DEVELOPMENT OF A MODAL ANALYSIS SOFTWARE PLATFORM

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

ONUR ÖZGELEN

IN PARTIAL FULLFILLMENT OF REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
MECHANICAL ENGINEERING

FEBRUARY 2014

Approval of the thesis:

DEVELOPMENT OF A MODAL ANALYSIS SOFTWARE PLATFORM

submitted by **ONUR ÖZGELEN** in partial fullfillment of the requirements for the degree of **Master of Science in Mechanical Engineering Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen _______________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Suha Oral _______________
Head of the Department, **Mechanical Engineering**

Asst. Prof. Dr. Gökhan O. Özgen _______________
Supervisor, **Mechanical Engineering Department, METU**

Examining Committee Members:

Prof. Dr. Mehmet Çalışkan _______________
Mechanical Engineering Department, METU

Asst. Prof. Dr. Gökhan O. Özgen _______________
Mechanical Engineering Department, METU

Assoc. Prof. Dr. Ender Ciğeroğlu _______________
Mechanical Engineering Department, METU

Asst. Prof. Dr. Yiğit Yazıcıoğlu _______________
Mechanical Engineering Department, METU

Prof. Dr. Altan Kayran _______________
Aerospace Engineering Department, METU

Date: 03.02.2014

I hereby declare that all information in this document has been obtained and presented inaccordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last Name : ONUR ÖZGELEN

Signature :

ABSTRACT

DEVELOPMENT OF A MODAL ANALYSIS SOFTWARE PLATFORM

ÖZGELEN, Onur

M.S. , Department of Mechanical Engineering

Supervisor : Asst. Prof. Dr. Gökhan O. Özgen

February 2014, 95 pages

Modal identification techniques are used for identifying modal parameters of the system via frequency response functions of the system obtained from modal testing. In this master study, it is aimed to develop a software platform in which different modal identification techniques can be implemented. For this purpose, many modal identification techniques available in the literature are investigated and some of them are selected to be used in the software platform.

Besides the investigation on modal identification techniques, commercially available modal analysis softwares are researched in order to get the recipe for what a modal analysis software is to be included. According to the findings of the recipe, the parts of modal analysis software are constructed.

LabVIEW is chosen for environment to develop the software. It is preferred for that it has programming advantages for the developer over other programming languages. Furthermore, creating graphical user interfaces for the users using LabVIEW is by far easier compared to other programming languages.

In order to validate that the software works properly, two case studies are carried out. The result of first case study is taken from a finite element software package. Numerically generated frequency response functions are taken from an arbitrary structure by using the finite element software. These FRFs are processed in the developed software. As for the second case study, experimentally gathered FRFs are processed in the software. Thus, it is shown that the software works properly for FRFs which is obtained by either analytical or experimental means.

Keywords : Experimental Modal Analysis, Modal Parameter Estimation, Modal Identification Techniques, Curve Fitting

ÖZ

MODAL ANALİZ YAZILIM PLATFORMU GELİŞTİRİLMESİ

ÖZGELEN, Onur
Yüksek Lisans , Makina Mühendisliği Bölümü
Tez Yöneticisi : Yard. Doç. Dr. Gökhan O. Özgen

Şubat 2014, 95 sayfa

Modal tanımlama teknikleri sistemlerin deneysel elde edilen frekans tabanındaki sonuç fonksiyonlarından sistemlerin modal parametrelerini belirlemek için kullanılırlar. Bu yüksek lisans çalışmasında, içinde farklı modal tanımlama tekniklerinin uygulanabileceği bir yazılım platformu geliştirilmesi amaçlanmıştır. Bu amaçla, literatürde mevcut olan birçok modal tanımlama teknikleri incelenmiştir ve içlerinden bazıları yazılımda kullanılmak üzere seçilmiştir.

Modal tanımlama tekniklerinin araştırılmasına ek olarak, ticari olarak kullanılan modal analiz yazılımları da bir modal analiz yazılımının neleri içermesi gerektiği konusunda reçete çıkarabilmek için araştırılmıştır. Reçetedeki bulgulara göre, modal analiz yazılımının bölümleri oluşturulmuştur.

Yazılımı geliştirme ortamı olarak LabVIEW seçilmiştir. LabVIEW, yazılım geliştiricisi açısından diğer programlama dillerine göre programlama avantajları olduğu için tercih edilmiştir. Bunun yanısıra, LabVIEW kullanarak kullanıcılar için grafik arayüzleri hazırlamak diğer programlama dillerine göre çok daha kolaydır.

Yazılımın düzgün çalıştığını onaylamak için iki tane örnek çalışma gerçekleştirilmiştir. Birinci örnek çalışmanın sonuçları sonlu eleman yazılım programından alınmıştır. Sonlu eleman yazılımı kullanılarak herhangi bir yapı üzerinden sayısal olarak oluşturulmuş frekans sonuç fonksiyonları alınmıştır. Bu frekans sonuç fonksiyonları geliştirilen yazılımda işlenmiştir. İkinci örnek çalışma için ise deneysel olarak elde edilen frekans sonuç fonksiyonları yazılımda işlenmiştir.

Böylece, yazılımın analitik veya deneysel yollarla elde edilen frekans sonuç fonksiyonları için doğru bir şekilde çalıştığı gösterilmiştir.

Anahtar Kelimeler : Deneysel Modal Analiz, Modal Parametre Tahmini, Modal Tanımlama Teknikleri, Eğri Uydurma

To My Family & To My Grandfather

ACKNOWLEDGMENTS

The author wishes to express his deepest gratitude to his supervisor Asst. Prof. Dr. Gökhan O. Özgen for his guidance, advice, criticism, encouragements and insight throughout the study.

The author would also like to thank Bilgehan Erdoğan for his help, suggestions and comments about developing the software.

The author also wishes to express his gratitude to his colleagues from ASELSAN for their support throughout the study. The author especially thanks to his manager Serdar Terzi for his tolerance throughout the study.

Finally, the author would like to thank his beloved family, Mustafa Kemal, Bilge Özgelen and his grandfather Bedri Özgelen for their support and faith throughout his life.

TABLE OF CONTENTS

LIST OF TABLES

TABLES

LIST OF FIGURES

FIGURES

CHAPTER 1

INTRODUCTION

Modal analysis serves as a tool to define the dynamic behaviour of structures in real engineering applications. Modal parameters can be determined analytically or experimentally. Experimental modal analysis (EMA) is the extraction of modal parameters in terms of damped frequencies, damping factors, modal vectors and modal scaling using experimentally identifed frequency response functions (FRF) of the system of interest. The identification of modal parameters is accomplished by curve-fitting of FRFs or impulse response functions (IRFs). Curve-fitting is based on minimizing an error function (i.e. least-squares approach) between the analytical function and the measured data.

Experimental modal analysis is comprised of three stages: Modal data acquisition, Modal parameter estimation and Modal data verification. Focus of this thesis study is on modal parameter estimation and modal data verification stages. There are various options of modal parameter estimation techniques for someone who wishing to apply these techniques for a structure using experimental modal analysis. There exists some advantages and some constraints of these techniques that will lead the user towards which one should be used for a certain testing application. Therefore, a good background of the theory behind these techniques provide the user to evaluate different options of methods to be used for a given problem.

In the scope of the thesis work, various modal identification techniques are investigated. Introductory information is given about almost all of these techniques. The formulations & features of some of these techniques are presented. The features & sections of the developed software are determined by investigating the commercial modal analysis softwares.

Implementation of the most commonly used modal parameter identification techniques is another focus of this thesis. LabVIEW is selected as environment for the development of the software to build up a know-how & user-experience about LabVIEW in the department. The experience gained during this thesis work can be a good starting point for the new developers.

With this thesis work, the main purpose is to generate a know-how about available modal identification techniques and modal analysis procedure. Of course, one can find entire modal analysis software packages, which is linked to modal testing systems and doing not only modal parameter estimation but also pre- and post-processing calculations. However, these softwares are very expensive and are not easily affordable for engineers wishing to work in vibration and related fields. Therefore, it is very important to gain experience about this issue. In the near future, the developed software may be an initial step or an prototype to originally develop national modal analysis software.

The thesis involves five chapters. In the second chapter, a brief information about modal analysis and modal identification techniques is given. Moreover, important concepts in modal parameter estimation are introduced and modal data verification stages are explained. The features of commercial softwares are presented. Consequently, previous studies similar to this study are discussed. Third chapter is about the development procedure of the software. The detailed formulations & expressions of selected modal parameter estimation methods for the software are given and graphical user interface (GUI) design of the software is introduced. In the fourth chapter, the results obtained from the developed software are presented and discussed. Two case studies are performed analytical & experimental, respectively, in order to verify that the developed software works properly. In the last chapter, future work suggestions for the study are discussed.

CHAPTER 2

LITERATURE SURVEY

2.1. Introduction

In literature survey of the study, the theory required for developing a modal parameter estimation software is investigated. Firstly, a general overview modal analysis and modal parameter estimation techniques is presented. Then, important concepts in modal parameter estimation are found out. Obtained modal parameters also require verfication process. Therefore, the ways of modal data verfication are introduced. Commercial softwares related to modal analysis are discussed in order to have an idea about of what a modal analysis software should have. Finally, some similar studies to my study are discussed.

2.2. Modal Analysis

Modal analysis identifies the modal parameters of structures in real engineering applications. Experimental modal analysis requires measured vibration data in order to obtain modal parameters. There exists frequency and time domain modal analysis methods which use frequency response and impulse response functions, respectively.

The Frequency Response Function (FRF) is basically a transfer function between two points on a structure as a function of frequency. Figure 2.1 shows a block diagram representation of an FRF. An FRF may also be defined as a degree of measure which shows that how much response of a structure in terms of displacement, velocity, acceleration is at an output DOF per unit of excitation force at an input DOF.

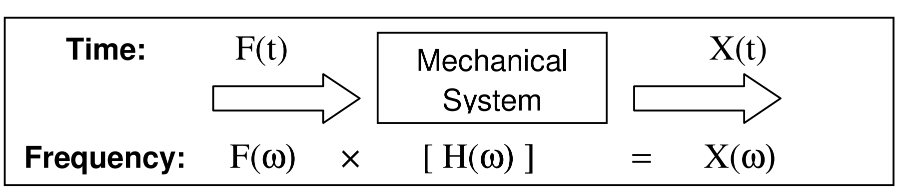


Figure 2.1. Block Diagram of an FRF

While frequency domain modal analysis use measured FRF data, time domain modal analysis utilizes from time response data. Time response data or impulse response functions (IRF) are related to FRFs through inverse Fourier transformation. Impulse response information can be obtained either from measured time responses or the FRF data. Time domain methods has several advantages over frequency domain methods. Firstly, time domain methods requires fewer response data. They can also be quite effective for analysing closely spaced vibration modes. On the other hand, vibration modes are not visible when time domain methods are used, which makes these methods more user dependent [3].

2.3. Modal Testing

Modal testing is the act of performing vibration tests on a structure for identifying FRFs or IRFs which will then be used to extract modal parameters of the same structure. At least one row or column of the FRF matrix must be constructed to carry out modal testing. There are common ways to measure a single row or a single column of the FRF matrix. These are called a roving hammer impact test and a shaker test, respectively. By a roving hammer test, the output remains fixed and FRFs are measured from multiple inputs. Inversely, the input remains fixed and FRFs are measured from multiple outputs by shaker test. Both impact and shaker test can be grouped as single reference (or SIMO) testing.

When an FRF matrix has more than one column or row, this means that two or more fixed inputs or outputs are used. This is called as Multiple Reference or MIMO (Multiple Input – Multiple Output) testing. MIMO testing may be necessary for the following reasons:

- All modes of interest of the structure can not be excited from one reference.
- The structure has repeated roots (or closely spaced modes) [1].

Figure 2.2 is a good representation of constructing FRF or IRF matrix with modal testing [1].

2.4. Experimental Modal Analysis

Experimental Modal Analysis (EMA) definitely plays a key role in the analysis of structural dynamics. Experimental modal analysis is the extraction of modal parameters (damped frequencies, damping factors, modal vectors and modal scaling) from experimental FRFs or IRFs. Modal parameters are found by fitting the curves of FRFs or IRFs obtained from modal testing.. Curve-fitting means to determine suitable mathematical expression to the measured data points. Generally, an error function is defined between assigned analytical function to the FRF or IRF and measured data points. Then, this error function is tried to be minimized. Modal parameters can be also determined by analytical means. Figure 2.3 shows the different ways how modal parameters are obtained, both analytically and experimentally.

Experimental modal analysis may be used to verify the results of the analytical solution of modal parameters for a structure. Additionally, it has a wide range of uses such as structural modification, troubleshooting vibration problems, finite element analysis correction, development of mathematical models, the evaluation of design changes.

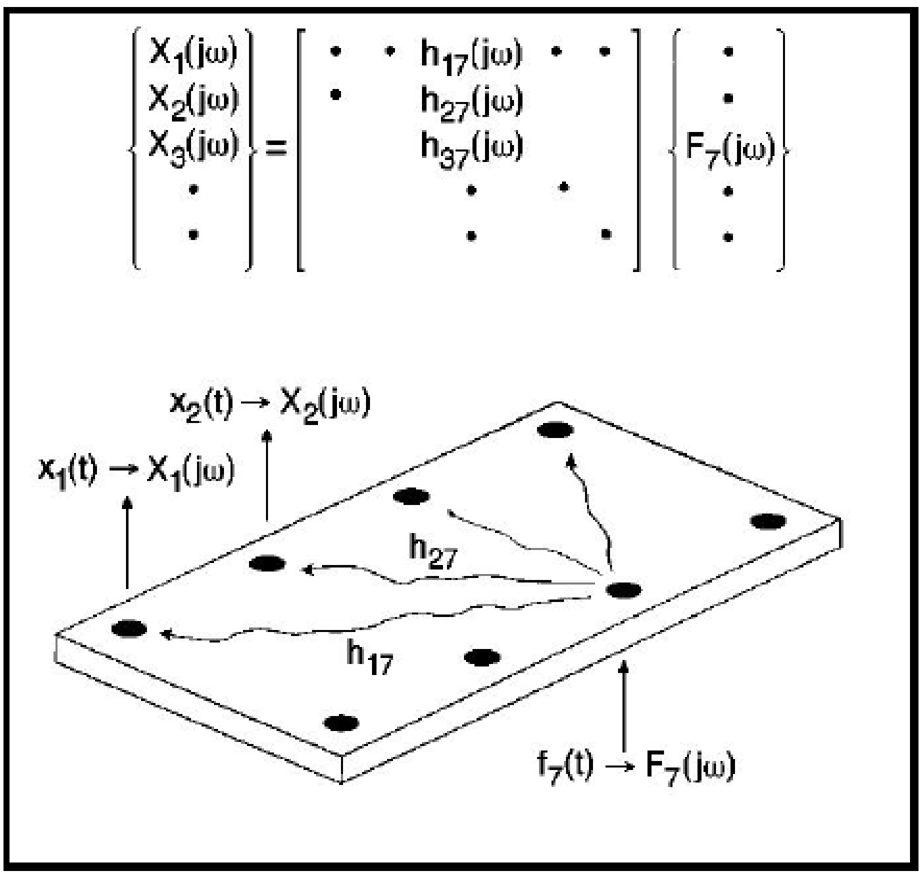


Figure 2.2. Measuring FRFs or IRFs on a Structure [1]

In order for experimental modal analysis results to be viable, four basic assumptions should be valid for the structure of inerest: linearity, time invariancy, reciprocity i.e.($H_{pq} = H_{qp}$), and underdamped system behavior.

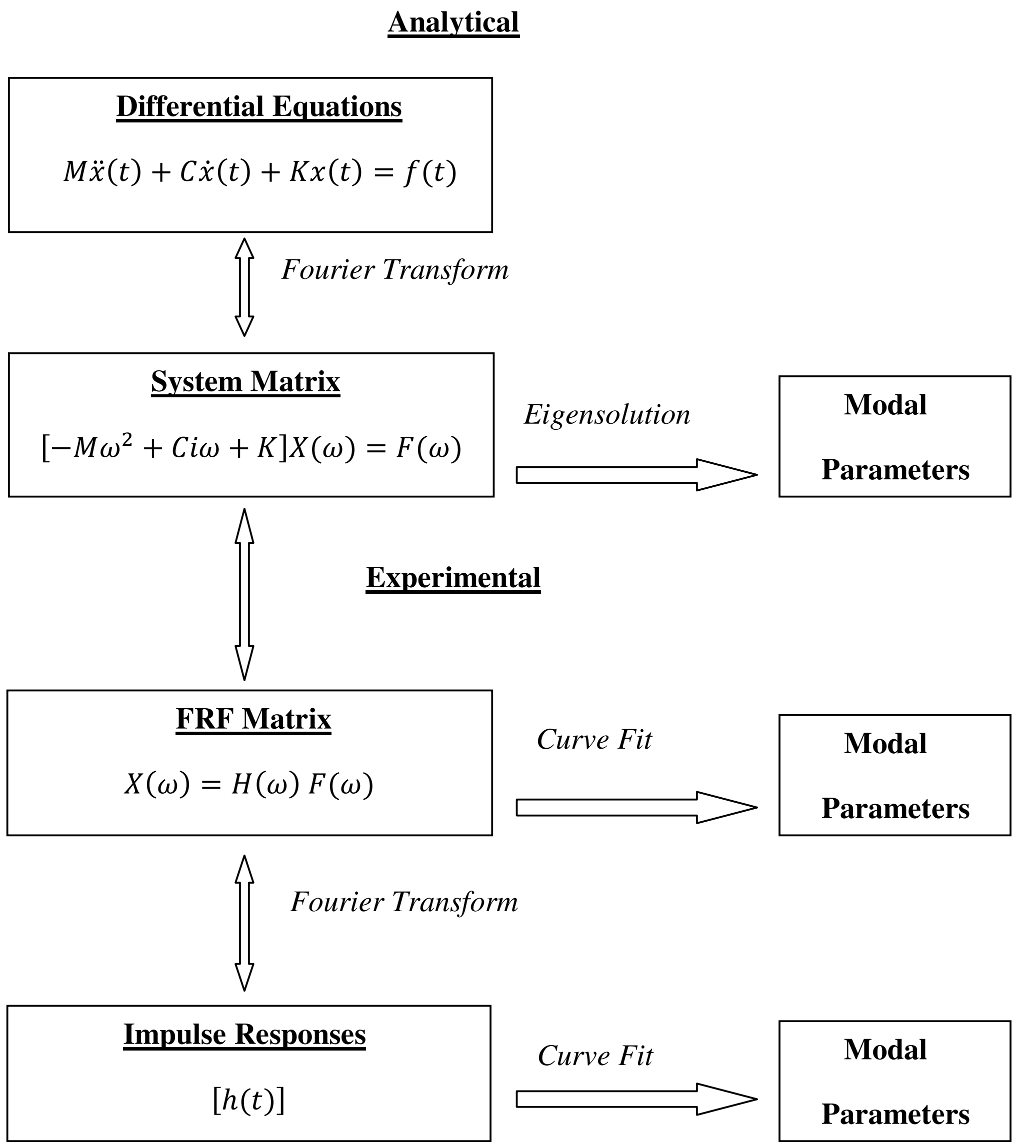


Figure 2.3. Analytical and Experimental Modal Parameter Extraction [1]

Experimental modal analysis can be mainly described as three stages: Modal Data Acquisition, Modal Parameter Estimation, and Modal Data Verification. Main concern of this study is on modal parameter estimation and modal data verification stages.

2.5. Modal Data Acquisition & FRF Estimation

Data acquisition is the initial step of getting modal parameters to be estimated. It is mainly related with digital signal processing. Modal data acquisiton has some techniques in order to minimize random or deterministic (bias) error in nature. The accuracy of frequency response measurements is directly related to errors that may occur due to misuse of digital signal processing. There are two categories of the error in FRF estimation: variance and bias. The variance is an indication of the distribution of each sample data from the mean value. The bias error is the difference between the expected and the true value of the estimated parameter. Aliasing and leakage are the two most commonly faced errors that would be classified as bias error. Aliasing arises from the discretization of the original continuous time history. If the sampling rate is not chosen properly, the original signal is distorted both interms of frequency & phase. The minimum sampling rate is at least two times the signal frequency in order to avoid aliasing. This phenomenon is known as “Nyquist-Shannon Sampling Theory”. The distortion of the continuous input signal from periodicity can cause leakage. Windowing functions, which is multiplied by time signals, can be utilized in order to minimize leakage problem.

2.6. Modal Parameter Estimation Techniques

Modal parameter estimation is the process of estimating modal parameters utilizing a suitable mathematical model to fit to the measured FRF or IRF data. It plays a key role in experimental modal analysis and is also often called “curve fitting” in literature. A vast amount of literature exists about the curve-fitting subject. In this section different types of curve-fitting applications will be categorized and discussed. As discussed earlier, at least one row or column of frequency response matrix is required to identify complete set of modal parameters. After measuring complete set of frequency responses, the frequency and damping for each mode are firstly estimated. Then, the residues can be calculated for each measurement point. Finally, mode shapes are scaled from residue terms. These main steps of modal parameter estimation can be also illustrated in Figure 2.4 [2].

Modal parameter estimation techniques work with frequency-response functions (FRFs) and impulse-response functions (IRFs) working in the frequency and time domain, respectively. The IRFs are obtained from inverse Fourier transform (IFFT) of FRF's. FRF and IRF can be written as follows:

$$[H(s)] = \sum_{k=1}^{N}\left(\frac{[A_k]}{(s-\lambda_k)} + \frac{[A_k^*]}{(s-\lambda_k^*)}\right) \tag{2.1}$$

where N is the number of modes of vibration, $[A_k]$=(n x n) residue matrix for the kth mode, λ_k is pole location for the kth mode, σ_k is damping coefficient for the kth mode, ω_k is damped natural frequency for the kth mode, and "*" is the operator for complex conjugate.

$$[h(t)] = \sum_{k=1}^{N}\left([A_k]e^{\lambda_k(t)} + [A_k^*]e^{\lambda_k^*(t)}\right) \tag{2.2}$$

where [$h(t)$] is (p x q) the IRF matrix.

Main parameter for the classification of identification methods is whether the data is used in time domain or frequency domain. The number of response and force locations may also change for each domain, which leads to the following classification:

- single-input single-output (SISO)
- single-input multiple-output (SIMO)
- multiple-input multiple-output (MIMO)
- multiple-input single-output (MISO)

One more classification criterion for both domains relates with the terms of estimated dynamic properties. While direct methods estimate the dynamic properties in terms of stiffnesses, masses, damping coefficients; indirect methods estimate the modal parameters: resonant frequency, damping ratio and modal constants. Figure 2.5 shows the overall classification of modal identification techniques:

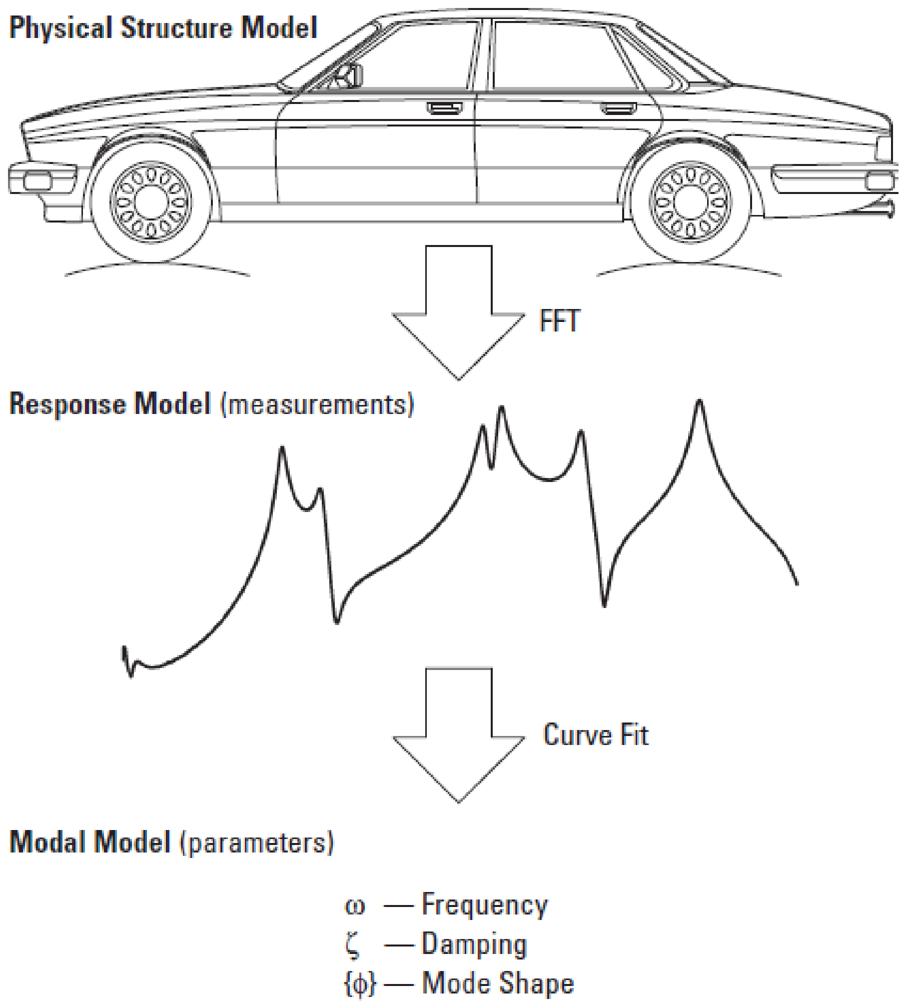


Figure 2.4. Main Steps of Modal Parameter Estimation [2]

The peak-picking method is a frequency domain SDOF method. The natural frequency of each mode is estimated from the peak values of the FRF. The damping is then identified using half power point method. Finally, the modal constant can be easily calculated from the known natural frequency and damping values. This method works fine for structures with well-separated modes. The structure should neither be so lightly-damped nor so heavily-damped [4].

The Quadrature Response and Maximum Quadrature Component methods differ from peak-picking method when identifying natural frequencies of the structure. The Quadrature Response states that natural frequencies occur at the points where real part of the response is zero. Whereas, the Maximum Quadrature Component method considers the natural frequencies occur at the points where imaginary part of the response is maximum [4].

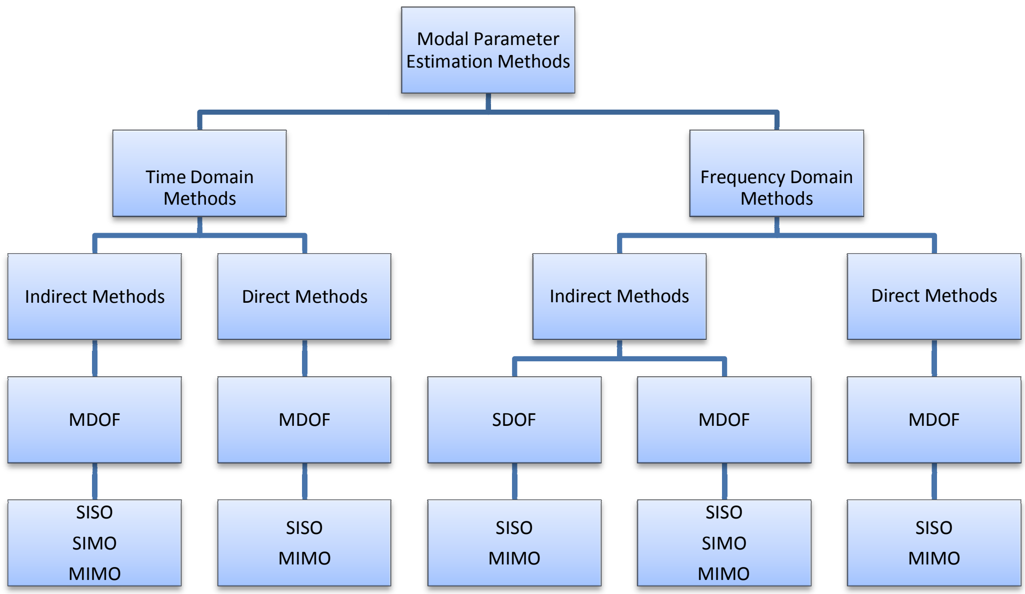


Figure 2.5. Classification of Modal Parameter Estimation Methods [4]

The work of Kennedy&Pancu, also called Circle Fit method, is also another early and basic modal parameter identification method. The method offers to represent FRF in the complex plane, called Nyquist plot. The plot seems like circle around the resonant frequency. In fact, the shape of the plot is exactly circular for a single-degree-of-freedom (SDOF) system. Resonant frequencies and the dynamic properties associated with each of resonances are identified through a circle-fitting method utilizing Nyquist plot. The basic assumption of the model is that there is a complex constant contributing the selected mode by other modes. This contribution is represented as mass and stiffness term separately. Two or even more data points are required from outside of the experimental frequency range to identify mass and stiffness term [4].

Another SDOF method is the inverse FRF method [5]. The method accepts that the inverse of the FRF data is linear. Therefore, the real and imaginary parts of inverse FRF are fitted with straight lines. The natural frequency is determined from the intercept of the real plot. The phase of the modal constant can be estimated from the ratio of two slopes. Then the modulus of the modal constant is found out from either slope of straight lines.

Dobson [6] developed a method, which can be regarded as the extension of the inverse method. It is based on curve-fitting straight lines like the inverse FRF method. A series of straight lines can be obtained for both the real and imaginary parts of function by selecting a frequency around the natural frequency. Fitting the slopes of these straight lines with respect to square of the frequency around the natural frequency, two slope plots are obtained. Modal parameters can then be identified from the slopes and intercepts of the two straight lines.

The complex exponential (CE) method is a time-domain SISO method. IRFs are used for this technique. The algorithm of the method is based on Prony's equation. Prony equation leads the user to the coefficients of the charasteristic polynomial. Modal parameters can be identified from the roots of characteristic polynomial [4].

Least-squares complex exponential (LSCE) is the SIMO version of CE method, proposed in [7]. IRFs are used simultaneously for this case, generating a set of global parameters (natural frequencies, damping ratios). Therefore, the variations in the same parameters are prevented. As for LSCE, the coefficients β of charasteristic polynomial are global quantities. In other words, they are not changing for every IRF used. Approaching the solution in the global sense generates an overdetermined system of equations, solved by least-squares sense. The rest of the procedure is same as CE method.

Polyreference complex exponential (PRCE) is a MIMO method in time domain, developed by Vold et. al. [8,9]. It is actually the extension of LSCE method, including IRFs from several input reference points. PRCE was really a breakthrough in modal parameter estimation since identifying closely-spaced or even repeated modes of complex structures has become possible. PRCE is also called as polyreference time domain (PTD) in literature. Zhang et. al. [10] assert that the identification accuracy of the method can be improved by applying correlation filter to noisy time response data.

Ibrahim time domain method (ITD) is a SIMO method, developed by Ibrahim & Mikulcik [11]. Instead of IRFs, the method utilizes from free acceleration responses. The method uses state vectors. Displacement and velocity responses are also required, calculated by the integration of the free acceleration responses. All modal parameters can be identified in one step by eigenvalue problem. Estimating the correct number of modes is a problem. Eigenvalue problem works based on the number of chosen responses. If these chosen responses are greater than the true number of resonances, some of these solutions are not physically meaningful. Therefore, it is difficult to understand whether they are the true solutions or not. There is a quality check for the results proposed in [12], called modal confidence factor, comparing eigenvectors in the subsequent time intervals. The ITD method is also extended into MIMO version [13], called as Multi-reference Ibrahim Time Domain (MRITD) or Enhanced Ibrahim Time Domain (EITD) method.

Eigensystem Realization Algorithm (ERA) is a time domain MIMO method, developed by Juang&Pappa [14]. It is based on general state-space description of linear dynamic systems. State-space model comes from system realization used in system/control engineering and it is adapted to modal identification in aerospace, mechanical and civil engineering applications. A frequency domain algorithm that follows mainly the same steps as for the time domain version is also developed (ERA-FD) [15]. Furthermore, the general realization algorithm (GRA) [16] is also developed to identify modal parameters of linear multi-degree-of-freedom (MDOF) dynamic systems. The GRA is an extension of the well known ERA. In addition to ERA case, the GRA offers a refinement of state-space realization through least-squares optimization in order to get the minimum prediction error for the response. It is alleged in the study that the GRA gives very accurate estimates of the modal parameters in case of noise-free input-output data or low output noise. The GRA is recommended for realization of linear dynamic systems subjected to short-duration or nonbroadband excitations such as eartquake and shake table excitations.

The Autoregressive Moving-Average method (ARMA) is also proposed as a tine-domain method.. The behaviour of a linear system is described with single input-single output. MIMO versions of the method are also available. They are called as the Autoregressive Moving-Average with exogenous variables (ARMAX) and the Autoregressive Moving-Average Vector (ARMAV) [17]. The principles are the same as for ARMA. However, time series is expressed in terms of matrix difference equations. These methods must be used when the input is not known, for example; bridges subjected to wind excitation [4].

The Rational Fraction Polynomial (RFP) is a SISO method in frequency domain [18]. The FRF used in the method is expressed as the ratio of two polynomials. The coefficients of two polynomials are calculated by the method. Orthogonal polynomial coefficients are calculated from ordinary polynomial coefficients to overcome ill-conditioned numerical problems. A global version of this technique is also available, called as Global Rational Fraction Polynomial (GRFP). This extension of the RFP is developed in [19,20]. GRFP method is also extended into MIMO version to cope with multiple input locations by different authors [21,22]. Instead of using orthogonal polynomials in order to obtain well-conditioned set of linear equations, a technique based on RFP representation of FRF is developed in [23]. The method can be adapted to SISO, SIMO & MIMO systems. The method solves set of linear equations in total least-squares sense in the Z-domain, which enables the user to achieve modal parameters in a numerically efficient way.

Polyreference frequency domain method (PFD), developed by Zhang *et. al.* [24-26], was the first MIMO method in frequency domain. In addition to receptance function, it makes also use of mobility function. A similar version of this method based on direct parameter identification was also developed in [27]. This method is also called as frequency domain parameter identification (FDPI) in literature.

Peeters B. *et. al.* [28] proposed a new frequency-domain parameter estimation method. The method is called as PolyMAX or polyreference least-squares complex frequency-domain estimator. The method is based on a weighted least-squares approach and uses MIMO FRFs. The method follows so-called right-matrix fraction model in Z-domain model. In the study, it is shown that when PolyMAX method is compared with commercially available methods concerning stability, accuracy of the estimated modal parameters and quality of the frequency response function synthesis, it yields superior results. Very clean stabilisation diagrams are obtained by the implementation of the method, which enables the user to select the best structural system poles.

2.6.1. Comparison of Modal Parameter Estimation Methods

It is difficult to determine the requirements for each method and to find the best one due to the large amount of literature on currently available algorithms. There is no ideal solution for a specific case. Many of the methods are similar to each other.

In fact, some methods are just extensions of basic curve-fitting algorithms. Although there are several ways to categorize curve-fitting methods, all can be grouped into four categories:

- Local SDOF
- Local MDOF
- Global (SIMO testing applications)
- Poly-reference (MIMO testing application)

While local methods work with one FRF at a time, global and poly-reference methods work with entire set of FRFs. Local SDOF methods are easiest to use and work efficiently with lightly damped structures. When the system is highly damped, MDOF methods must be preferred. Global methods give better results than local MDOF methods. However, global methods have some problems in solving repeated-roots, which means very closely coupled modes. Poly-reference methods serve the best estimates for obtaining modal parameters. Two or more columns and rows of the FRF matrix can be handled by these algorithms. Table 2.1 [20] shows a comparison table of all curve-fitting methods in terms of pros and cons.

As a final remark, there is no such a suggestion as the best solution. Some methods work better for some applications. The choice of the method depends upon:

- Available resources
- Available time
- Object of the study
- Personal experience of the user

2.6.2. Transfer & Frequency Response Function Relationship

The transfer function correlates input and output of a physical system, mathematically. There is a relationship between input and output of a physical system and can be defined as in (2.3). This is the ratio of output divided by input in Laplace transform.

$$G(s) = \frac{Y(s)}{X(s)} = \frac{Output}{Input} \tag{2.3}$$

Furthermore, the frequency response function is actually a transfer function which is used to solve vibration problems. The relationship between transfer and frequency response functions can be adapted to modal parameter estimation process. Either frequency domain or time domain method is used, modal parameter estimation routine consequently reaches the poles of the system, as illustrated in Figure 2.6 [29]:

After finding the poles of the system, natural frequencies and damping ratios can be easily found using the equations below. A conversion (z-transformation) is necessary if a time domain method is used to estimate modal parameters.

$$\lambda_k = \frac{ln z_k}{\Delta t} \tag{2.4}$$

$$\lambda_k = -\sigma_k + i\omega_k \tag{2.5}$$

Table 2.1. Comparison Table of Modal Parameter Estimation Methods [20]

Method	**Advantages**	**Disadvantages**
Local SDOF	* Fast * Easy to use	* For lightly damped structures
Local MDOF	* heavily damped structures * good with noise	* requires skill to choose the # of modes * poor results
Global	* global frequency & damping estimation	* errors with poorly excited modes
Poly-reference	* good for repeated roots	* poor results with inconsistent measurements

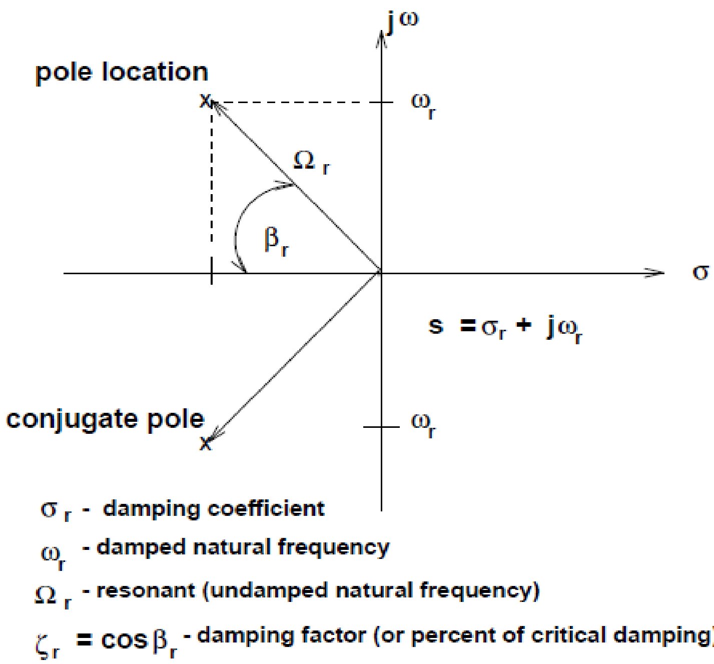


Figure 2.6. Pole Location of a Transfer Function [29]

$$\Omega_k = |\lambda_k| = \sqrt{|\sigma_k|^2 + |\omega_k|^2} \tag{2.6}$$

$$cos\beta_r = \varepsilon_r = \frac{|-\sigma_k|}{\Omega_k} \tag{2.7}$$

where λ_k is pole location for the kth mode, ω_k is damped natural frequency for the kth mode, Ω_k is undamped natural frequency for the kth mode and ε_r is damping ratio for the kth mode.

As seen in (2.4), a conversion is necessary for time domain methods to identify the pole of the system. Δt term is necessary in this formulation. The user may sometimes not have time history data. Since most of the time domain methods work with IRFs obtained by inverse FFT, the user may perform time domain analysis with only frequency data. For such cases, the following relation arising from Sampling Theory must be used in order to obtain time interval(Δt):

$$\Delta t = \frac{1}{2F_{max}} \tag{2.8}$$

where F_{max} is maximum frequency and Δt is sampling time.

2.7. Important Concepts in Modal Parameter Estimation

2.7.1. Mode Indicator Functions

Mode indication functions (MIF) indicate possible existence points of modes in frequency domain. These functions provide deeper insight into the user about the number of poles in the frequency range of interest. Many tools are available for the pole estimation. Main tools used today as mode indicator functions are:

- Multivariate Mode Indicator Function (MvMIF)
- Complex Mode Indicator Function (CMIF)

2.7.1.1. Multivariate Mode Indicator Function (MvMIF)

The modes are the points which have the lowest value in the selected frequency range of interest in Multivariate mode indicator functions. MvMIF may not give accurate results when non-real or complex modes exist in the system [30]. The number of inputs of modal testing determines the number of mode indicator functions to be plotted. Although the primary MIF exhibits local minima at the natural frequencies of the system, the higher order ones have local minima in case of repeated roots.

Since the real part of the FRF is much smaller than the imaginary part of the FRF, a minimization problem is offered [31] in order to take advantage of this fact:

$$\underset{\|F\|=1}{min} = \frac{\{F\}^t[H_{real}]^t[H_{real}]\{F\}}{\{F\}^t\left([H_{real}]^t[H_{real}]+\left[H_{imag}\right]^t\left[H_{imag}\right]\right)\{F\}} = \lambda \tag{2.9}$$

The solution of the minimization problem is to find the smallest eigenvalue λ_{min} and the corresponding eigenvector $\{F\}_{min}$:

$$[H_{real}]^t[H_{real}]\{F\} = \lambda\left([H_{real}]^t[H_{real}] + \left[H_{imag}\right]^t\left[H_{imag}\right]\right)\{F\} \tag{2.10}$$

The above eigenvalue problem is solved at each frequency. The resulting eigenvalues are plotted with respect to frequency range of interest as a plot of MvMIF. Figure 2.7 shows as an example of MvMIF.

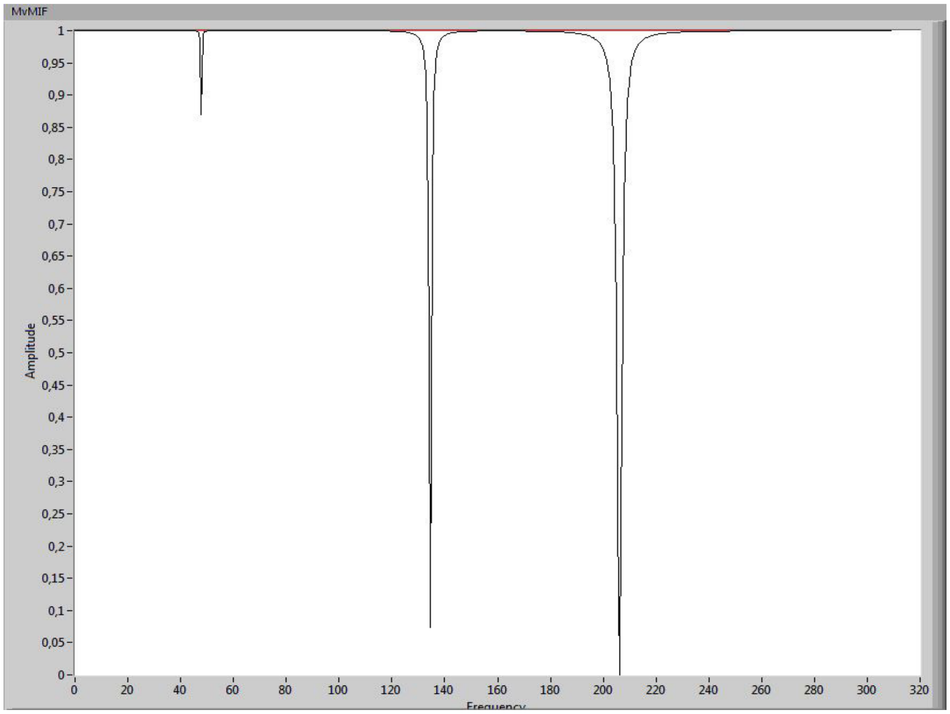


Figure 2.7. An example of MvMIF

2.7.1.2. Complex Mode Indicator Function (CMIF)

CMIF is another mode indicator function available in literature. The peaks in the CMIF plot give the damped natural frequencies for each mode. By taking the singular value decomposition of the FRF matrix at each spectral line, an equation is as follows:

$$\underset{(N_o \times N_i)}{[H(\omega)]} = \underset{(N_o \times N_e)}{[U(\omega)]}\underset{(N_e \times N_e)}{[S(\omega)]}\underset{(N_e \times N_i)}{[V(\omega)]^H} \tag{2.11}$$

where N_o is number of outputs, N_i is number of inputs, N_e is number of effective modes, $[U(\omega)]$ is left singular matrix cor. to the matrix of mode shape vectors, $[S(\omega)]$ is singular value matrix and $[V(\omega)]$ is right singular matrix cor. to the matrix of modal par. vectors.

The eigenvalues of normal matrix is another mathematical expression of CMIF. The normal matrix is written as below :

$$[H(\omega)]^H[H(\omega)] = [V(\omega)][S^2(\omega)][V(\omega)]^H \tag{2.12}$$

The CMIF plot is the plot of eigenvalues of the normal matrix on a log scale in frequency domain :

$$CMIF_k(\omega) = \mu_k(\omega) = S_k(\omega)^2 \qquad k = 1,2, \dots \dots, N_e \tag{2.13}$$

where $CMIF_k(\omega)$ is k^th^ CMIF function of frequency, $S_k(\omega)$ is k^th^ singular value of the FRF matrix at frequency ω, $\mu_k(\omega)$ is k^th^ eigenvalue of the normal matrix of FRF matrix at frequency ω.

Figure 2.8 is an example of complex mode indicator function. The frequencies where more than one curve approaches the same maximum are repeated modal frequencies.

Apart from being a mode indicator function, CMIF may serve as a powerful parameter estimation method. In addition to identifying modes, CMIF also gives global modal parameters, damped natural frequencies, mode shapes and modal participation vectors. Unscaled mode shape vectors and modal participation factors are used to obtain the enhanced FRF for each mode *(r)*. It is formulated as:

$$eFRF_k(\omega) = \{U\}_k^H [H(\omega)] \{V\}_k \tag{2.14}$$

where $eFRF_k$ is enhanced FRF function for k^{th} mode

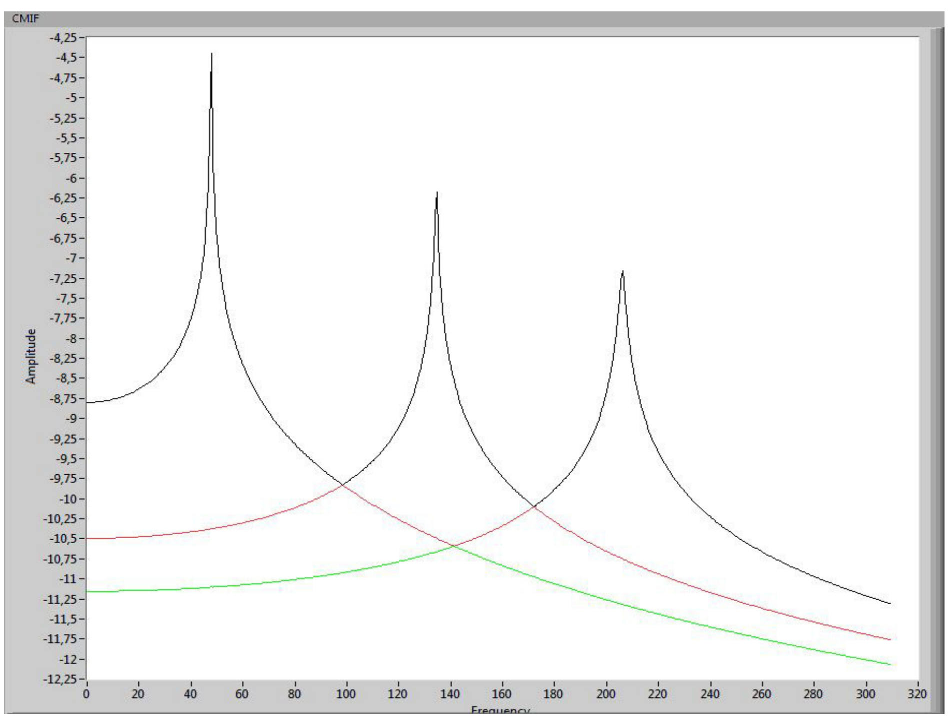


Figure 2.8. An example of CMIF

Since mode shape vectors and modal participation factors are normalized by SVD, the enhanced FRF is actually decoupled single mode response function:

$$eFRF_k(\omega) = \frac{Q_k}{\omega - \lambda_k} \tag{2.15}$$

where Q_k is the scaling factor.

At this stage, a single degree of freedom method(such as peak picking, circle-fitting) can be applied to enhance the accuracy of the natural frequency estimate. Then, damping values and scaling factors for the mode shape can be found.

2.7.1.2.1. Cross Eigenvalue Effect

Peak values of the CMIF curve may not sometimes represent damped frequencies of the structure. Errors such as noise, leakage, nonlinearity and cross-eigenvalue effect can also make a peak. Cross eigenvalue effect may occur due to the equal contribution of two modes. The eigenvectors of adjacent spectral lines must be compared to ensure that the observed peaks are physically meaningful modes [32].

Another way of understanding whether the peaks in CMIF functions are due to cross eigenvalue effect or not is to calculate MAC matrix by using data from different CMIF functions. Two arbitrarily frequency points are chosen for both CMIF functions. MAC value is checked for these frequency points. When this MAC matrix is close to the unity matrix, this means that the peak in CMIF functions really represents a resonance peak [33].

$$\begin{bmatrix} MAC(1a, 1b) & MAC(2a, 1b) \\ MAC(1a, 2b) & MAC(2a, 2b) \end{bmatrix} \cong \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{2.16}$$

where *a* & *b* represents frequencies around the peak in CMIF functions.

2.7.2. Stability Diagram

In modal analysis, stability diagram is an important tool which guides the user to identify physical poles from mathematical ones. The chart is obtained by repeating the analysis for increasing model order of characteristic polynomial equation. When the estimates of physical modal parameters stabilize, it means that the assigned model order is sufficient. Whereas the physical poles tends to be at nearly identical locations for each estimation order, non-physical ones do not stabilize during the analysis, which must be extracted out of modal parameter data. The data which shows inconsistencies is just a mathematical poles and they basically arise from noise, leakage errors etc. in the measurements. Figure 2.9 is an example of stability diagram.

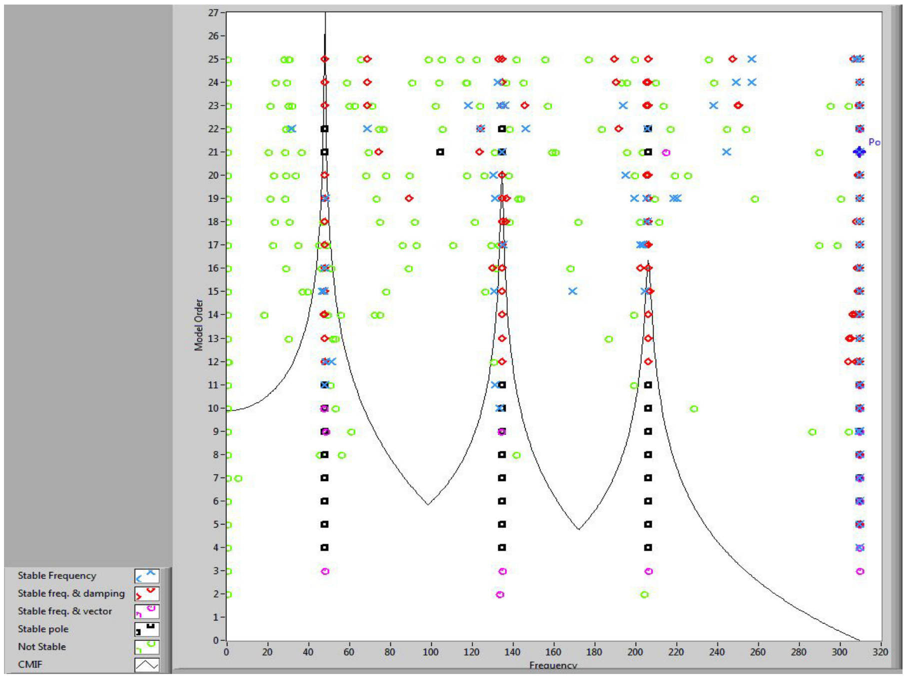


Figure 2.9. An example of Stability Diagram

2.8. Modal Data Verification

Modal data verification process enables the user to check whether modal parameters obtained through modal analysis is meaningful or not. This process usually involves numerical tabulation, plotting or animation of modal parameters. There are some ways available in the literature to validate the correctness of modal parameters.

2.8.1. FRF Synthesis

FRF Synthesis is the re-plot of the FRF based on identified modal parameters using the measured data. The plots of synthesized and measured frequency response functions shoul be look similar at all frequency points. Figure 2.10 shows a typical FRF synthesis.

Apart from visual check, a numerical value can also be given to check the validity of any FRF estimates. This numerical value is called as 'Synthesis Correlation Coefficient' and should be close to unity, if the estimate is correct.

$$COR_{pq} = \frac{\left|\sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)\widehat{H}_{pq}(\omega)\right|^2}{\sum_{\omega=\omega_1}^{\omega_2} H_{pq}(\omega)H_{pq}(\omega)\sum_{\omega=\omega_1}^{\omega_2} \widehat{H}_{pq}(\omega)\widehat{H}_{pq}(\omega)} \quad (2.17)$$

where $H_{pq}(\omega)$ is measured FRF, $\widehat{H}_{pq}(\omega)$ is synthesized FRF.

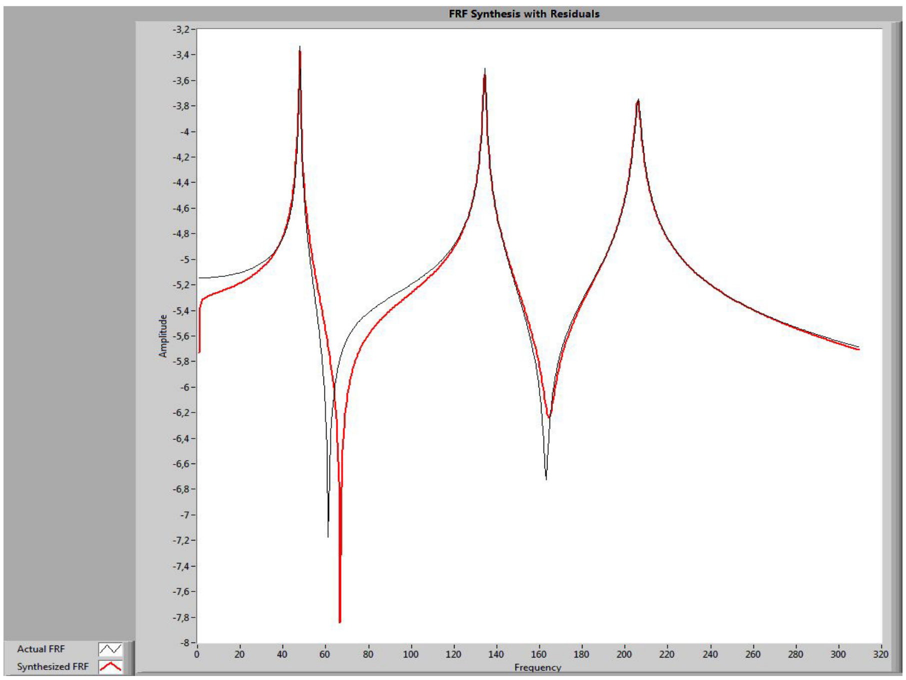


Figure 2.10. Typical FRF Synthesis

2.8.2. Modal Assurance Criterion (MAC)

MAC is a degree of measure of similarity between two modal vectors. This criterion is formulated by [29] :

$$MAC = \frac{|\{\Psi_1^{*t}\}\{\Psi_2\}|^2}{(|\{\Psi_1^{*t}\}\{\Psi_1\}|)(|\{\Psi_2^{*t}\}\{\Psi_2\}|)} \quad (2.18)$$

where $\Psi_{1,2}$ is mode shape vector obtained from method 1 & 2 and "$*$,^{t}" are the operators for complex conjugate and transpose, respectively.

MAC value is always between 0 and 1. Being MAC value unity, mode shape vectors obtained by different solution aproaches is fully correlated to each other. If that number approaches to zero, it can be inferred that correlation between vectors diminishes.

2.8.3. Modal Phase Colinearity (MPC)

The estimated mode shapes should be normal for lightly damped systems, which means that the phase angle between the real & imaginary part of mode shape coefficient should be either 0°, 180° or -180°. Modal Phase Colinearity is an index which indicates the linear functional relationship between the real & imaginary parts of each mode shape coefficient. The index should be unity for a real normal mode. A low value of MPC means that there exists error in the measured data or modal parameter estimation.

The variance and the covariance of real & imaginary parts of mode shape vectors are computed to quantify the complexity of a mode shape vector. All the formulas given below are taken from [34]:

$$\begin{aligned} S_{xx} &= \Psi_r^t \Psi_r \\ S_{yy} &= \Psi_i^t \Psi_i \\ S_{xy} &= \Psi_r^t \Psi_i \end{aligned} \tag{2.19}$$

The eigenvalues of the covariance matrix must then be calculated:

$$\begin{bmatrix} S_{xx} & S_{xy} \\ S_{xy} & S_{yy} \end{bmatrix}$$

The eigenvalues of the covariance matrix can be calculated by given formulas:

$$\begin{gathered} \lambda_{1,2} = \frac{S_{xx} + S_{yy}}{2} \pm S_{xy}\sqrt{\eta^2 + 1} \quad with \\ \eta = \frac{S_{yy} - S_{xx}}{2S_{xy}} \end{gathered} \tag{2.20}$$

The MPC index is expressed as:

$$MPC = \left(\frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2}\right)^2 \tag{2.21}$$

2.9. Commercial Softwares

2.9.1. LMS Software

LMS software is a very eficient tool to perform experimental modal analysis. It is widely used especially by defense industry. The software is able to plot mode indicator functions and stabilization diagram for mode selection. Modal parameter estimation algorithms from simple to poly-reference are included in the software.

The superiority of the software over other softwares is that it has its unique estimator which is developed in LMS test laboratories. The estimator is named as PolyMAX. PolyMAX is able to obtain very clean stabilization diagrams. Additionally, it is capable of dealing with structures which have high-damping.

The software has also automatic modal pole selection (AMPS) feature which provides substantial guidance for new users.

The software is also capable of performing operational modal analysis. The software can deal with complex structures with sufficient efficiency for modal analysis.

2.9.2. ME'scopeVES

ME'scopeVES (Visual Engineering Series) is a family of software packages which enables the user to analyze vibration problems in machinery or structures. The software can also perform analytical modal analysis apart from experimental one. ME'scopeVES has the capability for performing:

- FRF-Based Modal Analysis
- Operational Modal Analysis
- Vibro-Acoustic Analysis
- Dynamics Modelling & Simulation
- Structural Dynamics Modification
- Experimental FEA
- FEA Model Updating

ME'scopeVES estimates modal parameters (frequency, damping and mode shapes) by curve-fitting of a set of FRFs. ME'scopeVES has three kinds of modal analysis options as follows:

- Basic Modal Analysis
- Multi-Reference Modal Analysis
- Operational Modal Analysis

As stated before, multi-reference modal analysis is used with FRFs obtained from more than one reference in modal testing. Closely coupled modes can be identified with multi-reference modal analysis option. Multi-reference option modal analysis option of the method has three algorithms:

- Complex Exponential
- Z-Polynomial (LSCF) frequency domain curve fitting
- Patented Alias Free (AF) Polynomial method

The software is also able to plot mode indicatior functions and stability diagram as for LMS software.

2.9.3. Pulse Analyzer Platform (Brüel & Kjaer)

Pulse is a Brüel & Kjaer’s platform for noise and vibration analysis. It is actually hardware and software family. My main focus is on the software part of the platform, which is able to perform experimental modal analysis.

The software has various to perform modal analysis such as mode indicator functions, curve-fitters and validation tools.

As for LMS software, automated mode selection procedure is a provided option to the user. It is able to draw complex mode indicator function (CMIF) & multivariate mode indicator function (MvMIF). Various SDOF & MDOF curve fitters can be implemented by the software. SDOF curve fitters are least-squares global fraction and quadrature picking. MDOF curve fitters are RFP-Z, Polyreference Frequency, Polyreference Time, Eigensystem Realisation, Alias-free Polyreference.

The software has the enhanced mode selection algorithm in order to obtain clean stabilisation diagrams. Pole cluster diagram and pole density plots are provided to the user for mode selection in addition to stabilization diagram.

FRF Synthesis, mode shape animation, AutoMAC, CrossMAC tables and 3D plots are validation tools of the software. The software is able to perform correlation analysis between FEM vs. Test, Test vs. Test and FEM vs. FEM results.

The software has also reporting capabilities directly related to Microsoft Office applications such as Word, Excel, PowerPoint.

In summary, Pulse Reflex allows the user to perform a complete modal analysis in five main steps:

- Measurement Validation
- Parameter Estimation Setup
- Mode Selection
- Analysis Validation
- Reporting

2.9.4. STAR Modal

Star Modal is an another commericla software package for modal analysis. It provides you to analyze and present the modal results with confidence. Key features of the software can be summarized as follows :

- Modal Analysis
- Time Domain Analysis and Animation
- Operating Deflection Shapes
- Multi-Reference Curve-Fitting
- Modal Assurance Criterion
- FRF Synthesis
- Structural Dynamics Modifications
- Forced Response Simulation
- Sinusıidal Response
- Measurement Calculator
- Geometry Tools

STAR Modal software utilizes from commonly used curve-fitting algorithms in the industry in order to estimate modal parameters precisely. For simple analysis, the software has various SDOF and MDOF curve fitters.

Least Squares Complex Exponential is the multi-reference curve-fitter of the software.

The software uses MAC tool and FRF Synthesis to check the quality of your analysis.

2.9.5. Common Features of Commercial Softwares

Evaluating the overall features of the commercial softwares, common features of those of softwares can be listed as follows:

- Plotting Mode Indicator Functions
- SDOF/MDOF – Local / Global / Polyreference Curve Fitters
- Stability Diagrams for mode selection
- Validation Analysis
    - ✓ FRF Synthesis
    - ✓ Modal Assurance Criterion (MAC)

2.10. Previous Studies

There are lots of research related to modal parameter estimation. In this part of the thesis, the previous researches which have close scope to my study are investigated.

Iglesias [35] investigated various modal parameter estimation techniques to see how much accurate the damping ratios are estimated with respect to various parameters and conditions. Four modal parameter estimation techniques were selected to perform this investigation. These techniques are Complex Exponential Method (CE), which is also called as Prony algorithm, the Ibrahim Time Domain Method (ITD), the Rational Fraction Polynomial Method (RFP) and Hilbert Envelope Method. Both simulated analytical and experimental data were used during the analysis. The simulated analytical data was analyzed because the exact values of modal parameters are known, which enables the user to see the exact error in the estimation. As for the experimental data, the exact values are not known. However, the results obtained from different estimation techniques were compared for this case. RFP method is said to give the most accurate damping ratio estimate. Moreover, Hilbert Envelope Method is claimed to give more reliable results of damping ratios compared to CEM and ITD. CEM and ITD method require more user judgment because when the FRF is truncated or the mode is isolated the FRF loses information which generates leakage error in the calculated IRF. Finally, CEM and ITD method are found to be more appropriate for heavily damped structures.

Beskhyroun [36] developed a graphical user interface (GUI) based software toolbox to estimate modal parameters. The software was developed in MATLAB. The software is capable of doing modal analysis in three frequency domain and two time domain techniques. While frequency domain ones are peak picking (PP) method, the frequency domain decomposition (FDD) and the enhanced frequency domain decomposition (EFDD), time domain ones are the eigensystem realization algorithm (ERA) combined with natural excitation technique (NExT) and stochastic subspace identification (SSI) method. The software can work with either experimental (input-output data) or operational (only output) data. Identified modal parameters can be visualized in tabulated form and the results obtained from different techniques can be easily illustrated in the same fashion for identified modal parameters. In order to test the functionality of the software, a case study was performed with different data sets. It was observed that the ERA technique was the least accurate of whole techniques, for almost all data sets especially for higher modes. For some data sets, PP, FDD and EFDD techniques gave consistent results for all modes. Despite some inconsistencies in the results, the software has been proven to provide effective modal analysis for real structures.

Allemang R.J. and Brown D.L. [37] developed a formulation of modal parameter estimation algorithms into a singular mathematical representation. This is called as Unified Matrix Polynomial Approach (UMAP). The modal parameter estimation methods are categorized as zero, low, high order algorithms, respectively. This approach is used to unify the algorithm of current modal parameter estimation methods such as the least-squares complex exponential (LSCE), the polyreference time domain (PTD), Ibrahim time domain (ITD), eigensystem realization algorithm

(ERA), rational fraction polynomial (RFP), polyreference frequency domain (PFD) and the complex mode indicator function (CMIF). UMAP concept reformulates different classes of modal parameter estimation techniques into a structure of matrix polynomial model.

In the lights of above discussion, an experimental modal analysis software package is developed in Structural Dynamics Research Lab at the University of Cincinnati, whose name is X-Modal II. The software functions efficiently to determine modal parameters of a structure. The software works based on the principles of matrix polynomial approach. The software can utilize from the FRF data provided in Universal File Format (UFF). It has a graphical user interface developed in MATLAB. The solutions of all the major modal parameter estimation techniques can be handled by the software.

Peeters B. & Ventura C.E. [38] presented the results of a comparative study of various techniques in order to investigate bridge dynamic properties from experimental data. Seven different research teams came together for the study applying various types of modal analysis techniques. Selected techniques for the study are Peak-Picking (PP), complex mode indication function (CMIF), also called frequency domain decomposition (FDD), rational fraction polynomial (RFP), also called orthogonal polynomial (OP), as frequency domain methods. As for time domain methods, Ibrahim time domain (ITD), subspace identification methods (SUBSP). Variants of these methods are eigensystem realization algorithm (ERA), canonical variate analysis (CVA) and balanced realization (BR). Finally, two-stage least squares method (2LS), also called ARMAV technique. Vibration data was taken in three ways: forced, free, ambient. The Z24-Bridge, a three-span reinforced concrete bridge in Switzerland, was selected as a case study. Comparing the results of different modal analysis techniques from different excitation techniques fro large civil engineering structures was the objective of the study.

Three stabilisation diagram was obtained by applying subspace identification to the shaker, free, ambient data, respectively. The results has shown that the number of estimated poles is different for different excitation techniques, which indicates that the excitation type significantly affects the quality of identification results. In addition to this factor, applied modal analysis techniques also affect the quality of the results. Furthermore, the experience of the modal analyst, time spent to analyse the data and the capacity of the analysis computer directly play an important role in the accuracy of results.

Evaluating the overall results of modal identification techniques, researchers concluded that the multiple-input RFP method applied to the FRFs obtained from shaker and subspace methods applied to all data sets gave the the most consistent modal parameter estimates. It is also possible to obtain less consistent results with the subspace methods, if the experience of the user is not enough. Another observation of the researchers is that the stabilisation diagram is a very powerful tool to detect resonant frequencies, even for FRFs with unclear peaks.

Jianping H. *et. al.* [39] compared the identification results obtained from RFP in frequency domain and ERA in time domain for the data taken from shaker on a 12-storey reinforced concrete frame model. The results has shown that frequency values between identification methods and Finite Element Analysis (FEA) are in good agreement. On the other hand, it is observed that there exists discrepancies in damping ratios for higher modes. ERA has also a satisfactory accuracy for mode shapes compared to FEA results.

CHAPTER 3

DEVELOPMENT OF THE SOFTWARE

3.1. Introduction

The task of the developed software starts after gathering modal test data written in Universal File Format (UFF). The software enables the user to identify modal parameters of a structure utilizing from available curve-fitting algorithms operating in the frequency domain or time domain. Curve-fitting methods in time domain work based on the Inverse Fourier Transform of a FRF, corresponding to an impulse response function (IRF). The calculation of inverse FFT may cause a leakage problem. Some methods directly use the force and response measurements to overcome the leakage problem. Generally, time domain methods are chosen when dealing with large frequency range of data because better results can be obtained from time domain methods working with large number of modes in the data. When the frequency range of interest is limited, frequency domain methods should be used. However, frequency domain methods have recently become more popular than time domain methods since frequency domain methods are able to account for residual terms in the data, which improves the accuracy of the results, contrary to time domain methods. Under the lights of facts discussed above, some curve-fitting methods are chosen for the developed software.

3.2. Universal File Format (UFF)

The software is able to read UFF file. This is a special text file which includes necessary information about modal test data. The use of Universal File Format provides standardization between experimental modal analysis softwares. These formats are still being used by the experimental modal analysis community. The specific ASCII structure of several existing Universal File Formats used for experimental modal analysis are:

- UFF Dataset 151: includes Header information
- UFF Dataset 164: includes Units information
- UFF Dataset 15: includes Nodes information where the nodes attached on the structure in terms of x,y,z coordinates.
- UFF Dataset 58: includess Nodal Function information
- UFF Dataset 82: includes Trace Lines information

There exists guide file which says which line provides which information [40].

3.3. Preliminary Checks of FRF Data

Before identifying modal parameters, the software gives the user to carry out some preliminary visual checks of measured FRF data. These initial checks are advised to take in order not to waste time on data which subsequently gives inadequate results. There are some points on which the user should pay attention to the measured data.

A log-log plot of the modulus of the measured FRF is used to carry out most of the necessary checks. The measured FRF may be in the form of receptance, mobility or accelerance. The characteristics at very low frequencies is the first thing to be checked on the measured data. The user should see the trend in the data which corresponds to the support condition of the test for low frequencies of the measured data. The FRF curve should appear as asymptotic to the stiffness line at low frequencies, of which the magnitude of the curve is almost equal to the static stifness of the structure provided that test condition of the structure is grounded. As for free condition testing case, a mass-line asymptote in the curve is expected at low frequencies. If these expected trends are not observed, it means that the required support conditions have not been achieved.

The upper end of the frequency range of the measured data is an another checkpoint especially for mobility measurements. The curve may seem like asymptotic to a mass line or to a stiffness line. Such a tendency in the measured data reflects that the excitation is applied at a point of very high mass or flexibility, which makes the modal analysis process by far more difficult. If the tendency discussed above is observed, the user must choose a different excitation point to analyze.

Clearly observed resonance and antiresonance characteristics is another important point for the FRF data. Failure in sharp characteristics of resonances in the FRF data may result in poor modal parameter estimation [41].

3.4. Selected Modal Parameter Estimation Methods for the Software

Based upon literature survey, the most popular and widely used modal parameter estimation methods both for frequency and time domain are selected for the developed software.

3.4.1. Frequency Domain Methods

3.4.1.1. The Rational Fractional Polynomial Method (RFP)

The RFP is one of the most widely used MDOF frequency domain method. It is implemented in many commercial modal analysis softwares. It is a local curve-fit method and works with single FRF. The mathematical model of the RFP method is in the rational fraction polynomial form instead of general frequency domain curve fitting model, which is partial fraction form of the FRF. These two models can be seen in 3.1 and 3.2.

- Partial Fraction Form:

$$[H(s)] = \sum_{k=1}^{N}\left(\frac{[A_k]}{(s-\lambda_k)} + \frac{[A_k^*]}{(s-\lambda_k^*)}\right) \tag{3.1}$$

- Rational Fraction Form:

$$[H(s)] = \frac{\sum_{k=0}^{2N-1} \alpha_k (s)^k}{\sum_{k=0}^{2N} \beta_k (s)^k} \tag{3.2}$$

$where:$

$\alpha_k = numerator\ polynomial\ coefficients$

$\beta_k = denominator\ polynomial\ coefficients$

The unknown numerator and denominator polynomial coefficients of set of linear equations are solved by the RFP method. Knowing polynomial coefficients, poles and residues can be determined by numerical root solving and partial fraction expansion, as illustrated in Figure 3.1.

The whole procedure of the method is fully referenced by [18]. First thing to do is to define an error function getting into problem formulation. Error function is defined as a measure of the error at each value of frequency and obtained from rational fraction polynomial formula (3.2):

$$e_i = \sum_{k=0}^{m} a_k (j\omega_i)^k - h_i \left[\sum_{k=0}^{n} b_k (j\omega_i)^k + (j\omega_i)^n \right] \tag{3.3}$$

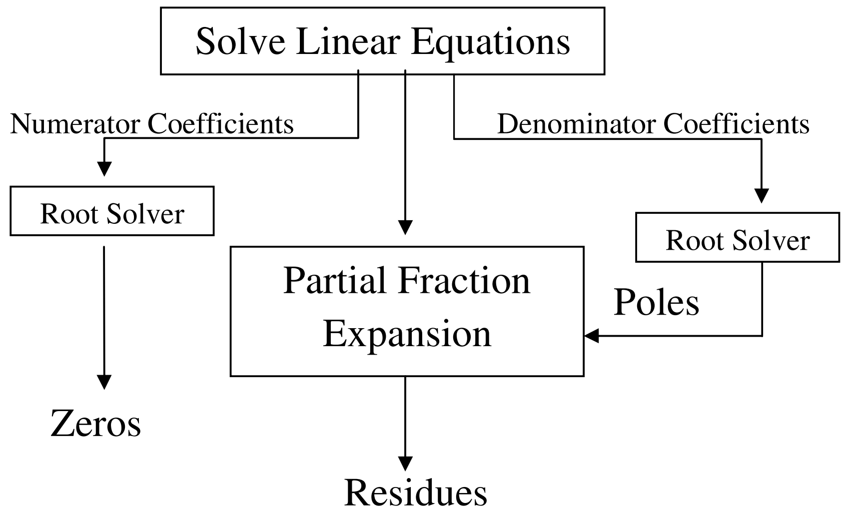


Figure 3.1. RFP Solution Procedure [42]

Error function can then be reformulated by an entire vector of error:

$$Error\ Vector = \{E\} = \begin{Bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{Bmatrix} \tag{3.4}$$

The error vector can be rewritten in matrix form:

$$\{E\} = [P]\{A\} - [T]\{B\} - \{W\} \tag{3.5}$$

$where:$

$$[P] = \begin{bmatrix} 1 & \cdots & (j\omega_1)^m \\ \vdots & \ddots & \vdots \\ 1 & \cdots & (j\omega_L)^m \end{bmatrix} \qquad (\text{L} \times \text{m+1})$$

$$[T] = \begin{bmatrix} h_1 & \cdots & h_1(j\omega_1)^{n-1} \\ \vdots & \ddots & \vdots \\ h_L & \cdots & h_L(j\omega_L)^{n-1} \end{bmatrix} \qquad (\text{L} \times \text{n})$$

$$\{A\} = \begin{Bmatrix} a_0 \\ a_1 \\ \vdots \\ a_m \end{Bmatrix}, \quad \{B\} = \begin{Bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \end{Bmatrix}, \quad \{W\} = \begin{Bmatrix} h_1(j\omega_1)^n \\ h_2(j\omega_2)^n \\ \vdots \\ h_L(j\omega_L)^n \end{Bmatrix}$$

A squared error criterion can be written from error function. Main idea is to try to minimize that equation.

$$J = \sum_{i=1}^{L} e_i{}^* \, e_i = \{E^*\}^t\{E\} \tag{3.6}$$

The polynomial coefficients can be obtained by minimizing equation (3.6). However, Miramand N. *et. al* [43] claim that solutions obtained from these equations are generally ill-conditioned. Therefore, reformulation is required as follows:

$$H(\omega_i) = \frac{\sum_{k=0}^{m} c_k \varphi_{i,k}{}^+}{\sum_{k=0}^{n} d_k \theta_{i,k}{}^+} \qquad i = 1, \dots, L \tag{3.7}$$

$where:$

$\sum_{i=1}^{L} \left(\varphi_{i,k}{}^+\right)^* \varphi_{i,j}{}^+ = \begin{cases} 0, k \neq j \\ 0.5, k = j \end{cases}$

$\sum_{i=1}^{L} \left(\theta_{i,k}{}^+\right)^* |h_i|^2 \theta_{i,j}{}^+ = \begin{cases} 0, k \neq j \\ 0.5, k = j \end{cases}$

$|h_i|^2: weighting\ function(square\ of\ the\ FRF\ in\ magnitude)$

After finding orthogonal polynomial coefficients, next step is to turn back to the ordinary polynomial coefficients. Then, modal parameters can easily be calculated.

In Richardson & Formenti's study, orthogonal polynomials are generated by Forsythe method. Orthogonal polynomials can be also obtained by other orthogonal polynomial approaches like Legendre, Chebyshev polynomials etc.

3.4.1.2. The Global Rational Fractional Polynomial Method (GRFP)

This method is the extension of the RFP method to analyze modal parameters in global sense, using a set of FRFs in one single input reference. Identifying modal parameters in global sense was developed in references [19] and [20]. Therefore, all formulas used below are taken from references [19] & [20].

Using several FRFs to determine the modal frequency and damping ratios causes overdetermined set of equations (3.8), which means that the number of equations is greater than the number of unknowns.

$$\begin{bmatrix} U_1 \\ \dots \\ U_2 \\ \dots \\ \vdots \\ \dots \\ U_P \end{bmatrix} \{B\} = \begin{bmatrix} V_1 \\ \dots \\ V_2 \\ \dots \\ \vdots \\ \dots \\ V_P \end{bmatrix} \quad (n \times p)\ equations \tag{3.8}$$

Least squared error solution:

$$\sum_{k=1}^{P} [U_k]^2 \{B\} = \sum_{k=1}^{p} [U_k] \{V_k\} \tag{3.9}$$

Solving (3.9) by pseudo-inverse gives (3.10):

$$\{B\} = ([U_k]^t [U_k])^{-1} [U_k]^t \{V_k\} \tag{3.10}$$

After solving (3.10), polynomial coefficients for the denominator can be found. Then a polynomial solver can be used to determine modal frequency and damping estimates in global sense.

3.4.1.3. The Polyreference Frequency Domain Method (PFD)

The governing equation for a linear mechanical system with N degrees of freedom and N_i inputs can be described by a set of N second order differential equations [27]:

$$[M]\{\ddot{x}(t)\} + [C]\{x(\dot{t})\} + [K]\{x(t)\} = \{f(x(t)\} \tag{3.11}$$

At this point, the state space expansion is implemented to reduce the system into a first order system and formulate a new eigenvalue problem:

$$\{z\} = \begin{bmatrix} \dot{x} \\ x \end{bmatrix} \tag{3.12}$$

The system of second order differential equation can be rewritten in terms of the state space response vector as follows:

$$\begin{bmatrix} [M] & [C] \\ [0] & [I] \end{bmatrix} \{\dot{z}\} + \begin{bmatrix} [0] & [K] \\ -[I] & [0] \end{bmatrix} \{z\} = \begin{Bmatrix} \{f\} \\ \{0\} \end{Bmatrix} \tag{3.13}$$

The response vector turns out to be impulse response function for unit impulsive loads. The state space form of the equations of motion becomes:

$$[A]\begin{Bmatrix}\{\ddot{h}\}\\ \{\dot{h}\}\end{Bmatrix}+[B]\begin{Bmatrix}\{\dot{h}\}\\ \{h\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix} \tag{3.14}$$

Finite difference form of the above equation is as follows:

$$[A_1]\begin{Bmatrix}\{h_{m+2}\}\\ \{h_{m+1}\}\end{Bmatrix}+[A_0]\begin{Bmatrix}\{h_{m+1}\}\\ \{h_{m+0}\}\end{Bmatrix}=\begin{Bmatrix}\{0\}\\ \{0\}\end{Bmatrix} \tag{3.15}$$

Taking Laplace transform of the above equation and considering the harmonic response of the system:

$$[A]\begin{Bmatrix}-\omega^2\{X(\omega)\}\\ j\omega\{X(\omega)\}\end{Bmatrix}+[B]\begin{Bmatrix}j\omega\{X(\omega)\}\\ \{X(\omega)\}\end{Bmatrix}=\begin{Bmatrix}\{F(\omega)\}\\ \{0\}\end{Bmatrix} \tag{3.16}$$

The above equation comes out to be for unit harmonic response at a given point:

$$[A]\begin{Bmatrix}-\omega^2\{H(\omega)\}_i\\ j\omega\{X(\omega)\}_i\end{Bmatrix}+[B]\begin{Bmatrix}j\omega\{H(\omega)\}_i\\ \{X(\omega)\}_i\end{Bmatrix}=\begin{Bmatrix}\{1\}_i\\ \{0\}\end{Bmatrix} \tag{3.17}$$

A more general form of previous equation can be written as:

$$[A_1]\begin{Bmatrix}-\omega^2\{H(\omega)\}_i\\ j\omega\{H(\omega)\}_i\end{Bmatrix}+[A_0]\begin{Bmatrix}j\omega\{H(\omega)\}_i\\ \{X(\omega)\}_i\end{Bmatrix}=(\cdots+[B_0]+j\omega[B_1]+\cdots)\begin{Bmatrix}\{1\}_i\\ \{0\}\end{Bmatrix} \tag{3.18}$$

A & B matrices are calculated from FRF and frequency information. Either A_0 & A_1 has to be assumed to be identity matrix to solve the system of equations. Assuming A_1 is equalto identity and reformulating (3.18) with FRF information at each frequencies, following equation is written:

$$\begin{bmatrix}[A_0] & [B_0] & [B_1] & \cdots\end{bmatrix}\begin{bmatrix} j\omega_1\{H(\omega_1)\}_i & \cdots & j\omega_k\{H(\omega_k)\}_i \\ \{H(\omega_1)\}_i & \cdots & \{H(\omega_k)\}_i \\ \{1\}_i & \cdots & \{1\}_i \\ \{0\} & \cdots & \{0\} \\ j\omega_1\{1\}_i & \cdots & j\omega_k\{1\}_i \\ \{0\} & \cdots & \{0\} \\ \vdots & \cdots & \vdots \end{bmatrix} \tag{3.19}$$

$$=\begin{bmatrix} -\omega_1{}^2\{H(\omega_1)\}_i & \cdots & -\omega_k{}^2\{H(\omega_k)\}_i \\ j\omega_1\{H(\omega_1)\}_i & \cdots & j\omega_k\{H(\omega_k)\}_i \end{bmatrix}$$

Since PFD works with large amount of spatial information, condensation must be applied to reduce data to be processed. Singular value decomposition is an efficient tool for condensation.

$$[H(\omega)]_{condensed} = [U]_{1\ldots n}^{t}[H(\omega)] \tag{3.20}$$

$where:$

$[H(\omega)] = [U(\omega)][S(\omega)][V(\omega)]^H$

$n = number\ of\ modes$

Once $[H]$ is condensed, parameter estimation procedure as for the full data set is followed. The poles of the condensed data are the same as the poles of the full data set. However, modal vectors are not the same. They must be transformed back into full space.

$$[\Psi] = [U]_{1\ldots n}[\Psi'] \tag{3.21}$$

$where:$

$[\Psi] = full - space\ modal\ vectors$

$[\Psi'] = condensed - space\ modal\ vectors$

Above all, the identification procedure can be summarized as follows:

1. First step is to discover how many modes exist in the system. Mode indicator functions can be utilized for this purpose.

2. Secondly, frequency range of interest, the number of modes & the number of B terms are selected. Then, condensation is applied.

3. A_0 matrix is computed using equation (3.19) and an eigenvalue problem is formed to calculate Modal Frequencies & Modal Vectors.

$$[A_0 - \lambda I] = 0 \tag{3.22}$$

4. The whole three steps are repeated until all Modal Frequencies & Modal Vectors are identified.

The limitation criterion of the method is that since the estimated matrix A_0 is $N_e \times N_e$ matrix, which is equal to number of effective modes, the number of independent response measurements must be higher than the number of effective modes of the system.

3.4.2. Time Domain Methods

3.4.2.1. The Complex Exponential Method (CE)

The CE method works with one impulse response function (IRF) at a time obtained from inverse fourier transform of a FRF. The algorithm of the method is taken from [45].

$$h_{jk}(t) = \sum_{r=1}^{2N} {A_{jk}}^{r} e^{\lambda_r t} \tag{3.23}$$

Real-valued time response at a series of L equally spaced time intervals Δt is written as:

$$\begin{aligned} h_0 &= h(0) = \sum_{r=1}^{2N} A_r{}' \\ h_1 &= h(\Delta t) = \sum_{r=1}^{2N} A_r{}' \, e^{\lambda_r \Delta t} \\ &\vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ &\vdots \qquad \vdots \qquad \vdots \qquad \vdots \\ h_L &= h(L\Delta t) = \sum_{r=1}^{2N} A_r{}' \, e^{\lambda_r (L\Delta t)} \end{aligned} \tag{3.24}$$

or:

$$\begin{aligned} h_0 &= \sum_{r=1}^{2N} A_r{}' \\ h_1 &= \sum_{r=1}^{2N} A_r{}' \, V_r \\ &\vdots \qquad \vdots \qquad \vdots \\ &\vdots \qquad \vdots \qquad \vdots \end{aligned} \tag{3.25}$$

$$h_L = \sum_{r=1}^{2N} A_r{}' \, V_r{}^L$$

$where:$

$V_r = e^{\lambda_r \Delta t}$

Unknown variables of $A_r{}'$ and V_r can be calculated using the Prony's method. Prony method states that an underdamped system has always the roots λ_r in complex conjugate pairs. Thus, a polynomial with V_r of order L with real coefficients β can always be written as:

$$\beta_0 + \beta_1 V_r + \beta_2 V_r{}^2 + \cdots + \beta_L V_r{}^L = 0 \tag{3.26}$$

After some mathematical manipulations, the coefficients β can be calculated by the following equation:

$$\begin{bmatrix} h_0 & \cdots & h_{2N-1} \\ \vdots & \ddots & \vdots \\ h_{2N-1} & \cdots & h_{4N-2} \end{bmatrix} \begin{Bmatrix} \beta_0 \\ \vdots \\ \beta_{2N-1} \end{Bmatrix} = - \begin{Bmatrix} h_{2N} \\ \vdots \\ h_{4N-1} \end{Bmatrix} \tag{3.27}$$

or simply:

$$\underset{(2N \times 2N)}{[h]} \underset{(2N \times 1)}{\{\beta\}} = \underset{(2N \times 1)}{\{h'\}} \tag{3.28}$$

From (3.28), the coefficients β can be found. The roots V_r can be calculated after obtaining the coefficients β. Since the roots V_r are directly related to the roots λ_r, natural frequencies and damping ratios can be calculated using complex conjugate pairs of λ_r.

3.4.2.2. The Least-Squares Complex Exponential Method (LSCE)

Exactly same case is valid between CE & LSCE as in the case of RFP & GRFP. The LSCE is the extension of CE method. It performs the analysis using several IRF's at one single reference. Therefore, it overcomes the variation problem in the results obtained from the CE method.

Equation (3.28) in CE method provides the solution for modal parameters. As for this case, they must be same for every IRF used. Therefore, equation (3.29) must be written as:

$$\begin{bmatrix} [h]_1 \\ [h]_2 \\ \vdots \\ \vdots \\ [h]_p \end{bmatrix} \{\beta\} = \begin{bmatrix} \{h'\}_1 \\ \{h'\}_2 \\ \vdots \\ \vdots \\ \{h'\}_p \end{bmatrix} \tag{3.29}$$

or simply:

$$\underset{(2Np \times 2N)}{[h_G]} \underset{(2N \times 1)}{\{\beta\}} = \underset{(2Np \times 1)}{\{h_G{}'\}} \tag{3.30}$$

Since the number of equations is more than the number of unknowns, equation (3.30) can be solved by pseudo-inverse technique:

$$\{\beta\} = \left[[h_G]^t [h_G]\right]^{-1} [h_G]^t \{h_G{}'\} \tag{3.31}$$

The rest of the solution is same as CE method. After finding coefficients β, the values of V_r can be obtained.

The estimation of the correct number of modes is stil a problem, as for the CE method. The rank of matrix $[h_G]$ can be used as a sign to estimate the correct number of modes [7]. The procedure of the CE and LSCE method can be summarized in Figure 3.2.

3.4.2.3. Ibrahim Time Domain Method (ITD)

The ITD method was firstly presented as identification method from free decay responses. Later, it evolved to a IRF-driven modal parameter estimation method. As in the case of PFD, a second order differential equation for a linear mechanical system can be written in terms of state response vector:

$$\begin{bmatrix} [M] & [C] \\ [0] & [I] \end{bmatrix} \begin{Bmatrix} \dot{z} \\ z \end{Bmatrix} + \begin{bmatrix} [0] & [K] \\ -[I] & [0] \end{bmatrix} \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix} = \begin{Bmatrix} \{f\} \\ \{0\} \end{Bmatrix} \tag{3.32}$$

$where:$

$\{z\} = [\dot{x}]$

Multiplying (3.32) by $[M]^{-1}$:

$$\begin{bmatrix} [I] & [0] \\ [0] & [I] \end{bmatrix} \begin{Bmatrix} \dot{z} \\ z \end{Bmatrix} + \begin{bmatrix} [M^{-1}C] & [M^{-1}K] \\ -[I] & [0] \end{bmatrix} \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix} = \begin{Bmatrix} [M]^{-1}\{f\} \\ \{0\} \end{Bmatrix} \tag{3.33}$$

Rewriting the above equation:

$$[A_1]\{\dot{q}\} + [A_0]\{q\} = [B_0]\{f\} \tag{3.34}$$

$where:$

$\{q\} = \begin{Bmatrix} \dot{x} \\ x \end{Bmatrix}$, $\{\dot{q}\} = \begin{Bmatrix} \dot{z} \\ z \end{Bmatrix} = \begin{Bmatrix} \ddot{x} \\ \dot{x} \end{Bmatrix}$

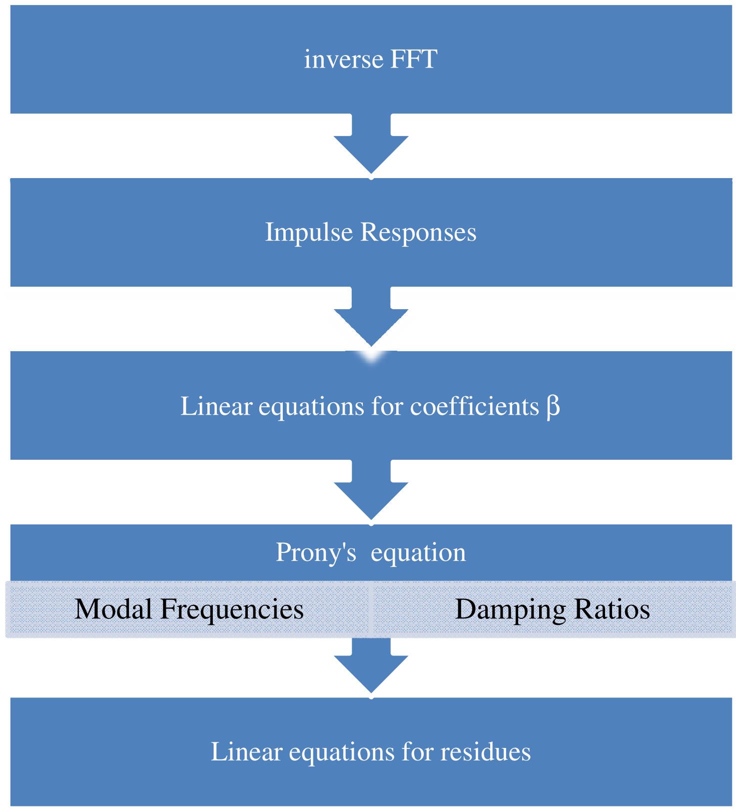


Figure 3.2. Procedure of the CE and LSCE method [3]

Considering free decay condition, the equation turns out to be a eigenvalue problem:

$$[A_1]\{\dot{q}\} + [A_0]\{q\} = 0 \tag{3.35}$$

$$[A_1]\begin{Bmatrix} x_{m+2} \\ x_{m+1} \end{Bmatrix} + [A_0]\begin{Bmatrix} x_{m+1} \\ x_{m+0} \end{Bmatrix} = 0 \tag{3.36}$$

$where:$

$[A_1] = [I]$

$[h_{m1}] = \begin{Bmatrix} x_{m+2} \\ x_{m+1} \end{Bmatrix}$

$[h_{m0}] = \begin{Bmatrix} x_{m+1} \\ x_{m+0} \end{Bmatrix}$

$[A_0] = -[h_{m1}][h_{m0}]^{-1}$

Finally, equation (3.36) comes out to be:

$$\left[[I] + [A_0]\right]\{x_z\} = 0 \tag{3.37}$$

The eigenvectors of the above equation are the modal vectors of the system. Modal frequencies can be found by applying z-transform to the eigenvalues of the above equation.

3.4.2.4. The Polyreference Complex Exponential Method (PRCE)

This method is the MIMO version of previously discussed LSCE method. The method identifies the modal parameters several IRFs from several output & input locations. The full algorithm of the method is referred to [45]. Extending the equation in CE & LSCE for every input & output locations leads to:

$$\underset{(qxLq)}{[B_T]} = \underset{(qxLq)}{[h_T{}'][h_T]^t}\underset{(LqxLq)}{([h_T][h_T]^t)^{-1}} \tag{3.38}$$

Knowing the coefficient matrix $[B]$, equation (3.81) can now be used to solve the eigenvalues $[V]$. The matrix in the following equation is called as companion matrix in literature:

$$\underset{(LqxLq)}{\begin{bmatrix} -[\beta_{L-1}] & -[\beta_{L-2}] & \vdots & -[\beta_1] & -[\beta_0] \\ [I] & [0] & \vdots & [0] & [0] \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ [0] & [0] & \vdots & [I] & [0] \end{bmatrix}} \underset{(Lqx1)}{\begin{Bmatrix} \{z_{L-1}\} \\ \{z_{L-2}\} \\ \vdots \\ \{z_0\} \end{Bmatrix}} = V_r \underset{(Lqx1)}{\begin{Bmatrix} \{z_{L-1}\} \\ \{z_{L-2}\} \\ \vdots \\ \{z_0\} \end{Bmatrix}} \tag{3.39}$$

The modal frequencies and damping ratios can be calculated from the eigenvalues and eigenvectors of the companion matrix.

3.4.2.5. Eigensystem Realization Algorithm (ERA)

This is a MIMO method in time domain, developed by Juang & Pappa [14]. Clear explanation of main steps is discussed below [45]. The algorithm is based on a state-space formulation. The basic equation of the method is:

$$\left[[h(t_{i+0})][h(t_{i+1})]\right]_{N_i \times 2N_o}[\alpha_o]_{2N_o \times 2N_o} = -\left[[h(t_{i+1})][h(t_{i+2})]\right]_{N_i \times 2N_o} \tag{3.40}$$

The Hankel matrix is formed, which is composed by Markov parameters:

$$\underset{(prxqs)}{[H(k-1)]} = \begin{bmatrix} [X(k)] & \cdots & [X(k+j)] \\ \vdots & \ddots & \vdots \\ [X(k+i)] & \cdots & [X(k+i+j)] \end{bmatrix} \tag{3.41}$$

$where:$

$i = 1, \dots, r-1 \;\; and \;\; j = 1, \dots, s-1 \; as \; integers.$

$$\underset{(pxq)}{[X(k)]} = \underset{(px2N)}{[R]}\underset{(2Nx2N)}{[A]^{k-1}}\underset{(2Nxq)}{\{B\}} \tag{3.42}$$

$where$:

$\{x(t)\} = [R]\{u(t)\}$

$[A] = e^{[A']\Delta t} \quad , \quad [A'] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}$

$[B] = \int_0^{\Delta t} e^{[A']\tau'} d\tau' [B'] \quad , \quad [B'] = \begin{bmatrix} [0] \\ -[M]^{-1}[F] \end{bmatrix}$

The pseudo-inverse of $[H(0)]$ can be calculated by the singular value decomposition(SVD):

$$\underset{(prxqs)}{[H(0)]} = \underset{(prx2N)}{[U_{2N}]}\underset{(2Nx2N)}{[\Sigma_{2N}]}\underset{(2Nxqs)}{[V_{2N}]^t} \tag{3.43}$$

After some mathematical manipulations, realized matrices can be calculated by equations:

$$\begin{gathered} [R] = \left[E_p\right]^t [U_{2N}][\Sigma_{2N}]^{1/2} \\ [A] = \left[[\Sigma_{2N}]^{-1/2}[U_{2N}]^t[H(1)][V_{2N}][\Sigma_{2N}]^{-1/2}\right] \\ [B] = [\Sigma_{2N}]^{1/2}[V_{2N}]^t\left[E_q\right] \end{gathered} \tag{3.44}$$

$where$:

$$\underset{(pxpr)}{\left[E_p\right]^t} = \left[\underset{(pxp)}{[I]} \quad \underset{(pxp)}{\cdots} \quad \underset{(pxp)}{[0]}\right]$$

$$\underset{(qsxq)}{\left[E_q\right]} = \begin{bmatrix} [I] \\ \vdots \\ [0] \end{bmatrix}$$

Then, modal parameters can be derived from an eigenproblem based on matrix $[A]$:

$$[A]\{\Psi_u\} = \lambda\{\Psi_u\} \tag{3.45}$$

The following relation is used in order to obtain the mode shapes in terms of physical coordinates of the system:

$$\underset{(px1)}{\{\Psi_x\}} = \underset{(px2N)}{[R]}\underset{(2Nx1)}{\{\Psi_u\}} \tag{3.46}$$

In summary, a flowchart of the procedure to perform modal parameter estimation with the ERA is presented as follows in Figure 3.3 [14].

Modal Amplitude Coherence [14], defined as the coherence between each modal amplitude, is the suggested check method to distinguish between system and noise modes. However, stability diagram is used in the developed software as a tool to identify computational modes for all the methods.

3.5. Summary of Modal Parameter Estimation Procedure

Selected modal parameter estimators for the software have different features. At least one local, global & poly-reference curve-fitter method for both frequency & time domain is included for the software. Mathematical models of the methods can be grouped into two as polynomial & state-space models. Local, global & poly-reference curve-fitters give different sizes of polynomial coefficients to solve the poles of the system. Depending on the size of polynomial coefficients, modal parameters are determined by finding the roots of the polynomial equation or solving an eigenvalue problem.

The construction of stabilization diagram also varies due to the sizes of polynomial coefficients. While the construction of stabilization diagram is based on frequency & damping estimates for local & global methods, it is based on frequency, damping & modal vectors estimates for poly-reference ones. The size of polynomial coefficients can also vary with respect to output & input locations. Table 3.1 shows a comparison table of selected modal parameter estimators for the software.

Although some steps can vary with different algorithms, the whole modal parameter estimation procedure can somehow be unified and summarized as follows:

- The matrix polynomial coefficients are determined in a least-squares sense from a set of linear equations obtained by measured FRFs.
- Stability Diagram is constructed by finding the roots of polynomial coefficients or solving an eigenvalue problem with respect to increasing model orders of polynomials or different subspace model sizes. This

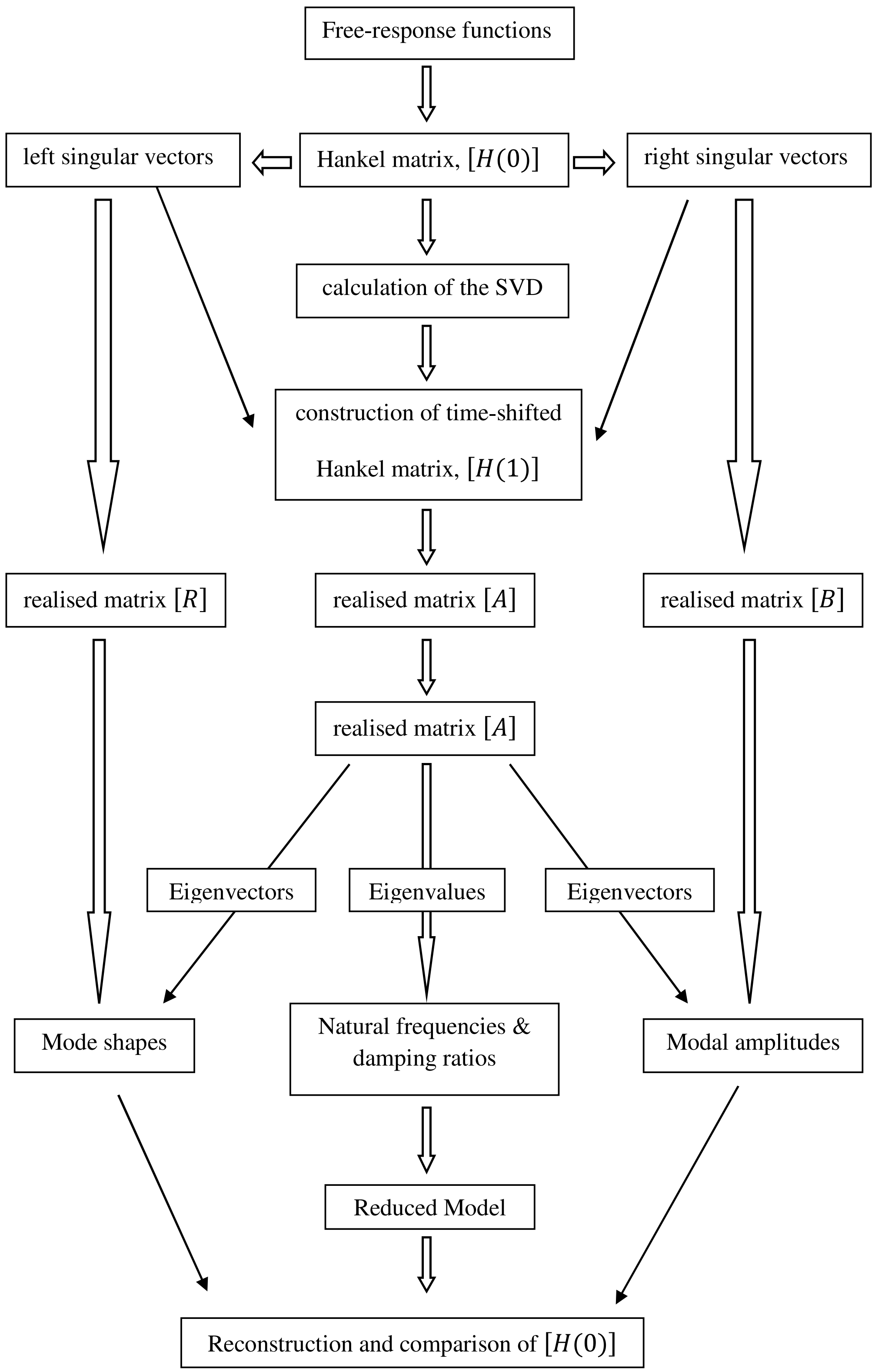


Figure 3.3. Flow Chart of ERA [14]

diagram involves the estimate information of modal frequencies, damping ratios, modal vectors or modal participation factors.

- Based on the user-interpretation of the stability diagram, modal frequency estimates which have physical meaning for the structure are selected. Damping ratios & modal vectors can also be determined from the selected modal frequencies provided that the size of polynomial coefficients is a matrix. Otherwise, only damping ratios can be determined.
- Residue terms, the lower and upper residuals of FRFs can then be calculated by solving equation (3.47) in a least-squares sense.

$$\underset{(pxq)}{[H(\omega)]} = \sum_{r=1}^{N} \frac{A_{pqr}}{j\omega - \lambda_r} + \frac{A^*_{pqr}}{j\omega - \lambda^*_r} - \frac{LR_{pq}}{\omega^2} + UR_{pq} \tag{3.47}$$

- If the size of polynomial coefficients is qxq , modal vectors can not be directly determined since stability diagram contains the information of modal participation factors. An additional computation (3.48) is required to calculate modal vectors or mode shapes.

$$A_{pqr} = L_{qr}\Psi_{pr} \tag{3.48}$$

$where:$

$$[\Psi_{pr}] = \begin{Bmatrix} \Psi_{p1} \\ \Psi_{p2} \\ \vdots \\ \Psi_{p2N} \end{Bmatrix} \qquad [L_{rq}] = \begin{bmatrix} L_{11} & L_{12} & \vdots & \vdots & L_{12N} \\ L_{21} & L_{22} & \vdots & \vdots & L_{22N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ L_{q1} & L_{q2} & \vdots & \vdots & L_{q2N} \end{bmatrix}$$

$\Psi_{pr} = modal\ vectors$

$L_{qr} = modal\ participation\ factors$

3.6. Graphical User Interface (GUI) Design of the Software

LabVIEW is chosen as software development environment. A graphical user interface (GUI) is designed for the software. The software serves as a modal parameter estimation toolbox for the user. Being compact, efficient, user-friendly and easy to handle are taken into consideration while developing the software by the developer. Therefore, LabVIEW is chosen as the development environment since it is the easiest one among programming languages to handle the purposes of the developer which are discussed above.

Table 3.1. Comparison Table of Selected Modal Parameter Estimators

	Domain		Coefficients		Stability Criteria	
Algorithm	**Frequency**	**Time**	**Scalar**	**Matrix**	**Freq. & Damp.**	**Freq. & Damp. Vector**
RFP	•		•		•	
GRFP	•		•		•	
PFD	•			$p \times p$		•
CE		•	•		•	
LSCE		•	•		•	
ITD		•		$p \times p$		•
PTD		•		$q \times q$		•
ERA		•		$p \times p$		•

where p,q are the number of output & input locations, respectively.

It is very easy to create GUIs by using LabVIEW. When the developer wants to create a control or an indicator object on the interface, the only thing to do is to pick up the control unit from controls palette of the software and place onto the front panel. Its icon is automatically created on the block diagram of the software. While front panel serves as a graphical user interface, block diagram serves as a programming environment. Contrary to other text-based programming languages, LabVIEW has graphical based programming (GBP) technology. Thanks to GBP, the developer does not have to correlate a control unit on the front panel and its icon on the block diagram with the code. They are already correlated automatically. Programming is done by wiring different icons or symbols on the block diagram to each other. Another advantage of LabVIEW, maybe the most important one, is not to worry about the memory of the computer. In C/C++, C# etc. programming languages, the user must use pointers in order not to occupy the memory of the computer so much by the defined variables. However, LabVIEW manages memory for you and you do not have to think about arranging the memory by pointers.

The software involves five tabs. Each tab accomplishes a different defined task for the modal parameter estimation. In this section, the usage of each tab of the software is introduced.

3.6.1. Test Parameters Tab

The first tab in the modal parameter estimation toolbox is *"Test Parameters"* tab, as shown in Figure 3.4. First thing to do of the user is to open the input file which includes FRFs obtained from modal testing. The software is able to read UFF file, so the input file must be in UFF. The user must give the address from *"Open Input File"* path control and press the *"Open"* button. As soon as the user presses the *"Open"* button, he will see output & input degree of freedoms of the structure, where the FRFs are taken from the structure. A degree of freedom is identified by a node number and its direction.

The purpose of this tab is to give the user a visual check of gathered FRF files. The user must select output & input DOFs from the given listboxes and click the *"FRF"* button to plot the selected FRFs. The user can see the different versions of FRFs such as magnitude, real part, imaginary part & phase of FRFs by selecting *"FRF Plot Type"* from menu ring control. If the user wants to see impulse responses, *"IRF"* button must be used.

If the user does not want to include all the frequencies in the analysis, he can use *"Lower & Upper Limit"* cursors seen on FRF plot. By moving these cursors, frequency range of interest can be adjusted. The location of the cursors can be followed by numeric controls entitled as *"Lower Limit"* & *"Upper Limit"*. Furthermore, the user can plot the mode indicator functions by clicking *"CMIF"* & *"MvMIF"* buttons. The user can add or subtract FRFs by choosing from listboxes. When the user decides the FRFs to be involved in the analysis, he must choose them from listboxes and click *"Pick Data"* button.

3.6.2. Pole Results Tab

Once the user picks the FRF that is to be included in the analysis, the user must move to the *"Pole Results"* tab, shown in Figure 3.5. Firstly, the user must select the method from *"Select Method"* menu ring control. After selecting the method, the user enters the necessary inputs for the selected method from string or numeric controls entitled as *"Model Order"*, *"Number of Iterations"*. The input varies with the selected method. Then, the user clicks the *"Plot"* button to plot stability diagram to see the different sets of pole estimates.

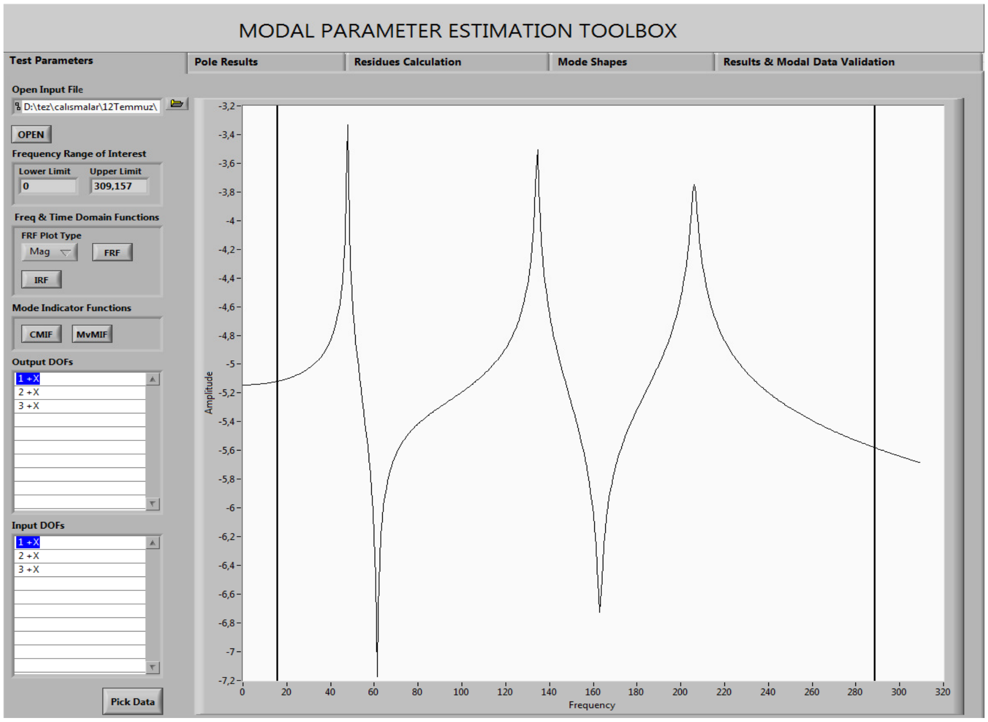


Figure 3.4. Test Parameters Tab

Once the stability diagram is plotted, a cursor is seen on the diagram named as *"Pole Detector"* sticking to stable freq.& damping or stable pole estimates depending on the selected method. Using this detector, modal frequencies of the structure can be identified by the user. The user can pick the identified modal frequencies by moving the cursor on stable poles estimates. Whatever the user makes choice, *"Add"* is to be pressed and the identified frequency value can be seen on *"Selected Modes"* listbox. When the user picks the unwanted result, he can delete the result by clicking *"Delete"* button. Similarly, *"Delete All"* button deletes the whole result in the listbox. When the number of modes to be picked is complete, the user can click *"Save"* button to save modal parameters in the memory.

There is a button on the bottom left corner of the *"Pole Results"* tab named as *"Stability Criteria"*. Clicking on this button, a pop-up window opens (Figure 3.6). The values seen on Figure 3.6 are the default values for stability criteria of stable pole estimates in terms of frequency, damping & vector. Stability Criteria affects the stability check to construct stability diagrams. Upon request, the user can change the stability criteria for different type of estimates by changing their numeric control values. Whenever the user clicks on *"Cancel"* button, the pop-up window closes up and the program returns to *"Pole Results"* tab.

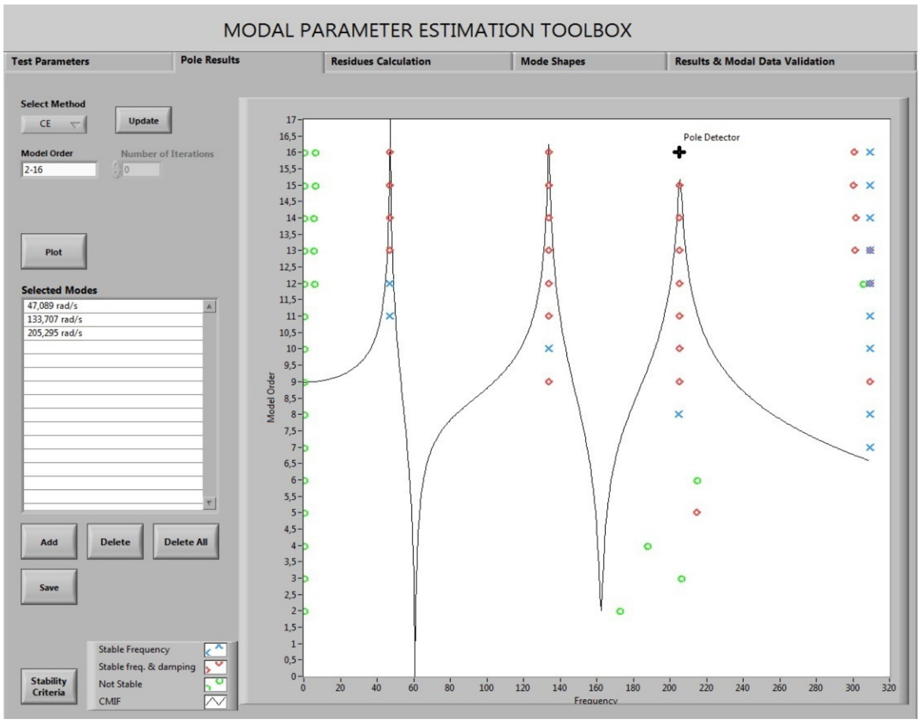


Figure 3.5. Pole Results Tab

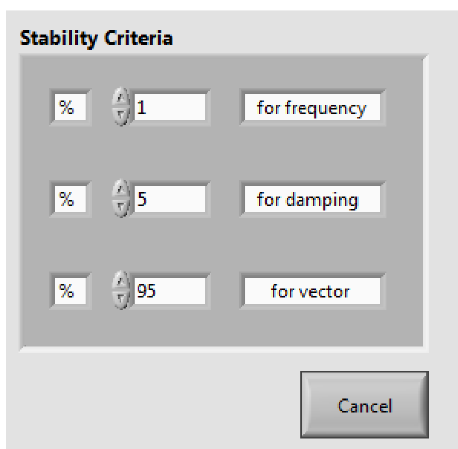


Figure 3.6. Pop-Up Window of Pole Results Tab

Finally, *"Update"* button is added to *"Pole Results"* tab. Using this button, The user refreshes his memory about selected modes before and he can see the results in the listbox.

3.6.3. Residues Calculation Tab

Having identified the modal frequencies or the poles of FRFs, next step is to calculate residue terms of FRFs. Then, FRFs are again plotted up to identified modal parameters. Firstly, the user chooses the method whose solution he wants to use from *"Choose Method"* menu. Then, the user sees the picked results with the selected method from *"Modal Frequencies"* listbox. Subsequently, the user decides whether the residual terms are added to FRF synthesis calculation by selecting *"Yes"* or *"No"* from *"Residual Terms"* menu ring control. As in the case of test parameters, the user must select output & input DOFs to perform FRF synthesis from the given listboxes. Synthesized FRF is plotted by clicking on the *"Plot"* button. *"Synthesize_COF"* can also be seen for the selected FRF via a numeric indicator at the left bottom of *"Residues Calculation"* tab (Figure 3.7).

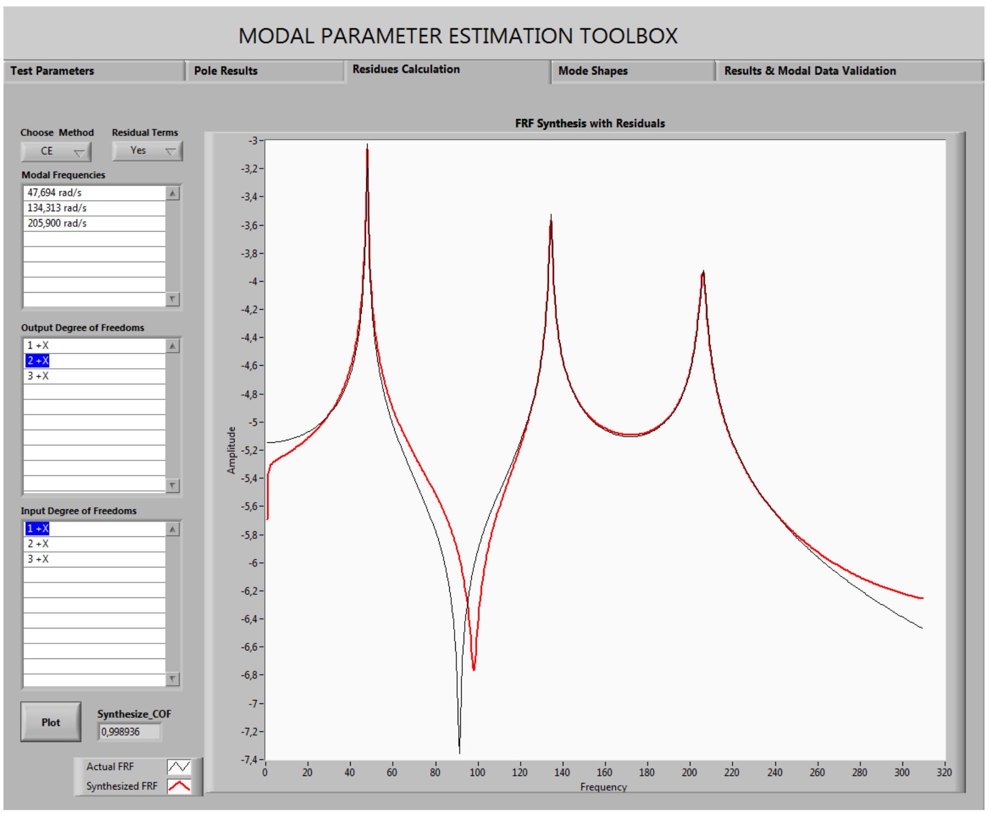


Figure 3.7. Residues Calculation Tab

3.6.4. Mode Shapes Tab

"Mode Shapes" tab (Figure 3.8) enables the user to see the mode shape results. The coordinates & trace line information of nodes on the structure must be input as

an initial point. Then, the geometry of the structure is visualized by clicking on *"Draw Geometry"* button. After the geometry is drawn, mode shapes can be plotted by giving *"Mode Number"* & *"Magnification Factor"* for any method. *"Milisecond Timer"* slider control provides the user somehow to adjust the frames per second of mode shape animation. *"Start/Stop Animation"* vertical toggle button either starts or stops mode shape animation.

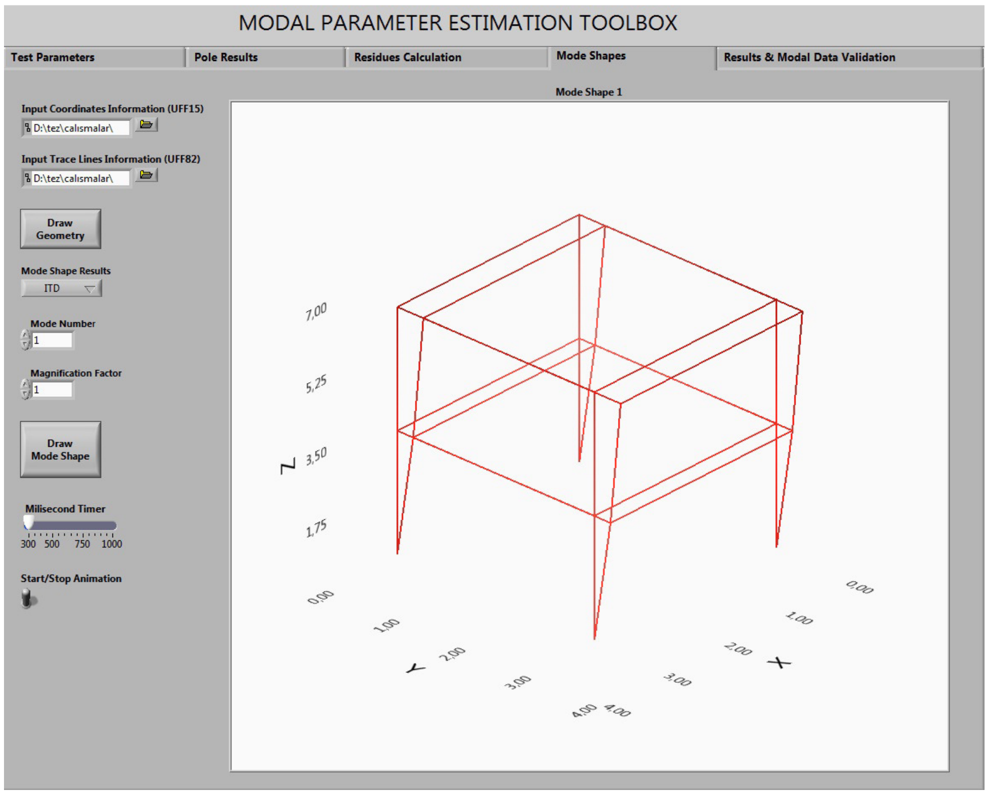


Figure 3.8. Mode Shapes Tab

3.6.5. Results & Modal Data Validation Tab

"Results & Modal Data Validation" tab (Figure 3.9) offers some useful tools to validate modal data from different methods. Modal Frequencies & Damping Ratios can be seen for different methods by clicking *"Show Results"* button. Mode shape results obtained from different techniques can be compared in MAC bar plot using *"Compare Results"* button. *"Get Image"* button gets the image of MAC bar plot in jpeg format into current VIs path. "Save Modal Parameters" button saves all modal parameters from every method. Also, it saves the MAC bar plot results in tabulated form and all saved parameters are exported to Excel datasheet.

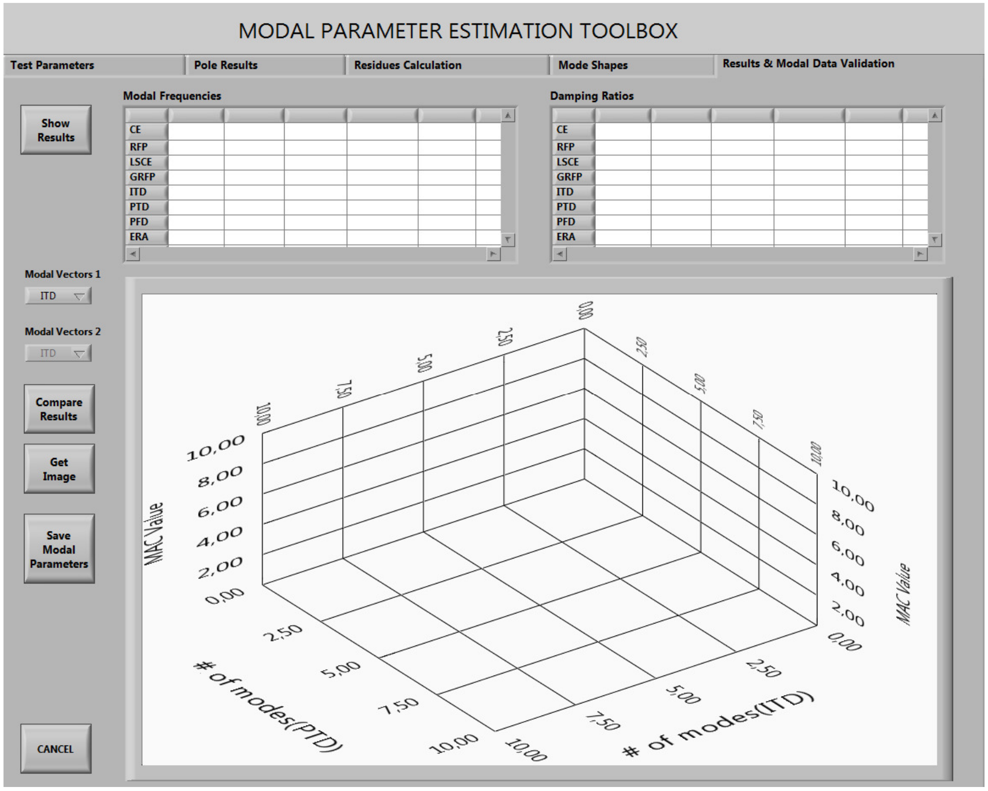


Figure 3.9. Results & Modal Data Validation Tab

3.7. Modular Structure of the Software

That the software has modular structure is very important for upcoming developers of the software. The code of the software is divided into small sub-groups as much as possible to provide penetrability to the code. The software is composed of one main VI (main function) & sixty sub VIs (sub-functions). All the algorithms of selected modal parameter estimators are written as sub VIs in order to facilitate the work of new developers. When a new developer of the software wants to add a new MPE estimator, the only thing he needs to do is to write his own sub VI for the estimator and connect to the main structure without trying to comprehend the whole frame of the software. Figure 3.10 shows an illustrative description of what a developer must do to add a new estimator to the software.

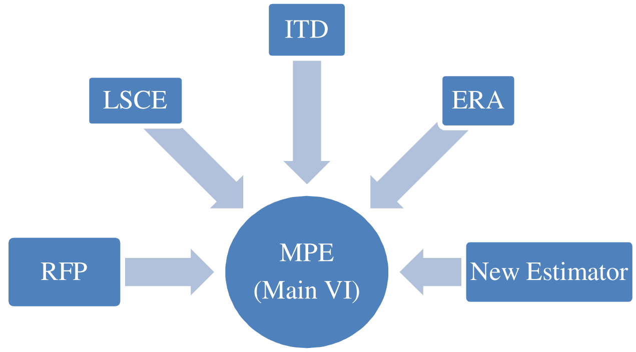


Figure 3.10. Modular Structure of the Software

CHAPTER 4

RESULTS & DISCUSSION

In this part of the thesis, that the software working properly is validated by analytical and experimental means. Two case studies are implemented. Firstly, a theoretical modal analysis will be performed by using a finite element software (ANSYS). Then, FRFs obtained from modal testing are processed in the software. Modal parameters of the structures for both cases are estimated by using the developed software in the study. The results of modal parameters obtained from different cases are compared and discussed.

4.1. Analytical Case Study

The main goal of the case study is to show that the software can sense and work with FRFs in x,y,z directions at the same time. In order to perform case study analytically, a finite element software (ANSYS) is used. A structure is modeled (Figure 4.1), whose mode shapes can be sensed in 3-D (x,y,z) directions. Firstly, modal analysis is performed in order to identify modal frequencies. Seven modes of the structure is used in the analysis. Knowing modal frequencies enables the user to determine the frequencies of the cyclic load for the harmonic analysis. Subsequent to modal analysis, harmonic analysis is performed in order to extract FRFs of the structure. The cyclic load is defined one by one in 3-D directions.

The model is composed of 12 nodes. All degree-of-freedoms of displacements of four nodes are constrained at the bottom of the structure. Unit force in x,y,z directions is applied for each analysis, respectively. 24 FRFs are obtained from each analysis and 72 FRFs in total. Thus, FRFs in x,y,z directions are achieved for each node.

The structure is composed of line elements. BEAM4 is suggested element type of line elements in literature for 3D analysis. All elements are 3 meters in length. A square shape cross section (0.3m x 0.3m) is defined for the element (Figure 4.2).

Structural steel is chosen as the material of the element type. The necessary geometric properties for steel is given as an input to the software in Table 4.1.

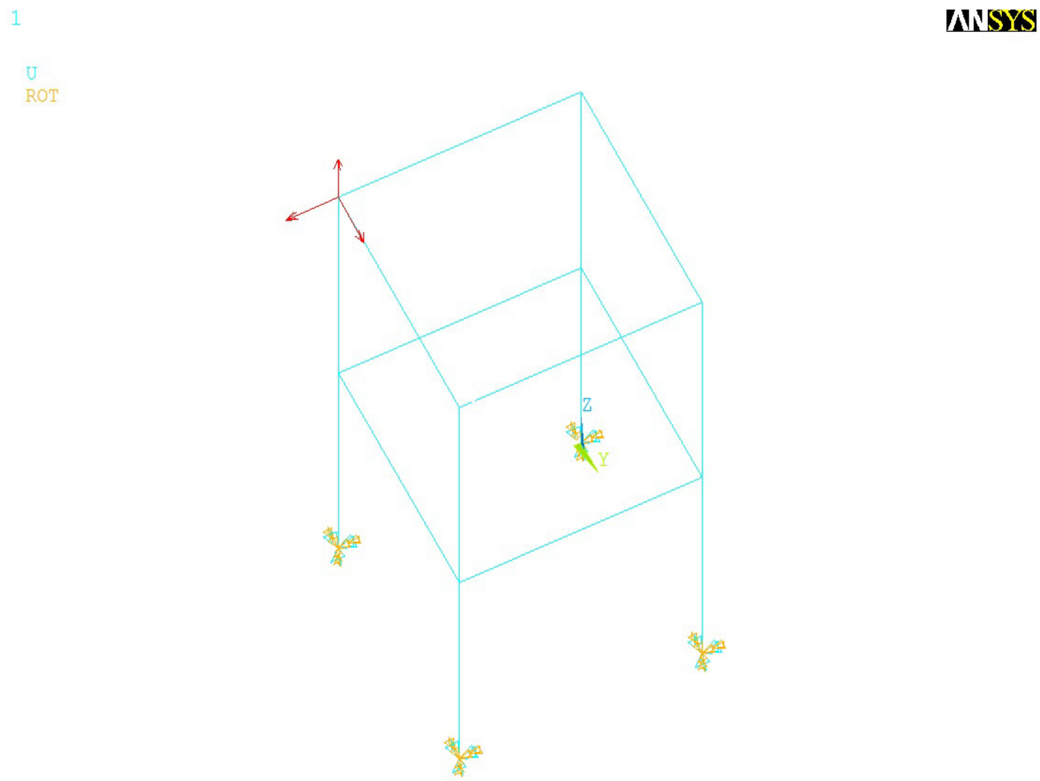


Figure 4.1. Geometry of the Model

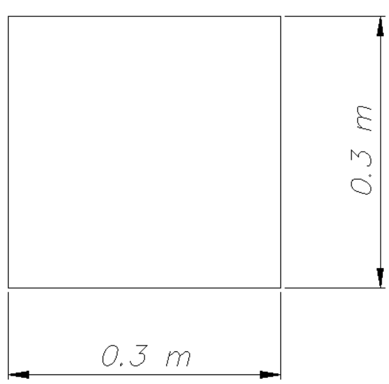


Figure 4.2. Cross section of elements for the structure

Table 4.1. Geometric Properties for Steel

Young's Modulus (E)	200 GPa
Poisson's Ratio	0.33
Density	7800 kg/m^3

The damping is given as an input to the structure. The damping matrix C in ANSYS is calculated by given formula in harmonic, damped modal and transient analysis [46]:

$$[C] = \alpha[M] + \beta[K] + \sum_{j=1}^{N_{mat}} \beta_j[K_j] + \beta_c\,[K] + [C_\zeta] + \sum_{k=1}^{N_{ele}} [C_k] \tag{4.1}$$

$where$:

$\alpha = constant\ mass\ matrix\ multiplier$

$\beta = constant\ stiffness\ matrix\ multiplier$

$\beta_j = constant\ stiffness\ matrix\ multiplier$

$\beta_c = variable\ stiffness\ matrix\ multiplier$

$\beta_c = \frac{\zeta}{\omega}\quad where\ \ \zeta = constant\ damping\ ratio$

$[C_\zeta] = frequency - dependent\ damping\ matrix$

$[C_k] = element\ damping\ matrix$

Constant mass & stiffness matrix multiplier α & β and constant damping ratio ζ are specified for the model (Table 4.2). Of course, the specified damping has no physical meaning. Yet, the aim of the case study is to calculate the damping for the structure and compare with the results of the developed software.

Table 4.2. Specified Constants for Damping in ANSYS

$\boldsymbol{\alpha}$	0.00011
$\boldsymbol{\beta}$	0.00017
$\boldsymbol{\zeta}$	0.01

ANSYS has seven types of eigenvalue solvers for modal analysis, which can be listed as Block Lanczos, PCG Lanczos, Reduced, Unsymmetric, Damped, QR

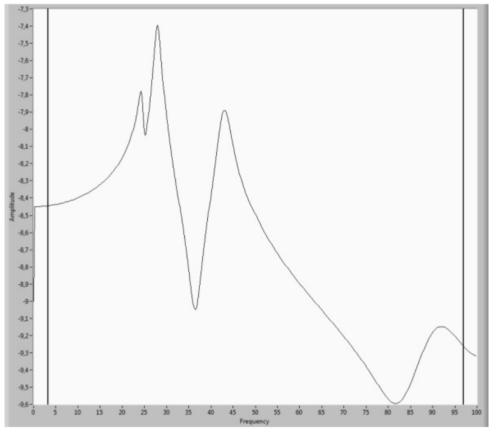
Damped and Supernode. Except for Damped & QR Damped cases, other methods are not suitable for damped cases. Therefore, QR damped mode extraction method is selected to perform modal analysis for the structure.

Performing modal analysis, seven modes are found in between 0-100 rad/s (Table 4.3). However, these seven modes can not be excited in all FRFs while performing harmonic analysis with different directions of excitation forces. Figure 4.3 shows the FRF curves of different nodes obtained from harmonic analysis. As seen from FRF curves, seven modes of the structure can not be detected. Various number of modes are excited with respect to different FRFs. Fortunately, mode indicator functions exhibits all modes of the structure and gives the user an initial idea how to be advanced through the analysis. Figure 4.4 & 4.5 indicates all the identified modes from mode indicator functions. Last mode of the structure can be more clearly seen with MvMIF compared to CMIF plot.

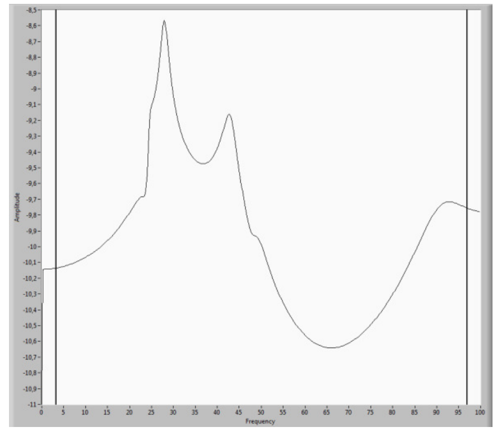
After investigating FRFs and mode indicator functions, modes are estimated with all the methods available in the software. There are SISO methods like RFP, CE etc. which can work with single FRF or IRF in the software. Therefore, there is a risk of not identifying all modes of the structure. When working with this type of methods, FRF is chosen for the analysis, in which all the modes are excited.

Table 4.3. Modal Frequencies by ANSYS

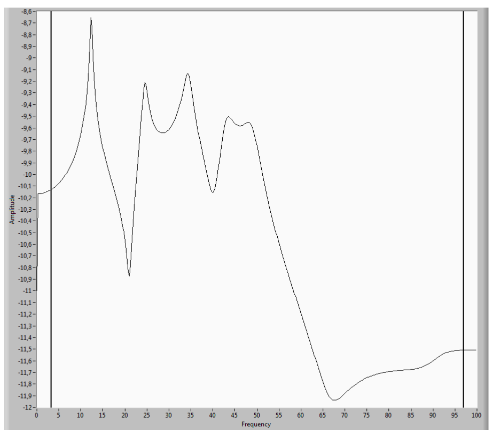
Mode	Natural Frequencies (rad/s)
1	12.330
2	24.455
3	27.963
4	34.373
5	42.951
6	48.842
7	90.637



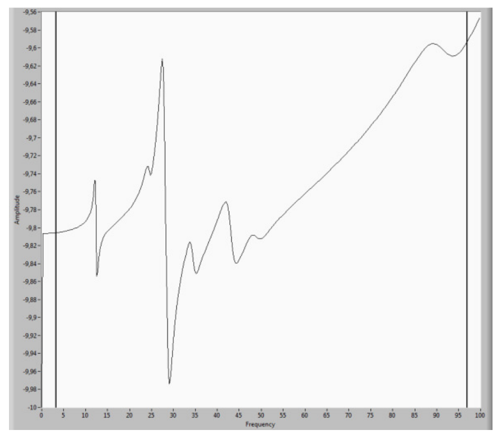
(1)



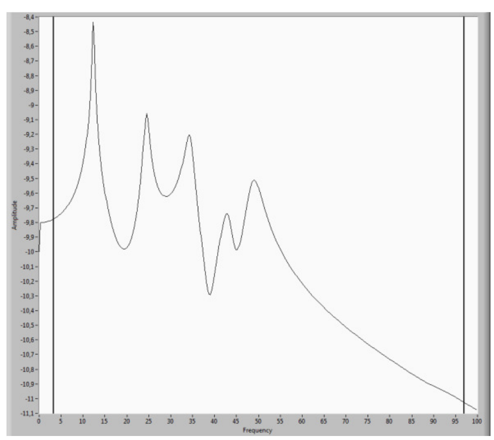
(2)



(3)



(4)



(5)

Figure 4.3. FRF curves of different nodes from harmonic analysis – (1) 11Z-9X, (2) 9X-9X, (3) 8Y-9Z, (4) 4Z-9Z, (5) 9Z-9Y

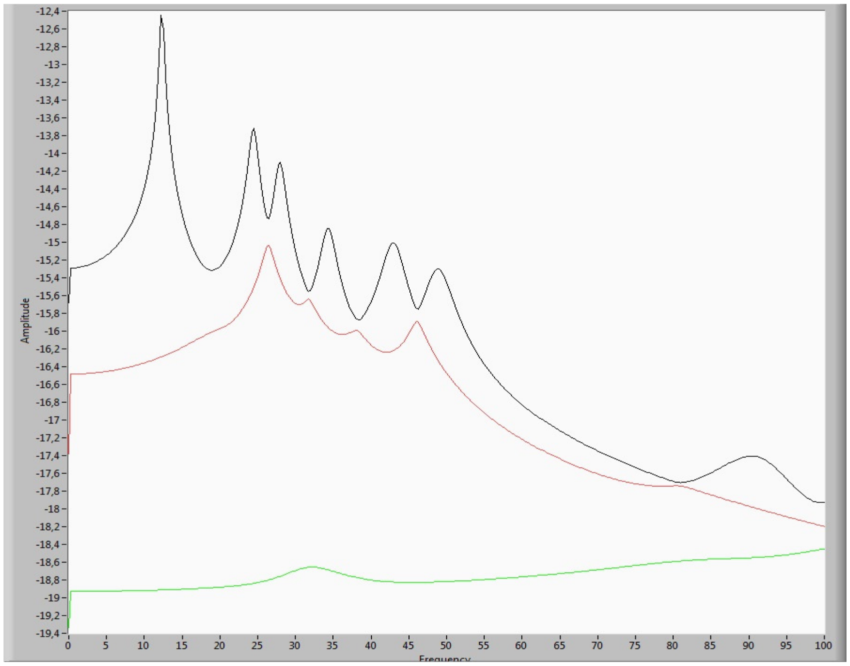


Figure 4.4. CMIF of all DOFs from harmonic analysis

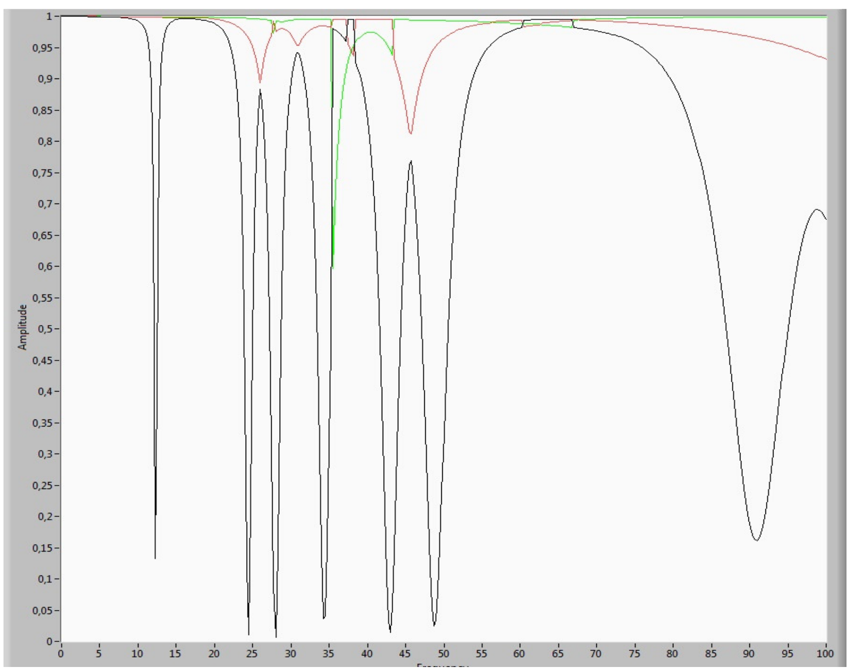


Figure 4.5. MvMIF of all DOFs from harmonic analysis

Modal parameters are estimated utilizing from stability diagrams for each method. These stability diagrams are presented in following figures.

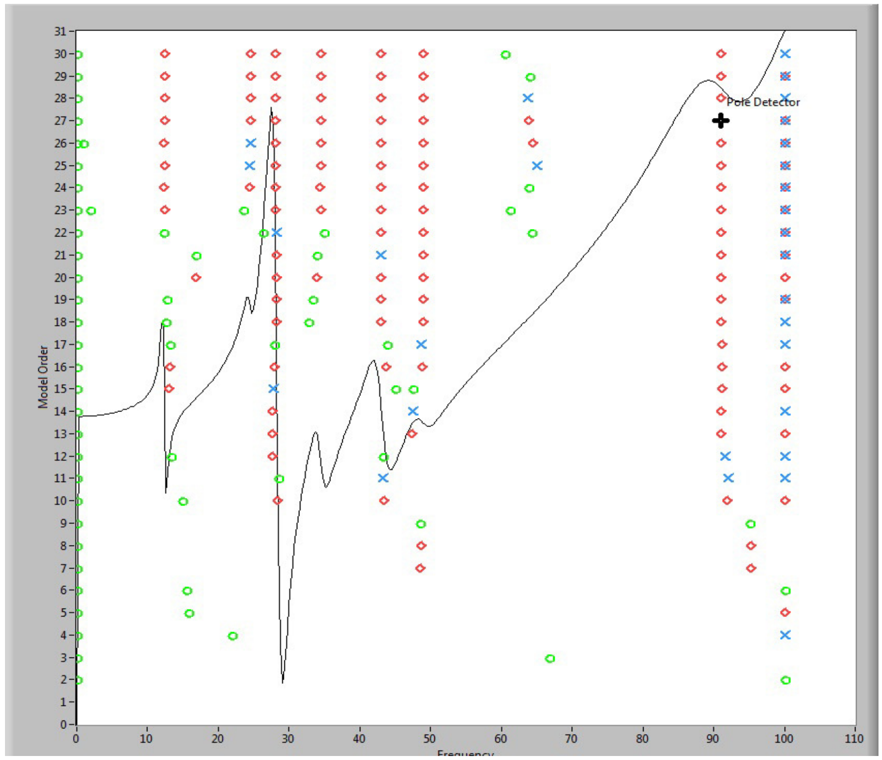


Figure 4.6. Stability diagram obtained by CE

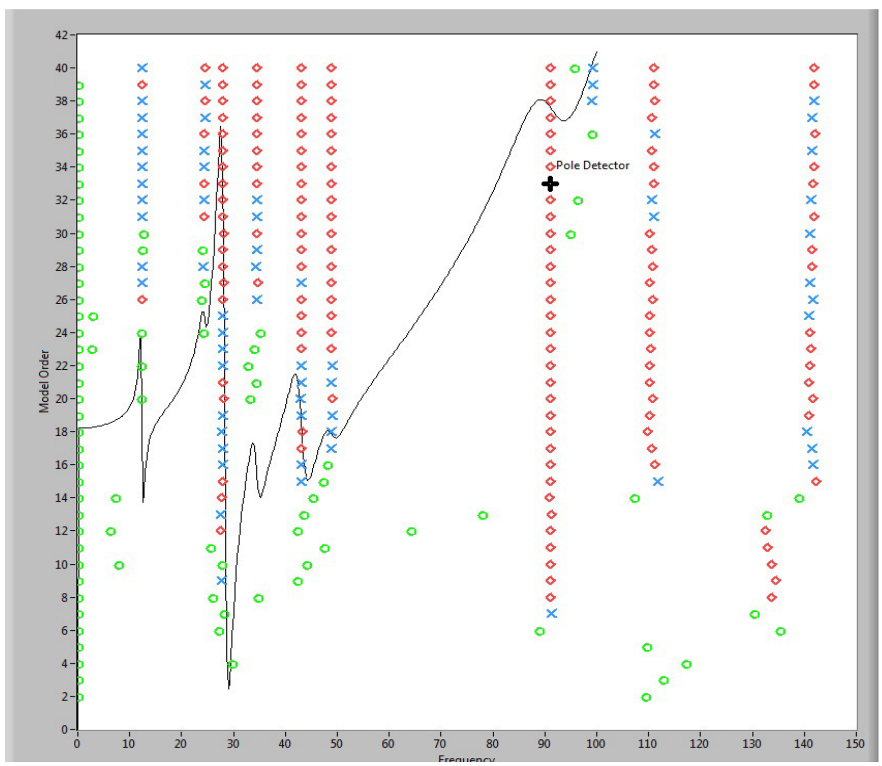


Figure 4.7. Stability diagram obtained by RFP

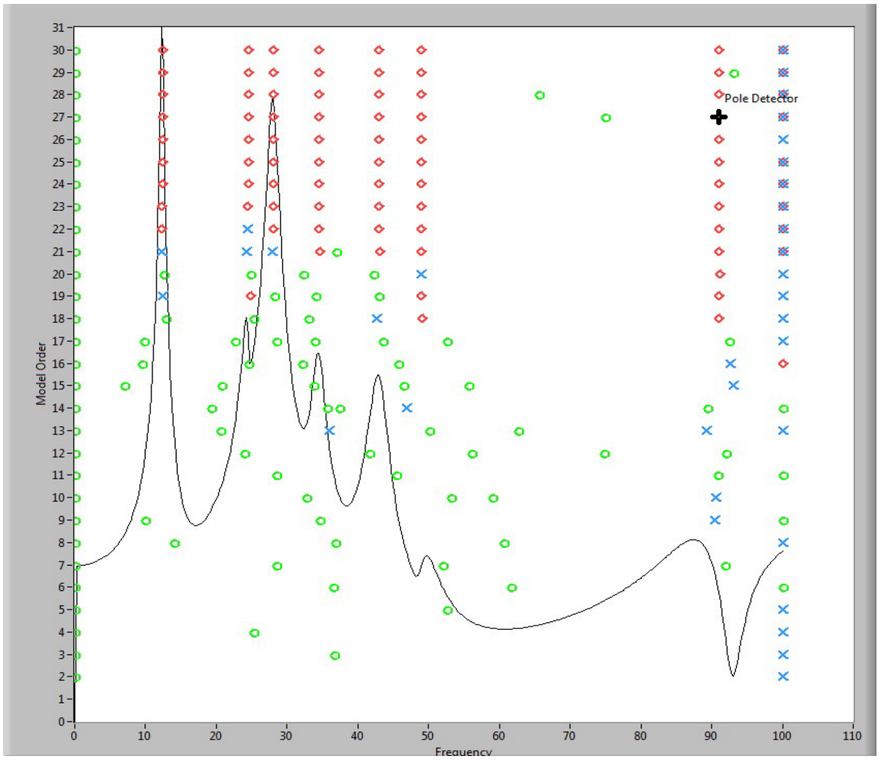


Figure 4.8. Stability diagram obtained by LSCE

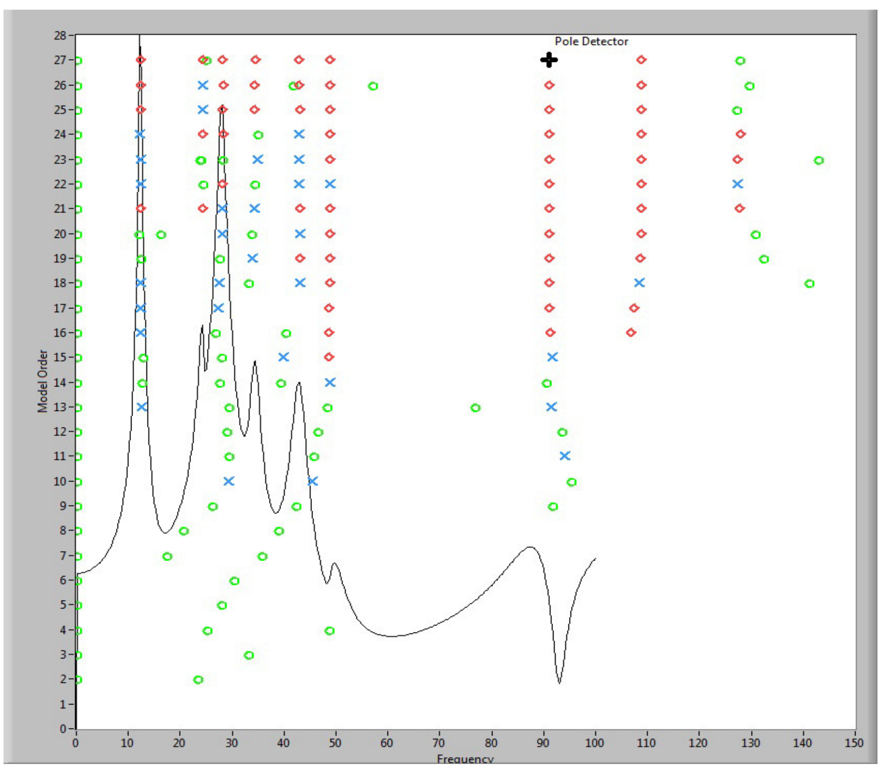


Figure 4.9. Stability diagram obtained by GRFP

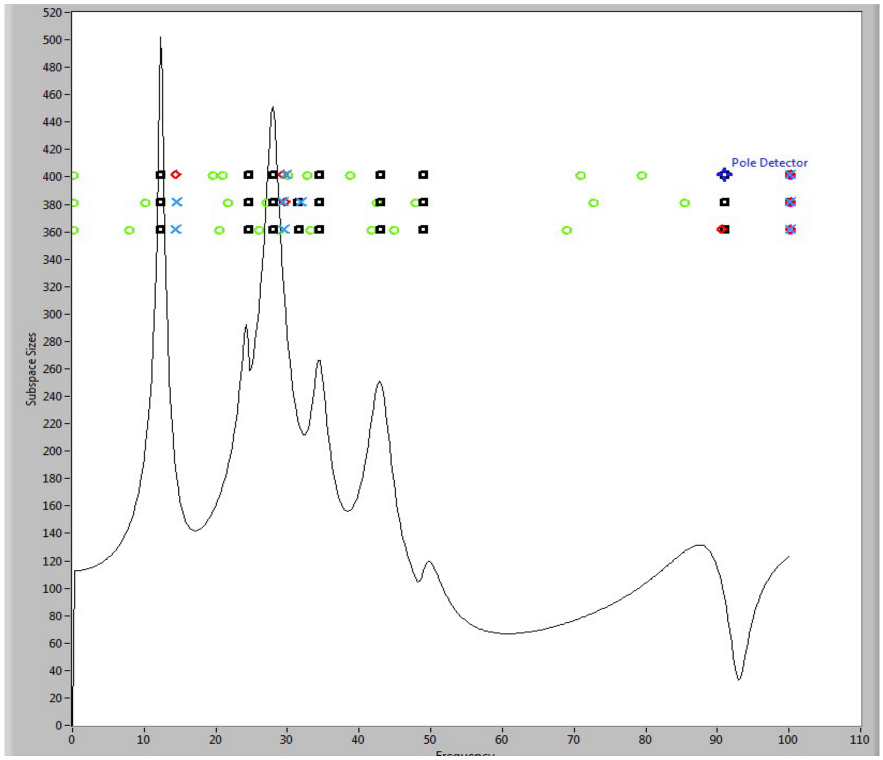


Figure 4.10. Stability diagram obtained by ITD

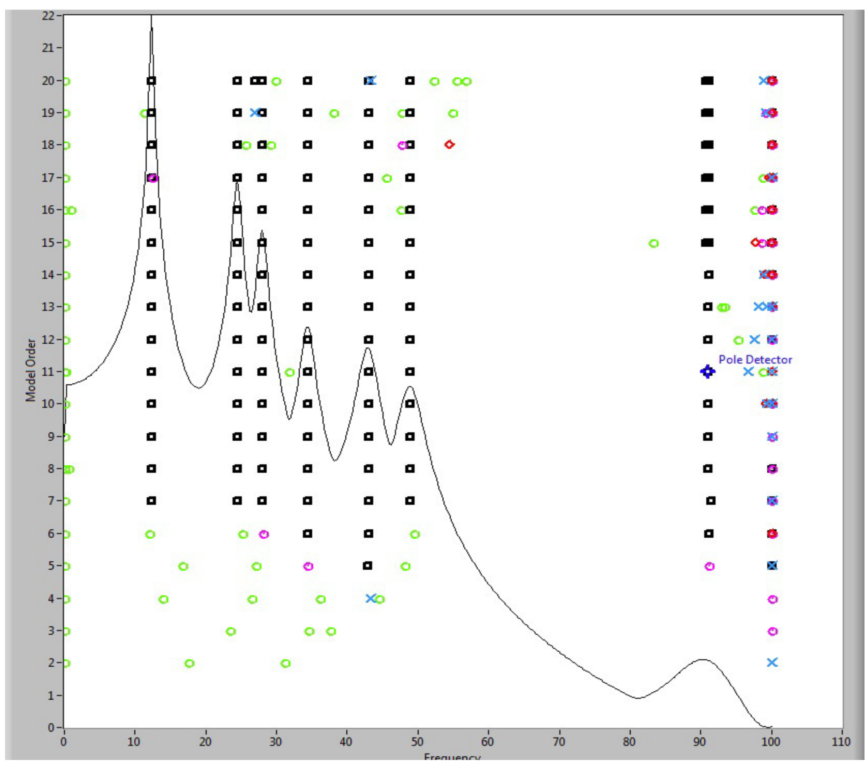


Figure 4.11. Stability diagram obtained by PTD

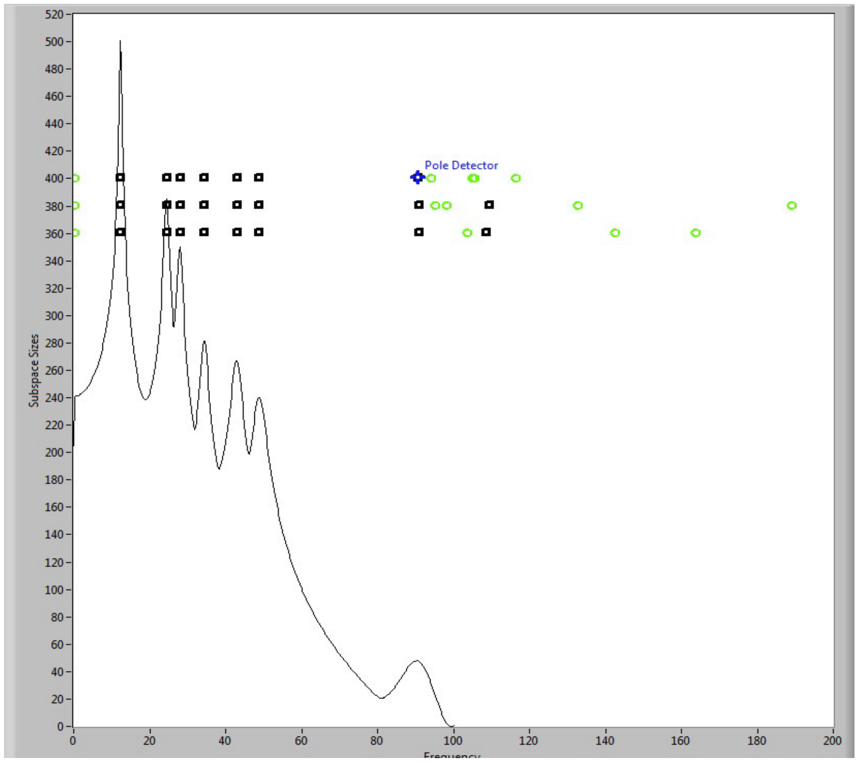


Figure 4.12. Stability diagram obtained by PFD

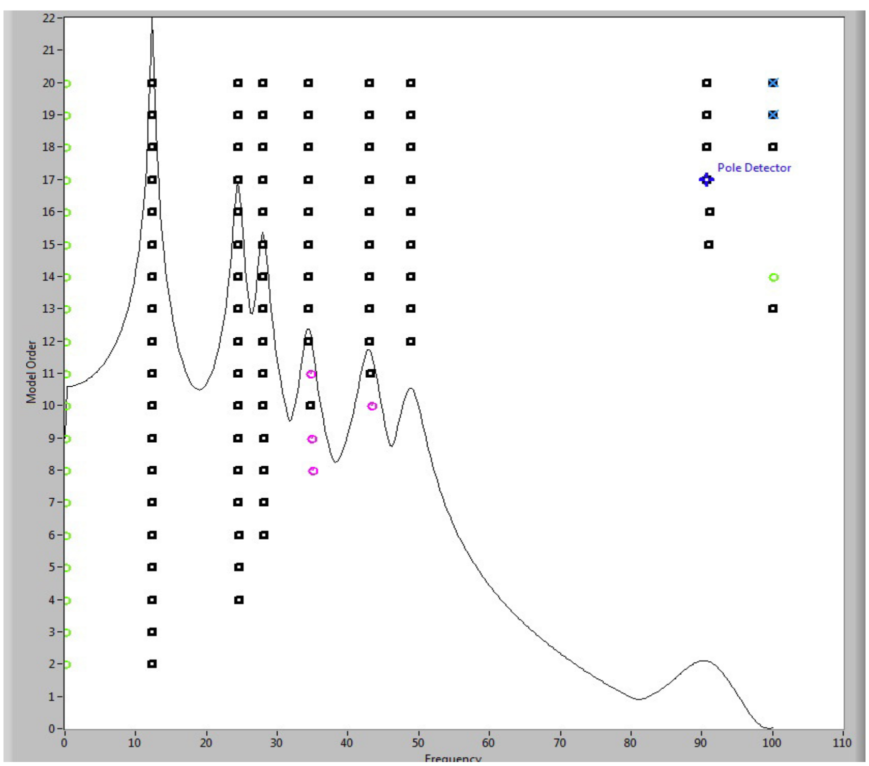


Figure 4.13. Stability diagram obtained by ERA

When two SISO methods compared in frequency & time domain, it is observed that CE works more efficient than RFP. As it seen in Figure 4.6 & 4.7, model order is increased more for RFP compared to CE to reach stable poles. Moreover, first mode of the system is not so clearly defined with RFP.

During the analysis, it is seen that GRFP method is not an efficient method when many degree of freedoms exist for the system. It occupies computer memory too much and decreases the speed of the analysis. Stable poles are achieved with lower level of model order compared to RFP since it is working with many measurements. As in the case of SISO methods, modes are more clearly defined with LSCE compared to GRFP. GRFP is not recommended when working with large engineering structures since it requires so much computation effort.

ITD & PFD methods are based on state space models. Since they work with large amount of spatial information, stable poles are estimated with fewer runs compared to other methods. PFD method also requires much computation effort. PTD & ERA methods estimate the stable poles with lower model orders compared to other modes. Since the data do not contain noise inside, the methods start to reach stable poles with so low model orders up to seven. Above all, the results of modal parameters by each method & ANSYS are given in Table 4.4 & Table 4.5.

Table 4.4. Modal frequencies by each method & ANSYS

Mode	Modal Frequencies (rad/s)								
	CE	**RFP**	**LSCE**	**GRFP**	**ITD**	**PTD**	**PFD**	**ERA**	**ANSYS**
1	12.341	12.332	12.346	12.299	12.345	12.345	12.332	12.345	12.329
2	24.488	24.420	24.475	24.318	24.484	24.484	24.455	24.485	24.453
3	27.996	27.970	28.000	28.189	27.996	27.996	27.959	27.997	27.960
4	34.411	34.381	34.414	34.253	34.412	34.412	34.378	34.414	34.367
5	42.996	42.943	42.996	42.940	42.995	42.995	42.939	43.001	42.940
6	48.888	48.830	48.890	48.838	48.889	48.889	48.838	48.900	48.826
7	91.066	90.951	91.058	90.949	91.068	91.053	90.937	91.029	90.531

Table 4.5. Damping ratios by each method & ANSYS

Mode	Damping ratios (%)								
	CE	**RFP**	**LSCE**	**GRFP**	**ITD**	**PTD**	**PFD**	**ERA**	**ANSYS**
1	1.640	1.683	1.683	1.584	1.658	1.658	1.649	1.658	1.665
2	2.278	1.666	2.331	2.395	2.305	2.306	2.309	2.305	2.306
3	2.496	2.498	2.504	2.396	2.493	2.493	2.496	2.493	2.493
4	2.839	2.850	2.836	2.712	2.835	2.835	2.818	2.832	2.835
5	3.295	3.291	3.293	3.173	3.293	3.293	3.308	3.281	3.293
6	3.611	3.610	3.608	3.606	3.608	3.608	3.555	3.586	3.608
7	5.873	5.862	5.868	5.860	5.859	5.887	5.856	6.451	5.840

Comparing the results with the modal parameters by ANSYS, all methods give very consistent results except for the damping ratio of second mode obtained by RFP method. Furthermore, ITD & PTD methods almost give same results. The closest results in modal frequencies to the analytical ones are found by PFD. On the other hand, ITD & PTD methods almost give same damping ratios with analytical ones. ERA method can not find accurate damping ratio for the seventh mode (Table 4.6 & 4.7).

Table 4.6. Percentage error of modal frequencies between each method & ANSYS

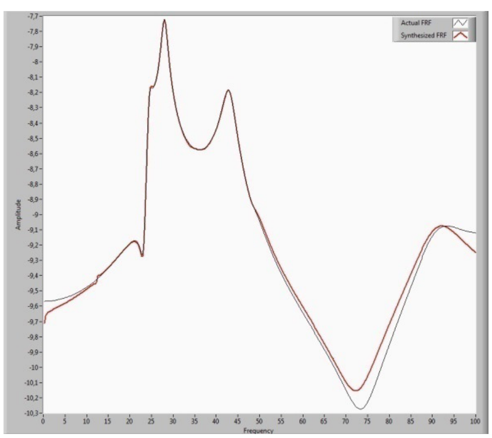
Mode	Percentage error (%)							
	CE	**RFP**	**LSCE**	**GRFP**	**ITD**	**PTD**	**PFD**	**ERA**
1	0.097	0.024	0.138	0.243	0.130	0.130	0.024	0.130
2	0.143	0.135	0.090	0.552	0.127	0.127	0.008	0.131
3	0.129	0.036	0.143	0.819	0.129	0.129	0.004	0.132
4	0.128	0.041	0.137	0.332	0.131	0.131	0.032	0.137
5	0.130	0.007	0.130	0.000	0.128	0.128	0.002	0.142
6	0.127	0.008	0.131	0.025	0.129	0.129	0.025	0.152
7	0.591	0.464	0.582	0.462	0.593	0.577	0.448	0.550

Table 4.7. Percentage error of damping ratios between each method & ANSYS

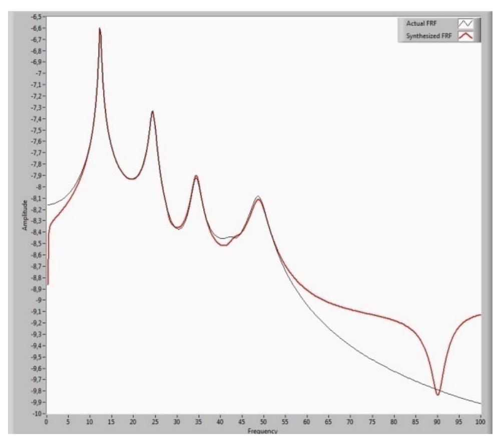
Mode	Percentage error (%)							
	CE	RFP	LSCE	GRFP	ITD	PTD	PFD	ERA
1	1.502	1.081	1.081	4.865	0.420	0.420	0.961	0.420
2	1.214	27.754	1.084	3.859	0.043	0.000	0.130	0.043
3	0.120	0.201	0.441	3.891	0.000	0.000	0.120	0.000
4	0.141	0.529	0.035	4.339	0.000	0.000	0.600	0.106
5	0.061	0.061	0.000	3.644	0.000	0.000	0.456	0.364
6	0.083	0.055	0.000	0.055	0.000	0.000	1.469	0.610
7	0.565	0.377	0.479	0.342	0.325	0.805	0.274	10.462

After modal parameters are estimated, arbitrary samples of FRFs are chosen and recalculated based on estimated modal parameters. These synthesized FRFs are given in Figure 4.14. Also, modal vectors found by each method are compared with MAC bar plots (Figure 4.15). Discrepancies between results are observed. It may occur due to inconsistencies between FRFs. Reliable mode shapes can not be found with PFD method.

Mode shape plots obtained from the developed software & ANSYS are compared. Except for the seventh mode, satisfactory match between results is obtained (Figure 4.16).

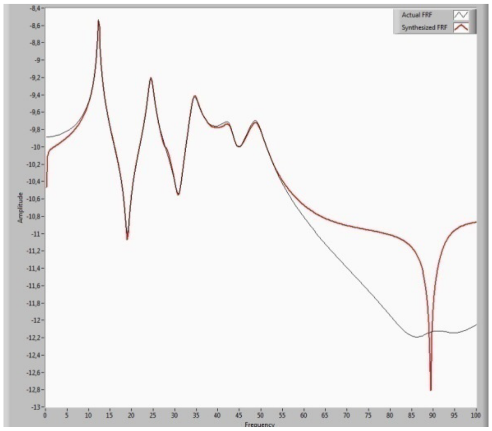


(1)

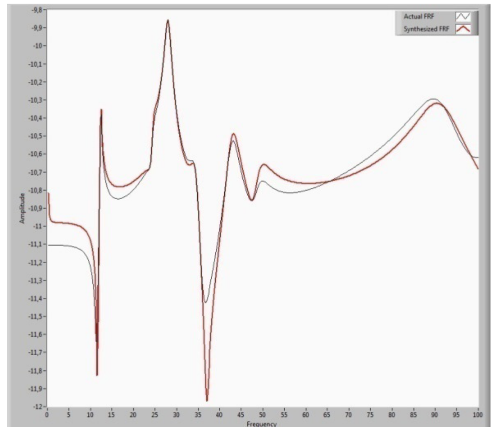


(2)

Figure 4.14. Synthesized FRFs of node points between - (1) 6X-9X, (2) 10Y-9Y, (3) 2Z-9Y, (4) 6Z-9Z

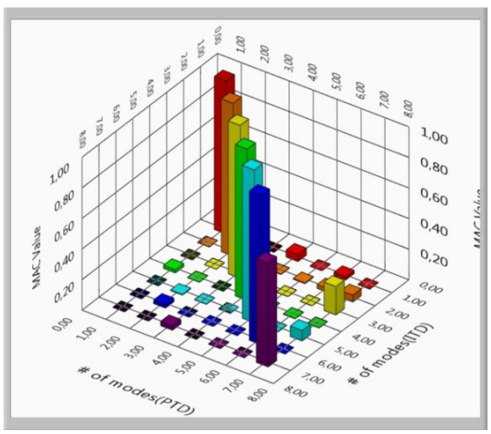


(3)

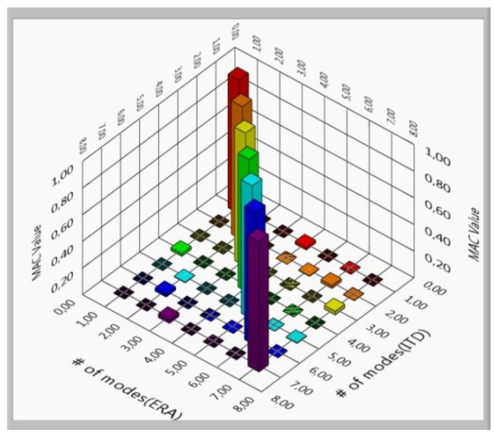


(4)

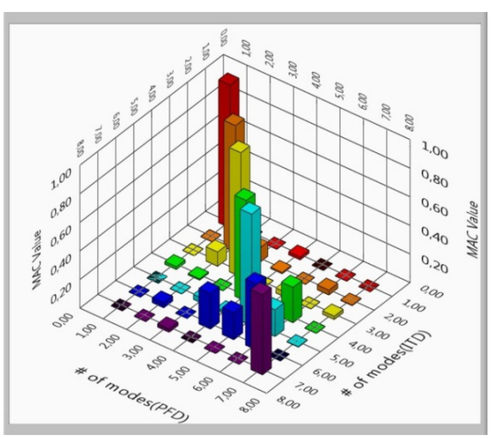
Figure 4.14. Continued



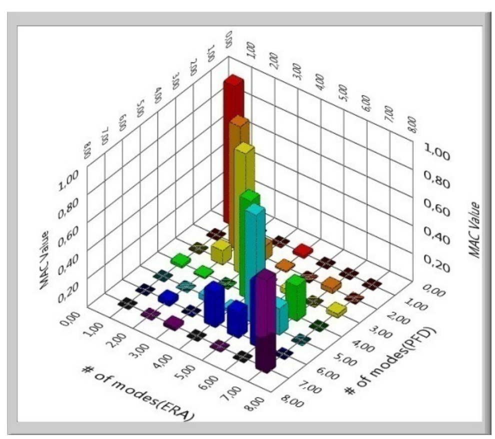
(1)



(2)

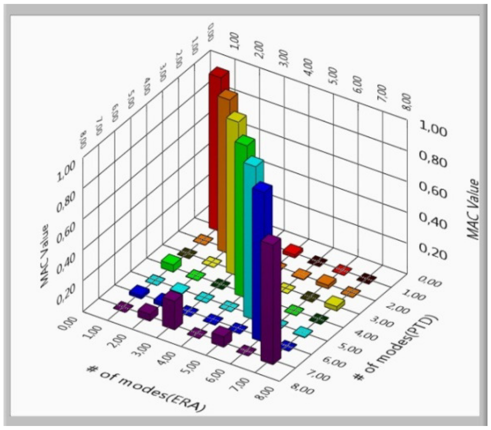


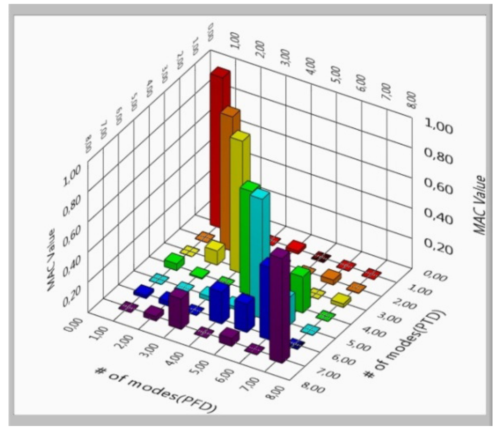
(3)



(4)

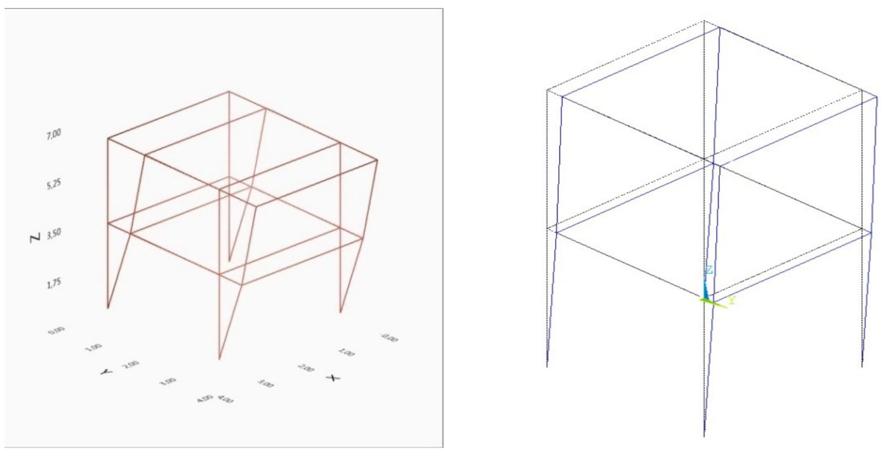
Figure 4.15. MAC bar plots between - (1) ITD-PTD, (2) ITD-ERA, (3) ITD-PFD, (4) PFD-ERA, (5) PTD-ERA, (6) PTD-PFD



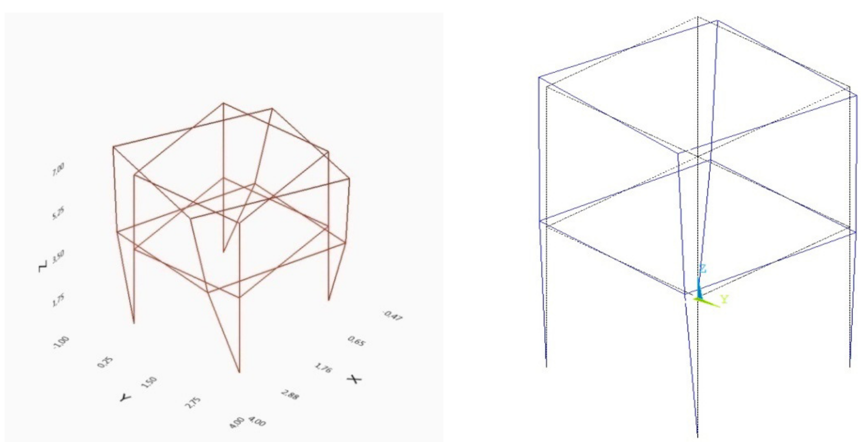


(5) (6)

Figure 4.15. Continued

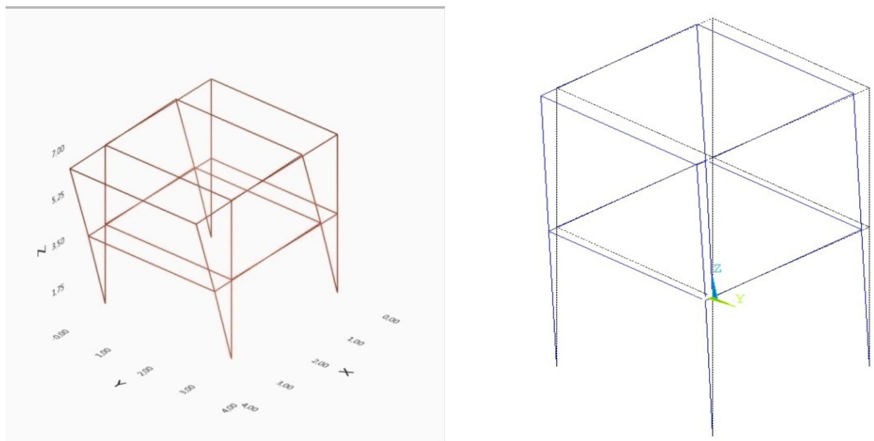


(1)

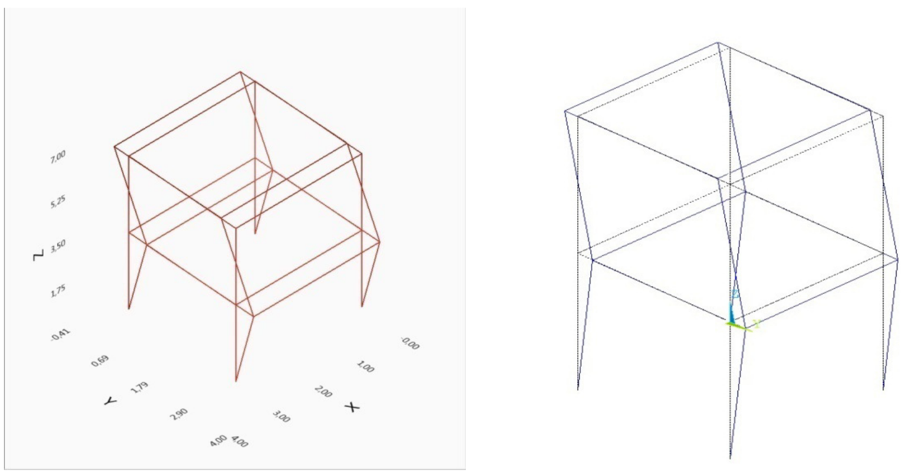


(2)

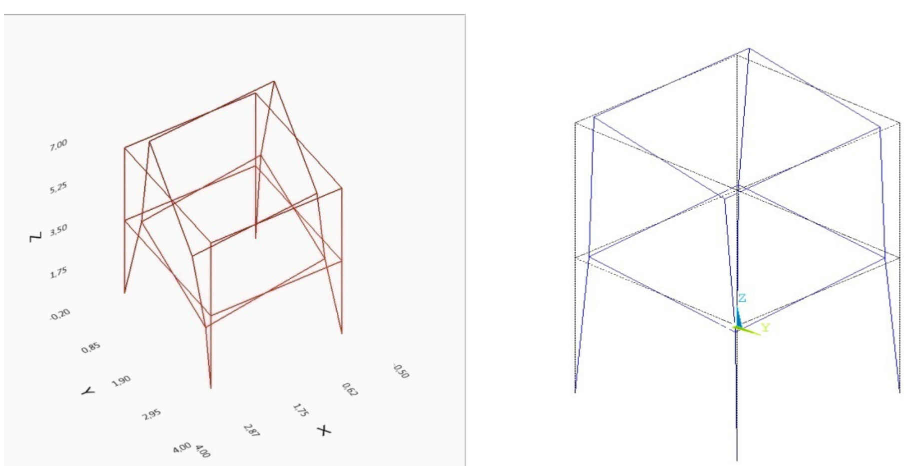
Figure 4.16. Mode shapes from the developed software & ANSYS – (1) 1st mode, (2) 2nd mode, (3) 3rd mode, (4) 4th mode, (5) 5th mode, (6) 6th mode, (7) 7th mode



(3)

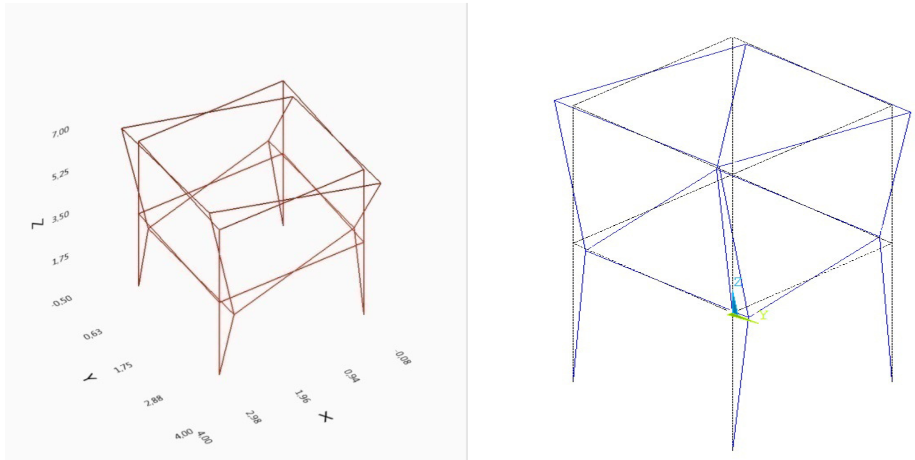


(4)

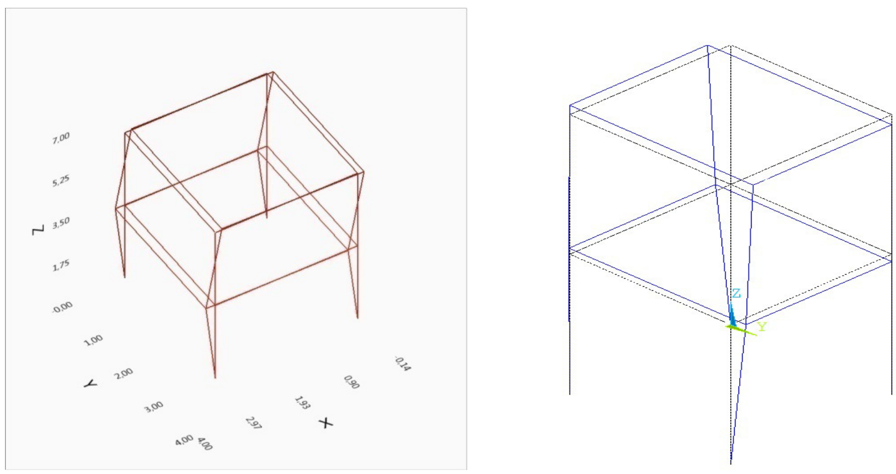


(5)

Figure 4.16. Continued



(6)



(7)

Figure 4.16. Continued

Inconsistency in seventh mode can also be seen in MAC table (Table 4.8) between the results of ANSYS & ITD method of the developed software. Other modes are in perfect match with each other.

Reliability of the mode shape results can also be checked by MPC table obtained by different methods. Table 4.9 shows MPC results obtained from different methods. Being close to unity of table results means that the results are as expected. Low value of MPC results of PFD explains the inconsistency in MAC bar plots between PFD & the other methods.

Table 4.8. MAC table between the mode shape results of ANSYS & ITD method of the software

		ITD method of the software						
	Mode	1	2	3	4	5	6	7
ANSYS	1	0.999	0.000	0.000	0.020	0.000	0.000	0.000
	2	0.000	1.000	0.000	0.000	0.003	0.022	0.000
	3	0.000	0.000	1.000	0.000	0.000	0.000	0.027
	4	0.020	0.000	0.000	1.000	0.000	0.000	0.000
	5	0.000	0.003	0.000	0.000	1.000	0.001	0.000
	6	0.000	0.022	0.000	0.000	0.001	1.000	0.000
	7	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 4.9. MPC table obtained from different methods

Mode	**1**	**2**	**3**	**4**	**5**	**6**	**7**
ITD	1.000	1.000	1.000	1.000	1.000	1.000	0.999
PTD	0.999	0.999	0.999	0.979	0.995	0.908	0.486
PFD	0.999	0.628	0.567	0.275	0.807	0.341	0.017
ERA	0.999	0.999	0.999	0.999	0.999	0.999	0.940

Since the 7th mode does not match at all for the mode shape results, modal analysis by ANSYS is re-evaluated and repeated. The analysis is extended to eight modes. It is realized that modal frequencies of 7th & 8th mode obtained by ANSYS is so close to each other (Table 4.10).

Table 4.10. Modal frequencies of 7th & 8th mode obtained by ANSYS

Mode	**Modal Frequencies(rad/s)**
7	90.531
8	90.945

Later, it is understood that 8th mode of the structure is estimated as 7th mode by methods of the developed software. This idea is also verified by the results between so called 7th mode of the developed software & 8th mode obtained by ANSYS (Figure 4.17). The satisfactory match between mode shape results is also obtained between the modes.

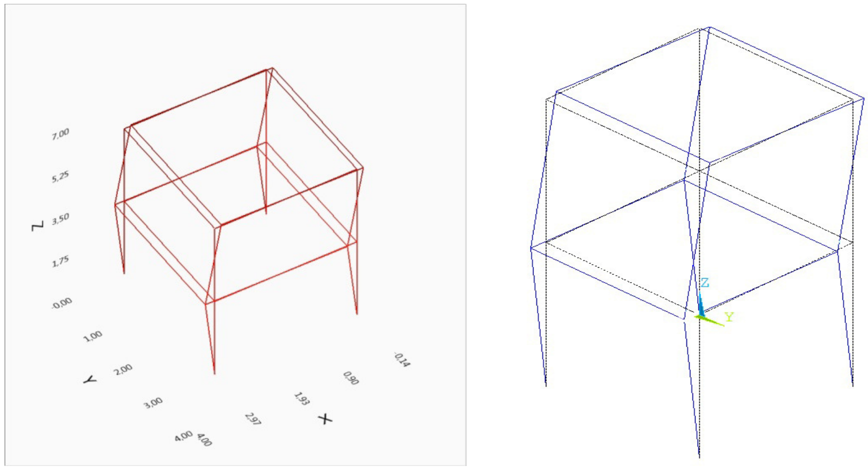


Figure 4.17. Mode shape results from 7th mode of the developed software & 8th mode of ANSYS

4.2. Experimental Case Study

The main purpose of the case study is to show that the developed software can give accurate modal parameter results while working with FRFs obtained from modal testing. For this purpose, the results of modal testing on an aluminum circular plate structure having two input and thirty output degrees of freedom are used. The results of the developed software are compared with the results of modal analysis software X-Modal II developed by SDRL at the University of Cincinnati.

4.2.1. Test Structure

The test object is a ¼ inch circular aluminum plate with a small hole at its center. The outer & inner diameter of the circular plate are 30 inches & 2 inches, respectively. It is supported by three soft springs to obtain free-free boundary condition simulation. The outputs were taken from thirty points while the system was excited simultaneously at two points (Figure 4.18).

The instrumentation setup is composed of a laptop computer, a Hewlett Packard VXI mainframe, two sixteen channel ICP supplies, two amplifiers, two voltage feedback shakers, 30 ICP accelerometers, and 2 ICP load-cells. A detailed layout of the instrumentation setup is given in Figure 4.19. The summary of final measurement configuration is given in Table 4.11.

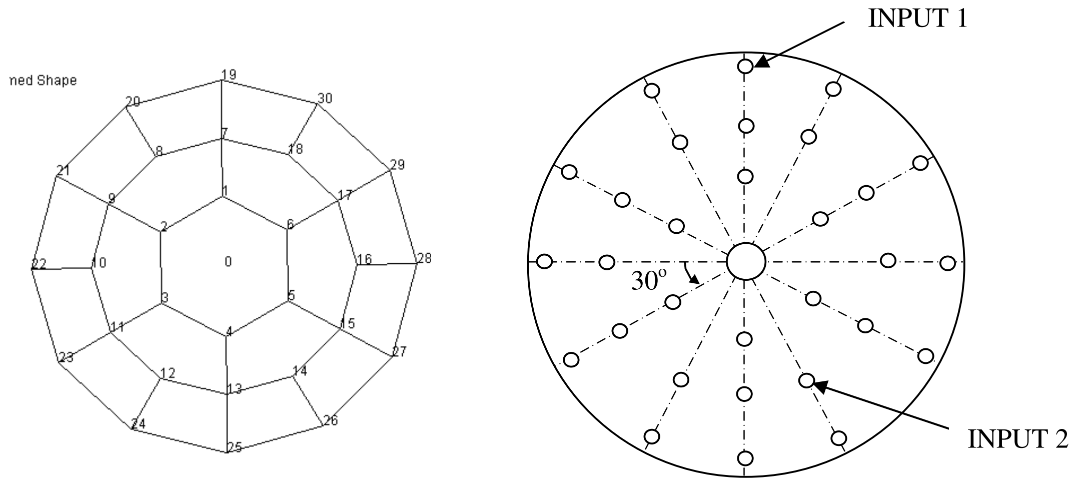


Figure 4.18. Test Structure – Circular plate

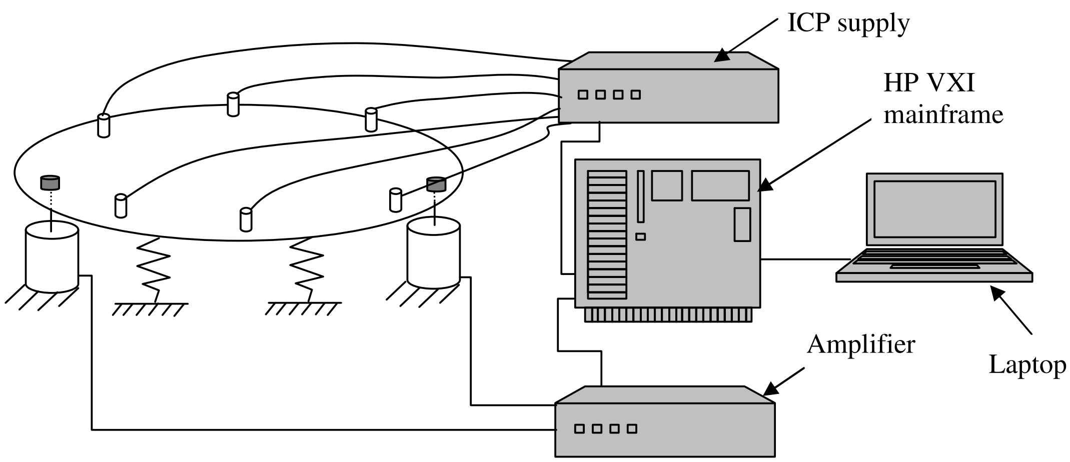


Figure 4.19. Detailed layout of the instrumentation

Table 4.11. Test Configuration

# of inputs	2
# of outputs	30
Test method	MIMO
Excitation	Burst random (%40)
FRF Estimator	Hv
# of cyclic ave.	6
# of spectral ave.	15
Windowing Type	Uniform
Frequency Range	0-800 Hz
# of spectral lines	3200

4.2.2. Modal Parameter Estimation Results

CMIF plots are utilized to estimate the number of modes correctly. Two reference measurements yield two CMIF plots. CMIF plots indicate the presence of six pairs of repeated roots and three single roots between 0-500 Hz frequency range which means the existence of fifteen modes in the selected frequency range (Figure 4.20). Table 4.12 shows the approximate estimated frequencies and the order of their roots. The number of modes is also estimated by MvMIF plots. However, repeated roots can not be clearly seen with MvMIF plots for this test configuration (Figure 4.21).

Table 4.12. Inferences from CMIF plots

Estimated frequency	Number of roots
≈ 60	2
≈ 100	1
≈ 135	2
≈ 225	2
≈ 230	2
≈ 350	2
≈ 375	2
≈ 420	1
≈ 500	1

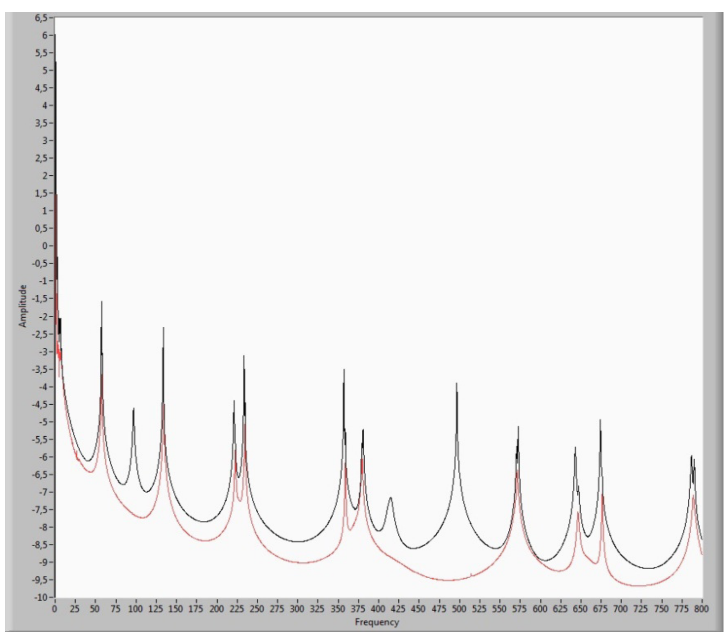


Figure 4.20. CMIF plots

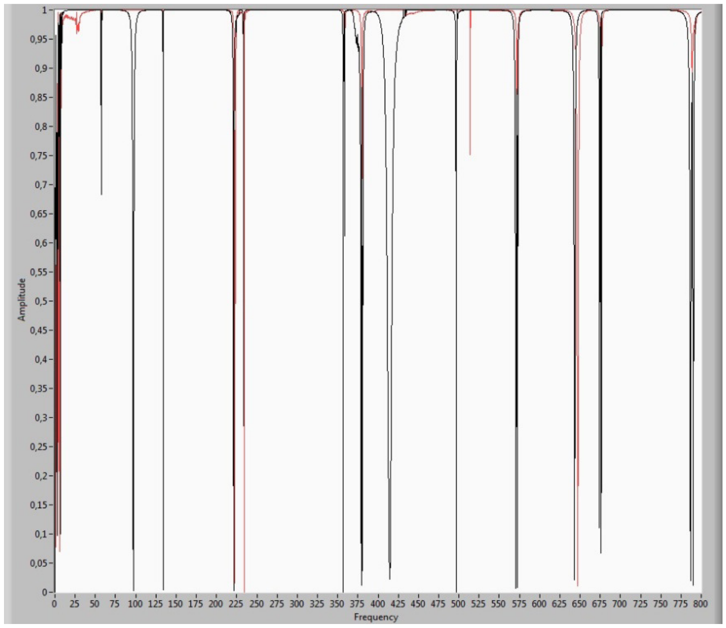


Figure 4.21. MvMIF plots

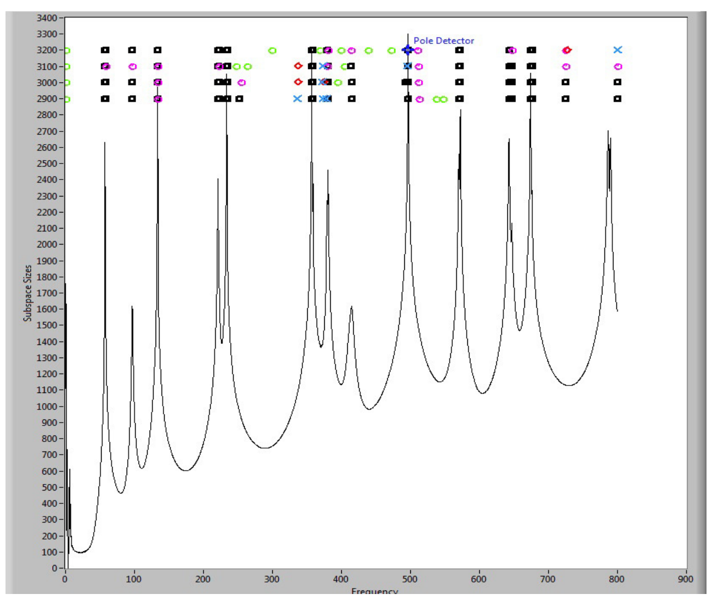
Parameter estimation results are obtained using X-Modal II. MIMO version of the RFP method was used for this analysis. Final results of each mode obtained from X-Modal II & the developed software are given in Table 4.13.

The methods which are not MIMO methods in the software can not identify repeated roots of the structure. Therefore, their results are not presented. Modal parameters are estimated by using ITD, PTD & ERA. Stability diagrams obtained by those methods are presented in Figure 4.22. One pair of the repeated roots can not be estimated by ITD & ERA. PTD can find all the modes which are estimated by X-Modal II using the RFP method. The results of modal frequencies are very close to the results of X-Modal II. However, damping ratios do not perfectly match. The damping ratios found by ERA are not correct. Although some variations exist in the results, it can be concluded that PTD is the best option to identify repeated roots in the structure.

Also when mode shape plots compared, satisfactory match between results of X-Modal II & the developed software is observed (Figure 4.23).

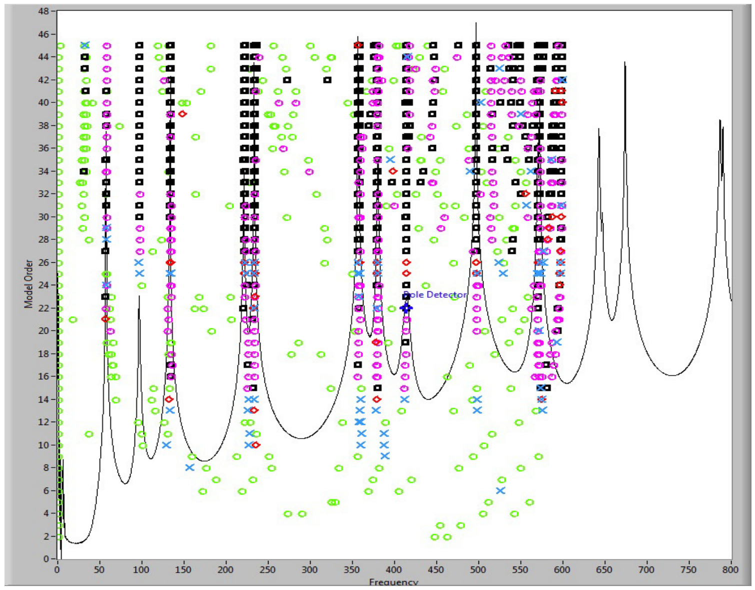
Table 4.13. Parameter estimation results

Mode	X-Modal II		The developed software					
	RFP		ITD		PTD		ERA	
	Freq(Hz)	ξ (%)	Freq(Hz)	ξ (%)	Freq(Hz)	ξ (%)	Freq(Hz)	ξ (%)
1	57.229	0.034	56.963	0.042	56.802	0.032	56.919	0.151
2	57.861	0.030	57.577	0.058	57.531	0.179	57.628	0.253
3	97.062	0.560	96.736	0.709	96.724	0.589	96.949	1.212
4	133.373	0.024	133.090	0.025	133.077	0.024	-	-
5	134.013	0.025	-	-	133.729	0.022	133.720	0.035
6	221.273	0.148	220.961	0.156	221.009	0.161	221.050	0.127
7	222.992	0.115	222.442	0.139	222.487	0.140	222.445	0.404
8	233.560	0.034	233.262	0.032	233.252	0.033	233.318	0.039
9	234.531	0.031	234.209	0.039	234.221	0.035	234.270	0.047
10	357.000	0.032	356.692	0.032	356.695	0.033	356.683	0.031
11	358.974	0.025	358.558	0.027	358.547	0.023	358.572	0.034
12	379.203	0.116	378.954	0.109	378.753	0.176	378.690	0.183
13	380.603	0.130	380.323	0.124	380.119	0.159	380.126	0.138
14	414.344	0.668	414.318	0.770	413.960	0.605	413.900	0.618
15	496.754	0.034	496.424	0.034	496.422	0.035	496.459	0.034

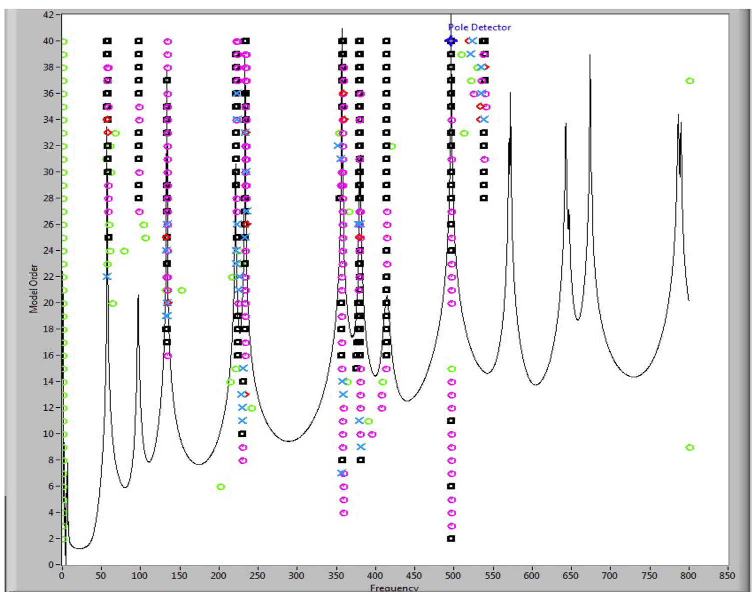


(1)

Figure 4.22. Stability diagrams obtained by different methods – (1) ITD, (2) PTD, (3) ERA

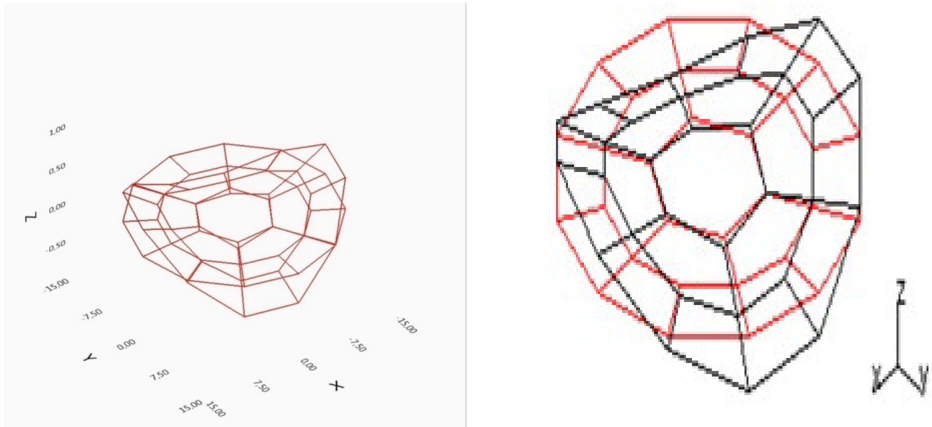


(2)

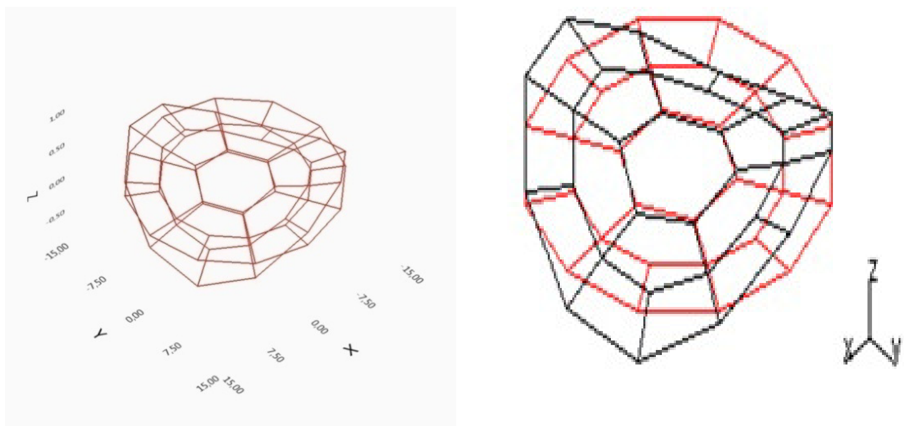


(3)

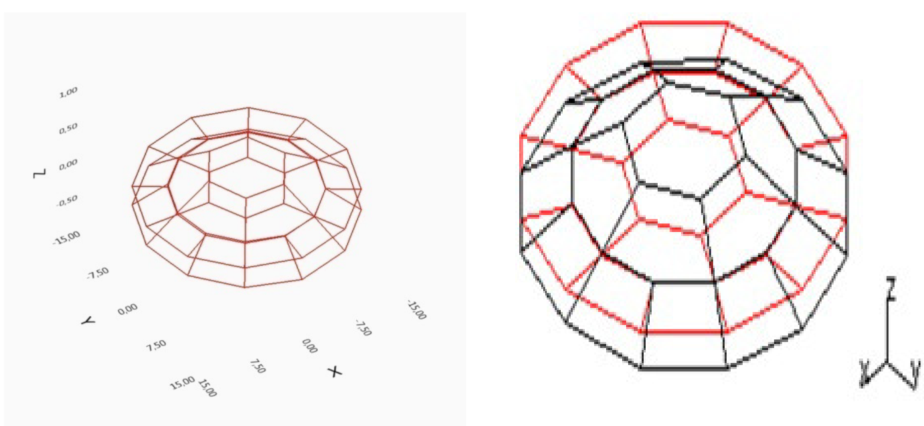
Figure 4.22. Continued



(1)

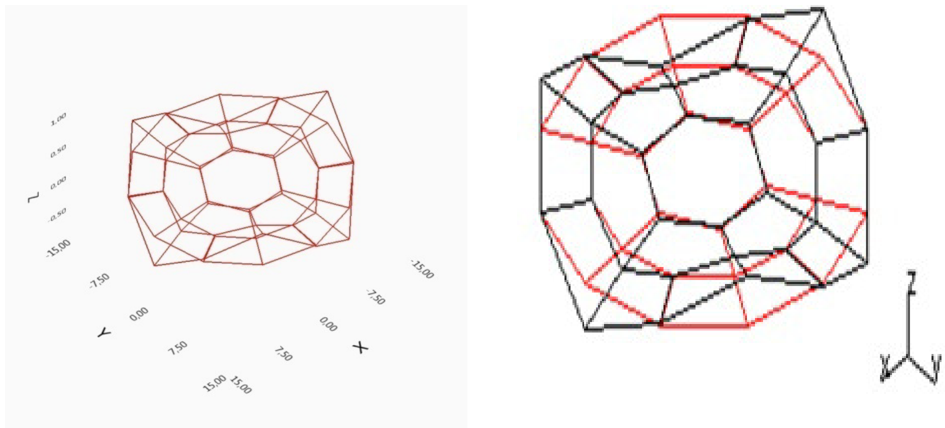


(2)

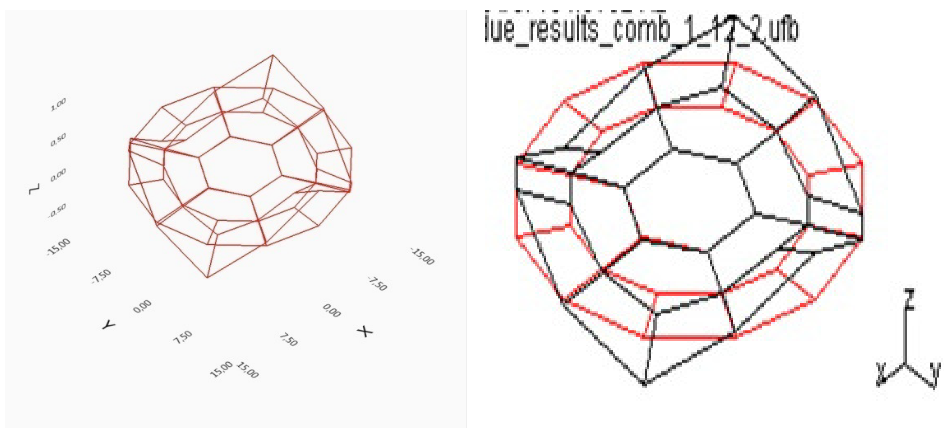


(3)

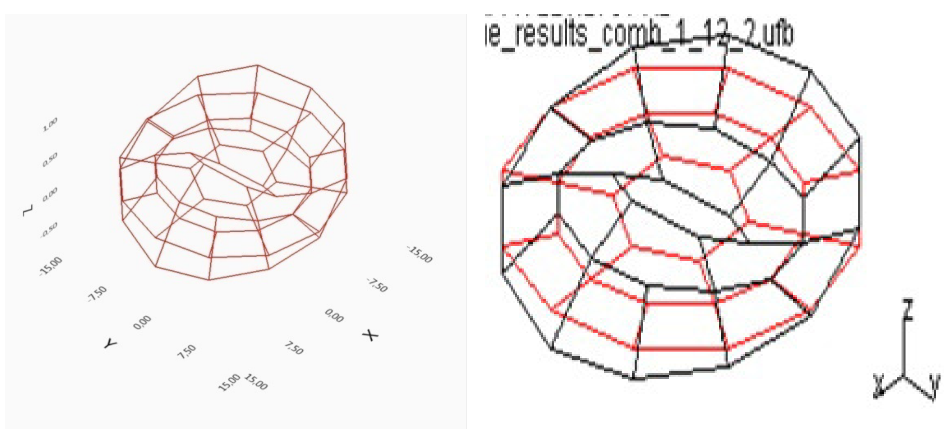
Figure 4.23. Mode shape plots obtained from the developed software & X-Modal II – (1) 1st mode, (2) 2nd mode, (3) 3rd mode, (4) 4th mode, (5) 5th mode, (6) 6th mode, (7) 7th mode, (8) 8th mode, (9) 9th mode, (10) 10th mode, (11) 11th mode, (12) 12th mode, (13) 13th mode, (14) 14th mode, (15) 15th mode



(4)

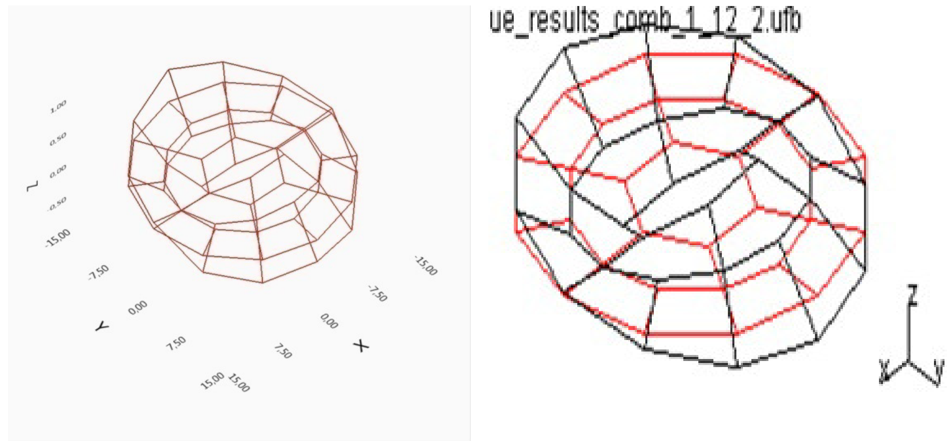


(5)

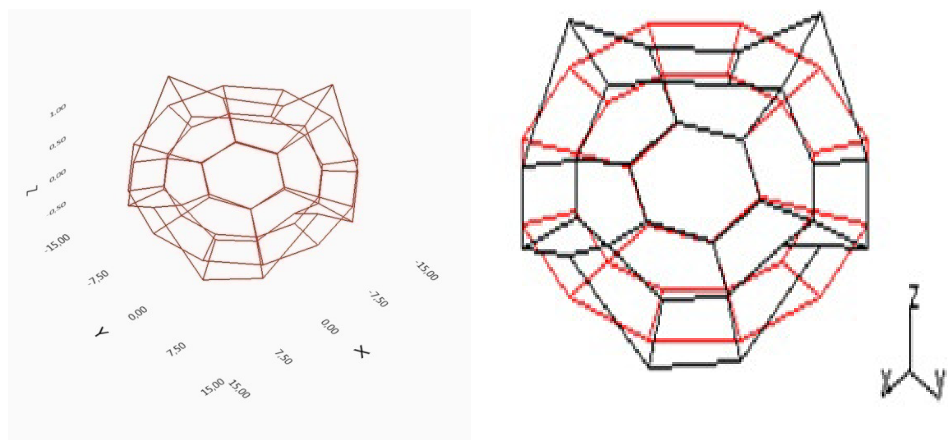


(6)

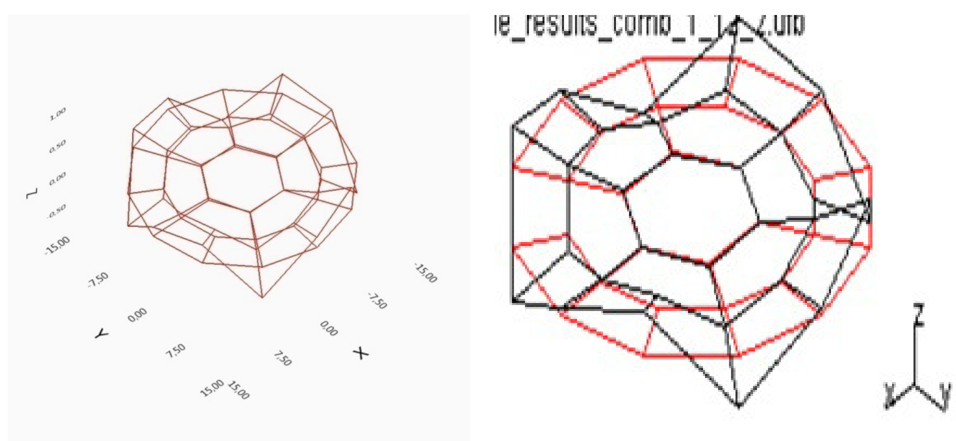
Figure 4.23. Continued



(7)

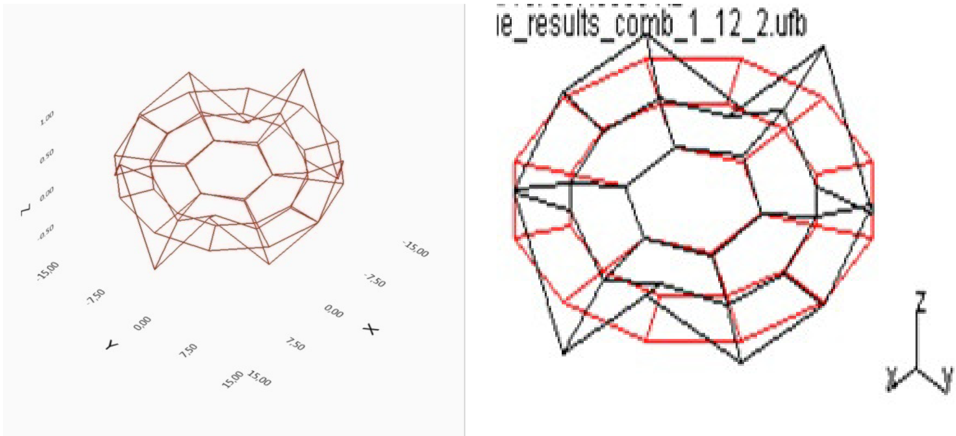


(8)

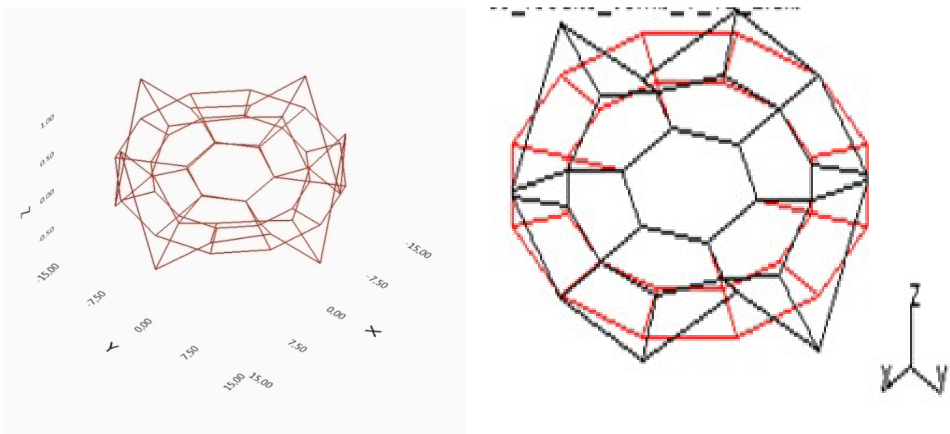


(9)

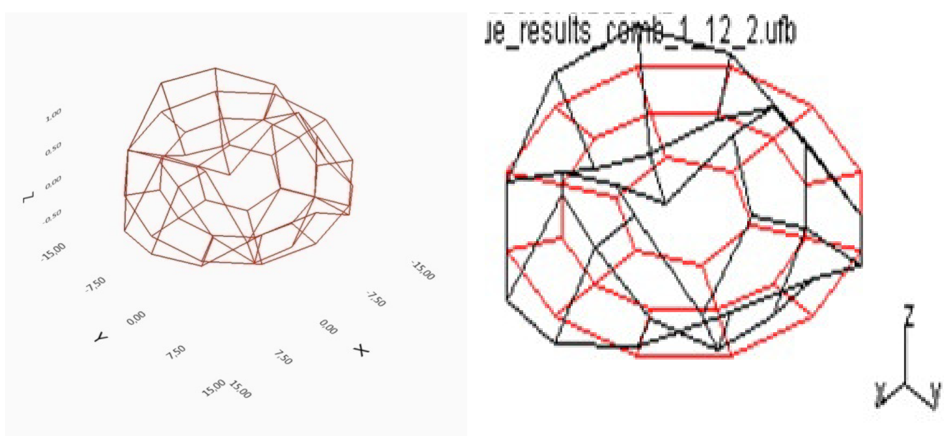
Figure 4.23. Continued



(10)

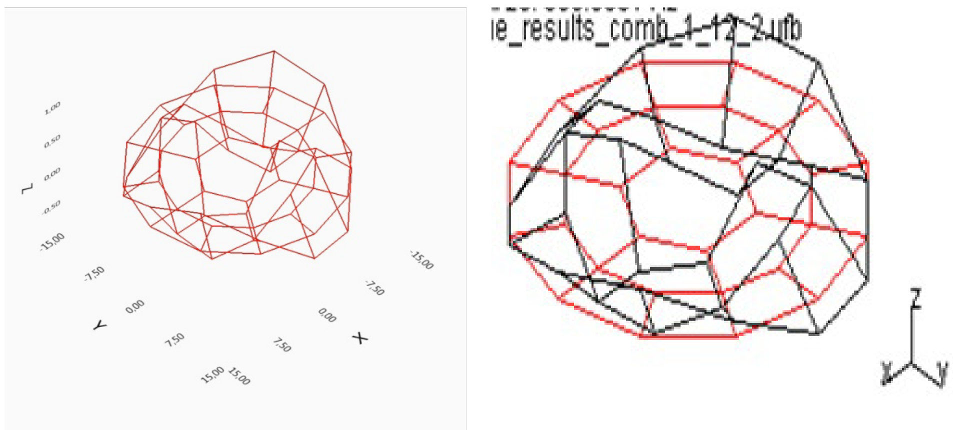


(11)

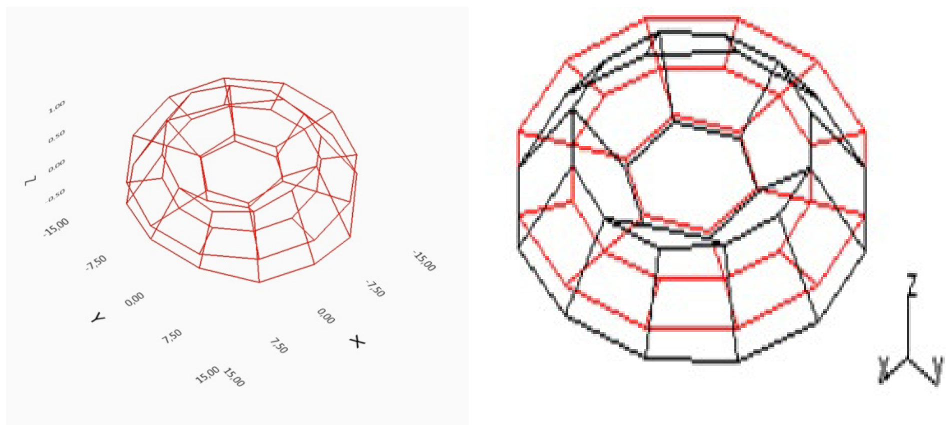


(12)

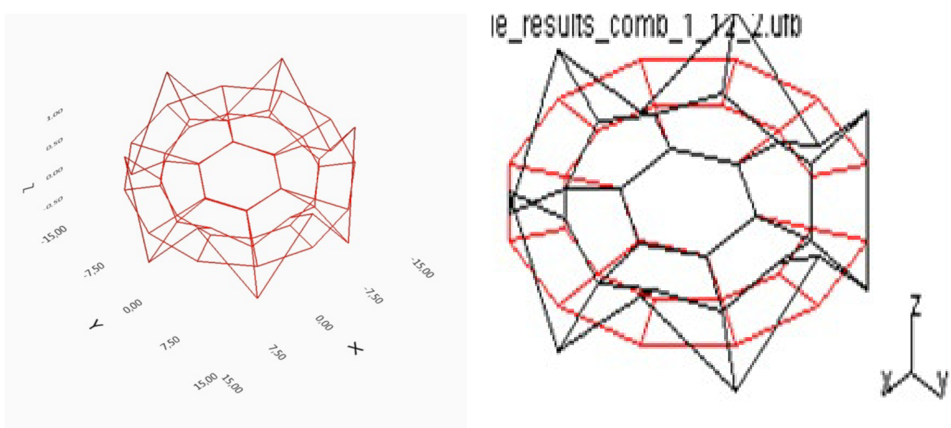
Figure 4.23. Continued



(13)



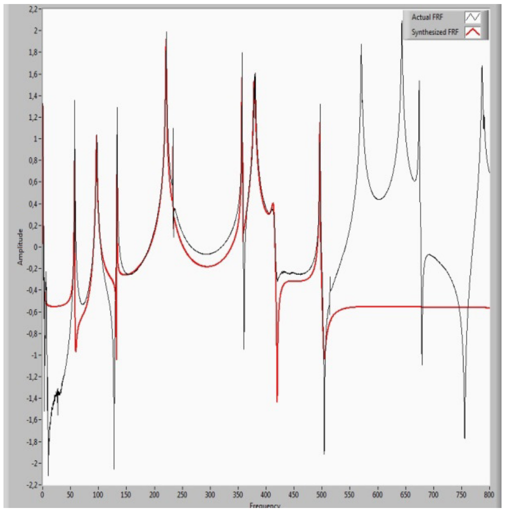
(14)



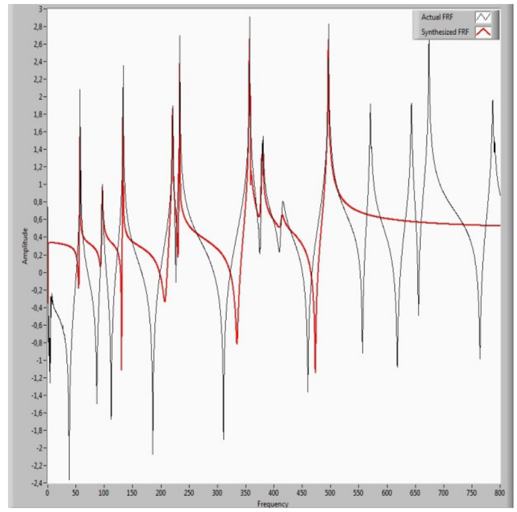
(15)

Figure 4.23. Continued

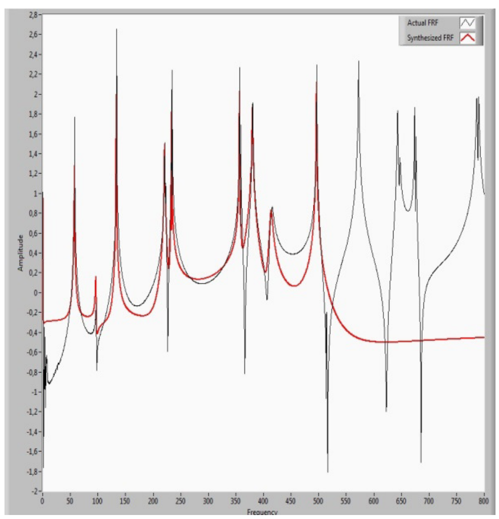
Finally, synthesized FRF results are presented in Figure 4.24.



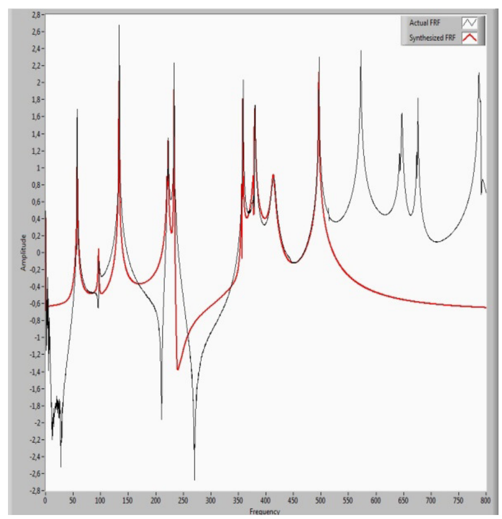
(1)



(2)



(3)



(4)

Figure 4.24. Synthesized FRFs of node points between (1) 4Z-19Z, (2) 19Z-19Z, (3) 20Z-14Z, (4) 22Z-14Z

CHAPTER 5

CONCLUSION & FUTURE WORK

In the scope of the thesis, a modal analysis software platform is developed which is mainly based on experimental modal identification techniques. Various modal parameter estimation techniques are investigated and selected for the software to be implemented. At least one time and frequency domain method, from simple to complex, is included for the software.

LabVIEW is used to develop the software. Whether the software gives the correct results is checked by one analytical & experimental case study. FRFs are gathered by harmonic analysis using ANSYS to perform the analytical study. Moreover, experimental results taken from a circular plate are processed. The validity of the results of the software can be checked since the correct results of analytical & experimental case studies are known. However, the software gives reasonable results, each method chosen for the software does not work with same efficiency. For instance, GRFP method occupies computer memory too much working with large amount of FRFs since it requires a transformation in between ordinary & orthogonal polynomial coefficients to overcome ill-conditioned numerical conditions. Furthermore, although PFD method identifies modal frequencies correctly, it can not find the correct mode shape vectors. In experimental case study, the structure has several repeated roots. PTD method is the only one which can identify all the modes of the system.

One of the objectives of the study is to get familiar with modal parameter identification techniques and understand the main idea to estimate modal parameters through these techniques. This background may lead new researchers to develop an original modal parameter estimation algorithm within the department. Another objective is to clearly define the sections of a modal analysis software. In other words, which features a modal software is to be include. Getting know-how regarding LabVIEW is an another motivation of the research.

In conclusion, modal analysis is a research field which is open to further discussion and development. Hopefully, this thesis work will make great contribution to upcoming researchers within the department.

Since the scope of the modal analysis is so wide, there are still much more concept that is to be investigated and discussed following this master study.

First of all, this study is mainly focused on modal identification techniques working with input & output measurements. However, there also exists output only modal identification techniques in literature. They are also called as Operational Modal Analysis (OMA) techniques. OMA has been growing its popularity in recent years since conducting a forced excitation tests are very difficult for some engineering structures such as large aerospace structures, civil engineering constructions like bridges. For such cases, modal parameter extraction which is possible directly from operational data where the input forces can not be measured is a great use for engineers. OMA techniques mainly utilize from auto- and cross-correlation functions instead of FRFs.

Several primary modal parameter estimation techniques that have been proposed as OMA techniques are Auto-Regressive Moving Averaging Models (ARMA), the Natural Excitation Technique (NExT), subspace identification methods which can be subdivided as Balanced Realization (BR) & Canonical Variate Analysis (CVA) as time domain methods. As for frequency domains, peak-picking can be adapted to OMA case. Furthermore, CMIF was reformulated and given a new name as the frequency-domain decomposition (FDD) method [47]. A new version of the RFP method which can used for OMA cases is also introduced by Schwarz & Richardson [48].

The available curve-fitting algorithms in literature can be grouped under the name of least-squares (LS) estimators. When the data obtained from modal testing is too noisy, LS estimators may not give satisfactory results. Maximum likelihood estimator (MLE) must then be used. This estimator identify modal parameters together with their confidence intervals. MLE is based on statistical framework. On the other hand, LS estimators has deterministic approach. Yet, the framework of the LS estimators can be turned into statistical, and called as Weighted Generalized Total Least Squares (WGTLS) or Bootstrapped Total LSE. The examples of these implementations can be found in [49]. Although MLE can tackle with noisy measurements, it has some drawbacks. MLE requires much more computational effort and initial estimates for the modal parameters, which are obtained by LS estimators [50].

Unfortunately, pole selection of the software is based on the user interpretation. However, an automated modal analysis procedure can be developed in order to overcome the problems of inexperienced analysts. Automating the modal parameter estimation process also aims to prevent inconsistency between estimates of different operators. J. Lanslots *et. al.* [51] presents an approach for automating pole selection. Similarly, P. Verboven *et. al.* [52, 53] develops a clustering algorithm for the poles which divides the poles into physical and mathematical ones without any user interaction.

To summarize up the points discussed above:

- More identification techniques can be added to the software.
- OMA techniques can be added to the software.
- MLE can be an additional assisting tool for the software.
- An automated mode selection procedure can be developed for the software

REFERENCES

[1] Brian J.Schwarz, Mark H.Richardson: "Experimental Modal Analysis" Vibrant Tech. Inc., Orlando FL, October 1999.

[2] Fundamentals of Modal Testing Application Note 243-3. Agilent Technologies

[3] H, Jimin. & F, Zhi-Fang. “Modal Analysis”. Oxford;Boston:Butterworth–Heinemann, pp.158-196.

[4] Maia, N. M. & Silva, J. M. Modal Analysis Identification Techniques. Phil. Trans. R. Soc. Lond. A 2001 359, 29-40.

[5] Dobson, B.J. Modal Analysis Using Dynamic Stiffness Data. Royal Naval Engineering College (RNEC), TR-84015, June 1984.

[6] Dobson, B.J. A straight-line technique for extracting modal properties from frequency response data. Mechanical Systems and Signal Processing, Vol. 1, 1987, pp. 29-40.

[7] Brown, D., Allemang, R.J., Zimmerman, R., Mergeay, M. Parameter Estimation Techniques for Modal Analysis. SAE Technical Paper Series, No.790221, 1979.

[8] Vold, H., Kundrat, J., Rocklin, G.T., Russel, R. A Multi-Input Modal Estimation Algorithm for Mini-Computers. SAE Technical Paper Series No. 820194, 1982.

[9] Vold, H. & Rocklin G.T. The Numerical Implementation of a Multi-Input Modal Estimation Method for Mini-Computers, Proceedings of the 1st International Modal Analysis Conference (IMAC I), Orlando, Florida, USA, 1982, pp.542-548.

[10] Zhang, L. *et. al.* An Improved Time Domain Polyreference Method for Modal Identification. Mechanical System and Signal Processing, v1, n4, 1987.

[11] Ibrahim, S.R. & Mikulcik, E.C. A Method for the Direct Identification of Vibration Parameters from the Free Response. The Shock and Vibration Bulletin, Vol.47, No.4, 1977, pp.183-198.

[12] Ibrahim, S.R. Modal Confidence Factor in Vibration Testing. The Shock and Vibration Bulletin, Vol.48, No.1, 1978, pp.65-75.

[13] Fukuzono, K. Investigation of Multiple Reference Ibrahim Time Domain Modal Parameter Estimation Technique. MSc. Thesis, Department of Mechanical and Industry Engineering, University of Cincinnati, 1986.

[14] Juang, J.N., Pappa, R.S. An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction. Journal of Guidance, Control, and Dynamics, Vol. 8, No. 5, Sept-Oct. 1985, pp. 620-627.

[15] Juang, J.N. & Suzuki, H. An Eigensystem Realization Algorithm in Frequency Domain for Modal Parameter Identification. Transactions of the ASME, Journal of Vibrations, Acoustics, Stress, and Reliability in Design, Vol. 110, 1988.

[16] De Callafon, R.A. *et al.* General Realization Algorithm for Modal Identification of Linear Dynamic Systems. Journal of Engineering Mechanics © ASCE, September 2008.

[17] Andersen, P. 1997 Identification of civil engineering structures using vector ARMA models. PhD Thesis, Aalborg University, Denmark.

[18] Richardson, M. H. & Formenti D. L. Parameter Estimation From Frequency Response Measurements Using Rational Fraction Polynomials. In 1st IMAC Conference, Orlando, FL, 1982.

[19] Richardson, M. H. & Formenti, D.L. Global Curve Fitting of Frequency Response Measurements using the Rational Fraction Polynomial Method. Proceedings of the 3rd International Modal Analysis Conference, Orlando, Florida, January 28-31, 1985.

[20] Richardson, M. H. Global Frequency & Damping Estimates From Frequency Response Measurements. In 4th IMAC Conference, Los Angeles, CA, 1986.

[21] Van Der Auweraer, H., Leuridan, J. Multiple Input Orthogonal Polynomial Parameter Estimation. Mechanical Systems and Signal Processing, Vol.1, No.3, 1987, pp.259-272.

[22] Shih, C.Y., Tsuei, Y.G., Allemang, R.J., Brown, D.L. A Frequency Domain Global Parameter Estimation Method for Multiple Reference Frequency Response

Measurements, Mechanical Systems and Signal Processing, Vol.4, No.2, Oct. 1988, pp.349-365.

[23] Fasana, A. Modal Parameters estimation in the Z-domain. Mechanical Systems and Signal Processing 23 (2009), 217-225.

[24] Zhang, L., Kanda, H. The Algorithm and Application of a New Multi-Input-Multi-Output Modal Parameter Identification Method. The 56th Shock and Vibration Symposium, October 1985.

[25] Zhang, L., Kanda, H., Brown, D., Allemang R.J. A Polyreference Frequency Domain Method for Modal Parameter Identification. ASME Paper, No.85-DET-106, 1985, pp. 1-6.

[26] Zhang, L., Kanda, H., Lembregts, F. Some Applications of Frequency Domain Polyreference Modal Parameter Identification Method. Proceedings of the 4th International Modal Analysis Conference (IMAC IV), Vol. II, Los Angeles, California, USA, 1986, pp. 1237-1245.

[27] Lembregts, F., Leuridan, J., Van Brussel H. Frequency Domain Direct Parameter Identification for Modal Analysis: State Space Formulation. Mechanical Systems and Signal Processing, Vol. 4, No. 1, 1990, pp.65-75.

[28] Peeters, B. *et. al.* The PolyMAX frequency-domain method: a new standard for modal parameter estimation? Shock and Vibration 11, 2004, pp.395-409.

[29] Allemang R.J. Experimental Modal Analysis. UC-SDRL-RJA-CN-20-263-663/664, Revision: June 7,1999+.

[30] Avitabile, P. Modal Space In Our Own Little World. Modal Analysis and Control Laboratory University of Massachusetts Lowell.

[31] Williams, R., Crowley, J. & Vold, H. Multivariate Mode Indicator Function in Modal Analysis. Proceedings of International Modal Analysis Conference III, January 1985.

[32] Shih C.Y., Tsue Y.G., Allemang R.J., Brown D.L. Complex Mode Indication Function and its Applications to Spatial Domain Parameter Estimation. IMAC VII, January 30, 1989.

[33] LMS International, The LMS Theory and Background Book, Leuven, Belgium, 2000.

[34] Vacher, P., Jacquier B., Bucharles A. Extensions of the MAC Criterion to complex modes. Proceedings of ISMA 2010 including USD2010, 2713-2726.

[35] Iglesias M.A. Investigating Various Modal Analysis Extraction Techniques to Estimate Damping Ratio, Msc Thesis, Mechanical Engineering, Virginia Polytechnic Institute and State University, 2000.

[36] Beskhyroun S. Graphical Interface Toolbox for Modal Analysis. Proceedings of the Ninth Pacific Conference on Earthquake Engineering, 14-16 April, 2011, Auckland, New Zealand.

[37] Allemang R.J. & Brown D.L. A Unified Matrix Polynomial Approach to Modal Identification. Journal of Sound and Vibration 211(3), 301-322, 1998.

[38] Peeters, B. & Ventura, C. Comparative Study of Modal Analysis Techniques for Bridge Dynamic Characteristics. Mechanical Systems and Signal Processing, 2003.

[39] Jianping H., Xilin L., Feixing W. Comparison of Modal Parameter Identification Algorithms based on Shaking Table Model Test data. International Conference on Experimental Mechanics, 2008.

[40] Structural Dynamics Research Lab at the University of Cincinnati. Universal File Formats for Modal Testing. Available at www.sdrl.uc.edu.

[41] Ewins, D. J. 1984 Modal testing: theory and practice. Taunton: Research Studies Press.

[42] Richardson, M. & Schwarz, B. Modal Parameter Estimation from Operating Data. Vibrant Technology, Inc., Jamestown, California.

[43] Miramand, N., Billand, J.F., LeLeux, F. and Kernevez, J.P. Identification of Structural Modal Parameters by Dynamic Tests at a Single Point, Nov., 1982.

[44] Kelly, L.G. Handbook of Numerical Methods and Applications, Chapter 5 Curve Fitting and Data Smoothing. Addison-Wesley Pub.Co.Inc., 1967.

[45] Maia, N. M. M. & Silva, J. M. M. 1997 Theoretical and experimental modal analysis. Baldock, UK: Research Studies Press (distributed by Wiley, Chichester).

[46] C, Cai. *et. al.* Modeling of Material Damping Properties in ANSYS. Defense Systems Division, Institute of High Performance Computing 89C Science Park Drive.

[47] Brincker, R. *et. al.* Modal identification from Ambient Responses using Frequency Domain Decomposition. Proceedings of IMAC 18 2000, San Antonio, TX, USA , 625-630.

[48] Schwarz, B. & Richardson, M. Modal Parameter Estimation from Ambient Response Data. Presented at IMAC 2001, Feb 5-8.

[49] Verboven, P. Frequency-Domain System Identification for Modal Analysis. PhD Thesis, Dept. of Mechanical Engineering, Vrije University, Belgium, 2002.

[50] Zhang, L. An Overview of Major Developments and Issues in Modal Identification. Institiute of Vibration Engineering, Nanjing University of Aeronautics & Astronautics.

[51] Lanslots, J. *et. al.* Automatic modal analysis: reality or myth? LMS International.

[52] Verboven, P. *et. al.* User-assisting tools for a fast frequency-domain modal parameter estimation method. Mechanical Systems and Signal Processing 18 (2004), 759-780.

[53] Vanlanduit, S. *et. al.* An automatic frequency domain modal parameter estimation algorithm. Journal of Sound and Vibration 265 (2003), 647-661.