, you see. Just use whatever feels easier. Was this the same with yours?
32	S8: Yes teacher exactly.
33	ST: Let's write an example. What is the 14th term of an arithmetic sequence where the common difference is 7 and the sum of the 6th and 14th terms is 64?
34	T: Let's wait for a while give chance to our friends to think about it. Still we have multiple ways and use whatever you feel like to use. My suggestion here, since my own studentship I always prefer to use the general term. This has happened to be like that all times. If you use the general term a1 will come from all
35	S9: Yes it comes from the general term easily.
36	T: I always preferred that since my studentship. Ok let's do it. You choose one of your friends to solve it? [*To St*]
37	ST: It is so hard…
38	S7: 60, is it 60?
39	Ss: Yes, 60.
40	S6: [comes to the board and finds 1st term and d by using the general term formula she solves and explains the procedure while solving].
41	T: Now since the 1st term and d is evident, we can find 14th or 114th term either.
42	S6: [*continues solving by using the general term formula and finds 14th term.*]
43	T: Is it clear? Is there anyone who did not understand? The silent ones. Ok. Let' move on
44	St: I will move on with a new property again. In one arithmetic sequence every term is equal to the arithmetic mean of the terms which are at the equal distance from that term.

Table A.3 (Continued)

45	T: We call that the arithmetic mean property, let's write it alongside. The term in between is the half of the sum of the terms which are at the equal distance of that term. That's the arithmetical mean property. This provides easiness at the questions.
46	ST: $a_8 + a_{20} = 90$ and the sum of the 14th and 25th term is 125 in an arithmetic sequence. What is the 25th term?
47	T: what do you say fellows? Can you think about the terms in the middle of the 8th and 14th terms? Just in the middle.
48	Ss: 14
49	T: Is it 14th term?
50	Ss: Yes
51	T: then you can find that by…
52	S8: using the arithmetic mean.
53	T: S5 You go and solve the problem, ok then we will do it together. If we are to sort the terms fellows, the right hand side and the left hand side of the 14th term, it goes 6 terms from the left hand side to the a_8 and from the right hand side it goes 6 terms again toa_{20}. This means a_{14} is at the middle of the 8th and 20th terms. Since it is an arithmetic sequence every term is equal to the half of the sum of the terms which are at the equal distance to itself, means what is a_{14} ?
54	Ss: 45
55	T: It is 45.
56	ST: Let's move on to another property. [*She reads the property that states the sum of the first n terms* S_n *and the rest of the class writes*] an example…
57	T: Ok lets think about it for a while, listen to me please I will ask you a question. For instance if you are given the sum of the first 60 term of an arithmetic sequence, by adding the 61th term, do we find the sum of the 61 terms?
58	Ss: yes
59	T: From here, when the sums are given how can we switch off to the terms how can we connect? Please think about it.
60	S5: We can subtract s_8 from s_{10} and can find $a_9 + a_{10}$
61	T: very good
62	S5: or the difference between s_9 from s_{10} gives me a_{10}
63	T: Yes, What S5 says is by taking the difference of the sums we can switch off to terms
	Observation #12: 4th April 2012 Mathematics lesson
	T: What is the 6th term of the sequence whose general term is $\sum_{k=1}^{n}(\frac{1}{2})^{k-1}$ What does it mean here the 6th term fellows?
1	Ss: a_6
2	T: Ok how do you get that?
3	S1: We may subtract those which are till a_5 from those which are a_5?
4	S2: It comes directly from the formula.
5	S3: Isn't it to write 6 in the place of n ?
6	Ss: mumbling
7	S1: But there is sum there
8	T: Yes but we will be finding its result.
9	S1: Just a while ago when I wrote 6 it gave me the sum of those that are from 1 to 6. If I wrote…

Table A.3 (Continued)

10	T: I don't want the sum of the terms from 1 to 6. I want the 6^{th} term.
11	S1: I got that but I will write those which from 1 to 6 and subtract those which are till 5 so that only 6^{th} term remains. Otherwise it doesn't.
12	S4: Ok teacher when we do from 1 to 6 us will find the sum of the terms up to 6.
13	S5: No, it says sum, then it would not have said sum.
14	S6: Can't it be like that?
15	T: Fellows who thinks like that please don't confuse it with the sum of the terms. a_n has been given as the sum symbol. We will put directly 6 to n 's place.
16	S1: I don't understand that teacher, should not it be like from 1 to 6 since there is a sum symbol
17	T: But what you wrote here are not the terms of the sequence it is the sum of the k's. Ok let's do and see what it is like. I mean a_5 is from 1 to 5, a_6 is from 1 to 6. If my lower limit was a_6 then what you said would be correct but the thing which is stated here is different.
18	S7:[*Does the operations on the board and explains what she does simultaneously*] I give values on k from 1 to 6.
19	S8: and this is 63 over…
20	S7: ok now, let me finish!
21	T: Do we have an induction formula here or am I wrong?
22	S7: Yes we have… 1 minus r to the power n over 1 minus r. I write ½ in the place of n and [*completes the rest of the operations by telling and writing]*
23	T: we can find the solution directly from here. Is it clear? Now Let's make that part clear. It as $\sum_{k=1}^{n} a_k$ then it would be the sum from k is equal to 1 from n
24	S9: Does it ask only the a_6 or the sum?
25	S4: Would not it be ok if we find 6 and then subtract 5?
26	T: Your friend has said the same thing, but it asks only the 6^{th} term not the sum. Let us articulate that when we give the sum of the first n terms at the arithmetic and geometric sequences.

APPENDIX F

INTERVIEW TEXTS AS INTERPRETIVE STORIES

Pre study interview

Story 1: There is no reason to unsuccessful if you listen like that…

> We already set the rules at the beginning of the year. First and above all, attendance is a must…Arriving the lesson on time, taking notes within a neat and systematic manner, listening to the teacher very carefully, asking what one did not understand instantly, not after some time has passed on it just at while we are solving the problem: asking like ‘teacher I could not understand that point. Can you repeat once more?’ Then there is no reason to be unsuccessful if you listen like that and do your homeworks.
>
> Of course sometimes students may not be like this and we may not be at the same performance (level) every time. We are actually living through an orientation period of… like two weeks. These result from the other teacher`s attitudes. For instance I say that none of my students will come to the class after me. Everyone will be at the classroom before me. If someone knocks the door after I started the lesson or talking or working out a problem; we are distracted and lose our attention. That’s why, if students are on time for the lesson, listen to their teacher with all ears and ask questions when needed, in my opinion they are ready for learning everything.

Story 2: It is the teacher`s responsibility…

> Teaching, in my opinion, should be the responsibility of the teacher in general. You give essential definitions, solve sample questions. Is it possible to develop different solution techniques? Yes. In doing these, teacher should be the leader. She must lead students to alternative thinking processes. This is what I understand from saying student centered or that student is active. It is teacher`s responsibility to give the essence of the subject. Teacher usually transmits but students may generate different solutions. Switching between prior and the present knowledge should be left to students and the teacher should be the one who gives rise to ideas.
>
> In order to become ready to learn maths one must be able to understand mathematical expressions, use them, explain the procedures that is made, show what comes from where... For these things to occur there must be some sort of accumulation. Like we’ve said before it is the teacher’s responsibility… Teacher is important in these areas where students bridge prior and new knowledge. Teacher is the one who have them to make this. There is a lot for the teacher to be done. Thus far it is under teacher`s responsibility. For instance while teaching polynomials if they say: “We have seen those at functions, we were also doing like that at the

functions” then teacher could have managed to have them to gain something. Students’ activation should come after this. Then they must solve polynomial questions very well. And until that part it is teacher’s turn.

We exactly indicate our expectations from the students. I also ask students in that period: What do you expect from me? How do you like me to be? How do you like that our lessons to be? If they say “in a different way”, “we would like to treat lessons that way this year”. I say: Let’s talk and discuss it’s appropriateness. Can we manage to do that? If it is appropriate, I try to implement it.

If I want them to be on time in the classroom I have to be on time also. If you continue teaching except with situations of exceptions like you know, you make jokes or give pauses but everything has certain limits. If you don’t go too far, students position you at a very good place.

Interviewer: Do you mean when you use the time effectively?

Yes the time and the lesson effectively. In choosing our examples we move from simple to complex then we choose ÖSS exam questions.

First of all, in my classes students should know about where mathematics or the subject being taught… should be used. Not because they are required to learn but according to where they will use them. Our classes are comprised of SM (Science-Mathematics) or TM (Turkish-Mathematics) branch students hence they should acknowledge the priority of mathematics considering the most important exam they will take in their lives…At least what I`m trying to do is to bring them into a position of achieving their ideals considering the priority of mathematics. Otherwise, ways for learning are various, but as I said before I prefer students love mathematics rather than feeling obliged to do it. To feel the pleasure of….Being aware of the fact that knowing mathematics will grant privilege to them….My aim is to make them love mathematics, without being bored and getting out of the door with nothing remained vague and unclear…

Story 3:Everything is hidden in definitions.

Everything is hidden in definitions. Generally they are neglected by the students. Being able to understand them and mathematical language is of high importance. The more they are able to use these definitions well; more they understand the mathematical definitions when they open up mathematics book by themselves. Since everything is hidden in definitions; students knowing the meaning of, for instance, counting numbers, natural numbers and an integer and the differences between them and writing these with a mathematical language lead them to understand easily and realize the difference between various questions by asking “what is this meant to be” while reading a mathematics textbook. But students mostly cannot use them [definitions] correctly.

Like the case in complex numbers. We started by saying that “i” is a complex number unit, but beforehand we made some revisions regarding prior subjects. In geometry lessons we started with polygons and we have told there too. All *n* are elements of natural numbers we've said. Now students are able to understand what it means but cannot write it. We were dealing with these since last week and will deal for one week more but then I am not going to write on the board I will tell them verbally and they will write. At first some students resisted. But as they use the mathematical expressions both in written and verbal forms they become able to explain what they did on the on the board… Generally they are very bad at listening and writing. Unfortunately our students' expressions especially in SM branches are really weak… One of them asked lately "May I not talk, teacher, I can solve the problem but cannot explain". But as he go on using mathematical expressions he will realize that he both solved and explained the problem. He will surprise to himself. By using the definitions and one more thing… Having them to underline some important points. You will see in my lessons: I have them to take notes on regular basis and to underline important points to put exclamations and stars. Having notebooks and using color pencils are very important for me because notes that we have students to take includes the summary of 5 or 6 resources rather than one textbook. Students must have that kind of resource. I am struggling with these issues for about first two weeks. Then they slowly get used to it and at the end of the year they are then…Like our professor told us at graduation. "You are now ready to learn mathematics!" I tell this to my students at the end of the year 11 or 12. You are newly becoming quantitative students. Newly become able to learn mathematics.

Story 4: You release your ideals at some point

Interviewer: What was the most compelling factor while teaching mathematics during your professional life?

Not now, I cannot say that I struggle too much nowadays but in my first years, you know we begin with idealism. Everything would be perfect and we would work under perfect conditions, our relationship with the students would be perfect…but when you begin nothing is like as you thought of. Your relationships with the administration are not like you wanted. You are not given an appropriate schedule, like you can feel yourself fine, sometimes I had to teach 8 hours successively. This is very exhaustive. Especially at the 7^{th} and 8^{th} hours are the moments that the teacher is washed out. I wish I could have given more to those classes. Presently, every new system or new regulation makes the life more difficult for us. You release your ideals after some point. For instance, when a student gives you a blank paper, in the name of that classes' success rate should be 50 %, you… the administration makes you to do that since it is their obligation. If student does not want to learn, you should have told her that you head towards different areas, if you like there are other options and you work for to learn the best… I would like to work in such systems but no unfortunately every new coming regulation oppresses us a little more…We are, as teachers exhausted financially and emotionally, they are changing the curriculums and the systems on continuous basis. Things turn into a great mess in every 4 or 5 years. Apart from those I don't have too much problems with my students, in my 19 years of experience, only solitary things…But everyone lived through things like that and we solved them by talking after a certain point.

Story 5: I also care about mathematical expressions in terms of self expression

I also care about mathematical expressions regarding students being able to use their oral language, in terms of self expression, nowadays one can be an architect or an engineer for instance designing a plan or a project but what if he can not present it expresses him thoroughly, advertise himself then it means nothing. On the other hand, some one being no better than this one who is able to advertise or present himself very well…isn't he more likely to be chosen? Then I say you should do your best but you should be able to present that best… I would personally prefer the second one, then you have to come to the fore, people like you should increase. My efforts are for these. Otherwise 15th or 20th years are not important. The important thing is for students to express themselves correctly, experience the priority of both knowing mathematics and showing this to his environment. That he should both know and be able to use mathematics.

Story 6: There must be something for my share.

I don't have any expectations for myself I am at where I can reach. Can I make more students to love mathematics; can I give them more of mathematical knowledge? In that sense there must be something for my share, that`s why we are working together…Loving and doing mathematics is a great priviledge. I sometimes tell to my students that one who knows maths is the king of that place, in a community there are lots of people who know geographical or general knowledge or verbal subjects but there are limited amount of people who knows mathematical subjects and the one who knows them best is the king of that community. You can be the king or the queen of where you occupy. Why not should they live the privilege of learning mathematics? Why not should they earn the reward of it? You see, we should live and learn it and come to such a position that to get to the best in our profession.

Interview #2

Aylin :

We have solved the prior university examination questions then made an exam and they asked "why could nobody solve those easy questions?" And I said "if you can remember this knowledge when you come to the senior year which is very good, we are striving to do that…Else there is nothing in the exam questions"…

We had a similar discussion about taking the exponential of the polar form too, were you at the class? No? There we had a longer one tough, we have stated clearly according to what we learnt first. We can take exponentials as the powers of 1-I and 1+ I it was the easiest way. But when a complex number is not given in that form then how can we take the exponential of it? Then we head into taking the exponential of the polar form. First we have to transform it into polar form. It is much easier. We had an exam question just like that …then they understood the difference in between. But of course you cannot transform every number into polar form so this procedure gets a little bit longer.

We treat it as if we had transformed it then it gets longer and students get bored after some while. They say "teacher this takes too much time. Might a procedure take so much time? Yes it might…Sometimes we might confront with these kinds of procedures so we have to know them also. But under normal circumstances, exponential in the polar form is always the easiest.

Story 1: Now they are started to say I have done the solution, I can explain it right away.

Students come to understand mathematical expressions, explain their solutions or which operation comes from where, make alternative interpretations when solving equations. We can see these in the lessons better; we have seen limited episodes here. Students were initially hesitating to come to the board, but now we have gradually overcame that. They come to board and they are gradually able to explain their solving techniques. Now they started to say I have done the solution I can explain it right away. Then they say, I can explain the second way. I think, from now on we are slowly able to succeed in doing this. It has been a while after the school has been opened. Like one and a half month. We are transmitting knowledge intensively then and we will receive from them back but how come?

Logarithms turned up a little bit different to them. But I told that we will be in such a position that taking the logarithm of two sides will be among the first methods that comes to our mind when we are presented with a question regarding exponentials within forthcoming days. Just as that taking the square of both sides when solving an equation with a root comes to our mind right away. But when?After finishing the logarithm completely, after giving all of it's properties. This is my objective. They must solve equations with logarithms just like they solve any other equation. That's my objective.

Theorizing:

Interviewer: What do you say going one step further? Like doing some proofs especially in those SM classes? Would you consider that?

There are of course the proofs of the formulas. Although the books are directing us towards these, the total amount of time of each geometry class per week is limited, like 2 or 3 hours and the content are very intense. We put 7 or 8 properties if we deal with all of these proofs then we don`t have enough time to make practice and solve problems, thus I prefer to pick 1- 2 important one to justify in simple terms while noting the others are rooted in them. For instance we use triangles most as solving problems, "We say we are using this because of the congruency in triangles. This property comes from there". This is more practical I believe. We try to give in a fastest and a more practical manner since we need time for solving questions and their geometrical background are not very strong sometimes I have to repeat one question three times and spent almost an hour for it. On the other hand, if I spend so many time on each question then not any chance remains to solve other important problems which I call them key questions since I have to move on to a new topic on the next lesson. That`s the reason that I cannot allow so much time for the proofs in my lessons. But this is due to the reasons beyond my control.

Argumentation:

Absolutely, I believe that such things- understanding mathematical expressions and articulating them- are the descriptive parts of mathematics and I place great importance on them. But when I make a proof in the classroom the first question arose is “will you ask this in the exam?” This is the biggest problem; though I prefer doing those rather than memorizing…There are lots of formulas in their minds attached with different subjects which they can easily mix up. Yet I make them to prepare formula boards for logarithm, complex numbers and so on. We make them face with a mess of formulas. For instance this is mostly true for trigonometry. They confuse the sine cosine and tangent values of 30, 60 and 90 degrees, I tell them to draw an acute triangle but they don’t want to deal with reasoning behind them. As things piles up everything can be a mess. Actually proofs are good at that point, I mean why this formula? Or what is the reasoning behind the formula? How can I solve this without memorizing? If a student would think about those without dealing with a mess of formulas he can apply this. But we are wasting so much time with it since our time is limited.

Story 2: Doing these proofs or simply giving the reasoning behind these formulations are important but I usually do not have time for these

In fact grades 11 might be convenient for such things. In the topic of induction we are making the proofs of all formulas for instance. But they stumble “How may I continue after that? Can’t you give the formula directly, is this that much long?” but if they would knew where that formula came from or under which circumstances we shall use which formula this would ease our job. They will decide this after all. I think that doing these proofs or simply giving the reasoning behind these formulations are important but I usually do not have time for these. But while solving problems or giving the formulations I certainly emphasize the reasoning orally by saying this comes from that, this is resulted from that. Otherwise if we give the formulas directly they are permanently alienated from them. They don’t want to memorize in any case, they have this thought laziness. How can we overcome that? I try to ask questions requiring multiple knowledge bases so as they can switch from that knowledge to the other. Or by checking whether they can accomplish asking questions to themselves like “is it possible?” and answering it themselves.

Story 3: You have to work cooperatively, you have to form study groups

I like this very much. Can I think about or solve it differently? These questions are asked by students themselves mostly in geometry lessons…This is not very much appropriate for mathematics lessons…In geometry lessons we see that too much. Ways to solution is not one. The answer is unique, not the number of ways. I try to show them every possible solution that I can for that moment, but as I said before in EA classes we have only 2 hours for the geometry and have the same curriculum with SM classes. We solve 15 or 20 questions generally but in EA classes only 5 or 6. In SM classes they have only one or 2 or 3 questions that they could not have done. There are 10 -12 tests after each unit, we are moving on while scanning each test. Do you have unsolved questions in the first? No. The second? We are moving like that at the SM groups but in EA’s for instance they might say I could not do 3 -

10 and 13 in the first test. We would like to solve each of them but then I say you have to work cooperatively, you have to form study groups. If you solve that you explain to your friend and if you still have questions then we will solve them at the class. Or I ask for instance could not we solve that with parallelogram or with a trapezoid. We have some answers sometimes but not in EA classes.

Story 4: Due to system deficits…

Interviewer : Which possible way or method can be followed to help these EA students to close this gap?

Uhmmm. For instance I had a two hour lesson to them on Friday. They neither have any triangle knowledge, nor any angle knowledge. Nothing! I'm not sure if it is on the record now, but this is due to system's deficits. As I told before, when they became 11th grade they had been learnt triangles very well, knew angles and we were teaching quadrilaterals upon that. They were learning very well without deficiencies. Now we start with vectors its vectors at 10th and 9 th grades. At the second semester there are still vectors and vector related operations they forgot what geometry is. I'm definitely against that. A student at 11th grade and there is almost nothing in her about triangles and angles.

Theorizing:

That's why it's revision time at first two weeks. I tell them that "You are going to study triangles". Before beginning the quadrilaterals, in order make connections between them…Sometimes I connect on questions, problems. For instance if we used medians or angle bisectors I explain those. Mostly we use congruence when calculating areas for example I say ribbon, I draw with the markers in a way that they can understand most easily. But making this transition is very difficult for them, actually after understanding it is easy but they are very slow, slow at solving questions, they think very slow, very very slow. They might be aware that it is congruency but they have to look at it for 10 minutes. We have to accelerate them at that point.

Story 5: Students today are not like our studentship

Interviewer: Is it because of the conceptual deficits?

Yes but this is not the only thing. They have knowledge deficits, they can't see some certain things, and how can we overcome those? With practicing, solving lots of questions. The more we practice the more they are able to see well. What is given, which to apply in which situation? As I told before there was no note taking regularly at first couple of weeks, now they do. And one more thing as I told before, I usually have them to take notes of important points like if we draw a straight angle originated from a straight angle then you use Euclid if there is a median who originates from a straight angle then you use "the incredible three" so on and so forth….if you don't give this to students and have them to take notes they don't do it by themselves. Students today are not alike our studentship. I used to take excellent notes, using arrows writing somewhere else like this comes from there and that from there and this has served me a lot when solving different kind of problems. I wish them to do in this way as well but unfortunately they don't. Both in EA and SM classes this is the case, you know why? It has been told to these kids to wrote then they have written otherwise they don't. They remember why they did like this for that moment but at the next lesson or at the exam, he forgets where has that came from? Thus the habit of note taking is very important… I make them to take notes whatever is implemented in each question. I tell them like we use Pythagoras if it is like that in triangles for instance or Euclid in that situation. I try to make to take notes of everything. After doing these in EA classes, solving problems increases a little bit.

Augmentation:

I had them to take notes of those…quadrilaterals..if the diagonals are perpendicular then it is a rectangle but if there is both perpendicularity and equality then it is a square… if I did not do that students cannot decide whether it is a rectangle or a square…I have seen this through experience, now I dictate all and this simplifies our job, because they have the habit of memorization you know, ok this is a rectangle he says for instance , he constructs a mental picture from his notes then now I have to behave according to that rules,…Actually he doesn't do it by thinking but I obligatorily made him to, because otherwise he does not think anything. Some say let it go, let's not deal with these just solve the questions.

Story 6: Now they say, I don't care about the definitions, I think on the questions

Interviewer: What about making a little bit more discussions?

It's ok when they are at 12^{th} grade since seniors are more open to interpretations but for 11th graders these moments of discussion are counted as spare time like a break! It might be possible in the eleventh grades' second term. Now they say, I don't care about the definitions, I think on the questions. Then, for instance I ask students to comment on finding the inverse of an exponential function. What do you have to do then? One says "I will think about it on the question"... I also made them to note that as a warning: The inverse of a logarithmic function is exponential; and vice versa. At first stage, when a student has a problem he/she has to decide on that. Then if he should find the logarithm function then he should. I say that now you are able to do these now but when I give them after one month after the second exam say, and ask the inverse of the logarithm function then you give it back to me still as a logarithm function. Do you realize that you are contradicting with yourselves? Now it seems that they are aware of it but more than 50 percent will do the same at the exam. They say outspokenly that they don't care about that part; they will do on the questions. But he is memorizing and everything is bound to memorizing. What kind of connection there is among them? This is our biggest problem. You know, that question: how will this serve me, where will we use this? I think, he should know where comes from where in mathematics, know its antecedents so that he could deal with their children, but students are not into that point. I don't want to be unfair to some of my students since they care about those things, but I still deal with these issues.

Story 7: Now my objective is for my students to solve the problems by working out their knowledge

Interviewer: How might they present this knowledge by solving the problems and making interpretation?

> Now my objective is for my students to solve the problems by working out their knowledge…Now they are aware of the fact that logarithm is a function, they have these properties, and from now on they should know the differences between the logarithmic function and the other functions…
>
> Of course knowing is not sufficient they have to present, put into practice don't care about that part, they do on the questions. How might she/he present this knowledge? By solving the problems or making interpretations and students who are able to make interpretations are more successful than the others…Or they should interpret the logarithmic function`s graphics, how to say, while finding the extended range he should notice the difference between the ordinary function and logarithmic function. If that function was a polynomial instead of logarithmic function how would the extended range be? They should interpret these. These are my objectives…
>
> For the geometry lessons we will be finished up with the rectangular possibly we will be started to the square by the end of the month. I am thinking for this month by facilitating students making the transitions between the antecedents like trapezoids or parallelogram and making comparisons between different quadrilaterals to solve the problems to generate different solution techniques and to make connections in between one another. Just as we use triangles most then they should solve problems using quadrilaterals than triangles. For instance using trapezoids rather than triangles. Or a parallelogram. It's ok using triangles but they should be able to tell me that it is possible to solve that question by using a parallelogram. Let these be my objectives and let me try to reach them.

Interview 3:

> I don`t enjoy these 12 geometry classes even a bit. I have three of them. I have particularly requested not to give me senior geometry but they have. They are so called preparing for the exam and I don't enjoy a bit, I hate it. I teach mathematics more delightfully, love more than geometry. And if the class is good then it is even better, I have the chance to ask various questions have them to reason. But in weak classes…I made the second exam, by the way. Did I give it to you? I have asked 3 questions from the complex numbers but 2 more logarithms. And those were substitute questions.. They could do whatever they like logarithm or complex numbers.

Story 1: You have to develop yourself in all aspects but I prefer mathematics more

> In geometry I don't have much opportunity having them to reason. It is related with being able to see. We use parallelogram, rhombus but you cannot think much over. This is not the case in mathematics. We had a calculus professor back in the college and he was saying: Mathematics is like a playing a violin, the more you play the

more you take the pleasure in and be delightful. Mathematics is really like that. But some people are more assertive in geometry. This might be because of the reason that they feel more confident in it. Frankly speaking, I am not very assertive in geometry. But I think that you have to develop yourself in all aspects and if you have shortfalls you complete them, thus I believe that a mathematics teacher should both give math's and geometry lessons. But I prefer mathematics more. Do you know that students are also aware of it ? They can judge it very well. When I came to that school from a vocational school the vice president told me that we can give you the lessons that you are more confident with, I mean do you prefer math's or geometry? And I responded this is unacceptable for me, if I think like that I would be accepted my deficiencies, actually what I want from you is to give me a variety of classes and lessons so that I would not stay at one section which is the point presently that I cannot agree with my colleagues, for instance they say I am predominantly in geometry section I don't want the math section, give me only geometry lessons. But what if a student ask you mathematics question? That must be like that you have to develop yourself in all areas, if you have shortfalls than you can complete, this is a different issue, after all the former vice president told me "I appreciate that madam". He was giving us his due.

Story 2: This is a technique which I have learned from my mentor

[She reflected on an episode from her 11th mathematics video recordings and indicated a rule that she has learnt from her mentor as a pre service teacher which simplifies students to fill in the table of inequality in identifying the domain of the logarithm function.]

This is not known by many teachers but I did not find it by myself this is a technique that I have learnt from an experienced teacher at the beginning years of my teaching. Thus I never give a formula jam to the students. Like I said before students confuse where was the same sign with a or the inverse of a. This is the case for 2^{nd} degrees and what are we going to do if it's 5^{th} 6^{th} or 7^{th} degrees? Shall we give a rule for each of them? I give only one rule, not the products or divisions, there are lots of tables on many books. I don't give any of them. This is the most eligible one. I was also a student when I have seen that from my mentor…I have made lots of tries with the formula and I reached the same results both by the longer method so this method is advantageous in time management. I also get students to take notes of it as warning. What to do for instance when it is double root? etc…I make them to write down one by one. I try to proceed by filling the shortfalls in students' knowledge for instance later on we will come across with exponentials and root numbers in logarithm which they nearly don't know anything. But this rule is among one of my best rules.

Description :

Now we will move on to logarithmic inequalities from now on actually we are done but I am taking extra time now since my colleagues could not finished yet. We have common examinations on January you know. From now on we will continually solve problems and make practices, and when we get in to inequalities we will solve inequalities and repeat them once more.

Theorizing:

> They are shier in geometry, because they don't have any backgrounds about it: New curriculum has settled their hash. In 11th geometry subjects are related with triangle knowledge but they don't know much about it because they overwhelm them with vectors in 10th geometry and they cannot learn thoroughly or properly because this topic is at the end of the first term which is the time that the common exams starts so and the important points are being missed. Since they don't have the necessary prior background when they start to the 11th grade they are shier.

Description:

[After watching another episode from her 11th geometry class she describes how she makes the comparison between geometric concepts]

> Last lesson I have had them to make notes one by one again. I made the comparison myself but I asked them first. Think a while…For instance in which quadrilateral are the diagonals vertical? They can answer like rhombus and square. First, some said rectangle, then we said no their diagonals were not vertical , they were same as length, then some said parallelogram some said it is wrong, and then we ended up with …I made them to write it down. Quadrilaterals with vertical diagonals were rhombuses and squares. Then I asked one more thing: Which quadrilaterals have the same diagonal length and then we moved on to squares as we have seen rhombuses beforehand. I reminded that all of the properties of the square exists in the rhombus as well except for that the corner angles being acute. And what else there we have discriminating properties of square from rhombuses so that we called it a square? There were ones who answered right but it was two or three, in a class of 30 and more students only a few can answer that question.

Argumentation: But they started to use the mathematical language better

> But they started to use the mathematical language better, they might say since we have stated that it is an angle bisector since we have a property that comes from the parallelogram if it is an angle bisector than the other angle must be absolutely 90 degrees. When I asked why are the diagonals vertical, they started gradually saying that. Then after this point he manages to comprehend the stuff.
>
> Did I give you the second exam paper? I drew a rhombus without drawing the diagonals vertical in which they have to put it themselves. Students who could see the verticality solved the problem. Or I might give the same length now we will use squares and I will say that diagonals have the same length in square then while giving one of them I will make them to solve problems which uses "the other one has the equal length". I plan to ask a question like that in the 3rd exam for instance.

Story 3: The aim was students to make interpretation and getting them to use prior topics

> As far as I remember, in mathematics lessons and for the specifics of logarithms we aimed students to make interpretations Them to be aware of the differences of the logarithm function and the other functions, we started with the domain. Now they are able to do that. Since they have problems with the graphs so as the graphs of the

logarithmic functions, I have solved more graphical problems including the university examination questions. Apart from those we have solved more complicated problems but I asked much easier in the exam. I tried to cover all, that's why I asked 12 questions, two of them were up to their choice one from complex numbers and the other from logarithms they preferred logarithm. They can make that interpretation at least much better than geometry though.

In geometry my aim was getting them to use the prior topics rather than using triangles. For instance, when solving a problem with rhombuses students can say that "I have used the property of parallelogram or rectangle". We tried to have them to make connections when we were explaining the rhombus proceeding with bridging connections with getting them to write down the important areas. I always tried to teach like that so they won't see the parallelogram and rhombus as separate quadrilaterals. I have taught like this throughout my 19 years career. And I am not even aware of what the curriculum is or is not like. I see that my students are successful because of my strategy.

Story 4: I try to give my students in a way that I learnt: if it is short it is easier

I used rectangle for instance while solving the problem if I solve like this students will solve like this too…I try to give my students in a way that I learnt when I was at their ages…for instance in trigonometry, my teacher was using the same right corner of the board to explain the signs of the trigonometric functions and used a mnemonic code for it. I think it is very useful. Students do not ever forget the region where function is negative or positive. I use another one for tangent 2a formula: tan plus tan divided by 1minus tan tan. That was how we memorized the formula. If it is like a roll even if it is short it is easier. I have learnt like that maybe it helps. Yes since we have learnt like that otherwise if you would follow the general procedures…for instance where does those formulas come from, ok it is nice, but one hour of two will let go for drawing out one of them.

Story 5: I'm trying to teach that in 9^{th} grade but I could never had the chance in that school

I am telling that "you have to use a proper language first". In these SM classes students are like: "we don't need our language related skills to be very well we don't need to read much". No, if you are a good SM student first you need to read a lot of books, understand what you read, interpret it, and solve problems. That's why, students are to explain the procedures of what they did, articulate it and in doing that they should use mathematical symbols and expressions rather than daily life language. I'm trying to teach that from 9^{th} grades but I could never have the chance in that school. No. Never had. But if I had the chance they could have been using these expressions better…But unfortunately this never happens in this school, you take one class you make a certain effort to carry them to another point then you leave them and start with a new class from zero. This tires a teacher very much.

Augmentation:

For instance, 11 c is a good class but yet the success rate was 30 percent in the first exam. The reason is the students could not express themselves; they had never had the chance. Until that only writing the result was sufficient. Students have found something from there and something from there and there. This is also related with the teacher. I should first use the board properly myself then students should take notes properly. I should be first like I told then students should be like me.

They are taking our so called petitions and telling us to indicate our requests. I have written the 10 and 11th grades mathematics first, of course the geometry lessons are ours too. But what they give is completely different; these kinds of things tire out you. If I could teach to a class which I have taught previously I would see and know the shortfalls of the students, I would say "I could not taught this and that but did this and that". For instance when teaching complex numbers where they have the biggest shortfalls, in the arguments they are stucked then we make revisions of trigonometry. I do it for one or 2 hours but it is not enough you have to do it for 5 or 6 hours.

Argumentation:

Can you imagine the nonsense in the geometry curriculum? We teach trapezoids before parallelograms and while teaching trapezoids we give a property of parallelograms. Two years ago we were first teaching parallelograms and the trapezoids then proceeded but now trapezoids come first. I have to go parallel with the order necessarily because we have common examinations we cannot meet in the middle. But the 9 and 11th geometry kills students and it is not able to teach something. In 9th geometry there is everything I don't know very well since I did not see the annual plan but there are solids areas lots of stuff and in 10th geometry you drawn in vectors and not enough time for the triangles when he comes to 11th grade how can I teach them without knowing these? Things are like this nonsense. But, unfortunately since the intention is different…

Interview #4

Actually if they have left the curriculum to us, if the things in the reports we wrote each year would have considered important, things would be more effective and well. Especially we are complainant about the geometry as mathematics teachers. The geometry knowledge is sacrificed and students are in a situation of not being able to learn geometry. Look at the grades of the 9th and 10th grade geometry throughout the school, they are all disaster. And they cannot carry any 11th year geometry knowledge. They put vector knowledge unnecessarily and meaninglessly into students' minds. You might teach those as well but as a topic not for semesters and semesters. A semester! Not a month. Presently the vectors continue until the midst of the second semester at 10th grade. This is unnecessary. We keep telling that for two years but no one cares and I don't think that it will be taken into consideration anyhow. From now on this will be like that because the intention is different, I don't want to step in to the politics here….we would reach our objectives much more better if they have left those things to us. Especially in terms of university exam success rate. They should do correctly 20 out of 25 question in the exam but now you are lucky if they solve 10 of them. The distortions of the system influence our motivation negatively and obstruct us in reaching our objectives.

Interviewer: How did you perceive this study in terms of reaching your objectives?

Story 1: I realized that in the past I was not listening to what student`s say

This was asked on the notes you gave me. It had very positive effects on me, actually I always determine some objectives at the beginning of each year as I keep saying these are my objectives for the first term and those are for the second term…The disadvantaged part was I have only one mathematics class this year I could not make comparisons between classes. This is a science and mathematics class and they are relatively good I think mostly I reached my objectives for them. But now I realized that in the past I was not listening to what student`s say impatiently completing their answers or comments. Now I am more cautious to let them to finish their words. Yet we have the time constraint that's why I was completing student's sentences, but now I don't.

I wait and see whether he will be able to finish his words or not or how will he complete his expression? In the past, I was impatient I did not wait for them to finish; now I care about listening. By allowing more time to student`s opinions or ideas, focusing more on what they say or you know, I realized in myself that I am trying to activate students more in my lessons. But especially in mathematics lessons…In geometry lessons we have to be in a little bit more hurry that's why I could not give so much opportunity to students…In those lessons I am the one who is more active. In mathematics this year I feel more comfortable and asking students' questions in an ease and calm manner waiting for and listening to their responses or ideas until they finish or letting them to make more comments and allowing more time for all these during my lessons. And I think this has been very effective this year.

Now I`m aware of this fact, after all student's achievement is considerably high. I generally hear very positive things from my students about lessons and I had many compliments from the parents this year, I get many things from my students this year at the 11 th grades. I really say that with all my heart I have discovered this year at my 19th year that how impatient I were in the past I should give more turns to speak and more time to listen to them. I was indeed giving time but it was very limited now I wait more and now I feel less exhausted.

When we decided to work on mathematical communication the first thing that came to my mind that teacher leads the flow of communication when students are active. Teacher is a resource who they will consult when they feel stuck and frustrated. On the contrary I tell and show everything then… you are so wear out. Now I feel more comfortable and contended. But you need students with well backgrounds so that they won't have problems with the operations because in that case they cannot progress that much (on their own). One might say: "I cannot do it" and waits. My advantage is the 11th grade mathematics class has a relatively good background. In fact the success rate was 78 % I guess. Is not there someone else, very bad, like zero so to speak? Yes, these are the students who changed their sections. Some students have changed their classes and some of them came from social science classes, and their (mathematics) background is very weak and they are still, how to say "closed"…I will pay more attention to them in the second semester…These 22 percent is very important to me, since the rest is set out they can now use the mathematical expressions, write them express them understand what it is written in a math book and can make interpretations, They can also connect prior topics to present one: This was my objective. I think we can see the difference way better if we work together next semester.

Theorizing:

I want to add something more. The students of this generation have always looking for your mistakes or deficiencies. Most students who go to dershane claim that you are doing something wrong or sometimes deficient. However, they are not aware of the fact that I'm giving the same formulas maybe in another way or modality…They know something but that knowledge are floating in the air they cannot put them in the right place.. "Ooo you wrote it wrong teacher this is not what they`ve told to us in dershane"…On the other hand the other students who do not go to dershane thinks that this is the only place that I can learn maths so I have to learn from her sooner or later. Sometimes students say that "there were 15 formulas at dersane why did not give those to us?" I get angry and say "because I don't want you to learn!".

They give so many formulas at the dersanes. Students don't want to memorize formulas or learn any rules anyway. Why am I giving that rule at the inequalities? To ensure them not to memorize the rules since they cannot memorize evenly, they get confused. Was it positive or negative? Just give the basic rules at logarithm and let them to extract the others.

My aim is to reach the students who do not go to dersane. Students attending to a dershane are trying to trip me up or they want us to move the lesson quickly, one of them for instance says "you are teaching very slowly or solving very small amount of problems.The system brings forth that students are to learn by themselves the major aim is to terminate us….

Story 2: This is how we connect in mathematics

When we head into the sequences in the second term these are the questions that I ask mostly to the students: Does the constant sequence reminds you something? Constant function. We have given the same rules when teaching the functions. In the types of functions, you know how is it like when the function is constant or for two polynomials…in the polynomials when you are equating them for instance how is it like the equality of functions. You have to use the transitions between them like a bridge so that they don't think of the other as a different thing, he can think abstractly, how do we transfer how do we do the operations this is how we connect in mathematics. You know trying to get them familiarize with this system of course not all of them will be mathematician but they need to learn this way of thinking. Students in EA might not deal with a lot of mathematical operation when they enter university but this kind of thinking will carry them to somewhere else very easily.

Story 3: Your sentences must be rational comprehensible and clear

If you are good in math's then you are also good in social science or literature…We should not have ambiguity in our words because our deficiencies or mistakes cannot be retreated. That is your sentences must be rational comprehensible and clear. My students have these fallacies too and I cannot put up with these. I feel a strong desire to interrupt. I do this a lot in my daily life too; it's because of my identity. I suppose I'm a little bit impatient I want things to be done urgently because I'm very busy I want everything to be done in a minute, but I listen to my students more carefully, I listen to them more, really I do…

Story 4: Unless any interpretations has been made students cannot manage the transitions

Sometimes this could happen: Things being expressed by students does not match up with what they were doing, it is because of the insufficiency in their expression.When they try to explain what they did they say different things talk about different things…I am the person who will bridge these two I am the resource or facilitator .

In doing these I pay extra attention to students to finish their words for about two months in case they can speak because in the social classes there is no one speaking and I cannot ask so many questions. You might ask why because they have a very weak or empty background that`s why students cannot generate any comments, make any connections. In other classes the knowledge is floating in the air but in the EA classes mostly there is nothing. If not any interpretations has been made, students unfortunately can't manage the transitions from topic to topic or make connections between concepts, but not all of my colleagues are able to do these interpretations even if they know… this is the truth.

Argumentation:

I was allowing less time for listening, I am on this for about two months. In the second semester I plan to have students to present their term projects which we have chosen from the topics of the second semester, they will make individual presentations. In the mean time they start studying, I warn them before 3 or 4 weeks like we are coming close to your topic. I want them to make a short presentation which lasts them 1 0r 2 lessons. They come and ask if there is somewhere that they did not understand and I explain it to them, actually I also support them. Students never miss a question from that subject ever. For instance I will teach sum and product symbols and the proofs of inductions. Proofs are relatively new to students so…After each students making their presentations I will then solve problems. Since I know that only students' presentations are not effective. I know because I do this for years they say that "we understand better from you can you repeat once more?" But I believe that this is a wonderful chance for the student who does the presentation. Since their classmates can ask questions in any moment and they could also ask questions to them … I mean they make their presentations precisely like I teach in my lessons. Making a presentation could be a problem for some of them at first but after that the other students try to do what they see from their friends the positive and they try to omit the negative parts. I think this is very favorable.

Augmentation:

My aim for this class is after logarithm I aim them to make more comments and make interpretations. Until that I was asking questions about to what extent they can carry things they have learnt, from now on both of these types of questions can be asked: Both questions aimed at prior knowledge and more interpretive questions. I take a lot from student's perspective…Students can take such various strategies or ways in order to reach solution that sometimes I could not think. But the important thing is giving chance to them. Solving problems all the time or writing a problem and asking them to solve it, it is no good. Students might not learn very well when everything is under my control. We can see that, if I'm not mistaken.

Story 5: I try to have them communicate via problem solving

Communication between students is limited when they became seniors they happen to see themselves as opponents or rivals, they do not tell what they know to the closest friend, and he should go and learn like I have learnt, he ought to not so solve the questions, I ought to they say… Thus communication in senior classes is very limited. But we are trying to provide this in 10^{th} and 11^{th} grades. I try to have them to communicate via solving questions in the tests. In geometry for instance I try to make them to ask questions to each other buy not answering all of the questions in the class. For instance in test 3 there is a question unsolved number 5. I ask "Is there no one else who solved that question", someone says yes I did, then I tell that "you are going to tell it to your friend". That's how I try to increase the flow of communication among students.

In some topics or some subjects I can have them to comment more on but in some topics it is not very possible to have students to reason. For instance in the sum and product symbols we have certain rules in order to do the operations and we will give these rules but in sequences it is possible that you can make more reasoning or probing and it can be easier that you may direct students. Another topic is trigonometry or parabolas. There can be very well comments or contributions from students. We have said sometimes solving fewer questions can be better, remember. In trigonometry I solve very few questions for instance. Talking about the concepts or the topic can help students to learn better because they are prejudiced about this topic. I try to have the class to reason and talk about it supporting with graphics of trigonometric functions than trying to give it very theoretically. Actually I don't go deep with graphics I usually prefer as a visual and brush over like this graphic increases or decreases periodically like this. If you teach all of these slowly and piece by piece then they will understand very well. For instance, the 10 th grade curriculum is very nice this year. You only teach trigonometry for the whole second semester. So you have the chance to teach like I described before. Sometimes I finish lessons with solving two or one question but sometimes there remain unsolved questions, and we solve them on the next lessons. I'm trying to solve minimum amount of questions that helps to make interpretations, summing ups the subject, and helping them to be aware of the it's differences. Students will redeem the rest with homework which I check afterwards.

In sum I pace along slowly trying to solve minimum amount of problems as far as possible trying to have them to make more contribution in terms of reasoning and interpretations teaching the subject and have students to notice the differences and similarities…from that point on students become able to retrieve by themselves with homework or with extra work. I check homework and I assign one verbal note from this homework so students are practicing at home while solving those tests from his/her book. Like we told before let's solve the key questions, open that door and let he move ahead, if he stumbles we may help. This must be our situation in here I think.

Interview #5

Description :

> Yes, I tried to make students to come to the board and to present and organize a one or two hour lesson for the subjects they chose as their annual projects... This is easier to do in the science/ math classes but, the TM classes have a bit more difficulties especially in geometry and I have to be more active in these classes so I did not prefer this method in those classes. At least students learn their project topics very well. Now in the 11th geometry it has been added some analytical geometry topics like circle equations, closed form analytical forms…These were analytical geometry topics previously, given as a separate lesson but now it has been added into geometry courses where the analytical geometry lessons will not be opened next year in senior year. These subjects are very well presented by the students in the science or math's classes and I do the presentation or teaching in EA groups.

Description:

> There is no problem with these classes when they listen properly, there are no problems remained related with my communication with students. I just think that SM classes make a lot of noise in lessons, talk a lot which roots in their self confidence compared to EA groups (regarding maths). They do not talk about or discuss things about maths, this noise is much about other stuff between them, laughing and talking about other stuff, this is generally our problem in spring terms, after the first exam they usually think that everything is over but it is not. There still remains lots of important topics to be covered after that…I do not know why this is the case…but it happens each and every year, but apart from these I do not have any problem with students or my lessons…

Returning to the video: Pupil solving the problem on the board this is another pupil not the one presenting the topic.

Interviewer : How do you find the progress of that student? Was she confident at that level previously or?

Argumentation: Becoming more active after presentation

> She is a very good student, for sure…The weaker one is the one who makes this presentation, and students who are weaker choose these projects after all, in order to get an additional mark to support their final grades. Generally they do, for example learn the sum symbol by preparing the project and don't ever miss any questions or problems at University entrance exams. If I did not have them make presentations,

> like I was doing in my earliest years in the profession and just give them the subject students come up only with writing, they don't learn the subject. But, I say them "you are going to make a presentation along with solving examples and questions from the university entrance exams just like I do, write problems to your classmates and have them to solve those questions". In that case students necessarily…for instance this pupil had came and asked me questions many times before he did the presentation… "Teacher, I`ve read these from the book but I did not understand what does it say here". As I say, slowly they are coming to understand the language of math or the mathematical expressions, previously they say they did not look or understand anything from the written statements from the book but now we are able to understand little by little and come to ask questions about the places we did not make account of. After making the presentation she become more active in lessons she participates more… This was the one who made his presentation before that, his activity has increased after presentation, he presented the introduction of the sum symbol and this friend of him did the properties of the sum symbol. Look he went to the whiteboard to solve the problem, you see…

Augmentation: Major shortcomings of students

> But there are lots of shortcomings within their backgrounds though this class can be considered as a good class, there are many still. But the EA classes… oh god… their situation is how to say… pathetic. These students neither know numbers nor functions, polynomials or parabolas. Nothing…The most important gaps in this class is trigonometry, they confirm that also. And also "the knowledge of functions". It was also in our examination, the Anatolian high schools teachers' selection exam, the inequalities, functions; graphics…Functions will show up everywhere. Still as a teacher I'm asked to present my knowledge about functions. So this is very important.

> There are complainants of course.. They say 'it is not as effective as you tell or teach' but necessarily seeing the student performance is like that. You have to put up with this (to students). Ask when you did not understand and I explain. Moreover, we go like this one lesson and then I make the other lesson as usual. It is like this student activity on Monday and I teach again on Friday, my performance. I usually ask them whether there is a problem with what their friend explains, and still we are going over again and again on each question or problem but I want them to tell me on Friday if something remained unclear and I'll get over it again. Then, there was not any problems..Some students say that we don't want this method, but I said no way I decide this not you, if there is a problem we will handle it together.

Story 1: I know I have to wait and I wait

Interviewer: How do you think about students' activity or speech this semester? Do you think that they are talking more than first semester?

> They are talking really good, especially students who make the presentations, I feel that there is a different light in their eyes; they became very happy taking the role of the teacher…Especially, there are students whom voices I've never heard of, she is for example a very silent student, I have been heard her voice. At the very beginning

they start with low voice, like five minutes then they open themselves, then they become very good through the end of it.

Interviewer: You have said that I listen more carefully and patiently to students

I care a lot. For instance students doing the presentation makes uhmm mmm…. Ehhh…a lot of silences exist sometimes he needs 5 minutes to extract a sentence but I'm listening with patience now, because I know that I have to wait, I wait…But I feel sad sometimes to tell the truth that lessons are wasted because as the more problems we solve in 40 minutes the better it is for us. But now I`m taking care of it.

Story 2: It is very well you go faster and get practical but do not solve it like that in my exam

Timing is very important you know sometimes when we are solving a problem students going to dershane; most of the students in SM classes are going to dershane; solve by omitting lines, or 3 or 4 operations. We are making our assessments in open ended question forms not in multiple choice test item forms; you should do this in University Entrance trial exams in dershane, it is very well you go faster and get practical but do not solve it like that in my exam, all stages or parts have certain points in the answers key when you write only the answer you get only 2 points. I'm struggling with these students. They are generally have seen the topic previously in dershane and want to solve the questions on the whiteboard but when these students start omitting 3 lines or operations the rest of the class do not comprehend, they ask me where did this came from, how did this lead to that, and when I insist them to explain or write it more clearly, they might say 'why bother with writing these middle stages'..Sometimes they even criticize me, "look how much time you are having us to waste? Don't you think we are pacing so slowly?"

Description:

She compares me with her teacher at dershane who teaches her to solve each question in 1 minute maximum. Hence, while struggling with those sort of students, telling that we will not keep up with your speed, but the average class speed, if you are done with your problem you can solve another one, but do not interfere with my speed and my teaching. Sometimes they might say just skip these… For instance while we were doing the induction proofs and she said "we did not see them at dershane so just skip them". I am telling them that I will not repeat the explanation again so you should be carefully listening to your friends who is doing the presentation. I' m not going to repeat all those through like this is because, of this property, so on I only solve extra problems. So listen her as serious as you listen to me ask her whenever something is not clear for you, she is responsible for articulating and clarifying the issue for you if still there is a roblem then I will step in (and participate.)

Story 3: In the 11th grade I think efficiency is increase and this is supported by the presentations

I am always there, the biggest concern of who make the presentations is that also. What if I can not explain clearly and they do not understand? I say then I will step in since I'm physically present anyway. Some students attempted to chose their projects from other lessons initially since they had concerns that they could not do a presentation. They told I can not talk like you talk and explain with long sentences, actually new generation students are a bit…speech impaired..I think…They are very bad at verbal expression. Yet they are very ineffective at using mathematical expressions and language and also at writing them. But this is not their offence, it results from the teaching experiences they have, everything should occur by writing in front of their eyes, they have always focused on visual learning, there is no audio learning in them, teacher will write and they will write right after her, no symbols. When I say sum from K=1 to n. I don't need to write it in the earliest times they were writing just as I say, and this is a huge problem. In the 11^{th} grade I think efficiency is increased and this is supported by the presentations.

Argumentation: Listening to the presenter very carefully

From now on, whole subjects are under student`s responsibility for instance the next topic is sequences, operations on sequences it is given to a student…But of course I am always there for back up. Since students may skip and stutter and sometimes construct wrong sentences in terms of mathematical knowledge then I immediately step in. So I have to listen to the presenter very carefully. May I focus on something else while she is doing her presentation? No, this is not possible. Necessarily in case that there is a wrong expression I have to correct it since students do not forget wrong info, they can forget the right one but not the wrong one. Sometimes very weak students are doing presentations, for instance the girl who will be doing her presentation today is very bad, bad at mathematics. And I can guess how her performance will be more or less. But let's see. Don't give up I said and we will see her today.

Interviewer : Do you make them to make their presentations individually and do you plan to do it that way?

Description:

No. But in geometry for instance in one class there are 12 students who take term project from the topic of circles since EA classes are of 50 students they are very crowded. I give the circle equations to 3 students; one will present the closed form the other will the parametric forma and the other will be presenting the properties of tangent or chord`s properties…I spread out the subjects to students like that. What if say a circle equation comes to two students? They one will present the subject the other will be asking questions and solve it. But of course I will be there for back up.

Post study interview

Description:

We keep going with the presentations. On the last lesson two students were also prepared visually. They have made their presentation via a projector like teachers do. Students liked it very much and it was very effective.

Interviewer :How come?

At least it was different for them and the exam results are fine. 88 % success..

Story 1: But in these presentations actually I get more tired

Interviwer :How would you evaluate students' situation from the beginning of the year?

> There has not been radical changes in my mathematics lessons for students because they were good after all, but in geometry and especially in TM classes since the second term topics are related with circle, it has been a bit easy and fun for them that's why there were more students who proceeded to attack in geometry. They have also made presentations for their projects…but in these presentations actually I get more tired. Even though the students do the presentations still I have to repeat once more in order to provide student learning or they ask what they did not understand to me not to his friend. Even if they would asked to the student doing a presentation I had to step in since these students are generally weak they cannot answer properly. So I get exhausted.
>
> But that student never misses any question regarding his subject. Since I tell them that "You have to solve the ÖSS exam questions or similar level questions you have to make them practicing then they study obligatorily and never misses that question in that subject"

Story 2: It occurred to me that…we can set a better student as a mentor.

> While weak students are doing the presentation neither the presenter nor listeners do not understand what is being done. But this is my aim. I have to make them to accomplish that! They are getting used to as time passes minute by minute they feel more ease and when it is the second hour then they became more comfortable doing that. Thus I warn the weaker students to not to take these projects as means of survival, since it does not help them to survive.

Interviewer :What if you might think another option to include these students to projects? How would you design it?

> Ok let me think, it occurred to me that… We can set a better student as a mentor or teacher to them and then you can say that study with your fiend for a couple of weeks and I will let's say have you to make a presentation. Or like "you prepare a project and your friend might be your coordinator I will evaluate all of you"…I have not done that before but it could be a useful idea, and in order to motivate the better one also, the coordinator student might evaluate the group of students he trained. But there is also that, in geometry for instance in the circles there is not much need for prior knowledge, we need more in quadrilaterals and it is much more fun to them…The coordinator students might evaluate her trainee students, like we make some students to check the classes' homework. We can do that on next year's.

Verbal assessments can be very good indeed; I will have this in my mind. But these things can be applicable for 9. 10 and 11the grades most unfortunately due to the examination system.

Story 3: The number issues are the biggest problem for the students

I could not do much on second term it is mostly comprised of student performance. Look Tansu will come to the board she might do a induction proof...good student very good indeed...very excited personality..Wants to do first, answer first that's why she makes a lot of mistakes.

Previous proofs were easy for them but this was a bit difficult. There is factorial you know, their biggest lack of knowledge, 9 th grade knowledge…Quantitative, I mean number issues…Their biggest problem is with the number area, like factorial concept. Then the functions. Trying to support them within different topics …

[While video is streaming… teacher is directing students with her questions and when students ask something to her she says you are going to say it after I tell you to do. Then it is seen on the video that one student interrupts and asks can I say something and say…Teacher says I was about to say it…then student says ok teacher I apology. Her comments about this situation:]

How quick she is to apology very cute…but I think her apologizing that much is too much for me… She has multiplied that before but did not add k+1 to this side, and this confused her about when to add that K+1..They don't do such proofs very frequently; in fact they have never come across like, there are no graphical interpretations like… whether they have drawn the graphs of a function? I don't know. It is a 9^{th} grade subject actually. Like the linear function graphics, when the graphics is given this must the associated equation etc.

Argumentation :

When they got stucked I have to move on. They cannot do those kinds of comparisons. This is a special situation. It is already given that k is bigger than 1 but they cannot think that k+1 is bigger than two. I do these because they are quantitative students, if they were EA students I would not do that much. Since they will be studying science and mathematics related subjects I want them to see that proofs are done like that at least. Or they might develop their proving techniques a little bit…In the dersanes they have given formula portfolios to students. There is for example a formula for the arithmetic sequences and 20 formulas are given like that… especially for the series. They have given that the sum of a series is found like that, but students don't know whether the first term of the series begins with r or m? They say I did it teacher that's how they taught us in the dersane. We also deal with correcting those shortfalls or completing these deficiencies. It would be better if they were not going to anywhere else and listen from me directly.

Theorizing:

Interviewer: Do you worry about they can't do if I don't intervene?

I'm worrying about the others won't understand I think they won't understand when a student with a weak background what if she explains superficially and obstruct others to understand fully or if there are shortfalls this is what I worry and why I interfered a lot.

There is a lot been going on in the school today, negatively for the last three months. The vice principal comes and demoralize us in the breaks and we go to the lesson with that psychology. Can you imagine? Sometimes he come during the lessons and say I will sign your lecture records and check it. He himself comes just for demoralizing me. We were in a psychological war but we had to continue lecturing. Just imagine that how did these three months pass? I don't even want to remember and I did not want to come to the school either…

Interviewer: Here students are trying to make a point?

Of course not everyone (she tells 4 or 6 student's name) not everyone talks only these students sometimes ozge and gulsah, but these don't speak until they are certain. But Tansu and yasin for instance speak even if they are not sure. "Is it possible teacher?".Very nice. "I think like that. Is it appropriate?" Thinking aloud …

Story 4: I interfere because if I don't it turns out to be a big thing

Interviewer: These kinds of interactions don't occur frequently, am I wrong?

Actually it occurs a lot but there is no when the camera is recording they are afraid of saying or doing something wrong. I interfere because if I don't it turns out to be a big thing, noise increases and it becomes like you cannot understand anything, they don't listen anything whatever you say the other continues to speak and turns out to be a quarrel. Not any consensus is made up, only a junk noise. This won't get anywhere or no result. Ok they should talk let's see if they can come up with a result. "Ok friend your idea is right".There is no such thing. Then it is finished and we end up with nothing. I have to put things together again…

Argumentation:

If you have followed the latest ÖSS exam questions and especially in the second level we see lots of interpretation questions like graphics regions they increase or decrease…Were you there in that lesson, when I was teaching about the extended range of the logarithm function telling that if the function increases the graphic would be or decreases…they told wy bother these give the formulas and then we will solve the problems..Do you remember?

While we were solving an ÖSS exam question. There was a rectangle put under a decreasing logarithm function, it moves as tangent at the base and they ask the area of that rectangle. I told those students "Ok? How am I going to find that area? We know how to find the area of a rectangle but how do I find that a on the long side and b on the other side of the rectangle? How do I without thinking?"And they said "Now we understand what you say."

Story 5: I learned that I have to take the position of listening students more

> Being able to make more use of mathematical knowledge, using more mathematical expressions and signs, developing mathematical thinking procedures I have focused on these this year on my lessons, I believe I have accomplished to a large extent. I tried to provide students abstractions and visualize things which they could not see with their eyes but within their minds. I learned that I have to take the position of listening students more. I have listened them all the time this term, they presented and I listened, interfered when appropriate or it might have been the very end of the lesson and that I did not want students go without solving a problem, to solve the question since the lesson would finish. I really care more about students talking with more patience and much more listening, allowing them more time to talk.I wish I had more time to that more because we have to catch up with the plan…

Argumentation: Turning things into action

Interviewer : You said I try to turn things into actions. Can you explain it further?

> Mathematics should not be an uninviting lesson; they should not take mathematics as a lesson but take it as a necessary thing throughout his whole life. They should not learn it obligatorily, but with love and curiosity. “Hey, look! that comes from there and it is possible to go from there to there or there is a relationship between them.” I am trying to maintain that. For instance we give functions until 12^{th} degree but there are few students who are able to connect these.
>
> And I am a little bit technology impaired person and opposed to these smart boards. Writing is a must for mathematics, working together on the board with students or without explaining it to them right away or patting them on the back this lesson can neither be taught nor get loved. Otherwise all of them have computers and cd’s , they can watch and learn, if it were like that it would. Teacher should take care students from both tactile, audial, visual all aspects…

Description:

> We have multiple roles, at the beginning of the topic we are teachers, but as the subject progresses while solving exercises we undertake the role of a friend, whenever they cannot we approach them with a mother`s compassion in order to have them continue trying .

Story 6: There is only one option, one person that I can do this with but not with others

Interviewer: Do you think you can repeat this kind of study with a colleague?

> We are 16 in this school and If I think for my school there is only one option one person that I can do that with but not with others… and this study can be done better with a mate from the same school because there might be some disadvantages like I might have a lesson on the time that she has lessons, I should be free in order to observe her…But I could do it with Muge that she is the only colleague that I trust now. If we are to observe each other about one hour each month. For instance when I cannot figure out something I ask muge and we try to find the solution together.I don’t trust the others so much. Sometimes she asks me to take a look at a problem and I ask her too… the others they never approach this way they also do not ever

look at students problems, for instance if I am not there today and my students could not find me they don't ask anybody because they think like: if he could not manage to learn then she should take a private lesson from me. There are so many teachers thinking like that so nobody trusts each other anyway.

But I would do it with a colleague whom I can trust. I'm open to anything that might develop me.

Theorizing: Future objectives

I am choosing necessarily similar objectives each and every year. Our school policy does not allow us to take the same classes we have lectured on the previous year. So we have to take different classes every New Year and as from the beginning of the year I have to deal with student's mathematical language and writing. They don't even know how to write mathematically… Then I aim to emphasize mathematical expressions and develop mathematical thinking system and methods, connecting the topics and subjects and have them make interpretations and reasoning respectively…Because I cannot teach them one more year I can not set a new objective and set these objectives every year.

CURRICULUM VITAE

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EDUCATION

Degree	Institution	Year of Graduation
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PUBLICATIONS

Çelebi İlhan, E. Gül; Tekin, N.,& Karaçam, S. (2008).*Okul Temelli Mesleki gelişim programının verimliliğinin belirlenmesi raporu,* Ankara: Milli Eğitim Bakanlığı Retrieved June 10, 2013 from http://otmg.meb.gov.tr/belgeler/otmg/2008_otmg_raporson.pdf

Çelebi İlhan, E. Gül (2012). A functional perspective on examining the mathematical teacher talk in a multilingual secondary classroom. In L. G. Chova, I. C. Torres, A. L.Martínez (Eds.) *Proceedings of the International Conference on Education and new learning Technologies 12 IATED* (pp, 2207-2214) Barcelona, Spain.

HOBBIES

Cycling, Swimming, Dance, Movies and Creative writing