AN INVESTIGATION OF PROSPECTIVE SECONDARY MATHEMATICS TEACHERS' THINKING ABOUT MATHEMATICAL MODELING AND PEDAGOGY OF MODELING THROUGHOUT A MODELING COURSE

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

AN INVESTIGATION OF PROSPECTIVE SECONDARY MATHEMATICS TEACHERS' THINKING ABOUT MATHEMATICAL MODELING AND PEDAGOGY OF MODELING THROUGHOUT A MODELING COURSE

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The purpose of this study is to investigate the evolution of prospective secondary mathematics teachers' thinking about mathematical modeling and pedagogical knowledge of mathematical modeling throughout a modeling course designed for teachers. This study utilizes case study as a research design. The participants were 25 prospective secondary mathematics teachers enrolled in a course entitled "Mathematical Modeling for Prospective Teachers" in a public university, in Ankara during the spring semester of 2011-2012. Data were collected through six modeling activities, participants' working sheets on the modeling activities, pre- and post-survey forms related to pedagogy of modeling, semi-structured interviews, field notes, videotaped classroom discussions, classroom observation notes, and students' way of thinking sheets. The data were analyzed by using qualitative methods.

The results of the study demonstrated that prospective teachers developed positive views about mathematical modeling and about the use of mathematical modeling activities in the classroom settings. Prospective teachers' conceptions about mathematical modeling changed from modeling as concrete materials and visualization to modeling as relating mathematics to real life. Another important finding is that prospective teachers developed significant ideas about the knowledge and skills needed for teachers to teach mathematics through modeling. The prospective teachers also emphasized the importance of teachers' role in the modeling process. It was concluded that the course contributed significantly to the prospective teachers' thinking about pedagogy of modeling.

Keywords: Prospective Secondary Mathematics Teachers, Mathematical Modeling, Mathematics Teacher Education, Pedagogy of Mathematical Modeling.

ÖZ

ORTAÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSEL MODELLEME VE MODELLEME PEDAGOJİSİ ÜZERİNE DÜŞÜNCELERİNİN BİR MODELLEME DERSİ SÜRESİNCE İNCELENMESİ

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Bu çalışmanın amacı matematik öğretmenleri için tasarlanan matematiksel modelleme dersi kapsamında ortaöğretim matematik öğretmen adaylarının matematiksel modellemenin kullanımı ile ilgili pedagoji bilgilerindeki değişimi ve matematiksel modellemenin kullanımına yönelik düşüncelerindeki değişimi ve gelişimi incelemektir. Bu çalışmada örnek durum çalışması araştırma deseni olarak kullanılmıştır. Bu araştırmaya Ankara'daki bir devlet üniversitesinde öğrenim gören ve 2011-2012 akademik yılının ikinci döneminde "Öğretmen Adayları için Matematiksel Modelleme" dersine kayıt yaptıran 25 ortaöğretim matematik öğretmen adayı katılmıştır. Bu ders kapsamında uygulanan altı modelleme etkinliği, katılımcıların modelleme etkinlikleri ile ilgili çalışma kağıtları, modelleme pedagojisi ile ilgili ön- ve son-tarama formları, yarı-yapılandırılmış görüşmeler, alan notları, video kaydına alınmış sınıf içi tartışmalar, sınıf içi gözlem notları, öğrenci düşünme şekilleri raporları çalışmanın veri kaynaklarını oluşturmaktadır. Veri analizi nitel yöntemler kullanılarak yapılmıştır.

Araştırmanın sonuçları öğretmen adaylarının matematiksel modelleme ve sınıf içinde kullanımı hakkında olumlu görüş geliştirdiklerini göstermiştir. Öğretmen adaylarının modellemeye yönelik düşünceleri somut materyal kullanımı ve görselleştirmeden matematiği gerçek yaşamla ilişkilendirme yönünde değiştiği görülmüştür. Araştırmanın diğer bir önemli bulgusu da öğretmen adaylarının öğretmenlerin matematiği modelleme yoluyla öğretmeleri için sahip olmaları gereken bilgi ve beceriler hakkında önemli fikirler geliştirdikleri görülmüştür. Ayrıca, öğretmen adayları öğremenin modelleme sürecindeki rolünün önemini vurgulamışlardır. Araştırma sonunda modelleme dersinin öğretmen adaylarının modelleme pedagojisi hakkındaki düşüncelerine önemli katkı sunduğu sonucuna varılmıştır.

Anahtar Kelimeler: Ortaöğretim Matematik Öğretmen Adayları, Matematiksel Modelleme, Matematik Öğretmen Eğitimi, Matematiksel Modellemenin Pedagojisi.

To my family with all of my heart

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CHAPTER 1

INTRODUCTION

In recent decades, mathematical modeling has been on the agenda of mathematics education community as an aim for mathematics teaching (Blomhøj ve Jensen, 2007; Blum, Galbraith, Henn & Niss, 2002; Crouch ve Haines, 2004; Haines ve Crouch, 2001; Lingefjärd, 2002a; Lingefjärd & Holmquist, 2005) or as an alternative method for teaching and learning of mathematics (Gravemeijer, 2002; Gravemeijer & Stephan, 2002; Lesh & Doerr, 2003a). Many researchers put great emphasis on the importance of mathematical modeling in the teaching and learning of mathematics (Blum & Niss, 1991; Doerr & Lesh, 2003, 2011; Lesh & Zawojewski, 2007). Although mathematical modeling has been underlined and integrated in many countries' secondary school curricula (Burkhardt, 2006; Lingefjärd, 2002b, 2006; Stillman, 2010), the implementations of mathematical modeling and modeling activities in classrooms are still scarce (Burkhardt, 2006; Frejd, 2012; Henn, 2007; Maaβ, 2005) due to several reasons such as hinders related to teachers' negative conceptions about mathematical modeling and its use in the teaching and learning of mathematics (Blum & Niss, 1991; Kaiser & Maaβ, 2007; Maaβ, 2011; Schmidt, 2011; Siller, Kuntze, Lerman, & Vogl, 2011), obstacles related to the domination of existing educational system, chaotic situation and messiness of the real world, and limited professional development of teachers (Blum & Niss, 1991; Burkhardt, 2006; Kuntze, Siller, & Vogl, 2013).

Since limited professional development of teachers in mathematical modeling was indicated as an obstacle that hinders teachers in using mathematical modeling in their classrooms, many studies were conducted to design a mathematical modeling courses for both prospective teachers and in-service teachers by various researchers (e.g., Barbosa, 2001; Blomhøj & Kjeldsen, 2006; Holmquist & Lingefjärd, 2003; Jiang, McClintock, & O'Brien, 2003; Lingefjärd, 2006; Kaiser & Schwarz, 2006; Borromeo Ferri & Blum, 2009; Maaß & Gurlitt, 2011). Although these efforts were valuable and important steps for ensuring teachers' professional development in mathematical modeling and its teaching, there is a need for an enhanced and reinforced professional development in modeling-specific pedagogical content knowledge (PCK) both in initial mathematics teacher preparation and professional development programs for in-service teachers (Kuntze et al., 2013). Many researchers indicated that there is a need for further studies on what qualifications teachers need related to mathematical modeling and pedagogy of modeling in order to implement mathematical modeling and modeling activities in their classrooms effectively (Blum et al., 2002). Therefore, the components of teachers' modeling-specific PCK should be determined by researchers through new studies on it and the results of these studies need to be evaluated in order to improve professional development of teachers in the domain of modeling-specific PCK (Kuntze et al., 2013).

1.1 Mathematical Modeling and Its Importance in the Teaching and Learning of Mathematics

Many countries have been updating their school mathematics curricula in accordance with the requirements of the new era. In the recent decades, a remarkable interest has been growing on the use of mathematical modeling in the teaching and learning of mathematics. Many researchers underlined the pivotal role of mathematical modeling and its key roles in the teaching of mathematics (Blum & Niss, 1991; Doerr & Lesh, 2011; Kaiser & Maaβ, 2007; Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007; Lingefjärd, 2000; 2002a). Although the importance of mathematical modeling has been emphasized strongly in mathematics education community, there is not a common understanding of mathematical modeling and its components (Crouch & Haines, 2004; Doerr & Lesh, 2003; Lingefjärd, 2002a; Pollak, 2003). The theoretical consideration of mathematical modeling differentiates in terms of its definition (Lesh & Doerr, 2003a; Lingefjärd, 2002a), pedagogical goals involving its role and place in the teaching of mathematics (Blum & Niss, 1991; Kaiser & Sriraman, 2006), modeling processes (Borromeo Ferri, 2006), mathematical modeling abilities and competencies (Haines & Crouch, 2007), and assessing the performance of students in the modeling process (Borromeo Ferri & Blum, 2009; Galbraith, 2007).

Crouch and Haines (2004) described mathematical modeling as "moving from a real-world situation to a model, working with that model and using it to understand and to develop or solve the real-world problems" (p. 197). According to Lesh and

Lehrer (2003), mathematical modeling is a process of developing models that are used to describe particular situations for specific goals, involves testing, revising, and refining procedures. A significant role has been attributed to mathematical modeling in the teaching of mathematics in recent years. The teaching of mathematics through modeling activities involves local conceptual development for both students and teachers (Lesh & Doerr, 2003a). By local conceptual development, we mean that students and teachers need to develop constructs and conceptual systems when they deal with model-eliciting activities. This process includes similar "operational/relational schemas (or cognitive structures) that developmental psychologists have investigated for children's mathematical judgements about underlying ideas" that result in conceptual development in students' mind locally (Lesh & Harel, 2003, p. 161). In addition, mathematical modeling activities provide students learning contexts for meaningful learning of mathematical concepts (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Lesh & Doerr, 2003a; Lingefjard, 2002; Zbiek & Conner, 2006).

Although mathematical modeling is highlighted in the teaching and learning of mathematics, there is not an agreement among the issues and concepts of mathematical modeling in the literature (Kaiser & Sriraman, 2006). For instance, the relationship between mathematical modeling and problem solving is one of the controversial issues. Several researchers pointed out the similarities and differences between mathematical modeling and problem solving (e.g., Erbaş et al., in press; Lesh & Yoon, 2007; Lesh & Zawojevski, 2007; Zawojevski & Lesh, 2003; Zawojevski, 2010). Researchers proposed that traditional problem solving included in *model-eliciting activities* (Lesh & Doerr, 2003a) and the general characteristics of model-eliciting activities are identified (Lesh, Hoover, Hole, Kelly, & Post, 2000). Furthermore, mathematical modeling can be confused with the use of concrete manipulatives and visual models in elementary school levels (Erbaş et al., in press; Gravemeijer, 2002; Lesh et al., 2003). For instance, using "arithmetic blocks" in the teaching of mathematics can be regarded as concrete manipulatives to create constructs for forming a basis for mathematical reasoning (Lesh et al., 2003). Whereas, mathematical modeling involves more than using concrete manipulatives as defined by various researchers (Lesh & Doerr, 2003a; Lesh & Lehrer, 2003; Pollak, 2003; Verschaffel et al., 2002).

Many national or international organizations published standards and reports for school mathematics emphasizing mathematical modeling. National Council of Teachers of Mathematics (NCTM) (1989) reported that students in grades 9-12 should be able to “apply the process of mathematical modeling to real-world problem situations (p. 137)” as one of the standards in the *Curriculum and Evaluation Standards for School Mathematics*. In the *Principles and Standards for School Mathematics*, NCTM (2000) underlined the importance of mathematical modeling in the teaching and learning of mathematics starting from earlier grades. In addition, some international organizations like *Organisation for Economic Co-operation and Development* (OECD), which conduct educational assessment programs and surveys, pointed out the need for fostering students’ mathematical modeling competencies (OECD, 2009). In recent years, the Common Core State Standards (CCSS) announced in the United States by the Common Core State Standards Initiative (CCSSI) also emphasize the importance of mathematical modeling as both process and mathematical content.

> Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions (CCSSI, 2010, p. 72).

At national level in Turkey, Ministry of National Education [MEB] (2011a) identified some mathematical skills and with secondary mathematics curriculum, “the development of some important mathematical skills have also been targeted. These skills are; reasoning, problem solving, communication, association and modeling” (p. 4). MEB (2011a) described mathematical modeling as a skill that needs to be developed throughout the mathematics curriculum for secondary school. In recently updated mathematics curriculum, mathematical modeling is identified as a competency and skill with problem solving that should be fostered in students (MEB, 2013). One of the matters that should be taken into consideration when preparing materials and learning environments in the implementation of mathematics curriculum is the following: “Learning environments which is relevant to students’ levels and intersts and based on realistic problem solving and modeling activities by ensuring the active participation should be preferred” (MEB, 2013, p. ii). In general, a great emphasis has been put on mathematical modeling and problem solving in the latest secondary mathematics curriculum. Moreover, mathematical modeling and problem solving are used commonly in the objectives of subject matters throughout the curriculum. For instance, in the teaching and learning of "Equations and Inequalities", the following objective (9.2.4.2) is aimed: "Students will be able to use equations and inequalities in modeling and problem solving of real/realistic life situations" (MEB, 2013, p. 6).

As pointed out by many researchers, several countries have integrated mathematical modeling in their school mathematics curriculum (e.g., Burkhardt, 2006; García, Maaβ, & Wake, 2010; Lingefjärd, 2002b, 2006; Stillman, 2010). In addition, OECD (2003, 2009) reports and *Standards* (CCSSI, 2010; NCTM, 1989, 2000) addressed the significance of mathematical modeling. Turkish Secondary School Mathematics Curriculum (MEB, 2011a, 2013) also put emphasis on the mathematical modeling and modeling abilities as one of the goals of mathematics education in high schools. However, it can be seen that the implications for the importance of modeling in teaching and learning of mathematics have not been put into practice in curricular documents and it is far away from the desired level.

1.2 Teachers' Pedagogical Knowledge of Mathematical Modeling and Their Professional Developments

Many researchers pointed out that teachers play a key role in the implementation of the modeling process (Chapman, 2007; Doerr, 2006; 2007; Lingefjärd & Meier, 2010; Ng, 2010; Stillman, 2010). The research studies revealed that teachers avoid of implementing modeling activities in their classrooms due to the various reasons (Bisognin & Bisognin, 2012; Burkhardt, 2006; Blum & Niss, 1991; Ikeda, 2007; Kaiser & Maaβ, 2007) such as not knowing how to act as managers of classroom during the implementation of modeling activities (Lingefjärd & Meier, 2010), not knowing how to respond students' questions during the implementation, not feeling themselves comfortable in such a process (Ng, 2010). Doerr (2007) indicated that teachers rarely use mathematical modeling in their teaching and she pointed out the lack of knowledge about the pedagogy of mathematical modeling as a possible reason.

Subject matter knowledge and pedagogical knowledge of teachers has a strong impact on their teaching of mathematics (Grossman, 1990; Marks, 1990; Shulman, 1986, 1987). The same is true for the use of mathematical modeling in teaching (Doerr, 2007; Stacey, 2008). Stacey (2008) asserted that the knowledge of mathematical modeling is one of the components of pedagogical content knowledge (PCK) of teachers. For the pedagogy of mathematical modeling, Doerr (2007) identified that teachers need to have some qualifications such as being aware of students' ways of thinking, listening the students and providing them valuable feedbacks for fostering their emergent models, providing and offering them useful representations appropriate for their models. Doerr (2007) also suggested that both subject matter knowledge and pedagogical knowledge should be included in teacher education programs in regard to mathematical modeling. Additionally, Burkhardt (2006) mentioned about some obstacles that prevent teachers from implementing mathematical modeling in the classroom. These obstacles are being resistant to change of habits, beliefs, teaching skills coming from ancestors, the messiness of real world, and restricted level of professional knowledge about mathematical modeling. Similar obstacles have been mentioned by many other researchers (e.g., Bisognin & Bisognin, 2012; Burkhardt, 2006; Blum & Niss, 1991; Ikeda, 2007; Kaiser & Maaβ, 2007).

Knowledge about mathematical modeling has been considered as an important part of professional knowledge of mathematics teachers. Although Conference Board of the Mathematical Sciences (CBMS) underlined the importance of modeling course that "prospective teachers should have experience modeling rich real-world problems", there is not adequate information about which contents should these courses include or how these courses should be implemented (CBMS, 2012, p.60). Recently, The National Council for Accreditation of Teacher Education (NCATE) and NCTM has announced the *Program Standards for Mathematics Education* for teacher preparation programs in terms of elementary, middle, and secondary levels. Mathematical modeling has been emphasized in the *Program Standards for Mathematics Education* for all levels in the *Standard 2*. It is indicated by the *Standard* that prospective mathematics teachers should be able to "Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems" (NCATE/NCTM, 2012, p. 1) in order to become effective mathematics teacher. In the same document, *Standard 3* is strongly associated with prospective teachers' knowledge of content pedagogy. That is, the standard includes sub-dimensions such as prospective secondary mathematics teachers "apply knowledge of curriculum standards for secondary mathematics and their relationship to student learning within and across mathematical domains" and "provide students with opportunities to communicate about mathematics and make connections among mathematics, other content areas, everyday life, and the workplace" which are also strongly related with pedagogy of mathematical modeling (NCATE/NCTM, 2012, p.2). By these standards (*Standard 2 & 3*), knowledge and pedagogy of mathematical modeling has been considered as a qualification for being secondary mathematics teacher. Therefore, mathematics teacher preparation programs gain more importance for fostering pre-service teachers in terms of the pedagogy of mathematical modeling.

In the context of Turkey, there are several efforts to describe both elementary and secondary mathematics teacher competencies (MEB, 2008, 2011b). Although mathematical modeling has not been mentioned as a mathematics teacher competency in the competencies prepared for elementary mathematics teachers by MEB (2008), it has been indicated intuitively in the competencies of secondary mathematics teacher as teachers of mathematics "apply basic concepts and subjects in mathematics to other disciplines and real life situations" (MEB, 2011b). While mathematical modeling has been highlighted in the MEB's curricular documents, it is important to revise mathematics teacher education curriculum nationally and globally in order to reveal the connection and coordination between secondary mathematics curriculum and secondary mathematics teacher training programs.

Several researchers also indicated the need for professional development of teachers in order to implement mathematical modeling process in classroom environment (Doerr, 2006; 2007; Kuntze et al., 2013; Lingefjärd & Meier, 2010; Maaβ & Gurlitt, 2009; Ng, 2010). Moreover, it has been documented that the number of undergraduate courses on mathematical modeling in the teacher education programs are not adequate (Borromeo Ferri & Blum, 2009; Lingefjärd, 2007). Recent studies show that there are some research orientation in designing professional development courses and/or programs on mathematical modeling for pre-service teachers (Barbosa, 2001; Borromeo Ferri & Blum, 2009; Holmquist & Lingefjärd, 2003, 2006; Jiang, McClintock, & O'Brien, 2003; Kaiser & Schwarz, 2006; Lingefjärd, 2002b; Maaβ & Gurlitt, 2011) and in-service teachers (Blomhøj & Kjeldsen, 2006). Most of these courses have an aim of teaching of modeling rather than teaching of mathematics through modeling. In the related literature, these courses were designed and developed according to researchers' or teachers' previous experiences rather than based on literature or theoretical approaches. From this perspective, results and implications for pedagogy of mathematical modeling have been scarce. Kaiser, Blomhøj, and Sriraman (2006) stated that there is much more need for more studies about mathematical modeling and its use in the teaching of mathematics in the classroom. The researchers pointed that a general theory about teaching and learning of mathematical modeling has not been proposed yet (Kaiser et al., 2006). Blum and others (2002) indicated that there existed many initiatives proposed and implemented mathematical modeling courses and applications involving distinct techniques of instructions from most traditional to innovative teaching approaches underlining the group work. Blum and others (2002) posed the following questions to reveal the gap in the related literature:

> *What are appropriate pedagogical principles and strategies for the development of applications and modelling courses and their teaching? Are there different principles and strategies for different educational levels? (p. 164)*

At this point, this study will try to address these questions, which Blum and others (2002) asked on the basis of thinking of prospective mathematics teachers. In this study, the evolution of prospective teachers' thinking about mathematical modeling and pedagogy of modeling were investigated throughout a modeling course.

1.3 Teachers' Conceptions of Mathematical Modeling and Its Use in the Classroom Settings

Teachers' beliefs and conceptions play significant role in the application of their previous acquired knowledge and they influence teachers' decisions whether teachers implement them or not (Pajares, 1992). Many researchers carried out studies on the teachers' beliefs about mathematics and its teaching and learning (e.g., Beswick, 2005, 2007, 2012; Carpenter, & Loef, 1989; Cooney, Shealy, Arvold, 1998; Ernest, 1989; Perry, Howard, & Tracey, 1999; Peterson, Fennema, McLeod & McLeod, 2002) and influences of teachers' conceptions in their teaching prectices (e.g., Agudelo-Valderrama, Clarke, & Bishop, 2007; Beswick, 2005; Cross, 2009; Dougherty, 1990; Kaplan, 1991; Raymond, 1997; Speer, 2005; Thompson, 1984; Wilkins, 2008). Richardson (2003) indicated the importance of beliefs in teacher education because of the reasons containing philosophical and psychological aspects. Prospective teachers bring teacher preparation programs their beliefs about mathematics and its teaching that have been acquired during previous educational experiences when they were students such that some of them tend to be central beliefs that are resistant to change (Ball, 1988, 1990a; Lampert & Ball, 1998; Richardson, 2003) and these conceptions influence what and how they learn to teach from teacher education programs (Calderhead & Robson, 1991; Richardson, 1996). There are many studies about teachers' conceptions of mathematical modeling (e.g., Gould, 2013; Kuntze, 2011; Verschaffel, De Courte, & Borghart, 1997), and about teaching and learning of mathematics through mathematical modeling (e.g., Bisognin & Bisognin, 2012; Kaiser & Maaβ, 2007; Maaβ & Gurlith, 2011; Schorr & Lesh, 2003; Yu & Chang, 2011). The result of these studies demonstrated that teachers' conceptions about mathematical modeling and about its use in teaching of mathematics show differences. For example, on the one hand, Kaiser and Maaβ (2007) identified that teachers' beliefs about mathematics are main obstacles that dissuade teachers from using mathematical modeling in their classrooms. On the other hand, Yu and Chang (2011) found that teachers believed that mathematical modeling enables students to make connection between mathematics and real life situations, increases students' mathematical skills, improves students' communication abilities by sharing ideas within the group work. Similarly, Schorr and Lesh (2003) reported that teachers who participated in their study changed their perceptions and views about the use of mathematical modeling in the classroom setting in a positive direction.

Although there exist many studies which report difficulties in changing the conceptions of prospective teachers about mathematics and about its teaching, Borasi, Fonzi, Smith, and Rose (1999) noted that beliefs about the experience of teachers with new activities is very significant in the period of attending pre-service teacher education programs before they experience them as in-service teachers. Borasi and his collegues (1999) expressed that there were many evidence about the importance of prospective teachers dealing with new activities during the pre-service teacher education in their project on professional development. Moreover, many researchers indicated that prospective teachers come to teacher education programs with their previously held conceptions about mathematics and its teaching that were brought from their personal experiences when they were students (Ball, 1990a, 1990b). By the researchers' point of views, mathematics teacher preparation programs present an opportunity to change prospective teachers' conceptions about mathematics and teaching of it.

Since many researchers pointed out the importance and potential influences of pre-service teacher education programs on prospective teachers' conceptions about mathematics and its teaching, the studies about the effects of undergraduate courses (like a modeling course for prospective teachers) on prospective teachers' conception of mathematics and teaching of it may serve as a valuable contribution to mathematics education literature.

1.4 Statement of the Problem

Professional development of mathematics teachers and mathematical modeling are two subjects that have attracted attention of mathematics education community in recent years. Several researchers pointed out the role of modeling on teacher development (Doerr & Lesh, 2003; Schorr & Lesh, 2003). Although many foundations and organizations underlined the significance of mathematical modeling in school curriculum (CCSSI, 2010; MEB, 2011b; NCTM, 1989, 2000; OECD, 2003, 2009) and for professional standards that teachers should possess (NCATE/NCTM , 2012; NCTM, 1991), there are limited number of studies related to development of prospective teachers' thinking about mathematical modeling and what knowledge prospective teachers need to learn in order to implement mathematical modeling process in their in-service practice effectively. Many researchers indicated that more studies are needed on these subjects in order to reveal undetermined issues or clarify tentative findings of previous studies (Blum et al., 2002; Doerr, 2007; Kuntze et al., 2013).

1.5 Research Questions

In this study, prospective mathematics teachers' developing ideas about the pedagogy of mathematical modeling was investigated throughout the implementation of a course designed for prospective mathematics teachers in order to provide them with knowledge on using mathematical modeling in the teaching of mathematics; to improve their mathematical modeling skills; and to guide them in developing positive views on mathematical modeling. The following research questions guided this study:

- How did prospective secondary mathematics teachers' conceptions about mathematical modeling change throughout the implementation of the designed course?

- How did prospective secondary mathematics teachers' conceptions about use of mathematical modeling in teaching change throughout the implementation of the designed course?

1.6 Significance of the Study

Many studies were carried out about the use of mathematical modeling in teaching of mathematics through modeling (Crouch & Haines, 2004; Burkhardt, 2006) which emphasize the significance of mathematical modeling not only in conceptual development of students, but also in conceptual and professional development of teachers (Doerr, 2006; 2007; Doerr & Lesh, 2003; Shorr & Lesh, 2003). Furthermore, many organizations such as NCTM, OECD, and CCSSI underlined the importance of mathematical modeling in their reports and suggested standards for school curricula (CCSSI, 2010; NCTM, 1989, 2000; OECD, 2003, 2009).

Teacher education programs and professional development programs for prospective mathematics teachers and in-service teachers have become vital for developing and improving their subject matter knowledge and pedagogical knowledge of teaching mathematics (Doerr, 2007). Educational organizations and institutions published several documents to determine the qualifications and standards for teachers' knowledge. *Professional Standards for Teaching Mathematics* (NCTM, 1991) indicatied what knowledge teachers need to have in order to teach mathematics efficiently and *Program Standards for Mathematics Education* (NCATE, 2012) proposed standards for initial preparation of teachers of mathematics for all levels (elementary, middle, and secondary). These *Standards* stressed the significance of modeling knowledge of mathematics teachers in order to become effective and qualified mathematics teachers.

Since mathematical modeling are highlighted more commonly, there are various theoretical and empirical studies on the nature of mathematical modeling (Carrejo & Marshall, 2007; Kaiser & Sriraman, 2006), on modeling competencies (Blomhøj & Jensen, 2007; Jensen, 2007; Kaiser, 2007; Maaβ, 2006;), on modeling process (Borromeo Ferri, 2006; Galbraith & Stillman, 2006), on the classroom implementations of modeling (Burkhardt, 2006; Blum & Niss, 1991; Chapman, 2007; Stillman, 2010), on the role of teachers in modeling process (Chapman, 2007; Doerr, 2006; Lingefjärd & Meier, 2010; Ng, 2010), and so on. Researchers pointed out that there are very limited studies about what knowledge teachers need to have in order to implement mathematical modeling process effectively in their classrooms and announced that international studies are needed in the preparation and development of mathematics teachers in this domain (Blum, 2002; Doerr, 2007). There has been existed very restricted number of theoretical (e.g., Antonius, Haines, Jensen, Niss, & Burkhardt, 2007) and empirical studies (e.g., Doerr, 2006, 2007; Lingefjärd & Meier, 2010) on the pedagogical knowledge of mathematical modeling. Kuntze et al. (2013) underlined the need for professional development of both prospective and in-service teachers in the pedagogy of modeling more intensely. Therefore, there are calls for further studies on this issue in order to reveal distinct types of teacher implementations of modeling activities according to various conditions in terms of modeling pedagogy (Blum et al., 2002; Doerr, 2006, 2007; Stillman, 2010). However, it is clear from the related literature that there have been few mathematics education programs that include mathematical modeling course (Lingefjärd, 2007). There are some studies on designing and developing a modeling course for professional development for pre-service teachers or in-service teachers (Barbosa, 2001; Blomhøj & Kjeldsen, 2006; Borromeo Ferri & Blum, 2009; Lingefjärd, 2006; Kaiser & Schwarz, 2006; Maaß & Gurlitt, 2011). The aims of these courses changed according to target population, teachers' needs, and experiences of researcher. The main goal of these courses was to develop modeling competencies of prospective teachers rather than to teach them how to implement mathematical modeling tasks or how to facilitate the modeling process.

This study investigates how prospective mathematics teachers' thinking about mathematical modeling and pedagogy in the use of mathematical modeling evolve throughout the implementation of the designed modeling course. Many researchers point out the importance of studies on pedagogical knowledge of mathematical modeling that provide prospective teachers fundamental knowledge about mathematical modeling as well as pedagogical knowledge of modeling (Blum et al., 2002; Doerr, 2007; Kuntze et al., 2013) before they work as in-service teachers. The results of the study may serve as a step for further studies on what knowledge teachers need to possess in order for conducting mathematical modeling process and how to act during the modeling process. The findings of this study might serve as a base for developing prospective teachers' pedagogical knowledge on mathematical modeling.

1.7 Definition of Important Terms

Conception

In this study, the term of conception is used and defined as “a general notion or mental structure encompassing beliefs, meanings, concepts, propositions, rules, mental images, and preferences” (Philipp, 2007, p.259).

Models

Models defined as follows: “Models are conceptual systems (consisting of elements, relations, operations and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other systems-perhaps so that the other systems can be manipulated or predicted intelligently.” (Lesh & Doerr, 2003a, p.10)

Mathematical Models

Mathematical models are purposeful descriptions or explanations focusing on patterns, regularities, and other systemic characteristics of structurally significant systems (Lesh & Doerr, 2003a).

Modeling

In this study, Lesh and Lehrer’s (2003) definition of modeling is used. It is defined as “a process of developing representational descriptions for specific purposes in specific situations. It usually involves a series of iterative testing and revision cycles in which competing interpretations are gradually sorted out or integrated or both—and in which promising trial descriptions and explanations are gradually revised, refined, or rejected” (p. 109).

Model-Eliciting Activities

The activities used in this study defined as "the descriptions, explanations, and constructions that students generate while working on them directly reveal how they are interpreting the mathematical situations that they encounter by disclosing how these situations are being mathematized (e.g., quantified, organized, coordinatized, dimensionalized) or interpreted" (Lesh, Hoover, Hole, Kelly, & Post, 2000, pp. 592-593).

Teachers' conceptions about mathematical modeling

In this study, "teachers' conceptions about mathematical modeling" refers to teachers' general notions or mental structure about mathematical modeling in terms of its meaning, importance, and place in their mind. Teachers' thinking about anything is important because it influences teachers' responses (Ball, 1990a).

Teachers' conceptions about pedagogy of modeling

For this expression, an operational definition is used in this study. Teachers' conceptions about pedagogy of modeling means teachers' general notions about what qualifications and knowledge should be acquired and how these knowledge should be used in the modeling process in order to conduct modeling activities effectively.

Pedagogical knowledge of modeling

In this study, "pedagogical knowledge of modeling" refers to the pedagogical knowledge that teachers need to have in order to carry out mathematical modeling process effectively, and that it involves various characteristics, which are: "(1) to be able to listen for anticipated ambiguities, (2) to offer useful representations of student ideas, (3) to hear unexpected approaches, and (4) to support students in making connections to other representations" (Doerr, 2007, p. 77); to follow students' solution processes and give instant feedback if it is needed, and to manage classroom during the modeling process, to know how and when to intervene solution process (Antonius et al., 2007) etc.

CHAPTER 2

LITERATURE REVIEW

In this section, I will briefly mention previous research about the relationship between mathematical modeling and mathematics teachers under three subtitles. Firstly, studies on teachers' experiences of implementation of modeling tasks will be documented cohesively. Secondly, studies on pre-service and in-service modeling courses, which were designed and developed throughout several implementations in teacher education programs or professional development programs, will be examined. Thirdly, studies on professional development of mathematics teachers according to modeling perspective will be discussed. Teachers' views, beliefs, and conceptions about mathematics, about its teaching, and about mathematical modeling will be discussed in the light of several studies. Lastly, the conceptual framework of the current study will be provided at the end of this chapter.

2.1 Mathematical Modeling and Its Importance for Teaching and Learning of Mathematics

In the last few decades, mathematical modeling and its educational implications have gained more interest in the mathematics education society. Although mathematical modeling evoke the feeling that it is associated with mathematics only, it has many application fields as a part of distinct strands such as natural sciences including physics, chemistry, biology; engineering, architecture, economics, pure mathematics and sure for educational purposes (Haines & Crouch, 2007). In general sense, expressions of model and modeling has been used in daily life commonly. For example, people mentioned about models when they talk about a certain type of cars or computers. Students take their favorite teacher as a model and try to behave like their teachers for modeling him. It is possible to give more examples from the real life for modeling. In mathematical sense, mathematical modeling denoted as the process of transferring real life situations into mathematical language and expressing them mathematically.

In recent decades, it has been given more emphasis on the mathematical modeling in the teaching and learning of mathematics through K to higher education and suggested to be an integral part of mathematics courses due to the assumption that mathematical modeling helps students to learn mathematics more meaningfully and associated with real life. There have been studies on the mathematical modeling applications in recent years (Doerr & English, 2006; Greer, 1997; Lingefjärd, 2002b; Schukajlow, Leiss, Pekrun, Blum, Müller, & Messner, 2012) and studies on the integration of the mathematical modeling into the changing curriculum all over the world (*Department for Education* [DFE], 1997; *Ministere de l'Education Nationale*, 1997; NCTM, 1989, 2000; NGACBP & CCSSO, 2010; *Ministry of National Education* [MEB], 2011a, 2013). Although it was given more emphasis on the mathematical modeling such that school mathematics curriculum included mathematical modeling and modeling activities, researchers pointed out that there is no common agreement among researchers for the answers of the following questions such as what mathematical modeling means, what mathematical modeling aims, how mathematical modeling should be presented? How mathematical modeling should be integrated into school mathematics curriculum? How mathematics teachers should be educated to implement mathematical modeling?" etc. in the school environment (Erbaş et al., in press; Kaiser, Blomhøj, & Sriraman, 2006; Niss, Blum & Galbraith, 2007). Because of the importance attributed to mathematical modeling, it can be quite beneficial to begin with reviewing the related literature about the definitions, meanings, and approaches of mathematical modeling.

2.1.1 What Does the "Mathematical Model(s)" and "Mathematical Modeling" Mean?

It is clear from the related literature on mathematical modeling that notions of "mathematical model(s)" and "mathematical modeling" have been widely used in various meanings in accordance with distinct modeling approaches (Blum & Niss, 1991; Garcia, Pérez, Higueras, & Casabó, 2006; Haines & Crouch, 2007; Lesh & Doerr, 2003a, 2003b; Pollak, 2003; Lingefjärd, 2006; Verschaffel, Greer, & De Corte, 2002). For mathematical models, Lesh and Doerr (2003a) proposed a complicated definition as

> Models are conceptual systems (consisting of elements, relations, operations, and rules governing interactions) that are expressed using external notation systems, and that are used to construct, describe, or explain the behaviors of other system(s)—perhaps so that the other system can be manipulated or predicted intelligently (p. 10).

The researchers put emphasis on the mathematical models such that these are dealt with structural properties of related conceptual system. Lesh and Lehrer (2003) also defined the mathematical models as "purposeful mathematical descriptions of situations, embedded within particular systems of practice that feature an epistemology of model fit and revision" and mathematical modeling as "a process of developing representational descriptions for specific purposes in specific situations" (p. 109). Verschaffel and others (2002) illustrated the mathematical model as a description of properties of elements and associations contained in the real life situation. Similarly, Pollak (2003) gave the definition of a mathematical model as a description and representation of real life situation related with any mathematical structure included mathematical concepts and tools.

When it is looked through the literature on mathematical modeling, it is seen that there exist no common agreement on the definition of mathematical modeling due to aforementioned issues related to mathematical modeling. Nevertheless, there have been many attempts for defining model and modeling according to distinct perspectives of mathematical modeling. The notion of mathematical modeling as a term has been used in a variety of meanings (Galbraith & Stillman, 2006). According to Gravemeijer (2002), mathematical tools that are found in the mind and are used for explanation of real life situations constituted mathematical models. Lingefjärd (2006) stated the definition of mathematical modeling as "a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyzes (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model (Swetz & Hartzler, 1991)" (p. 16) by emphasizing the steps involved in it. Verschaffel, Greer and De Corte (2002) briefly summarized the mathematical modeling as using mathematics in order to find solutions to the problems emerged in the real life situations and gave a definition for mathematical modeling by putting emphasis on its phases as follows:

> a complex process involving a number of phases: understanding the situation described; constructing a mathematical model that describes the essence of those elements and relations embedded in the situation that are relevant; working through the mathematical model to identify what follows from it; interpreting the outcome of the computational work to arrive at a solution to the practical situation that gave rise to the mathematical model; evaluating that interpreted

> outcome in relation to the original situation; and communicating the interpreted results (pp. 257-258).

The researchers defined the mathematical modeling as a complicated process that includes a variety of phases and these phases have distinct but related goals in each of them in order to constitute the process as a whole. Pollak (2003) also denoted the mathematical modeling as "the process of creating, applying, refining, and validating [developed model]" (p. 653, cited in Gould, 2013). In his study, Pollak mentioned about steps to be followed in mathematical modeling process as other researchers did. In another research, Haines and Crouch (2007) stressed the cyclic nature of and phases included in the process of modeling by stating definition of mathematical modeling as

> a cyclic process in which real world problems are abstracted, mathematised, solved and evaluated in order passing through six stages: real world problem statement; formulating a model; solving mathematics; interpreting solutions; evaluating a solution; refining the model, before reconsidering the real world problem statement again and repeating the cycle (p. 418).

According to Haines and Crouch (2007), the operations of abstraction, mathematization, solution, and evaluation are applied to the daily life problems in the process of modeling and these operations carried out in the six phases. The authors of the study also added seventh phase to the mentioned phases as reporting what was done throughout solution process. NCTM (1989) indicated the steps of the mathematical modeling process including understanding real life problem situation, formulating the problem, forming mathematical model for the solution, finding the solution by using formed model, explicating the obtained solution according to given real problem articulation, and conforming the solution with regard to original context of the problem. According to NCTM (1989), Mathematical modeling process is illustrated in the Figure 1.

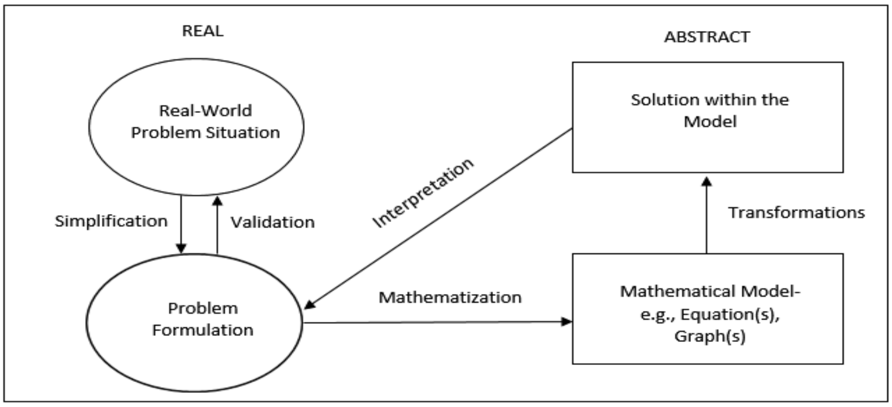


Figure 1 The process of mathematical modeling (NCTM, 1989, p. 138)

As indicated in the study of Haines and Crouch (2007), many researchers point to the cyclic nature of mathematical modeling processes (Burkhardt, 1994; Greer, 1997; Lesh & Doerr, 2003a; Lesh & Lamon, 1992 cited in Verschaffel et al., 2002; NCTM, 1989). Furthermore, mathematical modeling is also defined as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGACBP & CCSSO, 2010, p. 72). According to *Common Core State Standards for Mathematics* (CCSSM), mathematical models can be very simple, for example, "writing total cost as a product of unit price and number bought" (p. 72).

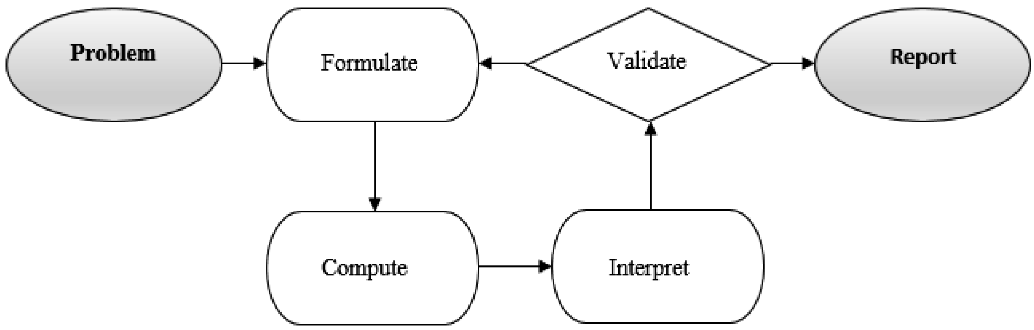


Figure 2 Mathematical modeling cycle (NGACBP & CCSSO, 2010, p. 72).

The author of CCSSM mentioned about the cycles in the modeling process (see Figure 2) and outlined steps of the process. According to CCSSM, these steps include as follows:

(1) identifying variables in the situation and selecting those that represent essential features,

(2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relations between the variables,

(3) analyzing and performing operations on these relationships to draw conclusions,

(4) interpreting the results of the mathematics in terms of the original situation,

(5) validating the conclusions by comparing them with the situation, and then either improving the model or if it is acceptable,

(6) reporting on the conclusions and the reasoning behind them (NGACBP & CCSSO, 2010, pp. 72-73).

The process of mathematical modeling includes cycles in which descriptions, explanations, and interpretations are revised, refined, or refused (Lesh & Lehrer, 2003). Apart from other researchers (e.g. Haines & Crouch, 2007; Verschaffel et al., 2002) and educational organizations (e.g. NCTM, 1989; NGACBP & CCSSO, 2010), Lesh and Doerr (2003a) offered modeling cycle include four steps which are description, manipulation, translation, and verification (see Figure 3).

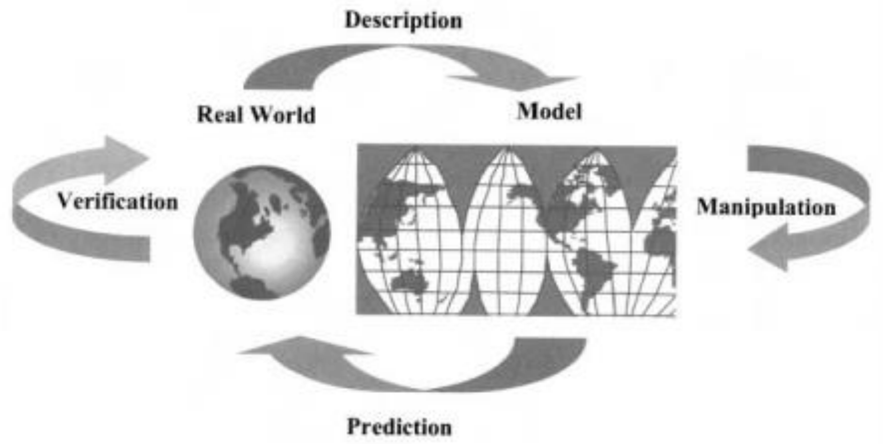


Figure 3 Mathematical modeling cycle (Lesh & Doerr, 2003a, p. 17)

According to proposed modeling cycle, description stage includes forming a relation between real world and model world. In the manipulation stage, using and manipulating offered model to give rise to put forward more predictions and actions about solution of real world problem situation. Translation stage means transferring the suitable outcomes from model world to real world. Eventually, verification stage contains the procedure of confirming the obtained outcomes and predictions in terms of usefulness (Lesh & Doerr, 2003a).

It is seen from the proposed descriptions for mathematical modeling by different researchers pointing to the similar expressions such as it has a cyclic structure rather than a linear sequence and that includes distinct stages and these stages cover different actions (Haines & Crouch, 2007; Lesh & Doerr, 2003a; NCTM, 1989; NGACBP & CCSSO, 2010). Although researchers have similarities in their descriptions for mathematical modeling, some distinctions emerged in their perspectives to models. Gravemeijer (2002) stated that mathematical models are the mathematical tools that are in the mind and used to express and explain encountered real life situations in terms of mathematics. On the contrary, mathematical models are the conceptual systems that describe and explain actions of other systems (Lesh & Doerr, 2003a).

Another issue appeared in the literature that mathematical modeling has been mixed with using concrete materials. Researchers indicated that mathematical modeling has been comprehended commonly in elementary stages as the use of concrete materials (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003).

2.1.1.1 Differences between mathematical modeling and problem solving

When it is looked through the mathematics education studies, there have been various studies on the problem solving. Researchers expressed that problem solving activities need to be separated from the problem solving activities and exercises related to traditional word problems (English, 2003; Schoenfeld, 1992). With the advent of emergence of mathematical modeling, there has been existed a debate on the distinctions between mathematical modeling and problem solving in the mathematics education society (Lesh & Doerr, 2003a) and how models and modeling perspectives describe problem solving (Zawojevski & Lesh, 2003). Lingefjärd (2002b) indicated that problem solving existed in the process of modeling as sub-processes so that it is not meaningful that making comparisons between problem solving and mathematical modeling. However, there has been existed many studies about the distinctions between problem solving and mathematical modeling (Lesh & Yoon, 2007; Zawojevski, 2010; Zawojevski & Lesh, 2003). In order to compare mathematical modeling and problem solving, it is need to look through their definitions and descriptions. Blum and Niss (1991) explained what they meant by a *problem* as

> a situation which carries with it certain open questions that challenge somebody intellectually who is not immediate possession of direct methods/ procedures/ algorithms etc. sufficient to answer the questions (p. 37).

Although the problem defined in the previous quote include the conditions such as being challenging and having no existing knowledge about how to solve, traditional word problems are away from that definition. Problem solving is described as "a process of getting from givens to goals when the path is not obvious" (Lesh & Yoon, 2007, p. 166). Polya (2004) defines the process of problem solving as consisting of four steps:

- Understanding the problem by identifying unknowns, given data, and condition for the problem,
- Devising a plan for solution by recognizing similar problem statements and finding a connection between givens and asked unknowns,
- Conducting the plan by checking the each step in the solution procedure and ensuring the correctness of the operations,
- Looking back to revise solution the process and examine the results in terms of correct application of the plan.

When it was looked at the process of problem solving suggested by Polya, procedural and linear structure of the process was prominent. The characteristics of problem solving process expressed by Lesh and Yoon (2007) in their paper.

In their study, Lesh and Yoon (2007) stated the basic properties of problem solving process. First one is that the process begin with identification of unknowns, given data, and conditions and it is clear that what was asked in the problem statement in the problem solving. Secondly, the intended conclusion is obtaining an obvious mathematical result within the mathematical domain. Thirdly, the process of problem solving is based on finding the answer by following the givens to intended goals within the mathematical world. On the contrary, mathematical modeling seeks “the development of meaning and usefulness for powerful mathematical concepts or conceptual systems” (Lesh & Yoon, 2007, p. 166). Mathematical modeling includes real life situations and the statements of the mathematical modeling activities are problematic in order to make students find distinct useful ways for solution. The solution does not need to be simple and provide a mathematical answer. Rather, the solution is a mathematical tool that describes and explains similar situations. Lesh and Yoon (2007) also indicated that mathematical modeling process contains test, revise, and refine procedures in order to reach more explaining model as a result.

Zawojewski (2010) claimed that one of the main differences between problem solving and mathematical modeling is “in the emphasis on and the importance of the nature of the task posed, and therefore task design becomes an important feature of research” (p. 239). According to the researcher, the person who solve problems try to find a correct way to follow from given data to the goals of the problem (Zawojewski, 2010; Zawojewski & Lesh, 2003). Conversely, in mathematical modeling, the person engaged in the modeling activity need to construe the given data and asked results for modeling the real life situation mathematically. In problem solving, correct solutions are the target and removing the wrong steps from the solution procedure. Nevertheless, mathematical modeling requires constructing mathematical models that fits the best given real life situations and can be generated to similar situations. Wrong parts are the parts of modeling process in the period of test, revise, and refine procedures (Zawojewski, 2010). With the help of ideas emerged in the literature on problem solving and mathematical modeling, Erbaş and his collegues (in press) suggested the following table (see Table 1) for the comparison of problem solving and mathematical modeling in their papers.

Table 1 A comparison of problem solving approaches and mathematical modeling (Erbaş et al., in press)

Traditional Problem Solving Approaches	**Mathematical Modeling**
The process of reaching to specific goal by using givens	Multiple cycles, different interpretations
Real or realistic life situations with idealized problem context	Authentic real life context
It is expected from students to use taught structures such as formulas, algorithms, strategies, mathematical ideas etc.	Students experience development of significant mathematical ideas and structures, revision, and refinement steps in the modeling process
Individual working stands in the forefront	Group work is stressed. (e.g., social communication, sharing mathematical ideas etc.)
Abstracted from real life	Associated with real life and possessing interdisciplinary nature
It is expected from students to give meaning to mathematical symbols and constructs	Students try to describe real life situations mathematically
Teaching of specific problem solving strategies (e.g. developing distinct approach, transferring it on a shape etc.) and using it in the solution of similar problems	It involves more than one inconspicuous solution strategies developed by students consciously that are specific to certain situation
There is unique correct solution	There are more than one solution strategies and solutions (model)

According to given table above (see Table 1), mathematical modeling involves more complex procedures and cycles consisting of distinct ideas and solution ways in the process of testing, revising, and refining the obtained product (model) which is reusable and shareable for the similar kind of situations (Lesh & Doerr, 2003a, 2003b; Lesh & Yoon, 2007). On the other hand, traditional problem solving perspectives include the notion that reaching to goals from givens by using previously taught formula, algorithms, procedures, and techniques which are supposed that students make sense of mathematical symbols and structures and underlines individual working in its essence (Erbaş et al., in press). Lesh and Doerr (2003a) identified the traditional problem solving as a subset of *model-eliciting activities* (Lesh, Hoover, Hole, Kelly, & Post, 2000) in the process of mathematical modeling. That is, model-eliciting activities included the traditional problem solving as a particular situation in them and much more extended than problem solving. Reusser (1995) encouraged previous statements by describing the relationship between word problems and mathematical modeling such that word problems constitute “an important part of mathematics education, inasmuch as they represent the interplay between mathematics and reality, and they give a basic experience in mathematical modelling” (p. 1). Lesh and Doerr (2003a) identified the main characteristics of traditional word problems such that traditional word problems highlight the students’ skills related to computation and it is expected from students “to make meaning of symbolically described situations” (p. 3). On the other hand, students are expected to explain and describe the meaningful real life situations mathematically by dealing with model-eliciting activities that resemble daily life situation. Another distinction between mathematical modeling activities and traditional word problem is there exists more wealthy learning processes for students in which they gain experience in mathematical modeling activities than traditional word problems include. Yu and Chang (2011) identified all model-eliciting activities as open-ended problems including more data.

It is indicated in several studies that mathematical modeling has perceived as using concrete manipulative in order to teach mathematical subject matters, especially in the elementary school levels (Abramovich, 2010; Erbaş et al., in press; Lesh et al., 2003). The use of concrete materials in mathematics instruction based on the theoretical considerations of Zoltan Dienes (1960) (cited in Thompson, 1994). Dienes (1960) built his theory on concrete-to-abstract aspects of conceptual development of children by designing activities that include concrete manipulatives (e.g. arithmetic blocks) and developed principles for instruction (cited in Lesh et al., 2003). According to Dienes’ instructional design principles, concrete manipulatives that are beneficial for students to create constructs for employing a basis for their mathematical reasoning. Dienes (1960) called the use these purposeful concrete manipulatives in instruction as "*embodiment*" and the use of embodiments is regarded as significant in the mathematical instruction for the conceptual development of children when they are used according to instructional principles (Lesh et al., 2003).

There existed studies that explain the transition from the use of concrete materials (i.e. manipulatives) to modeling. Gravemeijer (2002) pointed out the change in approaches to mathematics education from the use of concrete materials to mathematical modeling by the use modeling activities as

> what is called symbolizing and modeling nowadays differs significantly from the use of manipulative materials and visual models – often generically referred to as 'manipulatives'–that has been common practice for a long time in mathematics education (p. 7).

According to instructional design that include the use of manipulatives and visual representations (proposed by mathematics educators and scholars who use the information-processing theory and Gal'perin's theory), the main goal is to "make the abstract mathematics to be taught more concrete and accessible for the students" (Gravemeijer, 2002, p.8). The intended goal seems to be admitted at first, but serious critics have been risen by theorists of constructivism. Constructivists critique is that "… external representations do not come with intrinsic meaning, but that the meaning of external representations is dependent on the knowledge and understanding of the interpreter" which leads to doubt about the use of manipulatives in the mathematics teaching (Gravemeijer, 2002, p.8). Lesh and others (2003) reported that many studies demonstrated that teachers might have difficulties in using embodiments so that they tend to use activities with concrete manipulatives rarely. Erbaş and others (in press) indicated that labeling the use of concrete materials in the teaching of mathematics caused to perception of mathematical modeling as designing and using concrete manipulatives. In fact, mathematical modeling involves more meaning than designing and using concrete materials. Mathematical modeling is "a non-linear process that involves elements of both a treated-as-real world and a mathematics world" (Zbiek & Conner, 2006, p. 91). As there existed confusion between mathematical modeling and the use of concrete manipulative among teachers and students at elementary school level (Erbaş et al., in press; Lesh et al., 2003), similar confusion arises between mathematical models and representations. Mathematical models have been regarded as representations that include "written symbols, spoken languages, pictures or diagrams, concrete manipulatives or experience-based metaphors" (Lesh & Doerr, 2000, p. 363). According to Lesh and Doerr (2000), the distinction between mathematical models and representations explained such that mathematical models are the essence of general characteristics of modeled conceptual systems. On the other hand, representations comprise the objects in conceptual system rather than being as a system.

When it is looked through the modeling literature, it can be seen that there have been many conceptions about the terminology of the mathematical modeling activities. Some researchers preferred to use the expression "*model-eliciting activities*" offered by Lesh and others (2000) as those activities elicit models in the process of mathematical modeling as a result of expressing, testing, revising, and refining steps (Doerr & English, 2006; Lesh & Doerr, 2003a; Lesh & Yoon, 2007; Zawojewski & Lesh, 2003). English (2003) used the "student modelling activities" for the modeling activities and described modeling activities as "…, [these activities] are designed explicitly to reveal children's various ways of thinking, the ways in which they document their thinking, and the nature of their conceptual development during problem solution" (p. 230).

Model-eliciting activities (Lesh et al., 2000) have been elucidated in-depth as

> … thought revealing activities that focus on the development of constructs (models or conceptual systems that are embedded in a variety of representational systems) that provide the conceptual foundations for deeper and higher order understandings of many of the most powerful ideas in precollege mathematics and science curricula. Therefore, the activities that are emphasized herein are not only thought revealing, but also model-eliciting. That is, the descriptions, explanations, and constructions that students generate while working on them directly reveal how they are interpreting the mathematical situations that they encounter by disclosing how these situations are being mathematized (e.g., quantified, organized, coordinatized, dimensionalized) or interpreted (pp. 592-593).

Lesh and his collegues (2000) proposed six principles for designing model-eliciting activities: the model construction principle, the reality principle, the self-assessment principle, the construct documentation principle, the construct shareability and reusability principle, and the effective prototype principle. In order to design and develop qualified model-eliciting activities, aforementioned principles should be fulfilled. By the model construction principle, modeling activity intended to be designed need to permit to create a model that explains, describes the related real life situation. The reality principle, also called as meaningfulness principle, accounts for the problem situation that really exists in the real life so that students comprehend the situation meaningfully. According to the self-assessment principle, the problem context needs to include relevant standards in order to evaluate the obtained and possible solutions in terms of its usefulness. Students need to reflect their thinking explicitly on the problem statement and solution process by documenting their developed models in order to fulfill the construct documentation principle. The developed constructs (models) need to be shareable and reusable for the similar real life situations to satisfy the construct shareability and reusability principle. The obtained solution need to provide sufficient prototype in order to interpret similar situations and others can use that solution in the same purposes which explains the effective prototype principle. These principles need to be followed in order for designing and developing more productive thought revealing activities (Lesh et al., 2000). Stillman (2010) explored the modeling task conditions for carrying out mathematical modeling successfully in her study. The task conditions include helping students go beyond the target achievement; encouraging students' interests and curiosity; permitting students to select appropriate technological tools freely; letting students to benefit from multiple representations in the modeling process; providing students to reply questions that requires interpretation; and helping students to improve keeping records in the process of application and modeling. These conditions explain the pedagogical sides of six principles for designing model-eliciting activities (Lesh et al., 2000) in the action.

2.1.1.2 The use of technology in mathematical modeling

In many studies, the use of technological tools in the teaching of mathematics emphasized in terms of its help in problem solving and providing a learning environment to explore the mathematical conceptions in detail, and knowledge about using technology identified a professional standard for mathematics teachers (NCTM, 1991). Technology also specified as a principle for school mathematics and underlined its significance in mathematics education as "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24). Teachers can pose "more activities and projects that include exploration, investigation, and modelling" (Da Ponte, Oliveira, & Varandas, 2002). The knowledge about use of technology is seen as a characteristic of mathematics teachers' that they need to have if they teach the mathematical modeling (Lingefjärd & Holmquist, 2001). The technological tools in mathematical modeling used to produce and confirm as models of real life phenomena (Zbiek, 1998).

However, the use of technological tools such as computer simulations and applets is more beneficial in estimation and experimentation rather than conforming mathematical models (Doerr, 1994/1995; cited in Zbiek, 1998).

The use of technological tools in mathematics education and mathematical modeling, as part of mathematics education, has always been on the agenda of international conferences such as *Psychology of Mathematics Education* (PME), *International Commission on Mathematical Instruction* (ICMI), and especially in *the International Community of Teachers of Mathematical Modelling and Applications* (ICTMA). In the proceedings of the ICTMA conferences, one of the main sections has been dedicated to technology in mathematical modeling that demonstrated the fact that technology and technological tools accepted as an important part of mathematical modeling process (Blum & Leiß, 2007; Greefrath, 2011). According to Greefrath (2011), the significance of the use of technological tools in modeling process as "Using digital tools broadens the possibilities to solve certain mathematical models, which would not be used and solved if digital tools were not available" (p. 301).

There are many studies on the use of technological tools in mathematics education and mathematical modeling. In the study of Zbiek (1998), pre-service secondary mathematics teachers' ways of developing and confirming mathematical models with the use of technological tools in mathematical modeling investigated within an elective mathematics course in which 13 pre-service secondary mathematics teachers enrolled. The researcher collected data including interviews and observations carried out in classroom and laboratory throughout the semester. The results of the study demonstrated that most of the pre-service teachers who attended the course tried to use technological tools in the development and testing of mathematical models. Preferences of pre-service teachers' about strategy was affected by the features of modeling activities and interactions within the classroom.

Another study conducted by Lingefjärd and Holmquist (2001). The researchers carried out a modeling course for undergraduate students who studied mathematics and science education that aimed to show how the use of technology in modeling influences teaching and learning of mathematics in the classroom setting and provide students to conceptualize mathematical subject matters with use of technological tools. The researchers assigned two modeling tasks that required the use of technological tools to solve and analyzed students' responses to these tasks. The findings indicated that students used technological tools and this permitted them to develop accurate models for the tasks. The researchers reported that most of the students did not check the correctness of their generated models via technological tools with mathematical structures.

In recent decade, many papers presented in ICTMA conferences which discuss the use of technology and technological tools in mathematical modeling process and the relationship between technology and modeling in various perspectives (e.g. Campbell, 2010; Confrey & Maloney, 2007; Fuller, 2001; Galbraith, Renshaw, Goos, & Geiger, 2003; Geiger, Galbraith, Renshaw, & Goos, 2003; Henn, 2001; Keune & Henning, 2003; Kissane, 2010; Pead, Ralph, & Muller, 2007; Sinclair & Jackiw, 2010). The studies on the mathematical modeling and technology demonstrated that technology has been an integral part of mathematical modeling process as a vehicle or way of mathematical modeling.

2.1.2 The Importance of Mathematical Modeling for Teaching and Learning of Mathematics

The role of mathematical modeling on the teaching and learning of mathematics has been gaining more importance in mathematics education society. With the advent of technological developments, nations want to become frontier in the following years. Due to this, growing individuals who solve problems concerning with real life situations successfully becomes crucial for nations in order to hold a leader position and perpetuate this position for the following years, even decades all over the world. This depends on how well nations educate their students in significant domains such as mathematics, science, technology etc. From this point of view, researchers asserted that the goal of mathematics education has to be making students gain problem solving skills in real life situations and mathematical modeling can be used in order to reach this goal (Erbaş et al., in press; Gravemeijer & Stephan, 2002; Lesh & Doerr, 2003a). Many researchers put emphasis on the key role of mathematical modeling in the teaching of mathematics and mentioned its importance (Blum et al., 2002; Blum & Niss, 1991; Doerr & Lesh, 2011; Kaiser & Maaβ, 2007; Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007; Lingefjärd, 2000, 2002a) and its importance in the school curricula (Blum & Niss, 1989; Niss, 1989). Several researchers asserted that mathematical modeling need to be taught and learned as a subject matter that rather than a teaching method (Burkhardt, 2006; Holmquist & Lingefjärd, 2003; Lingefjärd, 2006).

Various international and national communities and organizations declared reports and standards for school mathematics underlining mathematical modeling. NCTM (1989) pointed out the mathematical modeling as an ability for students need to have such that students should be able to "apply the process of mathematical modeling to real-world problem situations" (p. 137) in the *Curriculum and Evaluation Standards for School Mathematics*. NCTM (2000) emphasized the significance of the mathematical modeling in the teaching and learning of mathematics in which beginning from the preschool to high school education according to students' cognitive levels in the *Principles and Standards for School Mathematics*. Moreover, as an international organization, *OECD* have been carrying out an international assessment program for students (PISA) which aims to measure knowledge of students about the specific domains such as reading, mathematical and scientific literacy at a specific age in order to monitor students' readiness for the challenges of today's societies (OECD, 2003) since 1997. Mathematical literacy is one of the domains assessed in PISA which mathematical modeling cycle is used in mathematics framework to describe the levels students go through in solving contextualized problem situations (OECD, 2009). In the report, modeling is also underlined as a mathematical competency for mathematical literacy described its significance as "It is critical to mathematical literacy since it underpins the capacity to move comfortably between the real world in which problems are met and solutions are evaluated, and the mathematical world where problems are and solved" (OECD, 2009. p. 32). As in the national level, *CCSS* for mathematic has been declared in United States by CCSSI as unifying curriculum standards which reflects expectations from students, teachers, even parents providing students successful in college and their careers. CCSS for mathematics put emphasis on mathematical modeling in both the standards for mathematical practice as a process and standards for mathematical content as a high school strand. In the *Standards for Mathematics Content*, mathematical modeling described as follows:

> Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. (CCSSI, 2010, p. 72).

In a real world setting, daily life situations are complicated, sophisticated, and dynamic. The problems emerged in these conditions such as designing a workplace, arranging flights, or arranging timetables for traffic lights in a city also tend to be more complicated, difficult, and sophisticated rather than traditional word problem arise in the mathematics textbooks. Resolving problems emerged in real life situations and interpreting these complex systems include significant mathematical processes that has been not given enough emphasis by policy makers and curriculum designers in many countries (English & Watters, 2005). As mathematical modeling involves "processes such as constructing, explaining, justifying, predicting, conjecturing, and representing, as well as quantifying, coordinating, and organising data are becoming all the more important for all citizens" (English & Watters, 2005, p. 58). Thanks to mathematical modeling activities, students develops their argumentation skills due to the reason that modeling activities accepted as social experiences included in the modeling process (Zawojewski, Lesh, & English, 2003) and these activities promote students' communication skills, provide students to work as a team, and make reflections about themselves and their developed model (English & Watters, 2005). Researchers indicated that students at any level demonstrated that they could handle with mathematics and achieve more things when they engaged in mathematical modeling activities (Doerr & English, 2003; Lesh & Doerr, 2003a). The group work in mathematical modeling activities highlighted by English and Watters (2005) and explained its importance and functioning as follows:

> The modelling activities are specifically designed for small-group work, where children are required to develop sharable products that involve descriptions, explanations, justifications, and mathematical representations. Numerous questions, conjectures, conflicts, resolutions, and revisions normally arise as children develop, assess, and prepare to communicate their products. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team members (p. 61).

Antonius and others (2007) also indicated the role of group work in the process of modeling as "Group work contributes significantly to the engagement of students, increasing motivation, and leading to better understanding of both the real world context and the mathematical concepts and techniques required for success" (p. 298). Ikeda and Stephens (2001) found that there existed advances in the performance of group members when discussion arose among them.

In many studies, it is pointed out that students make connections between mathematics and real life which leads them to learn mathematics meaningfully and students think mathematically when they tried to elicit products (models) in the mathematical modeling process which leads to conceptual and cognitive development of students and also teachers (Doerr & Lesh, 2003; Lesh & Doerr, 2003a, 2003b; Lesh & Lehrer, 2003). Skolverket (1997) mentioned about the significance of mathematical models and modeling and underlined the mathematical modeling as an integral part of mathematics needed to be taught in schools (cited in Lingefjärd, 2006).

Because of the significance of mathematical modeling in mathematics education, the school curricula comprise mathematical modeling as compulsory part in many countries (Borromeo Ferri & Blum, 2009; Lingefjärd, 2006). For the school curricula in USA, Zbiek and Conner (2006) stated the importance of mathematical modeling in the instruction such that

> … extensive student engagement in classroom modeling activities is essential in mathematics instruction only if modeling provides our students with significant opportunities to develop deeper and stronger understanding of curricular mathematics (p. 90)

rather than identifying the mathematical modeling as one of mathematical strands. The significance of using mathematical modeling in the classroom setting expressed as "Making modelling, generalization, and justification an explicit focus of instruction can help to make big ideas available to all students at all ages" (Carpenter & Romberg, 2004, p. 5). In the mathematics curriculum for secondary school (MEB, 2011a) in Turkey, mathematical modeling denoted as one of the skills that aimed to be developed throughout the program. Mathematical modeling described as "a dynamic method which enables us to observe the relationships within the nature of problems from every field in the life; to express the relationships among them with mathematical terms by discovering them; to classify them; to generate them, and facilitates us to draw conclusion" (MEB, 2011a, p. 10). It is emphasized the importance of mathematical modeling as a skill that needs to be gained in the throughout the implementation of the program. In the latest secondary school mathematics curriculum, mathematical modeling expressed as not only as a skill, but also a competency with problem solving (MEB, 2013). The significance of mathematical modeling that consistent with previous program underlined and it was mentioned about gains that are supposed to students have with mathematical modeling.

2.1.2.1 Mathematical modeling for what? Classification of modeling perspectives

Even though many researchers have mentioned about the significance of mathematical modeling in mathematics education (Blum & Niss, 1991; Doerr & Lesh, 2011; Kaiser & Maaβ, 2007; Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007; Lingefjärd, 2000, 2002a), there have been existed a debate on the perspectives of mathematical modeling in mathematics education (Kaiser, 2005; Kaiser-Messmer, 1986; Kaiser & Sriraman, 2006) and it is evident from the studies on mathematical modeling that there has been no common ground on the perspectives of modeling (Kaiser, Blum, Borromeo Ferri, & Stillman, 2011). There existed few studies about the modeling approaches for mathematics education (e.g., Erbaş et al., in press; Kaiser, 2005; Kaiser & Sriraman, 2006). The distinct perspectives on mathematical modeling explained and distinguished firstly by Kaiser-Messmer (1986) within two categories. These are *pragmatic perspective*, which is based on "utilitarian or pragmatic goals, the ability of learners to apply mathematics to solve practical problems" and *scientific-humanistic perspective* that is inclined to "mathematics as a science and humanistic ideals of education with focus on the ability of learners to create relations between mathematics and reality" (Kaiser, 2005, p. 1613). In recent years, categorization of perspectives extended with the studies of Kaiser (2005) and Kaiser and Sriraman (2006) (see Table 2).

Table 2 Categorization of perspectives on modeling (Kaiser & Sriraman, 2006, p. 304)

Name of the perspective	**Central aims**	**Relations to earlier perspectives**	**Background**
Realistic or applied modelling	Pragmatic-utilitarian goals, i.e.: solving real world problems, under- standing of the real world, promotion of modelling competencies	Pragmatic perspective of Pollak	Anglo-Saxon pragmatism and applied mathematics
Contextual modelling	Subject-related and psychological goals, i.e. solving word problems	Information processing approaches leading to systems approaches	American problem solving debate as well as everyday school practice and psychological lab experiments
Educational modelling; differentiated in a) **didactical** modelling and b) **conceptual** modelling	Pedagogical and subject-related goals: a) Structuring of learning processes and its promotion b) Concept introduction and development	Integrative perspectives (Blum, Niss) and further developments of the scientific-humanistic approach	Didactical theories and learning theories

Table 2 (continued)

Socio-critical modelling	Pedagogical goals such as critical understanding of the surrounding world	Emancipatory perspective	Socio-critical approaches in political sociology
Epistemological or theoretical modelling	Theory-oriented goals, i.e. promotion of theory development	Scientific-humanistic perspective of "early" Freudenthal	Roman epistemology

Kaiser and Sriraman (2006) added the "cognitive modeling" as a "meta-perspective" apart from above table. In cognitive modeling, investigation of cognitive processes of students during the mathematical modeling process and efforts to comprehend these processes are involved based on cognitive psychology. The studies that attempt to categorize the mathematical modeling perspectives in mathematics education are very limited and comprise the papers presented in the *ICMI* and *the ICTMA* in terms of their aims and the theoretical approaches used within them. The categorization of modeling perspectives conducted according to the researchers' point of views rather than scientific classification principles and the researchers admitted that the categorization of perspectives on modeling were made superficially in order to attract notice to the use of concepts modeling concepts with their theoretical backgrounds and suppositions (Kaiser & Sriraman, 2006). The authors recommended, "a precise clarification of concepts is necessary in order to sharpen the discussion and to contribute for a better mutual understanding" (p. 308).

The recent studies on mathematical modeling approaches demonstrated that each modeling perspective has different goals on mathematics education in accordance with how they define mathematical modeling and the way of implementation of mathematical modeling involved in school mathematics (Erbaş et al., in press). For instance, Lesh and Doerr (2003b) asserted that mathematical modeling is a paradigm that goes beyond the constructivism "for thinking about the nature of children's developing mathematical knowledge and abilities and about mathematics teaching, learning, and problem solving" (p. 516). On the other hand, several researchers explained mathematical modeling as using mathematics in order to solve problems that emerged in real life (Verschaffel et al., 2002); expression of real life problems in terms of mathematical symbols or representations and the application of mathematical structures such as formulas into real life (Haines & Crouch, 2007).

According to several researchers (Erbaş et al., in press; Galbraith, 2012), there have been two distinct perspectives on the use of mathematical modeling in mathematics education. These are teaching of mathematical modeling as a goal and a mathematical tool or method for teaching and learning of mathematics. In the former approach, mathematical modeling is seen as a topic or domain in mathematics and ought to be taught as one of the significant goals of mathematics education in order

> … to make students properly aware of the value of mathematical modelling in a wide range of situations, and to train them how to apply IT [information technology] tools most effectively. The benefits that will accrue are essential for the survival and future growth of commerce, industry and science, and there are opportunities for them to be realized at every level of employment (Lingefjärd, 2006, p. 98).

In the latter modeling approach to the use of mathematical modeling in mathematics education, mathematical modeling is seen as a way of teaching and learning mathematics by using real life situations in which powerful mathematical constructs created and developed in the process (Lesh & Doerr, 2003a, 2003b) and make learners discover the mathematical concepts intuitively by involvement of them in real life situations as an integral part of designed instruction (Gravemeijer, 2002; Gravemeijer & Stephan, 2002).

Niss and his collegues (2007) characterized the former approach as "*application* of mathematics" in which there existed a movement from mathematics to reality, that is, using mathematical structures in order to in order to real life situations and included "*objects*" are underlined (p. 10). However, in the latter approach that denotes the mathematical modeling as "vehicle" (Galbraith, 2012) for teaching of mathematics, it is centered on the move from reality to mathematics and the modeling process highlighted. *Models and Modeling Perspectives* (MMP) (Lesh & Doerr, 2003a, 2003b; Lesh & Lehrer, 2003; Lesh & Yoon, 2007) and *Emergent modeling* (Doorman and Gravemeijer, 2009; Gravemeijer, 2007) perspectives are exemplary of modeling approaches that mathematical modeling as vehicle.

2.1.2.2 Studies on the use of mathematical modeling in the classroom settings

Even though there has been a great emphasis on the importance of mathematical modeling in mathematics education, many studies indicated that the use mathematical modeling in the school mathematics is scarce (Burkhardt, 2006; Henn, 2007; Maaβ, 2005) and there existed difficulties and obstacles in the integration of mathematical modeling into mathematics lessons (Bisognin & Bisognin, 2012; Blum

& Niss, 1991; Burkhardt, 2006; Ikeda, 2007; Kaiser & Maaβ, 2007). Although the use mathematical modeling and modeling activities in mathematics lessons seems to be less than desired, many researchers reported successful implementations of mathematical modeling in classroom settings in primary school level (English, 2004; English, 2006; English & Watters, 2005), middle school level (Lesh & Harel, 2003; Schukajlow et al., 2012), and secondary school levels (Busse, 2011; Lingefjärd, 2011).

In the study of Blum and Niss (1991), the main difficulties related to implementing a mathematical modeling based on teaching were discussed in terms of various point of views. According to teachers' perspective, mathematical modeling and applications of mathematics bring about the instruction more prone to open-ended and requiring more effort to cope with it because it requires more qualifications rather than mathematical ones and it is too hard to assess students' successes. In addition, Blum and Niss (1991) stated that many teachers do not feel themselves safe while coping with situations from real world. The reasons for this stemming from either their restricted knowledge about mathematical modeling or its applications, or not having adequate time to develop a teaching plan for it. From instruction perspective, allocating time seems to be troublesome. Many teachers of mathematics worried about not possessing adequate time to handle with mathematical modeling and its applications beside school mathematics curriculum. Additionally, some of mathematics teachers suspect that formal and context-free nature of mathematics may disappear via using mathematical modeling tasks and applications.

Burkhardt (2006) critically analyzed that although the importance of mathematical modeling emphasized in mathematics education, "Why then do we not have modelling as an integral part of mathematics curricula worldwide?" (p. 189). According to the author, there existed some hinders in the implementation of mathematical modeling in classroom settings. Burkhardt expressed possible obstacles as the domination of existing education system and its resistance to change described as "*systemic inertia*", the complicatedness and messiness of the real world and situations in it, limited professional development of teachers, and gaining slow advances in educational system due to the nature of educational research. Another study associated with possible obstacles for implementation of mathematical modeling reported Kaiser and Maaβ (2007). In the study, results of two distinct researchers provided. In the first study, it was conducted with students; the results indicated that students' beliefs on mathematical modeling might preclude teachers to apply mathematical modeling activities in their classrooms. According to second study carried out with teachers, the results demonstrated that mathematical modeling had little effect on the beliefs of teachers about mathematics and its teaching. Ikeda (2007) investigated the obstacles for the implementation of mathematical modeling with respect to different countries around the world. According to the results of the study, parallel hinders were identified. These were not seeing the mathematical modeling as a significant component of school mathematics curricula; controversial situation between the central emphasis on the modeling in the curricula and teachers' preferences and applications; including modeling in mathematics curricula, but not as an integral part; teachers' perceptions and comprehensions of mathematics; deficiencies in curriculum materials (e.g., textbooks, modeling activities) and measurement and evaluation techniques; and appropriate mathematical modeling activities or problems for high stake general exams. Ikeda (2007) underlined the importance of teacher education as a crucial and pivotal role that needs to be taken into account for providing efficient implementation of mathematical modeling in classrooms.

Many studies indicate the usefulness and appropriateness of the use of mathematical modeling in classroom settings in various perspectives and levels. For example, English (2004) carried out a research on the processes that a class of primary school students and their teachers engaged in during the mathematical modeling activities and their constructions of products (models). This study was a part of a longitudinal study designed according to multilevel research in which researchers, pre-service teachers, class teachers, and students in primary level and only reported the processes of students' model construction. A sequence of mathematical modeling activities implemented in the classroom. Before the each implementation, arranged meetings held with the teacher. The lesson format included announcement of the modeling activity by the teacher, then small-group working started on the modeling activity. Teacher and researchers remained as observers during the group working. At the end of the lesson, each group presented their products and asked for feedbacks from other group of students. The lesson ends with general classroom discussion on the obtained mathematical models in terms of their similarities and differences. Data comprised of video tape records of all implementation of modeling activities classroom discussions and small-group working periods, field notes, students' working sheets, and final reports that each group provided. English (2004) indicated as a result of the study that students displayed higher cognitive skills in spite of being primary students such as interpreting the modeling activity contexts, specifying the aim, making decisions about the solution strategies, implementing and testing of these strategies, making assumptions about the problem situations, developing and refining models according to group discussions, thinking all aspects of the problem situation, and implementing, justifying, revising and refining their developed mathematical models according to feedbacks.

Another study on the use of mathematical modeling in the very beginning of the primary school years was carried out by English and Watters (2005) in which they investigated the development of mathematical knowledge of primary school students (grade 3) their reasoning during the implementation of mathematical modeling activities. The participants of the study were four classes of primary school students at grade 3 and their teachers that accompanied with the program for professional development of teachers. The data sources of the research were video tape records of teachers' implementation of modeling activities and focus group, field notes, students' reports that involved written and oral explanations about their works. The findings of the research demonstrated that primary school students developed significant mathematical ideas in the modeling processes through they had not received any instruction about modeling before. The results also suggested that mathematical modeling activities helped students develop mathematical illustration, interpretation, validation, revision of significant mathematical ideas.

In the middle school level, Lesh and Harel (2003) carried out a research on comparison of modeling processes that students went through in the period of implementation of model-eliciting activities with their natural developmental stages of mathematical constructs throughout their educational lives. The research was conducted with three groups of middle school students who enrolled the remedial mathematics classes due to their poor grades in their previous educational backgrounds. Three mathematical modeling activities were used in the study and each group worked on only one of them for around 90 minutes. Their works recorded with video cameras and data sources were transcripts obtained from these video records. The results of the study showed that students constructed mathematical models that include more complicated mathematical ideas about the real life situations that they dealt with in modeling activities by using their previous knowledge and experiences. These constructs were more complicated than their previous ones that were included in traditional teaching materials such as textbooks and tests. The findings revealed that students who were weak according to traditional education system developed more powerful constructs than those who were successful in traditional system. In addition, researchers reported that students demonstrated similar conceptual development in modeling cycles as part of model-eliciting activities with developmental stages that students passed through while constructing mathematical ideas that observed by developmental psychologists (Lesh & Kaput, 1988, cited in Lesh & Harel, 2003).

Busse (2011) carried out a research on upper secondary school students' comprehension of modeling activities that include real life contexts. The researcher selected four pairs of students who came from four distinct upper secondary schools. Three modeling tasks assigned to each pairs of students to solve them and their solution processes recorded with video cameras. After the solution, each student from pairs watched the recorded tapes with researcher such that researcher used playback interruptions in order to give more time to students to tell his/her considerations about the modeling tasks. After this procedure, researcher made interviews with students in order to obtain more information about the modeling processes and real life contexts. The findings of the study demonstrated that students' individual perspectives play significant role when interpreting the real life situation of a modeling task. The researcher suggested that teachers ought to notice that each student might have different conception about the real life context of a modeling activity.

To sum up literature on the use of modeling activities in classroom settings, various studies on distinct levels including primary, middle, and secondary demonstrated that the use of mathematical modeling in educational environment provide students to develop their cognitive skills such as interpretation of modeling activities, specifying the goals that asked to reach, constructing models, testing, revising, and refining of these models (English, 2004; Lesh & Doerr, 2003a, 2003b) according to given feedback from their friends; creating more powerful mathematical constructs that provide students local conceptual development that is similar to their natural developmental stages that they passed through suggested by developmental psychologists (Lesh & Harel, 2003), and having different thoughts on the same real life context of modeling activities (Busse, 2011).

2.2 What Knowledge Teachers Should Have to Teach Mathematics?

In today's world, education has gained remarkable significance in natural sciences that are related to engineering, medicine, and technology. Nations who wants to become forward and pioneer in these areas need to grow qualified individuals. At this point, teachers and teacher education becomes vital for the growth of qualified students who will become qualified engineers, doctors, and technology developers in following years for a nation. Therefore, the subject of professional development of teachers is very significant. In the sense of mathematics, which is crucial for aforementioned areas of natural science, mathematics teachers' preparation and their professional development, is important for obtaining the needed knowledge in order to educate students according to national and international goals and increase their mathematical knowledge (Sowder, 2007) in at least other branches of natural sciences.

There have been many studies on the development of mathematics teachers in various dimensions such as what necessary knowledge needed for mathematics teachers (Fennema & Franke, 1992; Kahan, Cooper, & Bethea, 2003), mathematical content knowledge and pedagogical content knowledge of mathematics teachers (Shulman, 1986, 1987; Grossman, 1990; Marks, 1990), pre-service mathematics teacher education (Ball, 1990a, 1990b; Brown & Mayor, 1958; Chapman, 2007; Ensor, 2001; Manouchehri, 1997; Ponte & Chapman, 2008; Sowder, 2007), and so forth. In this sense, some of the professional institutions published standards and qualifications for mathematics teachers (CBMS, 2001; NCTM, 2000).

Nurturing knowledgeable teachers from teacher preparation and professional development programs has been one of the important goals in order to advance education in national and international level that is documented by several organizations such as NCTM and OECD. In *Principles and Standards for School Mathematics*, six principles were identified in order to carry out qualified mathematics education. One of the principles for school mathematics is the teaching principle stated as "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well." (NCTM, 2000, p. 16). To carry out effective mathematics teaching, teachers need to know mathematics and school mathematics and develop their understandings according to *Standards* expressed above. From this point of view, knowledge of prospective mathematics teachers and their professional development is one of the critical aspects of effective mathematics teaching that should be taken into account. What do mathematics teachers need to know in order to conduct effective mathematics teaching? How do mathematics teacher develop their knowledge? These significant questions should be discussed under previous studies about teachers' knowledge and their professional development.

Shulman (1986) proposed a categorization on teachers' knowledge, which are subject matter *content knowledge*, *pedagogical content knowledge* (PCK), and *curricular knowledge*. Subject matter or content knowledge includes knowledge of facts and concepts of a domain, understanding the structures of subject matter. Shulman expresses the importance of the subject matter knowledge for a teacher as

> The teacher need not only understand that something is so; the teacher must further understand why it is so, on what grounds its warrant can be asserted, and under what circumstances our belief in its justification can be weakened and even denied (p. 9).

PCK defined as "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (Shulman, 1987, p. 8) and the pedagogical knowledge that exceeds the subject matter knowledge such that it includes subject matter knowledge and its teaching (Shulman, 1986). PCK contains various themes such as the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations. "An understanding of what makes the learning of specific topics easy or difficult" (Shulman, 1986, p. 9), the conceptions and misconceptions of students about taught topics and lessons are also included in PCK. In the third category of teachers' knowledge is curricular knowledge. This knowledge type includes knowledge about curriculum that involves

> … the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances (Sulman, 1986, p. 10).

Shulman (1987) extended the classification of teacher knowledge, called as "*Knowledge Base*", and explained what constituted this knowledge base. According to Shulman, the following least categories need to be included in this base:

> – content knowledge;
>
> – general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;

> – curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers;
>
> – pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
>
> – knowledge of learners and their characteristics;
>
> – knowledge of educational context, ranging from the workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures; and
>
> – knowledge of educational ends, purposes, and values, and their philosophical and historical grounds (p. 8)

Teachers need the knowledge of the strategies in order to overcome misconceptions that stem from preconceptions. PCK is broadened and combined with four components which are (1) knowledge of students' understanding, (2) knowledge of curriculum, (3) knowledge of instructional strategies, and (4) purposes for teaching by Shulman (1987) and his colleagues (e.g. Grossman, 1990; Marks, 1990) (cited in Graeber & Tirosh, 2008). The knowledge of mathematical modeling is addressed as another component of PCK and it provides wisdom to teachers about how mathematics embedded in real life situations (Stacey, 2008).

Fennema and Franke (1992) contributed to research on teacher knowledge by suggesting a framework for the teacher knowledge that contains content (subject matter) knowledge, pedagogical knowledge, beliefs, knowledge of students' mathematical cognitions, and knowledge of the context. The context delineated as "the structure that defines the components of the knowledge and beliefs that come into play" (p. 162).

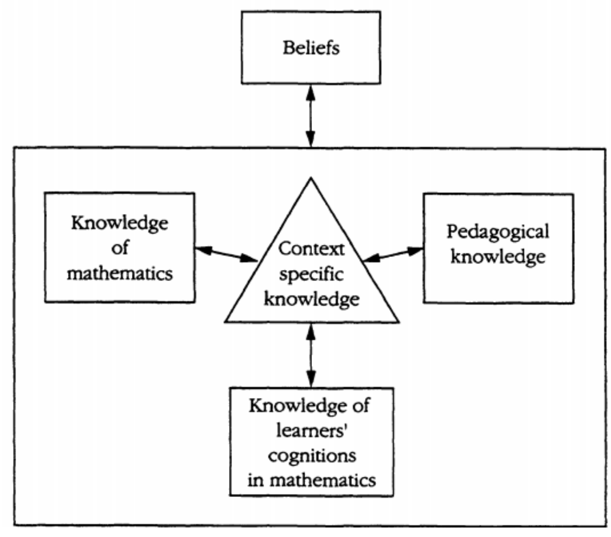


Figure 4 Teachers' knowledge model (Fennema & Franke, 1992, p. 162)

According to model in Figure 4, teachers' knowledge, that is interactive and dynamic according to context, involves knowledge of mathematics and pedagogical knowledge both included in Shulman (1987) as content knowledge and general pedagogical knowledge. Apart from Shulman (1987), context specific knowledge, knowledge of learners' cognitions in mathematics, and beliefs are contained in the model as components of teachers' knowledge (Fennema & Franke, 1992). The context specific knowledge means "teachers' knowledge and beliefs in context or as situated" (p. 162). The knowledge of learners' cognitions comprises the knowledge about the way of student thinking and learning of any mathematical concept within its specificity. Even and Tirosh (1995) extended the knowledge of learners as "*knowing that*" and "*knowing why*". *Knowing that* forms the fundamental level of content knowledge which "includes declarative knowledge of rules, algorithms, procedures and concepts related to specific mathematical topics in the school curriculum" (Even & Tirosh, 1995, p. 7). *Knowing why* is defined as "Knowledge which pertains to the underlying meaning and understanding of why things are the way they are, enables better pedagogical decisions" (Even & Tirosh, 1995, p. 9).

Lappan and Theule-Lubienski (1994) also offered a model for domains of teachers' knowledge with designated spheres for each domain. According to the model, teacher knowledge displayed with three spheres of knowing which formed intersections with each other. These three spheres are pedagogy of mathematics, students, and mathematics. Those spheres characterizes

> knowledge of the mathematics content; knowledge of students' cognition, knowledge of students' difficulties with concept domains, and how to motivate and facilitate learning; and finally knowledge of how to orchestrate pedagogy of mathematics that empowers learning and students involvement (Fi, 2003, p. 27).

The model suggested by Lappan and Theule-Lubienski (1994) is adapted from the theoretical model that Shulman (1986, 1987) proposed.

Another framework for teacher knowledge suggested by Ball and Bass (2003) according the interpretations of teachers' work. In this interpretation, researchers concentrated on the teachers'

> …representing and making mathematical ideas available to students; attending to, interpreting, and handling students' oral and written productions; giving and evaluating mathematical explanations and justification; and establishing and managing the discourse and collectivity of the class for mathematics learning (p. 6).

The framework comprises two integral parts, which are content (subject matter) knowledge and PCK. In this framework, subject matter knowledge involves “common content knowledge” which means the knowledge that one can know whose mathematical background good enough (Ball & Bass, 2003) and “specialized content knowledge” which is described as the mathematical knowledge that is utilized for teaching of mathematics, but this knowledge is not transferred to learners in a direct way (Hill, Sleep, Lewis, & Ball, 2007).

Another study on teacher knowledge was carried out by Borko et al. (2000) and described the teacher knowledge in domains. Researchers pointed out that domains of teacher knowledge are subject matter knowledge, general pedagogical knowledge, PCK, and knowledge of students proposed by Shulman (1986) and their perspective on the teacher knowledge has different categorization. Borko and others (2000) noted that subject matter knowledge considered as a distinct knowledge domain. The rest of the Shulman’s teachers’ knowledge categories included into a more expanded category called as “*mathematics-specific pedagogy*” (Borko et al., 2000). According to situative perspective, it is emphasized “… knowledge is inseparable from the physical and social context in which it develops and is used” (p. 197). This leads to the situated view such that mathematics-specific pedagogy involves three of the categories proposed by Shulman other than subject matter knowledge due to the reason that “… general pedagogical knowledge and knowledge about students, although theoretically distinct from pedagogical content knowledge, are inseparable in practice” (Borko et al., 2000, p. 197). Since teachers’ identities have influence on the teachers’ decision-making process and practices in classroom setting according to situative view on the cognition (Greeno, 1998; Greeno & MMAP, 1998, cited in Borko et al., 2000), the researchers specified the teachers’ professional identity as a domain of teacher knowledge.

2.3 Modeling Perspectives on Teacher Knowledge

In general, teacher knowledge and its characterization attracted many researchers due to the fact that teachers play key role in the teaching and learning situations. Many researchers proposed general theoretical models (e.g. Ball & Bass, 2003; Borko et al., 2000; Fennema & Franke, 1992; Lappan & Theule-Lubienski, 1994; Shulman, 1986, 1987) in order to explain the nature of teachers’ knowledge according to various perspectives in mathematics as well as other areas. Since modeling has emerged as an alternative approach to traditional teaching perspectives of mathematics (Lesh & Doerr, 2003a) with great emphasis on the relationship between mathematics and reality (Haines & Coruch, 2007), researchers proposed their own perspectives about the use of mathematical modeling in the teaching of mathematics. As indicated in the section 2.1.2, there have been many modeling approaches according to their description of mathematical modeling and their goals (Kaiser & Sriraman, 2006) and these approaches differentiated according to how do researchers see mathematical modeling, as a vehicle for teaching of mathematics (Gravemeijer, 2002; Gravemeijer & Stephan, 2002; Lesh & Doerr, 2003a, 2003b; Lesh & Lehrer, 2003; Lesh & Yoon, 2007; Zawojevski & Lesh, 2003) or as a subject matter need to be learned and taught (Lingefjärd, 2000; Lingefjärd, 2002a, 2002b; Lingefjard & Holmquist, 2001).

Although there have been many studies on the development of teacher education and teacher development programs which concentrated on supplying teachers in the curriculum implementation and pedagogical issues which centered on the learners' thinking and understanding (Fennema & Franke, 1992; Fennema, Franke, Carpenter, & Carey, 1993; Fennema, Carpenter, Franke, Levi, Jacobs, & Empson, 1996), "However, these programs for teachers' professional development have not been grounded in a similarly extensive research base on the nature of teachers' knowledge and its development" (Doerr & Lesh, 2003, p. 128), and it is still very new for the field. The studies on the cognitive science and situative perspective demonstrated that "knowledge is situated and grounded in the context and constraints of practice (e.g., Borko, Mayfield, Marion, Flexer, & Combo, 1997; Lave & Wenger, 1991; Leinhardt, 1990, and others)" (Doerr & Lesh, 2003, p. 128). The researchers described the nature of teachers' knowledge as *pluralistic*, *multidimensional*, *variable*, *contextual*, and *continual* (Doerr & Lesh, 2003). Knowledge is *pluralistic* due to the reason that many constructed models might be suitable for any similar given situations and any person might possess many thinking ways about any given real life situation; similarly many people might possess many thing ways about the given real life situation (Doerr & Lesh, 2003). Knowledge described as *multidimensional* such that

> Meanings, descriptions and explanations [which constituted models] evolve along a number of dimensions such as concrete-to-abstract, simple-to-complex, external-to-internal, sequential-to-simultaneous, discrete-to-continuous, particular-to-general, and static-to-dynamic (Doerr & Lesh, 2003, p. 129).

Teacher knowledge is *variable* due to the reason that any individual produced the knowledge from the given real life situation and that knowledge is subjected to change along the dimensions (such as concrete-to-abstract, simple to complex, etc.) and according to individuals (Doerr & Lesh, 2003). Knowledge of teachers' is said to be *contextual* because many researchers revealed that "a students' ability to use a given conceptual model often differs a great deal from one situation to another, depending on a variety of contextual factors and student characteristics (Carraher, Carraher, & Schliemann, 1985; Lave, 1988)" (p. 129). Knowledge is said to be *continual* because of the reason that knowledge is subjected to change and develop over time due to conceptual development with "acquisition of some general, all-purpose cognitive structure" (pp. 129-130).

Doerr and Lesh (2003) indicated that modeling perspective focus on teachers' actual knowledge to use them in expressing, testing, revising, and refining their subject matter knowledge and extending that knowledge to stronger and more useful artifacts for teaching in classroom settings rather than researching and detecting deficiencies in teachers' subject matter knowledge and suggesting ways to improving these deficiencies as many studies has been centered on recent decades. Doerr and Lesh (2003) described the teachers' knowledge according to modeling perspective as follows:

> The knowledge that teachers need consists of at least the mathematical understanding of the idea, an understanding of how children's thinking might develop, and a knowledge of pedagogical strategies in relationship to both the mathematical development and psychological development (Doerr & Lesh, 2003, p. 131).

It was emphasized in the above excerpt that teachers' knowledge need to include mathematical comprehension of mathematical conception, a comprehension of the ways of students' thinking improve, and pedagogical knowledge which involves both mathematical and psychological aspects of development (Doerr & Lesh, 2003).

There has been a great emphasis on the knowledge that teachers need in the teaching of mathematics through mathematical modeling (Blum et al., 2003; Niss et al., 2007). Although teachers' knowledge about using mathematical modeling underlined, there have been existed very few studies in the related literature (Oliveira & Barbosa, 2009, 2010). Many researchers put emphasis on the teachers' role in the mathematical modeling (Blum & Leiß, 2007; Doerr & Lesh, 2003, Schorr & Lesh, 2003). Several studies were conducted about the knowledge that teachers need in the teaching of mathematics implementation via mathematical modeling (e.g. Burkhardt, 2006; Doerr, 2006, 2007; Doerr & English, 2006; Lingefjärd & Meier, 2010; Ng, 2010; Stillman, 2010; Zawojewski et al., 2003). In the modeling process, all of the domains of teachers' knowledge proposed by several researchers (e.g., Grossman, 1990; Marks, 1990; Shulman, 1986, 1987) involved with the knowledge of mathematical modeling suggested as a component of PCK (Stacey, 2008). Apart from teachers' knowledge categories, teachers' role played in the modeling process described as modeling pedagogy (Blum et al., 2003; Niss et al., 2007). Niss and his collegues (2007) explained the modeling pedagogy as "The pedagogy of applications and modelling intersects with the general pedagogy of mathematics instruction in many respects, but simultaneously involves a range of practices that are not part of the traditional mathematics classroom" (p. 21).

Researchers found evidence that mathematical modeling has an influence on the development of mathematical knowledge of teachers' such that it extends teachers' mathematical knowledge, improves teachers' skills in solving applied problems, and affect their beliefs about mathematics and its teaching (Barbosa, 2007; Holmquist & Lingefjärd, 2003).

In general, several questions arise about the acquisition of the knowledge that teachers need to have, but the main question is the following: How do teachers acquire the knowledge that they need to possess in order to carry out modeling process effectively and successfully? At this point, development of teachers' knowledge in mathematical modeling in terms of professional development and teacher education become more apparent for replying this question. In this study, knowledge that teachers need to use mathematical modeling in the classroom setting is investigated specifically.

2.3.1 What Knowledge Do Teachers Need to Have for Teaching Mathematics through Mathematical Modeling?

Many studies underlined the importance of teachers' role in the modeling process (e.g. English & Watters, 2005; Niss, 1988; Zawojewski et al., 2003; Zbiek, 1998). Teachers' pedagogical knowledge of modeling becomes significant in the process of mathematical modeling so that teachers become in a central position during the implementation of mathematical modeling activities although mathematical modeling has not covered the significant place in mathematics teacher education (Antonius et al., 2007; Niss, 1988, cited in Zbiek, 1998). Several studies pointed out that mathematical modeling have become an integral part of school curricula in many countries (Burkhardt, 2006; Lingefjärd, 2002a, 2007; Schukajlow et al., 2012; Stillman, 2010). Zbiek (1998) indicated the main rationale for involvement of mathematical modeling in school curriculum as "to encourage deeper student understandings of mathematics through developing connections between mathematics and the real world" (p. 185). Since teachers are the implementers of school curricula, the knowledge of teachers about and their professional developments on mathematical modeling becomes significant in order to carry out modeling process in their classrooms. Yet, Lesh, Yoon, and Zawojewski (2007) cautioned teachers about effectiveness of mathematical modeling activities that rests on when to use these activities. According to Lesh and others (2007), using MEAs in the beginning of the lesson serves an aim that supporting students develop their own understanding of any subject matter or mathematical concept. On the other hand, using MEAs after the lesson meant they were application of what they were taught (Lesh et al., 2007; Yoon, Dreyfus, & Thomas, 2010). Therefore, teachers' knowledge is important for implementation of mathematical modeling activities effectively. There are several studies on teachers' implementation of modeling tasks in schools (Doerr, 2006, 2007; Lingefjärd & Meier, 2010; Ng, 2010; Stillman, 2010; Aydoğan-Yenmez, 2012; Şen-Zeytun, 2013) and professional developments of teachers on mathematical modeling (Maaβ & Gurlitt, 2009, 2011; Schorr & Lesh, 2003). The findings of these studies indicated that teachers needed to have particular knowledge about the use of mathematical modeling in order to implement modeling process successfully and efficiently.

Teachers orchestrate what is going on in lessons. In traditional educational system, lessons are teacher-centered and direct teaching methods are used in teaching of mathematics like other branches. In modeling process, the roles of teachers have been subject to change into more of an organizer, facilitator, and complex than that is in traditional system. With the advent of mathematical modeling as part of school curricula, knowledge of teachers has been subject to a debate in modeling literature (Burkhardt, 2006; Doerr, 2007).

In his study, Burkhardt (2006) examined the development of modeling approach in the learning and teaching of mathematics by describing the developmental stages in terms of fundamental features attributed to mathematical modeling. The teaching of mathematical modeling is emphasized such that teachers of mathematics should possess distinct teaching styles and strategies rather than what a traditional teachers have and do. The author pointed out that teachers need more abilities to teach their students mathematics (via) modeling. According to the author, the extra skills are "handling discussion in the class in a *non-directive* but supportive way, giving students time and confidence, providing strategic guidance, and finding supplementary questions" (Burkhardt, 2006, p. 188). The researcher implied that mathematical modeling process includes active engagement, distinct tasks, responsibility of students for their solution procedure, discussions occurred in the process, and qualified teachers in terms of modeling activities. Burkhardt (2006) stated that the qualified teachers' features in modeling activities are managing discussions in the classroom occurred during the modeling process by providing support and assistance, providing students appropriate time and encouragement to implement their solution approaches, guiding and supporting students in strategic way that not interfering their solution procedure and giving more detailed hints, and questioning students in order to improve them.

Burkhardt (2006) mentioned that mathematics education community acknowledged "the need for students to learn to model with mathematics" (p. 189) and many suggestions appeared in order to implement integration of modeling in school curriculum. The researcher pointed out to the obstacles that prevent the implementation of modeling in broad sense. These are existing and previous teaching systems' effects such as habits of implementation of practices, core beliefs of teachers, students, and even society, teaching strategies etc., messiness of real world, restricted professional development, ineffectiveness of studies on the development of education, and so on. It addresses the need that "a firmer research base and good presentation, in general and in the context of each particular modeling course" (p. 191) in order to convince teachers, students, policy makers to integrate mathematical modeling in school curriculum.

In the study of Antonius et al. (2007), pedagogical issues in the use of mathematical modeling activities were identified. The researchers stated the ways of using modeling activities (e.g. working individually or working in-group) providing the advantages and drawbacks of group work. The researchers emphasized the role of the teacher during the implementation of modeling activities. It is underlined that teachers' guidance need to be centered on strategic questions including *more metacognitive prompts* (e.g. "What did you find", "What are you going to try next", etc.), *some prompts focused on specific strategies* (e.g. "Have you looked at some specific cases?", "Did you see any patterns that you recognize?", etc.), and *little detailed guidance* (e.g. Why don't you try a linear fit?", "That's wrong", etc.) (Shell Centre, 1984, cited in Antonius et al., 2007, p. 301). The researchers suggested principles adapted from Steen and Forman (2001) for effective teachers to have pedagogical knowledge of modeling during the implementation that are *active*, *student-centered*, and *contextual* (cited in Antonius et al., 2007). The researchers indicated the question "How much guidance?" as the fundamental of the teaching of mathematical modeling and that depend on the teachers' approaches. The researchers suggested that teachers ought to have "a deep understanding of many different kinds of subject matter that allows them to predict the possible obstacles and outcomes of different paths students may follow" (Antonius et al., 2007, p. 308). The pedagogical knowledge of mathematical modeling described as "a mathematics teacher competence" which could be gained not only in teacher preparation programs, but also in it needs to be acquired and improved in in-service period of teaching. The researchers strongly recommended that pedagogical knowledge of modeling should be seeded in pre-service teacher preparation programs.

Similar issues were voiced by Wake (2011) in a study where teachers' professional learning in mathematical modeling was investigated. The participants of the study were five teachers and two mathematics specialists who formed a teacher development group. Teachers conducted a series of lessons in mathematical modeling within study. The results revealed the importance of changing roles of teachers during the implementation of modeling activities. The findings showed that teachers need professional learning in order to broaden their background about both subject matter knowledge and pedagogy that is particular to mathematical modeling.

Doerr (2007) put emphasis on the nature of teachers' knowledge about implementing modeling processes in terms of teaching mathematics. The teachers' knowledge about teaching mathematics via modeling approach includes creating, selecting, and conducting modeling tasks, interpretation of the tasks, models that generated by teachers for teaching (Doerr & Lesh, 2003), and the way of using these models in teaching mathematics. Doerr conducted two studies on subject matter knowledge of pre-service mathematics teachers and pedagogical knowledge of an in-service secondary mathematics teacher. In the first study, the researcher designed a modeling course for undergraduate pre-service teachers, which aimed to introduce fundamentals of mathematical modeling by involving them in modeling process.

Modeling tasks from distinct domains such as physics, biology, and mathematics constituted the content of the course. Pre-service teachers studied in small groups and five modeling tasks were accomplished throughout the semester. Data sources of the study were pre-service teachers' group works in classroom, classroom discussions, and assignments. The results of the study demonstrated that pre-service teachers have serious misconception about binomial distributions and probabilities of independent events (Doerr, 2007). The researcher stated that pre-service teachers corrected their misconception by the process of group discussion and providing reasonable explanation to each other. This situation shows that "mathematical modeling is a potentially powerful context for the mathematics learning of pre-service teachers." (Doerr, 2007, p. 72). Another finding is the perceptions and beliefs of pre-service teachers about the modeling process changed throughout the semester as a result of the development in their experiences.

In the second study, the researcher investigated pedagogical knowledge of an experienced in-service secondary teacher through the implementation of a sequence of modeling activities on the subject of exponential functions. The results showed that teachers should possess extensive and profound understanding of distinct perspectives that students could consider. The researcher explained the four characteristics of the pedagogical knowledge of teachers as "(1) to be able to listen for anticipated ambiguities, (2) to offer useful representations of student ideas, (3) to hear unexpected approaches, and (4) to support students in making connections to other representations" (p. 77). Doerr (2007) noted that researchers should explore how teachers obtain pedagogical knowledge for modeling in their undergraduate or in-service practice.

In the study of Stillman (2010), the conditions for conducting applications and mathematical modeling successfully in secondary school were investigated which is based on Singapore syllabus in two Australian States, namely Queensland and Victoria. The study includes results of small part of very large projects. Stillman examined the several conditions for carrying out applications related to real world and modeling tasks in the secondary school classroom efficiently. These are conditions associated with tasks, students, and teachers in the process of modeling. Stillman described the student condition as "developing understanding of situation in groups", "using physical activities related to the task to develop domain knowledge", and "participating in rich dialogue and discussion with peers and the teacher" (pp. 314-315).

Stillman (2010) expressed the teacher conditions as (i) being aware of the time to cut in; (ii) hoping students participation of modeling process positively; (iii) having knowledge of the nature of tasks, welcoming students' distinct developments; (iv) and summarizing and completing the task. Stillman (2010) claimed that teachers could develop themselves by experiencing to conduct more applications of modeling activities on their own and sharing knowledge and experiences with their colleagues. These assertions need to be proven by future studies.

Doerr (2006) carried out a research on the way of teachers' listening to students' ideas in the process of modeling and the types of teachers' responses to students' developing models in order to foster their emerging models. The researcher worked with four experienced secondary teachers who implemented a modeling task about exponential growth in a case study. The researcher attended the lessons as a non-participant observer and took field notes, and all implementations of four teachers recorded with videotapes. The results of the study showed that teachers have various approaches in listening to students' thinking during the modeling process ranging from refraining from direct expressing the next steps to students to listening to students' distinct ways of solutions, but not allowing enough time students to develop their ideas. This result displayed that sophistication of the knowledge domain in the teaching expertise. Doerr suggested that "The goal for the teacher is not to classify that thinking as an end in itself, but rather to have a broad schema for ways that students might think about a task in order to provide the students with conflicts that need to be resolved or alternatives that can be tested" (p. 267).

In another study, Lingefjärd and Meier (2010) focused on the teachers' role in the modeling process. The sample of the study consists of two-experienced teacher, one from Sweden and the other was from Germany as a part of large project. In the study, each teacher implemented a modeling task in his or her own classrooms. Lessons were observed by researchers as non-participant observer. The data analyzed according to frame analysis of Goffmann (1974/1986) and concept of teacher intervention model of Leiβ (2007) (cited in Lingefjärd & Meier, 2010). The results of the study demonstrated that teachers used diagnostic questions in the modeling process in order to support students' thinking and provided content-related help for students in understanding and interpreting the problems situation. Another finding was that teachers made affective intervention to students even if students can develop a solution by themselves. Lingefjärd & Meier suggested that teachers need more knowledge, practice, and supervision about what to do and how to move in particular conditions of modeling process.

In the study of Ng (2010), first practice of primary school teachers in mathematical modeling is reported. Teachers engaged in three mathematical modeling tasks actively by forming groups that include four teachers in each. Peer evaluation approach was used in the assessment that is based on three principles: the way of representation, validity of developed models, and reusability of these models in other similar situations that are denoted as main properties of modeling process (Kaiser, 2005, cited in Ng, 2010). There are several findings of that teachers were uncomfortable against the open-ended property of modeling tasks even though teachers were receptive to modeling activities in real world context. Teachers showed that they preferred to get unique answers from the solution process. Ng suggested that "teacher educators have to bridge the gaps between the potentials of such tasks and teachers' current levels of expertise (i.e. content and pedagogy), as well as raise their comfort levels to such open-ended contextualized tasks." (p. 142).

Several studies were conducted about prospective teachers (e.g., Şen-Zeytun, 2013) and about in-service teachers (Aydoğan-Yenmez, 2012) regarding to modeling perspective. In the study of Aydoğan-Yenmez (2012), the change in in-service secondary mathematics teachers' knowledge investigated through activities related to professional development on the basis of lesson study design by using modeling approach. The researcher examined the development of teachers' knowledge on questioning styles, assessment techniques, classroom arrangement, and management with the sample consisting four experienced secondary mathematics teachers. The findings of the study indicated that the professional development program applied in the study had an influence on the teachers' PCK and pedagogical knowledge involving instructional practice with student-centered and classroom arrangement and management.

Şen-Zeytun (2013) investigated prospective teachers' ways of model development through involving in modeling activities and their views about the causes that affect their modeling process. Six prospective teachers participated in the study that was designed as case study. The results of the research demonstrated that various factors such as educational system based on exams, inadequate practice with modeling activities, time limitations etc. had influence on prospective teachers' modeling process.

To sum up, many studies put emphasis on the importance of teachers' position in the modeling process (Niss, 1988; Zawojewski et al., 2003; Zbiek, 1998). Although teachers encounter many obstacles that may deter them from using modeling activities in the classrooms (Blum & Niss, 1991; Burkhardt, 2006; Ng, 2010; Şen-Zeytun, 2013), teachers need to have particular knowledge about the use of mathematical modeling (i.e., pedagogy of modeling) to carry out mathematical modeling activities and modeling process as a whole effectively (Antonius et al., 2007; Aydoğan-Yenmez, 2012; Doerr, 2006, 2007; Lingefjärd & Meier, 2010; Stillman, 2010; Wake, 2011).

2.4 Teachers' Beliefs and Conceptions about Mathematics and Its Teaching

Since teachers occupy a very significant place in educational society due to having direct connection with learners, especially with students from kindergarden to higher educational levels in terms of teaching, guiding and changing of them. Teachers also have been seen as "important agents of change in the reform effort current under way in education and thus are expected to play a key role in changing schools and classrooms" (Prawat, 1992, p. 354). Due to the reason that teachers play a central role in the teaching and learning, their worldviews and considerations about teaching and learning become crucial in order to support the changing efforts in education and development of both students and themselves.

Many constructs influence the teachers' worldviews and considerations about any subject matter or teaching and learning of it. Teachers' beliefs and conceptions are the constructs that affect their applications in the educational context (Pajares, 1992). There are many definitions of beliefs according to distinct researchers (e.g., Pehkonen & Törner, 1996; Richardson, 1996; Thompson, 1992). The more common and agreed definition of beliefs is "Psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259; Richardson, 1996, p. 103; Richardson, 2003, p. 2). Beliefs also defined from another perspective as "an individual's understanding and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior" (Schoenfeld, 1992, p. 358). Schoenfeld (1985) described belief systems as "belief systems are one's mathematical world view" (p. 44). Green (1971, cited in Thompson, 1992) identified several aspects of beliefs involved in belief systems, which is "a metaphor for examining and describing how an individual's beliefs are organized" (p. 130). According to the belief systems, there exists three aspects in this system. In the first category, some beliefs are more basic in comparison with others. These basic beliefs called as *primary beliefs*. The other beliefs are said to be *derivative beliefs* that are derived from primary beliefs. Another aspect of beliefs system contains some beliefs are more resistant to change which are *central* and others are not strong as *central* ones that are *peripheral*. The beliefs that are said to be *peripheral* more subject to change rather than *central* ones. The last aspect of belief systems is that beliefs are contained in *clusters* so that there is no association between other belief families (Thompson, 1992).

In recent decades, there have been many studies on the teachers' beliefs about mathematics and its teaching and learning (e.g., Beswick, 2005, 2007, 2012; Cooney, Shealy, Arvold, 1998; Ernest, 1989; McLeod & McLeod, 2002; Perry, Howard, & Tracey, 1999; Peterson, Fennema, Carpenter, & Loef, 1989; Roesken, Pepin, & Törner, 2011; Shilling-Traina & Stylianides, 2012), and the effects of teachers' beliefs on teachers' practice in the classroom setting (e.g., Beswick, 2005; Cross, 2009; Dougherty, 1990; Raymond, 1997; Speer, 2005; Wilkins, 2008). Many researchers stated that teachers' beliefs about mathematics and its teaching and learning have an impact on forming teachers' considerations about instructional practice (Artzt, 1999; Dougherty, 1990; Pajares, 1992; Richardson, 1996; Thompson, 1984; Wilkins, 2008). Ernest (1989) pointed out that "The teacher's mental contents or schemas, particularly the system of beliefs concerning mathematics and its teaching and learning" (p.1) identified as one of the factors that affect teachers' practice of teaching of mathematics. Beswick (2005) crystallized the classification of teachers' beliefs about the mathematics, its teaching and learning by utilizing the propositions that are suggested by Ernest (1989) and Van Zoest, Jones, and Thornton (1994) (see Table 3).

Table 3 Classification of Teachers' Beliefs (Beswick, 2005, p.40)

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989)
Instrumentalist	Content focussed with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content focussed with an emphasis on understanding	Active construction of understanding
Problem solving	Learner focused	Autonomous exploration of own interests

In the first category, Ernest (1989) illustrated teachers' beliefs about the nature of mathematics in terms of three perspectives, which are instrumentalist, Platonist, and problem solving. According to instrumentalist perspective, mathematics seen as "an accumulation of facts, skills and rules to be used in the pursuance of some external end." (p. 250). This perspective of mathematics suggests that distinct kinds of topics that contained in mathematics are diverse. Platonist perspective described the mathematics as "a static body of unified, preexisting knowledge awaiting discovery" (Beswick, 2012, p.129). According to this perspective, there is a significant relationship between mathematical knowledge structures and various subjects. The problem solving perspective suggests that mathematics is a kind of dynamic process that is subject to human creation and discovery. This perspective views mathematics as a process instead of product (Ernest, 1989).

Conceptions are more general constructs than beliefs. According to Thompson (1992), teachers' conceptions are "a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences, and the like" (p. 130). Teachers' conceptions of mathematics involve "teachers' conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics" (Thompson, 1992, p. 132). Thompson's (1992) explanations of teachers' conceptions about the nature of mathematics, teachers' conceptions about teaching and learning of mathematics also fitted into the classification of teachers' beliefs suggested by several authors (Ernest, 1989; Van Zoest et al., 1994) that was summarized by Beswick (2005). Thomson (1992) pointed out the perspectives for conceptions of the nature of mathematics stated by Lerman (1983, cited in Thompson, 1992). In the first perspective, *absolutist*, emphasized "universal, absolute, certain, value-free" nature of mathematics. In the second perspective, *fallibilist*, advocated the "developing through conjectures, proofs, and refutations" nature of mathematics (Thompson, 1992, p. 132). Several studies reported that there exist consistencies between teachers' conceptions about mathematics and their instructional practice (Agudelo-Valderrama, Clarke, & Bishop, 2007; Dougherty, 1990; Kaplan, 1991; Thompson, 1984) and some researchers indicated variability and inconsistencies between teachers' conceptions about mathematics, its teaching and learning and their instructional practice (Andrews & Hatch, 1999; Pepin, 1999; Raymond, 1997).

Although there existed attempts to define psychological constructs such as beliefs, conceptions, views, perceptions etc., when it is looked through the related literature, there is no common ground and clear cut distinctions between the terms *beliefs, conceptions,* and *views* and these terms have been used interchangeably (Pehkohnen & Törner, 1996; Shilling-Traina & Stylianides, 2012; Törner, 2002). It is reported that teachers' beliefs about mathematics and about mathematics teaching and learning are accepted as significant indicators for comprehending teachers' instructional practices in classroom settings (Pajares, 1992; Skott, 2001). The relationship between teachers' beliefs and their instructional practice emphasized in many studies (e.g., Ball, 1990a, 1990b; Thompson, 1992). Many researchers documented the relationship between teacher change and beliefs according to various levels from elementary to high schools (e.g., Borasi et al., 1999; Even, 1999; Even & Tirosh, 1995; Lloyd, 1999; Vacc & Bright, 1999; Wood & Sellers, 1997).

It is indicated that the significance of beliefs, as a form of cognition, stressed in the teacher education due to reasons that include philosophical and psychological dimensions (Richardson, 2003). According to philosophical aspect, beliefs are regarded as the center of change in the teacher preparation programs. In the second dimension, prospective teachers carry their beliefs, especially strong ones that are some of them said to be central beliefs about mathematics teaching, to teacher preparation programs (Ball, 1988, 1990a; Borko et al., 1992; Lampert & Ball, 1998; Richardson, 2003) and these conceptions affect how they learn to teach from these programs (Calderhead & Robson, 1991; Richardson, 1996). It is evidenced from the study of Ross, Johnson and Smith (1991) that pre-service teachers' entering beliefs found to be the most significant factor that influence what and how they learn in teacher preparation program and also influence the way that they teach in classroom settings (Richardson, 2003). Pre-service teacher programs suggest prospective teachers "unique opportunities between the pre-service teacher's school experience and future teaching practice to pause and reconsider their affective dispositions towards mathematics teaching and learning" (Grootenboer, 2003, p. 42). It is pointed that pre-service teachers' beliefs are resistant to change at the teacher preparation programs due to the several reasons (Ball, 1990a) and researchers documented that some conceptions of in-service mathematics teachers are strong and resistant to change (Wilson & Goldenberg, 1998), another finding is that prospective teachers' conceptions about mathematics did not change in the period that they conducted a teacher preparation course (Foss & Kleinsasser, 1996).

Since teachers' beliefs about mathematics and its learning affects their practices, teachers' beliefs about mathematical modeling and its use in teaching mathematics have the potential to influence their practices about modeling (Chapman, 2007; Doerr & Lesh, 2003; Kuntze, 2011; Maaβ & Gurlith, 2011).

2.4.1 Teachers' Conceptions about Mathematical Modeling and Its Use in Teaching of Mathematics

There exist many studies about the teachers' conceptions about mathematical modeling (e.g., Frejd, 2012; Gould, 2013; Kuntze, 2011; Maaβ, 2011; Verschaffel, De Courte, & Borghart, 1997; Ärlebäck, 2010), about modeling activities (Eraslan, 2011; Kayhan-Altay, Yetkin-Özdemir, & Şengil-Akar, 2014; Kuntze, 2011), and about teaching and learning of mathematics via mathematical modeling (e.g., Bisognin & Bisognin, 2012; Chapman, 2007; Kaiser & Maaβ, 2007; Kuntze et al., 2013; Maaβ & Gurlith, 2011; Schmidt, 2011; Siller, Kuntze, Lerman, & Vogl, 2011; Yu & Chang, 2011).

In the study of Verschaffel and others (1997), prospective teachers' conceptions and beliefs about the role of real world knowledge in mathematical modeling involved in traditional word problems investigated. A large number of prospective teachers from four different teacher preparation programs participated in the study. Prospective teachers administered a test including 14 word problems (seven of them include real life context). Prospective teachers solved these problems on their own and then they made evaluation about four distinct responses from students for each word problem. The results of the research demonstrated that prospective teachers have an inclination to keep out the real life knowledge from their solutions and they did so for the students' solutions that they were assigned to make evaluations.

Chapman (2007) examined six experienced in-service mathematics teachers' conceptions about mathematical modeling and their practices through mathematical modeling in high school. The researcher reported the findings from data of a large research and concentrated on the high school teachers and their teaching of mathematics via mathematical modeling. Data obtained from a variety of sources including open-ended interviews, classroom observations, classroom discussions, and role-play scenarios. As a result of the study, the researcher indicated that teachers emphasized the significance of real world connection in their conceptions about mathematics, word problems and problem solving and they pointed out the mathematical modeling as a necessity in teaching and learning of mathematics.

Kuntze (2011) explored the views of mathematics teachers (including both of in-service and pre-service) about mathematical modeling activities in the classroom settings. In the study, 230 prospective and 79 in-service teachers participated and they were asked their views about instructional practices, containing their views about mathematical modeling and about properties of distinct mathematical modeling activities concerning to the degree of their demands. The participants were asked to complete a questionnaire on characteristics of modeling tasks (i.e., demanding high or low requirements) and their global views. The results of the study demonstrated that pre-service teachers chose the modeling activities with low demands rather than ones that are more demanding. In contrast to pre-service teachers' preferences, in-service teachers selected more complicated and demanding modeling activities. Although prospective teachers showed higher fear of incompatibility of modeling tasks with the aim of mathematical exactness than in-service teachers. The findings showed that prospective teachers stressed that prospective teachers need to work more on the modeling tasks in mathematics teacher education programs.

Another study conducted by Gould (2013) investigated prospective secondary mathematics teachers' conceptions and misconceptions about mathematical models and modeling. It was used both quantitative and qualitative approaches in the study. The researcher collected quantitative data from 274 in-service and prospective secondary mathematics teachers by using online survey form and qualitative data collected from case studies of a group of mathematics teachers attended to a professional development course on mathematical modeling. The findings demonstrated that secondary mathematics teachers have some misconceptions about mathematical models and modeling such as not comprehending the process of mathematical modeling and its aspects; believing mathematical models to be formed of concrete manipulatives, visual representations; having beliefs about situations of mathematical modeling such that these situations might become unrealistic scenarios. The researcher indicated that teachers believed that to carry out mathematical modeling successfully, it ought to be implemented with the help of other teachers and teachers need to make collaboration for developing ideas and strategies. Gould (2013) mentioned that a majority of teachers pointed out the some issues related to mathematical modeling implementation such as taking much time.

Several researchers reported that teachers' conceptions about the use of mathematical modeling in the teaching and learning of mathematics did not show any significant change during and after the implementation of their treatments (Maaß & Gurlitt, 2009; Maaβ & Gurlitt, 2011) and teachers' mathematical beliefs found to be the main hinders to deter teachers from using mathematical modeling in classroom settings (Kaiser & Maaβ, 2007). Several studies indicated that there were several factors that influence teachers' conceptions about the use of mathematical modeling activities in their actual classrooms (Schmidt, 2011; Yu & Chang, 2011). For example, Schmidt (2011) found that lack of time and absence of assessment method for students' performance in mathematical modeling appeared as hinders preventing teachers from using mathematical modeling in their classrooms. Similar findings reported in the study of Yu and Chang (2011). Additionally, Yu and Chang (2011) indicated that teachers had both positive and negative perceptions about using mathematical modeling activities in classroom setting and designing such activities. The researchers reported that teachers identified making connections between mathematics and real life situations, increasing students' mathematical skills, developing their communication skills and sharing ideas through group work as the advantages of using modeling activities in classrooms. Yu and Chang (2011) also stated that teachers mentioned that mathematical modeling was not included in the curriculum and content of the entrance examinations and took much time as the drawbacks.

Siller and others (2011) explored the views of pre-service mathematics teachers about the importance of mathematical modeling in the classroom instruction. The sample of the study was 117 German and 42 Austrian pre-service teachers. A developed questionnaire within a large project was applied to participants. The results indicated that pre-service teachers viewed importance of mathematical modeling at average as an important big idea for classroom instructional strategy. However, pre-service teachers perceived mathematical modeling unimportant in comparison with other big ideas.

In the study of Kuntze and others (2013), pre-service and in-service teachers' perceptions about their PCK associated with modeling during the modeling process and views about their professional development at undergraduate level investigated in terms of mathematical modeling. The sample of the study involved 38 pre-service and

48 in-service teachers. A questionnaire was applied to both pre-service and in-teachers. The results revealed that both pre-service and intervice teachers had negative views about their modeling-specific pedagogical development. The results of the study indicated that teachers' self-perceptions about PCK associated with mathematical modeling displayed a necessity for professional development for both PCK associated with modeling and pedagogical modeling based self-efficacy of teachers. Another finding was that teachers' explanations of self-perceptions associated with modeling suggested that practical experience with mathematical modeling activities was underlined. The researchers point out that the results of the study stated the need for the further studies about structure of teachers' modeling knowledge.

Maaβ (2011) conducted a qualitative research on the effects of teachers' beliefs on their professional development within the modeling context. Six in-service secondary school teachers participated in professional development course and took part in the interviews after the implementation of the course. The results of the study showed that teachers identified three drawbacks of mathematical modeling, namely: (i) time is not enough for doing mathematical modeling, (ii) modeling is not included in external assessment, and (iii) students dislike modeling or are not able to solve modeling tasks. The analysis of data provided researcher to determine two opposite teacher types, the *Static Type* and the *Process Type*. Although teachers who were in the first category (the *Static Type*) seemed to avoid from using modeling in their lessons by thinking of drawbacks of modeling, teachers who were in the second category (the *Process Type*) seemed to integrate modeling in their lessons by developing strategies to overcome the obstacles. The findings suggest that teachers' beliefs have an influence on their actions whether trying to exclude from or involve in their mathematics lessons.

In the study of Frejd (2012), upper secondary school teachers' the ways of teaching mathematical modeling and their conceptions about mathematical models and modeling were examined. Eighteen teachers participated in the study from 12 distinct secondary schools. Teachers responded a questionnaire that aimed to collect data about teachers' thinking about mathematical modeling and characteristics of schools. Interviews were held with each of the participants after application of questionnaire. The results revealed that half of the teachers become aware of the concept of mathematical modeling before joining the study. Most of the teachers defined the mathematical modeling as describing or simplifying something with mathematics. The findings indicated that teachers' conceptions about mathematical modeling were associated with building mathematical model founded on a situation. Moreover, the results demonstrated teachers' experiences of the concept of mathematical modeling were inadequate. Teachers did not pay more importance to include modeling in their everyday mathematics lessons.

Ärlebäck (2010) studied teachers' beliefs about mathematical models and modeling. The author tried to apprehend teachers' beliefs related to mathematical models and modeling within a framework that includes teachers' beliefs about the real life, the nature of mathematics, school mathematics, and implementing and implementations of mathematics. The researchers carried out a case study with two teachers. The findings suggest that teachers did not have an apparent mathematical models and modeling conception. The researcher found that teachers participated in the study did not possess any well-constructed beliefs about mathematical models and modeling and there were discrepancies between these beliefs.

In the context of Turkey, several researchers conducted studies about prospective teachers' conceptions about mathematical modeling and modeling activities in the teaching and learning of mathematics. Eraslan (2011) conducted a study on prospective teachers' perceptions on MEAs and their influences on learning of mathematics. The researcher selected six prospective mathematics teachers from 45 prospective teachers who were taking "Modeling in Teaching Mathematics" course and these prospective teachers were from two focus groups in the implementation of MEA. These groups of prospective teachers were interviewed and videotaped. The findings of the study showed that prospective teachers indicated the ambiguity of MEAs as a general character of these activities. Prospective teachers thought that MEAs have positive influences on the learning of mathematics. The findings suggest that prospective teachers stated important ideas about the use of MEAs such as using MEAs with participation of all class or small groups rather than individually because these activities require higher thinking skills. In antoher study, Türker, Sağlam, and Umay (2010) explored prospective mathematics teachers' performances in the modeling process and their views of modeling process throughout implementation of four modeling activities. Sixty prospective teachers were participated in the study. Data were collected through four modeling activities and semi-structured interviews. The results of the study demonstrated that prospective teachers stated that they did not experience modeling activities before and they identified that these activities demanded higher thinking skills. In the process, the researchers indicated that prospective teachers developed positive ideas about mathematical modeling in relation with developing solutions to real life problems. Another finding of the research was that prospective teachers declared that the lack of courses that they could progress modeling skills in teacher preparation programs.

Kayhan-Altay and others (2014) studied prospective elementary teachers' views of MEAs and the nature of these activities in the teaching and learning mathematics throughout "Modeling in Teaching Mathematics Course" that was offered as an elective course at a public university in Ankara, Turkey. Prospective elementary teachers who enrolled the course were the participants of the study. The data sources of the study were the documents of four MEAs collected during the implementation of the course, semi-structured focus group interview, and field notes. The results of the study indicated that prospective elementary teachers developed positive ideas about MEAs although they did not have any knowledge about MEAs and modeling process before the implementation of the course. Similar to the findings of Türker et al. (2010), prospective elementary teachers stated that there was an absolute requirement for a course that they could progress their modeling skills in the initial teacher preparation programs. The findings of the study showed that prospective teachers thought that teaching and learning mathematics via MEAs was pleasant and meaningful because these activities demand of students to relate mathematical concepts with real life situations. Prospective teachers also indicated that the most suitable method for using MEAs was group working due to observing more solution strategies. Even though prospective teachers indicated positive feelings about the use of MEAs in teaching and learning of mathematics, they mentioned about possible difficulties that may deter them from using these activities in their future classrooms. Kayhan-Altay and others (2014) underlined that the development of modeling skills takes more time and therefore, prospective teachers need to have more experience.

To summarize, several studies demonstrated that both prospective and in-service teachers had various conceptions about mathematical models and modeling (Frejd, 2012; Gould, 2013; Ärlebäck, 2010), about modeling activities (Eraslan, 2011; Kayhan-Altay et al., 2014; Kuntze, 2011; Türker et al., 2010), about the use of mathematical modeling in the teaching and learning of mathematics (Chapman, 2007; Eraslan, 2011; Gould, 2013; Kuntze et al., 2013; Maaβ, 2011; Schmidt, 2011; Siller et al., 2011; Yu & Chang, 2011). The findings of these studies revealed that prospective teachers developed positive conceptions about mathematical modeling and modeling activities and they pointed out the obstacles that may hinder the use of these activities in the classrooms. However, pedagogical side of mathematical modeling and teachers' conceptions about pedagogy of modeling is still scarce.

2.5 Initial Preparation and Professional Development of Mathematics Teachers

Teachers have been playing a key role in almost every level of education and reform efforts in the educational system, and the significance of their role in the classroom emphasized in many studies (e.g. Franke, Kazemi, & Battey, 2007; Sowder, 2007). Since the significance of their roles, preparation and professional developments of teachers becomes crucial for acquisition of their needed knowledge for teaching of mathematics (Opfer & Pedder, 2011; Sowder, 2007) and for improvement of education (Borko, 2004; Guskey, 2002). Sowder (2007) pointed out that many researchers stated differences in the descriptions of the terms teacher education, professional development, and teacher change. For instance, teacher education is linked to initial teacher preparation which involves courses and practice experience of prospective teachers, on the contrary, professional development of teachers is correlated with programs towards teachers who are currently teaching which includes attending projects, completing offered courses and their responsibilities (Lerman, 2001; Ponte, 2001, cited in Sowder, 2007). However, Sowder (2007) noted that this distinction is not clear due to the fact that "much of what is true for professional development is also true for teacher preparation" (p. 158).

Various studies addressed the significance of professional development of teachers and school administrators such that advancement in instruction and increase in students' success rely heavily on their professional development (Ball & Cohen, 1999; Elmore & Burney, 1999; Sykes, 1999; Thompson & Zeuli, 1999). The importance of professional development for advancement in education illustrated as a "significant lever for education improvement" (Sykes, 1999, p. 151). Sowder (2007) identified six general aims of professional development which are, namely (1) developing a shared vision, (2) developing mathematical content knowledge, (3) developing an understanding of how students think about and learn mathematics, (4) developing pedagogical content knowledge, (5) developing an understanding of the role of equity in school mathematics, and (6) developing a sense of self as a teacher of mathematics. Many studies were conducted on the professional development of teachers in line with these aims of professional development. For example, several organizations and institutions (e.g. NCTM, OECD, and NCATE) published several documents which identify teacher qualifications and standards, and criteria for professional development programs in order to maintain and develop a shared vision. The OECD (2009) published *Teaching and Learning International Survey* (TALIS) that gives a comparison of 30 distinct member countries in terms of their educational conditions for teaching and learning. In the TALIS, professional development of teachers analyzed with respect to attending to professional development and pointed out the importance of participating professional development activities. On the other hand, many studies carried out on the mathematical and PCK of teachers and teacher learning (e.g. Ball, 1990b; Ball & Mosenthal, 1990; Fuerborn, Chinn, & Morlan, 2009; Hawley &Valli, 1999) which are said to be consisted of some of the aims proposed by Sowder (2007). Studies based on teacher knowledge about learners' thinking (Fennema et al., 1993; 1996) can be considered as an example of the developing a comprehension of students' thinking and learning.

Sowder (2007) sorted out the professional development types according to framework for the types of teacher knowledge offered by Cochran-Smith and Lytle (1999). The framework consists of three categories, which are *knowledge-for-practice*, *knowledge-in-practice*, and *knowledge-of-practice*. According to Cochran-Smith and Lytle (1999), *knowledge-for-practice* means that "formal knowledge and theory (including codifications of the so-called wisdom of practice) for teachers to use in order to improve practice" that obtained from teacher preparation programs and courses taken in university level (p. 250). *Knowledge-in-practice* refers to "practical knowledge, or what very competent teachers know as it is embedded in practice and in teachers' reflections on practice" and *knowledge-of-practice* refers to

> the knowledge teachers need to teach well is generated when teachers treat their own classrooms and schools as sites for intentional investigation at the same time that they treat the knowledge and theory produced by others as generative material for interrogation and interpretation. In this sense, teachers learn when they generate local knowledge of practice by working within the contexts of inquiry communities to theorize and construct their work and to connect it to larger social, cultural, and political issues (Cochran-Smith & Lytle, 1999, p. 250).

Sowder (2007) stated the ways of acquiring knowledge-for-practice classified into four categories. These are perspectives that centered on student thinking, perspectives that centered on curriculum, perspectives that centered on case studies, and perspectives that centered on formal course work.The researcher pointed out that courses suggested in initial teacher preparation programs ought to be designed according to principles of professional development suggested by Loucks-Horsley, Love, Stiles, Mundry, and Hewson (2003). As Sowder (2007) identified formal course works in undergraduate and graduate levels in university within the ways of acquisition of *knowledge-for-practice*, initial teacher programs and their contents become more important for the quality of prospective teachers' professional development and establishing a powerful knowledge base for being effective teachers. Many studies questioned the influences of teacher education programs on teacher learning to teach (National Center for Research on Teacher Education, 1988; Wilson, Floden, & Ferrini-Mundy, 2002) and teachers' professional careers (DeAngelis, Wall, & Che, 2013), and many researchers criticized prospective teacher preparation programs for several reasons such as deficiencies in courses and connections between courses, forming standards and setting obvious aims (Feiman-Nemser, 2001; Zeichner, 2006).

When it is looked through the literature on prospective mathematics teacher education, most of the studies are about prospective teachers' beliefs and experiences about the nature of mathematics and its teaching and learning (e.g. Cooney, Shealy, & Arvold, 1998; Frykholm, 1996, 1998, 1999; Kelly, 2001; Kinach, 2002; Langford & Huntley, 1999; Langrall, Thornton, Jones, & Malone, 1996; Mewborn, 1999). Most of these studies demonstrated that method courses have positive influence on prospective teachers' beliefs about mathematics and its teaching and learning. Besides, Kelly (2001) reported that prospective teachers who attended method courses displayed strong confidence in the teaching of mathematics. Clift and Brady (2005) briefly summarized the studies on the method courses and field experiences of prospective teachers in the period of 1995 to 2001. Brown (2010) investigated the influence of high-stakes reforms on the teaching and prospective teacher education. Zeichner and Conklin (2005) reviewed the studies on teacher education programs within the period from 1986 to 2002 in terms of various aspects and the researchers categorized the studies on teacher education programs into three categories which are 4-year versus 5-year programs, alternative versus traditional, and the case studies of teacher preparation programs. In general, the researchers indicated that there is no ground to compare or contrast distinct prospective teacher education programs that have distinct characteristics, complexities and aims due to their different natures including cultures, educational policies, contexts, etc. Several researchers pointed out that there exists no common knowledge base for constructing effective pre-service teacher education programs (Hiebert, Morris, & Glass, 2003). Nevertheless, especially in mathematics education, studies on the courses and practical experiences of prospective teacher education programs reported that the method courses have an impact on professional development of prospective teachers and their way of teaching (Ensor, 2001; Ball, 1990b) and these courses need to include several forms of knowledge for teaching (Graeber, 1999; Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992).

In recent decades, there has been a strong emphasis by educational organizations on what qualifications and standards teachers need to have in order to teach mathematics effectively. In teacher preparation and professional development programs, the knowledge of mathematical modeling included both in *Professional Standards for Teaching Mathematics* (NCTM, 1991) and in *Program Standards for Mathematics Education* (NCATE/NCTM, 2012) and stressed its importance for teachers' teaching of mathematics and their professional development. According *Standard 2* towards mathematical practices, it is expected from

> Effective teachers of secondary mathematics solve problems, represent mathematical ideas, reason, prove, use mathematical models,", and prospective mathematics teachers "Formulate, represent, analyze, and interpret mathematical models derived from real-world contexts or mathematical problems (NCTM/NCATE, 2012, p. 1).

More specifically, knowledge of mathematical modeling is underlined as a competency for mathematical content knowledge (MEB, 2011a).

In order to make teachers gain knowledge of mathematical modeling and its pedagogy, professional development programs might involve courses and seminars to fulfill this goal. Studies on teachers' professional development demonstrated that interventions committed in professional development give rise to changes providing that the courses, which include both teaching and reflection, are long-term, and factors related to teachers' teaching were controlled (Tirosh & Graeber, 2003; Wilson & Cooney, 2002). Hill and Ball (2004) reported similar findings that teachers who attended the professional development program displayed improvements in their performance in knowledge for teaching mathematics.

Many researchers emphasized the importance of such courses for acquisition knowledge of mathematical modeling (Blum & Niss, 1991; Kaiser, Blomhøj, & Sriraman, 2006; Lingefjärd, 2007). However, although many countries involve mathematical modeling as a part of their school curricula, researchers indicated that mathematical modeling has not been covered in the prospective teacher education courses in university level (Lingefjärd, 2007; Borromeo Ferri & Blum, 2009).

2.5.1 Studies on the Professional Development of Mathematics Teachers in the Context of Mathematical Modeling

Since mathematical modeling has been becoming as an integral part of school curricula in many countries and emphasized its importance in teaching and learning of mathematics (Blum & Niss, 1991; Lingefjärd, 2000; 2002a; Blum et al., 2002; Lesh & Doerr, 2003a; Kaiser & Maaβ, 2007; Lesh & Zawojewski, 2007; Doerr & Lesh, 2011) and its significance for school mathematics curricula (Blum & Niss, 1989; Niss, 1989), there has been many efforts to design and implement mathematical modeling courses for pre-service (Barbosa, 2001; Holmquist & Lingefjärd, 2003; Jiang et al., 2003; Lingefjärd, 2006; Kaiser & Schwarz, 2006; Borromeo Ferri & Blum, 2009; Maaß & Gurlitt, 2011) and in-service (Blomhøj & Kjeldsen, 2006) teacher education in the field of mathematics education in recent decades. The studies on mathematical modeling courses illustrated in the following paragraphs in terms of their significance for professional development of teachers.

In the study of Barbosa (2001), the features of a prospective teacher education program on mathematical modeling with the perspective of the mathematics teaching is investigated. The researcher developed a course titled as "Modelling and Mathematics Education" and offered for undergraduate level. Eight prospective teachers and one in-service teacher was enrolled in the course. The data sources of the study included observations, open interview, and written documents. The results of the study suggested that students expressed their satisfaction with the course in terms of content of the course. Students had difficulties in working as groups that affected the processes of mathematical modeling. Another result was that students elaborated a broad conception about mathematical modeling in mathematics teaching. On one hand, some of the participants stated warning about the use of mathematical modeling in lessons such as modeling could be used as a practice for checking the results. On the other hand, some of the students stated that they were always willing to use modeling in their lessons. Modeling course evolved students' conceptions about teaching of mathematics such as relating mathematics with real life rather than considering as abstract and not connected to real life. The researcher suggested that more studies needed to reveal the role of mathematical modeling in the preparation of prospective teachers and clarify possible techniques to implement in classroom settings.

Holmquist and Lingefjärd (2003) designed a mathematical content course on modeling applications which is content was designed to make prospective teachers learn how to deal with modeling tasks with the help of technological tools such as distinct software and graphing calculators. In this study, researchers used an assessment technique such that assessment of prospective teachers and teaching process used simultaneously. Sample of the study involved 11 prospective teachers for a semester. The results of the study demonstrated that prospective teachers had a great confidence with technological tools and did not incline to check the correctness of the obtained results. Another result of the study showed that students might become confused even if they used their own developed models.

In the study of the Jiang and others (2003), a mathematical modeling course designed for prospective secondary mathematics teachers in order to gain knowledge and experience about solving real life problems by giving motivation and encouragement to provoke to engage. The course was designed according to NCTM Standards (1989, 2000) using the modeling process diagram (see Figure 1) and consisted of problem solving related with modeling, mathematical inquiry and the use of technological tools. As a result of the implementation of the modeling course, the researchers indicated that the modeling course could support prospective teacher in gaining a conceptual understanding of concepts and advanced their problem solving skills by dealing with real life related modeling tasks. The researchers suggested that assessment of these courses could be accomplished by using alternative assessment techniques such as project work, observations, and individual interviews that have superior features to traditional assessment methods.

Lingefjärd (2006) reported that the modeling courses for teachers that he carried out altered in terms of many dimensions such as the way of assessing students, the courses content, and even their introduction parts according to personal and cultural preferences. Lingefjärd (2007) stated that most of the mathematics teacher education programs did not offer mathematical modeling courses in Sweden universities.

In the study of Kaiser and Schwarz (2006), a research was carried out on mathematical modeling seminars that was held in university by the collaboration of departments of mathematics and mathematics education and pre-service teachers and high school students participated. It is important that pre-service teachers need to develop an understanding of mathematical modeling and learn how to conduct mathematical modeling in school environment constitutes a basis for designing mathematical modeling courses in teacher education (Kaiser & Schwarz, 2006). It is reported that modeling examples, one example to each group, were given to groups of high schools that were supervised by pre-service teachers and each group would develop a solution to modeling examples. This was the general scope of the course content. The aim of the course was to promote pre-service teachers developing an understanding of mathematics with respect to modeling and teaching modeling competencies for conducting modeling processes in classrooms (Kaiser & Schwarz, 2006).The study demonstrated that a change was occurred in the beliefs of pre-service teachers about using mathematical modeling in mathematics teaching from more conservative to use ordinary mathematics (pure) to more eager to use modeling examples. Furthermore, pre-service teachers evaluated the course positively. The other results of the study were that the lessons were very difficult and that took much more time to complete works.

Borromeo Ferri and Blum (2009) conducted a study about investigating characteristics of a modeling course for teachers such as contents, methods, etc. The authors designed a seminar course for pre-service teachers and its content based on four modeling competencies which are theoretical competency (knowledge about the nature of mathematical modeling), task related competency (solving, analyzing and developing task skills), teaching competency (knowledge about implementing modeling in classroom environment), and diagnostic competency (ability to follow the modeling processes and identifying students' difficulties in the implementation period). The authors of the study reported that they identified these competencies from their previous modeling seminar experiences. Borromeo Ferri and Blum suggested the aforementioned competencies for forming contents such a modeling course. The researchers pointed out that theory and practice should be considered together while designing the content of mathematical modeling course from their experiences.

Apart from previous studies mentioned above paragraphs, Blomhøj and Kjeldsen (2006) researched in-service teachers' experiences from planning, conducting, and evaluating an in-service course on modeling for teachers through based on project. The main aim of the mathematical modeling course was to teach in-service teachers mathematical modeling through project based learning by help teachers in preparing lesson plans, implementing, and making evaluation of these lessons based on project work. The study showed that in-service teachers can carry out problem oriented modeling projects in high schools and their roles are very significant during the implementation of the modeling activities. It is reported that teachers need to pay attention to modeling processes includes sub-processes in order students develop modeling competencies. The researchers indicated that the way of teachers' presentation of mathematical ideas might influence students' engagement in modeling activities negatively.

Maaβ and Gurlitt (2009) proposed qualifications for teachers in order to conduct mathematical modeling activities in their classrooms which are having the knowledge of fundamental concepts of mathematical modeling, altering their beliefs about the nature of mathematics if they are inconvenient to philosophy of mathematical modeling, and noticing their own power to implement mathematical modeling activities in their classrooms.

Similar to the study of Borromeo Ferri and Blum (2009), Maaβ and Gurlitt (2011) mentioned about much larger project (LEMA), which has aim to design a common professional development course for pre-service teachers to teach them modeling. In their study, the researchers presented framework of the project including theoretical backgrounds, design process of the course, implementation, and results. In order to constitute theoretical basis, the researchers reported that they tried to find the answer of the question: What knowledge does a teacher need in order to teach modeling? This question demonstrated the focus of the research. Maaβ and Gurlitt (2011) stated that the theoretical model comprised the knowledge of modeling, modeling tasks, lessons, and assessment parts in a modeling course. The needs analysis of teachers, which included teachers' beliefs about the aforementioned parts were also taken into account together with theoretical background. The results of the study demonstrated that the implemented course had no impact on teachers' beliefs. Nevertheless, it influenced the teachers' PCK of modeling and self-efficacy positively.

2.6 Summary of the Literature Review

To summarize this entire chapter in terms of what previous studies told us, teacher preparation programs are very significant for prospective teachers (Ball, 1990; Brown & Mayor, 1958; Chapman, 2007; Ensor, 2001; Manouchehri, 1997; Ponte & Chapman, 2008; Sowder, 2007) in order to gain mathematical knowledge (Sowder, 2007), PCK (Shulman, 1986, 1987; Grossman, 1990), and necessary knowledge needed for mathematics teachers (Fennema & Franke, 1992; Kahan et al., 2003). Mathematical modeling and its importance in the teaching and learning of mathematics stressed in the previous studies (Blum et al., 2002; Blum & Niss, 1991; Doerr & Lesh, 2011; Kaiser & Maaβ, 2007; Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007) and documents of several institutions and organizations (CCSSI, 2010; NCTM, 1989, 2000; OECD, 2003, 2009). Moreover, many studies showed that prospective and in-service teachers had various conceptions about mathematical modeling (Frejd, 2012; Ärlebäck, 2010) and the use of mathematical modeling in the teaching and learning of mathematics (e.g., Chapman, 2007; Eraslan, 2011; Kaiser & Maaβ, 2007 Kuntze et al., 2013; Maaβ & Gurlith, 2011; Schmidt, 2011; Siller et al., 2011; Yu & Chang, 2011). Therefore, introducing mathematical modeling to teachers is significant as soon as possible, even it would be better in teacher preparation programs. Some researchers indicated that there should be mathematical modeling courses in teacher preparation programs (Lingefjärd, 2007). Many researchers attempted to design a mathematical modeling course and investigate its influence on prospective teacher' knowledge and beliefs (Barbosa, 2001; Holmquist & Lingefjärd, 2003; Jiang, McClintock, & O'Brien, 2003; Lingefjärd, 2006; Kaiser & Schwarz, 2006; Borromeo Ferri & Blum, 2009; Maaß & Gurlitt, 2011) Nevertheless, these previous studies were far away from revealing all of undetermined or unclarified issues about prospective teachers' conceptions about mathematical modeling and its use in the class (Blum et al., 2002). According to these implications, the current study would contribute to the literature in terms of development of prospective teachers' thinking about mathematical modeling and its use in their future classrooms.

CHAPTER 3

METHODOLOGY

The purpose of the current study is to investigate the evolution of prospective secondary mathematics teachers' conceptions about mathematical modeling and pedagogical knowledge of mathematical modeling throughout a mathematical modeling course offered to prospective mathematics teachers. More specifically, the following research questions are explored within the study:

- How did prospective secondary mathematics teachers' conceptions about mathematical modeling change throughout the implementation of the designed course?
- How did prospective secondary mathematics teachers' conceptions about use of mathematical modeling in teaching change throughout the implementation of the designed course?

The study was carried out with 25 prospective secondary mathematics teachers who enrolled in an elective course entitled "Mathematical Modeling for Prospective Teachers" in a public university in Ankara, Turkey. The course was designed and offered as part of a larger research project supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the grant number 110K250. The project was comprised of two fundamental parts: One of them was developing a mathematical modeling training module for in-service mathematics teachers and the other one was designing and developing an academic mathematical modeling course for prospective teachers. Main purposes of the project were (i) to develop mathematical modeling tasks and activities that can be used with both secondary school students and pre-service and in-service teacher education programs; (ii) to develop an in-service mathematics teacher professional development program about mathematical modeling and to investigate how the program would affect teachers' beliefs, knowledge and practices; (iii) to develop an academic course for pre-service mathematics teachers and investigate how the course would affect pre-service teachers' knowledge, competencies and attitudes in terms of mathematics, mathematical modeling and using mathematical modeling in mathematics education. As a part of the larger project, the current study was carried out for 14 weeks in Spring semester of 2011-2012. Much more information about the research was provided in the following sections.

In this chapter, I discuss design of the study, participants, detailed research procedure including design process of the course with theoretical considerations, data collection procedure, and data analysis.

3.1 Design of the Study

In this study, case study was selected as the research design from qualitative research approaches. According to Creswell (2007), a case study is "an in-depth exploration of a bounded system (e.g., activity, event, process, or individuals) based on extensive data collection" (Creswell, 2011, p. 465). A case study was also defined as "an empirical inquiry that investigates a contemporary phenomenon in depth and within its real life context, especially when the boundaries between phenomenon and context are not clearly evident" (Yin, 2009, p. 18). In this study, the case was the phenomenon, which was the development of prospective secondary mathematics teachers' thinking about mathematical modeling and about pedagogy of mathematical modeling who enrolled the "Mathematical Modeling for Prospective Teachers" course for a semester.

3.2 Participants of the Study

The participants of the study were 25 prospective mathematics teachers who enrolled the elective course "Mathematical Modeling for Prospective Mathematics Teachers" in a state university as a part of much larger project. While the majority of the prospective teachers were 3rd-year students (n=16), seven of them were 4th-year, and only two of them were 5th-year students in their 5-year teacher education program. The majority of prospective teachers were female (n=18) and seven of them were male. The participants' ages ranged from 19 to 22 years old. Participants' GPAs ranged from 2.06 to 3.62 out of 4.00 ($\bar{X} = 2.66$, $SD = 0.37$).

Throughout the program, prospective secondary mathematics teachers have to complete both compulsory and elective courses in order to graduate from the program. Participants of the mathematical modeling course (4th-year and 5th-year students) completed the compulsory courses, which were “Discrete mathematics, Linear algebra, One variable calculus, Multiple variable calculus, Analytic geometry, Differential equations, Introduction to algebra, Set theory, and Topology, Euclidean geometry”, in their first 3 years. During the implementation of the modeling course, these prospective teachers were also taking “Transformation geometry, Abstract Algebra, and Number Theory” courses. At the same time, 4th-year and 5th-year prospective teachers took pedagogical courses like “Measurement and Evaluation, “Approaches and Theories of Teaching and Learning” and pedagogical content courses related to mathematics education (i.e., “Teaching Methods in Mathematics Education” and “Problem Solving”). Moreover, although 3rd-year prospective teachers who enrolled the course completed some pedagogical courses such as “Introduction to Educational Sciences”, “The Psychology of Development”, and “Guidance and Counseling”, they did not take any courses related to mathematics education different from 4th-year and 5th-year prospective teachers. The implemented course within the current study was their first mathematics education course. Only four of participants of the study stated that they had an experience with mathematical modeling within other courses.

The prospective teachers were divided into seven groups according to their own preferences. Four of them included four prospective teachers and the remaning groups consisted of three prospective teachers. Demographic data pertaining to prospective teachers was given in Table 4.

Table 4 Characteristics of participants with their group numbers and pseudonyms

Group No.	**Prospective Teachers (PTs)**	**Class**	**Gender**	**Age**
1	PT1*	5	Female	24
7	PT2	3	Female	20
7	PT3	3	Female	21
7	PT4	3	Male	21
3	PT5	3	Female	21

Table 4 (continued)

Group No.	**Prospective Teachers (PTs)**	**Class**	**Gender**	**Age**
3	PT6	3	Female	21
5	PT7	3	Female	22
5	PT8	3	Female	21
5	PT9	3	Female	21
4	PT10	3	Female	21
4	PT11	3	Female	21
4	PT12	3	Female	21
2	PT13	3	Female	22
2	PT14	3	Female	21
2	PT15	3	Female	21
2	PT16	4	Male	22
1	PT17	4	Female	23
1	PT18	4	Male	22
3	PT19	3	Male	21
4	PT20	4	Male	22
6	PT21	4	Female	23
6	PT22	4	Female	22
6	PT23	5	Male	24
7	PT24	3	Male	21
5	PT25	3	Female	21

*: PT1 represents the coded prospective teacher numbered as 1.

3.3 Conceptual Framework of the Study

In this section of the study, the conceptual framework that is used in this study will be discussed and explained in detail. Conceptual frameworks are very significant on the construction of research, planning the research, implementation, analysis, and interpretation of the results in a coherent and meaningful frame. Throughout this part of the section, *MMP* on teacher development will be discussed in details with implications from distinct scholars working within this framework.

3.3.1 Models and Modeling Perspective on Teacher Development

This study investigates both the change in prospective teachers' thinking about mathematical modeling and pedagogical knowledge of modeling about the use of mathematical modeling activities in the classroom setting throughout the implementation of a designed undergraduate course for teachers called as "Mathematical Modeling for Prospective Teachers". Since one of the main aims of the course is to provide professional development for prospective teachers about mathematical modeling and its use in the classroom environment, a conceptual framework, which is *MMP* on teacher development, adopted in this study in the investigation of prospective teachers' professional development in the use of mathematical modeling activities in classroom environment. Lesh and Doerr (2003) proposed the MMP theoretical framework for different research purposes. It is underlined the MMP research that it involves investigation of "what it means to "understand" important concepts and abilities" (Doerr & Lesh, 2011, p. 248). In MMP research indicates that teaching from a modeling perspective includes designing teaching and learning environments according to MMP, implementation of MEAs in modeling process demonstrated the how crucial the role teachers play. MMP suggests a perspective for the development of teachers (Doerr & Lesh, 2003; Doerr & Lesh, 2011; English, 2003; Lesh & Lehrer, 2003). According to MMP, teachers need to have experiences about mathematical modeling and modeling activities by involving the these processes as their students engaged in order to

> deepen, extend and share their own knowledge and understanding of the content of mathematics, the ways in which students build mathematical ideas, and the pedagogical implications of teaching mathematics in a manner which encourages the development of powerful mathematical models (Schorr & Lesh, 2003, p. 143).

Koellner-Clark and Lesh (2003) pointed out that teacher development is similar to student development such that mathematical modeling activities need to be prepared according to the same principles used for modeling activities for students. Apart from the student development, teachers focused on the following aspects while dealing with modeling activities.

> … clarifying and or elaborating their own ideas including mathematical content, pedagogy, and knowledge of student thinking while at the same time making connections among and between their previous models and elaborating on each other's thoughts and ideas (p. 165)

It has been indicated that teachers' involvement in the modeling processes and activities like solving mathematical modeling problems like their students was an integral component of the professional development of teachers and understanding of students' thinking (Koellner-Clark & Lesh, 2003; Schorr & Lesh, 2003, English, 2003). Lesh and Doerr (2003) emphasise the interdependent nature of development existing between students and teachers by means of emergence of MMP as a conceptual lens. As they develop, curriculum materials and instructional programs evolve accordingly (Lesh & Lehrer, 2003).

It is adamantly emphasized that for their pedagogical development, teachers and prospective teachers also need to resolve the questions themselves like students which are used in modeling activities related to using mathematical modeling in the process of mathematics teaching (Doerr & Lesh, 2003; Lesh & Doerr, 2003a; Schorr & Lesh, 2003). Both experiencing the modeling process by solving related modeling questions and watching the processes that students go through during classroom applications provide teachers with rich learning environment for their professional development (Doerr & Lesh, 2003; English, 2003). In mathematical modeling process, depending on various presumptions, as more than one solution approach and method may come up, teacher needs to have enough pedagogical qualifications in order to manage to understand and evaluate these various methods in the process and give correct feedbacks when necessary. In order to do that, first, they need to experience this process as a person to be able to have deep knowledge on the processes that students will go through in a mathematical modeling activity solution process. An emphasis was put on the "… expertise in teaching is reflected not only in what teachers can "do," but also what they "see" in teaching, learning, and problem solving situations" (p. 111) about the development of mathematics teachers and their training (Lesh & Lehrer, 2003). Therefore, teachers' knowledge about mathematical modeling and modeling pedagogy (Niss et al., 2007) comprise in the teachers' professional development about mathematical modeling.

According to Schorr and Lesh (2003), a number of problems in math teaching stem from teacher-student communication. A teacher is expected to have deep information on students' thinking processes such as how students learn a concept and which mental processes these students go through while learning it. Lack of this information, it causes the conduct of teaching students a concept in a specific way and expecting them to learn it in the same way, which is the paramount source for the lack of communication between the student and the teacher. Because the student may have understood highly different things from what the teacher explains and reports for a concept. Thus, he may have structured that concept in a different way in his mind. Schorr and Lesh (2003) asserted that modeling activities need to occupy an important position in teacher training to resolve this problem. In these activities, teachers or prospective teachers will learn via experiencing which thinking processes students go through for a mathematical model, concept to be produced and how this process should be evaluated (Doerr & Lesh, 2003). However, the result of a study as small groups with students by Schorr ve Lesh (2003) indicated that there were considerable changes in the opinions of teachers on: (a) in their perceptions about the paramount behaviors that calls for observation on students in problem solution activities (b) their opinions about the points that need to be evaluated as weak and strong in students' answers and (c) evaluation-assessment.

It is asserted that modeling activities offer significant opportunities for mathematics teachers' professional developments (see Doerr & Lesh, 2003). In traditional method, the teacher expresses the various models that she possesses. As there may not be a compelling reason why the teacher should develop the models he already possesses, similarly, he may not feel such an anxiety. Because while the teacher is the source of knowledge, the students are in the receiver role. However, in modeling approach the teacher will have to force his own mental modeling borders in order to evaluate and improve the various solution ways, models, and interpretation of real life situations that students created. Through the various models that students have developed, teachers will have improved their own model images. The mental models of the teachers need to possess a broader perspective than those the students have. Doerr and Lesh (2003) asserted that teachers' models involve "the ways of seeing and interpreting situations for particular purposes" (p. 126) and these models are developed, expanded, revised, and implemented in various classroom situations. The researchers stated that teachers' models need to involve not only a comprehension of students' models, but also a comprehension of how students' models developed, conceptions about "curricular development of the concepts" and "pedagogical strategies for teaching the concept in various settings to students with varying backgrounds" (Doerr & Lesh, 2003, p. 134). According to MMP, teachers' knowledge include distinct classroom contexts and aims of teaching, powerful sides and weak sides of teachers' analysis about the contexts and aims, and going on revising and refining those considerations about teaching.

Mathematical modeling activities for teachers were highlighted in terms of possible contributions to teacher knowledge. Doerr and Lesh (2003) stated the general purposes of model-eliciting activities for teachers as (1) making teachers discover their thinking ways, (2) testing, revising, and refining their thinking ways for specific aims, (3) sharing with their teacher friends for repetition, and (4) reusing their thinking was in more situations. The authors suggested that modeling activities for teachers "should provide teachers with concrete opportunities to explore mathematical meanings, interpret students' thinking, plan for instructional activities, select materials, structure activities and assess student performance" (Doerr & Lesh, 2003, p. 135). According the MMP conceptual framework, studies include "on-the-job classroom based professional development activities" containing the mathematical modeling activities for students furnish rich contexts for the experience of teachers that encourage development of teachers (Koellner-Clark & Lesh, 2003, p 172; Schorr & Lesh, 2003, p. 157). Therefore, in this study, mathematical modeling activities for students included provide prospective teachers to gain more experience about mathematical modeling activities that they can use their future classroom.

Carpenter, Fennema, and Romberg (1993) demonstrated that helping teachers acquainted with their students' thinking about significant mathematical conceptions and skills which are supposed to developed by their students accepted as a kind of fruitful approach to assist teachers to improve their teaching (cited in English, 2003). Developing a comprehension for the ways students' of thinking becomes more important in order to make accurate interpretations about students' mathematical ideas and possible misunderstandings of them for teachers (Doerr & Lesh, 2003; Schorr & Lesh, 2003; Niss et al., 2007). As mathematical modeling process involves various assumptions and solution processes and so differentiates from conventional problems, naturally teacher's role change during the process as well. According to English (2003), during a modeling activity practice process, the teacher should be able to follow students' thinking processes, determine questioning strategies that will assist them to develop correct mathematical ideas and guide them correctly, and lastly, the teacher should provide a discussion environment that will enable all students to provide the development of a correct mathematical opinion. To manage to do that, a teacher ought to be able to understand students' thinking ways rapidly and guide them correctly. However, on the other hand, according to Blum and Niss (1991), one of the paramount reasons why teachers avoid using modeling activities within classroom environment is that teachers feel unconfident against the questions students may ask during the solution process. Upon a classical problem, the questions that student may ask are limited and the teacher's answer for them is already ready. However, because the nature of the probable questions to be asked during modeling process is quite complicated, it does not only lead to a compelling phenomenon for the teacher, but also creates a perfect environment to improve themselves pedagogically (Doerr, 2006; English, 2003; Doerr & Lesh, 2003). As a result of these studies, the produced information indicate that analyzing and interpreting students' thinking manners play an important role upon the development of teachers' (prospective teachers) knowledge related to mathematical modeling and in-class practices. Hence, study of student s' ways of thinking has been evaluated as a component of the class.

To sum up, in this study, the mathematical modeling course designed according to suggestions and implications of MMP on teacher development and principles of professional development (Loucks-Horsley et al., 2003) were taken into account during the design process. Figure 5 illustrates the components of the modeling course that are significant for professional development of prospective teachers about mathematical modeling and its use in the classroom setting. The mathematical modeling course for prospective teachers involves mathematical modeling activities, students' way of thinking works, use of technology, group work, designing and preparing modeling activities, and preparing an implementation plan for designed modeling activities and carrying out the implementation for having experience as the integral components. The details of the course design process and its theoretical foundation was provided in the following section.

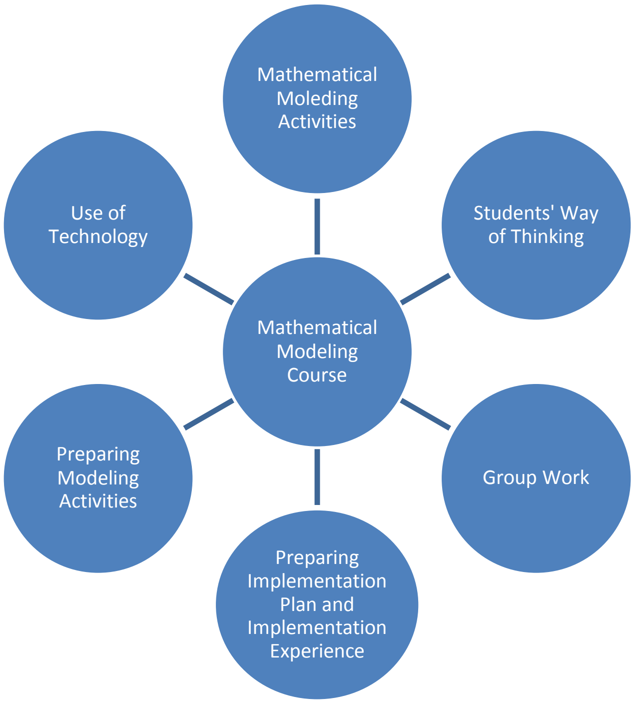


Figure 5 Components of the mathematical modeling course designed for prospective teachers in this study

3.3.2 Course Design Process with Theoretical Considerations

In this section, course development processes within the project are explained in detail including determination of the components of the course by using various studies in the related literature and pilot study.

Within the scope of prospective teacher education component of the project, a course planning at undergraduate level was pledged in that the prospective mathematics teachers who attend the 3rd or the 4th grade will improve their knowledge about mathematical modeling and gain the necessary information, skills and attitudes to be able to use mathematical modeling in mathematics teaching after they launch their profession. Within this scope, a course called "Mathematical Modeling for Prospective Mathematics Teachers" was contemplated and from Fall semester of 2010-2011 academic year on, each semester (three semesters) the content was revised again and again, improved and executed with prospective teachers who attend Elementary School Mathematics Education Program of a public university. The course was executed in pilot study throughout the three semesters. The final outline of the course was drawn and in Spring semester of 2011-2012, it was made to be ready for pre-service component of the much larger project which was executed with the prospective teachers who attend a different public university Secondary School

Mathematic Teacher Education Program. In this section, the course planning process that was used as a model to improve prospective secondary mathematics teachers' conceptions about mathematical modeling and pedagogy of modeling is going to be reported. Within this frame, initially the theoretical basis that the components which are determined while the lesson content is made up of is going to be mentioned. Afterwards, the pilot study and the basic impressions gained from them will be explained. Finally, in the light of the findings that are observed at the end of the pilot study, how the final lesson content was drawn was explained in detail.

3.3.2.1 Description of the course

The planning process of a course directed to field education (i.e., mathematics education) involves sub-processes such as determining the aims and objectives of the course, determining the key components (subjects) that will make up the content of the course in accordance with these aims and objectives, carrying out an appropriate, and time planning for the subjects to dealt with (see Table 5). In this section, in the planning process of the course determining the key components that creates the lesson content, lesson content/definition and ranging and timing of the subjects to be handled will be considered in detail.

Table 5 Definition, aim, and objectives of the planned course

The Course Definition and Content: Model and modeling approach in solving mathematical problems, mathematics education, and training. Using technology in modeling process. Mathematics applications in real life situations. Teacher training in modeling process: Understanding real life situations, producing presumptions, transfering the problem situation into mathematics language and solve it, interpreting the solution, confirming the model, reporting depending on the model, explanation and estimations. Using mathematical modeling in teaching process and teacher training related to real classroom applications.
Aims and Objectives of the Course: The prospective teachers who completes this course successfully;
• will be able to use their modeling skills such as understanding real life problems (or realistic problems), building a mathematical model which appropriate for the problem context and producing a mathematical solution, interpreting the solution by regarding its real situation, (if necessary) expanding the solution and deciding to change it etc.
• will be able use their mathematical knowledge and skills to solve his real life problems (or realistic problems).
• will be able to express their mathematical knowledge both orally and written effectively by using various displays as mathematics language, symbolic system, diagram and graphic.
• in mathematics learning, especially in problem solving, will be able to use technology.
• will be able to explain the nature of modeling process and the characteristics of modeling activities.

Table 5 (continued)

• will be able to discuss the importance of mathematical modeling in teaching and learning of mathematics. • will be able to interpret students' ways of thinking and processes within mathematical modeling context. • will be able to design modeling activities that can be used in math teaching individually or as a group. • shall be able to practice mathematical modeling activities.

3.3.2.2 Determining the key components of the course

The course "Mathematical Modeling for Prospective Teachers" was designed to improve prospective teachers' mathematical modeling skills; to provide them with knowledge on using mathematical model in math teaching, and develop a positive view. The general outcome of the studies carried out for professional development of in-service mathematics teachers and prospective teachers indicate that it is not so easy to make radical changes on their thinking structures and beliefs directed to learning and teaching. (e.g., Llyod, 1999; Schorr & Koellner-Clark, 2003; Schorr & Lesh, 2003). The content creation process of this course, which aimed at improving prospective teachers' knowledge, skill and a positive view about using mathematical modeling upon mathematics teaching, was begun by the awareness of these difficulties. During the planning of the lesson, primarily, studies about teacher education on the basis of mathematical modeling approach were investigated. In this context, in mathematical modeling approach and in studies on teacher training within the scope of other programs, it becomes apparent that it is important for teachers to learn through living and involving in the process (e.g., Doerr & Lesh, 2003; Schorr & Lesh, 2003). In this context, it comes to the fore that in such a course, it is essential to have prospective teachers gain a foreseeing upon subjects like what a mathematical modeling is, what the in-class modeling activities include, how the in-class modeling activities implement how and in what subjects students will think about, the importance of group work and teacher's role. In this context, getting the prospective teachers to experience the modeling processes and stages that students will go through makes up the key philosophy of the lesson content.

Table 6 displays the key components and the course of lesson process of "Mathematical Modeling for Prospective Teacher' which was designed within the project. The basic components of this course which are designed within the scope of the literature review are as follows: a) individual and group work upon mathematical modeling activities, b) group work during modeling process, active participation and discussion, c) using technology during modeling process, d) through the modeling activities which are resolved, analyzing students' ways of thinking, e) developing a modeling activity, creating a implementation plan and having implementation experience. Moreover, as a part of the studies carried out within the scope of the course established upon these components; made up of another aspect of the course, classroom discussions, reflection papers written after the implementation of each modeling activities, and individual interviews about the issues asked in the reflection paper guide. Mathematical modeling process, nature of modeling activities, the importance of mathematical modeling in the teaching and learning of mathematics, role of instructor in the use of mathematical modeling were included in the framework of the couse (see Figure 6). Components, which are determined for a class, designed within a comprehensive literature framework and the detailed information about how to handle in the course of lesson is explained in the following.

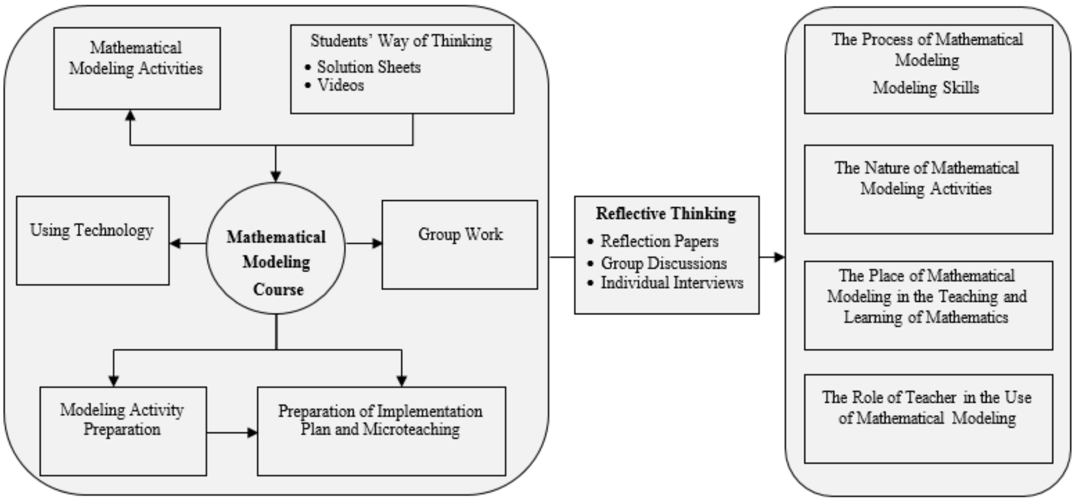


Figure 6 The framework of mathematical modeling course for prospective secondary mathematics teachers

Mathematical modeling activities

It is adamantly emphasized that for their pedagogical development, in-service and prospective mathematics teachers also need to resolve the modeling activities themselves like students which are used in modeling activities related to using mathematical modeling in the process of mathematics teaching (Doerr & Lesh, 2003; Lesh & Doerr, 2003a; Schorr & Lesh, 2003). Both experiencing the modeling process by solving related modeling activities and observing the processes that students go through during classroom applications provide teachers with a very rich learning environment for their professional development (Doerr & Lesh, 2003; English, 2003). In mathematical modeling process, depending on various presumptions, as more than one solution approach and method may come up, teachers need to have enough pedagogical qualifications in order to manage to understand and evaluate these various solution strategies in the process and give correct feedbacks when they are necessary. In order for that, first, they need to experience this process as a person to be able to have deep knowledge about the processes that students will go through in a mathematical modeling activity solution process.

According to Schorr and Lesh (2003), a number of problems in teaching of mathematics stem from teacher-student communication. A teacher is expected to have deep knowledge about students' ways of thinking such as how students learn a concept and which mental processes these students go through while learning it. Lack of this knowledge cause the conduct of teaching students a concept in a specific way and expecting them to learn it in the same way, which is the paramount source for the lack of communication between the student and the teacher. Because the student may have understood highly different things from what the teacher explains and reports for a concept. Thus, he or she may have structured that concept in a different way in his or her mind. Schorr and Lesh (2003) asserted that modeling activities need to occupy an important position in teacher training to resolve this problem. In these activities, in-service teachers or prospective teachers will learn via experiencing which students' ways of thinking go through for a mathematical model, concept to be produced and how this process should be evaluated (Doerr & Lesh, 2003). However, the result of a study as small groups with students by Schorr ve Lesh (2003) indicated that there were considerable changes in the opinions of teachers on: (a) in their perceptions about the paramount behaviors that calls for observation on students in problem solving activities (b) their opinions about the points that need to be evaluated as weak and strong in students' answers and (c) evaluation-assessment.

It is asserted that mathematical modeling activities offer significant opportunities for mathematics teachers' professional developments (see Doerr & Lesh, 2003). In traditional teaching method, the teachers express the various models that they possess. As there may not be a compelling reason why the teachers should develop the models they already possess, similarly, they may not feel such an anxiety. Because while the teacher is the source of knowledge, the students are in the receiver role. However, in modeling approach the teacher will have to force his own mental modeling borders in order to evaluate and improve the various solution ways, models, and interpretation of real life situations that students created. Through the various models that students have developed, teachers will have improved their own model images. The mental models of the teachers need to possess a broader perspective than those the students have. As a result of the experiences that the activities had during the solution process, working on modeling activities was included in the lesson content as the paramount component foreseeing that prospective teachers develop ideas about phenomena which will be theoretically discussed and handled, like students' ways of thinking, which are the other components of the course at the same time, teacher's role, group work's role, the nature of modeling.

Analyzing studies of students' ways of thinking

A professional development approach is that it focuses on students' ways of thinking, in addition to the ones produced out of research studies, upon the solutions produced in the context of modeling activities which directly come from students and regards examining and interpreting students' ways of thinking structures systematically as a basis. Teachers' knowledge about students' thinking structures is one of the paramount components of the field of pedagogy that Shulman (1986) defined. In recent years, it is also one of the most emphasized subjects related to teacher education and professional development of teachers (Kieran, 2007; Sowder, 2007). Students can have various ways of thinking. A learning environment where these various thinking structures is made to come to the fore and supported will play an important role in growing individuals who own the knowledge and skills that are foreseen within contemporary teaching approaches framework. The studies carried out emphasize the importance of students' having knowledge about different thinking structures and more importantly, shaping their own knowledge, beliefs and teaching plans (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Franke, Carpenter, & Carey, 1993; Nathan & Koedinger, 2000a; 2000b). In international professional development programs like "Cognitive Guiding Training" fulfilled about teacher education (Cognitively Guided Instruction, Carpenter, Fennema & Franke,

1996) and “Multi Level Program Development” (Multi-tier Program Development, Koellner-Clark & Lesh, 2003), teachers who were trained through students’ ways of thinking could include this knowledge in their teaching plans and contributed to students’ success (Carpenter et al., 1989; Fennema et al., 1993).

When looked into the studies of using mathematical modeling in the teaching of mathematics, it is seen that understanding students’ ways of thinking have become more important (Lesh & Doerr, 2003a; Schorr & Lesh, 2003; Niss, Blum & Galbarith, 2007). As mathematical modeling process involves various assumptions and solution processes and so differentiates from conventional problems, naturally teacher’s role change during the process as well. According to English (2003), during a modeling activity implementation process, the teacher should be able to follow students’ ways of thinking, determine questioning strategies, which will assist them to develop correct mathematical ideas and guide them correctly, and lastly, the teacher should provide a discussion environment that will enable all students to provide the development of a correct mathematical concept. To manage to do that, a teacher should be able to understand students’ ways of thinking rapidly and guide them correctly. However, on the other hand, according to Blum and Niss (1991), one of the paramount reasons why teachers avoid using modeling activities within classroom environment is that teachers feel unconfident against the questions students may ask during the solution process. Upon a classical problem, the questions that student may ask are limited and the teacher’s answer for them is already ready. However, because the nature of the probable questions to be asked during modeling process is quite complicated, it does not only lead to a compelling phenomenon for the teacher, but also creates a perfect environment to improve themselves pedagogically (Doerr, 2006; English, 2003; Doerr & Lesh, 2003). As a result of these studies, the produced knowledge indicate that analyzing and interpreting students’ ways of thinking play an important role upon the development of teachers’ (prospective teachers) knowledge related to mathematical modeling and in-class practices. Hence, the study of students’ ways of thinking research has been evaluated as a component of the class.

Using technology in modeling

The mathematical modeling of real life situations process is a process that can be a highly difficult for both teachers and students. According to Johnson and Lesh (2003), models make up of two main structures as interior – structures (conceptual systems) and exterior representational system. Models are used for interpreting, explaining, and defining the systems and during this process, multiple interactions among external representational systems exists. Technology enables richness in that it provides with alternative representational systems and new means of communication for individuals.

As the nature of modeling activities necessitates, as it keeps various solution approaches out of various perspectives, that it does not include algorithms which are not pre-determined, and determining the mathematical operations to be done according to student's solution approach, using technology is frequently needed for complicated mathematical operations and calculations (Johnson & Lesh, 2003). In one of his studies, Lingefjärd (2000) reported that as a result of the modeling activities carried out within a context that was enriched through technology teachers comprehended mathematics and their mathematical modeling skills improved.

A successful/efficient mathematical modeling process relies on using multiple representations flexibly and fluently (Lesh & Doerr, 2003a). According to Goldin (1998), for conceptual development, multiple representations occupy an important position on math teaching. Tables, graphics, diagrams or pictures, concrete models, metaphors, the spoken language and written symbols are the basic various representational systems (Lesh & Doerr 2003a). While each representation system brings the different aspect of a concept to the fore, it may disregard the other aspects. Hence, within a mathematics teaching process, for students to comprehend a concept with different aspects, it is recommended that multiple representations be taken advantage (Goldin, 1998; Kaput, 1987). In this context, the potential of technology that can present different representations simultaneously and easily comes to the fore. That is why, use of technology has been determined to be one of the fundamental components of the designed course; depending on the content of modeling activities, a lesson planning which enables using technological tools appropriately was paid attention.

Group work

Group work occupies a highly important position in education. After Piaget handled the development within learning, which Vygotsky referred to interaction with the social environment can be stated to be the basis of group work method is being thought to be an important component of teaching environments. In development and learning, it is stated that social, sensational, and cognitive faces interaction is important and that using group activities in teaching environments is beneficial for multiway development of children's thinking skills, past experiences, learning capacities from their friends and peers (Blachford, Kutnick, Baines, & Galton, 2003). In group work, there are some benefits as students with various interests and skills will be able to teach a lot to each other and hence, they will be able to learn more effectively by teaching each other. However, especially in the context of mathematics education, group work is not a method that both students and teachers are very familiar with. In using mathematical modeling within teaching of mathematics, that the group work is an indispensable method is one the points where the studies done in this field come together (Blum & Niss, 1991; Lesh & Doerr, 2003a; Zawojewski et al., 2003).

Group works are accepted to be significant in education in terms of social development of individual. When looked from the perspective of mathematics teaching, in addition to discussing and refusing a number of mathematical ideas which are not formal; the group work is the process of creating, developing a mathematical model or idea and commenting on that idea and combining this idea which is discussed as a group with its own idea infrastructure (Hoyles, 1985; cited in Ubuz & Haser, 2002). The mathematical semantics that composed in this way is the product of social interaction and discussion. During the discussion, two different aspects of the speech may be mentioned: They are those that a person can mention his opinions obviously and express them in the way that other group members can understand. These two aspects assist in that a group working explains the work on which they study to other group members and reach a consensus (Hoyles, 1985). Upon mathematical modeling activities, the identical role of group work is mentioned as well. According to Zawojewski and others (2003), in traditional problem solving activities, since what is expected to be resolved is mathematical (numerical) result, there is no need to be shared and thus the social aspect is very poor. However, the principles of modeling and model generalization principles within mathematical modeling activities enable the model developed to be shared and reused. That the modeling activities have social functions, which were mentioned above, expresses the notion that it should be done as a group work with a social aspect. Within modeling activities, during group work process, in general sense each student is interpreting the problem with his own external representation and these interpretations are discussed as a group. The most appropriate model is made up of only after each model that every individual asserts is discussed and evaluated. As the model developed during this process shall be used by others, students need to explain each process, method, and strategy. With the group members' evaluation of each other in group work, the teacher furthers away being the sole evaluation source. Of the reasons cited above, group work were determined as one of the components of the course and were taken into consideration upon planning in-class modeling activities.

Microteaching

Microteaching can also be defined as a teaching method that comes true by the practice of the knowledge related to teaching profession that prospective teachers have theoretically gained and used to enable prospective teachers to obtain teaching skills and improve them as well.

Microteaching technique offers a real teaching opportunity for prospective teachers that the real classroom environment confusion is minimized. The researchers define the microteaching variously. For instance, Allen and Eve (1968) define it as "a scaled down teaching encounter" and emphasize that "normal teaching encounter have been reduced and the level of feedback to the teacher has been greatly increased" in the microteaching (p. 181). However, these definitions include common phenomenon such as 'the prospective teachers teach in order to get a certain teaching skill gained upon a certain subject with a limited number of students within a limited time span (between 5 and 10 minutes) (e.g., Allen, 1980; McAlesse & Unwin, 1971).

Microteaching is one of the most fundamental elements in teacher education programs. During microteaching, while a prospective teacher takes the role of the teacher, the others take the student role. By creating a real classroom environment, the prospective teacher taking the teacher's role explain the subject determined by himself/herself by asking questions within a time span like 5-to-20 minutes. Meanwhile, the other prospective teachers taking the student role participate in the course process actively. The video recording of the microteaching, objective evaluations of the microteaching performance by the other prospective teachers and the instructor of the course, and the feedback provided instantly make up an important part of this process. The prospective teacher then is expected to revise his/her plans after these feedbacks , contemplate a new way of teaching, and practice it (Singh & Sharma, 2004). In addition to providing prospective teachers with learning experience via practicing and living, microteaching gets prospective teachers to gain teaching experience and skills in that it provides feedback and self-evaluation opportunities in the view of prospective teachers see their deficiencies and errors (Lakshmi, 2009). In a course planned to improve the pedagogical knowledge of prospective teachers' related to modeling in teaching process, with the idea that microteaching must be an indispensable component; it was decided that the content of this course must be a subject that needs to be focused on.

3.4 The Research Procedure

Under this section, the research procedure was reported in details in accordance with the key components of the course that were mathematical modeling activities, students' ways of thinking works, the use of technology, group work, and microteaching carried out by researchers and participants.

3.4.1 Pilot Study

"Mathematical Modeling for Prospective Teachers" course was offered every semester starting 2010-2011 Fall semester at Secondary Science and Mathematics Education Department of a public university In the first two semesters, implementations that new ideas may be developed upon determining the key components in the content of the course first two semesters and how to deal with these components in the course of lesson were fulfilled. In these two semesters both, 7 to 8 modeling activities were implemented and they served to the targets of developing and revising the implementations, and choosing the activities to be used in Fall semester of 2011-2012 (pilot study) and in the original study. In all three semesters in the context of pilot studies, great experiences were gained about the subjects such as managing the solution process of modeling activities, determining the phenomenon to focus on in reflection papers, maturing the ideas that prospective teachers developed during the process by creating efficient discussion environments. In the pilot study, 18 third and fourth grade BA students and three graduate MA students who study at elementary mathematics education program, totally 21 prospective mathematics teachers enrolled in the "Mathematical Modeling for Prospective Teachers" course. Twenty-one prospective teachers were divided into seven groups with three members in each. Throughout the fall semester, the planned course program (see Appendix A) was implemented by the course instructor and three researchers; one of them was the author of this dissertation. Survey forms, mathematical modeling activities and students' solution sheets, reflection papers written after the implementation of each modeling activity, focus group and individual interviews (semi-structured), observations supported with audio and video recordings, lesson plans, and presentations of prospective teachers through implementation experiences (microteaching) were the data resources of the pilot study. After the pilot study, the obtained data analyzed by three researchers collaboratively. According to results of the pilot study, some improvements and arrangements were made in the original study plan (see Appendix B).

3.4.2 Implementation Process of Designed Course

In the theory part and in the planning and the application parts of this chapter, the how of the determining process of the main components of the modeling class for the prospective teachers and the types of the applications in the curriculum were explained in detail. In this section, the core implementation processes of the designed course were presented in details. The core components of the course and the implementations were explained first and afterwards the overall flow of the course were reported.

3.4.2.1 Studies on solving mathematical modeling activities

For the main study, six modeling activities, "The Summer Job", "The Ferris Wheel", "The Street Parking", "The Bouncing Ball", "The Free Roller Coaster", and "The Water Tank", are planned to be worked on with prospective teachers (see Appendix C). As the attendance was low on the first week of the class (3-4 students), the modeling activities were delayed for a week just like other implementations; however there were no problems in the implementations afterwards.

In the pilot study, there was a serious timing problem in the classroom implementations of modeling activities; the time was not enough for group presentations and for the classroom discussions; hence, the planned theoretical presentations and discussions were not held. As a result, for each activity specifically detailed implementation plans were developed for the original study, and the timing was made more realistic (see Appendix D). Even though the specific implementation time of each implementation is different from the rest, a general timing organization was prepared such as five minutes for individual work, 90-120 minutes for group work and 30-50 minutes for classroom discussion. Thanks to the detailed preparation of the implementation plans and the creation of a more realistic time scheme by means of increasing implementation durations, there were no serious timing issues during the classroom implementation of modeling activities. Group presentations and the classroom discussions were held during the implementation of modeling activities.

During the group presentations, a documenting camera was used. By reflecting from the documenting camera, the group solution paper was shared with the class. Thus, the presenter was able to show their solution steps to the class more easily while their classmates were able to follow the solutions of the groups more clearly. When the solution papers of the groups were reflected on the screen during the presentation, they were scanned at the same time by using the documenting camera to have them computerized. At the end of the class, an e-mail of all the group solutions with the computerized copy attached was sent to their relative groups. The computerized attachment of the solution papers was quite beneficial for those students who are at the stage of writing a reflection paper. Compared to the implementations at the pilot studies, we can say that the group presentations and the classroom discussions were more systematical and less prone to errors in the original study. During the implementations process of activities, the researcher acting as the instructor tried to play his/her role in the best possible way and the implementation plans, which were prepared in great detail to complement the plans, guided the researcher well (see Appendix D).

3.4.2.2 Group work

Twenty-five students, nineteen of whom was on their third and six of whom was on their 4th or 5th years, enrolled to the course. For the class, from the first activity on, the prospective teachers were grouped in three and in four members in each group and scheduled to work with the same group for the modeling and the students' ways of thinking activities together. The grouping process was left to the preferences of the students at the beginning and afterwards the newcomer students randomly grouped by themselves. As the attendance was at its lowest at the first week, the 25 students, who registered for the class at the second week, grouped in three and four members in each groups to form seven groups in total. Three of the groups contained three members and four of them contained four members each. Students carried on all of the group activities with their group mates during the semester.

3.4.2.3 Technology usage in the modeling

For the class, at the second week, a general presentation about the usage of technology in mathematics education was held and it was followed by a small workshop on the MS EXCEL, a spreadsheet software, for an hour (see Appendix B). For the implementation of the following modeling activity, enough graphic calculators (Casio ClassPad) were brought to the class and given to the students who needed them. The researchers, including the author of the current study, acted as observers and helped the students when they encountered problems in using the calculators.

3.4.2.4 Classroom works on students' ways of thinking

Among the six modeling activities, the prospective teachers chose four called "The Free Roller Coaster", "The Bouncing Ball", "The Street Parking", and "The Water Tank", and planned group and class discussions using the classroom video episodes and student solution papers. The planned activities on students' ways of thinking were implemented on the 6th, 8th, 11th and on 13th weeks (see Appendix B). As mentioned before, due to not having classes on the first week, these activities were applied with a one-week delay.

Analyzing students' ways of thinking activities contains studies including both the written and the visual-auditory students' ways of thinking in the form of "student solution papers and video episodes of the student solution papers". In accordance with the purpose of analyzing students' ways of thinking activities, written and visual work of the students are gathered from in-service part of the same project. These works are chosen and organized according to some specific directives in order to be used in these studies. Below, the preparation and the application process of the student solution papers and video feeds are given in detail.

Preparation of student solution papers and video episodes

For each activity that was planned to be used in the class, all student solution papers gathered at the classroom implementations of in-service part of the project were examined (approximately from 10 – 12 different groups) and for every activity, solution papers from 4 or 5 different student groups that contain different approaches to the same problem (whether they are right or wrong) were chosen. The reason why the number of the student solution papers was limited to four or five is eliminating the sheer number of papers would allow prospective teachers to work and analyze in more detail as solution papers were supported with video episodes. Thus, in the elimination process of the papers, variety in the understanding and comprehending the question; the usage of different mathematical subjects, concepts and signifiers; the mistakes in arithmetical, geometrical, logical and intuitive areas; trouble with some concepts and processes and finally representing the thought patterns of students were some of the points taken into consideration. In addition to the written work of the students, video episodes that support these works were also supplied. Before editing the videos, two researchers watched all of them repeatedly with great care in order to create a consensus on which parts of the videos would be used. The editing of the students' videos was done in three different ways. The first type of the video episodes belonged to student solutions and they are derived from videos of student presentations during classroom implementations. The second type of the video episodes were the videos of a focused group during the classroom implementations. In other words, these are the videos showing the processes of students' mathematical thinking in more detail. The type of the video episodes were the videos showing the general a modeling process of implementation of modeling activity. These videos showed the solution processes of groups (their first approaches to the solution, their arguments during the process, the solutions they came up with, the student mistakes and difficulties they had faced), the solution approaches they developed and reflected the role of the instructor during the implementation of the modeling activity.

"Windows Movie Maker" and "Wondershare Video Editor" are the programs used in the video editing process. In accordance with the criteria defined above, the selected parts of the classroom implementations are cut with these programs and combined in a logical way to create 7-8 minutes of video clips.

3.4.2.5 Presentations and classroom discussions

It was previously mentioned that during the modeling course, the six modeling activities, which the prospective teachers worked on with their group mates, would help the development of important ideas and capabilities of those teachers about the classroom implementation of mathematical modeling as it was stated in the objectives of the course. As it was in the pilot study, in addition to the knowledge developed in this practice, it was decided that the classroom discussions supported with theoretical presentations on classroom implementations and discussions on the nature of mathematical modeling would be effective in providing prospective teachers with theoretical knowledge as well. Thus, on the 5th week "Classroom discussion on the pre-evaluation of modeling activities", on the 5th and on the 9th weeks "Presentation and discussion on the nature of modeling activities", on the 6th week "Presentation and discussion on the effective group work", and on the 7th week "presentation and discussion on the role of the teacher in modeling activities" were planned (see Appendix A).

As there were no classes on the 1st week, all applications had shifted a week. "The pre-evaluation of modeling activities" and "Presentation and discussion on the nature of modeling activities" were planned to be held on the 5th week. In the regular plan, the classroom discussion on these two subjects was to be held on the 6th week; however, since the "Work on students' ways of thinking with *the Street Parking*" took longer than a class hour, no other applications could be held that week. As a result, "The pre-evaluation of modeling activities" and "Presentation and discussion on the nature of modeling activities" could only be held at the 7th week, after the implementation of "*the Bouncing Ball*" activity (see Appendix B). In these discussions, prospective teachers sharde their thinking on the nature of the modeling activities in accordance with the four modeling activities they have solved and three modeling activities they have studied, the modeling process, definition of modeling, and how the classroom implementations should be in addition to the weekly reflection papers that they have written.

Again in the original study plan, the discussion and the presentation on "Effective group work" was planned to be held on the 6th week, the discussion and the presentation on "the Teacher's role in the implementation of modeling activities" was planned to be held on the 7th week, and the discussion on "The Nature of modeling activities and the modeling process" was planned to be held on the 9th week; however, these discussions was held in one single session in the original study. At the 8th week, a two-hour discussion, where each and every one of these topics were mentioned, and spontaneously theoretical presentations were held. The researchers decided to carry out all these presentations, which were planned to be held in three different weeks, in one single week.

3.4.2.6 Microteaching

Since the role of the teacher is vastly different compared to the traditional methods in the classroom implementation process of the modeling activities, it was thought that it would be beneficial for the prospective teachers if they presume the role of the teacher in a modeling implementation of their own design. In the original plan, it was planned that the three weeks of the class (the 12th, 13th and the 14 weeks) were to be reserved for implementation experience of prospective teachers. However, since there were no classes on the first week, the 10th week was reserved for the midterm exams, and the school was vacant at that week, we lost two weeks of the planned program. Hence, we were able to reserve only a week to microteaching. Since it was impossible for seven groups to present their applications in one-week, after arranging with students, we had extra classes after the semester was completed. This week, which is shown as the 16th week of the program schedule at Appendix B, was used for implementation experinces of prospective teachers. In other words, the implementation experience that was planned to be three weeks could only be done for two weeks due to the lost time. On the 15th week, three groups and on the 16th week four groups implemented their modeling activity in 45 to 60 minutes.

Since the groups consisted of three to four members, groups were asked to be organized in manner at which one of the group members was in the role of teacher during the implementation process while the others observed the groups and took notes. Prior to the implementation, the prospective teachers created an implementation plan as a group and during the implementation, they were asked to bring the plan to the class so that the researchers could have a copy of it. Besides, the prospective teachers wrote individual reflection papers on the implementation experience (see Appendix M).

3.4.2.7 Revisions in course schedule

Even though the changes are mentioned briefly above under for each implementation, it could be list the changes in the program schedule in short as follows, (see Appendix A and B):

- On the first week of the semester, the class was not held due to low attendance. As a result, all implementations were conducted with one-week delay.
- Because of the delay, the “The pre-evaluation of modeling activities” and “The nature of modeling activities” presentations and discussions was supposed to be held on the 6th week. However, the implementation of “*the Street Parking*” activity on students’ ways of thinking took longer than expected; it was not

possible to hold the previously mentioned presentations and discussions. This theoretical presentation and discussion studies took place at the 7th week, after the implementation of "*the Bouncing Ball*" activity.

- The theoretical presentation and classroom applications that were supposed to take place on the 6th, 7th and on the 9th weeks of the planned program were held on the 8th week and accomplished in one session. The reason of this change in the program was that the students' ways of thinking activities took longer than expected. During the planning, activities of analyzing students' ways of thinking were given 100 minutes each and expected to be followed with theoretical presentations and classroom applications immediately afterwards.
- The fact that during the midterm exams, the classes are not held in the university where the implementations take place was not foreseen during the planning process. Thus, in the original study, classes could not be held during the 10th week. For this class, at the final exam week, a make-up class was held in a date arranged and decided with all students.
- "The General Evaluation" (Survey forms and discussion) study was planned to be held on the last week of the classes; however, it took place on the 14th week, before the implementation experiences of prospective teachers in the actual implementation. The reason why the "General Evaluation Study" took place two weeks before the planned time as it was considered that with the approaching final exams, there could have been a decline in the attendance.
- Even though the implementation experiences were planned to be held for three weeks on the 12th, 13th and the 14th weeks, due to the loss of time on the first week, the actual implementation took place on the 15th and the 16th weeks for two weeks.

3.4.2.8 The role of the researcher

Since this study was carried out as a part of much larger project, pre-service teacher education part of the project were conducted by four researchers. In addition to the instructor of the course, three researchers (scholarship students/bursars), one of them was the author of the current study, also participated in the class for data collecting (taking observation notes about the application) and for technical help (videotaping, maintenance of voice equipment, technical help with projectors etc.). The author of the current study and other observers did not take part in the teaching process or facilitating in the group discussions and in the class presentations; whereas they observed the class or a specific group. Each researcher was also in charge of a camera and did not take part in any other activity than observing the groups they taped. In the pilot study process, some decisions were made concerning how researchers should act during the class. These are;

- The researchers will plan the class extensively and in the day before the class will meet with the instructor of the course to share the planned process with them with all details.
- The solutions of the activities that will be applied and different solution approaches (if any from the previous applications) will be presented to the researcher conducting the class extensively.
- As long as it becomes necessary, nobody other than the instructor will participate in-class activities.
- The students will ask their questions only to the instructor.
- During the application of the activities, nobody other than the instructor will interact with the students, including the groups they observe.

These decisions were conducted by the researchers with great care. Preparations for the class and the planning of the class were completed by the three researchers before the semester started. Besides, the lesson plan and a class preparation meeting about what would be done that week was conducted with the researcher conducting the class every week, a day before the class and before almost all classes. In these meetings, the planned contents of the class were examined and the implementation plans were finalized. In addition, after every class, an evaluation session was held and the plans of the next class, as well as the semester plan was overviewed.

3.4.2.9 Classroom organization

Since the classroom, in which the implementation was carried out, was suitable for group work, on the first week no additional organization was necessary. Seven of the eight tables in the class was given to allot for the students' use. The distance between the groups was adjusted in a manner that will minimize the interaction between the groups. After organizing the class, the camera was placed in the best possible place to record the interaction between the group members and groups. The three groups to be recorded was decided during the introduction meeting at the second week and during the semester, these same three groups were recorded. In addition, a fourth camera was placed to oversee the general state of the class and to record the classroom discussions. In addition to the video recording, the discussions of all groups was recorded with a voice recorded.

3.5 Data Collection

Since the current study was conducted as a part of much larger project, the data used in this study was collected with the project data synchronously. The data used to achieve the goals of the study were gathered using (i) survey forms (pre- and post-), (ii) studying papers that prospective teachers used during the modeling activities, (iii) observations supported with video feeds and voice recordings, (iv) reflection papers that prospective teachers wrote down after each modeling activity, (v) semi-structured interviews, (vi) students' way of thinking sheets, (vii) the lesson plans prepared by the prospective teachers, and (viii) the presentations of the prospective teachers (microteaching). Below, it was explained in detail how each of these data resources are used during the application process.

3.5.1 Data Collection Tools and Procedures

Under this section, instruments that were used in collecting data and how data were gathered throughout the study were described in details.

3.5.1.1 Survey Forms

In addition to the classroom observations, the change in students' thinking about and ideas on the usage of mathematical modeling was planned to be recorded with two survey forms at the beginning and at the end of the semester and the plan was conducted (see Appendix G and Appendix H). On the pre-survey form, the prospective teachers were asked to evaluate mathematical modeling in general, their knowledge on mathematical modeling activities, their ideas on the advantages and disadvantages of using modeling activities in mathematics classes and the role of mathematical modeling in students' comprehension of mathematical structures. These pre-survey forms took place on the 2nd week of the semester. The post-survey form took place at the “General Evaluation” meeting at the 14th week of the semester. The questions of the post-survey were similar to that of the first one; however, this time the students also included a general evaluation of the class. Both surveys took approximately 60 minutes. During the classroom discussions and with reflection papers, the prospective teachers thought on the subjects given in the survey forms and explained their opinions. It can be said that the surveys were an important source of data in terms of letting the teachers present their thinking in a compact manner.

3.5.1.2 Modeling Activities

The solution processes and the reports of the modeling activities were the core element of the content of the course and they were designed as a basis for prospective teachers to develop important ideas concerning other components. For mathematical modeling and the classroom implementations, six activity implementations, which were predicted to develop the basic knowledge and the skills, were planned. These six modeling activities (see Appendix C) were chosen from activities that were tried out as the pilot of the project in high schools and undergraduate level primarily. On the order of the implementation of the modeling activities, factors such being consecutive in nature and reflecting the nature of modeling process were taken into consideration. Thus, the activity called “*The Summer Job*”, which we thought to reflect the modeling process and its nature the best, as the first activity. After that, activities called “*The Ferris Wheel*” and “*The Street Parking*” were chosen in relation to the trigonometry subject. “*The Bouncing Ball*” activity was chosen as it was more structured and the solution path was more defined. “*The Free Roller Coaster*” and “*The Water Tank*” activities were chosen as the last two activities as they were related to graphic reading, functions, and derivatives.

All of these activities were also applied on high school level. Four of the activities that were used on high school level were used to make the prospective teachers examine and analyze the students’ ways of thinking documents. The implementation duration of the activities were shown on the implementation plans that were prepared specifically for each activity (see Appendix D). The first five minutes were allotted for individual reflection and working on the activity, the next 90-120 minutes were allotted for group solution of modeling activities and the last 40 minutes were allotted to the students’ presentations about modeling activities with the class and to the classroom discussion. The instructor used the previously prepared activity implementation plans as a guide at the implementation of the activities (see Appendix D). The group solution papers of the modeling activities were collected at the end of each implementation. Additionally, the solution papers that were reflected on the screen via documentary camera were electronically saved. The group solution papers will be used during the data analysis process together with the other data sources.

3.5.1.3 Reflection Papers

The prospective teachers were asked to write a report (reflection paper) after each activity in accordance with the reflection paper directive and the plans (see Appendix I). For each one of six the implemented modeling activities, the prospective teachers wrote reflection papers. After the implementation of first activity, the papers were read by the three researchers and the students were given detailed feedbacks. After the first reflection paper, the instructor gave a general evaluation presentation about developing the missing points in their reflection papers (see Appendix J). The later redlection papers were graded out of twenty and the students received feedbacks weekly. In addition, the students wrote a reflection paper after the activities of analyzing students' ways of thinking in accordance with the previously prepared directives (see Appendix F). After analyzing students' ways of thinking, which was conducted with four activities in total, reflection papers on students' ways of thinking were graded out of 10, and the students were given feedbacks.

3.5.1.4 Focus Group Meetings

For the prospective teachers, focus group meetings about the reflection papers, which they prepared about the activities they conducted, were planned. These discussions took place in the class for 15-25 minutes with the participation of all the students after each activity. In these discussions, subjects like mathematical concepts embedded in each activity, possible solution approaches and how, when and on which level they can be used at high school level were discussed. Additionally, at the 7th and the 8th weeks, a general classroom discussion following presentations on "the nature of modeling activities/The modeling process" and "The role of the teacher during the implementation of modeling activities". After the implementation experiences of prospective teachers, 10-15 minute meetings were held with the group about the implementation they carried out. In these meetings, the groups were asked about their expectations before the implementation process and their opinions on the implementation experience. The aim was to let prospective teachers reveal what they learned and developed during the classroom implementations of modeling activities in theory and in practice. All meetings were recorded with a voice recorder so that they could be used in the data analysis process to answer the research questions.

3.5.1.5 Observations Supported with Audio/Video Recordings

Three researchers (one of them was the researcher of the current study) other than the researcher who were the instructor of the course participated in the class as observers. During the research, four cameras and seven voice recorders were used. Two of the researchers observed two groups chosen at the beginning of the semester while the other overviewed the general atmosphere of the class with a camera recorder. The author of the current study used one the cameras to capture the general atmosphere of the classroom and to observe the actions of instructor overlooking the groups. A forth camera was used to record the studies of other groups without the consulting of a researcher. In addition to the cameras, the working process of each group was recorder with voice recorders, as well. The observations took place as two researchers observing the same group during the semester while the researcher of the current study observing whole class and taking notes.

3.5.1.6 Semi-Structured Interviews

Individual interviews that aim to reveal the group study process and the individual thinking processes had been held (see Appendix K). In the choosing process of the students to be met, two principles were taken into account, which were having a representative from each group and volunteering. With every student that had been chosen in accordance with these principles, ten meetings were held during the semester. Six of these meetings happened after the implementation of modeling activities and four of them happened after activities on students' ways of thinking. The timing of the meetings was decided at the beginning of the semester for the free time of both the researchers and interviewees. Moreover, all along the semester, these meetings were done with the same prospective teachers by the three researchers. With the permission of the prospective teachers, the meetings were recorded. Each meeting took approximately 30 minutes.

3.5.1.7 Lesson Plans

In the original course plan, after the implementation of six modeling activities, preparing lesson plan activities were planned. To achieve this, the prospective teachers held a group for the activity called "*the Water Tank*", which they have solved before, and created a lesson plan to implement the modeling activity to 10th grader students (see Appendix B, 13th week). During this study, implementation plan developing form was used (see Appendix L). The implementation plans created by the groups were shared with the class and a class discussion was held.

3.5.1.8 Microteaching experience

The preparing implementation plan activity at the 13th week of the class was an important experience for the students' creation of lesson plans for the implementation of activities that they have developed as a group. The last two weeks of the class was reserved for the microteaching activities in which the students were to implement the modeling activities they created as a group in class. The groups first developed a modeling activity. Before the developed activities were applied, the groups studied on an implementation plan and they submitted the implementation plans to the course instructor. After the implementation experience, each group wrote an individual reflection paper according to a directive (see Appendix M). Additionally with every group, a 10 – 15 minute meeting, which was recorded, about the implementation experience was held. The implementation experience plans, the observations during this implementation process, the individual reports of the prospective teachers and the meetings after will be essential data sources for the research questions on the classroom implementations of mathematical modeling and the pedagogical knowledge of the prospective teachers.

The data were collected from all of the participants except semi-structured individual interviews. Interviewees were selected voluntarily among each group who were representatives of their groups in order to reveal the group study process and individual thinking processes (see Table 6). That is, researcher interviewed with seven prospective teachers from each group after the implementation of each modeling activities.

While reporting the results obtained from data analysis, it is not possible to report the development of all prospective teachers by presenting each stages the prospective teachers passed through due to the nature of case study research.

Therefore, four groups (Group 1, Gorup 2, Group 5, and Group 7) were selected from seven groups by mathching the groups in which prospective teachers stated similar views and showed similar developmental stages in their thinking. In order to reflect the evolution of prospective teachers' thinking about mathematical modeling and modeling activities, the researcher presented excerpts and episodes mostly from the representatives of four groups (PT17, PT14, PT9, and PT24) to present the change in other prospective teachers' thinking about mathematical modeling and modeling activities for answering the first research question. Prospective teachers who indicated different views apart from others about the issues also designated.

Table 6 Interviewees selected from each group

Groups	**Group representative**	**Class**	**Number of members**
Group 1	PT17	4	3
Group 2	PT14	3	4
Group 3	PT5	3	3
Group 4	PT10	3	4
Group 5	PT9	3	4
Group 6	PT23	5	3
Group 7	PT24	3	4

In order to demonstrate and gain an insight about what was going on about discussions occured in each group, the views and thinking of group representatives were presented in the tables and episodes as much as possible. Evidence that reflects almost all of the prospective teachers' thinking were presented throughout the results chapter.

3.6 Data Analysis

In general, qualitative data analysis consists of four stages, which are (1) coding data, (2) establishing themes, (3) arranging data with respect to themes and codes, and (4) interpreting obtained findings (Yıldırım & Şimşek, 2008). In the analysis of the data, conceptual framework was developed through the analysis of previous studies and the grounded theory approach (Glaser & Strauss, 1967; Strauss & Corbin, 1998).

The data were coded by “creating provisional start list of codes prior to field work” (Miles & Huberman, 1994) and using open-coding technique (Glaser & Strauss, 1967; Creswell, 2006).

3.6.1 Arranging and Coding Data

The data analysis illustrated as “There is no particular moment when data analysis begins. Analysis is a matter of giving meaning to first impressions as well as to final compilations” (Stake, 1995, p. 71). Analyzing process began with organization. The data obtained by using different methods were primarily organized. The written documents that were belonging to students’ classroom works were transferred to electronic environment. Solution reports for the modeling activities, lesson plans, works for the nature of mathematical modeling activities, and students’ reflection papers were scanned and their electronic copies were created. Classroom video records and interviews were transcribed. After the creation of electronic copies of the data, these were transferred to a qualitative data analysis software Nvivo (v. 8) (QSR Int., 2008). Then, the coding stage started with the coordination of three researchers, whose one them was the researcher of the current study. Coding defined as “the process of segmenting and labeling text to form descriptions and broad themes in the data” (Creswell, 2011, p. 243). The following way was pursued when coding and in-depth analysis: First, to answer each research question, data resources and instruments were determined for each research question. By reviewing the available conceptual frameworks and used terminologies that were related to how to analyze the data and can be used in naming codes, estimated codes and themes that might arise in relation with each research questions were identified. Then, the subject of each research question to be associated with which dimension of the research and what would be the subcomponents of each dimension, the data were restarted to be analyzed after the determination of conceptual frameworks. The researcher classified the data resources according to research questions by reviewing the relevant data resources.

The analysis of the data began after the classification of data resources and determination of conceptual framework. Since almost all of the data sources included qualitative data, qualitative data analysis conducted. While making qualitative analysis, data were analyzed according to data sets one by one. For instance, first, prospective teachers’ reflection papers were analyzed. Because of this primary analysis, researchers created temporary code list that came from the profound investigation of prior research and field. Then, transcribed interviews were analyzed and new codes were combined with the list of codes from previous data sets. The final version of code list obtained after the analysis of all of the data sets. In order to establish a consensus on naming codes, each researcher examined the same data set individually. Then, researchers conducted the works together like determining repeated codes and specifying new codes for newly arising situations and naming codes. In the parts that researchers had different thoughts, the arising situations were analyzed in-depth by researchers together and researcher coded these situations when they reached 80% agreement level with other two Ph. D. students in the same project.

3.6.2 Establishing and Organizing Themes

Code list that were obtained after the analysis of all data sets were examined by the researcher. Before starting to data analysis, related codes in the code list were grouped up under the themes by taking the themes that were emerged during the literature review into account. The work of naming themes was carried out by researcher and two Ph. D. students who worked for the same project. The findings of the research were interpreted by using themes, which were obtained after the data analysis, and its sub-dimensions.

3.7 Reliability and Validity Issues

Reliability and validity concerns are not only in quantitative research terminology, but also in qualitative research designs. Nevertheless, these issues have not the same meanings in quantitative and qualitative research approaches. Validity in qualitative research designs defined as checking for correctness of the findings that involves a set of procedures within and the reliability in qualitative approaches means that “the researcher's approach is consistent across different researchers and different projects” (Gibbs, 2007; cited in Creswell, 2009, p. 190). In order to maintain reliability in qualitative research designs, for example in case study research, several recommendations expressed by various researchers. For instance, Yin (2003) offers that procedures of case studies ought to be documented and these ought to be captured in a database. Gibbs (2007) recommended various reliability methods. These are controlling transcribed data whether there exist any apparent mistakes in the process of transcription, ensuring that there is no problem with codes in terms of definition and meaning, inter-reliability checks (crosscheck) between coders (cited in Creswell, 2009). Miles and Huberman (1994) suggested that it is need to be at least 80% of time in the agreement in order to maintain satisfactory qualitative reliability.

Although many researchers mentioned about reliability and validity issues using different terminologies (e.g., Stake (1995) and Patton (2002) uses "triangulation", Yin (2009) prefers "a chain of evidence"), Lincoln and Guba (1985) opted credibility, transferability, dependability, and confirmability instead of internal validity, external validity, reliability, and objectivity respectively. Lincoln and Guba (1985) indicated the reason for their preferences, as suggested terminologies are more appropriate for qualitative research approaches concerning their natures. Several validity strategies proposed in order for supplying and showing evident for the principles. According to Lincoln and Guba (1985), *prolonged time in the field*, *data triangulation*, and *member checking* are suggested procedure in order to fulfill credibility principle. Transferability principle is supposed to be supplied with using *rich*, *thick description* of settings. The dependability principle need to be evidenced with presenting *dependability audit*. As the last principle, confirmability, ought to be supported with *confirmability audit*.

Within the current study, *prolonged time in the field* was satisfied by pursuing the following procedure: The study with its piloting phase took almost one year (two semesters) to complete. In this period, the researcher built a profound understanding of the phenomenon and detailed information about the setting and participants. Since the current study carried out with much larger project and the researcher had dual role within these studies, the researcher became familiar with participants in their real settings.

In the study, distinct data resources (e.g., survey forms, transcriptions obtained from semi-constructed one-to-one interviews and classroom discussions audio and video recordings, focused group interviews, reflection papers, students' solution papers for modeling activities, lesson plans, presentation reports, field notes, observation notes, etc.) were utilized to satisfy the *data triangulation*. This provided the researcher to construct a sound justification for themes and codes.

After the implementation of "Mathematical Modeling for Prospective Teachers" course, the data analysis was started. The researcher established themes by using open-coding method. When coding session completed and themes got the latest version, the report written by the researcher shared with some of the prospective teachers (due to the difficulty to reach all participants in summer holiday), who attended the implemented course, in order to get their comments about the correctness of themes and about interpretation made by the researcher (*member checking procedure*). The participants responded the reports positively and most of them commented that established themes and the findings of the study reflected their thinking as it was like in the transcripts of audio and video records. The field notes and observations made by the researcher also checked out by the participants in order to eliminate possible misunderstandings.

In the results chapter, in order to reflect the development of prospective teachers' thinking about mathematical modeling and its use in the classroom setting, *rich and thick descriptions* of findings made by the researcher. When it is looked the findings, direct participants' quotes, and one-to-one interviews transcripts were utilized to support the findings. In this wise, the audience of the current study can follow the findings one their own and make their own conclusions by interpreting the given data.

In order to ensure the reliability of the data collection procedure and coding process, two researchers who worked in the same project as Ph. D. students in mathematics education checked. Gibbs' (2007) suggested reliability procedures applied in the period. The researcher and his two colleagues coded the analyzed data independently and they developed crosscheck codes. According to inter-rater dependability, it was reached 84% intercoder agreement (Creswell, 2009; Miles & Huberman, 1994) on these codes (*dependability audit*). Specifically, the coders disagreed in coding and categorizing the prospective teachers' thinking about the use of mathematical modeling activities in their future classrooms in terms of the aim of use (cognitive-affective), the frequency of use, the place of use (before the subject matter or after etc.), and the method of use (individual-group). Afterwards, all researchers reached the agreement on the non-agreed codes by comparing and contrasting by describing how they define and establish their codes.

Two researchers were also confirmed the obtained raw data in order to validate the findings, interpretations, conclusions, and recommendations in the current study that could be deduced the consequences from the obtained raw data (*comfirmability audit*).

CHAPTER 4

RESULTS

In this chapter, the results of the current case study research that were obtained from an in-depth data analysis procedure are presented. In reporting, findings were presented according to chronological order in order to reflect prospective teachers' progress.

Since we are dealing with the prospective secondary mathematics teachers who took the "Mathematical Modeling for Prospective Teachers" course as a class, my research questions are mostly associated with tracking their progress. Our unit of analysis is to note if there are any changes in conceptions of prospective teachers about the use of mathematical modeling in the teaching of mathematics. We also note whether prospective teachers' thinking about pedagogical knowledge of mathematical modeling developed or not during spring semester in 2012. In order to demonstrate the overall changes and progress, evidence was presented by giving direct quotations and episodes are shared from the selected participants in interviews and from other prospective teachers.

Firstly, the findings about the change in conceptions are presented in chronologic order providing that the change in prospective teachers' conceptions about mathematical modeling and its use was observed by the audience. Secondly, the findings about the development of prospective teachers' thinking about pedagogical knowledge of mathematical modeling of prospective teachers in classroom environment will be presented again in chronological order to reflect the development in the process.

4.1 The Evolving Conceptions of Prospective Secondary Mathematics Teachers on Mathematical Modeling and about the Nature of Modeling Activities

It is crucial that prospective teachers' conceptions and descriptions of mathematical modeling and their conceptions about the use of mathematical modeling in the classroom setting are understood in order to conduct modeling activities successfully. Therefore, prospective teachers' preexisting knowledge and conceptions about mathematical modeling were determined by using pre-survey forms including open-ended questions at the beginning of the semester. The results are reported according to how conceptions of prospective secondary mathematics teachers about mathematical modeling and its use in classroom evolve throughout the semester. Rich details are also provided from their written documents, interview transcripts, transcripts obtained from audio and video records, and episodes from recorded videos in chronological order.

4.1.1 Prospective Teachers' Conceptions of Mathematical Modeling

Two survey forms were given to prospective teachers before and after the course implementation. Prospective teachers' descriptions about mathematical modeling were analyzed according to their answers on both longitudinal surveys. The findings showed that almost all of the prospective teachers had a conception that mathematical modeling is related to using concrete manipulatives and visualization of abstract mathematical concepts before taking the mathematical modeling course. Few prospective teachers who enrolled in the course indicated that they had taken a modeling course that was different from the current course in terms of instruction methodology, content, and purposes. Even though the content and purposes of the courses are distinct, the prospective teachers who had attended modeling courses before had their own description of the mathematical modeling concept.

Prospective teacher, PT17, was a member of group 1 in the classroom. The members of group 1 said that they had taken a course on modeling. Therefore, prospective teachers in this group showed that they had a preexisting conception about mathematical modeling. In an open-ended pre-survey, PT17 wrote, "Mathematical modeling is the exemplification of a mathematical concept by relating it to real life." This quote demonstrates that PT17 had a clear description of mathematical modeling at the very beginning of the semester. After the implementation of the modeling course, post-survey forms were given to prospective teachers in order to identify what they learned from the course about mathematical modeling. PT17 illustrated the mathematical modeling as “making a relationship between a mathematical problem and a situation in daily life, and the object or situation which is associated with it is called as model.” It is understood from the quote that PT17 developed her modeling description from exemplification to relating mathematical problems to real life situations by adding the new model definition. This situation shows that PT17 changed her thinking and definition of mathematical modeling after the implementation of the course.

PT14 was one of the prospective secondary mathematics teachers and participated in the study as a 3rd grade student. She was a member of group 2 that included four members. In the pre-survey form, PT14 indicated that she had often heard about mathematical modeling as a concept and had encountered it a lot. She illustrated mathematical modeling as follows:

> I think mathematical modeling contains the methods and techniques which are used for facilitation, using concrete manipulatives, and visualization of teaching and comprehensibility in mathematics education. For example, a mathematics problem can be rendered as more meaningful and interpretive by using a computer. It will be important for teachers to facilitate understanding and give form to mathematics by using techniques like these (*PT14, pre-survey form*).

From this quote, for PT14, mathematical modeling is using concrete manipulatives and visualization of mathematical concepts rather than relating real life situations to mathematical concepts. It is indicated in the section above that most of the prospective teachers denoted mathematical modeling as using concrete manipulatives and visualization of mathematical concepts. PT14 was one of those prospective teachers who agreed with the mathematical modeling illustration given in the quote above.

Although PT14 depicted the mathematical modeling as using concrete manipulatives and visualization of mathematics and mathematical concepts, it is understood from her comments in the post-survey form that she changed her conception of mathematical modeling to include the relation between mathematical concepts and real life situations.

> Mathematical modeling is the process of finding a solution for situations or events that creates a problem for teachers in daily life by using mathematical concepts and mathematical ideas. Nevertheless, we could not find the solution for every event, situation, or concept by approaching it mathematically. That is, we could not produce a mathematical model for each different kind of event. The problems that we considered mathematically or found solutions for by incorporating the mathematics itself are models for us (*PT14, post-survey form*).

In the comment above, PT14 points out the relation between the mathematical concepts and real life situations. It is clear from the above excerpt that PT14 changed her conception of using concrete manipulatives and visualization of mathematical concepts to making parallels between mathematical concepts and real life situations. She also stated that there are situations that arise where no solution could be found for that specific event using mathematical modeling activities. It can be drawn from PT14's statement that she developed an idea in the process and by the way she defined the mathematical model and gave an example of it.

Another one of the prospective secondary mathematics teachers who enrolled in the course was PT9. She was a member of group 5. This group consisted of four members and none of them had taken courses about mathematical modeling. Before the implementation of the course they were asked, "Have you ever heard of the expression 'mathematical modeling' before? Explain what you understand from this expression by giving examples. PT9 responded as follows: "Two things come to my mind. First, it is the expression of one event mathematically. The other is that it is the modeling of a mathematical problem visually, that is, preparing materials." It can be drawn from her quote that PT9 thought of mathematical modeling as visualizing the mathematical problems by using concrete materials, that is, by using concrete manipulatives of the mathematical problems and concepts.

After the implementation of the modeling course, PT9 reported in the post-survey form that she described the mathematical modeling as follows: "The presentation of a mathematical subject as it relates to problems from daily life and noticing that there are solutions for many real life problems that we experience associated with mathematics". From her quote, we see that she altered her previous conception of mathematical modeling with a new conception that relates mathematics to real life situations.

PT24 was another participant. He was a member of group 7 and attended the mathematical modeling course. As other prospective teachers mentioned above, he described mathematical modeling as using concrete manipulatives for the mathematical concepts in order to comprehend them easily: "The expression of 'mathematical modeling' is the work of displaying how a mathematical expression functions in a real life context and establishing more concrete materials that facilitate the understanding of a mathematical concept by reducing from the abstract to the concrete. Nevertheless, I do not have enough knowledge about this subject since I have not taken this course." From PT24's statement, it can be said that even though he had not taken any course about mathematical modeling, he commented that mathematical modeling was related to embodying the mathematical concepts in order to facilitate their understanding. On the other hand, in the post-survey form, he explained mathematical modeling as being associated with a real life context: "Mathematical modeling is the embodying of abstract mathematical concepts in a student's mind by relating mathematical subjects to real life." This quote demonstrates that there was a change in his conception of mathematical modeling, but he still puts emphasis on using concrete manipulatives for teaching mathematical concepts, but in the minds of the students.

To summarize, the results demonstrate that prospective mathematics teachers' conceptions about mathematical modeling have been subject to a change throughout the implementation of mathematical modeling course. Although almost all of them had a conception that mathematical modeling was the using of concrete manipulatives and visualization of mathematical concepts before the implementation of the modeling course, they changed their conceptions about mathematical modeling by making the relation between mathematical concepts and real life situations. Since prospective teachers reflected their groups' considerations, more examples can be provided which reflect the change in the conceptions of the other prospective teachers. For example, PT23 described mathematical modeling before the modeling course as follows: "We dealt with mathematical modeling in the course on 'Teaching Methods in Mathematics Education'. Mathematical modeling in some ways is the work of displaying embodiment. I can express a square with one side length x by the expression of x^2 and a rectangle with lengths x and y by the expression of xy ". From this quote, PT23 perceived mathematical modeling as using concrete manipulatives for teaching mathematical concepts. After the implementation of the course, PT23 defined mathematical modeling as follows: "In my opinion, a mathematical model is associating a mathematical problem with an object in real life. However, mathematical modeling is the application of mathematical problems to real life problems". This statement shows that the conception of PT23 about mathematical modeling changed.

Another example was the case of PT3. Before the implementation of the modeling course, she explained the mathematical modeling with her own words as follows: "We embodied our mathematical subjects and shaped them in order to be comprehended well by students in our last term's multivariable calculus course. That is, mathematical modeling means ease of comprehensibility for me." The same prospective teacher indicated her thoughts about mathematical modeling at the end of the term as follows: "In my opinion, mathematical models and modeling is the presenting of a particular mathematical subject to students in order to understand whether students understand the subject or have deficiencies in which mathematical concepts are explained by relating real life to mathematics". The previous excerpt demonstrated that PT3 changed her conception about mathematical modeling after the implementation of the course.

The results indicated above clearly demonstrate that prospective teachers' conception of mathematical modeling has been subject to change due to the implementation of mathematical modeling course. Prospective teachers developed a conception of mathematical modeling such that most of them made connections between real life situations and mathematical concepts. In order to display the change based on group members, the following table (see Table 7) shows the change in terms of codes obtained from prospective teachers' surveys applied before and after the implementation of the course.

Table 7 Change in the conceptions of the prospective teachers about mathematical modeling

Prospective Teacher	**Before the Implementation of Designed Course**	**After the Implementation of Designed Course**
PT17 (Group 1)	Relating mathematics with real life	Relating mathematics with real life
PT14 (Group 2)	Using concrete manipulative and visualization	Relating mathematics with real life
PT5 (Goup 3)	Using concrete manipulative and visualization	Relating mathematics with real life
PT10 (Group 4)	Using concrete manipulative and visualization	Relating mathematics with real life
PT9 (Group 5)	Using concrete manipulative and visualization	Relating mathematics with real life
PT23 (Group 6)	Using concrete manipulative and visualization	Relating mathematics with real life
PT24 (Group 7)	Using concrete manipulative and visualization	Relating mathematics with real life

*: PT5, the number 5 represents the coded prospective teachers.

The table shows that, prior to the course, while six of the seven prospective teachers who were selected as representatives of their groups described the mathematical modeling as using concrete manipulative and visualization, only one of them described it as relating mathematics with real life before the implementation of the course. In general, at the beginning of the course, while fifteen of the 25 prospective teachers described mathematical modeling as using concrete manipulative and visualization, the remaining prospective teachers expressed mathematical modeling as relating mathematics with real life situations. At the end of the course, however, almost all of them described mathematical modeling as relating mathematics with real life after the implementation. Only five prospective teachers' conceptions about mathematical modeling and descriptions of mathematical modeling had not changed their conception prior to the course that is modeling as "relating mathematics with real life". At the beginning of the course, when they were asked, "Have you ever solved/applied any modeling activity? Can you give examples?" almost half of the prospective teachers (n=12) stated that they did not have any prior experience with mathematical modeling. The remaining prospective teachers (n=13) had experience with mathematical modeling in the sense of "using concrete manipulative and visualization." It was also observed that only three prospective teachers had experience with mathematical modeling in the context of the real life situations.

At the beginning of the course, prospective teachers were asked whether they had ever heard of the expression of mathematical modeling before and what they understood from that expression via pre-survey form. By this question, it was aimed to reveal what knowledge prospective teachers had beforehand about mathematical modeling and its characteristics. They were also asked whether they had solved any mathematical modeling activities before in order to determine their prior experience about modeling and modeling activities. These data were analyzed and some results were displayed in the below table (see Table 8). According to prospective teachers' responses to these questions, it was detected that prospective teachers had superficial mathematical modeling knowledge and almost no one had solved or applied any modeling activity before at the beginning of the modeling course. As it is seen from the Table 8, prospective teachers stated that they described mathematical modeling as producing concrete manipulatives. For example, PT24 defined mathematical modeling as "creating materials that enable the students to understand the subjects more easily by making that statement more concrete and to demonstrate how a mathematical statement works in real life.” He mentioned concrete manipulatives for explaining the concept of convergence of sequences as a prior experience for mathematical modeling. PT14 said that she had heard of mathematical modeling before and also described the mathematical modeling as techniques or strategies that included the concrete manipulatives and visualizations in order to facilitate teaching and comprehensibility of mathematics. She gave examples about visualizations and concrete manipulatives that she made in the previous courses. As indicated in previous sections of the results chapter, it was observed that most of the prospective teachers had very narrow knowledge about mathematical modeling and modeling activities and their modeling knowledge was limited to producing concrete manipulatives and visualization. Moreover, it was seen that they had quite inadequate knowledge about what mathematical modeling meant and how mathematical modeling activities should be. For instance, PT9 was unclear about her description of mathematical modeling. She wrote, “Two things come to my mind. First is an expression of an event or situation mathematically and the other one is modeling of mathematical problem visually, namely; preparation of material”. According to the previous quote, PT9 gave the sense that she knew about modeling in the first statement, but the following statement and the example she provided about a mathematical modeling activity disproved this sense. In fact, she defined mathematical modeling as producing concrete manipulative to present a subject matter.

Table 8 Prospective teachers’ expressions about foreknowledge and prior experience on modeling and modeling activities

Prospective Teachers	**Foreknowledge** Have you ever heard of the expression “Mathematical modeling” before? Please explain what you understand from this expression with examples.	**Prior Experience** Have you ever solved/applied any modeling activities? Can you give examples?
PT17 (Group 1)	Mathematical modeling is exemplification of mathematical concept by correlating with life.	I did not solve and implement.

Table 8 (continued)

PT14 (Group 2)	Mathematical modeling is a term that is often heard and confronted. I think that mathematical modeling comprises techniques and methods that can be used to simplify, concretize and visualize teaching and comprehensibility during the education of math. For example, a math problem can be more comprehensible and easier to interpret by the use of a computer. By the use of such techniques, simplifying the comprehension and being able to make math more concrete will be very important for a teacher.	We visualized the solution phases of any mathematical problem or proof technique by using computer programs. There were activities that we discovered especially in computer classes and we prepared a material to simplify display about the subject given to us in multivariable analysis lesson last year. Our subject, for example, was substitution in multiple integral and we developed a material suitable for this subject.
PT5 (Group 3)	We concretized our math subjects and shaped it to make it more comprehensible in “Multivariable Analysis” course last year. Namely, this statement means easy comprehensibility.	No.
PT10 (Group 4)	I heard about it in many lessons. It enables math’s being more comprehensible. It enables three dimension is being apprehended better.	I prepared a material that shows three-dimensional curve’s tangent and normal to curve in “Multivariable Analysis” course.
PT9 (Group 5)	Two things come to my mind. First is expression of an event or situation mathematically and the other one is modeling of mathematical problem visually, namely; preparation of material.	We, as a group, prepared a material about “polar coordinates in double integrals” in “Multivariable Analysis” course, but I cannot say we were good at this work. Because I do not think, we could convey the subject into the material fully.
PT23 (Group 6)	We dealt with mathematical modeling in the course of ‘Teaching Methods in Mathematics Education’. Mathematical modeling in some ways is the work of displaying embodiment. I can express a square with one side length x by the expression of x^2 and a rectangle with lengths x and y by the expression of xy	Yes, I mentioned before.
PT24 (Group 7)	The expression of mathematical modeling is creating materials that enable the students to understand the subjects more easily by making that statement more concrete to demonstrate how a mathematical statement works in real life.	In the second term of the second class, I made a modeling about “convergence of series.” for “Multivariable Analysis” course.

After the implementation of the modeling course, post-survey forms were applied to prospective teachers in order to reveal the change in perspective of prospective teachers’ thinking about mathematical modeling and modeling activities. Some of the results were illustrated in Table 9. It was observed that prospective teachers who considered mathematical modeling as showing mathematical concepts through concrete manipulatives before taking the modeling course developed important ideas on modeling and modeling activities throughout the implementation process. For example, PT3 illustrated the change in her perspective as follows:

> When modeling was uttered, explaining a mathematical concept or subject visibly, tactually with the help of three-dimensional object would come to my mind. But after this lesson, after discussing and solving six modeling questions, not only three-dimensional shapes come to my mind. I think that problems of daily life can be solved by math and in order to explain math better and make it clear, examples from daily life can be used and by this way, math will be loved by students. Such questions are modeling questions and when the problems in daily life are adjusted to math, they will be modeling examples for us (*PT3, post-survey form*).

In the preceding except, PT3 stated that she considered the mathematical modeling before as teaching mathematics with the help of three-dimensional objects and concrete manipulatives. In the process of that course, PT3 expressed that she came to understand that mathematical modeling was a tool for solving problems encountered in real life and a teaching tool for teaching and learning of mathematics enriched with examples from real life. Similar to expressions of PT3, by saying "However, I think that modeling questions asked towards the end of the term have a positive effect on students 'understanding of concepts about which I think students have problems and insufficiencies", PT15 stressed the pedagogical gains provided by the use of mathematical modeling in the process of mathematics teaching. PT12 confessed that she did not know about the concept of modeling, or even that she understood it incorrectly:

> Initially, I noticed that I do not know the concept of modeling fully and I even misunderstood it. But now I know what a modeling question means. After the questions that I have discovered in this lesson, questions of daily life situations revive in my mind. For example, I would not think like this as a mathematician when I went to an amusement park; now I can look and think more as a mathematician (*PT12, post-survey form*).

In the process of implementation, she realized the meaning of mathematical modeling and changed her perspective on real life situations. She expressed that she looked at real life situations more mathematically, like a mathematician.

Table 9 Views of some of the prospective teachers about the change in their approaches to modeling and modeling activities

Prospective Teacher	Views (Excerpts from Post-Survey Forms)
PT17 (Group 1)	I had no idea about modeling activities' measurability, namely; I had doubts about whether I could have feedbacks when I direct these questions to the students. But when we analyzed the solutions of the students, I noticed that I can have an opinion about which questions the students know or where the students have difficulty.
PT14 (Group 2)	Any student having studied the modeling activity should be able to interpret both usage area of the concepts and concepts to be questioned within other situations.
PT5 (Group 3)	We made lots of modeling activities from the beginning of the term till now. We discovered that some questions have more than one correct answer and learned to look from different perspectives in our first work, *The Summer Job*. In the other activities following *The Summer Job*, the questions have only one answer. We paid attention what to consider and how to start a modeling activity during this process in which we did these activities. While evaluating a question from only one perspective and way in the past, we tried to approach modeling activities from different perspectives in this process.
PT10 (Group 4)	When the term started and when I tackled modeling questions, at first I was attempting to find formulas and exact solutions to them but in the activities done by the end of the term, I started to evaluate a question intuitively by using interpretation method. I gained different point of views.
PT9 (Group 5)	I was trying to think different ways of the solution during the first weeks, but for the last weeks, I did not think to break out of the ways we had found. Initially how to approach the students about making them realize their mistakes was only in my mind, I could not express it verbally, but now I have an idea about this subject.
PT23 (Group 6)	When I look through to the process, I see that modeling is a matching of daily life problems with mathematical a problem. Maybe I saw that my opinion was verified. But also I may have attained this: Not only does it make the problem solved in daily life but also has an aim to help the other solutions of the problem.
PT24 (Group 7)	We tried to have approaches that will bring the easiest and comprehensible solutions for modeling questions from the outset. We formed opinions about every question and we tried to choose the most suitable one among these opinions.
PT12 (Group 4)	Initially, I noticed that I do not know the concept modeling fully and I even miss know it. But now I know what a modeling question means. After the modeling questions that I discovered in this lesson, questions of daily life situations revive in my mind. For example, I would not think like this as a mathematician when I went to an amusement park; now I can look and think more as a mathematician.
PT15 (Group 2)	I would not have an approach at the beginning of the term as I have no knowledge about modeling. However, I think that modeling questions asked towards the end of the term have much positive effect on students and beneficial for them on 'understanding of the concepts about which I think students have problems or insufficiency. For example, I practiced "*The Ferris Wheel*" activity on my brother and waited for his solving. After that, we discussed the solution and talked about the concepts. In the end, my brother told me "he understood this subject in that way but now I know what it means". Furthermore, he said, "I did not know it works in these subjects". Thereby my perspective on modeling developed and changed.
PT2 (Group 7)	It was clear that I did not know much about modeling questions in the beginning of the term. In other words, I used to think it must be like this by interpreting what I heard before. When modeling is said, an idea of harmonization of daily life with math used to come to my mind and not a

Table 9 (continued)

	different idea formed in my mind in time. What has changed was my believing much more in their being useful and understanding after seeing how these questions were transferred and what kind of questions were used. There is no difference in my approach to these questions, as I have never encountered with modeling questions. But it is clear that the activities that we did and elapsed time have made a great contribution to my understanding the modeling and realizing it more in daily life. I could say I consider and think mathematical concepts in every event and situation. Although I did not take such a lesson before and though I took it for the first time, I understood the logic of the subject quite well and I think that it is useful.
PT8 (Group 5)	We did not have sufficient knowledge about requested subjects or how to approach them. (In the field of application) At first, I used to evaluate the questions with classical test mentality, assuming that there is an exact solution and it is necessary to reach numeric data. Then questions that are without numeric data require directive and based on interpretation (like *The Water Tank*) were given to us and we saw that there are many different ways of solution. In fact, I saw and understood that there can be real life math questions that are without numeric data and have no exact solution.

As can be seen in Table 9, prospective teachers evaluated themselves in terms of what they knew about modeling and modeling activities before taking the current modeling course and what they learned in the process of implementation. According to above table, prospective teachers indicated that they did not have enough knowledge about mathematical modeling in the beginning, but they developed significant ideas about it in the process of implementation of the course. From these findings, although prospective teachers had very narrow knowledge about mathematical modeling before the implementation of the modeling course, it can be interpreted that there was a significant change in the conception and description of mathematical modeling throughout the implementation of the course.

4.1.2 Prospective Teachers' Conceptions about the Nature of Mathematical Modeling Activities

It is clear that prospective teachers' thinking about the knowledge of mathematical modeling activities needed to be brought up in order to talk about the development in their thinking on knowledge of the use of mathematical modeling activities. Within the modeling course, prospective teachers' developed ideas were questioned after the implementation of the first four modeling activities (see Appendix B). This investigation was made before the "Presentation and discussion on the nature of modeling activities" was done. After the implementation of the first four modeling activities, all groups were asked to determine the general properties of modeling activities and specify the differences between modeling activities and traditional word problems and write it down on a sheet. After the analysis, the results were displayed in Table 10.

Table 10 Views about general characteristics of modeling activities and their comparison with traditional word problems

Groups	General Characteristics of Modeling Activities	Comparison with Traditional Word Problems	
		Similarities	**Differences**
Group 1	Being associated with different mathematical concepts Having no chance to verify resolution of some modeling activities Not foreseeing the solution process of some modeling activities	The Bouncing Ball activity was similar to traditional word problems	The remaining e activities were different than traditional word problems The solution procedure is not clear
Group 2	Requiring methods by using givens. Including more than one mathematical concept Suitable for group working Open-ended and detailed Requiring the use of technological tools	-	Having more results Including more than one mathematical concept Taking more time Requiring discussion Providing freedom of thinking Using data from real life Not giving the result directly
Group 3	Including situations that can be faced in real life Including more than one mathematical concept	-	Being different as a problem structure Using data from real life Including more than one mathematical concept Taking more time
Group 4	Qualified to be thought-provoking Including more than one mathematical concept Not having a specific pattern Including situations that can be faced in real life Showing the real life applications of mathematics	Mathematical operations	Including more than one mathematical concept Containing of detailed information

Table 10 (continued)

Group 5	Showing the real life applications of mathematics Containing of detailed information Having diverse solution procedures Including more than one mathematical concept	-	Style Showing the real life applications of mathematics Having diverse solution procedures Putting forward the thinking on problems
Group 6	Being open to interpretation as the structure of question Putting forward the thinking on problems Showing the real life applications of mathematics Including situations that can be faced in real life	Mathematical operations	Putting forward the thinking on problems Suitable for group working Requiring caution
Group 7	Showing the real life applications of mathematics Having diverse solution procedures Salient Including more than one mathematical concept Suitable for group working Including situations that can be faced in real life Providing distinct perspectives	-	Putting forward the thinking on problems Containing of detailed information Not having obvious solution procedure Having diverse solution procedures

As can be seen in Table 10, prospective teachers put forward the general characteristics of modeling activities, including more than one mathematical concept: situations that can be faced in real life, showing the real life applications of mathematics, and what is suitable for group work. Furthermore, prospective teachers stated that although modeling activities had different properties (e.g. including more than one mathematical concept, putting forward the thinking on problems, having diverse solution procedures), one of the modeling activities (*the Bouncing Ball*) had common properties with traditional word problems (see Table 10).

Some of the prospective teachers' ideas about general characteristics of modeling activities, and similarities and differences between modeling activities and traditional word problems, were analyzed in chronological order in order to demonstrate their developmental process. The findings demonstrated that prospective teachers identified seven general characteristics of mathematical modeling activities. They characterized the modeling activities as real; open-ended; including more than one mathematical concept; having diverse solution procedures and cyclic structure of solution process; feeling the need; generalization and being prototype; and diversity from traditional word problems . These properties were illustrated by using evidence from the obtained data analysis.

4.1.2.1 The characteristics that prospective teachers attributed to mathematical modeling activities

Throughout the implementation period of the course and the current research, prospective teachers indicated their thinking about the mathematical modeling activities and ideas on what kind of characteristics they possess. The characteristics depicted by prospective teachers are reality, open-endedness, including more than one mathematical concept, having diverse solution procedures and cyclic structure of solution process, the need for a solution or model, generalizability and being prototype, and having distinctions from traditional word problems. The findings were illustrated in the following sections in detail.

Reality

Prospective teachers saw reality as a character of modeling activities in relation with daily life situations and meaningfulness. As can be seen from Table 10, almost every group agreed that modeling activities included real situations from daily life. Most of prospective teachers reported in their reflection papers and interviews that mathematical modeling activities were directly related to real life situations. They thought that the context of modeling activities were directly taken from real life and that these activities dealt with reality indeed. For example, after the implementation of *the Ferris Wheel* activity, PT17 wrote:

> After solving this question, I noticed how to use trigonometric attainments that I had acquired in math lessons. In other words, I noticed that these attainments are not only for being successful in exams but also for making the life easier. I was also curious about what the use of trigonometry was in daily life like many students, during the process in which I learned trigonometry. It was a kind of problem that I can use to reply to a student who has such a curiosity when I am a teacher (*PT17, reflection paper for the Ferris Wheel activity*).

In the above quote, she said that she noticed the relationship between mathematical subjects (e.g., trigonometry) and real life situations (designing a Ferris Wheel for entertainment). She mentioned meaningful mathematics teaching explicitly because she wanted use this modeling activity in order to show students how mathematics can play a role in real life. In the post-survey form, she emphasized reality as a central property for modeling activities in the following excerpt: “First of all, its relation with daily life should be stated. Namely, there should be an explanation for whether something exists indeed or not. Its directives should be clear and comprehensible. It should be realistic and consistent.”

PT14 emphasized that modeling activities include more than one solution. It was understood from the following quote that she understood some general properties of modeling activities apparently. For example, modeling activities were related to real life situations and finding solutions to these situations that fit them best.

> We noticed in the process of solving this question that more than one solution can be found for a problem and with many different perspectives one can reach the solutions of the problems. But with this problem I understood that the most important thing is to find the most practical and suitable one for daily life; namely, to choose the most useful one and to be able to determine a solution in that way (*PT14, post-survey form*).

In the post-survey form, PT14 put emphasis on reality as one of the properties of modeling activities: “It should also reflect the problems in daily life, namely it should not be isolated from real life.” This evidence showed that she preserved her idea about reality property after the implementation of modeling course.

In the below episode, PT9 stated that modeling activities were realistic problems that related mathematics with real life as follows:

> Interviewer: Can you comment on the modeling questions? Are they different from earlier questions, or are they similar?
>
> PT9: Well, the construction of the problem. You gave us a test last week, the questions were very realistic, and they were similar to modeling questions. I mean, in real life, mathematics is very cool, applicable to real life. We do not see that very often, but when we do, we realize the mathematics. I mean, I love mathematics.

It is understood from the above excerpt that she got an impression that the context of modeling activities ought to be from real life intuitively. In the following activities and at the end of the implementation of the modeling course, it was observed that PT9 developed her ideas about the properties of modeling activities.

In the subsequent modeling activities, PT24 identified the property of modeling activities such that it was related to real life situations and how mathematics were integrated in daily life. He wrote: “Our attainments for this week as in every week are like this: We learned how to use math in daily life. Furthermore we acquired attainments about how to use an area efficiently and which processes to do while placing different components into a component." It is understood from the preceding quote that all mathematical modeling activities were directly related to real life situations and mathematics in them. PT24 figured out that they had learned how to complete the puzzle by using the information given in real life situations.

From the examples given in these episodes and quotes, prospective teachers identified the property of reality in modeling activities in the implementation period of the first four modeling activities. This situation also demonstrated that prospective teachers could interpret the modeling activities as making connections between modeling activities and real life situations intuitively and developed their ideas about reality property.

Open-endedness

Another property that was mentioned by prospective teachers was open-endedness of modeling activities. As seen in Table 10, members of group 2, group 4, and group 6 indicated that modeling activities were open-ended and they had no specific pattern in order to predict the ways of solution. Prospective teachers referred the open-ended property of modeling activities in the process of implementation of the modeling course. For example, in the interview after *the Free Roller Coaster* activity, PT14 mentioned the open-endedness property as follows:

> Interviewer: How would you evaluate this problem in general?
>
> PT14: It was open-ended, leaving all the work to us. I mean, OK, I draw this on the paper, and then I have to think what to do next. With *The Bouncing Ball* activity, we were done after finding the coefficient; we did not have to do anything else. With this problem, we have to consider how applicable this is to real life. I mean, it should not be extreme, because it is something we use in real life. I think those were the differences.

PT14 stated that *the Free Roller Coaster* activity needed a design and many factors needed to be taken into consideration in the period of design and therefore the activity could be denoted as considerably open-ended. She compared *the Free Roller Coaster activity* with *the Bouncing Ball* activity in terms of open-endedness; she concluded that *the Free Roller Coaster* activity was more open-ended than the letter one. PT20 expressed how a good modeling activity ought to be in the following excerpt.

> A good modeling question should be comprehensible but at the same time, it should not be too directive. It should be a bit open-ended. Namely, it should not have only one correct solution. It should have more than one solution. Modeling question should be correlated with daily life so that the student both understand the subject easily and see that math is pretty much within the daily life

> indeed. Moreover, solution of the question should not be easy. The students should have difficulties in some points and by this way; they can give much more thoughts to the points that we desire to attract their attention (*PT20, post-survey form*).

As it is seen in the preceding excerpt, PT20 indicated that a modeling activity ought to be open-ended as well as other properties. According to PT20, modeling activities needed to have diverse ways of solution. Most of the prospective teachers suggested that modeling activities ought to have diverse solution strategies and be open to diverse interpretations in the process and post-survey forms.

PT24 commented on the differences between traditional word problems and modeling activities in the following quote.

> The problems given us till now were absolute problems which have unique solutions based on variables under control. However, it was an open-ended question that requires our accepting something to reach a solution and that enables us to solve it upon this assumption. Therefore, it was a problem which has solutions shaped by the resolvent's (person) idea rather than having one and absolute solution (*PT24, reflection paper for the Street Parking activity*).

In the preceding quote, PT24 compared the modeling activity with traditional word problems. According to his comment, traditional word problems had few variables that were clear and a certain solution. On the other hand, modeling activities were open-ended problems that required assumptions to proceed to distinct solutions that were structured to one's assumptions. It is understood from this comment that PT24 noticed the some differences between modeling activities and traditional word problems after the implementation of the first modeling activity and gave emphasis on the open-endedness of the modeling activities.

The excerpts given in above paragraphs were exemplary that reflected prospective teachers' ideas about the open-endedness of modeling activities. It can be interpreted that most of the prospective teachers noticed the property of open-endedness of modeling activities and accepted as a necessary condition for being a good modeling activity.

Including more than one mathematical concept

Most of the prospective teachers emphasized that mathematical modeling activities contained distinct mathematical concepts in the same activity in the period implementation of the modeling course. According to Table 10, almost all of the groups (except groups 1 and 6) indicated that including more than one concept was a general property of modeling activities and this differentiated them from traditional word problems.

In the reflection paper, PT9 expressed that many mathematical concepts were used in *the Ferris Wheel* activity such as ratio, trigonometric concepts (e.g., cosine theorem, properties of circle), and velocity. She wrote:

> In this question, we used the concept of ratio and proportion, trigonometry, cosine law, characteristics of circle and concept of velocity. We made transformation between unities (min. - sec.). In this problem, math and physic lessons were like mingled. Because when I saw the movement of the circle, the concept that came to my mind was circular movement. We used math and geometry in a mingled way (*PT9, reflection paper for the Ferris Wheel activity*).

From above excerpt, PT9 emphasized that modeling activities could have interdisciplinary an approach in the solution process such as the concept of circular motion was associated with physics and also mathematics. PT8 mentioned about mathematical ideas and concepts included in *the Water Tank* activity in the reflection paper as follows:

> We can call this problem as a pool problem because interpretation of graphics, point-tangent, and point-slope, volumes of geometrical figures are the kinds of problems that are similar to pool problems. Furthermore, we used height concept covered by quantity of water filling in the unit volume as basis and thereby we reached to a solution (*PT8, reflection paper for the Water Tank activity*).

According to above excerpt, PT8 stated that they used many mathematical concepts together such as graph interpreting, slope at a point, and height variation in volume. PT8 also mentioned that modeling activities ought to include more than one concept when describing modeling activities in the post-survey. She wrote: "A good modeling question should consist of more than one approach of solution and mathematical subject. It should be generalizable, clear and comprehensible, data should be useful and within the life and should be valid". In the post-survey form, PT14 also stated that modeling activities ought to query many mathematical concepts in the same activity as follows:

> A good mathematical modeling should be usable primarily for the other situations. Namely, method of the question's solution should be valid for all the situations that have similar characteristics with this question. Furthermore, it should reflect the problems of daily life more; that is, it should not be abstracted from daily life. Moreover, a good modeling question should be able to make mathematical concepts examined and be open to different solution strategies and comments (*PT14, post-survey form*).

Only two of the prospective teachers (PT5 and PT23) asserted that including more than one mathematical concept was not suitable for objective based teaching and learning approach. In the reflection paper, PT5 underlined that modeling activities ought not to include more than one subject matter or concepts in the following: "Use of all the subjects and use of the concepts together should be avoided. In other words when we want to use more than a few (three or five) subjects together the students get back and get confused". Other prospective teachers delivered positive opinions on the property of including more than one mathematical concept and indicated that it was affirmative for conjecturing among subject matters and concepts. In the following exemplary excerpt, PT14 emphasized that including more than one mathematical concept in a modeling activity could help students gain more knowledge about using these concepts and subject matters interchangeably.

> Mathematical concepts that we used were triangle geometry, trigonometric functions, ratio, and proportion. This problem in which we use many mathematical concepts may bring the students in lots of things mathematically. After solving this question, I think a student must be able to answer the following questions: Where is scanned (shaded) angle in circular area? When do I need to use sinus function and when do I need to use cosine function or in which circumstances which the trigonometric function does resolve the question easily? (In our solution, in the section of ground clearance, using cosine instead of sinus was easier). How can the changes of signals in sinus and cosine functions be observed while the scanned angle is expanding? What does the expression of an angle as time- dependent mean? How can it be written for a triangle dependent on any cosine or sinus functions? What is referred by instantaneous change mathematically? (*PT14, interview for the Ferris Wheel activity*).

In summary, most of the prospective teachers stated that modeling activities included more than one mathematical concept that could be interpreted as a general property of modeling activities. The given excerpts were exemplary for most of the prospective teachers' opinion accepted as a property for modeling activities.

Having diverse solution procedures and cyclic structure of solution process

Another finding was having distinct solutions of modeling activities and cyclic structure of solution process. This emerged as another property of modeling activities that prospective teachers formed in the implementation of process. As it is seen in Table 8, members of some groups reported that modeling activities could have various solutions and the solution process might be unclear, therefore, these situations could be regarded as properties of modeling activities. For example, after the implementation of the first modeling activity (*the Summer Job*), PT17 compared the modeling activities with traditional word problems in terms of their differences. She indicated that there was a unique solution to traditional word problems and everyone tried to find this solution. Modeling activities, on the other hand, required them to produce solutions rather than just finding a unique solution. The following episode illustrated her thinking about the differences between modeling activities and traditional word problems.

Interviewer: What can you say if you compare this problem to other problems, you have seen so far?

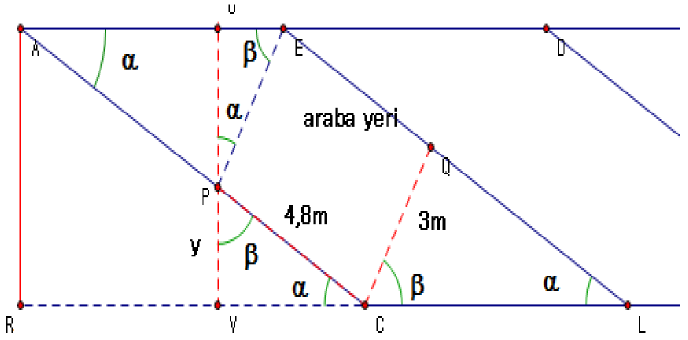
PT17: During K-12 years, we have seen problems that had only one solution. We have tried to find that specific solution instead of finding different solutions. This changed a little bit during university years and we started finding one or more solutions. This is a problem that can have multiple solutions. I can solve this by considering being steady or time or money. So, it was different in that way.

PT17 also expressed that the modeling activities were different from traditional word problems because traditional word problems contained only one solution. On the other hand, modeling activities included more than one level, for example comparing the solution with previous ones. These expressions demonstrated that she noticed the some properties of modeling activities intuitively.

Again, as indicated in Table 10, the cyclic structure of the solution process of modeling activities was seen as unclear during the solution process and its complexity was also noted by some of the prospective teachers. This cyclic structure appeared in the data resources of prospective teachers when they illuminated their way of solution procedures. For example, PT5 told their general solution procedure after implementation interview as follows:

Nevertheless, we made many calculation errors. We used different wrong ways of solutions, we had great difficulties at first and for this reason, and we went back to beginning because of calculation errors. We handled the question and say “we found the correct answer” all the time but our supervisor came and told us that there was a mistake in somewhere within the question. Then we handled the question again and there were some problems in minor things like multiplying and thus we went back to beginning (*PT5, interview for the Street Parking activity*).

In the preceding excerpt, as describing the process of solution in *the Street Parking* activity, PT5 explained that several times they needed to return from coming to a point in the solution to the beginning after the teacher asked them to check their errors. A member of group 5, PT15 also explained why they needed to change their solution strategies several times in the same activity in the following quote:



$$\sin\alpha = \frac{y}{4.8} \Rightarrow y = 4.8\sin\alpha$$
$$y = 4.8\sin\alpha < 4.5$$
$$\Rightarrow 4.8\sin\alpha < 4.5$$
$$\Rightarrow \sin\alpha < 0.9375$$
$$\Rightarrow \alpha \leq 70°$$

Figure 7 A solution for *the Street Parking* activity presented by PT15

We got $\alpha \leq 70°$. Instantly we tried to prove this result. When we took $\alpha = 69°$, we found the length of AP as 1.1 m in the AEP triangle that is shown in the Figure 7. Hence, we got the length of AC as $AC = 4.8 + 1.1 = 5.9$ m . Then we apply this in ARP triangle and put it in the sine law, we found that the length of $AR = 5.5$. That is, the length was 5.5 m, but it ought to be 4.5 m. Therefore, the solution was wrong. I told my friend PT13 that the length of y should be smaller than $4.5 - UP$ which resulted in wrong solution. She asked “Why?” I told her that when the length of y became 4.5 , then $\alpha = 90°$. This may lead to contain more high values in the interval. Hence, we observed that our method was not appropriate for the solution (*PT15, reflection paper for the Street Parking activity*).

In the preceding excerpt, PT15 explained their approach to solving the problem firstly by providing figures (see Figure 7) in the solution process. PT15 illustrated how they obtained an equation and found the angle 70 degrees by using this equation. Then PT15 noticed the error when they put the angle in the equation and got the result 5.5 meters that was actually 4.5. After they found a contradiction in the result, they decided to change their approach. Similarly, PT9, as a member of the same group, tried to solve the problem by using the idea of area. She said why they gave up the idea of area in the solution process in the following episode:

PT9: There was a huge difference. Then, we went back to starting point, to the equation. We decided to take derivative of the equation. Then we solved the equation.

Interviewer: After that, you solved the equation, not using differentiation method.

PT9: Yes, like solving an equation. No, differentiation. The hypothesis collapsed. We got 20.65 by solving the equation. Other group friends calculated the angle that corresponds to 20.65. Approximately, the obtained valued were close. Therefore, we checked that we solved correctly.

Interviewer: So, what was the point that you got difficulty the most in terms of solution method PT9?

PT9: We went back to the starting point two times. In the first trial, we struggled to solve the problem situation with PT14. Namely, we obtained many equations and the result was complicated.

Interviewer: You said that, $\frac{c}{\sin \varphi}$, the point you made mistake.

PT9: Yeah, it was the incorrect part.

Interviewer: Then, you turned back to the starting point.

PT9: Yes, we turned back to the starting point again.

In the above episode, she tried to figure out that they intended to solve the problem by using the idea of derivative, but they observed that this was not possible. Therefore, they changed their solution strategy and attempted instead to solve by using equations. When the interviewer asked her where they encountered the most difficulties she point out the situations when they needed to restart twice although they were far along in the process, but they had not found any results. Prospective teachers described the cyclic structure of solution process as being unclear regarding ways of finding a solution, complexity, and open to more solution strategies generally.

The need for a solution or model

Some of the prospective teachers pointed out the need for a solution or a model. They noted that the mathematical concepts to be taught and the building of a model in order to find a solution ought to make students feel it is a necessity about the nature of mathematical modeling activities. The feeling of this need can be observed in the prospective teachers' responses to the question "How should be a good modeling activity in your opinion? What kind of properties does it possess?" in the post-survey form. For example, PT10 underlined the feeling the need property in the following excerpt:

> In a good modeling question, a student should feel that s/he needs to build a new structure that will be an answer to the question asked to the student's himself or herself and then the student should form a model. The modeling question with the knowledge and experiences that students have should enable a meaningful real life problem to be solved. The students should be able to express their ideas freely. As it is possible for a student to test his or her results' and interpretation's accuracy, the student should reach to the judgment that his/her modeling needs to be developed or retrieved on his/her own. Modeling question should be generalizable by other questions. It should be in a way that enables the student to form a model whose explanatory power is great and which can be used to interpret structurally similar other situations. It should be clear and comprehensible (*PT10, post-survey form*).

In the preceding excerpt, PT10 stated that students needed to feel that they built a structure that may result in a solution for the modeling activity and students ought to create a model. She also said that students could evaluate their models in terms of its correctness and the need for correction. PT11 also commented that modeling activities made students feel that a model for a solution needed to be created, developed, and corrected if it needed. She wrote:

> A good modeling question should make the student feel that s/he needs to form, develop, or edit a model that will be a solution for the situation; and this modeling question should enable the students to solve a meaningful real life question with their own knowledge and experience. Concrete materials, calculators, computer software should be enabled to help the students test their results. The solution of the question should be able to be used in interpreting other situations (*PT11, post-survey form*).

It is clear from the given excepts that PT10 and PT11 put emphasis on the need that a modeling activity ought to direct students to feel the need for creating models and structures that could be a solution for these activities. Moreover, prospective teachers declared that modeling activities ought to be qualified to question intended mathematical constructs and concepts. For instance, PT2 underlined this situation in the subsequent excerpt.

> A good modeling question should be in such a quality to be able to question desired concepts. But while doing this, it should not give this concept clearly. It should be able to make many concepts questioned. It should be valid and suitable for the level of students. It should not be complicated. Expressions of questions should be vivid and clear (*PT2, post-survey form*).

As it was understood from the preceding excerpt, PT2 stressed the property of inquiring about the mathematical concepts as well as modeling activities possessing a distinct type of properties such as relevancy with real life, validity for other similar situations, etc.

In summary, the need for a solution or model was another property of modeling activities that ought to be taken into account. The prospective teachers suggested that modeling activities inquired about the mathematical concepts and ought to be suitable for real life situations.

Generalizability and being prototype

Another property of modeling activities that emerged for prospective teachers was using the same developed models to explain other similar real life situations. In the period of implementation of modeling activities, prospective teachers were asked to discuss the generalizability and validity of solutions or models in terms of usability in similar situations in interviews and write down these discussions on their reflection papers. In addition to this, prospective teachers were asked to evaluate the solution or model according to generalizability and validity properties. The findings demonstrated that most of the prospective teachers emphasized the generalizability and validity of solutions or models as one of the general properties of modeling activities and they indicated that almost all of the implemented modeling possessed the generalizability and validity properties. For example, after the implementation of the first modeling activity, *the Summer Job*, PT17 underlined that the solution could be reused in the similar situations in the following interview excerpt.

> Interviewer: Do you think can you generalize the solution to the similar situations in daily life?
>
> PT17: Yes, it can be generalized because that is a general solution method. That is, if there exist a firm like that and this firm can hire employees for the work.

She also indicated that the solution for *the Water Tank* activity was generalizable for other similar situations in the following transcript.

> Interviewer: Can you generalize you developed solution method to the similar situations in real life?
>
> PT17: So I think it is generalizable. I think it would be beneficial in volume related problems, area of slices while drawing graphs.

Similar to the ideas of PT17, PT14 also expressed that the solution of modeling activities could be reused in the similar situations. For example, she said that their group solution for *the Summer Job* activity might be generalizable to similar situations in the following interview excerpt.

> Interviewer: OK, well, do you think you can use this solution for other similar situations in real life? Is the idea generalizable?
>
> PT14: Well, for example, there were three variables intense, medium, and daily. If you can address these issues, I think it is applicable. I mean, it is also about what kind of a problem it is. I cannot think of any examples right now, but maybe it is implemented now.

Although she expressed the possibility of generalization in the above excerpt, she stressed the importance of generalizability and validity of modeling activities when she answered the following question in the post-survey form: "How should a good modeling activity be in your opinion? What kind of properties does it possess?" She wrote, "A good mathematical modeling activity; first of all, should be able to be usable for other situations. Namely, method of solution should keep its validity for all other situations that have similar characteristics with this question." It can be interpreted that she accepted the generalizability and validity as properties of modeling activities in the implementation process.

Another prospective teacher, PT24, stated that his or her solution for *the Summer Job* activity could be generalizable to similar situations and gave a concrete example for it in the following episode:

> Interviewer: Well, can you tell a little bit about how your solution strategy may be applied to other situations?
>
> PT24: I think it can be. For example, production at a factory, or turning off a machine. Also, how does the machine work, how long it takes for it to get ready, after how long does it have to be stopped etc. This method can be used in similar situations.

In the preceding excerpt, PT24 made a connection between workers in *the Summer Job* activity and types of machines for a factory in terms of efficiency. In both situations, efficiency was at the center. He pointed out this relationship and the solution strategy that could be used in the second situation that was similar. PT24 also stated the generalizability of their solution for *the Water Tank* activity that was the last implemented activity in the process. He wrote in his reflection paper as follows, "Our solution was like this. It was quite simple and useful solution. With the directive we formed, it was a directive that would enable any person to form graphics of quantity of water and water level." From the quote, PT24 expressed that their solution or model could be used for all kind of water tanks and valid for similar situations.

It is evident from the examples given that prospective teachers considered the generalizability and validity of solutions or models as properties of good modeling activities. It can also be observed from the findings that prospective teachers had different views on the generalizability and validity of solutions or models. For example, although PT17, PT14, and PT24 said that solutions for almost all of the implemented modeling activities could be generalizable to other similar situations in the above excerpts, PT10 stated that she did not think that the solution for *the Free Roller Coaster* activity could be generalizable to any other situation. She explained this situation as follows:

> If it is necessary to interpret the way of solution that we formed by drawing tangent to curve, in the utmost point of tangent's slope, tension increases. This was what was wanted from us and according to this, by making interpretation on shape. We produced indefinite solutions. I do not think there is another situation that can generalize this situation (*PT10, interview for the Free Roller Coaster activity*).

In the preceding quote, PT10 indicated that the solution strategy for the modeling activity included the idea that excitement increased the most when the slope was at a maximum. Hence, she thought that the idea was not applicable to other situations.

When the preceding excerpts were examined, it was seen that prospective teachers declared the idea that a solution of a modeling activity ought to be used in the similar situations and this was a necessary condition to be a good modeling activity.

Distinctions from traditional word problems

Most of the prospective teachers noted the differences between modeling activities and traditional word problems according to their own experiences and educational backgrounds. This situation was the most frequently encountered throughout the implementation of modeling course. Since most of the prospective teachers introduced the modeling and modeling activities, they often tried to compare modeling activities with traditional word problems that they were accustomed to in their previous educational lives. As summarized in Table 10, almost all of the groups identified the differences between the modeling activities and traditional word problems. The most distinct differences were that modeling activities have an unclear solution procedure; included more than one mathematical concept; have diverse solution strategies; are suitable for group work due to requiring discussion; include situations from real life; and take more time. On the other hand, traditional word problems have a clear solution procedure; a unique solution; are not suitable for group working; require the memorization of formulas; do not provoke the mathematical thinking, etc. These findings were illustrated with prospective teachers' experiences throughout the implementation period. For example, PT14 expressed her first feelings after the implementation of *the Summer Job* activity. She criticized the traditional word problems she experienced in the high school level and University Entrance Exam (ÖSS). For example, in the following episode, she mentioned the negative features of traditional word problems such as including memorization rather than thinking, givens and solutions were clear, stereotypes etc.

> Interviewer: OK. Now, we want you to compare this problem to other problems you have seen so far. I mean back to middle school and then high school etc.
>
> PT14: Well, I mean, when you say problem, I think of problems like in ÖSS. Problems that never push you to think, you just need to memorize the formula and then plug the numbers in. Well, honestly, I thought we were rote learners.

In the following weeks, she stressed that modeling activities included more variables and more situations that were needed to take into consideration. For example, in *the Bouncing Ball* activity, she compared the previous modeling activities in terms of similarities and differences.

> Interviewer: Good. I want you to compare this problem to those you solved in earlier activities, and in general.
>
> PT14: Well, when you say compare, I understand like, I mean. Well, you reach to bad solutions on both, I mean; I think there is a little bit difference in solutions. When you are working on, you of course think about something, you relate things. Well, there is not much to relate here, but, for example, there are many concepts in *the Street Parking* activity. I mean, there was also something about writing, everybody got stuck on writing.

According the above interview excerpt, she observed that modeling activities could include more mathematical concepts in the same activity and these concepts were related, or need to be related, which meant modeling activities were more complex.

Another prospective teacher PT9 expressed her thinking about modeling activities by comparing them with her previous experiences in high school. For example, in the interview after the implementation of *the Summer Job* activity, she mentioned about their previous education and its applications.

> Interviewer: Well, you have seen different mathematical problems beginning from earlier grades to your university years. Thinking about those problems and this problem together with similarities and differences, what can you say if you compare them all?
>
> PT9: The problems we have encountered since elementary school were mostly multiple-choice test items; so, a problem like this is too long and open-ended for me. I mean, it can be interpreted in different ways, so, I do not remember any problem like this. They were problems with certain results. And everybody had an idea, and we tried to do something considering those ideas. We all had different solutions, not individually, but discussing on each other's' ideas.

In the above situation, she was used to multiple-choice types of question in previous educational background. Hence, she denoted the modeling activities as being lengthy and open-ended questions. She said that she never met any problem like that. She mentioned that traditional word problems were more likely certain and led to a more accurate solution.

PT24 stated that they encountered difficulties when they were faced with modeling activities because of they were accustomed to multiple-choice tests and questions. He talked about the basic characteristics of his previous educational background in mathematics in the following episode from the interview after the implementation of *the Summer Job* activity.

> Interviewer: OK. What about your first impressions about the activity? Like, is it an appropriate question?
>
> PT24: Not like "is it an appropriate question?", but, it is different from the problems we are familiar with, so this type of problems are harder for us.

From the above scenario, it can be interpreted that prospective teachers' educational backgrounds might influence their approaches to modeling activities and take time to adapt to mathematical modeling and modeling activities.

PT24 also compared the modeling activities with previous ones according to structural properties. For example, he noted *the Bouncing Ball* activity that was similar to traditional word problems in terms of easiness and complexity. He stated in the reflection paper as follows, "This week's activity was quite simple compared to the other questions we solved in the other weeks, and it was a question that did not require much time for solution. When I read this question for the first time, it aroused a feeling of being series question on me". From the quote, it can be interpreted that modeling activities can be different in terms of their structures.

In summary, the findings showed that prospective teachers intuitively noticed and identified the differences between modeling activities and traditional word problems. Moreover, prospective teachers identified some general properties of modeling activities in the process of implementation.

> Interviewer: OK, well, if you compare this problem to other mathematics geometry problems you have seen so far, what can you say about similarities and differences?
>
> PT10: The results would be the same, but there would be vast differences in terms of solution methods. We have done proof with other problems we have seen so far.
>
> Interviewer: What about the high school mathematics problems?
>
> PT10: In high school…
>
> Interviewer: You can think of your whole education life in general.
>
> PT10: Well, we have always given inputs and asked for outputs, never asked to reason about something. For example, when we were given a geometry problem, we could only solve it if we saw it. I mean, we have never seen a problem where we need to use sine, cosine, speed-time formulas, the length of a circle etc. It has been either cosine or sine theorem. I have never seen a problem that I had to think deep, even at the university. I will not lie about this, because we never learned geometry. We have just been introduced geometry this year. It has been mathematics usually, like derivatives, integral, proof, theorem etc. So, I have seen this kind of things during university years.

In the preceding interview excerpt, PT10 expressed that modeling activities and traditional world problems could have the same results, but there were differences between them with respect to the ways they are solved. PT10 also mentioned that traditional world problems did not require more thinking, and the ways of solution were obvious. PT10 told that she had never been faced with the problems that had more thinking process before. PT15 indicated that it could be encountered the situation like in *the Street Parking* activity, but the style of activities were different. PT15 wrote, "This problem is a kind of problem that can be encountered in nearly all geometry books in terms of solution. But the style of question's being given us, along with adding difference to the question also can make the question more complicated with the concepts like "angle's being widest"". In the previous quote, PT15 put emphasis on the style of modeling activities that were distinct from traditional word problems. It was inferred from the previous excerpt that although the modeling activity seemed to be like geometry problems in lesson books, but its style was different and more complicated.

To sum up, most of the prospective teachers stressed the differences between modeling activities and traditional word problems by giving examples from their previous experiences in mathematics courses. They expressed these differences by indicating open-endedness, complexity, having distinct solution strategies, the way of enforce to think, including more than one mathematical concept etc. These differences were explained in terms of prospective teachers' specifications gained throughout the implementation process. It is evident from the given excerpts and other examples that prospective teachers improved their knowledge regarding the nature of mathematical modeling and modeling activities.

4.2 Prospective Teachers' Conceptions of Modeling-Specific Pedagogical (Content) Knowledge

This part of the dissertation will report the development of prospective teachers' thinking about the modeling-specific pedagogical knowledge of modeling activities in the classroom environment throughout the implementation process. The main goal of the implementation of the modeling course was to encourage prospective teachers to gain knowledge about the use of mathematical modeling and modeling activities. In order to reveal and demonstrate the development of prospective teachers' thinking throughout the implementation process about what teachers need to know and what qualifications teachers need to possess in order to implement modeling activities in the classroom environment successfully and effectively, individual semi-structured interviews, reflection papers written after each implementation, classroom discussions, pre- and post-survey forms were analyzed and in the light of emergent findings pedagogical knowledge, group working, students' way of thinking, classroom management, the relationship between modeling and technology, and practical experience formed as codes.

4.2.1 Prospective Teachers' Views about Qualifications Needed for Teachers to Conduct Modeling Process

In this part, the pedagogical ideas that participants developed regarding the knowledge and qualifications that a teacher needs to be able to apply modeling activities were reported. The results revealed that prospective teachers developed some ideas about pedagogical equipment for the teacher by observing the role that the instructor played in the modeling course. Prospective teachers commented on the role of the instructor played during the implementation of modeling activities. For example, PT24 compared the role of the teacher in terms of the roles the teachers played in the traditional teaching process and in the implementation of modeling activities.

Interviewer: Well, PT24, I want you to evaluate the role of teacher regarding to both teacher in traditional teaching and teacher in modeling process. Can you compare two roles of a teacher?

PT24: Now, in traditional teaching, teacher teaches and the students follow him or her. This continues like that. However, if teacher, like in the modeling process, do not show the solution way and prepare the conditions for students for finding their ways in the process, the development of students show progress. Therefore, the development of students is very important issue.

Interviewer: OK, What should be the role of teacher in the process of modeling activity implementation?

PT24: In my opinion, the role of teacher should be a guide or giving inquiry to students. In the process of modeling, if students do not show any effort, she or he direct students to thinking about the situation, not telling them to the solution. Thinking is significant for the development of students.

In the preceding excerpt, PT24 gave an example from his previous high school experience and stated that teachers in the traditional teaching process only gave lecture and students followed their teacher's instructions without asking any questions. However, teachers should have served as a guide for students by encouraging them to think more during the modeling process.

In the same interview, PT24 evaluated the role of the modeling course instructor in the latter episode.

Interviewer: So, what do you think about the role of your course instructor?

PT24: Our instructor?

Interviewer: Yes, he who carries out this modeling course. I think he came to your group during the solution process. What did he do?

PT24: He came to us; he asked us what we were doing. He listened to us about our ideas on the solution. He observed which level we were in the solution process. He asked questions about our solution method. He asked questions like "What will you do if that is happened?", "What happens if this goes like this?" These questions were about to direct us to understand the situation and the context very well. He avoided from directing one way and tried to question the way of our solution method. His questions provided us to check our solution method in the process. According to me, he was good.

According to preceding excerpt, PT24 talked about the instructor's approach and questioning style. He stressed that the instructor questioning style such that instructor helped them to find their way by asking appropriate questions rather than telling them the correct way. In another interview, PT24 also discussed the instructor's approach during the solution process of *the Ferris Wheel* activity.

Interviewer: Are there any factors that influence the solution process positively?

PT24: OK, an event happened in the solution process of the modeling activity. We were trying to solve the problem with our group members and our instructor was checking our solution method. I believed that we could solve the problem. However, we got stucked. As the times passed, there was no solution at all. The solution period was extended. In the process, the instructed asked some questions in order to move us to think about the situation. That was good. He asked, “Why did you think like that at this point?” By asking questions that move us to think more, he guided us to find the correct solution method without directly telling the method. If he said, like “That was wrong!” I would be unmotivated. Some of friends could be unmotivated too.

In the above episode, PT24 put emphasis on the questioning style of the instructor and considered this approach positive for students during the implementation of the modeling activity. He indicated that guiding students by asking appropriate questions led them to think more and reach the correct solution. He also said that students would be unmotivated if instructor said “That is wrong!” or “That is not the case”.

After the implementation of *the Free Roller Coaster* activity, PT9 drew attention to the instructor’s intervention during the solution process. She advocated the instructor’s intervention in the following excerpt.

With the help of this question, I saw how a teacher should interfere to the students when the students start to solve a question in a wrong way or cannot reach solution. This was a kind of interference that I had also thought, but when it came to giving examples, no certain examples had come to my mind. I saw the examples by experiencing. Change of slope question asked on drawn curve and the slope of this curve and furthermore questions on which points the curve must be maximum enabled me to perceive the question correctly. When I look at the shape, I think about how I could not realize characteristic of the shape that I drew in the classroom. Unintentionally, I had already drawn desired shape but the role of the teacher was essential for me to realize the last characteristic we had found (*PT9, reflection paper for the Free Roller Coaster activity*).

In the preceding quote, PT9 pointed out that she learned how to intervene in the solution process when students could not reach any solution or went in wrong direction and she agreed with the style of intervention. It was understood that questioning during the intervention helped her to understand the problem

PT9 also evaluated the role of the instructor after the implementation of *the Water Tank* activity. She defended the idea that teachers ought to intervene students if they did wrong in the following episode.

Interviewer: Hmm, OK. So, PT9, I want to ask you, Can you evaluate the role of the instructor during your group working? That is, How can you describe the role of the instructor while carrying out the modeling process and managing the classroom?

PT9: He listens to us. He never directed us so far. In addition, the instructor comes and listens to us. He asks questions about our solution process. For example, in this week, he asked us a question. We said that the point should be at the maximum. He asked us which point we meant particularly. We could not answer the question completely.

Interviewer: In your opinion, how it should be?

PT9: According to me, teacher should intervene the students when they did a mistake. That is, I do not mean that it is wrong, maybe teacher want students to summarize the steps of the solution process in order to give feedback to them about their possible mistakes and make them notice their own. Teacher should ask questions about the mistakes that students did.

Interviewer: OK, has this happened so far? According to you, did the instructor do this or not?

PT9: The instructor did not do this. He intervened us about the subject that where place should be taken as 100 meters. He asked questions such as "Which place can it be?" He explained the situation to the class also.

In the preceding episode, PT9 asserted that the teacher ought to intervene during the solution process if the students were solving the problem incorrectly. She also stated that teacher ought not to tell the correct solution. Instead, teacher could summarize the problem situation by pointing out the errors they made and ask questions to make them find their errors.

For the same modeling activity as PT9 mentioned in the previous episode, PT14 put emphasis on the role the instructor played during the modeling process in the following episode.

Interviewer: Well, have you thought anything about the solution?

PT14: Did I think anything about the solution? [*Thinking*]. Well, I pay attention more on the independent and dependent variables of the graph while I am interpreting particularly the graphs like that. For example, even when we discussing with the instructor about the variables. I said that the amount of the water was the dependent variable, and then I said it was the independent variable. But the instructor did not say anything about the situation as always. The instructor confused our minds by doing like that.

Interviewer: What did he say that you made confused?

PT14: The instructor asks something that make us confused, but it is good for us. I say the instructor is coming again and he will make us confused.

In the preceding episode, she indicated that the instructor avoided directing students during the solution process and asked students to understand the problems situation. She denoted the instructors' questions as "confusing", but she admitted that these questions were useful for them. The same interview was continued with her evaluation of the role of course instructor.

Interviewer: OK, at this point, PT14, can you evaluate the role of the course instructor? For example, what does he do in the class? How does he plays the role while walking through groups and asking you about the solution process?

PT14: Actually, I learned a lot from the instructor. He asks very logical questions such that he helps you to find the mistakes on your own. For example, he comes to us, he asks, "What are finding at that moment?", "Can you draw the graph of that?" Sometimes I imitate him during the group discussion when the instructor comes to our desk. He asks that I am doing like this.

Interviewer: You mean questioning style.

PT14: Yes, his style is very good. He makes our mind confused by asking question, but these questions enforce us to think more and more about the situation and the modeling activity.

In the preceding part of the interview, PT14 stressed the significance of the questioning style of the course instructor in revealing the students' errors and unperceived points of the problem situation. She stated that the instructor asked logical questions to help students find their mistakes.

Interviewer: Doe he say something clearly? Alternatively, he says, "That is correct!" or "That is wrong!" like that.

PT14: No, absolutely. Even no, we do not get any meaning from his gestures and mimics. When he laughs, it would be wrong in generally. He asks qualified questions such that he make you find not only your mistakes, but also the correct ways for the solution.

Interviewer: Can you give a specific example for it?

PT14: I do not remember much really. For example, we compared the volumes by considering the heights. The instructor asked us "Draw a graph providing different conditions". By the question, he helped us to observe different types of the graphs according to the shapes of the water tank.

In the above except from the same interview, interviewer asked PT14 about whether the instructor directed prospective teachers by saying "wrong" or "correct". She stated that they tried to learn if their solution was correct or incorrect by observing the instructor's gestures. She was also asked to give a specific example of the instructor's questions that made students find their errors and notice the correct approach, but she expressed that she did not remember a specific situation for it. When videotaped records were examined, the situation during their group study on *the Water Tank* activity expressed by PT14 is observed in the following episode:

Instructor: I see that you are drawing three graphs.

PT16: Yes.

Instructor: What kind of decisions you take while drawing graphs? According to, what you have drawn?

T16: We take their shapes into consideration.

Instructor: You have drawn according to their shapes. So, for example this (Water Tank 3), have you drawn this? How have you drawn this, please tell me.

PT16: For example, is this a triangle teacher? That is, the lower one is not the same with the upper one.

Instructor: It is not. It is like cone. Is not it?

PT14: Yes.

PT16: Teacher, first of all, when we fill the hemisphere with water, since it expands transversally (the lower case of the tank), as the amount of water increases, level of water (h) will increase slowly, that is, it will increase with decreasing rate. So, the graph will demonstrate an increase with decreasing rate. The graph of that (see Figure 8).

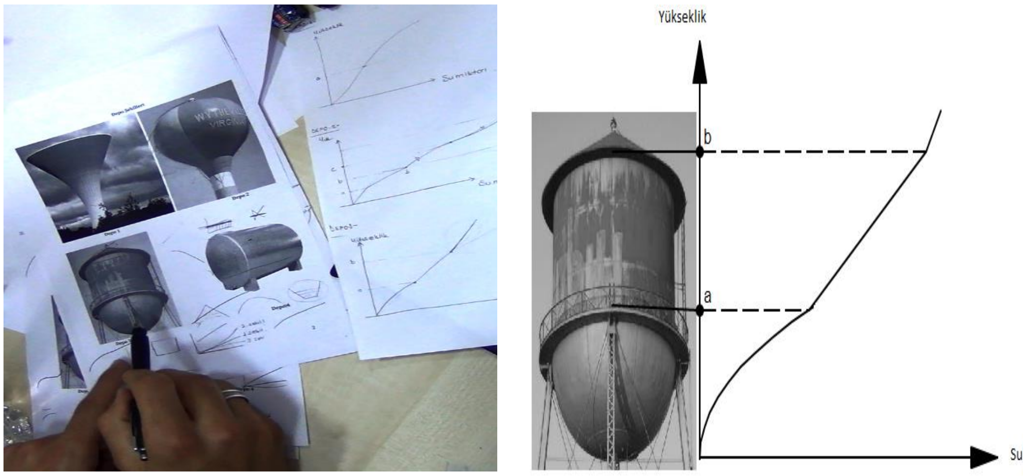


Figure 8 A video still from group 2’s solution and graph drawn on the right side is the recreation of original sketch for tank 3 in *the Water Tank* activity.

Instructor: You say graph display an increase with decreasing rate.

PT16: It will increase slowly after a degree. Since here is constant (the cylindrical part of the tank), let us say cylinder to that, the amount of water increased to the point a.

Instructor: OK.

PT16: Since the shape of the tank from point a to b is cylinder and flow rate is also constant, the level of the water (h) increases constantly. This part of the water tank (the upper part) is in the shape of cone. The cone is one third of a cylinder. However, there exist a thing here. The slope here is decreasing continually; therefore, the slope of the curve with respect to previous one is more.

Instructor: Is it more?

Figure 9 A video still from group 2’s discussion on solution of upper part of tank 3

PT16: slightly higher, no, it will be like that.

PT13: In my opinion, it will be like that!

Instructor: Come on, there are different drawings here.

PT15: I do not think so.

Instructor: Wow! You all fall into a serious disagreement now. Four distinct idea emerged.

In the preceding example, the instructor tried to learn how the group reached their solution. PT16 explained that they divided the tank into three parts that were semi-hemisphere, cylinder, and the upper part (see Figure 8). It was understood from the above conversation that they had a problem identifying the upper part of the tank and PT16 asked the instructor whether it was a triangle. This situation showed that PT16 thought the upper part of the tank was a triangle. In fact, it was a cone shape and the following part of the conversation demonstrated this resulted in a series of mistakes in the solution process. It was clear that the group members agreed on the graph of the first two parts of the tank. However, all members had different ideas regarding the shape of the graph of the upper part of the tank (see Figure 9). This situation was explained by the instructor.

PT13: Teacher, here is linear. We do not decide whether this line will go over according to previous part of the graph or it will go down. That is, will the slope of that line be higher or lower than the previous drawing?

Instructor: Where? Is it at this point or that point?

PT13: That point [*pointing the third part of the graph, after the point b*]

Instructor: Ok.

PT13: We have not decided yet.

Instructor: Think about that critical point here. Any pencil? [*Drawing a cone*]

PT14: Is it a cone?

Instructor: If we assume that part as a cone, let us suppose the shape like that (*referring to the shape in Figure 10 below*).

PT14: But, is there any increase here?

Instructor: If you want to draw the graph of this, how do you draw it? What is the graph of this?

PT13: The amount of water versus height of water level.

Instructor: The amount of water or briefly, we can say it volume of the water. Is not it?

PT13: Yes.

Instructor: And height. Let us denote that with h. Yes.

In the above conversation, PT13 explained the problem they had faced during the solution procedure. PT13 pointed out the transition point in the graph. The transition point was at the end of the cylindrical part of the tank and at the beginning of the part shaped like cone. By drawing a sketch (see Figure 10), the instructor asked the group members to visualize the upper part of the tank from a different perspective. After drawing the cone sketch, the instructor asked the question “If you want to draw only this graph, what kind of graphic would you draw? What graph is this and whose is this graph?” in order to help students overcome the difficulty they were having with the problem.

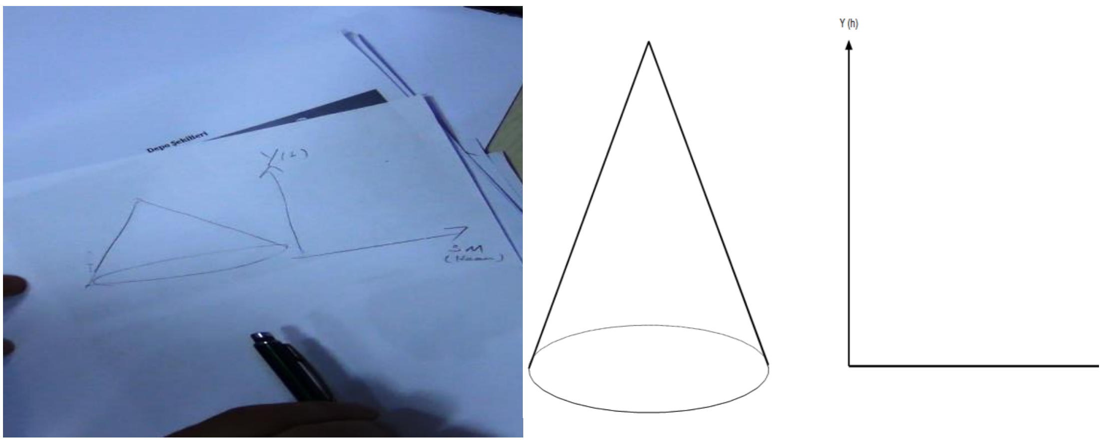


Figure 10 A video still from the instructor’s sketch and the drawing on the right side is the recreation of the instructor’s original sketch

Instructor: How would you do that?

TC16: I would do like this (see Figure 11). There exist a similarity in the triangle. If I say S, 3S, 5S. What happens?

Instructor: What did you do? Did you divide into volumes?

PT16: Yes, I divided into volumes. Totally, I got 9S. First of all, it will fill the 5S part of the cone, then.

PT15: What will be these heights? [Instructor pointing out the heights of pars of cone]

PT16: All of them are equal.

Instructor: Have you taken the heights as equal, have not you?

PT16: Yes, I have.

Instructor: How do volumes change?

PT13: S, 3S, 5S. No.

Instructor: Do volumes change like that?

PT16: You know there exists triangle similarity; I think it is valid for cone also.

Instructor: What do you compare with by using triangle similarity? What do you mean from 3, 5?

PT16: You know it happens in the triangle.

Instructor: Areas?

PT16: Yes, areas. I think areas of the triangle are related to volumes of a cone.

Instructor: OK, I understand. You say they are associated.

PT16: Yes, everything like pi square.

PT15: For instance, the line will go like that. In normal, it was like that [*Showing the drawing style*].

PT13: Yes, it will like that. Let us delete here.

Instructor: How did it happen? Do you all agree with it? There is no problem if you all agree with it.

In the preceding conversation, the instructor asked the group members how they drew the graph of the sketch to encourage them to predict the shape of the graph. PT16 asserted the way of his solution strategy with the help of triangle similarity. PT16 used proportions with area for triangle similarity by denoting S, 3S, and 5S for areas slices of the triangle (see Figure 11).

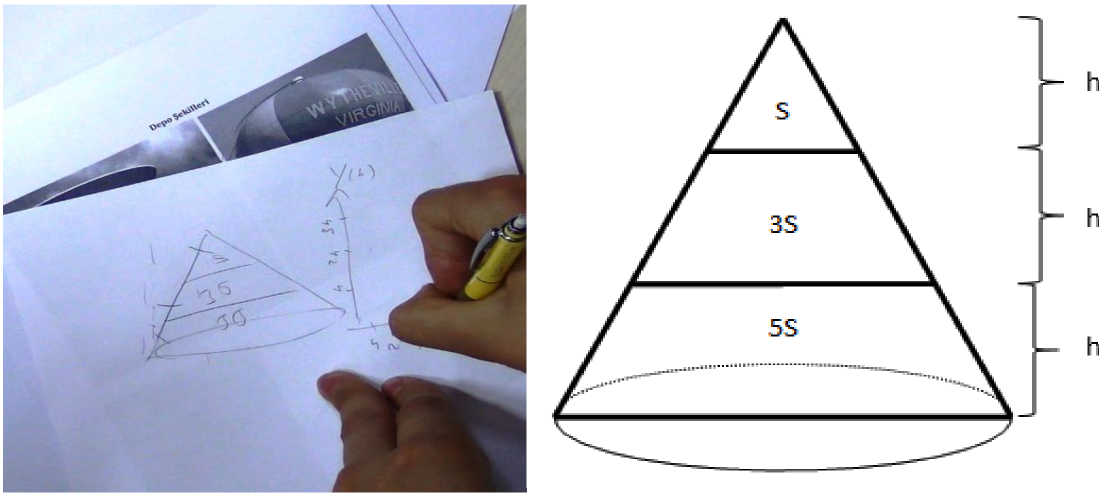


Figure 11 A video still from the solution of PT16 and the drawing on the right side is the recreation of the instructor's original sketch

The instructor asked several questions on the solution strategy of PT16 and tried to draw attention to the conceptual mistakes that might stem from confusing the proportions with area and volume. From above conversation, it was clear that PT16 used triangle proportions similar with area to interpret the volume of the cone. PT16 stated that area of the triangle was related to volume of the cone. The instructor tried to help him find his error by asking questions "What are you comparing with in triangle similarity? What do you mean by 3, 5?" and "Are they areas?" The instructor also tried to involve other members of the group in the discussion by asking the question "How did it happen? Do you agree with it?"

Instructor: OK, if we take equal intervals on that axis [*y-axis*], there exist different values on that axis [*x-axis*] will change accordingly. OK, you went from there, let us look here more closely. What are these S, 3S, and 5S?

PT16: Teacher, they were obtained from triangle similarity.

Instructor: What is 3S in the similarity?

PT16: For example, let us take 1 to 2.

Instructor: What do you mean 1 to 2?

PT16: OK, let me show it.

Instructor: If this and that are equal, where is 1 and 3?

PT16: Teacher, their ratio of areas are proportional to the square of their edges. If the area here is S, then all of them is 4S. Therefore, 3S remains to the area. Then, if we draw the same triangle, 5S remains there.

Instructor: OK, you mentioned about the area here. Is not it? You say it will be equal when you multiply edges with height. I do not understand, by volume.

PT16: Yes, length of there x, then here is $2x$. Then, these areas will be proportional to squares of these edges.

Instructor: Let us think a cone like that. The other sides need to be equal, a cone like that. It can be continued like that (see Figure 12). Do you say the volume of this part is one third of that part?

PT16: Can I try?

Instructor: OK, try then.

In the above conversation between the instructor and PT16, it was understood that PT16 continued to believe that proportions with areas could be used for proportions with volumes. The instructor also continued to pose questions about the strategy that PT16 used to solve the problem. It was evident from the above part of the conversation that PT16 proposed that the volumes of the cone slices were proportional to the square of their areas with ignoring the heights of triangle pieces. Again, the instructor tried to draw attention to the ratio of the volume of cone slices by drawing a new sketch and underlining the cone shape and its volume (see Figure 12). The instructor continued asking questions about the new sketch and volume of slices of the cone with equal heights h. From above conversation, PT16 did not understand the point the instructor was trying to make but PT16 decided to try by considering the volumes and their heights.

PT16: If here was 1 (assigned value to edge of inner triangle), then here 2 (assigned value to edge of exterior triangle). Okay, let me draw a triangle here. Then we get 1, and four by squaring each. What will be total area? Hmm, I will multiply this (assigned value 1) with h, and multiply this (assigned value 4) with 2h.

PT14: Yes.

PT16: Yes, so if here is S, then total will be 8S.

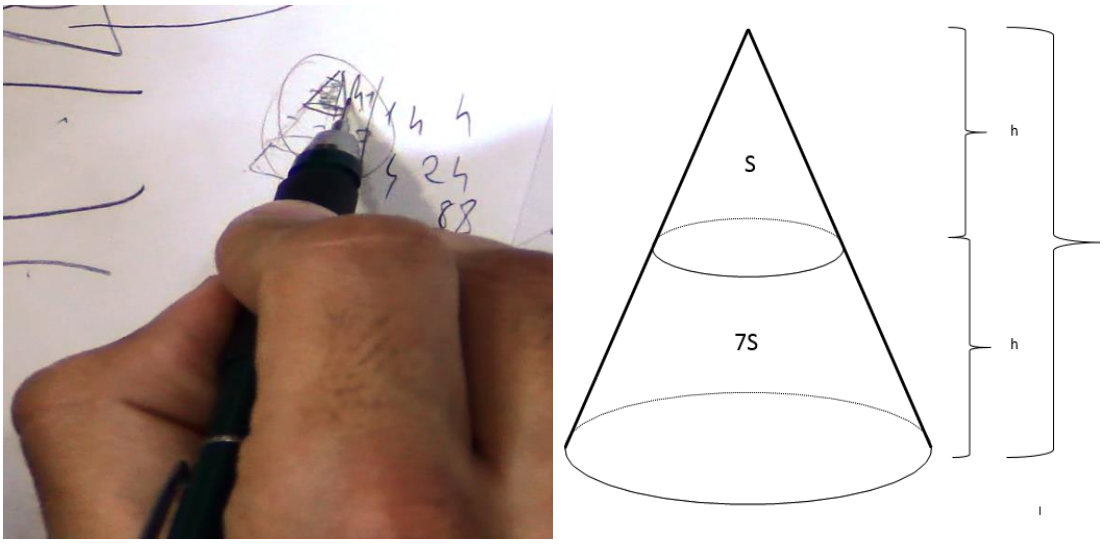


Figure 12 A video still from the instructor's sketch of the upper part of tank 3 and the drawing on the right side is the recreation of the instructor's original sketch

PT14: Yes, that is true.

PT16: Hmm, maybe it will increase like that.

PT14: It will increase; I think it will increase by increasing rate. Because when it goes up to here, assume that it will increase the volume 5V. If we think, here will be 8V. We add a little volume to that.

Instructor: OK, now PT16 tell your friends what is happening. Tell your friends the last point. Maybe they would disagree with you. They find a mistake again.

PT16: Now, I think, I assumed that cones are similar to triangles. When I tried again, I found that the volume of the inner part is S and of the exterior part 7S.

In the preceding conversation, it was understood that PT16 realized the mistake he made when relating the triangle with the cone in computing the volumes of cone slices and corrected it. This finding showed that the instructor's role in the modeling process is important as well as how the instructor helped students by asking appropriate questions to help them find their errors and reach a correct solution. This view was expressed by several prospective teachers and they put emphasis on the role of the teachers during the implementation of modeling activities. It can also be interpreted the instructor's played a role in noticing the problem in the solution strategy and helped students by asking appropriate questions to ensure that students notice their conceptual errors. This situation provided prospective teachers with the opportunity to revise their solution steps and comprehend the problem situation more conceptually. Similar conversations between prospective teachers and the instructor in other groups were observed during the solution process of modeling activities, and the instructor showed the similar approach for other groups' members like in the case of group 2.

Prospective teachers' responses to the question "What kind of knowledge and skills do teachers need in order to implement modeling activities in the classroom setting effectively?" were analyzed. As a result of the analysis, prospective teachers indicated that teachers need to possess mathematical content knowledge, pedagogical knowledge of modeling, knowledge of modeling, knowledge of the nature of modeling activities, and knowledge of classroom management in order to carry out modeling activities effectively and successfully in the classroom environment. The findings demonstrated that prospective teachers developed significant ideas about the knowledge that they thought that teachers need to have to conduct mathematical modeling activities in their classrooms successfully.

When prospective teachers' responses were reviewed, almost all of the prospective teachers declared their opinions about what teachers need to know. Some of the prospective teachers drew attention to the pedagogical knowledge of modeling in their answers to the question in the post-survey form. For example, PT9 pointed out that the role of the teacher in the classroom during the modeling process. She wrote:

> A teacher should determine his or her role well in a classroom. The teacher should be like a guide and she or he should not give every information instantly. She or he should enable the students to discover that information on their own. She or he should understand the students' way of thinking and should lead them and their mistakes according to it (*PT9, post-survey form*).

In the preceding quote, she emphasized the role of the teacher as a guide and suggested that the teacher ought not to tell every detail related to the subject matter, but rather she or he should help students access the required knowledge by himself or herself with comprehending their students' thinking. Another prospective teacher PT14 expressed her opinions about the pedagogical knowledge of modeling needed for teachers as follows:

> A teacher also be able to predict possible mistakes of the students or problematic subjects for students and the teacher should be a leader that enables students to notice their own mistakes. When the students make mistake in any phase of the solution, the teacher should interfere but this interference should be used to help them find their own mistakes (*PT14, post-survey form*).

In the above excerpt, PT14 stated that teachers ought to predict the possible mistakes and difficulties students may have. In addition to this, they should assist students in recognizing their own mistakes during the solution process. She also indicated that teachers ought to intervene to the solution process, but it should be in the direction that helps students find their own mistakes. PT17 expressed similar opinions with PT14 for teachers about predicting difficulties that students might encounter and having an idea of how to approach to these difficulties.

Another required quality needed for teachers to implement modeling activities concerned content knowledge. Prospective teachers put an emphasis on the content knowledge of mathematical subject matters during the mathematical modeling process. For instance, PT12 pointed out the content knowledge that teachers needed during the implementation of modeling activities. She wrote, "First, a teacher needs to be mathematically well equipped. She or he needs to answer the questions correctly asked in the process of problem so that the students are not misleaded? For this reason, she or he needs to be well equipped". In the previous quote, PT12 underlined the content knowledge for teachers during the modeling process such that the teacher ought to have enough content knowledge in order to reply to students' questions and not mislead them. Similarly, PT24 asserted that teachers who wanted to implement modeling activities in the classroom setting ought to have mastered content knowledge of subject matters. He explained his opinion as follows: "A teacher should also have a grasp of the mathematical concept that is analyzed in question. Because they should be able to find solutions and be able to direct the students to the right way".

Prospective teachers suggested that knowledge of modeling and the nature of mathematical modeling activities were prerequisites that teachers needed to possess. It was evident from the prospective teachers' responses that knowledge about modeling and modeling activities was significant for teachers in order to implement these activities in the classroom environment successfully. For example, PT5 pointed out teachers' knowing what modeling really meant in the following except:

> A teacher needs to know what the modeling is in order to be able to implement modeling question in a classroom. Alongside this, how to help to the students must be determined. A teacher should also have sufficient knowledge about the activity that he will implement and should be open to ways of solution. S/he should have an idea about possible questions and problems (*PT5, post-survey form*).

In the above excerpt, PT5 asserted that teachers ought to know the meaning of mathematical modeling before the implementation of any modeling activity. PT5 also noted down that teachers should determine how to help students during the modeling process and have enough knowledge about the nature of the modeling activity including possible solutions, questions from students, and any difficulties that students might face.

In addition to prospective teachers' thinking about the knowledge that teachers needed to possess, prospective teachers mentioned classroom management during the implementation of modeling activities. Prospective teachers reported that teachers ought to know how to manage classroom while carrying out the modeling activities. For instance, PT9 stated that teachers ought to be aware of how to manage classroom and preclude possible noise during the group presentations as follows: "A teacher should maintain class management; because she or he needs to avoid possible noise in group presentations". Similarly, PT5 discussed classroom management in terms of time allocation in the following: "Teachers should use time effectively and should give enough time to the students for the activity". It was seen that other prospective teachers suggested that classroom management ought to be taken into consideration during the modeling process. In light of the prospective teachers' expressions on the post-survey forms, interviews, and classroom discussions the knowledge that teachers need to know in order to implement modeling activities in the classroom environment are illustrated in the following table (see Table 11).

Table 11 Views of prospective teachers about the needed knowledge for using modeling activities in classroom setting

Content Knowledge	**Pedagogical Knowledge of Modeling**	**Knowledge of Modeling**	**Knowledge of the Nature of Modeling Activities**	**Knowledge of Classroom Management**
• Having mastered the concepts that were investigated • Having equipped with subject matter knowledge	• Knowing where and how to intervene • Provoking students to think • Knowing how to guide students • Asking leading questions • Knowing the role in the modeling process • Being in the guide position • Providing students to access to the knowledge on their own • Having the knowledge of the students way of' thinking • Using the modeling activities for the intended aim	• Having the knowledge of modeling • Having taken similar courses with modeling	• Activities ought to be solved before • Having foresight about the activities (Solution ways, student questions, possible difficulties etc.) • Having no suspect about the activity • Having the knowledge of developing modeling activities	• Giving enough time • Knowing classroom management • Preparing application plan and following it • Preparing condition for free expression of ideas • Being tolerant, understanding, and patient

4.2.1.1 Prospective teachers' thinking on knowledge about the students' ways of thinking when solving a modeling task

After the analysis of prospective teachers' responses to the post-survey form, individual semi-structured interviews, field and observation notes, and reflection papers the findings demonstrated that prospective teachers should try to understand the students' ways of thinking to observe different students' ways of solutions; to be informed about possible mistakes that students could do; to see what and how students think about mathematical concepts; to see what kind of difficulties students suffer in the process. For example, PT16 pointed out that the students' ways of thinking was significant in revealing the diversity of students' approaches to the solution as revealed in the following quote:

> When they deal with an activity that they have never seen or done and if they are aware of the daily life problem, incredible solutions, which also have never come to my mind, occur and come. Like scaling in roller coaster and trying to use mgh, $\frac{1}{2}mv^2$. The student tries all the ways to reach the solutions. But when I analyzed solution papers of the students till now and when I watched videos, I noticed that they have weak proving skills (*PT16, reflection paper for the Free Roller Coaster activity*).

Similarly, PT11 indicated the same function of students' way of thinking in the post-survey form as follows:

> A far as I saw in students' videos and solution papers of the students, the students can see the question in its different aspects. The ones that reach the solution are quite a lot. Modeling questions are quite effective to understand the mathematical thinking ways of the students. Their solutions for the question show us what they know or do not know (*P11, post-survey form*).

In the preceding excerpt, PT11 stressed the significance of the modeling activities in understanding the students' way of thinking and stated that students could look through the modeling activities from different perspectives in order to solve them. As indicated in the application plan (see Appendix B) students' ways of thinking were implemented after four modeling activities.

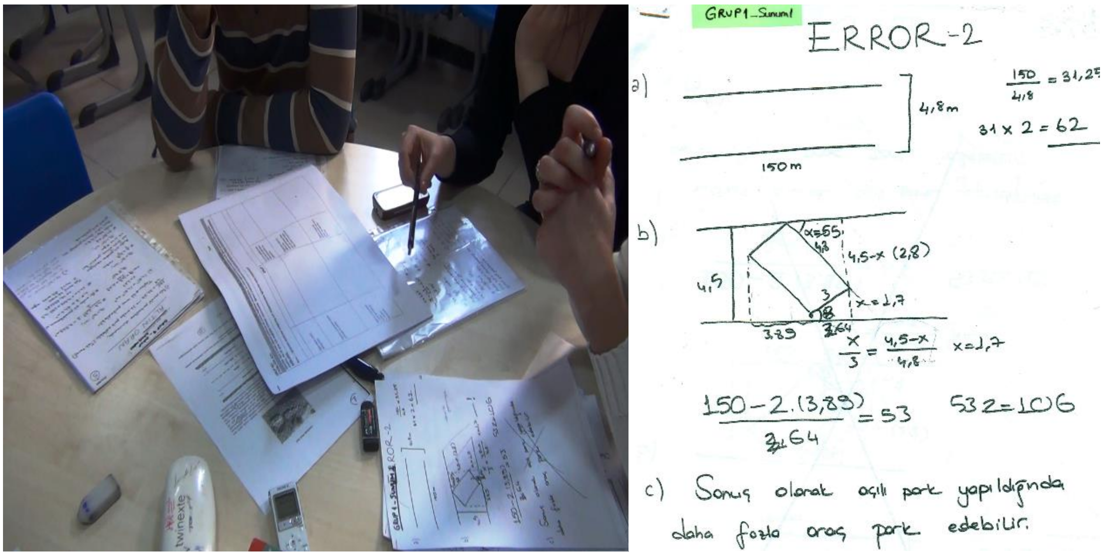


Figure 13 A view from prospective teachers' works on the students' ways of thinking sheets

Prospective teachers interpreted students' way of thinking in terms of how students' work gathered in the in-service part of the same project. Several prospective teachers said that they felt astonished when they saw the solutions and students' the way of thinking in the solution procedure (see Figure 13). For example, PT2 and PT3 expressed their ideas to the instructor about the students' way of thinking during the examination of the students' work in the following episode:

PT3: For example, after completing the activity, we went to our homes. We wrote reflection papers. When writing the report, there existed a question "When you looked through the eyes of a teacher, how students approach to the question?"

Instructor: Yes.

PT3: Obviously, I wrote the same thing in all reflection papers. I always used to think the same way. I used to assume that students think like me. I think they would make the same mistakes, but I am surprised and do not expect that. I am surprised when I see these mistakes. They thought differently. I mean, they made interpretation according to their own drawing. They saw wrongly, so they draw a perpendicular line. They made foolish mistakes. I do not think they made these mistakes.

PT2: They constructed a geometric shape and then they interpreted it. For example, one group members did not think whether a car could park the place or not. They parked the car with by the length 2.64 meters. Ok, by establishing a geometrical shape, then what can I do?

PT3: I mean, they did not see it as a problem, but they saw it as a geometry question.

Instructor: Ok, Other students also told what you told about. That is, these are to be investigated.

PT3: That is to say, obviously we had not have any activity like that so far. Since we had not been any setting, so we saw a classroom setting in this activity. I mean we had been in classroom as students and we did similar mistakes. These mistakes can change person to person, so we did not see anything in general. I wrote this on my report in the yesterday evening.

PT2: I will change this in my reports accordingly too. I always write positively like I will do like that and that. In my opinion used to say mistakes could be very simple. For example, I said that they could not see that the angle was 360 degrees in *The Ferris Wheel* activity. However, I observed that they could do very simple mistakes that I could not guess. I noticed that.

PT3: For example, I do not see no more things as being a teacher. As if, the problem is clear and obvious. It seemed like they saw hints clearly, but there existed more things. That is, there are more duties for teacher in the modeling process.

In the preceding conversation, PT2 and PT3 mentioned the difference between the possible outcomes of students' way of solutions and thinking they envisioned when they were writing reflection paper and the real documents retrieved from the in-service part of the project. PT3 indicated that a difference between the envisioned situation and the real situation in terms of students' way students were thinking was surprising. PT3 expressed that students could think differently from them and make simple mistakes in the solution process. PT3 noted that exploring students' work revealed that teachers had more responsibilities during the modeling process than they considered.

Prospective teachers suggested that students' way of thinking provided a way for teachers to understand how students think about mathematical concepts. They could observe which point they did not comprehend in the subject matter and hereby teachers had a chance to revise and refine the teaching of the related point. For example, PT6 illustrated this approach as follows:

> I think modeling activities are useful in understanding mathematical thinking ways. Because instead of solving the question with a formula in a normal mathematical question, we try to make the question solved by students' questioning. In this process, we see to what extend they have understood or not and how to fix these points (*PT6, post-survey form*).

In the preceding quote, PT6 stated that the modeling activities were beneficial for understanding students' mathematical way of thinking, and teachers could detect the concept that students did not understand and how to refine it with the help of students' way of thinking works. PT20 also stressed the importance of students' way of thinking works in terms of revealing the missing situations related to the knowledge that students learned in the process. PT20 wrote:

> With the studies that students have carried out, mathematical thinking ways of the students help us a lot to see whether the students have understood the mathematical subjects, false ideas about the subject and also helps to see if there are problems in practice even though the subject is understood (*PT20, post-survey form*).

Prospective teachers emphasized that students' ways of thinking about work helped teachers in many ways such as providing information about whether students understood the subject matter or not, observing the application of the gathered knowledge on the activities, and detecting the misunderstanding of the concepts during the implementation of the modeling activities. For example, in the following quote, PT8 compared the modeling activities with traditional word problems in terms of their characteristics in assessing students' knowledge.

> We cannot understand fully how the students actually think about that mathematical subject with the exam questions during the lessons. We can have detailed information about thinking ways of the students as we can see and evaluate clearly with modeling questions both concretely (papers of solution) and verbally (presentations) what and how the students think, what the students know or do not know, among which subjects they establish connections and which mathematical concepts they use (*PT8, post-survey form*).

According to preceding excerpt, PT8 asserted that traditional word problems were insufficient to understand how students thought about the related subject matter. Nevertheless, it was possible to observe how students' think, what they know or not, to which subjects they make connections and which concepts they used during the modeling process. PT8 indicated that they discovered knowledge about students' way of thinking with the help of modeling activities. In addition, PT14 suggested that students' way of thinking works would be helpful for understanding students' difficulties, deficiencies, and misunderstandings.

> As I mentioned before, indeed, with help of this lesson I realized for first time that I am a prospective teacher. Because in the simplest term, I had never tried to guess possible mistakes of the students by putting myself in students' place. Seeing the subjects misperceived by the students, their difficulties, inadequacies, and strength will make great use and contribution in the future while performing my job. In this sense, this course was a guide for me (*PT14, post-survey form*).

In the preceding excerpt, PT14 stated that she realized the fact that they were prospective teachers thanks to the modeling course. She expressed that comprehending students' way of thinking would be beneficial for her when she became a teacher.

In summary, almost all of the prospective teachers emphasized the significance of the students' way of thinking about their work. They also indicated that this work would be helpful for teachers in noticing the distinct way of students' solutions, detecting students' deficiencies, and difficulties they had. It would also help them become aware of how students think about the mathematical concepts and subject matters.

4.2.1.2 Prospective teachers’ views about knowledge about classroom management during the modeling process

In general, classroom management is a required process in order to carry out teaching and learning in a smooth manner in the classroom setting. In this part, “classroom management” refers to managing and operating activities including what teachers should do prior to the implementation of a modeling activity (e.g. selecting modeling activity, preparing implementation plan), how teachers implement modeling activity in the implementation process (e.g. implementing the activity by grouping or individually), how teachers play the role in the process, and what teachers do after the implementation of the activity. Since this part of the study included interrelated issues with previous findings, situations that prospective teachers experienced before and after their implementation of modeling activities, ideas they grasped about knowledge that during the implementation period about the use of modeling activities in the classroom setting were reported in this sub-section. Individual interviews, project reports written after the implementation experience, transcripts from video and audiotaped records, field, and observation notes constituted the main source of this section.

The findings demonstrated that most of the prospective teachers preferred group work as a method for implementation (see Table 18, section 4.2.2.3) and designed a plan with their group members for their implementation experience (see Appendix L). Some of the prospective teachers believed that they could use group or individual work methods according to the properties of modeling activities. Prospective teachers stated that they tried to comply with the implementation plan that they prepared, but they had difficulty allocating enough time for the implementation process. As a result, they said that they revised the implementation plan according to deficiencies they faced during the implementation. Prospective teachers stressed the importance of the implementation plan for conducting modeling activities successfully. For example, PT12 shared her ideas about classroom management during the modeling process with emphasis on the implementation plan in the following excerpt:

> First of all, teachers should know characteristics of a class. Then, they should prepare implementation plan according to the level of the class and they should manage the plan accordingly the level of the class. At first, the teacher should learn whether the question is understood or not by going near each of the groups. If there are problematic parts in the question, the teacher should help the students by guiding them. After that, the teacher should give priority

> to the groups whose solutions are different in the process of the students' presentations of their solutions and the teacher should pass to phase of summing up after the students' presentations are completed. Finally, in this phase, ideas and opinions of the students should be reminded and correct or incorrect ones should be shared with students and then the lesson should be finished (*PT12, post-survey form).*

In the preceding quote, PT12 stated that the implementation plan ought to be prepared according to the general characteristics of the class. PT12 also summarized how teachers manage the classroom according to possible events such as walking around the groups and guiding them through the activity if they an encounter any parts that are difficult. In general, the instructor asked the prospective teachers whether they understood the context of the problem after giving them enough time to read the given activity sheets. For example, instructor asked students what they understood from the activity sheet during the implementation of *the Free Roller Coaster* activity. The following episode illustrated the situation that occurred between the instructor and prospective teachers.

> Instructor: Have everyone read the activity sheet?
>
> [*No reply, everyone is discussing*]
>
> Instructor: I should think so. Even they have already started to produce solution. OK, what did you understand from the text of activity?
>
> PT21: I think it is confusing.
>
> Instructor: It is confusing. What does it ask?
>
> PT21: I understand that we need to design a railway such that it has slopes and fluctuation in three parts. Therefore, some values are given. It should be more than these values or we need to design the railway between the intervals.
>
> Instructor: Which part of the text is confusing?
>
> PT21: The railway need to have three fluctuation, for aught I know.
>
> Instructor: Let us read the text again. Okay, you read quickly. Let us read again please.

In the preceding episode, the course instructor asked prospective teachers whether they had any problem in understanding the problem situation or not. In order to clarify what was understood from the given sheet, instructor randomly selected a student and asked her to explain what she understood from the text. It was evident from the above episode that PT21 implied that the text of the activity was confusing and instructor wanted all prospective teachers to read the given sheet carefully. The role played by the instructor during the implementation was paid attention by prospective teachers. For instance, PT15 illustrated suggested role for teachers in the implementation period in terms of classroom management in the following excerpt:

> I believe that this process needs to be done by the teachers who are educated and knows the subject. Because I think that, a teacher should be a very good guide and be able to guess the possible questions in this activity's practice process except from the class management... For this reason while preparing implementation plan, first of all teachers should adjust time in accordance with students' level. Furthermore, a teacher should make sure of using this implementation plan and proceeding gradually. While planning this plan she should be cautious about all kinds of obstacles and she should make his plan flexible enough (*PT15, interview for the role of teacher*).

According to above quote, PT15 point out the characteristics of the teachers who wanted to implement modeling activities and offered several suggestions for teachers to carry out modeling activities successfully and effectively. PT15 highlighted the importance of implementation plan and complying with the plan. PT2 also pointed out the significance of the implementation plan in classroom management during the use of modeling activities in the following quote:

> I think that a teacher should be able to control the class but she or he should avoid seriousness and tension that can bore the students... She or he should not give the answer directly to the students but s/he should ask and answer questions that lead them to the solution... While summing up the lesson, one should pay attention to form an implementation plan in which all the ideas are presented, questions are asked by the teacher and heard by everyone in the class, the subject is summed up in general after all the opinions and ideas are taken without any chaos, correct and incorrect ones are distinguished and finally a plan which is visible both for teacher himself or herself and the students (*PT2, interview after implementation experience*).

In the preceding quote, PT2 mentioned the balance of the teachers' authority in regard to classroom management during the implementation of modeling activities. PT2 also put emphasis on the teachers' role in responding to students' questions as well as ideas concerning the solution process during the modeling process.

To sum up, prospective teachers emphasized the importance of classroom management during the implementation of modeling activities. Prospective teachers paid attention to preparing an implementation plan. They focused on time allocation, group or individual working styles, checking and revising solution procedures and the role of teachers during the implementation period. In the light of obtained findings from prospective teachers' project reports and transcripts obtained from video and audiotaped records, prospective teachers suggested that teachers ought to pay attention to the following situations during the modeling process:

Teachers should be able to

- prepare an implementation plan according to the characteristics of the class,
- prepare a flexible implementation plan and comply with the plan,

- ask questions that will help students find their mistakes and guide them to the correct solution,
- walk around the groups and be aware of solution processes,
- allocate suitable time for each section,
- dominate the classroom,
- arrange groups homogeneously and voluntary,
- avoid making certain judgments (e.g., “That is correct or wrong!”),
- make members of groups who had different ways of solving the problem present,
- summarize the ideas developed during the modeling process.

Prospective teachers indicated that implementation of modeling activities would reach the intended aim if the above bulleted situations were taken into account by the teachers regarding classroom management during the implementation process. The findings demonstrated that prospective teachers took the “Implementation Guide for Modeling Activities” (see Appendix M) into consideration, but some unforeseen problems could arise in the process, and they hoped to maintain a positive classroom environment by effectively revising and re-defining the problems that emerged during the implementation process.

4.2.1.3 Prospective teachers’ thinking about their own implementation experience

As part of “modeling course applied plan” (see Appendix B), prospective teachers were asked to develop and implement a mathematical modeling activity as a group. Participants developed a modeling activity with their group members and designed an implementation plan in order to carry out the process effectively. The findings showed that prospective teachers paid attention to the implementation time, intended gains, and potential students’ mistakes while preparing the implementation plan. For example, as a member of group 4, PT10 illustrated their process of preparing an implementation plan in the project as follows:

> While forming implementation plan, we had already had an implementation plan and we headed away from explanations for this plan. We determined to which classes this implementation would be implemented. We calculated practice time by considering lesson duration in a way the students would not be bored. As we will perform an implementation, we separated total time into small pieces. For example, we gave five minutes to explain the question and for the presentations, we gave half an hour. We gave these minutes according to the circumstances we experienced. We formed an implementation plan by considering what kind of things a student should acquire with this question. We thought about what kind of skills a student should use for this question. We decided which tools and equipment the students need to use and tried to procure them (*PT10, report written after the implementation experience*).

In the preceding excerpt, PT10 expressed that they used the given sample implementation plan during the preparation and divided the implementation process into sections. They devoted suitable time for each section such as giving 5 minutes for explanation of the problem and 30 minutes for the presentation part. PT10 indicated that they prepared the implementation plan according to objectives of the modeling activity and what students gained from that implementation. PT23, a member of group 6, stated that they took the intended objectives into consideration while preparing their implementation plan. PT23 wrote:

> While forming an implementation plan, there were ideas such as for which level of class, in which learning environment we can place our activity. According to this, we formed our plan and then we thought about what we can add to each space in this plan and considering this, we formed our plan (*PT23, report written after the implementation experience*).

In the above quote, PT23 mentioned that students' levels, learning domain, and intended objectives were taken into consideration while developing the implementation plan. PT24 described the process of preparing the implementation plan for their groups (group 7) on the basis of potential students' thinking about the ways of solution in the following excerpt.

> While forming the implementation plan, we put emphasis on ideas such as how the students can see many more ways of solutions, how the students see or grasp all the concepts in the question, what kind of attitudes the students have for different solutions in different discussion environments. Moreover, this plan was prepared considering high school students and it was a method that can be liked by them. However, when this plan was implemented to students in our class who are above high school level and as the time was limited; unfortunately, we could not get efficiency that we had expected (*PT24, report written after the implementation experience*).

In the preceding quote, PT24 indicated that they focused on how students choose different solution strategies and the mathematical concepts embedded in the modeling activity in the modeling process. He further stated that they prepared their implementation plan for secondary school students, but they did not effectively implement the plan due to time constraints. Other prospective teachers also expressed this issue as indicated in the above paragraphs.

As indicated in the given exemplary situations about the importance of preparing an implementation plan and which points needed to be considered while preparing it, prospective teachers developed ideas about the points that ought to be taken into account when teachers prepared implementation plan. The points that prospective teachers suggested as groups are illustrated in the Table 12.

Table 12 The issues that PTs considered while preparing an implementation plan for the modeling activities they developed as groups

Groups	**The points taken into account in the preparation plan**
Group 1	Student ideas/thinking for solution
	Potential ways of solution
Group 2	Students' thinking processes
	Potential student mistakes
	Determining duration of the implementation
	Feasibility of the plan
Group 3	Determining duration of the implementation
	Target objectives
	Potential student mistakes
Group 4	Determining duration of the implementation
	Target objectives
	Materials would be used in the process
	Distribution of tasks
	Classroom organization
Group 5	Determining duration of the implementation
	Target objectives
	The way of reaching objectives
	Potential student questions and their answers
Group 6	Level of the class
	Learning domain
	Target objectives
Group 7	Perspectives of students
	Potential attitudes and behaviors of students
	Implementation ought not to be boring

As it was seen from the Table 12, prospective teachers suggested mostly that duration of the implementation and target objectives ought to be determined while the implementation plan is being prepared. In addition to these points, prospective teachers put forward different ideas such as students' thinking processes, students' potential ways to come up with a solution, ways to identify students' mistakes classroom organization, levels of students, and feasibility of the plan etc. It can be interpreted that prospective teachers noticed the significance of the implementation plan and developed ideas regarding what should be included in the plan.

Prospective teachers evaluated the situations they faced throughout the implementation experience in both project reports and interviews after the implementation experience. In this sense, prospective teachers reported that students produced used many creative tactics for solving the problems. Participants stated that they were sometimes able to predict the techniques students would use to solve the problem but they were surprised by some of the steps students used to find a solution during the modeling activity. For example, members of group 4 indicated that students displayed both expected and unexpected ways of solution for their developed modeling activities. In the following episode, prospective teachers discussed the expected solution ways.

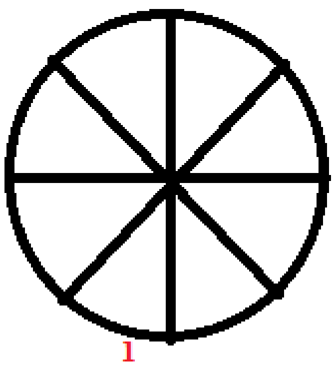
> PT12: They used inverse proportion.
>
> PT10: Yes, we certainly thought that they would find the length d and e. We also considered that they would get distinct solutions. The results confirmed us. That is, we expected these solutions. It was good.
>
> Interviewer: So, was the aim of this activity to reach distinct solution methods?
>
> PT10: Yes, the aim of the activity was so. I told you that we wanted to prepare a modeling activity that involves multiple ways of solution techniques. We saw from our friends, and we tried different solution ways after establishing the modeling activity.

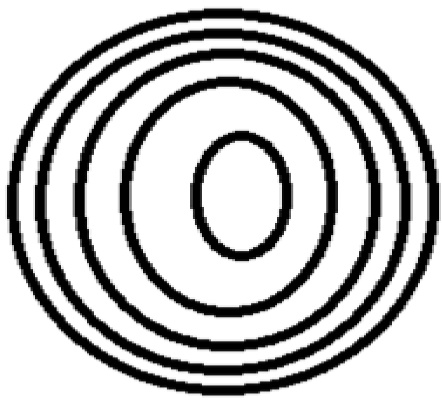
In the above episode, prospective teachers expressed that the solution to the problem varied from group to group during the modeling activity. They indicated that the purpose of the modeling activity was to show students that modeling activity might have more than one way to find a solution. In the same implementation experience, prospective teachers stated that they encountered unexpected ways of solving the problem for their modeling activity. For example, group members indicated in the following episode that students presented unexpected strategies for the solution.

> Interviewer: So, actually I want to ask something that you answered unintentionally. What did you expect before the implementation experience? What did you find after the implementation? Did unexpected situations happen?
>
> PT11: Yes, it happened.
>
> PT10: I thought that they would sum up and use proportion, but they thought that dividing money according to the number of person, that was a quick solution rather than following the operation one by one.
>
> Interviewer: There existed an inverse proportion between them.
>
> PT10: Yes, they thought that. The number of person will increase; the ratio will increase that influenced the sequence. Some of the groups used method of giving points.
>
> Interviewer: They used unexpected approach for the solution. Did not they?
>
> PT10: Yes, I did not expect that approach.

In the preceding episode, prospective teachers expressed that students presented different ways of finding a solution during the solution process, and some of the groups used unexpected strategies in order to find a solution during the modeling activity. This situation demonstrated that it might not possible to foresee the all the possible ways of solution for a modeling activity. Similarly, PT17 stated that students found unexpected solutions during the modeling activity. She wrote: “Well, I think at lead sprinkler can be put in a different way instead of putting them in quadratic form. Well, an idea came to my mind. Why do we ignore sprinklers' being stacked up on top of each other and their getting wet?” It was understood from the previous quote that students could think different from the instructor’s expected way of solution in the modeling process. Another unexpected exemplary solution way was reported by PT9 for her group (group 5) in the following excerpt.

> Solution approaches for the question were really different and fine. We had already predicted possible a few situations as we practiced this question in many different groups. But in classroom practice solutions different from our predictions came. Main solution idea was the situation of arranging the equal chances that correspond to the equal points to equal areas. Solutions that verifies this would be correct and valid solutions.


1


2

> The most likely solution we expected was their allocation of circle into ten equal areas by drawing radius from the center (Drawing 1). Of course, another solution was also possible with equaling areas and changing radius of annuluses (Drawing 2). While the first situation came to one's mind for the implementations in outdoors, in classroom implementation groups usually took the second situation into consideration. Of course, there were different solutions, too. For example, a Sudoku like board design offer in which score system was thought from one to nine came and it was quite well received situation for us as a group. Indeed, probabilities of the scores would be equal and sometimes next to number 1, number 9 would come and excitement and tension would increase (*PT9, report written after the implementation experience*).

In the preceding excerpt, PT9 indicated that there existed distinct ways of solutions in their implementation experience. Students thought different approaches for the solution of the modeling activity. PT9 gave two approaches from different groups, the first solution strategy was the expected one (Drawing 1), but most of the groups preferred the second way (Drawing 2) which was not expected solution method. Other groups reported similar results for students' ways of solution.

The findings demonstrated that prospective teachers observed that students had difficulties during the implementation stage stemming from their own developed modeling activities. Prospective teachers reported that some difficulties emerged in understanding of the problem situation during the period of implementation experience. These difficulties were caused by structural problems of the developed activities, problems related with linguistics and ambiguity, incomprehensibility about what was asked, and not making relationship between givens and what was being asked In other words, students had problems making relational and logical connections. For example, PT23 stated that their modeling activity had problems related to linguistics and ambiguity in the interview after the implementation experience in the following episode.

> Interviewer: We stayed at here, what did you say about the activity?
>
> PT23: Now, here it is, creative thinking like those. Which one is seen by more people, the idea of target audience can be reached more. Some of the groups understood the context of the modeling activity, but some of the groups misunderstood the activity due to reason that we expressed wrongly or they misunderstood. We had two schedule lists for the work. We wanted them to use these five times for each situation. The point we meant, only one group understood correctly. Other groups did not understand completely. Maybe it could be confusing for them. The process of understanding of modeling activity was as I expressed.

In the preceding episode, PT23 discussed a problem with modeling activity text during the implementation experience such that the modeling activity included two different timetables and students were asked to use the timetable five times, but only one group properly understood the instructions. This problem can be linked to linguistic ambiguity. PT14 noted in the project report that some of the groups had difficulty with understanding the data. PT14 wrote:

> After reading the question for the first time, some students had hesitations how and according to what they needed to draw graphics of burning tree depending asked time in question. Especially, the students could not make sense of the numbers written on the sides of the shapes and on which heights are showed. What kind of relation these numbers, namely slope can have or whether there is a connection between these numbers and graphics was another point in which the students have difficulty in understanding. Alongside these things, we especially wanted to mention about density of trees. Increase in numbers of trees and to draw attention to decrease in increase of trees

> depending on this increase in numbers, we placed our trees namely density of trees in such a way to increase with increasing rate or increase with decreasing rate. However, there were also some friends who could not correlate this situation with density of trees (*PT14, interview after the implementation experience*).

In the preceding excerpt, PT14 gave some examples of difficulties that students experienced during the implementation exercise. It was understood from the above quote that students had difficulties in understanding the situation described in the problem and the figures attached to the text. PT14 discussed the problem between the scenario described in the text and what students understood from it. Problems concerned with developed modeling activities appeared in the implementation experience are summarized in the following table (see Table 13).

Table 13 Problems emerged during the implementation of modeling activities developed in each group

Groups	**Structural problems**	**Linguistic and ambiguity problems**	**Relational and logical problems**	**No problem**
Group 1	X		X	
Group 2		X	X	
Group 3		X		
Group 4				X
Group 5				X
Group 6		X		
Group 7	X			

As illustrated in the above table, the problem most frequently encountered was related to the development of modeling activities. The group members of group 2, group 3, and group 6 reported that students had difficulty understanding the problem due to linguistic ambiguity. Members of groups 1 and group 7 asserted that students had difficulty understanding the problem because of structural problems during the modeling activities.

It was understood from the above table (see Table 13), relational and logical problems emerged during the implementation experience of group 1 and group 2. The group members of group 4 and group 5 stated that they did not face any problems associated with their own developed modeling activities in the implementation experience period.

In the interviews held after the implementation stage, prospective teachers were asked how to revise and refine their developed modeling activities with regard to problems that emerged during the modeling activity as well as their own observations. Prospective teachers stated that they needed to revise their modeling activities. For example, PT23, a member of group 6, indicated that there was a problem related to the incomprehensibility of the activity.

Interviewer: Then, you reminded the difficulties stemming from modeling activity. What kind of difficulties you experienced that stemmed from the implementation and classroom today?

PT23: the difficulties stemming from classroom?

Interviewer: the ones that stemming from the activity itself.

PT23: The difficulty was related to its incomprehensibility.

Interviewer: Yes.

PT21: We thought that it could be understandable, but it is not understandable.

Interviewer: OK, you will revise and refine the modeling activity.

PT23: Yes, we thought to write down notes to explain the situation.

In the preceding episode, prospective teachers explained that they had encountered problems stemming from incomprehensibility of the modeling activity during the implementation process. In order to make the correction, they added explanatory notes to the modeling activity worksheets. Members of group 2 mentioned the problems with linguistic ambiguity associated with their modeling activities.

Interviewer: What did you expect before the implementation experience? What did u find after the implementation?

PT15: OK, for example, we determined a point and then we used the expression “at the latest”. After the implementation, feedbacks were given. I considered doing some changes according to feedbacks. However, it contradicted with the expression “at the latest”. In the end, when the fire was started from here, the expression “at the latest” became bizarre.

Interviewer: OK.

PT15: To what, to whom, why?

Interviewer: Yes.

PT15: Since the questions asked in the modeling activity was inadequate, the implementation of modeling activity did not reach its aims. Our goal was to comprehend the critical point in the graph and observing the increase with increasing rate, and solving the problem situation by using the concept of critical point and the behavior of the graph. The goal was not achieved by the students. The only thing was that we drew a graph according to given shape.

PT14: We assumed that if the fire started at the middle point of the area, so our assumption was very logical. We need to rethink the situation. For example, we asked students to draw two graphs of the forest. We thought that the fire would expand when it started at middle point. If the fire grows, there will be an increase in the fired place. We made this for students to see the dimension of the increase clearly. We observed some deficiencies in the modeling activity. We need to improve it by removing these deficiencies. Apart from that, we observed clearly, there existed some parts of the activity that were not understood by the students.

In the above episode, prospective teachers stated that their modeling activity had problems with linguistic ambiguity. PT15 asserted that they tried to explain the condition for the problem context, but the expression of “at latest” was perceived differently from what they had intended. PT15 also indicated that their assumptions for the solution method were not considered by the students. Moreover, they thought the students would use the figures in order to draw the graphs; however, students started from a different point that they did not foresee. Therefore, PT15 and PT14 pointed out that they needed to revise the text and figures used in their modeling activities in order to provide comprehensibility of the problem. Similarly, members of the group 3 stated that they had to make revisions in the text and figures of their modeling activities in the following interview episode.

Interviewer: So, what did you change in if you will implement this activity again?

PT5: Hmm, we would change the explanation part of the activity.

Interviewer: it is the point that can be drawn, so this point need to be checked again.

PT6: Yes, it can be drawn in the activity sheet. That is, it can be drawn in order to show that comes to that point. We do not have enough time for that. After changing these deficiencies, the activity can be very good enough.

Interviewer: What will you change in the text of the activity?

PT5: Namely, it is not understood when we say the “top”.

Interviewer: You say text of activity.

PT5: We need to change text of the activity. Besides, we can demonstrate the drawn figure at the end of the presentations. In my opinion, both of them are required.

In the preceding episode, prospective students expressed that there were deficiencies between the drawing of visual figures and text of the modeling activity. They stated that if they made these corrections, their modeling activity would improve in terms of comprehensibility. The members of PT17’s group (group 1) also stated in the interview that only one group member had a problem in understanding the context of the problem.

Interviewer: Okay, was there any problem with comprehensibility of the activity?

PT1: Yes, for example, members of one of the groups put forth a claim that "how much area needed in order to grow a vegetable" when saying a vegetable and a fountain.

PT18: We need to say that the volume of vegetables is ignored at that point.

PT1: Yes, as if they understood that there exist a vegetable for each unit square. Some of the students proposed an idea like that, but almost all of the groups had no problem with watering the vegetables twice. Even some of them who did not understand understood the point.

In the preceding episode, prospective teachers indicated that students had difficulty in understanding the logical reasoning in the text of their modeling activities. PT18 suggested that they had to indicate the ignored situations on the activity sheet.

To sum up, many of the group members who participated in the implementation experience indicated that they needed to revise the modeling activities. For the problems associated with the development of modeling activities indicated in the previous table (see Table 13).

Prospective teachers proposed various suggestions to eliminate problems emerged in modeling activities that prospective teachers developed for implementation experience. Accordingly,

- using more simple and comprehensible expressions in order to eliminate linguistic and ambiguity problems,
- expressing the unclear representations more clearly,
- placing explanations if needed,
- removing information which is leading and include more expressions.

The previous suggestions recommended for refining the modeling activities. Participants declared that if they applied these recommendations during the development of modeling activities the implementation stage would improve.

Prospective teachers who played the role of the teacher during the implementation exercise were asked to evaluate role of the teacher in the modeling process. They were asked to consider their knowledge gathered throughout the modeling course implementation and their implementation experience. They reported in the interviews and project reports that students had difficulties with the modeling process during the implementation experience. Several difficulties were indicated by prospective teachers including prolongation of the solution process due to not using given materials and wavering in making a choice in the solution stage. They also mentioned that they had problems because they did not read the graphs. Because of this, they were not able to grasp the context of the situation. Participants explained in the following exemplary situations how they approached these problems and how they behaved in unexpected situations. Prospective teachers indicated that they tried to observe students by walking around the groups, giving appropriate answers to the students' questions, by following solution strategies. For example, PT17 expressed in the latter episode that students would neglect the given materials used in the modeling activity sheet. Because of this students, encountered problems during the solution process.

> PT17: But teacher we already delivered the activity sheets. What we wanted was observing the crux of activity by drawing picture of it. My group members tried to solve the problem situation like the activity of group 2 by ignoring the given handouts. They struggled to solve much, I did not intervene the process, only watch them. After the intervention such that I said them to use the given handouts, they were able to solve the problem. Namely, we encouraged them to do the activity.
>
> Interviewer: You mean visual drawing is important.
>
> PT17: Yes, even that means nothing is not in vain. They said that if you delivered this sheet, we would solve on this. I am sure that if I do not deliver the sheet, they will not solve it.

According to the preceding episode, PT17 stated that students ignored the given material such as the graphing sheet, and they were not able to solve the problem. PT17 stated that assisted the group by encouraging them to use the given documents in order to advance in the solution process. PT23 illustrated in the following except how he played the role in the implementation experience.

> I was the one who practiced the activity in the group. I gave some time to the students to reach solution on their own after making necessary explanations and having completed drawing attention phase of the question. I tried to take part in the students' solution process and tried to answer the questions asked during this process (*PT23, interview after the implementation experience*).

In the preceding quote, PT23 explained that he played the role of the teacher during the implementation process. PT23 expressed that he made explanations about the modeling activity after drawing students' attention to the problem. It was understood from the quote that PT23 tried to engage with students during the modeling process by walking around and observing the groups and answering students' questions. In another exemplary situation, PT6 described her role as a teacher as follows:

> Throughout the activity, first of all I tried to enable them understand the question correctly and in the same way with my friends in group we, both by asking questions and answering each groups' questions, enabled them to understand the question. We made explanations about whether every information must to be used or not. We asked questions on the basis of their mistakes and asked their reasons for the answers or if they were wrong answers we, tried to make them understand their mistakes (*PT6, report written after the implementation experience*).

According to above excerpt, PT6 indicated that she tried to help students understand the problem by asking and answering questions to the groups. PT6 explained the situation in the text during the modeling activity and questioned their solutions. If their solutions were wrong, PT6 said that she helped them notice their mistakes by asking relevant questions. PT16 also illustrated in the following quote that what happened and how he engaged with students during their implementation experience.

> In the practice process of this activity, my role was being a teacher. Firstly, I went near all the desks and asked what they understood from the activity. By this way, I would be able to get the record straight or want them to read the question again. Then I asked questions to make students think about higher order thinking. For example, "What did you understand from the expression drawing graph of burned trees? What did you understand from the expression" the latest? What kind of changes can difference of slope between mountains cause in our graphic?" Furthermore, if there are different ideas in a group, I listened their friends in other groups and their opinions related to these ideas (*PT16, interview after the implementation experience*).

In the preceding excerpt, PT16 told that he walked around the groups in order to understand what students understood from the modeling activity. He also wanted to determine if any distinct ideas emerged in group discussion. PT16 noted that he made students present their solution process and gathered feedback from these presentations.

In general, it was observed that prospective teachers who played the teacher role tried to present skills about what a teacher ought to do formally or informally in the process of modeling as a part of the course. As indicated by the prospective teachers, PT9 and PT14, it was observed that the role played by the course instructor drew prospective teachers' attention and they tried to carry out modeling process by using their gathered knowledge and observations from the role played by the course instructor.

4.2.1.4 Prospective teachers' impressions for their own implementation of modeling activities

After the implementation experience, prospective teachers were asked to write a report about the modeling activities they developed. Prospective teachers were assigned a guide for the modeling project (see Appendix M). Participants were asked to answer the question, "What kind of impressions did you get about the use of modeling activities on the teaching of mathematics in the period of implementation experience? Explain with giving examples" was analyzed. According to the findings, most of the prospective teachers had a positive response to the use of modeling activities in the teaching of mathematics and declared that its use would be beneficial.

For example, PT14 stated that her impressions during the implementation process in the following excerpt.

> We had a chance to see by experiencing and practicing how the students can think certain things so differently, they can ask different questions, and they can correlate different concepts with the questions. Misperceived points or points that are not understood by the students can be observed and determined more clearly during the activity. For example, we could not write so many things on the section "possible questions by the students" while preparing this question. But after the implementation, we encountered with some questions which did not even come to our minds. These modeling activities will both broaden point of views of teachers and enable the teachers to determine not fully understood concepts or points in a clear way and also will develop thinking skills of the students (*PT14, report written after the implementation experience*).

In the preceding quote, PT14 indicated she observed that students had different considerations and asked distinct questions. Students made connections between different concepts and questions in the process by applying the modeling process. She asserted that points that were incomprehensible or misperceived were identified explicitly in the process by giving an example from their implementation experience. She claimed that modeling activities would broaden teachers' perspectives and help them to identify the incomprehensible points and concepts clearly. She also stated that modeling activities would help develop students' thinking skills. PT24 mentioned the characteristics of modeling activities in the teaching of mathematics as follows:

> I think modeling implementations are quite efficient and necessary tools in mathematics education. It helps the students to make abstract concepts concrete in their minds, to make connections between concepts and to practice and experience the subjects that they normally ask where to use. For example in our study, the students made connections between ellipse and parabola, and circle and parabola. Alternatively, they saw concretely that ways of the curves in the space are independent from coordinate axis. Furthermore, they had a chance to witness changing and unchanging concepts about translation in curves. Making students acquire all these attainments is quite difficult task by presentation method. For this reason, I am of the opinion that modeling should take place in education of math as much as possible (*PT24, report written after the implementation experience*).

In the above quote, PT24 emphasized the importance of mathematical modeling as teaching tools. He pointed out that modeling activities provided students with an introduction to the concepts. These activities also gave students the opportunity to observe the implementation of these concepts in the context of real life, and they were also able to make connections between concepts by receiving concrete examples from their implementation experience. In contrast to this, some of the participants mentioned the difficulty of implementing modeling activities. For example, PT17 explained the difficulties of implementing modeling activities in the classroom setting in the following excerpt.

> First of all, I realized that implementation of such an activity is difficult because one can encounter with different ways of solution or different approaches. Understanding, interpreting and interfering them can be difficult, too. I think implementation of such activities can be useful to be able to see different points of view and approaches (*PT17, report written after the implementation experience*). .

According to preceding excerpt, PT17 stated that implementing the modeling activity like the one they developed could be difficult for students to understand and interpret. Nevertheless, she admitted that using these activities were advantageous in terms of observing distinct approaches and perspectives of students. Similarly, PT15 pointed out both the difficulties and the benefits of using modeling activities in the classroom environment in the following excerpt.

> Implementation of modeling activity is rather difficult but useful as well. I find this activity difficult because the students are of the level who can wander the subject and aim of the activity. The students may not understand seriousness of this activity and I think that supervisor or a teacher implementing this activity should be qualified and educated enough to deal with all these problems and trouble. For these reason this activity seems to me difficult (*PT15, report written after the implementation experience*).

In the preceding quote, PT15 mentioned the difficulties regarding the implementation of modeling activities in the classroom using several different reasons In order to overcome these problems; PT15 suggested that teachers ought to be qualified and well educated. Similarly, other prospective teachers highlighted similar issues in their project reports.

4.2.2 Prospective Teachers' Thinking about the Use of Mathematical Modeling Activities in the Classroom

Since changes in the conceptions of the use of mathematical modeling in the classroom environment emerged throughout the implementation, it is important to present the findings in a chronological order according to what they did each week, particularly in modeling activities. In the previous section, it was shown that most of the prospective teachers did not have experience with mathematical modeling and described it as "using concrete manipulative and visualization of mathematical concepts", hence, most of them did not have any idea how a mathematical modeling activity could be used in teaching and learning.

After the first modeling activity, that is *the Summer Job*, PT17 said that she had experienced the modeling activity differently than she did before in mathematics courses. She made a distinction between modeling activities and traditional word problems. For example, she said that traditional word problems had only one solution and she tried to find the solution, but in this modeling activity, there was a need for producing multiple solutions rather than just finding one solution. Although she experienced modeling the activity quite differently from previous problem solving processes, she was positive about the use of mathematical modeling activities in the classroom setting and also described how she wanted to apply these activities in the episode given below.

> Interviewer: Well, as a prospective mathematics teacher, if you could use this activity in your class, what do you expect students to gain?
>
> PT17: I think that I would apply this activity for reasoning and making connections since, as I mentioned before, reasoning existed in this activity. That is, making connections and reasoning skills existed together here.
>
> Interviewer: Well, mathematically?
>
> PT17: Ratio could be here mathematical. As I mentioned before, since we were accustomed to short questions and direct solution procedures I could not relate this question to mathematical concepts. For example, if you brought any question related to traditional word problems, I could say, “that is integration question”, but I could not relate this question to any mathematical concept clearly.
>
> Interviewer: How could you apply this question in a classroom?
>
> PT17: I could use group work since this question was implemented by way of group work or at least I thought that it would be better if it were done so.

The second modeling activity prospective teachers worked in the course was *the Ferris Wheel*. After they worked on *the Ferris Wheel* activity, PT17 expressed positive opinions about the modeling activity in terms of its meaningfulness. She illustrated her opinion about the modeling activity as follows:

> I noticed that I could use the trigonometric gains that I obtained in my mathematics education in daily life. That is, I realized that the objectives we obtained were not only for being successful in exams, but also for facilitating real life. Because I had always wondered, as many students do, which situations trigonometry might be useful for in real life. When I became a mathematics teacher, it was the kind of problem that I would use to explain this question to students who were wondering about it. Therefore, I think the problem is beneficial (*PT17, reflection paper for the Ferris Wheel activity*).

The above quotation demonstrates that PT17 was developing positive thinking and beliefs about mathematical modeling activities due to their relevancy and the need for students to understand mathematical concepts more meaningfully. She also indicates her willingness for application of the modeling activity in the classroom during the interview as follows: “I wanted students to use trigonometric expressions in order to understand whether they understand and apply or not. Therefore, I could also apply this activity to my students”.

The Street Parking activity was applied to prospective teachers in the fifth week. Looking through the reflection papers and interview transcripts, there is no direct implication and thinking for using the modeling activity in a classroom setting. Rather PT17 used the probabilistic statements such as "If I applied this activity to my class as a teacher, I would do it in the following manner." After the fourth modeling activity, *the Bouncing Ball*, she thought that she could use the modeling activity in her class when she became a teacher.

PT17 rarely stated her unwillingness to use modeling activities. For instance, after the implementation of *the Free Roller Coaster* activity, since she did not comprehend the purpose of the modeling activity, she commented her unwillingness to use the modeling activity in the classroom setting. She said:

> I would not want to apply this activity to my class until you explain the purpose of the activity, because I did not understand the focus here. If I gave any problem to my students, I would want them to understand the mathematical concepts on the basis of the given problem (*PT17, reflection paper for the Free Roller Coaster activity*).

After she understood the focus of the activity, she stated her willingness to implement the modeling activity in the classroom when she became a teacher. The last modeling activity presented to the prospective teachers was *the Water Tank*. In the reflection paper, she indicated her willingness to use the modeling activity as follows:

> If I look at the activity through the eyes of a teacher, I appreciate the activity. I think that this activity would be beneficial in the interpretation of graphs and things like that. That is, we can see how mathematical concepts like 'increasing with increasing slope' emerge in the mind of students (*PT17, reflection paper for the Water Tank activity*).

PT17 positively evaluated the change in her conceptions about the use of mathematical modeling activities in the classroom setting at the end of the course as follows:

> I have not got any idea about the measuring property of modeling activities. That is, I had doubts that I could get feedback when assigning these activities to students, but I realized that I would have opinions about which mathematical concepts students know and don't know and/or where students could have difficulties (*PT17, post-survey form*).

When PT17's statements and comments were analyzed throughout the implementation of modeling activities, it was seen that her conceptions about the use of mathematical modeling activities in classroom environments were positive and relevant to her purposes of using these activities, that is, to provide meaningful understanding and strengthening students' knowledge about mathematical concepts.

In the case of PT14, she had never taken any course related to mathematical modeling and its relevant subject matters. After the implementation of the first modeling activity, *the Summer Job*, she indicated her considerations about the usefulness of that activity in her future classrooms in observing to what extent students apply their previous mathematical knowledge when they encounter such problems. The dialogue below shows her thoughts about the modeling activity and her intentions to use it.

Interviewer: Ok, now I want you to look through the eyes of a teacher.

PT14: I have already written it in my reflection paper.

Interviewer: If you applied this problem in a classroom setting, which objectives would you expect students to gain?

PT14: … you know we have been learning much more mathematics thus far. But, I wish I had applied that activity in order to see to what extent students apply their mathematical knowledge. Which knowledge did they use where? How did they solve the problem? That is, I would want to see what they did in the process. They have reached a certain level of mathematical knowledge up until now, but how is their knowledge reflected in their writings when they face a problem.

Interviewer: Well, which related subject matter do you choose to apply to this activity?

PT14: Which related subject matter is it? That could be the very thing. For example, what does it mean? What does average mean and what does it mean in daily life? For example, I teach a mathematics course, an ordered sequence was given, and some student finds its average but he or she is not aware of what he or she is doing, that is only memorization. The sum of all given entities is divided by the number of entity, that is all. There is no idea in the mind of the students that all these entities accumulate at that point and approximate that number. I want to learn how mathematical concepts like finding the average or mean exist in daily life rather than just ordered numbers and sequences.

PT14 stated her willingness to use another modeling activity, *the Ferris Wheel*, in teaching trigonometry by comparing her previous experience in high school. In the reflection paper, she wrote, “Remembering from my high school years (private training center, school etc.), trigonometry was not a favorite course for students. Thus, I think that an activity like that may impact or change their views about trigonometry positively”. From the quote, it can be said that she wanted to use the modeling activity in order to make students like the subject matter that was previously seen as unfavorable by students. The subsequent modeling activity implemented in the prospective teachers’ class was *the Street Parking*. It was understood from her transcribed statements that some modeling activities like *the Street Parking* should not be used in vocational high schools or general high schools due to a lack of adequate mathematical knowledge and thinking capacity in order to implement these activities. It is illustrated in the dialogue below:

PT14: In my opinion, students need to have a certain level in order to produce solutions for these activities. That is, they need to have sufficient mathematical knowledge and mental capacity in order to apply modeling activities. At the very least, they need to know where to use what. Think of triangle congruence, I could not ask students problems like that. The problems like this can be applied to students who are genuinely interested in mathematics. As one of my friends said, I think a mathematics club could be formed and these problems might be put to them. I cannot apply this activity to just any students in the vocational high school.

Interviewer: If you applied it, perhaps different things might emerge.

PT14: No! It is absolutely impossible.

Interviewer: Perhaps students would show more initiative for learning and asking themselves "how can we solve this?"

PT14: No, I don't think so.

Interviewer: Anyway, it needs to be tried to see the result.

As is seen from the episode above, the prospective teacher, PT14, had a belief that some modeling activities are not applicable to students (e.g., *The Street Parking* activity). The reasons she gave were not having enough mathematical knowledge, mental capacity, and not being on a certain level. It was seen that PT14 developed a conceptual plan for the application of the next modeling activity that was *the Bouncing Ball*. She expressed her willingness to implement the modeling activity in her future classrooms. Specifically, she described how she would use the modeling activity as follows:

Interviewer: Well, how would you apply this activity in a classroom?

PT14: How would I apply this activity in a class, namely?

Interviewer: Think about a class with 30 students.

PT14: We might imagine a class with 30 students, each student thinking about the subject matter on her or his own. That is ok; I have an idea about the subject matter. For instance, I could say the thing that was not remembered by my friend and vice versa. But what does s/he think about the problem? So, therefore I would give the modeling activity a day before the implementation of the activity in order to make him/her think about the activity. I would ask them what they think about the activity, what comes to mind, which mathematical concept comes to mind that is related to this activity.

We can see from the above episode that PT14 dealt with the purpose and the related mathematical concepts that the modeling activity had in the background. It can be implied that PT14 was developing her own implementation plan for the modeling activities rather than discussing whether or not to apply them in her future classrooms. In the following modeling activity, *the Free Roller Coaster* activity, she stated her intention to use this modeling activity by expressing possible benefits of using it as follows:

> … On the other side, students who experienced this modeling activity (read it before) would make sense of givens and requested data more easily. For this reason, I asked students to ride a roller coaster (we had experienced and seen where excitement existed exactly) or explore a roller coaster in Luna Park or examine pictures and videos of roller coasters from the Internet. This process would either aid them in their understanding the activity or expand their imagination during the railway designing (*PT14, reflection paper for the Free Roller Coaster activity*).

It is understood from the above quotation that PT14 made a connection between the modeling activity and its real life counterpart in order to ensure that students could understand what the modeling activity aimed at if they had experienced such a real life situation. In the last modeling activity, *the Water Tank*, PT14 expressed her willingness to use the modeling activity in her future classrooms by emphasizing it strongly as follows:

> Interviewer: Well, lastly looking from a teacher's perspective PT14, just before you mentioned it, how would you use this activity in the classroom setting? Would you use this activity when you become in-service teacher?
>
> PT14: I definitely could use this activity. I never saw these activities in high school. I wonder if I was just lazy or if the teacher taught this so that I had a problem with it. These things were not taught in high school. In general, the only things we learned were velocity versus time graphs and location versus time graphs. Velocity versus time graphs was usually linear.

In the post-survey, PT14 evaluated the change in knowledge and beliefs about mathematical modeling from beginning of the course to end of the term in the following quote:

> Before I took this course, I knew that there was a course called "mathematical modeling" but I did not have enough knowledge about its content. After I took this course, my perspective of the world around me changed. Because I wrongly supposed that mathematics was being used only in such areas as space sciences, astronomy, engineering etc. However, mathematics is in every area of our life. This conception contributed much to identifying and questioning mathematical situations in the world around me (*PT14, post-survey form*).

From the preceding quote, we see that PT14 had no information about mathematical modeling other than just knowing its course name. After the implementation of the modeling course, she described the change that took place in her mind as mathematics was suddenly all around her in her daily life. She also stated that she thought that she would use mathematical modeling activities in her future classrooms. To sum up, PT14 demonstrated gradual development regarding the use of mathematical modeling activities in the classroom setting from the beginning of the course to the end. She developed a conception that mathematical modeling was significant for learning mathematics meaningfully. Although she said some of the modeling activities were not applicable for students in some high schools, such as vocational high schools, she generally indicated that she wanted to use modeling activities in her future classrooms.

PT9 was one of the prospective mathematics teachers who took the modeling course as a member of group 5. It was documented from the obtained results that she developed a positive conception about the use of mathematical modeling activities in the classroom setting. From her writings and transcribed expressions, it is understood that she intended to use modeling activities to help students comprehend the mathematical concepts meaningfully and conceptually. These findings were illustrated with implemented modeling activities during the implementation of the modeling course.

For *the Summer Job* activity, PT9 thought that this modeling activity could be used to show students that there could be distinct solutions according to assumptions and that there was no unique solution to these kinds of problems. For example, the following interview episode illustrated this evaluation:

> Interviewer: … There is a section in the report guide that looks from the teacher's perspective. When you use this activity in your classroom, when you become a teacher, which objectives do you expect your students to reach when you apply this activity?
>
> PT9: You know this question is an open-ended one. Everyone says different things. There were very different solution strategies in our class. I think that there would be very distinct ways to find a solution when I apply it to my future classroom. That is, I think that this activity would help students while discussing the situations and show them that there are no uniquely correct answers, and also students will see different types of correct answers according to their thinking and they would reach the goal as a result. Everyone defended and appropriated their own answers as if there was a unique answer. When I think of that situation, since there was no unique answer, they did not need to appropriate their answer as unique.

In the second activity, *the Ferris Wheel*, she indicated in her reflection paper that she desired to use the modeling activity in her future classrooms. She described how to use this activity as follows:

> I think that this activity should be implemented in the classroom individually. I would apply this activity like that. I would observe students during the solution procedure and try to understand what the difficulties were and what they lacked. If there was a problem with the individual problem solving process, I would form groups (*PT9, reflection paper for the Ferris Wheel activity*).

In the above quote, she stated how to apply the modeling activity in the classroom environment and what she would do during the solution process of the modeling activity. After the third activity applied to prospective teachers of mathematics, *the*

Street Parking, she continued to express her ideas about the use of the modeling activity in her future classroom. For example, she wrote:

> I want my students to notice parking areas that they see and notice how vehicles park in these areas. They would think differently of parking styles. I want them to consider that when they choose a parking space areas result the less area they cover. After that, I would say that we could show this mathematically and which one is relevant to this and give them the hand out (*PT9, reflection paper for the Street Parking activity*).

It was understood from the above quote that PT9 wanted her (future) students to make a connection between real life and its mathematical meaning by experiencing the situation cognitively. She explained the way of using the modeling activity in the classroom setting. This can be interpreted as her wanting to use *the Street Parking* activity in her future classrooms. In the subsequent weeks, another modeling activity was implemented. This modeling activity, *the Bouncing Ball*, was different from previous modeling activities, as was stated in the methodology chapter. Almost none of the selected modeling activities were associated in terms of the same mathematical subject matter. However, it was intended that prospective teachers would see distinct modeling activities so that they would be familiar with these activities and develop ideas about how a modeling activity could be. In her reflection paper, she expressed her ideas about using the modeling activity in her future classrooms as follows: "I think that we can take the students inside the concept of geometric sequences with this question. The students can discover this concept on their own even if they do not know what the geometrical sequence is. They can form the geometric sequence formula." She explained that she could use the modeling activity in order to form a mathematical concept (geometric sequences) in students' minds so that they could discover it by themselves.

In the following activities, it was clear that she wanted to use the remaining two mathematical modeling activities in her future classrooms. For example, she wanted to use *the Free Roller Coaster* activity in order to teach the concept of "slope of curves". The following transcribed episode illustrated this:

> Interviewer: Then let us move to the assessment by the teacher. If you implement this problem in the classroom, which acquisitions would you expect students to have?
>
> PT9: The concept of slope.
>
> Interviewer: What kind of slope? Slope of a line or slope of a curve?

PT9: Slope of a curve because when it is a line there is no safety. Because it has to be safe and more applicable to real life; when it is linear the slope of a line is itself. The slope of itself you know from the slope of a curve. I wrote something there.

She also indicated her willingness to use that activity in her reflection paper by stating her aim as follows:

I would implement this question to help my students understand the points where the slope will be at a maximum and where it will be minimum and what the characteristics of these points are. I could implement this activity before introducing derivatives. I think that the students will be able to make sense of this information better by drawing parallels between critical points acquired from this activity and critical points in theory (*PT9, reflection paper for the Free Roller Coaster activity*).

It is understood from the above quotation that she wanted to use the modeling activity to help students comprehend the concept of slope and to make a connection between the situation in the activity and its mathematical meaning cognitively. Hereby, she thought that students would learn the slope concept meaningfully. After the implementation of the last modeling activity, *the Water Tank*, PT9 described her approach of using modeling activities at the beginning of the lesson. She stated her intention to implement the last modeling activity in the following excerpt:

Interviewer: I got it. Well, PT9, by applying this activity in the classroom, which acquisitions would you like your students to have or would you like to apply it in all? Let us start from there.

PT9: Actually, I think it is a good activity for students to interpret the graphs because we noticed that some groups could not interpret the graphs drawn in last week's activity. Here, it would be good for them to interpret the graphs they drew. For example, in terms of comparison, in store 2.

Interviewer: For the transition between second and third zone in storage 2.

PT9: Yes, for that transition. Those zones, for instance, were both increasing by decreasing, but we commented on what the difference was. I think it is useful for students in terms of interpreting these and understanding their differences.

In previous excerpt, she expressed that *the Water Tank* activity could be helpful for students in the interpretation of graphs and fluency parts in the graphs. It was also understood from the above episode that each modeling activity would serve a mathematical idea and goal. In the post-survey form, she indicated her positive opinion about using mathematical modeling activities when she became an in-service teacher as following:

I consider using it. I handle these questions like this: What if the students solves the question without knowing the subject and by this way, I can enable the students to realize the subject on their own but indeed, I realized this in two or three activities throughout our activities. Doing the modeling activity before the subject seems to me more attractive for now. However, I think that

> implementation just after the subject is necessary especially for subjects like graphic interpretation and derivatives (*PT9, post-survey form*).

In summary, PT9 developed positive conceptions about the use of mathematical modeling activities in the classroom setting when she became a mathematics teacher. It was evident from her interview transcripts, writings in reflection papers, and survey forms that she indicated her thinking about using mathematical modeling activities in her future classrooms by emphasizing why she wanted to use them to teach mathematical concepts and ideas.

PT24 was another volunteer prospective mathematics teacher who attended the modeling course class and participated in interviews carried out after each activity implementation. In general, PT24 displayed gradual development in the conceptions about using mathematical modeling activities in the classroom environment. It was seen that he actively participated in the six modeling activities throughout the semester and tried to engage in all sessions of the course such as group discussions, classroom discussions, interviews, etc. The analysis of the gathered data demonstrated that he developed positive ideas about the use of mathematical modeling in the classroom setting. The change or development in his conceptions will be illustrated by giving direct evidence from the process.

After the implementation of the first modeling problem, *the Summer Job*, PT24 stated in the interview that these kinds of problems should be integrated in the school curriculum according to students' levels. He indicated that these activities ensure students produce new ways of finding solutions. These ideas were stated as follows:

> Interviewer: Good, well, do you think these kinds of problems should be in the high school curriculum or in the university level? If so, how?
>
> PT24: I think these kinds of problems should be, why? Because, they encourage students to look for different approaches and different results. I mean, instead of grade 4, it can start in later grades at low levels. We can asks others' opinions and increase the levels gradually. If a high school graduate encounters a problem in daily life, s/he should at least be able to come up with three different solutions.

In the second problem, *the Ferris Wheel*, it was understood from the interview excerpt that PT24 developed an idea that the implemented modeling activity was suitable for students in making sense of mathematics in real life. That is, mathematics was in daily life and served specific purposes. The following episodes describe the situation:

> Interviewer: From a teacher's perspective, which acquisitions would you expect your students to have when you use this problem?

PT24: This problem, like I said before, is an application of mathematics in a real life situation. I would expect students to answer these questions. I mean, that mathematics really works, other than simple operations; higher mathematics is also useful for something. We would expect them to know it can be used in daily life. We always questioned how mathematics is used in real life. We asked what logs or complex number system are used for. Now students see that they are used in trigonometry and they think if they can be used here we can also use them in other places, including real life. I would expect students to understand how mathematics really works.

From the above quotation, it can be interpreted that he thought of modeling as serving the purpose of providing meaning to mathematics in relation with daily life and so that students could think that mathematics was useful and meaningful as an important branch of science. In addition, he also shared his opinions in the reflection paper as follows: "The students can find answers to the question asked all the time about mathematics: "What use will it be for me?" Furthermore, they can see the role of mathematics in real life and can be more interested in mathematics."

In the reflection paper, PT24 wrote his thoughts about the utility of *the Street Parking* activity in the classroom environment. He stated:

> When it comes to evaluation of this question from the point of view of a teacher, this question can be used in an activity about rectangles because we used expressions like parallelogram, rectangle, triangle, and their characteristics in the solution of the question. Thus, this can be an enjoyable activity by which the students use these expressions and reinforce what they know (*PT24, reflection paper for the Street Parking activity*).

In the above quote, he expressed that this kind of modeling activity was suitable for geometric terms such as rectangles, triangles, and parallelograms. It was understood from what he said that he had developed positive considerations about using this modeling activity in his future classrooms. PT24 also indicated his willingness to implement the fourth modeling activity, *the Bouncing Ball,* in his future classrooms. He said that he wanted to use these kinds of activities in order to show students where mathematics appears in real life and that made sense of the role of mathematics in daily life.

For the fifth modeling activity, *the Free Roller Coaster*, PT24 thought that the modeling activity was not an easy task for students due to the open-ended and unclear way it was asked. He thought that students could encounter difficulties when doing this activity. He illustrated these difficulties and explained how to overcome these issues in the following:

> The students may not understand what the question is asking them to solve. I stated how to prevent this situation. Furthermore, if they understand fully what the question requires, they may be confused between the slope of the curve and the slope of the straight line and then would not grasp the notion of the slope of the curve. Even if the student understood the notion of the curve, then

> he or she may have problems defining the inflection point. In short, this week's question was rather difficult for the students to solve.
>
> I would take precautions that would avoid the aforementioned mistakes if I implemented this question in the classroom environment. I would delineate the problem and present precisely what is expected and required. After making this correction to the question, I would ask them to solve the question in groups before starting on the subject of derivation, or just after starting this question, I would expect something to form in the minds of the students about the notion of derivative, the notion of the slope of the curve, or the notion of derivative function. I would try to build subsequent subjects on this concrete notion formed in the minds of the children. It is clear from the quote that he wanted to use the modeling activity in his future classrooms by taking precautions about it. He expected that students might not understand the modeling activity due to reasons stemming from its context. Nevertheless, he wanted to explain the modeling activity in detail so that students could conceptualize the situation in their minds. He also mentioned the subject he wanted to use the modeling activity for (*PT24, interview for the Free Roller Coaster activity*).

In the last activity, *the Water Tank* activity, he stated that he wanted to use that modeling activity before introducing the graphing of functions so that students would comprehended the basic properties of the graphs of functions. He also indicated that he wanted to use the modeling activity in order to measure whether they understood it or not.

> The main subjects that can be analyzed mathematically are the concepts about the characteristics of curves, the reasons for increasing with increasing rate and decreasing with increasing rate, how unity changes, and how increase or decrease in a function affects the graphic of the function. For this reason, if I were the teacher, I would use this question to measure if the subject is understood in the end or to enable something concrete in the minds of the students before starting to explain the solution of graphics (*PT24, post-survey form*).

At the end of the term, PT24 demonstrated that he developed positive conceptions and ideas about the use of mathematical modeling activities in the classroom setting. He wrote, "Yes, I would consider implementing mathematical modeling in the future because I think that the students will love math, the fears will diminish with these modeling activities, and they will understand the subjects better." When the developmental period is investigated thoroughly, the presented evidence proves that PT24 gained positive conceptions about the modeling activities and their usage in the classroom environment after he completed his undergraduate education.

To sum up, the in-depth analysis of the process of selecting four prospective mathematics teachers and asking them what they thought and would like to do in the future in their profession throughout the semester demonstrated that although they had very restricted knowledge about mathematical modeling and its usage in teaching mathematics before, they developed very positive conceptions about mathematical modeling activities and the appropriateness of their usage in the classroom setting. The obtained evidence shows that prospective mathematics teachers gradually developed such a positive conception that they had a tendency to want to use mathematical modeling activities in the teaching and learning of mathematics when they become in-service mathematics teachers. It was clear from the previous passages that each prospective teacher had different purposes in using these activities. For example, although PT17 and PT24 thought that they could use modeling activities for meaningful mathematical teaching and learning, PT9 thought that she wanted to use modeling activities in order to teach a specific mathematical concept such as slope. In a general sense, in order to illustrate the development of the prospective teachers from each group that are represented throughout the implementations of six modeling activities, it is summarized in the below table (see Table 14). This table demonstrates the evolving conceptions about the use of mathematical modeling activities. When the post-survey form and ideas of prospective teachers were evaluated together, it was observed that the majority of prospective teachers had a positive point of view about the use of modeling activities in the classroom. It may be understood from the given table that almost all the selected prospective teachers from each group developed positive conceptions about the use of modeling activities in the classroom setting. In the period of the implementation of these activities, it was observed that the prospective mathematics teachers differed in the way they would use modeling activities in the classroom environment, which might have resulted from their group experiences and classroom discussions that took place after each implementation of mathematical modeling activity. The results also demonstrated that prospective teachers' preferences changed in the aim of use (cognitive-affective), the frequency of use, the place of use (before the subject matter or after etc.), and the method of use (individual-group). Evidence and results will be reported in the following sections which are associated with prospective teachers' choices about how to use, the purpose of use, and where and when to use mathematical modeling activities in the classroom setting.

Table 14 The views of prospective teachers about the use of modeling in the classroom with students

Prospective Teacher (Group #)	Name of the Modeling Activity					
	The Summer Job	**The Ferris Wheel**	**The Street Parking**	**The Bouncing Ball**	**The Free Roller Coaster**	**The Water Tank**
PT17 (Group 1)	If I look through the eyes of a teacher, I do not think that I will apply this kind of activity every high school. (Negative) (Reflection paper)	Since we applied more mathematical operations in this activity, I think of the modeling activities are more applicable to the students at the school level. (Positive) (Interview)	It seems that it was more applicable to me. (Positive) (Interview)	I would use this activity on application step of objective that was already gained. (Positive) (Reflection paper)	That is, I wish I would use that activity when I mastered the subject matter. (Positive) (Interview)	It seems that I would use these kinds of activities in the application level. (Positive) (Interview)
PT14 (Group 2)	If I want to arrange a modeling activity like that, I would divide students groups consist of 3 or 4 students just as we did and I wish that they resolved the problem by discussing together. (Positive)(Reflection paper)	Remembering from my high school years (private training center, school etc.), trigonometry was not a liked course by students. Thus, I think that an activity like that may impact on changing their views about trigonometry positively. (Positive) (Reflection paper)	Besides any student's self-confidence and motivation is positively affected by solving and interpreting these similar problems. (Positive) (Reflection paper)	… So, therefore I would give the modeling activity a day before the implementation of the activity in order to make him/her thought. (Positive) (Interview)	In my opinion, this activity should be applied exactly after teaching the derivative subject matter. (Positive) (Interview)	It can be helpful for students that seeing use of different variables on the plotted curves and, getting knowledge about these curves and their analysis. (Positive) (Reflection paper)

Table 14 (continued)

PT5 (Group 3)	I tell them to work in groups by applying this activity in the classroom and help them to establish healthy relations among them. (Positive)(Reflection paper)	According to me, this activity can be implemented after lecturing a subject matter in order to conceptualize it, before and after the exercises. (Positive) (Interview)	… I think activity like that should be implemented because it might be beneficial, but not harmful for them. (Positive) (Interview	Yes, exactly it can be applicable to students on teaching and examination. This activity is can be applicable as either an activity or problem. (Positive)(Intervie w)	… it was an activity that I could implement in the classroom for me. (Positive) (Interview)	Yes, these activities seem that they were applicable easily… (Positive) (Interview)
PT10 (Group 4)	For example, I could apply this in order to show how average problems or the way of solution would work. (Positive) (Interview)	Teacher, students understand concretely by applying this activity. I will be reified the concepts when I applied such an activity. (Positive) (Interview)	I would apply this activity with technique of group work by forming groups consisting at least after this preliminary study finished. (Positive) (Reflection paper)	- (Not attended to activity)	High probably I would apply this activity after teaching derivative and curves. (Positive) (Interview)	I would devote 2 hours to implement that activity… (Positive) (Reflection paper)
PT9 (Group 5)	I think that there would be very different resolution ways when I applied this activity in the class. (Positive)(Interview)	I think that this activity is more suitable for individual working. I would apply like that. (Positive) (Reflection paper)	I would say students “Let’s show that mathematically which one is more relevant” and then I apply the activity. (Positive) (Reflection paper)	I would use this activity in order to make students learn concept of geometric sequence. (Positive)(Intervie w)	I think that this activity could be applied before derivative is not taught. (Positive) (Interview)	This activity will be helpful for making students think on and conceptualize the concept of increased by decreasing and increased by increasing. (Positive) (Reflection paper)

Table 14 (continued)

PT23 (Group 6)	That is, average, arithmetical average, geometrical average like that, I approve to apply when these concepts are held. (Positive)(Interview)	I would use this activity after giving objectives related to subject matter. (Positive) (Interview)	It seems to be more applicable. (Positive) (Interview)	I would use this activity at level of application of grasped knowledge. (Positive)(Reflection paper)	That is, I would use this activity when I mastered the related subject matter. (Positive) (Interview)	It seems that I would use these activities during the application level. (Positive) (Interview)
PT24 (Group 7)	I think that activities like that should be in the teaching. (Positive)(Interview)	I expect from students to find answer the following question when applying the activity. (Positive) (Interview)	… this activity could be used in the subject-related to quadrilaterals in the geometry. (Positive) (Reflection paper)	Yes, this activity can be applicable in the classroom, even in elementary schools. (Positive)(Interview)	This activity can be used in order to create something in students' minds, not only for solving the problem. (Positive) (Interview)	…I would use this activity to measure that whether the subject matter is understood or not. (Positive) (Reflection paper)

4.2.2.1 Prospective teachers' thinking about the aim of using modeling activities

The purposes of prospective teachers using mathematical modeling activities were examined in detailed by using different data sources with document analysis technique. Data used in this context included the responses of prospective teachers to the question, "Do you intend to implement modeling activities in your classroom when you become an in-service teacher? Explain with reasons." These were in the post-survey forms, field and observation notes taken by researcher, reflection papers that were written throughout the term, and transcripts of individual interviews that took place after each implementation of the modeling activities. As a result of the analysis, it was found that prospective students' purposes for the use of modeling activities in classroom are twofold; cognitive and affective. Cognitive purposes included *teaching mathematics meaningfully, seeing students' thinking processes, measurement and evaluation, reinforcing concepts,* and *displaying the relation between mathematical concepts and real life*. Affective goals were *increasing students' motivation through learning mathematics, disposing students' prejudices against mathematics, attracting students to the lesson, reducing students' mathematical anxiety,* and *providing self-confidence in using mathematical expressions*.

It was found that the purpose of using modeling activities differed according to prospective mathematics teachers' characteristics (e.g., personal background, educational background, experiences, etc.). Prospective teachers said that they could use the same modeling activity (*the Ferris Wheel*) for different purposes. The following excerpts illustrate prospective teachers' purposes of using mathematical modeling activities in the classroom.

> I wish s/he used trigonometric expressions. Did s/he understand this or not and can s/he implement it? For instance, I could implement this activity for students because correlations with daily life attract more attention and acclimatize children much more to the lesson (*PT17, reflection paper for the Ferris Wheel activity*).

> As far as I remember from my high school years, (school, private course etc.) trigonometry was not a popular subject. Therefore, I think that such an activity may have positive effects on the students' changing their views on this subject. Their discovering the usage areas of math in daily life may enhance their interest in the lesson. Furthermore, self-confidence and motivation of a student who is able to solve and interpret this and that kind of question will be affected positively (*PT14, interview for the Ferris Wheel activity*).

> As I mentioned before, this was a question taken from daily life and it was like an adaptation of math to current life. I expect students to find answers to these questions. Math is useful; not only simple math like adding, subtracting, but also sophisticated math (called second phase of math formerly) has a function and it is useful too. We can expect students to find answers to questions about the use of math in daily life. (*PT24, reflection paper for the Ferris Wheel activity*)

According to above excerpts, in *the Ferris Wheel* activity, although PT14 and PT24 thought that it was suitable for meaningful mathematics teaching, PT17 considered using the modeling activity for measurement and evaluation in order to determine what students learn, and in motivating students to learn mathematics subjects using real life situations. In order to figure out the prospective teachers' goals in using modeling activities in the classroom, three activities were selected according to chronological order (each one from the beginning, in the middle, and at the end of semester) and prospective teachers preferred aims in using modeling activities were presented in the Table 15.

Table 15 Prospective teachers' thinking about aims of using modeling activities

Prospective Teacher	**Name of the Modeling Activity**		
	The Ferris Wheel	**The Bouncing Ball**	**The Water Tank**
PT17 (Group 1)	Measurement and evaluation	Reinforcing concepts	Reinforcing concepts
PT14 (Group 2)	Meaningful mathematics teaching	Meaningful mathematics teaching	Meaningful mathematics teaching
PT5 (Group 3)	Reinforcing concepts	Reinforcing concepts	Reinforcing concepts
PT10 (Group 4)	Meaningful mathematics teaching	-	Meaningful mathematics teaching
PT9 (Group 5)	Meaningful mathematics teaching	Meaningful mathematics teaching	Reinforcing concepts
PT23 (Group 6)	Reinforcing concepts	Reinforcing concepts	Reinforcing concepts
PT24 (Group 7)	Meaningful mathematics teaching	Reinforcing concepts	Measurement and evaluation

In *the Bouncing Ball* activity, more than half of the prospective teachers commented that they intended to use the modeling activity for reinforcing mathematical concepts in relation with the modeling activity after teaching the subject matter.

PT17: That is all I wanted to say, I mean, I would absolutely use this activity in my classroom. It was a more applicable problem for students.

Interviewer: OK, then, where would you like to use it in the curriculum?

PT17: Exponentials, exponential. If it was before, then series.

Interviewer: Yes, we said that, and then we changed the topic.

PT17: I can make some changes and use the problem in series as well, but using it in exponentials will be better, because students are used to seeing this type of problem in series. For instance, they learn exponentials in 9th grade and series in 11th. Instead of 9th grade, I can use this problem in 11th grade. Let us see how many there are like me.

> Interviewer: Or, you can use it for 9th graders and 11th graders.
>
> PT17: I can do that. Ha ha. I can use it for both to see how they are doing.

In the above episode, PT17 showed great enthusiasm for implementing the modeling activity when she became an in-service mathematics teacher. She stated that *the Bouncing Ball* activity could be useful in the subjects of exponents and series. She wondered how students would think during the solution procedure. That is, it was understood from what she said that she wanted to implement the activity in order to observe how students do and how well they would apply their learning in the activity. That would reinforce students learning the concepts. In addition, she indicated that since the modeling activity was very close to traditional word problems, students would not be down in the dumps.

PT24 also claimed that *the Bouncing Ball* activity would improve and reinforce students' conceptual knowledge about the related sequences and series. He wrote the following passage in the reflection paper to express his ideas about his intention to use the modeling activity.

> When it comes to evaluation of this question from the point of view of a teacher, this is a question applicable only after the subject of sequence series (though not existing in the new curriculum) in high schools or it can be an activity to be implemented in the subject of rational numbers. (In the solution if it is taken as x , it can be a relevant question with rational numbers). I think that it is a question that can be solved easily by high school level students. I am of the opinion that the students who understand the essence of series in some degree will come to a solution without any difficulty and who does not understand it will be able to come to a solution with their own logic and rational operations (*PT24, reflection paper for The Bouncing Ball activity*).

From the above quote, PT24 tried to explain his aim to use *the Bouncing Ball* activity such that it could be applied to students after the sequence and series subjects and even rational numbers subject in order to help students understand the concepts related to these subjects.

When we look at the overall prospective teachers' aims in using the modeling activities that were implemented throughout the semester, it was found prospective teachers tended to note cognitive purposes mostly (see Table 16). Prospective teachers stressed the aims that showed the relationship between mathematical concepts and real life and provided meaningful mathematics teaching.

Table 16 Prospective teachers' ideas about the use of mathematical modeling activities

The Usage of Modeling Activities					
Negative Ideas (n=2)	**Positive Ideas (n=23)**				
Reasons	**The Aim of Usage**		**The Place of Usage**	**The Frequency of Usage**	**The Method of Usage**
It requires more time (n=2)	*Cognitive Aims*	*Affective Aims*	Before subject matter (making students feel the mathematical concepts as necessity, students notice these concepts) (n=1)	Rarely (2-3 in a year) (n=3)	I use these activities due to the fact that group working is useful. (n=1)
It is not established relationship between curriculum adequately (n=1)	Meaningful mathematics teaching (Providing persistence in the mind and recovering from memorizing) (n=7)	Providing students motivation to learn mathematics (n=3)	After subject matter (measurement and evaluation, reinforcing the learned concepts) (n=5)	Sometimes (when it is required) (n=7)	I use these activities since it was a king od student-centered approach. (n=1)
There is no clear objectives (n=1)	Observing students' thinking process, providing students uncover distinct ideas (n=3)	Breaking the prejudices about mathematics (n=2)		Often (n=15)	
	Measurement and evaluation, reinforcing concepts (n=4)	Attracting students attentions to lesson, Making the lessons fun (n=2)			
	Displaying the relationship between mathematical concepts and real life (n=2)	Reducing anxiety against the mathematics (n=1)			
		Increasing the self-confidence on mathematical expressions (n=2)			

4.2.2.2 Prospective teachers’ thinking about the place of using modeling activities

Prospective teachers’ responses to semi-structured interviews, reflection papers, field and observation notes, and post-survey responses were analyzed together. As a result of this analysis, the views about timing in using modeling activities and how this differs according to the levels of students emerged as codes. The timing of using modeling activities meant that some prospective students preferred to use the same modeling activity before the subject matter and some of them preferred to implement it after lecturing on the subject matter. Moreover, it was examined how prospective teachers’ views about the timing of using modeling activities changed according to their level of knowledge throughout the semester. Prospective teachers’ ideas about the timing of using selected modeling activities were illustrated on the following table (see Table 17). According to given table, four prospective teachers considered using the given modeling activities after teaching the subject matter. This situation can be interpreted as meaning that these prospective teachers might use these activities in order to review what is learned and to assess how students apply their knowledge on a given activity related to the subject matter. This interpretation was supported by the previous results about prospective teachers’ aims of use. That is, the prospective teachers who use modeling activities after teaching subject matter preferred to use modeling activities in order to reinforce mathematical concepts and for measurement and evaluation purposes mostly. For example, PT17 wrote in her reflection paper about *the Ferris Wheel* activity: “When I consider this problem from the point of view of a teacher, I can have an idea about whether trigonometric concepts are acquired or not when I apply this problem to my students”. She considered using the modeling activity to measure and evaluate students learning after lecturing on the subject matter. PT24 also expressed his thinking as follows: “As a teacher if I wanted to apply this question in a classroom environment, I would ask the students to solve it individually after my explaining the angles in a circle and solving a few questions about angles in circles and trigonometric calculations”. According to PT24’s expression, he thought to use the same modeling activity to reinforce students’ conceptual knowledge about the subject matter.

Table 17 Views of prospective teachers about the place of using the modeling activities

Prospective Teacher	Name of the Modeling Activity		
	The Ferris Wheel	***The Bouncing Ball***	***The Water Tank***
PT17 (Group 1)	After subject matter	After subject matter	After subject matter
PT14 (Group 2)	Before subject matter	Before subject matter	Before subject matter
PT5 (Group 3)	After subject matter	After subject matter	After subject matter
PT10 (Group 4)	Before subject matter	NA*	Before subject matter
PT9 (Group 5)	Before subject matter	Before subject matter	After subject matter
PT23 (Group 6)	After subject matter	After subject matter	After subject matter
PT24 (Group 7)	After subject matter	After subject matter	After subject matter

*NA: Not attended.

On the contrary, almost all of the other prospective teachers who consider using modeling activities before teaching the subject matter wanted to use these activities in order to teach mathematics meaningfully. For instance, PT14 wrote the following in her reflection paper:

> As far as I remember from my high school years, (school, private course etc.) trigonometry was not a popular subject. Therefore, I think that such an activity may have positive effects on the students changing their views on this subject. Their discovering the usage areas of math in daily life may enhance their interest in the lesson (*PT14, reflection paper for the Ferris Wheel activity*).

She meant that modeling activities were useful for meaningful mathematics teaching. It can be interpreted as students could learn mathematics meaningfully in this way. PT9 also indicated the importance of meaningful mathematics teaching with the help of modeling activities in the interview.

> Interviewer: Hmmm. Well PT9, what did you learn from this activity?
>
> PT9: The angle with sine.
>
> Interviewer: Things that you already knew.
>
> PT9: Well, actually, I cannot say it give me too much, but, I can say I will remember that sine formula and will not forget easily.

These evidences supported the interpretation that prospective teachers who aimed to use modeling activities for reinforcing concepts, measurement, and evaluation had a tendency to use these activities after teaching the related subject matter. PT23 stated his preference for using modeling activities after teaching related subject matter, specifically about three aforementioned activities in the Table 15. He wrote in the reflection paper for *the Ferris Wheel* activity: “Namely, I would use this after giving attainment which I will use at work.” He expressed his ideas about where he considered using it as follows for *the Bouncing Ball* activity: “If I handle this from the point of view of a teacher, I do not apply this activity with the idea of making students have attainments. I use acquired attainments in implementation step.” He also wrote, “It seems to me that I could use these in the implementation phase” for *the Water Tank* activity. To sum up, prospective teachers perspectives became more dominant about why and where to use the modeling activities.

4.2.2.3 Prospective teachers’ thinking about the method of using modeling activities

When prospective teachers’ views about using mathematical modeling activities in the classroom setting were examined, it was revealed that prospective teachers had inclinations to use group work rather than individual work or a combination of group and individual work in using modeling activities. The views of seven prospective teachers who were selected from their groups were investigated throughout the implementation of six modeling activities. The findings demonstrated that prospective teachers had an agreement about using group work when they would implement *the Summer Job* and *the Free Roller Coaster* activities. It was observed that most of group representatives preferred group work for all applied modeling activities (see Table 18).

Table 18 Prospective teachers’ views about suitability of modeling activities for group vs. individual work.

	Name of the Modeling Activity					
Prospective Teacher	**Summer Job**	**Ferris Wheel**	**Street Parking**	**Bouncing Ball**	**Free Roller Coaster**	**Water Tank**
PT17 (group 1)	Group work	Group work	Individual work	Individual work	Group work	Group/ Individual
PT14 (group 2)	Group work	NC*	Group work	Group work	NC	NC
PT5 (group 3)	Group work	NC	Group work	Group/ Individual	Group work	Group work
PT10 (group 4)	Group work	Group work	Group work	NA*	Group work	Group work
PT9 (group 5)	Group work	Individual work	Group work	Group work	Group work	Group work

Table 18 (continued)

Prospective Teacher	Name of the Modeling Activity					
	Summer Job	**Ferris Wheel**	**Street Parking**	**Bouncing Ball**	**Free Roller Coaster**	**Water Tank**
PT23 (group 6)	Group work	Group work	Group work	Group work	Group work	Group work
PT24 (group 7)	Group work	Individual work	Group work	Group work	Group work	NC

*: NC: No comment, NA: Not attended

According to above table (see Table 18), some of the prospective teachers expressed their ideas. They considered using only group work as an implementation method with respect to each of the modeling activities (e.g., PT14, PT10, and PT23). For example, PT23 explained his ideas about group work as a method of application in the following episode after *the Summer Job* activity. He stated the advantages of group work in modeling activities such as emergence of realistic ideas, finding the solution in a short time, interaction of group members etc.

Interviewer: You decided to use this activity. How would you do it?

PT23: I would use group study.

Interviewer: Why group study?

PT23: Well, more realistic ideas come out with group study. Students can criticize each other's ideas and find better and more realistic solutions. Easier solutions in shorter times.

Interviewer: You are saying that you would use group study for this type of problems.

PT23: I mean, group study can be better for students to collaborate, listen and understand each other, and find a solution.

Similar to PT23's expressions, PT14 stated her comments about why group work was so important and why she considered selecting group work as an implementation method for *the Street Parking* activity in the following excerpt.

Interviewer: How would you implement this whole activity in your classroom?

PT14: Well, to begin with, I think group study is very important. I understood this very well with this activity, because you may not see that you are doing it wrong, but your friends could show you the right way. I can see Mehmet's solution and it makes sense. I can see different solution methods with group study and I can be aware of my mistakes. I think learning about your mistakes is as important as finding a solution. For these benefits, I would use group study, because at some point, you cannot go further individually.

Although most of the prospective teachers preferred the group work method in their future implementations of modeling activities, some of the prospective teachers (PT17, PT9, and PT24) indicated that they could use individual work with respect to the nature of modeling activities and related subject matter. For instance, PT17 stated her preference in *the Bouncing Ball* activity as follows: “If I were a teacher, I would implement this problem individually. I think that students can do it on their own too.” PT24 also indicated his preference as individual work while implementing *the Ferris Wheel* activity by giving reasons. He wrote: “No use of technological programs, it is not having so much open-ended point, it is being parallel to former questions and its being solved only by mathematical process make this question more suitable to use for an individual study”. Alike PT17 and PT24, PT9 also expressed her preference to use modeling activity (*the Ferris Wheel*) individually.

It is evident from the above quotes that some prospective teachers mentioned the reasons such as structural properties of modeling activities (e.g., open-endedness, requiring assumptions), aiming to measure and evaluate students’ learning, and seeing the similar activities in previous lessons or courses.

4.2.2.4 Prospective teachers’ thinking about the frequency of using modeling activities

Prospective teachers’ responses to the following question in the post-survey were analyzed and coded in terms of qualitative data analysis: “Do you think to implement modeling activities in your classroom when you become an in-service teacher? Explain it with reasons.” Process analysis was also carried out throughout the semester. As a result, prospective teachers’ responses to post-survey about the frequency of using modeling activities were parallel to their answers throughout the implementation of the modeling course. For example, in the very beginning of the modeling course, PT24 stressed the significance of mathematical modeling activities in high school and undergraduate levels without indicating any frequency after the implementation of the first modeling activity (*the Summer Job*). He emphasized that these activities ought to be in the high school and undergraduate school curricula.

> Interviewer: Good, well, do you think these kind of problems should be in high school curriculum or in university level? If so, how it should be?
>
> PT24: I think these kind of problems should be, why? Because, they have students look for different approaches and different results. I mean, instead of grade 4, it can start in further grades as low levels. We can asks others’ opinions and increase the levels gradually. If a high school graduate encounters a problem in daily life, he/she should at least be able to come up with three different solutions.

Since there was not any question related frequency of using modeling activities in the process (in reflection paper guideline and interview questions), it is not possible to display developmental process in frequency of using these activities. Rather, there were evidences about prospective teachers' thinking about using modeling activities when they become mathematics teachers actually. In the post-survey form, PT24 indicated his opinion about using modeling activities as follows: "Yes, I consider implementing mathematical modeling in the future because I think that the students will love math, the fears will diminish with these modeling, and they will understand the subjects better." Aforementioned in the previous parts, more than half of the prospective teachers stated that they thought to use modeling activities frequently. For example, PT10 wrote: "When I am a teacher, I will implement these activities quite often because I mentioned about my own attainments above. I want my students to have similar attainments". PT5 also intended to use modeling activities in order to demonstrate to students the relationship between mathematics and real life situations in the following excerpt from her post-survey form.

> I am thinking of implementing this modeling in the future when I am a teacher. Because with these modeling activities, first of all, they will enable the students to find answers to questions like "Sir, Where will we use this and what is the use of this?" and they will show to the students math is frequently used in daily life. What is more, alongside common math questions, I will check and see if they understood the subject fully or not and even if they understood it, I will have seen whether they could use it in an effective way or not. And these activities will enable me to understand which subjects I should or should not repeat considering their final solutions (*PT5, post-survey form*).

Even though more than half of the prospective teachers stated a frequency about using modeling activities in their future classrooms, some of them declared that they preferred to use modeling activities when they were needed. For example, PT2 pointed out that modeling activities would be helpful when students did not understand any mathematical concept rather than using in every unit or teaching mathematical concepts. She also mentioned that using more modeling activities would bore students. She wrote:

> I cannot tell you, I quite often include modeling activities when I am a teacher. Because I think that modeling activities for every subject can bore the students. But I think that for the subjects which are not understood fully or seem illogical to the students and moreover which may bring to mind questions like" why do we learn this? Where will we use this?" these activities will be useful and will make a great contribution to their learning. Furthermore especially for the subjects that consist of parrot fashion, I implement this activity occasionally to make my students find out what they know and to make the subject catchy and then by this way I want to make my students think. An activity where an everyday situation is blended with thinking and knowing could be implemented in a classroom and I think of using such activities in my future lessons (*PT2, post-survey form*).

PT23 wrote in the post-survey form as "By the time I am a teacher I think of using modelings for some subjects when they are appropriate. Because I hope that this will be useful to revive some subjects in the minds of the students." In the previous quote, PT23 said that using modeling activities in some topics would be beneficial for students in stimulating the cognitive processes of concepts in their mind. Although PT2 and PT23 declared that they considered using modeling activities when they were needed, some prospective teachers put forward an opinion that there would be less time for these activities due to the educational system and therefore, they could use a few activities when they become in-service teachers.

> I do not implement much, 2 or 3 times at most in a year. Because, unfortunately the system of education requires this. But if I notice that the students did not understand the subject of curves, then I would do an activity of roller coaster with the aim of helping them think about the subject differently. Also I could evaluate their knowledge by this activity instead of quizzes (*PT16, post-survey form*).

For instance, PT16 implied in the above quotation that he could get only a few chances to implement these activities because of the educational system (2-3 activity in a year). He also expressed that he could use these activities when students did not understand a subject matter as a last help.

To summarize this section, the findings demonstrated that more than half of the prospective teachers who took the modeling course thought to use modeling activities frequently in their future classrooms. Some of the remaining prospective students reported that modeling activities could be used when needed. Very few prospective teachers said that very few modeling activities could be used in the classroom because of some reasons such as educational system, or not enough time allocation. It is significant that prospective teachers developed an idea about using modeling activities whereas they did not have any opinion about using modeling activities in classroom setting before they took the modeling course.

4.2.3 Prospective Teachers' Thinking on Group Work in the Modeling Process

In the light of the analysis of reflection papers, individual interviews, videotaped records, and observations during the modeling process, the findings demonstrated that most of the prospective teachers viewed the group work process as positive and effective. Some of the participants pointed out that they would have encountered difficulties if they solved the modeling activities individually instead of in a group. For example, PT24 indicated both the advantages and the disadvantages of group work in the interview after the implementation of first modeling activity (*the Summer Job*).

Interviewer: Okay, PT24, how was the group working for you?

PT24: Group working was good. I like to work with group because distinct ideas, different perspectives emerges in the group working. One of my friends sees the point that I do not see. I could complete the missing of my friends or vice versa, thus the solution could be found easily and quickly. Sometimes the solution could be found late because I could not see the point and so my friends which results in failure. There exists a time difference between individual working and group working. In this activity, we produced a good solution.

In the preceding episode, PT24 elaborated on the benefits of group work such as exploring different ideas and perspectives, which allowed them to find the solution quickly. PT24 indicated that sometimes group work resulted in failure because of a breakdown in communication between group members. PT14 evaluated their group work process in the interview after the implementation of *the Ferris Wheel* activity.

Interviewer: Can you evaluate the process of group working during the implementation? Was it useful?

PT14: Of course, yes. The group working was good and enjoyable. Namely, in order to solve the problem situation, you should be eager. If I was on my own, I would not be eager to solve. In group working, I would be eager to engage in the activity and solve it. It was also enjoyable with your group members.

Interviewer: Do you enjoy with it?

PT14: Yes, I enjoy it.

Interviewer: The communication in the group is good by then.

PT14: Yes, our communication is good and therefore, one of us always discards an idea. When one of us do not find any solution, others develop solutions for the problem situation. That is good.

In the above episode, PT14 stated that the group work gave them the opportunity to more easily understand the problem. She compared group work with individual work in terms of motivation. She concluded that she would not be motivated if she worked individually. She also put emphasis on the communication in the group work. Communication was important because it allowed them to discuss ideas regarding the solution. For instance, the following group working episode (group 2) transcribed from videotaped records reflected communication in the group. It also showed how prospective teachers discussed the problem in order to understand it.

PT13: There will be a sudden descent at least three places.

PT15: It is not asked us to design a way.

PT13: So, what is expected from us?

PT15: The only thing, no, it is not wanted from us to design. Here is 6 meters, here is 9 meters. There are three ups and downs at most from here.

PT13: OK, we will do this part.

PT15: We will find in which parts the excitement will increase in this section. In which part will the excitement decrease?

PT13: The excitement will increase here. Will we say this?

PT15: Yes, when we assumed that the slope is 5.67 mathematically, namely, when it was go up here, we will compare the slopes at distinct positions. For example, as if the excitement increased here, in fact, it would be increase more at that point.

PT14: This graph will be distance versus height.

In the above part of the episode, members of the group 2 tried to understand problem in *the Free Roller Coaster* activity. As it was understood from the conversation between group members, they tried to understand the problem and develop solution strategies.

PT14: Let us do like that. Now is here 3 meters? Okay, here is 3 meters. What slope means, I will draw a tangent line.

PT15: It says at least. That means we can do more if we want. It does not matter how many they are. According to me, the most important thing is at which height the slope is more. Namely, it is need to check out how many ups and downs would be when the slope chosen as 5.67.

PT14: OK, then. Why the a hundred meters has been given? Where will we use it?

PT15: Here, therefore, what distance it covers when the slope is 5.67?

PT14: Just like that.

PT15: OK, then we will look how many ups and downs could be made according to it.

PT14: Okay, Let us do like that. Will we take the slopes from here? Or the slope of here. What are the givens? 9 meters, 6 meters, and 100 meters are known. Then, I will fix the value of tangent angle with 5.67. Think about I draw a line from here. Now, I know that here is 3 meters. Is it here a positive angle with x-axis?

PT15: Yes.

PT14: By using it, I will find the tangent value of that angle. I will say here 3 meters. I will find the distance here, under the 100 meters.

PT15: Just like that. We will find here.

In the preceding part of the group discussion, prospective teachers told each other what they understood from the problem and clarified the possible solution methods by making assumptions such as fixing the value of tangent angle on the potential graph for the roller coaster. It can be interpreted that group members tried to convince each other how to solve the problem by asking questions or claiming new ideas. As it was seen from the above conversation, the discussion had by two prospective teachers was observed other members of the group. For example, PT15 sketched a drawing for the way of roller coaster (see Figure 14) and made assumptions from that drawing. Meanwhile, PT16 also participated in the discussion by drawing attention to the text of the problem in the following part of the conversation.

PT14: So we found here. How will find there?

PT15: We will think of all like that. Accordingly how many times…

PT16: But, the finishing height will be 9 meters. Is there like that?

PT15: No.

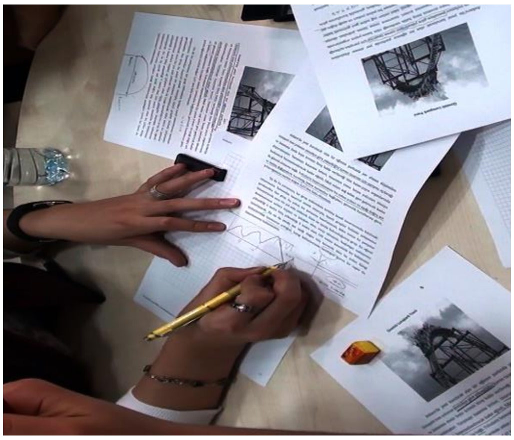
PT16: I understood that it will start from a point, it may go up, but it will finish at 9 meters level at the end.

PT15: Yes, correct!

PT14: You say it does not need to be fixed at 9 meters level.

PT16: No, it does not.

PT14: It is really very logical. Let us draw here again.



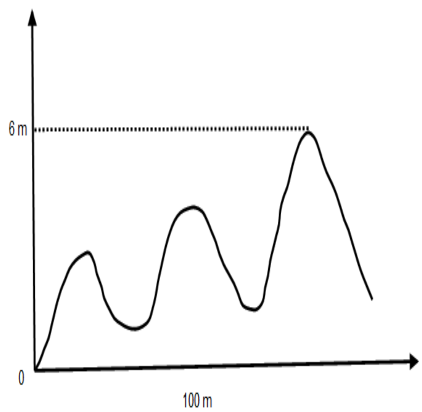


Figure 14 A video still from the solution of the group 2 for *the Free Roller Coaster activity* and the drawing on the right side is the recreation of the students' original sketch

According to preceding part of the episode, PT16 asserted that the height needed to be 9 meters. It was understood from the conversation that other members of the group accepted this suggestion and they were convinced to revise their first drawing of the graph as displayed in Figure 15.

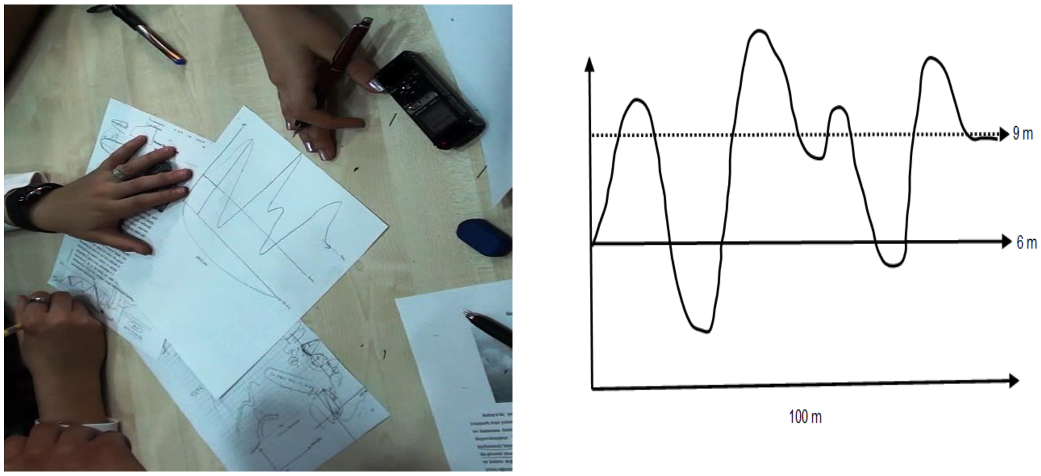


Figure 15 A video still from group 2’s solution and the drawing right side is the recreation of the students’ original sketch

The evidence presented in the above section demonstrated that prospective teachers could put forward distinct ideas and other group members checked the correctness of these ideas. They might also suggest a more accurate way for the solution as indicated by another prospective teacher in the beginning of the quote. Similarly, PT10 contended that group work was effective in the process of solution in the following episode.

> Interviewer: Okay, if we go on with the perspective of group working, do you think that the group working is efficient?
>
> PT10: Definitely, that is, it is really efficient. With the modeling activity, it is expected from us to think about and chew over the problem situation. In my opinion, this type of activities require group working precisely. I found the cosine theorem, but if my friends did not tell me that the sine theorem or how to calculate the height there, I could not do anything about the solution. I think like that. I could solve only second part of the problem situation. I could not solve the remaining parts. I achieved that with the help of group working. I got my friends’ opinions, we discussed the ways of solution, and then we solved the problem.

In the above episode, PT10 stated that group work was necessary for modeling activities because it required more thinking. She said that she would have been able to solve very little of the problem if she worked individually. She also explained the importance of discussion in the group work process.

In summary, most of the prospective teachers indicated that group work was effective in terms of developing distinct ideas, communicating among group members, learning from each other, and solving the modeling activity quickly when compared to working individually. In addition, some of the prospective teachers stated that working in groups provided them with the opportunity to help each other during the solution process. In order to better understand prospective teachers' ideas about group work in the implemented activities in terms of effectiveness, the following table (see Table 19) illustrates PTs' opinions according to the modeling activities.

Table 19 Prospective teachers' views about effectiveness of their own group processes

	Name of the Modeling Activity					
Prospective Teacher	**Summer Job**	**Ferris Wheel**	**Street Parking**	**Bouncing Ball**	**Free Roller Coaster**	**Water Tank**
PT17 (Group1)	Efficient	Efficient	Inefficient	Inefficient	Efficient	Efficient
PT14 (Group 2)	Efficient	Efficient	Efficient	Efficient	Efficient	Efficient
PT5 (Group 3)	Efficient	Efficient	Efficient	Efficient	Efficient	Inefficient
PT10 (Group 4)	Efficient	Efficient	Efficient	NA	Efficient	Efficient
PT9 (Group 5)	Efficient	Inefficient	Efficient	Efficient	Efficient	Efficient
PT23 (Group 6)	Efficient	Inefficient	Efficient	Efficient	Efficient	Efficient
PT24 (Group 7)	Efficient	Efficient	Efficient	Efficient	Efficient	Efficient

According to table above, most of the prospective teachers who represented their groups declared the process of group work in all implemented activities as efficient. This outcome correlated with the views of prospective teachers about the methods preferred when using modeling activities as indicated in section 4.2.2.3. In addition, some of the prospective teachers thought that they would encounter difficulties if they worked individually. As can be seen in from the above table (see Table 19), some of the prospective teachers asserted that their group work was inefficient in some of the implemented activities.

Prospective teachers were asked how they would implement the modeling activities they learned throughout the semester in interviews and reflection papers after the implementation of each modeling activity. Their preferences are indicated in section 4.2.2.3. According to these findings, prospective teachers expressed that they were considering using group work when they became teachers. The findings demonstrated that differences existed in prospective teachers' considerations about forming and managing groups. When the Table 18 was examined, it was clear that prospective teachers had an inclination to use group work during the modeling process. When the responses of the participants who considered using group work were analyzed differences about the formation of groups with respect to intended aim and individual preferences played a role in the determination of the number of members in each group. For example, PT17 claimed that she preferred to form groups with only two members each. Her explanation is provided in the following episode.

> Interviewer: You told that if group working selected, there would be three members for each group. So, what do you think about the number of member in each group for being appropriate in the implementation?
>
> PT17: I always think there should be two members for each group. I think about the students, for example. If they were three or four members in each group, they do not respect each other due to the reasons such as not mating enough. One of them could be dominant or diffident. Therefore, if there are two members in each group, they communicate each other easily. If there existed four members in each group, two of them communicate with each other and the remaining members could drop back. So, two members in each group is much better. It also depends on the number of students in the class. For example, if there are 24 students in the classroom, dividing students into groups with two members could be difficult. In this case, three members in each group is better according to me.

In the preceding episode, PT17 stated that she could determine the number of members in each group according to students' characteristics (e.g., not respecting each other, not maturing enough), potential dynamics in emerged in the group (e.g., dominating each other's' ideas, being ineffective) or class size. PT9 pointed out in the following quote that grouping students' according to the way they think would be effective in the modeling process.

> I would want my students to pay attention to settlements of vehicles, to think about parking areas around them. They would think different settlements and then I would ask them to think which of these settlements takes smaller space. After doing these things, I would say "Let's show this mathematically now; which one will take smaller space" and I would hand out the questions. After having first ideas of the students, I would put the students who have chosen the same settlements in the same group. I would form these groups with three or four people each. These groups would try to prove their own hypothesis (*PT9, interview after the implementation experience*).

In the previous excerpt, PT9 noted that forming groups according to students who had similar considerations about the possible solution of the modeling activity would be effective. She also stated that groups would discuss a possible solution and therefore, they reached the correct solution. Moreover, PT24 emphasized the importance of group work in order to produce new ideas, "Implementing this question in a classroom as a group may be more efficient. Because it is a kind of question that can be solved with different ideas, they can compare their ideas with their friends and can think up an idea together". In the preceding quote, PT24 kept in the forefront the function of group work rather than grouping style and the number of members in each group.

According to the findings obtained from the analysis of post-survey responses, prospective teachers underlined the significance and relevance of group work in the implementation of modeling activities. For instance, PT17 stated that she learned a lot in the process of group work in the following excerpt: "I learned to combine different ideas with ours with the help of group work. We also tried to overcome the difficulties that we encountered together. It was also useful in terms of this point." According to previous quote, she expressed the contribution of group work because it helped her to learn how to combine different ideas with her own ideas. She also learned how to deal with difficulties during the solution process. In addition to this, PT24 supported PT17's consideration as follows: "I also discovered how personal opinions formed by group work, it helps students to have or catch different points of views". In the preceding quote, PT24 indicated that group work helped students to notice the different perspectives during the modeling process. PT3 suggested that group work ought to be done while implementing modeling activities in the following quote:

> While implementing modeling questions in teaching of mathematics, group work absolutely should be done. The students should discuss their ideas among themselves and reach solution by this way. Because one can compensate the other one's incorrect idea with his/her correct idea and they can reach solutions more apprehensibly and correctly (*PT3, report written after the implementation experience*).

In the preceding excerpt, PT3 mentioned the importance of discussion during group work such that students could correct their wrong ideas during the process, and they could reach comprehensible and correct solutions. Other prospective teachers expressed similar opinions about the importance of group work during the implementation of modeling activities.

To sum up, prospective teachers declared their opinions about group work as follows:

- group work was important and relevant in the implementation of modeling activities,
- group work revealed the distinct ideas and solution approaches,
- members of the groups filled in the missing parts of each other in terms of knowledge,

- group work enabled students to evaluate all of the proposed ideas and then, they reached the solution.

In addition to the advantages of group work, some of the prospective teachers mentioned the limitations of it. For example, PT23 expressed that dominant member in any group could cause other members of the group to become passive in the group work process. As a result, it was observed that group work was an efficient method in the implementation of modeling activities and approached positively by the prospective teachers.

4.2.4 Prospective Teachers' Conceptions about the Relationship between Technology and Mathematical Modeling

The findings acquired from the analysis of reflection papers and individual interviews after the implementation of modeling activities demonstrated that prospective teachers thought that the use of technological tools accelerated the process of finding a solution by providing fast computation in the operations. In addition to this, it was helpful for computing the results of difficult equations. For example, PT24 underlined the importance of technological tools in the solution process in the following episode.

> Interviewer: What did you do in order to overcome these difficulties? That is, how did you eliminate the errors in the operations?
>
> PT24: We eliminated our errors by using calculators. We tried to reach the precise solution with the help of calculator. We used calculator for calculating the areas. Although we used the calculator, we got mistakes. We corrected later. We eliminated the mistakes that stemmed from operations.

In the preceding episode, PT24 told said they used a calculator check their operations. He stated that they eliminated the mistakes in the operations by using calculator and this provided them with accurate results. Similarly, PT9 mentioned the importance of technological tools in the modeling process in the following episode.

> PT9: I think we will get difficulty when we calculate with paper and pencil method.
>
> Interviewer: However, your strategy determined the operations in the solution steps.
>
> PT9: Yes, but when we denoted here by " a ", that is, when we used *Pythagoras theorem*, it resulted in complicated. Namely, OK, may be either delete or …
>
> Interviewer: Namely, it requires technological assistance.
>
> PT9: Yes, it is needed.

…

Interviewer: The result was an equation with second order. The roots were not familiar, they were really strange.

PT9: Yes, that would be a problem for us. It was not solved by paper and pencil. Namely, we solved it with the help of technological tool.

In the above episode, PT9 expressed that using technological tools facilitated their computation. She stated that they would have difficulty in computing if they used paper and pencil rather than a technological tool such as a calculator. She also indicated that finding the roots of some quadratic equations could not be resolved by paper and pencil. Therefore, technological assistance was needed in these situations. Some of the prospective teachers stated that using technological tools such as a calculator speed up the solution process. For instance, PT8 wrote on the post-survey form as follows:

> Technologically, we used calculators for the solution of the problem. If we had not used calculators, no matter how the equation was with one unknown in order to solve such an equation whose quadratic multiples were not whole numbers, we would have needed to spend too long time on the problem. In this respect, calculator's contribution was great and at the same time, we made advantage of calculators to calculate trigonometric values (*PT8, post-survey form for modeling-technology relationship*).

In the preceding excerpt, PT8 compared computation by paper and pencil with technological tools, and he concluded that the paper and pencil method was not an efficient use of time. She expressed that using a calculator facilitated their solution process by finding accurate trigonometric values.

Besides positive contributions from technological tools, some of the prospective teachers mentioned the limitations associated with them. For instance, PT5 expressed the difficulty that she had when she tried to use calculator during the solution process as follows:

> I encountered with many difficulties while studying on the problem. The most important one among these problems was to calculate an angle whose sinus value was known. Because some calculators while giving results as radian, it was necessary to turn the result into degree and use of calculators was quite complicated (*PT5, post-survey form for modeling-technology relationship*).

In the preceding quote, PT5 mentioned the difficulties that she experienced in using a calculator. It was understood from the quote that using technological tools could be difficult because prerequisite knowledge of the calculator was required in order to use it properly.

In summary, prospective teachers stated that using technological tools could be beneficial in the modeling process, but it could be difficult if using these tools required extra knowledge to use them properly.

4.3 Prospective Teachers' Conceptions about the Importance of Mathematical Modeling in Teaching and Learning Mathematics

In the previous sections, it is reported that prospective teachers developed positive conceptions about the use of mathematical modeling activities in the classroom setting. Most of the prospective teachers indicated that they wanted to use these modeling activities when they became in-service mathematics teachers. The findings demonstrated that prospective teachers stressed the importance of mathematical modeling and modeling activities in the teaching and learning of mathematics. Most of them indicated that modeling activities were useful tools to use explaining subject matters used related real life situations. It also served as a tool to help motivate students to focus on the subject matter. In order to reflect prospective teachers' positive point of view mathematics, it was focused on how prospective teachers developed positive perspective about the use of mathematical modeling and modeling activities in the classroom settings as exemplary for reflecting other teachers' opinions about the place and significance of mathematical modeling in teaching and learning

In the case of PT14, after the implementation of the first two modeling activities, she discussed the importance of modeling and modeling activities.

Interviewer: Well, have you learned anything new here?

PT14: Of course! Actually, I thought like, we have been here for 3 years, but I feel like they were floating in the air. I mean, it is like there is never any application. Well, we have been proving things for years. I wonder about something, for example, I studied Topology last semester.

Interviewer: It is a really good course.

PT14: It is good, I really liked it and I had a perfect grade, but I keep thinking about it and I say, "How can I adopt Topology?" I cannot concretize some of the things; they are just floating in the air. OK, I know what Topology is, I know about the surfaces and everything, but how can I use it in everyday life? I passed the course, but I know that I will definitely forget about it. I mean, it does not fit anywhere, like I said, we know about everything, but when it comes to application, we have problems.

Interviewer: What if we had modeling activities that we can apply to those subjects?

PT14: I passed Topology course, I passed it, and we have had courses with single variable and multiple variables. We just got out of high school and we were introduced to everything. I mean, it was really hard, but everything makes sense now.

In the preceding episode, PT14 expressed that she felt "knowledge gathered from undergraduate mathematics courses seemed to be up in the air" after taking these course by giving examples from some of them. For this reason, she declared that they were difficulties in perceiving and embodying the concepts in some mathematics courses that were taken in undergraduate education. She stated that these concepts became logical and meaningful via mathematical modeling.

Interviewer: OK, did the previous activities contribute the solution of this activity? P

PT14: Thinking style is changing. For example, the used mathematical concepts are different from each other. We used exponentials here; we used different concepts in the previous activities. Our perspective is changing after solving different types of modeling activities. I try to think more widely with the influence of previous activities although there were no common points between the activities. We obtained different kind of solutions such as equations, intervals, etc. Another example is that we thought there was only one solution or more by thinking of the first modeling activity (*The Summer Job*). We try to think that there can be more solutions according to the nature of modeling activity and mathematical concepts within. In my opinion, each modeling activity has an influence on our thinking style and perspective.

Interviewer: You said they changed out perspectives.

PT14: Yes, it happens like that. For instance, I said myself that we could not solve this by using that. However, students from different groups could solve the problem by relating different concepts.

In the interview held during the midterm with the prospective teacher PT14, she stated in the preceding excerpt that the implemented modeling activities changed their perspective by including distinct concepts and contexts. Due to this, they learned how to look at the situations from a wide perspective and understood cause-effect relationships easily.

PT14: … For example, I have a private student at grade seven, she always think everything linearly. She do not understand the difference between proportion and inverse proportion in the daily life examples. Therefore, I teach the proportion by giving example from real life and also for the inverse proportion. Then I ask her to give one example for each about the proportion and inverse proportion. I grasped this technique from the modeling course. Because our perspectives are changing and it changes your approach to mathematics also.

Interviewer: it is good for you. Relational understanding you say.

PT14: Yes, that is right. This course taught me the concept of awareness, then you notice something different that were not noticed before.

In the interview with PT14 after the implementation of the modeling activity, *the Water Tank*, she stressed the importance of mathematical modeling in the course. She applied the information she gained from the modeling course with her private student.

In the case of PT17, it was observed that she made evaluations specific to each activity regarding the place and significance of mathematical modeling and modeling activities throughout the implementation process. In the beginning of the semester, she indicated the importance of modeling activities for introducing and motivating students to relatively difficult subject matter such as trigonometry.

Interviewer: If you look through eyes of a teacher, what do you expect from students to gain if you implement his activity?

PT17: I want students to use trigonometric identities in the activity. I want to observe whether students understand or not. I want to use this activity in my class in the future. Because when relating mathematical concepts with real life, students could pay more attention to the lesson and they could adapt to lesson more easily.

In the preceding excerpt, PT17 stated that using modeling activities motivates students to comprehend the subject matter, and it enables them to adapt to the lesson. She also mentioned that these activities could be used for measurement and evaluation after the teaching of a subject matter.

> I noticed how to use my trigonometric attainments, which I acquired from mathematics education, in daily life. Namely, I discovered that these attainments are not only to have success in exams but also to make life easier. Because like many students, I was also curious about what the use of trigonometry was in daily life, during the process in which I learned trigonometry. It was a kind of problem that I can use to reply to a student who has such a curiosity when I am a teacher. For this reason, I think that this problem is useful (*PT17, reflection paper for the Ferris Wheel activity*).

It was understood from the above excerpt that modeling activities helped to explain the relationship between the mathematical concepts and real life situations. She also pointed out that she noticed the knowledge gathered from subject matter helped during daily life. Moreover, these activities were useful for teachers when they explained subject matter used in real life.

In the middle of the implementation period, she compared the previous modeling problems with *the Street Parking* activity in terms of context and difficulty. In the following episode, PT17's thought about the modeling activities were given.

Interviewer: Did the previous modeling activities contribute to this process of solution?

PT17: OK, let me think about it. For example, in this activity we understood that we need to construct a system. In the previous activity (*The Ferris Wheel*), we designed a system also. That is, we got familiar to these types of activities. In traditional word problems, an angle of a triangle was given, then we put the values on the triangle, then we get the correct answer. But in modeling activities, we could not find the unknowns easily. There is no exact and obvious solution path. We experienced with modeling activities and we are practical now.

Interviewer: You mentioned about the "we used equation, trigonometric identities". Have you know these concepts before? Have you learned new things? Is there any change in your knowledge?

PT17: All of these concepts are known. Maybe if we were high school students, I could say that we did not know to solve these equations or we did not know the relationship between the value of angle and of sine. Since we examined all these concepts, there is no concept that we learned newly.

In the previous excerpt, she indicated that they were accustomed to the style of modeling activities. She asserted that previous modeling activities were hard, but they were also practical. She stated that she did not learn any new subject matter, but she admitted that she learned how to make relationships between mathematical concepts and real life situations.

After the implementation of the last modeling activity, *the Water Tank*, she evaluated the change in her perspective on classroom discussions carried out after each modeling activity.

Interviewer: So, what can you say about the contributions of these activities other than mathematical ideas?

PT17: For example, I can say about the classroom works. I used to see the classroom works as only presentations. However, I observed in this activity that I saw how students see the concept of slope and curve. I thought of common ideas about how to teach these mathematical concepts. I envisioned that if I teach like that, what images emerge in the students' minds. Therefore, classroom discussion was useful for me.

According to the above excerpt, she observed other prospective students' considerations on the slope and curve. She expressed that classroom discussions were useful in helping to understand what students thought about mathematical concepts.

In the case of PT9, after the implementation of first modeling activity, she expressed the enthusiasm about the modeling course and modeling activity as follows: "These are good. I like the course because I liked the activities within the course". She also expressed that *the Ferris Wheel* activity helped her understand the *sine theorem*.

Interviewer: Yes, OK, What did you gain from this activity?

PT9: I grasped the use of *sine theorem*.

Interviewer: In fact, you have this knowledge before.

PT9: Namely, I cannot say that I have learned a lot from this activity, but I can say that how to use sine theorem. Hereafter, I can say that I do not forget this no longer easily.

It was evident from the above episode that modeling activities provided her with the opportunity to internalize the *sine theo*rem. She said that she learned to apply the *sine theorem* to real life situations although she knew the theorem before.

In the reflection paper she wrote after the implementation of fourth modeling activity, she noted teacher intervention during the application of modeling activity. She indicated the importance of teacher intervention and its timing as follows:

With the help of this question, I saw how a teacher should treat a student when the student cannot reach the solution or reaches with wrong way. This was the kind of treatment I had thought but when I needed to give examples, certain examples did not come to my mind. I saw this by experiencing. The question asked us “which point the slope will be maximum”, drawn curve and slope change question asked upon this curve helped me to comprehend the question rightly. I think of how I could not notice the characteristic of the shape when I had a look on it. I had already drawn the requested shape unintentionally, but for me, my teacher’s role was in the forefront in noticing the last characteristic we found (*PT9, reflection paper for the Free Roller Coaster activity*).

In the preceding excerpt, PT9 stressed the significance of the role of the teacher in the implementation period. She expressed that the teachers’ questions about the slope of the curve enabled her to comprehend the main scope of the modeling activity. For this reason, she emphasized the role of the teacher during the implementation of modeling activities.

In the interview held after the implementation of last modeling activity, she analyzed the modeling activity and indicated where to use it.

Interviewer: OK, I see, if you implement this activity in your classroom in the future, which objectives do you want your students to reach? Or, would you implement it?

PT9: Namely, I think it would be good for students’ interpretation of graphs because we observed that some of the groups did not interpret their drawings. This activity could be beneficial for students to interpret their own graphs.

In the preceding episode, PT9 identified the purpose of the modeling activity and specified its place for use by observing the other groups’ presentation in the previous modeling activities. From the episode, it can be interpreted that she determined where to use the modeling activity according to the students’ needs.

At the beginning of the term, PT24 indicated that he learned where to use the *sine theorem* in the interview that was held after the implementation of the second modeling activity.

Interviewer: So, did you know the mathematical concepts and ideas involved in the modeling activity? Or, is there any change in your knowledge about these concepts? Did you learn new things in this process?

PT24: In the process, I ought to learn the sine and cosine theorems, but I have not learned these theorems. I should learn these theorems because these are could be needed.

Interviewer: In fact, do you know them or?

PT24: I know these theorems and their definitions and formula. However, we do not make practice about how to use these theorems. We have been away from making practice for three years. We dealt with proofs and theorems. We need to make practices about them to show progress. There is no unknown concepts here, we know all of these. However, we do not have any idea about how to use these concepts in real life. I always wonder about the cosine theorems and about where we can use it in daily life. Because these theorems are fundamentals of mathematics. We can face with these concepts almost in every part of

> mathematics. I realized that these concepts can be used in daily life and they are very useful for us.

In the above episode, PT24 stated that the modeling activity gave him the opportunity to remember the sine and cosine theorems. Equally important, he was able to see where these theorems were used in the real life that. He declared that he did not know how to apply the sine or cosine theorem to real life situations although he knew these theorems theoretically. He said that he used to question why these theorems were used so much. Then, realized how they were used in real life situations via mathematical modeling activities.

PT24 mentioned the contributions of modeling activities during the middle of the implementation process. He stressed the importance of modeling activities in learning the subject matter as a whole and conceptually by observing the real life applications. For example, he said that he comprehended the role of derivative during the investigation of functions and their characteristics as follows:

> I saw concretely with the help of this question what the concept of the role of derivative is in characters of derivative that I had seen abstractly and in analysis of these characters. Information about derivative and instant derivative shaped better in my mind. Namely, I can say that I experienced quite difficult but efficient solution process (*PT24, reflection paper for the Free Roller Coaster activity*).

It is clear from his quote that the modeling activity helped him to understand the subject matter in-depth. It also helped him form strong schemas in his mind about the subject matter.

In the reflection paper written after the implementation of the last modeling activity, which was *the Water Tank,* PT24 specified the mathematical concepts in the modeling activity and made a plan regarding how and when to use that modeling activity in the classroom setting.

> Mathematically main subjects in this question were these concepts, characteristics of curves, what causes the function that increase with increasing rate or increase with decreasing rate; how an increase or decrease in unit's change in a function affects the graphic of function. For this reason, If I were a teacher, before explaining solutions of graphics to the students, I would use this question to enable something concrete in the minds of the students or to measure whether the subject is understood or not at the end of the subject (lesson) (*PT24, reflection paper for the Water Tank activity*).

In the preceding excerpt, PT24 understood mathematical concepts such as the characteristics of curves, increase, decrease, and concavity of function graphs etc. in the context of a modeling activity. He also made a decision on how to use the modeling activity when he became an in-service mathematics teacher

In summary, as it is seen from exemplary evidence collected from the some of the prospective teachers' experiences, prospective teachers realized the significance of mathematical modeling and modeling activities throughout the implementation period. Most of them suggested that modeling activities were very useful tools for teaching and learning mathematics. As a result, they believe that they provide meaningful mathematics teaching and learning in the classroom environment.

4.3.1 Prospective Teachers' Thinking about the Advantages and Disadvantages of Using Mathematical Modeling Activities

Throughout the implementation period, it was reported that prospective teachers developed positive conceptions and beliefs about using mathematical modeling activities in the classroom setting. Most of the prospective teachers stated that they wanted to use modeling activities in their classrooms when they became in-service mathematics teachers. They mentioned the advantages and disadvantages of using modeling activities in the classroom environment.

In the analysis, teachers' responses to the question "What can be the advantages and disadvantages of using modeling activities in mathematics lesson?" in the post-survey form, reflection papers written after each modeling activity, individual semi-structured interviews were taken into account. The findings demonstrated that the advantages of using modeling activities were motivating students, maintaining the persistency of learning, meaningful learning, including more than one mathematical subject and concept, allowing learning with group work, facilitating learning mathematics expressed frequently by the most of the prospective teachers. Prospective teachers stated the similar advantages of using modeling activities throughout the implementation of modeling activities and modeling course load. For example, after the implementation of the second modeling activity *the Ferris Wheel*, PT17 wrote in the reflection paper as follows:

> After solving this activity, I discovered how to use trigonometric attainments that I acquired during my education in daily life. In other words, I realized that these attainments were not only for being successful in exams but also for making our life easier. Because like many students, I was also curious about the use or benefit of this subject in daily life during learning process of trigonometry subject. This activity was one that I would use to answer one of my curious students when I could become a teacher. Therefore, this question (activity) was quite useful (*PT17, reflection paper for the Ferris Wheel activity*).

In the preceding excerpt, PT17 mentioned daily life applications of modeling activities. She discussed how these activities could motivate students to facilitate meaningful learning of mathematics based on her previous high school experience. She also indicated that modeling activities could be the answer for students who wondered how mathematics was used in real life.

Similar to views of PT17, PT14 pointed out the daily life applications of modeling activities. In the interview after the same activity, she said:

> Yes, I think that I learn many things here every week for example about which trigonometric function I should use. Namely, I know what trigonometric function is but this place helps me to learn how and where I should implement this function and it is really useful and good in terms of seeing and exploring (*PT14, interview for the Ferris Wheel activity*).

In the above excerpt, she indicated that she thought that she learned many things from the modeling course every week. She stressed the importance of modeling activities such that these activities enabled them to combine theoretical and practical knowledge of any subject matter. This situation was also described as a meaningful mathematics learning experience.

Another advantage of using modeling activities expressed by prospective teachers was that modeling activities help motivate students in learning mathematical subjects and concepts. PT2 illustrated this situation in the following quote: “I would think that if the students saw math and geometry were used in daily life, especially maybe in the most enjoyable part of the life in terms of children, their interest in math would enhance”. She pointed out that students would be motivated and more interested in mathematics if they observed that mathematics and geometry were used in real life situations.

Prospective teachers stressed that modeling activities allow students to work in groups and hereby students learned from each other. For instance, after the implementation of the last modeling activity (*the Water Tank*), PT9 stated in her reflection paper group work was one of the advantages of modeling activities.

> I think that this question is suitable for group solution and it is open-ended and a question to be discussed. I think that discussion on this question can be useful for the students. This is a question that can help the students to think and make sense of the concepts like increase with decreasing rate or increase with increasing rate that are mostly confused by students (*PT9, reflection paper for the Water Tank activity*).

According to above except, she declared that since the modeling activity was open-ended, it was suitable for group work and discussion. She thought that the modeling activity would be beneficial for students because it required students to think about concepts that they often found confusing.

Furthermore, including more than one mathematical subject and concept was seen as one of the advantages of modeling activities by prospective teachers. For example, PT8 emphasized this situation in the post-survey form as follows: "A convenience can be provided to reinforce a few subjects in a single question or in order to determine in which subjects the students have insufficiency; instead of examining all the subjects one by one, a modeling question that includes a lot of subjects will help us". It is clear from the quote that including more than one subject in the same modeling activity might be helpful for enforcing the subject matter and finding students' deficiencies in these subject matters.

On the other hand, prospective teachers asserted that using mathematical modeling activities took a lot of time which caused them to fall behind in their lesson plan. They also stated that implementing modeling activities was difficult in the crowded classrooms. For example, PT8 stressed that using modeling activities took much time and that might cause some problems related to schedule and other issues. She wrote in the post-survey form at the end of the course as follows:

> Difficulties could be experienced in terms of time. Because teachers have problems like completing the curriculum in time. The students may not want to do this activity or even if this activity is implemented in a crowded class, there can be problems in terms of evaluation and feedback (*PT8, post-survey form for difficulties of using mathematical modeling*).

In the preceding quote, she expressed that modeling actives generally took more time with respect to other activities and that might result in teachers not being able to keep up with the class schedule. She also indicated that using modeling activities in crowded classrooms might not be effective for students in terms of not being able to give enough feedback and assessment.

Another prospective teacher PT5 discussed the same issue in the interview held after the implementation of *the Street Parking* activity. PT5 stated her thinking about disadvantage of taking much time in the following episode.

> PT5: Only time can be a problem if I implement the activity in high school. The allocated time for a lesson in high schools is about 40 minutes. Students could be bored with the modeling activity or they could be uninterested in these activities. Other than these possibilities, there could be no problem about the implementation of these activities in high school classrooms.
>
> Interviewer: Namely, do you think it can be applicable at the high schools?
>
> PT5: I think it can be applicable, but there is an extensive time allocation for that. Now we spend four hours for these activities [making attribution to their own course]. We cannot allocate four hours in high schools; they might not complete their works in two hours. Perhaps they could complete their works, but they have no more time to present their works.

In the preceding episode, PT5 discussed possible negative scenarios that could occur if the modeling activities took too much time and compared their modeling course experience with the possible use of modeling activities in a high school classroom. She said that students could get bored due to the long implementation time and teachers could not devote much time for modeling activities like those that the current modeling course used.

To sum up, prospective teachers mentioned the advantages and disadvantages of using modeling activities in a classroom setting. The ideas regarding the advantages and disadvantages of using modeling activities were displayed in the following table (see Table 20).

Table 20 Views of groups on advantages and disadvantages of using modeling activities

Groups	**Advantages**	**Disadvantages**
Group 1	Providing motivation Maintaining persistency about grasped knowledge Higher cognitive gains	Choosing relevant activity Difficulty in ensuring compliance with level of students Difficulties in implementing in the crowded classrooms
Group 2	Maintaining persistency about grasped knowledge Being effective in challenging subject matters Embodying mathematics	Taking time Causing to fail to keep to course schedule
Group 3	Meaningful learning Providing motivation Daily life applications Using previous knowledge Providing learning with group work Making students gain different perspectives	Taking time Facing with operational errors Possibility in having lower group levels
Group 4	Usability for measurement and evaluation Thinking mathematically Maintaining persistency about grasped knowledge Including more than one subject and concept	Taking time Causing to fail to keep to course schedule
Group 5	Including more than one subject and concept Maintaining persistency about grasped knowledge Usability for measurement and evaluation Being effective in challenging subject matters Providing learning with group work Daily life applications	Taking time Difficulties in implementing in the crowded classrooms Incomprehensibility of the activity
Group 6	Maintaining persistency about grasped knowledge	Difficulty in finding and choosing suitable activity

Table 20 (continued)

Groups	Advantages	Disadvantages
Group 7	Providing motivation Providing learning with group work Facilitating learning Maintaining persistency about grasped knowledge Providing motivation Embodying mathematics Facilitating learning	Taking time Having students lack of subjects Inconvenience for frequently use Causing to fail to keep to course schedule Failing to meet responsibilities for teachers

According to preceding table (see Table 20), most of the groups indicated that providing motivation, maintaining persistency about grasped knowledge, daily life applications, allowing to group working were commonly identified as the advantages of using modeling activities in classrooms. Other expressed advantages were providing higher order cognitive gains, being effective in challenging subject matters, meaningful learning, making students gain different perspectives, and facilitating learning. Conversely, taking much time and causing to fail to keep to course schedule most frequently were expressed as disadvantages of using modeling. Some of the other mentioned advantages of using modeling in classroom were difficulty in finding and choosing suitable activity, difficulties in implementing in the crowded classrooms, difficulty in ensuring compliance with level of students, facing with operational errors, incomprehensibility of the activity, and inconvenience for frequently use.

CHAPTER 5

DISCUSSION, CONCLUSION, AND IMPLICATIONS

In this study, it is aimed to investigate prospective secondary mathematics teachers' thinking about knowledge about mathematical modeling and knowledge about the pedagogical issues with regard to the usage of mathematical modeling in the classroom setting throughout the implementation of the designed course. In this chapter, first, the findings related to pre-service teachers' existing conceptions about mathematical modeling and the nature of modeling activities with regard to the use of mathematical modeling in teaching were discussed by comparing with the current body of literature. Eventually, developments in pre-service mathematics teachers' conceptions about knowledge about the mathematical modeling and the pedagogical issues with regard to the use of modeling in classrooms were discussed. It is followed by the conclusions drawn, implications, limitations, and suggestions for future studies.

5.1 Developing Prospective Teachers' Conceptions of Mathematical Modeling and the Nature of Mathematical Modeling Activities

The results showed that prospective secondary mathematics teachers had very little knowledge about mathematical modeling and the nature of mathematical modeling activities at the beginning of the course. Prior to the course, almost all of them shared the conception that mathematical modeling is associated with the concrete manipulatives and visualization of abstract mathematical concepts. It can be stated that the knowledge of pre-service teachers enrolled in the current study about mathematical modeling was quite limited at the beginning. The same findings were expressed in several studies conducted in different countries (Abramovich, 2010; Borromeo Ferri & Blum, 2009; Maaβ & Gurlitt, 2011, Gould, 2013). For example, in the study of Abramovich (2010), most of the teachers believed that mathematical modeling involves concrete materials and manipulatives. In another research carried out by Borromeo Ferri and Blum (2009), it was demonstrated that pre-service teachers had very little knowledge about mathematical modeling. Similarly, in the study of Gould (2013), it was found that secondary mathematics teachers possess misconceptions about mathematical modeling such as believing that mathematical models are concrete materials, types of representations including graphs, scaled maps, or formula. The results of the current study imply that pre-service and in-service teachers have very limited knowledge about mathematical modeling. This may stem from the lack of courses on mathematical modeling in the teacher education programs (Lingefjärd, 2007). Although mathematical modeling has been widely underlined in school mathematics curricula, it has been also indicated that the sources and materials about mathematical modeling that can be used in the classrooms are not adequate (Blum & Niss, 1991; Burkhardt, 2006; Kaiser & Maaβ, 2007; Ikeda, 2007). The inadequacy of sources and materials that can be used in the classrooms by teachers or that can be used in teacher education programs have also been emphasized for our country (Erbaş et al., in press; Türker et al., 2010). Most of the pre-service and in-service teachers in Turkey are not aware of what mathematical modeling is, and they do not have any experience of solving modeling activities (Kayhan-Altay et al., 2014; Kertil, 2008).

Eventually, almost all of the prospective secondary mathematics teachers' conceptions about mathematical modeling evolved from "using concrete manipulatives and visualization" to "relating mathematics to real life situations". Additionally, pre-service teachers' conceptions about mathematical modeling deepened and they provided more comprehensive descriptions about mathematical modeling. For instance, prospective teachers started to indicate the relationship between mathematics and real life in the subsequent phases of the course. They also started to see mathematical modeling as a vehicle for teaching of mathematical topics and concepts meaningfully. These results are consistent with some studies in the literature (Chapman, 2007; Kuntze, 2011; Gould, 2013). For instance, Chapman (2007) reported that in-service teachers underlined the importance of real world association in mathematical modeling. In contrast to the findings of the current study and the study of Chapman (2007), Gould (2013) indicated that teachers had a belief that mathematical models and situations of mathematical modeling were unrealistic scenarios rather than real life situations.

The analysis of the data showed that prospective secondary mathematics teachers did not have adequate knowledge about mathematical modeling at the beginning of the course. In the progress of the course, they developed important ideas about mathematical modeling. At the beginning of the study, almost all of the prospective teachers described mathematical modeling as “using concrete manipulatives and visualizations” and most of them expressed that they never faced and solved mathematical modeling activities before taking this course. Nevertheless, during the course, they displayed appreciable developments in general knowledge about mathematical modeling. Prospective teachers emphasize that teachers need to have a strong mathematical content knowledge (Grossman, 1990; Marks, 1990; Shulman, 1986, 1987) in order to understand, assess, and give feedback to unexpected ideas that come from students during solution process of mathematical modeling activities. These ideas were verified by several researchers. For example, Doerr (2007) found that prospective teachers have serious misconceptions and drawbacks about mathematical content knowledge and stated that prospective teachers showed progress in improving their misconceptions and deficiencies about mathematical content knowledge by passing through mathematical modeling processes.

The findings of this study showed that prospective teachers indicated the significance of the knowledge about the nature of mathematical modeling activities for teachers who want to use those activities in their classrooms. Prospective teachers explained the importance of knowledge about the nature of modeling activities in a way that it is not possible to attain the general pedagogical goals if these activities were implemented like traditional word problems. In other words, the use of mathematical modeling in a classroom setting is strictly different from solving traditional word problems and prospective teachers underlined that teachers ought to use these activities without guiding students explicitly and giving the correct answer directly. Lesh and Doerr (2003a) stated that mathematical modeling had emerged as an alternative perspective to traditional teaching approaches with putting emphasis on the association between mathematics and real life (Haines & Coruch, 2007). The findings of the current study demonstrated that teachers’ suggestions for having strong knowledge about mathematical content knowledge in order to use mathematical modeling in classroom which also has been voiced by Blum and others (2002). These findings also in line with previous studies (Blum & Niss, 1991; Doerr, 2007; Lesh & Lehrer, 2003).

The findings also revealed that prospective teachers developed significant ideas about the nature of mathematical modeling activities in the process. Prospective teachers identified the general characterisitics of mathematical modeling activities intuitively throughout the implementation of modeling activities. For instance, almost all of the prospective teachers expressed the idea that mathematical modeling activities involve authentic real situations which is the common characteristic indicated by different researchers (e.g., Crouch & Haines, 2004; Lesh et al., 2000). Prospective teachers indicated that mathematical modeling activities ought to make feel the students for a need for solution to understanding of a mathematical concept. Therefore, they expressed that modeling activities should attract students' attention to the need for a solution or model so that they construct a structure as a solution or model for them that is the property indicated by Lesh et al. (2002) as the "model construction" principle. Other than these aforementioned properties underlined by prospective teachers, they indicated the followings as the general properties of modeling activities: open-ended, including more than one mathematical concept, having diverse solution methods and the cyclic structure of solution processes, and being generalizable and prototype. The properties for modeling activities expressed by prospective teachers were similar to the principles determined by Lesh et al. (2000) for developing *model-eliciting activities*. According to Lesh and his collegues (2000), a good *model-eliciting activity* should carry out the properties and principles that are the model construction principle, the reality principle, the self-assessment principle, the construct documentation principle, the sharebility and reusability principle, and the effective prototype principle. In the current study pre-service teachers mentioned about almost all of these principles with different terminologies. For example, the ideas coded as "reality" in modeling activities described by prospective teachers correspond to *the reality principle*. Prospective teachers also suggested that modeling activities should make students feel that there is a need for a solution or model which is the idea consistent with *the model construction principle*. These findings demonstrated that implemented modeling activities and classroom discussions throughout the course enabled prospective teachers to develop fundamental ideas about the nature of mathematical modeling activities. It can be argued from the findings that prospective teacher reached to the basic ideas commonly expressed in the literature about mathematical modeling and its usage in the teaching process. Living the experience of solving modeling activities for pre-service teachers as if they were students is critical here. They not only were provided with theoretical explanations, but also developed their ideas about mathematical modeling in practice. As indicated by many researchers (e.g., Doerr & Lesh, 2003; Schorr & Lesh, 2003), pre-service teachers should be provided learning environments in which they can learn by doing and applying various modeling activities in order to foster their knowledge about mathematical modeling and its usage in the teaching of mathematics.

Prospective teachers indicated that knowledge about the nature of mathematical modeling activities is important for teachers who want to implement these activities in their classrooms. As a reason for this, they noted that the use of mathematical modeling activities should not be handled as if they were traditional word problems. In other words, when modeling activities were implemented as if they were classical word problem solving activities, prospective teachers pointed that the benefits expected from modeling activities such as feeling a need for a concept and giving opportunity to form the concept intuitively could not be actualized. At this point, prospective teachers expressed that if teachers had the knowledge about the nature of mathematical modeling activities, this would let them have the knowledge about distinctions between using mathematical modeling activities and the application of traditional word problems. These findings were consistent with the previous studies that mathematical modeling activities have been seen as an opportunity for professional development of teachers (Doerr & Lesh, 2003; Koellner-Clark & Lesh, 2003; Schorr & Lesh, 2003).

In addition, prospective teachers developed ideas about distinctions of mathematical modeling activities from traditional word problems. They indicated ideas about the modeling activities such as unclearness of solution process, including more than one mathematical concept, having multiple ways of solutions, being suitable for group works, involving a real life situation, and taking more time. These findings indicate that prospective teachers developed ideas about the nature of mathematical modeling activities. The related literature indicate similar differences between modeling activities and traditional word problems (e.g. Kayhan-Altay et al., 2014; Lesh & Doerr, 2003a; Lesh & Zawojewski, 2007; Türker et al., 2010; Verschaffel et al., 2002; Zawojewski & Lesh, 2003). For instance, Kayhan-Altay and others (2014) and Türker and her collegues (2010) found that prospective teachers indicated that MEAs did not resemble traditional word problems because they required students to think more. Lesh and Doerr (2003) indicated that students were needed to be challenged with complicated problems throughout the teaching process. Additionally, the same researchers expressed that traditional word problems were far from satisfying that need. It was declared that the nature of mathematical modeling activities, which were far from artificiality, are more complicated and authentic, and these activities ought to be used in the teaching process. As pointed out by Lesh and Doerr (2003), prospective teachers also stated this property of modeling activities as having multiple solutions and unclearness of the solution process.

5.2 Developing Prospective Teachers' Conceptions of Pedagogy of Modeling

In this section, the results related to development of prospective teachers' conceptions about pedagogy of modeling were discussed in the light of previous research. The results were examined according to what kind of qualifications and knowledge teachers need to have in order to conduct modeling activities effectively in the classroom settings.

The data of the current study showed that, before the implementation of the course, prospective teachers did not have any conception about the use of mathematical modeling in teaching mathematics. In the progress of the course, it was observed that prospective teachers gradually developed important ideas about the use of mathematical modeling in the classroom setting as they experienced different mathematical modeling activities. The results suggested that majority of prospective teachers developed positive views about the use of mathematical modeling and they expressed that they would like to use mathematical modeling in the teaching of mathematics when they become in-service teachers. According to Pehkonen and Törner (1996), teachers' conceptions about teaching and learning of mathematics influence their practice. This implies that developing positive ideas about the use of mathematical modeling in the classroom setting can be interpreted as an important advancement for the prospective teachers. Several researchers reported similar findings about the views of teachers about the use of mathematical modeling in the teaching and learning of mathematics (Blomhøj & Kjeldsen, 2006; Burkhardt, 2006; Eraslan, 2011; Kaiser & Schwarz, 2006; Kaiser & Maaβ, 2007; Kuntze et al., 2013; Maaβ & Gurlitt, 2009, 2011; Siller et al., 2011; Yu & Chang, 2011). For instance, in the study of Yu and Chang (2011), participants expressed their positive perceptions about using mathematical modeling activities in the classroom and planning and carrying out these activities after the implementation. Kuntze and his collegues (2013) showed that both pre-service and in-service teachers had negative perceptions about their modeling-specific PCK. Maaβ and Gurlitt (2009) showed that teachers have very little knowledge about mathematical modeling, especially about modeling cycles, at the very beginning of their study, as a part of a much larger study, i.e. LEMA project. They reported that teachers developed positive beliefs about mathematical modeling in the period of their research.

The results of the current study demonstrated that prospective teachers also developed conceptions about the issues of why to use, where to use, how to use, and when to use mathematical modeling activities in the classroom setting. The conceptions indicated by pre-service teachers showed some differences from person to person. It was obviously observed that prospective teachers formed broad and profound views on the use of mathematical modeling in classroom. Besides these findings are parallel to previous studies (e.g., García et al., 2010), for example, some participants indicated their preferences for using mathematical modeling at the beginning of the lesson in order to attract students' attention to the topic. On the other hand, some of the prospective teachers told that they wanted to use these activities after the lesson aiming at assessing and measuring students' levels of understanding. The timing of using modeling activities in the classroom setting with the related goals was emphasized by Lesh and others (2007). According to Lesh and his friends, using mathematical modeling activities in the beginning of the topic or beginning the topic with these activities provides students an opportunity to construct their own understanding of the mathematical concepts. When modeling activities are used at the end of the topic, it serves the goal of applying already taught concepts in the topic (Haines & Crouch, 2007; Lesh et al., 2007). In the current study, pre-service teachers also indicated similar ideas about the timing of modeling activities. The results of current study are in line with Lesh and others' (2007) explanations and theoretical expressions and others (Yoon et al., 2010). Moreover, prospective teachers expressed their opinions on using mathematical modeling as supporting meaningful mathematics teaching, observing students' thinking processes, assessing students' performance, reinforcing mathematical concepts, and relating mathematical concepts with real life. These ideas were labeled as cognitive goals of using mathematical modeling and these ideas were frequently declared by several researchers about the requirements of using mathematical modeling in the teaching process within the fundamental arguments in the literature (Doerr & Lesh, 2011; Lesh & Harel, 2003; Lesh & Lehrer, 2003).

Although many prospective teachers expressed their positive considerations and conceptions about the use of mathematical modeling activities when they become in-service, they mentioned about difficulties in using mathematical modeling in the teaching and learning of mathematics. Some of the difficulties indicated by pre-service teachers were using mathematical modeling activities takes much time; having difficulties in managing the group works in crowded classrooms; and causing not to get the current school mathematics curriculum done in time for teachers. The difficulties of using modeling activities in the teaching indicated by prospective teachers in the current study were also observed in the previous studies (Blomhøj & Kjeldsen, 2006; Blum & Niss, 1991; Burkhardt, 2006; Gould, 2013; Kayhan-Altay et al., 2014; Schmidt, 2011; Yu & Chang, 2011). For instance, Burkhardt (2006) mentioned that several obstacles, some of which were in common with the findings of the current study, might deter teachers from using mathematical modeling activities in their classrooms. Blum and Niss (1991) also discussed the fundamental difficulties in using mathematical modeling in the classroom settings such as not feeling comfortable with mathematical modeling activities, having not enough time for these activities due to curriculum, and difficulties related to assessing students with modeling activities. Kaiser and Maaβ (2007) also demonstrated in their study that teachers' beliefs emerged as the main obstacles that discourage teachers from using mathematical modeling in classroom settings. The results involving pre-service teachers' ideas about the difficulties of using mathematical modeling in the teaching process resonate with the difficulties indicated by many studies (e.g., Blum & Niss, 1991; Kaiser & Maaβ, 2007). This shows that pre-service and in-service teachers share common perceptions about the difficulties of using mathematical modeling in the classroom all around the world.

5.2.1 The Importance of Pedagogical Knowledge of Modeling

Data analysis demonstrated that prospective teachers developed significant ideas about the pedagogy of modeling throughout the implementation of modeling course. It should be noted here that the teacher role demonstrated by the instructor while implementing the modeling activities provided valuable knowledge for pre-service teachers about the pedagogy of mathematical modeling. They frequently supported their arguments by providing examples from the applications of the instructor. Most of the prospective teachers indicated that the use of mathematical modeling activities would be beneficial for students and for teachers and they developed important ideas about changing roles of teachers in the process of implementation of modeling activities. Prospective teachers emphasized that teachers were required to guide students with probing questions that examine students' way of thinking without giving the correct answer during the implementation of mathematical modeling activities in the classroom setting. This finding of the study is highly correlated with the study of Kuntze and others (2013) in terms of modeling-specific PCK of teachers. Besides, prospective teachers expressed that teachers need to make students realize their mistakes and provide students to understand the underlying mathematical idea on their own via these probing questions. The results showed that prospective teachers provided examples about the role of teacher during the implementation of modeling activities from the classroom implementations of their instructor of the course in order to support their ideas. This situation shows that knowledge related to the pedagogy of modeling in pre-service teachers can be fostered by directly applying the necessary knowledge in the courses and this might lead them to gain the pedagogical knowledge by imitating from the instructor. This finding also demonstrated that prospective teachers examined and appreciated the teacher role played by the instructor while implementing the modeling activities. That is, prospective teachers took the role of instructor of the implemented course as a model for them. The point that prospective teachers underlined was indicated by several researchers as one of the important qualifications that teachers should have (Antonius et al., 2007; Barbosa, 2001; Doerr, 2006, 2007; Lingefjärd & Meier, 2010). For instance, Antonius and others (2007) put emphasis on the teachers' guidance during the modeling process in such a way that there is a need for asking strategic probing questions. Doerr (2007) identified the four characteristics of pedagogical knowledge about mathematical modeling for teachers as "(1) to be able to listen for anticipated ambiguities, (2) to offer useful representations of student ideas, (3) to hear unexpected approaches, and (4) to support students in making connections to other representations" (p. 77). Additionally, prospective teachers mentioned that teachers should know about the nature of mathematical modeling problems and have the ability to select appropriate modeling activities according to the topic and by considering the needs of students. Various researchers like Doerr (2007) expressed similar findings that support the importance of modeling-specific PCK of teachers in the implementation of modeling activities (Kuntze, 2011; Kuntze et al., 2013; Wake, 2011).

Prospective teachers' thinking about the pedagogical knowledge of mathematical modeling was coded according to their descriptions. The results suggested that prospective teachers offered several characterizations for teachers' pedagogical knowledge of modeling. According to the prospective teachers, the teachers who would use mathematical modeling should know where and how to intervene the process of modeling. Teachers should provoke students to think by asking leading and probing questions in order to find their own solution way or their own mistakes if they did. Prospective teachers suggested that teachers should know how to guide the students in the modeling process without directing to correct solution. That is, teachers should be aware of their roles in the modeling process and try to be in the guide position. Prospective teachers offered that teachers ought to have the knowledge of students' way of thinking so that teachers can easily detect the possible mistakes made by the students and understand how students could think in different situations. Lastly, prospective teachers recommended that teachers should use the modeling activities for an intended aim such as getting students to comprehend any mathematical concepts or reinforcing some mathematical concepts that were learned previously.

The above characterizations that prospective teachers made were in line with previous studies (Antonius et al., 2007; Aydogan-Yenmez, 2012; Burkhardt, 2006; Doerr, 2007; Lingefjärd & Meier, 2010; Stillman, 2010). These findings show that prospective teachers developed fruitful ideas about what pedagogical knowledge of modeling teachers need to have in order to carry out the modeling process effectively and successfully. The findings also demonstrate that prospective teachers could develop significant ideas about pedagogical knowledge of teachers related to modeling if appropriate conditions are provided for them during their undergraduate education which is in line with "intensified professional development in the domain of modelling-specific PCK" is required at university level (Kuntze et al., 2013, p. 324). In the light of the results of the current study, it can be argued that prospective teachers' observations during the implementation period would be valuable for their future implementations when they teach in-service courses including mathematical modeling. Since prospective teachers came from high schools and they observed their teachers many years in terms of "how to be a teacher" and brought mathematical understandings with them to teacher education programs (Ball, 1990a, 1990b), it is quite difficult to change their core beliefs about teaching of mathematics and adapt correspondingly new mathematics teaching methods. Nevertheless, participations of prospective teachers in modeling courses and experiencing them can lead them to realize the importance of mathematical modeling more effectively. From the results, it can be interpreted as almost all of the prospective teachers developed positive ideas about the use of mathematical modeling and how a teacher should behave during the modeling process by participating the modeling activities actively and experiencing the all phases of modeling process. These ideas were voiced by several researchers many times in their studies (Doerr & Lesh, 2003; Doerr & Lesh, 2011; Lesh & Lehrer, 2003).

5.2.2 The Importance of Classroom Management and Group Work during Modeling Process

The results obtained from the analysis showed that prospective teachers developed ideas about the characteristics of the teachers who wanted to use modeling activities and recommended various suggestions for teachers to conduct modeling activities effectively and successfully. Most of the prospective teachers indicated that they would choose group work as a method for implementation of their designed plan. It was observed that almost all of the prospective teachers who declared to use group work moved in their implementation experience in the same way. In order to carry out the modeling process effectively and successfully, prospective teachers emphasized that teachers ought to prepare a well-organized implementation plan and they should obey the plan during the implementation. The findings showed that prospective teachers prepared implementation plans before the application and tried to implement their plans, but they expressed that they encountered difficulties when applying the plan because they did not allocate enough time to parts of the process. This situation shows that time arrangement emerges as an important issue in implementation of mathematical modeling. Similar issues were noted in the studies of Burkhardt (2006) and Doerr (2006). According to Burkhardt (2006), qualified teachers' characteristics for implementing mathematical modeling were illustrated as managing the discussions emerged in the classroom while implementing the modeling activities by giving support and help, allocating enough time for students to solve the problem situation, encouraging them to use their own solution methods, guiding and giving support students in strategic way such that they do not interfere students solution process. Like Burkhardt's (2006) point of views, Stillman (2010) described the conditions for tasks, students, and teachers in order to carry out modeling activities effectively in classroom settings.

About classroom management during the modeling process, prospective teachers underlined that teachers ought to be a good organizer as Lingefjärd and Meier (2010) expressed the same situation such that teachers were the manager of modeling process. Prospective teachers asserted that the conditions emerged during the group work sessions were not appropriate for traditional discipline approach, therefore teachers could not get used to these conditions easily. The chaotic conditions mentioned by prospective teachers supported the findings indicated in the related literature that teachers had deficiency in managing classroom and felt this situation as a threat (Blum & Niss, 1991; Burkhardt, 2006). Prospective teachers pointed out that students engaged in the mathematical modeling activities actively so that teachers could feel this situation more complicated in comparison to traditional teaching methods. Lesh and Doerr (2003) underlined this finding that students participated mathematical modeling process actively compared to traditional teaching methods in which teachers played transmitter role in teaching and students were receivers. Prospective teachers indicated that they learned from each other via active participation of modeling process.

Throughout the implementation process of modeling course, prospective teachers developed significant ideas about the classroom management during the mathematical modeling activities such as trying to understand the ways of students thinking by walking around the groups in the classroom, guiding students by asking appropriate questions, and managing classroom and classroom activities nicely during the group work. These findings were supported by several studies in relation with the role of teachers in mathematical modeling process and classroom management (e.g., Blum & Leiß, 2007; Doerr, 2007; Doerr & Lesh, 2003; Lingefjärd & Meier, 2010; Schorr & Lesh, 2003). From this perspective, the implemented mathematical modeling course enabled prospective teachers to develop fundamental ideas about how to manage classroom during the modeling process.

At the end of this study, prospective teachers formed a view about how mathematical modeling activities should be implemented in the classroom with students. Most of them stated that they prefer getting students collaborate and work in groups when they implement mathematical modeling activities in the classroom (see section 4.2.2.3, Table 18). The necessity of idea that mathematical modeling activities should be implemented in the form of group work by prospective teachers as a result of efficiency of their group work sessions and from the emphasis on the fact that they learned a lot from each other in the process. Similarly, many researchers in the related literature underlined that mathematical modeling activities ought to be implemented as group work (Aydoğan-Yenmez, 2012; Borromeo Ferri & Blum, 2009; Ikeda & Stephens, 2001; Maaß & Gurlitt, 2011; Zawojewski et al., 2003). Prospective teachers commented that there were several points before establishing the groups and the number of members in each group ought to be between three and five based on classroom conditions. This situation can be interpreted as prospective teachers could be influenced from their course instructor's applications in forming groups and they extended their ideas about the number of members in each group by adding classroom conditions such as the number of students in the classroom.

5.2.3 The Role of Microteaching Experiences in Modeling Courses for Prospective Teachers

When the prospective teachers' views about their implementation experience in the classroom were considered, they indicated that they had a chance to carry out a mathematical modeling activity such that they experienced possible difficulties and problems during the implementation experience. The results showed that this implementation experience provided prospective teachers to have an experience even if it was slight. As a common problem emerged in several countries, many researchers indicated the fact that teachers hesitated to and even preferred not to use mathematical modeling and modeling activities (Blum & Niss, 1991; Maaβ & Gurlitt, 2011; Niss et al., 2007). One of the major reasons would be not providing teachers with a chance to have experience of usage of mathematical modeling activities in the classroom environment when they were prospective teachers at teacher preparation programs (Kuntze et al., 2013). However, when it was looked through the existing studies about designing and developing mathematical modeling courses and contents of these courses, it was acknowledged that there existed no implementation experiences about the use of mathematical modeling activities for prospective teachers. Making prospective teachers have experience at implementation of mathematical modeling in their classrooms was emphasized by researchers like Niss and others (2007) and Blomhøj and Kjeldsen (2006).

As indicated in the paragraph above, the emphases on the notion that prospective teachers need to have experience of using mathematical modeling activities in the classrooms had taken into consideration during formation of implemented mathematical modeling course within the larger project and in the current research involved in that project. These implementation experiences made prospective teachers gain valuable ideas about possible problems and conditions while applying these activities in the classroom. Prospective teachers stated that there had to be more implementation experiences in order for them conceptualize knowledge about the use of mathematical modeling and acquiring the knowledge as skills. Nevertheless, the results of the current study showed that this research could be one of the primary efforts that involve microteaching for making prospective teachers gain experience about the use of mathematical modeling activities in classroom settings stressed by several researchers (Blum, 2002; Doerr, 2006, 2007; Kuntze et al., 2013). Prospective teachers declared that they had an opportunity to learn how to conduct a modeling activity in a classroom by doing via implementation experience. From this perspective, implementation experience matters for prospective teachers in developing, designing and carrying out a mathematical modeling by themselves and this provides an opportunity to prepare themselves when they become in-service. Besides, implementation experience help prospective teachers conceptualize the role of the teacher and classroom management during the modeling process.

5.2.4 The Role of the Course on Developing Teacher Conceptions' about Mathematical Modeling

In this study, almost all of the prospective teachers indicated that mathematical modeling course was important for them and they developed significant ideas about the connections between mathematics and real life in general. A recent study carried out by Kuntze and others (2013) showed the importance of the focus of the current study. According to Kuntze and his friends (2013), "intensified professional development in the domain of modelling-specific PCK appears to be needed, both in initial teacher education at the university level …, even in the case of existing modelling courses at universities" (p. 324). In the study of Verschaffel and others (1997), it was found that students could not make connections between reality and mathematics. The results of the current study demonstrated that implemented mathematical course changed prospective teachers' conceptions about teaching of mathematics. That is, prospective teachers thought that teaching of mathematics via modeling enables students to relate mathematics with real life rather than thinking of mathematics as abstract. Barbosa (2001) found similar results in her study and indicated that the participants of the study stated their satisfaction with the course concerning with its content and relating mathematics with real life. However, although there has been some efforts to design and implement mathematical modeling course in the related literature (e.g., Blomhøj & Kjeldsen, 2006; Borromeo Ferri & Blum, 2009; Kaiser & Schwarz, 2006; Maaß & Gurlitt, 2009; Lingefjärd, 2006), the designed courses on mathematical modeling showed differences with respect to aim, content, and target population (for prospective or inservice teachers). While some of these studies aimed to develop prospective teachers' modeling competencies in order to teach mathematical modeling in schools (e.g., Borromeo Ferri & Blum, 2009; Maaß & Gurlitt, 2009), some of them intended to design professional development course on mathematical course in order for advancing prospective teachers' understanding of mathematics in regard to mathematics (Kaiser & Schwarz, 2006). The content of the modeling course developed and implemented within the large project and current study as a part of this large project involves both similar and distinct components compared to the designed courses in previous modeling courses. Similar components are involving modeling tasks, using technology during modeling process, and group working. Apart from previous designed modeling courses, the implemented modeling course within the current study involves components such as analyzing studies of students' ways of thinking, developing and implementing modeling tasks that provide prospective teachers gain experience about the implementation of modeling activities in classroom. Prospective teachers indicated their impressions about the implementation experience. In implementation experience, prospective teachers faced with unexpected ways of solution and tried to understand these solutions. Moreover, prospective teachers had difficulties during the implementation stemming from their own developed activities.

Since prospective teachers developed positive conceptions about the "Mathematical Modeling for Prospective Teachers" course, the issue of offered courses in elementary and secondary mathematics teacher education should be discussed in order to revise the current situation. When it is looked through mathematics teacher education programs and their curricula in Turkey, it can be observed that there have been separate curricula framework for prospective elementary and secondary mathematics teachers that include compulsory and elective courses and these curricula were determined by the Turkish Higher Education Council (YÖK) until Fall semester of 2014-2015 academic year. The curricula of mathematics teacher education programs were similar to the programs in European countries (Binbaşıoğlu, 1995; cited in Çakıroğlu & Çakıroğlu, 2003). There has been a debate about the types and relevancy of courses offered in these programs and this issue appeared as the problems of teacher preparation programs in Turkey (Çakıroğlu & Çakıroğlu, 2003). According to several researchers, there existed inconsistencies between the courses that prospective teachers took in the undergraduate level and the implementations they encountered in the real classrooms (Bulut, Demircioğlu, & Yildirim, 1995; cited in Çakıroğlu & Çakıroğlu, 2003). Furthermore, it is indicated that there exist a deficiency in collaboration and contact between the staff in faculty of education and in-service teachers (Binbaşıoğlu, 1995; cited in Çakıroğlu & Çakıroğlu, 2003). When it is looked at the curriculum of mathematics teacher education programs, there are very limited numbers of practice courses that prospective teachers gain experience of teaching in elementary or secondary schools. Although collaboration between faculty of education and schools of MEB still continues, the effectiveness of the courses that require prospective teachers to practice in schools is being questioned due to various reasons (YÖK, 2007). Starting from this point, prospective teachers need to have more experience about mathematical modeling in order to develop their knowledge and skills related to pedagogy of modeling so that these experiences may influence their instructional practices in a positive way. Educational authorities and policy makers should take this point into consideration.

The findings of the current study showed that distinct components of the designed and implemented modeling course focused on pedagogy of mathematical modeling that was not much involved in previous modeling courses mentioned in the literature. Therefore, the findings of the study is valuable for the further research on the development of prospective teachers' pedagogical knowledge of mathematical modeling.

5.3 Conclusions

The results of the current study revealed that some of the findings of the study are in line with previous research about the pre-service teacher education related to mathematical modeling and the use of mathematical modeling activities in classroom settings. Conceptual framework of the current study was MMP on teacher development (Doerr & Lesh, 2003). This approach suggested a perspective for the development of teachers (Doerr & Lesh, 2003; Doerr & Lesh, 2011; English, 2003;

Lesh & Lehrer, 2003) that involves designing teaching and learning settings according to MMP, implementation of mathematical modeling activities (e.g., MEAs) in modeling process that is similar to student development. Nevertheless, apart from the student development, teachers centered on "clarifying and or elaborating their own ideas including mathematical content, pedagogy, and knowledge of student thinking" (Koellner-Clark & Lesh, 2003, p. 165). This study filled some of the gaps in the mathematical modeling literature dealing with designing and conducting a modeling course for prospective teachers at undergraduate level and how implementation of modeling course influence prospective teachers' thinking about mathematical modeling and its use in the teaching of mathematics in their future classrooms. Although there were not adequate modeling courses in teacher preparation programs (Lingefjärd, 2007), it was evident from the related literature that there existed many research on designing and developing modeling courses for both prospective teachers (Barbosa, 2001; Borromeo Ferri & Blum, 2009; Kaiser & Schwarz, 2006; Maaß & Gurlitt, 2009) and in-sevice teachers (Blomhøj & Kjeldsen, 2006). Nevertheless, teaching of mathematical modeling and improving modeling competencies of teachers was in the foreground in these studies. As distinct from these studies, development of prospective teachers' pedagogical knowledge of mathematical modeling included in the implemented modeling course within the current study as a part of much larger project. The pedagogical side of mathematical modeling and prospective teachers' thinking about that was the scope of this study.

This study presents how a designed and implemented mathematical modeling course influences prospective teachers' thinking about mathematical modeling and knowledge about its pedagogical aspects for their future classroom applications and what ideas prospective teachers developed throughout the implementation period. The results of the current study shed light on the points that had been discussed by mathematics education community about mathematical modeling and its use in the teaching of mathematics, especially the change in prospective teachers' thinking about mathematical modeling and pedagogical knowledge of modeling after taking mathematical modeling course. The overall conclusions for the results of the research demonstrated that

- Prospective teachers' conceptions about mathematical modeling changed from "using concrete manipulatives" to "relating mathematics with real live" that involved giving in-depth definitions of mathematical modeling.

- Prospective teachers developed ideas about the mathematical modeling and the nature of mathematical modeling activities about which they did not have adequate knowledge before taking the course.

- Prospective teachers developed positive ideas about the use of mathematical modeling activities in the classroom settings although they did not have any idea or conception about it before they had not taken the course.

- Prospective teachers developed significant ideas about the knowledge that teachers need to have in order to carry out mathematical modeling activities effectively and successfully in their future classrooms under the themes content knowledge, knowledge about using mathematical modeling, and knowledge of modeling and nature of modeling activities.

- Prospective teachers noted that they had an opportunity to learn the role of teacher in mathematical modeling process with the implementation experience and developed ideas about practice. They proposed suggestions to teachers who want to use mathematical modeling activities in their classrooms.

In conclusion, the implemented "Mathematical Modeling for Prospective Teachers" course enabled prospective secondary mathematics teachers to change their thinking about mathematical modeling and the use of mathematical modeling in the teaching of mathematics. The course implementation influenced prospective teachers' thinking about the use of mathematical modeling activities in classrooms positively and helped them develop significant ideas what teachers should know in order to conduct a modeling activity in a classroom setting effectively and successfully.

The conclusions mentioned above demonstrated that a mathematical modeling course for prospective teachers enable them to develop important ideas about mathematical modeling, about its use in the teaching and learning of mathematics, about what knowledge teachers need in order to carry out (Doerr, 2007) mathematical modeling activities effectively and successfully before they work in-service, especially pedagogy of modeling which were underlined by several authors (Antonius et al., 2007; Blum, 2002; Doerr, 2006, 2007).

5.4 Suggestions, Limitations, and Implications

In this part of the last chapter, suggestions for the audience of the current study was provided. Limitations of the study and implications for the further research also mentioned under this section.

5.4.1 Suggestions

Suggestions for teachers who want to use mathematical modeling

The findings obtained from the current study suggested that the developed and implemented "Mathematical Modeling for Prospective Teachers" course could impact upon prospective teachers' thinking such that they developed significant ideas about mathematical modeling and its use in the classroom environments. The findings of the study suggested that teachers who want to use mathematical modeling should have various kind of knowledge such as content knowledge, pedagogical knowledge of modeling, knowledge of modeling, and knowledge of the nature of mathematical modeling activities. In addition, the findings offer that teachers need to have some qualifications for the implementation of mathematical modeling in their classrooms. These are having knowledge about the students' way of thinking by exploring and understanding previous students' works, having knowledge about classroom management during the modeling process, and having implementation experience of mathematical modeling activities. In order to use mathematical modeling mathematical modeling in their classrooms effectively and successfully, suggestions for teachers was illustrated according to findings of the current study as follows:

The teacher who will carry out mathematical modeling in his or her classroom

- should develop mathematical modeling according to six principles that were proposed by Lesh and his collegues (2000) and/or select appropriate modeling activity that was found on the grade level of the students according to aim, place, and method of use.
- should design an implementation plan for determined modeling activity in order to carry out the modeling process systematically and successfully,
- should solve the mathematical modeling activities that he wants to use in the classroom before the implementing them and reveal all-possible solutions so

that teachers decrease the chance of unexpected solutions and have a confidence about the implementation.

- should be aware of distinct solutions by walking around the groups during the implementation of modeling activities, by this way teachers give chances to the groups to present distinct solutions in the presentation sessions.

- should make students discuss distinct solution methods of groups in terms of the way of solution and conclude the implementation with general expressions that summarize the all over the process.

Even though prospective teachers developed significant ideas about the use mathematical modeling in the teaching of mathematics within the implemented course, it is should be noted that prospective teachers need to put these ideas and knowledge acquired from the course into practice in order to build them as skills. Therefore, prospective teachers' acquired theoretical knowledge and developed ideas can be put into practice by accompanying with the undergraduate courses like "School Experience" and "Teaching Experience". In this way, prospective teachers can gain more experience about the use of mathematical modeling in real classroom environment and they had a chance to observe mathematical modeling process by engaging in it with real high school students. Besides, this will provide more contributions to prospective teachers' knowledge about the use of mathematical modeling in the teaching of mathematics and help them to internalize that knowledge in order for using mathematical modeling in their future classrooms.

Suggestions for instructors who will carry out modeling course

Although instructors of mathematical modeling course were out of scope of this study, the suggestions for the teachers are also valid for instructors of modeling course in some respects with extra reserved requirements pertaining to the modeling course rather than any mathematics course in high school level. Distinct from the suggestions for teachers, instructors can follow content of the modeling course that was designed and implemented within the much larger project supported by TUBITAK under the grant number 110K250 which involved the current study as a part. The instructors who want to carry out the modeling course should take the aims and objectives of the course (see Table 5) as a whole and adhere to these aims and objectives. The role of the instructor played during the implementation of the course is very significant for prospective teachers because the prospective teachers take role of instructor as a model for them for their future implementations. Therefore, instructors should be aware of their role during the modeling course and try to do their best in this regards.

5.4.2 Limitations of the Study

This study investigated developments in prospective teachers’ thinking about knowledge about mathematical modeling and pedagogical issues during the implementation of a course. It should be indicated that mathematical modeling in the context of the current study is a broad topic involving the technical competencies and pedagogical issues. Because it is impossible to cover all these issues detailly in only such a course, this can be accepted as the main limitation of the study. Most of the prospective teachers who participated to the study heard about mathematical modeling first time. Therefore, although the main aim of the course was pedagogical issues about mathematical modeling an important portion of the course was devoted for the basic technical issues. Prospective teachers should be taught about the basic issues of mathematical modeling such as modeling processes and phases, modeling abilities, and the nature of modeling tasks in previous courses. Then the courses focusing on more specific issues as the pedagogical issues on mathematical modeling can be more effective.

Secondly, the developments in prospective teachers’ thinking about knowledge about mathematical modeling and its classroom implementation were mostly obtained from their reflections and thoughts. This can be seen as another limitation of the study. They lived the experience of implementing a modeling activitiy only one time for a limited period in the course. Prospective teachers’ thinking about the pedagogical issues were formed mostly what they observed from the practice of the instructor. Therefore, prospective teachers’ practical knowledge about the pedagogical issues of mathematical modeling should be considered in detail.

Lastly, although the relationship between mathematical modeling and technology was emphasized and expressed as the use of technological tools during the modeling process, the time that was allocated to introduce and teach the technological tools used in the modeling process was not adequate. This might prevent students from using technological tools effectively during the modeling processes. This can be another limitation of the study.

5.4.3 Implications for Further Research

There are several implications for future research in this study. After the conclusions of this study, some questions appeared to be investigated in the further studies. In this study, the change and development in prospective teachers' thinking about the knowledge about mathematical modeling and its use in the classroom setting were examined. However, the answer for the questions "How do prospective teachers acquire and develop knowledge about mathematical modeling and about its use in the classroom?" and "How do prospective secondary mathematics teachers' pedagogical knowledge of mathematical modeling evolve throughout the implementation of designed course?" is still scarce. Because this study investigated the development in the prospective teachers' thinking about mathematical modeling and pedagogical knowledge of mathematical knowledge throughout a mathematical modeling course. That is to say, the answers of the above questions are not covered within this study. In order to mention about the development of prospective teachers' knowledge about mathematical modeling and its use in the classroom, there is a need for an extended research that focus on knowledge base of prospective teachers and their works that they show their knowledge on. These issues can attract other researchers' attention for further studies.

As it was mentioned in the section 5.4.1, even though prospective teachers developed significant ideas and opinions about mathematical modeling and its use in the classroom, prospective teachers did not have more chance to make their gains from this course into practice in real classrooms. In the future studies, it can be investigated that how prospective teachers implement modeling activities in real classrooms after gaining theoretical knowledge about mathematical modeling and its use in the class. Since several researchers emphasized the need for more research about the pedagogy of mathematical modeling (e.g., Blum & Niss, 1991; Blum, 2002; Doerr, 2006, 2007), future research can be on the investigation of prospective teachers' knowledge about the use of mathematical modeling in classroom environment accompanying with teacher experience courses. In addition, there is also a need for a longitudinal research in order to observe the relationship between prospective teachers' acquired knowledge from the modeling course and the knowledge they put into practice when they become in-service teachers. The results of this research would reveal the fact that how much teachers internalize and make into practice the knowledge that they gain from the modeling course and reflect in their school mathematics when they become in-service.

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APPENDIX A

MATHEMATICAL MODELING COURSE PLAN (PILOT STUDY)

Table 21 A plan of mathematical modeling course for pilot study

Hafta	Tarih	Konu ve Temalar	Ödev
1	29 Eylül 2011	• Tanışma • Dersin amacı, kapsamı ve süreci hakkında bilgilendirme	-
2	6 Ekim, 2011	• "Yaz İşi" başlıklı etkinlik uygulaması • Mini atölye çalışması: MS Excel elektronik tablo yazılımı	Etkinlik sonrası düşünce raporu
3	13 Ekim 2011	• Modelleme ve Teknoloji Kullanımı: ClassPad mini atölye çalışması • Modelleme soruları ile ilgili ön değerlendirme ve modelleme sorularının doğası üzerine tartışma (yaz işi ve üç modelleme etkinliği incelenecek)	
4	20 Ekim 2011	• "Dönme Dolap" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
5	27 Ekim, 2011	• "Caddede Park Yeri" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
6	3 Kasım 2011	• "Caddede Park Yeri" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri çalışması	
7	17 Kasım 2011	• "Su Deposu" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
8	24 Kasım, 2011	• "Gelecek Yüzyılda Türkiye" başlıklı etkinlik uygulaması • "Gelecek Yüzyılda Türkiye" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri çalışması	Etkinlik sonrası düşünce raporu
9	1 Aralık 2011	• "Acile Gelen Yüksek Tansiyon Hastası" başlıklı etkinlik uygulaması • Modelleme etkinliklerinin doğası ve modelleme süreci ile ilgili sunum/tartışma	Etkinlik sonrası düşünce raporu
10	8 Aralık 2011	• "Lunapark Treni" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu

Table 21 (continued)

Hafta	Tarih	Konu ve Temalar	Ödev
11	15 Aralık 2011	• "Doğa Yürüyüşü Parkuru" başlıklı etkinlik uygulaması • Modelleme sürecinde grup çalışması üzerine tartışma ve teorik sunum	Etkinlik sonrası düşünce raporu Proje taslaklarının teslimi
12	22 Aralık 2011	• Modelleme sürecinde öğretmenin rolü ve soru sorma şekilleri üzerine sunum/tartışma • "Dönme Dolap" etkinliği için lise düzeyinde bir sınıf Uygulaması Planı hazırlama	
13	29 Aralık 2011	• Gruplarca tasarlanan modelleme etkinliklerinin sınıf uygulamaları (mikro-öğretim)	
14	6 Ocak 2012	• Gruplarca tasarlanan modelleme etkinliklerinin sınıf uygulamaları (mikro-öğretim)	-
15	13 Ocak 2012	• Genel Değerlendirme	Proje raporlarının teslimi (18 Ocak)

APPENDIX B

MATHEMATICAL MODELING COURSE PLAN (ORIGINAL STUDY)

Table 22 A plan of mathematical modeling course for original study

Haftalar	Tarih	Konu	Ödev
1	10.02. 2012	• Tanışma • Dersin amacı, kapsamı ve süreci hakkında bilgilendirme	-
2	17.02.2012	• "Yaz İşi" başlıklı etkinlik uygulaması • Mini atölye çalışması: MS Excel elektronik tablo yazılımı	Etkinlik sonrası düşünce raporu
3	24.02.2012	• "Dönme Dolap" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
4	02.03.2012	• "Caddede Park Yeri" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
5	09.03.2012	• "Caddede Park Yeri" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri üzerine tartışma • Modelleme soruları ile ilgili ön değerlendirme • Modelleme sorularının doğası üzerine sunum/tartışma	Öğrenci düşünme şekilleri değerlendirme raporu
6	16.03.2012	• "Zıplayan Top" başlıklı etkinlik uygulaması • "Etkili grup çalışması" ile ilgili tartışma ve sunum	Etkinlik sonrası düşünce raporu
7	23.03.2012	• "Zıplayan Top" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri çalışması • "Modelleme Ekinliklerinde Öğretmenin Rolü ve Soru Sorma/Yönlendirme Şekilleri" üzerine sunum/tartışma	Öğrenci düşünme şekilleri değerlendirme raporu
8	30.03.2012	• "Lunapark Treni" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu
9	06.04.2012	• "Lunapark Treni" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri üzerine tartışma • Modelleme etkinliklerinin doğası ve modelleme süreci ile ilgili sunum/tartışma	Öğrenci düşünme şekilleri değerlendirme raporu
10	13.04.2012	• "Su Deposu" başlıklı etkinlik uygulaması	Etkinlik sonrası düşünce raporu Proje taslaklarının teslimi

Table 22 (continued)

11	20.04.2012	• "Su Deposu" başlıklı etkinlik bağlamında öğrenci düşünme şekilleri üzerine tartışma • Modelleme etkinlikleri için Uygulama Planı hazırlama	Öğrenci düşünme şekilleri değerlendirme raporu
12	27.04.2012	• Gruplarca tasarlanan modelleme etkinliklerinin sınıf-içi uygulamaları	-
13	04.05.2012	• Gruplarca tasarlanan modelleme etkinliklerinin sınıf-içi uygulamaları	-
14	11.05.2012	• Gruplarca tasarlanan modelleme etkinliklerinin sınıf-içi uygulamaları	-
15	18.05.2012	• Genel Değerlendirme	Modelleme etkinliklerinin ve proje raporlarının teslimi (28 Mayıs)

APPENDIX C

MATHEMATICAL MODELING ACTIVITIES

C.1 The Summer Job

Arzu geçen yaz Vahşi Doğa eğlence parkında yiyecek satışı yapan bir firmanın temsilcisi olarak çalışmaya başlamıştır. Arzu'ya bağlı olarak çalışan elemanlar seyyar arabalarla parkın içerisinde turlamakta ve sakız, sandviç, çerez, su, meşrubat vb. satmaktadırlar.

Geliri tatmin edici olduğu için önceki yaz çalışan elemanların tamamı bu yaz da çalışmak için Arzu'ya tekrar başvurdu. Fakat park yöneticileri Arzu'ya, bu yaz geçen yılki kadar çok satıcının parka alınamayacağını söylediler. Buna göre Arzu geçen yıl çalışan dokuz elemanın yalnızca üçte birini tam zamanlı çalışmak üzere, üçte birini ise yarı zamanlı çalışmak üzere işe alabilecektir. Kalan üçte birini ise işe alamayacaktır. Ancak, Arzu geçen yıl çalışan bu dokuz elemandan hangilerini tekrar işe alacağına karar verememekte ve bu konuda yardımınıza ihtiyaç duymaktadır.

Arzu, geçen yaz çalışan dokuz kişinin kayıtlarını gözden geçirerek, parkın "çok yoğun, orta yoğunlukta ve durgun" olduğu zamanlara göre her bir satıcı için toplam çalışma saatlerini ve satışlardan elde ettikleri toplam geliri hesaplayarak aşağıdaki tabloyu oluşturmuştur. Arzu kendisine en çok para kazandıran elemanları tekrar işe almak istemekte, ancak elemanları performanslarına göre nasıl karşılaştıracağını bilememektedir. Çünkü her bir eleman farklı sürelerde çalışmıştır. Ayrıca, parkın yoğun olduğu saatlerde daha çok satış yapmanın mümkün olduğu gerçeği de göz ardı edilmemelidir.

Gelecek yaz çalışmak üzere kimin işe alınması gerektiğine karar vermek için bir yöntem bulmaya çalışınız. Bu çerçevede, Arzu'ya geçen yaz kendisi için çalışan bu dokuz elemanı nasıl değerlendirebileceğini ve hangilerini tam zamanlı, hangilerini yarı zamanlı olarak işe alabileceğini anlatan bir rapor yazınız. Yönteminizin tabloda gösterilen dokuz kişi için nasıl işlediğini gösteriniz. Yönteminizi anlayabilmesi için Arzu'ya yeterince ayrıntı vermelisiniz. Ayrıca Arzu'nun yönteminizin onun için iyi bir yöntem olup olmadığına karar verebilmesi için açıklamanızın anlaşılır olmasına dikkat ediniz.

Geçen Yaz Çalışılan Saatler

	HAZİRAN			TEMMUZ			AĞUSTOS		
	Yoğun	Orta	Durgun	Yoğun	Orta	Durgun	Yoğun	Orta	Durgun
MELEK	12.5	15	9	10	14	17.5	12.5	33.5	35
KÜBRA	5.5	22	15.5	53.5	40	15.5	50	14	23.5
TÜLAY	12	17	14.5	20	25	21.5	19.5	20.5	24.5
JALE	19.5	30.5	34	20	31	14	22	19.5	36
ÇETİN	19.5	26	0	36	15.5	27	30	24	4.5
CAN	13	4.5	12	33.5	37.5	6.5	16	24	16.5
REMZİ	26.5	43.5	27	67	26	3	41.5	58	5.5
TEKİN	7.5	16	25	16	45.5	51	7.5	42	84
VELİ	0	3	4.5	38	17.5	39	37	22	12

Geçen Yaz Toplanan Para

	HAZİRAN			TEMMUZ			AGUSTOS		
	Yoğun	Orta	Durgun	Yoğun	Orta	Durgun	Yoğun	Orta	Durgun
MELEK	690	780	452	699	758	835	788	1732	1462
KÜBRA	474	874	406	4612	2032	477	4500	834	712
TÜLAY	1047	667	284	1389	804	450	1062	806	491
JALE	1263	1188	765	1584	1668	449	1822	1276	1358
ÇETİN	1264	1172	0	2477	681	548	1923	1130	89
CAN	1115	278	574	2972	2399	231	1322	1594	577
REMZİ	2253	1702	610	4470	993	75	2754	2327	87
TEKİN	550	903	928	1296	2360	2610	615	2184	2518
VELİ	0	125	64	3073	767	768	3005	1253	253

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250) **"The Summer Jobs"** activity adapted from "Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In A. E. Kelly, & R. A. Lesh (Eds.), Handbook of research design in mathematics and science education (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers."

Tag: Mathematical concepts under the MEA are mean, ratios, and proportions that take place in 9th grade in the mathematics curricula published by Turkish Ministry of National Education (MEB, 2011a).

C.2 The Ferris Wheel

İngiltere'nin başkenti Londra'daki "London Eye" ismiyle bilinen dönme dolap Londra'yı kuşbakışı izlemek isteyenler için tavsiye edilmektedir. 1999 yılında inşa edilen ve dünyanın en büyük dönme dolaplarından birisi olan yapı, yıllık 4 milyon civarında ziyaretçisiyle Londra'nın önemli turizm kaynaklarından biri haline gelmiştir. 135 metre yüksekliğindeki bu dönme dolap her biri 25 kişi kapasiteli, içinde insanların rahatça dolaşabileceği genişlikte 32 kapsülden oluşmaktadır. Dönme dolabın bir diğer özelliği de hiç durmadan hareketine devam etmesidir. Yani yolcu indirmek ya da bindirmek için durmayan dolap, insanların yer seviyesinde kapsüllere rahatlıkla inip binebileceği kadar yavaş hareket etmektedir.

Londra'daki bu yapıyı inceleyen ve müşteri potansiyelinden etkilenen bir yatırımcı, benzer bir dönme dolabı İstanbul'da Çamlıca tepesine yapmaya karar veriyor. Çapı 140 metre olması planlanan dönme dolap, yerden yüksekliği 4 metre olan bir platform üzerine kurulacaktır. Dönme dolap üzerine eşit aralıklarla her biri 25 kişi kapasiteli 36 kapsülün yerleştirilmesi düşünülmektedir. Dönme dolabın bir tam turunu tamamlama süresi 30 dakika olarak planlanmaktadır. Kapsüllerin içerisine yerleştirilecek olan elektronik göstergelerde müşteriye anlık olarak aktarılması planlanan bilgiler şunlardır:

- *Yerden yükseklik,*
- *Kapsüle bindikleri noktaya olan uzaklık,*
- *Hız,*
- *Bir tam turun tamamlanmasına ne kadar zaman kaldığı* (bir tam turun bitmesine 1 dakika kala yolcuların iniş hazırlığı için erken uyarı devreye girecektir).

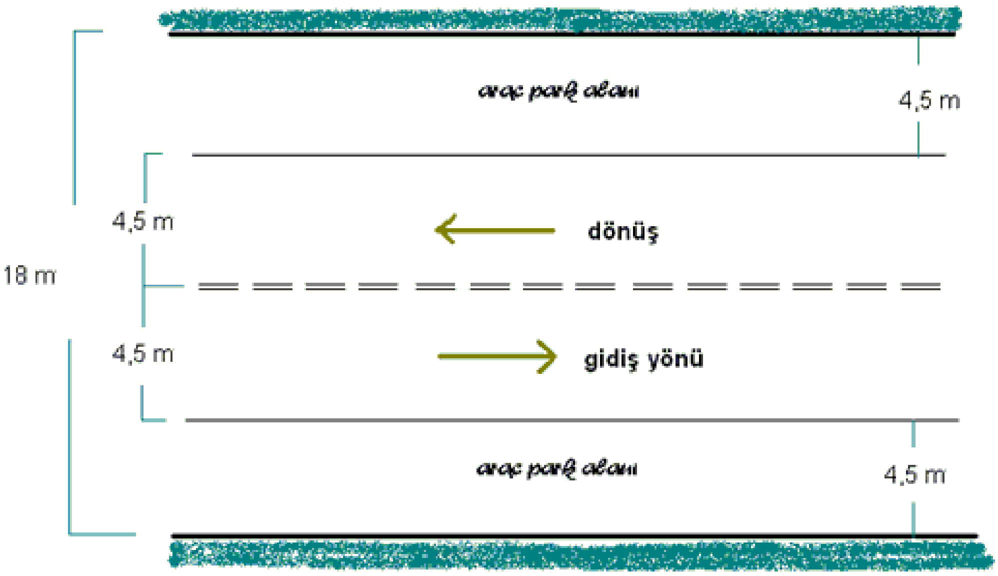
Bu bilgileri anlık hesaplayabilecek yazılımı geliştirecek bilgisayar programcısına yardımcı olmanız istenmektedir. Bu çerçevede, programcıya bu bilgilerin matematiksel olarak nasıl hesaplanabileceği konusunda bir yöntem öneriniz.

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250)
"The Ferris Wheel" activity adapted from "Lesh, R., & Lehrer, R. (2000). Iterative refinement cycles for videotape analyses of conceptual change. In A. E. Kelly, & R. A. Lesh (Eds.), Handbook of research design in mathematics and science education (pp. 665-708). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers."

Tag: Mathematical concepts under the MEA are trigonometric functions that take place in 10th grade in the mathematics curricula published by Turkish Ministry of National Education (MoNe, 2011).

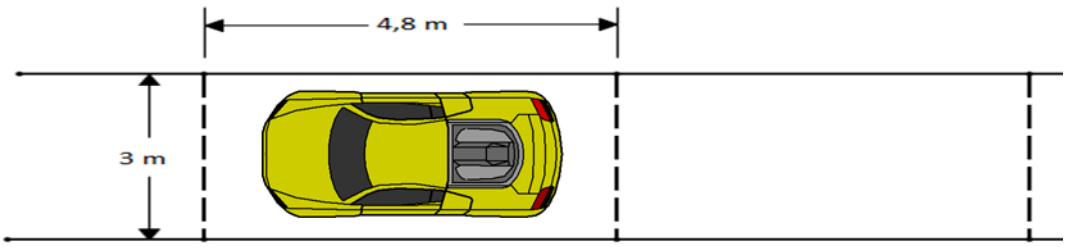
C.3 The Street Parking

Bir şehir planlamacısı iki yönlü bir yolun kenarında, evlerin önünde araba park yeri tasarlamak için sizden yardım istiyor. Şehir plancısının amacı caddede park edilebilecek araç sayısının en fazla olacağı düzeni sağlamaktır. Park edilecek yer yolun 150 metrelik kısmını oluşturuyor. Yolun toplam genişliği aşağıdaki çizimde görüldüğü gibi 18 metredir. Bu yolda hem iki yönlü trafik işlemeli, hem de iki tarafında arabalar park edebilmelidir. Şekil 1'de görüldüğü gibi yolun bir şeridi şerit çizgisi dâhil 4,5 metre ve yolun kenarındaki bir araç park alanının genişliği de 4,5 metredir. Bir arabanın güvenli bir şekilde park edilebilmesi için şerit çizgileri dâhil 3 m genişliğinde 4,8 m uzunluğunda bir alan ayrılmalıdır. Bu alan, yola paralel olabileceği gibi (bkz. Şekil 2a) açılı olarak da tasarlanabilir (bkz. Şekil 2b) ancak bu durumda araçlar yola taşmamalıdır.

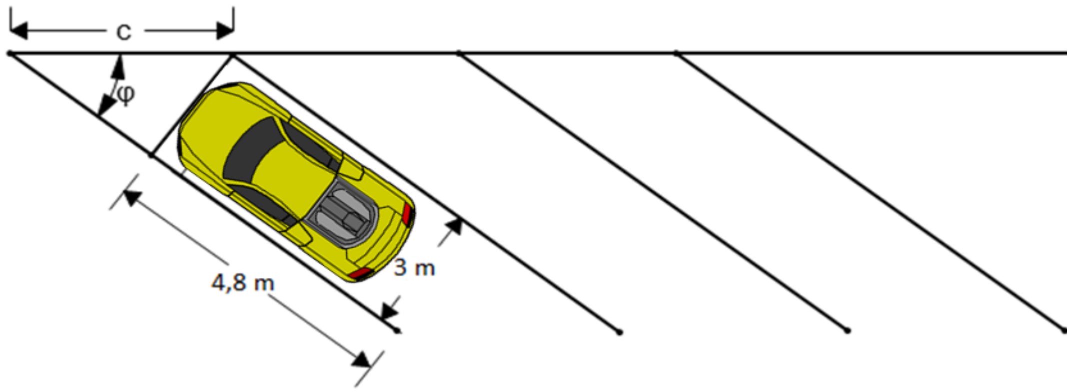


Şekil 1. Araba park alanı ve yol planı

Sizden istenen yolun bu 150 m'lik kısmına en fazla sayıda araç park edilebilecek şekilde yola paralel veya açılı park yerleri tasarlamanızdır. Araba park yeri tasarımınızda aşağıdaki çizimlerden yararlanabilirsiniz.



Şekil 2a. Paralel araba park yeri tasarımı



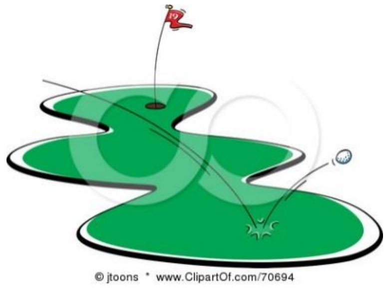
Şekil 2b. Açılı araba park yeri tasarımı

Eğer araç park alanının genişliği için verilen 4,5 metre sınırlaması olmasaydı şehir planlamacısına en fazla sayıda araç park edilebilmesi için nasıl bir park tasarımı önerisinde bulunurdunuz? Nedenleriyle açıklayınız.

As a part of a project supported by TUBITAK (Grant no 110K250) **"Street Parking"** activity was adapted from "Swetz, F. & Hartzer, J.S., (1991). Mathematical modeling in the secondary school curriculum: A resource guide of classroom exercises. Reston, VA: NCTM. (pp. 71)"

Tag: Mathematical concepts under the MEA are the geometry of the triangle and trigonometric functions that take place in 10th grade in the mathematics curricula published by Turkish Ministry of National Education (MEB, 2011a)

C.4 The Bouncing Ball



Birçok popüler spor dalı bir çeşit top kullanımı gerektirir. Spor dallarında kullanılan topları tasarlarken göz önünde bulundurulması gereken en önemli etkenlerden birisi de topun iyi zıplayabilmesi, yani esnekliğidir. Örneğin, bir golf topu sert bir yüzeye çarptığında düştüğü yüksekliğin yaklaşık 2/3 ’ü kadar sıçramalıdır.

Çeşitli spor dallarında kullanılmak üzere toplar üreten bir firmanın ARGE birimi çalışanları, esnekliğini test etmek için yeni geliştirdikleri bir topu, 52 metre yüksekliğindeki bir binanın çatısından aşağı doğru bırakıyor. Binanın bir katında gözlem yapan bir görevli de topun, yerden 15 metre yüksek olarak belirlenen gözlem seviyesinden 17 kez geçtiğini rapor ediyor. ARGE bölümünün matematikçisi olarak sizden, bu verileri kullanarak test edilen topun zıplama oranının ne olabileceğini bulmanız istenmektedir. Bunu yaparken, topun düz bir zemine çarparak her zıplayışta bir önceki yüksekliğinin belli ve sabit bir oranına ulaştığını varsayın.

As a part of a project supported by TUBITAK (Grant no 110K250) **“The Bouncing Ball”** activity was adapted from “http://intermath.coe.uga.edu/topics/nmcncept/ratios/r16.htm (Source: Mathematics Teaching in the Middle School, Feb 1999)”.

Tag: Mathematical concepts under the MEA are exponential functions, exponential inequalities that take place in 11th grade in the mathematics curricula published by Turkish Ministry of National Education (MEB, 2011a).

C.5 The Free Roller Coaster

Ankara'da yeni kurulacak olan bir eğlence parkında yer alması düşünülen lunapark tren yolunun mesafeye göre yüksekliğini içeren tasarımı için yarışma açılacağı ve kazanana ömür boyu ücretsiz biniş hakkı verileceği tüm basın-yayın organlarında duyurulmuştur. Yarışmayı kazanma kriteri, tasarımın trene binen yolcuları ölesiye korkutarak heyecanlandıracak kadar eğimli, fakat onları sağ salim geri getirecek kadar da güvenli olmasına bağlı. Yolcuların heyecanlanması bu yolun yukarı ve aşağı doğru ani ve keskin değişimlerle harekete imkan vermesine bağlıyken, güvenlik kurallarına göre, yolun eğiminin mutlak değeri 5,67 den fazla olmamalı.

Siz de bu yarışmaya, bir grup mühendisle birlikte kendi tasarımınızla katılmak istiyorsunuz. Zamandan tasarruf etmek amacıyla, üçerli gruplar halinde çalışmanız gerekmekte. Her grup, bu yolun bir parçasını tasarlayacak, daha sonra bu parçalar birleştirilerek uzun bir yol elde edilecek. Sizin de içinde bulunduğunuz grup, bu eğimli demiryolunun sadece inişleri ve çıkışları olan, virajı olmayan, başlangıç noktasının yüksekliği 6 metre bitiş yüksekliği 9 metre olan 100 metre mesafelik bir bölümünü tasarlayacak. En az üç yerde ani aşağı doğru iniş içerecek olan bu yolun hangi bölümlerinde heyecanın arttığını, hangi bölümlerinde azaldığını içeren bir rapor da hazırlamanız beklenmekte.

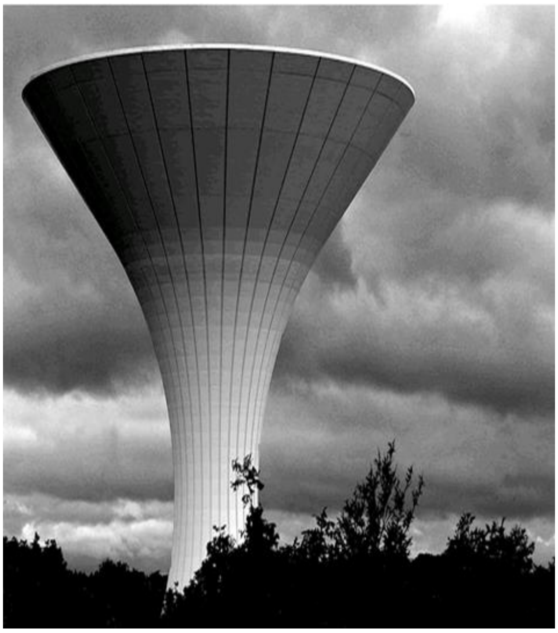
As a part of a project supported by TUBITAK (Grant no 110K250) **"Free Roller Coaster"** activity was adapted from "Cabana, Cooper, Dietiker, Douglas, Gulick, Simon, & Thomas (2000). *College preparatory mathematics calculus.* First Edition. (pp.162-163)."

Tag: Mathematical concepts under the MEA are slope of tangent lines and curve analysis that take place in 12th grade in the mathematics curricula published by Turkish Ministry of National Education (MEB, 2011a).

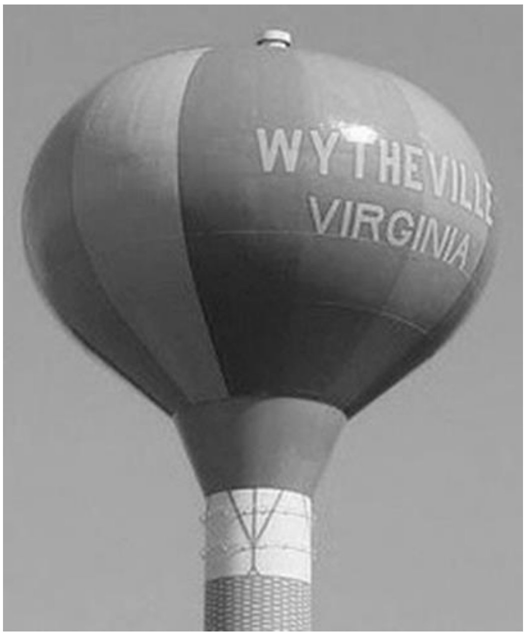
C.6 The Water Tank

Bir şirket bilgisayar destekli eğitim amaçlı yazılımlar hazırlamaktadır. Şirketteki bir ekibe öğrencilerin grafik çizme ve yorumlama becerilerini geliştirmeye yardımcı olacak bir su deposu doldurma animasyonu üzerinde çalışma işi verilmiştir. Ekibin bu animasyonu oluşturabilmesi için depo suyla dolarken depoda biriken su miktarına bağlı olarak suyun yüksekliğini gösteren bir grafiğe ihtiyacı bulunmaktadır.

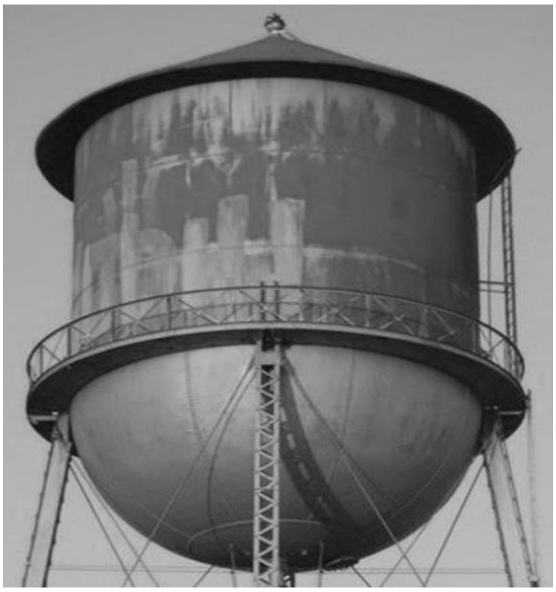
Ekibin matematikçi üyesi olarak sizden istenen ekte verilen örnek depolar için istenen türden bu grafikleri yaklaşık olarak çizmeniz ve sonrasında herhangi bir şekle sahip bir su deposu için su miktarına bağlı olarak suyun yüksekliğini gösteren grafiğin nasıl çizileceğini açıklayan bir yönerge hazırlamanızdır.



Tank 1



Tank 2



Tank 3 **Tank 4**

As a part modeling task development stage of a project supported by TUBITAK (Grant no 110K250), "**The Water Tank**" activity was adapted from "Carlson, M. P. (1998). A cross-sectional investigation of the development of the function concept. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics education, III* (CBMS Issues in Mathematics Education, Vol. 7, pp. 114 –162). Washington, DC: Mathematical Association of America." and "Carlson, M., Larsen, S., &Lesh, R. (2003). Integrating models and modeling perspective with existing research and practice. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modeling perspective* (pp. 465-478). Mahwah, NJ: Lawrence Erlbaum Associates."

Tag: mathematical concepts under the MEA are change, concave up, and concave down functions that take place in 10th grade in the mathematics curricula published by Turkish Ministry of National Education (MEB, 2011a).

APPENDIX D

SAMPLE MODELING ACTIVITY IMPLEMENTATION PLAN

CADDEDE PARK YERİ ETKİNLİĞİ UYGULAMA PLANI

Modelleme Sorusunun Adı: Caddede Park Yeri

Öğrenme Alanı: Geometri, Trigonometri

Toplam Süre: 140 dk

Sorunun çözülmesi için süre: 100 dk

Sunumlar için süre: 40 dk

Araç ve Gereç: A3 ve A4 kâğıdı, etkinlik kâğıdı, hesap makinesi

UYGULAMA

1. Öğrencilere derste yapılacakların açıklanması

Bireysel çalışma, grup çalışması ve grup sunumlarının işleyişi aşağıdaki hususlar hatırlatılacak.

Bireysel Çalışma: Bireysel okuma ve çözüm yaklaşımları üzerinde düşünme

Grup Çalışması: Zamanı etkili kullanmaları, çözüm sürecinde raporu nasıl yazacaklarını planlamaları gerektiği

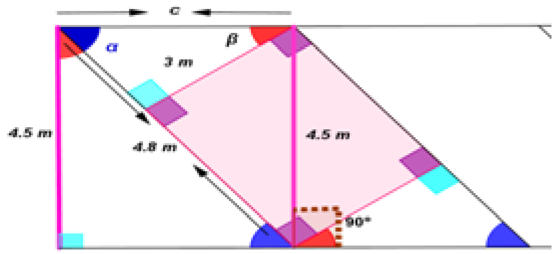
Grup Raporları: çözüm sürecinin matematiksel olarak ayrıntılı açıklanması, varsa çözüm yaklaşımlarındaki değişikliklerin gerekçesiyle rapora yansıtılması, grup ismi ve üyelerin isimleri rapora yazılması

Grup Sunumları: dersin sonunda grup sunumlarının yapılacağı

2. Sorunun anlaşılmasını sağlamak ve soruya ısındırmak için yapılabilecekler

Bireysel çalışmadan sonra sorunun nasıl anlaşıldığı veya anlaşılmayan noktaları üzerine sınıf tartışması yapılabilir.

Sorunun anlaşılmayacağı yer: Şekil-1 de görüldüğü gibi, 4,8 metre uzunluğundaki yerin neresi olduğu; yani 4,8 metre olacak yerin park yeri çizgisinin toplam uzunluğu mu olacaktı yoksa 3 metreye 4,8 metrelik bir dikdörtgensel park alanı mı? (Aşağıdaki şekil-1 yanlış algılanan durumu örneklemektedir)



3. Uygulamada öğrencilerin kullanabilecekleri çözüm stratejileri

Paralel Park Tasarımı İçin;

ÇÖZÜM YAKLAŞIMI:

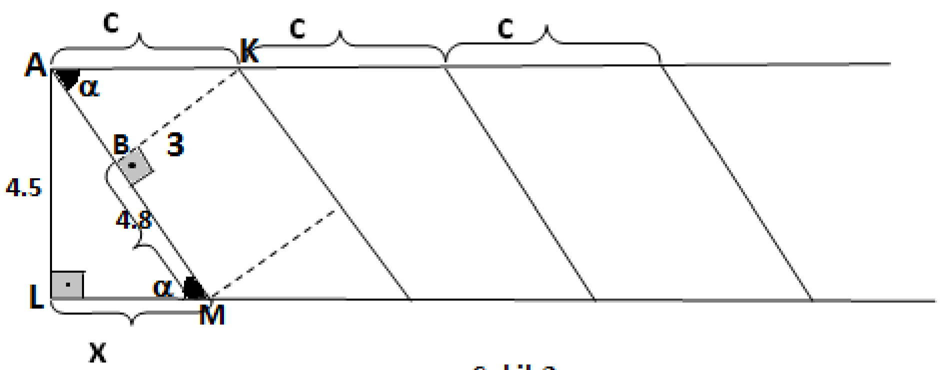
150 ÷ 4,8 = 31, 25 araba (yolun bir kenarına) 0,25 araba olamayacağı için; 31 × 2 = 62 araba (yolun 2 kenarına)

Açılı Park Tasarımı İçin;

ÇÖZÜM YAKLAŞIMI 1:

Şekil-2 görüldüğü gibi;

- Park alanlarının genişliği " c " olsun.
- Araçlar park açılı park ettiğinde artan yol " x " metre olsun (LM uzunluğu).



Şekil-2

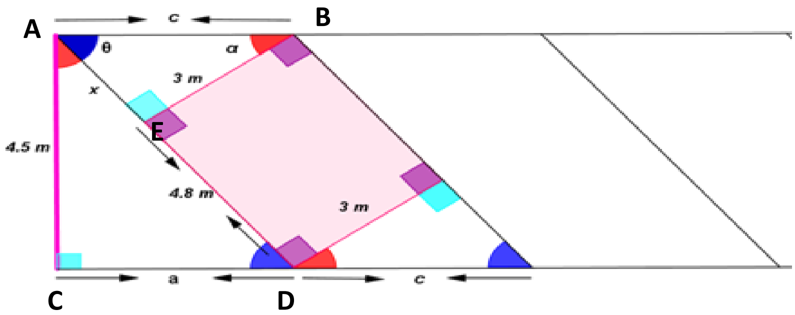
Şekil-2 de
görüldüğü gibi, öğrenciler ABK üçgeni ve ALM üçgenini kullanarak, x ve c bilinmeyenlerini α açısına bağlı fonksiyon olarak yazabilirler (formül oluşturma):

- $c = 3/\sin\alpha$ (ABK üçgeninden)
- $x = 4{,}5/\tan\alpha$ (ALM üçgeninden)
- $(150 - x)/c$ ise $\left[150 - (4{,}5/\tan\alpha)\right] : 3/\sin\alpha$ olur.

Oluşan bu formüldeki α yerine çeşitli açı değerleri vererek değişik açı değerlerinde kaç aracın park edilebileceğini bulma.

Yapılabilecek Hata: α açısına değerler verirken sonucunda park alanının uzunluğunun 4,5 metreyi aşıp aşmadığını hiç göz önünde bulundurmama.

ÇÖZÜM YAKLAŞIMI 2:



Şekilde belirtilen uzunlukları ve açıları kullanarak denklemler oluşturulabilir:

1. $x^2 + 3^2 = c^2$ (AEB üçgeninden pisagor teorimi ile)
2. $(4,5)^2 + a^2 = (x + 4,8)^2$ (ACD üçgeninden pisagor teoremi ile)
3. $(1,5)c = 4,8 + x$ (AEB ve ACD üçgenleri arasında benzerlikten)
4. $\frac{3}{x} = \frac{4,5}{a} = \tan\theta \rightarrow 1,5x = a$ (AEB ve ACD üçgenleri arasında benzerlikten)

Ve bu denklemlerden c'ye bağlı ve x'e bağlı aşağıdaki gibi ikinci dereceden denklemler elde edebilirler.

$1,25c^2 - 14,4c + 32,04 = 0$ ve $1,25x^2 - 9,6x - 2,79 = 0$

Bu denklemleri çözerek, x ve c değerlerini bulup;

- $\tan\theta = \frac{3}{x} = \frac{3}{7,98} = 0,375$
- $\arctan(0,375) = 20,556^\circ$ açı değerini ve $150/c$ değerinden park edilecek araba sayısını bulma.

Yapılabilecek Hata (Eksiklik): Soruda istenen "değişik açı değerleri için" park edilecek araç sayısını denememe ve sadece bir açı değeri bulma.

ÇÖZÜM YAKLAŞIMI 3: Farklı açı değerleri için (özellikle bilinen açılardan yola çıkarak) araç sayısını deneme

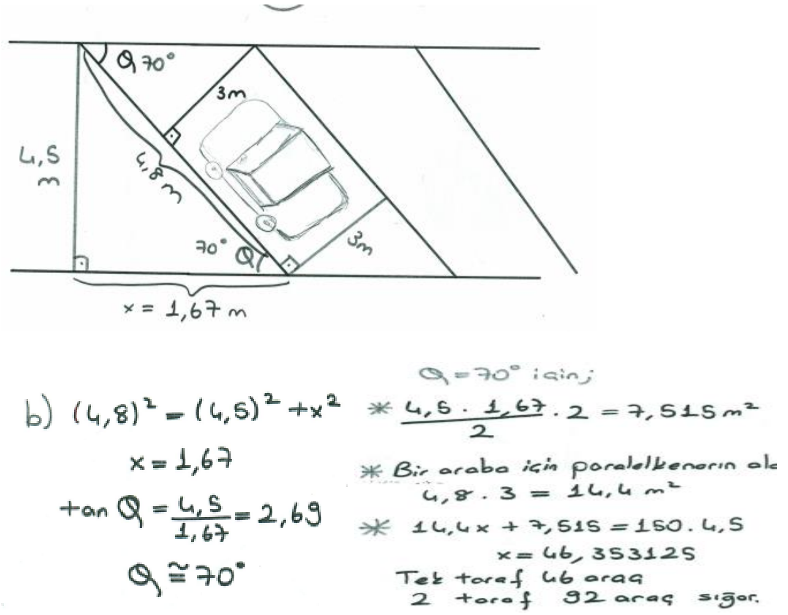
Örneğin;

30° olarak düşündüğümüzde yolun bir tarafına 25 araç 45° olarak düşündüğümüzde yaklaşık olarak 32 araç, 90° 'de ise 50 araç sığabiliyordu.

Yapılabilecek Hata: α açısına değerler verirken sonucunda park alanının uzunluğunun 4,5 metreyi aşıp aşmadığını hiç göz önünde bulundurmama.

ÇÖZÜM YAKLAŞIMI 4: (Tamamen yanlış yaklaşım)

4.8 metrenin yanlış yerde almaları ve yanlış sonuca ulaşma ve bu durumda en idealini açılı park olarak bulma



4. **Uygulamada öğrencilerin nerelerde ve ne tür hatalar yapabilecekleri ve yapabilecekleri hataların üstesinden gelmek için kullanabilecek yöntemler**

   - Açılı park etmesi konusunda. 4,8 metre uzunluğunun neresi olacağının anlaşılmaması ve 4.8m uzunluğu yanlış yerde alma.
   - Park alanının uzunluğunun 4,5 metreyi aşıp aşmadığını hiç göz önünde bulundurmama.
   - Açılı park edilme esnasında ölü alanı dikkate almama.

5. **Öğrencilere soruyu çözerken sorulabilecek sorular ve bu soruları sormadaki amaçlar**

Gruplar dolaşılırken genel olarak sorulacak sorular:

- Ne tür çözüm yaklaşımları düşündünüz?
- Hangi yaklaşımla çözmeye karar verdiniz? Neden? (Gerekirse vazgeçtikleri diğer yaklaşımlardan neden vazgeçtikleri?)

- Hangi varsayımlarda bulunarak bu yönteme karar verdiniz?
- Nasıl çözdünüz? Açıklar mısınız?

Etkinliğe özel sorular:

- Arabanın güvenli bir şekilde park edilebilmesi için şerit çizgileri dâhil 3 m genişliğinde

4,8 m uzunluğunda bir alan olarak nereyi aldınız?

- Farklı açıları deneyerek yapan gruplara:

Hangi açıları kullandınız? Neden bu açıları denediniz? Bu açıları denerken göz önünde

bulundurduğunuz kriterler var mı? (4.5 m geçmeme koşuluna dikkat edip etmediklerini sorgulamak.

6. Çözümlerin hangi sıraya göre, ne şekilde sundurulacağı ve nedenleri

- Derste grupların çözümleri ve zaman gibi unsurlar göz önünde bulundurularak ders esnasında karar verilecek.

Örneğin; farklı çözüm yaklaşımlarına sahip tüm gruplara sunum yaptırılması beklenmektedir.

7. Bu sorunun uygulanmasında öğretmenin dikkat etmesi gereken diğer hususlar

- Sorunun çözümünde öğrenciler açılı park tasarımını yaparken, park edilecek açıyı bulmayan park edecek araç sayısını bulan gruplara
  - ➢ Bulduğunuz açı değeri nedir?
  - ➢ Bu açıdan büyük ve küçük açı değerleri için ne söyleyebilirsiniz?

Şeklinde sorular sorarak öğrencilere buldukları araç sayısına karşılık gelen ideal açı değerini ve neden o açı olduğunu sorgulatabilir.

- Bu etkinlikte öğrenciler trigonometrik ilişkiler için hesaplama yapacaklardır. Bunun için öğretmen sınıfta bilimsel hesap yapabilen hesap makinası bulundurmalıdır. Hesap makinasının kullanımı için öğrencilerin yardıma ihtiyacı olabilir.
- Problemde verilen 3 x 4,8 m'lik park alanının güvenli park edebilme şartının, giriş ve çıkış esnasındaki manevra mesafelerini de göz önüne alarak belirlendiği öğrencilere açıklayabilir.
- Öğretmen, problemin anlaşılması için uygun bir ölçekte kâğıt modeller kullanımını önerebilir.

APPENDIX E

AN EXAMINATION GUIDE OF STUDENTS' WAYS OF THINKING

ÖĞRENCİ DÜŞÜNME ŞEKİLLERİ İNCELEME REHBERİ

Amaç: Öğretmen adaylarının, öğrencilerin matematiksel düşünme süreçlerini ***fark etme***, anlama ve ***yorumlamalarını*** (becerilerini) arttırmak (öğrenmesi).

Süre: 100dk

Roller:

A. Sunucu ve Yönetici

- Kuralların hatırlatılması
  - Gruplar işbirliği ve fikir birliği içinde çalışması
  - Her bir bölüm için ayrılan sürenin hatırlatılması
- Sürecin yönetilmesi ve rehberlik
  - Soruların sorulması
  - Öğretmen adaylarının öğrenci kâğıtlarına verilen grup numaralarını belirterek açıklama yapmasını sağlama
  - Öğrenci kâğıtları ile videoların nasıl eşleşmiş olduğunu açıklar.
- Öğrenci çalışmaları ile ilgili belli bilgilerin verilmesi
  - Kaçıncı sınıf, öğrenci çalışmalarının oluştuğu ortam, ortalama ne kadar zamanda çözdükleri

B. Araştırmacı

- Öğrenci çözüm kâğıtlarının öğretmen adaylarına dağıtılması
- Öğrencilerin not almaları için amaç doğrultusunda hazırlanmış "öğrenci düşünmeleri şekilleri değerlendirme formu" öğretmen adaylarına dağıtılması

(*öğrenciler bunu öğrenci kâğıtlarının analizi sürecinde doldurması)

- Video kesitlerinin hazırlanması ve sunulması

YÖNTEM/İZLENECEK YOL

1. **ADIM: Başlama/Giriş ve Öğrenci çalışmalarının sunulması (5dk)**

- *Yönetici/Sunucu kısaca kuralları açıklar ve zaman hakkında bilgi verir.*
- *Araştırmacılar tarafında*n:
  - **Öğrenci kâğıtları**: Seçilmiş, 4-5 farklı gruba ait öğrenci kâğıtları gruplara dağıtılır.

Uyarı: Öğrenci çalışmaları hakkında başlangıçta sadece bazı bilgilerin verilmesi (sınıf seviyesi, grupça çalışmaları ve grupların kaç kişilik olduğu (öğrenci çalışmalarının oluşturulduğu ortam), hangi başarı düzeyinde hangi okullardan olduğu başlangıçta değil, 4. adımda konuşulmalıdır.

2. **ADIM: Belirlenmiş öğrenci kâğıtları ile ilgili, öğrenci sunumlarına ait videoların gösterilmesi (10dk) ve öğrenci kâğıtlarının ön analizi (10dk)**

- **Video Kesitleri:** Öğrenci düşünme süreçlerini yansıtan hazırlanmış kâğıtlarla ilgili video kesitlerinin gösterilmesi (her biri 5-7dk)
- Araştırmacı***, "sunumlara ait"*** video görüntülerini gösterir (ortalama 10dk).
- Gruplar kâğıtlarla ilgili öğrenci sunumlarını dinler ve kendilerine dağıtılan öğrenci düşünme şekilleri değerlendirme formuna (genel) notlar alır.
- Öğretmen adayları video görüntülerinden elde ettikleri notlarla birlikte öğrenci kâğıtlarına bakarak, öğrenci kâğıtları ile ilk izlenimleri ile ilgili notlar alırlar.
- Gruplar bu süreçte mümkün olduğu kadar öğrenci çalışmalarından bilgi toplamaya çalışırlar.

3. **ADIM: Öğrencilerin neler yaptığının tanımlanması**

Sunucu/Yönetici sorar; "Öğrenci kâğıtlarını ve ilgili videoları incelediğiniz".

- Neler görüyorsunuz, neleri fark ettiniz, neler söyleyebilirsiniz?

!! Öğrenci çalışmalarından örnekler (yerler) göstererek bu söylediklerinizi destekleyebilir misiniz?

Uyarı:

- Grupların ilk izlenimleri
- Çalışmanın kalitesi hakkında (iyi/kötü/başarılı/başarısız vs.) mümkün olduğunca yorumsuz, yorum gelirse bu düşüncesi ile ilgili ***öğrenci kağıdından kanıt/açıklama sunması.***

- **ADIM: Odak grup'a ait video görüntüsünün izlenmesi ve öğrenci kâğıtlarının detaylı analizi Video Kesitleri** : Öğretmen adayları, odak gruba ait video görüntülerini izler ve kendilerine dağıtılan öğrenci düşünme şekilleri değerlendirme formuna (genel) notlar alır (bu süreçte notları bireysel alırlar)

- Öğretmen adayları video görüntülerinden elde ettikleri notlarla birlikte aralarında

tartışarak (grupça) öğrenci kağıtları üzerinde çalışırlar ve verilen kağıtlar üzerindeki istenenler doğrultusunda notlarını detaylandırırlar.

Uyarı:

- Gruplar bu sürede mümkün olduğu kadar öğrenci çalışmaları ile ilgili bilgi toplar.

4. **ADIM: Öğrenci çalışmalarının yorumlanması**

Uyarı: Gruplar (Öğretmen adayları) bu süreçte, öğrencinin ne yaptığını, nasıl yaptığını ve niçin yaptığını yorumlamaya çalışacaklardır. Grupların (öğretmen adaylarının) görevleri: öğrencilerin gördüğü/düşündüğü gibi görmek/düşünmek. Bu süreçte gruplar aşağıdaki sorulara cevap verecek şekilde öğrenci çalışmalarını yorumlayacaklar ve öğrenci çalışmalarından yorumlarını destekleyecek kanıtlar sunacaklardır.

1. Öğrenciler soruyu ne kadar iyi anlamış?
2. Öğrencilerin kullandıkları ***farklı çözüm yolları*** nelerdir? (her bir öğrenci kâğıdı için)
   - Hangi matematiksel konu ve gösterimlerden yararlanmışlar?
   - (Varsa) Sorunun çözümünde kullanabilecekleri hangi gerekli matematiksel bilgi/beceri/konuyu göz ardı etmişler?
3. Öğrencinin çözüm ve düşünme süreçlerinde güçlü gördüğünüz yerler nerelerdir?
   - Öğrenciler sorunun hangi kısmında/kısımlarınsa en fazla çaba göstermişler? (öğrenci çözümlerinin güçlü yönleri)
4. Öğrenci çözüm ve düşünme süreçlerindeki zayıf gördüğünüz yönler/problemler nelerdir?
   - Öğrenciler sorunun hangi kısmında/kısımlarınsa en az çaba göstermişler?
   - Nerede zorlanmışlar/ ne tür hatalar yapmışlardır? Hangi kavramlar/matematiksel süreç onlar için zor gelmiştir?
5. Öğrencilerin çözümlerinden/düşünme süreçlerinde size ilginç gelen /şaşırtan bir yaklaşım var mı?
6. Sizin beklentilerinizden/tahminlerinizden farklı öğrenci düşünme şekilleri (ve hatalar, zorluklar) nelerdir?

!! Öğrenci çalışmalarından örnekler (yerler) göstererek bu söylediklerinizi destekleyebilir misiniz?

5. **ADIM: Sürecin Değerlendirilmesi (15dk.)**

*4-5 farklı grup öğrenci kâğıtları ve ilgili video görüntülerini incelediniz ve üzerinde tartıştınız***.**

- **“** Bu öğrencilerin düşünme süreçleri arasında herhangi bir ilişki görebiliyor musunuz? (ne gibi bir ilişki görüyorsunuz/benzerlikler ve farklılıklar?
- “Öğrencilerin bu çalışmaları onların matematiksel düşünme süreçleri hakkında size ne söylüyor?”

APPENDIX F

EVALUATION REPORT OF STUDENTS' WAYS OF THINKING

Öğrenci Düşünme Şekillerini Değerlendirme Raporu

Öğrenci düşünme şekilleri sürecindeki tartışmalarınızı tekrar düşünerek, aşağıdaki sorulara cevap veren, mümkün olduğu kadar **detaylarıyla ve örneklendirerek** açıklayan bir rapor yazmanız beklenmektedir. Listedeki soruların hepsinin cevaplanmasına özen gösteriniz, ancak, listedeki sıralamayı takip etmek zorunda değilsiniz. Bununla birlikte***, sorulara karşılık gelmeyen ekleyeceğiniz başka düşünceleriniz olursa*** kendinizi bu sorularla kısıtlamadan, etkinlikle ilgili her türlü düşünce ve eleştirilerinizi de yazabilirsiniz.

1. Öğrencilerin çalışmalarını yani öğrenci çözüm kâğıtlarını ve bu çözüm kâğıtları ile ilgili video kesitlerini incelerken ve değerlendirirken çözüm kâğıtlarında ve videolarda öncelikli olarak nelere dikkat ettiniz (odaklandınız)? Açıklayınız.
2. "Etkinlik sonrası düşünce raporunuzda, öğrencilerin bu soruya getireceği farklı çözüm yaklaşımlarını, öğrencilerin ne tür zorluklar yaşayacağını ve yapabilecekleri olası hatalar ile ilgili beklenti ve tahminlerinizi" ifade etmiştiniz. Öğrenci çözüm kâğıtlarını ve video görüntülerini incelemeden önceki sizin beklentileriniz/tahminleriniz ile inceledikten sonraki gördüğünüz "öğrencilerin çözüm yaklaşımları", "sorunun çözümünde karşılaştıkları zorluklar" ve "yaptıkları hatalar" arasında farklılıklar var mıydı? Varsa, bu farklılıkları ***çözüm kâğıtlarından ve videolardan örneklerle destekleyerek açıklayınız.***
3. Öğrencilerin ortaya koyduğu bu çözüm yollarından, "*bu şekilde düşüneceğini gerçekten de düşünemezdim; beni çok şaşırttı."* dediğiniz bir çözüm yaklaşımı (matematiksel düşünme süreci) var mıydı? Varsa, hangi çözüm yaklaşımı olduğunu ***nedeniyle birlikte açıklayınız***.
4. İncelediğiniz tüm öğrenci çözüm kâğıtlarını ve video görüntülerini göz önüne aldığınızda, öğrencilerin matematiksel olarak nasıl düşündüğü, neler bildiği ve bilmediği hakkında neler öğrendiniz? Öğrenci çözümlerinden (kâğıtlardan ve videolardan) ***örneklerle*** açıklayınız?
5. Ders sürecinde incelediğiniz kâğıtları ve videoları değerlendirdiğinizde;

   a. Öğrenci çözüm kağıtları *hangi yönleri* ile sizin öğrencilerin düşünme süreçlerini anlamanıza ve yorumlamanıza yardımcı oldu?

   b. Öğrenci videoları *hangi yönleri* ile sizin öğrencilerin düşünme süreçlerini anlamanıza ve yorumlamanıza yardımcı oldu?
6. Grup ortamında çalışmanızın (grup içi tartışmaların) öğrencilerin düşünme süreçleri ile öğrendiklerinize katkı sağladığını düşünüyor musunuz? ***Nasıl ve Neden***?
7. Sınıf tartışmalarınızın öğrencilerin düşünme süreçleri ile öğrendiklerinize katkı sağladığını düşünüyor musunuz? *Nasıl ve Neden?*

APPENDIX G

PRE-SURVEY FORM

Adı Soyadı: **Tarih:**
Doğum Yılı:
Genel Not Ortalaması:

1) Kendinizi kısaca tanıtınız. Akademik özgeçmişiniz (mezun olduğunuz lise, üniversitede aldığınız matematik, matematik eğitimi, diğer eğitim dersleri ve staj dersleri); ilgi alanlarınız; varsa çalışma deneyiminiz ve deneyim yılınız (özel ders, dershane ve ya diğer özel kuruluşlar) hakkında kısaca bilgi veriniz.

2) Matematiği nasıl tanımlarsınız? Sizce matematik nedir?

3) Matematik eğitiminde problem çözmenin yeri nedir? Sizce matematiksel bir problem nasıl olmalıdır?

4) Matematik dersi bilgi ve becerilerinizi nasıl değerlendirirsiniz? İyi ya da zayıf olduğunuzu düşündüğünüz dersler/konular var mı?

5) "Matematiksel modelleme" ifadesini daha önce duydunuz mu? Bu ifadeden ne anladığınızı örneklerle açıklayınız.

6) Hiç modelleme etkinliği çözdünüz mü/uyguladınız mı? Örnekler verir misiniz?

7) Gerçek hayatta karşınıza çıkan problemleri çözmenizde matematik bilginizin size nasıl bir katkı sağladığını düşünüyorsunuz? Örneklerle açıklayınız.

8) Matematik dersinde gerçek hayat problemi kullanımının sağlayacağı kolaylıklar ve zorluklar neler olabilir?

APPENDIX H

POST-SURVEY FORM

Adı-Soyadı: ……………………………………

Genel Değerlendirme Soruları

1. Sizce matematiksel model ve modelleme nedir?
2. Dönem boyunca yaptığımız aktivitelerden en çok hangilerini sevdiniz? Neden?
3. Dönem boyunca yaptığımız aktivitelerden en az hangilerini sevdiniz? Neden?
4. Sizce iyi bir modelleme sorusu nasıl olmalıdır? Ne tür özellikler taşımalıdır?
5. Dönem başından sonuna kadar dersteki yaptığınız çalışmaları düşünerek, modellemeye ve modelleme sorularına yaklaşımınızdaki değişimi değerlendiriniz.
6. Dönem başından sonuna kadar dersteki yaptığınız çalışmaları düşünerek modelleme sorularını çözme becerilerinizdeki gelişimi değerlendiriniz.
7. Bu derste matematik ve matematiksel düşünme adına neler öğrendiniz? Bu öğrenmede modelleme etkinliklerinin nasıl bir rolü oldu?
8. Öğretmen olduğunuzda sınıfınızda modelleme aktivitelerini uygulamayı düşünür müsünüz? Sebepleriyle açıklayınız.
9. Modelleme sorularının sınıf ortamında verimli bir şeklide uygulanabilmesi için öğretmenlerin ne tür bilgi ve becerilere sahip olması gerekir?
10. Matematik dersinde modelleme aktivitelerinin kullanımının sağlayacağı kolaylıklar ve zorluklar neler olabilir?
11. Modelleme sorularını öğrencilerin matematiksel düşünme şekillerini anlama bakımından değerlendiriniz.
12. Bu derste modellemenin matematik öğretiminde kullanımı ile ilgili neler öğrendiğinizi ders kapsamında yapılan çalışmalara da değinerek değerlendiriniz.
    a. Modelleme soruları uygulamaları

b. Grup çalışması

c. Öğrenci düşünme şekillerinin incelenmesi

d. Modelleme sorusu geliştirme

e. Teknoloji kullanımı

f. Uygulama planı hazırlama ve mikro-öğretim

13. Bu dersin geliştirilmesine yönelik neler önerirsiniz?

APPENDIX I

REFLECTION PAPER

Etkinlik Sonrası Düşünce Raporu

Etkinlik sürecinde yaşadıklarınızı tekrar düşünerek, grupça yaptığınız çözüm sürecini mümkün olduğu kadar detaylarıyla ve örneklendirerek açıklayan bir rapor yazmanız beklenmektedir. Raporunuzda problemin çözümünde kullandığınız ***grafik, tablo ve denklemleri*** kullanabilirsiniz. Raporunuzu hazırlarken aşağıdaki soru listesini kullanınız. Listedeki soruların hepsinin cevaplanmasına özen gösteriniz, ancak listedeki sıralamayı takip etmek zorunda değilsiniz. Bununla birlikte***, sorulara karşılık gelmeyen ekleyeceğiniz başka düşünceleriniz olursa*** kendinizi bu sorularla kısıtlamadan etkinlikle ilgili her türlü düşünce ve eleştirilerinizi de yazabilirsiniz.

1. Üzerinde çalıştığınız problemin veya incelediğiniz durumun tanımı:
   Çalıştığınız problem (veya durum) neydi? Bu çalışmadaki amacınız neydi?

2. Problemi çözmeye başlamadan önceki bireysel düşünceleriniz.
   *Problem hakkında ne düşündünüz? Problem durumunu tam anlayabildiniz mi? Problemin çözümü için ilk aklınıza gelen yol neydi (**yanlış da olsa belirtiniz**)? Problemi okuduktan sonra soruyu çözerim ya da çözemem diye düşündünüz mü? Böyle düşünmenizin sebepleri nelerdir? vb.*

   3., 4. ve 5. sorularda çözüm süreciyle ilgili sizin düşünceleriniz sorulduğundan, cevaplarken grup çalışma sürecinde geçtiğiniz aşamalara ve önemli dönüm noktalarına vurgu yaparak yanıtlayınız. Burada sizden beklenen grup çözüm ve tartışma sürecini raporunuza en iyi şekilde yansıtmanızdır.

3. Problemin çözüm süreci ve bu süreç hakkındaki düşünceleriniz. Çözümün başından sonuna kadar geçtiğiniz süreçleri, grubunuzda ortaya çıkan farklı düşünme biçimlerini tanımlar mısınız? (***yanlış da olsa belirtiniz***)

   a. Problemin ne olduğunu anladıktan sonra soruyu çözmeye nasıl başladınız?

   b. Problemin çözümünü veya durumun analizini nasıl yaptınız? Problem üzerinde uğraşırken karşılaştığınız zorluklar ve kolaylıklar nelerdi? Tıkandığınız yerler var mıydı? Bunlar nelerdi? Tıkandığınız noktaları aşmak için ne yaptınız? Okuyup anladığınızı düşündüğünüz fakat çözerken takıldığınız ve soruyu tekrar okuyup anlamaya çalıştığınız noktalar var mıydı? Bunlar hangileriydi açıklayarak yazınız.

   c. Problemi çözerken problem durumu ile ilgili dikkate aldığınız durumlar ve varsayımlar nelerdi? Bu varsayımları nasıl belirlediniz? Belirlemede neler etkili oldu (grup tartışması, önbilgiler vs.)?

   d. Problemi çözerken kullandığınız matematiksel kavramlar, fikirler ve stratejiler nelerdi?

   e. Problemin anlaşılması, çözümü ve doğrulanması aşamalarında matematiksel gösterimlerden (grafik, tablo, resim, vs…) nasıl yararlandınız?

4. Çözüme ulaşamadığınızda çözüm yolunu nasıl değiştirdiniz? Ne yapıyorum ve nasıl yapıyorum diye durup çözüm basamaklarını kontrol ettiniz mi? Açıklayınız.

5. Bulduğunuz sonucu nasıl yorumlarsınız? Çözümünüzün geçerliliğini veya başka durumlar için kullanılabilir olduğunu nasıl gösterirsiniz?

6. Bu soruyu çözdükten sonra neler öğrendiniz? Soru ve çözüm yollarınız hakkında kendi performansınızı nasıl buldunuz? Kısaca yazınız.

7. Diğer grupların çözüm yaklaşımlarını dikkate aldığınızda kendi çözümünüzü nasıl geliştirirdiniz? Açıklayınız.

8. Bu derste daha önce yapılan etkinlikler bu haftaki çözüm yaklaşımınıza nasıl katkı sağladı? Açıklayınız (*İkinci modelleme sorusu çözüldükten sonra cevaplanacak*).

9. Bu problemin çözümü sürecinde ön plana çıkan matematiksel kavram ve fikirler nelerdi?

    a. Bunlar önceden bildiğiniz kavramlar mıydı? Yeni bir kavram ve fikir öğrendiniz mi?

    b. Bildiğiniz bir kavram ise bu kavramlarla ilgili bilgilerinizde bir değişiklik oldu mu? Ne tür değişiklikler oldu?

10. ***Bir öğretmen gözüyle bakmanız gerekirse;***

    a. Bu problemi sınıf ortamında uygularsanız öğrencilerin hangi kazanımlara ulaşmasını beklersiniz?
    b. Bu soruya öğrencilerin getireceği çözüm yaklaşımları neler olabilir?
    c. Bu problemi sınıf ortamında nasıl uygularsınız?
    d. Böyle bir sınıf uygulamasında öğrenciler
        i. Nerelerde ve ne tür zorluklar yaşayabilirler?
        ii. Ne tür hatalar yapmasını beklersiniz?
    e. Öğrencilerin yaptıkları hataları ya da yaşadıkları zorlukları aşması için neler yaparsınız?

11. Problemi modellerken/çözerken öğrendiğiniz teknolojileri/yazılımları kullandınız mı? Kullandıysanız hangilerini, nasıl kullandınız? Teknolojinin katkısı ve/ya sınırlılıkları hakkında neler söyleyebilirsiniz?

12. Bu etkinlikte grup çalışmasının sizin açınızdan verimli olduğunu düşünüyor musunuz? Nasıl? Soruyu bireysel çözmeye çalışsaydınız çözümünüzde nasıl bir farklılık olurdu?

13. Problemi çözmeye çalışırken sizi bir sonuca ulaştırmayan farklı yollar denemiş olabilirsiniz. Bu yollar bazen problemi anlama ve çözüme gitmede yardımcı da olabilmektedir. Böyle ilginç yan yollar var ise raporunuza EK olarak koyunuz.

14. Bu problemi, bu güne kadar gördüğünüz problem türleri ile benzerlikleri farklılıkları açısından değerlendiriniz.

APPENDIX J

A GENERAL EVALUATION PRESENTATION OF REFLECTION PAPERS

1. Dosya adı aşağıdaki gibi verilmeli:

ÖdevAdi_Ad_Soyad_OgrenciNumarasi

Örnek: DusunmeRaporu_1_Sukran_Yilmaz_Caglayan_987654321.doc

2. Düşünce raporunun başına raporun hangi etkinlikle ilgili olduğu ve tarih bilgileri eklenmeli.

Ad Soyad:
Etkinliğin Adı:
Tarih:

3. Rapor soru-cevap şeklinde değil düz metin olarak hazırlanmalı. Raporda, metnin organizasyonuna ve ifadelerin doğruluğuna dikkat edilmeli. Örneğin, metinde farklı konulara geçiş yapıldığında paragraf başı yapma ve yazım yanlışlarını kontrol etme.

4. Rapor, size verilen soru listesindeki tüm sorulara cevap verilecek şekilde hazırlanmalı. Listedeki sorulara karşılık gelmeyen ekleyeceğiniz başka düşüncelerinizi de yazabilirsiniz.

5. Sorunun çözümüyle ilgili olarak önce bireysel olarak ne düşündüğünüzü, hangi çözüm yöntemini geliştirdiğinizi belirtiniz. Daha sonra grup çözüm sürecini açıklayınız.

6. Ortaya atılan düşünce ve iddialar açıklayıcı ifadeler ve örneklerle ile desteklenmeli.

> "Grup çalışması olması, farklı bakış açılarını görüp soruya değişik açılardan yaklaşım sağlaması açısından çok yararlı oldu."
>
> "Bu soruyu çözdükten sonra bu tarz problem temelli sorulara nasıl yaklaşmam gerektiği konusunda daha iyi fikirler edindim. Örneğin, farklı tablolar elde edip farklı düşünceler geliştirmem gerektiği ve bunu karşımdaki insanları ikna edecek şekilde açıklamam gerekliliğinin ne kadar önemli olduğunu gördüm."

7. Varsayımların gerekçeleri açıkça ifade edilmeli.

8. Grup tartışma süreci, hangi noktalarda nasıl karar verildiği, neden karar değiştirildiği, farklı fikirlerin neler olduğu ve nasıl fikir birliği sağlandığı gibi konular daha detaylı yazılmalı.

> "Grup çalışmasının çok faydalı olduğunu düşünüyorum çünkü bazen tıkandığımız yerlerde birimizin ortaya attığı bir fikri diğer iki kişinin bilgisi sayesinde daha kullanışlı bir hale getirerek çözüme yönelik büyük adımlar atmış olduk."
>
> "İlk olarak 3 ayı da bir bütün olarak incelemeyi düşünmüştük fakat sonradan bunun yanlış olacağını anlayıp her birini ayrı ayrı düşünüp yoğun, orta ve durgun zamanlarla beraber incelemeye karar verdik."

9. Teknolojinin sağladığı kolaylıkların neler olduğu spesifik olarak yazılmalı

> "Sorunun çözümünde Excel kullandık ve bu bize inanılmaz kolaylık sağladı"

10. Matematiksel denklem, grafik vs. gibi Word üzerinde yazmakta zorlandığınız şekil veya grafikler varsa size e-mail ile gönderilen çözümünüzden kesip kullanabilirsiniz.

11. Öğretmen gözüyle yapılan değerlendirmede, öğrencilerin kazanımları ve yaşayacakları zorluklar konusu ele alınmış. Bununla birlikte matematiksel açıdan kazanımlar ve zorluklarla ilgili durumlar yeterince irdelenmemiş.

APPENDIX K

SEMI-STRUCTURED INTERVIEW QUESTIONS

Sorular

1. Bu haftaki soru ile ilgili genel olarak ne düşünüyorsunuz?
2. Problemin çözüm süreci ve bu süreç hakkındaki düşüncelerinizi almak istiyoruz. Çözümün başından sonuna kadar geçtiğiniz süreçleri anlatır mısınız (*yanlış da olsa belirtiniz*)?
   a. *Problem durumunu tam anlayabildiniz mi? Eğer anlayamadıysanız, anlamak için neler yaptınız?*
   b. *Problemin çözümü için ilk aklınıza gelen yol neydi (yanlış da olsa belirtiniz)?*
   c. Problemi formüle ederken problem durumu ile ilgili dikkate aldığınız durumlar ve varsayımlar nelerdi? Bu varsayımları belirlemede ne etkili oldu?
   d. Matematiksel kavramlar, fikirler ve kullandığınız stratejiler nelerdi?
   e. (Raporlar ve çözüm kâğıtları incelendikten sonra) Şu yöntemi kullanmışsınız, neden bunu kullandınız? …
   f. Problem üzerinde uğraşırken karşılaştığınız zorluklar nelerdi?
   g. Bunları aşmak için ne yaptınız?
   h. Çözüm sürecinde grubunuzda ortaya çıkan farklı düşünceleri anlatır mısınız?
   i. Derste geliştirdiğiniz çözüm ile ilgili şuanda ne düşünüyorsunuz? Diğer grupların çözüm yaklaşımlarını nasıl değerlendirdiniz. Kısaca açıklar mısınız?
3. Size göre çözüm sürecinizi olumlu ya da olumsuz yönde etkileyen faktörler nelerdi?
4. Bu derste daha önce yapılan etkinlikler bu haftaki çözüm yaklaşımınıza nasıl katkı sağladı? Açıklayınız (*Etkinlik sonrası düşünce raporunda yetersiz veya eksik ifadelerin anlaşılması için sorulacak*).
5. Bu probleme getirdiğiniz çözümü ve matematiksel fikri benzer başka durumlara genelleyebilir misiniz? Örnek verir misiniz?
6. Tüm grup çözümlerini de göz önüne aldığınızda ve bu problemde ön plana çıkan matematiksel kavram ve fikirleri düşündüğünüzde;
   a) Önceden bildiğiniz kavramlar mıydı?
   b) Yeni bir kavram ya da bir fikir öğrendiniz mi?
   c) Bu kavramlarla ilgili sizin bilgilerinizde bir değişiklik oldu mu?
7. Bu etkinlikte grup çalışmasının sizin açınızdan verimli olduğunu düşünüyor musunuz? Nasıl?
   a) Soruyu bireysel çözmeye çalışsaydınız çözümünüzde nasıl bir farklılık olurdu? (Opsiyonel)
8. Bu problemi bu güne kadar gördüğünüz problem türleri ile benzerlikleri farklılıkları açısından değerlendiriniz. *Etkinlik sonrası düşünce raporunda yetersiz veya eksik ifadelerin anlaşılması için sorulacak*).

9. *Bir öğretmen gözüyle bakmanız gerekirse;*
   f. Bu problemi sınıf ortamında uygularsanız öğrencilerin hangi kazanımlara ulaşmasını beklersiniz?
   g. Bu problemi sınıf ortamında nasıl uygularsınız?
   h. Bu soruya öğrencilerin getireceği çözüm yaklaşımları neler olabilir?
      i. Nerelerde ve ne tür zorluklar yaşayabilirler?
      ii. Ne tür hatalar yapmasını beklersiniz?
   i. Öğrencilerin yaptıkları hataları ya da yaşadıkları zorlukları aşması için neler yaparsınız?

10. Eklemek istediğiniz başka bir şey var mı?

APPENDIX L

APPLICATION PLAN FOR PROSPECTIVE TEACHERS

UYGULAMA PLANI

Uygulama planını hazırlayanlar:
Modelleme Sorusunun Adı:
Sınıf:
Öğrenme Alanı:
Alt Öğrenme Alanı:
Toplam Süre:

- Derste yapılacakların açıklanması için süre:
- Sorunun çözülmesi için süre:
- Sunumlar için süre:
- Dersi toparlamak için süre:

Kazanımlar:

Öğrenciler tarafından kullanılması beklenen beceriler:

Araç ve Gereç:

HAZIRLIK (Sorunun uygulama öncesi ile ilgili plan aşağıdaki başlıklar çerçevesinde oluşturulabilir.)

- *Soruda öne çıkan matematiksel kavramlar ve bu kavramlar arasındaki ilişkiler*
- *Sorunun uygulamasından önce öğrencilerin soruda öne çıkan matematiksel kavramları anlayabilmesi için gerekli olan ön bilgiler*
- *Hazırlık aşamasında dikkat edilmesi gereken diğer hususlar*

UYGULAMA (Sorunun uygulaması ile ilgili plan aşağıdaki başlıklar çerçevesinde oluşturulabilir.)

- *Gruptaki kişi sayısının ve gruptaki kişilerin nasıl belirleneceği*
- *Öğrencilere derste yapılacakların açıklanması*
- *Sorunun anlaşılmasını sağlamak ve soruya ısındırmak için yapılabilecekler*
- *Uygulamada öğrencilerin kullanabilecekleri çözüm stratejileri*
- *Uygulamada öğrencilerin nerelerde* ve ne tür hatalar yapabilecekleri ve *yapabilecekleri hataların üstesinden gelmek için kullanabilecek yöntemler*
- *Uygulamada öğrencilerin nerelerde* ve ne tür zorluklar yaşayabilecekleri ve yaşanabilecek zorlukların *üstesinden gelmek için kullanabilecek yöntemler*
- *Öğrencilere soruyu çözerken sorulabilecek sorular ve bu soruları sormadaki amaçlar*
- *Öğrenciler soruyu çözerken kişileri ve grupları puanlandırma kriterlerinin neler olabileceği ve bu kriterlerin öncelik sırası*
- *Çözümlerin hangi sıraya göre, ne şekilde sundurulacağı ve nedenleri*
- *Öğrenciler çözümlerini sunarken kişileri ve grupları puanlandırma kriterlerinin neler olabileceği ve bu kriterlerin öncelik sırası*
- *Sorunun çözülmesinin ve çözümlerin sunulmasının ardından dersin nasıl toparlanılacağı*
- *Bu sorunun uygulanmasında öğretmenin dikkat etmesi gereken diğer hususlar*

APPENDIX M

MICROTEACHING (IMPLEMENTATION EXPERIENCE) PRESCRIPTION

Etkinliği Uygulaması Sürecinde Yapılacak İşler

Uygulama Öncesinde;

- Modelleme sorusu uygulama planını hazırlayınız ve çıktısını derse getiriniz.
- Modelleme sorusunun son hali üzerinde görüş almak isteyen gruplar uygulama öncesinde bizimle iletişime geçiniz.
- Derste kullanılacak teknoloji, materyal vs. gibi konularla ilgili hocaları dersten önce bilgilendiriniz.

Uygulama Esnasında;

- Her bir grubun uygulama planı 60 dakikalık bir ders süreci için yapılmalı.
- Grup elemanları etkinlik uygulaması için görev dağılımı yapmalı (uygulamayı yapma, uygulama ile ilgili sorunları not alma, sorunun çözümü ile ilgili ortaya çıkan yaklaşımları ve fikirleri not alma v.)

Uygulama Sonrasında;

- Uygulamada çıkan sonuçlara göre modelleme sorusu ve uygulama planını gözden geçirerek son halini geliştiriniz.
- Proje raporunu ekteki (diğer sayfada) yönergeye göre <u>bireysel olarak</u> hazırlayınız. Proje raporu ile birlikte teslim edilecek dokümanlar: Modelleme sorusu, sorunun künyesi, sorunun çözümü, uygulama planının son hali (Bu belgelerin grup elemanlarından sadece bir kişi tarafından teslim edilmesi yeterlidir).

Proje Raporu Yönergesi

1. Modelleme sorusunu geliştirme sürecinin anlatılması;
   - Sorunuzu hazırlarken nasıl bir yöntem izlediniz (*bir matematiksel fikirden yola çıkma, bir gerçek hayat durumundan yola çıkma vs.*)?
   - Süreçte bu soruya karar vermeden önce başka hangi soru veya fikirler üzerinde çalıştınız?
   - Bu soruya nasıl karar verdiniz?
   - Geliştirdiğiniz sorunun modelleme sorusu olmasını sağlamak için göz önünde bulundurduğunuz başlıca kriterler nelerdi? Bunlara nasıl karar verdiniz?
   - Soruyu geliştirme sürecinde grup olarak nasıl çalıştınız?
   - Soruyu geliştirme ve örnek çözümler hazırlama sürecinde ne tür zorluklar yaşadınız? Bu zorlukları nasıl aştınız?
2. Sorunun uygulama süreci;

   Uygulama öncesi
   - Uygulama planınızı nasıl oluşturdunuz? Planı oluştururken nelere dikkate ettiniz?

   Uygulama esnasında
   - Sorunun anlaşılması ile ilgili ne tür sorunlar çıktı? Bunları nasıl aştınız?
   - Öğrenciler nerelerde ve ne tür zorluklar yaşadılar?
   - Öğrencilerin yaptıkları hataları ya da yaşadıkları zorlukları aşması için neler yaptınız?
   - Sorunun çözümü ile ilgili ne tür yaklaşımlar ve fikirler çıktı?
   - Grupları dolaşırken ne tür sorular geldi? Bu soruları nasıl cevaplandırdınız?

Uygulama sonrası

- Sorunuzda ne tür değişiklikler yapmaya karar verdiniz? Neden?
- Uygulama planınızda nasıl değişiklikler yaptınız?

3. Genel değerlendirme;
    - Etkinlik uygulama sürecinde uygulamayı yapan kişi/grup olarak rolünüzü değerlendiriniz. Örneklerle açıklayınız.
    - Sınıf uygulaması sürecinde modelleme etkinliklerinin matematik öğretiminde kullanılması ile ilgili ne tür izlenimler edindiniz? Örneklerle açıklayınız.
    - Sınıf uygulama sürecinizi düşündüğünüzde bu tür uygulamalar yapacak öğretmenlere neler tavsiye edersiniz (*uygulama planı hazırlarken nelere dikkat etmeli, uygulama sürecini nasıl yönetmeli ve nelere dikkat etmeli, dersin toparlanmasında nelere dikkat etmeli vs.*)?

APPENDIX N

CODE LIST AND SAMPLE EXCERPTS

Table 23 List of codes with their definitions and sample excerpts

Codes with Their Definitions	Sample Excerpts
Conceptions about Mathematical Modeling and about the Use of Modeling Activities: this primary code illustrates evidence of the prospective teachers' conceptions about mathematical modeling and the use of mathematical modeling activities in the classroom.	
Conceptions and definitions of mathematical modeling: conceptions and definitions of mathematical modeling describe prospective teachers' conceptions and definitions of mathematical before and after the implementation.	
Using concrete manipulative and visualization: *evidence that the prospective teachers' perceived mathematical modeling as using concrete objects and visualization for teaching and learning mathematics (Before implementation).*	"I think mathematical modeling contains the methods and techniques which are used for facilitation, using concrete manipulatives and visualization of teaching and comprehensibility during the mathematics education."
Relating mathematics with real life: *evidence that the prospective teachers conceived mathematical modeling as teaching and learning mathematics relating with daily life (After implementation).*	"The presentation of a mathematical subject with daily life problem and noticing that there are solutions of many real life problems that are encountered us associated with mathematics."
Conceptions about the use of mathematical modeling activities: this code describes prospective teachers' conceptions about the use of mathematical modeling activities for their future classrooms.	
Why to use modeling activities: *evidence of prospective teachers' aims for using mathematical modeling activities.*	"It seems that I would use these kinds of activities in the application level."

Table 23 (continued)

Where to use modeling activities: *evidence of the place that prospective teachers think to use these activities.*	"As a teacher if I wanted to apply this question in a classroom environment, I would ask the students solve it individually after my explaining the angles in circle and solving a few questions about angles in circle and trigonometric calculations"
How to use modeling activities: *evidence that the method of prospective teachers think to use these activities.*	"Interviewer: You decided to use this activity. How would you do it? TC23: I would use group study."
When to use modeling activities: *evidence that prospective teachers think to use these activities at a certain frequency.*	"When I am a teacher, I implement these activities quite often. Because I mentioned about my own attainments above. I want my students to have similar attainments"
Thinking about Knowledge of Mathematical Modeling and Modeling Activities: this primary code describe evidence of the prospective teachers' thinking about the knowledge of mathematical modeling and mathematical modeling activities and their comments about what that knowledge include.	
Thinking about general knowledge of mathematical modeling: this code describes prospective teachers' opinions on what mathematical modeling knowledge involves in general.	"This is creating materials that enable the students to understand the subjects more easily by making that statement more concrete and to demonstrate how a mathematical statement works in real life" (Before the implementation). "Initially, I noticed that I do not know the concept modeling fully and I even misknow it. However, now I know what a modeling question means. After the questions that I have discovered in this lesson, questions of daily life situations revive in my mind. For example, I would not think like this as a mathematician when I went to an amusement park; now I can look and think more as a mathematician" (After the implementation).
Thinking about the knowledge about the nature of mathematical modeling activities: this code describes prospective teachers' ideas about the nature of mathematical modeling activities and their properties.	
Reality: *evidence that prospective teachers considered this as a property of mathematical modeling activities in relation with daily life situations and meaningfulness.*	"It should also reflect the problems in daily life, namely it should not be isolated from real life".

Table 23 (continued)

Open-Endedness: *evidence that prospective teachers thought that mathematical modeling activities are open-ended.*	"The problems given us till now were absolute problems which have unique solutions based on variables under control. However, it was an open-ended question that requires our accepting something to reach a solution and that enables us to solve it upon this assumption. Therefore, it was a problem which has solutions shaped by the resolvent's (person) idea rather than having one and absolute solution".
Including more than one mathematical concept: *evidence that prospective teachers believed that these activities involve more mathematical concepts in their structure.*	"Mathematical concepts that we used were triangle geometry, trigonometric functions, ratio, and proportion. This problem in which we use many mathematical concepts may bring the students in lots of things mathematically".
Having diverse solution procedures and cyclic structure of solution process: *evidence that prospective teachers thought that these activities could have more ways of solutions and these solution ways may consist many "returns".*	"Nevertheless, we made many calculation errors. We used different wrong ways of solutions, we had great difficulties at first and for this reason, and we went back to beginning because of calculation errors. We handled the question and say "we found the correct answer" all the time but our supervisor came and told us that there was a mistake in somewhere within the question. Then we handled the question again and there was some problems in minor things like multiplying and thus we went back to beginning".
The need for a solution or model: *evidence that prospective teachers believed that mathematical concepts should be taught with the condition that students feel the necessity for these concepts.*	"In a good modeling question, a student should feel that s/he needs to build a new structure that will be an answer to the question asked to the student's himself or herself and then the student should form a model".
Generalizability and being prototype: *evidence that prospective teachers thought that the solutions of these activities should be replicable and reused for similar situations.*	"Interviewer: Do you think can you generalize the solution to the similar situations in daily life? PT17: Yes, it can be generalized because that is a general solution method. That is, if there exist a firm like that and this firm can hire employees for the work".
Distinctions from traditional word problems: *evidence that prospective teachers thought that there existed many differences between mathematical modeling activities and traditional word problems. These differences might be a property of modeling activities.*	"This problem is a kind of problem that can be encountered in nearly all geometry books in terms of solution. But the style of question's being given us, along with adding difference to the question also can make the question more complicated with the concepts like " angle's being widest" ".

Table 23 (continued)

Thinking about Knowledge about Implementation of Modeling Activities in the Classroom Setting: this primary code illustrates evidence of the prospective teachers' ideas, opinions, thinking about the knowledged that is needed to implement mathematical modeling activities in the class with their own effectively.	
Views about knowledge and qualifications needed for teachers: this code describes prospective teachers' ideas, opinions, and comments about what knowledge and qualifications teachers need to carry out mathematical modeling activities in classroom environment.	
Views about the knowledge about the students' way of thinking: *evidence that prospective teachers thought that teachers should know students' way of thinking.*	"As far as I saw in students' videos and solution papers of the students, the students can see the question in its different aspects. The ones that reach the solution are quite a lot. Modeling questions are quite effective to understand the mathematical thinking ways of the students. Their solutions for the question show us what they know or do not know".
Views about the knowledge about classroom management: *Prospective teachers' ideas about what knowledge teachers should have when they conduct a mathematical modeling activity.*	"First of all, teachers should know characteristics of a class. Then, they should prepare implementation plan according to the level of the class and they should manage the plan accordingly the level of the class. At first, the teacher should learn whether the question is understood or not by going near each of the groups. If there are problematic parts in the question, the teacher should help the students by guiding them. After that, the teacher should give priority to the groups whose solutions are different in the process of the students' presentations of their solutions and the teacher should pass to phase of summing up after the students' presentations are completed. Finally, in this phase, ideas and opinions of the students should be reminded and correct or incorrect ones should be shared with students and then the lesson should be finished".
Ideas about implementation experience: *Prospective teachers' comments and ideas about their own implementation experience with their developed mathematical modeling activity.*	"Solution approaches for the question were really different and fine. We had already predicted possible a few situations as we practiced this question in many different groups. But in classroom practice solutions different from our predictions came. Main solution idea was the situation of arranging the equal chances that correspond to the equal points to equal areas. Solutions that verifies this would be correct and valid solutions".

Table 23 (continued)

Impressions for the implementation of modeling activities: *Prospective teachers' feelings about conducting an own developed modeling activity and the process of mathematical modeling.*	
Opinions on group work: this code describes prospective teachers' comments and ideas about group working in the modeling process.	
Efficient: *Prospective teachers believed that group work in modeling activity is effective.*	"Interviewer: Okay, PT24, how was the group working for you? PT24: Group working was good. I like to work with group because distinct ideas, different perspectives emerges in the group working. One of my friends sees the point that I do not see. I could complete the missings of my friends or vice versa, thus the solution could be found easily and quickly. Sometimes the solution could be found late because I could not see the point and so my friends which results in failure. There exists a time difference between individual working and group working. In this activity, we produced a good solution".
Inefficient: *Prospective teachers' thought that group work in modeling process is not effective.*	"If they were three or four members in each group, they do not respect each other due to the reasons such as not mating enough. One of them could be dominant or diffident".
Views about the relationship between technology and modeling: this code describes prospective teachers' ideas about the association between technology and mathematical modeling.	
Contributions of technology: *evidence that prospective teachers believed that technology assisted students in modeling process.*	"Technologically, we used calculators for the solution of the problem. If we had not used calculators, no matter how the equation was with one unknown in order to solve such an equation whose quadratic multiples were not whole numbers, we would have needed to spend too long time on the problem. In this respect, calculator's contribution was great and at the same time, we made advantage of calculators to calculate trigonometric values".
Obstacles of technology: *evidence that prospective teachers conceived that technology hinders the modeling process.*	"I encountered with many difficulties while studying on the problem. The most important one among these problems was to calculate an angle whose sinus value was known. Because some calculators while giving results as radian, it was necessary to turn the result into degree and use of calculators was quite complicated".

Table 23 (continued)

Views about the place and importance of mathematical modeling in teaching and learning mathematics: this primary codes describes prospective teachers' ideas and views about the mathematical modeling in the teaching and learning of mathematics.	
Advantages of mathematical modeling: *evidence that prospective teachers believed that mathematical modeling has benefits in mathematics teaching and learning.*	"After solving this activity, I discovered how to use trigonometric attainments that I acquired during my education in daily life. In other words, I realized that these attainments were not only for being successful in exams but also for making our life easier. Because like many students, I was also curious about the use or benefit of this subject in daily life during learning process of trigonometry subject. This activity was one that I would use to answer one of my curious students when I could become a teacher. Therefore, this question (activity) was quite useful".
Disadvantages of mathematical modeling: *evidence that prospective teachers thought that mathematical modeling has drawbacks in the teaching and learning of mathematics.*	"Difficulties could be experienced in terms of time. Because teachers have problems like completing the curriculum in time. The students may not want to do this activity or even if this activity is implemented in a crowded class, there can be problems in terms of evaluation and feedback".

CURRICULUM VITAE

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EDUCATION

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Master of Science and Bachelor of Science (Integrated), Mathematics Education, Boğaziçi University, Istanbul. July, 2005.

WORKING EXPERIENCE

Years	**Place**	**Enrollment**
2007-…	Adıyaman University, Adıyaman	Research Assistant

PUBLICATIONS

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