STUDENTS' INTUITIVELY-BASED MISCONCEPTIONS IN PROBABILITY: TEACHERS' AWARENESSES AND TEACHING PRACTICES IN MIDDLE AND HIGH SCHOOLS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

MEHMET FATİH ÖÇAL

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY
IN
SECONDARY SCIENCE AND MATHEMATICS EDUCATION

AUGUST 2014

Approval of the thesis:

STUDENTS' INTUITIVELY-BASED MISCONCEPTIONS IN PROBABILITY: TEACHERS' AWARENESSES AND TEACHING PRACTICES IN MIDDLE AND HIGH SCHOOLS

submitted by **MEHMET FATİH ÖÇAL** in partial fulfillment of the requirements for the degree of **Doctor of Philosophy in Secondary Science and Mathematics Education Department, Middle East Technical University** by,

Prof. Dr. Canan Özgen __________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Ömer Geban __________
Head of Department, **Secondary Science and Math. Edu.**

Assoc. Prof. Dr. Ayhan Kürşat Erbaş __________
Supervisor, **Secondary Science and Math. Edu. Dept., METU**

Examining Committee Members:

Assoc. Prof. Dr. Erdinç Çakıroğlu __________
Elementary Edu. Dept., METU

Assoc. Prof. Dr. Ayhan Kürşat Erbaş __________
Secondary Science and Math. Edu. Dept., METU

Assoc. Prof. Dr. Bülent Çetinkaya __________
Secondary Science and Math. Edu. Dept., METU

Assoc. Prof. Dr. Emin Aydın __________
Secondary Science and Math. Edu. Dept., Marmara University

Assist. Prof. Dr. Ömer Faruk Özdemir __________
Secondary Science and Math. Edu. Dept., METU

Date: 29.08.2014

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Mehmet Fatih Öçal

Signature:

ABSTRACT

STUDENTS' INTUITIVELY-BASED MISCONCEPTIONS IN PROBABILITY: TEACHERS' AWARENESSES AND TEACHING PRACTICES IN MIDDLE AND HIGH SCHOOLS

Öçal, Mehmet Fatih

Ph.D. Department of Secondary School Science and Mathematics Education
Supervisor: Assoc. Prof. Dr. Ayhan Kürşat Erbaş

August 2014, 254 pages

The purpose of this study was to investigate 8th and 11th grade students' intuitively-based misconceptions in probability and to what extend their teachers' awareness and teaching practices in their regular instructions resolve these misconceptions. This study was designed as a multiple case study with middle and high school mathematics teachers and their students as cases. The participants were two middle school teachers and their 59 students in the first case, and three high school teachers and their 59 students in the second case.

Data were collected through interviews, classroom observations, and pre- and post-tests consisting of open-ended diagnostic questions about intuitively-based misconceptions in probability. While the interviews and the classroom observations were analyzed through content analysis method, students' responses in the diagnostic test were analyzed through descriptive analysis method. Frequency tables were also provided for the findings from the test.

Students' responses to open ended questions showed that students had various intuitively-based misconceptions including availability and representativeness heuristics, simple and compound events, conjunction fallacy, time-axis probability, and misconceptions from Stavy and Tirosh's theory of intuitive rules.

The findings gathered from interviews indicated that teachers had awareness of and knowledge of methods for teaching probability, students' difficulties in probability, and the possible reasons for their difficulties. Based on the classroom observations, it was found that what teachers aware of and what teaching practices they performed in the classrooms were contradictory. Although there were many teaching practices such as developing concepts for the content, constructing relation between the probability and other topics, using physical materials, giving related examples, solving related questions, and constructing shortcuts for the event types and formulas, it was observed that teachers teaching practices was not effective in resolving students' intuitively-based misconceptions. According to the post-test results, it was observed that some misconceptions appeared in the pre-tests slightly decreased among both middle and high schools, while some others stayed still. In the case of outcome approach misconception, the occurrence frequency among students increased slightly after students received regular instruction.

In conclusion, teachers in this study focused on the high school and university entrance exams in their instructions and did not have much effort on resolving students' intuitively-based misconceptions. These findings implied that teachers focused on completing the curriculum (or course content) before the academic year finishes instead of considering students' comprehension of the ideas and concept, and possible misconceptions in probability. Teachers should be equipped with the knowledge of students' cognition and prepare their instructional practices accordingly.

Keywords: Mathematics Education, Intuitively-based Misconceptions, Probability.

ÖZ

ÖĞRENCİLERİN OLASILIKLA İLGİLİ SEZGİ TEMELLİ KAVRAM YANILGILARI: ORTAOKUL VE LİSE MATEMATİK ÖĞRETMENLERİNİN FARKINDALIKLARI VE ÖĞRETME PRATİKLERİ

Öçal, Mehmet Fatih
Doktora, Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü
Tez Yöneticisi: Doç. Dr. Ayhan Kürşat Erbaş

Ağustos 2014, 254 sayfa

Bu çalışmanın amacı 8 ve 11. sınıf öğrencilerinin, olasılık konusundaki sezgi temelli kavram yanılgılarını ile öğretmenlerinin farkındalıklarının ve olağan derslerindeki öğretme pratiklerinin bu kavram yanılgılarını ne derece çözdüğünü belirlemektir. Bu çalışma çoklu örnek olay (durum) çalışması olup, durumlar ortaokul ve lise öğretmenleri ile öğrencilerinden oluşmaktadır. Birinci durum iki ortaokul öğretmeni ve 59 öğrenciden oluşurken, ikinci durum üç lise öğretmeni ve 59 öğrenciden oluşmaktadır.

Veriler, görüşmeler, sınıf gözlemleri ve açık uçlu sorulardan oluşan ön ve son testlerden elde edilmiştir. Görüşmeler ve sınıf gözlemleri içerik analiz metodu ile analiz edilirken, öğrencilerin açık uçlu sorulardan oluşan soru kâğıdına verdikleri cevaplar betimsel analiz metodu ile analiz edilmiştir. Açık uçlu sorulardan oluşan soru kâğıdından elde edilen veriler için frekans tabloları oluşturulmuştur.

Öğrencilerin açık uçlu sorulara verdikleri cevaplar, öğrencilerin birçok sezgi temelli kavram yanılgılarının olduğunu göstermiştir. Bu kavram yanılgılarının içerisinde, hazır bulunma ve temsil etme sezgiselleri, basit ve bileşik olaylar, birleşme yanılgısı, zaman ekseni olasılığı ve Stavy ve Tirosh'un sezgi kuralları teorisindeki kavram yanılgıları gözlemlenmiştir.

Görüşmelerden elde edilen bulgular öğretmenlerin, öğretme metotları, öğrencilerin olasılıktaki zorluları ve bu zorlukların olası sebepleri hakkında bazı bilgilere sahip olduklarını göstermiştir. Sınıf gözlemlerine bağlı olarak, öğretmenlerin sahip oldukları bilgiler/farkındalıkları ile sınıftaki öğretme pratikleri arasında çelişki olduğu gözlemlenmiştir. Öğretmenlerin, içerik için kavram oluşturmaya, olasılık ile diğer konular arasında ilişki kurmaya, fiziksel materyal kullanmaya, ilgili örnek vermeye, ilgili soru çözmeye ve formüller ile olay çeşitleri için kısayollar oluşturmaya yönelik birçok öğretme pratikleri olmasına rağmen, öğretme pratiklerinin öğrencilerin sezgi temelli kavram yanılgılarını çözmede etkili olmadığı görülmüştür. Son test sonuçlarına göre, bazı kavram yanılgılarının hem ortaokul hem de lise öğrencileri arasında hafifçe azaldığı gözlemlenirken, bazı kavram yanılgılarında değişim olmadığı görülmüştür. Öğrenciler olağan derslerini işledikten sonra, öğrenciler arasında sonuç yaklaşımı kavram yanılgısının meydana gelme frekansının hafifçe arttığı gözlemlenmiştir. Sonuç olarak, öğretmenler olağan ders işleyişlerinde lise/üniversite giriş sınavlarına odaklanmışlardır. Ayrıca, öğrencilerin sezgi temelli kavram yanılgılarını çözmeye yönelik fazla çabalarının olmadığı gözlemlenmiştir. Çalışmanın bulgularına göre, öğretmenler, olasılıkta öğrencilerin ihtiyaçlarını, anlamalarını ve kavram yanılgılarını dikkate almak yerine akademik yılda müfredatı tamamlamaya odaklandıkları görülmüştür. Öğrencilerin bilişleri hakkında öğretmenlerin bilgi sahibi olmaları ve öğretmenlerin, öğretim sürecindeki uygulamaları bunlara göre hazırlamaları gerekmektedir.

Anahtar kelimeler: Matematik Eğitimi, Sezgi Temelli Kavram Yanılgıları, Olasılık.

To my wife, son, and parents.

ACKNOWLEDGMENTS

This dissertation is a culmination of so many people's thoughts, suggestions, influences, questions, and time that I don't wish take for it alone. I hope to acknowledge all with a fear that I will forget someone. Therefore, I owe my gratitude to all those people who made this dissertation possible.

Firstly, I would like to thank to my advisor Accos. Prof. Dr. Ayhan Kürşat Erbaş for his wonderful guidance and effective feedback on each manuscript of this dissertation. His patience, support and expertise helped me overcome many crisis situations and finish this dissertation.

Besides my dissertation advisor, I am also grateful to other jury members, Assoc. Prof. Dr. Erdinç Çakıroğlu, Assoc. Prof. Dr. Bülent Çetinkaya, Assoc. Prof. Dr. Emin Aydın, and Assist. Prof. Dr. Ömer Faruk Özdemir for their constructive and effective feedback that increased the quality of this research. Also with their encouragements, insights and expertise I was able to complete my dissertation.

Later, I have to thank to all principles of the schools, all teachers, and their students anonymously participated to my research. I really appreciate their help. Without this group of people, there could not have been a research.

I am also thankful to my parents Orhan and Fatma and my siblings Ferhat, Kevser, and Kübra for their unconditional love and support. I love each and every one of them dearly.

Last but definitely not least, I would like to give my special thanks to my wife Tuğba and my son Yunus Erkam whose patient love enabled me to complete this work.

I also appreciate the financial support I received from TUBITAK for my PhD study.

TABLE OF CONTENTS

APPENDICES

LIST OF TABLES

TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

> Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment (Alberta Education, 1996, p. 2).

Mathematics is a kind of area that is necessary for people during their lives. With the social and technological developments, many situations experienced in daily life are needed to be interpreted accordingly. These situations can be explored problem-solving situations and accommodating changing conditions as cited in the report of Alberta Education (1996). As a discipline, mathematics demands for systematic and organized structure. In order to be able to interpret, comprehend, and experience many situations arouse in daily life, everyone must have mathematical knowledge, at least, at basic level including mathematical operations, simple calculations, and reasoning for situations encountered in daily life (Güven, 2000, National Council of Teachers of Mathematics [NCTM], 2000; the Organisation for Economic Cooperation and Development [OECD], 1999; Ojose, 2011). Therefore, learning mathematics is essential for everyone. OECD (1999) gives evidence for the necessity of learning mathematics while defining the mathematical literacy as

> An individual's capacity to identify and to understand the role that mathematics plays in the world, to make well-founded mathematical judgments and to engage in mathematics in ways that meet the needs of that individual's current and future life as a constructive, concerned, and reflective citizen (OECD, 1999, p. 48).

Therefore, “students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 11).

One of the basic purposes of the mathematics education is that students in each grade level should understand the mathematical concepts and apply them in necessary areas (Riccomini, 2005) such as problem solving situations (Alberta Education, 1996), making judgment about the situations encountered (Kennis, 2006). As the required attention and importance were given to mathematics education in our nation for this and many other purposes, the Turkish Ministry of National Education (MoNE) (2005a; 2005b) went to large-scale curriculum revision both in middle and secondary schools in 2005. The aim was to make students learn the knowledge comprehensively and operate it instead of just memorizing the knowledge. Some of the general purposes of curriculum revisions in mathematics education were to teach and to learn mathematical thinking systems and to provide students with applying basic mathematical skills (problem solving, reasoning, relating, generalizing, communicating, sensual and psychomotor development) and related skills to real life situations.

Today, probability has become an integral component of everyday situations (Freudenthal, 1970; Kazak, 2009; Kvatinsky & Even, 2002; Way, 2003). Some of the application areas are games, data processing, insurance, economics and natural sciences (Kazak, 2008). Anastasiadou (2009) states the importance of probability topic by explaining interconnection with daily life situations, its instrumental role in other disciplines and especially its role in developing a critical reasoning. This is because people are always exposed to uncertainties (Andra, 2011) and chance factors (Way, 2003) in their lives. They judge events, make necessary decisions, and behave accordingly. In educational perspective, both international and national curricula emphasized probability teaching. For example, the NCTM (2000) expects students from pre-kindergarten through grade 12 to “understand and apply basic concept of probability” (p. 48). Its rationale is that “probability is connected to other areas of mathematics” and that “ideas from probability serve as a foundation to the collection, description, and interpretation of the data” (p. 51). In addition, one of the general aims of the Turkish Curriculum for Secondary School Mathematics is that students needed to be able to use prediction skills effectively (MoNE, 2011). In addition, MoNE (2011) requires students to learn the concept of probability, inclusive and mutually exclusive events, conditional probability, and the dependent and independent events. The middle school curriculum considers the probability as separate learning domain. The curriculum expects 8th grade students to determine the possible situations of an event and the events with different probabilities. In addition, students are required to investigate the equally probable events and calculate the probabilities of the events (MoNE, 2013).

Although students experience uncertainties and make judgments and decisions in their everyday lives (Kahneman & Tversky, 1982), understanding of probability concept is hard for students and even for adults (Kazak, 2009). This situation took mathematics education researchers' attention and they paid attention to identify and search for the reasons for misconceptions, how teachers, materials used, or the instructions intervene and resolve misconceptions encountered in probability topic (e.g., Fischbein, 1975; Fischbein & Schnarch, 1997; Pratt, 2000; Stohl & Tarr, 2002). Before introducing the reasons for the misconceptions, the meaning of the misconception would be presented. Misconception is defined as the incorrect concepts and conceptions that are perceived as correct by individuals. Accordingly, they use them in presenting different abilities and cannot develop sense of integrity among the meanings of the concepts (Koray & Bal, 2002). Individuals develop alternative definitions for the concepts in their minds and consider them as scientific sources (Tekkaya, Çapa, & Yılmaz, 2000). The common properties of the misconceptions are that misconceptions can be observed among many students, that alternative beliefs appear with them, that they may appear due to previous experiences and education, and they are resistant to change (Fisher, 1985).

The literature presents different reasons for these misconceptions. For example, potential role of teachers' pedagogical content knowledge in teaching and learning (Nakiboğlu, 2006; Rubel, 2002), inconsistencies in variety of mathematics contexts (Nakiboğlu, 2006), insufficiency in students' readiness (Skelly & Hall, 1993), and students' intuitions (Fischbein & Schnarch, 1997) might result in misconceptions in probability. In addition, traditional instructional methods are also considered as one of the reasons for the misconceptions (Marek, Cowan, & Cavallo, 1994; Ubuz, 1999).

With the development of probabilistic thinking among students, it is highly likely to have misconceptions about the topic or to do incorrect judgments when making decisions (Kennis, 2006). Since students use their intuitions to deal with problem situation or to solve probability questions, the misleading effect of intuition might result in misconceptions in students' mind (Myers, 2002). While explaining the close relation between intuition and probability subject, Fischbein and Schnarch (1997) stated as follow.

> Probability does not consist of mere technical information and procedures leading to solution. Rather, it requires a way of thinking that is genuinely different from that required by most school mathematics. In learning probability, students must create new intuition (p. 104).

Before dealing with the relation between the probability and individuals' intuition, the term intuition should be clarified. Basically, Fischbein (1987), one of the pioneers in the area of intuitive thinking, gave a definition to intuition or intuiting as immediate cognition that exceeds the given fact, as "a theory that implies an extrapolation beyond the directly accessible information" (p. 13). According to Fischbein (1987), intuition is kind of cognition that suddenly appears without depending on the formal knowledge or information while solving a problem. For example, after facing a problem, anyone in the world can experience a common "Aha" which is a kind of a flash of insight to go through the solution of the problem (Fererman, 2000). Newton's discovery of law of forces and motion, Archimedes' Eureka, unexpectedly developing a proof for mathematical theory could be given as examples of intuition and intuiting.

It is a general belief that mathematics can only be done by using precise and unambiguous definitions and formal notations (Weber & Alcock, 2004). In addition, mathematical practices or solving mathematical problems require formal reasoning including a set of well-defined and accepted procedures (Tall, 1989). Without using such mathematical notions or procedures, the solutions are generally not accepted by the others. However, Fiscbein (1982) insists that separation of formal mathematics thought and mathematical intuition is not desirable. This is because the mathematical intuition has positive impact on students' productive mathematical thinking. Weber and Alcock (2004) also support the idea that students benefit effectively from intuitive and non-formal representations while learning advanced mathematical concepts as well as scientific concepts. Fischbein (1999) also agreed and suggested the necessity of the intuitive thinking in intellectual cognition by indicating that intellectual cognitions could be represented in two basic forms: intuitive cognition which appears directly acceptable and logical, or logically based cognition which necessitates certain explicit and logical proof.

There are several reasons why intuition is important in learning mathematics. On the one hand, it may help students to comprehend and practice mathematical knowledge as well as science knowledge (Fischbein, 1999) including various mathematical subjects (Fischbein, 1987) and proofs of mathematical algorithms (Weber & Alcock, 2004); on the other hand, it may lead students to misconceptions while learning mathematics (Myers, 2002).

Myers (2002) mentions about the powers and perils of the intuitive thinking. Briefly, she summarized positive aspects of the intuition as follows.

a) Knowing without awareness. Intuition helps children learn and know without awareness. Myers (2002) explained it by stating that "some things we know we know, but we don't know how we know them" (p. 17). Language learning could be an example for this aspect.
b) Social intuition. With the help of the intuition, people can shape their future. Some events or things about a specific topic can be predictive of, and impact on the long-term impressions.
c) Intuitive expertise and creativity. When working on a problematic issue and not finding the solution, the solution may immediately appear.

Myers (2002) also gave three perils of intuitive thinking. These were;

a) Intuitions about our past and future. The belief about a concept from past experiences might affect negatively and mislead students' learning.
b) Intuitions about our competence and virtue, which include hindsight bias, self-serving bias and overconfidence bias.
c) Intuitions about reality. Students' knowledge may mislead to discover aimed concepts.

In addition, Fischbein (1987; 1999) asserted about drawbacks of using intuitive thinking. For example, a statement could be intuitively conflict with the formal proof. Moreover, there are some conflicting intuitions arising about the same mathematical statements, simultaneously. There is also a suggestion that removing intuitive conception is desirable (Fischbein, 1999). It is necessary to be aware of intuitive conceptions and to build new intuitions that are consistent with the formal explanations (Fischbein 1987).

Many researches about intuition and intuitive thinking have been conducted on wide variety of subjects in science education such as electrics (Levy, 1998 as cited in Stavy et al., 2006), heat (Hake, 1998), and mathematics education such as algebra (Rapaport, 1998 as cited in Stavy et al., 2006), geometry (Livne, 1996); probability (Babai et al., 2006); area/volume (Dooren et al., 2004). Considering these researches, although students were asked to answer to conceptually non-related task, their responses were similar to such tasks (Stavy et al., 2006). When comparing their responses, some common external features appeared (Babai et al., 2006). For example, students were asked to compare the areas of two rectangulars with different edges. Althought the areas were the same, students focused on the length of one edge and stated that the rectangular with longer edge had larger area (Livne, 1996). Similar feature takes place in Piaget's experiments related to some water in taller and shorter cups. As it is well known, pupils consider that there is more water in the taller cup, no matter whether they have same amount of water (Piaget, 1965).

Kazak and Confrey (2007) mention that while making decisions under uncertainty people use their intuitions in many fields in their lives such as sciences and sports. There are many interrelated topics that deal with uncertainty. Data and chance factors are two examples for uncertainty. Here, probability is a mean to deal with data and chance. While encountering with uncertain situations, we can mathematically resolve it with probability (Kvatinsky & Even, 2002).

Although national and international curricula give emphasis on increasing students' deep knowledge and conceptual understanding of subjects taught (MoNE, 2011; NCTM, 2000), this aim does not coincide with the actual situation.

Teachers and teaching practices play crucial role in doing them. It is expected from teachers to have sufficient knowledge and experience to teach concepts in probability. In addition, teachers are supported to make necessary repetations of the previous subjects during the lessons. Moreover, the effective instructional strategies should be practiced to teach probability (Memnun, 2008). Still, students have many difficulties in achieving MoNE's (2011) and NCTM's (2000) aims due to different reasons.

Zahner (2005) advocates that difficulty in solving probability questions is not merely related to computational procedure, in fact, it is related to cognitive process of understanding the problem, setting up the strategy, and basing the solution on appropriate solution method. Here, students base their probability understanding on their personal and experiential knowledge around their surroundings (Kazak & Confrey, 2007). Intuition, however, plays crucial role and sometimes contradicts with students' personal or experiential knowledge, because undeveloped probabilistic intuition based on their experience may not be enough to deal with more complex reasoning (Fischbein, 1975). Therefore, students' intuitions can be misleading while dealing with probability questions and result in misconceptions (Kazak, 2009), because probabilistic situations may result in many different disconnected or even conflicting intuitions during students' reasoning processes (Havill, 1998). The misconceptions are rooted from students' intuition and its misleading effects are called as intuitively-based misconceptions. Fischbein (1975) states that "undeveloped probabilistic intuition is not able to follow procedures of sophisticated reasoning nor to guide the selection of such procedures or evaluate the plausibility of obtained results" (p. 131). There are also various reasons for students' mistakes and misconceptions in mathematics subjects. Some of these reasons include students' difficulty to construct relations among mathematical concepts or between mathematics and other subjects in mathematics (Bills & Husbands, 2005), their different levels of receptivity (Keitel & Kilpatrick, 2005), the negative effect of previous knowledge on new knowledge and the effect of our mind on the subject to be learnt (Fischbein, 1987). These reasons are somehow interrelated. The main point that they are interrelated is students' intuition or their intuiting. In order to resolve students' misconceptions in general or their intuitively-based misconceptions, the teachers' role in teaching probability subject plays crucial role. Related to teachers' role, Stohl (2005) stated as follow.

> The success of any probability curriculum for developing students' probabilistic reasoning depends greatly on teachers' understanding of probability as well as a much deeper understanding of issues such as students' misconceptions and use of representations and tools (p. 351).

This statement indicates that there are important points to consider for students' success in probability. First of all, teachers should know the content. Secondly, they should have knowledge of students' misconceptions, previous knowledge and difficulties. Lastly, they should know how to teach the probability including different teaching strategies and the use of materials. In this study, teachers' awareness of such factors in teaching probability was investigated. In line with the Stohl's (2005) statement, what teachers' awarenesses in teaching probability include the knowledge of content, the knowledge of students' cognition, the knowledge of students' difficulties, the level of pre-knowledge, and possible misconceptions, and the knowledge of instructional methods and of how to use materials and resources.

Shulman (1987) summarizes the teacher's knowledge. These are mathematical content knowledge, pedagogical content knowledge, and knowledge of students' cognition. In this context, regarding probability, teachers' knowledge of randomness and basic concepts in probability can be related with their content knowledge of probability. In addition, teachers need to be aware of which instructional methods are more effective in teaching probability regarding pedagogical content knowledge in Shulman's (1987) classification. Lastly, teachers' knowledge of students' misconceptions, difficulties, and correct or incorrect intuitions in probability is an important issue to be considered for better teaching practices (Batanero, Godina, & Roa, 2004). In fact, Stohl (2005) summarized what teachers needed to know in line with Shulman's (1987) classification about the teacher's knowledge. These are the understanding of probability concepts, understanding of students' conceptions of probability, and means of instruction.

Starting from mathematical content knowledge, all mathematics teacher in middle and high schools need to know fundamental concepts of probability. According to Kvatinsky and Even (2002), these fundamental concepts include characteristic features of probability, which separates it from the other subjects such as chance factor and uncertain situations, and its relation with daily life situations. Although teacher-training programs need to ensure mathematics teacher to have comprehensive understanding of probability subject and its fundamental concepts, since teachers encounter with uncertain situation while teaching the topic, new teachers are not adequately successful in teaching it (Dollard, 2011). Moreover, teachers also bring some misconceptions into classroom environment about probability (Stohl, 2005). For example, Batanero, Godina, and Canizares (2005) found that most common misconceptions teachers have are representativeness heuristics, equiprobability and outcome approach.

Second important point is the understanding of students' conceptions and characteristics. According to Jones, Langrall, and Mooney (2007), teachers generally prepare their probability instruction according to their perception of how students learn and their knowledge of what students know and do not know. This situation brings the importance of the teachers' awareness about the students' conceptions and characteristics. Having information about students' previous knowledge, their levels of understanding, their previous learning and experiences, their ways of learning, and their misconceptions about probability would directly affect teachers' instruction and, as well as, students' learning of probability. Here, intuitively-based misconceptions are resistant to change (Fischbein, 1987). In addition, there are some discrepancies between intuitive reasoning and formal mathematical procedures in probability (Batanero & Diaz, 2012). Without the knowledge of students' intuitively-based misconceptions, for example, regular classroom instruction might not positively change the students' wrong cognitions. At this point, Batanero, Godino, and Roa (2004) consider "the prediction of students' learning difficulties, errors, obstacles and strategies in problem solving" and "interpretation of students' responses to the same problems" in probability teaching as complementary aspects of teacher's knowledge.

Lastly, instruction and tools used in teaching probability play crucial role in resolving students' intuitively-based misconceptions. Here, Steinbring (1991) advocates that representation of the subject and activities applied during the instruction are necessities for success in students' understanding of probability topic.

At this point, mathematics teachers need to know how to manage the instructional methods during instruction and what to use for sustaining students' comprehensive understanding of probability concepts. In order to reach this goal, many studies were conducted with different instructional methods and materials. Some of the materials include computer simulation software (Konold et al., 1993), handouts with large and small sample simulations (Polaki, 2002b), concept map (Gürbüz, 2006). On the other hand, exploration method (Yazıcı, 2002) and other special instructions (Aspinwall & Tarr, 2001) are used for teaching probability. Batanero and Diaz (2012) stress that the professional knowledge of teacher includes "whether they are able to or not to recognize what concepts can be addressed through a particular resource or task, and implement effective learning in the classroom with them" (p. 9). Lack of professional knowledge may influence teachers' lesson-planning task and may result in failure to present significant concepts and failure to differentiate the fundamental concepts with other ones, although valuable resources are available (Chick & Pierce, 2008).

1.1 Research Problem

Using intuition cannot be removed during students' learning processes, because it motivates students to learn and provides better comprehension of mathematical proofs with different representations or of mathematical concepts (Fererman, 2000). However, researchers also made us aware of the misleading effects of our intuition (Fischbein, 1987) such as the effects of previous knowledge on new one, perceptual effects of our mind. According to Fischbein (1987) intuition is self-evidence, which means a student may consider a mathematical statement as true without any justification and it has a characteristic of perseverance, which means that if intuitions are once established, they are very resistant to change. If the true knowledge about the subject is taught occurs via intuition, this helps students to understand it easily. New knowledge can be constructed on the previous true knowledge. However, the literature stated that students had many misconceptions related to intuitions based on their experiences and previous knowledge (e. g., Fischbein, 1975; Gal, 2005; Rubel, 2002). In addition, students are very prone to have intuitive misconceptions in probability subjects (Fischbein, 1975; Kazak, 2008). That means if students have intuitive misconceptions from their experiences, which Fischbein (1987) calls it primary intuition or from systematic instruction in their previous educational background, which Fischbein (1987) calls it secondary intuition, it would be hard for students to change their beliefs and they would continuously make mistakes or have misconceptions. On the other hand, Radakovic (2009) stated that intuitions are adaptable and systematic instruction can help students to change their misleading intuitions. Moreover, Nisbett et al. (1983) also states that formal training can change everyday inductive reasoning that students encounter. In addition, Havill (1998) suggests that teachers be aware of the interaction between the everyday intuitions and instruction related concepts of probability and prepare instruction method accordingly. Tirosh (2000) also emphasizes the potential role of teachers' pedagogical content knowledge in teaching while dealing with the misconceptions in probability. Teachers' awareness about students' intuitively-based misconceptions influences their teaching practices in their regular instructions and they prepare the instructional strategies and materials used accordingly. Here, the teachers' regular instruction means what teachers do in their regular probability lessons. It includes their teaching practices and instructional methods that they generally use in the lessons.

At this point, the research problem that mediates this study emerges. First of all, there is a necessity to determine students' intuitively-based misconceptions in probability. In classroom environment, on the other hand, investigating teachers' teaching practices to resolve students' difficulties and misconceptions gained importance for this study. Since teachers organize and process their teaching activities according to the knowledge of students' cognition, there is also a need to investigate teachers' awareness about students' misconceptions and the reasons behind them.

1.2 Purpose of the Study

The purpose of this study was to determine students' intuitively-based misconceptions in probability. In line with this purpose, the present study also tried to uncover whether teachers' regular instructions were enough for resolving students' intuitively-based misconceptions in middle and high schools. Another purpose was to investigate what teaching practices were handled to resolve these misconceptions. In addition, this study tried to determine teachers' awareness about intuitively-based misconceptions and the factors that might result in them.

1.3 Research Questions

The following research questions guided the study.

1- What are the middle and high school students' misconceptions in probability rooted from their intuitions?
2- What are the similarities and differences between middle and high school students' intuitively-based misconceptions in probability?
3- To what extent are middle and high school students' intuitively-based misconceptions related to probability change after the regular instructions?
4- To what extent are mathematics teachers aware of students' intuitively-based misconceptions and of the factors affecting them in middle and high schools?
5- What are the similarities and differences between middle and high school mathematics teachers' awarenesses about students' intuitively-based misconceptions in probability and of the factors that may result in them?
6- What teaching practices do middle and high school mathematics' teachers carry out to overcome intuitively-based misconceptions in 8th and 11th grades?

1.4 Significance of the Study

Mathematical standards changed both in national (MoNE, 2005a; 2005b) and international mathematics curricula (Common Core State Standards [CCSS], 2010) and they are in use today. The reason for this change was to keep up with new and rapid developments in educational areas and to provide students with better understanding of what was thought (NCTM, 2000). This change included the methods in instructions and teachers' role in teaching process.

It is stressed that probability is linked to other mathematical subjects such as counting techniques in numbers and operations, area concepts in geometry, binomial theorem, and the relationship between functions and the areas under their graphs in algebra (NCTM, 2000). Similar to NCTM (2000), the Common Core State Standards (2010) emphasizes to make connections between mathematics topics. The probability topic is recommended to be taught in 7^{th} grade. The emphasis is on chance processes (CCSS, 2010). In addition, students are expected to develop, use, and evaluate the probability models. In high school standards, the probability topic is considered under the statistics and probability strand. Instead of just teaching probability, CCSS (2010) emphasizes on using probability in order to make inferences and decisions, and to justify conclusions. The standards considered the probability as the tool to use in decision-making processes and in justifications of the situations encountered instead of just memorizing the rules in probability. In international curricula, importance of probability is emphasized in the principles and standards for school mathematics (NCTM, 2000). Standards for grade 6-8 indicate that middle school students "should understand and apply basic concepts of probability and should be able to compute probabilities for simple and compound events" (p. 248). In addition, standards for grades 9-12 state that "students should develop and evaluate inferences and predictions that are based on data, understand the concepts of conditional probability and independent events, and understand how to compute the probability of a compound event" (p. 324). In addition to the emphasized importance of the probability subject, one of the aims for NCTM (2000) is to support the research in mathematics education and to apply the findings of the research into teaching practices. In Turkish curriculum, on the other hand, students are expected to learn concept of probability, inclusive and mutually exclusive events, conditional probability and dependent and independent events (MoNE, 2011). In this program, the probability subject is taught to 11^{th} grade students. The changes in the curriculum also include the probability teaching (MoNE, 2011).

Studying probability as a research subject might give valuable data that would assist mathematics teachers and educators in maintaining effective instructions and teaching practices. Probability is very important subject for all students because it plays a crucial role in decision making while encountering with uncertainty situation in almost all areas of human activities. Therefore, students need to have mastery for the knowledge of the fundamentals of probability. However, it was indicated that students experienced difficulty in solving probability problems and they developed many misconceptions during even studying basic probability (Li, 2000). That means the instructions of probability might result in new misconceptions. Moreover, many others could be developed via experience (Fischbein, 1987). Therefore, their intuitions may also result in misconceptions in probability. Unlike the other subjects in mathematics, even the basics of the probability might sometimes be difficult for students (Radakovic, 2009). Although curriculum change aims to provide students with understanding of probability, Shaughnessy (1992) states that it is difficult to remove students' misconceptions without intense effort of instruction. At this point, the importance of the instruction and teachers' role becomes evident. In this study, whether the regular instruction in a public middle school and high school overcome the misconceptions developed by students was investigated. Especially the misconceptions based on students' intuition are hard to change because of conflict between their intuitions and formal solutions. This study investigated the effectiveness of the regular instruction over intuitively-based misconceptions. In addition, the findings of this study would indicate the possible intuitively-based misconceptions among students. The knowledge of students' possible misconceptions would help teachers in preparing their instructions.

This study was conducted in both middle and high schools. This was because Shaugnessy (1992) advocated that research in probability was generally conducted to middle school students and university level students. Shaugnessy also emphasized the importance of conducting research with high school students. In addition, regarding probability research, Jones, Langrall, and Mooney (2007) calls for research conducted with high school students.

> Although Shaugnessy's call for additional studies on secondary students' probabilistic conceptions has been heeded, much of the research has focused on their probabilistic reasoning prior to instruction. There is still a void in the kinds of classroom studies that investigates the effect of instruction on secondary students' probability learning (p. 944).

Today, the literature in this subject is generally based on what the probabilistic misconceptions are and what the reasons for these misconceptions (e.g., Fischbein, Nello, & Marino, 1991; Kennis, 2006). As it was indicated, students need intuitive thinking while dealing with probability questions and many misconceptions in probability arose due to misleading effect of or lack of necessary intuitive thinking. The gap in the literature is, on the other hand, to determine how teacher practices are shaped and how related instructions are appropriate for resolving students' misconceptions, especially the intuitively-based ones. This study tries to fill this gap in the literature.

In general, research on effect of instruction in teaching probability deals with special instruction programs (e.g., Polaki, 2002a; 2002b). With the specially prepared instruction programs and materials, students' growth in probabilistic thinking and resolving probability misconceptions were investigated. However, regular classroom practices of the mathematics teachers were not investigated in detail. In this study, however, the aim with the comparison of teaching practices and instructions used in middle and high school during teaching probability was to determine similarities and differences between them and to reveal the instructional reasons for students' intuitively-based misconceptions. Here, the misconceptions might arise from the previous knowledge and experiences. Therefore, the middle school practices were also investigated and the researcher tried to determine the possible reasons of the misconceptions in middle and high schools. In addition, general tendencies in teaching practices in both middle and high school probability teaching were investigated.

From the perspective of test developed for intuitively-based misconceptions, most studies investigated one or few parts of students' intuitively-based misconceptions. For example, Polaki (2002b) dealt with the sample size in probability, while Radakovic (2009) studied students' misconceptions related to outcome approach and randomness. In addition, Rubel (2002) looked at availability and representativeness heuristics in general. This test, on the other hand, included open-ended questions with most generally seen intuitively-based misconceptions. Shaugnessy (1992) also recommended for mathematics education researchers to develop standard and reliable tools in written format. In fact, the intuitively-based misconceptions were observed in the non-routine problems. The study included such type of questions. CCSS (2010) emphasizes the importance of solving non-routine questions and applying and adapting different kinds of appropriate strategies to solve problems.

Another important aspect of this study is investigating mathematics teachers' awareness about students' probability misconceptions. Teachers' knowledge should include meaning of the concepts to be taught, experience in teaching the subject to students' various levels of knowledge, ability to analyze course-books and other materials, prediction of students' learning difficulties, misconceptions and strategies in problem solving, ability to develop and evaluate assessment materials and good examples of teaching practices (Batanero, Godino, & Roa, 2004). Among them, studies with teachers' awareness of knowledge of students' learning difficulties, misconceptions were rare. Memnun (2008) stated that teachers' lack of knowledge and their awareness of student pre-knowledge might result in failure to comprehend the subject. In fact, there were studies with recommendations for teachers to teach probability (Papaieronymou, 2009) or studies with prospective mathematics teachers (Batanero & Diaz, 2012) and their conceptions about fundamental concepts of probability (Dollard, 2011). In this study, however, in-service mathematics teachers' awareness about students' misconceptions and related actions in regular classroom instruction were taken into account. Therefore, to what extent mathematics teachers considered these recommendations were investigated.

From the Turkey's point of view, literature in Turkey mentions about the possible materials to teach probability such as computer aided materials (Gürbüz, 2008), dramatization (Şengül & Ekinözü, 2004), concept map (Gürbüz, 2006). However, teachers do not search and apply these materials or teaching strategies. Especially in Turkey, teachers' teaching practices were parallel to the teacher book for the related course book. Memnun (2008) explains this situation by stating that teachers do not use common language that all students could understand to develop probabilistic thinking. This language stemmed from the use of course book. Therefore, teachers generally considered the course book as only necessary material to develop necessary probabilistic thinking for students. Here, there was a need for in-depth understanding of how teachers organized their teaching practices during teaching probability and whether they considered students' possible misconceptions during their instructions.

Moreover, research for instructional strategies in teaching probability was generally conducted to compare two teaching methods. In such research, the effectiveness of alternative method was compared to the traditional methods such as comparison between traditional and dramatization methods (Ekinözü, 2003), comparison between traditional and exploration methods (Yazıcı, 2002) and comparison between traditional and computer assisted methods (Dereli, 2009). However, regular or traditional instructions for teaching probability subject and teachers' practices during regular instructions were not studied alone. Here, this multiple case study investigated how teachers practiced their instructional strategies in regular classroom instructions, the points that teachers took into account during instruction, and what the relation was between teacher and students in regular classroom instruction.

In addition to the necessity for gap in literature, intuitively-based misconceptions were studied in different countries including the United States of America, Israel and Australia (e.g., Rubel, 1996; Watson & Kelly, 2007). Since each country has different culture and attitudes toward mathematics and probability, therefore it was also important to investigate whether intuitively-based misconceptions existed in Turkey and whether teachers in Turkey took precautions in resolving them.

Beside the cultural differences and student attitudes compared to the other countries, there was little in Turkish literature that empirically investigates the intuitively-based probabilistic misconceptions. For example, Kazak (2009) reviewed the literature for probability misconceptions including intuitively-based ones in which the studies mentioned in the review were generally conducted abroad. Therefore, a need to conduct an empiric study with Turkish students to get data for informing the mathematics education researchers about the situations of teacher practices and instructional strategies used during the probability teaching and resolving misconceptions arose that are based on students' intuitive thinking.

Another important point was that the researches generally deal with the types of misconceptions and strategies to overcome them (e.g., Fishcbein & Schnarch, 1997; Polaki, 2002b). Students' reasoning in solving probability questions was somehow missed. In fact, Jones, Langrall, and Mooney (2007) recommended exploring students' reasoning including their intuitive cognition in probability. This study provided in-depth investigation of how students think and reason when they try to solve probability questions by means of interviews. Therefore, the readers were provided with the reasons behind the misconceptions. This might help teachers to organize their instruction while teaching probability subject.

Moreover, the questions of tests used in the studies conducted in Turkey are generally chosen from the university preparation books or prepared similar to the questions asked in university entrance exam (e.g., Geçim, 2012; Mut, 2003). However, tests including questions related to intuitively-based misconceptions were not commonly used. In fact, the classical questions asked in exams and tests seek for numerical values of the probabilities of prospective events. On the other hand, intuitively-based misconceptions appear when students are affected by the previous events in the questions. In addition, these questions forces students to deeply think and compare the events that were already happened. In general, students generally experience contradiction between their intuitions and probabilistic thinking (Fischbein, 1987), because students can create wrong intuitions under the influence of daily life experiences and their incorrect intuitions are resistant to change (Fischbein, 1975). Such incorrect intuitions might lead students to misconceptions. Therefore, confronting students with such types of questions is important to make them face with their incorrect intuitions and comprehend the topic. In this study, non-traditional way was taken into account. With this study, test developed would help mathematics education researchers in Turkey to determine middle and high school students' intuitively-based misconceptions in probability.

1.5 Definitions of the Important Terms

a) Misconception: Certain conceptual relations that are acquired may be inappropriate within a certain context. We term such relations as misconceptions (Pines, 1985, p. 101). For example, students can think that the probability of getting a blue ball from an urn which includes eight blue and four red balls is higher than that of getting a blue ball from another urn which includes four blue and two red balls due to numbers of blue balls in the urns.
b) Intuition: "A feeling of knowing with certitude on the basis of inadequate information and without conscious awareness of rational thinking" (Shirney & Langan-Fox, 1996, p. 564). For example, without any formal proofs, many famous mathematicians, including Fermat, proposed theorems which were proved hundreds years later. However, they intuitively claimed that the theorems were correct. On the other hand, a mathematician works hard to

prove a theorem. At the end, he finds a way to prove it. This happens by basing his/her proof on formal knowledge and adequate information.

c) Intuitively-based misconceptions: The misconceptions that are rooted from students' intuition and its misleading effect (Fischbein, 1975; Havill, 1998; Kazak, 2009). For example, after getting heads in three consecutive throws of a coin, students may think that the next throw will be head again, since consecutive heads may mislead students' mind and they can think that the outcome of the next throw is dependent on the previous ones.

d) Teachers' awareness: In this study teachers' awareness refer to teachers' knowledge of content, students' cognition, students' difficulties, the level of students' pre-knowledge, and students' possible misconceptions in probability. In addition, teachers are expected to be aware of instructional methods and how to use materials and resources for effective teaching practices. For example, teachers are expected to know the common misconceptions among students in probability and prepare the instruction accordingly.

CHAPTER 2

LITERATURE REVIEW

This part includes previous studies related to the intuition and its characteristics, the probabilistic misconceptions related to students' intuitions, the learning and teaching of probability, and teachers' practices including instructional methods used in teaching probability. In the first part, intuition and its characteristics were investigated.

2.1 Intuition and Intuitively-based Misconceptions

In this part, literature related to intuition and intuitively-based misconceptions are presented. Firstly, the identification of the intuition and its characteristics are presented. Then, the literature findings related to students' intuitively-based misconceptions are presented.

2.1.1 Intuition and Its Characteristics

There are several definitions for the concept of intuition provided by psychologists, philosophers and education researchers. For example, Rorty (1967) briefly defines the intuition as "immediate apprehension". Therefore, the knowledge or the solution pursued comes immediately. This apprehension directly affects the students and teachers' educational practices in classroom (Diyarbekir, 2003). Instead of full memorization of the processes in the solution of the problems (Jung, 2002) and spending more time to comprehend the subject (Herman, 2007), students can immediately give meaning to what is studied in their lessons without waste of time.
Shirley and Langan-Fox (1996) state the intuition as "a feeling of knowing with certitude on the basis of inadequate information and without conscious awareness of rational thinking" (p. 564). From this definition, it is obvious that the intuition appears without conscious awareness (Fischbein, 1987; Stavy & Tirosh, 2000). This

property is our inevitable factor to live our daily life. We automatically learn from our surrounding unconsciously (Myers, 2002). The language acquisition of children can be given as an example. An average secondary school graduate student knows about 80.000 words. That means, we learn an average of 5.000 words each year, or 13 words each day (Myers, 2002).

From the educational perspective, Bruner (1962) also gave a definition for the intuition. It is "the act of grasping the meaning, significance, or structure of a problem without explicit reliance on the analytic apparatus of one's craft" (p. 60). The implicit learning takes a crucial role in intuitive cognition. This type of learning is directly related to the unconscious awareness of acquiring knowledge. Students may not be aware of the existence of this mechanism. However, it tacitly continues to happen in our mind and affects their reasoning in learning processes (Fischbein, 1987).

Dane and Pratt (2007) mentioned about three general characteristics of intuiting. They consider that intuiting is unconscious. It occurs without conscious thought. Secondly, it involves making holistic associations. Lastly, it is fast especially in the decision making process.

Based on the educational approach, intuitive cognition has basic characteristics presented in the Fischbein's (1987) book. Some of these characteristics are self-evidence, intrinsic certainty, perseverance, coerciveness, globality, and implicitness. In the immediate apprehension phase, people unconsciously and intrinsically adapt their behavior to their surroundings. These characteristics are also important in decision-making process, therefore, in making judgment while solving probability question. Such characteristics are as follows.

a) It is self-evident: Without any justification, one may consider a mathematical statement as true (e.g., whole is bigger than its each part)
b) Intrinsic certainty: Intuitive cognitions are accepted as certain. It is highly correlated with self-evidence; however, they are not the same. One is convinced that mathematical theorems are totally true, but most of them are not self-evidence.

c) Perseverance: Once established, intuitions are very resistant to change. (e.g., accepting the equality of natural numbers set and the set of even whole numbers)
d) Coerciveness: Reasoning that only one unique representation or interpretation for a statement is accepted and the others are ignored or considered as unacceptable.
e) Globality: If two situations are similar, one may be inspired to apply the same procedure to other situation. (e.g., formula for the volume of a cube may inspire to find a formula to find a formula for the area of square)
f) Impliciteness: One may not be aware of his perception. Intuitive cognition appears unconsciously (e.g., after several trials, one may intuitively reach a wrong perception of higher chance of getting tail) (Fischbein, 1987, p. 43).

2.1.2 Intuitively-based Misconceptions

Considering students' probabilistic reasoning, there are many misconceptions rooted from students' intuitions. As students' intuitions have positive consequences for individuals, sometimes they mislead their cognition and result in unfortunate misconceptions.

At the beginning, Tversky and Kahneman (1971) mentioned two main judgmental heuristics. These heuristics begin with students' intuitive predictions. These predictions may be in the form of the likelihood of an event or making decision. After conducting many studies related to uncertainty with students from different grade levels, Kahneman and Tversky (1982) found that people generally do not follow the principles of theory in judging the likelihood events and they generally use heuristics to judge uncertain events, which generally do not give correct solutions. In addition, Tversky and Kahneman (1974) stated as follows.

> People rely on a limited number of heuristics which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors (p. 1124).

There are two types of heuristics: representativeness and availability.

2.1.2.1 Availability Heuristic

According to Tversky and Kahneman (1983a), individuals judge the frequencies of the events in their lives, then, evaluate the probabilities of these events based on the ease of recalling the frequencies. Availability heuristic occurs when individuals see that the frequent events are easier to recall then infrequent events (Kennis, 2006). From students' perspective, they intuitively answer probability question by considering the events that are easier to come to mind. So, they evaluate that these events are more probable. For example, after having a car accident or after witnessing a traffic accident, an individual's judgment about the probability of traffic accidents increases.

In this type of the probabilistic misconceptions, students generally make decision according to the instances of the events that they easily remember (Shaughnessy, 1992). Many researchers studied the existence of availability heuristics in different age or grade levels (e.g., Çelik & Güneş, 2007; Fischbein & Schnarch, 1997; Fox & Levav, 2000; Kahneman & Tversky, 1972; Shaugnessy, 1992; Tversky & Kahneman, 1983a). In this part, four studies and their findings are introduced.
Starting from the Tversky and Kahneman's (1983a) study, the subjects were presented two paths A and B. Path A included three rows with eight connections (therefore eight columns) for each. Path B included nine rows with two connections for each. The researchers' explanation for the question was here.

> A path in a structure is a line that connects an element in the top row to an element in the bottom row, and passes through one and only one element in each row (p. 680).

The question is that which structure is more probable to have more paths. According to the findings, 46 of 54 subjects responded that there are more paths in A than B ($p<0.001$, by the sign test). In fact, both structures have the same number of paths. However, Tversky and Kahneman (1983a) explained this result is based on some factor. One is that subjects consider the number of the columns for their reasoning because their intuitions direct them to "most immediately available paths" (p. 680). There are 8 columns in A and 2 columns in B. Another one is that there are 8 more paths after crossing one row in A while there are only two paths in B. The correct answer, on the other hand, is that the number of the paths in both A and B structures is the same (i.e., $8^3=2^9$).

In the same study, subjects were asked to compare the probabilities that a word starts with R and that R is the third letter. In fact, this example is not appropriate for Turkish language. However, it is a good example to see how students' cognition is affected by the availability of the instances of the events. As it is well known, remembering the words starting with R is easier than remembering those with R in the third letter. Therefore, it was expected that subjects judged the letter's first position in the word is more likely. According to the results of the study, 105 out of 152 subjects judged the first position is to be more likely than the third position is. Only 47 subjects chose the third position to be more likely. Therefore, subjects' misconceptions were highly significant based on the sign test ($p<0.001$). On the other hand, this letter is more frequent in the words as the third letter.

Tversky and Kahneman (1983a) explained the misconception in the question with "the immediately available path." Students' cognition of immediately available situations observed in Fischbein and Schnarch's (1997) study. Fischbein and Schnarch (1997) found that students' misconceptions rooted from availability heuristics might increase with age. Students in 5th, 7th, 9th, and 11th grade levels were asked to compare the number of probabilities of choosing two-member teams from among 10 persons with that of choosing eight-member teams from among 10 people. As it is well known, there are the same number of two and eight-member teams when choosing them from 10 people. The results indicated that 10 % of 5th grade students answered that the number of probabilities of choosing two-member teams is higher than the other. On the other hand, 55 % of students did not respond to this question. The other percentages as follows, 20 % of 7th grade students gave the same answer while 40 % of them did not respond to it. 65 % of 9th grade students gave incorrect answer by indicating that there were more two-member teams. On the other hand, only 5 % did not give answer. Lastly, 85 % of 11th grade students fall into misconceptions. In this grade level, all students responded to question.

The same logic in Tversky and Kahneman's (1983a) and Fischbein and Schnarch's (1997) studies took different form in Keller, Siegrist, and Gutsher's (2006) study. Keller, Siegrist, and Gutscher (2006) who studied the social psychology of risk perception gave another good example of availability heuristics. 170 psychology students were presented scenarios of buying a house and they were given some information about the probabilities of flood risk. There are four versions.

> Version 1: On an average, there is a flood every hundred years.
> Version 2: Each year, there is a 1% probability of flood.
> Version 3: Within 40 years, there is a 33 % probability of flood.
> Version 4: Within 80 years, there is a 55 % probability of flood (p. 634).

It was also emphasized that the flood results in heavy damage and the damage was partly covered by insurance. They were expected to assess the risks by rating them from 1 (not risky at all) to 6 (very risky). According to the results, probability information for one year (second version) showed significantly lower risk ratings, then, the others. Among other versions, no significance was found. The reason is that students perceive that specific length of time was considered to be of small importance and they ignored the risk. The easiness of recalling the other situation resulted in the misconception as found in Tversky and Kahneman’s (1983a) study. Great rate of risk (availability of 33 % and 55 %) for longer period of time affected students and they perceived that the risk was higher for indicated longer time. Lower rate was ignored and rounded to zero.

2.1.2.2 Representativeness Heuristic

This type of heuristics took part in mathematics education literature after Tversky and Kahneman’s (1971) study. According to them, representativeness heuristic is related to the sample selected from the population. People believe that any randomly selected sample highly represents the population without considering the size of the sample. People generally make decision about the probability of events according to how similar the event is to the other events drawn in the same distribution (Shaugnessy, 1977). Shaugnessy also mentioned about the reasons for the misconceptions students fall into in predicting the probabilities of the events. However, individuals’ intuitions incorrectly affect their judgments (Kennis, 2006) that small sample size can also be applied to the representativeness of the population known as “law of small numbers” (Tversky & Kahneman, 1971). For example, it is very probable that we get about 500 heads after tossing 1000 trials of tossing a coin. However, the number of toss decreases to 10, for example, it is possible to get 7 heads and 3 tails, or 8 heads and 2 tails. However, people expect to get same number of heads and tails from these trials.

For the representativeness heuristics, misconceptions take different forms. These forms are negatively and positively recency effect (Fischbein, Nello, & Marino, 1991), outcome approach (Konord et al., 1993) and sample size (Fischbein & Schnarch, 1997).

2.1.2.2.1 Negatively and Positively Recency Effects

Negatively recency effect occurs with respect to incorrect use of *law of large numbers* in a population. Regardless of the size of the randomly selected sample from population, individuals may think that small number of samples is highly representative of the population. They have a belief that the trials of tossing a coin, for example, to have a corrective power to satisfy the properties of the population which they are drawn. This is also called *gamblers' fallacy* (Tversky & Kahneman, 1971). For example, if a fair coin is flipped four times and the outcome is TTTT, then people have a belief that the next outcome would again be tail, regardless the fact that the probability of getting tail is ½. This is called positively recency effect. On the other hand, some others think that the probability should be equal after several tosses, and the experiment has a corrective power, then they expect to get head. This is called negatively recently effect or some call it as gambler's fallacy (Shaugnessy, 1977).

This type of the misconception was also studied in middle, high school, and college level students. Some of the studies related to negatively and positively recency effects are the studies of and Chiesi and Primi (2008), Çelik and Güneş (2007), Fischbein (1975), Fischbein, Nello, and Marino (1991), Fischbein and Schnarch (1997), Shaugnessy (1992), Tversky and Kahneman (1971). In general, the studies investigated the evolution of the misconceptions across age. At this point, two of the studies and their findings are introduced.

The first example is again from Tversky and Kahneman's (1971) study. They shared a finding of an example to explain the meaning of the negatively recency effect. At the beginning of the example, Tversky and Kahneman (1971) insisted as follow.

> The heart of the gambler's fallacy is a misconception of the fairness of the laws of chance. The gambler feels that the fairness of the coin entitles him to expect that any deviation in one direction will soon be cancelled by a corresponding deviation in the other (p. 105).

As it is well known, however, fair coins may also represent different distributions in the small number of trials. In the example, individuals were informed that the mean IQ of the eight grade level students in a specific city was 100. In a study, 50 eighth grade students were selected for the achievement level. They asked that if the IQ of the first student was 150, what they expected the mean IQ of the sample selected. Although the correct answer was 101, the researchers indicated that large number of individuals responded it with the answer of 100. With explanation for this result, the researchers asserted that the process of sampling had power of self-correcting power. They also added that individuals had the intuition that the samples selected from the population were very similar to the others.

Behavior with respect to positively recency effect is opposite of the gambler's behaviors. If a coin, for example, is thrown four times and all of the outcomes become heads up, then, individuals, or more specifically students, think that the fifth throw will also be head up. However, the probability of getting head or tail after tossing a fair coin is ½.

In a cross-sectional study of Fischbein and Schnarch (1997), the purpose is to investigate the evaluation of probabilistic misconceptions based on intuition in accordance with the age. The sample of this study includes 5th, 7th, 9th, 11th grade, and college students. One of the questions asked in this study was in line with the purpose of Tversky and Kahneman's (1971) study. Fischbein and Schnarch (1997) studied the effect of positively and negatively recency effect while making probabilistic decision. The question is as follows,

> When tossing a coin, there are two possible outcomes: either heads or tails. Ronni flipped a coin three times and all cases heads came up. Ronni intends to flip the coin again. What is the chance of getting heads the fourth time? (p. 98)

The distribution of students' answers was given and they found that the main misconception occurred was the negatively recency effect. The researchers concluded that the negatively recency effect decreased with age while the positively recency was very rare among the subjects.

Table 2.1 Percentages of Students' Answers for Positively and Negatively Recency Effect Question (Fischbein & Schnarch, 1997, p. 98)

Students' Answers	Grades				
	5th	7th	9th	11th	CS[b]
Smaller than the chance of getting tails (Negatively recency)[a]	*35*	*35*	*20*	*10*	*0*
Equal to chance of getting tails? (Correct answer)	40	55	70	90	94
Greater than the chance of getting tails (Positively recency effect)	0	5	0	0	6
Other types of answers	25	5	10	0	0

[a]Negatively recency effect is highlighted, [b]CS means college students. N: 100 for each grade level

Chiesi and Primi (2008) investigated the evolution of probabilistic reasoning in accordance with the age. The topic to investigate in this study was the effect of positive and negative recency. Chiese and Primi (2008) tried to determine whether students were affected from the previously occurred events or not. With this purpose, this study was similar to Tversky and Kahneman's (1971), and Fischbein and Schnarch's (1997) studies. The sample of the study includes 25 third grade, 25 fifth grade and 35 college students. They were given outcomes of a sequence of independent events. Then they were asked to estimate the likelihood of next event. The question is as follows.

> Simon and John are playing together with a bag in which there are 15 Green and 15 Blue marbles. Simon drew marbles from the bag four times. Each time the drawn marble is put back into the bag. One after the other, Simon drew four Green marbles. What do you think is more likely Simon to draw next, a Blue or a red marble, or is each color marble just as likely? (p. 3).

Students' responses revealed that age factor was statistically significant on positively recency effect which was that after drawing four green marbles, the next outcome was also green. On the other hand, there was no significant effect of age between younger and older students when the negatively recency effect was considered. In the discussion part, Chiesi and Primi (2008) asserted that younger children generally relied on the sequence of previous outcomes without taking the base-rates into account. Therefore, the representativeness heuristics took important role in students' reasoning while estimating the probability and deciding the answers.

2.1.2.2.2 Outcome Approach

Another type of representativeness heuristics is outcome approach. For example, Konold et al. (1993) asked students to compare the probabilities of the outcomes after tossing a coin five times, the outcomes were given as HHHTT, THHTH, THTTT, HTHTH. Most of the students gave the true answer as the probabilities were equal. However, they also asked which one was less likely to occur. The correct responses decreased to 38 %. This was because students thought that some outcomes were less representative compared to other outcomes. In addition, students made their prediction according to the outcomes already happened.

Similar to the previous misconception, the studies in the literature investigated the frequency of appearance in different age groups or grade levels (e. g., Kahneman & Tversky, 1982; Konold et al., 1993; Lecoutre, 1992; Rubel, 2007; Shaugnessy, 1992).

In this part, two studies are discussed. In this type of the misconception, students interpret the uncertain event according to the most frequent probabilities. For example, people regard that the sequence of THTHHT is more probable that TTTHHH or TTTHTT when tossing a coin six times. This is because the first one seems more random when compared to the others. For example, Kahneman and Tversky (1982) asked the following question to students.

> All the families of six children in a city were surveyed. In 72 families, the exact order of births of boys and girls was GBGBBG. What is your estimate of the number of families surveyed in the exact order of births was BGBBBB? (p. 34)

As it is expected, students ignored the order of the births. The probabilities of birth order sequence for both situations are equal. However, individuals think that they are not equally representative. The results indicated that 75 out of 92 students indicated that the second sequence was less likely ($p<0.01$ by a sign test) (Kahneman & Tversky, 1972). The researchers' explanation for these results was that students ignored the order information and they reasoned that the second situation seems less random.

Secondly, Konold et al. (1993) used the flips of the coins instead of the sexes of the newborn babies used in Kahneman and Tversky's (1982) study.

HT-sequence problem.

> Part 1. Which of the following is the most likely result of five flips of a fair coin?
> a) HHHTT
> b) THHTH
> c) THTTT
> d) HTHTH
> e) All four sequences are equally likely
> Part 2. Which of the above sequences would be least likely to occur? (p. 397).

The sample was composed of 16 high school students who took summer-math course, 35 remedial mathematics class students from undergraduate degree, and 47 students who were enrolled in statistical methods course. According to the results, 72 % of all students correctly answered the part 1. Interestingly, 17.4 % of remedial class students chose the first option. The percentages for the correct answers were 60.8 % for remedial class students, 68.8 % for high school students, and 78.7 %. In the second part, on the other hand, the correct answer decreased to only 38 % for all students. Half of the students who correctly answered the first part could correctly respond to the second part, too. The other students chose one of the options from *a* to *d* as a least likely outcome. 43.4 % of remedial class students, 40 % of high school students, and 17.1 % of statistical methods course students chose the fourth option (HTHTH) as an event which could be least likely to occur (overall of 29.1 %). In addition, 22.8 % of all students chose the third option (THTTT). Again, students ignored the order of the occurrence and they immediately answered according to which option was less representative. For the fourth option, students thought that HTHTH was unrepresentative because it was very ordered. On the other hand, third option included many tails. Therefore, outcome approach affected students to choose the least likely event. Konold et al. (1993) indicated that non-random appearance of the sequences and excess of one outcome compared to the other affect students' reasoning in comparing the probabilities of events, because these factors were not consistent with the representative heuristics.

2.1.2.2.3 Sample Size

Fischbein and Schnarch (1997) conducted a study and asked students which one was more probable: getting two heads after three tosses or getting 200 heads after 300 tosses. Students' thoughts were affected by the equivalence of the ratios between them and they thought that the probabilities were equal. The important point is that students could not manage the sample size. This is called sample size misconceptions. Stavy and Tirosh's (2000) theory of intuitive rules also supports that this misconceptions are affected by students' intuitive cognition. The studies related to sample size are the studies of Fischbein and Schnarch (1997), Konold et al. (1993), Li and Periera-Mendoza (2002), Rubel (2002) and (2007). In general, the studies try to identify the existence of this type of misconception among students from different grade levels. Two studies and their findings related to this misconception are presented below.

Rubel (2002) investigated whether students neglected the size of the sample while estimating the probability of the events. Effects of age and ability were also assumed. In order to get answer for this question, Rubel (2002) asked two questions. These were Yankees Item and Coins Sample Size Item. These questions were as follows.

> Which is more likely:
> A) The Yankees win 80 out of 100 games
> B) The Yankees win 8 out of 10 games.
> C) Choice a and b are equally likely (p. 136).

The Coins Sample Size Item is similar to first one.

> Which is more likely:
> A) You get 7 tails on 10 tosses of a fair coin
> B) You get 700 tails on 1000 tosses of a fair coin
> C) Choice a and b are equally likely (p. 136).

Distribution of responses for Yankees Item and Coin Sample Size Item are given in the Tables 2.2 and 2.3 below.

Table 2.2 Distribution of the Responses for Yankees Item (Rubel, 2002, p. 138)

Responses\ Grade Level	5th (n=36)	7th (n=45)	9th (n=50)	11th (n=42)	Total (n=173)
The Yankees win 80 out of 100 games	8	8	18	16	50
The Yankees win 8 out of 10 games.	3	1	9	4	17
Equally likely[a]	*24*	*36*	*22*	*22*	*104*
No answer	1	0	1	0	2
Total	36	45	50	42	173

[a]Equally likely is highlighted

Rubel (2002) found that difference in terms of correct response was not statistically significant according to the age, but decrease in the answer for equal chance showed statistically significant results at the 0.01 level.

Table 2.3 Distribution of the Responses: Coins Sample Size Item (Rubel, 2002, p. 139)

Responses\ Grade Level	5th (n=36)	7th (n=45)	9th (n=50)	11th (n=42)	Total (n=173)
7 tails out of 10 tosses is more likely	6	6	22	17	40
700 tails out of 1000 tosses is more likely	0	0	5	2	7
Equally likely[a]	*28*	*39*	*35*	*23*	*124*
No answer	2	0	0	0	2
Total	36	45	50	42	173

[a]Equally likely is highlighted

Rubel (2002) found that the increase in the correct answer was not statistically significant at 0.01 level with respect to age level. On the other hand, the decrease was statistically significant for *equally likely* answer across age. Rubel (2002) pointed out that students who gave answer of *equally likely* made a justification that there was an equal ratio and fraction between the answers. It was also supported by the intuitive rules developed by Tirosh and Stavy (1999a; 1999b). For these questions, Same A, Same B rule existed. Therefore, students' justification was that since the ratios between the tails and total tosses were same, then, the probabilities were also the same.

Fischbein and Schnarch (1997) also had similar findings in line with Rubel's (2002) study. The question was comparison of getting heads at least twice when tossing three coins with getting heads at least 200 times out of 300 times. The distribution of the responses was indicated below in the Table 2.4. In both examples, students neglected the sample size during deciding the probabilities of the events.

Table 2.4 Percentages of Students' Answers for the Sample Size Question (Fischbein & Schnarch, 1997, p. 99)

Students' Answers	Grades				
	5th	7th	9th	11th	CS
At least 2 heads out of 3 tosses is more likely (correct)	5	5	25	10	6
Equally likely (main misconception)[a]	*30*	*45*	*60*	*75*	*44*
At least 200 heads out of 300 tosses is more likely (incorrect)	35	30	10	5	50
Other answers	5	10	0	0	0
No answer	25	10	5	10	0

[a]Equally likely is highlighted.

2.1.2.3 Simple and Compound Events

If students are given two or more compound events one by one or together, their predictions for the probability of compound events are fallacious (Kazak, 2008). Both national and international curricula expect students to have general understanding of simple and compound events (MoNE, 2011; NCTM, 2000). However, they could not manage the sample space while they encounter compound events. For example, Fischbein and Schnarch (1997) found that when two fair dice was thrown, students thought that the probability of getting two sixes was the same as getting a five and a six.

Again, there were many national and international studies conducted related to simple and compound events (e.g., Çelik & Güneş, 2007; Fischbein, Nello, & Marino, 1991; Fischnein & Schnarch, 1997; Lecoutre & Durand, 1988; Li & Periera-Mendoza, 2002; Rubel, 2002; 2007; Shaugnessy, 1992). The findings of two studies were presented below. They also presented the difference of the existence of this misconception among students from different grade levels.

From the Fischbein and Schnarch's (1997) study with the aim of investigating the probabilistic, intuitively misconceptions with regard to age which was mentioned above, and another question asked was related to simple and compound events. The question was that "suppose one rolls two dice simultaneously. Which of the following has a greater chance of happening?" (p. 98). The answers and the response distributions are given in Table 2.5 below.

Table 2.5 Percentages of Students' Answers for the Simple and Compound Events Question (Fischbein & Schnarch, 1997, p. 98)

Students' Answers	Grades				
	5th	7th	9th	11th	CS[b]
Getting the pair of 5-6 (Correct)	15	20	10	25	6
Getting the pair of 6-6	0	0	0	0	0
Both have the same chance (Main misconception)[a]	*70*	*70*	*75*	*75*	*78*
Other types of answers	15	10	15	0	16

[a]Main misconception is highlighted, [b]CS means college students.

The researchers indicated that this misconception was most frequent and most stable across age while compared to other seven questions resulting in intuitive based misconceptions.

Another study related to simple and compound event misconception was performed by Fischbein, Nello, and Marino (1991) whom asked the same question asked in Fischnein and Schnarch's (1997) study. In addition, the situation of throwing two coins was also asked to students in the study. The sample of this study was 600 students in middle and high school students. They were presented with four situations. These were coins-specific which was the comparison of getting one head and one tail with getting two heads after tossing the coin twice, coins-general which was the comparison of getting same faces with getting different faces after tossing the coin twice, dice-specific which was the comparison of getting one 5 and one 6 with getting 6 with both dice after throwing two dice, and, lastly, dice-general which was the comparison of getting same number with both dice with getting different numbers. The findings indicated that only 23 % of elementary school students answered the dice-specific question correctly and 34 % of them correctly answered the dice-general question. For the coins-specific question, 21 % of middle school students correctly answered it. In addition, 50 % of them correctly answered the coins-general question. The percentages of junior high students who correctly answered dice-specific, dice-general, coins-specific, and coins-general questions were 19 %, 43 %, 10 %, and 60 %, respectively. On the other hand, 46 % of middle school students and 56 % of junior high school students indicated that probability of getting 6-6 was equal to that of getting 5-6 or 6-5. Similarly, 40 % of middle school students and 42 % of junior high school students stated that probability of getting same number on both dice was equal to that of getting different numbers from these dice. Moreover, 41 % of middle school students and 43 % of junior high school students stated that probability of getting HH was equal to that of getting HT or TH.

2.1.2.4 Conjunction Fallacy

It is obvious that the probability of conjunction of two distinct events is less than the probability of either one of them. However, the conjunction fallacy appears when students have the belief of inverse thought. According to Tversky and Kahneman (1983b), conjunction of two events can be more representative that one of its constituents may sometimes become easier to imagine and remember. Here, Tversky and Kahneman (1983b) consider this fallacy as a type of representativeness heuristics.

The examples that investigated the existence of the conjunction fallacy among students were studied by Çelik and Güneş (2007), Shaugnessy (1992), and Tversky and Kahneman (1982; 1983b). Here, there are brief summaries of the studies of Tversky and Kahneman (1983b) and Shaugnessy (1992) below.

In this type of the misconception, the main erroneous judgment is that the conjunction of two events has higher probability of occurring when compared to any of these events has. As it is well known, the any of the two events has equal or higher probability than their conjunction. Again, Tversky and Kahneman (1983b) presented a study related to this misconception. According to their study, students were asked to compare the probabilities of an event and conjunction of this event with another. There were two questions asked to 88 undergraduate students in UBC. The structures of these questions were the similar. There were two scenarios.

> Scenario 1: Bill is 34 years old. He is intelligent, but unimaginative, compulsive and generally lifeless in school. He was strong in mathematics but weak in social studies and humanities (p. 297).

Then, eight statements were provided to students in order to describe Bill. These are “Bill is a physician who plays poker for a hobby, Bill is an architect, Bill is an accountant (A), Bill plays jazz for a hobby (J), Bill surfs for a hobby, Bill is a reporter, Bill is an accountant who plays jazz for a hobby (A & J), Bill climbs mountains for a hobby" (p. 297). Among these statements, three of them (A, J, and A & J) are emphasized.

> Scenerio 2: Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with the issues of discrimination and social justice, and also participated in anti-nuclear demonstrations (p. 297).

Again, there were eight statements to describe Linda. These are "Linda is teacher in an middle school, Linda works in a bookstore and takes Yoga classes, Linda is active in feminist movement (F), Linda is psychiatric social worker, Linda is a member of the League of Women Voters, Linda is bank teller (T), Linda is insurance salesperson, Linda is bank teller and is active in the feminist movement (T & F)" (p. 297).

Again three statements (T, F, and T & F) were emphasized for the readers of the study. The question for both scenarios was that "the degree to which Bill (Linda) resembles the typical member of that class." According to the answers of the students, 87 % predicted the order as "A>A & J>J" for Bill and 85 % predicted the order as "F>T & F>T" for Linda.

Instead of providing scenarios in Tversky and Kahneman (1983b), Shaugnessy provided students with two situations and expected from them to compare the probabilities. Shaughnessy (1992) asked students to compare the probabilities of that a person was 55 years old and had a heart attack with the probability of that a person had a heart attack (regardless of age). Surprisingly, most of the college students' choice was that first situation was more likely. This was because age was a kind of characteristics for having heart attack. However, regardless of the age, having heart attack is more probable.

2.1.2.5 Conditional Probability (Time-Axis Misconception)

This misconception is known as Falk phenomenon (Jones, Langrall, & Mooney, 2007; Shaugnessy, 1992) or time axis fallacy (Fischbein & Schnarch, 1997) in the literature. For the conditional probability P(A/B), if the event A precedes event B, students' thinking in the conditional probability would contradict with their intuitions (Kazak, 2008). This is because the dependent event occurs after the other event occurs; however, students consider the sequence as they happen regularly. Students' intuition contradicts with their reasoning and most of the students could not find the correct answer. There are many studies conducted in order to investigate the existence of this misconception among students (e.g., Bar-Hillen & Falk 1982; Falk, 1979; 1983; Fischbein & Schnarch, 1997; Fox & Levav, 2004; Rubel, 1996; 2002; Shaugnessy, 1992; Watson & Kelly, 2007).

To explain this misconception, a question asked in the study of Bar-Hillel and Falk (1982) can be given. The question is as follow.

> Mr. Smith is the father of two. We meet him walking along the street with a young boy whom he proudly introduces his son. What is the probability that Mr. Smith's other child is also a boy? (p. 109)

For this question, one mathematician stated that the probability was one-half, while the other one stated that the probability was one third. The cognitions varied in this situation. The first thought that being a boy (B) or a girl (G) was independent of the sibling. So, the other child was either boy or a girl. Therefore, the probability was one-half. On the other hand, the second cognition was about the condition. At the first time, the sample size was four including BB, BG, GB, and GG. Since it was stated that one child was boy, the sample size decreased to three the elements of which were BB, BG, GB, because the outcome of GG was deleted. Among them, the probability became one-half. Of course, one of these cognitions was fallacious.

In the study of Graberg and Brown (1995), a question which was published in an issue of *Parade* magazine by a writer named Savant was asked to 228 undergraduate students in two sections of the department of sociology class. The name of the question was Monte Hall Problem. It was asked as follow.

> Monte Hall presents you with three doors, behind one of which is a prize. You choose one of the doors. No matter which door you choose, Monte will always open one of the doors to reveal a goat. You, then, have the option to stick with your orginial choice or to switch to the unopened door. What should you do? (p. 712)

The first intuition states that the probability decreases from one-third to one-half. However, if you switch to the unopened door, the probability increases to two-third. Let's think that the doors are labeled as 1, 2, and 3. Let's think that door 1 is chosen.

If the prize is behind the door 1, then, Monte will open either door 1 or door 2 (first condition). If the prize is behind the door 2, then, Monte will open the door 3 (second condition). If the prize is behind the door 3, then, the Monte will open the door 2 (third condition). That means the sample is reduced to three, out of which the unopened door can be considered as two elements. Therefore, the probability increases to two-third instead of one-half.

Graberg and Brown (1995) indicated that there were more than 10000 letters were sent to Savant. Among them, 90 % of the answers were incorrect. Many of the readers sent negative and hostile responses to Savant's justification. It was also indicated that, 65 % of the letters were coming from academics among those who sent answers. The findings of the Graberg and Brown's study (1995) were that only 13 % of the undergraduate students switched the door.

In time-axis probability, the timing of first and second events changed. The question mentioned in Graberg and Brown's (1995) study was not directly related to time changes. According to different situations, the probabilities also changed. An example was from Rubel's (1996) study. The question asked expects students to differentiate the conditional in two situations. The question is as follows.

> There are two buckets, I and II, each with 1/3 of its marbles white. A coin will be flipped to determine which bucket will be selected, and then a marble will be randomly selected from the bucket. Frank bets that if bucket I is chosen, he will draw a white marble. Joe bets that if he chooses a white marble, then it will have been from bucket I (p. 75).

In this example, if the choosing a bucket was labeled as A and drawing a marble was labeled as B, it was expected from students to compare P(A/B) and P(B/A). Therefore, P(A/B) was one-third while P(B/A) was one-half. However, 55 out of 98 students who were enrolled in 9^{th} to 12^{th} grade level in an international school in Israel indicated that the probabilities were equal. The justifications for their answer, in general, were that both situations represented the same thing; therefore, the probabilities were the same. Their reasoning was that the same thing was presented with reverse order. Therefore, their intuitive reasoning misled them.

Similar to Rubel's (1996) study, Fox and Levav (2004) tried to investigate whether same cognition existed among undergraduate students. Fox and Levav (2004) asked three cards questions to the sample of 76 undergraduate students who were enrolled in the psychology department. The question below was asked to students.

> A box contains three cards. One card is red on both sides, one card is green on both sides, and one card is red on one side and green on the other. One card is selected from the box at random, and the color on one side is observed. If this side is green, what is the probability that the other side of the card is also green? (p. 630)

Among them, two-third (67 %) of all students stated that the probability was equal to $\frac{1}{2}$. In this question, the correct answer was found with the Bayes theorem. The probability was equal to $\frac{2}{3}$. Only 2.6 % (2 students) of all students stated that the probability was equal to $\frac{2}{3}$. The card red on both sides was stated as RR. There were there cards with the faces RR, GG, RG. It was known that the card was not the one with red on both sides. So the card was either GG or RG. It was clear that there were two sides with green color and one side with red color.

2.1.2.6 Stavy and Tirosh's Theory of Intuitive Rules

There were many studies conducted by different researchers about intuition and intuitive cognition in science and mathematics education, which surprisingly revealed that students responded to unrelated task in similar ways (e. g., Babai et al., 2006; Dooren et al., 2004; Fischbein & Grossman, 1997; Stavy et al., 2006; Stavy & Tirosh, 1996; 2000; Tirosh & Stavy, 1999a; 1999b). Interesting point was that students had similar ways of approaching to such unrelated tasks. From this point of view, Tirosh and Stavy (1999a; 1999b; 2000) investigated students' behaviours and developed a theory. This theory was constructing a based for students' intuition-related incorrect answers to unrelated tasks. The theory was called as theory of Intuitive Rules, which indicated that students' incorrect responses were of three forms. The rules were consistent in different topics in mathematics (Stavy & Tirosh, 2000; Tirosh & Stavy, 1999b). These are "Same of A – Same of B", "More of A – More B", where A and B are different quantities, and "everything can be divided endlessly". In fact, first two rules are related to purpose of this study. Therefore, the last one is not used in this study. One may argue that students' incorrect answers cannot be regarded as a theory. However, this theory has two important main strengths. These are (1) this theory explains most of the students' incorrect answers in science and mathematics education and (2) predictive power of this theory is strong, that is, students' incorrect responses can be predicted on a specific task (Stavy & Tirosh, 2000). Although its applications and existence are wide for most of the students' incorrect answers, Stavy and Tirosh (2000) also pointed out that all incorrect answers may not be explained by this theory. For example, the responses to questions such as "what is triangle?" cannot be affected by the applications of the intuitive rules. All in all, knowing such intuitive rules and students' possible misconceptions may help teachers and educators to minimize the possibility of such misconceptions.

There is a theory about the common external features among many students. This theory is called as "Intuition Rules" which was developed by Tirosh and Stavy (1999a, 1999b, 2000). According to this theory, there are three general intuitive rules that most of the students rely on. Consider that A and B are two different quantities. Then, the intuitive rules are

1- The same of A – The same of B. Comparing two objects which are equal with respect to a certain quantity A ($A_1 = A_2$). Then, students intuitively say that both objects are equal with respect to quantity B ($B_1 = B_2$).
2- The more of A – The more of B. Comparing two objects which are different with respect to a certain quantity A ($A_1 > A_2$). Then, students intuitively say that both objects are different with respect to quantity B ($B_1 > B_2$).
3- Everything can be divided endlessly.

Although there are three rules for the theory, the last one is not appropriate for the probability subject. Therefore, the last rule was not taken into consideration in the study. These rules are also directly related to the characteristics of intuitive cognitions. The studies about the intuitive cognition indicated that students' responses to the questions which revealed the intuitive rules explained the characteristics of intuitive cognition (Stavy et al., 1982).

2.1.2.6.1 The Misconception of the More of A – the More of B

When comparing two objects, if one quantity of these objects is different ($A_1<A_2$), students tend to give answers in similar ways. They think that the other quantities of these objects are also different ($B_1<B_2$). Stavy and Tirosh (2000), Tirosh (2000) and Tirosh and Stavy (1999a) calls students' similar responses to the unrelated tasks as "More A – More B" intuition rule.

To understand the misconceptions, the most apparent example was from Green's (1983) study. Green (1983) illustrated two sets of boxes which included one white and three black balls in the first box and two white and six black balls in the second one. Students were expected to choose one box with high probability of choosing a black ball. The question was asked to 5th, 7th, 9th, and 11th grade students. The total number of students in the study was 243. Although the number of the students who fell into this type of misconception decreased, interestingly, about half of the students chose the second one with the explanation that the second box included more black balls.

2.1.2.6.2 The Misconception of the Same of A – the Same of B

In our daily lives, there are many situations to compare two quantities whether they are equal or not. For example, even small children can differentiate two groups of identical balls, one of which includes more balls than the other directly from the visual information. However, there may not be enough clues to differentiate two quantities. Stavy and Tirosh (2000) explained that one of the students' behaviors to such kind of situations is that perceptually same quantity (A) can serve as a criterion to compare the other quantity (B). In the second intuitive rule, if a students are asked to compare two objects with respect to another quantity A which is equal for both situations ($A_1 = A_2$), students often incorrectly argue that these objects are equal with respect to other quantity B ($B_1 = B_2$). For example, students may perceive that a straight line and a wavy line both of which have the same end points are equal in length. This misconception was investigated in the studies conducted by Dooren et al. (2004), Fischbein and Schnarch (1997), Li & Periera-Mendoza (2002), and Stavy & Tirosh (2000). They all observed that students' intuitions were working in the same way in line with the intuitive rules (Stavy & Tirosh, 2000; Tirosh & Stavy, 1999a; 1999b). There were two studies discussed related to second intuitive rule given below.

In their studies, Dooren et al. (2004) studied first and second intuitive rules from Tirosh and Stavy's (1999a; 1999b; 2000) study. One question was about the probability. The question asked the whether the probability that Carmel's family who had two children had one son and one daughter was larger than/ equal to/or smaller than the probability that Levin's family who had four children had two daughters and two sons. The answers in line with respect to Same A – Same B revealed the percentages of 33 %, 24 % and 36 %, respectively in 10^{th}, 11^{th} and 12^{th} grade students.

Fischbein and Schnarch (1997) asked the question that whether the probability of getting heads at least twice after three trials is smaller than/equal to/ or greater than the probability of getting heads at least 200 times out of 300 times. This question had the same logic with the sample size effect investigated in Çelik and Güneş's (2007) study. According to the results of this study, respectively, 30 %, 45 %, 60 % and 75 % of 5^{th}, 7^{th}, 9^{th} and 11^{th} grade students answered as their probabilities are equal. However, the probability of the first situation is greater due to the law of large numbers.

2.2 Teaching and Learning of Probability

This part of the study includes the literature related to teachers' teaching practices, their awareness of students' difficulties and misconceptions. In addition, the reasons for students' difficulties and misconceptions were also presented in line with the literature. First of all, the national and international curricula for probability topic are investigated. Accordingly, the teaching methods and practices for probability are presented.

2.2.1 Curriculum and Teachers' Teaching Practices for Probability

Beginning with the content taught in the middle and high school, there were slight differences between the curricula. Both international and national curricula were parallel in contents taught. NCTM (2000) and MoNE (2009; 2011) indicated that both middle and high school curricula included the basic concepts in probability, the event types such as inclusive-mutually exclusive events, simple probability, the probability types, and the calculations of the values of the probabilities. The differences were as follows. The middle school curriculum included the theoretical, experimental, and subjective probabilities which were not included in the high school curriculum (MoNE, 2009; 2011). On the other hand, the conditional probability was only taught in high schools (MoNE; 2011).

The general focus both in middle and high school curricula were *explaining* the concepts, *determining* the event and probability types, and *calculating* the probability values (MoNE, 2009; 2011). It was expressed to make students discover, explain, evaluate the facts taught. Students were expected to use them in their lives. Similarly, CCSS (2010) and NCTM (2000) standards were focused on the *understanding and the interpretation* of the probability concepts and *the computation* of the probability values for interpreting data, using to solve problems, and evaluating the outcomes for making decisions.

In the practice, however, the general trend in teaching probability did not coincide with the expected outcomes of the national and international curricula. Instead of making students comprehend the concepts and how to determine probability types and compute the values, teachers generally briefly explained the basic concepts (Çelik & Güneş, 2007) and focusing on the determination of the event types and on the calculation of the probability values asked in the questions (Bulut, 2001). In fact, the types of questions asked were also important in understanding the probability (Fox & Levav, 2000). In the study of Riccomini (2005), it was recommended to provide students with familiarity with probability situations. Therefore, in case that students experienced unfamiliar situations in probability, they seeked for solution methods (Havill, 1998). In Turkey, however, this was not provided for students in teaching probability. Köğce and Baki (2009) found that the criteria for asking questions in high schools was generally in line with the question types asked in university entrance exams. Papaieronymou (2009) also found that students were not prepared for unfamiliar questions in probability. This situation could also be attributed to the teaching of probability in middle schools.

From the perspective of teaching practices in schools, Polaki (2002a; 2002b) used special instructional method in teaching probability. The aim was to make students discover the rules and create desired fulfillment in understanding the topic. In fact, this aim was parallel to the aims of both national and international aims (MoNE, 2009; 2011; NCTM, 2000). However, the teaching practices were different. Teachers were trying to make students determine the event type at the first stance, then, determine the sample size (Çelik & Güneş, 2007). While doing so, teachers led students to memorize the ways to determine the event types (Memnun, 2008) with if-then statements (Rubel, 1996). Although some memorizations helped students to understand the topic, the excessive memorizations influence the learning negatively (Gürbüz, 2008). However, Kazak (2009) stated that students had to understand the probabilistic situations and developed thinking on them. From the perspective of students' cognition in learning probability, the excessive amount of memorizations influenced students' intuitions in negative ways (Fischbein, 1987; Myers, 2002; Stavy & Tirosh, 2000) and students generally used their intuitions in dealing with the problems instead of constructing logical structures (Fischbein & Schnarch, 1997). In general, the intuitions mislead (Shaugnessy, 1992). Therefore, it is essential to create correct intuitions. Fischbein (1987) also stated that students should create new intuitions to construct understanding of probability topics.

Continuing with the teaching practices, the selection and the use of the methods in the teaching of probability also influence students to learn and understand the topic or develop positive or negative attitude toward the learning of probability (Bulut, 2001; Bulut, Yetkin, & Kazak, 2002; Çelik & Güneş, 2007; Gürbüz, 2007). At this point, Gürbüz et al. (2010) found that direct teaching was dominant in schools, where teachers were generally active while students were passive listeners. The general trend was to follow the course book and prepare the instruction parallel to course book (Memnun, 2008). In addition, Rubel (1996) stated that the focus in the teaching of the probability was generally on the mathematical operations, formulas, and rote memorizations. Therefore, students were lost while dealing with the tasks given in probability subjects. It was found by Bulut, Ekici, and İşeri (1999) that students were exposed to the memorizations of the formulas and rules instead of making them think about the questions. Therefore, students did not have opportunities to synthase facts and construct patterns while solving probability questions. These situations were affecting students' intuitions and resulted in misconceptions (Fischbein, Nello, & Marino, 1991).

There were many studies with different instructional methods in order to make students comprehend the probability and to resolve probability misconceptions (Babai et al., 2006; Brunner, 1997; Chiese & Primi, 2008; 2009; Dereli, 2009; Gürbüz, 2005; 2007; Gürbüz et al., 2010; Nicolson, 2005; Polaki; 2002a; 2002b; Şengül & Ekinözü, 2004; Tirosh; 2000; Watson, 2001; Yazıcı, 2002). Some of these studies were related to provide students with better understanding of the probability subjects, while some others were trying to resolve existing misconceptions some of which were intuitively ones.

For the teaching of probability, Gürbüz (2008) and Dereli (2009) compared traditional method with computer-aided method. Şengül and Ekinözü (2004) and Ekinözü (2003) used dramatization against traditional method in teaching probability. Moreover, Yazıcı (2002) compared exploration method with traditional one. In all studies, the achievement levels were higher in the experiment groups.

On the other hand, some studies were related to resolution of probability misconceptions, some of which were intuitively-based ones. Among them, Chiese and Primi (2008; 2009) created gaming situation in order to resolve recency effects among primary and college level students. This misconception was one of the intuitively-based misconceptions. Similar to Chiese and Primi's (2008; 2009), Polaki (2002a; 2002b) also tried to resolve intuitively-based misconceptions among middle school students. Since these studies were important, these studies were introduced widely.

Polaki's two studies (2002a; 2002b) are directly related to the teachers practices related to teaching probability. In both studies, Polaki investigated students' growth in probabilistic thinking after the instructions given. During the instructions, major practices were examined.

Polaki (2002b) studied students' growth in probabilistic thinking, especially for sample space and the probability of events. The sample of this study was composed of 12 students from 4th and 5th grade who were assigned to two instructional groups A and B according to achievement and grade levels. They were assessed via interviews. The instructional time for sample space and probability of event was 70 % while probability comparison took 30 % of it. The general instructional strategy began with easy one-dimensional problems, then, continued with two-dimensional problems. The problems were open-ended and whole-group discussion was used. Then, students made conjectures about the solutions to the opening problem. The findings of this study indicated that major mathematical practices for sample space were odometer strategy which listed complete set of outcomes in either one or two dimensional problems and multiplication rule. For the probability of an event, the findings indicated that mathematical practices included "use of invented informal language to describe probabilities" and "use of sample space composition as a basis for probability predictions" (p. 357). In addition, it was observed that there were minimal student to student interactions in the classroom (Polaki, 2002b).

Polaki (2002a) again investigated the growth of 4th and 5th students' probabilistic thinking by means of two versions of teaching experiments. The first version included small sample experimental data and sample space composition for dealing with probability problems. On the other hand, second version included large sample with software simulations after students were provided with sample space experimental data. 12 students were purposively selected for this study. Quantitative findings indicated no significant difference in probabilistic thinking of the students for these groups while each teaching experiment showed noticeable influence on students' growth in probabilistic thinking. The important finding related with present study here is that the researcher observed minimal student-to-student interaction and that students generally relied on procedures teachers provided.

From different instructional methods in the literature considered above, it was observed that the tests or the questions asked to determine the achievement levels of the students were in line with the instructional methods. Therefore, students were acquired familiarity with the questions asked in the studies.

Turning back to students' cognition, in addition to their intuitions in learning probability, previous experiences in daily life (Fischbein, 1975) and in school learning (Kvatinsky & Even, 2002), previous knowledge, and types of instruction methods positively or negatively shapes students' intuitions (Fischbein, 1987). Therefore, their learning of probability was also affected by these factors. In fact, the teachers should prepare their instructions according to such factors. As it was mentioned, however, teachers generally prepared their instructions parallel to course books (Memnun, 2008).

Students make generalizations from experiences in daily life (Fischbein, 1975) and from experiences in previous learning (Kvatinsky & Even, 2002). They try to relate their experience with the newly encountered tasks. In some cases, this relation becomes incorrect. In relation with the learning of the probability, the daily life experiences are important factors due to the nature of the topic, which includes instances from the daily life (Fischbein, 1987; Kazak; 2009; Tirosh & Stavy, 1999a). Considering the previous learning, wrong learning and intuitions become barriers for students to learn probability (Çelik & Güneş, 2007), because new knowledge is built on the previous ones (Papaienymou, 2009).

In line with the previous knowledge, there were many topics for learning probability including fractions and its comparison with percentage (Memnun, 2008), sets (Bar-On & Or-Bach, 1988), permutation and combination (Jones, Langrall, & Money, 2007; Yazıcı, 2002). Therefore, it was necessary for students to have pre-knowledge about such topics. However, most of the students came with the lack of pre-knowledge in the process of learning probability (Bulut, 2001).

2.2.2 Students' Learning Difficulties and Underlying Reasons in Probability

The probability topic is considered as one of the hardest topics in middle and high school mathematics (Kazak, 2008; 2009) even for adults (Kazak, 2009). Therefore, the appearance of students' difficulties and misconceptions was inevitable in such grade levels. Before introducing general difficulties and misconceptions that appears among students, there was a need to uncover the underlying reasons for them in probability.

Considering the reasons for students' difficulties and misconceptions, the literature focused on four main categories, which were student related reasons (e.g., Bulut, Ekici, & İşeri, 1999; Çelik & Güneş, 2007; Fischbein, 1975; Güven, 2000; Memnun, 2008; Papaieronymou, 2009; Rubel, 1996), teacher related reasons (e.g., Bulut, 2001; Gürbüz, 2007; Köğce & Baki, 2009; Memnun, 2008; Polaki, 2002a; 2002b; Rubel, 1996), task related reasons (e.g., Fox & Levav, 2000; Havill, 1998; Kazak, 2008; Papaieronymou, 2009; Riccomini, 2005), and students' attitudes towards probability (e.g., Fererman, 2000; Fischbein, Nello, & Marino, 1991; Kazak, 2008; Shaugnessy, 1992).

For the student related reasons, students' insufficiency in readiness plays crucial role (Fischbein, Nello, & Marino, 1991). Students do not have necessary pre-knowledge before beginning to learn probability (Stavy & Tirosh, 1996; Gürbüz, 2005; Memnun, 2008). As it is known, mathematics is built on previously learnt knowledge (Papaieronymou, 2009). If students are lack of pre-knowledge that is necessary for probability, they will not understand the concepts and applications in probability (Çelik & Güneş, 2007). Therefore, students may rely on their wrong intuitions and fall into misconceptions (Fischbein, 1987), because students could develop incorrect intuitions without necessary pre-knowledge (Fischbein, 1987). Another student related reason is that students are focused on the memorization of the rules and formulas instead of comprehending the topic (Bulut, Ekici, & İşeri, 1999; Memnun, 2008). So, students may lose the main points and move away from the aim of the tasks asked and the content taught (Rubel, 1996). In fact, the memorization can also be attributed to teacher related reasons. It is possible that teachers lead students to memorize the facts and formulas (Memnun, 2008). The excessive memorization may result in misconceptions in students' minds (Gürbüz, 2008). In addition, teachers' knowledge is also another reason for the misconceptions (Batanero, Godina, & Roa, 2004; Memnun, 2008). If teachers' knowledge is not enough, the development of new concepts in students' mind becomes insufficient (Bulut, 2001). Teachers' knowledge includes the content knowledge (Batanero, Godina, & Roa, 2004) and the use of methods (Bulut, Yetkin, & Kazak, 2002; Gürbüz, 2007) and materials (Gürbüz, 2008). It is possible that teachers' mislead students to learn probability (Bulut, 2001).

For the task related subjects, students need to be familiar with different types of tasks and problems (Fox & Levav, 2000). If students are not prepared for unfamiliar questions, it is highly possible that students experience difficulties (Havill, 1998; Papaieronymou, 2009). In Turkey, the questions asked in classrooms are generally consistent with the university entrance exams (Köğce & Baki, 2009). Therefore, students are not much exposed to unfamiliar situations.

Lastly, students' fears (Memnun, 2008; Shaugnessy, 1992) and their lack of encouragement (Kazak, 2008) to deal with mathematics are considered as the reasons for students' misconceptions. Students' fear influences their intuitions in a negative way and may lead students to fall into misconceptions (Fischbein, Nello, & Marino, 1991). In relation with teacher and students' fears, teachers can also promote fear over students (Memnun, 2008).

The students' general difficulties were observed in determining the sample size and the set of the expected elements, in determining the event types (Çelik & Güven, 2007), in differentiating dependent and independent events (Kazak, 2008), in the interpretation of the chance, and in the interpretation of the concepts of possible and impossible (Li & Periera-Mendoza, 2002). Students did also subjective judgments, trying to develop their own solution. Moreover, students had confusion in simple and compound events. They also ignored the size of the samples in the questions (Memnun, 2008). All difficulties and misconceptions that students had were not considered as the intuitively-based ones. However, most of the misconceptions in the probability could be considered under the intuitively-based misconceptions (e.g., Çelik & Güven, 2007; Fischbein & Schnarch, 1997; Kazak; 2009; Li & Periera-Mendoza, 2002; Memnun, 2008; Stavy & Tirosh, 2000).

2.2.3 Teachers Awareness and Knowledge of Students' Misconceptions

In Stohl (2005) and Batanero, Godina, and Roa's (2004) studies, what teachers' knowledge should include was clarified. Accordingly, teachers should have the content knowledge of probability, pedagogical knowledge including instructional methods, strategies in problem solving processes, ability to analyze course books and other materials, and their use in teaching practices. In addition, teachers' knowledge of students' misconceptions (Stohl, 2005) and thinking (Rowan et al., 2001) are also considered as the necessary knowledge that teachers need to have. For students' thinking, Rowan et al. (2001) refers to "the knowledge of likely conceptions, misconceptions, and difficulties that students at various grade levels encounter when learning various fine-grained curricular topics" (p. 5).

> Even if a teacher possesses a sophisticated understanding of specific conceptual obstacles and their causes, such awareness may not be prioritized during teaching (Bayazit & Gray, 2006, p. 121).

From the quotation given above, even if teachers are equipped with the necessary knowledge, it will not necessarily mean that teaching practices become successful in classroom environment. However, having necessary knowledge was crucial in the way through successful teaching practices (Shulman, 1986).

Although teachers need to beware of the possible misconceptions in probability before beginning to the teaching practices, the literature indicates that even teachers have misconceptions including intuitively-based ones (e.g., Begg & Edwards, 1999; Fischbein & Schnarch, 1997; Ilgun & Işık, 2012; Jendraszek, 2008; 2010; Watson, 2001). For example, Jendraszek (2008) states that teachers have intuitively-based misconceptions such as availability and types of representativeness heuristics (Begg & Edwards, 1999; Fischbein & Schnarch, 1997), and time-axis probability (Carnell, 1997). In addition, Ilgun and Işıksal (2012) found that pre-service elementary school teachers had misconceptions of conjunction fallacy, and the positively and negatively recency effect.

Continuing with the knowledge of students' misconceptions, Watson (2001) found that teachers were aware of the difficulties of finding probabilities, interpretation of the data and outcomes, conceptual understanding of theoretical and experimental probabilities, as well as conditional probability. In addition, Papaieronymou (2009) stated that teachers needed to acquire common difficulties and misconceptions including learning of the probability concepts, law of large numbers, representativeness, and biases.

In order to resolve misconceptions, the necessity to use hands on and practical activities (Watson, 2001) including visual (Batanero & Diaz, 2012) and physical materials (Gürbüz, 2005) were important knowledge that teachers needed to know. Memnun (2008) and Nicolson (2005) implied the importance of the necessary repetitions which were not sufficiently performed during the teaching practices.

Batanero, Godina, and Roa (2004) found that teachers were lack of the knowledge about students' difficulties and misconceptions, teaching approaches, resources, and the use of materials and technologies in the classrooms. Another important point to provide students with understanding of the probability was students' involvement (Ojeda, 1999). Watson (2001) found that teachers were doing surveys and projects, playing chance games for getting their involvement.

Although daily life examples were important to imagine the probability concepts (Fischbein, 1987), teachers were using verbal expressions in order to explain probability concepts including "certain", "probable", impossible (Paparistodemou, Potari, & Pitta, 2006). However, verbal expressions were not persistent in students mind (Gürbüz, 2008). In fact, teachers theoretically knew the importance of giving daily life examples, the students' involvement, the use of tools; they were not transferring them into practice in the teaching of the probability (Paparistodemou, Potari, & Pitta, 2006). One of the reasons for this situation was that teacher training in probability did not satisfy the demands of classroom practices (Ojeda, 1999).

Lastly, it was found that teachers gained awarenesses and knowledge of students' difficulties and misconceptions via experience (Memnun, 2008). In the early years of teaching profession, teachers cannot combine their awareness and knowledge with the practice (Paparistodemou, Potari, & Pitta, 2006).

2.3 Summary of the Literature

In this chapter, the current literature about intuition, its characteristics, its relation with probability, its role in learning probability, common intuitively-based misconceptions among students in probability, teachers' teaching practices in teaching probability, and students' difficulties and underlying reasons in learning probability were reviewed before beginning to collect data. In addition, national and studies related to teaching of probability were also reviewed. The summary of the literature review was given as follows.

- Intuition has many definitions (e.g., Bruner, 1962; Fererman, 2000; Fischbein 1987; Shirley & Langan-Fox, 1996) one of which is immediate apprehension without conscious awarenss (Rorty, 1967; Stavy & Tirosh;

2000). In addition, it has many characteristics including unconscious apprehension (e.g., Herman, 2007; Jung, 2002; Myers, 2002), affecting people's reasonings (e.g., Dane & Pratt, 2007; Fischbein, 1987).

- Probability topic has many application areas in daily life (Anastasiadou, 2009; Andra, 2011; Freudenthal, 1970; Kazak, 2008; 2009; Kahneman & Tversky, 1982; Kvatinsky & Even, 2002; Way, 2003) and students can easily be affected by their intuitions in learning it (Fischbein, 1975; Fischbein & Schnarch, 1997; Langrall, Jones, & Mooney, 2007; Pratt, 2000). Students need to use correct intuitions during the learning processes (Fischbein, 1999; Weber & Alcock, 2004).
- Using intuitions has positive impact on learning probability. On the other hand, sometimes, it may behave as an obstacle to learn it. (Babai et al., 2006; Fischbein, 1975; 1982; 1987; 1999; Havill, 1998; Kazak & Confrey, 2007; Myers, 2002; Stavy et al., 2006; Weber & Alcock, 2004)
- Regarding probability, students have intuitively-based misconceptions while solving problems. Common intuitively-based misconceptions among students in probability are availability heuristics (Çelik & Güneş, 2007; Fischbein & Schnarch, 1997; Fox & Levav, 2000; Kahneman & Tversky, 1972; Shaugnessy, 1992; Tversky & Kahneman, 1983a), representativeness heuristics (including positively and negatively recency effects, sample size effect, and outcome approach) (Batanero, Godina, & Canizares, 2005; Chiese & Primi, 2008; Fischbein, 1975; Fischbein, Nello, & Marino 1991; Konold et al., 1993; Lecoutre, 1992; Li & Periera-Mendoza 2002; Rubel 2002; Shaugnessy, 1992; Tversky & Kahneman, 1971; 1982), simple and compound events (Çelik & Güneş, 2007; Fischbein, Nello, & Marino, 1991; Fischnein & Schnarch, 1997; Lecoutre & Durand, 1988; Li & Periera-Mendoza, 2002; Rubel, 2002; 2007; Shaugnessy, 1992), conjunction fallacy (Çelik & Güneş, 2007; Shaugnessy, 1992; Tversky & Kahneman, 1982; 1983b), and conditional probability misconceptions (Bar-Hillen & Falk 1982; Falk, 1979; 1983, Fischbein & Schnarch, 1997; Fox & Levav, 2004; Jones, Langrall, & Money, 2007; Kazak, 2008; Watson & Kelly, 2007). In addition, Tirosh and Stavy (1999a; 1999b; 2000) developed a theory of intuitive rules.

Tirosh and Stavy observed that students gave similar answers to unrelated tasks. Two rules were taken into consideration in this study. These were the misconceptions of “same of A – same of B” and “more of A – more of B”.

- There are many reasons of students’ difficulties in probability. These difficulties are stemmed from content related factors (Bills & Husbands, 2005; Fox & Levav, 2000; Keitel & Kilpatrick, 2005; Memnun, 2008; Papaieronymou, 2009; Riccomini, 2005), teacher related factors (Köğce & Baki, 2009; Marek, Cowan, & Cavallo, 1994; Nakiboğlu, 2006; Polaki, 2002a; 2002b; Rubel, 2002; Stohl, 2005; Ubuz, 1999), and student related factors (Bulut, Ekici, & İşeri, 1999; Çelik & Güven, 2007; Güven, 2000; Memnun, 2008; Skelly & Hall, 1993). Among them, the insufficiency of regular instruction (Batanero & Diaz, 2012; Batanero, Godina, & Roa, 2004; Chich & Pierce, 2008; Gürbüz, 2007; 2009, Yazıcı, 2008) and students’ incorrect intuitions (Fischbein, 1987; 1992; Fischbein & Schnarch, 1997; Fox & Levav, 2004; Memnun, 2008; Myers, 2002) are crucial for students’ difficulties and misconceptions in probability.
- Instead of only memorizing the rules and doing calculations, national (MoNE, 2005a; 2005b) and international curricula (CCSS, 2010; NCTM, 2000) emphasized understanding and interpreting probability concepts and computing probability values for interpreting data, using to solve problems, and evaluating the outcomes for making decisions.
- However, the aims of teachers’ teaching practices are not consistent with the aims of the national and international curricula (Bulut, 2001; Çelik & Güneş, 2007; Gürbüz, 2008; Havill, 1998; Köğce & Bal, 2009; Memnun, 2008; Papairenymou, 2009; Shaugnessy, 1992; Riccomini, 2005).
- The direct instruction method was dominant among mathematics teachers and they followed course books while teaching probability (Bulut, 2001; Gürbüz et al., 2010; Rubel, 1996; Memnun, 2008; Yetkin & Kazak, 2002). Teachers focused on formulas, types of events, rote memorizations. This situation directed students to misconceptions including intuitively-based ones (Bulut, Ekici, & İşeri, 1999; Fischbein, 1987; 1999; Fischbein, Nello, & Marino, 1991, Tirosh & Stavy, 1999a; 1999b).

- There were many studies that investigated the effect of different instructional methods to make students comprehend the topic and resolve their misconceptions (Babai et al., 2006; Brunner, 1997; Chiese & Primi, 2008; 2009; Dereli, 2009; Gürbüz, 2005; 2007; Gürbüz et al., 2010; Nicolson, 2005; Polaki; 2002a; 2002b; Şengül & Ekinözü, 2004; Tirosh; 2000; Watson, 2001; Yazıcı, 2002). Some of them specifically investigated the alternative instructional methods to resolve intuitively-based misconceptions (Babai et al., 2006; Chiese & Primi, 2008; 2009; Polaki; 2002a; 2002b).
- As for teachers' awarenesses, teachers need to know content knowledge of probability, pedagogical knowledge including specific instructional methods for probability, strategies in problem solving processes, and have ability to analyze resources to use in teaching practices (Batanero, Godina, & Roa, 2004; Stohl, 2005). In addition, teachers need to know students' thinking and misconceptions in probability (Batanero, Godina, & Roa, 2004; Bayazit & Gray, 2006; Stohl, 2005; Papaireonymou, 2009; Rowan et al., 2001; Watson, 2001).
- However, even teachers have misconceptions in probability (Begg & Edwards, 1999; Fischbein & Schnarch, 1997; Ilgun & Işık, 2012; Jendraszek, 2008; 2010; Watson, 2001) including intuitively-based ones (Begg & Edwards, 1999; Carnell, 1997; Fischbein & Schnarch, 1997; Ilgun & Işıksal, 2012; Jendraszek, 2008).
- In order to resolve misconceptions in probability, there are many hands on and practical activities (Batanero & Diaz, 2012; Fischbein, 1987; Gürbüz, 2005; Memnun, 2008; Nicolson, 2005; Ojeda, 1999; Watson, 2001). However, teachers did not transfer them into practice (Bayazit & Gray, 2006; Ojeda, 1999; Paparistodemou, Potari, & Pitta, 2006).

CHAPTER 3

METHODOLOGY

In this chapter, methodology of the research is described. This chapter includes the design of the research, participants, instruments, and data analysis procedures. Then, the reliability, validity, and ethical issues are discussed.

3.1 Design of the Study

The purpose of this study was to investigate middle and high school students' intuitively-based misconceptions, and teachers' knowledge about the misconceptions and teaching practices in resolving them. In line with the purpose of the study, since the aim of qualitative approach was to understand individuals' behaviors in their natural environment from a different perspective (Yıldırım & Şimşek, 2006), the qualitative approach was used in this study. This study included semi-structured and unstructured interviews with participants, the observations of teachers' regular instructions in the classrooms, and the open-ended tests subjected to the participants.

The multiple case study method was used in the study because the procedures can be replicated and the results among the cases can be compared (Stake, 1995) while investigating a concern or issue (Cresswell, 2007). The concern in this study was intuitively-based misconceptions and teachers' practices to resolve them. With this method, more generalized findings can be reached (Merriam, 1998). There was a need to bring standards to the aim of data collection, the data collection tools, and data analysis for the same research problem while studying with different cases. Therefore, holistic-multiple case study method was used to compare the findings gathered from the cases separately (Yin, 2003). So, more generalized findings were presented (Yıldırım & Şimşek, 2006).

In line with the procedures, benefits, and the requirements of multiple case study design, there were two cases in this study. The cases were middle and high school mathematics teachers and their students. In the first case, there were two middle school mathematics teachers who were teaching to 8^{th} grade students. Second case was including three high school mathematics teachers who were teaching to 11^{th} grade students. In addition, the procedures for the holistic multiple case study design were used because the same data collection tools were used and the same procedures were followed in the data collection and the data analysis. At the end, the results gathered from the cases were compared with each other.

3.2 Participants

In general, the purpose of a case study is to investigate and understand the characteristics of a unit (Cohen, Manion, & Morrison, 2000). Here, the unit can be a person, a class or a community. In this study, the units to be investigated were middle and high school teachers and their students. The participants were selected purposively.

In qualitative studies, the general concern is to gather maximum information from the cases while choosing a case (Stake, 1995). It is very beneficial for the researcher to choose "information rich cases" while collecting data from them (Merriam, 1998). Since the research questions of this study were comparing similarities and differences between middle and high school mathematics teachers' knowledge of students' intuitively-based misconceptions, their students' misconceptions, and their teaching practices in resolving them, the cases were considered as middle and high school teachers and their students. For selecting the participants for each case, the researcher conducted interviews with six school principals about the purpose of the study, about what was needed for the research, and about the ways to collect data in the schools. In addition, the researcher showed the permission letter obtained from Provincial Directorate of National Education (see Appendix E). After that, the researcher made informal interviews with three middle school teachers and five high school teachers in a city center located in the eastern part of Turkey. The researcher informed teachers about the purpose of the study, about what was needed to do during the study, and the ways to collect data.

To get richer information from the teachers, the researcher asked questions in a way that whether teachers were willingly to attend the study or not. For example, they were asked whether they are willing to be interviewed several times during the data collection processes or not. Their moods during the interviews were also important for the researcher. For example, a middle school teacher expressed the high workload in the school. While talking about the data collection, one science high school teacher did not want to be observed and recorded with the camera in the classroom. Therefore, the researcher had to eliminate these two teachers from the research.

Merriam (1998) asserted about the necessity of the other criteria for selection of the cases. The criteria should depend on the purpose of the study. Accordingly, the researcher considered the achievement levels of the schools because teachers' teaching practices might change while teaching the probability topic. In addition, students from different school types might have different intuitively-based misconception. The researcher wanted to choose one teacher from science high school which had high achieving students, one teacher from vocational high school which had low-achieving students, and lastly one teacher from Anatolian high school which had both high and low achieving students. Although both middle schools were located in the city region, one school was located in the city center while the other school was located around the city. The researcher learnt in the interviews with the principals that the school in the city center was very popular and the families were very interested in their children's education. On the other hand, the other school was located in the village-like suburban area; therefore, students who were living around this school attended the study.

Another important point was to get the same curriculum for the classrooms in which the study conducted. Although there were two teachers in vocational high school to teach probability, there were only two hours per week in one teacher's program, and four hours per week in the other teachers' program. To satisfy this criterion, the researcher eliminated the teacher who had two hours per week in the program. Therefore, two cases including five teachers were chosen purposively for the research. Each teacher was from different schools.

Another criterion was that teachers must have at least one-year experience (completing internship as a teacher) and must teach the probability subject before. The reason was that teachers needed to get in touch with students, to have information about students' needs, and to get familiarities about what the important points were to teach in teaching processes. Among five teachers, all satisfied this criterion.

The researcher preferred to use pseudonyms for each teacher. This part includes some background information about the teachers. The Table 3.1 below indicates some basic information for the teachers.

Table 3.1 Teachers' Demographic Information

Cases	Teachers	Gender	Age	Experience (years in teaching)	Graduation Year and Department
Middle School Teachers	Ahmet*	M	25	2.5	2009 – Department of Middle School Mathematics Education.
	Barış	M	30	6	2005 – Department of Middle School Mathematics Education.
High School Teachers	Cihan (Vocational)	M	27	2	2009 – Department of Mathematics 2010 – Teaching Certificate
	Doğan (Anatolian)	M	32	8	2004 – Department of Mathematics 2006 – Teaching Certificate
	Erdal (Science)	M	39	15	1998 – Department of Mathematics Education

*: All names are pseudonyms.

In this part of the study, the cases were presented. The general information was given about the teachers in the cases, the general environment of their classrooms, and what they generally did in a regular lesson. In addition, there was also information about student participants for the interviews.

3.2.1 Introduction of the Case 1: Middle School Teachers and their Students

The case one was including two middle school teachers (Ahmet and Barış) and their students. The general information about the teachers, their students, and classroom settings were presented in this section.

Ahmet was 25 years old and had 2.5 years teaching experience. He offered private preparatory courses in a private institution (dersane) to middle school students for a while, then, began working in the middle school located in the city center. In addition, he was also giving private courses to students. He has worked in this school since September 2012. He graduated from the department of middle school mathematics education in 2009. He got two probability and statistics courses during the teacher training program. He taught probability subject to middle school students. He was very anxious about the data collection procedures. To overcome this problem, the researcher followed his classes twice before beginning to the actual classroom observations.

Beginning with the classroom environment, the classroom was located in the ground floor. The walls were painted into white color and the classroom got sun lights during the lessons. The classroom was very clean. Students were getting full time education in the school. The lessons were beginning at 8:20 and ending at 15:20. The mathematics lessons were at 8:20 to 9:50 in Tuesday in two sections and at 10:00 to 11:30 in Thursday in two sections. Each class hour lasted 40 minutes. Therefore, all mathematics lessons were before the noon.

The sitting arrangement in the classroom was given below in the Figure 3.1. The boys and girls were shown as “B” and “G”, respectively. There were 10 boys and 12 girls in the classroom. There were more desks for students than their needs in the classroom. Although the general appearance of the sitting arrangement was seen in the Figure 3.1, sometimes students were changing their desks before, between or during the lessons.

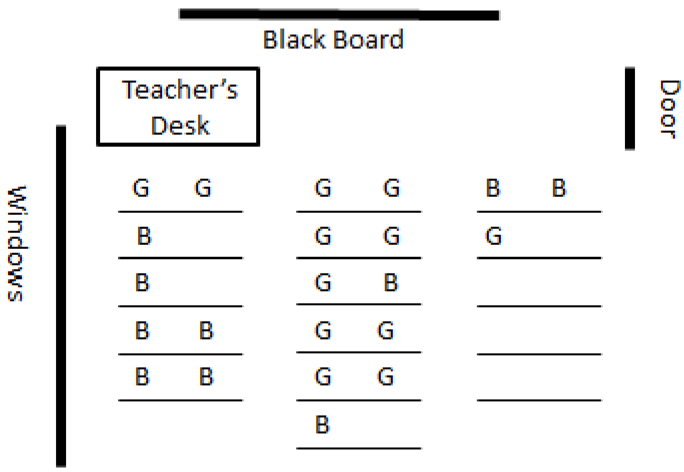


Figure 3.1 General appearance of Ahmet’s classroom

In his regular lessons, Ahmet generally began with the roll call. After the roll call, teacher tried to manage the classroom. There was a classroom management problem in the classroom. Students were going from one row to another every time. The classroom was becoming very noisy in many times. Teacher sometimes spent times for making the classroom be quiet and for making students sit on their desks.

Ahmet was stick to the course book. He always followed it and did not use any other sources. He wrote everything in the book to the blackboard and expected students to write it on their notebooks. He was using direct instruction method. He did not ask anything to measure students' readiness or he did not do formative test before teaching the probability. Students were taught basic concepts and some probability topics in 6^{th} and 7^{th} grades. However, they did not consider whether they forgot the topics or not. He wrote the concepts and definitions on board and explained them verbally. In the question solving process, he expected some students from front rows to write the question on the board. He sometimes made students solve the questions on the board. In general, he explained the solution on the board and passed to the next question. Between the questions, he spent too much time for writing the question to the board, for waiting for students' solutions, and for waiting students to write the solutions to their notebooks. He was always asking whether students understood the question or not. During the question solution processes, students faced with some concepts that they had never learnt before or they could not remember. However, the teacher did not give pay attention to these concepts. Sometimes, he briefly said the meaning. The student-teacher interaction was very limited during the observations.

He taught dependent/independent probabilities and theoretical/practical and subjective probabilities in six class hours. He did not mention about the previous topics.

Barış, on the other hand, was 30 years old and had 6-year teaching experiences. His teaching profession began in the MoNE. First of all, he worked in a different city for a year. Then, he began working in the city where the study was conducted since September 2008. The school he was working at was located in the suburban part of this city. In addition to lessons in this school, he was also teaching mathematics in a

SODES (Social Support) project conducted in the city center for students who were studying for SBS exam and were financially in a bad condition. Moreover, he was preparing students for the regional workshop of TÜBİTAK (Scientific and Technological Research Council of Turkey). He was very relaxed about the data collection procedures.

He had been teaching in his school for four years. The classroom was in a bad condition compared to Ahmet's classroom. The wall paints were not good enough. However, the classroom was clean enough and got sun lights. Students were getting half time education in the school. They were coming only in the morning. The lessons in the school were starting at 6:40 and ending at 11:40. The mathematics lessons were from 8:10 to 9:35 on Monday and from 6:40 to 8:05 on Wednesday in two sections. Each section lasted 40 minutes. There were five-minute-breaks between lessons.

Although the classroom was one of the less crowded classrooms in the school, there were 37 students. Among them, 20 students were male and 17 students were female. General appearance of the sitting arrangement was shown below in the Figure 3.2. Sometimes students were changing their desks. However, it was very limited. In addition, some students were coming a little late on Wednesdays because the lessons began very early in the morning.

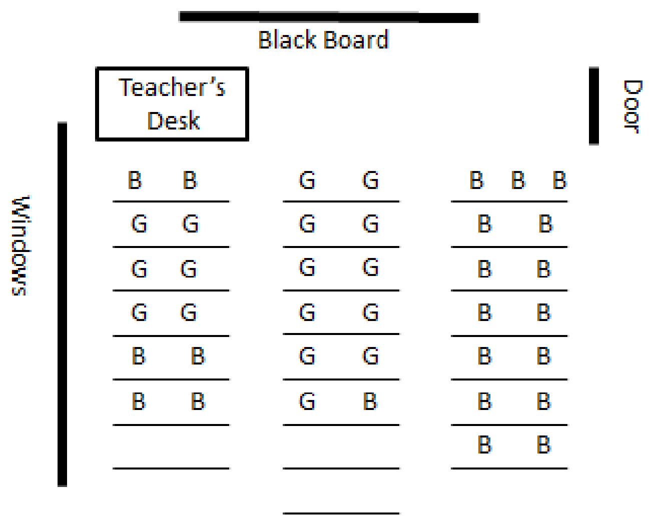


Figure 3.2 General appearance of Barış's Classroom

In his regular lessons, Barış also began with the roll call. Then, he mentioned about what was learnt in the previous lessons and related it with the following concepts or topics. He tried to eliminate the readiness and to explain the concepts and application according to students' needs before or during the lessons. He was using direct instruction method. In addition, he was questioning students with question-answer method and evaluating their answers. While beginning to the probability subject, he briefly mentioned about the previous topics in the probability. He explained the concepts and the necessary knowledge for preparing students to 8^{th} grade probability curriculum. He also used materials and gave daily life examples in the lesson. He was also stick to the course book. However, he rarely used different sources and materials in his lessons.

He was also writing whole question to the blackboard. However, he sometimes used abbreviations related to questions and solved the questions based on them. He was giving enough time to students to solve the questions. Then, he directly solved the question or expected one student to solve it on the board. When new or possible forgotten concepts appeared in the question solution, he explained it by using different methods such as giving daily life examples. There were good interaction between the students and the teacher. Students could easily ask questions to teachers and the teacher was giving satisfactory answers. In addition, the teacher was also asking some questions to students to make them understand the subject comprehensively.

Although the curriculum was including only the topics of the dependent/independent probabilities and theoretical/practical and subjective probabilities, he also taught the probability of simple events and the inclusive-mutually exclusive events. He also solved questions for the topic of the infinite probabilities. He allocated six class hours for the probability subject. During six hours, he also briefly explained the basic probability subjects, formulas and previous lessons in the first class hour.

3.2.2 Introduction of the Case 2: High School Teachers and their Students

The case two was composed of three high school teachers and their students. This section of the study gives general information about these teachers, their students, and the classroom settings.

Cihan was 27 years old and had two-years of teaching experience. He graduated from mathematics department of a science and art faculty in 2009. Then, he continued to a non-thesis master degree for the certificate of teacher education. He completed this degree in 2010. In 2012, he was continued to the master degree with thesis in the department of computer education technologies. He was working in the vocational high school. This school was the first place for his teaching experience. He was also working as the chairman of the mathematics department in his school. Therefore, he was responsible to prepare the basics of the teaching program. Similar to Ahmet, he was also excited when he heard the observations in the classroom. Before beginning the data collection processes in the study, the researcher observed two hours of his instructions.

He taught two years in this school. The classroom was in a good condition, clean and had white painted walls. It got sun lights during the lessons. All students were getting full time education. The lessons were from 8:00 and ending at 16:10 on Mondays and 15:20 on the other week days. The mathematics lessons were from 10:30 to 12:10 on Tuesday and from 13:30 to 15:10 on Wednesday in two sections. Each section lasts 45 minutes. There were ten-minute-breaks between lessons.

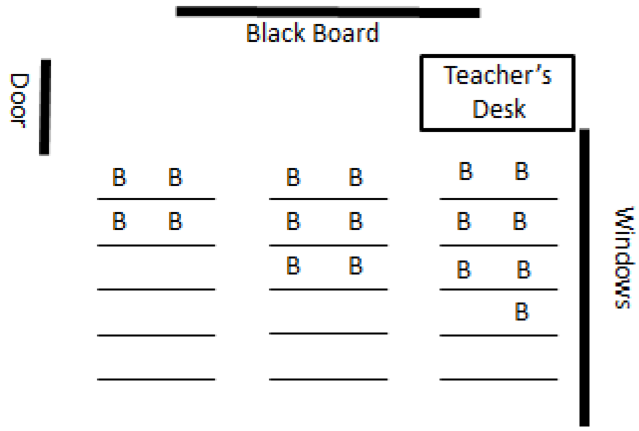


Figure 3.3 General appearance of Cihan's classroom

The classroom was less crowded one among the other classrooms in the study. There were 17 male students. There were no female students in this classroom. The general arrangement of the classroom was shown in the Figure 3.3.
Since the classroom was not crowded. The roll call was always skipped. He usually asked one of the students whether all students are present or not. Then, he mentioned about what was done in the previous lessons. He was also mentioning about what would be learnt in two class hours in that day. He was using direct instruction method in the lessons. He was always making students remember the concepts and was always repeating the meanings of the concepts, formulas, and the ways of solving the questions during the lessons. Question-answer method was widely used in the lessons. When he began to teach probability subject, he mentioned about the relations among the topics of permutation, combination, and probability. He spent considerable amount of time for concept teaching. He tried to eliminate the inadequacy of students' readiness for the probability subject. During the lessons, he was stick to the text book. However, this text book was not one of the MoNE's course books; instead, it was for preparatory book for university entrance exam. He was tried to solve as many questions as possible. He did not use any materials for teaching other than course book. Instead, he tried to give daily life examples.

In general, he was solving the questions written on the blackboard. However, he was giving permission to the students to solve the questions on the board. He was directing students and telling the ways of solving it while students were on the board.

He was explaining the solutions at least two times. Before beginning to solve the question, he mentioned about the type of the question and the ways to solve it. He was very stick to the formula. He was trying to transform the given information into formulas. If any new concept appears in the question, he was explaining it or asking students for explanation. He was not giving enough time to students to find the answer. There were student-teacher interactions. However, it was not that much as in the Barış's or Doğan's classrooms.

In the curriculum, there were 10 class hours for probability subject. Cihan used all of them. He completed whole subject in these lessons, then, he used last two hours for solving mixed questions. He also taught the infinite sample space subject.

Doğan was 32 years old and had eight years of teaching experience. He was graduated from mathematics department from science and art faculty in 2004. Then, he continued to non-thesis master degree for getting the teaching certificate in educational institutions. Before working for MoNE, he also offered private preparatory courses (dersane) in private institutions for three years. He got the certificate while working in the private institutions. Then, he began working in different high schools in the city where the study was conducted. He has worked in Anatolian high school since September 2010. He stated that he was very flexible and relaxed in the classroom; therefore, he was open to be observed in the classroom and to be interviewed.

Doğan was a teacher in Anatolian high school. He has taught in this school for three years. His classroom was in good condition and painted to white. It was clean and getting sun light during the lessons. The classroom included two boards, white and black. However, the blackboard included smart board behind it. So, teachers could use smart board whenever they wanted. Doğan did not use the smart board any time during the observations. Students were coming to school both before and after noon. The lessons in the school were beginning at 8:00 and ending at 16:05. The mathematics lessons were from 13:30 to 15:10 on Tuesdays and from 9:50 to 11:30 on Wednesdays in two sections. Each section lasted 45 minutes. There were ten-minute-breaks between the lessons.

The class was not very crowded. There were 21 students in his classroom. There were 15 male students and six female students. During the observations, some students were attending a kind of competitions among the high schools. Therefore, two or three students could not attend to some lessons. In addition, there were very few changes in sitting positions among the students during the observations. The general sitting appearance is shown in the Figure 3.4 below.

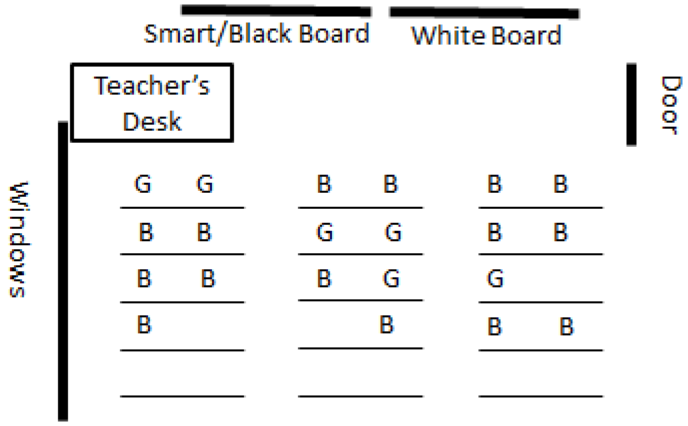


Figure 3.4 General appearance of Doğan's classroom

In his regular lessons, Doğan also began with the roll call. After then, he briefly mentioned about what would be learnt in the lesson. He was also mentioning about the previous lessons' content. He was asking about whether there were any points students could not understand in that content. He was generally using the direct instruction.

He began to teach probability subject with the concept definitions. While doing this, he was stick to a book which was a preparatory book for university entrance exam. He was writing the concepts to the board and explained them by using daily life examples. He did not spend much time on concepts. However, he asked students the meanings of them and expected examples from them. Although the classroom was provided with a smart board, he never used it during the observations. The general trend of his lessons was to solve as many question as possible. He was also trying to choose the questions from different types. He did not use any material other than the supplementary book.

He was writing the questions to board and giving enough time to the students for solving it. While waiting them, he was checking their solutions on their notebooks and showing the incorrect points if including. He was asking students to solve the question on the board. If they were stuck on the solutions, he helped to the students. Moreover, he was solving questions as many possible ways as he could. First of all, he was solving the questions in formal solution with formulas and concepts, then, he was trying to show the shortcuts. In addition, if there was any point which was not understood, he was giving satisfactory explanations. The student-teacher interaction was very high during the observations.

Although the time allotted for probability subject was 10 class hours, he completed the subject in seven hours. He did not teach the topic of the infinite probabilities. He completed the subject in six hours and he spent one hour for solving mixed questions from a test sheet including 16 multiple-choice questions.

Erdal was 39 years old and had 15-year teaching experiences. He was working in science high school since September 2010. He graduated from the department of mathematics education from faculty of education in 1998. He stated that he completed the university in 4.5 years. He began working in public high school in February 1998. He worked in two different high schools in the city center where this study was conducted. After working three years, he gave resignation and began working in private institution in a different city. He was offering preparatory courses to students both in private institutions and in his home office. He started to work in a town of a different city. One year after he started to work in private institutions, he moved to that city's center. He worked five years in private institutions. Then, he decided to work in public schools. He moved to city where the study was conducted, again. He had been there since September 2006. He worked three different high schools. In the meanwhile, he was also working in private institutions. At the end, he had his own office for offering private courses. He was very interested in new applications and materials in mathematics. He was also very active in TMOZ (an internet platform -*google group*- for mathematics teachers, mathematics learners, and students in order to share ideas, and questions and to interact with others in Turkey). He was very interested in finding and solving different questions in mathematics. He was also preparing students for the regional workshop of TÜBİTAK. This teacher was also the only teacher who used smart board in classroom. He was very relaxed while the observations.

Erdal's school was very prestigious in the city where the study was conducted; therefore, the physical condition of the school was very good. The classroom was provided with the smart board similar to that in Anatolian high school. The students were coming to school before and after noon. The school was a boarding school both for girls and boys. Therefore, most of the students were staying in the dormitory. The school was starting at 7.30 and ending at 15.35 in the school. The mathematics lessons were from 7.30 to 9.10 on Wednesdays and from 13.00 to 14.40 on Thursdays in two sections each. Each section lasted 45 minutes. There were ten-minute-breaks between lessons.

In general, the classrooms in science high school were not very crowded. There were 21 students in Erdal's classroom. Among them, six students were girls and 16 students were boys. The researcher did not see any changes in the desk arrangements. Since it was boarding school, all students were ready in the lessons during the observations. The general appearance of the seat arrangement in Erdal's classroom was as follow.

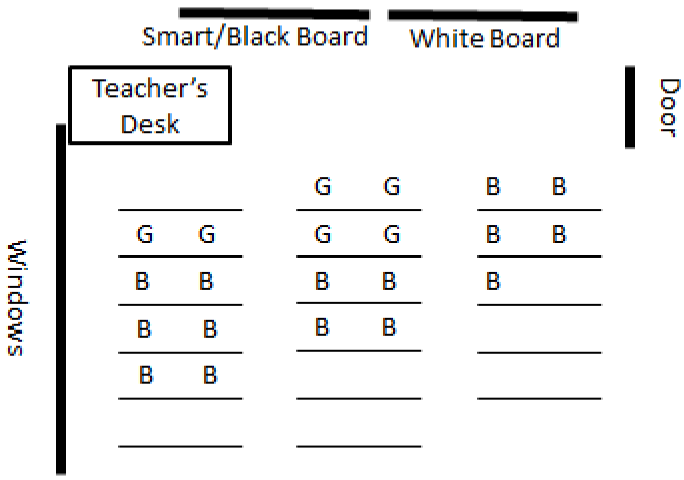


Figure 3.5 General appearance of Erdal's classroom

The lesson began with the roll call. Then, teacher swithed on the smartboard to teach probability. In concept teaching process, teacher explained the concept verbally. He did not pay much attention to them. He was considering that students' readiness for probability was enough and they did not need extra effort for it. He was only giving the basics of each topic in the probability subject. In general, he was explaining the formulas for independent probability, conditional probability. These were written on the board as small notes. Then, he was solving questions related to the topics taught. While doing it, he was using the smart board in order to reflect the pdf-file formatted preparatory books for university entrance exam. He used two different books during the observations. He was trying to solve different type of questions. He was also searching for interesting questions. Other than reflecting the questions and writing short notes, he did not use the smart book for any other purposes.

If there appeared any points which were not understood, he was explaining verbally by giving daily life examples. He did not use any materials during the observations other than these books. During the question solving processes, he waited for students to solve it. While waiting for them, he was checking students' answers and showing the incorrect points if it included. After showing the way of solving the question, he did not wait for the exact answer and passed to the another one.

Although the time allotted for probability subject was 10 class hours, he completed the subject in six hours. He completed the subject teaching processes in five hours and used last hour for solving unfamiliar probability questions and mixed ones. Although he did not mention about the infinite sample size topic, he solved such unfamiliar questions with this topic.

3.2.3 Student Participants for the Interviews

The study also included interviews with the students selected from the classrooms of each teacher. In the first stage, since the criteria were easy to satisfy, the convenience sampling method which provides researcher with saving time and effort was used (Patton, 2002). At the beginning, all students in the selected classrooms took the Probability Test of Intuition (PTI) (see Appendices C and D) test before and after the regular instructions. Then, intensity sampling method was used to choose students participants for the interviews. This type of method "consists of information-rich cases that manifest the phenomenon of interest intensely, but not extremely" (Patton, 2002, p.234). Patton gives examples of sample for the information-rich cases for this method as below average/above average students. In this study, the instruction affected students' knowledge of probability, therefore, the misconceptions that they had. It was expected that some students would change the solution methods in the second test, while others would not. In addition, it was also expected that some students would give correct answers while others would not. Here, the interest was whether students' intuition misleaded them in solving questions and whether their intuition changed in accordance with the given instruction or not. For the information-rich cases, two students from each classroom (total of 10 students from five classrooms) were selected for interviews. In the selection procedure, students' responses to the pre- and post- tests of the PTI were considered. According to the results, students who intuitively responded to question incorrectly in both tests and those who changed their answers after instruction were selected for the interviews. The purpose of these interviews was to investigate underlying reasons of the intuitively-based misconceptions in probability. Students' thoughts about the misconceptions and reasons behind their intuitively-based misconceptions were examined during the interviews. The interview questions were the same questions in the PTI. In line with the purpose of semi-structured interviews, their reasonings were seeked with "how" and "why" questions, so students' logic about the misconceptions was uncovered.

3.3 Procedure and Data Collection Tools

The data were gathered from different sources. The sources were the probability test which was prepared by the researcher, as a result of interviews with teachers and students, classroom observations, and field notes during the observations. Various sources of data help the researcher to develop overall organized data analysis and interpret data properly (Fraenkel & Wallen, 1996). As the purpose was to seek for patterns and common themes about the topic studied, triangulation method was a useful tool for this study (McMillan & Schumacher, 2010). In triangulation, the researcher uses two or more methods to collect data for studying on human behaviors and to find similarities about individuals' behaviors (Cohen, Manion, & Morrison, 2000). Similarly, this study was seeking pattern from their responses to open-ended questions to determine students' intuitive based misconceptions from different perspectives such as the answers to test, interviews and classroom observation.

Related to the topic studied, probability was taught to 8th grade students under the probability and statistics topic which was one of the five learning domains in middle school curriculum. The time allotted for probability was six class-hours for 8th grades (MoNE, 2005a).

On the other hand, the probability is taught in the 11th grade of students' education life in high school. There are three learning domains that students are responsible for. These learning domains are algebra, probability and statistics, and trigonometry. The probability is one of the four sub-domains under the probability and statistics learning domain. Permutation, combination, binomial expansion and probability are sub-domains under this learning domain. The total time allotted for probability and statistics learning domain is 28 hours. However, only 8 class hours were allotted for probability sub-domain. In the weekly course plan for 11th grade students, there are four hours mathematics lessons. Therefore, the time duration for the instruction for the probability sub-domain is two weeks (MoNE, 2011). However, teachers did not consider the time interval. Researcher attended teachers' all lessons in probability.
Due to the requirement of triangulation, there were many data collection tools used in the study, such as the test conducted to students, two semi-structured interviews for teachers and one for students, and field notes.

The study was conducted in two stages. In the first stage, the classes of 8th and 11th grade students were subjected to the PTI two times (before and after regular instructions). The reason of the administration of the test to the whole class was that conducting the study in natural setting was one of the most important characteristics of qualitative research (Creswell, 2009; Fraenkel & Wallen, 1996; Patton, 2002). Then, purposively selected participants were interviewed in the second stage. In addition to interview with students, all teachers from each grade levels were interviewed before and after the regular classroom instruction. Moreover, their instructions were observed during the process of teaching probability. The aim of the interviews with teachers and video-taped observations was to determine the differences and similarities of the teachers' awareness and knowledge about the intuitively-based misconceptions and the factors that might result in such misconceptions. In addition, teachers' awareness about students' cognitions in probabilistic thinking (e.g. their awareness about students' misconceptions, conceptions and mistakes in probability), how they organize the instruction to resolve the perceived misconceptions (e.g. the use of materials or instructional methods and strategies), and the possible teaching practices were also investigated.

3.3.1 Probability Test of Intuition

A test was prepared by the researcher to determine 8^{th} and 11^{th} grade students' common intuitively-based misconceptions in probability. It measures whether students fall into intuitively-based misconceptions and students' thinking about the misconceptions. The test included seven open-ended questions. In the preparation of the test, the researcher benefited from the current literature. The questions in the test were either adapted from the literature or prepared directly by the researcher. This test was administered twice. The first administration was at the beginning of the research, before the instruction was provided. After the instruction finished, the same test was administered to students, once again. The tests took one class hour and administered by the researcher himself.

Before administering the test, it was subjected to expert opinions for determining the suitableness of the content of the test for the participants of this study, its appropriateness for the curricula of 8^{th} and 11^{th} grade, and for the purpose of this study. Necessary rubric was provided to experts who evaluated the content of the test. The experts were doing their PhD in mathematics education and were research assistants in middle school mathematics education department. In the rubric, the experts were expected to indicate if the test items were suitable for middle and high school curricula and if the test items were measuring what they were supposed to measure (the existence of intuitively-based misconceptions) in order to satisfy content and face validity. Before getting expert opinions, they were provided with the probability content in middle and high school curricula and the explanation for each intuitively-based misconception. Before using the test during the study, it was administred to students in a high school classroom twice during under the same conditions during the pilot study. Pilot study held in a vocational high school with 24 students. The test-retest reliability coefficient was found as Pearson's r=0.86 (p<0.01), which indicated the strong positive correlation between students' scores in two occassions of the test (Cohen, 1992). Scoring criteria for all questions were presented in Appendix G. After preparing the last version of the test, it was used during the study.

Each question in the test was prepared according to different types of students' common intuitively-based misconceptions presented in the literature. This part of the study explained what misconceptions were matched with the specific questions in the test. Although expected misconceptions occurred among 8^{th} and 11^{th} grade students, there appeared other intuitively-based misconceptions. There are two misconceptions that were explained in the Stavy and Tirosh's (1999a; 1999b; 2000) theory of intuitive rules. The questions and expected intuitively-based misconceptions were explained below.

The first question in the PTI was developed by the researcher according to the features of the availability heuristics. The type of misconception occurs when the most readily available choice exist. Students fall into misconceptions by choosing the most remembered choice. The question is stated below.

> In a chance game, six numbers are chosen from numbers between 1 and 49 (1 and 49 are included). Somebody who correctly predicts these numbers wins the game. Çiğdem, Merve, and Hakan's predictions are given below:
>
> Çiğdem : 1, 2, 3, 4, 5, 6
> Merve : 44, 45, 46, 47, 48, 49
> Hakan : 39, 1, 17, 33, 8, 27
>
> Compare these persons' probabilities of winning the game. Explain your answer.

Here, the Çiğdem's probability of winning the game is shown as P(Ç). Similarly, Merve's and Hakan's probability of winning the game are shown as P(M) and P(H). The answer of the question is P(Ç)=P(M)=P(H). No matter what they predict, since all chose six from 49 numbers, their probabilities of winning the game is equal. The probability is equal to $\frac{1}{49}\cdot\frac{1}{48}\cdot\frac{1}{47}\cdot\frac{1}{46}\cdot\frac{1}{45}\cdot\frac{1}{44}$.

What is expected in this question as intuitively-based misconception is that students consider that Hakan's probability of winning (P(H)) the game is higher than Çiğdem's and Merve's, because the numbers that Hakan chose are mixed in order. Therefore, the most available choice for students is that P(H) is higher than P(M) and P(Ç). Here, availability heuristics is expected outcome from students' responses to the questions. If students give answer as P(Ç)=P(M)=P(H) or use the probability value as $\frac{1}{49}\cdot\frac{1}{48}\cdot\frac{1}{47}\cdot\frac{1}{46}\cdot\frac{1}{45}\cdot\frac{1}{44}$, their responspes are considered as "correct." If the answers indicate that P(H) is higher than others' probability of winning the game due to the shuffled order of numbers, these answers are considered as "availability heuristics misconception." The answers similar to that Merve's probability of winning the game is higher due to highness of the numbers selected are considered as "the more of A – the more of B" misconceptions." Students' other answers which are irrelevant of previously mentioned answers are considered as "incorrect."

The second question in the PTI is adapted from the question in the study of Fischbein and Schnarch (1997). The question is related to intuitively-based misconception of one type of the representativeness heuristics. There are three types of representative heuristics, which are *positively and negatively recency effect*, *sample size effect*, and *outcome approach*. The question is about the first one. This misconception occurs when there are events already happened. Students think that the outcomes of the previous events affect the future outcomes. However, each event occurs independent of others. The second question in the PTI is stated below.

> An unbiased coin is flipped three times and the outcome for each flip is head (H). If the coin is flipped fourth times, what is the most probable outcome between head (H) and tail (T)? Justify your answer.

Here, the correct answer is that head and tail have equal probability. That means the previous events do not affect the future outcomes. However, students are expected to think that the previous events affect the future outcomes. At this point, *positive recency effect* appears when students think that the probability of getting head is higher than that of getting tail. They think that this trend continues similarly. Therefore, they think that the next outcome would be the same as the previous ones which is head again. The answers indicating this situation is considered as “positively recency effect misconceptions.” Similarly, *negatively recency effect* appears in inverse situation. The latter is also called as *gambler’s fallacy* which means that the events have corrective power to 50 percent for each. Students think that the probabilities should be equal. Therefore, the number of the heads and tails should also be similar or same. They think that the next outcome would be tail, so, the number of heads and tails in the trials becomes closer. Such answers are considered as “negatively recency effect misconception.” If students mention about the independence of the events and state that the probability is ½, their answers are considered as “correct.” Students’ other answers which are irrelevant of previously mentioned answers are considered as “incorrect.”

The third question in the PTI is related to sample size effect, one type of the representative heuristic misconceptions. The question is adapted from the study of Fischbein and Schnarch (1997). For this type of intuitively-based misconception, students make their predictions according to the distribution of occurrences in a sample. For example, if somebody listens to the prediction of forecast in TV as 80 % of rain today, s/he thinks that there must be rain today. Mainly, representations of data mislead students to answer the questions. The third question in the PTI is given below.

> Compare the probability that there are at least two boys among three new-born babies and the probability that there are at least 200 boys among 300 new-born babies in a hospital. Justify your answer.

Here, the correct answer is that probability of the preceding event is higher hand the further one. The sample size for the first event is eight. (Here, the boys are shown as B, and the girls are shown as G) The sample is as follows. E={BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}. If we call first event as event A, then A={BBB, BBG, BGB, GBB}. Therefore, $P(A)=\frac{s(A)}{s(E)}=\frac{4}{8}=\frac{1}{2}$. On the other hand, finding the probability of the second event is different. The formal solution is as follows. Let’s we call the second event as event B. The sample size of the event B is 2^{300}. On the other hand, the number of element in the event B can be found as follows.

$P(B) = \frac{\binom{300}{200} + \binom{300}{201} + ... + \binom{300}{300}}{2^{300}}$ which is less than $\frac{1}{2}$. It is hard for students to apply this formula and to compare the probabilities. However, students can interpret that the probability becomes lower when the number of babies increases. This type of reasoning and students' corresponding responses are considered as "correct" answers. What is expected from students, on the other hand, is that students look at the ratios between the number of boys and the number of babies. It is expected that students cannot manage the sample size. Therefore, the intuitively-based misconception that students fall into is that the probabilities are the same. Students fall into the misconception of sample size which is one type of representativeness heuristics. Students' answers stating the equality of the probabilities are considered as "sample size effect misconception." Since same logic is valid for "the same of A – the same of B" misconception, the frequencies of the students who fall into sample size effect misconceptions are also indicated in the findings for the same of A – the same of B misconception. Moreover, a few students look at the numeric values of the situations and state that the latter event has greater probability due to greater numbers in the event. Their answers are considered as "the more of A – the more of B misconception." Students' other answers which are irrelevant of previously mentioned answers are considered as "incorrect."

The fourth question in the PTI was adapted from the question asked in Kahnemann and Tverstky's (1972) study. It is asked to determine students' misconception of the outcome approach which is one of the misconceptions related to the representativeness heuristics. This is similar to the sample size effect misconception. However, the outcomes that already happened gain more importance. In this type of the misconception, students neglect the sample size and the formal probabilities of the event or different distributions of the events and they give answers to the questions asked according to the other factors such as representations of the outcomes in the questions and the representations of the distributions. They gave more importance to these factors and fell into the intuitively-based misconception. The question in PTI related to the outcome approach is as follows.

16 game cards were randomly distributed to a group of players a few times. In two distributions, the numbers of cards that each student gets were given as below. Which does distribution have higher probability of occurrence among these two distributions? Justify your answers.

	Distribution I	Distribution II
Ayşe	4	4
Mehmet	4	3
Fatma	4	4
Hüseyin	4	5

Although it is hard to find the correct answer by using formulas and calculating the probabilities, it is obvious that each distribution has the same probabilities which are similar to the other distributions. Students' responses relevant to this situation are considered as "correct" answers. However, students look at the distributions and they try to predict the correct answer by comparing the differences between these distributions. It was expected from students that the probability of occurrence of the Distribution I (DI) was higher than that of the Distribution II (DII). Students intuitively think that each player gets the same number of cards no matter whether the distributions have the same probability of occurrences or not. Students' responses relevant to these reasonings are considered as "outcome approach misconception." On the other hand, students' answers that indicate the correctness of inverse situation (the probability of DII is higher than the probability of DI) due to shuffled numbers in distribution are considered as "availability heuristics misconception." A few students compared the players' cards without carefully reading the question. Accordingly, students' answers that indicated equality of the probability of Ayşe's and Fatma's card distributions due to the same number of cards that these players got were considered as "the same of A – the same of B misconception." Similarly, students' answers indicating that Hüseyin has higher probability of winning the game due to more cards he gets in the second distribution and that Mehmet has lower probability are considered as "the more of A – the more of B misconception." Students' other answers which are irrelevant of previously mentioned answers are considered as "incorrect."

The fifth question in the PTI was adapted from the question asked to 9 to 14 age old students in Fischbein, Nello, & Marino's (1991) study. It is related to the misconception of the simple and compound events. This intuitively-based misconception appears when students confuse the probabilities of simple and compound events. Students may try to find the probability of a simple event in accordance with the procedures used in finding the probability of a compound events, or vice versa. The fifth question asked in the PTI is given below.

> Compare the probability of getting a pair of 4-4 and a pair of 4-3 after throwing a dice twice. Justify your answer.

In this question, what is expected from students is to consider 4-3 and 4-4 outcomes as one after another. Instead, the question is asking the probabilities of getting pairs of 4-4 and 4-3. Here, the correct answer is that the probability of getting a pair of 4-3 is higher than that of getting a pair of 4-4, because 4-3 and 3-4 can be considered as a pair of 4-3. However, they may consider that the probabilities of getting pairs of 4-4 and 4-3 are equal without considering the other outcomes of any pairs. Related answers are considered as “simple and compound events misconception.” Students’ answers indicating the equality of the probabilities are considered as “correct” answers. Students’ other answers which are irrelevant of these answers are considered as “incorrect.”

The sixth question in the PTI was developed by the researcher. It is specifically asked to determine whether students have intuitively-based misconception of the conjunction fallacy. This misconception is based on the different events that either one covers the other. Therefore, the probability of one event would always be higher than the other. If students could not determine this situation and asserts that the inverse is true, they fall into this type of misconception. For this misconception, the question presented below was asked to students.

> Ayşe, Ali, and Ahmet are playing a game. In this game, a coin is thrown continuously until one player wins the game. Each player selects an order of consecutive heads and tails in a row. The first arrangement that appears in consecutive throws of the coin wins the game. Ayşe, Ali, and Ahmet’s arrangements were given below.
>
> Ayşe : HTHHH
> Ahmet : THTHHH
> Ali : HTHHHH
>
> According to these arrangements, put the probabilities of winning the game in order from higher to lower. Justify your answer.

As it is seen in the arrangements, Ayşe chooses an ordering of five throws which is included in both Ahmet’s and Ali’s arrangement. Therefore, Ayşe’s probability of winning the game is higher than Ali's and Ahmet's probabilities of winning the game. On the other hand, Ali's and Ahmet's probabilities of the winning the games are equal to each other. This is because they choose an ordering of six throws. In addition, Ayşe's probability of winning the game is theoretically higher than the others' probabilities. This is because Ayşe needs only five throws instead of six throws. It was expected from students to ignore the number of throws and that of heads and tails in players' selection and to assert that the probabilities are equal. If students saw the relation among the players' selection and stated that P(Ayşe)>P(Ahmet)=P(Ali), their answers were considered as "correct." Students' answers indicating the equality of the probabilities were considered as "conjunction fallacy misconception." On the other hand, a few students just looked at the numbers of heads and tails in the arrangments. Accordingly, since there were more heads in Ali's arrangement, they stated that Ali's probability of winning the game was higher than the other players' probabilities. These answers were considered as "the more of A – the more of B misconception." Students' other answers which were irrelevant of previously mentioned answers were considered as "incorrect."

The aim of the last question is to determine whether students have intuitively-based misconception of conditional probability or not. It was adapted from the question asked in Watson and Kelly's (2007) study. The time interval between two events plays important role in this question. Although the preceding event may affect the further one, the inverse situation is incorrect. However, it is possible that students think in that way. That is why this misconception is also called as *time-axis* probability. The question is presented below.

> There are equal numbers of blue and red balls in an urn. The ball chosen is not put into urn. For two balls chosen one by one, compare the probabilities of the situations stated below. Justify your answers.
>
> a) To be given that the first ball chosen is blue, the probability of the second ball to be blue
> b) To be given that the second ball chosen is blue, the probability of the first ball to be blue

Part (a) in the question is regular conditional probability question and the probability becomes less than one half. The formal solution is as follow. If there are n blue and n red balls, the probability of getting ball in the second choice is $\frac{n-1}{2n-1}$. This answer or students' answers indicating that the probability was less than one-half was considered as "correct", while the other answers were considered as "incorrect."

For the part (b), second event does not affect the first one. Therefore, the probability of getting blue ball in the first chose is one half. This answer was considered as "correct" for part (b). What is expected from students is that they confuse the time interval of the events happening. Therefore, they were expected to think that the further event affected the preceding event. Therefore, they might state that the probability was not equal to one-half. Similar justifications were considered as "conditional probability misconception." Students' other answers which were irrelevant of previously mentioned answers were considered as "incorrect."

3.3.2 Semi-structured Interviews

Two students from each classroom were subjected to the interview in order to get more information about the topic studied. In addition, mathematics teachers were interviewed before and after their instructions (see interview protocols in Appendices A and B). There were also unstructured interviews during the breaks between lessons. Clement (2000) asserted that the researcher could gather and analyze the data about the participants' ideas and mental processes in understanding of a situation that they encountered by means of interviews. In addition, the interview uncovers the ideas, structures and methods in resolving a problem experienced behind individuals' thoughts. Instead of just identifying what participants do for the given task, as well as, researcher tries to identify how and why they do it (Güven, 2006). In this study, the task for the interviews with students was the test questions that were asked to students. So, the researcher tried to identify how students found the answers to questions asked, how they organized the solution process, and why they used such methods in solution process. Moreover, how the instruction changed students' ideas and misconceptions in probability was investigated. Therefore, interview was suitable method to uncover students' intuitions, thoughts and other factors in solving probability questions. Again, the questions of this semi-structured interview were subjected to expert opinions. Their opinions were considered and the necessary changes were made.

The interview questions were prepared by the researcher in accordance with the purpose of the study. In the first interviews with teachers, for example, teachers were asked to answer to the questions “what are the general reasons for students’ difficulties in mathematics” or “what are the instruction related reasons for students’ difficulties in probability” in order to identify teachers’ knowledge about the reasons for misconceptions. In addition, the question “what are the students’ difficulty in probability topic” was asked in order to get evidences about teachers’ knowledge about students’ intuitively-based misconceptions. In the final interviews with teachers, they were asked about the possible misconceptions that might occur among students for the test questions. They were also asked about students’ difficulties in the instructions and the possible ways to resolve them. In general, the findings of interviews were used to identify teachers’ knowledge of students’ cognition, the teachers’ awarenesses and knowledge of the factors that might result in intuitively-based misconceptions, and to identify the teaching practices with the rationales of doing these. Moreover, especially second interviews were used to reveal teachers’ awareness about intuitively-based misconceptions.

The time duration for each interview was between 30 to 55 minutes. Two students from each classroom were selected according to their test results. After applying the test at the end of the instructions, the interviews were immediately held with both students and their teachers. This was because students might forget what they did in the test and teachers might forget the reasons of the strategies and tools used during their instruction. Each interview was audiotaped and transcribed.

3.3.3 Videotaped Classroom Observations

The researcher observed teachers’ teaching practices in the probability topic during the 2012-2013 academic years. During the instructions, the researcher attended the lessons allotted for the probability subject. According to teachers’ program, the topic was taught either in the fall or the spring semester. The researcher had a role of participant observer. The researcher overtly observed the lesson (Fraenkel & Wallen, 1996) that the teachers and the students knew that the researcher was observing the classroom. During the observations, the researcher sat at the back row of the classroom, followed procedures done in the instructions, and observed students' behaviors during the activity and teacher's responses to students' questions. The lessons were both video-taped and audio-taped. In addition, the researcher took field notes about teachers' directions, instructional procedures, students' behaviors and questions, and teachers' responses.

From teacher to teacher, the number of lessons allotted for teaching probability changed. In fact, the number of class-hour allotted for probability was six in the middle school and eight in the high school curricula. The middle school teachers followed the curriculum. However, Cihan, Doğan, and Erdal taught the lessons in ten, seven, and six class-hours, respectively.

3.3.4 Other Data Collection Tools

Other data collection tools were field notes that were taken during the observation, unstructured interviews with both the teacher and the students during the breaks, and also textbooks used for the lessons. They were considered to support the data gathered from other sources, to understand students' natural settings, and to what other factors that might positively or negatively affect the study. In addition, the course and supplementary materials were also used as data collection tools. Since the teachers were stick to the coursebook or the supplementary books during the instructions, the examples were taken from them. Therefore, the questions that teachers asked from books could have relation with students' intuitively-based misconceptions. Therefore, these questions were also investigated.

At last, there is a link between each research question and the necessary data collection tools. The Table 3.2 indicates this link between them below.

Table 3.2 Research questions and corresponding data sources

Research Questions	Data Sources
1- What are the middle and high school students' misconceptions in probability rooted from their intuitions?	Pre-test and post-test of PTI, Interviews with students
2- What are the similarities and differences between middle and high school students' intuitively-based misconceptions in probability?	Pre-test and post-test of PTI, Interviews with students
3- To what extent do middle and high school students' intuitions related to probability change after the regular instructions?	Pre-test and post-test of PTI, Interviews with students
4- To what extent do mathematics teachers aware of students' intuitively-based and of the factors that may result in them misconceptions in the middle and high school?	Interviews with teachers Classroom Observations
5- What are the similarities and differences between middle and high school mathematics teachers' awareness about students' intuitively-based misconceptions in probability and of the factors that may result in them?	Interviews with teachers Classroom Observations
6- What teaching practices do middle and high school mathematics' teachers carry out to overcome intuitively-based misconceptions in 8th and 11th grades?	Interviews with teachers Classroom Observations

3.4 Data Analysis

As it is mentioned above, observations, test results, interviews, and other sources such as field notes were the data collection tools for this study. As it is well known, human activities cannot be observed or measured directly. Even getting information directly the individuals observed is not always possible (Fraenkel & Wallen, 1996). Therefore, different data types were analyzed with different methods.

3.4.1 Data Analysis of the Pre- and Post-Implementations of the PTI

In the analysis of the pre- and post-tests of the PTI, the descriptive analysis method was used (Fraenkel & Wallen, 1996; Yıldırım & Şimşek, 2006). In the descriptive analysis, the data gathered were summarized and interpreted according to the themes which were already determined. In such process, the direct quotations were presented in order to reflect the participants' thoughts (Yıldırım & Şimşek, 2006). The procedure in data analysis was to form a frame for descriptive analysis. Then, the data gathered was analyzed and interpreted according to these thematics frame. (Yıldırım & Şimşek, 2006). In order to enrich the findings from descriptive analysis, the frequencies can be given for the themes (Cohen, Manion, & Morisson, 2000).

In the analysis of the pre- and post-test of the PTI, the intuitively-based misconceptions that appeared in each question of the PTI were considered as the main themes (see Appendix F). The themes were availability heuristics, types of representativeness heuristics (negatively and positively recency effect, sample size effect, and outcome approach), simple and compound events, conjunction fallacy, and conditional probability. In addition to these themes, two intuitive rules (more of A – more of B and same of A – same of B) from Tirosh and Stavy's (1999a; 1999b; 2000) were also considered as the themes. The findings from the analysis of the observations and interviews were used to support these themes. For example, teachers' knowledge of students' cognition and their teaching practice to resolve misconceptions were presented under each theme. Moreover, the frequencies tables for each misconception were presented. The frequency tables were presenting the number of students who fell into specific type of intuitively-based misconceptions for each case. In the administration of the pre-test, 59 students from middle schools and 59 students from high schools attended the test. There were also 59 students from middle schools and 59 students from high schools attended to the post-test. In order to reflect students' thoughts about the questions and their justifications for the intuitively-based misconceptions, the direct quotations were presented from students' responses to answers in the PTI and also in the interviews with students.

3.4.2 Data Analysis of the Interviews and Classroom Observations

The data that were gathered from interviews and classroom observations were analyzed according to content analysis method (Fraenkel & Wallen, 1996; Yıldırım & Şimşek, 2006). In this type of analysis, the purpose is "to reach appropriate concepts and relations to explain the data gathered" (Yıldırım & Şimşek, 2006, p.227). The procedure in content analysis is firstly to conceptualize the data gathered, to organize the data logically based on concepts appeared and, then, to determine the general themes that explain the data gathered (Yıldırım & Şimşek, 2006). In the first part, all data gathered from different data collection tools were coded. The researcher's data analysis method is similar to that of the procedures as Goetz and LeCompte (1981) states. It is as:

> As social phenomena are recorded and classified, that is, hypothesis generation begins with the analysis of initial observations, undergoes continuous refinement throughout the data collection and analysis process, continuously feeding back into the process of category coding. As events are constantly compared with previous events, new typological dimensions, as well as, new relationships may be discovered (Goetz & LeCompte, 1981, p. 58).

In this study, the general concepts were determined before the findings gathered from the interviews with students. These were the general misconceptions in solving probability questions which are availability and representativeness heuristics, simple and compound events, conjunction fallacy, and conditional probability. According to general concepts, the codes were prepared from the data collected from different sources.

On the other hand, new concepts could be created in coding process according to the findings of interviews with teachers. This was because different teaching strategies, teachers' awareness of students' misconceptions, teachers' perspective of how they perceive the students and different materials used may result in emergence of new concepts to study. For example, others such as the misconceptions in Tirosh and Stavy's (1999a; 1999b; 2000) theory of intuitive rules emerged. In addition to the main themes which were the intuitively-based misconceptions, the teachers' awareness and knowledge of reasons for the misconceptions were also emerged as the new themes.

These were types of the coding procedures mentioned in Strauss and Corbin's (1998) book. The codes can be determined before the research begins or while all data are analyzed. Or lastly, the general frameworks can be determined and the codes can be created under them during the data analysis procedures. The last one is the method that is most suitable for this study. This is because the general frameworks are the types of the intuitively-based misconceptions in probability, related teaching practices, and teachers' knowledge of students' cognition related to these misconceptions, which appeared during the data analysis procedures. In addition, the codes under each framework also appeared. According to the data gathered from test, observations, interviews; the codes were created. The main themes, categories and codes can be found in the Appendix F.

In the interviews, the main categories appeared as teachers' awareness and the knowledge of students' pre-knowledge. In addition, the ways of determining deficiencies, the knowledge of activities in teaching probability, the reasons for students' difficulties and misconceptions, and the knowledge of students' misconceptions were the main categories that appeared in the analysis of the data gathered from interviews and classroom observations. In the observations, main categories were appeared as concept teaching, relating the probability with other subjects, activities done in order to resolve misconceptions such as giving key points, rote memorizations and shortcuts, and usage of resources and materials. The categories and codes appeared in the analysis of the interviews and classroom observations were embedded in the main themes, which are students' intuitively-based misconceptions. For example, either in the interviews or observations, if teacher indicated a misconception that was consistent with the availability heuristics, this code was used in this theme to support it.

3.5 Reliability and Validity

In any study, there is a need to satisfy the persuasiveness, for which two main criteria are validity and reliability (Creswell, 2009; Yıldırım & Şimşek, 2006). For validity concern, the researcher need to reflect the case or phenomenon investigated as accurately as it happens (Yıldırım & Şimşek, 2006). In addition, the researcher needs to be objective while reflecting it (Cohen, Manion, & Morisson, 2000). While conducting a research, the researcher reaches too many findings from different data collection tools and conclusions at the end. In order to create accurate and holistic conclusions for the research, the researcher needs for confirmatory means or strategies. At this point, the validity of the study takes place. Cresswell (2009) recommended the use of multiple strategies in order to satisfy the accuracy of the findings and to convince the readers for this accuracy. From Cresswell's (2009) recommendations, this study utilized multiple strategies for validity including "triangulation, rich and thick description, clarifying the bias, presenting negative or discrepant information, prolonged time, peer debriefing" (p. 191-192).

First of all, *triangulation* necessitates that different sources of information and different perspectives from participants are taken into account in order to reach holistic conclusions and coherent justifications. In this study, the data were gathered from the observations of the classrooms, test results, interviews with ten students and five teachers, and field notes.

In this study, *rich and thick description* about natural settings and the participants of the study was provided. It was expected that detailed descriptions became more realistic and understandable by the reader. In addition, direct quotes from interviews and conversation between teacher and students were also used to give evidences about participants and classroom environments. Quotes from students' responses to open-ended questions were also included in this study. Therefore, the reader can have information about student participants.

Another strategy for validity of the study is *clarifying the biases.* In any research, there may occur biases rooted from the researcher himself, the data collection tool or the method used. To overcome this situation, the researcher gave detailed information about his position during the study, participants' background information, the procedures in applying data collection tools and the method used during the data collection and data analysis processes. In addition, the researcher informed the cases about the purpose of the study the procedures that were followed during the study. The researcher attended a few lessons before the actual observations in order to make students get used to the existence of the researcher in their natural settings. Students also knew that the tests and the interviews conducted were for research purposes, not for the grading purposes.

Another strategy to add to the validity of the study *is spending prolonged time* in the research site. Therefore, the researcher can get in-depth understanding of the physical status of the site, personal characteristics of the participants. This may also help researcher to understand the reasons behind the participants' behaviors. In this study, the researcher attended all probability classes. Therefore, he observed the research site and the participants. During the observation, the researcher also interacted with the participants. The researcher also recorded *the negative or discrepant data* that may influence the study. These were presented in the limitations.

In the study, *peer debriefing* strategy was also used. The peer stated the over-emphasized, under-emphasized points or missed parts that were very crucial for the study. In this study, the peer was a research assistant in the mathematics education department and doing her PhD in the department of teaching mathematics in middle schools. First of all, she was provided with information about the purpose of the study, the procedure, data collection tools, and data analysis methods. She investigated the transcripts of interviews and the students' test results, then, informed the researcher about the over- emphasized, under-emphasized, and missed parts of the study.

External validity is about whether the findings of the study are generalizable to other situations (Meriam, 1998). Among the strategies for external validity, replication strategy which requires the similar results in different situations was used (Yin, 2003). In this study, five teachers were interviewed and their students' were subjected to tests and interviews. In addition, their classrooms were observed during the teaching of the probability. The findings were compared at the end.

McMillan and Schumacher (2010) also suggest these strategies to enhance the validity of the study. In addition, they suggest using "mechanically recorded data", considering "the participant language" (p. 330-332).

Second important criterion for the creditability and persuasiveness of the study is reliability. Reliability is about whether "the researcher's approach is consistent across different researchers and different projects" (or studies) (Gibbs, 2007 as cited in Creswell, 2009, p. 190). Cohen, Manion, and Morisson (2000) consider the reliability as "a synonym for consistency and replicability over time, over instrument and over groups of respondents" (p. 117). For the qualitative researchers, they suggest to use "stability of observations and parallel forms" (p. 119). The researcher observed the participants at different times, and searched whether the same observation and interpretations were made. The same interview form and questionnaire were administered to all participants, and the interviewer for all the interviews was the same person, who was the researcher.

In order to increase the reliability, the researcher made a record of all data collection procedures and activities. The records included interviews, observations in the classrooms, and field notes for coding and data analysis procedures. In addition, the field notes included the some parts of the unstructured interviews with students and teachers.

Reliability is widely used in qualitative research if comparison of codes created by several coders is needed for data sets (Cresswell & Clark, 2007). In order to increase the reliability of the study, inter-rater reliability can be calculated (Fraenkel & Wallen, 1996). Marques & McCall (2005) stated that the inter-rater reliability can be calculated as follows: "100×(Total number of agreements)/(Total number of observations)". In addition, the acceptable level of inter-rater reliability level was considered as above 80 % (Marques & McCall, 2005). In the study, a research assistant doing her PhD in the field of mathematics education who had experience in descriptive and content analysis method also studied over students' pre- and post-test results of a one randomly selected classroom, one randomly selected teacher's interviews which were administered before and after the instructions, and total of five randomly selected observations. First of all, the researcher explained how to analyze the transcription of the interviews and the observations.

For the pre- and post-test of the PTI, the second coder was provided with the misconceptions observed in the tests. There were ten misconceptions appeared among students in the tests. The second coder analyzed the pre- and post test results of Doğan's classroom. The total numbers of occurrence frequencies in the pre- and post-test were 83 and 69, respectively, according to the misconceptions that students fell into. At the beginning, the total numbers of agreements between the researcher and the second coder were 77 and 65, respectively. The inter-rater reliability rates were 93 % and 90 % for the pre- and post-test results, respectively. These codes that the researcher and second coder disagree appeared due to students' unclear answers to the questions. While an answer was considered as specific type of intuitively-based misconception by researcher, the second coder was disagreed with researcher's opinion, or vice versa. After the discussion between the researcher and the second coder, they reached consensus on all codes. These codes were used in the results of the study.

In the interviews with teachers, teachers' knowledge about students' pre-knowledge, how to determine students' misconceptions, the material use, possible activities in resolving them, the reasons for the misconceptions, and students' misconceptions were investigated. There appeared 59 codes in the interviews. However, the second coder analyzed the transcription of Cihan's interview. In Cihan's interview, there were 51 codes appeared. There were 42 codes that the researcher and the second coder agreed on. Therefore, the inter-rater reliability rate was 82 % for the interview. In general, the second coder disregarded the codes under necessary pre-knowledge for students and considered them as one code. In addition, she also combined some of the students' difficulties and misconceptions from teachers' point of view. At the end, the researcher and the second coder discussed and reached to the consensus either on researcher or on second coder's thoughts. Accodingly, the results were presented on the codes that both researcher and the second coder reached the consensus.

Lastly, the occurrences of the misconceptions and teachers teaching practices were observed in the classrooms. Teaching practices included the constructing solution methods, using visual or physical materials, giving examples, and constructing shortcuts for solving questions. There were total of 35 video-recordings in the classrooms. One classroom observation from each teacher's instructions was randomly selected and the second coder watched total of five video-recordings. These were observation-2 in Ahmet's classroom, observation-4 in Barış's classroom, observation-4 in Cihan's classroom, observation-6 in Doğan's classroom, and observation-3 in Erdal's classroom. There were total of 24 observations in these video-recordings. The second observer indicated 21 observations. The inter-rater reliability rate was 88 % for the video-recordings. This rate is above acceptable level for inter-rater reliability rates. After the discussion between the researcher and second coder, it appeared that the second coder combined two shortcuts under one code (rote memorizations for independent events) and missed two intuitively-based misconceptions that appeared while teachers were solving questions. The codes that the researcher and the second coders agreed were used in the results of the study.

3.6 Delimitations and Limitations

In this case study, the grade levels were 8^{th} and 11^{th} grades. In the curriculum of middle school education, probability was taught to students from grades 6 through 8 (MoNE, 2005b). Therefore, this grade level has probability knowledge in this grade level. In high school level, on the other hand, the probability subject was taught in 11^{th} grade level (MoNE, 2005a). As the aim of this study was to determine teachers' teaching practices and their awareness of students' cognition that their students in these grade levels might have probability misconceptions which may appear due to previous knowledge, experiences in daily life and therefore students' intuitions. Therefore, these grade levels were appropriate level to study.

The study was mainly based on the misconceptions in probability subject. As it is well known, previous knowledge, experience gained in the daily life and misconceptions that appears by means of previous knowledge and experience are important factors because the further learning is positively or negatively affected by them (Tirosh, 2000). If students were aware of their misconceptions and searched for some solutions for them, they could better understand the subject taught. On the other hand, they might construct new knowledge on their previous misconceptions. This study searched for the intuitive based misconceptions. During the construction of misconceptions, students' intuitions were important to consider (Fischbein, 1987). Students might easily comprehend the subject taught due to their intuition, but they might also create new misconception.

There were five objectives for the probability subject in mathematics curriculum for high schools. Some of these objectives include the basic concepts related to probability. However, this study mainly dealt with the probability problems. Therefore, the objectives that require computations, reasoning during solving problems were taken into consideration. The topics to consider in this study were finding probabilities of simple, inclusive-mutually exclusive events, conditional probability, dependent and independent events. Different than the high school curriculum, middle school curriculum proposed (MoNE, 2005a) that students have knowledge of simple and compound event, inclusive-mutually exclusive events except for conditional probability, except for conditional probability before beginning to the 8th grade. They would learn theoretical, experimental, and subjective events and dependent-independent events in the 8th grade. Therefore, delimitation for the sub-topics in probability was those indicated above.

For the limitations, this study might include some elements that influence the internal validity of the study. Some of these elements might be due to the methodology of this study. Firstly, there was a test administer whom students did not know. Therefore, there was a direct incursion in the natural settings of the classroom (Fraenkel & Wallen, 1996). Students completed inventory and the test at this situation. In addition, they were observed during the lessons. Therefore, their natural behaviors were affected. However, they were debriefed about the purpose of the study and the confidentiality concerns were informed.

During the student interviews, students faced with the researcher whom they did not know before. Therefore, their responses to questions asked might be affected. They might not specifically express their thoughts about the questions. Since their thoughts were indicator for intuitively-based misconceptions, they should be relaxed during the interviews. To overcome this situation, the researcher was talking to different students during the time breaks between regular instructions. The researcher tried to gain students' confidence. Before doing interviews, he also chatted with the students who were interviewed. He debriefed students about the purpose of the study. He also informed students about confidentiality concerns and the test was not for grading purpose. After getting students' permissions for interviews and having them relaxed, the researcher beginned to the interviews.

There were at least four weeks between the administration times of pre- and post tests. Middle and high school students in the study groups took the same test before and after they received regular instructions. It is possible that students could remember the questions while solving the questions in the PTI. However, time interval between the pre- and post-tests was sufficient in the qualitative research (Fraenkel & Wallen, 1996).

At last, the possibility of biased description by the researcher might occur. This was due to the researcher's interest in probability misconceptions based on their intuitions. Therefore, during the interview, researcher might focus on such concepts and ignore participants other views and thoughts. In addition, researcher might not see all dimensions during the data analysis and coding processes. To minimize these threats, the second coder strategy was very suitable (Creswell, 2009; McMillan & Schumacher, 2010). Researcher might reduce the possibility of biased descriptions; and wrong or lacking coding pattern.

Another limitation that could not be dealt with was about the teacher participants. All teacher participants were male in the present study. It could be better to include female participants as well. However, it was not possible for the researcher due to their willingness to attend the study. It was possible that female teachers could focus on different points in the interviews and regular instructions. Moreover, their relations with students might be different, so, their instructions could be different than the male teachers.

3.7 Ethical Issues

In an educational research, the researchers must consider some ethical issues to protect participants from harm, to ensure confidentiality of the research data, and undeceive participants (Fraenkel & Wallen, 1996). The researcher was also careful about these issues in the present study.

First of all, the necessary permissions were taken from the Provincial Directorate of National Education in the city where this study was conducted. With the permission letter, the researcher talked to school principals and teachers about the purpose of the study and the procedures for data collections. School principals also gave permission to conduct the study in their schools. The study was conducted only with volunteer teachers. Then, the teacher participants talked their students about the purpose of the study. Before starting to observe classroom, the researcher also talked with students about the purpose of the study and tried to diminish their curiosity and concerns. In order to ensure confidentiality, the researcher did not use students' and teachers' real names. All names used in this study were pseudonyms.

In the application of the questionnaire, students were informed about its purpose and their permissions were taken. It was ensured that any of students' results would not be shared with third persons with specific names. They also knew that the results would not influence their grades in their mathematics lessons.

The researcher interviewed teachers in their free times. In addition, there was no annoying or inconvenient questions asked to teachers. During the observations, the researcher did not talk and interact with students. The researcher sat on a chair at the back of the classrooms while observing and video-taping teachers' teaching practices. There was no incursion into organizations and applications of the teaching practices and activities. The researcher did not help or interact with students while they are doing activities or solving questions. Lastly, the researcher was honest both in data collection and in data analysis processes. Teachers were informed about the results of the studies.

CHAPTER 4

RESULTS

In this part of the study, the findings from the analysis of the data gathered from different data collection tools were presented. The data collected consisted of the semi-structured interviews with teachers before and after the probability topic taught, the findings from the test with open-ended questions, subjected to students before and after the teaching of probability, classroom observations during the teaching of probability subjects, and also interviews with students.

As it was stated in the introduction chapter, this study seeks for answers to six research questions which are related to students' cognition, the role of the instruction, and the role of teacher in resolving intuitively-based misconceptions. These research questions' answers were shared in this chapter.

Each part of this chapter was related to intuitively-based misconceptions appeared in the study. The answers for the first, second, and third research questions were presented under the first sub-headings of each part of this chapter. The research questions related to teachers' awarenesses (fourth and fifth research questions) and the one related to teaching practices to resolve misconceptions (sixth research question) were answered in the second sub-headings of each part. Different than the parts from 4.1 to 4.9, the last part of this chapter (4.10) presented the findings gathered from the interviews. This part included answers for the fourth and fifth research questions related teachers' awarenesses.

4.1 Availability Heuristic as an Intuitively-based Misconception

In the following parts, the main intuitively-based misconceptions among middle and high school students were presented. The pre- and post-test results, teachers' awareness of these misconceptions and how to resolve them, the teaching practices including activities done and examples provided which were specific to availability heuristics were presented in the next parts. This part and the following ones through 4.9 include two sub-headings. The first sub-heading gives answers to second and third research questions. The pre- and post-test results and the data gathered from interviews with students were presented under this sub-heading. The second sub-heading gives answers to fourth, fifth, and sixth research questions. Data gathered from interviews with teachers and classroom observations were used to answer these research questions.

4.1.1 Pre- and Post-Test Results

Availability heuristics misconception was observed in both 8th and 11th grade levels. The pre-test results that showed the frequencies of the students who fell into this type of misconception for each classroom were given in the Table 4.1 below. This misconception was observed in the first and fourth questions in the PTI.

Table 4.1 Frequencies of students' answers reflecting the misconceptions of availability heuristics in the pre- and post-tests

Questions in PTI	Middle School Teachers						High School Teachers							
	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1st Question	11	8	6	10	17	18	5	5	3	2	14	4	22	11
4th Question	9	6	7	5	16	11	5	8	6	5	13	7	24	20

*: All names are psydonyms.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of availability heuristic counted only once.

Table 4.1 indicated that availability heuristics was seen in all grade levels. This misconception is the intuition that the consecutive numbers does not appear in chance games. Therefore, this intuition misleaded students in the way that Hakan's probability of winning the game was higher than the others' probability of winning the game because Hakan chose the numbers mixed in order. There were 17 students in the middle school classrooms. This means 29.82 % of all middle school fell into this misconception. Some of the assertions from the middle school students for this misconception were as follows.

> It is very hard that the numbers come over and over (from pre-test administred to Ahmet's classroom).
> Hakan's probability of winning the game is higher, because the numbers that he chose does not come one after another. Çiğdem's and Merve's probability of winning the game are lower. (from pre-test administred to Ahmet's classroom).
> Hakan chose mixed numbers. Merve and Çiğdem (the probabilities) are equal, because the numbers are close to 1 and 49 (from pre-test administred to Barış's classroom).
> When Hakan chooses the numbers, he may increase the probability of winning the game (from pre-test administred to Barış's classroom).

Similar justifications were given for this misconception among high school students. One different justification mentioned about Hakan's random choices. According to this student, Merve and Çiğdem's choices were not random. He considered that their choices were consecutive numbers, so the probability of winning the game was lower than Hakan's probability of winning the game. This justification was stated as below.

> Hakan's probability of winning the game is higher. His numbers are random. Çiğdem's and Merve's probabilities of winning the game are the same. Both of them chose the numbers one after another, but their probabilities are lower (from pre-test administred to Cihan's classroom).

22 high school students fell into this misconception in high schools. This was equal to 37.29 % of all high school students. Interestingly, more than half of the students in science high school fell into this misconception. There were 14 students out of 23 incorrectly stated that the Hakan's probability of winning the game was higher than others, while the Çiğdem's and Merve's probability of winning the game were equal. Another interesting finding was that two students in the vocational high school stated in both tests that the Merve's and Çiğdem's probability of winning the game was equal and higher than Hakan's probability of winning the game. They thought that consecutive numbers resulted in higher possibility to appear when it was compared.

The number of students who fell into this misconception decreased in the post-tests, when it was compared to the total number of students joined the tests. However, there was slight increase in the number of middle school students while there was a sharp decrease in the number of high school students.

Test results indicated that there was slight difference among the middle school students to resolve the availability heuristics misconception after getting instruction. However, the findings indicated that instruction given did not resolve this type of intuitively-based misconception. The number of the middle school students was 18 in post-test, which was equal to 31.03 % of all middle school students.

For the first question, number of students' correct answers increased sharply from 23 students to 39 students. High school students tried to use formulas for this question. However, there were still 11 students (18.64 %) who fell into availability heuristics misconception in the post-test. Therefore, the main difference between the answers before and after getting regular instruction was that students were more get used to use the formula for the question. However, some of them could not use the formula correctly.

For this type of misconception, there was almost no change in the number of students in vocational and Anatolian high school. However, the number of the students who fell into this misconception in science high school decreased sharply from 14 students to 4 students in the first question and from 13 to 7 in the fourth question. Most of the students gave correct answer.

Students who fell into availability heuristics misconception in both middle and high schools gave similar justifications for their answers. They mentioned about the shuffled order of the numbers in the questions. They stated that this Çiğdem's and Merve's orders were very strict and are hard to win the game.

4.1.2 Teachers' Awareness and Teaching Practices for Availability Heuristics Misconception

This part presents the results related to teachers' awareness about availability heuristics and their recommendations for resolving it. Their teaching practices and the examples given for resolving the misconception were also included in this part of the study.

Availability heuristics appeared in the first and fourth questions. In the interview-2, teachers were asked about the possible misconceptions that students could fall into in the PTI. Findings for availability heuristics indicate teachers' awareness about this misconception in the Table 4.2 below.

Table 4.2 Teachers' awareness about students' misconception of the availability heuristic in the PTI

Questions in PTI	Middle School Teachers		High School Teachers		
	Ahmet*	Barış	Cihan	Doğan	Erdal
1st Question	√**	√	√	√	√
4th Question			√	√	

*: All names are psydonyms.
**: "√" indicates that the teacher expected to observe the intuitively-based misconception in students' responses to questions in the PTI.

In fact, the first question was specifically asked to determine students' misconception of availability heuristics. For the first question, all teachers stated that irregular order of numbers might affect students' cognition. According to them, students might state that Hakan's probability of winning the game was higher than the other's. Some of their justifications were as follows.

> Students may think that the probability of choosing consecutive numbers repeatedly is lower than the probability of Hakan's choices (Interview-2 with Barış).
> Here, students may think that Hakan's choices may increase the probability of winning the game, because his choices are the numbers in different interval (Interview-2 with Doğan).

On the other hand, only high school teachers stated that there might appear the misconception of availability heuristics in the fourth question. Cihan and Doğan gave similar explanations for this misconception.

> Students may think that the probability of Distribution II is higher, because there are different numbers of cards for each player (Interview-2 with Cihan).
> Students, here, get confused because of the difference in the number of cards that each player gets (Interview-2 with Doğan).

Teachers presented recommendations to resolve this misconception in the interview-2. All teachers mentioned about the importance of finding probabilities of each events in the questions. They focused on how effectively use the general probability formula. In addition, all middle school teachers and Cihan stated that students could be informed about events' independence of consecutiveness and steadiness in order. Ahmet also mentioned about chance factor. He stated that chance games had effect on the students. His recommendation was that students should be informed about the ineffectiveness of the lucky numbers in chance games.

During the observations, teachers did some activities to solve this kind of misconception. In general, learning the basic concepts in probability comprehensively helps students not to fall into this misconception. Students use their intuitions to solve questions instead of using the knowledge of the basic concepts. Therefore, concept development in the lessons plays crucial role. In fact, Doğan stated in the interview-1 that a difficulty that students encountered in probability was that students were unable to understand the basic concepts in probability. During the observations, Ahmet was very stick to textbook. He did not mention about the basic concepts of probability which were learnt before. For example, students were not provided with the meaning of probability, simple or compound events. He used course book and directly gave the meaning of dependent and independent probabilities according to the definition given in the course book. On the other hand, Barış began the lesson by explaining the concepts learnt in the 6^{th} and 7^{th} grades. He gave answers from dice and coins. However, he also supported the understanding of the concepts by using materials brought to the classroom. He did experiments by choosing balls from urn, by throwing dice and coins. He also stated that students would learn the probability and event types according to the general logic of probability. In the experiments, teacher showed that there could be conflict with students' intuitions and the probabilities of the events.

The observation results indicated that high school teachers generally tried to explain the basic concepts in probability with students-teacher interactions. They all began with the meaning of the probability. Students gave answers to teachers' questions. Then, teachers evaluated students' answers. They either widened the meaning or gave some examples for the concepts and passed to another concept. The same procedure continued for the concepts of experiment, outcome, certain and impossible events, and sample size. Therefore, teachers tried to eliminate students' incorrect intuitions which might lead to availability heuristics. A conversation between Cihan and students showed what procedure Cihan followed in the teaching of the concepts below.

Cihan : What comes to your mind about probability?

Student 1 : The maximum cases that may appear
Student 2 : The chance, the probability of winning a game
Cihan : Chance games!
Student 3 : Throwing a coin
Cihan : Yes. We can combine them. After throwing dice, coins, we can talk about the possibilities or occurrences of the choices of the cases. We can use probability in chance games, in insurance (Observation-1 in Cihan's classrom).

On the other hand, Erdal directly passed to the topic of sample space after giving short explanations and examples related to basic concepts. He did not write the meanings of the concepts. Instead, he considered that students already knew these concepts. He wrote the formulas for different types of probabilities (e.g. independent probability).

During the observations, there appeared many situations that might result in this type of misconceptions. First of all, teachers were stating that they would deal with the questions related to dice, coin, and urns in the introduction of probability. Students could think that the probability was all about coins, dice, and urns. To resolve these thoughts, teachers were trying to solve different kinds of questions. Ahmet, Doğan, and Erdal were curious about this situation. Especially Erdal solved questions that were not similar to those asked in the university entrance exam. He also stated that there was no limit in the type of questions that could be asked in probability. In fact, both middle and high school teachers thought that students experience difficulty in determining which formula to use. They thought that solving as many questions as possible might help students to resolve this difficulty.

Secondly, Barış and Cihan mentioned the relation between the chance games and probability. In addition, Ahmet stated that the availability heuristics could be resolved by explaining that the statistical information in chance games did not coincide with the actual probabilities in the last interview. As it is known, the televisions and advertisements always impose the lucky numbers. This might result in that some numbers could be luckier than the others. Both Barış and Cihan explained the probability of winning a lottery (Milli Piyango). In addition, they both stated that each person had equal chance of winning the lottery and it was about one in a million. They also stated that there was no lucky number in the lessons.

As one of the most important basic concepts in probability, all teachers emphasized and repeated that the value of probability of the events was between “1” and “0” during the observations. This repetition continued many times in different lessons in different classrooms. Although all teachers considered this situation as one of the students’ difficulties in probability, only Erdal did not give emphasis on it. This repetition helped students not to fall into misconception. This was because some students found the value of the probabilities more than one according to available values in some questions. To overcome this misconception, especially high school teachers provided a property for the probabilities. After indicating that $0 \leq P(A) \leq 1$, teachers gave examples for the property. Two of them were as follows.

> If $P(A) = 1$, then, it is impossible event. The probability of getting seven after throwing a die is impossible. Why? Because there is no seven in the universal set (Observation-2 in Doğan’s classroom).
> $P(A) = 1$, then, it is a certain event. The probability of getting less than seven after throwing a die is certain, because all elements in universal set are also expected elements (Observation-2 in Doğan’s classroom).

The subjective probability is related to this misconception which is in the middle school curriculum. Middle school teachers explained the differences among theoretical, experimental, and subjective probabilities. In fact, Ahmet stated in the interview-1 that students experience difficulty while differentiating theoretical, experimental, and subjective probabilities. During the observations, both Ahmet and Barış asked questions related to subjective probabilities. For example, Barış asked the probability of championship of Fenerbahçe at the end of the season. Students gave different probabilities such as 100%, 20%, and 0% according to the students’ favorite teams. Then, Barış stated that the theoretical probability was different than students’ thoughts. He stated that they needed to consider the actual probabilities instead of what they thought. This type of misconception was generally observed in middle schools. Some of the examples observed in middle schools were as follows.

Table 4.3 Examples and students' availability heuristics misconceptions

Examples	Student Answers	Explanation
There are x red and y blue balls in a box. If the probability of getting a red ball in random choice is 1/3, then what is the ratio between x and y?	Student: 3 times (Observation-3 in Ahmet's classroom)	Student just looked at the ratio of 1/3 and indicated the answer as 3 times.
Nur is taking a multiple choice exam. There are four choices for each question in 100-question-exam. What is the probability that Nur chooses the correct answers for all questions?	Student 1 : ½ Student 2 : 4/100 Student 3 : 1/25 (Observation-4 in Ahmet's classroom)	Easily available facts were four choices and 100 questions. So, they comined these facts and gave answers.
What is the probability of getting 5 after throwing a die?	Student : 5/6 (Observation-2 in Barış's classroom)	Easily available facts are five and six outcomes. Student gave answer accordingly.

In general, teachers tried to resolve these misconceptions by doing formal solutions for each question during the observations. In case students did not understand the solution, they repeated the formal solution on the board or tried to give different examples. In such situations, both Ahmet and Barış tried to direct students to the probability formula. They first found the sample size, then, they solved questions according to algorithm taught before. These situations were not observed in high schools.

Another misconception related to availability heuristics was that both students and teachers considered the sample size as the universal set. At this manner, both middle and high school teachers stated that students face difficulty in determining the sample size and the set of expected elements in the events. The reason was that teachers were explaining the direct relation between probability and set subjects. However, neither teachers nor students were uncomfortable with this situation. Only one student in Erdal's classroom stated that they had to say sample size instead of universal set. Erdal confirmed it and continued to solve the questions.

What was observed especially among high school students about availability heuristics was that they could not differentiate the dependent-independent events and inclusive-mutually exclusive events. They were using the readily available formula for the questions related to these topics. In such situations, high school teachers used shortcuts and rote memorizations. One of the rote memorizations was that "the sample size is different for the former (dependent-intedendent); the sample size is the same for the latter (inclusive-mutually exclusive)."

Similar situation also existed for the differentiation between the use of permutation and combination in high school classrooms. Again, the teachers used rote memorizations to resolve this situation during the observations. One of the rote memorizations was that "the arrangement requires permutation, the selection requires the combination."

Overall, middle school teachers used questions directly related to this misconceptions due to the topics covered in the curriculum. On the other hand, this misconception was indirectly observed among high school teachers' classrooms. According to test results, the occurrence of this misconception decreased slightly both in middle and high schools. The decrease was sharp only in Erdal's classroom. Erdal was always directing students to use formula in solving the questions. This situation might be reason for this decrease. Teachers' teaching practices, warns, and examples helped students to resolve this misconception in both middle and high schools. However, quarter of all students fell into this misconception after the regular instruction.

4.2 Representativeness Heuristics: Negatively and Positively Recency Effects as Intuitively-based Misconceptions

There are three types of representativeness heuristics. The second question of the PTI was asked specifically for this misconception. The expected misconceptions were observed among both middle and high school students. The findings related to the first type of representativeness heuristics misconceptions, which was negatively and positively recency effect, were presented as follows.

4.2.1 Pre-and Post-Test Results

According to the test results for the second question, there exist two main intuitively-based misconceptions. As the question is related to representativeness heuristic, first main misconception is called as positively recency effect. In this type of misconception, students thought that the fourth trial in the experiment would become head after getting three consecutive heads. On the other hand, the second intuitively-based misconception is called as negatively recency effect. Students' intuition misleaded students in such a way that the trials in the experiments had corrective power, so the fourth flip would become tail to reach the 50 % of heads and tails. The correct answer in this question is that the previous trials do not affect the future ones, so the probability of getting head is 50 %. This is also same for the probability of getting tail. Pre-test results of the second question in the PTI are given in the Table 4.4.

Table 4.4 Frequencies of students' answers reflecting the misconceptions of positively and negatively recency effects in the pre- and post-tests

	Middle School Teachers						High School Teachers							
Mis-conception	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Positively Recency Effect	14	13	12	13	26	26	7	8	1	3	10	1	18	12
Negatively Recency Effect	8	7	9	6	17	13	4	2	1	0	8	0	13	2

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the the misconception of positively and negatively recency effects counted only once.

Majority of the students fell into the intuitively-based misconception of positively recency effect in the pre-test. Almost half of the students in middle school classrooms answered in the same way. 12 students from Ahmet's classroom and 14 students from Barış's classroom gave answers which were parallel to the misconception of positively recency effect. Therefore, 45.61 % of all middle school students fell into this type of misconception. They thought that the outcomes of the trials would not change and the next trail would be the same. Some of them thought that there was a bias on the dice, therefore, the trend would continue in the same way. Some of the justifications given by different students in the middle schools for this misconception were as follows.

> Head is always appearing, so it is head again. It is hard to get tail, because it is always coming head (Pre-test administred to Ahmet's classrom).
> 4/4, the result is head. The probability of getting tail is lower (Pre-test administred to Barış's classroom).
> It is head, because it always appeared. Maybe it comes again (Pre-test administred to Barış's classroom).

High school students generally fell into the misconception of positively recency effect in the pre-tests. For this misconception, there were 18 students (30.51 %). Among them, seven students were from vocational high school, one student was from Anatolian high school and ten students were from science high school. Here, there was an interesting finding that there were ten students from science high school. It was expected that most of the students in science high school could find the correct answer in this question. High school students gave the similar justifications for this misconception. Most of the students preferred to use verbal justifications. Some of these justifications given by different students were stated below.

> There is a probability of getting head again, because the first three trials are the head. Or this man made a trick, and it is providing to bring head at each trial (Unstructured interview with a student from Erdal's classroom).
> If the first three trials are head, the fourth trial can also be head, because there is a bias (Pre-test administred to Doğan's classroom).

Some students thought that there is a bias in the experiments despite the fact that the question indicated that the coin used was unbiased. Some other students thought that the trials would continue in the same way and the next trial would become head, again. Therefore, they fell into the misconception of positively recency effect.

Overall, 44 students from middle and high school students fell into the misconception of positively recency effect. This was equal to 37.93 % of all students observed. On the other hand, there were 30 students (25.86 %) in middle and high schools who fell into the misconception of negatively recency effect. Most of them were from middle schools.

Almost one third of all middle school students fell into the misconception of negatively recency effect in the pre-tests. Students thought that there should be tail after getting three consecutive heads. They though that there should be equality between the number of heads and tails. According to the test results, 17 students in middle schools (29.82 %) fell into this type of misconception. Among them eight students were from Ahmet's classroom and nine students were from Barış's classroom. Some of the justifications for the answers were stated below.

> If there are three heads, the probability of getting tail is higher in the fourth trial. HHH+T (Pre-test administred to Ahmet's classroom).
> If the coin is thrown fourth time, the probability of getting head becomes lower, because it comes head three times. The next trial is most probably tail (Pre-test administred to Barış's classroom).
> For me, if there are three consecutive heads and if there is no bias, the fourth trial becomes tail (Pre-test administred to Barış's classroom).

There were also interesting answers. Two of them were as follows.

> An unbiased coin is thrown three times and it is coming head. At this trial, there is a probability of getting tail. The probability of getting head is lower, because it is always coming head. At this trial, there is a probability of 50 % head and 50% tail (Pre-test administred to Ahmet's classroom).
> Coin → HHH→3T. 4th coin →HHHT→3H+1T→ Tail is higher (Pre-test administred to Barış's classroom).

Here, the student knew that the probability of getting head or probability of getting tail was equal and was 50 %. However, s/he generalized the situation and said that the total trials had to have the same number of heads and tails. Therefore, s/he thought that 50 % of getting head or of getting tail was also valid for getting the same number of heads and tails in the experiments.

Similar findings also appeared in high schools in the pre-tests. There were 13 students (22.03 %) who fell into this type of misconception. Among them, four students were from vocational high school, one student was from Anatolian high school, and eight students were from science high school. They also gave similar justifications. Here, students generally preferred the verbal justifications instead of formal solution. Some of the students' answers were given below.

> The probability of getting head is lower. There are three heads and they came one after another. This lowers the probability of getting head (Pre-test administred to Cihan's classroom).
> The probability of getting tail is higher, because, in general, getting head increases the probability of getting tail and getting tail increases the probability of getting head in the next trials (Interview with a student from Doğan's classroom).

After students got regular instruction in middle schools, the number of students who fell into the misconception of positively and negatively recency effect decreased in the post-tests. Table 4.4 shows the frequencies of the students who fell into these types of the misconceptions. Although the number of students who correctly answered the question increased in the post-test, almost half of the students fell into the first type of intuitively-based misconceptions in middle schools. According to the test results about this misconception, 11 students (18.97 %) correctly answered the question. Getting regular classroom instruction affected students' answers. Their answers included argumentations about experimental and theoretical probability. Some of the justifications by students were as follows.

> There is 50 % probability, because we can see what will come by trying (Post-test administred to Ahmet's classrom).
> The probability of getting either head or tail is equal to ½. So, it is 50 % probability (Post-test administred to Barış's classrom).

Similarly, after getting the regular classroom instruction, correct answers in high schools increased to 43 students (72.88 %) in the post-tests. Among them, six students in vocational high school, 16 students in Anatolian high school and 21 students in science high school gave correct answers. The major increase was observed in the science high school. Although the number of correct answers in science high school was low, it increased to 21 students. Almost all students in science high school correctly answered the question. Some of the justifications for correct answers were given below.

> Since the coin is unbiased, no matter how many times that you throw the coin, since the next trial is independent of previous trials, the probabilities of getting head and tail are equal (Post-test administred to Cihan's classroom).
> Previous data does not affect the further event (Post-test administred to Cihan's classroom).
> They all have the same probability, because these events are independent of each other (Post-test administred to Erdal's classroom).
> <u>H</u> <u>H</u> <u>H</u> __. The fourth trial can be head or tail. So, they are equal (Post-test administred to Doğan's classroom).

General thought about correct answers was related to the independence of the events. They stated that the probabilities of events occurring were independent of each other. One student underlined first three trials in the experiment. Then, s/he drew one more space for the fourth trial. Then, s/he explained that the fourth space might be either head or tail. S/he also indirectly mentioned about the independence of the events occurring.

Almost half of the students in middle schools fell into the misconception of positively recency effect in the post-tests. The number of students who fell into this type of misconception was 26 (44.83 %). Although the regular classroom instruction was given, the number of incorrect answers did not change. However, some students used mathematical terms learnt in the instruction such as theoretical and experimental probability. Some of the justifications for this type of misconception from students' answers are given below.

> The probability of getting head is higher, because if we look at the past, we can see that there are more heads in the trials (Post-test administred to Barış's classroom).
> Since it is experimental probability, it is higher to get head (Post-test administred to Barış's classroom).
> It was thrown three times. Head comes. I think the chance is 50 % and it comes head again (Interview with a student from Ahmet's classroom).

As it was seen from the justifications, a student thought about the effect of previous trials. On the other hand, one student thought about the experimental probability and s/he felt that this trend continues similarly in the post-test. Lastly, one student knew the probability. However, s/he was still under the effect of previous trials. Another student differentiated the theoretical and experimental probabilities. This student's answer was as follows.

> Experimentally, it is higher to get head if the coin is thrown fourth time. However, the probabilities for both of them are ½ theoretically (Post-test administred to Barış's classroom).

After the regular instruction was given, the mostly observed intuitively-based misconception was positively recency effect in high schools in the post-tests. There were 12 students (20.34 %) who fell into this type of misconception. Majority of them were from vocational high school. There were three students who gave answers related to this misconception in Anatolian high school. The answers were given with the thought that the trend of getting heads should continue. Some of the justifications related to this type of misconception were given below.

> If it is thrown fourth time, the probability of getting head is higher again, because heads come three times. It comes head, again (Post-test administred to Cihan's classroom).

> Since head is appearing in each trial, the probability of getting head in the fourth trial is 99 % (Post-test administred to Cihan's classroom).

Students generally thought that this trend would continue in the same way. Therefore, they expect that the fourth trial would be head, again. However, they did not consider the independence of the events occurring. One student was very sure that the fourth trial would be head. This is because s/he thought that the probability of getting head was 99 %.

One student considered the total number of the trials and said that the probability of getting head in is 75 %. S/he considered that three trials out of four became head. Therefore, the last one had the probability of 75 % head.

Overall, the total number of students who fell into this type of misconception decreased to 38 students (32.48 %). However, this decrease was due to the decrease in the number of incorrect answers among high school students. On the other hand, there were 15 students (12.82 %) who fell into the misconception of negatively recency effect among all students. There were only two students in high schools for this misconception. Both of these students were from vocational high school.

Number of students who fell into the misconception of negatively recency effect decreased. Despite this decrease, there were still 13 (22.41 %) students in middle schools who gave answer which fitted this type of misconception in the post-tests. Among them, seven students were from Ahmet's classroom and six students were from Barış's classroom. They also gave answers similar to the pre-test questions. They also mentioned about the mathematical terms learnt in the regular instruction. Some of the students' justifications were given below.

> The probability getting tail was higher, because it came head three times. Now, it is tail's turn. (The probability of getting) tail is higher (Interview with a student from Barış's classroom).
> Tail comes, becaue it does not always appear head (Post-test administred to Ahmet's classroom).
> The probability of getting tail is higher, because it approaches to tail in each step (Post-test administred to Barış's classroom).

In general, high school students did not fall into the misconception of negatively recency effect. There were only two students (3.39 %) gave answers which were related to the misconception of the negatively recency effect. Both of the students were from the vocational high school. There were no students in Anatolian and science high schools who answered the question in accordance with the second type of intuitively-based misconception.

Another finding was that students preferred to give verbal justifications to the answers. However, some students tried to give answers with formal explanations and with formulas. Although some students used them correctly, some could not manage the formulas for the question.

4.2.2 Teachers' Awareness and Teaching Practices for Positively and Negatively Recency Effect

The second question was asked to determine students' misconceptions of positively and negatively recency effects. Similarly, it was observed in the interview-2 that teachers identified these misconceptions as seen in the Table 4.5 below.

Table 4.5 Teachers' awareness about the misconceptions of positively and negatively recency effects in the PTI

Misconceptions	Middle School Teachers		High School Teachers		
	Ahmet*	Barış	Cihan	Doğan	Erdal
Positively Recency Effect	√**	√	√	√	√
Negatively Recency Effect	√	√	√		√

*: All names are psydonyms.
**:"√" indicates that the teacher expected to observe the intuitively-based misconception in students' responses to questions in the PTI.

As seen in the Table 4.5, all teachers stated that students might fall into the misconception of positively recency effect in the interviews-2. In addition, Doğan was the only teacher who did not mentioned about the misconception of negatively recency effect. Some of the justifications were as follows.

> It can be advocated that the probability of getting head is lower … On the other hand, students may think that the coin is biased or that the person who throws the coin may act as biased, so they may think the inverse situation (Interview-2 with Barış).
> Students may think that getting the same results again and again may increase the probability of getting head again. Students generally ignore the mutually exclusive events in such type of questions (Interview-2with Doğan).

There were different suggestions from different teachers for resolving this kind of misconception in the interviews-2. Barış mentioned about the importance of the biasness of the coin. He suggested that student should be exposed about the biasness of the coin. Cihan and Doğan suggested that students must be taught how to distinguish the dependent and independent events. Erdal stated that students should be exposed with such kind of questions. Observation findings indicated that both middle and high school teachers used *if-then* statements. Students were expected to memorize them and use when necessary. The shortcuts were related to independence of the events. In fact, both middle and high school teachers stated that students' experience difficulty in determining whether the event is dependent or independent in the interviews-1. The following statements were observed in different classrooms to distinguish the independent events from the other event types and to put it into algorithm to solve the questions.

> If the ball is released into the urn, then, it is independent.
> If the event A and B are independent, then, the probabilities are multiplied.
> They are not affecting each other, so it is independent.
> The difference between inclusive-mutually exclusive events and dependent-independent events is that the sample sizes are same for former and they are different for the further.

This misconception appears when the existence of the previous events affect students' mind. In general, since the questions were asking the present event, this misconception did not appear in observations. However, the experimental probability topic in middle schools was directly related to resolution of this type of misconception. Due to the existence of the topic, middle school teachers stated in the observations that whatever the results of the previous events, the theoretical probability did not change. The examples given related to experimental questions were as follows.

> Can is throwing a coin 10 times and getting eight heads and two tails. Uğur is throwing a coin 50 times and getting 30 heads and 20 tails. Şule is throwing a coin 100 times and getting 53 heads and 43 tails. Let's look at the experimental probabilities and compare it with the theoretical one (Observation-4 in Ahmet's classrom).
> If I increase the trials, the probability is approaching to the theoretical one. For example, if I try it 1000 times, I can get 510 heads and 490 tails. I mean it approaches to theoretical probability (Observation-2 in Barış's classrom).

> A coin is thrown 100 times and it appears 90 heads and 10 tails. Ayşe is predicting that the 101th trial is head. What probability type is it? (Observation-4 in Ahmet's classrom).

In fact, it was leading students that the probability always approaches to the theoretical one. In the last example, Ahmet stated that the probability was 90% head by looking the previous events. Therefore, he leaded students to this type of misconception. He did not mention about the biasness. Although the trials might appear in such way, it did not guarantee that the probability was 90%.

In one example, Ahmet fell into this type of misconception. When explaining the relation between experimental and theoretical probabilities, he leaded students to the misconception of negatively recency effect. In the interview-1, however, he stated that one of the difficulties that students experience was difficulty in seeing difference among the theoretical, experimental, and subjective events. His statement was as follow.

> You can predict the theoretical probability, I mean, the result of the experimental probability, but you cannot predict it at the beginning of the experiments. For example, I threw 20 coins and I got 16 heads and four tails. If we continue throwing, the number of tails will increase (Observation-3 in Ahmet's classroom).

For high schools, there were no such topic and question. The questions were asking the probability of the event happening. Since the questions did not include the previous events which were already happened, such misconception was not observed. In addition, teachers did not consider that this type of misconception could exist. Therefore, they did not solve related questions.

However, there was indirect relation with the questions that require arrangement. For example, if the question asking the probability of getting two red (R) and two blue (B) balls after choosing four balls from the urn one after another, students needed to predict the possible arrangement such as RRBB, BRBR. In high schools, these types of the questions also had shortcuts during the observations. Teachers showed the shortcuts and rote memorizations for the arrangement in different lessons. Then, students used permutation for this type of the questions. Some of them were as follows.

> The permutation is for arrangement.
> If the arrangement is given, multiply the events.
> If the arrangement is not give, use permutation.
> If you take the balls together, then, use permutation and it is related to selection.

The use of permutation and combination in probability questions were also considered as one of the students' difficulties. All high school teachers and Barış mentioned about this difficulty in the first interviews with teachers. However, Barış did not use these topics in his teaching practices.

Indirectly, this topic was related to the independent of the events due to the independence of occurrences of the events from the previous ones. In the concept development process, Ahmet gave the definition of the dependent-independent events without giving examples or explaining the concept during the observations. Students were not satisfied with the explanation. The course book definition and a quotation between a student and Ahmet were as follow.

> (Definition given in the classroom) If the occurrences of two or more events are independent to each other (e.g. if the ball chosen from urn is released back into urn again), that means if the result of the one event is not affecting the other event, this type of events is called as independent (Observation-1 in Ahmet's classroom).

Teacher explained how to determine independent and dependent events by giving examples from dice and coin, then,

> Ahmet : For example, I throw a coin and head appeared, then, I throw a die, then 5 appeared. Is getting head from coin affecting getting 5 from die?
> Student : Yes
> Ahmet : Why?
> Student : You are throwing the coin in the first place and the die in the second place. I think if the dice is thrown in the first place, we may get different outcomes
> Ahmet : If the first did not affect we call this kind of events as independent events (Discussion ends here) (Observation-2 in Ahmet's classroom).

On the other hand, Barış explained the concept by giving examples different examples and by using materials such as coin and dice. For example, he threw a coin, then, he threw again. While throwing it second time, he asked whether the previous event was affecting the second throw or not. Different than the middle school teachers, high school teachers gave formal definition and the formula needed for the dependent-independent events.

Overall, these misconceptions appeared among both middle and high school students before the regular instruction. In general, the positively recency effect was dominant among students before and after the regular instructions. After regular instruction, however, these misconceptions stayed still among middle school students and in Cihan's classroom. On the other hand, there was sharp decrease in Doğan's and Erdal's classrooms. There were only two students for positively recency effect and two for negatively recency effect in Doğan's and Cihan's classrooms.

4.3 Representativeness Heuristics: The Sample Size Effect as an Intuitively-based Misconception

This part of the study presents the results specific to the intuitively-based misconception of the sample size effect. The findings related to the occurrence of the misconception among students, teachers' knowledge about the misconception and teaching practices were presented as follows.

4.3.1 Pre- and Post-Test Results

In the pre- and post-test of the PTI, the event of having at least two boys among three new-born babies was shown as A, so the probability of the event A was shown as P(A). Similarly, the event of having at least 200 boys among 300 new-born babies was shown as B. So, its probability was shown as P(B).

The misconception of the sample size effect appeared in the third question was as follows. Students thought that the probabilities of the events A and B were equal, because the ratios between the number of boys and total number of babies were equal. They could not manage the sample size in this question. This question was searching whether this misconception was common in both middle and high schools. The pre- and post-test results indicated that majority of the students fell into this misconception both in middle schools and high schools. Table 4.6 indicated the frequencies of the students who fell into this misconception in the pre- and post-tests.

Table 4.6 Frequencies of students' answers reflecting the misconceptions of sample size effect in the pre- and post-tests

Mis-conception	Middle School Teachers						High School Teachers							
	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Sample Size Effect	18	18	16	13	34	31	14	13	11	13	10	6	35	32

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of sample size effect counted only once.

As it was expected from students, majority of the students fell into this type of misconceptions in the pre-test. Total of 34 students (59.65 %) in middle schools thought that the probabilities of the events A and B were equal. In both teachers' classroom, most of the students answered accordingly. There were 18 students in Ahmet's classroom and 16 students in Barış's classroom who thought that the probabilities were equal. In general, students constructed ratios for the events A and B. Then, they equated the ratios. This ratio was generally shown as $\frac{2}{3} = \frac{200}{300}$. Some of the students' answers were as follows.

> They are both equal to the same number, because the numbers increased. However, the numbers are increasing in the same amount. Both events are look like to each other (Pre-test administred to Barış's classroom).
> The probability of having at least three boys among three babies and of having at least 200 boys among 300 babies is equal to $\frac{2}{3}$ (Pre-test administred to Barış's classroom).

Again, majority of the students in high schools gave answers that fitted to this misconception in the pre-tests. There were total of 35 students (59.32 %) who fell into this misconception. Their justifications were also similar. They considered the equality of the ratios. Therefore, this equality misleaded students in solving the question. In addition, students were very sure in their answers. Some of the justifications were given below.

> Both are the same. I think there is no need to compare them (Pre-test administred to Cihan's classroom).

> It is obvious that if there are at least two boys among three babies, it is normal to have at least 200 boys among 300 babies (Pre-test administred to Doğan's classroom).
> The probabilities for both of them are equal. Since the ratios are equal to each other, the probabilities are also equal. $\frac{2}{3} = \frac{200}{300}$ (Pre-test administred to Doğan's classroom).

The total number of students who fell into this misconception was 69 (60 %) in both middle and high schools in the pre-test. This means more than half of the students fell into this type misconception. They justifications in their answers were also similar.

After students received regular instruction, the number of the correct answers increased in the post-tests. The answers included more calculations. In addition, students' justifications also included mathematical terms learnt in the instructions.
Although student received instruction, there were still 31 students (53.45 %) who fell into the misconception of the sample size effect in middle schools. They explained their answers according to the equality of the ratios. They observed that there were ratios between the number of boys and the total number of babies, and that they were equal. Therefore, they stated that the probabilities of the events A and B were equal. However, the answers included more calculations and they tried to justify their answers by using the terms such as theoretical and experimental probabilities. Some of their answers were stated below.

> Both ratios are equal. $\frac{2}{3} = \frac{200}{300}$ (Post-test administred to Ahmet's classroom).
> No matter how big the numbers, it should not belie us. $\frac{2}{3} = \frac{200}{300}$. They are equal (Post-test administred to Barış's classroom).
> Both become equal, because if we simplify the numbers, both numbers become equal (Post-test administred to Barış's classroom).
> They are equal, because the probabilities of having at least two boys among three babies and of having at least 200 boys among 300 babies are experimentally equal (Post-test administed to Barış's classroom).

As it was seen from the answers, their justifications were similar to those answers given in the pre-test. However, students' answers also included some mathematical terms learnt in the lesson.

In general, this type of misconception was dominant among the high school students in the post-tests. Majority of the students fell into this misconception. There were 32 students (54.24%) who gave answers related to this type of misconception. Among them, the number of the science high school students decreased sharply to six students. On the other hand, there were 13 students in Cihan's classroom and 13 students in Doğan's classroom. Their justifications were again the same. They considered the equality of the ratios. Therefore, they stated that the probabilities of the events A and B were equal. Some of the students' answers were as follows.

> Since it says at least, it can be at least two boys or three boys. The probabilities become equal if we increase the numbers (Post-test administred to Cihan's classroom).
> The solution method and the logic for both of them are almost equal, because both are similar to each other and they are equal (Post-test administred to Cihan's classroom).
> They are equal, because first is two out of three and the second is 200 out of 300. As a result the head numbers are equal (Post-test administred to Doğan's classroom).

Students were successful in finding the probability of the event A in the post-tests. However, since they could not find the probability of the event B, they relied on the equality of the ratios. For example, one student stated his/her answer as follow.

> Four situations among eight are equal to having at least two boys. Therefore, it is $\frac{4}{8}$. So, it is equal to $\frac{1}{2}$. It is also equal in the situation of having at least 200 boys out of 300 babies. Both are equal. Therefore, the probabilities of both events are equal (Interview with a student from Doğan's classroom).

In general, the total number of the students who fell into this misconception was 63 (53.39 %). Still, more than half of the students fell into this type of misconceptions. In general, students tried to solve the question by using formulas and by doing calculations. However, their intuition was dominant. This misconception was observed both among middle and high school students.

4.3.2 Teachers' Awareness and Teaching Practices for Sample Size Effect

The third question aimed at determining students' misconceptions of the sample size effect. In the first place, middle school teachers and Doğan stated that the probabilities were equal in the interview-2. Therefore, they also fell into this type of misconception. At the end, all teachers stated that students could fall into the sample size effect in the 3rd questions in the PTI.

While Cihan and Erdal saw the question, they wanted to be sure about their answers and compared whether the probabilities were equal or not in the interview-2. Even Erdal wrote the general formula of the probability of having at least 200 boys out of 300 new-born babies. They stated that this misconception should be common among students. In addition, other teachers also agreed about this type of misconception. In this problem, *the same – the same* and *the sample size effect* misconceptions had the same logic. Therefore, it was considered that teachers were also aware of both types of the misconceptions. For this question, some of the justifications were as follows.

> Ratios are equal. Students can ignore the sample size and they just look at the ratios (Interview-2 with Barış).
> Here, students think that the numbers are multiple of each other. So, they consider they are equal (Interview-2 with Erdal).

In order to resolve this misconception, middle school teachers suggested that students the sample size should be emphasized to students in third questions in the interview-2. Barış directly stated that teachers should consider the sample size and teach the topic accordingly. On the other hand, Cihan indirectly stated the importance of the sample size. He suggested that the number of trial might be increased, for example, to six times. He also stated that the trials might be performed in the classroom with real coin. In addition, Barış also mentioned about the usage of visual materials and technological tools. Doğan stated that the misconception could be resolved with the help of comparisons of the probabilities. In addition, he also mentioned about the importance of the related subjects such as permutation, cyclic permutation, and combination. He suggested that students should be provided with better understanding of these subjects instead of just going through the probability subject in the interview-2. Erdal mentioned about the importance of solving as many questions as possible, students could understand how to find the sample size of any event, no matter whether the questions were familiar or unfamiliar to students. In practice, teachers were careful about the importance of the sample size.

This type of misconception was related to the determination of the sample size of the events asked. Students needed to manage the sample sizes for any question in probability. Findings from classroom observations indicated that all teachers were curious about this issue and stated that determining the sets of expected elements and the sample size in the events were among students' main difficulties in the probability. High school teachers, especially Doğan and Erdal, emphasized to find the sample sizes for the events asked. In addition, when giving the algorithms for solving different types of the questions, all teachers included "finding the sample size" as one of the basic steps. Therefore, there were numerous examples solved to find the sample size.

In the concept development phase, all teachers began with the simple questions in order to make students find the sample size for the events correctly during the observations. Then, especially high school teachers gave more emphasis on finding sample size and the sample space of equally probable elements. For example, Erdal stated in the interview-1 that he solved 20 to 40 questions about this concept. During the observations, he really solved more than 20 questions related these topics. In addition, Cihan and Doğan constructed a relation between sample size and the permutation and combination concepts. They emphasized to use permutation while finding sample size in the probability questions. On the other hand, middle school teachers gave more emphasis on the general formula for probability.

For managing the sample sizes in the events happening, there were also some shortcuts and memorizations observed in the classrooms. One basic example that all teachers except Ahmet used was to find the sample sizes if a coin or a die was thrown n times. The shortcut was that the sample size is 2^n for coin and 6^n for die.

During the observations, there were also some other shortcuts about finding sample sizes. For example, all teachers stated that there might appear 4-2 and also 2-4 after throwing two dice. This situation was directly related to this type of misconception. In addition, only high school teachers expected from students to memorize the statement of "the sample size for throwing two dice (coins) and for throwing a dice (a coin) twice are equal."

All teachers provided relation between the probability and set topics during the observations. While showing the samples sizes and the expected elements, they were directly using the sets or the number of elements in a set. In addition, only high school teachers used permutations and combinations for finding samples sizes in the questions. All high school teachers used same procedure for using permutation and combination. In general, the determination of where to use the permutation and combination was also considered as one of students' difficulties by teachers while solving probability questions. They used *if-then* type of statements. For example, they expected students to use combination, if the question was about the selection and permutation if the question was about the arrangement. Doğan and Erdal directly used the permutation and combination in the questions without explaining the logic behind the usage. They followed the procedures of using permutation and combination only when needed. On the other hand, Cihan explained the reasons for using them. Even in the questions of choosing a ball from urn, he showed that the possible outcomes were $\binom{n}{1}$ if n was the number of the balls in an urn. For example, when choosing two students from three, he both used $\binom{3}{2}$ and showed possible outcomes. He numbered students from 1 to 3. Then, he showed the possible outcomes by writing 1-2, 1-3, and 2-3 during the observations.

It was observed that the determinations of sample size in dependent-independent events and inclusive-mutually exclusive events were hard for students. They could not manage the sample sizes. For such situations, teachers used different rote memorizations to determine the event types. In addition, teachers provided algorithms for different types of the questions.

Another point that was observed in classrooms was that students could not manage and associate the set of expected elements and the sample size. For example, one question was as follows.

> There are three red (R), three blue (B), and four yellow (Y) balls in an urn. Three of them are randomly chosen. What is the probability that only one ball among the chosen balls is red (Observation-5 in Cihan's classroom).

In this question, students could easily find the sample size as C(10,3). However, they could not manage how to find the set of expected elements in the first moment during the observations. Then, Cihan stated that the possible outcomes were RBB, RBY, RYY, and their different arrangements. He explained that they needed to add the possibilities and their different arrangements that could be found with permutation. Therefore, they needed to predict different situations, and their arrangements, and to find the sample size. After that, teacher showed another way of solving the question by using combination. Overall, teachers tried to show little visualization for the predictions and they also made students remember the necessary knowledge for the questions. In general, they were stick to the algorithm for the solutions of the questions.

Another important issue to find the sample size appeared in infinite probability topic. During the observations, Barış and Cihan provided students with the general probability formula and explained that the sample size was the total area, length or volume. Erdal directly stated it after writing a question related to this topic. None of the students were uncomfortable with such explanations. In fact, Ahmet, Cihan, and Erdal considered that students experienced difficulty in relating other subjects (e.g. geometry) with probability.

Erdal solved unfamiliar questions in the classroom. In such questions, students could not develop either the sample size or the set of expected elements. He showed the solution method at the end. However, the aim of asking such questions was that there was no limit to ask questions related to probability. One of the questions was as follow.

> A glass rod falls into ground. It is broken from two points. What is the probability that joining the broken points of three pieces generates a triangle? (Observation-6 in Erdal's classroom).

In this question, Erdal constructed a relation between probability and geometry. He showed the sample size in analytic plane. The sample size and the set of expected elements were shown with areas in the analytic plane.

In fact, the geometry was used by Barış, Cihan, and Erdal in the lessons. The topic of infinite probabilities was taught by Barış and Cihan. Erdal directly used geometry when needed in the unfamiliar questions. Barış and Cihan gave the definition from the preparatory book for university entrance exam and the general formula for it. The formula was given as $P(A) = \frac{the\ measure\ of\ A}{the\ measure\ of\ universal\ set}$. Cihan explained the formula as follow.

> Here, different than the other topics, we have measures such as area, volume, and angle. They include infinite points. So, we can find the answer by dividing the expected measure by the whole measure. You will understand it in the examples (Observation-7 in Cihan's classroom).

He directly began to solve questions. One of the questions asked was as follow.

> What is the probability of choosing a point from a circle with the radius of r that is closer to its center than its perimeter? (Observation-7 in Cihan's classroom)

In general, high school teachers were curious about how to find the sample size in any questions. Although middle school teachers showed the ways to find it, high school teachers were more focused on this issue. However, the regular instructions in both middle and high schools did not have positive impact of resolving students' misconceptions. The numbers of students who fell into this misconception stayed almost the same before and after they received regular instruction both in middle and high schools.

4.4 Representativeness Heuristics: Outcome Approach as an Intuitively-based Misconception

This misconception is again a type of representativeness heuristics. It occurs when students decide the probabilities according to the appearances of already happened events. The findings related to the pre- and post-test results, teachers' awareness and teaching practices about outcome approach were presented in this part of the study.

4.4.1 Pre- and Post-Test Results

The fourth question in the PTI was specifically asked to determine students' intuitively-based misconception of the outcome approach. The outcome approach appeared when students were affected by the outcomes or distributions of the events. The main point was that the outcomes already happened in the events. They tried to solve questions under the effect of the outcomes of the events in the questions.

What was expected from students to do in this question was that they chose the "Distribution I" (DI). Since there were 16 game cards distributed and four players, students were expected to distribute 16 game cards to four players equally.

Table 4.7 Frequencies of students' answers reflecting the misconceptions of outcome approach in the pre- and post-tests

Mis-conception	Middle School Teachers						High School Teachers							
	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Outcome Approach	8	11	5	9	13	20	3	4	3	4	1	7	7	15

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of outcome approach counted only once.

According to the pre-test results, total of 20 students (17.39 %) in middle and high schools fell into this type of misconception. Among them, 13 students were from middle schools and 7 students were from high schools for this misconception. The test results revealed that there were less students for this misconception compared to the other intuitively-based misconceptions.

Students generally thought that the players should get the same number of cards. Their justifications also supported the idea of this misconception. In this type of misconception, middle and high school students' justifications were similar. Some of the students' justifications were as follows in the pre-tests.

> Because we are giving the cards to players one by one. So, they will get equal number of cards (Pre-test administred to Ahmet's classroom).
> Because 16 cards are distributed to four persons, so each gets four cards. (Pre-test administred to Barış's classroom).
> Because the numbers are more regular (Pre-test administred to Barış's classroom).
> Because the probability of distributing four cards for each person is higher. (Pre-test administred to Doğan's classrom).

They all mentioned about the equality of the number of the cards given to each player in the pre-tests. Despite the other justification, one student mentioned about the regularity of the numbers.

After students got regular instruction, the number of the students who fell into this type of misconception increased. In fact, the number of correct answers increased. However, since students fell more into *the more of A – the more of B* and *the availability heuristics* misconceptions in the pre-test, the increase in this type of misconception appeared in the post-test.

When compared to the other intuitively-based misconceptions, students who fell into the outcome approach misconceptions were still low in the post-tests. There were total of 35 students (30.43 %) who fell into this misconception in middle and high schools. Among them, 20 students, which were equal to 35.09 % of all middle schools, were from middle schools. The rest (15 students) were from high schools. This was equal to 25.86 %.

Similar to the justifications given in the pre-test, students' answers involved the assertion of the equality of numbers for each player. Especially middle school students supported this idea. Some of the justifications from middle school students were as follows.

> It comes 4 4 4 4 steadily (Post-test administred to Barış's classroom).
> The order is collapsed in DII (Post-test administred to Ahmet's classroom).
> The cards were distributed in wrong way (Post-test administred to Barış's classroom).

Students thought that each player should get four cards. For this thought, one student asserted that the order in the second distribution was incorrect. In addition, one student thought that the cards were distributed in wrong way in the post-test.

In high school, the justifications changed the form in the post-tests. Especially in science high school, the number of students who fell into this misconception increased from one student to seven. The main reason for this situation was that they tried to find the sample size and to solve the question in formal way with formulas. However, some of them could not manage the formula. Other than the wrong use of the formula, one student asserted this justification.

> A M F H $\Rightarrow$ If the order of distributing the cards is in this way, then, the probability of distribution I is higher, because each player should have equal number of cards (Post-test administred to Cihan's classroom).

On the other hand, one student who gave correct answer in science high school also had the same thought. This student's assertion was as follows.

> The distribution is random. It begins from a player, but it follows a system. In such situation, the probability of distribution I is higher (Post-test administred to Erdal's classroom).

One student in Anatolia high school had the same idea as the middle school students. S/he mentioned about the necessity of the equality of the number of the cards that each player should get.

4.4.2 Teachers' Awareness and Teaching Practices for Outcome Approach

In the fourth question of the PTI, whether students had the misconception of the outcome approach was investigated. When asked to the teachers about the occurrences of this misconception among students, all teachers except for Erdal stated that they might have the misconception of the outcome approach in the interviews-2. They gave similar justifications. For example, Ahmet stated that "students might think that each player got the same number of cards, so they could fall into misconception" (Interview-2 with Ahmet).

Erdal gave a different explanation for this question. His statement was about the understanding of the basics of the probability. The following quotation was about this issue.

> A student who knows the basics of probability can easily state that the probabilities are equal. ... They can calculate the probability. I do not think that students will fall into a misconception in this question (Interview-2 with Erdal).

There were no specific suggestions for this misconception in the interviews-2. Instead, teachers stated that students should solve more questions and they needed to know how to understand the topic of the elements of equal probabilities in the sample size. They stated that they could give more emphasis on this issue. In practice, however, teachers did not give emphasis on this misconception.

This misconception appears when comparing the already happened events. Since the teachers asked questions based on the high school/university entrance exams, such kind of questions were not observed in the classrooms. However, middle school

teachers were closer to diminish their students' misconception of outcome approach due to the curriculum. The curriculum included experimental and subjective probabilities. The teaching of these topics had indirect effect on students' possible misconception of outcome approach.
During the observations, for example, Ahmet stated that after 20 trials he could get 16 heads and four tails. In another experiment, he stated that he could get 510 heads and 490 tails. At the end, he stated that if the trials increased the outcomes approach to the theoretical probability. In such situation, students could be familiar with the different events with different outcomes. Then, they could create an understanding for resolving this misconception. However, the examples given in the classrooms were not directly related to this misconception.

For the classrooms in high schools, there may be indirect relation with the misconception of outcome approach and the dependent events. Since students learnt sample space of equally probable events in the beginning of the probability subject, students could think that the sample sizes of the dependent events were the same. They might consider that the probabilities for taking one ball from an urn after another are the same. For such questions, teachers developed algorithms for question types and shortcuts for such situations. During the observatrions, the algorithm that all teachers used to solve questions was as follows. The algorithms changed according to question type. For Cihan, for example, the necessary steps to follow were as follows.

- Determine the event type
- Determine the events of A and B, and write them separately.
- Determine the sample size and the number of expected elements
- Use the necessary formula according to the even type
- Find the answer

In general, outcome approach was related to concepts of sample size and dependent events. The general trend among teachers was that they gave course book definition for the dependent event and explained the concept with simple examples during the observations. The most apparent example among all teachers was about choosing a ball from an urn. If the ball was put back into the urn, then, the event was dependent. All high school teachers stated that determination of which formula to use was considered as one of the students' difficulties in probability. However, all teachers used the same formula by relating the concept with the set topic. The formula was P(AUB)=P(A)+P(B)-P(A∩B). They also used shortcuts to renew formula for the specific type of the events. Again, determining event type was also considered as one of the students' difficulties by all teachers in the interviews-1. These shortcuts were generally in the form of if-then statements during the observations. Students were provided with many shortcuts for each event types, so, they confused which shortcut to use in a specific question. The shortcuts observed related to dependent events were as follows.

- If the ball is not released into the urn, then, it is dependent.
- If the first event reduces the number of elements in the second event, it is dependent.
- They are affecting each other, so it is dependent.
- If you take the balls one by one, then, use multiplication rule and it is dependent.
- "and" means union and addition.
- "or" means intersection and multiplication.

After students received regular instructions, the numbers of students who fell into this misconception did not change in Cihan's and Doğan's classroom in the post-tests. However, the occurrence of this misconception increased in middle school and Erdal's classrooms. Especially in Erdal's classroom, this misconception increased sharply. In fact, teaching practices for resolving this misconception were not observed much in the classrooms. Due to the findings, it could be inferred that teachers' teaching practices promoted this misconception in students' minds. Here, the quarter of the students fell into this misconception after the regular instruction.

4.5 Simple and Compound Events as an Intuitively-based Misconception

This part of the study presents the results of pre- and post-tests, teachers' awareness and teaching practices specific to the misconception of the simple and compound events. In general, this misconception appears when students confuse to differentiate the probabilities of the simple and compound events.

4.5.1 Pre- and Post-Test Results

The national and international curricula gave importance to the topic of the simple and compound events in teaching probability. Students were expected to comprehend the differences between the simple and compound events. From this point of view, it was obvious that students could easily experience difficulty in differentiating simple and compound events. In some cases, this problem could be considered as one of the intuitively-based misconceptions. If students did not take the sample size and its elements into consideration, they might fall into this type of misconception. The fifth question in the PTI was specifically asked to determine whether this type of misconception was common among middle and high school students.

The main problem in this type of misconception was that students did not realize the outcomes of any pairs in the pre- and post-tests. A pair of 4-3 included both the outcomes of 4-3 and 3-4 if a die was thrown twice. However, they gave the emphasis only on the outcome of 4-3 and ignored the other outcome. Therefore, they fell into misconception.

Table 4.8 Frequencies of students' answers reflecting the misconceptions of simple and compound events in the pre- and post-tests

	Middle School Teachers						High School Teachers							
Mis-conception	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Simple and Compound Events	12	16	13	13	25	29	7	10	19	16	11	16	37	42

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of simple and compound events counted only once.

The pre-test results indicated that the misconception of the simple and compound events was very common in both middle and high schools. In every classroom, this misconception was extensively observed in the pre- and post-tests. There were 25 students who fell into this misconception in middle schools in the pre-test. This was equal to 43.86 % of all middle school students. In fact, there were also many students who fell into "the more of A– the more of B" misconception. Therefore, this percentage was very high among middle school students. On the other hand, there were 37 high school students for this misconception in the pre-test. This was equal to 63.79 % among high school students. There were total of 62 students, which was equal to 53.91 % of all students in the pre-test. Among them, almost all students in Anatolian high school and more than half of the students in science high school fell into this misconception.

Students in middle schools generally considered the outcomes of the 4-4 and 4-3 as separate ones; they asserted that the probability of getting any of them was equal in the pre-tests. While students gave this answers, they supported their ideas with the following justifications.

> The probabilities are equal, because we are choosing one from each (Pre-test administred to Ahmet's classroom).
> I think both faces are involved in a die. I mean both can appear. As a result, the probability of getting any face is equal if we throw a die. Therefore, the probability of getting any of the events is equal (Barış's classroom).
> Because the dice are unbiased. Both can come (Pre-test administred to Barış's classroom).

One student noticed that there were two outcomes for the pair of 4-3. However, he could not manage that there were only one outcome for the pair of 4-4. His justification was given below.

> $\frac{2}{36}=\frac{2}{36}$. They are equal, because two dice are becoming 36 (*sample size*). There are two 4-4s. $\frac{2}{36}$. Similarly, there are two 4-3s. It is again $\frac{2}{36}$ (Pre-test administred to Barış's classroom).

High school students also gave similar justifications for this misconception in the pre-tests. Students generally ignored the possible outcomes of the pairs of 4-4 and 4-3. One student mentioned about the ratio. One other student stated that there were one 4-4 and one 4-3. So the probabilities were equal. Their answers were as follows.

> The likelihood of both of them are equal, because there is only one number that is the same for two dice. So, the probabilities are equal. The probabilities of getting a pair of 4-4 and a pair of 4-3 are proportional (Pre-test administred to Doğan's classroom).

What was different between the answers of high school and middle school students was that high school students were more dependent on the formal solutions. They tried to use the general formula of the probability. However, since they ignored the elements in the events, they fell into this misconception.

After students received regular instruction, the number of the students who fell into this misconception increased sharply in all classrooms in the post-tests. As it is observed in the Table 4.8, the number of students in both middle and high schools increased in the post-tests. After they received regular instruction, some middle school students who fell into "the more A – the more B" misconception changed their answers and they fell into the misconception of the simple and compound events. 29 students fell into this type of misconception. This was equal to 49.15 % of all middle school students. Almost half of the students fell into this misconception. On the other hand, there was a slight decrease among Anatolian high school students. The total number was 42 students in high schools in the post-test. This was equal to 71.18 % of all high school students. As it was seen in the findings, the regular instruction had negative effect on resolving students' intuitively-based misconceptions of the simple and compound events, especially in high schools.

Students in middle schools tried to use formula for the probabilities in this question in the post-tests. They tried to support their answers by using mathematical expressions and by doing calculations. Some of their justifications were as follows.

> The die 1 {1,2,3,4,5,6} $= \frac{2}{12}$ for the outcome of 4-4. The die 2 {1,2,3,4,5,6} 4-3. Same (Post-test administred to Barış's classroom).
>
> Because if we compare both of them, the numbers for each die is equal to each other. Therefore, the probabilities are also the same (Post-test administred to Barış's classroom).
>
> Both of them are equal. In two dice $\Rightarrow$ the probability of getting 3 or 4 are equal to each other. First situation $\frac{1}{6} \cdot \frac{1}{6}$ and for the second situation $\frac{1}{6} \cdot \frac{1}{6}$. The probability is $\frac{1}{36}$ (Post-test administred to Ahmet's classroom).
>
> For me, 4-4 and 4-3 are equal because it is not clear which one will appear. The probability of getting 4-4 is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$. The probability of getting 4-3 is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$. As it is shown, the probabilities of getting the pairs of 4-4 and 4-3 are equal (Theoretically) (Pos-test administred to Barış's classroom).

One student also used mathematical expression in the post-tests. He used the word "theoretically" in his solution. This was learnt in the topic of theoretical, experimental and subjective probability.

Similarly, high school students also tried to support their answers with the appropriate formula. However, they also fell into the same misconception. They ignored one outcome.

Different than the middle school students, some students in high schools mentioned about the dependent and independent events in the post-tests. They considered the 4-4 and 4-3 pairs as independent events. Some of the justifications were as follows.

> They are independent of each other (Post-test administred to Cihan's classroom).
> Since the dice are independent of each other the probabilities of getting 4-4 and 4-3 are equal (Post-test administred to Cihan's classroom).
> Since they are independent, the probability of getting 4 in the first die and 4 in the second is equal to $\frac{1}{36}$. The probability of getting 4 in the first one and 3 in the second is $\frac{1}{36}$. They are equal (Post-test administred to Erdal's classroom).

One student in vocational high school mentioned about the expected outcomes and sample size. He stated that there was one expected outcome among 36 possible outcomes for both pairs of 4-4 and 4-3 in the post-test. Some students also compared the probabilities of any other outcomes such as 2-3 or 5-6. While students in middle schools mentioned more about the theoretical probabilities, students in high schools mentioned more about the independence of the events. However, all of them ignored the possible outcomes in the expected pairs. Therefore, they all fell into the same misconception.

4.5.2 Teachers' Awareness and Teaching Practices for Simple and Compound Events

The main aim of the fifth question was to determine students' intuitively-based misconception of the simple and compound events. In this question, Barış and Cihan also fell into this misconception in the interview-2 with teachers. In addition, Ahmet was not sure about whether the question asked pairs or only 4-3. He explained students' misconceptions accordingly. He stated that if the arrangement was important, students might state that the probabilities were equal. While teachers deeply investigated the question, they all agreed that students might fell into this type of misconception in the interviews-2.

In general, teachers mentioned about the precautions similar to those stated for the misconception of the sample size effect in the interviews-2. They stated that they needed to focus on the sample sizes of any event. Especially middle school teachers stated that the use of materials such as coins, dice, and computer programs might help students to resolve this kind of misconception. Doğan also stated that students needed to be careful about the comparison of the probabilities of the events. In general, they suggested that students needed to solve different kind of questions in the interviews-2. In practice, high school teachers solved questions related to this misconception in their lessons.

According to the findings of the interview-1 with teachers, students' difficulties in line with simple and compound event misconception were as follows. All teachers mentioned about students' difficulties of determining the sets of expected elements and the sample spaces for the events in the probability questions. Moreover, all teachers stated that students might experience difficulty in determining whether the event was dependent or independent and simple or compound. In addition, all teachers except for Ahmet stated that students were unable to relate and use permutation and combination when needed in probability questions in the interview1.

This misconception appears when two or more events can be considered as one event. Therefore, the probabilities could be calculated either with the methods for independent events or with the general probability formula for simple events. For example, throwing two coins could be considered as two separate events or one simple event. In such situations, especially high school teachers were solving the questions in both ways during the observations. One question is as follow.

> What is the probability of getting the same outcome after throwing the two dice? (Observation-2 in Erdal's classroom)

In similar questions, it was considered as simple event for every teacher during the observations. They found that the sample size was $6^2 = 36$. Then, they stated that the set of expected elements is {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)}. Therefore, so the probability is $\frac{6}{36}$. On the other hand, especially high school teachers showed the alternative way. The first event was getting 1 and the second event must have been the same. So, they showed that the probability of getting (1,1) is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$. At the end, they stated that there could be six outcomes. So, the probability is $6 \cdot \frac{1}{36} = \frac{6}{36}$.

The basic misconception comes from the understanding of the pairs of coins or dice. The sample size for two coins was {TT, TH, HT, HH} and for two dice was {(1,1), (1,2), … , (3,3)…, (6,5), (6,6)}. Teachers directly stated that if there was a pair of TH, then, there must be HT during the observations. Similar situation was also mentioned for throwing a pair of dice. Therefore, they stated that the arrangement was important. The only explanation for this situation was that the different arrangements changed the sample size. Same situation was valid for the dice, too. However, none of the teachers explained why, for example, TT, HH, (1,1), and (3,3) were written only one time in sample size during the observations. On the other hand, all students were comfortable and satisfied with the situation. Students were writing the set of expected elements of event asked.

In general, all teachers used similar teaching practices. For such situations, teachers constructed some shortcuts. They expected from students to remember while solving questions. Their practices indicated that students could consider the occurrences of two or more events as one event during the observations. All teachers, for example, emphasized that there might appear 2-3 (or other combinations) and also 3-2 after throwing two dice. Therefore, teachers promoted that two events could be considered as one event. Another shortcut was that the sample sizes for throwing two dice and for throwing a die twice were equal.

Erdal created a table for the questions of throwing two dice in the lesson. In the table provided, the numbers of outcomes were matched with the addition of the values that appeared after throwing two dice. Instead of finding the expected elements one by one, students used the table and finding the number of outcomes. The rule table for two dice given in the classroom was presented in the Figure 4.1 below.

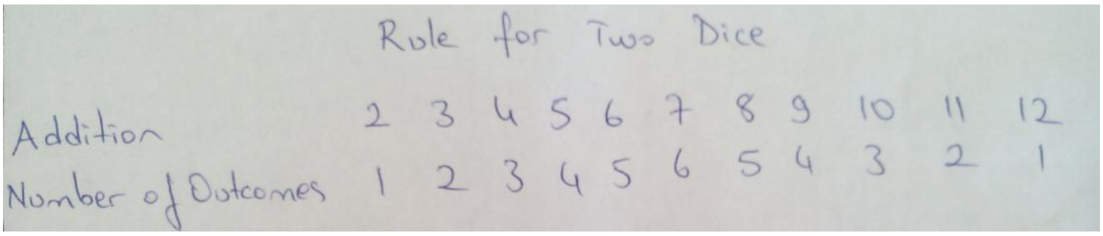


Figure 4.1 Table for number of addition of two dice observed in Erdal's classroom

Observation findings indicated that the concepts of simple event and compound events were taught separately. All teachers taught the general formula for the probability. However, they solved questions related to simple events while explaining the formula. On the other hand, the independent or dependent events were taught as separate topics in probabilities. Related to this issue, while solving mixed questions in the last lessons of the probability, Doğan and Cihan showed alternative solution methods for some questions if appropriate. Doğan solved the question below.

> What is probability of getting two heads after throwing two dice?
> (Observation-2 in Doğan's classroom)

Doğan solved it in two ways. The first one was considering each throw as separate event. Then, he found answer as $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$. He also solved the question by showing the sample size for throwing two dice. Then, he indicated that there was only one element that was expected in the question.

Overall, this misconception was observed excessively both in middle and high school students. Although there were teaching practices such as the examples given, the shortcuts to memorize in order to resolve this misconception, regular instructions did not have positive impact on students' minds. The number of students who fell into this misconception increased from 25 and 37 to 29 and 42 in middle and high school classrooms, respectively. Here, this misconception was dominant among high school students. More than half of the students both in middle and high schools fell into this misconception.

4.6 Conjunction Fallacy as an Intuitively-based Misconception

If an event already happens when another event occurs, this is called conjunction fallacy. At this point, students may ignore this situation and consider them separately. Therefore, students may fall into the misconception of the conjunction fallacy.

4.6.1 Pre- and Post-Test Results

Sixth question in the PTI was specifically asked to students whether they considered the conjunction or not. There were numerous answers in this question. In this question, students who did not realize the conjunction or ignored the order of the events happened were considered that they fell into this type of misconception.

The pre-test results indicated that many students fell into the misconception of the conjunction fallacy. Especially those who said that Ayşe's probability of winning the game was lower than the others were considered as ones who fell into this type of misconception. Table 4.9 indicated the frequencies of the students who fell into this misconception.

Table 4.9 Frequencies of students' answers reflecting the misconceptions of conjunction fallacy in the pre- and post-tests

	Middle School Teachers						High School Teachers							
Mis-conception	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Conjunction Fallacy	15	12	14	10	29	22	8	5	7	3	8	5	23	13

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of conjunction fallacy counted only once.

According to the results shown in the Table 4.9, this misconception was very common among middle school students. On the other hand, the number of correct answers was high among high school students, especially in Anatolian high school in the pre-tests. The percentage for vocational high school was similar to that for middle schools. The total number of students who fell into this misconception among middle schools was 29 in pre-test, which was equal to 50.87 %. This percentage was 39.65 % for high schools. There were 23 students who fell into this misconception among high school students in pre-test. As it was stated, the percentage was high in vocational high school. There were 8 students who fell into this misconception in the vocational high school. This was equal to 44.45% for this classroom. This percentage was close to the middle school students' percentage. There were total of 52 students who fell into this misconception in the pre-test. This was equal to 45.21 %.

In general, students in middle schools ignored the players' choices. Although Ayşe chose an order of five throws while the others chose an order of six throws, they did not consider this situation and looked at different points. For example, they looked at the number of heads and tails, the biasness of the dice in the pre-tests. Some of the justifications for this misconception were as follows.

> Because the coin is biased, there are more heads. Therefore, Ayşe has more heads. The probability is also higher (Pre-test administred to Ahmet).
> It does not appear four heads one after another (Pre-test administred to Barış).

Similar justifications were seen among high school students in pre-test, too. They observed that the numbers of heads were high in the sequences. Therefore, they gave answers on the basis of the heads and tails. For this misconception, middle school students excessively fell into this misconception. On the other hand, the number of high school students lower when compared to middle school students. However, justifications of the high school students who fell into this misconception were similar to that of middle school students.

After students received regular instruction, there was slight decrease in both of the school types in the post-tests. There was a sharp decrease only in the number of Anatolian high school students.

The Table 4.9 indicated that the regular instruction helped students to resolve the misconception. However, there were still many students who fell into this type of misconception. The number of middle school students who fell into this misconception decreased to 22 students, which was equal to 37.29 % for middle school in the post-test. On the other hand, there were only 13 students in high school.

This was equal to 22.03 %. Total number of students who fell into this misconception was 35 in the post-test. This was equal to 29.66 %.

When considering students' justifications, there was almost no change for both middle and high school students in the post-tests. They ignored the sequences of the players' choices and focused more on the biasness of the dice and the number of heads and tails in the sequences. For this misconception, there was no difference between middle and high school students' justifications. The only difference was that this misconception was very common among the middle school students. On the other hand, there were lesser students who fell into this misconception among high school students. In addition, the regular instruction had small impact on resolving this misconception in both middle and high schools.

4.6.2 Teachers' Awareness and Teaching Practices for Conjunction Fallacy

The fifth question was aiming to determine students' misconception of the conjunction fallacy. For this question, all teachers except for Barış stated the possibility of appearance of this misconception among students in the interviews-2. In addition, Cihan determined all kinds of misconceptions that appeared in the pre- and post-tests conducted to the students.

For *the conjunction fallacy,* Doğan's explanation was useful in the interview-2. He stated as follows.

> Ali's first five predictions and Ayşe's predictions were the same. So, if the first five trials are as those in Ayşe's prediction, there will be no need to do one more trial. Ayşe wins. I mean Ali's probability of winning the game is zero. Here, students may fall into misconception by relating the events which are independent (Interview-2 with Barış).

For the question related to this misconception, there were different suggestions from the teachers in middle and high schools in the interviews-2. Middle school teachers and Doğan stated that they could calculate the probabilities separately and compare them. On the other hand, high school teachers could give emphasis on the independence of the events. Teachers followed their suggestions in teaching probability in their teaching practices.

Observation findings indicated that the misconception of conjunction fallacy was observed in three ways in the classrooms. Two of them were related to the topics of inclusive-mutually exclusive events and conditional probabilities. In fact, both topics were in the high school curriculum. However, Barış taught inclusive-mutually exclusive events. Therefore, it was possible that Barış's students got familiarity for this type of misconception. The last one was about the topic of infinite probabilities. In concept development phase, teachers used verbal explanations examples for the inclusive-mutually exclusive events during the observations. Then, they focused on the dictionary meaning of the concept. Following quotation was related to the teaching of the concept of mutually exclusive events.

> Cihan : As we understand from its name, mutually exclusive events are separated events. They have no relation. They are separated. Here, we need at least two events. Say that they are A and B. If they do not have common points between them, what can we say?
> Student : Their intersection is empty set (Observation-5 in Cihan's classroom).

Then, teacher explained the concept with example from the probabilities of getting odd and even numbers after throwing a die. The same example was asked by all high school teachers. Differently, Doğan showed the difference between inclusive and mutually exclusive events by giving example from the probability of getting even and prime numbers and of getting even and odd numbers during the observations.

To differentiate the inclusive and mutually exclusive events, teachers provided some statements to memorize. This was because both middle and high school teachers stated that students experience difficulty in determining whether the event in the probability questions was inclusive or mutually exclusive in the interviews-1 with teachers. These memorizations were in the form of *if-then* statements. The following statements were observed in the classroom. Most of the memorizations were observed especially in high school teachers' classrooms. Some of them are given below.

- If the intersection is not empty, then, the events are inclusive.
- If the intersection is empty, then, the events are mutually exclusive.
- If it is not mutually exclusive event, add P(A∩B) into the formula

Although these topics were related to the misconception of conjunction fallacy, it only helped students to get familiarity for it during the observations. Beginning with the inclusive-mutually exclusive events, since conjunction fallacy was about two events of A and B one of which was the subset of another, inclusive events had direct relation. In such questions, students needed to find the intersections of A and B. In some cases, the requirements of conjunction fallacy existed. However, such situation was observed only in Barış's classroom. The question was as follow.

> After throwing a die, what is the probability of getting two or even number? (Observation-5 in Barış' classroom).

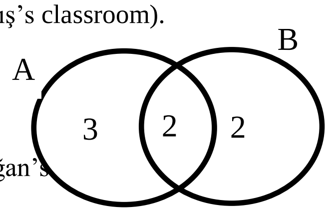
In this question, two was even number. Barış wrote the events A and B for these events and stated that the set of A is subset of B. However, he directly leaded students to the formula for inclusive events, which was P(AUB)=P(A)+P(B)-P(A∩B). On the other hand, all teachers except for Ahmet solved questions related to inclusive events. They also explained the inclusive and mutually exclusive events with examples. In fact, only high school teachers stated that students experience difficulty in determining which formula to use in the probability questions in the interviews-1 with teachers. The Table 4.10 below showed the formulas for the events and examples given by teachers during the observations.

Table 4.10 Formulas used and examples given for inclusive-mutually exclusive events.

Formulas	Examples
P(AUB)=P(A)+P(B) (If A and B are mutually exclusive) A∩B=Φ	Let's ask the probability of getting head or tail (after throwing a coin). The probability of getting head is ½. The probability of getting tail is ½. They are mutually exclusive. As in the addition of the elements in mutually exclusive sets, the probabilities were also added. If the events are not mutually exclusive, we use different formula (Observation-4 in Doğan's classroom).
P(AUB)=P(A)+P(B)-P(A∩B) (If A and B are inclusive)	If the events are mutually exclusive, then, the intersection is empty set. But if it is not, we are using this formula. It is similar to addition of the number of elements of the sets A and B. For example, if it is asked to find the probability of getting even numbers or of getting prime numbers, we can use it, because two is both even and prime. They are not mutually exclusive (Observation-5 in Cihan's classroom).

They generally showed algorithm for solving the questions and directed students to the formula during the observations. When explaining the intersections of the events, all teachers used small visualizations. According to the table or the Venn schemas created by teachers, they put the data into the formula and solved the questions. When explaining the formula, Barış, Cihan, and Doğan stated that the intersections were counted two times and one should be deleted. They showed it on the list of the sets and Venn schemas. They emphasized that the last part of the formula while explaining why they subtracted the intersection. The question types and the small visualizations were given as follows during the observations.

Table 4.11 Examples asked in the classroom and small visualizations for them

<table>
<tr><th>Examples for question types</th><th>Small Visualizations</th></tr>
<tr><td>What is the probability of getting odd numbers or prime numbers after throwing two dice?</td><td>A={1,3,5} and B={2,3,5}. So, the intersection A∩B={3,5} (Observation-2 in Cihan’s classroom).</td></tr>
<tr><td>There are 12 women, seven of whom have glasses and nine men, six of whom have glasses. What is the probability that one chosen person is either man or with glasses?</td><td><table><tr><th></th><th>Women</th><th>Men</th></tr><tr><td>w/ glasses</td><td>7</td><td>6</td></tr><tr><td>w/out glasses</td><td>5</td><td>3</td></tr></table>(Observation-3 in Barış’s classroom).</td></tr>
<tr><td>The numbers from 1 to 9 are written in small papers and put in a box. If a paper is chosen from the box, what is the probability that is less than 6 or even numbers?</td><td>s(A)=5
s(B)=4
s(A∩B)=2
(Observation-3 in Doğan’s classroom)
</td></tr>
</table>

Secondly, the conditional probability had relation with the conjunction fallacy. This was because the sample sizes were obtained according to the conditions given in the questions. This meant that the sample size in the event was the subset of the sample size without condition. Some of the examples observed were given in the Table 4.12 below.

Table 4.12 Teachers' use of the sample sizes of the questions with and without the condition

Question	Sample size with condition	Sample size without condition
A die is thrown. If it is known that the number is prime, what is the probability of getting even number?	E={2,3,5} s(E)=3	E={1,2,3,4,5,6} s(E)=6 (Observatin-4 in Erdal's classroom)
It is known that addition of the outcomes of throwing two dice is less than 4, what is the probability that the addition can be divided by 3?	E={(1,1), (1,2), (2,1), (2,2), (3,1), (1,3)} s(E)=6	E={(1,1), (1,2), … , (6,6)} s(E)=36 (Observatin-5 in Doğan's classroom).

In such situations, high school teachers showed the key points in the question and directly wrote the sample size with condition during the observations. Among the elements in the sample size with condition, teachers wrote the expected elements. The aim of solving these questions was not to resolve the misconception of conjunction fallacy. However, it might give familiarity to students about the conjunctions. From teachers' point of view, teachers stated in the interviews-1 that the important point was that students need to be aware of the condition and to develop the sample size accordingly.

Indirectly, there was indirect relation between the topic of infinite probability and conjunction fallacy. Students were suggesting that the event was happening under the given facts and solving the questions accordingly.

There was only limited number of questions asked to students in the observations.
For example, one question asked in Barış's classroom was as follow.

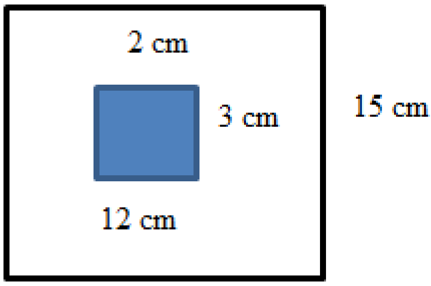


Figure 4.2 Figure for the parachute jumper question

> According to the given facts, what is the probability that a parachute jumper is landing in the shaded area? (Observation-5 in Barış's classroom)

Here, the condition was that the parachute jumper landed inside the bigger square. However, Barış did not give emphasis on this situation. He stated that the sample size was the area. He gave the formulas for square and the probability. Then, he solved it as a probability of the simple event.

Overall, both middle and high school teachers provided many examples and visualizations that were related to conjunction fallacy. In fact, teaching practices were in line with the curriculum followed. However, the questions asked were parallel to resolve this misconception. After students got regular instruction, the numbers of students who fell into this misconception decreased slightly in both middle and high school classroom. In the post-test, this misconception observed among more than quarter of all middle and high school students.

4.7 Conditional (Time-Axis) Probability

The misconception of the conditional probability is called as *the time axis probability* in some sources. The misconception occurs when students think that the further event affected the preceding event. The second part of the question was related to this misconception. The results related to this misconception are presented in this part of the study.

4.7.1 Pre- and Post-Test Results

The last question in the PTI was specifically asked to determine students' intuitively-based misconception of conditional probability. This question is composed of two parts one of which is classical conditional probability question. On the other hand, the second part includes time-axis condition. When students answered this question, most of the students tried to solve the question by considering that there were two blue and two red balls. Similar behaviors were also observed. Some others also chose five blue and five red balls when they calculating the probabilities. Some other students considered *x* number of blue and *x* number of red balls in their calculations. Students who did not use the number of balls in calculations tried to explain their answers with verbal justifications.

The conditional probability topic was not in the middle school curriculum. Therefore, this question was not asked to middle school students. Only the high school students answered this question. In addition, high school students did not learn this topic before.

Although students did not learn the conditional probability topic before, the number of students who gave correct answer to first part of the question was high in the pre-tests. In the pre-test, students preferred to give answers with verbal justifications. The answers of the students who answered that the probability of getting blue was low, they were considered as correct in the first part. Similarly, the answers like "the probabilities were same." were considered as correct in the second part. The Table 4.13 indicated the frequencies for the correct answers in the first part and the misconceptions in the second part.

Table 4.13 Frequencies of students' answers reflecting the misconceptions of conditional probability in the pre- and post-tests

Question in the PTI	Students' Responses	High Schools Teachers							
		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
		Pre	Post	Pre	Post	Pre	Post	Pre	Post
Question 7	Correct Answer (from the first part)	6	11	17	17	11	22	34	50
	Misconception (from the second part)	6	8	8	12	10	6	24	26

*: All names are psydonym.
Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of conditional probability counted only once.

As it was observed in the Table 4.13, there were 34 students who gave correct answers to this misconception in the pre-test. This was equal to 59.65 % of all high school students. Although they did not learn the conditional probability, they used their logic and found the correct answer. When considering the time-axis misconception in this question, the number of correct answer was important. The ratio between the number of students who fell into misconception in the second part and the number of students who gave correct answer in the first part was equal to 70.59 %. Therefore, almost three fouth of all students who gave correct answer in the first part did not differentiate the time elapse of picking the balls. They could not manage that the further event did not affect the preceding event.

In general, students who fell into this misconception gave the correct answer in the first part of the question in the pre-tests. Only one student from Anatolian high school who could not give correct answer in the first part fell into this misconception in the second part. From this point of view, all students who gave correct answer in the first part fell into this misconception in vocational high school. Similar situation was valid among science high school students. On the other hand, almost all students in Anatolian high school gave the correct answer in the first part. However, eight students among them fell into this misconception.

The justifications for the misconception were generally similar to each other in the pre-tests. They thought that the further event affected the preceding event similar to the first part of the question. Some of the justifications were as follows.

> If the second ball chosen is blue, then, the probability that the first ball chosen is blue is lower. Simply, since the balls are equally distributed in each urn, the probability of getting red is higher in the first selection (Pre-test administred to Doğan's classroom).
> If the second ball chosen is blue, the probability of getting blue ball in first selection is lower (Pre-test administred to Erdal's classroom).
> Let's say there are two white and two blue balls. If the second ball is blue, there will be two red and one blue ball in the first selection. Therefore, the probability of getting blue ball in the first selection is $\frac{1}{3}$ (Pre-test administred to Doğan's classroom).

After students received regular instruction, almost all students found the correct answer in the first part of the question. On the other hand, there was a slight increase in the number of students who fell into the misconception in the post-test.

Among the high school students, there were 50 students who found the correct answer in the first part of the post-test. This was equal to 84.75 %. Students who could not realize the interdependence of the preceding event from the second event stated that the probabilities are equal in both situations. The number of students who fell into the misconception increased to 26 students in the post-test. Although the number of students who fell into misconception increased with the correct answers in the first answer in the vocational high school, there was an inverse situation among students in science high school. The number of students who gave correct answer increased to 11 students in vocational high school, while it increased to 22 students in science high school in the post-test. However, the number of students who fell into the misconception increased to eight students in vocational high school and it decreased to six students in science high school. From this findings, it could be argued that regular instruction helped students to calculate the classical conditional probability problems in vocational and in science high schools. However, it did not helped students to realize the time axis and the interdependence of the preceding event in vocational high school, while students in science high school were successful in doing so.

Similar justifications were given in the post-test, too. However, they generally tried to use probability formula and calculate the probabilities. They still ignored the importance of the happening of the precedence of the events. They tried to calculate the probability by accepting that one of the blue balls is taken away. Only the science high school students were careful about the time axis situation. On the other hand, the regular instruction was not effective in vocational and Anatolian high schools.

4.7.2 Teachers’ Awareness and Teaching Practices for Conditional Probability

The last question in the PTI was only asked to high school teachers. They all mentioned about the misconception of *the time axis* in the interviews-2 with teachers.

Their explanations about the existence of this misconception were given in the quotations below.

> Students may experience difficulty because the color of second ball taken was given. So, they may experience in calculating the probability (Interview-2 with Cihan).
> Students need to know that the probability of further event happening did not affect that of the preceding one (Interview-2 with Doğan).
> Students may ignore that the second situation did not affect the first one (Interview-2 with Erdal).

All high school teachers stated that they could make students aware that the further events did not affect the preceding ones in the seventh question in the pre-tests. Doğan also stated that this misconception could affect their real life and make mistakes in their real lives. Moreover, Cihan stated that they should give examples from real lives instead of just using question-answer interaction between students and teachers in classrooms.

The topic of the conditional probability was only in the high school curriculum. Therefore, it could only be observed in the high school. In concept development phase, although they gave formal definitions from text books, teachers used shortcuts and keywords to determine whether the questions were related to conditional probability during the observations. All high school teachers gave emphasis on the statements such as “known to be”. Although there were many shortcuts for conditional probability, they did not help students to resolve time-axis situation because teachers chose the questions which were similar to those asked in the university entrance exams. The shortcuts observed in all high schools for conditional probability were as follows.

- If there are statements like “known to be”, it is conditional probability.
- The sample size of conditional probability is the sample size of event B.
- The expected elements of the conditional probability are the intersection of the events A and B.
- If the occurrence of the event A is dependent on the event B, it is conditional probability.

With the shortcuts given, students could determine whether the event was conditional or not during the observations. In addition, they could also determine the condition in the questions. These two situations were considered as students’ difficulties by high school teachers in the interview-1 with high school teachers. In fact, all teachers solved different questions related to conditional probability in classrooms. However, the key point in the misconception of conditional probability was the time-axis. If the preceding event was given and the further event was asked, there was no problem for both students and teachers. However, the inverse situation was not observed very much in the classrooms.

During the observations, there were four questions asked related to conditional probability in Cihan's and Doğan's classrooms. Although there was no question related to time-axis situation, there was only one question that might lead students to fall into this type of misconception. The question was as follows.

> There are two yellow and three red balls in the und I and three yellow and four red balls in the urn II. It is known that a ball taken is red, what is the probability that the urn is taken from urn I? (Observation-6 in Doğan's classroom).

Different than routine questions asked about conditional probability, the second event was given here. Both Doğan and his students experienced difficulty in solving this question in the lesson. Before solving in front of students, Doğan looked at the textbook for the solution. He did not explain the time of events happening. He directed students to the formula for conditional probability taught before. He did not do anything other than putting the given facts into the formula.

Erdal solved ten questions related to conditional probability during the observations. One was similar to Barış's questions above (only the numbers of balls were different). He directly used the formula and passed to the other questions.

Among the questions asked in Erdal's classroom, there were three similar questions which required a little more thinking. In fact, these questions were not about the time axis. It was possible that if students understood the logic behind the questions, they could solve the questions related to time-axis situations. The questions were as follows.

> A coin is thrown two times. It is known that one outcome is head. What is the probability of getting tail in the second outcome? (Observation-4 in Erdal's classroom)

> A die is thrown two times. It is known that one outcome is four. What is the probability of getting odd number in the second outcome? (Observation-4 in Erdal's classroom)
> There are five red and three white balls in an urn. It is known that a ball selected among two balls is red. What is the probability that the other ball is white? (Observation-5 in Erdal's classroom)

Observation findings indicated that Erdal showed the possible outcomes for the questions and solved questions by following the steps in the algorithm shown before. For example, he showed the sample size for the first question as {HH, HT, TH} and the set of expected elements as {HT, TH}. Then, he used general formula for the probability.

Overall, teachers' aim was to solve questions about conditional probability according to the university entrance exam. They did not give emphasis on different situations such as time-axis. In line with these findings, the pre- and post-test results indicated that occurrence of the time-axis probability stayed still among high school students. After students received regular instruction, the number of students who fell into this misconception was almost the same when compared to the pre-test results.

4.9 Theory of Intuitive Rules: The More of A – The More of B as an Intuitively-based Misconception

According to Stavy and Tirosh's (1999a; 1999b; 2000) theory, what is more in one quantity can be attributed to other quantities. Students can construct direct relation between two quantities which are compared. If one quantity increases, students can think that the other quantity also increases. The findings specific to this misconception were presented below.

4.8.1 Pre- and Post-Test Results

In the PTI, students' responses to the questions gave evidence about the existence of this type of misconception in the pre- and post-tests. What is more in one quantity or in one happening was attributed to the answers to the questions.

Many students were observed that they fell into the first misconception in Stavy and Tirosh's (1999a; 1999b; 2000) theory of intuitive rules. Students' answers were appropriate to the requirements of this type of misconceptions in five questions in the PTI. How this misconception occurred was explained as below.

In the first question, the numerical values of Merve's choice in the game were higher than those of Çiğdem and Hakan's choices. Therefore, students who fell into this type of misconception thought that Merve's probability of winning the game was higher than Çiğdem's and Hakan's probabilities of winning the game.

In the third question, there were new-born babies (300 babies) and there were more boys (200 boys) to be expected in the second experiment. Therefore, students thought that the probability of second experiment was higher than the first one.

In the fourth question, some students did not read the question well or they did not read the question at all. They only focused on the persons who got the cards instead of on the distributions. Here, they stated that Hüseyin has the great probability of winning the game, because he got the more cards in both distributions when compared to others' cards.

In the fifth question, students thought that 4-4 was higher than 4-3. Therefore, the first pair had greater probability to happen. They either added or multiplied the outcomes.

In the sixth question, students collected the number of heads in each player's rows. They stated that there were five heads in Ali's row; therefore, Ali's probability of winning the game was higher than the others' probabilities of winning the game. The Table 4.14 indicated the frequencies of the students who fell into this type of misconception in each question in the pre- and post-tests.

Table 4.14 Frequencies of students' answers reflecting the misconceptions of the more of A– the more of B in the pre- and post-tests

Questions in the PTI	Middle School Teachers						High School Teachers							
	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1st Question	10	14	14	9	24	23	3	2	0	0	0	0	3	2
3rd Question	3	1	4	3	7	4	1	0	4	0	4	1	9	1
4th Question	3	9	4	8	7	17	5	3	1	0	2	0	8	3
5th Question	5	4	5	7	10	11	2	1	0	0	1	0	3	1
6th Question	9	12	10	10	19	22	7	4	1	1	5	0	13	5

*: All namer are psydonyms.
Note: Numbers represent individual students as multiple answers by the same student reflecting the the misconception of the more of A – the more of B counted only once.

Considering pre- and post-test results, there was a considerable decrease among high school students in the number of students who fell into this misconception after they received regular instruction. On the other hand, the number of students who fell into this misconception in middle school increased slightly in the first and third questions, while there was slight increase in the fourth, fifth and sixth questions. Overall, there was an increase only in fourth question.

In general, middle school students were very prone to fall into this type of misconception, while high school students did not fall much into this type of misconception. Therefore, the misconception of the more of A – the more of B misconception was common among middle school students both in pre and post tests. According to the Table 4.14, there were total of 27 students (23.48 %) who fell into this misconception in the first question of the pre-test. Excessive numbers of students were from middle schools. There were 24 students (42.10 %) who fell into this misconception among middle schools. There were only three students who fell into this misconception in high schools. These three students were from vocational high school. There were no students in Anatolian or science high schools in the pre-test.

In the first question, while there were only two students who fell into this misconception in vocational high school, there were 23 students in middle schools in the post-test. This number was equal to 20 % for all of the middle school students.

For the third question, there total of 17 students who fell into this misconception in the pre-test. This was equal to 17.78 %. Among them, 7 students were from middle school and 10 students from high schools. Since Anatolian and science high school students tried to solve the question in formal way by using formulas, they gave incorrect answers. They found that the numerical value of the probability in the second experiment was higher than that in the first experiment. The number of students who fell into this misconception decreased after students' received regular instruction. The reason why it happened was that more students fell into the same of A – the same of B or sample size effect misconception. There were four students in middle schools and one student in high schools.

There were 15 students (13.04 %) who fell into this misconception in fourth question of the pre-test. Again, most of the students were from middle schools and vocational high schools in this question. There were seven students (12.28 %) in middle schools and eight students in high schools in the pre-test. Among the students who fell into this misconception in high schools, five of them were from vocational high school.

For the fourth question, there were only three students who fell into this misconception in high schools in the post-test. These students were, again, from vocational high school. There were 17 students out of total of 20 students fell into this misconception in middle schools in the post-test. They did not focus on the distributions. Instead, they focused more on the total cards for each player. They gave their answers accordingly.

For the fifth question, most of the students who fell into this misconception were from middle schools in the pre-test. There were total of 13 (11.30 %) students who fell into this misconception. Among them only three were from high schools. In the post-test, there were 12 students who fell into this misconception. Among them 11 students were from middle schools, which was equal to 9.32 %. Students gave their answers by basing their justifications on the chance.

For the sixth question, there were 32 students (27.83 %) who fell into this misconception in the pre-test. Most of them were from middle schools. Among the high school students, seven out of 12 students were from vocational high school. In this question, while middle school students and vocational high school students were more focused on the number of heads in each row, science high school students explained their justifications by stating the possible biasness of the dice. There was only one student who fell into this misconception in Anatolian high school in the pre-test.

The number of students who fell into this misconception increased when comparing it with the pre-test results. There were 27 students who fell into this misconception. This was equal to 22.89 % of all students. Among them, 22 of them were from middle schools. This was 37.39 % of all middle school students. While high school students did not counted the number of heads and tails in each row, middle school students did.

After students got regular instructions, middle school students mentioned about the theoretical and experimental probability especially in third and fifth questions in the post-tests. Even four students stated two answers for one question. According to them, the answer was something theoretically but it was something else experimentally.

In the third question, some of the students mentioned that the experimental probability got closer to theoretical probability in the post-tests, while high school students tried to solve the question by using formulas and by doing calculations. Similar answers were also given in the fifth question. In addition, some students in middle school based their answers to chance factor. They stated that if the player was lucky, s/he got 4-4, if not it was impossible for him/her to get 4-4.

What was different between middle and high school students was that middle school students were more inclined to fell into the more of A – the more of B misconception in general. Students in high schools did not fell more into this misconception. Among the high schools students, vocational high school students' answers were parallel to those in middle schools. In general, Anatolian and science high school students did not fell into this misconception in the post-tests. For those who fell into this misconception in Anatolian and science high schools, students based their answers to formal justifications and calculations.

According to the test results, regular instruction had slight effect on resolving students' misconception of the more of A – the more of B. In general, this misconception was dominant among middle school students. On the other hand, this misconception was not observed very much among high school students.

4.8.2 Teachers' Awareness and Teaching Practices for the More of A – the More of B

This misconception appeared among students in five questions. According to the findings from interview-2 with teachers, the Table 4.15 below presents the misconceptions appeared in the test results and teachers' awareness about this misconception.

Table 4.15 Teachers' awareness about students' misconception of the more of A – the more of B in the PTI

Questions in PTI	Middle School Teachers		High School Teachers		
	Ahmet*	Barış	Cihan	Doğan	Erdal
1st Question			√**		
3rd Question					
4th Question					
5th Question		√			
6th Question	√	√	√	√	

*: All names are psydonyms.
**: "√" indicates that the teacher expected to observe the intuitively-based misconception in students' responses to questions in the PTI.

Although this misconception appeared generally among middle school students, neither middle nor high school teachers mentioned about the possibility of occurrence of this misconception in the interviews-2 with teachers. For example, the more of A – the more of B misconception appeared generally among middle school students and vocational high school students in the first question, there were only one teacher (Cihan) who stated that students might fall into this type of misconception

Similarly, this misconception generally observed among middle school students in the third and fourth questions in the pre- and post-tests. However, neither middle school teachers nor the other teachers mentioned about this misconception in the interviews-2 with teachers.

Similarly, only one middle school teacher stated that students might have this kind of misconception in the fifth question in the interview-2. According to him, students might think that the probability of getting 4-4 had higher probability.

Different than the other questions, middle school teachers and two high school teachers stated that students might fall into this misconception in the interviews-2. Some of the justifications were as follows.

> HTHHHH distribution is steadier. There are more heads in it. Students may say either the probability is higher because of more heads or the probability is lesser because of steadiness (Interview-2 with Barış).
> Students may think that getting more head is more advantageous (Interview-2 with Cihan).

During the observations, all teachers showed some properties of probabilities in line with the set topic. Related to this misconception, there were two main properties. Teachers gave examples for explaining these properties. For the first property, the numbers of elements in A and B sets could be considered as the number of expected elements in the probability. Student might confuse the number of expected elements and the sample size, so they might fall into misconception. However, all teachers explained the properties with examples and emphasized that the elements in the sets A and B are the expected elements in the probabilities of the events A and B. These properties and the examples were given in the Table 4.16 below.

Table 4.16 Properties in the probability and the examples given.

Properties	Examples
If $A \subset B$, then, $P(A) \leq P(B)$.	(This was explained from the number of elements in the sets the events A and B) (Observaton-1 in Cihan's classroom)
$P(A')=1-P(A)$	In sets, addition of a set and its complement is equal to 1. Similarly, probability of an event and of its inverse is equal to 1. So, we can find if one of them is given. For example, if the probability of getting five is $\frac{1}{6}$, then, we can say that the probability of not getting five or of getting 1,2,3,4,6 is equal to $1-\frac{1}{6}=\frac{5}{6}$ (Observation-1 in Doğan's classroom).

In the classroom observations, there was no question in the high schools that were related to this misconception. In fact, this misconception was related to the comparison of the events. The questions asked in the university entrance exams were not asking the numerical values of the events. Since the high school students were expected to find the numerical values of the probabilities asked in the questions in line with the questions in the university entrance exam, teachers did not emphasis on the questions that required comparisons. Moreover, the high school curriculum did not include objectives for comparisons, high school teachers were not curious about students' possible misconception of the more of A – the more of B misconception during the observations.

On the other hand, middle school curriculum included comparison of the probabilities of the events. The existence of the theoretical, experimental, and subjective events, middle school teachers exposed students to the comparison of the events. Inability to make comparison between the probabilities of the events was also considered as one of the difficulties that students experience by middle school teachers in the interviews-1 with teachers.

Among middle school teachers, Barış solved two questions that are directly related to this misconception. These questions were as follow.

> There are two blue and three red balls in an urn. Another urn includes eight blue and 12 red balls. In which urn is the probability of choosing blue higher? (Observation-2 in Barış's classroom).
> There are three students who wear glasses among 20 students in 8A and six who wear glasses among 40 students in 8B. In which classroom is the probability of choosing a student who wears glasses higher? (Observation-2 in Barış's classroom)

When asked, students fell into this misconception. After teacher suggested students to calculate the probabilities, students found the correct answer. At the end, Barış explained the situation and solved the misconception during the observations. He asked the other questions in different class hour. However, students were careful about the trick. The conversations between Barış and students were as follow.

Student 1	: For me, it is the second urn.
Barış	: Why?
Student 1	: Because there are more blue balls.
Barış	: Use your pencils. Is there any other answer?
Student 2	: They are same.
Barış	: Why are they the same?
Student 2	: If we convert it to percentage, they both are 40 %.
Barış	: Numbers should not deceive you. I can say the second has higher probability due to eight balls. Or I can say there are less blue balls in the first one, so, the probability is also less.

> However, we need to solve it with pencil. Pencil never lies. No matter whether there are 200 blue and 300 red balls, their probabilities are theoretically the same (Observation-2 in Barış's classroom).

In teaching experimental probability, teachers might lead students to this misconception. For example, Barış explained the difference between the experimental and theoretical probabilities. He wrote the results of the experiments and explained the experimental probability as follow.

> The probability of getting head is 0.2 in 20 trials, 0.4275 in 32 trials, and 0.48 in 50 trials. What happened? As the number of trials increased, the probabilities also increased. We call them experiments and the probability is called as experimental probability (Observation-5 in Barış's classroom).

In such situations, the values of the probabilities were increasing gradually. It might result in students' mind that if the number of trials increased, the probabilities also increased. However, Barış stated at the end that the as the number of trials increased, the value of the probability got closer to the value of the theoretical probability during the observations. It was ½ for the coin. Ahmet solved the question below in the lesson. It was also similar to the question in Barış's classroom.

> Aslı is doing an experiment. A dice is thrown five times, 25 times, 75 times, and 200 times. Then, the results are recorded. In which experiment is the probability of getting five closer theoretically? (Observation-5 in Ahmet's classroom)

Similarly, the students could think that as the number of the trials increased, the probability got closer to that of getting five. In fact, the correct answer was 200 trials due to rule exposed by both Ahmet and Barış. Observation findings indicated that they both stated that excessive number of experiments turned into theoretical probability.

Indirectly, some questions might lead to this misconception in students' minds. However, the focus of the questions was not about the comparison, instead, it was finding the numerical value of the probability of the events. These questions were observed in both middle and high school classrooms. However, the misconception was not observed. Two of the examples asked were as follows.

> There are three oranges and six apples in a basket. Another basket includes five oranges and two apples. One fruit is taken from each basket. What is the probability that both fruits are apple? (Observation-4 in Cihan's classroom)
> There are 120 animals in a farm 20 of which are cows. Another farm includes 40 cows among 200 animals. One animal from each farm is chosen. What is the probability that the chosen animals are not cow? (Observation-5 in Ahmet's classroom)

In such question, students might consider the probabilities of choosing apple or cow by just looking at the numbers. According to the numbers, they might compare the probabilities. However, the focuses in the question were not on the comparison. Therefore, students tried to find the numerical values of the sample size, the expected elements, and the probability. At this point, finding sample size and the set of expected elements were considered as students' main difficulties in probability by all teachers in the interviews-1 with teachers. However, in case they had this misconception in mind about the comparisons of the probabilities, it was not observed. Teachers showed the formulas and formal solution methods in the questions.

Overall, this misconception was observed excessively among middle school students. Although this misconception was generally observed in middle schools and teachers tried to resolve it when observed, the difference between the numbers of the students who fell into this misconceptions in the pre- and post- tests was very slight.

4.9 Theory of Intuitive Rules: The Same of A – The Same of B as an Intuitively-based Misconception

Second intuitive rule in Tirosh and Stavy's (1999a; 1999b) theory is called as the same of A – the same of B. The logic behind this misconception is similar to the first one. In this misconception, there are two objects, quantities or phenomena. If there is equality between these objects or quantities, students might think that the results or the phenomena are also the same.

4.9.1 Pre- and Post-Test Results

In fact, there was no specific question asked for this misconception. However, third question was directly related to it. In addition, there were some evidences in students' responses to the three questions in the PTI that students fell into this type of misconception.

According to students' responses to the questions in PTI, there were three questions that gave evidence for the existence of this type of misconception in the pre- and post-tests. Among them, the third question was directly related to it. Although this question was asked to determine students' misconception of the sample size effect, the same logic was valid for this misconception. In this question, expecting at least two boys out of three babies has the same ratio for expecting at least two hundred boys out of three hundred babies. Since students found the ratio of $\frac{2}{3}$ for both events, they stated that the probabilities for each event were equal to each other in the test results.

In the fourth question, students did not read the question well and they directly looked at the players and the number of their cards. According to players' cards, they stated that Ayşe and Fatma got the same number of cards in each distribution. Therefore, students' stated that Ayşe and Fatma had same probability of getting four cards.

In the sixth question, students looked at the number of heads or tails in each row. Then, they stated that Ayşe and Ahmet had the same probability of winning the game, since they had four heads in their rows. In addition, one student stated that Ayşe and Ali had the same probability due to the number of tails in their rows.

Table 4.17 Frequencies of students' answers reflecting the misconceptions of the same of A – the same of B in the pre- and post-tests

Questions in the PTI	Middle School Teachers						High School Teachers							
	Ahmet* (n=22)		Barış (n=37)		Total (n=59)		Cihan (n=17)		Doğan (n=21)		Erdal (n=21)		Total (n=59)	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
3rd Question	18	20	16	11	34	31	14	13	11	13	10	6	35	32
4th Question	2	0	1	1	3	1	1	1	0	0	0	0	1	1
6th Question	2	1	2	0	4	1	0	0	1	6	0	3	1	9

*: All namer are psydonyms.

Note: Numbers represent individual students as multiple answers by the same student reflecting the misconception of the same of A – the same of B counted only once.

For the third question, the same logic was valid with the sample size effect. As it was seen in the frequency Table 4.17, the misconception was observed in 8th and 11th grade students. There were 69 students who fell into this misconception in the pre-test. Among them, 34 students were from middle schools and 35 students were from high schools.

In the fourth and fifth questions, the number of the students who fell into this misconception was very rare in both pre- and post tests. However, students in middle schools were more prone to fall into this misconception. In both questions, there were students in each classroom in middle schools; however, there were only one student in high schools for each question.

After students received regular instruction, the number of students in middle schools decreased in the fourth and sixth question. Students' misconception transformed to the more of A – the more of B misconception. For high school students, there was only one student for the fourth question. This student was from vocational high school.

Interestingly, the number of students in Anatolian and science high schools increased sharply in the post-test. Students who fell into this misconception were asked about why they answered in such way. One student from Anatolian high school stated that "We are get used to multiple choice format. First of all, we are reading the last part in the question. Then, if necessary, we are reading the explanation in the question" (from unstructured interview). Therefore, it was observed that students did not pay attention to the explanations in the question. Another statement was that "We are trying to solve it with formula. We need to be quick" (from unstructured interview). With this statement, it was understood that students considered themselves as they were ready to solve any question after getting the instruction.

4.9.2 Teachers' Awareness and Teaching Practices for the Same of A – the Same of B

In the interview-2 with teachers, they were asked about the possible misconceptions that students might fall into. In general, teachers stated which misconception the questions were aiming to determine among students. This misconception appeared in three questions. The Table 4.18 below presents the misconceptions appeared in the test results and teachers' awareness about this misconception for each question.

Table 4.18 Teachers' awarenesses about students' misconception of "the same – the same" in the PTI

Questions in PTI	Middle School Teachers		High School Teachers		
	Ahmet*	Barış	Cihan	Doğan	Erdal
3rd Question	√**	√	√	√	√
4th Question					
6th Question			√		

*: All names are psydonyms.
**: "√" indicates that the teacher expected to observe the intuitively-based misconception in students' responses to questions in the PTI.

In the third question, there was a same logic for the sample size effect and the same – the same misconceptions. For the sample size effect, representation of the numbers 200 out of 300 and 2 out of 3 are similar, therefore, students might think that the probabilities for both situations were equal. On the other hand, the misconception of the same – the same can also appear among students due to the equivalence of the ratios.

For the fourth question, on the other hand, teachers did not mention about the misconception in the interviews-2 with teachers. This misconception appeared especially among middle school students. There were also a few students in vocational high school. However, neither middle school teachers nor Cihan or other high school teachers mentioned about such misconceptions. Teachers generally focused on the outcome approach. For the last question, only Cihan mentioned about the possibility of appearance of the same of A– the same of B misconception among students in the interview-2.

Similar to the misconception of the more of A – the more of B, this misconception was also related to the comparison of the events. During the observations, this misconception was observed slightly in high school due to the existence of the university entrance exam and the high school curriculum, which did not include objectives related to comparison. However, the middle school curriculum included the topics of the experimental, theoretical, and subjective probabilities. This misconception was generally observed in middle school classrooms.

In teaching experimental probability, Ahmet gave contraversy situation to distinguish the experimental and the theoretical probabilities during the observations. The question and the conversation between students and teacher were as follows.

Burak is throwing a coin 70 times. He gets 30 head and 40 tails. What is the probability of getting a head experimentally?

Student : $\frac{1}{2}$

Ahmet : But theoretically it is $\frac{1}{2}$. Is'nt it? It was half and half.

Student : It is asking experimentally (Observation-6 in Ahmet's classroom).

In case it was asking the theoretical probability, students might be affected by the numbers and give the same. Here, Ahmet showed the difference.

Again in Ahmet's classroom, this misconception was observed in the questions asked below. In this question students looked at the numerical values and gave answers according to these numbers.

There are *x* red, y white balls in a box. It is known that the probability of choosing red ball from the box is $\frac{1}{3}$, how many *x*'s of y are there in the box?

Student : Three times.

Ahmet : (With anger) First, try to solve it. Don't answer it without hassle (Observation-5 in Ahmet's classroom).

After discoursing students, he showed the formal solution by using general formula and found the answer. He also asked whether students understood the solution. Similar situation was observed in the question below in Ahmet's classroom.

The probability of Ali's success in an exam is three times higher than his failure. What is the probability in percentage of Ali's failure in the exam?

Student : 35 % (Observation-5 in Ahmet's classroom).

Student's answer was according to the ratios between the probabilities of Ahmet's success and failure. Again, Ahmet showed his anger and expected students to solve the question on their notebooks. Then, he solved question by indicating that the Ahmet's success as *x* and his failure as *y*. He wrote as $\frac{x}{y}=3$. Then, he wrote $\frac{y}{x+y}=\frac{y}{3y+y}=\frac{1}{4}=\frac{25}{100}$. He also explained the solutions two times. However, he did not try to resolve student's misconception.

The following question also resulted in this type of misconception. It was observed in Ahmet's classroom, again. The question was as follows.

> Nur is taking a multiple-choice exam. There are four choices for each question in 100-question-exam. What is the probability that Nur chooses the correct answers for all questions? (Observation-4 in Ahmet's classroom)

Students' answers were $\frac{4}{100}$, $\frac{1}{25}$, and 50 %. First two answers were given according to the ratio between the number of choices in each question and the number of questions. In fact, all teachers indicated that students experience difficulty in finding the sample size and the set of expected elements. First of all, Ahmet explained the mistakes in students' justifications. He used visualization while finding the probability of marking correct answers in the first two questions. Then, he constructed a pattern to find the correct answer. The visualization was as follows.

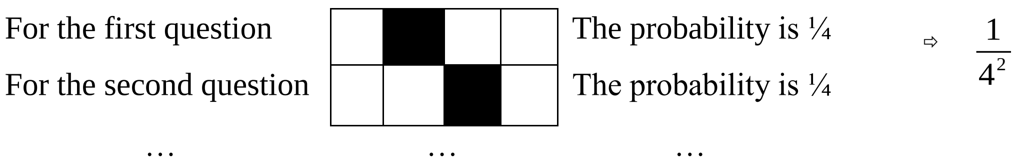


Figure 4.3 Visualization for the solution of the exam question (Observation-4 in Ahmet's classroom)

> Ahmet : There are four choices in the first question and second questions. So, there is one correct choice for each question. We can say that the correct answers can be AD, AB, …, AC. If we combine 100 questions, the correct answer becomes $\frac{1}{4^{100}}$ (Observation-4 in Ahmet's classroom).

In high schools, this misconception appeared in the infinite probabilities topic during the observations. Cihan asked the following question.

> What is the probability of choosing a point from a circle with the radius of r that is closer to its center than its perimeter? (Observation-8 in Cihan's classroom)

In this question, teacher drew two circles with the radius of r and $\frac{r}{2}$. Then, he showed that the expected area is the inside of the inner circle. A few students stated that the probability is $\frac{1}{2}$. However, Cihan directed students to the formula and showed that the probability is equal to $\frac{1}{4}$. At the end, he warned students to use the general formula in solving questions. In this question, students could not relate the probability with geometry. This situation was considered as one of the students' difficulties in probability by all teachers expect for Ahmet in the interviews-1 with teachers.

Similar to the situation in the more of A – more of B misconception, students might think that the probabilities were the same in some questions. The questions including two urns with the same number of same colored balls might mislead student and result in misconception. However, this effect was indirect, because the focuses in such questions were finding the numerical values of the probabilities. Therefore, this misconception was not observed in such questions. There was one example for this type of misconception below.

> There are two urns including four white and three black balls. One ball is taken from the first urn and released into the second one. Then, a ball is taken from the second urn and related into the first one. What is the probability that the urns take the same situation in the first case? (Observation-6 in Cihan's classroom)

Overall, teaching practices to resolve this misconception were generally observed in middle schools. In fact, this misconception had the same logic with the misconception of the sample size effect in the third question of the PTI. There were lots of students who fell into this misconception in the third question. On the other hand, this misconception was not observed much in the other questions both in middle and high schools.

4.10 Teachers' Opinions regarding Students' Difficulties and Misconceptions in Probability

In this part of the study, general considerations related to students' difficulties and misconceptions in probability were presented. The general considerations included teachers' knowledge and awareness about the reasons for students' difficulties and misconceptions, the activities to determine their misconceptions, and the material use. This part of the study was answering the fourth and fifth research questions related to teachers' awareness about the misconceptions, reasons for misconceptions, and possible instructional methods to resolve them. The data gathered from interviews-1 with teachers and related classroom observations were presented in this part of the study.

4.10.1 Teachers' Opinions about Reasons for Students' Difficulties and Misconceptions in Probability

Findings gathered from interviews-1 with teachers indicated that there were various reasons for difficulties and misconceptions in probability. Although these reasons were general reasons for students' difficulties and misconceptions in probability, they were also directly related to the reasons for intuitively-based misconceptions in probability. These reasons and whether the teachers agree with such reasons were given in Table 4.19.

Table 4.19 General reasons for students' difficulties in probability from teachers' point of views

Reasons for Intuitively-based Misconceptions	Middle School Teachers			High School Teachers	
	Ahmet*	Barış	Cihan	Doğan	Erdal
Insufficiency in readiness	√**	√	√	√	√
Insufficiency in reading comprehension	√	√	√	√	√
Rote memorization	√		√	√	√
Unable to imagine	√	√	√	√	√
Unable to relate with daily life			√		
Low level of students' understanding	√	√	√		√
Necessity of thinking				√	√
Not being open to interpretation					√
Unable to synthase the facts	√				√
Unable to construct patterns	√	√			
High school or university entrance exams	√	√	√	√	√
Carelessness	√				
Not studying regularly			√		
Insufficient course book		√			
Unable to understand the logic of the probability		√		√	√
Fear	v	√		√	√

*: All names are pseudonyms.

**: "√" indicates that the teacher considered the issue as a reason for students' difficulty in probability

Although teachers mentioned about different reasons for students' difficulties and misconceptions, some of them coincided each other's assertions in the interviews-1 with teachers. Among the reasons, both middle and high school teachers stated in the interviews-1 that "insufficiency in readiness", "unable to think abstractly", "unable to imagine the situation", and "existence of high school/university entrance exams" were the reasons for students' difficulties and misconceptions in probability. Although Doğan and Erdal did not see "the meeting the readiness" as necessary in classroom activities, they also stressed this issue in the interviews-1 with teachers. At this point, the importance of the students' pre-knowledge appeared while investigating the reasons for students' difficulties and misconceptions in probability.

In the interviews-1, teachers' stated that one of the reasons for students' intuitively-based misconceptions was that students did not have enough pre-knowledge to learn probability. They might base their knowledge on incorrect knowledge. Based on this issue, Ahmet, Doğan and Erdal did not specifically indicate which knowledge was necessary for learning probability in the interviews-1. However, all said that they should know factorial concept, permutation and combination subject in the interview-1. Since these subjects were taught before going through probability, all teachers considered these subjects as the necessary pre-knowledge for probability. On the other hand, Barış and Cihan specifically stated some necessary pre-knowledge in the interviews-1. Barış indicated that students should know fractions, simplification, percentage, sets and numbers, while Cihan stated that they should know operations and rational numbers. In addition, Ahmet and Barış mentioned that some topics in probability were taught in 6^{th} and 7^{th} grades. What Barış said about the students' pre-knowledge was important in the interview-1. Barış stated that

> Mathematics is ongoing lesson which the subjects are built on one another. Therefore, they should be aware of some subjects before beginning to the probability and also to other subject (Interview-1 with Barış).

Interview findings indicated that the main difference between middle and high school teachers was that middle school teachers considered the previously taught probability topics as the pre-knowledge necessary for students. On the other hand, one middle school teacher and two high school teachers did not mentioned about the necessary pre-knowledge in the interviews-1.

During the observations, middle school teachers and Cihan experienced students' deficiencies on fractions, and simplifications while all teachers related the probability with sets. They briefly explained the necessary parts of these subjects for solving the questions during the lessons. Although they did not mentioned the necessary pre-knowledge that students need to know in the interviews-1, all teacher gave brief explanations for non-probability concepts that appeared in the teaching processes during the observations.

Among the reasons for students' difficulties and misconceptions in probability, high school teachers gave emphasis on "rote memorization", "unable to relate with daily life", "necessity of thinking", "not being open to the interpretation", and "unable to understand the logic of the probability" in the interviews-1. On the other hand, middle school teachers stressed the existence of "unable to construct patterns", "careless" in solving questions and "insufficient course book." Among them, one middle school teacher also mentioned about "the rote memorization" and "unable to understand the logic of the probability" as reasons for students' difficulties in the interview-1. There were only one high school teacher mentioning "not being open to interpretation" in the interview-1.

There were also some reasons for students' difficulties which were stated by both middle and high school teachers. For example, "insufficiency in reading comprehension" and "unable to construct facts" were stated by a middle school teacher and a high school teacher in the interviews-1. There were only one high school teacher stating "not studying regularly" as a reason for students' difficulties and misconceptions.

All teachers indicated that probability subject include all subjects inside in the interviews-1. Therefore, it makes the probability subject harder, at least from students' eyes. All teachers expect Cihan indicated that one of the reasons which create prejudgment and fear to mathematics or probability among students was that the mathematics was built on previous knowledge. Therefore, students were afraid of this property of the mathematics.

From the findings above, high school teachers gave more emphasis on thinking. On the other hand, middle school teachers were more interested in patterns, attention, and materials in learning probability.

4.10.2 Teachers' Opinions about How to Determine Students' Understanding of Probability

Another key issue for resolving students' intuitively-based misconception is the awareness of students' deficiencies in probability subject. This part also gives answers to the research questions related to teachers' awareness about students' understandings. Findings gathered from interviews-1 indicated that activities done differed from teachers to teachers and from school type to another. Although teachers' mentioned different methods to determine students' deficiencies in the interviews, the practice was different than what they said during the observations. The ways of determining students' deficiencies in probability was as follows.

Table 4.20 How to determine students' understandings from teachers' point of views

Characteristics	Middle School Teachers			High School Teachers	
	Ahmet*	Barış	Cihan	Doğan	Erdal
From students' responses to questions	√**	√	√	√	√
Expecting students' own definitions and explanation				√	
Conducting diagnostic tests	√	√	√	√	
Conducting formative tests		√		√	
Conducting Summative tests		√			
Asking students' understanding					√
Students' attendance to lesson	√	√	√	√	

*: All names are pseudonyms.

**: "√" indicates that teacher considered the issue as a way to determine students' understandings

First of all, the general approach to determine students' deficiencies was "students' responses to teachers' questions" in the interviews-1 with teachers. All teachers stated that they were using this method in the interviews. For example, Doğan stated what he did in a regular class hour for determining the deficiencies by this method.

> I'm directly going through the questions. I'm continuing with the questions of "what do you think about this question?", "For you, what is asked in this question?" or "what is expected?" According to students' responses, I'm

> trying to determine the problems and trying to resolve them by concentrating on these problems (Interview-1 with Doğan).

Observation findings indicated that Cihan and Doğan really used this ways. On the other hand, middle school teachers and Cihan generally showed the key points on the questions. Another category similar to this one is "asking students' understanding" found in the interviews-1 with teachers. In fact, only Erdal mentioned about this issue. However, all teachers used this method in practice. Here, a problem appeared in the observations. Although some students did not understand the solution methods of the questions, they even said that they understood.

In the interviews, Doğan stated that he expected students to state their own definitions and explanations for the concepts and solutions. In practice, Doğan and Cihan used this method. On the other hand, neither middle school teachers nor Erdal used it. To support this idea, Doğan explained the reason why he expects them from students.

> First of all, students attend the lesson not just physically but also mentally. I believe that students should digest (comprehensively understand) the lesson. So they try to find and construct their own sentences. I think that students' knowledge is not in depth while they were taught the lesson. If they do so, they begin to understand the lesson. If there exists a problem, it appears while students are constructing their own sentences (Interview-1 with Doğan).

Interview findings indicated that all teachers except for Erdal stated that they use diagnostic tests for determine students' deficiencies in their previous experiences. In addition, Barış and Doğan mentioned about the formative tests and Barış also said that he uses summative test in the interviews. Summative test means the test administred at the end of teaching probability to see students' understanding in this subject. During the observations, however, nobody used diagnostic test during the teaching processes in order to determine students' deficiencies. Cihan stated that the importance of conducting diagnostic test, but he indicated that he did not use it in his previous experiences. From the Table 4.20, interview findings indicated that middle school teachers were more prone to use tests to determine the deficiencies. Here, Barış stated the reason why they mentioned about the tests.

> The textbooks include pre-tests before each subject. … They are very beneficial for us. Whether students are ready or not? They are published as

> preparation. They are giving pre-information about the subject taught (Interview-1 with Barış).

Observation findings indicated that Barış really used this test in the beginning. On the other hand, Ahmet skipped it. For summative tests, Barış distributed a test to all students in the last lesson and solve the questions with students. Cihan and Erdal solved mixed questions in the last lessons with students. Barış distributed a test as homework. Ahmet did not do anything. Therefore, high school teachers used summative tests and tried to eliminate the misconceptions. On the other hand, middle school teachers did not use the summative tests for determining students' deficiencies.

Lastly, all teachers except Erdal stated that they determined the deficiencies according to students' attendance to lessons in the interviews-1. The number of students who wanted to solve the questions was one indicator for students' understanding for him. According to the number of students, they could determine whether they understood the topics in probability or not.

4.10.3 Teachers' Opinions about the Use of Material and Resources

From Stohl's (2005) quotation, how to use the materials and resources in the lessons was considered as the knowledge that teachers need to know. Therefore, this part gives answers to the research questions related to teachers' awarenesses. This part includes teachers' opinions about the use of materials and resources in their lesson. The findings gathered from interviews-1 with teachers and classroom observations were presented in this part of the study. It is not enough to indicate the materials and resources. However, the reasons that teachers' proposed for material and resource usage give implications for their knowledge of the ways to resolve misconceptions.

4.21 Teachers' opinions about necessary materials and resources during teaching probability

Materials used	Middle School Teachers		High School Teachers		
	Ahmet*	Barış	Cihan	Doğan	Erdal
Course Books	√**	√	√	√	√
Test Books	√	√	√	√	√
Visual Materials	√	√		√	√
Teachers' Own Questions			√	√	

*: All names are pseudonyms.

**: "√" indicates that the teacher considered the issue as a material that he uses in his classroom

In general, interview findings indicated that all teachers use course books and test books as resources for teaching probability. However, usage of course book is very limited for high school teachers during the observations. Teachers stated the reasons why the usage of course book was limited. Two high school teachers (Cihan and Erdal) complained about the redundancy of verbal expressions and explanations, the limited number of questions and question types, and unnecessary knowledge including proofs of theorems. All high school teachers also stated that the course book was not appropriate for the university entrance exam in the interviews-1. They indicated that they merely use the questions in course book during their instruction in the interview-1. One interesting point was that Cihan stated that the course book was appropriate for higher-level students including Anatolian and science high school in the interview-1. However, both science and Anatolian teachers (Doğan and Erdal) asserted the inverse statement. Ahmet complained about the limited content. He also asserted that the course book sometimes did not give necessary formula for the expressions. Among the teachers, Barış stated that there was a gradual improvement in course book. He stated that the concepts were simplified and the number of question types was increased in the interview-1.

They also stated the reasons why they prefer to use supplementary books in the interview-1. The supplementary books include several questions with different types, explanations with only necessary knowledge, and they are appropriate for the high school and university entrance exams. In addition, Doğan and Erdal (high school teachers) asserted that the test books give necessary formulas and shortcuts for solving questions in the interviews-1. Moreover, Erdal also stated that they were prepared specifically for each grade level.

Observation findings indicated that all middle school teachers were stick on the course books, while all high school teachers used supplementary book as their teaching material. Cihan and Doğan followed only one supplementary book. On the other hand, Erdal used two different supplementary books. He was choosing questions for each topic in the probability subject and solved them.

All teachers except Cihan stated that they used simple materials for visualizing the events in the interviews-1. The materials used are dice, coin, and boxes with marbles. Ahmet stated why he uses materials.

> I generally use materials to visualize the situation and to concretize the topics. For example, I'm bringing coin and throwing it. Sometimes, I bring dice and see what comes after throwing it. My purpose is to show what the dice and coin looks like. I'm trying to visualize the situations (Interview-1 with Ahmet).

During the observations, only Barış brought different materials and tried it in the classroom. He brought urn with marbles of different colors. He also brought dice and played with students. He asked students the probability of getting head after throwing coins. On the other hand, Ahmet only used coins. High school teachers did not use any materials to visualize the situations. Although Doğan and Erdal had smart board provided in the classroom, they did not use it.

Lastly, two high school teachers stated that they prepare their own questions for their students in the interviews-1. They stated that they tried to prepare the questions according to their students' levels. During the observations, all teachers used course or supplementary books. It was also observed that they did not ask their own questions.

CHAPTER 5

CONCLUSIONS, DISCUSSIONS, AND RECOMMENDATIONS

This chapter includes the discussion of the findings of the study. In addition, the important points in the study are given in the conclusion part.

5.1 The Discussion and the Conclusion of the Findings

This part includes the discussions of the misconceptions appeared among middle and high school students and the effect of the regular instruction over students' intuitively-based misconceptions. In addition, the teachers' awareness of the issues related to intuitively-based misconceptions and the teachers' teaching practices in order to resolve students' intuitively-based misconceptions were also discussed in this chapter. Instead of discussion teachers' awareness and teaching practices specific to each intuitively-based misconception, the general discussion was presented in this part of the study.

5.1.1 The Effect of Regular Instruction to Resolve Students' Intuitively-based Misconceptions

The first research question in the present study was about what type of intuitively-based misconceptions 8^{th} and 11^{th} grade students have. For this research question, possible intuitively-based misconceptions were investigated among 8^{th} and 11^{th} grades students. The results of the study showed that both middle and high school students had intuitively-based misconceptions in probability subject. Among the misconceptions, the findings indicated that students had the misconceptions of availability and representativeness heuristics, simple and compound events, conjunction fallacy, and conditional probability. In fact, the questions in the PTI were asked in order to determine whether these misconceptions existed among middle and high school students. There were three types of representativeness heuristics which were the positively and negatively recency effect, the sample size effect, and outcome approach. These misconceptions were also observed among students. In addition to these misconceptions, two more misconceptions also appeared. These were related to the Tirosh and Stavy's (1999a; 1999b; 2000) theory of intuitive rules; the misconceptions of the more of A – the more of B and the same of A – the same of B.

The pre- and post-test findings of the study indicated that teachers' regular instructions both in middle and high schools did not effectively resolve students' intuitively-based misconceptions. For example, there were very slight differences between the misconceptions of availability heuristics and sample size effect, and for the intuitive rules. On the other hand, there was slight decrease in the misconceptions of positively and negatively recency effect and conjunction fallacy both in middle and high school students. Moreover, there was slight increase for the misconception of the simple and compound events. One interesting finding was that students' misconception of outcome approach increased sharply after students get regular instruction. Overall, there was an ignorable change for 8^{th} and 11^{th} grade students' misconceptions between before and after receiving regular instruction.

Beginning with availability heuristics, there was slight difference between pre- and post-test results among middle school students. On the other hand, the number of high school students who fell into this misconception decreased sharply in the post-test. This situation can be attributed to many occasions observed in the classrooms. Especially, high school teachers mentioned about the chance games and independence of the events. They also solved many questions related to independent events. On the other hand, middle school teachers used textbook definitions and did not provide students with real life situations about the independence of the events. They also focused on routine questions provided in the textbooks and skipped to the other topics in probability. Another reason could be that high school teachers directed students to use general formula for probability. However, middle school students used the knowledge learnt pragmatically in the post-test to explain their incorrect intuitions (Evans, 2006).

Similar situation was observed for the misconception of negatively and positively recency effects. Again, there was slight change among the number of middle school students and sharp decrease among that of high school students for these misconceptions. In fact, the middle school curriculum included experimental probability, therefore, students were provided with questions to resolve such kind of misconceptions. However, middle school teachers generally expected from students to memorize the rules of this topic. In the post-tests, students' answers included pieces of knowledge learnt during the instructions. However, this knowledge was in the form of rules proposed by teachers. In fact, it was expected from student to analytically apply the knowledge while solving questions, they just wrote the rule and continued to write the answers according to their incorrect intuition. This situation brought a discussion of how teachers provide comprehensive understanding of the topics. They were expecting students to use the rules when "needed". However, they did not show which occasion is "needed" time while solving questions. Teachers did not help to solve the contradiction between incorrect intuitions in students' minds and formal solutions. Therefore, students relied on their heuristics instead of analytically investigating the occasions and using the rules in appropriate places.

Although both middle and and high school teachers emphasized the importance of sample sizes of the probabilities of the events and solved and explained the questions similar to one asked in the PTI, more than half of both middle and high school students fell into the misconception of sample size effect. The main point that students focused on was the ratios between the numbers in the event. Instead of understanding the key points in the question, students focused on the representation of the data presented (Kahneman & Tversky, 1982). Therefore, both middle and high school students' heuristics were dominant while solving the related question in the PTI. Especially middle school teachers provided questions related to comparison of the probabilities of the events. However, they promoted incorrect intuitions to students with their statements during the observations. This situation negatively affected students. Instead of resolving students' intuition-related misconception in their minds, teachers also added new incorrect intuitions. Therefore, the occurrence of this misconception stayed still both among middle and high school students. This misconception also had similar logic for the misconception of same of A-same of B.

One interesting finding was that number of middle and high school students who fell into outcome approach increased. This misconception was about the comparison of the probabilities of events already happened. Both middle and high school teachers did not give emphasis on such situation. In general, they focused on finding the numerical values of the probabilities of events. Although some questions related to comparisons of the events were solved in middle school classrooms during the observations, the post-test results indicated that occurrence of this misconception increased among students. In fact, the numbers of incorrect answers in pre- and post-test were still high. The misconceptions changed the form. In the pre-tests, students mainly fell into more of A-more of B and availability heuristics. Their answers changed in the post-test and students fell into outcome approach misconception. Considering teachers' instructions, teachers were blaming students about their misconceptions. Teachers stated that students did not listen to the lesson, solve questions, and study enough, there were only very few occasions observed in order to resolve the misconception of outcome approach. Instead of considering students' reasoning and misconceptions (Kazak, 2009), they generally followed the textbook or supplementary books.

For the misconception of simple and compound events, especially high school teachers solved related questions in two ways: by considering the event as simple event and by considering the event as compound events. The instructions included related examples, shortcuts to resolve misconception, but the regular instruction was not effective in solving the misconception. In fact, students did not have chance to work on the questions. In general, teachers were writing the question on the table, waiting for a while, and solving it without considering students' understanding. In case, students stated that they did not understand, teachers verbally explained the situation in the same way as solved before. However, teachers verbally emphasized this situation. The post-test findings indicated that teachers' instructions did not have enough effect on resolving this misconception. The main point was that students ignore the concept of "pair" (Tirosh & Stavy, 1999a). In fact, both middle and high school teachers showed it by using the sample sizes for coins and dice. For example, they emphasized that if there was an outcome of 2-4, there might also an outcome of 4-2. However, many students ignored this situation and considered the outcome of 4-3 as a single outcome. Teachers did not include students into instruction. Therefore, the occurrence frequency of this misconception appeared high both among middle and high school students.

After students received regular instructions, the occurrence of conjunction fallacy decreased among both middle and high school students in the post-test. Although the regular instruction had impact on resolving students' intuitively-based misconception, this effect was small. This impact could be attributed to teachers' instructional practices during the observations. Related to conjunction fallacy, both middle and high school teachers provided many examples, illustrations, verbal explanations, and different solutions for questions asked. The main reason for such instructional practices was that curriculum included the related topic for this misconception. As this misconception was related to inclusive-mutully exclusive events, especially high school teachers taught the topic by giving daily life examples, using visualisations, and providing different types of questions during the observations. In fact, this misconception was observed more among middle school students. This was because the topic was not included in the middle school curriculum. However, Barış taught the lesson during the observations.

For the conditional probability misconception, the related topic was taught only in high school classroom. Since the first research question was about students' common misconceptions in probability, this misconception was also presented in this study. Previous literature indicated that this misconception was also observed among middle school students (Rubel, 1996; Watson & Kelly, 2007). Since the first part of the question was routine type of question that students could easily take one ball away and calculate the probability of simple events, the number of students who gave correct answer to first part was high both in pre- and post-test. The second part was of great importance for the conditional probability misconception. Since the further event happens before the preceding event, the time-axis situation contradicted with students' intuitions. Therefore, the occurrence of this misconception among high school students was high. In fact, no high school teacher considered this situation as important. Therefore, they did not give emphasis on resolving this misconception. From these observations, it was obvious that teachers did not search for students' possible misconceptions and did not take their incorrect intuitions into consideration (Memnun, 2008). They did not even have awareness about this type of misconception. Therefore, they did not have any precautions for this type of misconceptions in their instructional practices. Instead, they followed the curriculum and focused on the routine type of questions. Only science high school students were successful in solving the question asked in the PTI. The main reason was that students got familiarity with non-routione questions, which forced them to think about the question and prepare appropriate solution method.

It was expected that students confront their incorrect intuitions and develop new ones while solving questions. Students' intuitions gained through experiences and with prior knowledge are resistant to change (Fischbein, 1987). However, teachers used different strategies and solution methods to make students analyze the questions analytically and approach the solutions correctly during the observations. In line with this aim, it was observed that teachers followed procedures to solve questions. These procedures included correct understanding of what the question is asking, basing the solution method on the sample size and the set of expected elements, and finding the numerical value of the probabilities for the events asked. Instead of approaching to the solution, students pragmatically used what teachers taught to develop analytic thinking on the questions to support their incorrect intuitions (Evan, 2006). They used the knowledge learnt in the classrooms to explain their incorrect answers (intuitively-based misconceptions) to the questions asked in the PTI. On the other hand, it was expected that teachers convey students from intuitive thinking to analytic one by means of regular instructions. However, it was observed that teachers promoted students' intuitive thinking which are generally incorrect during the instructions (Evan, 2008). For example, many middle school students used the rule learnt for the relation between experimental and theoretical probabilities to explain the correctness of their misconception of negatively or positively recency effects. Similarly, high school students tried to find the sample size and the set of expected elements in the question related to sample size effect in the PTI by using combinations. However, they still relied on the ratios of between the baby-boys and new-born babies for each event.

The main reason for this finding could be that students dealt with non-traditional tasks in the PTI. Although students were exposed similar questions in regular questions, the questions in the PTI are not similar to those solved in the classrooms. Teachers in the study also asserted that these questions were very different than the questions in the textbooks. Similar finding were found in the study of Havill (1998). Students were more successful in solving questions if the questions were familiar for students. In the inverse situation, they experienced difficulties.

Teachers in this study also stated that the aim of the regular lessons was to prepare students for the exams in the interviews. These exams were in the form of either regular written exams in schools or high school/university entrance exams. It was also found that teachers were solving questions which were asked in the nationwide exams such as high school or university entrance exam. Therefore, teachers were making students gain familiarity to the questions similar to those asked in the exams. In Köğce and Baki's (2009) study, questions asked in the written exams in the high schools were compared with the questions asked in the university entrance exam. Although they found that questions asked in Anatolian and science high schools were consistent in terms of Bloom taxonomy, those asked the written exams in vocational high school were not consisted with the questions in the university entrance exam. On the other hand, it was observed that the questions asked in all classrooms in the present study were consistent with both the written exam and the high school/university entrance exam. Therefore, students were generally prepared for such exams. They were not prepared for unfamiliar situations (Papaieronymou, 2009). This situation could be the explanation for the slight changes for misconceptions between before and after regular instructions given. Among the teachers in the present study, only Erdal solved unfamiliar questions. The success in the post-test in the PTI was higher when compared to other teachers' classroom. This could be because they were ready for different types of tasks, so, they could easily adapt to the questions asked in the PTI.

Another reason that might explain ineffectiveness of the regular instruction in resolving intuitively-based misconception was that since students were prepared for the exams, they were exposed more with the rote memorizations. As it was found in this study, students were expected to memorize *if-then* statements. Rubel (1996) explained that *if-then* statements do not improve students' understandings of the probability. Instead, they kept students from thinking about the questions or situations in the tasks asked. It was suggested that students needed to understand the tasks and develop thinking on them (Kazak, 2009). Rubel's (1996) study also explained why there was a sharp increase in the misconception of outcome approach. Students were more focused on the operations, formulas, and rote memorizations; then, they become lost in dealing with the tasks. In fact, all high school teachers were trying to solve question related to outcome approach according to the formula provided before.

Instead of unfamiliar situations in probability, teachers stated that they were trying to follow the curriculum in the interviews. Therefore, they were stick to the course or supplementary books. In the observations, middle school teachers were using course book and high school teachers were using supplementary books from different publishers. Teachers stated that students were familiar with the multiple choice questions and the questions in the PTI were not classical questions asked in the written exams and in high school/university entrance exams. Only Erdal was solving unfamiliar questions in the classroom. Therefore, they stated that some misconceptions could appear when non-traditional questions were asked to students (Havill, 1998). In practice, teachers were solving classical type of questions in the classrooms. In general, if the curriculum included content related to the intuitively-based misconceptions, teachers were focusing on these issues. If not, they were not providing students with such situations.

Throughout the observations, teachers consistently used the same books. In general, middle school teachers used course books and high school teachers used supplementary preparatory books for university entrance exam. Since these books were including questions which were parallel to those asked in the written exams in the schools or in the high school/university entrance exams, students did not deal with unfamiliar situations. Therefore, students experience difficulty and fell into misconceptions (including intuitively-based ones) while they encountered with unfamiliar situations in probability questions.

Polaki (2002a; 2002b) found that the frequency of occurrences of the intuitively-based misconceptions among students decreased. However, the main difference between the present study and Polaki's studies (2002a; 2002b) was that Polaki used special instruction methods to resolve intuitively-based misconceptions. From this point, there was a need for special instruction methods to resolve students' intuitively-based misconception or diminish the occurences frequencies. Therefore, the regular instructions were not effective in doing so. Although teachers needed to prepare their lessons according to students' difficulties and misconceptions including intuitively-based ones, they did not pay attention to students' cognition and followed only a few resources.

One important result was that the existence of these misconceptions varied according to the age. In fact, there are studies (e.g., Fischbein & Schnarch, 1997; Li & Pereira-Mendoza, 2002; Tirosh & Stavy, 1999a; 1999b) which investigated the evolution of intuitively-based misconceptions among students with age. The age factor was related to the second research question which was about the similarities and differences between 8th and 11th grade students' intuitively-based misconceptions. When considering the pre-test results only, there appears age factor between the cases. As the cases included 8th and 11th grade students, pre-tests results indicated some similarities and differences for intuitively-based misconceptions in probability. For example, positively and negatively recency effect, outcome approach, conjunction fallacy, the rules in the Stavy and Tirosh's (2000) theory were observed more among middle school students. On the other hand, the misconception of simple and compound event was observed more among high school students. Lastly, availability heuristics and sample size effect were observed similar among both middle and high school students. Fischbein and Schnarch (1997) conducted a study that investigated the evaluation of probabilistic intuitively-based misconceptions with age. Similar findings were also found in this study. For example, they found that the existence of the misconceptions of the negatively and positively recency effect and conjunction fallacy decreased with age, as found in this study. It was also found in Fischbein and Schnarch's (1997) study that the availability heuristics were observed in all age groups, as similar to the findings of this study. On the other hand, while Fischbein and Schnarch (1997) found that the existence of the sample size increased with age, the findings of this study showed that it did not change with age. In addition, the existence of the misconception of the simple and compound events increased with age in this study, while it was found in some other studies that it did not change with age (Fischbein & Schnarch, 1997; Li & Pereira-Mendoza, 2002). Although the misconception of conditional probability was found only among high school students, Fischbein and Scharch (1997) also found that this misconception existed among middle school students. For the outcome approach, the misconception decreased with age as found in the study of Li and Pereira-Mendoza (2002). Considering Stavy and Tirosh's theory, both intuitive rules decreased with age (Tirosh & Stavy, 1999a; 1999b).

Of course, the age is important factor for the intuitively-based misconceptions; different factors may also lead students to fall into these misconceptions or may resolve the possible misconceptions. For example, students' familiarity with mathematical operations (Riccomini, 2005), with probability or with different types of tasks (Fox & Levav, 2000), their formal knowledge about the probability (Stavy & Tirosh, 1996) and generalization from experiences in daily life (Fischbein, 1975) or in school situations (Kvatinsky & Even, 2002) may also resulted in the differences for 8^{th} and 11^{th} grade students' intuitively-based misconceptions. The other findings in this study also supported differences in the misconceptions. For example, some external factors such as the characteristics of the task itself (Kazak, 2008) and instruction the students get (Tirosh & Stavy, 1999a) might also result in the intuitively-based misconceptions among students. In fact, the effect of instruction given to students was another aim that this study seeks for. This study was investigating whether the regular instruction resolved 8^{th} and 11^{th} grade students' intuitively-based misconceptions or not. In this manner, teachers' awareness and knowledge of students' difficulties and misconceptions played crucial role in resolving students' intuitively-based misconceptions.

5.1.2 Teachers' Awareness about Students' Difficulties and Misconceptions in Probability

The discussion continues with teachers' awarenesses about students' intuitively-based misconceptions and reasons for them. The research questions related to teachers' awarenesses were discussed in this part of the study.

Beginning with the teachers' awarenesses about students' misconceptions in probability, teachers stated many misconceptions and students' difficulties in the interviews. Teachers checked the curriculum and stated students' difficulties according the main headings in probability. The misconceptions that teachers stated in the interviews were in the form of, for example, "distinguishing the event types between dependent and independent events" by basing their awarenesses on their experiences. However, the misconceptions and difficulties they stated were the most general ones. Only Erdal mentioned about specific misconceptions and difficulties in the interviews. From the interview findings, teachers did not mention about the intuitively-based misconceptions. At this point, it was obvious that teachers did not prepare their instructional practices according to students' difficulties and misconceptions. Neither had they searched for students' possible misconceptions in probability. Without knowing students' cognitions and difficulties, it was not possible to expect from them to resolve specific types of students' misconceptions including intuitively-based ones. Therefore, it was also not possible for students to reach comprehensive understanding of probability with the help of teachers' teaching practices. Teachers only focused on the curriculum and textbook. Observation findings indicated that teachers considered the textbooks or supplementary books as the fundamental resources for teaching probability. Only Erdal was curious about giving additional information and helping students to investigate the topic analytically.

In the interviews-2 with teachers, teachers were asked about possible misconceptions that might appear among students in the questions of the PTI. They did not name the misconceptions, but they found them. They proposed the misconceptions only when they saw the related questions. This situation indicated that teachers did not experience such type of students' difficulties and misconceptions in their teaching practices. As the experience is important factor for effective teaching practices (Watson, 2001), it takes time to fulfill it. During the observations, however, teachers did not have efford to uncover students' difficulties and misconceptions. Teachers were questioning students' understanding with "do you understand?" question. In case, students did not understand the solution methods of the questions asked, they just repeated the solution verbally. They did not show alternative solution methods. In addition, they did not try to uncover the reasons behind their difficulties and misconceptions appeared while solving questions.

From the interview findings, Table 4.19 indicated the possible reasons behind students' difficulties and misconceptions in probability. When investigating the codes in the Table 4.19, teachers considered that the main reasons for students' difficulties and misconceptions were about student-related factors. Instead of judging their teaching practices or considering the possibilities of teacher or curriculum-related factors, teachers simply blamed students for their difficulties and misconceptions in probability. From these findings, teachers inferred that they provided necessary teaching practices during the instructions and they were succesful to teach probability. However, the observations indicated that teacher also fell into intuitively-based misconceptions in some cases and promoted incorrect judgment and intuitions related to probability during the instructions.

The interview results indicated that the knowledge of students' readiness for probability was important issue to consider. Before learning the probability, all teachers stated that students needed to have the necessary pre-knowledge for learning probability in the interviews. The mathematics was built on previously learnt knowledge (Papaieronymou, 2009) and if there were missing points on students' previous knowledge or if students did not understand the concepts that are necessary for further learning, the learning of new concepts and topics are affected in negative way (Çelik & Güneş, 2007). For example, sample size effect requires the knowledge of set concept which was directly related to the misconception of sample size effect and simple and compound events. In addition, Fischbein (1987) stated that students' intuitions could be shaped negatively with the wrong understanding of the previously learned concepts. Moreover, if students had lack of necessary knowledge for probability, their intuitions were shaped according to their experiences (Fischbein, 1987). From this point, all teachers were aware of the importance of the students' readiness for probability. Although the necessary pre-knowledge for probability were sets, sample size (Bar-On & Or-Bach, 1988), fractions, percentages (Carpenter et al., 1981), permutations and combinations (Yazıcı, 2002), only one middle school teacher (Barış) and one high school teacher (Cihan) specifically mentioned about the necessary knowledge for probability. Among them, Barış stated that students needed to know fractions, simplification, percentage, sets and numbers while Cihan mentioned about the operations and rational numbers. At this point, the knowledge of set topic could be considered as fundamental for solving any type of intuitively-based misconceptions. In addition, permutation and combinations were used to find the sample sizes in the questions. Therefore, the knowledge of these topics helps to manage the sample sizes of the events, which was important for resolving sample size effect and simple and compound events misconceptions.

In practice, however, teachers did not try to identify students' readiness before teaching probability. Instead, they were briefly explaining the necessary knowledge when it appears. As Bayazit and Gray (2006) asserted that teachers might know students' conceptual difficulties and their causes about any topic, it is possible that teachers do not use this awareness during teaching. In this study, what teachers were aware of and what they really did in their teaching practices contradicted. In fact, they implied many reasons for this situation. For example, they mentioned about the workload, unexpected seminars and meetings with administration, programs with students during the academic years. In addition, they considered some topics as more important than others. They gave more time to such topics and ignored the others. Overall, these reasons and many others prevented students to follow the curricula. Therefore, they stated that they could not practice their awareness. Instead, they tried to complete the basics of the topics and teach it based on the textbooks followed. Although teachers were expected to show different types of questions, use different instructional methods appropriate for the topic, and provide students with comprensive understanding of the topic, they could not even cover the topic completely.

In order to be successful in doing mathematics, Güven (2000) stated that students needed to construct relations with other subjects and with other disciplines. In addition, while solving questions, constructing patterns promotes students' thinking and improves their intuitions. In line with Güven's (2000) assertion, especially middle school teachers mentioned about the importance of these issues. They also stated that they had to prepare an environment that help students to discover the interactions within the topics in probability and between probability and other subjects in mathematics. However, the practice did not concide with their awareness. Instead of making students discover the interactions, they generally used direct teaching and provided the keypoints directly. This brought a problem of excessive numbers of memorizations. Students were expected to remember such interactions and keypoints when needed. However, they experienced difficulties especially in probability problems and fell into intuitively-based misconceptions.

One of the reasons that the intuitively-based misconceptions appeared among students was that they did not comprehensively understand the probability subject (Fischbein, 1987). Incorrect understanding of the concepts or topics in probability might lead to misconceptions in students' minds. In fact, the main of teaching practices in classroom was providing students with full understanding of the subject. From this point, teachers proposed different ways. Among them, teachers stated that the mostly used method was to evaluate students' responses to the questions, to follow students' attendance to the lesson, and to conduct diagnostic tests during the lessons. Some other ways are to expect students' own definitions and explanations about the questions, to conduct summative and formative tests, and to ask students' understanding. In the practice, however, the main trend was not seeking for students' understanding; instead, they were trying to follow the topics in books. In general, teachers were just asking whether they understand the content or not. On the other hand, Barış, Cihan, and Doğan used summative tests. None of the teachers used formative or summative tests. From the teaching practices, a deficiency observed was that teachers were not reaching to all students. During the limited time of teaching practice, they tried to complete the topics proposed in the curriculum. Therefore, insufficiency in understanding of the probability subject may lead to students' misconceptions. This situation may be one of the reasons for the slight differences between students' pre- and post-test results.

Teachers stated that there were many activities to fulfill students' understanding. Some of these assertions were directly related to the development of students' intuitions. Fischbein (1987) indicated that the daily life experiences, previous knowledge, and the instructions given were important factors to shape students' intuition in positive or negative ways. To shape students' intuitions in a positive way, all teachers stated that they gave daily life examples and provide them with daily life experiences. In practice, the examples were generally used in the first lesson while explaining the probability concept. In resolving the availability heuristics, for example, teachers mentioned about the probabilities of winning a chance game. They mentioned about the equality of the probabilities of the situations in chance games and how hard to win it. Then, teachers became more focused on the topics and questions in the probability. They were following the curriculum. During the teaching of sub-topics in the lessons, they sometimes processed activities that helped students to resolve intuitively-based misconceptions. For example, the middle school teachers asked questions about the theoretical and experimental probabilities. Some questions were directly related to resolution of the misconceptions of negatively and positively recency effect, sample size effect, simple and compound events, and conjunction fallacy. In addition, they emphasized the importance of visual stimulant in teaching probability. Especially middle school teachers stated that they needed to meet students' readiness, so, they would not experience difficulty due to the lack of previous knowledge. All teachers stated that they make students encounter with different type of questions and situations. In practice, however, they all solved questions mainly related to dice, coins, and urns. This situation contracted an intuition that all questions would be in the form of only dice, coins, and urns questions. This was directly related to availability heuristics. In fact, it was good to use traditional and non-traditional probability contexts in order to develop students' intuitive cognition (Havill, 1998), only Erdal used unfamiliar questions in the classrooms. In general, the questions asked in the PTI were not similar to those asked in the high school/university entrance exams. These unfamiliar situations helped students' in Erdal's classroom. The post-test resulted also showed that students in Erdal's classroom were more successful in diminishing the intuitively-based misconceptions.

Although teachers mentioned about the possible activities to do in order to develop students' understanding of probability subject, teachers generally used the direct teaching method in teaching probability. Teachers also stated this issue in the interviews. About the usage of visual materials, only Barış brought urns with balls inside, coins, and dice in the first lesson of the probability. Since the type of the questions in the high school/university entrance exams are about these, he wanted to emphasize that it is important to visualize the context in probability questions about urns, coins, and dice.

When the teachers were asked about the possible misconceptions among students in probability subject, they gave similar answers. Some of the difficulties that teachers presented were related to the specific types of intuitively-based misconceptions. Teachers indicated that the main difficulties were seen in determining the sample size and the set of expected elements. In the study of Çelik and Güneş (2007), it was reported that the misconceptions of determining sample size and the set of expected elements were observed in all grade levels. In resolution of the misconceptions of sample size effect, outcome approach, simple and compound events, and the misconceptions in Stavy and Tirosh's (1999a; 1999b; 2000) theory of intuitive rules, the determination of the sample size was of great importance. They also stated that students have difficulty in simple and compound events and in intersection and union of the sets in probability questions, which were directly related to the misconceptions of simple and compound events and conjunction fallacy. They stated that the other difficulties were determining the type of events, which were inclusive-mutually exclusive events, dependent-independent events, and the events in conditional probability. In fact, the difficulty in determining the event types might promote students' intuitively-based misconceptions of the negatively and positively recency effect, outcome approach, simple and compound events, conjunction fallacy, and time-axis probability. Teachers also stated that students could not determine which formula to use. However, the last one was related to the determination of the event types. The reason why they had these difficulties could be because of over explosion of rote memorizations and rules in probability. Teachers had lots of rules for topics of probability and they were giving these rules with *if-then* statements. In fact, teachers had necessary content knowledge for probability. However, they could not put this knowledge into practice. Mathematical knowledge for teaching was not sufficient during the observations. As Aslan-Tutak and Ertaş (2013) proposed that specialized mathematical knowledge is necessary but not sufficient condition for pedagogical content knowledge, their practices did not reflected their content knowledge to students. In practice, all teachers constructed rote memorizations for each type of difficulties and misconceptions that they proposed. The other reason for this situation may be using their intuitions instead of constructing logical structures (Fischbein & Schnarch, 1997). Most of the times, these intuitions were misleading (Shaugnessy, 1992). In addition, students might have inability to make reasoning (Fischbein & Schnarch, 1997), so they could not develop these logical structures during solving probability questions.

Especially middle school teachers and Cihan attributed students' misconceptions to some other factors such as low level of students' understanding, insufficient course book, and students' careless while solving questions. What all teachers stated was that they were preparing students to the high school/university entrance exams. This situation prevented teachers to make students encounter with different situation in probability. In fact, it was observed in the classrooms that teachers were solving questions according to these exams or to the written exams. In the pre- and post-test of the PTI also indicated that they fell into misconceptions when they encountered with the unfamiliar situations.

Another important factor that the teachers were uncomfortable was that students were memorizing the formulas and rules in the probability. Memnun (2008) indicated this situation as one important factor for students' difficulties in understanding probability. However, it was observed that teachers were providing shortcuts and rules to memorize. They were using *if-then* statements for each topic, which was not always helpful for students' understanding of probability (Rubel, 1996). Various kinds of rote memorizations and shortcuts were observed in the teaching practices parts of the study. In addition, it was observed that students were experiencing difficulty in using which rule for which type of questions. Erdal was also aware of students' difficulty of determining which formula or rule to use.

Although there were many misconceptions that teachers were aware of in the present study, they did not mention about the possible intuitively-based misconceptions. In the last interviews with teachers after the observations, they were asked the possible misconceptions that might appear among students in the questions of the PTI. As it was known, each question in the PTI was asked to determine specific type intuitively-based misconceptions. However, some other misconceptions also appeared among students. Although teachers indicated what was expected from students, they failed to determine the other misconceptions that appeared in the pre- and post-tests. In addition, teachers also fell into a few of these misconceptions when they saw the questions in the PTI at the first time. From these findings, teachers were not expected to organize the lessons according to these misconceptions. As it was expected, however, teachers were stick on book and followed the topics in the middle and high school curricula. Although they had recommendations for resolving these misconceptions, these were either the general recommendations or the ones that even they did not practice during the teaching practices. For example, middle school teachers suggested that students should be provided with the understanding of the relation between theoretical and experimental probabilities as they always repeated during the classroom observations.

In addition to teachers' knowledge and awareness about students' difficulties and misconceptions, the discussion continued with teaching practices. Teachers processed many situations and activities during the probability lessons. Some of their activities were consistent with the resolution of intuitively-based misconceptions. Instead of explaining the activities which were specific to each intuitively-based misconception, the activities were taken into consideration in general.

It is very obvious that teacher plays a crucial role in providing students with the understanding of the probability subject. Teachers may influence students' attitude and learning towards probability (Fischbein, Nello, & Marino, 1991). In addition, the selection and use of method in teaching probability also influenced students to learn and to understand the subject or develop positive or negative attitude to the learning of probability (Bulut, Yetkin, & Kazak, 2002; Çelik & Güneş, 2007; Gürbüz, 2007). In addition, if teachers' knowledge is not enough to teach probability, the development of new concepts in students' mind becomes insufficient (Bulut, 2001) and may result in misconceptions. Overall, there are many teacher related factors that influence students' learning of probability.

Since mathematics was built on previous knowledge, students need to construct a base for further learning in probability (Bulut, 2001). Therefore, teachers needed to create this base for students by teaching the basic concepts in probability. The present study, firstly, investigated how teachers developed concepts in probability. In such processes, middle and high school teachers behaved differently. As it is known, the 8^{th} grade middle school curriculum included theoretical, experimental, and subjective probabilities and dependent-independent events. Ahmet was stick to the course book. He did not make students remember the previous topics in probability. For example, the basic concepts in probability were taught in 6^{th} and 7^{th} grades. The basic concepts were crucial especially for preventing students from the intuitively-based misconception of availability heuristics. Instead, he directly began to teach the 8^{th} grade content in the curriculum. He gave the definitions of the concepts which were taught in 8^{th} grade from the course book. In some cases, he misleaded students by explaining the concepts incorrectly. For example, he directed students to fall into negatively and positively recency effect, while explaining the prediction of the outcome in the consecutive trials of throwing a coin during teaching the experimental probability. This might result in difficulties among students in understanding the subject (Bulut, 2001). On the other hand, Barış summarized the necessary concepts for understanding the probability. He brought visual materials to explain the subjects. Gürbüz (2008) states that instead of using traditional methods, which are insufficient in resolving the difficulties in learning probability, visual materials were very helpful for students to develop concepts in probability. On the other hand, high school teachers used student-teacher interaction while teaching the concepts. They generally used the meaning of the concepts. In addition, the real life examples were given to explain the concepts. In fact, teachers indirectly helped students to resolve some of the intuitively-based misconceptions by giving daily life examples for the concepts. For example, while explaining the independent events and experimental probability with examples, they also helped students to understand the logic behind the negatively and positively recency effect and simple, conjunction fallacy, and compound events. They gave similar examples for the concepts such as throwing dice, coin, and taking balls from the urn.

Comparing their awareness and practices, they all mentioned about the importance of using visual materials and the meaning of the concepts in developing them. However, only Barış brought materials in such processes. What they generally did was using question-answer method and developing the concepts from the meanings and the daily life examples. Although the daily life examples were important in imagining the concepts in mind (Fischbein, 1987), the verbal explanations were not persistent in students' minds (Gürbüz, 2008). It is suggested that students should be exposed to many stimulants to improve their intuitions (Shaugnessy, 1992). So, they can diminish the frequency of falling into misconceptions.

5.1.3 Teachers' Teaching Practices to Resolve Students' Intuitively-based Misconceptions

The presentation of the relation between probability and other subjects was investigated. As it is known, the probability is related with some other subjects such as fractions, permutation, and combination (Jones, Langrall, & Mooney, 2007). Therefore, teachers needed to meet students' readiness for teaching probability. In teaching practices, the most apparent topic that was related to probability was the sets. The knowledge of set topic was directly or indirectly related to any type of the intuitively misconceptions from case to case. All teachers used the properties of the sets in teaching probability. Teachers provided relation between these properties and the probability. They related the union and intersection of the sets, the complement of a set with the necessary topics in probability such as dependent-independent and inclusive-mutually exclusive events. Therefore, teachers helped students to resolve the misconceptions of negatively and positively recency effects, outcome approach, simple and compound events, conjunction fallacy, and conditional probability. While relating them, teachers provided examples generally from throwing dice, coins, and taking balls from urn. Only Erdal wrote the formulas and considered that students had enough knowledge for the relations between the set and probability subjects.

Although the permutation and combination topics were taught before the probability, only high school teachers used these subjects in teaching probability. Among the high schools, Cihan was very dependent on the permutation and combinations while selecting something or arranging the elements. He used them even in the easiest questions. Considering the relation between determining the sample size and the misconceptions of sample size effect, and simple and compound events, high school teachers' use of permutation and combination helped students to resolve these types of misconceptions. On the other hand, Barış, Cihan, and Erdal gave relation between geometry and probability. In fact, the infinite probability was included in the high school curriculum. However, Doğan did not mention about this topic. In addition, Barış and Erdal solved questions related to this topic without teaching it. The general probability formula was adapted to such topics while solving the questions. It was observed in the classrooms that students were indirectly affected by the relation between the geometry and probability topics. It was observed that the conjunction fallacy and conditional probability misconceptions were indirectly related to this relation. On the other hand, the more of A – the more of B misconception was appeared in Cihan's classroom while solving a probability questions which required the knowledge of geometry. The other subjects that teachers related with the probability were the fractions, the comparison of the fractional numbers, percentages. Memnun (2008) mentioned about them for the difficulties in learning probabilities.

In general, students' readiness is important to learn probability (Gürbüz, 2005). They needed to have necessary knowledge for solving questions that requires the knowledge from other subjects. Without the necessary pre-knowledge, students could develop incorrect intuitions and fall into misconceptions (Fischbein, 1987). In meeting the readiness, the method that teachers used in general was that they briefly explained or made students remember the necessary knowledge from other subjects only when the knowledge was needed. For example, if the area of the circle was needed, Cihan briefly stated that the area is π times radius squared. They did not try to determine or explain the necessary knowledge from other subjects. It was good when solving the provided questions in the classroom. However, it might create problems especially when students experienced irregular or unfamiliar situation or questions. These situations or questions could require students' intuitions to solve.

In the literature, the main factor for the comprehensive understanding of probability and for the reasons for the possible misconceptions was attributed to the teaching methods and type of instruction (Babai et al., 2006; Brunner, 1997; Gürbüz, 2005; 2007; Gürbüz et al., 2010; Nicolson, 2005; Polaki, 2002a; 2002b; Tirosh, 2000; Watson, 2001). In accordance with the importance of the instructions, teachers' teaching practices were investigated in the observations. The findings from the analysis of the observations indicated that there were giving key points, rote memorizations and shortcuts, using resources and materials, and student-teacher interactions.

The main method in the instructions was direct teaching. Teachers were the only authority throughout the lessons. Although they sometimes gave emphasis on student-teacher interactions with question-answer method, teachers were active during the lessons while the students were passive listeners. This situation was considered as one of the reasons for difficulties in effective teaching of the probability (Gürbüz et al., 2010).

With the applications of the direct teaching, all teachers in the present study were careful about showing the crucial points in the probability subjects or in the questions asked. Among the key points, all teachers in the present study were trying to make students memorize the algorithms, some shortcuts, formulas, and determine the key words in solving questions. For each type of intuitively-based misconception, specific shortcuts were observed in the classrooms.

Related to the memorizations, all teachers had rules for different concepts or event types in probability. There were lots of rules and shortcuts that teachers imposed. Among them, most of the shortcuts were related to the determination of event types and necessary algorithms to follow for them. These rules were about the general concepts, the sample size, the event types, the use of combination and permutation, conditional probability, and other sub-topics.

In fact, teachers were giving such shortcuts for sub-topics in the probability. However, some shortcuts could be used to resolve some intuitively-based misconceptions. For example, one shortcut was that the sample sizes for throwing a die twice and for throwing two dice are equal. This shortcut indirectly gave evidence to resolution of simple and compound events. Teachers were emphasizing the key words in the questions. Then, they were emphasizing which shortcuts to use for specific questions.

The difference between middle and high school teachers' practices were observed only when there was difference between the middle and high school curricula. For examples, there were rules for theoretical, experimental, and subjective probabilities which were observed only in middle school teachers' classrooms. In fact, the existence of these sub-topics helped students to resolve some intuitively-based misconceptions including outcome approach, positively and negatively recency effect, and intuitive rules. On the other hand, the rules for the use of permutation and combinations and for conditional probability were observed only in high school teachers' classrooms. It was expected from students to memorize these rules and use when necessary. However, there was a big problem when using these rules. Since there were numerous rules, students had difficulty to find which rule to use in the questions asked. Instead of expecting from students to memorize them, teachers needed to provide them with comprehensive understanding of the topics. The practices for generalizing the rote memorizations and explaining why high school teachers used one formula for different event types were useful in high schools for this purpose. However, even high school teachers were imposing that students are in a rally for university entrance exams and they need to spend time effectively in the exam. Therefore, they were imposing that they needed to memorize rules, use when needed, and pass to the other questions in the exams, instead of providing fulfillment in understanding the subject. In fact, Polaki (2002a; 2002b) tried to make students discover the rules and to create desired understanding of the subject in his studies.

Different than middle school teachers, high school teachers made some teaching practices for biasness-unbiasness of the events, giving counter examples, making generalization from the rote memorization, and using same formula for different event types. Overall, general trend was making students memorize the algorithms, rules, keywords. Although some memorizations help students to understand the subject, the excessive memorizations influence the learning negatively (Gürbüz, 2008). Therefore, students might create new incorrect intuitions and fall into intuitively-based misconceptions. In addition, Fischbein (1987), Myers (2002), Stavy and Tirosh (2000) indicated the negative relation between the memorization and students' intuitions. Instead of understanding the concepts, memorizing the rules do not improve their intuitions. Therefore, students continued with their incorrect intuitions which were gained from their previous learning or previous experiences in daily life.

All teachers were giving emphasis on the importance of the sample size. Especially high school teachers solved many questions related to the determination of the sample size. They stated that the main point in the whole probability is about the sample size. The first thing that Cihan and Doğan were doing in the solution of the questions was to determine the sample size. The literature was also emphasizing its importance in resolving the misconceptions of sample size effect and simple and compound events in probability (Fischbein & Schnarch, 1997; Kazak, 2008; Shaugnessy, 1992).

Different than middle school teachers, high school teachers were trying more to convince students about their learning. For example, Erdal mentioned about the biasness of the material used in the experiments. He stated that the results change if the coins or dice are biased. Therefore, he was making students aware of the different situations in probability subject. In fact, the biasness of coins or dice could create the availability heuristics misconception and the misconceptions in Tirosh and Stavy's (1999a; 1999b; 2000) theory in students' minds. Moreover, especially Cihan and Doğan were using counter examples in order to make students compare the events and determine the differences. One interesting finding was that high school teachers were using same formula for different event types. According to the properties of the events, teachers were renewing the formula and continuing to solve questions with new versions. As it was found, the determination of the events are related to simple and compound events, conjunction fallacy, negatively and positively recency effects. In fact, this helped students to resolve their misconceptions. This is because teachers were creating an environment for understanding the logic behind the formula. It might brake students' wrong intuitions and develop them in positive way.

Andra (2011) studied with pre-service teachers about the intuitive use of representations in teaching probability, it was indicated that the type and the way to use different representations influence students' learning of the subject. In addition, Batanero and Diaz (2012) also implied the importance of the use of visual materials in teaching any subjects. Moreover, Primi (2008) studied the resolution of one type of intuitively-based misconceptions by means of gaming situation. Gürbüz (2005) stated that one reason for students' difficulty in probability was that there were not enough visual or physical materials and resources to teach it. Taking them into consideration, the use of materials and resources in the instructions is one factor to help students not to fall into misconceptions and to resolve the existing ones. In the present study, the use of materials and resources was also investigated in the observations. Although, in line with the literature, teachers stated that the visual or physical materials are important to fulfill students' understanding, only Barış used visual materials in the first lessons of the observations. Although Doğan and Erdal were provided with smart board in their classrooms, Doğan never used it and Erdal used it only for reflecting the pdf-format supplementary book on the board. The general trend in the observations was that teachers were stick to either supplementary or course books throughout the lessons. In general, middle school teachers were using course books and high school teachers were using supplementary books for preparing students to the exams.

Overall, teachers were aware of the situations that might influence students in negative way in learning probability or of the factors that may lead students to misconceptions; they did not always behave in the observations accordingly. There appeared a contradiction between what they said and what they did in the classrooms (Bayazıt & Gray, 2006).

In the last part of the study, examples and questions asked in the regular instructions related to students' intuitively-based misconceptions were investigated. Solving as many questions as possible provided students with familiarity for different types of questions in probability. It also gave courage to them. They got a feeling of doing mathematics. In addition, if they solved many questions, they encountered with different and unfamiliar questions. If the intuitively-based misconceptions were considered as unfamiliar ones, solving many questions also helped students to resolve their existing misconceptions. In the practice, all teachers were mentioning about the importance of solving as many questions as possible. In addition, they were expecting students to do homework. Students were responsible to solve questions from textbooks as homework. In line with this situation, the numbers of questions solved in the classrooms were more in high schools when compared to middle school classrooms. Compared to Cihan, Doğan and Erdal solved more questions in the classroom. Among the questions solved, all teachers gave importance to basic concepts and the probabilities of simple events. The distribution of the questions asked in the classrooms was according to the topics taught. While high school teachers solved different questions for each topic, Ahmet solved questions only about the basic concepts, simple events, theoretical, experimental, and subjective events, and dependent-independent events. On the other hand, Barış taught all topics expect for conditional probability. Different than the other teachers, Erdal also solved unfamiliar questions that were different that the questions asked in the university entrance exams or in the preparatory tests for university entrance exams. These questions might widen students' intuitions. This is because their success was higher in the post-test of the PTI when compared to the other classrooms.

Teachers did not consider the intuitively-based misconceptions when asking questions in the classrooms. The main criteria for asking questions was whether the type of questions were similar to those asked in high school/university entrance exams or in written exams in the schools (Köğce & Baki, 2009). In fact, there were questions which had relations with the intuitively-based misconceptions appeared among students. However, these questions were asked due to the content of the topics taught. For example, the questions related to availability heuristics and the positively and negatively recency effects were asked in the middle schools. In fact, the topic of theoretical, experimental, and subjective probability is directly related to these misconceptions. On the other hand, high school teachers gave examples from lottery and chance games, which were related availability heuristics. For the sample size effect, all teachers were giving importance to sample sizes of the events. In the algorithms followed, the first thing they did in solving questions was to find the sample size. Although students were successful to find the sample size for small samples, they could not manage bigger sample size and they fell into this type of misconception. Since the outcome approach is related to making predictions from already happened events, these kinds of questions were not asked in the classrooms due to the existence of the high school/university entrance exams. However, a few indirectly related questions were asked in the middle school classrooms because they learnt experimental probability. In high schools, the topic of the inclusive-mutually exclusive events was taught. The questions in such topic were indirectly related to this type of misconceptions. If the set of event A is sub-set of event B, this was about conjunction fallacy. There were only a few number of questions asked in high schools. To solve such questions, high school teachers used small visualizations to make students understand the situations in the questions. On the other hand, all questions in the topic of conditional probability include a condition and it was indirectly related to conjunction fallacy. In the solution of the questions, teachers indicated that the sample size is gathered from the set of "the condition".

The conditional probability subject was taught only in high school. However, conditional probability misconception was about the timing of the events. There were only one question asked by Doğan and a few questions asked by Erdal, which were related to conditional probability. However, teachers focused on the formula for conditional probability and did not explain the logic to solve the questions.

In general, there were many activities done and questions asked in teachers' regular instructions. There were many teaching practices from teachers to teach probability in order to provide students with understanding of the subject. However, there were also many factors that affected students' understanding and that lead students to misconceptions. It was found in the present study that the regular instructions had slight effect on resolving students' intuitively-based misconception.

5.2 Suggestions

The findings of the study indicated the 8^{th} and 11^{th} grade students' intuitively-based misconceptions, 8^{th} and 11^{th} grade mathematics teachers' awareness about the factors that result in such misconceptions and of such misconceptions. The teachers' teaching practices for resolving such misconceptions were also investigated in the study. In the lights of the findings of the study, there were some suggestions for practice and for the further studies.

5.2.1 Suggestions for Practice

In this part of the study, the suggestions were for the teacher educators and practitioners of mathematics education area. These suggestions were as follows.

- The findings of the study indicated that intuitively-based misconceptions existed among both middle and high school students. In addition, these misconceptions continued to exist after regular instructions. When asked to teachers, they indicated students' possible misconceptions by means of their experiences. Before teaching the probability, teachers should have knowledge about the possible misconceptions and prepare the lessons accordingly. In doing so, teacher training programs should provide pre-service teachers with the knowledge of possible misconceptions, the factors causing difficulties in learning the subject, and the methods to resolve these misconceptions.
- Based on the findings, students' had many misleading intuitions before learning probability. Teachers were directly beginning to teach it. Without examining students' pre-knowledge for probability and wrong intuitions about probability, they struggle to learn it and easily fall into intuitively-based misconceptions. Teachers should be encouraged to perform formative tests to see students' readiness and wrong intuitions. So, teachers can specifically pay attention to students' wrong intuitions and keep them from felling into misconceptions while solving probability questions.
- The teachers in the present study used limited number of teaching techniques, which were generally the traditional ones. It was observed that such techniques were not sufficient to resolve students' misconceptions. In teaching probability, the mathematics teachers should be trained about different types of teaching methods and techniques.
- Teachers mentioned about the difficulty to follow the curricula during the academic years. Among the difficulties, they stated that the existence of high school/university entrance exams, the work load, the number of students in

the classrooms, and the limited number of lessons allocated for probability were obstacles for meeting students' readiness, for preparing the lessons according to different dimensions, and even for completing the curriculum. To overcome these problems, teachers' work load should be decreased and the content of the curriculum should be simplified to provide students with comprehensive understanding of the topic. So, teachers can have extra time to search for new trends, teaching practices, and materials in teaching probability. They can benefit from effective instructional methods practiced in national or international schools.

- The understanding of the basic concepts in probability may reduce the possible misconceptions among students. If basic concepts are not learnt well enough, students generally use their intuitions that were gathered from previous experiences instead of thinking critically and developing reasoning on the questions asked. In general, these intuitions are misleading. At this point, teachers should seek for the techniques to satisfy full understanding of the basic concepts. These can be done via different visual or physical materials and resources. Teachers should also seek for the techniques for developing students' reasoning skills in order not to be influenced by the negative effects of the intuitions.
- Due to the existence of the high school/university entrance exams, teachers were organizing the lessons accordingly. It was observed that teachers were solving questions similar to those asked in such exams. Students were not exposed to different and unfamiliar situations and questions in the lessons. Since the intuitively-based misconceptions were appearing in the unfamiliar situations, the regular instructions were not sufficient to resolve them. It can be suggested that teachers provide unfamiliar situations and questions in the lessons to make students see that the probability is not limited to dice, coins, and urns. To do this, teachers can be encouraged to follow professional journals in mathematics education for new situations and problems in teaching probability. Instead of just relying on traditional methods, they should seek for alternative ways. Professional journals are including specific situations and examples encountered in classroom environments. These situations were can also be observed in our classrooms. For example,

teachers can read articles about intuitively-based misconceptions appeared in the classroom and what teachers did to resolve them. Therefore, teachers can have knowledge about the misconceptions and methods to resolve them. With the appropriate teaching practices, students can get correct intuitions about such misconceptions. For example, the misconceptions found in Crawfold (1997) stated probability misconceptions that were observed in his classrooms. Some of these misconceptions were related to intuitively-based ones. Teachers can benefit from Crawfold's personel views related this misconception and prepare their instructions accordingly.

- From the observations and interviews, teachers were knowledgeable about students' basic difficulties and misconceptions. They were generally following a course or supplementary book in their instructions and ignoring the specific misconceptions. To be knowledgeable about students' cognitions, difficulties, misconceptions, teachers can use *google* groups such as TMOZ (branch of Turkish mathematics teachers) to share the teaching experiences with mathematics teachers, to determine students' difficulties and misconceptions in probability and in other subjects. Teachers can create a discussion about students' difficulties and solution methods in probability. In addition, they can also share teaching methods and materials used, and discuss their experiences. With the knowledge of students' difficulties and misconceptions and by using other teachers' experiences, teachers can shape their instructions accordingly. So, they can use the teaching practices in resolving students' possible misconceptions.
- During the observations, teachers were generally followed textbooks or supplementary books in teaching practices. Therefore, they only solved the questions presented in the textbooks or supplementary books. Questions presented in such books were generally in the form of routine questions which were asked to high school/university entrance exams. As the questions asked in the PTI were non-routine types of questions, students in the observation groups could not encounter with non-routine questions. Since misconceptions in probability were generally intuitively-based types, students' intuitively-based misconceptions stayed still after they received regular instruction. From these findings, textbooks and supplementary books

can include non-routine questions which were in line with intuitively-based misconceptions. As the textbooks are prepared according to the present curricula, similar suggesstions can be made for them, too. The curricula can emphasize the solution of non-routine questions. Therefore, teachers can help to resolve students' intuitively-based misconceptions in probability.

5.2.2 Suggestions for Further Studies

This part of the study presented suggestions for the further studies. The suggestions were for the researchers from mathematics educations. The suggestions for further studies were as follows:

- This study was conducted only in five classrooms which were not randomly selected. In addition, only 8th and 11th grade students were participants of this study. This situation might yield a generalizability concern. The results generated from a larger sample of randomly selected participants from different grade levels could be more generally applicable to mathematics education community. A study can investigate students' intuitively-based misconceptions from different age groups or grade levels with increased number of students. Therefore, more comprehensive understanding of students' intuitive thinking in different grade levels can be identified.
- This study did not focus on gender differences while determining students' intuitively-based misconceptions. However, Kennis (2006) found gender difference for students' probabilistic misconceptions. This yield a discussion of whether regular instruction increases or decreases gender differences in resolving students' intuitively-based misconceptions or not. A study can focus on gender difference about students' intuitively-based misconceptions before and after students receive regular instruction.
- In the present study, the participants were from 8th and 11th grade levels. However, students' intuitive cognitions were not investigated in other grade levels. It is known that previous knowledge affects students' intuitions (Fischbein, 1987). With the new knowledge in different grades, their intuitions are also shaped. After students learn probability in 8th grade, a

longitudinal study can investigate students' intuitively-based misconceptions according to the changes in their intuitions throughout the grade 12.

- In order to resolve students' intuitively-based misconceptions, different teaching methods can be compared with the regular instruction in experimental studies. For example, Polaki (2002a) conducted an experimental study for sample size misconception. In this study, it was found that the regular instructions were not effective in resolving intuitively-based misconceptions. An instruction can be organized specially for different types of intuitively-based misconceptions. The effective methods in resolving such misconceptions can be determined via comparing the methods.
- This study seeked for teachers' knowledge about students' misconceptions and the factors resulting in them. So, they organized their instructions accordingly. However, teachers' beliefs about probability topic and students' thinking processes were missed in this study. It is another factor that influences teachers' teaching practices. If their beliefs are consistent with students' thinking processes and intuitions, students can benefit from appropriate teaching practices to get rid of incorrect intuitions. In line with this situation, teachers' beliefs about the topic and students' intuitive thinking can be investigated.
- There is a difference between theoretical knowledge and the knowledge gathered from experience in many situations. In the profession of teaching, this difference becomes apparent. Although they theoretically learn the knowledge of students and pedagogical content knowledge in teacher training departments, they should also learn when and how to use this knowledge via experience. In the present study, the focus was not on teachers' experiences. However, the most succesful classroom after instructions was Erdal's classroom. Although success level of his classroom was not high at the beginning, it was not a coincidence that the most experienced teacher was Erdal in this study. From this point of view, the in-service and pre-service teachers' awareness about the intuitively-based misconceptions and the factors related to them can be different. Their perceptions can be gathered and compared to understand the importance of experience.

- The theoretical and experiental probabilities were emphasized in NCTM's (2000) standards and in national curriculum (2005b), especially in lower grades, because the values are more realistic to daily life situations. In the observations, teachers did not give in depth information about these topics. In addition, the findings indicated that middle school teachers used theoretical and experimental probabilities while solving question in PTI. However, their incorrect interpretation of this knowledge leaded them to fall into intuitively-based misconceptions. Studies of how best to organize lessons to teach probability and of their effect on resolving related intuitively-based misconceptions could be helpful for teachers and mathematics educators.
- This study focused on the probability topic. It was found that students' incorrect intuitions resulted in misconceptions. There may appear different kinds of intuitively-based misconceptions related to other subjects. At this point, Tirosh and Stavy (1999a; 1999b; 2000) indicated that students behave similarly regardless of differences in tasks. Students can show similar behaviors in other topics in mathematics. Tirosh and Stavy called it as "intuitive rules". The misconceptions of "same of A – same of B" and "more of A – more of B" are example for these rules. Therefore, different intuitively-based misconceptions in other subjects can be investigated. Different than the regular misconceptions, intuitively-based ones necessitate more thinking and are directly related to students' incorrect intuitions. Accordingly, similar study can also be conducted for other topics in mathematics.

REFERENCES

Alberta Education. (1996). *Alberta Program of Studies for K-9 mathematics: Western Canadian protocol for collaboration in basic education*. Edmonton, Canada: Alberta Education Branch

Anastasiadou, S. (2009). Greek students' ability in probability problem solving. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *The proceeding of 6th conference of the European Society for Research in Mathematics Rducation* (pp. 404-412). Lyon, France: Institut National de Recherche Pedagogique.

Andra, C. (2011). Pre-service primary school teachers' intuitive use of representations in uncertain situation. In M. Pytlak, E. Swoboda, & T. Rowland (Eds.), *The proceeding of 7th conference of the European Society for Research in Mathematics Education* (pp. 715-724). Rzeszow, Poland: University of Rzeszow.

Aslan-Tutak, F., & Ertaş, F. G. (2013). Practices to enhance preservice secondary teachers' specialized content knowledge, In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *The proceeding of 8th conference of the European Society for Researh in Mathematics Education* (pp. 2917-2926). Antalya, Turkey: Middle East Technical University.

Aspinwall, L., & Tarr, J. E. (2001). Middle school students' understanding of the role sample size plays in experimental probability. *Journal of Mathematical Behavior, 20*, 1-17.

Babai, R., Brecher, T., Stavy, R., & Tirosh, D. (2006). Intuitive inference in probabilistic reasoning. *International Journal of Science and Mathematics Education, 4*, 627-639.

Begg, A., & Edwards, R. (1999). Teachers' ideas about teaching statistics. *The paper presented at the combined meeting of Australian Association for Research in*

Education and New Zealand Association for Research in Education, (November 29-December 2), Melbourne, Australia.

Bar-Hillen, M., & Falk, R. (1982). Some teachers concerning conditional probabilities. *Cognition*, *11*(2), 109-122.

Bar-On, E., & Or-Bach, R. (1988). Programming mathematics: A new approach in introducing probability to less able pupils. *Journal of Mathematics Education in Science and Technology, 19*(2), 281-297.

Batanero, C., & Diaz, C. (2012). Training school teachers to teach probability: Reflections and challenges. *Chilean Journal of Statistics, 3*(1), 3-13.

Batanero, C., Godino, J. D., & Roa, R. (2004). Training teachers to teach probability. *Journal of Statistics Education*, *12*(1). Retrieved from www.amstat.org/publications/jse/v12n1/batanero.html

Batanero, C., Godino. J. D., & Cañizares, M. J. (2005). Simulations as a tool to train pre-service school teachers**.** *The proceedings of first ICMI African Regional Conference* (pp. 176-192). Johannesburg, South Africa: International Comission on Mathematics Instruction.

Bayazit, İ., & Gray, E. (2006). A contradiction between pedagogical content knowledge and teaching indications. In Novotna, J., Moraova, H., Kratka, M. &Stehlikova, N. (Eds.), *The proceeding of 30th conference of the International Group for the Psychology of Mathematics Education* (vol. 2, pp. 121-128). Prague, the Czech Republic: International Group for the Psychology of Mathematics Education.

Bills, L., & Husbands, H. (2005). Values education in mathematics classroom: Subject values education values and one teacher's articulation of her practice. *Cambridge Journal of Education, 35*(1), 7-18.

Bruner, J. (1960). *The process of education: A landmark in educational theory*. Cambridge, the UK: Harvard University Press.

Brunner, R. B. (1997). Numbers, please. *Mathematics Teacher*, *6*(4), 704-709.

Bulut, S. (2001). Investigation of performances of prospective mathematics teachers on probability. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 20*, 33-39.

Bulut, S., Ekici, C., & İşeri, A. İ. (1999). Bazı olasılık kavramlarının öğretimi için çalışma yapraklarının geliştirilmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi. 15*, 129-136.

Bulut, S., Yetkin, İ. E., & Kazak, S. (2002). Matematik öğretmen adaylarının olasılık başarısı, olasılık ve matematiğe yönelik tutumlarının cinsiyete göre incelenmesi. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 22*, 21-28.

Carnell, L. J. (1997). *Characteristics of reasoning about conditional probability (preservice teachers)* (Unpublished doctoral dissertation). University of North Carolina, Greensboro, NC.

Carpenter, T. P., Corbitt, M. K., Kepner, H. S., Linquist, M. M., & Reys, E. R. (1981).What are the chances of your students knowing probability? *Mathematics Teacher, 73*, 342-344

Çelik, D., & Güneş, G. (2007). 7, 8 ve 9. sınıf öğrencilerinin olasılık ile ilgili anlama ve kavram yanılgılarının incelenmesi. *Milli Eğitim Dergisi, 173*, 361–375.

Chick, H. L., & Pierce, R. U. (2008). Teaching statistics at the primary school level: Beliefs, affordances, and pedagogical content knowledge. In C. Batanero, G. Burrill, C. Reading, & A. Rossman (Eds.), *The proceedings of the ICMI Study 18 and 2008 IASE Round Table Conference* (pp. 671-676). Monterrey, Mexico: International Commission on Mathematics Instruction and International Association for Statistical Education.

Chiese, F., & Primi, C. (2008). Primary school children's and college students' recency effects in a gaming situation. *Paper presented at 11th international congress on Mathematical Education, (*July 6-13), Monterrey, Mexico. Retreived from http://iase-web.org/documents/papers/icme11/ICME11_TSG13_05P_chiesi.pdf.

Chiese, F., & Primi, C. (2009). Recency effects in primary-age children and college students. *International Electronic Journal of Mathematics Education, 4*(3), 259-274.

Clement, J. (2000). Analysis of clinical interviews: Foundations and model viability. In R. Lesh, & A. Kelly, (Eds.), *Handbook of research methodologies for science and mathematics education* (pp. 341-385). Hillsdale, NJ: Lawrence Erlbaum.

Cohen, L., Manion, L., & Morrison, K. (2000). *Research methods in education* (5th ed.). London, the UK: Routledge Falmer.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics.* Washington, D. C.: Council of Chief State School Officers and National Governors Association.

Crawfold, D. (1997). Learning probability: Misconceptions and all. *Mathematics Teaching, 159*, 23-29.

Creswell, J. W. (2007). *Research design: Qualitative, quantitative, and mixed methods approaches* (3rd ed.). Thousand Oaks, CA: Sage.

Creswell, J. W., & Clark, V. L. (2007). *Designing and conducting mixed methods research*. Thousand Oaks, CA: Sage.

Dane, E., & Pratt, M. G. (2007). Exploring intuition and its role in managerial decision making. *Academy of Management Review, 32*(1), 33-54.

Dereli, A. (2009). S*ekizinci sınıf öğrencilerinin olasılık konusundaki hataları ve kavram yanılgıları*. (Unpublished master's thesis). Osmangazi University, Eskişehir, Turkey.

Diyarbekir, G. (2003). *The effectiveness of demonstrative computer animation in developing intuition: A case for gravitational acceleration.* (Unpublished master's thesis). Boğaziçi University, İstanbul, Turkey.

Dollard, C. (2011). Preservice elementary teachers and the fundamentals of probability. *Statistics Education Research Journal, 10*(2), 27-47.

Dooren, W. V., Dirk, D. B., Weyers, D., & Verschaffel, L. (2004). The predictive power of intuitive rules: A critical analysis of the impact of “more A – more B” and “same A – same B”. *Educational Studies in Mathematics, 56*, 179-207.

Ekinözü, İ. (2003). *İlköğretimde permütasyon ve olasılık konusunun dramatizasyon ile öğretiminin başarıya etkisinin incelenmesi*. (Unpublished master’s thesis). Marmara University, Istanbul, Turkey.

Evans, J. S. B. T. (2006). The heuristic-analytic theory of reasoning: Extension and evaluation. *Psychonomic Bulletin & Review, 13*(3), 378-395.

Evans, J. S. B. T. (2008). Dual-processing accounts of reasoning, judgment, and social cognition. *Annual Review of Psychology, 59*, 255-278.

Falk, R. (1979). Revision of probabilities and the time axis. In D. Tall (Ed.), *The proceedings of the third international conference for the Psychology of Mathematics Education* (pp. 64–66). Warwick, UK: University of Warwick.

Falk, R. (1983). Children’s choice behavior in probabilistic situations. In D. R. Grey, P. Holmes, V. Barnett, & G. M. Constable (Eds.), *The proceeding of the first international conference on Teaching Statistics* (pp. 714-716). Sheffield, the UK: Teaching Statistics Trust.

Fererman, S. (2000). Mathematical intuition vs. mathematical monsters. *Synthese, 125*, 317-332.

Fischbein, E., & Grossman A. (1997). Schemata and intuitions in combinatorial reasoning. *Educational Studies in Mathematics, 34*, 27–47.

Fischbein, E., & Schnarch, D. (1997). The evolution with age of probabilistic: Intuitively-based misconceptions. *Journal of Research in Mathematics Education, 28*(1), 95-106.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children.* Dordrecht, The Netherlands: Reidel Publishing.

Fischbein, E. (1982). Intuition and proof. *For the Learning of Mathematics, 3*(2), 8-24.

Fischbein, E. (1987). *Intuition in science and mathematics: An educational approach.* Dordrecht, The Netherlands: Reidel Publishing.

Fischbein, E. (1999). Intuition and schemata in mathematical reasoning. *Educational Studies in Mathematics, 38,* 11-50.

Fischbein, E., Nello, M. S., & Marino, M. S. (1991). Factors affecting probabilistic judgments in children and adolescents. *Educational Studies in Mathematics, 22,* 523-549.

Fisher, K. (1985). A misconception in biology: Amino acids and translation. *Journal of Biology Education, 22*, 53-62.

Fox, C. R., & Levav, J. (2000). Familiarity bias and belief reversal in relative likelihood judgment. *Organizational Behavior and Human Decision Processes, 82*(2), 268-292.

Fox, C. R., & Levav, J. (2004). Partition-edit-count: Naïve extensional reasoning in judgment of conditional probability. *Journal of Experimental Psychology: General, 133*(4), 626-642.

Fraenkel, J. R., & Wallen, N. E. (1996). *How to design and evaluate research in education* (3rd ed.). New York, NY: McGraw-Hill Inc.

Freudenthal, H. (1970). The aims of teaching probability. In L. Rade (Ed.), *The teaching of probability and statistics* (pp. 151-167). Stockholm, Sweden: Almqvist & Wiksell.

Gal, I. (2005). Towards "probability literacy" for all citizens: Building blocks and instructional dilemmas. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 39-63). New York, NY: Springer.

Geçim, A. D. (2012). *The effect of creative drama-based instruction on seventh grade students' mathematics achievement in probability concept and the* *attitudes towards mathematics*. (Unpublished master's thesis). Middle East Technical University, Ankara, Turkey.

Graberg, D., & Brown, T. A. (1995). The monty hall dilemma. *Personality and Social Psychology Bulletin*, *21*(7), 711-729.

Green, D. R. (1983). A survey of probability concepts in 3,000 pupils aged 11-16 years. In D. R. Grey, P. Holmes, & G. M. Constable (Eds.), *Proceedings of the first international conference on Teaching of Statistics* (pp. 766-783). Sheffield, the UK: Teaching Statistics Trust.

Gürbüz, R. (2005). Olasılık kavramlarının öğretimi için örnek çalışma yapraklarının geliştirilmesi. *Çukurova Üniversitesi Eğitim Fakültesi Dergisi, 31*(2), 111-123.

Gürbüz, R. (2006). Olasılık konusunun öğretiminde kavram haritaları. *Yüzüncü Yıl Üniversitesi Eğitim Fakültesi Dergisi*, *3*(2), 133-151.

Gürbüz, R. (2007). The effects of computer aided instruction on students' conceptual development: A case of probability subject. *Eurasian Journal of Educational Research*, *28*(8), 75-87

Gürbüz, R. (2008). Olasılık konusunun öğretiminde kullanılabilecek bilgisayar destekli materyal. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi, 8*(15), 41-52.

Gürbüz, R. (2008). Olasılık konusunun öğretiminde kullanılabilecek bilgisayar destekli bir materyal. *Mehmet Akif Ersoy Üniversitesi Eğitim Fakültesi Dergisi, 15*, 41-52.

Gürbüz, R., Çatlıoğlu, H., Birgin, O., & Erdem, E. (2010). Etkinlik temelli öğretimin 5. sınıf öğrencilerinin bazı olasılık kavramlarındaki gelişimlerine etkileri: Yarı deneysel bir çalışma. *Kuramdan Uygulamaya Eğitim Bilimleri, 10*(2), 1021-1069.

Güven, Y. (2000). *Sezgisel düşünme ve matematik: Ev ve okul ortamında uygulama örnekleriyle*. İstanbul, Turkey: Ya-Pa Yayınları.

Güven, S. (2006). *Toplumbilimde araştırma yöntemleri.* Bursa, Turkey: Ezgi Yayınevi.

Hake, R. (1998). Interactive-engagement vs traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*, 64-74.

Havill, D. E. (1998). *Traditional and nontraditional probability contexts: The role of instruction-related intuitions and everyday intuitions in students' reasoning about sequences of events*. (Unpublished doctoral dissertation). University of California, Santa Barbara, CA.

Herman, M. (2007). What students choose to do and have to say about use of multiple representation. *The Journal of Computers in Mathematics and Science Teaching, 26*(1), 27-54.

Ilgun, M., & Işıksal, M. (2012). Pre-service elementary mathematics teachers' misconceptions regarding probability concepts. *Paper presented at the European conference on Educational Research 2012,* (September 18-21), University of Cadiz, Cadiz, Spain.

Jendraszek, P. (2008). *A study on misconceptions of probability among future teachers of mathematics.* (Unpublished doctoral dissertation). Columbia University, Graduate School of Arts and Science, New York, NY.

Jones, G. A., Langrall, C., & Mooney, E. S. (2007). Research in probability: Responding to classroom realities. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 909-955). Greenwich, CT: Information Age Publishing, Inc. and National Counsil of Teachers of Mathematics.

Jung, I. (2002). *Student representation and understanding of geometric transformations with technology experience*. (Unpublished doctoral dissertation). University of Georgia, Athens, GA.

Lecoutre M. P. (1992). Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics, 23*, 557-568.

Lecoutre, M. P., & Durand, J. L. (1988). Probabilistic judgments and cognitive models: Study of a random situation. *Educational Studies in Mathematics, 19*, 357-368.

Kahneman, D. & Tversky, A. (1972). Subjective probability: A judgment of representativeness. *Cognitive Psychology, 3*, 430-454

Kahneman, D., & Tversky, A. (1982). Subjective probability: A judgment of representativeness. In D. Kahneman, P. Slovic, & A. Tversky (Eds.), *Judgment under uncertainty: Heuristics and biases*. (pp. 32-47). Cambridge, the UK: Cambridge University Press.

Kazak, S., & Confrey, J. (2007). Elementary school students' intuitive conceptions of random distributions. *International Electronic Journal of Mathematics Education, 2*(3), 227-244.

Kazak, S. (2008). Öğrencilerin olasılık konularındaki kavram yanılgıları ile öğrenme zorlukları. In M. F. Özmantar, E. Bingölbali, & H. Akkoç (Eds.), *Matematiksel kavram yanılgıları ve çözüm önerileri.* (pp. 121-150). Ankara, Turkey: PegemA Yayınevi.

Kazak, S. (2009). Olasılık konusu öğrencilere neden zor gelmektedir? In E. Bingölbali, & M. F. Özmantar (Eds.), *İlköğretimde karşılaşılan matematiksel zorluklar ve çözüm önerileri* (pp. 216-239). Ankara, Turkey: PegemA Yayınevi.

Keitel, C., & Kilpatrick, J. (2005). Mathematics education and common sense. In J. Kilpatrick, C. Hoyles, & O. Skovsmose (Eds.), *Meaning in mathematics education* (pp. 105-128). Dordrecht, The Netherlands: Kluwer.

Keller, C., Siegrist, M., & Gutscher, H. (2006). The role of the affect and availability heuristics in risk communication. *Risk Analysis, 26*(3), 631-639.

Kennis, J. R. (2006). *Probabilistic misconceptions across age and gender*. (Unpublished doctoral dissertation). Graduate School of Arts and Sciences, Columbia University, New York, NY.

Köğce, D., & Baki, A. (2009). Matematik öğretmenlerinin yazılı sınav soruları ile ÖSS sınavlarında sorulan matematik sorularının Bloom taksonomisine göre karşılaştırılması, *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi, 26*, 70-80.

Konold, C., Pollatsek, A., Well, A., Lohmeier, J., & Lipson, A. (1993). Inconsistencies in students' reasoning about probability. *Journal for Research in Mathematics*, *24*, 392-414.

Koray, Ö., & Bal, Ş. (2002). Fen öğretiminde kavram yanılgıları ve kavramsal değişim stratejisi. *Gazi Üniversitesi Kastamonu Eğitim Fakültesi Dergisi, 10*(1), 83-90.

Kvatinsky, T., & Even, R. (2002). Framework for teacher knowledge and understanding about probability. In B. Phillips (Ed.), *The proceedings of the sixth international conference on the Teaching of Statistics* (CD). Cape Town, South Africa: International Statistical Institute. Retrieved from http://iase-web.org/documents/papers/icots6/6a4_kvat.pdf.

Li, J., & Pereira-Mendoza, L. (2002). Misconceptions in probability. In B. Phillips (Ed.), *The proceedings of the sixth international conference on Teaching of Statistics* (CD). Cape Town, South Africa: International Statistical Institute. Retrieved from http://iase-web.org/documents/papers/icots6/6g4_jun.pdf.

Li, J. (2000). *Chinese students' understanding of probability.* (Unpublished doctoral dissertation). National Institute of Education, Nanyang Technological University, Singapure.

Livne, T. (1996). *Examination of high school students' difficulties in understanding the change in surface area, volume and surface area/volume ratio with the change in size and/or shape of a body*. (Unpublished master's thesis). Tel Aviv University, Tel Aviv, Israel.

Marek, E. A., Cawon , C. C., & Cavallo, A. M. L. (1994). Students' misconceptions about diffusion: How can they be eliminated. *The American Biology Teacher, 56*(2), 74-77.

Marques, J. F., & McCall, C. (2005). The application of interrater reliability as a solidification instrument in a phenomenological study. *The Qualitative Report, 10*(3), 439-462.

McMillan, J. H., & Schumaher, S. (2010). *Educational research: Evidence based inquiry*. Boston, MA: Pearson.

Memnun, D. S. (2008). Olasılık kavramının öğrenilmesinde karşılaşılan zorluklar, bu kavramların öğrenilmeme nedenleri ve çözüm önerileri. *İnönü Üniversitesi Eğitim Fakültesi Dergisi, 9*(15), 89-101.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass Inc.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2005a). *Milli Egitim Bakanligi Talim ve Terbiye Kurulu Baskanligi orta öğretim matematik (9-12. sınıflar) dersi ögretim programı.* Ankara, Turkey: MEB Yayınları.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2005b). *6-8. sınıf matematik programı*. Ankara, Turkey: MEB Yayınları.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2009). *İlköğretim matematik dersi 6-8. sınıflar öğretim programı ve klavuzu.* Ankara, Turkey: MEB Yayınları.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2011). *Ortaöğretim matematik dersi öğretim programı*. Ankara, Turkey: MEB Yayınları.

Milli Eğitim Bakanlığı [Ministry of National Education]. (2013). *Ortaokul matematik dersi (5, 6, 7 ve 8. Sınıflar) öğretim programı*. Ankara, Turkey: MEB Yayınevi.

Mut, A. İ. (2003). *Investigation of students' probabilistic misconceptions.* (Unpublished master's thesis). Middle East Technical University, Ankara, Turkey.

Myers, D. G. (2002). *Intuition: Its powers and perils*. New Heaven & London, CT: Yale University Press.

Nakiboğlu, C. (2006). *Fen ve teknoloji öğretimi: Fen ve teknoloji öğretiminde yanlış kavramalar.* Ankara: Pegema Yayıncılık.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Virginia, VA: NCTM.

Nicolson, C. P. (2005). In chance fair? One student's thoughts on probability. *Teaching Children Mathematics, 12*(2), 83-89.

Nisbett, R., Krantz, D., Jepson, C., & Kunda, Z. (1983). The use of statistical heuristics in everyday inductive reasoning. *Psychological Review, 90*(4), 339-363.

Ojeda, A. M. (1999). Training and practice of teachers of probability: An epistemological stance. In Bills, L. (Ed.), *The proceedings of the British Society for Research into Learning Mathematics* (pp. 55-60). Nottingham, the UK: University of Nottingham.

Ojose, B. (2011). Mathematics literacy: Are we able to put the mathematics we learn into everyday use? *Journal of Mathematics Education, 4*(1), 89-100.

Organization for Economic Cooperation and Development. (1999). *Measuring student knowledge and skills: A new framework for assessment*. Paris, France: OECD.

Papaieronymou, I. (2009). Recommended knowledge of probability for secondary mathematics teachers. Working Group 3. *The proceedings of 6th congress of European Research in Mathematics Education* (p. 358-367). Lyon, France.

Paparistodemou, E., Potari, D., & Pitta, D. (2006). Prospective teachers' awareness of young children's stochastic activities. *The proceeding 7th international conferece on Teaching Statistics*, Salvador, Bahia, Brazil. Retrieved from http://iase-web.org/documents/papers/icots7/2A1_PAPA.pdf.

Patton, M. Q. (1987). *How to use qualitative methods in evaluation.* California: Sage Pub.

Piaget, J. (1965). *The child's conception of number*. New York, NY: Norton.

Pines, A. L. (1985). Towards a taxonomy of conceptual relations. In L. West and A. L. Pines (Eds.), *Cognitive structure and conceptual change* (pp.101-116). New York, NY: Academic Press.

Polaki, M. V. (2002a). Using instruction to identify key features of Basotho elementary students' growth in probabilistic thinking. *Mathematical Thinking and Learning*, *4*, 285-314.

Polaki, M. V. (2002b). Using instruction to identify mathematical practices associated with Basotho elementary students' growth in probabilistic thinking. *Canadian Journal for Science, Mathematics and Technology Education, 2*, 357-370.

Pratt, D. (2000). Making sense of the total of two dice. *Journal for Research in Mathematics Education*, *31*(5), 602-625.

Radakovic, N. (2009). *Elementary school students' understanding of randomness*. (Unpublished master's thesis). University of Toronto, Toronto, Canada.

Riccomini, P. J. (2005). Identification and remediation of systematic error patterns in subtraction. *Learning Disability Quarterly, 28*, 233-242.

Rorty, R. (1967). Intution. In P. Edwards (Ed.), *The encyclopedia of philosophy* (Vol. 4, pp. 204-212). New York, NY: McMillan Publishing Company.

Rowan B., Schilling, S. G., Ball, D. L., & Miller, R. (2001). *Measuring teachers' pedagogical content knowledge in surveys: An exploratory study.* Ann Arbor, MI: Consortium for policy research in education, Study of Instructional Improvement, University of Michigan. Retrieved from http://www.sii.soe.umich.edu/documents/pck%20final%20report%20revised%20BR100901.pdf

Rubel, L. H. (1996). *Conditional reasoning: How are if-then statements understood by high school students?* (Unpublished master's thesis). Tel Aviv University, Tel Aviv, Israel.

Rubel, L. H. (2002). *Probabilistic misconceptions: Middle and high school students' mechanisms for judgments under uncertainty*. (Unpublished doctoral dissertation). Columbia University, New York, NY.

Rubel, L. H. (2007). Middle school and high school students' probabilistic reasoning on coin tasks. *Journal for Research in Mathematics Education, 38*(5), 531-556.

Shaughnessy, J. M. (1977). Misconceptions of probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level. *Educational Studies in Mathematics, 8*, 295-316.

Shaughnessy, J. M. (1992). Research on probability and statistics: Reflections and directions. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494). New York, NY: McMillan Publishing Company.

Shirley, D., & Langan-Fox, J. (1996). Intuition: A review of the literature. *Psychological Reports, 79*, 563-584

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-31.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Skelly, K. M., & Hall, D. (1993). The development and validation of a categorization of sources of misconceptions in chemistry. *The proceeding of the third international seminar on Misconceptions and Educational Strategies in Science and Mathematics* (pp. 1496-1535), Ithaca, NY.

Stake, R. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.

Stavy, R., & Tirosh, D. (1996). Intuitive rules in mathematics and science: The case of "the more of A – the more of B". *International Journal of Science Education, 18*(6), 653-667.

Stavy, R., & Tirosh, D. (2000). *How students (Mis-)understand science and mathematics: Intuitive rules*. New York, NY: Teacher College Press.

Stavy, R., Babai, R., Tsamir, P., Tirosh, D., Lin, F. L., & Mcrobbie, C. (2006). Are intuitive rules universal? *International Journal of Science and Mathematics Education, 4,* 417-436.

Stavy, R., Strauss, S., Orpaz, N., & Carmi, G. (1982). U-shaped behavioral growth in ratio comparisons, or that's funny I would not have thought you were U-ish. In S. Strauss & R. Stavy (Eds.), *U-shaped behavioral growth* (pp. 11-36). New York, NY: Academy Press.

Steinbring, H. (1991).The theoretical nature of probability in classroom. In R. Kapadia & M. Borovcnik (Eds.), *Chance encounters: Probability in education* (pp.135-168). Dordrecht, The Netherlands: Kluwer.

Stohl, H. (2005). Probability in teacher education and development. In G. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 345–366). New York, NY: Springer.

Stohl, H., & Tarr, J. E. (2002). Developing notions of inference with probability simulation tools. *Journal of Mathematical Behavior, 21*(3), 319-337.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory.* (2nd ed.). London, the UK: Sage.

Şengül, S., & Ekinözü, S. (2004). Permütasyon ve olasılık konusunun öğretiminde canlandırma kullanılmasının öğrenci başarısına ve hatırlama düzeyine etkisi**.** *XIII. Ulusal Eğitim Bilimleri Kurultayı,* İnönü Üniversitesi, Eğitim Fakültesi, Malatya, Turkey.

Tall, D. (1989). The nature of mathematical proof. *Mathematics Teaching, 127*, 28-32.

Tekkaya, C., Çapa, Y., & Yılmaz, Ö. (2000). Biyoloji öğretmen adaylarının genel biyoloji konularındaki kavram yanılgıları. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 18*, 140-147.

Tirosh, D., & Stavy, R. (1999a). Intuitive rules: A way to explain and predict students' reasoning. *Educational Studieps in Mathematics, 38*, 51–66.

Tirosh, D., & Stavy, R. (1999b). Intuitive rules and comparison tasks. *Mathematical Thinking and Learning, 1*(3), 179-194.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*, 5–25.

Tversky, A., & Kahneman, D. (1971). Belief in law of small numbers. *Psychological Bulletion, 76(*2). 105-110.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science, 185*, 1124-1131.

Tversky, A., & Kahneman, D. (1983a). Availability: A heuristic for judging frequency and probability. In B. J. Baars, W. P. Banks, & J. B. Newman (Eds.), *Essential sources in the scientific study of consciousness* (pp. 677-696). Cambridge, MA: The Massachusetts Institute of Technology Press.

Tversky, A., & Kahneman, D. (1983b). Extensional versus intuitive reasoning: The conjuction fallacy in probability judgment. *Psychological Review*, *90*, 293-315.

Ubuz, B. (1999). 10. ve 11. sınıf öğrencilerinin geometride kavram yanılgıları ve cinsiyet farklılıkları. *Paper presented at Öğretmen Eğitiminde Çağdaş Yaklaşımlar Sempozyumu*, (March 18-24), Dokuz Eylül Üniversitesi, İzmir.

Watson, J. M. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education, 4*, 305-337.

Watson, J. M., & Kelly, B. A. (2007). The development of conditional probability reasoning. *International Journal of Mathematical Education in Science and Technology, 38*(2), 213–235.

Way, J. (2003). The development of young children's notions of probability. *The proceeding of 3rd conference of the european society for research in mathematics education,* Thematic Group 5. Bellaria, Italy. Retrieved on February 14, 2012, from http://www.dm.unipi.it/~didattica/CERME3/proceedings/Groups/TG5/TG5_way_cerme3.pdf.

Weber, K., & Alcock, L. (2004). Semantic and syntactic proof productions. *Educational Studies in Mathematics, 56*, 209-234.

Yazıcı, E. (2002). *Permütasyon ve olasılık konusunun buluş yoluyla öğretilmesi*. (Unpublished master's thesis). Karadeniz Teknik University, Trabzon, Turkey.

Yıldırım, A., & Şimşek, H. (2006). *Sosyal bilimlerde nitel araştırma yöntemleri* (5nd ed.). Ankara: Seçkin Yayıncılık.

Yin, R. K. (2003). *Case study research design and methods* (3rd ed.). San Francisco, CA: Sage Publications.

Zahner, D. C. (2005). *Using clinical interviewing and problem solving protocols to uncover the cognitive processes of probability problem solvers.* (Unpublished doctoral dissertation). Columbia University, New York, NY.

APPENDIX A

PROTOCOL FOR INTERVIEW I

1. Öğretmenin kendisini tanıtmasına yönelik sorular. (Demografik bilgiler)
    - Kaç yıllık öğretmensiniz?
    - Mezuniyet veya formasyon eğitiminiz var mı?
    - Daha önce nerelerde çalıştınız?
2. Üniversitede veya başka bir yerde özel olarak olasılıkla ilgili bir çalışma yapıldı mı?
3. Olasılık konusu için öğrencilerde olması gereken özelliklerden bahseder misiniz?
4. Olasılık öğrenimine başlamadan önce öğrencilerin hangi konuları bilmeleri gerekmektedir? Olasılık konusuna geçmeden önce öğrencilerin iyi olduğunu düşündüğünüz konular nelerdir? Eksiklik gördüğünüz yerlerde nelerdir? Bu eksiklikler için neler yapmayı planlıyorsunuz?
5. Öğrencilerin eksikliklerini belirlemeye yönelik yaptığınız şeyler var mı?
6. Olasılık konusu anlatırken hangi kaynaklardan yararlanıyorsunuz? Bu kaynakları nasıl kullanıyorsunuz?
7. Konu anlatımında, kavram öğretiminde, problem çözümünde, öğrenciler zorluklarla karşılaştıklarında nelere dikkat ediyorsunuz?
8. Daha önceki deneyimlerinize dayanarak, öğrenciler hangi noktalarda zorlanmaktadırlar? Zorlukları nasıl belirliyorsunuz? Konu anlatımında bunları dikkate alıyor musunuz? Bu zorlukları nasıl önlemeye çalışıyorsunuz? Hangi strateji, yöntem veya teknikleri kullanıyorsunuz? Varsa neler?
9. Daha önceki deneyimlerinize dayanarak, öğrenciler genellikle hangi eksikliklerle veya kavram yanılgılarıyla gelmektedirler? Bunları önlemeye yönelik uyguladığınız yöntemler var mı? Varsa neler? Neden?

APPENDIX B

PROTOCOL FOR INTERVIEW II

1. Öğrenciler olasılık konusunda en çok hangi noktalarda zorlandılar? Sebepleri nelerdir?
2. Öğrencilere uygulanan olasılık testindeki birinci soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
3. Testteki ikinci soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
4. Testteki üçüncü soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
5. Testteki dördüncü soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
6. Testteki beşinci soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
7. Testteki altıncı soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
8. Testteki yedinci soruda, öğrencilerin düşübileceği hata veya kavram yanılgıları nelerdir? Bu hata veya kavram yanılgılarını gidermeye yönelik ne yapılabilir?
9. Belirttiğiniz önerilerin bir kısmını sınıfta yapamamanızın sebepleri nelerdir?

APPENDIX C

PROBABILITY TEST OF INTUITION (MIDDLE SCHOOL VERSION)

Sevgili öğrenciler,

"Aşağıda matematik programınızda yer alan İstatistik ve Olasılık öğrenme alanındaki Olasılık konularla ilgili sorulardan oluşan bir test yer almaktadır. Vereceğiniz cevaplar sadece bilimsel bir araştırmada veri olarak kullanılacak ve tamamen gizli tutulacaktır. Bu nedenle soruları kaygılanmadan ve içtenlikle cevaplamanız yapılan bu çalışmanın doğru bir şekilde değerlendirilmesi açısından önem taşımaktadır."

1- Bir şans oyununda 1'den 49'a kadarki sayılardan (1 ve 49 dahil) altı rakam seçiliyor. Bu sayıları doğru tahmin eden kişi yarışmayı kazanmaktadır. Çiğdem, Merve ve Hakan'ın tahminleri aşağıdaki gibidir.

Çiğdem	: 1, 2, 3, 4, 5, 6
Merve	: 44, 45, 46, 47, 48, 49
Hakan	: 39, 1, 17, 33, 8, 27

Oyuncuların oyunu kazanma olasılıklarını karşılaştırınız. Cevabınızı açıklayınız.

2- Hilesiz bir para üç defa atılıyor ve her seferinde "tura" geliyor. Eğer para dördüncü defa atılırsa, en olası sonuç ne olur? Cevabınızı açıklayınız.

3- Bir hastanede yeni doğan 3 çocuğun en az 2'sinin erkek olma olasılığı ile yeni doğan 300 çocuğun en az 200'ünün erkek olma olasılıklarını karşılaştırınız. Cevabınızı açıklayınız.

4- 16 tane oyun kartı, bir grup öğrenciye rastgele birkaç defa dağıtılmıştır. İki farklı dağılım aşağıdaki gibidir. Bu dağılımlardan hangisinin ortaya çıkma olasılığı daha yüksektir? Cevabınızı açıklayınız.

	Dağılım I	**Dağılım II**
Ayşe	4	4
Mehmet	4	3
Fatma	4	4
Hüseyin	4	5

5- İki zar atılıyor. Üst yüze gelen sayı çiftleri için 4-4 çiftinin gelme olasılığı ile 4-3 çiftinin gelme olasılığını karşılaştırınız. Cevabınızı açıklayınız.

6- Ayşe, Ali ile Ahmet bir oyun oynuyorlar. Bu oyunda, bir para oyunculardan biri oyunu kazanana kadar devamlı atılmaktadır. Her oyuncu, yazı (Y) ve turadan (T) oluşan sıralamalar seçmiştir. Buna göre, hangi sıralama en önce gelirse oyunu o kişi kazanmaktadır. Oyuncuların seçtikleri sıralamalar aşağıdaki gibidir.

Ayşe	: TYTTT
Ahmet	: YTYTTT
Ali	: TYTTTT

Bu sırdalamalara göre, oyuncuların oyunu kazanma olasılıklarını en yüksekten en düşüğe doğru sıralayınız. Cevabınızı açıklayınız.

APPENDIX D

PROBABILITY TEST OF INTUITION (HIGH SCHOOL VERSION)

Sevgili öğrenciler,

"Aşağıda matematik programınızda yer alan İstatistik ve Olasılık öğrenme alanındaki Olasılık konularla ilgili sorulardan oluşan bir test yer almaktadır. Vereceğiniz cevaplar sadece bilimsel bir araştırmada veri olarak kullanılacak ve tamamen gizli tutulacaktır. Bu nedenle soruları kaygılanmadan ve içtenlikle cevaplamanız yapılan bu çalışmanın doğru bir şekilde değerlendirilmesi açısından önem taşımaktadır."

1- Bir şans oyununda 1'den 49'a kadarki sayılardan (1 ve 49 dahil) altı rakam seçiliyor. Bu sayıları doğru tahmin eden kişi yarışmayı kazanmaktadır. Çiğdem, Merve ve Hakan'ın tahminleri aşağıdaki gibidir.

Çiğdem	: 1, 2, 3, 4, 5, 6
Merve	: 44, 45, 46, 47, 48, 49
Hakan	: 39, 1, 17, 33, 8, 27

Oyuncuların oyunu kazanma olasılıklarını karşılaştırınız. Cevabınızı açıklayınız.

2- Hilesiz bir para üç defa atılıyor ve her seferinde "tura" geliyor. Eğer para dördüncü defa atılırsa, en olası sonuç ne olur? Cevabınızı açıklayınız.

3- Bir hastanede yeni doğan 3 çocuğun en az 2'sinin erkek olma olasılığı ile yeni doğan 300 çocuğun en az 200'ünün erkek olma olasılığını karşılaştırınız. Cevabınızı açıklayınız.

4- 16 tane oyun kartı, bir grup öğrenciye rastgele birkaç defa dağıtılmıştır. İki farklı dağılım aşağıdaki gibi oluşmuştur. Bu dağılımlardan hangisinin ortaya çıkma olasılığı daha yüksektir? Cevabınızı açıklayınız.

	Dağılım I	**Dağılım II**
Ayşe	4	4
Mehmet	4	3
Fatma	4	4
Hüseyin	4	5

5- İki zar atılıyor. Üst yüze gelen sayı çiftleri için 4-4 çiftinin gelme olasılığı ile 4-3 çiftinin gelme olasılığını karşılaştırınız. Cevabınızı açıklayınız.

6- Ayşe, Ali ile Ahmet bir oyun oynuyorlar. Bu oyunda, bir para oyunculardan biri oyunu kazanana kadar devamlı atılmaktadır. Her oyuncu, yazı (Y) ve turalardan (T) oluşan sıralamalar seçmişlerdir. Buna göre, hangi sıralama en önce gelirse oyunu o kişi kazanmaktadır. Oyuncuların seçtikleri sıralamalar aşağıdaki gibidir.

Ayşe	: TYTTT
Ahmet	: YTYTTT
Ali	: TYTTTT

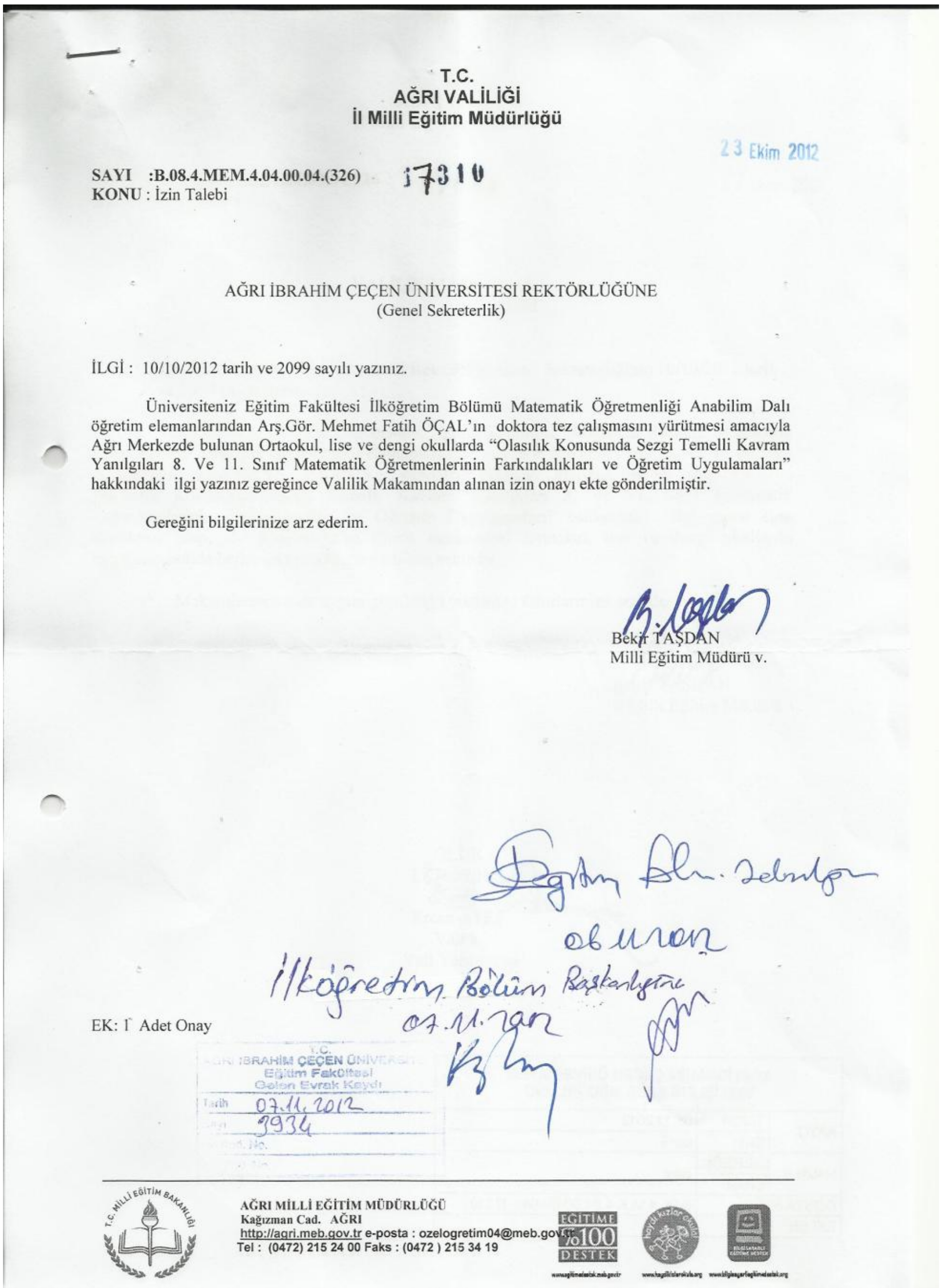
Bu göre, oyuncuların oyunu kazanma olasılıklarını en yüksekten en düşüğe doğru sıralayınız. Cevabınızı açıklayınız.

7- Bir torbada eşit sayıda mavi ve kırmızı toplar bulunmaktadır. Çekilen top tekrar torbaya atılmamaktadır. Çekilen iki top için aşağıdaki durumların olasılıklarını karşılaştırınız. Cevabınızı açıklayınız.

a) Çekilen ilk top mavi ise ikinci topun mavi olma olasılığı
b) Çekilen ikinci top mavi ise ilk topun mavi olma olasılığı

APPENDIX E

PERMISSION LETTER FROM PROVINCIAL DIRECTORATE OF NATIONAL EDUCATION



T.C.
AĞRI VALİLİĞİ
İl Milli Eğitim Müdürlüğü

23 Ekim 2012

SAYI :B.08.4.MEM.4.04.00.04.(326) 17310
KONU : İzin Talebi

AĞRI İBRAHİM ÇEÇEN ÜNİVERSİTESİ REKTÖRLÜĞÜNE
(Genel Sekreterlik)

İLGİ : 10/10/2012 tarih ve 2099 sayılı yazınız.

Üniversiteniz Eğitim Fakültesi İlköğretim Bölümü Matematik Öğretmenliği Anabilim Dalı öğretim elemanlarından Arş.Gör. Mehmet Fatih ÖÇAL'ın doktora tez çalışmasını yürütmesi amacıyla Ağrı Merkezde bulunan Ortaokul, lise ve dengi okullarda "Olasılık Konusunda Sezgi Temelli Kavram Yanılgıları 8. Ve 11. Sınıf Matematik Öğretmenlerinin Farkındalıkları ve Öğretim Uygulamaları" hakkındaki ilgi yazınız gereğince Valilik Makamından alınan izin onayı ekte gönderilmiştir.

Gereğini bilgilerinize arz ederim.

Bekir TAŞDAN
Milli Eğitim Müdürü v.

EK: 1 Adet Onay

T.C.
AĞRI İBRAHİM ÇEÇEN ÜNİVERSİTESİ
Eğitim Fakültesi
Gelen Evrak Kaydı
Tarih 07.11.2012
Sayı 3934

AĞRI MİLLİ EĞİTİM MÜDÜRLÜĞÜ
Kağızman Cad. AĞRI
http://agri.meb.gov.tr e-posta : ozelogretim04@meb.gov.tr
Tel : (0472) 215 24 00 Faks : (0472) 215 34 19

T.C.
AĞRI VALİLİĞİ
İl Milli Eğitim Müdürlüğü

SAYI : **B.08.4.MEM.4.04.00.04.326** 17049 22 Ekim 2012
KONU : İzin talebi

VALİLİK MAKAMINA
AĞRI

İLGİ : Ağrı İbrahim ÇEÇEN Üniversitesi Rektörlüğü Genel Sekreterliğinin 10/10/2012 tarih ve 2099 sayılı yazısı.

Ağrı İbrahim ÇEÇEN Üniversitesi Eğitim Fakültesi İlköğretim Bölümü Matematik Öğretmenliği Anabilim Dalı öğretim elemanlarından Arş.Gör. Mehmet Fatih ÖÇAL'ın doktora tez çalışmasını yürütmesi amacıyla Ağrı Merkezde bulunan Ortaokul, lise ve dengi okullarda "Olasılık Konusunda Sezgi Temelli Kavram Yanılgıları 8. Ve 11. Sınıf Matematik Öğretmenlerinin Farkındalıkları ve Öğretim Uygulamaları" hakkındaki ilgi yazısı ekte sunulmuş olup, söz konusu tezin İlimiz merkezdeki Ortaokul, lise ve dengi okullarda uygulanmasında herhangi bir sakınca görülmemektedir.

Makamlarınızca da uygun görüldüğü takdirde ; Olurlarınıza arz ederim.

Bekir TAŞDAN
İl Milli Eğitim Müdürü V.

OLUR
1.8/10/2012
Ercan ATEŞ
Vali a.
Vali Yardımcısı

AĞRI MİLLİ EĞİTİM MÜDÜRLÜĞÜ
Kağızman Cad. AĞRI
http://agri.meb.gov.tr e-posta : ozelogretim04@meb.gov.tr
Tel : (0472) 215 24 00 Faks : (0472) 215 34 19

APPENDIX F

CATEGORIES AND CODES APPEARED IN THE ANALYSIS OF TESTS RESULTS, CLASSROOM OBSERVATIONS AND INTERVIEWS

Table F.1 Main Themes Appeared in Students' Responses to PTI Items and in Observations

	Themes for Students' Intuitively-based Misconceptions
Students' Intuitively-based Misconceptions were	Availability Heuristics Representativeness Heuristics: Negatively and Positively Recency Effect Representativeness Heuristics: Sample Size Effect Representativeness Heuristics: Outcome Approach Simple and Compound Events Conjunction Fallacy Conditional Probability Intuitive Rules: the More of A – the More of B Intuitive Rules: the Same of A – the Same of B

Table F.2 Categories and Codes for Teachers' Awareness Appeared in the Interviews

Category I. Teachers' Opinions about Necessary Pre-Knowledge for Students	
The necessary pre-knowledge for students were	Fractions Simplifications Percentages Arithmetic Operations Sets Rational Numbers Factorial Concept Permutations Combinations
Category II. Teachers' Opinions about How to Determine Students' Understandings	
Teachers determined students' understanding by	Evaluating students responses to questions Expecting students' own definitions and explanations Conducting diagnostic tests Conducting formative tests Conducting summative tests Asking whether students understand the concept/solution Believing their experiences Considering students' attendance to lessons

Table F.2 (Continued)

Category III: Teachers' Opinions about the Reasons for Students' Difficulties in Probability	
The reasons for students' difficulties in probability were	Insufficiency in readiness Insufficiency in reading-comprehension Rote memorizations Unable to imagine Unable to relate with daily life Low level of students' understanding Necessity of thinking Not being open to interpretation Unable to synthase the facts Unable to construct patterns Existence of high school or university entrance exams Carelessness Not studying regularly Insufficient course book Unabile to understand the logic of the probability Fear
Category IV. Teachers' Opinions about the Use of Materials and Resources in Lessons	
The materials and resources used were	Course books Supplementary books Visual materials Teachers' own questions
Category V. Teachers' Opinions about Possible Teaching Practices in Teaching Probability	
The teaching practice to be done were	Meeting students' readiness Direct instruction Giving daily life examples Using visual materials, visualizing the events (dice, coin, boxes with marbles) Peer to peer instruction Showing the shortcuts Solving different types of questions Providing students with real life experiences Directing students to think
Category VI. Teachers' Opinions about Students' Difficulties and Misconceptions in Probability	
Students were unable to	Understand the basic concepts Determine the set of expected elements Determine which formula to use Determine sample size Determine whether the event is dependent or independent Determine whether the event is independent or mutually exclusive Dee the difference among theoretical, experimental and subjective probabilities Relate the other subjects with probability (e.g. geometry) Understand the property that the value of probability is between 0 and 1 Relate and use permutation and combination in probability questions Determine the conditional events in conditional probability

Table F.3 Categories and Codes for Teaching Practices appeared in the Observations

Category I. Teaching Practices about Keypoints given	
Teachers' teaching practices about keypoints given in probability were	Giving examples from die, coins, and urns Emphasizing the keywords in the questions Emphasizing biasness or unbiasness of the die/coins in the questions Repeating necessary rote memorizations Writing the events A and B separately Emphasizing and showing the sample sizes of the events Showing how to follow algorithms in solving questions Developing concepts by giving examples from daily life. Generalizing the rules from rote memorizations Giving controversy examples Explanatining the logic in the questions Explaining the shortcuts Showing the appropriate formula for different question types
Category II. Rote memorizations that Teachers Expect Students to Remember	
Rote memorizations in probability were	<u>About the general concepts in probability</u> The value of probabilities of the events is between "1" and "0". If it is empty set, it is impossible event. If the set is universal set, it is certain event. At least two heads means two or more heads <u>About the sample size</u> Sample size of throwing n dice (coins) is 6^n (2^n). E.g. There may appear 4-2 and also 2-4 after throwing two dice. The sample size for throwing two dice and for throwing a die twice is equal. <u>About the theoretical, experimental, and subjective probabilities</u> Excessive number of experiments turns into theoretical probability The probability according to a person is called as subjective probability The usage of formula is about theoretical probability, the experiments that I did or saw are experimental probability. <u>About the use of permutation and combination</u> The permutation is for arrangement. The combination for selection. The arrangement is given, so multiply the events. The arrangement is not given, so use permutation (We need to do arrangement) If you take the balls together, then, use permutation and it is related to selection. <u>About the dependent- independent events</u> If the ball is released into the urn, then, it is independent. If the ball is not released into the urn, then, it is dependent. If the events A and B are independent, then, the probabilities are multiplied. If the first event reduces the number of elements in second event, it is dependent. They are not affecting each other, so it is independent. They are affecting each other, so it is dependent If you take the balls one by one, then, use multiplication rule and it is dependent. <u>About the inclusive-mutually exclusive events</u> If the intersection is not empty, then, the events are inclusive. If the intersection is empty, then, the events are mutually exclusive If it is not mutually exclusive event, add $P(A \cap B)$ into the formula <u>About the conditional probability</u> If there are statements like "known to be", it is conditional probability.

	The sample size of the conditional probability is the sample size of the event B. The expected elements of the conditional probability are the intersection of the events. If the happening of the event A is dependent on the event B, it is conditional probability About other topics “and” means union and addition. or” means intersection and multiplication. The difference between inclusive-mutually exclusive events and dependent-independent events is that the sample sizes are same for former and they are different for further The number of outcomes for addition of two dice can be found with the table below There is no triangle that can be constructed on three points on a line

APPENDIX G

SAMPLE RESPONSES AND SCORING CRITERIA FOR THE QUESTIONS IN THE PROBABILITY TEST OF INTUITION

Table G.1 Sample Responses and Scoring for the Questions in the PTI

Question Number	Correct Response		Partially Correct Response		Incorrect or No Response	
	Score	Justification	Score	Justification	Score	Justification
1	2	Players' probabilities of winning the game are equal, because they all choice six numbers. Or Players' probabilities of winning the game is equal to $\frac{1}{49}\cdot\frac{1}{48}\cdot\frac{1}{47}\cdot\frac{1}{46}\cdot\frac{1}{45}\cdot\frac{1}{44}$ So, they are equal.	1	Players' probabilities of winning the game are equal. (Correct answer with no justification) Or Hakan's probability of winning the game seems greater, but they all have same number of choices. (Incorrect answer with correct justification)	0	Hakan's probability of winning the game is higher than other players' probabilities of winning the game. Or No response
2	2	Getting head or tail has equal probability, because previous events do not influence the further events. Or Their probabilities are equal to ½, because the event is independent of the others.	1	Getting head or tail has equal probability. (Correct answer with no justification)	0	Probability of getting head is higher, because the outcomes of the previous events were also heads. Or No response
3	2	Probability of getting two baby boys out of three new-born babies is equal to ½. However, the probability of getting 200 baby boys out of 300 new born-babies is lesser. Or The probability for the second event is equal to $\frac{\binom{300}{200}+\binom{300}{200}+\cdots+\binom{300}{300}}{2^{300}}$ which is less than first one.	1	Probability of getting two baby boys out of three new born babies is greater than that of getting 200 baby boys out of 300 new-born babies. (Correct answer with no justification)	0	Two events have same probabilities, because the ratios are equal. Or No response

Table G.1 (Continued)						
4	2	Both distributions have equal probabilities, because they are randomly distributed.	1	Both distributions have equal probabilities. (Correct answer with no justification)	0	The distribution I has greater probability, because all players get the same number of cars. Or No response
5	2	Probability of getting a pair of 4-3 has greater probability than that of 4-4, because there are two outcomes for the pair of 4-3.	1	Probability of getting a pair of 4-3 has greater probability than that of 4-4. (Correct answer with no justification)	0	Probability of getting any of the pairs of 4-4 and 4-3 are equal. Or No response
6	2	Ayşe's probability of winning the game is higher, because her choice requires five throws. Other players' probability of winning the game are equal, their choices require six consecutive throws	1	Ayşe's probability of winning the game is higher. (Incorrect comparison for other players' probabilities or correct answer with no justification)	0	All players have the same probability. Or No response
7a	1	Let's say there are n blue and n red balls. The probability getting blue ball after picking a blue ball without replacement is less than one half. Or The probability is equal to $\frac{n-1}{2n-1}$	0.5	The probability is less than one half. (Correct answer with no justification)	0	The probability is equal to ½. Or No response
7b	1	Since the further event does not affect the previous one, the probability of getting blue in the first picking is equal to ½. Or The previous event is independent of the further one, so the probability is equal to ½.	0.5	The probability of getting blue ball is equal to ½. (Correct answer with no justification) Or Probability is less than one half, but the event is independent of the other one. (Incorrect answer with correct justification)	0	Probability is less than one half Or No response

APPENDIX H

INTERVIEW TRANSCRIPT OF ONE TEACHER

Interviewer (I) : Öncelikle sizi tanımaya yönelik birkaç soru olacak. Kaç yıldır öğretmenlik yapmaktasınız.
Cihan (C): 2009 mezunuyum. X Üniversitesi, Fen-Edebiyat Fakültesi, Matematik Bölümü mezunuyum. 2010'de de formasyon eğitimi aldım. Bir süre dersanede çalıştıktan sonra atamam yapıldı. Toplamda iki yıldır öğretmenlik yapmaktayım. Onun dışında hep özel ders verdim.
I: Üniversite de iken veya daha sonrasında olasılık ve istatistik dersleri ile alakalı yaptığınız özel bir çalışma var mıydı?
C: Mezun olduğum için ve yüksek lisans yaptığım için üniversite seviyesinden ders alanlar daha çok bölüm dersleri alıyorlardı. Ama üniversiteye hazırlanan öğrencilerde benden ders alanlar hep meslek lisesi öğrencileriydi. Dolayısıyla meslek lisesi öğrencilerine sınavda en fazla soru çıkan konuları anlattım. O yüzden öğrencilere daha çok sayılar, oran orantı ve problemler üzerine yoğunlaştık. Yani olasılık konusu, temeli olmayan bir öğrenci için ağır gelen bir konu olduğundan dolayı ben kısa sürede temel oluşturabileceğim ve en kısa sürede işlerine yarayabilecek, daha net anlayabilecekleri konular üzerinde durdum.
I: Özel dersler dışında olasılık konusu ile alakalı özel çalıştığınız bir durum var mıydı?
C: Üniversite de istatistik dersi almadık. Bunun sebebi belki olasılık alanında çalışan hocamızın olmayışı olabilir. Seçmeli derslerde de topolojiye yönelik dersler almaya çalıştım. İstatistik olasılık dersi seçmeli varsa da ben dikkat etmedim. Lisanstan itibaren topolojiye ağırlık verdim. Lisans seçmelilerim de metrik uzay, topolojik uzay üzerineydi. Olasılık konusu üzerine çalışma yapmadım özel olarak.
I: Olasılık konusunun ağır olduğunu söylediniz? Sebebinden bahsedebilir misiniz?
C: Öğrencilere ağır gelme sebebi şöyle. Öğrenciler genel olarak ezbere dayalı yapmaktalar. Öğrenci gözünden "bana bir kalıp verilsin ben onu kullanarak soruları çözeyim" mantığı var. Olasılık konusu ise çok düşündürücüdür. Neyin ne olduğunu düşünmesi gerekiyor. Aslında olasılığında bir yapısı var. Yani bir örnek uzayınız var. İstenilen durum ve tüm durumlar var. Aslında basit bir mantığı var. Ama dediğim gibi bu mantığı kurgulamak öğrencilere ağır geliyor.
I: Niçin öğrencilere kurgulamak zor geliyor?
C: İlk sebebi matematik eğitiminin baştan beri ezbere verilmesi. Bir kalıba sokulabilir bir ders olarak anlatılıyor olması.
I: Öğrenci açısından düşünürsek?
C: Aslında olasılık öğrencilere çok garip görünüyor. Olasılık deyince bile korkuyorlar. Onlar için olasılık onları düşünmeye zorlayacak bir konu olarak algılıyorlar. Açıkçası ben şu andaki öğrencilerimde de görüyorum. Düşünmeyelim. Hazır gelsin. Yapalım.

I: Düşünmeyi biraz daha açabilir misiniz?
C: Öğrenci mesela bir parayı atıyor. Bunu hayal etmesi lazım. Yazı ve tura gelir. Bu basit geliyor öğrencilere. Ama diyelim ki bir zar atılıyor. Ve bu zarın üst yüzüne gelen sayıların asal olma olasılıkları, veya üst yüze gelen sayıların toplamının 5'ten küçük olması tarzındaki sorularda zorlanıyorlar. Bunu kurgulamaları gerekiyor. Yani öğrencilerin bunu kafalarında canlandırmaları gerekiyor. Orada bir sıkıntı oluşuyor. Ya da boyalı bir küpün yeşil yüzünün ne kadar gelmesi falan kurgulamak gerekiyor. Ama öğrenci bunları kurgulamaktan kaçıyor.
I: Öğrencilerin özelliklerinden devam edelim. Olasılık konusu için öğrencilerde olması gereken özellikler nelerdir?
C: Öğrencilerde olması gereken en önemli özellikler arasında, bence öğrencilerin düşünmeye yatkın, mantığını çalıştıran, kendini çok fazla kalıba sokmayan olmalarıdır. Hani size belirli bir kalıp veriliyor, diyelim ki ben şu şekilde anlatıyorum. Bu farklı bir şekilde olsaydı nasıl olurdu? Diyelim ki para atılmış ama zar atılsaydı ne olurdu? Ya da altı yüzlü değil de zar sekiz yüzlü olsaydı nasıl olurdu? Hani biraz daha örneklendirebilen, biraz daha farklı düşünebilen öğrenciler başarılı olurlar.
I: Geniş çaplı düşünebilen öğrenci derken neyden bahsediyoruz?
C: Ezberin dışına çıkabilen olmalı. Farklı durumları düşünebilmeli öğrenciler.
I: Bunu nasıl sağlarız öğrencilerde?
C: Ben mesela ders anlatırken öğrencileri düşünmeye yönlendirmeye çalışırım. Diyelim ki ben bir cümle söylüyorum. Öğrencilere "bu cümle başka nasıl ifade edilebilir?" diye soruyorum. Ya da "benim kurduğum cümleyi bir de sen kur." "Ben anlattım bir de sen anlat" şeklinde sağlamaya çalışıyorum. Bunun sebebi kişi anlatırken konuyu daha iyi anlar. Çünkü anlatırken öğrenci kendi cümlelerini kurar. Ve o cümleleri kurarken öğrencinin düşünmesi gerekir. Bana ifade edebilmesi için onu düşünmesi gerekeceği için daha ayrıntılı düşünür belki. Ben özel derste de bunu yapardım. Ben sana anlattım. Akşamda sen babana anlat. Ya da arkadaşına anlat şeklinde yaklaşımlarda bulunuyorum. Akran eğitimi ile iyi olabilir diye düşünüyorum.
I: Olasılık konusuna başlamadan önce illaki bir ön bilgiye ihtiyaç vardır. Hangi konuları bilmelidir ki bu konuyu iyi kavrayabilsin öğrenci?
C: Permütasyon ve kombinasyon konuları bu konudan önce anlatılır. Neden? Çünkü seçim ne demektir? Grup nasıl yapılır? Bu bilinsin. Ya da çarpım ve toplam şeklinde. Öncelikle bunlar anlatıyoruz. Mesela iki gömlek ve üç kravat kaç değişik şekilde giyilebilir. Ya da bir fotoğraf çekilecek, kaç değişik şekilde 5 kişiyi sıralayabiliriz. Ya da doktor ve öğretmen grubundan şu kadar doktor bu kadar öğretmen kaç değişik şekilde seçilebilir. Önce o seçim ve gruplandırmaları gösteriyoruz. Daha doğrusu aslında örnek uzay kurmayı öğretiyoruz.
I: Yani permütasyon ve kombinasyonu bunun için öğreniyorlar.
C: Ve bunun üzerine bir temel ediniyorlar. Daha sonra olasılığı kurabiliyoruz. Bunlar üzerine inşa edebiliyoruz.
I: Permütasyon ve kombinasyon zaten olasılık konusu öncesinde anlatılıyor. Bunların gerektiğini zaten sizde söylüyorsunuz. Peki bunlardan önce en başta hangi konular gerekiyor. Meslek lisesi olarak düşünürsek?
C: Permütasyon ve kombinasyonda öncelikle faktöryel kavramı vardır. Bunlar için faktöryeli bilmeleri lazım. Öncelikle faktörleyin anlatılmış olması gerekir. Daha da öncesinde çarpma, yani dört işlemin iyi bilinmesi gerekmektedir. Çarpma ve bölmede sorunsuz olacak öğrenci. Üzerine faktöryel kavramının ne olduğu ve

faktöryellerle ilgili soru çözümü yapabilmeli ve çok iyi bilmesi lazım. Bunlar yapıldığı zaman öğrenci hangi soruda permütasyon, hangi soruda kombinasyon kullanıldığını öğrenecek. Üzerine artık olasılık anlatılabilir.
I: Hangisinin kullanılacağını öğrenci nasıl anlıyor?
C: Onu da sorularda şöyle ayırt ediyoruz. Eğer bir sıralama yapılıyorsa bu permütasyondur. Seçim yapılıyorsa bu kombinasyondur. Yani problem içerisinde belirlenmesi gerekiyor. O yüzden düşünebilme gerekiyor. Yapılan olay seçim mi sıralama mı? İlk önce bunu belirlemesi gerekiyor. Daha sonra o problem verildiğinde seçim mi yapılıyor, yoksa bir topluluk içerisinde bir seçim mi yapılıyor. Sıralama yapılıyorsa permütasyon kullanılacak. Seçim yapılıyorsa bir gruplaştırma yapılıyorsa kombinasyon kullanılır. O ayrımı yaptıktan sonra hangisi kombinasyon hangisi permütasyon olduğunu belirlenebilir.
I: Olasılık konusuna gelmeden önce öğrenciler açısından hangi konularda iyidirler, hangi konularda eksiklikleri vardır?
C: Öğrenciler faktöryel konusunda mesela 5! 4! Nedir denildiğinde yapılabiliyor. Bunlarda problem yok. Çünkü ard arda çarpılıyor. Şöyle bir durum oluştuğunda sıkıntı çıkıyor. n! Veya (n-1)! Denildiğinde sıkıntılar ortaya çıkıyor. Bir de soruda seçimden mi gruptan mı bahsediliyor? Belirleme de problemler çıkıyor.
I: Bu eksiklikleri gidermek için ne yapmayı planlıyorsunuz.
C: Bunları engellemeye yönelik soruların ayırt edilebilirliklerini arttırmaya çalışıyorum. Özellikle soruda "burada ne yapılıyor" diye soruyorum. Diyelim ki fotoğraf çektiriyoruz. Biz yan yana geldiğimizde ne yapıyoruz. Sıralıyoruz. Ama ben sınıf başkanı seçerken ne yapıyorum. Aranızdan tek tek seçiyorum. Tek tek sıralamıyorum. Seçiyorum. İşte o seçimi ben kombinasyonla yaparım. Fotograf çekiliyorsam veya yan yana oturtuyorsam bu da sıralamadır. Permütasyon kullanılır. Ben orada problemi daha algılanabilir hale getirmeye çalışıyorum.
I: Belirli bir farkındalığınız var bu eksikliklere yönelik. Peki onları belirlemeye yönelik yaptığınız bir şeyler var mı konuya gelmeden önce?
C: Ben direk soruları sorarak soru üzerinden gidiyorum. Öğrencilere "soru hakkında ne düşünüyorsun?" sorusu sorarak ilerliyorum. "Sence bu problemde ne yapılıyor?" "Ya da bizden ne istenmiş?" Öğrencinin cevabına göre de problemleri ortaya çıkartıp üzerine yoğunlaşmaya çalışıyorum.
I: Olasılık konusunda kullandığınız kaynaklar nelerdir?
C: Ben daha çok ders kitaplarından yararlanmıyorum. Milli Eğitimin verdiği kitaplardan yararlanmıyorum. Çünkü kitaplar biraz sözel yapıdalar. Çok fazla konu anlatımına ağırlık verilmiş. Çocuklar okurken kitap hikaye kitabı gibi geliyor ve sıkılıyorlar. Bu yüzden test kitapları ve üniversiteye dönük kitaplardan ya da hiç olmadı kendim sade soru hazırlamaya çalışıyorum.
I: Kitaplar hikaye kitabı gibi dediniz. Peki bu öğrencileri düşünmeye yönlendiriyor mu?
C: Son hazırlanan kitaplar düşünmeye yönlendirmesi için hazırlanmış. İşte ders öncesi hazırlık soruları, ders sonrası soruları falan. Ama buna alışkın öğrenci yapısına sahip değiliz. Yani kendimizden de düşünürsek, biz hiçbir zaman ders öncesi hazırlık soruları ve ders sonrası hazırlık soruları cevaplanmaz. Yani o sayfayı geç denir. Daha çok biz kalıpsal öğrenmeye ve anlamaya o kadar çok alışkınız ki o sorular bize zor geliyor. Ve hiç bakılmıyor açıkçası. O yüzden hikaye gelen kısmı konu anlatımı. Orayı da ben kullanmıyorum açıkçası. Kendi yöntemimle ilerliyorum. Zaten o kitap tüm öğrenciler için hazırlanmış. Ama benim öğrencilerimin bir alt yapı sorunu varsa ya da benim öğrencilerimin çok iyi değilse ben onların seviyesine göre

başlamam gerekir. Yeri geliyor ben permütasyon konusunda ben faktöryel konusundan bahsetmek zorunda kalıyorum. Yani ben normal müfredatı işleyemiyorum zaten. Şartlar uygun olmuyor. Ben gerekli olan yerden başlayıp o noktaya getirmeye çalışıyorum. Özellikle ben bulunduğum okuldan hiçbir zaman müfredattaki gibi işleyemem zaten. Eksiklikleri var. Öğrencilerin eksikliklerini öncelikle kapatmam gerekiyor. Sonra gereken konuları tekrar anlatıyorum. Ama normal zaman diliminde değil de önce alt yapıyı hazırladıktan sonra o konuya sıra geliyor.
I: Yeri gelmişken öğrenci seviyelerinden bahsedelim. Burada öğrencilerin seviyelerinin düşük olduğunu biliyoruz.
C: Evet. Öğrencilerin bazılarında toplama, çıkarma bile sorunlu. Mesela 9. Sınıflarda ben üslü sayıları anlatmam gerekiyor şu an. Ama ben dört işlem çalışıyorum. Ve doğal sayılara biraz başladık. Çözümleme yapıyoruz. İşte AB iki basamaklı sayısının ne olduğu 10A+B. Bunu yapıyoruz. Ya da 369 sayısını çözümleme. Birler onlar yüzler basamağı. Bunlara çalışıyoruz. Ya da -7+5. Tam sayılarda dört işlemlerle uğraşıyoruz. Yani benim normal müfredata öğrencinin hazır bulunuşluğuna göre işleyebilirim. Ama diyelim ki ben şu an ilk konu olasılık olsaydı, benim olasılıktan başlamam imkansızdı. Benim önce onlara çarpmayı öğretmem, sonra faktöryele geçmem, faktöryelden sonra permütasyon, kombinasyon sonra da olasılığa geçebilirdim. Yani ekimde yapılması gereken şeyi ben ancak kasım ayı sonunda ya da aralıkta işleyebileceğim. Bana göre matematik dersi alttan üste çıkarak ilerler. Mesela tarih konusunda ben işte Osmanlı tarihini bilmeden inkılap tarihini az çok anlayabilirim. Çünkü orada bir Atatürk dönemi vardır. Öncesinde Kanuni dönemini bilmenize gerek yoktur. Az çok Osmanlı kurulmuş ve yıkılmıştır. Bunu bilirsiniz. Ama dediğim gibi olasılık için çarpmanızda bile probleminiz varsa olasılık inşa edilmiyor. Bu da ileri ki öğrenmelerini etkiliyor. Yani benim çarpmadaki sıkıntıyı çözmem sonra üzerine olasılık konusunu inşa etmem gerekiyor. Yani olasılıkta çıkan sorunların sebeplerinden bir tanesi geçmiş öğrenmelerindeki eksikliklerinden dolayıdır. Bizim lisede işlediğimiz müfredatın başarıya ulaşmamasının en büyük sebebi ortaokul veya ilkokul temelinin sağlam atılmamış olması. Yani 9. Sınıfta bana gelen öğrencilerin seviyesinin ilkokul 2. Sınıf olmasıdır.
I: Aslında ortaokulda öğrencilerin koşullu olasılık haricinde bilmedikleri olmamasına rağmen liseye geldiklerinde sıfırlıyorlar.
C: Evet. Ben olasılık deyince öğrenciler benim yüzüme bakıyorlar. Hiç duymamış gibiler. Olasılık ne? Sanki bu çocuklara ortaokulda hiç bir şey verilmiyor. Ortaokul hocalarına olumsuz eleştiri vermiyorum ama öğrenciler hiç haberdar olmamış gibi 3 seneyi atlatıp liseye bu çocuklar nasıl geliyor ben onu anlamıyorum. Belki de sınıfların kalabalık oluşu, hocaların tek tek ilgilenemiyor olmaları ya da "anlayın, anlamıyorsanız siz bilirsiniz" tipi yaklaşımlar gibisinden. Yani herkes kişiseldir. Ortaokul hocalarına bu eleştirileri yöneltemeyiz ama şu an benim elimdeki öğrencilerin ilkokul hocalarında bir problem olduğu kesin. Yani o çarpmadaki problem nasıl olmuşta ilkokulda atlanılmış, ortaokulda atlanılmış taki bana gelmiş. Ben şu an problem yaşıyorum. Neden? Üslü sayılarda 2^3 dediğimde 2.3 deniyor bana. Ya da 2.2.2 ye hala 6 diyenler var. Yan yana 2.2.2 yazdığımda bile hala 6 diyen öğrenci var.
I: Altta yatan sebebi nedir bunların?
C: İşte ilkokul hocasının çarpmanın nasıl olduğunu yüzeysel mi geçti bilmiyoruz. Diyelim ki iki basamaklıya iki basamaklı çarpılıyor. Nedir? İşte ilk basamağı çarptıktan sonra ikinci basamakta bir basamak kaydırırsınız. Birçok öğrenci o

basamak kaydırmayı hayatlarında hiç duymamış tamamen alt alta yazıyorlar. O kadar korkunç bir görüntü ki. Biri de öyle anlamış ki iki basamak birden atlıyor. Artık sorun ilkokuldaki o kalabalık sınıftan mı yoksa başka bir şekilde mi olmuş bilmiyorum. Ama sanki çarpım tablosu ezberlenmiş o da unutulmuş onu anlarım. Ama dediğim gibi yöntemin nasıl olacağı, nerede ne yapılıyor, işte burada basamak atlanıyor, ya da ne bileyim toplanırken şöyle toplanıyor tarzından tane tane anlatılmıyor sanki. Çünkü öğrenciler bunların uzaydan yeni gelmiş gibi bakıyorlar suratıma. Diyorum ki bir basamak atlanacak, haaa diyorlar. Daha önce hiç mi duymadın sen. Sıkıntı alt kademelerde var.
I: Konu anlatımına gelirsek. Siz neler yapıyorsunuz? Kavram öğretimi, problem çözümü vb..
C: Terimler noktasında, deney, çıktı, örnek uzay. Terimler işin içine girince öğrencilerde o terimlere takılıp kalma ortaya çıkıyor.
I: Öncelikle kavram öğretiminden başlarsak…
C: Kavram öğreniminde sıkıntı var. En basitinden geometri dersinde nokta dediğimde öğrenci bunu cümle sonuna konan nokta aklına geliyor. Ya da doğru diyorum. Yanlışın zıddı diyor. Ben doğruyu anlatmama rağmen ilk yaptığım quizde halen doğrunun tanımına "yapıldığında doğru görünen şeydir" diyor. Yani kavramlarda çok büyük sıkıntımız var. Bunları zaten görmeden geliyorlar. Yani öğrenci bir noktanın geometrideki varlığından habersiz ki ben nokta dediğimde en basitinden izdir diyecek kağıt üzerindeki. Boyutu olmayan. Ama öğrenci cümle sonuna konan noktadır diyor. Bu ders matematik, Türkçe dersindeki noktadan bahsedecek değilim ki. Yani öğrenci matematikteki kullanımı haricindeki her şeyden bahsediyor. Olasılık konusunda da deney ve çıktı diyorum. Deney deyince akıllarına fen bilgisinde yaptıkları geliyor. Az çok benziyor diyorum. Evet. bizim deneyle oradaki deney birbirlerine benziyor. Ya da çıktı deyince, öğrenci yazıcıdan alınan çıktıdan bahsediyor öğrenci. Tamam diyorum. Bizim çıktı da bunun mantığında. Ben daha çok çağrışım yapılabilecek şeyler yapmaya çalışıyorum. Mesela doğrudaş noktaları anlatırken vatandaştan çağrışım yaptırıyorum. Vatandaş nedir? Aynı vatanı paylaşan insanlara denir. Aynı doğruyu paylaşan doğrulara da doğrudaş denir. Hani oradaki -daş eki ne anlama geliyor. Onu oradan çağrışım yaptırıyorum. Çıktıyı benzer şekilde yazıcıdan aldığım kağıtsa, verdiğiniz kağıt bir şeyler oluyor ve size olmuş bir şekilde geri dönüyor. İşte çıktı olasılıkta ona benzer gibisinden. Yani kavram anlatılırken çağrışım yaptırılabilecek şeyler kullanıyorum.
I: Daha sonraki aşamaları nelerdir peki?
C: Kavramları tanıttım mesela. Örnek uzaydan bahsettim. Ve daha sonra olay çeşitleri, imkansız ve kesin olay. Ben derste daha çok hayatı aktarmaya çalışıyorum ki akıllarında kalsın. Kesin olay, imkansız olay nedir? İmkansız ne demektir diyorum öncelikle. Bir şeyin imkansız olması için ne olması gerekir. O imkansız olabilecek bir durumu bana söylüyor. Sonra ben diyorum ki para dik gelir mi? Veya bir zar da altı yüzü varsa 6 gelebilir ama 7 gelemez. Öğrencilere sorarım. "zarın 7 gelme olasılığı var dersem ne dersiniz?" diye sorarım. Var mı zarda 7. Yok. O zaman bu imkansızdır şeklinde. Yani daha görsel, daha somut, daha hayatta gördüğü, elde tutulur, canlandırabileceği şekilde anlatmaya çalışıyorum. Ayrık ve ayrık olmayan olaylar için. Ayrık nedir diye soruyorum. Farklı. Ayrık. Tamamen birbirinden ayrılmış. Yani iki olay birbirini etkilemiyor. Bu şekilde anlatıyorum. Mesela koşullu. Bir şeyin koşulu nedir diye soruyorum. Ve bu bir şarta bağlanıyor. Diyorum ki "şöyle olursa şu olacak" Diyelim ki " sınıfı geçersen baban sana şunu alacak" işte diyorum koşullu olasılıkta bu diyorum. Bunun olması buna bağlıysa demek diyorum

bu olaylar birbiriyle alakalı. Şu olduğu zaman bu olacak. Öğrencileri doğru düşünmeye yönlendirmeye çalışıyorum. Yani bu kelimelere göre nasıl düşünürsün. Ya da nasıl düşünmen gerekiyor. Biraz daha ezbercilikten çıkartıp çocukların zihnini kullandırmaya yönlendirmeye çalışıyorum.

I: Olasılık sorularını çözerken nelere dikkat ediyorsunuz?

C: Öncelikle basit ve algılanması daha kolay olan sorularla başlayıp sonrasında kademe kademe arttırıyorum. Mesela para görseldir ve yazı tura gelme olasılığı kolaydır. Daha sonra zar üzerinden deneyler yapıp o tür sorular çözülüyor. Yani kademe kademe gidiyoruz. Bir problemi çözerken, problemi algılayabildiklerinde, yani o algı düzeylerine geldiklerinde örneklerin zorluklarını arttırıp kolaydan zora doğru.

I: O algıyı nasıl algılıyoruz hocam.

C: Derste ben genellikle öğrencilerle iletişim halindeyim. Yani "bu anlaşıldı mı?" Diyelim ki anlaşıldı diyenler var ama diğer taraftan sen çıkarmayanlar var. O ses çıkarmayanlara dönüp "neden anlamadın?" ya da "ne anladın?" "anladığını bana anlat" derim. Anladığını ya da anlayamadığını ifade edemediği zaman anlıyorsunuz ki o noktada bir problem var. Tekrar ediyorsunuz. Ya da algılanmayan kısmı açıklamaya ve daha da ayrıntılı anlatmaya çalışıyorsunuz. Sonra bakıyorsunuz. Gözlerinden anlaşılıyor zaten. Bedenleri onu gösteriyor zaten.

I: Öğrenci anladım diyor ama aslında anlamamış?

C: Öğrenciler dersi hemen geçirmeye çalışıyorlar. Sırf ders geçsin diye anladık diyorlar. Yoksa hoca ilerlemeyecek. Anladım denir ama bunu gözler ifade eder. Boş boş bakar. Ya da anlamsız bakar. Siz oradan anlarsınız zaten anlamadığını.

I: Her sınıfa aynı anlatımı mı yapıyorsunuz?

C: Hayır. Her sınıf için kişilere göre ders planı çıkartıyoruz. Sınıfın düzeyi hangi durumda ise ona göre. Mesela benim meslek sınıflarında bilgisayar bölümünde bu dersi anlatacağım. Ya da tesisat bölümünde bu dersi anlatacağım. Bilgisayar bölümü nisbeten daha iyiler. Çünkü bileşim çalışıyorlar. Ya da tesisat bölümünde daha çok bedensel işler yaptıkları için çok fazla seviye olarak ileri olmayan öğrencilerden oluşuyor. Bu durumda diğerlerine daha ayrıntılı sorular çözebiliyorsunuz. Yani bilgisayar bölümüne daha karmaşık ve ayrıntılı sorular çözerken diğerlerine daha yüzeysel sorular çözüyorsunuz. Onların anlayabileceği düzeyde anlatıyorsunuz. Dersin konusu aynı ama içeriği farklı olmak zorunda.

I: Meslek lisesi olmasaydı da başka bir yer olsaydı nasıl davranırdınız? Nasıl anlarım bunların seviyesini?

C: Öncelikle hazır bulunuşluklarını ölçerdim. Bunu bir seviye belirleme testi uygulardım. Sadece başlangıç olarak. Ne kadar biliniyor. Ne kadar bilinmiyor? Belirli bir yüzde ortaya çıkar. Sonra biraz konuyu işlersiniz. Diyelim ki bir noktaya kadar geldiniz. Kavramları verdiniz. Başlangıç problemleri çözersiniz. Bir izleme testi yaparsınız. Ne kadar ilerliyorum. Acaba izlediğim yöntem ağır mı gelmiş. Yoksa yerinde mi?

I: Peki izleme testlerini ve başlangıç testini neye göre hazırlarsınız.

C: Seviye belirleme testini hazırlarken çocukların seviyelerini bilmediğiniz için orta düzeyde bir sınav uygulanabilir. Ya da kolay orta zor karışımı yapılabilir. Hangi sorular yapılabilmiş, hangileri yapılamamış. Oradan düzeyleri fark edilebilir. İzleme testinde neler yaptıysanız o. Benim düşüncem odur. Ben neler işlediysem benzerlerini sorarım ki o işlediğim konu anlaşılabilmiş mi?

I: Kolay orta zor dediniz? Bunları neye göre belirlersiniz?

C: Mesela koşullu olasılıkta durum bir koşula bağlı olarak verilir. Aynı şeyde iki farklı durumu düşünmesi gerekir. Bu bana göre çocukların anlama düzeylerine zor gelen bir durum. Tek durum olduğunda onu daha kolay irdeliyorlar. Diyelim ki bir zarın üst yüzüne gelen asal ve çift olması veya asal ve üçle bölünebilmesi. Bu sadece üçtür. Düşünmesi gerekir öğrencinin. Asal sayı olması mesela. Bu durumda matematiğin diğer konularından da bilgi çağırması gereken sorularda öğrenciler daha zorlanmaktadırlar. Ama direk paranın yazı veya tura gelme olasılıkları, ya da zarın 5 gelme olasılığı. Bunlarda daha basit sorulardır.
I: Farklı sınıflarda farklılıklar oluyor demiştiniz? Nedir bu farklılıklar?
C: Algılama düzeyine oluşan farklılıklar vardır. Ya da sınıfta dersi dinleme dinlememe düzeyinde farklılıklar vardır. O sınıfta dersi rahat işlersiniz. Ya da zor işlersiniz. O durumda da konu istenilen düzeye gelmiyor. Derse 15 dakika geç başlamak zorunda kalıyorsunuz. Geriye kalan 25 dakikada dersi ne kadar iyi işleyebilirsiniz? Böyle farklılıklar oluşabiliyor. Ya da o sınıfta 5 soru çözebilirsiniz anlıyorlarsa, ama anlamıyorlarsa maksimum 2 soru çözebilirsiniz. O tür farklılıklar oluşabiliyor. Hem çocukların sınıf içerisindeki farklılıkları hem de zihinsel durumları farklılıklara neden olabiliyor.
I: Peki bu konuyu anlatırken kullandığınız özel yöntem, teknikler stratejiler nelerdir?
C: Görselleştirme, günlük hayatta kullanım ve somutlaştırma kullanmaya çalışıyorum daha çok. Bunun için mesela zar getiriyorum. Çocuklar gerçekten o zarın durumunu görebilsinler diye. Ya da para kullanabilirsiniz. Bu şekilde daha çok görselleştirmeye yönelik materyal kullanabiliyorum. Ben daha çok öğrencilere anlattırmayı seviyorum. Ben anlatıyorum anladığını bana anlat. Anlatırken kendi cümlelerini kullanmaya çalışıyorlar. Daha çok bire bir ezberden ziyade üretime geçmiş oluyoruz.
I: Anlattırmanızın sebebi nedir?
C: Öğrenci derse katılmış oluyor. Böylelikle sadece bedenen değil zihnen de derste olmuş oluyor. Dersi sindirebilmesi için gerekli olduğunu düşünüyorum. Böylelikle öğrenci kendi cümlelerini bulmaya çalışıyor. Ders anlatıldığında yüzeydedir bilgiler. Öğrenci kendi anlatmaya çalıştığında ise sindirmeye başlıyor ve içine kendisinden de bir şeyler katmaya çalışıyor. Siz bir şeyler katmaya başlıyorsunuz. Hem daha iyi anlama açısından hem de öğrencinin hem bedenen hem zihnen orda bulunması açısından anlattırma yöntemini uygun buluyorum. Bir de tahtaya kalkan öğrencinin sürekli aynı öğrenciler olasından ziyade diğer öğrencileri seçiyorum. Parmak kaldırmıyorum. Kendim seçiyorum. Ve benim için önemli olanın sadece tahtaya kalmaları olduğunu belirtmeye çalışıyorum. Biliyorsun ya da bilmiyorsun. Bu benim için önemli değil. Gel ben sana ipuçları vereceğim. Ben seni sorularla yönlendireceğim ve sen doğruyu bulacaksın. Hem tahtaya kalma psikolojisini yenmiş oluyorlar hem de bir şeyleri yapabileceklerine inanıyorlar. Yeni bir şeyler yapmak istiyorlar. Evet ben bu soruyu çözebiliyormuşum. Ben bu tepkiyi çok görüyorum. İşte “ilk defa matematikten bir soruyu çözdüm.” “Ben tahtada bir şeyler yapabiliyormuşum”. Özgüvenleri gelmiş oluyor. Zaten matematiğin en büyük sıkıntısı da odur zaten. Çocuklarda özgüven sıkıntısı var. Ön yargıları vardır. İşte “bu yapılamaz”. Ben bunu yapamam diye düşünüyorlar. Yaptıklarını gördüklerinde özgüven oluşuyor ve daha fazlasını yapmaya çalışıyorlar. Sınıfın en yaramaz öğrencisinin bile dersle ilgilendiğini görüyorsunuz bir süre sonra.
I: Özgüvenle öğrencilerde illa bir geri dönüt oluşuyor?
C: Böyle olunca şaşılacak derecede geri dönütler oluşuyor. Dersle hiç alakası olmayan, dersi nasıl kaynatabilirim diye düşünen öğrencilerin bir süre sonra dersle

ilgilendiklerini ve sorulara cevap vermek için yarıştıklarını görüyorsunuz. Hani cevap vereyim ki hoca benimle de ilgilensin. Beni de tahtaya kaldırsın. Böylece sınıftaki sessizliği de sağlamış oluyorsunuz. Huzuru bozanları da elemiş oluyorsunuz. Tüm sınıfın dersle ilgilenmelerini ve dersten zevk almalarını sağlamış oluyorsunuz. Hani o zil çaldığında zilin çalmasına kaç dakika kaldı muhabbeti kalkmış oluyor bir süre sonra. Bir süre sonra fark etmiyor zamanın nasıl geçtiğini. Çünkü birebir o işin içinde. Yani seyirci değil artık öğrenciler. Oyuncu olmuş oluyorlar. Mesela bir örnek vereyim. Bir soru sordum. Öğrenciler çözdü. Sonra öğrencilerden biri söylüyor ve ben duyuyorum. "Soru mu çok kolaydı yoksa hoca mı çok iyi anlatıyor?" "Anladım yani diyor" Anladığı için çok mutlu öğrenci. "Hayatımda ilk defa bir soruyu anladım" diyor öğrenci. "Aman Allah'ım kolaymış matematik" dediklerini duyuyorum bazen. Daha önce matematikle ilgisi olmayan öğrencinin bunu söylemesi çok güzel. Matematiği kolay buluyor. "Niye bize kolay olduğunu söylemediler" diyorlar.
I: Öğrenciye daha önce zor geliyor ki bunu söylüyor öğrenciler. Bunun sebebi nedir?
C: Mesela çarpmada problemi var öğrencinin. Siz o çocuğa üstlü sayılarda çarpma gösteriyorsunuz. Ya da denklem çözdürüyorsunuz. Orada x'ler sayılar var. Çocuk daha sayının anlamını bilmiyor ki. Siz buna denklem çözdürüyorsunuz. Artı eksilerden bahsediyorsunuz.
I: Öğrenci zor olduğunu neden düşünüyor?
C: Bazı öğretmenler matematiğin zor olduğunu özellikle bastırıyorlar. Bakın bu ders çok zor aşırı çalışmanız lazım. Çok çalışma şey oluyor. Sevmiyorlar ya çalışmayı, siz ona daha çok çalışmaları için empoze etmeye çalışıyorsunuz. Çocuk nefret ediyor zaten zamanla. Ve gözünde kocaman bir şey oluyor matematik. Aşılması zor bir dağ gibi geliyor onlara. Daha çok dış etkenlerden dolayı öğrenci böyle düşünüyor. Mesela benim fizik korkum öyle oluşmuştur. Ben hep yapamayacağıma inanmıştım. Hiç uğraşmadım bunla yani. İçinde hiçbir istek kalmıyor ki. Dersi gerçekten hoca sevdiriyor ya da nefret ettiriyor. Sırf hocanın bir davranışından dolayı dersten nefret edebiliyorsunuz. Derslerdeki başarısızlığı daha çok dış etkenlere bağlıyorum.
I: Olasılık konularına bakarsak, öğrenciler hangi noktalarda daha çok zorlanıyorlar?
C: Bağımlı olaylarda daha çok zorlanıyorlar. Koşullarda iki durumu aynı anda düşünmeleri gerektiğinden dolayı sıkıntı yaşıyorlar. Mesela tek ve asal gelme olasılığı düşünüldüğünde hem tek hem de asal gelme olasılıklarını öğrenci düşünmek zorunda. Alttaki eksiklikler etkiliyor. Öğrenci tek sayının veya asal sayının ne olduğunu bilmiyorsa bu soruyu zaten çözemiyor. Olasılığın "istenilen durumlar bölü tüm durumlar" olduğunu biliyor. Ama istenilen durumları bulamıyor ki. Tek sayıları biliyor mesela ama asalları bilmiyor. Koşullu olasılığın sıkıntısı orada. Tek bir durum değil. Aynı anda birden çok şeyi bilmesi gerekiyor. Daha önce öğrendiklerini birleştirmesi gerekiyor. Sıkıntı orada oluşuyor. Yani istenen durumları bulsa olay bitecek. Olasılık basit. Diyoruz ki zaten istenen bölü tüm durumlar. Evet bu mantık kolay ama istenen durumları irdelemek zor. Onu oluşturmak çok zor.
I: Ayrık ve ayrık olmayan olaylarda sıkıntı oluyor mu?
C: Eğer kavramları düzgün verirseniz sıkıntı olmuyor. Belki Türkçe'de problemi varsa kelimelerin anlamlarını iyi bilmiyorsa o zaman problem çıkıyor. Eğer kavramlarda sıkıntı yoksa bu konularda sıkıntı yok. Koşullu olaylarda ise birden çok durumu birleştirmek zorunda kaldıklarından dolayı problem çıkıyor.
I: Bunları nasıl belirliyorsunuz?
C: Soruyorsunuz öğrenciye soruyu. Önce tekleri düşünüyor. Sonra asalları düşünmeye çalışıyor. Ama içinden çıkamıyor öğrenci. Burada sıkıntı var. İki durumu

aynı anda düşünme sıkıntıları var. Yani sadece bir durumu düşünüyor. Ve o düşündüğü yoldan gidiyor. Onunla kalıyor. Ekstra bir şeyler çıkabiliyor. Mesela 2 hem tek hem asal bir sayı.

I: Düşünüyor ve orada kalıyor diyorsunuz. Yani bir yere yönleniyor sadece.

C: Aslında aradan gelecek yolları hiç hesaba katmıyor. Dümdüz gidiyor. Bunun sebebi de öğrencileri düşünmeye yönlendirmiyoruz. Anlattırmaya yönlendirmiyoruz. "Sen beni dinle" yaklaşımındayız. Öğretmenin lider olduğu durumlarda sıkıntı çıkıyor. Öğrencilere "ne dersim onu dinleyeceksin" yazacaksın çıkacaksın bitti. Ama ben bu durumu sevmiyorum. Sen benimle konuş. Dersle ilgili ilgisiz konuş. Bir şekilde kendini ifade etmeyi öğren. Kendi cümlelerin olsun. Mesela tanım sorarız diyelim ki. Doğru nedir? Mesela. Benim verdiğim bir tanım var. Ama sen bunu algılayıp başka türlü ifade edebiliyorsan işte öğrendiğini öyle gösteriyorsun bana.

I: Tekrar öğrencilerin düşündüğü bir şey var ve ondan başka bir yol kullanmıyor. Bunun altta kalan sebepleri neler olabilir?

C: Hayatta da öyle değil mi? Çocuklara diyoruz ki bu doğru bu yanlış. Bu doğrunun niye sini kimse irdelemiyor. "Niye doğru?" Geçmiş eğitimlerinden gelen bir sebepleri var yani. "Annem doğru dediği için bu doğrudur." "Babam buna yanlış dediği için yanlıştır." Niye yanlış olduğunu irdelemiyoruz. Mesela 2 asal sayı. Niye asal sayı olduğunu irdelemiyoruz. Mesela 4 asal sayı ama niyesini irdelemiyoruz.

I: Öğrenciler niye bunu irdelemiyor?

C: İrdelemeyi öğretmiyoruz ve yönlendirmiyoruz. Bizde biraz öyleyiz. Bize bir şey doğru deniyorsa doğrudur. Bunu yap deniyorsa yapıyoruz. Yapma deniyorsa yapmıyoruz. Niyesini kimse sormuyor. Sorunca zaten seni o ortamda istemiyorlar. Sorun çıkıyor.

I: Bunları önlemeye yönelik ne yapıyorsunuz? Teknik materyal?

C: Ben hep çenemi kullanıyorum. Öncelikle çocuklara özgüven yüklemeye çalışıyorum. Bana güvenmelerini sağlamaya çalışıyorum. Bana çocuklar rahatlıkla bilmiyorum diyebiliyorlar. Ya da anlamadım diyebiliyorlar. Mesela aynı şeyi 3 defa söylediğinde gücenmiyorum. Çünkü Saliha hoca tepki vermeyecek. Bizi eleştirmeyecek. Bizimle alay etmeyecek. Bu özgüveni önce oluşturmaya çalışıyorum. Öğrencilere "çarpmanızda sorun varsa onu bile söyleyin" diyorum ki oracıkta halledelim. Çünkü matematik tek başına yapılabilecek bir şey değildir. Buraya çalışırsınız ama teki bilmez asalı bilmezsen diğer kısımlar hep havada kalır. Ama biri siz soruyu çözerken, düşündüğünüz anda bile ben fark ederim ki orada teki düşünüyorsun. Ben özel ders verdiğim içinde hani o avantajım var. Ben soruları çocuklara çözdürürdüm. Mesela gel bunu sen çöz. Çözmeye başladı okudu bir şeyler yapıyor. Kalem bir duraksar. Düşünüyor çünkü. O kalem durdu. O kalem nerede durdu, teki düşünürken. Demek ki o tekte problem var. Neyin tek neyin çift olduğu noktada sıkıntısı var. Hemen müdahale ediyorsunuz. Sonra hemen kalem oynamaya başlıyor. Fark ediyorsunuz oradaki eksikliğini. Konuşturuyorum ki öğrenciyi nerelerde eksikliği var anlamaya çalışıyorum. Hangi kelime de takılıyor. İşlemi yaparken acabası oluşuyor. Ben düşüncelerini dışa yansıtmaya çalışıyorum. Sesli düşünmelerini sağlamaya çalışıyorum. Sesli düşünün ki düşünemediğiniz yeri ben fark edeyim. Yargılamıyorum öğrencileri. "Liseye gelmişsiniz ama hala bilmiyorsunuz çarpım tablosunu" demiyorum. Çok kızdığım zaman söylüyorum da. Bazen çıldırtıyorlar.

I: Öğrencilerin olasılık konusunda sahip oldukları kavram yanılgıları, hataları nelerdir?

C: Kavramlardan başlarsak, örnek uzayı evrensel küme ile bağdaştırdığınızda rahatlıkla kavrayabiliyorlar. Evrensel küme konusu yerleşmişse diyorsun ki bu bunun evrensel kümesi. Bunda pek sıkıntı olmuyor. Örnek uzayla bağdaştırdığımızda olabilecek durumların hepsi olarak söyleyebiliyoruz. Bunu kavrayabiliyorlar. Mesela çok büyük şeyden bir kutudaki topların örnek uzayına geçerken sıkıntı olmuyor. Çünkü tümden gelimde sıkıntı olmuyor bizim öğrencilerimizde. Tümevarımda sıkıntı oluşuyor. Büyüğü gördüğü için onu küçültebiliyor. Tümden gelebiliyor. Ama küçükten büyüğe geçişte bu durumu hayal edemiyor. Bunun sebebi de öğrencilere genel kurallardan özel durumlara geçirebiliyoruz algılarında bebeklikten itibaren. Ama ufacık bir şeyin büyüyebileceği veya o kuralın genişletilebileceğini, herkese uyarlanabileceğini gösteremiyoruz. Ya da göstermiyoruz. Yetiştirme tarzı, çocukluktan gelen bir eğitim tarzı belki de. Tümden gelimde ben öyle gördüm öğrencilerimde. Örnek uzayda ben öğrencilerimde sıkıntı görmedim. İmkansız ve kesin olaylarda da sıkıntı yaşamadım. İmkansız ne demek olduğu bilindiği için bunu algılayabiliyorlar. Ya da kesinlik nedir algılanabiliyor. Biraz ayrık olaylarda sıkıntı var. Aslında bunu bir şeylerle bağdaştırdığınızda çözüyorsunuz. "Ayrık olması ayrı olması. Birbirinden ayrı durması. Örnekler veriyorsunuz. İki şey birbirinden nasıl ayrı tutulabilir?" Bunların ortak olmayan bir özellikleri var. Ama ayrık tamamen ayrı. Hiçbir ortak özelliği yok. Aslında burada kümeler konusu anlaşıldığında çok rahat anlaşılabilecek bir kısım. Çünkü kümeler konusunda kesişim, birleşim kümeyi gösteriyoruz. Ve kümeler biraz daha görseldir. Bir yuvarlak çizer, içine elemanları yerleştirir. Orada kesişim, birleşim ve evrensel kümeleri anlatmışsanız hemen oradan çağrıştırabiliyorsunuz. Ayrık küme neydi iki farklı küme, yuvarlakları birbirine bitiştirmiyorduk. Biri mesela elmalar kümesi biri armutlar kümesi. Birbirleri ile ortak noktaları yoktur. Eğer meyve olmaları düşünülmüyorsa bunlar tamamen birbirinden ayrılar. Yani matematikteki daha önce öğrenilenleri buraya uyarlayacak şekilde çağrıştırıyoruz. Basit olasılıkta da sıkıntı yok. En büyük problemimiz koşullu olasılıkta. Aynı anda iki şeyi düşünme ve irdeleme, bunların özelliklerini bilme noktasında problem yaşanabiliyor. O problemde soruyu çözerken karşımıza çıkıyor. En başta kavramsal olarak çıkıyor. Daha önceki anlatılanlardan eksik kalınması, her şeyin tam olarak anlaşılmaması. Bundan dolayı matematik öğretmenlerinin yapması gereken en önemli şey "bu konu her anlamda tam anlaşılmış mı?" olduğunu belirlemektir. En ufak bir açık bırakırsanız o bir sonraki konuda sizi buluyor. Orada o konu ile yine uğraşıyorsunuz. İzleme testleri çok önemli işte. İzleme testinden aldığınız başarı %50'nin altında ise o konu kesinlikle tekrar anlatılmalı. %70'lerde ise sıkıntı nerede yoğunlaşıyor, bunu çıkarıp ona çözüm aranmalıdır. Sonra yeni konuya geçilmelidir. Zaten %80-90'larda ise amaca ulaşılmıştır. Anlamayanlarda siz anlatırken bir şekilde kapar o konuyu. Çünkü zaten temeli oluşmuş ve çok küçük eksiklikleri var.

I: %80-90 derken niçin bu oranları aldınız?

C: Sebebi şöyle. %80'i ben %100 yapamam. Tam öğrenmeyi sağlayamıyorsunuz hiçbir zaman. Çünkü gerçekten matematiğe eğilimi olmayana öğrencilerde vardır. Aslında %80'i bulduğunuzda tam öğrenmeyi sağlamış oluyorsunuz. Zaten 30 kişilik sınıfın tamamının öğrenmesini de bekleyemezsiniz. Bazı öğrenciler sözel eğilimlidir. Sayılarla arası yoktur. Bu zihinsel bir şey. Ben bunu ne kadar zorlarsam da onun kapasitesi bu. Fazlası gelmiyor ki. Anlayabilme kapasitesi bu kadar. Yeteri kadar da anlamıştır aslında % 80 ile. Ben o sınıfı %100'e çıkarmaya çalışırsam diğer konulara geçemem. Bunu zamana yaymak gerekir. Bir de bazı konuların biraz sindirildikten sonra anlaşıldığını düşünüyorum. Sayılar konusu mesela. Tüm konular bittikten

sonra tam manasıyla anlaşılır. Çünkü içesinde denklemler, üstlü sayılar, oran orantı vardır. En büyük sıkıntı zaten sayıların başta verilmesidir. O sıralamayı da sevmiyorum. Başta veriyoruz ama çocuklar hiçbir şey bilmezken her şeyi veriyorsunuz. Hiç bir şey anlamıyor. Tamamen özgüvenini baştan yitirtiyorsunuz. Bundan dolayı ben sayılar konusunu yüzeysel geçerim. Sayıları tanıtırım geçerim. Basamaklardan bahsederim maksimum. En son üstlü sayıları, köklü sayıları ve oran orantıları veririm, denklem çözmeyi veririm. Daha sonra sayılar konusunun problemlerine geçerim.
I: Çok teşekkür ederiz. Eklemek istediğiniz bir şey var mı?
C: Matematik sevdirilmeli. Matematik artık öcü olmaktan çıkıp sevimli bir şey haline getirilmeli. Bunun için ise öğretmenlerin daha sevimli olmaları gerekmektedir. Matematiğin zor olması sadece öğretmenin egosunu tatmin eder. Çocukların gözünde şöyle oluyorsun. Matematik yapan bir insan. Ne kadar deha bir insan. Ama benim deha olup olmamam önemli değil. Sizin de o dehadan biri olmanız önemli. Siz yeri geldiğinde beni uyarın. Hocam orada yanlış yaptın? Niye yaptın? Beni sorgulayan öğrenciler istiyorum. Açıkçası öğretmenlerimizin en büyük işi dış etkenleri engellememizdir. Olumsuz dış etkenleri ortadan kaldırmalıyız.
I: Dış etkenlerden kasıt nedir?
C: İşte bu “Eleştirme” “Soru sorma” “ Ne diyorsam onu yap” ya da “matematik zor” “ sen bunu yapamazsın” “senin buna kafan basmaz” tarzındaki yaklaşımlarımız. Bunlar yerine “matematik kolaydır”, “düşünün” demeliyiz. Bize ortaokul hocamızın söylediği çok güzel bir söz vardı. “Düşündüğünüzde kafanızdan beyninizden bir şey eksilmez, aksine artar” derdi. Hani “paslanmazsınız” “daha da çalışır hale gelir”. En çok başın ağrımaya başlar. Azıcık ısındığını fark edersiniz. Ama bir süre sonra görüyorsun ki evet gerçekten çok çalışıyor. Hatta matematikte çok çalıştığınız zaman hayattaki her şeyde algılarınızın daha da yükseldiğini fark edersiniz. Farklı açılardan bakarsınız. Problem çözme yeteneğiniz günlük hayata da yansıyor. Hayatınız kolaylaşıyor. İnsan ilişkileriniz rahatlıyor. Çünkü rasyonel yaklaşıyorsunuz. Duygularınızdan arınıyorsunuz. Daha kolay problemlerin üstesinden gelebiliyorsunuz. Matematiğin kolay olduğunu öğrencilere aşılamalıyız. Olasılık konusunun kolay olduğunu öğrencilere aşılamalıyız diye düşünüyorum.

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Öçal, Mehmet Fatih
Nationality: Turkish (TC)
Date and Place of Birth: 3 March, 1983, Arhavi
Maritial Status: Married
Email: fatihocal@gmail.com

EDUCATION

Degree	Institution	Year of Graduation
MS	Boğaziçi University	2007
BS	Boğaziçi University	2007
High School	İvriz Anatolian Teacher High School	2001

WORK EXPERIENCE

Year	Place	Enrollment
2011-Present	Ağrı İbrahim Çeçen Üniversity	Lecturer

PUBLICATIONS

Öçal, M. F., & Güzel, A. (2010). Investigation of effectiveness of the pedagogical education from matematics teachers' perceptions, *The International Journal of Research in Teacher Education, 1*(3), 25-31.

Öçal, M. F., & Güler, G. (2010). Pre-service mathematics teachers' views about proof by using concept maps, *Procedia - Social and Behavioral Sciences, 9,* 318-323.

Öçal, M. F., & Yalçın, T. (2010). İlköğretim dördüncü sınıfların alanlar konusunu anlama düzeyleri: durum çalışması, *E-Journal of New World Sciences Academy, 5*(3), 1107-1118.

Yalçın, T., & **Öçal, M. F.** (2010). Sınıf öğretmeni adaylarınınm, matematik öğretimine yönelik öz yeterlilik inançları: nitel bir çalışma. *E-Journal of New World Sciences Academy, 5*(3), 1119-1125.

Durkaya, M., Aksu, Z., **Öçal, M. F**., Özge, Ş., Konyalıoğlu, A. C., & Hızarcı, S. (2011). Pre-service mathematics teachers' multiple representation competencies about determinant concept. *Procedia - Social and Behavioral Sciences, 15,* 2554-2558.

Durkaya, M., Özge, Ş., **Öçal, M. F.,** Aksu, Z., Konyalıoğlu, A. C., & Hızarcı, S. (2011). Secondary school mathematics teachers' approaches to students' possible mistakes. *Procedia - Social and Behavioral Sciences, 15,* 2569-2573.

Kar, T., Işık, A., **Öçal, M. F.**, Çiltaş, A., Güler, G. & Işık, C. (2011). Prospective mathematics teachers abilities' to construct relations between the different representations of series with complex terms, *Procedia - Social and Behavioral Sciences, 15,* 356-360 .

Güler, G., **Öçal, M. F.**, & Akgün, L. (2011). Pre-service mathematics teachers' metaphors about mathematics concept. *Procedia - Social and Behavioral Sciences, 15,* 327 - 330.

Şimşek, M., **Öçal, M. F.,** Öçal, T., & Deniz, D. (2013). Pre-service mathematics teachers' problem solving strategies with GSP: Mirror Problem. 2*.Uluslararası Avrasya Matematik Bilimleri ve Uygulamaları Konferansı (IECMSA)* (August 26-29), Saraybosna, Bosna-Hersek: International University of Sarayevo.