AN INVESTIGATION OF PRE-SERVICE MATHEMATICS TEACHERS' MATHEMATIZING DURING A MATHEMATICAL MODELING TASK

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name: Murat Kol

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ABSTRACT

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The purpose of this study was to investigate pre-service mathematics teachers' mathematization process in a mathematical modeling activity. The process was investigated in two different levels of mathematization based on Realistic Mathematics Education (RME) framework namely, horizontal mathematization and vertical mathematization.

This study designed as a case study. The participants of the study were six pre-service elementary mathematics teachers enrolled in a course entitled "Mathematical Modeling for Teachers". One of the activities covered in the course was about the real life situations emerged in the context of Ferris wheel and selected for this study. The participants worked on the activity in groups of three. The source of data comprised of reflection papers written by the participants, transcription of videotapes during the group discussions, group presentations and worksheets.

The results of this study revealed that during both horizontal and vertical mathematization process, pre-service teachers had difficulties and misconceptions while understanding the problem, determining the independent variable of the function and forming the function.

Keywords: Mathematics education, mathematical modeling, mathematization, trigonometry, functions, pre-service mathematics teachers.

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMEN ADAYLARININ MATEMATİKSELLEŞTİRME SÜRECİNİN BİR MATEMATİKSEL MODELLEME ETKİNLİĞİ SÜRESİNCE İNCELENMESİ

Yüksek Lisans, Orta Öğretim Fen ve Matematik Alanları Eğitimi Bölümü

Tez Yöneticisi: Doç. Dr. Ayhan Kürşat Erbaş

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Bu çalışmanın amacı ilköğretim matematik öğretmeni adaylarının bir matematiksel modelleme etkinliği içinde yer alan matematikselleştime sürecini incelemektir. Bu süreç Gerçekçi Matematik Eğitimi (GME) teorik çerçevesinde yer alan, matematikselleştirmenin iki boyutu olan yatay ve dikey matematikselleştirme temel alınarak yapılmıştır.

Araştırma deseni olarak bu çalışmada, örnek durum deseni kullanılmıştır. Çalışmanın katılımcıları "Öğretmenler için Matematiksel Modelleme" isimli dersi alan altı ilköğretim matematik öğretmeni adayıdır. Ders kapsamında gerçekleştirilen modelleme etkinliklerinden birisi bir dönme dolap ve buna bağlı gerçekçi durumların incelendiği bir problem içermektedir ve bu çalışma için seçilmiştir. Katılımcılar üçerli gruplar halinde çalıştılar. Çalışmanın verileri öğrencilerin etkinlik sonrası yazdıkları düşünce raporlarını, grupların sonunun çözümü sırasındaki tartışmalarını ve sınıfta çözümlerinin sunulmasını içeren video kayıtlarının yazıya dökümü ve çalışma kâğıtlarını içermektedir.

Araştırmanın sonuçları, öğretmen adaylarının yatay ve düşey matematikselleştirme sürecinde, problemi anlamada, istenilen fonksiyonun değişkenini belirlemede ve fonksiyonu yazmada zorluklara ve kavram yanılgılarına sahip olduklarını göstermiştir.

Anahtar kelimeler: Matematik eğitimi, matematiksel modelleme, matematikselleştirme, trigonometri, fonksiyonlar, matematik öğretmeni adayları.

To My Family.

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LIST OF SYMBOLS AND/OR ABBREVIATIONS

MEA: Model-Eliciting Activities

PST: Pre-service Teacher

PSTAn: n-th Pre-service Teacher in Group A

PSTBn: n-th Pre-service Teacher in Group B

RAn: n-th Research Assistant

CHAPTERS

1. INTRODUCTION

Development in technology, especially in computerized instruments, affects the life style of citizens all over the world. Besides the dazzling speed in the development of technological tools, their effects in educational settings are still limited. Attaining information related to any subject is just on a "single click". However, the important issue is not the ways of getting that information. The problem here is to learn how to use the attained information when it is needed. Therefore, the citizens of the digital world should be educated in accordance with this harmony. At this point, the educators are to shoulder such a heavy load. Educating people includes the adaptation of them to their environment and the needs of life (Ruediger, 1910). Another important aspect of education is to improve individual's abilities in such a way that he can use them properly during his life (Minnick, 1921). Hence, educating our students according to the needs of our social life should be in the center of education system. Gravemeijer (1994) state the importance of starting mathematics lessons with real life problems.

In the light of preparing students aligned with the needs of life, mathematics education programs were developed to emphasize the importance of mathematical modeling (National Council of Teachers of Mathematics [NCTM], 1989; Ministry of National Education [MoNE]-Board of Education, 2013; National Council of Educational Research and Training [NCERT], 2005). In these programs, it is stressed that individuals in the society should be equipped with the ability of solving the problems faced in their life. One of the major equipment that students are supposed to use is mathematical modeling. A didactical approach based on mathematical modeling had a key role of those reforms in mathematics education.

It was a good step for our country to increase the emphasis on the importance of mathematical modeling in national mathematics program. In fact, mathematical modeling was mentioned as one of the main mathematical skills to be developed in the mathematics program (MoNE, 2013, p. IV). The lack of sense of connecting mathematics to real life is evidently seen in TIMMS and PISA results. According to Yayan and Berberoğlu (2008), Turkey had fallen behind 36 countries among 57. Another important determination stressing the mathematical literacy was done in Program for International Student Assessment [PISA] study by Organisation for Economic Co-operation and Development [OECD] as "capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to engage in mathematics, in ways that meet the needs of that individual's life as a constructive, concerned, and reflective citizen." (OECD, 2000, p.50). Hence, one of the major aims of education should be to prepare mathematically literate individuals. As emphasized in the new national mathematics program, if mathematical modeling can be applied in schools, it can be a cure to the lack of connecting mathematics to real life.

1.1. Mathematical Modeling and Mathematizing

Mathematical modeling as being one of the popular trends affecting mathematics education in all over the world had great influence on community of mathematics education research. There are several mathematical modeling approaches in the literature for different purposes (Borromeo-Ferri, 2006; Kaiser et al., 2006). In all of those different approaches, mathematizing plays a crucial role in mathematical modeling. Besides the mathematical modeling, Freudenthal (1973) states that "there is no mathematics without mathematizing". Therefore, mathematization is an important ability to gain deep competence for students.

Different forms of mathematization were mentioned in Freudenthal's (1991) and Treffers's (1987) studies to separate mathematization into two different types of mathematical activity. One of them involves mathematization from real world to mathematics, namely horizontal mathematizing. The other one includes movement within the mathematics with symbols, namely vertical mathematizing. Actually, the distinction between those two forms was not so clear and they have equal value

(Freudenthal, 1991). Modeling activities comprises many different processes, which will be later examined in detail in the literature review chapter. However, in this study our emphasis focuses on particularly pre-service teachers' mathematization process during a mathematical modeling activity.

1.2. Purpose of the study

The purpose of this study is to investigate pre-service mathematics teachers' mathematization process during a mathematical modeling activity. Investigating pre-service mathematics teachers' both horizontal and vertical mathematization may contribute to the literature in terms of examining the differences on these processes and allow us to observe the pre-service teachers' difficulties during these phases.

1.3. Research questions

Yoon et al. (2010) defined mathematizing as "interpreting the structural aspects (i.e., the objects, relations, actions, patterns, regularities, assumptions, etc.) in a real world system, and expressing this structure in a mathematical model using mathematical representations such as symbols, text, graphs, diagrams, and so forth." According to this classification, data is examined and the following research questions related to the mathematization process were determined. It was realized that the first character of the mathematization in this definition was aligned with the horizontal mathematization definition of RME. Hence, the first research question was about PSTs' the horizontal mathematization process. Similarly, the second character exhibits the properties of vertical mathematization. Therefore, the second research question and sub-questions were about the vertical mathematization process. The clarifications about the research questions were given below when it is supposed to be needed.

a) How do pre-service mathematics teachers interpret structural aspects of a real life situation mathematically during a modeling activity?

b) How do pre-service mathematics teachers express the inner structure of a model of a real life situation in the Ferris wheel activity?

i) How do pre-service mathematics teachers use diagrams while mathematizing the situation during Ferris wheel activity?

ii) How do pre-service mathematics teachers form functions while mathematizing the situation during Ferris wheel activity?

In the light of this research question, how PSTs formed functions from verbal definitions were investigated.

iii) How do pre-service mathematics teachers write formulae while mathematizing the situation during Ferris wheel activity?

Under this research question, the process of formulating a pattern and using the mathematical symbols while writing the formula were investigated.

iv) How do pre-service mathematics teachers apply previously learnt topics while mathematizing the situation during Ferris wheel activity?

1.4. Significance of Study

Although the mathematization process is at the core of mathematical modeling, there are limited number of studies in the literature focusing on this crucial step (Wheeler, 2001). On the other hand, mathematical modeling is one of the important approaches steering the educational policies. Since mathematics teachers are the implementers of mathematical modeling tasks in high schools, it is important for them to comprehend the processes during mathematical modeling.

In the limited number of previous studies (e.g. Keisoglou, & Spyrou, 2003; Molnar, 2008; Murata, & Kattubadi, 2012) mathematization process was investigated as a whole. In this study, some specific components of mathematization defined by Lesh and Doerr (2003) and Yoon et al. (2010) were investigated.

It was aimed to contribute the “cognitive modelling perspective” (Kaiser & Sriraman, 2006) of mathematical modeling by investigation of the mathematization process in detail. By understanding pre-service teachers’ cognitive processes during the mathematical modeling, researchers may serve more appropriate modeling tasks and courses offered for pre-service teachers to prepare them for applying mathematical modeling activities in high schools.

1.5. Definition of terms

The following important terms are associated with this study.

Mathematical modeling:

Mathematical modeling has various definitions. In general, it can be defined as using mathematics for describing, investigating, and explaining a real world phenomenon. A model as the product of a mathematical modeling activity is described by Lesh and Doerr (2003, p.3) as conceptual tools which are “sharable, manipulatable, modifiable, and reusable for constructing, describing, explaining, manipulating, predicting, or controlling mathematically significant systems”.

Modeling activity:

Modeling activity is used instead of model-eliciting activity, which is defined by Lesh et al. (2000) as tools helping students to reveal their way of thinking and their learning process. Model-eliciting activities are designed to imitate the real-world problems encountered in many different fields of life. Hence, they are more productive didactical activities than ordinary application problems since they enhance conceptual understandings by mathematizing real world situations (Lesh et al., 2000; Lesh & Doerr, 2003). Modeling activity and modeling task were used interchangeably in this study.

Mathematization:

Describing real life situation mathematically by "quantifying qualitative information, dimensionalizing space coordinatizing locations, algebratizing, systematizing relevant objects, relationships, actions patterns and regularities" (Lesh & Doerr, 2003, p.16).

2. LITERATURE REVIEW

In line with the purpose of this study, this chapter was made up to analyze the previous studies about the modeling and mathematization process. According to the research questions of the study, the literature review is divided into three sections: mathematical modeling and different approaches, mathematization as a part of mathematical modeling process, and functions and trigonometry.

2.1. Mathematical Modeling and Different Approaches

According to previous experiences and researches, students have different "heads" in school and in their real-life. Combining these two heads should be a crucial goal for education, and mathematical modeling can be a good tool for this aim (Lesh & Doerr, 2006). The connection between the real-life and school life is lost. Many students think that they are very distinct and behave as actors of these distinct lives. Probably, after realizing the potential of this important issue in education, many countries (USA, Australia, India, and Turkey and probably many others) emphasized the importance of mathematical modeling.

Mathematical modeling has various meanings in different subject areas like natural sciences, engineering, social sciences, and education. In the scope of this study, only mathematical modeling in mathematics education will be mentioned. Besides the different meanings of mathematical modeling in other disciplines, it was even described in different ways by mathematics education researchers. There is no consensus about the meaning, goal, usage, application in classroom settings, and the integration in mathematics education curriculums of mathematical modeling (Gravemeijer & Stephan, 2002; Kaiser, Blomhøj & Sriraman, 2006). However, transferring real-life situations to mathematical language and interpreting them mathematically was mentioned in most of these definitions.

> *"Mathematical modeling can be defined as a mathematical process that involves observing a phenomenon, conjecturing relationships, applying*

mathematical analyses (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model" (Swetz & Hartzler, 1991 cited in Lingefjärd, 2006, p. 1)

Kaiser and Sriraman (2006) stated six different perspectives on mathematical modeling in their extensive article. Those six perspectives were listed as realistic or applied modelling, contextual modeling, educational modeling, socio-critical modeling, epistemological or theoretical modeling, and cognitive modeling. The last one is listed as a kind of meta-perspective. The research goals of this cognitive perspective were described as follows: "analysis of cognitive processes taking place during modelling processes and understanding of these cognitive processes". Galbraith (2012) classified the different modeling perspectives into two genres to compromise the "confusing voices" about the modeling perspectives in educational settings. According to this classification, modeling is used as a "vehicle" to enhance the learning of a mathematical concept or as "content" to develop modeling abilities for solving real problems without the emphasis of a mathematical concept as a prior aim of modeling.

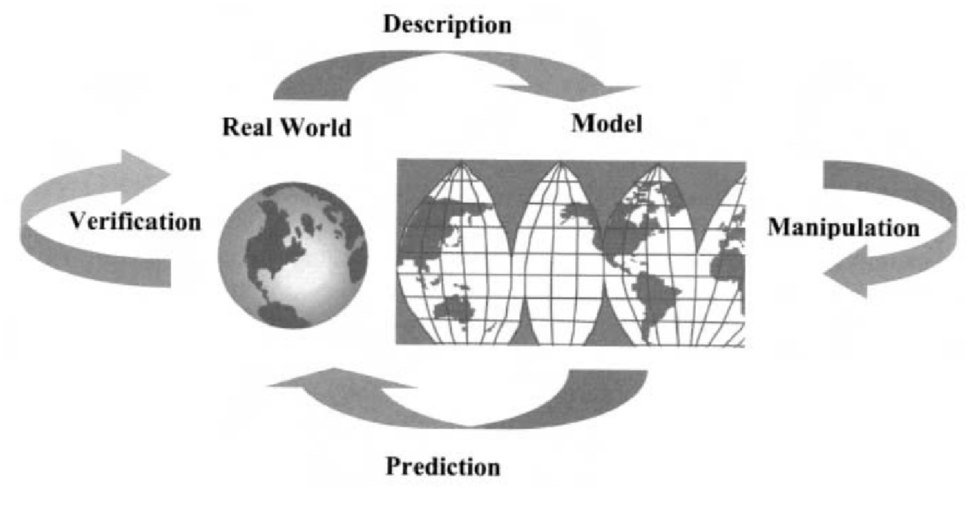

Figure 1 Four- step modeling cycle (Lesh & Doerr, 2003, p.17)

In contrast to traditional word problems in textbooks, the process in mathematical modeling process is not linear (Lesh & Harel, 2003). The cyclic nature was stressed in most of the definitions of mathematical modeling process. Although Lesh and Doerr (2003) mentioned four basic steps (see Figure 1) in a mathematical modeling process, by interpreting the Blum and Leiss's (2007) definition of modeling cycles, Ferri (2006) used a six-step modeling cycle while explaining the modeling cycle under a cognitive perspective (see Figure 2). All of these steps have important effects on students' learning since the process is the product in model-eliciting activities. Traditional problem solving was interpreted as a special case of mode-eliciting activities (Lesh & Doerr, 2003). The difference between the traditional problem solving and MEAs was explained more precisely as:

> *"in traditional problem solving, the goal is to process information using procedures associated with a fixed construct (that simply needs to be identified, retrieved, and executed correctly), whereas, in model-eliciting activities, it is the constructs themselves that need to be processed." (Lesh & Doerr, 2003, p.22)*

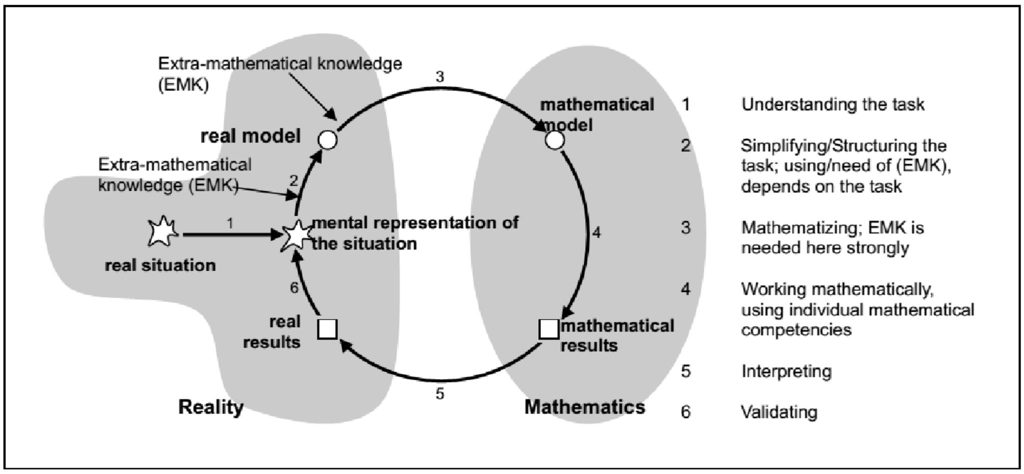

Figure 2 Modeling cycle under a cognitive perspective (Ferri, 2006, p.87)

According to Lesh et al. (2007), the timing of implementation of MEAs has substantial impact on effectiveness of instruction and mathematical modeling activity. In order for MEAs to serve their intended role of encouraging students, they should be implemented before any direct instruction. Otherwise, it resembles the application problems those can be found in most of the textbooks. In application problems, students can apply what they have been taught during the instruction. Yoon et al. (2010) later examined this order and concluded that applying MEA's after an instructional unit can be even more fruitful. By doing so, "students may have the opportunity to deepen their understanding of mathematical topics by mathematizing, as well as an opportunity to apply their knowledge."

2.2. Mathematization as a Part of Mathematical Modeling Process

Mathematization is a key ability for students to do mathematics. National curriculum of India (NCERT, 2005, p.127) emphasized the focus of their mathematics program on mathematization as follows:

> *"Mathematization (ability to think logically, formulate and handle abstractions) rather than 'knowledge' of mathematics (formal and mechanical procedures) is the main goal of teaching mathematics."*

According to Freudenthal (1973), "there is no mathematics without mathematization". In addition to this important anchoring, Wheeler (1983) made another important assignation as "more useful to know how to mathematise than to know a lot of mathematics". Almost all of the researchers emphasized mathematization as a must process in their cycle of process in MEA. Lesh and Doerr (2003, p.5) define this process of mathematising in MEAs as:

> *"Model-eliciting activities usually involve mathematizing – by quantifying, dimensionalizing, coordinatizing, categorizing, algebratizing, and systematizing relevant objects, relationships, actions, patterns, and regularities."*

Modeling, especially mathematization, plays an important role in RME as a vehicle for conceptual knowledge (Artigue & Blomhøj, 2013). The importance of mathematization for RME stated by Doorman & Gravemeijer (2009) as "the core mathematical activity of RME is mathematizing". In RME, reinvention of mathematics by students is substantial, and mathematization is a good way to do this reinvention (Freudenthal, 1991). Mathematization is categorized into two forms as horizontal and vertical mathematization (Treffers, 1987) in RME. While horizontal mathematization focused on transition from real world situation to mathematics terms and models, vertical mathematization deals with the reflection and work with associated techniques and semiotic tools within mathematics itself (Artigue & Blomhøj, 2013). Horizontal mathematization includes schematizing and ordering in order to build a model of reality in such a way that it becomes easier to cope with. A mathematical treatment can access the problem by the help of horizontal mathematization. In vertical mathematization, a sophisticated mathematical processing takes place to build the model related to the reality. However, the difference between these two mathematization is not so clear and they are of equal value (Freudenthal, 1991). Quantifying qualitative information, dimensionalizing space, coordinating locations, and investigating patterns take place in mathematization process (Lesh & Doerr, 2003). Similar to the separating the mathematization process in horizontal and vertical mathematization, Yoon et al. (2010) defined the mathematization process as follows:

> *"we characterize the process of mathematising as interpreting the structural aspects (i.e., the objects, relations, actions, patterns, regularities, assumptions, etc.) in a real world system, and expressing this structure in a mathematical model using mathematical representations such as symbols, text, graphs, diagrams, and so forth."*

The first part of their definition, interpreting the structural aspects, resembles the horizontal mathematization while the second part, expressing the structure in a mathematical model, resembles the vertical mathematization distinction of Treffers (1987). Students involved in mathematization process need mathematical resources beyond arithmetic, including visualization, schematization, and data handling (Lehrer & Schauble, 2003). Similarly, the second part of Yoon and others' (2010) definition includes some of the characteristics of the vertical mathematization.

2.3. Functions and Trigonometry

Most of the modeling activities involve mathematizing by writing a suitable function to represent the desired real-life situation. Therefore, the literature related to process of mathematization including writing functions exploits literature of functions. In traditional approach, linear functions are the first function type that students encounter during their mathematics education years, which may result in some sort of difficulties like illusion of linearity (Dooren et al., 2004; Michelsen, 2006). Later on, students tend to use linear function everywhere whenever they need a function to represent a situation. In order not to face this difficulty, other type of functions like exponential and trigonometric functions can be used to give the concept of smooth change and develop the concept of variable (Michelsen, 2006). Hence, the sequence of types of functions becomes important and should be selected carefully when it is introduced to students.

Although the concept of variable is a key concept in algebra, most of the textbooks do not even mention it (Akgün & Özdemir, 2006; Skemp, 1986, p.213). Previous studies show that students have difficulties about the concept of variable, which may limit the learning of other concepts of algebra (Akgün & Özdemir, 2006; Bardini et al., 2005; Dede, 2004; Michelsen, 2006;). This difficulty emerges more clearly, when students confront with a case of understanding variations in a form of relating a variable to another (Malisani & Spagnolo, 2009). The experience that most of the students have, before they were introduced functions about variables, is the distinction between the concept of given and unknown quantities. However, some researchers (Dede, 2004; Feeley, 2013; Sierpinska, 1992) supposed that starting the concept of variables as literal symbols instead of unknowns may facilitate the understanding of the concept of function.

While forming the functions expressing the real-life situation in modeling activities, students deal with independent and dependent variables. Therefore, modeling activities may be used as a facilitator to solve the difficulty mentioned above since students gain practice in identifying and representing variables (Michelsen, 2006). However, it should be a well-planned procedure since previous studies showed that students had difficulties to recognize variables and parameters while they were solving modeling tasks (Huang, 2012). Without such important knowledge, students may suffer from difficulties in modeling activities especially during the mathematization process.

Trigonometric functions, besides carrying all the properties of concept of functions, involve cognitive obstacles for students to see them as functions. Breindenbach et al. (1992) stated that since trigonometric functions cannot be expressed as algebraic formulae derived from arithmetic operations, students have difficulty to recognize them as functions (cited in Weber, 2005). According to Weber (2005), trigonometric functions may play a crucial role for understanding the concept of function. Thompson (2007) states that since modeling activities serve a realistic environment, they can be used to develop more connected and meaningful understanding of trigonometric concepts.

When trigonometric concepts are first introduced, most of the textbooks use the right triangle approach. After learning trigonometric functions as ratios of the length of sides of a right triangle, the transition from right triangles to unit circle approach is followed. However, this transition may bring cognitive difficulties with its nature (Brown, 2005; Keisoglou & Spyrou, 2003; Swetz, 1995). According to previous experience, students had difficulty when they faced the trigonometric values of obtuse angles.

Another important issue concerning the trigonometric functions is about the domain of them. Students have cognitive obstacles to gain the concept of real-valued functions (Fi, 2003; Steckroth, 2007). Studies showed that many students have difficulties to understand the radian concept. Although the radian concept can be used as a bridge between the domain of trigonometric functions involving angles and those involving real values, it becomes another source of misconceptions (Akkoç, 2008; Steckroth, 2007). Therefore, the misconceptions about the radian concept should be taken into consideration when it is used to introduce the trigonometric functions as real-valued functions.

Ferris wheel is a rich context for introducing the concepts related to trigonometric functions. It was used frequently in many different studies and textbooks (Doerr & Zangor, 2000; Larson & Edwards, 2007; Larson & Edwards, 2013; Moore, 2010; Streefland, 2003; Thompson, 2007). There were also two questions based on the Ferris wheel context in PISA 2012 main survey (PISA, 2012). In those questions, the height of an object, the vertical position of a point and periodicity were to be examined.

3. METHODOLOGY

The aim of this study is to investigate and understand the difficulties of PSTs in mathematization process in a mathematical modeling task. In this chapter, the methodology of this research was described in terms of the context, participants, data collection procedures, data analysis, and settings of the study.

3.1. Design of the Study

This study aimed to get in-depth understanding of pre-service teachers' mathematization process in a mathematical modeling task based on a Ferris Wheel context including mainly trigonometric functions (see Appendix A). This modeling task investigates the basic properties of trigonometric functions, expressing the functions representing the real life situation. PSTs' mathematization process is examined in their written reports and interviews. Hence, case study as a qualitative research approach was used. Creswell (2008) defined the case study as "a strategy of inquiry in which the researcher explores in depth a program, event, activity, process, or one or more individuals" (p. 13). The case in this study comprised of two groups of three PSTs. The n^{th} pre-service teacher in groups A and B were coded as PSTAn and PSTBn respectively. Besides these groups, pre-service teachers involved in none of the groups A and B, but participated in group presentation session that were coded as PSTn. The mathematization processes of PSTs in a mathematical modeling task that were investigated and analyzed deeply in this study.

The groups were selected from pre-service teachers who were enrolled in the "Mathematical Modeling for Teachers" course in 2011-12 fall semester at a public university in Ankara, Turkey. The first five weeks of this course were a kind of pilot study that is an implementation of a model development unit. The required data in this study were collected from the "Ferris wheel activity" applied in the fourth week of the course. In this activity, PSTs were asked to develop mathematical formulas giving information in the capsules of the Ferris wheel. In each capsule, there is a screen displaying the following information: 1. the height from ground level, 2. velocity, 3. the distance from the boarding point, and 4. remaining time to complete the revolution.

The framework used to analyze participants' mathematization process in this study is based on the process of mathematization occurred within MEAs. The distinction of mathematization process as horizontal and vertical mathematization in RME is also used to support the MEA framework. Since the cognitive processes took place during the modeling activity were analyzed in the study, cognitive modeling perspective (Kaiser & Sriraman, 2006) can be accepted as the modeling perspective of the study.

Similar to Yoon and others' study (2010), since the pre-service teachers in the study had been taught trigonometry and other relevant concepts before the activity, participants may approach the activity as an application problem and a modeling task.

3.2. Setting and the Procedures

Pre-service teachers worked as groups in all activities. The groups were formed by PSTs according to their willingness. The sitting plan during the activity is given in the figure 3. As seen in the figure, the desks were arranged in a way to increase the interaction among them. There were also enough spaces between the groups so that the interference was not allowed.

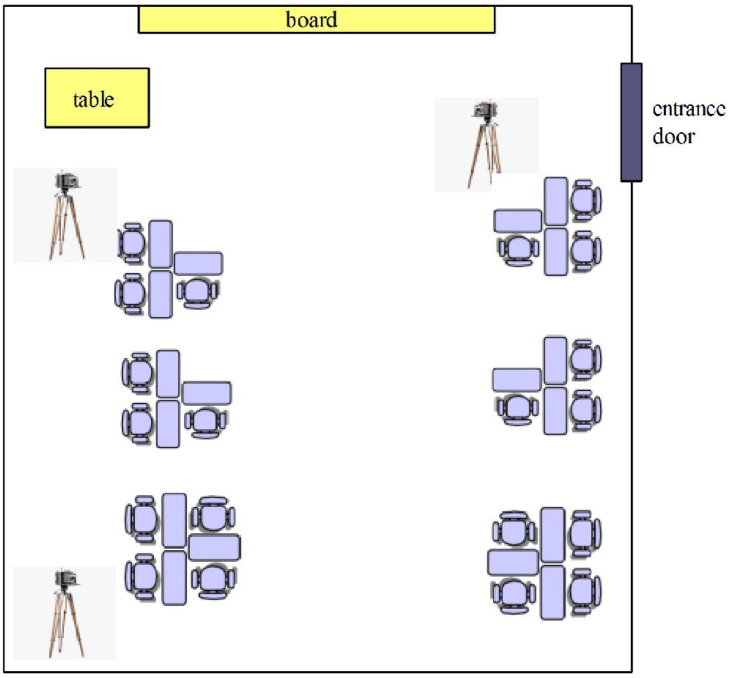

Figure 3 Allocation plan during the data collection

During the activities in each session, researcher and four research assistants were present in the class. Three of the research assistants were recording the solution process of the groups during the activity by video camera. On each group of desks, a voice recorder was also placed to record the voice of participants.

3.2.1. The structure of the course

PSTs were informed about the schedule of course and data collection for the aim of a research in the first lesson. It was also mentioned that the names of the participants would be kept confidential and never be used in any publication. After these confirmed information about the project, the consent letters were taken from the participants (see Appendix B). The instructor of the course was a member of the project team.

Table 1 Schedule of "Mathematical Modeling for Teachers" course

Week	Day	Topic	Assignment
1	Sep. 29, 2011	Overview and organization of the course Concept Map-I	
2	Oct. 6, 2011	"Summer Job" Activity Technology and mathematical modeling: An overview of MS Excel Discussion of first impressions	
3	Oct. 13, 2011	Technology and mathematical modeling: An overview of ClassPad. Nature of Mathematical Modeling (3 modeling activities will be studied by groups)	Reflection papers on Summer Job
4	Oct. 20, 2011	"Ferris Wheel" Activity Nature of mathematical modeling	
5	Oct. 27, 2011	"Street Parking" Activity	Reflection Papers on Ferris Wheel
6	Nov. 3, 2011	Students Thinking Styles related to "Street Parking" Activity Questionnaire-I	Reflection Papers on Street Parking
7	Nov. 17, 2011	"Water Tank" Activity	
8	Nov. 24, 2011	"Population of Turkey in Next Century" Activity	

(Table 1 continued)

Week	Day	Topic	Assignment
9	Dec. 1, 2011	Discussion on the nature of mathematical modeling, the characteristics of mathematical modeling activity and principles that have to be considered in developing a modeling activity.	Reflection Papers on Population of Turkey
10	Dec. 8, 2011	"Roller Coaster" Activity	
11	Dec. 15, 2011	Classroom Discussion on Roller Coaster activity Students' ways of thinking	Reflection Papers on Roller Coaster Project drafts
12	Dec. 22, 2011	"Tracking Track" Activity Discussion on nature of mathematical modeling and role of group work	
13	Dec. 29, 2011	Questionnaire-II Discussion on the roles of teachers in modeling process Developing plan for classroom application of a modeling activity	Reflection papers on Tracking Track Report on Models and Modeling
14	Jan. 06, 2012	Presentation of projects.	
15		Presentation of projects. The general evaluation of the semester.	Projects

3.3. Participants of the study and data sources

The aim of this study is not to reach statistically valid generalizations, it was targeting a particular group of pre-service teachers. Therefore, the sampling method can be accepted as a non-probability sample (Cohen, Manion, & Marrison, 2007). There were 20 undergraduate and graduate students taking the course. They were enrolled in elementary mathematics teacher education program in the university. Seventeen of them were senior students who had already taken the courses Calculus-I, Calculus-II, Differential Equations, and Linear Algebra from the department of mathematics. Seventeen participants were students enrolled in elementary mathematics education department in the university. Three of the remaining participants were graduate students, one of whom was an in-service teacher. The ratio of females to males was 13 to 7. Participants had taken most of the must courses offered by department of elementary teacher education program. Besides the courses offered in the university, they have been taught trigonometric ratios and functions including trigonometric equations during their high school years. In elementary school years, the trigonometry was first introduced as ratios related to right triangles. Then trigonometric functions based on unit circle were described in high school.

In order to investigate mathematization process deeply, purposive sampling (Merriam, 1998) was used to decide the richest cases, which may gave generous information. Therefore, the PSTs in the selected groups served a good opportunity to investigate the mathematization process. In contrast to the selected groups, the other groups were more successful in modeling process. Besides the groups A and B, some PSTS's reflections and discussion during the group presentation were also included in the investigation. The selected groups comprised of three pre-service teachers. In group A, two of three PSTs were male while one of three PSTs was male in group B. It is understood from the discussions that none of these pre-service teachers faced modeling activities before this course.

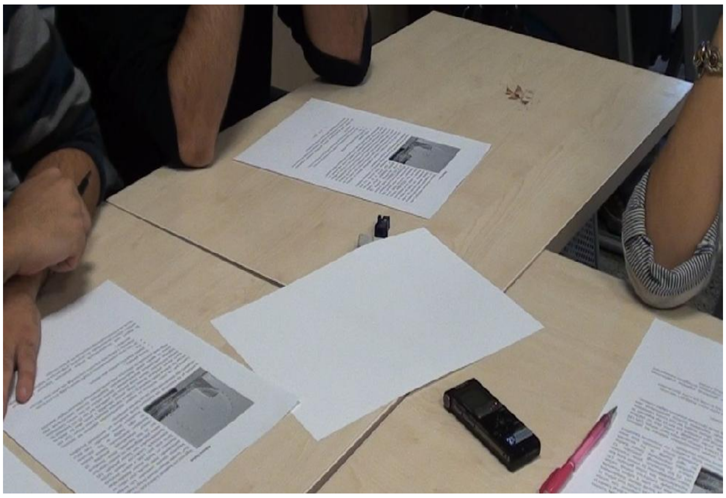

Figure 4 Snapshot during individual work in group A

During the first phase of the activity, the participants worked on the problem and tried to find out a solution within the groups (see Figure 4). All the group discussions were videotaped and also their voices were recorded. The second phase was about the discussion and presentation about group solutions. In this phase, the groups presented their solutions and answered the questions addressed from the other participants. These group presentations were also videotaped. After the group presentations, group solutions and reports about the procedure were collected. Papers including hand writings were collected after class sessions. Each pre-service teacher was also responsible for writing a reflection about the activity.

3.3.1. Worksheets and group reports

After the modeling activity, not only individual worksheets and reports but also group worksheets and reports were collected and transferred into digital form. Then, necessary figures and passages were captured using digital image processing software.

3.3.2. Reflection papers

Each pre-service teacher was responsible for writing a reflection about the activity. In these reflection papers, PSTs were expected to emphasize the redefinition of problem, personal ideas before group work, explanation of problem solving process, evaluation of their solution, comparison of their solutions to others', and looking from teacher perspective while applying the problem in classroom settings.

3.3.3. Audio and video recordings of the group discussions

Throughout the activity, group discussions were videotaped. When a close up was needed, the research assistant using video camera focused on the papers (see Figure 5).

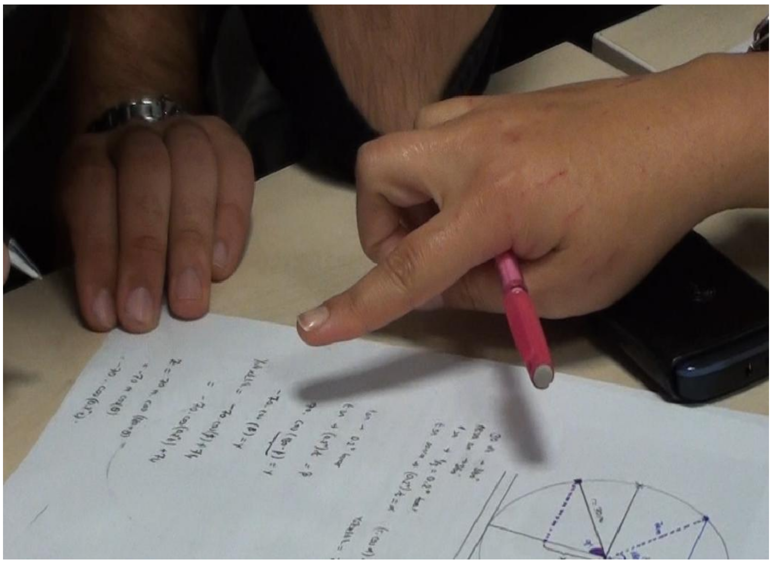

Figure 5 Close up during group discussion

Besides the videotape, a voice recorder was placed on desk (see Figure 6) to record the PSTs' voices during the group discussions.

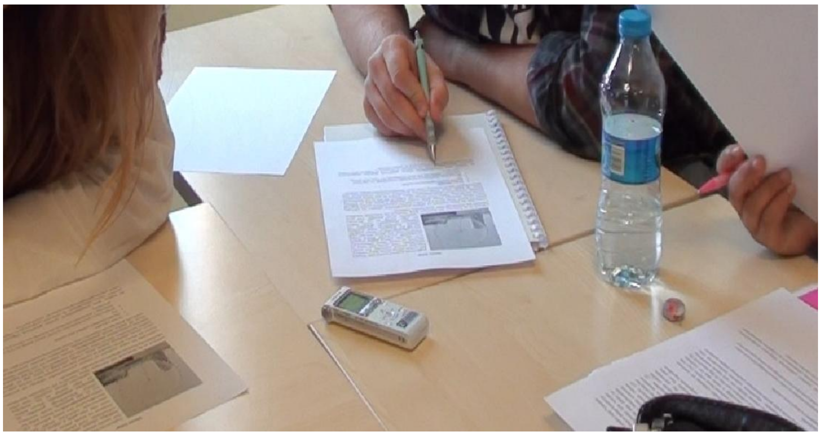

Figure 6 Snapshot during discussion of group B

3.3.4. Video recordings of the group presentations

Similar to the video recordings of the group discussions, the group presentations were also videotaped.

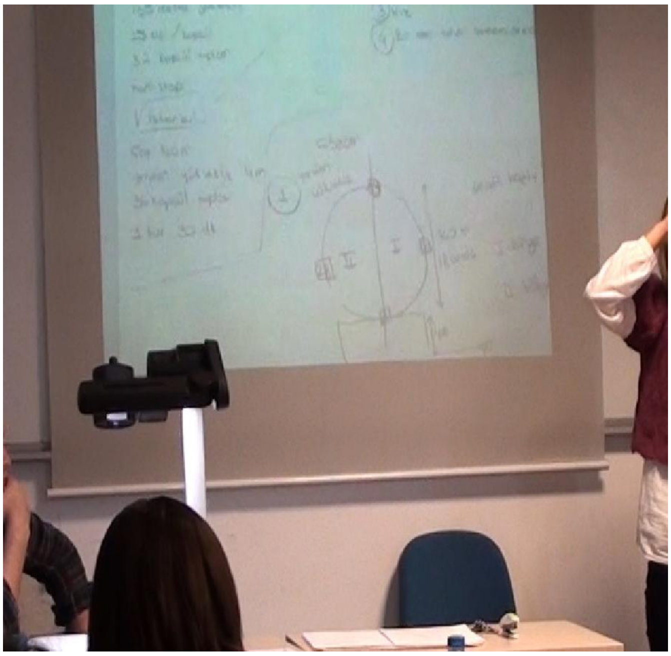

Figure 7 Snapshot during group presentation

3.5. Data Analysis

In this section, the approach to analyze the gathered data including how pre-service mathematics teachers use mathematization during the Ferris wheel modeling activity is presented. The collected data including videotapes, voice recordings, worksheets, group reports, and reflection papers about the activity were first organized. After the initial organization of data, an initial analysis of videos by viewing all videotapes was roughly completed. During this phase, the notes related to students' behaviors and responses in group discussion were taken.

After the initial analysis, two groups were selected to be transcribed. The group discussion video in group A was transcribed in detail and reanalyzed together with worksheets of PSTAs and their reflection papers. After the reanalysis of transcription of video of group A, themes and categories of the data were formed by comparing group work papers. In order to decide themes and codes, a framework for mathematization process according to Yoon et al. (2010) was used.

> *"Thus, we characterize the process of mathematising as interpreting the structural aspects (i.e., the objects, relations, actions, patterns, regularities, assumptions, etc.) in a real world system, and expressing this structure in a mathematical model using mathematical representations such as symbols, text, graphs, diagrams, and so forth."*

The codes were determined by the characterization of mathematizing above and divided into two main groups. These groups were also convenient with classification of mathematization process by Treffers (1987) as horizontal mathematization and vertical mathematization. When a clarification was needed about a theme, video was viewed again. According to the identified themes from discussion within group A, the selected transcription of the group B was completed. (See Appendix C)

Table 2 Table of codes used in data analysis

	Code description		Description	Example from data
Interpreting structural aspects	Understanding problem	Verbal definitions	Any utterance while expressing the meaning of the situation.	While dealing with the distance to the ground PSTA1 said, "we shall work on constant speed. I mean average speed."
		Sense of connection to real life	Any utterance about the relationship between the real-life situation and mathematical world.	Explaining the vertical distance between consecutive capsules PSTB1 stated that "One of the capsules will be here, closer to that one. I mean the arc length here is longer".
		Assumptions	Any utterance or worksheet that take place while PSTs suppose an underlying knowledge to build their solutions on it.	When PSTAs were trying to find the distance between two consecutive capsules PSTA2 said, "The circumference is 440 m. 36 many capsules…We will divide the circumference by 36. Or 35? I think 35."
	Visualization		Any graphical representation attempt to understand or extend the solution.	PSTBs' drawing to understand the height of a capsule from ground (see Figure 9)
Expressing structure in a mathematical model	Writing formula		Any utterance about the inference in formulating of a pattern	PSTB' attempt for recognizing the pattern "PSTB1: We cannot formulate it, sir. We cannot relate the values in the pattern."

Table 2 (continued)

Code description		**Description**	**Example from data**
Using diagrams		Any figure or drawing to illustrate needed to clarify the situation.	The drawing to clarify the usage of cosine theorem for finding the distance to initial point. (see Figure 15)
Functions	Identifying variables	Any utterance about deciding the variables of functions representing the desired situation.	PSTA3: Time is important. If I can write a formula for the arc length for any arbitrary time, then it will always be valid.
	Trigonometric functions	Any utterance about the concept of trigonometric functions.	PSTA3: OK! What is $\cos\alpha$? $\cos\alpha = \frac{a}{r}$. We can find a, using this expression. PSTA2: But you cannot use it. It is valid for a right triangle.
Applying previously learnt topics		Any utterance about the application of other topics in the mathematization process.	While PSTB1 presents their solution PST1 rejected as "But we cannot form proportion. They are not proportional. It decreases gradually."

In order to increase the validity of the study, triangulation by using different data sources was used. When a finding was needed to be clarified and supported, other sources were taken into account and investigated deeply. For each finding to be reported, reflection papers, worksheets and transcripts were analyzed for triangulation to assure the validity of the study.

4. RESULTS

This chapter answers the research questions of the study by reporting the results from the process of mathematization in modeling task. The detailed descriptions of mathematization process of group work and passages from the reflections of pre-service teachers were selected and presented in the chapter. The results obtained from data were arranged according to the different forms of mathematizing classified by Treffers (1987) as horizontal and vertical mathematization. Similar to the interpretation of Yoon et al. (2007) for the forms of mathematization, the results were classified into two groups as the way of understanding the real life situation (interpreting the structural aspects) and expressing the inner structure of a model (expressing the structure in a mathematical model using mathematical representations). This classification is also suitable with the research questions of the study.

4.1. Description of PSTSs' Interpretations of Structural Aspects of Ferris wheel Activity

In this part of the results section, the first research question, "How do pre-service mathematics teachers interpret structural aspects of a real life situation mathematically during a modeling activity?" was explained.

4.1.1. The Way of Understanding the Ferris wheel Activity of PSTs

Understanding the real life situation here means to perceive the written text in the problem, to establish the connection of problem with real life and to use the correct assumptions by utilizing their own verbal definitions of the situation.

After reading the problem, PSTs were tried to explain the problem situation by their own words to the other group members. During the discussion for understanding the modeling problem, PSTs were made to reread the question and state the givens. They tried to express the distance and time according to a well-known formula

$\text{Distance} = \text{Speed} \times \text{Time}$. However, they could not concentrate on the linear speed but on angular velocity.

As it can be observed in the following episode from group discussion, the pre-service teachers' difficulty with discriminating angular and linear speed concepts appeared. This also appeared in other group discussions. The following excerpt from the discussion of group 8 with four members shows how participants concentrated on the speed concept and how they made a distinction between two types of speeds.

> *PSTA3: What you said is correct! Look! If we think the speed as… Is it in the same direction with rotation? Yes! I have a speed like that which give me the distance from the base and one that is horizontal speed.*
>
> *PSTA1: Doesn't* $\text{Speed} \times \text{Time}$ *give us the distance?*
>
> *PSTA3: Yes*
>
> *PSTA1: Which distance does it give? The one it takes around the circumference. OK then! How can we convert it into the distance from ground?*
>
> *PSTA3: You will take vertical speed.*

After talking about the components of speed, they decided to concentrate on average speed. When

> *PSTA1: We shall work on constant speed. I mean average speed. Total distance divided by total time.*
>
> *PSTA2: Speed is already constant.*

PSTs were then focused on the central angle formed by capsules. However, they could not decide which angle they considered easily.

PSTA1: Do we know its angle? The angle is always changing. There is no constant angle but instantaneous angles. The angle stands here.

PSTA3: Let me tell you that… This angle never changes.

PSTA1: What does never changes mean? Take this, 90 degree… Take this one, 0 degree.

In order to state their solution, PSTs stated their assumptions. Assumptions were very important to decide the starting point for solution. For finding the speed of the Ferris wheel, they decided to find the number of distances from two consecutive capsules.

PSTA2: The circumference is 440 m. 36 many capsules… We will divide the circumference by 36. Or 35? I think 35.

PSTA1: OK then. The answer is 12.57 m. I mean the distance between these two. But, wait! We cannot say that it is 12.57 m. Because there are also places that cover themselves. It becomes messy.

The second group did similar false assumption while they were trying to understand the distance between two consecutive capsules.

PSTB1: There are 36 capsules. How many intervals are there? Ten fingers and how many intervals?

PSTB3: 9

PSTB1: 9 intervals. 36 capsules, 35 intervals.

PSTB2: No. Wait, wait! What do you do?

PSTB3: Oh! It is circular. So n – 1.

PSTB1: It turns to be a circular table problem.

In relation to consecutive distance, when the Researcher asked PSTs to summarize what they had done so far, they insisted on the same assumption.

Researcher: OK! What about the distance?

PSTB3: We thought is as… It resembles the circular table problem. One interval and it becomes n-1 in circular table.

The same assumption was continued when they wanted to find the measure of the central angle formed by two consecutive capsules and the radii.

PSTA2: There are 36 capsules, right! What if we find the angle here (By pointing out the central angle) Like that… $36°$ *is to 30 min as* $10°$ *is to x min?*

PSTA1: $\frac{36 \cdot 60}{10} = 216$ *seconds.*

PSTA3: Are you sure about $10°$ *? Because there are 35 many of this.*

PSTA2: Right! So, $\frac{360}{35} = 10,285714$

When second group worked on the vertical distance from any capsule to ground, they used the assumption that the capsules distributed evenly when their vertical distances take into account.

Researcher: How did you find that?

PSTB1: We found it like that… As the vertical distance between any two consecutive capsules is 7.78 m… This is the first position; this is the second position… What do we do to find the distance of the second one? We write $(2-1) \cdot 7,78$ *m.*

Researcher: Is this 7.78 m the distance from ground? Or what? Or the distance between any two?

PSTB1: The vertical distance between any two capsules, sir.

Researcher: Not the distance between two capsules, right?

At this point, PSTs assumed that the vertical distance between two consecutive capsules, which are equidistant from each other on the circumference of the circular path, is equal. Researcher was trying to emphasize this assumption.

Researcher: The word "distance between two" means…

PSTB1: This height is exactly 140 m. There are 18 many intervals here. Hence each interval is 7.78 m long.

Although the researcher was trying to give a clue about the false assumption, they insisted on the equal vertical distances.

Researcher: (By joining the capsules which are equally distributed on circumference by horizontal line segments)

PSTB1: We assume that these distances are equal.

Researcher: Your assumption… I am trying to understand that. However, why those are equal?

PSTB2: They are distributed equally.

Researcher: Yes, they are distributed equally on circumference. As I understood, you

PSTB1: This was our assumption.

Researcher: No problem in case you can explain the reason.

After this conversation, PSTB1 caught the mistake and tried to explain the misassumption to their friends.

PSTB1: We made a mistake!

Researcher: Why do you think that you made a mistake? You can convince me, I am so open-minded.

PSTB1: Maybe but I tried something like that. Take the angle $45°$ *, the midpoint. Those two angles are same but these are definitely different. This is longer than this one.*

Researcher: What is your opinion?

Although the explanation about the non-equality of distances was correct, the group members were not convinced.

PSTB1: When we find the vertical projection, this is absolutely greater than this one.

PSTB2: Do you mean those two?

PSTB1: Yes

PSTB2: Equal! Because they are both radii.

Researcher: She does not talk about that. She means those two.

PSTB1: Yes, sir.

PSTB3: Equal! Because when you draw from center, they are all radii.

Researcher: She does not mean that. What did you do there? You meant the distances from the capsules to diameter.

PSTB3: Yes, they are equal!

At this point, the researcher gave them the last chance to understand the hint and divided the quarter into three equal arcs. However, this was not enough for them to understand the mistake.

PSTB2: But it is true for $45°$ *. Let us draw* $45°$ *.*

PSTB3: No! We should not think like that.

Researcher: When the angle is $45°$*, think about these lengths. This is the midpoint. If it is twice this length, then you are right. Is it true?*

PSTB1 drew the point related to $45°$ and calculated the corresponding lengths. She then concluded that $\sqrt{2}-1 \neq 1$ to support fallacy of the assumption. Her drawing was given in the figure 8.

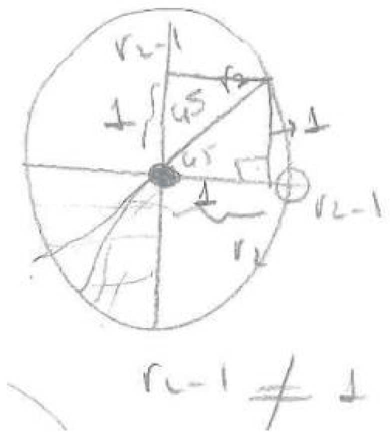

Figure 8 PSTBs drawing for understanding proportionality of vertical distances

Another important finding during the understanding the question was the lack of sense of connection to real life. When the group A discussed the length of the interval between two consecutive capsules, they took the length of the touching point of the capsules into account.

PSTA1: Distance between two capsules is 12.57 m. But, wait. We cannot say that it is 12.57 m. Because the capsules themselves cover an area. Then it becomes confusing. How many capsules do we have in total?

PSTA2: 36 capsules.

PSTA3: Then $\frac{440-36x}{35}$.

PSTA1: What does x stand for here? What is y?

PSTA3: x is thickness.

PSTA1: Thickness of what?

PSTA2: But we cannot know it.

In class discussion episode, PSTB1 was trying to explain assumption related to the vertical distances in group presentation. One of the PST's objected this assumption by asking questions. Throughout this narrative, a lack of sense of connection to real life was obvious.

PST2: The distances between the capsules are not equal according to the projections. Aren't they? Because this Ferris wheel is continuously rotating. Otherwise, there will be an unbalance. If the intervals are not evenly distributed, its speed will not be constant. Sometime it will be faster and sometimes slower.

PSTB1: I would just explain this situation. When one of the capsules stands here, the 7,78 m distance will be kept. One of them will be here closer to that one. I mean the arc length here is longer.

PST2: What happens when it rotates?

PSTB1: When it rotates… It is another assumption. When it rotates, the bars here were designed to be stronger. I mean it is constructed like that.

4.1.2. Visualizing to Understand the Situation

During the understanding stage of the modeling question, PSTs were frequently used drawings for the sake of easiness and arrangement for understanding the case. In this section, pre-service teachers' the visualization attempts during the horizontal mathematization were presented.

PSTBs' proportional angle and vertical distance assumption were stated after they drew the figure showing the settlement of the capsules on the Ferris wheel.

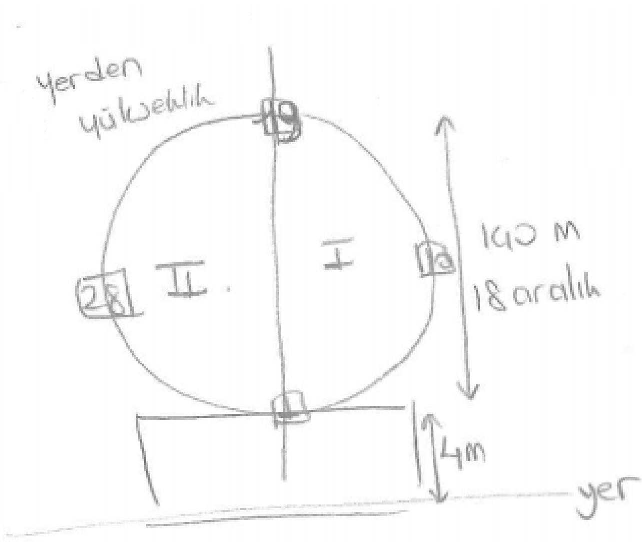

Figure 9 PSTBs' visualization to understand the height of a capsule from ground

This drawing probably misled them to use the proportionality, which will be investigated later in detail. PSTs also visualized the possible positions of capsules and their relative positions in order to understand the speed of the Ferris wheel.

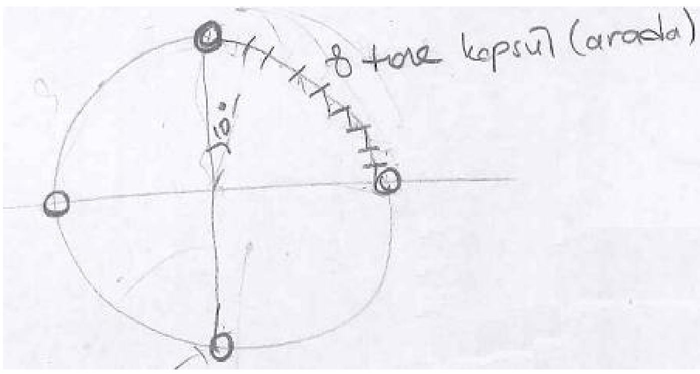

Figure 10 Visualization for distance between consecutive capsules

4.2. Characterizing PSTs' the Way of Expressing the Inner Structure of a Model

This section is about the findings related to the second research question; "How do pre-service mathematics teachers express the inner structure of a model of the real life situation in the Ferris wheel activity?" These findings were related to vertical mathematization of the modeling process. The results were categorized according to components of vertical mathematization.

4.2.1. Using Diagrams

> *PST5: While we are solving questions, we draw some figures to understand it better. I can give an example of this kind of figures as the figure below.*

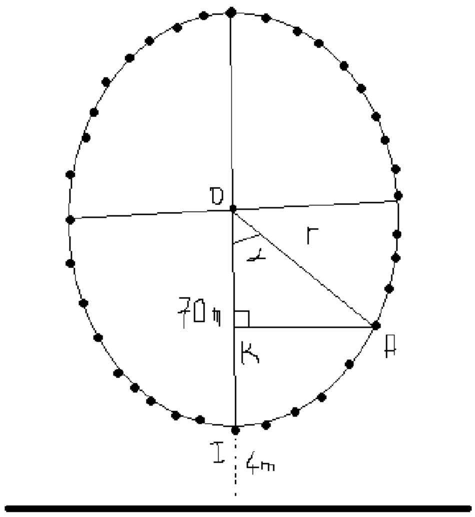

Figure 11 PST5's diagram to understand the height of a capsule from ground

When PSTs in group A investigated the situation, they considered the capsules in different positions with different subintervals. They divided the case into four parts. To explain these parts, they used the diagram to depict the case (see Figure 12).

> *PSTA2: Then how much does it remaining to complete a full revolution? The total time 30 min. How much did it take? How much is remaining?*
>
> *PSTA1: OK! Let us suppose that it is here. 15 minutes remain to complete the revolution. When it is exactly here, 22.5 minutes is remaining.*

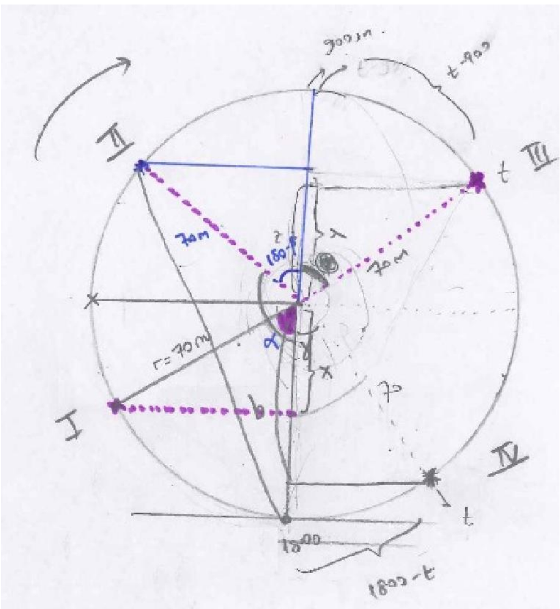

Figure 12 PSTAs' drawing for illustrating different positions of capsules

PSTs in Group B drew their assumption of equal vertical distances by using the following figure (see Figure 13).

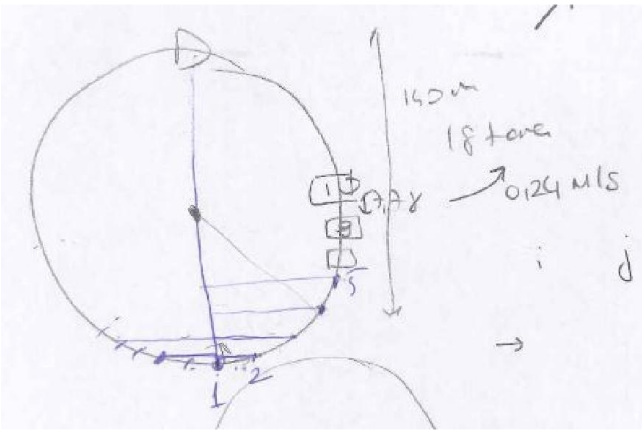

Figure 13 PSTBs' drawing for understanding vertical distances

During group presentations, PST3 used the following diagram to explain the usage of proportional vertical distance, which will be investigated later in detail.

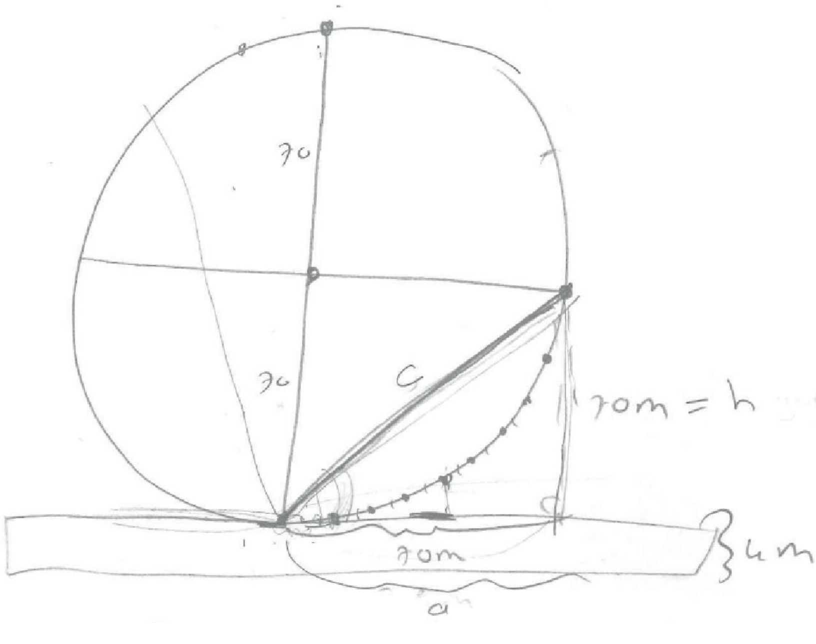

Figure 14 PST3's drawing for explaining the proportionality of vertical distance

PSTs were made to utilize different approaches and assumptions while they were finding the distance from the departure point for an arbitrary moment. One of them used cosine theorem for triangles to find the length of a side when the lengths of two sides and the angled between them. In order to explain the reason to their group members, the diagram was used (see Figure 15).

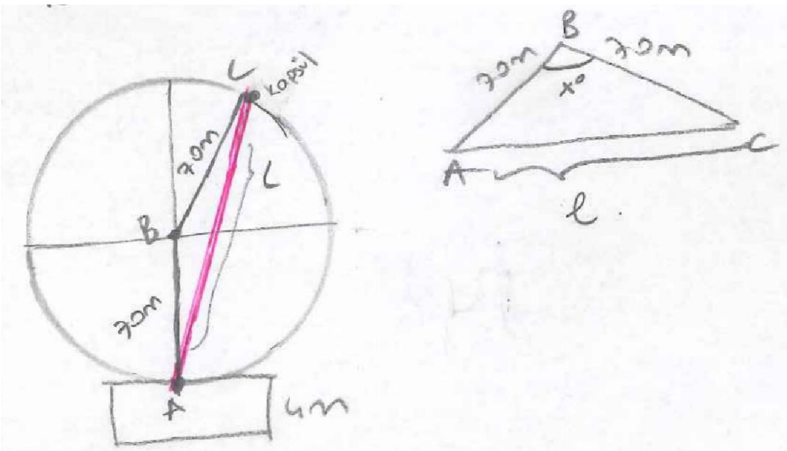

Figure 15 Drawing for explaining the usage of cosine theorem to find the distance to departure point

For the same part of the modeling task, another PST used the figure 16 and come to a conclusion that twice the sine of the half of the central angle multiplied by radius would give them the answer.

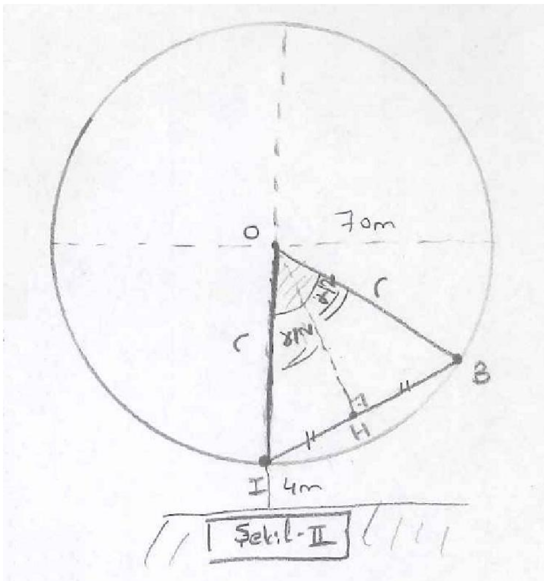

Figure 16 Drawing for explaining the usage of sine of an angle in a right triangle to find the distance to departure point.

4.2.2. Forming Functions to Represent the Situation

In many modeling activities, writing function representing the real life situation is the most essential part. This section was aimed to report the process of mathematization related to concept of functions.

4.2.2.1. Identifying the Variables

The first theme about the functions to be reported is identification of variables. While writing the function representing the desired real life situation, PSTs needed to determine the variables of functions. They were sometimes confused to choose the variable to have a continuous function. Since the desired data on the screen should be continuous, they should choose a correct variable.

The first attempt to choose the independent variable was generally assigning the independent variable as the central angle formed by two capsules and the center of Ferris wheel.

PSTA2: We know the arc length. If w know it, we can find the angle.

PSTA3: How?

PSTA1: We can do it with angle. Because, from the circumference… Can we find the height if we know the angle?

PSTA2: Yes! First, let us talk about what happens when we find the angle. If you know the angle, can you find the height?

This approach can be seen in the reflection papers of PSTs. They emphasized the priority of assigning the independent variable for the desired function.

PST4: I offered that we could place the capsules according the arc length corresponding to angle measured by $10°$*. Therefore, we could use only angle for all desired expressions.*

Although they realized the importance of the time as an important factor, they continued to try to use the angle as the central variable.

> *PST: This Ferris wheel completes the revolution in 30 minutes in other words 1800 seconds. This means it spans* $360°$ *in 1800 seconds. Hence, it spans* $0,2°$ *in 1 second.*

After discussions on the functions to be written, some of the group members realized that time should be the independent variable of the desired function.

> *PSTA3: Look! I will tell you something important. All the touching points are same, right? What happens when it moves? It changes. Come here! You know the length of this arc. Don't you?*
>
> *PSTA2: If it is in the middle, then we can find it.*
>
> *PSTA3: No! Can we find for an arbitrary point?*
>
> *PSTA2: In fact, you can. I will find on 1 cm. Then we can find when it moves for each 1 cm.*
>
> *PSTA3: Time is important. If I can write a formula for the arc length for any arbitrary time, then it will always be valid.*

This progress was stated also in some reflection papers of PSTs. After the modeling activity, they reported the solution process and realized the transition from angle to time as independent variable.

> *PST1: We formed the function representing the vertical distance of a capsule at an arbitrary position depending on the angle. Therefore, we thought that we completed the solution. However, we later on realized that writing the function with time, as the independent variable would be better. Hence, we changed all the angles to time.*

PSTA1 stated the reason of decision to choose time as the independent variable of the function as follows in his reflection paper.

PSTA1: We prioritized time as the independent variable of the function since the text paid attention to the word "instantaneous".

PSTBs used another approach to assign the independent variable for the function. They used "numbered positions of capsules" instead of a continuous variable while PSTB1 was explaining their process of selecting independent variable in their group presentation.

PST2: You said that n is the number of position. Did you take it 36? I mean, is the interval for n from 1 to 36?

PSTB1: Yes! This is the first position and this is 36^th^ position.

PST 2: OK! Instantaneous information available for the passengers in the capsule is asked in the question. Let us suppose that the capsule is in the fifth position. You gave the information in the fourth position. Will you wait until it moves to the fifth position? What about any position between 4^th^ and 5^th^? For example, do you substitute 4,5 into n to find the information in 4,5?

PSTB1: I did not think about it. However, it can be given for any position.

At this point, an interesting conversation between PSTs was observed. The point of view was focused on from the independent variable to concept of function itself.

PST1: There is also an interesting situation here. It can give information to just one capsule. The one at the bottom. Say that, it moved to the second position. Since the arc lengths are not same, the values are not valid for the others.

PST2: You should not consider n from 1 to 36 then. Say, n is 4! You should give many different values to n like 4.1, 4.2 and so on.

PST3: It means there is no function. It is not continuous!

Continuity was accepted as a necessary condition to be a function here.

4.2.2.2. Trigonometric Functions

This section focused on the difficulties related to trigonometric functions in the mathematization process of Ferris wheel activity. PSTAs were probably insisting on assigning the central angle as the independent variable unconsciously while forming the functions.

PSTA1: Which angle will I use know? This one! How can I explain this angle? Let the angle spanned this length be β *. Therefore, it is* $180-\beta$*. OK! How can I find* β *now? It spans* $0.2°$ *in a second. It came here in t seconds and spanned* $0.2 \cdot t°$ *.*

PSTA2: We can use the same way.

PSTA1: $70 \cdot \cos(180-\beta)$*. What is this now?*

PSTA2: It is y.

PSTA1: Then, what can I write instead of $\cos(180-\beta)$*? Can I say* $-70 \cdot \cos\beta$ *?*

PSTA2: This will always be same. No need to say beta or theta. Look it is also true here. Because it is depended on t.

PSTA3: Look! According to my idea it is $\cos(360-\alpha) \cdot 0.2 \cdot t$*. Isn't it? We need to write a formula depending on just t. There will be no alpha.*

While PSTs in group A expressing the vertical distance of a capsule to the ground, they were discussing on trigonometric functions defined on right triangles. They tried to use trigonometric function for angles less than $90°$.

PSTA3: OK! What is $\cos\alpha$ *?* $\cos\alpha = \frac{a}{r}$*. We can find a using this expression.*

PSTA2: But you cannot use it. It is valid for a right triangle.

A similar approach can be seen in the discussion carried in group A. PSTA3 had the difficulty to express the distance when angle was not acute.

PSTA3: We should now find the distance to the departure point. Now, we have this distance. If we find alpha and a, then we can find this distance.

PSTA1: We can find it using Pythagorean Theorem.

PSTA3: Do you know what the problem is? The problem is… What happens if this point somewhere here? (By showing a point resulting in an obtuse triangle)

In the group presentations episode, while PST3 was presenting their solution, a typical mistake and difficulty was observed.

PSTA1: I wonder if we substitute 450 for t, do we get 70?

PST3: Our formula for the height is valid for the angles from 0 to 90. Because $\tan 90°$ *is 0. Since height will be 0 and our period is* 0.2 *…*

PSTB2: $\tan 90°$ *is infinity.*

PST3: Undefined! That means we cannot calculate anything there. Since our period is 0.2, the formula is valid until $89.8°$ *.*

The only confusion about the period of a trigonometric function is not the coefficient of α in $\cos\alpha$ but they had some difficulties about the circular pattern of trigonometric functions.

PSTA2: But wait! The distance will not change if the capsule goes from this point to this one (By pointing out two symmetric points with respect to the peak point of the Ferris wheel)

PSTs were also not capable of guessing the function directly by just looking at the periodic pattern of the height of a capsule in a Ferris wheel. The circular figure of the wheel should remind them to use trigonometric function, but they could not test it first. Although they guessed the circular pattern, they could not go one more step further to make a guess about trigonometric function. PST4 mentioned the process of finding the function in his reflection.

PST4: We wanted to find the information shown on a screen in each capsule to show distance, time left and height at current position. In order that screen to show the desired data consistently, we need to write a continuous function. However, the graph of height-time is not an increasing function.

PSTs' difficulties while deciding the type of the function expressing the model of the reality were observed in data. They first tried using the other types of functions like linear functions, radical functions and quadratic functions.

4.2.3. Writing Formulae

Choosing the function best fitting the given situation was very important as seen in the previous sections. After choosing the function to represent the situation, writing their formulae was another obstacle on PSTs' solution process. PST5 emphasized this difficulty in her reflection paper.

PST5: We had some difficulties while formulating the results. We had also been stuck when we are writing general formulae representing the desired data.

PSTAs had first worked on formulae by dividing the Ferris wheel into four parts. For each quarter, they wrote a different formula. After discussion on the formulae and working on cosine function and its properties, they realized that writing the piecewise function with 4 subintervals was unnecessary. (see Figure 18)

PSTA1: If it spans $360°$ *in 30 minutes or 1800 seconds, it spans* $0.2°$ *in a second. Right? Let us suppose that t seconds later it spans* $0.2t°$ *wherever it is. Radius is 70 meters. We know alpha. Then the formula is* $r \cdot \cos\alpha$ *and alpha is* $0.2t°$*. Is it equal to x? Then what about this distance?*

PSTA3: Wait a minute! What did you do? R times cosine alpha… Is it x?

PSTA1: Now, what is the height? It is 74 meters totally. We should subtract this. This is our height formula. $74-70\cdot\cos(0.2t)$ *but where is it valid? What is the interval for t? It is between 0 and 450 seconds. Is it OK? Now we came here?*

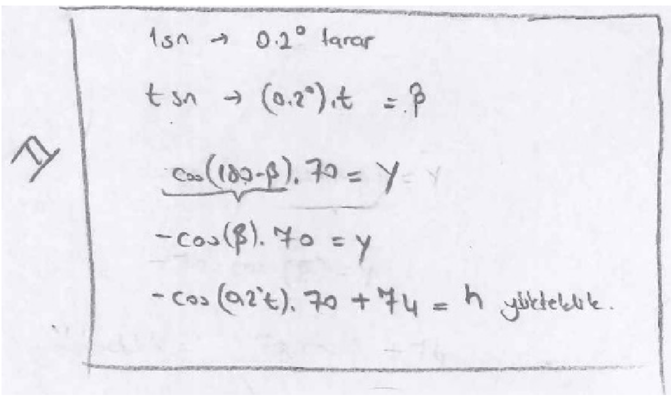

Figure 17 PSTAs' first attempt of usage of piecewise defined functions to find the height of a capsule

They first changed their mind to write a piecewise function with two subintervals. (see Figure 17)

PSTA3: We do not need to divide into four. Just divide into two.

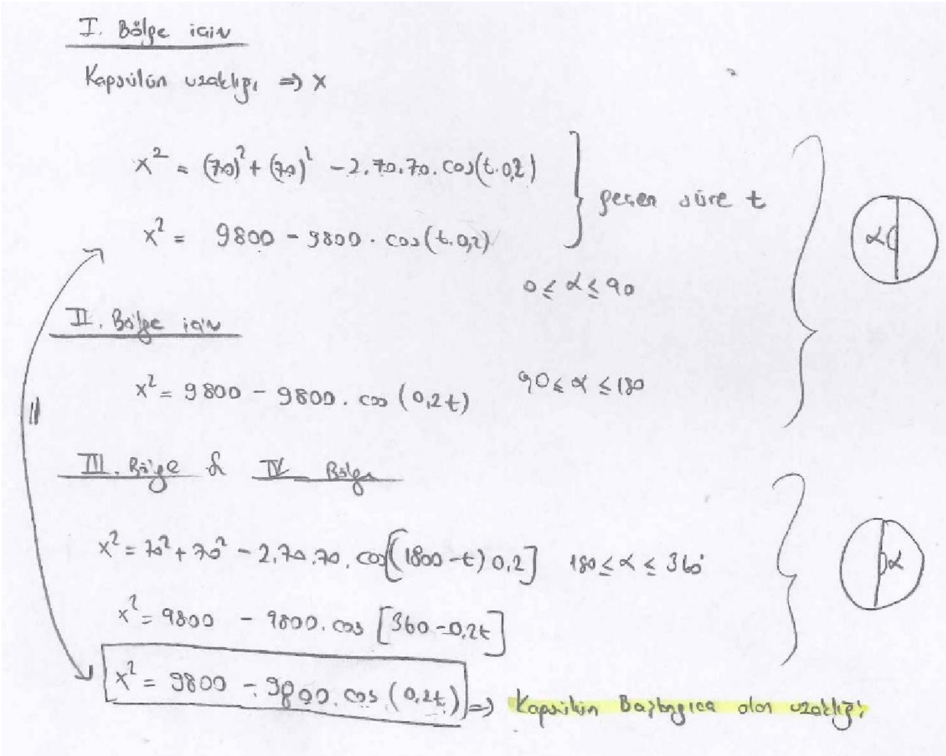

Figure 18 PSTAs' second attempt of usage of functions defined piecewise to find the height of a capsule

PSTA1: Now, how will I write theta? How long does it take to come here? $15 \cdot 60 = 900$ *seconds. Then I will write* $(t-900)$.

PSTA2: But it becomes another t.

PSTA1: We take always t from the initial point. Don't we?

PSTA2: Yes.

PSTA1: Why did I subtract 900? I take 15 minutes out. I need this part (by pointing out the third part of the Ferris wheel).

PSTA2: I see but we have just use different variable for each quarter. Then, why do we use t again here?

At the end of discussion, PSTAs concluded that only one formula would be enough to represent the desired height for any arbitrary time (see Figure 19).

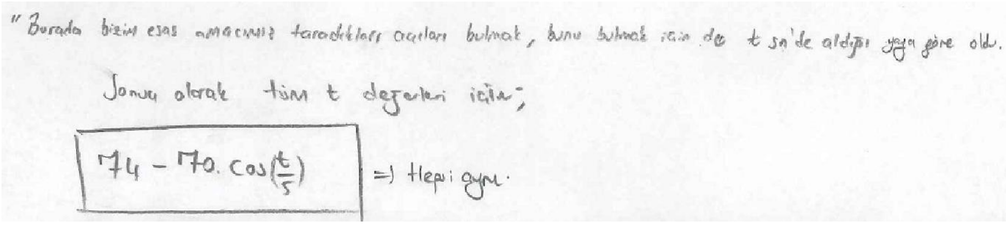

Figure 19 PSTAs' decision of usage of function to find the height of a capsule

In order to find the height of any capsule in the Ferris wheel, PSTBs had used a different approach. They supposed positions for capsules and decided to find the height of them using proportion. Besides the fallacy of their assumption, they also had difficulty to formulate the situation.

> *PSTB2: Look! A formula for 11th, another one for 12th... For 19th... That is to say, let us take 7.78 meters constant. I can formulize all of them with $(n-1)$... However; I need a formula that gives 3 for 11 and 5 for 12. I could not formulate them.*
>
> *PSTB1: Right!*
>
> *PSTB2: After that we will not find a new one. While we are programming we will equalize 18 to 20. Just equalization in these two quarters.*
>
> *PSTB3: Let us find the relation between them.*

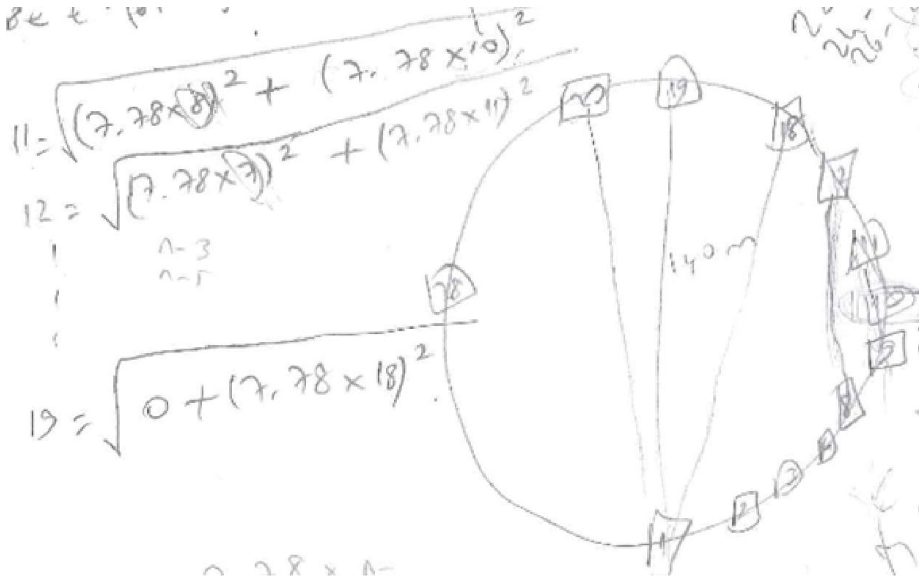

Figure 20 PSTBs' drawing to help for writing the formulae

They tried to recognize a sort of pattern to find a formula explaining all the cases. However, this was not an easy process for them.

PSTB3: Let us find the relationship between them. Is it $2n+1$ *?*

PSTB2: No! When we take $n=11$*, it will give us 8. What if it would be* $n-3$ *... No! it is not true again. At 11, it will decre3ase by 3... Decrease by 5 at 12.*

PSTB1: We can check from reverse! When it decreases by one, this one increases by one.

PSTB2: Or one increase versus one decrease.

RA1: What happened friends?

PSTB1: We cannot formulate it, sir. We cannot relate the values in the pattern. Oh! I found something! What about this $\pm(n-19)$ *? We take the positive one.*

PSTB2: Wait a minute! Substitute 11.

PSTB1: We substitute 11 and get – 8 but take the positive one. Substitute 12 and find -7 but take the positive one.

Figure 21 PSTBs' attempt for recognizing the pattern for writing the formulae

4.2.4. Applying previously learnt topics

One of the biggest difficulties that students experienced in the mathematics was the difficulty about lack of the ability in applying the previously learnt topics and integrating them to new topics. Although PSTs in the study had taken courses covering the sufficient knowledge for application in the Ferris wheel activity, they could not use this knowledge in a reasonable manner.

Both PSTs in groups A and B confused the interval with consecutive terms in a linear arrangement and a circular arrangement. They even confused this number with circular permutation of n different objects as $(n-1)!$.

PSTA1: It spans $360°$ *in 30 minutes. Alpha is about* $10.28°$.

Researcher: What number you divide it by?

PSTA1: Divided by 35.

PSTA2: We divided 360 by 35. There are 35 many of this angle.

Researcher: Really?

PSTA2: Yes! Because there are 36 capsules.

PSTA1: There are 35 intervals between them.

The combination of these two different confusions was investigated in group B. They misused both the number of intervals in linear arrangement and circular permutation.

PSTB1: There are 36 capsules. How many intervals are there? How many intervals between 10 fingers?

PSTB3: 9

PSTB1: 9 intervals. Then, 36 capsules and 35 intervals.

PSTB2: No! Wait! What are you doing?

PSTB3: It is circular. Therefore, the answer is $(n-1)$.

PSTB1: Hence, it becomes a circular permutation problem.

It was also observed that the PSTs had difficulties related to the tendency to relate linear characteristic to any two magnitudes. They thought that the central angle (arc length) and vertical (or horizontal) distance of capsules were in a linear relationship. In other words, arc length and vertical distance were proportional.

PSTB1: This height is exactly 140 m. There are 18 many intervals here. Hence, each interval is 7.78 m long.

Researcher: (By joining the capsules which are equally distributed on circumference by horizontal line segments)

PSTB2: They are distributed equally.

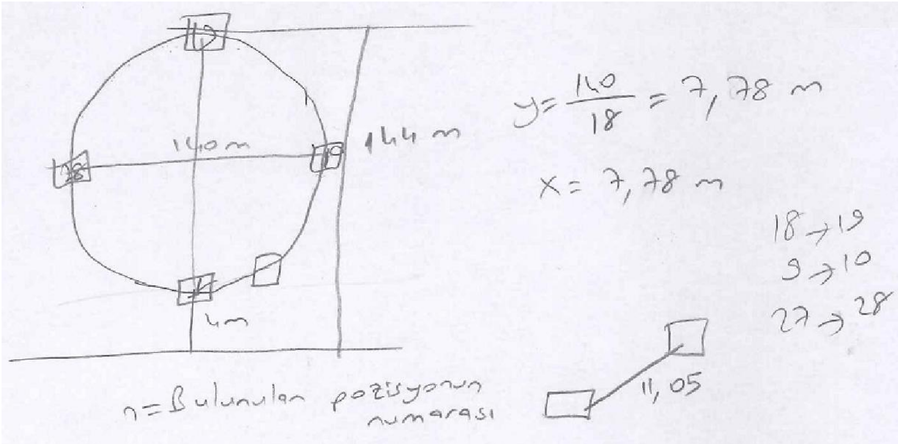

Figure 22 PSTBs' drawing for understanding the usage of proportion

PST3 fell into the same mistake when he was presenting their group solution. He explained that they formed an equation from direct proportion to find the change in the vertical distance based on the central angle.

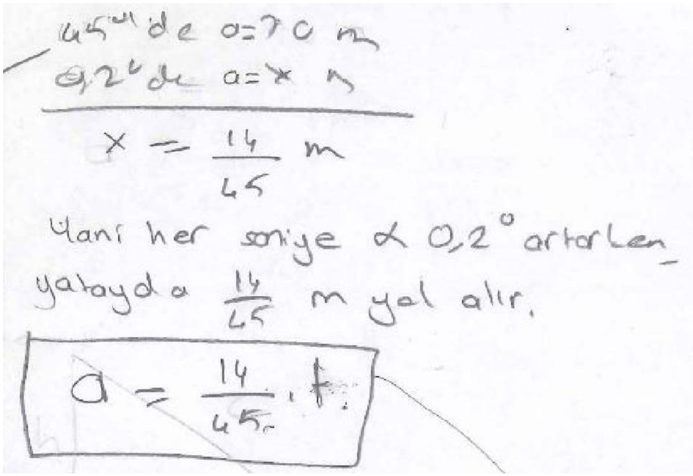

Figure 23 PST3's usage of proportion

> *PST3: We found* $\frac{14}{45}$ *meter by forming the direct proportion by writing* $45°$ *is to 70 meters then* $0.2°$ *is to x.*

PST1: But we cannot form proportion. They are not proportional. It decreases gradually.

When PST1 mistakenly used the proportionality between the angle and the vertical distance in their group presentation, PSTB2 offered him to use a correct way of proportion.

PSTB2: I agree with PST1. If you want to use proportion, I can suggest you a way. You can use the ratio of times instead of angles. I can explain it to you if you want.

Researcher: But angle and time are already proportional.

PST1: Yes

PSTB2: No! Not like that! We cannot form a proportion like what is the height corresponding to $0.2°$ *if* $45°$ *corresponds to that height. However, we can form like if takes* $45°$ *in this much seconds…*

Researcher: But they have same meaning.

PSTB2: No! Not same.

Researcher: Why? Time and angle are proportional.

PSTB2: (After some calculations) I can prove, they are same! I got the same result in my way. The ratio is again $\frac{4}{45}$ *but in my way.*

5. DISCUSSION AND CONCLUSION

This chapter is about the general findings of the study in detail. After initiating with a brief summary of explanations about the results, major findings were discussed in categories according to the research questions of the study. Then, implications were explained for future studies and practical applications. They were followed by limitations of the study.

5.1. The nature of pre-service teachers' mathematization process

The Ferris wheel modeling activity consists of many different didactical features related to trigonometric functions besides the obstacles obscured in the nature of kinematic objects. As McDermott et al. (1987) described in their extensive study, the difficulties of students in making connections between the kinematical objects and their representations, the PSTs in this study demonstrated some difficulties in explaining different representations of mathematical relations relevant to a Ferris wheel. The data analysis revealed that PSTs demonstrated some difficulties related to the nature of modeling problems. These observations were examined in mainly two sections aimed to investigate the research questions of the study.

In the literature review, there was surprisingly no evidence in most of the studies about the mathematization and modeling. Therefore, in case of the lack of similar studies in modeling literature, other studies focusing on the findings were discussed throughout the chapter.

5.2. The nature of pre-service teachers' horizontal mathematization

During the transition from the real world to the mathematical terms and mathematical model which is described as horizontal mathematization (Treffers, 1987), PSTs exhibited difficulties on the process of making sense of the desired information presented in the modeling task.

5.2.1. Pre-service teachers' way of understanding modeling task

Understanding and expressing the real world mathematically was a little bit problematic for PSTs. During the understanding of the problem stage, PSTs used verbal representations frequently while they were discussing the problem. This was consistent with the previous studies (Özaltun et al., 2013) exploring the process of modeling. However, their false assumptions during those discussions that were centered on verbal explanations of the situation verbally yield misunderstandings about the situation. They spent so much time on these false assumptions. The proportionality of vertical distance with the central angle was a good example illustrating the false assumptions. Two of the groups constructed their solutions on this assumption in a wrong way to find the height of any arbitrary capsules in the Ferris wheel.

It was extensively seen in the group discussions within group A that PSTs had difficulty to distinguish the angular and linear speed of a moving object. Previous studies have been emphasizing similar findings that students have had difficulty in relating the topics from different disciplines and this difficulty can be prevented by starting the process of teaching in that context (Delice & Kertil, 2013; Doorman & Gravemeijer, 2009). By following this way, the PSTs' sense of connection to real life can be developed. The lack of sense of connection produced difficulties during the understanding of modeling task. Taking the thickness of the capsules into account as unknown while they were finding the consecutive distance between the capsules, stable positions of capsules around the rotating Ferris wheel were visible in the study. Those were probably the result of insufficient sense of connection to real life.

5.2.2. Pre-service teachers' visualization during mathematization

There were some studies in problem solving literature in which visualization is an important step in the solution of problem (Doerr, 2007; Lowrie, 2001). Similar to the findings in problem solving literature, visualization was a commonly used first step for mathematization. However, the difficulties in visualization produced false point of views for the rest of mathematization process. Two different situations were observed related to the connection between assumptions and visualizations. Both visualizations based on assumptions and assumptions based on visualization were exist in the study. Sometimes they constructed their assumptions by looking at the figure they draw. On the other hand, they also draw figures according to their assumptions.

5.3. The nature of pre-service teachers' vertical mathematization

After transition from real world to the language of mathematics, PSTs had difficulties in expressing the inner structure of the model. Mathematization within mathematics, namely vertical mathematization, was more problematic for PSTs in mathematization process.

5.3.1. Pre-service teachers' usage of diagrams

Whether consciously or not, almost each PST started to draw a realistic picture to perceive the situation. Since their drawings scaffold the process in mathematization, this step was so important. However, similar to the results of Doerr's study (Doerr, 2007), some of pre-service teachers were not picturized the case correctly. Hence, their assumptions based on these diagrams that brought them to wrong conclusions. Hence, for drawing out the mathematical meaning, decision of the correct assumptions and the visualization seem to be important in mathematization process.

In his extensive study about the effects of visualization on problem solving abilities of students, Lowrie (2001) stated that students using visual methods predominantly in their solutions were more successful. Except the difference in the context, similar to the results of Lowrie's study, PSTs using the diagram frequently and effectively outperformed in their groups in this study.

5.3.2. Identification of functions representing the real life situation

The data analysis revealed the fact that PSTs slog on recognizing the independent variable for the function indicating the desired situation in the modeling task. When PSTs had failures in deciding the independent variable of the function, the rest of the mathematization process became troublesome. In his study, Huang pointed out this difficulty of recognizing the independent variable of the function and stated the implication that educators should not ignore this insufficient ability of students (Huang, 2012). PSTs could not decide the needed independent variables for the function in Ferris wheel activity. Their assignment of central angle between two capsules and numbers of positions as the independent variables lead them to incorrect responds. It was expected in the modeling task that the continuous information needed to display on the screen in the capsules led them to guess the time as a needed independent variable.

Another interesting finding in the mathematization process related to concept of function in the study was related to the necessary conditions for an expression to be a function. During the group presentations, pre-service teachers stated that the related expression could not be a function since it was not continuous. This consequence stated by the PST was a misconception about the concept of a function. This was probably because of the emphasis on only the continuous functions during the high school years. In Turkish high school mathematics curriculum, the concept of functions was taught by initiating with the set theoretic approach. Only a few examples of functions defined on discrete sets were given in most of mathematics textbooks. After that, they saw frequently only continuous functions in the rest of the mathematics lessons. Underlying reason for the mentioned difficulty can be this order of teaching function concept. However, it is not in the scope of this study.

The next difficulty observed during the mathematization process is again related to independent variable of function. However, it can be discussed in other context, namely independent variables of trigonometric functions. Most of the PSTs insisted on writing the trigonometric function representing the desired situation with central angle as the independent variable. Once they needed to use a trigonometric function they believed that independent variable must be an angle. There is probably a gap in their mind about the concept image of trigonometric functions defined for angles or any arbitrary real number. The reason for this obstacle can be their way of learning trigonometry. Most of the textbooks start with the famous conversion formula $\frac{D}{180^o} = \frac{R}{\pi}$ to relate angle measures in degree and radian. However, this can be a major obstacle for students to think the trigonometric functions as real-valued functions. Previous studies (Akkoç, 2008; Steckroth, 2007) emphasized the learning difficulties related to the radian concept.

During the group presentations, PST3 was trying to explain the height of a capsule at an arbitrary time he used the function $h = \tan(0.2 \cdot t)\frac{14t}{45} + 4$. However, he claimed that the period of the function was 0.2. The possible reason for this mistake is the way that he has learnt the trigonometric functions. In high school years, trigonometry is given engaged with full of formulas in most of the textbooks. They just memorized that the period of function of the form $f(x) = a \cdot \tan(bx + c) + d$ was $\frac{\pi}{|b|}$. He probably remembered the relationship of a trigonometric function with the coefficients but made wrong conclusion.

Trigonometric functions were associated with ratios of the lengths of the sides of a right triangle in the mathematics curriculum of Turkey. The main emphasis is based on the right triangle. However, this approach accommodates some difficulties in transition from trigonometric ratio to trigonometric functions (Kendal & Stacey, 1998; Swetz, 1995). It is evidently seen in the analysis of data that PSTs had difficulties while they were faced with trigonometric ratios when the angle was not an acute angle. They asked each other what to do in case the angle was obtuse not an acute one.

Another difficulty related to trigonometric functions as ratios was seen while pre-service teachers were explaining the value of $\tan 90°$. They stated that the value was "zero". The main idea here was $\tan x = \frac{\sin x}{\cos x}$ and he thought that $\sin 90° = 0$. Then he concluded that the value of $\tan 90°$ is zero since he used the trigonometric ratio unconsciously. Whereas the real-valued tangent function was explained in textbooks with the values on unit circle (see Figure 24). Indeed, he could recognize that the tangent function was not defined at $\frac{\pi}{2}$ radian since the domain and graph of tangent function were actually explained in high school years in detail. However, after these explanations, the main emphasis was again on the trigonometric ratios.

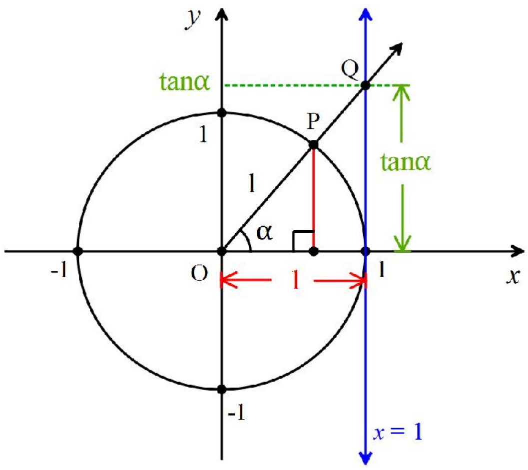

Figure 24 Trigonometric functions on unit a circle

Another pre-service teacher did interesting objection to this mistake, as $\tan 90°$ is "infinity" which contains another misconception, namely division by zero. Many studies in the literature (Ball, 1990; Simon, 1993; Tsamir & Tirosh, 2002) state the difficulties of students' and pre-service teachers' difficulties by division. They could not conceptualize the division by zero. The response of infinity maybe as a reason of calculus courses including the limits of functions. They see that the limit of rational functions at a point where the denominator becomes zero approaches "infinity".

Although the nature of the Ferris wheel activity incorporates the periodicity idea, the PSTs could not recognize to write a periodic function representing the situation.

The most common periodic function that they had learnt before were trigonometric functions. Hence, at first glance, they could think that the situation could be expressed by the help trigonometric function. This can be a result of their way of learning of types of functions. After completing everything about functions, they learnt trigonometric functions. Hence, it was an expected behavior that PSTs were worked on other type of functions initially.

5.3.3. Pre-service teachers' formulation of functions

When the turn came to write the general formula of the function representing the situation, there were some problems. As mentioned in reflection papers written by pre-service teachers they stuck at writing a formula for each quarter of the Ferris wheel. They divided the formula into four parts. Actually, it was piecewise function defined by different formulas in subintervals. The reason underlying this difficulty may be related to the right angles or the way of learning the trigonometry in unit circle, because they just decided that the cosine function should be chosen to represent the situation. After that, they decreased the number of intervals to two. Finally, they reached the correct formula written as a single expression.

5.3.4. Pre-service teachers' application of previously learnt topics

During the mathematization process of Ferris Wheel activity, PSTs had also difficulties while applying the previously seen topics of mathematics. They could not relate them in an efficient way. They sometimes misused the topic and sometimes could not remember to use them in necessary conditions.

First, the tendency to use proportion frequently is obvious in the analysis of data. They misused the proportion to find the vertical distance between each consecutive capsules. However, this was not the only remarkable case about the proportion. They had also difficulty to relate the variables in a compound proportion by offering to use time in the proportion for the vertical distance instead of angle. Actually, time and angle were directly proportional. Although other PSTs reminded this feature of time and angle, she insisted on the statement that time could be used instead of angle for the proportion. The compound proportion was extensively learnt in high school years. However, when it comes to use the topic as needed it becomes hard.

A different point of view can be stated here about the proportionality of vertical distance and the angle. PSTs had learnt previously how the values of sine function changes according to change in angle by examining the sine values on a unit circle. In this context, sine function was defined as exactly the vertical component of the point on the unit circle. Hence, they should know that the change is not proportional.

The number of different arrangements of n different objects around a circular table is $(n-1)!$. This is a well-known formula, but PSTs in both groups tried to use this knowledge in wrong context. When they determined the numbers of intervals between each consecutive pairs of capsules on Ferris wheel, they concluded that that there are 35 intervals between 36 capsules by stating the similarity of the case with circular permutation. In most of the textbooks the conceptual base of the circular permutation was not given. The most common way that the pre-service teachers learned these topics includes the statement of the rules and solving the questions based on the rules. Since the conceptual understanding was not sufficient even for the pre-service teachers, they were probably using these false assumptions during the solution.

5.4. Conclusions and Implications

This study investigated the pre-service teachers' mathematization process in a modeling activity. During both horizontal and vertical mathematization, PSTs had difficulties to use different components of mathematization process. After presenting data analysis and literature review parts, in this section, the conclusions will be put forward

5.4.1. Conclusions

In this era, it is easy to reach any information by searching on different tools. Hence, it is not so important for a student as an individual to know so much. Instead, the important ability is to handle this knowledge and be prepared for life. Therefore, the focus of education should not be on the quantity but quality. Applicability of the knowledge interdisciplinary is much more important. Mathematical modeling can be an effective device to reach this goal.

The emphasis of mathematical modeling on mathematics education is growing and can be seen in some mathematics education programs (MoNE-Board of Education, 2013; National curriculum framework, India, 2005; NCTM, 1989). However, this importance was not taken into the consideration as supposed. Instead of mathematical modeling activities, many textbooks include only application problems, but they are not solely enough to construct the bridge between real life and mathematics. Therefore, programs and textbooks should especially emphasize the importance of modeling activities and teachers should be educated and encouraged to apply these activities in classrooms.

Well-prepared modeling activities according to six principles for designing MEA (Lesh et al., 2000) including functions can be used for teaching concept of function in order to avoid some misconceptions about functions. Illusion of linearity (Dooren et al., 2004) can be reduced if the first function that students faced is not a linear function but a trigonometric or an exponential function. As seen in Weber's (2005) study, students had difficulty to consider trigonometric functions as functions since they cannot be expressed as a combination of arithmetic procedures. Therefore, the mathematization process in a modeling task including a non-linear function (e.g. trigonometric functions or exponential functions) may construct a better conception of function.

Although the participants in this study had taken trigonometry courses in their high school years and calculus courses (at least two must courses) in their graduate years they still had difficulties while writing trigonometric functions related with real life situation. The underlying reason for this difficulty may be explained by taking this modeling course as the first attempt to solve modeling questions. This study revealed that the modeling activities should take place as an essential part of the curriculum in order to ensure that students can relate the real life with the mathematics taught in schools.

Before the mathematization process, the current study showed that the visual iconic representation (picture) of the situation played a crucial role. Almost all the participants started their solution with a realistic picture in order to see the situation, but only a small percentage of the pictures were correct. The pictures they draw scaffold what they do next in the mathematization process, which involves reaching any form of mathematical representations, objects, relations and operations. Therefore, visualization ability seems to be important at deciding the picture for the situation and then extracting the mathematical meaning from it. (Delice & Kertil, 2013).

The difficulties and misconceptions emerged in mathematization process were similar to those in other type of mathematical activities. When the mathematical modeling tasks were used after direct instructions they may reveal some misconceptions and difficulties about the related context. Hence, mathematization can also be used as a diagnostic tool for mathematics educators. In order to remedy those difficulties and misconceptions some detailed studies focusing on the cognitive processes can be used.

5.4.2. Suggestions for future studies

This study investigated the mathematization process in unique mathematical modeling activity. More modeling activities covering different contexts in mathematics curriculum may provide a broader perspective related to mathematization process. Application of different modeling tasks related to different context may reveal other mathematization components.

Yoon et al. (2007) discussed timing of the mathematical modeling task in their study by pointing out the order of direct instruction and mathematical modeling task. They suggested applying mathematical modeling after the direct instruction. A study focusing on the mathematization process within two different groups getting the modeling activity before and after the instruction can be investigated.

A study comparing the mathematization process of pre-service teachers and high school students can be carried out. The similarities and difference between their process of mathematization and reasons for these possible differences can be investigated. In this setting, the comparison of mathematization processes between groups of participants with different level of knowledge related to context may be investigated. How mathematization process changes according to participants' level of knowledge could be a good question to be answered. By comparing pre-service teachers' and high school students' mathematization process, we may have a chance to evaluate the change throughout the years after a massive course load.

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APPENDICES

A. Ferris Wheel Activity

London Eye is a giant 135-meter tall Ferris wheel situated on the banks of the River Thames in the English capital. It is the tallest Ferris wheel in Europe, and the most popular paid tourist attraction in the United Kingdom, visited by over 4 million people annually. When erected in 1999, it was the tallest Ferris wheel in the world. The wheel carries 32 sealed and air-conditioned passenger capsules. Each capsule holds 25 people, who are free to walk around inside the capsule, though seating is provided. The wheel does not usually stop to take on passengers; the rotation rate is slow enough to allow passengers to walk on and off the moving capsules at ground level.

A businessman viewed this construction impressed from the customer potential, and decided to build such a Ferris wheel at the top of Çamlıca Hill in Istanbul. The Ferris wheel is planned as having 140-meter diameter, and will be mounted on a 4-meter height platform. 32 equidistant passenger capsules each having 25 people capacity will be mounted on the wheel. One revolution is planned to be 30 minutes. The followings are some of the information that is planned to be displayed for passengers instantaneously with an electronic sign chart.

The height from the ground level

Velocity

The distance from the boarding point

The time remained to complete one revolution

The technical staff realized that they need help from a mathematician about the mathematical background for computing these values instantaneously and developing the required software program. So, it is expected from you developing a mathematical procedure and explaining this procedure to the technical staff in detail.

B. Consent Form

GÖNÜLLÜ KATILIM FORMU

Bu ders, Doç. Dr. Ayhan Kürşat Erbaş tarafından yürütülen "Ortaöğretim Matematik Eğitiminde Matematiksel Modelleme: Hizmet İçi ve Hizmet Öncesi Öğretmen Eğitimi" projesi kapsamında içeriği oluşturulmuş matematiksel modelleme konusunda hizmet öncesi öğretmen eğitimini amaçlamaktadır. Matematik öğretmen adaylarının matematik öğretiminde matematiksel modelleme kullanımı ile ilgili bilgi, beceri ve tutumlarını ortaya çıkarma ve bunlardaki gelişimi ve değişimi tasarlanan hizmet öncesi eğitim programları aracılığıyla inceleme proje çalışmasının konularını oluşturmaktadır. Bu amaçlar için tasarlanan ders kapsamında 14 hafta sürmesi planlanan çalışma süresince (i) modelleme testi, (ii) anket, (iii) kavram haritası, (iv) modelleme etkinlikleri için grup çalışma raporları, (v) devam soruları için bireysel çözüm kâğıtları, (vi) ses kayıt ve video kayıt cihazlarıyla desteklenmiş gözlemler, (vii) görüşmeler, (viii) etkinlik sonrası düşünce raporları, (ix) gruplarca hazırlanan modelleme soruları ve bu soruların uygulama planları (x) öğretmen adaylarının sunumları (mikro-öğretim) temel veri kaynakları olacaktır. Bu kapsamda türev konusu ile ilgili toplanacak veriler Araş. Gör. Murat Kol'un yüksek lisans tez çalışmasında kullanılacaktır.

Çalışma süresince toplanacak veriler tamamıyla gizli tutulacak ve sadece araştırmacılar tarafından değerlendirilecektir. Elde edilecek bulgular tez çalışmasında ve bilimsel yayımlarda kullanılacaktır. Çalışmaya katılım tamamıyla gönüllülük temelindedir. Çalışma süresince katılımcılar için potansiyel bir risk öngörülmemektedir. Ancak, katılım sırasında farklı amaçlarla toplanan veya alınan dersin gerekleri olarak toplanacak verilerin bilimsel çalışma ve tez çalışması amaçları çerçevesinde kullanılmamasını isteyebilirsiniz. Bu durum ders performansınızın değerlendirilmesinde kesinlikle negatif bir durum oluşturmayacaktır.

Çalışma hakkında daha fazla bilgi almak için ODTÜ Eğitim Fakültesi Ortaöğretim Fen ve Matematik Alanları Eğitimi Bölümü öğretim üyeleri Doç.Dr Ayhan Kürşat Erbaş (kursat@gmail.com), Y. Doç. Dr. Bülent ÇETİNKAYA (Tel: 210 3651; e-posta: bcetinka@metu.edu.tr) ve yüksek lisans öğrencisi Mahmut Kertil (e-posta: mkol@metu.edu.tr) ile iletişim kurabilirsiniz. Bu çalışmaya katıldığınız için şimdiden teşekkür ederiz.

Bu çalışmaya tamamen gönüllü olarak katılıyorum ve istediğim zaman yarıda kesip çıkabileceğimi biliyorum. Verdiğim bilgilerin bilimsel amaçlı yayımlarda kullanılmasını kabul ediyorum. (Formu doldurup imzaladıktan sonra uygulayıcıya geri veriniz).

İsim, Soyisim	Tarih	İmza	Alınan Ders
	----/----/-----		