PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' CONCEPTION OF DERIVATIVE FROM COMMOGNITION PERSPECTIVE

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ABSTRACT

PRE-SERVICE ELEMENTARY MATHEMATICS TEACHERS' CONCEPTION OF DERIVATIVE FROM COMMOGNITION PERSPECTIVE

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The purposes of this study were to investigate pre-service elementary mathematics teachers' discourse on derivative in group, classroom and individual discussions and determine their conception of derivative. In order to examine pre-service teachers' discourse on derivative communicational approach to cognition (commognition) was applied. Pre-service teachers' words and their uses, visual mediators, narratives and routines were analyzed.

This study was designed as a qualitative study. Data were collected from pre-service mathematics teachers in the fall semester of the 2009-2010 academic year. Pre-service mathematics teachers were freshmen students enrolled in a public university. Derivative test results, group discussion records, classroom discussion records and interview records were analyzed to determine pre-service teachers' discourse on derivative concept and their conception of derivative.

According to the results of the study, group discourse reveals that pre-service mathematics teachers in the observed group have the conception of derivative as slope. However, individual discourse shows that interviewed pre-service teachers had the conception of derivative as limit of the slopes. Some pre-service teachers have common usages related to the tangent value in the group and classroom discourses.

Group discussions develop pre-service teachers' discourse on the rate of change of a function in the observed group.

Analysis of the words, visual mediators, endorsed narratives and the routines of the group, classroom and individual discussions reveal that pre-service teachers have some difficulties related to the concept of derivative. Some of them have problems to understand the instantaneous rate of change. They can not differentiate it from the average rate of change. Some of them have difficulty to understand the relation between the function properties and the first and second derivatives of the function. They also have problems to give meaning to the relations between the first and second derivatives. Most of the pre-service teachers have tendency to depend more on rules related to the concept of derivative. Some pre-service teachers have problems for the transition of one form of the representation into another form. Moreover, visual mediators are useful to understand what pre-service teachers mean in their words and endorsed narratives. The fact that what some pre-service teachers actally say and what they mean are totally different from each other is observed.

Findings obtained from this study showed that pre-service teachers have some deficiencies related to the concept of derivative. Therefore, in calculus and mathematics teaching method courses, these deficiencies should be emphasized more. Besides, it is found that there can be differences what pre-service teachers say and what they actually want to say. Therefore, what pre-service teachers would like to say in their words, endorsed narratives, visual mediators and routines should be paid more attention. Group discussions enable pre-service teachers to improve their ideas in terms of the rate of change. Therefore, pre-service teachers have the chance of sharing and developing their ideas with the help of group, classroom and individual discussions. And also analyzing these discussions could enable us to notice the problems that pre-service teachers have while learning the subject of derivative.

Keywords: communicational approach to cognition, commognition, derivative, mathematics education

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMENLİĞİ ADAYLARININ TÜREVİ KAVRAYIŞLARININ BİLİŞE İLETİŞİMSEL YAKLAŞIM AÇISINDAN İNCELENMESİ

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Çalışmanın amaçları ilköğretim matematik öğretmen adaylarının türev üzerine söylemlerini grup, sınıf ve bireysel tartışmalarda incelemek ve türevi nasıl kavradıklarını belirlemektir. Öğretmen adaylarının türev üzerine söylemlerini incelemek için bilişe iletişimsel yaklaşım (commognition) kullanılmıştır. Öğretmen adaylarının kelime kullanımları (word use), görsel mediyatörleri (visual mediators), anlatımları (endorsed narratives) ve rutinleri (routines) analiz edilmiştir.

Çalışma nitel bir çalışma olarak tasarlanmıştır. Veriler 2009-2010 akademik yılı güz döneminde matematik öğretmen adaylarından toplanmıştır. Matematik öğretmen adayları bir devlet üniversitesinde öğrenim gören birinci sınıf öğrencileridir. Matematik öğretmen adaylarının türev üzerine söylemlerini ve türevi kavramalarını incelemek için türev testi sonuçları, grup ve sınıf içi tartışma kayıtları ve görüşme kayıtları incelenmiştir.

Araştırmanın sonuçlarına göre, grup içi söylemleri o gruptaki öğretmen adaylarının türevi eğim olarak algıladıklarını göstermiştir. Bununla birlikte bireysel söylemleri de görüşme yapılan öğretmen adaylarının türevi eğimlerin limit olarak algıladıklarını ortaya koymuştur. Grup içi ve sınıf içi söylemlerine göre bazı

öğretmen adaylarının teğetle ilgili ortak kullandıkları ifadeler vardır. Grup içi tartışmalar öğretmen adaylarının fonksiyonun değişim oranı ile ilgili söylemlerini geliştirmiştir.

Grup içi, sınıf içi tartışmalarda ve bireysel görüşmelerde kullanılan kelimelerin, görsel mediyatörlerin, anlatımların ve rutinlerin analizi, öğretmen adaylarının türev kavramı ile ilgili çeşitli zorluklar yaşadıklarını göstermiştir. Bazıları anlık değişim oranını anlamayla ilgili problemler yaşamışlardır. Anlık hızı, ortalama değişim oranından ayırt edemişlerdir. Bazıları fonksiyonun özellikleri ile birinci ve ikinci türevi arasındaki ilişkiyi anlamakta zorlanmıştır. Ayrıca birinci ve ikinci türev arasındaki ilişkiyi de anlamlandırmakta problemler yaşamışlardır. Öğretmen adaylarının büyük çoğunluğu türevle ilgili kural kullanmaya eğilim göstermektedirler. Bazı öğretmen adayları bir gösterimden diğer gösterime geçerken problemler yaşamaktadırlar. Ayrıca, görsel mediyatörler öğretmen adaylarının kullandıkları kelimelerde ve anlatımlarında ne demek istediklerini anlamakta faydalı olmuştur. Öğretmen adaylarının gerçekten söyledikleriyle demek istediklerinin birbirinden tamamen farklı olduğu gerçeği gözlemlenmiştir.

Bu çalışmadan elde edilen sonuçlar öğretmen adaylarının türev kavramıyla ilgili çeşitli eksiklikleri olduğunu göstermiştir. Bu nedenle, analiz derslerinde ve matematik öğretimi derslerinde, bu zorlukların üzerinde durulması gerekmektedir. Bunun yanında, öğretmen adaylarının söyledikleriyle söylemek istediklerinin farklı olabildiği farkedilmiştir. Bu nedenle analiz ve matematik eğitimi derslerinde öğretmen adaylarının kullandıkları kelimelerde, görsel mediyatörlerinde, anlatımlarında ve rutinlerinde ne demek istediklerine dikkat edilmelidir. Grup tartışmaları öğretmen adaylarının değişim oranı ile ilgili bilgilerini geliştirdiklerini göstermiştir. Bu nedenle, öğretmen adaylarına grup içi, sınıf içi ve bireysel tartışmalar yardımıyla fikirlerini geliştirmeleri için sanş verilmiş olacaktır. Ayrıca, bu tartışmaların incelenmesiyle öğretmen adaylarının karşılaştıkları problemlerin belirlenmesi mümkün olacaktır.

Anahtar Kelimeler: Bilişe iletişimsel yaklaşım, türev, matematik eğitimi

To my son BARIŞ

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CHAPTER 1

INTRODUCTION

Mathematics and mathematics education communities consider Calculus as the most important course according to pedagogy and curriculum (Ferrini-Mundy & Graham, 1991). Therefore improving calculus has a crucial importance for the future development. Calculus Reform Movement began in the mid eighties aimed giving importance to understanding of concepts rather than applying rules and procedures to provide students insight into the mathematical connections and real world applications (Ferrini-Mundy & Graham, 1991). Therefore, there is a developing attempt for the research related to calculus concepts of function, limit, derivative and integral (Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2008; Gravemeijer & Doorman, 1999; Habre & Abboud, 2006; Thompson, 1994; White & Mitchelmore, 1996).

A research in mathematics education begins with answering the questions of “what it means to understand the concept?” and “how that understanding can be constructed by a learner?” (Asiala, Cottrill, Dubinsky & Scwingerdorf, 1997). Understanding occurs if an individual construct his/her meaning of the concept (Dubinsky, 1991; Hiebert & Carpenter, 1992; Skemp, 1976; Tall & Vinner, 1981; Vinner, 1997). Individuals’ mental structures, such as definition, properties actions, process of an object and the connections of these structures form this meaning. How a student constructs the knowledge and how this construction process can be developed are the basic questions that the mathematics educators and the researchers have been investigating. While working on these questions, researchers developed new theories explaining the learning of mathematical concepts.

Learning can be perceived in various ways from different perspectives. There are two metaphors which characterize learning from different perspectives: learning-as-acquisition and learning-as-participation (Sfard, 2008). Learning-as-acquisition metaphor is the result of monological discourses of the traditional psychology and its narratives are presented as if they are natural constructs. Moreover, the follower of this metaphor is unlikely to deal effectively with the metaphor of the object. On the other hand, for the metaphor of learning-as-participation, individual and collective forms of doing are presented as different indicators of the same type of processes (Sfard, 2008). This metaphor was a part of dialogical approach. Dialogical approach recognizes the dialogical nature of research and defines some of the discursive constructs again (Sfard, 2008). This approach puts an end to research supporting "mind without behavior" or "behavior without mind".

Sfard (2008), follower of the metaphor of learning-as-participation, perceives cognitive processes as individualized forms of interpersonal communication and comprehends thinking and communicating together to stress their unity to form the term commognition. She states that thinking can be considered as a human activity that emerges when individuals can communicate with themselves. She argues the distinct characteristics of communicational actions. Human communicational actions are rule driven, function of voluntary decisions, implemented with the help of specific perceptual mediators and about certain object (Sfard, 2008).

According to Sfard (2008), discourses are differentiated by their respective objects. Therefore, mathematical discourse is about mathematical objects such as numbers, functions, sets and geometrical shapes. As being an autopoietic system, mathematics contains the objects of talk and grows when new objects are added one after another (Sfard, 2008). Learning mathematics is similar to having, altering and enlarging one's discourse (Sfard, 2007). When one have competence to be part of a mathematical communication with herself and others, her mathematical discourse becomes individualized and she or he learns mathematics. When a person learns about any mathematical subject, his or her discursive skills changes and uses this new communication format in solving mathematical problems. Discursive development of a person can be defined as determining the changes in the discursive characteristics: the use of words, visual mediators, endorsed narratives and routines. Being familiar with the discourse is a precondition for the participation of a discourse and also such familiarity emerged from the participation in this discourse (Sfard, 2008). Threfore, it is important to talk about verbally and visually related to the concept which the one is learning. This present study also aims to search pre-service teachers' discourse on derivative concept. Therefore, this study can reveal pre-service teachers' verbal and visual perception and their relations on derivative concept.

When the research studies on learning derivative were considered as a whole, it was realized that most of them depended on the metaphor of learning-as-acquisition and elaborated cognitive processes (Aspinwall, et. al, 1997; Baker, Cooley & Trigueros, 2000; Gravemeijer & Doorman, 1999; Habre & Abboud, 2006; Thompson, 1994; White & Mitchelmore, 1996). Some of them focused on students' concept images related to derivative (Aspinwall, et. al., 1997; Berry & Nyman, 2003; Thompson, 1994). Some of them elaborated students' conceptual knowledge of derivative concept (Asiala, et. al., 1997; Borgen, & Manu, 2002; Habre & Abbout, 2006; Tall, 1986). Some of them figured out students' misconceptions of derivative (Orton, 1983; Ubuz, 2001; Ubuz, 2007). It was also detected from these research studies that they focused on students' individual constructions of mathematical concepts. One of the aims of this study was to determine pre-service teachers' conception of derivative focusing on the group, classroom and individual discourses. Therefore, this present study would reveal participants' conception of derivative from learning-as-participation metaphor. In order to determine pre-service mathematics teachers' discourse on derivative, Sfard's communicational approach to cognition would be employed.

Derivative concept is represented graphically, verbally, physically and symbolically (Zandieh, 2000). It means graphically as the slope of the tangent line to a curve at a point or as the slope of the line a curve seems to approach under magnification, verbally as the instantaneous rate of change, physically as speed or velocity, and symbolically as the limit of the difference quotient. Therefore learners of the derivative concept would develop different approaches while learning derivative. This study also aims to investigate pre-service teachers' conception of the derivative concept but from different approach as communicational approach to cognition (commognition).

In most of the research studies related to derivative concept, participants were chosen from engineering, mathematics or science majors (Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Bingölbali & Monaghan, 2008; Ubuz, 2001; Ubuz, 2007). As calculus was thought to be main area of engineering, mathematician and physicist candidates, teacher candidates were ignored for their conception of derivative unless the study was related to pedagogical content knowledge. Therefore, there is an urgent need for the studies to explain pre-service mathematics teachers' conception of the mathematical concepts. As one of the aims of this study is to express pre-service mathematics teachers' conception of derivative, this study can provide us to get information related to pre-service mathematics teachers' conception of derivative.

In studies focusing on participants' discourses related to limit (Güçler, 2013) and derivative (Park, 2013), discourse of the instructor was determined only in the classroom settings as the students did not contribute the discussion so much. The present study aims to determine pre-service teachers' discourse on derivative in group, classroom and individual settings. Thus, this study can contribute educational research a lot by explaining the pre-service teachers' group and classroom discourse.

In Turkey, curriculum of high school mathematics contains some of the subjects of calculus such as limit, derivative and integral (MEB, 2011). Therefore, pre-service mathematics teachers cover the derivative concept before they attend to the university. As they are acquainted with the derivative concept, they develop a discourse on derivative during the high school years. Therefore, knowing pre-service teachers' discourse before the instruction will provide calculus instructors to determine pre-service teachers' discourse beforehand and to plan their instructions accordingly. This study can also provide us to determine pre-service elementary mathematics teachers' discourse on derivative before attending to the university in the group and classroom discussions.

One of the standards of the elementary mathematics curriculum is communication (MEB, 2009). Talking, writing and listening about mathematics develop the communication skills and also help learners to understand mathematical concepts better. This research can also contribute pre-service teachers to develop their communication skills, so they can talk and develop ideas about the derivative concept in the group and classroom discussions. Moreover, pre-service teachers can have chance to experience a learning environment with group and classroom discussions. Therefore, this experience will also provide pre-service teachers to have an idea related to this learning environment.

Consequently, this present study aims to investigate pre-service elementary mathematics teachers' group, classroom and individual discourse on derivative with respect to communicational approach to cognition (commognition). Moreover, it aims to put forward pre-service elementary mathematics teachers' conception of derivative concept with respect to group, classroom and individual discourses.

1.1 Purpose of the Study

The aims of this study were to investigate pre-service elementary mathematics teachers' discourse on derivative in group, classroom and individual settings from communicational approach to cognition (commognition) perspective and determine their conception of derivative.

1.2 Research Questions

This study aims to answer the following question:

How do pre-service elementary mathematics teachers explain the concept of derivative in group, classroom and individual discourses from commognition perspective?

a) How do pre-service elementary mathematics teachers explain the concept of derivative in group discourse from commognition perspective?
b) How do pre-service elementary mathematics teachers explain the concept of derivative in classroom discourse from commognition perspective?
c) How do pre-service elementary mathematics teachers explain the concept of derivative in individual discourse from commognition perspective?

1.3 Definition of Important Terms

The research questions consist of several terms that need to be defined constitutively and operationally.

Discourse

Any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system (Sfard, 2001, p. 28).

Mathematical Discourse

Patterned ways of using questioning, explaining, listening and various modes of communication in the classroom to promote conceptual understanding in mathematics (Blanton, Berenson & Norwood, 2001).

Classroom Discourse

Nuthall, Graesser and Person (2013) define classroom discourse as the language that was used to communicate with each other by teachers and students in the classroom. As talking and conversation is the way that teaching progress, the study of classroom discourse means studying the face-to-face classroom teaching. Classroom discourse support mathematical thinking.

Group Discourse

According to Nuthall, Graesser and Person's (2013) definition of classroom discourse, group discourse would be defined as the language that was used to communicate each other by the students in a group.

Commognition

Commognition is a word composed of two words communication and cognition (Sfard, 2008). This word defines Sfard's framework about mathematical thinking especially "thinking about thinking". According to Sfard thinking is a form of communication and it is the individualized version of interpersonal communication (Sfard & Kieren, 2001; Sfard, 2007; Sfard, 2008; Sfard, 2012). She accepts the cognitive processes and interpersonal communication processes to be different expressions of the same fact.

Discourses are made distinct according to some features (Sfard, 2007; Sfard, 2008; Sfard, 2010). For the mathematical discourse, these features are word use, visual mediators, endorsed narratives and routines.

Word Use:

As for all discourses, mathematical discourse should have its own words. (Sfard, 2007; Sfard, 2008). In mathematical discourse, these words are numerical, geometrical, signifying quantities and shapes.

Visual Mediators

Visual mediators are the symbolic artifacts that are used in special forms. Numerals, algebraic formulas, algebraic notation, graphs, drawings and diagrams are the most used examples of visual mediators in mathematics (Sfard, 2007; Sfard, 2008).

Narratives

Written or spoken texts which are the explanation of objects or relations between objects or activities with or by objects (Sfard, 2007). It is any sequence of utterances framed as descriptions of objects, of relations between objects, or of processes with or by objects (Sfard, 2008).

Routines

They are repetitive patterns in communicators’ activities (Sfard, 2007; Sfard, 2008). These repetitive patterns are determined while giving attention to the use of mathematical words and mediators or narratives.

CHAPTER 2

LITERATURE REVIEW

In this chapter, a review of the literature was prepared to determine the information previously documented on the theory and the applications of this research was constructed on. Literature review consists of three parts: the conceptual framework of communicational approach to cognition (commognition), the notion of derivative and research studies related to the students' learning of derivative concept.

2.1 Communicational Approach to Cognition (Commognition)

Commognition is a new word composes of two words communication and cognition (Sfard, 2008). This word is offered by Anna Sfard to name her framework related to mathematical thinking especially "thinking about thinking". Sfard sees thinking as a form of communication and defines thinking as the individualized version of interpersonal communication (Sfard & Kieren, 2001; Sfard, 2007; Sfard, 2008; Sfard, 2012). She accepts the cognitive processes and interpersonal communication processes to be different expressions of the same fact.

Sfard (2001) builds her framework of communicational approach to cognition on saming communication to thinking. She also grounded this framework on the metaphors of "learning-as-acquisition" and "learning-as-participation." Sfard explains "learning-as-acquisition" metaphor that this metaphor conceptualizes learning as storing information in the form of mental representation. Acquisition of knowledge would be by passive reception or by active construction of this knowledge. Thus, this active construction would result in a personalized version of concepts and procedures. Personal construction of the concepts would not always result in conceptions. Sometimes, they would be misconceptions.

The theories of conceptual development depend on Kantian/Piagetian concept schemes organizing mental structures for one from former conceptions (Sfard, 2001). According to cognitive psychology, learning with understanding occurs when one relates new knowledge to already possessed knowledge. Sfard (2001) gave the definition of understanding within the acquisionist framework as a mode of knowledge, this knowledge is conceptualized as a certain object which a person either possess or not and learning is regarded as a process of acquiring this object. Therefore, when this knowledge is possessed, it is used whenever needed and it is appropriate and carried from one situation to another.

Sfard (2001) points out that as a researcher or a practitioner, the notions related to acquisition metaphor, are too crude an instrument for some more advanced needs. She emphasized that acquisition based theories tell us only a restricted part of the story of learning. The researchers that are following acquisition based theories do not consider the important points for understanding. She insists on that these missing parts are significant enough to change the picture. She also states that participationist approach to learning and understanding is not taking the role of acquisition metaphor. On the contrary it has a complementary role with the more traditional theories.

Sfard (2001) constructs the bases of communicational approach to cognition on partipationist approach to cognition which grows from the sociocultural tradition. She begins emphasizing the difference between acquisitionists and the participationists as defining learning. Sociocultural psychologists view learning as becoming a participant in certain distinct activities rather than becoming a possessor of generalized, context-independent conceptual schemes. For the followers of the participationist framework, learning is about the development of ways in which one participates in well established communal activities. The participationist researcher interested in the growth of mutual understanding and coordination between the individual and the others in the community.

Sfard summarized the difference how followers of these two metaphors perceive learning in the following sentences (2001). While followers of acquisitionist framework focus on not changing variants of learning; followers of participationist framework focus on the activity itself and its changing context-depended dimensions. The community affects the change in the learner's activities. However, the change in the individual should be important focus of the study, when learner is regarded as an isolated entity but as a part of a larger community many other elements should be considered as a part of a new much broader unit of analysis. Describing what was happening between the interlocutors accordingly only cognition (abilities and contents of their minds) and ignoring many aspects and factors of change would lead insufficient and unhelpful picture of learning.

As the way people behave would change from one situation to another, success in problem solving not only prove highly sensitive context of the activity. As participationists focus on the situation and the behavior of the individual, they prefer cognitive apprenticeship as a mode of learning. They believe that abstract scholarly learning may have a theoretical advantage but apprentice-like participation in specific activities is more effective than scholarly learning.

There is an argument between acquisitionists and participationists concern the nature and sources of human knowing. Acquitionists are interested in human-independent circumstances of learning, such as direct encounter between the individual and the world and a range of biological determinants, from inheritance to physiological growth and to the structure of human brain. They seem insensitive to social, cultural, historical and situational context. On the other hand, participationists are interested in mostly in human practice and as society produces and sustains this practice, they give emphasis to the society. And also participationists view learning as a human activity beginning and ending in society. Thus there is a need for interaction and communication and its continual growth. Participationists propose that much attention should be given to the contextual factors before assessing one's performance in terms of permanent quality such as the learner has mathematical ability or not.

Sfard constructed the framework of communicational approach to thinking on the roots in Vygotskian writings and with its branches in contemporary philosophical-sociological thought and in recent advances in linguistics. Sfard explains the basic tenets of this communicational approach to the study of human cognition as thinking may be conceptualized as a case of communication, as one's communication with oneself. Sfard perceives thinking as a dialogical endeavor, where we inform ourselves, we argue, we ask questions and wait for our own responses. Thinking is a private version of interpersonal communication as also Vygotsky believes. Thinking also our communicating with ourselves not necessarily is inner or verbal. Sfard points out elements of this framework as communication may be defined as a person's attempt to make an interlocutor act, think or feel according to her intentions, research that looks at cognition as a communicational activity focuses, in fact, on the phenomenon of mutual regulation and of self-regulation. The dichotomy/thought communication practically disappears and speech is no longer considered as a mere "window to the mind".

In order to understand the commognition framework well, it is important to define some certain terms such as thinking, communication, discourse and mathematical discourse. "Thinking" is a variety of the activity of communicating (Sfard & Kieran, 2001; Sfard, 2007). It is an individualized form of communication (Sfard, 2008, p.82), especially an activity of an individual communicating with herself or himself (Sfard, 2001; Sfard, 2012). Although, thinking is individualized, it needs outside support but not have to be interpersonal. It is prior to the activities of communication. It does not have to have vocal or visual elements or be in words (Sfard, 2007; Sfard, 2008; Sfard & Kieren, 2001). Moreover, thinking is dialogical as we argue, ask questions, and wait for our own responses (Sfard, 2007).

Another term that should be defined is communication. Communication is defined as the importing or exchanging information by speaking, writing or using some other medium in the Oxford Dictionaries (2013). In Webster's New World Dictionary of American English (1988) communication is defined as "giving or exchanging of information, signals or messages as by talk, gestures or writing. Sfard defines communication as an activity that an individual's action is followed by an action of another individual. First individual's action should be well defined communicational actions and the second individual's actions are reaction to the former action (Sfard, 2008). For the effectiveness of the communication, it is important to be sure about the sameness of the send and the received messages of an idea, meaning or feeling (Sfard, 2001).

Communication has two elements; objects and mediators (Sfard, 2001; Sfard, 2008). The object of a communicational act is used to draw the attention of the re-actors by the actors. For example, when an actor mentions about a property of a function, this function is the object of this communicational act. However, sometimes actors or re-actors would understand different things from this communicational object. The other element of communication is the communication mediators (Sfard, 2001; Sfard, 2008). Communication mediators are objects which help the interlocutors to communicate. They can be vocal, visual or even concrete. Communication mediators can be adopted to perform this role or they can be produced by people.

Communications differ from each other according to the communication objects or mediators used to provide communication or the rules followed by the interlocutors. People would be a part of some communications however, does not take roles in others. Different types of communication that draw some individuals together while excluding some others are called discourses (Sfard, 2008 p.98). Discourse is communication of ideas, information,
etc, especially by talking; conversation (Webster's New World Dictionary of American English, 1988).

Again Sfard (2001; 2007) defines discourse as any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself whether predominantly verbal or with the help of any other symbolic system. It develops as a reaction to specific duties (Sfard, 2010). People are parts and members of different activities throughout their life time and so they participate in different communicational activities. Therefore, they are part of specific discourses changing according to their activity. The participants of the same discourse don't need to face to or communicate with each other. To be a member of any discourse, one needs to participate in the communicational activities of that discourse (Sfard, 2007). Personal discourses are hard to investigate as they consist of individual's thinking so they are silent and inner (Sfard, 2008).

The common point of the research studies from topics focused on the development of algebraic and geometrical thinking, dependence of mathematics on language, curricular implementation, interaction between children trying to learn mathematics in collaborative groups and the affective domains of mathematics learning is the need to communicate with one another (Sfard, 2012). They need a common discourse which has integrated system of tools and grounded in a set of foundational assumptions. Moreover, this common discourse should meet the same aspects of the teaching and learning processes such as cognitive and affective, intra-personal and inter-personal (individual and social). Owing to this common discourse, researchers having different research interests find common points to understand each other and ways of talking.

As thinking has been defined as self-communication (Sfard, 2001; Sfard, 2008; Sfard, 2012), it is not easy to investigate and understand personal discourses. Moreover, to be a part of a mathematical discourse always does not mean that participants are aware of mathematical self-communication (Sfard, 2008). It is not possible for every participant to be able to manage mathematical self-communication. Learning mathematics is the same as having, altering and enlarging one's discourse (Sfard, 2007). When one has competence to be part of a mathematical communication with others and with herself or himself, then her or his mathematical discourse becomes individualized and she or he learn mathematics. When a person learns about any mathematical subject, his or her discursive skills change and uses this new communication format in solving mathematical problems. Discursive development of a person can be defined as determining the changes in the discursive characteristics: the use of words, the use of mediators, endorsed narratives and routines.

As we mentioned mathematics learning is altering discourse, there are two types of learning: object-level learning and meta-level learning (Sfard, 2007). Object-level learning occurs when enlarging the existing discourse by learning the new vocabulary, constructing new routines and producing new endorsed narratives. Meta-level learning occurs when meta-rules of the discourse changed. The meta-level learning is most likely to originate in the learner's direct encounter with the new discourse. Some familiar tasks such as defining a word or identifying geometric figures will now be done in a different, unfamiliar way. We can discriminate discourses from each other according to their objects (Sfard, 2008). For example; we can categorize mathematical discourse by recognizing the mathematical objects, such as numbers, functions, sets, geometrical shapes, three dimensional mathematical objects, algebraic expressions, etc.

Discourses are made distinct according to some features (Sfard, 2007; Sfard, 2008; Sfard, 2010). These features that differentiate mathematical discourse from other discourses are word use, visual mediators, endorsed narratives and routines. In this present study, these features of mathematical discourse were used for the analysis of the data to determine pre-service elementary mathematics teachers' discourse on the derivative concept.

1) **Word Use**: Keywords are one of the characteristics that make the discourses different from each other (Sfard, 2008). Discourses should have their own words (Sfard, 2007; Sfard, 2008). It is also crucial for mathematical discourse. Mathematical discourse should have mathematical words, such as numerical, geometrical, signifying quantities and shapes. New words which are only belong to mathematics or new uses of formerly used words are learned while becoming a participant of this discourse. Word use is important as it gives clues about how the user perceives the world (Sfard & Lavie, 2005; Sfard, 2007; Sfard, 2008).

2) **Visual Mediators:** They are visible concrete objects that are real or imaginary. They are the symbolic artifacts that are used in special forms. Numerals, algebraic formulas, algebraic notations, graphs, drawings and diagrams are the most used examples of visual mediators in mathematics (Sfard, 2007; Sfard, 2008). By using these visual mediators, the participants of this certain discourse define the objects of the discourse and arrange their communication (Sfard, 2005). Colloquial discourses are mediated by concrete objects coordinating communication. They are named with nouns and pronouns. Literate discourses consist of visually mediated symbolic artifacts and icons which are conventionally or individually designed diagrams, graphs and other drawings.

Most of the mathematicians use visual imagery in advanced and abstract discourses. They sometimes draw these pictures and sometimes just imagine. These drawings in some cases mean nothing to others but the mathematician uses them to keep his discourse focused.

3) **Narratives:** Written or spoken texts which are the explanation of objects or relations between objects or activities with or by objects are the narratives (Sfard, 2007). It is any sequence of utterances framed as descriptions of objects, of relations between objects, or of processes with or by objects (Sfard, 2008). Narratives are called true or false according to the approval or disapproval. The criteria of approval would change according to the discourse. Mathematical theories, definitions, proofs and theorems are the narratives of a mathematical discourse (Sfard, 2007; Sfard, 2008). Mathematical narratives would be considered in two categories: Object level and meta-level. Object level narratives are the stories about mathematical objects. For example $4 + 7 = 11$; $(a - b)^2 = a^2 - 2ab + b^2$ or the sum of the angles in a square is 360^0. Meta-level narratives are stories about how mathematics is done. For example while taking the derivative of polynomial functions take the power of x as the coefficient of x and subtract 1 from the power.

4) **Routines**: They are regularities in communicators' activities (Sfard, 2007; Sfard, 2008). These repetitive patterns are determined while giving attention to the use of mathematical words and mediators or narratives. Routines could be seen in any form of mathematical discourse, such as categorizing and comparing the sameness or difference. The form of routines depends on the participants' ability to apply mathematical discourse. Routines include word use, visual mediator use and endorsing narratives and they are much more than these three. There are two types of routines as in the narratives. These are object- level and meta-level rules. For example numerical calculations made according to the properties of associativity, commutivity and distributivity of addition and multiplication are the object level rules. In this type of routine, rules are obvious. The other type of routine is the rules understood from the communicators' activities and in most cases communicators are less aware of the rules (Sfard, 2007). This routine is called meta-level routine.

Realization of the signifier is an object which may be used to produce or substantiate narratives about the signifiers. Signifiers are the words or symbols which participants of the discourse use as a noun. For example, "Slope of a function g" is the signifier and "5" is the realization of this signifier. Another example is "solution of the equation $7x + 4 = 5x + 8$" is the signifier of the realization "the x-coordinate of intersection of the two straight lines that realize $7x + 4$ and $5x + 8$", respectively. And also, 123 is the realization of the signifier "$86 + 37$". Different types of signifiers' realization in mathematical discourse were given in the Figure 2.1. These realizations were firstly grouped in two forms. The first one was vocal realization consisting of verbal-spoken words. The second type of realizations was visual realizations consisting of verbal, iconic, concrete and gestural realizations. Verbal realizations were also categorized as written words and algebraic symbols.

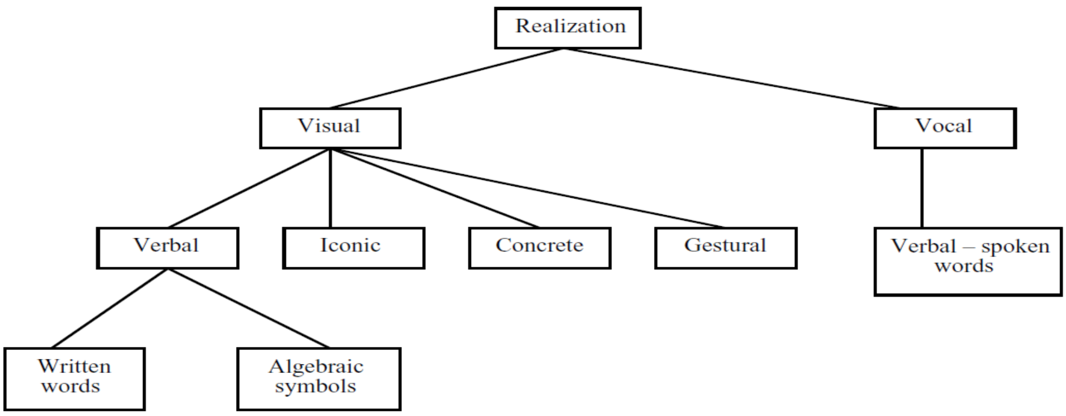


Figure 2.1 Different types (modalities) of signifiers' realization in mathematical discourse (Sfard, 2008 p.155)

Realization of one signifier would lead to the realization of other signifiers. For example, realizing the "function g" as $5x - 5$ would lead to the realization of

$g(6)$. Moreover, realization of a signifier would not only depend on visual mediation. For instance, one could use a table, algebraic formula and a graph to realize a "function g". Most of the case, the signifiers and the realization relation is symmetrical. For example, "$86 + 37$" is signifier of the realization "123", also it could be assigned in the reverse order and 123 "" becomes the signifier and "$86 + 37$" becomes the realization. As in the case of the realization of the signifier "function g", the transition from signifier to realization would be immediate. On the other hand, this transition would be mediated like the transition of the signifier "$7x + 4$" to the realization "a particular straight line".

There would be transitions from one medium to another while realizing a signifier. This diversity of visual realizations makes the communication more effective and makes the people to express themselves more appropriately. Some narratives would be defined in different ways and certain ways would be easier to use and construct for some people. On the other hand, usage of some realizations would be preferred for the specific discursive rules. For example, iconic and concrete realizations would facilitate production of factual narratives; mathematicians prefer symbolic realizations as they find this way more reliable for the endorsement of the narratives. For example, one should use symbolic realizations for finding the intersecting point of the lines of $7x + 4$ and $5x + 8$, although he would use or imagine the graphs of these lines for the solution considered appropriate for the mathematicians.

Realization procedures require a combination of verbal actions, visual scanning and physical manipulation and the amount of these processes differs according to medium in which the process takes place. For example, operating on symbols is mostly linguistic activity, on the other hand iconic and concrete procedures entails relatively small amount of verbalization. For the implementation of the iconic and concrete procedures one uses her eyes for the scan and sometimes her hands for physical transformation as it happens in the addition process. In the addition process one puts the two sets of objects together and counts the elements of this new set. On the other hand, symbolic realizations require sequential discursive procedures. These procedures need one's memory more than the iconic and concrete processes.

Frequently repeated realization procedures may become embodied and automated. The necessary scanning and physical actions are remembered by our bodies rather than our minds as a series of discursive moves such as swimming, bicycling or typing. This procedure is called embodiment. Moreover, in some cases all components of procedures are performed without verbal descriptions without an explicit thought about the connection and without thinking what the next step is. This procedure is called automation. Embodiment and automation of realization procedures are very common in colloquial discourses. For example, for the workers of a warehouse using mental arithmetic while visualizing the container of different shapes and sizes while how to arrange the certain amount of milk. This process is embodied and automated for the workers.

In literate discourse, embodiment and automation also occur. For example, when asked to perform the addition procedure of two fractions given in the form of $(3, 5) + (7, 12)$ we would stop and think and would try to revert this form into the classical form $\frac{3}{5} + \frac{7}{12}$ as we get used to. The fraction symbol of $\frac{a}{b}$ became the leading signifier for the simple fraction through the years of practice. The canonical vertical form becomes our second chose. It is similar for the example of identifying functions. We use algebraic symbols to identify functions rather than using the graph of this function. As it is explained in the examples, embodied and automated realizations direct us to the leading signified object and the other representations would be accounted as the trivial representation of the object.

Algebraic symbolic realization has an important feature especially for the literate mathematical discourse. Algebraic symbolic realization like $\frac{134}{29}$ and $2x - 5$ are shortcuts for verbal expressions. With symbolic expressions, spoken discourse becomes permanent and the different discursive elements become simultaneously present. The symbolically expressed mathematical discourse is more appropriate than its spoken or even written expressions to become an object of metadiscursive activity. When symbolic medium is used, process of realization turn into their outcome. A single symbolic discursive expression becomes as both a series of actions and a noun which generates this process's realization. Symbolic realizations also save time and labor. Moreover, when symbolic realization is used, discourse becomes more effective and applicable. Concrete realization is used only when the material is present and the person is able to perform the task with the material. However, when we are associated with the symbolic system, it becomes our own property and it is always with us.

As human communication is a rule-regulated activity, it is essential to define object-level and meta-level rules. Object-level rules are narratives about the regularities in the behavior of objects of the discourse. On the other hand metarules define patterns in the activity of the discursants trying to produce and substantiate object-level narratives (Sfard, 2008, p.201). For example, the mathematical narratives on geometrical shapes "the sum of the angles in a polygon with n-sides equals "$(n-2)x180$" is an object-level rule. "To multiply a sum of two numbers by a third number one can first multiply each addend and then add the products" is a metarule of arithmetic. As mathematics is an autopoietic system which grows by adding its own metadiscourses, metarules in one mathematical discourse will turn to an object-level rule. For example, the metarule of arithmetic "to multiply a sum of two numbers by a third number one can first multiply each addend and then add the products" becomes an object-level rule "$a(b+c)=ab+ac$" which express the relation between three algebraic objects, the variable a, b, c (the domain is real numbers).

Metadiscursive rules have some characteristics which are differentiated from other rules. Metadiscursive rules (metarules) may evolve over time. The activities of defining, substantiating, recording are arranged by the metarules of mathematics. The aim of school learning is to make students have the metarules appropriately for development of their mathematical discourse. However, students' mathematical discourse shaped by the experts especially the teachers that teach the mathematics. Students' metarules changes according to their teacher or any expert they study. Therefore, metarules show variability.

Metarules that are accepted as a person's own is called endorsed while how an observer interpret this person's action is called enacted. According to the observer; enacted metarules are described how they occur, however the endorsed metarules are explained by the discursants. Therefore, they could be different. For example, a students' enacted metarule is "use concrete materials while calculating", as she was observed counting finger while calculating two numbers. But when she asked to show how she counted fingers, she refused and said "I do it silently, so that people won't see". She was counting finger as opposing to the arithmetic behavior. Probably the reason for the difference between the endorsed and enacted metarules would be the students could not give up the rules and habit they had by their experiences. Metarules should base on a standard which are expected by the experts of the community. For a metarule to be a norm, the rule should have two properties. First of all, it should be enacted by the community and then it must be endorsed by almost everybody. This metarule must be accepted as one of the defining and characteristics of the given type of discourse. Metarules make communication possible and these rules prevent countless possible discursive alternatives and make the interlocutors to be in the borders of the actual discourse. For example, if a mathematics teacher say "investigate the function $f(x) = 3x^3 - 2x + 5$" the students would not be sure what to investigate the graph or the real life applications of the given function. Likewise, when one asked "find x" the discursants of mathematics would not saying anything rather than solving the equation and finding the value of x.

These repetitive discursive actions are defined by two subsets: how routine and when routine. The how routine is the metarules that identify the way of the discursive action (course of action). The "when routine" is the metarules that identify the cases which the discursant would accept processes and actions as appropriate. Routines are general to the most of the discourses but some would be specific to the certain discourses like anthropology or sociology investigating community specific metarules. In the how routine, metarules define the actions that are activated in an order as a response to a question. It is not easy to determine the when routines of the discourse of a group or a person. In this case, anomalies are watched and listed rather than the normal patterns. For both how and when routines, past experiences are the clues for the predictions of the future actions. These identified metarules provide the observer for the discursive development.

Applicability conditions, the course of action (procedure) and the closing conditions of the routine are the subsets of the set of metarules which constitute the routines. Applicability conditions and the closing conditions generate when routine and the course of action (procedure) form the how routine. Two identical students in their performance would differ in the applicability and the closing conditions of their routines as how routine is obvious besides, when routine is constituted by a work through a whole life.

2.2 The Notion of Derivative

The notion of speed especially the speed of an object at an instant time was problematic through history (Hughes-Hallett, Gleason & et al., 1992). There was a paradox in trying to quantify the property of motion at a particular instant in time, since by focusing on a single instant you stopped the motion. Problems of motion were in the middle of the interest of Zeno and other philosophers of 5^{th} century BC. However, Newton's calculus produced a modern perspective and gave up looking for a simple notion of speed at an instant and began to look at speed over small intervals containing the instant.

Calculus books gave the definition of derivative in the following manner by using different notations:

The derivative of a function f is another function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

at all points x for which the limit exists. (Adams, 1995, p.98)

In different Calculus books different approaches were used for the derivative concept. For example, Adams introduced the concept of derivative with the geometrical approach by using the tangent line and its slope (Adams, 1995). Then he gave the above definition and introduced the function form of the derivative. He followed by the differentiation rules and application procedures. On the other hand, Silverman (1985) introduced the derivative concept using the velocity, rate of change and instantaneous rate of change by elaborating some physical problems of velocity of a stone dropped into a deep dry and average density of piece of rods. After giving the definition of derivative and some differentiation rules, he introduced the tangent line and continued other differentiation rules and finished with applications. Hughes-Hallet, Gleason and et al. (1992) followed a different approach guided by two principles. The first one was "every topic should be presented geometrically, numerically and algebraically and the second one was the way of Archimedes "formal definitions and procedures evolved from the investigation of practical problems" (p. v). They firstly studied average and instantaneous velocity. They mentioned about the derivative function after giving the average rate of change and slope of tangent line. Then they gave the differentiation rules and the application of derivative.

2.3 Research Studies Related to Students' Learning of Derivative Concept

Research studies related to students' learning of the derivative concept mentioned in this section would be grouped in three categories. The first group is the studies investigating students' reasoning of the derivative concept according to the multiple representations (Amoah & Laridon, 2004; Habre & Abboud, 2006; Zandieh, 1997; Zandieh, 2000; Zandieh & Knapp, 2006). The second categorization contains the studies related to students' understanding of the rate of change and the relation between the concept of derivative and the tangent line (Bezuidenhout, 1998; Orton, 1983; Tall, 1986; White & Mitchelmore, 1996). The last category is related to the students' graphical understanding of a function and its derivative (Asiala, Cottrill, Dubinsky & Scwingerdorf, 1997; Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Habre & Abboud, 2006).

Studies emphasizing multiple representation of the derivative concept analyzed students' graphical, numerical and algebraic understanding of the derivative concept. According to these studies teaching concepts using different representational methods and making connections between these representations increased students' understanding of the concept of derivative (Amoah & Laridon, 2004; Habre & Abboud, 2006). Most of the students participated in these studies had problems moving different representational modes such as symbolic equations, tables of values and graphs for the derivative concept. These students also had troubles finding the derivative at a point graphically. Their tendency was finding the algebraic equation of the function and then finding the derivative value by using the differentiation rules. Algebraic representation of a function was used mostly by the students.

In their study, Amoah and Laridon (2004) investigated students' graphical, numerical and algebraic understanding of the derivative concepts after differential calculus course. In this study, in the teaching approach of the five groups, the emphasis was on concepts. This teaching approach aimed at mathematical sense making. All groups used the same worksheets containing numerical, graphical and elementary applications of the derivative. And also, students' written work and mathematical discussions between peers and with the facilitator/lecturer were used to identify students' ideas. They also developed a test to obtain information on the students' conceptual understanding of differential calculus. 150 students took the test. According to the results of the study, students have difficulties to move comfortably among the different representational modes as in symbolic equations, tables of values and graphs for the derivative concept. Most of the students could not find the derivative at a point from the graph. Only 39 (26 %) students out of the 150 students were able to find the derivative at a point graphically. Incorrect answers given because some of the students confused the derivative at the point with y-value of the point of tangency or some students had difficulty in computing the gradient of the tangent to the curve. Others tried to find an equation for the function which only graphically represented.

In another study on multiple representation of the concept of derivative, Habre and Abboud (2006) analyzed calculus students' understanding of the function concept and its derivative in a non-traditional calculus course emphasizing graphical, numerical and symbolic notions of the concept of derivative. In this course, the concept of derivative was taught by first discussing the rate of change of a function at a given point as the limit of average rate of change, proceeded to relate the result to the slope of a tangent line, to arrive finally at the analytic definition of the derivative. Technology was also employed in well chosen problems as a tool assisting in the exploration of problems, allowing students to visualize, reflect, analyze and modify their thinking until an appropriate conclusion was reached. According to the interview and students' performance on exam questions, some of the students thought function as a graph; very few of them related graphical representation with a function. Some students could not visualize the functions without seeing the appropriate examples and they applied prototypical examples to construct the concept in their mind. Most of the students had a complete understanding of the concept of derivative geometrically, as the idea of instantaneous rate of change and the slope of a curve at a given point, but they could not define the concept geometrically. Very few of the students employed mechanical methods for finding derivative and algebraic representation of a function still dominated their thinking.

Another approach focusing on multiple representations was defined by Zandieh (1997; 2000). In the Figure 2.2, Zandieh (1997) explains understanding the concept of derivative as understanding the concept in three forms (Zandieh, 1997; Zandieh, 2000); a ratio, a limit and a function. For a ratio form, the derivative is a slope or a rate of change in y divided by the change in x. In the limit form, derivative is the limit of the slopes of secant lines or the limit of difference quotients. For the function form, each input has a meaning as an output such as the slope of the tangent line.

Zandieh (1997; 2000) explained the concept of derivative in two main components. The first one was the multiple representations or context and the other one was the layers of process-object pairs. According to Zandieh's outline of the framework for the concept of derivative was given. In this framework, the concept of derivative was represented by three forms (Zandieh, 2000). Derivative concept was represented graphically as the slope of the tangent line to a curve at a point or as the slope of the line a curve seems to approach under magnification; verbally as the instantaneous rate of change; physically speed or velocity and symbolically as the limit of the difference quotient. In the second component, Zandieh explained the concept of derivative in three aspects. These are ratio, limit and function and called as the layers of the framework. Zandieh (2000) defined the derivative of f, f', as a function whose value at any point is defined as the limit of a ratio. She explained these three layers by the process object duality and structured the concept of derivative in the following matrix (Zandieh, 2000). In her framework, Zandieh added a third dimension and explained the notions ratio, limit and function in two forms as process and object (Zandieh, 2000). A ratio would be thought as division as a process and a pair of integers as an object. In the same way limit would be thought as a process as approaching the limiting value and an object with the definition of epsilon delta. Also, function would be a process taking an element and producing another one and an object a set of ordered pairs.

			Contexts		
	Graphical	Verbal	Paradigmatic Physical	Symbolic	Other
Process-object layer	Slope	Rate	Velocity	Difference Quotient	
Ratio Limit Function					

Figure 2.2 Zandieh's outline of the framework for the concept of derivative.

Zandieh and Knapp (2006) explained students' reasoning of the derivative concept by examining the roles of metonymy by using the Zandieh's framework of three layers. They took into consideration of Lakoff's description of a metonymy as "either easier to understand, easier to remember, easier to recognize, or more immediately useful for the given purpose" (Zandieh & Knapp; 2006, p.14). They concluded that there were three metonymies in students' reasoning of the derivative concept. The first one was students preferred to use only one column or context to explain the whole concept of derivative, although it would be more useful and easier to explain the whole concept by using another context or representation such as using

the graphical representation instead of physical context like velocity. In the second one, students used single row or layer to express again the whole concept. Students used the rate of change layer despite it only defined the average
rate of change not the limiting process of this average rate of change. The third and the most complex metonymic relationship was seen in the chaning of the process-object pairs. It means that one part of the derivative structure was used instead of another part of the derivative structure.

Tall (1986) explains the importance of discussion and negotiation of the meaning of the complicated situations through studying appropriate examples and non-examples between teacher and students for abstraction of mathematical notions (p.70).

> Mathematicians analyze concepts in a formal manner, producing a hierarchical development that may be inappropriate for the developing learner. Instead of clear, formal definitions, it may be better for the learner to meet moderately complicated situations which require the abstraction of essential points through handling appropriate examples and non-examples. Such complexity requires discussion and negotiation of meaning between teacher and pupils.

Tall (1986) investigated whether the interactive computer programs, encouraging teacher demonstration and pupil investigation of a wide variety of examples and non-examples would help students develop a richer concept image. In this purpose, three experimental classes of sixteen year-olds were taught using computer packages to form the relationship between gradient and tangent. Five other classes were taught traditional methods for comparison. In these three experimental groups, students were encouraged to work with the computer in small groups after teacher's demonstration to lead a discussion centered on the computer. The aim of these activities was to discuss the meaning of the tangent using the computer to sketch a line through two very close points on the graph as a part of the notion of gradient of a graph. The control classes followed a more traditional strategy assuming an intuitive knowledge of the meaning of a tangent. As a result of this study, Tall concluded that experimental groups were better able to explain the tangent/gradient at a point where the formula changed but left and right gradients were the same. This result supported the theory that software provided the students to manipulate examples and non-examples of the concept in complex situations. On the other hand, the notion of genetic tangent (an imagined line touching the graph at only one point) persisted in both groups.

To be able to symbolize the derivatives of the given problems require forming the relationships between concepts and this process indicates the conceptual knowledge of the aforementioned concept. According to their study, White and Mitchelmore (1996) reported that some students had difficulties to symbolize rate of change in complex situations. They reached this result after studying twenty four hour concept-based calculus instruction with the sample of fourty first year calculus students. Some students had difficulties in the development of the concept of variable. They struggled applying procedures related to the concept of derivative on the given variable and constructing the meaning or the relationship of these variables to the concept of derivative (White and Mitchelmore, 1996). They had problems to identify and symbolize an appropriate variable by translating the given quantities in the items and also symbolizing the quantities in an appropriate form as they considered the symbols as the objects that well known manipulation rules could be applied.

All cognitive structure of an individual for a concept was determined by the images of this concept (Tall & Vinner, 1981). These cognitive structures of a concept was called concept image. This image included all the mental pictures and associated properties and processes. For each learner concept image was determined by his/her concept definition. Concept definition was the form of words used to specify the considered concept (Tall & Vinner, 1981). As the information was constructed by the individual, personal concept definition would be different from the formal definition of the concept. These personal concept images would provide some conflict for the conceptualization of the concepts.

Some studies related to the concept of derivative revealed that students had some common misunderstandings because of insufficient concept image of average rate of change and tangent (Bezuidenhout, 1998; Orton, 1983). All cognitive structure of an individual for a concept was determined by the images of this concept (Tall & Vinner, 1981). These cognitive structures of a concept was called concept image. This image included all the mental pictures and associated properties and processes. For each learner concept image was determined by his/her concept definition. Concept definition was the form of words used to specify the considered concept (Tall & Vinner, 1981). As the information was constructed by the individual, personal concept definition would be different from the formal definition of the concept. These personal concept images would provide some conflict for the conceptualization of the concepts.

One of the early studies investigating students' understanding of the concept of derivative was conducted by Orton in 1983. Orton (1983) administered a clinical interview with 110 students aged between 16-22 years to reveal their understanding of rate of change and differentiation. The results of the interview showed that students had common errors in understanding the concepts related to derivative such as tangent line and rate of change. Most of the students thought that the tangent line was the limit of the secant lines. They gave the answers of "the line gets shorter", "it becomes a point", "the area gets smaller", and "it disappears" related to the secant line (Orton, 1983, p.237). Another result of this study was some students had difficulty with the graphical understanding of the rate of change. These students thought average rate of change was calculated in the same way for a curve and for a straight line. Thus, students had difficulty to understand the difference of the average rate of change of a curve and a straight line. These results revealed that they could not make sense of average rate of change was the same for a line in every interval. On the contrary, they supposed that average rate of change should change in every interval.

In another study, Bezuidenhout (1998) investigate students' errors and misconceptions related to the concept of derivative. The results of this study revealed that some students have deficiencies related to the concept images of the graphical representation of the rate of change. Besides, test and interview results revealed that some students had confusion related to the average rate of change and arithmetic mean (Bezuidenhout, 1998). They confused the meanings of "average rate of change", "average value of a continuous function" and "arithmetic mean". Bezuidenhout (1998) connected this result that students memorized the rules without thinking the conceptual meaning. They did not know the meaning of average rate of change and they just knew the rule and applied when it was asked.

Hiebert and Carpenter (1992) defined mathematical understanding as "a mathematical idea or procedure is understood if it is a part of an internal network. The degree of understanding is determined by the number and the strength of the connections. A mathematical idea, procedure or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections" (p. 67).There is a strong link between conceptual knowledge and mathematical understanding according to the Hiebert's and Carpenter's definition of the mathematical understanding. Conceptual knowledge is "the knowledge that is rich in relationship" (Hiebert and Lefevre, 1986). Procedural knowledge is "the set of symbols and algorithms, where the essential features include actions, transformations that are connected and executed in a linear or sequential fashion" (Hiebert and Lefevre, 1986). Vinner (1997) perceives the conceptual understanding similar as the Skemp's relational understanding; knowing what to do and why (Skemp, 1976). Students apply procedures mechanically without thinking about the related conceptual knowledge. Instruction should give emphasis more on procedural knowledge, although both conceptual and procedural knowledge was important (Hiebert & Carpenter, 1992).

As mathematical understanding requires constructing stronger or numerous connections between concepts (Hiebert & Carpenter, 1992; Tall, 1986), the relationship between the derivative of a function at a point and the slope of the line tangent to the graph of the function at that point forms the basis for understanding the derivative as a function (Asiala, et. al, 1997). Also this relationship gives the corresponding value of the slope for each point in the domain of the derivative. In their study, Asiala et. al. (1997) explored 41 calculus students' graphical understanding of a function and its derivative. They designed an instructional treatment called ACE teaching cycle (Activities, Class and Exercises) based on the genetic decomposition of the mathematical concepts. The main strategy of this instructional method was to provide students to construct mathematical ideas on the computer using a mathematical programming language. In this study, students investigated the mathematical concepts using computer system and engaged in problem solving activities and discussions working in cooperative groups. They analyzed the students' understanding according to the Action-Process-Object-Schema (APOS) theoretical framework. They reported some difficulties students had related with graphical understanding of a function and its derivative. For example, some students had tendency to equate the derivative function to the equation for the line tangent to the graph at a given point. Some students stated correct formula for finding the slope by using the differentiation rules but did not compute it correctly. Moreover, they reported that in some cases students associated the function underlying the original function and in other cases they identified it with the derivative of that function. They also revealed that some students had tendency to have the expression for the function to differentiate rather than using the given data.

Students had difficulty in conceptualizing the derivative as a function (Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Thompson, 1994; Ubuz, 2001). Thompson (1994) analyzed a teaching experiment with 19 senior and graduate mathematics students enrolled in a course on computers in teaching mathematics. This group composed of 7 senior mathematics majors, 1 senior elementary education major, 10 master students in secondary education and 1 master student in applied mathematics. In this study, Thompson (1994) investigated students' understanding of the concepts of derivative and integral according to the Piaget's notion of internalization of objects and actions. Results of the study revealed that many students had a figural image of function. They suggest that a function was an image of a short expression on the left and a long expression on the right, separated by an equal sign. Many students referred to the visual object, the graph of the function not to the covariation of two variables. And also, students' images of Riemann sums were insufficient to support their reasoning about sums of rate of change. Students' images of Riemann sum seemed not to have entailed a sense of motion, either its argument or its value. Results of the analysis of the study also showed that students did not have operational schemes for average rate of change. For the operational scheme of the average rate of change, Thompson (1994) meant that if a quantity were to grow in measure at a constant rate of change with respect to a uniformly changing quantity. For example, an average rate speed of 40 km/hr on a trip means that if we were to repeat the trip travelling at a constant rate of 40 km/hr, then one would travel precisely the same amount of distance in the same amount of time as same as it was in the first case. This notion of the derivative is related to the Mean Value Theorem which means that all differentiable functions do have an average rate of change over an interval and it is equal to some instantaneous rate of change within that interval.

In their study Baker, Cooley and Trigueros (2000) analyzed 41 engineering, mathematics and science students' understanding of the calculus concepts used in solving non-routine calculus graphing. They wanted the students to sketch the graph of the function whose analytic properties such as first and second derivatives, limits and continuity were given on a specific interval. They investigated students' conceptualizations of the graphical implications of the first and second derivatives, continuity and the value of limits and how students used these components to sketch the graph of the function. They analyzed the detailed students' responses in both oral and written interviews according to APOS theory. Results of this study revealed that students sketched the graph mostly relying on the first derivative of the function and they had weak conception or misconception of the first derivative as a function. Analysis also revealed that students had trouble to understand the vertical tangent at x=0 and the limit on the derivative.

In another study, Berry and Nyman (2003) observed students' understanding about the link between the graph of the derived function and the original function. Students were asked to sketch the original graph of the four graphs of the derived functions and then walk these graphs as if they were displacement-time graphs. To reveal the results students' discussions were audio recorded and their walks were captured using their data logging equipment. All these data and the students' paper and pencil notes were analyzed. Then they concluded that the students have an algebraicsymbolic view of calculus and find it difficult to make connections between the graphs of a derived function andthe function itself.

Ubuz (2001) found that students both who used computer applications and who didn't had common mistakes before and after the calculus course. Students had difficulties related to the derivative as a function. They thought that the derivative at a point gives the function of a derivative or tangent equation was the derivative function. Moreover, they had misconceptions related to the tangent line and its equation. They supposed that derivative at a point was the tangent equation and derivative at a point is the value of the tangent equation at that point. Another dimension of the same study with 147 first year engineering students who studied calculus with or without computer from four universities investigated the conception and misconceptions of the concept of derivative and sketching the graph of a function and its derivative graph (Ubuz, 2007). Analysis of the answers to the test questions and responses to the follow-up interviews of with and without computer groups revealed that students used prototypes; they had weak understanding of the limit concept and confused the process-product. Moreover, they had problems using graphical information.

Another study related to the students' understanding of the graphical connections between a function and its derivative was conducted by Aspinwall et.al. (1997). They investigated one students' use of imagery in understanding the graphical connections between a function and its derivative using the case study method. Student who had completed a year of study of elementary calculus were employed 20 tasks. He was given non-routine problems to determine the graphs of the derivatives. Results revealed that he sketch cubic function as the derivative graph of the polynomial function having vertical asymptotes because of his mistaken imagery. Moreover, he thought that the derivative of the quadratic function should be a line as he had analytic knowledge of the derivative.

Students' understanding of the concept of derivative is influenced by their department and by the perspectives they were taught the concept of derivative such as rate of change, tangent, function or limit. For example, the study conducted by Bingölbali & Monaghan (2008) with 50 mechanical engineering and 32 mathematics students by administering pre-test, post-test and delayed post-tests, questionnaires and interviews and observing lessons and coffee house discussions revealed that mechanical engineering students' concept images developed in the direction of rate of change and the mathematics students' concept images developed in the direction of tangent aspects. They concluded that these results were because of their department's perspective and their practice they got from their department (Bingölbali & Monaghan, 2008).

2.4 Summary

The areas of research on the concept of derivative were basically related to students' understanding of rate of change, graphical representation of rate of change (Bezuidenhout, 1998; Orton, 1983; Tall, 1986; White & Mitchelmore, 1996), relation between the tangent line and the derivative concept and the derivative function (Asiala, Cottrill, Dubinsky & Scwingerdorf, 1997; Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Habre & Abboud, 2006). According to the results of these studies, similar errors, misconceptions or weak concept images related to the concept of derivative were determined. Results of these studies indicated that most of the students had difficulty with graphical understanding of the rate of change (Bezuidenhout, 1998; Orton, 1983; Tall, 1986; White & Mitchelmore, 1996). They had problems with moving different representational modes of the derivative concept (Amoah & Laridon, 2004; Habre & Abboud, 2006). They had difficulty in using graphical information to find the derivative value at a point or derivative function of the given function (Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Thompson, 1994; Ubuz, 2001). Students also tend to use the algebraic equation of the function and then finding the derivative value by using the differentiation rules. Another problem that students encountered in terms of the derivative concept was that they had trouble in conceptualizing the derivative as a function. Research results showed that derivative learners had problems, deficiencies and misconceptions about rate of change, increasing and decreasing function and its relation to derivative function, function graph and the graph of the derivative function in classroom, group and individual settings. In this present study, pre-service elementary mathematics teachers' conception of the derivative related to these notions will be tried to be put forward.

In most of the studies related to derivative concept, it was noticed that students' understanding of the derivative concept was determined by analyzing the test results of the students or their individual performances of written tasks (Amoah & Laridon, 2004; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Habre & Abboud, 2006; Orton, 1983; Thompson, 1994; Ubuz, 2001; Ubuz, 2007; White & Mitchelmore, 1996). Very few of these studies focused on the performances of the students in small or large groups and in most of these studies, students worked in pairs. Very few of these studies investigated the notions of students' understanding of the concept of derivative in the group or classroom discussions. Therefore, there is an urgent need to reveal learners' conception of the derivative concept in the group and classroom discussions. This present study also aims to determine and explain pre-service elementary mathematics teachers' conception of derivative in group, classroom and individual discourses.

In most of the studies searching students' understanding of the derivative concept, participants are the students of engineering, mathematics or science majors (Aspinwall, Shaw & Presmeg, 1997; Baker, Cooley & Trigueros, 2000; Berry & Nyman, 2003; Bingölbali & Monaghan, 2008; Ubuz, 2001; Ubuz, 2007) Very few of the participants are from mathematics teacher education majors (Thompson, 1994). Thus, studying the discourse of pre-service elementary mathematics teachers on derivative and conception of derivative concept will enable the instructors of calculus and mathematics teaching method courses to learn more about teacher candidates and their discourse and to plan instruction accordingly. In this present study, pre-service elementary mathematics teachers' discourse on derivative and their conception of derivative will be determined with group, classroom and individual discourses.

According to the followers of the socio-cultural tradition, the community affects the change in the learner's activities (Sfard, 2001). As the way people behave would change from one situation to another. Observing the students in the different settings will provide researchers to identify the understanding of students more efficiently. As human practice is produced and sustained in the society, there is a need for interaction and communication. Researches related to the concept of derivative shows that there is a less emphasize on the students' interaction between each other in small and large group settings. In some studies as there is less interaction between the students and the instructors, individual interviews are constructed to show students’ understanding of the concept of derivative. As Sfard (2008) states that to be a part of any discourse, one needs to participate in the communicational activities. Analyzing pre-service teachers’ words, visual mediators and narratives will provide the researcher to understand the students’ conceptualizations of the concepts. Therefore, there is a need to provide pre-service teachers’ understanding of mathematical concepts in the different settings such as group, classroom discussions and individual discussions and to provide information about how students behave in these settings.

CHAPTER 3

METHODOLOGY

In this section, methodology of this study was discussed. The design of the study, the participants of the study, the procedures of data collection and data analysis were described.

3.1 Design of the Study

In order to determine pre-service elementary mathematics teachers' discourse on derivative in group, classroom and individual settings from communicational approach to cognition (commognition) perspective, qualitative research methodologies (Creswell, 2007) were used.

There are different definitions for the qualitative research study. Marshall and Rossman (2011) define the qualitative research as a broad approach to the study of social phenomena, it is naturalistic and interpretive and they employ multiple methods of inquiry. Denzin and Lincoln (2005) accept qualitative research as a field of inquiry in its own right. Qualitative research is consisted of a complex, interconnected family of terms, concepts and assumptions. Denzin and Lincoln (2005, p.3) offer another initial and generic definition:

> Qualitative research is a situated activity that locates the observer in the world. It consists of a set of interpretive, material practices that make the world visible. These practices transform the world. They turn the world into a series of representations, including field notes, interviews, conversations, photographs, recordings and memos of the self. At this level qualitative research involves an interpretive, naturalistic approach to the world. This means that qualitative researchers study things in their natural settings

attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them.

Besides, Creswell (2007) defines qualitative research from the point of the design of the research and the use of distinct approaches to inquiry. He proposes that qualitative research begins with assumptions, a worldview, the possible use of theoretical lens and the study of research problems inquiring into the meaning of individuals or groups give to the problem. Moreover, researchers use a qualitative approach to inquiry, they collect data in natural setting and analyze data inductively and establish the patterns or themes. The results of the study presents the thoughts of the participants, the reflexivity of the researcher complex description and interpretation of the problem and it contributes to the literature or comments on the further action.

Bogdan and Biklen (1998) mention about five characteristics of qualitative study. It is *naturalistic* as it has natural settings as the direct source of data and the research is the key instrument. Qualitative research is *descriptive*. Qualitative researchers are *concerned with the process* rather than simply outcomes or products. They analyze their data *inductively.* They don't search out data or evidence to prove or disprove hypotheses they hold before entering the study. For qualitative researchers, *meaning* has importance. They are interested in the participant perspective, how participants make sense of their lives.

Considering these definitions and the characteristics of the qualitative research, it was determined for the design of this research to accomplish the proposed aim. Freshmen pre-service mathematics teachers of the department of elementary education of a university in Central Anatolia region were chosen for the participants of the study. Main focus for the participant selection was being pre-service mathematics teachers. As the freshmen did not have the course covering derivative concept at the university level, they were determined as the participants of this study. In the following part of this section, the procedures for the participant selection and the characteristics of the participants will be explained.

3.2 Participants of the Study

Freshmen pre-service elementary mathematics teachers were the participants of this study. Purposeful sampling method was used for the sampling procedure to reach the purpose of this research and answer the research questions. As the aim of this research was to investigate the pre-service elementary mathematics teachers' conception of derivative concept, freshmen of elementary mathematics teacher education department of a university in Central Anatolia region were chosen for the sample. While selecting the participants, the criteria were being candidates of pre-service mathematics teachers and not before accounted with derivative concept at university level. After determining the characteristics of the participants, I had contacted with the head of mathematics education departments of two universities to conduct the research. One of them did not allowed to conduct such a study for freshmen as the researcher would be the instructor. The other one accepted to conduct the study in the course that the pre-calculus concepts were thought. Thereon I contacted with the instructor of the course and we designed the course that in the first six weeks the instructor covered the pre-calculus subjects. In the remainder weeks I would conduct the study. In the first two weeks, I covered the limit concept as it was the basis for derivative concept. The last five weeks of the fall semester of 2009-2010 education year, I conducted the study.

There were 61 pre-service teachers enrolled the course; 16 of them were male and 45 were female. 45 of them took both first and last applications of derivative tests. They were grouped consisting of three, four or five pre-service teachers at the beginning of the course and they studied within their groups in each class sessions. Pre-service teachers were allowed to form their groups, there wasn't any criteria for the grouping process considering students would study efficiently with people that they chose. Therefore, the class was consisting of fifteen groups. Two groups were consisting of three students, ten groups were consisting of four students and three groups were consisting of five students. For group discussions, I chose a group consisting of 4 freshmen who were all female. I chose this group as each pre-service teacher of this group were interested in the course, very active and also enthusiastic to learn. Moreover, during the pilot study, I had chance to get acquainted with the participants, so I thought that I could get deep understanding of their conception of derivative concept while observing their group discussions.

An interview with six pre-service teachers was conduted. These six pre-service teachers were determined according to the results of the derivative test. Derivative test was conducted to the pre-service teachers as a pre-test at the beginning of the class which cover the derivative concept and as a post-test at the last lesson of the class. It took 60 minutes to complete the test for the participants for both pre and post-test. 45 students took both tests. Descriptive statistics of the pre-test and post-test scores of the derivative test is given in the Table 3.1. Possible minimum score was 0 and possible maximum score was 140 for the test. The minimum score was 27 and the maximum score was 98 for the pre-test and the minimum score was 52 and the maximum score was 124 for the post-test. The mean was 55,44 for pre-test and 91,78 for post-test.

Table 3.1

Descriptive statistics of the pre-test and post-test scores of the derivative test

Descriptive Statistics of Pre-Test and Post-Test Scores of Derivative Test		
	Pre-Test	Post-Test
Mean	55,44	91,78
Median	53,00	94,00
Std. Deviation	17,52	16,76

Possible maximum and minimum scores for both exams: 140 and 0.

In Figure 3.1 box plot shows the increase in the means of the pre-test to post-test scores of the derivative test. Minimum score of the test increased from 27 to 52.

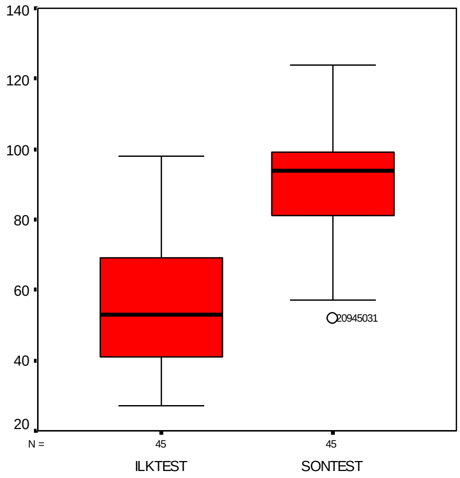


Figure 3.1 Box plot displays of the pre-test and post-test derivative scores

Differences of the scores of first and last applications of derivative test determined. Pre-service teachers were arranged according to these differences of the scores in an ascending order. Pre-service teachers were divided into three groups consisting of equal number of people in each group to determine small change, average change and big change in their scores. Two pre-service teachers were chosen from each group. Two of them were male and four of them were female. They were chosen because I thought that I could get deep understanding of their conception of derivative in individual discourse. Moreover, as choosing the ones who were interested in the course and enthusiastic to learn, I could get more information about their individual discourse on derivative.

Six pre-service teachers were chosen for the individual interviews. These pre-service teachers were Sezen, Semra, Yasin, Yakup, Meral and Suzan. These names are pseudonyms. Yasin, Yakup, Meral and Suzan were gradute d from Anatolian Teacher High School. Sezen and Semra were graduted from Anatolian High School. Their pre-test and post-test scores were: Sezen got 61 from the pre-test and 121 from the post-test. Semra got 38 from pre-test and 105 from the post-test. Meral got 64 from pre-test and 79 from post-test. Yasin got 83 from pre-test and 106 from post-test. Suzan got 67 from pre-test and 96 from post-test. Yakup got 64 from pre-test and 99 from post-test. Their definitions of the derivative in the pre-test were: Sezen and Semra didn’t defined derivative in the pre-test. Yakup defined derivative as “to reduce the higher order functions to fewer orders”. Yasin defined derivative as “it is a new form of an expression that changed according to certain rules”. Meral defined derivative as “Derivative of a function like $y = ax^2 + bx + c$ would be found by multiplying the power of the x values with this x value and reducing the power 1 degree. It is $y' = a.2.x^{2-1} + b.1.x^{1-1} = 2ax + b$”. Suzan defined derivative as “to find the slope of the tangent sketchedn to the graph of a function”.

All participants were at their first year at university and were taking the course first time except one participant. He was not from Turkish Nationality, he was from Turkmenistan. He was taking the course second time. Because of the language problems he wasn’t active in group and classroom activities.

3.3 The Research Procedure

Data was collected from November 2009 to January 2010 and February 2010 to March 2010. Data for this study was collected in four steps. The first step included pilot study conducted in November 2009. The second step included application of derivative test. The third step included group and classroom discussions. The fourth step included individual interviews. A timeline for the data collection procedure used in four steps is given in Table 3.2.

Table 3.2

Timeline of data collection

Step	Week	Date	Data Collection Procedure
1	1	December 2 December 3	Pilot Study
	2	December 9	Pilot study
2	2	December 10	Derivative pre-test was given
3	2	December 10	Group discussion on 1st worksheet Classroom discussion on 1st Worksheet
	3	December 16	Group discussion on 2nd worksheet Classroom discussion on 2nd Worksheet
		December 17	Group discussion on 3rd worksheet
	4	December 23	Classroom discussion on 3rd worksheet
		December 24	Group discussion on 4th worksheet Classroom discussion on 4th worksheet
	5	December 30	Group discussion on 5th worksheet Classroom discussion on 5th worksheet
		December 31	Group discussion on 6th worksheet Classroom discussion on 6th worksheet
	6	January 6	Group discussion on 7th worksheet Classroom discussion on 7th worksheet
		January 7	Derivative post-test was given
4		February 22- March 26	Individual interviews

3.4 Data Collection

The primary sources of the data for this study consisted of video records of the each class session, responses to the derivative test and the task-based interviews including students' written work. The data was collected at the last five weeks of the fall semester and the spring semester of 2009-2010 academic year.

3.4.1 Derivative Test

The derivative test was developed by the researcher to evaluate the pre-service elementary mathematics teachers' conception of the derivative concept. It was administered to the participants twice. First administration was at the beginning of the course. It was applied as a pre-test to reveal the pre-service teachers' prerequisite knowledge about the derivative concept. The second administration was at the end of the course. The results of the first and second administrations were used to select the participants to interview. Answers given in the second application of the test was used in the interview.

The test was developed by the researcher by examining the related literature. The objectives of the derivative test were designed based on the concepts related to derivative. For the face validity of the test objectives were grouped according to the subjects related to derivative concept. The specification table of the derivative test for the objectives and number of items was given in the Appendix B. For the content validity, the test was checked by six university calculus instructors according to the appropriateness of the content, format, language. Test was revised according to their comments. Description of the questions was given in the Table 3.2. Test consisted of 15 items related to the definition of derivative, daily life applications of derivative, increasing and decreasing functions, local minimum and local maximum points. For the reliability of the test, pilot study was conducted. Test was piloted to 116 pre-service mathematics teachers of mathematics education department of a university. According to the pilot study, Cronbach alfa coefficient was 0.85 which implies a sufficient reliability for the test (Fraenkel & Wallen, 2000). Turkish version of the test was given in the Appendix C. Derivative test was partially graded. Grading criteria used for each question were given in the derivative test scoring rubric in Appendix D.

The first, third, seventh and tenth questions of the second application of the derivative test were analyzed to reveal individual discourse of pre-service teachers on the derivative concept and their conception of the derivative, the rate of change and the increasing and decreasing functions. These questions were chosen as they provided more conceptual knowledge on derivative and in other questions answers of pre-service teachers repeated themselves. If there was any different answer it was included into the appropriate question. First question wanted the pre-service teachers define the derivative concept and explain their answers. The second question was related to the rate of change. In this question pre-service teachers wanted to find the derivative value at the intended point according to the graph of the function without knowing the algebraic equaition of the function. The aim of this question is to determine knowledge of pre-service teachers about the rate of change. The seventh question was related to the increasing and decreasing functions and their relation to the first derivative of the function and also the extremum points of a function. This question wanted pre-service teachers to find the intervals where the function was increasing and decreasing and the extremum points of the function by evaluating the given graph of the derivative function. The tenth question wanted pre-service teachers to sketch the graph of the derivative function according to the graph of the function

Table 3.3

Description of the derivative test questions

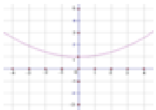
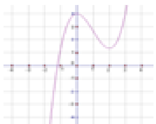
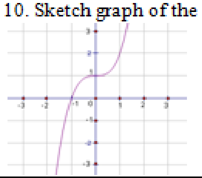
<table>
<tr><th>Question</th><th>Information about the question</th></tr>
<tr><td>1. What is derivative? Explain your answer.</td><td>It was prepared to reveal pre-service teachers' conception of derivative concept</td></tr>
<tr><td>2. Where do we acquaint with derivative concept in daily life? Explain your answer.</td><td>It was prepared to determine whether pre-service teachers have any knowledge related to daily life applications of derivative concept.</td></tr>
<tr><td>3.

<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y</td><td>1,0</td><td>1,1</td><td>1,4</td><td>1,9</td></tr></table>Find an approximate derivative value at x=2 for the function of that graph and some values were given in the table above.</td><td>It was taken and adapted from Garner and Garner (2001). In this question whether pre-service teachers comprehended the derivative concept as limit of the average rate of change was examined. Whether pre-service teachers would use the graph of the function and some of its values given in the table rather than using the algebraic expression of the function to find the derivative value was also investigated.</td></tr>
<tr><td>4. Find the derivative of the function $f(x) = x + \frac{1}{x}$ by using the definition of derivative.</td><td>It was prepared to reveal pre-service teachers' conception of derivative concept as limit of the difference quotients.</td></tr>
<tr><td>5. Find the first derivative of the given functions.
a) $y = (3x^2 + 1)^3$, b) $y = xe^{2x}$, c) $y = \frac{e^{sin2x}}{cos^2(x)}$, d) $y = (\ln(3^x + x^3))^4$,
e) $y = tan^3(x)\sec(2x) + \arctan(x^4)$</td><td>It was prepared to reveal how pre-service teachers apply the differentiation rules for the polynomial functions, exponential functions, rational functions, logarithmic functions and trigonometric functions.</td></tr>
<tr><td>6. Find the values according to the function graph given below.
a) $f(0)$, b) $f(3)$, c) $f'(0)$, d) $f'(3)$, e) $\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$
f) Does $f'(1)$ exist? If it does, find it. If it does not, explain why.</td><td>It was taken and adapted from Hartter (1995). Pre-service teachers' understanding of the continuity of a function graph and right and left derivative was examined. The graph of a piecewise function was given and the pre-service teachers were wanted to find some derivative values and to explain their answers.</td></tr>
<tr><td>7.

a) Find the intervals where the function $f(x)$ is increasing and decreasing according to the derivative graph of f given above.
b) Find the points at which local minimum and local maximum of f occur.</td><td>It was taken and adapted from Garner and Garner (2001). Pre-service teachers' conception of the increasing and decreasing intervals, local extremum points of a function and their relation to first derivative was investigated.</td></tr>
<tr><td>8. Consider the function $f(x) = \frac{2}{3}x^3 - 2x^2 + 4$.
a) Determine whether the function f is increasing or decreasing at x = 1
b) Determine the points at which the function f has local maximum or local maximum values.</td><td>It was taken and adapted from Garner and Garner (2001). Pre-service teachers' conception of increasing and decreasing function and local extremum points was analyzed.</td></tr>
</table>

Table 3.3 (cont'd)

Description of the derivative test questions

Question	Information about the question
9.Find the equation of the tangent line at the point $(0,1)$ for the function $y=\frac{x+1}{x-1}$.	It was taken and adapted from Hartter (1995). In this question pre-service teachers' understanding of the tangent line and equation of it was examined.
10. Sketch graph of the derivative function for below given function graph. Explain your answer. 	It was prepared to reveal whether pre-service teachers understand the relation between first derivative, second derivative and the function. Moreover, whether they would analyze the function graph and sketch its derivative function graph. In this question, the function graph was given and pre-service teachers were wanted to sketch the derivative function graph.
11 $f(0)=0, f(2)=3, f(3)=7,$ $f'(0)=0, \lim_{x\to-\infty} f(x)=0, \lim_{x\to\infty}(f(x)-2x)=0, \lim_{x\to-1^+} f(x)=\infty, \lim_{x\to-1^-} f(x)=-\infty$ For $x\in(-\infty,-1)$ and $x\in(-1,0)$ $f'(x)<0$, for $x\in(0,\infty)$ $f'(x)>0$, For $x\in(-\infty,-1)$ and $x\in(2,3)$ $f''(x)<0$, for $x\in(-1,2)$ and $x\in(3,\infty)$ $f''(x)>0$ According to the function which satisfies the above conditions a) Find the intervals in which function f is increasing or decreasing. b) Find the intervals in which function f is concave or convex and the inflection points.	It was prepared to reveal pre-service teachers' understanding of the relation between derivative and the function was investigated. In this question, properties such as some values, limit values and derivative values at some points of a function were given and pre-service teachers were wanted to determine the intervals increasing or decreasing, concave or convex and the inflection points.
12. The revenue function G of a candy company was given, p represents the selling price. $G(p)=-500p^2+4500p$, Find the maximum revenue.	It was taken and adapted from Garner and Garner (2001). In this question, pre-service teachers' understanding of local maximum value was investigated.
13. The height of a ball thrown vertically upward (after t seconds) was given to be $s(t)=-16t^2+128t+320$. a) Find the maximum height the ball attains. b)How fast, and in what direction, is the ball traveling at t=4 s.	It was taken and adapted from Hartter (1995). In this question, again pre-service teachers' understanding of local maximum value and movement of an object was investigated.
14. Find the closest point of the hyperbola $y^2-x^2=4$ to the point $P(2,0)$.	It was prepared to reveal pre-service teachers' conception of derivative and their skills applying derivative concepts into problems.
15. Find the volume of the perpendicular circular cylinder which is placed in a sphere with radius R.	It was prepared to reveal pre-service teachers' conception of derivative and their skills applying derivative concepts into problems.

3.4.2 Interview

I conducted an open ended and focused interview (Yin, 2003). I interviewed with the participants approximately one hour period, and also I followed an interview protocol. I had interviews with six pre-service teachers. These six pre-service teachers were determined according to the results of the derivative test. Participants took the derivative test twice at the beginning and at the end of the lesson. Differences of the scores of first and last applications of derivative test determined. Pre-service teachers were arranged according to these differences of the scores in an ascending order. Pre-service teachers were divided into three groups consisting of equal number of people in each group to determine small change, average change and big change in their scores. Two pre-service teachers were chosen from each group. Two of them were male and four of them were female. They were chosen because I thought that I could get deep understanding of their conception of derivative in individual discourse. Moreover, as choosing the ones who were interested in the course and enthusiastic to learn, I could get more information about their individual discourse on derivative. The main aim of interviewing is to get information about what is in others' mind and think about the concerned issue (Patton, 2002). Well informed respondents can provide important insights into a situation (Yin, 2003).

An interview protocol was developed and used to reveal the participants in-depth understanding of the derivative concept. Interview protocol is developed by the researcher and the derivative test was taken into consideration while developing this protocol. Interview questions were developed according to what each test item aims to evaluate and reveal the acquisition of the intended concept (Appendix F). Interview protocol was checked by two university calculus instructors. One of them suggested asking the meaning of increasing and decreasing function. The other one suggested requiring pre-service teachers to explain derivative also on a graph. I added their suggestions to the interview protocol.

Interviews were conducted in a silent place and the interview days were chosen according to the appropriate days and hours of the participants. The participants were contacted via telephone. Interviews took approximately one hour.

The researcher conducted the interviews and each interview was video and audio recorded. At the beginning of the interview, participants were informed that their names would not be mentioned anywhere in the thesis and all the information they gave would not be given to anybody. Permission was taken from each interviewer for the video and audio records.

3.4.3 Worksheets

Worksheets were prepared to study in the class sessions in the group discussions and the classroom discussions. They were prepared according to the subjects and objectives of the each class sessions. The textbook Calculus Preliminary Edition (Hughes- Hallet et. al., 1992) was used while preparing the worksheets. Questions and examples were chosen from this textbook. The authors of the book stated that they prepared this book according to three rules: Every topic should be presented geometrically, numerically and algebraically (Hughes- Hallet et. al., 1992, p. v). Therefore, while preparing the worksheets these three rules were considered. Worksheets were prepared according to multiple representations and each subject represented geometrically, numerically and algebraically in the worksheets. There were four worksheets and they were prepared by the researcher. They were checked by two university calculus instructors for the appropriateness of the objectives. They were revised according to their suggestions. These worksheets were given in the Appendix E. The worksheets were on the subjects; rate of change, average velocity, instantaneous velocity, average rate of change, definition of derivative. Objectives of each worksheet were given in the Table 3.4.

Table 3.4

Objectives of the worksheets

Worksheet	Subject	Objectives
1	• Rate of change	• Interpret the change with respect to time at the given table • Interpret the change with respect to time at the given graph • Understand the average rate of change • Understand the relation between average rate of change and slope of the curve • Understand the limit of the average rate of change of a function gives the slope of the tangent line at a given point
2	• Average Velocity • Instantaneous Velocity • Average Rate of Change	• Interpret the change in the velocity with respect to time • Understand the average velocity between given certain times • Understand the average rate of change • Comprehend the instantaneous velocity at a certain time • Interpret the average velocity and instantaneous velocity at the given height-time graph
3	• Average rate of change • definition of derivative • increasing and decrasing functions • derivative functions, derivatives of the given functions at certain points	• Interpret the average and instantaneous rate of change of a function from the given values and graphs of the functions • Understand the relation between average rate of change of a function and the definition of the derivative • Comprehend the instantaneous rate of change gives the derivative of a function at a certain point • Interpret the sign of the derivative of a function in an interval where the function is increasing or decreasing. • Sketch graphs for the functions whose derivative is positive • Sketch graphs for the functions whose derivative is negative • Finds the sign of the average rate of change of a function whose values are given • Finds the slope of the tangent line at a certain point
4	• Maximum and minimum points of a graph of a function • Convex and concave graphs, inflection points • Minimum and maximum problems, equations of tangent line and normal line	• understand the relation between minimum and maximum points of a graph of a function and its derivative these points • find the minimum and maximum points of a graph of a function • understand the critical points of a function • Understand minimum and maximum problems • Solve minimum and maximum problems • Find the equation of tangent line • Find the equation of normal line • understand the relation between the second derivative and the convex and concave graphs • understand the inflection points • find the inflection point

Worksheets were prepared to make group discussion and classroom discussions focus on them. They were also used to determine the boundaries of these discussions. Moreover, in the analysis of the group and classroom discussions these worksheets were used. These worksheets were distributed to each group at the beginning of the each class session and appropriate time was given for these groups to discuss each question, come to an agreement after the discussion and write their answers to the worksheets. After these group discussions, classroom discussion was started and all participants and the researcher discussed the questions on the worksheets. All the worksheets were collected after completing the group and class discussions and they were used for the analysis.

3.4.4 Pilot Study of Instruction

Two week, eighteen class hours pilot study was conducted, before the instruction. In the pilot study limit concept was covered. The aims of the pilot study were two fold. The first one was to provide researcher to get used to the instruction and the second one was to provide the participants eliminate the deficiencies of the limit concept. Also, the participants got used to be video recorded in these prior applications.

Pilot study was conducted to the pre-service teachers who were the participants of the study. There were 61 pre-service teachers enrolled the course; 16 of them were male and 45 were female. At the first day of the pilot study they were grouped consisting of three, four or five pre-service teachers and they studied within their groups in each class sessions. Pre-service teachers were allowed to form their groups, there wasn’t any criteria for the grouping process considering students would study efficiently with people that they chose. Therefore, the class was consisting of fifteen groups. Two groups were consisting of three students, ten groups were consisting of four students and three groups were consisting of five students.

The method of the instruction in the pilot study was same as the instruction of the study. Therefore, it is explained in the following 3.3.5 Instruction section.

3.4.5 Instruction

Instruction took totally five weeks and thirty class hours between December 10th, 2009 and January 7th, 2010. At the last day of the instruction, the derivative test was administered to the participants. Participants didn't know that they were administered the tests at those days. Each class session was video recorded. At the first day of the instruction, pre-service teachers signed a participant permission form given in the Appendix A. Pre-service teachers gave permission to the researcher for all the classroom applications to be video recorded by signing this form.

There were 61 pre-service teachers enrolled the course; 16 of them were male and 45 were female. And also same pre-service teachers studied in the same groups. As in the pilot study the class was consisting of fifteen groups. Two groups were consisting of three students, ten groups were consisting of four students and three groups were consisting of five students. The instruction took place in a big classroom. In this classroom, the seats were placed back to back and alongside like an amphitheater. In group discussions, group members were sitting side by side. There were empty seats between the different group members. In the classroom discussions, group members were also sitting together.

Instruction was designed according to the leaning-as-participation metaphor. Pre-service teachers studied in groups on the worksheets to provide them to discuss on the concepts and improve their interaction between group members. After the group discussions, all pre-service teachers and the instructor discussed altogether each questions given on the worksheets. These classroom discussions provided them to develop their interaction between each pre-service teacher and develop ideas related to derivative concept.

Pre-service teachers discussed the questions given in the worksheets initially. In the group discussions, pre-service teachers discussed the questions on the worksheets. After each group completed discussing the questions, classroom discussion was started. They were not allowed to make any changes what they write on the worksheet when the classroom discussion was started. Worksheets were collected from the researcher when the classroom discussions finished. Group members wrote their names on the worksheets in order to identify which group the worksheets were belonged to. One group's discussions were video recorded. This group was consisting of four female pre-service teachers. The aim of these group discussions were to reveal the pre-service teachers' group discourse on derivative and determine how pre-service teachers affect each other in group settings.

In the classroom discussions, pre-service mathematics teachers and the instructor discussed the questions on the worksheets altogether after the group discussions. The researcher was the instructor of the classroom discussions. They covered each question and discussed mathematical reasons, relations between the concepts whether they were asked or mentioned on the worksheets. The aim of these classroom discussions was to reveal pre-service teachers conception of derivative and to determine how they affect each other in classroom settings.

Average rate of change, average velocity, instantaneous velocity, instantaneous rate of change, definition of derivative, increasing or decreasing of the graph of the function at certain intervals, derivative functions of given functions, derivatives of the given functions at certain points, maximum and minimum points of a function, convex and concave graphs, inflection points, minimum and maximum problems, equations of tangent line and normal line were covered through these discussions. Each class took approximately 50 minutes. The researcher was the instructor.

3.5 Data Analysis

For the analysis of the quantitative data descriptive and inferential statistics were used. Paired samples t-test was used for comparing the results of the first and last application scores of the participants. Moreover, mean and standard deviation was used to summarize and organize derivative scores of the participants. All quantitative data analysis was conducted using SPSS software.

Descriptive and content analysis was used for the qualitative data. Video transcripts of the classroom discussions to reveal classroom discourse, video transcripts of one group discussions to reveal group discourse and video and audio transcribes of the interviews with six pre-service teachers to reveal individual discourse on derivative and six pre-service teachers' answers to the derivative test were analyzed.

Video transcripts of one group's discussions on rate of change, average rate of change, instantaneous rate of change, increasing, decreasing functions, concavity of the function. Pre-service teachers' group, classroom and individual discourse were analyzed according to the word use, visual mediators, narratives and routines.

The word use of the pre-service teachers in group discussions, classroom discussions and interviews was determined according to the analysis of the transcripts. Discourses should have their own words (Sfard, 2007; Sfard, 2008). It is also crucial for mathematical discourse. The words related to these concepts were determined and categorized according to relevant mathematical notion. Moreover, the used words were categorized according to colloquially used words, operationally used words and objectified used words. The words were categories colloquially if they were specific to this group's and classroom's discourse and only the members of the discourse would understand in which purpose they were used and what they meant such as "upwards". The words were categorized as operationally used words if they were used to refer process such as "approach". The words which were used to identify or define objects were categorized as objectified used words such as "slope".

Pre-service teachers' visual mediators were also analyzed. Visual mediators were the symbolic artifacts that were used in special forms. Pre-service teachers' visual mediators' used in the group discussions and interviews, instructors' used visual mediators in the classroom discussions were analyzed. They were categorized as the written words, graphs, algebraic notations, diagrams, etc.

Pre-service teachers' narratives used in the group and classroom discussions and interviews and instructors' narratives used in the classroom discussion were analyzed. Narratives were written or spoken texts which were the explanation of objects or relations between objects or activities with or by objects (Sfard, 2007). It was any sequence of utterances framed as descriptions of objects, of relations between objects, or of processes with or by objects (Sfard, 2008). Narratives were grouped in two categories: Object level and meta-level. Narratives used to refer the mathematical objects were object level narratives such as 4+7=11 or the sum of the angles in a square was 360^0. Meta-level narratives were explanations about how mathematics was done. For example “while taking the derivative of polynomial functions take the power of x as the coefficient of x and subtract 1 from the power”.

Pre-service teachers’ regularities in their group, classroom and individual discourses and the instructors’ regularities in the classroom discourse were analyzed. Regularities seen in the use of mathematical words and mediators or narratives were called routines. Routines were grouped in two categories. These were object- level and meta-level rules. For example numerical calculations made according to the properties of associativity, commutivity and distributivity of addition and multiplication were the object level rules. In this type of routine, rules were obvious. The other type of routine was the rules understood from the communicators’ activities and in most cases communicators were less aware of the rules (Sfard, 2007). This routine was called meta-level routine. Pre-service teachers’ regularities while using the words and visual mediators were determined and categorized as object-level or meta-level routines.

3.6 Researcher’s Background, Role and Biases

It is important to state the role of the researcher as researcher is the key instrument for qualitative research as examining documents, observing behavior, and interviewing participants (Cresswell, 2007). This part of this section will mention about the researcher’s role and possible bias throughout the study.

The researcher got her B.S. degree from Mathematics department and M.Sc. degree from Secondary Science and Mathematics Education department of Middle East Technical University in Ankara. After graduating from the B.S. program, she started to work at Elementary Mathematics Education program of Başkent University as a research assistant. She was teaching assistant of the recitation hours of Calculus and Advanced Calculus courses for seven years. Moreover, she was teaching assistant of Special Teaching Methods courses of Elementary Mathematics Education program and Mathematics Teaching Methods courses of Primary Education program for seven years. She has been the instructor of Special Teaching Methods and Mathematics Teaching Methods courses since 2010.

During the study, the researcher was the instructor of the course. In order to get acquainted with the participants, she observed the class during first five weeks period. And also, two week, twelve class hours pilot study was conducted, before the instruction on derivative. Thus, the researcher and the participants found chance to know each other. During the data collection procedure there was no problems between the researcher and the participants. The researcher arranged the interview days and times according to the participants' appropriate times. At the beginning of the study, the researcher took permission from the participants for the audio and video records of group and classroom discussions and the interviews. During the interviews the researcher was sensitive for the confortability of the participants.

3.7 Validity and Reliability of the Study

In this section, measures taken during data collection and analysis to increase the credibility of study will be explained. Creswell (2007) explained eight procedures for validaton of the qualitative study. These procedures are triangulation, disconfirming evidence, clarifiying researcher biases, member checking, prolonged engagement in the setting, audit trail, thick and rich description, and peer debriefing. Creswell (2007) suggests using at leat two of these methods for the validity of the study. In this study most of the validation procedures that Cresswell mentioned were used for the validation process.

In triangulation, Creswell (2007) suggest to use different methods to provide corroborating evidence to shed light on a theme or perspective. In this study, several methods were used for data collection. Pre-service teachers' answers to the derivative test, transcripts of their classroom and group discussions, transcripts of the interviews were analyzed. Group discussion records and answers to the worksheets were used. Interview records and answers to the derivative test were analyzed for individual discourse. A doctorate student in elementary mathematics education and experienced in the recitation sessions of the calculus course also analyzed and categorized data. The second coder recorded the group and classroom discussions. Therefore, she was acquinted with the participants and the instruction took place. Throughout the study the researcher and the second coder were in cooperation. She also observed the classroom and group discussions. Data analysis procedures were explained to the second coder. The researcher and the second coder analyzed all the data separately. Then they compared each categorization of data and discussed if there was any inconsistency. They agreed on all the data.

For member checking, the researcher should consider the participants' views of the findings and interpretations (Creswell, 2007). In this study, in the interview sessions pre-service teachers wanted to explain their answers to the pre-test and elaborate what they want to say.

To clarify the resarcher bias to understand the researcher's position and any factors affecting the inquiry, researchers' role ad biases should be explained (Creswell, 2007). In this study, in the previous section, resercher's background, role and biases were explained to eliminate the biases.

Creswell (2007) explained prolonged engagement in the field necessary for the researcher bilding trust with participants and learning the culture of the setting. For this study, for prolonged engagement in the setting, the researcher attended the course for the first six weeks of the fall semester. And also the researcher had twelve class hours pilot study with the participants covering limit concept.

Rich and thick description of the data would allow the reader of the research to transfer information to other settings and decide whether the findings ccould be transferred to other researches as they have similar charactersitics (Creswell, 2007). In this study detailed description of the setting, the participants, and the themes were given for the thick and rich description of the research.

For the reliability of this study, Creswell (2007) also suggested using multiple coders and importance of agreement of these coders while analyzing data. The second coder and the coding and agreement processes were mentioned in the triangulation part of the validation processes. Therefore, reliability process was explained in this part.

3.8 Limitations of the study

As present study was designed as a case study, it is not possible to generalize the findings to all pre-service elementary mathematics teachers. In this study, only one group's discourse on derivative was examined. Therefore, the findings would also change according to other groups. Study was conducted using the communicational approach to cognition. However, if another framework was applied, there would be different point of views related to discourse on derivative. Another researcher using a different framework could see the classroom in different ways than I did. And also the researcher was the insructor. If the researcher was only the observer of the course, the group and classroom discussions would be observed more objective way and different findings would be figured out.

CHAPTER 4

RESULTS

This chapter summarized the findings of this research study. Sections in this chapter were organized in the order of research questions. Each section dealt with one of the research questions. In the first section, pre-service teachers' explanations of the derivative concept in the group discourse were investigated. In the second section, pre-service teachers' explanations of the derivative concept in the classroom discourse were analyzed. In the third section, pre-service teachers' explanations on the derivative concept in individual settings were explained.

Research questions that are dealt with in this chapter are:

How do pre-service elementary mathematics teachers explain the concept of derivative in group, classroom and individual discourses from commognition perspective?

a) How do pre-service elementary mathematics teachers explain the concept of derivative in group discourse from commognition perspective?
b) How do pre-service elementary mathematics teachers explain the concept of derivative in classroom discourse from commognition perspective?
c) How do pre-service elementary mathematics teachers explain the concept of derivative in individual discourse from commognition perspective?

Pre-service teachers' group, classroom and individual discourse on derivative was analyzed according to rate of change and increasing and decreasing functions from commognition perspective. Their used words, visual mediators, endorsed narratives and routines were analysed in order to determine their discourse on the derivative concept. Transcripts of the pre-service teachers' discussions on the questions of the worksheets in group and classroom settings and the individual interviews and also their written materials were analyzed according to the four elements of mathematical discourse from the commognitive framework: word use, visual mediators, endorsed narratives and routines. All the words that pre-service teachers used in the discussions and the interviews to define derivative and explain their perception of derivative listed and categorized in three dimensions in Zandieh's (2000) framework which categorizes derivative according to the representations (graphical-slope, verbal-rate, paradigmatic physical-velocity, symbolic- difference quotient) and layers (ratio, limit and function) and Sfard's (1991) process-object duality which categorizes the word use as operational and objectified. Objectified word use referred to "the whole cluster of internal representations and associations evoked by the concept" (Sfard, 1991, p.3). Besides, "processes, algorithms and actions" reflected an operational conception of a notion (Sfard, 1991, p.4). The words reffering to the operational conception of a notion will be used as operationally used words and objectified used words will be used as objectified words while representing the results of this study.

Pre-service teachers' visual mediators were analyzed according to visual realizations (Sfard, 2008) which are verbal, iconic, concrete, written words and algebraic symbols. Vocal realizations were not taken into account as pre-service teachers' used words were analyzed according to their spoken words. Pre-service teachers' narratives were categorized as object-level and meta-level. The narratives that pre-service teachers explained the properties of rate of change, increasing and decreasing functions, derivative, derivative function, second derivative function were categorized as object-level narrative and the narratives that explained how some procedures were done were categorized as meta-level narratives. Pre-service teachers' regularities in their actions in the group discourse and classroom were explained as routines. Examples of routines in pre-service teachers' actions while studying on the worksheets in the group and classroom discussions related to rate of change and increasing and decreasing functions were also explained in the result part.

4.1 Pre-service Teachers' Group Discourse on Derivative

In this section, the research question "How do pre-service elementary mathematics teachers explain the concept of derivative in group discourse from commognition perspective?" will be explained. Pre-service teachers' group discourse on derivative was determined according to discussions of one group consisting of four pre-service teachers working on the worksheets. All the participants in this group were female. All the group discussions were video recorded. Transcripts of these records and the written materials were used to determine pre-service teachers' group discourse on derivative. Written materials were consisting of pre-service teachers' answers and explanations to the questions on the worksheets.

In the next section pre-service elementary mathematics teachers' group discourse on and their conception of rate of change will be explained from commognition perspective.

4.1.1 Pre-service Teachers' Group Discourse on Rate of Change

In this section, in order to reveal pre-service elementary mathematics teachers' discourse on and conception of rate of change in group settings transcripts of the group discussions and their written answers on the worksheets were analyzed. To determine pre-service teachers' discourse and conception their word use, visual mediators, narratives and routines were examined.

Words used in group discourse on rate of change

Pre-service teachers' transcripts of the group discourse and written materials were analyzed to determine the used words related to rate of change. They were categorized according to the mathematical notions and Sfard's process-object duality which categorized the word use as operational and objectified. Categories of the used words and their types were given in the Table 4.1. Categories related to mathematical notions were rate of change, average rate of change, slope, instantaneous rate of change and limit. According to the process-object duality the words used in group discourse were mostly objectified. The operationally used words were connected with the mathematical notion of limit.

The words related to rate of change were "weight over week", "weight lost per week", "kilograms per week", "weight lost per unit week", "weight lost per week". They were all categorized as objectified.

The words related to average rate of change were "average weight", "average weight that she lost per week", "average weight that she lost per one week", "average weight gained between 5-7", "average velocity". They were all categorized as objectified.

The words related to slope were "rate in five weeks", "slope of line", "slope is zero, slope of the tangent". All these words were objectified.

The words related to instantaneous rate of change were "instantaneous velocity", "change in position over time", "velocity for one second". They all were objectified.

The words related to limit showed differences according to process-object duality. The words "limit of the slopes", "limit of the slope of the tangent", "limit of the slope of the line segments", "limit of change" were objectified. The words "approach", "move away", "approach from left and right" were operational.

Table 4.1

Word use of group discourse on rate of change

Category	Words	Type
Rate of change	weight over week	Objectified
	weight lost per week kilograms	Objectified
	per week	Objectified
	weight lost per unit week	Objectified
	weight lost per week	Objectified
Average rate of change	average weight	Objectified
	average weight that she lost	Objectified
	per week	Objectified
	average weight that she lost	Objectified
	per one week	Objectified
	average weight gained	Objectified
	between 5-7	Objectified
	average velocity	Objectified
Slope	rate in five weeks	Objectified
	slope of line	Objectified
	slope is zero	Objectified
	slope of the tangent	Objectified
Instantaneous rate of change	instantaneous velocity	Objectified
	change in position over time	Objectified
	velocity for one second	Objectified
Limit	limit of the slopes	Objectified
	limit of the slope of the tangent	Objectified
	limit of the slope of the line segments	Objectified
	limit of change	Objectified
	approach	Operational
	move away	Operational
	approach from left and right	Operational

When whole discussion related to rate of change was considered, it was seen that there was a development of pre-service teachers' conception of the rate of change. At the beginning of discussion on rate of change, they used the words "weight over week" which only represented the units, not the change in weight or time. Pre-service teachers defined the meaning of that rate of the change in weight to the time passed as "weight lost per one week", "kilograms per week", "weight that lost per unit week" and "weight lost per week". In all these expressions, they wanted to explain how much weight was lost in one week period. Therefore, they used the expressions as "per one week", "per week" and "per unit week". The rate was the change of weight over the passed time (week), so they found the unit of that rate as "kilogram per week".

In the following dialogue, Özgü explained the rate of change of weight in five weeks period as "something that is lost in one week". She tried to explain that it gave the lost weight per one week period. She also used the words "weight over week" to explain this rate of change as they divided the change of weight in five weeks period to the time period passed over.

Suzan: What does that rate mean?

Özgü: It is something that is lost in one week. It is weight over week. It is something like average weight.

As the discussion progressed, they started to consider the change in time as it was understood from the words "something lost in one week." In these words, the change in time came into consideration. Then they found the rate of change algebraically as $\frac{8}{5}$, in which 8 represented the change in weight and 5 represented the change in time (weeks).

Suzan divided 8 by 5 to answer question. She gave this answer as 8 kilograms were lost over 5 weeks period. In the following dialogue, Özgü wrote the unit of this rate as the kilograms per week which was lost weight over the time passed.

Suzan: Ok then. 8 is divided by 5.

Özgü: Write kilograms per week

Towards the end of the group work, they realized the change in weight and came to a conclusion that this rate meant "lost weight in five weeks period". These words implied both change in weight and time and also rate of change. At the end of the discussion, they concluded that this rate means average weight that lost in five weeks period. They associated this rate of change with average rate of change.

Özgü and Derya made comments on rate of change. Özgü said that “weight that lost per unit week” which meant lost weight was divided by the time, 5 weeks passed over. She defined the unit time as week. Derya made comment on this rate as “weight lost per week”. This rate meant the lost weight in 5 weeks period and the lost weight was divided by this passed time. Therefore, she used “per week”. Their dialogue was given below.

Derya: Average weight that she lost per one week

Özgü: Weight that lost per unit week

Derya: Weight lost per week

Group members commented on that rate of change in weight to the week passed over was the average rate of change. At the beginning of the discussion, Özgü said that this rate was “average weight”. She said only the weight was average. In the following part of the discussion, Özgü said that rate of change was “average weight that she lost per week”. In this expression she also added the time period. Derya also commented on this rate as “average weight that she lost per one week”. In this expression she emphasized that the average weight fell to one week period. She emphasized that the relation between the slope of the line segment and the rate was “average weight gained in 5-7 [between 5^{th} -7^{th}weeks]”. They found the average rate of change of weight by dividing the change in the weight by the passed time. They also found the average velocity of the ball by dividing the change in position by the change in time. They commented that negative sign of the average velocity meant that ball had slowed down.

Özgü explained the rate of change in weight in five weeks period as “average weight”. Suzan tried to understand the rate of change in weight in five weeks period. Özgü continued to define this rate as “average weight that she lost per week”. They found eight kilos lost in five weeks period. Özgü emphasized that the rate gave the average weight that was lost in one week. Therefore, she explained the rate as the average weight that she lost per week. Their explanations were given below.

Suzan: What does that rate mean?

Özgü: It is something that is lost in one week. It is weight per week. It is something like average weight.

Suzan: She lost 8 kilos in five weeks. What this rate means? Look at that.

Özgü: Average weight that she lost per week.

Suzan: Rate, slope in five weeks change here.

Derya also defined the rate that was 8 divided by 5 as "average weight that she lost per one week". The difference between Özgü's definition and Derya's definition was "per one week". Özgü defined it as "per week" but Derya defined it as "per one week". However, they defined that lost weight in five weeks period as the average weight. Their dialogue was given below.

Suzan: 8 is divided by 5.

Özgü: Kilograms per week.

Derya: Average weight that she lost per one week

Özgü: Weight that lost per unit week

Derya: Weight lost per week

In the following dialogue Suzan explained the relation between the slope of the line segment sketchedn between the 5^{th} and the 7^{th} weeks and average weight that lost in two weeks period as "average weight gained in 5-7 (between 5^{th} and 7^{th} weeks)".

Derya: What is the relation between the average rate of change and the slope of the line segment. Doesn't the slope of the line segment give the average weight lost in two weeks period from fifth to seventh weeks?

Suzan: It gives the gained weight.

Derya: It gives the gained weight. It asks the relation.

Suzan: Average weight gained in 5-7 (between 5^{th} and 7^{th} weeks)."

Members of the observed group explained the average velocity as "total change in position over change in time". They found the average velocity in this time interval as -13.2 by applying their definition for the average velocity as total change in position over change in time: $\frac{31.8-45}{5-4} = -13.2$. They associated the negative sign with the velocity was decreasing, not the direction of the ball.

According to Suzan, the rate of change in five weeks period was the slope. Derya commented on the relation as “the slope of the line segment gives the average weight lost in two weeks period from 5^{th} to 7^{th} weeks”. Derya also compared the values of the slope of the line segment sketchedn between 5^{th} to 7^{th} weeks which was 0 and the rate of change which was also 0. After this comparison, Derya commented on the relation according to these findings as “the slope of this line segment gives the rate of change” and they wrote this answer on the worksheet.

Suzan made the comment on the rate of the weight lost şn two weeks period to the five weeks as “rate in five weeks slope change”. Although the relation between the slope of the line segment sketched between the 5^{th} and the 7^{th} weeks and the average weight that lost in five weeks period wasn’t asked she used rate and slope in one utterance. Their dialogue was given below.

Suzan: What is this rate means?

Özgü: It is something that is lost in one week. It is weight per week. It is something like average weight.

Suzan: She lost 8 kilos in five weeks. What this rate means?

Özgü: Average weight that she lost per week.

Suzan: Rate in five weeks slope change here.

Derya explained the relation between the slope of the line segment sketched between the 5^{th} and the 7^{th} weeks and the average weight that lost in this two weeks period as “the slope of the line segment gives the average weight lost in two weeks period”. Her explanations were given below dialogue.

Derya: What is the relation between the average rate of change and the slope of the line segment. Doesn’t the slope of the line segment give the average weight lost in two weeks period from 5^{th} to 7^{th}weeks?

Suzan: It gives the gained weight.

Derya: It gives the gained weight. It asks the relation.

Derya explained the relation between the slope of the line segment and the average rate of change to Suzan by giving the value of the slope of the line segment and the value of the rate of change. She said that “slope of the line segment is 0” and “The rate of change is also 0”. Therefore, she emphasized the relation between the slope of the line segment and the average rate of change. But she did not use the average rate of change. She only used the words rate of change.

Suzan: What is the relation between the line segment given in the question and the average rate of change?

Derya: Slope of it, slope of the line segment is 0, isn't it? The rate of change is also 0. The slope of this line segment gives the rate of change.

Members of the observed group thought that to find the instantaneous velocity finding average velocity was not enough. They thought that to find the velocity of the ball at $t = 1$ second, they should find instantaneous velocity at that second. Suzan said that to find the instantaneous velocity, finding average velocity was not enough. Özgü stated that to find the instantaneous rate of change, they should use $\Delta changeinposition$ over time. However, they concluded that it would be the same thing with the average velocity.

They were asked to define the instantaneous rate of change for a function at a point in the third worksheet. They discussed on this question and thought that the limit of the slope of the tangents gave the instantaneous rate of change, but they were not sure about it. Then they looked at their notebook and realized that limit of the slope of the secant lines gave the instantaneous rate of change. But they did not use the words secant lines.

They defined the instantaneous rate of change as the limit of change of the given function (x^2) for an answer to the question: "find instantaneous rate of change of function$f(x) = x^2$ at $t = 1\ s$." Close values of the function were given in the question. They also explained how they would find the instantaneous velocity as finding the limit of the tangent line at the intended point.

Their minds were confused while analyzing the question of "find differentiation rule for the function $f(x) = x^2$ using the table of function values at the close points of x values 1, 2 and 3." In this question, they tried to relate the values to the derivative $2x$ of the function x^2 as they know the differentiation rule beforehand. However, they could not come to a conclusion and did not answer this question.

Visual mediators used in group discourse on rate of change

Pre-service teachers used visual mediators to answer and explain the questions asked in the worksheets. Visual mediators they used about rate of change were grouped in three categories such as graph, algebraic symbols and written words. Graphs that they were given on the worksheets and their comments on them or graphs that they sketched to study will be discussed first. Then, algebraic symbols that they used and their written words to answer the questions on the worksheets will be analyzed.

Students in this group sketched the weight versus time graph and the line segments between the specified points given in the question as visual mediators. They sketched weight versus time graph given in the Figure to understand the change of the kilos for each week and also to see the continuity of the graph with respect to the weight change. They accepted the domain as real numbers. Therefore, they sketched a continuous graph.

Moreover, they were asked to sketch the line segments on the weight versus time graph to relate the average rate of change with the slope of the line segments sketched between the given points in the question. The graph that they sketched was given in Figure 4.1.

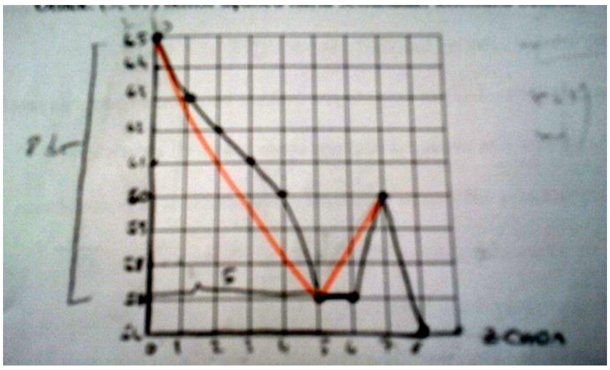


Figure 4.1 Graph shows the relation between slope of the line segment and average rate of change

Members of the observed group sketched the line segment between the first week and the fifth week, and the fifth week to seventh week. They found the slope of these line segments and compared the findings with the average rate of change in weight per week. In the group work on the d part of the fourth question and the d part of the sixth question, they discussed the meaning of these relations.

Members of the observed group began to study instantaneous rate of change with the instantaneous velocity of the ball at $t = 1$ s. They did not decide how to find the instantaneous velocity of the ball at $t = 1$ s before using the graph of the function representing the motion of the ball. When they started to study on the graph, they thought that they should find the instantaneous velocity by finding the slope of the line tangent to the curve which represented the motion of the ball at the point $(1, 27)$ and they sketched this tangent line. However, they could not find a way that is different from finding the average velocity. Although they thought that instantaneous velocity and average velocity were different things and they should find the slope of the tangent line at the intended point, they could not find any way rather than finding the average velocity.

They also sketched the line segments on the height-time graph represented the movement of the ball given in the second worksheet. They showed the relation between the slope of the line segments and the average rate of change on the graph given in the Figure 4.2.

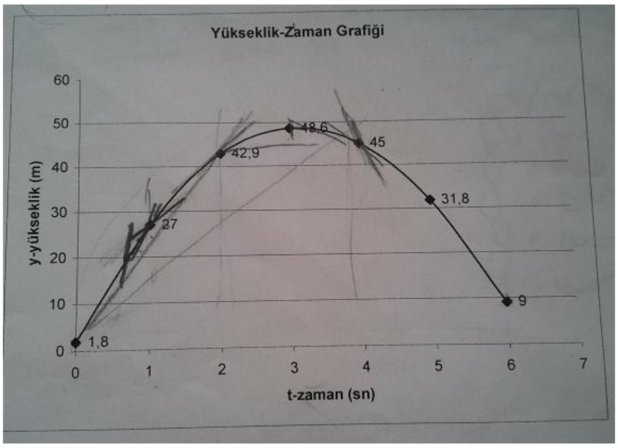


Figure 4.2 Height versus time graph

Then they tried to remember the rule that they used in physics course, but they could not remember. Then they decided to study on a graph representing the motion of the ball. They realized that the graph given in the 11^{th} question in the worksheet was the graph they needed. They studied on this graph thereafter (Figure 4.2).

After the classroom discourse on average and instantaneous velocity, in the group discourse, they were asked to find the instantaneous velocity of an object by using its movement function graph. However, they thought and also wrote on the worksheet as an answer that they should sketch a tangent line to the graph at the intended point and find the slope of this line, they did not find the instantaneous velocity using the graph given in Figure 4.2.

When they were working on the graph given in the Figure 4.2, they remembered that the instantaneous velocity was the slope of the line tangent to the graph at the intended point. They sketched a line tangent at the point (1, 27). They tried to find the slope of that tangent line as if the line was passing through the point (0, 1.8) by applying the formula as 27 minus 1.8 over 1. They decided that the instantaneous velocity should be found by this formula. Then they wrote on the worksheet as the answer of this question: “Finding the average velocity is not enough. We should find the instantaneous velocity. Instantaneous velocity is the slope at t=1 s. Here $\frac{27-1.8}{1-0} = 25.2$." They applied this formula for all questions asking the instantaneous velocity at any second.

In the next question they were given the positions of the ball for the very close time to t=1 s. and again asked to find the velocity of the ball at t=1 s. They applied the formula they used in the former question again to find the instantaneous velocity. They wrote on the worksheet "the slope of the graph at t=1 s gives the velocity at t=1 s. Again $\frac{27-1.8}{1-0} = 25.2$."

According to the given graph in the 11^{th} question given in the Figure 4.2, they thought that the velocity of ball increased when they approached to t=1 s from left since the slope of the tangent line increased. They also thought that velocity would decrease when they approached from right since the slope of the tangent decreased. They also concluded that the average velocity was found for an interval such as$(2, 4)$; on the other hand the instantaneous velocity was found for the value $t = 2$s.

Members of the observed group explained the meaning of the quotient $\frac{f(a+h)-f(a)}{h}$, if f is a function by sketching an increasing function graph given in Figure 4.3. They assigned the points (a, f(a)) as A and (a+h, f(a+h)) as B and sketched a line segment between these points A and B and wrote on the worksheet as an answer that this expression represented the slope of the line segment $[A, B]$.

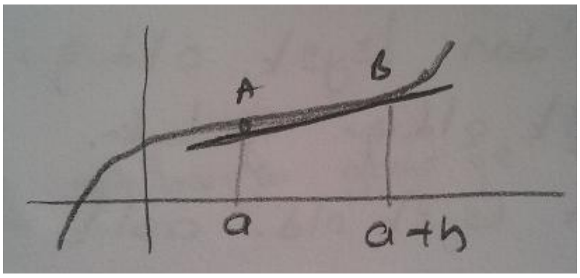


Figure 4.3 Graphical representation of $\frac{f(a+h)-f(a)}{h}$

The second category of the visual mediators that pre-service teachers used while answering or explaining their answers was algebraic symbols. They used these expressions to represent the symbolic representation of the mathematical notation such as they defined average velocity as $\frac{total\ change\ in\ position}{total\ time}$. They also used the algebraic symbols to the calculations and got the intended value such as $\frac{31.8-45}{5-4} = -13.2$ and $\frac{27-1.8}{1-0} = 25.2$.

In another question, they were asked to express the instantaneous rate of change for a function at $x = a$. They represented this instantaneous rate of change as the limit of the function while approaching to a as in the symbolic expression $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$. They wrote that "the slope of the tangent line at $x = a$ gives us the instantaneous rate of change". However, when they were asked to find the derivative of the function by using the given function values for some points they thought that they should use the formal definition of derivative in the symbolic form $f'(x) = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h}$. But they could not find the derivative function using the given values.

The third category of the visual mediators was written words which were written on the worksheet as an answer. For example, they wrote that "the limit of the slope of the tangent at the point a gives the instantaneous change." In this example they explained the instantaneous change of a function at an apsis value a. The other written words that they used were given in the Table 4.2.

Table 4.2

Written words pre-service teachers wrote related to rate of change

Written words
"The rate of change gives the slope of that line"
"It gives lost weight in each week"
"It is not enough to find average velocity. We should find instantaneous velocity. Instantaneous velocity is the slope at t=1 s."
"The slope of the graph at t=1 s gives us the velocity at t=1 s."
"We can calculate instantaneous velocity for a value. This is equal to the slope of the tangent sketched at that point. Instantaneous velocity cannot be calculated for an interval."
"Limit of the change in x^2"

Members of the observed group explain the relation between the rate of change and theslope of the line tangent to the graph at the intented point in the expression "The rate of change gives the slope of that line". In the expression "It gives lost weight in each week", they meant that the rate of change between the given weeks represented the lost weight in

each week. They meant that to find the velocity at the second t=1, they should find the instantaneous velocity and the average velocity was not enough to find the instantaneous velocity, the slope of the line tangent to the graph gave the instantaneous velocity in the expressions "It is not enough to find average velocity. We should find instantaneous velocity. Instantaneous velocity is the slope at t=1 s" and "The slope of the graph at t=1 s gives us the velocity at t=1 s". In the expression "We can calculate instantaneous velocity for a value. This is equal to the slope of the tangent sketched at that point. Instantaneous velocity cannot be calculated for an interval", they explained that instantaneous velocity would be calculated for a value not for an interval. The expression "Limit of the change in x^2" meant that to find the instantaneous rate of change they should find the limit of the change in function x^2.

There were differences between the use of words and the written words. The written words were the results of pre-service teachers' thought process and their concluded ideas related to the mathematical notions of the group work. Therefore, they used more formal words to express these ideas on the worksheets. However, in their word use they felt comfortable and did not think on the words they use, so they did not choose the words carefully. When they used these words, they discussed on the questions or studied on the visual mediators and developed ideas related to the mathematical notions. Sometimes, they tried to remember the relations or rules. They also tried to refute or understand the group members' ideas.

Narratives used in group discourse on rate of change

Pre-service teachers' narratives related to rate of change were mostly object level as they explained mathematical notions slope, average velocity, rate of change, instantaneous velocity, limit and relations between these notions in these narratives. Narratives that members of the observed group used while studying on the worksheets related to rate of change were given in the Table 4.3. In this table the types of the narratives were also specified.

The object-level narratives were related to where to find the average and instantaneous velocity, relation between the slope of the tangent line and the rate of change, limit of the average velocity and the instantaneous velocity. The meta-level narrative on rate of change was on how to find the slope of the tangent lines.

Table 4.3

Narratives pre-service teachers used on rate of change

Narrative	Type
We find average velocity for an interval	Object-level
We find instantaneous velocity for an exact value such as t=1 s.	Object-level
The slope of the line gives the rate of change.	Object-level
The rate of change gives the slope of the line.	Object-level
Average velocity is total change in position over total time.	Object-level
Instantaneous velocity is the slope at t=1 s.	Object-level
The slope of the tangent line gives the instantaneous velocity.	Object-level
The slope of the line segments gives the instantaneous rate of change.	Object-level
The limit of the average velocity gives the instantaneous rate of change.	Object-level
When we approach a point we find the limit of the slope of the tangents at the intended point.	Meta-level

Routine of group discourse on rate of change

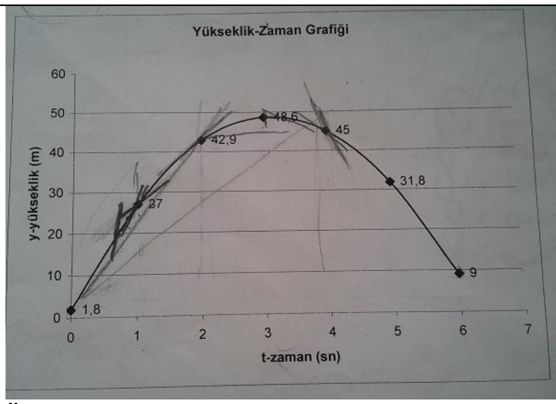
In the group discussion on instantaneous velocity, pre-service teachers' routine was given in the Table 4.4. As they worked on the worksheets the prompt of the routines of the members of the observed group was the questions asked on the worksheets. The prompt of the example routine given in the Table 4.4 was the question given in the worksheet "Find the velocity of ball at t=1 s. Finding the average velocity was enough or not?"This question started the discussion.

The "how routine" was the development part of the discussion. In the how routine part they studied and discussed on the questions and find answers to these questions. In the example routine they worked on the graph given on the worksheet and decided that instantaneous velocity is the slope of the tangent line. They found the average velocity in the interval (0,1).

In the closure part of the routine they concluded the discussion. They decided what the answer should be and write the answers on the worksheets. In the given example routine they concluded and ended the discussion on instantaneous velocity by writing “Finding the average velocity is not enough. We should find the instantaneous velocity. Instantaneous velocity is the slope at t=1 s. Here $\frac{27-1.8}{1-0}=25.2$” on the worksheet as an answer to the question.

Table 4.4

Routine of group discourse on instantaneous velocity

Prompt	Starting discussion on instantaneous velocity	Question: Find the velocity of ball at t=1 s. Finding the average velocity was enough or not?
How routine	They worked on the graph and sketched tangent line to the graph at t=1 s	
	Deciding the instantaneous velocity is the slope of the tangent line	Özgü: slope of this (sketched tangent line to the graph) Derya: Instantaneous velocity is the slope here (shows the tangent line)
	Finding the average velocity in the interval (0, 1)	Derya: It says 27. 27 minus 1.8 over 1. Özgü: It is the same as the previous one. Let's do like this
Closure	Concluding the discussion	Derya: Instantaneous velocity is the slope. Özgü: Explain that it is the instantaneous velocity.
	Write their answer on the worksheet	"Finding the average velocity is not enough. We should find the instantaneous velocity. Instantaneous velocity is the slope at t=1 s. Here $\frac{27-1.8}{1-0} = 25.2$."

4.1.2 Group Discourse on Increasing and Decreasing Function

In this section, members of the observed group discussed the increasing and decreasing function and its relation to the sign of the derivative function. Their group discourse was analyzed according to the elements of commognition. Their word use, visual mediators, endorsed narratives and routines will be discussed in this section.

Words used in group discourse on increasing and decreasing function

Transcripts of the group discussions of members of the observed group and written materials were analyzed to determine the used words related to ncreasing and decreasing function. They were categorized according to the mathematical notions and Sfard's process-object duality which categorized the word use as operational and objectified. Their word use on increasing and decreasing function was grouped in five categories. These categories were "derivative", "function", "interval", "graph", and "slope". Words were also categorized as colloquial, operational and objectified. Colloquial words referred to the words that were used specific to this discourse. Objectified words referred to the words that identify an object. Operational words referred to the words that identify a process. Used words and categories were given in the Table 4.5.

In the "derivative" category, objectified words were "derivative is greater than zero", "derivative is positive", "derivative is in positive direction", "derivative is negative". Besides, "derivative is increasing" and "derivative is decreasing" were operationally used words. "Derivative is positive everywhere" was colloquially used. In the "function" category, there was one objectified word "decreasing function". "Increasing", "decreasing" and "y values were decreasing" were operationally used. In the "graph" category, "curve positive", "positive decreasing curve", "increasing graph" were objectified words. "Decreasing everywhere", "going upwards", "decreasing", "curve is decreasing" were operational words.

In the "slope" category, "slope was positive", "slope is negative", "slope is not in positive direction" were objectified words.

In the interval category, "interval function is increasing", "increasing till 1", "decreasing after 1", "function f is increasing in that interval" were objectified words. “Increasing there”, “curve positive and decreasing everywhere”, “derivative is negative everywhere” and “after four” were colloquially used words.

Table 4.5

Words used in group discourse on increasing and decreasing function

Category	Words	Type
Derivative	derivative is greater than zero	Objectified
	derivative is positive	Objectified
	derivative is in positive direction	Objectified
	derivative is negative	Objectified
	derivative is increasing	Operational
	derivative is decreasing	Operational
	derivative is positive everywhere	Colloquial
Function	decreasing function	Objectified
	increasing	Operational
	decreasing	Operational
	y values were decreasing	Operational
Graph	curve positive	Objectified
	positive decreasing curve	Objectified
	increasing graph	Objectified
	decreasing everywhere	Operational
	going upwards	Operational
	decreasing	Operational
	curve is decreasing	Operational
Slope	slope was positive	Objectified
	slope is negative	Objectified
	slope is not in positive direction	Objectified
Interval	interval function is increasing	Objectified
	increasing till 1	Objectified
	decreasing after 1	Objectified
	Function f is increasing in that interval	Objectified
	Increasing there	Colloquially
	curve positive and decreasing everywhere	Colloquially
	derivative is negative everywhere	Colloquially
	after four	Colloquially

They discussed the relation between the notion of derivative and the increasing and decreasing functions. They analyzed whether the derivative was positive or negative in the intervals that the function was increasing and decreasing. They checked for the slope of the tangent lines positive or negative for the derivative values. Then they concluded that in the interval $(-\infty, 1)$ the function was increasing and the slope of the tangent lines were positive. In the interval $(1, \infty)$, the function was decreasing and the slope of the tangent lines were negative. Moreover, they related the slope of the tangent lines to the derivative of the function. They concluded that when the derivative values were positive the function was increasing and when the derivative values were negative the function was decreasing.

They started to study on the questions, related to increasing and decreasing functions, first derivative and second derivative. They discussed on the question that the graph of the function $f(x) = x^2$ were given and it was asked to verify the relation for the first derivative $f'(x) = 2x$, $f'(x) > 0$, $x > 0$ and $f'(x) < 0$, $x < 0$, and for the second derivative $f''(x) = 2, f'(x) > 0$, $\forall x \in IR$, analyzing the graph of the function f.

Suzan made mistake related to increasing and decreasing functions. She commented that $f(x) = x^2$ was increasing for all values of x but Dilek corrected her that the function was decreasing for the values $x < 0$ and increasing for the values $x > 0$. Suzan insisted that when x was -10, the function value 100 and it was also same for $x = 10$. She says that the function value increased. Dilek said that when the x values increased, the function values decreased for the interval $(-\infty, 0)$. Then Suzan agreed Dilek according to her explanation.

Then Suzan analyzed slopes of the tangent lines to verify the intervals where the function was increasing and decreasing. She sketched two tangent lines to the curve, one in the interval $(-\infty, 0)$ and one in the interval $(0, \infty)$. After analyzing the slopes of the tangent lines, they decided that the function was increasing in the interval $(0, \infty)$ and decreasing in the interval $(-\infty, 0)$. They wrote their decision on the worksheet as "$(0, \infty)$ increasing function, $(-\infty, 0)$ decreasing function". They also wrote "When we sketch tangents to the function graph, if the slope of the tangents is positive, function is increasing, if negative, function was decreasing" as an explanation for their answer.

Then they discussed another similar question that the graph of the function $f(x) = -x^2$ were given and asked to verify the relation for the first derivative $f'(x) = -2x$, $f'(x) < 0$, $x > 0$ and $f'(x) > 0$, $x < 0$, and for the second derivative $f''(x) = -2, f'(x) < 0$, $\forall x \in IR$, analyzing the graph of the function f. They analyzed the graph of the function by sketching tangent lines as they did in the former question and stated that the function was decreasing in the interval $(0, \infty)$ and increasing in the interval $(-\infty, 0)$. They wrote on the worksheet "$(0, \infty)$ increasing function, $(-\infty, 0)$ decreasing function."

In both questions they verified the relation between increasing and decreasing functions, first derivative and second derivative of the given function. They found if the function was increasing or decreasing by analyzing whether the first derivative was positive or negative rather than analyzing x and y values of the function. They agreed on $f'(x) > 0$ for increasing function and $f'(x) < 0$ for decreasing function and used this relation to answer these questions. However, they could not relate first derivative to second derivative. They also could not verify the first and second derivative for the function given in another question.

They constructed the relation between positivity of the function and the increasing and decreasing function for the first and second derivative in the fifth question. They wrote on the worksheet that "In the interval if $f''(x) > 0$, then $f'(x)$ increasing and if $f''(x) < 0$, then $f'(x)$ decreasing." However, they could not relate the graph of a function with its second derivative.

In another question, they were asked to find the minimum and maximum points of the function f. They used the table that they assigned the increasing and decreasing intervals and the sign of the derivative function. They again determined the increasing and decreasing intervals according to the slopes of the tangent lines rather than analyzing the curve. They tried to find the maximum and minimum points of the second derivative function by sketching tangent lines. Özgü stated that the points that the second derivative was zero would be the points that the slope of the derivative graph was zero.

In the seventh question they remembered the relation of the function graph and the second derivative relation. They answered this question according to the concavity of the function graph. Özgü stated that where the function graph was concave, the second derivative was negative and where the function graph was convex in that the second derivative was positive. Although they thought the relation, they did not found the intervals. Suzan sketched a convex graph and asked whether this graph was convex or concave. They could not give an answer.

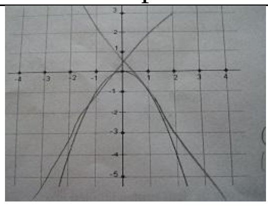
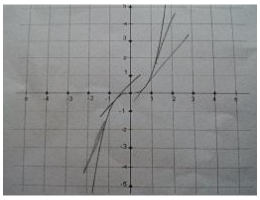
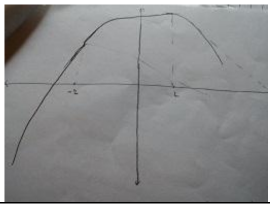
They discussed another question in which information was given for a function $f: R \to R$. It asked to sketch the function graph considering this information. They analyzed the given information and interpreted it. For the information "in $x < -2$, $f'(x) > 0$ and the derivative was increasing", they said that the graph should be increasing and angles of the slopes should be acute angles as the derivative was greater than zero and the derivative was increasing.

Visual mediators used in group discourse on increasing and decreasing function

Members of the observed group, instructor used three types of visual mediators on increasing and decreasing functions: graphs, algebraic symbols and written words. Pre-service teachers sketched tangent lines to the graphs given on the worksheets. And also, they sketched a graph according to the given properties. Three examples of graphs were given in the Table 4.6.

Table 4.6

Graphs sketched as visual mediator in group discourse on increasing and decreasing function

Graph	When it was sketched
	Tangent lines sketched to show relation between increasing and decreasing function and sign of derivative
	Tangent lines sketched to show relation between increasing function and positive sign of derivative
	Graph sketched according to the given properties of the function

Members of the observed group sketched tangent lines to show relation between increasing and decreasing function and the sign of the derivative. They decided the sign of the derivative function was positive in the interval where the function was increasing and negative in the interval where the derivative function was decreasing.

They sketched a graph according to the given properties of the function in the worksheet. They sketched an increasing concave curve for the first part of the graph, for the interval $(-\infty, -2)$. For the information "in $-2 < x < 1$, $f'(x) > 0$ and the derivative was decreasing, they said that the function graph would increase but the slopes would decrease. Derya sketched a decreasing curve in this interval but the others corrected her and said that it was decreasing. Özgü said that function might be decreasing but the derivative might be increasing. She misunderstood the given information. Suzan said that the slopes would be positive but decreasing. They discussed on the curve that Derya sketched. Özgü said that it would be correct. However, Suzan refuted her and said that if this part of the graph decrease than the slopes would be negative but in the question the slopes were positive. They sketched a concave curve in the interval $-2<x<1$. Although the derivative function value was zero at $x=1$, they sketched straight curve near $x=1$. They completed the graph with decreasing concave curve for $x>1$. At the end they sketched a concave parabolic graph (figure 4.4).

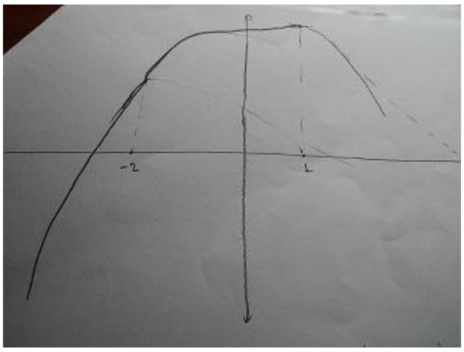


Figure 4.4 Concave parabolic graph that pre-service teachers sketched

They explained their graph on the worksheet that:

"$(-\infty, -2)$ When we sketch tangents to the graph the slopes are positive. Increasing function."

"(-2, 1) when we sketched tangents to the graph the slopes are positive. Increasing function.

However, the slopes values decreases and the slope becomes zero at $x=1$."

"$(1, \infty)$ Tangent lines are sketched to the graph; the slopes of the tangent lines are negative. The function is decreasing."

They did not relate the increasing or decreasing derivative function with the second derivative. Therefore, they did not use the concavity of the function while sketching the graph. And also they did not relate the increasing or decreasing derivative function to the slopes of the tangents.

Members of the observed group also used written words in their discussions while answering the questions given in the worksheets. Examples of pre-service teachers' written words were given in the Table 4.7. In these written words they explained the relation between the increasing and decreasing functions and the positive or negative values of the derivative functions.

Table 4.7

Pre-service teachers' written words on increasing and decreasing functions

Written Words
"The intervals where the derivative is greater than zero are the intervals where the slope is greater than zero."
"The interval where the derivative is greater than zero in $(-\infty, 1)$"
"The intervals where the derivative is less than zero, the sign of the slope is negative, derivative is less than zero."
"In the interval that the function f is increasing, the slope is positive. Slope means derivative."
"In the interval that the function f is decreasing, the slope is negative."

Pre-service teacher also used algebraic symbols to indicate the properties of derivative in the group discourse. These algebraic symbols were given below. They represented the value of the derivative function.

$f' > 0, f' = 0, f' < 0$

Narratives used in group discourse on increasing and decreasing function

All the narratives pre-service teachers used were object level narratives. They were used to define the relation between increasing and decreasing function and first derivative. Pre-service teachers' narratives used in group discourse on increasing and decreasing function were given in the Table 4.8.

Table 4.8

Narratives used in group discourse on increasing and decreasing functions

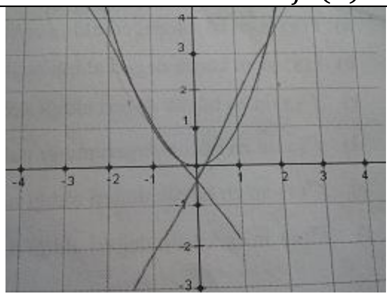
Narratives	Type
In the interval that the function f was increasing, the derivative was increasing.	Object-level
Function f is decreasing in the interval that the derivative was less than zero.	Object-level
f is increasing if derivative is less than zero.	Object-level
If derivative of f is less than zero, f is decreasing.	Object-level
When the slope is positive, then the derivative is positive.	Object-level
If the slope was positive, function is increasing.	Object-level
Increasing if the derivative is greater than zero.	Object-level

Routine of group discourse on increasing and decreasing functions

In the group discussion on increasing and decreasing function, pre-service teachers' routine was given in the Table 4.9. The prompt of the routine was the questions given in the worksheet "what would you say for function f in the interval where $f'(x) > 0$?" and "What would you say for function f in the interval where $f'(x) < 0$?" These questions started the discussion. Then they worked on the graph given on the worksheet and examined the slope of the tangent lines. They related the sign of these slopes of tangent lines to the sign of the derivative in those intervals. They concluded and ended the discussion on increasing and decreasing function as "when the derivative values were positive the function was increasing and when the derivative values were negative the function was decreasing." They wrote on the worksheet that “in the interval that the function was increasing, slope is positive. Slope means derivative. In the interval that the function was decreasing, slope is negative.

Table 4.9

Routine of group discourse on increasing and decreasing function

Prompt	Starting discussion on increasing and decreasing function	Question1: What would you say for function f in the interval where $f'(x) > 0$? Question2: What would you say for function f in the interval where $f'(x) < 0$?
How routine	Working on the graph Sketching lines tangent to the graph	
	Relating the sign of the slopes to the derivative	Suzan: First of all derivative means slope
	Examining the slopes of the tangent lines positive or negative	Suzan: Slope means that if x is greater than 0, then, when we sketch a tangent line to function, the graph, let's say we see that the slope is positive.
Closure	Concluding the discussion	Özgü: When the derivative values were positive the function was increasing and when the derivative values were negative the function was decreasing.
	Write their answer on the worksheet	In the interval that the function was increasing, slope is positive. Slope means derivative In the interval that the function was decreasing, slope is negative

4.2 Classroom Discourse on Derivative

In this section, the research question "How do pre-service elementary mathematics teachers explain the concept of derivative in classroom discourse from commognition perspective?" would be answered.

4.2.1 Classroom Discourse on Rate of Change

Pre-service teachers' classroom discourse on rate of change was analyzed according to rate of change, average rate of change and instantaneous rate of change.

Words used in classroom discourse on rate of change

Pre-service teachers' classroom discourse was analyzed to determine the used words in classroom discussions. They were categorized according to the mathematical notions and Sfard's process-object duality. Therefore categories related to mathematical notions were rate of change, average rate of change, slope, instantaneous rate of change, limit and derivative. According to the process-object duality, the words used in group discourse were mostly objectified. The operational words were related to the mathematical "notion of limit. Pre-service teachers' words used related to rate of change in classroom discussion and their categories were given in the Table

"Minus eight divided by five" and "three divided by two" were operationally used words connected with rate of change. Other words used connected with rate of change were "rate of change", "rate of change for each week", "rate of change per week" and "kilogram over week", "rate of change of function". They were objectified words.

"Average rate of change in first five weeks", "average velocity", "rate of total change in position to passed time", "total change in position to time", "average lost weight", "slope between points on the curve" and "slope of rated parts of function" were the words defining average rate of change. These words were objectified."Total change in position is divided by passed time" and "slope of the rated parts of function" were the operationally used words.

"Slope of the tangent line" was the word related to slope. It was operationally used. "Limit of the derivative", "limit of slopes", "limit of average velocity" were the objectified used words related to limit notion of derivative. Also, “approach” and “approach from left and from right” were operationally used words.

“Instantaneous velocity” and “at t=1” were words connected with instantaneous rate of change. They are objectified words.

Table 4.10

Words used in group discourse on increasing and decreasing function

Category	Words	Type
Rate of change	Minus eight divided by five	Operational
	Three divided by two	Operational
	Rate of change	Objectified
	Rate of change for each week	Objectified
	Rate of change per week	Objectified
	Kilogram over week	Objectified
	Rate of change of function	Objectified
Average rate of change	Average rate of change in first five weeks	Objectified
	Average velocity	Objectified
	Rate of total change in position to passed time	Objectified
	Total change in position to time	Objectified
	Average lost weight	Objectified
	Slope between points on the curve	Objectified
	Slope of rated parts of function	Objectified
	Total change in position is divided by passed time	Operational
	Slope of the rated parts of function	Operational
Slope	Slope of the tangent line	Operational
Instantaneous rate of change	Instantaneous velocity	Objectified
	At t=1	Objectified
Limit	Limit of the derivative	Objectified
	Limit of slopes	Objectified
	Limit of average velocity	Objectified
	Approach	Operational
	Approach from left and from right	Operational

In the classroom discussion, pre-service teachers explained the rate of change of the weight as “the rate of change in weight with respect to the change in time passed” as $-\frac{8}{5}$. By finding this rate, they divided the change in weight in five weeks period as 8 by the period of 5 weeks. As the weight was lost in that 5 weeks period, the sign of the rate was minus. There were some students who gave the answer of $\frac{8}{5}$ but they corrected their answers as $-\frac{8}{5}$ after the discussion on how to find the answer of that question. How to find the answer to the question was explained by the instructor at the end of the discussion as “What will I write? Last weight minus first weight over z last (time last) minus z first (time first) (at the same time she wrote on the board $\frac{weight_{last}-weight_{first}}{time_{last}-time_{first}}$. What is this rate meant? (She asked the rate of the change of the weight given in part a to the change of the time given in part c).

B: 8 over 5

C: minus 8 over 5

R: What will I write? Last weight minus first weight over z last (time last) minus z first (time first) (at the same time she wrote on the board $\frac{weight_{last}-weight_{first}}{time_{last}-time_{first}}$. What is this rate?

Pre-service teachers discussed the meaning of the rate of change of the weight in first five weeks in the classroom discussion. Yasin commented on that rate as “rate of change”. Suzan answered as “rate of change for each week”. Yeliz and Mahmut said that “rate of change per week”. Yasin focused only on the rate of change in his answer not the past time. However, Suzan, Yeliz and Mahmut thought about the past time and gave the answer as that rate meant how much change in weight for each week. Actually, that rate was average rate of change per week. Filiz saw that truth and answered the instructor’s question of “what we found by this rate” as “Average rate of change in weight for first five weeks”. Their dialogue was given below.

R: What is this? 1 point 6 (She wrote on the board $-\frac{8}{5}=-1.6$). Ok we found this. What does this rate means?

Yasin: Rate of change.

Suzan: Rate of change for each week.

Yeliz: Rate of change per week.

Mahmut: Rate of change per week

R: What we found by this rate?

Filiz: Average rate of change in weight for first five weeks

Derya found the average rate of change in weight from 5th week to 7th week. Derya answered that question as "$\frac{3}{2}$" and "one and half". After the instructor explained how to find the answer of that question, Yeliz commented on the answer as "kilogram over week". She would give that answer because the rate was found by dividing the change in the weight in kilograms to the passed time, two weeks passed.

Derya: 3 over 2

R: 3 over 2 ($\frac{3}{2}$). Then.

Derya: one and half

R: 1 point 5. (She wrote $\frac{60-57}{7-5} = \frac{3}{2} = 1.5$ on the board)

Yeliz: kilogram over week

Filiz explaned the meaning of the rate of change in weight to passed time as "average rate of change in weight for first five weeks". Their dialogue was given below.

R: What is this? 1 point 6 (She wrote on the board $-\frac{8}{5} = -1.6$). Ok we found this. What does this rate means?

Yasin: Rate of change.

Suzan: Rate of change for each week.

Yeliz: Rate of change per week.

Mahmut: Rate of change per week

R: What we found by this rate? Let's say again.

Filiz: Average rate of change in weight for first five weeks

Instructor and the pre-service teachers discussed on the average velocity. The instructor asked what they should understand from average velocity in a time interval. Pre-service teachers answered this question as "the rate of total change in position to passed time", "total distance over total time" and "change in position over time". Their discussion was given below.

Instructor: What you understand from average velocity in a time interval?

Yeliz: The rate of change in position to passed time.

Mahmut: Total distance over total time.

Özgü: Total change in position over time.

They also discussed the relation between the average rate of change and the slope of the line that is sketched between the points of a given interval. The instructor asked whether they could find the slope or not? Yasin said that "the slope gives the instantaneous velocity".

After Yasin's answer, instructor sketched a graph on the board and sketched a secant line between two points on the curve. Pre-service teachers agreed that the slope of this line did not give the instantaneous velocity. Then Emel defined the instantaneous velocity as "the slope of the line sketched at a point on the curve". Then Yasin also agreed. The instructor explained how to find the average velocity as "I find average velocity, I look at where the ball is at the third second and where it is at the first second and I divide it by how much time passed."

Then they find the average velocities in the intervals $4<t<5$, and decided that the negative sign represented the ball was moving to downwards. Yakup said that the unit of that average velocity was meter over second (m/s).

They discussed if this average velocity was the average rate of change or not? Pre-service teachers said that the change was in the velocity. The instructor corrected them and explained that the change was in the position of the ball, not the velocity, and explained the movement of the ball through six seconds. She explained how they could find the change in position as they look at where the ball was at the beginning and at the end of the time interval and how much change occurred. She said "I found the rate of this change to the passed time".

The instructor made pre-service teachers remember the average change in weight and how they found this average rate of change. Yeliz said that "we found the rate of change in weight to the week". Then the instructor explained that in this situation the rate of change was the average velocity.

They discussed what that quotient mean $\frac{f(a+h)-f(a)}{h}$ given in the third worksheet. Ayşe explained this quotient as "the slope between the points a and $a+h$". In her expression, Ayşe intended to mean that this quotient represented the slope of the line segment sketched between the points of x values a and $a + h$. However, she did not use the words “line”, “line segment” or “secant line” in her expression and she called the apsis values a and $a + h$ as point. Özgü also explained quotient that it was the slope between the points A and B on the curve and she sketched a line in the air. In her expression Özgü also did not use the words “line”, “line segment” or “secant line” to explain which slope was that. Therefore, in Ayşe’s and Özgü’s explanations what this slope was related to was implicit. However, they intended to mean the slope of the line sketched between two points on a curve of a function. Ali commented on the meaning of this quotient as the slope of the rated parts of the function and explained this expression by the limiting process.

Instructor: In the first question, f was a function, what this quotient meant. (shows the quotient $\frac{f(a+h)-f(a)}{h}$ that she wrote on the board).

Ayşe: There is a point $a + h$ and there is also a point a. It represents the slope between them.

Instructor: It represents the slope between them.

Özgü: As we did a curve like that (she sketched a curve in the air). There is a point A and a point B, the slope between them.

Instructor: All of you think like that.

Ali: It gives the rated parts of the function

Instructor: It gives the rated parts of the function. What does it mean?

Ali: They approach.

Then the instructor explained what this quotient meant. She added that this quotient was not aboutapproaching; limit is not the focus here. She sketched a positive convex curve on the graph. She pointed the x values of a and $a + h$ on the x axis and pointed to the corresponding points of these x-values on the curve, then explained that the function values were subtracted from each other ($f(a + h) - f(a)$) and divided by the difference between the x-values ($a + h - a = h$). Therefore, this quotient meant the rate of change in the function f.

Pre-service teachers had conflicts on some mathematical notions related to instantaneous rate of change. One of these conflicts was if the average velocity and instantaneous velocity were same or not. The instructor and the pre-service teachers discussed on the instantaneous velocity of the ball, instantaneous rate of change of a function, and the relation between instantaneous rate of change and derivative. At the beginning of the discussion, a few pre-service teachers considered average velocity and instantaneous velocity were the same. However, five other pre-service teachers refused this idea as reasoning that average velocity was defined in an interval. In this interval, the velocity changed continuously, therefore it was not possible to find the velocity at a point by using average velocity. The second conflict was on how to find the instantaneous velocity. They agreed that the instantaneous velocity of the ball was equal to the slope of the line tangent to the curve at the intended point. However, they had different ideas on how to find the slope of this tangent line. One of them suggested finding the slope as dividing the position of the ball at the intended point by time (1. second). One of them proposed to find the algebraic expression for the function of the graph. Another one said that they would find the algebraic expression for the parabola. The third conflict was finding the limit of the average velocities. Some of them thought that they found the limit of the derivative; a few of them thought that they found the slopes and the velocity-time graph.

According to these discussions, there were some situations where pre-service teachers had some common usage such as "the slope of the function" and most of them understood what they meant. They explained this expression as the slope of the function meant that slope of the tangent line. There were some implicit expressions in pre-service explorations. For example, one pre-service teacher used the expression "slope between two points". In this expression, it was not clear that this slope was related to a line segment, a tangent line or a secant line.

Classroom discussion was sometimes directed by the instructor and sometimes by the pre-service teachers. In some cases, the instructor asked a question about thenotion and they started a new discussion. In some other cases, pre-service teachers asked a question or used an expression, and then they discussed the answer or what this expression meant.

They discussed the instantaneous velocity of the ball at t=1 s. The instructor asked whether finding the average velocity was enough to find the velocity at t=1s. Yasin answered this question as the velocity between 0 and 1 second was equal to the average velocity, but they could not say this for the graph. He added that they could say that the average velocity between 0 and 1 seconds was equal to the velocity at t=1 second. Deniz refused Yasin's ideas that as the velocity changes continuously in the interval (0, 1), so they could not find the velocity by finding the average velocity. Emel and Yeliz also approved Deniz. Emel stated that there were parts between 0 and 1 second such as 0.25 seconds and 0.5 seconds, thus there were velocities at those seconds. Therefore the average velocity and the instantaneous velocity were different. Suzan also agreed with her friends Deniz, Emel and Yeliz and supported her idea by giving example of the weight change in the first worksheet." As one week consisted of seven days, in the first day one would lost 600 gr then would not loose weigth". She added that lost weight was the average lost weight.

Mahmut approved Yasin and said that the average velocity was equal to instantaneous velocity. Yeşim refused Mahmut and Yasin by explaining that values of average rate of change in the interval (0,1) and the instantaneous velocity at t= 1s. were different. Tamer also applied a wrong method to find the instantaneous velocity as $\frac{V_{last}-V_{first}}{t_{last}-t_{first}}$ where V_{last} represented the velocity at t=1s. and V_{first} represented the velocity at t=0 second. t_{last} represented time when the movement ended and t_{ilk} represents time when the movement started. He took the V_{last} velocity at t=1s. as 25.2 as the average velocity. Emel refused him as that velocity was the average velocity, we could not know if that was the velocity at t=1s. The instructor asked to whole class as if they agreed that the average velocity and instantaneous velocity were different things. More than half of the pre-service teachers agreed. Then she explained the meaning of the velocity and the difference between instantaneous velocity and average velocity.

Pre-service teachers said that they should find the slope at t=1s. to find the instantaneous velocity. Instructor sketched the graph representing the movement of the ball and a tangent line at t=1s. They all agreed that the slope of that tangent line would be the instantaneous velocity at t=1s. However, pre-service teachers could not decide how to find the slope of that tangent line. Ali said that they would find the slope using the tangent value of the angle; however he could not answer the question of where that tangent line passed through the x-axis. Emel said that it was distance over time. The instructor answered her that there was no change in position. Yeliz said that they should find the limit. Selin suggested finding the algebraic expression for the graph. Selim agreed with her and he also suggested finding the algebraic formula for the parabola as the graph was a concave parabola.

After the discussions on finding the algebraic formula for the graph, Eda suggested determining close values to the x value 1. Then the instructor wrote on the board "The average velocity is not enough. We find the slope of the tangent line at that point" as the answer of the seventh question.

After deciding on average velocity was not enough for finding the velocity at a certain value such as t=1s, they started to discuss on instantaneous velocity. Emel and Sezen agreed that there should be "limit". Yasin mentioned about approaching 1 considering the given close points in the eighth question of the second worksheet. They examined the given close values to 1. They realized that these given apsis values approach 1 from left and from right. The instructor assigned these given x values on the graph represented at the movement of the ball. Şenay suggested finding the slope of the line segment but Yasin said that they need close values, the slope of this line segments was not enough. Then they agreed to consider the close x values to 1. They decided to find the average velocities between these x values and the instructor found the average velocities for each interval such as (0.9, 1), (0.99, 1), (1, 1.001), (1, 1.01).

After finding these average velocities, they found an approximate value for the velocity of the ball at t=1s. Pre-service teachers commented on this procedure as finding the limit value.

Emel: We approached like limit from left and from right.

Yakup: Did we take the limit?

Emre: It goes to limit.

Emel: We come from left and from right for derivative.

Özgür: We found the derivative.

Emel: We looked for the limit of the derivative

Eda: We looked for the limit of the slopes

İlker: We looked for the limit at 1.

They commented on finding the limit of the derivative, slopes, velocity-time graph.

Emel: Limit of the derivative.

Mine: Limit of the slopes.

Emre: Limit of the velocity-time graph.

Emel insisted on finding the limit of the derivative, although she thought that finding the limit, she also got the derivative.

They came to the conclusion of finding this limit value of the average velocities. Then they found the instantaneous rate of change. Pre-service teachers commented on how they found this instantaneous velocity. They made that approach by taking limit, although they found the limit by approaching. Then they related this instantaneous velocity to the slope of the tangent line at t=1s. At the end of the lesson they come to a conclusion that they found the limit of the slopes.The instructor expressed this symbolically and wrote on the board $\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$ as $\frac{f(x)-f(x_0)}{x-x_0}$ gave the slope. Eda said that it was the definition of derivative.

In the next lesson, they discussed on how they would represent this instantaneous velocity for any function. Pre-service teachers studied in their groups and decided that $\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0}$ represented the instantaneous rate of change for a function. Pre-service teachers commented that to find the instantaneous rate of change of a function was what they needed.

Şule: $\frac{f(x)-f(x_0)}{x-x_0}$ represented the limit, we find the limit of the slope.

Ayça: Slope of the function.

Özgü: There would be x and x_0 points.

Şeyda: We need the slope of the tangent.

Mahmut: The limit while approaching to x_0

Pre-service teachers developed a common language for the slope of the tangent line as "slope of the function". Mahmut explained this expression as "slope at a point, slope of the tangent"

Semra showed this quotient $\frac{f(x)-f(x_0)}{x-x_0}$ by sketching a line between two points A and B on the curve and said that this quotient meant the slope of the line segment AB. While the instructor was explaining the meaning of $\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$, she referred the pre-service teachers expressions. She asked "what does " $\lim\limits_{x\to x_0}$ " means? Then she explained what instantaneous rate of change meant for a function. She said that she should approach x_0 as the difference between x and x_0 would be nearly zero as $\lim\limits_{x\to x_0}$ represented. Then she added that this quotient meant the slope of the line segment sketched between the point $(x, f(x))$ and $(x_0, f(x_0))$. Then she wanted the pre-service teachers explain what the whole expression $\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$ meant. Hakan said that it was the derivative at x_0. Sezin said that it was the slope of the line passing through the point x_0. Yavuz said that it meant approaching to x_0. Aslı said that it was the slope at x_0. Yeliz said that when x was equal to x_0, as x approached to x_0, the slope of the lines approached that point. Then the instructor explained that the x value represented different apsis values in the domain of the function.

They concluded that $\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$ represented the limit of the slopes of the line segments sketched between $(x, f(x))$ and $(x_0, f(x_0))$. And as x represented different apsis values in the interval and when this x values approached to x_0, the slope of the lines became the slope of the tangent line at x_0 and the slope of this tangent line was the derivative.

After explaining the meaning of $\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0}$, they discussed the meaninng of $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. First of all they discussed the meaning of h and come to conclusion of h meant the difference between the values x and x_0. Asx values approach to x_0, h also approach to 0.

Then they found the instantaneous rate of change for the function x^2 at the x value 1 by using x values and $f(x)$ values given for the close values to $x = 1$. They used the quotient $\frac{f(a+h)-f(a)}{h}$ to find the average rate of change of the function for the close x values to 1. When they analyzed this rate of change values, they concluded that these values approached to 2 when these close x values approached to 1.

In the fifth question of the third worksheet, they used the graph to find the instantaneous velocity of the ball at t=1s. They chose close x values to 1 and found the average velocities for those intervals, then found the limit of these average velocities as these apsis values approached to 1. They applied the same procedures to find a formula for the function x^2 by using the close x values to 1, 2, and 3 given in the table.

Visual mediators used in classroom discourse on rate of change

In classroom discourse instructor used visual mediators to answer and explain the questions asked in the worksheets. Visual mediators they used to express rate of change were grouped in three categories such as graph, algebraic symbols and written words.

She used graphs to explain the average velocity, average rate of change, instantaneous velocity and instantaneous rate of change. She sketched increasing and convex graphs in her explanations given in the Figure 4.5.

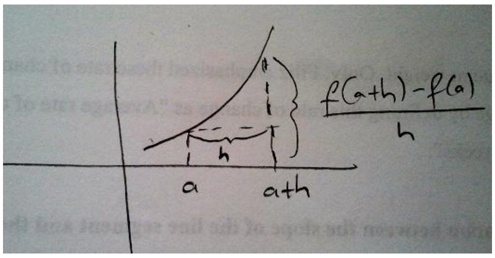


Figure 4.5 Graph represents difference quotient $\frac{f(a+h)-f(a)}{h}$

She sketched the graph in figure 4.6 to explain the instantaneous rate of change of function of x^2.

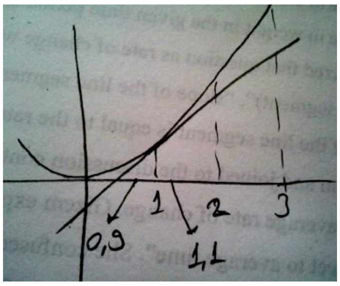


Figure 4.6 Graph represents instantaneous rate of change

She also used symbolic expressions to explain average rate of change of function and used algebraic symbols to find average rate of change values. Algebraic symbols that instructor used: $\frac{f(a+h)-f(a)}{h}$ and for the interval $0.998 < x < 1$, $\frac{1-0.996004}{1-0.998} = 1.998$.

Endorsed narratives used in classroom discourse on rate of change

In classroom discourse, instructor and pre-service teachers discussed on rate of change, average rate of change and instantaneous rate of change. They used endorsed narratives to explain the relation between rate of change and slope of a line segment, instantaneous rate of change and slope of the tangent line to a graph at any point. Endorsed narratives they were used were listed in the Table 4.11.

Pre-service teachers used both object-level and meta-level endorsed narratives. Their object level narratives were on "the slope of the line segment and rate of change" and "instantaneous rate of change and slope of the tangent line". Meta level narratives were on "how to find the instantaneous rate of change". Instructor also used both object-level and meta-level endorsed narratives. She used object-level narratives to explain "relation between limit of the slope of secant lines, slope of the tangent line and derivative." She used meta-level narratives to explain "how to find derivative at a point.

Table 4.11

Instructor's and pre-service teachers' narratives on rate of change

Instructor's narratives	Type
As limit of the slopes gives the slope of the tangent, this gives me the derivative	Object-level
I found the limit of average rate of change, limit of slope of the line segments to find the derivative	Meta-level

Pre-service teachers' narratives	Type
The slope of the line segment gives the rate of change of weight	Object-level
The slope of the line segment is equal to the rate of change	Object-level
The slope is instantaneous velocity	Object-level
The slope of the tangent to the graph at any point (instantaneous velocity)	Object-level
I found slope of the tangent and limit of it	Meta-level
I found the instantaneous rate of change by finding the limit as h goes to zero	Meta-level
I found the instantaneous velocity by finding the limit of average velocities	Meta-level

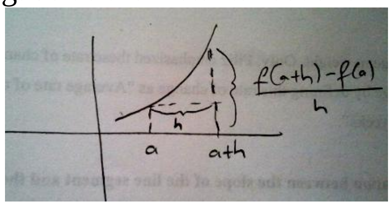
Routine of classroom discourse on instantaneous rate of change

Pre-service teachers' and instructor' discussions on instantaneous rate of change in the classroom discussions had a repetitive pattern. They discussed the instantaneous velocity and instantaneous rate of change in the same way. Therefore, I explained the routine on how

they discussed instantaneous rate of change in Table 4.12. In this routine, the instructor started the discussion on instantaneous rate of change of a function by asking the question of "How do you explain instantaneous rate of change of a function? What do you do to find it?" They discussed the slope of the tangent line and instantaneous velocity. The instructor sketched a graph and explained the difference quotient on it. Then they determined the slope of the secant lines. The instructor closed the discussion by giving the algebraic symbol of the instantaneous rate of change and defining derivative as instantaneous rate of change.

Table 4.12

Routine of classroom discourse on instantaneous rate of change of a function

Prompt	Starting the discussion Asking question related to instantaneous rate of change of a function	I: How do you explain instantaneous rate of change of a function? What do you do to find it?
How routine	Pre-service teachers answered her	Yeliz: I find the limit Özgü: I find the slope of the tangent; I find the limit of it.
	Mentioned the slope of the tangent line	I: I approach to a. The slope of the tangent line gave me the instantaneous rate of change.
	They referred to the previous notion of instantaneous velocity	I: How did we find the instantaneous velocity Pre-service teachers: We found the limit of average velocities. I: We looked for the average velocities in an interval. The limit of the average velocities gave us the instantaneous rate of change
	Sketched a graph on the board Assign the x values a, a+h and showed the differences between x values and y values	
	Determined the average rate of change for a function	I: Average rate of change in that interval (She sketched secant lines between points on the curve)
	Determine the slope of the secant lines	I: I find the slope of the secant lines
	Determine the limit of the slope of the secant lines	I: I approach to a from left and from right and look for the slopes of the lines. What this limit of the slopes gave me? Pre-service teachers: Instantaneous velocity I: Instantaneous velocity, instantaneous rate of change for a function
Closure	Write the algebraic formula Gives the definition of derivative	$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

4.2.2 Classroom Discourse on Increasing and Decreasing Function

In this section, pre-service teachers and the instructor discussed the increasing and decreasing function and its relation to the sign of the derivative function. Their classroom discourse was analyzed according to the elements of commognition. Their word use, visual mediators, endorsed narratives and routines will be discussed in this section.

Words used in classroom discourse on increasing and decreasing function

Transcripts of the pre-service teachers' and the instructors' classroom discourse and written materials were analyzed to determine the used words related to increasing and decreasing function. They were categorized according to the mathematical notions and Sfard's process-object duality which categorized the word use as operational and objectified. Categories of the used words and their types were given in the Table 4.13.

Pre-service teachers' and the instructor's used words in the classroom discourse related to increasing and decreasing functions were grouped into four categories. These categories were slope, derivative, function and interval. Instructor used the words related to function to express the properties of a function related to the first and second derivative function. Properties of the graph of the function were also grouped in this category. Used words grouped in this category were also classified as operational and objectified. Objectified words in the "function" category were "rate of change", "change in f", "change in x", "change was 0", "h", "extremum points", "maximum point", "minimum point", "local minimum", "local maximum", "critical point", "derivative function", "second derivative function", "origin", "graph of derivative function", "curve graph", "line graph", "positive values", "concave", "convex", "values where the function were positive". Operationally used words were "increasing", "decreasing", "x values increase" and "y values increase". Used words in this category were also classified in four subcategories that function was related to: slope, first derivative, second derivative and graph. Categories and subcategories and the words appeared in these categories were listed in the Table 4.13.

Words in the category of slope used by the instructor were "average rate of change", "slope", "point where the slope was 0", "positive slope", "negative slope", “points where slope was greater than 0”, “tangent at a point”, “angle of inclination”, “acute angle”, “obtuse angle”, “angle values were increasing”, “angle values were decreasing”, “slope was increasing”, “slope was decreasing”. These words were also classified in three subcategories that words were related to: sign of slope, angle of inclination, function graph (Table 4.13). “Angle values were increasing”, “angle values were decreasing”, “slope was increasing” and “slope was decreasing” were operationally used words which were used to explain the change in angle values and slope of the tangent lines sketched to the function graph. “Average rate of change”, “slope”, “point where the slope was 0”, “positive slope”, “negative slope”, “points where slope was greater than 0”, “tangent at a point”, “angle of inclination”, “acute angle” and “obtuse angle” were the objectified words.

Words used in the “derivative” category were also classified in two subcategories that words were related to: first derivative and second derivative (Table 4.13). “Derivative is decreasing”, “derivative is increasing”, “derivative is positive and increasing”, “derivative is positive and decreasing”, “derivative is negative increasing”, “increasing as x decreases” and “decreasing as x increases” were operationally used words that implied the increasing and decreasing of the function and the derivative of it. “Interval that derivative is greater than zero”, “derivative is positive”, “derivative is negative”, “derivative is zero”, “derivative is less than zero”, “derivative is greater than zero”, “derivative is greater than zero everywhere”, “derivative is negative everywhere”, “derivative function”, “second derivative”, “second derivative function”, “second derivative is greater than zero”, “graph of derivative function”, “acceleration”, “acceleration is positive” and “acceleration is negative” were objectified words as they identified the first and second derivative and their relation to the function.

In the fourth categorization, the words were used related to the intervals where the function and the derivative function were increasing. Words used in this category were also categorized according to colloquially and objectified words. “Points where the slope was greater than zero”, “minus infinity to one”, “one to infinity”, “all real numbers in that interval”, “for x was greater than zero” were the objectified words as they infer the intervals that the derivative was positive and negative and the function was increasing and decreasing. “Derivative was negative everywhere”, “derivative was increasing everywhere”, “increasing everywhere”, “positive everywhere” and “upward” were colloquially used words. In these expressions “above” implied the points above x axis in the coordinate system and “everywhere” implied the x values in the domain of the function. The instructor used these expressions to make the pre-service teachers form the relation between the increasing and decreasing function and the first derivative of the function or the concavity of the function and the second derivative. These expressions were also used by pre-service teachers. These expressions were specific for this classroom discourse. For someone not a member of this discourse, these words would not have meanings.

Pre-service teachers’ used words in the classroom discourse related to increasing and decreasing functions were also grouped into four categories: Slope, function, derivative and interval. Used words were classified in the function category were also grouped in subcategories as slope and rate of change, first derivative, second derivative and graph of the function. The categories and the used words were given in the Table 4.8. Used words grouped in this category were also classified as operational and objectified. Objectified words in the “function” category were “rate of change was zero”, “change in y values”, “1+h”, “change was zero”, “critical point”, “function graph was concave”, “function graph was convex”, “the function looks upward”, “extremum points”, “function was positive”, “function was decreasing”, “function was increasing”, “increasing”, “decreasing”, “increasing curve” and “decreasing as x increases”. Operationally used words were “function was increasing”, “increasing”, “function value was decreasing”, “function was decreasing”, “function was increasing”, “increasing”, “decreasing”, “increasing curve” and “decreasing as x increases”.

Pre-service teachers used words in the “slope” category were also classified in three subcategories: sign, angle of inclination, graph. Categories and subcategories and the used words appeared in these categories were listed in the Table 4.8. “Points where the slope was positive”, “positive”, “slope was positive”, “slope was negative”, “angle of inclination was zero”, “slope of the tangent”, “less than 90^0”, “alfa is greater than 90^0” and “slope was constant” were the objectified words. “Angle was increasing”, “slope was increasing”,

“slope was decreasing” and “tangent of the angle was decreasing” were operationally used words. “Slopes were big”, “slopes were small” and “slope was vertical” were colloquially used words as these words were specific to this classroom discourse.

There were two subcategories related to “derivative” category of the pre-service teachers’ used words. These subcategories were first derivative and second derivative. Used words and the related to slope category and its subcategories were listed in the Table 4.8. “Derivative is greater than zero”, “derivative is positive”, “derivative is less than zero” and “derivative of the derivative” were the objectified words. “Derivative is increasing”,

“derivative is negative increasing”, “derivative of the derivative”, “velocity is increasing” and “velocity is decreasing” were the operationally used words. “Derivative is negative on the left” and “derivative is positive on the right” were colloquially used words. The listed words in the first derivative subcategory were the words used to explain the relation between the first derivative and the increasing and decreasing of the function. On the other hand, words in the second derivative subcategory used to explain the relation between second derivative and derivative function.

In the “interval” category, “minus infinity to one”, “one to infinity”, “on the positive side” and “above the x axis” were the objectified words. “Increasing curve in the first quadrant” and “goes to infinity” were operationally used words. Moreover, “derivative is negative on the left” and “derivative is positive on the right” were colloquially used words.

There were some similarities and differences between the words instructor and pre-service teachers used in the classroom discourse on increasing and decreasing function. In the function category and second derivative category, instructor used the “second derivative function” expression. But pre-service teachers did not use. On the other hand, pre-service teachers used “the function looks upward”, but the instructor did not use this expression. Moreover, pre-service teachers did not use the expression “first derivative function” in the first derivative while the instructor used it. It implies that pre-service teachers avoided using “first derivative function” and “second derivative function” expressions.

Pre-service teachers found the increasing and decreasing intervals of the function by using positive and negative values of slope of tangent lines. They inferred first derivative a few times. Therefore, they used only five words in the “first derivative” subcategory of derivative category. On the other hand, instructor emphasized the relation between increasing and decreasing function and sign of the derivative of the function. Thus, instructor used much more words connected to first derivative than pre-service teachers.

Table 4.13

Word use of pre-service teachers and instructor on increasing and decreasing function

		Instructor		Pre-service teacher	
Function	Slope	• rate of change • change in f • change in x	• change was 0 • h	• rate of change was zero • change in y values	• 1+h • change was zero
	First derivative	• extremum points • maximum point • minimum point • local minimum • local maximum • derivative function	• critical point • increasing • decreasing • x values increase • y values increase	• function was increasing • function value was decreasing • critical point • extremum points • function was positive	• function was decreasing • function was increasing • decreasing • increasing curve • decreasing as x increases
	Second derivative	• second derivative function	• concave • convex	• function graph was concave • function graph was convex	• the function looks upward
	Graph	• origin • graph of derivative function • curve graph • line graph	• positive values • concave • convex • values where the function were positive	• function graph was concave • function graph was convex • the function looks upward	
Slope	Sign	• Point where the slope was 0 • positive slope	• negative slope • points where slope was greater than 0	• Points where the slope was positive • positive	• slope was positive • slope was negative
	Angle of inclination	• angle of inclination • acute angle • obtuse angle	• angle values were increasing • angle values were decreasing	• angle of inclination was zero • slope of the tangent • less than 90^0	• alfa is greater than 90^0 • angle was increasing
	Function graph	• slope was increasing • slope was decreasing		• Slope was increasing • Slope was decreasing • Slopes were big • Slopes were small	• Slope was vertical • Tangent of the angle was decreasing • Slope was constant

Table 4.13 (cont.'d)

Word use of pre-service teachers and instructor on increasing and decreasing function

		Instructor		Pre-service teacher	
Derivative	First derivative	• derivative is greater than zero • derivative is positive • derivative is negative • derivative is zero • derivative is less than zero • derivative is greater than 0 • derivative is decreasing • derivative is increasing • graph of derivative function	• derivative is positive and increasing • derivative is positive and decreasing • derivative is greater than 0 everywhere • derivative is negative everywhere • derivative is negative increasing • derivative function	• Derivative is greater than zero • Derivative is positive • Derivative is less than zero • Derivative is negative on the left • Derivative is positive on the right	
	Second derivative	• second derivative • second derivative function • second derivative is greater than zero • increasing as x decreases	• decreasing as x increases • increases acceleration • acceleration is positive • acceleration is negative	• Derivative is increasing • Derivative is negative increasing • Derivative of the derivative	• Velocity is increasing • Velocity is decreasing
Interval		• Points the slope was greater than 0 • Minus infinity to one • One to infinity • Derivative was negative everywhere • Upward	• Derivative was increasing everywhere • All real numbers in that interval • Increasing everywhere • Positive everywhere • For x was greater than zero	• Minus infinity to one • One to infinity • Derivative is negative on the left • Derivative is positive on the right	• Goes to infinity • On the positive side • Above the x axis • Increasing curve in the first quadrant

Pre-service teachers and the instructor discussed increasing decreasing functions, concavity of the function graph, local extremum points and their relation to first and second derivatives. Pre-service teachers had some problems and difficulties to make the connections between properties of the function, its first derivative and second derivative. They sometimes confused the relations and considered them in reverse way. They had tendency to employ the rules that they know from high school and the algebraic expression of the function rather than finding or using the relations.

Some of the pre-service teachers tried to use the rules that they remembered from high school in the classroom discussions. For example, when they were working on the relation between the first and the second derivative of a function they tried to use the algebraic expression of the function and the differentiation rules. For example Tekin considering the graph of x^2 said that as the graph was convex therefore the second derivative should be greater than zero. Büşra also tried to use the algebraic expression of the derivative function of f and define the relation. However, the instructor refuted them and insisted on finding the relation by analyzing the given graph of the function f.

On the other hand, Meryem suggested sketching the graph of first derivative function of f. She explained the relation of the graph of f and the first derivative of that function as the graph decreasing in the interval $(-\infty, 0)$, then the derivative function should be negative. And also as the graph increasing in the interval $(0, \infty)$, then the derivative function should be positive. Moreover, at x value 0 the derivative of function f was zero.

The instructor explained Meryem's suggestion and said that they could sketch a similar graph having the properties of the derivative function of f to comment on the second derivative of function f. Therefore, the instructor sketched a graph on the board. She checked if the graph provided the features of the derivative graph. She explained the derivative function why it was a function as it had all the x values and corresponding y values in the interval. Therefore, it represented by an algebraic equation of a function. She also explained that the second derivative was also a function. They verified the relation between the function, the first derivative function and the second derivative function by analyzing the function graph.

Some of the pre-service teachers had problems related to the increasing and decreasing functions. They could not distinguish whether the function was increasing or decreasing. Instructor explained increasing and decreasing functions on the graph several times. She examined whether the y values increase or decrease while x values were increasing and decreasing. Some pre-service teachers also had problems related to the increasing or decreasing of the derivative function although they did not have information relevant to derivative function was increasing or decreasing. They knew that the derivative function was positive for all $x \in R$.

They analyzed the graph that was given in the former questions verifying the relation if the second derivative was greater than zero then the first derivative was increasing and the function graph was convex and if the second derivative was less than zero then the first derivative was decreasing and the function graph was concave. They sketched the graph of the first derivative function and analyzed it to reveal the relation between the first and second derivative functions. Meryem explained the relation considering the first derivative.

They discussed the maximum and minimum points of the function, first and second derivative functions according to the derivative function graph of f. Meryem found the maximum and minimum points of the function f considering the function should be increasing as the derivative graph was positive. They also analyzed the given derivative graph if it had a point where the derivative was zero.

Pre-service teachers had difficulty to analyze the derivative graph and relate it to the maximum and minimum points of the graph of second derivative function and they considered the relation increasing and decreasing function and the sign of the function in reverse way for the first and second derivative functions. They stated that if the first derivative value was positive then the second derivative function was increasing and if the first derivative value was negative then the second derivative was decreasing.

Deniz suggested analyzing the graph of the derivative function in the table. However, she confused the first and second derivative and made conclusions for the second derivative although they were related to the first derivative. Then Yeliz commented that the maximum points for the second derivative function should be between the points where the second derivative function was positive. The instructor also added the points that the second derivative function value was zero. She explained that the points where the graph of the derivative function had the extremum points then the second derivative had the value zero at the x values of these points. Derivative function graph had the extremum points where the derivative of the derivative was zero. Müge said that the second derivative function was decreasing where the first derivative was negative. She formed the relation in reverse way.

They sketched the graph of the second derivative function approximately according to the intervals that the second derivative function was positive or negative and points where the second derivative was zero. Then they determined the maximum and minimum points.

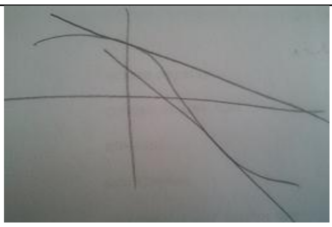
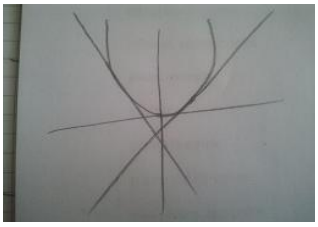
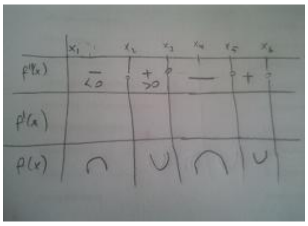
They discussed on the distance-time graph of a moving object. They determined the direction of its movement in which interval it went to right and left. Selin asked whether it slowed down. Then they discussed that they should consider the derivative function to decide whether the velocity of the object decreased or increased. Then they analyzed the graph according to the acceleration of the object and they checked for the concavity of the function graph and they related it to the second derivative of the function. They also used information related to first and second derivative of a function and intervals that the first and second derivatives were positive and negative to sketch the graph of the function without considering the algebraic equation of the function.

Visual Mediators in classroom discourse on increasing and decreasing function

In the classroom discourse, instructor used three types of visual mediators on increasing and decreasing functions: graphs, tables, algebraic symbols and written words. Instructor sketched graphs to show the relations between increasing and decreasing functions and the first derivative and, the first derivative and the second derivative. She sketched the graphs given in the worksheets or sketched new graphs. She also sketched the tangent lines to relate the slope of these lines to the derivative of the function at those points. Instructor also used a table to show the relation between the first derivative, second derivative, increasing-decreasing and concavity of function. Therefore, she determined the intervals that the function was increasing and decreasing, concave and convex and also the critical and inflection points. Three examples of graphs and the table were given in the Table 4.14

Table 4.14

Visual mediators used to explain connections between first derivative, second derivative and function

Visual mediator	When it was sketched
	Graph sketched to show relation between decreasing function and negative sign of derivative
	Graph sketched to show relation between increasing function and positive sign of derivative
	Graph sketched to show relation between increasing-decreasing function and first derivative; concavity of the function and second derivative
	Table showed the relation between first derivative, second derivative and the function

Instructor wrote some expressions on the board after the classroom discussion on the related notions. These written words were on the connection between sign of the derivative value and interval where the function was increasing and decreasing; the sign of the second derivative and interval where the derivative function was increasing and decreasing; extremum points and the value of function f. The written words and when they were written on the board were given in the Table 4.15.

Table 4.15

Instructors' written words related to increasing and decreasing function

Written words	**When they were written**
"In the interval $f^{'}(x) > 0$, function f was increasing." "In the interval $f^{'}(x) < 0$, function f was decreasing" "It was increasing for $x \in$ R" "If the derivative was less than zero, the function was decreasing"	To explain the connection between increasing and decreasing function and the sign of the derivative.
"In the interval $f''(x) > 0$, $f'(x)$ was increasing." "In the interval $f''(x) < 0$, $f'(x)$ was decreasing."	To explain the connection between increasing and decreasing of derivative function and the sign of the second derivative.
"In the interval $f''(x) > 0$, $f^{'}(x)$was increasing, the function graph was looking upwards (convex)." "In the interval $f''(x) < 0$, $f^{'}(x)$ was decreasing, the function graph was looking downwards (concave)."	To explain the connection between first and second derivative and the function graph.
"The point where f(x) gets the maximum value; $(x_6, f(x_6))$" "The point where f(x) gets the minimum value; $(x_1, f(x_1))$"	To find extremum points of the function with respect to the given derivative function graph.
"The point where $f^{'}(x)$ gets the maximum value $(x_3, f(x_3))$" "The points where $f^{'}(x)$ get the minimum value $(x_2, f(x_2))$, $(x_5, f(x_5))$".	To find extremum points of the derivative function with respect to the given derivative function graph.
"At extremum points, the derivative is 0"	To explain extremum points and derivative relation
"In the intervals (0,2/3) and (2,t), it goes to right"	To determine the intervals that the object moves to right
"In the intervals (2/3, 2), it goes to left"	To determine the intervals that the object moves to right

Instructor also used symbolic notations to explain the relations between sign of the derivative value and interval where the function was increasing and decreasing; the sign of the second derivative and interval where the derivative function was increasing and decreasing; extremum points and the value of function f.

Some examples of symbolic notations were given in the Table 4.16.

Table 4.16

Symbolic notations instructor used

Symbolic Notations	**What for they were used**
For x>0 $f^{'}(x) > 0$ For x<0 $f^{'}(x) < 0$	Intervals where the function was increasing and decreasing
$x \in R,\ f^{''}(x)$ >0	Interval where the second derivative function was greater than zero, the function graph was convex
$\beta < 90^{0}\ f^{'}(x) > 0$	Relation between the slope of the tangent line and the derivative value
In $\left(0, \frac{4}{3}\right)\ s^{''(t)} < 0$ In $\left(\frac{4}{3}, t_1\right)\ s^{''(t)} > 0$	Intervals where acceleration function was greater and less than zero

Narratives used in classroom discourse on increasing and decreasing function

Instructor and pre-service teachers used narratives in the classroom discussions to interpret the relations and rules. They used both meta-level and object level narratives. Examples of their narratives and their types were listed in the Table 4.17. Most of the narratives both for instructor and the pre-service teachers were object level. Instructor used three meta-level narratives how to find the derivative function, relation between the first derivative and slope, the relation between the derivative function and increasing-decreasing function. Pre-service teachers used one meta-level narrative how to find the second derivative by using the slope of the tangent lines.

Table 4.17

Instructor's and pre-service teachers' narratives on increasing and decreasing functions

Instuctor's Narratives	Type	Pre-service teachers Narratives	Types
As the function was decreasing for x was less than zero then the derivative should be less than zero	Object-level	The slope is negative between $-\infty$ and 0, so the derivative of the function will be negative.	Object-level
When you find the derivative of the function, you get the derivative function.	Meta-level	Angles are less than 90^0, so the derivative of f is greater than zero. In the other quadrant, angles are greater than 90^0, so the derivative of f is less than zero	Object-level
When I am finding the first derivative, I look for whether the function is decreasing	Meta-level	y values decrease when x values decrease then the function is increasing	Object-level
As the alfa is less than 90^0so the derivative is greater than zero	Object-level	The first derivative is less than zero where x is greater than zero. Then it decreases.	Object-level
As the alfa is greater than 90^0so the derivative is less than zero	Object-level	It is increasing where it is greater than zero	Object-level
y values increase when x values increase then the function is increasing	Object-level	There is transition from increasing to decreasing and decreasing to increasing. So there are extremum points	Object-level
y values decrease when x values increase then the function is decreasing	Object-level	As it decreases (first derivative), the second derivative is less than zero	Object-level
As it is increasing, the derivative should be positive	Object-level	As the graph looks upward, second derivative is greater than zero	Object-level
As it is decreasing, the derivative should be negative	Object-level	We find the second derivative looking for the slope.	Meta-level
When you find the derivative you get the slope	Meta-level	The acceleration is the rate of change of velocity to time	Object-level
The derivative function is decreasing so second derivative is less than zero	Object-level		
As the angle of inclination is greater than 90^0, the tangent of this angle is negative, so the slope is negative.	Object-level		
The derivative is less than zero as β is greater than 90^0	Object-level		
If the acceleration is greater than zero then the velocity is increasing.	Object-level		
If the acceleration is less than zero then the velocity is decreasing.	Object-level		
Maximum and minimum points are the critical points; the points where the second derivative is zero are the inflection points.	Object-level		

Routines of classroom discourse on increasing and decreasing function

Pre-service teachers' and instructor' discussions on increasing and decreasing function in the classroom discussions had a repetitive pattern. They discussed the increasing and decreasing function in the same way. Therefore, I explained the routine on how they discussed increasing function in Table 4.18. It is an example of thr routine of the pre-service teachers in the classroom discussions. In this routine, the instructor started the discussion on increasing function by asking the question of "What would you say for function f in the interval where $f'(x) > 0$?" This is the prompt of the routine. The "how routine" of the routine was developed according to participated pre-service teachers' varying answers. Instructor explained the relation between the sign of the first derivative and the increasing function using the sign of the slopes of the tangent lines. The instructor concluded the discussion by stating the relation "when the derivative values were positive the function was increasing". Then she wrote "In the interval $f'(x) > 0$, function f was increasing" on the board. This was the closure part of the pre-service teachers' routine.

Table 4.18

Routine of classroom discourse on increasing and decreasing function

Prompt	Starting the discussion Asking question related sign of the first derivative	I: What would you say for function f in the interval where $f'(x) > 0$?
How routine	Pre-service teachers answered her	Tekin considering the graph of x^2 said that as the graph was convex therefore the second derivative should be greater than zero. Büşra also tried to use the algebraic expression of the derivative function of f and define the relation. Meryem suggested sketching the graph of first derivative function of f. She explained the relation of the graph of f and the first derivative of that function as the graph decreasing in the interval $(-\infty,\ 0)$, then the derivative function should be negative. And also as the graph increasing in the interval $(0, \infty)$, then the derivative function should be positive.
	They worked on the graph Instructor sketched an increasing graph	
	Related the sign of the slope of the tangent lines to the derivative	I: Slope of the lines is positive as the tangent of the angles less than 90^0. Thus the derivative is also positive
	Related the sign of the derivative to the increasing and decreasing function	I: The function is increasing and the derivative is greater than zero.
Closure	Concluding the discussion Write on the board	I: When the derivative values were positive the function was increasing In the interval $f'(x) > 0$, function f was increasing.

4.3 Pre-Service Teachers' Individual Discourse on Derivative

In this part, pre-service teachers' answers to the first, third, seventh and tenth questions of the derivative test and explanations of their answers to these questions in the interview were analyzed. In this section the research question "How do pre-service elementary mathematics teachers explain the concept of derivative in individual discourse from commognition perspective?" would be answered.

4.3.1 Individual Discourse on First Question on Definition of Derivative

The first question required pre-service teachers to define "derivative" and explain their answers. The first question was "What is derivative? Explain your answer". Pre-service teachers' answers to this question were given in the Table 4.19. According to the first application of derivative test, 10 students defined derivative as differentiation rule for polynomial function, 8 of them defined as slope between points / slope of a function /slope of a point/curve/line/number. 2 of them used the symbolic definition $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$, 2 of them defined as slope of tangent and 2 of them as slope. 1 of them gave the definition of limit. Moreover, 8 of them answered the question by unrelated expressions and 19 of them did not answer it.

According to the second application of the derivative test, pre-service teachers also gave varying answers. I would mention the most popular ones. 14 of the pre-service teachers answered this question as "slope of tangent", 8 of them used the symbolic definition of derivative $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and 8 of them defined as limit of slopes and 6 of them answered as limit of average rate of change.

Pre-service teachers' answers to this question in the post-test were analyzed according to the four elements of discourse from the commognitive framework: word use, visual mediators, endorsed narratives and routines. All the words that pre-service teachers used in the interview to define derivative and explain their perception of derivative listed and categorized in three dimensions in Zandieh's framework which categorizes derivative according to the representations (graphical-slope, verbal-rate, paradigmatic physical-velocity, symbolic- difference quotient) and layers (ratio, limit and function) and Sfard's process-object duality which categorizes the word use as operational and objectified.

Table 4.19

First and second application results of first question on definition of derivative

Answers	Number of pre-service teachers	
	First Application	Second Application
$\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$	2	8
Limit of average rate of change	-	6
Limit of slopes	-	8
Limit of slopes of tangents	-	3
Slope of tangent	2	14
Slope of the tangent line on the graph	-	7
Limit of average speed / instantaneous speed	-	5
Rate of change of a function	-	3
Limit of slope of a line / Slope of tangent	-	3
Definition of Limit	1	5
Tangent of a function at point x=0		1
Differentiation rule for polynomial function	10	1
Slope between points / Slope of a function /slope of a point/curve/line/number	8	3
Slope equation at point x_0	-	2
Derivative = Slope	2	2
Rate of change of a function + Average rate of change	-	4
Not related expression	8	-
No Answer	19	4

Pre-service teachers' word use of the first question on definition of derivative

In this part, each pre-service teacher's word use in the first question is explained. The words that pre-service teachers used in the interview to define derivative and explain their perception of derivative were listed and categorized in three dimensions according to Zandieh's framework which categorizes derivative according to the representations (slope, rate, velocity, difference quotient) and layers (ratio, limit and function) and Sfard's process-object duality which categorizes the word use as operational or objectified.

Sezen's word use for the first question of definition of derivative:

Sezen could not answer this question in the pre-test but she wrote two definitions in the post-test.

The first definition is:

"The derivative is the slope of the line sketched tangent to the function f(x) at the point$(x_0, f(x_0))$."

The second definition is:

"The limit of the slope of the lines passing through the points coming closer to the point $(x_0, f(x_0))$ gives the derivative at the point $(x_0, f(x_0))$".

Sezen defined the derivative concept as slope, namely the slope of the tangent line. In the definition "the derivative is the slope of the line sketched tangent to the function f at the point $(x_0, f(x_0))$", she also perceived derivative as a ratio. She defined the derivative as the slope of the line tangent to the graph of the function at the point $(x_0, f(x_0))$. She said that she could find the slope of the tangent line by the tangent value of the angle formed between the x-axis and the tangent line. $tan \propto$ gives the slope of the tangent line at the same time. According to her first definition and the explanation of this definition she perceived derivative as a slope and a ratio.

Her second definition for the derivative concept was "the limit of the slope of the lines passing through the points closer to $(x_0, f(x_0))$gives the derivative at the point of $(x_0, f(x_0))$". She explained this definition as the limit of the slope of the lines close to the point $(x_0, f(x_0))$. She said that the limit of the slopes of the lines tangent to the graph at the points very close to the point of $(x_0, f(x_0))$gave the derivative of the function at the point $(x_0, f(x_0))$. She indicated that, to find the derivative value at the point $(x_0, f(x_0))$, she should approach this point from left and from right in small intervals. Sezen's second definition of derivative revealed that Sezen still perceived derivative as slope. On the other hand, there is a difference that she perceived derivative also as limit, namely limit of the slope of tangents.

She couldn't find the relation between the symbol $\frac{f(x_1)-f(x_0)}{x_1-x_0}$ which gives the slope of the secant line passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$.
She gavethe definition correctly but she thoughtthat she should take the limit of the slopes of the tangent lines passing through the points very close to the point of the $(x_0, f(x_0))$.

In order to explain her answer to the first question, Sezen used the words "tangent", "tangent alfa", "slope of the tangent", "derivative at the point of x_0", "divide distance to difference", "limit", "close point", "approach", "from left and from right", "slopes approaching to one point" and "small intervals". The words "tangent", "tangent alfa", "slope of the tangent" and "derivative at the point x_0", "divide distance to difference" were connected with the slope notion of derivative. Besides "close point", "approach", "from left and from right", "slopes approaching one point" and "small intervals" were the words indicating the limit notion of derivative.
She used the word "tangent" to define derivative and explain her definition. She defined the derivative as "for the function f(x), the derivative is the slope of the tangent at the point of $x = x_0$". She explained her definition in the following sentences:

Sezen: Here, for example, at this x graph (she sketched a concave and positive graph) for example this point of x_0 (shows the point of x value x_0), let's sketch a *tangent* at this point.

She also used the words "tangent alpha" while explaining her second definition of derivative, she sketched a concave positive function graph and a tangent line at the point $(x_0, f(x_0))$ that she determined on the graph and she intersected this line with the x-axis and she named the angle constructed between the x-axis and this tangent line as alpha (α). Then she explained that she could use the tangent value of this angle to find the slope of the tangent line and she called the derivative as tangent value of the angle alpha. She said that

Sezen: I said *tangent alpha* is the derivative here.

She also used the word “slope of the line” for two purposes, the first one was to define the tangent alpha and the second one was to explain the symbolic formula $\frac{f(x_1)-f(x_0)}{x_1-x_0}$.

She explained tangent alpha as the slope of the line passing through the point of x valuex_0:

R: For this line, what does tangent alpha mean at the same time? (Shows the tangent line sketched at x_0)

Sezen: The *slope of the line* at the same time.

Sezen used the word “limit” to explain her second definition “The limit of the slope of the lines passing through the points coming closer to the point of $(x_0, f(x_0))$ gives the derivative at the point $(x_0, f(x_0))$.” She explained her definition as the limit of the slope of the tangent lines sketched at the points close to the point $(x_0, f(x_0))$.

Sezen: …of the line sketched while getting closer to pointx_0. I tried to express here by *limit*. For example, the slope is this at the point so close (she determined a point close to the point that she sketched the tangent) then I get closer both from right and from left. While getting closer, the slopes get closer to a point. This point, where the slopes get closer, gives again the slope at $(x_0, f(x_0))$.

Sezen also used “difference (of the function values)” which would be related to average rate of change for function values. She used this expression to explain how to find the slope of the lines that are tangent to the points getting closer to the point of $(x_0, f(x_0))$ that she used to define the derivative in her second definition and to explain the limiting process.

Sezen: The limit of the slopes.

R: How we could find the slope of this line (shows the line that Sezen sketched)

Sezen: We were dividing the distance to the difference.

R: Which distance you were dividing?

Sezen: What was it? Not $(x_1 - x_0)$. $\frac{f(x_1)-f(x_0)}{x_1-x_0}$.

R: You were finding the slope by using this.

Sezen: Yes.

Sezen: Of the difference of the functions

Sezen perceived derivative concept as slope of the tangent line to a curve at a point. In the first definition she accepted this slope as a ratio and in the second definition as limit. She expressed the limiting process to find the slope of the line tangent to any function graph at the point of x valuex_0 In her explanation of this definition, she mentioned about taking the

limit of the slopes of the tangent lines passing through close points to the point of x value x_0 instead of the secant lines passing through the points of $(x_0, f(x_0))$ and $(x_1, f(x_1))$. Sezen also said that she should find the slope of the lines passing through the points of the $(x_0, f(x_0))$ and $(x_1, f(x_1))$ but she was confused while trying to find this slope or to explain what it meant as she stated that she would find the slope of the tangent line by using the value of $tan \propto$. Moreover, she made some mistakes about the expression of some mathematical terms. She used "$f(x)$ function" to represent "function f", "tangent" to express "line tangent at any point to the graph of a function" or point for only x value like "point x_0 " or "point x_1".

Semra's word use of the first question on definition of derivative:

Semra didn't answer the question in the pre-test. She answered this question in the post-test by giving the symbolic definition of derivative $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and by sketching a graph given in the Figure 4.7 and a line tangent to this graph at the point $(x_0, f(x_0))$ Then she wrote that the slope of this line gives the derivative.

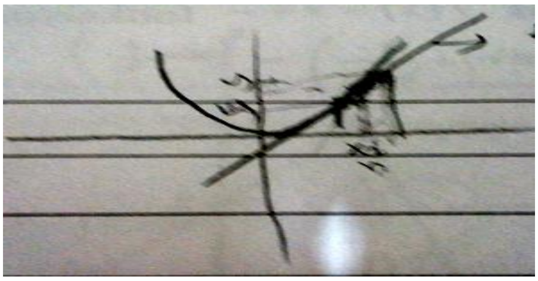


Figure 4.7 Graph that Semra sketched to answer the first question on definition of derivative

In the interview, she defined the derivative as the slope of the tangent line. To find the slope of the tangent line at a point, she said that she needed two points. She would use the values of these points as the difference between x values and difference between y values.

In the symbolic definition $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, h gets closer to 0 as x values of the points get closer to each other. As x values get closer to each other, y values get closer to each other. The points become a point and the slope of the line tangent to the graph at this point gives the slope and the derivative value.

According to the answers that she gave in the post-test and her explanations that she gave in the interview, Semra perceived derivative as a difference quotient and also limit of this difference quotient as she answered this question of the symbolic definition of derivative, $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$. According to her second definition and the explanation of this definition, Semra also perceived derivative as slope and a ratio.

Semra used the words “slope of the line”, “limit”, “difference between x’s and y’s”, “distance gets closer to 0”, “y’s corresponding to this point”, “close two points” and “distance gets smaller” to explain her answer. She used “slope of the line” to explain her answer for derivative as the graph showing a line tangent at a point. She said:

Semra: The derivative is slope of a line. While I was finding the slope of line, for example finding the slope of this line (She shows the graph that she sketched).

Then she continued her explanation.She combined this definition as a slope to the symbolic definition of $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ as limit of the slopes of the secant lines. She used the expression "difference between x's and y's" and "limit" to explain how she could find the slope of the line. She said:

Semra: How can I say while I was finding the slope? I will say the difference between x's and y's. This will not be the exact explanation; I will say I will find the limit of this.

Semra used the expression "distance gets closer to 0 on the parabola", "y's corresponding to this point", "close two points" and "distance gets smaller" to explain how the secant lines come closer to the point of the line tangent to the graph and how she could find the slope of the tangent line and also to explain the symbolic definition $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

Yakup's word use of the first question on definition of derivative:

In the pre-test, Yakup defined derivative as "to reduce the higher order functions to fewer orders." In the post-test, Yakup answered this question as "limit of the change of the function values".

He explained the derivative as the limit of the change of the function values in the post-test and he intended to take the limit of the quotient $\frac{f(x+h)-f(x)}{h}$ as it is the change of the function values.

He sketched the graph of the function of x^2 to explain his definition of the derivative (Figure 4.8). He emphasized that if the graph of the function is a line, then he could find the derivative by finding the slope of this line. However, if the graph of the function was a curve then he should find the derivative by finding the limit of the change of the y values. He found the limit of the change of the y values and the change of the x values. Therefore, the points become one point. The slope of the line sketched tangent to the curve at that point gives the derivative of the function at that point.

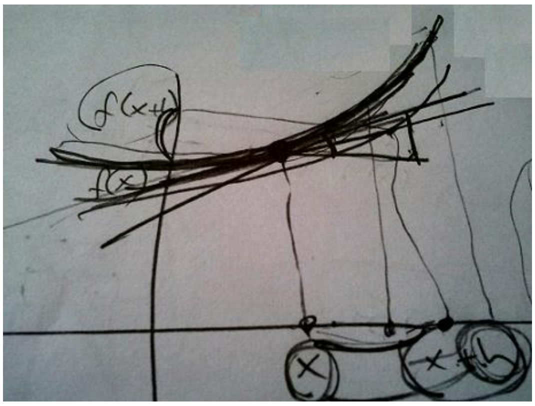


Figure 4.8 Graph Yakup sketched the line tangent to the curve

He used the words “limit of the change”, “h goes to zero”, “change as well as h”, “becomes point”, “slope of the curve” , “tangent” and “slope of the line” to explain the definition he answered in the post-test. Yakup explained his definition of the derivative as “the limit of the change”. He explained the derivative as the limit of the quotient $\frac{f(x+h)-f(x)}{h}$ that is the change of the function values. He also used the words “h goes to zero” to explain the derivative as the limit of the change of the function values and he symbolized the derivative as the limit of $\frac{f(x+h)-f(x)}{h}$ as h goes to zero. He said that:

Yakup: I said that (in the post-test) limit of the change of the values of a function gives us the derivative.

R: What did you want to say here?

Yakup: We know the definition of derivative.

R: You can sketch here (on a paper).

Yakup: The definition of the derivative is this for different values as h goes to zero.

R: ...

Yakup: It was $\frac{f(x+h)-f(x)}{h}$. This means that there is change as h. The limit of the changes gives us the derivative. I expressed verbally as the definition of the derivative is that.

Yakup used the word "change as well as h" to explain the quotient $\frac{f(x+h)-f(x)}{h}$ and said that there is "change as well as h". However, it couldn't be referred from his explanation that if he used this amount of change, h for the change of the x values or y values. However, in the following part of the conversation instructorasked "where is the change" and "you divide by h, what is this" to understand what he meant by "change as well as h". He talked about the change in the x values. He said that:

Yakup: Here, when it asked the slope at the point of x, we should take changes. These changes, we will take x the first one and x+h the second one. We should take as small as this h.

Yakup also used the words "becomes point" to explain his definition by using the graph of the function x^2. Here, he means that as the change of the x values gets closer to zero, the values will come closer to each other and become a point. He said that:

Yakup: That is the point of x so it will become f(x). For example, there will be x+h. It comes to f(x+h) from there. When this goes like that I will combine like that. Sure for this function the linerity and the curvature will change. For example, if it is a linear function, if it is x, if it is a first degree function, this will be a straight line. But if it is x^2 , then it will become a curve. We will take this change in a small degree as we said 0. They will come very close to each other and they will become a point, ... we will study for this curve.

Yakup thinks that if the graph of the function is a line, we could find the slope of this line easily. However, he thinks that if the graph is a curve, he should use the limiting process to find the "slope of the curve". Moreover, he used the word "tangent" while he was explaining when he must use the limiting process for finding the slope of the curve. He said that:

Yakup: ... If it was a line we could find the slope directly. For a curve, we will come such closer that when it becomes a point, we will say that the *slope of this* curve at this point is this. But by sketching a *tangent.*

Yakup also used the words “slope of the line” to explain the meaning of the symbol $\frac{f(x+h)-f(x)}{h}$ as the slope of the line passing through the points $(x, f(x))$ and $(x+h, f(x+h))$.

Yakup: .. the change is here. There will be a triangle here.

R: What will be this slope equal to?

...

Yakup: It will be equal to the opposite over the neighbor. This angle.

R: What does it mean?

Yakup: ... it willexpress the slope of the line passing through this point. Shortly, the slope of the line at this point.

Yakup defined derivative as “the limit of the change of the function values”. According to his definition and the words used to explain this definition he perceived the derivative concept as limit of the difference quotient of a function’s values.

He used some wrong expressions such as “the slope of the curve” to explain the definition he stated. It revealed that he was confused with the slope of a line or a curve. He also made mistake while expressing points, he calledthe x value as a point, says “point x”.

Yasin’s word use for the first question on definition of derivative:

Yasin answered the first question “what is derivative” in the pre-test as “It is a new form of an expression that changed according to certain rules”. In the post-test he defined derivative as “In some circumstances, it requires to investigate instantaneous change of a case. But we don’t investigate the change by concentrating on only one point. Therefore, we find the limit of the average rate of change at that point by approaching this point. This gives us the instantaneous change of the case, namely the derivative”. In his definition in the post-test he defined derivative as the instantaneous rate of change. He stated that, in order to find the instantaneous rate of change, he should find limit of the average rate of change getting close to the intended point. He said that, this limit gave the instantaneous rate of change, namely the derivative at the intended point.

In the interview, he explained his answer gave in the post-test. He said that, to find the derivative value of a function at a point, he needed to find the instantaneous rate of change. However, it is not possible to find the instantaneous rate of change without finding the average rate of change at very close points of the intended point as the graph of the function always change. Therefore, it is important to find the average rate of change. Moreover, it is also necessary to find the limit of the average rate of change to find the instantaneous rate of change. Therefore, the instantaneous rate of change gives the derivative at the intended point. He says that:

Yasin: …I said that in some conditions, instantaneous change of some cases should be examined. Because if we should examine in a certain interval, I thought that I couldn’t reach any certain value. There would be average rate of change because the graph will show constant change, the slope would change, we would think like that. I thought that we should look at the instantaneous rate of change; we should find which value we reach by approaching from right and left.

He explained his thought by sketching an increasing convex graph of a function (Figure 4.9). He pointed 1 on the curve and points near to this value 0.9 and 1.1 to defend his explanation. He used these points to explain the average rate of change from these points to the x value 1.

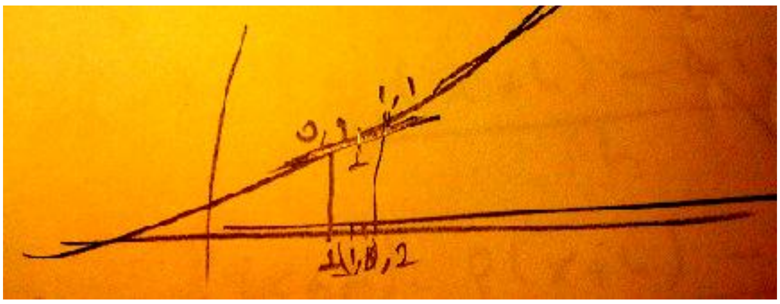


Figure 4.9 Graph Yasin showed the average rate of change

Most of the time, he used the words average change and instantaneous change instead of average rate of change and instantaneous rate of change. But he meant average rate of change and instantaneous rate of change.

He used the words of "instantaneous change", "exact value", "average rate of change of function", "graph change continuously", "slope changes", "limit of average rate of change", "approaching from left and from right" and "close intervals" to express his thoughts, knowledge about the answer of the question and his answer gave in the post test.

He used the words "instantaneous change", "exact value", and "average rate of change of function", in his answer to explain that to find the derivative it is important to find the instantaneous change as the curve always changes. Without analyzing the instantaneous rate of change, he thought that he could not get an "exact value" for an interval, as the "graph changes continuously". He stated that to find the instantaneous rate of change he should find the average rate of changes and take the limit of these rates of changes in an interval. He used the word "slope" to explain his thought that the curve always changed and the slope of the curve also changed. He used the word limit to explain how to find the instantaneous rate of change, for this he should find the limit of the average rate of change of the function. He used all these words in the flowing utterances.

Yasin: …I said that, in some conditions, instantaneous change of some cases should be examined. Because if we should examine in a certain interval, I thought that I couldn't reach any certain value. There would be average rate of change because the graph will show constant change, the slope would change, we would think like that. I thought that we should look at the instantaneous rate of change; we should find which value we reach by approaching from right and left.

He also used the words "instantaneous value" to explain that he could not find an instantaneous value as the function always changes. He said also that he should take some intervals very close to the point that he find the derivative and find the average rate of these values and take the limit of these values. He said that:

Yasin: Because we cannot look at instantaneous value, how I can tell, because the graph is always changing, I will sketch the following, let's think that it changes like that (he sketched a convex curve). Graph changes always, it takes certain values for certain intervals. That is for 1, let's assume that there is 0.9 (he assigned 1 for the x-value and assigned 0.9 on the curve for the y value). For 1.1 the graph will change, it will always change but there will be an average value for these two values. There will be values very close to each other…

According to the words that Yasin used in his explanation of the first question of "what is derivative" he represented the derivative concept as verbally instantaneous rate of change and he perceived the derivative concept as the limit of rate of change of function values. Moreover, his explanations and the words "average rate of change of function" and "close intervals" he used revealed that, this instantaneous rate of change is a function. Because, he emphasized that he should find the average rate of change of the function and finding these rate of change would not give the exact value, therefore he should approach by the close intervals to find the instantaneous rate of change.

Meral's word use for the first question on definition of derivative:

Meral answered the first question in the pre-test as "Derivative of a function like $y = ax^2 + bx + c$ would be found by multiplying the power of the x values with this x value and reducing the power 1 degree. It is $y' = a.2.x^{2-1} + b.1.x^{1-1} = 2ax + b$". In this answer she assumed the function as a polynomial function and used the differentiation rule of the polynomial function to define derivative. In the post-test she answered this question as "derivative is the limit of the slopes that was sketched at the point of $a$of a function." She used the words "point a", "from left and right", "limit of slopes", "slope of the tangent" and "approached limit" to explain her answer. She used the words "point a" to explain that she take the slope of the lines that are tangent to the function at a point and she called this point "a". However, it does not represent a point; it is the apsis value of the point that the lines are tangent to the graph of the function.

She used the words "from left and right" to explain that she take the limit of the slopes of the tangent lines approaching to the determined point from left and from right. The x value for this point is called as a in Meral's definition. The use of the word "limit" represents the limit of the slope of the lines that are tangent to the graph of the function at the point$(a, f(a))$. She also used the word "slope" to explain that she find the limit of the slope of the tangent lines. However, she used these word in the expression of "derivative is the limit of the slopes that was sketched at the point of a of a function". Therefore, it is understood from this sentences that she took the limit of the slopes and these slopes are sketched at the intended point. According to this sentence, we would think that sheconfused the meaning of the slope. However, while she was explaining her answer, she corrected her expression and said that she should take the limit of the slopes of the tangent lines. She said that:

Meral: It says what is derivative.

R: What did you say?

M: Derivative is the limit of the slopes that was sketched at the point of a of a function.

R: What did you mean here?

Meral: What did I mean? I thought that for example, let $f(x)$ be a function, there is a point x_0 here. We sketch tangents while approaching this point from right and left, the limit that the slope of these tangents, this point, gives us the derivative.

According to the answer she gave to this question in the post-test and her explanation of her answer reveals that Meral perceived the derivative as slope of the tangent line and limit of the slope of the tangent lines.

Suzan's word use for the first question on definition of derivative:

Suzan answered the first question in the pre-test as "to find the slope of the tangent sketched to the graph of a function". In the post-test she answered this question as "the slope of the tangent that was sketched to the graph of a function gives us the derivative at that point. Sketching tangent to the graph at an exact value x_0, the limit of the slopes gives us the derivative at that point".

She used the words "tangent", "slope of tangent", "limit of slopes" and "close values" to explain her definition of the derivative concept. The word "slope" is used in the definition of the derivative. In these utterances the student defines the derivative as the slope of the line which is tangent to the graph of the function at a point $(x_0, f(x_0))$. She also used the word "tangent" to explain how she would find the derivative at a point. She said that she should sketch a tangent line at the value of

x_0 on the graph and took the limit of these slopes to find the derivative at that point. She said that:

Suzan: I said derivative…the slope of the tangent that was sketched to the graph of a function gives us the derivative at that point. Sketching tangent to the graph at an exact value of x_0, the limit of the slopes gives us the derivative at that point.

She used the words “close values” to explain that the derivative was the limit of the slopes of the tangents which were sketched at the x values very close to the x value to find the derivative at a point.

Suzan’s word use and her explanation revealed that, Suzan represented derivative as slope of the tangent line. Suzan perceived derivative as both ratio and limit.

Summary of the word use of first question on definition of derivative

Pre-service mathematics teachers’ interview results of the individual discourse showed that, they gave varying answers to the question of “what was derivative?” Words that pre-service teachers used when describing and explaining derivative, consisted of the words related to the notions of slope as both ratio and limit, difference quotient as limit, rate as function and limit. Words pre-service teachers used in their individual discourse and the related notions are listed in the Table 4.20.

Pre-service teachers’ definitions given in the post-test and the explanations of these definitions revealed that three pre-service teachers, Sezen, Meral and Suzan perceived derivative as *slope*. Two pre-service teachers, Semra and Yakup perceived derivative as *difference quotient*. At the same time, Yakup perceived derivative as the *limit* of this difference quotient and Semra as *slope*. Besides, accepting derivative as slope, Suzan and Sezen perceived derivative as both *ratio* and *limit*. Meral perceived derivative as also *limit* and Semra as *ratio*. Yasin perceived derivative as *limit* of the *difference quotient*.

Pre-service teachers’ word use to explain the derivative concept showed two characteristics. The first one is operational word use and the second one is the objectified word use. The operational word use is the result of the consideration of the derivative as a process such as the limit of the difference quotients or limit of the slope of lines (secant lines). On the other hand, the operational word use is the result of the consideration of the derivative concept as the slope of the tangent line or average rate of change.

Pre-service teachers' operational word use was observed if they perceive derivative as the limit of slopes, the limit of the difference quotient or limit of rate of change. They regarded the limit notion in their definitions and explanations as a process rather than a number or value. In their explanations, Sezen, Suzan, Meral used mostly the words "limit of slopes", "approach", "from left and right" as they perceived derivative as the limit of the slopes of the lines passing through the points close to the intended point where they found the derivative. For example, Sezen perceived derivative as the limit of the slope of the tangent lines. She defined derivative as "the limit of the slope of the lines passing through the points coming closer to the point of x value x^2 gives the derivative at the point of x value x^2". She explained her definition as the limit of the slope of the tangent lines sketched at the points close to the point $(x_0, f(x_0))$. She said that:

Sezen: Of the line sketched while getting closer to points x_0. I tried to express here by *limit*. For example, the slope is this at the point (she determined a point close to the point that she sketched the tangent) then I get closer both from right and from left, While getting closer the slopes get closer to a point. This point that the slopes get closer gives again the slope at the point atx_0.

Moreover, Yakup perceived derivative as the limit of the difference quotient. He defined derivative as "limit of the change of the function values" and intended to take the limit of quotient $\frac{f(x+h)-f(x)}{h}$ that is the change of the function values. He used the words "limit of the change", "h goes to zero", "change as well as h" and "becomes point" to explain his definition and how he perceived derivative. He said that:

Yakup: I said that (in the post-test) limit of the change of the values of a function expresses us the derivative.

R: What did you want to say here?

Yakup: We know the definition of derivative.

Yakup: Definition of the derivative is this for different values as h goes to zero.

Table 4.20

Words pre-service teachers used to define derivative in their individual discourse and the related notions

DERIVATIVE									
Slope						Rate		Difference quotient	
Limit			Ratio			Function	Limit	Limit	
Sezen	Suzan	Meral	Sezen	Semra	Suzan	Yasin	Yasin	Semra	Yakup
Small intervals (ob) Limit (op) Close point (ob) Approach (op) From left and right (op) Slopes approaching to one point (op)	Limit of slopes (op) Close values (ob)	Point a (ob) From left and right (op) Limit of slopes (op) Slope of the tangent (op) Approached limit (op)	Tangent(ob) Tangent alpha (ob) Slope of the tangent (ob) Divide distance by difference (op)	Slope of the line (ob)	Tangent (ob) Slope of tangent (ob)	Average rate of change (ob) Close intervals (ob)	Exact value (ob) Instantaneous change (op) Graph change continuously (op) Slope changes (op) Limit of average rate of change (op) Approaching from left and right (op)	Distance gets closer to zero (op) Distance gets smaller (op) Limit (op) Distance between x's and y's (ob) Close two points (ob)	Limit of the change (op) h goes to zero (op) Change as well as h (op) Becomes point (op) Slope of the curve (ob) Tangent (ob) Slope of the line (ob)

Yakup: It was $\frac{f(x+h)-f(x)}{h}$. That means that there is change as much as h. The limit of the changes gives us the derivative. I expressed verbally the definition of the derivative like that.

In another example, Yasin perceived derivative as the limit of the rate of change, namely instantaneous rate of change. He defined derivative as "in some circumstances, it requires to investigate instantaneous change of a case. But we don't investigate the change by concentrating on only one point. Therefore, we find the limit of the average rate of change at that point by approaching this point. This gives us the instantaneous change of the case, namely the derivative". He used the words "instantaneous change", "graph change continuously", "slope changes", "limit of average rate of change", "approaching from left and right" to explain his definition and how he perceivedderivative.

Yasin: …I said that in some conditions, instantaneous change of some cases should be examined. Because if we should examine in a certain interval, I thought that I couldn't reach any certain value. There would be average rate of change because the graph will show constant change, the slope would change, we would think like that. I thought that we should look at the instantaneous rate of change; we should find which value we reach by approaching from right and left.

Pre-service teachers objectified word use emerges if they perceive derivative as a mathematical object rather than a process such as the slope of the tangent line, a ratio or they accepted rate of change as a function. Yasin, Sezen, Semra and Suzan's words were mostly objectified as Sezen, Semra and Suzan perceived derivative as the slope of the tangent line and Yasin perceived rate of change as a function.

Sezen, Semra and Suzan used the words "tangent", "tangent alpha", "slope of lines" and "slope of tangents" to define derivative and explain their perceptions of derivative. For example, Sezen defined derivative as "derivative is the slope of the line sketched tangent to the function $f(x)$ at the point of x value x_0" and she explained her definition and the perception of derivative as:

Sezen: Here, for example, at this x graph (she sketched a concave and positive function graph) for example this point of x_0 (shows the point $(x_0, f(x_0))$, let's sketch a *tangent* at this point.

Sezen: I said *tangent alpha* is the derivative here.

R: What does tangent alpha mean at the same time for example for this line? (shows the tangent line sketched at x_0)

Sezen: The *slope of the line* at the same time.

In another example, Yasin used objectified words to define and explain his perception of derivative. He perceived derivative as rate of change and accepted this rate of change as also a function. The words "average rate of change" and "close intervals" were the objectified used words and also revealed Yasin's perception of derivative as function. He said that:

Yasin: …I said that in some conditions, instantaneous change of some cases should be examined. Because if we should examine in a certain interval, I thought that I couldn't reach any certain value. There would be average rate of change because the graph will show constant change, the slope would change, we would think like that. I thought that we should look at the instantaneous rate of change; we should find which value we reach by approaching from right and left.

Pre-service teachers' visual mediators of first question on definition of derivative

Visual mediators are real or imagery concrete objects or symbolic artifacts that are used to define the objects of the discourse and arrange communication. In mathematics, numerals, algebraic formulas, algebraic notation, graphs, Sketchings and diagrams are the most used examples of visual mediators. In this part, visual mediators that pre-service teachers used to define derivative and explain their perception of derivative was examined.

Sezen's visual mediators of first question on definition of derivative

Sezen used graphs and symbolic notations to explain her definition and perception of derivative. She defined derivative as in two forms. The first definition is "the derivative is the slope of the line sketched tangent to the function $f(x)$ at the point of x value x_0". The second one is "the limit of the slope of the lines passing through the points coming closer to the point of x value x_0 gives the derivative at the point of x value x_0."

She sketched the graph given in the Figure 4.10 to explain her definition of derivative was the slope of the line sketched tangent to the graph at the point

$(x_0, f(x_0))$. She said that she could find the slope of the tangent line by the tangent value of the angle formed between the x-axis and the tangent line.

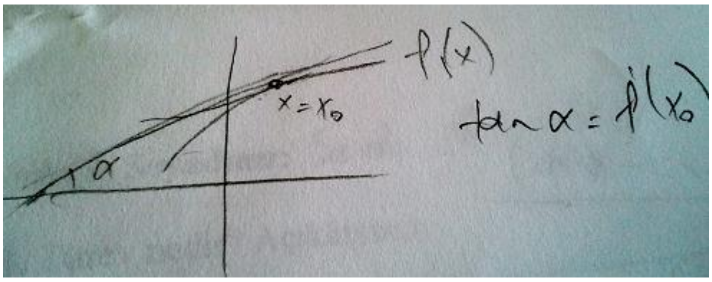


Figure 4.10 Sezen sketched line tangent to the graph

She also used the symbolic notation $tan \propto = f'(x_0)$ which gives at the same time the slope of the tangent line and this tangent value is equal to the derivative value at x_0 . She triedto explain her second definition as the limit of the slope of the lines close to the point of $(x_0, f(x_0))$ on the graph that she sketched. In figure 4.11., she showed the points close to the point $(x_0, f(x_0))$.

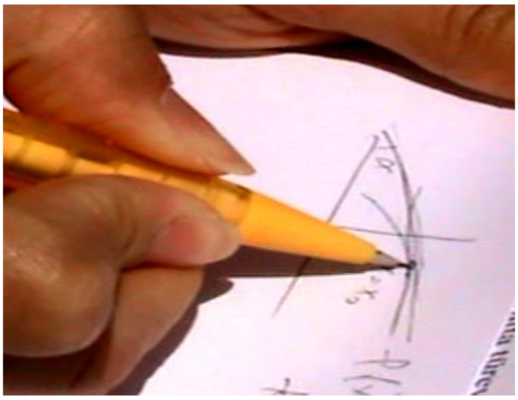


Figure 4.11 Sezen shows the points close to the point $(x_0, f(x_0))$

She said that, the limit of the slopes of the lines tangents to the graph at the points very close to the point $(x_0, f(x_0))$gives the derivative of the function at the point $(x_0, f(x_0))$. She tried to explain for the lines sketched tangent to the graph given in the Figure 4.12. She sketched another tangent line passing through the point $(x_1, f(x_1))$.

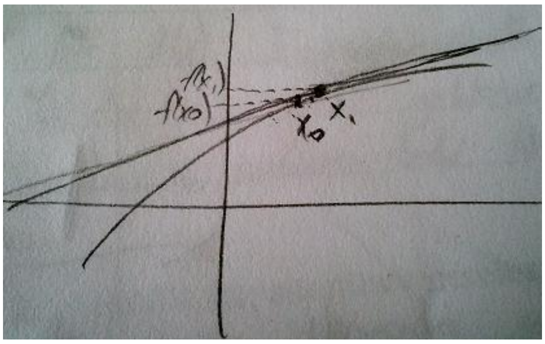


Figure 4.12 Sezen sketched tangent line passing through the point $(x_1, f(x_1))$

She also used the symbolic notation $\frac{f(x_1)-f(x_0))}{x_1-x_0}$ to find the slope of the line segment passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$. However, she couldn't explain what this notation represents. She said:

R: What does this represent? (Shows the symbolic expression $\frac{f(x_1)-f(x_0))}{x_1-x_0}$ that Sezen wrote before hand) Would you show this (symbolic expression) here (on the graph that Sezen sketched)?

Sezen: I wrote that but now.

R: What did you wanted to express?

Sezen: I tried to find the *slope* of that (tangent line) but.

R: …

Sezen: That slope

R: Whose slope is this?

Sezen: It won't be.

R: Whose slope is this? Which line or line segment?

Sezen: I tried to write the slope of (the line) atx_1 but.

Semra's visual mediators of first question on definition of derivative

Semra defined derivative by using visual mediators of graph and symbolic notation. She wrote the symbolic definition of derivative as the first definition: $\lim_{h\to 0}\frac{f(x+h)-f(x))}{h}$. In the symbolic definition $\lim_{h\to 0}\frac{f(x+h)-f(x))}{h}$ h gets closer to 0 as the x values of the points gets closer to each other. As the x values gets closer to each other the y values get closer to each other. The points become a point and the slope of the line tangent to the graph at this point gives the slope and the derivative value. She said that:

R: What do you mean here? What $\lim_{h\to 0}\frac{f(x+h)-f(x))}{h}$ mean?

Semra: …

R: … You can explain on a graph…

Semra: this h means that distance on the parabola approaches zero. So the distance decreases.

She sketched the graph given in the Figure 4.13 as the second definition of derivative to show the slope of the tangent line gives the derivative at the intended point.

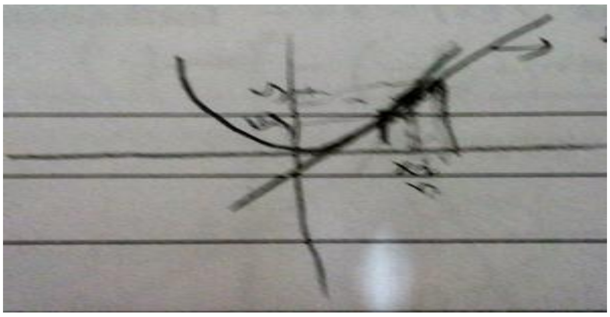


Figure 4.13 Graph that Semra sketched to answer the first question on definition of derivative

In the interview she defined the derivative as the slope of the tangent line. To find the slope of the tangent line to the graph at a point, she said that she needed two points. She would use the difference between x values and difference between y values.

Semra combined the symbolic definition of $\lim_{h\to 0}\frac{f(x+h)-f(x))}{h}$ as limit of the slopes of the secant lines. She used the expression "difference between x's and y's" and "limit" to explain how she could find the slope of the line. She said that;
Semra: How can I say while I was finding the slope? I will say the difference between x's and y's. This will not be the exact explanation; I will say I will find the limit of this.

Yakup's visual mediators

Yakup used symbolic notation and graph to explain his definition and how he perceivedderivative. He defined derivative as "limit of the change of the function values”. He used the symbolic definition of derivative to explain his definition: $\lim_{h\to 0}\frac{f(x+h)-f(x))}{h}$. He explainedthe derivative as the limit of the quotient $\frac{f(x+h)-f(x)}{h}$ that is the change of the function values. He symbolized the derivative as the limit of $\frac{f(x+h)-f(x)}{h}$ as h goes to zero. He said that:

Yakup: I said that (in the post-test) limit of the change of the values of a function expresses us the derivative.

R: What did you want to say here?

Yakup: We know the definition of derivative.

R: You can sketch on (this paper).

Yakup: The definition of the derivative is this for different values as h goes to zero.

Yakup: It was $\frac{f(x+h)-f(x)}{h}$. That means that there is change as h. The limit of the changes gives us the derivative. I expressed verbally as the definition of the derivative is that.

He sketched the graph of the function of x^2 to explain his definition of the derivative.He emphasized thatif the graph of the function is a line then he could find the derivative by finding the slope of this line. However, if the graph of the function is a curve then he should find the derivative by finding the limit of the change of the y values. Therefore, the points become one point. The slope of the line sketched tangent to the curve at that point gives the derivative of the function at that point. In the figure 4.14, the graph that he sketched to explain the difference of the function values. Also he explained the limit of the difference of the function values.

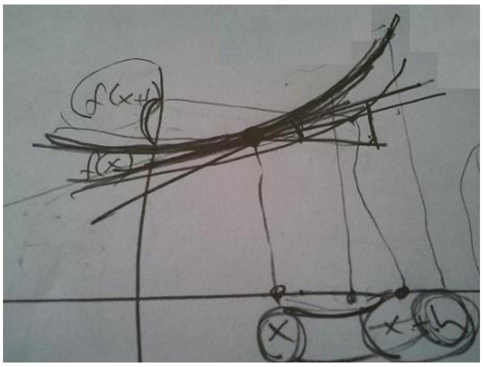


Figure 4.14 Yakup sketched a line tangent to the curve

In this graph he also marked the x values and the corresponding y values and explainedthe change in the function values. He also sketched the tangent line at the point $(x_0, f(x_0))$ to show how the slope of the curve would be the slope of the tangent line.

Yasin's visual mediators of first question on definition of derivative

Yasin used a graph to explain his definition and how he perceivedderivative as a visual mediator. Yasin perceived derivative as the limit of average rate of change, namely instantaneous rate of change. He said that, to find the instantaneous rate of change he has to find the average rate of changes and take the limit of themas the values of the curve change and also the slopes of the lines sketched tangent to the curve always changes. Therefore to find an exact value, it is important to find the instantaneous rate of change. So to support his thought, he sketched a convex, increasing curve, seen in the Figure 4.15 and marked the points close to the point of x value.

He took an x value 1 on the curve and points closeto this x value 0.9 and 1.1 to defend his explanation. He usedthese points to explain the average rate of change from these points to the x value 1.

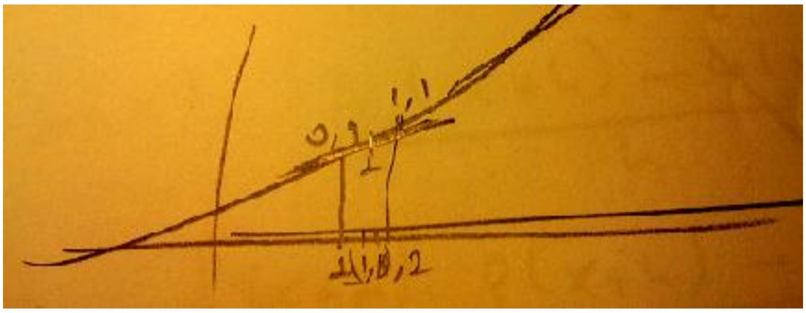


Figure 4.15 Yasin sketched a convex, increasing curve

Meral's visual mediators of first question on definition of derivative

Meral used symbolic notation and graph to explain her definition and perception of derivative. Meral defined derivative as "derivative is the limit of the slopes that was sketched at the point a of a function." She perceived derivative as the limit of the slopes of the tangent lines". She sketched an increasing function graph, seen in the Figure 4.16 to explain her perception of derivative. She marked an x value a on the x-axis and sketched a line tangent to this curve.

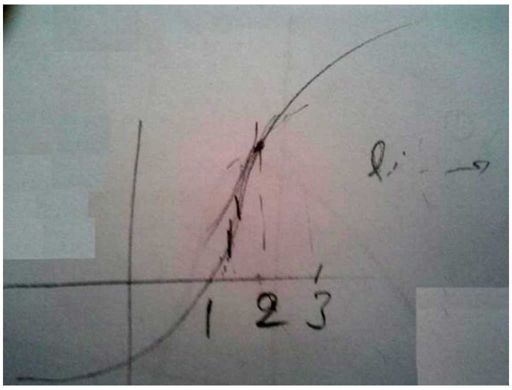


Figure 4.16 Graph that Meral sketched to explain the first question on definition of derivative

She explained how she could find the slope of the tangent line as using the symbolic notation $\frac{f(2)-f(1)}{2-1}$ and she marked the x values 1 and 2 on the x axis and saidthat “I say that when x is between the points 1 and 2, I find by using $\frac{f(2)-f(1)}{2-1}$,” and she also used the symbolic notation representing the interval of the x values “1<x<2”. She continuedto explain how she could find the slope of the line as “I gave 3 here, I look at the x values between 2 and 3 and I guess an approximate value”.

Suzan’s Visual Mediator

Suzan used graph to explain her definition and how she perceivedderivative. She defined derivative as “the slope of the tangent that was sketched to the graph of a function gaveus the derivative at that point. Sketching tangent to the graph at an exact value of x_0, the limit of the slopes gives us the derivative at that point”. She perceives derivative as slope of the tangent line and also both ratio and limit.

She sketched an increasing function graph, seen in the Figure 4.17 to explain her definition. She marked a point on the curve and sketched a line tangent to the curve at this point. She said that:

Suzan: I said derivative…the slope of the tangent that was sketched to the graph of a function gives us the derivative at that point. Sketching tangent to the graph at an exact value of x_0, the limit of the slopes gives us the derivative at that point.

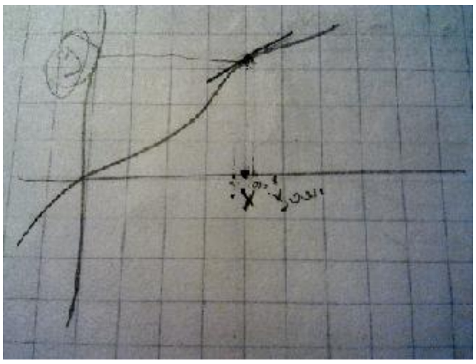


Figure 4.17 Graph that Suzan sketched to explain her definition

She explained that she foundthe slope of the tangent line by finding the slopes of the lines tangent to the curve at the close points such as points of the x values 0.009 and 0.001. The limit of the slopes of these tangent lines sketched at those close points would give the slope of that tangent line and that would give the derivative.

Summary of pre-service teachers' use of visual mediators of first question on definition of dervative

Pre-service teachers used symbolic notation and graph to explain the definitions that they stated in the post-test and how they perceived derivative. Sezen, Semra, Yakup and Meral used both graph and symbolic notation to explain their perceptions. On the other hand, Yasin and Suzan only used graph. There was a tendency between these pre-service teachers that most of them sketched an increasing graph. Sezen and Semra sketched a polynomial graph. Sezen's graph was convex and Semra's was concave. All of them sketched a line that was tangent to the graph at a point that they named differently. Meral called this point as a, Yasin 1, Sezen and Semra x_0, Yakup and Suzan x. All of them marked one or more points close to the point that the line was tangent.

Sezen, Semra, Yakup and Meral used symbolic notation besides graph to explain their perceptions. Semra and Yakup used the symbolic notation $\lim_{h \to 0} \frac{f(x+h)-f(x))}{h}$ as the definition of derivative. Sezen used the symbolic notation $\frac{f(x_1)-f(x_0)}{x_1-x_0}$to find the slope of the line segment passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$. She also used the symbolic notation $tan \propto = f(x_0)$ which gives at the same time the slope of the tangent line and this tangent value is equal to the derivative value at the point of $(x_0, f(x_0))$. Meral used the notation $\frac{f(2)-f(1)}{2-1}$ to find the slope of the line passing through the points of $(1, f(1))$ and $(2, f(2))$.

Pre-service Teachers' Endorsed Narratives Used in Individual Discourse of First Question on Definition of Derivative

Narratives are written or spoken texts which are the explanation of objects or relations between objects or activities with or by objects (Sfard, 2007). It is any sequence of utterances framed as descriptions of objects, of relations between objects, or of processes with or by objects (Sfard, 2008). Narratives are called true or false according to the approval or disapproval. Mathematical narratives would be considered in two categories: Object level and meta-level. Object level narratives are the stories about mathematical objects. Meta-level narratives are stories about how mathematics is done.

There are two types of endorsed narratives that pre-service teachers used: meta-level and object level. Object level narratives are all related tothe definition of derivative. Meta-level narratives are all abouthow pre-service teachers would find the derivative from participants' perspective. In this part, pre-service teachers' narratives connected with the definition and their perception of derivative will be examined.

Sezen's endorsed narratives of first question on definition of derivative

Sezen used three endorsed narratives to define and explain derivative. The first narrative she used to define derivative is "The derivative is the slope of the line sketched tangent to the function f(x) at the point of x value x^2." It is an object level narrative as it defines what derivative is and she used objectified words to state it.

The second narrative that she used is "the limit of the slope of the lines passing through the points coming closer to the point of x value x^2 gives the derivative at the point of x value x^2." This narrative is also used to define derivative. Although it is used for defining derivative, it is a meta-level narrative as it defines how derivative at a point is found and operational words are used to state it.

The third narrative is also a meta-level rule as it explains how the slope of the tangent line is found. It is "…the slope is this at that close point. Then I approach from left and from right, the slopes approach a certain point. This point (means value) the slopes approaching give the slope at the point of x_0."

Semra's endorsed narratives of first question on definition of derivative

Semra used two endorsed narrative to define and explain her perception of derivative. The first one used to define derivative: "Derivative is the slope of a line". It is an object-level narrative as it defines derivative as an object. The second narrative is used to explain how she would find this slope as mentioned in the first narrative. "While finding this slope, I choose two close points to this point (the point where the slope tangent to the function graph) and I take the corresponding y's. While finding limit I will say difference between x's and y's, I will take the limit of this (she mentions about the ratio)." This is a meta-level narrative as it explains how to find the slope using the difference quotient.

Yakup's endorsed narratives of first question on definition of derivative

Yakup used two endorsed narratives to define and explain derivative. The first one is "the limit of the change of the function values expresses us derivative." It is an object level narrative as it define derivative.

The second narrative is on how he could find this limit. Therefore, it is a meta-level narrative. This narrative is "that is the point of x, so it will become $f(x)$. For example, there will be $x + h$. It comes to $f(x + h)$ from there. When this goes like that I will combine like that. Sure for this function the linerity and the curvature will change. For example, if it is a linear function, if it is x, if it is a first degree function, this will be a straight line. But if it is x^2, then it will become a curve. We will take this change in a small degree as we said 0. They will come very close to each other and they will become a point, ... we will study for this curve."

Yasin's endorsed narratives of first question on definition of derivative

Yasin used two endorsed narrative to define and explain derivative. The first one is "we will look for the limit of average rate of change". It is an object level narrative as it represents the definition of derivative.

The second narrative is onhow he could find the limit of average rate of change and importance of close intervals. Therefore, it is a meta-level narrative. This narrative is "Close points, very close points, not for these points, very close intervals for this point. We could not mention derivative for a point, I think. We mention very close intervals. We could not investigate each one by one, we should find the average rate of change and investigate for the limit, I think."

Meral's endorsed narratives of first question on definition of derivative

Meral used two endorsed narrative to define and explain derivative. The first narrative is object-level endorsed narrative as it defines derivative "For a function $f(x)$, while approaching a point a from left and from right, limit of the slopes that was sketched at that point is called derivative."

The second narrative is meta-level narrative as explain how to find the derivative. "There is a point a here. We sketched tangents while we approach from left and right. The limit that the slopes of these tangents approach gives us the derivative.

Suzan's endorsed narratives of first question on definition of derivative

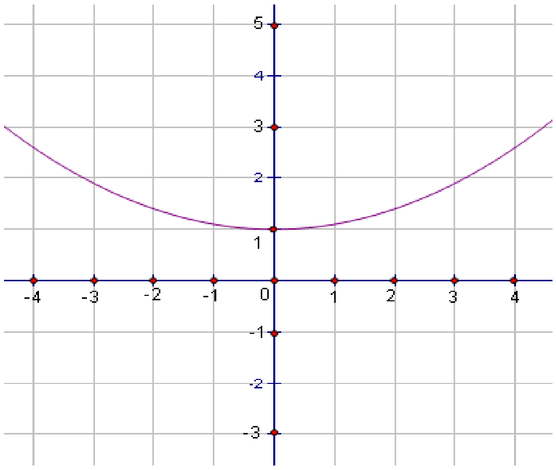
Suzan used two endorsed narrative to define and explain derivative. The first one is "the slope of the tangent that was sketched to the graph of a function gives us the derivative at that point". It is an object-level narrative as it explains the definition of derivative.

The second derivative is "sketching tangent to the graph at an exact value of x_0, the limit of the slopes gives us the derivative at that point". It is a meta-level narrative as it explains how he could find the derivative.

4.3.2 Individual Discourse of Third Question on Rate of Change

The third question demands pre-service teachers to find the approximate value of derivative at $x = 2$ for the function f whose graph and some x values and corresponding y values are given in the table. This question requires participants to find the derivative value at the interest point using the limit of the rate of change or the limit of the slopes of the secant lines by analyzing the graph of function f and by using the function values given in the table rather than using the algebraic expression of the function and derivative function. The third question is given below.

Third question:



x	0	1	2	3
y	1.0	1.1	1.4	1.9

Figure 4.18 Graph given in the third question

Find the approximate value of derivative at $x = 2$ for the function f whose graph and values are given in the Figure 4.18.

Pre-service teachers' answers to the third question given in the pre-test and post-test were given in the Table 4.21. In the pre-test 24 of the 52 pre-service teachers took the derivative test before the instruction did not answer this question. 5 of them gave unrelated answer. 1 of them answered this question finding the limit of average rate of change of a function. 2 of them gave the answer of rate of change. 15 of them answered this question by finding an algebraic equation of a function and 5 of them answered as finding the slope of the tangent line. In the post-test 5 of them did not answer this question. 6 of them gave unrelated answer. 24 of them found the limit of average ate of change of the function. 6 of them tried to find the function value. 4 of them found the rate of change. 2 of them related the rate of change to the slope pf the tangent line. 3 of them found the limit of the function. 4 of them tried to find the algebraic equation of the function and 1 of them found the slope of the function.

Table 4.21

Pre-service teachers' answers to the third question in pre-test and post-test

Pre-service teachers' answers	Pre Test	Post Test
No Answer	24	5
Not related expression	5	6
Limit of average rate of change of a function	1	24
Function value		6
Rate of change	2	4
Rate of change is related to slope of the tangent		2
Limit of the function		3
Algebraic equation of function	15	4
Slope of the tangent	5	1

Sezen's answer to third question on rate of change

Sezen did not answer the third question in the pre-test. She answered the question in the post-test using the symbolic definition of the derivative. She found the slope of the secant lines sketched between the points (1, 1.1) and (2, 1.4) as 0.3, and (2, 1.4) and (3, 1.9) as 0.5. Then she found the limit of these slopes as x goes to 2. Then she found 0.4 as the derivative of the given function at the point (2, 1.4).

Sezen's words used while explaining her answer were "close points", "closer from left", "average value", "closer from right", "limit at 0.4", "slope of the line", "middle value".

She explained the reason for choosing these two points as to find close points to the point of x value 2 in order to find the derivative of the function at the point of x value 2. She explained the formula that she used to find the slope of the lines dividing the difference between the function values bythe difference between the x values. She explainedthat her intention was to find an average value. She said that by choosing the point of x value 1 to reach the point of x value 2 from left and the point of x value 3 to reach the point of x value 2 from right. She foundthe limit value 0.4 the value between the slopes 0.3 and 0.5.

She could not find the relation between the slope of a line and the formula that she used as the difference between the function values divided by the difference between x values. The instructor told her to sketch any line and to write the formula to find the slope of this line. Then she realized that she did the same thing by using this formula which is the difference between the function values divided by the difference between x values. She realized that she found the slope of the line passing through the points of x values 1 and 2 by using the formula $\frac{f(2)-f(1)}{2-1} = \frac{1,4-1.1}{1} = 0.3$ and the slope of the line passing through the points of x values 2 and 3 by using the formula $\frac{f(3)-f(2)}{3-2} = \frac{1.9-1.4}{1} = 0.5$.

She used algebraic symbols and graphs to explain her answer to the third question. She used $1 < x < 2 \quad \frac{f(2)-f(1)}{2-1} = \frac{1,4-1.1}{1} = 0.3$ to find the slope of the secant line passing through the points $(1, 1.1)$ and $(2,\ 1,4)$ and the symbols $2 < x < 3 \quad \frac{f(3)-f(2)}{3-2} = \frac{1,9-1.4}{1} = 0.5$ to find the slope of the secant line passing through the points $(2,\ 1,4)$ and $(3,\ 1,9)$ to find the derivative of the given function at the point $(2,\ 1,4)$.

Sezen found the slope of the secant line passing through the points $(1, 1.1)$ and $(2,\ 1,4)$ and $(2,\ 1,4)$ and $(3,\ 1,9)$. Then she found the limit of these slopes to find the derivative of the given function at the point $(2,\ 1,4)$ by using the algebraic expression $\lim_{x\to x_0} \frac{f(x)-f(x_0)}{x-x_0} = \lim_{x\to 2} \frac{f(x)-f(2)}{x-2} = 0.4$.

Semra's answer to third question on rate of change

Semra tried to answer this question in the pre-test by finding the algebraic equation of the function. She found the algebraic equation as $f(x) = x^2 + 1$ and the derivative as $f^{'}(x) = 2x$. Then she found the derivative at the point of x value 2 as 4.

In the post-test she again tried to find the algebraic equation of the given function. She found the algebraic equation as $f(x) = x^2 - 2x + 1$. She found the derivative of function f as $2x - 2$and found at the point of x value 2 as 4.

She used the words "approximate value" and "slope of the lines" to explain her answer to he question. In the interview, to explain her answer, she analyzed the given table in the question and found x values corresponding to y values. She chose two x values 1 and 3 to find the derivative of the given function at the point of x value 2. She showed these points on the given graph in the question. She chose the points (1, 1.1) and (3, 1.9) firstly to find the derivative at the point (2, 1.4) as these points were closer to the point (2, 1.4). Therefore, by using the closer points she tried to find the derivative of the function.

In order to find the derivative of the given function in the question at the point of x value 2, she used closer points to the point of (2, 1.4). First of all she chose the points (1, 1.1) and (3, 1.9) and found the slope of the line sketched between these points as 0.4 Then she chose the points of (0, 1) and (1, 1.1) and found the slope as 0,1 The choosen points (2, 1.4) and (3, 1.9) gave the slope as 0.5. The slope for points (1, 1.1) and (2, 1.4) was 0.3. She tried to find the derivative of the function at the point of (2, 1.4) but she usedall the points given in the table and foundthe slope of the lines sketched between these points.

As she could not find the exact value of the slope of the line tangent to the function at the point (2, 1.4), she used the slope of the lines passing through the points closer to the point (2, 1.4). Therefore, she could not find the exact value of the slope and also the derivative, she found the approximate value of the slope and derivative. If two points on the line would be given, she could find the exact value of the slope of the function. Moreover, the points were on the parabola, so she could not find the exact value.

She explained what she found by searching the slope of the lines passing through the points that she chose on the graph given in Figure 4.19 as visual mediator. She found the slope of the line passing through the points (1, 1.1) and (3, 1.9). However, she said that she found the slope of the line tangent to the graph at the point of x value 2.

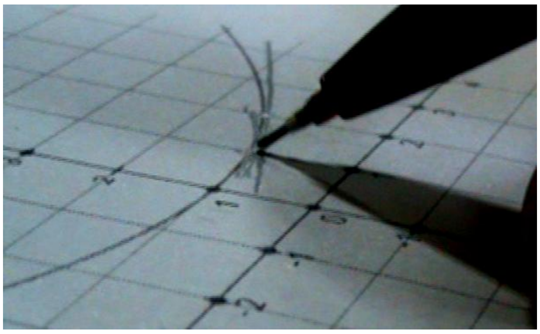


Figure 4.19 Semra sketched lines tangent to the graph

She explained what she found by using $\frac{f(x+h)-f(x)}{h}$ as she found the slope of a line by passing through the points $\left(x+h, f(x+h)\right)$ and $(x, f(x))$. She showed the points on the graph that she sketched (Figure 4.20). She explained that h gives the difference between the x values that the line passing through x and $x+h$, and the $f(x+h)-f(x)$ gives the difference between points $f(x+h)$ and $f(x)$. Therefore, she found the approximate value of the slope of the line tangent to the graph at a point. As h goes to 0, the difference between the points becomes 0. The apsis value of the point $(x+h, f(x+h))$ becomes x.

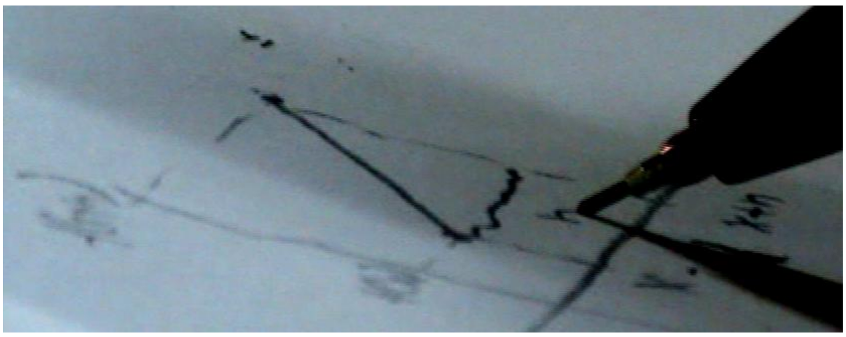


Figure 4.20 Semra explains $\frac{f(x+h)-f(x)}{h}$ on the graph

Yakup's answer to third question on rate of change

In the pre-test Yakup found the algebraic expression of the value according to the given values of the function at certain values. Then he found the derivative of the function by using the rule for the polynomial functions and he found the answer by using the x value of the point (2, 1.4). He wrote that:

$$f(x) = \frac{x^2}{10} + 1$$

$$f'(x) = \frac{2x}{10} = \frac{x}{5} \rightarrow f'(2) = \frac{2}{5}$$

In the post-test he did not answer the question. In the interview he used the words "derivative", "slope", and "standard change" to explain his thought related to the third question. He also used the algebraic symbols $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and $\frac{f(2+h)-f(2)}{h}$ as visual mediator. He also used the narrative "slope of the line sketched tangent to the graph of the function at the point of x value 2 gives the derivative of the function" to explain the relation between the slope of the tangent line and the derivative value at that point.

In the interview, first of all he decided to find the derivative of the function by using the symbolic definition of the derivative ($\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$). He used 2 instead of the x value and again tried to find the derivative value by using the symbolic definition of the derivative $\frac{f(2+h)-f(2)}{h}$. However, Yakup realized that he did not know the function. He said that if he knew the function, he could find the derivative function by using the symbolic definition and he could get the uncertainty as the values of $f(x+2)$ and $f(2)$ be the same and the h value goesto zero.

He changed his mind to the graph of the function and the function values given at the x values of 0, 1, 2 and 3 given in the table in the question. He said that in the table the change in the y values are given. He realized that the change of the y values according to the x value is 0. The y value changed 0.1 from 0 to 1 of x value, 0.4 from 0 to 2 and 0.9 from 0 to 3. After he analyzed the table and the graph given in the question he realized that the values of $f(x+2)$ and $f(2)$ don't become the same and the h value don't get to zero as he find the limit of this rate. Then he realized that he could use the values given in the table in the symbolic definition and first of all he used the x values 0 and 2. Then he found the rate of change between the x values of 0 and 3 as 0.3.

The instructor asked how he could use the graph to find the derivative of the function. He said that he could find the slope of the line tangent to the graph at the point of x value 2. He realized that by using the formula $\frac{f(x+h)-f(x)}{h}$, he found the slope of the secant lines between the points of x values 0 and 2. He concluded that, to find the exact value he should choose the points very close to the point of (2, 1.4).

Yakup explained that $f(x+h)-f(x)$ gives the change of the y values. $\frac{f(x+h)-f(x)}{h}$ figured out the slope of the secant line sketched between any two points on the graph of the function. The limit gave the slope of the tangent line sketched at the point where the derivative is found.

Yasin's answer to third question on rate of change

Yasin did not answer the third question in the pre-test. He answered this question in the post-test as the x values gets closer to the value of 2, the y values got closer to 1.4, and thus the answer was 1.4.

Yasin's answer given in the post-test revealed that Yasin took the limit of the function instead of the derivative of the function at the point of x value 2. But he thought that he found the derivative of the function at the point of x value 2.

He used "close from right and left", "limit from right and left" and "limit of the average rate of change" to explain his answer to the third question. The instructor asked if the limit of the function was asked, then what he would do. He said that he would approachfrom right and from left to the point and reach to the point of y value 1.4. He got the same answer while explaining the derivative of the function.

Yasin**:** …now we couldn't look at this point directly at $x = 2$ (shows the x value 2) we should approach from left and from right approximately. We don't have an exact value here. Here let's assume it is 1.3. When we approach from the other side, let's assume that we go from 3 to 2. When we approach in small intervals the graph goes down… I thought that 1.3.

The instructor asked if it is same for the derivative and the limit of the function. He answered this question as "we should find the limit of the average rate of change". He found the average rate of change as $\frac{2+1}{2} = 1.5$ for the y values of the function taking the points of y value 1 and 2.

R: If the limit of the function at $x = 2$ is asked, what would you say?

Yasin: We will look at the limit at $x = 2$ from right and from left. We will find the point that these two valuesareequal.

R: For example.

Yasin: The exact value is not obvious.

R: approximate

Yasin: It would be 1.4. Let's say 1.5, for example. It would change if it is open or closed interval.

R: Okay, if it asks the derivative, is it mean the same?

Yasin: We should look at the limit of the average rate of change in this interval (He shows the x values 1 and 3 on the graph). Here $\frac{2+1}{2}$ is the average rate of change, for y values $\frac{3}{2} = 1.5$, we will look at the limit, the limit of the average rate of change.

R: How would you find the limit? It says for $x = 2$.

Yasin: … I assume that I approach in small intervals (he shows the close x values 2 from right and from left). Graph also approaches in small intervals (he shows the close values to the point (2, 1.4) on the graph).

He knew what he should do to find the derivative of the function.He found the limit of the average rate of change between the points close to the point $(2, f(2))$. However, he could not apply this to the given function in the question. He found the average rate of change by taking the values of $f\ (1)\ =\ 1$ and $f\ (3)\ =\ 2$ and found that $\frac{2+1}{2} = 1.5$. He said that the rate of change is 1.5. He said that the y values change from 1 to 2, however the rate of change is 1.5.

The instructor asked that whether hefound the middle value of the y values of 1 and 2. He answered this question as yes.

R: How would you find the limit? It says for $x = 2$.

Yasin: … I assume that I approach in small intervals (he shows the close x values 2 from right and from left). Graph is also approach in small intervals (he shows the close values to the point (2, 1.4) on the graph).

He defended his thought as the limit of the function and the limit of the average rate of change are not the same. He said that he found the limit of the average rate of change by finding the middle value of the two y values of the function.

He used algebraic symbols and graph as visual mediator to explain his thought related to third question. The fourth question asked for the derivative of a function by using the definition of the derivative. Yasin found the derivative of the given function by using the definition of the derivative as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and also he explained this definition by using the graph of a function and marking on the graph $f(x+h)$, $f(x)$, $x+h$, x and h values. After his explanation, the instructor asked the difference between the derivative he found in the fourth question and the derivative he found in the third question. In the fourth questionhe used the formula $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ and in the third one he used the formula $\frac{f(1)+f(3)}{2}=\frac{2+1}{2}=1.5$. After this question he realized his mistakeand corrected by using the formula $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. First of all, he wrote the formula for the points of x values 1 and 0. Then he took the points of x values 3 and 2. After the instructor took the attention to a reference point that he found the derivative at, in this question this point is (2, 1.4). Then he realized that he should take the points close to this point where he would find the derivative at. Therefore, he found the average rate of changes between the points (1, 1.1) and (2, 1.4) from left and (2, 1.4) and (3, 1.9) from right. He found the average values as $\frac{1.4-1.1}{2-1}=0.3$ and $\frac{1.9-1.4}{3-2}=0.5$. He said that he would try to find and average value.

He sketched the graph and a secant line between the points (1, 1.1) and (3, 1.9) given in the Figure 4.21 to explain how to find the derivative value at the point (2, 1.4).

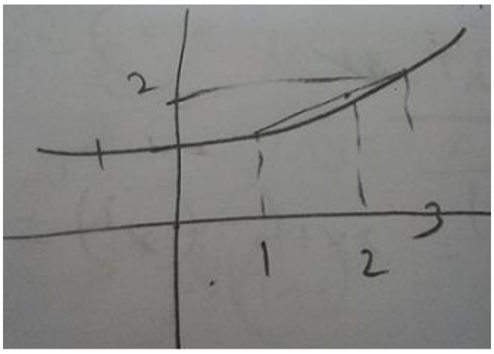


Figure 4.21 Yasin sketched graph to explain how to find the derivative value at the point (2, 1.4)

Meral's answer to third question on rate of change

Meral answered the third question as "the slope that is tangent to the point $x = 2$ gives the derivative of the function $f(x)$ at the point $x = 2$" in the pre-test. In the post-test, she found the slopes of the line segments between the points of x values 1 and 2, 2 and 3 by using the difference quotients of $\frac{f(2)-f(1)}{2-1}$ and $\frac{f(3)-f(2)}{3-2}$. She stated that the slope of the line tangent at the point of x value 2 should be between the slopes of these line segments 0.3 and 0.5. Therefore, she found the derivative value as 0.4. Meral found the derivative value at the point that has$x$ value 2 by using the slopes of the secant lines sketched between the point of value 2 $(2, f(2))$and close points.She chose the points of x values 1 and 3 at whichcorresponding y values are given. She gave the following answer in the post-test:

$$\left.\begin{array}{l} 1 < x < 2\ \frac{f(2)-f(1)}{2-1} = \frac{1.4-1.1}{1} = 0.3 \\ 2 < x < 3\ \frac{f(3)-f(2)}{3-2} = \frac{1.9-1.4}{1} = 0.5 \end{array}\right\} \text{would be a value between these values}$$

We could say derivative is 0.4, for example.

She used the words "approaching", "slope of the tangents", "slope", "line passing through 1 and 2" to explain her answer to the third question.

Although she used the secant lines to find the derivative value at the intended point, she stated that she found the derivative value at that point using the slopes of the tangent lines. She said that:

Meral: It asks the approximate value at 2. As I said (for the first question) I find the slopes while I'm coming here. I found the value that would be between these two (these two close points).

R: Why do you express a value that would be between these two points?

Meral: Because from there to here the slope increases. I thought that it should get smaller value here and larger value there.

R: Okay, whose slope is this $\frac{f(2)-f(1)}{2-1}$?

Meral: The line that is passing through 1 and 2.

R: This? (shows the line passing through the x values 2 and 3).

Meral: This is 3 and 2.

She used algebraic symbols $\frac{f(2)-f(1)}{2-1}$ and $\frac{f(3)-f(2)}{3-2}$ as visual mediators. She did not used narratives while explaining her answer to this question.

Suzan's answer to third question on rate of change

Suzan answered the third question in the pre-test as $\frac{1.4}{2} = \frac{1.4}{10} . \frac{1}{2} = \frac{7}{10}$. She also sketched a right triangle and named one angle as $\propto$ and wrote 1.4 at the opposite side and 2 at the near side. She answered the third question as "when we sketched a tangent at the point $x = 2$ approximately $\frac{1.4-1.1}{2-1} = \frac{0.3}{1} = 0.3$ 0.3 gives us the slope and the slope is the derivative." She also sketched a line tangent to the curve at the point of (2, 1.4)

She used the words "close points" and "tangent" to explain her answer to third question. In this answer, considering derivative as the slope of the line tangent to the interest point, she answered this question as the slope of the secant line sketched between the points (1, 1.1) and (2, 1.4). In the interview when she was asked whether this value is the slope of the tangent line, she realized her mistake. Then she explained that she should find the slope of the tangent line by using the close values. In this purpose, she said that she would use the slope of the lines sketched between the points (1, 1.1) and (2, 1.4) and the points (1, 1.1) and (3, 1.9) to find the approximate value of the slope of the line tangent at the point of (2, 1.4). She said that this value approximately would be 0.35.

Suzan: Now I would say that I look at 1and 2 (x values) for example, here it is 0.3/1, 0.3. Then I will look at 3 and 1.

R: …Why did you choose this value?

Suzan: I use the values that I have. I will look at the values of 1 and 3. 1.9-1

R: Which value you try to get?

Suzan: 2. when I approach 2 and when I look at 1 and 3 if I divide 0.8 by 2, I get 0.4. When I look at between 1 and 3,   0.4, if I look a little bit, I would say 0.35. When I look at here I approach 0.3.

She also used algebraic symbols and graph as visual mediators. She used $\frac{1.4-1.1}{2-1} = \frac{0.3}{1} = 0.3$ as algebraic symbols and she sketched the graph given in the Figure 4.22 to show the line tangent to the graph at the intended point.

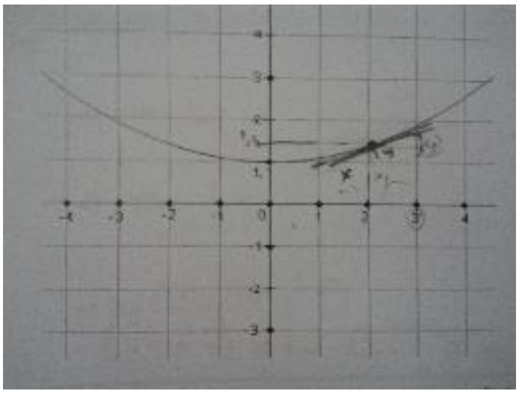


Figure 4.22 Graph Suzan sketched to show the tangent line

4.3.3 Individual Discourse of Seventh Question on Increasing and Decreasing Function

Derivative function graph is given for the first part (a) of the seventh question and the participants are required to find the intervals where the function f is increasing and decreasing. In the second part (b), participants are expected to find the points where the function f has extremum values according to the given graph of the derivative function. For the first part of the question, pre-service teachers are expected to analyze the given graph and decide in which intervals function is increasing and decreasing where the derivative function has positive or negative values. For the second part of the question, they were required to find the points where the function has extremum values according to the points that the derivative function value was zero. Moreover, the points where derivative value is zero are local minimum or local maximum points. Seventh question is given below.

Seventh question:

$f^{'}(x)$

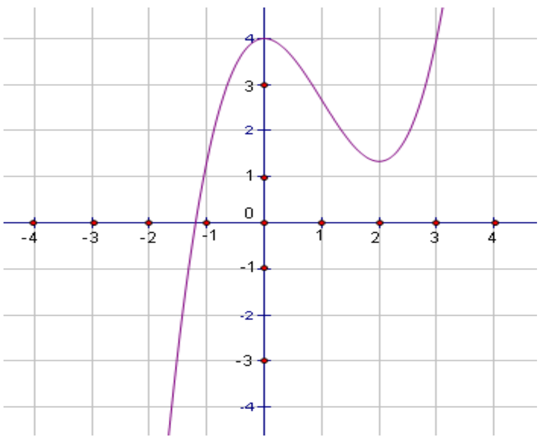


Figure 4.23 Graph of the seventh question

a) Find the interval where the function f, whose derivative graph is given in the Figure 4.23, is increasing and decreasing.

b) At which points function f has extremum values.

Pre-service teachers' answers to the seventh questions' part a was given in the Table 4.22. For the part a 10 pre-service teachers gave no answer in the pre-test. 5 of them gave wrong related answer. 19 of them found correct relation between the increasing and decreasing function and the sign of the derivative function. 18 of them found correct relation for the given function f. In the post-test 5 pre-service teachers gave no answer. 2 of them constructed wrong relation. 37 of them found correct relation between increasing and decreasing function and the sign of the derivative function.

Table 4.22

Pre-service teachers' answers to the seventh question part a on increasing and decreasing function

Pre-service teachers' answers	Pre Test	Post Test
No Answer	10	5
Wrong related expression	5	2
Correct relation for the derivative function (Increasing decreasing function and sign of the derivative function)	19	37
Relation for function f	18	11

Pre-service teachers' answers to the b part of the seventh question were given in the Table 4.23. In the pre-test 12 pre-service teachers gave wrong no answer. 4 of them constructed wrong related expression. 14 of them found extremum point according to derivative function. 22 of them found extremum point according to function f. In the post-test 6 pre-service teachers gave no answer. 1 of them constructed wrong relation. 31 of them found extremum point according to derivative function. 17 of them found extremum point according to function f.

Table 4.23

Pre-service teachers' answers to the seventh question part b on extremum points

Pre-service teachers' answers	Pre Test	Post Test
No Answer	12	6
Wrong related expression	4	1
Extremum point according to derivative function	14	31
Extremum point according to function f	22	17

Sezen's answer to seventh question of part a on increasing and decreasing function

Sezen did not answer the seventh question of part a in the pre-test. In the post-test she gave the answer as: "If $f'(x) > 0$ f is $i$ncreasing in $(-\frac{4}{3}, \infty)$ and if $f'(x) < 0$ f is decreasing in $(-\infty, -\frac{4}{3})$".

She used the following words to explain her answer to the seventh question part a: "positive derivative", "f is increasing", "negative derivative", "f is decreasing", "positive slope", "negative slope" and "obtuse angle".

In the interview, she explained her answer again as the function is decreasing in the interval $(-\infty, a)$ as the derivative of the function is negative. The value for a was the x value that passed through the x axis and at that point, the derivative value was 0. In the interval (a, ∞) the function was increasing as the derivative of the function was positive.

She sketched graphs and used algebraic symbols as visual mediators. She explained the relation between the sign of the derivative function and the increasing and decreasing of the derivative by sketching graphs. First of all she sketched a convex increasing graph. For this graph she sketched lines tangent to the graph and she realized that the slope of these tangent lines are positive. Therefore she concluded that as the slope of the tangent lines gave the derivative of the function, the derivative of the function is positive and the function is increasing. So she showed the relation if the derivative of the function was positive then the function was increasing. The increasing graph that she sketched was given in the Figure 4.24.

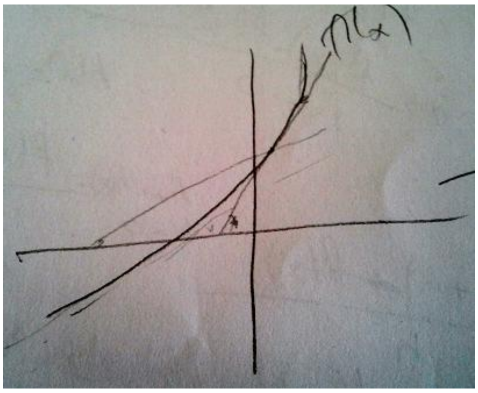


Figure 4.24 Sezen sketched increasing, convex graph

Then she sketched a convex decreasing graph and lines tangent to the graph (Figure 4.25). She realized that the slopes of these tangent lines are negative and so the derivative of this function was negative too. Therefore, the function was decreasing and the derivative function was negative. She sketched the following graph to explain the relation if the derivative of the function was negative, the function was decreasing.

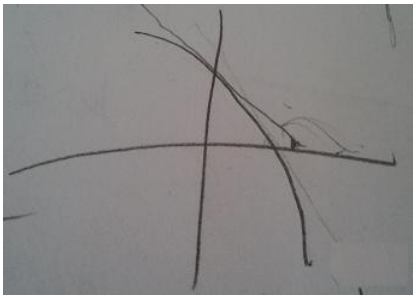


Figure 4.25 Sezen sketched decreasing concave graph

She used algebraic symbols to explain the relation between the sign of the derivative and increasing and decreasing of the function. She used the algebraic symbols “$f'(x) > 0 \quad f(x)$ increasing” and “$f'(x) < 0$ f(x) decreasing”. She also used $(-\frac{4}{3}, \infty)$ and $(-\infty, -\frac{4}{3})$ to represent the increasing and decreasing intervals of the function.

She used narratives to explain the relation between the sign of the derivative function and the increasing and decreasing of the function. “If the function f is increasing then $f'(x)$ is greater then 0” and “If the function f is decreasing then $f'(x)$ is less then 0”. She also explained the increasing function using the narrative “if y values increased while x values were increasing, then the function was increasing”. And she also explained the decreasing function by using the narrative “if y values decreased while x values were increasing, then the function was decreasing”.

Sezen's answer to the seventh question part b on extremum points

Sezen answered this question in the pre-test as "the points where $f'(x) = 0$ is the local minimum or local maximum points. In this question the point $x = -1, \ldots$ is the local minimum point." In the post-test she answered this question as "the point $x = -\frac{4}{3}$ is the local minimum point". She used words "local maximum" and "local minimum" to explain her answer to the seventh question of the part b. In the interview she explained that the local minimum and local maximum points were the points where the roots of the derivative function. For the question she said that the point of x value a was the local minimum point as the derivative function was negative till the point of x value a and positive after this point. Therefore, the function was decreasing till this point and increasing after this point. So the point of x value a was the local minimum point. She analyzed the local minimum point at the Figure 4.26.

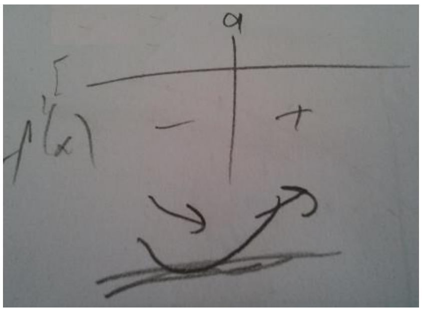


Figure 4.26 Table Sezen sketched to show the local minimum point

She sketched a graph as visual mediator in Figure 4.27. She also showed the local minimum points on a graph that she sketched.

Figure 4.27 Sezen shows local minimum points on the graph

She explained the local minimum point as the point that got the minimum value in a specific interval. However, she could not explain the minimum point of a function. She could not differentiate the local minimum value and the minimum value of a function.

For the local maximum points she also said as a narrative that the local maximum points were the points that the roots of the derivative and the point that got the minimum value in a specific interval. She also could not differentiate the local maximum value and minimum value of a function. She explained why she got the derivative value 0 as the slope of the line tangent to the graph at this point was 0 therefore the derivative was 0 too.

Semra's answer to seventh question part a on increasing and decreasing function

In the pre-test, Semra answered the 7th question as thinking that the given graph was the function graph. She wrote that in the intervals $(-\infty, 0)$ and $(0, 2)$ the function was decreasing and in the interval $(2, \infty)$ the function was increasing. If the given graph was the function graph instead of derivative function graph, the answer that she gave in the pre-test was true. But according to the derivative graph, the answer was not true.

In the post-test, Semra answered this question as if $f'(x) > 0$ the function was increasing, for this derivative graph the function was increasing in the interval (a, ∞), a was the x value of the point that the derivative function was passing through the x axis. And the function was decreasing if $f'(x) < 0$, for this derivative graph in the interval $(-\infty, a)$, the function was decreasing.

She used the words "increasing function", "decreasing function", "positive angle" to explain her answer to the question. In the interview, she explained the relation between the increasing function and the first derivative by the following narratives: "If the first derivative of the function was greater than 0, then the function was increasing". And for the decreasing function, "if the first derivative of the function was less than 0, then the function was decreasing". For the function whose derivative function graph was given in the question, "in the interval $(-1, \infty)$, the function was increasing and in the interval $(-\infty, -1)$, the function was decreasing". She explained the relation between the sign of the slope of the tangent line and the first derivative by sketching a graph and lines tangent to this graph as visual mediator (Figure 4.28).

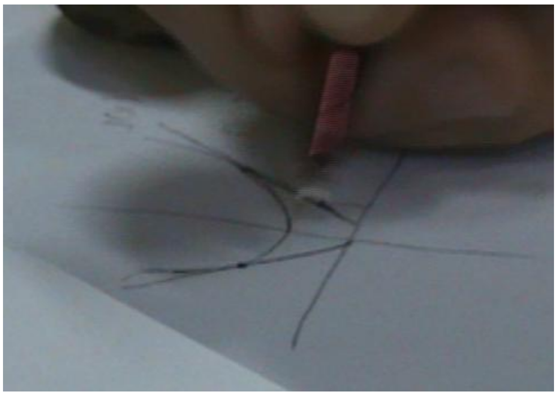


Figure 4.28 Semra sketched lines tangent to the function graph

She stated that in the right part of the graph, the slope of the tangent line was positive as it had an acute angle between the x-axis. Therefore, the slope was positive and the function was increasing. In the left side of the graph, the slope of the tangent line was negative as it had obtuse angle between the x-axis. Therefore, the slope was negative and the function was decreasing. She defined the increasing function as, if the y values increased while the x values was increasing, then the function was increasing. And she defined the decreasing function as, if the x values were decreasing and the function values were increasing then the function was decreasing.

Semra's answer to the seventh question part b on extremum points

In the pre-test she answered this question in the graph but she thought that the graph is the graph of the f function (Figure 4.29). She assigned the point (0, 4) the local maximum point and the point (2, 1.2) the local minimum point.

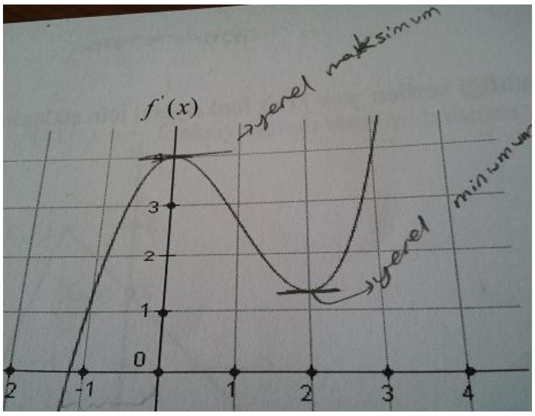


Figure 4.29 Semra assigned the local minimum and local maximum points

In the post-test, she answered this question by using more explanations. She wrote that the points that $f'(x) = 0$. She analyzed these points on the following Figure 4.30 according to the sign of the first derivative.

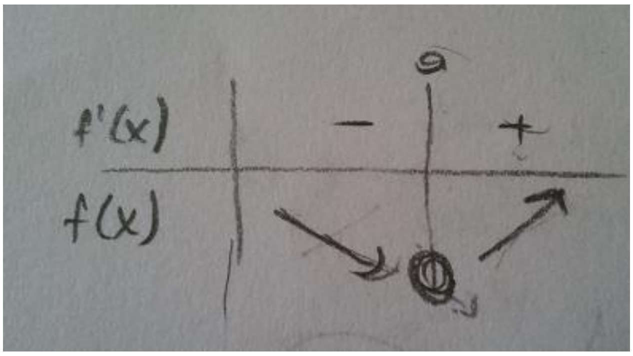


Figure 4.30 Semra found the local minimum point on the table

She used the words “roots of the derivative”, “local minimum point”, “function change direction” to explain her answer to the question. She named the x value of the point that the curve was passing through the x-axis as a. She wrote that, at the point of x value a. $f^{'}(a) = 0$, the point $(a, 0)$ is the local minimum point. However, she missed the truth that $(a, 0)$ is not the point that the function has the local minimum value. The local minimum point is the point that the x value was$a$, but we could not know the y value. 0 was the derivative value of the function at the point of x value a.

In the interview, she explained what she did to find the local minimum and local maximum values of the function. She said that she found the roots of the derivative of the function and check for the increasing and decreasing of the function. She explained how to find the local minimum point as narrative: “The function firstly decreased till the point of x value a (the root of the derivative function) and after this point the function increased. Therefore, this point is the local minimum point”.

She analyzed the graph that she sketched as if it has a local minimum point or local maximum point and she concluded that it has a local minimum point as the graph of the function change direction from decreasing to increasing as visual mediator (Figure 4.31).

Figure 4.31 Function change direction from decreasing to increasing.

She could not answer the question of why she found the roots of the derivative to find the local minimum and local maximum points.

Yakup's answer to seventh question part a on increasing and decreasing function

In the pre-test he named the point that passing through the x axis as a and he wrote that the function wasincreasing on the right side of the x value a and decreasing in the left side of the x value a. And he also analyzed where the derivative function waspositive or negative and where the function was decreasing or increasing on the table.

In the post-test he gave the value of $-\frac{4}{3}$ to the point where the graph of the derivative function passedthrough the x axis instead of a. He again analyzed where the derivative function waspositive or negative and where the function was decreasing or increasing on the table. He concluded that the derivative function was negative and the function was decreasing in the interval $(-\infty, -\frac{4}{3})$ and the derivative function was positive and increasing in the interval.

He used the words "sign of the derivative", "sketched point", "derivative is zero", "derivative is positive", "derivative is negative", "increasing in the right" and "decreasing in the left" to explain his answer to the part a of the seventh question. In the interview, he explained how he decided the increasing and the decreasing intervals. He found the sign of the derivative according to the point where the graph of the derivative function was passing through the x-axis and got the y value 0. He said that the function wasincreasing where the derivative of the function was greater than 0 and decreasing where the derivative function was less than 0.

He used graph as visual mediator. Yakup sketched the table given in the Figure 4.32 to show the intervals where the derivative function is positive and negative and for the function increasing and decreasing.

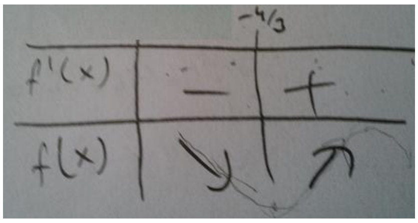


Figure 4.32 Table shows the intervals function was increasing and decreasing

He explained the relation between the sign of the derivative function values and the increasing or decreasing of the function by given the example of the function $f(x) = x^2 - 1$. He sketched the graph of this function as in the Figure 4.33.

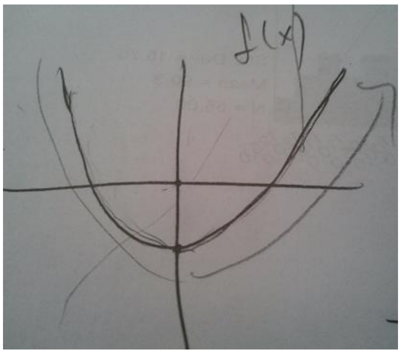


Figure 4.33 Yakup shows the intervals that the function was increasing and decreasing

Then he sketched the graph of the derivative function as the following one (Figure 4.34).

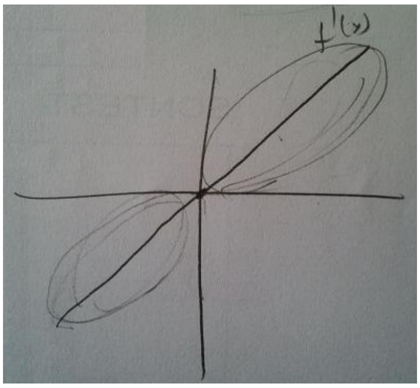


Figure 4.34 Yakup sketched the graph of the derivative function

Then he explained the relation by using the graph of the function and the graph of the derivative function. He showed the derivative function is negative in the interval $(-\infty, 0)$ and the function was decreasing in this interval. Also, he showed the derivative function was positive in the interval $(0, \infty)$ and the function was increasing in this interval. And he also explained these relations by using the narratives “the function is decreasing where the derivative is negative and increasing where the derivative is positive”, “if the y values increase while the x values increase, then the function is increasing” and “if the y values decrease while the x values increase, then the funciton is decreasing”.

Yakup’s answer to seventh question part b on extremum points

Yakup answered this question in the pre-test that the point the curve pass through had the local minimum and did nothave local maximum. He also gave the same answer in the post-test. He wrote that there was no maximum point. There was local minimum point $(-\frac{4}{3}, y)$.

He used the words “local minimum”, “local maximum”, “increasing” and “decreasing” to explain his answer to the b part of the seventh question. In the interview, he explained his answer at the table that he sketched as visual mediator to answer the part b of the question given in the Table 4.35.

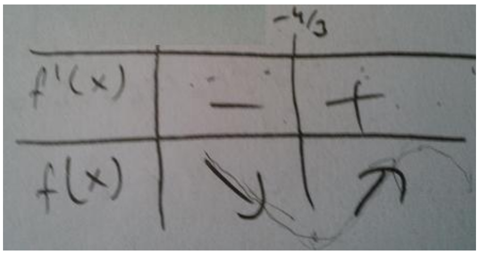


Figure 4.35 Yakup analyzed the local minimum and local maximum points in the table

According to the table, he said that the function was decreasing in the interval $(-\infty, -\frac{4}{3})$ and an increasing at the points after the x value $-\frac{4}{3}$ in the interval $(-\frac{4}{3}, \infty)$. Therefore, he said that there was a turn at the point of x value $-\frac{4}{3}$ because of this decrease and increase. But there was no turn because of increase till a point then decrease after this point. Therefore, there was a local minimum point and there wasno local maximum point. He determined this point as the derivative had a change at that point. The derivative value was negative on the left side of this point and positive on the right side.

He also explained his answer on a graph that he sketched as visual mediator given in the Figure 4.36.

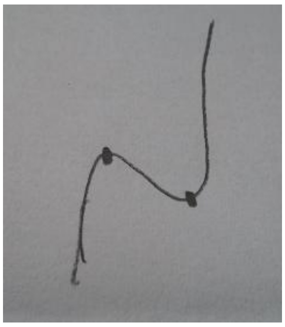


Figure 4.36 Yakup analyzed the local minimum and local maximum points on the graph

He said that to have a local maximum value a curve should be like a wave consisting of increasing then decreasing parts and to have a local minimum value curve should be like a curve consisting of decreasing and increasing parts. He used the narratives “the function decreases where the derivative function is negative” and “at the local minimum, derivative is positive in the left and negative in the right”.

Yasin's answer to seventh question part a on increasing and decreasing function

Yasin, answered this question in the pre-test as "in the intervals $(-\infty, 0)$ and, (2,∞) increasing and in the interval $(0, 2)$ decreasing" and "$(x_1 < x_2 \wedge f(x_1) < f(x_2)) \Rightarrow f(x)$ is increasing".

For this question, he gave the true answers, however he thought that the given graph wasthe graph of the function f although the graph wasthe graph of the derivative function of the function f.

In the post-test he gave the following answer

"If the function wasincreasing then $f'(x) > 0$, if the function is decreasing then $f'(x) < 0$". "In the interval$(-\infty, -1)$, the function wasdecreasing, in the interval$(-1, \infty)$, the function was increasing".

He used the words "increasing", "decreasing" and "positive slope" to explain his answer to the a part of the seventh question. In the interview after the post-test, he answered the question as he gave the answer like he gave in the pre-test and he said that in the intervals $(-\infty, 0)$ and, $(2, \infty)$ increasing and in the interval $(0, 2)$ decreasing.

After the instructor warned him, he realized that the given graph was not the graph of the function f, he changed his mind and answered the question as "then we would look where the y values were positive or negative." He gave the answer in the interval $(-\infty, -1)$ where the derivative of the function was negative; the function was decreasing, in the interval $(-1, \infty)$ where the derivative of the function was positive; the function wasincreasing. He showed the interval on the graph given in the question by his hand. He showed the interval where the derivative of the function was negative on the graph given in the question.

Yasin explained the relation between the derivative of the function and the function was positive or negative by sketching a concave parabola given in the Figure 4.37 as visual mediator. As the function was increasing, then the slope of the lines sketched tangent to the graph of the function was positive. Therefore the derivative of the function was positive, too.

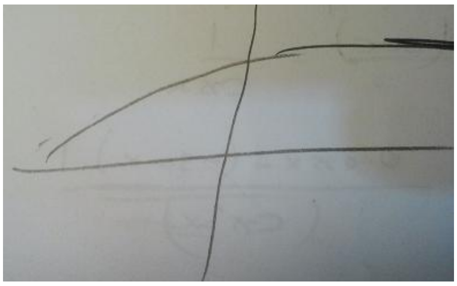


Figure 4.37 Parabola that Yasin sketched

He sketched a concave decreasing graph (Figure 4.38) as visual mediator to show that the function was decreasing if the derivative is negative. As the function was decreasing then the slope of the lines sketched tangent to the graph of the function was negative. Therefore the derivative of the function was negative, too.

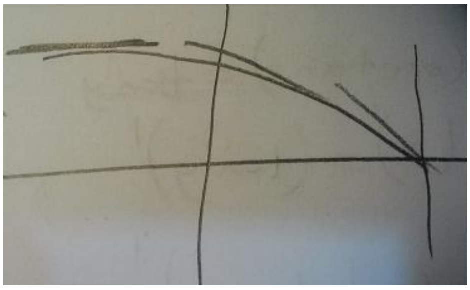


Figure 4.38 Decreasing part of concave parabola

He explained the relation between the increasing and decreasing function and the sign of the derivative and the slope of the tangent lines by the narratives. He used the narratives that "The intervals where the function was increasing or decreasing, if the function was increasing, if $x_1 < x_2$ ise $f(x_1) < f(x_2)$", "if the slope is positive then the function is increasing" and "if the slope is negative then the function is decreasing".

Yasin's answer to the seventh question part b on extremum points

In the pre-test he answered this question as "the points where the function change sign is the extremum points. Therefore, the points of 0 and 2 are the extremum points." In the post-test he said that the extremum points are the points where the derivative of the function waszero. He explained his answer as the graph of the function passed from increasing to decreasing or decreasing to increasing. Therefore, the slope of the lines tangent to the graph changed positive to negative or negative to positive at these points. As this graph was the graph of the derivative function of the function f, the extremum points of this graph were at the points where the graph passes from positive to negative and he sketched the graph given in the Figure 4.39 as visual mediator to explain his answer.

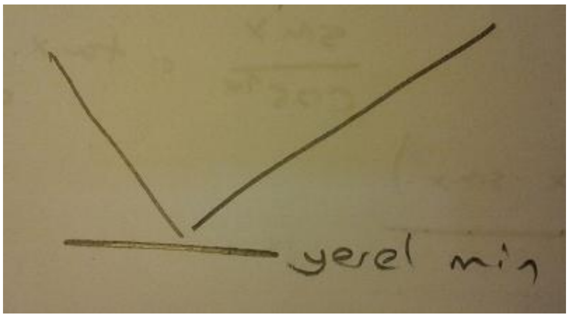


Figure 4.39 Graph of the derivative function

He also sketched the graph given in the Figure 4.40 to how the maximum points of the derivative function.

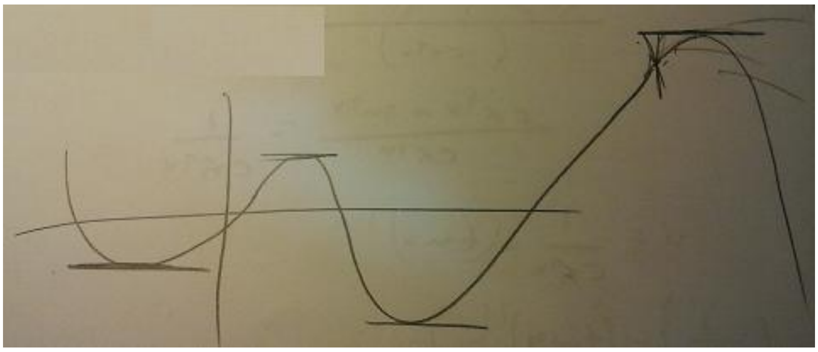


Figure 4.40 Graph Yasin sketched to show the local minimum and local maximum points

As the graph passes from decreasing to increasing, this graph has a local minimum value at this point. After this point as the graph was always increasing there wasno other local minimum or local maximum points.

He used the words “local minimum”, “local maximum”, “absolute minimum” and “absolute maximum” to explain his answer to the b part of the seventh question. Yasin explained the minimum point of a function that the point had minimum y value. There would be many local minimum points however; the minimum point of the function had the least y value. This minimum point was called the absolute minimum point. Moreover, the maximum point wasthe point where the function has the maximum y value at this point between the other local maximum points. This point was called the absolute maximum point.

Meral’s answer to the seventh question of part a on increasing and decreasing function

Meral didn’t answer the seventh question in both pre-test and post-test. In the interview, when she was asked to explain this question she said that she didn’t remember this subject. The instructor insisted on her to think about the question. She used the words “derivative function graph”, “function is positive”, “increasing” and “decreasing” to explain her thought related the a part of the seventh question. She said that the function was increasing where the derivative was positive and decreasing where the derivative was negative. She showed the interval where the function is increasing and decreasing on the given derivative graph of the function.

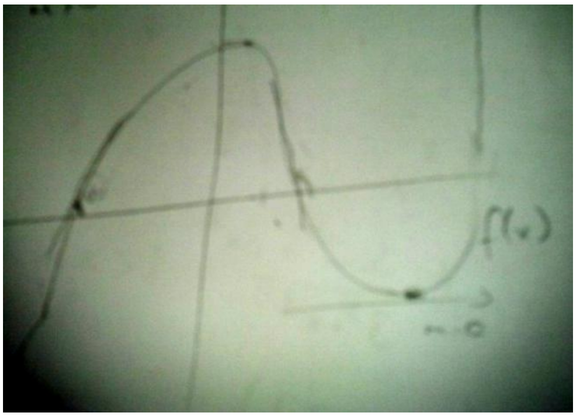


Figure 4.41 Meral sketched this graph to explain her thought

She explained the relation between the function was increasing and decreasing and the sign of the derivative of this function by sketching the graph as visual mediator given in the Figure 4.41. She sketched lines tangent to the graph at some points where the graph increasing and explained that the slopes were positive in these intervals. She repeated this procedure for the points in the interval where the function was decreasing. Again she concluded that in this interval the slopes of the tangent lines were negative.

She explained the relation between the increasing and decreasing function and the sign of the derivative by using the narratives. She used the narratives “points where the derivative was negative than the function was decreasing”, “points where the derivative was positive than the function was increasing”, “y values increase while x values increase” and “ y values decrease while x values increase”.

Meral's answer to the seventh question part b on extremum points

Meral didn't answer this question in both pre-test and post-test. However, in the interview she answered this question as "the minimum and maximum points were the points where the value of the derivative function is zero." She showed the local minimum point on the given derivative graph of function f. She used the words "local minimum", "minimum", "maximum", "increasing" and "decreasing" to explain her thought related to the b part of the seventh question. She explained that it was the local minimum point because at that point the function turns from decreasing to increasing. She also examined the extremum points in the table. She explained why function has local minimum and local maximum points at the points where the derivative value of the function was zero on the graph that she sketched as visual mediator given in the Figure 4.41.

She also used the narratives to explain the local minimum and local maximum points. Her narratives are "local minimum and local maximum points are the points where the derivative value is zero", "here decreasing and here increasing, decreasing and increasing than this point is local minimum", "the point where the y value takes the maximum value then it is the maximum point; takes the minimum value then it is minimum point".

Suzan's answer to seventh question part a on increasing and decreasing function

In the pre-test Suzan answered this question considering the given graph wasthe function graph rather than the graph of the derivative function. Therefore she wrote that "between 2 - 4 increasing, between 2 - 0 decreasing, between -3 – 0 increasing". In the post-test, she realized that the given graph wasthe graph of the derivative function. She answered this question in the post-test that, "Above graph is belong to $f'(x)$. Above the x axis is the part the function is increasing and below the x axis is the part the function is decreasing. $In\ (-\infty, -1.1)\ f\ is\ decreasing\ \ in\ (-1.1, \infty)\ f\ is\ increasing$"

Suzan used the words "increasing", "decreasing", "increasing function", "decreasing function", "obtuse angle" and "acute angle". In the interview, Suzan determined the relation between the decreasing function and the negative slope when she was helped. But she could not relate negative slope and tangent value.

She sketched an increasing function graph and a line tangent at any point as visual mediator given in the Figure 4.42. She reached a conclusion that if the slope of the line was positive, the derivative was positive.

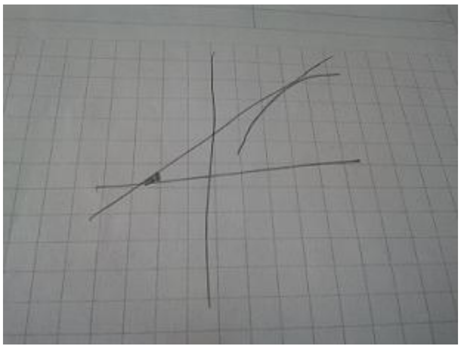


Figure 4.42 Suzan sketched an increasing function graph

Suzan defined the increasing function as the x values were increasing the function values were increasing and the decreasing function as the x values were decreasing the function values were decreasing and showed the angles on the graph as visual mediator given in the Figure 4.43.

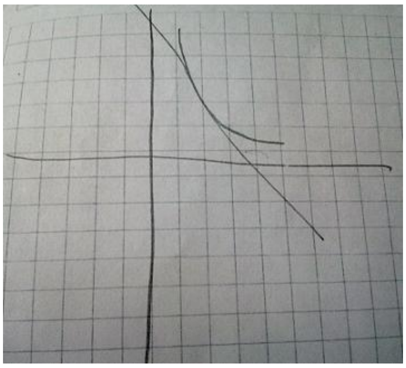


Figure 4.43 Suzan sketched a decreasing graph

She used narratives “if the slope of the tangent line sketched to the graph is negative then the function is decreasing”, “the function values increase if the x values increase for the increasing function”, “the function values decrease if the x values decrease for the decreasing function” to explain the relations between the increasing and decreasing function and the function values and the slope of the lines tangent to the graph of the function.

Suzan’s answer to seventh question part b on extremum points

Suzan answered this question in the pre-test as “0 max, 2 min” and she sketched a table and examined the increasing and decreasing intervals by considering the given graph was the function graph. In the post-test, she answered this question as “-1.1 is the minimum point”. She also investigated the increasing and decreasing intervals on the table and signed the interval before the x value -1.1 as negative indicating decreasing function and after this value as positive indicating increasing function. She used the words “maximum”, “minimum”, “local maximum”, “local minimum” and “derivative is zero” to explain her answer to the b part of the seventh question. She explained how she would find the increasing and decreasing points in the interval as she would look for the points where the derivative value was zero. She added that at the point of local minimum the function change direction from decreasing to increasing. Reverse situation was valid for local maximum point that the function change direction from increasing to decreasing. She explained her thought on the graph seen in the Figure 4.44 as visual mediator.

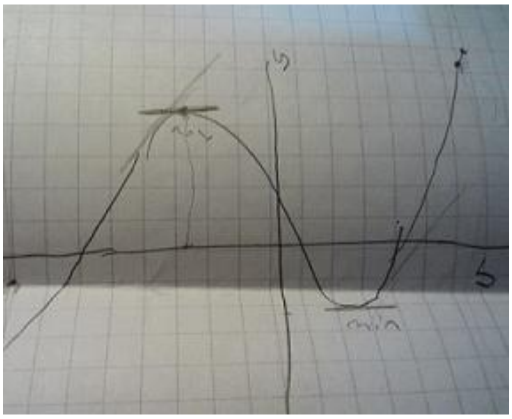


Figure 4.44 Graph that Suzan explains extremum points

She also analyzed the local maximum points and the local minimum points on the table using the increasing and decreasing intervals on the table given in the Figure 4.45 as visual mediator.

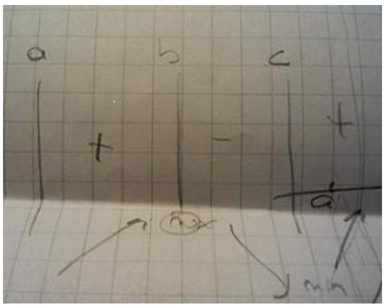


Figure 4.45 Table that Suzan explains extremum points

4.3.4 Individual Discourse of Tenth Question on Derivative Function Graph

Tenth question of the derivative test demands the participants to sketch the derivative function of f function according to its given graph in the question. In this question pre-service teachers were expected to analyze the function graph according to the intervals the function wasincreasing and decreasing related to the sign of the first derivative or according to the concavity of the function related to the sign of the second derivative. Tenth question was given below.

Tenth Question

Sketch the graph of the derivative function of the function f whose graph was given below (Figure 4.46). Explain your answer.

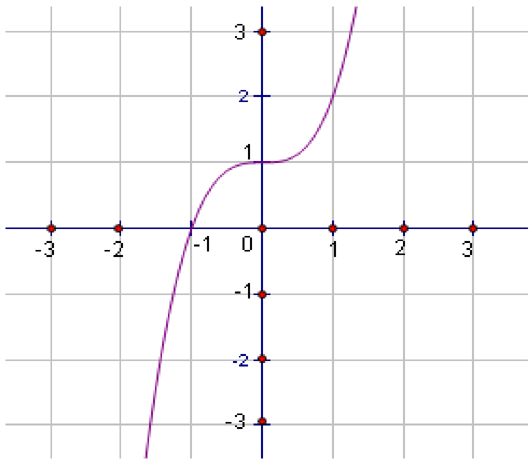


Figure 4.46 Graph of the tenth question

Pre-service teachers' answers to the 10th question were given in the Table 4.24. In the pre-test 38 pre-service teachers didn't answer the question. 7 of them answered with wrong related expression or graph. 1 of them answered with correct graph and correct relation. 2 of them answered with correct relation and wrong graph. 1 of them constructed correct relation but could not sketch a graph. 1 of them sketched a correct graph but mentioned no relation. 2 of them found the algebraic expression of the given function graph. In the post-test 6 of them gave no answer. 3 of them constructed wrong relation or sketched wrong graph. 21 of them gave correct answer. 15 of them constructed correct relation but sketched wrong graph. 10 of them constructed correct relation but sketched no graph.

Table 4.24

Pre-service teachers' answers to tenth question on derivative function graph

	Pre Test	**Post Test**
No Answer	38	6
Wrong related expression/graph	7	3
Correct graph and relation	1	21
Correct relation, wrong graph	2	15
Correct relation, no graph	1	10
Correct graph, no relation	1	
Algebraic expression of the function	2	

Sezen's answer to the tenth question on derivative function graph

Sezen didn't answer this question in the pre-test. In the post-test, she sketched the graph given in the Figure 4.47. She explained her answer in the post-test as;

- $f'(x) = 0$
- In the interval $(-\infty, 0)$ derivative is positive and decreasing
- In the interval $(0, \infty)$ derivative is positive and increasing

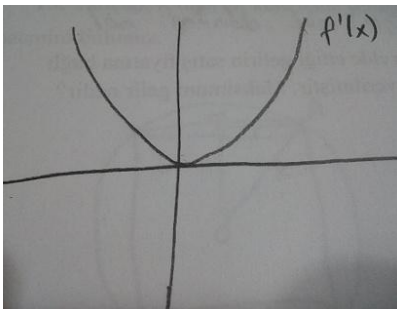


Figure 4.47 Sezen’s answer to tenth question on derivative function graph

She used the words “slope of the tangent”, “derivative is greater than zero”, “function is positive and increasing”, “derivative is decreasing”, “derivative is positive and decreasing” to explain her answer to tenth question.

In the interview, she explained her answer as the derivative of the function was positive as the function value was increasing everywhere. The derivative value was decreasing as the slopes of the lines sketched tangent to the graph in the interval $(-\infty, 0)$. She concluded this result according to the tangent values of the angles for these lines decreasing till the point of x value 0. In the interval $(0, \infty)$, the derivative is increasing.

She also used the narrative “as the angle gets small the tangent value gets small” to explain the relation between the slope of the angle and the tangent value.

Semra’s answer to the tenth question on derivative function graph

In the pre-test, Semra did not answer the 10th question. In the post-test, she answered the question as when $x > 0$, $f^{'}(x) > 0$ and when $x < 0$, $f'(x) > 0$. For $\forall x \in R$ except $x = 0$, $f'(x) > 0$. And she sketched the following graph (Figure 4.48) as the derivative graph of the given function.

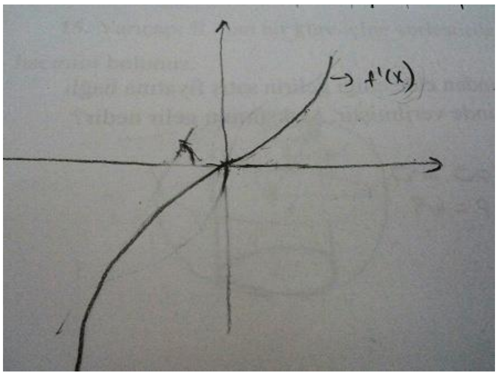


Figure 4.48 Semra's answer to tenth question on derivative function graph

She wrote that the derivative was greater than 0 for every value of the x, but she sketched the left side of the y-axis as negative. For this graph the right side of the y-axis was true but the left side was not true.

She used the words "derivative is less than zero", "derivative is greater than zero" to explain her answer to the tenth question. In the interview, she explained that the derivative function was positive on the right side of the y-axis and the tangent lines to this function graph on the right side were positive. And it was positive on the left side of the y-axis as the tangent lines on the left side werepositive. Therefore, she concluded that the derivative function waspositive everywhere.

She realized the relation between the second derivative and the first derivative function. Then she explained that as the right side of the graph was convex, the second derivative wasgreater than 0 and the first derivative wasincreasing. And also, as the left side of the graph was concave the second derivative was less than 0 and the first derivative was decreasing.

Therefore, she sketched the derivative graph in the following form given in the Figure 4.49 according to the analysis of the first derivative and the second derivative. It is also the visual mediator that Semra used. First of all she decided that the derivative graph should be positive everywhere. And then she analyzed the function graph according to the concavity. As the function graph was convex in the right side of the y axis, then the second derivative was positive. Therefore, the derivative function was increasing in the interval $(0, \infty)$. As the function graph was concave in the left side of the y axis then the second derivative was negative. Therefore, the derivative function was decreasing in the interval $(-\infty, 0)$.

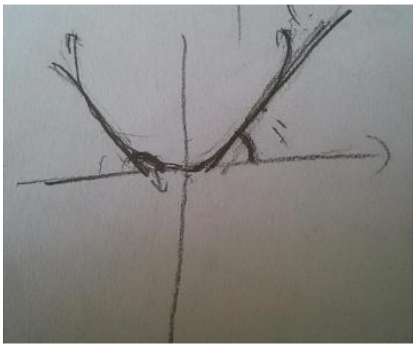


Figure 4.49 Graph Semra sketched in the interview

She also used the narratives "as the right side of the graph is concave up, then the second derivative is greater then 0 and the first derivative is increasing" and "as the second derivative of the function is less then 0, then the first derivative is decreasing" to explain her answer.

Yakup's answer to the tenth question on derivative function graph

Yakup did not answer in the pre-test but in the post-test he sketched the following graph as the derivative graph of the function. In the Figure 4.50 the graph that he sketched was given as a visual mediator. He explained why he sketched this graph as the function is increasing everywhere therefore the derivative graph should be positive everywhere.

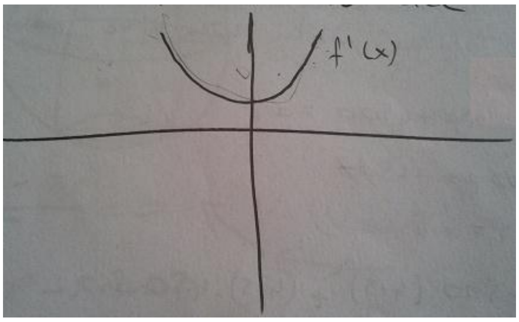


Figure 4.50 Yakup's answer to the tenth question on derivative function graph in the post-test

In the interview, he explained his answer as he wrote in the post-test. He used the words "increasing function", "sign of the derivative", "convex" and "concave" to explain his answer to the 10th question. However, he didn't mention about the derivative at the point of the x value 0. Therefore, he sketched this graph but he didn't realize that the derivative of the function should be zero. The instructor asked if the graph given in the figure 4.51 would be the answer of the tenth question. He answered this question that it could be the graph of the derivative function as it had the positive values.

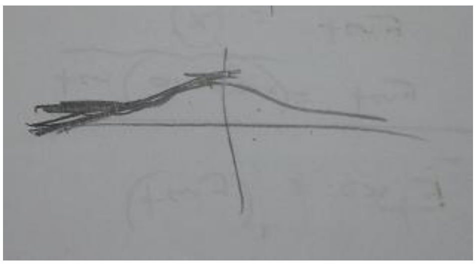


Figure 4.51 Graph instructor asked whether it would be the answer of the tenth question

Yakup explained the relation between the concavity and the function behavior as the derivative function graph looks up where the function was increasing. He made a mistake that where the graph looks up, the second derivative of the function waspositive and if the graph looks down, the second derivative of the function wasnegative. He also couldn't relate the second derivative of the function and the first derivative of the function.

He used the narratives to explain how she sketched the graph of the derivative function. He used the following narratives: "The derivative function is positive where the function is increasing" and "the derivative of the function is positive where the function is increasing".

Yasin's answer to the tenth question on derivative function graph

In this question the graph of the derivative function of the given function was asked. Yasin could not answer this question in the pre-test. In the post-test he sketched the graph given in the Figure 4.52. .

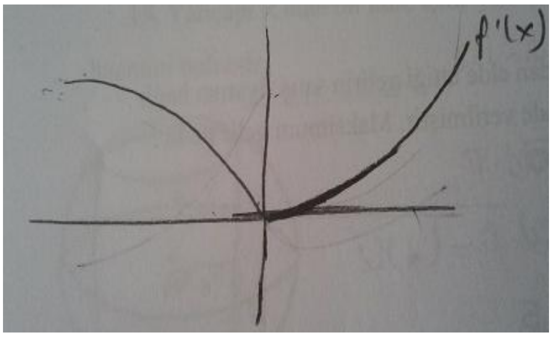


Figure 4.52 Graph Yasin sketched in the post-test

In this answer, he sketched the right side of the graph correctly. However, he made mistake in the left side of the graph. He should sketch a convex graph in the both sides, but he sketched a concave graph in the left side of the graph. He should also sketch a convex up graph in the left side of the graph. He made this mistake as he didn't consider the concavity of the given graph of the function. He should use the concavity of the function according to the given graph and make connection between the second derivative and first derivative of the function. He should use the second derivative of the function as the first derivative of the function.

He analyzed the graph of the given function and made conclusions according to the given graph of the function. He wrote on the paper in the post-test that "the function wasdecreasing in the interval $(-\infty, 0)$, so $f'(x) < 0$" and "the function wasincreasing in the interval $(0, -\infty)$, so $f'(x) > 0$" and "at the point of x value 0, the slope of the function is 0, so $f'(x) = 0$".

In this explanation, he also made a mistake as he wrote that "the function is increasing in the interval $(0, -\infty)$, so $f'(x) > 0$". According to the graph of the function given in the question, the function wasincreasing everywhere, therefore it does not decreasing in the interval $(0, -\infty)$. He also made the same mistake in the interview while he was explaining his answer to the question. I think he made this mistake unconsciously because in the following sentences he corrected his mistake and said that the function was positive everywhere.

He used the words "increasing", "decreasing", "increasing slope", "decreasing slope" and "slope is zero" to explain his answer to the tenth question. In the interview, he decided that the function was positive everywhere, he said that he should decide if the slope of the lines tangent to the graph was increasing or decreasing. In the interval $(-\infty, 0)$ he decided that the slope of the lines tangent to the graph was decreasing as the slope comes to zero at the point of the x value 0. He said that the function was increasing while the slope was decreasing and the graph should be same as the following one that he sketched in the post-test given in the figure 4.50 as visual mediator.

The instructor sketched two other graphs asked whether these two graphs would be the graph of the derivative functions. The first one is given in the figure 4.53.

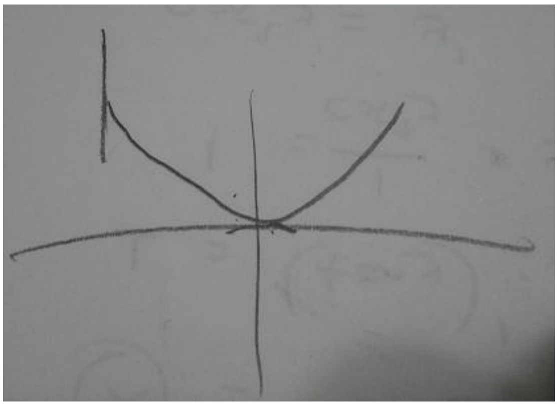


Figure 4.53 First graph that instructor asked whether this would be the answer of the tenth question

He confirmed that this graph would be the derivative graph of the given function. The second graph is given in the Figure 4.54. He didn’t confirmed that the second given graph would not be the derivative graph of the given function.

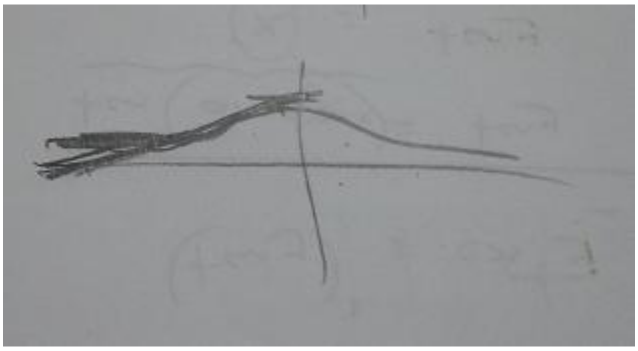


Figure 4.54 Second graph that instructor asked whether this would be the answer of the tenth question

He thought that this graph wasnot the graph of the derivative function as the slope of the lines sketched tangent to the graph increases and then decreases from left to the point of the x value zero and become zero at that point. Therefore, this graph would not be the graph of the derivative function of the graph given in the question.

He also saidthat the slope of the line sketched tangent to the graph at the point of x value 0 is 0. Therefore, the derivative of the function at that point should also be zero.

Meral's answer to the tenth question on derivative function graph

Meral didn't answer the tenth question in both pre-test and post-test. In the pre-test she didn't write anything on the paper. However, in the post-test she analyzed the interval where the function and the derivative of it was positive or negative given in Figure 4.55.

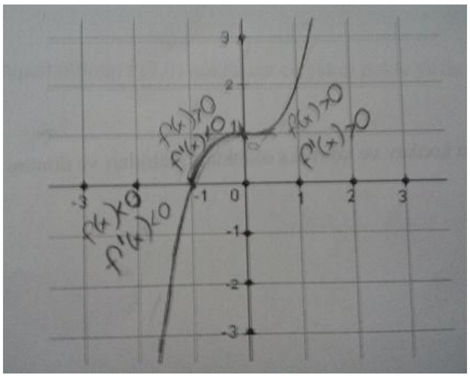


Figure 4.55 Meral analyzed the function is increasing or decreasing

She indicated that for the interval $(-\infty,-1), f(x) < 0, f'(x) < 0$ and in the interval $(-1,0), f(x) > 0,\ f'(x) < 0$ and in the interval $(0,\infty), f(x) > 0, f'(x) > 0$.

Meral used the words “graph of the derivative function”, “increase after zero”, “increasing” and “decreasing” to explain her thought related to tenth question. In the interview, Meral said that in order to find the derivative function of the function f of which graph was given in the question, the intervals that the function was increasing or decreasing should be determined. She added that, so it would be possible to specify where the derivative function was positive and negative. She deduced that the function should be above the x-axis as the function was increasing after the x value 0 and as a result the derivative function was positive. She said that:

Meral: Here it is increasing (shows the first part of the coordinate system), as it is increasing after zero, it is positive. The graph should be above the x-axis.

Instructor asked whether the function was decreasing before the x values zero. She said that it was decreasing before zero and she followed the curve in the third part of the coordinate system. Then she changed her mind and said that the function was also increasing for the interval $(\infty, 0)$. According to this analysis she concluded that the function was increasing for all the values in the domain of the function. Therefore, she decided that the graph of the derivative function should be above the x-axis.

She stated that the derivative value would be 0 at the point (0, 1) if she sketched a tangent line to the curve at that point. According to her analysis she concluded that the graph of the derivative function should be the graph of the function x^2 where it was positive for every value of the domain of the function and she sketched the graph given in the Figure 4.56. This graph is the visual mediator that Meral used in her explanations.

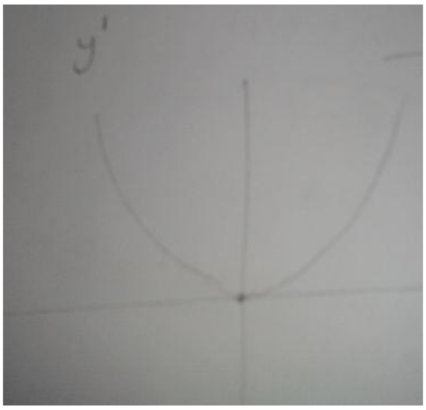


Figure 4.56 Derivative function graph that Meral sketched in the interview

She also used the narrative “I know where the graph is positive and negative according to the increasing and decreasing intervals of the function” to explain how she decided that this graph is the derivative function graph.

Suzan’s answer to the tenth question on derivative function graph

Suzan sketched a correct graph as an answer to tenth question. She checked for derivative of the function if it was positive or negative. Therefore, she checked for the slopes of the lines tangent to the graph of the given function. Then she decided that the slope of the lines tangent everywhere to the function graph has acute angle then the slope waspositive. Therefore, the derivative function was positive everywhere. Moreover, the derivative of the function gets zero value at the x value 0. Therefore, she sketched the graph of the derivative function graph given in the Figure 4.57. Her answer of the sketched graph is the visual mediator that she used in her answer to the question.

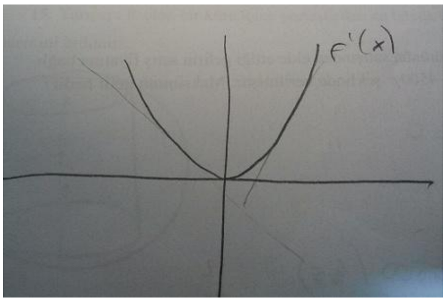


Figure 4.57 Suzan's answer to tenth question on derivative function graph

The instructor asked if the graph would be in the following form. She insisted that the graph of the derivative function should pass through the point (0, 0) as the slope of the line tangent to the graph of the function at the x value 0 is zero. Therefore, she decided that the graph couldn't be the function given by the instructor.

They also talked about the concavity of the graph and the relation between the second derivative of the function and the concavity of the graph of this function. She used the words "slopes", "acute angle", "positive tangent" and "slope is zero" to explain her answer to the tenth question. Suzan remembered the relation between the second derivative of the function and the concavity of the graph of the function as if the second derivative was positive then the graph of the function was convex. If the second derivative was negative then the graph of the function was concave. But she could not form the relation on her own and could not use this relation while sketching the derivative graph of the function. She sketched tangent lines on the function graph given in the tenth question given in the Figure 4.58.

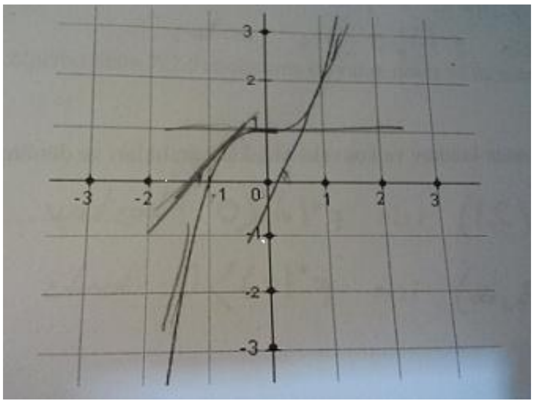


Figure 4.58 Suzan sketched the tangent lines to decide increasing and decreasing intervals of the given function

She also used narratives to explain the relation between the slope of the lines tangent to the given graph and the sign and the concavity of the derivative function. These narratives are "for the acute angle the tangent value is positive", "if the second derivative of the function is positive then the graph looks up, if the second derivative of the function is negative then the graph looks down."

4.4 Summary of the Results

4.4.1 Summary of the Results of Group Discourse

When whole discussion was considered related to the rate of change, it was seen that there was a development related to the pre-service teachers' perception of the rate of change. At the beginning of discussion on the rate of change, they used the words "weight over week" which only represented the units, not the change in weight or time. Throughout the discussion, they started to consider the change in time as it was understood from the words "something lost in one week." In these words, the change in time came into consideration. Then they found the rate of change algebraically as $\frac{8}{5}$, in which 8 represented the change in weight and 5 represented the change in time (weeks). Through the end of the group work, they also realized the change in weight and came to a conclusion that this rate meant "lost weight in five weeks period". These words implied both change in weight and time and also the rate of change. At the end of the discussion they concluded that this rate means the average weight that lost in five weeks period. They associated this rate of change with average rate of change.

Although pre-service teachers thought that instantaneous velocity and average velocity were different things and they should find the slope of the tangent line at the intended point, they could not find any way to find the slope of the tangent line rather than finding the average rate of change. They studied instantaneous rate of change with the instantaneous velocity of the ball at t=1 s. They did not decide how to find the instantaneous velocity of the ball at t=1 s. before using the graph of the function representing the motion of the ball. When they started to study on the graph, they thought that they should find the instantaneous velocity by finding the slope of the line tangent to the curve which represented the motion of the ball at the point (1, 27) and they sketched this tangent line. However, they could not find a way that is different from finding the average velocity.

They represented instantaneous rate of change as the limit of the function while approaching to the x value a as in the symbolic notation $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$. They wrote that "the slope of the tangent line at x=a gives us the instantaneous rate of change". However, when they were asked to find the derivative of the function by using the given function values for some points they thought that they should use the formal definition of derivative in the symbolic notation $\mathrm{f}'(\mathrm{x}) = \lim_{h \to 0} \frac{(x+h)^2-x^2}{h}$. But they could not find the derivative function using the given values.

Words used on rate of change were categorized as rate of change, average rate of change, slope, instantaneous rate of change and limit. According to the process-object duality the words used in group discourse were mostly objectified in the categories of rate of change, average rate of change, slope, and instantaneous rate of change. The operational words were mostly related limit. Pre-servce teachers used operational words while referring limit.

Words used by pre-service teachers in group discourse on increasing and decreasing were grouped in five categories. These categories were "derivative", “function”, “interval”, “graph”, and “slope”. The words representing the graph property such as “increasing” and “decreasing” were operational words. “Increasing there”, “curve positive and decreasing everywhere”, “derivative is negative everywhere” and “after four” were colloquially used words. The others were objectified.

Visual mediators they used were grouped in three categories such as graph, algebraic symbols and written words. In group words, visual mediators were used to express and develop ideas.

There were differences between the use of words and the written words. The written words were the results of pre-service teachers’ thought process and their conclutions related to the mathematical notions of the group work. Therefore, they used more formal words to express these ideas on the worksheets. However, in their word use they felt comfortable and did not think on the words they use, so they did not choose the words carefully. When they used these words, they discussed on the questions or studied on the visual mediators and developed ideas related to the mathematical notions. Sometimes, they tried to remember the relations or rules. They also tried to refute or understand the group members’ ideas.

Pre-service teachers narratives related to rate of change were mostly object level as they explained mathematical objects such as slope, average velocity, rate of change, instantaneous velocity, and limit. There was also one meta-level narratives related to finding the limit of slope of the tangent l. All the narratives pre-service teachers used on increasing and decreasing functions were object level narratives. They were used to define the relation between increasing and decreasing function and first derivative.

In the group discourse routines prompts of the discourse were mostly the questions asked on the worksheets. They started the discussions on the related mathematical notions. Pre-service teachers used visual mediators, especially graphs to understand the relation, to develop thought and express ideas while working on the question of worksheets. Most of the times, they wrote narratives conclude discussion or answer questions.

4.4.2 Summary of the Results of Classroom Discourse

Pre-service teachers had conflicts on some mathematical notions related to instantaneous rate of change. One of these conflicts was if the average velocity and instantaneous velocity were same or not. The instructor and the pre-service teachers discussed on the instantaneous velocity of the ball, instantaneous rate of change of a function, and the relation between instantaneous rate of change and derivative. At the beginning of the discussion, a few pre-service teachers considered average velocity and instantaneous velocity were same. However, five other pre-service teachers refused this idea as reasoning that average velocity was defined in an interval. In this interval, the velocity changed continuously, therefore it was not possible to find the velocity at a point by using average velocity. The second conflict was on how to find the instantaneous velocity. They agreed that the instantaneous velocity of the ball was equal to the slope of the line tangent to the curve at the intended point. However, they had different ideas on how to find the slope of this tangent line. One of them suggested finding the slope as dividing the position of the ball at the intended point by time (1. second). One of them proposed to find the algebraic expression for the function of the graph. Another one said that they would find the algebraic expression for the parabola. The third conflict was finding the limit of the average velocities. They thought that they found the limit of the derivative, the slopes, velocity time graph.

According to these discussions, there were some situations where pre-service teachers had some common usage such as they use “the slope of the function” and most of them understood what they meant. They explained this expression as the slope of the tangent meant that slope of this function. There were some implicit expressions in pre-service explorations. For example one of them used the expression “slope between two points”. It was not clear that this slope was related to a line segment, a tangent line or a secant line.

Classroom discussion was sometimes directed by the instructor and sometimes by the pre-service teachers. In some cases, the instructor asked a question about the notion and they started a new discussion. In some other cases, pre-service teachers asked a question or used an expression, and then they discussed the answer or what this expression meant.

Pre-service teachers had some problems and difficulties to make the connections between properties of the function, its first derivative and second derivative. They sometimes confused the relations and considered them in reverse way. They had tendency to employ the rules that they know from high school and the algebraic expression of the function rather than finding or using the relations.

Pre-service teachers had problems related to the increasing and decreasing functions. They could not distinguish whether the function was increasing or decreasing. Instructor explained increasing and decreasing functions on the graph several times. She examined whether the y values increase or decrease while x values were increasing and decreasing.

Pre-service teachers also had problems related to the increasing or decreasing of the derivative function although they did not have information relevant to derivative function was increasing or decreasing. They knew that the derivative function was positive for all $x \in R$.

Pre-service teachers had difficulty to analyze the derivative graph and relate it to the maximum and minimum points of the graph of second derivative function and they considered the relation increasing and decreasing function and the sign of the function in reverse way for the first and second derivative functions. They stated that if the first derivative was positive then the second derivative function was increasing and if the first derivative was negative then the second derivative was decreasing.

Categories in word used related to mathematical notions were rate of change, average rate of change, slope, and instantaneous rate of change, limit and derivative. According to the process-object duality, the words used in group discourse were mostly objectified. The operational words were related to the mathematical "notion of limit.

The pre-service teachers' and the instructor's used words in the classroom discourse related to increasing and decreasing functions were grouped into four categorization. These categories were slope, derivative, function and interval.

Instructor used the words related to function to express the properties of a function related to the first and second derivative function.

There were some similarities and differences between the words instructor and pre-service teachers used in the classroom discourse on increasing and decreasing function. In the function category and second derivative category, instructor used the "second derivative function" expression. But pre-service teachers did not use. On the other hand, pre-service teachers used "the function looks upward", but the instructor did not use this expression. Moreover, pre-service teachers did not use the expression "first derivative function" in the first derivative while the instructor used it. It implies that pre-service teachers avoided using "first derivative function" and "second derivative function" expressions.

Pre-service teachers found the increasing and decreasing intervals of the function by using positive and negative values of slope of tangent lines. They inferred first derivative a few times. Therefore, they used only five words in the "first derivative" subcategory of derivative category. On the other hand, instructor emphasized the relation between increasing and decreasing function and sign of the derivative of the function. Thus, instructor used much more words connected to first derivative than pre-service teachers.

In classroom discourse instructor used visual mediators to answer and explain questions asked in the worksheets. Visual mediators they used to express rate of change were grouped in three categories such as graph, algebraic symbols and written words. Instructor used graphs to explain the average velocity, average rate of change, instantantous velocity and instantaneous rate of change. She sketched increasing and convex graphs in her explanations.

Instructor used three types of visual mediators in the classroom discourse on increasing and decreasing functions: graphs, tables, algebraic symbols and written words. Instructor sketched graphs to show the relations between increasing and decreasing functions and the first derivative and, the first derivative and the second derivative.

They used endorsed narratives to explain the relation between rate of change and slope of a line segment, instantaneous rate of change and slope of the tangent line to a graph at any point. Pre-service teachers and the instructor used both object-level and meta-level endorsed narratives. Pre-service teachers object level narratives were on "the slope of the line segment and rate of change" and "instantaneous rate of change and slope of the tangent line". Meta level narratives were on "how to find the instantaneous rate of change". Instructor used object-level narrative to explain "relation between limit of the slope of secant lines, slope of the tangent line and derivative." She used meta-level narrative to explain "how to find derivative at a point.

Most of the narratives both for instructor and the pre-service teachers were object level. Instructor used three meta-level narratives how to find the derivative function, relation between the first derivative and slope, the relation between the derivative function and increasing-decreasing function. Pre-service teacher's used one meta-level narrative related to how to find the second derivative by using the slope of the tangent lines.

In routine, the instructor started the discussion on instantaneous rate of change of a function or increasing and decreasing functions by asking the questions given on the worksheets. They altogether discussed these questions. Then the instructor sketched graphs and explained these mathematical notions and relations on these graphs. Then they came to a conclusion. The instructor closed the discussion by object level or meta-level narratives; giving definition, rule or any relation between mathematical objects.

4.4.3 Summary of the Results of Individual Discourse

Pre-service mathematics teachers'interview results of the individual discourse showed that, they gave varying answers to the question of "what was derivative?". Words that pre-service teachers used when describing and explaining derivative, consisted of the words related to the notions of slope as both ratio and limit, difference quotient as limit, rate as function and limit.

Pre-service teachers' operational word use was observed if they perceive derivative as the limit of slopes, the limit of the difference quotient or limit of rate of change. They regarded the limit notion in their definitions and explanations as a process rather than a number or value.

Pre-service teachers objectified word use emerges if they perceive derivative as a mathematical object rather than a process such as the slope of the tangent line, a ratio or they accepted rate of change as a function. Yasin, Sezen, Semra and Suzan's words were mostly objectified as Sezen, Semra and Suzan perceived derivative as the slope of the tangent line and Yasin perceived rate of change as a function.

Pre-service teachers used symbolic notation and graph to explain the definitions that they stated in the post-test and how they perceived derivative. Sezen, Semra, Yakup and Meral used both a graph and symbolic notation to explain their perceptions. On the other hand, Yasin and Suzan only used graph. There was a tendency between these pre-service teachers that most of them sketched an increasing graph. Sezen and Semra sketched a polynomial graph.

Semra and Yakup used the symbolic notation $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ as the definition of derivative. Sezen used the symbolic notation $\frac{f(x_1)-f(x_0)}{x_1-x_0}$ to find the slope of the line segment passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$. She also used the symbolic notation $tan \propto = f'(x_0)$ which gives at the same time the slope of the tangent line and this tangent value is equal to the derivative value at the point of $(x_0, f(x_0))$. Meral used the notation $\frac{f(2)-f(1)}{2-1}$ to find the slope of the line passing through the points of $(1, f(1))$ and $(2, f(2))$.

Pre-service teachers' narratives used in the individual discourse were both object level and meta-level. They used object level narratives to define and explain derivative by using objectified words to state it. They used meta-level narratives defining how derivative at a point was found and used operational words to state it.

CHAPTER V

CONCLUSION, DISCUSSION, IMPLICATION, AND RECOMMENDATION

The purposes of this study were to investigate pre-service elementary mathematics teachers' discourse on derivative in group, classroom and individual discussions from communicational approach to cognition (commonition) perspective and their conception of the derivative concept. In these purposes, this study also aims to answer the following research questions:

How do pre-service elementary mathematics teachers explain the concept of derivative in group, classroom and individual discourses from commognition perspective?

a) How do pre-service elementary mathematics teachers explain the concept of derivative in group discourse from commognition perspective?
b) How do pre-service elementary mathematics teachers explain the concept of derivative in classroom discourse from commognition perspective?
c) How do pre-service elementary mathematics teachers explain the concept of derivative in individual discourse from commognition perspective?

According to these purposes, this chapter deals with the discussion and the conclusion of the results, educational implications, recommendations for future research studies and the limitations of the research study.

5.1 Explanations of Pre-service Teachers on the Concept of Derivative in Group Discourse

Taking group discussions into consideration, it could be seen that there was an improvement in pre-service teachers' perception of rate of change. At the beginning of the group discussion on rate of change, pre-service teachers used the words “weight over week” which represented any change in the quantities. These words only indicated the units. Then throughout the discussion, pre-service teachers used the words “something lost in one week” which signifies the change only in time. At last they used the words “lost weight in five weeks period” implying both change in weight and time. This usage shows the progress to an image of the covariation of two quantities. However, this improvement indicated different progress from Thompson’s (1994) explanation. Thompson (1994) explains the development of images of rate as starting with children’s image of change in some quantity for example displacement of position or increase in volume. Second step in development of this image is the progress of images of two quantities such as displacement of position and duration of displacement. Third step is the progress to an image of the covarition of two quantities. At last, covariation of these two quantities remains in constant ratio. And also the development of mature images of rate requires a schematic coordination of relationships among accumulations of two quantities and accruals by which the accumulations are constructed as it is seen in the case of constant speed, the total distance travelled in relation to the duration and accruals of time. So the accrual of distance in relation to the accrual of time is the same at any time during the trip the total distance travelled at that moment in relation to the total time of the trip. This development also supports suggestions of commognition framework such that understanding occurs and grows by the coordination between the individual and the others in the community (Sfard, 2001). As a result of this coordination, community affects the change in the learner’s activities.

Furthermore, for the development of the image of rate, Thompson (1994) adds that there is a need for further abstraction in covaration of two non-temporal quantities such as volume and surface area and the notion of average rate of change of some quantity over some range of an independent quantity. According to the analysis of the group discussions, members of the observed group defined average rate of change properly. They thought that average rate of change and instantaneous rate of change were different things and they should find the slope of the tangent line at the intended point to find the instantaneous rate of change. They could not find the slope of the tangent line rather then finding the average rate of change. They also could not find any way to find the instantaneous velocity of the ball before analyzing the graph of the function while they were studying on the graph representing the motion of the ball. While they were scanning the graph, they realized that they should find the slope of the line tangent to the curve at the intended point. However, they could not find any way to find the slope of this tangent line rather than finding the average velocity of the ball. The analysis of the classroom discussions presents similar results. That most of the pre-service teachers taking this course have the same conflicts related to instantaneous rate of change. Most of them tended to construct the relation between the average rate of change and instantaneous rate of change as members of the observed group. They claimed that they should find the instantaneous rate of change same as they find the rate of change. These findings coincide with the findings of the research studies on students' understanding of the derivative concepts. The results of these studies revealed that students have little "intuitive" understanding of the derivative and have fundamental misconceptions (Ferrini-Mundy & Graham, 1991). Orton (1983) found that students had difficulty related to the tangent as the limit of a set of secants and to the ideas of rate of change of a straight line versus rate of change of a curve and rate of change at a point versus rate of change over an interval in his study related to students' misconceptions on derivative concept.

Analysis of the group discussions on increasing and decreasing functions showed that some pre-service teachers had problems related to verifying whether the given function was increasing or decreasing. One of the pre-service teachers of the observed group confused whether the function $f(x) = x^2$ was increasing or decreasing. She decided that $f(x) = x^2$ is increasing for all values of x. She concluded that the function values increases for all positive and negative values. Moreover, in the group discussions, members of the observed group had tendency to decide whether any given function was increasing or decreasing by checking the positivity of the first derivative function values rather than analyzing the function values for increasing or decreasing x values or the curve is increasing or decreasing.

Pre-service teachers defined derivative in different forms. In group discourse, they defined derivative as a slope, their word use was mostly objectified.

5.2 Explanations of the Pre-service Teachers' on the Concept of Derivative in Classroom Discourse

Analysis of the classroom discussions put forward that pre-service teachers applied different ways to find the instantaneous rate of change. There were two suggestions how to find the instantaneous rate of change. The first one was to divide the position of the ball to the time at which the instantaneous velocity was asked. The second suggestion was to find the algebraic expression of the graph representing the motion of the ball and then to find the derivative value at the intended point using the differentiation rules.

According to the classroom discussions, some pre-service teachers had conflicts with which notion they should take the limit value to find the limit of the average velocities. They thought that they should find the limit of the "derivative", "slopes" and "velocity-time graph."

Understanding the role of derivative in comprehending the properties of a function has been investigated in different studies. In Ferrini-Mundy and Graham's study (1994) many students tried to find an algebraic expression to sketch the derivative graph of a function given only graphically. Thompson (1994) and Berry and Nyman (2000) also found that students had difficulty in conceptualizing the derivative as a function. Research has shown that students had difficulties in working with the properties of second derivative (Baker, Cooley & Triguros, 2000). Some ignored the second derivative or some would confuse the first derivative graphical implications with the properties of second derivative and also they were unable to coordinate the first derivative and second derivative conditions cross intervals. Most students in this study showed little understanding of the relationship between the first and second derivatives and so they made few comments about this relationship. Researches revealed that most students had an algebraic symbolic view of calculus. In classroom discussions of this study, some pre-service teachers confused these relations between the function properties and the first derivative and also the relations between the increasing and decreasing of the first derivative function and the positivity of the second derivative function. Sometimes, they thought these relations in a reverse way such as "if the first derivative was positive then the second derivative function was increasing and if the first derivative was negative then the second derivative was decreasing."

According to the analysis of the classroom discussions, some pre-service teachers tended to employ rules in terms of function properties such as increasing and decreasing functions and the positivity of the first derivative rather than finding and using these relations. As pre-service teachers studied the derivative concept in high school, they got images get familiar to this notion. For example, some of the pre-service teachers defined derivative as the differentiation rule, and some others as the slope of the tangent line. The discourse they had before their university education affected improvement of their discourse on derivative. They mostly referred to former discourse while studying in group or in classroom. For example, they remembered the rules of "if the derivative was greater than zero for an interval, the function was increasing" or "if the derivative was less than zero, the function was decreasing." It was also observed that they remember certain rules from their former discourse. They also tried to remember the explanations rather than discovering them.

Analysis of the classroom discussions also revealed similar problems related to increasing and decreasing functions. Some pre-service teachers also had problems to determine whether the function was increasing or decreasing and also they confused the relations between the increasing and decreasing of the function and the positivity of the first derivative.

In classroom discourse they defined derivative as slope of the tangent lines, limit of the difference quotient, instantaneous rate of change of function f and limit of the slope of the secant lines. However, they mostly elaborated the limit notion of derivative, therefore they mostly used operational words while defining derivative.

In the classroom discourse, as being the researcher, the instructor of the course, was the one side of the communication developed in the classroom. It was realized that the researcher used more formal language when she concluded the discussion about any topic and expressed the definition or the certain rule of something. Actually, she developed more literate discourse in those situations. On the other hand, while they were discussing any topic, she developed colloquial discourse. The reason of developing colloquial discourse was her demand to express these mathematical notions in any way that pre-service teachers understand. In those cases, it was noticed that the researcher usually used the words or expressions that they used in their utterances or questions. Therefore, she tried to make the conversation they developed to be at the same level and understand the same things from these expressions.

5.3 Explanations of the Pre-service Teachers on the Concept of Derivative in Individual Discourse

In this study, one students' confusion related to average rate of change and average mean was also determined according to the individual discussions. In the individual discussion, he tried to find the average rate of change by using the average mean. Although, he defined derivative as the limit of the difference quotient and explained it as the limit of the average rate of the change of function values, he confused the average rate of change and average mean. Similar result was seen in Bezuidenhout's (1998) study. Bezuidenhout (1998) identified students' deficiencies related to the concept images of the graphical representation of the rate of change. Students had confusion with the average rate of change and arithmetic mean.

In the individual discourse, words pre-service teachers used while describing and explaining derivative consisted of the words related to the notion of slope as both ratio and limit, difference quotient as limit, rate as function and limit. Pre-service teachers' word use was mostly operational when they perceived derivative as the limit of the slope of the tangent lines or the slope of the difference quotients. They mostly used the words "approach", "approach from right and from left", "approaching to one point", "h goes to zero". Their word use was mostly objectified as they perceived derivative as slope and ratio. They used the words "slope of the tangent", "slope of the line", "average rate of change" to define derivative.

In the individual discourse pre-service teachers used graphs to explain their answers to the questions. They used these graphs to elaborate their explanations. Also in group discourse, they used graphs to understand the questions and answer them. They used the graphs to show the relations between the notions of derivative; instantaneous rate of change and limit of slopes;

In the individual discourse pre-service teachers were used graph and symbolic notation while defining or explaining derivative. They absolutely used the graph as visual mediator. They mostly sketched an increasing or polynomial graph. It also corresponded to the results of the studies that "the graph of the function x^2" was the prototype of the visual representations (Habre & Abboud, 2006). It was mostly preferred graphical expression. This would be the result of using increasing function graphs in the group and classroom discussions. Another reason would be using the graph of x^2 mostly in the worksheet questions. Pre-service teachers also used symbolic notations and algebraic symbols as visual mediator. They mostly used the difference quotient in symbolic form or algebraic form such as $\frac{f(x+h)-f(x)}{h}$, $\frac{f(2)-f(1)}{2-1}$ and $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$.

In Habre and Abboud's (2006) study interviewees having complete understanding of the derivative concept geometrically as the idea of instantaneous rate of change. Unlike Habre and Abboud's study, this study showed that pre-service teachers had deficiencies related to these notions. They had problems with choosing the lines to find the limit of slopes to determine the instantaneous rate of change of a function. They used narratives such as "limit of the tangent" or "limit of the tangent lines". From these expressions it could be inferred that they found the limit of the tangent lines at close points to the intended point rather than the limit of the secant lines. Moreover, the narrative "the limit of the average velocity gives the instantaneous rate of change" referred that pre-service teachers did not consider that the independent variable took several values while approaching the intended point. Therefore, there were lots of average velocity values.

Using multiple representations and making connections between these representations increase the students' understanding of the concept. However, students have difficulties in moving comfortably among the different representational modes as in symbolic equations, tables of values and graphs for the derivative concept (Amoah & Laridon, 2004). Also in this study for some pre-service teachers' transition from one representation to another caused some problems. For example, they were asked to express the instantaneous rate of change of a function in any form and according to the given x and corresponding y values in a table, to find instantaneous velocity of an object at any second according to the graph of its movement. They were wanted to find similar things using different representations. They had difficulty to use the table and graph forms rather than the algebraic one.

In the individual discourse pre-service teachers used graphs to explain their answers to the questions. They used these graphs to elaborate their explanations. Also in group discourse, they used graphs to understand the questions and answer them. Results of individual discussions and the graphical representations revealed that some pre-service teachers had some conflicts related to the concept of derivative. It was understood from their explanations of their answers to the questions. For example some of them defined derivative as the limit of the slopes of the lines sketched close to the point where the derivative value was found. However, while they were explaning their answers on the graph, it was seen that although their words directing us to the limit of the slope of the secant lines, it was seen that they were talking about the slope of the lines sketched tangent to the graph. Therefore, this result reveals the importance of interaction and assessing one's performance using different discursive characteristics such as the use of words, mediators, endorsed narratives and routines. Besides, results of the individual discourse revelaed that some pre-service teachers had some deficiencies to manage mathematical self-communicaton which would be the result of not possessing enough mathematical discourse (Sfard, 2008).

There were also some expressions that pre-service teachers used in the group and classroom discourse. They were implicit expressions that could not give the meanings of the mathematical expressions: "the slope between the points a and $a+h$","limit of the derivative", "slope of the function", "slope of the curve", "limit of the tangents".

In the group and classroom discourses, graphs, symbolic notations and written words were used as visual mediators. The graphs that were given in the worksheet were the most used graphs. Other than these graphs pre-service teachers and the instructor used mostly increasing function graphs in the classroom discussions if there was no specification. Moreover, in the worksheets in most questions graph of the function x^2 was used. Written words were mostly endorsed narratives. The symbolic notations were the ones that represent the definition of derivative as limit of the difference quotient and the algebraic expressions used to find the average rate of change of functions or average velocities.

Pre-service teachers used both meta-level and object level endorsed narratives in their group discourse, classroom discourse and individual discourse. They used meta-level endorsed narratives especially in explaining the limit notion of derivative.

5.4 Implications

This study provides information about implementing useful applications for mathematics teacher education. According to the findings of this research and the review of the literature, educational and pedagogical suggestions can be presented.

Recent study supported the findings in the literature that learners had difficulty related to the tangents as the limit of the set of secants (Ferrini-Mundy & Graham, 1991; Orton, 1983). Recent study showed that pre-service teachers had difficulty related to the instantaneous rate of change. Therefore, calculus instructors and mathematics teacher educators should emphasize the definition of the derivative concept more. And also, Turkish Secondary School Mathematics curriculum covers the concepts related to derivative, secondary school mathematics teachers should also pay more attention to the definition and the meaning of the concepts.

According to the results of this study, pre-service teachers had difficulties in the transition from one form of the representation to another one such as from graphical form to algebraic form or vice versa. This result coincides with the findings of the literature that learners of the derivative suffered from reading the graphs and commenting on these graphs and finding the derivative value without using the algebraic expression of the function (Amoah & Laridon, 2004). Therefore, mathematics instructors should pay more attention to the multiple representation of the concept of derivative. They would use different representations related to the mathematical concepts and emphasize the transition from one representation to other one.

Research showed that pre-service teachers had problems to understand the role of first and second derivative in understanding the properties of a function. They could not manage the transition of the relations between the function and the first derivative function and to the relation between the first derivative function to the second derivative function which was also seen in the studies of Ferrini-Mundy and Graham (1994), Baker, Cooley and Triguros (2000), Berry and Nyman (2000) and Thompson (1994). Pre-service teachers had also tendency to depend on rules related to the relation between the function properties and the first derivative function. Therefore, calculus instructors should give more emphasis to the meanings of the rules and let the learners to discover these rules.

As pre-service teachers developed their perception of the rate of change in the group discussions, learners should be enabled to study in the groups so as to join discussions related to mathematical subjects. With this chance, the learners would find the opportunity to develop ideas and express them to their friends. Classroom discussions would be both beneficial and crucial for the pre-service teachers and the learners to develop ideas and see their problems in terms of the mathematical subjects in general. Classroom discussions would also provide the instructors to determine the learners' deficiecncies, problems and thought processes. Therefore, group and classroom discussions should be part of instuctional process.

Analysis of the words, visual mediators, endorsed narratives and the routines of the pre-service teachers in group, classroom and individual discussions revealed that there were differences between what pre-service teachers said and what they actually meant. There were differences between pre-service teachers said in their used words and narratives and how they explained them using their visual mediators. Therefore, calculus instructors and mathematics educators should consider the learners' words, narratives, visual mediators and routines to determine what they say and what they want to say.

This study had also contributions for the teaching and learning applications. First of all, the participants could have the chance of experiencing group and classroom discussions and seeing how ideas were developed in these settings. They could also determine their deficiencies in derivative concept and expressing their ideas. Besides, thinking that the current perspective of each depertment in the universities effected the conception of derivative, this study clarified us the conception of mathematics teacher candidates.

5.5 Recommendations for the Further Research Studies

This research study aimed at understanding pre-service elementary mathematics teachers' discourse on derivative in group, classroom and individual discussions. Findings of this study revealed useful implications for mathematics educators and calculus instructors. According to these findings some related research studies were suggested.

As the results of this study revealed that group, classroom and individual discussions developed pre-service teachers' discourse on derivative concept, other studies searching learners' group, classroom and individual discourse on other mathematical concepts would also beneficial in group and classroom settings.

The focus of this research was pre-service elementary mathematics teachers. Further studies related to discourse on derivative and other mathematical concepts of pre-service secondary mathematics teachers and students from other majors would also provide mathematics educators and calculus instructors to determine these learners' discourse on these subjects. Moreover, to investigate secondary school students' mathematical discourse on mathematical concepts would provide the calculus instructors and mathematics educators to learn about the future students' mathematical discourses related to these subjects.

In this research study, learners' discourse on derivative concept was examined. Investigating effects of instructors' and other additional materials' such as curriculum and textbooks on students' mathematical discourse would be beneficial to understand learners' mathematical discourse deeply.

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APPENDIX A

PARTICIPANT PERMISSION FORM

Sevgili Öğretmen Adayları,

Yürütmekte olduğum tezimin amacına ulaşabilmesi için sınıf içi katılımlarınızın videoya çekilmesi ve yaptığınız her türlü çalışmanın incelenmesi gerekmektedir. Çekilen video görüntüleri ve yaptığınız her türlü çalışma gizli tutulacak ve sadece araştırmacı tarafından incelenecektir. Yapılan analizlerde isminiz kesinlikle kullanılmayacaktır. Bütün bu uygulamaları kabul ettiğinize dair aşağıdaki formu imzalamanız gerekmektedir. Forma eklemek istediğiniz herhangi bir görüşünüz ya da isteğiniz varsa lütfen aşağıda ayrılan "Not" kısmına yazınız.

Yardımlarınız ve katılımız için teşekkür ederim.

Özge Yiğitcan Nayir

Yürütülmekte olan tezin uygulaması olarak, sınıf içi katılımlarımın videoya kaydedilmesinde, dersle ilgili yaptığım her türlü çalışmanın veri olarak kullanılmasında hiçbir sakınca yoktur.

Tarih:

Adı-Soyad:

İmza

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APPENDIX B

DERIVATIVE TEST SPECIFICATION TABLE

Table A.1
Derivative test specification table

# of questions	Subject	Objective	Question No
1	Definition of derivative	Defines the derivative	1
1	Daily life applications of derivative	Knows the daily life applications of derivative	2
1	Derivative value at one point	Finds the derivative value at one point using the graph of a function	3
		Finds the derivative value at one point using the values of the function	3
5	Derivative of functions	Finds the derivative of power function	4a
		Finds the derivative of exponential functions	4b
		Finds the derivative of rational functions	4c
		Finds the derivative of logarithmic functions	4d
		Finds the derivative of trigonometric functions	4e
1	Points that the derivative of the function not exist	Finds the points which not have derivatives according to the given graphs.	5
2	Increasing and decreasing intervals of a function	Determines the increasing and decreasing intervals of a function using the graph of the function	6a
		Determines if the function is increasing or decreasing at the given point	7a
2	Local minimum and maximum points	Determines the local minimum and local maximum points using the graph of the function	6b
		Finds the local minimum and local maximum points	7b
1	Equation of tangent line	Finds the equation of the tangent line	8
1	Graph of the derivative function	Sketchs the graph of the derivative function of a function whose graph is given	9
2	Graph of functions	Sketchs the graph of the functions whose some qualities are described (function values at some points, derivative values at some points, asymptotes, increasing and decreasing intervals)	10,11
4	Minimum and maximum problems	Solves the minimum and maximum problems	12,13,14,15

APPENDIX C

DERIVATIVE TEST

Sevgili arkadaşlar,

Orta Doğu Teknik Üniversitesi, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümünde yürütmekte olduğum tez çalışmam kapsamında aşağıda verilen soruları cevaplamanız beklenmektedir. Cevaplarınız sadece araştırmacı tarafından incelenecek ve tez çalışması dışında hiçbir yerde kullanılmayacaktır. İsimleriniz kesinlikle tezin hiçbir bölümünde kullanılmayacaktır. Test 15 sorudan oluşmaktadır. Sorular sizin türev konusuyla ilgili bilgi seviyenizi belirlemek amacıyla oluşturulmuştur. Lütfen her bir soruyu cevaplamaya çalışınız.

Yardımlarınız ve işbirliğiniz için teşekkür ederim.

Özge YİĞİTCAN NAYİR

ODTÜ OFMAE

Doktora Öğrencisi

Adınız Soyadınız:

1. Türev nedir? Açıklayınız.

____

____

____

____

2. Günlük hayatta türev nerelerde karşımıza çıkmaktadır? Açıklayınız.

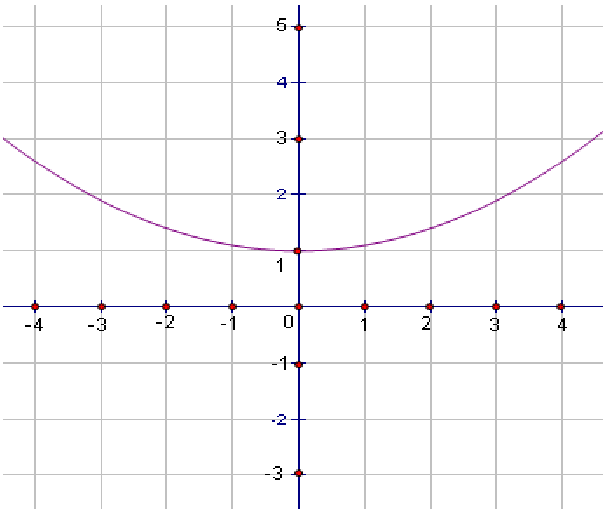
____

____

____

____

3.



x	0	1	2	3
y	1,0	1,1	1,4	1,9

Yukarıda grafiği ve tabloda değerleri verilen f fonksiyonunun x = 2'deki türevini yaklaşık olarak bulunuz.

4. Türevin tanımını kullanarak $f(x) = x + \frac{1}{x}$ fonksiyonunun türevini bulunuz.

5. Aşağıdaki fonksiyonların birinci türevlerini bulunuz.

a) $y = (3x^2 + 1)^3$

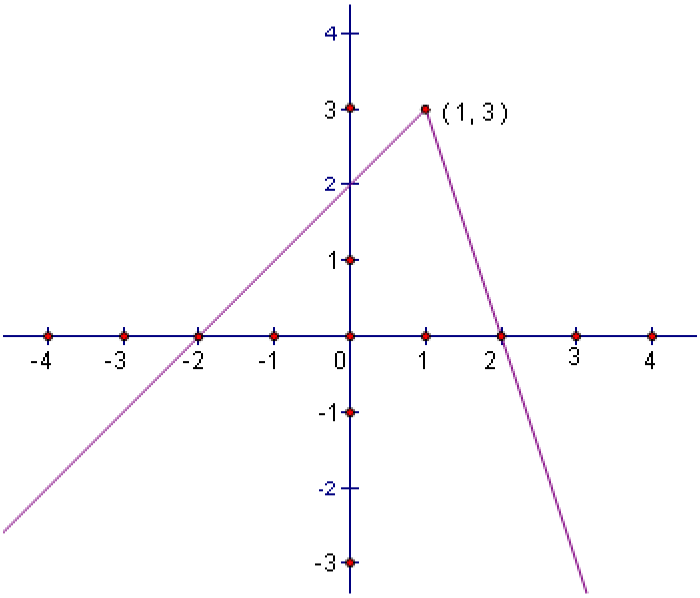
b) $y = xe^{2x}$

c) $y = \dfrac{e^{\sin 3x}}{\cos^2 x}$

d) $y = (\ln(3^x + x^3))^4$

e) $y = \tan^3 x \sec(2x) + \arctan(x^4)$

6. Aşağıda grafiği verilen $y = f(x)$ fonksiyonu için şıklarda verilen değerleri bulunuz.



a) $f(0)$

b) $f(3)$

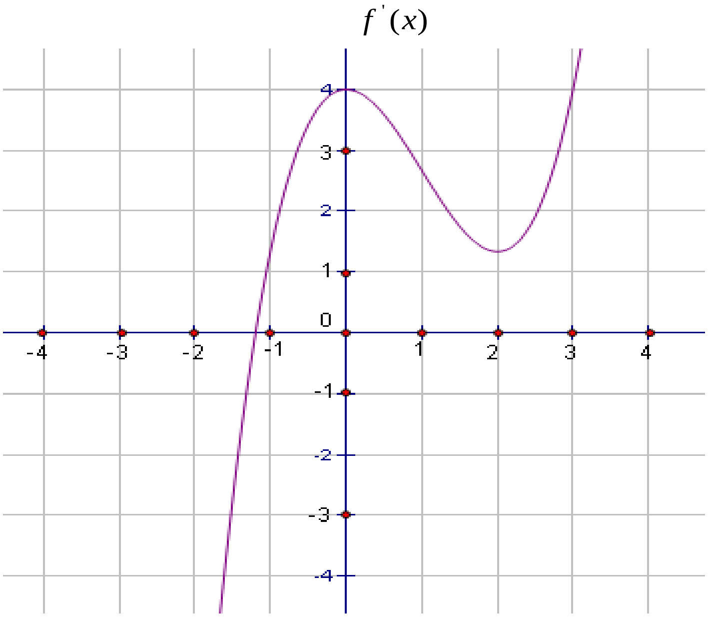
c) $f'(0)$

d) $f'(3)$

e) $\lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

f) $f'(1)$ var mıdır? Eğer varsa değerini bulunuz. Eğer yoksa, neden olmadığını açıklayınız.

7.



a) Yukarıda türev grafiği verilen f(x) fonksiyonunun artan ve azalan oldukları aralıkları bulunuz.

b) Fonksiyonun hangi noktaları yerel maksimum ve yerel minimum noktalarıdır?

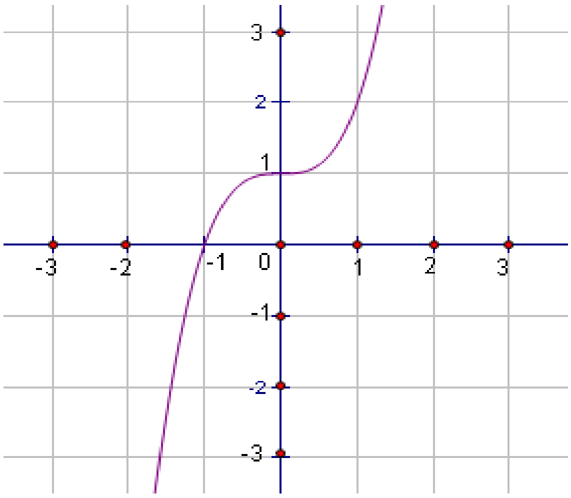
8. $f(x)=\frac{2}{3}x^3-2x^2+4$ fonksiyonunun

a) x = 1 noktasında artan mı yoksa azalan mı olduğunu bulunuz.

a) Yerel minimum ve yerel maksimum noktalarını bulunuz.

9. $y=\frac{x+1}{x-1}$ fonksiyonunun (0, -1) noktasındaki tanjant (teğet) doğrusunun denklemini bulunuz.

10. Aşağıda grafiği verilen f fonksiyonun türevinin grafiğini çiziniz. Cevabınızı açıklayınız.



11. f fonksiyonunun özellikleri aşağıda verilmiştir.

- $f(0)=0$, $f(2)=3$, $f(3)=7$, $f'(0)=0$
- $\lim_{x\to-\infty} f(x)=0$, $\lim_{x\to\infty}(f(x)-2x)=0$, $\lim_{x\to-1^+} f(x)=\infty$, $\lim_{x\to-1^-} f(x)=-\infty$
- $(-\infty,-1)$ ve $(-1, 0)$ için $f'(x)<0$
- $(0, \infty)$ için $f'(x)>0$
- $(-\infty, -1)$ ve $(2, 3)$ için $f''(x)<0$
- $(-1, 2)$ ve $(3, \infty)$ için $f''(x)>0$

Bu özellikleri göz önünde bulundurarak aşağıdaki soruları yanıtlayınız.

a) f(x) fonksiyonunun artan ve azalan olduğu aralıkları bulunuz.

b) Fonksiyonun konkav ve konveks oldukları aralıkları ve dönüm noktalarını bulunuz.

12. Bir şeker fabrikasının, bir ürünün satışından elde ettiği gelirin satış fiyatına bağlı fonksiyonu, $G=-500p^2+4500p$ şeklinde verilmiştir. Maksimum gelir nedir?

13. Belli bir yükseklikten yukarıya doğru atılan topun t sn deki yüksekliğini veren fonksiyon $s(t) = -16t^2 + 128t + 320$ dir.

a) Topun ulaşabileceği maksimum yüksekliği bulunuz.

b) $t = 4sn$ de top hangi hızda ve hangi yönde hareket etmektedir?

14. $y^2 - x^2 = 4$ hiperbolünün P(2,0) noktasına en yakın nokta ya da noktalarını bulunuz.

15. Yarıçapı R olan bir küre içine yerleştirilen en büyük hacimli dik dairesel silindirin hacmini bulunuz.

APPENDIX D

DERIVATIVE TEST SCORING RUBRIC

1. soru

(5) Türevin ne demek olduğunu tam olarak açıkladıysa

(4) Limitten bahsettiyse ama eksikleri varsa

(3) Açıklamada eksiklikler, hatalar var ise (Limit değerinden bahsetmediyse)

(0) Açıklama yanlış ise

(0) Hiçbir açıklama yapmadıysa

2. soru

(5) Doğru ve yeterli örnekler verdiyse

(3) Örnekleri doğru fakat yetersizse

(1) Örnekler yanlışsa

(0) Hiç örnek vermemişse

3. soru

(5) Grafiği ve tabloyu kullanarak, ortalama değişim oranlarını inceleyip, limit değerini bulup, doğru sonuca ulaştıysa

(4) Grafiği ve tabloyu doğru yorumlayıp, ortalama değişim oranlarına bakıp yaklaşık değer bulamadıysa (limit değerine bakmadıysa)

(3) Grafik ve tabloyu kullanıp ortalama değişim oranlarını incelemediyse

(3) Fonksiyonu yazmaya çalışıp hata yaptıysa

(2) Grafiği ve tabloyu doğru yorumladıysa

(1) Sadece grafiği ya da tabloyu doğru incelediyse ama ortalama değişim oranlarına bakmadan sonuca ulaşmaya çalıştıysa

(0) Hiçbir sonuca ulaşmadıysa; hiçbir yorum yapmadıysa

4. soru

(5) Türevin tanımını doğru kullanarak, işlemlerde hata yapmadan doğru türeve ulaştıysa

(4) Türevin tanımını kullanıp, işlemlerde küçük hatalar yaptıysa

(3) Türevin tanımını kullanıp, işlemlerde önemli hatalar yaptıysa

(2) Türevin tanımını doğru yazıp, tanımın uygulamasında hata yaptıysa

(1) Türevin tanımı hatalı ya da eksikse

(0) Hiçbir yorum ya da işlem yapılmamışsa

(0) Türev tanımını kullanmadan sonuca ulaştıysa

5. soru a şıkkı

(5) Üssün türevi ve zincir kuralını doğru bir şekilde uygulayıp, türevini doğru bulduysa

(4) Kuralları doğru uygulayıp, işlem hatası yaptıysa

(3) Kurallardan birini yanlış uyguladıysa

(1) Kuralların ikisini de yanlış uyguladıysa

(0) Hiçbir yorum yapmadıysa

5. soru b şıkkı

(5) Çarpım türevi, üstel (e^x) fonksiyonun türevi ve zincir kuralını doğru bir şekilde uygulayıp, türevi doğru bulduysa

(4) Kuralları doğru uygulayıp işlem hatası yaptıysa

(3) Kurallardan birini yanlış uyguladıysa

(2) Kurallardan ikisini yanlış uyguladıysa

(1) Kuralları yanlış uyguladıysa

(0) Hiçbir yorum yapmadıysa

5. soru c şıkkı

(5) Bölüm türevi, trigonometrik fonksiyon (sinx ve cosx) türevi, üstel fonksiyonun türevi (e^x) ve zincir kuralını doğru uygulayıp, doğru sonuca ulaştıysa

(4) Kuralları doğru uygulayıp işlem hatası yaptıysa

(3) Kurallardan en çok ikisini yanlış uyguladıysa

(2) Kurallardan en çok üçünü yanlış uyguladıysa

(1) Kuralları üçünü yanlış uyguladıysa

(1) Hiçbir yorum yapmadıysa

5. soru d şıkkı

(5) Üssün türevi, logaritma fonksiyon türevi, üstel (e^x) fonksiyonun türevi doğru uygulayıp, doğru sonuca ulaştıysa

(4) Kuralları doğru uygulayıp işlem hatası yaptıysa

(3) Üssün türevi ve üstel (e^x) fonksiyonun türevini yanlış uyguladıysa

(2) Logaritma fonksiyonun türevini yanlış uyguladıysa

(1) Türev kurallarını yanlış uyguladıysa

(0) Hiçbir yorum yapmadıysa

5. soru e şıkkı

(5) Trigonometrik fonksiyonların türevi (tanx ve secx), ters trigonometrik fonksiyonların türevi (arctanx), üssün türevi, çarpım türevi, zincir kuralını doğru bir şekilde uygulayıp ve doğru sonuca ulaştıysa.

(4) Türev alma kurallarını doğru uygulayıp işlem hatası yaptıysa

(3) Üssün türevi ve/veya çarpım türevini ve/veya zincir kuralını yanlış uyguladıysa.

(2) Ters trigonometrik ve/veya trigonometrik fonksiyonun türevlerini yanlış uyguladıysa.

(1) Türev kurallarını yanlış uyguladıysa

(0) Hiçbir yorum yapmadıysa

6. soru a şıkkı

(5) Grafiği doğru yorumlayıp $f(0)$ değerini doğru bulduysa

(1) Grafiği yanlış yorumlayarak yanlış bir değer bulduysa

(0) Hiçbir yorum yapmadıysa

6. soru b şıkkı

(5) Doğrunun eğiminden yararlanarak $f(3)$ değerini doğru bir şekilde bulduysa

(4) Doğrunun eğiminden yararlanıp, işlem hatası yaptıysa

(3) Doğrunun eğimini hatalı bulduysa

(1) Doğrunun eğimini kullanmadan $f(3)$ değerini bulduysa

(0) Hiçbir yorum yapmadıysa

6. soru c şıkkı

(5) Doğrunun eğiminden yararlanarak $f'(3)$ değerini doğru bir şekilde bulduysa

(4) İşlem hatası yaptıysa

(3) Eğimi yanlış uyguladıysa

(2) Eğimle türev arasındaki ilişki kurmadan sonuca ulaşmaya çalıştıysa,

(0) Hiçbir yorum yapmadıysa

6. soru d şıkkı

(5) Doğrunun eğimi ve fonksiyonun bir noktadaki türevinin, fonksiyona o noktadaçizilen teğetin eğimi olduğu ilişkisini kurup $f'(3)$ değerini doğru bir şekilde bulduysa

(4) İşlem hatası yaptıysa

(3) Doğrunun eğimi ve türev arasında yanlış bir ilişki kurduysa

(2) Doğrunun eğimini bulduysa fakat bunun türevle ilişkisini kuramadıysa

(1) Doğrunun eğimini kullanmadan $f'(3)$ değerine ulaşmaya çalıştıysa

(0) Hiçbir yorum yapmadıysa

6. soru e şıkkı

(5) İstenen limit değerinin, fonksiyonun o noktadaki sağdan türevi olduğu bilgisini kullanıp, fonksiyonda (1,(3)) noktasının sağında kalan doğrunun eğiminin bu limit değerine eşit olduğundan yola çıkarak istenen limit değerini doğru bir şekilde bulduysa. (fonksiyonu bulup, doğrunun eğimine ulaştıysa)

(4) Fonksiyonu yanlış bulduysa ya da işlem hatası yaptıysa

(3) Fonksiyonu doğru bulup, eğimi yanlış bulduysa

(2) Fonksiyonu ve eğim değerini bulup ilişkilendirmediyse

(1) Yanlış yorumlayıp yanlış çözüm yaptıysa

(0) Hiçbir yorum yapmadıysa

6. soru f şıkkı

(5) Bir fonksiyonun bir noktada türevinin olması için sağdan ve soldan türev değerlerinin eşit olması gerçeğinden yola çıkarak doğru cevaba ulaşıp, doğru açıklamaları yaptıysa

(5) Birden fazla teğet çizebilir yorumunu yaptıysa

(4) Sağdan ve soldan türevlere bakmak gerektiğini bilip, işlem hatası yaptıysa

(3) Sağdan ve soldan türevleri bulup hiçbir ilişkilendirme yapmadıysa

(2) Sadece bir yönden türevi bulup bunu yeterli kabul ettiyse

(2) "Evet" ya da "Hayır" deyip açıklama yapmadıysa

(1) Sağdan ve soldan türev ve o noktadaki türev arasında yanlış bir ilişki kurduysa

(0) Hiçbir yorum yapmadıysa

7. soru a şıkkı

(5) Fonksiyonun türev grafiği olduğunu anlayıp, grafiği doğru yorumlaması "$f'(x) > 0 \Rightarrow f(x)$ fonksiyonu artandır", "$f'(x) < 0 \Rightarrow f(x)$ fonksiyonu azalandır" yorumunu yapıp doğru aralıklara ulaşması

(4) Fonksiyonun türev grafiği olmasına göre "$f'(x) > 0 \Rightarrow f(x)$ fonksiyonu artandır", "$f'(x) < 0 \Rightarrow f(x)$ fonksiyonu azalandır" yorumunu yapıp uygulamada küçük bir hata yaptıysa

(3) Bütün bu yorumları fonksiyon grafiğinde uyguladıysa

(2) Uygulamada ciddi hatalar yaptıysa

(1) Cevabı belirtip açıklama yapmadıysa

(0) Hiçbir yorum yapmadıysa

7. soru b şıkkı

(5) $f'(x) = 0$ olduğu nokta yerel minimum ya da yerel maksimum noktalarıdır,azalandan artana geçiyorsa yerel minimum noktasıdır, artandan azalana geçiyorsa yerel maksimum noktasıdır yorumlarını yapıp doğru sonuca ulaştıysa

(4) $f'(x) = 0$ olduğu nokta yerel minimum ya da yerel maksimum

noktalarıdır yorumunu yapıp küçük hatalarla noktaları bulduysa.

(3) Noktaları bulup yerel minimum ya da yerel maksimum olup olmadıklarında hata yaptıysa

(2) Noktaları hiçbir açıklama yapmadan bulduysa

(1) Yorumu ve sonuçları yanlışsa

(0) Hiçbir yorum yapmadıysa

8. soru a şıkkı

(5) Bir noktada fonksiyonun; artan olması için o noktadaki türevin pozitif, azalan olması için ise o noktada negatif olması gerektiğinden yola çıkıp, fonksiyonun x=1 noktasındaki türevini bulup, pozitif ya da negatif olmasını inceleyip doğru sonuca ulaştıysa

(4) Yorumlarını ve uygulamalarını doğru yapıp işlem hatası yaptıysa

(3) Yorumlarını doğru yapıp uygulamada hata yaptıysa

(1) Yorumları ve uygulaması hatalıysa

(1) Türevini alıp bıraktıysa

(0) Hiçbir yorum yapmadıysa

8. soru b şıkkı

(5) Yerel minimum ve yerel maksimum noktalarını bulabilmek için fonksiyonun türevinin 0 olduğu noktaları bulmak gerektiği gerçeğinden yola çıkıp, türevin 0 olduğu noktaları bulup, bu noktaları tabloda inceleyip azalandan artana geçiyorsa yerel minimum ya da artandan azalana geçiyorsan yerel maksimum noktası olduğu yorumunu yapıp doğru sonuca ulaştıysa

(5) Türevi 0 yapan noktaları bulup, ikinci türev bu noktalarda pozitifse yerel maksimum, negatifse yerel minimum noktasıdır yorumunu yapıp doğru sonuca ulaşıyorsa

(4) Uygulamada işlem hatası varsa

(3) Noktaların minimum ya da maksimum olmasını doğru yorumlayıp, noktaları bulmada hata yapıyorsa

(2) Noktaları bulup, minimum ya da maksimum olarak yorumlamasında

hata varsa

(1) Noktalar ve yorumlar yanlışsa

(0) Hiçbir yorum yapmadıysa

9. soru

(5) Fonksiyonun herhangi bir noktadaki teğet doğrusunun denklemini bulabilmek için, doğrunun eğimine, fonksiyona teğet olduğu noktaya ihtiyaç olmasından yola çıkıp, doğrunun eğimini de fonksiyonun o noktadaki türevi ile bulup, teğet doğrusunu bulduysa

(4) Yorumları doğruysa, eğimi doğru bulunduysa fakat teğet doğrusunun denklemini bulunmakta işlem hatası yaptıysa

(3) Yorumlar doğru, eğimin ve doğru denkleminin bulunmasında hata varsa

(2) Tanjant doğrusunun bulunmasıyla ilgili yorumda hata varsa

(1) Yorum ve yapılan işlemler hatalıysa

(0) Hiçbir yorum yapmadıysa

10. soru

(5) Verilen grafiği doğru yorumlayıp, grafiği doğru çizip, açıklamalarını doğru yaptıysa

(4) Açıklamalar doğruysa fakat grafiğin çiziminde küçük hatalar yaptıysa,

(3) Grafiği doğru çizip açıklamalarda hatalar yaptıysa,

(2) Grafikte de, açıklamalarda da hatalar yaptıysa

(1) Grafikte de, açıklamalarda da hatalar varsa

(0) Hiçbir yorum yapmadıysa

11. soru a şıkkı

(5) $f'(x) < 0$ ise azalan, $f'(x) > 0$ ise artandır yorumunu yapıp, fonksiyonun verilen özelliklerine göre doğru sonuca ulaştıysa

(3) Artan ya da azalan olduğu aralığı doğru bulmuş ve açıklamada hata yapmışsa

(1) Aralıkları yanlış bulmuş ve açıklamaları da hatalıysa

(0) Hiçbir yorum yapılmadıysa

11. soru b şıkkı

(5) $f''(x) < 0$ ise fonksiyon konkavdır, $f''(x) > 0$ ise fonksiyon konvekstir, $f''(x) = 0$ olduğu noktalar dönüm noktalarıdır açıklamasını yapıp, fonksiyonun verilen özelliklerini kullanıp doğru sonuca ulaştıysa

(4) Aralıklardan ya da noktalardan herhangi birini yanlış bulup ve açıklaması hata yaptıysa

(3) Aralıklarda ya da noktalarda birden fazlası hata yaptıysa

(2) Açıklamaları yanlış yapmışsa

(1) Aralıkları, noktaları ve açıklamaları yanlış yapmışsa

(0) Hiçbir yorum yapmadıysa

12. soru

(5) Hangi satış fiyatında maksimum gelire ulaşıldığını bulmak için verilen fonksiyonun türevi alıp, satış fiyatına ulaşılır, maksimum geliri verip vermeyeceğini inceleyip, geliri bulduysa,

(4) Yorumları doğru yapıp, işlem hatası yaptıysa

(4) Yorumları ve işlemleri doğru fakat maksimum satış fiyatı olup olmadığını incelemediyse

(3) Maksimum satış fiyatını bulmuş, maksimum geliri verip vermediğini incelemiş, ama geliri bulmamışsa

(2) Satış fiyatını bulduysa, geliri bulamadıysa ve satış fiyatının maksimum geliri verip vermediğini incelememişse

(1) Türevini bulup bıraktıysa

(1) Yapılan yorumlar ve işlemler yanlışsa

(0) Hiçbir yorum yapılmadıysa

13. soru a şıkkı

(5) Konum fonksiyonunun türevini 0'a eşitleyip, kaçıncı saniyede maksimum yüksekliğe ulaştığını bulunup, bu zamanın gerçekten maksimum yüksekliği veripvermediğini kontrol edip ve maksimum yüksekliği bulduysa

(4) Maksimum yüksekliğe ulaştığı saniyeyi ve maksimum yüksekliği bulduysa,maksimum yükseklik olduğunu kontrol etmediyse

(3) Maksimum yüksekliğe ulaştığı saniye bulup ve maksimum yüksekliği bulmadıysa

(2) Konum fonksiyonunun türevini almadan, maksimum yüksekliğe ulaştığı saniyeyi bulduysa

(1) Uyguladığı tüm yöntemler hatalıysa

(0) Hiçbir yorum yapılmadıysa

13. soru b şıkkı

(5) t=(4)sn'yi konum fonksiyonunda yerine yerleştirip hangi konumda olduğunu bulup hangi yönde hareket ettiğine karar verdiyse

(4) İşlem hatası yaptıysa

(3) Topun konumunu bulup, yönünü yanlış ya da hiç bulamadıysa

(1) Topun konumunda ve yönünde hata yaptıysa

(0) Hiçbir yorum yapmadıysa

14. soru

(5) İki nokta arasındaki uzaklığı fonksiyon halinde belirtip, bu fonksiyonun türevinin olduğu nokta ya da noktaları bulup, bu noktanın en yakın nokta olduğunu kontrol ettiyse

(4) İşlem hatası yaptıysa

(4) Sadece bir noktayı bulup bıraktıysa

(4) Nokta ya da noktaların en yakın nokta olduğunu kontrol etmediyse

(3) Sadece bir noktayı bulup, bu noktanın minimum uzaklıkta olup olmadığını kontrol etmediyse

(2) Fonksiyonu yanlış ifade ettiyse

(1) Hatalı yöntem kullanarak, en yakın nokta ya da noktaları bulmaya çalıştıysa

(0) Hiçbir yorum yapmadıysa

15. soru

(5) Silindirin hacmini doğru ifade edip, hacim formülünün türevinin 0 olduğu değeri uygulayıp silindirin hacmini doğru bulduysa

(4) Silindirin hacmini yanlış bulduysa

(3) Fonksiyonu yanlış ifade ettiyse

(2) Silindir hacmini veya r,R,L arasındaki ilişkiyi yazdıysa ve silindirin hacminin türevini aldıysa

(1) Hatalı yöntem kullanıp sonuca ulaşmaya çalıştıysa

(1) Silindir hacmini veya r,R,L arasındaki ilişkiyi yazdıysa

(0) Hiçbir yorum yapmadıys

APPENDIX E

INSTRUCTIONAL MATERIALS

1. Application:

Subjects: Average rate of change

Time: 2 lesson (2 x 50 min)

Objectives:

At the end of the lesson students will be able to,

- Interpret the change with respect to time at the given table
- Interpret the change with respect to time at the given graph
- Understand the average rate of change
- Understand the relation between average rate of change and slope of the curve
- Understand the limit of the average rate of change of a function gives the slope of the tangent line at a given point

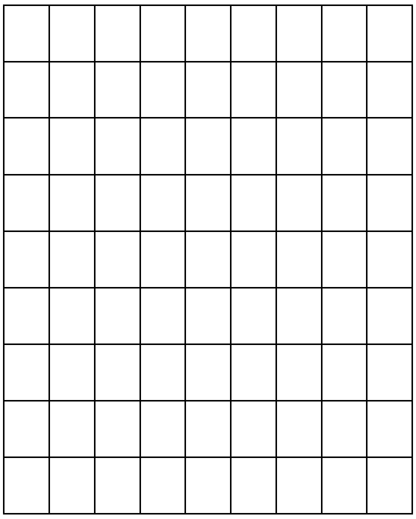
1. Application Worksheet

Bir sağlık ve spor kulübü üyesisiniz. Kulübün diyetisyeni ve spor faaliyetlerinde size yardımcı olan çalıştırıcınız size 8 haftalık bir diyet ve egzersiz programı hazırladılar. Aşağıdaki tablo kilonuzu zamana bağlı olarak değişimini 8 haftalık periyotta göstermektedir.

Zaman ve Kilo Tablosu

Zaman (hafta)	0	1	2	3	4	5	6	7	8
Kilo (kg)	65	63	62	61	60	57	57	60	56

1. **a.** zaman-kilo (z,k) ikilileri şeklinde verileri çiziniz.
   Örnek: (3, 61) ikilisi üçüncü hafta sonundaki kilonuzu belirtir.



   b. Bu fonksiyonun tanım kümesi nedir?
   c. Bu fonksiyonun değer kümesi nedir?
2. **a.** Programın başlangıcında kilonuz nedir?
   b. İlk haftanın sonunda kilonuz nedir?
3. Sağlık programının sizin için faydalı olup olmadığını anlamak için haftalık kilonuzun değişimini sekiz haftalık periyotla inceliyorsunuz.
   a. Hangi haftalar boyunca ağırlığınız artıyor?

b. Hangi haftalar boyunca ağırlığınız azalıyor?

c. Hangi haftalar boyunca ağırlığınız değişmiyor?

4. Kilonuz ilk beş hafta boyunca azalıyor.

a. Beş hafta boyunca kilonuzdaki değişimi bulunuz.

b. Cevabınız pozitif mi yoksa negatif mi? Bu işaretin anlamı nedir?

c. İlk beş haftada z-değerindeki değişimi belirleyin.

d. a şıkkında verilen kilonuzdaki değişimin, c şıkkında verilen zamandaki değişime oranını yazınız. Bu oran ne anlama gelir?

5. **a.** Problem 1 deki grafikte (0, 65) noktasıyla (5, 57) noktasını bir doğru parçasıyla birleştiriniz. Doğru parçasını soldan sağa doğru takip ettiğinizde artıyor mu, azalıyor mu ya da yatay bir şekilde sabit mi kalıyor?

b. İlk beş haftadaki ortalama değişim oranını göz önünde bulundurduğunuzda problem 5a da çizilen doğru parçası size ortalama değişim oranı hakkında ne söyler?

6.a. 5. haftadan 7 haftaya kadar kilonuzda meydana gelen ortalama değişim oranını belirleyiniz. Uygun işareti ve birimi ekleyiniz.

b. Diyetinize göre a şıkkındaki oranı belirtiniz.

c. Problem 1 deki grafikte, (5, 57) ve (7, 60) noktalarını bir doğru parçasıyla birleştiriniz. Soldan sağa doğru takip ettiğinizde doğru parçası artıyor mu, azalıyor mu ya da yatay olarak sabit mi kalıyor?

d. Bu iki haftalık periyotta kilodaki ortalama değişim oranının c şıkkında çizilen doğru parçasıyla ilişkisi nedir?

7.a. 6. haftada kilonuzdaki değişim oranı nedir? (h=5 ten h=6 ya)

b. a şıkındaki değişim oranını diyetinize göre belirtiniz.

c. (5, 57) ve (6, 57) noktalarını bir doğru parçasıyla birleştiriniz. Bu doğru parçası artıyor mu azalıyor mu yoksa yatay bir şekilde sabit mi kalıyor?

d. c şıkkında çizilen doğru parçasının ortalama değişim oranıyla ilişkisi nedir?

8. **a.** 4. haftadan 7. haftaya kilonuzdaki ortalama değişim oranı nedir?

b. a şıkkındaki değişim oranı diyetinizdeki bu üç haftalık gelişmeyi nasıl yansıtır?

2. Application:

Subjects: Average Velocity, Instantaneous Velocity, Average Rate of Change

Time: 2 lesson (2 x 50 min)

Objectives:

At the end of the lesson students will be able to,

- Interpret the change in the velocity with respect to time
- Understand the average velocity between given certain times
- Understand the average rate of change
- Comprehend the instantaneous velocity at a certain time
- Interpret the average velocity and instantaneous velocity at the given height-time graph

2. Application Worksheet

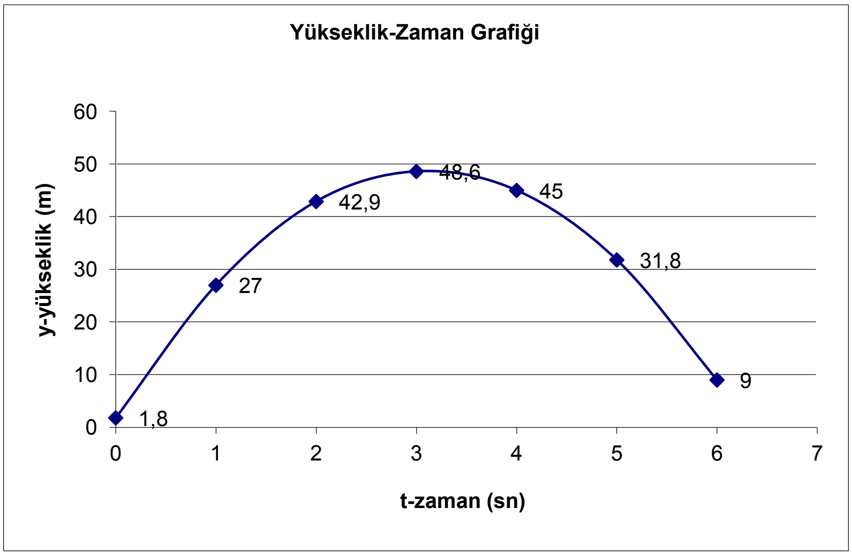
Bir top 1,8 m yükseklikteki bir duvardan yukarıya doğru kuvvetli bir şekilde atılıyor. Aşağıda verilen tablo topun 6 sn boyunca yüksekliğindeki değişimi ifade etmektedir.

t (sn)	0	1	2	3	4	5	6
y (m)	1,8 m	27 m	42,9 m	48,6 m	45 m	31,8 m	9 m

Aşağıda verilen soruları bu tabloya göre cevaplayınız. Cevaplarınızı açıklayınız.

1. Top ilk sn de ne kadar hareket etmektedir?
2. Top 2. sn de ne kadar hareket etmektedir?
3. Top hangi sn de daha hızlı hareket etmektedir?
4. Ortalama hızdan ne anlıyorsunuz?
5. Topun $4<t<5$ zaman aralığındaki ortalama hızı nedir? İşaret bize neyi ifade eder?
6. Topun $1<t<3$ zaman aralığındaki ortalama hızı nedir?
7. Topun t=1 sn de ki hızını nasıl bulabiliriz? Ortalama hızı bulmak yeterli midir?
8. Top t = 0.9 sn de 24,912 m., t=1.1 sn de 28,992 m.de, t=0.99 sn de 26.7954 m., t=1,01 sn de 27.2034 m. de t=0.999 sn de 26,796 m de, t=1,001 sn de 27.204 m dedir.

   Topun t=1 sn deki hızını bulunuz. Cevabınızı açıklayınız.
9. t=1 sn.'ye ye daha yakın çok küçük zaman aralıkları alınırsa topun hızı hakkında ne söyleyebilirsiniz?
10. Topun t=1 anındaki hızına anlık hız denir. Topun t=1 anındaki anlık hızını nasıl bulabilirsiniz?



11. Grafikte 2<t<4 sn lerindeki ortalama hızı ve anlık hızı nasıl gösterebilirsiniz.

3. Application

Subjects: Average rate of change, definition of derivative, tendency (increasing or decreasing) of the graph of the function at certain intervals, derivative functions of given functions, derivatives of the given functions at certain points

Time: 2 lesson (2 x 50 min)

Objectives:

At the end of the lesson students will be able to,

- Interpret the average and instantaneous rate of change of a function from the given values and graphs of the functions
- Understand the relation between average rate of change of a function and the definition of the derivative
- Comprehend the instantaneous rate of change gives the derivative of a function at a certain point
- Interpret the sign of the derivative of a function in an interval where the function is increasing or decreasing.
- Sketch graphs for the functions whose derivative is positive
- Sketch graphs for the functions whose derivative is negative
- Finds the sign of the average rate of change of a function whose values are given
- Finds the slope of the tangent line at a certain point

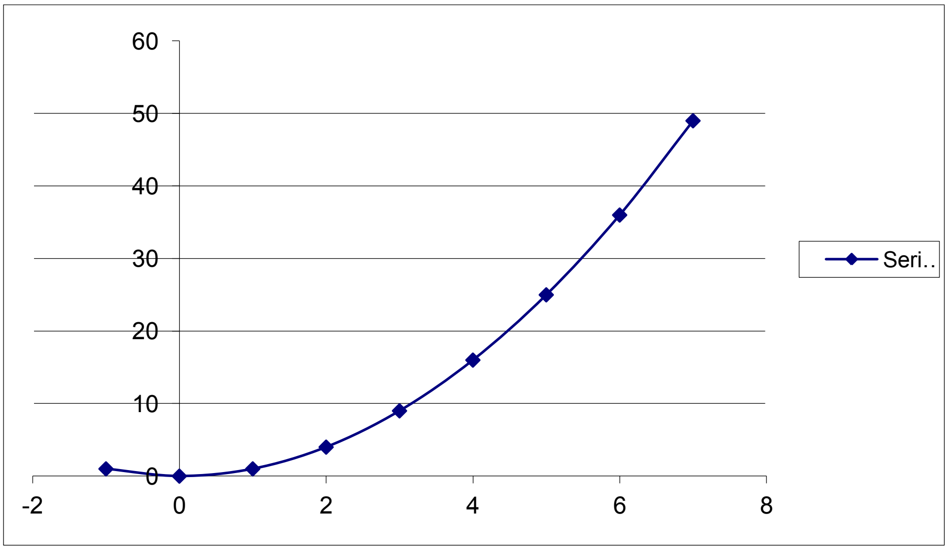
3. Application Worksheet

Aşağıdaki soruları cevaplandırınız. Cevaplarınızı ve açıklamalarınızı not alınız.

1. f bir fonksiyon olmak üzere, $\dfrac{f(a+h)-f(a)}{h}$ oranı neyi ifade eder? Açıklayınız.
2. a) x’e bağlı bir fonksiyon için x’deki küçük bir değişim f’de büyük bir değişime neden oluyorsa bu değişim hakkında ne söyleyebilirsiniz?
   b) Aynı şekilde x’deki büyük bir değişim f’de küçük bir değişime neden oluyorsa bu değişim için ne söyleyebilirsiniz?
3. Bir fonksiyonun a noktasındaki anlık değişim oranını ifade ediniz.
4. Aşağıdaki tabloyu kullanarak $f(x)=x^2$ fonksiyonunun x = 1 noktasındaki anlık değişim oranını bulunuz?

x	x^2	x^2 değerindeki değişim
0.998	0.996004	
		0.001997
0.999	0.998001	
		0.001999
1.000	1.000000	
		0.002001
1.001	1.002001	
		0.002003
1.002	1.004004	

5. $f(t)=t^2$ fonksiyonunun grafiğini aşağıda görüyorsunuz. t =1 noktasındaki anlık hızını grafiği kullanarak nasıl bulabilirsiniz?



6. Aşağıdaki tabloları kullanarak $f(x) = x^2$ fonksiyonunun türevi için formül bulmaya çalışınız.

x=1 civarında	
x	x^2
0,999	0,998
1,000	1,000
1,001	1,002
1,002	1,004

x=2 civarında	
x	x^2
1,999	3,996
2,000	4,000
2,001	4,004
2,002	4,008

x=3 civarında	
x	x^2
2,999	8,994
3,000	9,000
3,001	9,006
3,002	9,012

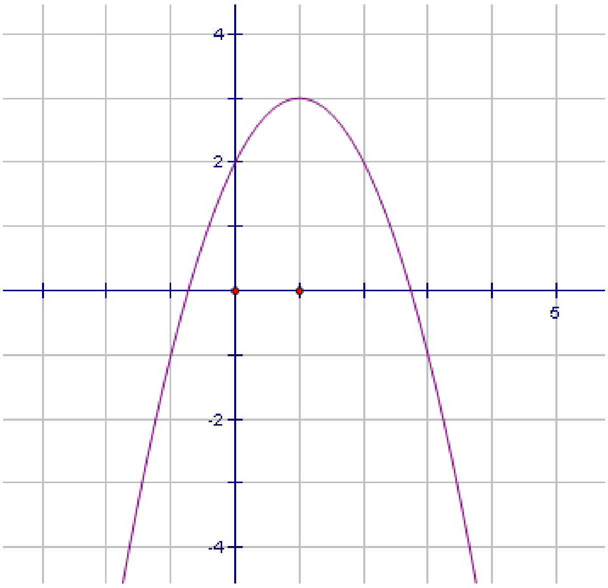
7. Aşağıda grafiği verilen $f(x) = -(x-1)^2 + 3$ fonksiyonunun türevinin 0 olduğu noktayı bulunuz.

Türevinin 0'dan büyük olduğu aralığı bulunuz.

Türevinin 0'dan küçük olduğu aralığı bulunuz.

f fonksiyonun artan olduğu aralıkla türevi arasında nasıl bir ilişki vardır?

f fonksiyonun azalan olduğu aralıkla türevi arasında nasıl bir ilişki vardır?



Herhangi bir f fonksiyonu için $f' > 0$, $f' < 0$ ve $f' = 0$ olduğu aralıklarda f fonksiyonu artan mı, azalan mı yoksa sabit midir? Açıklayınız.

8. **a)** Türevi her yerde pozitif olan ve artan bir eğri çiziniz.

b) Türevi her yerde pozitif olan ve azalan bir eğri çiziniz.

c) Türevi her yerde negatif olan ve artan bir eğri çiziniz.

d) Türevi her yerde negatif olan ve azalan bir eğri çiziniz.

9. Aşağıda verilen tabloya göre f(x) fonksiyonunun türevinin ortalama değerini bulunuz. f(x) in değişim oranı nerelerde pozitiftir? Nerelerde negatiftir? f(x) in değişim oranı nerede en büyüktür?

x	0	1	2	3	4	5	6	7	8
f(x)	18	13	10	9	9	11	15	21	30

10. **a)** $f(x) = \sin x$ fonksiyonunun $x = \pi$ deki türevi pozitif mi yoksa negatif midir? Neden?

b) $f(x) = \sin x$ fonksiyonunun (0,0) noktasındaki türevini tahmin ediniz. Cevabınızı açıklayınız.

11. **a)** $f(x) = x^2 + 1$ fonksiyonunun x=3 teki türevini bulunuz.

b) Teğet doğrusunun eğimini bulunuz.

c) Teğet doğrusunun denklemini bulunuz.

4. Application

Subjects: Maximum and minimum points of a graph of a function, Convex and concave graphs, inflection points, Minimum and maximum problems, equations of tangent line and normal line

Time: 2 lesson (2 x 50 min)

Objectives:

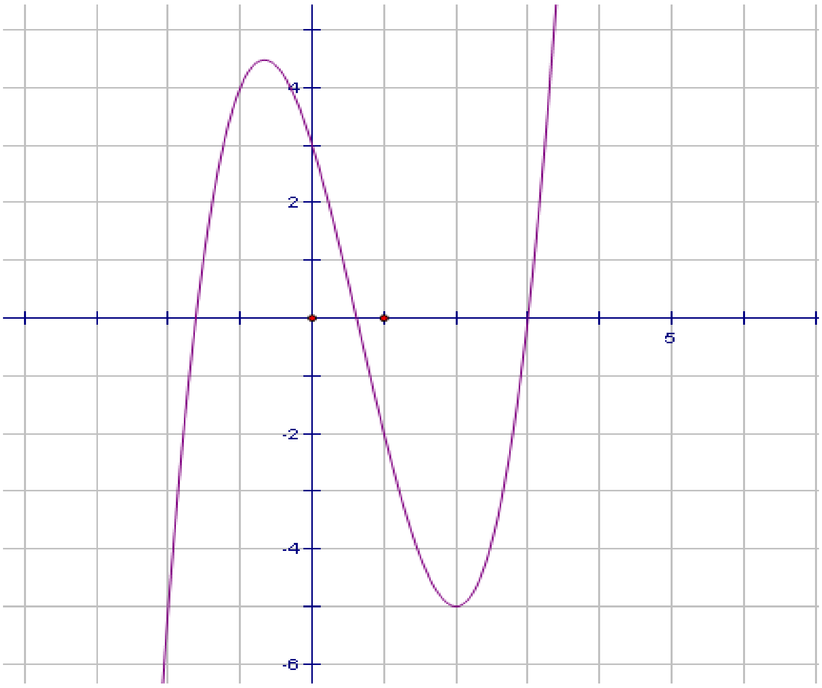
At the end of the lesson students will be able to,

- understand the relation between minimum and maximum points of a graph of a function and its derivative these points
- find the minimum and maximum points of a graph of a function
- understand the critical points of a function
- Understand minimum and maximum problems
- Solve minimum and maximum problems
- Find the equation of tangent line
- Find the equation of normal line
- understand the relation between the second derivative and the convex and concave graphs
- understand the inflection points
- find the inflection point

4. Application Worksheet

1) $f(x) = \dfrac{x}{x^2+1}$ fonksiyonunun yerel minimum, yerel maksimum, büküm noktalarını, artan, azalan, konveks ve konkav oldukları aralıkları bulunuz.

2) $f(x) = x^3 - 2x^2 - 4x + 3$ fonksiyonunun grafiği aşağıda verilmiştir. Grafiği inceleyerek aşağıdaki soruları cevaplandırınız.



a) $x = \dfrac{2}{3}$ noktasında grafik nasıl bir değişim gösterir? Açıklayınız.

b) f fonksiyonun yerel minimum ve yerel maksimum noktaları hangi noktalardır? Minimum ve maksimum noktaları hangileridir?

c) f fonksiyonunun ikinci türevini bulunuz.

$f''(x) > 0$ olduğu aralıkta fonksiyonun grafiği nasıl bir özellik gösterir? Belirtiniz.

- $f''(x) < 0$ olduğu aralıkta fonksiyonun grafiği nasıl bir özellik gösterir? Belirtiniz.

- $f''(x) = 0$ olduğu x noktasının özelliği nedir?

d) Fonksiyonun aşağı bakmasıyla (konveks) ikinci türevi arasında nasıl bir ilişki vardır?

Fonksiyonun yukarı bakmasıyla (konkav) ikinci türevi arasında nasıl bir ilişki vardır?

e) $x = -\frac{2}{3}$ noktası için $f'(-\frac{2}{3}) = 0$ ise, $f''(-\frac{2}{3})$ ü bulunuz.

$x = -\frac{2}{3}$ noktasının özelliği nedir? $f''(-\frac{2}{3})$ pozitif mi yoksa negatif midir?

f) $x = 2$ noktası için $f'(2) = 0$ ise, $f''(2)$ yi bulunuz.

$x = 2$ noktasının özelliği nedir? $f''(2)$ pozitif mi yoksa negatif midir?

g)f ve g şıklarında verdiğimiz cevapları göz önünde bulundurarak $f''(x)$ in yerel minimum ve yerel maksimum noktalarını belirlemedeki rolünü açıklayınız.

h) $f'(a) = 0$ ve $f''(a) > 0$ ise, $x = a$ noktası f fonksiyonunun

_________________________ noktasıdır.

$f'(b) = 0$ ve $f''(b) < 0$ ise, $x = b$ noktası f fonksiyonunun

_________________________ noktasıdır.

3) R yarıçaplı bir çember içine çizilebilen bir ikizkenar üçgenin alanı en fazla ne olabilir?

4) $y = \frac{1}{3}x^3 - 1$ eğrisinin x=-1 noktasındaki teğet doğrusunun denklemini bulunuz.

Normal doğrusunun denklemini bulunuz.

$x^5 + y^5 - 2xy = 0$ eğrisinin A(1,1) noktasındaki teğet ve normal doğrularının denklemlerini bulunuz.

5) A(2,0) noktasının $y = \sqrt{x}$ eğrisine olan uzaklığını hesaplayınız.

a. İki nokta arasındaki uzaklık nasıl bulunur?

b.Bu uzaklığın minimum mu yoksa maksimum mu olması beklenir?

c. A noktası ve eğri arasındaki uzaklığı bulunuz.

6) İçine k cm yarıçaplı bir küre yerleştirilen bir dik dairesel koninin hacmi en az kaç cm^3 olur?

a. Koninin hacmini nasıl ifade edersiniz?

b. Kürenin hacmini nasıl ifade edersiniz?

c. Hacimler arasındaki ilişki nedir?

d. Koninin hacmi ne olur?

APPENDIX F

INTERVIEW GUIDE
QUESTIONS RELATED TO THE DERIVATIVE TEST

1. Soru

- Cevabınızı açıklayınız.
- Cevabınızı grafik üzerinde açıklayınız.

3. Soru

- Cevabınızı açıklayınız.
- x=2 deki türevi bulmak için grafik nasıl kullanılabilir?
- x=2 deki türevi bulmak için tablodaki değerler nasıl kullanılabilir?
- Fonksiyonun değerindeki değişimin x deki değişime oranı ne anlama gelir?
- Sadece bir noktadaki değişim oranını bulmak türevi bulmak için yeterli olur mu? Neden?

7. Soru

a şıkkı

- Cevabınızı açıklayınız.
- Fonksiyonun artan olması ne demektir?
- Fonksiyonun azalan olması ne demektir?
- Fonksiyonun türeviyle artan olduğu aralık arasında nasıl bir ilişki vardır?
    - ✓ Neden böyle bir ilişki vardır?
- Fonksiyonun türeviyle azalan olduğu aralık arasında nasıl bir ilişki vardır?
    - ✓ Neden böyle bir ilişki vardır?

b şıkkı

- Cevabınızı açıklayınız.
- Yerel minimum noktası ne demektir?
- Yerel maksimum noktası ne demektir?
- Fonksiyonun minimum ve maksimum noktaları ne demektir?
- Yerel minimum ve yerel maksimum noktaları nasıl bulunur?
- Türev grafiğini kullanarak yerel minimum ve yerel maksimum noktaları nasıl bulunur? Neden?

10. Soru

- Cevabınızı açıklayınız.
- Fonksiyonun artan olmasıyla türev grafiğinin nasıl bir ilişkisi vardır?
- Fonksiyonun azalan olmasıyla türev grafiğinin nasıl bir ilişkisi vardır?
- Fonksiyonun konkav ve konveks olduğu yerlerin birinci türevle ne ilgisi vardır

CURRICULUM VITAE

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High School	Turgut Reis Lisesi (Süper Lise)	1998

WORK EXPERIENCE

Year	**Place**	**Enrollment**
2003- Present	Başkent University	Research Assistant

FOREIGN LANGUAGES

Advanced English

PUBLICATIONS

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