AERODYNAMIC PARAMETER ESTIMATION OF A MISSILE

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ABSTRACT

AERODYNAMIC PARAMETER ESTIMATION OF A MISSILE

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Aerodynamic characteristics of missiles depend strongly on wind angles, that is, angle of attack and sideslip angle. However it is impractical to measure these angles during missile testing. Therefore, without direct information of the wind angles, it becomes a difficult problem to be able to accurately estimate the missile aerodynamic parameters from flight tests. This thesis addresses this problem and suggests an approach to estimate missile aerodynamic parameters successfully without wind angles measurements. Instead of reconstructing wind angles with post-process calculations prior to estimation, reconstruction process is handled within the estimation. The algorithm developed is tested with simulated missile data. Results are compared with true values used in simulation. It is demonstrated that suggested approach can provide accurate and reliable estimations without wind angles measurements. The approach is also applied to real flight test data of a missile with success.

Keywords: Missile, Open Loop Simulation, Parameter Estimation, Maximum Likelihood

ÖZ

BİR FÜZENİN AERODİNAMİK PARAMETRE TAHMİNİ

Aksu, Arda
Yüksek Lisans, Havacılık ve Uzay Mühendisliği Bölümü
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Füzelerin aerodinamik karakteristikleri baskın olarak rüzgar açılarına bağlıdır. Fakat uçuşlu testler sırasında bu açıların ölçümü pratik olmamaktadır. Bu yüzden bir füzenin aerodinamik parametrelerinin uçuşlu testlerden düzgün bir şekilde tahmin edilebilmesi de zor bir problem haline gelmektedir. Bu tez, bu problemi konu alarak bir füzenin aerodinamik parametrelerinin rüzgar açıları ölçümü olmadan başarılı bir şekilde tahmin edilebilmesi için bir yaklaşım önermektedir. Rüzgar açıları verilerinin tahmin çalışmasında kullanılmak üzere yeniden yapılandırılması yerine, bu yapılandırma tahmin sırasında ele alınmaktadır. Geliştirilen tahmin algoritması modellenen bir füzenin benzetim sonuçları üzerinde denenmiştir. Tahminden elde edilen sonuçlar benzetimde kullanılan gerçek değerler ile karşılaştırılarak, önerilen yaklaşımın rüzgar açılarının mevcut olmadığı durumda doğru ve güvenilir sonuçlar verebileceği gösterilmiştir. Yaklaşım aynı zamanda gerçek bir atışlı test verisine de başarıyla uygulanmıştır.

Anahtar Kelimeler: Füze, Açık Döngü Benzetim, Parametre Tahmini, Maksimum Olasılık

To all who has ever taught me

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NOMENCLATURE

$p,\ q,\ r$	body axis angular rates (roll, pitch, yaw)
$a_x,\ a_y,\ a_z$	body axis translational accelerations
$u,\ v,\ w$	body axis velocities
$x,\ y,\ z$	earth axis positions
$\phi,\ \theta,\ \psi$	Euler angles (roll, pitch, yaw)
$\alpha,\ \beta$	wind angles (angle of attack, sideslip angle)
$\delta_e,\ \delta_r,\ \delta_a$	control surface deflections (elevator, rudder, aileron)
V	total velocity relative to air
$C_X,\ C_Y,\ C_Z$	body axis non-dimensional aerodynamic force coefficients
$C_l,\ C_m,\ C_n$	body axis non-dimensional aerodynamic moment coefficients
$J_{xx},\ J_{yy},\ J_{zz},\ J_{xz}$	mass moments of inertia
b	reference span
l	reference length
S	reference area
ρ	air density
$\bar{q}$	dynamic pressure
a_{sound}	sound speed
m	mass
g	gravitational acceleration

CHAPTER 1

INTRODUCTION

Flight vehicle systems are designed with initial predictions based on similar systems mostly. Throughout the design stage characteristics of the system are needed to be represented with higher fidelities as the design evolves. One of the most difficult parts of the modeling involves postulating an accurate aerodynamic model for successful evaluation of system behavior.

Aerodynamic modeling starts with analytical calculations and continues with wind tunnel tests for fine tuning of aerodynamic parameters. In the end, postulated model is verified through flight tests. The easiest and most straight forward way of aerodynamic model validation is comparing the simulation results with real flight tests carried out for performance demonstration. However those tests are usually held with the autopilot in the (closed) loop. Match between simulation and flight test results does not necessarily mean that postulated model is accurate enough. This brings the necessity of separate flight tests specifically designed for aerodynamic model validation. For this reason, estimating aerodynamic parameters from flight tests has always been a major interest for flight vehicles.

1.1. Literature Review

Aerodynamic parameter estimation methods have been extensively applied to flight tests for decades. The various parameter estimation methods can be broadly classified into three categories: equation error, output error, and filter error methods [4].

In equation error method, aerodynamic parameters are achieved with a classical regression technique such as least square estimation. Synthesis of aerodynamic forces and moments through Taylor series expansion leads to a model that is linear in parameters. Aerodynamic coefficients are computed from linear and angular accelerations measurements and parameters of linearized model are obtained with least square fits to coefficients. Popularity of this method comes from its simplicity. For a given model structure, estimations are easily obtained with minimal computation in one shot. Due to the presence of measurement errors however, estimations might be asymptotically biased, inconsistent, and inefficient.

Output error method, as the name suggests, aims to minimize the error between the model outputs and measurements. This method is a nonlinear optimization method that has been most widely used for aerodynamic parameter estimation studies ever since its introduction around the seventies. Cost function is usually obtained from likelihood function so that the method is also referred as maximum likelihood estimation. The main advantage of this method over equation error method is that aerodynamic parameters can be implemented in state equations while minimizing the error. This in turn results with more accurate models.

In output error method, process noise in states is neglected and only measurement noise is accounted. The filter error method on the other hand accounts for both process and measurement noises and is the most general stochastic approach to aerodynamic parameter estimation. Process noise is included in state equations so that minor errors in system model can be eliminated with a state filter. In the presence of atmospheric turbulence this method is known to yield accurate results [10].

In addition to methods above, frequency domain approaches might be preferable over time domain approaches for rotorcraft identification [4]. Since no integration is involved in the frequency domain, method becomes suitable for unstable systems for which numerical integration in time domain can lead to problems. Moreover, without affecting the estimation results, the zero frequency can be neglected in evaluation, which can be advantageous in eliminating the need to bias parameters.

Other approaches appear in the literature are filtering approach which provides real time estimations and neural network based methods for highly nonlinear aerodynamic models.

1.2. Problem Definition and Contribution of Thesis

It can be seen that most of the studies appeared in the literature involve aircraft systems. The advantage of studying such systems is having reliable sensors in addition to Inertial Measurement Unit (IMU) such as airflow angle vane, integrating gyro and dynamic pressure sensor [12].

This is not the case for missile applications. For practical reasons, most of the time missiles have only IMU which measures translational accelerations and angular rates only. The required states are obtained by integrating IMU measurements during flight. Bias and scale errors in IMU measurements however, cause the integrated data to drift. Launch angles may also have uncertainty or IMU may not be able to detect attitude and velocity changes with enough accuracy during launch. These errors can be either neglected when their affect is minimal [1] or handled with post process data reconstruction techniques [12],[13]. Nevertheless, they can cause a poor representation of the true states.

Morelli has recently suggested a more reliable way of estimating aerodynamic parameters without wind angles measurements [16]. It was demonstrated that high frequency content of both reconstructed wind angles and real measurements are almost same. Making use of this information, wind angles were calculated with integrating IMU measurements, passed through a high pass filter and then used in frequency domain estimation.

This thesis focuses on an alternative solution in time domain and proposes an approach with output error method to estimate aerodynamic parameters of a missile from control surface deflections and IMU measurements only. Instead of reconstructing wind angles with post-process calculations prior to estimation, reconstruction process is proposed to be handled within the estimation. Output error method is utilized for this purpose. Efficiency of the algorithm developed is demonstrated with both simulation and real flight test data.

1.3. Scope

In order to evaluate the parameter estimation algorithm, a missile simulation is developed in Chapter 2. Javelin missile system is taken as an example and modeled in MATLAB. Aerodynamic model is obtained from Missile Datcom with assumed dimensions from Javelin pictures.

Necessary steps of an estimation study taking place prior to the flight test are explained and detailed in Chapter 3. These steps are postulating an aerodynamic model, preparation of test scenario and input design.

Chapter 4 gives mathematical details about the estimation algorithm to be tested. Practical considerations for real life applications are given. Two different system models to be used in estimation algorithm are proposed. Implications for advantages and disadvantages of both models are also discussed.

In Chapter 5, the algorithm developed is tested with both simulated data and real flight test data. Suggestions are also given about practical applications of the algorithm.

Finally, in Chapter 6, all results are discussed with conclusions and possible future works.

CHAPTER 2

MISSILE FLIGHT SIMULATION

2.1. Javelin ATGM

Javelin Anti-Tank Guided Missile (ATGM) [6],[14] is a man-portable, fire-and-forget system designed specifically to hit and destroy armored tanks and fighting vehicles. The project has been managed by Texas Instrument (later changed as Raytheon) and Lockheed Martin. Production phase has started in 1996 and missile has been used in field since then.

Javelin is fired by gunner after a target is locked with infrared (IR) seeker of the missile. The missile has two separate propulsion units, namely launch motor and flight motor. The launch motor provides soft launch to eject the missile from launch tube with approximately 13m/s velocity. Once the missile clears the tube, 8 mid-body wings and 4 tail fins flip out. Missile travels with this velocity to a safe distance of around 5 meters, then flight motor ignites and provides thrust for propelling the missile to its maximum velocity. A launch of Javelin ATGM is shown in Figure 2.1.

Figure 2.1 - Launch picture of Javelin ATGM

After burn-out of the flight motor, Javelin missile has approximately 190m/s velocity and continues the flight without thrust. Mass specifications of Javelin ATGM after burn-out [5] are given in Table 2.1.

Table 2.1 - Javelin ATGM Specifications After Burn-out

Mass	10.15 kg
Diameter	0.127 m
Length	1.081 m
CG (from nose)	0.446 m
J_{XX}	0.023 kg m/s^2
J_{YY} , J_{ZZ}	0.914 kg m/s^2

2.2. Reference Frames and Modeling Assumptions

In order to supply data for the estimation study, response of Javelin ATGM is simulated with six degrees of freedom (6-DOF) including inertial positions x,y,z and Euler angles ϕ,θ,ψ. The two reference frames used in simulation, namely inertial frame and body frame are defined in Figure 2.2.

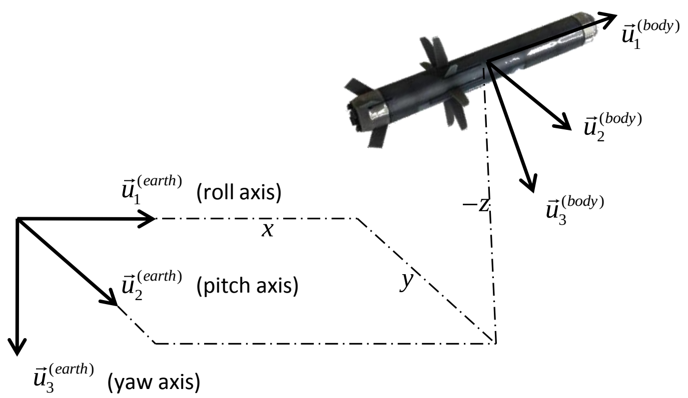

Figure 2.2 - Reference frames

Inertial frame is fixed with respect to earth at launched position of the missile with z axis pointing the same direction as the gravity vector, x axis pointing the direction of the missile and y axis pointing the right side of the initial orientation of missile. Body frame origin is at the missile center of gravity (CG), with x axis pointing forward through the nose of the missile, y axis pointing the right side of missile and z axis pointing through the underside. Rotation of the body frame from fixed earth frame is determined with 3-2-1 Euler sequence (ψ,θ,ϕ respectively).

Simulation is started at burn-out with 190 m/s initial velocity and physical specs are held constant during the simulation due to burn-out. Since the main focus in this thesis is estimating the aerodynamic parameters of the missile while it is in open loop, simulation of the missile before the burn-out and design of a proper controller are not in the scope of this study. It is assumed that missile is brought to a desired height and attitude at burn-out after launch. After that open loop control surface deflections are applied in order to excite the missile. Only the response of the missile to those excitations is simulated.

Further discussions of the assumptions made are given in the following sections.

2.3. Equations of Motion

The collected equations of motion of the missile to be used in the simulation are summarized below [12]:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \frac{\underline{F}}{m} + g \begin{bmatrix} -\mathrm{s}\theta \\ \mathrm{c}\theta\mathrm{s}\phi \\ \mathrm{c}\theta\mathrm{c}\phi \end{bmatrix} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.1}$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}^{-1} \left(\underline{M} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right) \tag{2.2}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{s}\theta\mathrm{s}\phi/\mathrm{c}\theta & \mathrm{s}\theta\mathrm{c}\phi/\mathrm{c}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \mathrm{s}\phi/\mathrm{c}\theta & \mathrm{c}\phi/\mathrm{c}\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2.3}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \mathrm{c}\psi\mathrm{c}\theta & -\mathrm{s}\psi\mathrm{c}\phi + \mathrm{c}\phi\mathrm{s}\theta\mathrm{s}\phi & \mathrm{s}\psi\mathrm{s}\phi + \mathrm{c}\psi\mathrm{s}\theta\mathrm{c}\phi \\ \mathrm{s}\psi\mathrm{c}\theta & \mathrm{c}\psi\mathrm{c}\phi + \mathrm{s}\psi\mathrm{s}\theta\mathrm{s}\phi & -\mathrm{c}\psi\mathrm{s}\phi + \mathrm{s}\psi\mathrm{s}\theta\mathrm{c}\phi \\ -\mathrm{s}\theta & \mathrm{c}\theta\mathrm{s}\phi & \mathrm{c}\theta\mathrm{c}\phi \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{2.4}$$

where F and M are the resultant aerodynamic force and moment vectors acting on the missile center of gravity expressed in the body coordinate frame. Note that *sine()* and *cosine()* functions are denoted with *s* and *c* for simplicity. Aerodynamic forces and moments are expressed in terms of non-dimensional aerodynamic coefficients as follows:

$$\underline{F} = \frac{\rho V^2}{2} \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} S \tag{2.5}$$

$$\underline{M} = \frac{\rho V^2}{2} \begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} Sl \tag{2.6}$$

Gravitational acceleration, g and air density, ρ in the above equations are assumed not to vary and used as constant in simulation. Considering the duration of the simulation and total change in the altitude, this is a reasonable assumption.

State equations given above are numerically integrated in MATLAB using 2nd order Runge-Kutta integration with 1ms time step.

States and inputs of the missile model are defined as:

$$\underline{x} = [u \quad v \quad w \quad p \quad q \quad r \quad \phi \quad \theta \quad \psi \quad x \quad y \quad z]^T \tag{2.7}$$

$$\underline{u} = [\delta_e \quad \delta_r \quad \delta_a]^T \tag{2.8}$$

The applied integration formula then can be shown as:

$$\underline{x}(k+1) = \underline{x}(k) + f\left[\underline{x}(k) + \frac{f\left[\underline{x}(k), \underline{u}(k)\right]dt}{2}, \hat{\underline{u}}(k)\right]dt \tag{2.9}$$

where f is represented for the state function and $\hat{\underline{u}}$ is represented for average value of the current *(k)* and future *(k+1)* points.

2.4. Aerodynamic Model

Non-dimensional aerodynamic force and moment coefficients in equations (2.5) and (2.6) are calculated using MISSILE DATCOM [2]. Geometric information of Javelin ATGM obtained through a reference picture is given in Figure 2.3. Since the missile is symmetric in XZ and XY planes, same coefficients are used for both planes.

As mentioned before, only the perturbed response of the missile in open loop is simulated in a relatively small time interval. This is the key point for most of the assumptions made especially in the aerodynamic model. Since the missile flies in a close vicinity of the ballistic trajectory when small perturbations are given to the control surfaces, a small space around reference condition is needed for the aerodynamic database. Vector of input breakpoints used to determine the space of aerodynamic database are given in Table 2.2.

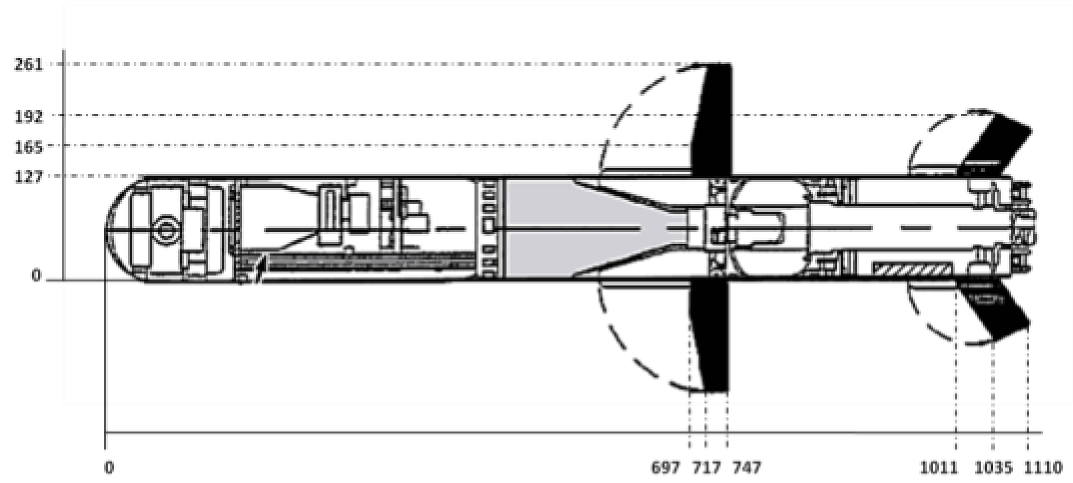

Figure 2.3 - Dimensions (in mm) of Javelin ATGM

Table 2.2 - Input vectors of aerodynamic database

Parameter	**Inputs**
Mach	[0.3 0.4 0.5 0.6]
$\alpha, \beta, \delta_e, \delta_r, \delta_a$	[-5 -4 -3 -2 -1 0 1 2 3 4 5] deg

Aerodynamic model using the parameters given in Table 2.2 are obtained from MISSILE DATCOM in the following form:

$$C_X = C_X^{static}(M, \alpha, \beta, \delta_e, \delta_r, \delta_a) \tag{2.10}$$

$$C_Y = C_Y^{static}(M, \beta, \delta_r) + C_{Y_r}^{dynamic}(M)\frac{rl}{2V} \tag{2.11}$$

$$C_Z = C_Z^{static}(M, \alpha, \delta_e) + C_{Z_q}^{dynamic}(M)\frac{ql}{2V} \tag{2.12}$$

$$C_l = C_l^{static}(M, \alpha, \beta, \delta_a) + C_{l_p}^{dynamic}(M)\frac{pl}{2V} \tag{2.13}$$

$$C_m = C_m^{static}(M, \alpha, \delta_e) + C_{m_q}^{dynamic}(M)\frac{ql}{2V} \tag{2.14}$$

$$C_n = C_n^{static}(M, \beta, \delta_r) + C_{n_r}^{dynamic}(M)\frac{rl}{2V} \tag{2.15}$$

Using the states and control surface deflection inputs in the simulation, each coefficient is calculated with linear interpolation from the aerodynamic database at every time step. Total velocity and wind angles are calculated from the states for no-wind condition as follows:

$$V = \sqrt{u^2 + v^2 + w^2} \tag{2.16}$$

$$\alpha = \tan^{-1}(w/u) \tag{2.17}$$

$$\beta = \tan^{-1}(v/u) \tag{2.18}$$

Note that the flank angle representation is used for sideslip angle. This is a fair assumption due to low attack angles.

One last assumption in the simulation is taking the speed of sound to be constant. Mach number is then calculated as follows:

$$M = \frac{V}{a_{sound}} \tag{2.19}$$

CHAPTER 3

EXPERIMENT DESIGN

An aerodynamic parameter estimation study starts before the flight test. First an aerodynamic model whose parameters are to be verified is determined. Then an appropriate test scenario is prepared in which missile is held as long as possible in the region where the determined model remains valid. Finally inputs whether in open loop or in close loop are designed so that missile supplies rich content of information in its response. These are all parts of the aerodynamic parameter estimation work that take place prior to flight. Before developing an estimation algorithm, these steps are explained in detail below.

3.1. Aerodynamic Model Verification

In this study, identification of the aerodynamic model is restricted to Y and Z axes force and moment coefficients. In other words only C_Z and C_m coefficients are identified. Note that, since missile is modeled as symmetric in pitch and yaw planes, C_Y and C_n are identical to C_Z and C_m in absolute values. There is only sign difference due to convention with:

$$C_Z^{static} = C_Y^{static} \tag{3.1}$$

$$C_{Z_q}^{dynamic} = -C_{Y_r}^{dynamic} \tag{3.2}$$

$$C_m^{static} = -C_n^{static} \tag{3.3}$$

$$C_{m_q}^{dynamic} = C_{n_r}^{dynamic} \tag{3.4}$$

Aerodynamic model is generally identified through parameters of linear expansion of the model at a reference Mach number:

$$C_Y = C_{Y_\beta}\beta + C_{Y_{\delta_r}}\delta_r + C_{Y_r}r \tag{3.5}$$

$$C_Z = C_{Z_\alpha}\alpha + C_{Z_{\delta_e}}\delta_e + C_{Z_q}q \tag{3.6}$$

$$C_m = C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_q}q \tag{3.7}$$

$$C_n = C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r + C_{n_r}r \tag{3.8}$$

Since the aerodynamic models in pitch and yaw planes are same, common aerodynamic derivatives are used also in linearized equations:

$$C_Y = C_{Z_\alpha}\beta + C_{Z_{\delta_e}}\delta_r - C_{Z_q}r \tag{3.9}$$

$$C_Z = C_{Z_\alpha}\alpha + C_{Z_{\delta_e}}\delta_e + C_{Z_q}q \tag{3.10}$$

$$C_m = C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_q}q \tag{3.11}$$

$$C_n = -C_{m_\alpha}\beta - C_{m_{\delta_e}}\delta_r + C_{m_q}r \tag{3.12}$$

The goal of a parameter estimation study is to find the unknown parameters of a known mathematical model. Here, the nonlinear aerodynamics of the missile is approximated by a linear model and the parameters of this model will be estimated. The purpose of this section is to find the linear model that best fits the actual nonlinear database. The linear model fitted to the database will be used to evaluate the performance of the estimation methods in the following sections.

Aerodynamic derivatives in linearized models are first evaluated from nonlinear database directly as reference values. A reference Mach number is selected with considering the mean velocity of the missile during the perturbations and nonlinear model is linearized around a reference point. Most general way of doing this is taking central difference around zero points for near ballistic flights. While the linear model obtained from central difference method can approximate the aerodynamic data well within a small neighborhood around the reference point, approximation becomes less and less accurate as the operation point moves away from the reference point. To overcome this inaccuracy, linear model can be determined with a least square fit of database values within a region that will be explored during the excitations, rather than at a single point at the reference flight condition. Parameters of a linear model obtained with two approaches for 0.5 Mach number are given in Table 3.1. It can be seen that relative error between two approaches are nearly %5 for control derivatives.

Table 3.1 - Aerodynamic moment derivatives at 0.4M

	Central difference approach	Least square approach
Czα (stability derivative)	-18.489	-19.183
Czδ (control derivative)	-2.573	-2.666
Cmα (stability derivative)	-30.894	-31.781
Cmδ (control derivative)	-12.118	-12.557

Note that linear model obtained with central difference approach is exact for [-1,+1] degrees angle of attack and [-1,+1] degrees elevator deflection intervals. On the other hand least square approach fits the nonlinear model with a better coverage. Linear models are plotted on aerodynamic database values in Figure 3.1. It can be seen that linear model obtained with the central difference approach has larger approximation error at points far away from the reference point compared to the linear model obtained with the least square approach.

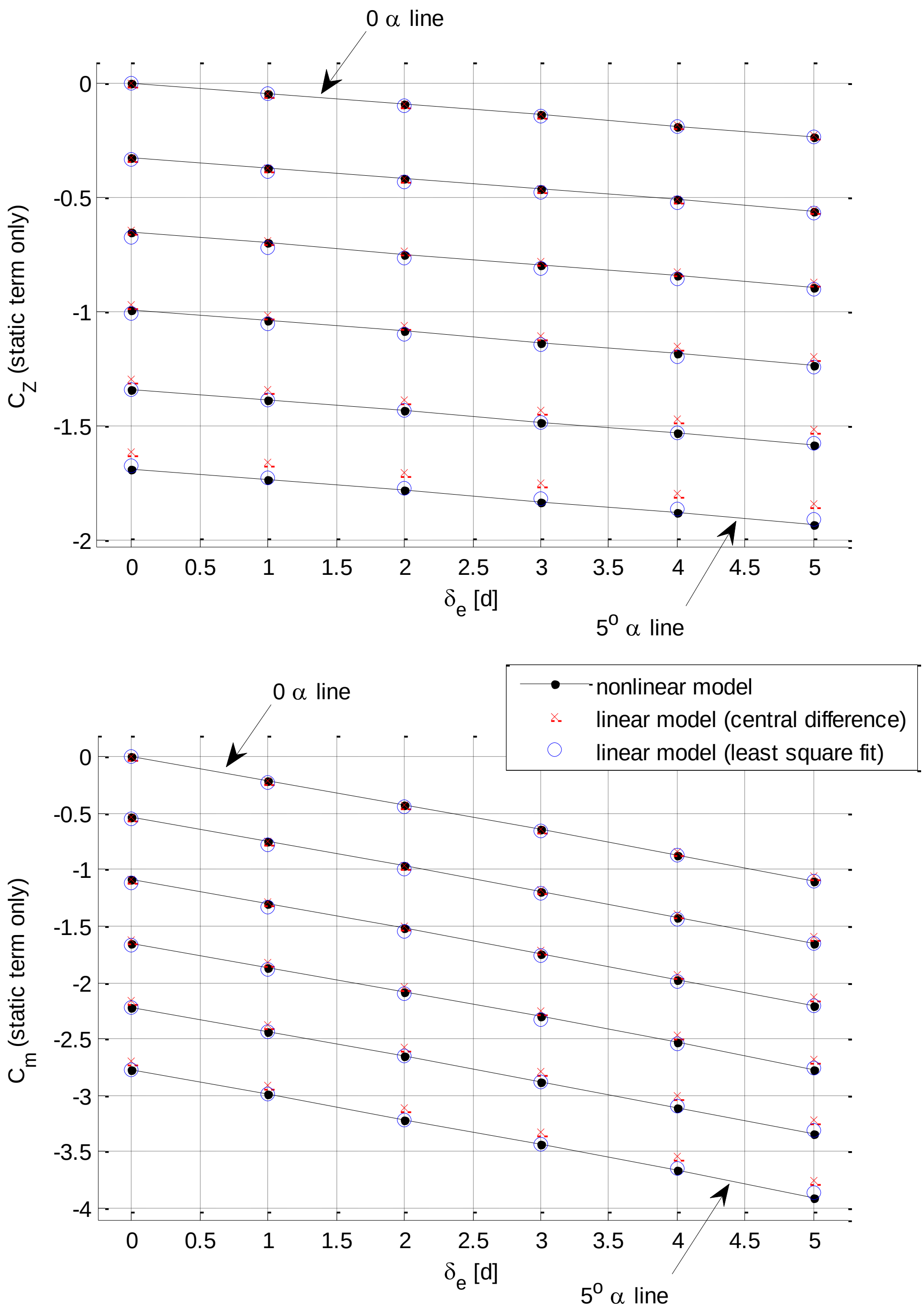

Figure 3.1 - Pitch plane static coefficients

One of the linear models should be selected in the estimation studies by considering the focus of the estimation. In the case of building a database through estimated values instead of verifying an existing one, central difference approach becomes a better choice over least square approach. Parameters obtained through multiple flight test cases are put all together and a final curve fit can be used for determination for a database model.

On the other hand, least square approach is better for studies that involve verification of an existing model. Using the control surface deflections and angle of attack obtained -directly or indirectly- from flight test as regressors, a least square fit is applied to nonlinear model for determining the reference linear model. So that values obtained from estimation can be compared with this reference model.

Either way, missile response must stay on the linear side of the real aerodynamic model as much as possible during the perturbations applied for exciting the missile. So that time invariant aerodynamic model used in estimation algorithm holds true in practice as well. That being said, least square approach is preferred in this study for the verification of the aerodynamic model used in simulation with estimation results.

Note that the static term in aerodynamic model created with Missile Datcom is linearized with respect to wind angles, control surface deflections and Mach number while dynamic term is linearized with respect to Mach number only.

3.2. Test Scenario

As explained before, estimation is applied to open loop response of the missile which flies close to the ballistic trajectory, in other words around zero angle of attack and sideslip angle after burn-out. Initial condition of this phase needs to be carefully determined with focusing to create an interval with minimum velocity change during the perturbations. This ensures that aerodynamic model can be estimated with a linearized expansion around a reference Mach condition.

Altitude of the missile from ground level at burn-out is taken as 100m [5]. Simulation is started from burn-out of the missile at that altitude with 190m/s velocity. Initial attitude of missile -more specifically pitch angle- is needed to be determined next. Using a negative pitch angle around -20 degrees provides the desired interval with minimum velocity change. Unfortunately that does not appear to be a realistic scenario since the missile might hit the ground too soon in such a trajectory. Instead, a relatively high pitch angle should be selected so that missile can climb more and longer intervals to be used in estimation might be obtained. Sample free flight without any control surface deflections are simulated for different initial pitch angles and results are shown in Figure 3.2. It can be seen that the tradeoff for longer intervals is to apply the estimation in lower velocity regions. Moreover velocity change increases for longer intervals as well. Therefore a suitable region must be selected where the time invariant aerodynamic model assumption holds and a reasonable amount of time exists for system identification excitations.

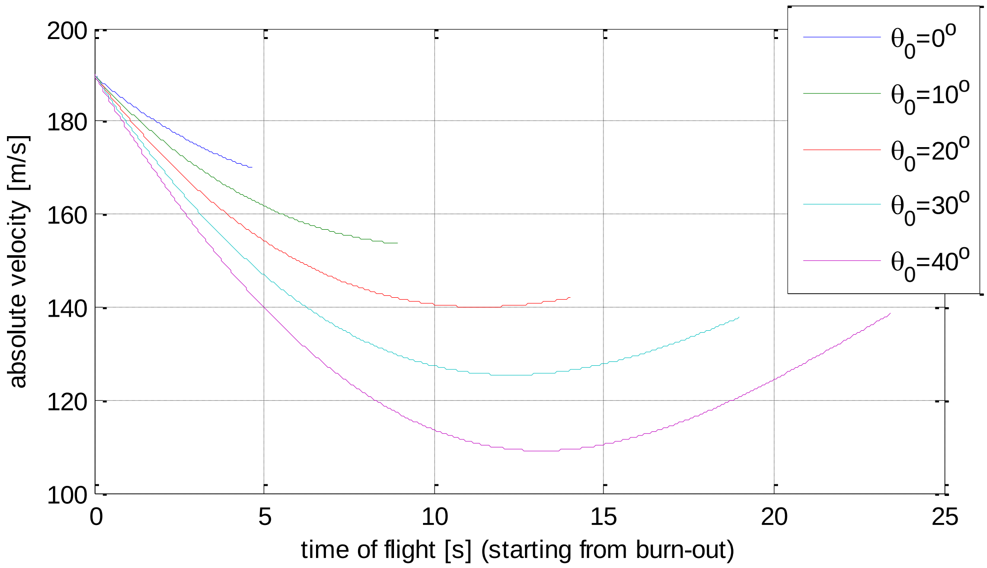

Figure 3.2 - Velocity plots for free flights with different initial pitch angles

Although it appears that in lower velocities there are longer intervals which might promise better estimations naturally, it is critical to verify the aerodynamic model close to the velocities which missile normally operates as far as possible. Therefore initial pitch angle is selected as 30 degrees. Perturbations are applied between seconds 7 and 19 in 30^{o} theta plot in Figure 3.2. The difference between minimum and maximum velocity during the interval is 12.5 m/s which is represented with 0.037 Mach number in simulation. This should be an acceptable change for the time invariant aerodynamic model assumption and is checked in the next section.

3.3. Input Design

The objective of the input design is to excite the missile so that measurement data contains sufficient information for a successful estimation. Since measurements are noisy, higher excitations yields better signal to noise ratios. The practical difficulty that must be taken into consideration while exciting the missile with higher amplitudes is to ensure that states stay in the region required by the aerodynamic model used in estimation.

Note that dynamic terms in the aerodynamic model are already linear. Static terms are linearized to be used in estimation. Those linear models are functions of wind angles and control surface deflections. These parameters must stay on the linear region of real aerodynamic model around zero points in order that linear aerodynamic model retains the validity in that region and so it can give the same results with measurements. If the parameters drift away from the linear region, no single linear model can closely approximate the database anymore. This means there exists no parameter set that will cause the model output to match the real missile behavior closely. In this case the estimation process will produce inaccurate results or no results at all.

Local derivatives of pitch plane aerodynamic coefficients at 0.37 and 0.42 Mach numbers (which are the minimum and maximum velocities within the test interval selected for excitations) are given in Figure 3.3 with respect to angle of attack and elevator deflection. In Y axis of plots, relative errors with respect to derivatives at zero angle of attack, zero elevator deflection and mean velocity are also included.

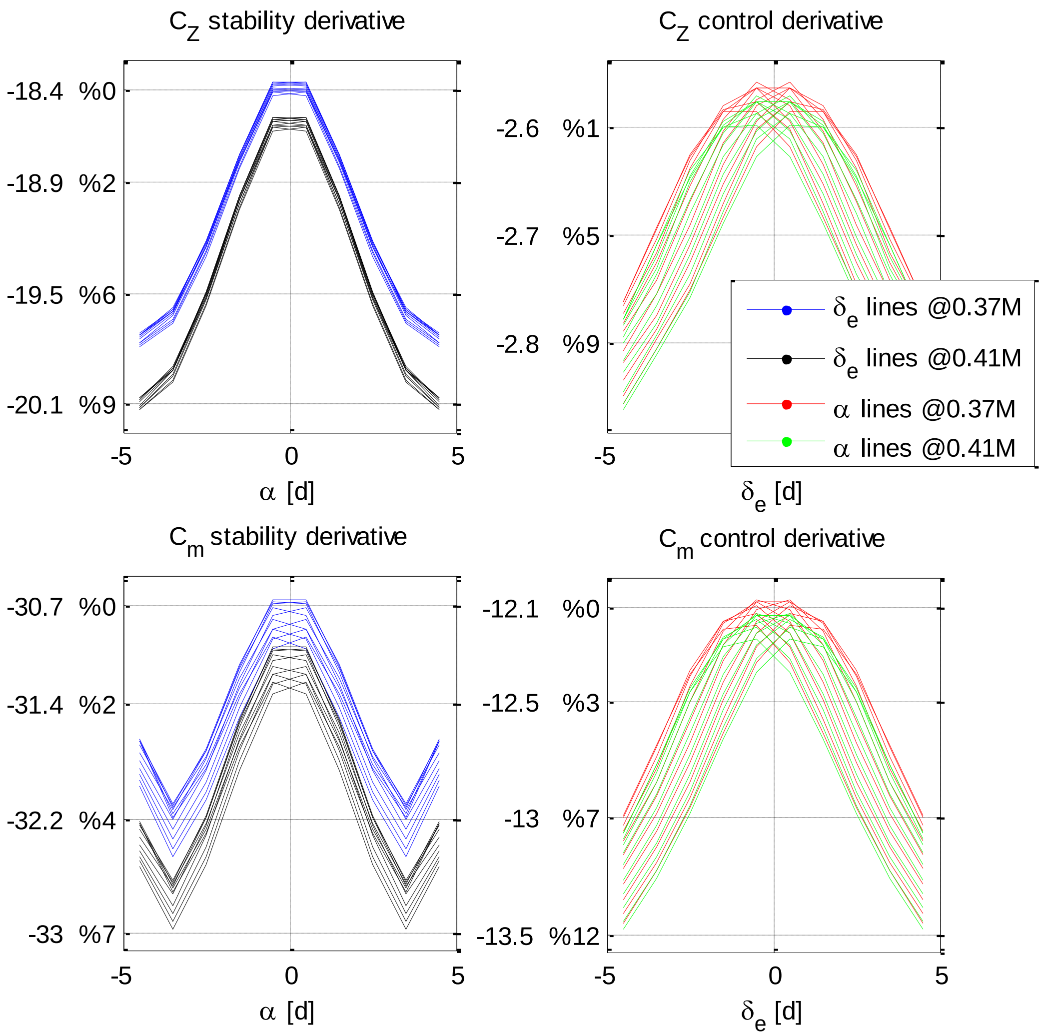

Figure 3.3 - Aerodynamic force and moment derivatives

It can be seen that dominant relative error is in control derivatives due to angle of attack and elevator deflection. After 4 degrees for both of those parameters, relative errors exceed the %10 bands, above which linearity assumption might fail. Note that 0.04M velocity change has a negligible effect on relative errors. In fact due to this reason, 12 sec interval discussed above might be stretched little more with starting excitations earlier if it is necessary to increase observability.

In order to provide high signal to noise ratio at measurements, missile should be excited near the natural frequency of dominant dynamic mode while keeping the states in linear region at the same time. If a parameter estimation study is intended to be applied without any prior information about the aerodynamic model, missile should be excited over a broad frequency range with nearly constant power for all frequencies. However this study focuses on verification of an existing model so that all the information available prior to the estimation can be used in input design stage.

Natural frequency of dominant dynamic mode must be determined first. The easiest and practical way of doing this without analytical calculations is to apply frequency sweep input in simulation and analyzing the frequency response of the missile. The frequency having the highest amplitude in measurements is the dominant dynamic mode. Exciting the missile in near frequencies provides better signal to noise ratios in measurements hence better observability of aerodynamic parameters. Applying perturbed control surface deflections with frequency which is same as the dynamic mode frequency and a suitable amplitude chosen based on the linearity concerns gives a good starting point. Further fine tunings should be made for better usage of linear model limits. Both frequency and amplitude of the control surface deflections can be adjusted appropriately in order to design an optimal test case. For example the dynamic mode of the Javelin ATGM at 0.4M is approximately 3.5Hz. Applying control surface deflections with that frequency and 3 degrees amplitude to missile causes wind angles to exceed 5 degrees which should not be accepted due to the limits of the linear region. Either amplitude of the inputs should be lowered or frequency should be moved further from dynamic mode to resolve this issue. First choice lowers the signal to noise ratio obviously if control surface deflection measurements are noisy. Therefore changing the input frequency is a better choice.

Designed inputs are shown in Figure 3.4. Multi-step inputs in 2-1-1 pulses [12] are applied with 0.56 sec period (nearly 2.5Hz) and 2.5 degrees amplitude. The reason of multi-step input choice over classic doublet inputs is to make the states to be uncorrelated with inputs [12]. In order to catch the free response of the system zero input intervals are also included which also aids to lower correlations. Note that square wave inputs generated in MATLAB are filtered with 20 Hz low pass filter which is implemented as control actuator dynamics to be more realistic.

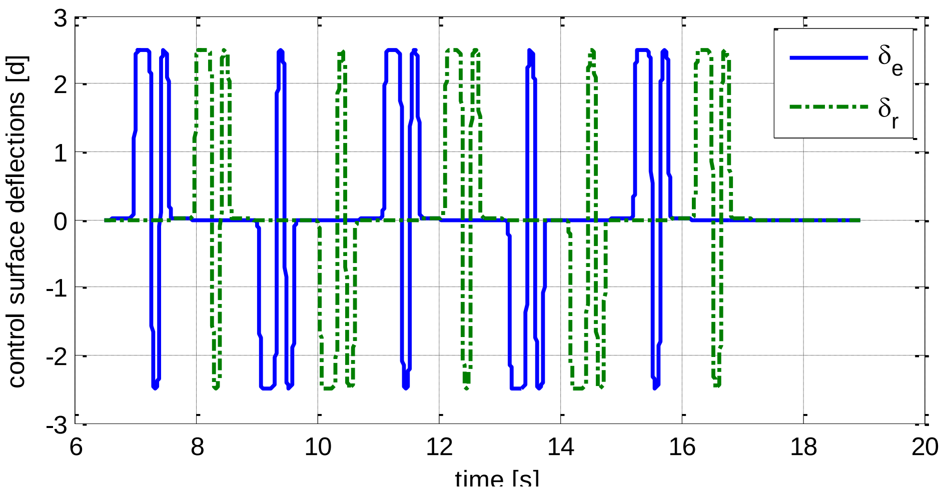

Figure 3.4 - Control surface deflections in studied test case

Wind angles responses of the missile to the applied perturbed inputs are given in Figure 3.5. It can be seen that wind angles do not exceed 4 degrees region in which aerodynamic model can be accepted as linear.

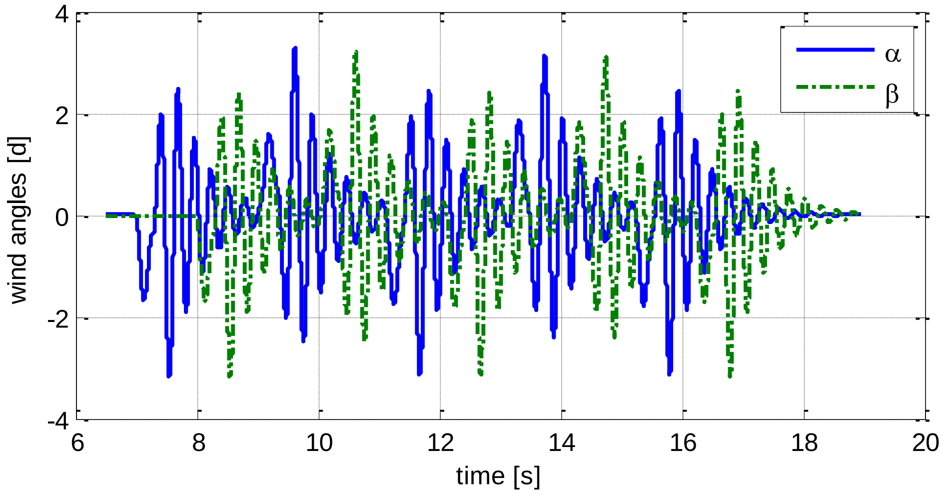

Figure 3.5 - Wind angles in studied test case

Parameters of the linear model obtained by applying a least square fit to the region experienced by missile are given in Table 3.2. Central difference values of nonlinear aerodynamic model at mean velocity during the perturbations are also given in the same table for reference values.

Table 3.2 - Parameters of linear model obtained with least square fit

Parameter	**Value**	**Central difference value**
$Cz\alpha$	-18.628	-18.421
$Cz\delta$	-2.611	-2.568
Czq	-0.048	-0.048
$Cm\alpha$	-31.031	-30.774
$Cm\delta$	-12.246	-12.097
Cmq	-0.121	-0.121

Comparisons of nonlinear model with both central difference and least square models are given in Figure 3.6. It can be seen from error plots that even though errors of central difference approach are minimal in free response regions, least square model minimizes the error in overall response.

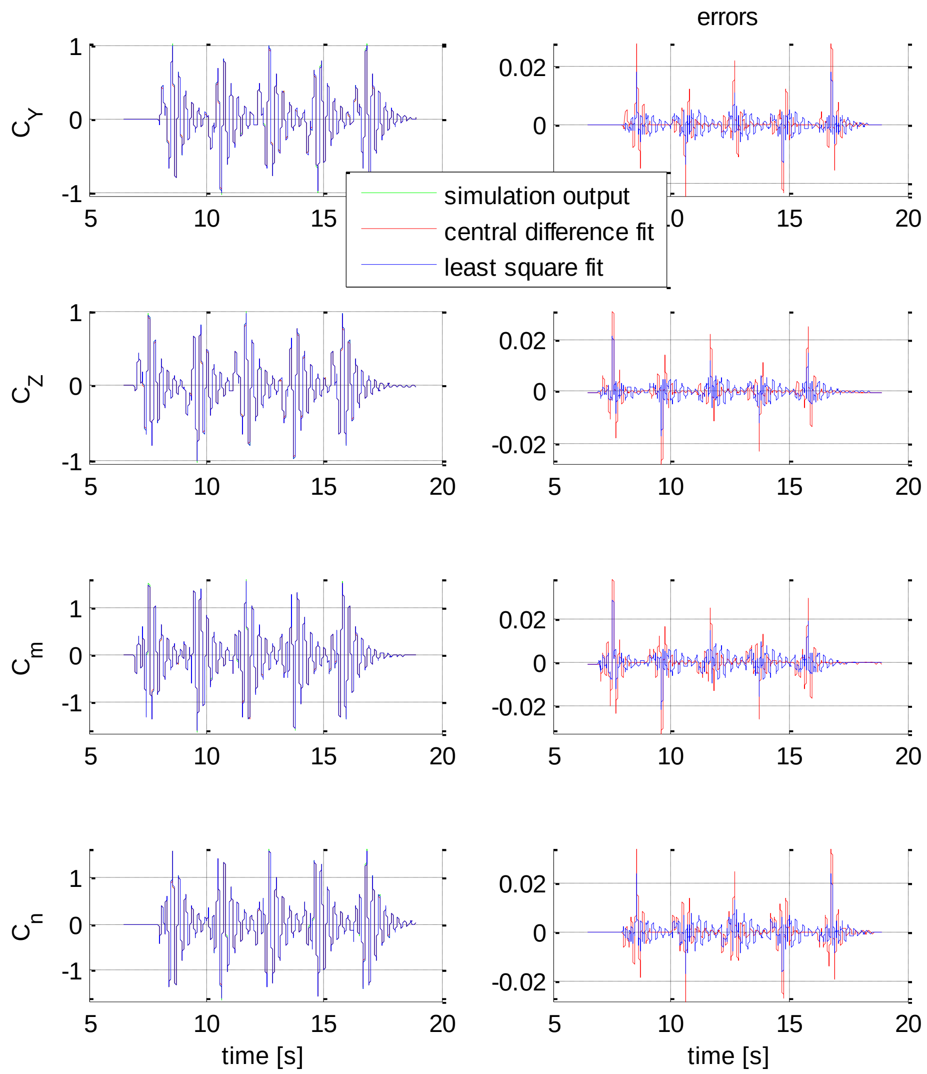

Figure 3.6 - Comparison of aerodynamic models

Note that results of the linear models are not outputs from simulation. Those results are obtained by applying the linear models using outputs of the simulation with the nonlinear aerodynamic model. If somehow, all the states of missile during a flight test are available without any biases and noises, the optimal linear aerodynamic model -covering the real response in a limited region as best as it possible- can easily be obtained with a least square fit like this. This is the reason why least square fit results rather than central difference values are used for the verification of the aerodynamic model.

CHAPTER 4

ESTIMATION ALGORITHM

In general mathematical model of a dynamic system whose unknown parameters are to be estimated is given by:

$$z = y(\underline{\theta}) + v \tag{4.1}$$

where *z* is the observation or measurement, θ is the (n_p x 1) unknown parameter vector, *y* is the output function of the system model and *v* is the measurement noise. Then, based on the Fisher estimation theory [11], likelihood function of independent random observations can be defined as:

$$L\left(z;\underline{\theta}\right) = p\left(z \mid \underline{\theta}\right) = \prod_{i=1}^{N} p\left(z_i \mid \underline{\theta}\right) \tag{4.2}$$

Above function is probability density of measured data as a function of system parameters, θ. In other words, it is probability of measurements given system parameters. The likelihood function gets the maximum value for true parameters. Therefore maximum likelihood estimator for parameter vector θ is equal to θ that maximizes the likelihood function for *N* measurements. This can be shown as follows:

$$\hat{\underline{\theta}} = \max_{\underline{\theta}} \left[\prod_{i=1}^{N} p\left(z_i \mid \underline{\theta}\right) \right] \tag{4.3}$$

It is generally preferred to minimize the negative logarithm of likelihood function rather than to maximize the likelihood function [8]. So that a suitable optimization technique can be applied to the negative logarithm of the likelihood function which represents the cost function:

$$\hat{\underline{\theta}} = \min_{\underline{\theta}} \left[-\ln \prod_{i=1}^{N} p\left(z_i \mid \underline{\theta}\right) \right] = \min_{\underline{\theta}} \left[\sum_{i=1}^{N} -\ln p\left(z_i \mid \underline{\theta}\right) \right] \tag{4.4}$$

Minimizing the negative logarithm instead of maximizing also comes with great numeric stability. Since the maximum value of the likelihood function is between zero and one, multiplying the likelihood functions of different measurements with each other repeatedly gives eventually a small number which can’t be represented with enough precision in a computer. To resolve this, scaling might be applied after each multiplication. However this complicates the optimization technique to be used and brings computational burden as well. Instead, the logarithm scales the result at each step naturally and increases the precision. Yet the logarithm function is monotonic, so maximizing the logarithm functions can also be achieved with minimizing the negative of it. Since minimization of a cost function is an easier procedure than maximizing, negative logarithm of likelihood function is by far advantageous over likelihood function alone.

Maximum likelihood estimation can be applied to any form of probability density function. One of the most widely used density distribution for likelihood functions is Gaussian distribution:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right] \tag{4.5}$$

where *m* is the expected value (mean) of *x* and σ is the covariance of *x*. Likelihood function defined above was probability density of one observation parameter as a function of unknown parameters. In case of observing parameters more than one, likelihood function becomes the joint probability distribution of observations. Joint probability density function of *n* Gaussian distributed random variables is given by:

$$p(\underline{x}) = \frac{1}{\sqrt{(2\pi)^n |\underline{R}|}} \exp\left[-\frac{1}{2}(\underline{x}-\underline{m})^T \underline{R}^{-1} (\underline{x}-\underline{m})\right] \tag{4.6}$$

$\underline{R}$ given above is the (*n* x *n*) covariance matrix of random variable vector. Assuming that random variables are uncorrelated with each other, covariance matrix can be stated as diagonal. Since mean values of the random variables represent the true outputs, joint probability distribution of observations are stated as:

$$p(\underline{z}) = \frac{1}{\sqrt{(2\pi)^n |\underline{R}|}} \exp\left[-\frac{1}{2}(\underline{z}-\underline{y})^T \underline{R}^{-1} (\underline{z}-\underline{y})\right] \tag{4.7}$$

where $\underline{z}$ and $\underline{y}$ is the (*n* x 1) measurement and output vectors respectively. Note that (*n* x *n*) covariance matrix is now represented for measurement noise $\underline{z}-\underline{y}$. Likelihood function of *n* independent Gaussian distributed random variables as *N* many observations each is then given by:

$$L(\underline{z};\underline{\theta}) = \prod_{i=1}^{N} p(\underline{z}_i \mid \underline{\theta}) = \left(\frac{1}{\sqrt{(2\pi)^n |\underline{R}|}}\right)^N \exp\left[-\frac{1}{2}\sum_{i=1}^{N}(\underline{z}_i-\underline{y}_i)^T \underline{R}^{-1} (\underline{z}_i-\underline{y}_i)\right] \tag{4.8}$$

By taking the negative logarithm of above function, cost function to be minimized for estimating unknown parameters is obtained:

$$J\left(\underline{\theta}\right) = -\ln\left[L\left(\underline{z};\underline{\theta}\right)\right] \tag{4.9}$$

$$J\left(\underline{\theta}\right) = \frac{nN\ln\left(2\pi\right)}{2} + \frac{N\ln\left(\left|\underline{R}\right|\right)}{2} + \frac{1}{2}\sum_{i=1}^{N}\left(\underline{z}_i - \underline{y}_i\right)^T \underline{R}^{-1}\left(\underline{z}_i - \underline{y}_i\right) \tag{4.10}$$

4.1. Optimization

Minimization of the above function can be satisfied by setting the gradient to zero:

$$\frac{\partial J\left(\underline{\theta}_0 + \Delta\underline{\theta}\right)}{\partial\underline{\theta}} = 0 \tag{4.11}$$

Using the first order Taylor series expansion as an approximation, the gradient of the cost function is given by:

$$\frac{\partial J\left(\underline{\theta} + \Delta\underline{\theta}\right)}{\partial\underline{\theta}} = \left.\frac{\partial J\left(\underline{\theta}\right)}{\partial\underline{\theta}}\right|_{\underline{\theta}=\underline{\theta}_0} + \left.\frac{\partial^2 J\left(\underline{\theta}\right)}{\partial\underline{\theta}\partial\underline{\theta}^T}\right|_{\underline{\theta}=\underline{\theta}_0} \Delta\underline{\theta} \tag{4.12}$$

Setting the right hand side of above equation to zero and solving for $\Delta\underline{\theta}$ gives:

$$\Delta\underline{\theta} = -\left[\left.\frac{\partial^2 J\left(\underline{\theta}\right)}{\partial\underline{\theta}\partial\underline{\theta}^T}\right|_{\underline{\theta}=\underline{\theta}_0}\right]^{-1} \left.\frac{\partial J\left(\underline{\theta}\right)}{\partial\underline{\theta}}\right|_{\underline{\theta}=\underline{\theta}_0} \tag{4.13}$$

Above change in parameter vector makes local gradient zero at that point. With an initial guess, iterative solution of above equation provides the parameter vector for the minimum value of cost function. This approach is commonly known as Newton-Raphson optimization in the literature.

The difficulty of applying this technique is that the covariance matrix given in the cost function depends also on unknown parameters. This fact complicates the optimization algorithm while taking the derivatives of cost. In fact mathematically speaking there is no closed form solution of this problem. Instead of applying the minimization for the unknown system parameters all at once, relaxation technique can be used. In this technique, covariance matrix is assumed not to be affected by change in system parameters. It is used as constant in cost function and after each parameter update it is updated independently for the new parameters.

The procedure of the relaxation technique can be summarized as follows:

1. Set initial values for parameters.

2. Find system outputs for selected parameters and estimate the noise covariance matrix from measurement errors.
3. Apply the optimization to minimize the cost function and update the unknown parameter vector.
4. Iterate step 2 and 3 until convergence.

4.1.1. Noise Covariance Matrix

Estimation of the covariance matrix is obtained similarly. The gradient with respect to the covariance matrix is set to zero and then solved for the covariance matrix. The first term in the cost function has no effect on minimization. Dropping that term and rearranging the cost function as follows as a function of covariance matrix makes easier to take the derivative.

$$J\left(\underline{R}\right)=\frac{N\ln\left(\left|\underline{R}\right|\right)}{2}+\frac{1}{2}\left[\underline{R}^{-1}\sum_{i=1}^{N}\left(\underline{z}_i-\underline{y}_i\right)\left(\underline{z}_i-\underline{y}_i\right)^T\right]^T \tag{4.14}$$

Then partial derivative with respect to covariance matrix is obtained as:

$$\frac{\partial J\left(\underline{R}\right)}{\partial\underline{R}}=\frac{N}{2}\underline{R}^{-1}-\frac{1}{2}\underline{R}^{-1}\sum_{i=1}^{N}\left(\underline{z}_i-\underline{y}_i\right)\left(\underline{z}_i-\underline{y}_i\right)^T\underline{R}^{-1} \tag{4.15}$$

Setting the gradient to zero and solving for $\underline{R}$ gives the estimate of the noise covariance matrix for the current values of parameters at that step:

$$\hat{\underline{R}}=\frac{1}{N}\sum_{i=1}^{N}\left(\underline{z}_i-\underline{y}_i\right)\left(\underline{z}_i-\underline{y}_i\right)^T \tag{4.16}$$

After each parameter update, covariance matrix is calculated again. Estimated covariance matrix is then used as constant while finding the parameter update.

4.1.2. Parameter Update

Since covariance matrix in the cost function is fixed at each step during the optimization, cost to be minimized reduces to:

$$J\left(\underline{\theta}\right)=-\frac{1}{2}\sum_{i=1}^{N}\left(\underline{z}_i-\underline{y}_i\right)^T\underline{R}^{-1}\left(\underline{z}_i-\underline{y}_i\right) \tag{4.17}$$

The gradient of the cost function with respect to the parameter vector

$$\frac{\partial J(\underline{\theta})}{\partial \underline{\theta}} = \frac{1}{2}\sum_{i=1}^{N}\left\{\left[\frac{\partial(\underline{z}_i - \underline{y}_i)}{\partial \underline{\theta}}\right]^T \underline{R}^{-1}(\underline{z}_i - \underline{y}_i) + (\underline{z}_i - \underline{y}_i)^T \underline{R}^{-1}\frac{\partial(\underline{z}_i - \underline{y}_i)}{\partial \underline{\theta}}\right\} \tag{4.18}$$

Measurement vector is also independent from parameters. Simplifying above equation using this gives:

$$\frac{\partial J(\underline{\theta})}{\partial \underline{\theta}} = \sum_{i=1}^{N}\left\{\left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right]^T \underline{R}^{-1}(\underline{z}_i - \underline{y}_i)\right\} \tag{4.19}$$

The second order gradient of the cost function is given by:

$$\frac{\partial^2 J(\underline{\theta})}{\partial \underline{\theta}\partial \underline{\theta}^T} = \sum_{i=1}^{N}\left\{\left[\frac{\partial^2 \underline{y}_i}{\partial \underline{\theta}\partial \underline{\theta}^T}\right]^T \underline{R}^{-1}(\underline{z}_i - \underline{y}_i) + \left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right]^T \underline{R}^{-1}\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right\} \tag{4.20}$$

The partial derivative of outputs with respect to the parameter vector is called response gradient or output sensitivity. This is an (n x n_p) matrix with n is the number of output or measurement variables and n_p is the number of unknown parameters or in other words length of the parameter vector, $\underline{\theta}$. [i,j] element of the matrix is quantifies the change in ith observation due to the change in jth parameter.

The first term in the summation above includes the second order gradient of the response. This gradient is computationally expensive to obtain and generally suggested to be neglected. Yet summation of products of the second order response gradients with residuals converges to zero for the true parameters. For that reason neglecting this term is a good approximation near the final solution. The algorithm obtained with this simplification is known as Gauss-Newton method in the literature.

Combining the cost gradients in parameter change equation gives:

$$\Delta\underline{\theta} = -\left[\sum_{i=1}^{N}\left\{\left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right]^T \underline{R}^{-1}\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right\}\Bigg|_{\underline{\theta}=\underline{\theta}_0}\right]^{-1} \sum_{i=1}^{N}\left\{\left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}}\right]^T \underline{R}^{-1}(\underline{z}_i - \underline{y}_i)\right\}\Bigg|_{\underline{\theta}=\underline{\theta}_0} \tag{4.21}$$

In order the inversion of the second order cost gradient in the parameter change equation to be successful, the necessary condition is having a full rank matrix inside. Since $\underline{R}$ is taken diagonal as explained before, response gradients must be linearly independent to satisfy that condition. In other words both rows and columns of the response gradients must be linearly independent with each other.

This is only possible when system parameters to be estimated must have a unique impact on outputs and those outputs are not correlated with each other. Otherwise second order cost gradient becomes singular and inversion might simply fail.

Numerical errors might also be accounted for a nearly singular matrix. If inversion does not fail, parameter change might result in with divergence in cost function. In order to prevent this, inversion with singular value decomposition [15] might be a better way instead of direct inversion.

Gauss-Newton method explained above is an unconstrained optimization starting from an initial point based on a quadratic cost function assumption. In some circumstances such as when the cost function is highly nonlinear or initial parameters are far away from the true values, the step size of parameter vector might be too large during the iteration. Singular value decomposition helps to detect the directions of large parameter changes [12]. Defining an upper limit for the change helps the cost function to converge. However one limit might not be applicable for all directions due to the difference of parameter scales. Instead, a simpler approach based on heuristic considerations is commonly preferred. If the cost function diverges at any step during the iteration, parameter update size is reduced by halving each time until reduction in cost function is satisfied [8]. This is applied by implementing a weight factor in parameter change equation as follows:

$$\Delta\underline{\theta} = -2^{-(k-1)} \left[\sum_{i=1}^{N} \left\{ \left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}} \right]^T \underline{R}^{-1} \frac{\partial \underline{y}_i}{\partial \underline{\theta}} \right\} \Bigg|_{\underline{\theta}=\underline{\theta}_0} \right]^{-1} \sum_{i=1}^{N} \left\{ \left[\frac{\partial \underline{y}_i}{\partial \underline{\theta}} \right]^T \underline{R}^{-1} \left(\underline{z}_i - \underline{y}_i \right) \right\} \Bigg|_{\underline{\theta}=\underline{\theta}_0} \tag{4.22}$$

where k is used as one at each iteration. If the cost increases, k is also increased by one until the cost decreases during the iterations.

There are other methods also to prevent divergences such as bounding the parameters [7] and switching to simplex method in cost increase [18] but halving approach is found to work fine and is preferred in this study for its simplicity.

Parameter update process is repeated until a convergence criterion is satisfied. Assessment of final convergence should be made for both relative changes in cost function and in parameters at the same time. Estimation loop including parameter halving procedure is presented in Figure 4.1.

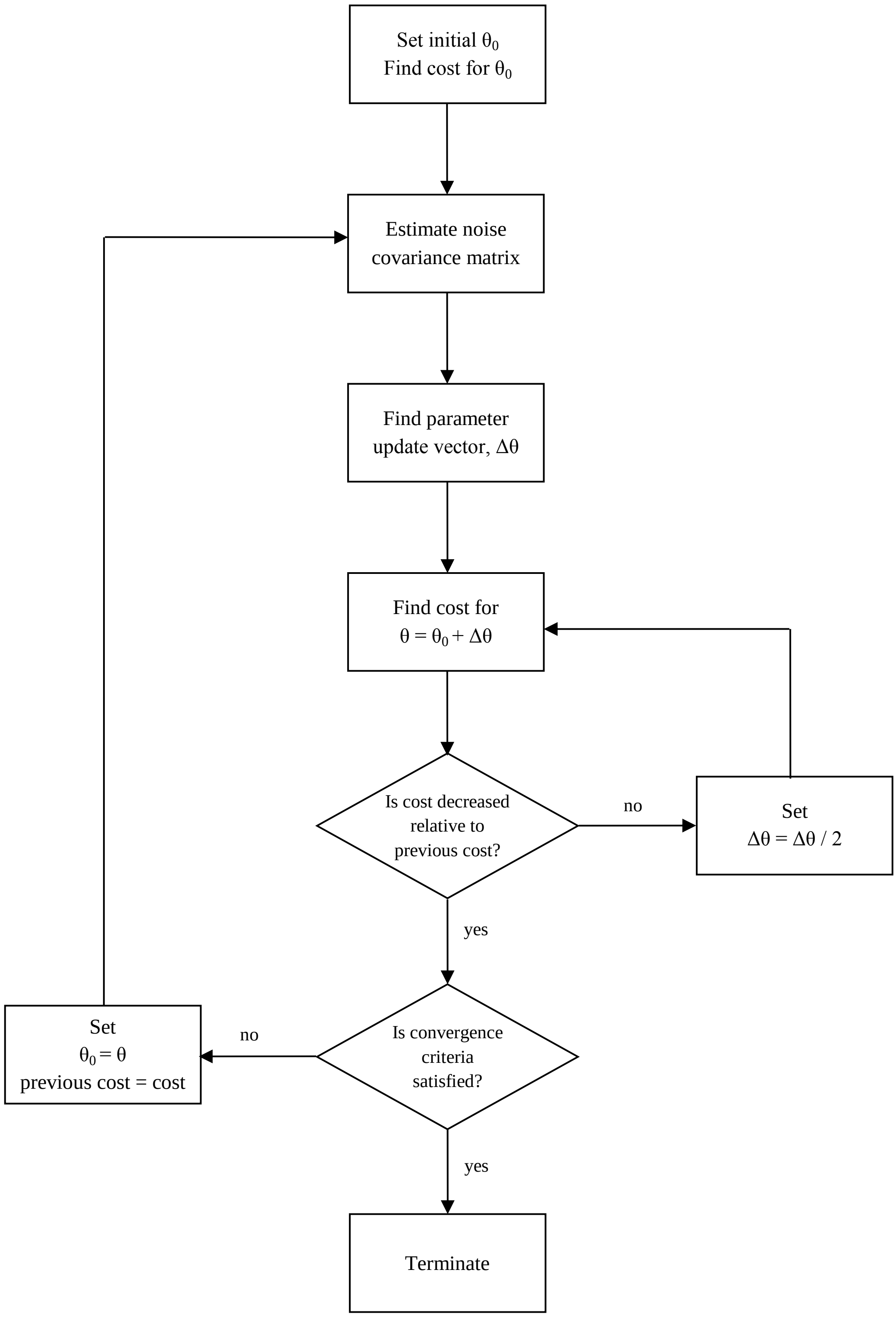

Figure 4.1 - Maximum Likelihood estimation loop

4.1.3. Output Sensitivities

Output sensitivities both in first and second order cost gradients can be found analytically by taking partial derivatives of outputs with respect to unknown parameters. However in the case of nonlinearity involved in the postulated output model analytical calculations might be really complicated. Yet derivatives must be found again even if a minor modeling change in the postulated model occurs. In order to deal with this difficulty, output sensitivities are approximated with numerical differences.

Using central difference approximation for each parameter, j^{th} column of the output sensitivity matrix can be found as:

$$\frac{\partial \underline{y}}{\partial \theta_j} = \frac{\underline{y}(\underline{\theta} + \delta\theta_j \underline{e}^j) - \underline{y}(\underline{\theta} - \delta\theta_j \underline{e}^j)}{2\left|\delta\theta_j\right|} \tag{4.23}$$

$\delta\theta_j$ given below is the perturbation step for the j^{th} parameter. $\underline{e}^j$ denotes a vector with one in the j^{th} element and zeroes elsewhere.

Perturbation size is generally chosen relative to the parameter to be perturbed. 0.1% of the nominal value is found to be a reasonable choice. Also in case of the nominal value is too small to be perturbed with this relative size, perturbation size must be limited with an absolute lower bound.

4.2. System Models

As already stated in previous chapters, wind angles are one of the most important inputs for the estimation process. Aerodynamic response depends strongly on angle of attack and sideslip angle. Since these angles are not measured directly, they must be properly represented in system model. To do so linear body velocities are reconstructed in system model during the iterations and wind angles are computed from those parameters. The assumption for that representation to be true is that there is no wind acting on the system. Therefore flight test must be carried out in steady atmosphere with low level wind condition.

Since the focus is verifying the aerodynamic parameters through IMU measurements only, outputs of the system model used in estimation must be restricted with translational accelerations and angular rates.

Two different system models, namely implicit and explicit models are postulated to be used in optimization of the maximum likelihood function. System models represented here are continuous state equations and are numerically integrated to states with 2^{nd} order Runge-Kutta integration method. Details of the models are given below in this section.

4.2.1. Implicit Model

Angle of attack and sideslip angle are determined implicitly in this model. Response of the system to control surface deflections is obtained using dynamic system equations in pitch and yaw planes whereas roll plane response is obtained by integrating measurements using kinematic equations. Initial states are chosen as unknown with prior values and determined through the optimization routine together with aerodynamic parameters. Bias errors in input and output variables are also introduced as unknown parameters in the estimation.

For aerodynamic parameter estimation studies in the literature -conducted for aircraft mostly- state models used in estimation algorithms are in similar forms [8],[12]. It can be seen that almost every model uses the aerodynamic parameters to be estimated in state equations. So that output of the estimation model becomes more reliable when matched with real measurements. However, that would be a good approach only if there are several measurements to verify the outputs with. In the case of limited number of measurements, integration of states obtained purely from system model lacks robustness in estimation. Since model outputs at any time involve historic data which might have been affected from any modeling errors, observability problems may appear while trying to match them with measurements. Since wind angles which are one of the most valuable information are missing in this study, it is very likely that this model fails.

Input vector is given by:

$$\underline{u} = [\delta_e \quad \delta_r \quad p \quad a_x]^T \tag{4.24}$$

States of the model are defined as:

$$\underline{x} = [q \quad r \quad u \quad v \quad w \quad \phi \quad \theta]^T \tag{4.25}$$

Model outputs in order to match with measurements are:

$$\underline{y} = [a_y \quad a_z \quad q \quad r]^T \tag{4.26}$$

State equations of the postulated model can be shown as follows:

$$\begin{aligned}\dot{q} &= (1 - J_x / J_y)\,pr \\ &\quad + \rho(u^2 + v^2 + w^2)Sl(C_{M_\alpha} \tan^{-1}(w/u) + C_{M_{\delta_e}}\delta_e + C_{M_q} q)/2/J_y\end{aligned} \tag{4.27}$$

$$\begin{aligned}\dot{r} &= (J_x / J_y - 1)\,pq \\ &\quad + \rho(u^2 + v^2 + w^2)Sl(-C_{M_\alpha} \tan^{-1}(v/u) - C_{M_{\delta_e}}\delta_r + C_{M_q} r)/2/J_y\end{aligned} \tag{4.28}$$

$$\dot{u} = -qw + rv - g\sin\theta + a_x \tag{4.29}$$

$$\dot{v} = -ru + pw + g\cos\theta\sin\phi \\ \quad + \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(v/u) + C_{Z_{\delta_e}}\delta_r - C_{Z_q}r)/2/m \tag{4.30}$$

$$\dot{w} = -pv + qu + g\cos\theta\cos\phi \\ \quad + \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(w/u) + C_{Z_{\delta_e}}\delta_e + C_{Z_q}q)/2/m \tag{4.31}$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \tag{4.32}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{4.33}$$

Note that same aerodynamic derivatives are used for both pitch and yaw planes as explained in Section 3.1. Observation equations can be defined using inputs and states at each sample point:

$$a_y = \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(v/u) + C_{Z_{\delta_e}}\delta_r - C_{Z_q}r)/2/m \tag{4.34}$$

$$a_z = \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(w/u) + C_{Z_{\delta_e}}\delta_e + C_{Z_q}q)/2/m \tag{4.35}$$

$$q = q \tag{4.36}$$

$$r = r \tag{4.37}$$

Previously stated disadvantage about error propagation can be clearly seen from equations above. In order to represent the angle of attack and sideslip angle, integrated velocities are used in observation equations. However neither the air flow angles nor those velocities are available independently. This is the difficulty itself of estimating aerodynamic parameters without any air flow angle information. The weighted parts of outputs come from the air flow angle product term and for this reason air flow angle data must be carefully constructed. However this model suffers from sensitivity since the integration in state equations highly depends on the aerodynamic characteristics of model. Any modeling errors or relatively high disturbances at any time in data might affect the whole optimization process.

Bias errors in measurements mentioned before are not included in both state equations and observation equations. Since model is already nonlinear, those errors can be placed as unknown parameters in inputs and outputs instead of using directly in equations to preserve the simplicity of equations. Then, input and output vectors must be represented in the following form:

$$\underline{u} = [\delta_e \quad \delta_r \quad p - b_p \quad a_x - b_{a_x}]^T \tag{4.38}$$

$$\underline{y} = [a_y + b_{a_y} \quad a_z + b_{a_z} \quad q + b_q \quad r + b_r]^T \tag{4.39}$$

Note that control surface deflection measurements are assumed to be true without any bias errors.

Flowchart of this model can be seen in Figure 4.2.

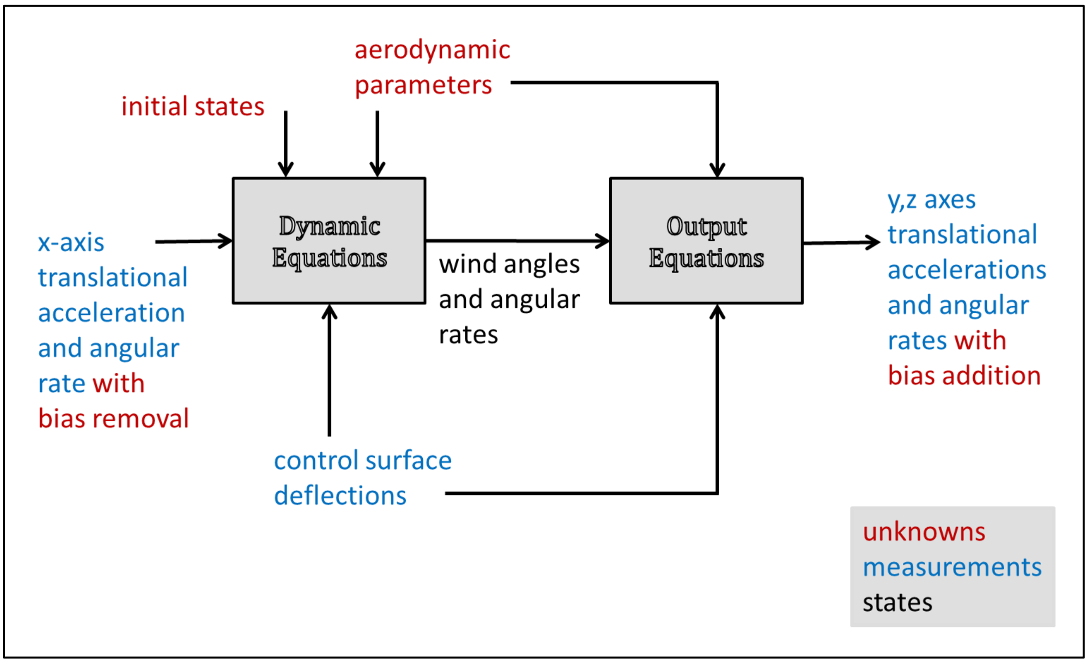

Figure 4.2 - Flow chart of implicit system model

4.2.2. Explicit Model

In order to deal with the air flow angles reconstruction nicely, an alternative approach in model definition is studied. Rather than obtaining the angle of attack and sideslip angle by using pure response of parameters to be estimated, response of the system from measurements is used. In other words kinematic equations are preferred instead of dynamic equations. So that angle of attack and sideslip angle are explicitly evaluated from input data and take place in observation equations. Again initial states are also chosen as unknown. Input and output biases for this model are more important and have more impact on estimation results than before.

Inputs are control surface deflections, translational accelerations and angular rates:

$$\underline{u} = [\delta_e \quad \delta_r \quad p - b_p \quad q - b_q \quad r - b_r \quad a_x - b_{a_x} \quad a_y - b_{a_y} \quad a_z - b_{a_z}]^T \tag{4.40}$$

IMU measurements used for verifying the model outputs are also used as inputs in this model. The reason for this approach is to carry some information with input data and decouple the effects of control surface inputs and body motion in outputs.

States of the model are linear body velocities and Euler angles:

$$\underline{x} = [u \quad v \quad w \quad \phi \quad \theta]^T \tag{4.41}$$

Model outputs are also different from the previous model:

$$\underline{y} = [a_y + b_{a_y} \quad a_z + b_{a_z} \quad \dot{q} \quad \dot{r}]^T \tag{4.42}$$

In order to increase convergence and gain robustness account for estimating moment coefficients, it is more appropriate to use angular accelerations, derivatives of angular rates, as observation variables. This is evident from the fact that equations of the derivatives themselves are directly related with aerodynamic parameters. Using the integrated data to match with the measurement, involves propagation of errors from both minor model differences and possible flight disturbances. This creates similar problems mentioned for implicit model.

Only problem of using angular accelerations as measurements is taking derivative of noisy measurements. Since angular accelerations are not directly measured, these variables can only be obtained by differentiating gyro measurements. This is generally handled by applying a digital filter [8] before differentiation.

Note that bias terms are not included in angular accelerations because they are automatically eliminated in derivatives of gyro measurements.

State equations of the model can be defined as:

$$\dot{u} = -qw + rv - g\sin\theta + a_x \tag{4.43}$$

$$\dot{v} = -ru + pw + g\cos\theta\sin\phi + a_y \tag{4.44}$$

$$\dot{w} = -pv + qu + g\cos\theta\cos\phi + a_z \tag{4.45}$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \tag{4.46}$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{4.47}$$

Same observation equations are used as before:

$$a_y = \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(v/u) + C_{Z_{\delta_e}}\delta_r - C_{Z_q}r)/2/m \tag{4.48}$$

$$a_z = \rho(u^2 + v^2 + w^2)S(C_{Z_\alpha}\tan^{-1}(w/u) + C_{Z_{\delta_e}}\delta_e + C_{Z_q}q)/2/m \tag{4.49}$$

$$\dot{q} = (1 - J_x / J_y) pr + \rho(u^2 + v^2 + w^2) Sl(C_{M_\alpha} \tan^{-1}(w/u) + C_{M_{\delta_e}} \delta_e + C_{M_q} q) / 2 / J_y \tag{4.50}$$

$$\dot{r} = (J_x / J_y - 1) pq + \rho(u^2 + v^2 + w^2) Sl(-C_{M_\alpha} \tan^{-1}(v/u) - C_{M_{\delta_e}} \delta_r + C_{M_q} r) / 2 / J_y \tag{4.51}$$

The major difference of this model lies in the way the velocities are handled. Instead of using the aerodynamic characteristics of model alone in order to obtain velocities, system response is included in the equations. Now even if there is a behavior which is not included in aerodynamic models above, that behavior sneaks into velocities through both translational accelerations and angular rates measurements. So there wouldn't be any accumulative modeling errors in velocities which are used to represent air flow angles.

The challenge in explicit model comes from a different side unfortunately. Instead of accumulative modeling errors, this model suffers from measurement errors since only measurements are used in state equations. In practice accelerometer and gyro sensors contains bias errors which causes drift in data due to the integration of these measurements. Using the measured accelerometer and gyro data without any correction causes improper representation of states of the model. In order to prevent this, bias errors are introduced for all sensor measurements. So that, with this model not only the aerodynamic parameters are estimated but also other variables used in observation equations are intended to be properly reconstructed from measured gyro and accelerometer data. Now aerodynamic parameters which are the main interest of the estimation process, depend only on observation equations. This fact significantly increases the convergence of the optimization routine at the cost of a risk to decrease the identifiability of the true model parameters.

Flowchart of this model can be seen in Figure 4.3. It can be seen that now aerodynamic parameters are only used in output equations and possible errors in postulated aerodynamic models are not integrated.

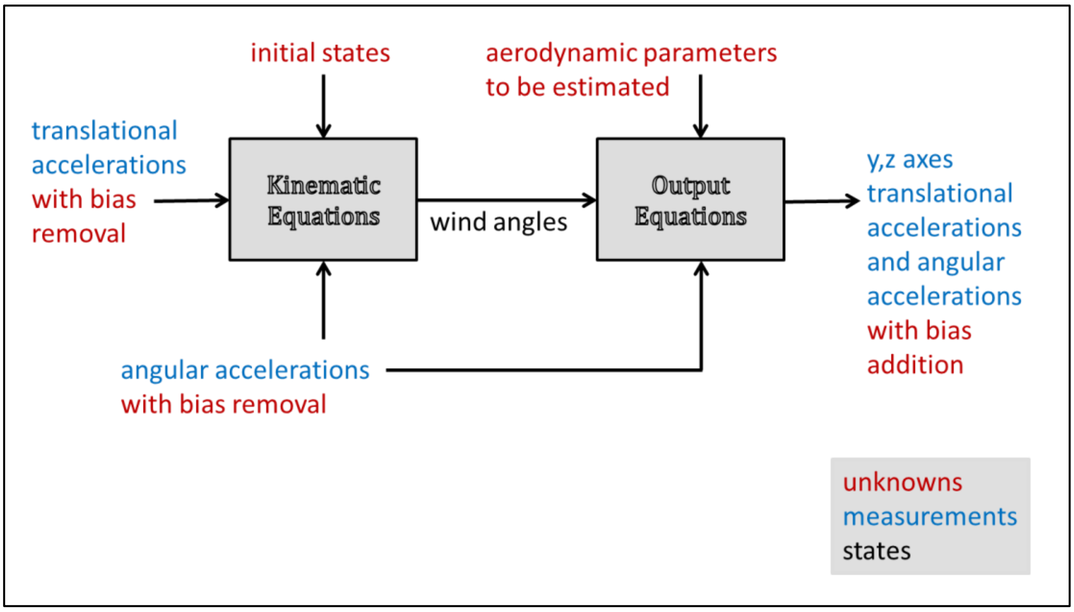

Figure 4.3 - Flowchart of explicit system model

CHAPTER 5

ESTIMATION RESULTS

5.1. Sample Test Case

Simulated response of Javelin ATGM is used for testing the estimation algorithm with both implicit and explicit system models. In the estimation algorithm, elevator and rudder deflections and IMU measurements obtained from simulation are used only. Control surface deflections are used as true values without including any biases or noises. IMU measurements are generated within simulation by correlating translational accelerations and angular rates with biases and zero mean white Gaussian noise sequences. As stated earlier, initial values of states are also assumed to be unknown and included in unknown parameter vector. Aerodynamic parameters, initial states, and bias values are all treated as unknowns and included in the unknown parameter vector in the estimation algorithm. Initial values of these parameters are randomly selected with realistic uncertainties. True values together with selected initial values are given in Table 5.1. Same measurements with same initial values are used to test the implicit and explicit models accordingly.

Table 5.1 - True values and initial estimates used in sample test case

Parameters	**True values**	**Initial value for estimation**
Cza	-18.628	-17.952
Czd	-2.611	-2.135
Czq	-0.048	-0.058
Cma	-31.031	-31.317
Cmd	-12.246	-9.906
Cmq	-0.121	-0.142
q_0	-3.98 d/s	-4.25 d/s
r_0	0 d/s	0.21 d/s
u_0	138.81 m/s	142.88 m/s
v_0	0 m/s	0 m/s
w_0	0.036 m/s	0 m/s
ϕ_0	0 d	0 d
θ_0	8.67 d	6.45 d
a_x bias	-0.266 m/s^2	0 m/s^2
a_y bias	-0.411 m/s^2	0 m/s^2
a_z bias	-0.131 m/s^2	0 m/s^2
p bias	-0.096 d/s	0 d/s
q bias	0.047 d/s	0 d/s
r bias	0.020 d/s	0 d/s

Outputs of implicit and explicit models with defined initial set of unknown parameters are given in Figure 5.1 and Figure 5.2 respectively. The difference between the two models can be clearly seen from the graphs. In implicit model, there are both amplitude and phase differences between the measurements and model outputs. This is typical because states are integrated using the unknown parameters which have incorrect initial values. In explicit model there is no phase error on the other hand. Since IMU measurements are integrated, phases of outputs are matched with measurements without any difference. Obviously amplitude difference still exists due to incorrect usage of aerodynamic parameters. However this time there are also drift errors in outputs due to IMU biases. All of those errors appeared in two models are intended to be eliminated within the estimation algorithm. Model outputs with final values obtained from estimation algorithm are shown in Figure 5.3 and Figure 5.4. Results with both implicit and explicit system models are presented in Table 5.2. Relative errors from reference aerodynamic parameters are also included here. It is observed from the results that estimated values are more accurate for explicit model.

Table 5.2 - Estimated values with implicit and explicit models

Parameters	**True values**	**Final estimates with implicit model**	**Final estimates with explicit model**
Cza	-18.628	-19.541 (%4.9)	-18.806 (%1.0)
Czd	-2.611	-2.986 (%14.4)	-2.647 (%1.4)
Czq	-0.048	-0.060 (%25.0)	-0.049 (%2.1)
Cma	-31.031	-32.473 (%4.6)	-31.304 (%0.9)
Cmd	-12.246	-12.850 (%4.9)	-12.344 (%0.9)
Cmq	-0.121	-0.126 (%4.1)	-0.122 (%0.8)
q_0	-3.98 d/s	-4.00 d/s	NA[a]
r_0	0 d/s	0.31 d/s	NA
u_0	138.81 m/s	136.21 m/s	138.425 m/s
v_0	0 m/s	0.00 m/s	-0.004 m/s
w_0	0.036 m/s	0.03 m/s	0.028 m/s
ϕ_0	0 d	3.90 d	0.169 d
θ_0	8.67 d	9.83 d	8.743 d
a_x bias	-0.266 m/s^2	-0.344 m/s^2	-0.240 m/s^2
a_y bias	-0.411 m/s^2	-0.408 m/s^2	-0.411 m/s^2
a_z bias	-0.131 m/s^2	-0.136 m/s^2	-0.132 m/s^2
p bias	-0.096 d/s	-0.290 d/s	-0.133 d/s
q bias	0.047 d/s	0.129 d/s	0.064 d/s
r bias	0.020 d/s	-0.342 d/s	0.007 d/s

[a] These parameters are not included in unknown parameter vector of this model

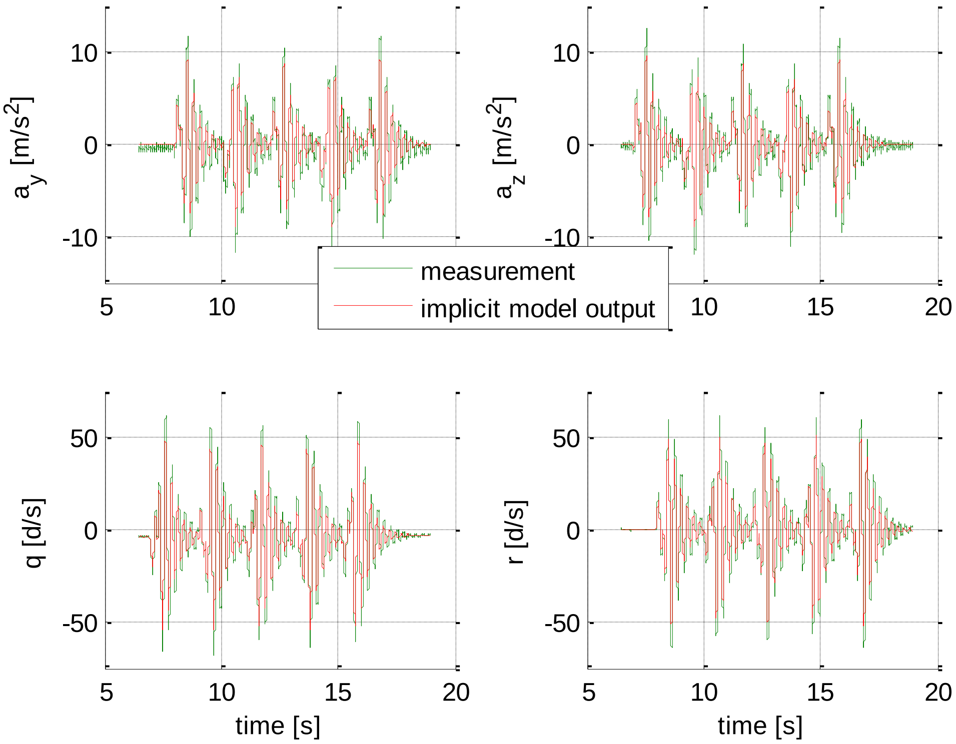

Figure 5.1 - Implicit model response with initial unknowns

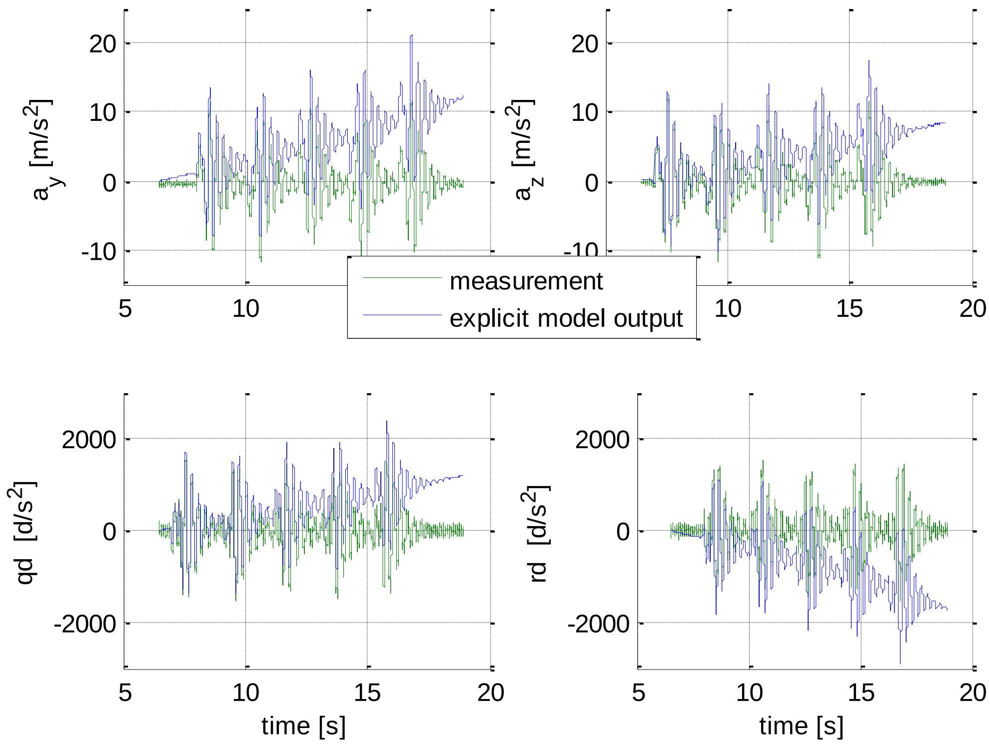

Figure 5.2 - Explicit model response with initial unknowns

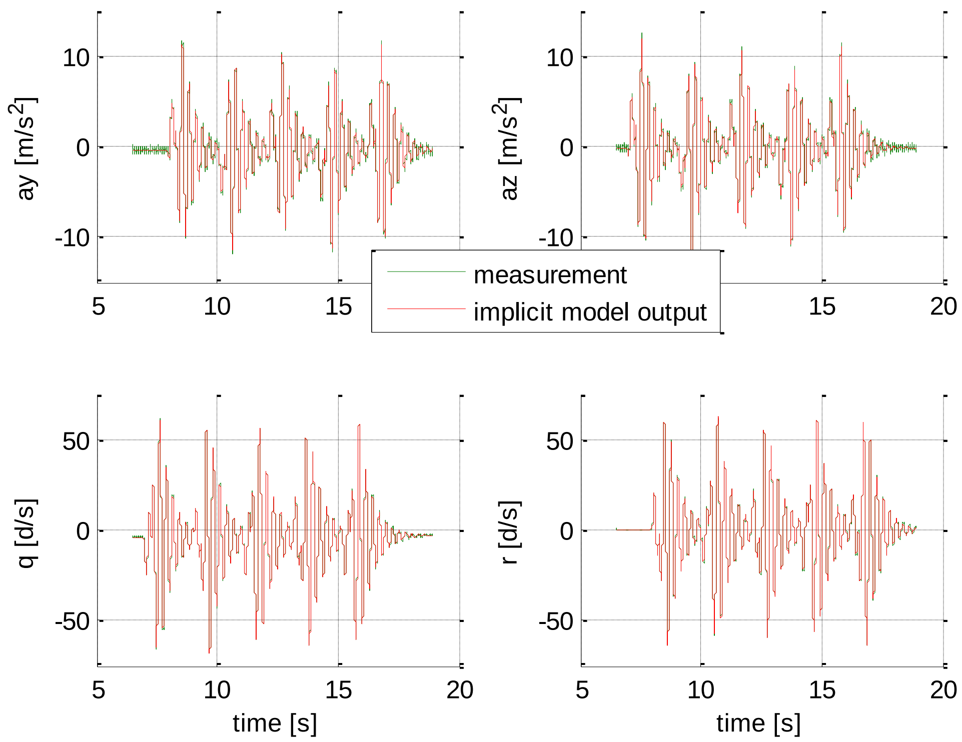

Figure 5.3 - Implicit model response with final estimates

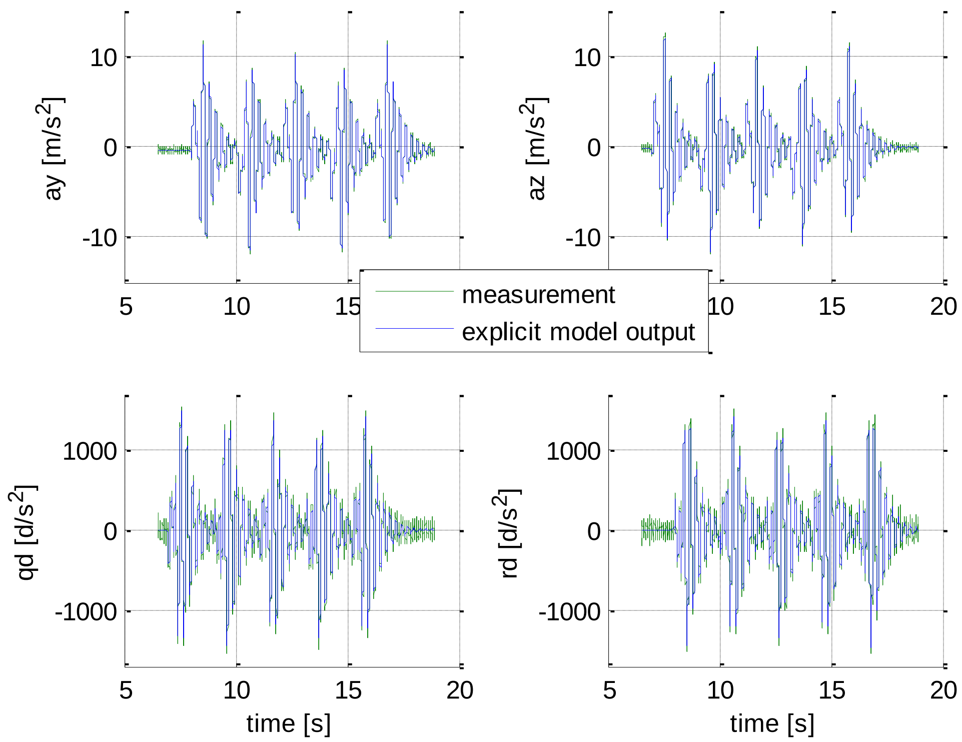

Figure 5.4 - Explicit model response with final estimates

It can be seen from final estimation graphs that outputs of both models are successfully matched with measurements without any visible error. Residual plots of translational accelerations given in Figure 5.5 prove that as well. Errors in both models are in similar noise levels without any deterministic behavior.

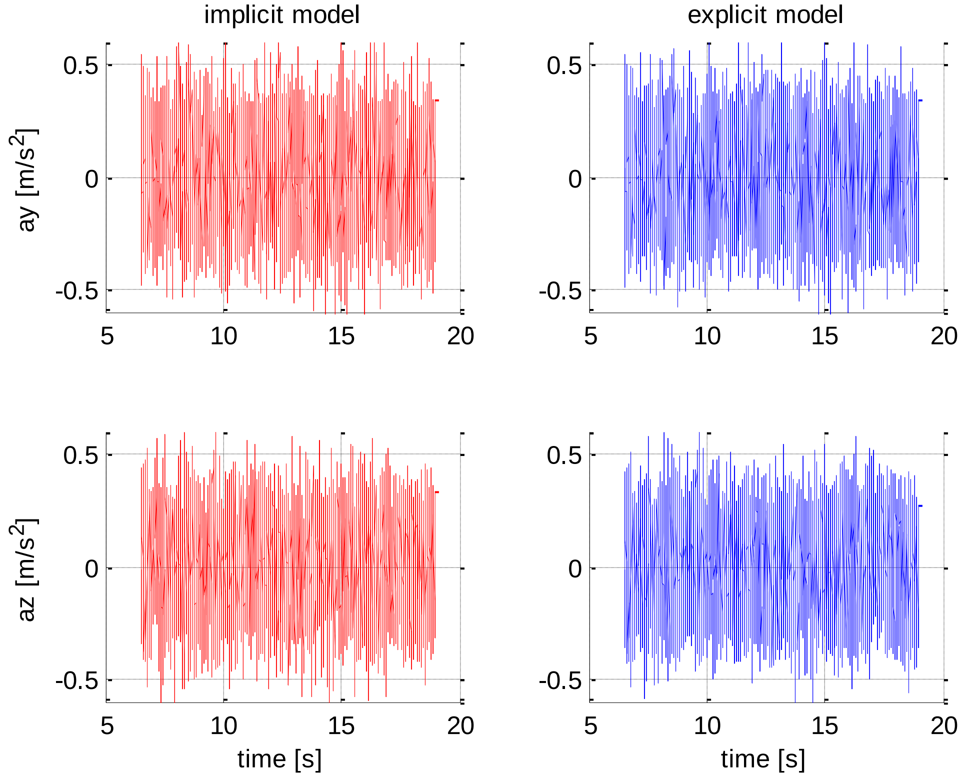

Figure 5.5 - Translational acceleration errors of implicit and explicit models

Correlation coefficients higher than 0.9 in implicit model results are given in Table 5.3. Aerodynamic parameter estimates obtained with implicit model appear to be highly correlated with each other. This fact reduces the reliability of implicit model significantly. Note that correlation coefficients for explicit model are all below 0.9 and hence not given here. This indicates that parameters are not significantly correlated with each other.

Table 5.3 - Correlations higher than 0.9 in implicit model results

	u_0	θ_0	Cz_α	Cm_α	Cm_δ
u_0	1	0,908	0,967	0,999	0,992
θ_0	0,908	1	0,865	0,893	0,885
Cz_α	0,967	0,865	1	0,968	0,956
Cm_α	0,999	0,893	0,968	1	0,993
Cm_δ	0,992	0,885	0,956	0,993	1

This problem can also be verified from the residual plots given in Figure 5.6. Errors of wind angles reconstructed within the estimation process from real simulation data are presented here. Even though errors are relatively small, explicit model appears to match better the real data. Yet the essential difference is how the errors appear for each model. Residuals are in random noise level for explicit model. In implicit model however, there is a deterministic behavior that couldn't be caught in all data. This is undesirable for reliable results.

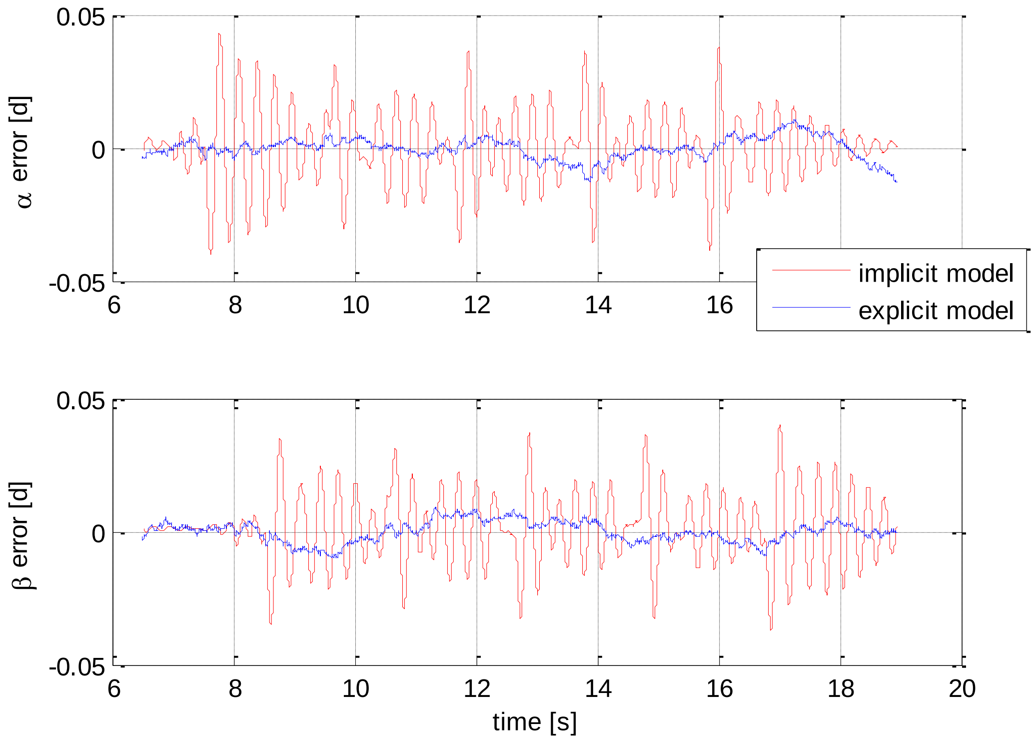

Figure 5.6 - Wind angle errors of implicit and explicit models

Convergence plots of aerodynamic parameters are also given in Figure 5.7. Updates indicated by cross markers are given with one sigma error bands obtained from Dispersion matrix. Estimation algorithm is converged within 4 iterations only with explicit model for this sample test case. However it takes additional 6 steps to converge with implicit model.

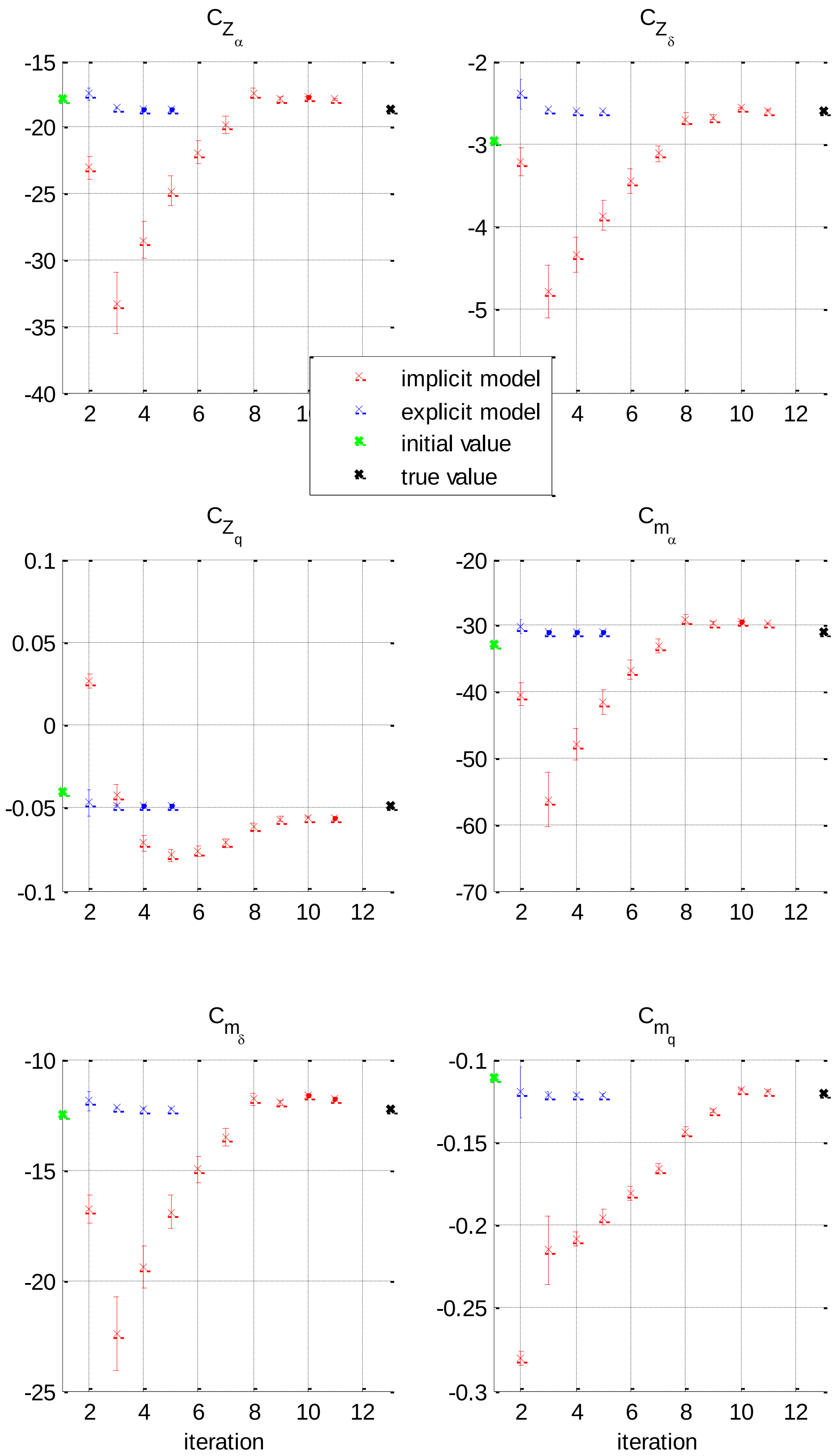

Figure 5.7 - Update of aerodynamic parameters in sample case

To sum up, implicit model within maximum likelihood estimation gives biased results due to the correlations of unknown parameters. Explicit system model on the other hand, seems to provide accurate and reliable results for the sample test case.

However, an important fact was also realized during the study. In order for explicit model to be successful, it is really important not to restrict aerodynamic parameter estimation in one plane of the system studied on. In other words it would be much better that estimation algorithm uses aerodynamic models of both pitch and yaw planes so that wind angles reconstruction becomes more reliable. This can be demonstrated with same test case analyzed above. This time however, estimation algorithm with explicit model is applied to pitch plane response alone. State equations in explicit system model remain same. Translational acceleration at Z axis and angular acceleration at Y axis are used as model outputs only to match with measurements, so that output vector given in Equation (4.42) becomes:

$$\underline{y} = [a_z + b_{a_z} \quad \dot{q}]^T \tag{5.1}$$

Unlike before, local divergences occur during the optimization. These divergences are handled by halving the parameter changes as explained in the previous chapter. In addition, it takes 10 steps to converge while it takes only 4 steps in estimation with both pitch and yaw planes. The results are given in Table 5.4. Even though aerodynamic parameters are still good enough, other parameters contains high errors.

Table 5.4 - Estimated values with explicit model

Parameters	**True values**	**Both pitch and yaw plane responses**	**Pitch plane response only**
Cza	-18.628	-18.806 (%1.0)	-18.382 (%1.3)
Czd	-2.611	-2.647 (%1.4)	-2.589 (%0.8)
Czq	-0.048	-0.049 (%2.1)	-0.049 (%2.1)
Cma	-31.031	-31.304 (%0.9)	-30.609 (%1.4)
Cmd	-12.246	-12.344 (%0.9)	-12.072 (%1.4)
Cmq	-0.121	-0.122 (%0.8)	-0.121 (%0.0)
u_0	138.81 m/s	138.425 m/s	140.48 m/s
v_0	0 m/s	-0.004 m/s	6.033 m/s
w_0	0.036 m/s	0.028 m/s	0.037 m/s
ϕ_0	0 d	0.169 d	-21.405 d
θ_0	8.67 d	8.743 d	7.265 d
a_x bias	-0.266 m/s^2	-0.240 m/s^2	0.080 m/s^2
a_y bias	-0.411 m/s^2	-0.411 m/s^2	3.402 m/s^2
a_z bias	-0.131 m/s^2	-0.132 m/s^2	-0.133 m/s^2
p bias	-0.096 d/s	-0.133 d/s	-0.697 d/s
q bias	0.047 d/s	0.064 d/s	-0.277 d/s
r bias	0.020 d/s	0.007 d/s	3.100 d/s

Once again, the real focus here is to estimate aerodynamic parameters and others are irrelevant for this study. That means accuracies of other parameters are not important. The real problem is on the other hand, correlations of aerodynamic parameter estimations. Aerodynamic parameters are now highly correlated (Table 5.5). This reduces the reliability of estimations even if the estimated values seem to be true. In the presence of more disturbances and modeling errors, these correlations might cause biased estimations or even total failure in convergence.

Table 5.5 - Correlations higher than 0.9 in pitch plane explicit model results

	u_0	a_y bias	Cz_α	Cz_δ	Cm_α	Cm_δ
u_0	1	0,967	0,989	0,938	0,984	0,955
a_y bias	0,967	1	0,958	0,911	0,953	0,926
Cz_α	0,989	0,958	1	0,953	0,993	0,963
Cz_δ	0,938	0,911	0,953	1	0,942	0,915
Cm_α	0,984	0,953	0,993	0,942	1	0,971
Cm_δ	0,955	0,926	0,963	0,915	0,971	1

5.2. Monte-Carlo Analysis

Although it appears that explicit system model promises accurate and reliable results, it is important also to show that this is still true when bias values change or estimation is started from different initial values. These parameters are randomly changed according to the criteria given in Table 5.6 and estimation is repeated every time.

Note that since the system is excited in open loop, response of the system is independent of IMU bias values and therefore same simulation outputs are used by changing measurement errors only in each run. Bias values presented in Table 5.6 are typical errors for a commercial IMU. Initial estimates of these bias values are taken to be zero at each run. Initial values of states u and θ are assumed to be obtained by integrating IMU measurements from launch of the missile up to the excitations. Because of the accumulated errors in integrated states, initial values of these states are selected with an error from true value with intervals of ±10m/s and ±5d. Since excitations are started to be applied when missile is in ballistic trajectory, initial values of ϕ, v and w are selected as zero with zero wind angles approximation. Lastly, initial values of angular rates q and r are selected by including an error interval to true values in bias error amplitudes.

Table 5.6 - True values and initial errors of unknown parameters

Parameters	**True values**	**Initial errors for estimation**
Cza	-18.628	%20 [b]
Czd	-2.611	%20
Czq	-0.048	%20
Cma	-31.031	%20
Cmd	-12.246	%20
Cmq	-0.121	%20
q_0	-3.98 d/s	±0.5 d/s [c]
r_0	0 d/s	±0.5 d/s
u_0	138.81 m/s	±10 m/s
v_0	0 m/s	=0 [d]
w_0	0.036 m/s	=0
ϕ_0	0 d	=0
θ_0	8.67 d	±5 d
a_x bias	±0.5 m/s^2	=0
a_y bias	±0.5 m/s^2	=0
a_z bias	±0.5 m/s^2	=0
p bias	±0.5 d/s	=0
q bias	±0.5 d/s	=0
r bias	±0.5 d/s	=0

Histogram plots of 200 Monte-Carlo runs are presented in Figure 5.8. Using explicit model in estimation algorithm provides accurate and reliable results within less than %3 error bands. However estimation algorithm fails to converge in nearly %25 of total runs with implicit model and does not give any results at all. In addition, the results are biased when the algorithm converges. Unfortunately the estimations are always correlated as detailed above which simply means that implicit model is not reliable.

[b] Relative error band
[c] Absolute error band
[d] Exact value

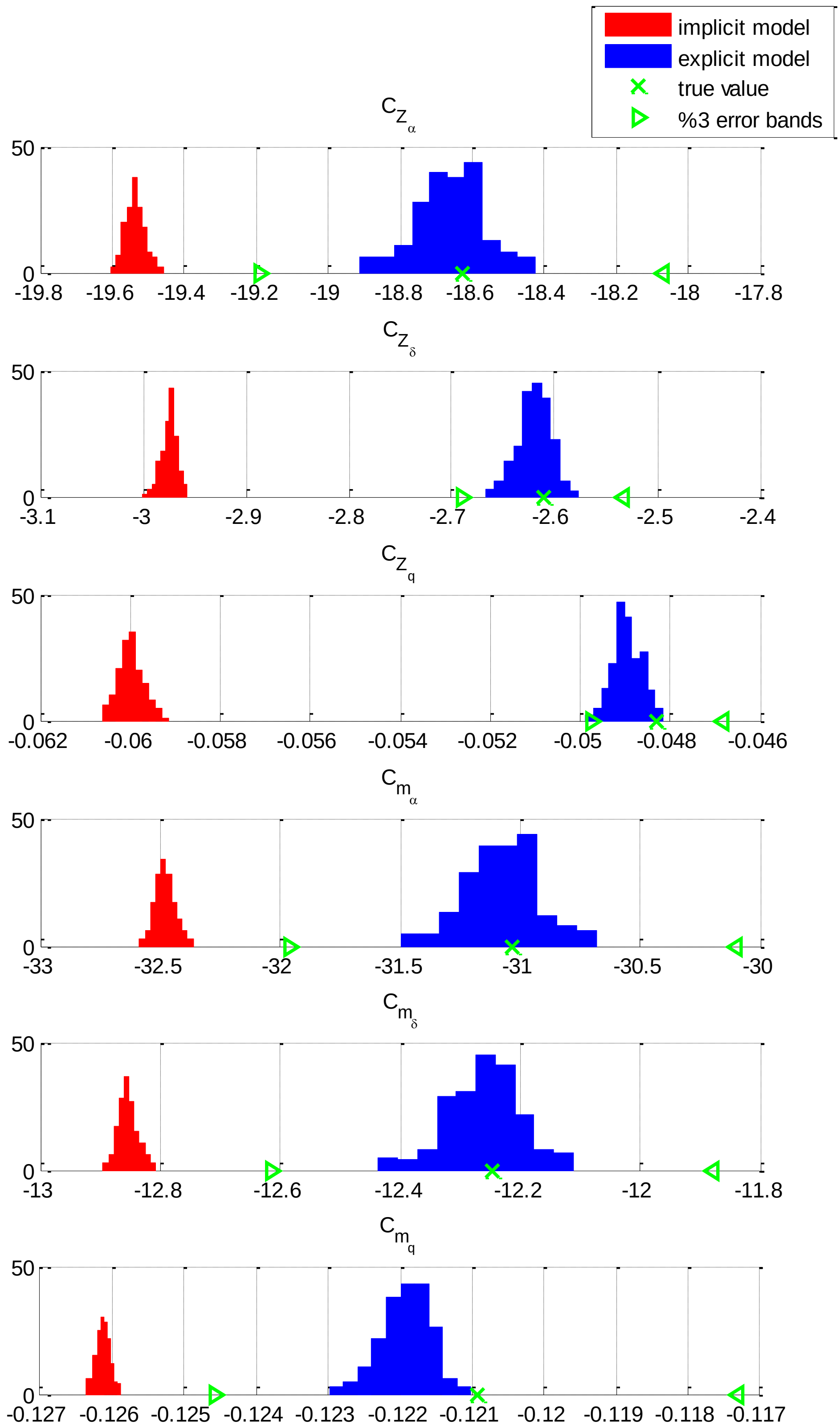

Figure 5.8 - Monte-Carlo results of implicit and explicit models

5.3. Real Flight Test

Suggested approach with explicit system model is also tested on real flight test data of a surface to surface missile that is researched and developed in Roketsan Missiles Industries. Flight test was designed specifically for aerodynamic parameter estimation. Square wave inputs with modal frequency which was determined prior to test relying on the wind tunnel tests were applied to control surfaces to excite missile in pitch and yaw planes while the control system was in open loop. Inputs were chosen to provide enough signal to noise ratio and also to keep missile close to reference flight condition in order to ensure that system stays in the linear region of the aerodynamic model.

The convergence plots of the estimation procedure are given in Figure 5.9. Plots are scaled independently according to the final estimated values so that final values of results appear as one and other values present the relative errors. Initial values which are indicated by red points in plots are selected from wind tunnel database. Updates indicated by blue points are given with one sigma error bands. As in the simulation results, estimated values goes in the destination without shifting much and converges to the global minimum of the cost function. Correlation coefficients are also checked and it is seen that all values are below 0.9 which indicates that there is not a serious correlation issue.

Match between model outputs and measurements can be seen from Figure 5.10 with a scaled view. Note that angular accelerations given as measurements are locally smoothed derivatives of IMU angular rate outputs. It is observed that navigation errors are successfully corrected during the estimation without any other information except IMU measurements. Errors of model outputs from measurements are also plotted in Figure 5.11. These errors are typical due to flight disturbances and minor model differences in pitch and yaw planes.

As studied in simulation, same aerodynamic derivatives in pitch and yaw planes are used in estimation model. Even though the missile tested here is symmetric in those planes, independent aerodynamic parameters may also be included in estimation model. Since there are slight offset errors due to the production in both control surfaces and mid-body wings, using different aerodynamic parameters in pitch and yaw planes might result better matches of model outputs with measurements. Similarly constant terms may also be included in the aerodynamic model to correct the bias errors between measurements and model outputs. This error can be seen from yaw angular acceleration plot of estimated model in Figure 4.15. Postulated model doesn't involve any constant aerodynamic terms as it should be however it can be clearly seen that there is constant moment acting on the system in yaw plane originated from the offset errors just mentioned.

Since the main focus of the estimation is verifying the aerodynamic model used in simulations, it is more appropriate to use the model as it is and searching for the best values of its parameters. Therefore minor modeling errors such as the ones mentioned above are neglected in this study since they don't risk the convergence of unknown parameters.

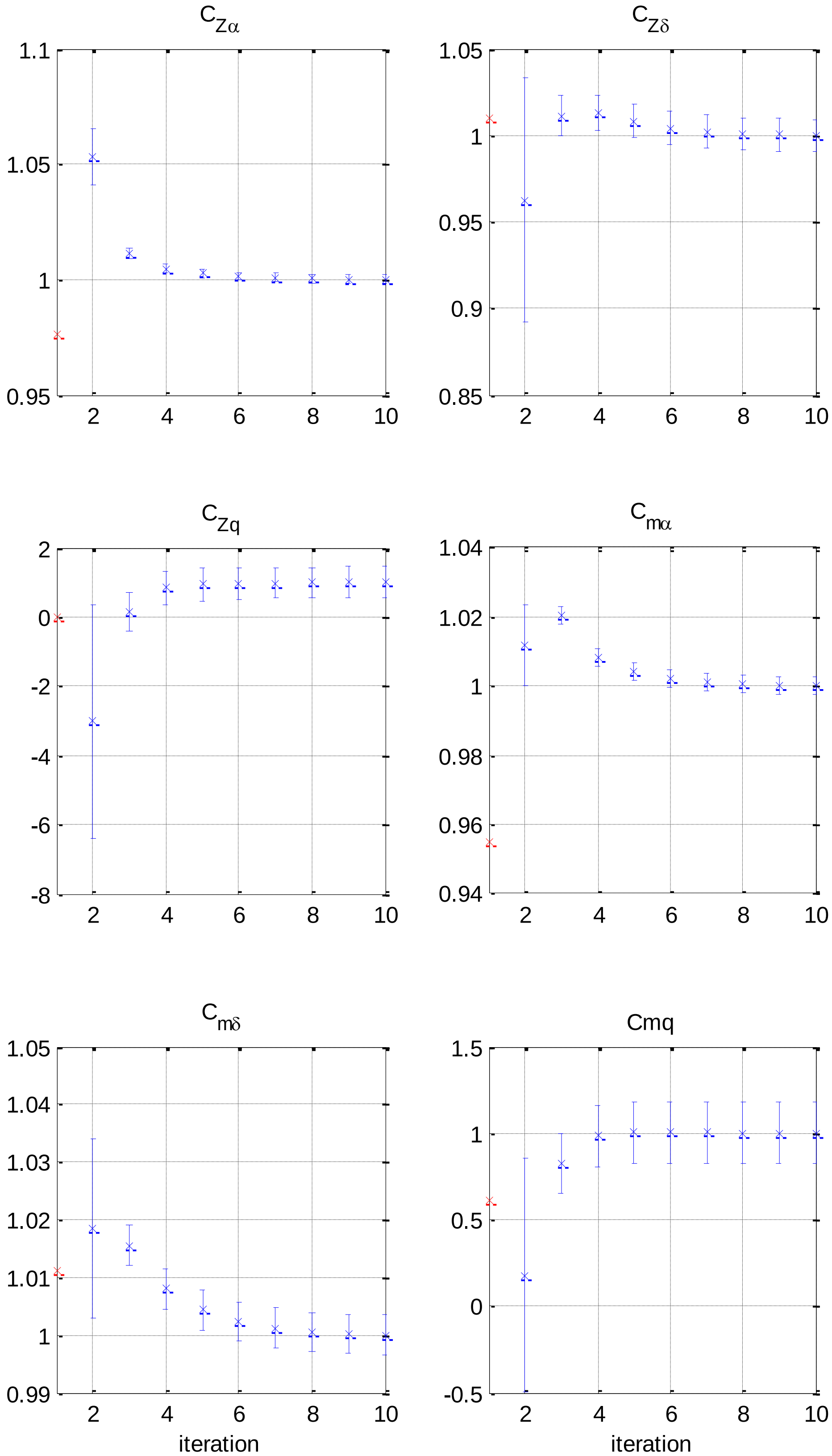

Figure 5.9 - Convergence plots of flight test estimation

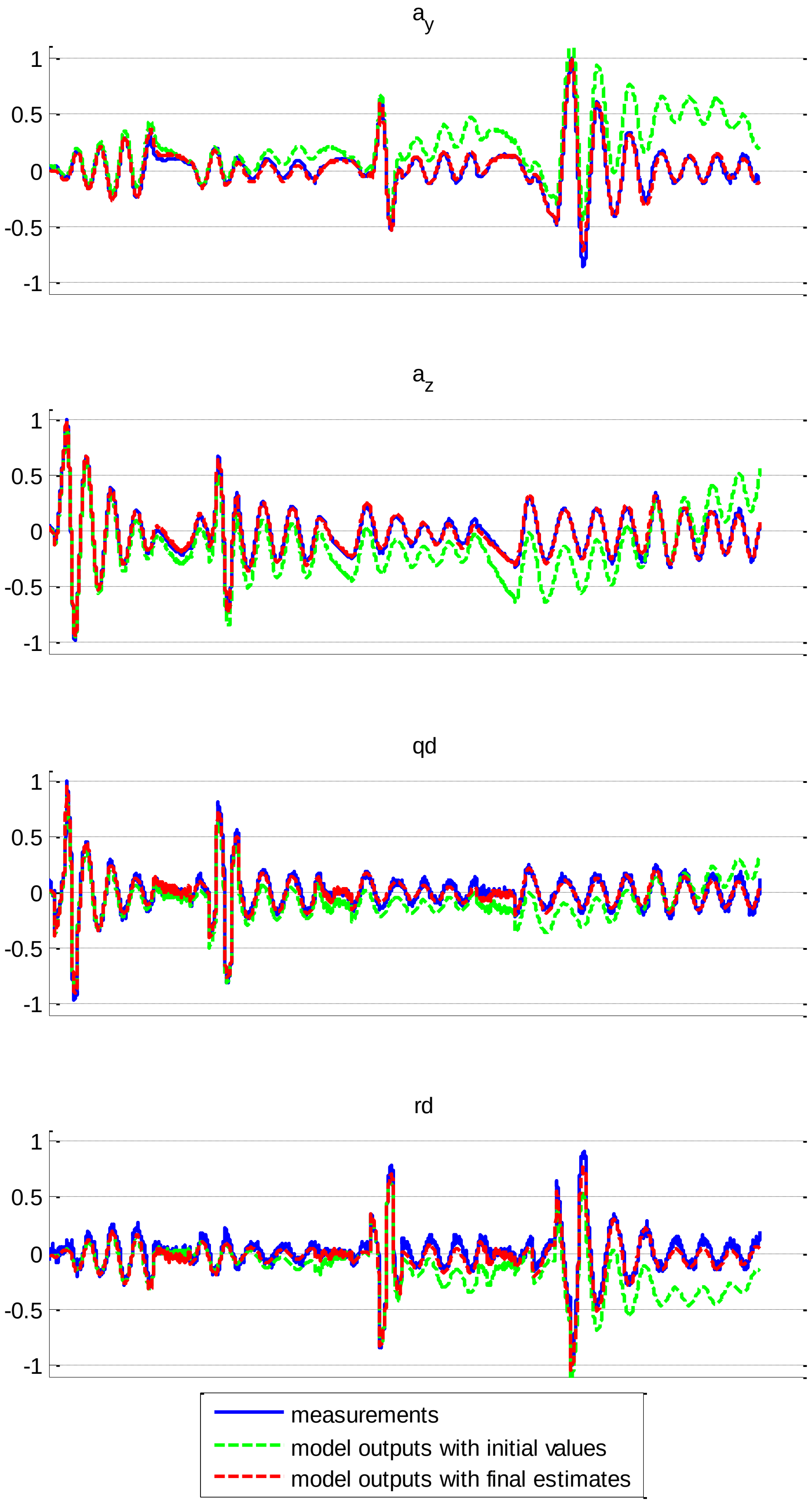

Figure 5.10 - Comparisons of flight test measurements and model outputs

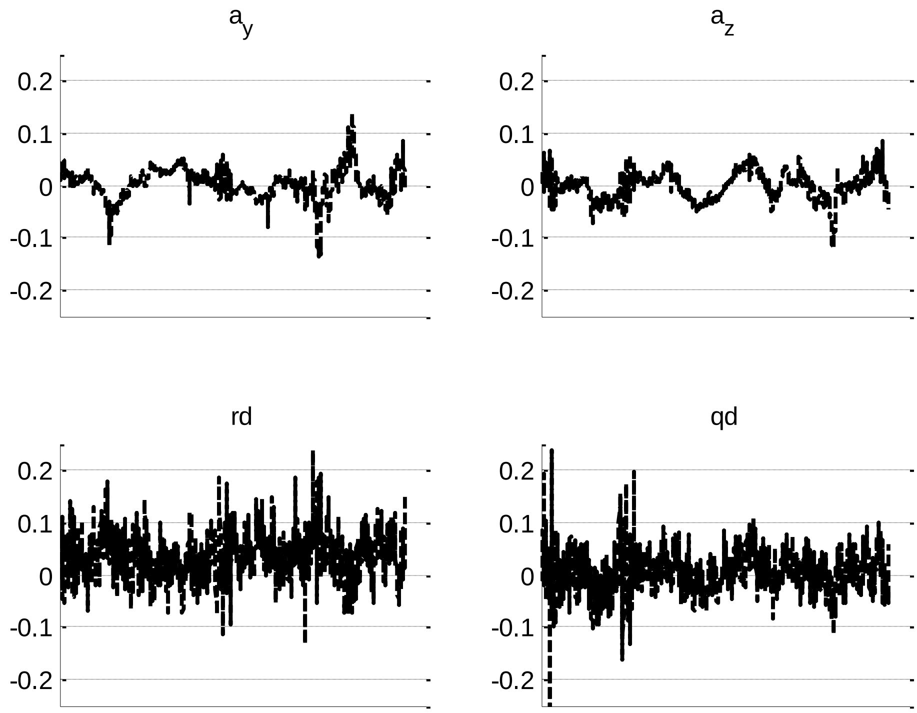

Figure 5.11 - Errors from measurements

It was also observed that estimation algorithm using the implicit model fails to converge and doesn't match the measurements as it should. This is obviously because of the modeling errors involved in the estimation model. Considering that implicit model fails to give accurate and reliable results even in simulations, this failure is not a surprise.

CHAPTER 6

CONCLUSIONS

Estimating aerodynamic parameters of flight vehicles from real tests has always been a great interest. There are numerous studies that focuses on this subject in the literature. However only a limited number of references exist for missiles compared to aircraft. The natural problem that arises while estimating aerodynamic parameters of missiles is lack of wind angles measurements. This problem is usually handled by calculating wind angles from flight test measurements during post processing prior to the estimation study. When only IMU measurements are available for the response of the system, post processed data is subjected to drift errors because of biased measurements. This brings a need of alternative approach for estimating aerodynamic parameters when wind angles measurements are not available.

In this thesis aerodynamic parameter estimation of a missile in the absence of wind angles measurements is studied in detail. Output error method is utilized for the estimation of aerodynamic parameters. Two different system models are proposed to be used in this method. Without using any post process calculations, wind angles are determined within the estimation. As for the measurements; translational accelerations, angular rates and control surface deflections are used.

First model which is named implicit model uses the equations of motion of the system as state equations. In other words unknown aerodynamic parameters are used in state equations. Outputs of the model are selected as translational accelerations and angular rates. Using this model in output-error method is like simulating the response of the system with current values of unknown parameters during the iterations. In fact output-error method is generally preferred to be able to use the aerodynamic parameters in state equations for more reliable results [4]. Therefore implicit system model is the very first approach that naturally comes to mind.

Unfortunately, it is demonstrated with simulated data that aerodynamic parameters are simply not observable with this approach. Estimation results appear –when algorithm converges– to be biased and highly correlated with each other. This is a typical problem when the model structure includes either too many terms or too few terms relative to the information content in the data [17]. In this case there are too many unknowns with limited information about response. This approach is also tested with data of a real flight test which is designed specifically for estimation. As a proof of simulation study, estimation algorithm doesn’t converge to any results in real flight test. If wind angles measurements were available, this might have not happened [12] or alternatively the problem could be easily eliminated by using those measurements as inputs in state equations [8].

In the second system model which is named explicit model, state equations are made free of unknown system characteristics. Translational accelerations and angular accelerations, which are obtained from locally smoothed derivatives of angular rates, are used as inputs and state equations are built from kinematic equations. Bias values of measurements are used as unknowns in state equations in order to be able to eliminate drift errors. Aerodynamic parameters are now included in output equations only. Thus this approach can be thought as a nonlinear equation-error method instead. If translational accelerations and angular rates measurements were somehow bias free, the easiest way of estimating aerodynamic parameters would be integrating those measurements to obtain all the states (with no wind assumption) and then applying a least square fit which is known as equation-error method. However measurement bias cause accumulated errors when integrated, which cannot be corrected easily when the exact bias value is not known. Using proposed explicit system model with output-error method provides a way of correcting those errors iteratively. This is online reconstruction of states during the estimation. Then in the last step, method gives the maximum likelihood estimates using reconstructed states, which are essentially the same as least square estimates [8].

Trials with simulation data show that second approach, estimation with explicit system model, provides reliable and accurate results. In addition, algorithm is applied to real flight test data with success. The results of this study demonstrate that with this suggested approach aerodynamic parameter estimation can be accurately done without measurements of wind angles. However an important fact must be emphasized here again. This approach is meant to work only when the system is excited in both pitch and yaw planes. Otherwise online reconstruction mentioned above fails to give accurate results. This is demonstrated with simulated data by applying the estimation in pitch plane only.

Studied missile in this thesis is symmetric in pitch and yaw planes therefore it has the same aerodynamic model in both planes. This is clearly a benefit for estimation with limited measurements. Yet there is opportunity to apply the suggested approach to asymmetric systems. In Appendix A, suggested approach is tested with real flight data of a research aircraft without going too much detail. It is apparent from the results that it might be possible to accurately estimate aerodynamic parameters of aircraft with this approach suggested here, as well.

The first assumption made in this study is no wind condition. Even though wind is excluded in this study, suggested approach would still work in the case of a steady wind. Including additional constant terms in aerodynamic model might help algorithm to model the effect of wind in these terms. Nevertheless, this should be tested in detail as a future work. In addition, an effort should be directed to apply this approach with a state filter to eliminate flight disturbances. This would be necessary to obtain more accurate estimations in the case of turbulent atmosphere.

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APPENDIX A

AIRCRAFT APPLICATION

In this thesis, estimation of a symmetric missile is studied in detail and suggested approach is verified with real flight test data of another missile which is also symmetric in pitch and yaw planes. Having the same aerodynamic model in pitch and yaw planes is clearly a benefit for estimation with limited measurements. It is observed that wind angles can be easily reconstructed from IMU measurements when wind is excluded. However, method is not tested yet for systems having different models in pitch and yaw. Here, without putting too much detail, suggested approach is tested with real flight data of a research aircraft, ATTAS [9]. Test data is obtained from support materials of "Flight Vehicle System Identification: A Time Domain Methodology" [8].

Figure A 1 - ATTAS Research Aircraft [3]

On the aircraft, there were additional sensors that were readily available to measure wind angles, true airspeed and Euler angles. These measurements are used first to find a reference set for unknown parameters. Aerodynamic coefficients are calculated from measurements and least square fits are applied directly to those coefficients in order to find constant terms and derivatives as a reference set. Then, without using attack and sideslip angles measurements, aerodynamic parameters are estimated again and compared with the reference values.

As demonstrated before, suggested approach works properly if estimation is applied to both pitch and yaw planes. Therefore available flight test data of three different maneuvers with elevator, aileron and rudder inputs from same flight are merged into one. Merged data is presented in Figure A 2. Since intervals between the maneuvers are absent, initial states of all three maneuvers are included in the unknown parameter vector. Of course, it would be much better if test data had no gaps between the maneuvers so that only the initial states at the beginning would be included in unknowns and wind angle reconstruction would be more reliable.

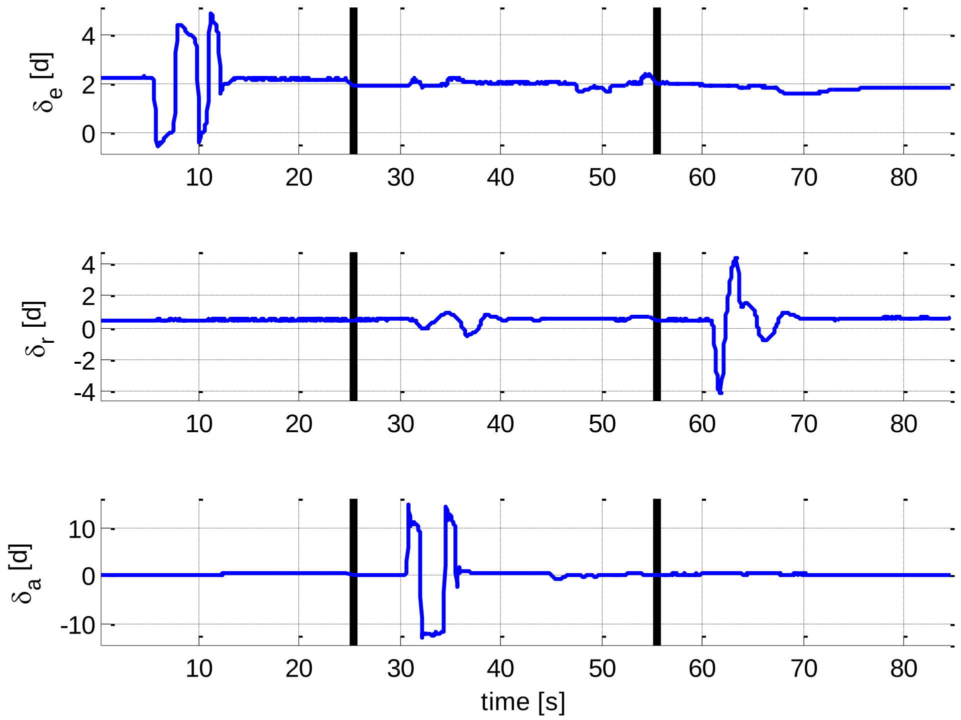

Figure A 2 - Merged maneuvers

Before testing the estimation algorithm, reconstruction from IMU is tested in estimation algorithm with other measurements. Explicit system model is modified so that outputs are composed of wind angles, Euler angles and altitude above ground. Initial states together with IMU bias values are used as unknown parameters to be estimated iteratively. Using the same estimation algorithm described in Chapter 4, it is checked that reconstruction can be successful when properly applied. Note that, this is nothing but a preliminary check to see whether or not IMU is capable of reconstructing wind angles within the sampling frequency (25 Hz) of available test data.

The results of reconstruction process are plotted in Figure A 3. Minor errors appeared in reconstructed data seems acceptable. Possible reasons for these errors are neglected scale factor errors in IMU, disturbances occurred during flight and numerical errors involved in post-process calculations. Yet, overall fit of reconstructed data to measurements are successful.

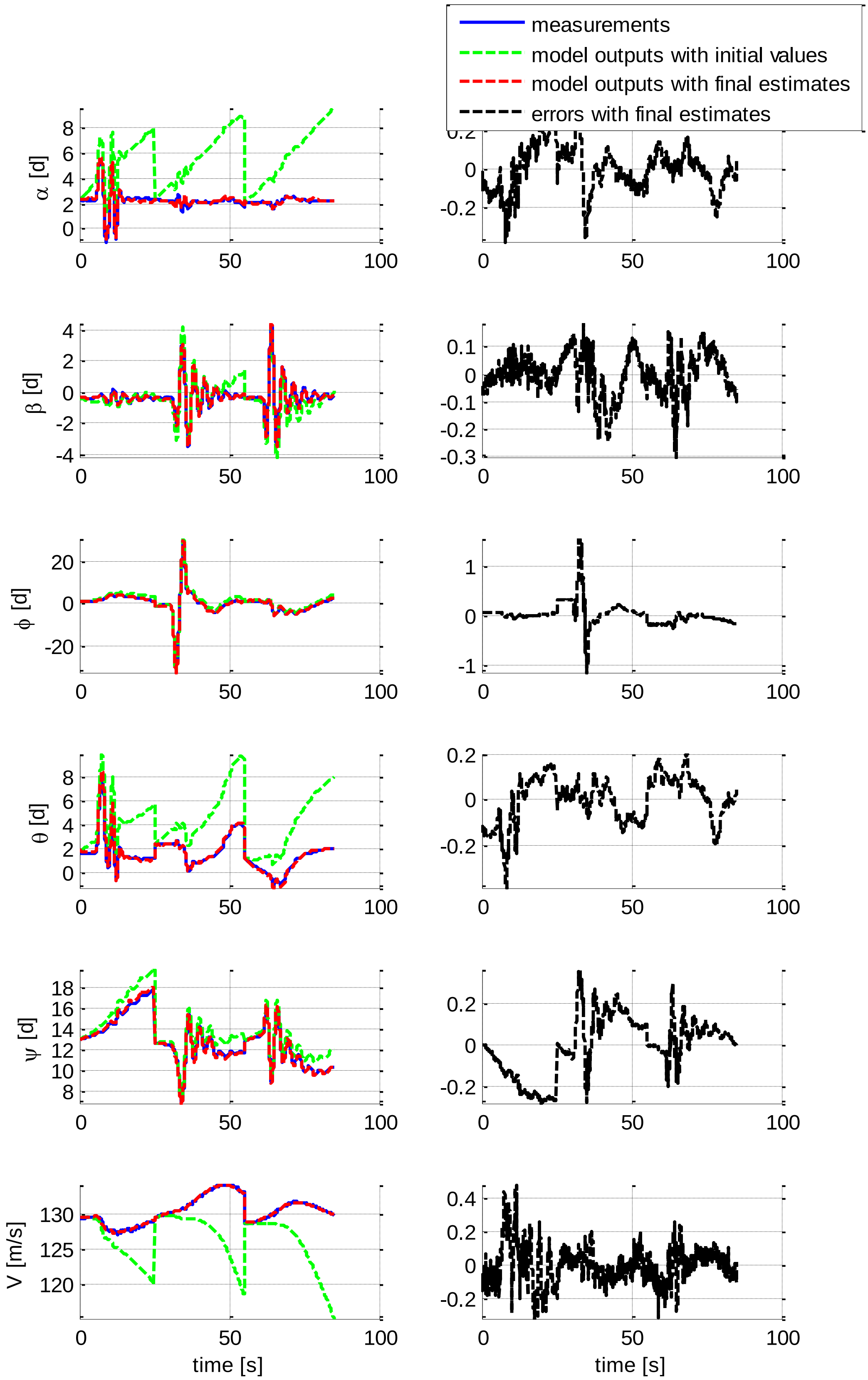

Figure A 3 - Flight path reconstruction from IMU measurements

Aerodynamic models of the aircraft at the center of gravity are postulated in the following form [8]:

$$C_X = C_{X_0} + C_{X_\alpha}\alpha \tag{A.1}$$

$$C_Y = C_{Y_0} + C_{Y_\beta}\beta \tag{A.2}$$

$$C_Z = C_{Z_0} + C_{Z_\alpha}\alpha \tag{A.3}$$

$$C_l = C_{l_0} + C_{l_\beta}\beta + C_{l_{\delta_a}}\delta_a + C_{l_p}p^* + C_{l_r}r^* \tag{A.4}$$

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e + C_{m_q}q^* \tag{A.5}$$

$$C_n = C_{n_0} + C_{n_\beta}\beta + C_{n_{\delta_r}}\delta_r + C_{n_p}p^* + C_{n_r}r^* \tag{A.6}$$

where normalized angular rates are given by:

$$p^* = pb/V \qquad q^* = qc/V \qquad r^* = rb/V \tag{A.7}$$

Aerodynamic force and moment coefficients given above are calculated using IMU and thrust measurements:

$$\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} = \left(m \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - \begin{bmatrix} F_{thrust} \\ 0 \\ 0 \end{bmatrix} \right) \frac{1}{\bar{q}S} \tag{A.8}$$

$$\begin{bmatrix} C_l \\ C_m \\ C_n \end{bmatrix} = \frac{1}{\bar{q}S} \begin{bmatrix} b & 0 & 0 \\ 0 & l & 0 \\ 0 & 0 & b \end{bmatrix}^{-1} \left(\underline{J} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} + \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \underline{J} \begin{bmatrix} p \\ q \\ r \end{bmatrix} - \begin{bmatrix} 0 \\ z_{encg}F_{thrust} \\ 0 \end{bmatrix} \right) \tag{A.9}$$

where inertia matrix is defined as:

$$\underline{J} = \begin{bmatrix} J_{xx} & 0 & J_{xz} \\ 0 & J_{yy} & 0 \\ J_{xz} & 0 & J_{zz} \end{bmatrix} \tag{A.10}$$

Note that these coefficients are obtained at CG of the aircraft. Therefore translational accelerations are first transformed to CG from sensor position and then used in above equations. In addition angular acceleration measurement was not available; therefore angular rates are smoothed and numerically differentiated to obtain angular accelerations.

Using least square estimation, aerodynamic parameters in postulated linear models are achieved. Then translational accelerations and angular accelerations are evaluated from estimation results by going backwards in Equations (A.8) and (A.9). Comparison of actual measurements and model fits are given in Figure A 4.

Aerodynamic parameters are also estimated without wind angles measurements. Maximum likelihood estimation is applied with explicit system model. However output equations of the system model are now modified as:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \frac{\rho V^2 S}{2m} \begin{bmatrix} C_{X_0} + C_{X_\alpha} \alpha \\ C_{Y_0} + C_{Y_\beta} \beta \\ C_{Z_0} + C_{Z_\alpha} \alpha \end{bmatrix} - \frac{1}{m} \begin{bmatrix} F_{thrust} \\ 0 \\ 0 \end{bmatrix} \tag{A.11}$$

$$\begin{aligned} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = +\underline{J}^{-1} \begin{bmatrix} b & 0 & 0 \\ 0 & l & 0 \\ 0 & 0 & b \end{bmatrix} & \begin{bmatrix} C_{l_0} + C_{l_\beta} \beta + C_{l_{\delta_a}} \delta_a + (C_{l_p} p + C_{l_r} r) b / V^2 \\ C_{m_0} + C_{m_\alpha} \alpha + C_{m_{\delta_e}} \delta_e + C_{m_q} q l / V^2 \\ C_{n_0} + C_{n_\beta} \beta + C_{n_{\delta_r}} \delta_r + (C_{n_p} p + C_{n_r} r) b / V^2 \end{bmatrix} \frac{\rho S}{2} \\ & - \underline{J}^{-1} \begin{bmatrix} 0 \\ z_{encg} F_{thrust} \\ 0 \end{bmatrix} - \underline{J}^{-1} \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \underline{J} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \end{aligned} \tag{A.12}$$

where total velocity, angle of attack and sideslip parameters appeared in above output equations are evaluated from states:

$$V = \sqrt{u^2 + v^2 + w^2} \qquad \alpha = \tan^{-1}(w/u) \qquad \beta = \tan^{-1}(v/u) \tag{A.13}$$

This change is necessary due to the difference between Javelin and ATTAS aerodynamic models and inertia matrices. Note that translational acceleration and angular acceleration at X axis are also included to output vector. Since state equations of explicit model do not involve any aerodynamic parameters, those are remained same.

Comparison of explicit model outputs with final estimates and real measurements are given in Figure A 5. Correlation coefficients are also checked and it is observed that aerodynamic parameters are not correlated with any other parameter. Unfortunately estimations of initial conditions appear as highly correlated which might affect the estimation results. This is most likely because three different maneuver sets are used as one and two additional initial condition sets are included to unknown parameters.

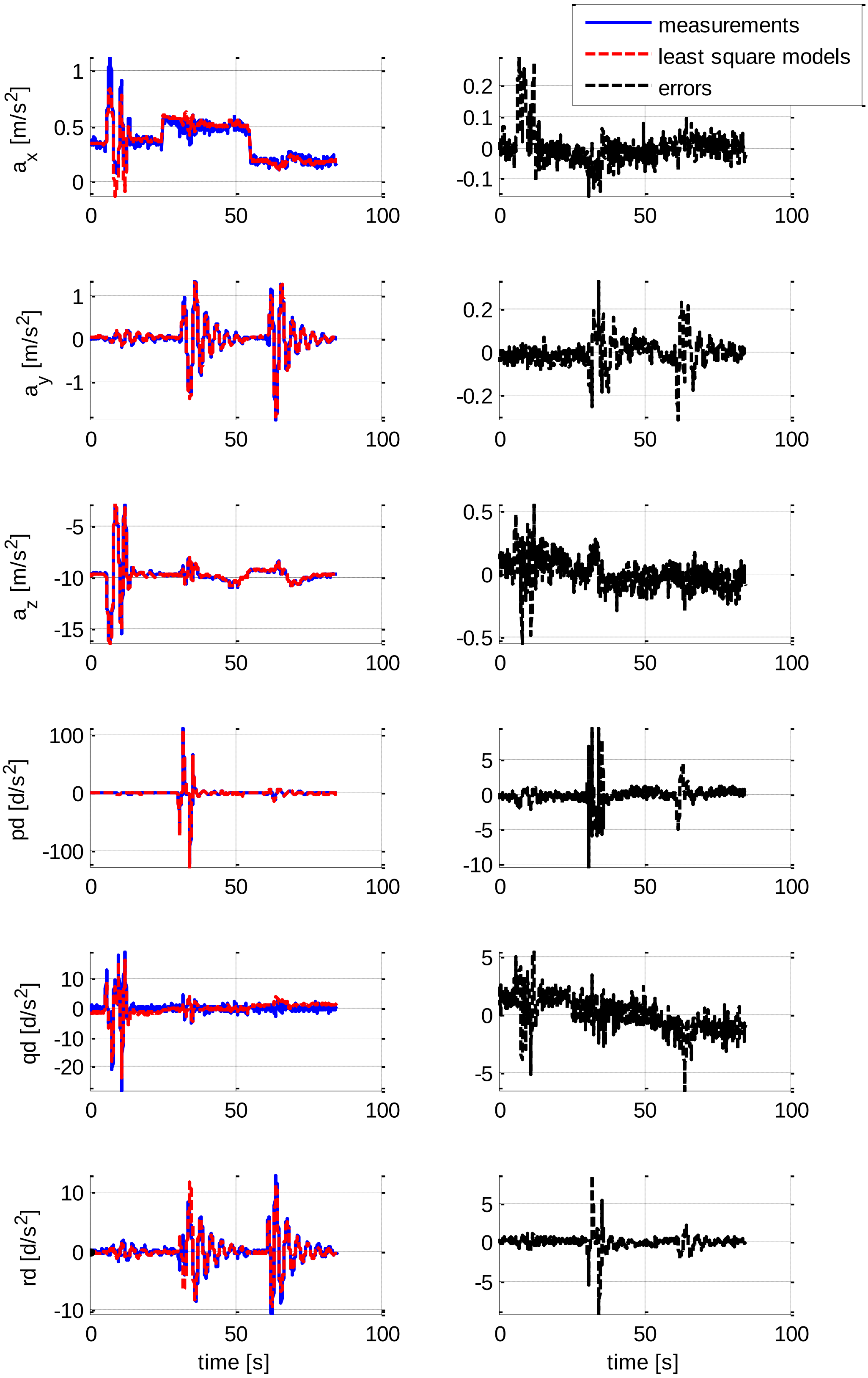

Figure A 4 - Least square estimation results

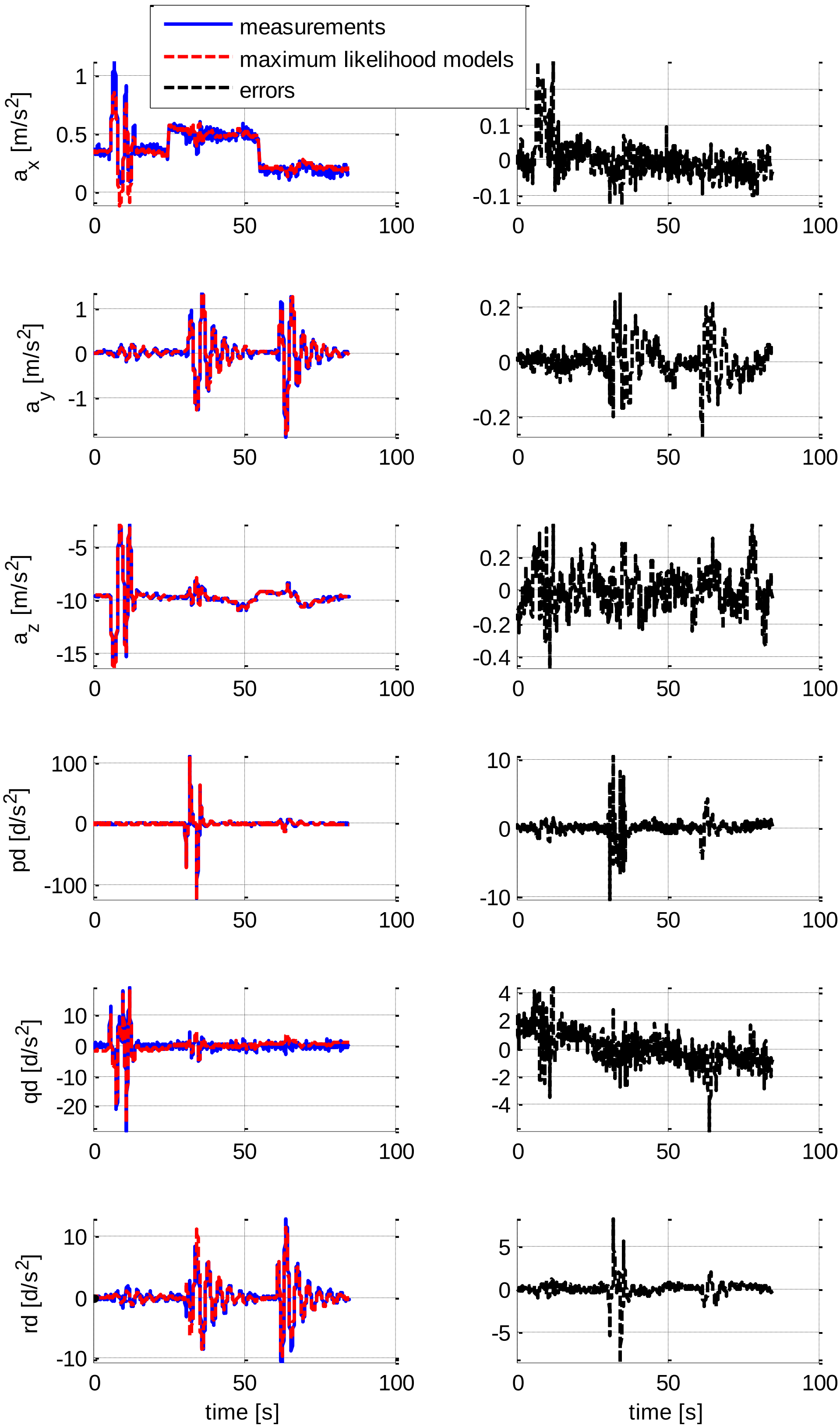

Figure A 5 - Maximum likelihood estimation results

Results of least square estimation and maximum likelihood estimation are compared in Table A 1. It can be seen that aerodynamic parameters obtained with two different estimations are close to each other. Note that, as mentioned before, this test data is not perfect due to the obligation of additional two initial condition sets. Therefore results should even be better for a more appropriate flight test containing sufficient excitations in all planes without gaps between them.

Table A 1 - Estimation results

	Maximum Likelihood	**Least Square**
Cx0	-0,048	-0,047
Cxa	0,401	0,389
Cy0	0,003	-0,005
Cyb	-1,015	-1,005
Cz0	-0,231	-0,230
Cza	-5,458	-5,252
Cl0	0,002	0,001
Clb	-0,100	-0,100
Cld	-0,194	-0,194
Clr	0,256	0,246
Clp	-0,765	-0,766
Cm0	0,088	0,081
Cma	-0,991	-0,920
Cmd	-1,142	-0,993
Cmq	-7,870	-5,989
Cn0	0,001	0,003
Cnb	0,235	0,229
Cnd	-0,150	-0,140
Cnr	-0,125	-0,108
Cnp	-0,040	-0,041

	Maximum Likelihood	**Measurement**
u1	127,177	129,350
v1	0,324	-0,919
w1	4,897	5,104
phi1	-0,028	0,016
the1	0,042	0,029
u2	130,088	129,406
v2	0,478	-0,840
w2	5,189	5,195
phi2	-0,074	-0,022
the2	0,075	0,041
u3	126,824	128,658
v3	0,154	-0,910
w3	4,805	4,627
phi3	-0,021	0,021
the3	0,050	0,021
bax	-0,201	NA
bay	0,632	NA
baz	0,048	NA
bp	0,001	NA
bq	0,004	NA
br	0,009	NA

Errors are also compared in Figure A 6. There are deterministic errors suggesting the aerodynamic model inaccuracies. Yet these errors appear almost same for both estimations. Other than that, maximum likelihood estimation seems to provide slightly better fits.

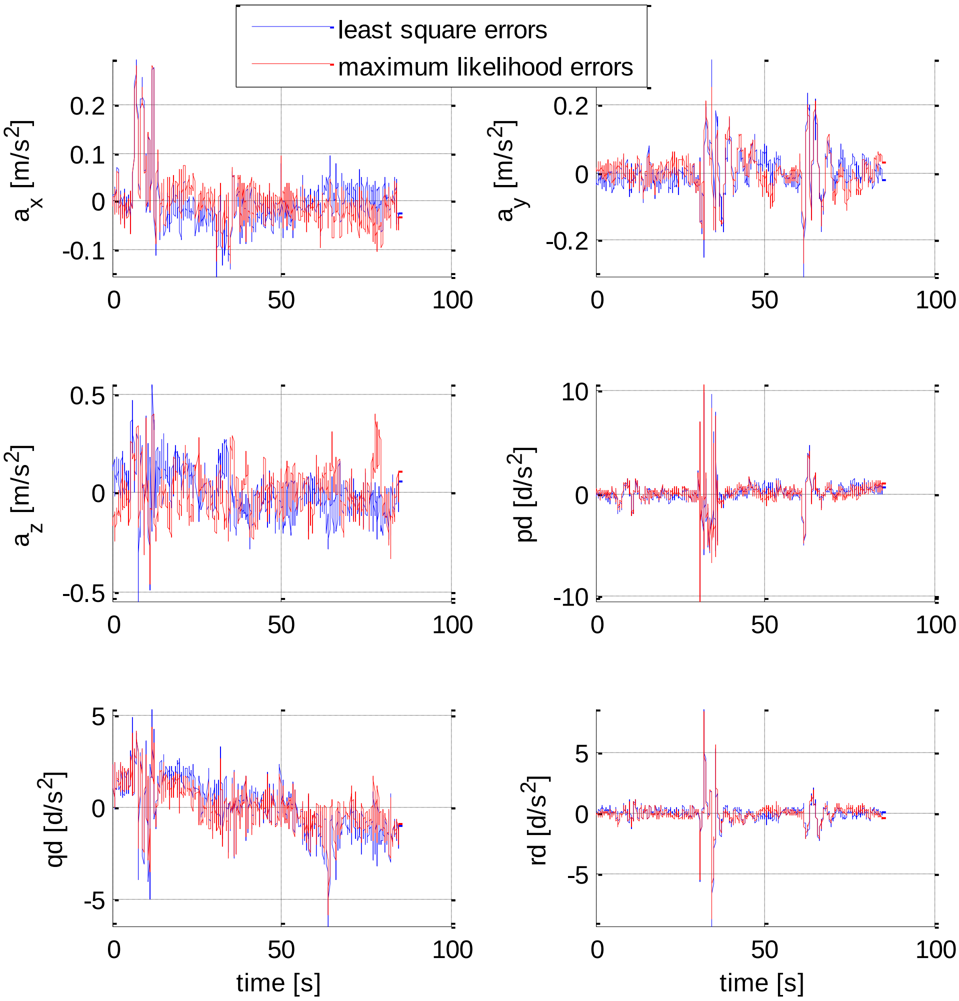

Figure A 6 - Error comparison of estimations

To sum up, maximum likelihood estimation without wind angle measurements can also be applied to aircrafts under appropriate circumstances. Limitations given for a symmetric missile still holds here. Test data to be used in estimation must contain sufficient information for all planes so that wind angles reconstruction can be done without any correlations. Moreover wind must not be existed during the excitations of system. Otherwise reconstructed wind angles will not be exact and resulting parameter estimates will be biased.

Even if above conditions cannot be met exactly, maximum likelihood estimation with explicit system model suggested in this thesis may be used to obtain rough models for aircrafts.