AN ANALYSIS OF BIST30 INDEX OPTIONS MARKET

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ABSTRACT

AN ANALYSIS OF BIST30 INDEX OPTIONS MARKET

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Index Options have been used more than forty years in international financial markets. These types of options have been used in Turkey since December 21, 2012. They are very important derivatives as they cause leverage effect on returns, and they also help to get information about the investors' expectations related to future prices of the financial assets. In addition to these features they are perfect instruments in order to hedge future risks.

In this thesis, we analyze the very new index option markets in Turkey. The analysis is performed for European options which exist for only one index (BIST30). The extension to American Options may be done by using similar methods used in this thesis. We compare observed and theoretical values of options and construct well known portfolios (Put-Call Parity) in order to check for arbitrage opportunities. We show that there were arbitrage opportunities in the market and they began to diminish after some point. We also argue that Black-Scholes-Merton model does not price efficiently index options in Turkey or the pricing of options in Turkey is not efficient due to these differences between observed and theoretical values and arbitrage opportunities at some data points. We believe that this first analysis of index options market in Turkey and hopefully proceeding papers will help to increase people's awareness of options and consequently increase the volume of the market.

Key words: Option Pricing, Observed Option Prices, Theoretically Calculated Option Prices

ÖZ

BIST30 ENDEKS OPSIYONLARI ÜZERİNE İNCELEME

Akyapı, Berkay
Yüksek Lisans, İktisat Bölümü
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Opsiyonlar 40 yıldan fazla bir zamandır finansal marketlerde kullanılmaktadır. Türkiye'de ise 21 Aralık 2012 tarihinden itibaren kullanılmaya başlanmışlardır. Opsiyonlar yüksek kaldıraç efekti ve hisse senetlerinin gelecekteki hareketlerini tahmin etmemize olanak sağladıkları için çok önemli türev araçlarıdır. Bu özelliklerinin yanında opsiyonlar muhtemel riskleri önlemek için de çok verimli finansal araçlardır.

Bu çalışma Türkiye'de çok yeni kullanılmaya başlanmış olan opsiyon piyasasının durumunu incelemek üzere yapılmıştır. Analiz, şu an için sadece bir endekste yazılabilen (BIST30) avrupa tipi opsiyonlar için yapılmıştır. Amerikan tipi opsiyonlar için aynı analiz benzer şekilde yapılabilir. Marketi analiz edebilmek için öncelikle piyasada bilgileri yer alan gerçek fiyatlarla teorik fiyatlar karşılaştırılmış ve risksiz kazanç olanağının varlığının kontrol edilebilmesi amacıyla basit portfolyolar oluşturulmuştur. Bu çalışma ile, Türkiye'de opsiyon piyasalarında risksiz kazanç olanağı bulunduğunu ve bunun belli bir noktadan sonra azalmaya başladığı gösterilmektedir. Ayrıca teorik ölçüm ile gözlemlenen ölçümlerde fark görülen noktalarda Türkiye'de Black-Scholes-Merton modelin verimli fiyatlama yapmadığı veya marketlerde gözlenen opsiyon fiyatlarının verimli olmadığı anlaşılmıştır. İlk kez yapılacak olan bu analizle birlikte Türkiye'deki yatırımcıların opsiyonlara olan farkındalığının ve opsiyon piyasalarındaki hacmin arttırılması hedeflenmektedir.

Anahtar Kelimeler: Opsiyon Fiyatlandırması, Gözlemlenen Opsiyon Fiyatları, Teorik Olarak Hesaplanan Opsiyon Fiyatları

To My Family

ACKNOWLEDGMENTS

During my undergraduate years in Electrical and Electronics Engineering in Bilkent University I had always been busy. Sometimes we asked ourselves the reason and our motivation, however new tasks arrived so that we stopped thinking. After graduating and beginning to work, I realized that we were inside a river that flows intensively. Yet, now that I have a more straightforward life in which it is nearly all defined what I will do for the rest of my life, it felt like I am sitting in the border of the river and watch it flow. This thesis is for me a pebble that I can throw into that river and this makes me believe that it will continue to flow with that river even when I stop to watch. Therefore, I want to express my gradidutes to those people who helped me throw this pebble.

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CHAPTER 1

1. INTRODUCTION

Finance derivatives are becoming more and more important presently in the daily and academic life. It is becoming important in daily life because small investors can take higher risks (hence higher returns) with smaller amounts of money thanks to derivatives. It can also be used for institutions in order to hedge future risks. For example assume that you own BIST30 index and you want to sell it one year later. You are not sure if the asset price will move up or down in the future. In order to hedge the risk of a down movement you can long a put option, in which you have the right to sell the stocks at a determined price even if it goes below to that predetermined price. Hence, you can determine your minimum cash flow in the future. If the price goes up than that determined price you have the right (as you longed the put) to not to exercise your option, hence your maximum gain may go as much as the stock price increment. However, due to the leverage effect options contain higher risks for the investors using it, that is why some people see it as "civilized gambling" [1]. Especially when the investors are not aware of the risks, this concern may not be an exaggeration anymore. We may give the Barings Bank as an example to this risk [2]. Barings bank was an old bank founded in 1762 and was a strong bank which has even financed Louisiana Purchase [3], however it went to bankruptcy due to misuse of option trading.

Options are mostly being priced by using the Nobel Prize winning Black-Scholes-Merton formula [4] [5]. Nevertheless there are studies going on to improve option pricing. In order to find out a faster way to solve the equation, there are studies which use the Fourier Transform like [6] and [7] for option valuation. On the other hand, Black-Scholes-Merton model assumes that the volatility of the stock price stays the same until the expiration date and as it is explained in this thesis option prices are very sensitive to volatility. However, we know that the volatility will change frequently even in one week. So as to prevent this effect and capture the change in volatility researches are being done to price the option with Hidden

Markov Models. See, for example, [9], [10] and [11] for more details about these models.

Index Options market has recently become popular in Turkish Financial Markets. We care about functioning and efficiency of markets as economists. The main question of this thesis is: Is the Options market efficient in Turkey? To answer this question, we first analyze the observed data and compare the observed prices with the theoretically calculated Black-Scholes-Merton prices. We also analyze deviations from put-call parity to show that there were arbitrage opportunities in the market and these opportunities began to diminish after some point. These deviations can be used in order to have additional information about future price movements of BIST30 index as well as riskless profits. These finding imply that the options market may not be efficient in Turkey.

The rest of the thesis is organized as follows. Firstly, some background information is given for options and Turkey's options market. Chapter 2 provides the theoretical model and related information in order to completely specify our theoretical benchmark, the Black-Scholes-Merton approach to pricing options. Chapter 3, introduces the real data that is used to analyze the market. In Chapter 4, our analysis comparing observed and theoretically calculated prices for BIST30 options is given using options with different strike prices. Chapter 5 contains concluding remarks and discussion.

1.1. OPTIONS

In this subsection a brief textbook introduction about options is provided. We refer the reader to [11] and [12] for more detailed introduction. Options are derivatives used in the stock markets. A relatively simple derivative is a forward or future contract in which parties basically agree to buy or sell an asset at a certain future time for a certain price. In the case of forward contracts the parties have to buy or sell at the agreed price on the agreed date. In the options the difference is that the holder has "the right" to do something. That is the holder may or may not exercise the option at the strike date. As a result acquiring an option has a price whereas forward contracts are free. The thesis study is about finding if this pricing is done accurately in Turkey's financial markets. Firstly, let's summarize some basic concepts about options:

Strike (Exercise) Price: The price in the contract that the holder of the option can buy or sell the asset.

Maturity (Expiration Date): The date specified in the contract for the realization of buying or selling.

Call Option: Gives the holder the right to buy the underlying asset at the specified strike price in the expiration date.

Put Option: Gives the holder the right to sell the underlying asset at the specified strike price in the expiration date.

Option Positions: There are two parties in an option contract: a buyer and a seller. The buyer gives the price for the option and buys the right. In this case, the holder is in a ***long position.*** The seller receives the price for the option and gives the right for his/her asset to be sold or bought to the buyer. In this case, seller is said to be in a ***short position.*** To sum up, there are four types of position for options: A long position in a call option, a short position in a call option, a long position in a put option and a short position in a put option. These positions and option types are analyzed more deeply with graphical representation of profits in the following subsections. For more detailed definitions and explanation related to option mechanics, see [11] and [12].

1.1.1. Factors Affecting Option Prices

Option prices are affected by six factors [11]:

1. S_0: Current Stock Price
2. K: Strike Price
3. T: Time to expiration
4. σ: Volatility of the stock price

5. r: The risk-free interest rate
6. Dividends expected during the life of the option.

For this thesis, we assume for simplicity that there are no dividends during the life of the option.

The effects of the factors can be seen in the table shown below [11], where '+' indicates that as the variable increases the price of the option increases and '-' indicates that as the variable increases the option price decreases:

Table 1: Summary of the effects of one variable on the stock price holding all the other variables fixed

Variable	European Call	European Put
S_0	+	-
K	-	+
T	Uncertain	Uncertain
σ	+	+
R	+	-
Dividends	-	+

Source: [11]

Now, we analyze the profits for different option types and positions against the final stock price in the next subsection.

1.1.2. Payoffs from Positions in European Options:

In the graphs below it is assumed that the stock price when the option is settled is 90 TL. The strike price is 100 TL, and assume that the option price calculated with some known time to expiration, risk-free interest rate, volatility and given current stock price and strike price is 5 TL. We also assume for simplicity that the option contract is signed for only one share of the stock. With this given parameters, we can now analyze the payoffs from different European options.

Long Call:

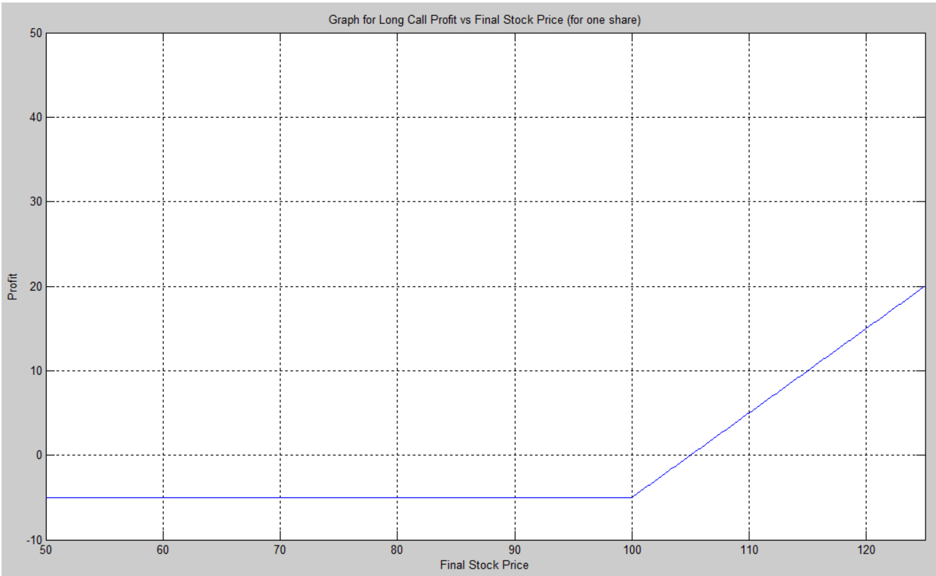

Figure 1: Plot for long position in a European call option profits versus final stock price (Source: Author's Calculations)

Figure 1 shows that below the strike price the holder of the option does not exercise his right to buy the share, because he or she can buy it at a smaller price from the market (and he or she paid for this right while settling the option). As a result below the strike price the holder will have a loss equal to the option price (5 TL in our case). Above the strike price the holder will buy share by the predetermined strike price (100 TL in our case). Above the strike price as the stock price increases the profit will increase for the long call position. Therefore, the payoff from a long European call option is $max(S_T - K, -Option\ Price)$.

Short Call:

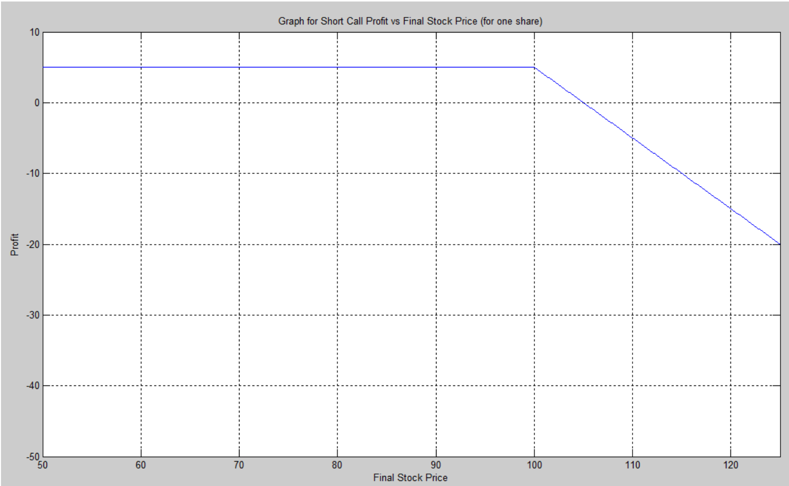

Figure 2: Plot for short position in a European call option profits versus final stock price (Source: Author's Calculations)

Figure 2 shows that the investor in the short position will have a profit equal to the amount he received (5 TL in our case) for giving the right to buy the stock at the maturity by the strike price regardless to the actual stock price at that time (S_T), because as we argued above the investor of the long position will not buy the stock at a price below the strike price. For higher values than the strike price (100 TL in our case) as the price of the stock increases the profit decreases for the investor in a short position in a European call option. Therefore, the payoff from a short European call option is *min*(K-S_T, Option Price).

Long Put:

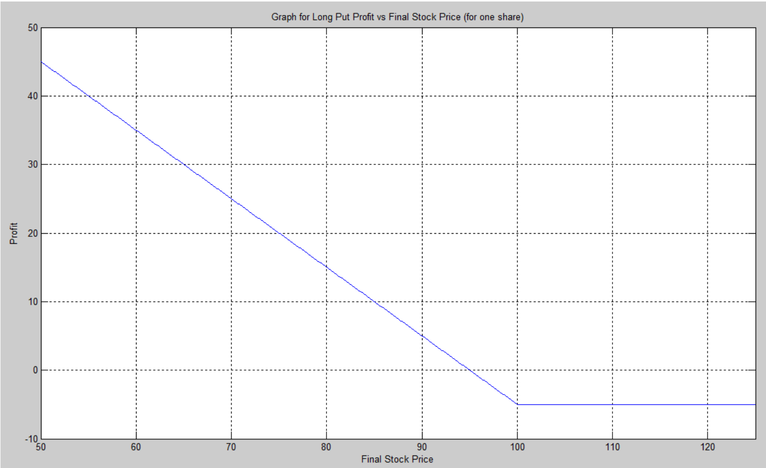

Figure 3: Plot for long position in a European put option profits versus final stock price (Source: Author's Calculations)

From Figure 3 we can infer that the investor in a position of a long put option will not sell the stock at a price above the strike price as he or she can sell it at a higher price in the market (again the investor paid a price (5 TL in our case) for this right). When the stock price at the maturity S_T is lower than the strike price (100 TL in our case) will sell the stock. Hence, the payoff for a long position in a European put option is *max* (K-S_T, -Option Price).

Short Put:

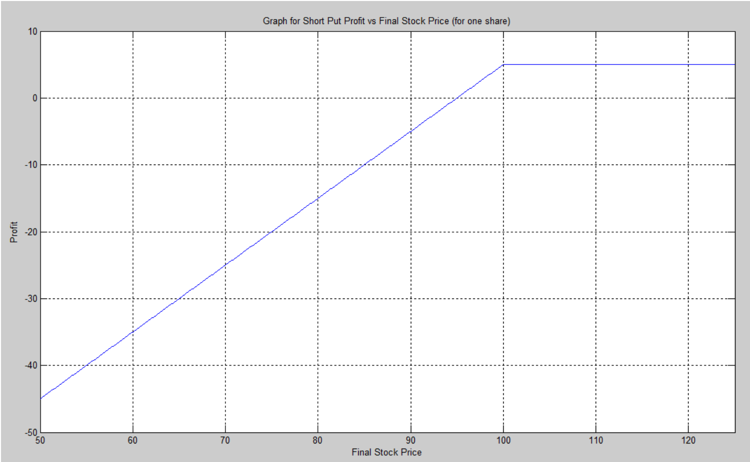

Figure 4: Plot for short position in a European put option profits versus final stock price (Source: Author's Calculations)

As it can be seen from Figure 4, the investor in a short position in a European put option will have a profit equal to the amount he received (5 TL in our case) for giving the right to the other investor for selling the stock to him/herself when the stock price the maturity, S_T, is above the strike price, because the investor in the long position will not sell the stock at that price. When S_T is below the strike price the investor in the short position will have to buy the stock at the specified strike price, hence the investor will be in a loss, as normally he or she could buy the stock at a cheaper price at the maturity time in the market. To conclude, the payoff for a short position in a European put option is *min*(S_T-K, Option Price).

The MATLAB codes for the graphs shown in this section can be found in Appendix B.

1.1.3. A Summary about Turkey's Options Market

Options are being use in Turkey since 21 December 2012 [13]. There are two types of options used presently in Turkey: single stock options and equity index options [14]. Presently, the underlying securities that are used for settling option contracts are:

T.Garanti Bankası A.Ş.	GARAN	Türk Hava Yolları A.O.	THYAO
T. İş Bankası A.Ş.	ISCTR	Ereğli Demir ve Çelik Fabrikaları T.A.Ş	EREGL
Akbank T.A.Ş.	AKBNK	H.Ö. Sabancı Holding A.Ş.	SAHOL
Türkiye Vakıflar Bankası T.A.O.	VAKBN	Turkcell İletişim Hizmetleri A.Ş.	TCELL
Yapi ve Kredi Bankası A.Ş.	YKBNK	Tüpraş- Türkiye Petrol Rafinerileri A.Ş.	TUPRS

Figure 5: Underlying securities for single stock options in Turkey (Source: [15])

The rules for a contract of single stock option in Borsa İstanbul are given below [16]:

Option Class: *Call and put options*

Exercise Style: *American i.e. exercisable on or before the expiry date*

Contract Size: *100 shares of underlying stock per contract*

Tick Size: *Prices are offered for the premium value of one underlying share. TL 0.01 per share = TL 1.00 (contract size 100 shares)*

Contract Months: *Current month, the next calendar month and the next cycle month. Cycle months are March, June, September and December*

Settlement Method: *Physical delivery of the underlying security. No automatic exercise on expiration day.*

Trading Hours: *Continuous trading from 09:15 to 17:40 (local time) with a non-trading period between 12:30-14:00 hours.*

Settlement Period: *T+3 (3 business days after the exercise assignment)*

Daily Settlement Price: *Daily settlement prices will be determined at the close of the trading session as follows:*

- Weighted average price of all trades performed within the last 10 minutes before the closing of the trading session,

- If number of trades performed within the last 10 minutes before the closing of the trading session is less than 10, the weighted average of the last 10 trades before the closing will be set as the settlement price.

Expiry Date: *Last business day of the contract month.*

Last Trading Day: *Last business day of the contract month.*

Daily Price Limit: *None.*

Margins:

- *Initial Margin: It is set by the SPAN portfolio margining method.*
- *Required Collateral: The sum of initial margin and physical delivery collateral.*
- *Maintenance Margin: 75% of the required collateral*
- *Physical delivery collateral will be taken only in cases of option exercises*

The thesis study is about European option pricing, therefore single stock options are not in the scope of this master's thesis study.

The only underlying security involved with the second type of the option used in Turkey (equity index option) is BIST 30 Price Index. The rules for a contract of equity index options in Borsa İstanbul are:

Option Class: *Call and put options.*

Exercise Style: *European*

Contract Size: *Underlying security is the 1/1000 of the index values. Contract size for the index options is 100 underlying securities.*

Tick Size:*Prices are offered for the premium value of one underlying security. TRY0.01 per underlying security=TRY1.00 (contract size 100 underlying securities).*

Contract Months: *Current month, the next calendar month and the next cycle month. Cycle months are March, June, September, and December.*

Settlement: *Cash settlement.*

Trading Hours: *Continuous trading from 9:15 to 17:45 (local time) with a non-trading period between 12:30-14:00 hours.*

Settlement Period: *T+1 (first day following the expiry date)*

Daily Settlement Price: *Daily settlement prices will be determined at the closing of the trading session as follows:*

- Weighted average price of all trades performed within the last 10 minutes before the closing of the trading session,

- If number of trades performed within the last 10 minutes before the closing of the trading session is less than 10, weighted average of the last 10 trades before the closing will be set as the settlement price.

Final Settlement Price: *Index values that will be the basis for final settlement price will be closing index values that are calculated by the closing price in the closing session of the underlying securities in the index.*

Final settlement price for the call option contracts is the difference between 1/1000 of closing index values of underlying index and option strike price rounded to the minimum price tick. If the difference is negative or zero, then the final settlement price is minimum price tick.

Final settlement price for the put option contracts is the difference between option strike price and 1/1000 of closing index values of underlying index rounded to the minimum price tick. If the difference is negative or zero, then the final settlement price is minimum price tick.

In the last trading day, in case index values could not be calculated, final settlement price will be determined by Settlement Price Committee.

Expiry Date: *Last business day of the contract month.*

Last Trading Day: *Last business day of the contract month.*

Daily Price Limit: *Such a limit does not exist.*

Margins:

Initial Margin (Required Collateral): It is set by the SPAN portfolio margining method.

Maintenance Margin: 75% of the required collateral.

As stated above the thesis study is about European option pricing, therefore this type of options (equity index option) is in the scope of this master's thesis. The Black & Scholes option pricing formula does not change when equity index options or single stock options are used. The differences between these two types are mostly about regulations. The differences can be summarized as [17]:

1. Index options have multiple underlying stocks whereas single stock options are based on a single one.
2. Index options are settle in cash whereas single stock options have physical delivery.

3. Most index options are European whereas single stock options are mostly American.

In the next chapter the pricing methodology for European Options and the necessary mathematical tools in order to understand the methodology.

CHAPTER 2

2. BACKGROUND AND RELATED WORK

In this section, we provide some background information used while deriving the Black-Scholes-Merton formula. We especially focus on Markov Chains in order to better understand the "random walk" concept that is used both in binomial pricing model and Black-Scholes-Merton model. Later on, a summary about the derivation of the Black-Scholes-Merton model is given.

2.1. MARKOV CHAINS

The information in this section is mostly a summary of [18]. We focus on discrete-time Markov chain where the state changes at discrete time instants. Firstly, we introduce some definitions that we are going to use at the rest of the thesis.

Let $X_n$be the state of the chain at time step n. Define also a finite set S, called the state space where S = {1,…,m} for some positive integer m. Let also $p_{ij} \in [0,1]$ be a transition probability of going from state i to state j . That is,

$$p_{ij} = \mathrm{P}(X_{n+1} = j \mid X_n = \mathrm{i}) \text{ where i, j } \epsilon \text{ S.}$$

The key assumption in Markov chains is that the next state only depends on the previous one, that is the next state j is independent of which states we pass until we come to state i which is named as Markov property. We assume that stock prices have this Markov Property, that is the stock price only depends to its previous instant value.

We can summarize the Markov property mathematically in Eq. 2.1.1:

$$\mathrm{P}(X_{n+1} = j | X_n = \mathrm{i},\, X_n = i_{n-1},\, \ldots,\, X_0 = i_0) = \mathrm{P}(X_{n+1} = j \mid X_n = \mathrm{i}) = p_{ij}\ ,\text{(Eq. 2.1.1)}$$

where i_{n-1} through i_0 are the earlier states.

The transition probabilities satisfy the general rule for probability measures. That is,

$$0 \leq p_{ij} \leq 1 \text{ and } \sum_{j=1}^{m} p_{ij} = 1 \; \forall i.$$

Note that it is also possible to have p_{ii}, that is to stay in the same state.We can also define the **transition probability matrix** such that

$$\mathbf{P} = \begin{matrix} p_{11} & \cdots & p_{1m} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mm} \end{matrix}$$

Given the Markov chain model, we can compute the probability of a sequence of future states by:

$$\mathrm{P}(X_0 = i_0, X_1 = i_1, \ldots, X_n = i_n) = \mathrm{P}(X_0 = i_0)\, p_{i_0\, i_1}\, p_{i_1\, i_2} \,\ldots\, p_{i_{n-1}\, i_n} . \quad \text{(Eq. 2.1.2)}$$

It is easy to see that in order to find the final probability of a model we need to specify a probability for the initial state, or we can take the initial state as given.

We can prove Eq. 2.1.2 by using the property that the next state depends only on the last state and it is independent of what happened before that. That is:

$\mathrm{P}(X_0 = i_0, X_1 = i_1, \ldots, X_n = i_n)$

$= \mathrm{P}(X_n = i_n \mid X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1})\, \mathrm{P}(X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1})$

$= p_{i_{n-1}\, i_n}\ \mathrm{P}(X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1})$,

and then by doing the same thing to $\mathrm{P}(X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1})$.

While pricing options with binomial trees and Black-Scholes-Merton model, stock prices are assumed to have the Markov property. This property emerge in the pricing concepts while stating that stock prices make a random walk, as it is explained in 2.2. Below an example of a random walk, birth-death process, is given.

Birth-Death Processes: A birth-death process is a Markov-Chain in which the states are linearly queued and we either can move to a neighbored state or stay in the same state. We can visualize this process in Figure 6:

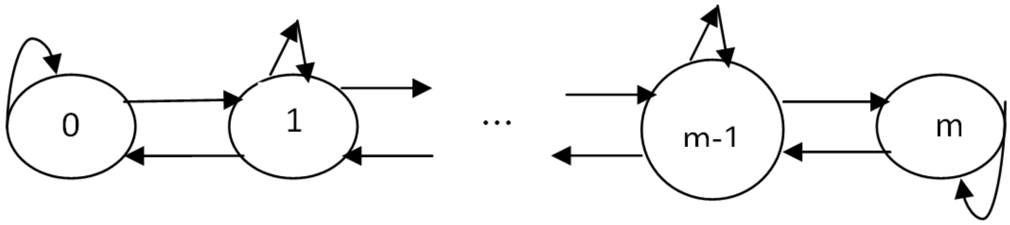

Figure 6: Transition probability graph of a birth-death process
(Source: Author's Drawing)

Let's define $b_i = P(X_{n+1} = i + 1 \,|X_n = i)$ as the "birth probability at state i" and define
$d_i = P(X_{n+1} = i - 1|X_n = i|X_n = i)$ as the "death probability at state i". We know that for a transition from i to i+1 for a second time we need a transition from i+1 to i between the first and second move between i to i+1. Hence the expected frequency of transition from i to i+1 is equal to the expected frequency of transition from i+1 to i. That is $\pi_i b_i = \pi_{i+1} d_{i+1}$ for i = 0,1, … ,m-1. From here we can obtain:

$$\pi_i = \pi_0 \frac{b_0 b_1 \ldots b_{i-1}}{d_1 d_2 \ldots d_i} \text{ for i = 0,1, … ,m} \quad \text{(Eq. 2.1.7)}$$

In order to understand the birth death process we will give an example from [16] at page 360:

Example: Random Walk with Reflecting Barriers

A person walks along a straight line and, at each time period, takes a step to the right with probability b, and a step to the left with probability 1-b. The person starts in one of the positions 1,2,…,m and when he is in position 1 or m, he will state in that position with corresponding probability 1-b and b, respectively. The transition probability graph of the chain is given in figure 4.

The local balance equations are

$$\pi_i b = \pi_{i+1}(1-b), for\ i = 1, \dots, m-1$$

Thus, $\pi_{i+1} = \rho\pi_i$, where

$$\rho = \frac{b}{1-b}$$

and we can express all the π_j in terms of π_1, as

$$\pi_i = \rho^{i-1}\pi_1 \text{ for I = 1, … , m}$$

Using 1 = $\pi_1 + \cdots + \pi_m$ we obtain

$$1 = \pi_1(1 + \rho + \cdots + \rho^{m-1}),$$

which leads to

$$\pi_i = \frac{\rho^{i-1}}{1+\rho+\cdots+\rho^{m-1}} \text{ for I = 1, … , m}$$

Note that if left and right probabilities are equally likely, then $\pi_i = {}^1/_m$ for all i.

2.2. BLACK-SCHOLES-MERTON MODEL:

We now introduce the well-known Black-Scholes-Merton Model. We follow the standard notation and definitions used in the literature ([4], [5], [11] and [12]).

We make the well-known assumptions stated below to derive Black-Scholes-Merton differential equation:

- The stock price follows wiener process explained below.
- The short selling of securities with full use of proceeds is permitted
- There are no transaction costs or taxes. All securities are perfectly divisible.
- There are no dividends during the life of the derivative.
- There are no arbitrage opportunities.
- Security trading is continuous.
- The risk-free rate of interest is constant and the same for all maturities.

In order to derive the Black-Scholes-Merton formula besides the Markov Property we use the tools and logic explained below. Firstly, we explain how options are priced with binomial trees as it has similar and more basic principals with Black-Scholes-Merton formula. Later on, we explain Wiener Process and Ito's Lemma which is used in the model. Together with the Markov process explained in the previous subsection these will be sufficient to understand Black-Scholes-Merton model.

2.2.1. Binomial Trees:

In order to understand the logic behind the Binomial trees let's consider a portfolio consisting of a long position in Δ shares of a stock and a short position in one call option. Assume that the stock price is S_0 at time 0. Assume further that there are two possibilities for the price change of the stock. It can be either move up by *u* (u>1) or move down by *d* (d<1). The price movement and the price of the option for both cases is shown in the figure below:

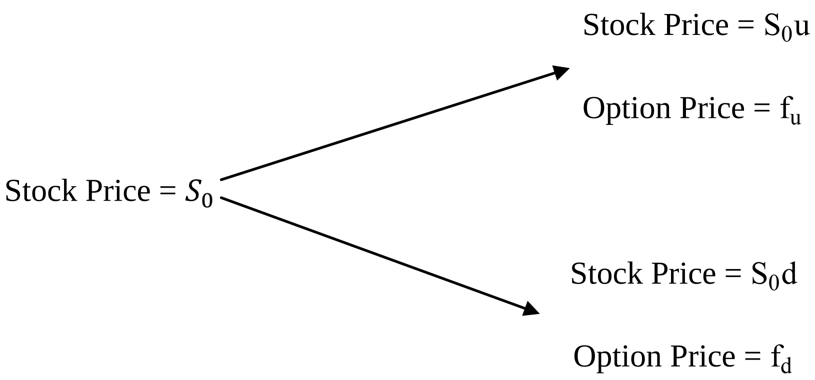

Figure 7: Stock and option price movement

We want to find Δ that makes the portfolio riskless. That is:

$$S_0u\Delta - f_u = S_0d\Delta - f_d$$

Then,

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d} \quad \text{(Eq. 2.2.1)}$$

The reason why we have a negative sign in front of f is that we short the option. In other words we receive the money.

If Eq. 2.2.1 is satisfied the portfolio is riskless, therefore the return should be equal to the return of the risk-free interest rate (*r*).

In the light of this logic if we denote the option price paid when setting up the portfolio as *f*:

$$S_0\Delta - f = (S_0 u\Delta - f_u)e^{-rT},$$

where T denotes the time elapsed.

Hence,

$$f = S_0\Delta(1 - u\,e^{-rT}) + f_u e^{-rT}.$$

If we combine the above equation with the Δ found in Eq. 2.2.1 we get:

$$f = e^{-rT}[pf_u + (1-p)\,f_d], \quad \text{(Eq. 2.2.2)}$$

where

$$p = \frac{e^{-rT} - d}{u - d}. \quad \text{(Eq. 2.2.3)}$$

We can interpret p as the probability for an up movement. In this case:

$$E[S_T] = pS_0u + (1 - p)\,S_0d.$$

If we combine the above equation with Eq. 2.2.3 we get:

$$E[S_T] = S_0e^{rT}. \quad \text{(Eq. 2.2.4)}$$

The above equation comes with the assumption of a “risk-neutral world”. That is people do not need compensation for taking risk and the expected return from the securities is the risk-free interest rate.

The above derivations are done for a one step binomial tree. For more steps we can iterate in the same way. However, instead of using T we use Δt to denote the time steps between each node. In addition in real word, we assume that the expected return on a stock μ and the volatility is σ. Therefore the expected stock price in the end of the first time step is $S_0e^{\mu\Delta t}$.

Therefore, the real world probability of an up movement p* is:

$$p^* = \frac{e^{-\mu\Delta t} - d}{u-d}. \quad \text{(Eq. 2.2.5)}$$

It will be explained later that the standard deviation of the return of the stock price in a short period of time with length Δt will be defined as $\sigma\sqrt{\Delta t}$. As a result the variance
($E[X^2] - E[X]^2$) is equal to:

$$p^*u^2 + (1-p^*)d^2 - [p^*u + (1-p^*)d]^2 = \sigma^2 \Delta t. \quad \text{(Eq. 2.2.6)}$$

Combining Eq.1.5 with Eq. 2.2.6 we get:

$$e^{\mu\Delta t}(u+d) - ud - e^{2\mu\Delta t} = \sigma^2 \Delta t.$$

Using the series expansion $e^x = 1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ and ignoring Δt^2 and higher terms as Δt is small we get:

$$u = e^{\sigma\sqrt{\Delta t}} \text{ and } d = e^{-\sigma\sqrt{\Delta t}} \quad \text{(Eq. 2.2.7)}$$

We provided the explanation of relatively simple binomial option pricing model. The arguments used to derive Black-Scholes-Merton model are similar to the use of binomial trees. However, there is one important difference between the Black-Scholes-Merton and the model developed above. The position in the stock and the derivative is riskless for only a short period of time. To remain riskless we should change and balance our position repeatedly.

The option price for different number of states in binomial model versus Black-Scholes-Merton model can be seen in the figure below:

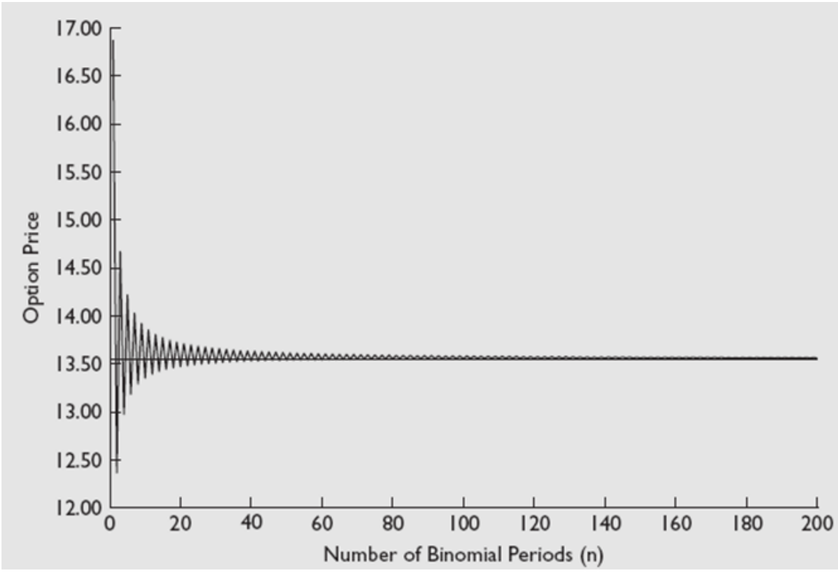

Figure 8: Binomial Option Prices for Different Number of Periods (Source: [13])

In the figure above, the line that is close to 13.5 is an option price calculated with the Black-Scholes-Merton model for a given strike price, time to expiration, volatility, risk free interest rate and stock price. The oscillating line that converges to it, is the value of the call option with the same variables for different number of binomial periods. That is, if we use the binomial model provided above with sufficiently large steps we will reach the same result as the Black-Scholes-Merton formula explained below.

In the next subsection, we explain other critical mathematical tools to understand Black-Scholes-Merton model.

2.2.2. Wiener Processes and Itô's Lemma:

We summarize and define here some stochastic processes that are used to explain the changes on stock prices.

Wiener Process: A variable z follows a Wiener process if:

Property 1: The change Δz during a small period of time Δt is

$$\Delta z = \epsilon\sqrt{\Delta t}, \quad \text{(Eq. 2.2.8)}$$

where ϵ has a standardized normal distribution $\phi(0, 1)$.

Property 2: The values of Δz for any two different short intervals of time Δt are independent. That is z follows a Markov process. See subsection 2.1. for a detail analysis on Markov Chains.

The mean change per unit time for a stochastic process is known as the drift rate and the variance per unit time is known as the variance rate. Given this definitions a generalized Wiener process can be written as

$$dx = adt + bdz, \quad \text{(Eq. 2.2.9)}$$

where a (drift rate) and b (variance rate) are constant.

In a small time interval Δt, we can combine Eq. 2.2.8 and Eq. 2.2.9, and get:

$$\Delta x = a\Delta t + b\epsilon\sqrt{\Delta t}.$$

In this example Δx has a normal distribution such that:

$$\text{mean of } \Delta z = a\ \Delta t,$$

$$\text{standard deviation of } \Delta x = b\sqrt{\Delta t}.$$

Itô Process: It is the same as Wiener Process with a difference such that a (drift rate) and b (variance) rate are not constant, but they depend on the underlying variable x and time, t. Hence Itô Process can be written as:

$$dx = a(x, t)dt + b(x, t)dz. \quad \text{(Eq. 2.2.10)}$$

And in a small time interval Δt,

$$\Delta x = a(x, t)\Delta t + b(x, t)\epsilon\sqrt{\Delta t}.$$

The Process for a Stock Price: It is not assumed that stock price follows Wiener process because constant drift rate and variance rate suggest that an investor expects the same return (ceteris paribus) independent of the amount. In other words people may want to expect higher returns when the make higher investment. That is if S is the stock price at time t, then the expected drift rate in S is assumed to be μS, where μ is a constant. Assuming that volatility is zero, in a small time interval Δt we have:

$$\Delta S = \mu S \Delta t.$$

As Δt→0 we have dS = μSdt. Solving this differential equation and integrating it from 0 to T, we get:

$$S_T = S_0 e^{\mu t}. \quad \text{(Eq. 2.2.11)}$$

Of course, we have volatility in real life. Hence the equation becomes:

$$\frac{dS}{S} = \mu dt + \sigma dz, \quad \text{(Eq. 2.2.12)}$$

where μ is the expected rate of return and σ is the volatility.

The model developed is known as geometric Brownian motion. The discrete time version of Eq. 2.2.12 is:

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \epsilon \sqrt{\Delta t}, \quad \text{(Eq. 2.2.13)}$$

where ϵ has the standard normal distribution and ΔS is the change in the stock price in a small time interval Δt. In other words $\frac{\Delta S}{S}$ is normally distributed with mean μΔt and standard deviation $\sigma\sqrt{\Delta t}$. That is:

$$\frac{\Delta S}{S} \sim \phi(\mu \Delta t, \sigma\sqrt{\Delta t}). \quad \text{(Eq. 2.2.14)}$$

Itô's Lemma: Itô's Lemma shows that a function G of x and t follows the process

$$dG = \left(\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2} b^2\right)dt + \frac{\partial G}{\partial x} b dz, \quad \text{(Eq. 2.2.15)}$$

where dz follows the same Wiener process as in Eq. 2.2.10. Here a and b both depend on x and t. Hence G follows an Itô process with drift rate of $\frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}$ and variance rate of $(\frac{\partial G}{\partial x} b)^2$.

The Lognormal Property: We use Itô's lemma to see the process follows by lnS where S follows the process of Eq. 2.2.11. Hence we have:

$$G = \ln S \Rightarrow \frac{\partial G}{\partial S} = \frac{1}{S} \Rightarrow \frac{\partial^2 S}{\partial S^2} = -\frac{1}{S^2} \text{ and } \frac{\partial G}{\partial t} = 0.$$

Using Itô's formula to Eq. 2.2.12 and combining with the equations found above we get:

$$dG = (\mu - \frac{\sigma^2}{2})dt + \sigma dz. \quad \text{(Eq. 2.2.16)}$$

G = lnS follows a generalized wiener process with a constant drift rate $\mu - \frac{\sigma^2}{2}$ and constant variance σ^2. The change of lnS between time 0 and T is therefore normally distributed such that:

$$\ln S_T - \ln S_0 \sim \phi\, [(\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}]. \quad \text{(Eq. 2.2.17)}$$

With the tools developed above we can now understand the Black-Scholes-Merton model. We now define μ as expected return on stock per year and σ as volatility of the stock price per year. The mean of the percentage change in the stock price in time Δt is μΔt and the standard deviation is $\sigma\sqrt{\Delta t}$ so that we get $\frac{\Delta S}{S} \sim \phi(\mu\Delta t, \sigma\sqrt{\Delta t}.)$ as Eq. 2.2.14.

We showed that with Itô's lemma this equation implies Eq. 2.2.17. We can also write Eq. 2.2.17 as:

$$\ln S_T \sim \phi\, [\ln S_0 + (\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}]. \quad \text{(Eq. 2.2.18)}$$

From Eq. 2.2.18 and the properties of the lognormal distribution it can be shown that the expected value of S_T fits with the definition of μ as the expected rate of return as it can be found as:

$$E[S_T] = S_0 e^{\mu T} \quad \text{(Eq. 2.2.19)}$$

Also the variance of S_T can be found as:

$$\text{var}(S_T) = S_0^2 e^{2\mu t}(e^{\sigma^2 T} - 1). \quad \text{(Eq. 2.2.20)}$$

We can use the lognormal property of the stock prices to provide information about the probability distribution of the continuously compounded rate of return earned on a stock between 0 and T. Let's define the continuously compounded rate of return per annum realized between 0 and T as x, then

$$S_T = S_0 e^{xt},$$

so we have:

$$x = \frac{1}{T} ln\frac{S_T}{S_0}. \quad \text{(Eq. 2.2.21)}$$

Therefore from Eq. 2.2.17 (using the fact that $ln\frac{S_T}{S_0} = \ln S_T - \ln S_0$) x follows the process shown below:

$$x \sim \phi \left[\mu - \frac{\sigma^2}{2}, \frac{\sigma}{\sqrt{T}}\right]. \quad \text{(Eq. 2.2.22)}$$

In Eq. 2.2.14 we found that $\mu\Delta t$ is the expected percentage change in the stock price in a very short period of time. We should not mistake it with x, the continuously compounded rate of return as the expected value of x is less than μ (it is $\mu - \frac{\sigma^2}{2}$).

2.2.3. Derivation of the Model:

We developed the stock price process (Eq. 2.2.12) as:

$$dS = \mu S dt + \sigma S dz. \quad \text{(Eq. 2.2.23)}$$

Or in discrete version (Eq. 2.2.13)

$$\Delta S = \mu S \Delta t + \sigma S \epsilon \sqrt{\Delta t}. \quad \text{(Eq. 2.2.24)}$$

Assuming that f is the price of a call option or other derivative of S we can find by Itô's formula that these equations follow the same process as the equations below for continuous or discrete time, respectively:

$$df = \left(\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2\right)dt + \frac{\partial f}{\partial S}\sigma S dz, \quad \text{(Eq. 2.2.25)}$$

$$\Delta f = (\frac{\partial f}{\partial S}\mu S + \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)\Delta t + \frac{\partial f}{\partial S}\sigma S\Delta z. \quad \text{(Eq. 2.2.26)}$$

We tried to find a portfolio with an option and a share that makes our portfolio riskless in order to find the price of the option (binomial trees). With the same logic we will try to eliminate the Wiener process underlying f and S. The appropriate portfolio is: -1 derivative (short one derivative) and $\frac{\partial f}{\partial S}$ shares of the stock.

Define the value of the portfolio as Π. From the above portfolio this value is:

$$\Pi = -f + \frac{\partial f}{\partial S}S. \quad \text{(Eq. 2.2.27)}$$

The change in ΔΠ in a small time interval Δt is therefore:

$$\Delta\Pi = -\Delta f + \frac{\partial f}{\partial S}\Delta S. \quad \text{(Eq. 2.2.28)}$$

Combining Eq. 2.2.24 and Eq. 2.2.26 with Eq. 2.2.28 we get:

$$\Delta\Pi = (-\frac{\partial f}{\partial t} - \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)\Delta t. \quad \text{(Eq. 2.2.29)}$$

As we state before the assumption that the portfolio is riskless holds for a short period of time and it is equal to the same rate of return as other short term risk free securities. That is it should be equal to:

$$\Delta\Pi = r\,\Pi\Delta t, \quad \text{(Eq. 2.2.30)}$$

where r is the risk free interest rate. Combining Eq. 2.2.27 with Eq. 2.2.30, and equating the solution to Eq. 2.2.29 we get:

$$(\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2)\Delta t = r\,(f - \frac{\partial f}{\partial S}S)\Delta t,$$

hence

$$rf = \frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial S^2}\sigma^2 S^2 + rS\frac{\partial f}{\partial S}. \quad \text{(Eq. 2.2.31)}$$

Eq. 2.2.31 is the Black-Scholes-Merton differential equation. The solution depends on the boundary conditions. For example the boundary conditions for a European call option is:

$$f = \max(S-K, 0) \text{ when } t = T.$$

Any function that satisfies Eq. 2.2.31 is the theoretical price of a derivative that could be traded. That is it would not create any arbitrage opportunities.

The Black-Scholes formulas for a European call and European put option on a non-dividend-paying stock are shown below respectively:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2) \quad \text{(Eq. 2.2.32)}$$

and

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1), \quad \text{(Eq. 2.2.33)}$$

where

$$d_1 = \frac{ln\left(\frac{S_0}{K}\right) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, \quad \text{(Eq. 2.2.34)}$$

$$d_2 = \frac{\ln\left(\frac{S_0}{K}\right) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}, \quad \text{(Eq. 2.2.35)}$$

where N(x) is the cumulative distribution function for a standardized normal distribution.

2.3. THE GREEKS

Eq. 2.2.32 through Eq. 2.2.35 show that option prices depend on Strike Price, Stock Price, Time to Expiration, volatility and the risk free interest rate. The sensitivity of the option prices to these variables (except the Strike Price as it does not change once the contract is settled) can be found by differentiating Eq. 2.2.32 and Eq. 2.2.33 with these variables. Each result is named with a Greek letter (except Vega) and we summarize them in this section. These results are used to hedge the risk on portfolios containing options. The analysis is done for each of the variables *ceteris paribus*.

2.3.1. Option Price Sensitivity to Stock Prices: *Delta and Gamma*

For a call option, the price increases as the stock price increases. For a small increment in the stock price the option price will change by:

Call Delta = N(d_1). (Eq. 2.3.1)

As N(d_1) is a probability, the range is between 0 and 1.

Conversely a put option price moves in an opposite direction to the stock price and the put delta is:

Put Delta = 1 – N(d_1). (Eq. 2.3.2)

Again, put delta is between -1 and 0.

For small changes in stock prices delta is a very good measure to the change of the stock price, however as the movement in the stock price increases delta gets a worse approximation. Therefore, there is another measure called Gamma which is a measure for the relationship between Delta and stock price. Gamma is the same for a call and a put delta and it is shown in Eq. 2.3.3:

Call and Put Gamma = $\frac{e^{d_1^2/2}}{S_0\sigma\sqrt{2\pi T}}$. (Eq. 2.3.3)

2.3.2. Option Price Sensitivity to Risk-Free Rate: *Rho*

The sensitivity of the option prices for the risk free interest rate is called rho and it is given in Eq. 2.3.4 for call options and in Eq. 2.3.5 for put options:

Call Rho = TK$e^{-rT}N(d_2)$, (Eq. 2.3.4)

Put Rho $=$ -TK$e^{-rT}[1 - N(d_2)]$. (Eq. 2.3.5)

Eq. 2.3.4 and Eq. 2.3.5 show that Call rho is always positive (that is if the interest rate increases, the option price will increase) for call options and it is always negative (that is if the interest rate increases, the option price will decrease) for put options, relatively. The logic can be understood simply by realizing that the payment for a long position in call options is postponed whereas in a long position in put options the received payment is postponed.

2.3.3. Option Price Sensitivity to Volatility: *Vega*

The relationship between the option prices and volatility is named as Vega and it is the same for call and put options:

$$\text{Call and Put Vega} = \frac{S_0\sqrt{T}e^{-d_1^2/2}}{\sqrt{2\pi}}. \quad \text{(Eq. 2.3.6)}$$

Both call and put option are highly sensitive to volatility. As volatility increases the price of the options increases. The reason can be seen in the graphs in section 1.1.2. The loss for the long positions is constant, that is a high movement to the side where there are losses (for long positions) does not change the total amount we lose, however the gains increases (for long positions) as much as we move further than the strike price.

2.3.4. Option Price Sensitivity to Time to Expiration: *Theta*

The relationship between the option prices and time to expiration is given in the equations shown below:

$$\text{Call Theta} = -\frac{S_0\sigma e^{-d_1^2/2}}{2\sqrt{2\pi T}} - rKe^{-rT}N(d_2), \quad \text{(Eq. 2.3.7)}$$

$$\text{Put Theta} = -\frac{S_0\sigma e^{-d_1^2/2}}{2\sqrt{2\pi T}} + rKe^{-rT}(1 - N(d_2)). \quad \text{(Eq.2.3.8)}$$

The above equations show the rate of change of the value of the options with respect to the passage of time with all other values remaining the same.

The hedging process against the change in the variables are named with these Greek letters. For example, if an investor wants to protect herself against the change on stock prices she can make delta-hedging and/or gamma-hedging. It may not make sense to make theta-hedging because although volatility, risk-free interest rate and stock prices are uncertain for the future, there is no uncertainty about the passage of time. In spite of this, traders see theta as a descriptive statistic for a portfolio.

To understand better how a hedge could be done let's analyze the example given below:

Example for delta-hedging process:

Assume you own same 1000 call options with a delta of 0.3727 and you want to construct a portfolio such that it is delta-neutral. There exists a put option on the market with a delta of -0.6273. So as to make a portfolio that is delta-neutral (in other words the delta of the portfolio should be 0) you may long (buy) 594 number of put options and your portfolio's delta would be 1000 x 0.3727 – 594 x 0.6273 ≅ 0.

2.4. PUT-CALL PARITY

The prices of European calls and puts with the same exercise price and same expiration date have a relationship that has to hold. That relationship can be summarized in the formula given below:

$$S_0 + P = C + Ke^{-rT} \quad \text{(Eq. 2.4.1)}$$

where P and C are the put and call price respectively, of the same underlying asset with the same strike price and maturity date. According to this formulation, today's price of a portfolio consisting of a stock and a put option written on that stock, has to be the same as today's price of a Call option and a bond with face value equal to the strike price of both options.

The logic behind Eq. 2.4.1 can be explained by analyzing the table given below:

Table 2: Put Call Parity

Payoff From	Current Value	Payoffs from Portfolio Given Stock Price at Expiration	
		$S_T \leq K$	$S_T > K$
Stock	S_0	S_T	S_T
Put	$P(S_0, T, K)$	K-S_T	0
Payoff from Portfolio A at Expiration		**K**	$\boldsymbol{S_T}$
Call	$C(S_0, T, K)$	0	$S_T - K$
Bonds	$K(1+r)^{-T}$	K	K
Payoff from Portfolio B at Expiration		**K**	$\boldsymbol{S_T}$

(Source: [12])

In the table given above Portfolio A consists of a stock and a put option whereas portfolio B consists of a call option and a bond with a face value of K, which is the exercise price of the two options. Note that in the right hand side of the table that at the expiration date -no matter what the stock price is- both portfolios have the same return. As their returns in the future are the same, their prices today have to be the same in order to prevent arbitrage. Otherwise an arbitrageur could buy the portfolio with a smaller value and short sell the portfolio with higher value and get a riskless gain.

2.5. T-TEST

We use the t-test in order to test the hypothesis that observed and theoretically calculated option prices are equal, as a result we included a small explanation about the test and the p value. While comparing two data, looking only to the mean differences can be misleading because we also have to take into account the variances. That is what t-test does and the equation for the t-test is:

$$t = \frac{\overline{X_O} - \overline{X_T}}{\sqrt{\frac{var_O}{n_O} + \frac{var_T}{n_T}}}$$

We also use the p-value for our hypothesis testing. The p-value is the probability of obtaining the observed sample results when the null hypothesis cannot be rejected. If the p-value is less than or equal to a threshold value, the null hypothesis is rejected. As the p value gets larger we have a lower incentive to reject the null hypothesis.

We have studied in this chapter the tools that we used in order to analyze BIST30 index options. In the proceeding chapters we explain how the data is extracted and the results found.

CHAPTER 3

3. THE DATA

In this chapter, we summarize the data used in order to compare the observed option prices with the theoretically calculated option prices. In other words, we define what is "observed option price" and how the variables to calculate "observed option prices" are chosen. Firstly, we explain how the observed option prices are collected and then the approach of choosing variables for the theoretical calculation (unless otherwise stated theoretical calculation means Black-Scholes-Merton model in the rest of this thesis).

An European Option contract can be signed for only on one index in Turkey: the BIST30 index. The price movement of BIST30 index for the last 5 years is presented in the following figure:

Figure 9: BIST30 Price movements from June 2009 until May 2014 (Source: [19])

Figure 9 show that the price of BIST30 had a tendency to increase beginning from 2009. It reached it maximum value of 115.341 at May 22, 2013 and began to decrease from that day. As of May 2014 BIST30 index is composed from the following companies' stocks [20]:

Table 3: BIST30 Companies

Company Name	**Weight**
T. GARANTI BANKASI A.S.	13.5%
AKBANK T. A.S.	10.36%
T. HALK BANKASI A.S.	7.42%
BIM BIRLESIK MAGAZALAR A.S.	7.36%
HACI OMER SABANCI HOLDING A.S.	6.95%
T. IS BANKASI A.S.	5.93%
TURKCELL ILETISIM HIZMETLERI A.S.	5.59%
TUPRAS-TURKIYE PETROL RAFINELERI A.S.	4.81%
KOC HOLDING A.S.	4.54%
EMLAK KONUT GAYRIMENKUL YATIRIM ORTAKLIĞI A.S.	4.52%
TURK HAVA YOLLARI A.O.	3.82%
EREGLI DEMIR CELIK FABRIKLARI A.S.	2.75%
VAKIFLAR BANKASI A.S.	2.38%
TURK TELEKOMUNIKASYON A.S.	2.32%
ENKA INSAAT VE SANAYI A.S.	2.05%
TAV HAVALIMANLARI HOLDING A.S.	1.99%
YAPI VE KREDI BANKASI A.S.	1.92%
ARCELIK A.S.	1.89%
ULKER BISKUVI	1.72%
TOFAS TURK OTOMOBIL FABRIKASI A.S.	1.25%
T. SISE VE CAM FABRIKALARI A.S.	1%
PEGASUS HAVA TAŞIMACILIĞI A.S.	0.86%
PETKIM PETROKIMYA HOLDING A.S.	0.86%
KARDEMIR KARABUK DEMIR CELIK SANAYI VE TICARET A.S.	0.80%
KOZA ALTIN ISLETMELERI A.S.	0.73%
ASYA KATILIM BANKASI A.S.	0.63%
TEKFEN HOLDING A.S.	0.59%
ASELSAN ELEKTRONIK SANAYI VE TICARET A.S	0.55%
MIGROS TİCARET A.S.	0.55%
KOZA ANADOLU METAL MADENCILIK ISLETMELERI A.S.	0.36%

3.1. OBSERVED OPTION PRICES

Contract data for options and futures market can be downloaded from [22] for each working day. In the document downloaded, one can found among other information the "last settlement" price at the end of each day for a given exercise price and maturity date. The settlement price calculation is explained in BIST web site as:

"*At the end of the session, the daily settlement price is calculated as follows and rounded to the nearest price tick:*

a) The weighted average price of all the trades performed within the last 10 minutes of the normal session,

b) If less than 10 trades were executed in the last 10 minutes of the session, the weighted average price of the last 10 trades performed during the session,

c) If less than 10 trades were performed during the session, the weighted average price of all the trades performed during the session,

d) If no trades were performed, theoretical prices calculated in consideration prices of underlying asset and other contracts based on the same underlying asset will be determined as the daily settlement price.

If the daily settlement price cannot be calculated in accordance with the above methods by the end of the session, or it is decided that the prices calculated do not reflect the market correctly, the Exchange may determine the daily settlement price in consideration of theoretical price, spot price of the underlying asset, the previous day's settlement price or the best bid and ask prices at the end of the session."[16]

We used the options with the expiration date June 30, 2014 since the thesis is intended to be presented in June. The contracts with maturity date of June 30, 2014 appear in the market from the date December 31, 2013. In December 31, 2013 there were 8 different Strike Prices and they range from 76 to 90 with an increment of 2 TL for both put and call options. As time goes on different Strike Prices appear however only these data are used for comparison. Information about used observed data is summarized in the following table:

Table 4: A summary for the used observed data

Option Type	Underlying Index	Maturity Date	Strike Prices
European Option	BIST30	30.06.2014	76, 78, 80, 82, 84, 86, 88, 90

3.2. THEORETICALLY CALCULATED OPTION PRICES

We showed in Chapter 2 that in order to calculate the option prices using Black-Scholes-Merton model we need to know five variables at the contract date: Strike Price (K), Stock (or index) Price, Time to Expiration (T), risk-free interest rate (r) and volatility of the stock (or index) (σ). Below, the methods for choosing each variable are explained.

Strike Price: As it is stated in 3.1. Observed Option Prices strike prices extracted from observed data are 76, 78, 80, 82, 84, 86, 88 and 90. As a result, for theoretical calculations these values are used.

Index Price: The index price for BIST30 is extracted from [23]. The data extracted contains these information for each day: lowest, highest, close price for 2 sessions. Session 1 is before the break and Session 2 is after the break. As the quotation from [16] states, observed prices are normally calculated for the last traded contracts. As a result, we used the average of the lowest and highest price of Session 2 for each date as BIST30 price.

Time to Expiration: The maturity date is chosen to be 30.06.2014. As a result T is calculated as (Days Left to 30.06.2014)/(365) for each date.

Risk-Free Interest Rate: Debt Securities Market Daily Bulletins can be extracted from [24]. A perfect match for the same maturity date could not be found for any government traded bonds. As a result the government bond interest rate and price information for "TRT110614T13" is used. Date to maturity is 18 days ahead of the option contract and this is the smallest difference for the government bonds trading. The risk free rate information is get again from the document downloaded from [24] which is given in the column of "Compounded Yield" (weighted average) for “TRT110614T13” because in Black-Scholes-Merton formula the interest rate is annually compounded interest rate.

Volatility: Historical volatility data for BIST30 can be extracted directly from [23]. Each day the volatility is calculated for different number of dates (21, 42, 63, 126, and 252). As in Black-Scholes-Merton formula annual volatility is needed, we use the information given for 252 days. The volatility calculation is said to be done in [25] as it is quoted below:

> “*The historical volatility of BIST 100 and BIST 30 Indices are calculated daily by “close-close volatility” method. Therefore, the following formula is used in calculating the realized volatility of an index for an n number of trading days (including t day) as of the t day:*
>
> $$vol_{t,n} = \sqrt{252x\frac{1}{n}x\sum_{i=1}^{n}(R_{t-i+1} - r_{t,n})^2}$$
>
> $$R_t = lnE_t - lnE_{t-1}$$
>
> $$r_{t,n} = \frac{1}{n}x\sum_{i=1}^{n}R_{t-i+1}$$
>
> $Vol_{t,n}$*=the realized volatility of the Index for an n number of trading days (including t day) as of the t day*
>
> E_t *=Closing value of the Index on t day*
>
> *n =Number of days for which volatility is calculated*
>
> *Historical volatility data are calculated on a daily basis for 21, 42, 63, 126, and 252 trading days.*”

With the data explained in this chapter, we begin our analysis of BIST30 index option market in the next chapter.

CHAPTER 4

4. ANALYSIS

With the tools explained in Chapter 2 and 3, we can now start to analyze the options market in Turkey. The data begins from December 31, 2013 and ends at May 16, 2014. In Subsection 4.1, we simply look for the difference of the observed option values versus theoretically calculated values. We also calculate the mean, sum of squares, variance and standard deviation of the differences. In Subsection 4.2. we analyze deeply some options and see if put-call parity is satisfied in the options market to check for an arbitrage opportunity in this market. In subsection 4.3. we try to explain possible reasons and potential results of this deviation.

4.1. COMPARISON OF OBSERVEDAND THEORETICALLY CALCULATED VALUES

We present the difference between observed values and theoretically calculated values in the figures plotted in MATLAB for 8 different strike prices (see Chapter 3) in this section. The code for the plots can be found in Appendix C. In each figure there are two plots. The left hand side plots show the observed values (blue line) and the theoretical values (red line), whereas the right hand side plots show the difference between those values. A more detailed analysis for strike prices 90 (There is not a very special reason for analyzing this exercise price. We did not include all of the strike prices for the detailed explanations because they have very similar principals.) Note that x axis for the plots begin from 0 (date December 31, 2013) and end at 96 (date May 16, 2014). Note also that the count of 96 does not include weekends. In other words although there are in total 136 days between December 31, 2013 and May 16, 2014, there are only 96 working days.

4.1.1. Comparison for Call Options:

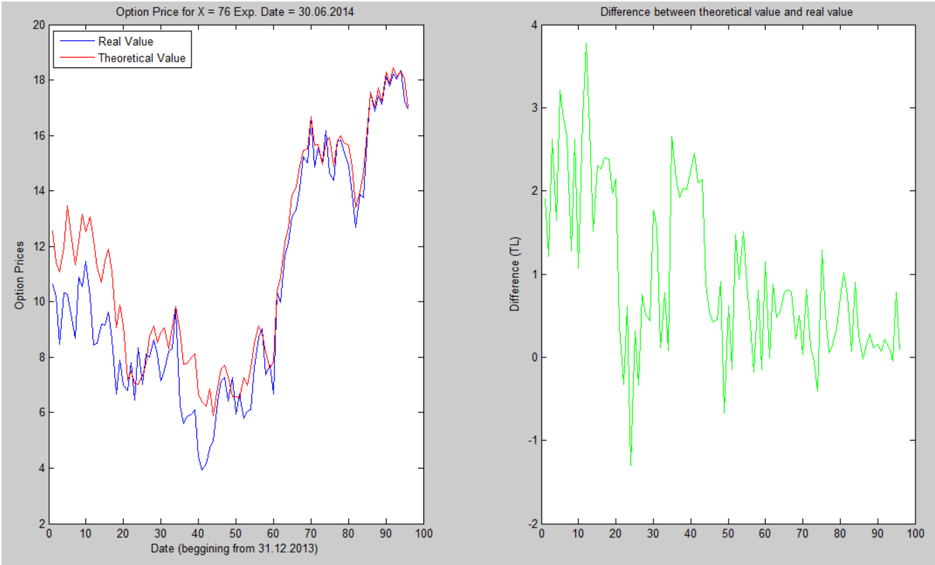

Figure 10: Comparison of Observed and Theoretical Values for call options with Strike Price=76 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

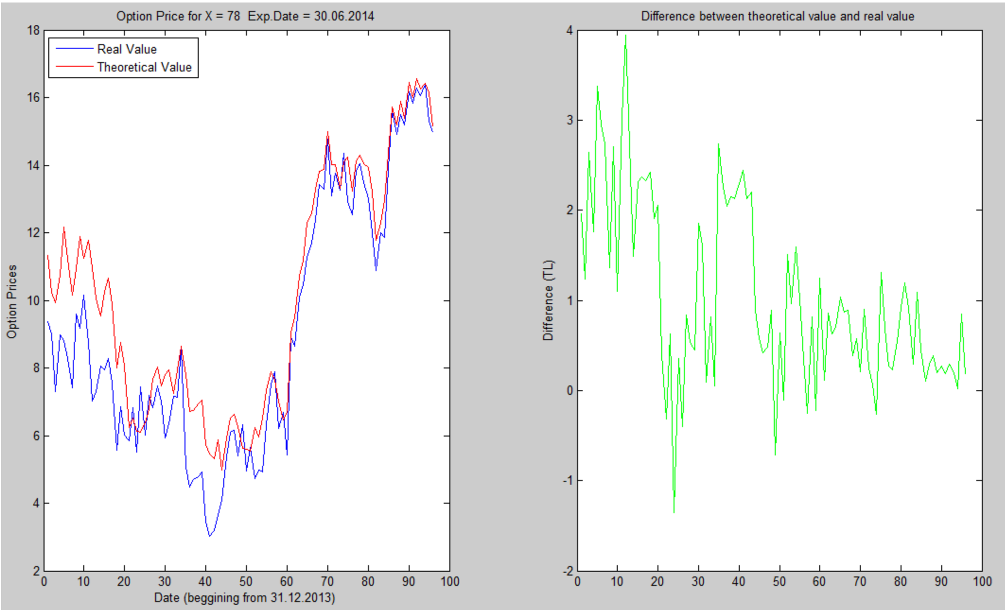

Figure 11: Comparison of Observed and Theoretical Values for call options with Strike Price=78 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

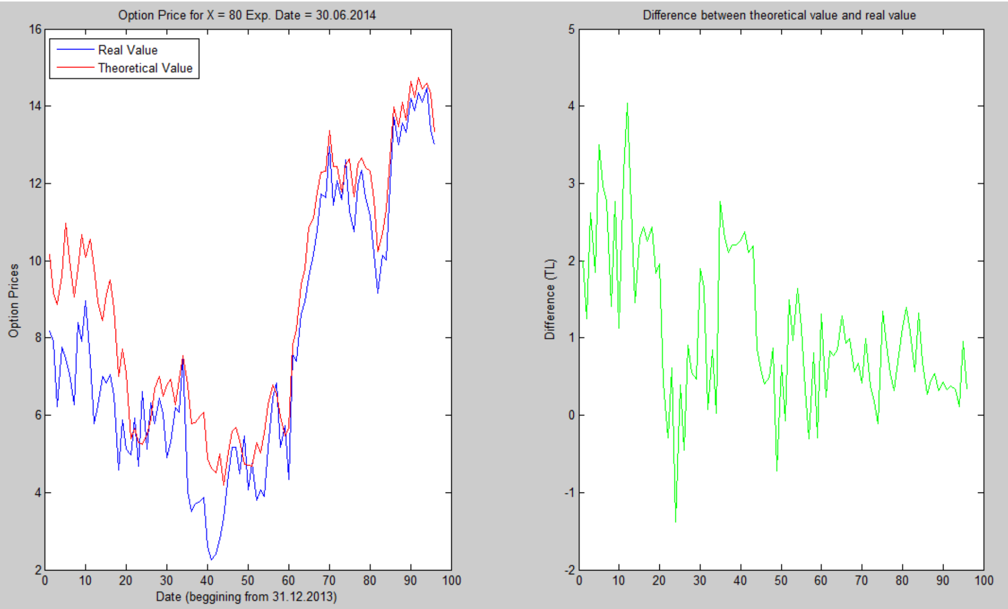

Figure 12: Comparison of Observed and Theoretical Values for call options with Strike Price=80 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

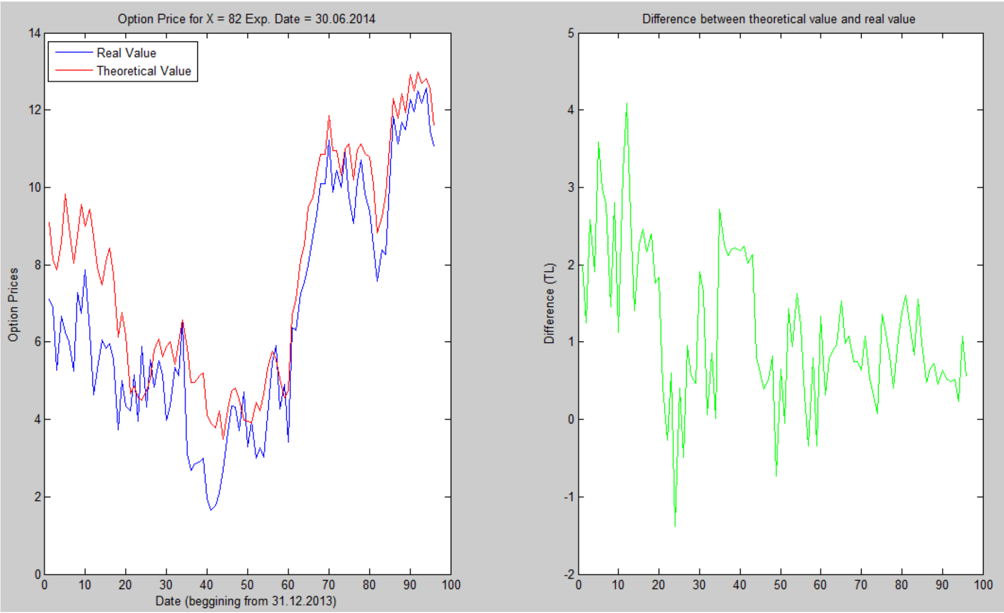

Figure 13: Comparison of Observed and Theoretical Values for call options with Strike Price=82 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

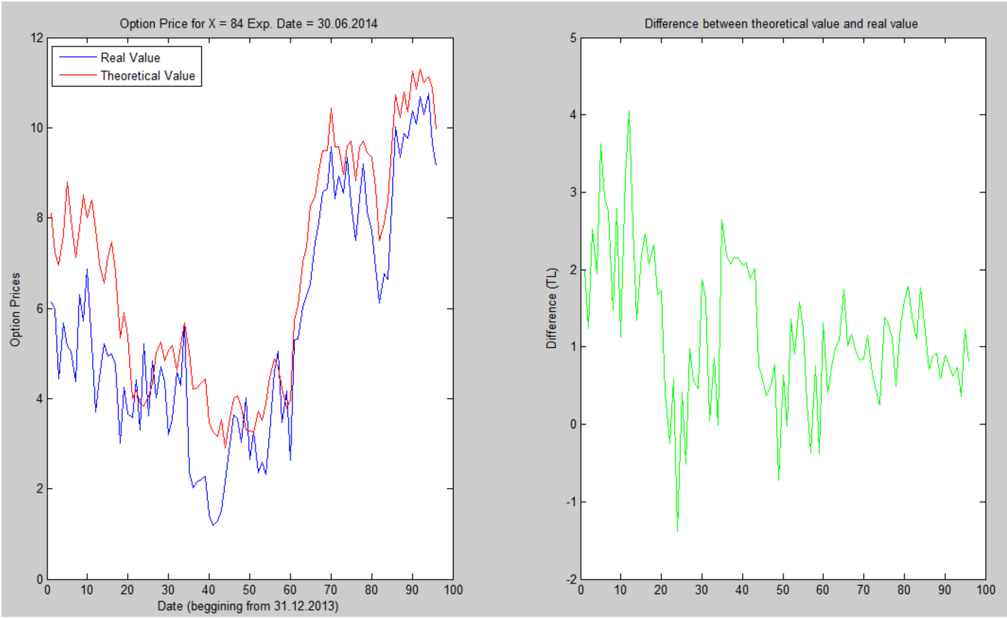

Figure 14: Comparison of Observed and Theoretical Values for call options with Strike Price=84 (Blue Line: [21], Red Line and Green Line: Author’s Calculations)

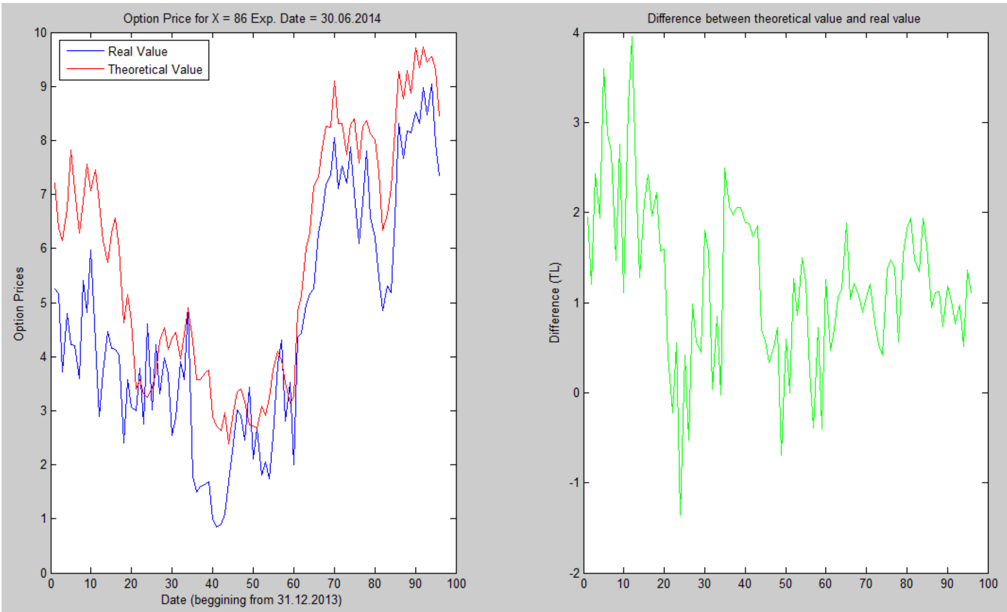

Figure 15: Comparison of Observed and Theoretical Values for call options with Strike Price=86 (Blue Line: [21], Red Line and Green Line: Author’s Calculations)

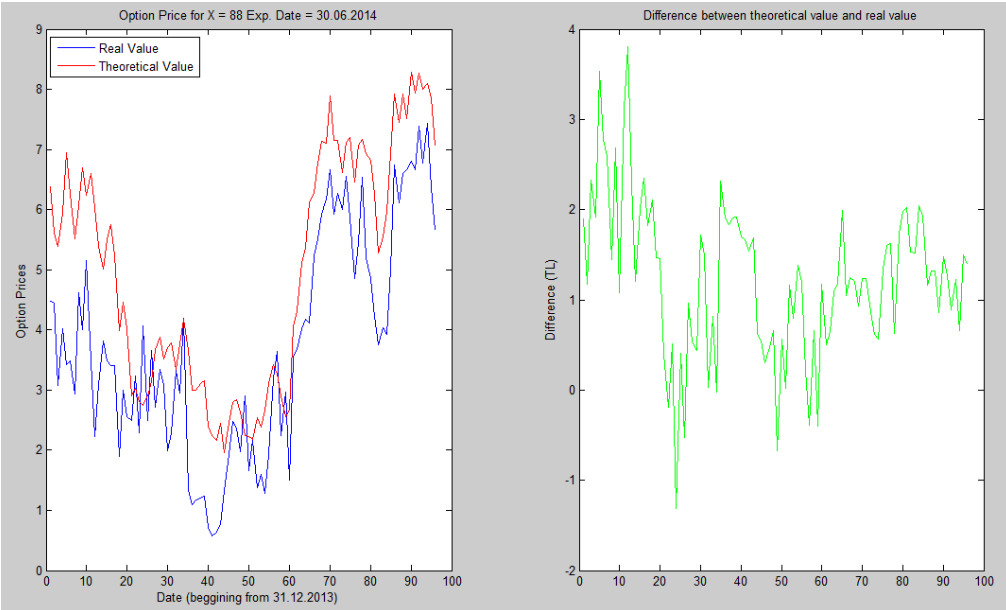

Figure 16: Comparison of Observed and Theoretical Values for call options with Strike Price=88 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

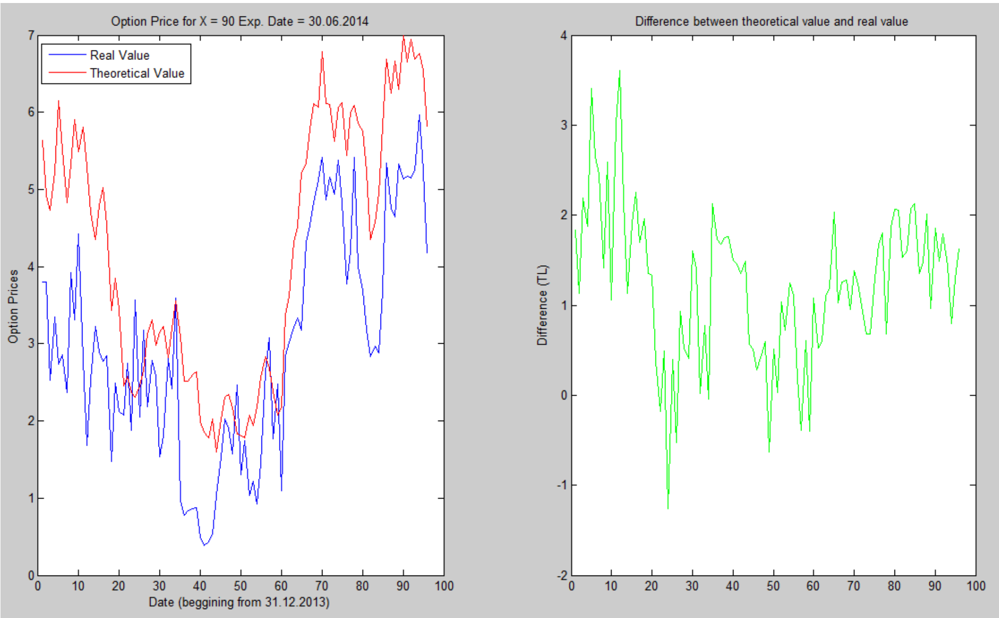

Figure 17: Comparison of Observed and Theoretical Values for call options with Strike Price=90 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

Figure 10 through Figure 17 show that the theoretical and observed values differ across time for call options. The mean, sum of squares, the variance and standard deviations of the differences are presented in Table 5. Note that most of the times the observed values are below the theoretical values, that is the difference is mostly positive. Hence, buying the call option may result in an arbitrage opportunity (we also see this while analyzing the put call parity in section 4.2.) As it is stated before more detailed analysis for the plots is done in subsection 4.2.

Table 5: Statistical Data for Differences between Observed and Theoretically Calculated Call Option Prices

Call Option with Strike Price:	Mean	Sum of Squares of the differences	Variance of the differences	Standard Deviation of the difference
76	0.5727	46.5419	0.4899	0.6999
78	0.6342	51.4511	0.5416	0.7359
80	0.6878	55.1983	0.5810	0.7623
82	0.7111	58.8316	0.6193	0.7869
84	0.7501	62.3498	0.6563	0.8101
86	0.7557	66.3500	0.6984	0.8357
88	0.7636	72.2783	0.7608	0.8723
90	0.7354	77.9435	0.8205	0.9058

(Source: Author's Calculations)

Our observations from the figures are also supported when we test with t-test the null hypotheses that the differences have a normal distribution with mean equal to zero. The hypotheses are rejected for all exercise prices at 1% significance level. In other words, the hypotheses that observed call option prices and theoretically calculated call option prices are the same, are rejected.

4.1.2. Comparison for Put Options:

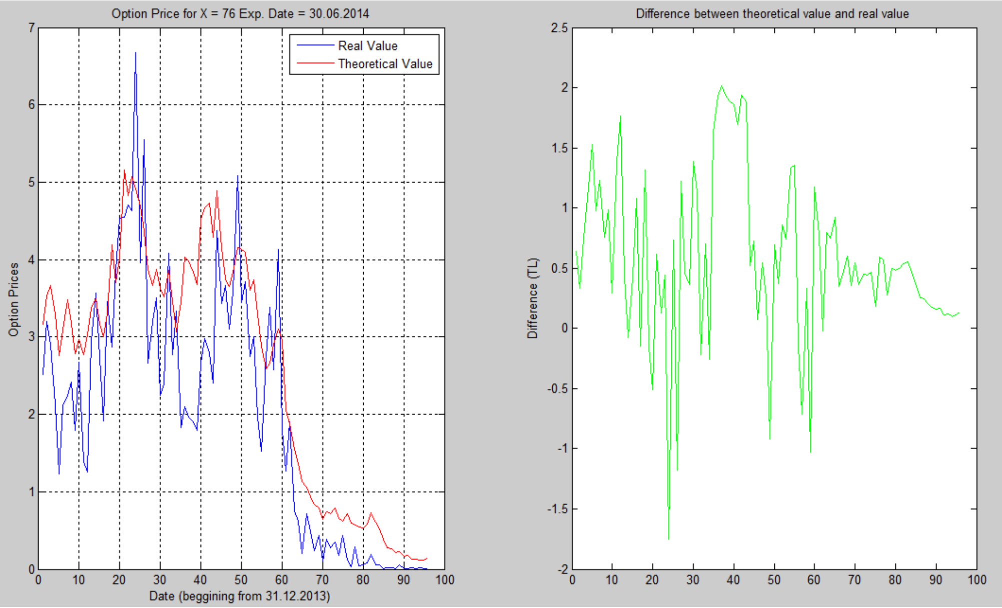

Figure 18: Comparison of Observed and Theoretical Values for put options with Strike Price=76 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

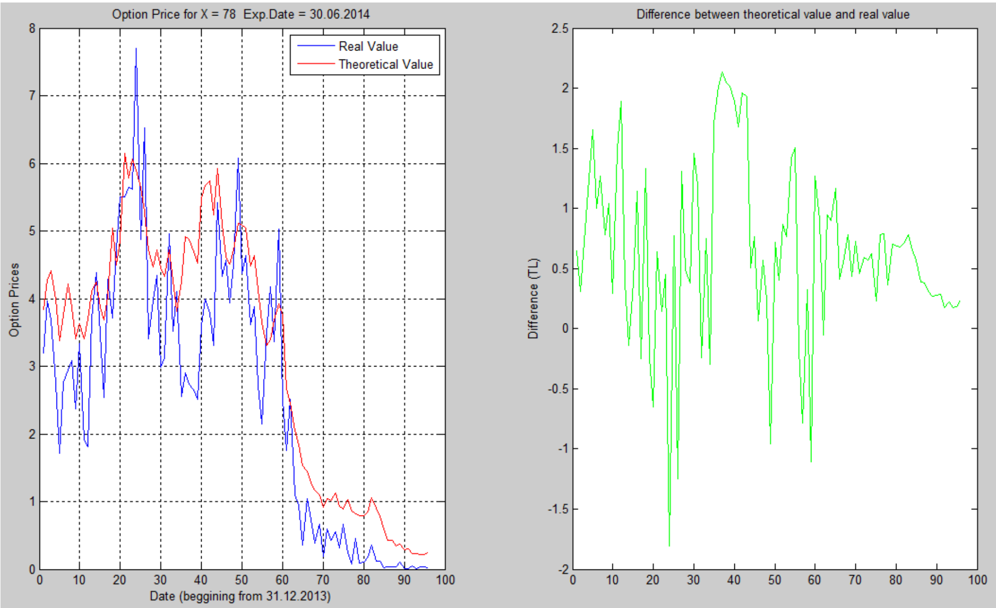

Figure 19: Comparison of Observed and Theoretical Values for put options with Strike Price=78 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

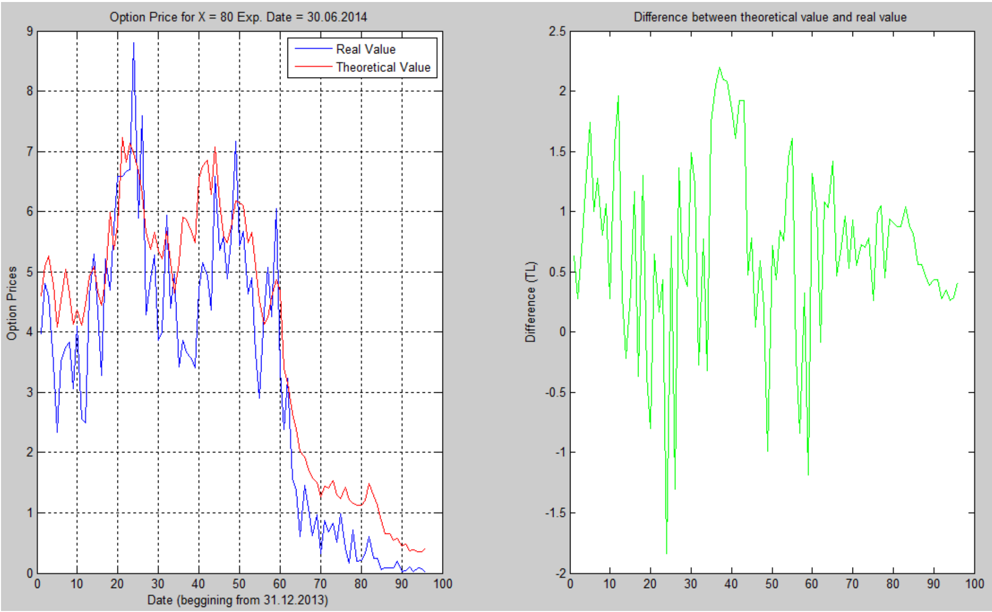

Figure 20: Comparison of Observed and Theoretical Values for put options with Strike Price=80 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

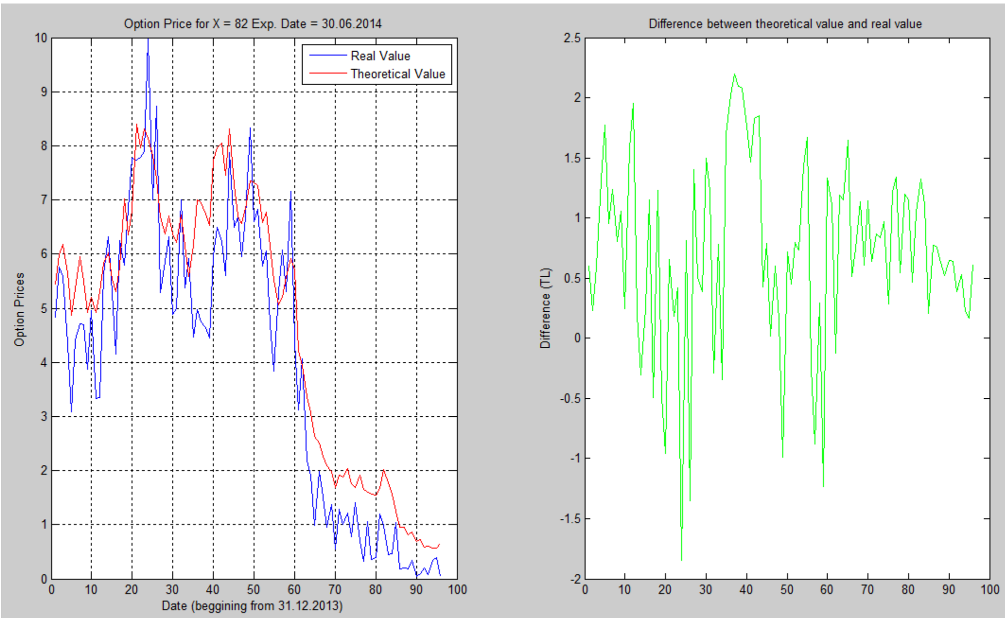

Figure 21: Comparison of Observed and Theoretical Values for put options with Strike Price=82 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

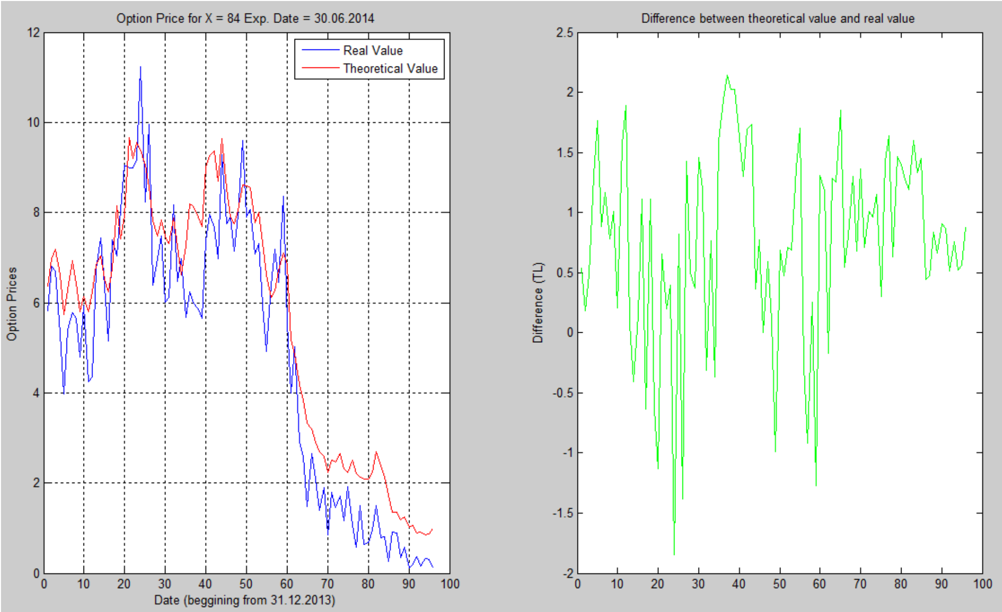

Figure 22: Comparison of Observed and Theoretical Values for put options with Strike Price=84 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

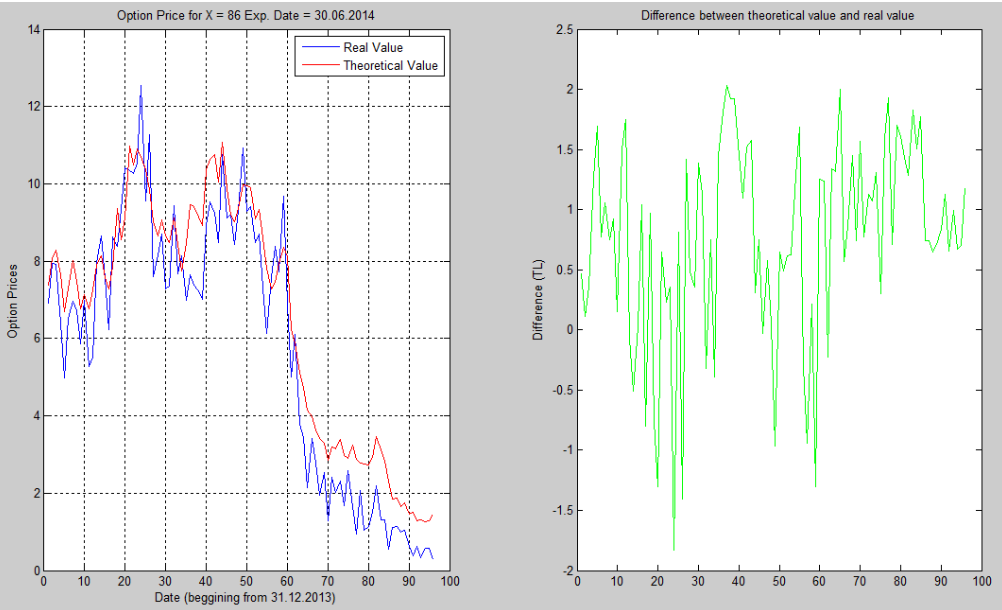

Figure 23: Comparison of Observed and Theoretical Values for put options with Strike Price=86 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

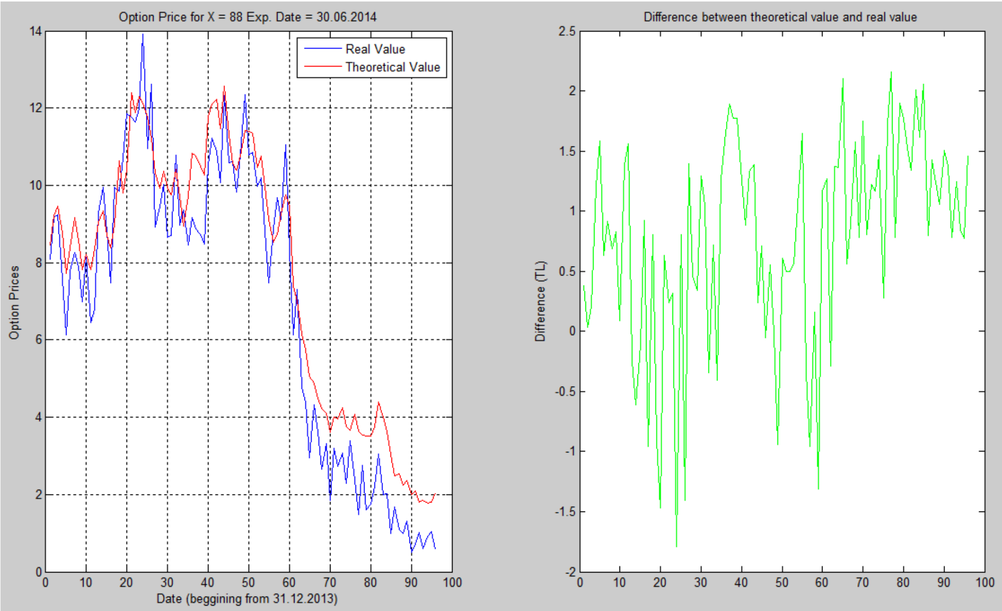

Figure 24: Comparison of Observed and Theoretical Values for put options with Strike Price=88 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

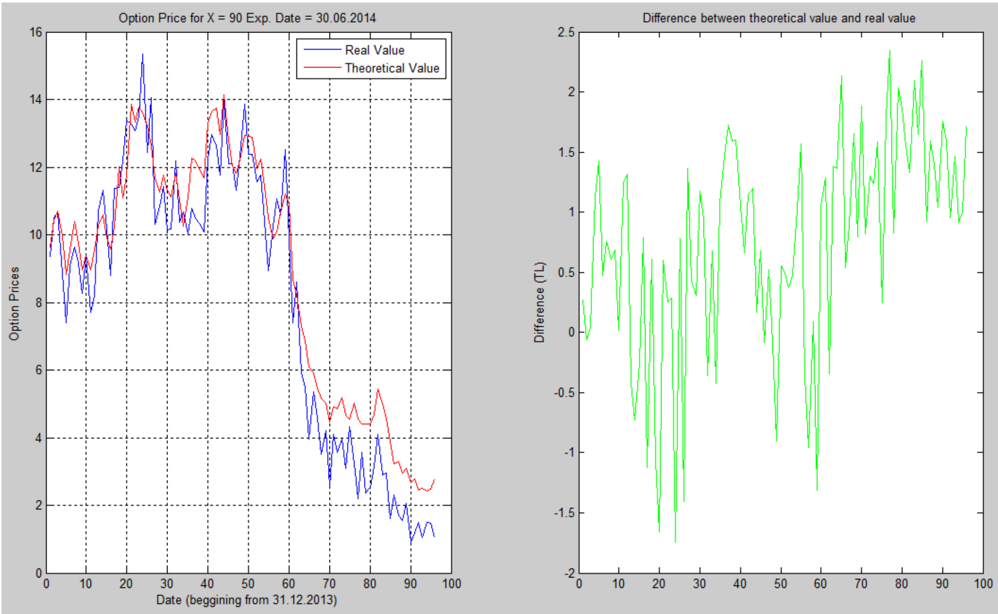

Figure 25: Comparison of Observed and Theoretical Values for put options with Strike Price=90 (Blue Line: [21], Red Line and Green Line: Author's Calculations)

Figures 18 through 25 show that the theoretical and observed values differ across time also for put options. The mean, sum of squares, the variance and standard deviations of the differences can be seen in Table 6. As it is stated before more detailed analysis for the plots will be done in subsection 4.2.

Table 6: Statistical Data for Differences between Observed and Theoretically Calculated Put Option Prices

Put Option with Strike Price:	**Mean**	**Sum of Squares of the differences**	**Variance of the differences**	**Standard Deviation of the difference**
76	0.6383	49.7857	0.5747	0.7581
78	0.6821	53.2029	0.6450	0.8031
80	0.7084	55.2553	0.6986	0.8358
82	0.7163	55.8721	0.7363	0.8581
84	0.7035	54.8700	0.7642	0.8742
86	0.6720	52.4131	0.7830	0.8849
88	0.6231	48.5981	0.7947	0.8915
90	0.5574	43.4805	0.8025	0.8958

We also test the hypotheses that the differences have a normal distribution equal to zero with t-test. Our observations from the figures are also supported by the fact that all the hypotheses are being rejected at 1% significance level. In other words, the hypotheses that observed put option prices and theoretically calculated put option prices are equal, are rejected.

4.2. DEEPER ANALYSIS OF OPTION PRICES WITH STRIKE PRICE EQUAL TO 90

Without any specific reason and without loss of generality we provide a deeper analysis for the case where the exercise price is 90 for call and put options. During the analysis we construct the portfolios explained in Subsection 2.4 and see if there is or there was an arbitrage opportunity in the market. If there was an arbitrage opportunity and it disappeared, we find the exact date of it and see what happened to option prices. We also present in this section *"The Greeks"* for the options in Turkey and see if the observed option prices that are different from the theoretically calculated values are above the theoretical minimum value.

Again the plots contain the observed (blue line) and theoretically calculated (red line) prices. In addition to them we have also the theoretical minimum value (green line) in the figures. Note that in the proceeding plots the x axis is different from the previous graphs. In this case instead of counting from 0 to 96, we count backwards. The x axis begins from 180 and continues until 44. In other words the x axis shows the number of days till the maturity date. As it was explained before, 96 days are the working days between December 31, 2013 and May 16, 2014, whereas 136 days (180-44) are total number of days between those dates. The codes for this section can be found in Appendix D. It is important to point out that "observed value" is used as a definition of the observed market prices in Turkey for the options.

4.2.1. Call Options with Strike Price Equal to 90:

Call option prices (observed and theoretical) can be seen in the figure shown below more closely:

Figure 26: Call Option Prices (Observed and Theoretical) for Strike Price = 90 and Minimum Theoretical Value (Blue Line: [21], Red Line and Green Line: Author's Calculations)

Firstly, we show how the minimum theoretical value is calculated. For this purpose we again construct a portfolio [12]. Portfolio A consists of only one stock and portfolio B consists of a call option written on that specific stock with strike price K and a government bond with face value K and containing the same maturity T with the call option. For different values of the stock at time T, both portfolios' return can be seen in the table shown below:

Table 7: Portfolio Determining the Minimum Value of a Call Option

Portfolio	Current Value	Payoffs from Portfolio Given Stock Price at Expiration	
		$S_T \leq K$	$S_T > K$
A	S_0	S_T	S_T
B	$C(S_0, T, K) + K(1+r)^{-T}$	K	$(S_T - K) + K = S_T$

(Source: [12])

From Table 7 we can observe that Portfolio B has a value greater than Portfolio A when the stock price at time T is smaller than the exercise price (when the call option is not exercised, that is when it is out of the money) and both portfolios have the same value when the stock price at time T is larger than the exercise price (when the call option is exercised, that is when it is in the money). As a result the return of Portfolio B is always larger or equal to the return of Portfolio A in the future. As a result the value of Portfolio B should be greater or equal to the value of Portfolio A at time 0. Hence we have:

$$C + K(1+r)^{-T} \geq S_0 ,$$

or as the price of the call option cannot be smaller than 0:

$$C \geq \max(0, S_0 - K(1+r)^{-T}). \quad \text{(Eq. 4.2.1)}$$

Therefore, the green line in Figure 26 is actually Eq. 4.2.1 with BIST30 and K = 90 and this equation holds, that is the observed values of option prices are never smaller than the theoretical minimum value.

As it was pointed out before the theoretically calculated value is mostly greater than the observed values of the option prices. That could have several reasons. Firstly we use the approximation of taking the average value of the lowest and highest BIST30 price in the day that the option price was determined and may be the stock price determining the option price was the close price. However, by observation we can see that the close price on that day and the average price do not differ as much as to create that difference in Appendix E. We also see in Figure 27 how a price change in the stock price affects the option price. The second reason may be that people have bearish expectations (that is they expect the index prices to decrease) about the market and believe that the probability of being lower than 90 in the expiration date (that is the probability of the call being out of the money) is high. The reason may be that in those days there was a high fear of having an economic crisis due to the tension in politics because of the election on March 30, 2104 and due to the tapes appearing against the government claiming that the government was inside a corruption. This made people believe that the government will change and the "stability" of Turkey's economic situation would go away.

The data points in Figure 26 point out where the time to expiration is 152 (when the date is January 28, 2014). The reason will be explained while analyzing the put call parity.

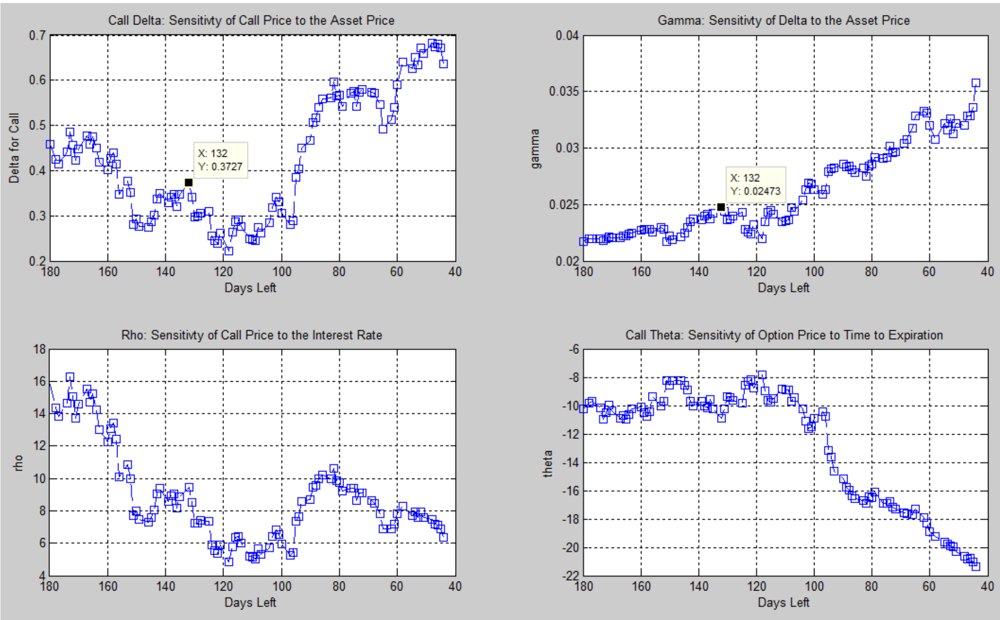

Figure 27: The Greeks (except Vega) for Call Option with Strike Price = 90 (Source: Author's Calculations)

Figure 27 shows the Greeks for the call options. Vega will be analyzed later after the analysis of the put options as it is the same for both call and put options.

If we wanted to hedge the call option position against the changes on the BIST30 index on February 17, 2014 (when there are 132 days to expiration) we had to create a portfolio with a delta of -0.3727 and a gamma of -0.02473. For more explanation about how this could be done, the example on subsection 2.3. can be studied. Note that as it was stated in Table 1, stock price and risk free interest rate effect positively the price of the call option.

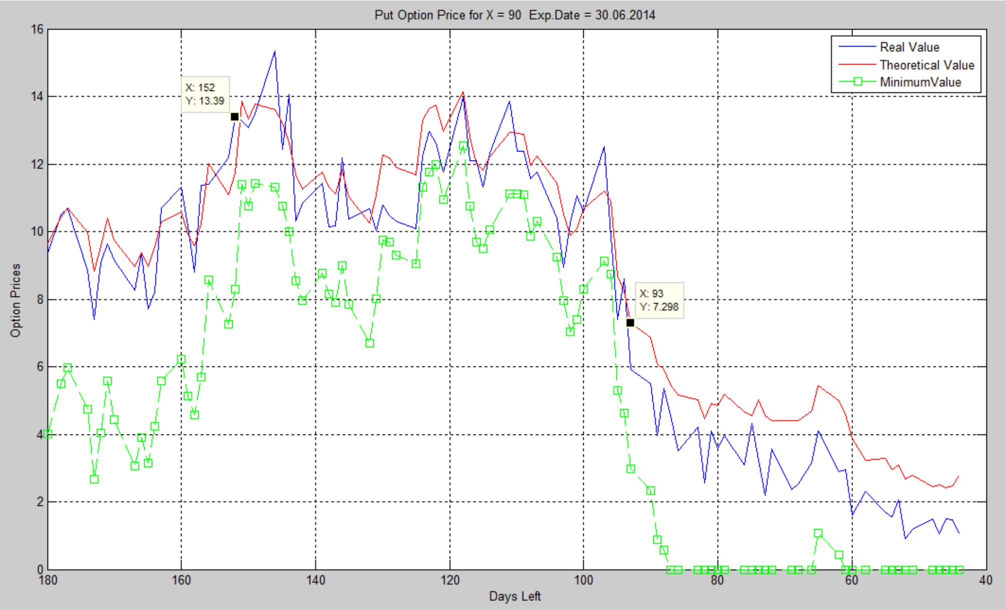

Figure 28: Put Option Prices (Observed and Theoretical) for Strike Price = 90 and Minimum Theoretical Value (Blue Line: [21], Red Line and Green Line: Author's Calculations)

Again we show below how the minimum theoretical value for put options is calculated. In Table 8, Portfolio A contains only one stock and Portfolio B consists of a put option bought (short position) on that specific stock with strike price K and a government bond with face value K and containing the same maturity T with the put option.

Table 8: Portfolio Determining the Minimum Value of a Put Option

Portfolio	Current Value	Payoffs from Portfolio Given Stock Price at Expiration	
		$S_T < K$	$S_T \geq K$
A	S_0	S_T	S_T
B	$K(1+r)^{-T} - P(S_0, T, K)$	K – (K-S_T) $= S_T$	K

(Source: [12])

Table 8 shows that no matter what is the stock price at time T Portfolio B has a return equal or higher than Portfolio A. As a result, at time 0 Portfolio B should worth at least equal to Portfolio A. Therefore:

$$K(1+r)^{-T} - P \geq S_0$$

or as the price of the put cannot be smaller than 0:

$$P \geq \max(0, K(1+r)^{-T} - S_0) \quad \text{(Eq. 4.2.2)}$$

The green line in Figure 28 represents Eq.4.2.2 and again the observed prices satisfy Eq. 4.2.2.

We can see in Figure 28 that until there are 93 days to expiration (that is until March 28, 2014) put option prices are close to observed option prices. However, after March 28, 2014 that is after the local elections held in Turkey in March 30, 2014 observed put option prices are below the theoretically calculated prices. This can be explained with the fact that the political party which is in the government got the majority of the votes. That may made people that the "stability" will continue in Turkish economic markets.

We stated that put option prices are close to observed prices before March 28, 2014. We can see this also with the t-test held in order to test this hypothesis. Figure 29 shows the p-value for different length of observations for the hypothesis testing of both (observed and theoretical) prices being the same. As we pointed out before there are 96 data points. The x line in Figure 29 shows the data not used for the testing. For example if x = 70, we do not look the last 70 data for the test, or similarly we only look for the first 26 data of theoretically calculated and observed prices and test the hypothesis that first 26 data points are equal.

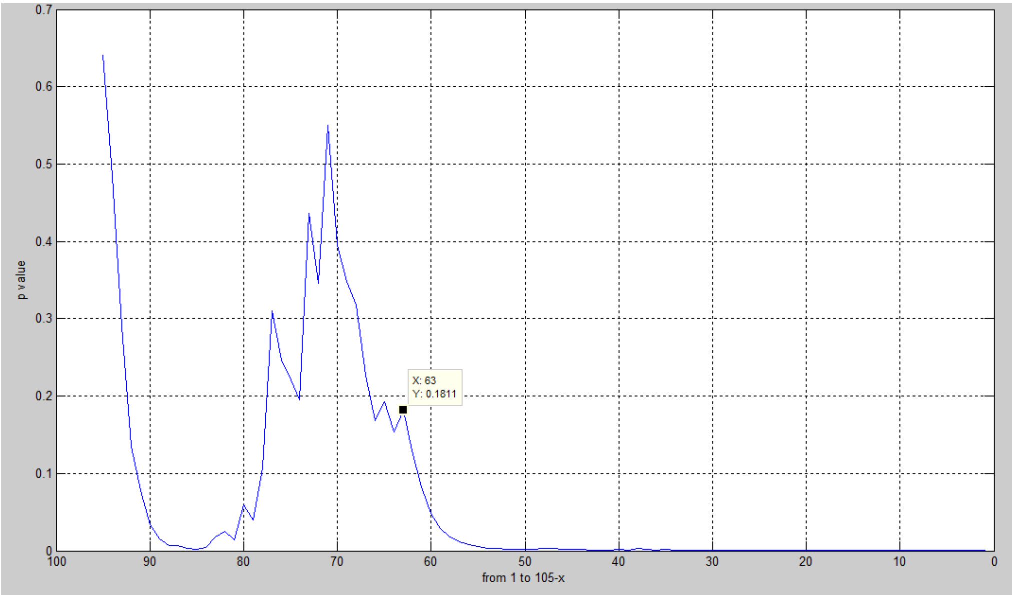

Figure 29: p value for the Hypothesis Testing of Observed and Theoretical Prices are Equal for Different Observation Lengths (Source: Author's Calculations)

The data point in Figure 29 shows the point where we omit the last 63 data (that is we omit the data after March 28, 2014) to test the hypothesis of both prices being equal. Note that before March 28, 2014 the p-value of the t-test is mostly above 10%, that is the test cannot reject the hypothesis of both prices being equal. However after the local elections (March 30, 2014) t-test begins to reject the null.

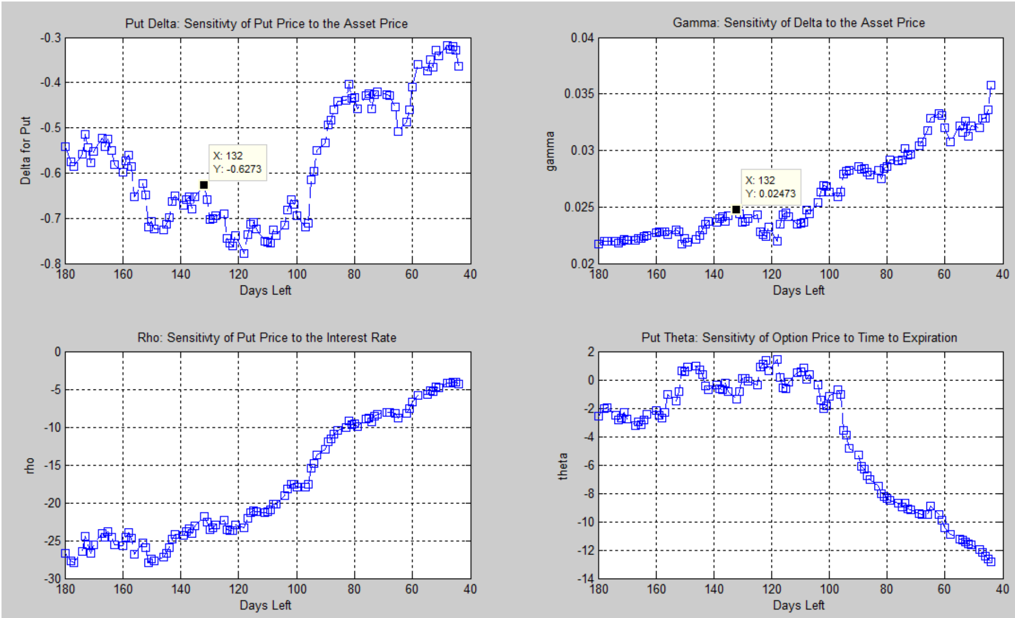

Figure 30: The Greeks (except Vega) for Put Option with Strike Price = 90 (Source: Author's Calculations)

If we wanted to hedge the put option position against the changes on the BIST30 index on February 17, 2014 (when there are 132 days to expiration) we had to create a portfolio with a delta of 0.6273 and a gamma of -0.02473. For more explanation about how this could be done, the example on subsection 2.3. may be studied. As it was stated in Table 1, stock price and interest rate effect positively put option prices.

Figure 31 shows one of the (normally no) Greeks –Vega- for both put and call options. Note that it has a very big value compared to other Greeks. As a result, assuming that the volatility does not change over the life of the option may cause inaccurate results. Therefore, there are ongoing researches to price options with Hidden Markov Models like [9], [7], [8].

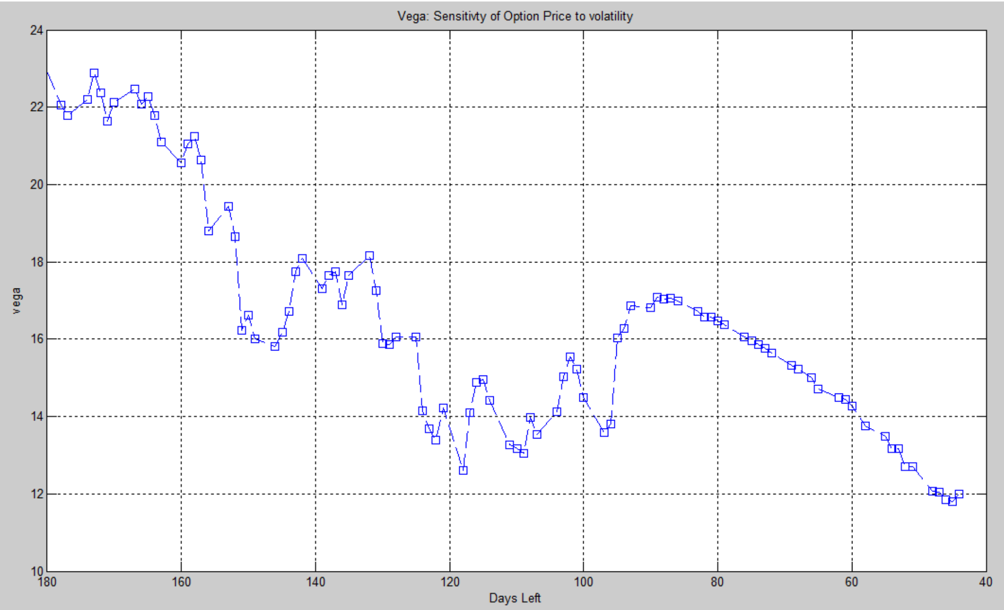

Figure 31: Vega for Options (Put and Call) (Source: Author's Calculations)

We found that option prices in Turkey are not following the theoretical value of the options. We have to point out that some of the days (for example January 28, 2014) the size of the option markets is 0 if we look to "Traded Value by Members" at [21]. That means, the option prices are calculated by determining "the daily settlement price in consideration of theoretical price, spot price of the underlying asset, the previous day's settlement price or the best bid and ask prices at the end of the session." [15] However, it is not zero all of the days (for example call options expiring at June have a volume of 1.124.000 TL at April 16, 2014 and a volume of 3.577.000 TL at May 15, 2014 (including all strike prices)) as a result an arbitrageur may still benefit from the financial markets in Turkey.

The graphs shown below plots the difference between the value of constructing a Portfolio (say Portfolio A) with BIST30 and a put written on BIST30 with exercise price of 90 and maturity date June 30, 2014 and a Portfolio (say portfolio B) consisting of call option with exercise price of 90 and the present value of a risk-free bond paying 90 both expiring at June 30, 2014. These portfolios are the ones defined on section 2.4. Put Call Parity, and the differences should be normally zero. The theoretical value (the red line) is actually zero for everyday, yet it is not zero when we use the observed values.

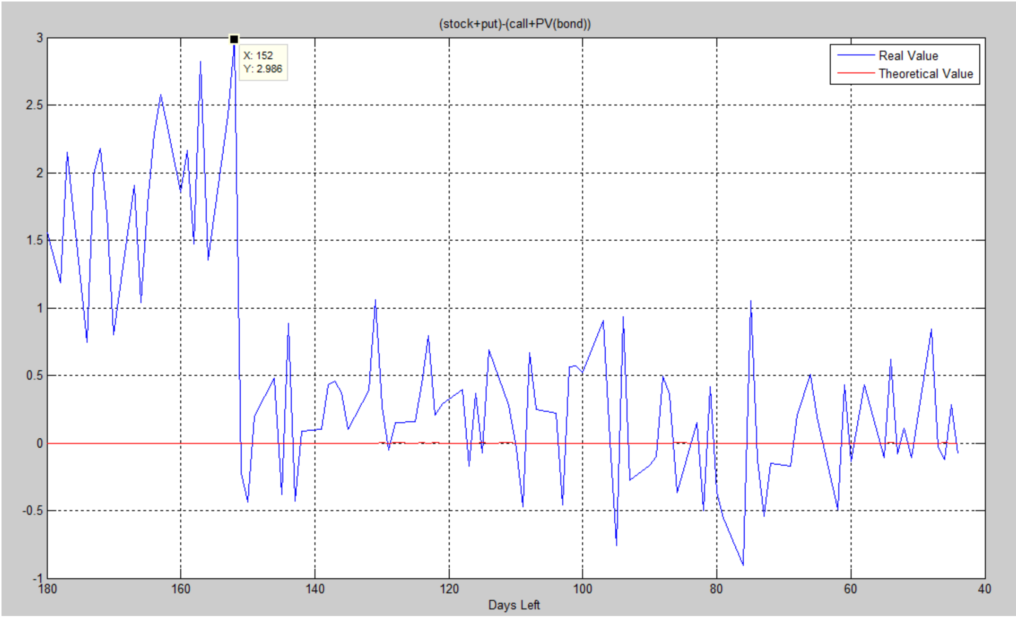

Figure 32: Put Call Parity Difference in Observed Market (Source: Author's Calculations)

Let's calculate what happens if we **sell** Portfolio A (call + bond) and **buy** Portfolio B (put + stock) at January 28, 2014 that is when there are 152 days to expiration. (The values can be seen in the graphs and in Appendix E).

Table 9: Value for Portfolio A and Portfolio B at January 28, 2014 (with observed values)

	Option Values (Call option for A, Put Option for B)	Stock Value for B, Bond present value for A	**Total**
Value of Portfolio A	2.12 TL	$90e^{-152/365*0.1031}$=86.22 TL	88.34 TL
Value of Portfolio B	13.39 TL	77.93 TL	91.32 TL

We showed in section 2.4 both have the same values at time to expiration. As a result net inflow or outflow is equal to 0 at June 30, 2014 from the sum of the portfolios as we long Portfolio A and short Portfolio B. In January 28 we give 88.34 TL and receive 91.32 TL hence we have a total inflow of nearly 2.98 TL. That is for one stock. A contract has 100 stock, therefore our net inflow will be 298 TL. If we do it for 100 contracts we make 29.800 TL without any risk! Of course, buying too much will push the call options up and selling too much will push put option down until the arbitrage opportunity disappears.

Note that from January 29, 2014 prices start to oscillate around 0, meaning that the price makers and/or new players noticed the arbitrage opportunity. Table 10 shows t-test statistics results for the hypotheses that put-call parity holds beginning from dates December 31, 2013 (when there are 180 days to expiration), January 29, 2014 (when there are 151 days to expiration) and February 19, 2014 (when there are 130 days to expiration). We can see this also in the Figure shown below.

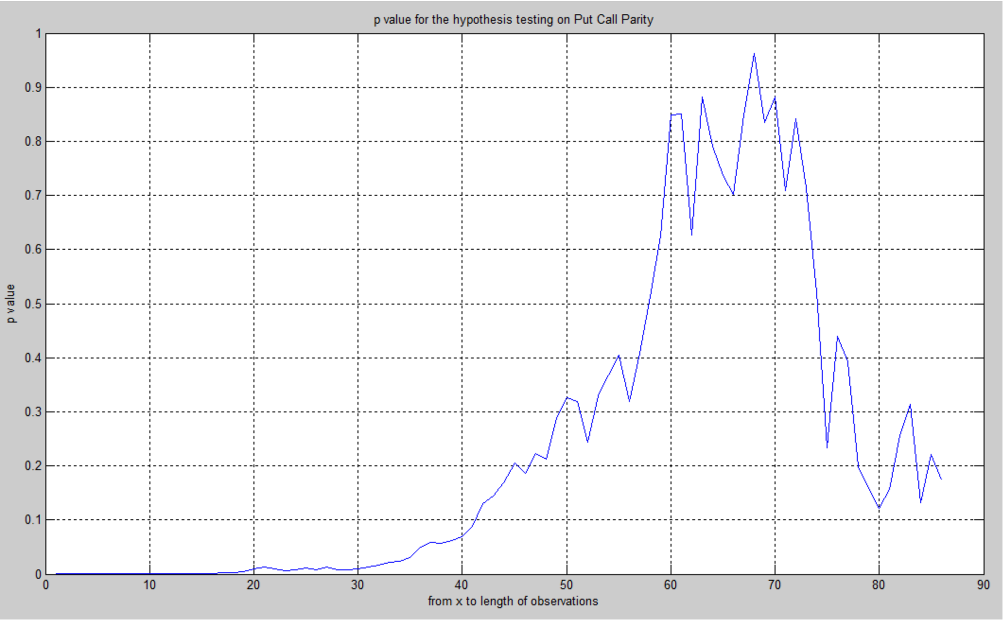

Figure 33: p value for the Hypothesis Testing that Put Call Parity Holds

(Source: Author's Calculations)

Figure 33 shows the p value for the t-test for the hypothesis testing that put call parity holds. The x line states the data point from which we look for the parity. That is if x = 10 we omit, first 10 observation and we make the test for the rest of observations (that is for 86 data points). Note that after 20th data point p value starts to increase, that is we begin to not be able to reject the null.

Table 10: Results for Testing the Hypothesis that Put-Call Parity Holds with t-Test

Number of days to expiration	**Result of the Hypothesis**	**Significance Level**
180	Rejected	%1
152	Cannot be rejected	%1
130	Cannot be rejected	%5

(Source: Author's Calculations)

Table 10 and Figure 33 support our observations that there was an arbitrage opportunity for 21 days and it began to diminish afterwards. In the next subsection, we argue the possible reasons and results of the deviations from put-call parity.

4.3. DEVIATIONS FROM PUT CALL PARITY

We summarized shortly in Chapter 2 the put-call parity firstly developed by Stoll in [25] European Options. Later on, put call parity was extended by Robert C. Merton in [26]. The original put-call parity was developed for over-the-counter markets and empirical study was made also by [27] and the put-call parity equation was mostly supported. However, legalized option markets are different from over-the-counter markets and different analyses were made in [28] and [29]. Both of the papers find small differences in the put-call parity, however in [28] they claimed that the reason was due to possible early exercise of the options for hedging risks and in [29] the reason was claimed to be because of transaction errors. Both of the explanation will not clarify the reason of the difference in Turkey's financial market when the volume of options market is 0 and the prices are determined by BIST itself as it is stated in [16]. However, they may explain the oscillations around zero after January 29, 2014.

Put-Call Parity equation was criticized by Dallas Brozik at [30] stating that it is not logical to accept the theory as true and try to find explanations for the deviation of it. He explains that interest rates are not a factor affecting the options prices in the short run (when time to expiration is less than 25 days). However, it is found that in the long run (when time to expiration is more than 30 days) put-call parity is close to hold. Yet, as it is stated before when options are priced by BIST itself, put call parity has to hold, because the reasons of not holding is due to transactional costs and/or different people's need of hedging, which are not possible when the volume is 0.

In [31] Easley, O'Hara and Srinivas state that some informed investors may choose to trade in options before trading in the underlying stocks and the option prices can carry information about future stock prices. As it is strongly possible that BIST30 has some more information than an investor, the deviations from put call parity may be due to this information and the deviation may carry some information about future stock movements. This information may be carried to the option prices by the volatility used during the calculations by BIST30.

To sum up, there are three possible reasons stated in the literature for the deviations from put call parity: transaction costs, different investor's needs and additional information. Small oscillations after January 29, 2014 may be explained by all of these three possible reasons when the volume of the market is different from 0. However, first 21 investment days for the June expiring options cannot be explained with transaction costs and people needs as the difference is large. As a result, index option market is not efficient according to Efficient Market Hypothesis due to these arbitrage possibilities.

CHAPTER 5

5. CONCLUSION

The main purpose of this thesis is to analyze the difference between observed and theoretical prices in the very new BIST30 index options market in Turkey. Firstly, we begin by explaining what options are basically and introduced the profit graphs and Turkey's Option Market's mechanics. Later on, we study the necessary tools to understand Black-Scholes-Merton Model, a Nobel Prize winner formula. After working on the theoretical side we introduced our approach to compare observed and theoretical data. Finally we show with the put-call parity that Turkey's option markets may be open for arbitrage opportunities. In subsection 4.3. we also argued that the deviations may include some information about future index prices as BIST may have information about firms that are in Bist30 hence we may use the deviation to predict future movements in BIST30.

In [33] it is argued that "Black-Scholes model does not fit properly during financial turbulences, when sudden changes in the most important input variable, the volatility on the underlying asset, occur". Especially beginning from May 31, 2013 (Gezi Protests) in Turkey, the financial markets were tense. The tension increased after operations began to the government beginning from December 17, 2013 and it reached to its top level before the elections on March 31, 2014. As a result, this "turbulences" may explain the difference between observed data and theoretical data. People were pessimistic about the financial situation of the country. In addition in the near past, in 2001, Turkey had a financial crisis. All of these reasons cause an increase and sudden changes in volatility. As a future work, Option Pricing with Hidden Markov Models may be studied in Turkey to be able to take into consideration these changes in volatility.

Besides the difference of observed and theoretical option prices we also study Put-Call Parity in order to analyze arbitrage effects. There are obvious arbitrage opportunities for the first twenty working days, yet arbitrage opportunities are slowly vanishing in the market. This shows that the market, even if it is slowly, is removing arbitrage opportunities. Option prices effect stock prices in the long period as much as stock prices effect options as option may be used as speculative tools. In [34], Martijn Cremers and David Weinbaum analyze the information that deviations from Put Call Parity contains for stock return predictability. This study can be made also in Turkey’s financial market as a future work.

To sum up, we find that observed option prices are not equal to the theoretically calculated option prices most of the time. In addition, there are deviations from put call parity although they are diminishing as we approach to the expiration date. The differences may be explained by the political situations in Turkey as we argue above or with the information and/or knowledge of the investors about option pricing. To conclude, we can strongly state that Black-Scholes-Merton model does not work for the option markets in Turkey and it is not efficient and/or observed prices are not efficient according to the Efficient Market Hypothesis as there are arbitrage opportunities.

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Appendix A: Tez Fotokopisi İzin Formu

TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü	
Sosyal Bilimler Enstitüsü	X
Uygulamalı Matematik Enstitüsü	
Enformatik Enstitüsü	
Deniz Bilimleri Enstitüsü	

YAZARIN

Soyadı : Berkay

Adı : AKYAPI

Bölümü : İktisat

TEZİN ADI : AN ANALYSIS OF BIST30 INDEX OPTIONS MARKET

TEZİN TÜRÜ : Yüksek Lisans [X] Doktora [ ]

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir.	X
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir.	
3. Tezimden bir bir (1) yıl süreyle fotokopi alınamaz.	

TEZİN KÜTÜPHANEYE TESLİM TARİHİ:

Appendix B: Code for Different Option Type's Profit vs. Final Stock Prices

```
clc
clear all
close all

S0 = 90;
K = 100;
OptionPrice = 5;
ST = [50:0.1:125];
n = length(ST);

Profit = zeros(1,n);

%Graph for Long Call Option Profit (One Share)
for a=1:n

    if (ST(1,a)<K)
        Profit(1,a) = -OptionPrice;
    else
        Profit(1,a) = ST(1,a)-K-OptionPrice;

    end
end

plot(ST, Profit)
hold on
grid on
xlabel('Final Stock Price')
ylabel('Profit')
title('Graph for Long Call Profit vs Final Stock Price (for one
share)')
ylim([-10 50])
xlim([50 125])
hold off

%Graph for Short Call Option Profit (One Share)
```

```
for a=1:n

    if (ST(1,a)<K)
        Profit(1,a) = OptionPrice;
    else
        Profit(1,a) = OptionPrice-(ST(1,a)-K);

    end
end

figure
plot(ST, Profit)
hold on
grid on
xlabel('Final Stock Price')
ylabel('Profit')
title('Graph for Short Call Profit vs Final Stock Price (for one
share)')
ylim([-50 10])
xlim([50 125])
hold off

% Graph for Long Put Option Profit (One Share)

for a=1:n

    if(ST(1,a)>K)
        Profit(1,a) = -OptionPrice;
    else
        Profit(1,a) = K-ST(1,a)-OptionPrice;
    end
end

figure
plot(ST, Profit)
hold on
grid on
xlabel('Final Stock Price')
```

```
ylabel('Profit')
title('Graph for Long Put Profit vs Final Stock Price (for one
share)')
ylim([-10 50])
xlim([50 125])
hold off

% Graph for Short Put Option Profit (One Share)

for a=1:n

    if (ST(1,a)>K)
        Profit(1,a) = OptionPrice;
    else
        Profit(1,a) = OptionPrice-(K - ST(1,a));
    end
end

figure
plot(ST, Profit)
hold on
grid on
xlabel('Final Stock Price')
ylabel('Profit')
title('Graph for Short Put Profit vs Final Stock Price (for one
share)')
ylim([-50 10])
xlim([50 125])
hold off
```

Appendix C: Code for Plotting the Differences between Observed and Theoretical Values

```
%-------------------------------------------------------------------
% Data beginning from 31.12.2013 and ends at 16.05.2014 for Call
% Options
%-------------------------------------------------------------------

clear all
close all
clc

load count.dat

addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\Contract Information\Merged');
addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\DataUsedinCode');

% Real Data for Option Prices

% Variables for Call Contract
StrPriceRangeC = 'C2:C1255';
OptPriceRangeC = 'F2:F1255';
DateRangeC = 'G2:G1255';

FileNameC = 'ExpirationTime_1406-Call';
SheetNameC = 'Call Contracts';

StrPriceC = xlsread(FileNameC, SheetNameC, StrPriceRangeC);
OptPriceC = xlsread(FileNameC, SheetNameC, OptPriceRangeC);
%DateC = xlsread(FileNameC, SheetNameC, DateRangeC); %Check this one

%Plotting Option Prices vs. Day for Call Options
n = length(StrPriceC);

count = ones(8,1);
```

```matlab
OptPriceC76 = zeros();
OptPriceC78 = zeros();
OptPriceC80 = zeros();
OptPriceC82 = zeros();
OptPriceC84 = zeros();
OptPriceC86 = zeros();
OptPriceC88 = zeros();
OptPriceC90 = zeros();

for a = 1:n

    if (StrPriceC(a,1)== 76)
        OptPriceC76(count(1,1),1) = OptPriceC(a,1);
        count(1,1) = count(1,1) + 1;
    elseif (StrPriceC(a,1)== 78)
        OptPriceC78(count(2,1),1) = OptPriceC(a,1);
        count(2,1) = count(2,1) + 1;
    elseif (StrPriceC(a,1)== 80)
        OptPriceC80(count(3,1),1) = OptPriceC(a,1);
        count(3,1) = count(3,1) + 1;
    elseif (StrPriceC(a,1)== 82)
        OptPriceC82(count(4,1),1) = OptPriceC(a,1);
        count(4,1) = count(4,1) + 1;
    elseif (StrPriceC(a,1)== 84)
        OptPriceC84(count(5,1),1) = OptPriceC(a,1);
        count(5,1) = count(5,1) + 1;
    elseif (StrPriceC(a,1)== 86)
        OptPriceC86(count(6,1),1) = OptPriceC(a,1);
        count(6,1) = count(6,1) + 1;
    elseif (StrPriceC(a,1)== 88)
        OptPriceC88(count(7,1),1) = OptPriceC(a,1);
        count(7,1) = count(7,1) + 1;
    elseif (StrPriceC(a,1)== 90)
        OptPriceC90(count(8,1),1) = OptPriceC(a,1);
        count(8,1) = count(8,1) + 1;
    else
        OptPriceC(a,1);
    end
```

```matlab
end

% Theoretical Calculation for Pricing Options with Black&Scholes
Model

%Getting bist30 data

FileName = 'Price_XU030';
SheetNameP = 'priceXU030';
PriceRange = 'G2:G97';

bist30Price = xlsread(FileName, SheetNameP, PriceRange);

%Getting volatility

FileName = 'volatility_XU030';
SheetNameV = 'volatility';
VolRange = 'D2:D97';

bist30Vol = xlsread(FileName, SheetNameV, VolRange);

%Getting Interest Rates

FileName = 'RiskFree';
SheetNameI = 'RiskFreeRate';
IntRange = 'B2:B97';

RFreeInt = xlsread(FileName, SheetNameI, IntRange);
NumberofDays = xlsread('RiskFree', 'RiskFreeRate', 'D2:D97');

% Calculation

CallPrice76 = zeros([length(OptPriceC76) 1]);
CallPrice78 = zeros([length(OptPriceC78) 1]);
CallPrice80 = zeros([length(OptPriceC80) 1]);
CallPrice82 = zeros([length(OptPriceC82) 1]);
CallPrice84 = zeros([length(OptPriceC84) 1]);
```

```
CallPrice86 = zeros([length(OptPriceC86) 1]);
CallPrice88 = zeros([length(OptPriceC88) 1]);
CallPrice90 = zeros([length(OptPriceC90) 1]);
differenceC76 = zeros([length(OptPriceC90) 1]);
differenceC78 = zeros([length(OptPriceC90) 1]);
differenceC80 = zeros([length(OptPriceC90) 1]);
differenceC82 = zeros([length(OptPriceC90) 1]);
differenceC84 = zeros([length(OptPriceC90) 1]);
differenceC86 = zeros([length(OptPriceC90) 1]);
differenceC88 = zeros([length(OptPriceC90) 1]);
differenceC90 = zeros([length(OptPriceC90) 1]);
Put = zeros(length(OptPriceC76));

for a = 1:length(OptPriceC76)

    [CallPrice76(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
76, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice78(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
78, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice80(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
80, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice82(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
82, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice84(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
84, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice86(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
86, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice88(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
88, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [CallPrice90(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
```

```
    differenceC76(a,1) = CallPrice76(a,1) - OptPriceC76(a,1);
    differenceC78(a,1) = CallPrice78(a,1) - OptPriceC78(a,1);
    differenceC80(a,1) = CallPrice80(a,1) - OptPriceC80(a,1);
    differenceC82(a,1) = CallPrice82(a,1) - OptPriceC82(a,1);
    differenceC84(a,1) = CallPrice84(a,1) - OptPriceC84(a,1);
    differenceC86(a,1) = CallPrice86(a,1) - OptPriceC86(a,1);
    differenceC88(a,1) = CallPrice88(a,1) - OptPriceC88(a,1);
    differenceC90(a,1) = CallPrice90(a,1) - OptPriceC90(a,1);

end

%-------------------------------------------------------------------
% Plot
%-------------------------------------------------------------------

% For Strike Price = 76
figure
subplot(1,2,1);
plot(OptPriceC76)
title ('Option Price for X = 76 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice76, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC76,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 78
figure
subplot(1,2,1);
plot(OptPriceC78)
title ('Option Price for X = 78  Exp.Date = 30.06.2014')
ylabel('Option Prices')
```

```
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice78, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC78,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 80
figure
subplot(1,2,1);
plot(OptPriceC80)
title ('Option Price for X = 80 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice80, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC80,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 82
figure
subplot(1,2,1);
plot(OptPriceC82)
title ('Option Price for X = 82 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice82, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC82,'g')
```

```
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 84
figure
subplot(1,2,1);
plot(OptPriceC84)
title ('Option Price for X = 84 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice84, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC84,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 86
figure
subplot(1,2,1);
plot(OptPriceC86)
title ('Option Price for X = 86 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice86, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC86,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% Fro Strike Price = 88
figure
subplot(1,2,1);
```

```
plot(OptPriceC88)
title ('Option Price for X = 88 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice88, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC88,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% For Strike Price = 90
figure
subplot(1,2,1);
plot(OptPriceC90)
title ('Option Price for X = 90 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(CallPrice90, 'r')
hold off
legend('Real Value', 'Theoretical Value')
subplot(1,2,2);
plot(differenceC90,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% Finding the mean of the differences
meanofDifC76 = mean(differenceC76);
meanofDifC78 = mean(differenceC78);
meanofDifC80 = mean(differenceC80);
meanofDifC82 = mean(differenceC82);
meanofDifC84 = mean(differenceC84);
meanofDifC86 = mean(differenceC86);
meanofDifC88 = mean(differenceC88);
meanofDifC90 = mean(differenceC90);
```

```
%Finding Sum of Squared Errors of the differences
deltaofDifference76 = 0;
deltaofDifference78 = 0;
deltaofDifference80 = 0;
deltaofDifference82 = 0;
deltaofDifference84 = 0;
deltaofDifference86 = 0;
deltaofDifference88 = 0;
deltaofDifference90 = 0;

for a = 1:length(differenceC76)

    deltaofDifference76 = deltaofDifference76 + (differenceC76(a,1)-
meanofDifC76)^2;
    deltaofDifference78 = deltaofDifference78 + (differenceC78(a,1)-
meanofDifC78)^2;
    deltaofDifference80 = deltaofDifference80 + (differenceC80(a,1)-
meanofDifC80)^2;
    deltaofDifference82 = deltaofDifference82 + (differenceC82(a,1)-
meanofDifC82)^2;
    deltaofDifference84 = deltaofDifference84 + (differenceC84(a,1)-
meanofDifC84)^2;
    deltaofDifference86 = deltaofDifference86 + (differenceC86(a,1)-
meanofDifC86)^2;
    deltaofDifference88 = deltaofDifference88 + (differenceC88(a,1)-
meanofDifC88)^2;
    deltaofDifference90 = deltaofDifference90 + (differenceC90(a,1)-
meanofDifC90)^2;

end

% Finding the variance of the differences
varofDifC76 = var(differenceC76);
varofDifC78 = var(differenceC78);
varofDifC80 = var(differenceC80);
varofDifC82 = var(differenceC82);
varofDifC84 = var(differenceC84);
varofDifC86 = var(differenceC86);
```

```
varofDifC88 = var(differenceC88);
varofDifC90 = var(differenceC90);

% Finding the standard deviation of the differences
stdofDifC76 = std(differenceC76);
stdofDifC78 = std(differenceC78);
stdofDifC80 = std(differenceC80);
stdofDifC82 = std(differenceC82);
stdofDifC84 = std(differenceC84);
stdofDifC86 = std(differenceC86);
stdofDifC88 = std(differenceC88);
stdofDifC90 = std(differenceC90);

% Testing the null hypothesis that real and theoretical option
values are
% equal. h = 1 if null is rejected, h = 0 otherwise

[h76,p76] = ttest(OptPriceC76, CallPrice76, 0.01);
[h78,p78] = ttest(OptPriceC78, CallPrice78, 0.01);
[h80,p80] = ttest(OptPriceC80, CallPrice80, 0.01);
[h82,p82] = ttest(OptPriceC82, CallPrice82, 0.01);
[h84,p84] = ttest(OptPriceC84, CallPrice84, 0.01);
[h86,p86] = ttest(OptPriceC86, CallPrice86, 0.01);
[h88,p88] = ttest(OptPriceC88, CallPrice88, 0.01);
[h90,p90] = ttest(OptPriceC90, CallPrice90, 0.01);

%------------------------------------------------------------------
% Data beginning from 31.12.2013 and ends at 16.05.2014 for Put
% Options
%------------------------------------------------------------------

addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\Contract Information\Merged');
addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\DataUsedinCode');

% Real data for option pricing
```

```
% Variables for Put Contracts
StrPriceRangeP = 'C2:C1255';
OptPriceRangeP = 'F2:F1255';
DateRangeP = 'G2:G1255';

FileNameP = 'ExpirationTime_1406-Put';
SheetNameP = 'Put Contracts';

StrPriceP = xlsread(FileNameP, SheetNameP, StrPriceRangeP);
OptPriceP = xlsread(FileNameP, SheetNameP, OptPriceRangeP);
%DateP = xlsread(FileNameP, SheetNameP, DateRangeP); %Check this one

%Plotting Option Prices vs. Time for Put Options
n = length(StrPriceP);

count = ones(8,1);
OptPriceP76 = zeros();
OptPriceP78 = zeros();
OptPriceP80 = zeros();
OptPriceP82 = zeros();
OptPriceP84 = zeros();
OptPriceP86 = zeros();
OptPriceP88 = zeros();
OptPriceP90 = zeros();

for a = 1:n

    if (StrPriceP(a,1)== 76)
        OptPriceP76(count(1,1),1) = OptPriceP(a,1);
        count(1,1) = count(1,1) + 1;
    elseif (StrPriceP(a,1)== 78)
        OptPriceP78(count(2,1),1) = OptPriceP(a,1);
        count(2,1) = count(2,1) + 1;
    elseif (StrPriceP(a,1)== 80)
        OptPriceP80(count(3,1),1) = OptPriceP(a,1);
        count(3,1) = count(3,1) + 1;
    elseif (StrPriceP(a,1)== 82)
        OptPriceP82(count(4,1),1) = OptPriceP(a,1);
```

```
        count(4,1) = count(4,1) + 1;
    elseif (StrPriceP(a,1)== 84)
        OptPriceP84(count(5,1),1) = OptPriceP(a,1);
        count(5,1) = count(5,1) + 1;
    elseif (StrPriceP(a,1)== 86)
        OptPriceP86(count(6,1),1) = OptPriceP(a,1);
        count(6,1) = count(6,1) + 1;
    elseif (StrPriceP(a,1)== 88)
        OptPriceP88(count(7,1),1) = OptPriceP(a,1);
        count(7,1) = count(7,1) + 1;
    elseif (StrPriceP(a,1)== 90)
        OptPriceP90(count(8,1),1) = OptPriceP(a,1);
        count(8,1) = count(8,1) + 1;
    else
        OptPriceP(a,1);
    end

end

% Theoretical Calculation for Pricing Options with Black&Scholes
Model

%Getting bist30 data

FileName = 'Price_XU030';
SheetNameP = 'priceXU030';
PriceRange = 'G2:G97';

bist30Price = xlsread(FileName, SheetNameP, PriceRange);

%Getting volatility

FileName = 'volatility_XU030';
SheetNameV = 'volatility';
VolRange = 'D2:D97';

bist30Vol = xlsread(FileName, SheetNameV, VolRange);
```

```
%Getting Interest Rates

FileName = 'RiskFree';
SheetNameI = 'RiskFreeRate';
IntRange = 'B2:B97';

RFreeInt = xlsread(FileName, SheetNameI, IntRange);
NumberofDays = xlsread('RiskFree', 'RiskFreeRate', 'D2:D97');

% Calculation

PutPrice76 = zeros([length(OptPriceP76) 1]);
PutPrice78 = zeros([length(OptPriceP78) 1]);
PutPrice80 = zeros([length(OptPriceP80) 1]);
PutPrice82 = zeros([length(OptPriceP82) 1]);
PutPrice84 = zeros([length(OptPriceP84) 1]);
PutPrice86 = zeros([length(OptPriceP86) 1]);
PutPrice88 = zeros([length(OptPriceP88) 1]);
PutPrice90 = zeros([length(OptPriceP90) 1]);
differenceP76 = zeros([length(OptPriceP90) 1]);
differenceP78 = zeros([length(OptPriceP90) 1]);
differenceP80 = zeros([length(OptPriceP90) 1]);
differenceP82 = zeros([length(OptPriceP90) 1]);
differenceP84 = zeros([length(OptPriceP90) 1]);
differenceP86 = zeros([length(OptPriceP90) 1]);
differenceP88 = zeros([length(OptPriceP90) 1]);
differenceP90 = zeros([length(OptPriceP90) 1]);
Put = zeros([length(OptPriceP76) 1]);

for a = 1:length(OptPriceP76)

    [Put(a,1), PutPrice76(a,1)] = blsprice(bist30Price(a,1)*10^-3,
76, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice78(a,1)] = blsprice(bist30Price(a,1)*10^-3,
78, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
```

```
    [Put(a,1), PutPrice80(a,1)] = blsprice(bist30Price(a,1)*10^-3,
80, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice82(a,1)] = blsprice(bist30Price(a,1)*10^-3,
82, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice84(a,1)] = blsprice(bist30Price(a,1)*10^-3,
84, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice86(a,1)] = blsprice(bist30Price(a,1)*10^-3,
86, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice88(a,1)] = blsprice(bist30Price(a,1)*10^-3,
88, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);
    [Put(a,1), PutPrice90(a,1)] = blsprice(bist30Price(a,1)*10^-3,
90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);

    differenceP76(a,1) = PutPrice76(a,1) - OptPriceP76(a,1);
    differenceP78(a,1) = PutPrice78(a,1) - OptPriceP78(a,1);
    differenceP80(a,1) = PutPrice80(a,1) - OptPriceP80(a,1);
    differenceP82(a,1) = PutPrice82(a,1) - OptPriceP82(a,1);
    differenceP84(a,1) = PutPrice84(a,1) - OptPriceP84(a,1);
    differenceP86(a,1) = PutPrice86(a,1) - OptPriceP86(a,1);
    differenceP88(a,1) = PutPrice88(a,1) - OptPriceP88(a,1);
    differenceP90(a,1) = PutPrice90(a,1) - OptPriceP90(a,1);

end

%-------------------------------------------------------------------
% Plot
%-------------------------------------------------------------------

figure
subplot(1,2,1);
plot(OptPriceP76)
title ('Option Price for X = 76 Exp. Date = 30.06.2014')
```

```
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice76, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP76,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP78)
title ('Option Price for X = 78  Exp.Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice78, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP78,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP80)
title ('Option Price for X = 80 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice80, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
```

```
subplot(1,2,2);
plot(differenceP80,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP82)
title ('Option Price for X = 82 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice82, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP82,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP84)
title ('Option Price for X = 84 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice84, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP84,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
```

```
subplot(1,2,1);
plot(OptPriceP86)
title ('Option Price for X = 86 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice86, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP86,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP88)
title ('Option Price for X = 88 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice88, 'r')
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP88,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

figure
subplot(1,2,1);
plot(OptPriceP90)
title ('Option Price for X = 90 Exp. Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Date (beggining from 31.12.2013)')
hold on
plot(PutPrice90, 'r')
```

```
hold off
legend('Real Value', 'Theoretical Value')
grid on
subplot(1,2,2);
plot(differenceP90,'g')
title('Difference between theoretical value and real value')
ylabel('Difference (TL)')

% Finding the mean of the differences
meanofDifP76 = mean(differenceP76);
meanofDifP78 = mean(differenceP78);
meanofDifP80 = mean(differenceP80);
meanofDifP82 = mean(differenceP82);
meanofDifP84 = mean(differenceP84);
meanofDifP86 = mean(differenceP86);
meanofDifP88 = mean(differenceP88);
meanofDifP90 = mean(differenceP90);

%Finding Sum of Squared Errors of the differences
deltaofDifference76 = 0;
deltaofDifference78 = 0;
deltaofDifference80 = 0;
deltaofDifference82 = 0;
deltaofDifference84 = 0;
deltaofDifference86 = 0;
deltaofDifference88 = 0;
deltaofDifference90 = 0;

for a = 1:length(differenceP76)

    deltaofDifference76 = deltaofDifference76 + (differenceP76(a,1)-
meanofDifP76)^2;
    deltaofDifference78 = deltaofDifference78 + (differenceP78(a,1)-
meanofDifP78)^2;
    deltaofDifference80 = deltaofDifference80 + (differenceP80(a,1)-
meanofDifP80)^2;
    deltaofDifference82 = deltaofDifference82 + (differenceP82(a,1)-
meanofDifP82)^2;
```

```
    deltaofDifference84 = deltaofDifference84 + (differenceP84(a,1)-
meanofDifP84)^2;
    deltaofDifference86 = deltaofDifference86 + (differenceP86(a,1)-
meanofDifP86)^2;
    deltaofDifference88 = deltaofDifference88 + (differenceP88(a,1)-
meanofDifP88)^2;
    deltaofDifference90 = deltaofDifference90 + (differenceP90(a,1)-
meanofDifP90)^2;

end

% Finding the variance of the differences
varofDifP76 = var(differenceP76);
varofDifP78 = var(differenceP78);
varofDifP80 = var(differenceP80);
varofDifP82 = var(differenceP82);
varofDifP84 = var(differenceP84);
varofDifP86 = var(differenceP86);
varofDifP88 = var(differenceP88);
varofDifP90 = var(differenceP90);

% Finding the standard deviation of the differences
stdofDifP76 = std(differenceP76);
stdofDifP78 = std(differenceP78);
stdofDifP80 = std(differenceP80);
stdofDifP82 = std(differenceP82);
stdofDifP84 = std(differenceP84);
stdofDifP86 = std(differenceP86);
stdofDifP88 = std(differenceP88);
stdofDifP90 = std(differenceP90);

% Testing the null hypothesis that real and theoretical option
values are
% equal. h = 1 if null is rejected, h = 0 otherwise

[h76,p76] = ttest(OptPriceP76, PutPrice76, 0.01);
[h78,p78] = ttest(OptPriceP78, PutPrice78, 0.01);
[h80,p80] = ttest(OptPriceP80, PutPrice80, 0.01);
```

```
[h82,p82] = ttest(OptPriceP82, PutPrice82, 0.01);
[h84,p84] = ttest(OptPriceP84, PutPrice84, 0.01);
[h86,p86] = ttest(OptPriceP86, PutPrice86, 0.01);
[h88,p88] = ttest(OptPriceP88, PutPrice88, 0.01);
[h90,p90] = ttest(OptPriceP90, PutPrice90, 0.01);
```

Appendix D: Code for Plotting Put Call Parity Graph

```
%-------------------------------------------------------------------
% Data beginning from 31.12.2013 and ends at 16.05.2014
% Put Call Parity Calculated for Strike Price = 90
%-------------------------------------------------------------------

% European Call Option for Strike Price = 90
clear all
close all
clc

load count.dat

addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\Contract Information\Merged');
addpath('C:\Users\Berkay\Desktop\Ders\Master of Science\Tez\Option
Pricing Turkey Data\DataUsedinCode');

% Real Data for Option Prices

% Variables for Call Contract
StrPriceRangeC = 'C2:C1255';
OptPriceRangeC = 'F2:F1255';
DateRangeC = 'G2:G1255';

FileNameC = 'ExpirationTime_1406-Call';
SheetNameC = 'Call Contracts';

StrPriceC = xlsread(FileNameC, SheetNameC, StrPriceRangeC);
OptPriceC = xlsread(FileNameC, SheetNameC, OptPriceRangeC);
%DateC = xlsread(FileNameC, SheetNameC, DateRangeC); %Check this one

%Plotting Option Prices vs. Day for Call Options
n = length(StrPriceC);

count = 1;
OptPriceC90 = zeros();
```

```
for a = 1:n

    if (StrPriceC(a,1)== 90)
        OptPriceC90(count,1) = OptPriceC(a,1);
        count = count + 1;
    else
        OptPriceC(a,1);
    end

end

% Theoretical Calculation for Pricing Options with Black&Scholes
Model

%Getting bist30 data

FileName = 'Price_XU030';
SheetNameP = 'priceXU030';
PriceRange = 'G2:G97';

bist30Price = xlsread(FileName, SheetNameP, PriceRange);

%Getting volatility

FileName = 'volatility_XU030';
SheetNameV = 'volatility';
VolRange = 'D2:D97';

bist30Vol = xlsread(FileName, SheetNameV, VolRange);

%Getting Interest Rates

FileName = 'RiskFree';
SheetNameI = 'RiskFreeRate';
```

```
IntRange = 'B2:B97';

RFreeInt = xlsread(FileName, SheetNameI, IntRange);
NumberofDays = xlsread('RiskFree', 'RiskFreeRate', 'D2:D97');

% Calculation

CallPrice90 = zeros([length(OptPriceC90) 1]);
differenceC90 = zeros([length(OptPriceC90) 1]);

Put = zeros(length(OptPriceC90));

for a = 1:length(OptPriceC90)

    [CallPrice90(a,1), Put(a,1)] = blsprice(bist30Price(a,1)*10^-3,
90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);

    differenceC90(a,1) = CallPrice90(a,1) - OptPriceC90(a,1);

end


%%
%-------------------------------------------------------------------
% European Put Option For Strike Price = 90
%-------------------------------------------------------------------

% Variables for Put Contracts
StrPriceRangeP = 'C2:C1255';
OptPriceRangeP = 'F2:F1255';
DateRangeP = 'G2:G1255';

FileNameP = 'ExpirationTime_1406-Put';
SheetNameP = 'Put Contracts';
```

```
StrPriceP = xlsread(FileNameP, SheetNameP, StrPriceRangeP);
OptPriceP = xlsread(FileNameP, SheetNameP, OptPriceRangeP);
%DateP = xlsread(FileNameP, SheetNameP, DateRangeP); %Check this one

%Plotting Option Prices vs. Time for Put Options
n = length(StrPriceP);

count = 1;
OptPriceP90 = zeros();

for a = 1:n

    if (StrPriceP(a,1)== 90)
        OptPriceP90(count,1) = OptPriceP(a,1);
        count = count + 1;
    else
        OptPriceP(a,1);
    end

end

% Theoretical Calculation for Pricing Options with Black&Scholes
Model

%Getting bist30 data

FileName = 'Price_XU030';
SheetNameP = 'priceXU030';
PriceRange = 'G2:G97';

bist30Price = xlsread(FileName, SheetNameP, PriceRange);

%Getting volatility

FileName = 'volatility_XU030';
SheetNameV = 'volatility';
VolRange = 'D2:D97';
```

```
bist30Vol = xlsread(FileName, SheetNameV, VolRange);

%Getting Interest Rates

FileName = 'RiskFree';
SheetNameI = 'RiskFreeRate';
IntRange = 'B2:B97';

RFreeInt = xlsread(FileName, SheetNameI, IntRange);
NumberofDays = xlsread('RiskFree', 'RiskFreeRate', 'D2:D97');

% Calculation

PutPrice90 = zeros([length(OptPriceP90) 1]);
differenceP90 = zeros([length(OptPriceP90) 1]);

Call = zeros([length(OptPriceP90) 1]);

for a = 1:length(OptPriceP90)

    [Call(a,1), PutPrice90(a,1)] = blsprice(bist30Price(a,1)*10^-3,
90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-
2);

    differenceP90(a,1) = PutPrice90(a,1) - OptPriceP90(a,1);

end

%%
%-------------------------------------------------------------------
% Calculation for Put Call Parity
% Put Call Parity: Stock + Put = Call + Bonds
% Contract Size of Options = 100
%-------------------------------------------------------------------
```

```
% Bond Data
FileName = 'RiskFree';
SheetNameI = 'RiskFreeRate';
BondPriceRange = 'C2:C97';
BondPrice = xlsread(FileName, SheetNameI, BondPriceRange);

% ParityDif = [100*Stock + Put] - [Call + 100*Bonds]

ParityDif_Real = zeros([length(BondPrice) 1]);
ParityDif_Theoretical = zeros([length(BondPrice) 1]);

for a = 1:length(BondPrice)

    ParityDif_Real(a,1) = (bist30Price(a,1)*10^-3 +
OptPriceP90(a,1)) - (OptPriceC90(a,1)+90*exp(RFreeInt(a,1)*(10^-
2)*(-NumberofDays(a,1)/365)));
    ParityDif_Theoretical(a,1) = (bist30Price(a,1)*10^-3 +
PutPrice90(a,1)) - (CallPrice90(a,1)+90*exp(RFreeInt(a,1)*(10^-2)*(-
NumberofDays(a,1)/365)));

end

figure
plot(NumberofDays, ParityDif_Real)
hold on
set ( gca, 'xdir', 'reverse' )
plot(NumberofDays, ParityDif_Theoretical,'r')
hold off
legend('Real Value', 'Theoretical Value')
title('(stock+put)-(call+PV(bond))')
grid on
set ( gca, 'xdir', 'reverse' )
xlabel('Days Left')

%%
%-------------------------------------------------------------------
% Calculation for Minimum Theoretical Value of the Call and Put
% Option
```

```
%-------------------------------------------------------------------
% Minimum Value of a Call Option
minimumValueC = zeros([length(OptPriceC90) 1]);

for a = 1:length(OptPriceC90)

    if(bist30Price(a,1)*(10^-3)>90*exp(RFreeInt(a,1)*(10^-2)*(-
NumberofDays(a,1)/365)))
        minimumValueC(a,1) = bist30Price(a,1)*(10^-3) -
90*exp(RFreeInt(a,1)*(10^-2)*(-NumberofDays(a,1)/365));
    else
        minimumValueC(a,1) = 0;
    end

end

figure
plot(NumberofDays, OptPriceC90)
title ('Call Option Price for X = 90  Exp.Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Days Left')
hold on
set ( gca, 'xdir', 'reverse' )
plot(NumberofDays, CallPrice90, 'r')
set ( gca, 'xdir', 'reverse' )
plot(NumberofDays, minimumValueC, '--gs')
set ( gca, 'xdir', 'reverse' )
hold off
legend('Real Value', 'Theoretical Value', 'MinimumValue')
grid on

% MinimumValue of a Put Option
minimumValueP = zeros([length(OptPriceP90) 1]);

for a = 1:length(OptPriceP90)

    if(bist30Price(a,1)*(10^-3)<90*exp(RFreeInt(a,1)*(10^-2)*(-
NumberofDays(a,1)/365)))
```

```
        minimumValueP(a,1) = 90*exp(RFreeInt(a,1)*(10^-2)*(-
NumberofDays(a,1)/365)) - bist30Price(a,1)*(10^-3);
    else
        minimumValueP(a,1) = 0;
    end

end

figure
plot(NumberofDays, OptPriceP90)
title ('Put Option Price for X = 90  Exp.Date = 30.06.2014')
ylabel('Option Prices')
xlabel('Days Left')
hold on
set ( gca, 'xdir', 'reverse' )
plot(NumberofDays, PutPrice90, 'r')
set ( gca, 'xdir', 'reverse' )
plot(NumberofDays, minimumValueP, '--gs')
set ( gca, 'xdir', 'reverse' )
hold off
legend('Real Value', 'Theoretical Value', 'MinimumValue')
grid on

%%
%-------------------------------------------------------------------
% Calculation of "The Greeks", delta, gamma, rho and vega
%-------------------------------------------------------------------

% Delta: the sensitivity of option value to change in the underlying
asset
% price

callDelta = zeros([length(OptPriceC90) 1]);
putDelta = zeros([length(OptPriceP90) 1]);

for a = 1:length(OptPriceP90)
```

```
    [callDelta(a,1), putDelta(a,1)] = blsdelta(bist30Price(a,1)*10^-3, 90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-2);

end

% Gamma: the sensitivity of delta to change in the underlying asset price.

gamma = zeros([length(OptPriceC90) 1]);
%******************************



for a = 1:length(OptPriceP90)

    gamma(a,1) = blsgamma(bist30Price(a,1)*10^-3, 90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-2);

end




% Rho: the sensitivity of option value to a change in interest rate

callRho = zeros([length(OptPriceC90) 1]);
putRho = zeros([length(OptPriceP90) 1]);

for a = 1:length(OptPriceP90)

    [callRho(a,1), putRho(a,1)] = blsrho(bist30Price(a,1)*10^-3, 90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-2);

end

% Vega: the sensitivity of option value to change in the underlying asset
```

```
% volatility

vega = zeros([length(OptPriceC90) 1]);
%*****************************

for a = 1:length(OptPriceP90)

    vega(a,1) = blsvega(bist30Price(a,1)*10^-3, 90,
RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365, bist30Vol(a,1)*10^-2);

end

% Theta: the sensitivity of option value to Time to Expiration

callTheta = zeros([length(OptPriceC90) 1]);
putTheta = zeros([length(OptPriceP90) 1]);

for a = 1:length(OptPriceP90)

    [callTheta(a,1), putTheta(a,1)] = blstheta(bist30Price(a,1)*10^-
3, 90, RFreeInt(a,1)*10^-2, NumberofDays(a,1)/365,
bist30Vol(a,1)*10^-2);

end

figure
subplot(2,2,1);
plot(NumberofDays, callDelta, '--bs')
set ( gca, 'xdir', 'reverse' )
title ('Call Delta: Sensitivty of Call Price to the Asset Price')
ylabel('Delta for Call')
xlabel('Days Left')
grid on
subplot(2,2,2);
plot(NumberofDays, gamma, '--bs')
title('Gamma: Sensitivty of Delta to the Asset Price')
ylabel('gamma')
```

```
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )
subplot(2,2,3)
plot(NumberofDays, callRho, '--bs')
title('Rho: Sensitivty of Call Price to the Interest Rate')
ylabel('rho')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )
subplot(2,2,4);
plot(NumberofDays,callTheta, '--bs')
title('Call Theta: Sensitivty of Option Price to Time to
Expiration')
ylabel('theta')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )

figure
subplot(2,2,1);
plot(NumberofDays,putDelta, '--bs')
title('Put Delta: Sensitivty of Put Price to the Asset Price')
ylabel('Delta for Put')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )
subplot(2,2,2);
plot(NumberofDays,gamma,'--bs')
title('Gamma: Sensitivty of Delta to the Asset Price')
ylabel('gamma')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )
subplot(2,2,3)
plot(NumberofDays,putRho, '--bs')
title('Rho: Sensitivty of Put Price to the Interest Rate')
ylabel('rho')
xlabel('Days Left')
```

```
grid on
set ( gca, 'xdir', 'reverse' )
subplot(2,2,4);
plot(NumberofDays,putTheta, '--bs')
title('Put Theta: Sensitivty of Option Price to Time to Expiration')
ylabel('theta')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse' )

figure
plot(NumberofDays,vega, '--bs')
title('Vega: Sensitivty of Option Price to volatility')
ylabel('vega')
xlabel('Days Left')
grid on
set ( gca, 'xdir', 'reverse')

%%
% Testing put call parity with t test
% Plotting p values

c=1;
realDiff = zeros();
h = zeros();
p = zeros();

for b = 1:(length(ParityDif_Real)-10)

    for a = b:length(ParityDif_Real)

        realDiff(c,1) = ParityDif_Real(a,1);
        c=c+1;
    end
    c=1;
    [h(b,1), p(b,1)] = ttest(realDiff,0.01);
    realDiff = zeros();
end
```

```
figure
plot(p)
xlabel('from x to length of observations')
ylabel('p value')
title('p value for the hypothesis testing on Put Call Parity')
grid on

figure
plot(h)
xlabel('from x to length of observations')
ylabel('h value')
title('h value for the hypothesis testing on Put Call Pairty (with
%1 significance level)')

c=1;
realDif = zeros();
h90 = zeros();
p90 = zeros();

for b = 0:(length(ParityDif_Real)-1)

    for a = 1:length(ParityDif_Real)-b

        realDif(c,1) = differenceP90(a,1);
        c=c+1;
    end
    c=1;
    [h90(b+1,1), p90(b+1,1)] = ttest(realDif);
    realDif = zeros();
end

figure
plot(p90)
xlabel('from 1 to 96-x')
ylabel('p value')
grid on
set ( gca, 'xdir', 'reverse' )
```

```
title('p value from 1 to 105-x for the hypothesis that put options
are equal')
```

Appendix E: Data Used During the Calculations

Table-a: Bist30 Index Value (Source: [22])

INDEX	DATE	SESSION	LOW	HIGH	CLOSE	AVERAGE
XU030	31.12.2013	2	81379,76	82938,86	82447,87	82159,31
XU030	02.01.2014	2	79840,81	81351,78	81351,78	80596,295
XU030	03.01.2014	2	79249,2	81141,41	79910,52	80195,305
XU030	06.01.2014	2	79994,57	83007,9	82574,31	81501,235
XU030	07.01.2014	2	82939,81	83787,58	83422,4	83363,695
XU030	08.01.2014	2	81735,32	82670,35	81865,26	82202,835
XU030	09.01.2014	2	80285,17	81196,14	80835,17	80740,655
XU030	10.01.2014	2	80995,65	82891,52	82891,52	81943,585
XU030	13.01.2014	2	83026,6	83706,73	83199,49	83366,665
XU030	14.01.2014	2	81807,29	83236,09	83236,09	82521,69
XU030	15.01.2014	2	82667,77	83824,17	83256,58	83245,97
XU030	16.01.2014	2	81652,84	82648,16	81652,84	82150,5
XU030	17.01.2014	2	80025,52	81678,32	80025,52	80851,92
XU030	20.01.2014	2	79810,05	80643,99	80133,25	80227,02
XU030	21.01.2014	2	80491,76	82126,77	80948,25	81309,265
XU030	22.01.2014	2	81484,83	82254,38	82226,13	81869,605
XU030	23.01.2014	2	79745,83	81709,5	79745,83	80727,665
XU030	24.01.2014	2	76950,73	78625,12	78359,92	77787,925
XU030	27.01.2014	2	78444,04	79727,72	78626,7	79085,88
XU030	28.01.2014	2	77008,04	78858,48	77047,89	77933,26
XU030	29.01.2014	2	73435,12	76219,34	75142,66	74827,23
XU030	30.01.2014	2	74685,17	76163,75	76031,78	75424,46
XU030	31.01.2014	2	74270,44	75279,24	74757,84	74774,84
XU030	03.02.2014	2	74461,74	75404,96	74661,2	74933,35
XU030	04.02.2014	2	74747,95	76207,18	76096,05	75477,565
XU030	05.02.2014	2	75612,66	76910,53	75613,76	76261,595
XU030	06.02.2014	2	77056,41	78388,44	78388,44	77722,425
XU030	07.02.2014	2	77704,56	78970,65	78496,18	78337,605
XU030	10.02.2014	2	77143,46	78056,09	77734,01	77599,775
XU030	11.02.2014	2	77755,27	78662,76	78039,38	78209,015
XU030	12.02.2014	2	78237,4	78724,16	78314,57	78480,78
XU030	13.02.2014	2	77160,63	77715,04	77313,06	77437,835
XU030	14.02.2014	2	78319,9	78894,46	78753,32	78607,18
XU030	17.02.2014	2	79476,61	80108,72	79690,98	79792,665
XU030	18.02.2014	2	77741,75	79366,62	77741,75	78554,185
XU030	19.02.2014	2	76362,89	77301,13	76823,38	76832,01

XU030	20.02.2014	2	76402,72	77343,05	77222,8	76872,885
XU030	21.02.2014	2	76819,18	77703,76	77420,03	77261,47
XU030	24.02.2014	2	77314,74	78031,38	77757,39	77673,06
XU030	25.02.2014	2	74933,01	75847,98	75207,68	75390,495
XU030	26.02.2014	2	74353,54	75639,5	74428,03	74996,52
XU030	27.02.2014	2	74362,63	75184,34	74824,35	74773,485
XU030	28.02.2014	2	75456,99	76306,19	75758,49	75881,59
XU030	03.03.2014	2	74119,47	74686,95	74172,73	74403,21
XU030	04.03.2014	2	75886,32	76610,5	76547,89	76248,41
XU030	05.03.2014	2	77059,6	77698,58	77090,23	77379,09
XU030	06.03.2014	2	77136,1	78049,81	77782,07	77592,955
XU030	07.03.2014	2	76258,84	77704,88	76474,43	76981,86
XU030	10.03.2014	2	75470,13	76509,06	75888,48	75989,595
XU030	11.03.2014	2	75724,02	76254,81	76238,16	75989,415
XU030	12.03.2014	2	75098,41	76774,72	76701,83	75936,565
XU030	13.03.2014	2	76769,03	77619,95	76810,7	77194,49
XU030	14.03.2014	2	76346,17	77170,17	76833,65	76758,17
XU030	17.03.2014	2	77657,77	78165,17	77991,67	77911,47
XU030	18.03.2014	2	78384,16	80183,61	79976,01	79283,885
XU030	19.03.2014	2	79782,1	80653,3	79887,03	80217,7
XU030	20.03.2014	2	79396,16	80357,72	79552,68	79876,94
XU030	21.03.2014	2	78722,28	79254,28	78722,28	78988,28
XU030	24.03.2014	2	77578,87	78927,2	77578,87	78253,035
XU030	25.03.2014	2	78028,92	79327,31	78887,4	78678,115
XU030	26.03.2014	2	80983,37	83330,02	83127,84	82156,695
XU030	27.03.2014	2	82095,39	83555,61	82120,18	82825,5
XU030	28.03.2014	2	83690,68	85262,43	85004,2	84476,555
XU030	31.03.2014	2	84373,77	86021,32	85615,98	85197,545
XU030	01.04.2014	2	86147,58	87273,5	87018,69	86710,54
XU030	02.04.2014	2	86383,12	87774,81	86781,39	87078,965
XU030	03.04.2014	2	87411,65	88586,03	87814,14	87998,84
XU030	04.04.2014	2	87995,15	89380,6	89206,3	88687,875
XU030	07.04.2014	2	88518	89359,29	88852,21	88938,645
XU030	08.04.2014	2	89840,06	90825,13	90825,13	90332,595
XU030	09.04.2014	2	88765,32	89947,86	88777,08	89356,59
XU030	10.04.2014	2	89005,87	89875,9	89624,36	89440,885
XU030	11.04.2014	2	88334,84	89193,37	89019,2	88764,105
XU030	14.04.2014	2	88809,61	90547,66	90422,83	89678,635
XU030	15.04.2014	2	88658,93	91190,46	88658,93	89924,695
XU030	16.04.2014	2	88375,99	89180,55	88709,94	88778,27
XU030	17.04.2014	2	89383,21	90302,37	90202,82	89842,79
XU030	18.04.2014	2	89814,63	90301,5	90046,03	90058,065

XU030	21.04.2014	2	89674,93	90126,39	89915,22	89900,66
XU030	22.04.2014	2	89460,34	90176,58	89460,34	89818,46
XU030	24.04.2014	2	88350,94	89750,83	88408,74	89050,885
XU030	25.04.2014	2	86956,81	87881,26	87118,72	87419,035
XU030	28.04.2014	2	87523,91	88760,29	88535,32	88142,1
XU030	29.04.2014	2	88407,02	89492,16	88426,52	88949,59
XU030	30.04.2014	2	89945,31	91106,59	90579,44	90525,95
XU030	02.05.2014	2	91693,47	92582,52	92327,46	92137,995
XU030	05.05.2014	2	91226,78	92116,52	91686,63	91671,65
XU030	06.05.2014	2	92161,32	92726,93	92292,12	92444,125
XU030	07.05.2014	2	91336,3	92557,78	91946,86	91947,04
XU030	08.05.2014	2	92745,8	93475,02	93040,84	93110,41
XU030	09.05.2014	2	92402,94	92901,22	92707,91	92652,08
XU030	12.05.2014	2	92824,11	93867,13	93194,92	93345,62
XU030	13.05.2014	2	92705,14	93421,2	93035,58	93063,17
XU030	14.05.2014	2	93050,54	93520,26	93357,2	93285,4
XU030	15.05.2014	2	92273,42	93788,64	92288,46	93031,03
XU030	16.05.2014	2	91624,6	92359,08	92002,48	91991,84

Table-b: Risk-Free Interest Rate and Time-to-Expiration for the option Data (Source: [23])

DATE	Risk Free Interest Rate	Bond Price (with Face Value 100)	Days Left (to expiration of the options)
31.12.2013	8,84	96,313	180
02.01.2014	9,09	96,255	178
03.01.2014	8,96	96,331	177
06.01.2014	8,92	96,415	174
07.01.2014	9,49	96,225	173
08.01.2014	9,02	96,418	172
09.01.2014	8,93	96,477	171
10.01.2014	8,83	96,538	170
13.01.2014	8,87	96,589	167
14.01.2014	8,94	96,589	166
15.01.2014	9,03	96,579	165
16.01.2014	9,08	96,586	164
17.01.2014	9,03	96,622	163
20.01.2014	9,15	96,652	160
21.01.2014	9,27	96,634	159
22.01.2014	9,35	96,631	158
23.01.2014	9,45	96,618	157

24.01.2014	9,67	96,57	156
27.01.2014	9,89	96,57	153
28.01.2014	10,31	96,462	152
29.01.2014	10,32	96,485	151
30.01.2014	10,55	96,44	150
31.01.2014	10,59	96,45184211	149
03.02.2014	10,68	96,50522811	146
04.02.2014	10,75	96,51047539	145
05.02.2014	10,76	96,53405071	144
06.02.2014	10,77	96,55651463	143
07.02.2014	10,8	96,57481171	142
10.02.2014	10,81	96,65632249	139
11.02.2014	10,87	96,66191481	138
12.02.2014	10,97	96,66204	137
13.02.2014	10,85	96,72693182	136
14.02.2014	10,86	96,75077694	135
17.02.2014	10,98	96,7987037	132
18.02.2014	10,87	96,85312734	131
19.02.2014	10,89	96,879	130
20.02.2014	11,04	96,86614286	129
21.02.2014	11,08	96,88130387	128
24.02.2014	10,88	97,0165	125
25.02.2014	10,96	97,02580171	124
26.02.2014	10,84	97,08191489	123
27.02.2014	10,98	97,0772353	122
28.02.2014	10,82	97,14380117	121
03.03.2014	10,68	97,25853556	118
04.03.2014	10,55	97,31464734	117
05.03.2014	10,37	97,38614939	116
06.03.2014	10,48	97,38708081	115
07.03.2014	10,72	97,35693782	114
10.03.2014	10,75	97,43210714	111
11.03.2014	10,87	97,433	110
12.03.2014	11,2	97,38833846	109
13.03.2014	11,21	97,41322805	108
14.03.2104	11,33	97,41866199	107
17.03.2014	11,3	97,50884974	104
18.03.2014	11,08	97,58108684	103
19.03.2014	11,04	97,619	102
20.03.2014	11,09	97,63709814	101

21.03.2014	11,16	97,64951316	100
24.03.2014	11,13	97,7427069	97
25.03.2014	11,09	97,77654762	96
26.03.2014	11,01	97,82125	95
27.03.2014	11,12	97,82946377	94
28.03.2014	11,22	97,83944586	93
31.03.2014	11,27	97,91631892	90
01.04.2014	11,06	97,97955859	89
02.04.2014	11	98,01777419	88
03.04.2014	10,96	98,05293166	87
04.04.2014	10,85	98,09860177	86
07.04.2014	10,45	98,245	83
08.04.2014	10,03	98,33785782	82
09.04.2014	9,32	98,47388337	81
10.04.2014	9,21	98,5144646	80
11.04.2014	8,59	98,632	79
14.04.2014	9,24	98,60529262	76
15.04.2014	9	98,6626	75
16.04.2014	9,08	98,67490476	74
17.04.2014	9,12	98,69241936	73
18.04.2014	9,2	98,70558333	72
21.04.2014	9,19	98,779	69
22.04.2014	9,26	98,794	68
24.04.2014	9,25	98,843	66
25.04.2014	9,46	98,843	65
28.04.2014	9,47	98,915	62
29.04.2014	9,51	98,936	61
30.04.2014	9,61	98,951	60
02.05.2014	9,4	99,02	58
05.05.2014	9,49	99,085	55
06.05.2014	9,55	99,105	54
07.05.2014	9,58	99,126	53
08.05.2014	9,46	99,161	52
09.05.2014	9,79	99,159	51
12.05.2014	9,76	99,237	48
13.05.2014	9,7	99,267	47
14.05.2014	9,43	99,311	46
15.05.2014	9,47	99,333	45
16.05.2014	9,59	99,35	44

Table-c: Volatility Data (Source: [24])

DATE	INDEX	NUMBER OF DAYS	VALUE
31.12.2013	XU030	252	31,72
02.01.2014	XU030	252	31,74
03.01.2014	XU030	252	31,79
06.01.2014	XU030	252	31,91
07.01.2014	XU030	252	31,92
08.01.2014	XU030	252	31,97
09.01.2014	XU030	252	31,98
10.01.2014	XU030	252	32,08
13.01.2014	XU030	252	32,05
14.01.2014	XU030	252	32,05
15.01.2014	XU030	252	32,05
16.01.2014	XU030	252	32,08
17.01.2014	XU030	252	32,13
20.01.2014	XU030	252	32,05
21.01.2014	XU030	252	32,05
22.01.2014	XU030	252	32,09
23.01.2014	XU030	252	32,22
24.01.2014	XU030	252	32,26
27.01.2014	XU030	252	32,25
28.01.2014	XU030	252	32,3
29.01.2014	XU030	252	32,33
30.01.2014	XU030	252	32,05
31.01.2014	XU030	252	32,06
03.02.2014	XU030	252	31,77
04.02.2014	XU030	252	31,83
05.02.2014	XU030	252	31,76
06.02.2014	XU030	252	31,97
07.02.2014	XU030	252	31,96
10.02.2014	XU030	252	31,98
11.02.2014	XU030	252	31,88
12.02.2014	XU030	252	31,87
13.02.2014	XU030	252	31,83
14.02.2014	XU030	252	31,87
17.02.2014	XU030	252	31,89
18.02.2014	XU030	252	31,98
19.02.2014	XU030	252	31,99
20.02.2014	XU030	252	32
21.02.2014	XU030	252	31,99
24.02.2014	XU030	252	31,99

25.02.2014	XU030	252	32,08
26.02.2014	XU030	252	32,03
27.02.2014	XU030	252	32,01
28.02.2014	XU030	252	32,01
03.03.2014	XU030	252	32,06
04.03.2014	XU030	252	32,19
05.03.2014	XU030	252	32,19
06.03.2014	XU030	252	32,19
07.03.2014	XU030	252	32,22
10.03.2014	XU030	252	32,19
11.03.2014	XU030	252	32,19
12.03.2014	XU030	252	32,1
13.03.2014	XU030	252	32,1
14.03.2014	XU030	252	32,1
17.03.2014	XU030	252	32,13
18.03.2014	XU030	252	32,22
19.03.2014	XU030	252	32,18
20.03.2014	XU030	252	32,18
21.03.2014	XU030	252	32,17
24.03.2014	XU030	252	32,19
25.03.2014	XU030	252	32,24
26.03.2014	XU030	252	32,68
27.03.2014	XU030	252	32,7
28.03.2014	XU030	252	32,85
31.03.2014	XU030	252	32,86
01.04.2014	XU030	252	32,87
02.04.2014	XU030	252	32,86
03.04.2014	XU030	252	32,89
04.04.2014	XU030	252	32,93
07.04.2014	XU030	252	32,9
08.04.2014	XU030	252	32,91
09.04.2014	XU030	252	32,95
10.04.2014	XU030	252	32,91
11.04.2014	XU030	252	32,91
14.04.2014	XU030	252	32,95
15.04.2014	XU030	252	32,93
16.04.2014	XU030	252	32,92
17.04.2014	XU030	252	32,96
18.04.2014	XU030	252	32,95
21.04.2014	XU030	252	32,95
22.04.2014	XU030	252	32,91
24.04.2014	XU030	252	32,93

25.04.2014	XU030	252	32,95
28.04.2014	XU030	252	32,96
29.04.2014	XU030	252	32,96
30.04.2014	XU030	252	33,05
02.05.2014	XU030	252	33,11
05.05.2014	XU030	252	33,09
06.05.2014	XU030	252	32,93
07.05.2014	XU030	252	32,93
08.05.2014	XU030	252	32,94
09.05.2014	XU030	252	32,94
12.05.2014	XU030	252	32,89
13.05.2014	XU030	252	32,87
14.05.2014	XU030	252	32,88
15.05.2014	XU030	252	32,89
16.05.2014	XU030	252	32,84

Appendix F: Turkish Summary

Bu tez, Türkiye'de 21 Aralık 2012'den itibaren kullanılabilmeye başlanan opsiyonların Türkiye'deki fiyatlandırılmasını incelemek üzere hazırlanmıştır. Opsiyonlar, opsiyona dayanak teşkil eden malı, opsiyon alıcısına sözleşme tarihinde belirlenen bir kullanım fiyatıyla satın alma veya satma hakkı tanıyan finansal türev araçlarıdır. Türkiye'de iki tip opsiyon kullanılmaktadır: Amerikan ve Avrupa tipi opsiyonlar. Amerikan tipi opsiyonlar alıcı tarafından vade tarihine kadar istenildiği zaman kullanılabilir. Avrupa tipi opsiyonlar ise sadece vade tarihinde kullanılabilir. Amerikan tipi Sözleşmeler genelde hisse senetleri için yazılmaktadır. Türkiye'de, Amerikan opsiyonu oluşturulabilen hisse senetleri Şekil 5'te verilmiştir. Avrupa tipi opsiyonlar ise genelde endeksler ile oluşturulabilmektedir. Türkiye'de sadece BIST30 endeksi için Avrupa tipi opsiyon mevcuttur. Amerikan tipi opsiyonlar ile Avrupa tipi opsiyonların çoğunlukla benzer özellikleri vardır, ancak Avrupa tipi opsiyonları incelemek ve anlamak bir nebze daha kolaydır ve aynı yöntemle Amerikan tipi opsiyon sözleşmeleri de incelenebilmektedir. Bu nedenle söz konusu tez Avrupa tipi opsiyonları incelemektedir.

Opsiyon alıcısı, opsiyonu kullanma veya kullanmama hakkına sahip olduğu için, opsiyon satıcısına sözleşmenin imzalandığı tarihte bir bedel ödemek zorundadır. Bu bedelin ne kadar olduğu çok önemlidir. Çünkü bedelin düşük (yüksek) olması durumunda markette alıcı (satıcı) açısından risksiz kazanç olanağı yaratılacaktır ve bu da marketin verimsiz olması anlamına gelir. Ayrıca opsiyonlar kaldıraç olanağı yaratan türev araçlarıdır. Yani düşük miktardaki bir parayla, büyük miktarda risk alınabilmektedir. Yanlış fiyatlandırma yapılması durumunda bu opsiyon taraflarının birine büyük yük getirebilmektedir.

Yukarıda da belirtildiği gibi opsiyonlarla satın alma veya satış hakkı elde edilebilmektedir. Örneğin, satış hakkı opsiyonuna sahip olan yatırımcı (uzun taraf), daha önceden belirlenmiş olan fiyatla (kullanım fiyatı), opsiyonun vade tarihindeki market fiyatı daha ucuz olsa dahi, opsiyonu satan tarafa (kısa taraf) satabilir. Alım ve satım opsiyonlarında uzun ve kısa taraflar için örnek kazanç grafikleri Şekil 1'den Şekil 4'e kadar verilmektedir.

Opsiyon fiyatları opsiyona dayanak teşkil eden malın opsiyon sözleşmesi oluşturulan tarihteki fiyatı, uzlaşma fiyatı, dayanak teşkil eden malın fiyatının volatilite değeri, vade tarihine kadar olan gün sayısı, opsiyon sözleşmesi imzalanan tarihteki faiz oranları ve temettülerden etkilenmektedir. Bu değişkenlerin opsiyon fiyatlarını nasıl etkilediği aşağıdaki tabloda gösterilmektedir (tez içerisinde yer alan Tablo 1 ile aynıdır):

Değişken	**Alım Opsiyonu (Avrupa Tipi)**	**Satım Opsiyonu (Avrupa Tipi)**
Opsiyona dayanak teşkil eden malın opsiyon sözleşmesi oluşturulan tarihteki fiyatı	+	-
Uzlaşma fiyatı	-	+
Vade tarihine kadar olan gün sayısı	Belirsiz	Belirsiz
Dayanak teşkil eden malın fiyatının volatilite değeri	+	+
Opsiyon sözleşmesi imzalanan tarihteki faiz oranları	+	-
Temettüler	-	+

Yukarıdaki tabloda "+" değişkenin değerinin artması (azalması) durumunda opsiyon fiyatının artacağını (azalacağını), "-" değişkenin değerinin artması (azalması) durumunda opsiyon fiyatının azalacağını (artacağını) belirtmektedir. Belirsiz ise değişkenin artmasının veya azalmasının opsiyon fiyatını tam olarak nasıl etkilediği bilinmediği durumu göstermektedir.

Opsiyonlar piyasalarda Nobel Ödülü kazanmış olan Black-Scholes-Merton formülüyle fiyatlandırılmaktadır. Söz konusu formülde yukarıdaki tabloda yer alan değişkenler (temettüler hariç) girdi olarak yer almaktadır. Formülde her bir değişken için türev alındığında, her bir değişkenin opsiyon fiyatının *ceteris* paribus nasıl etkilediği bulunabilmektedir. Belirli bir uzlaşma fiyatı için bu değişkenlerin Türkiye'deki opsiyon fiyatlarını nasıl etkilediği Şekil 27, Şekil 30 ve Şekil 31'de yer alan grafiklerde görülebilir.

Bu tezde Türkiye piyasalarında verilen fiyatların, formül (Black-Scholes-Merton) ile hesaplanan fiyatlarla paralel olup olmadığı araştırılmaktadır. Söz konusu araştırmanın yapılabilmesi amacıyla Haziran ayında vadesi gelen opsiyonlar kullanılmıştır (Avrupa tipi opsiyonlar için sadece Şubat, Nisan, Haziran, Ekim ve Aralık vadeleri mevcuttur). Bu opsiyonlar (Haziran 2014 vadeli opsiyonlar) için fiyatlama market tarafından 31 Aralık 2013 tarihinden itibaren yapılmaya başlanmıştır. Bu tarihte BIST30 endeks fiyatları 80 TL civarında olduğu için (BIST30'un son 5 yıllık değerleri Şekil 9'da görülebilir. Ayrıca BIST30'u oluşturan şirketler (2014 1. çeyrek ve 2. çeyrek itibariyle) Tablo 3'te verilmiştir) o dönemde uzlaşma fiyatı olarak 76, 78, 80, 82, 84, 86, 88 ve 90 TL mevcuttur. Karşılaştırma tüm bu uzlaşma fiyatları üzerinden yapılmıştır ancak yorumlar ve daha detaylı inceleme 90 TL'lik uzlaşma fiyatı olan opsiyon sözleşmeleri için yapılmıştır. Bunun nedeni bu uzlaşma fiyatlarına sahip olan opsiyonların benzer özelliklere sahip olmasıdır. Ölçümler 31 Aralık 2013 tarihinden 16 Mayıs 2014 tarihine kadar alınmıştır. Bu aralıkta BIST30 fiyatları 74 TL ile 99 TL arasında değiştiği için başka uzlaşma fiyatları da ortaya çıkmıştır. Ancak grafik sayısını çok arttırmamak ve en uzun veriye sahip olan opsiyonları kullanabilmek için 31 Aralık 2013'te var olan uzlaşma fiyatları kullanılmıştır. Tezde gözlemlenen veri olarak adlandırılan ve teorik veri ile karşılaştırılan ölçümler aşağıdaki tabloda özetlenmiştir. (Tez içerisindeki Tablo 4 ile aynıdır)

Opsiyon Tipi	**Opsiyona Dayanak Teşkil eden Mal**	**Vade Tarihi**	**Uzlaşma Fiyatları**
Avrupa Tipi	BIST30	30.06.2014	76, 78, 80, 82, 84, 86, 88, 90

Söz konusu özelliklere sahip olan Opsiyon Sözleşmeleri için Borsa İstanbul'un internet sitesinde veriler kısmında son uzlaşma fiyatı bulunabilmektedir. Son uzlaşma fiyatı şu şekilde hesaplanmaktadır [16]:

a) Seans sona ermeden önceki son 10 dakika içerisinde gerçekleştirilen tüm işlemlerin miktar ağırlıklı ortalama fiyatı,

b) Eğer son 10 dakika içerisinde 10'dan az işlem yapıldıysa, seans içerisinde geriye dönük olarak bulunan son 10 işlemin miktar ağırlıklı ortalama fiyatı,

c) Seans içerisinde 10'dan az işlem yapıldıysa, seans içerisinde gerçekleştirilen tüm işlemlerin miktar ağırlıklı ortalama fiyatı, *d) Seans içerisinde hiç işlem yapılmamışsa, dayanak varlık ve dayanak varlığa bağlı diğer sözleşmelerdeki fiyatlar dikkate alınarak hesaplanan teorik fiyatlar*

günlük uzlaşma fiyatı olarak belirlenir.

Gözlemlenen veriler ile karşılaştırılan teorik veriler ise Black-Scholes-Merton formülüyle hesaplanmıştır. Bu hesap için şu 5 bilgiye ihtiyaç duyulmaktadır: Dayanak teşkil eden malın opsiyon sözleşmesi oluşturulan tarihteki fiyatı, uzlaşma fiyatı, dayanak teşkil eden malın fiyatının volatilite değeri, vade tarihine kadar olan gün sayısı ve opsiyon sözleşmesi imzalanan tarihteki faiz oranları. Bu beş bilgiye nasıl ulaşıldığı aşağıda özetlenmiştir:

Vade Sonu Uzlaşma Fiyatı: Yukarıda da belirtildiği gibi gözlemlenen verilerin uzlaşma fiyatı 76, 78, 80, 82, 84, 86, 88 ve 90 TL olarak belirlenmiş olanlar incelenmeye karar verilmiştir. Bu nedenle teorik hesaplama da bu 8 değer için yapılmıştır.

Dayanak Teşkil Eden Malın (BIST30) Opsiyon Sözleşmesi Oluşturulan Tarihteki Fiyatı: Yukarıdaki alıntıda bahsedildiği üzere son uzlaşma fiyatı genel olarak son sözleşmelerin ağırlıklı ortalaması hesaplanarak yapılıyor. BIST30'un fiyatları yine Borsa İstanbul'un internet sitesinden elde edilebilmektedir. Veriler iki ayrı seans için (öğleden önce ve öğleden sonra) verilmektedir ve her iki seans için de en düşük fiyat ve en yüksek fiyat verilmektedir. Ayrıca o gün için kapanış fiyatı verilmektedir. Gözlemlenen veri olarak son sözleşmelerin ağırlığı verildiği için, BIST30 fiyatı şu şekilde hesaplanmıştır: İkinci seansın en düşük ve en yüksek fiyatının ortalaması.

Vade Tarihine kadar Olan Gün Sayısı: Daha önce de belirtildiği gibi 30 Haziran 2014'te vadesi sona eren opsiyon sözleşmeleri incelenmektedir. Dolayısıyla ölçüm alınan her gün için bu tarihe kadar olan gün sayısı hesaplanmıştır.

Volatilite Değeri: BIST30 için volatilite değerine yine Borsa İstanbul'un internet sitesinden erişilebilmektedir. Veriler 21, 42, 63, 126 ve 252 gün boyunca elde edilen veriler göz önünde bulundurularak hesaplanmaktadır. Teorik opsiyon sözleşmeleri değerini hesaplamak için kullanılan formül (Black-Scholes-Merton formülü) yıllık volatilite değeri ile kullanılmaktadır. Bu nedenle 252 günün verisi kullanılarak hesaplanan volatilite değeri kullanılmıştır.

Faiz Oranları: Formül için opsiyon sözleşmesinin vadesine kadar olan faizin bileşik getirisinin kullanılması gerekmektedir. Ancak tam 30 Haziran 2014'te mevcut olan bir borçlanma aracı bulunamamıştır. Yine de bu tarihe çok yakın bir tarihte vadesi dolan bir borçlanma aracının (TRT110614T13) bileşik faiz getirisinin ağırlıklı ortalaması kullanılmıştır.

Bahsedildiği şekilde elde edilen verilerle opsiyon sözleşmelerinin teorik değerleri hesaplanmış ve gözlemlenen değerlerle karşılaştırılmıştır. Söz konusu karşılaştırmalar (alım ve satım opsiyonları için) Şekil 10'dan Şekil 25'e kadar olan grafiklerde yer almaktadır. Teorik ve gözlemlenen verilerin aynı olmadığı gözle görülebilmektedir. Bununla birlikte eşitliğin sağlandığı hipotezi t-test ile test edilmiş ve hipotez reddedilmiştir. Ayrıca söz konusu şekillerin yanında yeşil ile çizilmiş bir grafik mevcuttur. Bu grafik, teorik değerlerle gözlemlenen değerlerin farkını göstermektedir. Tablo 5 (alım opsiyonları için) ve Tablo 6'da (satım opsiyonları için) teorik değerlerle gözlemlenen değerler arasındaki farkların istatistiksel bilgileri verilmektedir.

Alım opsiyonları için grafikler incelendiğinde gözlemlenen verilerin teorik verilerden çoğunlukla daha düşük olduğu görülebilmektedir. Örneğin, uzlaşma fiyatının 90 TL olduğunu varsayalım, opsiyonu satan yatırımcılar (yani 90 TL'nin üzerinde bir fiyatta olsa dahi BIST30 hissesini yine de 90 TL'ye satmak zorunda olan taraf. Ancak BIST30 değeri 30 Haziran 2014'te 90 TL'nin altındaysa alıcı taraf, hisseyi markette daha düşük bir fiyata almayı tercih edecek ve opsiyonu kullanmayacak.) BIST30 fiyatının 90 TL'nin altında kalacağını öngörüyor olabilir. Yani insanların hisse senet fiyatlarının daha çok geriye doğru gideceğine, yani fiyatların düşeceğine dair beklentileri var anlamına gelebilir. Bunun nedeni olarak o dönemlerde ülkenin politik olarak sancılı bir süreçten geçiyor olması gösterilebilir. Mayıs-Haziran 2013'te gerçekleşen gezi olayları, 17 Aralık 2013'ten itibaren başlayan yolsuzluk soruşturmaları ve sonrasında çıkan tapeler ülkede var olduğu söylenen istikrarın kaybolacağına dair bir beklenti oluşturdu. Bu nedenlerden ötürü yatırımcıların BIST30 hisse senet fiyatlarının düşeceğini öngördükleri ve opsiyon hakkını teorik değerden daha ucuza sattıkları söylenebilir.

Satım opsiyonu için grafikler incelendiğinde ise 28 Mart 2014 tarihine kadar satım opsiyonları için gözlemlenen fiyatların teorik değerlerle yaklaşık aynı olduğu görülebilir. Örneğin, uzlaşma fiyatı 90 TL olarak belirlenmiş olan satım opsiyonlarının ilk verileri için t-test yapıldığında teorik değerlerle gözlemlenen değerlerin eşit olduğu hipotezinin reddedilemediği görülmektedir (bkz. Şekil 29). Ancak 28 Mart 2014 sonrası verilerine bakıldığında hem gözle hem de t-test ile teorik değerlerle gözlemlenen değerlerin aynı olmadığı görülebilmektedir. Hatta gözlemlenen değerlerin teorik değerlerden daha düşük olduğu fark edilmektedir. Örneğin, 90 TL için satım opsiyon hakkını alan bir yatırımcı (yani opsiyon fiyatı olarak sözleşmenin imzalandığı tarihte verdiği değere karşılık, vade sonunda BIST30 hisse senet fiyatı 90 TL'nin altında olsa dahi hisse senedini 90 TL'ye satabilme hakkına sahip yatırımcı) BIST30 fiyatının 90 TL üzerinde olma olasılığını daha yüksek gördüğü için bu hakkı satın almaya teorik değerden daha az para veriyor Mart 2014 sonundan itibaren. Bu durum mevcut hükümetin hakkında çıkmış olan tapelere rağmen seçimde en yüksek oyu alan parti olmasıyla açıklanabilir. Ülkede var olduğu söylenen istikrarın devam edeceğine dair olan inanç arttığı için, yatırımcılar bu tarihten itibaren BIST30 hisselerinde bir artış bekliyor olabilir. Nitekim Şekil 9'da da görülebileceği üzere bu artış gerçekleşmiştir.

Opsiyon fiyatları için gözlemlenen değerlerle, teorik değerleri karşılaştırmanın yanında ayrıca gelecekti değeri (hisse senet fiyatı ne olursa olsun) eşit olduğu bilinen iki portfolyonun bugünkü değerleri karşılaştırılmıştır. Bilindiği üzere gelecekteki fiyatı eşit olacağı bilinen iki portfolyonun bugünkü fiyatları da eşit olmak zorundadır. Aksi takdirde ucuz olan portfolyo alınıp daha pahalı olan satılırsa bu gelecekte risksiz kazanç doğuracaktır. Söz konusu iki portfolyonun fiyatlarının karşılaştırıldığı Şekil 32'de de görülebileceği üzere teorik değer her zaman 0'a eşit iken gözlemlenen değerler her zaman böyle değildir, yani piyasada risksiz kazanç olanağı olmuştur. Ancak belli bir noktadan sonra gözlemlenen değerlerin 0'ın etrafında salınım yaptığı görülebilmektedir. Bu durum için t-test yaptığımızda baştaki veriler için farkın 0'a eşit olduğu hipotezi reddedilmektedir. Ancak zaman geçtikçe bu hipotez t-test için reddedilememektedir (Bakınız Tablo 10 ve Şekil 33).

Bahsi geçen portfolyoların farkının 0 olup olmadığı diğer ülkelerde de test edilerek sonuçlar literatüre başka araştırmacılar tarafından dâhil edilmiştir. Gelecekteki fiyatları eşit olduğu bilinen portfolyoların bugünkü değerleri arasında fark olması literatürde şu üç nedene bağlanmıştır: işlem maliyetleri, insanların olası ihtiyacı (örneğin paraya acil ihtiyaç duyan bir yatırımcı, portfolyoyu oluşturan türevlerden birini olması gerekenden daha ucuza satabilir) ve kimi yatırımcılarda olabilecek olan olası ilave bilgiler (örneğin yatırımcı BIST30 hisse değerinin artacağına bir nedenden ötürü eminse, alım opsiyonu için daha fazla para verebilir). Türkiye'de opsiyon piyasası çok yeni olduğu için hacim kimi zaman 0 olabilmektedir. Örneğin 16 Nisan 2014'te opsiyon piyasasının hacminin 1.124.000 TL olmasına, 15 Mayıs 2014 tarihinde 3.577.000 TL olmasına rağmen 28 Ocak 2014'te hacim 0'dır (Bu değerler tüm uzlaşma fiyatları dahil edilerek verilmiştir). Hacmin 0 olduğu zaman için iki portfolyo değerinin farkının 0'dan farklı olması Borsa İstanbul'un fiyatlama yaparkenki sahip olduğu ilave bilgilere bağlanabilir. Bu ilave bilgiler opsiyon fiyatlarına volatilite ile dahil olabilmektedir, çünkü volatilite opsiyon fiyatlamadaki tek gözlemlenemeyen değişkendir.

Özetlemek gerekirse Türkiye'de çok yeni kullanılmaya başlanmış olan opsiyonların teorik yollardan hesaplanmış değerlerle markette gözlemlenen değerlerin eşit olmadığı görülmüş ve bunun olası nedenlerinden birisinin ülkenin politik durumunun olabileceği tartışılmıştır. Literatürde araştırıldığında Black-Scholes-Merton modelin finans marketlerindeki kriz zamanlarında doğru sonuç vermediğine dair araştırmaların mevcut olduğu görülecektir. Yukarıda da bahsedildiği gibi ülke politikasındaki belirsizlikler Türkiye finans piyasalarına da yansımaktadır, bu nedenle teorik fiyatlarla gözlemlenen fiyat arasındaki farklılığın nedeni teorik fiyatların Türkiye'deki opsiyonları verimli olarak fiyatlandırmadığı da olabilir. Sonuç olarak, teorik fiyatların Türkiye'deki opsiyonları verimli olarak fiyatlamadığına veya Türkiye'deki gözlemlenen opsiyon fiyatlarının verimli olmadığına ulaşılmaktadır.

Teorik fiyatlar ile gözlemlenen fiyatlar arasında fark bulunması nedeniyle, bu farkın doğuracağı olası sonuçların (risksiz kazan olanağı) görülebilmesi amacıyla gelecekteki fiyatları eşit olduğu bilinen iki farklı portfolyonun bugünkü fiyatları karşılaştırılmıştır. Bu portfolyoların arasında farklar bulunduğu ancak bu farkın giderek azaldığı hem görsel olarak hem de t-test ile gösterilmiştir. Yani var olan risksiz kazanç olanağı beklenildiği gibi market tarafından yok edilmektedir.

Türkiye'de yeni kullanılmaya başlanmış olan opsiyon marketi için bu çalışmada kullanılan yöntem ile Amerikan tipi opsiyonlar için de benzer bir araştırma yapılabilir. Ayrıca daha farklı portfolyolar oluşturularak hem Amerikan Opsiyonlarında var olabilecek hem Avrupa tipi opsiyonlarından var olabilecek risksiz kazanç olanakları gözlemlenebilir.