NEAR AND FAR TRANSFER LEARNING IN MATHEMATICS LESSON DESIGNED BASED ON COGNITIVE LOAD THEORY PRINCIPLES:A CASE STUDY

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ABSTRACT

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The purpose of the study is to explore the contributions of learning tasks-involving worked examples, diverse worked examples, completion examples requiring backwards/forward fading, self-explanations and group discussions on near transfer and far transfer of learning towards learning, understanding and interpretation of mathematics. As a qualitative research approach, instrumental case study was used in this study. Qualitative data and quantitative data were collected from 30 students (female=19 and male=11) attending to Grade 11 and learning units of Derivatives within the curriculum of IB Mathematics Standard Level. The participants of this study were selected through convenience sampling method from a private school in Ankara. Students studied four main units. For each unit, firstly students learnt the subject through worked examples;then wrote self reflections and participated at group discussions at Moodle. Next, they studied at completion examples; then wrote self reflections and participated at group discussions at Moodle. Finally they completed practice problems. This cycle continued for four units for 12 weeks. In order to investigate the impacts of these learning activities, students' pre and post achievement tests' results were analyzed through paired sample t test and descriptive statistics. Moreover, students' self-reflections and group discussions were analyzed through content analysis. The opinions of students on the effects of worked examples, diverse worked examples, completion examples requiring backwards/forward fading, self-explanations and group discussions towards learning and understanding were gathered through semi structured interviews and student replies were analyzed through content analysis for main themes.

It could be interpreted from the results that there were indicators of near transfer and far transfer of learning in self reflections and group discussions. Near transfer indicators had higher frequencies compared to far transfer indicators'. There were agreements between the qualitative data analysis results and the comparison of pretests and post tests results.

This study may fill the gap in literature by concentrating on the implementation methods of these activities and their possible contributions towards near transfer and far transfer of learning in mathematics. The study focused on investigating possible reasons that may address to solve some educational problems in teaching derivatives units at grade 11.

Keywords: Near Transfer, Far Transfer, Worked Examples, Completion Examples, Self-Explanation

ÖZ

BİLİŞSEL YÜK KURAMI İLKELERİ TEMEL ALINARAK HAZIRLANMIŞ MATEMATİK DERSİNDE ÖĞRENMEDEKİ YAKIN TRANSFER VE UZAK TRANSFER: ÖRNEK OLAY İNCELEMESİ

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Bu çalışmanın amacı çözümlü örnekleri, geri ve ileri tamamlama örneklerini, yansıtma ve grup tartışmalarından oluşan öğrenme etkinliklerinin, öğrenmedeki yakın transfere ve uzak transfere, konuyu anlamaya ve yorumlamaya katkılarını incelemektir. Bu çalışmada nitel araştırma yaklaşımı olarak, örnek olay incelemesi kullanılmıştır ve veriler nicel ve nitel yöntemler kullanılarak toplanmıştır. Nitel ve nicel veriler, Ankara ilindeki özel bir lisede 11. sınıf düzeyine devam eden ve Uluslararası Bakalorya Matematik Standard düzey müfredatında Türev konularını öğrenen 30 (19 kız;11 erkek) öğrenciden toplanmıştır. Öğrenciler bu konu başlığı altında dört ana ünite üzerine çalışmıştır. Her ünite için öğrenciler sırasıyla, çözümlü örnekler üzerine çalışmış; konu hakkında yansıtıcı günlüker tutmuş ve Moodle'daki grup tartışmalarına katılmışladır. Daha sonar, aynı unite için geri ve ileri tamamlamalı örnekler üzerine çalışmış; konu hakkında yansıtıcı günlüker tutmuş ve Moodle'daki grup tartışmalarına katılmışladır. Ünitenin bitiminde alıştırmalarla öğrenme etkinliklerini tamamlamışlardır. Bu döngü oniki hafta süresince, dört ünite için aynı düzende devam etmiştir. Bu öğrenme etkinliklerinin öğrenmeye olan etkilerini araştırmak için, öğrencilerin ön ve son başarı testleri puanları çıkarsamalı ve betimleyici istatistiki yöntemler kullanılarak çözümlenmiştir. Ayrıca, öğrencilerin yansıtıcı günlükleri ve grup tartışmalarındaki cevapları içerik analizi yöntemi kullanılarak incelenmiştir. Öğrencilerin çözümlü, geri ve ileri tamamlama örnekli, kendine açıklamalı ve grup tartışmalı öğrenme etkinliklerinin öğrenmeye ve anlamaya yönelik katkılarına ilişkin düşünceleri, yarı-yapılandırılmış görüşmeler yoluyla toplanmış ve içerik analizi yöntemi kullanılarak incelenmiştir. .

Araştırma bulguları, Bilişsel Yük Kuramı temel alınarak hazırlanmış etkinlerin öğrenmede yakın transfer ve uzak transfere katkıda bulunduğunu göstemiştir. Ancak öğrenmedeki yakın transfer göstergelerinin uzak transfer göstergelerine gore daha yüksek olduğu ortaya çıkmıştır. Nitel verilerden elde edilen bulgular, nicel verilerden elde edilen bulgularla örtüşmektedir.

Bu çalışmadan elde edilen bulgular, sözkonusu etkinliklerin uygulama tekniklerini ve matematiği öğrenmedeki yakın transfere ve uzak transfere katkıları konularında alan yazındaki boşlukları doldurabilir. Çalışma, 11. sınıf düzeyinde öğretilen Türev konularındaki eğitim-öğretim problemlerinin olası sebeplerini araştırmaya ve eğitim problemlerinin çözümüne yoğunlanmıştır.

Anahtar Kelimeler: Öğrenmede Yakın Transfer, Öğrenmede Uzak Transfer, Çözümlü Örnekler, Tamamlamalı Örnekler, Yansıtma

To My Parents and My Brother
Zafer, Leman & Göksel Tüker

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CHAPTER 1

INTRODUCTION

This chapter consists of background of the study, problem statement, research questions, purpose of the study, significance of the study, the role of the researcher and definition of terms.

1.1 Background of the Study

So far, mathematics has been found to be one of the hardest courses in terms of learning, interpreting and understanding the concepts effectively to be able to integrate them in authentic contexts.

Mathematics topics are in close relation for development with many fields such as Economics, Physics, Physics, Engineering, Medicine and understanding key concepts in mathematics plays an important role in the improvements at those areas. Moreover, Calculus is one of the major topics that forms the basis of main subjects (Orton, 1983;Stewart, 1975) ; as it facilitates to form the fundamental principles at various discipline areas. However students face with many difficulties to interpret and understand the concepts of Calculus (Keith, 1989; Orton, 1980). Like every mathematics topics, interpretation of Calculus concepts well enough would facilitate problem solving in different contexts. Perkins & Salomon (1988) emphasize that students who do not understand the key mathematical concepts well enough, will not gain much enough from the general strategy of defining the problem and representing it well before the start. Thus, transfer of learning plays an important role in problem solving.

Transfer of learning has been defined in similar ways but in different wording by educational experts. One of the definitions that Erik de Corte (2003) suggested is that “transfer of learning is the broad, productive and supported use of acquired knowledge, skills, and motivations in new contexts and learning tasks” (p.143). Transfer is acknowledged as a fundamental theory in learning connecting process and outcome. Thinking critically, making plans, giving reasons, metacognition activities and problem solving strategies are all the results of transfer of learning. Hence transfer of learning is the core concept in understanding and interpretation. It is classified in categories due to learning outcome and application of knowledge. Perkins &Salomon (1988) state that transfer of learning is essential as the aim of education is not raising the students only for narrow school tasks but for authentic context. They also added that transfer of knowledge was a matter as students did not transfer knowledge into problem solving contexts easily when they had to think about new situations. There are two types of transfer of learning: near transfer and far transfer. *Near transfer* is the application of learned behavior, content knowledge, concepts, or skills in a situation similar to the original condition. Clark (1999) states that usually near transfer tasks are procedural. Same steps are applied in the same order. An example would be that applying the same steps for adding two three-digit numbers, after learning to add two two-digit numbers. On the other hand, *Far transfer* is the application of these skills in a situation that is different than the original learning context. Clark (1999) adds that far transfer is advantageous when the learner has to make judgments and to adapt to different situations. However, far transfer of knowledge is harder to teach and to transfer in learning.

It is found that traditional teaching methods limit students' thinking skills and learning in mathematics (Boaler, 1964; Major, Baden & Mackinnon, 2000; Tick, 2007; Ali, Hukamdad, Akhter & Khan 2010; Taj & Sivalingam, 2013) Boaler (1964) compared a school with the traditional skill based instruction where teachers introduced students to the different procedures in a structured and clear way with another school that focused on project based instruction. The author conclude that in mathematics traditional teaching methods resulted in limited knowledge that are ineffective in non-school settings . Ali et al. (2010) found that problem solving method enhanced the achievement of the students in mathematics compared to traditional instruction. The study of Taj et al. (2013) showed that students who learnt with computer assisted instruction performed better in achievement tests that students taught with traditional instruction. They conclude that students must be active participants of learning rather than "passive recipients of information". Previous studies aimed to find the answer to the question of how students can go beyond the limitations of their own ways of thinking and several methodologies has been developed to improve more powerful sense-making systems. It is acknowledged that the content of the learning tasks should support learning and understanding. To overcome the difficulties of learning mathematics, worked examples; completion examples; diverse worked examples; self-explanation and group discussions were used to facilitate transfer of learning due to Cognitive Load Theory principles.

To facilitate transfer of learning in mathematics, several studies are conducted on problem solving in class. One of them is the study of Kalyuga, Chadler, Tuovinen and Sweller (2001). They found that transition from worked examples to full problem solving facilitated learning for apprentices by gradually increasing the complexity. Proponents of Cognitive Load Theory claim that worked examples reduce the extraneous cognitive load on learner and they assist optimizing schema acquisition (Sweller & Owen, 1989; Sweller & Cooper, 1985). Yıldırım (2013) states that extraneous cognitive load which is not useful, prevents learning and during the learning process it should be reduced as much as possible. The author adds the studies of Clark, Nguyen and Sweller (2006); Renkl (2005) and Renkl and Atkinson (2010) showed that the aim of using worked examples lies under the principle of learners focusing on the method of solving problem through forming a schema, while they study on worked examples. The study of Schwonke, Renkl, Salden and Aleven (2011) also supported the previous findings of CLT on worked examples. Authors conclude that worked-example reduce extraneous cognitive load. Clark and Mayer (2011) claim that worked examples are powerful to build new cognitive skills and they are effective in sales lessons; mathematics probability problems and justification; and financial analysis. Another study of Renkl, Atkinson and Grosse (2004) on problem solving revealed the result that faded examples assist learning as they lead to correct type of self-explanations. Due to the result of their study, it was found that the faded steps were most effective towards learning regardless of where the fading was in a worked example. At many studies positive impacts of completion examples are found. Pass (1992) claimed that effective acquisition of cognitive skills were realized through completion examples. On the other hand, there are other studies showing that completion examples might not work effectively at all circumstances. For example, Renkl,Atkinson, Maier and Staley (2002) found that in some conditions completion example instructions were not as effective as worked example instructions. Renkl, Atkinson, Maier & Staley (2002) also mentioned that Paas (1992) did not find any difference in the performance of students who worked with complete and incomplete worked examples; however study of Stark (1999) showed that making examples incomplete can assist learning. Related to extraneous cognitive load Schwonke et al. (2011) add that “For conceptual knowledge, no ratio of worked steps and to-be-solved steps had an advantage over another” (p 61). Clark and Mayer (2011) report that there are still issues to be resolved, such as: how to best fade multi-step worked examples; when to stop working with worked examples and start practicing; and how the design of worked examples should be for ill-defined domains. Thus, there is still need of assessment and evaluation of worked and completion examples at special circumstances.

It is claimed that using *varied context examples* will assist students to concentrate on the structure rather than the covering story that will assist them for near and far transfer of the knowledge. *Varied context examples* are defined by Clark and his colleagues (2006) as: problems with similar structures with different cover stories. It is cited that “Ranzijn (1991) and Shapiro and Schmidt (1982) point out that increased variability of practice along the task dimensions is beneficial to schema acquisition and hence to transfer of acquired skills because it increases the chances that similar features can be identified and that relevant features can be distinguished from irrelevant ones” (Paas & Merrienböer,1994, p.124). Hence, the instructor has to be aware of these factors that would affect the transfer of learning in problem solving contexts.

Furthermore, the related approaches of constructivism such as inner speech, discourse as social interaction can also be integrated to the designed instruction for effectiveness. They can be used for facilitating higher order thinking skills which will be more effective to mostly actively participating learners, as Hannafin (1992) claims (Driscoll, 2005). Vygotsky (1978) states that human development starts from conversational and dialogical process; then moves inward becoming “inner speech” of thought. The development continuum of the process of speech internalization realized through dynamics according to Vygotsky (1986) where he explains the dynamics as “schema of development[where] first social, then egocentric then inner speech" (p. 35)" *Self-explanation* is one of the alternative ways that assist constructing knowledge through inner speech that leads to knowledge construction or schema formation. Also self explanation is one of the principles in Cognitive Load Theory that increases germane cognitive load. Renkl (2005) states that self-explanations are learners' own explanations that are mainly directed to themselves. Sweller and Mwangi (1998) mention that use of self explanation is an instructional technique that increases attention to relevant instructional area while is discourages irrelevant activities. Past studies showed that generation of more effective self-explanation facilitating understanding of worked solutions comes from good problem solvers compared to poor problem solvers. (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Recker & Pirolli, 1990). Moreover, Chi and Bassok (1989) claim that good problem solvers produce self explanations that involves linkages between the example statements themselves; and between the examples and previously covered principles in the text.

Additionally, Wong, Lawson and Keewes (2002) concludes that "a self-explanation procedure can be seen as a simple but potentially powerful technique for acquiring knowledge in the study"(p.254). Their study focused on geometry course at high school level. There was a significant difference in pre and post test results in the group of students who were facilitated self-explanation tasks .

Clark and his colleagues (2006) claim that *self-explanation* is a process of learners' effort to understand the solution rationale through finding the domain principle underlying the solution step. It is cited by Clark et al. (2006) that Renkl and his collegues (2004) claimed that "…faded worked examples help learners to incorporate more of the monitor and correct type of self-explanations that benefit learning" (p.229). Another study of Renkl and his colleagues (2002) also clarify that the quality of self-explanations to be especially important for far transfer and they suggested a further study on a combination of fading and self-explanation training to check for the facilitation of *both* near and far transfer of learning. Dunlosky, Rawson, Marsh, Nathan & Willingham (2013) agree with the previous studies and they add that self-explanation' facilitates understanding through generation of learners' own personal understanding of the to-be-learned information; since it assits to integrate new information with existing knowledge.

Students' reflective journal writings can facilitate *self-explanation* and it is one of the methods for construction of students' own knowledge in mathematics. Keith (1989) mentions that reflective journal writing assists students to explore and comprehend what is taught with their own perspectives; they facilitate learning and retention. Brandau (1989) is in agreement with Keith as he states that writing is central to understanding process; as we often do not realize what we think, until we discover to find the words put to paper.

In addition, social constructivism principles serve a variety of strategies so as to facilitate learning and understanding mathematics. "From a constructivist view, to gain deep understanding, students must link new learning to the ideas of they already have; otherwise they will simply learn verbalizations" (Leithwood, McAddie, Bascia & Rodrigue, 2006, p.19). According to social constructivism, knowledge is being constructed culturally and socially; learning takes place through interaction. Both Lave and Vygotsky put emphasis on combining social interaction and learning facility to foster best learning. Thus, this link is made through sharing ideas. Group discussions are one of the frequently used strategies in class to promote an educational atmosphere where learning is constructed through sharing knowledge.

As a result, mathematics teaching and learning is being affected from these recent developments in learning theories and technology integration; so it looks for alternative ways to help students for deep understanding. Students' perceptions of the situation, understanding of concepts and perspective to the events are shaped by the classroom community. Hodson (1998) mentions that "Even if they wished to do so, students could not isolate themselves from the classroom milieu; it continually impinges on their thoughts, emotions and relationships, interpreting messages they receive from teachers and other resources" (cited in Leithwood et al., 2006, p.22). Thus, classroom activities would facilitate students' critical thinking skills and improve metacognitive skills in mathematics, when learning theories are combined effectively.

1.2 Problem Statement

Basic derivatives and their applications are based on main principles of Calculus. Calculus is accepted as one of the important subject areas within modern mathematics (Stewart, 1975; Orton, 1983). It forms the roots of several applications in different discipline areas through formulating fundamental laws and principles. It is also known that calculus plays an important role in the improvement and reasoning of science. However, students face with difficulties while learning Calculus (Keith, 1989; Ubuz, 1996). Rasslan & Tall (2002) state that the fundamental units of calculus-functions, limit-continuity, derivatives and integral are found to be challenging for students to grasp and interpret.

Previous studies reveal that some students with different characteristics experience difficulties in learning mathematics. Miller & Mercer (1992); Mercer, Jordan & Miller (1994); Miller & Mercer (1998). Miller et al. (1998) listed these characteristics such as students with continuous failure, being a passive learner, having a math anxiety, attention problems, memory problems. These experts also revealed another type of student profile which is students who are not aware of how to become "metacognitive learners"; these types of students are hardly aware of selecting relevant problem solving strategy and adapting strategies as appropriately when needed. Hence, some studies aimed to address the needs of students by the analysis of efficiency in learning and new strategies have been developed in mathematics.

On the other hand, still there are unanswered questions on the metacognitive strategies that students apply while problem solving for deep understanding and learning. Efklides et al. (2006) added that students might think that they have understood the solution of the problem from worked examples; however metacognitive experiences such as feelings and judgments regarding to one's knowledge state might affect the understanding process.

Studying with worked examples, completion examples that facilitate backwards/forward fading, diverse worked examples and self-explanation are some of the recent strategies that might lead students towards effective learning by scaffolding and inner speech. By this methodologies, students are experiencing worked examples requiring fading some of the steps by gradually increasing its complexity leading to full problem solving. Efklides et al. (2006) mentioned that previous studies (Sweller, 1994; Kalyuga, Chandler, Tuovinen & Sweller, 2001) had shown that the effectiveness of worked examples depended on the learners' previous knowledge and expertise as well as on the cognitive load they impose. Hence it was claimed that worked examples efficiency is limited due to learner profile.

Renkl et al. (2002) mention that the use of worked examples as a learning methodology does guarantee effective learning. According to Atkinson, Derry, Renkl, and Wortham (2000), there are some factors moderating the effectiveness of worked examples towards learning: (a) self-explanations, (b) situational factors, and (c) example design. Within the study of Renkl et al. (2002) on the focus of example design, it is found that fading as a feature of example based learning has been found to be effective, at least due to near transfer. Moreover, backward fading manner has been found to be more favorable than forward manner. Hence a further study on type of fading used in mathematical problem solving and the proper use of worked examples in instruction would enable to make further generalizations in instructional design.

Mwangi & Sweller (1998) state that "Example based problem-solving instruction approaches have proved successful in Algebra (Sweller & Cooper, 1985; Zhu & Simon, 1987; Cooper & Sweller, 1987), Algebra word problems (Nathan, Mertz, & Ryan, 1994), Geometry (Zhu & Simon, 1987), computer programming (Trafton & Reiser, 1993), and Statistics (Paas, 1992; Paas &Van Merrienboer, 1994)...On the contrary, conventional problem solving encourages the use of heuristics such as means-ends analysis, which imposes greater cognitive demands on working memory" (p.174). However, Mwangi et al. (1998) add that it is proven that not all types of worked examples would be effective.If there are worked solutions requiring to split attention (at areas of Engineering, kinematics, psychological reports, geometry) between and to mentally integration of two or more resources of information, worked examples create high cognitive load and effectiveness of them decreases. Multiple sources of information integration is suggested to be embedded in instruction to eliminate such split of attention. Elimination of split of attention results can be gathered through self-explanations of learners that is one of the focus areas of Cognitive Load Theory. Hence, self-explanations do not only facilitate understanding through scaffolding; but also gives further analysis of high or low mental effort of learners during learning process.

Reflective journal writing is one of the tools for easing self-explanation as students are active learners. Countryman (1992) mention that students must construct knowledge by themselves through set of activities such as exploring, justifying, presenting, discussing and investigating; thus writing enables the realization of critical thinking skills. Writing contributes to scaffolding of ideas in interpretation of key concepts. However, although previous studies claim that writing reflective journals assists learning, there are also some conflicts due to the contribution of reflective journal writing towards learning. While there are some experts claiming the positive effects of reflective journal writing towards understanding (Countryman, 1992; Jurdak & Zein, 1998); on the contrary, there are also experts emphasizing that reflective journal writing has no effect on mathematics learning (Porter & Masingila, 2000; Croxton & Berger , 2003). Therefore further studies are needed for advance analysis of effective integration of reflective journal writing in mathematics instruction.

Though, there are some studies on teaching mathematics through worked examples, backwards/forward fading and self-explanation, the studies on math teaching by providing opportunities for scaffolding and inner speech; their impacts on near and far transfer in mathematics learning due to students' explanations as well as opinions are minimal especially at high school level. Hence, more research is needed in this area to investigate the metacognitive strategies that students apply while problem solving for deep understanding and learning.

1.3 Purpose of the Study

The purpose of the study is to explore the contributions of worked examples, completion examples, diverse worked examples, self-explanation and group discussions designed based on cognitive load theory principles and some approaches of constructivist view towards learning, understanding and interpretation of mathematics. In order to investigate the impacts of running a flexible learning environment through set of activities, the contributions of the learning tasks-involving worked examples, completion examples requiring backwards/forward fading, self-explanations and group discussions- on near and far transfer of learning were planned to be analyzed and the opinions of the students towards learning and understanding were to be gathered. Hence, study would focus on the contributions these learning activities on near and far transfer learning in mathematics at grade 11 mathematics level.

1.4 Research Questions

This study was conducted through integrating activities that were designed based on Cognitive Load Theory. The learning activities involved worked examples, diverse worked examples; completion examples that facilitated backwards and forward fading; practice problems; self-explanation through reflective journals and online group discussions at Moodle. The research questions guided this study are:

RQ1. Is there a significant mean difference between pre and post achievement test scores of students who were exposed to the learning activities in mathematics lesson designed based on cognitive load theory principles ?

RQ 1.1 Is there a significant mean difference between pre and post near transfer scores of students?

RQ 1.2 Is there a significant mean difference between pre and post far transfer scores of students?

RQ2. What are the indications of near and far transfer of learning in students' self-explanations and group discussions?

RQ3. What are students' opinions about the contributions of worked examples, diverse worked examples, completion examples, self-explanation, group discussions and practice problems towards learning mathematics and deep understanding?

1.5 Significance of the Study

Students have faced with challenges at transfer of learning and at problem solving while they have been studying mathematics (Thorndike & Woodworth, 1901; Luchins, 1942; Brown , 1989; Cummins,1992; Gordon, 2004; Zachariades,2007). Calculus is one of the main topics that has essential contributions towards innovations in different field areas; hence it plays an important role by its applications. Calculus involves variety of problem solving skills at different contexts with the requirement of critical thinking and successful transfer at various authentic settings. Hence, students studying Calculus and its sub topics should be guided towards successful transfer of learning with the tenets of Cognitive Load Theory (CLT) for crucial contributions at various fields.

The principles of Cognitive Load Theory suggest that the use of worked examples, diverse worked examples, backwards/forward fading and self-explanation for better learning transfer by facilitating related cognitive load. On the other hand, although it is claimed that worked examples, backwards/forward fading and self-explanation have positive effect in transfer of learning, their impacts in mathematics has not been investigated much at high school level in depth while teaching units of derivatives so far. There are still questions to be answered on to what extend near and far transfer learning differ depending on the level of mathematics learning. This study may contribute to reveal students' improvement via recognized learning activities with its qualitative structure and to establish alternative methodologies in the field of instructional technology along with students' opinions, while teaching derivatives units at high school level.

The study of Atkinson, Merill & Renkl (2003) revealed that backwards fading method fosters both near and far transfer and they found that there was a positive correlation between using fading and using self-explanation. However, the participants of these studies were college level students and high school students in experimental environment. Most CLT studies are experimental study and there are fewer qualitative studies in this area. It is important to investigate the findings in mathematics learning in depth that might lead to further studies on transfer of learning and problem solving strategies with less cognitive load in mathematics at high school level. The purpose of this study is to fill the gap in literature by concentrating on such instructional strategies based on CLT principles at high school level with qualitative content analysis. The discussion on the effectiveness of these strategies towards deep understanding and learning mathematical concepts are realized through the analysis of qualitative data along with the analysis of quantitative data coming from achievement tests during their implementation process. Thus, findings of this study may contribute to instructional technology field in regard to design and development of effective instructional/learning materials, and designing online group activities for mathematics learning.

Students usually have difficulties to transfer knowledge into problem solving situations. Practitioners are looking for effective strategies to facilitate meaningful transfer of learning. Also how to combine examples with self-explanation for promoting deep understanding is another problem for practitioners. The study on diverse worked examples, completion examples along with self explanations and group discussions are limited to particular mathematics subjects such as Geometry, Probability, Financial mathematics and Algebra in literature. There are few studies realized with teaching derivatives with CLT principles and group discussions. This study might contribute to the gap in the literature of instructional technology related to instructional design in teaching derivatives with CLT principles and integration of group discussions in computer environment.

This study might have a contribution to CLT principles with in-depth analysis of students' replies with its qualitative structure; whereas the quantitative data from the analysis of pre and post achievement test scores supporting the evidence of the qualitative data. Most research studies about CLT have experimental design and results are reported due to quantitative analysis only.

Finally, there are some studies about mathematics teaching realized in the schools of Turkey through the prementioned principles of Cognitive Load Theory. For example, the study of Takır (2011) claimed that designed Grade 7 Algebra lesson developed by the CLT principles increased achievements in tests and decreased the cognitive load. The author also stated that when students' perceptions are gathered about worked and completion examples, students thought worked examples had important positive effects in practive for learning Algebra; and they completion examples facilitated learning the units. However, there are very few studies at high school level involving mathematics lesson designed based on CLT principles in Turkey. This study also aims to collect students' opinions together with the analysis of students' reflective journal and forum discussions. Hence, this study has a significance to close the gap in literature about mathematics lesson designed based on CLT principles at high school level for Turkish students with its qualitative structure. Since there are not many studies conducted in Turkey referring to mathematics lessons designed basd on CLT principles.

This research has significance regard to practice (a) for teachers and (b) for instructional designers. The results of this study may guide teachers while implementing learning activities with CLT principles into practice with its findings and it may also assist instructional designers to develop effective mathematics lesson designs for the implementation of these learning activities while creating positive impact along with the perspectives of students towards improvement of these tasks. The results of this study can also be used by curriculum designers to reexamine the current status of problem solving activities and to revise related strategies in mathematics for the successful integration of blended setting tools .

1.6 Role of Researcher

The researcher of the study was the teacher of the students. The researcher aimed to be objective as possible while implementing learning activities and analyzing them. The roles of the researcher are explained below:

(a) The computer competency survey was developed through considering potential technological necessities of the study and the distribution of it realized before the study.

(b) The distribution and evaluation of MSQL Scale for the selection of interviewing students before the study.

(c) The development of materials for learning activities were realized through selecting IB question styles and writing questions.

(d) Student groups were formed for online forum discussion environment at Moodle.

(e) The interviews were conducted with 9 students at the end of the study.

(f) Students pretest and posttest scores in two achievement tests were analyzed through SPSS and quantitative data were interpreted along with qualitative data.

(g) Qualitative data were collected through students' reflective journals and online forum discussions. Collected data were analyzed through content analysis method by preparing Coding Schema. These data were interpreted and discussed by the researcher.

(h) Student interviews were transcribed. Content analysis was realized by developed Coding Schema.Results are interpreted and discussed together with the previous analysis results.

1.7 Definition of Terms:

Deep-Understanding: Perkins states that "as a mental process, understanding can be conceptualized as the ability to think and act flexibly with what one knows" (cited in Leithwood et al., 2006, p.27). Perkins et al. (2006) define performance conception of understanding as "understanding a topic is a matter of being able to think and act creatively and competently with one knows about the topic" (Perkins & Unger, 2006, p.97)

Near Transfer: Haskell (2001) defines near transfer as *"*This refers to when previous knowledge is transferred to new situations that are closely similar but not identical to previous situations" (p.29)

Far Transfer: Haskell (2001) defines far transfer as *"*This refers to applying learning to situations that are quite dissimilar to the original learning"(p.30)

Worked Example: "A worked example is defined as "step-by-step demonstration of how to perform a task or solve a problem" (Clark et al., 2006, p.190).

Completion Examples: Clark et al. (2006) mention that "In a completion example, some of the steps are demonstrated as in a worked example and the other steps are completed by the learner in a practice problem" (p.194).

Varied (Diverse) Context Examples are defined by Clark and his colleagues (2006) as: problems with similar structures with different cover stories.

Backwards Fading: According to Clark, Nguyen and Sweller (2006), "In backwards fading, the instruction fills the first steps and the learners finish the problem from starting to the end. Thus in a five step problem, first completion problem shows steps 1 through 4 worked out and the learner fills in Step 5. In the next completion example, steps 1 through 3 are worked out and the learner fills in Steps 4 and 5. At the end of the sequence learner is solving the full problem"(p.198).

Forward Fading: According to Clark, Nguyen and Sweller (2006), the instruction fills the last steps and the learners find the start of the solution to the problem from end to the beginning. Lines are faded gradually step by step.

Self-Explanation: Clark, Nguyen and Sweller (2006) defined that "A self explanation is a mental dialog that learners have when studying on a worked example that helps them to understand the example and build a schema from it"(p.226).

Online Group Discussion: An online discussion is exchanging ideas among participants in groups about a particular topic. Forums can be used to facilitate online group discussions and forums can enable participants to exchange ideas on a particular topic with open messages in online discussion groups.

Cognitive Load Theory: Clark et al. (2006) defined Cognitive Load Theory as "it is a universal set of learning principles that are proven to result in efficient instructional environments as a consequence of leveraging human cognitive learning processes"(p.6)

CHAPTER 2

LITERATURE REVIEW

This chapter provides base for this study in through discussion related to theoretical frameworks and research studies. There are six main sections within this chapter: (1) Learning Mathematics and (2) Problems at Mathematics Learning; (3) Transfer of Learning; (4) Online Group Discussions; (5) Cognitive Load Theory; (6) Conclusion

2.1 Learning Mathematics

For the last half of the century, mathematics learning activities has changed a lot. In the past, Thorndike and his colleagues claimed that mathematics is best learned through a drill and practice and demonstrated mathematics as a "hierarchy of mental habits or connections" (Thorndike, 1923, p. 52). Educators followed Thorndikes' Stimulus Response Bond Theory in mathematics education for a long time until a reform was suggested. Some of the new reforms shaped mathematics teaching and learning. Radical Constructivist Theory of knowledge which was against transmission of knowledge model in learning, affected traditional mathematical instruction with the suggestions of von Glasersfeld after 1984. Next, Holt's (1994) idea on mathematics education was shaped through progressive movement influenced by John Dewey (1899). Holt (1994) concluded that mathematics education "could be guided to expressive self-realization and social integration through scientific educational practices as they were evaluated and trained by experts according to their natural inclinations and abilities". Recently, there are several new trends and new forms of Mathematics activity influencing mathematics teaching. The new forms of Mathematics activities gaining significance are: "algorithms and programming, modeling, conjecturing, expository writing and lecturing" (Lovasz, 2006 p.2).

At high school, mathematics has been one of the challenging subjects for students to grasp and interpret the concepts into depth. There has been many studies conducted to identify the specific difficulties that students might have faced with during problem solving stages and about transfer of learning (Sweller & Cooper, 1985; Paas & Merrienboer, 1994; Jaworski , 2002;Kalyuga et al. 2003; Renkl & Atkinson, & Grosse, 2004; Clark, Nguyen & Sweller, 2006)

2.1.1 Strategies for Learning Mathematics

Within the previous studies, educational experts studied on effective strategies for permanent understanding transfer of the theories and their applications in the field. Different methodologies on problem solving are offered to guide students for permanent understanding of concepts with less cognitive load.

Educational experts looked for addressing students' interest areas and individuals' particular needs for learning. So as to interpret the concept of learning clearly, experts try to observe the way that individuals learn. Clark and his colleagues (2006) suggest the following for efficient learning by the support of worked examples, completion examples and fading strategy.

Replace Some Practice Problems with Worked Examples

A worked example is defined as "step-by-step demonstration of how to perform a task or solve a problem" (Clark et al., 2006, p.190). Renkl et al. (2004) conclude that a worked example is composed of a problem formulation, solution steps, and its final solution. Sweller and Cooper (1985) claim that studying on worked examples assisted learning effectively compared to working on practice problems right after getting familiarity on the topic. The study they conducted based on a methodology formed. Experiment group of students were introduced a topic and then they are given a worked example followed by a practice problem four times. Then they were asked to define the main principles of the problems step-by-step. The control group was just given eight practice problems immediately after the introduction of the unit. At the end a test was given on completion of problems. The researchers found that learning was facilitated through worked examples efficiently. The control group had more errors made on the test and they spent more time to finish. The researchers add that students grasp the conceptual knowledge in a shorter period of time compared to working on practice problems.

Additionally, Efklides et al. (2006) state that worked examples are most useful when the learner does not have the knowledge on the rules or the procedures for the solution of the problem. They add that at advanced stages of skill acquisition, learners' benefits come from problem solving and the study provided by experts on complex problem solving; so benefit did not come from worked examples. The studies of Kalyuga et al. (2001) show that after learners study on worked examples, active practice and in depth learning on one's own understanding of the problem facilitated learning.

Finally, Paas & Merrienboer (1994) conclude that "training with worked examples requires less time and is perceived as demanding less mental-effort than training with conventional problems. In addition, worked-example training leads to better transfer performance and is perceived as demanding less effort than training with conventional problems" (p.130).

Use Completion Examples to Promote Learning Processing

Yıldırım (2013) states that previous studies of Renkl (2005), Clark et al. (2006), and Renkl and Atkinson (2010) showed that the positive effects of worked examples might decrease when the learner has a prior knowledge on topic and experts called this effect as opposite effect of worked examples towards learning. Yıldırım (2013) adds that one of the solutions to this problem is decreasing the support of worked examples gradually that can be realized through using completion examples. According to Merrienboer et al. (2002) completion problems consist of requirement for learners to complete a partial solution, as they provide a given state, a goal state or a partial solution. They add that these problems set a bridge between worked and conventional problems. Clark et al. (2006) mention that "In a completion example, some of the steps are demonstrated as in a worked example and the other steps are completed by the learner in a practice problem" (p.194). It is mentioned by Clark et al. (2006) that Paas (1992) concluded that "training partially or completely worked-out problems leads to less effort-demanding and better transfer performance" (p.196). More experiments on worked examples and completion examples are conducted as explained below. It is argued by Clark and his colleagues (2006) that completion examples help reducing cognitive load as it enables providing schemas during the study with worked examples. It is added by the authors that completion examples lead to deep processing of information with its nature.

Transition from Worked examples to Problem Assignments with Backwards and Forward Fading

Clark, Nguyen and Sweller (2006) suggest techniques for efficiency of learning and backwards fading is one of them. According to Clark et al. (2006), "In backwards fading, the instruction fills the first steps and the learners finish the problem from starting the end. Thus in a five step problem, first completion problem shows steps 1 through 4 worked out and the learner fills in Step 5. In the next completion example, steps 1 through 3 are worked out and the learner fills in Steps 4 and 5. At the end of the sequence learner is solving the full problem"(p.198). Hence, when the students are assigned completion examples, they will be guided towards full problem solving by gradually fading the solution steps. Clark and his colleagues (2006) mention that Kalyuga et al. (2001) found that transition from worked examples to full problem solving facilitated learning for apprentices by gradually increasing the complexity.

According to Renkl et al. (2004), fading can be done in two ways: backwards fading and forward fading. For backwards fading procedure, first a complete worked example is introduced; then the last solution step is omitted in the second example; next, the last two steps are hidden in third example; in the final step last problem involves no solution steps. For forward fading procedure, first a complete example is given; then the first solution step is omitted in the second example; next, first two solution steps are faded out in the third example and the final example involves no solution steps (all three solution steps are missing).

Atkinson and his colleagues (2003) conclude that "their findings on the usefulness of a learning environment that combines fading worked-out steps with self-explanation prompts support the basic tenets of one of the most predominant, contemporary instructional models, namely the cognitive apprenticeship approach (Collins, Brown, & Newman, 1989). This approach suggests that learners should work on problems with close scaffolding provided by a mentor or instructor. This approach is characteristic of Vygotsky's (1978) *zone of proximal development* in which problems or tasks are provided to learners that are slightly more challenging than they can handle on their own" (p.782).

Use Diverse Worked Examples to Foster Transfer of Learning

It is claimed that using *varied context examples* will assist students to concentrate on the structure rather than the covering story that will assist them for near and far transfer of the knowledge. *Varied context examples* are defined by Clark and his colleagues (2006) as problems with similar structures with different cover stories. Past studies showed that diverse worked examples increase germane cognitive load through assisting forming schemas on problem solving steps.

It is cited that "Ranzijn (1991) and Shapiro and Schmidt (1982) point out that increased variability of practice along the task dimensions is beneficial to schema acquisition and hence to transfer of acquired skills because it increases the chances that similar features can be identified and that relevant features can be distinguished from irrelevant ones" (Paas & Merrienböer,1994, p.124). Yıldırım (2013) mentions Clark et al. (2006) stated that diverse worked examples and diverse problems facilitate forming flexible schemas that is not only helpful for learning principles and methods of solving problems; but also for learning the contexts and conditions where these problems should be used. Hence, in order to succeed at far transfer tasks, learners should be asked to explain the problem in its structural features that can be realized through conducting flexible schemas.

The study that was conducted by Paas & Merrienboer (1994) aimed to analyze the effects of learners studying on worked examples towards time and mental effort. The authors claim that "subjects in the worked condition would gain more from high practice variability than would subjects in the conventional condition" (p.124). Their results showed that "Practice-problem variability only had an influence in the worked conditions, such that high-variability practice resulted in better transfer performance than did low-variability practice" (Paas & Merrienboer, 1994, p.130).

2.2 Problems in Mathematics Learning

Previous studies showed that learners faced with transfer problems in mathematics. Barnett and Ceci (2002) mention about poor transfer examples that Thorndike & Woodworth (1901) came across within their study about the area of geometric shapes. Their results showed that in order to transfer knowledge, elements in the learning context had to exist in transfer context also.

Brown (1989) found that children could not form causal schemas to a subject if they knew nothing about it, for instance area. He concluded that when this criterion is met, children were able to transfer general principle to a novel context.

Study of Schliemann and Magalha˜es (1990) with uneducated Brazilian cooks, where they were given mathematical proportionality problems showed that they performed well on price problems since they were familiar with precise proportionality calculations; however they accomplished less well on recipe problems as they were acquainted with performing only with rough proportionality calculations.

There were also some cases where negative transfer had occurred, i.e. participants' performance was worse on the transfer task than they would have been. For example, Barnett et al. (2002) refer to the experiment of Luchins (1942).This experiment showed that "if participants were previously given training problems that required them to use more elaborate arithmetical processes, they were unlikely to solve a simpler transfer problem in the most straightforward manner"(Barnett et al., 2002, p.617). The authors added that according to Halpern et al. (1990), this type of overtransfer occurs in near contexts more than in far contexts.

Related to transfer of learning, effective strategies are looked for deep understanding in mathematics. Barnett et al. (2002) state the findings of Cummins Dellarosa (1992): "interproblem processing (focus on comparison questions) promoted more transfer than intraproblem processing (focus on specific wording or details)"(p.616). Similarly, the study of Needham and Begg (1991) resulted in that problem oriented training (e.g., trying to explain) leading to more transfer to a problem-solving task compared to memory-oriented training.

Calculus is one of the main subjects that has essential contributions to the other fields and as a result it is considered as an important area in mathematics. It forms the basis of topics that gains importance in different areas by its applications such as areas of science and even social sciences (Ferrini-Mundy & Lauten, 1994). Barnes (1992) argues that teaching calculus as a part of education might answer some conflicts of those learners who question "who uses mathematics" in real life through linking to authentic contents.

However, calculus has been one of the hardest topics for students to interpret its concepts. Leng (2011) mentions that "it is widely acknowledged that calculus concepts are abstract and complex for students and that teaching and learning these concepts can be challenging and even exasperating at times (Gordon,2004; Zachariades,2007)"(p.927).There has been good amount of research made recently to conclude that students face with difficulties while learning Calculus (Harel et al.,2006; Artigue et al., 2007). Gordon(2004) emphasizes the importance of generating underlying concepts of Calculus to make sense of algebraic practices in the area. Hence, experts agreed on teaching and learning concepts in Calculus is priority rather than simple calculation techniques for the improvement of student performance.

Understanding limits and derivatives are the elementary concepts in Calculus. Students need to interpret the idea of tangent line to the curve and its gradient to make a smooth transition to the applications of derivative. However, it might be still challenging students to transfer knowledge for the applications of derivatives to make a link. Naidoo and Naidoo (2007) mention that: “Differentiation assumes the understanding of function or more generally a curve (not all curves can be formulated by a function). There do not seem to be clear-cut characteristics that set advanced mathematical concepts from those in elementary mathematics. Each advanced concept is based on elementary concepts and cannot be grasped without a solid and sometimes very specific understanding of these elementary concepts. Thus the concepts of advanced mathematics carry an intrinsic complexity”(p.195). Authors emphasize that synthesis of appropriate mental frames for representation of concepts and procedures are essential for solution of problems in derivatives. Therefore, classroom activities should focus more on activities where students analyze more about the concepts and procedures in derivatives for sense making and permanent learning.

2.3 Transfer of Learning

Understanding, its indicators and transfer of learning have been studied by several educational experts. The concept of understanding has been interpreted in different ways by the experts. Perkins (1994) states that “as a mental process, understanding can be conceptualized as the ability to think and act flexibly with what one knows” (cited in Leithwood et al., 2006,p.27). Gardner defines understanding as “the capacity to use current knowledge, concepts, and skills to illuminate new problems or anticipated issues” (cited in Leithwood et al., 2006, p.29). Perkins and Unger (2006) state that understanding a topic well is a matter of knowing it well but knowledge by itself does not guarantee understanding. They also add that understanding a subject means forming a good schemata or mental model on the subject. Desoete et al. (2006) add that “Schemas or mental models are considered higher-order constructs that characterize on a conceptual level the integrated functioning of knowledge and beliefs” (p.86). According to Perkins and Unger (2006), forming a good mental model is not enough; learners need to demonstrate a good possession of schema or mental model.; hence a performance should be seen. Perkins et al. (2006) define performance conception of understanding as “understanding a topic is a matter of being able to think and act creatively and competently with one knows about the topic” (p.97). This can be understood through looking at learners’ interpretation on the subject; connections made by inner and outer life; strategies made; content of the discussions…etc. Chi et al. (1989) specified the operational assessment of understanding in the context of learning from examples under three measures: “ (1) Solutios to isomorphic problems; (2) Solutions to far transfer problems; (3) Explanations generated while studying examples” (p.149). This study also focused on the similar indicators derived from students' outcomes to assess understanding through transfer.

There are several definitions and theories on transfer of learning. Cormier and Hagman (1987) state that transfer of learning occurs when prior knowledge and skills create an affect on the performance of new knowledge and skills that are learned. According to Mayer and Wittrock (2006), transfer refers to the ability of learner in using prior knowledge in new situations. Barnett and Ceci (2002) stated nine dimensions for the taxonomy of transfer. For content dimensions which describe what is being transferred., they labeled as (a) learned skill, (b) performance change, and (c) memory demands. On the other hand, they groupped as (a) knowledge domain, (b) physical context, (c) temporal context, (d) functional context, (e) social context, and (f) modality are contextual dimensions of transfer that describe ''when and where learning is transferredfrom and to'' (p. 623). Forsyth (2012) mentioned that "transfer difficulty depends on how similar or different the target and base of a transfer problem is along each dimension" (p.517). According to Barnett and Ceci (2002) if the contextual dimensions has a high degree of similarity on the target and base, it is defined as''near transfer''; whereas if contextual dimensions have a high degree of difference on the target and base, it is defined as ''far transfer.''

However, there is not a common decion on whether transfer happens at all learning environments. Detterman (1993) mention that transfer is rarely happens and it is not easy to obtain transfer. On the contrary, some expert such as Bransford et al. (1999) and De Corte (1999) think that transfer occurs; the difficulty in identifying transfer depends on the perspective of the researcher.

In order to provide a meaningful understanding of transfer and its outcomes, the types of transfer are identified by experts. Near and far transfer are the most studied constructs in transfer of learning. The definition of near and far transfer tasks have a common consensus by educational experts.

Near Transfer

Near transfer was defined in parallel behaviors by educational expers. According to Haskell (2001), near transfer requires transfer of prior learning into slightly different situations. Jones,Antonentkot & Greenwood (2012) define that "near transfer is the epitome of applying classroom learning to real world situations" (p.481). Clark et al. (2006) mention that "Near transfer tasks are based on procedures that are done the same way each time they are performed" (p.220). Hence, near transfer involves knowledge demonstration in a new situation which is similar. For example, student learns the methods of finding derivatives and applies appropriate method to find slope a function that is similar to those which has been learnt.

The previous studies showed that individual difference might affect on near transfer performance (Goska & Ackerman, 1996; Sullivan, 1964; Woltz, Gardner & Gyll, 2000).

Woltz et al. (2000) mentioned that “attention processes related to disengagement from expected operations and resistance to interference from overlearned operations may be highly instrumental in the accuracy of cognitive skill performance and especially in the accuracy of near-transfer performance” (p.245).

Far Transfer

Clark et al. (2006) agree that “Far transfer tasks require the performer to adapt her skills to each new situation” (p.220). According to Jones et al. (2012) far transfer requires the use of analogical reasoning skills. Far transfer involves knowledge demonstation in a new and innovative situation. For example, student learns the methods of finding derivatives and makes generalizations on the graphs of functions and the derivative functions.

Some key points that can be examined in far transfer problems:

(1) Problems require additional solution steps than indicated in the problem or equation;

(2) Some initial thinking is needed before problem solution to extract needed variables from the word problem;

(3) Problems might require solving of multiple cases to be integrated to form the whole idea or generalization;

(4) Learners can be asked to explain the problem in its structural features that can be realized through conducting flexible schemas

According to the literature, far transfer comprises declarative knowledge construction with concepts and principles at an abstract level; so that arrays can become predictable across domains. When these arrays are applied to novel practice problems, it assists restoration of procedural steps (Peverly, 1991; Perkins, 1995; Anderson, Reder, & Simon, 1997; Phye, 1998; Schraw & Nietfeld, 1998).

It is considered that achieving far transfer performance is not high likely. Some recent studies showed positive far transfer outcomes. Jaeggi , Buschkuehl,Jonides &Perrig (2008) concluded that training on the N-back task advanced scores onthe Raven Progressive Matrices Task. Karbach & Kray (2009) determined a posive far transfer in training task-switching leading to performance on a working memory task. Chein et al., 2010 and Mackey et al. 2011 also found positive transfer at different tasks. On contrary, some studies failed to find far transfer outcomes although designed similarly to the earlier studies (Owen, Hampshire , Grahn, Stenton, Dajani, Burns & Ballard, 2010; Redick et al., 2013). Another example is the study of Kurtz, Boukrina & Getner (2013). Kurtz et al. (2013) studied on the effect of comparison on acquisition of relational understanding in a domain and its effects on further generalization of relational knowledge to a novel domain. They found that learners who did not grasp the source domain and did not learn well, had no chance to achieve far transfer.

2.3.1 Solutions to Facilitate Transfer

Van Merrienboer, Shuurman, De Crook & Paas (2002) mention that "Redirecting the learners' attention, by minimizing extraneous and simultaneously increasing germane cognitive load, is a very promising method to improve training efficiency and provides good opportunities to reach higher transfer performance" (p.36). To start with, the previous studies showed that training learners with worked and completion problems led to positive results on transfer performance. Pass (1992) concludes that "… cognitive structure resulting from instruction emphasizing practice with partly or completely worked-out problems is a more efficient knowledge base for solving transfer problems than one resulting from instruction emphasizing conventional problems. Training with partly or completely worked-out problems leads to less effort-demanding and better transfer performance" (p.433).The study results of also supported the conclusions of Paas (1992). Van Merrienboer et al. (2002) mention that "The completion effect indicates that solving completion problems yields higher transfer of acquired skills than conventional problem solving"(p.13). Furthermore, previous studies of Atkinson, Renkl & Merrill (2003) conclude that fading solution steps can facilitate both near transfer and far transfer.

Afterward, self-explanation has been found another effective strategy in problem based environment to provide transfer of learning. Gick and Holyoak (1983) suggest that if a more effective schema was generated by problem solvers, then transfer is enhanced. Clark, Nguyen & Sweller (2006) express *self-explanation* as "a mental dialog that learners have when studying a worked example that helps them to understand the example and build a schema from it" (p.226). To process the examples deeply, it is suggested by Clark et al. (2006) based on the studies of Chi and her colleagues (1989), self-explanation is necessary. Self-explanation has been found critical for successful learning with worked-out examples (Efklides,Konstantine & Kiosseoglou , 2006). During self-explanation, learner thinks aloud for reasoning for the choices and procedures in the solution of the problem.

Renkl et al. (2004) add that studying on worked examples and self-explanations would lead to germane load. *Germane load* is defined as that "it refers to demands placed on working memory that are imposed by mental activities that contribute directly to learning" (Renkl et al., 2004, p.60). They claim that *self-explanation* is a process of learners' effort to understand the solution rationale through finding the domain principle underlying the solution step. Roy & Chi (2005) conclude that "People learn more deeply when they spontaneously engage in or are prompted to provide explanations during learning. Thus, self-explaining on multirepresentational examples is a cognitively demanding but deeply constructive activity is contextualized in a specific domain (i.e., mathematics)" (p.280).

It is claimed by Renkl et al. (2004) that "…faded worked examples help learners to incorporate more of the monitor and correct type of self-explanations that benefit learning" (cited in Clark et al., 2006, p.229). Efklides et al. (2006) mention that the previous studies of Stark, Mandl, Gruber & Renkl (2002) showed that self-explanation that are linked to worked examples supported learning only when they contribute to the rationale of the solution and the procedure used.

Stark, Mandl, Gruber, and Renkl (1999) found that: "studying incomplete examples fostered significantly more the quality of self-explanations and the near and medium transfer of leamed solution methods than studying complete worked examples"(p.600). Furthermore, study of Atkinson, Renkl, Merrill (2003) demonstrated backward fading procedure together with self-explanation prompting significantly facilitated not only the near and medium transfer learning but also far transfer learning.

Finally, group discussions as a social constructivist view play an important role in transfer of learning. According to social constructivism, learning is being constructed by social interaction and knowledge is culturally and socially developed. Conceptual understanding of topics and mathematical thinking skills can be assisted through social interactions via discussions, group-work, sharing experiences, forums…etc in a math classroom. "Vygotsky (1978, 1986) believed that language was a psychological tool and that the usage of this tool invariably led to a series of inner or mental transformations such as the development of higher thought and concept development" (cited at Ehrich, 2006, p.13). This transformation in thinking is achieved through a process of internalization. It is mentioned that the *inner speech* comes from internalized social speech. It is cited that "Vygotsky (1986) perceives language development as a process which begins through social contact with others and then gradually moves inwards through a series of transitional stages towards the development of inner speech" (Ehrich, 2006, p.14) Thinking and language are dynamically related, since understanding and producing language are processes that transform the process of thinking" (Lindblom & Ziemke, 2002, p.2). Finally, it is concluded by Elrich (2006) that that inner speech results from higher thought and is developed through a set of developmental stages. He adds that these stages are thought to be from the external world that travels inwards; hence inner speech fosters deep understanding of the abstract concepts by its nature.

Consequently, group discussion enables learners to share ideas, construct knowledge through internalization socially on the transferred knowledge. Ernest (1998) summarizes the contribution of Vygotsky on scaffolding with his words: "Vygotsky offers a theory of genesis of higher level thought from language use" (Ernest, 1998, p.209). The teacher is a guider for learners to assist internalizing higher level functions. By scaffolding strategy, learners are encouraged to learn independently in a social environment. This study aims to integrate learning activities where worked examples students will provide individualized support related to problem solving strategies; whereas completion examples, self-explanations and group discussions, students will assist scaffolding ideas.

Therefore, this study aimed to combine CLT principles on worked examples, diverse worked examples, completion examples requiring backwards fading and self-explanation to the some approaches of constructivist view towards learning, understanding and interpretation of mathematics. With this goal, collaboration of individuals in pair works and group discussions of this study can be linked to social interaction principle of social constructivism. Also, studying on worked examples, completion examples; backwards/forward fading can be linked to Vygotsky's *scaffolding* strategy.

2.4 Online Group Discussion

As social constructivists claim, learning from social and interactive experiences is necessary, hence combining this perspective via online group discussions might lead to better transfer of learning in mathematics. Vygotsky's social constructivist approach denotes that learners going through the process of thinking together will experience self-actualization of the processes involved in individuals' thinking. Moreover, George Herbert Mead (1934) claims that individuals develop understanding through communion with others. According to the previous studies, cooperative learning is one of the useful methods to address to increasing coherence and understanding in mathematics. Hence, online group discussion strategy is one of the techniques that will offer learners an opportunity to think and interpret together and to realize self-actualization through unity of ideas.

From instructional technology perspective, Johnson & Aragon (2003) claim that technology integration in instruction has little impact towards learning with the conclusion of previous studies. Hence, the authors add that according to some proponents of instructional technology, Phipps and Merisotis (1999), the evidence of online technology on influencing the learning outcomes does not lead to essential impacts as much as other instructional factors such as pedagogy and course design. It is claimed that if instructional designers set clear learning goals and combine it with online environments, it would lead to effective learnig outcomes. Therefore, if online group discussion environment is designed due to aimed instructional goals combined with instructional methods with the tenets of CLT, it may result with desired impacts on learning. It is recognised that online group discussion is an effective method that fosters learning and understanding. Dosemagen (2007) mentions that online discussions can be helpful to solve logistical problems through written reflections derived from asked reflection questions for clarity. The author states that including a variety of opportunities for collaboration, providing an array of tools for students to choose, helping students to develop effective online communication skills, encouraging a face-to-face and online communication are some of the suggestions for settling an online environment for smooth transitions. Macdonald (2006) clarifies that online conferences in collaborative projects enable evaluation of process. He argued that so if a collaborative project was given online, teacher could assess the end product as well as process which would lead to much better understanding of students' learning facility in a blended setting.

Social Constructivist models stress the importance of collaboration among individuals. Collaboration and cooperative learning are the methods that facilitate learning through social constructivism. McInnerney & Roberts (2004) define collaboration as mutual engagement of individuals to solve a common problem or to achieve a common goal. The authors add that Paz Dennen (2000) claims that collaboration is a learning method that involves social interaction for knowledge building. McInnerney et al. (2004) state that there are four characteristics of a collaborative environment: shared knowledge between students and teachers, shared authority between students and teachers, teachers as mediators, heterogonous grouping of students. Hence heterogeneous grouping of students and providing discussion environment in class might lead to understanding through sharing knowledge.
McInnerney and Roberts (2004) argue that through collaborative environment, students teach each other to learn and to understand and collaborative learning can be effective towards learning if students share their thoughts, doubts, conflicts.

Not only the strategies are thought but also several guidelines are searched experts so as to enable effective use of online settings. Dosemagen (2007) suggests that setting a context for the studied procedures; using visual representations to clarify abstract concepts; using a variety of sample problems; relating concepts to mathematical contexts, visual and examples are the effective ways to facilitate learning in online settings.

Guidance plays an important role in learning through social interaction in class. Lave and Wenger (1991) claim that "knowledge of socially constituted world is socially mediated and open-ended" (cited in Ernest, 1998, p.231). As knowledge is constructed through interaction, monitoring and guiding the conversation plays an important role to make sure that effective learning takes place by realizing the desired objectives. Hence, if guidance is embedded in online group discussions, it will increase efficacy.

According to the previous study of Kramarski (2004), a mathematical discussion regarding online communication had at least four correlated roles to increase students' knowledge: "(1) mathematical discussion led students to discover all pieces of information related to the task; (2) mathematical discussion raised cognitive conflicts, which, in turn, encouraged students to discuss conflicts and suggest ways to resolve them; (3) cooperative work provided ample opportunities for students to participate in mutual reasoning; (4) mathematical discussion encouraged reflective discourse and collective reflection"(Kramarski & Mizrachi, 2006, p.601)

Kramarski et al. (2006) cited that: "Online discussion allows asynchronous exchanges and permits one-to-one and one-to-many interactions. Students exhibit motivation, learn independently, and transfer and apply knowledge to real-life situations (e.g., Deaudelin & Richer, 1999)"(p.219). The authors add that "Discussion mediates shared meaning. By critically examining others' reasoning and participating in the resolution of disagreements, students learn to monitor their thinking when they reason about important mathematical concepts (e.g., Artz & Yaloz-Femia, 1999; McClain & Cobb, 2001)"(p.218). Their study on online group discussion through metacognitive guidance training showed positive results in mathematics learning. These findings supported the previous studies conducted in mathematics learning at high school. Kramarski et al. (2006) reported that : “students who participated in successful group discussions most often responded to correct proposals by engaging them productively, whereas students in less successful group discussions frequently responded to correct ideas by rejecting or ignoring them (Barron, 2000a, 2000b; Kramarski, 2004)”(p.228)

Consequently, if mathematics classroom can be established regarding to the mentioned outcomes of the past research, then it will lead to create more flexibility towards learning that may result in efficient learning, deep understanding and higher achievement. This study aimed to promote an online learning environment with forum discussions and its activities for near and far transfer of learning.

2.5 Cognitive Load Theory

Cognitive Load Theory (CLT) focuses on the demands of the instructional material integration in order to reduce the cognitive load on working memory. Clark et al. (2006) defined Cognitive Load Theory as “it is a universal set of learning principles that are proven to result in efficient instructional environments as a consequence of leveraging human cognitive learning processes”(p.6). They add that CLT is about efficiency if efficiency is defined in two variables: learner performance and learner mental effort. Moreno (2006) mentions that “CLT explains learning outcomes by considering the strengths and limitations of the human cognitive architecture and deriving instructional design guidelines from our knowledge about how the human mind works (Paas, Renkl, & Sweller, 2003)”(p.171). Clark et al. (2006) claim that “instructional environments that result in higher learning outcomes with less mental effort are more efficient than environments that lead to lower outcomes with greater mental effort”(p.18). Hence, it is acknowledged that the aim of instruction concentrated on effective learning outcomes with less mental effort.

According to Cognitive Load Theory principles, when learners experience high and extraneous compulsory cognitive load, their performance was negatively affected (Scott & Schwartz, 2007). On the other hand, it is also claimed that performance is facilitated, if there is high and germane cognitive load. However germane cognitive load did not always enhanced performance. Grosse and Renkl (2006) declare that learner’s performance decreases if the germane cognitive load goes beyond the working memory capacity in mathematics learning.

The effects of instructional techniques increasing germane load that is directly relevant to learning are also explained through CLT. According to Merrienboer et al. (2002) construction of cognitive schemata are directly related to those processes. They add that “There are indications that variability over problem situations or other task dimensions, such as the manner in which the task is presented, the saliency of defining characteristics, or the context in which the task is performed, may increase germane cognitive load and improve schema construction (e.g. Paas & Van Merrienboer, 1994; Quilici & Mayer, 1996). The availability of generalized, more abstract cognitive schemata is then revealed by a higher performance on transfer tasks, which require the performance of the complex cognitive skill in new situations." (p.12). Therefore, schema construction has been one of the key concepts to reduce cognitive load and to lead to understanding.

Additionally, Pass et al. (2004) mention that "Cognitive load theory (CLT) is concerned with techniques for managing working memory load in order to facilitate the changes in long term memory associated with schema construction and automation" (p.2). Moreno (2006) agrees with Paas et al. and she adds that CLT enables predictions and explanations as to how learning can be effectively supported by teaching and instruction, by thinking the demands on the limited cognitive resources for schema acquisition and proceduralisation. So schema construction and its fostering activities can facilitate understanding with near and far transfer tasks.

Cooper &Sweller (1987) argue that worked examples facilitate rapid schema construction in problem solving compared to conventional problem solving. The study of Paas (1992) in statistical problem solving showed that same amount of mental effort was exposed while training with worked examples and completion examples. Moreover, he add that training learners with either one of them led to higher test performance with a lower cognitive load, compared to training with conventional problems. This result showed that superior cognitive schema to be formed by worked and completion example conditions.

2.5.1 Worked Examples

There are several strategies offered to reduce extraneous cognitive load while problem solving and learners studying on worked examples are one of the suggested techniques. Renkl et al. (2010) define that worked out examples include problem formulation, solution steps and final solution itself. Worked examples have been recognized with its advantages to mathematics learning (Sweller 1985; Cooper & Sweller, 1987) and there has been several studies in different areas of mathematics that demonstrated positive impacts towards learning: geometry (Zhu & Simon, 1987; Paas & Van Merrienboer, 1994); Algebra (Cooper & Sweller, 1987; Nathan, Mertz, & Ryan, 1994; Grosse & Renkl, 2006); Statistics (Pass, 1992)

Sweller and Cooper (1985) found within several experiments that working with multiple worked out examples resulted in more efficiency in near transfer and lead to better problem solving performance than studying with a single example followed by problem solving. Renkl (2005) also agrees with that working with worked examples results in better test performance of problem solving than solving the equal problems. Sweller (1988,1989) claims that "Early work on CLT has made clear that the reduction of extraneous cognitive load, for instance by studying worked-out examples or solving goal free problems, offers a more effective way of learning complex cognitive tasks than conventional problem solving" (cited in Pass et al., 2004 ,p.3). Gerven, Paas, Van Merrienboer & Schmidt (2002) are also in agreement with the efficiency of worked examples like the other experts and they conclude that “Worked examples are more likely to lead to an effective construction of cognitive schemata, because they focus the learner’s attention on problem states and operators, rather than on goals and subgoals”(p.90). Hence, worked examples are high likely recognized for permanent and effective learning with its promoting features.

Next, Renkl et al. (2004) state that “Research within the framework of cognitive load theory (CLT) has shown that worked examples should be provided initially followed by to-be-solved problems in order to foster cognitive skill acquisition in well-structured domains such as mathematics (Atkinson, Derry, Renkl and Wortham 2000; Kalyuga, Ayres, Chandler and Sweller, 2003)” (p.1). It is suggested worked examples to be used at well-structured domains such as in mathematics (Renkl et al., 2004). The authors add that “For fostering cognitive skill acquisition in well-structured domains, it is very effective to use a series of worked examples before problems to be solved or example-problem pairs as in the classical studies of Sweller and colleagues (e.g., Sweller and Cooper 1985; for an overview see Atkinson et al., 2000)” (p.61). Moreno (2006) mentions that “the worked-example effect can be explained as being the result of a practice method that makes a more efficient use of students’ limited cognitive resources than the one resulting from problem-solving practice... a direct implication of CLT assumptions for the design of worked examples is that example-based instruction should minimize students’ use of cognitive resources in activities that are not relevant to schema acquisition and automation and maximize students’ use of cognitive resources in germane activities within the limits of working memory capacity”(pp.171-172). Lastly, Miller (2010) made an experiment in a Calculus class at college level. He concludes that worked examples assist skill acquisition and provides forming schema. This helps students in the long run when faced with more general problem-solving situations.

On the other hand, the affects of worked examples on cognitive load in more complex domains might be different when students need to attain several sources of information involving diagrams, formulas and several solution steps. When the presentation of information in complex domains are not integrated, then the attention of students split in different information sources to integrate all mentally, then cognitive loads of students increase and it limits rule automization as well as schema acquisition (Tarmizi & Sweller, 1988; Ward & Sweller, 1990; Renkl, 2002).

Moreover, Renkl et al. (1998) found that students with less prior knowledge benefit more from worked examples on near and far transfer tasks compared to those who have already formed schema for solving the problem. This effect is due to the worked examples preventing mean-ends analysis and not reinforcing understanding the underlying principles. Students with background knowledge who have already formed schema on problem solving, do not benefit from worked examples, as they find worked examples to be redundant that leads to heavy cognitive load on working memory.

2.5.2 Completion Examples

Completion examples involve faded solution steps where learners are required to complete partially solved problems (Paas & Van Merrienboer,1994). Completion examples are found to assist towards near transfer by building and automating schemas when they are used along with worked examples (Clark et al. 2006). These examples also found to assist for reducing extraneous cognitive load. Pass (1992) argued that completion examples effectively support the acquisition of cognitive skills. Renkl et al. (2004) suggest a smooth transition from worked example to problem solving through omitting solution steps step by step after completing a worked example in the first step. The solution steps are gradually decreased and learners are guided towards full problem solving by working on completion examples. The authors claim that "by gradually increasing problem solving demands, the learners should retain sufficient cognitive capacity to successfully cope with these demands and, thereby, to focus on gaining understanding"(p.62). When completion examples are used along with self explanation, there has been efficiency in learning. Renkl & Atkinson (2010) found the substantial effects of fading and self explanation on near transfer and far transfer of learning while working with probability problems.

Additionally, Pass et al. (2004) added that the study of Renkl, Atkinson and Grosse showed that individuals learned most about those principles that were faded in worked out examples and fading solution steps led to fewer unproductive learning events. Renkl, Atkinson and Grosse (2004) conclude that "the faded steps were most effective towards learning regardless of where the fading was in a worked example. The step which is found to be related to the faded step was learnt the most. The learners' particular misunderstandings were identified through fading" (cited in Clark et al., 2006, p.229). Renkl et al. (2004) determine that "the backward procedure does not appear to offer any general advantage over the forward fading procedure"(p.80). Their study did not lead to any priori recommendation about how to sequence the fading procedure.

On the other hand, Renkl et al. (2003) acknowledge that: "From a cognitive load perspective, the backward-fading condition may be more favorable because the first problem- solving demand is imposed later as compared with forward fading. In the latter condition, the first to-be-determined step might come before the learner has gained an understanding of the step's solution, so that solving the step may impose a heavy cognitive load.It is more favorable to fade out worked-out solution steps in a backward manner as compared with a forward manner" (p.20). Moreover, Atkinson et al. (2003) stated that "Backwards fading procedure fosters the acquisition of rules that can be (more or less) directly applied (i.e., near transfer) as well as those that can be flexibly applied (i.e., far transfer). Backwards fading procedure can be combined with self-explanation prompting toproduce an effect that is both statistically and practically significant" (p.781). Therefore there is a need for a further study on how fading should be implemented and its effects.

The study results of Van Merrienboer et al. (2002) demonstrated that completion problems led to lower cognitive load than training with conventional problems; however training with completion problems resulted in at least equal transfer test performance and tendencies were more towards completion problems. Moreover, Van Merrienboer et al. (2002) determine that when completion problems were used in high contextual inference, training efficiency was positively affected; but the effect on transfer on test performance was not satisfactory. Therefore, according to them, not all methods to minimize extraneous load and increasing germane load would be equally effective. Renkl & Atkinson (2010) cited that the study of Salden,Aleven, Renkl and Schwonke (2008) demonstrated that adaptive fading of worked out examples lead to higher test performance on delayed post tests in Cognitive Tutors technology.Consequently, there is a need for further study on integration of completion problems in instruction that would reduce extraneous cognitive load as well as increasing near and far transfer in test performance

2.5.3 Diverse Worked Examples

Variability of examples has been found to be advantageous at far transfer within previous studies. Diverse worked examples consist of variety of worked out examples in diverse problem contexts. Diverse worked examples are found to be effective for far transfer by assisting towards building flexible schemas when they are processed through self explanations. Paas and Van Merrienboer (1994) claim that examples with variable surface features enhance learning compared with examples with similar features. Additionally, Study on geometrical problem solving done by Paas and Van Merrienboer (1994) showed that "students who studied worked examples gained most from high-variability examples, invested less time and mental effort in practice, and attained better and less effort-demanding transfer performance than students who first attempted to solve conventional problems and then studied work examples" (p.132).Renkl et al. (1998) concluded that "The only evidence that multiple examples are helpful is that learners with elicited self-explanation and multiple examples showed the highest performance on far-transfer problems "(p.105). On the other hand, guidance has been found essential to promote effective learning outcomes. The authors added that multiple examples without guidance for sophisticated self-explanations resulted in very low learning outcomes.

2.5.4 Self-Explanation

Clark, Nguyen & Sweller (2006) defined self explanation as "…a mental dialogue that learners have when studying with a worked example that helps them to understand the example and build a schema from it" (p.226). According to Sweller(2010), self-explanation prompts facilitates learners' thinking through guidance and increases awareness on what learners are doing. When learners need to formulate explanations, process of information relevant for learning is realized by learners. Thus self explanations enhance germane cognitive load. Also, Sweller (2011) mentioned that within Cognitive Theory Principle contexts, "self explanations require learners to establish interactions that relate various elements of worked examples both each other and previous knowledge" (p.188). Chi,Bassok, Lewis, Reimann & Glaser (1989) argued that self explanations were effective because they enable learners to generate inference rules from the principles provided in worked examples and when these rules were linked to appropriate actions at specific conditions, they later became procedural skills. It is cited that "Aleven and Koedinger (2002) obtained positive results with prompting for self-explanations during problem solving rather than during example study in an intelligent instructional environment. Specifically, they documented that problem-solving practice within such a learning environment can be enhanced with prompting the learners to self-explain by identifying the underlying problem-solving principles" (Atkinson, Renkl & Merill, 2003, p.775).

Anderson & Krathwohl (2001) and Marzano (2001) think that students engagement in metacognition realizes as soon as students' awareness of their thought process is recognized and when they start communicating their own thought process. Dosemagen (2007) emphasize that when students ask authentic questions, they demonstrate their ability to assess their own understanding and realize existing gaps and also within the question they refer to the area where they have confusion about. He adds that students can have a sense of what the answers might be due to their background knowledge and if they answer the question, they can be asked to indicate how they have arrived to the answer. He concludes that students can also be asked to define a specific problem solving strategy. All such activities lead to illumination of metacognition. In order to facilitate understanding process, students can be asked to create their own examples related to the topic learnt and they can explain solution steps with their own way.

Desoete et al. (2006) used in their studies that drawing conclusions, paraphrasing the answer, selecting relevant information, estimating the result, designing an action plan, adhering the plan, calculation correctness, noting problem-solving steps, monitoring a solution process, reflecting answers, and relating to earlier problems solved can be used techniques to evaluate metacognitive skillfulness. For the evaluation of understanding process, students can be asked to analyze the situation through communicating in a variety of ways, so then the teacher can observe their way of thinking clearly. Gardner state that "when ideas make sense to learners or when they get it, understanding is described from this representational view" (cited in Leithwood et al., 2006,p.30). Thus, students' learning can be derived from evaluation of organization of concepts that are embedded in invisible structures. Concept maps, setting a strategy for problem solving, grouping items…etc are some of the strategies that can bring into light those invisible structures. These strategies can be referred to students within student reflective journals. Reflective journal writing can be explained as the strategy preferred for self explanation in this study. Additionally, student reflective journals can be one of the ways to promote self-explanations for students and to assess students' learning outcomes through their commentaries. Reflective journals can be used by researchers to analyze and interpret the students' conclusions, evaluation and summaries about concepts and procedures.

Writing reflective journals can provide advantages when teacher aims to connect ideas. There are several studies about the effects and contribution of writing reflective journals

towards learning in mathematics (Rose,1989; Bagley& Gallenberger, 1992; Countryman, 1992; Jurdak & Zein, 1998). Moss, Sovchik, Dipillo (1997) categorized possible benefits of writing reflective journals into five groups: (a) reflective journals providing opportunities for expressing student feelings, (b) effective retention of information, (c) deeper understanding of mathematics, (d) stimulation of thinking about mathematics, and (e) improvement in writing about what they had learned. Ideas of Countryman (1992) are in the same path. He listed some of the advantages of reflective journal writing as follows:
"

- Writing reflective journals helps students become aware of what they know and what they do not know, can and cannot do…
- When students write, they connect the prior knowledge with what they are studying…
- They summarize their knowledge and give teachers insight into their understanding…
- They raise questions about new ideas…
- They reflect what they know…
- They construct mathematics for themselves…" (p.7)

Rose (1989) states that students are guided to write about specific concepts about what they have recently learned at most of the writing assignments in class; hence students had opportunities to reflect and communicate on learnt topic. As a result teaching is not one way delivery process. Educational experts agreed on when students summarize, organize, relate and associate prior knowledge it leads to better retention of mathematics (Bagley and Gallenberger, 1992; Moss et al. 1997).Quinn & Wilson (1997) mention that "Secondary students gained a better conceptual understanding of mathematics by writing hypotheses, sharing ideas, and summarizing their discoveries in writing (Wood 1992)"(p.14). They mentioned that Elliot (1996) stated "writing their own problems and discussing about the structures of problems and solving them, high school students gained insight in understanding algebra" (p.15).

Moreover, Pugalee (1997) claims that students reach an understanding of mathematical concepts through their own experiences while writing reflective journals. Countryman (1992) emphasizes that at grade 9-12 students should focus on reflection about clarified thinking on mathematical ideas and relationships and they should be able to make generalizations about investigations and they should be able to express ideas orally and verbally. He adds that as the emphasis at high school is more on understanding and communicating, writing in mathematics classes is one of the preferred ways to improve skills. Chapman (1996) states that reflective journals can also assist teachers to identify students' misconceptions. Hence, reflective journals are highly recognized to have advantages towards understanding in mathematics classrooms.

Reflective journals can have different advantages to guide student thinking about what mathematics is and how the instruction delivery should be. Countryman (1992) mentions that writing may prevent students to assume that mathematics is a collection of right answers posed to a question. Also, reflective journals can also be used to evaluate and improve instructional strategies when they ask students opinions about these techniques.

It is suggested that reflective journal questions should launch writing in mathematics classes. Some suggested reflective journal questions that can be directed to students are given as: describing what is covered in previous class; explaining what went wrong in problem 3 in the test (if student had errors in that problem); discussing the most difficult homework problem (Countryman,1992). The author adds that students can have free writing tasks; they have keep learning logs or they can write a formal paper in mathematics as reflective journals. Reflective journal activities should guide students to critically analyze and to communicate mathematics through reasoning as a result of their own experiences.

Finally, educational experts agree on that feedback is essential part of learning and for improvement of writing skills. While giving feedback to reflective journals teachers should facilitate construction of knowledge for effective communication in mathematics; rather than being judgmental or evaluative. (Gordon et al., 1993; Mayer et al., 1996). It can be concluded that if reflective journal writing can be implemented in classroom activities in the light of previous theories, it is expected to guide students towards deeper understanding of problem solving and the structures of the taught topic.

2.6 Summary

This chapter summarized the Cognitive Load Theory principles that aim to reduce extraneous cognitive load and increase germane cognitive load that will facilitate learning performance. Extraneous cognitive load can be reduced by worked and completion examples (Cooper and Sweller, 1987) ; whereas germane cognitive load can be enhanced through schema construction which can be achieved by self explanations and diverse worked examples (Renkl & Atkinson, 2001).

Understanding the core principles in mathematics, specifically Differential Calculus is complex for students. Several methodologies should be combined for permanent learning outcomes and their implications should be analyzed in depth for further discussion. The previous researches about worked examples are limited to college level and at specific topics. Moreover there are limited number of studies employed qualitative methodologies that investigate "how, why" issues. Not much study has been made at high school level about understanding main principles of derivatives and its applications with worked examples, completion examples, reflective journals and online group discussion.

Renkl et al. (2002) also mentioned that Paas (1992) did not find any difference in the performance of students who worked with complete and incomplete worked examples; however study of Stark (1999) showed that making examples incomplete can assist learning. Renkl et al. (2004) found that faded steps did not affect learning outcomes; individuals learned most about those principles that were faded. The authors also found that the backward procedure did not offer any general advantage over the forward fading procedure. Thus, there is still need of assessment and evaluation of worked and completion examples at special circumstances.

Moreover, while there are some experts claiming the positive effects of reflective journal writing towards understanding (Countryman, 1992; Jurdak & Zein, 1998); on the contrary, there are also experts emphasizing that reflective journal writing has no effect on mathematics learning (Porter & Masingila, 2000; Croxton & Berger, 2003). So as to uncover the effects of students' reflective journals, deep analysis of students' reasoning and their links to transfer of learning is needed. Thus, students' reflective journals and forums were used to uncover the main difficulties that students faced while learning derivatives and its application and to analyze the level of learning transfer.

This research aimed to combine Cognitive Load Theory principles on reducing extraneous cognitive load and increasing germane cognitive load to the suggestions on how to integrate group discussions . Moreover, self-explanation in this study can also be linked to Vygotsky's inner speech and scaffolding principles; where group discussion elements can be related to social constructivism principles. The research concentrated on deep understanding and near transfer and far transfer, hence the definitions of Perkins and Unger for performance conception of understanding, the principles of CLT will form the basis of the preparation of the learning tasks. These principles were embedded within the learning tasks to analyze students' level of efficiency in understanding.

CHAPTER 3

METHODOLOGY

This chapter denotes the research methodology used in this study. In the following sections, detailed information is provided about the design of study, context and participants, data collection instruments, procedures of the study, data analysis, validity and reliability of the study, limitations and delimitations of the study.

3.1 Design of the Study

The main aim of this study was to find the impacts of running a flexible learning environment in mathematics class through set of learning activities; involving worked examples, diverse worked examples, completion examples requiring forward/backwards fadings, self-explanation, online group discussion and diverse practice problems on near and far transfer in problem solving. There were two purposes of the study. Firstly, this study aimed to investigate the impact of the learning activities on near and far transfer in problem solving. Secondly, opinions of the students towards the effects of learning activities and their contributions towards deep understanding had been examined.

Under these conditions, this study aimed to investigate the answers for the following research questions:

RQ1. Is there a significant mean difference between pre and post achievement test scores of students who were exposed to due to learning activities in mathematics lesson designed based on cognitive load theory principles ?

> RQ 1.1 Is there a significant mean difference between pre and post near transfer scores of students?
>
> RQ.1.2 Is there a significant mean difference between pre and post far transfer scores of students?

RQ2. What are the indications of near and far transfer of learning in students' self-explanations and group discussions?

RQ3. What are students' opinions about the contributions of worked examples, diverse worked examples, completion examples, self-explanation, group discussions and practice problems towards learning mathematics and deep understanding?

In order to understand the transitions and students' progress towards deep understanding of mathematics topics throughout selected activities, quantitative and qualitative research methodologies were used.. The findings from qualitative data and quantitative data was used for triangulation Creswell (2007) defines qualitative research as "an inquiry process of understanding based on distinct methodological traditions of inquiry that explore a social or human problem. The researcher builds a complex, holistic picture, analyzes words, reports detailed views of informants, and constructs the study in a natural setting" (p. 15). Qualitative study design provides useful methods for educational research as it is flexible and adoptable to a wide range of contexts. Creswell (2007) mentions narrative research, phenomenological research, grounded theory research, ethnographic research, and case study research for the methodologies of qualitative research. Merriam (1998) states basic or generic qualitative study, ethnography, phenomenology, grounded theory, and case study as different approaches to qualitative research. Creswell (2007) and Merriam (1998) agree on that these methodologies were linked to each other and they worked in relation to each other. This study aimed to investigate for theoretically suggested effective ways to provide opportunities towards deep understanding of the mathematics. Hence, as a qualitative research approach, instrumental case study is used in this study.

Creswell (2007) states that "Case study research is a qualitative approach in which the investigator explores a bounded system or multiple bounded systems over a time, through detailed, in depth data collection involving multiple resources of information and reports a case description and case-based themes"(p.73). Green et al. (2006) suggests case study method when research reports descriptive or explanatory questions and intends to produce firsthand understanding of people and events. Creswell (2003) states that Stake (1995) mentions that case study is used to address exploratory, descriptive and explanatory research questions.

In this study, researcher aimed to understand and interpret the indicators of near transfer and far transfer of learning through student reflective journals and online group discussions. Yin (2003) mentions that types of data analysis of this data can be holistic analysis of the entire case or an embedded analysis of a specific aspect of a case. Hence holistic analysis of the embedded case is aimed to be realized. Stake (1995) categorizes case study in three kinds: intrinsic case studies, instrumental case studies and collective case studies. Stake (1995) suggested *instrumental case studies* when the researcher aimed to describe a specific case of a more general phenomenon. In instrumental case study, a set of predetermined criteria or a theory is explored and tested. Johnson et al. (2008) states that *instrumental case study* is more often used in exploratory research. Yin (2003) states that an instrumental case study is designed to "*…confirm, challenge, or extend the theory*" (p40).

This case study type aims to advance insights into a particular phenomenon, and there is an expectation of generalizing or developing a theory. In this study, the aim of researcher was to understand the holistic entity of problem solving on derivatives as well as understanding inner workings through deep understanding in IB Mathematics Standard Level course. Thus, this study aimed to form of generalization from the particular case to the wider context; thus the researcher aimed to use instrumental case study method in this study.

Creswell (2003) mentions that case study researchers have a tendency to be pragmatic and they support the use of multiple methods and multiple data resources; an eclectic approach is preferred to interpret the case through multiple resources. Participant observations, interviews, open ended questionnaires, reflective journals, documents are some of the instruments for data collection to reflect and analyze the case holistically. Data collection instruments for case studies may come from six sources: documents, archival records, interviews, direct observation, participant-observation, and physical artifacts (Yin, 2003). During this study, qualitative data were collected through students' reflective journals, online and semi-structured student interviews. Reflective journals and online group discussions assisted to uncover main concepts and substructures in student thinking during problem solving. They led to better grasping the roots of the subject and hence lead to deep understanding. Semi-structured interviews enabled to collect in-depth data from participants. The interview provided opportunity to reveal different perspectives of the participants about the effects of learning activities towards learning and understanding process. Students' scores in pretests and posttests were used to collect quantitative data and these data were used to support the findings of the qualitative data.

3.2 Participants of the Study

The participants of this study were grade 11 students who were taking International Baccalaureate (IB) Mathematics Standard Level in one of the International High Schools in Ankara, Turkey. There were 30 students at Grade 11 who were taking IB Mathematics Standard Level. There were 11 males and 19 females at the age of sixteen. They all took IGCSE Mathematics Extended 580 course from different mathematics teachers at grade 9 and 10. They were taught this course by the researcher at Grade 11. Students were attending to 6 hours of mathematics class every week. Each lesson was 45 minutes of length.
Hence the participants of this study were selected through convenience sampling method. Farrokhi & Mahmoudi- Hamidabad (2012) reference "Convenience sampling is a kind of non-probability or nonrandom sampling in which members of the target population, as Dörnyei (2007) mentions, are selected for the purpose of the study if they meet certain practical criteria, such as geographical proximity, availability at a certain time, easy accessibility, or the willingness to volunteer"(p785). Johnson et al. (2008) mentions that when convenience sampling method is selected it is important to mention the characteristics of participants in the research.

Students had an available computer lab at school where they could access at any time they want. They also have a computer access at their home. These students had a main mathematics textbook, a supplementary book where they could practice other than the distributed worksheets. The school also had a library where students could get more resource books if they needed to and when they could study or research on computers in their free periods.

The students were required to use TI 84 tools in IB Mathematics program. They had the requirement to learn instructions on how to use TI 84 and to use them during the mathematics assessments as required. They were shown how to use TI 84 along with the units. Thus, they used TI 84-Graphing Calculators during the pilot and this study.
With computer competency survey, more detailed information about technological skills of students on Word, Excel and Moodle was gathered and provided in Table 1. The computer competency survey showed that eighteen students defined themselves as they were competent to use Moodle, whereas only three of them did not define themselves at using Moodle. None of the students were very much interested to learn mathematics with Moodle. All students knew how to use Microsoft Word and majority were capable to use its basic tools such as choosing font size, arranging paragraphs, inserting images; however other than four students none of the others knew typing equations. Also only eight of them knew how to draw graphs. Students Microsoft Excel knowledge was more limited than Microsoft Word knowledge.

At Moodle, students were formed in groups of three to answer forum questions during the study. Students had an acquaintance to use Moodle since they had started high school. It had been used more frequently at science courses at grade 9 and 10. They had been using Moodle for following the pace of the course, downloading worksheets, accessing to resources that teacher made link to, reading announcements, contributing to forums for every course that they are taking. The frequency of integrating Moodle activities and contents changed from course to course. The computer competency survey showed that more than twenty-three students knew how to follow deadlines at Moodle as well as downloading documents. Only sixteen of them participated at group discussions at Moodle. Nineteen of them knew how to attach documents at Moodle.

Table 1 Demographic Information for Participants

Demographic	Frequency
Female	19
Male	11
Age 16	25
Age 17	5
Competency at using Moodle	18
Competency at participating forum discussions at Moodle	26
Competency at drawing graphs at Microsoft Word	8
Competency at typing equations at Microsoft Word	4
Competency at drawing graphs at Microsoft Excel	12

This survey was given to examine overall the technological skills of students so as to adjust forum questions and their requirements. Students who need assistance for minimum expectations of using Moodle such as following deadlines, participating into group discussion, attaching documents, sending emails, downloading documents were trained in class for 30-45 minutes as a group where explanations were provided as a seminar. Students were trained in an orientation program on those skills before the pilot study in October 2010. During the pilot and the study students were helped individually if they need to assistance to use these or more tools while answering forum questions.

Interview Participants

Among this group of students, nine students were selected for the interview through purposeful sampling technique. The criteria used for selection are students' Mathematics GPA and motivation levels. These students were selected by considering their differences in mathematics GPA and motivation towards learning mathematics. Selected students were also volunteered to participate to the interviews. Table 2 shows the motivation levels and the average of last 3 years' GPA in mathematics of selected students. The table of Motivation level vs. Mathematics GPA of all students are at Appendix A.

Table 2 Mathematics GPA and Motivation Levels of Selected Students (F: Female and M:Male)

			Motivation Survey Items		
Gender	Students	Math GPA (%)	Task Value/Intrinsic (%)	Self-Efficacy (%)	Test Anxiety (%)
F	$\mathbf{S_{2}}$	94	82	100	83
F	$\mathbf{S_{7}}$	89	34	41	29
F	$\mathbf{S_{14}}$	80	80	84	54
F	$\mathbf{S_{16}}$	78	32	37	20
F	$\mathbf{S_{17}}$	76	75	96	66
F	$\mathbf{S_{24}}$	67	61	73	60
M	$\mathbf{S_{27}}$	61	88	76	40
M	$\mathbf{S_{28}}$	51	43	43	49
M	$\mathbf{S_{29}}$	45	41	59	63

During this study, percentage norms of motivation levels were analyzed in depth. When Mathematics GPA scores of Grade 11 students were gathered due to last three years so as to categorize students in terms of mathematics achievements, the achievement levels of students were categorized as high achievement; average achievement and low achievement. Among each category of Math GPA, they were also categorized as High, Moderate and Low motivated due to the norms stated above for their motivation levels (Büyüköztürk, Akgün, Karadeniz, Çakmak and Demirel, 2007). If a student had two or more of the three motivation categories to be above 70% that student was recognized as High motivated, if s/he had two categories or more to be below 30%, s/he was recognized as Low motivated. Hence a total of nine students where three students in High Math GPA with different motivation levels; three students in Average Math GPA with different motivation levels and three students in Low Math GPA with different motivation levels were selected to interview at the end of the study. As a result 9 students were selected to interview at the end of the learning activities.

3.3 Data Collection Instruments

Computer Competency Survey:

Survey was realized at the beginning of the study. It aimed to check for students' basic skills and background knowledge about using Moodle, Word and Excel together with familiarity with their tools. There were 8 main questions (Appendix B). It has been reviewed with a peer to go over the content. The survey has been finalized with an expert through discussion before conducting.

Rating Scale on Motivation -MSLQ ITEMS

This scale aimed to investigate students' motivation towards achievement and to be aware of to what extend students is interested in learning mathematics. It is used to select interview students. Pintrich and Garcia (1996) had developed the original scale in 1993 by considering students' perspectives apart from in class observations with a constructivist perspective. It is used to measure of achievement motivation. The scale found to be having better criterion validity and reliability than most other objective measures. The scale has 31 items with five point Likert type scale. The MSQL rating scale of Garcia, T. & Pintrich, P. R. has been adapted by Büyüköztürk et al. (2007) in Turkish to explore students' motivation towards achievement, and to be aware of to what extend students were interested in learning mathematics effectively in 2007. The rating scale that had been used by Büyüköztürk et al. (2007) for a TÜBİTAK research in Turkey for grade 6 up to Grade 11. The adapted MSQL scale has 20 items with seven point Likert type scale. The adapted survey is considered in its motivation items separately for Task value, Self-Efficacy for Learning; and Performance and Test Anxiety for the aimed study (Appendix C). For realibility, the Cronbach Alpha values for the adapted scale referring to the age group 12-18 were calculated as follows: Task value, 0.72; Self-Efficacy:0.69; Performance and Test Anxiety, 0.66. (Büyüköztürk et al., 2007)

Percentage norm boundaries are defined by Büyüköztürk et al. (2007) to classify students due to their motivation levels such as high motivated, moderately motivated and low motivated. High motivation group in its category was defined as group of students that should be supported to enable keeping up with the qualifications towards effective learning with intrinsic motivation and using necessary strategies for learning. Moderate motivation group is defined as the group that needs support to go over the strategies towards effective learning with intrinsic motivation and close the gaps for using necessary strategies for learning. Low motivation group is defined as the group that requires immediate assistance towards effective learning with intrinsic motivation and to use necessary strategies for learning (Büyüköztürk, et al., 2007).

Students' Reflective Journals:

In order to realize the stages of construction of knowledge and increasing relevant cognitive load together with investigating the students' levels of understanding reflective journals were used within learning activities in Spring 2011. First, students had to study on assigned problems and then they wrote reflective journals to explain what they had learnt immediately and they summarized the concepts they learnt. Students completed 8 reflective journals during the study. Each reflective journal had a single question to be answered in class. The questions were in three types within reflective journals: mathematical problems in 4 reflective journals, interpretation-reflection questions on the introduction of theories in 4 reflective journals and questions that required proof in 4 reflective journals. Majority of reflective journals involved combination of these three question styles. Types of questions differed depending on the unit. The reflective journal questions had links to the derivative questions or concepts regarding to the solutions and the principles in worked examples and completion examples provided in class. In those reflective journals that required problem solving, students were asked to "*solve a problem*" and then were required to "*explain mathematical reasoning of the solutions step clearly through correct terminology and notations*" in their reflective journals. In reflective journals with interpretation-reflection questions, students were expected to give their *own examples with solutions and to explain the selected example through reasoning.* These examples aimed to explain an introduced theories to a classmate. Proof questions in reflective journals either aimed to prove a particular mathematics problem or a general principle at derivatives units. Table 3 gives information on the contents of the reflective journals. Appendix D gives the list of questions at reflective journals.

Students were allowed to check resources while writing reflective journals. Students used 90 minutes lesson to complete the reflective journal on the day it was assigned. Questions of reflective journals were prepared from various textbooks by discussing with another mathematics teacher for confirmation. Reflective journal question formats and duration were piloted with the same students in fall 2010 and necessary changes on durations in the implementation of learning activities and imposing worked examples with completion examples in the second part of activities were made in the study.

Table 3 Content of Reflective Journals per Week

Reflective Journal / Week	Interpretation-Reflection Questions	Mathematics Problems	Proof Questions
Week 1		✓	✓
Week 2			✓
Week 4		✓	✓
Week 5		✓	✓
Week 7	✓		
Week 8	✓		
Week 9	✓	✓	
Week 10	✓		

Online Forums:

Online discussion forums at Moodle were organized so as to better analyze the opinions of the students; to uncover the main principles of the topic; to see what the possible contributions of sharing ideas among students might be; to triangulate with the qualitative data gathered from reflective journals. Via forums, researcher aimed to learn more about the structure of students' thoughts.

Students completed 8 forums during the study from 1st week to 5th week and from 7th week to 11th week. They discussed forum questions in groups of three students. Each forum consisted of a single question that is either about concepts and theories regarding to the mathematics principles on derivatives used in worked examples and completion examples; or mathematical problems based on what is covered through learning activities in class. Forum questions did not necessarily required math equations to type or graphs to plot. Students were expected to answer verbally in order to uncover main concepts or to find proper method. There were *three interpretation and commenting questions on theories introduced in class, two proof questions on theories* and *last three forums consisted of challenging mathematics problems* to investigate further on what is introduced in class. Forum questions list is given at Appendix E. Questions are prepared from various textbooks by discussing with another mathematics teacher for confirmation on appropriateness. Students' forum discussion logs were kept in the server.

Achievement Tests:

The achievement tests developed for this study were used as pre and post achievement tests. During the study, there were two achievement tests, first achievement test involved near-transfer and far-transfer problems on derivatives unit. The second test involved near-transfer and far-transfer problems on applications of derivatives. Students took the first pretest before the study. Next, they took the first posttest and second pretest in the 6th week. Finally, they took the second posttest in the 12th week.

Achievement tests were structured based on International Baccalaureate (IB) format. Both were composed of Section1 and Section 2. Students were not permitted to use calculators for Section 1; whereas they were able to use TI-Calculators for Section 2. Within both achievement tests, the grading is equally balanced between near-transfer and far-transfer problems. In Section 1 of derivatives test consisted of *five questions: three of them were near-transfer problems* (16 points) *and two of them were far-transfer problems* (16 points). In Section 2 of derivatives tests *consisted of five problems: three of them were near-transfer problems* (19 points) *and two of them were far-transfer problems* (19 points) (Appendix F). The scoring of the tests were done through a rubric due to the standards of IB Marchemes of past exam papers on similar question styles.

In Section 1 of second test consisted of *four questions: two of them were near-transfer problems* (12 points) *and two of them were far-transfer problems* (12 points). In Section 2 of applications of derivatives test consisted of *four problems: two of them were near-transfer problems* (21 points) and *two of them were far-transfer problems* (21 points) (Appendix G). Table 4 and Table 5 shows distribution of marks in the first and second achievement tests.

Table 4 Distribution of Marks and Question Types of First Achievement Test

Questions	1	2	3	4	5	6	7	8	9	10
Transfer Types	Near	Near	Near	Far	Far	Near	Near	Near	Far	Far
Points	7	5	4	6	10	6	6	7	9	10

Table 5 Distribution of Marks and Question Types of Second Achievement Test

Questions	1	2	3	4	5	6	7	8
Transfer Types	Near	Near	Far	Far	Near	Near	Far	Far
Points	6	6	6	6	6	15	7	14

While preparing the questions, past IB mathematics standard level questions and their Table of specifications on marking were analyzed. Mathematics Textbooks written along with IB standards were examined. Appropriate questions were determined due their levels. Selected questions and weighting of these questions were arranged through discussion with another mathematics colleague. The two achievement tests had been prepared by the researcher and revised by another mathematics teacher. The two tests were piloted with Grade 11 students of 2009-2010 academic year and then tests were revised on wording and format.

The achievement test in pilot study was on trigonometry unit. The weighting of near-transfer and far-transfer problems was equal. However, it was discovered that using one achievement test to check for understanding of the unit was not very effective especially when there were more than two main chapters. For that reason, it was decided to prepare two achievement tests where each test assesses the understanding of two chapters. Moreover, it was discovered that students might have different background gaps in specific subjects such as Algebra and Geometry. These gaps had negative effects to answer the test questions correctly. If there were more Geometry based questions than questions requiring Algebra in the test, students who were weak in Geometry, were not able to express understanding of concepts, although they knew main concepts. Some of these students showed critical thinking skills in algebraic questions. Considering this fact, problems in achievement tests in this study were equally balanced between mathematics subjects where possible.

Semi-Structured Interview Protocol:

The interview questions were developed through considering literature review and research questions. Questions were prepared considering the results of the previous studies (Sweller & Cooper, 1985; Paas & Van Merrienböer, 1994; Kalyuga et al. 2001; Jaworski , 2002; Atkinson & Grosse, 2004; Renkl & Atkinson, & Grosse, 2004; Clark, Nguyen & Sweller, 2006). The aim of semi-structured interviews was gathering information about students' reflective opinions towards understanding mathematics units throughout worked examples, completion examples, practice problems, forums and reflective journals. Thus, a structured interview was considered to be limited to gather enough information. So as to provide flexibility, semi-structured interview was selected to be applied for further information.

In the interview protocol, there were six main questions (Appendix H). The questions were about the effectiveness of worked examples, completion examples with forward fading, completion examples with backwards fading, reflective journals, forums and practice problems towards understanding of derivatives and its applications. The main questions consisted of five sub questions. The questions were about positive and negative effects of each in class activity towards interpretation of the topic as well as learning the subject. If students mentioned about the contribution of the activity towards understanding, interview questions required to give a specific example and how it has contributed to learn the subject better. Also, if students had commented on negative effects of the learning activities, interview questions asked to student to give a specific example among the learning activities. The questions were about clarification on which method students had used while studying the worksheets and while they were replying to reflective journal and forums. Finally, their suggestions towards improving the material and their implementation in mathematics classes were asked so as to provide deep understanding.

Interview questions were revised several times with the advisor for structure and content. Interview was piloted with the same grade 11 students in Fall 2010. The piloting interview questions were on Trigonometry, and the interview protocol for this study was structured within the context of derivatives. Students' opinions and replies in pilot study assisted to improve the question formats and content in this study. Giving a specific example towards each student comment was found necessary to be included as sub-question in the interview protocol for this study. After revision made on the content of questions, expert opinion was taken for approval.

3.4 Procedures of the Study

Before this study, a pilot study was conducted to check if the procedures followed requires revisions with twenty-nine students at grade 11 in Fall 2010 and it lasted for five weeks other than interviews. In the following part, pilot study will be mentioned where it is necessary.
This study was conducted in Spring 2011 with the same students participated in the pilot study together with one new student. Based on the findings of the pilot study, some changes were made in this study in regard to duration given to students for worked examples, completion examples, reflective journal-forum completion; completion question types; combination of worked examples, completion examples and practice problems in worksheets; the system of selection of participants for the interview.

After the pilot study observations, some changes were considered to be made. It has been observed that running part 1, part 2 and part 3 in a week period was too much dense for the students so it has been decided to spend a week for each part inthis study applying to every subtopic of Mathematics Unit. Thus, after students completed part 1, they had written a reflective journal and completed a forum in the first week. Then, part 2 has started in the next week. They completed working on part 2, reflective journal and forum in the second week. In the third week they worked on part 3 and feedback given on part 3. Hence the duration of time has extended. Another considered change was to combine worked examples and completion examples in part 2. Hence one worked example was given and then a completion example had followed it in part 2. Part 3 involved worked examples, completion examples and practice problems together. The changes are made thinking that students might have forgotten what has been taught the week before. One other reason was to move on the unit. Spending time on the same topic for two or three weeks was longer that it was planned. Next change was to observe effects of completion examples requiring forward and backwards fading. Hence, part 2 of first two units of this study included backwards fading examples; whereas last two units included forward fading examples in part 2. The last change was referring to the content of part 1 and part 2. One more alternative method was shown to solve a problem rather than restricting with a single technique. This was given as a suggestion for some questions in part 1 and part 2.

Since the same students participated in this study, same computer competency survey was not given before the study, thinking that they had more familiarity to use required technological tools in the study. The new student was given the survey and trained for use of Moodle tools as necessary. See Table 6 below for procedures of the study.

Table 6 Procedures of the Study

Before the Study	**During the Study**				**End of the Study**
	Phase 1 (Week 1-5)	**Week 6**	**Phase 2 (Week 7 -11)**	**Week 12**	**Week 13**
MSQL Scale Pre-achievement test on derivatives	• Instruction of the topic by teacher • 10 worked examples study • 14 completion examples with backwards fading study • 20 practice problems • Writing 4 reflective journals • 4 forum discussions	Post-achievement test on derivatives Pre-achievement test on applications derivatives	• Instruction of the topic by teacher • 16 worked examples study • 15 completion examples with forward fading study • 18 Practice problems • Writing 4 journals • 4 forum discussions	Post achievement test on applications derivatives	Student interviews

- At the beginning of the study, adapted “Motivated Strategies for Learning Questionnaire” (MSQL), rating scale was used to check for motivation levels. The adapted MSQL items of Büyüköztürk et al. (2007) were used to determine motivation levels. The adapted MSQL items enabled to group students due to their characteristic profiles such as high motivated, moderately motivated and low motivated so then, at the end of the study, nine students were selected for interviews based on their motivation levels.
- Students were also given two pre-achievement tests in total for checking the prior knowledge on the subject aimed to be taught. First pre achievement test on derivatives was given before the study.
- For forum discussions at Moodle, students were grouped randomly in three students. The groups were composed of different members compared to pilot study groups. There were again ten groups formed.

This study continued for twelve weeks. During the first six-week period, students were taught with worked examples, completion examples with backwards fading, practice problems, writing reflective journals and Moodle forums on the topic. Students attended to mathematics class six times a week where each lesson was a double with 90 minutes.

- In phase 1, during the first week, students were briefly introduced unit 1, the fundamental principle of Calculus and simple rules of derivatives on the board for 90 minutes.
- Next class, students studied on worked examples for 90 minutes. Students were allowed to look at notes or textbook when they needed to recall the topic. They also had the opportunity to ask questions to their classmates when they had difficulty to understand the topic. The researcher, as teacher walked through the desks to monitor students’ study protocol. Some students worked on given problems with their individual solutions and then compared with the given solution. Teacher also answered their questions on interpretation of the solution when necessary.
- For the last double students wrote reflective journals for 90 minutes in class about the first unit topics, simple derivatives.
- They were assigned to participate at forum discussions during the following three days.
- In the second week, teacher introduced further rules on derivatives such as chain rule, product rule and quotient rule in 90 minutes.
- Students studied on completion examples in next class in 90 minutes and they completed a reflective journal during the next class for 90 minutes.
- They were assigned to participate at second forum discussion for the following 3 days. In forum discussions, students within the same group were able to see own group members’ replies. Teacher did not interfere to the comments of students before the deadline of forum. In the third week, students were given overall feedback on forums as necessary in 90 minutes and answered students’ questions in class.

- Teacher continued on explaining the topic further in the next 90 minutes. Students were assigned to study and complete practice problems in the third double class of the week. They were also allowed to keep up studying on practice problems during the weekend.

Same cycle continued for the fourth and fifth week for unit 2 in the same pace.

- In the six week students were assigned to complete practice problems for unit 2 in the first 90 minutes.
- Then, they took the first achievement test on derivatives as post-test for 90 minutes in next class.
- In the last double class of the week (Week 6), they took pre-achievement test on applications of derivatives.

Hence, students completed four reflective journals and participated at four forum discussions at the end of phase 1.

In phase 2, within the second five weeks, students were taught unit 3 and unit 4 on applications of derivatives with worked examples, completion examples with forward fading, practice problems, writing reflective journals and Moodle forums. Procedures and order of learning activities were the same as phase 1. In the second sixth week, they took achievement test on applications of derivatives as second post-test in 90 minutes. Hence, students completed four reflective journals and participated at four forum discussions at the end of phase 2.

During the study, some changes were made to complete reflective journals. First of all time has been given longer. Another change was related to the content of the questions in reflective journals. It was observed that students had spent long time to create their own examples and explain on it. Hence this type of question was asked in the reflective journal when the derivatives unit was easier and applicable. Moreover students were considered to spend more effort to understand derivatives units compared to Trigonometry as the units were completely new to them. It was harder for them to create own examples to address the expectations of the reflective journal in derivatives units. So rather than "creating an example" type of question, challenging mathematics problems that required deep understanding and proof, were preferred to ask in reflective journals.

In this study, reflective journals were checked immediately after completion and oral feedback is given. For the forums, students were given up to 3 days to reply to forum online. They were reminded to reply if they had not completed on time. It was expected that students to investigate each reply and to comment as necessary. Teacher did not interfere to correct the answers immediately but gave feedback at the end of 4 days.

During the study pre-achievement and post-achievement tests were given on Derivatives and Applications of Derivatives Units to check for learning and understanding. There were two achievement tests, first achievement test involved near-transfer and far-transfer problems on derivatives unit. The second test involved near-transfer and far-transfer problems on applications of derivatives. Pre-tests and post-tests were the same. The first achievement test was on the sixth week and the second achievement test was on the twelfth week. Both tests checked for understanding and learning with short and long response questions.

At the end of the study, nine students were interviewed in thirteenth week. Semi structured interviews for the study were conducted in the last week of the study so as to have more flexibility to realize potential outcomes where the researcher have not noticed. The interviews were conducted after all activities are finished.

3.4.1 Learning Activities Based on Cognitive Load Theory

Learning activities aimed to enable students to construct their own knowledge through the principles of scaffolding, inner-speech and sharing information and social interaction. During the study, students were given "Derivatives Unit" under four subtopics: Simple Rules of Derivatives; Derivatives of Exponential and Trigonometric functions; Curve Properties and Applications; Motion on a Straight Line and Optimization. For each subtopic, learning activities were divided into three parts again: Part 1 (worked examples); Part 2 (Diverse worked examples and completion examples) ; Part 3 (Practice Problems).

For each of the four units, first week, part 1 was assigned in class. For simple rules of derivatives (unit 1) worked examples had improved gradually. The very first examples required short calculations and the last examples required longer calculations. Students worked on part 1 for 90 minutes. Then, students had written reflective journals in class for 90 minutes individually, and then they completed group discussions at Moodle in 3 days on the studied topic. Appendix I provides the students' worksheets on worked examples.

Second week, part 2 was assigned in class, students worked individually. However, they had chance to ask questions to classmates when necessary. Part 2 questions started with 1 or 2 worked examples and then it consisted of completion examples involving backwards and forward fading. The completion examples were also improved gradually as more examples given. Appendix J provides the students' worksheets on completion examples for each unit. After students studied on part 2 in 90 minutes, they submitted the work to the researcher to get feedback within the same or next class. Researcher checked the completed part 2 worksheets and gave immediate feedback. Students completed reflective journals in class on the next day in 90 minutes and then they participated at group discussions at Moodle for the following 3 days. Refer to Figure 1 as an example of a forum discussion.

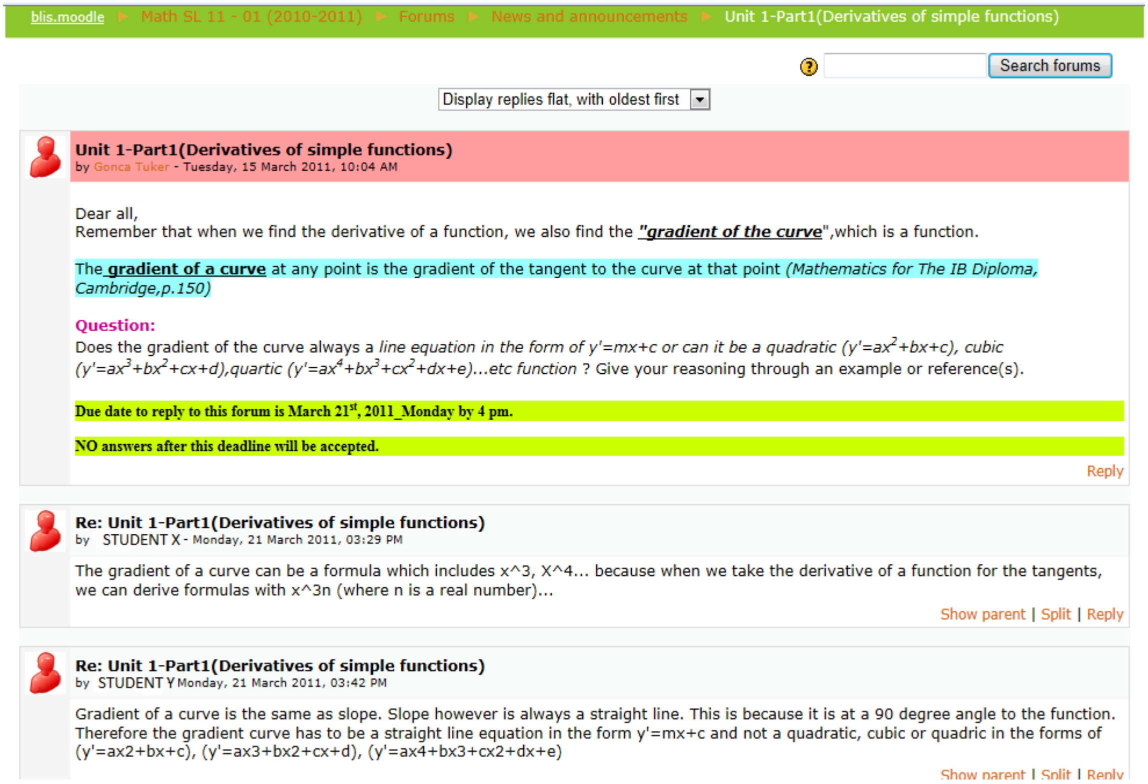

Figure 1. Forum Discussions at Moodle

Next week, Part 3 was assigned to students as individual assignment to be graded. Practice problems in part 3, first involved worked examples and then completion examples. When these two stages were observed, practice problems were given in exam format to be solved as homework. Duration was given for students to check their knowledge. Students stated from simple level and extended to harder level. Feedback on part 3 was given in the following week. The practice problem assignments were graded by the teacher due to marking schemes and feedback was given to the student during the week. This process was repeated in the same order and system for all four units. Part 3 worksheets are given at Appendix K.

At the end of first two units on derivatives, students took the achievement test, as post-test, on the first two topics of derivatives in 6^{th} week. At the end of third and fourth unit, students took second achievement test, as the second post-test, on the last two topics (Week 12).

All learning activity questions had been prepared from several resources, considering diverse examples and were discussed together with another mathematics teacher about applicability. These tasks formed the basis to analyze and observe transfer of learning and the level of understanding.

3.5 Data Analysis

Quantitative data was collected through analyzing descriptive statistics on achievement tests. Students' achievement test results were analyzed due to calculating descriptive statistics on near and far transfer of learning. Moreover, paired sample t-tests were conducted to understand students' level of near and far transfer through analysis of pre and post achievement test results. Paired sample t-tests were conducted to calculate mean difference of pre and post achievement test results of two tests separately and together on near and far transfer of learning. These data was used to combine with the results of qualitative data.

Qualitative data were collected through reflective journals, forums and interviews. While analyzing qualitative data, six steps are suggested by Marshall and Rossman (1999) such as organizing the data; generating categories, and themes; coding the data; testing the emergent understandings as considering students 'individual differences; searching for alternative explanations; and writing the report. Same steps were followed to analyze students' reflective journals, forums and interview results by focusing on research questions.

3.5.1 Analysis of Reflective Journals and Forums

Reflective journals and forums were analyzed based on Bloom's Taxonomies and IB objectives on Derivatives Units. First of all, six main Bloom taxonomies on cognitive domain, -which are Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation- were determined. Bloom (1976) defined six levels of cognitive domain classified hierarchically from simplest thinking acts to higher order thinking acts:

- *Knowledge* level of thinking refers to the recalling of specific items. It denotes to remembering material in the same form as it was taught. The learners can be asked to identify, to recall, and to show information that will enable to check for knowledge.

- *Comprehension* level of thinking demonstrates that the learner understands what has been taught. Learners are expected to restate, to rewrite, to give an example, to illustrate, to define, to summarize that will prove that the knowledge or concepts have been internalized.

- *Application* level of thinking refers to ability to use learned material in the given situation. Learners are expected to apply, to change, to compute, to demonstrate, to manipulate the learned material.

- *Analysis* level of thinking refers to the actions of learner examining the facts; making classifications; making comparisons; making contrasts; categorizing; or making explorations.

- *Synthesis* level of thinking is the ability to put together to form a new whole entity. Learners are expected to demonstrate the actions of making new connections. Learners are expected to demonstrate some process of adapting, designing, combining, inventing, predicting, and creating new entities.

- *Evaluation* level of thinking is the ability to judge the value of the materials accuracy, appropriateness, or applicability for a given situation to some standard. It is the most difficult to obtain in cognitive domain. Learners demonstrate actions of appraisal; making conclusions or generalizations; making criticisms on limitations of the material.

Next, the objectives of the course due to IB standards were analyzed in depth to define specific themes and key terms for each objective. After course objectives were defined more specifically under themes, indicators were defined. Each indicator of themes was classified in Bloom's taxonomies. Taxonomies were categorized as near and far transfer of learning outcomes. According to Hokanson and Hooper (2004), while designing an instruction Bloom's taxonomies can also guide through near and far transfer. The authors stated that *application* level involving simple drill and practice questions that are directly related to the learnt context can be categorized as near transfer; whereas any level above *application* level, which was called -*Extension* in their instructional design-, can be linked to far transfer as students will solve harder problems at different context wehere they will look for required principles. Hence, in this study, *knowledge, comprehension and analysis* levels were considered to indicate near transfer, if the students have learning outcomes straightly connected to content where principles were directly applied; and *analysis* and *synthesis* levels were considered to require far transfer of learning, when a problem is solved in a different situation, when a problem is harder that introduced problems and when students needs to think critically for generalizations, restrictions and judgements of validity. Therefore, five main categories were determined to summarize collected data. The coding schema was discussed with another subject area teacher during this categorization.

In the qualitative data analysis of reflective journals and forums, a combined technique of inductive and deductive thematic analysis was used. While coding reflective journals and forums, that week's student reflective journals and forums were read in depth. The technique of conceptualizing is used and patterns and themes were determined. According to Strauss & Corbin (1998), "Conceptualizing is the process of grouping similar items according to some defined properties and giving the items a name that stands for that common link."(p.121). Then links and through specifications patterns are built to form the structures in coding. "Axial coding is the process of relating categories to their subcategories, termed 'axial' because coding occurs around an axis of a category, linking categories at a level of properties and dimensions" (Strauss et al., 1998, p123). Axial coding technique enabled to relate categories to subcategories to explain the phenomena precisely. Hence it provided more power to explain the phenomenon in depth. Similar themes were combined by widening the definition of the theme in Coding Schema; whereas different themes were added in Coding Schema under appropriate taxonomies. Appendix L consists of the Coding Schema prepared for the analysis of reflective journals and forums.

Yıldırım and Şimşek (2006) emphasize that quantifying qualitative data increases the reliability, decreases subjectivity while providing opportunity to compare codes or themes. Therefore, the data gained from reflective journals and forums were quantified systematically. Frequencies due to that week's reflective journals were counted. Matrix tables were formed so as to examine the increase or decrease in indicators of near and far transfer due to the learning activities chronologically.

3.5.1.1. Description of Coding Scheme of Reflective Journals and Forums

Near Transfer

- Knowledge is one of the themes considered to indicate near transfer of learning of the subject. It involved use of *correct terminology, correct mathematical notation and knowledge of general rules in Calculus.*

By *correct terminology,* researcher looked for the key words that were introduced in the learning activities. Some examples of the required terminologies are derivative function, gradient or slope of tangent line, proper names of mentioned rules i.e. product rule, chain rule, quotient rule, second derivative, sign table, concave upwards, concave downwards, increasing/decreasing intervals...etc. students were also expected to use proper names of recalled functions such as exponential function, trigonometric function.

By *correct notation,* students were expected to use appropriate notations that will assist the flow of the mathematical explanation. Some examples of these notations are f'(x), y', $\frac{dy}{dx}$, $\sqrt{}$, labeling axes in graphs…etc. Students were expected to use proper mathematical notation in reflective journals; however they did not have any obligation to use math equation editor in forums. For example, if they had written "x/2" instead of "$\frac{x}{2}$" or if they used "^" notation to denote powers; there was no necessity to show "x cubed" as x^3 but with x^3, then they were noted down as using correct notation. Hence, there was flexibility in notations while coding online forums because students were not trained on how to use math equation editor but still there was an indication of they knew to express mathematical expressions.

By *knowledge of general rules*, students were expected to know facts or methods about derivatives. For example, students were expected to know that derivative function shows the slope function; or stationary points are found when derivative function equals to zero; or sign table of derivative function leading to show increasing/decreasing intervals…etc.

- Comprehension was analyzed under five headings: *understanding the problem; defining the rule/principle; identifying the correct method; outlining the steps of the problem solution; giving a similar example.*

By *understanding the problem,* students were expected to demonstrate some indicators about what the question is asking for. Paraphrasing the problem or choosing and/or solving through correct method were some of the criteria that were sought under this theme.*Defining the rule/principle* stood for statement of an introduced principle with individuals' own words or declaration of the introduced formula before the use of it. Identifying correct method referred to choosing the valid technique(s). That was usually recognized in the start of the problem solution. *Outlining the steps of solution* referred to solving problem step by step; or explaining the plan on how to solve the problem; or coming up with a summary about the order that should be followed to reach the result. *Giving a similar example* referred to stating a simpler but same type of example in the explanation of the answer to the question.

- *Organizing the given expressions; correct mathematical calculation; indication of breaking problem into parts and giving reasons of solution steps* were under application theme.

Any attempt to simplify the given function for the benefit of easier calculation steps to lessen algebra errors were indicators of *organizing the expression*. Simplifying the function in terms of smaller coefficients, rewriting rational function in product form for assistance to taking derivative with less steps or similar manipulations demonstrated organization. Moreover, *correct mathematical calculation* stood for finding the answer of the stated problem or individual's own example without any mathematical errors. *Indication of breaking problem into parts* referred to any case of splitting the given problem into smaller parts to analyze and then integrating cases to arrive the answer; or splitting given long functions in terms of addition or subtraction of shorter functions; or if a composition of functions is given, being able to derive the composed functions separately to solve the problem. Finally, the indicators of *giving reasoning of solution steps* were the statements on the aim of the solution step or an explanation of what the solution step showed. For example, if the question asked for the tangent line equation and if the student found the slope of tangent at given point and wrote a comment that it showed slope of tangent, and then s/he was accepted as giving reasoning of the calculation step.

Far Transfer

- There were six main themes under analysis: *Comparison of new problem with learnt problem; Differentiating the new problem; Solving a problem in new situations; Applying a new technique to solve a problem; Analyzing the problem further for more information; Correct interpretation of sign tables, graphs and results.*

By *comparison of a new problem,* researcher looked for a student comment on recalling the example that had been shown in learning activities and then looking for similarities on new problem with the old one. *Differentiating the new problem* was the consciousness of the student that the given problem was different than what had been introduced earlier. If the student made a comment on the difference of the problem or if there was an indicator of student attempt to restructure the problem in order to solve it in a different method other than what was introduced earlier, and then it had been coded under this theme. *Solving a problem in new situation* was rewarded for when a problem in a new situation was given by the researcher and the student successfully solved it. If the student had used a new and correct method different than what had been introduced in learning activities, then this working out had been coded under *applying a new technique* theme. When student analyzed the given problem further although it was not a requirement of the problem, then this was coded under *analyzing the problem further for more information* theme. For example, if the student had done additional sign diagrams to analyze concavity or checked for additional graphs for further interpretation of the question, then this code was given. Some questions required interpretation with first and second derivative sign tables for a conclusion or graphing through the analysis of sign tables and calculations. If the student successfully analyzed the question from these requirements, then this working out was coded under *correct interpretation of sign tables, graphs and results* theme.

- There were five main themes under synthesis*: Creating a new problem; generalizing a Calculus rule/fact; deriving abstract relationships; judging the reasonableness or validity of the results; stating limitations and restrictions.* Judging reasonableness or validity of the results and stating limitations and restrictions are actions that would result from *Evaluation* level of thinking due to Bloom's taxonomies; however they were coded under the title of Synthesis as none of the questions were planned to assess these cognitive thinking skills. Thus, they were expected to show up very low likely in the coding process.

Sometimes, student introduced a problem in a new situation and solved it to explain the question. This case had been also coded under c*reating a new problem* theme. At some cases, student realized a rule or a fact resulting from the pattern of solutions or the specific behaviors of functions that had not been visited in learning activities. Therefore, if there was a generalization about a fact, then *generalizing a Calculus rule/fact* theme had been used to code such circumstances. At some point, some students derived an abstract relationship between the functions as a result of applying derivation rules; *deriving abstract relationships* theme was used to code those cases. *Judging the reasonableness or validity of the results* stood for students' awareness of whether the result found was appropriate with the conditions of the question or not. Hence, if the student demonstrated a working out on validity of the calculated answer; or critically commented on the reasonableness of the conclusion, then this code was used. When the student stated restrictions of the results (if any) or critically interpreted the limitations of the final answer, it had been coded under *stating limitations and restrictions* theme.

3.5.2 Analysis of Semi-Structured Interviews

Open coding technique is used to code semi-structured interviews. Strauss and Corbin (1998) explain open coding as "the analytic process through which concepts are identified and their properties and dimensions are discovered in data" (p101). Data are broken down in to discrete parts for deeper inspection by comparison of similarities and differences. In this analysis, first of all conceptualization is made through reading the replies of the students.

After the accumulation of concepts was realized, categorization process was done to generalize concepts under abstract explanatory terms. Specific dimensions and properties were determined to identify terms precisely. Subcategories were formed to specify each category for further explanations. After coding was done, interviews were skimmed through again to update coding system as necessary. Appendix M consists of the coding Schema prepared for the analysis of interviews.

3.5.3 Validity and Reliability

In qualitative research, validity and reliability are necessary items to increase credibility of the results (Patton, 2001; Gay,Mills & Airasian, 2006; Yıldırım et al., 2006). Gay et al. (2006) defines validity such that it is the degree to which the qualitative data collected accurately measure what is aimed to be measured in qualitative research.

According to Johnson et al. (2004), "A qualitative research can be helpful to describe a causal phenomenon"(p.253). They add that internal validity referred to the degree to which a researcher is justified in concluding an observed relationship is causal. Methods of triangulation: the use of multiple methods and data triangulation: the use of multiple data resources can be conducted to promote internal validity. On the other hand, Merriam (1998) suggests six methods for validity process in qualitative research: i) Triangulation, ii) Checks, iii) Long-term observation, iv) Peer examination, v) Collaborative models of research, vi) Researcher‘s biases

Examples and questions in the data collection instruments were developed after doing a literature review and conducting pilot study. To realize internal validity of learning activities, reflective journals and forums, content questions were prepared together with an experienced mathematics teacher who had expertize in teaching the same course for more than 20 years and was working in the same school. With peer review, the clarity of these questions was revised several times. Multiple textbook resources and past exam papers were used to increase variability of questions in learning activities. Notations and graphs within learning activities were revised with peer.

Interview questions were prepared with the advices of the PhD. advisor. Clarity of questions was revised several times and necessary revisions were made after pilot study. With the agreement of the students, all semi-structured interviews were audio recorded. Interviews were transcribed by the researcher carefully. Tapes and transcripts are open to inspection by others. The data analysis process was reviewed with a peer.

Achievement test questions were prepared with an expert in the field of teaching IB mathematics standard level. Same achievement tests were implemented with 2009-2010 academic year's grade 11 standard level students to check for clarity and notations of questions. There were 32 students who took the tests in order for researcher to check for clarity and format. The tests were revised several times to make necessary changes.

For the realization of *construct validity* about transfer of learning, multiple source of evidence data were collected through pretest, posttest, reflective journals and online forum discussions. For example, the difference in mean scores in two pretests and two posttests were used to support construct validity. The themes in reflective journals and online forums discussions were supported with achievement test scores.

Gay et al. (2006) define *reliability* as "the degree to which research study consistently measure whatever they measure" (p. 407). Moreover, Bogdan & Biklen (1998) state that "Qualitative researchers tend to view reliability as a fit between what they record as data and what actually occurs in the setting under study" (p. 36). Interviews were recorded in order to not to avoid any important comment that can play an important role in descriptive reporting. Furthermore, data gathered through reflective journals and forums were explained in a descriptive way with clear examples of students through illuminating the main questions of "how" and "what they refer to" for deeper analysis.

Validity and reliability of coding in reflective journals and forums was reviewed through implementing coding process by another mathematics teacher, who was expert in the subject area and IB curriculum, for objectivity purposes. First of all, Coding Schema and their intended concepts were explained in details. From each eight reflective journals and forums, four or five pieces were selected randomly to be coded again. Krippendorff (2011) state that "Peer codes were compared with Percent or simple agreement, i.e., the proportion of the number of units of analysis on which two coders‟ categorizations, scale values, or measurements match perfectly to the total number of units coded, is easy enough to understand and obtain researcher's codes to check for the matching in terms of percentages"(p.4). He adds that "percent agreement can be calculated only for two coders (save for averaging percentages among different pairs of codes, which has questionable interpretations). It is limited to nominal or categorical data among which coders‟ distinctions can be determined as either same or different." (p.5). Researchers and peer's codes were in agreement with 82%. Although there is no specific distinctions made about percentages made, this percentage showed that codes were matched with a high percent. The confirmation on percentage match criteria was discussed and reviewed with Yıldırım (2011) who is a highly recognized expert in the field of qualitative research at METU.

For the validity of interviews, interviews were read line by line and coding process was done again with the same peer. Themes were defined and compared with the previous categories in coding. Revision of coding process for interviews was made as necessary. Coding was realized again with the peer for validity purposes. Percentage of code matching was analyzed. The overall theme codes matched with 93% and validity was satisfied.
The researcher's limitations and delimitations were clarified and results were compared with literature in chapter 5 to eliminate researcher's bias.

3.6 Limitations of Study

There were several limitations of the study:

- Validity of this study was restricted to the reliability of the data collection instruments used.

- Validity and reliability were limited to the honesty of the students' replies to reflective journals, online forums and semi-structured interviews

3.7 Delimitations of Study

- This study was delimited to Case Study. If the study was designed as an experimental study by having a control group and an experiment group with similar sample characteristics, more generalizations could have been made for the outcomes of learning activities.

- Students were following IB curriculum, hence the level of the questions had to match with the IB standards.

- For the check of learning materials, researcher had to explain the formats of worked examples, completion examples to another mathematics colleague and then content of learning materials were discussed for content validity.

- Students generally had negative attitude towards using Moodle in a course prior to the study and typing mathematics equations on computer. Hence, forum questions were adjusted due to the mathematical, technological and research skills of students.

- The duration of the study was limited to 13 weeks. More detailed observations could have been realized with a longer period of time.

- Sample size was limited to thirty students. If the sample size was larger, the results could have been analyzed and generalized with more confidence.

- This study results are limited to Derivatives and its Applications. Similar learning activities can be conducted with different areas of mathematics.

- This study was limited to Grade 11 students. Similar type of study can be conducted at different grade levels.

CHAPTER 4

RESULTS

This chapter denotes the results of the study that are arranged in the order of research questions. Before revealing the results of the research questions, the findings of pre achievement and post achievement test results, and near and far transfers results are given in the following section.

4.1. Students' Pre and Post Achievement Results

Table 7 below gives the summary of achievement scores of students from pre and post achievement tests together with gained scores that are calculated out of 100.

Table 7 Achievement Scores of Students from Pretests and Posttests

Student	Math GPA	Pre-tests	Post tests	Gained Scores
	100	**100**	**100**	**100**
S_1	95	5	91	87
S_2	94	10	96	89
S_3	93	10	87	79
S_4	92	9	90	83
S_5	91	9	83	77
S_6	89	9	82	75
S_7	89	1	77	76
S_8	87	5	85	82
S_9	86	11	88	79
S_{10}	84	4	71	68
S_{11}	84	3	88	85
S_{12}	83	5	62	58
S_{13}	82	9	82	75
S_{14}	80	7	76	71
S_{15}	79	9	80	74
S_{16}	78	4	53	50
S_{17}	76	10	95	87
S_{18}	75	6	74	70
S_{19}	73	5	63	59
S_{20}	72	4	82	79
S_{21}	71	4	77	75
S_{22}	70	4	81	78
S_{23}	68	2	53	51
S_{24}	67	12	90	81
S_{25}	63	0	46	46
S_{26}	61	4	59	56
S_{27}	61	7	65	60
S_{28}	51	0	55	55
S_{29}	45	3	47	45
S_{30}	45	3	58	56
Mean:		**6**	**75**	**70**
S.D.		**3**	**15**	**13**

4.1.1 Students' Level of Near Transfer

The findings for students levels of near transfer for the first achievement test (derivatives unit) together with students' Math GPA were provided at Table 8. The maximum possible marks that students would have achieved for near transfer questions in the first achievement test was 35. Therefore, pre and post achievement test scores on near transfer were calculated out of 100. As it is presented in table, students' pretest mean score for near transfer is M=3 and posttest mean score for near transfer is M=83 and near transfer gained mean score for the first achievement test is M=80. The findings indicated that students improved their near transfer learning in mathematics derivatives units as a result of mathematics lesson designed based on Cognitive Load Theory principles.

Table 8 Students' Level of Near Transfer from First Achievement Test

Students	Math GPA	Pre-test Near-Transfer Scores	Post-test Near-Transfer Scores	Gained Scores (Near-Transfer)
	100	**100**	**100**	**100**
S_1	95	0	94	94
S_2	94	6	97	91
S_3	93	6	100	94
S_4	92	3	100	97
S_5	91	6	86	80
S_6	89	6	100	94
S_7	89	0	100	100
S_8	87	0	97	97
S_9	86	6	97	91
S_{10}	84	0	97	97
S_{11}	84	0	94	94
S_{12}	83	6	46	40
S_{13}	82	6	80	74
S_{14}	80	6	69	63
S_{15}	79	0	100	100
S_{16}	78	6	86	80
S_{17}	76	14	97	83
S_{18}	75	0	83	83
S_{19}	73	6	97	91
S_{20}	72	0	94	94
S_{21}	71	0	71	71
S_{22}	70	6	97	91
S_{23}	68	6	63	57
S_{24}	67	6	94	89
S_{25}	63	0	37	37
S_{26}	61	6	66	60
S_{27}	61	0	83	83
S_{28}	51	0	74	74
S_{29}	45	0	46	46
S_{30}	45	0	66	66
Mean		3	83	80
S.D.		3	17	17

When the students' pre achievement test results on derivatives test was calculated, it was seen that students' scores varied between 0 to 14. 14 students got a score of zero and there was only one student who scored 14. It was also observed that almost half of the total students, 14 of them, got a score of six; this was because the beginning of the question linked to composition of functions (Appendix G; Question 8a). Since functions unit had been taught earlier as prerequisite knowledge, these students were able to answer composition of function question correct.

The post-achievement test on derivatives results showed that students' scores varied between 37 to 100. The mean score in near transfer problems was 83; and standard deviation was 17. This mean score for near transfer was evaluated as a high average. Even though the student who scored 37 as lowest mark had Math GPA of 61 and s/he was not the student with the lowest Math GPA.

From the post-test, five students scored full points from near transfer questions and the pre-test score of two of them, S_7 and $S_{15,}$ was zero points. Seven students scored 97; three out of whom also got zero or three from their pre-test. The score of 94 was achieved by four students, which was also one of the top scores in near transfer question points. Next, highest score was 86 that had been achieved by two students. Hence, 18 students out of 30 students (60 percent of the students) scored above or equal to 86, in other words they accomplished the attainment of getting 86 percent from near transfer problems. One student was below 45% (failing score) with the score of 37 and two other students scored 46.

Therefore, gained scores of students ranged between 37 and 100. Two students received gained score of 100; whereas one student got 37 as gained score. 15 students, half of them, accomplished to get above or equal to 89 as gained score; that was high likely to be recognized as successful outcome. Five of the students gained between 80 and 83 as gained score. Gained scores changed from 57 to 74 for seven students. Thus, other than three students scoring, 37, 40 and 46, rest of the students were above 57 as gained scores. From these three low scoring students, only the one who got 40 earned a score of two from pretest; rest of two got zero points from the pretest. Therefore, the near transfer gained percentage scores of 24 students were above 67; whereas the gained percentage scores of six students were between 33 and 67.

Second achievement test results were in the same trend as the first achievement test. The findings for students levels of near transfer for the second achievement test (applications of derivatives unit) together with students' Math GPA were provided at Table 9. The maximum possible marks that students would have achieved for near transfer questions in the second achievement test was 35. The scores were converted into out of 100. As it is presented in table, students' pretest mean score for near transfer is M=12 and posttest mean score for near transfer is M=91 and near transfer gained mean score for the second achievement test is M=82. The findings indicated that students improved their near transfer learning in mathematics application of derivatives units as a result of mathematics lesson designed based on Cognitive Load Theory principles.

Table 9 Students' Level of Near Transfer from Second Achievement Test

Students	Math GPA	Pretest Near Transfer Scores	Posttest Near Transfer Scores	Gained Scores (Near Transfer)
	100	**100**	**100**	**100**
S_1	95	18	100	82
S_2	94	21	100	79
S_3	93	15	85	70
S_4	92	18	100	82
S_5	91	12	100	88
S_6	89	6	94	88
S_7	89	0	88	88
S_8	87	12	100	88
S_9	86	15	88	73
S_{10}	84	18	82	64
S_{11}	84	12	100	88
S_{12}	83	6	100	94
S_{13}	82	15	85	70
S_{14}	80	12	94	82
S_{15}	79	18	94	76
S_{16}	78	0	76	76
S_{17}	76	21	100	79
S_{18}	75	12	97	85
S_{19}	73	12	85	73
S_{20}	72	15	94	79
S_{21}	71	6	97	91
S_{22}	70	12	94	82
S_{23}	68	0	79	79
S_{24}	67	18	100	82
S_{25}	63	0	100	100
S_{26}	61	0	94	94
S_{27}	61	12	88	76
S_{28}	51	0	79	79
S_{29}	45	12	79	67
S_{30}	45	6	91	85
Mean		12	91	82
S.D.		6	9	9

When the students' pre achievement test results on applications of derivatives test was observed, students' scores varied between 0 to 21. All the scores in the pretest were gathered from question 6; as this question was consisting of prerequisite knowledge on quadratic functions and taking derivative of a polynomial function where students had been taught earlier (Appendix G; question 6). Also, this question consisted of nearly 50 percent of far transfer score in the test test From the pretest, six students got a score of 0 , four students got a score of six; nine students got a score of 12; four of them got a score of 15; five students got a score of 18; and two students achieved a score of 21 as top pretest score.

The post-achievement test on derivatives results showed that students' scores varied between 76 to 100; that was considered to be a successful outcome from the near transfer problems. The mean score of near transfer problems was 91; while standard deviation was 9. This results clearly showed that the scores in near transfer problems in second test were quite high.

According to post achievement test, ten students were awarded full score and one student scored the lowest score, 76. There was not a pattern or a relation compared to Math GPA of students; thus full points were spread due to students with different Math GPAs. Number of students who got a score of 94 was six and it was higher than number of students who got a score of 97, which was achieved by two students. Both scores were recognized as quite favorable in near transfer problems by the researcher. Three students got the second lowest score, 79, which was still considered to be a good score.

Furthermore, the gained scores of students ranged between 64 and 100, which was a better range compared to the first achievement test range in near transfer problem scores. One student gained the score of 100; while two students got a score of 94 as gained score. 11 students accomplished to get above or equal to 85 as gained scores. Although it was observed that this frequency was less than the number of students who got above 89 in the first achievement test; the reason was reliant on question number six, where most of the students were able to earn more scores from this question in the pre-achievement test. 30 students (all) scored above 64 gained percentage score in the second achievement test for near transfer problems. 64 and 67 were scored by one student for each, where the gained scores percentages were quite adequate, although they were marked as lowest gained score percentages.

With the overall descriptive statistics in near transfer problem scores of both achievement tests, it was observed that the mean of second achievement test gained scores on near transfer problems were almost the same as the mean of first achievement test gained scores. Mean score of gained scores in second test was 82 and mean score of gained scores in first achievement test was 80%. However, the standard deviation of the first test was 17 and it showed more spread of data compared to second achievement test with the standard deviation of 9.

Table 10 below explains the overall scores in near transfer for both achievement tests. The maximum score for near transfer questions was 68. The scores were adapted to out of 100 in Table 10. The mean of overall near transfer scores in pretests is 7, which is a very low score. The mean of overall near transfer scores in post-tests is 88, which is quite high. The mean for gained scores is 81. This high mean indicates that there was a successful near transfer in general. The standard deviation for gained scores is 9 that shows that there was not a high spread of data; hence scores of students were almost balanced. One student had overall posttest score to be between 62; on the other hand 29 students had overall post test score on near transfer to be above 67. 28 students had gained scores to be above 67.

Table 10 Near Transfer Scores

Student	Math GPA	Near Transfer Scores in Pretests			Near Transfer Scores in Posttests			Near Transfer Results		
		Test1	Test2	Total Score	Test1	Test2	Total Score	Pretest Score	Posttest Score	Gained Score
	100	**35**	**33**	**68**	**35**	**33**	**68**	**100**	**100**	**100**
S_1	95	0	6	6	33	33	66	9	97	97
S_2	94	2	7	9	34	33	67	13	99	85
S_3	93	2	5	7	35	28	63	10	93	82
S_4	92	1	6	7	35	33	68	10	100	90
S_5	91	2	4	6	30	33	63	9	93	84
S_6	89	2	2	4	35	31	66	6	97	91
S_7	89	0	0	0	35	29	64	0	94	94
S_8	87	0	4	4	34	33	67	6	99	93
S_9	86	2	5	7	34	29	63	10	93	82
S_{10}	84	0	6	6	34	27	61	9	90	81
S_{11}	84	0	4	4	33	33	66	6	97	91
S_{12}	83	2	2	4	16	33	49	6	72	66
S_{13}	82	2	5	7	28	28	56	10	82	72
S_{14}	80	2	4	6	24	31	55	9	81	72
S_{15}	79	0	6	6	35	31	66	9	97	88
S_{16}	78	2	0	2	30	25	55	3	81	78
S_{17}	76	5	7	12	34	33	67	18	99	81
S_{18}	75	0	4	4	29	32	61	6	90	84
S_{19}	73	2	4	6	34	28	62	9	91	82
S_{20}	72	0	5	5	33	31	64	7	94	87
S_{21}	71	0	2	2	25	32	57	3	84	81
S_{22}	70	2	4	6	34	31	65	9	96	87
S_{23}	68	2	0	2	22	26	48	3	71	68
S_{24}	67	2	6	8	33	33	66	12	97	85
S_{25}	63	0	0	0	13	33	46	0	68	68
S_{26}	61	2	0	2	23	31	54	3	79	76
S_{27}	61	0	4	4	29	29	58	6	85	79
S_{28}	51	0	0	0	26	26	52	0	76	76

Table 10 continued

Student	Math GPA	Near Transfer Scores in Pretests			Near Transfer Scores in Posttests			Near Transfer Results		
		Test1	Test2	Total Score	Test1	Test2	Total Score	Pretest Score	Posttest Score	Gained Score
	100	**35**	**33**	**68**	**35**	**33**	**68**	**100**	**100**	**100**
S_{29}	45	0	4	4	16	26	42	6	62	56
S_{30}	45	0	2	2	23	30	53	3	78	75
							Mean	**7**	**88**	**81**
							S.D.	**4**	**11**	**9**

* Pre and post achievement test scores were adapted into out of 100.

4.1.2 Students' Level of Far Transfer

The findings for students levels of far transfer for the first achievement test (derivatives unit) together with students' Math GPA were provided at Table 11. The maximum possible mark for far transfer questions in the first achievement test was 35. This score was adapted into out of 100. As it is presented in table, students' pretest mean score for far transfer is *M*=9 and posttest mean score for far transfer is *M*=67 and far transfer gained mean score for the first achievement test is *M*=58. The findings indicated that there was an improvement in students' far transfer learning in mathematics derivatives units as a result of mathematics lesson designed based on Cognitive Load Theory principles.

Table 11 Students' Level of Far Transfer from First Achievement Test

	Students	Math GPA	Pre-test Far Transfer Scores	Post-test Far Transfer Scores	Gained Scores
		100	**100**	**100**	**100**
	S_1	95	3	74	71
	S_2	94	11	97	86
	S_3	93	20	94	74
	S_4	92	14	89	74
	S_5	91	17	91	74
	S_6	89	23	83	60
	S_7	89	6	89	83
	S_8	87	9	77	69
	S_9	86	23	97	74
	S_{10}	84	0	80	80
	S_{11}	84	0	100	100
	S_{12}	83	9	46	37
	S_{13}	82	14	89	74
	S_{14}	80	11	74	63
	S_{15}	79	17	86	69
	S_{16}	78	9	23	14
	S_{17}	76	6	94	89
	S_{18}	75	11	63	51
	S_{19}	73	3	29	26
	S_{20}	72	0	77	77
	S_{21}	71	9	66	57
	S_{22}	70	0	89	89
	S_{23}	68	3	46	43
	S_{24}	67	23	69	46
	S_{25}	63	0	14	14
	S_{26}	61	9	29	20
	S_{27}	61	14	49	34
	S_{28}	51	0	9	26
	S_{29}	45	0	9	26
	S_{30}	45	2	16	40
Mean			**9**	**67**	**58**
S.D.			**8**	**27**	**25**

Students' pre-test scores in far transfer were in between 0 to 23. Majority of these points came from question 5(Appendix F); within the first part of the question, students were asked to find intercepts and symmetry axis of quadratic function and then working with rearrangement in completed to square form. These were thought as prerequisite that some students were able to make the link to the previous topics to answer those parts.

Posttest scores differed from 14 to 100. The students who got these bottom and top scores achieved 0 points in the pretest. There was not a specific pattern observed on the post test scores compared to Math GPA of students; scores were spread randomly due to different Math GPA of students. The mean of scores in far transfer problems was calculated as 67. This mean indicated an improvement in far transfer of learning in derivatives unit. The standard deviation of scores in far transfer problems was 27. This standard deviation showed that students' scores in far transfer problems varied high likely.

Next, there were 11 students scoring above or equal to a score of 86 : very good score; whereas there were nine students who were below or equal to 46.

When the gained scores was observed, the range was 86 where the maximum gained score was 100 and the minimum gained score was 14 .The mean of the gained scores percentage in far transfer problems was 58 that was lower than the mean of gained scores percentage in near transfer problems, 80. Hence, students generally had difficulties to answer far transfer problems in the first test. The standard deviation of gained scores percentage was calculated as 25 this result was higher than the standard deviation of near transfer problems in both of the achievement tests. It was concluded that the data was more spread and students' skills at answering far transfer problems in the first test showed more diversity. 15 students had gained scores in far transfer to be above 67; whereas nine students had gained scores on far transfer to be between 33 and 67; and six students achieved below 33 in gained scores on far transfer.

The findings for students levels of far transfer for the second achievement test (applications of derivatives unit) together with students' Math GPA were provided at Table 12. The maximum score for far transfer questions in the second test was 33. The scores were adapted to out of 100 As it is presented in table, students' pretest mean score for far transfer is M=1 and posttest mean score for far transfer is M=55 and far transfer gained mean score for the second achievement test is M=55. The findings indicated that there was an improvement in students' far transfer learning in mathematics application of derivatives units as a result of mathematics lesson designed based on Cognitive Load Theory principles.

Table 12 Students' Level of Far Transfer From Second Achievement Test

Students	Math GPA	Pretest Far Transfer Score	Posttest Far Transfer Score	Gained Scores (Far Transfer)
	100	**100**	**100**	**100**
S_1	95	6	97	91
S_2	94	6	91	85
S_3	93	0	67	67
S_4	92	0	70	70
S_5	91	0	55	55
S_6	89	0	48	48
S_7	89	0	30	30
S_8	87	0	67	67
S_9	86	0	67	67
S_{10}	84	0	24	24
S_{11}	84	0	55	55
S_{12}	83	0	58	58
S_{13}	82	0	73	73
S_{14}	80	0	70	70
S_{15}	79	0	39	39
S_{16}	78	0	27	27
S_{17}	76	3	88	85
S_{18}	75	0	55	55
S_{19}	73	0	42	42
S_{20}	72	0	61	61
S_{21}	71	0	76	76
S_{22}	70	0	42	42
S_{23}	68	0	24	24
S_{24}	67	6	97	91
S_{25}	63	0	36	36
S_{26}	61	0	48	48
S_{27}	61	0	39	39
S_{28}	51	0	42	42
S_{29}	45	0	39	39
S_{30}	45	0	30	30
Mean		**1**	**55**	**55**
S.D.		**2**	**21**	**20**

The pretest scores changed from 0 to 6. Very few students were able to answer first part of question 8 (Appendix G). The content of this question was introduced in the previous semester; as a result some of the students remembered how to find the value of trigonometric function.

Posttest scores had the range score of 73 where maximum score was 97 and the minimumscore was 24. Two students achieved the highest score and two students got the lowest score. The mean score of post test results was calculated as 55. This average was lower than the average of near transfer problems in both achievement tests. Moreover, it was also lower than the average of far transfer problems in the first achievement test. This reflected that students, in general, were less successful in far transfer problems of the second test. The standard deviation of post test scores was 21. This result was lower than the standard deviation of the far transfer problems in the first test; however it was still higher than the standard deviation of near transfer problems in both achievement tests. Again, it demonstrated that the students' skills on solving far transfer problems varied more. Four students were above the score of 89 and 12 students were below the score of 46. Therefore, it also proved that students had more difficulties in far transfer problems of the second test.

The mean score of gained scores was 55 and the standard deviation was 20. These results were close to the result of far transfer problems in post-test, as their pretest score were 0 point for the majority.

When the gained scores observed, it was seen that 55 was the mean and 20 was the standard deviation. These two results were close to the far transfer results of first test. They also indicated that students had more difficulty in far transfer problems of both tests than the near transfer problems of both tests. This outcome was expected as it is known from the literature that far transfer cognitive thinking skills were harder to achieve than near transfer cognitive thinking skills (Bloom, 1976). The gainedscores differed from 24 to 91; that was a high variability. Two students achieved each of these gained scores. However, 11 students got lower than 45 in gained scores; failing score. Additionally, 11 students got gained scores on far transfer to be above 67; whereas 14 students got gained scores on far transfer to be between 33 and 67; and five students got percentage of gained scores on far transfer to be below 33.

Table 13 demonstrates overall results for far transfer in achievement tests. The score for pre and post achievement tests were 68 and this score was adapted into out of 100. The mean score of pretest results is 5; whereas the mean score of post test results is 61. This posttest mean for far transfer is lower than the mean of near transfer scores. The mean score of gained scores is 56. This average is lower than the mean for percentage of gained scores in near transfer. Hence there is less far transfer in general compared to near transfer. The standard deviation of gained scores is 19. That also shows that there is a larger spread of data in far transfer gained scores. 11 students got the percentage of far transfer gained scores to be above 67; whereas the far transfer gained percentage scores of 16 students were between 33 and 67; and three students had the far transfer gained scores to be below 33.

Table 13 Far Transfer Scores

Student	Math GPA	Far Transfer Scores in Pretests			Far Transfer Scores in Posttests			Far Transfer Results		
		Test1	Test2	Total Score	Test1	Test2	Total Score	Pretest Score	Posttest Score	Gained Score
	100	**35**	**33**	**68**	**35**	**33**	**68**	**100**	**100**	**100**
S_1	95	1	2	3	26	32	58	4	85	81
S_2	94	4	2	6	34	30	64	9	94	85
S_3	93	7	0	7	33	22	55	10	81	71
S_4	92	5	0	5	31	23	54	7	79	72
S_5	91	6	0	6	32	18	50	9	74	65
S_6	89	8	0	8	29	16	45	12	66	54
S_7	89	2	0	2	31	10	41	3	60	57
S_8	87	3	0	3	27	22	49	4	72	68
S_9	86	8	0	8	34	22	56	12	82	71
S_{10}	84	0	0	0	28	8	36	0	53	53
S_{11}	84	0	0	0	35	18	53	0	78	78
S_{12}	83	3	0	3	16	19	35	4	51	47
S_{13}	82	5	0	5	31	24	55	7	81	74
S_{14}	80	4	0	4	26	23	49	6	72	66
S_{15}	79	6	0	6	30	13	43	9	63	54
S_{16}	78	3	0	3	8	9	17	4	25	21
S_{17}	76	2	1	3	33	29	62	4	91	87
S_{18}	75	4	0	4	22	18	40	6	59	53
S_{19}	73	1	0	1	10	14	24	1	35	34
S_{20}	72	0	0	0	27	20	47	0	69	69
S_{21}	71	3	0	3	23	25	48	4	71	66
S_{22}	70	0	0	0	31	14	45	0	66	66
S_{23}	68	1	0	1	16	8	24	1	35	34
S_{24}	67	8	2	10	24	32	56	15	82	68
S_{25}	63	0	0	0	5	12	17	0	25	25
S_{26}	61	3	0	3	10	16	26	4	38	34
S_{27}	61	5	0	5	17	13	30	7	44	37
S_{28}	51	0	0	0	9	14	23	0	34	34
S_{29}	45	0	0	0	9	13	22	0	32	32
S_{30}	45	2	0	2	16	10	26	3	38	35
Mean							**5**	**61**	**56**	**Mean**
S.D.							**4**	**21**	**19**	**S.D.**

* Pre and post achievement test scores were adapted into out of 100.

4.2 Difference Between Pretest and Posttest Scores (RQ1)

For further analysis on the significance difference in pretest and post test scores due to learning activities in mathematics lesson designed based on related cognitive load principles, paired sample t test was conducted for both achievement tests. At first, results of descriptive statistics and paired t- test were presented for Achievement Test 2. Then, results of descriptive statistics and paired t- test were presented for Achievement Test 2. Finally, results of descriptive statistics and paired t- test were presented for the total scores of both achievement tests.

4.2.1 Overall Achievement Test Scores

Achievement Test 1

A paired-samples t-test was conducted to compare the mean differences between pretest and posttest of first achievement test overall mean scores. Table 14 shows the descriptive statistics for Achievement Test 1 on total scores; whereas Table 15 demonstrates paired samples t-test findings for Achievement Test 1 on total scores.

Table 14 Descriptive Statistics for Achievement Test 1

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Total_Pretest1	6.07	30	4.63	.85
Total_Posttest1	75.33	30	20.80	3.80

Results showed that there was a significant mean difference between pretest scores (*M*=6.07, *SD*=4.63) and posttest scores (*M*=75.33, *SD*=20.80); $t(29)= -19.55$, $p = 0.000$ (Table 15). These results suggest that learning activities in mathematics lesson designed based on cognitive load principles may have a positive effect on transfer of learning.

Table 15 Paired Sample T-Test For Achievement Test 1 (Overall)

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig.(2-tailed)
Pretest1 Posttest1	-69.27	19.41	3.54	-76.51	-62.02	-19.55	29	.000

Achievement Test 2

A paired-samples t-test was conducted to compare overall mean in pretest and posttest scores of second achievement test. Table 16 shows the descriptive statistics for Achievement Test 2 scores; whereas Table 17 demonstrates paired samples t-test findings for Achievement Test 2 scores.

Table 16 Descriptive Statistics for Achievement Test 2

Paired Samples Statistics				
	Mean	N	Std. Deviation	Std. Error Mean
Pretest2	5.81	30	3.98	.73
Posttest2	73.63	30	13.17	2.40

The findings showed that there was a significant mean difference between pretest scores (M=5.81, SD=3.98) and posttest scores (M=73.63, SD=13.17); $t(29)= -34.11$, $p = 0.000$ (Table 17). These results suggest that learning activities in mathematics lesson designed based on cognitive load principles may have a positive effect on transfer of learning.

Table 17 Paired Sample T-Test for Achievement Test 2

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pretest2 Posttest2	-67.83	10.89	1.99	-71.89	-63.76	-34.11	29	.000

Achievement Tests Overall

A paired-samples t-test was conducted to compare pretest and posttest overall achievement test scores of first and second achievement tests. Table 18 shows the descriptive statistics for achievement scores overall; whereas Table 19 demonstrates paired samples t-test for the same scores.

Table 18 Descriptive Statistics for Overall Test Scores

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretests	5.97	30	3.50	.64
Posttests	74.53	30	14.82	2.71

As indicated at Table 19, there was a significant mean difference between pretest scores (M=5.97, SD=3.50) and in posttest scores (M=74.53, SD=14.82); $t(29)= -30.01$, $p = 0.000$ (Table 19). These results suggest that learning activities in mathematics lesson designed based on cognitive load principles result with transfer of learning.

Table 19 Paired Sample T-Test for Overall Test Scores

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pretests Posttests	-68.57	12.51	2.29	-73.24	-63.89	-30.01	29	.000

4.2.2. Achievement Tests Results on Near Transfer (RQ1.1)

In this section, at first, descriptive statistics and paired t- test results on near transfer scores were presented for Achievement Test 1. Then, descriptive statistics and paired t- test results on near transfer scores were presented for Achievement Test 2. Finally, descriptive statistics and paired t- test results on near transfer scores were presented for the total near transfer scores of both achievement tests.

Achievement Test 1

A paired-samples t-test was conducted to compare near transfer pretest and posttest mean scores of first achievement test. Table 20 shows the descriptive statistics for Achievement Test 1 on near transfer scores; whereas Table 21 demonstrates the findings of paired samples t-test for Achievement Test 1 on near transfer.

Table 20 Descriptive Statistics for Achievement Test 1 (Near Transfer)

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretest1_NearTransfer	3.37	30	3.57	.65
Postest1_NearTransfer	83.70	30	18.16	3.32

According to these results, there was a significant mean difference in near transfer scores between pretest (*M*=3.37, *SD*=3.57) and posttest (*M*=83.70, *SD*=18.16); $t(29) = -24.45$, $p = 0.000$ (Table 21). These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles resulted with near transfer of learning.

Table 21 Paired Sample T-Test for Achievement Test 1 (Near Transfer)

Paired Samples Test

	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Differences Lower	Upper	t	df	Sig. (2-tailed)
Pretest1_NearTransfer Postest1_NearTransfer	-80.33	18.0	3.23	-87.05	-73.61	-24.45	29	.000

Achievement Test 2

A paired-samples t-test was conducted to compare near transfer mean scores in pretest and posttest of second achievement test. Table 22 shows the descriptive statistics for Achievement Test 2 on near transfer scores; whereas Table 23 demonstrates paired samples t-test findings for Achievement Test 2 on near transfer scores.

Table 22 Descriptive Statistics for Achievement Test 2 (Near Transfer)

Paired Samples Statistics

	Mean	N	Std. Deviation	Std. Error Mean
Pretest2_NearTransfer	10.80	30	6.80	1.24
Postest2_NearTransfer	92.10	30	7.82	1.43

As indicated at Table 23, there was a significant mean difference between pretest near transfer scores (*M*=10.80, *SD*=6.80) and posttest near transfer scores (*M*=92.10, *SD*=7.82); $t(29)= -53.25$, $p = 0.000$ (Table 23). These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles resulted with near transfer of learning.

Table 23 Paired Sample T-Test for Achievement Test 2 (Near Transfer)

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pretest2_NearTransfer Postest2_NearTransfer	-81.30	8.36	1.53	-84.42	-78.18	-53.25	29	.000

Total Near Transfer Test Scores

A paired-samples t-test was conducted to compare the total near transfer scores in pretest and posttest of first and second achievement test. Table 24 shows the descriptive statistics for Total Test scores on near transfer; whereas Table 25 demonstrates paired samples t-test for Total Test scores on near transfer.

Table 24 Descriptive Statistics for Total Test Scores (Near Transfer)

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretests_Total NearTransfer	7.00	30	4.07	.74
Postests_Total NearTransfer	87.83	30	10.58	1.93

According to the results, there was a significant mean difference between pretest near transfer scores (*M*=7.00, *SD*=4.07) and posttest near transfer scores (*M*=87.83, *SD*=10.58) ; *t(*29)= -49.93, *p*= 0.000 (Table 25). These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles result with near transfer of learning.

Table 25 Paired Sample T-Test for Total Test Scores (Near Transfer)

Paired Samples Test								
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Pretests_Total NearTransfer Postests_Total NearTransfer	-80.83	8.87	1.62	-84.14	-77.52	-49.93	29	.000

4.2.3 Achievement Test Results on Far Transfer (RQ1.2)

In this section, at first, descriptive statistics and paired t- test results on far transfer scores were presented for Achievement Test 1. Then, descriptive statistics and paired t- test results on far transfer scores were presented for Achievement Test 2. Finally, descriptive statistics and paired t- test results on far transfer scores were presented for the total far transfer scores of both achievement tests.

Achievement Test 1

A paired-samples t-test was conducted to compare far transfer pretest and posttest mean scores of first achievement test. Table 26 shows the descriptive statistics for Achievement Test 1 on far transfer scores; whereas Table 27 demonstrates paired samples t-test for Achievement Test 1 on far transfer.

Table 26 Descriptive Statistics for Achievement Test 1 (Far Transfer)

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretest1_FarTransfer	9.00	30	7.49	1.37
Posttest1_FarTransfer	67.07	30	26.66	4.87

The findings show that there was a significant mean difference between far transfer scores in pretest (M=9.00, SD=7.49) and in posttest (M=67.07, SD=26.66); $t(29)$= -12.86, p= 0.000 (Table 27). These results suggest that learning activities in mathematics lesson designed based on cognitive load principles may have a positive effect on far transfer of learning.

Table 27 Paired Sample T-Test for Achievement Test 1 (Far Transfer)

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pretest1_FarTransfer Posttest1_FarTransfer	-58.07	24.73	4.52	-67.30	-48.83	-12.86	29	.000

Achievement Test 2

A paired-samples t-test was conducted to compare far transfer pretest and posttest mean scores of second achievement test. Table 28 shows the descriptive statistics for Achievement Test 2 on far transfer scores; whereas Table 29 demonstrates paired samples t-test findings for Achievement Test 2 on far transfer scores.

Table 28 Descriptive Statistics for Achievement Test 2 (Far Transfer)

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretest2_FarTransfer	.70	30	1.88	.34
Posttest2_FarTransfer	55.23	30	21.41	3.91

The findings showed that there was a significant mean difference between pretest far transfer scores (*M*=0.70, *SD*=1.88) and posttest far transfer scores (*M*=55.23, *SD*=21.41); *t*(29)= -14.83, *p*= 0.000 (Table 29). These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles result with far transfer of learning.

Table 29 Paired Sample T-Test for Achievement Test 2 (Far Transfer)

Paired Samples Test								
				95% Confidence Interval of the Difference				
	Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t	df	Sig. (2-tailed)
Pretest2_FarTransfer Posttest2_FarTransfer	-54.53	20.14	3.68	-62.06	-47.01	-14.83	29	.000

Total Far Transfer Test Scores

A paired-samples t-test was conducted to compare the total far transfer pretest and posttest scores of first and second achievement test. Table 30 shows the descriptive statistics for Total Test scores on far transfer; whereas Table 31 demonstrates paired samples t-test for Total Test scores on far transfer.

Table 30 Descriptive Statistics for Total Test Scores (Far Transfer)

Paired Samples Statistics	Mean	N	Std. Deviation	Std. Error Mean
Pretests_Total Far Transfer	4.83	30	4.18	.76
Posttests_Total Far Transfer	61.17	30	20.78	3.79

Findings indicate that there was a significant mean difference between pretest far transfer scores (M=4.83, SD=4.18) and posttest far transfer scores (M=61.17, SD=20.78); $t(29)= -16.33$, $p = 0.000$ (Table 31). These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles result with far transfer of learning.

Table 31 Paired Sample T-Test for Total Test Scores (Far Transfer)

Paired Samples Test								
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Pretests_Total FarTransfer Posttests_Total FarTransfer	-56.33	18.89	3.45	-63.39	-49.28	-16.33	29	.000

Summary Table of Paired Sample T-Test Results

Table 32 represents the summary of analysis results for paired sample t- test conducted on achievement tests.

Table 32 Summary of Paired Sample t- test Results

	Mean	**Standard Deviation**	**t**	**Sig.(2-tailed)**
First Test Scores	-69.27	19.41	-19.55	0.00
Second Test Scores	-67.83	10.89	-34.11	0.00
Overall Total Scores	-68.57	12.51	-30.01	0.00
Near Transfer Scores in First Test	-80.33	18.00	-24.45	0.00
Near Transfer Scores in Second Test	-81.30	8.36	-53.25	0.00
Near Transfer Scores in Total Tests	-80.83	8.87	-49.93	0.00
Far Transfer Scores in First Test	-58.07	24.73	-12.86	0.00
Far Transfer Scores in Second Test	-54.53	20.14	-14.83	0.00
Far Transfer Scores in Total Tests	-56.33	18.89	-16.33	0.00

4.3 The Indicators of Near and Far Transfer of Learning (RQ2)

The near and far transfer of learning indicators of learning in self-explanations and group discussions were examined through reflective journals and online forums respectively. Near transfer of learning indicators and then far transfer of learning indicators are provided below.

4.3.1 Near Transfer of Learning Indicators

For the analysis of reflective journals and forums, Coding Schema is used. Coding Schema has three main themes together with their subcategories (Table 33) to derive near transfer outcomes through reflective journals and forums. Near transfer indicators were interpreted through percentages for reflective journals and forums. The percentages depended on the number of students who replied to the reflective journals and forums. Thus, if any two subcategories have same percentages, it may not show the same number of students' replies; because number of participating students might differ from activity to activity.

Table 33 Coding Schema for Near Transfer in Self-Explanations and Group Discussions

Knowledge	Comprehension	Application
• Correct terminology • Correct mathematical notation • Knowledge of general rules in Calculus	• Understands the problem • Defines the rule/ principle • Identifies the correct method • Outlines the steps of problem solution • Gives a similar example	• Organizes the given expressions • Correct mathematical calculation • Indication of breaking problem into parts • Gives reasons of solution steps

Indicators of Near Transfer in Reflective Journals

The indicators of near transfer had shown variety due to the reflective journal question. There was not an expectation that all sub categories under each three theme would be met by students; hence students' replies differed due to the explanation style of the student. Appendix N consists of some students' replies to reflective journals that will assist to give examples due to each near transfer theme at reflective journal. Due to themes, the percentages of of students who had indicators in reflective journals are shown at Table 34.

All reflective journals had indicators of Knowledge Theme when students replies are analyzed. The highest percentage of students who wrote reflective journals and had Knowledge Theme indicators in their replies was 100 % and the lowest percentage of students was 10.7% When Knowledge Theme was coded, it was detected that the student percentages for the *use of correct terminology*; *use of correct mathematical* notation and *knowledge of general rules in Calculus* were quite high. Student percentages of *Knowledge Theme* varied between 10.7 % and 100%. When it comes to *the knowledge of general rules in Calculus,* a student statement on recalling principles was expected to be observed within reflective journal in order to be coded with this subtheme. Since there were not many statements noticed in reflective journal 7; percentage was very low, 10.7%.

Table 34 Near Transfer Percentages for Each Reflective Journal

		N= 28	N= 29	N=24	N= 25	N=28	N= 26	N=28	N= 28
	Codes	**J_1**	**J_2**	**J_3**	**J_4**	**J_5**	**J_6**	**J_7**	**J_8**
KNOWLEDGE	Correct terminology	53.6 (15)	96.6 (28)	79.2 (19)	88.0 (22)	78.6 (22)	84.6 (22)	89.3 (25)	96.4 (27)
	Correct notation	57.1 (16)	58.6 (17)	75.0 (18)	76.0 (19)	78.6 (22)	80.8 (21)	53.6 (15)	71.4 (20)
	Knowledge of general rules in Calculus	89.3 (25)	100.0 (29)	95.8 (23)	84.0 (21)	92.9 (26)	100.0 (26)	10.7 (3)	64.3 (18)
COMPREHENSION	Understands the problem	100.0 (28)	100.0 (29)	100.0 (24)	100.0 (25)	0.0 (0)	100.0 (26)	92.9 (26)	100.0 (28)
	Defines the Rule or Principle	14.3 (4)	13.8 (4)	29.2 (7)	8.0 (2)	89.3 (25)	0.0 (0)	3.2 (1)	0.0 (0)
	Identifies correct method	89.3 (25)	93.1 (27)	95.8 (23)	80.0 (20)	92.9 (26)	100.0 (26)	92.9 (26)	100.0 (28)
	Outlines Step of Problem solution	0.0 (0)	3.4 (1)	100.0 (24)	0.0 (0)	21.4 (6)	11.5 (3)	7.1 (2)	67.9 (19)
	Gives a Similar Example	7.1 (2)	100.0 (29)	0.0 (0)	0.0 (0)	96.4 (27)	100.0 (26)	0.0 (0)	7.1 (2)
APPLICATION	Organizes the Given Expression	92.9 (26)	100.0 (29)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Correct Mathematical Calculation	100.0 (28)	93.1 (27)	87.5 (21)	72.0 (18)	92.9 (26)	100.0 (26)	89.3 (25)	100.0 (28)
	Indication of Breaking Problem into parts	53.6 (15)	100.0 (29)	100.0 (24)	16.0 (4)	21.4 (6)	53.8 (14)	0.0 (0)	0.0 (0)
	Gives Reasons of Solution Steps	78.6 (22)	82.8 (24)	83.3 (20)	100.0 (25)	50.0 (14)	50.0 (13)	21.4 (6)	67.9 (19)

J_1 to J_8= Reflective journal 1 to Reflective journal 8; *N= Number of Students ;*
Numbers in paranthesis show student frequencies

Under Comprehension Theme, lowest percentage of students was 0 % and highest percentage of students was 100 %. The students percentages changed due to reflective journals. There were overall indicators of comprehension at all reflective journals.

Understanding the problem was almost 100% for all reflective journals, since each reflective journal composed of a math problem or a proof; except reflective journal 5 where students were asked to explain the topic to a friend. Thus, explaining to a friend statement was not coded with *understanding the problem;* as it was a declaration rather than a problem to solve. The percentage range of *defining a rule or a principle* was not as high as understanding the problem; because some students recalled learnt rule before the solution to be integrated to the solution steps. At reflective journal 6 and reflective journal 8, there was not any statement seen linking to this subtheme. Hence, its percentage varied in between 0% and 89. 3%. Students mostly defined rules to be used at reflective journal 5, since they were explaining the topic to a friend through introducing the principles in the unit. *Identifying correct method* went along with understanding the problem. That might be the reason for its percentages were high, where the lowest percentage was 80% (at reflective journal 4). At reflective journals 6 and 8, students fully identified the correct method. However, at other reflective journals there were some minor misconceptions of students in the method. However, the percentages of this subtheme were quite satisfying for near transfer of thinking. *Outlining steps of solutions* was not compulsory for each reflective journal; however student were expected to outline steps in optimization problem at reflective journal 8. Even if this was the case, all students solved reflective journal 3 question step by step and were coded with 100%; whereas reflective journal 8 percentage was lower than reflective journal 3 percentage, 67.9%. This might be due to the optimization being a challenging topic. None of the students demonstrated a solution outline at reflective journal 1 and reflective journal 4. Researcher met with *giving a similar example* subtheme at all students' replies for reflective journal 2 and reflective journal 6; however there was not any stated example at reflective journal 3, reflective journal 4 and reflective journal 7. Reflective journal 6 was challenging students to give their own example to explain the unit; hence students had to give a similar example like in learning activities. Students were also expected to explain the unit at reflective journal 5 throughout own examples; henceforth its percentage there was also high, 96.4%.

Application Theme consisted of *organizing the given expression* that was only seen at reflective journals 1 and 2; because both reflective journals had problem with rational functions that can be arranged as product of two functions (where denominator can be expressed with negative powers). Within both reflective journals, students required using this technique, as they were not much familiar with quotient rule then. The two subtheme percentages of students were high: 92.9% and 100% respectively. Alternatively, *correct mathematical calculation* was required at all reflective journals. Its percentage of students varied between 72% and 100%, where the lowest percentage was at reflective journal 4 and reflective journal 1; reflective journal 6 and reflective journal 8 had the highest indicators of this subtheme. Students had the most difficulty at correct calculations while analyzing tangent line to trigonometric functions. The percentages of this subtheme were quite high in general and the results were satisfying to indicate near transfer. Indicators of *breaking problem into parts* were recognized with all students at reflective journal 2 and reflective journal 3. Nevertheless, it was not observed at all at reflective journal 7 and reflective journal 8. Outlining problem solution and breaking problem into parts were in agreement at reflective journal 3; therefore similar steps were coded with these two subcategories for some students. At other reflective journals, percentages were not very high. Very few students preferred to analyze the problem within smaller parts. *Giving reasons of solution steps* were highly recommended for students to include at reflective journals so that communication would be smooth. Its percentage range was 78.6 %. All students showed reasons of solution steps at reflective journal 4 with step by step mathematical calculations while finding tangent lines. At reflective journal 7, percentage of *reasoning solution steps* was very low, 21.4%. For this subtheme, it can be concluded that in general students were able to use and link reasoning of solution to the solution step; that might lead to efficient awareness of students on why that solution step was done.

Appendix N consists of examples from students' reflective journal entries. [1] is the reflective journal of S_{17} for Unit 1-part 1. It demonstrated *use of correct terminology and notation* in general. Student stated first principle and method to apply simple rule for derivatives; hence reply was coded with *Knowledge of general rules in Calculus*. The student *understood the problem* and *identified correct method* to solve the problem. There was *an organization of given expressions* while arranging powers and having fraction operations. S_{17} also simplified the solution and arrived at answer with *correct mathematical calculation.* S/he *broke the expression into parts* before applying simple rules to take derivative. Explaining steps in detail with math language; justifying abstract concept (0/0=undefined) through warning on a possible math error and giving reasoning of the arrangement of the solution were linked to *giving reasons of solution steps.*

[2] shows that S_{11} mostly used *correct terminology and notation* throughout the reflective journals as well while stating product and quotient rules. Knowledge of product rule and quotient rule was coded by *Knowledge of general rules in Calculus*. This student *understood the problem* and showed awareness on *identifying correct method* to solve the problem. The denominators were also organized in the form of powered expressions that was linked to *organizing the given expression*. The results out of product rule and quotient rule were successful; hence the student's *mathematical calculation was correct.* Showing detailed solution steps mathematically along with verbally was indicators of *giving reasons of solution steps.*

Example [3] consisted of *knowledge of general rule* where the inverse function of exponential e is of natural logarithm. S/he also demonstrated knowledge on derivative functions of exponential and natural logarithm functions. There is *correct use of terminology* however *mathematical notations* were problematic. The students followed *understood the problem* ands/he *used a correct method* since inverse of *f(x);* derivative of inverse function; derivative of *f(x)* and applying composition of function were observed in the reflective journal entry. Solving the problem was *broken into parts* as each part of the given formula were calculated in steps and then integrated to make up a whole. There results showed *correct mathematical calculation with giving reasons* or explanation of each step.

S_{18} at Example [4] had shown near transfer of thinking throughout the solution of the problem at reflective journal 4.

Within calculations the student used correct terminology and notation in general. S/he indicated *knowledge of rules in Calculus* by calculating derivatives of trigonometric functions. Student *understood the problem* by arriving at tangent line equations. These steps also indicated *correct mathematical calculations*. *Outlining steps of solutions* were also recognized at this reflective journal entry, throughout the calculation steps of tangent lines and also from the first paragraph of solution explanation. Reasoning of each step was also coded for this reflective journal, as verbal explanation and the plan on how to calculate tangent lines were given Within the graph plots of this reflective journal, student first calculated points to join to get the graph, that indicates *breaking problem into* parts and then s/he *identified correct method* to plot the graphs. Within the proof, student stated two ways to prove perpendicularity: Algebraically, Geometrically. S/he proved perpendicularity into ways, that verifies that s/he had knowledge on rules as well as *correct mathematical* calculation.

Next, other than involving *correct use of terminology; correct use of notation; knowledge on the methods of finding stationary points; understanding the problem; identifying correct method; giving an example to solve the problem; simplifying solution*, Example at [5] also demonstrated *outlining steps for problem solution* at the end of reflective journal 5. Example [8] also involved indicators of *outlining steps for problem solution* along with *correct use of terminology; correct use of notation; knowledge on the methods of finding stationary points; understanding the problem; identifying correct method; simplifying solution of mathematical results*; and *giving reasoning of solution steps.*

Indicators of Near Transfer in Group Discussions

The indicators of near transfer had shown variety due to the forum question. There was not an expectation that all sub categories under each three themes would be met by students; hence students' replies differed due to the explanation style of the student. Appendix O consists of some students' replies to forums that will assist to give examples due to each near transfer theme at forums. Due to themes, the percentages of students having indicators of near transfer in group discussions are shown at Table 35.

When the <u>Knowledge Theme</u> was observed, students' percentages were relatively higher in its sub categories compared to the subcategories of other two themes. All forums had indicators of *Knowledge Theme*. Among the participating students who replied to forums, the student percentages varied due to subthemes of <u>Knowledge</u>. Students percentages varied between 0 % and 100% depending of the subthemes <u>Knowledge Theme.</u> Between 69,6% and 100% of the students who participated at forums showed *use of correct terminology* in their replies. 95% of students also used *correct notation* in most of the forums. However, this subtheme was not applicable in forum 7 and forum 8; as both forums required verbal explanation and interpretation through sentences or short claims. Forums 2-3-4 necessitated knowledge of general rules in Calculus, hence students were awarded to get that subtheme code automatically.

[1] in Appendix O displays a student reply that showed that student *knowledge of the general rules in* Calculus by "Giving formal definition on how to find gradient and defining the functions in her examples as quartic, cubic, quadratic" Moreover, [2] illustrates another example of a student reply that had been coded with knowledge theme, where the student was aware of *knowledge of the general rules in Calculus,* through taking derivative as well as *use of correct terminology and notation.*

The percentage of students who replied to forums and had replies with Comprehension Theme indicators varied between 0% and 100%. However, all forums had indicators of comprehension. When it comes to Comprehension Theme, *understanding the problem* subtheme had quite high percentages. This subtheme percentage of students differed from 68.2% to 100%. This subtheme was very high likely coded when the student had demonstrated an attempt to use the correct method or used own word to paraphrase the given problem. On the other hand, *defining the rule or principle* was very rarely seen depending on the forum question; thus its percentage values were very low. Additionally, *identifying correct method* was seen more often as any attempt or expression leading to correct method of solving the problem was awarded with this subtheme. The student percentages of this subtheme were very high; ranging from 68.2% to 100% other than forum 5 (Unit 3-part 1). On the other hand, 4.3% of students were able to *identify correct method* at forum 5. *Outlining the step of problem solution* was not a very necessary attempt while answering forum question, hence very few students showed attempt to outline problem when a mathematical problem was asked in the forum. Next, percentage of *giving a similar example* differed due to question style. Most of the students preferred to explain the proof and interpretation questions (Forum 1 and Forum 3) in forums via their own examples, as they were not aware of formal proof techniques.

Table 35 Near Transfer Percentages for Each Forum

		N=26	N=23	N=22	N=20	N=23	N=22	N=23	N=22
	Codes	**F_1**	**F_2**	**F_3**	**F_4**	**F_5**	**F_6**	**F_7**	**F_8**
KNOWLEDGE	Correct terminology	80.8 (21)	100.0 (23)	95.5 (21)	90.0 (18)	78.3 (18)	100.0 (22)	69.6 (16)	95.5 (21)
	Correct notation	73.1 (19)	82.6 (19)	86.4 (19)	95.0 (19)	65.2 (15)	72.7 (16)	0.0 (0)	0.0 (0)
	Knowledge of general rules in Calculus	76.9 (20)	100.0 (23)	100.0 (22)	100.0 (20)	87.0 (20)	45.5 (10)	13.0 (3)	18.2 (4)
COMPREHENSION	Understands the problem	80.8 (21)	78.3 (18)	68.2 (15)	100.0 (20)	82.6 (19)	40.9 (9)	95.7 (22)	95.5 (21)
	Defines the rule or principle	11.5 (3)	4.3 (1)	4.5 (1)	0.0 (0)	26.1 (6)	0.0 (0)	0.0 (0)	0.0 (0)
	Identifies correct method	76.9 (20)	78.3 (18)	68.2 (15)	100.0 (20)	4.3 (1)	100.0 (22)	95.7 (22)	90.9 (20)
	Outlines steps of problem solution	0.0 (0)	17.4 (4)	4.5 (1)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Gives a similar example	76.9 (20)	8.7 (2)	95.5 (21)	5.0 (1)	30.4 (7)	9.1 (2)	4.3 (1)	0.0 (0)
APPLICATION	Organizes the given expression	3.8 (1)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Correct mathematical calculation	76.9 (20)	78.3 (18)	18.2 (4)	100.0 (20)	0.0 (0)	81.8 (18)	21.7 (5)	0.0 (0)
	Indication of breaking problems into parts	3.8 (1)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Gives reasons of solution steps	34.6 (9)	4.3 (1)	13.6 (3)	55.0 (11)	39.1 (9)	22.7 (5)	26.1 (6)	27.3 (6)

F_1 to F_8= Forum 1 to Forum 8; N= Number of Students; Numbers in paranthesis show student frequencies

One of the students' reply, [3] pointed out that the reply consisted of *understanding the problem*, *identifying correct method to solve the problem* and *giving own example to solve* in Forum 4 (Unit 2-part 2). Forum question was requiring to "give an example of a function where the second derivative of it is equal to itself and to prove." S_5 preferred to start by giving counter examples on polynomial and trigonometric functions; and then continued with an example of exponential function. The proof has ended with a generalization on this particular function.

On the other hand, [4] shows a student reply where she *defines the rule* between sign of derivative function to the function being increasing/decreasing while explaining the monotonicity question. [5] is another example of a student reply that consists of *understanding the problem* and *giving own examples*

Application Theme had lowest student percentages compared to the other two near transfer indicators. Student percentages in replies with Application Theme indicators varied between 0% and 100%. Within this theme, *organizing the given expression* was observed once in forums; that is only one student showed this attempt while answering Forum 1 question. [6] shows the student's reply to the first forum through his own examples. S_{27} gave his own examples with negative powered exponential functions and then he organized the answer in terms of positive powers. *Correct mathematical calculation* was observed nearly at all forums except Forum 8 and Forum 5; as students solved own examples to explain the question. This subtheme's student percentage range varied from 0% to 100% and there was not a particular pattern detected. The two highest percentages were recognized at Forum 4 and Forum 6-part 1. Other than forum 5 and forum 6, the lowest percentage of students was seen at forum 3; since students had difficulties to represent the solution steps fully correct. Most of them had algebra errors in their proofs with own examples, even if they understood what the question was asking for. Also, some students had challenge to interpret the notations at given formula at Forum 3. None of the students were able to answer fully correct; hence none of them were awarded with correct mathematical calculation code, though there were some correct calculations.

On the other hand, *breaking problem into parts* was not met at all at forums; this might have been caused due to question style of forums. There were three math problems (Forums 6; 7; 8) and they did not necessitate to breaking problem into parts. Thus, none of the replies consisted of coding with this subtheme. The student percentage of *giving reasoning of solution steps* changed from 4.3% to 55%. Students had most trouble to make connections at Forum 2 towards chain rule. They mostly had conceptual errors in understanding and interpretation of the given notation aimed at chain rule. Forum 4 indirectly required demonstrating reasoning of solution steps; since it was a proof question. That might be the reason for most of the reasoning of solutions steps was seen at the replies to Forum 4.

To illustrate some students' examples, [5] shows that S_{17} *gives reasoning of solution steps* by explaining the why that step was done and how it was linked to conclude increasing and decreasing functions at Forum 5. Next, [3] demonstrates S_5's reply to Forum 4 that had been coded to show *correct mathematical calculation* and *giving reasoning of solution steps*; as s/he explains the justification of solution steps towards replying the forum question. [7], the reply of S_6 to Forum 6, also shows that student does *correct mathematical calculation* and *gives reasoning of solution steps* by linking to learnt rules and principles.

4.3.2 Far Transfer of Learning Indicators

For the analysis of reflective journals and forums, Coding Schema is used. Coding Schema has two main themes together with their subcategories (Table 36) to derive far transfer outcomes through reflective journals and forums. There was not an expectation that all subcategories would be met by students; as students' method of explaining and answering the questions displayed variations. Far transfer indicators were interpreted through percentages for reflective journals and forums. The percentages depended on the number of students who replied to the reflective journals and forums. Thus, if any two subcategories have same percentages, it may not show the same number of students' replies; because number of participating students might differ from activity to activity.

Table 36 Coding Schema for Far Transfer in Self Explanations and Group Discussions

Analysis	**Synthesis**
• Comparison of new problem with learnt problem • Differentiates the new problem • Solves a problem in new situations • Applies a new technique to solve a problem • Analyzes the problem further for more information. • Correct interpretation of sign tables, graphs and results	• Creates a new problem • Generalizes a calculus rule/fact • Derives abstract relationships. • Judges the reasonableness or validity of the results • States limitations and restrictions

Indicators of Far Transfer in Reflective Journal

The indicators of far transfer had shown variety from one journal to another due to the reflective journal question structure. Appendix N consists of some students' replies that will assist to give examples due to each far transfer theme at reflective journals. The percentages due to themes are shown in Table 37.

The far transfer percentages of students in reflective journals were lower than near transfer percentages of students at reflective journals. There were two themes used for the analysis of far transfer. There were also subcategories for each theme. Not all the subcategories were met throughout the reflective journals; thus none of the reflective journals had indicators of *creating a new problem, deriving abstract relationships* or *stating limitations and restrictions* subcategories.

In Analysis Theme, there were some indicators for each its subtheme, only in some journals with very low frequencies. The student percentages varied between 0% and 100%. *Comparison of a new problem with learnt problem* was seen only in reflective journal 3, where all 24 students (100% of students) had this code when recall made on laws of logarithms for assistance of new problem solution in $f'(\ln x) = e^{\log_e x} = e^{\ln x}$ =x. This was coded with this code because composition of $f^{-1}(x) = \ln x$ with derivative function was new to the students; they managed to recall learnt law of logarithms to simplify the solution of the new problem by comparing the composition of functions outcome. *Differentiating the new problem* was also rarely detected at reflective journals; where only reflective journal 6 and reflective journal 7 had some clues. Reflective journal 7 had the highest percentage, where 75% of students *differentiated the new problem*. The problem structure at reflective journal 7 was brand new to the students; therefore it challenged for far transfer thinking. On the other hand, there were three students who demonstrated this attribute by analyzing the rational function behavior by finding turning points to sketch the graph that was not covered in learning activities for rational function.

Solving problem in new situations was one of the subcategories that was more frequently observed. When the students were able to solve reflective journal 4 question together with four correct graphs, they were coded with this subtheme. Its percentage at reflective journal 4 was 40%. This percentage was the highest among the other subthemes for that reflective journal. Therefore, less than half of the total students were able to demonstrate successful far transfer of thinking while problem solving in reflective journal 4. When the student gave new examples to explain in reflective journal 5 and reflective journal 6; and solved it fully throughout in his explanations, s/he also was coded with this subtheme. The percentage of this subtheme at reflective journal 5 was 39.3% and 26.9% at reflective journal 6. Again, less than half of the students were able to solve their own examples fully correct as they were explaining the topic that indicated far transfer of thinking. The structure of the question at reflective journal 7 was new to the students; therefore if they were able to solve that problem fully correct, then they were automatically coded with this subtheme. The success rate of students to answer this question correct was 78.6%, which was the highest percentage of this subtheme among all reflective journals. The second highest percentage for this subtheme was at reflective journal 8, where students were expected solve the given problem defined in a new context with a new structure. Although students were familiar with optimization problems, the generalization of optimization for the given rectangle was innovative investigation for them. That might be the reason for its student percentage was also high, 67.9%. Therefore majority of the students demonstrated far transfer of thinking while solving problems at reflective journal 7 and reflective journal 8.

Table 37 Far Transfer Percentages for Each Reflective Journal

		N=28	N=29	N=24	N=25	N=28	N=26	N=28	N=28
	Codes	**J_1**	**J_2**	**J_3**	**J_4**	**J_5**	**J_6**	**J_7**	**J_8**
ANALYSIS	Comparison of new problem with learnt problem	0.0 (0)	0.0 (0)	100.0 (24)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Differentiates the new problem	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	11.5 (3)	75.0 (21)	0.0 (0)
	Solves a problem in new situations	0.0 (0)	0.0 (0)	0.0 (0)	40.0 (10)	39.3 (11)	26.9 (7)	78.6 (22)	67.9 (19)
	Applies a new technique to solve a problem	0.0 (0)	0.0 (0)	0.0 (0)	12.0 (3)	0.0 (0)	3.8 (1)	0.0 (0)	0.0 (0)
	Analyzes the problem further for more information	10.7 (3)	3.4 (1)	0.0 (0)	4.0 (1)	25.0 (7)	0.0 (0)	0.0 (0)	0.0 (0)
	Correct interpretation of sign tables, graphs and results	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	7.1 (2)	0.0 (0)	64.3 (18)	82.1 (23)
SYNTHESIS	Creates a new problem	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Generalizes a Calculus rule/fact	0.0 (0)	6.9 (2)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	7.1 (2)
	Derives abstract relationships	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Judges the reasonableness or validity of the results	35.7 (10)	75.9 (22)	79.2 (19)	0.0 (0)	0.0 (0)	3.8 (1)	7.1 (2)	0.0 (0)
	States limitations and restrictions	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)

J_1 to J_8= Reflective journal 1 to Reflective journal 8; N= Number of Students;
Numbers in paranthesis show student frequencies

Applying a new technique to solve a problem was again very rarely detected at reflective journals. Very few students at reflective journal 4 and reflective journal 6 had this attribute. At reflective journal 4, three students either measured angles to be 90^o or benefitted from the reflection of *y*=*x* line to plot tangent line while solving the problem. This technique was a new approach to make a link to algebraic calculations to graphical analysis. At reflective journal 6, there was only one student who used a new method to find vertical asymptote.

Analyzing the problem further for more information was seen at four reflective journals, its percentage was also low for each reflective journal. For the first reflective journal, if three students gave a further example to examine simple rule with power=0. At reflective journal 2, one student showed two ways to apply product rule; one way with representing denominator with the product of an expression with negative powers. (i.e. $\frac{x}{x+1} = x.(x+1)^{-1}$). The other way was to apply product rule to numerator multiplied with a rational function with numerator. (i.e. $\frac{x}{x+1} = x.\frac{1}{x+1}$). At reflective journal 4, one student stated that there were two ways to find the slopes were perpendicular; algebraically and geometrically. For Reflective journal 5, seven students analyzed turning points and classified them before the sketch. They also investigated concavity of the function to get this subtheme code. The student percentage of this subtheme was highest at reflective journal 5, with 25%.

Correct interpretation of sign tables, results and graphs was detected at most at reflective journal 7 and reflective journal 8. Reflective journal 7 required students to interpret results to find how many times that the particle passed through origin. If the student understood the results of sign tables mostly correct to link to the solution, then s/he was coded with this subtheme. 64.3% of students had *correct interpretation of sign tables and results* in reflective journal 7. This meant more than half of the total students realized far transfer of thinking in reflective journal 7. Similarly, students had to interpret sign tables for derivative function for the optimization problem in reflective journal 8. 82.1% of students were able to comprehend the outcomes in their results to link to the solution. When the students had some minor conceptual errors to make a generalization, they were not awarded with *solving the problem at new situation;* where this code was strictly requiring full correct answers.

Students' examples show more limited indicators of far transfer than near transfer indicators at reflective journals. Example [1] at Appendix N involves the subtheme of *analyzing the problem further for more* information while examining reasons of how to find limit at rational functions, if the case is undefined for when limit of h approaches to zero. Another student related the new problem solution to the learnt laws of logarithms at $f'(\ln x) = e^{\log_e x} = e^{\ln x}$ =x; therefore *comparison of new problem with learnt problem* and then linking information exists at [3] at Appendix N. [4] demonstrated that the student plotted correct four graphs; hence she *solved the problem in new situation*. [4] also *applied a new technique* (graphical analysis-measuring angles to be 90^o) *to solve the problem* rather than algebraically. Example at [5] demonstrated that the student *solved a problem in new situation*; as finding stationary points in roots was a new situation and not only s/he made correct mathematical calculations but also made coherent and full problem solution with the analysis of sign tables. She also *analyzed the function further for more information* by finding and classifying inflection points as well as examining concavity intervals. *Correct interpretation of sign tables and results* also existed at [5]; with increasing/decreasing intervals; concavity intervals and critical points' classifications. [6] shows another example of a student's reply that involves *solving a problem in a new situation*, since the student analyzed the behavior of chosen rational function with its stationary points and asymptotes. S/he also sketched the graph successfully, although there were some missing labeling of axes and scales. Example [7] indicated that student *differentiated the new problem*; as the problem structure was new to them and s/he was able to *solve the problem in new situation* successfully. There was also *correct interpretation of* velocity-time *sign graph* and displacement analysis at critical times at that reflective journal entry. At Example [8], student successfully *solved the optimization problem in new situation* and made *correct interpretation of sign tables and results.*

The overall students percentages of <u>Synthesis Theme</u> was lowest. Its percentages varied between 0% and 79.2%. Under <u>Synthesis Theme</u>, *generalizing a Calculus rule/fact* was observed seldom. They were only seen at reflective journal 2 and reflective journal 8. Two students at reflective journal 2 and reflective journal 8 generalized a fact about each case as conclusion. *Judging the reasonableness or validity of results* was the subtheme that was most observed under Synthesis at reflective journals. The highest percentages of students were at reflective journal 2 and reflective journal 3, with 75.9% and 79.2% respectively. If the student made a comparison of product rule and quotient rule results at reflective journal 2 ; and if there was a comment on validity of the final solutions on the left and right hand side of the formula, $\left(f^{-1}(x)\right)' = \frac{1}{(f'(f^{-1}(x))}$ at reflective journal 3,then they were coded with this subtheme.

At [2] at Appendix N, student *judges the reasonableness or validity of the result* by comparing results from product rule and quotient rule; hence awareness exists on validity at proof. Student at example [3] also *justified validity of solution* by comparing with the previous solution (solutions matched). For the reflective journal entry, [7] and [8] *generalized a fact* stating that in order to get maximum area, rectangle must be a square.

Indicators of Far Transfer in Forums

The indicators of far transfer had shown variety due to the forum question. Appendix O consists of some students' replies to forums that will assist to give examples due to each far transfer theme at forums. The percentages due to themes are shown at Table 38.

Far transfer indicators were not as much as near transfer indicators. Within <u>Analysis Theme</u>, *comparison of new problem with learnt problem* were seen at Forums 1, Forum 3, Forum 4 and Forum 5. However, there was only one student at those forums which caused this fact; as a result the percentage of this subtheme varied from 3.8% to 5% at those forums. For example, Giving another function structure of a with negative powers, together with its gradient and then comparing with previously learnt polynomial function characteristics was awarded for [8] at Forum 1.

Differentiating the new problem was appreciated at three students at Forum1. The replies of these students were coded under this subtheme, because students made comments on what the gradient function would become for functions with negative powers or for fractions. These cases were not discussed in learning activities before the forum; thus students created a further problem in a new setup that was more challenging than pre-taught examples. [6] and [8] demonstrate two students' replies that had a discussion on functions involving terms with negative powers and the outcome in gradient function. It should be mentioned that these students were in different groups.

Solving a problem in new situations had the highest percentage score compared with the subcategories of Analysis Theme, 54.5%. This score was high at Forum 3 that required a proof and students gave their own examples. There were twelve students who were able to solve their own examples in this new situation.

[9] shows that S_{10} used two examples to prove the rule and successfully reached the result in the given new situation. [10] is another example that S_4 successfully answered *the given problem in new situation* at Forum 7. This type of problem was not introduced before in learning activities.

Table 38 Far Transfer Percentages for Each Forum

		N=26	N=23	N=22	N=20	N=23	N=22	N=23	N=22
	Codes	**F_1**	**F_2**	**F_3**	**F_4**	**F_5**	**F_6**	**F_7**	**F_8**
ANALYSIS	Comparison of new problem with learnt problem	3.8 (1)	0.0 (0)	4.5 (1)	5.0 (1)	4.3 (1)	0.0 (0)	0.0 (0)	0.0 (0)
	Differentiates the new problem	11.5 (3)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Solves a problem in new situations	3.8 (1)	0.0 (0)	54.5 (12)	0.0 (0)	0.0 (0)	22.7 (5)	47.8 (11)	27.3 (6)
	Applies a new technique to solve a problem	11.5 (3)	0.0 (0)	4.5 (1)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Analyzes the problem further for more information	3.8 (1)	8.7 (2)	9.1 (2)	0.0 (0)	0.0 (0)	4.5 (1)	21.7 (5)	18.2 (4)
	Correct interpretation of sign tables, graphs and results	69.2 (18)	52.2 (13)	13.6 (3)	0.0 (0)	43.5 (10)	27.3 (6)	65.2 (15)	68.2 (15)
SYNTHESIS	Creates a new problem	0.0 (0)	0.0 (0)	0.0 (0)	65.0 (13)	69.6 (16)	0.0 (0)	0.0 (0)	0.0 (0)
	Generalizes a Calculus rule/fact	30.8 (8)	43.5 (10)	18.2 (4)	20.0 (4)	17.4 (4)	0.0 (0)	0.0 (0)	0.0 (0)
	Derives abstract relationships	0.0 (0)	56.5 (13)	9.1 (2)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)	0.0 (0)
	Judges the reasonableness or validity of the results	19.2 (5)	47.8 (11)	63.6 (14)	80.0 (16)	8.7 (2)	9.1 (2)	0.0 (0)	0.0 (0)
	States limitations and restrictions	0.0 (0)	0.0 (0)	36.4 (8)	5.0 (1)	0.0 (0)	4.5 (1)	0.0 (0)	0.0 (0)

F_1 to F_8= Forum 1 to Forum 8;   N= Number of Students;
Numbers in paranthesis show student frequencies

Applies a new technique to solve a problem was also seen very rarely at forums. Only forum 1 and forum 3 had some indicators. As an example, S_{10}'s reply to Forum 3 coded with this subtheme; since he applied a new technique to solve a harder function (Cubic function) through finding its inverse function ([9]). He used the fact that if a function has an inverse, then f(a)=b indicates $f^{-1}(b)=a$

Analyzes the problem further for more information was observed except Forum 4 and Forum 5; however the highest student percentage was 21. 8%. Example [11] at Appendix O demonstrates a student's reply to forum 7. She made further analyze on acceleration-velocity relation to direction to increase of speed. This subtheme code was been awarded although there are some minor misconceptions on acceleration sign and decreasing speed. Additionally, within the student's reply in Example [12], it can be seen that student analyzed the graph's behavior and its relation to Economics with the statement of:

> "When our total revenue is the same as our cost, all the money from our revenue will be spent to the cost of production hence leaving no profit. After point A we can see that the cost graph decreases rapidly and the linear revenue graph increases hence making revenue more than the cost. This will give us profit because there is more revenue and less cost."

Correct interpretation of sign tables, graphs and results had the highest percentages; by way of it was given for those cases when student made a correct interpretation of any table, graph or result within the solution step that led to the final answer to be fully correct. Henceforth, this code was in agreement with *correct mathematical calculation* under "Application Theme" and *solves a problem in new situations* under Analysis Theme. At Example [2], student was able to made a *correct interpretation of previous calculations* as a meaningful conclusion of that *h(x)=(fog)(x)* with significant solution steps. That is the reason for he was assigned this subtheme. Example [9] illustrated a student reply with correct interpretation of the theoretical knowledge to be integrated in his new method of solving the problem; so he also recorded under this subtheme. Also, students' replies at [10], [11] and [12] were assigned this code as they made correct interpretations on analysis of graphs at Forums.

Finally, under Synthesis Theme, percentages of its subcategories were higher than majority of the percentages of subcategories under Analysis Theme. *Creates a new problem* subtheme had quite high student percentages such as 65% and 69.6% at Forums 4 and Forum 5 respectively. Other than these two forums, researcher did not come across with this subtheme at other forums. Forum 4 required students to "give an example of a function where the second derivative of it is equal to itself". Students were challenged to create a new example because this fact was not visited in learning activities before. Therefore, majority of the student who answered Forum 4 question correctly, were recorded with this subtheme. Forum 5, similarly, asked students to generate examples of monotonic functions. Consequently, they were assigned with this subtheme if they were successful at answering the forum question. *Generalizes a Calculus rule/fact* subtheme had changing percentages from 17.4% to 43.5% in the first five forums; however for the last three forums did not consist of any indications for this subtheme. This was due to the fact that, last three forums asked mathematical problems just to solve only. Subtheme of *deriving abstract relationships* was observed only at Forum 2 and Forum 3 where the highest percentage was 56.5% . Alternatively, *judging the reasonableness or validity of the results* had been observed more frequently at Forums. Other than Forum 7 and Forum 8, it was detected with the range of 70.9 %. There were three forums where *Stating limitations and restrictions* was observed. Students mostly made comments on this fact at Forum 3 with a percentage of 36.4%.

[13] is an example of a student reply that involves the subcategories of *creates new problem* with four examples*; generalizes that a fact* on the form of the exponential function that will give the second derivative to be the same as original function*; and justifying validity of solution* by comparing with the second derivative with the original function. Moreover, [14] shows another student's reply where the *correct interpretation of results* and *generalization of a fact* were done through backwards thinking in assistance of induction. S_{30} at [15] *generalized a fact* through realizing h'=(fog)'; *derived abstract relationships* by realizing chain rule is applied on composition of functions to derive; and *justified validity of solution* by comparing with the previous solution. Similarly, [9] demonstrates that student *stated restrictions* on when to apply the given rule with his first sentence; and he also made *justification of the solution* by comparing the result with the previous ends, in his check of solution section. As another example, [3] also shows that the *created her own three examples* to illustrate and *justification of the solution* was made by comparing the second derivative result with the original function within each example.

4.3.3. General Evaluation of Reflective Journals and Forums

Use of terminology improved as the number of reflective journals and forums increases. However, *use of mathematical notation* varied due to given task. It was observed that percentage of use of correct notation in forums was relatively higher than percentage of the correct notation in reflective journals; however, it was dependent on the sort of the question.

Knowledge of general facts in Calculus was usually high in both reflective journal and forum (> 84%, >76.9 % respectively). *Understanding the problem* was indeed in agreement with *using the correct method* for both forums and reflective journals. Except reflective journal 5, understanding the given problem was 100%. On the other hand, except forum 6, the frequency of this code was above 68.2%

Defining a rule was rarely observed so it was coded when a student defined a rule in Calculus. As reflective journal 5 involved them to summarize the unit, they had to refer to the theorems in Calculus; so they had to mention a rule. This code was above 8% in reflective journals.

Using correct method was more detailed observed in reflective journals. It was above 80% in reflective journals. *Outlining the steps of solutions* was not necessary to be used in every problem, so this code was formed to address to those who were using the method of listing the solutions first.

Giving a similar example was used mostly when the student was asked to write own example and to solve it through explaining solution steps. Hence, its use varied in reflective journals and forums. *Organization of solution steps* was again an individual choice. The first two reflective journals were necessitating them to organize powers so the percentages are high in the first two reflective journals(> 92.9 %).

Simplifying solution was observed that within reflective journals more than 72% of students found the answers correct. Within the forums, some forums did not necessitate a problem solution such as forum 5 (which was requiring interpretation) and this category was only applicable when student gave own example and solved his/her own example.

Breaking into parts was an individual choice except the requirement of reflective journal 2 and reflective journal 3. First six reflective journals had percentages above 16% for this subtheme; as most of the students solved the mathematics problems through breaking into parts. This subtheme was only observed at forum 1 with a low frequency among all forums.

Reasoning solution steps required showing solution steps in detail or stating why the next solution step was conducted. Again this was mostly seen in problem solution steps in reflective journal and all reflective journals had this code to be seen. Forums also involved this code as any reasoning regarding to interpretation of concepts was counted in this category.

When it comes to far transfer tasks, under <u>Analysis Theme,</u> *solving a problem in a new situation* was mostly had higher percentages at reflective journals than at online forums. There is not a trend of increase or decrease but for the last two reflective journals this subtheme corresponds an improvement together with *correct interpretation of sign tables, graphs and results.* On the other hand, *correct interpretation of sign tables, graphs and results* have higher percentages at online forums than reflective journals. Only forum 4 does not involve this subtheme. There are also indicators of s*olving a problem in a new situation* at online forums. *Analyzing the problem further for more information* is more often observed at forums than reflective journals. Six of the forums involved this subtheme.

Under <u>Synthesis Theme</u> *judging the reasonableness or validity of the results* and *analyzing the problems further* were the most seen codes within the reflective journals. On the other hand, forums involved more codes involving far transfer, as it was aiming to search for more. *Generalization of a fact or a rule in Calculus* and *correct interpretation of results, tables, graphs* were also another codes which were more frequently observed at students' replies in forums.

Students' self reflections and group discussion results revealed near transfer of learning at <u>Knowledge, Comprehension and Application Themes.</u> The average of percentages was higher at Knowledge Level (77.3%); whereas at Comprehension Level average was 52.9% ; and the average at Application Theme was 56.5% at reflective journals.The forum averages of near transfer indicators showed relatively close results at Knowledge Theme only. At Knowledge Theme average was 71.9%; at Comprehension Theme average was 38.9%; at Application Theme average was 19.0% at online forums. Thus self-reflections had more near transfer of learning indicators of participants.

Far transfer averages were lower than the percentages of near transfer indicators at self-reflections and group discussions. Students' self reflections and group discussion results revealed far transfer of learning at Analysis and Synthesis Themes. The average of percentages was 13.6 % at Analysis Theme (13.6%); whereas at Sythesis Theme average was 5.4% at reflective journals.The forum averages of far transfer indicators showed relatively close results. At Analysis Theme average was 12.6%; at Synthesis Theme average was 15.1%; at group discussions. Therefore, far transfer indicators were rarely observed at self reflections and group discussions. Alhough percentages are low, there were more indicators of far transfer at different subthemes at forums than reflective journals.

4.4 Student Opinions About Learning Activities and Their Impacts Towards Understanding (RQ3)

Students' Opinions About Worked Examples

The interview results about the worked examples revealed three main themes namely, Understanding and Learning; Reinforcing and Confusion. Table 39 shows the main themes and subcategories drawn from students' responses about worked examples used in the study. Understanding and Learning referred to increasing awareness on the new knowledge and interpretation on new concepts. Reinforcing stood for strengthening solving problems by adding extra support. Confusion was coded for leading to no clarity to think or understand with clarity.

Table 39 Themes and Sub-categories for the Coding of Interview Comments on Worked Example

Understanding & Learning	**Reinforcing**	**Confusion**	**Suggestion for Effective Usage**
• similar examples leads to long term memory (3) • solutions steps helps understanding reasoning (4) • recalls background knowledge (builds on the top of prior knowledge) (1)	For: • practicing (9) • especially for harder topics (2) • the concepts for known topics. (1)	• understanding every solution steps led to confusion (3) • examples had more solutions steps than needed (1) • student had trouble to remember background knowledge (1) • solution techniques were limited. More variety needed.(1)	• students explains solution steps verbally.(1) • having a space for students' intended solution (1) • seeing all variety of solution techniques(1)

Numbers in paranthesis show student frequencies, N=9

By the first question, researcher aimed to gather opinions on the effectiveness of various worked examples towards understanding and interpretation on the derivative units. Eight students stated their positive effects towards *understanding and learning*. Three students mentioned about seeing similar examples led to long term memory (S_2, S_{14}, S_{27})
One of them (S_{27}) stated that:

> "I think it actually helped me because as you see how the problem is solved by looking at the solutions and its method; it actually gets into your long term memory so you can easily remember those."

Four students mentioned that seeing step by step solutions helped for understanding the reasoning.

One student (S_{17}) stated that:

> "I think seeing like how you solved the question or how someone solved the question is helpful because you get to see how someone solved it and then you can learn from that."

One student mentioned about effectiveness of worked examples towards understanding as it builds on the top of background knowledge (S_{16}).

Almost all students mentioned that worked examples were helpful for all the subjects and they stated that worked examples were *reinforcing* to learn the subjects. Two students stated that worked examples were more helpful for harder subjects and another student stated that those examples to be more useful for unit 3 where it required to interpret graphs of functions. S_{24} claimed that worked examples helped if the subject was known. This student said that s/he studied the units earlier and hence s/he recognized some of the subjects in the unit.
On the other hand there were also negative comments on effectiveness of worked examples. More specifically some students found worked examples *confusing*. Three students stated that understanding solution steps were confusing.(S_2, S_{24}, S_{16})

S_2 mentioned that:

> "I think they are not as effective as doing them on the board. Because sometimes when I am doing, going for the worked problems, I get confused and I do not know where to look back to. But the board may be more effective but other than that if it's followed by a question that we have to do, it kind of helps us know the steps."

Moreover another student (S_{24}) stated that that solution steps had more steps than needed:

> "For time, they helped me when I knew the subject well because I could understand what the steps were going to be. But in these cases, my mind did not maybe work as slow as the examples went. Sometimes I deduced the information quicker. So they confused me a little bit"

One student (S_{16}) stated that although worked examples were useful through combining background knowledge, they were confusing as that student had trouble to remember background knowledge. Another student mentioned that worked examples should show all alternative solution techniques as student's solution steps might differ than suggested method of solving problem.
She (S_{16}) stated that:

> "Like I do not understand math by reading, I understand it by doing; by solving questions on my own. I do not really understand. Because everyone has different techniques and you have on your own technique. Those papers want me to get your own technique but I always wanted to solve it with my technique so I ca not cope with that paper."

Students followed similar *methods while studying* on worked examples. All interviewing students read the solution steps of easy questions. Five of them tried to solve the problem before seeing the solution and then compared their solution (if there is) with the given steps. One student (S_2) first read the steps and then tried to solve the same problem for repeating the method.

There were different *suggestions* made by the students to improve the implementation of worked example studying in class. The suggestions are "*seeing all alternative solution methods; having a space elsewhere for students who intents for solving the problem* before seeing solution steps for a comparison later on*; student guided towards explaining the reasoning of each solution step verbally in their own words,* while reading the solution steps to facilitate thinking*"*.

All interviewing students thought on the gradual enhancement level of questions and the difficulty level of questions appropriateness.

Students' Opinions About Completion Examples

The interview results about the completion examples revealed four main themes namely, Guidance, Reinforcing, Challenging and Confusing. Table 40 shows the main themes and subcategories drawn from students' responses about completion examples used in the study. Guidance referred to leading, directing, or advising the proper method. Reinforcing was coded for strengthening solving problems by adding extra support. Challenging was used for to code when students had difficulty to address the demand for explanation or justification. Confusion was coded for leading to no clarity to think or understand with clarity.

Table 40 Themes and Sub-categories for the Coding of Interview Comments on Completion Examples

Guidance	**Reinforcing**	**Challenging**	**Confusion**	**Suggestion for Effective Usage**
• c.e.b.f was helpful for understanding the unit: - guidance was there (5) • c.e.b.f . was less confusing, (once started follow up was there) (1) • c.e.b.f. is easier to understand than c.e.f.f. (more guidance given) (1) • c.e.f.f was helpful for understanding the unit (3) • • c.e.f.f. was less helpful as it involved answers for checking (1) • c.e.f.f. was more comfortable as it answers to check (1)	• c.e.b.f. helped more for harder subjects (2) • c.e.f.f. encouraged thinking more. (3) • c.e.f.f. is harder : how to start makes you think more (2) • c.e.f.f. more helpful to learn the subject (2)	• c.e.f.f. was challenging at starting the solution step. (3) • c.e.f.f. was harder: expectation how to fill the blanks was subjective (3) • c.e.f.f. was more challenging: hard to see where the mistake is (1)	• expectation on what to write or how much work should be shown were not clear. (2) • sometimes mixing up mind if the topic was studied earlier. (1) • understanding the solution steps was hard.(2)	• more solution steps can be given to fill (1) • different methodologies of solutions can be given alternatively (1)

c.e.f.f.: completion example with forward fading; c.e.b.f: completion example backwards fading; Numbers in paranthesis show student frequencies ; N=9

Second and third questions were about the effectiveness of various completion examples with backwards and forward fading towards understanding and interpretation on derivatives units.

Students stated that completion examples were effective on *guidance*; however opinions had changed on which type of completion examples was more helpful and in which subjects examples they helped most. By *guidance* srudents referred to directing towards the solution steps of the problem and advising proper method for the solution.

Although eight students thought that any completion examples were helpful towards understanding through *guidance*, five students (S_2, S_{14}, S_7, S_{17}, S_{28}) found completion examples with backwards fading to be more effective in guiding compared to the completion examples with forward fading. Five students found completion examples with backwards fading helpful for different reasons such as completing the last solution steps with less errors as guidance was given; encouraging thinking last steps leading to learning proper method; and reinforcing the necessity to read first solution steps. Three students (S_{27}, S_{16}, S_{29}) found completion examples with forward fading to be more helpful in its guidance for having the answer for checking; and encouraging thinking on proper method to start with . Some of the comments about the guidance of completion examples with backwards fading made by students were as follows:

S_{14} stated that:

> "…I think those (*referring to completion examples with backwards fading*) were easier to do because you had already everything done for you; so there is no way that you make mistakes. You just follow one by one. You do the last bit."

S_{29} said that:

> "I think the one which we had the first steps (*referring to completion examples with backwards fading*) were more useful because the first steps were easier, taking the derivative and other stuff. Last steps were harder. So doing harder part, I think, is more useful than doing the easy part."

S_7 explained that:

> "When you gave the last steps, it was not helpful but when you gave the first steps it helped better…because when you gave the last steps, when I was done the first steps, I did not look at the last steps. But if you gave the first steps then I had to read the first steps and then do the last steps."

On the other hand, three students (S_{27}, S_{16}, S_{29}) mentioned that completion examples with forward fading were more useful in its reinforcement on guidance than completion examples with backwards fading.
S_{16} explained that:

> "Not giving the solution but the answer, it was way better. you just gave the answer so that we can just check".

One student (S_{29}) explained as follows:

> "When you start the question, you guide us through the question, so it is easier to do the question afterwards. But when you gave us the answer and we have to guide ourselves towards was harder and it was more for our benefit than the other… It is easy to do but the optimization questions were really hard and it was really good to have the answer and the last part of the solution. Working towards it was more helpful."

Six students stated that completion examples were *reinforcing* to grasp the concepts and applications of theorems. Two students (S_{24}, S_{16}) thought completion examples with forward fading were very helpful for interpretations in harder subjects; especially at units 3 and 4; where students were studying on applications of derivatives. Two students (S_{27}, S_{24}) emphasized those completion examples with forward fading encouraged thinking more.

S_{24} stated that:

> "…I felt like I was doing more of the work. I had to think of how the problem was supposed to go. In the other one *(referring to completion examples with backwards fading*), because the paper was leading you to the answer, you did not feel like you were learning and doing that much."

There were also comments about the *challenge* of completing the solution steps in examples. Three students (S_{14}, S_{17}, S_{27}) commented on completion examples with forward fading were challenging as you had to think more on how to start the problem and the beginning solution steps might differ than what was expected to be presented as solution method.

S_{14} mentioned that:

> "I think they (*referring to completion examples with forward fading*) are harder but they are more helpful for the test…because I guess you make small mistakes and you have to think. First you have step one, step two and three, you have to think it by yourself; instead of they are on the paper so it makes you think more."

Five students(S_2, S_{17}, S_{24}, S_{16}, S_{28}) thought completion examples to be *confusing* for several reasons: (1) expectation on what to write or how much work should be shown were not clear; (2) sometimes mixing up mind if the topic was studied earlier; (3) understanding the solution steps was hard.

Related with *confusion*, S_2 expressed her idea as follows:

> "Completion kind of confused me. Because there were some parts for example where I did not know what parts of the answers that I had to write. So with completions, sometimes it confuse what I knew."

S_{17} declared that:

> "I think it's challenging to complete because sometimes the completion exercise is different than how you would have solved the questions. Sometimes it was hard to understand but when you get used to it, you just fill in the blanks."

There were similar *methods while studying* on completion examples. Eight of interviewing students showed effort to complete solution steps on their own. One student (S_{16}) preferred to complete the steps on her own if the topic was easy to understand; otherwise she asked to a friend to explain verbally how to fill the blanks, when the topic was a harder subject to interpret.

There were two *suggestions* made for the improvement of implementation of completion examples: "*more solution steps can be given to fill;different methodologies of solving the problems can be alternatively"* given so that student can fill in the blanks of the method that she has in her head.

Students' Opinions About Practice Problems

The interview results about the practice problems revealed five main themes namely, Reinforcing; Understanding and Learning; Mental Model; Learning; and Time Taking Table 41 shows the main themes and subcategories drawn from students' responses about practice problems used in the study.

Reinforcing was coded for strengthening solving problems by adding extra support. Understanding and Learning referred to increasing awareness on the new knowledge and interpretation on new concepts. Mental Model was used for to describe students' thinking procedure about how the solution steps of problem form. Learning was used to code for the development of gaining knowledge or dexterity. Time taking was used to code spending long time to complete.

Table 41 Coding Schema for Interview Comments on Practice Problems

Reinforcing	**Understanding & Learning**	**Mental Model**	**Learning**	**Time Taking**	**Suggestions for Effective Usage**
• tested knowledge (2) • asked main points to emphasize learning the subject(2)	• developed critical thinking skills with variety (3) • challenge made to analyze more (4) • facilitated revision with similar problems (4) • no guidance facilitated thinking deeply (2) • practice helped for harder topics (2) • assisted permanent learning (1) • lead to effective interpretation of reasoning steps(3)	• prepared for the test style questions by encouraging forming schema to solve problems (1)	• reduced calculation mistakes by practicing with similar problems. (1) • harder questions helped to realize the expected level of knowledge (2) • duration helped to pace up (1) • Students get prepared for the test (9)	• when there are harder problems (4) • prior knowledge is combined (1)	• allocate more time (1) • allocate answers at the back of the worksheet (1)

Numbers in paranthesis show student frequencie, N=9

Majority of the interviewing students commented on the positive effects of practice problems. To start with, two students mentioned about that practice problems were *reinforcing* as it tested knowledge and asked main points to emphasize learning the subject.

One of the students (S_2) stated that:

> "I think in part 3-since we had to do everything on our own- it just tests our knowledge which is good. Because we need to know what we did wrong what we did not. Because usually, we get to part 3 we know everything except that couple of key points that we forgot. It just asks the key points, the details that we need to remember. I think we learn in part 3."

Most of the comments made about the practice problems were on its effects on *understanding and learning.* All interviewing students taught that practice problems were effective towards understanding the subject. Three students expressed that practice problems developed critical thinking skills with the variety of hard problems.

A student (S_{24}) expressed this point with the following statement:

> "I think they are good as well because they had some questions that I had not seen before in textbook. So they helped me to think the subject in a different aspect. I think I learnt from Part 3s"

Four students (S_2, S_{17}, S_{27}, S_{29}) thought that challenge and no guidance made to think and analyze that assisted for learning.

One of them(S_{17}) commented that:

> "I think that was the best part of the worksheet because you gave hard questions on those on those test. We, I know everyone struggled at one particular part of that worksheet. And when we learnt how to solve those, it's really helpful."

Another one (S_2) mentioned that:

> "I think they are hard but you can do them if you actually think about them. It takes time but you can actually understand them when you get into the problem"

The other two students (S_{27}, S_{29} respectively) said that no guidance facilitated thinking deeply. They stated that:

> "Since it does not have any solutions in it, it forces me. I am doing them more carefully because since you did not give the answers, I am trying to solve them as much as possible."

> "Part 3 of course was more useful, since we did not have anything to start with. We just had the question and we need to work everything out of itself."

Moreover, four students mentioned that practice problems aided revision with similar problems and they were supportive to understand and learning the topic through revision.

One of them(S_{16}) said that:

> "…They were good and it was good that you kind of asked the same type of questions so we practiced by doing the same thing over and over again. At the end of it I kind of solved it. "

Another one (S_{28}) added that:

> "I benefitted from them. It makes me repeat. I read the topics. when I re do it, I think I have a better understanding."

A student (S_{27}) who made similar explanations as above, made an additional comment on that practice problems assisted permanent learning through practice. Student said that:

> "They really helped actually because when you solve it by yourself and when you see your mistake without the solutions, then it makes the question permanent."

Next, two students reported that practicing with problems helped them to understand challenging topics. One of them stated that:

> "well in part 3, I really improvement my derivatives: applications, finding stationary points, inflection points, maximum ,minimum, curve properties. I really had problems with them. I really do not know why. Probably I was confusing if the derivative is equal to zero then you find stationary point, if the second derivative was zero then you find inflection point . like I really had confusions about that topic. But as I solved in part 3 as I solved by myself, it actually improved. I can just remember it now. I do not confuse them now."

Finally, three students (S_2, S_{27}, S_{29}) mentioned that practice lead to effective interpretation of reasoning solution steps. One of them (S_2) explained that:

"… because it(*referring to part 3*) helps you to understand why you apply that step and why is the solution step has been chosen."

Another one of them(S_{27}) expressed that practice problems made him realized that he had to be careful in every step of solution as to eliminate follow through error.

One major theme stated by a student (S_{14}) was about *mental model*. She thought that practice problems prepared to test style questions by encouraging forming schema on how to solve problems.

Student said that:

"I mean not memorizing, I go through and then I say OK I have to do step one, step two, step three for this question. I mean I memorize the methods but not the solutions I guess."

Another theme that was deducted from the comment of students were practice problems effects on *Learning.* At first, one student (S_{27}) stated that practice problems reduced calculation mistakes by practicing with similar problems.

He mentioned that:

"…English teacher says, practice makes permanent, you do this kind of stuff without solutions, then you can see your mistake in the question. You just not repeat the mistake like there is a low chance of repeating the mistake so it *(referring to practice problems)* really helps to get the question"

Secondly, another two students (S_{17}, S_{24}) claimed that harder questions made them realize the expected level of knowledge in tests and in the curriculum.

One of them(S_{24}) said that:

"In the last part 3, they were IB style questions I think. So they helped me to understand what IB was looking for."

The other two students (S_{17}, S_{14}) mentioned:

"…I think that (*practice problems*) was the best part of the worksheet because you gave hard questions on those like on those test."

"…But they are more effective when it comes to quizzes and teats because its similar IB questions and so if you know how to do them, you can basically do the quiz."

Next, almost all interviewing students claimed that practice problems facilitated to get prepared for the test.

One student (S_{29}) stated that:

> "…they were just like tests and I like solving them. They are harder than all the other questions that we had done. If there are more test like questions then we can get ready for the test and we just solve the questions."

Another one (S_{17}) mentioned that:

> "…And when we learnt how to solve those, it's really helpful for the test because you ask similar not the same but basically like the same type of question so people benefited a lot from those."

She also added that she learnt better throughout challenge problems in practice problem worksheets and the success in tests came automatically.

Four students commented on that practice problems were *time taking* as they had a variety of harder problems and required to recall prior knowledge. One of them said that:

> "…but they are very long questions. They take a lot of time"

Another student (S_{14}) declared that when the question was combined with background knowledge like the unit of optimization combining geometry that she had difficulty to recall prior knowledge, hence it took her time to recall and then solve the problems built on the top of the prior knowledge.

There were several *methods* mentioned *while studying* on practice problems. Students preferred to look at notes, asking to friends, outlining solution steps for similar problems and studying on units before solving problems.

All students stated that the level of the difficulty was appropriate as well as gradual improvement of the questions. Students also made some suggestions to improve the efficiency of practice problems. One of the students suggested "*having the answers at the back of the worksheet".*

She (S_{14}) stated that:

> "…like I said if you put the answers on the back; but if you grade them as quiz grades, that would not work. So if the answers were there because we can check them on our own and if we did something wrong we can go back and then we could save time."

Another suggestion made by a student was to "*allocate more time*" to complete the worksheet (S_{28}).

As an additional comment, four students (S_2, S_{17}, S_{14}, S_{24}) compared practice problems with the previous learning activities. They thought that practice problems were the best part towards effective learning and preparation for test than the worked examples and completion examples.

Students' Opinions About Self-Explanations

The interview results about the self-explanations revealed five main themes namely, Thinking deeply; Mental Model; Learning; Challenging; and Not Helpful. Table 42 shows the main themes and subcategories drawn from students' responses about self explanations used in the study. Thinking deeply was coded for increasing awareness of proper expressions and organization. Mental Model was used for to describe students' thinking procedure about how the solution steps of problem form. Learning was used to code for the development of gaining knowledge or dexterity. Challenging was used for to code when students had difficulty to address the demand for explanation or justification. Not helpful stood for to explain cases when it does not assist understanding.

Table 42 Coding Schema for Interview Comments on Self Explanations

Thinking Deeply	Mental Model	Learning	Challenging	Not Helpful	Suggestion for Effective Usage
• makes think deeply (the proper expressions/examples to explain and the organization) (4) • prevents memorization(1) • assists grasping reasoning of solution steps (2)	• encourages forming schema for the solutions of problems (2) • assists summing up what is learnt (1) • makes you notice solution steps (5) • bringing together in different subjects(1)	• self-teaching helped learning (1) • analyzing cases in details helped learning (2) • terminology improvement(1)	• harder as it involves proper expressions to transmit knowledge (1) • when requires background knowledge(2) • when a new question style is asked (1)	• for understanding if it's done immediately after taught topic (1)	• reflective journals assigned as homework(1) • stating outline of expectations in reflective journals (1) • problems rather than summaries in reflective journals(1)

Numbers in paranthesis show student frequencies, N=9

Students generally mentioned that reflective journals had positive effects towards understanding and learning. All students thought that reflective journals were good practice for understanding. Four students emphasized that writing reflective journals made them *think deeply* about the proper expressions, terminologies and organization while transmitting knowledge (S_2, S_{17}, S_{24}, S_{29}).

One student stated that reflective journals prevented memorization (S_{17}):

> "reflective journals were definitely like helpful because you always wanted us to explain so people who just memorized the solution had a hard time to do those. Because you should not memorize and you should learn and that's why reflective journals were like helping people understand what they were doing instead of memorizing the problem what they had to do to solve the problem"

Another student mentioned that reflective journal writing facilitated thinking deeply on the details of procedures that should be used while solving the problem. Hence it facilitates understanding

S_{24} stated that:

> "In the reflective journals, we had to explain how we got the answer and how we were solving the question. So it kind of gave us the chance to think about how we were doing the question. Because usually, when you are doing the question, you are doing it quickly and do not think about the procedure used. But when you are trying to explain to someone else in a reflective journal, you understand how in kind of works in detail."

One student (S_{29}) mentioned about that he had problems on finding the reasoning of problem solving steps; however reflective journals assisted on grasping reasoning of solutions steps with its reinforcing nature of thinking requirement.

Eight students said that writing reflective journals facilitated creating a *mental model and integration with prior knowledge.* Two of the students stated that reflective journals made them notice that derivative problems had to follow a step by step solution in ordered manner so as to reach to the answer. Therefore, writing reflective journals helped to form a flow schema (S_{16}):

> "I figured out that I had to do step by step when we are doing in reflective journal. My answers were always correct but when I did a question my answer was not correct. So I just learnt that, in Calculus I have to solve step by step; and really organized . So reflective journals really taught me that … I made my own kind of flow. Like first do this ,

> second do that, third do that so reflective journal really made the awareness."

Five students stated that writing reflective journals increased awareness of solution steps (S_2, S_{29}, S_{27}, S_{16}, S_{24}).One student mentioned that reflective journals also assisted summing up the main ideas on what has been taught (S_{24}). Another student highlighted that writing reflective journals encouraged to recall prior mathematics topics or different subjects to combine with new information; hence it facilitated to make transfer knowledge into a new situation (S_{14}):

> "…it helps not only with terminology but having an overall understanding .Because most of the reflective journals interpreted a lot of different aspects into one. So I think it helped bringing things together instead of separate subjects."

Three students also found reflective journal writing to be useful towards *learning* the subject. One of the students (S_{17}) said that writing reflective journals in general reinforced to learn the details of subject effectively through its self-teaching nature.

One statement is that:

> "once you gave a reflective journal about the stationary points where you had to explain point of inflection and maximum-minimum. When we just started that topic, I had to open the textbook and I had to look at it. After that reflective journal I did not have to look at the textbook or the notebook again because I learnt."

Two students stated similar comments for the same naturalistic reason. They both thought that analyzing solution steps improved learning of the topic: (S_{29}, S_{28} respectively):

S_{29} said that:

> "writing reflective journals guided me and I taught to myself how I did everything step by step and by trying to tell other people. I think they are really beneficial for us."

S_{28} declared that:

> "writing the steps one by one and you are explaining them. So it is not just memorization but both teaching and learning. Teaching it to yourself and learning it by yourself."

One student clearly mentioned about that reflective journals also helped to learn the terminology (S_{14}):

> "…terminology! Because we use the terminology as well. When in the test a certain word comes up, I know I was like this is the word and I knew what to do."

There were also few negative comments on the effects of reflective journals towards understanding and learning. One student (S_{27}) stated about the *challenge* in reflective journals by saying that writing reflective journals was challenging especially when the subject is hard because even if the concepts are known, it was hard to transmit the knowledge with proper expressions. In other words finding proper terminologies and sentences to explain reasoning of solution steps were challenging. Another student mentioned that (S_{14}) writing reflective journals were *challenging* when it requires to integrate prior knowledge to the new subject, as it required more effort to remember the prerequisite topics through revision.

Another student (S_{24}) mentioned the challenge of writing reflective journals when there in an integration of prior knowledge especially when there was a new question style:

> "…That was the one that I had the most trouble with, not because it was hard but it was not what we were used to. It was a new question type."

One of the students(S_7) stated that writing reflective journals was not effective towards understanding especially when you needed to write immediately after the topic was taught; more time was needed to grasp the concepts and it was challenging to concentrate on writing in class environment.
She stated that:

> "…I do not know the effectiveness. I did not find them really effective. Because I usually got help while I am doing it. I really can not. When I am in class, I cannot really work on them. I need to go home and understand it better when I look at it by myself."

Students usually had similar *methods* while writing reflective journals. Six students checked notes, textbook when they had trouble with remembering proper terminologies, sentences and expressions. One of the students (S_{16}) preferred to get verbal help from her friend as she needed someone to explain the subject orally in order to understand the concepts. Two students preferred to write reflective journals from their minds as they were able to remember the topic.

There were also several suggestions made to improve the effectiveness of reflective journals towards understanding and learning. One student (S_7) preferred *reflective journals to be assigned as homework* so then they could be used to practice for the future assessments:

Student stated that:

> "If it was given as a homework assignment at the end of the unit, you know , to be as a revision before the test. If get reflective journal entries all in a stack or something as a revision, so that we can study that way for the exam, I think that would have been better."

Another student(S_{16}) suggested to *give expectations in reflective journals to be given in an outline format;* so that she would be aware of all requirements:

> "…because that not everyone understands the topics while they are doing reflective journals so maybe write the question and then bullet point. For instance one- find derivative, two-make a sign diagram, three-bla bla. That kind of easy kind of summary of steps that we have to do would be way easier to do it. That will actually help."

One of the students (S_2) preferred *mathematics problems rather than summaries in reflective journals* and she added that they involved too much verbal explanations. She added that if reflective journals involved more numeric calculations.

She said that:

> "…Because the summarization ones are too verbal I think. But mathematical ones, first we did it and then we wrote it so it was kind of like double doing it"

Students' Opinions About Online Group Discussions:

The interview results about the online group discussions revealed seven main themes namely, Motivation; Deep Thinking; Reflection Others' Thinking; Understanding; Competence; Learning Contribution and Not User-friendly. Table 43 shows the main themes and subcategories drawn from students' responses about online group discussions used in the study.

Motivation was coded for reinforcing elements that makes you enjoy the learnt concepts and skills. Deep Thinking was coded for increasing awareness of proper expressions and organization. Reflection Others' Thinking used for benefitting from other students' thinking styles. Understanding was coded for interpretation of concepts effectively. Competence was used when the student mentioned the well qualified skills for solving problems. No learning contribution used for when student stated that there was any meaningful influence towards learning the subject. Not user friendly stood for causing challenge to express opinions or mathematical ideas easily with its practical elements.

Table 43 Coding Schema for Interview Comments on Group Discussions

Motivation	Thinking Deeply	Reflection Others' Thinking	Understanding	Competence	No Learning Contribution	Not User-Friendly	Suggestion for Effective Usage
• made aware of different concepts (1) • linked to other subjects (3)	• seeing variety of questions extended thinking skills in different perspectives (3) • made to think more (3) • replies did not contribute much to deep thinking (4) • replies mixed up mind (2)	• reading friends' replies added knowledge(1)	• theories helped to interpret concepts (1)	• facilitated practice with further problems(1)	• prefers own solution (1) • theories did not add much to problem solving (1) • research problems not as helpful practice problems(1) • Can be learnt without forums(1)	• using computer to type equations is hard (1) • Words restrict discussion. Diagrams needed. Hard to plot diagrams on computer.so diagrams drawn by hand would help more to discuss. (1)	• more discussion provoking questions needed(1) • have discussions in class rather than online environment(1) • not to have online forums (1)

Numbers in paranthesis show student frequencies, N=9

Points of views regarding to the effects of online forum discussions towards understanding and interpretation of topics varied due to students expectations. Some students commented on positive effects of online forum discussions towards *motivation*. Three students found participating in forums increased motivation, since it enhanced awareness of different concepts and it made connection to different field areas. One student mentioned that some questions increased awareness of some facts about functions. She mentioned that (S_2):
She mentioned that (S_2):

> "I think there were interesting questions like there was one about "e" over "x"…I had to think a lot about that. I think those are good."

Other two students enjoyed combining derivatives with economics in the given profit problem at forums (S_{14}, S_{17}).
One of them (S_{17}) stated that:

> "…I like the Economy obviously but also because it was different. I had not seen the cost graph and the revenue graph so when I looked at it I had to think about it and I leant something from it."

Students had different opinions about forums effects on *deep thinking*. Three of the students agreed on seeing variety of questions facilitated thinking critically (S_{14}, S_{17}, S_{27}).

One of them expressed this situation as (S_{27}):

> "It really helped because when you give different questions, the student understands the basic of it because as you solve more problems in different varieties, you improve yourself. Because you get the basic of it, so in future when you see a question that you have not seen, you can just understand how to solve it very easily"

Three students approved that forums made them think more (S_2, S_{27}, S_{29}).One of the students said that problems made them think more compared to research questions.

He stated that (S_{29}):

> "…for the research questions, we just researched and looked through the internet. Just got the knowledge and wrote it down on the forum; but when we had to solve questions before posting it on the forum, that made us to think more."

Some students brought about that forums did not contribute much towards *deep thinking* or they mixed up mind when the replies of other group members' were read. Two students (S_{14}, S_7) mentioned that friends' replies mixed up mind at some cases.

One of them (S_7) declared that:

> "I looked at my friends' replies. I think those were even more confusing. I think the forums had confused me."

Four students (S_2, S_{17}, S_{16}, S_{28}) stated that friends' replies did not contribute much towards *deep thinking*. Some of them preferred to solve in their own way, some of them did not understand friends' replies to benefit from and some of them could not benefit from replies as there was not enough discussion environment.

One of them (S_{16}) said that:

> "I read my friends'. They had their own way so I could not really figured it out how they solved it so I just tried it on my own. "

One of the students mentioned about *the reflection on others' thinking*: (S_{24}) .She stated that:

> "In the forums, I read my friends' answers so that added new stuff. They helped me to find something that they have not already said. So it was good."

According to the same student, forums also assisted *understanding* of the subjects as the theories mentioned in the forums helped to interpret the concepts (S_{24}). She agreed on forums helping her to realize new methods of solutions or new perspectives on the top of what is stated.

She also said about the forum questions that:

> "…Some of them were theoretical, I think. Those helped me to understand what the subject was about".

Another student (S_{27}) talked about the contributions of forums towards practice, hence it facilitated *competence.* He said that:

> "It really helped because when you give different questions, the student understands the basic of it because as you solve more problems in different varieties, you improve yourself. Because you get the basic of it, so in future when you see a question that you have not seen, you can just understand how to solve it very easily. So, it's just practicing."

Three students (S_2, S_{17} and S_{16}) talked about that forums did not much *contribute towards learning.*

One of them stated that:

> ”I read my friends’. They had their own way so I could not really figured it out how they solved it so I just tried it on my own… I do not like other people’s method. Because I feel better, if I have my own method of solving it.”

One of them (S_{24}) mentioned about that visiting theories in forums did not add much to her problem solving skills. Another student explained that research question styles are not as helpful as practice problems towards problem solving (S_{29}).
He stated that:

> “Well the research forums, I do not want to say unnecessary. The other questions like the last graph question, they were more helpful…For the research questions, we just researched and looked through the internet. Just got the knowledge and wrote it down on the forum; but when we had to solve questions before posting it on the forum, that made us to think more.”

Only one could not participate at forum discussions he had problems with planning his time. He contributed 3 forums out of 8. That student mentioned that forums are neither effective nor necessary, hence he can learn without forums.

Two students (S_7 and S_{16}) commented about that forums at computers are *not user friendly.* One of them complained about the difficulty of typing equations at computer and she added that (S_{16}):

> ” I am against forums because you can give question homework for us or one question maybe as homework, we can solve it with our hands and stuff like that . ..Then, it could be easier. First of all, I hate typing math up…, it is really hard to explain math online, typing stuff...etc.”

The second student (S_7) talked about that she does not like technology in classes. Also she claimed that words restrict to explain a mathematical concept and she emphasized on the necessity of diagrams; however she added that plotting diagrams on computer is challenging.

She stated that:

> “…I do not like doing math with technology. I do not like using technology overall for my lessons. That is why I do not prefer to do it. I just do not like typing anything that has to do with math so I want to be able to draw diagrams and do all of those things. But words limits me…”

There were several *method used to follow* for the replies of forum questions. Four students gave their own answers and then did not check the . Two of them compared their answers after they replied. Two students checked friends' replies when they did not understand the question clearly and then gave their own answer.

Some *suggestions* were made to improve the forums. One of the students (S_2) mentioned that there should have been "*more discussion to take place"* for better improvement in problem solving and she also suggested more questions to provoke discussions. She recommended to motivate student with more open ended and/or interpretation questions that might motivate students to discuss more on the topic.

She said that:

> "I usually did it and then I waited for the replies and I read the replies so it was not a real discussion. Maybe the questions could be more discussion provoking…I do not know how it's going to motivate people but to motivate the discussion like more open-ended questions or more interpretation questions might be more like that. Also like the last one we also had to watch a video, that also kind of take me into the question. More questions like that. That could also motivate more."

Another student (S_{16}) suggested to "*have discussions in class rather than online environment".*

She mentioned that:

> "… I would prefer if you just gave small pink papers with the questions rather than putting it on forums, just give it to me. I can solve them and then bring back to you…If they were next to me , it will be way easier than just reading it . ıt is the same thing in papers too. Like if I had discussed it in classroom, that would be way better than trying to understand what they wrote the forum. "

One student (S_{28}) claimed "*not to have online forums in mathematic classes*"and he talked about the difficulties that he faced with to access internet when he needed to. Lastly, one student (S_7) recommended facilitating online experiments rather than online forums. She suggested online experiments instead of online forums if mathematics will be linked to technology.

She said that:

> "…When you get to experiment stuff. For math, when you had the fun things to do . when you need to stretch the corner of a triangle and test it; or when you put everything in a weighting machine and see how it affects the rest, that way that I can relate it to more to real life, then I will enjoy it more."

4.4.1 Additional Comments

Some students made additional comments on the comparison of learning activities. Four students thought that worked examples were more helpful towards learning in harder topics, like curve properties (Unit 3) and applications of derivatives (unit 4).

A student (S_{24}) mentioned that:

> "I think they helped me at tangents and normals because there are specific steps for those questions that you have to find the derivative, try to find the tangent line equation by substituting information. I could see and understand what the steps were…I was getting confused by some of the sign tables before but now I can guess after doing the examples. I got more practical and I can see that."

Another student (S_{17}) mentioned that she benefitted more from worked examples in visual problems where they were analyzing the relationship between the graphs of the functions, derivative function and second derivative function.

One of the students (S_7) thought that worked examples, completion examples and practice problems were more effective than reflective journals and forums.

She stated that:

> "I think they were very effective. For the Calculus unit, I only studied from the parts so I like all of the parts. Parts were more effective than reflective journals and forums."

Same student thought that completion example worksheets were the best learning activity compared to the rest. She added that:

> "Yes, that was the best part . I think part 2 (*referring to completion examples*) was the best out of all the other parts. Because I did not get completely confused because there was some clues, but still I had a chance to do it by myself because there were some blanks. So I think that was the best part."

One student (S_{16}) mentioned about that if she had known which steps to follow while answering a mathematical problem in reflective journal in harder units, she would have felt more comfortable to complete them.

4.4.2 Summary of Student Interviews

Table 44 shows the common codes that have appeared more than once while coding students' comments on learning activities. Reinforcing was the theme that had the highest frequency derived from students replies. The rest of the codes repeated twice among the replies to the five major interview questions.

Table 44 Interview Codes Repeating More Than Once

Code	Reinforcing	Understanding and Learning	Challenging	Confusion	Thinking Deeply	Mental Model	Learning
Frequency	3	2	2	2	2	2	2

In general it can be understood that students thought that they benefitted from the learning activities even though it varied which activity helped them most. For those students who learnt the topic for the first time, they found worked examples to be assistance towards learning and interpreting the subject. When it comes to the completion examples, opinions varied about which type of fading (backwards or forward) was more helpful than the other type; however the common idea was that completion examples were guiding towards thinking and understanding. Students generally thought that forward fading to be more challenging than the other fading style, as they had difficulties to start the solution of the problem. Some also had difficulties to understand the expectations on how to fill in blanks. Some suggestions on including all different methods of solution to the same problem were made on the improvement of worked examples and completion examples.

The students who actively thought and engaged in writing reflective journals believed that reflective journals assisted towards learning and understanding the subject. Some students had more challenge to find proper techniques, appropriate expressions or terminologies to be used in reflective journals; but they also agreed on the positive effects of writing reflective journals towards understanding and interpretation of the units.

Next, students usually had relatively negative comments on the process of online group discussions at Moodle forums. They agreed on variety of questions at forums facilitated thinking and awareness of the subject; however since not every student participated fully or on time, discussion environment was not so effective to run smoothly for better improvement on extending frames in thinking. Moreover, some students had faced with technical challenges to express diagrams or mathematical notations and they added that they felt more comfortable to explain on paper through writing but not typing.

Finally, most of the students decidedly thought that practice problems assisted them towards thinking deep and understanding. Hence they prepared them for the achievement tests through addressing possible students' outcomes.

4.5 Summary of the Findings

The summaries the findings of this study are given below under each research question. This section provides a brief information of the results chapter.Table 45 represents mean score and standard deviation of achievement tests about near transfer of learning. The gained scores of students' two post achievement tests (*M*: 81; *S.D.*:9) indicated there was an effective near transfer of learning. All students achieved above 56% of gained scores in near transfer.

Table 45 Descriptive Statistics for Achievement Test Scores on Near Transfer

	Pretest Score	Posttest Score	Gained Score
Mean	**7**	**88**	**81**
S.D.	**4**	**11**	**9**

Table 46 represents mean score and standard deviation of achievement tests about far transfer of learning. The gained scores of students' two post achievement tests (M: 56; S.D.:19) indicated far transfer of learning.also realized with lower gained scores All students achieved above 25 of gained scores in far transfer. The gained scores were not as high as near transfer gained scores'.

Table 46 Descriptive Statistics for Achievement Test Scores on Far Transfer

	Pretest Score	Posttest Score	Gained Score
Mean	**5**	**61**	**56**
S.D.	**4**	**21**	**19**

Research Question 1 Is there a significant mean difference between pre and post achievement test scores of students who were exposed to due to learning activities in mathematics lesson designed based on cognitive load theory principles ?

Table 47 represents paired sample t test results of achievement tests about the mean differences of pre and post test results.

Table 47 Paired Sample T-test Results

Paired Samples Test

	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
Total_Pretests Total_Posttests	-68.567	12.514	2.285	-73.240	-63.894	-30.011	29	.000

Research Question 2. What are the indications of near and far transfer of learning in students' self-explanations and group discussions?

Students' self reflections and group discussion results revealed near transfer of learning at Knowledge, Comprehension and Application Themes. The average of percentages of each level differed at self reflections and group discussions. Self-reflections and forums indicated near transfer of learning unders different subthemes. However, self reflections had more indicators of near transfer at some subthemes.

Far transfer indicators were observed at Analysis and Synthesis Themes at self reflections and group discussions. Self-reflections and forums indicated far transfer of learning under different subthemes. However, far transgfer indicators had less percentage averages than near transfer indicators' at self reflections and group discussions Furthermore, the averages of far transfer indicators at self reflections were lower than far transfer indicators of. group discussions; there were more far transfer indicators spread under different subthemes at group discussions.

Research Question 3 What are students' opinions about the contributions of worked examaples, diverse worked examples, completion examples, self-explanation, group discussions and practice problems towards learning mathematics and deep understanding?

The themes at Table 48 are the most frequently appearing themes about students' opinions about the effects and contribution of learning activities for learning and understanding

Table 48 Themes of Students Interviews

Worked Examples	**Completion Examples**	**Practice Problems**	**Self Reflections**	**Group Discussions**
• Understanding and Learning • Reinforcing • Confusion	• Guidance • Reinforcing • Challenging • Confusion	• Reinforcing • Understanding & Learning • Mental Model • Learning • Time Taking	• Thinking deeply • Mental Model • Learning • Challenging • Not helpful	• Motivation • Thinking deeply • Reflection others' thinking • Understanding • Competence • Not learning contribution • Not user-friendly

CHAPTER 5

DISCUSSION, CONCLUSION AND IMPLICATIONS

This chapter offers a discussion of findings and conclusion along with the implications of findings for theory, research and practice. The chapter aims to help for understanding and interpretation of the meanings of these findings and it provides recommendations for further research.

This chapter starts by a discussion and conclusion for near transfer of learning and it continues with the discussion and conclusion for far transfer of learning. Next, it provides recommendations by students for the improvement in implementation of worked examples, diversed worked examples, completion examples, practice problems, self reflections and group discussions. The chapter ends with implications and suggestions for practice and further research.

5.1 Near Transfer of Learning

In this section, the results of pre and post achievement tests due to near transfer are discussed along with the outcomes of self-reflection and group discussions. Finally, these results are discussed together with students' comments about the effects of implementation of worked examples, diversed worked examples, completion examples, practice problems, self reflections and group discussions towards near transfer of learning.

Previous studies showed that when the principles of CLT on reducing extraneous mental effort and increasing germane load implemented properly, they would increase students‘performance. (Sweller et al., 1998; Kalyuga et al., 2003; Tindall-Ford, Kalyuga, Chandler & Sweller, 1997). Previous studies focused on proper use of worked examples, completion examples, practice problems for facilitating transfer of learning while reducing extraneous mental load;whereas proper use of self-explanations along with worked examples and diverse worked examples leading to increase germane load.

Students in the current study had pre achievement test and post achievement tests before and after the implementation of learning activities respectively. When students' gained scores from near transfer problems in achievement tests were examined, it was seen that the means of gained scores from each achievement test were 80 and 82 respectively. These high means of gained scores indicated that students' scores were quite satisfying to conclude that there has been success on near transfer of learning. Furthermore, the standard deviation (17) of the gained scores from the first achievement test points out that there was a considerable difference in the spread of scores in near transfer problems. Therefore, there were some students who did not equally succeed in answering near transfer problems in the first test. On the other hand, the standard deviation (9) was relatively lower in the second test, that illustrated that students problem solving skills in near transfer were closely balanced in the second test. This less spread of data in the second achievement test may indicate that students' problem solving skills on near transfer problems showed an improvement in performance. The overall results in gained scores of near transfer indicates that there is a high evidence that near transfer of learning took place successfully with its overall mean, 81 and standard deviation, 9.

Along with the results of descriptive data from pre and post achievement tests, when a paired t- test was realized for pre and post achievement test 1, the result indicated that there was a significant mean difference between pre and post achievement near transfer test scores The paired sample t test result for pre and post achievement test 2 also showed a significant mean difference between pre and post test scores on near transfer. Additionally, the paired sample t-test of total pre and post achievement tests on near transfer showed that there was a significant mean difference between pretest near transfer scores and posttest near transfer scores. These results may suggest that learning activities in mathematics lesson designed based on the structures of cognitive load theory resulted with near transfer of learning. The statistics on achievement tests are parallel to the results of pre-mentioned studies where reducing extraneous mental effort and increasing germane load led to increasing student performance on near transfer problems in achievement tests. Additionally, Cooper and Sweller (1987) found that subjects trained by heavy emphasis on worked examples were better able to solve similar and transfer problems than those who were trained with conventional problems in algebra classes. They added that increasing the use of worked examples increased transfer performance in algebra. There might have been the similar effect of worked examples in increasing near transfer performance in differentiation.

Therefore, this achievement level in near transfer scores in achievement tests might have resulted due to designing an instruction based on CLT principles. However, since there was not a control group and an experiment group, the researcher is not sure whether these indicators were unique to achievement tests or not; and whether they directly came from the process of learning activities implementation or not. That is the reason for further analysis was made to examine near transfer indicators at self-reflections and group discussions to realize an overall picture.

Clark et al. (2006) suggested self-explanation to process the examples deeply based on the studies of Chi and her colleagues (1989). Eflides et al. (2006) declared that self-explanation has been found critical for successful learning with worked-out examples. Additionally, Renkl et al. (2004) added that self-explanations with worked examples would lead to increasing germane load. Therefore, integrating method of using self-explanations in the course was one of the operative methods that would lead to effective transfer of learning as this current study aimed and students wrote reflective journals along with worked examples and completion examples. The findings about near transfer of learning derived through reflective journals were parallel to the pre-mentioned literature outcomes.

When students' reflective journals were analyzed, it was observed that there were successful indicators of near transfer of learning within students' replies. The subtheme indicators of near transfer of learning, namely Knowledge, Comprehension and Application Themes had generally high frequencies in eight reflective journals. Almost all eight reflective journals had high percentages at Knowledge Theme at the use of correct terminologies (mean: 83.3%) and knowledge of general rules in Calculus (mean: 79.6%). Student percentages of Knowledge Theme varied between 10.7 % and 100%. This was not an unexpected result where students had to know the introduced principles and to be familiar with the proper use of terminology and notation in order to answer reflective journal question with appropriate expressions. Even if this was the case, if the student had made minor mistakes on mathematical notation or used improper terminology, this code was not awarded. It was determined at Knowledge Theme that students had more tendency to make minor mistakes on notation, rather than to use incorrect terminology.

At Comprehension Theme, most of the indicators were observed at understanding the problem (mean: 86.6%) and at identifies correct method (mean: 93.0%). These two subthemes were closely related as identifying correct method was followed after understanding the problem. On the other hand, researcher met with *giving a similar example* subtheme at all students' replies for reflective journal 2 and reflective journal 6; however there was not any stated example at reflective journal 3, reflective journal 4 and reflective journal 7. Also, reflective journal 6 might have been challenging students to create their own example to explain the unit; hence students had to give a very similar example like in learning activities.

Correct mathematical calculation was the subtheme that had highest percentage level under Application Theme (mean: 91.8%). The lowest standard deviation was at identifying correct method subtheme under Comprehension Theme at reflective journals with 6.4% ; whereas highest standard deviation was recognized again under Comprehension Theme at giving a similar example case at reflective journals with standard deviation 49.8%. These results indicate that students have similar thinking skills at near transfer of learning while identifying the correct method and when they find the correct method, there is more tendency to apply correct calculations in reflective journal problems. On the other hand, some students prefer to *give similar examples* depending on the question type of reflective journals. Again *giving similar example* subtheme might have this high spread due to the requirement of the reflective journal question.

It can be inferred that the findings of this study were in agreement with the previous studies declaring that self-explanations were assistance towards transfer of learning, when they were used along with worked and completion examples. Results pointed out that self-explanations lead to transfer of learning when they were used together with worked and completion examples. Therefore, students' replies to reflective journals indicated near transfer of learning through students' solutions and explanations generated while working on reflective journal questions.

Some of the near transfer indicators at self-explanations might be linked to several themes derived from students' interviews. For example when students commented on the effectiveness of self-explanations, eight students mentioned that self-explanation at reflective journals leading to *think deeply* on how to use proper expressions; how to reason solution steps and how to interpret for long term memory. Furthermore, nine interviewing students (all) stated that self-reflections assisted towards *Mental Model* for solution steps of a problem; summarizing the main idea and bringing together different subjects. Both themes are in agreement with the literature including the studies of Clark and his colleagues (2006) expressing that *self-explanation* as "a mental dialog that learners have when studying a worked example that helps them to understand the example and build a schema from it" (p.226). Students also made comments on self-reflections contribution to *Learning* on problem solving. This might be linked to the previous studies of Gick and Holyoak (1983) suggesting that if a more effective schema was generated by problem solvers, then transfer is enhanced. Moreover, the use of self–explanations along with worked examples might have created positive outcomes for learning in this current study as Crippen et al. (2005) claimed. According to their conclusion, "Worked examples alone may have a very refined impact on student performance...The combination of a worked example with a self-explanation prompt seems to produce a difference in performance, problem solving skill, and self-efficacy" (p.818). These three subthemes, mainly *Thinking deeply, Mental Model and Learning*, derived from the interviews can be linked to near and far transfer of learning from students' perspectives. Hence, these findings might indicate that students have an awareness on transfer with the assistance of self-explanations.

Group discussions have been determined as effective towards learning and higher order thinking according to the related structures of social constructivism. Students participated at group discussions mediated by online forums at Moodle and they completed eight forums during the study. Teacher did not interfere or to manipulate students' comments during the forum; instead she was an observer of the progress.

When students' replies to Moodle forums were analyzed through subtheme indicators of near transfer of learning, namely <u>Knowledge,</u> <u>Comprehension</u> and <u>Application Themes</u>, results indicated that these themes had generally high frequencies in eight forums. Hence, forum replies indicated positive improvement in near transfer of learning. On the other hand, frequencies of the themes in forums were lower than the frequencies of reflective journals on near transfer. <u>Knowledge Theme</u> at forum analysis had the highest frequencies compared with the other two themes of near transfer. In addition to that the highest average of near transfer indicator at forums was at use of correct terminology under <u>Knowledge Theme</u> with the mean of 88.7%. Thus, students have less problems to recognize the terminologies, notations and general rules in derivatives compared to comprehension and application of them at online forums.

The other highest percentages of near transfer of learning were recognized under Comprehension Theme.These subthemes were the same subthemes that had highest frequencies at reflective journals. *Understanding the problems* had mean: 80.2%, whereas *identifying correct method* had the mean: 76.8% at eight forums. On the other hand, 4.3% of students were able to *identify correct method* at forum 5. The low percentage value, 4.3% might have resulted due to the difficulty level or clarity of the question in that forum. Thus, students have higher tendencies to realise near transfer of learning at comprehension level compared to application level. However, this finding might depend on the difficulty level and clarity of the referred question.

Under Application Theme, *correct mathematical calculation* had the highest percentage mean with 47.1%; however this percentage was quite lower than the average of the reflective journals for the same subtheme. This subtheme also had the highest standard deviation with 41 %. There was a larger spread of variety at the replies to forums under that subtheme. That is because forum 5 and forum 8 did not have indicators of correct mathematical calculation. *Organizing given expression* and *indication of breaking problems into parts* were rarely observed at forums (once among eight forums); because forums usually concentrated on research based or conceptual type of questions rather than mathematical problems. Again, these findings on near transfer of learning might have changed due to the difficulty level and clarity of the referred question. Another reason of changing near transfer indicators might be due to the students' familiarity on using technology to explain ideas through mathematical notations and expressions. Technology might have caused difficulty in use compared to paper and pencil.

The positive near transfer improvement indicators of online forums support the literature findings on the effects of group discussion towards understanding and thinking; as Vygotsky (1978) and Driscoll (2005) claimed that group discussions highly recognized to facilitate higher order thinking skills that would be effective for actively participating learners. The results of the analysis of online forum also in agreement of Vygotsky's theories on the development of higher thought and concept development; since Elrich (2006) cited that "Vygotsky (1978, 1986) believed that language was a psychological tool and that the usage of this tool invariably led to a series of inner or mental transformations such as the development of higher thought and concept development" (p.13).

When students commented on the effectiveness of group discussions, there were some subthemes such as *Thinking deply, Reflection on others' Thinking,* and *Understanding* that can be related to near and far transfer themes that were coded for online group discussions. Students mentioned about positive impacts of online forums towards learning and understanding under these three subthemes. The facilitation of these mentioned higher order thinking skills might have resulted from the online group discussions as literature pointed out earlier. Past studies showed that the integration of the structures of social constructivism in instruction has been also highly recognized to facilitate higher order thinking skills that would be effective for actively participating learners, as Vygotsky (1989), Hannafin (1992) and Driscoll (2005) claimed. Additionally, Deaudelin & Richer (1999) mentioned that online discussion enables asynchronous exchanges and it assists one-to-one and one-to-many interactions;so then students are motivated for learning independently, they transfer and apply knowledge to real-life situations. Finally, the themes driven through students' interviews is in line with the outcomes of other studies as Kramarski et al. (2006) pointed out. The authors cited that "Discussion mediates shared meaning. By critically examining others' reasoning and participating in the resolution of disagreements, students learn to monitor their thinking when they reason about important mathematical concepts (e.g., Artz & Yaloz-Femia, 1999; McClain & Cobb, 2001)"(p.218). Thus, interviewing students also have an awareness of learning transfer with different benefits through forums.

It can be deducted that some students had technological difficulties, as two students found online forums at Moodle to be *not user-friendly*. Two students mentioned that they had hard times to reply as they wished to due to technological difficulties. They emphasized that typing and graphing at computer were taking time and it was not as easy as to present ideas with paper and pencil. These comments were expected as a result of technology survey that was given before the study. Although, some adjustments made on question styles, two students still faced with challenge at expressing themselves fully with mathematics language at computer. Another reason of these difficulties might be due to forums used for the first time in mathematics classes for learning.

Additional Notes on Self-reflections and Online Discussions about Near Transfer of Learning

When self-reflections and online forum discussions were compared on near transfer of learning, following outcomes were identified. Online forums consisted of lower percentages in near transfer than in reflective journals' might be due to the type of question addressed to the students. These results might have resulted due to the type of the forum question; as they might not have directed students towards these artifacts as much as reflective journals.

At Knowledge Theme, *use of mathematical notation* varied due to given task. If the given problems involve long operations, graphing and tables, notation errors were easily seen. Problems involving long calculations were preferred to ask in reflective journals rather than forums, that was because typing in Computer taking longer time than handwriting, so forums focused more on knowledge, interpretation, analysis of real facts rather than long calculations.

Under Comprehension Theme, *defining a rule* was rarely observed so it was coded when a student defined a rule in Calculus. It can be inferred from the comparison of percentages at Table 33 and Table 34, online forums led students more towards *defining a rule* more than reflective journals; this might be due to the fact that online forums focused more on interpretation of concepts and forums enabled them to mention the rule more than reflective journals. *Outlining the steps of solutions* was rarely used in reflective journals and online forums in general, except reflective journal 4 that was requesting students to outline the steps solutions

Under Application Theme, o*rganization of solution steps* was not observed frequently in every reflective journal and online forum. On the other hand, reflective journals 1 and 2 had higher percentages for this subtheme because they especially necessitated organization of solution steps. As there were less requirement for reorganization for the rest of the problems in reflective journals, it was not observed in rest of the reflective journals and forums. *Breaking into parts* was an individual choice except the requirement of reflective journal 2 and reflective journal 3. Forums were more concerned with interpretation of concepts so no *breaking into parts* of calculations were observed. *Reasoning solution steps* was mostly seen in problem solution steps in reflective journals and online forums.

Interview Themes for Worked example, Completion Examples and Practice Problems

From students' interviews, it can be seen that worked examples were positively recognized with its effects towards learning under the themes of *understanding& learning and reinforcing*. Most of the comments under *Understanding and Learning* theme for worked examples were at the common positive idea of solutions steps helping towards understanding the reasoning. These positive comments support the previous studies of Crippen et al. (2004) stating that" regarding impact on both performance and understanding, student survey respondents held positive perceptions of worked examples and solution strategies" (p.163). All students considered that worked examples *reinforced* practicing. Therefore, practice might have led students to near transfer of learning.

On the other hand, *Confusion* was another theme that came up from students' comments for worked examples. According to the opinions of students, it can be inferred that worked examples were most useful when learners did not know the subject. This inference is in agreement of the results found by Efklides et al. (2006) and Renkl et al. (1998).They concluded that worked examples were most useful when the learner does not have the knowledge on the rules or the procedures for the solution of the problem. According to experts, students with prior knowledge find worked examples to be redundant that leads to heavy cognitive load on working memory. The comment of a student supported the same idea; the methods in worked examples were sometimes confusing for her when the student studied the topic earlier; as she focused on her own method in the reference instruction. S/he found worked examples to be confusing when student deduced the information quicker than the solution steps of worked problems. Hence, at some cases, confusion might have negatively affected students' near transfer of learning in implications.

Related to near transfer of learning, the positive comments for completion examples can be grouped under two main themes: *Guidance* and *Reinforcing*. Under *guidance* theme, five students stated that backwards fading was helpful for understanding the unit; whereas three students stated that forward fading was helpful for understanding. Thus guidance might have facilitated near transfer of learning. Morever, two students mentioning that backwards fading *reinforced* learning for harder subjects and two other students stating forward fading *reinforced* more for learning show that students might have benefitted from completion examples for near transfer of learning.

For practice problems, the themes of *Understanding & Learning* and *Learning* can be related to near transfer of learning. All students thought that practice problems were useful towards *Understanding and Learning*. Under this theme, four students mentioned that practice problems facilitated revision with similar problems Under *learning* theme one student emphasized practice problems reduced calculation mistakes by practicing with similar problems. Therefore, reducing calculation mistakes and asssiting solving similar problems can be linked to comprehension and application themes of near transfer.

Conclusion Remarks

It can be concluded that based on the findings from achievement tests, reflective journals, online group discussions and interview that worked examples, diverse worked examples, completion examples, practice problems, self reflections and group discussions helped students learning the units of derivatives and application of derivatives in near transfer setting.However, technological barriers for online discussion and students' prior knowledge in solution steps should be considered in designing similar learning environment.

5.2 Far Transfer of Learning

In this section, the results of pre and post achievement tests due to far transfer are discussed along with the outcomes of self-reflection and group discussions. Finally, these results are combined with students' comments about the effects of implementation of worked examples, diversed worked examples, completion examples, practice problems, self reflections and group discussions towards far transfer of learning.

Pre and post achievement tests enabled to collect quantitative data on far transfer where students individually had to reflect problem solving skills in a determined time period. When students' gained scores from far transfer problems in achievement tests were examined, results showed that the means of gained scores from each achievement test for far transfer were 58 and 55 respectively. These means showed that far transfer of learning might have been realized by the assistance of designed learning activities. Nevertheless these means were lower than the means of near transfer gained scores in tests; this might be an indicator that students had more difficulty to solve far transfer problems in the achievement tests.

The standard deviations of gained score percentages in two achievement tests were 25 and 20 respectively. The results indicate that there was a larger spread of data; hence students' replies to far transfer problems differed due to their gained skills at transfer. The overall results in gained scores of far transfer indicates that there is a high evidence that far transfer of learning took place with its overall mean, 56 and standard deviation,19. These results support the findings of Clark(1999) in that students had more difficulties at far transfer of learning compared to near transfer of learning.

Along with the results of descriptive data from pre and post achievement tests, when a paired t- test was realized for pre and post achievement test 1, the result indicated that there was a significant mean difference between pre and post achievement far transfer test scores. The paired sample t test result for pre and post achievement test 2 also showed a significant mean difference between pre and post test scores on far transfer. Additionally, the paired t-test of total pre and post achievement tests on far transfer showed that there was a significant mean difference between pretest far transfer scores and posttest far transfer scores. These results may suggest that learning activities in mathematics lesson designed based on cognitive load principles result with far transfer of learning up to some extend. The statistics on far transfer in achievement tests showed that far transfer was facilitated through the learning activities of the current study and the quantitative data of the current study are parallel to the results of pre-mentioned studies on fading procedure, self explanation effect and diversity of worked examples. The analysis on achievement test results supports the study of Atkinson & Renkl & Merrill (2003) where they claimed fading procedure together with self explanations also enhanced far transfer.

The gained scores mean of far transfer problems in achievement tests are lower than the gained scores mean of near transfer problems in achievement tests.Moreover, the gained score standard deviation of far transfer problems in achievement tests shows a wider spread of data leading to more differentiation in students' skills at far transfer realization. These results of the test analysis are also in agreement with the conclusion of Clark (1999) in that far transfer of knowledge is harder to teach and to transfer in learning. For further verification for far transfer of learning, advance analysis was made to examine far transfer indicators at self-reflections and group discussions to see the overall picture.

Previous study of Atkinson, Renkl, Merrill (2003) demonstrated that backwards fading procedure together with self-explanation prompting significantly facilitated not only the near and medium transfer learning but also far transfer learning. Moreover, Renkl et al. (1998) declared that "The only evidence that multiple examples are helpful is that learners with elicited self-explanation and multiple examples showed the highest performance on far-transfer problems " (p.105).

The coding of self explanations mediated by reflective journals indicated positive improvement in far transfer of learning. Students had demonstrated indicators far transfer at analysis and synthesis levels of Bloom's taxonomies. On the other hand, the subtheme indicators of far transfer of learning, namely Analysis and Synthesis Themes had lower frequencies in eight reflective journals than near transfer indicators of reflective journals. The highest percentage average of far transfer indicator appeared under Analysis Theme with *solving a problem in new* situation (mean: 31.6%). This subtheme also consisted of the highest percentage in reflective journal 7 with 78.6 %. This code was awarded mostly for reflective journal 7 , since there was a requirement of solving a new problem. Thus, students demonstrated far transfer of learning when the question addressed to. The highest standard deviation of far transfer indicators of reflective journals was observed at *comparison of new problem with old problem* under Analysis Theme with 35.4%; because it was only seen at reflective journal 3. This result might resulted due to the requirement of the referred question.Although the average percentage of *correct interpretation of sign tables, graphs and results* is low due to the mean of eight reflective journals, it can be observed that students successfully analyzed graphs, tables and results for the last two reflective journals, since it was a requirement especially for these two journals.The percentages were 64.3% and 82.1 % respectively; therefore, students had shown successful far transfer of learning outcomes when the question addressed to interpretation of results under analysis level of Bloom's taxonomies.

Some of the far transfer indicators were not observed at reflective journals: namely *creating a new problem, deriving abstract relationships, stating limitations and restrictions*. Far transfer of learning were observed around at one tenths for reflective journals; however there were still some indicators of it. These outcomes might have resulted due to the type of the reflective journal question; as they might not have directed students towards these artifacts with its requirements and its fostering contents. Moreover, since it was found that near transfer of learning is relatively easier to realize compared to far transfer of learning, the findings of this study through students' self-explanations might be inferred as in agreement of the finding of Clark (1999) for far transfer of learning. Clark (1999) declared that far transfer is advantageous when the learner has to make judgments and to adapt to different situations; however, far transfer of knowledge is harder to teach and to transfer in learning.

Students generally enjoyed writing reflective journals and spending time on thinking proper expressions to answer the reflective journal question. It can be inferred from students' reflective journals that transfer of learning might have been realized through learning activities and self-explanation. Some of the themes derived from students' comments at interviews showed that students had some awareness on the effects of self reflection towards far transfer of learning. There were three main themes resulting from the students' comments that were along with transfer of learning: *Thinking deeply; Mental Model;* and *Learning*. Under *thinking deeply theme*, two students thought that self reflections assisted grasping reasoning of solution steps that can be linked to far transfer of learning outcome Under this theme, thinking genuinely on the proper expressions/examples to explain and the organization was the subtheme that had been mentioned the most. Another student mentioned that self reflections prevented memorization..

On the other hand, under *Mental Model* theme, self-reflections were highly recognized with its emphasis on in what way to organize solution steps. Students mentioning self reflections encouraging forming schema for the solutions of problems; bringing different subject areas to connect the topics and facilitating the main summary of the subject might demonstrate that students also have an awareness of the effect of self reflection towards far transfer of learning outcomes by assisting *Mental Model*. Under Learning Theme, two students stated that analyzing cases in details helped learning. Also, one student mentioned that self teaching helped learningunder *Learning Theme*. These comments supported the findings of Simon (1979). If it is considered that self-instruction was facilitated by self-explanation in this study, this conclusion might have provided an evidence to the previous study of Simon (1979) who stated that "students learn both by being taught and by self-instruction" (p.87) and the findings of Chi et al. (1989) who were in agreement of Simon's results. Hence, past study results and the results of this study do not only indicate far transfer of learning through the analysis of learning activities; but also students think parallel to the positive outcomes of far transfer through self reflections.

These comments were in harmonious with the claims in the study of Efklides et al. (2006) who concluded that during self-explanation, learner thinks aloud for reasoning for the choices and procedures in the solution of the problem. In addition to this, the three outcome themes from students' interview comments support the results of Crippen et al. (2005) from students perspective. Crippen et al. (2005) mentioned that when worked examples and self-explanations were used together, they increased in self-efficacy (motivation) and performance and together with self-explanations students learn a difficult content without overwhelmed; hence students might have understood and learnt a difficult subject like derivatives with the combination of worked examples and self-explanation. Hence, students had awareness on the effects of reflective journals towards understanding and learning; as the previous studies showed that self-reflection facilitates understanding when they were used with worked and completion examples (Simon, 1979; Chi et al. 1989; Paas 1992; Renkl et al. 2004; Efklides, 2006). These findings were in agreement with the students' comments in this study.

Group discussions mediated through online forums had relatively more indicators of far transfer of learning than reflective journals that might have resulted in the forum question type and its requirements. The subtheme indicators of far transfer of learning were Analysis and Synthesis Themes in eight forums. The highest percentage average was observed under Analysis Theme at *correct interpretation of signs, tables and results* (mean: 42.4%). Hence, students generally demonstrate far transfer of learning to *interpret sign tables, graphs and results correctly* at analysis level when the question requires to in group discussion.The second highest percentage of subtheme was *judging the reasonableness of the data* under Synthesis Theme with 28.6%. This average was closer to the reflective journals' percentage mean for the same subtheme (mean:25.2%). Hence, students demonstrate far transfer of learning as the question requires in technological environment in the same level as in paper pencil format; since the judgement is explained in words rather than mathematical notations.

In addition to that, although some of the far transfer indicators under Synthesis Theme were not observed at reflective journals: *creating a new problem, deriving abstract relationships, stating limitations and restrictions*, some forums revealed these subtheme indicators. For instance forum 5 had the highest percentage on creating a new problem under Synthesis Theme with 69.6%. On the other hand, forum 2 had the percentage of 56.5% for *deriving abstract relationships*. At forum 3, *stating limitations and restrictions* were observed with the percentage of 36.4%. Hence, it can be inferred that online forums reveled positive improvement in far transfer of learning due to learning activities. The highest standard deviation was observed at highest percentage on *creating a new problem* under Synthesis Theme with 31.2%. However, this subtheme was observed with high percentages at two forums, specifically forum 4 and forum 5, requiring creating a new problem in the question structure. Thus, it can be deduced that students might successfully create a new problem and demonstrate far transfer of learning outcomes at Synthesis Theme when the question required to as a result of the implemented learning activities. That is because percentages at this subtheme can be recognised as high 65.0% and 69.6% respectively.

It can be deduced that far transfer indicators at forums were not as much as near transfer indicators of forums. One of the reasons of this consequence might be due to the fact that far transfer outcomes were harder to achieve; it takes longer time to digest information efficiently which was parallel to the pre mentioned past study of Clark (1999). That might be the reason for the percentages of far transfer indicators were lower than the percentages of near transfer indicators. One other reason might be the question styles of forums addressing to far transfer outcomes. These outcomes of far transfer and lower percentages of indicators might have shown variability due to the question styles at online forums addressing to indicators of far transfer of learning.

Some of the students' comments were parallel to the findings of this study on far transfer of learning as a result of online forums. Within the interviews, the themes of *deep thinking, understanding, reflection of others' thinking*, *no learning contribution* and *not user friendly* might be related to above findings for forums. Under *thinking deeply* theme, three students thought that seeing variety of questions extended thinking skills in different perspectives and whereas three students stated that forums made them think more. One student mentioned that theories helped to interpret concepts under *understanding* theme. One student considered that reading friends' replies added knowledge under *reflection of others' thinking* theme. Hence, forums affected far transfer of learning positively with its assistance towards thinking and understanding for some students. On the other hand, four students considered that forum replies did not contribute much to *thinking deeply;* some of them preferred to solve in their own way; some of them did not understand friends' replies to benefit from; and some of them could not benefit from replies as there was not enough discussion environment. Two of the other students considered that friends' replies mixed up mind and they led to confusion rather than leading to *thinking deeply*. Another student thought that the theories emphasized did not help much to problem solving and thought it lead to *no learning contribution*. One student stated that she preferred own solution; whereas another student mentioned that students can learn without forums under the same theme.

Majority of the ideas were under *Deep Thinking* theme; nevertheless there were not any consistency on discussions' positive or negative effects. One third of students thought that seeing variety of questions extended thinking skills in different perspectives and forums made them think more. These comments were in favor of the literature findings, if it is also linked to self explanation effect on problem variability (Paas et al.,1994; Renkl et al., 1998). Paas et al. (1994) claimed that learners with elicited self-explanation and multiple examples showed the highest performance on far-transfer problems. Thus, students might have an awareness of problem variability extended their critical thinking skills on problem solving. Hence, the effects of forums towards problem solving and learning might be indirect by addressing the structure of concepts. As a result, perception of forums towards learning does not have any common consensus as well as not having a common outcome with its effects. Mentioned comments might explain the low percentages of far transfer of learning outcomes in online forums.

Researcher did not prefer to interfere to forum discussions much in order to prevent giving too much guidance that would disable student to generate own ideas. Instead there might be a slight conjecture driven from students' comments that decent guidance at forums might have eliminated confusions. Thus, the instructor should observe the learners and guide when necessary in online group discussion. Hence students' comments on guidance at online forum discussions support the findings of past study of Ernest (1998) asserting that "The learner may lack of confidence and need assurance or may not be able to make the transformations unaided in order to achieve the goal…Guidance helps her to extend her domain of competence so that ultimately she can undertake the challenging type of task unaided and complete the transformational sequence alone" (p.226). Thus, it might be inferred that replies of students' friends might have an effect on near transfer; but more discussion environment together with guidance should have been facilitated to reach far transfer of thinking.

In general, it can be inferred that analyzing online forums assisted to reveal far transfer of learning outcomes with its question structures. It can be deduced that group discussions had some positive impacts towards far transfer of learning; and there are some indicators of this result that are in agreement of the previous studies about problem solving activities harmonious with CLT principles.

Additional Notes on Self-reflections and Group Discussions about Far Transfer of Learning

When far transfer of learning outcomes are compared at reflective journals and online forums under Analysis and Synthesis Themes, it is observed that *judging the reasonableness or validity of the results* and *analyzing the problems further* were the most seen codes within the reflective journals. On the other hand, forums involved more codes involving far transfer, as it was aiming to search for more. *Generalization of a fact or a rule in Calculus* and *correct interpretation of results, tables, graphs* were also another codes which were frequently observed in students' replies in forums.The percentages were dependent on the question types and participation rates of students. The percentages might have also differed due to confidence of students with the use of mathematical notations, graphs and tables at replies on hard copy (self reflections) and on technology (group discussions). It might be infered by observation of near and far transfer that reflective journals might have assisted more towards forums. By reflective journals students might have discovered basis of the tasks evoking the skills on near transfer; and with forums they might have combined the taught lesson with reflective journal outcomes. This transition might have formed a schema in their interpretation of derivatives and their applications; hence student replies showed more indicators at far transfer of learning in online forums. It might be assumed that the mathematics lesson designed based on the principles of CLT and its fostering learning activities might have resulted in near and far transfer of learning.

Interview Themes for Worked example, Completion Examples and Practice Problems

The far transfer of learning indicators derived from students' comments about worked examples can also be linked to *Understanding and Learning*; and *Reinforcing* themes. These two themes were expected to be derived based on the findings of CLT principles in literature (Renkl et al., 1998; Crippen et al., 2004; Renkl et al., 2004). Under *understanding and learning* theme, four students mentioning solutions steps helping towards understanding the reasoning can be linked to far transfer of learning outcomes; since understanding and learning the reasons of solution steps can also facilitate flexible schema forming. In addition to that, under *understanding and learning* theme, three students mentioned that worked examples facilitated to store information in long term memory with its similarity in examples. Renkl et al. (2004) concluded that studying on worked examples and self-explanations together would lead to germane load that contributed directly towards learning. On the other hand, *confusion* theme and its reasons -mentioned in previous section- might also have produced some interruptions in far transfer for those students who knew the subject earlier, as Renkl et al. (1998) and Eflides et al. (2006) emphasized.

Related to far transfer of learning, the positive comments for completion examples can be related to two main themes: *Guidance* and *Reinforcing*. Merrienboer et al. (2002) found that completion examples resulted in higher transfer of acquired skills than conventional problem solving. Furthermore, Atkinson & Renkl & Merrill (2003) concluded that fading solution steps can facilitate both near transfer and far transfer. Students also thought that completion examples foster learning and understanding through its guidance and reinforcing elements towards thinking; they had to spend time on how to start or how to move on in the steps of problem solution. However, the ideas on which type of completion example type (backwards fading or forward fading) to be more useful varied due to the students. Under *Guidance* theme, five students thought that completion examples with backwards fading to be helpful for understanding the unit because guidance was given. Although, it is not clear whether students addressed usefulness of backward procedure on near and far transfer problems, these comments might be in agreement with the previous research of Renkl, Atkinson, Maier & Staley (2002) on far transfer effect. The experts claimed that backward fading has the potential to foster far transfer, whereas forward fading does not substantially reinforce the same accomplishment.On the other hand, under *Reinforcing* theme, three students agreed on completion examples with forward fading encouraged thinking more and two other students thought that completion examples with forward fading were to be more assistance towards learning the subject. This point of view of students revealed a different perspective of learners compared to the past studies of Renkl et al. (2002). On the other hand, one third of the interviewing students mentioned that completion examples with forward fading to be more *challenging* than completion examples with backwards fading.

Related to far transfer of learning, following comments are identified. All students thought that practice problems were useful towards *Understanding and Learning* and they all thought that practice problems prepared them to the achievement tests. Under this theme, one third of students generally thought that variety of practice problems challenged them and four students considered that they guided them towards critical thinking and analyzing. Three students also thought that practice lead to effective interpretation of reasoning solution steps. One student stated that practice problems prepared for the test style questions by encouraging forming schema to solve problems. Hence, practice problems also facilitates *Mental Model* for some students. These comments are in agreement of the past study of Renkl & Atkinson (2002) where they claimed that "Learning from examples is not the preferred method when learners reach the third stage (proceduralized rules), where problem-solving practice is the optimal instructional approach"(p.111). Thus students thought that practice prepared them for the tests after having learnt the subject through several reasons. Under *Learning* theme, seven students mentioned that practice problems was the best learning activity among the rest. That resulting derivation was not unexpected, because practicing with practice problems was a traditional system that has been followed for those students. These two themes of interviewing students were also in agreement of literature review. It is cited that "Ranzijn (1991) and Shapiro and Schmidt (1982) point out that increased variability of practice along the task dimensions is beneficial to schema acquisition and hence to transfer of acquired skills because it increases the chances that similar features can be identified and that relevant features can be distinguished from irrelevant ones" (Paas & Van Merrienböer, 1994, p.124). Also, these explanations of students might have come through the effect of already experienced situation and development of necessary skills while working with practice problems in the past. Additionally, students might have been enjoyed competency through worked examples and self reflections. As a result, they might be happy to solve the practice problems with a competence.

Conclusion Remarks

It can be concluded that based on the findings from achievement tests, reflective journals, online group discussions and interview that worked examples, diverse worked examples, completion examples, practice problems, self reflections and group discussions helped students learning the units of derivatives and application of derivatives in far transfer setting up to an extend. However, technological barriers for online discussion and students' prior knowledge in solution steps should be considered in designing similar learning environment.

5.3 Student Recommendations

In this section, the student recommendations about the effective implementation of learning activities driven from students' interviews are discussed. In general it can be understood that students thought that they benefitted from the learning activities even though their ideas varied which activity helped them most. Worked examples were positively recognized with *Understanding and Learning*; and *Reinforcing* themes by students. However, there were some cases where they led to *Confusion*. Hence it is recommended that worked examples should be integrated in instruction to promote understanding and learning and students are quite satisfied worked examples' reinforcing property. According to students' suggestions, worked examples can be improved by students' self-explanations to explain the given steps in their own words; having a space for them to try to solve the problem; and integrating variety of solution techniques. It is already known from literature that (Renkl et al. 1998, Renkl et al. 2004) when self explanations were used with worked examples,it enables schema formation; so one of the students also recommended to explain the given problem in their own words would increase its effect for learning.

Students thought that completion examples were effective towards learning and understanding. This can be understood through *Guidance and Reinforcing* themes derived from students' comments. When it comes to the type of completion examples, opinions varied about which type of fading (backwards or forward) was more helpful than the other type; however the common idea was that completion examples were guiding towards thinking and understanding. Thus, its use in instruction assisted learning and understanding through its elements. Additionally, five students thought completion examples to be *confusing* for several reasons: (1) expectation on what to write or how much work should be shown were not clear; (2) sometimes mixing up mind if the topic was studied earlier; (3) understanding the solution steps was hard. The comment at (2) can also be linked to the past study of Efklides et al. (2006) and Renkl et al. (1998) in its opposite effect like worked examples; completion examples may also lead to confusion if the topic is studied earlier. Instructors are recommended to use completion examples for addressing effective understanding; however they should be clear about the expectations on how to complete the fading steps.

As for students' recommendation on improvement of completion examples: one student suggested to have more solution steps can be given to fill;whereas another student suggested to have different methodologies of solutions can be given alternatively.Hence students preferred to have more variety in methods and more solution steps to fill for long problems.

When self-reflection recommendations are considered, there were different perspectives on the *challenge* of writing self-explanations. To start with, three students had been *challenged* at writing reflective journals when transmitting knowledge; when prior knowledge is integrated; and when a new problem style was asked. One of them found writing reflective journals to be stimulating, therefore they were to be useful towards learning. While the other two thought that integration of prior knowledge in new situation to be challenging. Thus, some students may be allocated more time for recalling prior knowledge before completing reflecive journals. One student suggested writing reflective journals after the new information was digested in a longer period of time; hence in that sense she thought that reflective journals did not help much. Another student suggested to have reflective journals as assigned homework. Thus, some students may take more time for forming schema through self-reflections and this process may take time. Another recommendation were stating outline of expectations in reflective journals;hence clear instructions should be given on expectations. This student comment may support the study of Renkl et al. (1998) emphasizing that guidance to be essential in promote effective learning outcomes while working with worked examples and self-explanations. The authors added that multiple examples without guidance for sophisticated self-explanations resulted in very low learning outcomes. On the other hand, this comment shows students not only need guidance in worked examples but also at the instructions in self explanations. One student recommended to have mathematics problems rather than summaries in reflective journals;this might be due to students habits in mathematics classes; participants were more used to solve mathematics problems than writing verbal explanations for mathematics as regular study habit.

Three students found online forums *motivating* towards learning when different field areas were linked to the derivatives units. Connecting mathematics units to other fields might have aroused intrinsic motivation of students that results in making more sense of what was learnt. Mueller et al. (2011) stated that intrinsically motivated students concentrate on understanding concepts. The authors concluded that “Intrinsic motivation fosters positive dispositions toward mathematics, which, in turn, encourage students to develop self-efficacy and mathematical autonomy as they discuss and share their understandings with their classmates”(p.42). One student believed that forums increased awareness of different concepts. Therefore, forums might be assistance towards learning with its intrinsic motivation property. As mentioned earlier as recommendation one student recommended to have more discussion provoking questions at forums; while another one stated that group discussions should take place in class rather than in online environments. Therefore, it can be inferred that some students are open to discuss mathematical ideas in groups when questions provoke and when they feel comfortable to discuss with immediate comments.

One of the students stated that theories examined at forums helped for understanding of concepts under *understanding* theme; whereas one student stated that forums fostered practice with further problems under *competence* theme. These comments were not unexpected, since the researcher aimed to increase awareness on the concepts of the units through forum questions. Thus, these students had awareness of the outcomes of the forums discussion through its reasoning and practice elements. The comments of students might show an evidence in that objective of group discussions were addressed to some students. Researcher aimed to look for to what extend the transfer would occur, if students were led towards further interpretation of the constructs through reasoning and justification. This aim was along with the past study of Schoenfield (1992) pointing out that asking why questions to students during the solution processes facilitated elaboration and retainment information. Moreover, the forums were in the line of the strategies of Kramarski et al. (2004) declaring that forum discussion is not sufficient for enhancing mathematical literacy; a structure for mathematical discussion is needed through the practice and reinforcement of giving reasons.

Finally, when it comes to practice problems, there were four positive main themes that resulted in through coding the students’ comments: *Reinforcing; Understanding and Learning; Mental Model;* and *Learning*.Under *reinforcing theme,* it is seen that two students thought practice problems tested knowledge while two other thought practice problems emphasizes main points. Thus, practice problems *reinforce* learning. As for recommendation on practice problems, four students believed that they needed more duration of time when there were challenging problems to analyze in depth and integrated prior knowledge . Hence time allocation should be considered more critically for those students who take more time to interpret the new knowledge depending of the work load. Answers of practice problems can be given to students; one student suggested to have answers at the back so then s/he can feel comfortable by checking the answers.

5.4 Implications and Suggestions for Practice

This study is a case study and therefore it is not possible to generalize the findings of the study.However based on the findings, some implications and suggestions for the similar settings are presented in the following section.

- The Cognitive Load Theory principles on reducing extraneous cognitive load and increasing germane load while problem solving are effective instructional methodologies towards learning to be implemented at high school mathematics classes. Teachers and instructional designers can work more on how to put them into practice by balancing the prior knowledge with the new information, giving appropriate duration to complete tasks and by proper guidance at high school level especially for a challenging subject, like derivatives.

- Teachers and instructional designers should be aware of that students would like to see variety of solution methods at worked examples and completion examples, and they would like to feel comfortable to follow the solution method that addresses their interpretation. In that situation confusion of students on understanding might be prevented more.

- This study result supports the literature findings on self-explanations. They were found to be effective towards transfer of learning if they were used along with worked and completion examples. However, it can be deduced from the findings that teachers and instructional designers should have awareness on the amount of challenge that students will be involved in when prerequisite knowledge and/or new problem style were given in self-explanations. Students comments on this study also enlightened that teachers and instructional designers should consider critically how to organize and to involve self-explanations with examples along with principles of CLT as well as designing an effective learning environment; teachers should also consider spending time in class for reflective journals as well as giving more time outside of the class for the completion of reflective journals.

- Teachers and instructional designers should consider critically how to integrate online group discussions and the structure of discussion questions in order to support learning and deep understanding with examples and problems in derivatives units. They should have an awareness on that students might necessitate more guidance to clarify concepts in discussions to create correct understanding. Students might be allocated more time to participate and discuss on online forums.

5.5 Implications and Suggestions for Further Research

- This study was conducted in thirteen weeks. Similar type of design can be used for a longer period of time to observe the float and the process in more details.

- Worked examples, diverse worked examples, completion examples and practice problems were found to be effective towards transfer of learning when implemented properly. However, a further research can be done on the proper level of questions in derivative units at high school and how to develop effective derivative questions, especially completion examples, that would gradually assist students towards smooth understanding.

- Group discussions were recognized as to foster learning and understanding through self-actualization. However, instructional designers should develop new instruments on facilitating online forums in mathematics while supporting deep thinking and intrinsic motivations of students.

- These learning activities should be implemented at different grade levels, at different field areas. Moreover, students should be guided towards more these types of learning activities in the tenets of CLT in mathematics classes and results should be examined further when students get used to example types and transitions.

- This study was done with participants selected via convenient sampling method. Same study design can be used through students with higher motivations or higher skills at using technology or with a larger sample size in order to see its possible outcomes compared with principles based on CLT.

- This study was a case study due to some limitations and restrictions. Same type of design can be implemented in an experimental study with larger samples in order to make more generalizations of the findings.

- Similar learning environment can be designed for realizing transfer of learning. However, the individual impacts of each learning activity regards to transfer of learning can be tested through excluding one of them, and then analyzing qualitatively the effects of isolation of the activity towards transfer of learning.

- The impact of the principles implemented can be tested in regard to retention of learning.

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APPENDIX A

PROFILE OF STUDENTS

Table 49 Mathematics CGPA and Motivation Levels of students (F: Female and M:Male)

			Motivation Survey Items		
Gender	Students	Math CGPA (%)	Task Value/Intrinsic (%)	Self-Efficacy (%)	Test Anxiety (%)
F	S_1	95	63	90	54
F	**S_2**	94	82	100	83
F	S_3	93	77	84	89
F	S_4	92	52	73	80
F	S_5	91	64	82	74
F	S_6	89	66	51	51
F	**S_7**	89	34	41	29
M	S_8	87	73	65	80
F	S_9	86	63	69	49
M	S_{10}	84	38	41	69
F	S_{11}	84	91	73	83
F	S_{12}	83	52	39	57
M	S_{13}	82	89	90	51
F	**S_{14}**	80	80	84	54
M	S_{15}	79	64	61	83

Table 49 (continued)

			Motivation Survey Items		
Gender	Students	Math CGPA (%)	Task Value/Intrinsic (%)	Self-Efficacy (%)	Test Anxiety (%)
F	**S_{16}**	78	32	37	20
F	**S_{17}**	76	75	96	66
F	S_{18}	75	39	35	26
F	S_{19}	73	79	73	14
M	S_{20}	72	75	84	97
M	S_{21}	71	91	88	37
M	S_{22}	70	34	31	66
F	S_{23}	68	29	57	57
F	**S_{24}**	67	61	73	60
F	S_{25}	63	23	24	54
F	S_{26}	61	63	57	37
M	**S_{27}**	61	88	76	40
M	**S_{28}**	51	43	43	49
M	**S_{29}**	45	41	59	63
M	S_{30}	45	64	67	46

APPENDIX B

MOODLE AND TECHNOLOGY SURVEY

Dear Students,
This questionnaire aims to find out the level of your knowledge about Moodle and Microsoft Office applications. This questionnaire will take approximately 5 or 10 minutes. There is no correct or wrong answer and you will **NOT** be judged by your answers. The answers of the questions will be confidential.
Thank you for your cooperation.

B.Gonca Tüker
Mathematics Teacher

SURVEY

Please provide the information about yourself.
Grade: ………………… Section: …………………,
Name/Last name:…………. Age: ………………

Please rate the given questions # 1 to #8.

1. How competent are you at using Moodle?
   1 (none)….. 2 (little)…… 3 (very much)……..
2. How much are you interested in learning mathematics with Moodle?
   1 (none)….. 2 (little)…… 3 (very much)……..
3. Do you think that math lessons should be supported with Moodle?
   1 (none)….. 2 (little)…… 3 (very much)……..

4. Do you use Microsoft Word?
   Yes………… No …….

5. If yes how competent are you at using Microsoft Word for the following?

Typing with a particular font size:	1 (none)…..	2 (little)……	3 (very much)……..
Arranging page layout:	1 (none)…..	2 (little)……	3 (very much)……..
Typing equations:	1 (none)…..	2 (little)……	3 (very much)……..
Drawing graphs:	1 (none)…..	2 (little)……	3 (very much)……..
Drawing shapes:	1 (none)…..	2 (little)……	3 (very much)……..
Inserting picture:	1 (none)…..	2 (little)……	3 (very much)……..

6.Do you use Microsoft Excel?

Yes………… No …….

7. If yes how competent are you at using Microsoft Excel for the following?

Typing summation formula:	1 (none)…..	2 (little)……	3 (very much)……..
Typing average formula:	1 (none)…..	2 (little)……	3 (very much)……..
Typing a formula involving 4 operations:	1 (none)…..	2 (little)……	3 (very much)……..
Drawing graphs:	1 (none)…..	2 (little)……	3 (very much)……..

8.How competent are you at realizing the following procedures at Moodle? Please rate the items.

Following Deadlines:	1 (none)…..	2 (little)……	3(very much)…….
Completing assignments with deadlines:	1 (none)…..	2 (little)……	3(very much)…….
Downloading worksheets:	1 (none)…..	2 (little)……	3(very much)…….
Downloading Videos/Powerpoints:	1 (none)…..	2 (little)……	3(very much)…….
Participating at group discussions at forum:	1 (none)…..	2 (little)……	3(very much)…….

Participating at individual forum discussions:	1 (none)…..	2 (little)……	3(very much)…….
Attaching documents :	1 (none)…..	2 (little)……	3(very much)…….
Sending e-mail:	1 (none)…..	2 (little)……	3(very much)…….

Other comments:

APPENDIX C

MOTIVATION SURVEY

Please provide the information about yourself.
Grade: ………………… Section: …………………,
Name/Last name:…………. Age: ………………
The following questions ask about your motivation and attitudes for this classs**.Again,there are no right or wrong answers.Answer the questions about how you study in this class as accurately as possible.Use the same scale to answer the remaining questions.** Please rate the given questions # 1 to #20.

		Not at all true of me	Usually not true of me	Sometimes but inferequently true of me	Occasionally true of me	Often true of me	Usually true of me	Very true of me
		1	**2**	**3**	**4**	**5**	**6**	**7**
1	In a class like this , I prefer course material that really challenges me so I can learn new things.							
2	If I study in appropriate ways, then I will be able to learn the material in this course.							
3	When I take a test I think about how poorly I'm doing compared with other students.(R)							
4	I believe I will receive an excellent grade in this class.							
5	When I take a test I think about items on other parts of the test I can not answer.(R)							
6	It is important for me to learn the course material in this class.							
7	When I take tests I think the consequence of failing.							
8	I'm confident I can understand the most complex material presented by the instructor in this course.							

9	In a class like this, I prefer course material that arouses my curiosity, even if it is difficult to learn.							
10	If I try hard enough, then I will understand the course material.							
11	I have an uneasy, upset feeling when I take the exam.(R)							
12	I'm confident I can do an excellent job on the assignments and tests in this course.							
13	The most satisfying thing for me in this course is trying to understand the content as thoroughly as possible.							
14	I think the course material in this course is useful for me to learn.							
15	When I have the opportunity in this class, I choose course assignments that I can learn from even if they do not guarantee a good grade.							
16	I like the subject matter of the course.							
17	Understanding the subject matter of this course is very important to me.							
18	I feel my heart beating fast when I take an exam.(R)							
19	I'm certain I can master the skills being taught in this class.							
20	Considering the difficulty of this course, the teacher, and my skills, I think I will do well in this class.							

Thank you for your time

REFERENCE:Garcia, T.& Pintrich, P.R (1996) *Assessing students' motivation and Learning Strategies in the classroom context: the motivated strategies for Learning questionnairre* in Birenbaum, M. & Dochy, F. Alternatives in assessment of Achievements, learning processes and prior Knowledge, by Kluwer Academic Publishers. (pp 320-339)

APPENDIX D

REFLECTIVE JOURNAL QUESTIONS

REFLECTIVE JOURNAL 1

Prove that derivative of f(x)= $2x^2 - \frac{1}{x}$ is $f'(x) = 4x - \frac{1}{x^2}$ from first rule and Fundemental Theorem of Calculus (First Principle)

REFLECTIVE JOURNAL 2

Prove that the derivative of y= $\frac{2x-1}{1+4x}$ is the same through product rule and quotient rule.

REFLECTIVE JOURNAL 3

If f(x)=e^x calculate $\frac{d(f^{-1}(x))}{dx} = (f^{-1}(x))'$ and $(f' o f^{-1})(x)$ and show that $\left(f^{-1}(x)\right)' = \frac{1}{(f'(f^{-1}(x))}$

REFLECTIVE JOURNAL 4

PART A.
Given that f(x)=*sin (x)* and g(x)=*cos (x).* Find tangent line to f(x) at (0,0).Find the tangent line to g(x) at $(\frac{\pi}{2}, 0)$. Graph the two tangent lines (of f(x) and g(x)), f(x) and g(x) on the same coordinate system. Prove that tangent line to f(x) is perpendicular to the tangent line to g(x).

PART B.
What are the difficulties that you faced with while solving questions on derivatives?

REFLECTIVE JOURNAL 5

Explain the topic to a friend who did not come to math class when Unit 3 was introduced including subtopics.
(Subtopics: increasing/decreasing intervals; stationary points and points of inflection.)

REFLECTIVE JOURNAL 6

Write your own rational function. Find stationary points and asymptotes of it. By analyzing the sign tables for stationary points, skecth the graph of your function.

REFLECTIVE JOURNAL 7

A particle moving along a straight line, has its displacement function, x meters from ma fixed point O, at time t seconds, governed by the function:

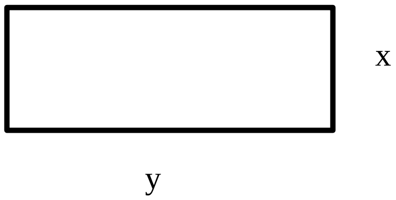
$$x = t^3 - 17t^2 + 80t - 100 \quad , t \geq 0$$

How many times will the particle pass through the origin.

REFLECTIVE JOURNAL 8

List down your steps to solve the optimisation problem below, then apply your steps to solve the problem.

<u>Problem:</u> A rectangle of area A cm^2 and dimensions of *x* cm and *y* cm. It has a constant perimeter of P cm.



a) Show that i) $y = \frac{P}{2} - x$

ii) $A = \frac{P}{2}x - x^2$

b) Find $\frac{dA}{dx}$ in terms of P and *x*.
c) Find maximum area in terms of P and *x*.

APPENDIX E

FORUM QUESTIONS

FORUM 1 (UNIT 1-PART 1)

Remember that when we find the derivative of a function, we also find the ***"gradient of the curve***",which is a function.
The **gradient of a curve** at any point is the gradient of the tangent to the curve at that point (Mathematics for the IB Diploma.Cambridge,p.150)

Does the gradient of the curve always a *line equation in the form of y'=mx+c or can it be a quadratic (y'=a*x^2*+bx+c), cubic (y'=a*x^3*+b*x^2*+cx+d),quartic (y'=a*x^4*+b*x^3*+c*x^2*+dx+e)...etc function* ? Give your reasoning through an example or reference(s).

FORUM 2 (UNIT 1-PART 2)

If f(x)=x^2 and g(x)=4x+3, then find f '(g(x)). g'(x) and discuss how this process is linked to chain rule by considering the derivative of h(x)=$(4x+3)^2$
Hint: Derivative of h(x) would give you the same result as f '(g(x)). g' (x)

FORUM 3 (UNIT 2-PART 1)

If f(x) is a differentiable function and if also the inverse of f(x) exists, so does the rule

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

hold for any function? Please show it by an example and discuss in between.

FORUM 4 (UNIT 2-PART 2)

Please give an example of a function where the second derivative of it is equal to itself. Please show the proof in your example.

FORUM 5 (UNIT 3-PART 1)

Some function either increase or decrease for all real x values. Give an example of a function with this property and relate this property of the curve to the gradient of the curve.Make a research on what the terminology used for such functions. (Remember to give a reference)

FORUM 6 (UNIT 3-PART 2)

Part 1

The population for a country is modeled by the equation

$P(t)=100+0.05\ t^3-0.0002\ t^4$

During what periods do you expect the **annual rate of increase** of the population **to be increasing**. Justify your answer by reasoning with calculations, sign tables and from the graph. Remember to explain the reasoning for each of your step.

Part 2:

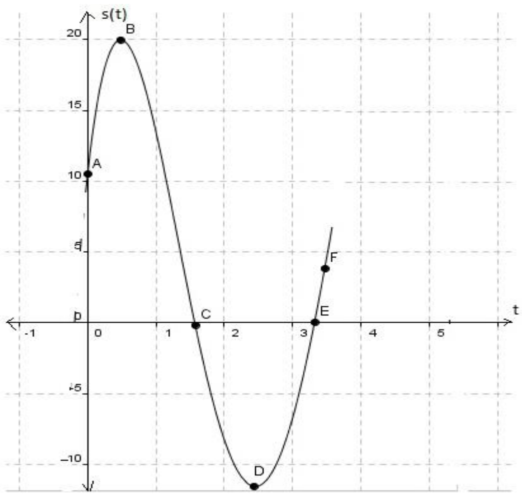
When we need to sketch the graph of a rational function, within the sign table of the derivative of the function, we also include the vertical asymptote values although the sign of the derivative function does not change.

What is the benefit of including x-values of the vertical asymptote lines?

Why does not the sign of derivative function change before and after x-coordinate of the vertical asymptote value?

FORUM 7 (UNIT 4-PART 1)

Interpret the direction and position of the particle in between the given five points A,B,C,D,E from the given displacement (s(t)) vs time (t) graph. The particle is thrown vertically upwards.



FORUM 8 (UNIT 4-PART 2)

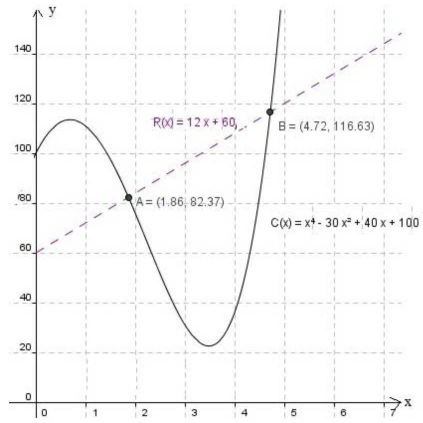
As you know of, we use optimization in Economic models. Given the graphs of cost function,

$$C(x)=x^4 - 30x^2 + 40x + 100 \text{ (in euros)}$$

and Revenue function , R(x)=12x+60 (in euros) where x shows number of units produced.

a) Interpret what happens to profit at points A and B. How will the profit change after points A and B

b)Interpret on the number of units to be produced to maximize the profit.



Hint: You may watch the video:
http://www.youtube.com/watch?v=LmftTXTE3Fw&feature=relmfu

APPENDIX F

DERIVATIVES TEST

GRADE 11 MATH SL

Name/Lastname :_______________ **Mark: / 70**

TIME 80 minutes

DATE: March 2011

INSTRUCTIONS TO CANDIDATES

Write your name in the space at the top of this page. Answer all questions in ***pen.***
You <u>*must*</u> *show all work where necessary. Remember marks are given for correct method, provided this is shown by written work. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. For π, use your calculator value.*

<u>*Make sure that your work is tidy*</u>

<u>Paper 1</u>*: Write your answers in the spaces provided on the question paper.*

NO CALCULATORS ARE ALLOWED

<u>Paper 2:</u> *Write your answers in the spaces provided on the question paper.*

CALCULATORS ARE ALLOWED

Duration:35 minutes

PAPER 1. *Show all your answers in the spaces provided. No calculators are permitted in this section. Remember to show all your working. Answers must be given to 3 significant figures unless otherwise stated.*

1. Differentiate each of the following with respect to *x*.

a) $y = 3x^2 - 2x + 5$

(1)

b) y=$-2x(\sqrt{x} - 1)$

(2)

c) $y = \frac{x^2+3x-\sqrt{x}}{5x}$

(4)

2. For each of these functions **f (*x*),** find ***x*** such that **f ′(*x*)** has the given value.

a) f(x)= x + $3x^2$ and f ′(*x*)=7

(2)

b) f(x)=(x+3)(2x-1) and f ′(*x*)= -3

(3)

3. Given the function *g* (*x*) = x^2 +4k*x* -(*m* + 2), determine the values of *k* and *m* such that *g* (2) = 14 and *g′* (-1) = 2.

(4)

4. Consider the function $f : x \mapsto 2x^2 + 3x + m$

(a) Write down $f'(x)$.

(1)

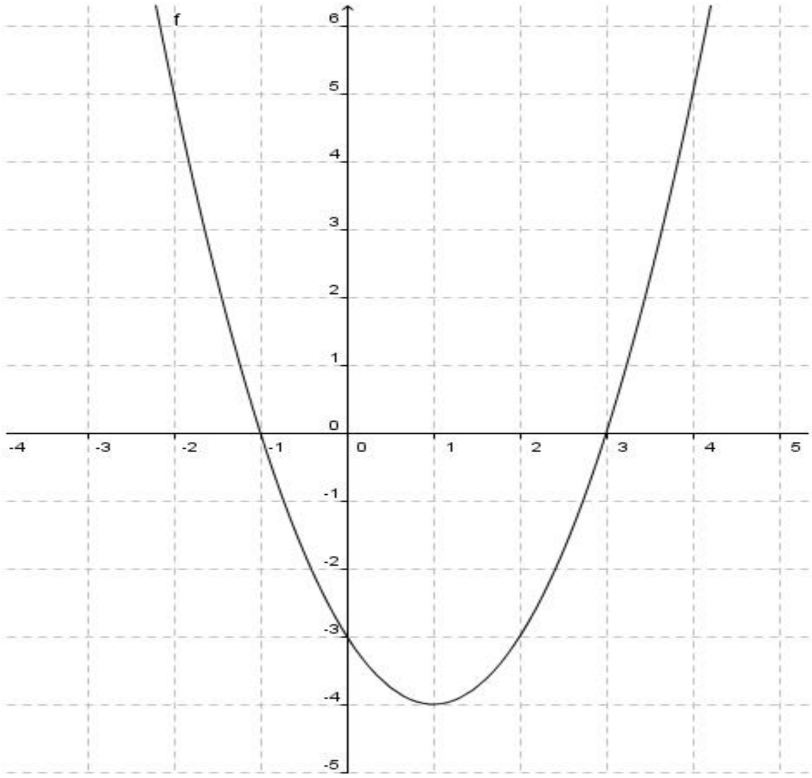
(b) The equation of the tangent to the graph of $f(x)$ at $x = a$ is $y = 4x + 1$. Find the value of a;

(2)

(c) m.

(3)

5. The following diagram shows part of the graph of a quadratic function, with equation in the form $y = (x - k)(x - t)$, where $k, t \in \mathbb{Z}$.



(a) Write down

(i) the value of k and of t;

(2)

(ii) the equation of the axis of symmetry of the curve.

(1)

(b) Find the equation of the function in the form $y = (x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(3)

(c) Find $\frac{dy}{dx}$.

(2)

(d) Let T be the tangent to the curve at the point (0, -3). Find the equation of T.

(2)

Duration: 45 minutes

Name/Last name:-______________________________

PAPER 2. *Show all your answers in the spaces provided. Calculators are permitted in this section. Remember to show all your working. Answers must be given to 3 significant figures unless otherwise stated.*

6. Given that $f(x) = (2x + 1)^3$ find

(a) $f'(x)$

(2)

(b) Find the equation of the normal to the curve at x= -1

(4)

7. Find the equation of the tangent line to the curve y=$x^2 - x + 1$ which is parallel to the line y=3x-5

(6)

8. $h(x)=(x+1)^3$ and $g(x)=\sin(2x)$

(a) Find $(h o g)(x)$

(2)

(b) Show that $\frac{d(h o g)(x)}{dx}$ (derivative of $(h o g)(x)$ with respect to x) is $6[\sin(2x)+1]^2\cos(2x)$

(4)

(c) Find the value of $\frac{d(h o g)(x)}{dx}$ at $x = \frac{\pi}{6}$

(1)

9. The line $y=x+6$ meet the curve $y=x^2$ at P and K

(a) Find the coordinates of P and K

(2)

(b) Find the equations of tangents to the curve at P and K.

(4)

(c) Find the points of intersection of tangents at P and K.

(3)

10. The equation of a curve may be written in the form $y = \frac{1}{2}x^2 - x - 4$.

The curve intersects the x-axis at A(–2, 0) and B(4, 0).

(a) (i) Find $\frac{dy}{dx}$.

(1)

(ii) A tangent is drawn to the curve at a point P. The gradient of this tangent is 5. Find the coordinates of P.

(3)

(b) The line L passes through B(4, 0), and is perpendicular to the tangent to the curve at point P.

(i) Find the equation of L.

(3)

(ii) Find the x-coordinate of the point where L intersects the curve again.

(3)

APPENDIX G

APPLICATIONS OF DERIVATIVES TEST

GRADE 11 MATH SL

Name/Lastname :_______________ **Mark: / 66**

TIME 70 minutes

DATE: April 2011

INSTRUCTIONS TO CANDIDATES

Write your name in the space at the top of this page. Answer all questions in ***pen.***
You must show all work where necessary. Remember marks are given for correct method, provided this is shown by written work. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. For π*, use your calculator value.*

Make sure that your work is tidy

Paper 1*: Write your answers in the spaces provided on the question paper.*

NO CALCULATORS ARE ALLOWED

Paper 2: *Write your answers in the spaces provided on the question paper.*

CALCULATORS ARE ALLOWED

Duration:25 minutes

PAPER 1. *Show all your answers in the spaces provided. No calculators are permitted in this section. Remember to show all your working. Answers must be given to 3 significant figures unless otherwise stated.*

1. The displacement *s* metres of a car, *t* seconds after leaving a fixed point A, is given by

$s = 16t - 2t^2$.

(a) Calculate the velocity when $t = 0$.

(3)

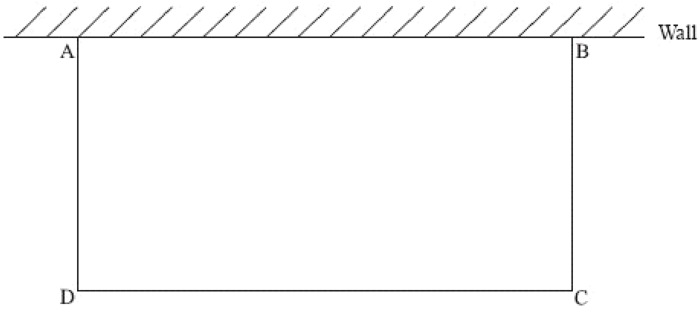
(b) Calculate the value of *t* when the velocity is zero.

(2)

(c) Calculate the displacement of the car from A when the velocity is zero.

(1)

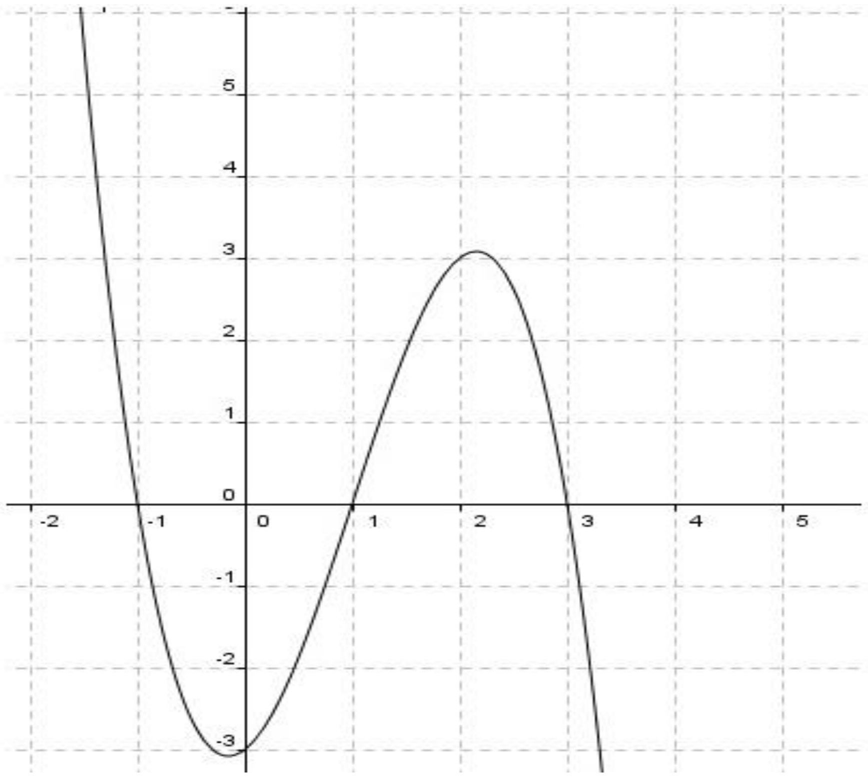
2. The following diagram shows a rectangular area ABCD enclosed on three sides by 80 m of fencing, and on the fourth by a wall AB.



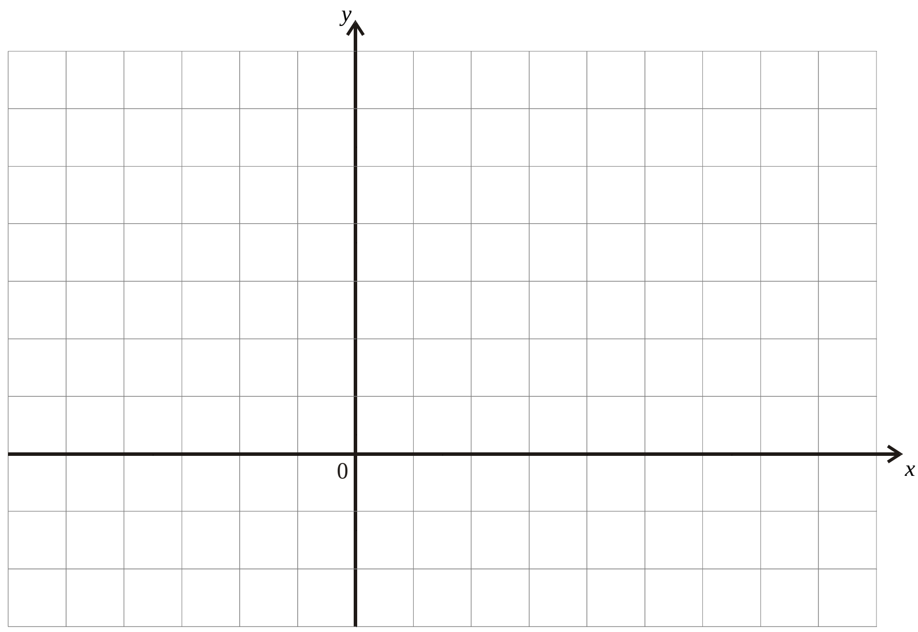
Find the width of the rectangle that gives its maximum area

(6)

3. The diagram shows the graph of $y = f(x)$.

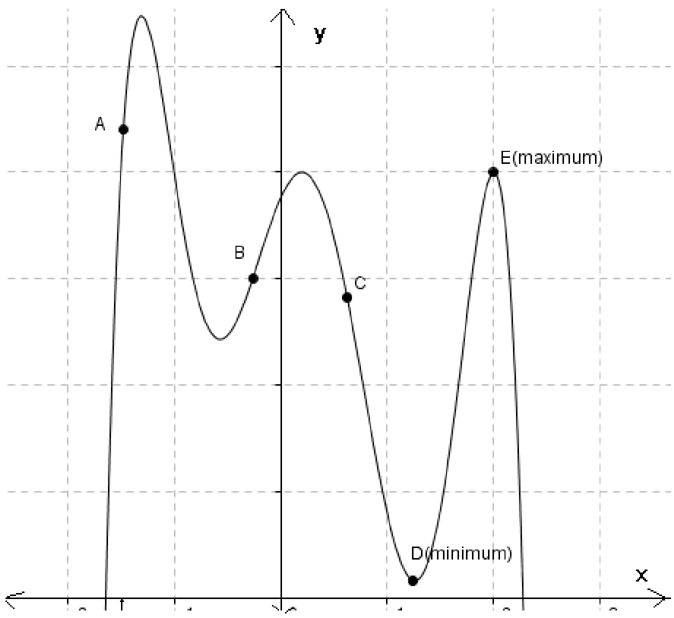


On the grid below sketch the graph of $y = f'(x)$. (Show intercepts and stationary points)



(6)

4. The graph of a function *f(x)* is given in the diagram below.



The curve has its maximum value at point E and its minimum value at point D. B and C are inflection points of *f (x)*

Complete the table below, by stating whether the first derivative *f ′* is positive, negative or zero, and whether the second derivative *f ″* is positive, negative or zero at the given points.

points	*f ′*	*f ″*
A		
B		
E		

(6)

Duration: 45 minutes

Name/Last name:-______________________________

PAPER 2. *Show all your answers in the spaces provided. Calculators are permitted in this section. Remember to show all your working. Answers must be given to 3 significant figures unless otherwise stated.*

5. The graph of $y = x^3 + 4x^2 - 12x + 2$ has a minimum point between $x = 0$ and $x = 2$. Find the coordinates of this minimum point algebraically

(6)

6. A ball is thrown vertically upwards into the air. The height, *h* metres, of the ball above the ground after *t* seconds is given by

$$h = 6 + 40t - 10t^2,\ t \geq 0$$

(a) Find the **initial** height above the ground of the ball (that is, its height at the instant when it is released).

(2)

(b) Show that the height of the ball after one second is 36 metres.

(2)

(c) At a later time the ball is **again** at a height of 36 metres.

(i) Write down an equation that *t* must satisfy when the ball is at a height of 36 metres.

(ii) Solve the equation **algebraically**.

(4)

(d) (i) Find $\frac{dh}{dt}$.

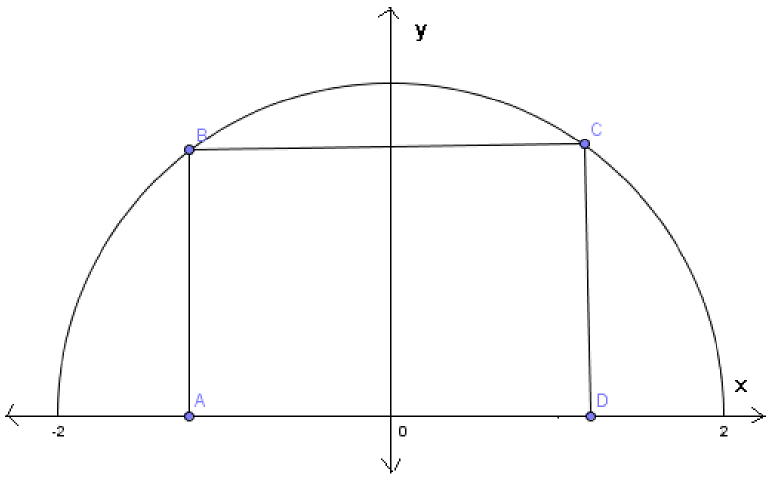
(ii) Find the **initial** velocity of the ball (that is, its velocity at the instant when it is released).

(iii) Find **when** the ball reaches its maximum height.

(iv) Find the maximum height of the ball.

(7)

7. Find the dimensions of the rectangle with the maximum area that is inscribed in a semi circle with radius 2 cm. The equation of the circle is $x^2+y^2=4$. Two vertices of the rectangle are on the semicircle and the two other vertices are on the x axis,as shown on the diagram.



(7)

8. An object is moving along a line such that its displacement,s metres ,from a fixed point at any time t,in seconds,is given by:

$$S(t)=\sin(t)+\cos(t) \text{ for the interval } 0\leq t\leq 6$$

(a)What is the object's initial displacement

(2)

(b)Find an expression for object's velocity

(2)

(c) At what time is the object's velocity maximum and what is maximum velocity?

(4)

(d) Find an expression for object's acceleration

(2)

(e) In the interval $0\leq t\leq 4$, at what time is the object's acceleration zero? Comment on the object's displacement and velocity when acceleration is zero

(4)

APPENDIX H

INTERVIEW QUESTIONS ON CALCULUS

1.What do you think on the effectiveness of various worked examples towards understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

1.1 (If there is a positive answer for 1)In what ways did various worked examples improve your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

1.2 (If there is a positive answer for 1)Can you give examples on how various worked examples contributed your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization? Please explain the reasons and the situations on how they contributed for your better understanding in details.

1.3 (If there is a negative answer for 1)In what ways did various worked examples affect your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

1.4 (If there is a negative answer for 1) Can you give examples on how various worked examples affected your understanding/interpretation product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

1.5 How could worked examples on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

2. What do you think on the effectiveness of various completion examples (backward fading) towards understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

2.1 (If there is a positive answer for 2) In what ways did various completion examples (backward fading) improve your understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

2.2 (If there is a positive answer for 2) Can you give examples on how various completion examples (backward fading) contributed your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization. Please explain the reasons and the situations on how they contributed for your better understanding in details.

2.3 (If there is a negative answer for 2) In what ways did various completion examples (backward fading) affect your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

2.4 (If there is a negative answer for 2) Can you give examples on how various completion examples (backward fading) affected your understanding/interpretation product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

2.5 How could completion examples (backward fading) on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

3. What do you think on the effectiveness of various completion examples (forward fading) towards understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

3.1 (If there is a positive answer for 3) In what ways did various completion examples (forward fading) improve your understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

3.2 (If there is a positive answer for 3) Can you give examples on how various completion examples (forward fading) contributed your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization? Please explain the reasons and the situations on how they contributed for your better understanding in details.

3.3 (If there is a negative answer for 3) In what ways did various completion examples (forward fading) affect your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

3.4 (If there is a negative answer for 3) Can you give examples on how various completion examples (forward fading) affected your understanding/interpretation product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

3.5 How could completion examples (forward fading) on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

4 What do you think on the effectiveness of writing reflective journals towards understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

4.1 (If there is a positive answer for 4) In what ways did writing reflective journals improve your understanding/ interpretation of trigonometric concepts on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle?

4.2 (If there is a positive answer for 4) Can you give examples on how writing reflective journals contributed your understanding/interpretation on sine-cosine rule/sector area-arc length/the graphs of trigonometric functions / solving trigonometric equations through unit circle? Please explain the reasons and the situations on how they contributed for your better understanding in details.

4.3 (If there is a negative answer for 4) In what ways did writing reflective journals affect your understanding/interpretations on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

4.4 (If there is a negative answer for 4) Can you give an example on writing reflective journals affected your understanding product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

4.5 How could writing reflective journals on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

5 How did using online group discussions affect your understanding product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

5.1 (If there is a positive answer for 5) In what ways did online group discussions improve your interpretation of product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

5.2 (If there is a positive answer for 5) Can you give examples on how online group discussions contributed your understanding/interpretations on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization? Please explain the reasons and the situations on how they contributed for your better understanding in details.

5.3 (If there is a negative answer for 5) In what ways did online group discussions affect your understanding/interpretations on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

5.4 (If there is a negative answer for 5) Can you give an example on group discussions affected your understanding product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

5.5 How could online group discussions on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

6. What do you think on the effectiveness of practice assignments towards understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

6.1 (If there is a positive answer for 6) In what ways did practice assignments improve your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization?

6.2 (If there is a positive answer for 6) Can you give examples on how practice assignments contributed your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization? Please explain the reasons and the situations on how they contributed for your better understanding in details.

6.3 (If there is a negative answer for 6) In what ways did practice assignments affect your understanding/interpretation on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization negatively?

6.4 (If there is a negative answer for 6) Can you give examples on practice assignments affected your understanding/interpretation product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization in a negative way? What experiences influenced this opinion? Please explain the reasons and the situations on how they affected your understanding negatively in details.

6.5 How could practice assignments on product rule/quotient rule, finding tangent and normal line equations, velocity-distance and rate of change problems, optimization be improved?

APPENDIX I

WORKED EXAMPLES

PART 1:WORKED EXAMPLES
UNIT 1-SIMPLE RULES OF DIFFERENTIATION

Example 1:

Find f '(x) for f(x) equal to

a) $5x^3-4x^2+x-7$

b) $8x-\frac{2}{x}-\frac{5}{x^2}$

c) $2\sqrt{x}-\frac{3}{\sqrt{x}}$

Solution 1:

Find f '(x) is the derivative function of f(x).

a) If f(x)= $5x^3-4x^2+x-7$, then by applying simple rules of differentiation, we can find

   f'(x)= $5(3x^2)-4(2x^1)+1\ (x^0)-0$. When the expression is simplified, the derivative of f(x) becomes

   f'(x)= $15x^2-8x+1$

b) If f(x)= 8x-$\frac{2}{x}$-$\frac{5}{x^2}$,then f(x) can be re-arranged as f(x)= 8x-2x^{-1}-5x^{-2}. When the derivative is taken,

f ’(x) = 8-2(-1x^{-2})-5(-2x^{-3}). When the expression is simplified, the derivative of f(x) becomes,

f ’(x) = 8+2x^{-2}+10x^{-3} . If we get rid of negative powers, the expression becomes,

f ’(x)= 8x+$\frac{2}{x^2}$+$\frac{10}{x^3}$

c) If f(x)= $2\sqrt{x} - \frac{3}{\sqrt{x}}$, then f(x) can be re-arranged as :

f(x)=$2x^{\frac{1}{2}}$ - $\frac{3}{x^{\frac{1}{2}}}$, that simplifies to

f(x)=$2x^{\frac{1}{2}}$ - $3x^{-\frac{1}{2}}$. The derivative becomes

f’(x)=2 ($\frac{1}{2}x^{-\frac{1}{2}}$)-3($-\frac{1}{2}x^{-\frac{3}{2}}$). This simplifies to:

f’(x)=$x^{-\frac{1}{2}}$+$\frac{3}{2}x^{-\frac{3}{2}}$. That equals to:

f’(x)=$\frac{1}{x^{\frac{1}{2}}} + \frac{3}{2x^{\frac{3}{2}}}$

f’(x)= $\frac{1}{\sqrt{x}} + \frac{3}{2x\sqrt{x}}$

Example 2:

Find $\frac{dy}{dx}$ for $y = \frac{1}{4x^3}$ and interpret its meaning.

Solution 2:

As $y = \frac{1}{4x^3}$, then

$y = \frac{1}{4}x^{-3}$.

$\frac{dy}{dx} = \frac{1}{4}(-3)x^{-4}$. It simplifies to:

$\frac{dy}{dx} = -\frac{3}{4x^4}$.

$\frac{dy}{dx}$ shows the derivative function of y with respect to x and it shows the instantaneous rate of change in y as x changes.

Example 3:

For each of the given functions, find x such that g'(x) has the given value.

a) g(x)=$(2x-1)^2$ and g'(x)=4

b) g(x)=$\frac{2x^3+1}{x^2}$ and g'(x)=-14

c) g(x)=$\sqrt{x}(3x-1)$ and g'(x)=0

Solution 3:

a) If g(x)=$(2x-1)^2$, then g(x)=$4x^2$-4x+1. When the gradient function is found for g(x),

g'(x)=4(2x)-4

g'(x)=8x-4. The condition given for g'(x)=4. So in order to find x for when g'(x)=4, we will equal both equations:

8x-4=4

8x=8

x=1

b) First we need to simplify g(x) before we find gradient function.

If g(x)=$\frac{2x^3+1}{x^2}$, then if the denominator is split for each of the term on numreator:

g(x)=$\frac{2x^3}{x^2}+\frac{1}{x^2}$. When the denominator simplifies with numerator, g(x) becomes

g(x)=2x+$\frac{1}{x^2}$. This simplifies to:

g(x)=2x+x^{-2}

When the derivative of g(x) is taken, g'(x)=2-2x^{-3}. When the given condition is matched with g'(x).

2-2$x^{-3}=-14$. When the equation is re-arranged:

16=2x^{-3}

8=x^{-3}. That is:

8=$\frac{1}{x^3}$. When cross multiplication is applied:

$8x^3=1$

x^3=$\frac{1}{8}$.when cube root of both sides is taken x=$\sqrt[3]{8} = \sqrt[3]{2^3}$. Then

x=2

c) If g(x)=$\sqrt{x}(3x-1)$, when the terms are multiplied, g(x)=3x$\sqrt{x} - \sqrt{x}$,

If g(x) is arranged in terms of powers:
g(x)=3$x^{\frac{3}{2}} - x^{\frac{1}{2}}$

Then derivative of g(x) is:

g'(x)=3($\frac{3}{2}x^{\frac{1}{2}}$) - $\frac{1}{2}x^{-\frac{1}{2}}$. It simplifies to:

g'(x)=$\frac{9}{2}x^{\frac{1}{2}} - \frac{1}{2}.\frac{1}{x^{\frac{1}{2}}}$. When the powers are converted to roots,

g'(x)= $\frac{9}{2}\sqrt{x} - \frac{1}{2}.\frac{1}{\sqrt{x}}$

If the condition is matched:

g'(x)= $\frac{9}{2}\sqrt{x} - \frac{1}{2}.\frac{1}{\sqrt{x}}$=0

$\frac{9}{2}\sqrt{x} = \frac{1}{2}.\frac{1}{\sqrt{x}}$.
When cross multiplication is realised,

$\frac{9}{2}\sqrt{x}\sqrt{x}.2 = 1$.This gives:
$9x = 1$

x=$\frac{1}{9}$

PART 1: WORKED EXAMPLES
UNIT 2-DERIVATIVES OF EXPONENTIAL AND TRIGONOMETRIC FUNCTIONS

Example 1:

Differentiate the following functions

d) f(x)=$e^{2x} - \ln(3x-1) + 4$

e) g(x)=$\frac{e^x - e^{-x}}{x}$

f) h(x)= $\frac{1}{\sqrt{e^x}}.(3 - x^2)$

Solution 1:

Find f '(x) is the derivative function of f(x).

a) If f(x)= $e^{2x} - \ln(3x-1) + 4$, then by applying rules of differentiation, we can find

f'(x)= $2e^{2x} - \frac{1}{3x-1}(3)$ When the expression is simplified,the derivative of f(x) becomes

f'(x)= $2e^{2x} - \frac{3}{3x-1}$

b) If g(x)=$\frac{e^x - e^{-x}}{x}$,then g(x) can be re-arranged as:

g(x)= $\frac{e^x}{x} - \frac{e^{-x}}{x}$.

When the derivative is taken by applying quotient rule to both terms in function g(x),

g'(x) = $\frac{e^x x - 1.e^x}{x^2} - \frac{e^{-x}(-1)x - 1.e^{-x}}{x^2}$.

When the expression is simplified,the derivative of g(x) becomes,

g'(x) = $\frac{e^x x - e^x + e^{-x}x + e^{-x}}{x^2}$.

c) If h(x)= $\frac{1}{\sqrt{e^x}}.(3 - x^2)$ then h(x) can be re-arranged as :

h(x)= $e^{-\frac{x}{2}}.(3 - x^2)$

The derivative can be taken through product rule

h'(x)=$-\frac{1}{2}e^{-\frac{x}{2}}.(3 - x^2) - (2x)e^{-\frac{x}{2}}$

Example 2:

Find $\frac{dy}{dx}$ for $y = \sin(2x) + \cos(lnx)$

Solution 2:

$$\frac{dy}{dx} = 2\cos(2x) - \frac{1}{\mathrm{x}}\sin(lnx)$$

Note that derivative of $\cos(lnx)$ is taken through chain rule.

Example 3:

For each of the given functions, find the slope of tangent line

d) g(x)=$\ln(\cos x)$ at x= $\frac{\pi}{3}$

Solution 3a:

If g(x)=$\ln(\cos x)$, then

g'(x)=- $sin\ x.\ \frac{1}{\cos(x)}$

g'(x)= $\frac{-\sin(x)}{\cos(x)}$ (gradient of the curve)

When the given x coordinate is substituted in g'(x), then gradient of the tangent line can be found as:

g'($\frac{\pi}{3}$)= $\frac{-\sin(\frac{\pi}{3})}{\cos(\frac{\pi}{3})}$=$\frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ = -$\sqrt{3}$

----

e) g(x)= $e^{3x} - \ln(2x - 1)$ at x=ln 2

Solution 3b:

First we need to simplify g(x) before we find gradient function.

If g(x)= $e^{3x} - \ln(2x - 1)$ then ,

g'(x)=$3e^{3x} - \frac{2}{2x-1}$ (gradient of the curve)

When x= ln 2 is substituted in g'(x), the slope of tangent line is found as:

g'(ln 2)=$3e^{3\ln(2)} - \frac{2}{2(ln2)-1}$

g'(ln 2)=$3e^{\ln(2^3)} - \frac{2}{ln2^2-1}$

g'(ln 2)=$3e^{\ln(8)} - \frac{2}{(ln4)-1}$

Note: $e^{\ln(8)}$simplifies to "8" because of the law of logarithms: $e^{log_e(b)} = b,$ where b is a positive real number

g'(ln 2)=$3(8) - \frac{2}{(ln4)-1}$

g'(ln 2)=$24 - \frac{2}{(ln4)-1}$≈18.8 (correct to 3 significant figures)

--

c)g(x)= tan(4x+1) at x=0

Solution 3c :

Given that g(x)= tan(4x+1).

Recall tan(x) = $\frac{\sin(x)}{\cos(x)}$, so g(x) can be simplified as:

g(x)=tan(4x+1) = $\frac{\sin(4x+1)}{\cos(4x+1)}$

Now, we can use quotient rule to differentiate g(x):

$$g(x) = \frac{\sin(4x+1)}{\cos(4x+1)}$$

$$g'(x) = \frac{4\cos(4x+1)\cos(4x+1) - (-4\sin(4x+1)\sin(4x+1))}{(\cos(4x+1))^2}$$

When the numerator is taken into 4 paranthesis:

$$g'(x) = \frac{4(cos^2(4x+1) + sin^2(4x+1))}{(\cos(4x+1))^2}$$

Note: $cos^2(\theta) + sin^2(\theta)$=1

$$g'(x) = \frac{4(1)}{(\cos(4x+1))^2}$$

To find the slope of tangent line, we need to replace 0 for x (given condition)

$$g'(0) = \frac{4(1)}{(\cos(0+1))^2}$$

$$g'(x) = \frac{4(1)}{(\cos(1))^2}$$

<u>**(make sure that calculators mode is in radians)**</u>

$g'(x) = \frac{4(1)}{(0.5403023059)^2}$≈13.7 (in 3 significant figures)

PART 1:WORKED EXAMPLES

UNIT 3-CURVE PROPERTIES & APPLICATIONS OF DERIVATIVES

Example 1:

Find the values of x for which the function $f(x)=2x^3-9x^2+12x+1$ is increasing and decreasing.

Solution 1:

From the definition, a function is increasing for when $f'(x)>0$; and it is decreasing for when $f'(x)<0$
$f'(x)=6x^2-18x+12$
In order to find the intervals where $f'(x)>0$ or $f'(x)<0$, we need to find critical points of the derivative function.
$0=6x^2-18x+12$
$0=6(x^2-3x+2)$. When both sides are divided by 6,
$0=x^2-3x+2$
$0=(x-1)(x-2)$
x=1 and x=2 are critical points of the derivative function.
Let's check the sign table for the derivative:

x	1	2	
f'(x)	+	-	+
f(x)	increasing	decreasing	Increasing

Hence, f(x) is increasing on $(-\infty, 1) \cup (2,\infty)$
f(x) is decreasing on (1,2)

Example 2 : Find the coordinates of stationary points on the curve $y=x^3-6x^2-15x+2$ and determine their nature. Hence sketch the curve

Solution 2:
From the definition, a function is increasing for when $\frac{dy}{dx}>0$; and it is decreasing for when $\frac{dy}{dx}<0$
$\frac{dy}{dx}=3x^2-12x-15$

In order to find the intervals where $\frac{dy}{dx}>0$ or $\frac{dy}{dx}<0$, we need to find critical points of the derivative function. A point where $\frac{dy}{dx}=0$ is called stationary point. So we need to solve for when $\frac{dy}{dx}=0$

$0=3x^2-12x-15$

$0=3(x^2-4x-5)$. When both sides are divided by 3,

$0=x^2-4x-5$

$0=(x+1)(x-5)$

x=-1 and x=5 are critical points of the derivative function.

Let's check the sign table for the derivative:

X		-1	5
$\frac{dy}{dx}$	+	-	+
y	increasing	decreasing	Increasing
y			

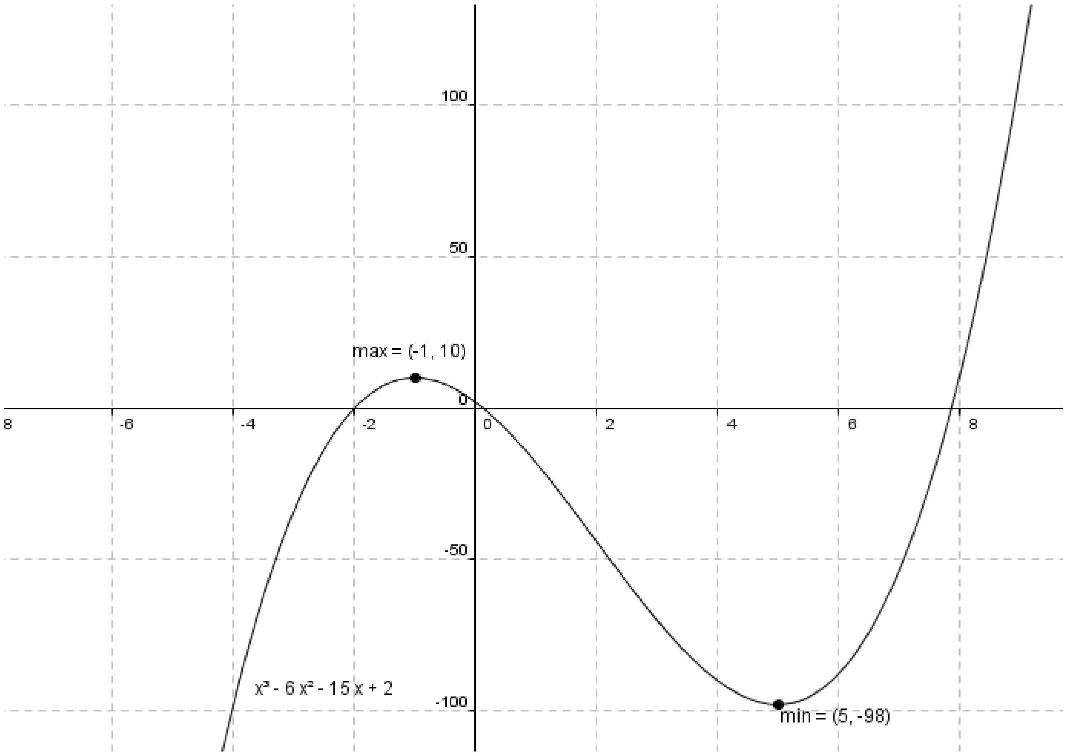
Hence, f(x) is increasing on (-∞, -1) U (5,∞)

f(x) is decreasing on (-1,5)

That means there is a local maximum at x=-1 and local minimum at x=5

Local maximum: (-1, 10) and local minimum: (5, -98).

So stationary points are (-1,10) and (5,-98)



Graph: Graph of $y=x^3-6x^2-15x+2$

Example 3: Determine the coordinates of local minimum for g(x)=x ln(x)

Solution 3: From product rule

u=x ***v=ln (x)***

u'=1 $\boldsymbol{v'=\frac{1}{x}}$

$g'(x)=1.\ln(x)+\frac{1}{x}x$

g'(x)=ln (x) + 1

If the critical points are found for g'(x):

g'(x)=0

ln (x) +1 =0

ln (x) =-1

$e^{-1}=x$

x=0.368

X	0.368	
g'(x)	-	+
g(x)	decreasing	increasing
g(x)		

If x=0.368

y= 0.368 ln (0.368)=-0.368

At (0,368, -0.368) there is a local minimum

Example 4: Find the points of inflection for y= x^4 -5x^3

Solution 4:

Point of inflection is at where $\frac{d^2y}{dx^2}$=0 and $\frac{dy}{dx} > 0$ smaller or larger values of x at $\frac{d^2y}{dx^2}$=0; OR $\frac{d^2y}{dx^2} = 0$ and when $\frac{dy}{dx} < 0$ for smaller or larger values of x at $\frac{d^2y}{dx^2}$=0

$$\frac{dy}{dx} = 4x^3 - 15x^2$$

$$\frac{d^2y}{dx^2} = 12x^2 - 30x$$

If $\frac{d^2y}{dx^2} = 0$, then

0=$12x^2 - 30x$

0=2x (6x-15)

x=0 and x=$\frac{5}{2}$ are the roots of the quadratic equation above.

It is not sufficient that both are inflection points, we need to check the sign table for $\frac{dy}{dx}$ for both of these x values

X	0	2.5	
$\frac{dy}{dx}$	-	-	-
y	decreasing	decreasing	decreasing
y			

If x=0, y=0 and if x=$\frac{5}{2}$, y= -39.01

Hence point of inflections are (0,0) and (2.5, -39.01)

Example 5: The estimated cost of a company in *t* years is given as *P(t)*=$3t^2$-18t+150 thousand euros.

a) Find the values of t where the profit will increase and decrease on the previous year.

b) Find the minimum profit of the company.

Solution 5:

a) In order to find the intervals for *t* where the profit will increase or decrease, the gradient function of P(t) must be investigated.

$$\frac{dP}{dt} = 6t - 18$$

We need to find the critical points for P(t).
0=6t-18
t=3 years
Let's check the sign table to find out increasing and decreasing intervals:

t	3	
$\frac{dP}{dt}$	-	+
P(t)	decreasing	increasing
P(t)		

P(t) is increasing on (3, ∞) and it is decreasing on (-∞, 3)

b) From the sign table in part (a), it can be observed that the minimum profit will occur in the 3rd year (t=3)
So minimum profit will be:
$P(3)= 3(3)^2-18(3)+150=123$ thousand euros

Example 6:

The volume of the remaining water of a pool (in m^3) is given as $V(t)=300(40-t)^2$ when the water is draining from the pool in t minutes.

a) Calculate the average rate at which the water leaves the pool in 10 minutes
b) Calculate the instantaneous rate at which the water is leaving the pool at t =10

Solution 6:

a) The average rate on $0 \leq t \leq 10$ is calculated as:
$\frac{V(10)-V(0)}{10-0} = \frac{300(40-10)^2-300(40-0)^2}{10}$=-21000 m^3 per minute
(21000 m^3 of water is leaving the pool per minute)

b) The instantaneous rate at 10th minute is the rate of change at that moment. Hence we need to take the derivative of V(t)

$\frac{dV}{dt}=300\ (2)(40-t)(-1)$…………(from chain rule)

$\frac{dV}{dt}=-600(40-t)$
When t=10

$\frac{dV}{dt}$ =-600(40-10)

$\frac{dV}{dt}$ = -18000 m^3 per minute
(18000 m^3 of water is leaving the pool per minute in the 10th minute)

PART 1:WORKED EXAMPLES
UNIT 4-MOTION ON A STRAIGHT LINE & OPTIMISATION

MOTION ON A STRAIGHT LINE:

Example 1:

A ball is thrown vertically upwards from the ground level. Its height is given as $h(t)=54t-2.7t^2$ metres per second. Show that its height is maximum when t=10 sec and hence find its maximum height.

Solution 1:

The ball is at maximum height when the velocity becomes zero (it is at the instant of changing direction from upwards to downwards) OR from the curve properties, maximum point can be found through when the derivative of the given function is zero.

h'(t)=54-5.4t
0=54-5.4t
t=10 sec
Let's prove at t=10 second , the ball is at its maximum height:

t	10	
h'(t)=v(t)	+	-
	Towards upwards	Towards downwards
h(t)	increasing	decreasing

When $0\leq t\leq 10$, h(t) function is increasing ; when $t\geq 10$, h(t) is decreasing. So there is a maximum point (maximum height) at t=10 seconds

So maximum height is:
$h(10)=54(10)-2.7(10^2)=270$ metres

Example 2:

A particle moves along a straight line. Its displacement from the origin is given by $s(t)=2t^3-54t+12$ cm where t is in minutes.
At what time the particle is at rest ? Find the acceleration at that time.

Solution 2:

The particle is resting when v(t)=0
$s'(t)=v(t)=6t^2-54$ cm/min
$0=6t^2-54$
$6t^2=54$
$t^2=9$
t= 3 minutes or t=-3 minutes

Time can not be negative .So the particle is at rest at t=3 minutes
The acceleration is :
$v'(t)=a(t)=12t$ cm/min^2

The acceleration at t=3 minutes is:
$a(3)=12(3)=36$ cm/min^2

Example 3:

A body moves on a straight line where its displacement is $s(t)=t^3-2t^2+t+10$ metres where t is in seconds ($t\geq 0$) . Find

a) the velocity and acceleration of the body and draw its sign diagram.
b) the conditions for the body when t=3 seconds
c) the time interval when the speed of the body is increasing

Solution 3:

a) Derivative of the dispacement (instantaneous rate of change of displacement with respect to time) shows the velocity of the object.

$s'(t)=v(t)=3t^2-4t+1$ m/sec

$0=3t^2-4t+1$

$0=(3t-1)(t-1)$

$t=\frac{1}{3}$ seconds or t=1 second

t		$\frac{1}{3}$	1
V(t)	+	-	+
	Towards right	Towards left	Towards right

Derivative of the velocity (instantaneous rate of change of velocity with respect to time) shows the acceleration of the object.

$s''(t)=v'(t)=6t-4$

$0=6t-4$

$t=\frac{2}{3}$ second

t		$\frac{2}{3}$
a(t)	-	+
	Towards left	Towards right

b) When t=3

$s(3)= 3^3-2(3^2)+3+10 =22$ m (body is 22 m on the right of origin)

$v(3)= 3(3^2)-4(3)+1= 7$ m/sec (body is moving to right with a speed of 7 m/sec)

$a(3)=6(3)-4=14$ m/sec^2 (body is accelerationg toward right)

c) We need the sign tables of v(t) and a(t) because when the signs of v(t) and a(t) are the same, speed is increasing no matter what the direction of the body is.

t		$\frac{1}{3}$	1
V(t)	+	-	+
	Towards right	Towards left	Towards right

t	$\frac{2}{3}$	
a(t)	-	+
	Towards left	Towards right

Condition 1:

When t $\geq \frac{2}{3}$ acceleration is positive; velocity is negative when $\frac{2}{3} \leq$ t$\leq$ 1 but it is positive for t$\geq$1

Hence, both acceleration and velocity is positive for t$\geq$1 second.

Therefore speed is increasing for t$\geq$1 (towards right)

Condition 2:

When t $\leq \frac{2}{3}$ acceleration is negative; velocity is negative when $\frac{1}{3} \leq$ t$\leq \frac{2}{3}$ but it is positive for t$\leq\frac{1}{3}$

Hence, both acceleration and velocity is negative for $\frac{1}{3} \leq$ t$\leq \frac{2}{3}$

Therefore speed is increasing for $\frac{1}{3} \leq$ t$\leq \frac{2}{3}$ (towards left)

Example 4:

A particle's displacement , x metres, from a fixed point is defined by the equation x(t)=sin (2t) –cos (2t) at time t seconds and 0≤ t ≤3

a) What is the maximum speed of the particle?
b) Describe particle's motion.
c)

Solution 4:

The speed can be found through the gradient of displacement function. The maximum speed can be found by analyzing for when the acceleration is zero

x'(t)=v'(t)=2. Cos (2t)+2 sin (2t) (from chain rule)

x''(t)=v '(t)=a(t)=-4sin(2t)+4 cos (2t)

When a(t)=0

0=-4sin(2t)+4 cos (2t)

0= -4(sin(2t)-cos (2t))

0= sin (2t)-cos (2t) .We need to solve for t. Take cos (2t) on the other side:

cos (2t)=sin (2t) .Divide both parts by cos(2t)

1=$\frac{\sin(2t)}{\cos(2t)}$

1=tan (2t)

So:

2t=$\tan^{-1}$(1). (Make sure that your mode is in ***radians*** if you are using calculators)

2t= $\frac{\pi}{4}$

OR

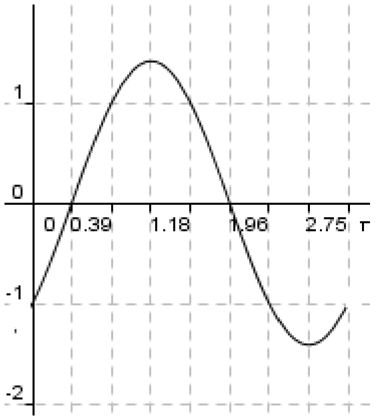
$2t = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

Then t= $\frac{\pi}{8}$ =0.393 seconds OR t= $\frac{5\pi}{8}$ =1.96 seconds.
When the graph of x(t) is drawn and analyzed on calculator:

t	0.393		1.96
v'(t)	+	-	+
v(t)	increasing	decreasing	increasing

So the maximum speed will be when t=0.393 sec
V(0.393)=2.83 m/sec

d) The displacement graph of the object shows that the particle is moving back and forth about the origin O in a periodic manner. That motion is knowns as simple harmonic motion.

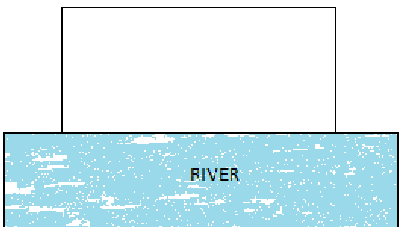


The extreme displacements of the particle is 1.41 metres (at t =1.18 sec) and -1.41 metres (at t=2.75 seconds). The motion is simple harmonic motion with a period of $\frac{2\pi}{2}$=π

OPTIMISATION (APPLIED MAXIMA AND MINIMA PROBLEMS)

Example 5:

Mario wants to eclose a rectangular garden next to river with a 60 m of rope. The river side will not be enclosed. Find the dimensions of the rectangular garden for a maximum land area.



Solution 5:

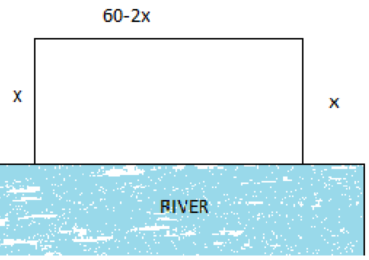
If we label width as x, the length will be:

Width+width+ length =60

x+x+lenght=60

2x+length =60

60-2x=length



Area of the rectangular garden is :

Area=A=width x lenght

A=x(60-2x)

A= $60x-2x^2$

To maximise the area of the garden we need to investigate when gradient of A is zero.

$\frac{dA}{dx}$ =60-4x

0=60-4x

X= 15 m

x	15	
$\frac{dA}{dx}$	+	-
A	increasing	decreasing

So, at x= 15, Area of the rectangle will have its maximum value. The dimensions of the rectangle are:

X=width=15 m

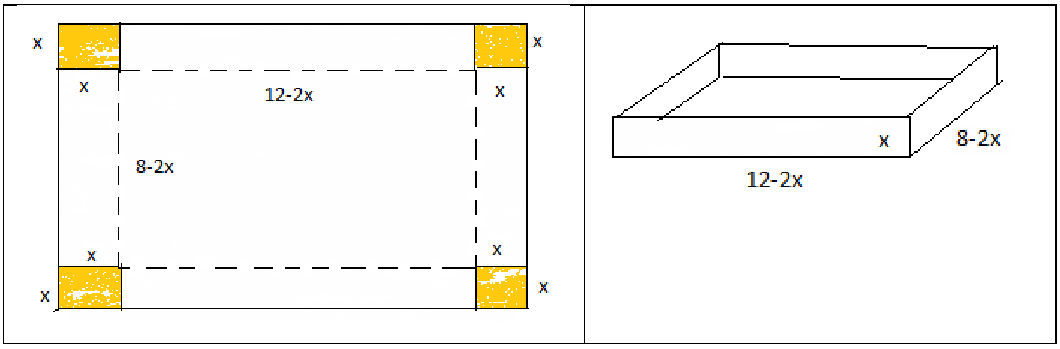
60-2x=60-2(15)=30 m (length)

Example 6:

A rectangular sheet measures 12 cm by 8 cm. Small squares of equal area are cut from each of four corners of the sheet. The remaining sides are folded to form an open box. Find the maximum volume that the box can have.

Solution 6:

If we label the lenght of cut piece by x, and if x is cut from each corner, lenght of the sheet will decrease by 2x units and width of the sheet will decrease by 2x units. (Lookj at the diagram below)



Since

The volume of a rectangular prism is :

V= heigth x length x width

V= x(8-2x)(12-2x)

$V=96x-40x^2+4x^3$

The maximum volume can be found by analyzing the condition for x that makes the gradient to be zero

$\frac{dV}{dx}=96-80x+12x^2$

$0=96-80x+12x^2$

$0=4(3x^2-20x+24)$. If quadratic formula is used to solve the equation:

X=1.57 cm or x=5.10 cm

Let'sinvestigate sign table for $\frac{dV}{dx}$ to decide on the maximum point.

x	1.57	5.10	
$\frac{dV}{dx}$	+	-	+
v	increasing	decreasing	increasing

So the volume is maximum when x=1.57 cm

$V_{maximum}=V(1.57)= 96(1.57)-40(1.57^2)+4(1.57^3)=156\ cm^3$

APPENDIX J

WORKED &COMPLETION EXAMPLES

PART 2: WORKED &COMPLETION EXAMPLES

UNIT 1-SIMPLE RULES OF DIFFERENTIATION

***Examples in odd** numbers are worked examples;**examples in even** numbers are completion examples that you need to complete*

Example 1: (Worked example) Find the gradient function of f(x) at x=4 where f(x)= $-\frac{2}{\sqrt{x}}$ +x

Solution : Gradient function of f(x) is the derivative function of f(x).

$$f(x) = -\frac{2}{\sqrt{x}} + x,$$

$$f(x) = \frac{-2}{x^{\frac{1}{2}}} + x$$

$$f(x) = -2x^{-\frac{1}{2}} + x$$

$$f'(x) = (-2)(-\frac{1}{2})x^{-\frac{1}{2}-1} + 1x^{1-1}$$

$$f'(x) = 1x^{-\frac{3}{2}} + 1$$

f '(x) = $\frac{1}{\sqrt{x^3}}$+1

at x=4

f '(4) = $\frac{-1}{\sqrt{4^3}}$+1

f '(4) = $\frac{-1}{8}$+1=$\frac{7}{8}$

Example 2: (Completion example) Find the gradient function of f(x) at x=1 where

a) f(x)= $\sqrt{x} + ex - \frac{1}{x}$

Solution 2a: Complete the following solution

Gradient function of f(x) is the derivative function of f(x).

f(x)= $\sqrt{x} + ex - \frac{1}{x}$

f(x)= $x^{\frac{1}{2}} + ex - x^{-1}$

f'(x)=$\frac{1}{2}x^{\frac{\ldots\ldots}{\ldots\ldots}}$ +.........+$x^{\ldots\ldots}$

f'(x)= $\frac{1}{2\ \sqrt{\ldots\ldots}} + \ \ldots\ldots\ldots + \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

at x=1

f'(1)= $\frac{1}{2\ \sqrt{\ldots\ldots}} + \ \ldots\ldots\ldots + \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

f'(1)= ………………………..

2b) f(x)=$\frac{3x^2-x+5}{x}$

Solution 2b: Complete the following solution

Gradient function of f(x) is the derivative function of f(x).If f(x) is rearranged by splitting the terms of the numerator:

f(x)= $3\frac{x^2}{x} - \frac{x}{x} + \frac{5}{x}$

f(x)=3x-1+$\frac{5}{x}$

f'(x)=$\ldots\ldots + \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

at x=1

f'(1)=... ... ...

Example 3 : (Worked example) Find the gradient of the curve of $y = (x^2 - x)^8$

Solution : If chain rule is used,then

f(x)= $(x^2 - x)^8$,then

f '(x)= 8. $(x^2 - x)^{8-1}.\frac{d(x^2-x)}{dx}$

f '(x)= 8. $(x^2 - x)^7.(2x-1)$ OR *f '(x)*= (16x-8). $(x^2 - x)^7$

Example 4: (Completion example)

Find $\frac{dy}{dx}$ using chain rule for

a) y=$(x - \sqrt{x})^6$

Solution 4a : Complete the following solution

y=$(x - \sqrt{x})^6$, then

$\frac{dy}{dx}$= .... $(x - \sqrt{x})^{\ldots\ldots-1}.\frac{d(x-\sqrt{x})}{dx}$

$\frac{dy}{dx}$= .... $(x - \sqrt{x})^{\ldots\ldots\ldots\ldots}$

4b) $y=\frac{1}{(2x^3-7)^2}$

Solution 4b: Complete the following solution

Given that

$y=\frac{1}{(2x^3-7)^2}$, then

$y=(2x^3 - 7)^{-2}$.

From chain rule:

$\frac{dy}{dx} = (\dots\dots.)(2x^3 - 7)^{\dots\dots\dots}\quad (\dots\dots\dots.)$

$$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots\dots.}{\dots\dots\dots\dots\dots\dots\dots}$$

4c) y=$\sqrt{3x - \pi}$

Solution 4c: Complete the following solution

Given that

y= $\sqrt{3x - \pi}$

$y = (3x - \pi)^{\frac{\dots\dots\dots\dots}{\dots\dots\dots}}$

$\frac{dy}{dx}$ =...................... So

$$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots}{\dots\dots\dots\dots\dots}$$

Example 5: (Worked example)

For each of the given functions, find gradient function by using product or quotient rule where appropriate.

a) y=$(10 + cx)(x^2 + 2)^5$

Solution 5a: *Product rule will be used as y is the product of two functions: (10+cx) and $(x^2 + 2)^5$*

a) y=$(10 + cx)(x^2 + 2)^5$

$$\frac{dy}{dx} = \frac{d(10 + cx)}{dx}(x^2 + 2)^5 + (10 + cx)\frac{d(x^2 + 2)^5}{dx}$$

(**Note that $\frac{d(x^2+2)^5}{dx}$:derivative of $(x^2 + 2)^5$ is found through chain rule)**

$$\frac{dy}{dx} = c\,(x^2 + 2)^5 + (10 + cx)5\,(x^2 + 2)^4.(2x)$$

$$\frac{dy}{dx} = c\,(x^2 + 2)^5 + (100x + 10cx^2)\,(x^2 + 2)^4$$

--

5b) y= $\pi x^2\sqrt{1+x}$

Solution 5b: *Product rule will be used as y is the product of two functions: πx^2 and $\sqrt{1+x}$*

Given that y= $\pi x^2\sqrt{1+x}$, then

$$y=\pi x^2(1+x)^{\frac{1}{2}}$$

$$\frac{dy}{dx}=\frac{d(\pi x^2)}{dx}(1+x)^{\frac{1}{2}}+(\pi x^2)\frac{d(1+x)^{\frac{1}{2}}}{dx}$$

$$\frac{dy}{dx}=2\pi x(1+x)^{\frac{1}{2}}+(\pi x^2)\frac{1}{2}(1+x)^{\frac{1}{-2}}.1$$

$$\frac{dy}{dx}=2\pi x\sqrt{1+x}+\frac{(\pi x^2)}{2\sqrt{1+x}}$$

5c) y=$\frac{1+x}{(4-3x^2)}$

Solution 5c: *Quotient rule will be used as y is a rational function*

$$\frac{dy}{dx}=\frac{\frac{d(1+x)}{dx}(4-3x^2)-\frac{d(4-3x^2)}{dx}(1+x)}{(4-3x^2)^2}$$

$$\frac{dy}{dx}=\frac{1.(4-3x^2)-(-6x)(1+x)}{(4-3x^2)^2}$$

$$\frac{dy}{dx}=\frac{4+6x-9x^2}{(4-3x^2)^2}$$

Example 6: (Completion example)

For each of the given functions, find gradient function by using product or quotient rule where appropriate.

a) y=$(\frac{1}{x^2}-1)\sqrt{2x+1}$

Solution 6a: Complete the following solution. *Product rule should be used.*

Given that y=$(\frac{1}{x^2}-1)\sqrt{2x+1}$.

So

$$y=(x^{-2}-1)(2x+1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{d(x^{-2}-1)}{dx}(2x+1)^{\frac{1}{2}} + (x^{-2}-1)\frac{d(2x+1)^{\frac{1}{2}}}{dx}$$

$$\frac{dy}{dx} = \ldots\ldots\ldots(2x+1)^{\frac{1}{2}} + (x^{-2}-1)\ldots\ldots\ldots\ldots\ldots..$$

$$\frac{dy}{dx} = \ldots\ldots..+\frac{\ldots\ldots\ldots\ldots\ldots\ldots\ldots..}{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..}$$

6b) y=$\frac{3\sqrt{x}}{(1-5x)^2}$

Solution 6b: Complete the following solution.

Quotient rule should be used as y is a rational function

$$\frac{dy}{dx} = \frac{\frac{d(3\sqrt{(x)}}{dx}(1-5x)^2 - \frac{d(1-5x)^2}{dx}(3\sqrt{(x)}}{((1-5x)^2)^2}$$

$$\frac{dy}{dx} = \frac{3\frac{\ldots\ldots.}{\ldots\ldots..}x^{\cdots\cdots}(1-5x)^2 - 2(\ldots\ldots\ldots.)(\ldots\ldots.)3\sqrt{x}}{(1-5x)^{\cdots\cdots}}$$

$$\frac{dy}{dx} = \frac{\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.}{\ldots\ldots\ldots\ldots\ldots}$$

PART 2: *WORKED & COMPLETION EXAMPLES*

UNIT 2- DERIVATIVES OF EXPONENTIAL AND TRIGONOMETRIC FUNCTIONS

***Examples in odd** numbers are worked examples;**examples in even** numbers are completion examples that you need to complete*

Example 1: (Worked example) Differentiate the following

a) $y = \frac{1}{x}\ lnx$

Solution 1a:

When product rule is applied,

u=$\frac{1}{x} = x^{-1}$ v=lnx

$\frac{du}{dx}$=-1x^{-2}= $\frac{1}{x^2}$ $\frac{dv}{dx}$= $\frac{1}{x}$

$$\frac{dy}{dx} = \frac{1}{x^2}\ln x + \frac{1}{x}\frac{1}{x}$$

$$\frac{dy}{dx} = \frac{lnx}{x^2} + \frac{1}{x^2}$$

b) Differentiate $y = e^{-2x+1}\sin 3x$

Solution 1b:

a)When product rule is applied

u=e^{-2x+1} v=sin 3x

$\frac{du}{dx}$=$e^{-2x+1}(-2)$ $\frac{dv}{dx}$=3cos (3x)

$$\frac{dy}{dx} = -2e^{-2x+1}\sin 3x + 3(cos3x)e^{-2x+1}$$

Example 2: (Completion example) Find the gradient of the curve

a) y = $\frac{e^x+x}{cos(2x-1)}$

Solution 2a: Complete the following solution.

If y = $\frac{e^x+x}{cos(2x-1)}$, then quotient rule can be applied to find the gradient of the curve

u=$e^x + x$ v=cos (2x-1)

$\frac{du}{dx}$=$e^x + 1$ $\frac{dv}{dx}$=-2sin (2x-1)

$$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots. - \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots}{(\dots\dots\dots\dots\dots\dots\dots\dots..)^2}$$

b)Find the gradient of the curve if f(x)= tan (5x-1)

Solution 2b: Complete the following solution

If f(x)= tan (5x-1), then f(x)= $\frac{\sin(5x-1)}{\cos(5x-1)}$ as *tan x*=$\frac{sin\,x}{cos\,x}$

Thus quotient rule can be applied to find the derivative of f(x) that is the gradient of the curve.

u=*sin (5x-1)* v=*cos (5x-1)*

$\frac{du}{dx}$=…………………… $\frac{dv}{dx}$=……………………………

$$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.. - \dots\dots.\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots}{(\dots\dots\dots\dots\dots\dots\dots\dots..)^2}$$

-----

c) Find the gradient of the curve *hog(x),* if *h(x)= cos x* and *g(x)=*e^{2x^2-x}

Solution 2c: Complete the following solution

First we need to find *hog(x)=* *h(g(x))=h(*e^{2x^2-x}*)* =……………………………………

Now, the derivative of hog(x)can be found through chain rule.

$$\frac{d(hog(x))}{dx} = \text{………………………………………………………………………….}$$

Example 3: Find the equation of tangent line if

y= sin (ln t) when t=1

Solution of 3:

This time the variable is t,that means we need to find the derivative of y with respect to t; that is $\frac{dy}{dt}$

STEP 1: Find derivative of y by chain rule

$\frac{dy}{dt}$*=cos(ln t)* $\frac{1}{t}$

STEP 2: Find derivative's value at t=1 to find the **slope of tangent line**

$\frac{dy}{dt}$*=cos(ln1)* $\frac{1}{1}$*=cos(0)=1*

STEP 3: Find y-value when t=1

y= sin (ln(1))=sin (0)=0

STEP 4: Find c in the equation of tangent line in the form of y=mx+c, where m is the slope of tangent line found in STEP 2

y=mx+c

0=1(1)+c , when t=1(given) and y=0 (found in STEP 3)

c=-1

STEP 5: substitute c and m into tangent line equation in the form: y=mx+c

y= 1x-1

y=x-1 (tangent line equation at t=1)

Example 4: Find the equation of normal line if

y= ln $(2e^2 \cos x)$ when x=$\frac{\pi}{3}$

Solution 4: Complete the following solution without calculator

STEP 1: Find derivative of y by chain rule

$\frac{dy}{dt}=-\frac{1}{e^2 \cos x}.e^2 \sin x = \frac{-e^2 sinx}{e^2 cosx} = -\tan x$

STEP 2: Find derivative's value at x=$\frac{\pi}{3}$ to find the **slope of tangent line**

$slope\ of\ tangent\ line = m_t = \frac{dy}{dt} = \frac{-sin\frac{\pi}{3}}{cos\frac{\pi}{3}} = -\tan\frac{\pi}{3} = \frac{\text{……………………………}}{\text{………………………}}$

STEP 3: Find the slope of normal line at x=$\frac{\pi}{3}$

$slope\ of\ normal\ line(m_N)x\ m_t = -1$

$m_N = \frac{\text{.................................}}{\text{...............................}}$

STEP 4: Find y-value when x= $\frac{\pi}{3}$

y= ………………………………….

STEP 5: Find c in the equation of normal line in the form of y=mx+c, where m is the slope of normal line found in STEP 3

y=mx+c

……………………=…………..($\frac{\pi}{3}$)+c , when x= $\frac{\pi}{3}$ (given) and y (found in STEP 4)

c=………………………..

STEP 6: substitute c and m into normal line equation in the form: y=mx+c

y=……………………………………………. (normal line equation at x= $\frac{\pi}{3}$)

PART 2

WORKED AND COMPLETION EXAMPLES

UNIT 3-CURVE PROPERTIES & APPLICATIONS OF DERIVATIVES

CURVE PROPERTIES

Example 1: (Worked example)

Find and classify stationary points of $y=x^3-3x+4$

Solution 1:

From the definition of stationary point we need to check for when $\frac{dy}{dx} = 0$.

$\frac{dy}{dx}= 3x^2 -3$

$0=3x^2 -3$

$x^2 =1$

$x=\pm1$

Let's check the sign table for the derivative:

x	-1	1	
$\frac{dy}{dx}$	+	-	+
y	increasing	decreasing	Increasing

Hence, y has a maximum at x=-1

y has a minimum at x=1

Let's find y values for x=1 and x=-1

$y= 1^3-3+4=2$ at x=1

$y= (-1)^3+3+4=6$ at x=-1

Thus, y has a local maximum at (-1,6)

y has a local minimum at (1,2)

Example 2 (Worked example) : Find intervals for when the function is concave upwards or downwards if $f(x)=2x^4 - 6x^3 + 7$

Solution 2: A curve is concave downwards if $f''(x) \leq 0$ for all x in an interval S.

A curve is concave upwards if $f''(x) \geq 0$ for all x in an interval S.

$f'(x)=8x^3-18x^2$

$f''(x)=24x^2-36x$. We need to solve for when $f''(x)=0$ to find critical points for the second derivative.

$0=12x(2x-3)$

x=0 and x=1.5 are the x-coordinates where there is a point of inflection.

When the sign table for f''(x) is examined, the concavity intervals for f(x) can be found

X	0	1.5	
f''(x)	+	-	+
f (x)	Concave upwards	Concave downwards	Concave upwards

f(x) is concave upwards on $(-\infty,0] \cup [1.5, \infty)$

f(x) is concave downwards on [0,1.5]

Example 3 : (Completion example) Find the coordinates of stationary points on the curve

$y=x^4-2x^2+10$ and determine their nature. Hence sketch the curve

Solution 3: *Complete the following solution*

From the definition, a function has stationary points for when $\frac{dy}{dx}=0$.

$\frac{dy}{dx}$=...

So we need to solve for when $\frac{dy}{dx}=0$

0=4x(...)

x=-1 or x=0 or x=1 are critical points of the derivative function.

Let's check the sign table for the derivative:

X	-1	0	1	
$\frac{dy}{dx}$	-	+	-	+
y	decreasing	increasing	decreasing	increasing
y				

Hence, y has a minimum at x=-1 and x=1

y has a maximum at x=0

Let's find y values for x=-1, x=0 and x=1

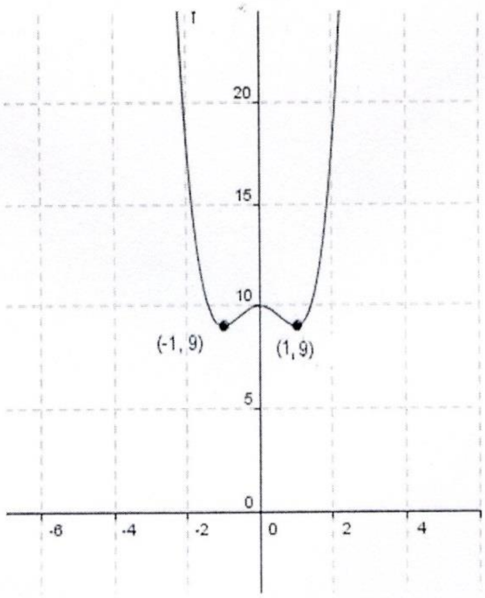
$y=(-1)^4-2(-1)^2+10$ =9 at x=-1

$y=(0)^4-2(0)^2+10$ =10 at x=0

$y=(1)^4-2(1)^2+10$ =9 at x=1

Thus, y has a local minimums at (-1,9) and (1,9)

y has a local maximum at (1,10)



Graph: Graph of $y=(x)^4-2(x)^2+10$

Example 4: (Completion example) Given that $g(x)= 2x^3+9x^2+1$

a) Determine and classify the coordinates of stationary points of g(x)
b) Determine and classify the coordinates of inflection point(s) of g(x)

Solution 3a: *Complete the following solution*

From the definition, a function has stationary points for when g'(x)=0

g'(x)=……………………………………………

0=……………………………………………

x=………….. or x=……………….

X	………	………	
g'(x)	…………………	…………………	
g(x)	………………………	………………..	……………………

If x=-3, then g(-3)= 28

If x=0, then g(0)= 1

Hence, g(x) has a local maximum at (-3,28)

g(x)has a local minimum at (0,1)

Solution 4b: *Complete the following solution*

From the definition, a function has inflection point for when g''(x)=0

g'' (x) =………………………………

0=…………………………………………

$x=\frac{\cdots\cdots\cdots}{\cdots\cdots\cdots}$ is the x-coordinate where the point of inflection exists.

Let's find whether it is a horizontal or non-horizontal inflection point by substituting this point into the first derivative:

g'(……….)= $\frac{-27}{2} \neq 0$ .So it is a non-horizontal inflection point

When we find the y-coordinate of this point

y= $\frac{29}{2}$

Thus $(\frac{-3}{2}, \frac{29}{2})$ is the non-horizontal inflection point.

Additional information:

X	$\frac{-3}{2}$	
g''(x)	-	+
g(x)	Concave downwards	Concave upwards

Example 5: (Completion Example) Find and classify stationary points and the points of inflection for

$$y= 3x^4 -8x^2 +1$$

Comment on the intervals for when y is concave upwards or downwards

Solution 5: *Complete the solution*

Stationary Points

$$\frac{dy}{dx} = \quad \ldots\ \ldots\ \ldots\ .\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots$$

0=...

x=..........., x=.................. and x=......................... are the roots of the derivative function

X	......	....	....	
$\frac{dy}{dx}$	....	....	....	....
y	....	....	....	....

If x=......., then y=...........

If x=......., then y=...........

If x=......., then y=...........

Hence y has .... points at (......, ...) and (....... , ...)

Hence y has a .... point at (......, ...)

Inflection point(s)

$\frac{d^2y}{dx^2}$= ....

0=...

x=........................ or x=................................. are the x-coordinates of inflection points.

Let's classify whether they are horizontal or non-horizontal inflection points

At x=............ , $\frac{dy}{dx}$ =................................

At x=............ , $\frac{dy}{dx}$ =................................

Hence, ...

....

Let's look at the sign table for $\frac{d^2y}{dx^2}$ to determine when y is concave upwards or downwards

X	....	....	
$\frac{d^2y}{dx^2}$	....	....	
y	....	....	....

Hence, y is concave upwards on (......., ...)

y is concave downwards on (......., ...)

APPLICATIONS OF DERIVATIVES

Example 6:

The height of a tree is given as H(t)=10 - $\frac{200}{t+2}$ metres, where t is the number of years after the tree was planted.

a) **(Completion example)** Find the rate at which the tree is growing at t=10 years.
b) **(Worked example)** Show that H'(t)> 0 for all t≥ 0. What is the significance of this result?

Solution 6:

a) *Complete the solution:*

Rate gives a hint about that you need to work with the derivative of the function, as derivative of H(t) at x=10 is the instantaneous rate of change at x=10.

H'(t)=$\frac{\dots\dots\dots\dots}{\dots\dots\dots\dots}$

H'(10)= $\frac{200}{(10+2)^2}$=1.39 m/year

b) H'(t)= $\frac{200}{(t+2)^2}$

0=$\frac{200}{(t+2)^2}$

No solution for t.

That means H'(t)= $\frac{200}{(t+2)^2}$≥0

t	0 +∞
H'(t)	+
H(t)	always increasing

Thus, the height of the tree is always increasing and the tree keeps up growing

Example 7 :(Completion Example) The quantity of a chemical in a cup is given by Q(t)=6t^2 +$\frac{1500}{t}$ cm^3 where t shows number of hours.

a) At what rate the quantity is changing at the hour of t=4.
b) At what time the quantity of the chemical will be minimum.

Solution 7a: *Complete the following solution*

Q'(t)=……………………………………………….

Q'(4)=……….. - $\frac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots\ldots}$=-45.8 cm^3 / hour

Solution 7b: *Complete the following solution*

Q'(t)=……………………………………………….

Solve for when Q'(t) =0 for the sign table

0=…………………………………………………….

t^3=…………

t=5 hours

Let's check the sign table to make sure that minimum exists at t=5 hours

Be careful that at t=0 there is a vertical asymptote:

t	5	
Q'(t)	-	+
Q(t)	dec	inc

Hence, minimum quantity occurs at t=5 hours

Example 8 (Completion example):

A ball is thrown; its height above the ground level is given as:

s(t)= 4.3 +12.4t-3.2t^2 meters , where *t* is time in seconds.

What is the maximum height of the ball?

Solution 8: *Complete the following solution*

s'(t)=……………………………………………….

0=……………………………………………….

t=………… Check the sign table

t	……..	
s'(t)	………….	……………
s(t)	…………….	………..

The height will be maximum at t=………………….. and the maximum height is : H(…………)=…………… m

PART 2:COMPLETION EXAMPLES

UNIT 4-MOTION ON A STRAIGHT LINE & OPTIMISATION
MOTION ON A STRAIGHT LINE

Example 1:

An insect moves on a straight line where its displacement is $s(t)=5t^2-12t+4$ milimetres where t is in seconds ($t\geq 0$) . Find the velocity and acceleration of the insect and draw its sign diagram. Hence, find the time interval when the speed of the insect is increasing.

Solution 1: *Complete the following solution*

Derivative of the displacement (instantaneous rate of change of displacement with respect to time) shows the velocity of the insect.

s'(t)=v(t)=................................ mm/sec

0=........................

t=1.2 sec

t	1.2	
V(t)	-	+
	Towards left	Towards right

Derivative of the velocity (instantaneous rate of change of velocity with respect to time) shows the acceleration of the insect.

s''(t)=v'(t)=a (t)=.......................... mm/sec^2

a(t)> 0

t	0 +∞
a(t)	+
	Towards right

When $t \geq 1.2$ acceleration is positive; velocity is also positive. (both a(t) and v(t) have the same signs.) Therefore speed is increasing for $t\geq 1.2$ seconds (insect is moving towards right).

Example 2: A body moves along a straight path where its displacement from the starting point is

$x(t)=25t^3-60t^2+45t+150$ metres

where t is in minutes ($t\geq 0$) . Find the time interval when the speed of the body is decreasing.

Solution 2: *Complete the following solution*

Derivative of the displacement (instantaneous rate of change of displacement with respect to time) shows the velocity of the body.

x'(t)=v(t)=................................ m/min

0=........................

t=.............min   or t= ....min

t	0		
V(t)	+	-	+
	Towards right	Towards left	

Derivative of the velocity (instantaneous rate of change of velocity with respect to time) shows the acceleration of the body.

x''(t)=v'(t)=a (t)=.......................... m/min^2

0=................

t=..........................

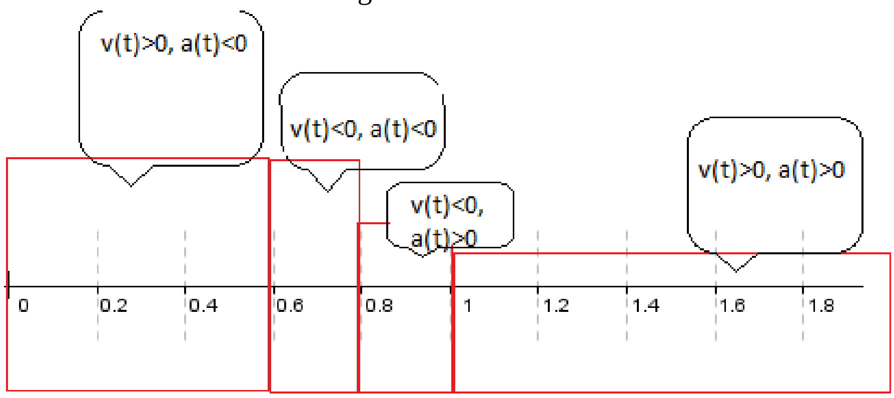
t	0	
a(t)	-	+
	Towards left	Towards right

When 0≤ t≤ 0.6 acceleration is negative; velocity is positive. Also when 0.8≤ t≤ 1 acceleration is positive but velocity is negative. (THEY MUST HAVE OPPOSITE SIGNS IN ORDER SPEED TO DECREASE)

Therefore, speed is decreasing for 0≤ t≤ 0.6 OR 0.8≤ t≤ 1

t is in [0,0.6] U [0.8, 1]

HINT : When the number line is investigated:



Example 3: A car moves on a straight road where its displacement from garage is $s(t)=t^4-8t^3-16t^2$ kms

where t is in hours (t≥0) . Find the time interval when the speed of the car is increasing.

Solution 3:

<u>OPTIMISATION</u>

Example 4:

A rectangular garden next to a wall is eclosed with a 80 m of wire. The wall side will not be enclosed. Find the maximum land area of the rectangular garden.

Solution 4: *Complete the following solution*

If we label width as x, the length will be=...........................

Area of the rectangular garden is :

Area=A=width x length

A=............................ *(in terms of x)*

To maximise the area of the garden we need to investigate when gradient of A is zero.

$\frac{dA}{dx}$ =80-4x

0=80-4x

X= 20 m

x	20	
$\frac{dA}{dx}$	+	-
A	increasing	decreasing

So, at x= 20, Area of the rectangle will have its maximum value. The dimensions of the rectangle are:

x=width=20 m

80-2x=40 m (length)

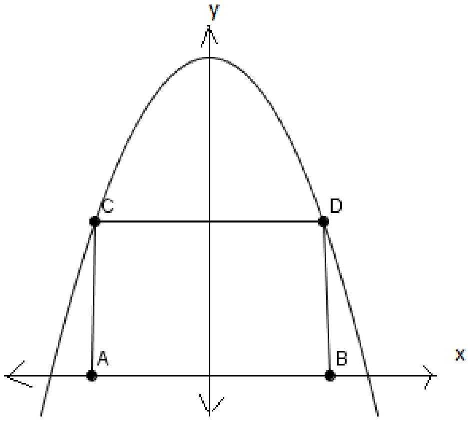
Area (max) =20 x 40= 800 m^2

Example 5.

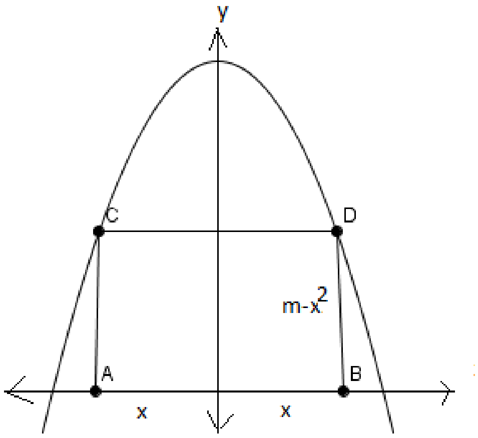
Rectange ACDB is inscribed within a parabola

$y= m-x^2$ and the x-axis, as shown.

If OB= x find m if the area of the rectangle ACDB is maximum when OB is $\sqrt{3}$, find "m".



Solution 5: *Complete the following solution*



If OB= x,
then DB=..................... (as D is on the parabola)

Area of the rectangle =A=..............................

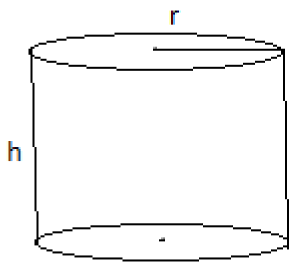
In order to find maximum point, we need to check for when gradient of A is 0.

$\frac{dA}{dx}$=...

0=..............................

It is given that when OB=x=$\sqrt{3}$, area is maximum,
So
$0=2m-6(\sqrt{3}^2)$
m=9

Example 6: The cost of metal used for surrounding a cylindrical tin cans is wanted to be minimised. The capacity of the tin can is 1 Litres (1000 cm^3). The radius of the base and top is “r” and the height of the cylinder is “h”.



a) Show that h = $\frac{1000}{\pi r^2}$ cm by using the volume of the tin can

b) Show that the total surface area is A=$2\pi r^2 + \frac{2000}{r}$ cm^3

c) Find r that makes “A” (surface area) as small as possible.

Solution 6: *Complete the solution*

APPENDIX K

PRACTICE PROBLEMS

PART 3:WORKED-COMPLETION EXAMPLES AND PRACTICE PROBLEMS

UNIT 1-WORKED AND COMPLETION PROBLEMS

Examples in odd *numbers are worked examples;* ***examples in even*** *numbers are completion examples that you need to complete*

Example 1: Find the slope of

a) tangent line to y= $\sqrt{x} + 3x^2 + 1$ at x=1

Solution 1a:

Slope of tangent line is found through substituting the given x-value into the gradient function. Hence,

y= $x^{\frac{1}{2}} + 3x^2 + 1$

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{-1}{2}}$ +6x

$\frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{x}}$ +6x .

So at x=1,

$\frac{dy}{dx} = \frac{1}{2}\frac{1}{\sqrt{1}}$ +6(1) =$\frac{13}{2}$ (slope of tangent line)

Example1b) Find the slope of normal line to y=$\frac{5x^2-x}{(3x+1)}$ at x=-1

Solution 1b:

Slope of tangent line is found through substituting the given x-value into the gradient function. The relationship between tangent line and normal line at the given point is they are being perpendicular to each other. Thus, slope of normal line is $\frac{-1}{slope\ of\ tangent\ line}$

We need to apply quotient rule so as to calculate the gradient of the curve:

$$\frac{dy}{dx}=\frac{(10x-1)(3x+1)-3(5x^2-x)}{(5x^2-x)^2}$$

So at x=-1,

$$\frac{dy}{dx}=\frac{(10(-1)-1)(3(-1)+1)-3(5(-1)^2-(-1))}{(5(-1)^2-(-1))^2}$$

$\frac{dy}{dx}=\frac{(-11)(-2)-3(6)}{(6)^2}=\frac{-40}{36}=\frac{-10}{9}$ (slope of tangent line)

As (slope of tangent line)x(slope of normal line) =-1,then

Slope of normal line=$\frac{-1}{\frac{-10}{9}}=\frac{9}{10}$

Example 2 : Find the gradient of

a) tangent line to y=(2x+1) $(x+5)^4$ at x=2

Solution 2a: Slope of tangent line is found through substituting the given x-value into the gradient function. Gradient of the curve found through product rule.

$$\frac{dy}{dx}= \dots\dots\dots\dots\dots\dots(x+5)^4+\ \dots\dots\dots\dots\dots\dots\dots\dots(2x+1)$$

At x=2,

$\frac{dy}{dx}=$ … … … … … … … … … … … … … … … = … … … … … (slope of tangent line)

Example 2b) Find the gradient of normal line to $y=(ax+2)^3$ at x=0 in terms of a (a is a real number)

Solution 2b: Slope of tangent line is found through substituting the given x-value into the gradient function. The relationship between tangent line and normal line at the given point is they are being perpendicular to each other. Thus, slope of normal line is

$$\frac{-1}{slope\ of\ tangent\ line}$$

Gradient of the curve found through chain rule

$$\frac{dy}{dx} = \ldots\ldots(ax+2)^2\ (\ldots.)$$

At x=0,

$\frac{dy}{dx} = \ldots\ldots\ldots\ldots\ldots\ldots = \ldots\ldots\ldots\ldots\ldots$ (slope of tangent line in terms of a)

As (slope of tangent line)x(slope of normal line) =-1,then

Slope of normal line=$\frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$ (in terms of a)

Example 3. Given that f(x) = -2x^3 and g(x) = 4-3x. Find the equations of

a) tangent line to *f(*x) passing through P(2,-16)

Solution 3 a:

The derivative of f(x) is : f ’(x) = -6x^2

at x= 2

f ’(2) = -6$(2)^2$=-24 (slope of tangent line)

When slope of tangent line, x and y values are substituted to the line equation in slope intercept form:

y= f’(2)x+c

-16 = -24 (2)+c

c=32

y=-24x+32 (equation of tangent line)

Example 3b) Find the equations of normal line to g(x) = 4-3x at Q(-1,7)

Solution 3b:

The derivative of g(x) is : g’(x) = -3

at x= -1

g ’(-1) = -3 (slope of tangent line)

slope of normal line =$\frac{-1}{-3} = \frac{1}{3}$

When slope of normal line, x and y values are substituted to the line equation in slope-intercept form:

y= mx+c

7= $\frac{1}{3}$(-1)+c

c=$\frac{22}{3}$

y=$\frac{1}{3}x + \frac{22}{3}$ or 3y= x+22 (equation of normal line)

Example 4: Given that f(x) = -4x^2+ 3and g(x) = 1-2x. Find the equations of

a) tangent line to *(fog)(*x) passing through A(1,-1)

Solution 4 a:

Firstly, *(fog)(x)* should be calculated:
(fog)(x)= - 4 (……..)^2 +3
The derivative of *(fog)(x)* can be calculated through chain rule:
(*fog)* '(*x*) = ……………………………….
at x= 1
(*fog) '(1)* = …………………………. (slope of tangent line)
When slope of tangent line, x and y values are substituted to the line equation in slope-intercept form:
y= mx+c
-1 = ……… (1)+c
c=……………………
y=……………………………… (equation of tangent line)

Example 4b) Given that f(x) = -4x^2+ 3and g(x) = 1-2x. Find the equations of normal line to (*gof)(*x) passing through B(-1,3)

Solution 4b: Firstly*, (gof)(x)* should be calculated:
(gof)(x)= 1- 2 (……..)
The derivative of *(gof)(x)*= can be calculated as
(gof)'(x)= = ……………………………….
at x= -1
(gof)(-1)= …………………………. (slope of tangent line)

Slope of normal =$\frac{-1}{slope\ of\ tangent\ line}$

When slope of normal line, x and y values are substituted to the line equation in slope-intercept form:
y= mx+c
3 = ……… (-1)+c
c=……………………
So y=……………………………… (equation of normal line)

UNIT 1-SIMPLE DERIVATIVES
PRACTICE PROBLEMS

Name/Lastname: **Mark : / 50**
Recommended time: *50 minutes*

1. Let $f(x) = 6\sqrt[3]{x^2}$. Find $f'(x)$.

(4)

2. Find the slope of tangent line to f(x) at given *x* values
$f(x)=\sqrt{3-2x}$ at x=1

(3)

3. Find equation of tangent line to $y=(x^2+1)^2$ at x= - 1

(5)

4. Given the function $f(x) = x^2 - 4mx + (n + 1)$, determine the values of *m* and *n* such that $f(1) = 0$ and $f'(2) = 0$.

(5)

5. Let $f(x) = \frac{2}{3}x^2 + 2x + 1$.
(a) Write down $f'(x)$.

(2)

(b) Find the x value for when gradient of normal line is zero

(3)

(c) Find the equation of the normal to the curve of *f (x)* at (-3, 1).

(4)

6. Let $f(x) = x^3 - 9x^2 + 24x - 12$

(a) Find $f'(x)$.

(2)

(b) Find the coordinates of point where the gradient of the curve is zero

(3)

(c) Find the equations of horizontal tangent lines to the curve.

(5)

7. The tangent to the curve y=2x $\sqrt{\mathbf{1 + 3x}}$ at the point (1,4) meets the x-axis at K, and the y axis at L. Calculate the area of the triangle OKL, where O is the origin.

(6)

8. The tangent of the curve $y=x^2(x+2)$ at the point (-1,1) meets the normal to the same curve at the point (-2,0), at the point Q. Find the coordinates of Q

(8)

PART 3:WORKED-COMPLETION EXAMPLES AND PRACTICE PROBLEMS

UNIT 2- DERIVATIVES OF EXPONENTIAL AND TRIGONOMETRIC FUNCTIONS

WORKED AND COMPLETION PROBLEMS

***Examples in odd** numbers are worked examples;**examples in even** numbers are completion examples that you need to complete*

Example 1: (Worked Example) The tangent to the curve *y=x ln(x)* at (e,e) meets the x-axis at A and y-axis at B. Find the distance between A and B.

Solution 1:

Step 1:Finding the derivative of y

$\frac{dy}{dx} = ln\,x + x\frac{1}{x} = lnx + 1$ (gradient of the curve)

Step 2: Finding slope of tangent at x=e

Substitute *"e"* where you see *x* in the derivative of y.

$\frac{dy}{dx} = lne + 1$=2 (slope of tangent line)

Step 3: Find c in the tangent line equation in the form y=mx+c

y=mx+c, where m is the slope of tangent line

e=2e+c

c= - e

Step 4: Substitute the found values in the equation of tangent line

y=2x-e (equation of tangent line)

Step 5: Find the coordinates of A and B

A shows the coordinates of x-intercept whereas B shows the coordinates of y-intercept.

To find A, we need to substitute 0(zero) for y in the tangent line equation, y=2x-e:

2x-e=0

2x=e

x=0.5e

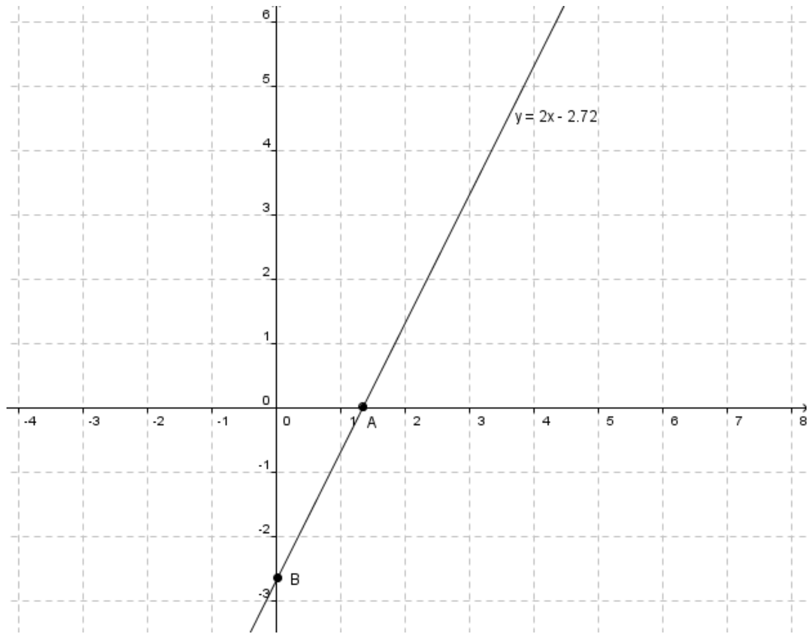
Hence, A(0.5e,0)

To find B, we need to substitute 0(zero) for x in the tangent line equation, y=2x-e:

y=2(0)-e

y=-e

Hence, B (0,-e)



Graph 1: Graph of y=2x-e

To find the distance between A and B, Pythagoras Theorem:

Distance=$d=\sqrt{|OB|^2+|OA|^2}$

$d=\sqrt{(e)^2+(0.5e)^2}$

d=3.04 units (3 significant figures)

Example 2: (Completion Example)

Given that $y = lne^{2x}$. The tangent line to the curve, y at (1,2) intersects with the normal line to the curve, y at (2,4) . Find the point of intersection.

Solution 2: *Complete the following solution*

Finding the equation of tangent line at (1,2)

$\frac{dy}{dx} = \frac{1}{e^{2x}}.e^{2x}.2 = 2$.

Thus slope of tangent line is always 2, no matter what the x coordinate is.

So we can find c by substituting (1,2) and slope of tangent line,2 into the line equation of tangent y=mx+c

2=2(1)+c

c=0

Thus the equation of tangent line is

y=2x

Finding the equation of normal line at (2,4)

$\frac{dy}{dx} = \frac{1}{e^{2x}}.e^{2x}.2 = 2$.

Thus slope of tangent line is always 2, no matter what the x coordinate is. We can find the slope of normal line by taking negative reciprocal of the slope of tangent line.

So the slope of normal line = $\frac{\ldots\ldots\ldots\ldots}{\ldots\ldots\ldots\ldots}$

So, we can find c by substituting (2,4) and the slope of normal line into the line equation of normal line y=mx+c

……….=……(…….)+c

c=…..

Thus the equation of normal line at (2,4) is

y=…………+………….

Now we need to find the point of intersection of the normal line and tangent line found above. We will make the y values to be equal to each other:

……………………..=……………………….

……………………..=……………………….

x=……

y=……

Example 3: (Worked Example)

Given that y= sin *x*+3 cos *x*, show that

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 10\sin x$$

Solution 3:

$$\frac{dy}{dx} = \cos x - 3sinx$$

$$\frac{d^2y}{dx^2} = -sinx - 3cosx$$

If $\frac{d^2y}{dx^2}$ and $\frac{dy}{dx}$ are substituted in

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$

$$-sinx - 3cosx - 3(cos\,x - 3sinx) + 2(sin\,x + 3\,cos\,x)$$

$$-\sin x + 9\sin x + 2\sin x - 3\cos x - 3\cos x + 6\cos x = 10\sin x$$

Hence

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$=10 sin x

Example 4: (Completion example)

Find the coordinates of the points on the curve $y = \frac{sinx}{1+\cos(x)}$ in the range $0 \leq x \leq 2\pi$, where the gradient is $\frac{2}{3}$

Solution 4: *Complete the following solution.*
From quotient rule, the derivative of y is:

$$\frac{dy}{dx} = \frac{cosx(1+cosx) - \sin x(-\sin x)}{(1+cosx)^2}$$

The derivative can be simplified as

$$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots\dots\dots\dots}{\dots\dots\dots\dots\dots\dots.}$$

It is given that gradient is $\frac{2}{3}$:

$\frac{dy}{dx} = \frac{\dots\dots\dots\dots\dots\dots}{\dots\dots\dots\dots\dots\dots} = \frac{2}{3}$

If the cross multiplication is conducted between two fractions
……………………………=…………………………………………
The quadratic equation above can be factorized as:

………………………………………=0

(…………………)(………………..)=0

Either cos x=$\frac{\dots\dots\dots}{\dots\dots\dots}$
or
cos x=…………………

x=…….. , ………. (from the first solution)
OR
x=……, ………(from the second solution)

PRACTICE PROBLEMS

Name/Lastname: **Mark : / 73**

Recommended time: *75 minutes*

1. The population *p* of bacteria at time *t* is given by $p = 200e^{0.02t}$.
Calculate
(a) the value of *p* when *t* = 0;

(b) the rate of increase of the population when $t = 5$.

(6)

2. Find the slope of tangent line to f(x) at given x values $f(x) = 2e^{\cos x}$ at $x = \frac{\pi}{2}$

(4)

3. Find equation of tangent line to y=1n(3x – 1) at x= 1

(6)

4. Let $y = e^{\frac{x}{3}} + 5\cos^2 x$. Find $\frac{dy}{dx}$

(6)

5. Differentiate each of the following with respect to x.

(a) $y = \cos 5x$

(1)

(b) $y = 3x \tan x$

(2)

(c) $y = \frac{\ln(x)}{2x}$

(3)

6. Let $f(x) = e^{2x} - x$.

(a) Find $f'(x)$.

(2)

(b) Find the coordinates of point where the gradient of the curve is zero

(3)

(c) Hence, find the equation of horizontal tangents to the curve

(5)

7. If y= 1n(3x – 1), then find $\frac{d^2y}{dx^2}$

(4)

8. Let $f(x) = e^{2x}$

(a) Evaluate $\frac{f(1+h)-f(1)}{h}$ for $h = 0.1$ by using your calculator

(b) What number does $\frac{f(1+h)-f(1)}{h}$ approach as h approaches zero?

(6)

9. Given that $y = e^x \cos x$.

(a) Show that $\frac{dy}{dx} = e^x (\cos x - \sin x)$.

(2)

(b) Find $\frac{d^2y}{dx^2}$.

(4)

10. Consider the function $g(x) = a \cos x - 6x$, where "a" is a constant.

(a) Find $g'(x)$.

(2)

(b) When $x = \frac{\pi}{6}$ the gradient of the curve of $g(x)$ is 4. Find the value of a.

(2)

11. Let $f(x) = 2 \cos 2x - \sin^2 x$.

(a) Show that $f'(x) = -5 \sin 2x$.

(3)

(b) In the interval $\frac{3\pi}{4} \leq x \leq 2\pi$, one tangent to the graph of f has equation $y = k$. Find the value of k.

(3)

(c) In the interval $\frac{3\pi}{4} \leq x \leq 2\pi$, one normal to the graph of f has equation $x = t$. Find the value of t.

(3)

12. Given that $y = \frac{e^x}{x^2-3}$, show that $\frac{dy}{dx} = \frac{e^x(x+1)(x-3)}{(x^2-3)^2}$. Hence find the coordinates of two points on the curve, $y = \frac{e^x}{x^2-3}$ where gradient is zero.

(6)

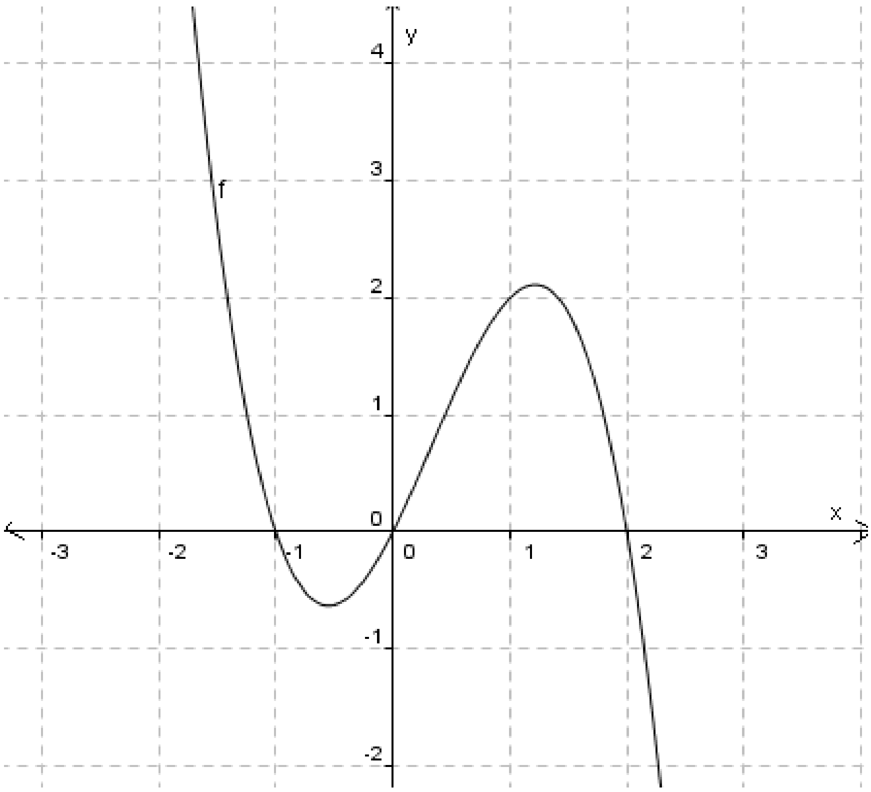
PART 3:WORKED-COMPLETION EXAMPLES AND PRACTICE PROBLEMS

UNIT 3- CURVE PROPERTIES AND APPLICATIONS OF DERIVATIVES

WORKED AND COMPLETION EXAMPLES

Example 1: (Worked example)

The diagram shows the graph of y=*f(x)*. Sketch the graph of *f'(x)*



Solution 1:

f(x) has a minimum in between $-1 \leq x \leq 0$, so f '(x)=0 at $x_{min.}$

f(x) has a maximum in between $0 \leq x \leq 2$, so f '(x)=0 at $x_{max.}$

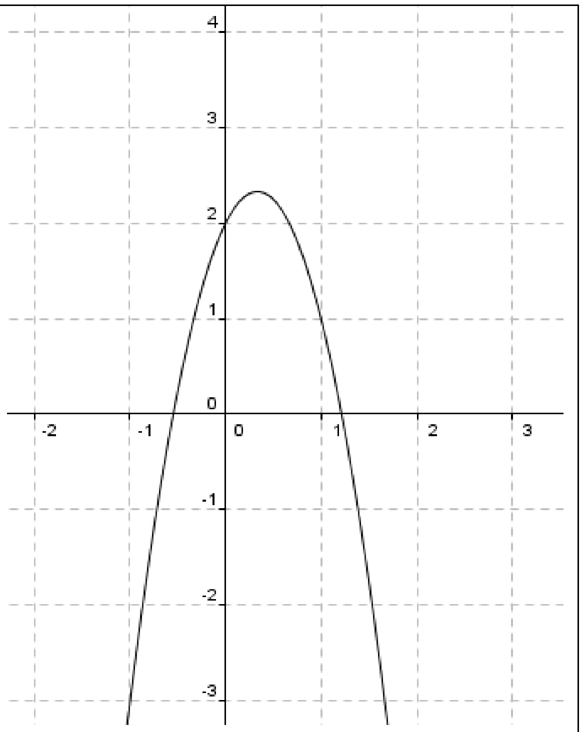
f(x) has three roots(three x-intercepts),hence f(x) is a cubic function where f'(x) has to be quadratic function (one degree less than f(x))

If f(x) is decreasing for $-\infty < x \leq x_{min}$, so f '(x) ≤ 0 there.

If f(x) is increasing for $x_{min} \leq x \leq x_{max}$, so f '(x) ≥ 0 there.

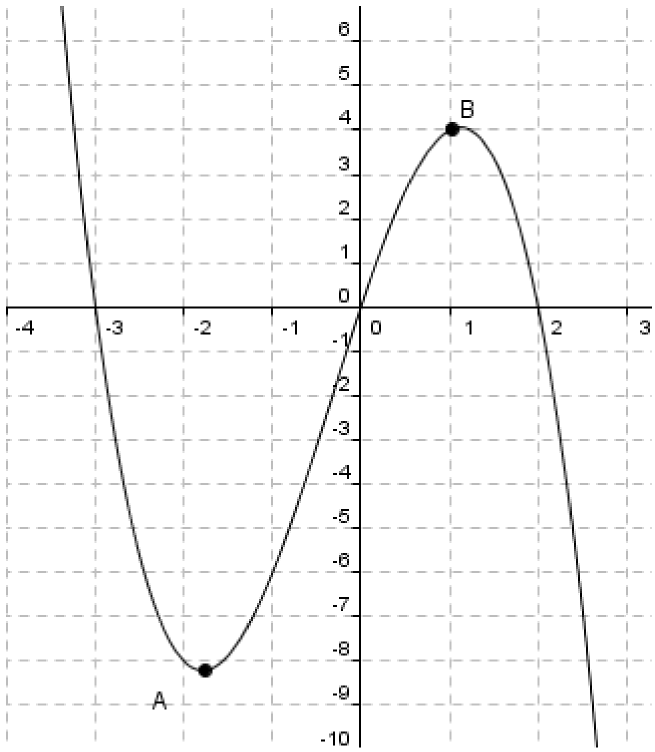
If f(x) is decreasing for $x_{max} < x < \infty$, so f '(x) ≤ 0 there.

So the graph of f '(x) will look like:



Example 2: (Completion Example)

The diagram shows the graph of g(x)=-x(x-m)(x-n). The graph cuts the x-axis at (-3,0), (0,0) and (2,0). There is a local minimum point at A and local maximum point at B.



a) Find m and n, where m<n

b) Find the exact values of x where g'(x)=0.

c) Hence, sketch the graph of g'(x)

Solution 2: *Complete the following solution*

a) If (-3,0), (0,0) and (2,0) are the x-intercepts, they are the roots of the g(x).

Thus:

g(x)= - (x-…) (x-…) (x-…)

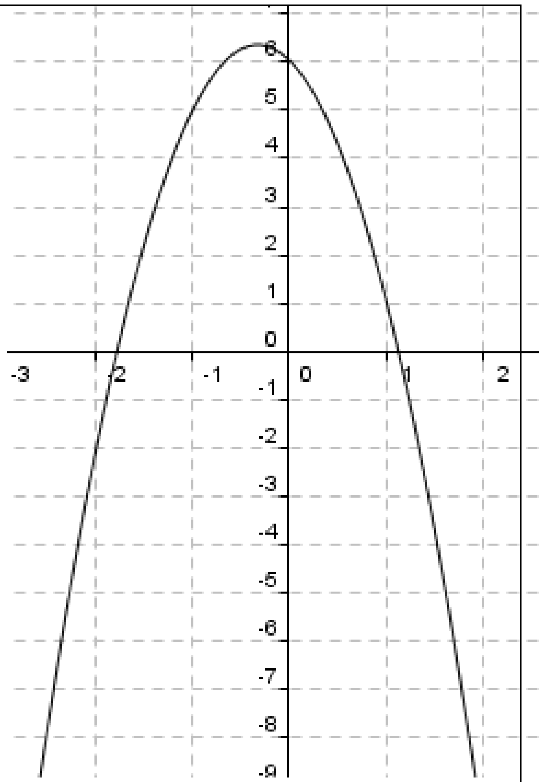
So m=-3, n=2

b) g '(x)=…………………..

If we need to find where g '(x)=0,
0=…………………………….. will give

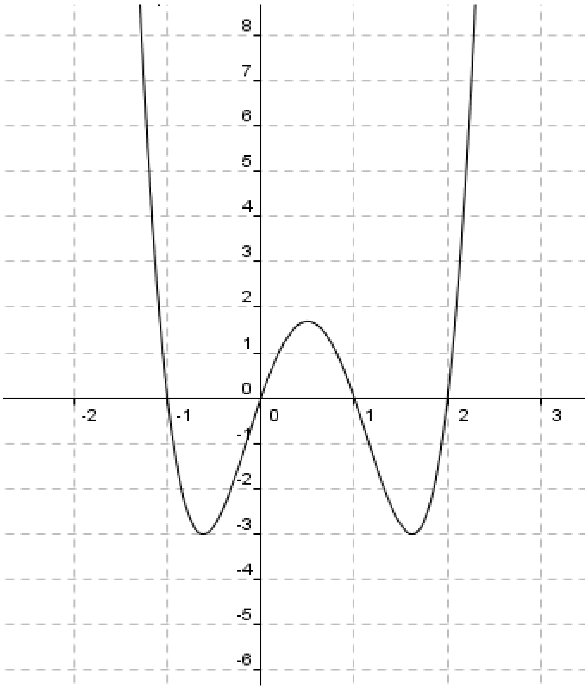
x=-1.12 or x=1.79

c) We have already found g'(x), but let's analyze the graph of g(x) further

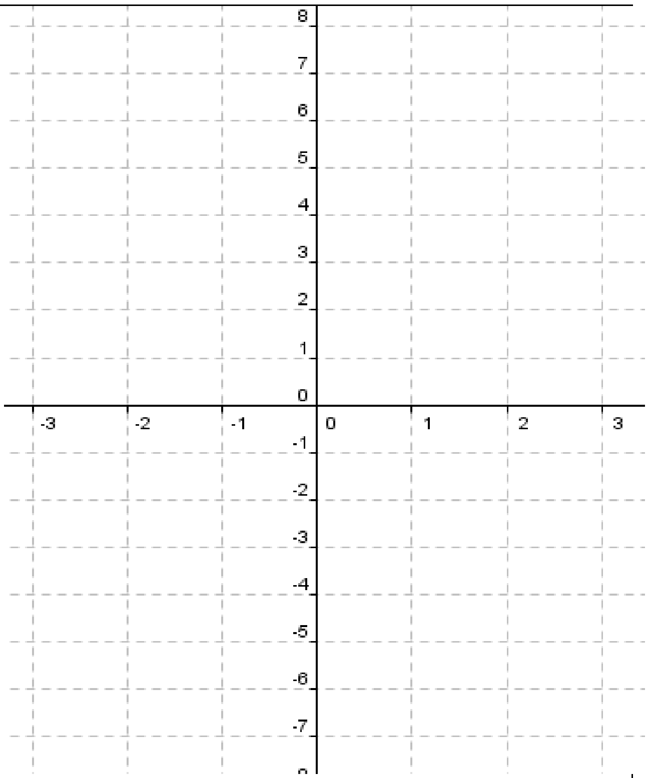
g(x) has a minimum at x=………., so g '(x)=0 at ………. (x_{min}) g(x) has a maximum at x=…………, so g'(x)=0 at x=…………… ($x_{max.}$) g(x) has three roots(three x-intercepts),hence g(x) is a cubic function where g '(x) has to be ……………… function (one degree less than g(x)) g(x) is decreasing for -∞<x≤ …….., so g'(x) ≤0 in that interval. g(x) is increasing for ……...≤ x ≤……, so g'(x) ≥0 in that interval. If g(x) is decreasing for …….≤x<∞, so g'(x) ≤0 in that interval. So the graph of g'(x) will look like:	

Example 3 (Completion example):

Given the graph of y, sketch the graph of $\dfrac{dy}{dx}$



Solution 3: *Complete the following solution*

y has minimums at x=……. and x=…….. so $\frac{dy}{dx}$ =0 at those x-coordinates y has maximums at x=……….. and x=…….. so $\frac{dy}{dx}$ =0 at those x-coordinates y has ……. roots, hence y is a …………..function where $\frac{dy}{dx}$ has to be ……………… function (one degree less than y) y is decreasing for ……….<x≤ …….. and ……….<x≤ …….., so $\frac{dy}{dx}$ ≤0 in those intervals. y is increasing for ……...≤ x ≤…… and ……...≤ x ≤…… . So $\frac{dy}{dx}$ ≥0 in those intervals.	

UNIT 3-CURVE PROPERTIES AND APPLICATIONS

PRACTICE PROBLEMS

Name/Lastname:
Date: **MARKS:** /72
Recommended Time: 75 minutes

1) Find and classify the stationary points for $f(x)=x^3+7x^2+8x-1$

(6)

2) Consider the function h(x)= $\frac{x^2}{1+2x}$.

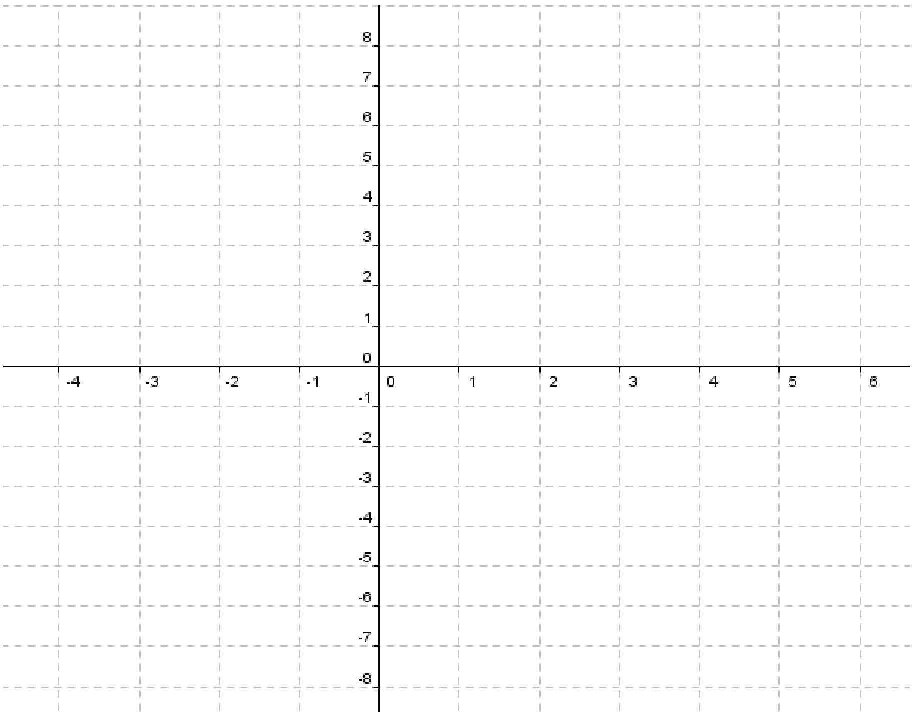
a) Find and classify the asymptotes.

(2)

b) Find and classify the stationary points.

(6)

c) Hence, sketch the graph of h(x) on the grid below by showing the coordinates of the stationary points and by drawing the asymptote lines.



(4)

3) Differentiate *y=x ln(x)***.** Hence find the coordinates of points (up to 1 decimal place) where y has a local minimum.

(6)

4) Find the coordinates of points of inflection for $0^o \leq x \leq 360^o$ for y= sin (2x)

(6)

5) The voltage in a power line t seconds after a power cut is modeled by the function

V(t)=100 e^{-2t} sin(50πt)

a) Find the voltage after t=0.02 seconds

(2)

b) Find the average rate of change in the voltage during the first hundredths of a second, giving your answer correct to three significant figures

(3)

c) Find the instantaneous rate of change at which the voltage is changing after $\frac{1}{200}ths$ of a second

(4)

6) The cost in dollars of x units of products is given by the equation
$C(x)=x^3-25x^2-100x+500, \quad x\geq 0$

Find the least number of units that should be sold to get maximum profit if each item is sold for $100.

(6)

7) Show that y= $2x^4-8x^3+1$ has a non-horizontal inflection point at x=2

(5)

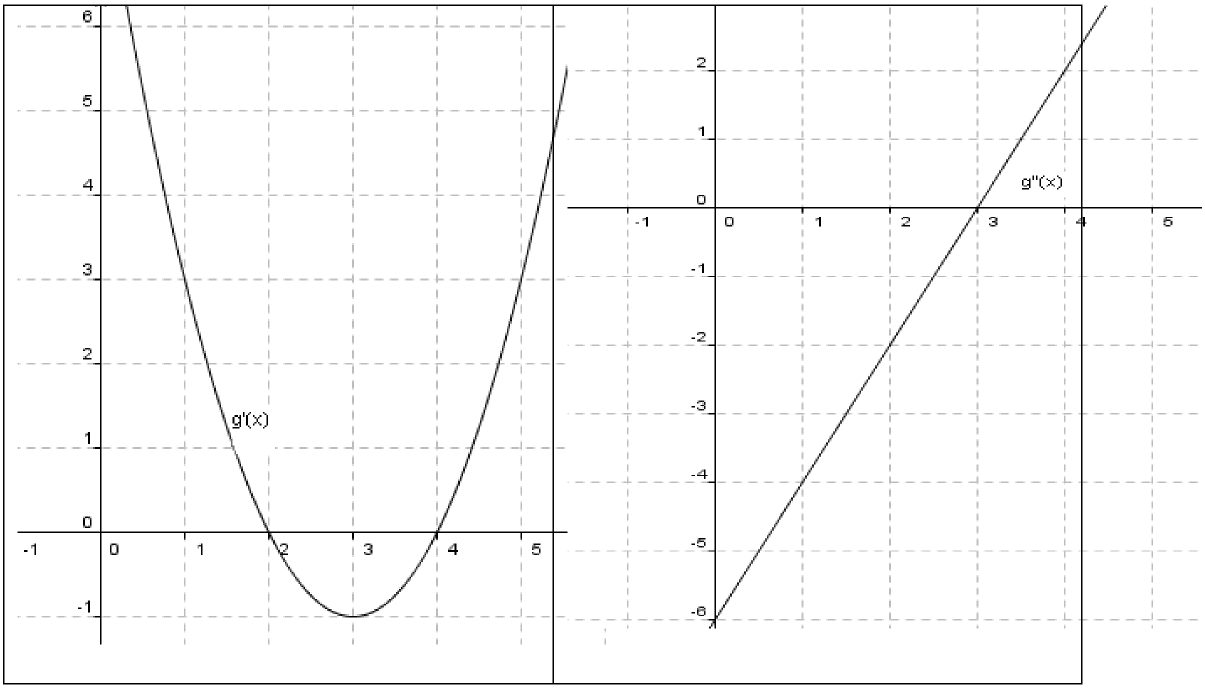
8) Find the local minimum point of y=$\frac{e^x}{2x}$

(4)

9) f(x)= $2x^3-Ax^2+1$, A is a real number. f(x) has a local minimum at x=$\frac{5}{3}$, find A and justify that the minimum is at x=$\frac{5}{3}$.

(6)

10) Let y= g(x) be a function of x. The graph of g(x) has an inflection point at K and a maximum point at L. Partial sketches of g'(x) (on the left) and g''(x) (on the right) are shown below.



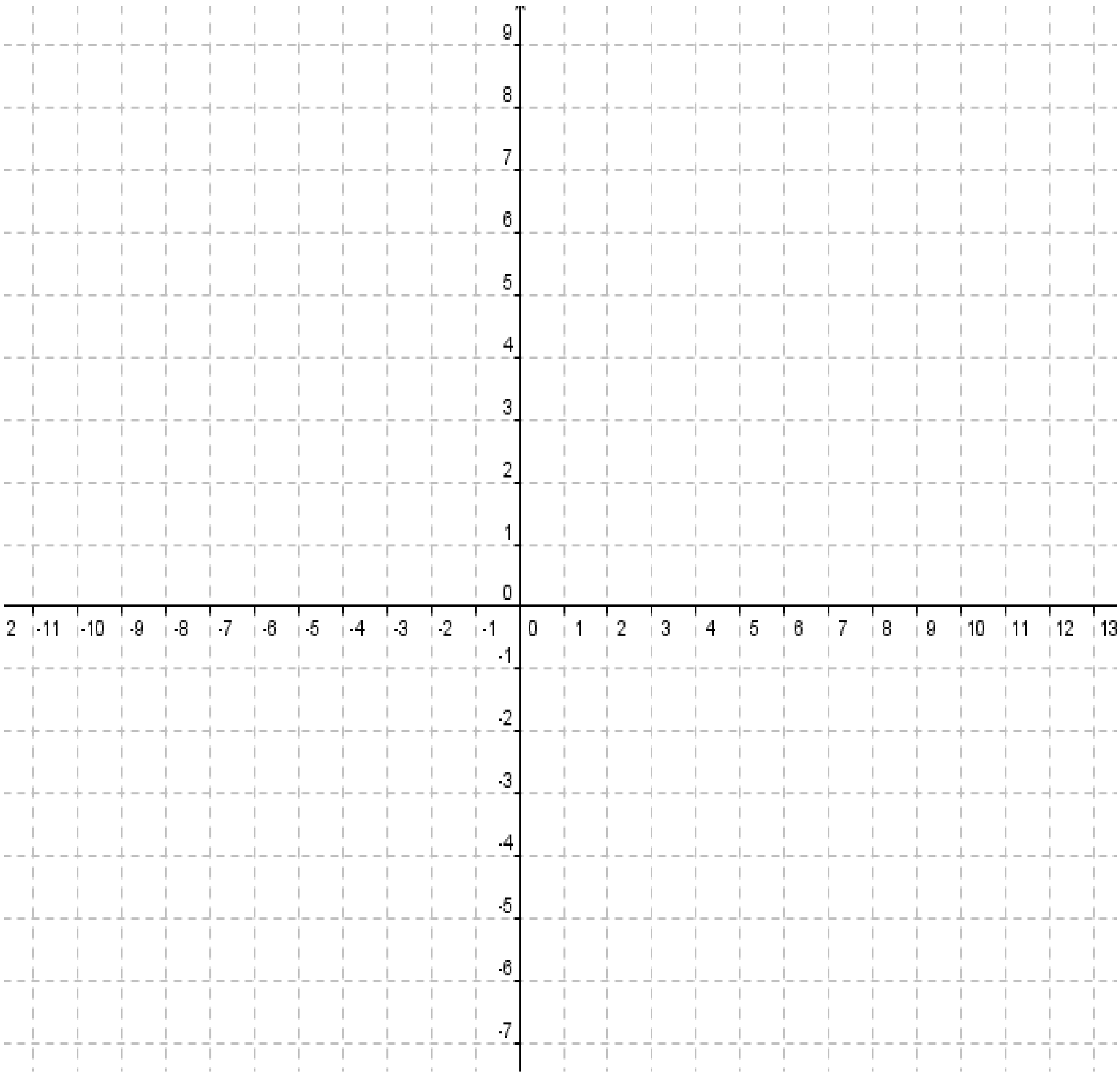
a) Write down the x-coordinate of K and justify your answer

(3)

b) Write down the x-coordinate of L and justify your answer

(3)

c) Given that g(3) =7. Sketch the graph of g(x) and mark the points K and L on your sketch.



(6)

PART 3:WORKED-COMPLETION EXAMPLES AND PRACTICE PROBLEMS

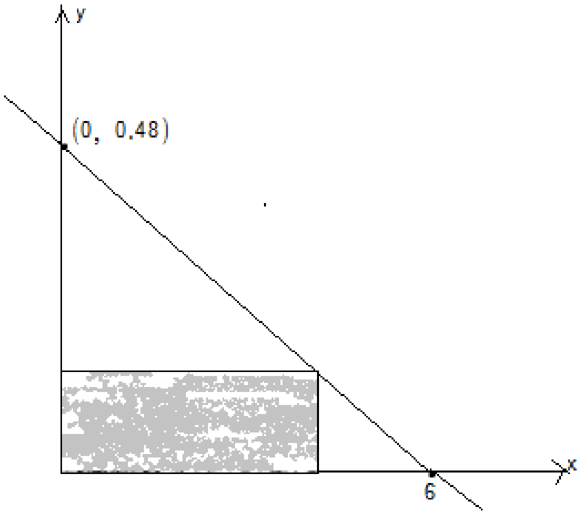
UNIT 4-MOTION ON A STRAIGHT LINE & OPTIMISATION

WORKED AND COMPLETION EXAMPLES

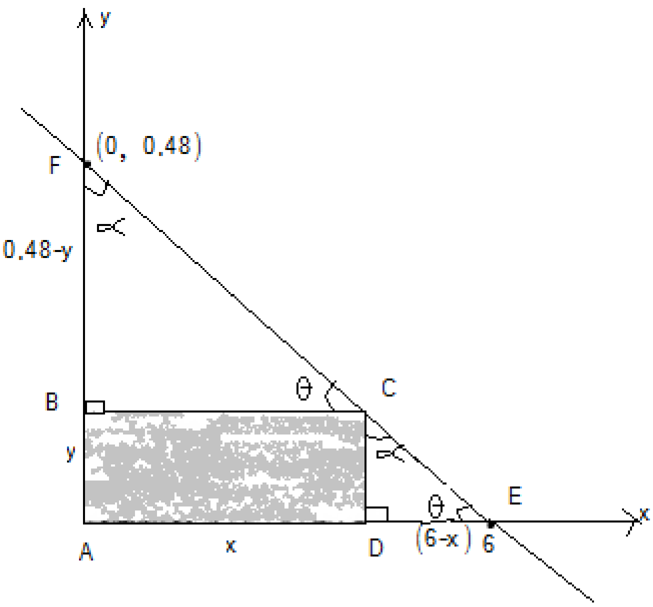
Example 1: (Worked example)

A rectangle is bounded by x- axis, y-axis and the line with equation:
y= $\frac{4}{50}(6-x)$

What are the dimensions of the rectangle with maximum area



Solution 1:



If |AD|=x, then |DE|=6-xIf |AB|=y, then |BF|=0.48-y
BCF and DEC are similar triangles if the angles are matched (AAA similarity)

So:
$\frac{|DE|}{|BC|} = \frac{|DC|}{|BF|} = \frac{|CE|}{|CF|} = k$, k constant

$$\frac{6-x}{x} = \frac{y}{0.48-y}$$

Cross multiplication will give:

2.88-0.48x+xy-6y=xy
2.88-0.48x=6y
y=0.48-0.08x

We would like to maximize area of the rectangle.

Area(A)=xy=x(0.48-0.08x)

A=0.48x-0.08x^2

$$\frac{dA}{dx} = 0.48 - 0.16x$$

$\frac{dA}{dx} = 0.48 - 0.16x$=0

x=3 cm

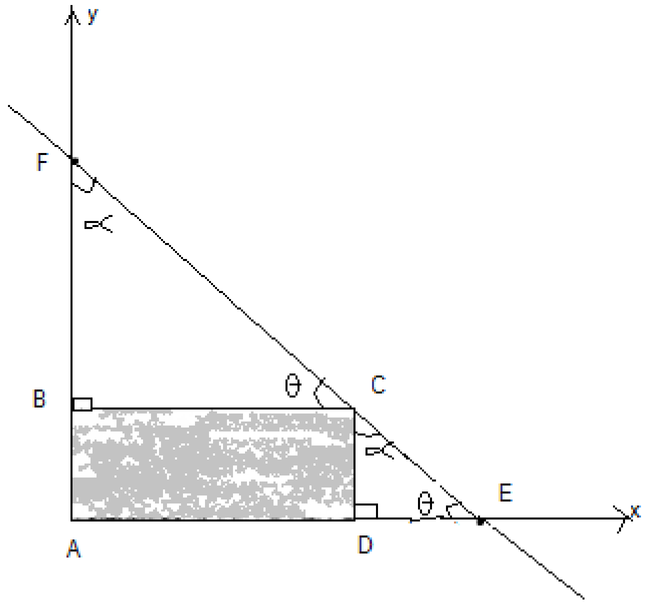
x	3	
$\frac{dA}{dx}$	+	-
A	increasing	decreasing

So at x=0.3 you have a maximum point

Length of rectangle=x=3 cm

Width of rectangle=y=0.48-(0.08(3))= 0.24 cm

Example 2: (Completion example)

A rectangle is bounded by x-axis, y-axis and the line with equation: y= $\frac{3}{5}(5-x)$ What are the dimensions of the rectangle with maximum area	

Solution 2 (Complete the solution):

If |AD|=x, then |DE|=……………………

If |AB|=y, then |BF|=…………………………

BCF and DEC are similar triangles if the angles are matched (AAA similarity)

So:
$\frac{|DE|}{|BC|}=\frac{|DC|}{|BF|}=\frac{|CE|}{|CF|}=k, \quad k$ constant

$$\frac{\dots\dots\dots\dots\dots..}{\dots\dots\dots\dots\dots\dots}=\frac{\dots\dots\dots\dots\dots\dots.}{\dots\dots\dots\dots\dots\dots..}$$

Cross multiplication will give:

15-3x+xy-5y=xy
15-3x=5y
y=3-0.6x
We would like to maximize area of the rectangle.

Area(A)=xy=x(3-0.6x)

A=3x-$0.6x^2$

$$\frac{dA}{dx} = 3 - 1.2x$$

$\frac{dA}{dx} = 3 - 1.2x$=0

x=2.5 cm

x	2.5	
$\frac{dA}{dx}$	+	-
A	increasing	decreasing

So at x=2.5 you have a maximum point

Length of rectangle=x=2.5 cm

Width of rectangle=y=3-(0.6(2.5))= 1.5 cm

UNIT 4- MOTION ON A STRAIGHT LINE & OPTIMISATION

PRACTICE PROBLEMS

Name/Lastname:
Date: **MARKS: /76**
Recommended Time: 80 minutes

1) The displacement *s* metres of a car, *t* seconds after leaving a fixed point A, is given by

$$s = 20t - 0.4t^2.$$

(a) Calculate the velocity when $t = 0$. **(3)**

(b) Calculate the value of *t* when the velocity is zero. **(2)**

(c) Calculate the displacement of the car from A when the velocity is zero. **(1)**

2) The displacement s metres at time *t* seconds is given by

$$s(t) = 4\cos 2t + t^2 + 8, \text{ for } t \geq 0.$$

(a) Write down the minimum value of *s*.

(1)

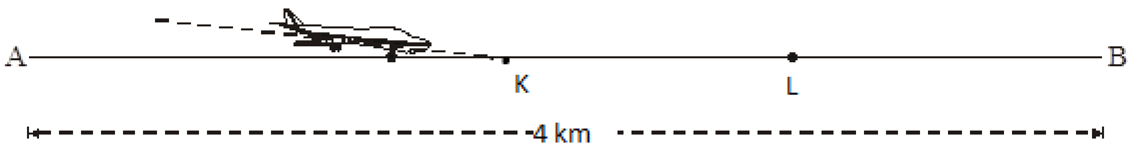
(b) Find the acceleration, *a*, at time *t*.

(2)

(c) Find the value of *t* when the **maximum** value of *a* first occurs.

(3)

3) The main runway at an airport is 4 km long. An airplane, landing at the airport, touches down at point K, and immediately starts to slow down. The point A is at the southern end of the runway. A marker is located at point L on the runway.



Not to scale

As the airplane slows down, its distance, *s*, from A, is given by

$$s = n + 200t - 8t^2,$$

where *t* is the time in seconds after touchdown, and *n* metres is the distance of K from A.

The airplane touches down 600 m from A, (*ie* $n = 600$).

(i) Find the distance travelled by the airplane in the first 3 seconds after touchdown.

(2)

(ii) Write down an expression for the velocity of the airplane at time *t* seconds after touchdown, and hence find the velocity after 3 seconds.

(3)

The airplane passes the marker at L with a velocity of 40 m s^{-1}. Find

(iii) how many seconds after touchdown it passes the marker;

(2)

(iv) the distance from L to A.

(3)

(b) Check whether the airplane touches down before reaching the point L, it can stop before reaching the northern end, B, of the runway.

(5)

4) A rock-climber slips off a rock-face and falls vertically. At first he falls freely, but after 1 second a safety rope slows him down. The height *h* metres of the rock-climber after *t* seconds of the fall is given by:

$h = 40 - 4t^2$, $\quad 0 \le t \le 1$

$h = 48 - 16t + 4t^2$, $\quad 1 \le t \le 3$

(a) Find the height of the rock-climber when $t = 1$.

(1)

(b) Sketch a graph of h against t for $0 \leq t \leq 3$.

(4)

(c) Find $\frac{dh}{dt}$ for:

(i) $0 \leq t \leq 1$

(ii) $1 \leq t \leq 3$

(2)

(d) Find the velocity of the rock-climber when $t = 1$.

(2)

(e) Find the times when the velocity of the rock-climber is zero.

(3)

(f) Find the minimum height of the rock-climber for $0 \leq t \leq 3$

(3)

5) A ball is dropped vertically from a great height. Its velocity v is given by

$$v = 30 - 30e^{-0.5t}, t \geq 0$$

where v is in metres per second and t is in seconds.

(a) Find the value of v when

(i) $t = 0$;

(ii) $t = 2$.

(2)

(b) (i) Find an expression for the acceleration, a, as a function of t.

(ii) What is the value of a when $t = 0$?

(3)

(c) (i) As t becomes large, what value does v approach?

(ii) As t becomes large, what value does a approach?

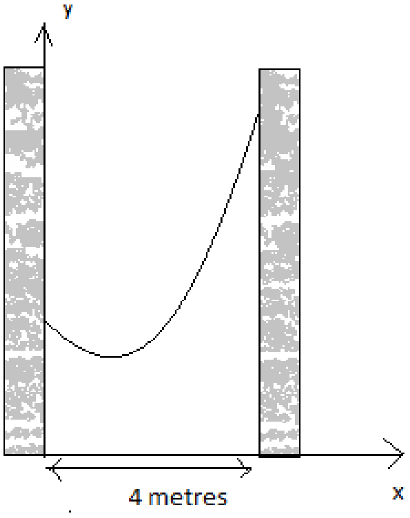
(iii) Explain the relationship between the answers to parts (i) and (ii).

(3)

6) A non-uniform metal chain hangs between two walls. The height above the ground level of this chain is given as

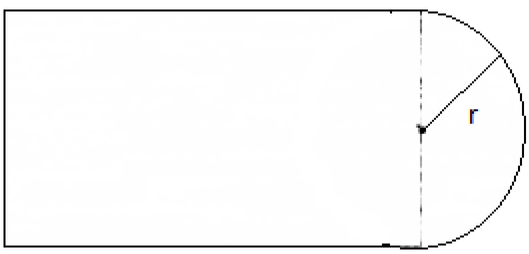
$H(x) = e^{-4x} + e^{x}$, $0 \leq x \leq 4$

Where x is the distance along the ground from the left wall. How close to the ground will chain get?



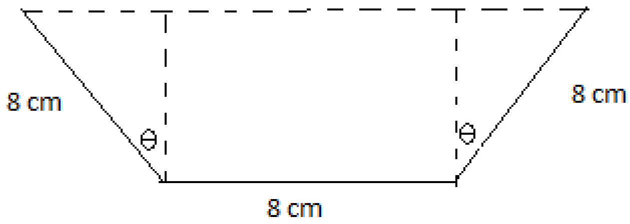
(6)

7) A pool is constructed as rectangle and a semicircle of radius r m. The perimeter of the pool to be 80 meters. Find the value of r and the dimensions of rectangular section to have maximum surface area of the pool.



(8)

8) A long flat sheet of tin 24 cm wide will be used to for a roof gutter. The sheet of tin will be turned up sides of 8 cm so that a trapezoidal cross-section will be formed as in the diagram. Find the value of θ to maximize the carrying capacity of the gutter



(13)

APPENDIX L

CODING SYSTEM FOR CALCULUS REFLECTIVE JOURNALS AND FORUMS

NEAR TRANSFER	**1. Knowledge**
	1.1 Correct terminology 1.2 Correct mathematical notation 1.3 Knowledge of general rules in calculus
	2. Comprehension
	2.1 Understands the problem 2.2 Defines the rule/ principle 2.3 Identifies the correct method 2.4 Outlines the steps of problem solution 2.5 Gives a similar example
	3. Application
	3.1. Organizes the given expressions 3.2. Correct mathematical calculation 3.3. Indication of breaking problem into parts 3.4. Gives reasons of solution steps

FAR TRANSFER	**4. Analysis**
	4.1. Comparison of new problem with learnt problem 4.2. Differentiates the new problem 4.3. Solves a problem in new situations 4.4. Applies a new technique to solve a problem 4.5. Analyzes the problem further for more information 4.6. Correct interpretation of sign tables, graphs and results
	5. Synthesis
	5.1. Creates a new problem 5.2. Generalizes a calculus rule/fact 5.3. Derives abstract relationships. 5.4. Judges the reasonableness or validity of the results 5.5. States limitations and restrictions

APPENDIX M

CODING SCHEMA FOR INTERVIEWS

Question 1. Worked examples

1. Understanding and Learning

1.1 Similar examples leads to long term memory
1.2 Solutions steps helps understanding reasoning
1.3 Recalls background knowledge (builds on the top of prior knowledge)

2. Reinforcing

2.1 For more for harder topics
2.2 For the concepts of previously learnt topics

3. Confusion

3.1 Caused confusion at understanding solution steps
3.2 There were more solutions steps than needed
3.3 Student had trouble to remember background knowledge
3.4 Solution techniques were limited. More variety needed.

Method used to Study

1. Reading Solutions

1.1 First read the solution then solved
1.2 Read the solution steps
1.3 Solved by own and then compared with the solution

Suggestion for Improvement

1 Students explain solution steps verbally
2. seeing all variety of solution techniques

Question 2. Completion examples (c.e.f.f.:forwad fading ; c.e.b.f: backwards fading

1.Guidance

1.1 c.e.b.f was helpful for understanding the unit: - guidance was there
1.2 c.e.f.f. was less helpful as it involved answers for checking
1.3c.e.f.f. was more comfortable as it answers to check
1.4 c.e.b.f . was less confusing, (once started follow up was there)
1.5 c.e.b.f. is easier to understand than c.e.f.f. (more guidance given)

2 Reinforcing

2.1 c.e.b.f. helped more for harder subjects
2.2 c.e.f.f. encouraged thinking more.
2.3 c.e.f.f. is harder : how to start makes you think more
2.4 c.e.f.f. more helpful to learn the subject

3 Challenging

3.1 c.e.f.f. was challenging at starting the solution step.
3.2 c.e.f.f. was harder: expectation how to fill the blanks was subjective
3.3 c.e.f.f. was more challenging: hard to see where the mistake is

4 Confusing

4.1 expectation on what to write or how much work should be shown were not clear.
4.2 sometimes mixing up mind if the topic was studied earlier
4.3 understanding the solution steps was hard.

Method used to Study

1. Read and completed
2. Asked a friend to explain

Suggestion for Improvement

1. Different methods should be stated
2. No completion examples should be given

Question 3 –Self-reflection

1. Thinking Deeply

1.1 Makes think deeply (the proper expressions/examples to explain and the organisation)
1.2 Prevents memorization

1.3 Assists grasping reasoning of solution steps

2. Mental Model

2.1 Encourages forming schema for the solutions of problems
2.2 Assists summing up what is learnt
2.3 Makes you notice solution steps
2.4 Brings together in different subjects

3. Learning

3.1Self teaching helped learning
3.2 Analyzing cases in details helped learning
3.3 Improves terminology

4. Challenging

4.1 Harder as it involves proper expressions to transmit knowledge

4.2 When requires background knowledge

4.3 When a new question style is asked

5. Not helpful

5.1 for understanding if it's done immediately after taught topic

Method used to Study
1.Checking notes and textbook

2. Received verbal help from a friend for explanation

Suggestion for Improvement:

1. Should consist more math calculation problems than summary
2. Should be assigned as homework at home
3. Teacher should give the outline of the reflective journal to clarify what is expected

Question 4 –Online Forum Discussions

1. Motivation

1.1. It made aware of different concepts
1.2. It enabled to link to other subjects

2. Thinking deeply

2.1 Seeing variety of questions extended thinking skills in different perspectives
2.2 Made to think more
2.3 Seeing friends' replied did not contribute much to deep thinking
2.4 Replies mixed up mind

3. Reflection Others' Thinking

3.1 Reading friends' replies added knowledge

4. Understanding

4.1 Theories helped to interpret concepts

5. Competence

5.1 Facilitated practice with further problems

6. No Learning Contribution

6.1 Prefers own solution (not friends' solutions)
6.2 Theories did not add much to problem solving
6.3 Research problems not as helpful as practice problems
6.4 Not needed. Someone can learn without forums

7. Not user friendly

7.1. Using computer to type equations is hard

7.2. Words restrict discussion. Digrams needed. Hard to plot diagrams on computer.so diagrams drawn by hand would help more to discuss.

Method used to Study

1. Gave own answer
2. Compared answer with friends'
3. When did not understand checked friends' replies

Suggestion for Improvement:

1. More group discussion needed
2. Discussion in class is better than online environment
3. Questions should be more discussion provoking
4. Instead of forums online experiments should be facilitated.

Question 5- Practice Problems

1. Reinforcing

1.1 Tested knowledge
1.2 Asked main points to emphasize learning the subject

2. Understanding & Learning

2.1 Developed critical thinking skills with variety of hard problems
2.2 Challenge made to think and analyze more
2.3 Facilitated revision with similar problems(long term memory)
2.4 No guidance facilitated thinking deeply
2.5 Practice helped to understand harder topics
2.6 Assisted permanent learning through practice
2.7 Practice lead to effective interpretation

3. Mental Model

3.1 Prepared for the test style questions by encouraging forming schema to solve problems

4. Learning

4.1 Reduced calculation mistakes by practicing with similar problems.
4.2 Harder questions helped to realize the expected level of knowledge
4.3 Duration helped to pace up
4.4 Get prepared for the test

5. Time Taking

5.1 When there are harder problems
5.2 prior knowledge is combined

Method used to study

1 Looked at notes
2 Asked to friends
3 Outlined solution steps for similar problems
4 Studied topics

Suggestion for Improvement:

1. Answers should be given at the back to check immediately
2. More time should be allocated as homework to complete the worksheet

APPENDIX N

SAMPLE WORKS OF STUDENTS AT REFLECTIVE JOURNALS

[1] Reply of S_{17} to Reflective Journal 1

Journal Entry:

Part 1 unit 1

1. Prove that; the derivative of $f(x) = 2x^2 - \frac{1}{x}$ is $f'(x) = 4x + \frac{1}{x^2}$ from simple rule & first principle

$f(x) = 2x^2 - \frac{1}{x}$

to find the derivative of f(x) we first need to take denominator x to the numerator. This will make calculations easier. since x has a power of 1, this power will become negative once it's taken to the numerator.

$f(x) = 2x^2 - x^{-1}$ ✓

simple rule tells us that if x has a power, while taking the derivative, we multiply the power & the coefficient of the x value & substitute 1 from the power. so,

$2(2)x^{2-1}$ is the derivative of the first part

$4x^1 = 4x$ ✓

For the next part we will use the same method.

$-x^{-1} \circled{=} -(-1)x^{-1-1} = +x^{-2}$

↑ no!

when combined we come up with the derivative as:

$4x + x^{-2}$

we don't want minus powers so we take x back to the denominator which is the opposite of our first step. This leaves us with:

$4x + \frac{1}{x^2}$ ✓,

not clear? ①

because x takes the denominator because (−) power suggest[s] that the # is divided by 1.

ow let's use the first principle to solve this example.

The first principle: $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

for $f(x+h)$ we write our function as

$2(x+h)^2 - \frac{1}{x+h}$, because we need to write (x+h) instead of x

$$\lim_{h\to 0} \frac{2(x+h)^2 - \frac{1}{x+h} - \left(2x^2 - \frac{1}{x}\right)}{h}$$ ← Brackets

is our equation when we write it according to the first principle, now let's try to simplify it.

$$\lim_{h\to 0} \frac{2(x^2+2xh+h^2) - \frac{1}{x+h} - \left(2x^2 - \frac{1}{x}\right)}{h}$$

$$\lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - 2x^2 - \frac{1}{x+h} + \frac{1}{x}}{h}$$

$$\lim_{h\to 0} \frac{4xh + 2h^2 - \frac{x + (x+h)}{x(x+h)}}{h}$$

Talk about common denominator

$$\lim_{h\to 0} \left(4xh + 2h^2 - \frac{x + x + h}{x(x+h)}\right) \cdot \frac{1}{h}$$

$$\lim_{h\to 0} \left(\frac{2h(2x+h)}{h}\right) + \left(\frac{\frac{-x+x+h}{x^2+xh}}{h}\right)$$ ← why is h cancelled?

$$\lim_{h\to 0} 2(2x)+h + \frac{h}{x^2+xh} \cdot \frac{1}{h} \quad \lim_{h\to 0} 2x+h + \frac{1}{x^2+xh} \rightarrow 2(2x)+0 + \frac{1}{x^2+0}$$

$$2(2x) + \frac{1}{x^2}$$

Looking at each step we can summarise this: You can never put zero to h in the first situation because it would make the solution undefined. Therefore it's important to say that if you look at first p. you have to simplify the answer until you don't find the answer to $\lim_{h\to 0}$ as zero or undefined. V. good

It is also important to encounter with plus & minus signs since they change and its also important to find common denominators here.

[2] Reply of S_{11} to Reflective Journal 2

Journal Unit 1 - Part 2 23 March

Prove that derivative of $y = \frac{2x-1}{1+4x}$ is the same through product rule & quotient rule.

Product Rule

if $y = u.v$

$\frac{dy}{dx} = u'v + v'u$

to use the product rule the function has to be rearranged to $y = u.v$ format.

$y = \frac{2x-1}{1+4x}$ we can also say that $y = (2x-1).\frac{1}{1+4x}$ to apply it to the format

in this case we can say that $y = (2x-1)(1+4x)^{-1}$

now the function is in $y = u.v$ format.

where $u = (2x-1)$ and $v = (1+4x)^{-1}$

to use the product rule, we should first find the derivative of u and v seperately:

$u' = 1.(2x-1)^{1-1} . \frac{d(2x-1)}{dy} = 1.2 = \boxed{2}$

$v' = -1.(1+4x)^{-1-1} . \frac{d(1+4x)}{dy} = -1.\frac{1}{(1+4x)^2}.4 = \boxed{-\frac{4}{(1+4x)^2}}$

now we can substitute the values into $\frac{dy}{dx} = u'v + v'u$

$\frac{dy}{dx} = 2(1+4x)^{-1} - \frac{4}{(1+4x)^2}(2x-1)$

Good!

$\frac{dy}{dx} = 2.\frac{1}{1+4x} - \frac{4(2x-1)}{(1+4x)^2} = \frac{2}{1+4x} - \frac{8x-4}{(1+4x)^2}$

we need to make the denominators equal to each other in order to come up with a single fraction.

$\frac{dy}{dx} = \frac{2+8x-8x+4}{(1+4x)^2} = \boxed{\frac{6}{(1+4x)^2}}$ → derivative of $y = \frac{2x-1}{1+4x}$

Quotient Rule

if $y = \frac{u}{v}$

$\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$

in this case quotient rule is easier to use than product rule since no re-arrangement is needed.

in this case: $u = 2x-1$ $v = 1+4x$

we find the derivatives of u and v seperately.

$\boxed{u' = 2}$

$\boxed{v' = 4}$

we can now substitute the values into the formula:

$\frac{dy}{dx} = \frac{2(1+4x) - 4(2x-1)}{(1+4x)^2} = \frac{2+8x-8x+4}{(1+4x)^2} = \boxed{\frac{6}{(1+4x)^2}}$

The results are the same in both cases.

1

[3] Reply of S_5 to Reflective Journal 3

Journal Entry
Unit 2 – Part 1

If $f(x)=e^x$, calculate derivative of $f^{-1}(x) = (f^{-1}(x))'$ and $(f' \circ f^{-1})(x)$ and show

$$(f^{-1}(x))' = \frac{1}{(f' \circ f^{-1})(x)}$$

First, we have to calculate the inverse of $f(x)$.

If $f(x)=e^x$, then

$y = e^x$

$\log_e y = x$ → $\boxed{\log_e = \ln}$

$\ln y = x$

$f^{-1}(x) = \ln x$

Next, we calculate the derivative of $f^{-1}(x) = \ln x$.

$\frac{dy}{dx} = \frac{1}{x} \cdot 1 = \frac{1}{x}$. $\boxed{\ln h(x) \rightarrow \frac{1}{h(x)} \cdot h'(x)}$

Therefore, the derivative of $f^{-1}(x) = \frac{1}{x}$.

~~Next, we calculate $(f' \circ f^{-1})(x)$.~~

~~$(f' \circ f^{-1})(x) = f'(e^x) = \frac{1}{x} \cdot 1 = \frac{x}{x}$~~

Explain each step

$f^{-1}(x) = \ln x$

$f'(x) = e^x$

Therefore

why?

$(f' \circ f^{-1})(x) = f'(\ln x) = e^{\ln x} = e^{\log_e x} = x$.

Therefore $(f' \circ f^{-1})(x)$ is the reciprocal of $(f^{-1}(x))'$.

$(f' \circ f^{-1})(x) \downarrow x$; $(f^{-1}(x))' \downarrow \frac{1}{x}$

(1)

[4] Reply of S_{18} to Reflective Journal 4

JOURNAL

UNIT 2 : Part 2

PART A

Given that $f(x)=\sin x$ and $g(x)=\cos x$, find the tangent line to $f(x)$ at $(0,0)$. Find the tangent line of $g(x)$ at $(\frac{\pi}{2}, 0)$. Graph the two tangent lines $f(x)$ and $g(x)$. Prove that the tangent of $f(x)$ is perpendicular to the tangent of $g(x)$.

Solution: In order to find the tangent line of f(x), we must find the derivative of this function. This will give us the slope (m). Since we are asked 2.4
to find the tangent line at point (0,0), we know that x=0 and y=0. All we need to do is give our x, y, m values in the formula in order to find c.

$f(x)=\sin x$ 1.3

$f'(x)=\cos x$ (This is a rule.)

Now, we substitute 0 where we see x: $\cos(0)=1$ Therefore our slope is 1. 1.1 1.3

$m=0$ 2.3 3.4

Now, let us submit the values in the formula: $y=mx+c$

$0=1\cdot 0+c$ 2.1 3.2

$c=0$ since $c=0$ and $m=1$, we can determine that $y=x$.

Therefore, the tangent line for f(x) is $\boxed{y=x}$

Now, we need to repeat the same steps in order to find the tangent line for g(x). 1.3

$g(x)=\cos x$

$g'(x)=-\sin x$ (This is also a rule)

$-\sin(90°)=-1$ → (Here we need to substitute the x-value which is $\frac{\pi}{2}$. We know that $\pi=180°$ so we can find $\frac{\pi}{2}$ by simply $\frac{180°}{2}$ which is 90°.) 1.1 1.3 2.3 3.4

Therefore; $m=-1$

Now, we need to substitute our values in the $y=mx+c$ formula.

$0=-1(\frac{\pi}{2})+c$ 2.1

$c=90$ Therefore, our formula is $\boxed{y=-x+\frac{\pi}{2}}$ 3.2

(1)

Now, we need to find the points in order to graph these lines.

$y = x$

$1 = 1$ (1,1)
$2 = 2$ (2,2)
(3,3) ...

$y = -x + \frac{\pi}{2}$ (We consider π as 3.14)

(1, 0.57)
(2, -0.43)
(3, -1.43)
(-1, 2.57)
(0, 1.57)

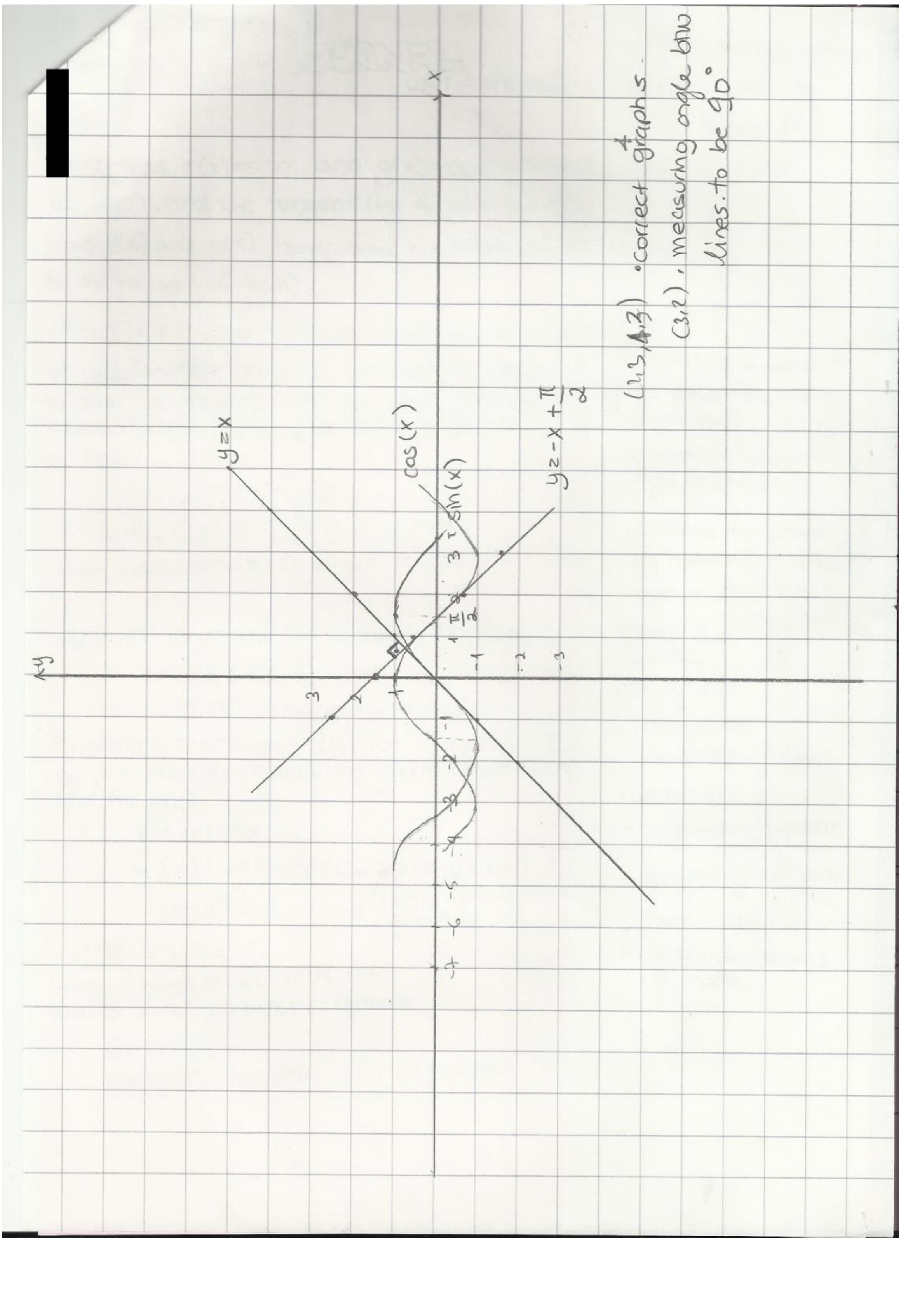
$y = \sin(x)$ 0,0
0.017

Now that we have drawn this graph, we need to prove that the tangent lines of $f(x)$ and $g(x)$ are perpendicular to eachother. We can do this algebraically or by measuring the angle between the two lines (which needs to be 90° in order for the lines to be perpendicular.) I chose to prove that they are perpendicular graphically with a protractor and the angle between the two lines is 90°. Therefore, the tangent of $f(x)$ and $g(x)$ are perpendicular to eachother.

PART B

In calculus, I feel strong in finding the derivatives and inverses of the functions because I have had a lot of practice in that area. I can also complete the steps for finding the equation of lines without any need of help. However, I have some difficulties in dealing with equations that include numbers such as ($\frac{\pi}{2}$, $3\pi/2$ ...) that refer to trigonometry. Also, I have some difficulties in graphing the sine and cosine graphs. This could be because I generaly have difficulties in using my knowledge of maths in different areas.

(2)


y=x
cos(x)
sin(x)
y=-x + π/2

[5] Reply of S_6 to Reflective Journal 5

Journal

Unit 3-part 1

Explain the topic to a friend who did not come to class.
Subtopics: increasing/decreasing intervals
maximum/minimum
point of inflection

When a question asks you to sketch a graph you have to first calculate certain properties. To do so we will first find the derivative and second derivative of the equation, then figure out the maximum and minimum point by finding the increasing and decreasing intervals. Assume $f(x) = 3x^3 - 6x$

① Stationary points: this is where $f'(x) = 0$

Find the first derivative of $f(x) \Rightarrow f'(x) = 3(3)x^{3-1} - 6(1)x^{1-1}$
$= 9x^2 - 6$

Now find x for $f'(x) = 0 \Rightarrow 9x^2 - 6 = 0$
$9x^2 = 6$
$x^2 = \frac{2}{3}$
$x = \pm\sqrt{\frac{2}{3}}$
$x = \sqrt{\frac{2}{3}}$ or $x = -\sqrt{\frac{2}{3}}$

*We can use these x values to find the intervals increasing and decreasing. Use a sign table as shown below:

x		$-\sqrt{\frac{2}{3}}$		$\sqrt{\frac{2}{3}}$	
$f'(x)$	+		−		+
$f(x)$	↗		↘		↗

From this table we can see that the maximum point of the curve has the x-value of $-\sqrt{\frac{2}{3}}$. The x-value of the minimum is $\sqrt{\frac{2}{3}}$. By putting these values into the function we can find the y-values.

When $x = -\sqrt{\frac{2}{3}}$
$f(-\sqrt{\tfrac{2}{3}}) = 3(-\sqrt{\tfrac{2}{3}})^3 - 6(-\sqrt{\tfrac{2}{3}}) = -1.63 + 4.89 = 3.53$
so: $(-\sqrt{\tfrac{2}{3}}, 3.53)$ maximum of curve

when $x = \sqrt{\frac{2}{3}}$
$f(\sqrt{\tfrac{2}{3}}) = 3(\sqrt{\tfrac{2}{3}})^3 - 6(\sqrt{\tfrac{2}{3}}) = 1.63 - 4.89 = -3.25$
so: $(\sqrt{\tfrac{2}{3}}, -3.25)$ minimum of curve

(1)

Inflection: Now you have to find out whether the inflection is horizontal or not. For this both $f''(x)$ and $f'(x)$ have to be equal to zero to be horizontal.

nding second derivative: $f'(x) = 9x^2 - 6$

$f''(x) = 18x \Rightarrow 18x = 0$

$x = 0$

heck $f'(0) \stackrel{?}{=} 0$

$9(0)^2 - 6 \neq 0$ ∴ at $x=0$ the inflection point is non-horizontal.

x		0	
$''(x)$	−	o	+
(x)	∩		∪

By looking at this table we can figure out that between the x-values $-\infty$ and 0 the graph will be concave down and between 0 and ∞ it will be concave up.

notice that "x" only had one value. If it is a quadratic or cubic derivative you will have to check each value one by one. Don't forget.

Intercepts: Lastly you have to calculate the y-intercept(s) and x-intercept(s), so you can draw the curve.

o to sketch a graph all you have to do is:

) Find the values of x for when $f'(x) = 0$.
) use this data in a sign table to find the maximum and minimum. Also use it to figure out the increasing and decreasing intervals.
) Find the value(s) of x for when $f''(x) = 0$.
) use this in a sign table to find if the inflection point is horizontal or non-horizontal.
Also find out the concave up and concave down intervals.
) Find the x-intercept and y-intercept.
Use all of the found above to sketch the graph.
use your TI to check your answer.

(2)

[6] Reply of S_{13} to Reflective Journal 6

Journal

Unit 3 – part 2

$f(x) = \frac{(x^2+5)}{(x^2-3)^2}$ Find stationary points and asymptotes.
By analyzing the sign tables for stationary points, sketch the graph of your function.

Let's first find the asymptotes.

Verticle asymptotes are found by makeing the denominator equal to 0.

$(x^2-3)^2 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

Horizantal asymptotes are found by limiting the rational function (where $x \to \infty$).
when we apply this to $f(x)$, we get:

$$\lim_{x\to\infty} \frac{(x^2+5)}{(x^2-3)^2} = \lim_{x\to\infty} \frac{(x^2+5)}{x^4-6x^2+9}$$

The solution is: $\lim_{x\to\infty} \frac{x^2(1+\frac{5}{x^2})}{x^2(x^2-6+\frac{9}{x^2})} \Rightarrow \lim_{x\to\infty} \frac{1}{x^2-6} \to \lim_{x\to\infty} \frac{x^2(\frac{1}{x^2})}{x^2(1-\frac{6}{x^2})}$

$= 0$

The horizantal asymphote is $y=0$

(1)

Next, let's find the stationary points.

First, we must find the derivative of f(x)

$f(x) = \frac{x^2+5}{(x^2-3)^2}$

$2x(x^2-3)-4x(x^2+5)$
$2x^3-6x-4x^3-20x$
$-2x^3-26x$

Using the Quotient rule,

$f'(x) = \frac{2x(x^2-3)^2 - 4x(x^2-3)(x^2+5)}{(x^2-3)^4} = \frac{-2x(x^2+13)}{(x^2-3)^3} = \frac{-2x^3-26x}{(x^2-3)^3}$

$2x(x^2-3)\left[(x^2-3)-2(x^2+5)\right]$
$2x(x^2-3)\left[-x^2-13\right]$
$\frac{-2x(x^2+13)}{(x^2-3)^3}$ ✓

Next, we need to find the 2nd derivative of f(x).

$f'(x) = \frac{-2x^3-26x}{(x^2-3)^3} = \frac{-2x(x^2+13)}{(x^2-3)^3}$

$f''(x) = \frac{(-2x\cdot 2)(x^2-3)^3 - 6(x^2-3)^2(-2x(x+13))}{(x^2-3)^6} = \frac{-4x(x^2-3)^2-\left[6(x^2-3)^2(-2x^2-26)\right]}{(x^2-3)^6}$

~~$= -4x(x^4-6x^2+9) - (6x^4-36x^2-54)($~~ $=$ ~~$-24x$~~ $= \frac{(x^2-3)^2\left[-4x-6(-2x^2-26)\right]}{(x^2-9)^6}$

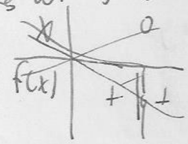
$= \frac{-4x+12x^2+156}{(x^2-3)^4} = \frac{4(3x^2-x+39)}{(x^2-3)^4}$ first we must make $f''(x)=0$

$3x^2-x+39=0$ $\quad 1 \pm \sqrt{1-4\cdot 3\cdot 39} = 1 \pm$ ~~$\sqrt{-467}$~~ Therefore, there aren't any

Inflection points

Sign diagram - $f'(x)=0$ the x value will gives us the sign diagram values.

$f'(x)=0$
$-2x^3-26x=0$
$-2x(x^2+13)=0$
$\boxed{x=0}$ or ~~$x=\sqrt{-13}$~~



x		0	
f'(x)	−	0	+

↘ ↗ minimum point

no asymptote values included

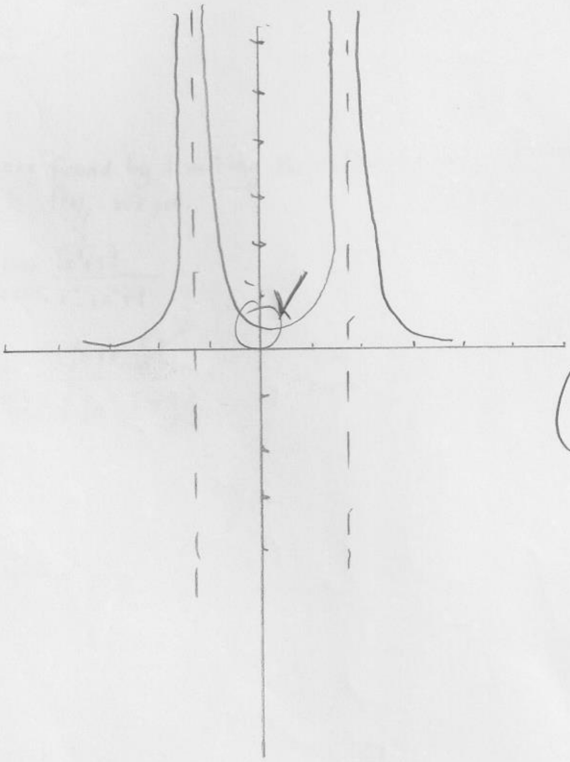
②

hen $X = 0$, we get the minimum point.

$f(x) = \frac{0+5}{(0-3)^2} = \frac{5}{9} = 0.55$

• correct y when $x = 0$ (3.2)

• Finds y-value when $x = 0$ (3.3)



(NO LABELLING MADE)

INCORRECT GRAPH (4.3)

3

[7] Reply of S_9 to Reflective Journal 7

Unit 4 - Part 1
Journal

A particle moving along a straight line has its displ... x metres from a fixed origin O, at time t seconds, govern... by the equation $x = t^3 - 17t^2 + 80t - 100,\ t \geq 0$

How many times will the particle pass through the origin? Prove by mathematical reasoning.

To find the number of times the particle passes through the origin, we need to find the equation of the velocity by taking the derivative of the displaceme... equation and make it equal to zero:

$v(t) = 3t^2 - 34t + 80$

$3t^2 - 34t + 80 = 0 \Rightarrow$ we need to use the quadratic formul...

$$\frac{34 \mp \sqrt{(-34)^2 - 4 \cdot 3 \cdot 80}}{6} = \frac{34 \mp \sqrt{196}}{6} \quad \frac{34+14}{6} = 8, \quad \frac{34-14}{6} = 3.\dot{3}$$

We then substitute the values we found in the displacement function to see where the object stops and if it chan... direction:

$s(8) = 8^3 - 17(8)^2 + 80(8) - 100 = -36\text{m} \Rightarrow$ left of origin

$s(3.3) = (3.3)^3 - 17(3.3)^2 + 80(3.3) - 100 = 14.8\text{m} \Rightarrow$ right of origin

We also need to substitute 0 into the displacement equation to see where the object started out:

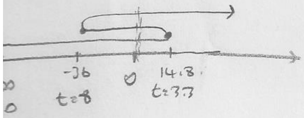
$S(0) = 0^3 - 17(0)^2 + 80(0) - 100 = -100\ \text{m} \Rightarrow$ left of origin

We make a sign diagram to see the direction of the p...

x		3.3		8	
$v(t)$	+		−		+
	right		left		right

①

astly, we plot all the points in the motion diagram.



Thus we see that the particle passes through the origin 3 times.

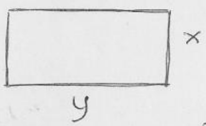
②

[8] Reply of S_2 to Reflective Journal 8

Unit 4 - Part 2

List down the steps to solve the optimization problems below and then apply your steps to solve the problem.

Problem → The rectangle of area A cm^2 and dimensions x cm and y cm. has a constant perimeter of P cm. ↳ $P \geq 10$



(a) Show that i) $y = \frac{P}{2} - x$

ii) $A = \frac{P}{2}x - x^2$

(b) Find $\frac{dA}{dx}$ in terms of P and x.

(c) Find maximum area in terms of P and x.

STEPS

1. First, we must find length y in terms of x and P. we know that $2x + 2y = P$ using this knowledge we can find y.
2. Then, we should find A in terms of x. The area is length times width. Width is in terms x, and we found length in terms of x above as well. Therefore, length x width we can find A in terms of x and P.
3. To find max area of the rectangle, we need to find derivative of Area function.
4. Once finding derivative we can find how it can be maximum by equalling the derivative to zero and drawing a sign diagram

Now let's actually do the steps above!

(a) i. $2x + 2y = P$

$2y = P - 2x$

$y = \frac{P-2x}{2} = \frac{P}{2} - x$

ii. $A = y \cdot x$

$y = \frac{P}{2} - x$

$A = \left(\frac{P}{2} - x\right)x = \frac{Px}{2} - x^2 \rightarrow \frac{P}{2}x - x^2$

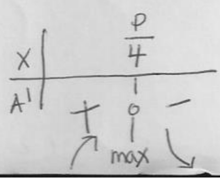
(b) $\frac{dA}{dx} = \frac{P}{2} - 2x$

(c) $\frac{P}{2} - 2x = 0$

$2x = \frac{P}{2}$

$P = 4x$

$x = P/4$



So there is a maximum at $\frac{P}{4} = x$

e know the x value to be $\frac{P}{4}$
o we should ~~to~~ find y value. we know
om before $y = \frac{P}{2} - x$. So $y = \frac{P}{2} - \frac{P}{4} = \frac{2P-P}{4}$

$$= \frac{P}{4}$$

Thus A maximum is $\frac{P^2}{16}$.

②

APPENDIX O

SAMPLE WORKS OF STUDENTS AT ONLINE FORUM DISCUSSIONS

[1] Reply of S_1 for Forum 1

Re: Unit 1-Part1(Derivatives of simple functions)
by [redacted] - Monday, 21 March 2011, 01:04 PM

The gradient of the curve in function form is equal to the derivative of a function, or the gradient of the tangent to that curve at that point.

The definition is either:
dy/dx = lim (dx→0) [(f(x+dx)-f(x)) / dx]

or according to the constant/power rule, the power of each term becomes the coefficient, and 1 is subtracted from the power. i.e.
If f(x) = c, then f '(x) = 0
If f(x) = x, then f '(x) = 1
If f(x) = x^2, then f '(x) = 2^x
If f(x) = x^3, then f '(x) = 3x^2
If f(x) = x^4, then f '(x) = 4x^3

Thus, in the last three examples, the derivative terms which will ultimately make up the gradient of the curve function, and since they have powers higher than one, the functions will become quadratic, cubic, quartic.

[2] Reply of S_{10} to Forum 2

Re: Unit 1-Part 2 (Chain Rule)
by [redacted] - Sunday, 27 March 2011, 04:54 PM

We will have to find the f'(x) and g'(x);
f'(x)=2x
f'(g(x))=2.(4x+3)
g'(x)=4
f'(g(x)).g'(x)=2.(4x+3).4
=8(4x+3)

h(x)=(4x+3)^2
h(x) is can be defined as the composite of the two functions which are f(x) and g(x)
Based on the fact that f(x) =x^2 and g(x) =4x+3; we will find the first derivative of h(x)
h(x)=2(4x+3).4
This will give us the result; 4 as the the first derivative of the g(x) and 2(4x+3) as the f'(g(x))
So we can say that h'(x)=2.(4x+3).4
=f'(g(x)).g'(x)
=(fog)'(x)
h(x)=(fog)(x)

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[3] Reply of S_5 to Forum 4

Re: unit 2-part2
by [redacted] - Saturday, 23 April 2011, 03:37 PM

I agree with [redacted]'s answer.
We can also find the answer if we do trial and error for different types of functions and their derivatives.

For example, for the function f(x)=5x^2, we know that f'(x) will be 10x and f''(x)=10. Therefore we know that it cannot be a simple function.

We can see if it is a trigonometric function, for example f(x)=sinx. f'(x)=cosx and f''(x)=-sinx. Therefore we know that it cannot be a trigonometric function.

If we try to use a logarithmic function, for example f(x)=e^x, then both the first and second derivative will be equal to e^x as well. This is because

f(x)-->f'(x)
e^h(x)-->e^h(x).h'(x)
Which is the same as saying it is the original function multiplied by the derivative of the exponent. If we ensure that the derivative of the exponent is 1, then the function will never change even if we take the first, or second derivative of it.

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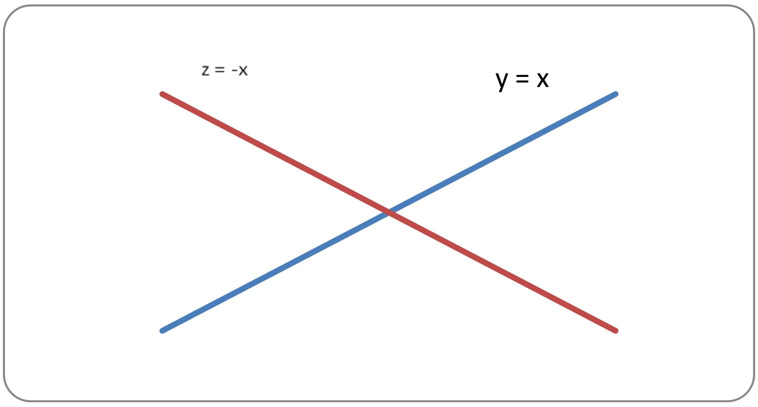
[4] Reply of S_2 to Forum 5 (Student typed at Microsoft Word)

“There are several number of function which satisfy these properties. One of the simplest function having the property of increasing for all real x values is “**y = x**”. On the contrary, simplest function decreasing for all real x values is “**z = -x**”. The hypothesis can be easily seen from the graphs of these functions:

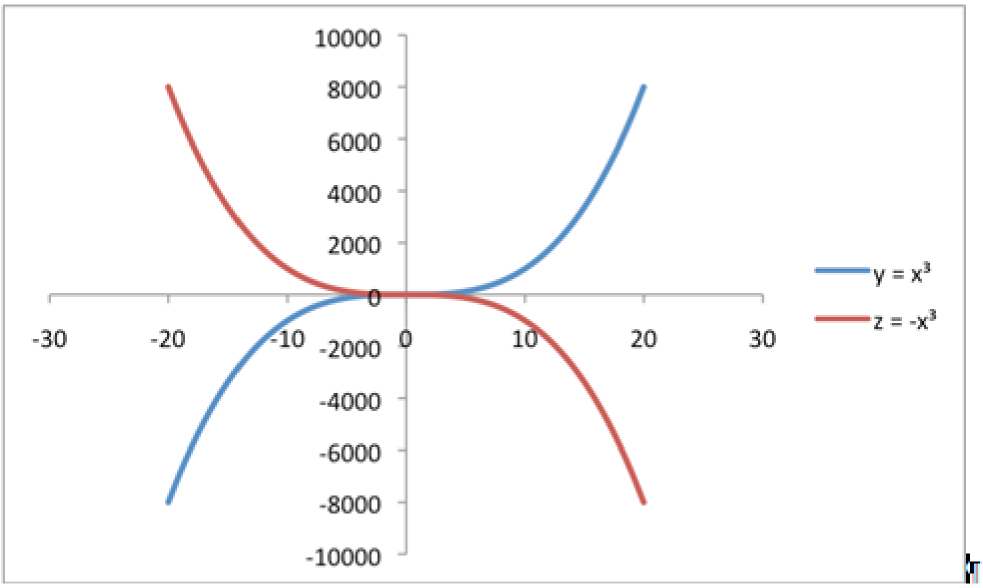


One level more complicated example may be the following function set:

“$y = x^3$”, “$z = -x^3$”

The function **y** is always increasing for all x values and **z** is always decreasing for all x values

These can be observed from the following graphs:



If a function is always increasing for all values of x, then it means; the slope of that function is always positive. Also, if a function is always decreasing for all values of x, then it means; the slope of that function is always negative.
Since the first derivative of a function shows the slope of the function; the following rules can be deducted:

> If a function is always increasing for all values of x, then i's first derivative is always positive for all values of x.
>
> Also, if a function is always decreasing for all values of x, then its first derivative is always negative for all values of x. "

[5] Reply of S_{17} to Forum 5

Re: Unit3-Part1
by [redacted] - Sunday, 1 May 2011, 05:03 PM

There are many examples we can give to functions that always increase or always decrease.

1. f(x)= e^x
This function always increases for all x values since f'(x)= e^x, which makes the derivate positive for all x. This suggests that f(x) is always increasing.

2. f(x)= -e^x
This function is always decreasing for all values of x because the derivative of this function is negative for all x values. Since the derivative is always negative, the function is decreasing for all x.

3. f(x)= e^-x
This function is always decreasing for all values of x because the derivative of this function is negative for all x values. Since the derivative is always negative, the function is decreasing for all x.

4. x^3+x^2+x+5
This function always increases for all x values since
f'(x)= 3x^2+2x+1. We can prove that it is function is always increasing by looking at delta. Delta= b^2-4ac = 4-4(3)(1)= 4-12=-8, which is smaller than zero.
This suggests that f(x) is always increasing.

5. f(x)= tanx
This function is also always increasing because derivative of this function is: 1/(cos^2x) which can never have a negative value.

6.f(x)= -tanx
On the other hand is always decreasing because derivative of it is-1/(cos^2x) which will always result in a negative value suggesting that for all x, this function is decreasing.

- These types of functions are said to be monotonic function as [redacted] had also previously stated. http://library.thinkquest.org/2647/algebra/ftmonoto.htm

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[6] Reply of S_{27} to Forum 1

Re: Unit 1-Part1(Derivatives of simple functions)
by [redacted] - Saturday, 26 March 2011, 11:50 PM

The same equation "f(x)=x^n" also works for the negative powered equations. However, when the power is negative, the equation will contain a negative power in the derivative form. Therefore, in order to get rid of that negative power, it is written in fraction form, in denominator so that the value gets rid of its negative power and becomes a power.

For example,

8x^(-3)-x^(2) - Numbers have been modified
When we take the derivative of the function:
f'(x) = -24x^(-4)-2x.

So instead of writing -24x^(-4), we can write it with a positive power such as -24/x^(4). It is the same expression but since positive powers are required, the reciprocal of it should be taken.

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[7] Reply of S_6 to Forum 6 (Student typed at Microsoft Word)

"Part 1

Since the questions is asking for the increase in the annual rate of increase, we have to find the interval at which the first derivative is increasing, meaning the second derivative is concave upwards.

1. Finding the derivative

P'(t) = 0.15t^2 – 0.0008t^3

Now this has to be equal to zero so we find at which points the function increases. So;

P'(t) = 0

0.15t^2 – 0.0008t^3 = 0

t=0 (double root) or t=187.5

t	0 187.5
P'(t)	+ ‖ + | -
P(t)	↑ ↑ ↓

We can deduce from the sign diagram that the curve is increasing between the x values – infinity and 187.5

Now check the second derivative:

P''(t) = 0.3t – 0.0024t^2

P''(t) = 0

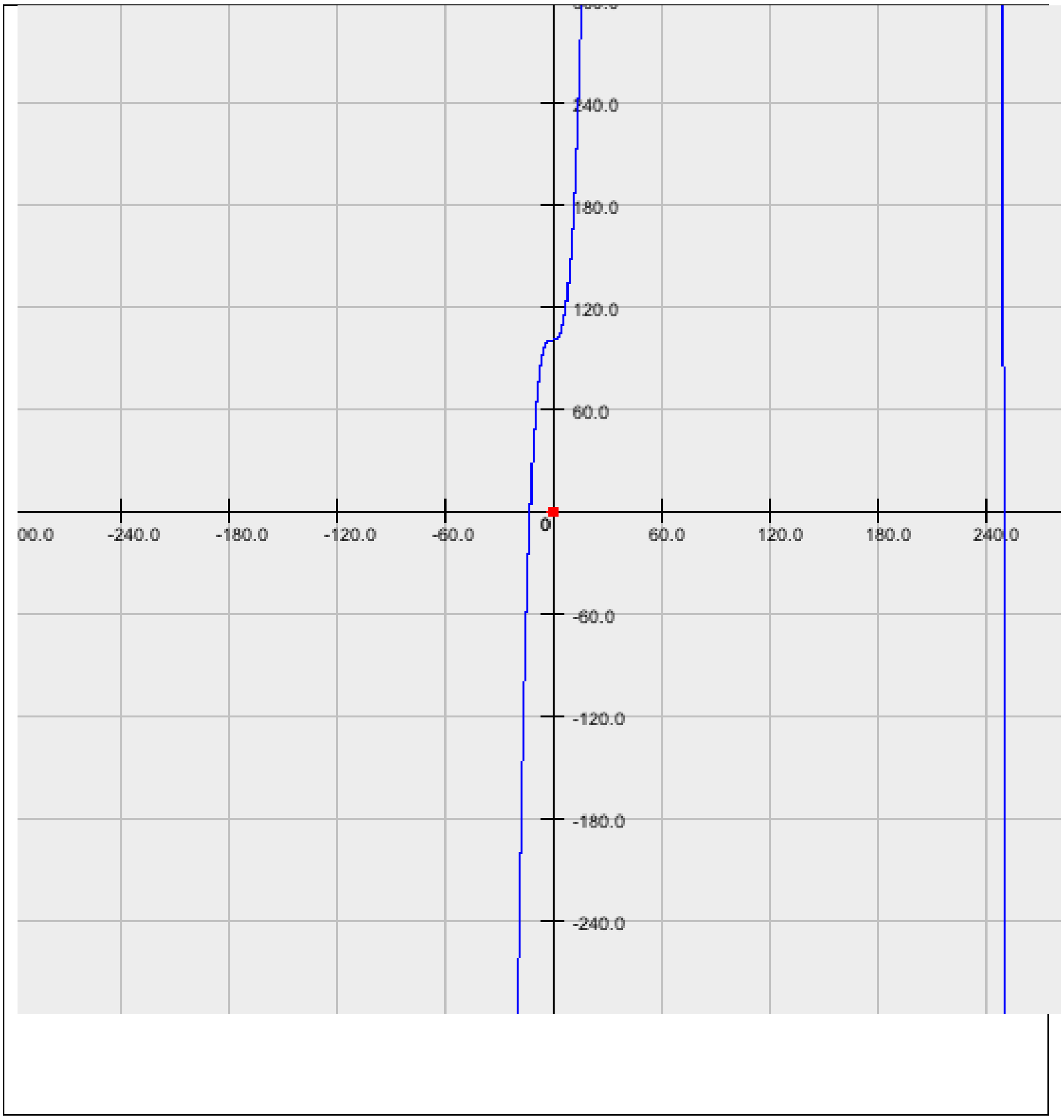
0.3t – 0.0024t^2 = 0

t = 0 or t = 125

t	0 125
P''(t)	- | + | -
P(t)	Concave downwards Concave upwards Concave downwards

When we put the information from both sign diagrams together we can say that:

0 < x < 125 is where we expect the rate of increase to be increasing. As you can see from the graph below this is the part of the curve on the right side of the y-axis, since the inflection is clearly increasing.



240.0
180.0
120.0
60.0
00.0
-240.0
-180.0
-120.0
-60.0
0
60.0
120.0
180.0
240.0
-60.0
-120.0
-180.0
-240.0

[8] Reply of S_{14} to Forum 1

Re: Unit 1-Part1(Derivatives of simple functions)
by [redacted] - Monday, 21 March 2011, 03:36 PM

According to the fact that the derivative will end up being equal to the gradient of the curve, the gradient of the curve does not have to be in the line equation form of y'=mx+c, it may also be a quadratic y'=ax2+bx+c, cubic y'=ax3+bx2+cx+d, or quartic y'=ax4+bx3+cx2+dx+e function. Therefore the the gradient function can be anything.

Example:
(using the simple rule as it is less time consuming and we end up with the same results)

f(x) = x^3 + 1
f '(x) = (3)x^(3-1) + 0
f '(x) = 3x^2

Therefore, the derivative has many forms and does not need to be in the y'=mx+c form.

If the gradient of the curve has a function with negative powers like
i.e.f(x)= x^(1/2) + x^(-2) does, then the solution will be

dy/dx= (1/2)(1/x^2) - 2 (1/x^3)

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[9] Reply of S_{10} to Forum 3

Re: Unit2-part1
by [redacted] - Monday, 4 April 2011, 06:10 PM

This function is valid when f'(x) exists and f(x) doesn't equal to 0 in a given domain and range.

Example:

f:R—>R If f(x)=3x+1 ,find $(f^{-1})'(x)$

First of all ,I will find the inverse function of f(x)

$f^{-1}(x)=(x-1)/3$

$(f^{-1})'(x)=1/3$ (1st equation)

Now I will use the formula to find the derivative of the inverse function and compare it with the above results.

$(f^{-1})'(x)=1/f'(f^{-1}(x))$ (2nd equation)

f'(x)=3 $f'(f^{-1}(x))=3$

I will insert these result in the 2nd equation so I will obtain:

$(f^{-1})'(x)=1/3$

Both of the results are the same

Example 2:

f:R—>R ,If $f(x)=x^3-3x^2+3x-2$ then find $(f^{-1})'(7)$

$f'(x)=3x^2-6x+3$

Now I will find the inverse of the function

$f(x)=x^3-3x^2+3x-1-1$

$= (x-1)^3-1$

$f^{-1}(x)=(x+1)^{1/3}$

$f'(f^{-1}(x))=3((x+1)^{1/3}+1)^{2}-6((x+1)^{1/3}+1)+3$

$f'(f^{-1}(7))=3((7+1)^{1/3}+1)^{2}-6((7+1)^{1/3}+1)+3$

$= 3.9-6.3+3$

$=12$

$(f^{-1})'(7)=1/f'(f^{-1}(7))=1/12$

Now I will check these results

$f(x)=x^3-3x^2+3x-2=y$

$f^{-1}(x^3-3x^2+3x-2)=x$

$(f^{-1})'(x^3-3x^2+3x-2).(3x^2-6x+3)=1$

$(f^{-1})'(x^3-3x^2+3x-2)=1/(3x^2-6x+3)$

$x^3-3x^2+3x-2=7$

$x^3-3x^2+3x-1=8$

$(x-1)^3=8$

$x=3$

$(f^{-1})'(7)=1/(3.3^2-6.3+3)=1/12$

I found the same results

[10] Reply of S_4 to Forum 7

Re: unit4_part1

by [redacted] - Sunday, 15 May 2011, 07:35 PM

At A the particle is 10 metres from the origin(probably on a hill) and is moving upwards. At B the particle is 20 metres from the origin and 10 metres upwards from where it was released, it is at this spot where the direction of the particle is about to change and hence will move downwards. At C the particle is at/or slightly on the left of the origin and is moving downwards. At D the particle is 10 metres downwards of the origin and is at the instant where it changing direction hence will be traveling upwards. At E the particle is at the origin traveling upwards. At F the particle is moving upwards and is about 4 metres above the origin.

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[11] Reply of S_5 to Forum 7

Re: Unit4_part1
by [redacted] - Tuesday, 24 May 2011, 06:39 PM

A-B
The ball's displacement indicates it has moved up 10 units.
The ball's velocity indicates that there is **positive** velocity; however, the ball is slowing down, which means acceleration is decreasing (or **negative**).
Point B is when velocity is zero, and the ball has reached its maximum point.

B-C
The ball's displacement indicates it has moved downwards, past the point of origin, for 20 units.
The ball's velocity indicates that there is **negative** velocity; moreover, the ball is speeding up, which means that acceleration is increasing (i.e. **negative**).

C-D
The ball's displacement indicates that it has moved downwards 12 units.
The ball's velocity indicates that there is **negative** velocity; however, the ball is slowing down, which means acceleration is decreasing (i.e. **positive**).
Point D is when velocity is zero, and the ball has reached its minimum point.

D-E
The ball's displacement indicates it has moved up for 12 units.
The ball's velocity indicates that there is **positive** velocity; moreover, the ball is speeding up, which means that acceleration is increasing (or **positive**).

E-F
The ball's displacement indicates that it has moved up for 4 units.
The ball's velocity indicates that there is **positive** velocity; morevoer there is constant velocity which means that acceleration at this interval is zero.

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[12] Reply of S_{17} to Forum 8

Re: Unit4_part2
by [redacted] - Wednesday, 25 May 2011, 08:56 PM

a) Since at points A and B the graphs are intersecting, the revenue function and the cost function are equal to each other the result of this will be zero profit. When our total revenue is the same as our cost, all the money from our revenue will be spent to the cost of production hence leaving no profit. After point A we can see that the cost graph decreases rapidly and the linear revenue graph increases hence making revenue more than the cost. This will give us profit because there is more revenue and less cost. This is the exact opposite case for after point B in which revenue is less than the cost which causes there to be a loss since more money is being spent then being earned.
b) Maximum profit will be earned when the cost is at a minimum. From the graph we can see that for cost to be minimum the x value is around 3. In order to find the minimum cost we need to find how many units should be produced to reach a minimum cost. To do this we will subtract C(x) from R(x) which will give us:
12x-40x-40-x^4+30x^2

When we say that this function is equal to zero we are left with:
4x^3-60x+28=0
x^4-15x+7=0
and hence; x= 3.61.

We can check our result by drawing the graph of C(x) and calculating the minimum point. The x coordinate of the point will give us the units that need to be produced to minimize the cost which will be given as the y-coordinate of the point.

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[13] Reply of S_{24} to Forum 4

Re: Unit 2-Part2
by [redacted] - Sunday, 24 April 2011, 10:31 PM

[redacted]s example is correct, the second derivative of e^x is e^x (itself).

Other examples of functions where the second derivative of it is equal to itself are:

1. e^(x+c) In this situation e has power x plus a constant. The derivative of the function is e^(x+c) and the second derivative is e^(x+c) again.

2. A*e^(x) In this example A is a constant such as 6. The derivative of 6e^(x+c) is 6e^(x+c), because the derivative of e^(x+c) that is multiplied with a constant value is the constant value multiplied by the derivativ of e^(x+c) which is itself. And the second derivative will give A*e^(x+c) again.

3. y=0 is a function. The derivative of this with respect to x is zero because x isn't present in the function. The second derivative of y=0 is zero again.

4. The second derivative of f(x)=Ae^(-x) gives the function itself. The first derivative:

f'(x)=A*e^(-x)*-1 = -Ae^(-x)

f''(x)=-A*e^(-x)*-1 = +Ae^(-x)   f(x)=Ae^(-x)

The second derivative of the function f(x)=Ae^(-x) is same as the function itself.

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[14] Reply of S_2 to Forum 1(Typed at Word)

No, the gradient of a curve is not always a line equation in the form of

$$y' = mx + c$$

It can be any degree function.
The degree of a gradient of a curve depends on the degree of the curve, such that;
The degree of the gradient is one (1) for a curve with a function of second degree,
The degree of the gradient is two (2) for a curve with a function of third degree,
The degree of the gradient is three (3) for a curve with a function of fourth degree,

....

The degree of the gradient is n for a curve with a function of n + 1 degree
This can be easily seen from the identity of derivative of a polynomial function as;

$$\frac{dy}{dx}\, ax^n = anx^{n-1}$$

The following are several examples for several curves of different degrees

$$\frac{dy}{dx}(2x^2 + 3x + 4) = 4x + 3$$

$$\frac{dy}{dx}(2x^3 + 3x^2 + 4x + 5) = 6x^2 + 3x + 4$$

$$\frac{dy}{dx}(2x^4 + 3x^3 + 4x^2 + 5x + 6) = 8x^3 + 9x^2 + 8x + 5$$

$$\frac{dy}{dx}(2x^5 + 3x^4 + 4x^3 + 5x^2 + 6x + 7) = 10x^4 + 12x^3 + 12x^2 + 10x + 6$$

[15] Reply of S_{30} to Forum 2

Chain Rule
by [redacted] - Wednesday, 30 March 2011, 11:36 AM

First we look at the two functions given f(x) and g(x):
f(x)=x2 and g(x)=4x+3
Where we are then asked to find: f '(g(x)). g'(x)

1st Step: We find the derivatives of the functions f(x) and g(x)
f(x) = x2
And to find the derivative the power of the x must be multiplied with x, causing it to 1 unit, now giving us:

f ' (x) = 2x
g(x)=4x+3
We use the same concept for the following function.
g' (x)= 4

2nd Step: Inserting the values we found in
f '(g(x)). g'(x), enabling us to find the solution:

=2(4x+3). 4
=(8x+6).4
=32x+24

Now we discuss how this process is linked to chain rule by considering the derivative of h(x)=(4x+3)2

We were given the information that:
f '(g(x)). g'(x) = h(x)

From this we can see that the first process used must be linked to the chain rule since they must be equal to eachother, to prove this we find h(x):

h(x) =(4x+3)2
Chain rule will tell us that(4x+3)2=u
h'(x)= 2u.u'
h'(x)=2(4x+3). 4
h'(x)=(8x+6).4
h'(x)=32x+24
This solution is the same from when we used the Chain Rule.

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APPENDIX P

CURRICULUM VITAE

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Interships	:	2003 ÖBL(Özel Bilkent Lisesi)/Bilkent (5 weeks) 2004 TED College/Ankara (13 weeks) 2004 ACI(American College of Izmir)/İzmir (2 weeks) 2004 BUPS/Bilkent (6 Weeks) 2005 IOWA STATE UNIVERSITY COLLAGE / USA (8 Weeks-Fulbright Scholarship)
Seminars and Courses	:	2001 - 2002 Deutsches Kultur – Instıtut / Ankara ***IB Seminars:*** 2007 IB Math SL-Level 1 Athens/Greece 2009 IB Math HL-Level 3 Oxford/England 2012 IB Math HL-Level 3 Ted Koleji/Ankara 2012 IB Math SL-Level 2 Florence/Italy

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Expertise	:	IB Math Studies - Grades 11/12 (9 years) IB Math Standard Level - Grades 11/12 (9 years) IB Math Higher Level - Grades 11/12 (9 years) IGCSE Extended Math (580) - Grades 9/10 (8 years) Cambridge International Mathematics (607) -Grades 9/10 (1 year) Middle School Mathematics - (2 years)
Experiences in Technology	:	- Web page design(front page) - Inspiration - Hyper studio - Geometer`s Sketchpad - GeoGebra - Autograph - TI (Texas Instruments Graphing Calculator) ve TI Smartview - C ve C++ Programming Languages - Database (SQL)
Foregin Languages	:	***English*** Writing:Very Good Speaking:Very Good Reading:Very Good ***German*** Writing:Good Speaking:Average Reading:Good
References	:	Available upon request