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## DESIGN OF COMPLIANT MECHANISMS FOR INDUSTRIAL APPLICATIONS

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ABSTRACT<br>\title{ DESIGN OF COMPLIANT MECHANISMS FOR INDUSTRIAL APPLICATIONS }<br>Yılmaz, Alikan<br>M.S, Department of Mechanical Engineering<br>Supervisor : Prof. Dr. Eres Söylemez<br>September 2013, 108 pages

Compliant mechanisms have the advantages of simplification in manufacturing and assembly, reducing cost, weight, wear, backlash and noise owing to their intrinsic structures. The aim of this study is to develop optimization methods to design reliable, high performance and easily manufacturable compliant equivalents of rigid body mechanisms used in industrial applications by utilizing the advantages of compliant mechanisms over rigid body mechanisms. Two particular mechanisms from the industry, namely a bistable field of view switch mechanism of optical systems and an adaptive finger mechanism for robot grippers are discussed in this thesis. The pseudo-rigid-body model is used for the design of bistable field of switch mechanism that is partially compliant while an analytical large deflection beam model is utilized to design the adaptive finger mechanism that is fully compliant. The optimized solutions satisfying the required motion conditions and desired force characteristics are also investigated through finite element analysis. The results are compared and the proposed designs are validated through physical prototypes.

Keywords: compliant mechanisms, bistable, pseudo rigid body model, robot grippers

# ENDÜSTRİYEL UYGULAMALAR İÇİN ESNEK MEKANİZMA TASARIMI 

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Esnek uzuvlu mekanizmalar yapıları itibariyle rijit mekanizmalara göre üretim ve montajının basit olması, düşük maliyet, hafiflik, daha az aşınma, boşluksuz ve sessiz çalışma gibi avantajlara sahiptir. Bu çalışmanın amacı esnek uzuvlu mekanizmaların sağladığı çeşitli avantajlardan faydalanarak sanayide kullanılan rijit mekanizmaların daha güvenilir, yüksek performanslı ve kolay üretilebilir esnek uzuvlu eşleniklerini tasarlamak için optimizasyon yöntemleri geliştirmektir. Bu çalışmada geliştirilen optimizasyon yöntemleri endüstride kullanılan iki farklı mekanizma üzerinden anlatılmıştır. Bunlar optik sistemler için iki konumda kararlı görüş açısı değiştirme mekanizması ve robot tutucular için adaptif parmak mekanizmasıdır. Kısmi esnek olan iki konumda kararlı görüş açısı değiştirme mekanizması için katımsı cisim modeli, tamamı esnek olan adaptif parmak mekanizması için büyük deformasyona uğrayabilen çubuk modeli kullanılmıştır. İstenilen kuvvet özelliklerini sağlamak ve amaçlanan hareketleri gerçekleştirmek üzere optimize edilen analitik modellerin ayrıca sonlu elemanlar yöntemi ile analizleri yapılmıştır ve elde edilen sonuçlar analitik çözümlerle karşılaştırılmıştrr. Aynı zamanda fiziksel prototipler üzerinden tasarımlar doğrulanmıştır.

Anahtar Kelimeler: esnek mekanizmalar, iki konumda kararlı, katımsı cisim modeli, robot tutucular

To my family and

My future wife, Ezgi

Thank you for your love and support

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## LIST OF SYMBOLS

| SYMB |  |
| :---: | :---: |
| $n_{\text {dof }}$ | Number of degree of freedom |
| $n_{\text {doa }}$ | Number of degree of actuation |
| M | Moment |
| $E$ | Modulus of elasticity |
| $E_{i}$ | Modulus of elasticity of the $\mathrm{i}^{\text {th }}$ link |
| I | Moment of inertia |
| $I_{i}$ | Moment of inertia of the $i^{\text {th }}$ link |
| $\rho$ | Radius of curvature |
| $\psi$ | Slope of the beam |
| $S$ | Arc length |
| $a$ | $x$-coordinate of the beam end at the deflected position |
| $b$ | $y$-coordinate of the beam end at the deflected position |
| $M_{0}$ | End moment |
| $\eta P$ | End force |
| $n P$ | Horizontal component of the end force |
| $P$ | Vertical component of the end force |
| $L$ | Length of the beam |
| $L_{i}$ | Length of the $i^{\text {th }}$ beam |
| $\gamma$ | Characteristic radius factor |
| 1 | Length of the flexible segment |
| $\theta$ | End angle of the beam |
| T | Torque |
| $\Theta$ | Pseudo-rigid-body angle |
| $K_{\theta}$ | Stiffness coefficient |
| K | Torsional spring constant |
| $K_{i}$ | Torsional spring constant of the $\mathrm{i}^{\text {th }}$ joint |
| $F_{x}$ | Force in x direction |
| $F_{y}$ | Force in y direction |
| $\eta(s)$ | Intrinsic curve for undeformed link |
| $\psi(s)$ | Intrinsic curve for deformed link |
| $h$ | Internal force along the positive x direction |
| $v$ | Internal force along the positive y direction |
| $u$ | Non-dimensional arc length |
| $A_{0}, B_{0}$ | Fixed pivot positions |


| $A_{i}, B_{i}$ | Moving pivot positions where $\mathrm{i}=1,2,3$ |
| :--- | :--- |
| $W$ | Center point coordinate vector |
| $Z$ | Circle point coordinate vector |
| $R$ | Coupler point coordinate vector |
| $\delta_{i j}$ | Displacement vector of the coupler point |
| $\beta_{i}$ | Prescribed input crank rotation |
| $\alpha_{i}$ | Coupler link rotation |
| $\psi_{i}$ | Prescribed output crank rotation |
| $\theta_{1 i}$ | Angle of the ith link with respect to ground |
| $r_{i}$ | Length of the it link |
| $V$ | Potential (strain) energy |
| $M_{c r}$ | Critical moment |
| $\Psi_{i}$ | Deflection of the ith spring |
| $S F$ | Safety factor |
| $S_{y}$ | Yield strength of the material |
| $S_{y} / E$ | Strength to modulus ratio |
| $w$ | Width of the beam |
| $t$ | Thickness of the beam |
| $F_{R}$ | Reaction force at the slider |
| $F_{A}$ | Actuation force |
| $L_{i m a x}$ | Maximum Actuation Displacement |
| $P_{C}$ | Contact pressure |
| $\Delta y$ | Actuation displacement |
| $\sigma_{i}$ | Bending stress of the ith beam |
| $\sigma_{a l l}$ | Allowable bending stress |
| $d_{o}$ | Diameter of the object |
| $n_{i}$ | ith $^{\text {th }}$ node |
| $x$ | Global coordinate |
| $y$ | Global coordinate |
| $c_{i}$ | Constraint function |
| $f$ | Objective function |

## CHAPTER 1

## INTRODUCTION

### 1.1 Motivation

Mechanisms, main components of the machines, are designed to transfer or transform motion, force and energy. Their performances affect the efficiency of the whole system. Therefore, it is significant to design these mechanisms properly to fulfill the desired tasks. Recently, there has been an intensive research on the traditional mechanisms to enhance their performance, decrease manufacturing and assembly costs and to design in micro scale. However, it is impossible to eliminate the problems such as backlash, wear and noise that result from the structure of the rigid joints found in these mechanisms. Although these problems have been minimized in the new designs to increase the reliability of the systems, their manufacturing costs have increased dramatically. Also, several machining techniques were developed to manufacture micro mechanisms but the difficulties in the assembly stage could not have been solved.

Scientists conducting research on mechanism science have utilized compliant mechanisms as an alternative solution to overcome the problems of traditional mechanisms. Their advantages over traditional mechanisms make them more preferable in many engineering designs and their outstanding examples are increasing gradually. Additionally, they offer a wide range of design alternatives. Hence, many traditional mechanisms have been replaced with the compliant ones as a result of studies in this field and improvements in material science.

Despite their advantages, the theory of compliant mechanisms is challenging and it requires intensive effort to model the systems and optimize the designs. Rigid body replacement and continuum methods were developed for the synthesis of compliant mechanisms. Unless a systematic procedure is followed in the design, it may take a long time to find the optimum solution. To decrease the modeling and optimization time in compliant mechanism design, there is a need to develop case-specific methods. Therefore, the complete domain of the problems may be searched for the best solutions.

### 1.2 Objective

The objective in this thesis is to design and validate the mechanisms for industrial applications using compliant mechanism theory. In this context, two problems from different application areas are selected. In both designs, it is intended to obtain reliable, simple and economical solutions. Also, it is aimed to develop and implement two different systematic design and optimization methods that may be used by the designers for similar compliant mechanism applications.

The first problem is about replacing a traditional field of view (FOV) switch mechanism in rifle-scopes with a compliant one to reduce the number of parts and to increase the reliability of the mechanism. This system has two FOVs and it requires a smooth transition in between. Therefore, bistable mechanisms can be considered to be the simplest way to do this. The design space and design requirements were defined previously. In this contribution, we propose a design and optimization procedure based on the rigid body replacement approach. Then, a bistable compliant four bar mechanism that meets the design requirements and accomplishes FOV switching function is designed. The performance of the mechanism is also be enhanced.

In the second problem, the complexity and expensiveness of the robot grippers used in industrial object handling are considered. When the reasons for the complex and expensive solutions are examined, it is revealed that what is desired is to increase the shape adaptivity. Therefore, in this study we aim to design a shape adaptive robot finger with the simplest structure. To have these properties at the same time, the monolithic structure of compliant mechanisms and infinite degrees of freedom in compliant members are utilized. For this design, we propose a procedure based on the finite element method and the analytical formulation of large beam deflections which are shown to be used interchangeably to design compliant finger mechanism. Furthermore, we develop an optimization code in Matlab that provides simple and effective way to optimize the finger geometry. Also, the prototype of the mechanism is manufactured to test its performance.

### 1.3 Organization of the Thesis

To reach the goals mentioned above the remainder of this thesis is organized as follows:

In Chapter 2, a detailed literature review related to this study is presented. This chapter is composed of three sections about compliant mechanisms, bistable mechanisms, and gripper mechanisms respectively. In each section, the theoretical background and the state-of-the-art designs are discussed to gain insight on the compliant mechanisms and to understand their working principles.

In Chapter 3, the useful mathematical models and theorems applied to the analysis and synthesis of compliant mechanisms are introduced. First, the difference between small deflection and large deflection theories is mentioned. Then, detailed derivations and formulations for large deflection analysis are presented. Also, the pseudo-rigid-body models that are commonly used and the shooting method that is used to solve the nonlinear differential equations numerically are explained. Finally, the stability phenomena and the theorems for bistability of four-bar mechanisms are stated. The design and optimization algorithms in the next chapters will be constructed according to the mathematical background supplied in this chapter.

In Chapter 4, the complete design process for bistable compliant four bar mechanism is explained. First of all, previously defined design space and design requirements are given, and problem is defined. Then, a design methodology which is convenient for the optimization is proposed. In the optimization algorithm, type synthesis, kinematic synthesis,
kinematic analysis and bistability analysis are performed simultaneously. After deciding on the final design dimensions and the material for the best performance, finite element analysis is performed to check the stress requirements.

In Chapter 5, design of a compliant shape adaptive gripper is performed. At first, a conceptual design is performed to come up with several alternatives and a few of them are considered to be appropriate to continue with further evaluation. Then, the selected alternatives are evaluated in detail with the help of a commercial finite element analysis program, ANSYS. Moreover, the beam elements in finger structure are modeled through analytical relations and obtained differential equations are solved using an easy but powerful numerical method called generalized multiple shooting method. After obtaining compatible results with FEM, the numerical method is utilized for the optimization of the finger geometry since it does not consume that much time required for remodeling in FEM. Finally, with the optimized dimensions a prototype of the gripper system is manufactured and tested for difficult tasks.

In Chapter 6, the overall contribution in this thesis is summarized and the important aspects in the implementation of compliant mechanisms to the conventional systems are outlined. Then, the results and improvements in this study are evaluated. Also, the limitations experienced during the prototyping of the finger mechanism are explained and the manufacturing options for the prototype are discussed. Finally, the topics which need to be further investigated are mentioned briefly.

## CHAPTER 2

## LITERATURE SURVEY

The intent of this chapter is to present a comprehensive review of the literature related to this study. The relevant literature is divided into three main sections, namely compliant mechanisms, bistable mechanisms and gripper mechanisms. Before introducing details about the case studies in this thesis, it is essential to provide background information about compliant mechanisms. First, the concept of compliant mechanisms including their classification, advantages and disadvantages, application areas and design methods is introduced. Then, important aspects about bistable mechanisms and gripper mechanisms are mentioned in Section 2 and 3, respectively. Also, in these sections the working principles of bistable mechanisms and gripper mechanisms are explained in detail, and some ingenious examples from the literature are presented. This chapter serves as a base for the design processes discussed in the following chapters.

### 2.1 Compliant Mechanisms

Formal definition of a mechanism was firstly pronounced by the father of modern kinematics, Franz Reuleaux as follows: a mechanism is an assemblage of resistant bodies, connected by movable joints, to form a closed kinematic chain with one link fixed and having the purpose of transforming motion [1]. Up to a few decades ago, designers and engineers stuck to this definition and all of the designed mechanisms were composed of links that were considered as perfectly rigid. Still, rigid-body mechanisms are designed with the same manner for low operating speeds and loads because this assumption greatly simplifies the analysis and design of mechanisms. Actually, none of the engineering materials is perfectly rigid, thus elastic deformation of the links is inevitable. At high operating speeds and loads, the resulting elastic deformations become larger and the performances of the mechanisms are influenced negatively. In order to compensate the adverse effects due to large deformations, structural rigidity of the links is increased as much as possible.

Despite it is avoided to have the links deformed in the former designs of mechanisms, the deformation capability of the members is considered to be a desired property in today's mechanisms. The first academic research on synthesizing mechanisms including flexible members started with the studies carried out by Burns, in 1964[2].He came up with the idea that designing a conventional rigid body mechanism with one of its link is flexible instead of using auxiliary springs to produce desired forces and motion. This study brought out a new point of view to the mechanism science, consequently another class of mechanism called flexible-link mechanism arose. He introduced a graphical method of flexible-link mechanism synthesis. In 1968, Burns and Crossley [3] presented a technique to obtain constant output torque from a four bar mechanism which consists of an axially flexible coupler link. They also demonstrated that the tip of a cantilever beam under large deflection traces an arc that has a center at distance of $1 / 6$ th span from the fixed end. As a result of further studies carried
out by Burns and the other researchers, it was shown that this type of mechanisms provides several advantages when compared to the rigid ones. Nevertheless, owing to the complexity of the mathematics involved in design and analysis, flexible-link mechanisms could not remain popular for a long time and the number of scientist studying in this research area decreased in 1970s.

In addition, a group of scientists inspired by previous improvements diverged to another research area, "kineto-elastodynamics of mechanisms" in 1970s. In dynamic analysis of high-speed machinery, the contribution of the mass of the links and also their elasticity formed the basic idea that constitutes the concept of the kineto-elastodynamics. This method provides a tool to analyze the effects of undesired elastic deformations such as positional errors in the mechanisms and fatigue failure due to introduced vibratory effects. Reader may refer to[4], [5]for further information.

In the first half of 1980s, Midha and his research group revisited the concept of flexible-link mechanisms. They proposed a new term, compliant, for mechanisms that were designed intentionally flexible and introduced the formal definition of a compliant mechanism as follows: a compliant mechanism is a new class of mechanism in which elastic deformation in at least one of its members is utilized to transmit motion and/or forces in a controlled manner[6]. This definition completely distinguishes rigid link mechanisms from the compliant mechanisms where elasticity effects are desired. Her and Midha determined several kinematic properties of compliant members and classified them according to their degrees-of-compliance. After this categorization, a systematic method for type synthesis of compliant mechanisms was developed[6]. In early 1990s, several methods and approximations employing traditional kinematics of rigid-link mechanisms were presented to analyze non-linear deflection of compliant members. The pseudo-rigid-body model is one of these techniques that greatly simplify the analysis of compliant mechanisms established by Howell and Midha[7]. Midha et al. furthered the study on the categorization of compliant mechanisms and presented the work done on the nomenclature, classification and abstraction of compliant mechanisms. Another research group headed by Kota focused on the distributed compliance in mechanisms and adapted the structural optimization techniques to design new compliant mechanisms[8]. Moreover, they presented several topology optimization routines and genetic algorithms. Howell et al. designed special purpose compliant mechanisms, e.g., compliant bistable mechanisms and compliant constant-force mechanisms, compliant parallel mechanisms, etc.[9]. Besides, many models for the dynamic analysis of compliant mechanisms were introduced by Howell et al.[10]. Lan and Lee emphasized on the significance of the computational methods to design compliant mechanisms. Lan et al. also applied powerful numerical methods, namely generalized shooting method and incremental linearization approach, to analyze the non-linear deflection of members in compliant mechanism[11],[12].

The theory and application have been developed well with the research carried by the university and industry over the past few decades. As a result, the number of products whose functionality depends on the flexible members has increased significantly. Many compliant mechanism designs can be found in the areas including MEMS, adaptive and smart structures, robotics, precision engineering, micro and nano devices, medical devices, hand-
held tools and everyday products. The field of compliant mechanisms will continue to grow as more advanced materials are developed[13].

Many designs of mankind took the advantage of elastic deformations throughout the history. Although the idea of using flexible members to store energy and create motion appeared thousands of years ago, intensive academic research has been done since 1960s.Unlike traditional mechanisms consisting of several rigid links connected with rigid joints; nowadays, this new type of mechanism, namely compliant mechanism, attracts a lot of attention from researchers and designers around the world. Compliant mechanisms gain at least a portion of their mobility from the flexibility of their members[9]. In other words, they are intentionally designed as flexible to achieve desired tasks. It may involve link(s) and joint(s) that all are flexible and no longer considered to be rigid. Also, a compliant mechanism may be composed of at least one component that is considerably deformable (flexible or compliant) compared to the other rigid links[14]. Hence, elastic deformation of materials is utilized to generate useful motions in numerous mechanisms. In Figure 2-1 the working principle of compliant mechanisms is illustrated.


Figure 2-1 Schematic representation of a compliant mechanism in terms of its energy [14]

### 2.1.1 Classification of Compliant Mechanisms

A brief discussion on the classification criteria would be useful at this point to inform the reader about the types of compliant mechanisms used in this thesis. Also, in order to facilitate the design process, it is worth to identify the functions of the compliant components explicitly.

In rigid body mechanisms, it is easy to distinguish the functions of the links and joints that constitute the whole system. In contrast, the classification process of the compliant mechanisms necessitates a comprehensive study owing to their sophisticated structure. Hence, there are numerous studies in the literature about their categorization. The first studies were limited to classify a special group of compliant mechanisms, e.g. flexible fourbar chains.


Figure 2-2 Classification of compliant mechanisms in terms of material distribution [9]

In 1994, Midha et al. suggested a standard terminology for the components of the compliant mechanisms and discussed the related issues in this process. They grouped compliant mechanisms into two main classes, namely partially and fully compliant mechanisms. Their study also includes the detailed classification of the compliant segments and the compliant links with respect to their functional and structural characteristics[13]. On the other hand, Ananthasuresh and Kota [8]offered to classify the fully compliant mechanisms according to the distribution of compliance in their members as lumped and distributed types. Figure 2-2 summarizes the form of classification accepted today.

Advances in the material and manufacturing technology allow the compliant mechanisms to be fabricated using various techniques. In terms of material continuity, they can be monolithic (single piece) or non-monolithic. Compliant mechanisms in the monolithic structure are named as "fully compliant mechanisms" while the ones in non-monolithic structure are named as "partially compliant mechanisms". A partially compliant mechanism has rigid-body joints while a fully compliant mechanism has not. The illustrative examples are given in Figure 2-3 and Figure 2-4 respectively. In fully compliant mechanisms, compliance may be distributed relatively equal through the entire mechanism or located at the discrete number of flexural pivots between rigid members. In this thesis, both partially and fully compliant mechanisms are investigated.


Figure 2-3 A fully compliant finger with distributed compliance [15]


Figure 2-4 A partially compliant slider-crank mechanism [16]

### 2.1.2 Advantages and Disadvantages

The usage of compliant mechanisms has become more common with the developed technology and the increased need for high precision and reliability. As compliant mechanisms offer several advantages over conventional rigid-body mechanisms, they started to take the place of their rigid counterparts. In manufacturing of a mechanical system, assembly costs account for nearly half of the total product cost. Therefore, designers tend to reduce the number of components to keep the assembly costs down. One of the most important advantages of compliant mechanisms is to manufacture as a single piece or with fewer parts. Besides, increased use of compliance in design of systems not only reduces the time and cost needed for assembly but also reduces the total weight. Thus, these advantages make the usage of compliant mechanisms appropriate for the production of micro mechanisms. Moreover, fewer rigid body joints help to reduce friction, wear, backlash, and noise[9]. Another advantage is that, the desired motion can be generated by means of elastic deformation of members and a variety of force deflection characteristics can be obtained with the least mechanical complexity. In addition, compliant mechanisms have the capability to store energy in their flexible members that helps to eliminate the need for additional energy storage elements such as springs used in rigid-body mechanisms.

On the other hand, there are several challenges of compliant mechanisms. The design freedom gained with the addition of compliance is often offset by difficulties encountered in the design and analysis of the compliant members. In contrast to the advances in the analysis software and the recently developed theories for compliant members to facilitate their design, there are other problems about strength limitations and low fatigue life. Thus, unlike rigid links, compliant links may experience stress relaxation or creep which remain under cyclic loading or work at high temperatures. Additionally, compliant mechanisms may have high stiffness values in the motion direction which necessitates a noticeable amount of force to make the system work. Also, in some applications, undesired energy storage in flexible segments may be a problem. Therefore, one should consider both the advantages and the disadvantages in the design of compliant mechanisms. Compliant mechanisms may not be the best solution for all applications.

### 2.1.3 Application Areas

Compliant mechanisms not only provide alternatives for many rigid link mechanisms but also provide innovative engineering solutions. That is why compliant mechanisms have become recently more popular in a wide range of engineering fields. Many examples of them can be found both in macro and micro scale applications. Especially in micro scale, compliant mechanisms promise lots of opportunities thereby permitting engineers to design simpler, more reliable and more economical products. Current application areas are mentioned below and examples can be seen in Figure 2-5.

- automotive and aerospace engineering
- bioengineering and biomimetics
- smart actuators and sensors
- precision engineering
- adaptive structures
- MEMS and NEMS
- robotics
- hand-held tools
- everyday products


Figure 2-5 Application areas of compliant mechanisms

### 2.1.4 Synthesis and Analysis Methods

Many design techniques are available for the synthesis and analysis of well-known rigid body mechanisms where kinematics and structural properties are considered separately. First, the link lengths are determined for the desired motion. Then, forces are calculated and the cross-sections of the links are determined with respect to the strength requirements.

However, it is not always possible to design compliant mechanisms by separating kinematics and deformation analysis. Motion of a compliant mechanism depends on location, direction and magnitude of applied forces, kinematics and energy storage must be considered together[9]. Therefore, it is a challenging task to synthesize and analyze compliant mechanisms when compared to traditional rigid-link mechanisms due to the introduction of nonlinear elastic deformation that adds a greater complexity[17]. According to Howell[9], these methods highly depend on the designer's intuition and experience. Also, it necessitates the knowledge of kinematics of mechanisms, multi-body dynamics, non-linear mechanics of materials, numerical optimization techniques, etc.[18].

This sub-section is devoted to summarize synthesis and analysis methods of compliant mechanisms, especially rigid-body replacement method utilizing pseudo-rigid body model and generalized shooting method that are utilized in this study.

### 2.1.4.1 Synthesis Methods

Many models and synthesis procedures for the compliant mechanisms have been developed over the past few decades. These approaches can be discussed in three main categories, namely kinematic approaches, building blocks approaches, structural optimization approaches as shown in Figure 2-6. Some of these methods are briefly introduced in the following.


Figure 2-6 Synthesis methods of compliant mechanisms

There are two approaches included in the rigid-body replacement method. One of them is the kinematic synthesis same as the one observed in the traditional rigid-link mechanisms. Other approach is the kinetostatic synthesis or synthesis with compliance whose name implies that a coupled problem of kinematics and statics are involved. Both of these methods use the pseudo-rigid body models extensively.

Kinematic synthesis uses pseudo-rigid-body models (PRBMs) that are the equivalent rigidbody mechanism of the compliant mechanism. This method is useful especially when a compliant mechanism is used to accomplish a conventional rigid-body mechanism task, e.g. path or motion generation without taking into account the storage of energy in the flexible members or the characteristics of the input/output force[9].

Kinetostatic synthesis also utilizes the PRBM concept. In this method energy storage in compliant members is taken into account and the related kinematic and static equations are
solved simultaneously. Howell applied the loop-closure technique including energy storage equations to synthesize the compliant mechanisms[19]. A compliant constant-force mechanism can be synthesized using this method.

Structural optimization approaches treat the compliant mechanisms as structures and model them using the methods of continuum solid mechanics incorporating the optimization techniques. According to the specified force and displacement constraints, geometry of the flexible material continuum is obtained. In order to solve the structural optimization problem, it is crucial to get familiar with the topology, shape and size optimization concepts. The next steps to be followed include the specification of the objective function, parameterization and application of solution techniques[18]. There are several formulations, parameterizations and solution methods in literature for the structural optimization.

There are also other synthesis approaches, namely freedom and constraints (FACT) method and the building blocks but they are out of the scope of this thesis.

### 2.1.4.2 Analysis Methods

Commonly used methods for the large deflection analysis of compliant members are the elliptic integrals, pseudo-rigid-body models and numerical methods such as chain algorithm, finite element method (FEM), finite difference method (FDM), shooting method and incremental linearization approach. Among these methods, elliptic integral solution method differs from the others being an exact method. Nevertheless, its derivation is more complex when compared to others regarding the mathematics involved. Additionally, it seems applicable for the analysis of simple geometries with simple loading conditions, e.g. cantilever beam. The formulas of pseudo-rigid-body models are approximated from the elliptic integral solutions for several loading conditions of compliant segments. In this thesis, pseudo-rigid body model and generalized shooting methods are implemented. They are briefly introduced in this section while the detailed information about these methods is given in Chapter 3.

## Pseudo-Rigid-Body Model

Systems with concentrated compliance are similar to classic rigid link mechanisms, where kinematic joints are replaced with flexure hinges. The most popular method to design compliant mechanisms with concentrated compliance is the rigid body replacement method, developed by Howell and Midha [20] and Howell[9]. This method utilizes the pseudo rigid body model that provides a tool used to simplify the analysis of the complex nonlinear deflections of many compliant mechanisms. PRBM unifies compliant mechanism theory with rigid-body mechanism theory. In other words, the PRBM concept allows compliant mechanisms to be analyzed using well-known rigid-body kinematics. The accuracy of the approximations in PRBM is stated as good enough to analyze the large deflection of the flexible members.

Salamon [21] is the first to introduce a methodology to model the compliance of flexible members as torsional and linear springs. These models can be considered much easier to analyze than the idealized ones that require finite element or elliptic integral solutions. Many

PRBMs for flexible beam can be developed based on the deflection curve of the beam tip subjected to given loadings. These deflection curves for flexible beams can be obtained by solving exact form of the Bernoulli-Euler beam equations. A classical method involves the solution of a second order non-linear differential equation using elliptic integrals of the first and second kind. Howell used closed-form elliptic integral solutions to develop deflection approximations for initially straight, flexible segment with linear material properties. However, this technique is limited to relatively simple geometries and loading. Also, it involves time-consuming derivations. Then, they investigated parametric approximation models for large deflection beams subjected to end forces and moments[7].
Saxena and Kramer [22] employed a numerical integration technique to solve large deflection Bernoulli-Euler beam equations whose implementation is simpler than the elliptic integral formulation and it also provides very accurate results for the tip deflection. Dado [23]repeated the same study in[7], but obtained a different curve fitting technique to parameterize the equivalent spring stiffness and the characteristic radius factor. Howell [19] and Edwards et al.[24], [25]developed PRBMs for beams in several configurations such as straight, initially curved with varying boundary and loading conditions.

Howell and Midha [20] modeled compliant mechanisms with small length flexural pivots and obtain PRBMs to analyze their behaviours. In this model, flexural segments are taken as small in length compared to the adjacent rigid segments. The proposed PRBM also consists of a pin joint at the center of the flexural segment and a torsional spring to represent the compliance. It is shown that the accuracy of this model and the relative length of the flexible segment are inversely proportional.

To conclude, PRBMs greatly simplify the non-linear deflection analysis of individual segments. This concept offers an easy method to model complex systems including flexible members. Moreover, it has proven to be an efficient approximation method for the analysis and synthesis of compliant mechanisms.

## Generalized Shooting Method

Generalized shooting method (GSM) is a more recently developed effective numerical solution technique to analyze the compliant mechanisms. Although shooting method (SM) has been used for a long time to solve nonlinear deflection of beams, Lan adapted this method for the solution of compliant mechanisms with multiple beams[11]. In SM, the principle idea is to convert the nonlinear boundary value problem into an initial-value problem. Moreover, when compared to the other numerical solution techniques specified in literature such as FEM, FDM, etc., it does not require the domain discretization and it provides more accurate solutions by utilizing an iterative solution technique. Besides, it is capable of analyzing deflections with nonlinear material properties while PRBM can be used only for the solution of the problems with linear material properties. Another advantage of SM over PRBM is that deflection on the whole link can be obtained. However, for SM to achieve the solution, initial guesses should be done properly. The multiple shooting method that applies SM on each subinterval can be utilized to overcome the instabilities due to numerical errors.

## Finite Element Method

Finite element method (FEM) can be considered as the most flexible method that is commonly used for the solution of the nonlinear differential equations in many engineering problems. In FEM, the spatial domain is discretized into finite sub-domains for which the deflections are approximated separately by satisfying the continuity between each element. While the methods introduced up to now are appropriate only for the simple beam models, FEM can handle irregular shapes. In literature, there exist several studies to approximate nonlinear deflections by utilizing FEM. To analyze the large systems many commercial software such as ANSYS, ABAQUS, NASTRAN, etc are also developed. Moreover, there are several special purpose FEM packages and codes developed by the researchers to analyze compliant mechanisms, e.g. CoMeT developed by Massachusetts Institute of Technology. Nevertheless, the nonlinearities and the need for using fine mesh to achieve a higher accuracy result in an increased computation time in finite element analysis.

### 2.2 Bistable Mechanisms

### 2.2.1 Definition and Application Areas

A bistable mechanism has two stable equilibrium positions within its motion range. Also, there exists an unstable equilibrium position between stable positions and the mechanism passes through it. Besides, a bistable mechanism has a smooth transition between these positions. The advantage of bistable mechanism is that it requires no energy input to remain stable in these stable positions. The energy is only required to switch between the stable positions. For the energy storage and bistable functionality, various flexible elements such as linear springs, flexure joints and compliant beams are utilized. These mechanisms can be observed in many application areas both in micro and macro domains such as switches, closures, relays, toggle mechanisms, etc.[9]. The light switch is a good example to demonstrate bistable behavior.

Bistable mechanism may be utilized to have another advantage such that when a bistable mechanism is restrained with a mechanical stop located at a position close to any of its stable position, it provides a contact force without the need for continued actuation force[26]. There are numerous studies in literature to verify the performance and reliability of bistable mechanisms.

### 2.2.2 Compliant Bistable Mechanisms

Recently, compliant members have replaced the auxiliary springs to obtain bistable functionality in many designs. The idea of designing for no assembly fits best for compliant mechanisms and increases the number of MEMS products such as micro switches, micro relays, thermal actuators, etc., designed for bistable characteristics. Young mechanism is the fisrt bistable compliant mechanism in MEMS applications shown in Figure 2-7. There are also compliant bistable mechanism designs in macro domain e.g., shampoo lids, light switches, furniture hinges, actuator mechanism for landing gear, door-lock mechanism for dishwashers.


Figure 2-7 Two configurations of Young mechanism [27]
In the desing compliant bistable mechanisms the energy storage and the motion characteristics are strongly coupled and must be considered simultaneously[9]. However, Jensen and Howell developed a theory to decouple energy storage requirements and motion characteristics of a four-link bistable compliant mechanism. They also investigated the mechanism configurations according to the spring positions in pseudo-rigid-body models resulting in bistable behavior and they proposed the results in tables for Grashof, nonGrashof and change-point mechanisms[28]. Moreover, they studied on the other mechanism types such as compliant slider-crank and double slider resulting in bistable behavior. Oh investigated the synthesis of multistable compliant mechanisms through combining bistable mechanisms [29]. Unverdi studied the design of compliant bistable lock mechanisms for dishwashers using pseudo-rigid-body model [30]. Buckling of beams is also utilized to obtain bistable or multistable characteristics.

### 2.3 Gripper Mechanisms

Robots have become common in use in all sectors of industry especially for the $21^{\text {th }}$ century owing to their priority over manpower in terms of time, quality and safety in automation. Hence, in the near future, it is not impossible that the robots will take the place of human labor completely when their capability to achieve dirty, risky and repetitive tasks is considered. As the robots are widely used in several testing and assembling processes, demand for the dexterous grippers to simulate the function of human hand has increased dramatically for the last decade.

The most highly developed natural gripper is the human hand [31]. Analogous to the human hand in handling and manipulation, robotic grippers play the same role for the robots being in contact with the objects[32]. Although there are advanced designs such as Utah/MIT hand, Stanford/JPL hand, Okada hand etc., dexterity of human hand has not yet been represented completely.

The future developments on robotic grippers depend on the variety of requirements, particularly simplicity, adaptivity and reliability. Besides, for low cost production, it is essential to increase the capability of the grippers while keeping their structure as simple as possible. Hence, use of underactuation concept and compliant mechanism theory in the design of grippers provides a solution to satisfy the requirements mentioned above with the least complexity.

### 2.3.1 Underactuated grippers

Mechanisms can be categorized in three classes with respect to the relation between the number of degrees-of-freedom $\left(n_{d o f}\right)$ and the number of actuators $\left(n_{\text {act }}\right)$ as follows:

- fully actuated mechanisms : $n_{d o f}=n_{\text {act }}$
- redundantly actuated mechanisms : $n_{d o f}<n_{\text {act }}$
- underactuated mechanisms : $n_{\text {dof }}>n_{\text {act }}$

The term underactuated refers to the fact that there are fewer actuators than the degrees of freedom in a system designed intentionally. The objective behind the underactuation is to keep the nature laws govern the mechanical device[33]. Moreover, underactuation in grasping aims to use an ingenious mechanical system that can adapt itself to the shape of the object automatically using a simple control. This mechanical intelligence is generally based on the principle of differential systems. These devices automatically distribute one input to several outputs, by determining the ratio between different outputs according to their design parameters and output states. Thus, the principle of underactuation leads to the shape adaptation of the hand or gripper.

Underactuation is a widely used and relatively old concept in robotics. Since the last decade, the interest focused on the design of underactuated mechanical systems for robotic grippers has increased. It has been revealed that the use of underactuated mechanisms in grippers yields promising results in terms of design simplifications and adaptive grasping. However, the special case in which the motion of all finger segments is mechanically coupled [34] cannot be considered as underactuated. This robotic finger has one actuator and one DOF. The motion is determined only by the design of its driving mechanism and there is no shape adaptation.


Figure 2-8 A two-finger planar underactuated hand [35]

Examples of underactuated mechanical hands designed by Laliberte and Gosselin [36] are shown in Figure 2-8. In their study the taxonomy of the grasp is reviewed and the principle of underactuation that leads to shape adaptation of the hand is introduced (Figure 2-9 and Figure 2-10). Then, they proposed architectures of two-degree-of-freedom underactuated fingers and developed a simulation tool to analyze their behavior.



Figure 2-9 Examples of underactuated two-degree-of-freedom mechanisms [36]




Figure 2-10 Closing sequence of an underactuated two-degree-of-freedom finger [36]
Birglen and Gosselin [37] proposed a three-phalanx underactuated finger and performed analysis on the grasp stability of two classes of them. These fingers have different transmission mechanisms based on either linkages or tendons and pulleys. The design technique that they have developed can be generalized to any number of phalanges.

Underactuation in robotic hands can be achieved by using differential, compliant and triggered mechanisms. Differential systems can be based on linkage systems or on tendonactuated mechanisms. However, tendon systems are limited to small grasp forces. They induce friction and elasticity. Linkage mechanisms are more efficient for applications with large grasp forces but relatively more bulky. In triggered mechanisms, once the torque exceeds a certain value, the joint locks. This is achieved in several hand configurations by using a transmission mechanism which can be disengaged or an automatic brake. On the other hand, it is possible to reduce the number of actuators by introducing compliance for each DOF. Many different kinds of compliant mechanisms are designed to obtain underactuation characteristics in robotic hands.

Moreover, the transmissions used in underactuated mechanisms can be divided into two classes based on the self-adaptive transmission used to route actuation to the various degrees of freedom, namely the single-acting transmission and double acting transmission. Single acting mechanisms can apply only unidirectional forces on the joints. On the other hand, a double acting mechanism can apply bi-directional forces on the joints such as pull and push[35].

Doria and Birglen [38] developed an underactuated compliant gripper for surgery with two fingers and five phalanges on each. In this gripper shown in Figure 2-11 underactuated
mechanisms are used in several configurations and have different functions such as transmission and driving. The geometry of the transmission mechanism was obtained using an optimization procedure. A driving mechanism that accomplishes the grasping of asymmetrical objects without requiring supplementary inputs was discussed in this study.


Figure 2-11 Model of a compliant underactuated finger using nitinol [38]
Steutel [15] designed and verified an underactuated finger with a monolithic structure and distributed compliance. Deciding on the general topology of the finger mechanism, different configurations based on the pseudo-rigid-body model were obtained and analyzed according to the maximum actuation displacement and transmission ratio of the actuation force with respect to the contact forces. Also, a finite element model was built to analyze the deflections and stresses. Finally, experimental results of the prototype and FEM results were compared.

(b)

Figure 2-12 PRBM (a), schematic representation (b) of a fully compliant underactuated finger [15]

### 2.3.2 Shape adaptation

In the last decade, there has been an extensive research on the development of innovative technologies in the area of robotic grippers to obtain a wide range of handling and to reduce the number of sensors and actuators. As the robotic grippers are widely used in the areas that necessitate sensitive and secure grasping, shape adaptivity has become a fundamental issue to be considered in the design of gripper mechanisms. Shape adaptive grippers have the capability to respond to new configurations with less control effort and apply enough pressure on gripping surfaces. There are several methods to design shape adaptive systems, examples of which are given in Figure 2-13.


Figure 2-13 Various shape adaptive gripper designs

### 2.3.3 Compliant grippers

Over the past decades, researchers have been working on creating robot hands that mimic dexterous and anthropomorphic properties of human hand. There are a lot of research focused on improving the appearance, functionality, and power-to-weight ratio. Nevertheless, these hands are still far from the desired level of dexterity, reliability and applicability. Instead of using rigid members connected at revolute joints, there has been research on using compliant components to serve as both the structure and rotational members of a robotic gripper. Many of the problems have been solved with the implementation of compliant mechanisms into robotic grippers during the past few years. Lightweight and compact grippers were designed while maintaining the major grasping functions. Reliable grippers are obtained to achieve adaptive grasping without using augmented sensors. Thus, compliant grippers have been more attractive than traditional grippers.


Figure 2-14 A shape memory alloy (SMA) wire actuated compliant finger [39]

### 2.4 Conclusions

In this chapter, the literature survey carried out during this thesis study is presented suitably This chapter provides the required knowledge about the compliant mechanisms design and the key issues in this study. These are bistability and shape adaptivity considered in Chapter 4 and Chapter 5 respectively. Moreover, the recent studies in the field of compliant mechanisms and the underactuated systems are summarized and visualized with examples.

## CHAPTER 3

## MATHEMATICAL BACKGROUND

The mathematical models and methods available in the literature to design and analyze compliant mechanism are mentioned in Chapter 2. In this chapter, we present the detailed formulation of two commonly used approximation models for the large deflection analysis of flexible members. These mathematical models are pseudo-rigid-body model (PRBM) and a compliant beam model formulated using shooting method. They are based on the non-linear Euler-Bernoulli beam theory. In order to reduce the time spent in pre-design stage and to gain insight about the physics of a compliant mechanism problem, expressing the deflection of flexible members in terms of mathematical or physical models is considered to be essential. Hence, in this chapter, models used in this study will be introduced.

### 3.1 Flexible Beam Model

In solid mechanics, theory of beams is widely used for the deformation and stress analysis of structures. It offers a simple and parameterized computation tool especially for the predesign stage. When it is compared to the finite element method, the time spent in the predesign stage is reduced and a valuable insight about the behavior of the model is obtained. There are several beam theories developed and used for the analysis of structures including beam elements. To simplify the analysis of sophisticated models in many engineering problems, these theories are based on various assumptions. The oldest and most commonly used one was established with the contributions from Daniel Bernoulli and Leonard Euler (1744) to model the bending of beams. It is called the Bernoulli-Euler beam theory, also known as the classical beam theory. It has the following three assumptions for a long, slender beam having isotropic material properties and solid cross-section throughout its length.

Assumption 1: The cross-section is infinitely rigid in its own plane.
Assumption 2: The cross-section of the beam remains plane after deformation.
Assumption 3: The cross-section remains normal to the deformed axis of the beam.


Figure 3-1 Bernoulli-Euler beam subjected to end-moments [40].

It is implied in the first assumption that the bending moment does not alter the length of the beam. In Figure 3-1, situations before and after the deformation are given to visualize the second and third assumptions of the classical beam theory. It can also be extracted from Figure 3-1 that the straight beam deforms into a curve of constant curvature, i.e., a circle centered at point O , under the action of the bending moment. According to the BernoulliEuler theory, the bending moment at arbitrary point on the beam is proportional to the curvature at that point. Hence, it is possible to write the moment-curvature relationship for a beam of linear elastic material as follows:

$$
\begin{equation*}
M=\frac{E I}{\rho}=E I \frac{d \psi}{d s} \tag{3.1}
\end{equation*}
$$

where $M$ and $d \psi / d s$ are the bending moment and the curvature at any point of the beam, respectively, $E$ is the modulus of elasticity and $I$ is the moment of inertia of the beam cross-section about the neutral axis. The curvature $d \psi / d s$ is usually called as the rate of change of angular deflection along the length of the beam. The exact expression for the curvature in rectangular coordinates is expressed as

$$
\begin{equation*}
\frac{1}{\rho}=\frac{d \psi}{d s}=\frac{\frac{d^{2} y}{d x^{2}}}{\left(1+\left(\frac{d y}{d x}\right)^{2}\right)^{3 / 2}} \tag{3.2}
\end{equation*}
$$

In Figure 3-2, deformed configuration of a cantilever beam subjected to an end force $\eta P$ and an end moment $M_{0}$ is shown. The components of the end force in horizontal and vertical directions are $n P$ and $P$, respectively. $\psi$ is the slope at any point $(x, y)$ and $S$ is the arc length.


Figure 3-2 A flexible beam model subjected to combined force and moment loads [41].
The governing beam deflection equation for the cantilever beam shown in Figure 3-2 can be obtained by substituting moment at point A into Bernoulli-Euler equation. The deflected shape of the beam for a given material and geometry depends on the moment $M$ given in Eq. (3.3).

$$
\begin{equation*}
M=E I \frac{d \psi}{d s}=P(a-x)+n P(b-y)+M_{0} \tag{3.3}
\end{equation*}
$$

Unless the loading conditions are simple, the above non-linear differential equation cannot be solved using the known solution techniques. Hence, it necessitates to make some assumptions on the deflections or to use different solution techniques for the nonlinear cases. The difference between small and large deflection assumptions lies in the treatment of the Bernoulli-Euler equation.

### 3.1.1 Small Deflection Analysis

For most of the cases, beam deflections are very small compared to the length of the beam. Then, the square of the slope, $(d y / d x)^{2}$, can be assumed to be small compared to unity in the denominator of the curvature formula. Based on this assumption, beam deflection equation becomes a linear differential equation which may be solved easily to obtain the beam deflection formulas given in mechanics textbooks. The linearized form given in Eq. (3.4) is termed as the elementary beam theory.

$$
\begin{equation*}
M=E I \frac{d^{2} y}{d x^{2}} \tag{3.4}
\end{equation*}
$$

For example, consider a uniform cantilever beam given in Figure 3-3 with a vertical force applied at its free end. The solution for the transverse deflection and the slope of the beam is obtained as follows:

$$
\begin{gather*}
\psi=\frac{d y}{d x}=\frac{P x}{2 E I}(2 L-x)  \tag{3.5}\\
y=\frac{P x^{2}}{6 E I}(3 L-x) \tag{3.6}
\end{gather*}
$$

For small deflections, the principle of superposition is also applicable to obtain the total deflection under combined loading. However, above assumption and solutions become invalid for the analysis of the members going under large deflections.


Figure 3-3 A cantilever beam with an end force and small deflections

### 3.1.2 Large Deflection Analysis

Generally, members in compliant mechanisms undergo large deflections which results in geometric nonlinearities. Hence, it requires special considerations in deriving equations and solutions techniques for their analysis [9]. Unless the deflections are small, the square of the slope in the curvature relation cannot be neglected. The elementary beam theory, therefore, becomes not applicable for the analysis of large deflection members.

In order to obtain a general equation that governs large deflections and express Eq. (3.3) explicitly in terms of $S$, it is differentiated with respect to $S$.

$$
\begin{equation*}
\frac{d}{d s}\left(E I \frac{d \psi}{d s}\right)=P\left(-\frac{d x}{d s}-n \frac{d y}{d s}\right) \tag{3.7}
\end{equation*}
$$

where $d x / d s=\cos (\psi)$ and $d y / d s=\sin (\psi)$. Note that, constant end moment $M_{0}$ vanishes after differentiation.

$$
\begin{equation*}
\frac{d^{2} \psi}{d s^{2}}=\frac{-P}{E I}(\cos (\psi)+n \sin (\psi)) \tag{3.8}
\end{equation*}
$$

Since Eq. (3.8) involves sine and cosine terms of the dependent variable $\psi$, it is a non-linear differential equation. To solve this second order differential equation, we need to specify two boundary conditions, which are:

$$
\begin{gather*}
\left.\psi\right|_{s=0}=0  \tag{3.9a}\\
\left.\frac{d \psi}{d s}\right|_{s=L}=\frac{M_{0}}{E I} \tag{3.9b}
\end{gather*}
$$

Several methods available to solve the resulting differential equations take into account the nonlinearities. Solution in terms of the elliptic integrals of the first and second kind [38] is the first one that comes to mind. Elliptic integrals provide closed-form solutions but their derivations are cumbersome and solutions exist for only simple geometries and loadings. To simplify the analysis of large deflection members, approximation models were also developed for such simple geometries. These models are called pseudo-rigid-body models (PRBMs) [7]. There are also other methods which are capable of handling complicated geometry and loading conditions. These are non-linear shooting method and Adomain decomposition method. Considering the scope of the thesis, pseudo-rigid body model concept and non-linear shooting method formulation will be provided in the following sections.

### 3.2 Pseudo-Rigid-Body Model (PRBM)

The analysis of systems that undergo large deflections is often complicated due to the geometric, material and boundary nonlinearities. Since it is required to perform many iterations in the preliminary design stage of such systems, formulation of the problem with
exact methods is cumbersome. Thus, simple models are useful in obtaining quick designs that can then be optimized using more exact methods. The pseudo-rigid-body model approximations offer such simplified means to parameterize the deflection path and forcedeflection relationships of large deflection members including geometric nonlinearities. After obtaining PRBM for a compliant link, it becomes convenient to unify compliant and rigid body mechanism theories. There are several PRBMs developed to approximate the deflection path and nonlinear force-deflection relationships of different types of flexible segments.

The PRBM of a cantilever beam given in Figure 3-4 consists of two rigid links joined at a pin joint and a torsional spring. The pin joint is referred as the "characteristic pivot" and it is located at $\gamma l$ from the beam tip in its undeflected position. Here, $\gamma$ is termed as the "characteristic radius factor" and $l$ is the length of the beam. The movable link can rotate about the pivot by an angle, $\Theta$ termed as the "pseudo-rigid-body angle". However, its rotation is restrained by a torsional spring having equivalent spring stiffness, $K$. This model predicts the deflection path of the beam end for a given end load within $0.5 \%$ error bound when compared to elliptic integral solutions for quite large deflections [9].


Figure 3-4 A cantilever beam and its pseudo-rigid-body model (PRBM) [9].
In the following sub-sections, the pseudo-rigid-body models that are utilized in this study to approximate large deflections are reviewed. Mathematical relations to calculate coordinates of the deflected beam tip and spring constant are presented for each model. Besides, rule of the thumb values for characteristic radius factor and stiffness coefficient are provided.

### 3.2.1 Small-Length Flexural Pivot

Small-length flexural pivot is one of the important flexible elements discussed in this study. Most of the compliant mechanism designs involve flexural pivots. In Figure 3-5, a fully compliant four-bar mechanism can be seen.

(a)

(b)

Figure 3-5 A compliant four-bar mechanism including flexural pivots and its PRBM [20].
A small-length flexural pivot can be specified as a beam that has two segments one of which is short and flexible in bending directions while the other is longer and rigid in those directions. Consider that the small section is significantly shorter and more flexible than the large segment, and then its structure is described by the following conditions.

$$
\begin{gather*}
L \gg l  \tag{3.10}\\
(E I)_{L} \gg(E I)_{l} \tag{3.11}
\end{gather*}
$$

The pseudo-rigid-body model for a small-length flexural pivot shown in Figure 3-6 is obtained as two rigid links connected by a pin joint where it is located at the centre of the flexural pivot. The $x$ and $y$ coordinates of the beam's end are approximated as

$$
\begin{align*}
& a=\frac{l}{2}+\left(L+\frac{l}{2}\right) \cos (\Theta)  \tag{3.12}\\
& b=\frac{l}{2}+\left(L+\frac{l}{2}\right) \sin (\Theta) \tag{3.13}
\end{align*}
$$

The beam resistance to deflection is modeled using a torsional spring with constant stiffness that is located at the characteristic pivot. The torque required to deflect the spring of an angle $\Theta$ is:

$$
\begin{equation*}
T=K \Theta \tag{3.14}
\end{equation*}
$$

where the torsional spring constant is equal to:

$$
\begin{equation*}
K=\frac{(E I)_{l}}{l} \tag{3.15}
\end{equation*}
$$

where $(E I)_{l}$ corresponds to the flexural rigidity of the small length flexural pivot.


Figure 3-6 A small-length flexural pivot and its PRBM [20].

### 3.2.2 Fixed-Pinned Segment

The other commonly used compliant member is a fixed-pinned segment whose PRBM is elucidated below. Actually, it is the case of a cantilever beam with force components at the free end. Since it is pinned at the free end, moment is equal to zero at this point. The geometric interpretation of the pseudo-rigid-body model for fixed-pinned segments is shown in Figure 3-7.


Figure 3-7 A fixed-pinned beam and its PRBM [9]
The coordinates of the deflected beam end are approximated as

$$
\begin{gather*}
a=l[1-\gamma(1-\cos \Theta)]  \tag{3.16}\\
b=\gamma l \sin \Theta \tag{3.17}
\end{gather*}
$$

For the torsional spring, stiffness constant is calculated as

$$
\begin{equation*}
K=\gamma K_{\Theta} \frac{E I}{l} \tag{3.18}
\end{equation*}
$$

where rule of the thumb values are $\gamma=0.85$ and $K_{\Theta}=2.65$.

### 3.2.3 Fixed-Fixed Segment

Some of the flexible segments have both of their boundary conditions fixed. Also, external loads applied at the boundaries differ from the pinned ones. Hence, its model is considerably different due to its complex loading condition. There are force and moment applied at both ends. It may be practical to represent such a segment with the simplified PRBM as given in Figure 3-8 [9]. Although this model is stated as less accurate when compared to other PRBMs, it can be useful for quick results at initial design stages.


Figure 3-8 A fixed-fixed beam and its simplified PRBM [9]
The coordinates of the deflected beam end are approximated as

$$
\begin{gather*}
a=l[1-\gamma(1-\cos \Theta)]  \tag{3.19}\\
b=\gamma l \sin \Theta \tag{3.20}
\end{gather*}
$$

For each torsional spring, stiffness constant is calculated as

$$
\begin{equation*}
K=2 \gamma K_{\Theta} \frac{E I}{l} \tag{3.21}
\end{equation*}
$$

where rule of the thumb values are $\gamma=0.8517$ and $K_{\Theta}=2.67$.

### 3.3 Shooting Method Formulation

In this section, shooting method formulation of a flexible beam used to analyze the large deflections in compliant mechanism will be introduced. In this approach, Bernoulli-Euler equation will be utilized according to the large deflection assumption. Consider the initially curved beam given in Figure 3-9. The figure shows the deflection of the beam under the action of external forces $F_{x}, F_{y}$ and moment $M_{0}$ applied at point C. The un-deformed and deformed shapes of the beam are described by the intrinsic functions $\eta(s)$ and $\psi(s)$ respectively. The bending moment $M$ at any point $(x, y)$ on the beam is given by

$$
\begin{equation*}
M=E(s) I(s)\left[\frac{d \psi}{d s}-\frac{d \eta}{d s}\right]=F_{y}(a-x)+F_{x}(b-y)+M_{0} \tag{3.22}
\end{equation*}
$$

Note that, the initial curvature of the beam $d \eta / d s$ has no effect on bending moment and it is subtracted from the Eq. (3.1). The moment of the area for non-uniform cross-section is defined by $I(s)$. For non-homogenous beams, modulus of elasticity can be defined as $E(s)$.


Figure 3-9 An Initially curved beam model in a compliant mechanism
In order to obtain all the terms in Eq. (3.22) in terms of $s$, it is differentiated with respect to $s$ resulting in the following second order differential equation:

$$
\begin{equation*}
E(s) I(s)\left[\frac{d^{2} \psi}{d s^{2}}-\frac{d^{2} \eta}{d s^{2}}\right]+\left[E(s) \frac{d I(s)}{d s}+I(s) \frac{d E(s)}{d s}\right]\left[\frac{d \psi}{d s}-\frac{d \eta}{d s}\right]=-v \cos (\psi)-h \sin (\psi) \tag{3.23}
\end{equation*}
$$

where $d x / d s=\cos (\psi), d y / d s=\sin (\psi), h=F_{x}$ and $v=F_{y}$.
Expressing Eq. (3.23) in terms of normalized arc-length parameter $u \in\left[\begin{array}{ll}0 & 1\end{array}\right]$ leads to general form of the governing differential equation.

$$
\begin{equation*}
\frac{d^{2} \psi}{d u^{2}}=\frac{d^{2} \eta}{d u^{2}}-\left(\frac{d \psi}{d u}-\frac{d \eta}{d u}\right)\left(\frac{1}{I(u)} \frac{d I(u)}{d u}+\frac{1}{E(u)} \frac{d E(u)}{d u}\right)+\frac{L^{2}}{E(u) I(u)}(-v \cos (\psi)-h \sin (\psi)) \tag{3.24}
\end{equation*}
$$

Special cases of Eq. (3.24) can be obtained when

- $\quad \eta(u)=0$, for initially straight beam.
- $I(u)=I$, for uniform cross-section along the beam.
- $E(u)=E$, for homogenous material property along the beam.

Since a closed-form solution is not available for Eq. (3.24) with normalized boundary conditions given by Eq. (3.25a) and Eq. (3.25b), it is solved numerically by using shooting method.

$$
\begin{gather*}
\left.\psi\right|_{u=0}=0  \tag{3.25a}\\
\left.\frac{d \psi}{d u}\right|_{u=1}=\frac{M_{0} L}{E I} \tag{3.25b}
\end{gather*}
$$

In shooting method the boundary value problem (BVP) is transformed into an initial value problem (IVP) by making guesses on the unknown initial condition. With the known and guessed initial conditions the second order differential equation is solved using fourth order Runge-Kutta method and the guessed initial condition is modified until the second boundary condition is satisfied. Once the IVP is solved, the deflected shape of the beam in rectangular coordinates can be obtained from the following equations.

$$
\begin{align*}
& x\left(u_{0}\right)=L \int_{0}^{u_{0}} \cos (\psi) d u  \tag{3.26}\\
& y\left(u_{0}\right)=L \int_{0}^{u_{0}} \sin (\psi) d u \tag{3.27}
\end{align*}
$$

where $u_{0}$ is any value between 0 and 1.

### 3.4 Stability and Multistability

Stability is a general term that refers to an equilibrium condition in different disciplines such as mathematics, engineering, natural sciences, etc. As its name implies, multistability is the term used to represent more than one equilibrium positions.

Generally, ball-on-the-hill analogy is used to illustrate the stability of a system (Figure 3-10). A system is at equilibrium when no external forces are required to maintain its position. An equilibrium position is stable if the system can recover after small disturbances but it is unstable if the system cannot maintain its undisturbed position[28]. Meanwhile, it is also important to emphasize the relation between the slope of the hill and the vertical force needed to move the ball upward.


Figure 3-10 An illustration of the "ball-on-the-hill" analogy. Positions A and C are stable, position B is unstable and position $D$ is neutrally stable [42]

Stability of the mechanism can be determined using the potential energy equation as in Eq. (3.28). In the potential energy function, stable positions are located at local minima while unstable positions are located at local maxima[42]. Furthermore, the information about where the equilibrium positions exist can be obtained from the first derivative of the energy equation. Then, according to the sign of the second derivative of the energy equation the stability of the equilibrium positions can be determined. The positive, negative and zero value of the second derivative corresponds to the stable, unstable, and neutrally stable equilibrium positions respectively.

$$
\begin{equation*}
V=\frac{1}{2} \sum_{i=1}^{n}\left(K_{i} \Psi_{i}^{2}\right) \tag{3.28}
\end{equation*}
$$

The study carried out by Jensen et al. includes more detailed information about the bistable compliant mechanism. They presented theorems for the guaranteed bistability according to the selected spring locations. The theory is based on the Grashof's criterion. According to the Grashof's law, four-link mechanisms are classified as follows:

- If $1+\mathrm{s}<\mathrm{p}+\mathrm{q}$, it is called as Grashof mechanism.
- If $1+\mathrm{s}>\mathrm{p}+\mathrm{q}$, it is called as non-Grashof mechanism
- If $1+\mathrm{s}=\mathrm{p}+\mathrm{q}$, it is called as change-point mechanism

After determining the type of the four link mechanism, the next step is to find the spring locations with the help of the theorems stated below [28].

Theorem 1: A compliant mechanism whose PRBM behaves like a Grashof four-link mechanism with a torsional spring placed at one joint will be bistable if and only if the torsional spring is located opposite the shortest link and the spring's undeflected state does not correspond to a mechanism position in which the shortest link and the other link opposite the spring are collinear.
Theorem 2: A compliant mechanism whose PRBM behaves like a non-Grashof four-link mechanism with a torsional spring at any one joint will be bistable if and only if the spring's undeflected state does not correspond to a mechanism position in which the two links opposite the spring are collinear.
Theorem 3: A compliant mechanism whose PRBM behaves like a change-point four-link mechanism with a torsional spring placed at any one joint will be bistable if and only if the spring's undeflected state does not correspond to a mechanism position in which the two links opposite the spring are collinear.

### 3.5 Conclusions

Analytical formulation and approximation models for deflection of elastic members are discussed. The theory of elastic deformation, small and large deflection assumptions are explained. To understand the challenges of compliant mechanism design, the derivation of governing equations for large deflections is illustrated in details. Although there are numerous approaches for the design of compliant mechanisms, the pseudo-rigid-body model approach and shooting method adapted to calculate compliant beam deflections are considered in this study. Besides, three useful PRBMs for small-length flexural pivot, fixedpinned segments and fixed-fixed segments are presented. Also, stability and multistability conditions are discussed and the mathematical definition for the mechanisms stability is explained. This chapter serves as the basis for the mathematical models utilized in this study.

## CHAPTER 4

## DESIGN AND ANALYSIS OF A BISTABLE COMPLIANT FOV SWITCHING MECHANISM FOR OPTICAL SYSTEMS

### 4.1 Introduction

In this chapter, design and optimization of a bistable compliant mechanism (BSCM) is introduced. The optimized mechanism is implemented in an optical system that provides switching between the two different field-of-views (FOVs). It is considered to exhibit better performance including reliability, repeatability and durability when compared to previously used rigid-body mechanisms for the same task.

### 4.2 Problem Statement

Optics is usually concerned with small measurements [43]. Hence, the tolerances for lenses and mechanical parts should match with the results of optical tolerance analysis. Repeatability is another concern for the mechanisms used in opto-mechanical systems because many mechanisms including conventional joints lose their repeatability and positioning accuracy after a number of cycles due to induced wear and backlash. Ultra precise and repeatable rigid mechanisms are found in optical alignment apparatus but generally, they are not simple and compact. A conventional bistable mechanism having a non-linear spring connected its crank is shown in Figure 4-1. It is composed of a lot of parts such as rigid links, spring, bearings, pins, nuts, snap rings, etc.


Figure 4-1 A bistable four-link mechanism used in an IR camera and its components

All the problems mentioned above may have practical solutions by utilizing the advantages of compliant mechanisms over conventional mechanisms. That's why flexures and compliant mechanisms need to be used in the design of opto-mechanical systems.

### 4.2.1 Field of View Switching Mechanisms in Optical Systems

Field of view is sometimes called angle of view describing the extent of a target or a scene that can be imaged by an optical system. It is inversely related to magnification. As magnification increases, field of view decreases. Optical systems may have one, two or multiple FOVs. Furthermore, some systems have infinite FOVs which are basically described as continuous zoom cameras. Hence, the FOV switching mechanisms used in scopes and cameras may differ from each according to the area of use. However, two field-of-view systems with a narrow and a relatively wide angle of view are the most commonly used ones. It is easy to accomplish the switching action in two FOV systems. The simplest method requires deviation of a lens or a lens group from the optical path completely. It works as "in and out" system. In the other method, this action is performed simply by the forth and back motions of a lens or a lens group in the aligned position with the optical path. In Figure 4-2, configurations of the lenses in an objective are shown. Note that, "in and out" switching method is considered in this design. Nevertheless, based on the optical design, there exist more complex layouts.


Figure 4-2 Position of the lenses (a) in narrow field of view, (b) in wide field of view

### 4.2.2 Design Objective

The objective of this study is to synthesize a bistable compliant four-bar mechanism that is used to rotate a rigid body about a fixed pivot through 90 degrees by providing stability at its initial and final positions. It is decided to implement compliant mechanisms since they assure an efficient and economic way to derive bistable behavior [27]. In this study, the synthesized mechanism is also optimized for providing high stiffness at the stable positions and obtaining moderate critical force to toggle between them. Moreover, the expected result of this study is the improvement in the performance of the systems utilizing bistable mechanisms. For the particular design considered in this chapter, the system requirements which are directly related to the performance of the FOV switching mechanism can be counted as follows:

Adjustability: The mechanism should allow adjusting the FOV concentricity precisely.

Repeatability: The mechanism must satisfy the adjusted concentricity value between FOVs at each working cycle.

Stability: The mechanism must be stable under shock and vibration such that no flicker is observed in the image, especially in narrow angle of view.

Durability: The mechanism must be capable of performing at least 20.000 cycles without any failure.

Simplicity: The mechanism should be as simple as possible for the ease of assembly and use.

Feasibility: The mechanism must be feasible in terms of manufacturing.

### 4.2.3 Description of the Design Domain and Physical Constraints

The design domain that is allocated for the FOV switching mechanism of a rifle scope can be seen in Figure 4-3. One of the fixed pivots is already located at the center which can be seen in Figure 4-4 clearly. All other joints and links must be placed inside the cross-hatched boundary and they must stay inside that boundary throughout the motion of the mechanism. In addition, the motion of the links should be checked in order to avoid overlapping conditions. In plane thickness of the links should be less than 4 mm due to space limitations. The input link must be on rigid part which holds the lens group owing to the physical restrictions. It must a continuous motion through 90 degrees and it cannot move beyond the specified limits which are restricted by two stop pins. The positions of the stop pins are adjustable for the alignment requirements.


Figure 4-3 Design space dimensions (all dimensions are in mm)

The mechanism is manually actuated using the arm shown in Figure 4-4. The length of the arm and the maximum actuation force applied on the arm are previously specified as 45 mm and $15_{-0}^{+5} \mathrm{~N}$ respectively. The variation of actuation force beyond the specified tolerances is not acceptable since the user should not have difficulty in moving between two positions.


Figure 4-4 3D model of the design space

### 4.3 Design Methodology

In a design process, it is essential to specify the procedure to be followed beforehand to proceed in a systematic way. The forthcoming sub-sections are organized according to the five steps of design procedure followed in this study. These steps are respectively

- type synthesis
- dimensional synthesis
- motion analysis
- analysis for bistability
- determination of spring constants


### 4.3.1 Type Synthesis

Type synthesis is simply defined as the determination of the most promising mechanism architectures that meet design requirements. It is similar for rigid and compliant mechanisms except some modifications. Type synthesis of a mechanism can be explained briefly in three steps. First, a basic topology of the mechanism is described. This basic configuration is then modified to enumerate new mechanism configurations. At this stage of type synthesis, enumerations not satisfying design requirements are eliminated. Finally, the designer decides on the configuration(s) to continue dimensional synthesis and analysis.

In this design, twenty-nine different configurations of four-link mechanism are generated by varying connection and segment types according to the design criteria. One of them is a rigid body mechanism which is the initial topology while the others are partially compliant mechanism employing flexural pivots and compliant segments. From Figure 4-5, it can be realized that the input links of the all enumerated mechanisms contain a rigid segment and conventional revolute joint at the ground connection. It is due to the requirement on the input link. In the enumeration process, there are other two restrictions taken into account which are clamped connection between two rigid segments and connection of flexural pivots to the compliant segments. Construction of such connections is not reasonable since the first connection just creates a longer rigid segment and the second one result in an undistinguished compliance.

### 4.3.1.1 Results of Type Synthesis



It is worth to mention that, type synthesis results are categorized into four groups so that, in group A one flexible beam is used as a compliant element, in group B two flexible beams are used, in group $\mathbf{C}$ only flexural pivot is concerned, and in group $\mathbf{D}$ both compliant elements are utilized. Each of these configurations are evaluated prior to the dimensional synthesis regarding the situations as follows,

- topology promising guaranteed bistable behaviour
- topology promising high stiffness in stable positions
- life expectancy of flexure hinges and compliant beam segments
- ease of manufacturing and assembly

As a result, configurations A2, A3, A5 and A6 are selected as the most promising topologies satisfying design requirements and considering above conditions. To proceed further in this work, the selected configurations for bistable compliant FOV switching mechanism are modeled using pseudo-rigid-body approach. All of them include two rigid links and one compliant segment in common. They only differ in the connection of compliant segment. The equivalent rigid-body models that have the same kinematic and force behaviors with the compliant mechanisms are generated using the simple PRBMs for the segments presented in Chapter 3. The resulting PRBMs for switching mechanism are presented in Figure 4-6. Note that, the same PRBM is obtained for configurations A2 and A5.


Figure 4-6 Pseudo-rigid-body models of the selected mechanism configurations

### 4.3.2 Dimensional Synthesis

Dimensional synthesis is the next step after obtaining pseudo-rigid-body model. There are two methods for the synthesis of compliant mechanisms using kinematics approaches namely rigid-body replacement and synthesis with compliance. However, the former is the simplest method to determine the geometry of the PRBM without concerning the energy storage in the mechanism using the traditional kinematic synthesis techniques. Treating the kinematics and the energy storage separately makes it straightforward to solve the equations but still in the optimization algorithm we relate the energy storage to the kinematics via theorems that state guaranteed bistable behavior.

In the following, the analytical technique used to synthesize the selected four-bar mechanism configurations is explained. This technique utilizes the complex numbers to represent the vector pairs in loop closures. Beforehand, it is required to define the details of the problem
formulation which will help us to understand the simplified vector diagrams, the loop closure equations, and the parameters free to choose. These details are as indicated below.

- The fixed pivot location, $A_{0}$, which belongs to input crank is defined previously at the center of the fixed coordinate frame.
- The moving pivot of the input crank can be either on the rotating lens holder or on its extension.
- The input crank is specified to make a total rotation of 90 degrees from initial to final position as shown in Figure 4-7.
- In this problem, the task can only be a function generation according to the specifications. Hence, it is formulated as a function generator for three positions which are two stable positions and one unstable position.
- In function generation we do not use extended coupler points. Thus, it is convenient to define $\boldsymbol{Z}_{\boldsymbol{A}}$ is equal to zero instead of $\boldsymbol{Z}_{\boldsymbol{B}}$.
- Since $A_{0}$ is at the center of the fixed coordinate frame, we can find $\boldsymbol{R}_{\boldsymbol{I}}=\boldsymbol{W}_{\boldsymbol{A}}$.
- In the synthesis for function generation, the input crank or output crank vector can be picked as an arbitrary choice because it only defines the scale and orientation of the mechanism [Howell]. Here, $\boldsymbol{W}_{\boldsymbol{A}}$ is picked as an arbitrary choice and the synthesis problem is reduced to synthesis of dyad B only.


Figure 4-7 Correlation of crank angles for three positions

After giving the details of the problem formulation, the solution procedure given below is followed. According to the vector diagrams seen in Figure 4-7, the loop closure equations are simplified as follows:

$$
\begin{align*}
& \boldsymbol{W}_{\boldsymbol{B}}\left(e^{i \psi_{2}}-1\right)+\boldsymbol{Z}_{B}\left(e^{i \alpha_{2}}-1\right)=\boldsymbol{\delta}_{2}  \tag{4.1}\\
& \boldsymbol{W}_{B}\left(e^{i \psi_{3}}-1\right)+\boldsymbol{Z}_{B}\left(e^{i \alpha_{3}}-1\right)=\boldsymbol{\delta}_{3} \tag{4.2}
\end{align*}
$$

where

$$
\begin{gather*}
\boldsymbol{\delta}_{2}=\boldsymbol{R}_{2}-\boldsymbol{R}_{I}=\boldsymbol{W}_{A} e^{i \beta_{2}}-\boldsymbol{W}_{A}  \tag{4.3}\\
\boldsymbol{\delta}_{3}=\boldsymbol{R}_{3}-\boldsymbol{R}_{1}=\boldsymbol{W}_{A} e^{i \beta_{3}}-\boldsymbol{W}_{A} \tag{4.4}
\end{gather*}
$$

The terms, $\beta_{2}$ and $\beta_{3}$ in Eq. (4.3) and Eq. (4.4), are the desired input crank rotations while $\psi_{2}$ and $\psi_{3}$ in Eq. (4.1) and Eq. (4.2) are the desired output crank rotations. Hence, these equations include eight scalar unknowns in total which are $\boldsymbol{W}_{\boldsymbol{B}}, \boldsymbol{Z}_{\boldsymbol{B}}, \boldsymbol{W}_{\boldsymbol{A}}, \alpha_{2}$ and $\alpha_{3}$. Anyone can solve these equations by specifying four of the unknowns. However, it is better to specify $\boldsymbol{W}_{\boldsymbol{A}}, \alpha_{2}$ and $\alpha_{3}$ as free parameters in order to obtain a linear equation set in terms of unknowns $\boldsymbol{W}_{\boldsymbol{B}}$ and $\boldsymbol{Z}_{\boldsymbol{B}}$. Then, it can be solved easily using Cramer's Rule.

$$
\begin{align*}
& \boldsymbol{Z}_{B}=\frac{\left|\left(\begin{array}{cc}
e^{i / /_{2}}-1 & \boldsymbol{\delta}_{2} \\
e^{i / \omega_{3}}-1 & \boldsymbol{\delta}_{3}
\end{array}\right)\right|}{\left|\left(\begin{array}{cc}
e^{i / \omega_{2}}-1 & e^{i \alpha_{2}}-1 \\
e^{i / \omega_{3}} & -1 \\
e^{i \alpha_{3}} & -1
\end{array}\right)\right|}  \tag{4.5}\\
& \boldsymbol{W}_{\boldsymbol{B}}=\frac{\left\lvert\,\left(\left.\begin{array}{ll}
\boldsymbol{\delta}_{2} & e^{i \alpha_{2}}-1 \\
\boldsymbol{\delta}_{3} & e^{i \alpha_{3}}-1
\end{array} \right\rvert\,\right.\right.}{\left|\left(\begin{array}{cc}
i \boldsymbol{e}^{i \gamma_{2}}-1 & e^{i \alpha_{2}}-1 \\
e^{i \mu_{3}}-1 & e^{i \alpha_{3}}-1
\end{array}\right)\right|} \tag{4.6}
\end{align*}
$$

Knowing link lengths and prescribed crank rotations, motion analysis of the obtained solution can be performed. Solving for $\boldsymbol{W}_{\boldsymbol{B}}$ and $\boldsymbol{Z}_{\boldsymbol{B}}$, link lengths are obtained as follows by utilizing the vector diagrams:

$$
\begin{gather*}
r_{1}=\left|\boldsymbol{W}_{\boldsymbol{A}}+\boldsymbol{Z}_{\boldsymbol{A}}-\boldsymbol{Z}_{\boldsymbol{B}}-\boldsymbol{W}_{\boldsymbol{B}}\right|  \tag{4.7}\\
r_{2}=\left|\boldsymbol{W}_{\boldsymbol{A}}\right|  \tag{4.8}\\
r_{3}=\left|\boldsymbol{Z}_{\boldsymbol{A}}-\boldsymbol{Z}_{\boldsymbol{B}}\right|  \tag{4.9}\\
r_{4}=\left|\boldsymbol{W}_{\boldsymbol{B}}\right| \tag{4.10}
\end{gather*}
$$

### 4.3.3 Motion Analysis

In this part, a method for position analysis of a four-bar mechanism is introduced. After all kinematic synthesis problems, synthesized mechanisms must be checked whether it is movable or not between the prescribed positions. Moreover, the design must also be checked in compliant mechanism synthesis for other reasons. Synthesis using rigid body techniques may yield configurations that are adequate for rigid-link mechanisms but are not acceptable for compliant mechanisms. For instance, a flexural pivot deflecting through a complete rotation is an unacceptable result [9]. Therefore, in compliant mechanism synthesis we need to perform more iteration on the free choices before obtaining an acceptable design.

There are so many methods to analyze a four-bar mechanism. However, we use a method that is defined in terms of complex numbers for the sake of convenience. Loop closure equations in complex form for the analysis of the mechanism are given below.

$$
\begin{equation*}
\overrightarrow{A_{0} A_{j}}+\overrightarrow{A_{j} B_{j}}=\overrightarrow{A_{0} B_{0}}+\overrightarrow{B_{0} B_{j}} \text { where } j=1,2,3 \tag{4.11}
\end{equation*}
$$

Inserting the dyad vectors into Eq. 4.11 and then rearranging in terms of link angles defined parametrically, we obtain

$$
\begin{equation*}
\boldsymbol{W}_{\boldsymbol{A}} e^{i\left(\theta_{12}\right)_{k}}+\left(\boldsymbol{Z}_{\boldsymbol{A}}-\boldsymbol{Z}_{\boldsymbol{B}}\right) e^{i\left(\theta_{13}\right)_{k}}=\boldsymbol{W}_{\boldsymbol{A}}+\boldsymbol{Z}_{\boldsymbol{A}}-\boldsymbol{Z}_{\boldsymbol{B}}-\boldsymbol{W}_{\boldsymbol{B}}+\boldsymbol{W}_{\boldsymbol{A}} e^{i\left(\theta_{12}\right)_{k}} \tag{4.12}
\end{equation*}
$$

To simplify Eq. 4.12, the terms G, and H are defined

$$
\begin{gather*}
\boldsymbol{G}_{\boldsymbol{k}}=\boldsymbol{Q}+\boldsymbol{W}_{A} e^{i\left(\theta_{12}\right)_{k}} \text { where } \boldsymbol{Q}=\boldsymbol{W}_{\boldsymbol{B}}+\boldsymbol{Z}_{\boldsymbol{B}}-\boldsymbol{Z}_{\boldsymbol{A}}-\boldsymbol{W}_{\boldsymbol{A}}  \tag{4.13}\\
\boldsymbol{H}=\boldsymbol{Z}_{\boldsymbol{B}}-\boldsymbol{Z}_{\boldsymbol{A}} \tag{4.14}
\end{gather*}
$$

In its simplest form, Eq.4.12 can be expressed as below

$$
\begin{equation*}
\boldsymbol{G}_{\boldsymbol{k}}-\boldsymbol{W}_{\boldsymbol{B}} e^{i\left(\theta_{14}\right)_{k}}=\boldsymbol{H} \boldsymbol{e}^{i\left(\theta_{13}\right)_{k}} \tag{4.15}
\end{equation*}
$$

Multiplying Eq.4.15 with its complex conjugate, $\overline{\boldsymbol{G}_{\boldsymbol{k}}}-\overline{\boldsymbol{W}_{\boldsymbol{B}}} e^{-i\left(\theta_{14}\right)_{k}}=\overline{\boldsymbol{H}} e^{-i\left(\theta_{13}\right)_{k}}$ results in

$$
\begin{equation*}
\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{G}_{\boldsymbol{k}}}-\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{W}_{\boldsymbol{B}}} e^{-i\left(\theta_{14}\right)_{k}}-\overline{\boldsymbol{G}_{\boldsymbol{k}}} \boldsymbol{W}_{\boldsymbol{B}} e^{i\left(\theta_{14}\right)_{k}}+\boldsymbol{W}_{\boldsymbol{B}} \overline{\boldsymbol{W}_{\boldsymbol{B}}}=\boldsymbol{H} \overline{\boldsymbol{H}} \tag{4.16}
\end{equation*}
$$

Note that, we eliminate $\theta_{13}$ from the equation. If both sides are multiplied with $-e^{i\left(\theta_{14}\right)_{k}}$, we obtain a quadratic equation in terms of $e^{i\left(\theta_{14}\right)_{k}}$ as given below.

$$
\begin{equation*}
\overline{\boldsymbol{G}_{\boldsymbol{k}}} \boldsymbol{W}_{\boldsymbol{B}} e^{2 i\left(\theta_{14}\right)_{k}}+\left(-\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{G}_{\boldsymbol{k}}}-\boldsymbol{W}_{\boldsymbol{B}} \overline{\boldsymbol{W}_{\boldsymbol{B}}}+\boldsymbol{H} \overline{\boldsymbol{H}}\right) e^{i\left(\theta_{14}\right)_{k}}+\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{W}_{\boldsymbol{B}}}=\boldsymbol{0} \tag{4.17}
\end{equation*}
$$

Above equation can be solved by using discrimination rule. First, it is necessary to define the following terms

$$
\left.\begin{array}{l}
\boldsymbol{A}_{\boldsymbol{k}}=\overline{\boldsymbol{G}_{\boldsymbol{k}}} \boldsymbol{V}_{\boldsymbol{B}}  \tag{4.18}\\
\boldsymbol{B}_{\boldsymbol{k}}=-\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{G}_{\boldsymbol{k}}}-\boldsymbol{W}_{B} \overline{\boldsymbol{W}_{B}}+\boldsymbol{H} \overline{\boldsymbol{H}} \\
\boldsymbol{C}_{\boldsymbol{k}}=\boldsymbol{G}_{\boldsymbol{k}} \overline{\boldsymbol{W}_{B}}
\end{array}\right\}
$$

Finally, the output crank angle is found as

$$
\begin{equation*}
\left(\theta_{14}\right)_{k}=\arg \left(e^{i\left(\theta_{14}\right)_{k}}\right)=\arg \left(\frac{-\boldsymbol{B}_{\boldsymbol{k}} \pm \sqrt{\boldsymbol{B}_{\boldsymbol{k}}{ }^{2}-4 \boldsymbol{A}_{\boldsymbol{k}} \boldsymbol{C}_{\boldsymbol{k}}}}{2 \boldsymbol{A}_{\boldsymbol{k}}}\right) \tag{4.19}
\end{equation*}
$$

Substituting $\theta_{14}$ into Eq. 4.15, the coupler link angle is obtained as

$$
\begin{equation*}
\left(\theta_{13}\right)_{k}=\arg \left(e^{i\left(\theta_{13}\right)_{k}}\right)=\arg \left(\frac{\boldsymbol{G}_{\boldsymbol{k}}-\boldsymbol{W}_{\boldsymbol{B}} e^{i\left(\theta_{14}\right)_{k}}}{\boldsymbol{H}}\right) \tag{4.20}
\end{equation*}
$$

As a result, for a given input crank angle, $\theta_{12}$, any value of the other angles, $\theta_{13}$ and $\theta_{14}$ can be determined. Then, $\theta_{13}$ vs $\theta_{12}$ and $\theta_{14}$ vs $\theta_{12}$ plots are obtained. Also, motion of the mechanism can be visualized with animation using $k$ number of points.

### 4.3.4 Analysis for Bistability

Analysis for bistability is the fourth step of the design methodology. Recall from Section 3.4 that, the concept of bistability can be demonstrated using "ball on a hill" analogy that is well suited for the equilibrium positions in potential energy curve of bistable mechanisms. As it is seen in Figure 4-8, stable positions A and E resemble local minima where unstable position C corresponds to a local maximum in a potential energy function.


Figure 4-8 A modified illustration of the "ball on a hill" analogy. Positions B and D are externally constraint stable [44].

According to the design requirements, the designed four-bar mechanism should to have two stable positions and move smoothly in between these positions. These positions are desired to be located similar to B and D which are externally constrained as shown in Figure 4-8. These constraints resemble adjustment pins on which mechanism always exerts force. However, in mathematical formulations for bistability positions A, C and E are considered.

## Potential Energy Function

For a general case, the PRBM of a fully compliant four-bar mechanism shown in Figure 4-9 can be considered. It is preferred to use a parametric formulation since each spring location is analyzed independently. Thus, it does not require reformulation of the problem. Only spring constants are modified by changing the non-zero spring constant. The total potential energy of the mechanism is given in Eq. (4.21) [9].

$$
\begin{equation*}
V=\frac{1}{2}\left(K_{1} \Psi_{1}^{2}+K_{2} \Psi_{2}^{2}+K_{3} \Psi_{3}^{2}+K_{4} \Psi_{4}^{2}\right) \tag{4.21}
\end{equation*}
$$

where

$$
\left.\begin{array}{l}
\Psi_{1}=\theta_{12}-\theta_{2 i}  \tag{4.22}\\
\Psi_{2}=\theta_{12}-\theta_{2 i}-\left(\theta_{13}-\theta_{3 i}\right) \\
\Psi_{3}=\theta_{14}-\theta_{4 i}-\left(\theta_{13}-\theta_{3 i}\right) \\
\Psi_{4}=\theta_{14}-\theta_{4 i}
\end{array}\right\}
$$



Figure 4-9 PRBM of a fully compliant four-bar mechanism at its initial configuration
Since all the kinematic relations for the motion of the mechanism are obtained in Section 4.3.3, potential energy, $V$, can be obtained as a function of input crank angle, $\theta_{12}$. When it is plotted, stable and unstable positions can be easily identified from the graph. However, it can be better to express local minima and local maximum analytically using first and second derivatives of the potential energy. This helps us to construct an algorithm to check equilibrium positions automatically. Taking the first derivative of $V$ with respect to $\theta_{12}$, we obtain

$$
\begin{equation*}
\frac{d V}{d \theta_{12}}=K_{1} \Psi_{1}+K_{2} \Psi_{2}\left(1-\frac{d \theta_{13}}{d \theta_{12}}\right)+K_{3} \Psi_{3}\left(\frac{d \theta_{14}}{d \theta_{12}}-\frac{d \theta_{13}}{d \theta_{12}}\right)+K_{4} \Psi_{4} \frac{d \theta_{14}}{d \theta_{12}} \tag{4.23}
\end{equation*}
$$

where the equations for the unknown derivatives, $\frac{d \theta_{13}}{d \theta_{12}}$ and $\frac{d \theta_{14}}{d \theta_{12}}$ are given below [45].

$$
\left.\begin{array}{l}
\frac{d \theta_{13}}{d \theta_{12}}=\frac{r_{2} \sin \left(\theta_{14}-\theta_{12}\right)}{r_{3} \sin \left(\theta_{13}-\theta_{14}\right)} \\
\frac{d \theta_{14}}{d \theta_{12}}=\frac{r_{2} \sin \left(\theta_{13}-\theta_{12}\right)}{r_{4} \sin \left(\theta_{13}-\theta_{14}\right)} \tag{4.24}
\end{array}\right\}
$$

As it is noted in Chapter 3, the first derivative of the potential energy function i.e. input torque applied to input crank must be equal to zero at equilibrium positions. Using the sign of the second derivative of the energy function, we may decide on the stability of these positions. Hence, the second derivative of $V$ with respect to $\theta_{12}$ is given as

$$
\begin{align*}
& \frac{d^{2} V}{d \theta_{12}^{2}}=K_{1}+K_{2}\left(1-2 \frac{d \theta_{13}}{d \theta_{12}}+\left(\frac{d \theta_{13}}{d \theta_{12}}\right)^{2}-\Psi_{2} \frac{d^{2} \theta_{13}}{d \theta_{12}^{2}}\right) \\
& +K_{3}\left(\left(\frac{d \theta_{14}}{d \theta_{12}}\right)^{2}-2 \frac{d \theta_{14}}{d \theta_{12}} \frac{d \theta_{13}}{d \theta_{12}}+\left(\frac{d \theta_{13}}{d \theta_{12}}\right)^{2}+\Psi_{3}\left(\frac{d^{2} \theta_{14}}{d \theta_{12}^{2}}-\frac{d^{2} \theta_{13}}{d \theta_{12}^{2}}\right)\right)  \tag{4.25}\\
& +K_{4}\left(\left(\frac{d \theta_{14}}{d \theta_{12}}\right)^{2}+\Psi_{4} \frac{d^{2} \theta_{14}}{d \theta_{12}^{2}}\right)
\end{align*}
$$

where the equations for the unknown derivatives, $\frac{d^{2} \theta_{13}}{d \theta_{12}^{2}}$ and $\frac{d^{2} \theta_{14}}{d \theta_{12}^{2}}$ are given below [45].

$$
\begin{align*}
& \frac{d^{2} \theta_{13}}{d \theta_{12}^{2}}=\frac{r_{2} \cos \left(\theta_{14}-\theta_{12}\right)+r_{3}\left(\frac{d \theta_{13}}{d \theta_{12}}\right)^{2} \cos \left(\theta_{14}-\theta_{13}\right)-r_{4}\left(\frac{d \theta_{14}}{d \theta_{12}}\right)^{2}}{r_{3} \sin \left(\theta_{14}-\theta_{13}\right)}  \tag{4.26}\\
& \frac{d^{2} \theta_{14}}{d \theta_{12}^{2}}=\frac{-r_{2} \cos \left(\theta_{13}-\theta_{12}\right)+r_{4}\left(\frac{d \theta_{14}}{d \theta_{12}}\right)^{2} \cos \left(\theta_{13}-\theta_{14}\right)-r_{3}\left(\frac{d \theta_{13}}{d \theta_{12}}\right)^{2}}{r_{4} \sin \left(\theta_{13}-\theta_{14}\right)}
\end{align*}
$$

The equilibrium positions where the first derivative equal to zero can now be stated as stable if the sign of the second derivative is positive or unstable if the sign of the second derivative is negative at corresponding positions. If it is also zero, the position is termed as neutrally stable.

## Bistability Criteria

Prior to discuss about the theorems for the bistability of four-link mechanisms in detail, it is required to identify the type of the synthesized mechanism since they are based on the Grashof's criterion. There are three theorems which are for Grashof, non-Grashof and change-point mechanisms, respectively. For the theorems and their proofs one can refer to [28]. These theorems developed by Jensen et al., may be used to help us to where to place a spring for the guaranteed bistable behaviour. Therefore, unnecessary calculations for all possible spring locations are eliminated. According to the type of the four-link mechanism, possible locations of the springs to obtain bistable behaviour are given in Table 4-1.

Table 4-1 The spring locations for bistable behaviour in four-link mechanisms [28].

| Mechanism Class | Location of Springs for Bistable Mechanism |
| :--- | :--- |
| Grashof Four-Link Mechanism | Either location opposite the shortest link |
| Change-Point Four-Link Mechanism | Any location |
| Non-Grashof Four-Link Mechanism | Any location |

Since the above-referred theorems are valid for mechanisms with one spring and they do not guarantee the behaviour of mechanisms with multiple springs. However, we may apply the principle of superposition when there exists more than one spring in the mechanism. In order to acquire miscellaneous bistable characteristics, mechanisms with multiple spring configurations can be preferred. In such an analysis, each spring location is considered independently. Then, their energy storages can be added up and final energy of the system is checked for bistable characteristics.

### 4.3.5 Determination of Spring Constants

The last step in the design procedure of the bistable compliant mechanism is to determine the spring constants of the PRBMs since in the investigation of the bistability of the mechanism the spring constants are taken as unity. As it is mentioned in Chapter 3, the spring constant of the PRBM depends on several parameters including the pseudo-rigid-body angle, the elastic modulus of the material and the link dimensions. Based on the PRBM of the selected mechanism configuration, Eq. (3.18) or Eq. (3.21) can be used to calculate the spring constants. Beforehand, the lengths of the flexible members are calculated using Eq. (4.27).

$$
\begin{equation*}
l_{i}=\frac{r_{i}}{\gamma} \text { where } \mathrm{i}=3,4 \tag{4.27}
\end{equation*}
$$

Since the pseudo rigid body angles and the lengths of the flexible members are obtained, in this part we decide on the material property and the cross-sectional dimensions of the members. Owing to the fact that bending is the most dominant loading condition in compliant mechanism theory, large deflection members need to be designed accordingly. Table 4-2 shows various materials used for compliant mechanisms and their properties such as $E, S_{y}$ and $S_{y} / E$. Strength-to-modulus ratio, $S_{y} / E$, is an important material property that represents the ability of bending without yield. In order to select the material used in this design, these ratios as well as the issues related to cost and manufacturability of the members with a selected material are considered. There is no other limitation on the material.

Table 4-2 Ratio of yield strength to Young's modulus for several materials [9]

| Material | $\mathbf{E}(\mathbf{G P a})$ | $\mathbf{S}_{\mathbf{y}} \mathbf{( M P a )}$ | $\left(\mathbf{S}_{\mathbf{y}} / \mathbf{E}\right) \times \mathbf{1 0 0 0}$ |
| :--- | :---: | :---: | :---: |
| Steel (1010 hot rolled) | 207 | 179 | 0.87 |
| Steel (4140 Q\&T@400) | 207 | 1641 | 7.9 |
| Aluminum(7075 heat treated) | 71.7 | 503 | 7.0 |
| Titanium (Ti-35A annealed) | 114 | 207 | 1.8 |
| Beryllium copper (CA 170) | 128 | 1170 | 9.2 |
| Polyethylene (HDPE) | 1.4 | 28 | 20 |
| Nylon (type 66) | 2.8 | 55 | 20 |
| Polypropylene | 1.4 | 34 | 25 |

The beam has rectangular section, and its moment of inertia, $I$, calculated from Eq. (4.28). The optimum value of $I$ is obtained from the maximum allowable stress equation and the critical torque requirements. The critical torque is the maximum absolute value of the torque that is applied to toggle between bistable positions. The input torque to the system can be derived from the derivative of the energy function. Thus, according to the most promising spring location for bistable characteristics Eq. (4.23) is rearranged and used to determine the moment of inertia of the beam. Additionally, design space limitation on the beam width, $w$, and minimum manufacturable beam thickness, $t$, are applied as constraints.

$$
\begin{equation*}
I=\frac{1}{12} w t^{3} \tag{4.28}
\end{equation*}
$$

Then, the maximum allowable stress given in Eq. (4.29) for flexible segments is required to be checked. Using Eq. (4.30), the safety factor which should be greater than 1 for working without static failure can be calculated.

$$
\begin{align*}
& \sigma_{b}=\frac{M t}{2 I}  \tag{4.29}\\
& S F=\frac{S_{y}}{\sigma_{b}} \tag{4.30}
\end{align*}
$$

In this design, both considering the durability and manufacturability of the mechanism it is revealed that stainless steel ( 4140 Q\&T@400) is the most appropriate material which has ratio $S_{y} / E=7.9$.

### 4.4 Dimensional and Functional Optimization

In this section, aforementioned design methodology is followed to optimize the dimensions and functions of the mechanism. A heuristic optimization procedure is used by changing arbitrary choices in the dimensional synthesis step until the design requirements are satisfied. The design parameters including arbitrarily selected ones, constraint functions given in Eq. (4.31) and objective function in Eq. (4.32) are described. The objective function $f$ is defined as deviation of the ratio of the absolute value of critical moments from unity and it will be minimized, thus the mechanisms may be actuated with same amount of force from both directions.

$$
\begin{align*}
& c_{1}:-25<A_{1 x}<27.5 \\
& c_{2}:-\sqrt{27.5^{2}-A_{1 x}^{2}}<A_{1 y}<\sqrt{27.5^{2}-A_{1 x}^{2}} \\
& c_{3}:-25<B_{1 x}<27.5 \\
& c_{4}:-\sqrt{27.5^{2}-B_{1 x}^{2}}<B_{1 y}<\sqrt{27.5^{2}-B_{1 x}^{2}}  \tag{4.31}\\
& c_{5}:-25<B_{0 x}<27.5 \\
& c_{6}:-\sqrt{27.5^{2}-B_{0 x}^{2}}<B_{0 y}<\sqrt{27.5^{2}-B_{0 x}^{2}}
\end{align*}
$$

$$
\begin{equation*}
f: 1-\left|\frac{M_{c r 1}}{M_{c r 2}}\right| \tag{4.32}
\end{equation*}
$$

For all the calculations in design methodology and the optimization part an optimization code is written in Matlab which applies space constraints, movability check and the bistability criteria. Moreover, for the mechanism configurations satisfying these conditions the objective function is evaluated. If any of these conditions is not satisfied, new design parameters are selected until the optimum values obtained for the objective functions are obtained. The analysis code is given in Appendix-A.


Figure 4-10 Design Space Check in Optimization Routine

### 4.4.1 Final Model

In this section, the validation process of the final BSCM design is presented. First of all, motion and bistability analyses are repeated with the optimized dimensions. Then, to predict the stress behaviour and the fatigue life of the flexible members so the fatigue life of the mechanism, a finite element analysis is performed. For the critical stress locations, the dimensions and geometry of the flexible member is refined regarding that its characteristic length remains same. Besides, a prototype of the mechanism is manufactured. Since, it is considered to be useful to check functionality and the other design requirements are satisfied by the design. Through simulations the movability, bistability are visualized and critical moments are controlled.


Figure 4-11 CAD model of the final design

The results for the optimized solution prior to finite element analysis are presented in Table $4-3$. In these figures movability, bistability and critical moments can be observed.


Figure 4-12 Bistability Control in Optimization Routine

Table 4-3 Optimization Results

| Optimized Parameters | Objective Function Value |
| :---: | :---: |
| $r_{1}=25.31 \mathrm{~mm}, r_{2}=9 \mathrm{~mm}, r_{3}=16.62 \mathrm{~mm}, r_{4}=11.2 \mathrm{~mm}$ |  |
| $l_{4}=13.15 \mathrm{~mm}, w=3.2 \mathrm{~mm}, t=0.65 \mathrm{~mm}$ | $f=0.0016$ |
| $\psi_{2}=-47.5 \mathrm{deg}, \psi_{3}=-90 \mathrm{deg}, \alpha_{2}=20 \mathrm{deg}, \alpha_{3}=45 \mathrm{deg}$ |  |

### 4.4.2 Finite Element Analysis of the Flexible Member

To realize the motion of the BSCM, the first step is to build the model of the mechanism using a CAD software. Then, this model is transferred to the commercial finite element analysis program, ANSYS. After defining the joints and boundary conditions, and assigning the selected material properties, the motion of the mechanism can be simulated using ANSYS. Also, the theoretical model and finite element model results such as moment and strain energy of the flexible member can be compared. It can be shown in Figure 4-14 and Figure 4-15 that, the analysis results are consistent with each other.


Figure 4-13 Finite Element Model of the Compliant Segment
Furthermore, according to the obtained stress results from the simulations, modifications can be made on the flexible member considering the previously defined PRBM properties are
preserved. Consequently, the flexible link geometry may be optimized by this approach for static failure and fatigue considerations. In the analysis, the flexible members are meshed using tetrahedron solid mesh elements while surface elements are used in the contact locations of the stop pins.


Figure 4-14 Strain Energy Results Obtained from FEA.


Figure 4-15 Required Torque Results Obtained from FEA.

### 4.5 Discussions

Design and analysis of bistable FOV switch mechanism that is a partially compliant four-bar mechanism is presented. Also, the mechanism is optmized for in terms of design requirements and structural properties. In this design, we consider the kinematics and kinetics separately. The design procedure is defined such that it allows searching the design domain for realizable solutions. Moreover, the mechanism is optimized for FOV switching forces the ratio of which is the objective function in this design. Finally, finite element analysis is performed for strength considerations. The the connections in CAD model are also optimized to decrase the stress concentration effects.

## CHAPTER 5

## DESIGN AND ANALYSIS OF A FULLY COMPLIANT SHAPE ADAPTIVE ROBOTIC GRIPPER

### 5.1 Introduction

In this chapter, the design and analysis of a fully compliant robotic gripper is explained. The purpose in this design is to develop a compliant gripper which can be easily mounted on the industrial robots which are used to pick and place the objects. Since its fingers are designed to be shape adaptive the size, the geometry and the orientation of the object would not be a problem unless the object size is out of the limits. Also, this design comes into prominence with its simple structure that can be manufactured economically. Only one actuator is implemented and multiple outputs are obtained at the finger contacts by taking the advantage of compliance in this study.

### 5.2 Preliminary Design Decisions

Although the conventional grippers are designed for specific tasks, their structures may also vary depending on the usage. The compliant gripper mechanism here is to be designed so that it can perform various tasks. Hence, there are some requirements which should be initially explained. Firstly, the gripper should be designed to be able to attach on any manipulator wrist to perform various tasks on a moving conveyor system or a stationary table. Secondly, considering that most of the manipulators in the automated industry are designed to work above the conveyor and manipulate the objects by reaching the objects from the top, it should be designed to grip objects from the top. Next, its dimensions are desired to be consistent with the dimensions of the human hand. However, depending on the application the number of fingers of the gripper can be two, three or four. Lastly, it should be easily manufacturable, inexpensive and space-saving to be used extensively for different purposes in the industry.

## Main Gripper Features

A review of the studies concerned with the design of the robotic grippers revealed that the gripper structure is composed of three main components that are actuation mechanism, transmission and finger mechanisms [35, 38]. In compliant robot grippers, the transmission mechanism and the finger mechanism may be observed to as an integrated structure. Nevertheless, the detailed work in this study is on the design and optimization of the structure of the compliant fingers, the actuation mechanism should be mentioned with its general properties. As mentioned in the literature, if the number of actuators decreases, the control of the system gets simplified. Also, the number of actuators used in the compliant gripper should be kept as small as possible to save the space. In this design it is decided to have one degree of actuation. The other expected features related to the finger mechanism can be stated as follows:

- It must adapt itself to the shape of the grasped object.
- It is intended to grasp any object with the size in the specified range.
- Its grip performance must be independent of the object's position and orientation.
- It must be able to be actuated with one actuator.
- It should have a monolithic structure with distributed compliance.
- Its length should be close to the length of the fingers of human hand.

In the following, the basic structure of the gripper to be designed is briefly discussed. The conventional method to obtain shape adaptation can be observed in many underactuated rigid grippers. They have two or more segments called phalanges in each finger to take the shape of the object. However, they are composed of many number of mechanical parts. The implementation of compliant mechanism into underactuated grippers can be seen in Figure 5-1.


Figure 5-1 Underactuated Compliant Gripper (Adapted from [46])
Intuitively, instead of using flexures to simulate the rotation of phalanges about pin joints, bending of a beam until it envelopes the object is considered as more simple. Accordingly, the basic structure shown in Figure 5-2 is proposed.

### 5.2.1 Description of the Design Domain and Physical Constraints

The boundaries for the design space and the physical constraints need to be defined at initial design stage. In this design, the maximum index finger length is specified as to be 100 mm considering the dimensions of the human fingers. In addition, the palm of the human hand can be defined by 80 mm X 80 mm area, thus this amount of region is reserved for the actuation system. The distance between the finger tips should be in between 50 mm and 80 mm , to match with the size of the objects that can be safely handled by a human hand. According to the requirements specified previously, the actuation mechanism should be adjacent to the fingers. Hence, the maximum dimensions of the design domain are chosen to be 180 mm in length, and 80 mm in width which can be seen in Figure 5-2. When the basic structure of the finger is considered it approximates a motion amplifier, i.e. with unit displacement of actuator the finger tips are displaced more than unity. Despite the fact that
for maximum displacement 40 mm of finger tips the required input displacement is less than 40 mm , it is limited to $\mathrm{L}_{\mathrm{imax}}=20 \mathrm{~mm}$ considering the strokes of the available miniature linear actuators [47].


Figure 5-2 Proposed Finger Structure and Dimensions of the Design Domain

### 5.2.2 Actuation Method

In this part, the decision on the actuation method is given. When the proposed design illustrated in Figure 5-2 is considered, the transmission mechanism is integrated into the fingers and the fingers are directly connected to the actuation system. At the beginning, it is decided to use one actuator. Hence, the fingers ends should be connected to each other to take the input from one actuator. To open and close the fingers, the simplest input can be the linear motion. This input motion can be by pulling or pushing the connected finger ends. If the inner ends are connected to each other, the pulling causes to close the finger while the outer ends are connected to each other it causes to open the finger as it is seen in Figure 5-3.

(a)

(b)

Figure 5-3 Two method of actuation (a) from inner ends (b) from outer ends

In the preliminary evaluations with a hand model, the configuration (b) provides good results in terms of shape adaptation compared to the configuration (a). However, it is found that there is a risk with configuration (b). The fingers tips may touch the ground. Thus, it is decided to use configuration (a) in next design steps.

### 5.3 Conceptual Design

In this section, the conceptual design process of the gripper finger is explained. This section consists of three main steps namely, concept generation, functional analysis and the evaluation of the concepts according to their performances. In conceptual design, most of time was devoted to the concept generation and functional analysis. It is not easy to come up with too different concepts for the finger structure and to estimate their behaviour of without a link between CAD and FEA programs. Thus, by linking these programs a lot of time was saved because the time required for analysis settings and post-processing in FEA was eliminated.

### 5.3.1 Design Concepts

Alternative finger structures are determined intuitively, starting from the proposed finger geometry and making modifications on that geometry regarding the design criteria. Firstly, the hand sketches are drawn considering the input location and the ground connections. Although many alternatives are developed, only eight of them are chosen to proceed with the functional analysis. The related sketches can be seen in Figure 5-4. Then, 3D models of these alternatives were obtained using a CAD program.


Figure 5-4 Sketches of the Selected Concepts for Functional Analysis
The design alternatives differ from each other with major modifications in their geometry or additional segments as it is seen from Figure 5-4.

### 5.3.2 Functional Analysis using Finite Element Analysis (FEA)

Functional analysis is performed for each alternative using finite element analysis tool ANSYS. The closing behaviour of the fingers is visualized and animated to decide on the shape adaptation property. Besides, some numerical results are obtained for contact pressure, actuation force, stresses and deformed node locations. There are some parameters and conditions which are kept unchanged during the analysis to compare the results of different alternatives objectively. The analysis conditions are mentioned as follows:

- The finger segments are modeled with the dimensions $\mathrm{w}=5 \mathrm{~mm}, \mathrm{t}_{1}=0.5 \mathrm{~mm}$ and $\mathrm{t}_{2}=1 \mathrm{~mm}$.
- The outer ends are specified as fixed joint.
- Mesh element size and type are kept constant for all flexible finger segments. Tetrahedron elements with a sizing factor 0.1 mm are chosen.
- Analysis and simulations are performed to grasp a cylindrical object with the diameter $d_{o}=40 \mathrm{~mm}$.
- The object is constrained to move in a plane.
- The object is taken as rigid (undeformable).
- Frictionless contact condition is specified on the surfaces which may experience contact with the object.
- The actuation input is provided from the inner ends until the object lost the contact with the ground.
- As a finger material polyethylene (PE) is assigned. The final material will be selected in the constructive design section.

In each analysis, the ability of the fingers to grasp a cylinder-shaped object is examined. Many FEM simulations are performed for each concept by making modifications on the dimensions of the fingers using. As a result, for each concept the optimum geometry is obtained, and then their performances are compared.

For shape adaptation, fingers are expected to envelope the object as it is seen in Figure 5-5a. However, there are also some results which show the shape adaptation could not accomplished properly. Such a case is illustrated in Figure 5-5b. Moreover, in some cases the finger rejects the object and loses contact with the object which yields unconverged solutions. The results for shape adaptation are given in Appendix-F for all concepts.


Figure 5-5 Shape Adaptation of the Finger a Stable Grasp (left), a Poor Grasp (right)

The numerical analysis results for actuation forces and contact pressures are illustrated in the Figure 5-6 and Figure 5-7 respectively. One can notice from the figures that if the contact pressure becomes constant the slope of the actuation force curve decreases. Also, it is found that their trends are closely related to each other except fluctuations in contact pressure curves.


Figure 5-6 Actuation Forces for All Concepts


Figure 5-7 Contact Pressures for All Concepts

The concepts which cannot grasp the object securely have very low actuation forces, accordingly very low contact pressures. It is revealed that the geometry of the Concept 5 is not suitable for shape adaptation since it couldn't be able to squeeze the object between two fingers. The finger slips over the object and the object is rejected. On the other hand, in Concept 3 the actuation force increase dramatically that is not a desired characteristic for the actuation mechanism and the object if it is fragile.

### 5.3.3 Concept Evaluation and Selection

The evaluation criteria to select the best concept are specified by taking into account the requirements of gripper finger, actuator and the safety of the work-piece. According to the evaluating criteria, the functional analysis results can be seen from Table 5-1. It can be observed that some concepts, i.e. Concept 6 and Concept 7, have similar characteristics. However, it makes Concept 7 a better choice to have the actuation displacement lower. Besides, Concept 3, Concept 4, Concept 6 and Concept 7 may experience static failure since the maximum allowable stress for Polyethylene (PE) is 25 MPa (Figure5-8). Moreover, another situation that is not mentioned in the design evaluation criteria is the noticeable upward movement of the work-piece after grasp. In case of grasping something fragile, there may be possibility for the work-piece to be damaged when it is released to the ground again. When compared to others, Concept 7 does not cause the object to move as much as the others. Considering all the advantages and the disadvantages of the alternatives, Concept 7 is determined to be the best conceptual design.

Table 5-1 Evaluation Criteria for Design Alternatives

|  | Evaluation Criteria |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Conceptual <br> Designs | Is the <br> grasp <br> stable? <br> (Y/N) | Does the <br> actuation <br> force <br> increase <br> dramatically? | Maximum <br> Contact <br> Pressure <br> (MPa) | Maximum <br> Actuation <br> Force <br> (N) | Maximum <br> Von-Misses <br> Stress <br> (MPa) |
|  | Yes | No | 0.155 | 0.864 | Required <br> Actuation |
|  | Yes | No | 0.180 | 0.670 | 9.10 |
| Concept 3 | Yes | Yes | 0.463 | 2.861 | 50.97 |
| Concept 4 | Yes | No | 0.337 | 1.707 | 33.44 |
| Concept 5 | No | No | 0.021 | 0.136 | 5.69 |
| Concept 6 | Yes | No | 0.448 | 2.276 | 34.65 |
| Concept 7 | Yes | No | 0.389 | 2.277 | 27.56 |
| Concept 8 | Yes | No | 0.250 | 1.172 | 13.48 |



Figure 5-8 Maximum Von-Misses Stress Results for All Concepts


Concept 7
Figure 5-9 Simulation Result of the Selected Concept

### 5.4 Constructive Design

The constructive design refers to the detailed modeling and analysis of the compliant fingers. In this design, this task is accomplished using two different approaches which are finite element and multiple shooting methods. Although models of the finger are obtained using different procedures they need to be consistent with each other. For the finite element method, CAD model is utilized while in multiple shooting method the finger geometry is decomposed into straight or curved beams which can be expressed parametrically. In the elaboration of the finger structure, the node locations shown in Figure 5-9 are taken to be same with respect to a common coordinate frame in both models. Hence, it makes the comparison of the model possible.

Initially, the models are created with rough dimensions satisfying the design aspects. In both methods, the geometric features such as node locations, lengths, angles and the crosssectional dimensions are defined parametrically and some of them are assigned as the optimization parameters. During the optimization process, the CAD model is updated by the ANSYS automatically if the design parameters are defined with a prefix common between two programs. The objective functions can be selected between the FEA results such as displacements, stresses and forces. On the other hand, Matlab optimization toolbox can be utilized to minimize same objective functions defined in ANSYS simulations.

### 5.4.1 Material Selection

In this design the material for the finger should be selected not only considering the results for the best force-deflection characteristic but also the manufacturing of the finger in an economical way. Also, the gripper must work in wide range of temperature without a loss in its performance. Moreover, the selected material should be suitable for molding process or sintering process in order to be manufactured in a monolithic structure. Hence, the criteria considered in material selection can be given as follows:

- strength-to-modulus ratio, $S_{y} / E$
- working temperature range
- manufacturability
- cost

Analyzing the available materials that can be used in compliant structures based on the decisions mentioned above, high density high density polyethylene (HDPE) is selected as the final material. Some of its mechanical and thermal properties are given below,

- Yield Strength : 30 Mpa
- Modulus of Elasticity : 0.8 Gpa
- Ultimate Tensile Strength : 50 Mpa
- Working Temperature Range : $-25^{\circ} \mathrm{C}$ to $70^{\circ} \mathrm{C}$
- Thermal Coefficient of Expansion : $22 \mu \mathrm{~m} / \mathrm{m}-{ }^{\circ} \mathrm{C}$ (linear)


### 5.4.2 Mathematical Modeling and Analysis

The mathematical model used in this design is based on the beam-type elements. It is a convenient to model compliant mechanism such elements since most of their members are straight or curved beams. Using the multiple shooting method formulation for the large beam deflections mentioned in Chapter 3, the governing equations are obtained where each beam element is expressed with a second order differential equation. Then, applying the constraint equations at the boundaries and the continuity equations between divisions of the beam elements, the differential equations are solved numerically to optimize the finger geometry. The GMSM is implemented in Matlab, and the source code is provided in Appendix-B-E.

The control nodes are assigned at the locations where the shape, material or cross-sectional discontinuities exist. The node assignment process is important since the monolithic finger structure is decomposed into simple beams which can be analyzed easily. In other words, the complex functions that determine the all properties of the finger are expressed as simple piece-wise functions.

In the following, the formulations and the constraint equations for the finger are illustrated. The sketch of the selected design and its initial dimensions are shown in Figure 5-10. The finger geometry is divided into three straight beam elements considering the shape and crosssectional discontinuities. Hence, we have three differential equations where the required boundary conditions are defined at the nodes $\mathrm{n}_{0}, \mathrm{n}_{1}, \mathrm{n}_{2}$ and $\mathrm{n}_{3}$. At $\mathrm{n}_{0}$ the first segment of the finger is clamped to the slider which is free to move in y direction only. The connection between the first and second segments of the finger is defined as floating clamped connection. The next connection between the second and the third segments is exactly same with the previous one. Finally, the connection between the third segment and the ground is specified as fixed clamped type connection.


Figure 5-10 Initial Dimensions of the Finger Geometry


Figure 5-11 Free-Body Diagram of the Finger
The free-body diagram of the finger is shown in figure 5-11. Without loss of generality, considering the principle of minimum potential energy the governing equations for the beam deflection can be expressed in the form of first order differential equation set for each beam as follows:

$$
\left[\begin{array}{l}
\psi_{i}  \tag{5.1}\\
\psi_{i}^{\prime} \\
x_{i} \\
y_{i}
\end{array}\right]^{\prime}=\left[\begin{array}{l}
\psi_{i}^{\prime} \\
\frac{L_{i}^{2}}{E_{i} I_{i}}\left(h_{i} \sin \psi_{i}-v_{i} \cos \psi_{i}\right)-\frac{I_{i}^{\prime}}{I_{i}}\left(\psi_{i}^{\prime}-\eta_{i}^{\prime}\right)+\eta_{i}^{\prime \prime} \\
L_{i} \cos \psi_{i} \\
L_{i} \sin \psi_{i}
\end{array}\right] \text { where } \mathrm{i}=1,2,3
$$

In order to solve Eq. 5.1 the boundary conditions defined at nodes are required. According to the specified connection type at the nodes we have the following boundary conditions

Table 5-2 Constraint Equations at the Nodes

| Nodes | Moment and Angle Constraints | Force and Displacement <br> Constraints |
| :---: | :--- | :--- |
| $\mathrm{n}_{0}$ | $\psi_{1}(0)-\eta_{1}(0)=c_{0}$ | $x_{1}(0)=x_{0}, \mathrm{y}_{1}(0)=y_{0}+\Delta y$ <br> $v_{1}+F_{R}=0$ |
| $\mathrm{n}_{1}$ | $\psi_{1}(1)-\eta_{1}(1)=\psi_{2}(0)-\eta_{2}(0)$ | $x_{1}(1)=x_{2}(0), \mathrm{y}_{1}(1)=y_{2}(0)$ |
| $h_{1}-h_{2}=0, v_{1}-v_{2}=0$ |  |  |
| $\mathrm{n}_{2} I_{1}$ |  |  |
| $L_{1}\left(\psi_{1}^{\prime}(1)-\eta_{1}^{\prime}(1)\right)=\frac{E_{2} I_{2}}{L_{2}}\left(\psi_{2}^{\prime}(0)-\eta_{2}^{\prime}(0)\right)$ | $\left.\begin{array}{l}\psi_{2}(1)-\eta_{2}(1)=\psi_{3}(0)-\eta_{3}(0) \\ E_{2} I_{2} \\ L_{2} \\ \hline\end{array} \psi_{2}^{\prime}(1)-\eta_{2}^{\prime}(1)\right)=\frac{E_{3} I_{3}}{L_{3}}\left(\psi_{3}^{\prime}(0)-\eta_{3}^{\prime}(0)\right)$ | $x_{2}(1)=x_{3}(0), \mathrm{y}_{2}(1)=y_{3}(0)$ <br> $h_{2}-h_{3}=0, v_{2}-v_{3}=0$ |
| $\mathrm{n}_{3}$ | $\psi_{3}(1)-\eta_{3}(1)=c_{3}$ | $x_{3}(1)=x_{3}, \mathrm{y}_{3}(1)=y_{3}$ |

The continuity equations each beam is divided into two sub-segments to increase the accuracy and between two sub-segments four continuity equations are defined. In total we have 12 continuity equations.

$$
\left.\begin{array}{rl}
\psi_{i 1}(1) & =\psi_{i 2}(0)  \tag{5.2}\\
\psi_{i 1}^{\prime}(1) & =\psi_{i 2}^{\prime}(0) \\
x_{i 1}(1) & =x_{i 2}(0) \\
y_{i 1}(1) & =y_{i 2}(0)
\end{array}\right\} \text { where } \mathrm{i}=1,2,3
$$

All the connections are the type of clamped joint. Hence, there are 24 unknown initial conditions and 2 unknown parameters for each beam element considering the unknowns of intermediate nodes. Additionally, there is one unknown parameter, $F_{R}$, the reaction force at the slider which needs to be considered. In total there are 31 unknowns. All of them need to be guessed to initialize the iterations. These unknowns are given in Table 5-3

Table 5-3 Unknowns Initial Values and Parameters

| Initial Conditions | Parameters |
| :---: | :---: |
| $\psi_{i j}(0), \psi_{i j}^{\prime}(0), x_{i j}(0), y_{i j}(0)$ where $\mathrm{i}=1,2,3$ and $\mathrm{j}=1,2$ | $h_{i}, v_{i}, F_{R}$ where $\mathrm{i}=1,2,3$ |

Then, the analysis is performed to obtain the deformed shape of the finger and the reaction force at the node $\mathrm{n}_{0}$. Since the gripper model has two symmetrical fingers, the analysis is carried out for one finger and then the reaction force at $\mathrm{n}_{0}$ is multiplied by two to obtain the total actuation force. Besides, stress due to the bending along the beam length is calculated for each beam segment. In Figure 5-12 closing motion of the fingers are visualized at each increment of the input displacement $\Delta y$. Figure 5-13 and Figure 5-14 show the analysis results for the path traced by the finger tip and the total reaction force respectively. Finally, in Figure 5-15 the stress distribution along the beam length is given.


Figure 5-12 Closing Motion of the Gripper with Two Fingers


Figure 5-13 Path Traced by the Finger Tips


Figure 5-14 Total Reaction Force at the Slider


Figure 5-15 Stress Distributions Along the Beam Length

### 5.4.3 Comparison of the Results with FEA Results

In order to compare the results, the finger geometry is modeled with exactly the same dimensions both in ANSYS and Matlab. Also, the material properties are selected to be
same. Since the interaction between the object and the finger is not handled in the mathematical model, the object used in the functional analysis part is omitted in these analyses. Thus, close results are obtained from both analyses. The comparison of the result for the deformed finger geometry, the actuation force, the path of the finger tip and stress distribution can be seen in Figure 5-16 to 5-21.


Figure 5-16 Closing Motion of the Gripper with Two Fingers


Figure 5-17 Path Traced by the Finger Tips


Figure 5-18 Total Reaction Force at the Slider


Figure 5-19 Stress Distributions Along the Beam-1


Figure 5-20 Stress Distributions Along the Beam-2


Figure 5-21 Stress Distributions Along the Beam-3
It is shown with these results that both in GMSM and FEA the same force-deflection characteristics are obtained for the beam type models. Since both models are validated through comparison of many results, one can use either of them. However, when the time for modeling and solution is considered GMSM provides a practical way for optimization.

### 5.4.4 Simulations for Grasp Performance

It is mentioned at the beginning that the gripper should be capable of grasping the objects with different size, shape and orientation within its design limits. To investigate the performance of the final design several analysis and simulations are performed in FEA program. In this part, the simulation results for the capability of the gripper to handle objects varying size shape are illustrated. Moreover, it is proved that the gripper worked well for each case.

### 5.4.4.1 Objects in Different Size

Obtained results show that the gripper with 60 mm opening between finger tips is able to grasp object with diameter of 10 mm to 50 mm safely. Some of the related simulation results can be seen in Figure 5-22.


Figure 5-22 Simulations for Object Size, $d_{o}=50 \mathrm{~mm}$ (left), $d_{o}=30 \mathrm{~mm}$ (middle),

$$
d_{o}=10 \mathrm{~mm} \text { (right) }
$$

### 5.4.4.2 Objects in Different Geometry

The gripper performance is also evaluated for the capability of grasping objects in arbitrary shapes. The edges of the objects smoothened since it is observed that the contact algorithm in FEA program yields unconverged solutions. The results for some of the selected geometries are shown in Figure 5-23.


Figure 5-23 Simulations for Arbitrary Object Shapes

### 5.4.5 Spherical (3D) Gripper

So far, all the analysis and simulations are done considering the planar gripper with two fingers. In these analyses, the object is restrained to move in a plane. However, in this part we model a spherical gripper including three fingers and remove the constraint on the objects motion in the analysis. It is realized that addition of the third finger increases the stability of the grasp. It is simulated that, the object is lifted up securely which is shown in Figure 5-24.


Figure 5-24 Spherical Gripper with Three Fingers
If the number of the finger is increased the stability to hold the object is increased, but the possibility to grip objects with arbitrary shapes may be decreased. Thus, as the final design of the gripper, three fingered structure is selected.

### 5.4.6 Physical Prototype and Testing

In order to validate the theoretical analysis and the proposed gripper performance, a physical prototype of the gripper with final finger dimensions needs to be manufactured and tested. However, the finger structure could not be manufactured as one piece, due to the limitation in time and budget.

To utilize the conventional techniques to manufacture the fingers, each of them is divided into three segments and their geometries are obtained precisely using laser cutting machine. As a material stainless spring steel (CS95) is used. Also, the design of the actuation system is canalized to use available materials which are Faulhaber miniature DC motor and its driver, gears, linear guide and lead screw mechanism. Additionally, some mechanical parts are manufactured to assemble the actuation and finger mechanism. The picture of the final prototype can be seen in Figure 5-25.


Figure 5-25 Final Prototype
After the real model is obtained, several adjustments are done on the prototype. First of all, fully opened and fully closed configurations are obtained and mechanical stops are placed into the actuation system in order not to damage the fingers during an unexpected motion. The limit configurations can be seen in Figure 5-26.


Figure 5-26 Limit configurations, Fully-opened (left) and Fully-closed (right)

Secondly, to control the motion of the fingers using the motion controller a simple macro code is written. It has several functions such as initialization of the motor encoders by finding limits, forward and backward motion, speed adjustment and fragile object option. For fragile objects, the current feedback of the motion controller is utilized such that the motor is stopped, if the current exceeds limit that is obtained experimentally.


Figure 5-27 Gripper with Its Controller
In the following figures, several tests performed to validate the prototype are given. Further test results are given Appendix-G.


Figure 5-28 Test for different object orientations


Figure 5-29 Test for objects with different size


Figure 5-30 Test for fragile object, an egg

### 5.5 Discussions and Conclusions

A robotic gripper design and analysis is presented. Two different approaches are used for the analysis of the gripper and it is shown that the results are in accordance with each other. Thus, analytical and finite element models are utilized interchangeably. In this study, it is assumed that there is no friction between the fingers and the object and analysis are done accordingly.

The design process is carried out systematically. First, a conceptual design step is performed to obtain design alternatives and select the best gripper topology. At this step, analyses are done using finite element model and the concepts are evaluated considering the finger and object interaction. The size and shape of the object and the finger boundary conditions are kept constant to visualize and interpret the performance of different finger topologies. Second, a constructive design step is fulfilled to obtain final and optimized dimensions of the finger geometry. During the optimization the object interaction is not considered. However, simulations are done for different object sizes and shapes with the final finger geometry. Finally, a prototype of the designed gripper is manufactured and its performance is compared with the analysis results. The performance of the prototype is considered as satisfactory under the difficult test conditions.

## CHAPTER 6

## CONCLUSION AND RECOMMENDATIONS

Design of compliant mechanisms requires a systematic study to investigate both their kinematical and structural behaviour. According to the type of the compliant mechanism the design procedures differ from each other. For partially compliant mechanisms, rigid body synthesis techniques can be applied with the addition of energy equations in the synthesis and analysis. However, fully compliant mechanism design needs structural optimization techniques utilizing the large deflection theory behind. Therefore, both in rigid-body replacement and structural optimization, iterations take much time to obtain an optimized solution.

In this study, two systematic analysis and design procedures which are convenient for optimization of partially and fully compliant mechanisms are developed. To take the advantages of compliant mechanisms over rigid mechanisms two important design examples are selected from industrial applications. One is designing a partially compliant bistable mechanism for FOV switch function in optical systems and the other is designing a fully compliant adaptive finger mechanism for robotic applications.

In the design of first example given in Chapter 4, a partially compliant four-bar mechanism is proposed as solution. Then, suitable compliant configuration is selected and dimensions are optimized. In this design, kinematic and force analysis are considered separately. However, when we consider the whole structure in the optimization routine the kinematics is linked to kinetics. The steps in the formulation of the problem and analysis are defined incisively to search the design domain for realizable solutions. Actually, the convenience in the use of this method depends on its parametric structure. Hence, any bistable compliant four link mechanism can be designed by redefining the design requirements, constraints and objectives in the developed code. The bistable field of view switch mechanisms can be considered as case study to validate the proposed design procedure. This mechanism is optimized for the specified task and a feasible and economic solution is obtained. Compared to the traditional example designed for the same purpose, the snap action is now performed using fewer components. Moreover, to actuate the mechanism equal force characteristic from both sides are obtained.

In the second design example, a fully compliant mechanism design for robotic grippers is presented. The main objective which is the adaptability of the finger for various object size and shapes with the simplest gripper structure is obtained. Firstly, the gripper having two fingers is designed and optimized. The actuator forces, actuator displacements are minimized while the ratio of actuator displacement to the tip displacement is maximized. Thus, the mechanical advantage is increased. In this design, two different approaches are utilized for the analysis of the gripper and it is found that the results are in accordance with each other. In this study, it is assumed that there is no friction between the gripper and the object and analysis are done accordingly.Then, the simulation results show that the three finger structure provides more stable grasp. Therefore as a final design three-finger gripper is
modeled and analyzed. Furthermore, a prototype of the designed gripper is manufactured and its performance is compared with the analysis results.

As a future work, the analysis of the contact between the object and the gripper may be studied using a friction model. The designed gripper may be manufactured in a single piece and fully tested in terms of performance and life requirements. Also, the performances of the gripper in different environmental conditions may be explored. Moreover, the idea of controlling the finger pressure with the motor torque can be studied and a control system can be designed.

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## APPENDICES

## APPENDIX A: OPTIMIZATION CODE FOR BISTABLE COMPLIANT MECHANISM DESIGN

```
function [obj]=main(d_par)
global a_2 theta20 beta2 beta3 R L
R=27.5;
L=25;
beta2=-55; %input crank rotation angle from position 1 to position 2
beta3=-90; %input crank rotation angle from position 1 to position 3
theta20=0;
a_2=8;
[ncons,ceq]=cons(d_par);
if ncons==0
obj=abs(bcm(d_par));
else
    obj=1;
end
%%%%
function[ratio]=bcm(d_par)
%clc;
%clear all;
%close all;
d_par_bcm=d_par
global a_2 theta20 beta2 beta3
%% DIMENSIONAL SYNTHESIS
% Design of a Compliant Mechanism with Rigid Body Replacement Method
% PROBLEM >> THREE POSITION SYNTHESIS USING CORRELATION OF CRANK
ANGLES
% Mechanism Design: Analysis and Synthesis 2nd ed. Vol }1\mathrm{ Erdman & Sandor
% Link Lengths of Four-Bar Mechanism
```

\% a1: fixed link
$\%$ a2 : crank (input)
\% a3 : coupler
$\%$ a4 : rocker (output)
$\% \%$ correlated angles (function generation)
\%angle conversion factors
ang2rad=pi/180;
rad2ang=180/pi;
\%d_par=[psi_2 psi_3 alpha_2 alpha_3];
$\% \mathrm{R}=45$;
$\% \mathrm{~L}=40$;
$\%$ theta20=130;
\% a_2=14.5;
$\%$ beta2=-57.5; \%input crank rotation angle from position 1 to position 2
$\%$ beta $3=-110 ; \%$ input crank rotation angle from position 1 to position 3
beta_2=beta2*ang2rad; \%input crank rotation angle from position 1 to position 2 beta_ $3=$ beta $3 *$ ang 2 rad ; \%input crank rotation angle from position 1 to position 3
psi_2=d_par(1)*ang2rad; \%output crank rotation angle from position 1 to position 2 psi_3=d_par(2)*ang2rad; \%output crank rotation angle from position 1 to position 3
$\% \%$ arbitrarily selected design parameters
theta_20=theta20*ang2rad;
$\mathrm{WA}=\mathrm{a} \_2 * \exp \left(\right.$ theta $\left.\_20 * 1 \mathrm{i}\right)$;
$\mathrm{ZA}=0$;
alpha_2=d_par(3)*ang2rad; \% coupler link rotation angle from position 1 to position 2 alpha_3=d_par(4)*ang2rad; \% coupler link rotation angle from position 1 to position 3
\%\% displacement vectors
$\mathrm{R} 1=\mathrm{WA}$;
$\mathrm{R} 2=\mathrm{WA} * \exp ($ beta_2*1i);
$\mathrm{R} 3=\mathrm{WA}$ * $\exp ($ beta_3*1i);
delta_2=R2-R1;
delta_3=R3-R1;
\%\% Three Position Synthesis
$\%$ since WA and $\mathrm{ZA}=0$; are specified, solve for WB and ZB using Cramer's Rule
\% Equations for dyad B

```
% WB*(exp(psi_2*1i)-1)+ZB*(exp(alpha_2*1i)-1)=delta_2
% WB*(exp(psi_3*1i)-1)+ZB*(exp(alpha_3*1i)-1)=delta_3
% from Cramer's Rule
ZB=\operatorname{det}([\operatorname{exp}(psi_2*1i)-1 delta_2;exp(psi_3*1i)-1 delta_3])/det([exp(psi_2*1i)-1
exp(alpha_2*1i)-1;exp(psi_3*1i)-1 exp(alpha_3*1i)-1]);
WB=det([delta_2 exp(alpha_2*1i)-1;delta_3 exp(alpha_3*1i)-1])/det([exp(psi_2*1i)-1
exp(alpha_2*1i)-1;exp(psi_3*1i)-1 exp(alpha_3*1i)-1]);
%Link Lengths
al=abs(WA+ZA-ZB-WB);
a2=abs(WA);
a3=abs(ZA-ZB);
a4=abs(WB);
%% Grashof's Criterion
G='Grashof'; %s+l < p+q (not including change-point)
nG='non-Grashof'; %s+l>p+q
cp='change-point'; %s+l=p+q
if isfinite([a1 a2 a3 a4])
if min([a1 a2 a3 a4])+\operatorname{max}([a1 a2 a3 a4])<(a1+a2+a3+a4)-(min([a1 a2 a3 a4])+max([a1 a2
a3 a4]))
type=G;
else if min([a1 a2 a3 a4])+\operatorname{max}([a1 a2 a3 a4])>(a1+a2+a3+a4)-(min([a1 a2 a3 a4])+max([a1
a2 a3 a4]))
type=nG;
    else
        type=cp;
    end
end
else
type='NA';
end
%% MOTION ANALYSIS
if strcmp(type,'Grashof')|strcmp(type,'change-point')
```

\%theta_2 $=0:\left(\operatorname{sign}\left(\operatorname{beta} \_3\right) *\right.$ ang2rad) $: 2 *$ pi $^{*} \operatorname{sign}($ beta_3); \% crank angle increment theta_2=0:(sign(beta_3)*ang2rad):beta_3; \% crank angle increment else
theta_2=0:(sign(beta_3)*ang2rad):beta_3; \% crank angle increment
end
\% LOOP CLOSURE EQUATION
$\% \mathrm{WA}^{*} \exp ($ theta_2(i)*1i)+(ZA-ZB)*exp(theta_3(i)*1i)=WA+ZA-ZB-
WB+WB*exp(theta_4(i)*1i)
$\mathrm{Q}=\mathrm{WB}+\mathrm{ZB}-\mathrm{ZA}-\mathrm{WA}$;
$\mathrm{H}=\mathrm{ZB}-\mathrm{ZA}$;
$\mathrm{G}=\mathrm{Q}+\mathrm{WA} * \exp \left(\right.$ theta $\left.\_2 * 1 \mathrm{i}\right)$;
\% loop closure equtaion in simplified form: $G(i)$ -
WB*exp(theta_4(i)*1i) $=\mathrm{H}^{*} \exp ($ theta_3(i)*1i)
\% by multiplying with its complex conjugate
$\mathrm{B}=-\mathrm{G} . * \operatorname{conj}(\mathrm{G})+\mathrm{H} * \operatorname{conj}(\mathrm{H})-\mathrm{WB} . * \operatorname{conj}(\mathrm{WB})$;
$\mathrm{A}=\mathrm{WB} . * \operatorname{conj}(\mathrm{G}) ;$
$\mathrm{C}=\mathrm{G} . * \operatorname{conj}(\mathrm{WB}) ;$
$\operatorname{disc}=B .{ }^{\wedge} 2-4 *$ A. ${ }^{*} \mathrm{C}$;
ang $=\left(\right.$ angle $(\mathrm{WB})$-angle(-ZB)) ${ }^{*}$ rad2ang
config $=\operatorname{sign}($ angle(WB)-angle(-ZB));
display(type);
display(config);
theta_3=zeros(1,length(theta_2));
theta_4=zeros(1,length(theta_2));
for $\mathrm{i}=1: 1$ :length(theta_2)
theta_4(i)=angle(((-B(i)-config*sqrt(disc(i)))/(2*A(i))));
theta_3(i)=angle((G(i)-WB*exp(theta_4(i)*1i))/H);
end
theta_12=theta_20+theta_2;
theta_13=angle(ZA-ZB)+theta_3;
theta_14=angle(WB)+theta_4;
\% \% ANALYSIS FOR BISTABILITY
\%\% Spring Constants according to Selected Configurations
if strcmp(type,'non-Grashof')||strcmp(type,'change-point')

```
%K=[[0114 0;0}0
K=[[00000ll
else if strcmp(type,'Grashof') && a1 == min([a1 a2 a3 a4])
    %K=[lllllllllll}
    K=[[\begin{array}{lllll}{0}&{0}&{0}&{1}\end{array}];
else if strcmp(type,'Grashof') && a2 == min([a1 a2 a3 a4])
        %K=[lllllllllll
        K=[[\begin{array}{lllll}{0}&{0}&{0}&{1}\end{array}];
    else
        K=[[\begin{array}{llll}{0}&{0}&{0}&{0}\end{array}];
    end
    end
end
```

\%\% Potential Energy Equation
\%\% Deflection Angles
JA1=theta_2;
JA2 $=$ theta_2-theta_3;
JA3 $=$ theta_4-theta_3;
JA4 $=$ theta_4;
ks $=\operatorname{size}(\mathrm{K})$;
for $\mathrm{i}=1: 1: \mathrm{ks}(1)$
$\mathrm{K} 1=\mathrm{K}(\mathrm{i}, 1)$;
$\mathrm{K} 2=\mathrm{K}(\mathrm{i}, 2)$;
$\mathrm{K} 3=\mathrm{K}(\mathrm{i}, 3)$;
$\mathrm{K} 4=\mathrm{K}(\mathrm{i}, 4)$;
$\mathrm{V}=(1 / 2) *\left(\mathrm{~K} 1 * \mathrm{JA} 1 . \wedge^{\wedge} 2+\mathrm{K} 2 * \mathrm{JA} 2 . \wedge 2+\mathrm{K} 3 * \mathrm{JA} 3 .{ }^{\wedge} 2+\mathrm{K} 4 * \mathrm{JA} 4 . \wedge 2\right) ;$
h32 $=(\mathrm{a} 2 * \sin ($ theta 14 -theta_12) $) /(\mathrm{a} 3 * \sin ($ theta_13-theta_14) $)$;
$\mathrm{h} 42=(\mathrm{a} 2 * \sin ($ theta_13-theta_12)$) . /(\mathrm{a} 4 * \sin ($ theta_13-theta_14) $)$;

```
dJA1= 1;
dJA2=(1-h32);
dJA3=(h42-h32);
dJA4= h42;
dV=K1*JA1*dJA1+K2*JA2.*dJA2+K3*JA3.*dJA3+K4*JA4.*dJA4;
dh32=(a2*}\operatorname{cos(theta_14-theta_12)+a3*h32.^2.*}\operatorname{cos(theta_14-theta_13)-
a4*h42.^2)./((a3*sin(theta_14-theta_13)));
dh42=(-a2*\operatorname{cos}(theta_13-theta_12)+a4*h42.^2.*\operatorname{cos(theta_13-theta_14)-}
a3*h32.^2)./((a4*sin(theta_13-theta_14)));
ddJA2=-dh32;
ddJA3=dh42-dh32;
ddJA4=dh42;
ddV =K1*dJA1+K2*(dJA2.^2+JA2.*ddJA2)+K3*(dJA3.^2+JA3.*ddJA3)+K4*(dJA4.^2+J
A4.*ddJA4);
PE(i,:)=V;
M(i,:)=dV;
dM(i,:)=ddV;
end
Range = -1:0.1:1;
X1 = theta_20*rad2ang*ones(size(Range));
X2 = (theta_20+beta_3)*rad2ang*ones(size(Range));
for i=1:1:ks(1)
[PEmax,PEimax,PEmin,PEimin]=extrema(PE(i,:));
[MEmax,MEimax,MEmin,MEimin]=extrema(M(i,:));
MEimax=sort(MEimax);
MEimin=sort(MEimin);
PEimax=sort(PEimax);
PEimin=sort(PEimin);
M1=findpeaks(M(i,:));
M2=findpeaks(-M(i,:));
if ~isempty(PEimax)& ~isempty(PEimin)& ~isempty(M1) & ~isempty(M2)
if M(i,PEimax(1))<1/100 & M(i,PEimin)<1/100
    mratio(i)=1-abs(M1/M2);
    display('***Bistability is Obtained***');
```

else
mratio(i) $=1$;
end
else
mratio(i) $=1$;
end
figure
plot(theta_12*rad2ang,PE(i,:),'-r','LineWidth',3);
hold on;
plot(theta_12(PEimax)*rad2ang,PEmax,'go',theta_12(PEimin)*rad2ang,PEmin,'yo','LineWid th',5);
hold on;
plot(theta_12*rad2ang,M(i,:),'-b','LineWidth',3);
hold on;
plot(theta_12(PEimax)*rad2ang,M(i,PEimax),'go',theta_12(PEimin)*rad2ang,M(i,PEimin),'y
o','LineWidth',5);
hold on;
plot(theta_12*rad2ang,dM(i,:),'-m','LineWidth',3);
hold on;
plot(theta_12(PEimax)*rad2ang,dM(i,PEimax),'go',theta_12(PEimin)*rad2ang,dM(i,PEimin ),'yo','LineWidth',5);
hold on;
plot(X1,Range*max(dM(i,:)),'--k',X2,Range*max(dM(i,:)),'--k','LineWidth',3);
hold on;
plot(theta_12(MEimax)*rad2ang,M(i,MEimax),'ko',theta_12(MEimin)*rad2ang,M(i,MEimi
n),'ko','LineWidth',5);
grid on;
hold on;
title(['Ratio is ',num2str(abs(mratio)),' ' 'Alpha3 = ',num2str(d_par(1,4))]);
pause(0.5);
end
ratio $=$ mratio;
\%\%\%\%\%\%\%\%\%
function [ncons, ceq] $=$ cons(d_par)
close all;
d_par_cons=d_par
global a_2 theta20 beta2 beta3 R L
\% Link Lengths of Four-Bar Mechanism

```
% a1 : fixed link
% a2 : crank (input)
% a3 : coupler
% a4 : rocker (output)
%% Three Position Synthesis
% correlated angles (function generation)
%angle conversion factors
ang2rad=pi/180;
rad2ang=180/pi;
%d_par=[psi_2 psi_3 alpha_2 alpha_3];
%R=45;
%L=40;
% theta20=130;
% a_2=14.5;
```

$\%$ beta $2=-57.5$; \%input crank rotation angle from position 1 to position 2
$\%$ beta $3=-110 ; \%$ input crank rotation angle from position 1 to position 3
beta_2=beta2*ang2rad; \%input crank rotation angle from position 1 to position 2
beta_3=beta3*ang2rad; \%input crank rotation angle from position 1 to position 3
psi_2=d_par(1)*ang2rad; \%output crank rotation angle from position 1 to position 2
psi_3=d_par(2)*ang2rad; \%output crank rotation angle from position 1 to position 3
\% arbitrarily selected design parameters
theta_20=theta 20 *ang2rad;
$\mathrm{WA}=\mathrm{a} \_2 * \exp \left(\right.$ theta $\left.\_20 * 1 \mathrm{i}\right)$;
$\mathrm{ZA}=0$;
alpha_2=d_par(3)*ang2rad; \% coupler link rotation angle from position 1 to position 2 alpha_3=d_par(4)*ang2rad; \% coupler link rotation angle from position 1 to position 3
\% displacement vectors
$\mathrm{R} 1=\mathrm{WA}$;
$\mathrm{R} 2=\mathrm{WA} * \exp ($ beta_2*1i);
$\mathrm{R} 3=\mathrm{WA} * \exp \left(\right.$ beta_- $\left.^{-} * 1 \mathrm{i}\right)$;
delta_2=R2-R1;
delta_-3=R3-R1;
$\%$ since WA and $\mathrm{ZA}=0$; are specified, solve for WB and ZB using Cramer's Rule

```
% Equations for dyad B
% WB*(exp(psi_2*1i)-1)+ZB*(exp(alpha_2*1i)-1)=delta_2
% WB*(exp(psi_3*1i)-1)+ZB*(exp(alpha_3*1i)-1)=delta_3
% from Cramer's Rule
ZB=\operatorname{det}([\operatorname{exp}(psi_2*1i)-1 delta_2;exp(psi_3*1i)-1 delta_3])/det([exp(psi_2*1i)-1
exp(alpha_2*1i)-1;exp(psi_3*1i)-1 exp(alpha_3*1i)-1]);
WB=det([delta_2 exp(alpha_2*1i)-1;delta_3 exp(alpha_3*1i)-1])/det([exp(psi_2*1i)-1
exp(alpha_2*1i)-1;exp(psi_3*1i)-1 exp(alpha_3*1i)-1]);
pos1=[0+0*1i WA WA-ZB WA-ZB-WB];
pos2=[0+0*1i WA*exp(beta_2*1i) WA*exp(beta_2*1i)-ZB*exp(alpha_2*1i)
WA*}\operatorname{exp(beta_2*1i)-ZB*exp(alpha_2*1i)-WB*exp(psi_2*1i)];
pos3=[0+0*1i WA*exp(beta_3*1i) WA*exp(beta_3*1i)-ZB*exp(alpha_3*1i)
WA*}\operatorname{exp(beta_3*1i)-ZB*exp(alpha_3*1i)-WB*}\operatorname{exp}(psi_3*1i)]
% Link Lengths
al=abs(WA+ZA-ZB-WB);
a2=abs(WA);
a3=abs(ZA-ZB);
a4=abs(WB);
% figHandle1 = figure(1);
% figure(figHandle1);
% plot(real(pos1),imag(pos1),'-ko','LineWidth',5);
% hold on;
% plot(real(pos2),imag(pos2),'-go','LineWidth',5);
% hold on;
% plot(real(pos3),imag(pos3),'-bo','LineWidth',5);
% hold on;
%
% x=-R:0.1:R;
% y 1 =sqrt(R^2-x.^2);
% y2=-y1;
% plot(x,y l,'--r','LineWidth',1);
% plot(x,y2,'--r','LineWidth',1);
%
% Y = -sqrt(R^2-L^2):0.1:sqrt(R^2-L^2);
% X = -L * ones(size(Y));
% plot(X,Y,'--r','LineWidth',1);
%
% axis equal;
% grid on;
%axis([-30 30-30 30])
%% Grashof's Criterion
G='Grashof'; %s+l < p+q (not including change-point)
nG='non-Grashof'; %s+l>p+q
```

```
cp='change-point'; %s+1 = p+q
if isfinite([a1 a2 a3 a4])
if min([a1 a2 a3 a4])+max([a1 a2 a3 a4])<(a1+a2+a3+a4)-(min([a1 a2 a3 a4])+max([a1 a2
a3 a4]))
type=G;
else if min([a1 a2 a3 a4])+max([a1 a2 a3 a4])>(a1+a2+a3+a4)-(min([a1 a2 a3 a4])+max([a1
a2 a3 a4]))
type=nG;
    else
        type=cp;
    end
end
else
type='NA';
end
%% MOTION ANALYSIS
if strcmp(type,'Grashof')||strcmp(type,'change-point')
theta_2=0:(sign(beta_3)*ang2rad):beta_3; % crank angle increment
%theta_2=0:(sign(beta_3)*ang2rad):2*pi*sign(beta_3); % crank angle increment
else
theta_2=0:(sign(beta_3)*ang2rad):beta_3; % crank angle increment
end
% LOOP CLOSURE EQUATION
% WA*exp(theta_2(i)*1i)+(ZA-ZB)*exp(theta_3(i)*1i)=WA+ZA-ZB-
WB+WB*exp(theta_4(i)*1i)
Q=WB+ZB-ZA-WA;
H=ZB-ZA;
G=Q+WA*exp(theta_2*1i);
% loop closure equtaion in simplified form:G(i)-
WB*exp(theta_4(i)*1i)=H*exp(theta_3(i)*1i)
% by multiplying with its complex conjugate
```

```
B=-G.*conj(G)+H*
A=WB.*conj(G);
C=G.*conj(WB);
disc=B.^2-4*A.*C;
% check if mechanism can be assembled or not
tol=0.0001;
if ((real(WA*exp(beta_2*1i)+(ZA-ZB)*exp(alpha_2*1i))-real(WA+(ZA-ZB)-WB*(1-
exp(psi_2*1i))))<tol)...
    & ((real(WA*exp(beta_3*1i)+(ZA-ZB)*exp(alpha_3*1i))-real(WA+(ZA-ZB)-WB*(1-
exp(psi_3*1i))))<tol)...
    & ((imag(WA*exp(beta_2*1i)+(ZA-ZB)*exp(alpha_2*1i))-imag(WA+(ZA-ZB)-WB*(1-
exp(psi_2*1i))))<tol)...
    & ((imag(WA*exp(beta_3*1i)+(ZA-ZB)*exp(alpha_3*1i))-imag(WA+(ZA-ZB)-WB*(1-
exp(psi_3*1i))))<tol)
ang=(angle(WB)-angle(-ZB)+2*pi)*rad2ang
ang2=angle(WB)*rad2ang
ang2=angle(-ZB)*rad2ang
config=sign(angle(WB)-angle(-ZB)+2*pi);
display(type);
display(config);
theta_3=zeros(1,length(theta_2));
theta_4=zeros(1,length(theta_2));
for i=1:1:length(theta_2)
theta_4(i)=angle(((-B(i)-config*sqrt(disc(i)))/(2*A(i))));
theta_3(i)=angle((G(i)-WB*exp(theta_4(i)*1i))/H);
end
% figHandle1 = figure(1);
% figure(figHandle1);
% plot(real(pos1),imag(pos1),'-ko','LineWidth',5);
% hold on;
% plot(real(pos2),imag(pos2),'-go','LineWidth',5);
% hold on;
% plot(real(pos3),imag(pos3),'-bo','LineWidth',5);
% hold on;
%
% x=-R:0.1:R;
% y1 =sqrt(R^2-x.^2);
% y2=-y1;
```

```
% plot(x,y1,'--r','LineWidth',1);
% plot(x,y2,'--r','LineWidth',1);
%
% Y = -sqrt(R^2-L^2):0.1:sqrt(R^2-L^2);
% X = -L * ones(size(Y));
% plot(X,Y,'--r','LineWidth',1);
%
% axis equal;
% grid on;
%axis([-30 30-30 30])
% for i=1:1:length(theta_2)
%
% pos=[0+0*1i WA*exp(theta_2(i)*1i) WA-ZB-WB+WB*exp(theta_4(i)*1i) WA-ZB-
WB];
% h1=plot(real(pos),imag(pos),'-mo','LineWidth',5);
% pause(0.05)
% set(h1,'Visible','off');
%
% end
% set(h1,'Visible','on');
%% Design Space Constaraint
A0X=0;
A0Y=0;
A1X=A0X+real(WA*exp(theta_2(1,:)*1i));
A1Y=A0Y+imag(WA*exp(theta_2(1,:)*1i));
B0X=A0X+real(WA+ZA-ZB-WB);
B0Y=A0Y+imag(WA+ZA-ZB-WB);
B1X=B0X+real(WB*exp(theta_4(1,:)*1i));
B1Y=B0Y+imag(WB*exp(theta_4(1,:)*1i));
if ~any(A1X(1,:)<-L | A1X(1,:)>R) & ~any(B0X(1,:)<-L | B0X(1,:)>R) & ~any(B1X(1,:)<-
L|B1X(1,:)>R)..
    & ~any(A1Y(1,:)<-sqrt(R^2-A1X.^2)| A1Y(1,:)>sqrt(R^2-A1X.^2)) & ~any(B0Y(1,:)<-
sqrt(R^2-B0X.^2)|B0Y(1,:)>sqrt(R^2-B0X.^2)) & ~any(B1Y(1,:)<-sqrt(R^2-B1X.^2)|
B1Y(1,:)>sqrt(R^2-B1X.^2))
    display('*******************************');
    display('Mechanism can be assembled');
    display('CONSTRAINTS OK, Check Bistability');
    display('*******************************');
    ncons=0;
else
    display('*******************************');
    display('Mechanism can be assembled');
```

```
    display('CONSTRAINTS NOT OK, Change parameters');
    display('*******************************');
    ncons=1;
end
else
    display('*******************************');
    display('Mechanism cannot be assembled');
    display('Change parameters');
    display('******************************');
    ncons=1;
end
ceq=[];
```

end

## APPENDIX B: CONSTRAINT FUNCTIONS FOR BEAM ELEMENTS IN COMPLIANT FINGER

```
function f= confun(p,L,EI,deltayslider,const)
%% Boundary (constraint) equations and force balance equations
q11_ini = p(1:4); % initial values of link 1
q12_ini = p(5:8); h1 = p(9); v1 = p(10);
q21_ini = p(11:14); % initial values of link 2
q22_ini = p(15:18); h2 = p(19); v2 = p(20);
q31_ini = p(21:24); % initial values of link 3
q32_ini = p(25:28); h3 = p(29); v3 = p(30);
Fr = p(31);
% multiple shooting N = 2
[tspan,q11] = ode45(@ss_eq,[0 0.5],q11_ini,[],h1,v1,L(1),EI(1)); % 1st subdivision of link
1
[tspan,q12] = ode45(@ss_eq,[0.5 1],q12_ini,[],h1,v1,L(1),EI(1)); % 2nd subdivision of link
1
[tspan,q21] = ode45(@ss_eq,[0 0.5],q21_ini,[],h2,v2,L(2),EI(2)); % 1st subdivision of link
2
[tspan,q22] = ode45(@ss_eq,[0.5 1],q22_ini,[],h2,v2,L(2),EI(2)); % 2nd subdivision of link
2
[tspan,q31] = ode45(@ss_eq,[0 0.5],q31_ini,[],h3,v3,L(3),EI(3)); % 1st subdivision of link
3
[tspan,q32] = ode45(@ss_eq,[0.5 1],q32_ini,[],h3,v3,L(3),EI(3)); % 2nd subdivision of link
3
```

$\%$ const $=\left[q 1 x 0 q 1 y 0\right.$ eta10 $d_{-}$eta 10 eta1n d_eta1n eta20 $d_{-}$eta20 eta2n d_eta2n q3xn q3yn
eta30 d_eta30 eta3n d_eta3n];
$\mathrm{f}=[]$;
\% 3 boundary constraint equations at the fixed clamped joint (0)
$\mathrm{f}(\mathrm{end}+1)=\mathrm{q} 11(1,3)-\operatorname{const}(1) ; \quad \% \mathrm{x}$ displacement
$\mathrm{f}($ end +1$)=\mathrm{q} 11(1,4)-($ const( 2$)+$ deltayslider $/ 1000) ; \%$ y displacement
$f($ end +1$)=q 11(1,1)-\operatorname{const}(3) ; \quad \quad \%$ angle if joint is clamped
$\% f($ end +1$)=-\operatorname{EI}(1) / L(1) *(q 11(1,2)-c o n s t(4)) ; \quad \%$ moment if joint is revolute
$\% 6$ boundary constraint equations at the 1st floating clamped joint (1)
$\mathrm{f}($ end +1$)=\mathrm{q} 12($ end, 1$)-\mathrm{q} 21(1,1)+(-\operatorname{const}(5)+\operatorname{const}(7))$;
$\%$ angle
$\mathrm{f}(\mathrm{end}+1)=\mathrm{EI}(1) / \mathrm{L}(1) *(\mathrm{q} 12($ end,2)-const(6))$)-\mathrm{EI}(2) / \mathrm{L}(2) *(\mathrm{q} 21(1,2)-\operatorname{const}(8)) ; \%$ moment
$\mathrm{f}($ end +1$)=\mathrm{q} 12($ end,3) $-\mathrm{q} 21(1,3)$; $\quad \% \mathrm{x}$ displacement
$\mathrm{f}($ end +1$)=\mathrm{q} 12($ end, 4$)-\mathrm{q} 21(1,4) ; \quad \%$ y displacement
$\mathrm{f}(\mathrm{end}+1)=\mathrm{h} 1-\mathrm{h} 2 ; \quad \%$ horizontal force
$f(e n d+1)=v 1-v 2 ; \quad$ \% vertical force
\% 6 boundary constraint equations at the 2nd floating clamped joint (2)
$\mathrm{f}($ end +1$)=\mathrm{q} 22($ end, 1$)-\mathrm{q} 31(1,1)+(-$ const $(9)+$ const(13) $) ; \quad \%$ angle
$\mathrm{f}(\mathrm{end}+1)=\mathrm{EI}(2) / \mathrm{L}(2) *(\mathrm{q} 22($ end,2)$)-c o n s t(10))-\operatorname{EI}(3) / \mathrm{L}(3) *(\mathrm{q} 31(1,2)-\operatorname{const}(14)) ; \%$ moment
$\mathrm{f}($ end +1$)=\mathrm{q} 22($ end,3) $-\mathrm{q} 31(1,3)$; $\% \mathrm{x}$ displacement
$\mathrm{f}($ end +1$)=\mathrm{q} 22($ end,4 $)-\mathrm{q} 31(1,4) ; \quad \%$ y displacement
$\mathrm{f}(\mathrm{end}+1)=\mathrm{h} 2-\mathrm{h} 3 ; \quad \%$ horizontal force

```
f(end+1)=v2 - v3; % vertical force
% 1 boundary constraint equation for slider reaction force
f(end+1) = v1 - Fr;
% vertical force at slider
% 3 boundary constraint equations at the fixed/revolute slider joint (3)
f(end+1) = q32(end,3) - const(11);
f(end+1) = q32(end,4) - const(12);
    %x displacement
f(end+1) = q32(end,4) - const(12); % y displacement
f(end+1) = q32(end,1) - const(15); % angle if joint is clamped
%f(end+1)=-EI(3)/L(3)*(q32(end,2)-const(16)); % moment if joint is revolute
%(N-1)*n*el = (2-1)*4*3 = 12 continuity equations at u}=0.
f(end+1)=q11(end,1)-q12(1,1); % psi
f(end+1) = q11(end,2)-q12(1,2); % psiP
f(end+1) = q11(end,3)-q12(1,3); % x
f(end+1) = q11(end,4)-q12(1,4); % y
f(end+1) = q21(end,1)-q22(1,1); % psi
f(end+1) = q21(end,2)-q22(1,2); % psiP
f(end+1) = q21(end,3)-q22(1,3); % x
f(end+1) = q21(end,4)-q22(1,4); % y
f(end+1)=q31(end,1)-q32(1,1); % psi
f(end+1) = q31(end,2)-q32(1,2); % psiP
f(end+1) = q31(end,3)-q32(1,3); % x
f(end+1)=q31(end,4)-q32(1,4); % y
```


## APPENDIX C: SET OF FIRST ORDER GOVERNING EQUATIONS FOR BEAM DEFLECTION

```
function dq = ss_eq(u,q,h,v,L_n,EI_n)
%% set of nonlinear governing equations
% the states are q(1) = psi;q(2) = psiP;q(3) = x;q(4) = y;
dq = zeros(4,1); % a column vector
dq(1) = q(2);
dq(2) = (L_n^2)/(EI_n)*(h*sin(q(1))-v*
dq(3) = L_n* 
dq(4) = L_n* *in(q(1));
```

```
APPENDIX D: OPTIMIZATION CODE FOR COMPLIANT FINGER MECHANISM
DESIGN
```

```
function []=compliant_gripper()
```

function []=compliant_gripper()
%% Sample GMSM Matlab code - Three Link Compliant Gripper
%% Sample GMSM Matlab code - Three Link Compliant Gripper
clc;
clc;
%close all;
%close all;
clear all;
clear all;
% INITIAL NODES
% INITIAL NODES
n0=[-2 0]/1000; % meter
n0=[-2 0]/1000; % meter
n1=[-35-80]/1000; % meter
n1=[-35-80]/1000; % meter
n2=[-31.25 -35]/1000; % meter
n2=[-31.25 -35]/1000; % meter
n3=[-25 40]/1000; % meter
n3=[-25 40]/1000; % meter
\% SHAPE FUNCTIONS for LINES

```
```

c1=atan2((n1(1,2)-n0(1,2)),(n1(1,1)-n0(1,1)));

```
c1=atan2((n1(1,2)-n0(1,2)),(n1(1,1)-n0(1,1)));
c2=atan2((n2(1,2)-n1(1,2)),(n2(1,1)-n1(1,1)));
c2=atan2((n2(1,2)-n1(1,2)),(n2(1,1)-n1(1,1)));
c3=atan2((n3(1,2)-n2(1,2)),(n3(1,1)-n2(1,1)));
c3=atan2((n3(1,2)-n2(1,2)),(n3(1,1)-n2(1,1)));
eta1 =@(U) c1+U*0; %where c1 is the slope of the line 1
eta2 = @(U) c2+U*0;%where c2 is the slope of the line 2
eta3 = @(U) c3+U*0;%where c3 is the slope of the line 3
d_eta1 =@(U) U*0; % 1st derivatives
d_eta2 = @(U) U*0;
d_eta3 = @(U) U*0;
% dd_eta1 = @(U) U*0; % 2nd derivatives
% dd_eta2 = @(U) U*0;
% dd_eta3 = @(U) U*0;
% SHAPE FUNCTIONS for CURVES
% eta1 = @(U) ?; % parametric equation of the curve 1
% eta2 = @(U) ?; % parametric equation of the curve 2
% eta3=@(U) ?; % parametric equation of the curve 3
% d_eta1=@(U) ?; % 1st derivatives
% d_eta2 =@(U) ?;
% d_eta3 = @(U) ?;
%
% dd_eta1 = @(U) ?; % 2nd derivatives
% dd_eta2 = @(U) ?;
% dd_eta3 = @(U) ?;
%% LINK PROPERTIES
% Line Length
```

$\mathrm{L} 1=\operatorname{sqrt}\left((\mathrm{n} 0(1,1)-\mathrm{n} 1(1,1))^{\wedge} 2+(\mathrm{n} 0(1,2)-\mathrm{n} 1(1,2))^{\wedge} 2\right) ; \%$ calculates line length 1
$\mathrm{L} 2=\operatorname{sqrt}\left((\mathrm{n} 1(1,1)-\mathrm{n} 2(1,1))^{\wedge} 2+(\mathrm{n} 1(1,2)-\mathrm{n} 2(1,2))^{\wedge} 2\right) ; \%$ calculates line length 2
$\mathrm{L} 3=\operatorname{sqrt}\left((\mathrm{n} 2(1,1)-\mathrm{n} 3(1,1))^{\wedge} 2+(\mathrm{n} 2(1,2)-\mathrm{n} 3(1,2))^{\wedge} 2\right) ; \%$ calculates line length 3

## \% Curve Length

\% calculates curve length 1
$\%$ calculates curve length 2
$\%$ calculates curve length 3
\% SECOND MOMENT OF AREAS of LINK CROSS SECTION (constant rectangular cross section)
$\mathrm{D}=10 / 1000 ; \quad \%$ meter out of plane thickness
$\mathrm{W} 1=0.5 / 1000 ; ~ \% ~ m e t e r ~ w i d t h ~ o f ~ t h e ~ 1 s t ~ l i n k ~$
$\mathrm{W} 2=0.5 / 1000 ; \%$ meter width of the 2 nd link
$\mathrm{W} 3=1 / 1000 ; \quad \%$ meter width of the 3rd link
$\mathrm{I} 1=\mathrm{D}^{*} \mathrm{~W} 1 \wedge 3 / 12 ; \quad \%$ meter $\wedge 4$
$\mathrm{I} 2=\mathrm{D}^{*} \mathrm{~W} 2^{\wedge} 3 / 12 ; \quad \%$ meter ${ }^{\wedge} 4$
$\mathrm{I} 3=\mathrm{D}^{*} \mathrm{~W} 3 \wedge 3 / 12 ; \quad \%$ meter ${ }^{\wedge} 4$
\% YOUNG'S MODULUS OF THE MATERIAL
$\mathrm{E}=1.1 \mathrm{e} 9 ; \quad \% \mathrm{~Pa}$ or $\mathrm{N} /$ meter $^{\wedge} 2$ (polyethlene)
$\% \mathrm{E}=1.93 \mathrm{e} 11 ; \quad \% \mathrm{~Pa}$ or $\mathrm{N} /$ meter $^{\wedge} 2$ (spring steel)
$\mathrm{EI}=\left[\mathrm{E}^{*}\right.$ I1 $\mathrm{E}^{*}$ I2 $\left.\mathrm{E}^{*} \mathrm{I} 3\right] ;$ \% Newton*meter^2
$\mathrm{L}=[\mathrm{L} 1 \mathrm{~L} 2 \mathrm{~L} 3] ; \quad \%$ meter
\%\% DIMENSIONLESS PARAMETERS
$\mathrm{a}=0$; \%lower boundary of u
$\mathrm{b}=1$; \%upper boundary of u
$\mathrm{N}=49$; \%steps
$\mathrm{h}=(\mathrm{b}-\mathrm{a}) / \mathrm{N} ; \%$ step size
$\mathrm{u}=\mathrm{a}: \mathrm{h}: \mathrm{b} ; \% \mathrm{u}=\mathrm{s} / \mathrm{L}$ dimensionless parameter
\% \% SHAPE INTEGRALS
s_x1 = @(U) $\cos \left(\mathrm{c} 1+\mathrm{U}^{*} 0\right)$;
$\mathrm{s} \_\mathrm{y} 1=@(\mathrm{U}) \sin \left(\mathrm{c} 1+\mathrm{U}^{*} 0\right)$;
s_x2 = @(U)cos(c2+U*0);
$\mathrm{s} \_\mathrm{y} 2=@(\mathrm{U}) \sin \left(\mathrm{c} 2+\mathrm{U}^{*} 0\right)$;
s_x3 = @(U) $\cos \left(\mathrm{c} 3+\mathrm{U}^{*} 0\right)$;
s_y3 = @(U) $\sin \left(\mathrm{c} 3+\mathrm{U}^{*} 0\right)$;
shape $=\left\{s \_x 1 ; s \_y 1 ; s \_x 2 ; s \_y 2 ; s \_x 3 ; s \_y 3\right\} ;$
shape_ini=[n0(1,1);n0(1,2);n1(1,1);n1(1,2);n2(1,1);n2(1,2)]; \% meter
x1_ud=zeros(1,N+1);
y1_ud=zeros(1,N+1);
x2_ud=zeros(1,N+1);
y2_ud=zeros(1,N+1);

```
x3_ud=zeros(1,N+1);
y3_ud=zeros(1,N+1);
shape_x1=cell2mat(shape(1));
shape_yl=cell2mat(shape(2));
shape_x2=cell2mat(shape(3));
shape_y2=cell2mat(shape(4));
shape_x3=cell2mat(shape(5));
shape _y3=cell2mat(shape(6));
% for n=1:N+1
%
% x1_ud(n)= shape_ini(1) + L1*(quadl(shape_x1,0,u(n)));
% x2_ud(n)= shape_ini(3) + L2*(quadl(shape_x2,0,u(n)));
% x3_ud(n)= shape_ini(5) + L3*(quadl(shape_x3,0,u(n)));
%
% y1_ud(n)= shape_ini(2) + L1*(quadl(shape_yl,0,u(n)));
% y2_ud(n)= shape_ini(4) + L2*(quadl(shape_y2,0,u(n)));
% y3_ud(n)= shape_ini(6)+ L3*(quadl(shape_y3,0,u(n)));
%
% end
%% Set initial configuration as initial guesses (estimates)
psi11 = c1;
psiP11 = 0;
x11 = n0(1,1);
y11 = n0(1,2);
psi12 = psi11;
psiP12 = 0;
x12 = (n1(1,1)+n0(1,1))/2;
y12=(n1(1,2)+n0(1,2))/2;
h1 = 0;
v1 = 0;
psi21 = c2;
psiP21 = 0;
x21 = n1(1,1);
y21 = n1(1,2);
psi22 = psi21;
psiP22 = 0;
x22 = (n2(1,1)+n1(1,1))/2;
y22 = (n2(1,2)+n1(1,2))/2;
h2 = 0;
v2 = 0;
psi31 = c3;
psiP31 = 0;
x31 = n2(1,1);
y31 = n2(1,2);
psi32 = psi31;
psiP32 = 0;
x32 = (n3(1,1)+n2(1,1))/2;
```

```
y32=(n3(1,2)+n2(1,2))/2;
h3 = 0;
v3 = 0;
Fr = 0;
% Initial Guess Vector el*(Nn+r)+1 = 3*(2*4+2)+1 = 31
ini_guess = [psi11 psiP11 x11 y11 psi12 psiP12 x12 y12 h1 v1\ldots
    psi21 psiP21 x21 y21 psi22 psiP22 x22 y22 h2 v2...
        psi31 psiP31 x31 y31 psi32 psiP32 x32 y32 h3 v3 Fr];
% Constraint Positions and Angles
q1x0=x11;
q1y0=y11;
eta10=feval(eta1,u(1));
d_eta10=feval(d_eta1,u(1));
eta1n=feval(eta1,u(N+1));
d_eta1n=feval(d_eta1,u(N+1));
eta20=feval(eta2,u(1));
d_eta20=feval(d_eta2,u(1));
eta2n=feval(eta2,u(N+1));
d_eta2n=feval(d_eta2,u(N+1));
q3xn=n3(1,1);
q3yn=n3(1,2);
eta30=feval(eta3,u(1));
d_eta30=feval(d_eta3,u(1));
eta3n=feval(eta3,u(N+1));
d_eta3n=feval(d_eta3,u(N+1));
const = [q1x0 q1y0 eta10 d_eta10 eta1n d_eta1n eta20 d_eta20 eta2n d_eta2n q3xn q3yn
eta30 d_eta30 eta3n d_eta3n];
%% Solve iteratively using Quasi-Newton method
display = 1; deltayslider = 0:1:11;
for i=1:size(deltayslider,2)
    %fprintf('*** displacement step = %d ---> delta_y slider = %.2f *** \n',i,deltayslider(i));
    [p,tol] = QNewton(@confun,ini_guess,display,L,EI,deltayslider(i),const);
    q1_ini = p(1:4); h1 = p(9); v1 = p(10);
    q2_ini = p(11:14); h2 = p(19); v2 = p(20);
    q3_ini = p(21:24); h3 = p(29); v3 = p(30);
    Fr = p(31);
    du = h; % integration step
    [tspan,q1] = ode45(@ss_eq,[0:du:1],q1_ini,[],h1,v1,L(1),EI(1)); % 1st link
    [tspan,q2] = ode45(@ss_eq,[0:du:1],q2_ini,[],h2,v2,L(2),EI(2)); % 2nd link
    [tspan,q3]=ode45(@ss_eq,[0:du:1],q3_ini,[],h3,v3,L(3),EI(3)); % 3rd link
    ini_guess = p; % set the current solution as the initial guess for the next step
```

figHandle $1=$ figure $(1)$;
figure(figHandle1);
$\mathrm{p} 8=\mathrm{plot}\left(\mathrm{q} 1(:, 3)^{*} 1000, \mathrm{q} 1(:, 4)^{*} 1000, \mathrm{r}^{\prime}\right.$,'LineWidth',3,'LineStyle','--'); \% plot the deformed
shape of the 1 st link
hold on;
p9=plot(q2(:,3)*1000,q2(:,4)*1000,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed
shape of the 2 nd link
hold on;
$\mathrm{p} 10=\mathrm{plot}\left(\mathrm{q} 3(:, 3)^{*} 1000, \mathrm{q} 3(:, 4)^{*} 1000\right.$, 'r','LineWidth', 5 ,'LineStyle','--'); \% plot the deformed shape of the 3rd link
hold on;
$\mathrm{p} 11=\operatorname{plot}\left(-\mathrm{q} 1(:, 3) * 1000, \mathrm{q} 1(:, 4)^{*} 1000, \mathrm{'r}^{\prime},{ }^{\prime}\right.$ LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 1 st link
hold on;
p12=plot(-q2(:,3)*1000,q2(:,4)*1000,'r','LineWidth',3,'LineStyle','--'); \% plot the
deformed shape of the 2 nd link
hold on;
p13=plot(-q3(:,3)*1000,q3(:,4)*1000,'r','LineWidth',5,'LineStyle','--'); \% plot the
deformed shape of the 3rd link
hold on;
$\mathrm{p} 14=\operatorname{plot}\left([\mathrm{n} 0(1,1)-\mathrm{n} 0(1,1)]^{*} 1000,[\mathrm{n} 0(1,2)+\right.$ deltayslider(i)/1000 -
$\mathrm{n} 0(1,2)+$ deltayslider(i)/1000]*1000,'r','LineWidth',5,'LineStyle','-');
hold on;
xlabel('x-coordinate (mm)');
ylabel('y-coordinate (mm)');
axis equal;
title('Deformation of the Fingers at Each Input Step');
grid on;
deformed=hggroup;
$\operatorname{set}([\mathrm{p} 8, \mathrm{p} 9, \mathrm{p} 10, \mathrm{p} 11, \mathrm{p} 12, \mathrm{p} 13, \mathrm{p} 14]$, 'Parent', deformed);
$\operatorname{tipx}(\mathrm{i})=\mathrm{q} 1(\mathrm{~N}+1,3)^{*} 1000$;
$\operatorname{tipy}(\mathrm{i})=\mathrm{q} 1(\mathrm{~N}+1,4) * 1000$;
$\mathrm{FR}(\mathrm{i})=-2 \mathrm{Fr} ;$
end
figHandle1 = figure(1);
figure(figHandle1);
$\mathrm{pl}=\operatorname{plot}\left([\mathrm{n} 0(1,1) \mathrm{n} 1(1,1)]^{*} 1000,[\mathrm{n} 0(1,2) \mathrm{n} 1(1,2)]^{*} 1000\right)$;
hold on;
$\mathrm{p} 2=\operatorname{plot}\left([\mathrm{n} 1(1,1) \mathrm{n} 2(1,1)]^{*} 1000,[\mathrm{n} 1(1,2) \mathrm{n} 2(1,2)]^{*} 1000\right)$;
hold on;
$\mathrm{p} 3=\operatorname{plot}\left([\mathrm{n} 2(1,1) \mathrm{n} 3(1,1)]^{*} 1000,[\mathrm{n} 2(1,2) \mathrm{n} 3(1,2)]^{*} 1000\right)$;
hold on;
$\mathrm{p} 4=\operatorname{plot}\left(-[\mathrm{n} 0(1,1) \mathrm{n} 1(1,1)]^{*} 1000,[\mathrm{n} 0(1,2) \mathrm{n} 1(1,2)]^{*} 1000\right)$;
hold on;
$\mathrm{p} 5=\operatorname{plot}\left(-[\mathrm{n} 1(1,1) \mathrm{n} 2(1,1)]^{*} 1000,[\mathrm{n} 1(1,2) \mathrm{n} 2(1,2)]^{*} 1000\right)$;

```
    hold on;
    p6=plot(-[n2(1,1) n3(1,1)]*1000,[n2(1,2) n3(1,2)]*1000);
    hold on;
    p7=plot([n0(1,1) -n0(1,1)]*1000,[n0(1,2) -n0(1,2)]*1000);
    hold on;
    set(p1,'Color','k','LineWidth',3,'LineStyle','-');
    set(p2,'Color','k','LineWidth',3,'LineStyle','-');
    set(p3,'Color','k','LineWidth',5,'LineStyle','-');
    set(p4,'Color','k','LineWidth',3,'LineStyle','-');
    set(p5,'Color','k','LineWidth',3,'LineStyle','-');
    set(p6,'Color','k','LineWidth',5,'LineStyle','-');
    set(p7,'Color','k','LineWidth',5,'LineStyle','-');
    undeformed=hggroup;
    set([p1,p2,p3,p4,p5,p6,p7],'Parent',undeformed);
    set(get(get(undeformed,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
    set(get(get(deformed,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
    legend('deformed','undeformed');
[def_x1,def_y1,def_x2,def_y2,def_x3,def_y3,tip_x,tip_y,f_r,t_step,stress1,stress2,stress3]=r
ead_ansys_-results();
    figHandle2 = figure(2);
    figure(figHandle2);
    p8=plot(q1(:,3)*1000,q1(:,4)*1000,'k','LineWidth',3,'LineStyle','-'); % plot the deformed
shape of the 1st link
    hold on;
    p9=plot(q2(:,3)*1000,q2(:,4)*1000,'k','LineWidth',3,'LineStyle','-'); % plot the deformed
shape of the 2nd link
    hold on;
    p10=plot(q3(:,3)*1000,q3(:,4)*1000,'k','LineWidth',5,'LineStyle','-'); % plot the
deformed shape of the 3rd link
    hold on;
    p11=plot(-q1(:,3)*1000,q1(:,4)*1000,'k','LineWidth',3,'LineStyle','-'); % plot the
deformed shape of the 1st link
    hold on;
    p12=plot(-q2(:,3)*1000,q2(:,4)*1000,'k','LineWidth',3,'LineStyle','-'); % plot the
deformed shape of the 2nd link
    hold on;
    p13=plot(-q3(:,3)*1000,q3(:,4)*1000,'k','LineWidth',5,'LineStyle','-'); % plot the
deformed shape of the 3rd link
    hold on;
    p14=plot([n0(1,1) -n0(1,1)]*1000,[n0(1,2)+deltayslider(i)/1000 -
n0(1,2)+deltayslider(i)/1000]*1000,'k','LineWidth',5,'LineStyle','-');
    hold on;
    Matlab=hggroup;
    set([p8,p9,p10,p11,p12,p13,p14],'Parent',Matlab);
```

p15=plot(def_x1,-def_y1,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 1st link ANSYS
hold on;
p16=plot(def_x2,-def_y2,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 2nd link ANSYS
hold on;
p17=plot(def_x3,-def_y3,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 3rd link ANSYS
hold on;
p18=plot(-def_x1,-def_y1,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 1 st link ANSYS
hold on;
p19=plot(-def_x2,-def_y2,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 2nd link ANSYS
hold on;
p20=plot(-def_x3,-def_y3,'r','LineWidth',3,'LineStyle','--'); \% plot the deformed shape of the 3rd link ANSYS
hold on;
xlabel('x-coordinate (mm)');
ylabel('y-coordinate (mm)');
axis equal;
ANSYS=hggroup;
$\operatorname{set}([p 15, \mathrm{p} 16, \mathrm{p} 17, \mathrm{p} 18, \mathrm{p} 19, \mathrm{p} 20]$, 'Parent',ANSYS);
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
legend('Matlab','ANSYS');
grid on;
title('Comparison of ANSYS and MATLAB Results');
figHandle3 $=$ figure(3);
figure(figHandle3);
plot(deltayslider,FR,'-.r','LineWidth',3);
hold on
plot(max(deltayslider)*t_step,f_r,':k*','LineWidth',3);
xlabel('\delta y (mm)');
ylabel('F_R (N)');
title('Reaction Force vs Input Displacement');
grid on;
axis([0 $\left.\begin{array}{llll}11 & 0 & 0.25\end{array}\right]$ );
legend('Matlab','ANSYS');
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on'); set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on'); legend('Matlab','ANSYS');
figHandle7 = figure(7);
figure(figHandle7);

```
p21=plot(tipx,tipy,'r','LineWidth',3);
hold on
p22=plot(-tipx,tipy,'r','LineWidth',3);
hold on
Matlab=hggroup;
set([p21,p22],'Parent',Matlab);
p23=plot(tip_x,-tip_y,'-.k*','LineWidth',3);
hold on
p24=plot(-tip_x,-tip_y,'-.k*','LineWidth',3);
ANSYS=hggroup;
set([p23,p24],'Parent',ANSYS);
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
legend('Matlab','ANSYS');
xlabel('x-coordinate (mm)');
ylabel('y-coordinate (mm)');
axis equal;
grid on;
title('Path of Finger Tip');
figHandle4 = figure(4);
figure(figHandle4);
sigma1=E*W1*abs(q1(:,2))/(2*L1)/1000000;
sigma2=E*W2*abs(q2(:,2))/(2*L2)/1000000;
sigma3=E*W3*abs(q3(:,2))/(2*L3)/1000000;
figHandle4 = figure(4);
figure(figHandle4);
p25=plot(u,sigma1,'r',u,sigma2,'b',u,sigma3,'g','LineWidth',3);
hold on;
Matlab=hggroup;
set(p25,'Parent',Matlab);
u1=0:1/(length(stress1)-1):1;
p26=plot(u1,stress1,'-.k*','LineWidth',3);
hold on;
ANSYS=hggroup;
set(p26,'Parent',ANSYS);
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
legend('Matlab','ANSYS');
xlabel('u');
ylabel('\sigma_{i} (MPa)');
axis([[0 1 1 0 7 7]);
grid on;
title('Bending Stress Distribution Along the 1st Segment ');
```

```
legend('1st Segment','2nd Segment','3rd Segment');
figHandle5 = figure(5);
figure(figHandle5);
p27=plot(u,sigma2,'b','LineWidth',3);
hold on;
Matlab=hggroup;
set(p27,'Parent',Matlab);
u2=0:1/(length(stress2)-1):1;
p28=plot(u2,stress2,'-.k*','LineWidth',3);
hold on;
ANSYS=hggroup;
set(p28,'Parent',ANSYS);
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
legend('Matlab','ANSYS');
xlabel('u');
ylabel('\sigma_{2} (MPa)');
axis([[0 1 0 4 4]);
grid on;
title('Bending Stress Distribution Along the 2nd Segment ');
figHandle6 = figure(6);
figure(figHandle6);
p29=plot(u,sigma3,'g','LineWidth',3);
hold on;
Matlab=hggroup;
set(p29,'Parent',Matlab);
u3=0:1/(length(stress3)-1):1;
p30=plot(u3,stress3,'-.k*','LineWidth',3);
hold on;
ANSYS=hggroup;
set(p30,'Parent',ANSYS);
set(get(get(Matlab,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
set(get(get(ANSYS,'Annotation'),'LegendInformation'),'IconDisplayStyle','on');
legend('Matlab','ANSYS');
xlabel('u');
ylabel('\sigma_{3} (MPa)');
axis([[0 1 1 0 4]);
grid on;
title('Bending Stress Distribution Along the 3rd Segment ');
```

end

## APPENDIX E: IMPLEMENTATION OF QUASI-NEWTON METHOD TO SOLVE NON-LINEAR ALGEBRAIC EQUATIONS

```
function [pGue tol] = QNewton(FUN, pGueIni, display, varargin)
% Based on the Quasi Newton Method, p.410, Faires Burden
pGue = pGueIni(:);
% Initialization
tol = 1; i = 1; alpha = 1e-4;
f = feval(FUN,pGue(:i),varargin {:});
A = Jacobian(pGue(:,i),f,ones(size(pGue,1),size(pGue,1)),FUN,varargin {:});
if display == 1; disp(' Iter. Fcount Tolerance'); end;
invA = inv(A);
while tol>1e-8
    Fcnt = 0;
    if i> 10000;
        msgbox('Iteration number > 1000','Warning Message','warn')
        break;
    end; % maximum iteration number is }100
    v=-invA*f(:); % DFP
    n_f= norm(f);
    n_g= n_f+1;
    beta =2;
    while (1-alpha*beta)*n_f < n_g %Armijio's rule
        beta = beta/2;
        if beta < 5e-2 % restart
            break
        end
        g = feval(FUN,pGue+beta*v,varargin {:});
        Fcnt = Fcnt + 1;
        n_g = norm(g);
    end
    gf = g(:)-f(:); betav = beta*v;
    pGue = pGue + betav;
    i= i + 1;
    invA = invA + (betav-invA*gf)*(betav'*invA)/(betav'*invA*gf); % DFP, p.413, Faires
Burden
    f=g; tol = norm(f);
end
if display == 1; disp(sprintf('%6.0f %6.0f %6.2e',i-1,Fcnt,tol)); end;
function Jstr = Jacobian(xcurr,valx,Jstr,fun,varargin)
%SFDNLS Sparse Jacobian via finite differences
%
% J = sfdnls(x,valx,J,group,[],fun) returns the
% sparse finite difference approximation J of a Jacobian matrix
% of function 'fun' at current point xcurr.
% Vector group indicates how to use sparse finite differencing:
% group(i) = j means that column i belongs to group (or color) j.
% Each group (or color) corresponds to a function difference.
% varargin are extra parameters (possibly) needed by function 'fun'.
%
```

$\% \mathrm{~J}=\operatorname{sfdnls}(\mathrm{x}$, valx,J,group,fdata,fun,alpha) overrides the default $\%$ finite differencing stepsize.
\%
$\%[\mathrm{~J}, \mathrm{ncol}]=\operatorname{sfdnls}(. .$.$) returns the number of function evaluations used$
\% in ncol.
\% Copyright 1990-2002 The MathWorks, Inc.
\% \$Revision: 1.9 \$ \$Date: 2002/03/12 20:36:20 \$
\% revised by Chao-Chieh Lan
\%
if nargin $<6$
error('Jacobian.m requires six arguments')
end
$\mathrm{x}=\mathrm{xcurr}(:) ; \%$ make it a column vector
$[\mathrm{m}, \mathrm{n}]=\operatorname{size}(\mathrm{Jstr}) ;$
alpha $=\operatorname{ones}(\mathrm{n}, 1)^{*} \operatorname{sqrt}(\mathrm{eps}) ;$
for $\mathrm{k}=1: \mathrm{n}$
$\operatorname{xnrm}=\max (\operatorname{abs}(\mathrm{x}(\mathrm{k})), 1)$;
$\operatorname{alpha}(\mathrm{k})=\operatorname{alpha}(\mathrm{k})^{*} \mathrm{xnrm}$; $\mathrm{y}=\mathrm{x}$;
$\mathrm{y}(\mathrm{k})=\mathrm{y}(\mathrm{k})+\operatorname{alpha}(\mathrm{k}) ;$
$\operatorname{xcurr}(:)=y ; \%$ reshape for userfunction
$\mathrm{v}=$ feval(fun, xcurr, varargin $\{:\}$ );
$\operatorname{Jstr}(:, \mathrm{k})=(\mathrm{v}-\mathrm{valx}) / \mathrm{alpha}(\mathrm{k})$;
end

APPENDIX F: RESULTS FOR FUNCTIONAL ANALYSIS OF CONCEPTS


APPENDIX G: FURTHER EXPERIMENTS ON THE GRIPPER PROTOTYPE


