MIDDLE SCHOOL STUDENTS' ACHIEVEMENT LEVELS, SOLUTION STRATEGIES, AND REASONS UNDERLYING THEIR INCORRECT ANSWERS IN LINEAR AND NON-LINEAR PROBLEMS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF SOCIAL SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

RUKİYE AYAN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
THE DEPARTMENT OF
ELEMENTARY SCIENCE AND MATHEMATICS EDUCATION

FEBRUARY 2014

Approval of the Graduate School of Social Sciences

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ABSTRACT

MIDDLE SCHOOL STUDENTS' ACHIEVEMENT LEVELS, SOLUTION STRATEGIES, AND REASONS UNDERLYING THEIR INCORRECT ANSWERS IN LINEAR AND NON-LINEAR PROBLEMS

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February 2014, 185 pages

Purposes of the study are three-fold. The first purpose is to investigate sixth, seventh, and eighth grade students' achievement levels in linear and non-linear problems regarding length, perimeter, area, and volume concepts. The second purpose is to determine correct solution strategies for these problems. The third purpose is to explore underlying reasons for incorrect answers in these problems. A mixed-method research design was utilized to reach these purposes.

Participants were selected through cluster random sampling. Data were collected during the spring semester of 2012-2013 academic year from 935 sixth, seventh, and eighth grade students enrolled in public middle schools in Yenimahalle District of Ankara. The achievement test including 10 open-ended questions was administered. Individual interviews were conducted with 12 participants to amplify their answers to the problems.

Findings indicated that achievement levels of the participants in linear problems were considerably high compared to those in non-linear problems. Findings also revealed that students used a limited number of strategies for linear and non-linear problems. These strategies were found to have lacked the argument of the linear and non-linear relationships for most of the participants' answers.

Underlying reasons for the incorrect answers of the participants were also explored. The common reasons for incorrect answers in linear and non-linear problems were inadequate knowledge in geometry or other mathematical concepts such as proportions. The main reason for the low achievement level in non-linear problems was assuming a linear relationship between the length and area or length and volume of geometrical figures.

Keywords: Illusion of linearity, additive and multiplicative reasoning, proportional reasoning, area and volume, middle school students

ÖZ

ORTAOKUL ÖĞRENCİLERİNİN DOĞRUSAL VE DOĞRUSAL OLMAYAN PROBLEMLERDEKİ BAŞARI DÜZEYLERİ, ÇÖZÜM STRATEJİLERİ VE YANLIŞ CEVAPLARININ NEDENLERİ

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Şubat 2014, 185 sayfa

Çalışmanın amaçları üç kısımdan oluşmaktadır. Çalışmanın birinci amacı, altıncı, yedinci ve sekizinci sınıf öğrencilerinin uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarı düzeylerini incelemektir. Çalışmanın ikinci amacı, bu öğrencilerin bahsedilen problemlerde kullandıkları çözüm stratejilerinin belirlenmesidir. Çalışmanın üçüncü amacı, öğrencilerin problemlere verdikleri yanlış cevapların nedenlerinin araştırılmasıdır. Bu amaçlara ulaşmak için karma bir araştırma yöntemi kullanılmıştır.

Veriler 2012-2013 öğretim yılı bahar döneminde Ankara'nın Yenimahalle ilçesindeki devlet okullarına devam eden 935 altıncı, yedinci ve sekizinci sınıf öğrencilerinden toplanmıştır. Katılımcılara 10 açık uçlu problemden oluşan başarı testi uygulanmıştır. Ek olarak, her sınıf seviyesinden toplam 12 katılımcı ile katılımcıların testteki cevaplarını açıklamaları amacıyla bireysel görüşmeler yapılmıştır.

Çalışmanın bulguları katılımcıların doğrusal problemlerdeki başarılarının doğrusal olmayan problemlerdeki başarılarına göre daha yüksek olduğunu göstermiştir. Ayrıca, bulgular öğrencilerin doğrusal ve doğrusal olmayan problemler için sınırlı sayıda strateji kullandıklarını göstermiştir. Birçok katılımcının cevaplarında doğrusal ve doğrusal olmayan ilişkilere dayalı bir akıl yürütme olmadığı görülmüştür.

Öğrencilerin sorulara verdikleri yanlış cevapların nedenleri de incelenmiştir. Doğrusal ve doğrusal olmayan problemlere verilen yanlış cevapların ortak nedenlerinin geometri ve ölçme alanında ya da orantı gibi diğer matematiksel kavramlardaki yetersiz bilgiler olduğu görülmüştür. Doğrusal olmayan problemlerdeki düşük başarının asıl sebebinin geometrik cisimlerin kenar uzunlukları ve alanları ya da hacimleri arasında doğrusal bir ilişki olduğu varsayımından kaynaklandığı görülmüştür.

Anahtar Kelimeler: Doğrusallık yanılsaması, toplamsal ve çarpımsal ilişkiler, orantısal düşünme, alan ve hacim, ortaokul öğrencileri

To My Family

ACKNOWLEDGMENTS

I would like to thank to my advisor Assoc. Prof. Dr. Mine IŞIKSAL-BOSTAN who always challenged me to do my best and kept this study in perspective. I have learnt so much about research and teaching from you. Every meeting and personal conversation with you broadened my horizon both about academic and personal issues. Thanks for always having your door open and sharing your support and intellect with me. I could have never made it to this point without you. Thanks a million.

I would also like to thank my committee members Assoc. Prof. Dr. Erdinç ÇAKIROĞLU, Assoc. Prof. Dr. Bülent ÇETİNKAYA, Assist. Prof. Dr. Didem AKYÜZ, and Assist. Prof. Dr. Elif YETKİN ÖZDEMİR for their feedback, contributions, and suggestions for my study.

I wish to express my sincere thanks and gratitude to my mother, my father, my sister, and my brother for their love, encouragement, and support throughout my life. You have always granted that you would always be by my side to support me. Without your love, tolerance, and patience I would have never finished this study.

I also extend my appreciation to my dear friends Murat Işıldak, Ayşe Yolcu, Bilal Özçakır, and Duygu Aydemir and my office-mates who volunteered to help me throughout every step of this study. I am so lucky that I have such life-saver friends like you.

Extra special thanks go to my dear friends who have always been there to share the most joyful and genuine times: Ayşegül Çabuk, Duygu Aydemir, Nagehan Orhan, Rezzan Doktoroğlu, Elanur Yılmaz, and Özlem Fatma Yıldırım.

Finally, I would like to thank TÜBİTAK for the scholarship which helped me pursue my master study.

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LIST OF ABBREVIATIONS

CCSSM	Common Core State Standards for School Mathematics
EARGED	Eğitim Araştırma Geliştirme Dairesi Başkanlığı
MoNE	Ministry of National Education
NAEP	National Assessment of Educational Progress
NCTM	National Council of Teachers of Mathematics
PASW	Predictive Analytics Software
PISA	Programme for International Student Assessment
RNP	Rational Number Project
SPSS	Statistical Package for the Social Sciences
TIMSS	Trends in International Mathematics and Science Study

CHAPTER 1

INTRODUCTION

Proportional reasoning lies at the heart of many mathematical structures, especially those included in the primary and middle school mathematics curricula. In addition to mathematical structures, proportional reasoning is also essential in understanding many situations in science and in everyday life (Cramer & Post, 1993). Hence, proportional reasoning is referred to as a watershed concept, a cornerstone of higher mathematics, a capstone of elementary concepts (Lesh, Post, & Behr, 1988) and a gateway to higher levels of mathematics success (Kilpatrick, Swafford, & Findell, 2001). Due to the importance attached to proportional reasoning, Common Core State Standards for School Mathematics (CCSSM, 2010) places extensive emphasis on proportional reasoning. Indeed, CCSSM identifies proportional reasoning as one of the key areas to which a significant portion of the instructional period should be devoted in sixth and seventh grade mathematics lessons. Similar to the American Standards, the revised Turkish Middle School Mathematics Curriculum highlights the importance of proportional reasoning and devotes a substantial portion of course time to it (MoNE, 2013).

Despite the great emphasis on proportional reasoning in mathematics curricula, several national and international studies reported low achievement and difficulties of students with respect to this concept (Kaplan, İşleyen, & Öztürk, 2011; Lobato & Thanheiser, 2002; Modestou & Gagatsis, 2007; Thompson & Preston, 1994). It is argued that the difficulties experienced by students in proportional reasoning might be derived from the fact that students' understanding of proportional reasoning tends to be superficial and limited since proportional reasoning involves, as conventionally referred to, solving missing-value problems (Post, Behr, & Lesh, 1988).

Nevertheless, there is a consensus in the mathematics education literature that solving missing-value problems cannot be regarded as an indicator of proportional reasoning (Cramer & Post, 1993; Post et al., 1988). In addition to solving missing-value problems, proportional reasoning is also related to the capabilities to compare quantities in a multiplicative rather than additive manner (Kestell & Kubota-Zarivnij, 2013) and to discern mathematical characteristics of proportional reasoning from those of non-proportional reasoning (Cramer & Post, 1993). However, the fact that teaching of proportional reasoning does not go beyond teaching of standardized rules for solving proportionality problems in a limited number of forms and applications might result in students' failure in proportional reasoning (Van Dooren, De Bock, Verschaffel, & Janssens, 2003).

As a result of a limited understanding of proportional reasoning, together with instruction focusing on procedural skills and solving missing value problems, students might not be able to develop different strategies for proportional reasoning problems. In fact, they might develop a habit of assuming a proportional relationship between any two quantities and applying proportional strategies even where they are not appropriate (Freudenthal, 1983). Applying proportional strategies where they are not applicable might occur in two ways: applying proportional strategies where additive strategies are required (Van Dooren, De Bock, & Verschaffel, 2010) and applying proportional strategies where non-proportional strategies are required (De Bock, Verschaffel, & Janssens, 1998). These two issues are considered as the reasons underlying students' difficulties in proportional reasoning and are to be investigated within the scope of this study. Furthermore, one of the areas in which these issues frequently constitute a problem is the area of geometry and measurement.

Kaput and West (1994) argued that geometry and measurement area is one of the most vulnerable areas to erroneous additive reasoning. Furthermore, NCTM (1989) pointed out that most students from grade 5 to grade 8 erroneously believe that "if the sides of a figure are doubled to produce a similar figure, the area and volume also will be doubled" (p.114–115). In other words, students believe that there is a proportional relationship between the sides of a figure and its area.

As opposed to students' beliefs, it is universally accepted that the valid principle for the relationships among length, area, and volume of reduced or enlarged geometrical figures is as follows: A linear enlargement or reduction by factor r multiplies lengths by factor r, areas by factor r^2, and volumes by factor r^3 (De Bock, Verschaffel, & Janssens, 2002). Nevertheless, students might not necessarily be awakened by their experiences in both real-life and mathematics lessons about the distinct growth rates of length, perimeter, area, and volume. Hence, students might develop tendencies to consider the relationship between length and area or between length and volume as proportional instead of quadratic and cubic. In the literature on mathematics education, this situation is referred to as "linear misconception", "linear obstacle", "linearity trap" and 'linear illusion" (De Bock et al., 1998, 2002; Freudenthal, 1983; Modestou & Gagatsis, 2007) or "illusion of proportionality" and "proportionality trap" (Behr, Harel, Post, & Lesh, 1992). However, in this study, the term 'illusion of linearity' will be used to refer to the students' belief that there is a linear relationship between the concepts of length, perimeter, area, and volume.

Proportionality is referred to as linearity, and linear and proportional relationships are used as synonyms in many studies (De Bock et al., 1998; 2002; Modestou & Gagatsis, 2007; 2010). Similarly, proportional relationships between length and perimeter are considered as linear relationships, and the terms proportional and linear are used interchangeably throughout the study.

1.1 Purposes of the Study

The purposes of this study are to investigate sixth, seventh, and eighth grade students' achievement levels in linear and non-linear problems regarding length, perimeter, area, and volume concepts and to determine students' correct solution strategies for these problems. This study also aimed at analyzing underlying reasons for students' incorrect answers in these problems.

1.2 Research Questions

- How successful are sixth, seventh, and eighth grade students at answering the linear and non-linear problems regarding length, perimeter, area, and volume concepts?

- Which correct solution strategies do sixth, seventh, and eighth grade students use to solve linear and non-linear problems regarding length, perimeter, area, and volume concepts?
- What are the underlying reasons for students' incorrect answers in linear and non-linear problems regarding length, perimeter, area, and volume concepts?

1.3 Significance of the Study

Proportional reasoning is related to many topics in mathematics and science, as well as situations in everyday life (Cramer & Post, 1993; Lesh et al., 1988). Basic scientific concepts related to proportional reasoning are temperature, density, concentrations, velocities, and chemical compositions (Karplus, Pulos, & Stage, 1983; Spinillo & Bryant, 1999). Everyday life situations include deciding on a best buy, grocery purchases, personal finances (Spinillo & Bryant, 1999), medicine dosages, and economic and sociological predictions (Valverde & Martínez, 2012). Moreover, proportional reasoning is an essential integrative concept which connects many mathematics topics in grades 6-8 (NCTM, 2000). Besides, it is a key and unifying concept in a wide variety of important topics beyond middle school (Van De Walle, Karp, Bay-Williams, & Wray, 2013). To begin with, Lamon (1999) stated that "proportional reasoning is one of the best indicators that a student has attained understanding of rational numbers" (p.3). Other mathematics topics related to proportional reasoning include ratios, fractions, percent, similarity, scaling, trigonometry (Beswick, 2011), basic algebra, geometry, problem solving (Empson, 1999; Fuson & Abrahamson, 2005; Hasemann, 1981; Saxe, Gearhart, & Seltzer, 1999), functions, graphing, algebraic equations, measurement (Karplus et al.,1983; Lamon, 2007; Vergnaud, 1988), probability and statistics, scale drawing, similar figures, measurement conversions (Greenes & Fendell, 2000) and steepness (Cheng, Star, & Chapin, 2013). In other words, proportional reasoning is a comprehensive, unifying, and integrative concept.

Similar to the relevance of proportional relationships to daily life, a large number of processes in daily life are also based on non-proportional situations (Ebersbach, Lehner, Resing, & Wilkening, 2008). Examples of situations involving non-proportional relationships include the acceleration of objects rolling down an inclined plane, the bounce height of a dropped ball, the temperature change in heating and cooling processes, the effects of compound interest rates, the spread of infections, the world's population growth, and-related to it-the decline of non-renewable natural resources (Ebersbach & Wilkening, 2007). Considering the geometry and measurement case, in the process of doubling or tripling each dimension of a geometrical figure non-proportional relationships occur among the length and area or length and volume of the figures. On the other hand, area is proportional to length only when width is held constant, and volume is proportional to length when width and height are held constant. Hence, students should be able to make inferences about distinct growth rates that occur among length, perimeter, area, and volume concepts when the figures are reduced or enlarged. Therefore, analyzing how Turkish middle school students make sense of proportional reasoning problems and proportional and non-proportional relationships in the context of geometry and measurement is essential. Moreover, considering the fact that there are a limited number of studies related to proportional reasoning skills in geometry and measurement area in related Turkish literature, conducting such a study seems significant.

Proportional reasoning is conventionally referred to as solving missing value problems. Nevertheless, it was mentioned that this definition does not include the essential components of proportional reasoning, which are the ability to distinguish additive and multiplicative structures and the ability to distinguish proportional situations from non-proportional ones. Therefore, it is significant to conduct a study by taking into consideration the essential components of proportional reasoning mentioned above. The results of such a study would give distinctive and valuable information related to the abilities and difficulties of students related to the components of proportional reasoning. Besides, the fact that the number of studies focusing on these components of proportional reasoning is highly restricted improves the significance of the present study.

NCTM (1989) states that "the ability to reason proportionally develops in students throughout grades 5-8" (p. 82). In other words, middle school years are the critical years for the development of proportional reasoning. Investigating students' development of proportional reasoning in years when they begin to form their proportional reasoning schemas is essential in order to understand how they make sense of proportional and non-proportional situations. Lamon (2006) stated that the number of people who are not proportional thinkers is even greater than half of the adult population. Therefore, it is of great significance that students' levels of achievements are investigated and the difficulties they experience related to proportional reasoning are addressed in early years before they constitute severe problems in later years (Fujimura, 2001).

Several studies revealed low student achievement in areas of proportional reasoning (Post, Behr, & Lesh, 1988), and geometry and measurement (Chappel & Thompson, 1999; Orhan, 2013; Sherman & Randolph, 2004; Tan-Şişman, 2010). In fact, Post et al. (1988) argued that "relatively few junior students of average ability use proportional reasoning in a consistent fashion" (p. 78). Furthermore, Sherman and Randolph (2004) reported low student achievement in geometry and measurement in international studies, such as TIMSS and PISA. This study tries to bind these two learning areas and aims to analyze students' achievement levels, correct strategies, and underlying reasons for their incorrect answers. Therefore, the results of this study might yield valuable information related to these aspects in these two specific areas and in the intersection point of these two areas.

Chapin and Johnson (2000) stated that students use distinct types of strategies for distinct types of problems. Moreover, the difficulties students experience in a subject might differ in its application to other contexts. To put it differently, students might make use of different solution strategies in proportional reasoning problems in geometry and measurement than they would in solving standard proportional reasoning problems. Similarly, they might experience distinct types of challenges in proportional reasoning problems related to length, perimeter, area, and volume concepts. Even though students' solution strategies for proportional word problems are documented in related literature, a detailed examination of their strategies for proportional and non-proportional problems related to concepts of length, perimeter, area, and volume have not been encountered in the available literature. Hence, investigating students' solution strategies and addressing their difficulties in a proportional reasoning task with a focus on areas of geometry and measurement and investigating their reasons have the potential to make a significant contribution to the existing literature in terms of not only identifying the difficulties students experience but also preventing and overcoming these difficulties by examining their implications to the classroom culture. On the other hand, the implications might also be extended to mathematics curriculum and teacher education by means of reconsidering the current curricula.

1.4 Definition of the Terms

Proportion: A proportion is defined as the multiplicative comparison of two equivalent ratios in the form $\frac{a}{b} = \frac{c}{d}$

Proportional reasoning: Proportional reasoning is defined as "making multiplicative comparisons between quantities" (Wright, 2005, p. 363) and "the ability to mentally process this relation" (Cheng et al., 2013, p. 23). It is also defined as "an aspect of mathematical reasoning used to make inferences and draw conclusions about multiplicative relationships involving direct variation" (Benson, 2009, p. 2).

In this study, proportional reasoning is defined as being able to apply proportional strategies only where they are applicable. Besides, it is also referred to as the ability of distinguishing additive and multiplicative structures as well as proportional and non-proportional situations.

Multiplicative Reasoning: Multiplicative reasoning is defined as "making multiplicative comparisons between quantities" (Wright, 2005, p. 363).

Additive Reasoning: In the additive reasoning "the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio" (Tourniaire & Pulos, 1985, p. 186).

Misinterpretation of additive and multiplicative reasoning is related to students' incapability to understand additive and multiplicative structures, their misinterpretation of additive and multiplicative relationships, and tendency to use them at inappropriate situations in this study.

Linearity: Linearity is referred to as the property of a function as homogenous and additive such that $f(ax) = a f(x)$ for all a and $f(x_1+x_2) = f(x_1) + f(x_2)$ mathematically. Particularly, the function $f(x)=ax$, with $a \neq 0$ satisfies both properties and can be seen to be representative of the terms "linear" or "linearity"; hence, the graph of a linear function is straight line passing through the origin (De Bock, Verschaffel, & Janssens, 2002). Looking at the inverse side, a proportion $\frac{a}{b} = \frac{c}{d}$ can be seen as a relationship of a linearity between two variables (Freudenthal, 1983). Hence, linearity is also referred to as the equality of a multitude of equal ratios as $\frac{a}{b} = \frac{c}{d}$.

In this study linearity refers to the proportionality in the form of $\frac{a}{b} = \frac{c}{d}$ and the function f (x) = ax.

Illusion of Linearity: In this study, illusion of linearity is defined as students' tendency to apply the model of linearity also in situations where it is not applicable. The term is also referred to as "linear misconception", "linearity obstacle", or "linearity trap" (De Bock et al., 1998) or "illusion of proportionality" and "proportionality trap" (Behr et al., 1992).

Within the scope of this study, linearity or a linear relationship is referred to as the type of a relationship in the same dimension for the length, perimeter, area and volume of geometrical figures. Also, illusion of linearity is referred to as ignorance of the fact that a linear enlargement or reduction by factor r multiplies lengths by factor r, areas by factor r^2, and volumes by factor r^3.

Area: Area is defined as "the two-dimensional space inside a region" (Van De Walle et al., 2013, p. 384)

Perimeter: Perimeter is defined as "a measure of the length of the boundary of a figure" (Ma, 1999, p. 84).

Volume: "Volume is defined as the size of three-dimensional objects" (Van De Walle et al., 2013, p. 391).

Linear Problems: These problems require a linear solution approach in this study.

Linear-Length Problems: These problems are related to the length concept and require a linear solution approach.

Linear-Perimeter Problems: These problems are related to the perimeter (or circumference) concept and require a linear solution approach.

Linear-Area Problems: These problems are related to the area concept and necessitate a linear solution approach.

Linear-Volume Problems: These problems are related to the volume concept and necessitate a linear solution approach.

Non-linear Problems: These problems require a non-linear solution approach.

Non-linear-Length Problems: These problems are related to the length concept and require a non-linear solution approach.

Non-linear-Perimeter Problems: These problems are related to the perimeter (or circumference) concept and require a non-linear solution approach.

Non-linear-Area Problems: These problems are related to the area concept and require a non-linear solution approach.

Non-linear-Volume Problems: These problems are related to the volume concept and necessitate a non-linear solution approach.

Questionable Proportion Strategy: This strategy included the ambiguity of whether the students considered the relationship between the variables given and asked in the problem or they just used the numbers given in the problem and directly wrote the proportion between these numbers for linear problems.

Reasonable Proportion Strategy: This strategy included an analysis of the problem statement, finding the related variables (i.e. perimeter), judging the type of the relationship between the variables, and then writing the direct proportion between the related variables for linear problems.

Length-Length-Area/Volume Relationship Strategy: This strategy included applying the linear relationships between the lengths of the figures and then finding the area or the volume of the second figure by using the relationship between lengths for non-linear problems.

Length-Area/Volume Relationship Strategy: This strategy included applying direct strategies for the relationship between the length and the area or volume of the figures, that is, anticipating that the area gets r^2 times larger than the length and the volume gets r^3 times larger than the length when the length increases by r for non-linear problems.

CHAPTER 2

LITERATURE REVIEW

The purposes of the current study include investigating the achievement levels of sixth, seventh, and eighth grade students in linear and non-linear problems regarding length, perimeter, area, and volume concepts. Besides, the study aims to analyze the strategies the participants of the study use for these problems and explore the reasons underlying their incorrect answers in these problems. In order to reach these purposes, it is initially essential to describe what proportional reasoning is, what strategies students use, and which difficulties they experience in solving proportions in broader means. Then, the theories and studies that explain students' strategies and reasons underlying their incorrect answers in proportional and non-proportional problems are stated. Lastly, concerns of the study related to areas of geometry and measurement are mentioned. Only then is it reasonable and possible to locate the goals of the study between the borders of proportional reasoning, and geometry and measurement.

2.1 Proportional Reasoning

A proportion is the multiplicative comparison of two equivalent ratios in the form $\frac{a}{b} = \frac{c}{d}$. Hence, proportional reasoning is related to "making multiplicative comparisons between quantities" (Wright, 2005, p. 363) and "the ability to mentally process this relation" (Cheng et al., 2013, p. 23). Moreover, proportional reasoning is referred to as reasoning in a system of two variables related by a linear function (Karplus et al., 1983). Lamon (1995) stated that a student has proportional reasoning skills if he makes reasonable judgments supporting the structural relationships which occur when two ratios are equivalent. For instance, a student who understands that \$2 for 5 apples is the same as \$4 for 10 apples and justifies that the four is the same multiple of two as ten is to five has proportional reasoning skills (Stemn, 2008). On the other hand, the student has to understand that the relationship between the quantities remains the same even when the quantities might change (Cramer, Post, & Currier 1993; Lobato & Ellis, 2010).

Proportional reasoning is referred to as a watershed concept, a cornerstone of higher mathematics, a capstone of elementary concepts (Lesh et al., 1988) and a gateway to higher levels of mathematics success (Kilpatrick et al., 2001). Parallel to the importance attached to proportional reasoning, it is referred to as one of the four key areas in sixth and seventh grade mathematics by The Common Core State Standards for School Mathematics (CCSSM, 2010). Similarly, a significant portion of importance is given to it in both current Turkish Middle School Mathematics Curriculum and the revised version (MoNE, 2008, 2013). Therefore, it can be inferred that proportional reasoning is one of the most crucial mathematical proficiencies for middle school students to be developed.

The importance of proportional reasoning is on account of the fact that it is related to many topics in mathematics and science, as well as situations in everyday life (Cramer & Post, 1993; Lesh et al., 1988). Basic scientific concepts related to proportional reasoning are temperature, density, concentrations, velocities, and chemical compositions (Karplus et al., 1983; Spinillo & Bryant, 1999). Everyday life situations include deciding on a best buy, grocery purchases, personal finances (Spinillo & Bryant 1999), medicine dosages, and economic and sociological predictions (Valverde & Martínez, 2012). Considering the mathematical concepts, NCTM (2000) stated that proportional reasoning is an essential integrative concept which connects many mathematics topics in grades 6-8. Besides, Van De Walle et al. (2013) indicated that it is a key and unifying concept in a wide variety of important topics beyond middle school. First, as indicated by Lamon (1999), "proportional reasoning is one of the best indicators that a student has attained understanding of rational numbers" (p. 3). Other mathematics topics related to proportional reasoning include ratios, fractions, percent, similarity, scaling, trigonometry (Beswick, 2011), basic algebra, geometry problem solving (Empson, 1999; Fuson & Abrahamson, 2005; Hasemann, 1981; Saxe et al., 1999), functions, graphing, algebraic equations, measurement (Karplus et al., 1983; Lamon, 2007; Vergnaud 1988), probability and statistics, scale drawing, linear functions, similar figures, measurement conversions (Greenes & Fendell, 2000) and steepness (Cheng et al., 2013).

Despite being such an inclusive, comprehensive, and essential concept in the curriculum, the definition of proportional reasoning is too superficial and restrictive in such a way that it is conventionally referred to as solving missing-value problems (Post et al., 1988). In other words, students who are able to solve problems asking for the fourth value when three values related to a situation are given are conventionally considered as reasoning proportionally. However, there is a consensus in the studies conducted in recent years that solving missing-value problems cannot be regarded as an indicator of proportional reasoning (Cramer & Post, 1993; Post et al., 1988).

2.1.1 Proportional Reasoning in the Turkish Mathematics Curriculum and Textbooks

The arguments made for the teaching of proportional reasoning with a focus on solving missing value problems might be considered valid when current Turkish Mathematics Curricula (2008) and the revised version (2013), and middle school mathematics textbooks are examined. For instance, within the context of the current Turkish Middle School Mathematics Curriculum (2008) students in grade six are expected to explain the concept of proportion and also the relationship between proportional situations. In the seventh grade, students are expected to solve problems including direct and inverse proportions. Even though there is no content area of proportion in the eighth grade, students are expected to determine the proportional lengths of similar figures.

The curriculum also stresses that proportion is not just related to writing two equivalent ratios and finding the missing value (MoNE, 2008), but requires a rich understanding of proportion in which proportional situations need to be recognized and the corresponding relationship needs to be investigated by means of numbers, tables, graphs, and equations. Hence, when the current curriculum is examined it is seen that it mentions some characteristics of proportional reasoning, such as recognizing proportional situations and the relationships. However, it is also seen that there is no explicit definition of and emphasis on proportional reasoning especially for the issues of distinguishing additive and multiplicative relationship or proportional and non-proportional situations. Moreover, the curriculum only gives place to direct and inverse proportions and not the non-proportional situations. On the other hand, when the revised version of Middle School Mathematics Curriculum (2013) is examined it is observed that there is no place for proportions in the sixth grade. However, an extensive emphasis is given to proportions and solving problems related to proportional situations in seventh grade. The curriculum also points out that students are expected to determine whether two quantities form a proportional situation or not and distinguish additive and multiplicative structures (MoNE, 2013). Similar to the arguments made for the current curriculum (2008), it is also seen that there is no explicit definition of and emphasis on proportional reasoning and the curriculum only gives place to direct and inverse proportions and not the non-proportional situations.

In addition to the mathematics curricula, sixth, seventh and eighth grade mathematics textbooks were also investigated in terms of the elements they included related to proportional reasoning and the sequence of these elements. To begin with the sixth grade textbook, the definition of ratio is given, and students are asked to compare the quantities multiplicatively and find several ratios between these quantities. An example of a problem related to finding the ratio between the number of quantities and comparing the two ratios in the sixth grade textbook is presented in Figure 2.1 below.

Bir simitçinin tezgâhında gün sonunda 5 simit, 7 poğaça ve 8 ay çöreği kalmıştır. Buna göre
a) Simitlerin sayısının ay çörekleri sayısına oranını farklı şekillerde yazalım.
b) Poğaça sayısının simit sayısına oranı ile simit sayısının poğaça sayısına oranını karşılaştıralım.

Figure 2.1 Sample problem related to ratio in sixth grade textbook (MoNE, 2012a, p. 126)

In addition to finding the ratios between various pairs of quantities, the concept of direct proportional relationships is introduced as “two quantities are directly proportional if one of the quantities is increased (or decreased) at the same ratio when the other quantity is increased (or decreased)" (MoNE, 2012a, p. 130). Then, some problems including direct proportional relationships are presented. A sample problem is given in Figure 2.2 below.

Bir otomobil fabrikasında 1 saatte 2 otomobil üretilmektedir. Bu fabrikada 6 saatte kaç otomobil üretilebileceğini bulalım.

Figure 2.2 Sample problem including direct proportional relationships in sixth grade textbook (MoNE, 2012a, p. 129)

The problem in Figure 2.2 asks the number of cars manufactured in six hours when the number of cars manufactured in one hour is given in the problem statement. For the solution of the problem, a table including the various number of cars according to the changing values of time periods is constructed, and students are required to find the answer by using this table. The solution to the problem in the textbook is presented in Figure 2.3 below.

Yandaki tabloda görüldüğü gibi süre; 2, 3, 4, ... kat artarken üretilen otomobil sayısı da aynı oranda 2, 3, 4, ... kat artar. Buna göre 6 saatte üretilen otomobil sayısı 12 olur.

Tablo: Otomobil Üretimi

Süre (saat)	1	2	3	4	5	6	7	...
Otomobil Sayısı	2	4	6	8	10	12	14	...

Figure 2.3 Solution to the problem including direct proportional relationships in sixth grade textbook (MoNE, 2012a, p. 130)

The instruction in the textbook continues with a definition of proportion as "the equality of two ratios" (MoNE, 2012a, p. 130). Then, the examples that require the examination of whether a pair of ratios form a proportion or not are provided.

In the mathematics curriculum, seventh grade students are expected to solve problems including direct and inverse proportions. In the seventh grade mathematics textbook, the definition for the proportions and direct proportional relationships are mentioned similar to the ones in the sixth grade textbook. In addition, the concept of scale factor is introduced. Some problems including direct proportional relationships are provided similar to the ones in sixth grade mathematics textbooks. A sample problem related to the proportional relationship is provided in Figure 2.4 below.

Okulumuzun dış cephesi boyanacaktır.

1) 2 kg boya ile 25 m^2 lik alan boyanabilmektedir. Buna göre yanda verilen tabloyu doldurunuz.

- Tabloyu yukarıdan aşağıya doğru okuduğunuzda boya miktarındaki değişim ile boyanan alan arasında nasıl bir ilişki olduğunu tartışınız.
- Tabloyu aşağıdan yukarıya doğru okuduğunuzda boya miktarındaki değişim ile boyanan alan arasında nasıl bir ilişki olduğunu tartışınız.
- Tabloda her bir satır için boya miktarı ile boyanacak alanın çarpımını bulunuz. Bulduğunuz oranlar arasındaki ilişkiyi söyleyiniz.

Boya Miktarı (kg)	Boyanacak Alan (m^2)
2	25
4	
6	
8	
10	
12	

Figure 2.4 Sample problem including direct proportional relationships in seventh grade textbook (MoNE, 2012b, p. 100)

The problem in Figure 2.4 is related to the direct proportional relationships between the amount of paint and the amount of the plane to be painted. A table including the various numbers for these two quantities is also provided. Another sample problem including direct proportional relationships is presented Figure 2.5 below.

2 kg elmadan 750 mL elma suyu elde ediliyor. 9 kg elmadan kaç mililitre elma suyu elde edileceğini bulalım.

Figure 2.5 Second sample problem including direct proportional relationships in seventh grade textbook (MoNE, 2012b, p. 132)

The problem in Figure 2.5 asks the amount of apple juice obtained from 9 kg when the amount of apple juice obtained from 2 kg is given. The solution to the problem is presented in Figure 2.6 below.

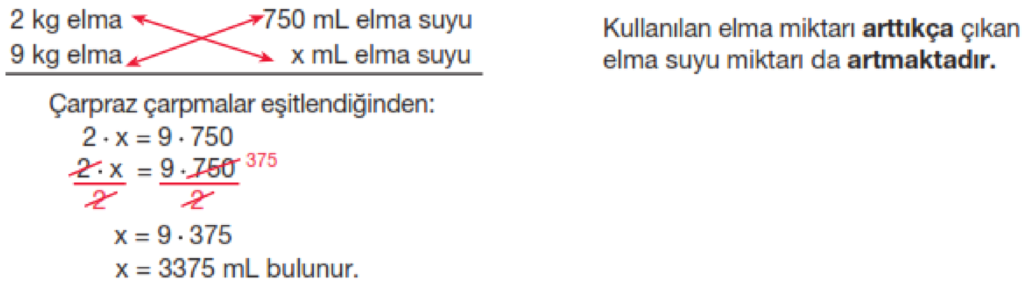


Figure 2.6 Solution to the second problem including direct proportional relationships in seventh grade textbooks (MoNE, 2012b, p. 101)

As seen in Figure 2.6, the problem is solved by cross-multiplication algorithm in which a direct proportional relationship is established between the number of apples and the amount of apple juice.

The teaching of proportional relationships in the seventh grade textbook continues with the definition of inverse proportion as "two quantities are inversely proportional if one of the quantities is decreased (or increased) at the same ratio when the other quantity is decreased (or increased)" (MoNE, 2012b, p. 102). Then, some problems related to the number of workers and work that is done is provided. A sample problem is presented in Figure 2.7 below.

Aşağıda verilen problemi okuyarak kendi cümlelerinizle ifade ediniz.
Bir halı dokuma atölyesindeki 3 işçi bir halıyı 12 günde dokuyor. Aynı halıyı, aynı hızla çalışan 5 işçi kaç günde dokur? Bulunuz.

Figure 2.7 Sample problem including inverse proportional relationships in seventh grade textbook (MoNE, 2012b, p. 105)

The problem in Figure 2.7 is related to the inverse proportional relationships between the number of workers and the time period required for manufacturing a carpet.

Even though there is no content area of proportion in the eighth grade, eighth grade students are expected to determine the proportional lengths of similar figures. In the meantime, the concept of scale factor is introduced and emphasized for the solution of the problems. A sample problem in the eighth grade textbook is provided in Figure 2.8 below.

Aşağıda verilen üçgenlerin benzer olup olmadıklarını inceleyelim:

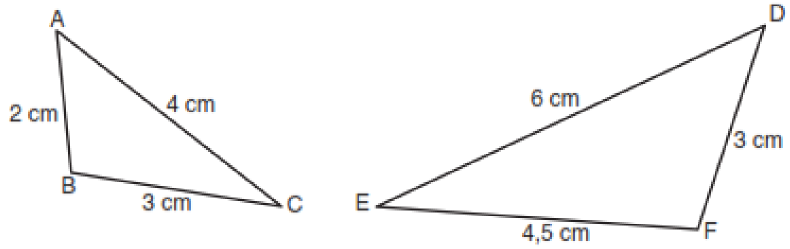


Figure 2.8 Sample problem related to determining the proportional lengths of similar figures in eighth grade textbook (MoNE, 2012c, p. 115)

The problem in Figure 2.8 asks whether the given two triangles are similar or not. The students are required to compare the ratios of the side lengths and determine whether these ratios are equivalent or not. The solution to the problem is provided in Figure 2.9 below.

İki üçgenin karşılıklı kenarlarını oranlayalım:

$$\frac{|AB|}{|DF|} = \frac{2}{3}$$

$$\frac{|AC|}{|DE|} = \frac{4}{6} = \frac{2}{3}$$

$$\frac{|BC|}{|FE|} = \frac{3}{4,5} = \frac{2}{3}$$

İki üçgenin karşılıklı kenarlarının oranı aynı olduğundan $\widehat{ABC}$ ve $\widehat{DFE}$ benzer üçgenlerdir. $\widehat{ABC}$'nin kenar uzunlukları, $\widehat{DFE}$'nin kenar uzunluklarının $\frac{2}{3}$' si oranında küçültülmüşüdür. Bu nedenle benzerlik oranı $\frac{2}{3}$' tür.

Figure 2.9 Solution to the sample problem related to determining the proportional lengths of similar figures in eighth grade textbook (MoNE, 2012c, p. 115)

The teaching of the similar figures in the eighth grade textbook continues with several examples related to determining whether two shapes are similar or not or finding the missing length of one of the two similar figures. Even though, the ratio of areas of two similar figures is not included in the teaching part a problem related to the ratio of the areas of similar figures is given place in assessment part of the section. The problem is provided in Figure 2.10 below. In the problem the ratio of areas of two similar figures is given as $\frac{16}{25}$, and the ratio of the corresponding side lengths of triangles are asked.

Benzer iki üçgenin alanlarının oranı $\frac{16}{25}$'tir. Bu iki üçgenin karşılıklı kenarlarının oranı nedir? Açıklayınız.

Figure 2.10 The problem related to the ratio of areas of similar figures (MoNE, 2012c, p. 121)

The examination of the curricula and textbooks revealed that proportions are introduced to the students by various problems including proportional situations. In the solution processes of these problems the use of tables, equality of ratios, and cross-product algorithm are emphasized. Then, situations including inverse proportions are introduced. Several problems related to direct and inverse proportions in limited contexts are presented.

2.1.2 Behaviors of a Proportional Thinker

It is discussed that conventional definitions of proportional reasoning is inadequate. However, defining proportional reasoning is rather difficult. One possible way to define proportional reasoning and to understand its components is classifying the characteristics of a person who reasons proportionally. To this purpose, Cramer and Post (1993) defined a set of behaviors of a proportional thinker. According to Cramer and Post, a *proportional thinker* should have the following characteristics as stated in Table 2.1 below:

Table 2.1 Behaviors of a Proportional Thinker (Cramer & Post, 1993, p. 342)

Behaviors of a Proportional Thinker
Knowing the mathematical characteristics of proportional situations
Being able to differentiate mathematical characteristics of proportional thinking from non-proportional contexts
Understanding realistic and mathematical examples of proportional situations
Realizing that multiple methods can be used to solve proportional tasks and that these methods are related to each other
Knowing how to solve quantitative and qualitative proportional-reasoning tasks
Being unaffected by the context of the numbers in the task

Cramer and Post (1993) emphasized that a proportional thinker should know the mathematical characteristics of proportional situations and discern proportional situations from non-proportional ones. In other words, being able to decide where to apply proportional reasoning and how to apply is a characteristic of a proportional thinker. Furthermore, developing different strategies for proportional reasoning problems is accepted as the sign of a proportional thinker.

Considering proportional reasoning in a narrower area, similarity concept is highly related to proportional reasoning in such a way that similar shapes are visual examples of a proportional situation (Van De Walle et al., 2013). Similarity in mathematical matters means that the shapes are the same even if they have distinct sizes. Similar figures have multiplicative relationships between the lengths and the widths (Lamon, 1999). Cox (2008) asserted that similarity is the unique geometric context for proportional situations, and children at or above 8 years of age can deal with similarity tasks by the help of visualization and pre-proportional strategies. Cox (2008) established a set of behaviors of a *geometric proportional thinker* based on the framework of Cramer and Post (1993) related to *proportional thinker*. Cox identified these behaviors in such a way that each behavior of a proportional thinker is adapted so as to be consistent with the terms of geometry, specifically the concept of similarity. The identified behaviors of proportional thinker (Cramer & Post, 1993) and geometrical proportional thinker (Cox, 2008) are provided in Table 2.2 below.

Table 2.2 Behaviors of a Geometric Proportional Thinker (Cox, 2008, p. 12)

Proportional Thinker	Geometric Proportional Thinker
Knowing the mathematical characteristics of proportional situations	Knowing the properties of similar figures
Being able to differentiate mathematical characteristics of proportional thinking from non-proportional contexts	Being able to recognize or surmise the presence and absence of distortion
Understanding realistic and mathematical examples of proportional situations	Understanding the principles of scale in both realistic and mathematical contexts
Realizing that multiple methods can be used to solve proportional tasks and that these methods are related to each other	Realizing that both within and between ratios can be used to differentiate figures and that these ratios also help judge the reasonableness of constructed figures
Knowing how to solve quantitative and qualitative proportional-reasoning tasks	Knowing how to scale images quantitatively and qualitatively and realizing the continuous nature of the scaling function
Being unaffected by the context of the numbers in the task	Being unaffected by the complexity or simplicity of the figure, the relationship of the labeled measurements, and the integral or non-integral nature of the numbers in the task

Cox (2008) maintained that a geometric proportional thinker should understand the principles of scale in different contexts and grasp the effects of scaling. Moreover, being able to understand principles of scale and effects of scaling in various complex figures is another characteristic of a geometric proportional thinker.

Cramer and Post (1993) and Cox (2008) stated the characteristics of proportional thinkers, more specifically geometrical proportional thinkers. However, literature on proportional reasoning seems to have a consensus that becoming a proportional thinker is not an easy and quick process; rather it requires a course of time period. In this period, students should develop their understanding of mathematical characteristics of proportional situations (Cramer & Post, 1993). One of the most important characteristics of proportional situations is the multiplicative relationship between them rather than the additive one. According to Cramer and Post (1993), a proportional thinker should be able to understand this multiplicative reasoning and apply it in various contexts and situations. Besides, a proportional thinker should determine whether a context requires a proportional reasoning or non-proportional reasoning. Hence, if students are provided with the opportunity to explore these mathematical characteristics, they might have a chance to learn and develop various essential strategies for solving proportional reasoning problems (Cramer & Post, 1993).

Literature review on proportional reasoning indicated that students' strategies develop throughout a period of time by following three stages. For instance, Inhelder and Piaget (1958) pointed out three levels of sophistication for proportional reasoning as additive, pre-proportional, and proportional. They asserted that the additive stage includes a partial awareness of proportionality with the ability to recognize only one relationship at a time. Children in the additive stage might concentrate on just one dimension and focus on the difference instead of a ratio. Pre-proportional level, which corresponds to concrete operational stage, includes dealing with ratio by means of additive strategies to build up patterns. Lastly, proportional stage, which corresponds to formal operations stage, involves students' abstractions of ratio concept and symbolic representations of second-order relationships. Taking Inhelder and Piaget's three levels of sophistication as a basis, several classifications of students' solution strategies for solving proportional tasks are mentioned in the following part.

2.2 Strategies for Solving Proportional Reasoning Problems

Several studies revealed alternative strategies which were used in solving proportions. In their study which investigated the correct strategies for solving proportions, Tourniaire and Pulos (1985) addressed two main strategies. These strategies were *multiplicative* and *building-up* strategies. Tournaire and Pulos stated that multiplicative strategies include a relation between the terms within a ratio, and the same relation is extended to another ratio. They also indicated that building-up strategies, which is a more elementary method, includes a relation within a ratio and extending the relation to another ratio by adding. For example given the problem "If

5 candies cost 3 Turkish Liras how much would 15 candies cost?" multiplicative strategies include understanding that the amount of candy gets three times as much and deciding that the amount of the money should also get three times as much. On the other hand, for the same problem, a student who uses a building up strategy might continue as follows: 15 is 10 more than 5 and 10 is two multiple of 5. Hence, two multiples of 3 is 6. So 3 + 6 is 9. Tourniaire and Pulos (1985) claimed that building-up strategies are predominant in students during childhood and the transition from building-up strategies to multiplicative strategies is slow and complex.

Lamon (1993) conducted a study with sixth graders in which she investigated the kinds of informal strategies children use for ratio and proportion problems prior to experiencing instruction related to proportion. Participants were administered a test including conventional ratio and proportion problems, and they were interviewed after the administration of the test. Students' solution strategies were coded as incorrect strategy, pre-proportional reasoning, qualitative proportional reasoning or quantitative proportional reasoning in terms of sophistication of the strategy. Pre-proportional reasoning was said to result in correct answers without an understanding of scalar or functional relations. These reasoning included direct modeling strategies, i.e. counting, matching, building-up, or pattern recognition. Furthermore, it was mentioned that proportional reasoning included an understanding of the equivalence of scalar ratios and the invariant ratio between the two measures. It was noted that the pattern building strategy included a lack of relational thinking. Lamon arranged these strategies hierarchically as presented in Table 2.3 below.

Table 2.3 Sixth-Grade Students' Strategies for Solving Ratio and Proportion Problems (Lamon, 1993, p. 46)

Strategies	Characteristics
Nonconstructive Strategies	
Avoiding	No serious interaction with the problem
Visual or Additive	Trial and error or Responses without reasons or Purely visual judgments ("It looks like…") or Incorrect additive approaches
Pattern Building	Use of oral written patterns without understanding numerical relationships
Constructive Strategies	
Preproportional Reasoning	Intuitive, sense-making activities (pictures, charts modeling, manipulating) and Use of some relative thinking
Qualitative Proportional Reasoning	Use of ratio as a unit and Use of relative thinking and Understanding of some numerical relationships
Quantitative Proportional Reasoning	Use of algebraic symbols to represent proportions with full understanding of functional and scalar relationships

Taking this framework of Lamon (1993) as a foundation, Cox (2013) identified the solution strategies of the students at ages 11-14 for geometric proportional reasoning problems by interviewing 21 students. The researcher identified seven strategies as avoidance, additive, visual, blending, pattern building, unitizing, and functional scaling for scaling figures. These strategies are presented in Table 2.4 below.

Table 2.4 Strategies Used by Students to Scale Geometric Figures (Cox, 2013, p. 13)

Strategy	Description
Avoidance (AV)	No serious interaction with the problem
Additive (AD)	Student determines scaled lengths by adding the scale factor to corresponding lengths in the original figure. Lengths are determined prior to drawing.
Functional Scaling (FS)	Application of scale factor as a functional ratio. Lengths are determined prior to drawing.
Visual (VI)	Student determines the size or placement of figures by sight or intuition rather than measurement or arithmetic calculation. Lengths are determined in the process of drawing.
Unitizing (UN)	Use of an original figure or component (length or angle) as one unit. Scale factor indicates the number of units in the corresponding image length. Lengths are determined in the process of drawing.
Pattern Building (PB)	Use of numeric patterns without understanding the functional nature of the scale factor. Lengths or angles are determined prior to drawing.
Blending (B)	Students either (1) use a numeric strategy first and then apply visual reasoning to "fix" a perceived distortion, or (2) they use a visual strategy and then apply numeric methods to test or evaluate their scale drawing. Lengths are adjusted throughout the process of drawing.

In the framework of Cox (2013), the first category included students' responses without a sense-making effort in the problem. The second category, which is the additive one, involved participants' efforts in scaling the lengths by taking the differences as in all the other reported additive approaches. Third, it was reported that students who used the functional scaling strategy multiplied the lengths by the scale factor in order to find the enlarged or reduced lengths before beginning to draw. Hence, in the additive and functional strategies students knew about the lengths before they began drawing. The fourth category, which included visual strategies, was related to visualization of the situations and was considered as more constructive than the additive strategy. The unitizing category, which is the fifth one, was about unitizing a figure in one or two dimensions taking the original figure as a unit and making an enlargement of the figure by tessellation. The last category was blending, which is related to either using a numeric strategy first and then applying visual reasoning or using a visual strategy first and judging the reasonableness of the image by applying numeric methods.

In another study by Ben-Chaim, Keret, and Ilany (2012), strategies for solving proportions were divided into two main categories as *pre-formal* and *formal* strategies. Pre-formal strategies included six sub-categories as *intuitive strategies*, *additive strategies*, *division by ratio*, *finding the unit*, *determining the part from whole*, and *missing value problems*. The first category, intuitive strategies, includes reaching the correct solution by direct experimentation but it does not involve the awareness of the equality between the two ratios. The additive strategies category, different from the building-up strategy in the study by Tourniaire and Pulos (1985), focuses on the differences between numbers rather than the multiplicative relationship between them. Ben-Chaim and colleagues claimed that additive strategies will fail when the total items in the problem are impossible to be divided equally into groups or the total is a huge number. The third strategy, division by ratio, involves awareness of the given ratio and using the multiplicative relationship between the numbers in the problem. It is stated that this strategy is a generalization of the former strategy in such a way that multiplication replaces the repeated additions. Both the fourth strategy “finding the unit” and the fifth strategy “determining the part from whole” require students to define the unit ratio in order to find the total amount or amount of each. Yet, the distinction between the two categories is students’ awareness of the parts making up the whole and, hence, the portion that each group receives in finding the unit strategy. Formal strategies are related to writing the proportion formula $\frac{a}{b} = \frac{c}{d}$ and solving the equation algebraically. It is claimed that this strategy is typical of abstract thinking mostly used by adults and adolescents.

Within the context of Rational Number Project (RNP), Harel and Behr (1995) conducted a study in order to investigate the strategies of in-service teachers, especially the ones who were successful, for solving multiplicative problems. Participants were interviewed with the aim of identifying and classifying their strategies by using a series of multiplicative problems. The main result of the study was that only the teachers integrating the concepts of ratio and proportion while solving the problems were the ones who correctly solved the problems. Moreover, the results of the study identified four categories of solution strategies as multiplicative, pre-multiplicative, operation search, and keyword strategies. They defined the multiplicative strategy as reasoning about the problem situation by using the concepts of ratio and proportion and writing the algebraic equation to find the unknown value. Furthermore, the pre-multiplicative strategy was defined as deriving an approximate answer by building up an additive relationship or by the help of a unit rate constructed multiplicatively. On the other hand, the operation strategy was referred to as a process of elimination until getting a reasonable answer, and the keyword search strategy was related to analyzing key words and deciding the procedure to be followed by the help of those key words. Harel and Behr concluded that the multiplicative strategy was the only correct strategy for solving multiplicative problems, yet the pre-multiplicative strategies involved a pathway to multiplicative reasoning.

In a recent study, Canada, Gilbert, and Adolphson (2008) have investigated 75 elementary pre-service teachers' conceptions of proportional reasoning and their approaches to proportional reasoning problems. The participants were asked to solve the proportional reasoning problems individually and write their approaches and explain their thinking. In line with the types of themes emerged from the literature and the findings of the study, the researchers categorized participants' explanations and approaches as *reasonable* or *questionable*. They defined reasonable approaches as considering a unit rate, demonstrating a between or within comparison or using multiplicative structures. Furthermore, questionable approaches were defined as including some level of confusion, lack of clarity, or erroneous thinking such as additive reasoning. The findings revealed that approximately 20.0% of the participants had given questionable responses or could not develop a proportional strategy, and, hence, demonstrated a limited understanding of proportional reasoning.

Literature review on students' and teachers' strategies for solving proportional reasoning problems demonstrated the use of various strategies, some of which included a deep understanding of proportionality and proportional reasoning; yet some of them were incorrect and some of them were primitive with little or no understanding of these concepts. Therefore, stating the difficulties of students in proportional reasoning would provide a better understanding of their development of proportional reasoning schemas. Thus, the difficulties of students in proportional reasoning are mentioned in the following part.

2.3 Difficulties in Proportional Reasoning

Resnick and Singer (1993) claimed that learning of the ratio and proportion concepts are challenging for students, and they constitute "one of the stumbling blocks of the middle school curriculum" (p. 107). Weinberg (2002) further claimed that these concepts are difficult especially for students who do not know what a specific proportional situation means or the reason that a solution strategy is effective for the given situation. Besides, Piaget and Inhelder (1975) indicated that proportional reasoning is a late achievement in the development of pupils since it includes higher order reasoning with an understanding of relations among relations. Several other studies addressed specific difficulties of the students for the concept of proportional reasoning. For instance, Thompson and Preston (1994) indicated that students experience challenges in covarying quantities while keeping the relationship the same while solving proportions. Besides, Lobato and Thanheiser (2002) stated that using incorrect or irrelevant data in computations when solving proportional problems is one of the common errors. Nevertheless, two other main difficulties, which are the foci of this study, are using additive reasoning instead of a multiplicative one (Harel, Behr, Lesh, & Post, 1994; Hart, 1984; Noelting, 1980), and inability to discern proportional situations from non-proportional ones, and applying proportional strategies for non-proportional situations (De Bock, Verschaffel, & Janssens, 1998; Freudenthal, 1983; Modestou & Gagatsis, 2007; Van Dooren, De Bock, Janssens, & Verschaffel., 2007). The former difficulty is referred to as misinterpretation of additive *and multiplicative reasoning*, whereas the latter one is referred as *illusion of linearity* in the present study. These two difficulties are stated and explained with an emphasis on the results of related studies in the following part.

2.3.1 Misinterpretation of Additive and Multiplicative Reasoning

Literature review on students' solution strategies for proportional problems revealed two main strategies as multiplicative reasoning and erroneous additive reasoning. Misailadou and Williams (2003) pointed out that additive reasoning is the strategy that was most commonly reported as an inappropriate strategy in solving proportional reasoning problems.

Multiplicative reasoning is used as a synonym for proportional reasoning in the current study and is defined as "making multiplicative comparisons between quantities" (Wright, 2005, p. 363). On the other hand, in additive reasoning "the relationship within the ratios is computed by subtracting one term from another, and then the difference is applied to the second ratio." (Tourniaire & Pulos, 1985, p.186). Additive reasoning is seen as a prior stage for multiplicative reasoning and, hence, the development of multiplicative reasoning is built on students' additive reasoning skills (Fernandez, Llineares, Van Dooren, De Bock, & Verschaffel, 2010). Throughout primary school and the early years of middle school, students' reasoning is expected to change from additive to multiplicative (NCTM, 2000; Harel & Confrey, 1994; Fernandez & Llinares, 2009). Fernandez and Llinares stated that discerning additive and multiplicative relationships from each other is a sign of mathematical maturity. Therefore, misinterpretation of additive and multiplicative reasoning is related to the incapability of students to understand additive and multiplicative structures, their misinterpretation of additive and multiplicative relationships, and tendency to use them in inappropriate situations.

Misinterpretation of additive and multiplicative reasoning might occur in two ways: either using additive strategies for multiplicative problems or using multiplicative strategies for additive problems. For instance, for the problem "Grandma adds 2 spoonfuls of sugar to the juice of 10 lemons to make lemonade. How many lemons are needed if 6 spoonfuls of sugar are used?" (Van Dooren et al., 2010, p. 362) students might erroneously think that the second mixture should include 6-2=4 more spoonfuls of sugar and, hence, it should include 10+4 = 14 lemons. On the other hand, for the problem "Sue and Julie were running equally fast around a track. Sue started first. When she had run 9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?" (Cramer & Post, 1993, p. 344) students might think that the correct answer is 45 by considering a multiplicative relationship as $\frac{a}{b} = \frac{c}{d}$. Nevertheless, it can be understood that the context of the problem requires an additive reasoning instead of a multiplicative one, and the result is 21 laps.

Van Dooren et al. (2010) conducted a study in order to investigate 325 third, fourth, fifth, and sixth grade students' additive strategies to solve proportional problems. The researchers also aimed at investigating the proportional strategies of the students in order to solve additive problems and also students' progress from additive to multiplicative ways of thinking. The researchers administered a test in which half of the problems required additive strategies and half of them required proportional strategies. The two sample questions, one of which is additive, and the other is multiplicative are provided in Figure 2.11 below.

Tom and his sister Ana have the same birthday. Tom is 15 years old when Ana is 5 years old. They are wondering how old Ana will be when Tom is 75… (additive situation) Rick is at the fish store to buy tuna. The customer before him bought 250 grams of tuna and had to pay 10 euro. Rick needs 750 grams of tuna, and he wonders what he will have to pay…(multiplicative situation)

Figure 2.11 Additive and multiplicative problems (Van Dooren et al., 2010, p. 361)

Van Dooren and colleagues described the first situation by a function f (x) = x+a and explained that the situation is additive since the numbers are related by addition and subtraction. Therefore, the correct solution strategy for the first problem is looking at the differences of the ages of the two persons and applying the difference to the second value. Moreover, they described the second situation by a function f (x) = bx and justified that the situation was multiplicative (proportional or linear) since the variables are related by multiplication and division. Thus, the correct solution strategy for the second problem is writing a proportion between the given variables or applying the ratio of the first two variables to the second variable. The researchers pointed out that the required reasoning in these situations is very distinct since the first one deals with a difference, and the second one deals with a ratio between the two values. The findings of the study revealed that students showed a tendency to use additive strategies for multiplicative problems. Specifically, 46.6% of the students in the third grade and 6.4% of the students in the sixth grade were additive reasoners. Another finding of the study was that the tendency to use additive strategies for multiplicative problems decreased with age, whereas the tendency to use multiplicative strategies for additive problems increased with age.

In another study conducted by Misailadou and Williams (2003), constructing an instrument to determine the misconceptions of the students in the domain of proportional reasoning was aimed at. The researchers hypothesized that misconception of the use of additive strategy for proportional problems would occur in students' answers frequently. 303 students between 10 and 14 years of age were given a test prepared to determine students' misconceptions related to proportional reasoning. Students' answers were coded as correct or erroneous for each item, and the results were analyzed by means of the Rasch model in order to scale most common errors. The findings of the study revealed that "tendency to additive strategy" was the strongest and the most frequent misconception.

Kaput and West (1994) asserted that the area of geometry and measurement was one of the most vulnerable areas to erroneous additive reasoning. This means that students might use additive strategies for geometry and measurement problems which are multiplicative in nature. To illustrate, a problem stated in the study of Kaput and West (1994) is presented in Figure 2.12 below.

The two sides of Figure A are 9 cm high and 15 cm long. Figure B is the same shape but bigger. If one side of Figure B is 24 cm high, how long is the other side?

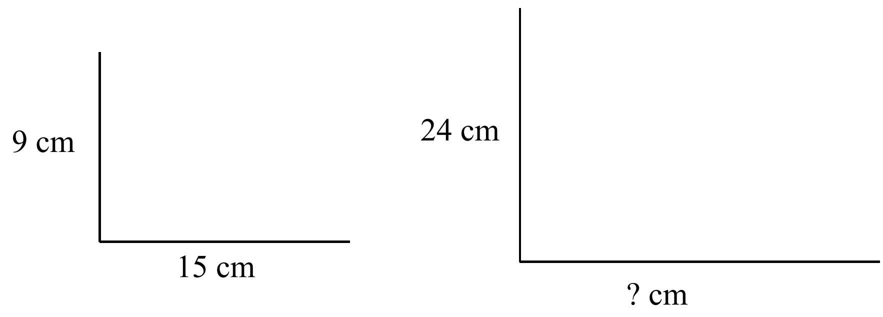


Figure 2.12 Missing value geometry problem (Kaput & West, 1994, p. 268-269)

A student who applies additive reasoning to this problem might think that since the height is increased by the amount of 15 (24 – 9 = 15), the length is also increased by 15 and, the result is 30 (15 + 15 = 30 cm). However, it is not the case for this problem since the shapes are similar and the lengths should increase by the same ratio, not the same amount. Therefore, the correct solution for the problem should be $\frac{9}{24} = \frac{15}{?}$ or $\frac{9}{15} = \frac{24}{?}$, that is 40 cm.

2.3.2 Illusion of Linearity

The second difficulty in proportional reasoning emphasized in the present study is illusion of linearity which is related to students' inability to discern linear situations from non-linear ones and apply linear strategies where non-linear strategies are needed or vice versa. Freudenthal (1983) brought out the issue of illusion of linearity in mathematics education literature and warned the practitioners of mathematics education about students' tendency to overuse properties of the linear model. Since then, the overgeneralization of linear methods in students' reasoning has been documented and discussed by several practitioners in the field of mathematics education. The samples of the studies included students from kindergarten to university, different countries having quite different mathematics curricula with didactic approaches and topics from a wide range of mathematical domains (elementary arithmetic, graphing, probability, and geometry and measurement).

In the problem “One kilogram of apples cost 3 Turkish Liras. How much money do 4 kilograms of apples cost?”, it is clear that a true linear relationship occurs between the weight of the apples and the cost, and the result is 12 Turkish Liras. However, in a question like “Two bottles are filled with water. If the temperature of the water in each bottle is 40 ^{o}C, what will be the temperature of the water if we combine the water in the two cups?” (Adapted from Stavy, Babai, Tsamir, Tirosh, Lin, & McRobbie, 2006), a student can immediately give the answer 80 °C considering that a linear relationship occurs as 2 x 40 = 80. Nevertheless, it is not the case for this question as it can be understood that the result is 40 °C considering the real life situation.

In addition to word problems, there are studies related to the graph domain. To illustrate, Hadjidemetriou and Williams (2010) conducted a study in which they attempted to analyze graphing practices of students in schools and how graphing practices come to privilege the linear model. They asked the participants to draw the graph of the following statement: “Draw the graph of a height of a person relative to his age in years”. Most of the students drew a graph of y= x with the explanation that “the older you are, the taller you are”. The remaining students had a tendency to draw a linear graph having distinct straight segments. The results of the study showed that illusion of linearity is also observed in the domain of graphs.

Illusion of linearity has also been investigated in the domain of probability. The difficulty in this area is related to the belief that a better chance occurs when an object is selected from a set containing more of that object than another set. To illustrate, in a study by Green (1982) the following counter problem is asked to the participants:

Two other bags have black and white counters

Bag J 3 black and 1 white

Bag K 6 black and 2 white

Which bag gives a better chance of picking a black counter?

(A) Same chance ☐

(B) Bag J ☐

(C) Bag K ☐

(D) Don't know ☐

Figure 2.13 Illusion of Linearity in Probability (Green, 1982, p. 20)

Results of the study revealed that 62.0% of the participants chose the option C, reasoning that Bag K has a higher number of black counters than does Bag J. Nevertheless, it can be understood that both bags give the same chance since the ratio of black balls to white balls is the same, and the answer is A.

The results of several studies conducted in recent years have indicated that illusion of linearity also constituted a challenge in the area of geometry and measurement. For instance, the results of these studies revealed that students thought that shapes with the same area should have the same perimeter or vice versa (Chapin & Johnson, 2000; Stavy & Tirosh, 2000) or a shape with smaller area should have smaller perimeter or vice versa (Marchett, Medici, Vighi, & Zaccomer, 2005; Stavy & Tirosh, 2000). Results of several studies revealed that understanding of both students and even in-service and pre-service teachers of the relationship between area and perimeter was very poor (D'Amore & Fandiño-Pinilla, 2006; Latt, 2007; Ma, 1999; Marchett et al., 2005; Menon, 1998; Rickard, 1996; Ryan & Williams, 2007). For instance, a study conducted by Tan-Şişman (2010) with 445 sixth grade public school students in Turkey aimed to investigate students' conceptual and procedural knowledge and word-problem solving in the domain of length (also perimeter), area, and volume concepts. Three types of achievement tests related to these concepts were administered. The results of the study revealed that 6.1% of the students stated that the areas of the figures were equal since their perimeters were equal. The findings also indicated that the students believed that if the perimeters were equal, the areas should also be equal.

Illusion of linearity was also seen in the area of geometry and measurement in such a way that students thought that there was a linear relationship between the lengths and the area or the lengths and the volume of the geometric figures (De Bock et al., 1998, 2002; Modestou & Gagatsis, 2007; Van Dooren et al., 2004, 2007). To illustrate, NCTM (1989) pointed out that most students from grade 5 to grade 8 erroneously believe that "if the sides of a figure are doubled to produce a similar figure, the area and volume also will be doubled" (p.114–115). In other words, students believed that doubling or tripling the dimensions of a figure resulted in a doubled or tripled area and volume. The early history also witnessed this challenge, and the origins of this challenge were reported in Plato's dialogue Meno. Socrates asked a slave boy to double the area of a given square and the following dialog occurred between the two:

> ***Socrates:*** If the length of this side is two feet and the length of this one two feet, how many feet would the area of the square be?
>
> ***Slave:*** Four, Socrates.
>
> ***Socrates:*** Could it be another ruled surface, double of this one, with all its sides equal?
>
> ***Slave:*** Yes.
>
> ***Socrates:*** How many feet would it be?
>
> ***Slave:*** Eight.
>
> ***Socrates:*** Now try to tell me how large would its side be? The side of this square is two feet. What would the side of the doubled be?
>
> ***Slave:*** It's obvious Socrates, that it would be doubled. (Fragkos, 1983, p. 70).

The slave boy immediately and spontaneously thought about doubling the sides of the square, applying the idea of a linear relationship between the side of the square and its area. However, it is universally accepted that there is not a linear relationship among the concepts of length, perimeter, area, and volume. Instead, the valid principle for the relationships among length, area and volume of reduced or enlarged geometrical figures is as follows: A linear enlargement or reduction by factor r multiplies lengths by factor r, areas by factor r^2 and volumes by factor r^3 (De Bock et al., 2002). Besides, this relationship between the length and area, or length and volume might be obtained by the following procedures stated in Figure 2.14, Figure 2.15, and Figure 2.16 below.

Original Area	$= 1 \times 1$	
Doubled Area	$= 2 \times 2$	
	$= (1 \times 2) \times (1 \times 2)$	double each side of the original square
	$= (1 \times 1) \times (2 \times 2)$	rearrange the order and grouping of the factors
	$= 1 \times 4$	original area times 4
	$= 4$	the new area is 4 times greater than the original area

This can also be shown by analyzing the following diagram:

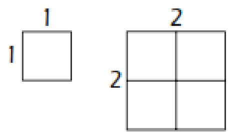


Figure 2.14 Doubling each side of a square (Chapin & Johnson, 2000, p. 179)

As illustrated in Figure 2.14, Chapin and Johnson explained that doubling the sides of a square results in a four times larger area, since the two dimensions are doubled at the same time. They further explained what happens to an area of a figure when the sides of a square is tripled and even, quadrupled as illustrated in Figure 2.15 below.

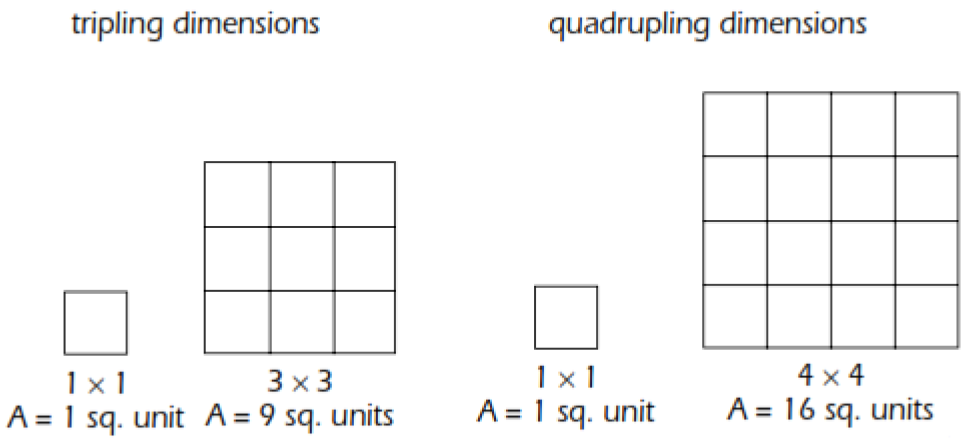


Figure 2.15 Tripling and quadrupling the dimensions of a square (Chapin & Johnson, 2000, p. 179)

As can be seen in Figure 2.15, Chapin and Johnson expressed that tripling the dimensions of a square results in a nine times larger area and, quadrupling the dimensions of a square results in a sixteen times larger area. Moreover, they stated that the same relationships can be observed for rectangles, triangles, and circles.

Chapin and Johnson also examined the change in the volume of a rectangular prism by unit cubes when one, two, or three dimensions are doubled. They took an example of prism having the dimensions as 1 x 2 x 3 units. Then the volume of the prism was 6 unit cubes. Doubling one dimension of the prism resulted in a two times (x 2) greater volume as 12 unit cubes, doubling two dimensions of the prism resulted in a four times (x 2 x 2) greater volume as 24 unit cubes, doubling three dimensions of the prism resulted in an eight times (x 2 x 2 x 2) greater volume as 48 unit cubes. Therefore, the final volume is multiplied by eight as a result of each dimension (length, width, and height) being multiplied by two (doubled), and then multiplied together. This procedure is illustrated in Figure 2.16 below.

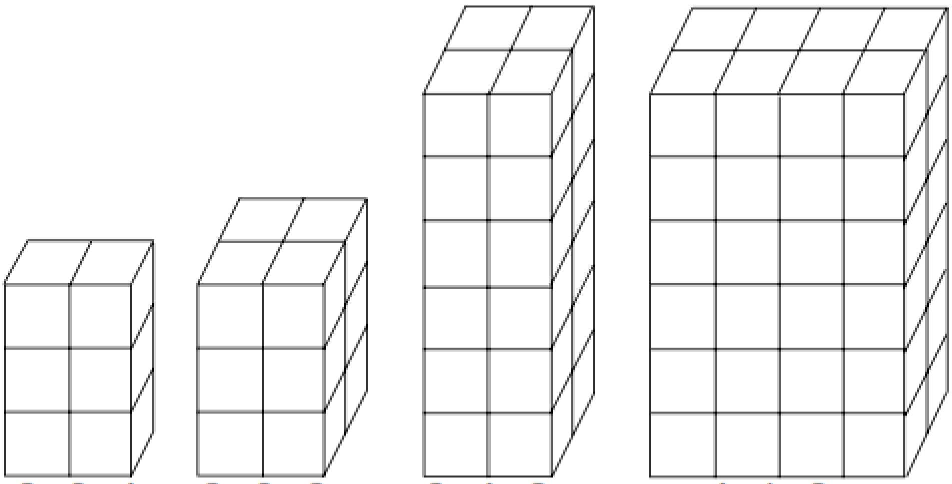


Figure 2.16 Doubling one, two, or all dimensions of a rectangular prism (Chapin & Johnson, 2000, p. 181)

Finally, as a generalization when all three dimensions of the rectangular prism are tripled, the volume gets 3x3x3 =27 times as much as that of the original one (Chapin & Johnson, 2000).

The valid principle and a series of practical examples have been provided for the relationship among length, area, and volume so far. Nevertheless, results of recent studies have indicated that students are not necessarily awakened about the distinct growth rates of length, area, and volume. Moreover, the results of these studies have revealed that they have tendencies to consider the relationship between length and area or between length and volume as linear instead of quadratic and cubic. Hence, the results of several studies concerning this issue are stated in the following parts.

Considering illusion of linearity in the area of geometry and measurement, De Bock and his colleagues conducted several studies with both elementary and high school students concerning the issue of linearity illusion or predominance with respect to problems related to length and area of geometrical figures beginning from 1990s.

To begin with, De Bock et al., 1998 conducted two parallel experimental studies with the aim of investigating the predominance of linearity in seventh and tenth grade students working on mathematical word problems related to length and area of square-shaped, circular, and irregular figures. The researchers also aimed at investigating the effects of ready-made or self-made drawings on overcoming the linearity illusion. In both studies, the subjects were separated into three groups; the first group was considered as the control group and the other two groups were considered as experimental groups. The study was conducted in two stages. In the first stage, all groups took the same pretest including items related to enlargements of similar geometrical figures. Six of these items were solvable and the remaining six were not solvable by a linear approach. In this stage, the problems did not include any pictures or drawings and the students were not given any clue or instruction. After some time, three versions of the pretest were prepared for the three groups. Each of these tests was administered to the three different groups. The first group students experienced the second test in just the same way as the pre-test, the second group students were required to make a self-drawing before answering the problems, and the third group students were given a correct ready-made drawing attached to the problems. The results of both experimental studies revealed that the misuse and overuse of the linear model were seen in a vast majority of students. That is, most of the students showed a tendency to apply a linear solution where it was not applicable. Furthermore, the results showed that the students' performance in the problems that were solvable by a linear approach was very high, whereas their performance in the problems that were not solvable by the linear model was very low. The researchers also reported that neither ready-made drawings nor self-made drawings were significantly effective in breaking the predominance of the linear model in both studies. The researchers also classified three correct solution strategies students used while solving non-linear problems. These strategies were as follows: the paving strategy which included paving the bigger shapes with little ones and examining the relationships between the areas, the strategy of computing and comparing the length or area of both figures in which finding the lengths or areas of both shapes and comparing them with each other, the strategy of applying the general rule which is when a side is multiplied by r, area is multiplied by r^2. The researchers concluded that the second strategy was the most frequently employed strategy.

Subsequently, a follow-up qualitative study was conducted by De Bock et al. (2002), in which the aim was to investigate the aspects of students' mathematical conceptions, beliefs, and habits that were responsible for the presence and strength of the illusion of linearity in problems of length and area of an irregular figure. To this purpose, the researchers conducted task-based interviews with 20 seventh and 20 tenth graders. During the interviews, the participants were initially presented with a non-linear problem, and then they were provided with clues in order to help them create conflicting ideas in their mind and discover the non-linear structure of the problem. The results revealed that the tendency to apply a linear solution process was frequently occurred in both groups. Only two students in tenth grade were able to arrive at the correct solution in their first attempt to solve the problem and the remaining thirty-eight students followed the linear method of solution. During the interview, with the help of the given clues, most of the students were able to realize the non-linear nature of the problem but at different phases with different number of clues. Nevertheless, four students from both groups insisted on their incorrect linear answers. The analysis of students' answers at each phase also provided significant information about the underlying reasons for applying straightforward linear calculation instead of a non-linear one. Firstly, the results indicated that one reason for this was that linear approach was deeply rooted in students' knowledge and students used the linear solution subconsciously or spontaneously. The second reason was that some students believed that the linear model was applicable to the situation and asserted this belief explicitly and deliberately, especially for the context of enlarging geometrical figures. The third reason stemmed from students' deficiency in geometrical knowledge, specifically about the effect of enlargement on the length and the area of a figure. The last reason indicated in the results of this study was students' poor habits in problem solving, which is related to the fact that students deal with problems superficially, just looking at the numbers and searching for keywords without making sense of the problem.

In another experimental study by Van Dooren, De Bock et al. (2004), the researchers aimed at designing a learning environment in line with the conceptual change theory that would be tested to overcome eighth grade students' intuitive tendency to answer the questions by linear applications instead of a non-linear one in the context of the relationships among perimeter, area, and volume of geometrical figures. To this purpose, the researchers developed 10 experimental class sessions to be conducted with the participants. One class experienced these 10 experimental lessons within two weeks while the other class (control group) experienced regular lessons at the same time period. Both groups of students were administered the pre-test at the beginning of the study, but only the experimental group was given the post-test. Besides, a retention test was applied to both groups of students after three months. The results of the study demonstrated that both groups of students performed better on the linear items than the non-linear items, and 80.0% of the students solved non-liner problems as if they were linear both in the pretest and the retention test. Besides, there was no significant change in the answers of control group students on the post-test and the retention test. On the other hand, the number of correct answers provided by students in the experimental group regarding the non-linear problems increased significantly from the pre-test to the post-test; yet this improvement was not observed in the answers on the retention test. The other finding of the study suggested that the number of correct answers of the experimental group students regarding the linear problems decreased from pre-test to post-test. The researchers concluded that the experimental lessons alerted students that there were some problems not solvable by a linear approach and this situation caused students to apply the non-linear solutions also to linear problems. Furthermore, the qualitative analyses of the study demonstrated that non-linear relations and the relationships among length, perimeter, area and volume of enlarged or reduced figures continued to be challenging for many students.

Lastly, Van Dooren, De Bock et al. (2007) continued to search for ways to overcome linear illusion for the problems related to length, perimeter, area and volume of geometrical figures. To this purpose, the researchers created performance tasks as alternatives for traditional school word problems and experimented whether experiencing linear and non-linear problems within the context of these performance tasks helped to prevent applying linear solutions for problems that needed to be solved by non-linear methods of solution. A pre-test including six traditional word problems was administered to 93 sixth graders. Then, the participants who applied a linear method of solution for a problem that needed to be solved by non-linear methods of solution were selected for the individual interviews. During the interviews, the control group students were presented with another non-linear problem just as the ones in the pre-test, yet the experimental group students were presented with a non-linear problem within a meaningful performance based task. After a couple of days, a post-test similar to the pre-test was applied to the participants. The results of the study claimed that the linearity illusion might be diminished when students deal with problems within a performance task compared with students who deal with traditional word problems.

Another study by Modestou, Gagatsis, and Pitta-Pantazi (2004) investigated the predominance of the linear model in 12-13-year old students while dealing with problems related to area and volume of rectangles. To this end, 307 sixth and seventh grade students were administered a pre-test including problems related to the area and volume of rectangles. Some of the problems in the test were to be solved by the linear approach and some of them were not. The instrument was designed in order to collect data on the frequency of using linear model when it was not appropriate. A second test including the same problems with a different data set (the length of the dimensions of the shapes) were implemented to the half of the students. The remaining half received a third test in which different presentations of the same problems were included. The results of the study depicted that there was a great discrepancy between students' success in the linear problems and non-linear problems in favor of linear problems. As regards the other research question, it was found that providing the length of the dimensions of the shapes and different problem representations helped in a small degree in overcoming the issue of illusion of linearity. The researchers concluded that students might have matched the area and volume tasks with the prototype of linear model; that is, they assumed that there was a linear relationship between the length and the area, or between the length and the volume. They further claimed that the idea of doubling, tripling etc. led students to use the operation of a linear multiplication with 2 or 3 etc.

Recent studies on illusion of linearity belong to Vlahovic-Štetic, Pavlin-Bernardic, and Rajter (2010, 2011), who studied the effects of two distinct variables on overcoming the misuse of linearity. The first variable was providing the answer which is obtained by the misuse of linear strategies among the options. The participants were 112 students at the ages of 15-16 and 18-19. One group of the participants solved non-linear problems in which the answer obtained by linear strategies was provided, and the other groups solved non-linear problems in which the answer obtained by linear strategies was not provided. The results of the study confirmed students' success in the linear problems and failure in the non-linear problems. Besides, the results showed that when the linear answer was not provided, students were more successful. The aim of the second variable was provide insight into the level of the students' performance on the pretest. To this end, the researchers implemented a pretest to 121 eleventh grade students. The test included problems for some of which the linear model was not suitable and for some of which the linear model was suitable. Afterwards, some of the students were given feedback related to their performance on the first test and there was no special treatment for the rest of the students. The results of the study showed that the students who were given feedback were more successful in the problems that were not suitable for linear solution than the rest of the students.

No study focusing on the illusion of linearity or the relationships among the length, perimeter, area, and volume of geometrical figures was encountered in the available Turkish literature. Yet two related studies on the misconceptions of students in the domain of ratio and proportion, and geometry and measurement are discussed below.

A case study exploring sixth grade students' misconceptions about ratio and proportion was carried out by Kaplan et al. (2011). The sample of the study consisted of 42 sixth grade students. A diagnostic test on misconception, which included 10 open-ended questions, was utilized for data collection, and follow-up interviews were conducted for further clarification. A related finding was that 63.0% of the students fell into "illusion of linearity" in a question requiring the exploration of effect of enlargement of a rectangular figure in its area. The researchers concluded that students had misconceptions related to considering the ratio as a real quantity, the formulation of a ratio from two given quantities, the misconceptions resulting from readiness of students and considering non-linear situations as linear. Besides, in the study of Tan-Şişman (2010) 74.3% of the students stated that when the volume of a prism is tripled, all the dimensions are also tripled. Therefore, these findings also revealed that students believe that if the sides of a rectangle are doubled, the area is also doubled and that when the volume of a prism is tripled all the dimensions are also tripled. In conclusion, illusion of linearity was a problem for the majority of the participants.

2.4 Summary of the Literature Review

Literature review highlighted the importance of proportional reasoning not only in mathematical and other school-related matters but also in daily life situations. Various definitions proposed for proportional reasoning were encountered in the literature. Among the related literature, the framework for behaviors of a proportional thinker by Cramer and Post (1993) sheds light on the essential components of proportional reasoning, which suggest that proportional reasoning is related to the ability to work with multiplicative relationships rather than additive relationships and to distinguish proportional situations from non-proportional relationships. In addition to the framework of Cramer and Post (1993), an adapted version of this framework named geometric proportional thinker by Cox (2008) was used in the study.

A number of solution strategies on various tasks related to proportional reasoning were also reported in several studies. Yet, most of these strategies showed similarity and revolved around the three levels of sophistication stated as additive, pre-proportional, and proportional by Inhelder and Piaget (1958).

The two issues that emerged from the two frameworks shed light on the difficulties students experienced in solving proportion problems, which were reported in several studies. The first difficulty is related to the inability to understand the characteristics of additive and multiplicative reasoning and to use additive reasoning where multiplicative reasoning was required or vice versa. The other difficulty is related to inability to comprehend the characteristics of proportional and non-proportional situations and to use proportional strategies where non-proportional strategies were needed or vice versa. The former difficulty is referred to as misinterpretation of additive and multiplicative relationships and the latter one is referred to as illusion of linearity in the present study. Illusion of linearity is considered in terms of two issues. The first one is related to the beliefs of students that there exists a linear relationship among these concepts, i.e. same perimeter same area/volume or larger perimeter larger area/volume. The other component of illusion of linearity is related to students' belief that doubling or tripling the dimensions of a figure results in a doubled or tripled area and volume. Several studies dealing with illusion of linearity were mentioned. The results of these studies revealed that illusion of linearity was a serious challenge for students. It was also noted that no study exactly dealing with this misconception was found in the available Turkish literature. It was concluded that proportional reasoning is a process that develops slowly over time.

All in all, the literature review provided us with the studies stressing the essential components of proportional reasoning and aiming at analyzing the solution strategies of students and their difficulties in proportional reasoning tasks in broader means. Some difficulties students experienced in proportional reasoning tasks related to the area of geometry and measurement and their reasons were also stated. However, no study bringing all these issues together was encountered in the available literature. Therefore, there is a gap in the literature in connection to research focusing on the essential components of proportional reasoning and analyzing the correct solution strategies and difficulties of middle grade students in line with these components. Results of such a research study would contribute much into the literature in order to observe the development of students' proportional reasoning.

CHAPTER 3

METHODOLOGY

This chapter is devoted to describe the research design, the population and the sample of the study with their major characteristics, data collection instruments and procedures, reliability and validity issues, analysis of the data, assumptions and limitations, and lastly the internal and the external validity of the study.

3.1 Research Design of the Study

The purposes of this study were to determine sixth, seventh, and eighth grade students' achievement levels in linear and non-linear problems regarding length, perimeter, area, and volume concepts and to investigate their correct solution strategies for these problems. This study also aimed to analyze underlying reasons for students' incorrect answers in these problems. Hence following research questions addressed this study:

1. How successful are sixth, seventh, and eighth grade students at answering the linear and non-linear problems regarding length, perimeter, area, and volume concepts?
2. Which solution strategies do sixth, seventh, and eighth grade students use for linear and non-linear problems regarding length, perimeter, area, and volume concepts?
3. What are the underlying reasons for sixth, seventh, and eighth students' incorrect answers in linear and non-linear problems regarding length, perimeter, area, and volume concepts?

Fraenkel and Wallen (2006) stated that if the aim is to determine the existent status of the population and to describe some aspects and characteristics of the population survey design might be adopted. Hence, in order to determine the frequencies of participants' correct and incorrect answers in linear and non-linear problems regarding length, perimeter, area, and volume concepts survey research design was utilized for the study. Particularly, the study was designed as a cross-sectional survey with the aim of collecting data at one point of time (Fraenkel & Wallen, 2006). More specifically, sixth, seventh, and eighth grade students' answers in the measuring instrument were graded as blank, correct, and incorrect in order to examine the first research question. Next, students' answers in the achievement test were examined in detail in order to understand the correct solution strategies that the participants used for solving the linear and non-linear problems and the underlying reasons for their incorrect answers. Lastly, some of the participants were selected to be interviewed in order to examine the second and third research question. Through these interviews, participants were asked to explain and clarify their correct solution strategies they used by the help of the semi structured interview protocol. The other aim of the interviews was to have an understanding of the probable underlying reasons for students' incorrect answers in linear and non-linear problems regarding length, perimeter, area, and volume concepts. Therefore, a mixed method research with both basic qualitative and quantitative approach was performed to address the three research questions.

3.2 Population and Sample

The target population for the study was determined as sixth, seventh, and eighth grade public school students in Ankara. The accessible population constituted sixth, seventh, and eighth grade public school students in Yenimahalle District of Ankara since it was not feasible to reach the whole target population.

Cluster random sampling method was used for selecting the sample of the survey study. Fraenkel and Wallen (2006) stated that clusters, schools in this case, are randomly selected in cluster random sampling method. In the sampling procedure of the study, firstly the number of the public middle schools was determined based on the information gathered from the Yenimahalle Directorate of National Education. It was seen that there were 97 public middle schools in the region. In order to ensure generalizability at least 10.0% of these schools had to participate in the study (Neuendorf, 2002). Thus, the names of all these 97 schools were listed randomly in Microsoft Excel program. By using RAND function of the Excel program, all schools were associated with a corresponding random number. These numbers were listed from the greatest to the smallest. Then 17 of these schools that had the greatest associated random numbers were contacted for getting permissions. Some of these schools were considered as substitutes in case of any probable problem. The contact information of the schools was obtained from the website of Yenimahalle Directorate of National Education. The schools were contacted with the aim of informing the principles about the purpose of the study and giving relevant details of the study. Besides, the school administrators were asked for the number of students in each grade in those schools so that enough copies of the achievement test could be made. Furthermore, a suitable time for test administration was negotiated with them. Some of the school administrators were not willing to take part in the study; but 11 schools (more than 10.0% of the population) were visited in the end. Another problem encountered related to data collection was that not all the students in these schools could be reached. This was due to the fact that some teachers did not allocate time for the data collection. Hence, the data were collected from the students of the schools whose administrators and teachers were willing to participate in the study. More detailed characteristics of the sample are displayed in Table 3.1 below:

Table 3.1 Major Characteristics of the Sample by Grade Level and Gender

Gender		Frequency	Percent
Male			
	6	154	32.6
	7	187	39.5
	8	132	27.9
	Total	473	100
Female			
	6	122	26.5
	7	196	42.5
	8	144	31.0
	Total	462	100

As can be seen from the Table 3.1, the number of the participants was 935. The number of the male participants was 473 (50.6 %) and the number of the female participants was 461 (49.3 %). Furthermore, 276 participants were at 6th grade (29.5 %), 383 participants were at 7th grade (41.0 %), and 276 participants were at 8th grade (29.5 %). Since all the schools were public schools in Yenimahalle District of Ankara the students were assumed to have moderate socio-economic status.

In addition, the sample of the interviews included 12 students: 7 girls (3 sixth graders, 3 seventh graders, and 1 eighth grader) and 5 boys (1 sixth grader, 1 seventh grader, and 3 eighth graders). There were 4 students from each grade level in total. Participants for the individual interviews were selected based on their achievement levels, solution strategies they used, and incorrect answers they gave in the achievement test. The sample of the interviews included students who used various correct solution strategies and others who gave incorrect answers due to various reasons in line with the pre-codes for solution strategies and reasons underlying incorrect answers obtained in the pilot study. Those students were approached to participate in the interviews. The interviews were conducted with the students who were willing to participate. The sample of the main study is summarized in Figure 3.1 below.

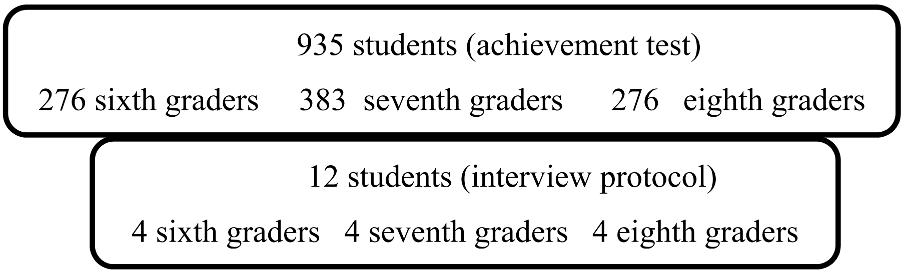


Figure 3.1 Sample of the main study

3.3 Data Collection Instruments

An achievement test was utilized in order to collect data related to participants' achievement in linear and non-linear problems regarding length, perimeter, area, and volume concepts and to investigate their solution strategies and underlying reasons of students' incorrect answers in these problems. The aim of the achievement test was to provide information related to all three research questions. This section describes the features of the achievement test and its development process.

3.3.1 Achievement Test

The instrument included ten open ended mathematical problems. Nine of these problems were adapted from the existing literature, and the last problem was written by the researcher. In the adaptation process of the items in the instrument, the objectives of national mathematics education curriculum were taken into account, and a table of specification was prepared (See Appendix E). Two of these problems, namely the sixth and ninth problems, included figures related to the problem statements. All the problems were categorized into two distinct groups as linear and non-linear. Six of these problems were linear, and four of them were non-linear. Secondly, the problems were divided into four categories as length, perimeter, area and volume with respect to the related concepts in the problem statement. Hence, the problems were subdivided into eight (4x2) subgroups as linear-length, linear-perimeter, linear-area, linear-volume, non-linear-length, non-linear-perimeter, non-linear-area, and non-linear-volume. The properties of these groups, the corresponding problems in these groups, and the changes made in the original problems are summarized below. The Turkish version of the achievement test is available at Appendix C.

Linear-Length: These problems are related to the length concept and need a linear solution approach. Two problems are included in this category. These problems are the second and the eighth problems of the instrument used in this study. The original versions and the changes made in the problems are mentioned below.

The second problem of the study was adapted from the study of Vlahovic-Štetic et al. (2010). The problem was related to the relationship between the time and the work done by a single person. The problem was very similar to textbook problems about the proportional situations, and similar problems can be found in sixth and seventh grade mathematics textbooks. The problem was a linear one and as follows:

> Peter walked 1 hour and during that course of time he passed 4 kilometers. How many kilometers will he pass in 2 hours if he keeps walking at the same speed? (Vlahovic-Štetic et al., 2010)
>
> a. 5 km b. 6 km c. 7 km d. 8 km e. 9 km

Figure 3.2 The original version of the second problem

The problem was related to the relationships among velocity, distance, and time contextually. This problem was a multiple choice item. It was directly translated into Turkish without any further modification but used as an open ended problem instead of a multiple choice item in the present study as follows:

Pınar walked 1 hour and during that course of time she passed 4 kilometers. How many kilometers will she pass in 2 hours if he keeps walking at the same speed? Show your solution way in detail.

a. 5 km b. 6 km c. 7 km d. 8 km e. 9 km

Figure 3.3 The second item in the instrument

The eighth problem was adapted from the study of De Bock et al. (1998). In the original problem, two maps of Belgium were mentioned. The side lengths of the maps were not given; yet the distance from one city to the second city and the distance from this city to another city were provided in the problem. The distance between the same two cities was provided on the second map that had a different scale, and the distance between the last two cities were asked. The problem required students to think about the fact that the real distance between the cities would not change. The distances were different because of the fact that the two maps had different scales; indeed the second one was four times enlarged version of the first one. The problem is a linear one and as follows:

On a map of Belgium in an atlas the distance from Genk to Leuven is approximately 5 cm and the distance from Genk to Ghent approximately 11 cm. On a map in front of the classroom the distance from Genk to Leuven is approximately 20 cm. How long is the distance from Genk to Ghent on this map? (De Bock et al., 1998)

Figure 3.4 The original version of the eighth problem

In order to use the problem in the present study, some changes were made in the original problem. Firstly, the names of the cities were replaced with the city names in Turkey. Then, the distances between the cities were written in accordance with the real distances. No further modification was made as in the following figure:

On a map of Turkey, the distance between Adana and Antalya is approximately 5 cm and the distance between Antalya and Muğla is approximately 3 cm. On another map of Turkey, the distance between Adana and Antalya is approximately 10 cm. How long is the distance from Antalya and Muğla on this map? Show your solution way in detail.

Figure 3.5 The eighth item in the instrument

Linear-Perimeter: These problems are related to the perimeter (or circumference) concept and need a linear solution approach. Two problems are included in this category. These problems are the first and the third problems of the instrument used in the present study. The original versions and the changes made in the problems are mentioned below.

The first question was adapted from the study of De Bock et al. (1998). The problem was related to the relationship between the side length and the perimeter of a square shape. The original problem is as follows:

Farmer Gus needs approximately 4 days to dig a ditch around a square pasture with a side of 100 m. How many days would he need to dig a ditch around a square pasture with a side of 300 m? (De Bock et al., 1998)

Figure 3.6 The original version of the first problem

A revised version of the problem was used in the present study. The nature of this relationship between the side length and the perimeter was kept the same; yet the problem situation was changed a little as in the following figure:

Farmer Ahmet wants to dig an irrigation canal around his square pasture. He can dig around this pasture with a side of 100 m in 4 days. How many days would he need to dig around a square pasture with a side of 300m if he continues to work with the same speed? Show your solution way in detail.

Figure 3.7 The first item in the instrument

The third problem was also adapted from the study of De Bock et al. (1998). The problem was related to the relationship between the distance and the time period contextually. The problem included the relationship between the perimeter of a circular shape and the time to pass around this shape. The problem is a linear one and stated below in Figure 3.8.

You need approximately 6 hours to sail around a circular island with a diameter of 70 km. How many hours would you need to sail around a circular island with a diameter of 140 km? (De Bock et al., 1998)

Figure 3.8 The original version of the third problem

The problem was related to the relationship between the time period and distance taken contextually. In the original problem, the time for a person to sail around an island is asked; yet the time for a sailing boat was asked in the problem that was adapted in order to be suitable for Turkish language. Besides, whereas the diameter of the circular shape was given in the original problem the radius of the circular shape was provided in the present study. The aim for this change was to prevent probable misuse of the circumference formula of the circle. The problem is adapted as in Figure 3.9 below.

A sailing boat needs approximately 6 hours to sail around a circular island with a radius of 35 m. How many hours would the same sailing boat need to sail around a circular island with a radius of 70 km if it keeps moving with the same speed? Show your solution way in detail.

Figure 3.9 The third item in the instrument

Linear-Area: These problems are related to the area concept and need a linear solution approach. One problem is included in this category. This problem is the seventh problem of the instrument used in this study.

The seventh problem was adapted from the study of Modestou, Elia, Gagatsis, & Spanoudis (2008). The problem was related to the amount of paint in order to paint all the faces of a cubic tank when the amount of paint for one face was given. The problem was related to the surface area of a cubic shape, and it was a linear one and as follows:

Mr Ben emptied all the water of an open cubic tank, in order to paint it. If he needs 10 L of paint to paint the bottom of the tank, how much paint will he need for the entire tank? (Modestou et al., 2008)

Figure 3.10 The original version of the seventh problem

Some changes were made in the problem. To begin with, the context was changed a little. Instead of a water tank, the object was chosen as a cubic money box which was considered as a more familiar shape to the students. Since the size of the object was changed the necessary amount of paint was also changed in accordance with the size. Besides, the object was mentioned as a regular cube instead of an open cube. The aim here was to prevent probable misuse of the surface area formula of the cube. The seventh problem adapted and used in the present study is as follows:

Aslı wants to paint the exterior surface of her cubic money box. She needs 10 ml of paint in order to paint one face of her money box. How much paint will she need for the entire faces of the money box?

Figure 3.11 The seventh item in the instrument

Linear-Volume: These problems are related to the volume concept and need a linear solution approach. One problem is included in this category. This problem is the tenth problem of the instrument used in this study.

A linear problem related to the volume of geometrical shapes was not encountered in the available literature. Hence, the last problem was written by the researcher. The problem was related to the effect of cutting off a rectangular prism from its half height and parallel to the base on the volume of that shape. The problem is a linear one and as follows:

A rectangular prism shaped box which has a volume of 60 m^3 is cut off from its half height and parallel to the base. What is the volume of this box after being cut? Show your solution way in detail.

Figure 3.12 The tenth item in the instrument

Non-linear-Length: These problems are related to the length concept and need a non-linear solution approach. One problem is included in this category. This problem is the sixth problem of the instrument used in this study.

The sixth problem was adapted from the study of De Bock et al. (1998). In the problem, two different maps of Belgium were mentioned. The side lengths of the maps were not given; yet the distance between the two cities and the area of the country on the first map were provided. The distance between the same two cities was provided on the second map that had a different scale, and the area of the country on the second map was asked to the students. The problem required students to think about the fact that the real distance between the cities and the real area of the country would not change. The distances were different because of the fact that the two maps had different scales; indeed the second one was two times enlarged version of the first one. The problem was a non-linear one and as follows:

On a map of Belgium in an atlas the distance from Genk to Tongeren is approximately 2 cm and the area of Belgium approximately 250 cm2. On a map in front of the classroom the distance from Genk to Tongeren is approximately 6 cm. How large is the area of Belgium on this map? (De Bock et al., 1998).

Figure 3.13 The original version of the sixth problem

Several changes were made to the problem in the adaptation process. Firstly, the figures related to the two maps were provided in order to help students to visualize the problem. The two figures were designed so as to be consistent with the given values in the problem. Besides, the numbers given were changed since the areas of Belgium and Turkey were different. The other reason was due to the fact that the figures of the maps would be provided, so the numbers had to be smaller. The sixth problem adapted and used in the present study is as follows:

On a map of Turkey given below as Figure 1, the distance from Ankara to İstanbul is 1 cm and the area of Turkey is 25 cm^2. How large is the area of Turkey on another map given below as Figure 2 in which the distance from Ankara to İstanbul is 2? Show your solution way in detail.

Figure 1

Figure 2

Figure 3.14 The sixth item in the instrument

Non-linear-Perimeter: These problems are related to the perimeter of geometrical figures and need a non-linear solution approach. One problem had been included in this category in the pilot study; yet the problem was deleted after the pilot study due to some reasons that will be discussed in the following parts. Therefore, no problem was included in the actual study for this category.

Non-linear-Area: These problems are related to the area of geometrical figures and need a non-linear solution approach. Two problems are included in this category. These problems are the fourth and the ninth problems of the instrument used in this study. The original versions and the changes made in the problems are mentioned below respectively.

The fourth problem of the study was adapted from the study of Modestou et al. (2008). The problem was related to the two rectangular shapes; one of which was the double of the other. The relationship between the side lengths and areas of these shapes was asked to the students. The area of one shape was given while side lengths were not provided in the question. The problem is a non-linear one and as follows:

George measured the surface of his classroom floor and found that its area is 25 m. The gym's floor has double the dimensions of the classroom. What is the area of the gym's floor? (Modestou et al., 2008).

Figure 3.15 The original version of the fourth problem

The problem was directly translated into Turkish without any modification as follows:

Özlem measured the surface of his classroom floor and found that its area is 25 m. The gym's floor has double the dimensions of the classroom. What is the area of the gym's floor? Show your solution way in detail.

Figure 3.16 The fourth item in the instrument

The ninth problem of the instrument was adapted from the study of Van Dooren et al., (2003). The problem was related to the amount of paint needed to paint a drawing. Two different situations were provided to the students together with the amount of paint given for the first situation. The drawings were in irregular shape forms; yet the drawing in the second situation was three times enlarged version of the first drawing. But this relationship was not given explicitly in the problem; instead the heights of the drawings were provided, and students were required to discover this relationship. Therefore, the problem asked students to analyze the relationship between the two shapes and then the amount of paint for the second drawing based on the amount of paint for the first drawing. The problem is non-linear and as follows:

Bart is a publicity painter. In the last few days, he had to paint Christmas decorations on several store windows. Yesterday, he made a drawing of a 56 cm high Father Christmas on the door of a bakery. He needed 6 ml of paint. Now he is asked to make an enlarged version of the same drawing on a supermarket window. This copy should be 168 cm high. Approximately how much paint will Bart need to do this? (Van Dooren et al., 2003)

Figure 3.17 The original version of the ninth problem

Several changes were made to the problem. The problem sentences were shortened in order to prevent students' hasty and careless readings. The story part in the problem was deleted. Instead, students were directly required to find the amount of paint necessary for the second painting based on the data given for the first painting. Numbers were changed with nicer numbers in order to help students see the relationship that the second shape was three times enlarged version of the first shape. The figures of the two paintings were provided in order to help students visually. The figures were designed so as to be consistent with the given numbers as follows:

6 ml of paint is needed to paint a Father Christmas picture with a height of 50 cm given in figure 1. How much paint will be needed to paint a copy of the same picture with a height of 150 cm given in figure 2? Show your solution way in detail.

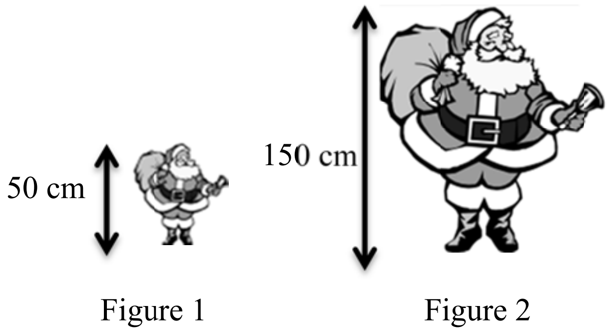


Figure 3.18 The ninth item in the instrument

Non-linear-Volume: These problems are related to the volume of geometrical figures and need a non-linear solution approach. One problem is included in this category. This problem is the fifth problem of the instrument used in this study.

The fifth problem of the study was adapted from the study of Modestou and Gagatsis (2007). The problem was related to the volumes of two swimming pools. The students were required to find the volume of another swimming pool whose dimensions were two times of the first swimming pool. The problem was considered as a non-linear one and as follows:

A gym's swimming pool has a rectangular shape and 70 m³ of water capacity. What is the water capacity of Nicosia's public swimming pool if its dimensions are two times the dimensions of the gym's swimming pool (Modestou & Gagatsis, 2007)

Figure 3.19 The original version of the fifth problem

In the adaptation process of the problem, it was emphasized in the problem statement that all the length, width, and height of the original shape were doubled whereas these three dimensions were not clearly stated in the original problem. The aim was to make students think about the effect of the enlargement on each dimension and ultimately on the volume. Also, it was emphasized that the pool was in the shape of a rectangular prism in the Turkish translation while it was stated as a rectangular shape in the original problem as follows:

> A swimming pool in a school is in the shape of a rectangular prism and has 70 m^3 of water capacity. What would be the water capacity of another swimming pool if its length, width and height were two times the dimensions of this swimming pool? Show your solution way in detail.

Figure 3.20 The fifth item in the instrument

The categories and the corresponding problems in each category are summarized in the Table 3.2 below.

Table 3.2 Table of Content for the Problems in the Achievement Test

Concepts	**Solution Approach**		
	Linear	**Non-Linear**	**Total Number**
Length	P2, P8	P6	3
Perimeter	P1, P3	-	2
Area	P7	P4, P9	3
Volume	P10	P5	2
Total Number	6	4	10

3.3.2 Interview Protocol

After the implementation of the achievement test, the answers of the students in the achievement test were investigated, and some pre-codes were determined for their correct solution strategies and underlying reasons for their incorrect answers.

Conducting interviews was essential in this study since it was not possible to clearly understand their correct solution strategies and underlying reasons for their incorrect answers in the problems. Indeed, the benefit that the interviews provided with was the direct observation of students when they were interacting with the ideas, hence diving into their strategies and intuitions (Cobb & Steffe, 1983). Therefore, semi structured interviews were conducted with the participants who took the achievement test in order to have an in-depth understanding of students' correct solution strategies and probable underlying reasons for their incorrect answers in the achievement test. The data collected through these interviews were used to compare and support the codes obtained from the investigation of students' work on the achievement test.

Prior to the interviews, the purposes of the study and interview were explained to the students. These interviews were conducted in available classrooms in which the participants felt comfortable. The interviews lasted approximately 30 minutes. Throughout these interviews, participants were asked to amplify or clarify answers to each item. In other words, they were required to explain and clarify solution strategies they used for the problems. The whole interview process was guided by open-ended questions that would yield participants' responses clarifying their correct solution strategies and revealing underlying reasons for their incorrect answers. Participants' initial responses to interview questions were probed by the interviewer with the kinds of questions "How did you obtain this answer? Why did you think like this?, What strategy did you use?". These kinds of questions were asked for each of the problems. All the interviews were audio-taped and transcribed. For the semi-structured interview protocol see Appendix D.

3.4 Validity and Reliability

Validity is the "appropriateness, correctness, meaningfulness, and usefulness of the inferences" (Fraenkel & Wallen, 2006, p. 151). In other words, validity is related to the consistency between the purposes of the study and the results drawn from the data. More specifically, the content validity is related to the content and format of the data collection tool (Fraenkel & Wallen, 2006). For the aim of ensuring content validity, national mathematics education curriculum for middle school was examined in terms of linearity and non-linearity; and also length, perimeter, area, and volume concepts. Related objectives were written down, and the problems were matched with those objectives based on the criteria whether the problems were intended to measure the objectives. The table of specification that was constructed based on the objectives of national mathematics education curriculum. For the table of specification see Appendix E.

Subsequently, the test was given to three mathematics education specialists for the expert opinions based on the table of specification. These three experts were experienced academic members at the department of elementary mathematics education from two public universities. The experts commented on the consistency of the problems with the national objectives, appropriateness for the grade levels, and the clarity of the items. Besides, one English teacher was consulted for the accuracy of the translation of the problems from English to Turkish since it is essential to take the cultural and psychological aspects into consideration while doing translations.

After taking the expert opinions and making the necessary revisions, a preliminary version of the data collection instrument was piloted with one class of each grade. Students were selected from public and private schools in Çankaya and Yenimahalle Districts of Ankara. These classes were selected based on the criteria accessibility and convenience. The aims for the pilot study were to check for the clarity and comprehensibility of the items, the appropriateness for each grade level, and also for deciding the average testing time. 17 students from sixth grade, 14 students from seventh grade, and 20 students from eighth grade participated in the pilot study, none of which would participate in the actual study. The researcher was present throughout the administration process of the pilot study which took approximately 40 minutes for each grade level. During the pilot study, it was observed that students from all grades were able to deal with the problems with the exception that sixth graders could not answer the non-linear problem related to the relationship between the area and perimeter of a circular region.

The feedbacks from the students during the administration process and the analysis of student answers on the pilot study provided considerable insight into the basic issues and the final version of the data collection tool before the actual data collection stage. Some modifications were done to the tool in line with those feedbacks. First of all, the non-linear problem related to the relationship between area and perimeter of a circular region was removed from the data collection tool since none of the sixth grade students was able to answer the problem. Hence, although the achievement test in the pilot study had included 11 problems the achievement test in the actual study included 10 problems. Second, it was observed that the students got bored of reading long sentences in the problems. Thus, the problems were shortened where possible. Also, it was observed that students experienced challenges while computing decimals and large whole numbers. Hence, whole numbers with small values were included instead of decimals and large numbers. Last, the analysis of the student work showed that there were three problems that were solved by few students. Hence, some pictures related to the problem situations were provided with the problems in order to help students visually.

Reliability is referred to as “the consistency of the scores obtained” (Fraenkel & Wallen, 2006, p. 157). To obtain reliability, internal consistency methods were used, firstly. After the pilot implementation of the instrument, students’ answers in the pilot version of the achievement test were analyzed by the researcher and a doctoral student in mathematics education department. Crocker and Algina (1986) claimed that Kuder-Richardson formulas can be used with items dichotomously scored to find the coefficient alpha. Hence, in order to apply Kuder-Richardson formulas students’ answers to the problems were coded 1 only when the answer was correct; it was coded as 0 in all other cases. The answers of the students were entered into computer by using PASW (Predictive Analytics SoftWare) 20 statistics program. Kuder-Richardson 20 formula was used to measure the internal consistency reliability of the scores. Reliability coefficient for the pilot study was computed as .77 by using the formula in Figure 3.21 below (Crocker & Algina, 1986, p. 139).

$$KR_{20} = \frac{k}{k-1}\left(1 - \frac{\Sigma pq}{\hat{\sigma}_X^2}\right)$$

Figure 3.21 Kuder-Richardson 20 formula (Crocker & Algina, 1986, p. 139).

In addition to the pilot study, the quantitative data collected for the actual study was evaluated for the Kuder-Richardson 20 formula by the same process. The reliability coefficient was found as .78. Fraenkel and Wallen (2006) mentioned that reliability measures above .70 can be considered as relatively high in educational sciences. Thus, analyses showed that scores were reliable.

Some procedures were also followed for the qualitative data of the study. A mathematics educator and a doctoral student in Elementary Mathematics Education department were asked to determine whether the interview questions work in line with the research questions and the purposes of the study. These two experts also checked the questions so that the questions were not biased or leading. Moreover, a pilot study of the interview questions was conducted with one student from each grade, none of which would participate in the actual study. During these pilot interviews, the researcher tried to detect whether there were any confusing, unclear, or irrelevant questions and also to determine whether the students understood the questions and gave relevant answers. Final version of the semi-structured interview protocol was constructed based on the feedbacks from the experts and the results of the pilot interview study.

After the implementation of the interview protocol for the actual study, the interviews of the actual study were also transcribed into the words. A doctoral student in Elementary Mathematics Education department was informed about the purposes and procedures of the study, and she was asked to analyze the interview transcripts in order to identify and comment on the themes that would come up. The co-coder did not see the actual names of the participants but their pseudonyms. The researcher and the co-coder independently read more than 10.0% of students' work on the achievement test in order to come up with common codes for students' correct solution strategies and underlying reasons for their incorrect answers in the problems. Similarly, the two readers independently worked on the interview transcripts in order to identify, compare, and support the common codes detected from the students' work on the achievement test. This process continued until 95.0% agreement was reached on the final versions of the codes. Then, these codes were saturated to form common themes related to solution strategies and reasons underlying incorrect answers and rubrics were prepared for the categories of solution strategies and underlying reasons for incorrect answers. In the meantime, it was also ensured that all the students' correct solution strategies and underlying reasons for their incorrect answers were attributed to a single theme.

3.5 Data Collection Procedures

In the fall of 2012, the achievement test was developed based on the relevant literature. Expert opinions were taken, and the necessary revisions were made by the end of the same semester. The necessary permissions were taken from Middle East Technical University Human Subjects Ethics Committee (see Appendix A) and the Head of Elementary Mathematics Education program of the university prior to the data collection process. Then, official permissions needed for conducting the main study were taken from the Ministry of National Education (see Appendix B). All the data were collected during spring semester of 2012-2013 academic year. A pilot study of the instrument was conducted to ensure the validity and the reliability of the instrument. After randomly selecting the schools, the school administrators were called and asked for an appropriate time schedule so that the implementation of the test did not coincide with students' own exams or any ceremony that would take place at the school. The students were administered the instrument during their regular class time by the researcher. All the necessary explanations about how to respond the items were made to the students at the beginning of the administration. The students were informed about confidentiality and willingness. In other words, it was announced that their answers on the items would be used only for the aims of the study and not for any other purposes. Besides, it was declared that they could refuse to participate in the study or withdraw at any point of the administration process.

After a couple of weeks later from the administration of the instrument, semi-structured interviews were conducted with some of the students participated in the first part of the study. The aim for conducting these interviews was to clarify correct solution strategies of the participants and to explore the reasons behind students' incorrect answers in the instrument. The time schedule for the data collection process is provided in the Table 3.3 below.

Table 3.3 Time schedule for the study

Date	Events
October 2012- February 2013	Development of the measuring instrument
March 2013	Pilot study and revisions of measuring instrument
April-May 2013	Data collection
May-June 2013	Data analysis

3.6 Analysis of Data

In order to reach the purposes of the study and have a complete and holistic understanding about the aims of the study, two different sets of data were analyzed. These two data sets were students' answers to the problems in the achievement test and written transcripts of the interviews. First, students' answers in the achievement test were analyzed in order to evaluate achievement level, correct solution strategies, and reasons underlying their incorrect answers. Then, the interview transcripts were analyzed and integrated to the categories obtained from the achievement test in order to support the findings.

In order to reach the first goal, students' answers in the achievement test were analyzed via an item-based analysis. A number of descriptive statistics in SPSS PASW 20 program were used. The frequencies of correct and incorrect responses to the items were evaluated by grade levels. Although the problems were open-ended, there was one right answer to each problem. Therefore, the answers were categorized as blank, correct, and incorrect unlike the pilot study in order to make a distinction between the blank, incorrect and correct answers. The answers of the students were coded as 0 if there was no answer at all, 1 if the student answered the question correctly, and 2 if the answer was incorrect. After entering participants' blank, correct, and incorrect answers in the PASW (Predictive Analytics SoftWare) 20 statistics program for each item, frequencies and percentages of these answers were calculated for the linear and non-linear items separately.

The second purpose of the study is to investigate students' correct solution strategies for the linear and non-linear problems in the achievement test. In order to reach this aim, students' correct answers in the problems were analyzed thoroughly. Related literature was reviewed first in order to reach relevant codes for students' correct solution strategies for linear and non-linear problems. However, some related data coding was encountered in the accessible sources. Then, the data collected in the pilot study were analyzed in order to determine some pre-codes.

The initial categories for the solution strategies in linear problems obtained from the pilot study were named as using direct proportion, finding the answer by calculating area or perimeter, and finding the answer without calculating the area or perimeter. Moreover, the initial categories for the solution strategies in non-linear problems obtained from the pilot study were named as using direct quadratic and cubic relationships and finding the side length and using the relationships between the side lengths to find the area or volume. Initial categories for the solution strategies obtained from the analysis of the data from the pilot study is presented in Table 3.4 below.

Table 3.4 Initial Categories for the Solution Strategies Obtained from the Analysis of the Data in the Pilot Study

Solution Strategies	
Linear Problems	Non-linear Problems
Using direct proportion	Using direct quadratic and cubic relationships
Finding the answer by calculating area or perimeter, and finding the answer without calculating the area or perimeter	Finding the side length and using the relationships between the side lengths

The initial categories regarding the underlying reasons for the incorrect answers in non-linear problems were also determined. These were instinctive application of linear strategies, not focusing on the concepts but only on the numbers, poor knowledge of geometrical concepts. On the other hand, there were no pre-codes for the underlying reasons for incorrect answers in linear problems and, hence, for the common reasons. Initial categories for the underlying reasons in non-linear problems obtained from the analysis of the data from the pilot study are presented in Table 3.5 below.

Table 3.5 Initial Categories for the Underlying Reasons for Incorrect Answers in Non-linear Problems Obtained from the Analysis of the Data in the Pilot Study

Underlying Reasons for Incorrect Answers in Non-linear Problems
Instinctive application of linear strategies
Not focusing on the concepts but only on the numbers
Poor knowledge of geometrical concepts

The data obtained from the pilot study gave essential clues for the data coding for correct solution strategies in linear and non-linear problems. Afterwards, students' answers in the measuring instrument in the actual study were analyzed thoroughly in order to have an understanding about students' correct solution strategies, the underlying reasons for their incorrect answers and, hence, to determine codes for the data analysis. Thus, categories obtained from the results of the pilot study were extended with the data collected during the actual study. The data set was listed under different categories for linear and non-linear problems. Categories were determined by considering familiar characteristics they shared. Before adding a new category, the strategy was compared with the previous strategies in order to avoid unnecessary categories. In the naming process of the categories of correct solution strategies for linear and non-linear problems regarding length, perimeter, area, and volume concepts terms from the literature were used in such a way that they would evoke the clues for correct solution strategy used.

Correct solution strategies for linear problems were listed in two categories upon the examination of the data. The first correct strategy for the linear problems was to use directly the numbers given in the problem and perform the operation by using these numbers without what was given and asked in the problem. This strategy is called *questionable proportion*. Students' answers which included the ambiguity of whether they considered the relationship between the variables given and asked in the problem and using the numbers given in the problem and directly writing the proportion between these numbers were placed in this category.

The second correct strategy for the linear problems was to analyze the problem statement, find the related variables, and then, use direct proportion between the related variables. This strategy is called *reasonable proportion*. Students' answers which included finding the related variables (i.e. perimeter) and writing the direct proportion between the related variables or judging the type of the relationship between the variables and determining that the same result would be reached by writing a proportion between the given variables (i.e. length) were placed in this category.

Correct solution strategies for non-linear problems were also listed in two categories upon the examination of the data. The first correct strategy for the non-linear problems was to obtain the area or volume of the second figure by using the relationships between the lengths of the two figures. This strategy is called *length-length-area/volume*. Students' answers which included finding arbitrary side lengths or dimensions for the figures by using the given area or volume and using the relationship between the lengths in order to find the area of the second figure were placed in this category.

The second correct strategy for non-linear problems was to use direct quadratic relationships between the areas of the two figures and direct cubic relationships between the volumes of the two figures. This strategy is called *length-area/volume*. Students' answers which included the argument that area would be multiplied by r^2 or the volume would be multiplied by r^3 when all the lengths or dimensions were multiplied by r were placed in this category.

Correct solution strategies of students for linear and non-linear problems are summarized in Table 3.6 below.

Table 3.6 Correct solution strategies for linear and non-linear problems

Linear Problems	Non-linear Problems
Questionable Proportion	Length-length-area/volume relationship
Reasonable Proportion	Length-area/volume relationship

The third purpose of the study was to analyze the underlying reasons for participants' incorrect answers in both linear and non-linear problems in the achievement test. Literature review provided some reasons and codes for students' incorrect answers for both linear and non-linear problems. In addition, the pilot study was helpful for spotting possible reasons for students' incorrect answers for linear and non-linear problems.

Analyses of the data in the pilot and actual study showed that some reasons for students' incorrect answers in linear and non-linear problems were common. The common reasons for students' incorrect answers are stated below once for both linear and non-linear problems. Later, specific reasons for students' incorrect answers in linear and non-linear problems are mentioned respectively. The first common reason for students' incorrect answers in linear and non-linear problems is *inadequacy in geometrical knowledge*. This category included students' inadequate knowledge in properties of figures or confusion of terms and concepts. For instance, students' answers which included such a statement that a cube has four sides were placed in this category. Besides, students' answers which included calculating the perimeter instead of area or vice versa or calculating the area instead of volume or vice versa were placed in this category. The second common reason for students' incorrect answers in linear and non-linear problems is *misinterpretation of additive and multiplicative reasoning*. This category included students' tendency to use additive reasoning for solving linear and non-linear problems where multiplicative reasoning was needed. For example, students' answers which included the argument that distances on a map should be increased by the same amount instead of the same ratio when the scale was changed or that volume should be multiplied by six when the dimensions of a figure are multiplied by two were placed in this category. In addition, taking the difference between the two variables and adding this difference to the second variable when a proportional situation exists were placed in this category. The third common reason for students' incorrect answers in linear and non-linear problems is *operational mistakes*. This category included students' mistakes in four basic operations while solving linear and non-linear problems. Students' answers which included computational mistakes with correct reasoning were placed in this category. In addition, students' answers which included a correct written explanation for the solution of the problem without finding the correct answer were placed in this category. The fourth and the last common reason for students' incorrect answers in linear and non-linear problems is *incomplete answers*. This category included students' incomplete answers with correct reasoning. For instance, writing the proportion correctly without calculating the answer or determining by which factor the area or volume would be multiplied without calculating the answer were placed in this category. Apart from common underlying reasons for students' incorrect answers in both linear and non-linear problems, the findings included some reasons that were specific to linear or non-linear problems. First, one reason that was only specific to linear problems is *misinterpretation of proportional situations.* This misuse included students' tendency to apply inverse proportion where direct proportion was needed. One reason that was only specific to non-linear problems is *illusion of linearity*. This misapplication included students' tendency to apply linear solution strategies where, actually, non-linear solution strategies were needed. To illustrate, students' answers including the argument that the area or volume would be multiplied by the same scale factor when the lengths or dimensions of a figure are multiplied by a scale factor were placed in this category. Besides, students' answers including writing a linear proportion between the variables in the problems where non-linear strategies are required were placed in this category. Rubrics were prepared for the categories of solution strategies and underlying reasons for incorrect answers in the achievement test. For the rubrics see Appendix F. Underlying reasons for participants' incorrect answers in both linear and non-linear problems in the achievement test were summarized in Table 3.7 below.

Table 3.7 Underlying Reasons for Incorrect Answers in Linear and Non-linear Problems

Underlying Reasons for Incorrect Answers in Linear Problems		Underlying Reasons for Incorrect Answers in Non-Linear Problems
	Common Underlying Reasons for Incorrect Answers	
1. Misinterpretation of Proportional Situations	1. Inadequacy in Geometrical Knowledge	1. Illusion of Linearity
	2. Misinterpretation of additive and multiplicative reasoning	
	3. Operational mistakes	
	4. Incomplete answers	

3.7 Assumptions and Limitations

In this part, the basic assumptions and limitations of the research study are stated. First of all, it was assumed that the achievement level of the participants in linear and non-linear problems regarding length, perimeter, area, and volume concepts can be measured by the prepared achievement test. It was also assumed that students paid close attention to each problem in the instrument, and that they were operative, sincere, and truthful while completing the achievement test and answering the interview questions.

In the survey part of this study, cluster sampling method was used to obtain the sample of the population. The survey part of the study aimed to determine participants' achievement level in linear and non-linear problems regarding length, perimeter, area, and volume concepts. Even though the findings of this study might be limited concerning its application to a more generalized population of sixth, seventh, and eight grade students, the results related to participants' achievement in the measuring tool can be generalized to students in similar contexts. On the other hand, 12 interview participants were selected purposively based on some criteria mentioned above. Therefore, the correct solution strategies of the participants and the underlying reasons for participants' incorrect answers in problems might be limited to those participants.

3.8 Internal and External Validity of the Study

Both internal and external validity matters are related to validity of any study. Thus, the two validity types are discussed in this part.

3.8.1 Internal Validity of the Study

Internal validity means that observed differences on the dependent variable are directly and solely related to the independent variable and not due to some other variable (Fraenkel & Wallen, 2006). There are several threats to internal validity related to the type of the study. Fraenkel and Wallen (2006) address three main threats to internal validity in a survey study as mortality, location, and instrumentation.

To start with, since the present study requires one-time data collection, mortality that is related to loss of participants was not an issue. Besides, the researcher got in touch with the school administrators about the time of data collection in order to ensure maximum number of participation.

Location which occurs as a threat when different individuals are tested in different locations (Fraenkel & Wallen, 2006) was controlled in this study since the tests were administrated in students' regular classrooms. It was assumed that the classroom environments of public schools in Yenimahalle District of Ankara are very similar.

Instrumentation threat is related to how the data are collected and used. Instrument decay, data collector characteristics, and data collector bias are threats constituting the instrumentation threat (Fraenkel & Wallen, 2006). First of all, instrument decay constitutes a problem when the instrument is changed during time or scored differently (Fraenkel & Wallen, 2006). Nevertheless instrument decay was not supposed to be a threat for the present study, since the instruments were implemented to the students just one time, and the answers of the participants were evaluated by two scorers in line with the pre-decided codes. Second, data collector characteristics might cause a threat to internal validity when the data are collected by different people (Fraenkel & Wallen, 2006). Lastly, data collector bias might occur when the data collector has a personal effect on the results either consciously or unconsciously (Fraenkel & Wallen, 2006). Yet these threats do not seem to have caused a problem since the data collection procedures were conducted by the researcher for all of the participants, and that the researcher administered the achievement test by herself. In addition, the researcher did not communicate with the participants other than merely explaining the expectations from the participants at the beginning.

Merriam stated that "the researcher is the primary instrument for data collection and analysis." (Merriam, 1998, p. 42). Therefore abilities, instincts, and expectations of the researcher might have an effect on the results of the study. Nevertheless, the researcher was aware of the biases, and she followed some procedures in order to reduce the biases. First, she tried to be sure that she understood participants' responses and explanations correctly by paraphrasing participants' responses and asking them to agree throughout the interviews. The researcher also conducted pilot interviews in order to gain experience related to conducting interviews. Besides, the researcher worked with another coder independently in order to reduce the researcher bias in the data analysis.

3.8.2 External Validity

External validity of the study is defined as "the extent to which the results of a study can be generalized from a sample to a population" (Fraenkel & Wallen, 2006, p.108). External validity involves two dimensions as population generalizability and ecological generalizability.

Population generalizability refers to "the degree to which a sample represents the population of interest" (Fraenkel & Wallen, 2006, p. 104). To ensure population generalizability in a study, the sample should represent the intended population. In the present study, the target population was sixth, seventh, and eighth grade public school students in Ankara. The accessible population was sixth, seventh, and eighth grade public school students in Yenimahalle District of Ankara. The achievement test was administered to 935 students who were selected from the public schools in a district of Ankara by cluster random sampling method. Hence, the results of the study which were related to sixth, seventh, and eighth grade students' achievement level might be generalized to the population under certain conditions since all the students participated in the study experience the same national curriculum. On the other hand, the interviews were conducted with 12 participants who were selected purposively. Therefore, the results related to participants' solution strategies and underlying reasons for their incorrect answers in problems might be generalizable only to a part of the population.

Ecological generalizability refers to "the degree to which the results of a study can be extended to other settings or conditions" (Fraenkel & Wallen, 2006, p. 106). The study was carried out with sixth, seventh, and eighth grade students in urban public schools where a national elementary mathematics education curriculum is implemented. Besides, it was assumed that most public schools have similar settings. Therefore, the results of the study might be generalizable to public elementary schools with similar settings.

CHAPTER 4

RESULTS

The purposes of this study are to determine sixth, seventh, and eighth grade students' achievement level in linear and non-linear problems regarding length, perimeter, area, and volume concepts, and to investigate students' correct solution strategies for these problems. This study also aims at analyzing the underlying reasons for students' incorrect answers in these problems.

In this chapter, the results of the data are presented in three main aspects. Each aspect is related to the research questions in order. Firstly, the achievement levels of the students in each of the linear and non-linear problems are presented in terms of the grade levels. Secondly, the correct solution strategies that students used for linear and non-linear problems are investigated respectively. Lastly, the underlying reasons for students' incorrect answers in linear and non-linear problems are explored respectively.

In the first section, descriptive information about students' achievement levels in the measuring instrument is presented. In subsequent sections, the codes and the related categories for students' correct solution strategies in linear and non-linear problems and the underlying reasons for their incorrect answers in these problems are explained based on students' answers in the achievement test. Then, these categories and codes obtained from the achievement test are compared and supported with the data obtained from the individual interviews.

4.1 Achievement Level in Linear and Non-linear Problems

The problem types in the instrument are categorized as linear and non-linear in terms of the solution strategy needed. There are six linear problems regarding length, perimeter, area, and volume concepts in the instrument. Problems 1, 2, 3, 7, 8, and 10 are linear problems. The answers of the students in these problems were coded as blank, correct, or incorrect at the first place as explained before. The corresponding frequencies and percentages for students' blank, correct, and incorrect answers were calculated for each of the linear and non-linear problems in order to determine their achievement level. Distribution of 935 students' answers in linear problems across problems and grade levels is presented in Table 4.1 below.

Table 4.1 Distribution of participants' blank, correct, and incorrect answers in linear problems across problems and grade levels

		Linear Problems					
		P1	P2	P3	P7	P8	P10
Grade 6	Blank	9 (3.3%)	19 (6.9%)	30 (10.9%)	46 (16.7%)	69 (25.0%)	108 (39.1%)
	Correct	**203 (73.6%)**	**237 (85.9%)**	**162 (58.7%)**	**162 (58.7%)**	**122 (44.2%)**	**127 (46.0%)**
	Incorrect	64 (23.2%)	20 (7.2%)	84 (30.4%)	68 (24.6%)	85 (30.8%)	41 (14.9%)
Grade 7	Blank	5 (1.3%)	7 (1.8%)	22 (5.7%)	47 (12.3%)	76 (19.8%)	143 (37.3%)
	Correct	**347 (90.6%)**	**360 (94.0%)**	**306 (79.9%)**	**288 (75.2%)**	**263 (68.7%)**	**203 (53.0%)**
	Incorrect	31 (8.1%)	16 (4.2%)	55 (14.4%)	48 (12.5%)	44 (11.5%)	37 (9.7%)
Grade 8	Blank	9 (3.3%)	2 (0.7%)	25 (9.1%)	27 (9.8%)	43 (15.6%)	75 (27.2%)
	Correct	**235 (85.1%)**	**261 (94.6%)**	**218 (79.0%)**	**211 (76.4%)**	**200 (72.5%)**	**180 (65.2%)**
	Incorrect	32 (11.6%)	13 (4.7%)	33 (12.0%)	38 (13.8%)	33 (12.0%)	21 (7.6%)
	Blank Total	23 (2.5%)	28 (3.0%)	77 (8.2%)	120 (12.8%)	188 (20.1%)	326 (34.9%)
	Correct Total	**785 (84.0%)**	**858 (91.8%)**	**686 (73.4%)**	**661 (70.7%)**	**585 (62.6%)**	**510 (54.5%)**
	Incorrect Total	127 (13.6%)	49 (5.2%)	172 (18.4%)	154 (16.5%)	162 (17.3%)	99 (10.6%)

Table 4.1 showed that the achievement level of sixth grade students in linear problems varied between 44.2% and 73.6%, the achievement level of seventh grade students in linear problems varied between 53.0% and 94.0%, and the achievement level of eighth grade students in linear problems varied between 65.2% and 94.6%.

Furthermore, Table 4.1 indicated that achievement levels for each linear problem varied in terms of the grade levels. Although eighth grade students and seventh grade students had closer achievement levels to each other, sixth grade students had lower achievement levels in all linear problems. Table 4.1 showed that achievement level in linear problems varied between 54.5% and 91.8% across problems. Moreover, it can be seen that problem 1 and 2 were mostly correctly answered problems whereas problem 8 and 10 were least correctly answered ones by all grade students.

In addition to linear problems, the achievement test includes four non-linear problems regarding length, perimeter, area, and volume concepts. Problems 4, 5, 6, and 9 are non-linear problems. Similar to the linear problems, the answers of the participants in these problems were coded as blank, correct, or incorrect at the first place. The corresponding frequencies and percentages for students' blank, correct, and incorrect answers were calculated for each of the non-linear problems. Descriptive analysis of 935 students' answers in non-linear problems in terms of grade levels is presented in Table 4.2 below.

Table 4.2 Distribution of participants' blank, correct, and incorrect answers in non-linear problems across problems and grade levels

		Non-linear Problems			
		P4	P5	P6	P9
Grade 6	Blank	56 (20.3%)	82 (29.7%)	81 (29.3%)	39 (14.1%)
	Correct	**33 (12.0%)**	**8 (2.9%)**	**10 (3.6%)**	**0 (0.0%)**
	Incorrect	187 (67.8%)	186 (67.4%)	185 (67.0%)	237 (85.9%)
Grade 7	Blank	78 (20.4%)	118 (30.8%)	84 (21.9%)	38 (9.9%)
	Correct	**71 (18.5%)**	**20 (5.2%)**	**5 (1.3%)**	**0 (0.0%)**
	Incorrect	234 (61.1%)	245 (64.0%)	294 (76.8%)	345 (90.1%)
Grade 8	Blank	43 (15.6%)	68 (24.6%)	47 (17.0%)	20 (7.2%)
	Correct	**109 (39.5%)**	**56 (20.3%)**	**29 (10.5%)**	**0 (0.0%)**
	Incorrect	124 (44.9%)	152 (55.1%)	200 (72.5%)	256 (92.8%)
	Blank Total	177 (18.9%)	268 (28.7%)	212 (22.7%)	97 (10.4%)
	Correct Total	**213 (22.8%)**	**84 (9.0%)**	**44 (4.7%)**	**0 (0.0%)**
	Incorrect Total	545 (58.3%)	583 (62.4%)	679 (72.6%)	838 (89.6%)

Table 4.2 showed that achievement level of sixth grade students in non-linear problems varied between 0.0% and 12.0%, the achievement level of seventh grade students in non-linear problems varied between 0.0% and 18.5%, and the achievement level of eighth grade students in linear problems varied between 0.0%

and 39.5%. Furthermore, Table 4.2 also indicated that students' achievement levels for each non-linear problem varied in terms of the grade levels. Although sixth grade students and seventh grade students had closer achievement levels to each other, eighth grade students had higher achievement levels in all problems except problem 9. These values indicated that eighth grade students performed better than sixth and seventh grade students for most of the problems. Table 4.2 also showed that achievement level in non-linear problems varied between 0.0% and 22.8% across problems. Moreover, it can be seen that problem 4 was mostly correctly answered problem by all grades whereas no student in any grade correctly answered problem 9.

Table 4.1 together with Table 4.2 showed that students' achievement in each of the linear problems was higher than their achievement in any of the non-linear problems regardless of their grade levels.

4.2 Analysis of Correct Solution Strategies for Linear and Non-linear Problems

The second research question of the study is related to investigating the correct solution strategies of sixth, seventh, and eighth grade students for linear and non-linear problems regarding length, perimeter, area, and volume concepts. Analysis of students' correct solution strategies for linear and non-linear problems in the achievement test supported with the analysis of interview transcripts are stated in the following parts respectively.

4.2.1 Analysis of Correct Solution Strategies for Linear Problems

In the achievement test, students were asked six linear problems related to length, perimeter, area, and volume concepts. These linear problems required students to analyze two cases and use a direct proportion between the related variables. The analysis of the achievement test supported by the analysis of the interview transcripts revealed that students used two main correct solution strategies for linear problems. These two strategies are named as questionable proportion and reasonable proportion, and are explained in the following parts.

4.2.1.1 Questionable Proportion Strategy

The first strategy for solving linear problems is questionable proportion strategy. It is related to using the numbers given in the problem directly and performing the operation by using these numbers without focusing on the relationship between the variables given and asked in the problem. This strategy was named as questionable proportion since it wasn't clear whether students considered the relationship between the variables given and asked in the problem. Instead, it was deduced that students used the numbers given in the problem and directly wrote the proportion between the given numbers. To illustrate, students' answers including the direct proportion between the length of a figure and the time period needed to pass around that figure were placed in this category since it was not clear that whether the students considered the relationship between the length and the perimeter. The students might have considered the linear relationship between the concepts and wrote the direct proportion without writing any indication of their thought process. Yet, it might also be the case that they skipped examining this relationship and wrote the proportion automatically. Indeed, both situations were encountered in the interview transcripts of the study. When the participants who directly performed the cross product rule were asked about whether they examined the relationship between the lengths and perimeter in the interview, some of their answers included an argument of the linear relationship between the length and the perimeter but some of them did not.

It was seen that students used this strategy for solving the first and third linear problems, which were categorized as linear-perimeter problems. These problems required participants to write a proportion between the perimeters (or circumference) of the shapes and the time periods needed to go around these shapes or to make an interpretation that using lengths instead of perimeters in the proportion would yield the same result. Students' answers in the first and third problems including a proportion between the lengths (or radii) and the time periods without reasoning the linear relationship between the length and the perimeter were included in this category. The frequencies of questionable proportion strategy for the first and third problems are provided in Table 4.3 below.

Table 4.3 Distribution of Questionable Proportion Strategy across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P1	177 (87.2%)	319 (91.9%)	210 (89.4%)	706 (89.9%)
P3	155 (95.7%)	292 (95.4%)	200 (91.7%)	647 (94.3%)

The analyses of the achievement test revealed that 706 students (89.9%) among 785 students who answered the first problem correctly used questionable proportion strategy for the first problem. This number was constituted of 177 sixth graders (87.2%), 319 seventh graders (91.9%), 210 eighth graders (89.4%). Besides, 647 students (94.3%) among 686 students who answered the third problem correctly used the mentioned strategy. This number was constituted of 155 sixth graders (95.7%), 292 seventh graders (95.4%), and 200 eighth graders (91.7%). In other words, most of the students used this strategy for solving the first and third problems.

Examples from students' answers in the first and third problems were given as illustration in Figure 4.1 and Figure 4.2 below.

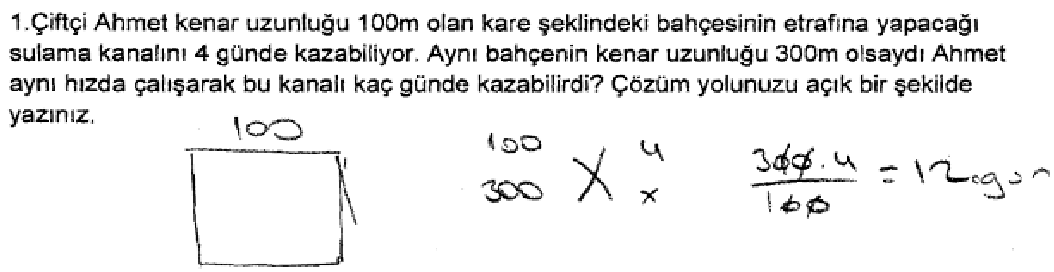


Figure 4.1 Questionable proportion strategy for the first problem

The given problem is related to the relationship between the side lengths and perimeters of two squares. As seen in Figure 4.1, the student found the correct answer without calculating the perimeter of the pasture or indicating the relationship between the side lengths and the perimeter. Instead, he wrote a proportion between the given quantities in the problem which were side lengths of the pastures and the time periods needed to dig around the pastures. It was not possible and reasonable to decide whether the student constructed the proportion blindly by looking at his work on the achievement test. However, his explanations for this solution strategy and reasons for using that strategy in the interview transcripts gave evidence for his thinking. In the interview, Participant 6 (Grade 7) explained that he wrote the proportion just by looking at the problem sentence and ignoring the related concepts in the problem. For instance:

> "…I wrote a ratio and proportion here. I thought that if he finishes 100 m in 4 days, I asked in how many days would he finish 300 m and found the answer 12 by writing a proportion. …I understood that I had to write a proportion by reading the question. The question is clear anyway. I looked at the sentences and the numbers in the question; it was very explanatory for me… The question is related to (distance in) meter concept of the pasture. The irrigation canal is digged around the pasture. But I didn't find the perimeter of the pasture because it is given that he finishes 100 m in 4 days, I used this information. I don't know whether the same result would be reached if I used the perimeter. I just used what was given in the problem." (Participant 6, Grade 7)
>
> […Oran orantı kurdum burada. 100 m'yi 4 günde bitirebiliyorsa 300 m yi kaç günde bitirir dedim oran orantı kullanarak 12 buldum… Soruyu okuyarak oran orantı kurmam gerektiğini anladım. Soru açık zaten. Sorudaki cümlelere ve sayılara baktım açıklayıcıydı benim için… Soru bahçenin metre kavramıyla ilgili… Sulama kanalı bahçenin çevresine kazılıyor… Ama ben çevreyi bulmadım zaten 100 m'yi 4 günde bitirdiği verilmiş ben bunu kullandım. Çevresini bularak yapsam aynı sonuç çıkar mıydı bilmiyorum. Ben soruda verilenleri kullandım sadece.]

Another solution for the third problem that was also coded as questionable proportion is presented in Figure 4.2 below.

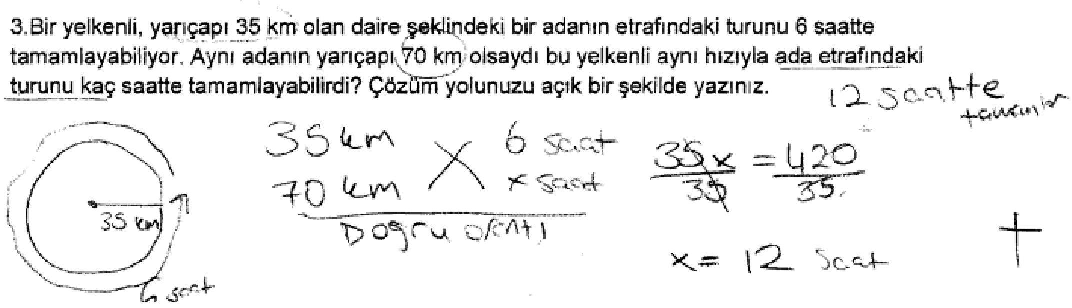


Figure 4.2 Questionable proportion strategy for the third problem

The third problem was related to the relationship between the circumference of a circle and the time period needed to go around this shape. As seen in Figure 4.2, the student found the correct answer without calculating the circumference of the circular island. Instead, he wrote a proportion between the given quantities in the problem which were the radii of the two islands and the time periods needed to sail around these islands. It was also hard to decide whether the student constructed the proportion blindly, or he considered the relationship between the radii and the circumference by looking at his work in the achievement test. However, his explanations in the interview gave essential evidence for his thinking and solution strategy. In the interview, Participant 1 (Grade 6) was asked about the related concepts in the problem and whether he considered these concepts in his solution. He stated that he did not consider those concepts since it was not asked in the problem. The participant stated as:

> "..In 12 hours…I used ratio and proportion here. I thought that the time should increase since the distance gets longer. The problem is related to (distance in) km of the island. I didn't find the perimeter of the island because it is not asked in the problem. So, I don't have to find it." (Participant 1, Grade 6)
>
> […12 saatte. Burada oran orantı kullandım. Yol arttığı için zaman da artmalı diye düşündüm. Burada da adanın km'si ile ilgili soru bu. Ben çevresini bulmadım çünkü çevresini istememiş. O yüzden bulmama gerek yok.]

4.2.1.2 Reasonable Proportion Strategy

The second strategy used for solving the linear problems is reasonable proportion. It is related to analyzing the problem statement, finding the related variables (i.e. perimeter), and then using a direct proportion between the related variables. This strategy was named as reasonable proportion since students focused on the relationship between the related variables in the problem. That is to say, they, as a first step, judged the type of the relationship between the variables, decided that a linear relationship exists, and lastly performed the proportion. The first and third problems which were categorized as linear-perimeter were solved by this strategy. The frequencies of reasonable proportion strategy for the first and third problem are provided in Table 4.4 below.

Table 4.4 Distribution of Reasonable Proportion Strategy across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P1	26 (12.8%)	28 (8.1%)	25 (10.6%)	79 (10.1%)
P3	7 (4.3%)	14 (4.6%)	18 (8.3%)	39 (5.7%)

The analyses of the achievement test revealed that 79 students (10.1%) among 785 students who answered the first problem correctly used reasonable proportion strategy. This number included 26 sixth graders (12.8%), 28 seventh graders (8.1%), and 25 eighth graders (10.6%). In addition, 39 students (5.7%) out of 686 students (i.e.7 sixth graders (4.3%), 14 seventh graders (4.6%), and 18 eighth graders (8.3%)) used the mentioned strategy for the third problem. In other saying, the number of the students who used this strategy for the first and third problem was very few. Examples from students’ answers in the first and third problems in the achievement test were given as illustration in Figure 4.3 and Figure 4.4 below.

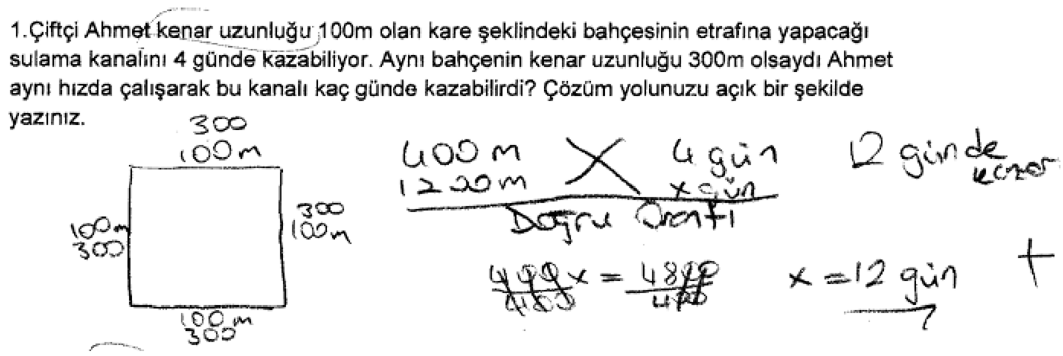


Figure 4.3 Reasonable proportion strategy for the first problem

As seen in the Figure 4.3, the student found the correct answer by finding the corresponding perimeters of the two pastures and then writing a direct proportion between the perimeters of the pastures and the time periods needed to dig around these pastures. Student's work on the achievement test was clear for placing into the category of reasonable proportion. However, the finding that Participant 4 (Grade 6) judged the relationship between the related concepts was also supported by the interview data such as:

> "I found the perimeter of the pasture… I found the perimeter of the pasture whose side length was 100 m and the other pasture whose side length was 300 m. I found how many times bigger the perimeter of the second pasture is than the perimeter of the first pasture. If he makes the irrigation canal around the first pasture in 4 days he makes the canal around the second pasture which is 3 times bigger than the first one in 12 days… One pasture is 3 times the size of the other; hence I thought that 3 of the same canal should be digged. Thus it should take longer…3 times longer…I wrote a ratio and proportion and I also used the perimeters of the two pastures. " (Participant 4, Grade 6)
>
> [Çevresini buldum bahçenin… Kenarı 100 m olan bahçenin çevresini buldum bir de kenarı 300 m olan bahçeninkini. Bu çevreleri birbirine bölerek kaç katı olduğunu bulmuşum. İlk bahçeye kanalı 4 günde yaptığına göre 3 katı olan ikinci bahçeye kanalı 12 günde yapar…Biri diğerinin 3 katı olduğu için aynı kanaldan 3 tane yapmak gerekir diye düşündüm…O

yüzden daha uzun sürmeli.. 3 kat daha uzun…Hem oran orantı kurdum hem de çevreleri kullandım.]

Another solution for the third problem that was also coded as reasonable proportion is presented in Figure 4.4 below.

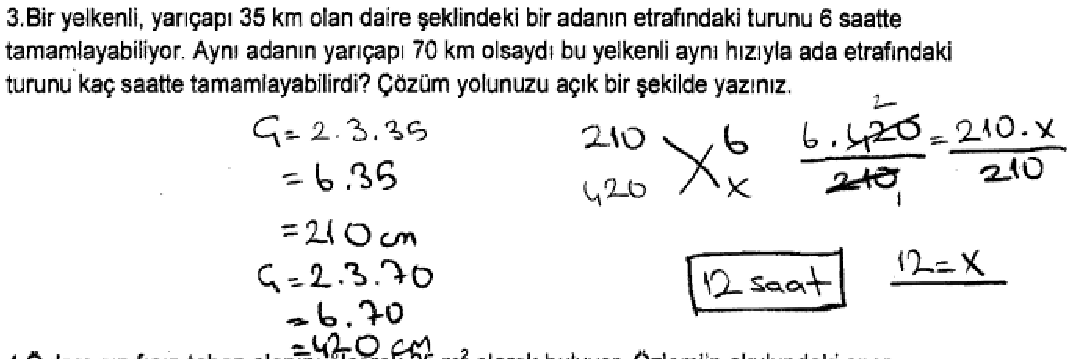


Figure 4.4 Reasonable proportion strategy for the third problem

As seen in Figure 4.4, the student found the correct answer by finding the corresponding circumferences of the islands and then writing a direct proportion between the circumferences of the islands and the time periods needed to sail around the islands. The thinking of Participant 4 (Grade 6) of the related concepts in solving the problem was supported by the interview data as follows:

> “First, I found the diameter by multiplying by 2…We find the perimeter of the island if we multiply the diameter with π. I found the perimeter of the second island in a similar way. It completes a tour in 6 hours for the first one; therefore, it completes a tour in 12 hours for the second one… Because when the perimeter is 210 the time is 6 hours hence when the perimeter is 420 then the time will be 12 hours. The perimeter is two times bigger than that of the first one. I used the cross product and found 12. The time increases directly proportional to the distance.” (Participant 4, Grade 6)
>
> [Önce çapı buldum… 2 ile çarparak bir de pi sayısı vardı çapla pi sayısını çarpınca çevreyi buluyoruz. İkinci adanın çevresini de buldum aynı şekilde. İlkini 6 saatte tamamlıyor o zaman ikincininki 12 saatte tamamlar… Çünkü

çevresi 210 ken 6 saatti 420 iken 12 saatte tamamlar. Çevresi 2 katı oluyor…içler dışlar çarpımı yaparak 12 buldum. Yol artınca zaman da aynı oranda artıyor.]

4.2.1.3 Strategies for the Remaining Linear Problems

The questionable and reasonable proportion strategies were categorized for the first and the third linear-perimeter problems. Since there were six linear problems the strategies for the remaining four linear problems, namely second, seventh, eighth, and tenth problems are mentioned in this part.

The first strategy for the second, seventh, and eighth problems was writing direct proportions between the variables in the problem and finding the answer. The use of this strategy was not observed for the tenth problem. For instance, writing a proportion between the time period and the distance in the second problem, between the number of faces of a cube and the paint needed in the seventh problem, between the corresponding distances on the map in the eighth problem were included in this category. Examples from students' answers in the achievement test are presented in Figure 4.5, Figure 4.6, and Figure 4.7 below.

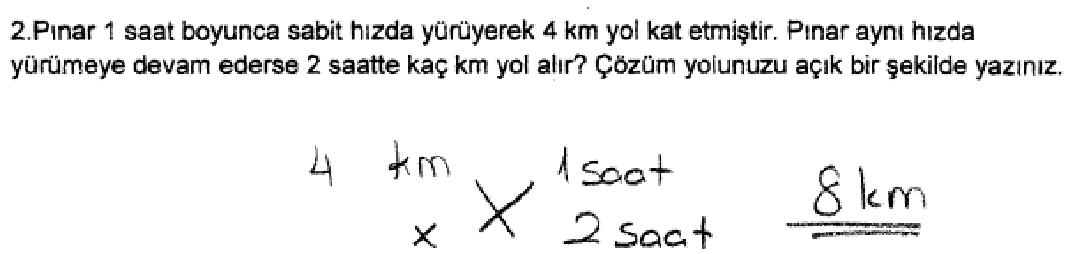


Figure 4.5 Using a direct proportion for the second problem

As seen in Figure 4.5, the student wrote a proportion between the distance taken and the time period to travel the distance and found the correct answer.

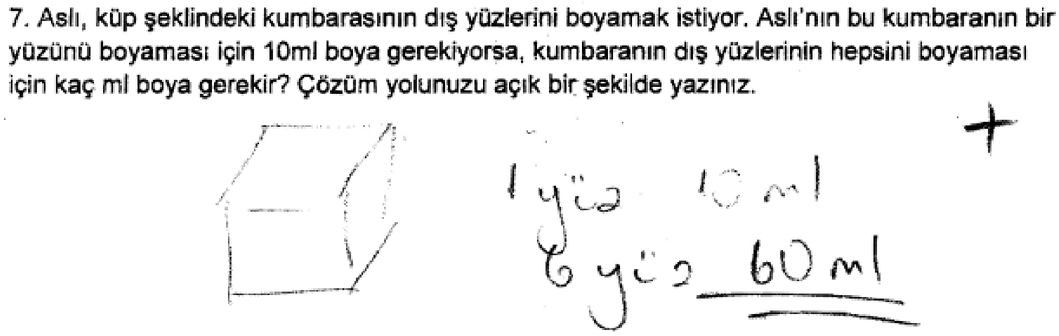

7. Aslı, küp şeklindeki kumbarasının dış yüzlerini boyamak istiyor. Aslı'nın bu kumbaranın bir yüzünü boyaması için 10ml boya gerekiyorsa, kumbaranın dış yüzlerinin hepsini boyaması için kaç ml boya gerekir? Çözüm yolunuzu açık bir şekilde yazınız.

Figure 4.6 Using a direct proportion for the seventh problem

As seen in Figure 4.6, the student wrote a direct proportion between the paint needed for one face and the paint needed for all the six faces of the cube.

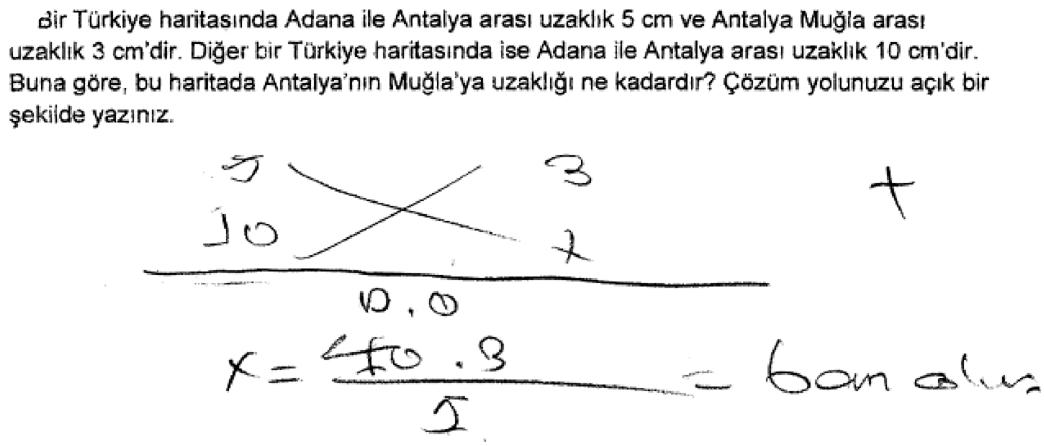
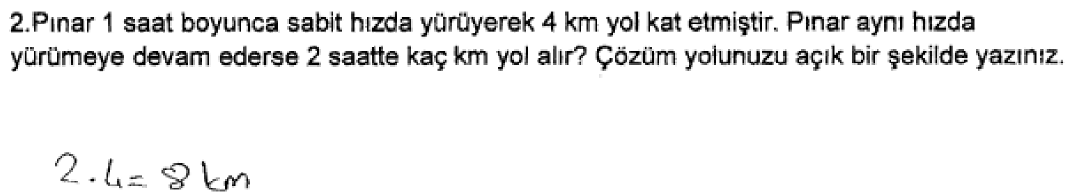

Bir Türkiye haritasında Adana ile Antalya arası uzaklık 5 cm ve Antalya Muğla arası uzaklık 3 cm'dir. Diğer bir Türkiye haritasında ise Adana ile Antalya arası uzaklık 10 cm'dir. Buna göre, bu haritada Antalya'nın Muğla'ya uzaklığı ne kadardır? Çözüm yolunuzu açık bir şekilde yazınız.

Figure 4.7 Using a direct proportion for the eighth problem

As can be seen in Figure 4.7, the student wrote a direct proportion between the distances on the first map and the distances on the second map.

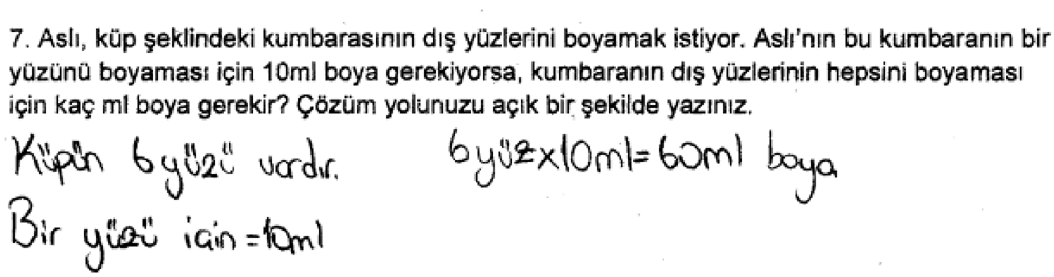
The second strategy for the second, seventh, eighth, and the tenth problem was finding the scale factor mentally and multiplying (or dividing) the second value by the scale factor without writing a proportion. Examples of multiplying (or dividing) directly by the scale factor for each of the second, seventh, eighth, and tenth problems are presented in Figure 4.8, Figure 4.9, Figure 4.10, and Figure 4.11 below.


2.Pınar 1 saat boyunca sabit hızda yürüyerek 4 km yol kat etmiştir. Pınar aynı hızda yürümeye devam ederse 2 saatte kaç km yol alır? Çözüm yolunuzu açık bir şekilde yazınız.

2.4= 8km

Figure 4.8 Multiplying the second value by the scale factor for the second problem

As seen in Figure 4.8, the student mentally calculated that the time period got two times as much as the first time period, hence determined that the scale factor was 2. Then, he directly multiplied the first distance taken by the scale factor and found the correct answer.


7. Aslı, küp şeklindeki kumbarasının dış yüzlerini boyamak istiyor. Aslı'nın bu kumbaranın bir yüzünü boyaması için 10ml boya gerekiyorsa, kumbaranın dış yüzlerinin hepsini boyaması için kaç ml boya gerekir? Çözüm yolunuzu açık bir şekilde yazınız.

Küpün 6 yüzü vardır. 6yüz x10ml=60ml boya

Bir yüzü icin =10ml

Figure 4.9 Multiplying the second value by the scale factor for the seventh problem

As seen in Figure 4.9, the student determined that the amount of paint should get six times as much as the first amount since a cube has six faces. Hence, he multiplied the first amount of paint by six and found the correct answer.

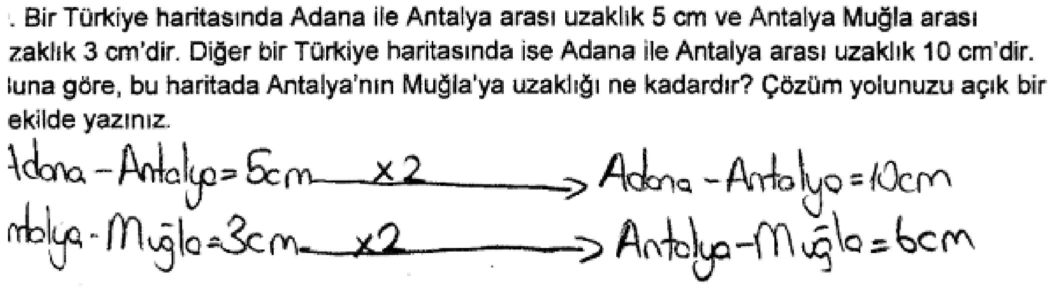


Figure 4.10 Multiplying the second value by the scale factor for the eighth problem

As seen in Figure 4.10, the student determined that the distance for the first two cities became two times longer on the second map and decided that the distance between the other two cities had to become two times longer as well. Hence, he multiplied the distance on the first map by two and found the correct answer.

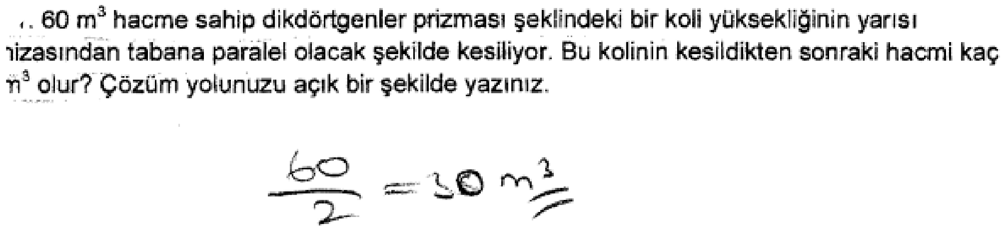


Figure 4.11 Dividing the second value by the scale factor for the tenth problem

As seen in Figure 4.11, the student divided the volume by two, hence determined the scale factor as $\frac{1}{2}$ since it was stated in the problem sentence that the box is cut off from its half.

Two other strategies were encountered for the tenth problem. The first strategy was finding the correct answer by giving arbitrary values to the dimensions of the rectangular prism. The second strategy was drawing a figure of the rectangular prism and observing what happens to the volume when the prism is cut off from its half. Examples from students' answers in the achievement test are presented in Figure 4.12 and Figure 4.13 below.

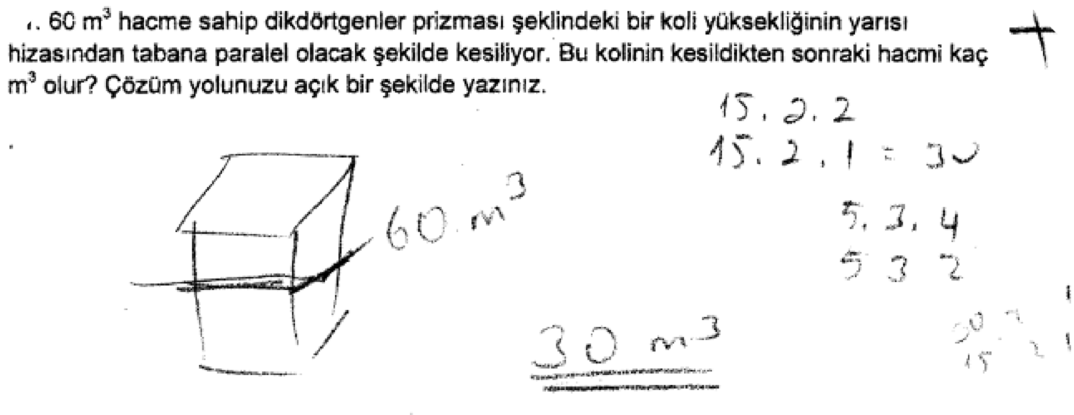


Figure 4.12 Giving arbitrary values to the dimensions of the rectangular prism for the tenth problem

As seen in Figure 4.12, the student gave arbitrary values to the dimensions of the rectangular prism in such a way that the volume would be equal to 60 m³. Then he divided the height by two and calculated the corresponding volume.

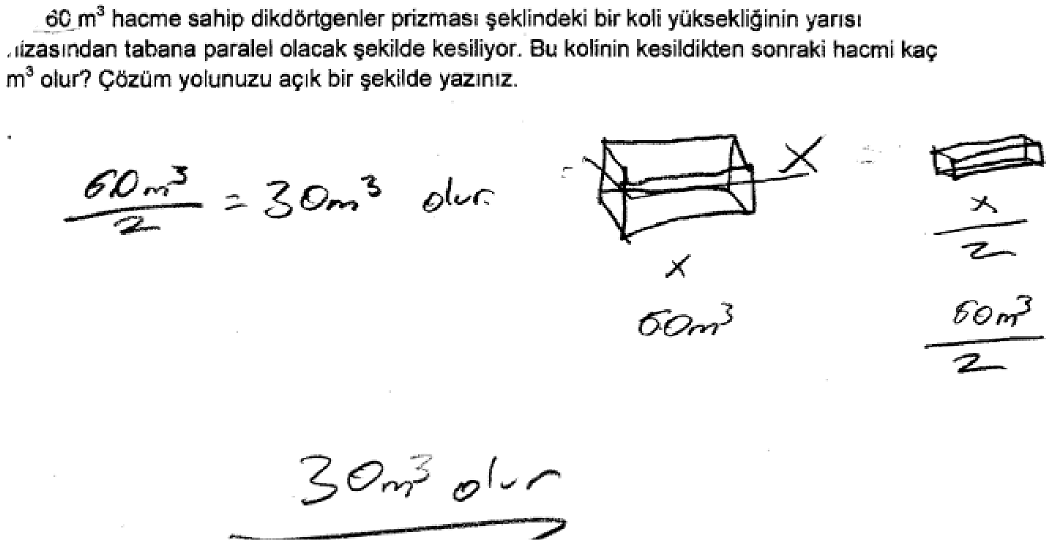


Figure 4.13 Drawing the figure of the rectangular prism for the tenth problem

As can be seen in Figure 4.13, the student drew a rectangular prism and cut it from its half and decreased the height to its half algebraically. Then, he determined that the volume would also be decreased to its half. Then, he divided the volume by two in order to find the corresponding volume.

4.2.2 Analysis of Correct Solution Strategies for Non-linear Problems

Students were asked four non-linear problems related to length, perimeter, area, and volume concepts in the achievement test. These non-linear problems required students to analyze two cases and apply non-linear solution strategies rather than using direct proportion. The analysis of students' answers in the achievement test supported by the analysis of the interviews revealed that the students used two main correct solution strategies for non-linear problems. These two strategies are named as length-length-area/volume relationships and length-area/volume relationships, and are explained in the following parts.

4.2.2.1 Length-Length-Area/Volume Relationships Strategy

The first correct strategy for the non-linear problems was to find the area or volume of the second figure by just using the relationships between the side lengths of the two figures. This strategy was named length-length-area/volume relationship since students moved from the side length of the first figure to the side length of the second figure and then to area or volume of the second figure. Similar to the above category, the fourth, sixth, and ninth problems which were categorized as non-linear area, and the fifth problem which was categorized as non-linear volume were solved by this strategy. Analysis revealed that 174 students (81.7%) among 213 students who correctly solved the fourth problem correctly employed this strategy. This number was constituted of 23 sixth graders (69.7%), 52 seventh graders (73.2%), and 99 eighth graders (90.8%) for the fourth problem. Besides, of 44 students who answered the sixth problem correctly 15 students (34.1%) (i.e. 1 sixth grader (10.0%), 2 seventh graders (40.0%), and 12 eighth graders (41.4%)) used this strategy. Besides, no student from any grade level used the mentioned strategy for the ninth problem. On the other hand, 34 students (40.5%) among 84 students who correctly answered the fifth problem implemented the mentioned strategy for fifth problem that was categorized as non-linear volume. This number included 3 sixth graders (37.5%), 10 seventh graders (50.0%), and 21 eighth graders (37.5%). The frequencies of length-length-area/volume relationships strategy for the four non-linear problems are provided in Table 4.5 below.

Table 4.5 Distribution of Length-Length-Area/Volume Relationships Strategy across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P4	23 (69.7%)	52 (73.2%)	99 (90.8)	174 (81.7%)
P5	3 (37.5%)	10 (50.0%)	21 (37.5%)	34 (40.5%)
P6	1 (10.0%)	2 (40.0%)	12 (41.4%)	15 (34.1%)
P9	0 (0.0%)	0 (0.0%)	0 (0.0%)	0 (0.0%)

Examples from students' answers in the fourth, fifth, and sixth problems are presented as illustration in Figure 4.14, Figure 4.15, and Figure 4.16 respectively.

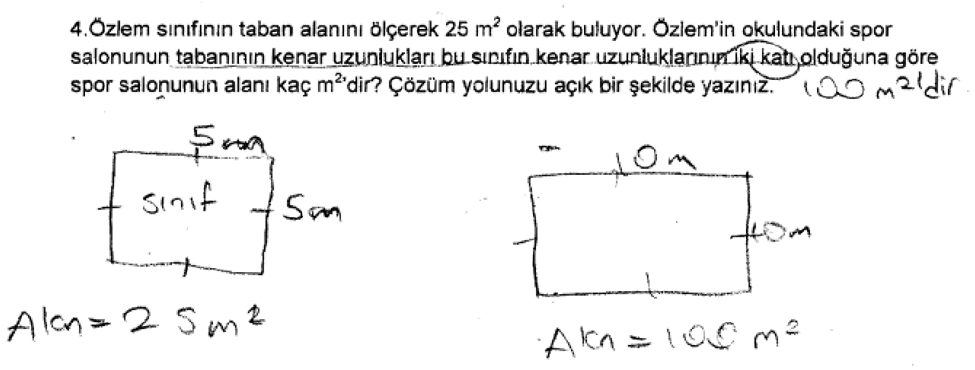
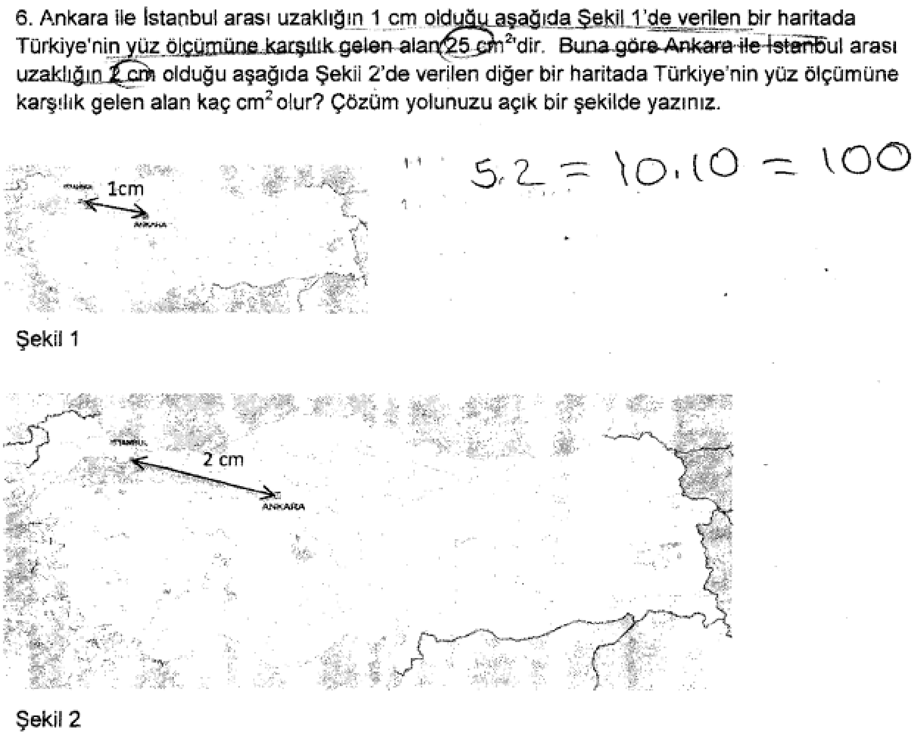


Figure 4.14 Length-length-area relationship strategy for the fourth problem

As seen in Figure 4.14, the student assumed that the classroom had a square shape and found the side lengths of the classroom as 5 m. Then he multiplied this length by two and got the side length of the second classroom as 10 m. Then, he found the correct answer for the area of the second classroom as 100 m². In other words, the student used the relationship between the side lengths of the two shapes and found the area of the second shape. Participant 2 (Grade 6) explained his strategy and what area meant to him in his own words as:

> "…We multiply the lengths to find the area. Area means its base or inside region. When I think of the class as a square, the side lengths of the square become 5. I thought the class as a square since thinking of it as a rectangle would be difficult. I multiplied 5 by 2 since it is stated that the side lengths become twice as much of the other. Then I multiplied the side lengths to find the area i.e. 10x10 and found 100." (Participant 2, Grade 6)
>
> […Alanı bulmak için kenarları çarparız… Alan cismin kapladığı taban...yüzey ya da iç kısmı. Sınıfı kare olarak düşününce bir kenar 5 m oluyor... Dikdörtgen olarak düşünmek zor olurdu diye kare olarak düşündüm. Kenar uzunlukları 2 kat dediği için 5'i 2 ile çarptım 10 oldu. Sonra alanı bulmak için kenarları çarptım yani 10 ile 10 u çarptım 100 oldu.]

Another student's answer in the sixth problem that was coded as length-length-area relationships strategy is presented in Figure 4.15 below.


6. Ankara ile İstanbul arası uzaklığın 1 cm olduğu aşağıda Şekil 1'de verilen bir haritada Türkiye'nin yüz ölçümüne karşılık gelen alan 25 cm^2'dir. Buna göre Ankara ile İstanbul arası uzaklığın 2 cm olduğu aşağıda Şekil 2'de verilen diğer bir haritada Türkiye'nin yüz ölçümüne karşılık gelen alan kaç cm^2 olur? Çözüm yolunuzu açık bir şekilde yazınız.

5.2 = 10.10 = 100

1cm

Şekil 1

2 cm

ANKARA

Şekil 2

Figure 4.15 Length-length-area relationship strategy for the sixth problem

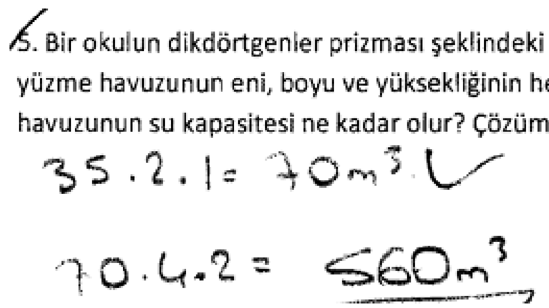
As seen in Figure 4.15, the student assumed that the maps had square shapes and found the side lengths of the map as 5 m. Then he multiplied the side length by 2 to get the side length of the second map. Then he found the correct answer for the area of the second map as 100 m^2. In other words, the student used the relationship between the side lengths of the two shapes to find the area of the second shape. Participant 11 (Grade 8) explained his thinking and solution strategy in the interview as follows:

> "…I tried to find the side lengths on the first map. Since the area is 25 the side length is 5. Then I multiplied this length by 2 in order to find the side length on the second map. Then I found the area of the second map as 100 by multiplying the two lengths. The distance between Ankara and İstanbul increased twice on the second map. Hence, all the lengths should increase twice." (Participant 11, Grade 8)

> […İlk haritadaki kenar uzunluklarını bulmaya çalıştım. Alan 25 olduğu için kenar 5 olur. İkinci haritadaki kenar uzunluğunu bulmak için bu uzunluğu 2

ile çarptım. Bu iki kenarı çarparak ikinci haritanın alanını 100 buldum. İkinci haritada Ankara ile İstanbul arası 2 katına çıkmış Bu yüzden tüm uzunluklar 2 katına çıkmalı.]

Another correct answer in the fifth problem that was coded as length-length-volume strategy is presented in Figure 4.16 below.


5. Bir okulun dikdörtgenler prizması şeklindeki yüzme havuzu 70 m^3 su kapasitesine sahiptir. Bu yüzme havuzunun eni, boyu ve yüksekliğinin her birinin 2 katı boyutlara sahip diğer bir yüzme havuzunun su kapasitesi ne kadar olur? Çözümünüzü açık bir biçimde ifade ediniz.

35 . 2 . 1 = 70 m^3

70 . 4 . 2 = 560 m^3

Figure 4.16 Length-length-volume relationship strategy for the fifth problem

As seen in Figure 4.16, the student assumed that the rectangular prism shaped pool had the dimensions as 35 m, 2 m, and 1 m. Then he multiplied all the lengths by two to get the dimensions of the second swimming pool as 70 m, 4 m, and 2 m. Then he found the correct answer for the volume of the second swimming pool as 560 m^3. In other words, the student used the relationship between the dimensions of the two shapes in order to find the volume of the second shape. In order to support this finding and ensure that the student used the strategy mentioned above, the explanations of Participant 10 (Grade 8) in the interview are stated as follows:

> "I gave values to dimensions as 35 m, 2 m, and 1 m so that the volume would be 70. Then I multiplied each of these lengths by 2 and found the new lengths. Then the volume became 560." (Participant 10, Grade 8)
>
> [Kenar uzunluklarına değer verdim uzunluk 35 m, 2 m ve yükseklik 1m olacak şekilde böylece hacim 70 oldu. Sonra bu uzunlukları tek tek 2 ile çarptım yeni uzunlukları buldum. Onun da hacmi 560 oldu.]

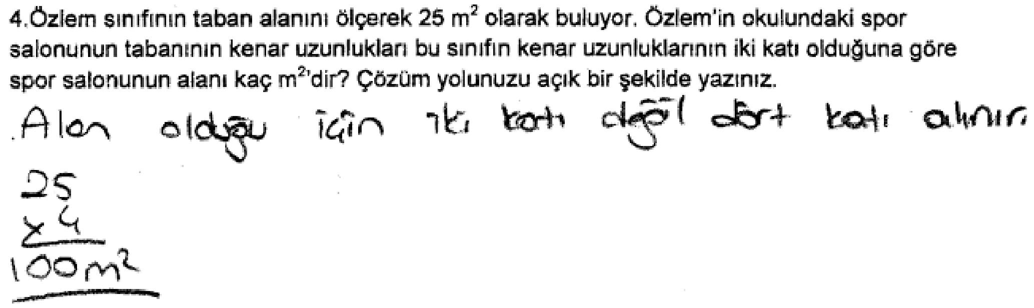
4.2.2.2 Length-Area/Volume Relationships Strategy

The second correct solution strategy for non-linear problems was to use direct non-linear relationships between the length and the area or between the length and the volume. This strategy was named as length-area/volume relationships since students used direct relationships between the length and the area or the length and the volume of geometrical figures. For instance, this category included students' answers implying that the area would be four times larger and the volume would be eight times larger when the side lengths of a figure gets twice as much. Analysis revealed that the students used this strategy for solving the fourth, sixth, and ninth problems which were categorized as non-linear area. Besides, the strategy was employed for the fifth problem which was categorized as non-linear volume. The analyses of the achievement test showed that few students used this strategy for solving the fourth, sixth, and ninth non-linear area problems. Specifically, 39 students (18.3%) among 213 students who answered the fourth problem correctly obtained the correct answer by this strategy. This number included 10 sixth graders (30.3%), 19 seventh graders (26.8%), and 10 eighth graders (9.2%). In addition, 29 students (65.9%) among 44 whose answers in the sixth problem were correct used the mentioned strategy. This number included 9 sixth graders (90.0%), 3 seventh graders (60.0%), and 17 eighth graders (58.6%). Besides, no student from any grade used the mentioned strategy for the ninth problem. Similarly, some students used this strategy for the fifth non-linear-volume problem. The number of students who used the mentioned strategy for the fifth problem was 50 (59.5%) among 84 students who answered the problem correctly. This number was constituted of 5 sixth graders (62.5%), 10 seventh graders (50.0%), and 35 eighth graders (62.5%). The frequencies of length-area/volume relationships strategy for the four non-linear problems are provided in Table 4.6 below.

Table 4.6 Distribution of Length- Area/Volume Relationships Strategy across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P4	10 (30.3%)	19 (26.8%)	10 (9.2%)	39 (18.3%)
P5	5 (62.5%)	10 (50.0%)	35 (62.5%)	50 (59.5%)
P6	9 (90.0%)	3 (60.0%)	17 (58.6%)	29 (65.9%)
P9	0 (0.0%)	0 (0.0%)	0 (0.0%)	0 (0.0%)

Examples from students' correct answers in the fourth, fifth, and sixth problems are presented as illustration in Figure 4.17, Figure 4.18, and Figure 4.19 respectively.


4.Özlem sınıfının taban alanını ölçerek 25 m² olarak buluyor. Özlem'in okulundaki spor salonunun tabanının kenar uzunlukları bu sınıfın kenar uzunluklarının iki katı olduğuna göre spor salonunun alanı kaç m²'dir? Çözüm yolunuzu açık bir şekilde yazınız.

Alan olduğu için iki katı değil dört katı alınır

25
x4
100m²

Figure 4.17 Length-area relationship strategy for the fourth problem

The given problem included two rectangular shapes; one of which had side lengths two times as much as the first one. The relationship between the areas of these shapes and their side lengths was asked to the students. The students were required to find the area of the second shape, which had sides two times as much as the first shape, by using the area of the first shape. The area of one shape was given while side lengths were not provided in the problem. As seen in Figure 4.17, the student figured out that the area of the second shape would be four times larger than the area of the first shape, and directly found the correct answer by multiplying the area of the first shape by four. When Participant 4 (Grade 6) was asked to further explain this solution and reasoning, it was ensured that the student multiplied the area of the first shape by four by understanding the relationship between the length and the area of the shape by the following statements:

> "…The side lengths of the sport hall are twice as much as those of the class. That is, all the sides are twice as much… The area would be four times bigger since it increases in two ways, the top and the side. That is, the side length gets two times larger and also the top gets two times larger. Therefore area becomes four times as much. So, four classrooms would fit into the sports hall." (Participant 4, Grade 6)
>
> […Spor salonunun kenar uzunlukları sınıfın 2 katıymış… Tüm kenarlar 2 katıymış… Alan 4 katına çıkar çünkü sadece bu kısmı artmıyor alt kısmı da üst kısmı da yandan da artıyor. O yüzden yan kenar 2 katına çıkıyor üst kenar da 2 katına çıkınca alan da 4 katına çıkmış oluyor. Böylece spor salonunda bu sınıftan 4 tane olmuş oluyor.]

Another participant's answer in the fifth problem which was coded as length-volume relationship is presented in Figure 4.18 below.

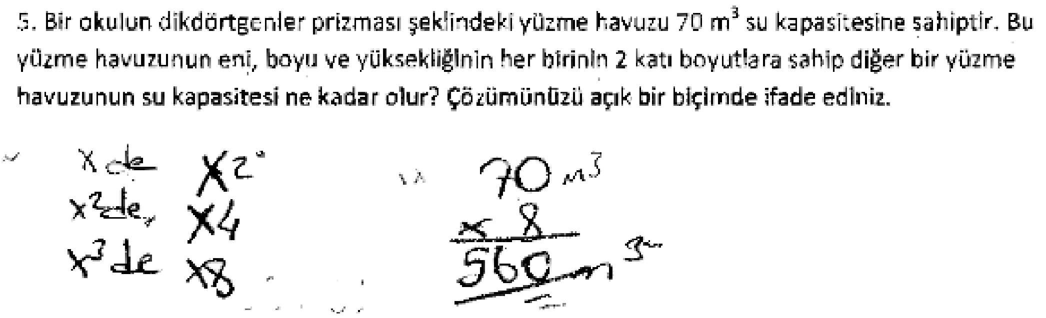


Figure 4.18 Length-volume relationship strategy for the fifth problem

The given problem was related to the volume of a swimming pool. The students were required to find the volume of another swimming pool whose dimensions were two times as much of this swimming pool. As seen in Figure 4.18, the student figured out that the volume of the second swimming pool would be eight times as much as the volume of the first swimming pool. He directly found the correct answer by multiplying the volume of the first swimming pool by eight. The answer of Participant 12 (Grade 8) was placed in length-volume relationship category since it was ensured that he considered the relationship between the length and the volume of the swimming pool as follows:

> "…The bottom becomes twice as much and the side becomes twice as much; hence the base becomes four times as much. In addition, the height becomes twice as much; therefore the volume gets eight times as much. It says the length, width, and height in the problem; hence there are 3 each 2. We have to multiply by 2 three times and it makes 8 times. I multiplied 70 by 8 and found 560. In fact, we have to take into consideration the depth since it is related to the volume and m³." (Participant 12, Grade 8)
>
> […Şurası 2 artıyor şurası 2 artıyor burası (taban) 4 katına çıkıyor. Bir de yüksekliği 2 katına çıktığı için 8 kat artıyor. Burada eni boyu ve yüksekliği dediği için 3 tane 2 var o yüzden 3 kere 2 ile çarpmamız lazım o yüzden 8 kat olur. 70 ile de 8 i çarparak 560 olarak buldum. Zaten hacim olduğu için m³ oluyor derinliği falan da düşünmemiz lazım]

Another participant's answer in the sixth problem which was coded as length-area relationship is provided in Figure 4.19 below.

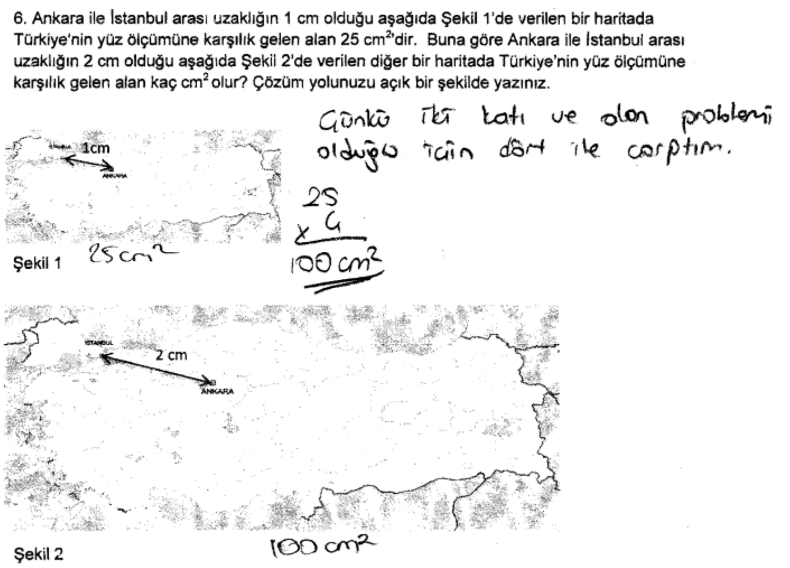


Figure 4.19 Length-Area relationship strategy for the sixth problem

The given problem was related to comparing the areas of the two maps based on the information implying that the distance between the two cities on the second map was two times as much as the distance between the same cities on the first map. The side lengths of the maps were not given; yet the distance between the two cities and the area of the country on the first map were provided. The distance between the same two cities was provided on the second map that had a different scale. The area of the country on the second map was asked to the students. Students were required to think about the fact that the real distance between the cities and the real area of the country would not change and to find the area of the second map by using area of the first map. As seen in Figure 4.19, Participant 8 (Grade 7) figured out that the area on the second map would be 4 times as much as the area of the first map and directly found the correct answer by multiplying the area of the first map by four. Interview data supported this finding as follows:

> "…The distances become twice as much since the scales of the maps are different. In other words, the scale is twice the size of the other. Hence all the lengths become twice as much. The side lengths of the maps became twice as much and hence the area will become four times as much." (Participant 8, Grade 7).
>
> [...Haritaların ölçekleri farklı olduğundan uzaklıklar 2 katına çıkmış. Yani ölçekleri birbirinin 2 katı. Tüm uzunluklar 2 katına çıkmış oluyor. Haritanın kenarları da 2 katına çıkar o yüzden alan da 4 katına çıkmış olur.]

Students' correct solution strategies for linear and non-linear problems were summarized in Table 4.7 below.

Table 4.7 Students' correct solution strategies for linear and non-linear problems

Linear Problems	Non-linear Problems
Questionable Proportion	Length-length-area/volume relationship
Reasonable Proportion	Length-area/volume relationship

4.3 Reasons for Incorrect Answers in Linear and Non-linear Problems

The third research question of the study is related to investigating the underlying reasons for students' incorrect answers in linear and non-linear problems regarding length, perimeter, area, and volume concepts. In order to answer this research question, each student's written solutions in the achievement test were analyzed, and the interview transcripts were explored in detail. These two sources of data were used so as to support each other. Analysis of the achievement test supported with the analysis of the interviews transcripts revealed that there were some common reasons and some reasons specific-to-linear and specific-to-non-linear problems underlying students' incorrect answers.

The common reasons for incorrect answers in linear and non-linear problems are operational mistakes, incomplete answers, misinterpretation of additive and multiplicative reasoning, and inadequacy in geometrical knowledge. The one and only reason specific to incorrect answers in linear problems is misinterpretation of proportional situations, and the one and only reason specific to incorrect answers in non-linear problems is illusion of linearity. These reasons with their frequencies and percentages compared to all students are presented in Table 4.8 below.

Table 4.8 Underlying reasons for students' incorrect answers in linear and non-linear problems

Underlying Reasons for Incorrect Answers in Linear Problems	Common Underlying Reasons for Incorrect Answers	Underlying Reasons for Incorrect Answers in Non-Linear Problems
1. Misinterpretation of Proportional Situations (at most 2.1%)	1. Inadequacy in Geometrical Knowledge (approximately between 3.7%% and 10.6%)	1. Illusion of Linearity (between 42.9% and 78.5%)
	2. Misinterpretation of Additive and Multiplicative Reasoning (between 4.0% and 4.5%5.0%)	
	3. Operational Mistakes (at most 1.9%)	
	4. Incomplete Answers (at most 2.5 %)	

As seen in Table 4.8, the percentage of misinterpretation of proportional situations is at most 2.1% among all students, the percentage of inadequacy in geometrical knowledge is between 3.7% and 10.6%, the percentage of misinterpretation of additive and multiplicative reasoning is at most 4.5%, and the percentage of operational mistakes is approximately at most 2.0%, the percentage of incomplete answers is at most 2.0%, and the percentage of illusion of linearity was approximately between 42.9% and 78.5% among all students.

The results of the analysis for the third research question are presented in three sections. In the first part, analysis of the reasons for students' incorrect answers that are common in linear and non-linear problems is stated. Later, analysis of the underlying reasons specific to incorrect answers in linear problems and non-linear problems is provided respectively.

4.3.1 Common Reasons for Incorrect Answers in Linear and Non-linear Problems

Content analysis of the achievement test supported by the analysis of the interview transcripts revealed that there were four underlying reasons for incorrect answers that are common in linear and non-linear problems. These common reasons were operational mistakes, incomplete answers, misinterpretation of additive and multiplicative reasoning, and inadequacy in geometrical knowledge. These reasons are explained in the following parts as parallel to their frequencies from the most frequent to the least frequent in detail.

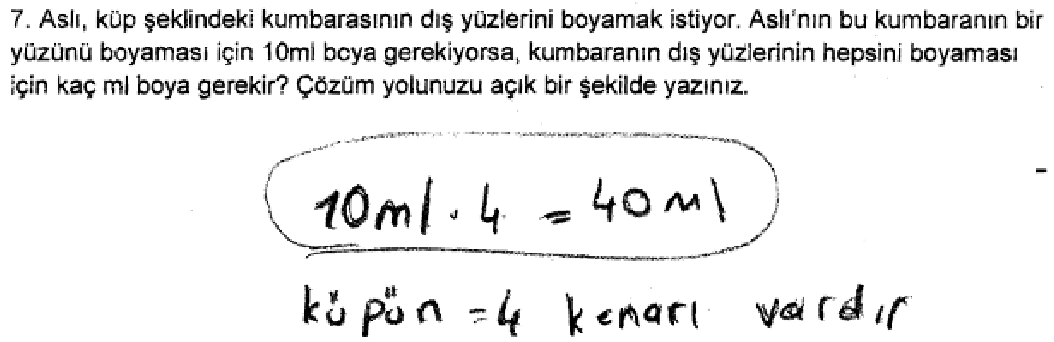
4.3.1.1 Inadequacy in Geometrical Knowledge

The first common reason for students' incorrect answers in both linear and non-linear problems was related to students' inadequate knowledge in geometry. This category was related to students' inadequate knowledge in properties of figures and their confusion of geometrical terms or concepts. It was observed that inadequacy in geometrical knowledge constituted a challenge mostly for the seventh linear and fourth non-linear problems. More specifically, the seventh problem was answered incorrectly due to students' inadequate knowledge in properties of figures, and the fourth problem was answered incorrectly due to confusion of geometrical terms area and perimeter. The number of students who answered the seventh problem incorrectly due to their inadequate knowledge in properties of figures was 99(64.3%) among 154 students who answered the problem incorrectly. This number was constituted of 43 sixth graders (63.2%), 37 seventh graders (77.1%), and 18 eighth graders (50.0%). Besides, 65 students (11.9%) among 545 answered the fourth item incorrectly due to confusion of the concepts of area and perimeter. The distribution of this number into the grade levels is as follows: 23 sixth graders (12.3%), 30 seventh graders (12.8%), and 12 eighth graders (9.7%). Since the percentage of the number of students whose answers were incorrect due to inadequacy in geometrical knowledge was calculated by dividing this number by the the number of students whose answers were correct for the problem, the total number of answers of sixth, seventh, and eighth graders are not equal to 100.0%. The frequencies of incorrect answers due to inadequacy in geometrical knowledge are provided in the Table 4.9 below.

Table 4.9 Distribution of Inadequacy in Geometrical Knowledge across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P4	23 (12.3%)	30 (12.8%)	12 (9.7%)	65 (11.9%)
P7	43 (63.2%)	37 (77.1%)	19(50.0%)	99(64.3%)

An example of incorrect answer in the seventh problem due to inadequacy in geometrical knowledge is provided in Figure 4.20 below.


7. Aslı, küp şeklindeki kumbarasının dış yüzlerini boyamak istiyor. Aslı'nın bu kumbaranın bir yüzünü boyaması için 10ml boya gerekiyorsa, kumbaranın dış yüzlerinin hepsini boyaması için kaç ml boya gerekir? Çözüm yolunuzu açık bir şekilde yazınız.

10ml · 4 = 40ml

küpün =4 kenarı vardır

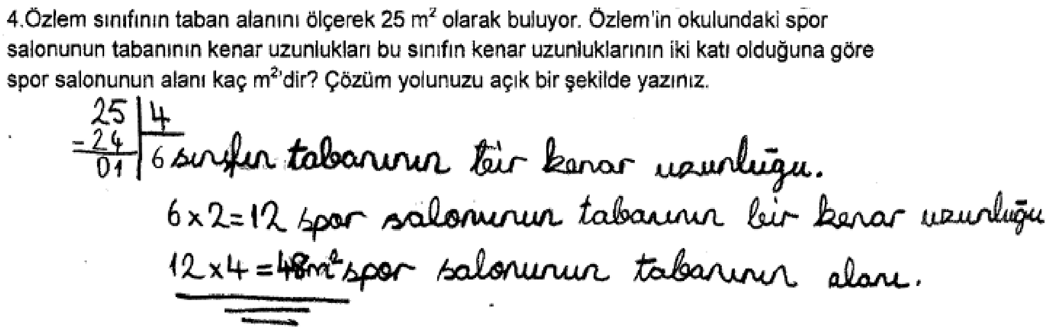
Figure 4.20 Inadequacy in geometrical knowledge (properties of figures) obtained from the solution of the seventh problem

The seventh problem was related to the amount of paint in order to paint all the faces of a moneybox when the amount of paint for one face was given. As can be seen in Figure 4.20, Participant 1 (Grade 6) thought that the number of faces of a cube equals to four. Therefore he multiplied the amount of paint for one face with four. This reflected his inadequate knowledge in properties of the cube. Furthermore, it was also seen in the interview transcripts that the student used the word “side of the cube” instead of “face of the cube”, and he confused the cube with a square. His sayings are stated as follows:

"…A cube has 4 sides and it is given in the problem that the paint for one side is 10 ml. I multiplied 10 by 4 to find the paint needed for all sides… Cube is similar to a square since it has 4 sides. All sides are equal to each other. If the moneybox were rectangular prism the answer would be the same, it is similar to a rectangle since it also has 4 sides." (Participant 1, Grade 6)

[…Küpün 4 kenarı vardır, problemde bir kenar için 10 ml boya gerekiyor diyor. 10 ile 4'ü çarparak tüm kenarlar için gereken boyayı 40 buldum. Küp işte kare gibi, 4 kenarı var o yüzden. Bu kenarlar da eşit birbirine. Kumbara dikdörtgenler prizması şeklinde olsaydı da cevap yine aynı olurdu, o da dikdörtgene benziyor 4 kenarı var.]

Another example of incorrect answers in the fourth problem due to confusion of terms is provided in Figure 4.21 below.


4.Özlem sınıfının taban alanını ölçerek 25 m^2 olarak buluyor. Özlem'in okulundaki spor salonunun tabanının kenar uzunlukları bu sınıfın kenar uzunluklarının iki katı olduğuna göre spor salonunun alanı kaç m^2'dir? Çözüm yolunuzu açık bir şekilde yazınız.

25 | 4
-24 | 6 sınıfın tabanının bir kenar uzunluğu.
01
6x2=12 spor salonunun tabanının bir kenar uzunluğu
12x4=48m² spor salonunun tabanının alanı.

Figure 4.21 Inadequacy in geometrical knowledge (confusion of terms) obtained from the solution of the fourth problem

As can be seen in Figure 4.21, the student found the side length of the classroom by dividing the area by four. Then, he multiplied the side length of the classroom by two in order to obtain the side lengths of the sport hall. Lastly, in order to find the area of the sport hall, he multiplied the length by four. In other words, the student applied the rules for calculating the perimeter rather than the area at two steps of his solution while calculating the side length of the first classroom and, also, while finding the area of the sport hall. Participant 3 (Grade 6) explained this solution strategy in the interview, too. Moreover, when he was asked questions about the related concepts in the problem in order to understand his knowledge about the area concept, it was seen that he did not understand the meaning of the concept of area. In addition, it was seen that he thought that the area could be calculated by multiplying the lengths by four. His words are stated below as:

> "I found the side of the classroom as 6 by dividing the area by 4. I multiplied 6 by 2 since it says the other classroom has lengths twice as much as this one. Then I multiplied 12 by 4 to find the area of the second classroom…I multiplied the length by 4 in order to find the area of the square. But the area is already given in this problem; hence I divided the area by 4 for finding the side…That is I did the reverse... Finally, I multiplied by 4 as usual. " (Participant 3, Grade 6).
>
> [Alanı 4'e bölerek sınıfın kenar uzunluğunu 6 buldum, 6'yı 2 ile çarptım çünkü ikincinin kenarı 2 katı diyor. 12 ile de 4 çarparak ikincinin alanını 48 buldum… Karenin alanını bulmak için kenarı 4 ile çarparım. Ama bize soruda alan verilmiş zaten o yüzden kenar için 4'e böldüm… Tersini yaptım yani… En sonda da bu sefer normal olarak 4 ile çarptım.]

4.3.1.2 Misinterpretation of Additive and Multiplicative Reasoning

The second common underlying reason for students' incorrect answers in linear and non-linear problems was misinterpretation of additive and multiplicative reasoning. This category was related to students' tendency to use additive reasoning for solving linear and non-linear problems where multiplicative reasoning was needed. Data analysis revealed that misuse of additive and multiplicative reasoning prevented students from answering the fifth and eighth problems correctly. The fifth problem was non-linear whereas the eighth problem was linear. The answers of 42 students (7.2%) among 583 students in the fifth problem were determined as incorrect since they used additive reasoning instead of a multiplicative one. This number was constituted of 11 sixth graders (5.9%), 26 seventh graders (10.6%), and 5 eighth graders (3.3%). Moreover, the eighth problem was answered incorrectly by 37 (22.8%) students among 162 students due to the misuse of additive reasoning instead of a multiplicative one. More specifically, 23 sixth graders (27.1%), 8 seventh graders (18.2%), and 6 eighth graders (18.2%) answered the eighth item incorrectly since they used additive reasoning inappropriately. These frequencies of incorrect answers due to misinterpretation of additive and multiplicative reasoning are provided in the Table 4.10 below.

Table 4.10 Distribution of Misinterpretation of Additive and Multiplicative Reasoning across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P5	11 (5.9%)	26 (10.6%)	5 (3.3%)	42 (7.2%)
P8	23 (27.1%)	8 (18.2 %)	6 (18.2%)	37 (22.8%)

An example of incorrect answer in the fifth problem due to misinterpretation of additive and multiplicative reasoning is given in Figure 4.22 below.

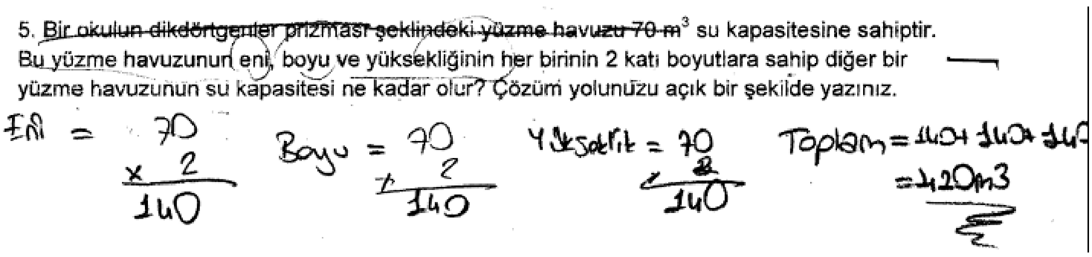


Figure 4.22 Misinterpretation of additive and multiplicative reasoning obtained from the solution of the fifth problem

As can be seen in Figure 4.22, the student thought the 70 m³ as the dimensions of the swimming pool and multiplied the dimensions by two for each dimension, and then added them up. Even though he misunderstood the problem, it was deduced that the thinking of the Participant 5 (Grade 7) was related to the additive reasoning which was supported by the interview transcripts:

> "Firstly, I found for one length by multiplying by 2. Since each dimension becomes twice as much we have to multiply the length 3 times by 2." (Participant 5, Grade 7)
>
> […İlk olarak, bir kenar için nasıl olduğunu buldum. Her boyut 2 katına çıktığı için uzunlukları 3 defa 2 ile çarpmalıyız.]

On the other hand, when the participant was asked to further explain and state the reasons for this solution strategy, he drew a diagram in which he started with an arbitrary length, multiplied it by two at the first step, and added two more shapes in the next two steps. For instance:

> "If the length is this (…draws the length and shows it) then it would be this (adds one more length) in the first step. Then I add two more lengths in the second step and also in the third step and get 6." (Participant 5)
>
> [Eğer uzunluk buysa (…uzunluğu çiziyor ve gösteriyor) ilk önce bu olur (aynı uzunluktan bir tane daha ekliyor). Daha sonra ikinci ve üçüncü adımda da 2'şer tane daha eklersek 6 tane olur.]

The drawing related to the explanation of the student is provided in Figure 4.23 below.

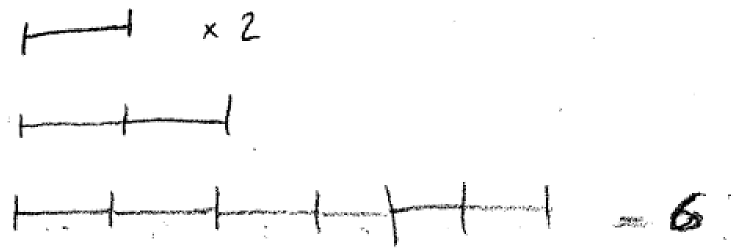
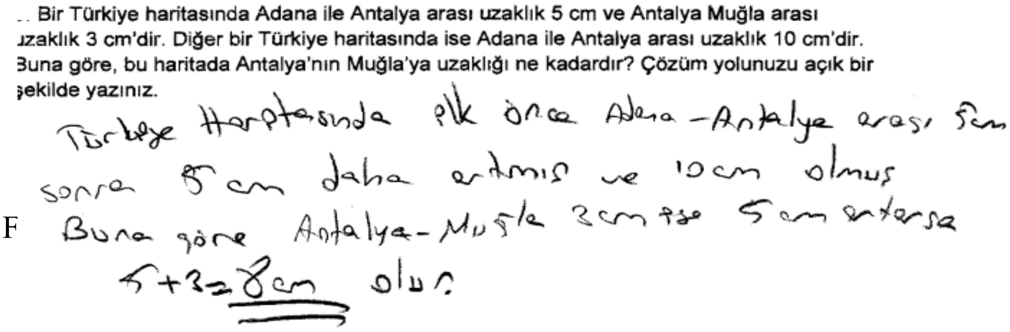


Figure 4.23 The drawing related to the explanation of the student in the fifth problem

Another example of incorrect answer in the eighth problem due to misinterpretation of additive and multiplicative reasoning is given in Figure 4.24 below.


.. Bir Türkiye haritasında Adana ile Antalya arası uzaklık 5 cm ve Antalya Muğla arası uzaklık 3 cm'dir. Diğer bir Türkiye haritasında ise Adana ile Antalya arası uzaklık 10 cm'dir. Buna göre, bu haritada Antalya'nın Muğla'ya uzaklığı ne kadardır? Çözüm yolunuzu açık bir şekilde yazınız.

Türkiye Haritasında ilk önce Adana-Antalya arası 5cm sonra 5 cm daha artmış ve 10cm olmuş Buna göre Antalya-Muğla 3cm ise 5cm artarsa 5+3=8cm olur

F

In Figure 4.24, it was seen that the student thought that the distance between Adana and Antalya on the first map was increased by 5 cm on the second map. Then, he decided that the distance between Antalya and Muğla on the first map had to be increased by the same amount on the second map. In other words, Participant 1 (Grade 6) used an additive reasoning where multiplicative reasoning was required. This finding also had a support from the interview transcripts:

> "… The distance between Ankara and Antalya became 10 cm on the second map whereas it was 5 cm on the first map. That means the distance became 5cm longer. Therefore the distance between Antalya and Muğla should be 8 cm by getting 5 cm longer as well……The distances get longer because the scale of the map grows. The answer is 8 since the lengths should increase by the same amount." (Participant 1, Grade 6)

> […İlk haritada Ankara Antalya arası uzaklık 5cm iken ikinci haritada 10 cm olmuş. Yani uzaklık 5 cm artmış. O zaman Antalya Muğla arası da 5 cm artarak 8 cm olmalıdır… Uzaklıklar artıyor çünkü haritanın ölçeği büyüyor. Aynı miktarda büyüme olmalı o yüzden 8 olur.]

4.3.1.3 Operational Mistakes

The third common underlying reason for students' incorrect answers in linear and non-linear problems was operational mistakes. This category was related to students' mistakes in four basic operations while solving the linear and non-linear problems. Students' incorrect answers due to only operational mistakes with correct reasoning were placed in this category. It was seen that operational mistakes constituted a barrier for students to answer the linear and non-linear problems correctly. These mistakes were more frequent in linear problems than in non-linear problems. Indeed, the third problem among the linear problems and the fifth problem among the non-linear problems were mostly incorrectly answered problems due to operational mistakes. Answers of 18 students (10.5%) among 172 whose answers were incorrect for the third problem were due to operational mistakes. This number included 6 sixth graders (7.1%), 7 seventh graders (12.7%), and 5 eighth graders (15.2%) among the ones who answered the problem incorrectly in terms of grade levels. On the other hand, 8 students (1.4%) among 583 who answered the fifth problem incorrectly made operational mistakes. This number was constituted of 1 sixth grader (0.5), 1 seventh grader (0.4%), and 6 eighth graders (3.9%). These frequencies of incorrect answers due to operational mistakes are provided in the Table 4.11 below.

Table 4.11 Distribution of Operational Mistakes across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P3	6 (7.1%)	7 (12.7%)	5 (15.2%)	18 (10.5%)
P5	1 (0.5%)	1 (0.4%)	6 (3.9%)	8 (1.4%)

An example of incorrect answer due to operational mistakes in the third linear problem is presented for illustration in Figure 4.25 below.

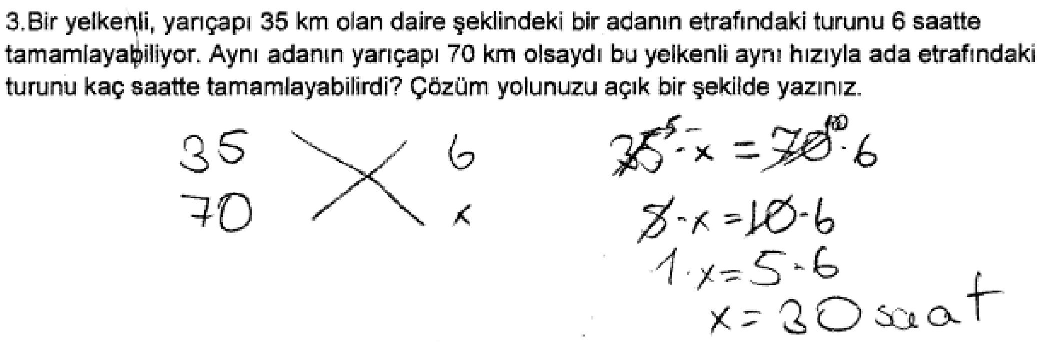


Figure 4.25 Operational mistakes obtained from the solution the third problem

As seen in Figure 4.25 the student wrote the proportion between the radii of the islands and the time periods needed to sail around those islands. Then he applied the properties of direct proportion but he made an operational mistake while simplifying 5 and 10 by dividing both by 5 at the end of the solution.

Another example of incorrect answer due to operational mistakes in the fifth non-linear problem is provided for illustration in Figure 4.26 below.

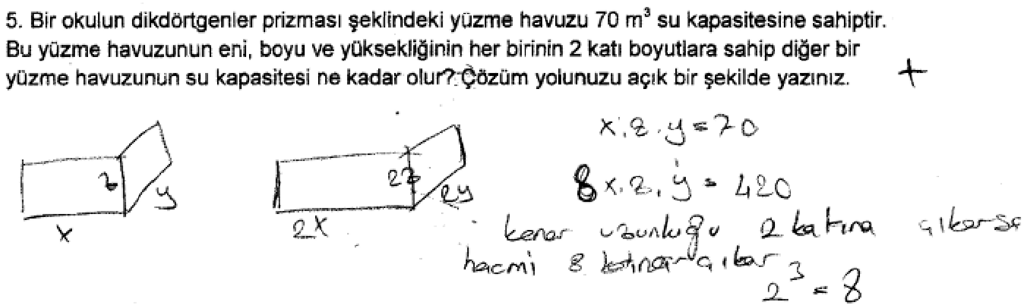


Figure 4.26 Operational mistakes obtained from the solution of the fifth problem

As can be seen in Figure 4.26, the student drew dimensions of a rectangular prism shape, found the lengths of the second swimming pool, and wrote that the volume increases by 8; yet he made an operational mistake while multiplying the volume of the first swimming pool by 8. He multiplied 70 by 8 and found the answer as 420 instead of 560.

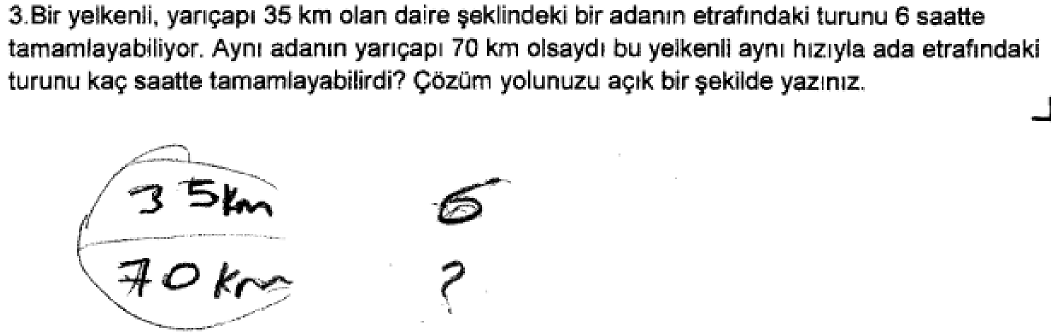
4.3.1.4 Incomplete Answers

The fourth and the last common underlying reason for students' incorrect answers in linear and non-linear problems is related to the incomplete answers of students. This category is related to students' incomplete answers with correct reasoning. In other words, the answers of students who attempted to solve the problem with a correct reasoning but left the problem unanswered were placed in this category. The analysis showed that 23 students (13.4%) among 172 students who answered the third problem incorrectly left the problem as incomplete. This number was constituted of 7 sixth graders (8.3%), 10 seventh graders (18.2%), and 6 eighth graders (18.2%). Moreover, the answers of 13 students (2.4%) among 583 who answered the fifth problem incorrectly were placed in the category of incomplete answers. This number was constituted of 1 sixth grader (0.5%), 5 seventh graders (2.1%), and 7 eighth graders (5.6%). These frequencies are summarized in Table 4.12 below.

Table 4.12 Distribution of Incomplete Answers across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P3	7 (8.3%)	10 (18.2%)	6 (18.2%)	23 (13.4%)
P4	1 (0.5%)	5 (2.1%)	7 (5.6%)	13 (2.4%)

An example of a student's answer in the third problem which was coded as incomplete answer is given below in Figure 4.27 below.


3.Bir yelkenli, yarıçapı 35 km olan daire şeklindeki bir adanın etrafındaki turunu 6 saatte tamamlayabiliyor. Aynı adanın yarıçapı 70 km olsaydı bu yelkenli aynı hızıyla ada etrafındaki turunu kaç saatte tamamlayabilirdi? Çözüm yolunuzu açık bir şekilde yazınız.

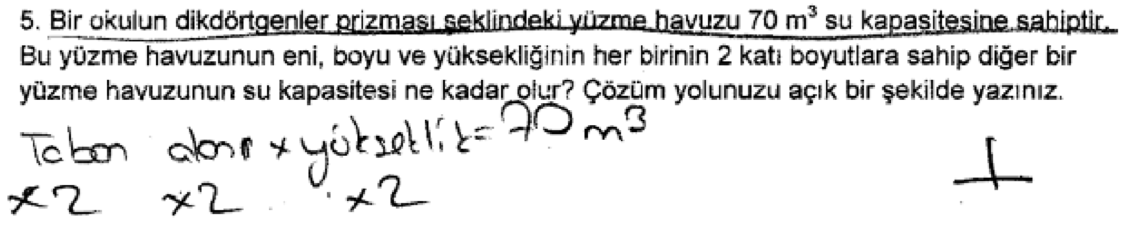
35km 6

70 km ?

Figure 4.27 Incomplete answer obtained from the solution of fifth problem

As seen in Figure 4.27 above, the student wrote a proportion between the radii of the circular islands and the time periods needed to sail around these islands. However, the student did not bring the solution to the end. He might have experienced challenges in applying the cross product algorithm or skipped the remaining part.

Another example of a student's answer in the fifth problem which was coded as incomplete answer is given below in Figure 4.28 below.


5. Bir okulun dikdörtgenler prizması şeklindeki yüzme havuzu 70 m^3 su kapasitesine sahiptir. Bu yüzme havuzunun eni, boyu ve yüksekliğinin her birinin 2 katı boyutlara sahip diğer bir yüzme havuzunun su kapasitesi ne kadar olur? Çözüm yolunuzu açık bir şekilde yazınız.

Taban alanı x yüksekliği = 70 m^3

x2 x2 x2

Figure 4.28 Incomplete answer obtained from the solution of fifth problem

As can be seen in Figure 4.28, the student wrote down the volume formula of the swimming pool as base times height. He thought that the volume is multiplied by 2 for each of the three dimensions. It is seen that the student used a correct reasoning but he did not complete his solution and find the correct answer.

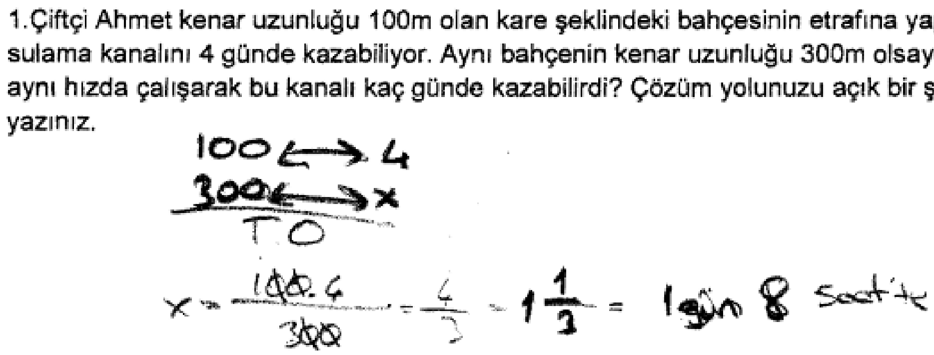
4.3.2 Reasons Specific to Incorrect Answers in Linear Problems

The analysis of the achievement test supported by the analysis of the interview transcripts revealed one and only one underlying reason that is only specific to incorrect answers in linear problems. The one and only reason specific to incorrect answers in linear problems was misinterpretation of proportional situations. This category included students' tendency to apply inverse proportion where direct proportion was needed based on the conventions in mathematics lessons. Data analysis showed that the third problem was the mostly incorrectly answered problem due to misinterpretation of proportional situations. The number of students who gave incorrect answers in the third problem due to misinterpretation of proportional situations was 20 (11.6%) out of 127 who answered the problem incorrectly. The distribution of this number to grade levels was as follows: 4 sixth graders (4.8%), 13 seventh graders (23.6%), and 3 eighth graders (9.1%). The frequencies are summarized in Table 4.13 below.

Table 4.13 Distribution of Misinterpretation of Proportional Situations across Grade Levels

Problem	Grade Levels			
	6	7	8	Total
P3	4 (4.8%)	13 (23.6%)	3 (9.1%)	20 (11.6%)

An example of incorrect answer in the first problem due to misinterpretation of proportional situations is presented in Figure 4.29 below.



1.Çiftçi Ahmet kenar uzunluğu 100m olan kare şeklindeki bahçesinin etrafına yapacağı sulama kanalını 4 günde kazabiliyor. Aynı bahçenin kenar uzunluğu 300m olsaydı Ahmet aynı hızda çalışarak bu kanalı kaç günde kazabilirdi? Çözüm yolunuzu açık bir şekilde yazınız.

100 ⟷ 4
300 ⟷ x
T.O

$x = \frac{100.4}{300} = \frac{4}{3} = 1\frac{1}{3}$ = 1 gün 8 saat'te

Figure 4.29 Misinterpretation of proportional situations obtained from the solution of the first problem

As can be seen in Figure 4.29, the student wrote a proportion between the side lengths of the pastures and the time periods needed to dig around these pastures. Then, he applied the properties of inverse proportion instead of the direct one. When Participant 1 (Grade 6) was asked about the reasons for his solution strategy in the interviews, it was understood that his understanding of proportion was only procedural. Besides, it was seen that he used inverse proportion since he was taught that work problems had to be solved by using inverse proportion. To illustrate:

> "It is written in the problem statement that Farmer Ahmet digs around the pasture with side length 100 m in 4 days. Hence in order to find in how many days he digs around the second pasture I used ratio and proportion. But I used an inverse proportion. Because, we use inverse proportion in work problems." (Participant 1, Grade 6)
>
> [Soruda Çiftçi Ahmet'in 100 m kenarı olan bahçenin etrafını 4 günde tamamlıyor diyor. Bu yüzden ikinci zamanı bulmak için oran orantı kullandım. Ama ters orantı kullandım. Çünkü işçi problemlerinde ters orantı kullanırız.]

4.3.3 Reasons Specific to Incorrect Answers in Non-linear Problems

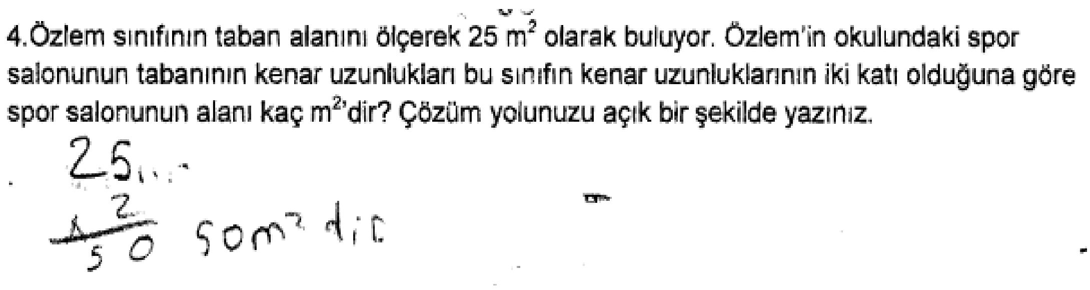
Analyses of data revealed one and only one underlying reason that was only specific to incorrect answers in non-linear problems. This reason was related to students' tendency to apply linear solution strategies where non-linear solution strategies were needed and named as illusion of linearity. The illusion of linearity was a major reason for incorrect answers in all of the four non-linear problems. The frequencies are summarized in Table 4.14 below.

Table 4.14 Distribution of Illusion of Linearity across Grade Levels

Problems	Grade Levels			
	6	7	8	Total
P4	129 (69.0%)	183 (78.2%)	89 (71.8%)	401 (73.6%)
P5	132 (71.0%)	172 (70.2%)	113 (74.3%)	417 (71.5%)
P6	157 (84.9%)	281 (95.6%)	187 (93.5%)	625 (92.0%)
P9	181 (76.4%)	319 (92.5%)	234 (91.4%)	734 (87.6%)

As seen in Table 4.14, 401 students (73.6%) among 545 answered the fourth problem incorrectly due to illusion of linearity. The distribution of this number into the grades was as follows: 129 sixth graders (69.0%), 183 seventh graders (78.2%), and 89 eighth graders (71.8%). Moreover, the answers of 417 students (71.5%) among 583 students in the fifth problem were found to be incorrect due illusion of linearity. This number was constituted of 132 sixth graders (71.0%), 172 seventh graders (70.2%), and 113 eighth graders (74.3 %). For the sixth problem, illusion of linearity constituted a challenge for 625 students (92.0%) among 679 students. The distribution into grade levels was as follows: 157 sixth graders (84.9%), 281 seventh graders (95.6%), and 187 eighth graders (93.5%). Lastly, the number of students who answered the ninth problem incorrect due to illusion of linearity was 734 (87.6%) among 838 students who answered the ninth problem incorrectly. This number was constituted of 181 sixth graders (76.4%), 319 seventh graders (92.5%), and 234 eighth graders (91.4%).

Examples of incorrect answers in fourth, fifth, sixth, and ninth problems due to illusion of linearity are presented in Figure 4.30, Figure 4.31, Figure 4.32, and Figure 4.33 respectively below.


4.Özlem sınıfının taban alanını ölçerek 25 m^2 olarak buluyor. Özlem'in okulundaki spor salonunun tabanının kenar uzunlukları bu sınıfın kenar uzunluklarının iki katı olduğuna göre spor salonunun alanı kaç m^2'dir? Çözüm yolunuzu açık bir şekilde yazınız.

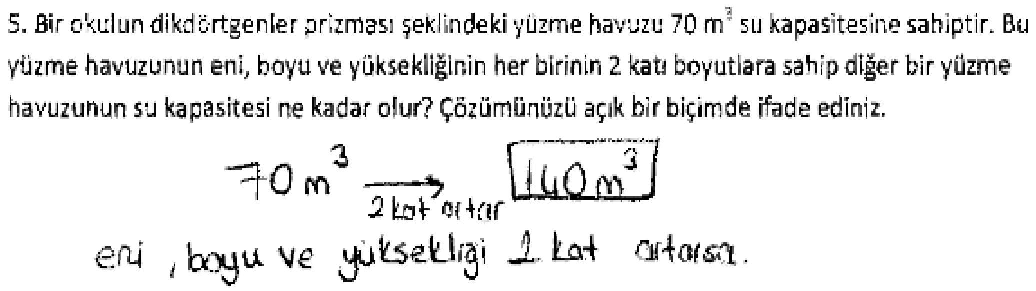
25
x 2
50 50m² dir

Figure 4.30 Illusion of linearity obtained from the solution of the fourth problem

As can be seen in Figure 4.30, the student thought that the area of the shape had to be multiplied by two since the side lengths were multiplied by two. In the interview, Participant 9 (Grade 8) explained that he took only the problem sentence into consideration. He said that he multiplied the area by two since the problem includes the word “two times”. To illustrate:

> “…The area of the classroom base is 25 m². There is a sport hall who had lengths 2 times of this classroom. The question asks how many m² the sport hall is. I multiplied the area by 2 since it says it is 2 times of the classroom. …The problem is related to the areas of the classroom and sport hall. I didn’t find the lengths since it says it is 2 times of the classroom. I don’t need to think anything else.” (Participant 9, Grade 8)
>
> […Sınıfın taban alanı 25 m² ymiş. Bu sınıfın kenar uzunluklarının 2 katı spor salonu varmış. Spor salonu kaç m² dir onu soruyor. 2 katı dediği için 2 ile çarptım… Soru sınıfın ve spor salonunun alanları ile ilgili. Kenar uzunluklarını bulmadım çünkü 2 katı diyor zaten başka bir şey düşünmeme gerek yok.]

Another example of incorrect answer due to illusion of linearity obtained from the solution of the fifth problem is presented in Figure 4.31 below.


5. Bir okulun dikdörtgenler prizması şeklindeki yüzme havuzu 70 m^3 su kapasitesine sahiptir. Bu yüzme havuzunun eni, boyu ve yüksekliğinin her birinin 2 katı boyutlara sahip diğer bir yüzme havuzunun su kapasitesi ne kadar olur? Çözümünüzü açık bir biçimde ifade ediniz.

70 m^3 → (2 kat artar) 140 m^3

eni, boyu ve yüksekliği 2 kat artarsa.

Figure 4.31 Illusion of linearity obtained from the solution of the fifth problem

As seen in Figure 4.31, the student thought that the volume of the swimming pool had to be increased at the same rate as the lengths, that is the volume had to be multiplied by two. When Participant 7 (Grade 7) was asked about the reasons for this solution, he implied that he did not focus on the volume concept but rather on the problem statement. He told that he had to multiply the volume by two since it is stated in the problem that the second swimming pool had lengths of the swimming pool were twice as much as the first one. He stated:

> "…The swimming pool has 70 m^3 water capacity. I multiplied this by 2 since it says 2 times of this swimming pool and I didn't consider anything else. The problem is related to m^3 of the swimming pool. Volume means the amount that an object can hold; like the water capacity in this problem. But I didn't consider the volume I just multiplied the given number by 2 since it says 2 times of this swimming pool." (Participant 7, Grade 7)
>
> […Havuz 70 m^3 lük su kapasitesine sahipmiş 2 katı dediği için 2 ile çarptım başka bir şeye dikkat etmedim. Soru havuzun m^3 ü ile alakalı yani hacmi ile alakalı. Hacim bir şeklin içine alabildiği miktar demek, bu sorudaki su kapasitesi gibi. Ama ben hacmi düşünmedim 2 kat dediği için verilen sayıyı direkt olarak 2 ile çarptım.]

Another example of an incorrect answer in the sixth problem due to illusion of linearity is given in Figure 4.32 below.

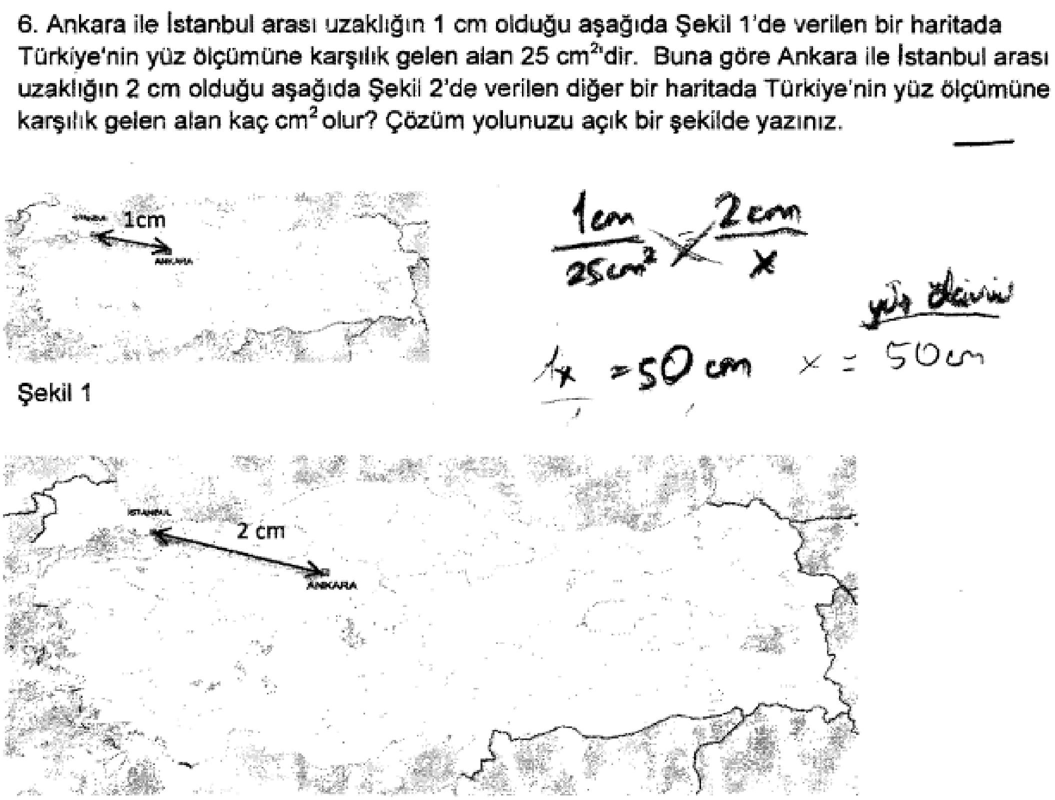


Figure 4.32 Illusion of linearity obtained from the solution of the sixth problem

As can be seen in Figure 4.32, the student thought that the area of the second map had to be multiplied by two since the lengths were multiplied by two. When the participant was asked to further explain and state his reasons for this solution strategy in the interview, he implied that the increase in the area should be the same as the increase in the lengths. Stated differently, Participant 3 (Grade 6) thought that the relationship between the areas of the two maps would be linear instead of quadratic. This idea found a place in the interview transcripts as follows:

> "…The lengths got 2 times longer here and became 2 cm. Both width and length increased… I wrote a proportion as if the area is 25 when the length is 1 then the area should be 50 when the length is 2 and found the answer as 50… I thought that the area would also increase by 2. Since the map increases by a certain ratio the area should increase by the same ratio. I used a direct proportion here." (Participant 3, Grade 6)

[…Burda şekiller büyümüş 2 katına çıkmış ve uzaklık 2 cm olmuş. En boy hepsi artmış… 2 cm 2 katı olduğu için 1 cm ken 25 ise 2 cm ken 2 ile çarptım 50 buldum. Orantı kurdum yine sonuçta 50 cm buldum… Alanı da 2 katına çıkar diye düşündüm Belli bir oranda büyütüyoruz haritalarda belli bir oranda büyüttüğümüz için o da o oranda büyüyecek. Doğru orantıdan yola çıkarak buldum.]

Illusion of linearity also constituted a barrier for students in answering the ninth problem. Most of the students thought that the amount of the paint needed to paint the second figure had to increase at the same rate as the height since the area would increase at the same rate. An example is given in Figure 4.33 below.

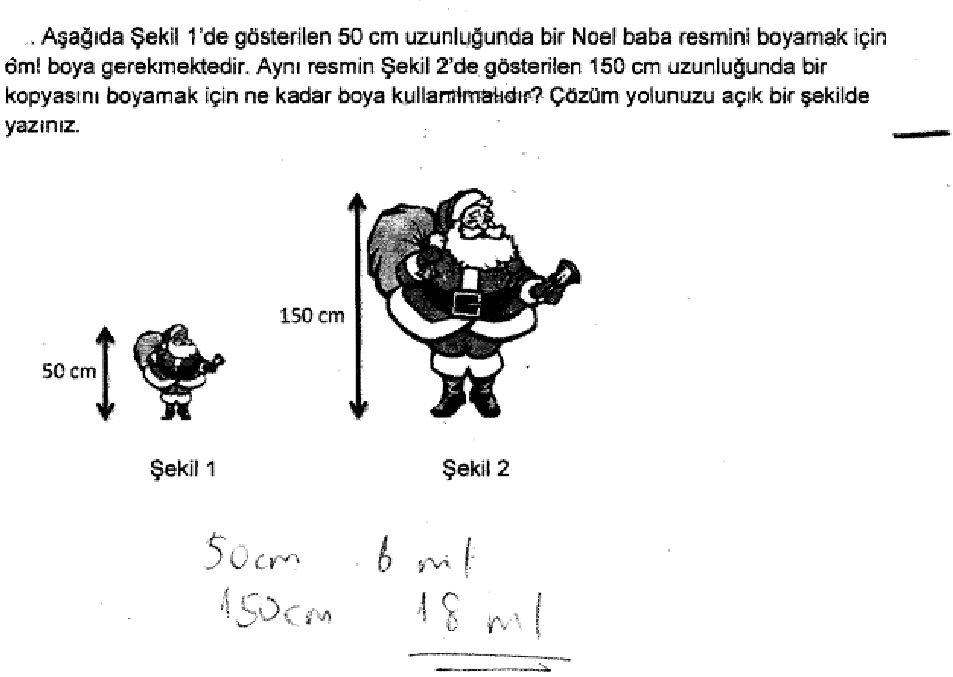


Figure 4.33 Illusion of linearity obtained from the solution of the ninth problem

Participant 9 (Grade 8) justified this solution strategy as stating that since the height became three times longer the area, also the amount of the paint, should become three times as much. This finding also had a support from the interview transcripts as:

"…I divided 150 by 50 in order to find the difference. It resulted in 3 times. That is, the second figure is three times bigger. I thought that if 6 ml is needed for 50 cm then 18 ml is used for 150 cm since when it becomes 3 times more ml should also increase 3 times as much. The length becomes 3 times much, hence paint should become 3 times as much. The area of the figures are painted. The area of the second figure is three times bigger. Hence, the amount of should also become 3 times as much. " (Participant 9, Grade 8)

[150 yi 50 ye böldüm arasındaki farkı bulmak için, 3 çıktı. Yani, 2. Şekil 3 kat daha büyük. 50 cm için 6 ml ise 150 cm için 18 ml boya kullanılır dedim çünkü 3 katına çıkıyorsa ml de 3 katına çıkar diye düşünmüştüm. Uzunluk 3 katına çıkıyor boya da 3 katına çıkar. Kat ilişkisi var. Uzunluğu 3 katına çıktığı için alan da 3 katına çıkar. Bu soru şeklin alanları ile ilgili alanlarını boyuyoruz. 3 kat daha büyük 2.'nin alanı. O yüzden boya miktarı da 3 kat artacak.]

4.4 Summary of the Findings

The aims of this study were three-fold. Determining the achievement level of the participants in linear and non-linear problems regarding length, perimeter, area, and volume concepts was the first aim of the study. Investigating the correct solution strategies of the participants for these problems was the second aim. The last aim was to explore the reasons behind participants' incorrect answers in these problems. The findings revealed that achievement levels of participants in all grades was higher in linear problems than that in non-linear problems with a little lower achievement of sixth graders compared to seventh and eighth graders. On the other hand, achievement level of the students in non-linear problems was lower than that in linear problems with a little higher achievement of eighth graders compared to sixth and seventh graders. As a matter of fact, students' achievement levels in each of the linear problems were higher than their achievement level in any of the non-linear problems regardless of their grade levels.

When the correct solution strategies of the participants were investigated in terms of linear problems, it was found that students implemented two main strategies for solving the linear problems. The first strategy was the questionable proportion which included using the numbers given in the problem and performing the operation directly by ignoring the relationship between the variables. It was seen that most of the students solved linear-perimeter problems by this strategy. The second strategy was reasonable proportion which was related to paying attention to the related variables by analyzing the problem sentence and using a direct proportion between the related variables. Results showed that few students employed this strategy for linear-perimeter problems.

The correct strategies for non-linear problems were also categorized into two groups. The first solution strategy for non-linear problems was the length-length-area/volume relationship which included finding the area or volume of the second figure by only using the relationships between the side lengths of the two figures. The second solution strategy for non-linear problems was the length-area/volume relationship which included establishing direct non-linear relationships between the length and area or between the side lengths and volume. It was noted that the first strategy did not include the direct relationships between the areas or volumes of the figures in contrast with the second strategy. Results indicated that students used length-length-area/volume relationships more frequently than the first strategy. In other words, they did not establish the direct relationships between the areas or volumes of the two figures; instead they needed to give arbitrary values to the side lengths and find the corresponding areas or volumes.

The correct solution strategies of the participants for both linear and non-linear problems are summarized in Table 4.15 below.

Table 4.15 Correct solution strategies for linear and non-linear problems

Linear Problems	Non-linear Problems
Questionable Proportion	Length-length-area/volume relationship
Reasonable Proportion	Length-area/volume relationship

The findings revealed that a few number of students could correctly answer the non-linear problems, and that they used a limited number of strategies similar to the linear problems. What is more, the strategies that most of the students came up with included linear relationships rather non-linear quadratic or cubic relationships between the length and area or length and volume. That is to say, the strategies of most participants lacked the argument of the linear relationships between the length and the perimeter or non-linear relationships between the length and area or length and volume for most of the participants' answers.

In line with the third aim of the study, the underlying reasons for students' incorrect answers in linear and non-linear problems were investigated. Some common reasons for both linear and non-linear problems, reasons specific to incorrect answers in linear problems, and lastly reasons specific to incorrect answers in non-linear problems were detected. Results revealed four common reasons which were inadequacy in geometrical knowledge, misinterpretation of additive and multiplicative reasoning, operational mistakes and incomplete answers. These four reasons followed an order of frequency respectively from the most frequent to the least frequent. Specifically, the one and only reason for incorrect answers specific to linear problems was misinterpretation of proportional situations which included applying inverse proportion where direct proportion was needed. Likewise, the one and only reason for incorrect answers specific to non-linear problems was illusion of linearity which included applying linear strategies where non-linear strategies were suitable.

There were many reasons discussed underlying the incorrect answers in non-linear problems. Mostly highlighted ones were misinterpretation of additive and multiplicative relationships and inadequate knowledge in geometry and measurement. Yet, the major finding of the study related to the reasons underlying incorrect answers in non-linear problems was due to illusion of linearity. The underlying reasons for students' incorrect answers in linear and non-linear problems are summarized in Table 4.16 below.

Table 4.16 Underlying reasons for students' incorrect answers in linear and non-linear problems

Underlying Reasons for Incorrect Answers in Linear Problems	Common Underlying Reasons for Incorrect Answers	Underlying Reasons for Incorrect Answers in Non-Linear Problems
1. Misinterpretation of Proportional Situations	1. Inadequacy in Geometrical Knowledge	1. Illusion of Linearity
	2. Misinterpretation of additive and multiplicative reasoning	
	3. Operational mistakes	
	4. Incomplete answers	

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The motivation for this study was to investigate students' achievement levels in linear and non-linear problems regarding length, perimeter, area, and volume concepts and to analyze their solution strategies for these problems. The study also aimed at revealing underlying reasons for students' incorrect answers in these problems. In Chapter I, the importance of proportional reasoning and the domain of geometry and measurement were stated. Besides, the need for analyzing students' solution strategies and reasons underlying the difficulties they experienced in a task on proportional reasoning in geometry and measurement was established. In Chapter II, definitions related to proportional reasoning in the geometry and measurement domain were provided. Furthermore, results of several studies related to students' solutions strategies and difficulties related to both proportional reasoning were mentioned. At this point, particular attention was paid to the studies that focused on "illusion of linearity" in the area of geometry and measurement. Next, Chapter III dwelled on the development of the achievement test in addition to research design and methodology. Both quantitative and qualitative findings of the study were presented in line with the research questions in Chapter IV. This final chapter will focus on the research questions in light of the quantitative and qualitative findings presented in Chapter IV. Furthermore, some implications for educational practices will be suggested and some recommendations will be given for future studies.

5.1 Discussion of the Findings

The purposes of this study were to determine sixth, seventh, and eighth grade students' achievement levels in linear and non-linear problems regarding length, perimeter, area, and volume concepts and to investigate their solution strategies for these problems. This study also aimed at analyzing underlying reasons for students' incorrect answers in these problems.

This chapter is organized in such a way that each section refers to the research questions in order. To be more specific, in the first section the achievement levels of the participants in linear and non-linear problems are discussed. Next, students' correct solution strategies for both linear and non-linear problems are discussed with an emphasis on their frequencies. Finally, underlying reasons behind students' incorrect answers in both linear and non-liner problems are discussed with reference to their frequencies. The findings are also compared and contrasted with previous research studies in the literature.

5.1.1 Discussion of Achievement Level

Quantitative analysis of the data revealed that the achievement level of students were higher for linear problems than those in non-linear problems regardless of their grade levels. In most of the linear problems, more than half of the students in all grade levels gave correct answers. This finding might be considered as consistent with previous research which reported students' higher achievement in linear problems (De Bock et al., 1998; Modestou & Gagatsis, 2009; Van Dooren et al., 2004; Vlahovic-Stetic et al., 2010). This higher achievement of students might be due to their experience and familiarity with linear problems beginning from the early years in mathematics lessons in school (De Bock et al., 1998) and also in other areas such as science (Vlahovic-Stetic et al., 2010). This might also be valid for the findings of the study since it is obvious that there is a strong emphasis on linear relationships when Turkish Mathematics and Science curricula are examined. For instance, proportionality problems and linear equations in mathematics or arithmetic problems regarding velocity and distance in science are related to the linearity concept.

Although the achievement level of all students in linear problems was found to be higher, there was a difference in terms of their grade levels in favor of seventh and eighth graders in all linear problems. Similar findings related to the increase in the achievement level in linear problems with higher grade levels were reported in previous studies (De Bock et al., 1998, 2002; Vlahovic-Stetic et al., 2010). Some explanations might be given for this finding. First, the finding might be due to the fact that students' knowledge and experience are accumulated throughout the grades; that is, they extend their knowledge throughout years. In other words, higher achievement in linear problems in upper grades might be resulting from students' cognitive development or maturity (Vlahovic-Stetic et al., 2010). Besides, the difference in seventh and eighth grade students' achievement level from that of sixth graders might be attributed to the differences in mathematics curriculum according to grade levels. It should be noted that students deal with proportional problems extensively in the seventh grade and afterwards, whereas less emphasis on proportions is given in the sixth grade mathematics curriculum (MoNE, 2008, 2013).

In addition to the grade levels, percentages of correct answers also showed variation with respect to the six linear problems. It was observed that problem 1 and 2 were the most correctly answered linear problems, whereas problem 8 and 10 were the least correctly answered ones by all grade students. This might be due to the fact that the first problem was related to the perimeter concept and the second problem was related to the length concept as Chapin and Johnson (2000) mentioned that length and perimeter concepts are easier than the area or volume concepts for students. It is anticipated that students experienced difficulty in problem 8 because although problem 8 was related to the length concept, the problem included scale drawings on a map and as Cox (2008) mentioned, scale drawings are difficult for students. Considering students' understanding of scale, the findings of the present study also revealed that even though the students used the term "scale" in solving the problems, it was seen that they used the term as a term specific to the maps without understanding its mathematical meaning, which is in line with the findings of Cox (2008). When the lower rate of correct answers in problem 10 is to be considered, it might be the case that students faced challenges in understanding the volume concept as Latt (2007) pointed out.

When the achievement level in non-linear problems is the point in question, it was found that achievement level of students in non-linear problems was very low, irrespective of their grade levels. A vast majority of the students gave incorrect answers in most of the non-linear problems. This finding is also consistent with the findings of previous research studies in which low achievement of students in non-linear problems were reported (De Bock et al., 1998; Modestou & Gagatsis, 2009; Van Dooren et al., 2004; Vlahovic-Stetic et al., 2010). This finding might have resulted from students' lack of familiarity with non-linear problems in school mathematics as De Bock and others (1998) argued. Considering the fact that the mathematics curricula and textbooks do not include adequate number of activities and problems regarding non-linear relationships this might be a reasonable explanation for students' low achievement level in non-linear problems in the present study. Moreover, since it was deduced that students considered all the problems in the test similar to each other and used the same strategies, such as cross multiplication, for most of the problems the explanations seems quite reasonable. That is to say, they might have assumed that all the problems in the test had similar structures.

When the achievement level of students in non-linear problems were taken into consideration in terms of grade levels, the findings revealed that there was an increase in the achievement level of students in most of the non-linear problems in favor of eighth graders. In other words, eight grade students' achievement level was higher than those of sixth and seventh grade students for most of the non-linear problems. This reason might be due to students' accumulation of knowledge throughout the grades and also their cognitive development as stated above for upper grade level students' higher achievement in linear problems. Hence, as a result of this cognitive development eighth grade students might have had a chance to develop different solution strategies and problem solving schemas as Vlahovic-Stetic and colleagues (2010) asserted. Unquestionably, another reason might be due to the fact that students deal with tasks and problems related to similarity concept in the eighth grade. Therefore, eighth graders might have experienced these kinds of problems while dealing with similarity tasks.

The percentage of correct answers in terms of the four non-linear problems proved to show variation. To be more specific, problem 4 was the most correctly answered problem by all grades, whereas no student in any grade correctly answered problem 9. The higher percentage of correct answers in the fourth problem might have resulted from the fact that the problem was related to the concept of area, and the concept of area was accepted to be easier than the concept of volume, which is similar to the arguments made for the linear problems. Another issue explaining the higher percentage of correct answers in the fourth problem than in any other non-linear problem might be due to using a square number as the area of the classroom. In this way, students might have had a chance to think of the classroom as a square and find the sides of the square and gave the correct answer easily. On the other hand, there might be some explanations underlying the finding that nobody was able to answer problem 9, which was related to the amount of paint needed to paint a picture whose sides were three times longer than those of the other picture. To begin with, the fact that the nature of problem 9 required an examination of the relationship between the height of an object and its area might have been challenging for students as Van Dooren and colleagues (2003) mentioned. In addition to the relationship between the height and the area, students had to examine another relationship between the amount of paint and the area of the figure. Hence, students might not have been able to consider these two relationships at the same time and carry out the necessary procedures. Furthermore, due to the fact that the figures did not have a regular shape like a square or a rectangle, students might not have been able to develop a strategy including the use of the relationships between the lengths. In order to solve this problem, the students had to know the quadratic relationships between the height of an object and the area of that object. In other words, length-length-area/volume relationship strategy was not applicable to the problem since the shape was irregular. Thus, students might not have been able to develop different strategies if they had not known this quadratic relationship.

To sum up, the findings indicated that students' achievement level in linear problems was higher than their achievement level in non-linear problems. In fact, the results revealed that students' achievement level in each of the linear problems was higher than their achievement in any of the non-linear problems regardless of their grade levels. Moreover, these findings are highly consistent with previous research studies, and several probable explanations are available for these findings.

5.1.2 Discussion of Correct Solution Strategies

Some strategies of students for solving linear and non-linear problems were detected by means of both quantitative and qualitative data analyses of the study. To begin with the linear problems, the findings revealed that students used two main correct strategies for linear problems. The first strategy was questionable proportion, which included the ambiguity of whether the students considered the relationship between the variables given and asked in the problem or they just used the numbers given in the problem and directly wrote the proportion between these numbers. The questionable strategy was also expressed in the study of Canada and colleagues (2008), in which they investigated pre-service teachers' strategies for solving proportions. The researchers included the strategies of the participants in the questionable category when their strategies involved some level of confusion, lack of clarity, or erroneous thinking, such as additive reasoning. Therefore, even though the definitions for the category questionable proportion in the study of Canada and colleagues (2008) and in the present study show some variance, they have a common point that these strategies involve some ambiguity. Therefore, the answers of some students in the present study lacked the clear indication of argument related to the linear relationships between the concepts, such as length and perimeter, which is similar to the findings of the study of Canada and others (2008). Some of these students whose answers lacked the argument related to the relationship between these concepts asserted that finding the perimeter was unnecessary since it was not asked in the problem statement. However, some of those students stated that they had had to take into account the perimeter, but they did not when answering the achievement test. Therefore, it might be deduced that these students had a tendency to apply the procedures for the given numbers in the problem just by looking at the problem sentences. It might be possible that students focused on the way that the problems were formulated; that is, the usual missing-value proportion problems. They might have immediately written the proportion algorithm without focusing on the related concepts such as perimeter since they thought that the problems were usual proportion problems. Another reason might be due to the fact that most of the students were not aware of the linear relationship between the length and the perimeter. Hence, they were not able to develop different strategies implying the relationship between the length and the perimeter.

The second correct strategy for linear problems in the present study is reasonable proportion strategy. This strategy included an analysis of the problem statement, finding the related variables (i.e. perimeter), judging the type of the relationship between the variables, and then writing the direct proportion between the related variables. Similarly, Canada and colleagues (2008) had the same category for pre-service teachers' strategies for solving proportions if their strategies involved considering a unit rate, demonstrating a between or within comparison or using multiplicative structures. Thus, even though the definitions for the category reasonable proportion in the present study and in Canada and others (2008) show some variance, they have a common point in that both categorizations include an understanding of the problem structure. It is likely that students in the present study were not aware of the linear relationship between the lengths and the perimeter. Nevertheless, they had a chance to calculate the perimeters of the two shapes and wrote a proportion between the perimeter and time periods since the lengths of the two figures were given in the problem.

Considering the frequencies of these two strategies, results revealed that a vast majority of the students in all grades used questionable strategies; that is, they wrote a direct proportion between the given numbers in the problem without analyzing the related concepts in the problem (i.e. perimeter or circumference), which is contrary to the finding of Canada and colleagues (2008) in which they found only one fifth of the participants using questionable strategies. The difference between the findings might have resulted from the fact that their participants were pre-service teachers. The higher frequency of the use of questionable strategies might be a result of students' belief that the variables that are not asked in the problem does not need to be considered as the analysis of the interview data revealed. Hence, they might have thought that they did not need to calculate the perimeter or circumference and carried out the operations with the given variables in the problems, such as side length or radius.

Analysis of the data also revealed that correct solution strategies for solving non-linear problems included two main strategies, which are length-length-area/volume relationships and length-area/volume relationships strategy. Students who used the length-length-area/volume relationships strategy applied only the linear relationships between the lengths of the figures and then found the area or the volume of the second figure by using the lengths they found as Ryan and Williams (2007) explained and illustrated. On the other hand, students who used length-area/volume relationships applied direct strategies for the relationship between the length and the area or volume of the figures. That is, they directly anticipated that the area gets r^2 times larger and the volume gets r^3 times larger when the lengths increase by r. It might be argued that the first strategy is related to linear relationships, whereas the second strategy requires an examination of non-linear relationships. When a problem related to the non-linear relationship between the lengths and the area of a square is correctly solved, the strategy mostly applied by the students was the first strategy, which is similar to the findings of the study of De Bock and colleagues (1998). This might be the case since using square shapes with an area of a square number allowed students to find the corresponding lengths more easily as stated before. Nevertheless, when irregular shapes like Santa are used in the problems, the students were not able to use length-length-area/volume strategy since it is not applicable to the problem. These problems could only be solved by the second strategy as De Bock and others (1998) claimed.

The findings revealed that the first strategy was the most frequently used one not only in problem 4 but also in other non-linear problems; that is, the frequency of using the second strategy was very low when compared to the first strategy. This might be deriving from the fact that the students were not able to establish the non-linear (quadratic or cubic) relationships among length, area, and volume. Instead, they needed to give arbitrary values to the sides or the dimensions of the figures in order to determine the effect of doubling or tripling the lengths of the figures on its area or volume. Therefore, it might be inferred that even though some students found the correct answers to the non-linear problems, most of them benefited from linear relationships and not from the quadratic or cubic ones.

5.1.3 Discussion of Reasons for Incorrect Answers

Analysis revealed four reasons that were common for students' incorrect answers in both linear and non-linear problems. Those were inadequacy in geometrical knowledge, misinterpretation of additive and multiplicative reasoning, operational mistakes and incomplete answers. Besides, one specific reason for each problem type was found: misinterpretation of proportional situations for linear problems and illusion of linearity for non-linear problems. These reasons are further explained and discussed in the following parts in order of most to least frequently observed.

To begin with, students' inadequate knowledge in areas of geometry and measurement seemed to be a problem that was hindering student achievement in both types of problems. Similarly, low achievement of students was documented in previous international and national studies (Clements & Ellerton, 1996; EARGED, 2003, 2005; Hart, 1987, 1993; Kouba, Brown, Carpenter, Lindquist, Silver, & Swafford, 1988; Orhan, 2013; Sherman & Randolph, 2004; Steele, 2006; Tan-Şişman, 2010; Thompson & Preston, 2004). In line with the results of these studies, the findings of the present study highlighted students' lack of geometrical knowledge regarding the properties of figures and also the concepts of length, perimeter, area, and volume. For instance, some students thought that a cube is similar to (or the same as) a square and that a cube has four sides. What's more, some students used the term "side" for the dimensions or faces of 3-D shapes. This inadequate knowledge in properties of figures constituted a handicap mostly in problem 7. The reason for this finding might be related to the fact that the teaching of the geometric shapes may not be laying emphasis on the similarities and differences among the shapes and that students may not be given the opportunity to gain practice in examining relationships among shapes.

Another issue related to inadequacy is the confusion of perimeter, area, and volume concepts. The findings of the present study revealed that students confuse the area of a shape with the perimeter of that shape or vice versa, which is a similar finding to the results of several studies (NAEP, 2007; Orhan, 2013; Ryan & Williams, 2007; Sherman & Randolph, 2004; Tan-Şişman, 2010). Findings of previous research studies have a consensus that the reasons for students' confusion might be due to their lack of conceptual knowledge (Kamii & Clark, 1997; Martin & Strutchens, 2000) and their reliance on formulas (Chappel & Thompson, 1999; Orhan, 2013). Similar to the findings of these studies, the results of the present study might be interpreted as indicating that students do not grasp the concepts of length, perimeter, area, and volume concepts especially while solving problems. They just apply the general formulas for these concepts without analyzing the conceptual structure of the problems and without giving much thought to deciding on which concept is applicable to the problem. This situation seems to have hindered students' achievement in both linear and non-linear problems. Still, the frequency of incorrect answers was found to be approximately between 4.0% and 10.0% among all students.

As a second common reason underlying incorrect answers in linear and non-linear problems, geometry and measurement areas are considered as the most vulnerable areas to erroneous additive strategies, as Kaput and West (1994) argued. Furthermore, the findings of previous research studies revealed that students use additive strategies where multiplicative reasoning is required (Harel et al., 1994; Hart, 1984; Noelting, 1980; Van Dooren et al., 2010). In line with the results of these studies, misinterpretation of additive and multiplicative reasoning was considered as a reason for students' incorrect answers in both linear and non-linear problems in the present study. An example for a linear problem is that some students thought that the distances on a map should increase by the same amount instead of the same ratio when the scale of the map is increased. A good example for a non-linear problem is that some students multiplied all three dimensions by 2 and then added them up and hence concluded that when the dimensions increased by 2, the volume also increased 6 times, as given in question 5.

Despite the major emphasis related to misinterpretation of additive and multiplicative reasoning given in the literature, the frequency of this reason in the present study was found to be as low as close to 5.0%. This might be due to the fact that additive strategies also worked for the two of the four non-linear problems, which asked for the effect of increasing the sides of a figure by 2 on the area of that figure. That is to say, if students thought that the area would increase by 4 since 2+2= 4 (additive) instead of 22 = 4 (quadratic), they could answer the problem correctly. Moreover, the same situation was also applicable to the linear problems, except for problem 8, since in most of the problems, the lengths get two times as much of the original ones and, additive, and multiplicative reasoning gave the same correct result.

Data analysis also showed that some students made operational mistakes in their calculations although their reasoning was correct. As Ryan and Williams (2007) pointed out, these mistakes do not stem from students' conceptual development but simple errors. This error was mostly observed in the linear problems in which the students used the cross-product algorithm, but the frequency of occurrence of this error was found to be as low as around 2.0%. This might be due to the fact that the numbers in the problems were chosen as numbers that were easy to work with.

It was observed in the data analysis that some students began to solve the problem with correct reasoning but could not pursue the correct reasoning till the end to find the correct answer. It is possible that students might have experienced difficulty at some point while solving the problem and left the problem incomplete, or they may have just passed the question without giving further consideration to the problem. The reason was incomplete answers and constituted the second common reason underlying incorrect answers in both linear and non-linear problems. In fact, the frequency of this reason was found to be as low as operational mistakes.

In addition to common reasons, one specific reason for incorrect answers in linear problems is stated as misinterpretation of proportional situations. Literature review revealed that learning the concept of proportion is highly challenging for students (Piaget & Inhelder, 1975; Resnick & Singer, 1993; Weinberg, 2002). In line with the findings of previous research, some students experienced difficulties in proportions in such a way that they applied inverse proportion where direct proportion was needed. However, the frequency of this error was found to be as low as approximately 5.0% despite the great emphasis on the difficulties of proportional reasoning. That is to say, the students could deal with the linear problems that required proportional situations, and few students experienced challenges. This finding might have resulted from the fact that the linear problems in the current study were conventional proportion problems that they encounter in school. In other words, students might have recognized the problem structures as usual problems that they deal with in school lessons. Hence, they might have applied previously-known strategies, such the cross multiplication algorithm. Considering the analysis of the interview data it might be inferred that students only focus on the problem sentences and the numbers in the problem. Similarly, Van Dooren and colleagues (2003) stated that students automatically apply proportional solutions when the problems are asked in a conventional missing-value proportion format. Therefore, while interpreting the results of the current study it is important to note that the higher achievement level in linear problems should not be understood as if students' understanding of proportion is conceptual. Similar to the findings of the current study, students' proficiency in procedural knowledge but failure in conceptual knowledge in proportional reasoning is a well-known phenomenon that was emphasized in previous studies such as Modestou and Gagatsis (2009).

Based on the findings of the present study, students' failure in conceptual knowledge of proportional reasoning might also be inferred to be related to the tendency of students to use proportional strategies where non-proportional strategies were required. Since Cramer and Post (1993) included the ability to discern proportional and non-proportional situations as one of the characteristics of a proportional thinker, the inability to differentiate between the two situations might be an indicator of failure in conceptual knowledge of proportional reasoning. Moreover, areas of geometry and measurement are stated as one of the most vulnerable areas to this inability to discern proportional and non-proportional situations from each other. This situation is referred to as illusion of linearity in the current study, and it constituted one single reason specific to incorrect answers in non-linear problems. The findings of the previous research studies revealed that students could not differentiate between linear and non-linear problems related to length, perimeter, area, and volume concepts and used linear strategies where non-linear strategies were required (De Bock et al., 1998; 2003; Modestou et al., 2007; Van Dooren et al., 2007; Vlahovic-Štetic et al., 2010, 2011). In line with the results of these studies, the findings of the present study also confirmed that students overused linear strategies for non-linear problems regarding length, perimeter, area, and volume concepts. That is to say, most of the students were not aware of the fact that increasing the lengths by r results in the increase in perimeter as r, area as r^2, and volume as r^3. Instead, most of them thought that the relationship among all these concepts were linear. To be more specific, most of the students thought that if lengths of a figure are doubled all of the perimeter, area, and volume are also doubled. Furthermore, illusion of linearity was a serious handicap hindering students' achievement in non-linear problems, and the frequency of illusion of linearity was very high with a frequency of approximately between 45.0% and 80.0% among all students according to the grade levels and problems. Hence, it might be deduced that it is the strongest reason among all reasons for incorrect answers in non-linear problems.

Several reasons for the illusion of linearity were mentioned in the literature. Firstly, the reasons related to the problem formulation and the test itself is to be discussed. To begin with, Van Dooren and colleagues (2003) argued that formulation of the problems as a missing-value format paves the way for automatically using a proportional or linear solution method. This might be due to the fact since students deal with a limited number of problem types (i.e. stereotypical linear problems) in schools, they might recognize the problem type and use the standard algorithm for solving that problem without deeply reasoning about the problems. This might be an issue in the present study also since most of the students used the cross product algorithm, which is one of the most preferred algorithms for solving proportional problems. Moreover, the fact that students implied in the interview that they decided on how to solve the problem by just looking at the problem sentence might be an indicator of the previously mentioned issue of problem formulation.

The second reason reported was related to students' educational background rather than the test itself. De Bock and colleagues (2002) argued that the linear relationships are applicable to many problems, especially the ones covered in middle school. Parallel to this usefulness, great emphasis is given to linear relationships both in curricula and lessons. However, Van Dooren and others (2003) stated that this great emphasis on linear strategies beginning from the early years of schooling prevents students from developing meaningful strategies to solve non-linear problems. In other words, since most of the problems in middle school are solvable by linear strategies, teachers might focus only on these strategies such as cross product algorithm. Consequently, this situation does not provide students with the opportunity to deal with and comprehend non-linear relationships. This reason might also be applicable for the findings of the present study when the important emphasis of proportions and no emphasis of non-proportional relationships in the Turkish Mathematics Curricula in any grade level are to be taken into consideration. Furthermore, considering the fact that there is no place for non-linearity in the area of either geometry or measurement, this reason seems to be very likely in the present study.

Last but not least, the simplicity and intuitiveness of linear strategies are seen as one of the sources of illusion of linearity (De Bock et al., 2002; Van Dooren et al., 2005). As argued by De Bock et al. (1998, 2002) students do not judge whether a direct proportion is applicable or not and do not hesitate to use them at any place due to its simplicity and wide applicability. Similar to these findings, it might be inferred that students in the present study used linear strategies where non-linear strategies were needed without further judgment. They automatically and intuitively used linear strategies inappropriately, and most of them had no idea about whether their strategies were correct or not as the analysis of the interview data revealed.

All in all, it was seen that the results of this study not only confirmed the findings of previous studies but also moved the discussions one step ahead. The present study not only examined students' achievement level in linear and non-linear problems but also dealt with how students solved these problems correctly and why they might not have solved them incorrectly. Therefore, this study took a deeper look at students' development of proportional reasoning, and hence obtained the elements of the bigger picture. Therefore, it would be very essential to provide some implications for educational practices and recommendations for further studies based on the findings of the present study and those of previous studies. Thus, the following two sections try to shed light into the practical and research-based issues in line with the findings of the present study together with the findings of previous studies.

5.2 Implications for Educational Practices

The findings of this study provides primary teachers, mathematics teachers, curriculum developers, and teacher educators with essential information related to students' achievement level in linear and non-linear problems regarding length, perimeter, area, and volume concepts, their solution strategies for these problems and the reasons underlying their incorrect answers in these problems. Therefore, some implications for these stakeholders are provided in this section.

First, the findings of the study revealed that students' levels of achievement in linear problems were higher than those in non-linear problems. However, despite this higher achievement in linear problems it was observed that students used a limited number of solution strategies, most of which were questionable strategies that focused on traditional algorithms. It was discussed that higher achievement of students in linear problems might be resulting from the fact that students deal with these types of routine problems in their mathematics lessons. Hence, it is highly probable that students recognize the structure of the problem and apply a familiar solution strategy. However, De Bock and colleagues (2007) claimed that high achievement in linear problems does not guarantee that students might be able to deal with linear problems asked in a more authentic and complex structure. Hence, enabling students with more authentic and enthusiastic linear problems in a variety of applications and contexts should be promoted in order to develop their conceptual understanding of proportional reasoning. Students should be aware of the linear relationships underlying the cross multiplication rule. This might be possible if the curriculum developers give a place to these types of problems in the mathematics curriculum beginning from the early grades of primary school. Yet, the teachers should also provide students with the opportunities to deal with these kinds of problems.

Findings of the present study also revealed that students' achievement level in non-linear problems were very low. In fact, the achievement level of students in non-linear problems was found to be so low that the findings of the study seem to be causing an alarm for further consideration. It was discussed in the previous section that students' failure in non-linear problems due to illusion of linearity might have stemmed from the novelty of the problems to the students. Although students might have encountered linear problems in their mathematics lessons beginning from the early grades, they might not have experienced non-linear problems prior to the present study. When the objectives of the middle school mathematics curriculum were analyzed, an explicit objective regarding the discrimination between linear and non-linear situations was encountered neither in the objectives related to ratio and proportion nor in geometry and measurement. Therefore, first of all, mathematics curriculum developers should give a place to not only linear investigations but also non-linear ones in order to enhance the conceptual understanding of linear and non-linear relationships and the discrimination between these relationships. It does not mean that these issues should be covered as separate objectives. On the contrary, these issues could be included in the objectives related to ratio and proportion concepts by modifying these objectives in order to highlight this discrimination.

In addition to the proportion objectives, the investigation of the linear and non-linear relationships among the concepts of length, perimeter, area, and volume might be integrated in the related objectives in the area of geometry and measurement area. In this way the teachers can identify the similarities and differences between these two types of relationships as linear and non-linear.

In addition to the integration of these issues into the curriculum, teachers should take most of the responsibility to improve students' understanding and achievement in linear and non-linear relationships. They should break the habit of putting most of the emphasis on the procedural skills rather than the conceptual ones and focusing on a single approach while solving linear problems.

There are various strategies by which teachers can help their students in understanding linear and non-linear situations and in discriminating one from the other. The first thing that might be done is that primary teachers and mathematics teachers could allow students to deal with more authentic linear problems in order to improve students' understanding of linear situations beginning from primary education. Furthermore, they could put an emphasis on the essence and the structure of non-linear problems. Another essential practice is that teachers could provide students with the opportunities to help them understand the differences between linear and non-linear situations not only in geometry and measurement but also in other areas.

All the remedial exercises mentioned above might be possible with the integration of modelling activities and investigations with technological tools into the mathematics curriculum and classroom discussions. In this way, students could experience more authentic and complex linear problems with the help of modelling activities that would be more meaningful, attractive, and challenging for students. Besides, they could have a chance to investigate the structure of the non-linear situations and to discriminate the properties of linear and non-linear situations with the help of technological devices. For the case of linear and non-linear relationships among length, perimeter, area, and volume Geogebra or Sketchpad tools might be helpful for providing students with various situations as enlarging and reducing geometrical figures. So, students might experience what happens to the area or volume when the lengths or dimensions are increased or decreased.

It could be possible to arrange classroom environment where students can find the opportunity to discuss with their peers and teachers about the fact that not every relationship should be linear. For instance, considering the case of geometry as in this study, students could investigate the effects of doubling or tripling the sides of a figure on its area and volume. What's more, they could investigate a number of different situations and reach the general rule that increasing the length by r results in an increase in the perimeter by r, area by r^2, and volume by r^3.

One possible explanation for the high frequency of illusion of linearity might be deriving from teachers' lack of knowledge and affects that they exhibit during the instructional period. Because of the fact that former elementary and high school curricula did not focus on the nature and structure of the non-linear problems, especially in the geometry and measurement domain, both primary and middle school mathematics teachers might not have adequate knowledge and experience in order to teach these topics. Therefore, in-service teacher trainings focusing on the linear and non-linear relationships especially in the geometry and measurement domain with their similarities and differences might be conducted. These trainings could also improve teachers' affect and insight related to the fact that every relationship does not need to be linear; hence, teachers could develop a deep understanding of whether a linear situation exists or not. In this way, their habits of putting emphasis on the procedural skills and sticking to a single solution approach might be broken down.

The implications related to improving teachers' knowledge and affect might also be extended to pre-service teacher training. Pre-service teachers could also experience some modelling activities and investigations with technological tools in order to form and improve their linear and non-linear schemas. For instance, they could explore the effects of doubling or tripling the lengths of a figure on its area or volume by the help of modelling activities or computer simulations in methods courses in order to improve their content knowledge related to linear and non-linear reasoning.

Furthermore, discussions related to how they could make their future students understand the structures of linear and non-linear relationships might improve their pedagogical content knowledge. These discussions could be beneficial for improving pre-service teachers' affects in relation to the fact that not every relationship is linear and there is no single solution way.

To sum up, implications for educational practices were mentioned in this section in line with the results of previous studies and those of the present study. Since some issues emerged from the findings of the present study, a number of recommendations are available in the following section.

5.3 Recommendations for Further Research Studies

The participants of the present study were selected based on random sampling method from the accessible population which consisted of public schools in Yenimahalle District of Ankara. First of all, some recommendations might be made considering the sample of the study. To begin with, the same study could be replicated with a larger sample randomly selected from nationwide schools in such a way that the sample would be representative of all sixth, seventh, and eighth grade students in Turkey. As such, the findings could be attributed to a wider range of students. Next, the same study could be extended to secondary school students in order to see the similarities and differences between the achievement levels, solution strategies, and reasons underlying incorrect answers of middle school students and secondary school students. Such a study might provide researchers with the opportunity to comprehend secondary school students' understanding of linear and non-linear relationships and ability to discriminate them. Besides, it might be possible to see whether secondary school students could be able to develop different strategies for both linear and non-linear problems owing to their cognitive development. It might also be possible to see whether secondary school students experience the same challenges as middle school students. Furthermore, the same study might be conducted with pre-service and in-service teachers since assessing their achievement level, solution strategies, and reasons for incorrect answers might give a clue to their concurrent knowledge and also their future instructional practices.

This study was designed as a survey method supported with individual interviews; hence, some changes might be done in the research methodology of the present study. In order to see the changes in students' achievement level, solution strategies, and reasons for incorrect answers, a longitudinal study investigating students' development of linear and non-linear relationships throughout a time period might be conducted. More specifically, a longitudinal study beginning with students in sixth grade and observing the same students' development of linear and non-linear relationships throughout their middle school education might be conducted. This kind of study might give essential information regarding at which grade level students experience difficulties, in which grade students develop an understanding of linear and non-linear relationships and how they develop these relationships. Besides, such a study might provide researchers with the opportunity to observe the changes in students' strategies for the problems including linear and non-linear relationships.

An experimental or an intervention study might also be conducted in order to see the differences in students' achievement level and their solution strategies in a different teaching environment focusing on conceptual understanding and supported by the use of models, manipulatives, and real-life applications. Conducting these kinds of studies especially with seventh grade students might give clues to the reasons underlying students' low achievement level since during seventh grade, students deal with proportional problems extensively. Besides, the results of such studies might give essential information regarding how to improve achievement of students in linear and non-linear problems, to help them develop various solution strategies, and to overcome the difficulties that prevent them from answering the items correctly. These experimental and intervention studies might be designed so as to focus on more authentic modelling activities or to integrate technological tools into the mathematics instruction.

Conducting intervention studies with the integration of technology is highly recommended since there is no related study in the available literature. Hence, the findings of such a study might help in terms of improving the achievement of students in linear and non-linear problems in the geometry and measurement domain or in any other domain, helping them to develop different solution strategies and to overcome their difficulties. This contribution could also be considered as essential since previous studies reported the resistance of students in applying linear strategies for non-linear problems. Thus, assessing how technologically supported instructions might have an impact on student achievement on helping students to develop new strategies and overcome their difficulties might make significant contributions to the current literature.

Developing another achievement test with the same purposes is highly recommended. To begin with, adding more problems into the sub-dimensions especially in the non-linear perimeter sub-dimension would increase the internal consistency of the achievement test. Besides, developing another achievement test with more authentic or modelling problems might significantly contribute to the literature since both the linear and non-linear problems in this study and in any previous study were word problems. Finally, developing another achievement test with the purpose of measuring the same research questions but in a different learning area rather than geometry and measurement is highly recommended. Even though there are some studies related to the domains of graphs and probability, a full test with an acceptable reliable score is missing in the available literature. Therefore, developing such a test dealing with other learning areas seem to contribute much into the existing literature Furthermore, developing such a test might help teachers in such a way that they could use the tests in their instructions.

Last but not least, studies might be conducted with the purpose of investigating middle school or secondary school students and pre-service or in-service teachers' affective domain related to linear and non-linear relationships. For example, conducting studies investigating the relationship between metacognitive behaviors or self-efficacy of students and the achievement in linear and non-linear problems is highly recommended since the findings of such a study have a potential to explain why students think that linear strategies are always applicable to any situation.

REFERENCES

Ben-Chaim, D., Keret, Y., & Ilany, B. (2012). *Ratio and proportion. Research and teaching in mathematics teachers' education*. Rotterdam: Sense Publishers.

Benson, S. L. D. (2009). *The Influence of Studying Students' Proportional Reasoning on Middle School Mathematics Teachers' Content and Pedagogical Content Knowledge* (Doctoral dissertation). Available from ProQuest Dissertations & Theses database. (UMI No. 3373689)

Beswick, K. (2011). Make Your Own Paint Chart: a Realistic Context for Developing Proportional Reasoning with Ratios. *Australian Mathematics Teacher*, *67*(1), 6-11.

Canada, D., Gilbert, M., & Adolphson, K. (2008). *Conceptions and misconceptions of elementary preservice teachers in proportional reasoning*. Paper presented at International Group for the Psychology of Mathematics Education PME-NA 32nd.. Retrieved from http://mikeandbarb.net/Articles/Conceptions%20and%20Misconceptions%20of%20Elementary%20Preservice%20Teachers%20in%20Proportional%20Reasoning.pdf

CCSSM (2010). *Common core state standards for mathematics*. Washington, DC. Retrieved from Common Core State Standards website: http://www.corestandards.org/assets/CCSSI_Math\%20Standards.pdf.

Chapin, S. H., & Johnson, A. (2000). *Math matters grades K-6: Understanding the math you teach*. Sausalito, CA: Math Solutions.

Chappell, M. F., & Thompson, D. R. (1999). Perimeter or area? Which measure is it? *Mathematics Teaching in the Middle School, 5*(1), 20–23.

Cheng, D., Star, J. R., & Chapin, S. (2013). Middle School Students' Steepness and Proportional Reasoning. *New Waves-Educational Research & Development*, *16*(1), 22-45.

Clark, H. J. (2008). *Investigating Students' Proportional Reasoning Strategies* (Master's Thesis). Available from ProQuest Dissertations & Theses database. (UMI No. 1453188)

Clements, M. A., & Ellerton, N. F. (1996). *Mathematics Education Research: Past, Present and Future*. Bangkok, Thailand: UNESCO.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. *Journal for Research in Mathematics Education, 14*(2), 83-94.

Cox, D. C. (2008). *Understanding similarity: Bridging geometric and numeric contexts for proportional reasoning.* (Doctoral Dissertation). Available from ProQuest Dissertations & Theses database. (UMI No. 3323517)

Cox D. C. (2013) Similarity in Middle School Mathematics: At the crossroads of geometry and number, *Mathematical Thinking and Learning, 15*(1), 3-23.

Cramer, K., & Post, T. (1993). Making connections: A case for proportionality. *Arithmetic Teacher, 60*(6), 342-346.

Cramer, K., Post, T., & Curier, S. (1993). Learning and teaching ratio and proportion: A research implication. In *Research ideas for the classroom: Middle school grades*, ed. Douglas Owens, 159– 78. New York: MacMillan.

Crocker, L., & Algina, J. (1986). *Introduction to modern and classical test theory*. FL: Holt Rinehart & Winston.

D'Amore, B., & Fandiño-Pinilla, M.I. (2006). Relationships between area and perimeter: beliefs of teachers and students. *Mediterranean Journal for Research in Mathematics Education*, *5*(2), 1-29

De Bock, D., Verschaffel, L., & Janssens, D. (1998). The predominance of the linear model in secondary school students' solutions of word problems involving length and area of similar plane figures. *Educational Studies in Mathematics, 35*(1), 65-83.

De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics, 50*(3), 311-334.

EARGED (2003). TIMSS 1999 Üçüncü Uluslararası Matematik ve Fen Bilgisi Çalışması Ulusal Rapor. Ankara: MEB.

EARGED (2005). PISA 2003 Projesi Ulusal Nihai Rapor. Ankara: MEB.

Ebersbach M., Lehner M., Resing W.C.M., & Wilkening F. (2008). Forecasting exponential growth and exponential decline: Similarities and differences. *Acta Psychologica, 127*, 247–257.

Ebersbach, M., & Wilkening, F. (2007). Children's intuitive mathematics: The development of knowledge about nonlinear growth. *Child Development, 78*(1), 296-308.

Empson, S. B. (1999). Equal sharing and shared meaning: The development of fraction concepts in a first-grade classroom. *Cognition and Instruction, 17*, 283–342.

Espejo, T., & Deters, A. (2011). Area or Perimeter: Using representations for the real world, *Ohio Journal of Mathematics, 63*, 11-16.

Fernandez, C, Llinares S., Van Dooren W., De Bock D., & Verschaffel, L. (2010). *The development of students' use of additive and proportional methods along primary and secondary school*. HUB Research Paper, 32. Retrieved from https://lirias.kuleuven.be/bitstream/123456789/407111/1/10HRP32.pdf

Fernández, C., & Llinares, S. (2009, October). *Understanding additive and multiplicative structures: The effect of number structure and nature of quantities on primary school students' performance*. Paper presented at First French-Cypriot Conference of Mathematics Education. Retrieved from http://rua.ua.es/dspace/bitstream/10045/13467/1/FernandezLlinares2.pdf

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education, 27*(3), 421-438.

Ferrer, B. B., Hunter, B., Irwin, K. C., Sheldon, M. J., Thompson, C. S., & Vistro-Yu, C. P. (2001, November). By the Unit or Square Unit? *Mathematics Teaching in the Middle School, 7*(3), 132-137.

Fraenkel, J., & Wallen, N. (2006). *How to design and evaluate research in education.* New York: McGraw-Hill.

Fragkos, P. C. (1983). *Pedagogical research and implementation*. Thessalonica, Greece: University Studio Press.

Freudenthal, H. (1983). *Didactical phenomenology of mathematical strategies*. D. Reidel, Dodrecht.

Fujimura, N. (2001). Facilitating children's proportional reasoning: a model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology, 93*, 589-603.

Fuson, K. C., & Abrahamson, D. (2005). Understanding ratio and proportion as an example of the apprehending zone and conceptual-phase problem-solving models. In J. Campbell (Ed.), *Handbook of mathematical cognition* (pp. 213–234). New York: Psychology Press.

Gay, A. S., & Aichele, D. B. (1997). Middle school students' understanding of number sense related to percent. *School Science and Mathematics, 97*, 27-36.

Gill, P. N. (1993). Non-linear proportionality in science problems. *International Journal of Mathematical Education in Science and Technology, 24*(3), 365-371. doi: 10.1080/0020739930240305.

Green, D. R. (1982). Probability concepts in 11-16 year old pupils. *Report of research sponsored by the Social Science Research Council. Loughborough: CAMET, University of Technology.*

Greenes, C., & Fendell C. (2000). *Groundworks: Algebraic puzzles and problems*. Chicago, IL: Creative Publications.

Hadjidemetriou, C., & Williams, J. (2010). The linearity prototype for graphs: Cognitive and sociocultural perspectives. *Mathematical Thinking and Learning, 12*(1), 68-85.

Harel, G., & Behr, M. (1995). Teachers' Solutions for Multiplicative Problems. *Hiroshima Journal of Mathematics Education, 3*, 31-51.

Harel, G., & Confrey, J. (Eds.). (1994).*The development of multiplicative reasoning in the learning of mathematics*. SUNY Press.

Harel, G., Behr, M., Lesh, R., & Post, T. (1994). Invariance of ratio: The case of children's anticipatory scheme for constancy of taste. *Journal for Research in Mathematics Education*, *25*(4), 324-345.

Hart, K. (1984). *Ratio*: *Children's strategies and errors. A report of the strategies and errors in secondary mathematics project*. London, United Kingdom: NFER-Nelson.

Hasemann, K. (1981). On difficulties with fractions. *Educational Studies in Mathematics, 12,* 71–87.

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence*. New York: Basic Books.

Kamii, C., & Clark, F. B. (1997). Measurement of length: The need for a better approach to teaching. *School Science and Mathematics, 97*(3), 116-121.

Kaplan, A., İşleyen, T., & Öztürk, M. (2011). 6. sınıf oran orantı konusundaki kavram yanılgıları. *Kastamonu Eğitim Dergisi, 19*(3), 953-968.

Karplus, R., Pulos, S., & Stage, E. K. (1983). Early adolescents' proportional reasoning on "rate" problems. *Educational Studies in Mathematics*, *14*(3), 219-233.

Kaput, J. J., &West, M. M. (1994). Missing-value proportional reasoning problems: Factors affecting informal reasoning patterns. In G. Harel & J. Confrey, (Eds.), *The development of multiplicative reasoning in the learning of mathematics* (pp. 235-287). Albany: State University of New York Press.

Kenney, P. A., & Silver, E. A. (1997). *Results from the Sixth Mathematics Assessment of the National Assessment of Educational Progress*. National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 20191-1593.

Kestell, M. L., & Kubota- Zarivnij, K. (2013). Proportional Reasoning in Number Sense and Numeration. *Abacus, The Ontario Mathematics Gazette, 52*(1), 1-8.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Kouba, V. L., Brown, C. A., Carpenter, T. P., Lindquist, M. M., Silver, E. A., & Swafford, J. (1988). Results of the Fourth NAEP Assessment of Mathematics: Number, operations, and word problems. *Arithmetic Teacher, 35*(4), 14-19.

Lamon S. J. (1993). Ratio and Proportion: Connecting Content and Children's Thinking. *Journal for Research in Mathematics Education, 24*(1), 41-61.

Lamon, S. (1995). Ratio and proportion: Elementary didactical phenomenology. In J. T. Sowder & B. P. Schappelle (Eds.), *Providing a foundation for teaching mathematics in the middle grades* (pp. 167–198). Albany, NY: State University of New York Press.

Lamon, S. (1999). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum.

Lamon, S. J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum.

Lamon, S. (2007). Rational numbers and proportional reasoning. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629-667). Charlotte, NC: Information Age Publishing.

Latt, W. E. (2007). *Pre-service Teachers' Conceptual Understanding of Perimeter, Area, and Volume*. (Master's Thesis, Lakehead University, Canada).

Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (Vol. 2, pp. 93-118). Reston, Virginia: Lawrence Erlbaum.

Lesh, R., Post, T., & Behr, M. (1988). Proportional Reasoning. In J. Hiebert & M. Behr (Eds.) *Number Concepts and Operations in the Middle Grades* (pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.

Lobato, J., & Thanheiser, E. (2002). Developing understanding of ratio as measure as a foundation for slope. In B. Litwiller, (Ed.) *Making sense of fractions, ratios, and proportions: 2002 Yearbook* (pp.162–175). Reston, VA: National Council of Teachers of Mathematics.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundemental mathematics in China and the United States*, Mahwah, New Jersey: Erlbaum.

Marchetti, P., Medici, D., Vighi, P., & Zaccomer, E. (2005, 17 - 21 February). Comparing perimeters and area. Children's pre-conceptions and spontaneous procedures. Paper presented at the Proceedings of the 4th Congress of the European Society for Research in Mathematics Education (Spain), Sant Feliu de Guíxols (pp. 766-776).

Martin, W. G., & Strutchens, M. E. (2000). *Geometry and measurement. Results from the seventh mathematics assessment of the National Assessment of Educational Progress,* 193-234.Menon, R. (1998). Preservice teachers' understanding of perimeter and area. *School Science and Mathematics, 98*(7), 361-367.

Merriam, S. B. (1998). *Qualitative research and case study applications in education* (Rev. ed.). Jossey-Bass, Inc., San Francisco.

Ministry of National Education [MoNE]. (2008), *İlköğretim Okulu Ders Programları: Matematik Programı 6-7-8 (Elementary Curricula Programs: Mathematics Curricula Program for Middle Grades)*. Ankara: MEB.

Ministry of National Education [MoNE] (2012a). İlköğretim Matematik 6 Ders Kitabı. [Middle School Grade 6 Mathematics Textbook]. Ankara: MEB.

Ministry of National Education [MoNE] (2012b). İlköğretim Matematik 7 Ders Kitabı. [Middle School Grade 7 Mathematics Textbook]. Ankara: MEB.

Ministry of National Education [MoNE] (2012c). İlköğretim Matematik 8 Ders Kitabı. [Middle School Grade 8 Mathematics Textbook]. Ankara: MEB.

Ministry of National Education [MoNE]. (2013), *Ortaokul Matematik Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı (Middle School Mathematics Curricula Program (for grades 5, 6, 7, and 8)*. Ankara: MEB.

Misailadou, C., & Williams, J. (2003, July 13-18). *Measuring Children's Proportional Reasoning, The" Tendency" for an Additive Strategy and The Effect of Models*. Paper presented at International Group for the Psychology of Mathematics Education, 293-300. Retrieved from http://files.eric.ed.gov/fulltext/ED501129.pdf

Modestou, M., Gagatsis, A., & Pitta-Pantazi, D. (2004, July 14-18). Students' improper proportional reasoning: The case of area and volume of rectangular figures. P*roceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education*, Bergen, Norway (Vol. 3, pp. 345-352).

Modestou, M., Elia, I., Gagatsis, A., & Spanoudes, G. (2007). Problem Solving in Geometry: The case of the illusion of proportionality. Paper presented at Fifth Congress of the European Society for Research in Mathematics Education, Cyprus, 22-26 February (pp. 1052-1062)

Modestou, M., & Gagatsis, A. (2007). Students' Improper Proportional Reasoning: A result of the epistemological obstacle of "linearity". *Educational Psychology, 27*(1), 75-92.

Modestou M., & Gagatsis A. (2009). Proportional Reasoning: The Strategies Behind the Percentages. *Acta Didactica Universitatis Comenianae, 9*, 25-40.

Modestou, M., & Gagatsis, A. (2010). Cognitive and metacognitive aspects of proportional reasoning. *Mathematical Thinking and Learning, 12*(1), 36-53.

Moyer, P. S. (2001). Using representations to explore perimeter and area. *Teaching Children Mathematics, 8*(1), 52-59.

National Assessment of Education Progress [NAEP] (2007). Long-term trend: National trends in reading by performance levels. Retrieved from http://nces.ed.gov/nationsreportcard/ltt/performance-levels.asp.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: The Council

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author

National Council of Teachers of Mathematics (2000). *Measurement standards for grades 3-5*. Retrieved from http://www.nctm.org/standards/content.aspx?id=3666

Noelting, G. (1980). The development of proportional reasoning and the ratio concept part I- Differentiation of stages. *Educational Studies in Mathematics, 11*(2), 217-253.

Ohio Department of Education, (2002). *Ohio Mathematics Content Standards*. Retrieved April 21, 2010, from Ohio Department of Education Web site: http://education.ohio.gov/GD/Templates/Pages/ODE/ODEDetail.aspx?page=3&%20TopicRelationID=1704&ContentID=801&Content=50719

Orhan, N. (2013). *An investigation of private middle school students' common errors in the domain of area and perimeter and the relationship between their geometry self-efficacy beliefs and basic procedural and conceptual knowledge of area and perimeter*. Unpublished Master's Thesis, Middle East Technical University, Ankara.

Paić-Antunović, J., & Vlahović-Štetić, V. (2011). The effect of feedback on the intensity of the illusion of linearity in high-school students' solving of geometry problems. *Review of Psychology, 18*(1), 23-32.

Piaget, J., & Inhelder, B. (1975). *The origin of idea of chance in children*. New York: Norton.

Resnick, L., & Singer, J. (1993). Protoquantitative origins of ratio reasoning. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 107-130). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Rickard, A. (1996). Connections and confusion: Teaching perimeter and area with a problem-solving oriented unit. *Journal of Mathematical Behavior, 15*(3), 303-327

Ryan, J., & Williams, J. (2007). *Children's mathematics 4-15: Learning from errors and misconceptions*. New York: NY: Open University Press.

Saxe, G. B., Gearhart, M., & Seltzer, M. (1999). Relations between classroom practices and student learning in the domain of fractions. *Cognition and Instruction, 17,* 1–24.

Sherman, H. & Randolph T (2004). Area and Perimeter: "Which is Which and How Do We Know?", *Research for Educational Reform, 9*(3), 25-36.

Spinillo A. G., & Bryant P. E. (1999). Proportional Reasoning in Young Children: Part–Part Comparisons about Continuous and Discontinuous Quantity. *Mathematical Cognition, 5(2),* 181–197.

Stavy, R., & Tirosh, D. (2000). *How students (mis-)understand science and mathematics*. New York: Teachers College Press.

Stavy, R., Babai, R., Tsamir, P., Tirosh, D., Lin, F. L., & McRobbie, C. (2006). Are intuitive rules universal? *International Journal of Science and Mathematics Education, 4*(3), 417-436.

Steele, M. D. (2006). *Middle grades geometry and measurement: Examining change in knowledge needed for teaching through a practice-based teacher education experience* (Doctoral dissertation, University of Pittsburgh). Available from ProQuest Dissertations & Theses database. (UMI No. 3224051).

Stemn, B. S. (2008) Building middle school students' understanding of proportional reasoning through mathematical investigation, *International Journal of Primary, Elementary and Early Years Education, 36*(4), 383-392.

Streefland, L. (Ed.). (1991). *Fractions in realistic mathematics education: A paradigm of developmental research* (Vol. 8). Springer.

Tan-Şişman, G. (2010). *Sixth grade students' conceptual and procedural knowledge and word problem solving skills in length, area, and volume measurement.* Unpublished Doctoral Dissertation, Middle East Technical University, Ankara.

Thompson, P. W., & Saldanha, L. A. (2003). Fractions and multiplicative reasoning. *Research Companion to the Principles and Standards for School Mathematics*, 95-113.

Thompson, T. D., & Preston, R. V. (2004). Measurement in the middle grades: Insights from NAEP and TIMMS. *Mathematics Teaching in the Middle School, 9*, 514-519.

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics, 16*(2), 181-204.

Valverde, G., & Martínez, E. C. (2012). Prospective Elementary School Teachers´ Proportional Reasoning. *PNA, 7*(1), 1-19.

Van De Walle, J. A., Karp, K. S., & Bay-Williams J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th Ed.). Boston, MA: Pearson Education, Inc.

Van Dooren, W., De Bock, D., Verschaffel, L., & Janssens, D. (2003). Improper applications of proportional reasoning. *Mathematics Teaching in the Middle School, 9(*4), 204-209.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2004). Remedying secondary school students' illusion of linearity: A teaching experiment aiming at conceptual change. *Learning and Instruction, 14*(5), 485-501.

Van Dooren, W., De Bock, D., Janssens, D., & Verschaffel, L. (2007). Pupils' over-reliance on linearity: A scholastic effect?. *British Journal of Educational Psychology, 77*(2), 307-321.

Van Dooren, W., De Bock, D. & Verschaffel, L. (2010). From addition to multiplication... and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*, 360–381.

Vergnaud, G. (1988). Multiplicative structures. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (Vol. 2, pp. 141-161). Reston, VA: National Council of Teachers of Mathematics.

Vlahović-Štetić, V., Pavlin-Bernardić, N., & Rajter, M. (2010). Illusion of linearity in geometry: Effect in multiple-choice problems. *Mathematical Thinking and Learning, 12*(1), 54-67.

Weinberg, S. (2002). Proportional Reasoning: One Problem, Many Solutions! In B.Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios,and Proportions: 2002 Yearbook*. Reston, VA: NCTM.

Wright, V. (2005). Proportional reasoning: Some rational thoughts. In J. Mousley, L. Bragg, & C. Campbell. (Eds.), *Mathematics: Celebrating achievement* (pp. 363-375). Melbourne: Mathematical Association of Victoria.

APPENDICES

APPENDIX A. PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

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Sayı: 28620816/115 -297

28 Mart 2013

Gönderilen: Doç.Dr. Mine Işıksal Bostan
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Fen ve Matematik Eğitimi Bölümü Yüksek Lisans öğrencisi Rukiye Ayan'ın "Ortaokul Öğrencilerinin Düşünmelerinde Doğrusallığın Baskınlığı: Geometrik Şekillerde Uzunluk, Çevre,Alan ve Hacim Durumu" isimli araştırmaları "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

28/03/2013

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

APPENDIX B. PERMISSION OBTAINED FROM MINISTRY OF NATIONAL EDUCATION

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

ÖĞRENCİ İŞ... DAİRESİ BAŞKA...ĞI
... Arş. Md. Sa... :

Sayı : 14588481/605.99/555225 11/04/2013
Konu: Araştırma izni

ORTA DOĞU TEKNİK ÜNİVERSİTESİNE
(Öğrenci İşleri Daire Başkanlığı)

İlgi: a) MEB Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu Genelgesi.
b) 08/04/2013 tarih ve 3898 sayılı yazınız.

Üniversiteniz Eğitim Fakültesi Araş. Gör. Rukiye AYAN' ın **"Ortaokul öğrencilerinin düşünmelerinde doğrusallığın baskınlığı: Geometrik şekillerde uzunluk, çevre, alan ve hacim durumu"** konulu tezi kapsamında çalışma yapma talebi Müdürlüğümüzce uygun görülmüş ve araşturmanın yapılacağı İlçe Milli Eğitim Müdürlüğüne bilgi verilmiştir.

Uygulama örneklerinin (5 sayfa) araştırmacı tarafından uygulama yapılacak sayıda çoğaltılması ve çalışmanın bitiminde iki örneğinin (cd ortamında) Müdürlüğümüz Strateji Geliştirme Bölümüne gönderilmesini arz ederim.

İlhan KOÇ
Müdür a.
Şube Müdürü

Güvenli Elektronik İmza
Aslı İle Aynıdır.
.../.../201...

15.04.2013 - 6397

SUBAŞI

APPENDIX C. ACHIEVEMENT TEST

Ad Soyad: **Okul:**
Sınıf: **Cinsiyet:** Kız☐ Erkek ☐

SORULAR

1.Çiftçi Ahmet kenar uzunluğu 100m olan kare şeklindeki bahçesinin etrafına yapacağı sulama kanalını 4 günde kazabiliyor. Aynı bahçenin kenar uzunluğu 300m olsaydı Ahmet aynı hızda çalışarak bu kanalı kaç günde kazabilirdi? Çözüm yolunuzu açık bir şekilde yazınız.

2.Pınar 1 saat boyunca sabit hızda yürüyerek 4 km yol kat etmiştir. Pınar aynı hızda yürümeye devam ederse 2 saatte kaç km yol alır? Çözüm yolunuzu açık bir şekilde yazınız.

3.Bir yelkenli, yarıçapı 35 km olan daire şeklindeki bir adanın etrafındaki turunu 6 saatte tamamlayabiliyor. Aynı adanın yarıçapı 70 km olsaydı bu yelkenli aynı hızıyla ada etrafındaki turunu kaç saatte tamamlayabilirdi? Çözüm yolunuzu açık bir şekilde yazınız.

4.Özlem sınıfının taban alanını ölçerek 25 m^2 olarak buluyor. Özlem'in okulundaki spor salonunun tabanının kenar uzunlukları bu sınıfın kenar uzunluklarının iki katı olduğuna göre spor salonunun alanı kaç m^2'dir? Çözüm yolunuzu açık bir şekilde yazınız.

5. Bir okulun dikdörtgenler prizması şeklindeki yüzme havuzu 70 m^3 su kapasitesine sahiptir. Bu yüzme havuzunun eni, boyu ve yüksekliğinin her birinin 2 katı boyutlara sahip diğer bir yüzme havuzunun su kapasitesi ne kadar olur? Çözüm yolunuzu açık bir şekilde yazınız.

6. Ankara ile İstanbul arası uzaklığın 1 cm olduğu aşağıda Şekil 1'de verilen bir haritada Türkiye'nin yüz ölçümüne karşılık gelen alan 25 cm^2'dir. Buna göre Ankara ile İstanbul arası uzaklığın 2 cm olduğu aşağıda Şekil 2'de verilen diğer bir haritada Türkiye'nin yüz ölçümüne karşılık gelen alan kaç cm^2 olur? Çözüm yolunuzu açık bir şekilde yazınız.

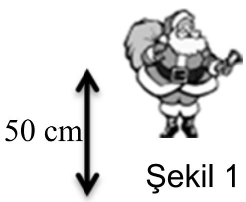
Şekil 1

Şekil 2

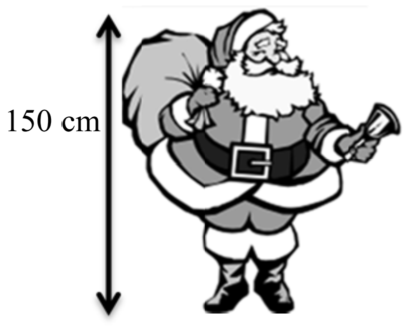
7. Aslı, küp şeklindeki kumbarasının dış yüzlerini boyamak istiyor. Aslı'nın bu kumbaranın bir yüzünü boyaması için 10ml boya gerekiyorsa, kumbaranın dış yüzlerinin hepsini boyaması için kaç ml boya gerekir? Çözüm yolunuzu açık bir şekilde yazınız.

8. Bir Türkiye haritasında Adana ile Antalya arası uzaklık 5 cm ve Antalya Muğla arası uzaklık 3 cm'dir. Diğer bir Türkiye haritasında ise Adana ile Antalya arası uzaklık 10 cm'dir. Buna göre, bu haritada Antalya'nın Muğla'ya uzaklığı ne kadardır? Çözüm yolunuzu açık bir şekilde yazınız.

9. Aşağıda Şekil 1'de gösterilen 50 cm uzunluğunda bir Noel baba resmini boyamak için 6ml boya gerekmektedir. Aynı resmin Şekil 2'de gösterilen 150 cm uzunluğunda bir kopyasını boyamak için ne kadar boya kullanılmalıdır? Çözüm yolunuzu açık bir şekilde yazınız.



Şekil 1



Şekil 2

10. 60 m³ hacme sahip dikdörtgenler prizması şeklindeki bir koli yüksekliğinin yarısı hizasından tabana paralel olacak şekilde kesiliyor. Bu kolinin kesildikten sonraki hacmi kaç m³ olur? Çözüm yolunuzu açık bir şekilde yazınız.

APPENDIX D. INTERVIEW QUESTIONS

Görüşme Soruları

1) Yaptığın çözüm yolunu anlatır mısın?
2) Neden bu şekilde yaptın (düşündün)?
3) Neden bu çözüm yolunu tercih ettin?
4) Problemden ne anladın?
5) Problemde hangi matematiksel kavramlar arasındaki ilişkiden bahsediliyor?
6) Problemi çözerken verilen bilgileri ve geometrik şekillerin özelliklerini inceledin mi?
7) Diğer kavramları (çevre, alan ya da hacim) düşünseydin sonuç aynı çıkar mıydı?

APPENDIX E. TABLE OF SPECIFICATION

Table of specification for the items based on the objectives of national mathematics education curriculum

Objectives (MoNE, 2008)	Grade Level	Related Problems
Solves and poses problems related to perimeter of plane figures [Düzlemsel şekillerin çevre uzunlukları ile ilgili problemleri çözer ve kurar.]	6	P1, P3
Solves and poses problems related to direct and inverse proportions [Doğru ve ters orantıyla ilgili problemleri çözer ve kurar.]	7	P1, P2, P3, P7, P8, P10
Solves and poses problems related to area of plane figures. [Düzlemsel bölgelerin alanları ile ilgili problemleri çözer ve kurar.]	6	P4, P6, P9
Solves and poses problems related to area of quadrilaterals [Dörtgensel bölgelerin alanları ile ilgili problemleri çözer ve kurar.]	7	P4, P6
Explains the relationship between side length and area of plane figures. [Kenar uzunluğu ile alan arasındaki ilişkiyi açıklar.]	7	P4, P6, P9
Solves and poses problems related to volume of rectangular prism, square prism and cube. [Dikdörtgenler prizması, kare prizma ve küpün hacmi ile ilgili problemleri çözer ve kurar.]	6	P5, P10
Estimates area of plane figures by using strategies. [Düzlemsel bölgelerin alanlarını strateji kullanarak tahmin eder.]	6	P4, P6, P9
Explains the relationship between quantities with direct and inverse proportion. [Doğru orantılı ve ters orantılı nicelikler arasındaki ilişkiyi açıklar.]	7	P1, P3, P7, P8, P10
Calculates surface area of rectangular prism, square prism and cube [Dikdörtgenler prizması, kare prizma ve küpün yüzey alanlarını hesaplar.]	6	P7
Solves and poses problems related to surface area of rectangular prism, square prism and cube [Dikdörtgenler prizması, kare prizma ve küpün yüzey alanı ile ilgili problemleri çözer ve kurar.]	6	P7

Solves and poses problems related to surface area of geometrical figures. [Geometrik cisimlerin yüzey alanları ile ilgili problemleri çözer ve kurar.]	8	P7
Estimates the volume of rectangular prism, square prism and cube by using strategies. [Dikdörtgenler prizması, kare prizma ve küpün hacmini strateji kullanarak tahmin eder.]	6	P5, P10
Solves and poses problems related to volume of rectangular prism, square prism and cube. [Dikdörtgenler prizması, kare prizma ve küpün hacmi ile ilgili problemleri çözer ve kurar.]	6	P5, P10
Estimates the volume of geometrical figures by using strategies. [Geometrik cisimlerin hacimlerini strateji kullanarak tahmin eder.]	8	P5, P10
Solves and poses problems related to volume of geometrical figures. [Geometrik cisimlerin hacimleri ile ilgili problemleri çözer ve kurar.]	8	P5, P10

*Translations are done by the researcher.

APPENDIX F. RUBRICS FOR THE CATEGORIES

Rubric for the Solution Strategies
Strategies for Linear-Perimeter Problems
Questionable Proportion Strategy

- Using the numbers (i.e. side lengths) given in the problem and writing a proportion between these numbers without calculating the perimeter
- Using the numbers (i.e. side lengths) given in the problem and writing a proportion between these numbers without an argument that writing a proportion would yield the same result as writing a proportion between the perimeters

Reasonable Proportion Strategy

- Finding the related variables (i.e. perimeters) and writing the direct proportion between the related variables
- Judging the type of the relationship between the variables and determining that the same result would be reached by writing a proportion between the given variables (i.e. length) and between the related concepts (i.e. perimeters)

Strategies for the Non-linear Problems
Length-Length-Area/Volume Relationships

- Giving arbitrary side lengths for the figures by using the given area and using the relationship between the lengths in order to find the area of the second figure
- Giving arbitrary dimensions for the figures by using the given volume and using the relationship between the dimensions in order to find the volume of the second figure

Length- Area/Volume Relationships

- Answers which included the argument that area would be multiplied by r^2 or the volume would be multiplied by r^3 when all the lengths were multiplied by r
- Answers which included the argument that volume would be multiplied by r^3 when all the dimensions were multiplied by r

Rubric for the Underlying Reasons for Incorrect Answers
Inadequacy in Geometrical Knowledge

- Answers which included such a statement that a cube has four sides
- Answers which included calculating the perimeter instead of area or vice versa or calculating the area instead of volume or vice versa.

Misinterpretation of Additive and Multiplicative Reasoning

- Taking the difference between the two variables and adding this difference to the second variable when a proportional situation exists
- Answers which included the argument that distances on a map should be increased by the same amount instead of the same ratio when the scale was changed
- Answers which included the argument that volume should be multiplied by six when the dimensions of a figure are multiplied by two

Operational Mistakes

- Answers which included computational mistakes with correct reasoning

Incomplete answers

- Writing the proportion correctly without calculating the answer
- Determining by which factor the area or volume would be multiplied without calculating the answer
- Answers' which included a correct written explanation for the solution of the problem without finding the correct answer

Misinterpretation of Proportional Situations

- Answers including the tendency to apply inverse proportion where direct proportion was needed

Illusion of Linearity

- Answers including the multiplying the area by the same scale factor when the lengths of a figure are multiplied by a scale factor
- Answers including multiplying the volume by the same scale factor when the dimensions of a figure are multiplied by a scale factor
- Answers including writing a linear proportion between the areas or volumes when the lengths or dimensions are multiplied by a scale factor

APPENDIX G. TURKISH SUMMARY

Giriş

Özellikle ilkokul ve ortaokul konuları olmak üzere birçok matematiksel durumun özünde orantısal düşünme yatmaktadır. Buna ek olarak, orantısal düşünme fendeki ve günlük hayattaki durumları anlamak için önemlidir (Cramer & Post, 1993). Orantısal düşünmeye verilen bu önem matematik müfredatlarında da görülmektedir. Örneğin, Amerikan Matematik Standartlarında orantsal düşünmeden 4 anahtar alandan biri olarak bahsedilmektedir (Common Core State Standards for School Mathematics [CCSSM], 2010). Benzer bir şekilde, Türk Ortaokul Matematik müfredatında bu konuya ayrılan sürenin çokluğu dikkat çekmektedir (Milli Eğitim Bakanlığı, [MEB], 2013).

Orantısal düşünmeye verilen bu önem ve ayrılan zamana rağmen birçok ulusal ve uluslararası çalışma öğrencilerin bu konu ile ilgili başarılarının düşük olduğunu ve onların bu konuda zorluklar yaşadıklarını göstermiştir (Kaplan, İşleyen, & Öztürk, 2011; Lobato & Thanheiser, 2002; Modestou & Gagatsis, 2007; Thompson & Preston, 1994). Düşük başarının ve yaşanan zorlukların sebebinin öğrencilerin bu konu ile ilgili sınırlı ve yüzeysel bilgiye sahip olmalarından kaynaklandığı öne sürülmektedir. Öyle ki orantısal düşünme birçok kişi tarafından verilmeyeni bulma problemlerini çözmek olarak algılanmaktadır (Post, Behr, & Lesh, 1988). Fakat, matematik eğitimi literatüründe bunun doğru olmadığına ve verilmeyeni bulma problemlerini çözebilmenin orantısal düşünmenin bir göstergesi olamayacağına vurgu yapılmaktadır (Cramer & Post, 1993; Post vd., 1988). Orantısal düşünme, bu problemleri çözebilmenin yanı sıra çoklukları toplamsal yerine çarpımsal olarak kıyaslama (Kestell & Kubota- Zarivnij, 2013) ve orantısal durumlar ile orantısal olmayan durumları birbirinden ayırt edebilmek ile ilgilidir (Cramer & Post, 1993). Fakat yüzeysel bilgi ve işlemsel becerilere önem veren öğretim yöntemlerinin uygulanması sonucuyla öğrenciler orantısal düşünme problemlerinde farklı stratejiler geliştiremeyebilirler. Hatta öğrenciler orantısal stratejilerin uygun olmadığı durumlarda bile orantısal çözüm stratejileri geliştirme ve kullanma alışkanlığı geliştirebilirler (Freudenthal, 1983). Bu durum iki farklı şekilde olabilir: toplamsal ilişkiler kullanmayı gerektiren problemler için çarpımsal (orantısal) stratejiler kullanmak (Van Dooren, De Bock, & Verschaffel, 2010) ve orantısal olmayan ilişkiler için orantısal stratejiler kullanmak (De Bock, Verschaffel, & Janssens, 1998). Bu iki durum öğrencilerin orantısal düşünmede yaşadıkları zorluklar olarak düşünülmekte ve bu çalışmanın araştırma kapsamına girmektedir.

Bu iki zorluğun en çok görüldüğü alanlardan birisi de geometri ve ölçme alanıdır. Öyle ki, 5. ve 8. sınıf aralığındaki öğrencilerin birçoğunun bir şeklin kenarları benzer bir şekil oluşturmak için iki katına çıkarıldığında alanının ve hacminin de iki katına çıkacağını düşünmektedirler (National Council of Teachers of Mathematics [NCTM], 1989). Yani öğrenciler bir şeklin kenarları ve alan ya da hacmi arasında doğrusal bir ilişki olduğunu düşünmektedirler.

Öğrencilerin bu düşüncelerinin aksine benzer şekillerin uzunluk, çevre, alan ve hacimleri arasındaki ilişkinin şu şekilde olduğu kabul edilmektedir: bir şeklin r oranıyla doğrusal olarak genişletilmesi ya da daraltılması uzunlukları aynı oranda, alanı r^2 olarak ve hacmi r^3 olarak etkilemektedir (De Bock, Verschaffel, & Janssens, 2002). Örneğin, bir şeklin tüm kenarları 2 katına çıktığında çevresi 2 katına, alanı 4 ($2^2 = 4$) katına ve hacmi 8 ($2^3=8$) katına çıkar. Fakat birçok öğrenci günlük hayattaki ve matematik derslerindeki deneyimlerinin eksikliklerinden dolayı bu durumun farkında olmayabilir ve bu kavramlar arasındaki tüm ilişkilerin doğrusal olduğunu düşünebilirler. Bu durum doğrusallık yanılsaması olarak adlandırılmaktadır (De Bock vd.,1998, 2002).

Çalışmanın Amaçları

Bu çalışmanın amaçları üç kısımdan oluşmaktadır. Çalışmanın birinci amacı, altıncı, yedinci ve sekizinci sınıf öğrencilerinin uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarı düzeylerini incelemektir. Çalışmanın ikinci amacı, bu öğrencilerin bahsedilen problemlerde kullandıkları çözüm stratejilerinin belirlenmesidir. Çalışmanın üçüncü amacı, öğrencilerin bu problemlere verdikleri yanlış cevapların nedenlerinin araştırılmasıdır.

Çalışmanın Önemi

Orantısal düşünme, matematik ve fendeki birçok konuyla ilgili olmasının yanı sıra günlük hayattaki birçok durumla da alakalıdır (Cramer & Post, 1993; Lesh vd., 1988). Sıcaklık, yoğunluk, kimyasal karışımlar, bileşikler ve hız konuları fendeki konulara örnek olarak verilebilir (Karplus, Pulos, & Stage, 1983; Spinillo & Bryant, 1999). Günlük hayattaki durumlara örnek olarak ise alışveriş, bütçe planlamaları ve en iyi fiyat araştırmaları (Spinillo & Bryant, 1999); ilaç dozları, ekonomik ve sosyolojik tahminler (Valverde & Martínez, 2012) verilebilir. Bunun yanı sıra, orantısal düşünme altı ve sekizinci sınıflar arası düzeydeki birçok matematiksel konu için (NCTM, 2000) ve ortaokul sonrasında da birçok önemli konu için (Van De Walle, Karp, Bay-Williams, & Wray, 2013) bütünleştirici bir kavramdır. Bu sebeple, ortaokul öğrencilerinin orantısal düşünme algılarını ve süreçlerini incelemek önemlidir. Bu konuda yapılan çalışmaların Türk alan yazınında yetersiz sayıda olduğu ise bu çalışmanın önemini artırmaktadır.

Orantısal düşünme verilmeyeni bulma problemlerini çözmek olarak algılanabilmektedir. Fakat bu tanım orantısal düşünmenin özünde yatan toplamsal ve çarpımsal ilişkileri ve orantısal durumlar ile orantısal olmayan durumları birbirinden ayırt etme becerilerini içermemektedir. Bu sebepten dolayı, bu çalışma orantısal düşünme kapsamında bu iki beceriyi içerdiği için önemli görülmektedir. Bu özellikleri öne çıkaran çalışmaların Türk alan yazınında yetersiz sayıda olduğu ise bu çalışmanın önemini artırmaktadır.

Orantısal düşünme 5 ve 8. sınıflar arasında gelişmektedir (NCTM, 1989). Yani, ortaokul yılları orantısal düşünmenin gelişmesi için kritik yıllardır. Bu sebepten dolayı öğrencilerin orantısal düşünme şemalarını oluşturduğu yıllarda bu konudaki başarı düzeyleri ve bu konu ile ilgili zorluklarını incelemek önemlidir.

Birçok çalışmanın sonuçları öğrencilerin orantısal düşünme ve geometri ve ölçme alanlarındaki düşük başarılarını ortaya koymuştur (Chappel & Thompson, 1999; Post vd., 1988; Sherman & Randolph, 2004; Tan-Şişman, 2010). Bu çalışma orantısal düşünme ve geometri ve ölçme alanları birleştiren bir yapıya sahip olduğu için bu çalışmanın sonuçlarının öğrencilerin başarı düzeyleri, çözüm stratejileri ve yanlış cevaplarının nedenleri ile ilgili her iki alanda ayrı olarak ve bu alanların kesişim noktalarında bilgi vermesi beklenmektedir.

Öğrenciler farklı problemler için farklı stratejiler kullanırlar (Chapin & Johnson, 2000). Bunun yanı sıra, öğrencilerin bir konudaki zorlukları o konunun diğer alanlardaki uygulamaları için değişiklik gösterebilir. Yani öğrenciler geometri ve ölçme alanındaki orantısal düşünme problemleri için farklı stratejiler kullanabilir ve farklı zorluklar yaşayabilirler. Öğrencilerin orantısal düşünme problemleri ile ilgili kullandıkları stratejiler ve yaşadıkları zorluklar araştırılmış olsa bile onların uzunluk, çevre, alan ve hacim ile ilgili doğrusal ve doğrusal olmayan problemlerle ilgili kullandıkları stratejiler ve yaşadıkları zorluklarla ilgili yapılan çalışmalar yetersizdir. Bu durum çalışmanın önemini artırmaktadır.

Önemli Terimlerin Tanımları

Orantı: Orantı, $\frac{a}{b}=\frac{c}{d}$ şeklindeki iki oranın eşitliği olarak tanımlanmaktadır.

Orantısal Düşünme: Orantısal düşünme çokluklar arasında çarpımsal kıyaslamalar yapmak (Wright, 2005) ve bu ilişkiyi farklı biçimlerde kullanabilmek (Cheng, Star, & Chapin, 2013) olarak tanımlanmıştır.

Bu çalışmada, orantısal düşünme ayrıca toplamsal ve çarpımsal ilişkileri ve orantısal durumlar ile orantısal olmayan durumları birbirinden ayırt edebilmek olarak tanımlanmaktadır.

Çarpımsal ve Toplamsal Düşünme: Çarpımsal düşünme çokluklar arasında çarpımsal kıyaslamalar yapmak (Wright, 2005) olarak tanımlanmaktadır. Diğer taraftan, toplamsal düşünme oranlar arasındaki ilişkiler bir terimden diğerini çıkarmak ve bu farkı ikinci orana uygulamak ile ilgilidir (Tourniaire & Pulos, 1985)

Bu çalışmada, çarpımsal ve toplamsal ilişkilerin yanlış anlamlandırılması öğrencilerin çarpımsal ve toplamsal ilişkileri birbirinden ayırt edememesi ve çarpımsal ve toplamsal stratejileri uygunsuz kullanmaları olarak tanımlanmıştır.

Doğrusallık: Doğrusallık, matematiksel olarak bir fonksiyonun homojen ve toplamsal olması, yani f(ax) = a f(x) (tüm a'lar için) ve $f(x_1+x_2) = f(x_1) + f(x_2)$ olmasıdır. f(x)=ax, (a ≠0) bu iki özelliği sağladığı için doğrusal ya da doğrusallığı temsil etmektedir. Diğer bir taraftan, $\frac{a}{b}=\frac{c}{d}$ orantısı iki değişken arasındaki doğrusal ilişki olarak tanımlanabilir (Freudenthal, 1983).

Doğrusallık Yanılsaması (Illusion of Linearity): Doğrusallık yanılsaması öğrencilerin doğrusal stratejileri doğrusal olmayan durumlarda da kullanması olarak tanımlanmıştır (De Bock, Verschaffel, & Janssens, 2002).

Bu çalışmada, doğrusallık ya da doğrusal ilişkiler uzunluk, çevre, alan ve hacim arasındaki aynı boyuttaki ilişkilere karşılık gelmektedir. Doğrusallık yanılsaması da öğrencilerin bir şeklin r oranıyla doğrusal olarak genişletilmesi ya da daraltılmasıyla uzunlukları aynı oranda, alanın r^2 olarak ve hacmin r^3 olarak etkilenmesi durumlarının farkında olmamaları ve tüm bu ilişkilerin doğrusal olduğunu düşünmeleri olarak tanımlanmıştır.

Doğrusal (Orantısal) Problemler: Bu problemler doğrusal (ya da orantısal) çözüm stratejileri gerektirmektedir.

Doğrusal (Orantısal) Olmayan Problemler: Bu problemler doğrusal (ya da orantısal) olmayan çözüm stratejileri gerektirmektedir.

Yöntem

Evren ve Örneklem

Bu çalışmanın örneklemini Ankara'nın Yenimahalle ilçesindeki devlet okullarına devam eden 935 altı, yedi ve sekizinci sınıf öğrencisi oluşturmaktadır. Bu öğrenciler Ankara'nın Yenimahalle ilçesindeki 97 devlet ortaokulundan küme örneklemesi yöntemiyle seçilmiştir. Bu öğrencilerin temel karakteristikleri aşağıda Tablo 1 de verilmiştir.

Tablo 1 Çalışmanın Katılımcıları ve Temel Karakteristikleri

Cinsiyet		Sayı	Yüzde
Erkek			
	6	154	32,6
	7	187	39,5
	8	132	27,9
	Toplam	473	100
Kız			
	6	122	26,5
	7	196	42,5
	8	144	31,0
	Toplam	462	100

Bu katılımcılar arasından her sınıf seviyesinden 4 öğrenci olmak üzere toplam 12 öğrenci ile bireysel görüşmeler yapılmıştır. Bu öğrencilerden 7'si kız (3 altıncı sınıf, 3 yedinci sınıf ve 1 sekizinci sınıf) ve 5'i erkektir (1 altıncı sınıf, 1 yedinci sınıf ve 3 sekizinci sınıf). Görüşmelerin katılımcıları başarı testindeki başarı düzeyleri, kullandıkları çözüm stratejileri ve testte verdikleri yanlış cevapların incelenmesi ve pilot çalışmadan elde edilen kodlara göre seçilmiştir.

Araştırma Soruları

Bu çalışmanın üç tane araştırma sorusu vardır.

1. Altı, yedi ve sekizinci sınıf öğrencilerinin uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarı düzeyleri nedir?
2. Altı, yedi ve sekizinci sınıf öğrencileri uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerde hangi stratejileri kullanmaktadırlar?
3. Altı, yedi ve sekizinci sınıf öğrencilerinin uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlere verdikleri yanlış cevapların nedenleri nelerdir?

Araştırma Yöntemi

Araştırmada nitel ve nicel araştırma yöntemlerini birleştiren karma bir araştırma yöntemi kullanılmıştır.

Veri Toplama Araçları

Çalışmanın verileri katılımcıların başarı testine verdikleri cevaplar ve bireysel görüşmeler aracılığıyla toplanmıştır.

Başarı Testi

Katılımcıların uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarı düzeylerinin, bu problemlerde kullandıkları çözüm stratejilerinin ve bu problemlerdeki yanlış cevaplarının nedenlerinin incelenmesi amacıyla bir başarı testi geliştirilmiştir. Bu başarı testi 10 adet açık uçlu matematiksel problemden oluşmaktadır. Bu problemlerden 9'u literatürden alınmış 1 tanesi ise araştırmacı tarafından geliştirilmiştir. Problemlerin adapte edilmesi sürecinde ortaokul matematik öğretim programında yer alan kazanımlar göz önüne alınmış ve belirtke tablosu hazırlanmıştır. Bu problemlerden ikisi (altıncı ve dokuzuncu) problem cümlesine yönelik figürler içermektedir. Tüm problemler doğrusal ve doğrusal olmayan problemler olmak üzere 2 gruba ayrılmıştır. Ayrıca, problemler ilgili oldukları kavramlara göre uzunluk, çevre, alan ve hacim olmak üzere 4 gruba ayrılmıştır. Yani, problemler 8 (4x2) alt gruba ayrılmıştır. Bu gruplar doğrusal-uzunluk, doğrusal-çevre, doğrusal-alan ve doğrusal hacim; doğrusal olmayan-uzunluk, doğrusal olmayan-çevre, doğrusal olmayan-alan ve doğrusal olmayan-hacimdir. Bu gruplar ve bu gruplarda bulunan problemler aşağıda Tablo 2'de belirtilmiştir.

Tablo 2 Başarı Testindeki Problemlerin Kategorileri ile İlgili İçerik Tablosu

Kavramlar	**Çözüm Yolu**		
	Doğrusal	**Doğrusal Olmayan**	**Toplam**
Uzunluk	P2, P8	P6	3
Çevre	P1, P3	-	2
Alan	P7	P4, P9	3
Hacim	P10	P5	2
Toplam	6	4	10

Başarı testi ve belirtke tablosu hazırlandıktan sonra üç uzmanın görüşü alınmış ve her sınıf seviyesinden bir sınıf ile pilot çalışma yapılmıştır. Pilot çalışmadan ve asıl çalışmadan elde edilen skorlar ile Kuder-Richardson-20 analizi yapılmıştır. Hem pilot çalışmanın hem de asıl çalışmanın skorları tutarlı bulunmuştur.

Bireysel Görüşmeler

Başarı testi uygulandıktan sonra katılımcıların teste verdikleri cevaplar derinlemesine incelenmiş, kullanılan çözüm stratejileri ve yanlış cevapların nedenleri ile ilgili ilk kodlar oluşturulmuştur. Bireysel görüşmeler için katılımcılar bu kodlara göre seçilmişlerdir. Bireysel görüşmelerden elde edilen veriler katılımcıların başarı testindeki cevaplarından elde edilen kodlarla kıyaslanmış ve desteklenmiştir. Görüşme sorularının araştırma sorularına yönelik olup olmadığını belirlemek için iki uzmandan görüş istenmiş ve her sınıf seviyesinden bir öğrenci ile pilot görüşmeler yapılmıştır.

Görüşmeler yaklaşık olarak 30 dakika sürmüştür ve bu süre içerisinde katılımcılardan başarı testine verdikleri cevapların açıklanması istenmiştir. Öğrencilerin açıklamaları "Bu sonuca nasıl ulaştın?", " Neden böyle düşündün?", "Nasıl bir strateji kullandın?" gibi açık uçlu sorularla ilerlemiştir.

Görüşmeler yapıldıktan sonra bu görüşmelerin transkriptleri yapılmıştır. İlköğretim bölümündeki bir doktora öğrencisi ile birlikte ortaya çıkan temalarla ilgili çalışmalar yapılmıştır. Ayrıca, öğrencilerin başarı testindeki cevaplarının en az %10'u ortak kodlayıcı ile birlikte incelenmiştir.

Veri Toplama Süreci

Çalışmanın verileri 2012-2013 eğitim öğretim yılının ikinci döneminde toplanmıştır. Gerekli etik izinler alındıktan sonra Mart ayında öncelikle pilot çalışmalar yapılmış, Nisan ve Mayıs aylarında asıl çalışmanın verileri toplanmıştır. Başarı testi öğrencilerin ders saatlerinde araştırmacı tarafından uygulanmıştır. Başarı testinin uygulamasından birkaç hafta sonra seçilen öğrencilerle bireysel görüşmeler yapılmıştır.

Veri Analizi

Çalışmanın amaçlarına ulaşması için iki farklı veri çeşidi analiz edilmiştir. Bunlar öğrencilerin başarı testindeki cevapları ve bireysel görüşmelerin yazılı transkriptleridir. Öncelikle öğrencilerin başarı düzeylerinin, çözüm stratejilerinin ve yanlış cevaplarının nedenlerinin belirlenmesi için başarı testindeki cevaplar incelenmiştir. Daha sonra başarı testinin incelenmesinden elde edilen kodların kıyaslanması ve desteklenmesi için görüşmelerin transkriptleri analiz edilmiştir.

Öncelikli olarak öğrencilerin başarı testindeki cevapları her problem için boş, doğru ve yanlış olarak kodlanmıştır. Daha sonra doğrusal ve doğrusal olmayan problemdeki bu cevapların sıklıkları ve yüzdeleri ayrı ayrı hesaplanarak başarı düzeyleri belirlenmiştir.

Katılımcıların çözüm stratejileri ve yanlış cevaplarının nedenlerinin belirlenmesi için öğrencilerin testteki cevapları ve görüşme transkriptleri derinlemesine incelenmiştir. Bunun yanı sıra alanyazındaki ilgili çalışmalardan kategoriler taranmıştır. Tüm bu süreçte diğer bir ilköğretim bölümü doktora öğrencisi ile çalışılmış ve uzlaşma şartı aranmıştır.

Araştırmanın Varsayımları ve Sınırlılıkları

Araştırmanın ilk varsayımı öğrencilerin uzunluk, çevre, alan ve hacim kavramları ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarılarının geliştirilen test aracılığıyla ölçülebileceğidir. Ayrıca öğrencilerin testi cevaplarken ve bireysel görüşmelerde içten, açık yürekli ve işbirlikçi oldukları varsayılmıştır.

Çalışmanın katılımcıları küme örneklemesi yoluyla seçilmesi sonuçların daha geniş bir popülasyona genellemesini sınırlandırmaktadır. Ayrıca görüşmeler için seçilen öğrencilerin amaca yönelik seçilmesinden dolayı görüşmelerden elde edilen veriler bu katılımcılarla sınırlı olabilir.

Bulgular ve Tartışma

Bu çalışmanın üç amacı bulunmaktadır. Katılımcıların uzunluk, çevre, alan ve hacim ile ilgili doğrusal ve doğrusal olmayan problemlerdeki başarılarının belirlenmesi birinci amaçtır. İkinci amaç ise öğrencilerin bu problemler için kullandıkları çözüm stratejilerinin belirlenmesidir. Üçüncü ve son amaç ise katılımcıların bu problemlerdeki yanlış cevaplarının incelenmesidir.

Çalışmanın bulguları öğrencilerin doğrusal problemlerdeki başarılarının doğrusal olmayan problemlerdeki başarılarına göre yüksek olduğunu göstermiştir. Bu bulgular geçmiş çalışmaların sonuçları ile tutarlılık göstermektedir (De Bock vd., 1998; Modestou & Gagatsis, 2009; Van Dooren vd., 2004). Doğrusal problemlerde doğrusal olmayan problemlere kıyasla görülen yüksek başarının sebebi öğrencilerin doğrusal problemlere olan aşinalığı ve doğrusal olmayan problemlere olan yabancılıklarından kaynaklanıyor olabilir (De Bock vd., 1998). Altıncı sınıf öğrencilerinin doğrusal problemlerdeki başarılarının yedi ve sekizinci sınıf öğrencilerinin başarılarına göre biraz daha düşük olduğu görülmüştür. Bunun yanı sıra, sekizinci sınıf öğrencilerinin doğrusal olmayan problemlerdeki başarılarının altı ve yedinci sınıf öğrencilerine göre biraz daha yüksek olduğunu göstermiştir. Sınıf seviyesine göre artan başarının sebebinin Vlahovic-Stetic ve arkadaşlarının (2010) belirttiği gibi öğrencilerin bilişsel gelişimleri ve bilgi birikimlerinden kaynaklandığı öne sürülebilir.

Öğrencilerin doğrusal problemlerdeki çözüm stratejileri incelendiğinde öğrencilerin çoğunlukla iki strateji kullandıkları görülmüştür. Birinci strateji problemdeki kavramlar arasındaki ilişkiyi düşünmeden verilen sayılar arasında direkt olarak orantı kurmayı içeren kuşkulu (questionable) orantıdır. Bu stratejiyi kullanan öğrencilerin bazıları görüşmelerde çevrenin ya da alanın bulunması gerekmediğini belirtirken bazıları bulmaları gerektiğini fakat problemi çözerken bunu düşünmediklerini dile getirmişlerdir. Buradan öğrencilerin yalnızca problem cümlesine odaklanarak önceden bilinen işlemleri direkt olarak uygulama eğilimi gösterdikleri yargısına varılabilir. Diğer bir sebep ise, öğrencilerin kenar ve çevre arasındaki doğrusal ilişkinin farkında olmamaları olabilir.

İkinci strateji problemdeki kavramlar ve değişkenler arasındaki doğrusal ilişkileri düşünüp bu değişkenler arasında orantı kurmayı içeren mantıklı (reasonable) orantıdır. Bu öğrencilerin kenar ve çevre arasındaki doğrusal ilişkinin farkında olmayabilecekleri fakat iki şeklin çevrelerini bularak bu çevreler arasında orantı kurabildikleri sonucuna varılabilir. Bu stratejilerin sıklıkları incelendiğinde ise birçok öğrencinin doğrusal-çevre problemleri için kuşkulu orantı stratejisini kullandıkları ve mantıklı orantı stratejisinin az sayıda öğrenci tarafından kullandığı görülmüştür.

Öğrencilerin doğrusal olmayan problemlerdeki çözüm stratejileri incelendiğinde ise öğrencilerin genellikle iki strateji kullandıkları görülmüştür. Birinci strateji alanı veya hacmi yalnızca iki şeklin kenarları arasındaki doğrusal ilişkilerden yararlanarak bulmayı içeren kenar-kenar-alan/hacim stratejisidir. İkinci strateji ise iki şeklin alanları ya da hacimleri arasındaki ikinci ya da üçüncü derece ilişkilerin direkt olarak kurulması ile ilgili olan kenar-alan/hacim stratejisidir. Birinci stratejinin ikinci stratejiden farklı olarak doğrusal olmayan ilişkileri içermediği görülmüştür. Öğrencilerin doğrusal olmayan problemler için çoğunlukla birinci stratejiyi kullandıkları görülmüştür. Yani, birçok öğrencinin geometrik şekillerin uzunlukları ve alanları ya da hacimleri arasındaki doğrusal olmayan ilişkileri kuramadıkları sonucuna varılabilir. Doğrusal olmayan ve karelerin alanları ile ilgili bir problem çözüldüğünde en çok kullanılan strateji kenar-kenar-alan stratejisi olmuştur. Bu sonuç karenin alanı olarak karesel bir sayının verilmesi ve dolayısıyla öğrencilerin karelerin kenar uzunluklarını bulabilmeye olanak sağlamasından kaynaklanabilmektedir. Fakat düzenli olmayan şekiller verildiğinde kullanılan strateji ise kenar-alan/hacim stratejisi olmuştur. Bunun sebebi ise birinci stratejinin bu tür problemlerin çözümünde geçerli olmadığından kaynaklıyor olabilir.

Doğrusal ve doğrusal olmayan problemler için kullanılan stratejilerle ilgili sonuçlar düşünüldüğünde, bulgular öğrencilerin doğrusal ve doğrusal olmayan problemler için sınırlı sayıda strateji kullandıklarını göstermiştir. Birçok katılımcının cevaplarında uzunluk ile çevre arasındaki doğrusal ilişkilere, uzunluk ile alan ya da uzunluk ile hacim arasındaki doğrusal olmayan ilişkilere dayalı bir akıl yürütme olmadığı görülmüştür.

Bu çalışmada öğrencilerin sorulara verdikleri yanlış cevapların nedenleri de incelenmiştir. Doğrusal ve doğrusal olmayan problemlere verilen yanlış cevapların ortak nedenleri ve doğrusal problemlerdeki yanlış cevaplara özel ve doğrusal olmayan problemlerdeki yanlış cevaplara özel nedenler bulunmuştur. Çalışmanın bulguları doğrusal ve doğrusal olmayan problemlerdeki yanlış cevapların ortak nedenleri geometrik bilgideki yetersizlikler, toplamsal ve çarpımsal ilişkilerin yanlış anlamlandırılması, işlemsel hatalar ve eksik cevaplardır. Bu nedenler en sık görülenden en az görülene doğru sıralanmıştır. Literatürdeki diğer çalışmalarla tutarlı olarak bu çalışmada da öğrencilerin geometrik bilgilerinin yetersiz olduğu görülmüştür (Clements & Ellerton, 1996; EARGED, 2003, 2005; Steele, 2006; Tan-Şişman, 2010). Bu yetersiz bilgilerin sebebi olarak geometri öğretiminin şekillerin özellikleri arasındaki ilişkilere önem vermemesi ve işlemsel becerilerin öne çıkarılması durumları gösterilebilir. Diğer çalışmaların sonuçları ile benzer bir şekilde bu çalışmada da öğrencilerin toplamsal ve çarpımsal ilişkileri anlamlandırmada zorluk çektikleri görülmüştür (Hart, 1984; Van Dooren vd., 2010). İşlemsel hatalar ve eksik cevaplar birer neden olarak görülse de sıklıklarının düşük düzeyde olduğu görülmüştür.

Doğrusal problemlerdeki yanlışlara özel bir neden doğru orantı yerine ters orantı kullanılması ile ilgili olan orantısal durumların yanlış anlamlandırılmasıdır. Alanyazında öğrencilerin oran ve orantı konuları ilgili yaşadıkları zorluklara birçok çalışmada yer verilmiştir (Resnick & Singer, 1993; Weinberg, 2002). Benzer olarak, doğrusal olmayan problemlerdeki yanlışlara özel bir neden doğrusal olmayan durumlar için doğrusal stratejiler kullanmayı içeren doğrusallık yanılsamasıdır. Alanyazındaki birçok çalışmanın sonuçlarına paralel olarak, doğrusallık yanılsaması yani geometrik cisimlerin kenar uzunlukları ve alanları ya da hacimleri arasında doğrusal bir ilişki olduğu varsayımı doğrusal olmayan problemlerdeki düşük başarının en büyük sebebi olarak bulunmuştur. (De Bock vd., 1998; 2002; Van Dooren vd., 2007; Vlahovic-Štetic vd., 2010). Doğrusallık yanılsamasının sebebi olarak ilk olarak soruların soruluş biçimi gösterilebilir. Van Dooren ve arkadaşlarının (2003) belirttiği gibi problemlerin verilmeyeni bulma problemi olarak sorulmasının öğrencileri otomatik olarak orantı kurmaya yönelttiği söylenebilir. İkinci sebep olarak ise ortaokulda görülen birçok durumun doğrusal stratejilerle çözülebilmesinden dolayı doğrusal ilişkilere verilen önemden kaynaklandığı söylenebilir. Fakat Van Dooren ve arkadaşlarının (2003) öne sürdüğü gibi bu durum öğrencilerin doğrusal olmayan problemler için farklı stratejiler geliştirmelerine engel olabilmektedir. Son olarak, doğrusal stratejilerin kolay ve sezgisel olması seebiyle öğrencilerin bu stratejileri kullandıkları öne sürülebilir (De Bock vd., 2002).

Doğurgalar

Bu çalışmanın sonuçları ilkokul öğretmenleri, matematik öğretmenleri, program geliştiriciler ve öğretmen eğiticileri için önemli bilgiler sunmaktadır.

İlk olarak öğrencilerin doğrusal problemlerdeki başarılarının doğrusal olmayan problemlerdeki başarılarına göre yüksek olduğu görülmüştür. Bu sebepten dolayı ilk olarak matematik müfredatında ve matematik derslerinde doğrusal olmayan durumlara değinilmesi önerilmektedir. Buna ek olarak, öğrencilerin doğrusal ve doğrusal olmayan problemlerde sınırlı sayıda strateji geliştirdikleri görülmüştür. Bu sebepten dolayı doğrusal ve doğrusal olmayan problemlerin anlamlandırılmasına ve bu problemler için farklı stratejiler geliştirilmesine önem verilmesi önerilmektedir.

Öğrencilerin doğrusal ve doğrusal olmayan durumları anlamlandırmaları ve bu durumların farklılıklarını anlamlandırmaları için modelleme etkinlikleri ve teknolojik araçlarla çalışmalar yapılması önerilmektedir.

Öğrencilerin düşük başarılarının ve sınırlı sayıda strateji kullanmalarının sebebi öğretmenlerin eksik bilgilerinden kaynaklanıyor olabilir. Bu sebepten dolayı okullardaki öğretmenlere ve öğretmen adaylarına yönelik seminerler organize edilebilir.

İleriki Çalışmalar için Öneriler

Aynı çalışma farklı katılımcılarla yürütülebilir. Örneğin, tüm ülkeden örneklem seçilerek daha fazla katılımcı ile çalışmalar yürütülebilir. Ayrıca, aynı çalışma lise öğrencileri, öğretmen adayları ya da öğretmenlerle yapılabilir.

Çalışmanın metodunda değişiklikler yapılarak farklı çalışmalar önerilebilir. İlk olarak, uzun süreli bir çalışma yürütülebilir. Bunun yanı sıra, kavramsal anlamaya önem veren, teknolojik araçların kullanıldığı, modeller ve modelleme etkinlikleriyle desteklenen deneysel çalışmalar yürütülebilir. Son olarak, çalışmada kullanılan teste soru eklemesi yapılabilir ya da farklı alanlarda (grafikler ya da olasılık) doğrusal ve doğrusal olmayan problemler içeren testler geliştirilebilir.

Kaynakça

CCSSM (2010). Common core state standards for mathematics. Washington, DC: Author. Re-trieved from Common Core State Standards website: http://www.corestandards.org/assets/CCSSI_Math\%20Standards.pdf

Chapin, S. H. & Johnson, A. (2000). *Math matters grades K-6: Understanding the math you teach*. Sausalito, CA: Math Solutions.

Chappell, M. F. & Thompson, D. R. (1999). Perimeter or area? Which measure is it? *Mathematics Teaching in the Middle School, 5*(1), 20–23.

Cheng, D., Star, J. R., & Chapin, S. (2013). Middle School Students' Steepness and Proportional Reasoning. *New Waves-Educational Research & Development*, *16*(1), 22-45.

Clements, M. A. & Ellerton, N. F. (1996). *Mathematics Education Research: Past, Present and Future*. Bangkok, Thailand: UNESCO.

Cramer, K., & Post, T. (1993). Connecting research to teaching proportional reasoning. *Mathematics Teacher, 86,* 404–407.

De Bock, D., Van Dooren, W., Janssens, D., & Verschaffel, L. (2002). Improper use of linear reasoning: An in-depth study of the nature and the irresistibility of secondary school students' errors. *Educational Studies in Mathematics, 50*(3), 311-334.

De Bock, D., Verschaffel, L., & Janssens, D. (1998). The predominance of the linear model in secondary school students' solutions of word problems involving length and area of similar plane figures. *Educational Studies in Mathematics, 35*(1), 65-83.

EARGED (2003). TIMSS 1999 Üçüncü Uluslararası Matematik ve Fen Bilgisi Çalışması Ulusal Rapor. Ankara: MEB.

EARGED (2005). PISA 2003 Projesi Ulusal Nihai Rapor. Ankara: MEB.

Ebersbach M., Lehner M., Resing W.C.M., Wilkening F. (2008). Forecasting exponential growth and exponential decline: Similarities and differences. *Acta Psychologica, 127*, 247–257.

Ebersbach, M., & Wilkening, F. (2007). Children's intuitive mathematics: The development of knowledge about nonlinear growth. *Child Development, 78*(1), 296-308.

Freudenthal, H. (1983). *Didactical Phenomenology of Mathematical Strategies*. D. Reidel, Dodrecht.

Hart, K. (1984). Ratio: *Children's strategies and errors. A report of the strategies and errors in secondary mathematics project*. London, United Kingdom: NFER-Nelson.

Kaplan, A., İşleyen, T., & Öztürk, M. (2011). 6. sınıf oran orantı konusundaki kavram yanılgıları. *Kastamonu Eğitim Dergisi, 19*(3), 953-968.

Karplus, R., Pulos, S., & Stage, E. K. (1983). Early adolescents' proportional reasoning on "rate" problems. *Educational Studies in Mathematics*, *14*(3), 219-233.

Kestell, M. L. & Kubota- Zarivnij, K. (2013). Proportional Reasoning in Number Sense and Numeration. Abacus, *Ontario Association for Mathematics Education, 52*(1).

Lobato, J., & Thanheiser, E. (2002). *Developing understanding of ratio as measure as a foundation for slope. Making sense of fractions, ratios, and proportions*, 162-175.

Milli Eğitim Bakanlığı [MEB], 2013. Ortaokul Matematik Dersi (5, 6, 7 ve 8. Sınıflar) Öğretim Programı. Ankara: MEB Yayıncılık.

Modestou M. & Gagatsis A. (2009). Proportional Reasoning: The Strategies Behind the Percentages. *Acta Didactica Universitatis Comenianae, 9*, 25-40.

Modestou, M., & Gagatsis, A. (2007). Students' Improper Proportional Reasoning: A result of the epistemological obstacle of "linearity". *Educational Psychology, 27*(1), 75-92.

National Council of Teachers of Mathematics (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: The Council.

National Council of Teachers of Mathematics (2000). *Principles and Standards for school mathematics.* Reston, VA : Author.

Post, T., Behr, M., & Lesh, R. (1988). Proportionality and the development of pre-algebra understanding. *The ideas of algebra, K–12*, 78-90.

Resnick, L., & Singer, J. (1993). Protoquantitative origins of ratio reasoning. In T. Carpenter, E. Fennema, & T. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 107-130). Hillsdale, New Jersey: Lawrence Erlbaum Associates

Sherman, H. & Randolph T (2004). Area and Perimeter: "Which is Which and How Do We Know?", *Research for Educational Reform, 9* (3), 25-36.

Spinillo A G. & Bryant P. E. (1999). Proportional Reasoning in Young Children: Part–Part Comparisons about Continuous an Discontinuous Quantity. *Mathematical Cognition, 5(2),* 181–197.

Tan-Şişman, G. (2010). *Sixth grade students' conceptual and procedural knowledge and word problem solving skills in length, area, and volume measurement.* Unpublished Doctoral Dissertation, Middle East Technical University, Ankara.

Thompson, T. D., & Preston, R. V. (2004). Measurement in the middle grades: Insights from NAEP and TIMMS. *Mathematics Teaching in the Middle School, 9*, 514-519.

Valverde, G., & Martínez, E. C. (2012). Prospective Elementary School Teachers´ Proportional Reasoning. *PNA, 7*(1), 1-19.

Van De Walle, J. A., Karp, K. S., & Bay-Williams J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th Ed.). Boston, MA: Pearson Education, Inc.

Van Dooren, W., De Bock, D. & Verschaffel, L. (2010). From addition to multiplication... and back: The development of students' additive and multiplicative reasoning skills. *Cognition and Instruction, 28*, 360–381.

Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2004). Remedying secondary school students' illusion of linearity: A teaching experiment aiming at conceptual change. *Learning and Instruction, 14*(5), 485-501.

Van Dooren, W., De Bock, D., Verschaffel, L., & Janssens, D. (2003). Improper applications of proportional reasoning. *Mathematics Teaching in the Middle School, 9*(4), 204-209.

Vlahović-Štetić, V., Pavlin-Bernardić, N., & Rajter, M. (2010). Illusion of linearity in geometry: Effect in multiple-choice problems. *Mathematical Thinking and Learning, 12*(1), 54-67.

Weinberg, S. (2002). Proportional Reasoning: One Problem, Many Solutions! In B.Litwiller & G. Bright (Eds.), *Making Sense of Fractions, Ratios,and Proportions: 2002 Yearbook*. Reston, VA: NCTM.

Wright, V. (2005). Proportional reasoning: Some rational thoughts. In J. Mousley, L. Bragg, & C. Campbell. (Eds.), *Mathematics: Celebrating achievement* (pp. 363-375). Melbourne: Mathematical Association of Victoria.

APPENDIX H. TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü	
Sosyal Bilimler Enstitüsü	X
Uygulamalı Matematik Enstitüsü	
Enformatik Enstitüsü	
Deniz Bilimleri Enstitüsü	

YAZARIN

Soyadı : Ayan
Adı : Rukiye
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI : (İngilizce): Middle School Students' Achievement Levels, Solution Strategies and Reasons Underlying Their Incorrect Answers in Linear and Non-linear Problems

TEZİN TÜRÜ : Yüksek Lisans [X] Doktora [ ]

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. [X]
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. [ ]
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz. [ ]

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: **İMZA:**