# A COMPARATIVE ANALYSIS OF MATHEMATICS TEACHER CONTENT KNOWLEDGE EXAMINATIONS IN TURKEY AND TEXAS

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**ABSTRACT**

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The purpose of this study was to investigate and compare elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas. In this study, the examinations were analyzed regarding at which curricular levels each item was, which field of mathematics was included in the item, at which component of cognitive domain the item was, and which domains of mathematics teacher knowledge were included in the items.

It is found that the items of the examinations in Turkey mostly included subject matter knowledge from high school and beyond, whereas the items of the examinations in Texas were mostly from elementary and high school levels. Moreover, items in the examinations in Turkey and Texas mostly included pedagogical content knowledge from below or at the level the prospective teacher would teach. It was found that subject matter knowledge items of the examination in Turkey were mostly from the fields of algebra, calculus, and geometry, whereas the subject matter knowledge items of the examination in Texas were mostly from the fields of algebra. Moreover, it was found that most of the items included identification and use of routine and familiar mathematical procedures in the examinations in Turkey and Texas. Although examinations did not display substantial differences regarding the inclusion of subject matter knowledge, there were some differences in the distribution of pedagogical content knowledge items. The study concludes with some useful suggestions regarding the inclusion of mathematical knowledge and skills in the mathematics teacher content knowledge examinations to have more competent mathematics teachers.

Keywords: Assessment of teacher knowledge, pedagogical content knowledge, subject matter knowledge, cognitive domain

**ÖZ**

TÜRKİYE VE TEKSAS MATEMATİK ÖĞRETMENLİĞİ ALAN BİLGİSİ SINAVLARININ KARŞILAŞTIRMALI BİR ANALİZİ

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Bu çalışmanın amacı, Türkiye ve Teksas'ta uygulanan ilköğretim ve ortaöğretim matematik öğretmenliği alan bilgisi sınavlarını incelemek ve karşılaştırmaktır. Bu çalışmada, belirtilen sınavlardaki soruların müfredat seviyelerinden hangisinde olduğuna, hangi matematik öğrenme alanını içerdiğine, bilişsel seviyelerin hangi bileşeninde olduğuna ve matematik öğretmeninin sahip olması gereken bilginin hangi bileşenlerini içerdiğine göre analiz edilmiştir.

Sonuçlara göre, Teksas'ta uygulanan sınavlardaki soruların çoğu öğretmen adaylarının ilköğretim ve lise seviyesindeki alan bilgisini içerirken, Türkiye'deki soruların çoğu ise öğretmen adaylarının lise ve daha ileri seviyelerdeki alan bilgisini içermektedir. Ayrıca, Türkiye ve Teksas'taki soruların çoğu öğretmen adaylarının öğreteceği seviyede ya da bu seviyenin altında bir pedagojik alan bilgisi içermektedir. Sınavlar matematik öğrenme alanlarına göre incelendiğinde, Teksas'ta uygulanan sınavlardaki alan bilgisi sorularının çoğu cebir öğrenme alanından, Türkiye'de uygulanan sınavlardaki alan bilgisi sorularının çoğu ise cebir, analiz ve geometri öğrenme alanlarındandır. Bunun yanı sıra, Türkiye ve Texas'ta yapılan sınavların içerdiği soruların çoğu bilinen matematiksel işlemlerin belirlenmesini ve uygulanmasını içermektedir.

Sınavlarda soruların alan bilgisinin bileşenlerine göre dağılımlarında oranlarında bir farklılık gözlenemezken, pedagojik alan bilgisinin bileşenlerine göre dağılım oranlarında bazı farklılıklar gözlenmiştir. Çalışma alanında gerekli yeterliliklere sahip matematik öğretmenlerinin yetişmesi için matematik öğretmeninin bilgisini ölçen sınavlarda hangi matematiksel bilgi ve becerilerin ne ölçüde olabileceği ile ilgili tavsiyelerle sonuçlandırılmıştır.

Anahtar Kelimeler: Öğretmen bilgisini ölçme, pedagojik alan bilgisi, alan bilgisi, bilişsel seviyeler

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# LIST OF ABBREVIATIONS

MONE: Ministry of National Education

ÖSYM: Ölçme, Seçme ve Yerleştirme Merkezi (The Center for Assessment, Selection, and Placement)

ÖABT: Öğretmenlik Alan Bilgisi Testi (Teacher Content Knowledge Examination)

TEA: Texas Education Agency

TExES: Texas Examination for Educator Standards

PCK: Pedagogical Content Knowledge

SMK: Subject Matter Knowledge

# CHAPTER 1

# INTRODUCTION

## 1.1 Background of the Study

"What do teachers need to know and be able to do in order to teach effectively? Or, what is required for effective teaching in terms of content understanding?" (Ball, Thames, & Phelps, 2008, p. 394). In order to identify and explain the frame of effective teaching, such questions have emerged. Teaching is defined as everything that teachers must do in order to promote student learning (Ball et al., 2008). Moreover, Shulman (1986) stated that teaching is a term including a teacher's understanding related to what is to be learned and how it is to be learned as key issues.

There have been several studies to promote teaching both as an activity and as a profession in order to make it "a more respected, more responsible, more rewarding, and better rewarded occupation" (Shulman, 1987, p. 3). As Knowles, Plake, Robinson, and Mitschell (2001) stated, what teachers should know and be able to do are issues which continuously change and develop as values of society undergo changes. As they clarified, the teaching profession requires highly complex and demanding tasks such as being responsible for the learning of students, motivating students with different backgrounds and with different learning styles, and improving their knowledge and skills in order to teach effectively. Therefore, teachers need different types of knowledge and skills in order to fulfill these expectations. In order to educate teachers so that they can be equipped with such capabilities, the concept of teacher competency has emerged in the field of teacher education. While teacher competency was defined as capabilities of teachers to provide rigorous and meaningful activities for the learning of students, recently it has mostly come to be associated with teacher characteristics and their technical proficiency (Knowles et al., 2001). For this reason, in order to educate qualified teachers, standards have been generated by national organizations such as the National Council of Teachers of Mathematics (NCTM), the Interstate New Teacher Assessment and Support Consortium (INTASC), the National Board for Professional Teaching Standards (NBPTS), and the Ministry of National Education (MONE) (INTASC, 1992; NBPTS, 2001; NCTM, 2000; MONE, 2008). Teacher knowledge is one of the aspects included in those standards (Hill, Schilling, & Ball, 2004). Teacher knowledge has a multidimensional nature (Kaiser & Blömeke, 2013). To illustrate, subject matter knowledge (SMK), pedagogical knowledge, and pedagogical content knowledge (PCK) are major components of teacher knowledge which are mentioned in the related literature (Ball et al., 2008; Cochran, DeRuiter, & King, 1993; Magnusson, Krajcik, & Borko, 1999; Marks, 1990; Shulman, 1987).

There seems to be agreement in teacher education literature that strong subject matter knowledge of teachers is a central component for teacher competency (Krauss et al., 2008). Subject matter knowledge refers to the knowledge of a subject and its constituent components (Ball et al., 2008). Shulman (1986) argued that knowledge of subject should include more than knowing its facts and rules. In addition, teachers must "… not only understand that something is so; the teacher must further understand why it is so" (Shulman, 1986, p. 391). "Teaching mathematics is not simply 'knowing' in front of students. Teaching requires making the content accessible, interpreting students' questions and productions, and being able to explain or represent ideas and procedures in multiple ways." (Hill, Sleep, Jewis, & Ball, 2007, p. 123). For this reason, merely having strong mathematics knowledge does not guarantee effective teaching. For this reason, teachers should have an additional knowledge component, namely pedagogical content knowledge (Ball et al., 2008; Kind, 2009).

Pedagogical content knowledge refers to the component of teacher knowledge by which a content specialist is distinguished from a pedagogue (Shulman, 1987). It is briefly defined as "the most useful ways of representing and formulating the subject that makes it comprehensible to others" (Shulman, 1986, p. 9). In addition, pedagogical content knowledge requires teachers to decide when to stop for more clarification, when to ask an appropriate mathematics question, and when to conduct a task to improve learning of students in a classroom session (Ball et al., 2008). By taking into consideration these definitions and studies of other researchers (Grossman, 1990; Magnusson et al., 1999, Marks, 1990), categories of mathematical pedagogical content knowledge for teaching of mathematics can be identified as knowledge of students' mathematical learning, knowledge of teaching mathematics, and knowledge of mathematics curriculum (Ball et al., 2008; Hill, Ball, & Schilling, 2008). Even if these two knowledge types (subject matter knowledge and pedagogical content knowledge) represent separate categories of teacher knowledge, subject matter knowledge can be accepted as a prerequisite for pedagogical content knowledge (Krauss et al., 2008). When they are combined, they form a single body of subject-specific knowledge of teachers (Hill, Schilling, & Ball, 2004; Krauss et al., 2008).

Studies providing definitions of what teachers should know and which skills they should possess contributed to the improvement of teacher knowledge assessment (Knowles et al., 2001; Hill et al., 2007). While studies have been conducted in order to improve the assessment of teacher knowledge, “What should be measured?” and “How should it be measured?” are questions which remain empirically unanswered (Hill et al., 2007, p.112). In addition, they argued that

> …how difficult the exams should be—and by extension, how many prospective teachers should be excluded on the basis of lack of knowledge; whether it is content knowledge or knowledge of methods that make a good teacher; whether conceptual or procedural knowledge should be assessed. The passage of 125 years has done little to bring closure to these important questions (Hill et al., 2007, p. 115).

Thus, teaching of mathematics is more than doing mathematics by standing on the board. It requires additional knowledge and skills of mathematics, namely mathematical knowledge for teaching (Ball & Bass, 2003). Mathematical knowledge for teaching is a construct which still has categories with undetermined borders. For this reason, no instrument is assumed to capture the knowledge for mathematics teachers adequately (Hill et al., 2007). Regarding the assessment of knowledge of teachers, there are questions that remain unanswered:

> "Who should control the licensing of teachers? Whether the profession or a public agency should control the process and standards", "Should it be based on a score on an examination? Should it be based on successful completion of an "approved" training program? Should it include both?", "What should be the elements of a course of training for teachers?", "How detailed and specific a licensing system should be" (Angus, 2001, p. 1-2).

In order to answer such questions, gaining insight into teacher assessment in different countries might be useful. According to the report of the Educational Testing Service (ETS), countries which stand at the top levels of the ranking of international studies like PISA and TIMMS generally have different licensure systems other than teacher certification after graduation from the teacher education program (Gonzales et al., 2008; Martin, Gregory, & Stemler, 2000; Mullis, Martin, & Foy, 2005; Wang, Coleman, Coley, Phelps, 2003). To illustrate, some countries like Korea, England, Singapore and Japan have teacher candidates take examinations before entering a teacher education program. For example, in Japan, this examination includes "Japanese language, foreign language, mathematics, the sciences, and social studies" (Wang et al., 2003, p. 18). Candidates who score the highest on this examination have the opportunity to attend the most prestigious teacher education programs. In addition, most universities in Japan have their own entrance examinations for teacher education programs. In Korea, a similar examination needs to be taken in order to enter a teacher education program, and additionally, recommendations of students' high school teachers are taken into consideration during the entrance procedure into teacher education programs. In Japan, Singapore, Hong Kong, Australia, and the Netherlands, there is no teacher licensure examination in order to be a teacher after graduation from the teacher education program since the diploma is accepted as a certification for being a teacher (Wang et al., 2003).

The United States is one of the countries which have a long history in teacher assessment, more than a hundred years (Hill et al., 2007). In general, a teacher has to complete an accredited teacher education program and also hold a major (for secondary education) or a minor (for elementary education) degree in the subject area they plan to teach. Then, they have to pass a state test, commonly PRAXIS or another exam (Roth & Swail, 2000). In detail, each state has its own teacher licensure system, which may be different from that in the other states of the United States. To illustrate, in order to be a teacher in California the requirements are the completion of an approved teacher education program including alternative certification programs, completion of course work in the teaching of reading, passing the California Basic Skills Test (CBEST), and taking an exam for subject matter competence (Roth & Swail, 2000). Similarly, in Texas, teacher preparation programs may require students to have a certification from an approved teacher education program and to pass the Texas Higher Education Assessment (THEA) basic skills exam in reading, writing, and mathematics. This test includes multiple choice test items within the reading and mathematics sections and open ended and multiple choice items for the writing section of the test. In addition, teacher candidates are required to pass the TExES examinations, which include multiple choice items including subject matter knowledge and pedagogical content knowledge (Texas Education Agency (TEA), 2014). In Michigan, teacher candidates have to pass the Michigan Test for Teacher Certification (MTTC) including the Professional Readiness Examination (PRE). PRE includes reading, mathematics, and writing sections similar to the basic skills test of Texas. MTTC includes multiple choice test items for subject matter knowledge and pedagogical content knowledge and PRE includes constructed-response assignments in addition to multiple choice test items (Michigan Teacher Test for Certification, 2014).

In Turkey, teacher candidates take examinations both before entering a teacher education program and after graduating from a teacher education program. Similar to other countries, mathematics teacher candidates are responsible for mathematics, Turkish language, social science, and science contents. After graduation from a teacher education program, they have to take a national exam including three main sections, namely educational sciences, basic knowledge and basic skills, and mathematics content knowledge examinations in order to be a teacher in public schools. Examination of mathematics content knowledge in Turkey includes subject matter knowledge and pedagogical content knowledge. Mathematics content knowledge section of the exam has been conducted for two years, while the other sections have a history of more than 10 years (Ölçme, Seçme ve Yerleştirme Merkezi (ÖSYM), 2014). It can be inferred that studies might be required in order to determine and improve the validity of the mathematics content knowledge examination in Turkey since it does not have a long history. As Blömeke and Delaney (2012) stated, the assessment of teacher knowledge is an indicator of teacher knowledge which reveals whether or not teacher education has an impact on knowledge or teacher. Since teacher knowledge is a construct which is continuously undergoing changes and development, instruments assessing the knowledge of mathematics teachers should be changed and developed in parallel with mathematics teacher knowledge (Hill et al., 2007). Moreover, there are still unanswered questions related to not only the structure of examinations assessing knowledge of teachers but also the preparation and application of these examinations (Angus, 2001). For this reason, studies might be conducted in the future regarding mathematics content knowledge examinations in Turkey in order to improve their structure and application in parallel with the developments in mathematics teacher knowledge in the teacher education literature. Furthermore, international studies are needed in teacher education literature to identify the requirements regarding the assessment of teacher knowledge (Brouwer, 2010; Wilson, Floden, & Ferrini-Mundy, 2001). For this reason, comparative studies could be conducted in order to examine the structure of teacher assessment examinations in Turkey with those of other countries which have a long history in the assessment of teachers' professional knowledge. Comparative studies such as TIMMS and PISA provide an opportunity to make basic reforms in school systems. Thus, studies on the assessment of teacher knowledge can be conducted across several countries for teacher education in order to experience such a reform in teacher education also (Blömeke & Delaney, 2012).

### 1.2. Problem Statement and Research Questions

The first purpose of this study was to investigate the mathematics teacher content knowledge examinations in Turkey and Texas based on four different dimensions: (i) the content and distribution of the items in relation to the components of knowledge for teaching mathematics, (ii) the content and distribution of the items regarding to the cognitive domain, (iii) the content and distribution of items with respect to the fields of mathematics and (iv) the content and distribution of the items based on the curricular levels. The second purpose of the study was to explore how the mathematics teacher content knowledge examinations are similar or different in consideration of the related four dimensions. The following research questions guided the study:

1. What are the structures of the mathematics teacher content knowledge examinations in Turkey and Texas in terms of the curricular levels, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching?

2. What are the similarities and differences between the teacher content knowledge examinations in Turkey and those in Texas in terms of the curricular levels, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching?

### 1.3. Significance of the Study

Shulman (1986) stated that the nature of placement of content and pedagogy had changed in teacher assessment tests throughout history. In the 1870s, pedagogy was essentially overlooked, and content was conspicuously absent in the 1980s. Although content and pedagogy were separately popular at different times in the history of teacher assessment, Shulman described them as an undistinguishable body of knowledge. As Shulman stated, both content knowledge and pedagogical content knowledge are crucial for teacher education. An investigation of teacher content knowledge examination in Turkey may reveal the structure of the teacher education system in Turkey on the basis of the components of teacher knowledge, which are initially described by Shulman (1986). Therefore, this study could provide insight into the distribution of the components of teacher knowledge in mathematics content knowledge examinations and might give suggestions related to how the distribution and balance of teachers' knowledge components should be provided. As a result, this study can provide suggestions for the body of professionals regarding the preparation of mathematics content knowledge examination in order to employ high quality teachers.

Teacher content knowledge examination in Turkey was conducted for the first time in 2013. There have been no studies investigating the structure of this examination; therefore, this study will be the first one analyzing the structure of this examination on the basis of mathematics teacher knowledge, cognitive domain, fields of mathematics, and curricular levels. Also, this study will be the first cross national study comparing the structure of teacher content knowledge examination in Turkey and Texas. Therefore, this study might contribute to the previous cross-national studies which investigated the similarities and differences of teacher assessment examinations of different countries on the basis of teacher knowledge. By this means, it is hoped that this study will reveal the strengths and weaknesses of the newly constructed mathematics teacher content knowledge examination in Turkey.

Olkun and Aydogdu (2003) conducted a study on geometry achievement of students in the international comparison study, TIMMS (Martin et al., 2000). According to the results of the study, students participating from Turkey demonstrated a very low performance with respect to geometry achievement. Olkun and Aydoğdu (2003) argued that one of the reasons of students being unsuccessful was teachers who introduced geometry and mathematics as a body of rules, procedures, and formulas which had to be memorized. Achievement of students might be an indicator of mathematics knowledge of teachers; therefore, teacher knowledge is an important criterion for student achievement (Hill et al., 2007; Olkun & Aydoğdu, 2003). For this reason, the investigation and comparison of different teacher assessment systems might give clues about how teachers are assessed and employed in terms of their knowledge of mathematics teaching in Turkey, which presumably affect the achievement of students.

Initially, Shulman (1986) highlighted the importance of content understanding as being a special key for the teaching profession. Then, teacher knowledge has been a popular issue within all areas in education as within mathematics education. What teachers need to know and be able to do or what effective teaching requires are issues which have been considered and discussed for many years in mathematics education (Ball et al., 2008). Since there has not been a concrete knowledge model for mathematics teachers, assessment of mathematics teachers' knowledge is a developing concept (Hill et al., 2007). Even today, the knowledge and ability that mathematics teachers should possess is not addressed evidently by educators and researchers. There is little agreement on how to and for what purpose to assess knowledge of teachers (Hill et al., 2007). Therefore, presenting the structure of the teacher knowledge model of related examinations may contribute to the literature by presenting the structure of contemporary examinations of different countries. There have been several models for knowledge of teachers in the literature (Ball et al., 2008; Cochran et al., 1999; Even, 1993; Ma, 1999; Magnusson et al., 1999). This study may contribute to the literature by revealing similarities and differences of teacher knowledge models from the literature and from the examinations in use. In other words, this study may visualize how teacher knowledge models mentioned in the literature are put into practice in mathematics teacher content knowledge examinations. Since examinations of different countries may use different teacher knowledge models, this study can represent how the teacher knowledge employed in these examinations are congruent with teacher knowledge models in the literature. In addition, this study may contribute to the literature by explaining how teacher assessment examinations of different countries resemble or differ in consideration to the teacher knowledge models they used.

Finally, examining the domains for teaching mathematics, the cognitive domain, fields of mathematics, and curricular levels of teacher content knowledge examinations of different countries provides educators, researchers, and legislators with information about the structures of those examinations. Therefore, this study aimed to give information to authorities who are responsible for the preparation and application of teacher assessment examinations about the similarities and differences of those examinations.

### 1.4. Definitions of Terms

**Knowledge for Teaching Mathematics:** Ball et al. (2008) defined knowledge for teaching mathematics as “the mathematical knowledge needed to carry out the work of teaching mathematics. They identified that this knowledge includes tasks related to teaching a mathematical concept and mathematical demands of these tasks. In this study, knowledge for teaching mathematics refers the knowledge including two main components as in the study of Ball et al. (2008), namely pedagogical content knowledge and subject matter knowledge.

**Pedagogical Content Knowledge:** Shulman (1987) defined pedagogical content knowledge as “special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding” (p. 8). In this study, pedagogical content knowledge refers to the construct including the knowledge specific to mathematics teachers, the knowledge of the features of students’ learning, the knowledge of teaching a specific subject in mathematics, and the knowledge of current mathematics curriculum and standards.

**Subject Matter Knowledge:** Shulman (1986) defined subject matter knowledge as the knowledge of a subject more than its facts and concepts. That is, knowledge of subject additionally includes organizing principles and structures of a concept. In this study, subject matter knowledge refers to the knowledge of mathematics which includes the knowledge of general mathematics, the specialized knowledge of mathematics which is unique to mathematics teachers, and the knowledge of mathematics which is related to nature and structure of mathematics (Ball et al., 2008).

**Cognitive Domain:** In this study, the cognitive domain refers to categories within a hierarchic framework which was adapted from Garden et al., (2006) and Grønmo, Lindquist, Arora, & Mullis (n.d). In this framework, categories are specified with respect to mathematical knowledge and skills required in the items. In consideration of mathematical knowledge and skills, cognitive domain was hierarchically separated into three categories, namely knowing, applying, and reasoning.

**Curricular levels:** In this study, curricular levels refers to a hierarchic framework of mathematical knowledge and skills with respect to the current mathematics curriculum of Turkey or Texas.

**Fields of mathematics:** In studies, such as Gonzales et al. (2008) and Tatto, Schwille, Senk, Ingvarson, Peck, and Rowley (2008), fields of mathematics were analyzed under four sections, number, geometry, algebra, and data and chance. In this study, fields of mathematics were described as basic domains of mathematics, namely algebra, calculus, applied mathematics, and geometry. In addition, framework of fields of mathematics was constituted in detail based on mathematics handbooks, Pearson (1990) and Rainbolt and Gallian (2010).

**Mathematics Teacher Content Knowledge Examination:** In this study, mathematics teacher content knowledge examinations refer to a section of examinations for the assessment of professional knowledge of teachers. This section includes items related to the knowledge for teaching mathematics which was comprised of subject matter knowledge and pedagogical content knowledge.

**Structure:** In this study, structure refers to the distribution of the items in mathematics teacher content knowledge examinations in Turkey and Texas, with respect to the specified variables, the curricular framework, the fields of mathematics, the cognitive domain, and the dimensions of knowledge for teaching. In addition, the word, structure, refers to the content of the items with respect to the variables identified above**.**

# CHAPTER 2

# LITERATURE REVIEW

The first purpose of this study was to investigate the structure of teacher content knowledge examinations of Turkey and Texas based on curricular level, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching. The second purpose of the study was to explore the similarities and differences of mathematics teacher content knowledge examinations of Turkey and Texas on the basis of curricular level, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching. The third purpose of this study was to investigate how congruent the teacher content knowledge examinations of Turkey and Texas are with respect to the teacher knowledge models in the teacher education literature. This chapter includes a review of the literature which is related to the study. Regarding the research questions of the study, the literature review has been categorized into four sections, namely mathematics teachers' competencies, mathematics teacher knowledge, and assessment of content knowledge domains of mathematics teachers.

## 2.1. Competency of Mathematics Teachers

It is found that mathematics teachers have a central role for the preparation and achievement of future generations K-12 students (An, Kulm, & Wu, 2004; Blömeke & Delaney, 2012). For this reason, teacher quality has become a major concern for policymakers and educators. There are several organizations developing teacher standards for establishing teacher quality such as Interstate New Teacher Assessment and Support Consortium (INTASC), National Council for Accreditation of Teacher Education (NCATE), and National Board for Professional Teaching Standards (NBPTS). To illustrate, "The teacher appropriately uses a variety of formal and informal assessment techniques (e.g., observation, portfolios of student work, teacher-made tests, performance tasks, projects, student self-assessments, peer assessment, and standardized tests)" is an example to standards of INTASC and "Teachers know the subjects they teach and how to teach those subjects to students." is an example to standards of NBPTS (Mitchell, Robinson, Plake, & Knowles, 2001, p. 27-28). NCTM (2001) stated that achieving a highly qualified mathematics instruction requires "solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence" (p. 3). Therefore, competency of teachers is one of the central concerns for the vision of high-quality mathematics education. Teacher competency is defined as the cognitive ability to develop solutions for problems concerning teaching profession and applying these solutions in various situations successfully (Weinert, 2001; Bromme, 1997). As it is modeled by Blömeke and Delaney (2012), teacher competency is categorized into two main topics, namely cognitive abilities and affective motivational characteristics.

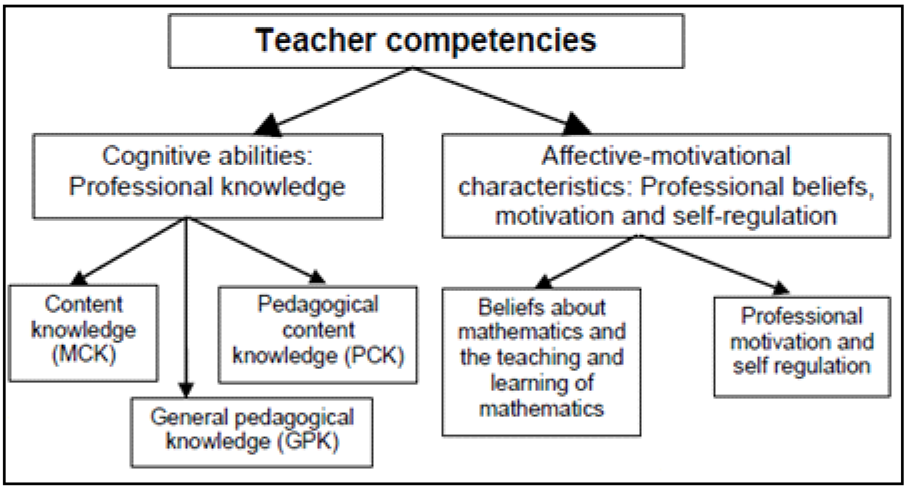



Figure 2. 1. Conceptual Framework of Teacher Competency (Blömeke & Delaney, 2012, p. 8)

Cognitive abilities stand for professional knowledge of teachers which can be divided into several categories as content knowledge, general pedagogical knowledge, and pedagogical content knowledge (Shulman, 1985) whereas affective-motivational characteristics stand for professional beliefs, motivation, and self-regulation of teachers (Richardson, 1996; Thompson et al., 1996). Richardson (2003) gave the definition of belief as “psychologically held understandings, premises, or propositions about the world that are felt to be true” (p. 2). Additionally, teacher beliefs can be defined as the link connecting knowledge and action since it is an indicator of the instruction that mathematics teachers will perform in the future (Brown & Rose, 1995). Beside teacher beliefs, motivation and self-regulation of teachers are also crucial factors for teacher competency. In the literature, self-regulated is the term identifying learners who are participating in learning meta-cognitively, motivationally, and strategically (Winne & Perry, 2000; Zimmerman, 1990). As Blömeke and Delaney (2012) points out teachers that regulate their behaviors will be able to observe and evaluate their actions systematically (Butler & Winne, 1995; Perry, Phillips, & Hutchinson, 2006; Boekaerts & Corno, 2005). Moreover, if teachers have professional motivation and self-regulation characteristics, they will be able to carry out their professional objectives and choose appropriate procedures in various classroom situations (Blömeke & Delaney, 2012). Apart from affective-motivational aspects of teacher competency, professional knowledge of teachers, the other category of teacher competency, is mainly investigated in this study. For this reason, professional knowledge of teachers will be discussed in detail in proceeding topics.

## 2.2. Knowledge of Mathematics Teachers

Elbaz (1983) stated that “the single factor which seems to have the greatest power to carry forward our understanding of the teacher’s role is the phenomenon of teachers’ knowledge” (p. 45). When the ranking of countries are observed based on the results of TEDS-M and TIMMS, it can be concluded that professional knowledge of mathematics teachers should be improved in order to promote achievement of students (Blömeke & Delaney, 2012; Schoenfeld, 2010; Wilson, 2007). Since there is a crucial relation between teacher knowledge and student achievement in mathematics (Baumert et al., 2009), teacher knowledge has attracted a wide attention of researchers and policy makers.

Despite its importance in educational territory, Shulman (1986) stated that there had not been a coherent theoretical framework for the professional knowledge of teachers for many years. In his stimulating study, Shulman (1987) explained professional knowledge of teachers under seven categories; namely content knowledge, general pedagogical knowledge, curricular knowledge, knowledge of learners and their characteristics, knowledge of educational contexts, knowledge of educational ends, purposes, values, and their philosophical and historical grounds, and pedagogical content knowledge. This study is generally focused on content knowledge (subject matter knowledge) and pedagogical content knowledge of mathematics teachers. Therefore, these facets of mathematics teacher knowledge will be given in more detail in the following topics.

### 2.2.1. Subject Matter Knowledge for Teaching Mathematics

As Askew (2008) stated: "…one thing is clear from the research evidence: many prospective and practicing primary teachers have, or express, a lack of confidence in their mathematical knowledge" (p. 16). This view coincides with the results of studies which are remarking that many teachers have inadequate conceptual understanding related to mathematics (Ball, Hill & Bass, 2005; Hill et al., 2008; Ma, 1999). In similar with the results of studies based on a range of mathematical topics, such as multiplication and place values (Ball et al., 2008, Ma, 1999), division (Borko et al., 1992), patterns and functions (Even, 1993), and geometry (Putnam, Heaton, Prawat, & Remillard, 1992), inadequate understanding of teachers regarding a particular mathematical content restricts them to explain and represent the content in a conceptual way. According to the study of Ma (1999) comparing teachers in China and the United States, Chinese teachers provide broader and more varied strategies while teaching mathematics thanks to their deep and flexible understanding of mathematics than teachers from the United States. The reason is better subject-matter preparation of teachers, one of the central concerns in order to improve teaching(Ball, 1990; Even, 1993).In teacher education literature, there has been an agreement that strong subject matter knowledge is a *core* component of teacher competence (American Council on Education, 1999; National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008). Teachers need to have such knowledge in order to create productive learning environments for their students, to develop students' mathematical understandings and to make them construct coherent frameworks on the basis of mathematical ideas

(Even, 1993). As NCTM (2000) reported, a teacher who has such knowledge will be able to give details to students, associate the subject with other areas, direct students several questions, and move out from the textbook. Since it is an important concern in mathematics education, there have been several studies related to this concern. Shulman (1986) defined subject matter knowledge as the "amount or organization of knowledge per se in the mind of the teacher" (p. 9).

As it is cited in the study of Shulman, a scholar Joseph Schwab structured subject matter knowledge as a subject constituted by both substantive and syntactic structures. Substantive structure refers to the organization of basic concepts, principles, and rules in a discipline whereas syntactic structure refers to the organization of proofs and procedures which are used for the exploration of the truth or falsehood of a subject in a particular discipline (Shulman, 1986). Shulman (1986) stated that a teacher should understand and explain *why* a particular proposition works in addition to understanding and explaining *that* the proposition does work. In other words, he asserted that just knowing the facts and concepts does not mean that the teacher has the required knowledge. Furthermore, a teacher must be able to explain why a particular proposition works, why it is worth knowing, and how it can be associated with other disciplines which is also asserted by mathematics educators as:

> …teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching . . . both prior to and beyond the level they are assigned to teach (National Mathematics Advisory Panel, 2008, p. 37).

Following the study of Shulman (1986), researchers have continued to study on subject matter knowledge of teachers. Ball (1991) defined the understanding of mathematical topic as an amalgam of knowledge, beliefs, and feelings about the subject. She took into account the mathematical knowledge on the basis of two dimensions, knowledge *of* mathematics and knowledge *about* mathematics. Knowledge of mathematics stands for:

> …understandings of particular topics (e.g., fractions and trigonometry), procedures (e.g., long division and factoring quadratic equations), and concepts

(e.g., quadrilaterals and infinity), and the relationships among these topics, procedures, and concepts (p.6).

The other dimension, knowledge about mathematics, covers the questions related to criterions which establish the validity of an answer, things that mathematicians do, origins of mathematical facts and evolution of mathematics. The knowledge about mathematics refers to:

…understandings about the nature of knowledge in the discipline--where it comes from, how it changes, and how truth is established (p. 6).

Among studies regarding subject matter knowledge of teachers, there is a common point that mathematics knowledge of teachers includes more than memorized facts and procedures in mathematics, but it also includes the knowledge of whys and hows of mathematical concepts; in other words, the knowledge of nature of mathematics (Ball, Thames, & Phelps, 2008; Even, 1993; Shulman, 1986). Discussions on what subject matter knowledge of teachers is get the researchers to generate different components for subject matter knowledge. Therefore, various models for mathematics content knowledge of teachers have been arisen in educational literature. Firstly, Even (1993) investigated mathematics knowledge of teachers based on a framework consisting of seven components while studying on subject matter knowledge and pedagogical content knowledge of teachers on the concept of function. The researcher identified the components of mathematics teachers knowledge related to the concept of function and identified the components of subject matter knowledge as follows: essential features - what is a function, different representations of functions, alternative ways of approaching functions, the strength of the concept – the inverse function and the composition of functions, basic repertoire – functions of the high school curriculum, different kinds of knowledge and understanding of function concept, and knowledge about mathematics. This was a detailed model including basic knowledge of definitions, properties, representations, and important points of a mathematical concept. The other mathematical content knowledge model includes four categories constructed by Krauss et al. (2008).These categories are proposed in a hierarchical order, namely the academic research knowledge generated at institutes of higher education, a profound mathematical understanding of the mathematics taught at school, a command of the school mathematics covered at the level taught, and the mathematical everyday knowledge that adults retain after leaving school. In this model, components of mathematics knowledge of teachers were constructed with respect to levels of mathematical knowledge from advance to elementary levels.

Another knowledge model for teaching, mathematics knowledge of teachers (MKT), is constructed by Ball, Thames, and Phelps (2008) which demonstrates the relationship of SMK and PCK. As it is seen in Figure 2.2, MKT model was divided into three categories, namely common content knowledge, specialized content knowledge, and horizon content knowledge. The left hand side of the oval called SMK which contains different components compared with common conceptualization of Shulman, common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK).

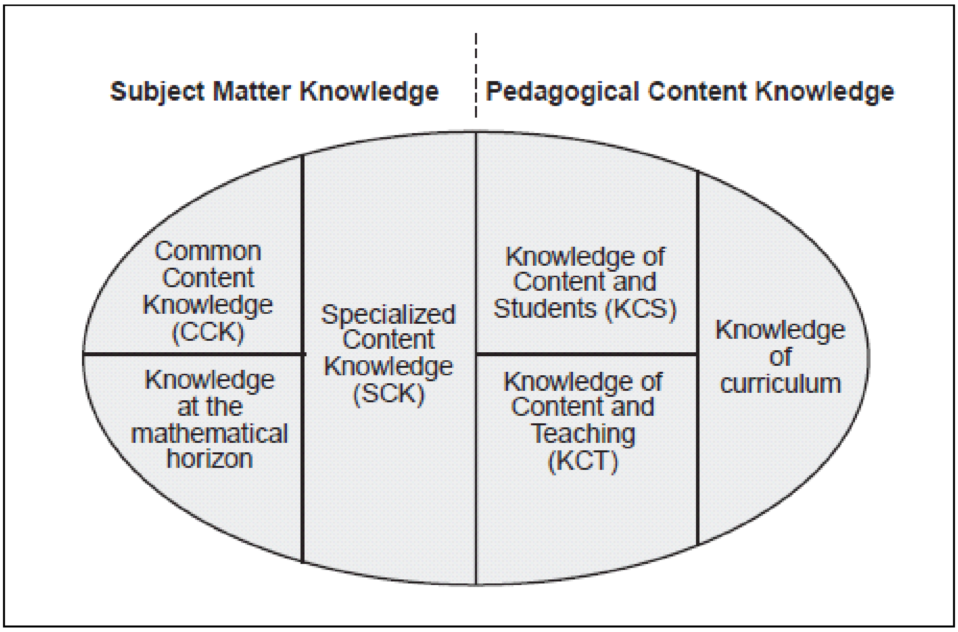



Figure 2. 2. Domain map for mathematical knowledge for teaching (Ball et al., 2008, p. 403)

Common content knowledge (CCK) is defined as making simple calculations and solving mathematical problems correctly. For example, “What is the number halfway between 1.1 and 1.11?”, “Can the number 8 be written as 008?”, and “What power of ten equals one?" are examples of questions related to CCK. This is the mathematical knowledge and skills used in other areas in addition to teaching. However, as researchers declared the statement "common" do not mean that this is a kind of knowledge that everyone has; that means, the knowledge can be used in various settings; in other words, it is not unique to teaching. While CCK corresponds to Shulman's subject matter knowledge, SCK is a new concept (Hill et al., 2008). It is defined as mathematical knowledge and skills in order to answer the question "why" in particular mathematical concepts. That is, SCK is a mathematical knowledge not required for purposes other than teaching. This is the conceptual knowledge which is beyond the level that is taught to students. It includes the knowledge of how mathematical language is used, how to use and construct mathematical representations effectively, and how to explain unseen mathematical facts behind procedures. To illustrate, conceptual knowledge related to mathematical procedures like invert and product for division of fractions are examples for which SCK is required. According to the study of Borko et al. (1992), a student teacher cannot make a correct representation to present division of fractions and cannot explain how the invert and multiply procedure works even though he could carry out these operations in procedural ways and even though he took several courses related to mathematics. Therefore, SCK is the knowledge which is far from CCK of mathematics, namely it is conceptual knowledge of mathematics which is mostly required in teaching. Researchers stated that sometimes it is difficult to separate CCK and SCK while classifying mathematical knowledge. In other words, it is ambiguous when CCK ends and SCK begins (Carrillo, Climent, Contreras & Muñoz-Catalán, 2013). Although there are concerns related to discrimination of CCK and SCK, there has been no empirical result on whether these two categories are distinguishable or not (Baumert et al., 2009). Lastly, horizon content knowledge (HCK) is the mathematical knowledge of teachers based on the order and relation of mathematical topics within mathematics curriculum and what will be put on existing mathematical knowledge in following grades. Having this sort of knowledge provides a teacher an insight regarding how to talk about a mathematical subject so that students appreciate the new knowledge and connect the previous knowledge to the new one. However, Ball et al. (2008) stated that they were not sure that whether HCK is a component of subject matter knowledge of teacher and whether categories of subject matter knowledge are established definitely. Therefore, it is stated that their categories will continue to be changed and revised.

The teacher knowledge model of Ball, Thames, and Phelps (2008) was studied and revised by researchers (Sosa, 2011, cited in Carreno, Rojas, Montes, & Flores, 2013). Sosa proposed descriptions for categories of mathematics teacher knowledge model of Ball et al. (2008). Sosa, defined CCK as a subcategory including definitions, rules, properties, and theorems related to a specific topic, use of mathematical notations, awareness of importance of mathematical items, and representations of mathematical concepts. The researcher stated that even though this knowledge can be required or used by other professions, it constitutes an integral part of the knowledge of mathematics teachers. According to descriptions of the researcher, the subcategory SCK includes complete and deep knowledge of mathematics, knowledge of unseen steps behind procedures, intuitive knowledge of students' mathematical errors. According to Sosa, horizon content knowledge (HCK), the third subcategory of subject matter knowledge, includes interrelations between general and specific mathematical concepts and awareness of interdisciplinary applications.

Finally, MKT model is discussed and revised by the research group headed by José Carrillo at the University of Huelva, Spain. This model was called as mathematics teachers' specialized knowledge (MTSK) including six sub-domains (Carreño et al., 2013; Carrillo et al., 2013). They identify sub-categories of MTSK as knowledge of topics (KOT), knowledge of structure of mathematics (KSM), knowledge about mathematics (KAM), knowledge of features of learning mathematics (KFLM), knowledge of mathematics teaching (KMT), and knowledge of mathematics learning standards (KCMLS).

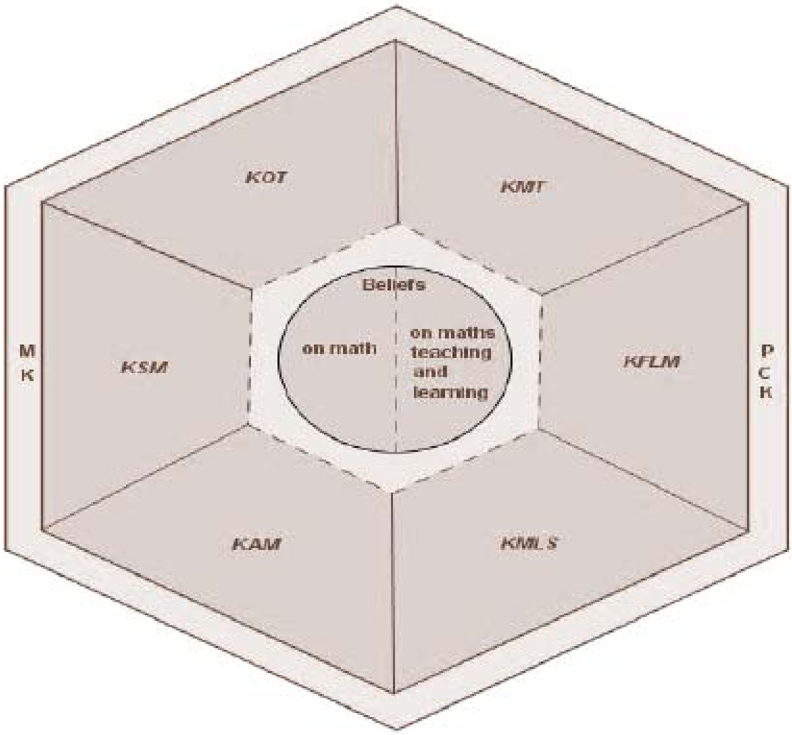



Figure 2. 3. Domain map for mathematical knowledge for teaching (Carrillo et al., 2013, p. 5)

As seen from the Figure, mathematics knowledge (MK) includes three components, KOT, KSM, and KAM. Firstly, KOT is defined as theoretical knowledge of mathematical concepts, procedures, and calculation methods. Also, it is defined as knowing a mathematical topic with different meanings and applications of this topic. For example, the concept of derivative can be used as the gradient of a curve or the limit of finite increments. KSM is defined as the knowledge of connections among prior and subsequent concepts in mathematics (Montes, Aguilar, Carrillo, & Muñoz-Catalán, 2013). However, this is a mathematical relation rather than being a curricular relation. For example, there is a mathematical relation between matrix algebra and geometry, but learning of geometry and matrix algebra does not have to be consecutive in mathematics curriculum. Also, it is described as a sense of larger mathematical environment. Therefore, teachers can make judgments about what is mathematically worthwhile even when they are not looking at the mathematics curricula (Carreno et al., 2013). The third sub-category is KAM which is described as mathematics knowledge, mostly specific to teaching profession. It requires mathematical knowledge such as what constitutes a definition and what the critical features of a definition are, when a demonstration has been completed, whether a proof or reasoning is valid, and identification of concepts and sub-concepts in a mathematical topic. This sub-category has a similarity with syntactic knowledge aspect of mathematical knowledge (Shulman, 1986) including knowing, creating or exploring in mathematics (Carrillo et al., 2013).

According to Kahan, Cooper, and Bethea (2003), strong mathematics knowledge is "a factor in recognizing and seizing teachable moments" (p. 245), but it does not ensure effective mathematics learning of students. In other words, subject matter knowledge of mathematics is a necessary prerequisite but not sufficient for an effective teaching (Krauss et al., 2008). At this point, a knowledge is required which combines the knowledge of mathematics, students, and pedagogy, namely PCK (Ball et al., 2001). In following topic, PCK of teachers will be described in detail.

### 2.2.2. Pedagogical Content Knowledge of Mathematics Teachers

Baumert et al. (2009) asserted that just knowing mathematics in an advanced level does not guarantee effective teaching. Rather, the important point is effective use of mathematics within the teaching profession. Shulman (1985) stated that "to be a teacher requires extensive and highly organized bodies of knowledge" (p. 47). And, he identified PCK as one of the most important categories of that knowledge. Shulman (1987) defined pedagogical content knowledge (PCK) as "the ways of representing and formulating the subject that makes it comprehensible to others" (p. 9). Also, he described PCK as a "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 8). This means, PCK refers to things that only teachers know and can do (Berry et al., 2008). In literature, there are several descriptions related to PCK. Firstly, Shulman (1986) described PCK as the 'capacity' of teachers in order to transform their particular content knowledge for the understanding of students in a pedagogically powerful and adaptive way throughout different ages, abilities, and backgrounds (Shulman, 1987). Moreover, Magnusson, Krajcik, and Borko (1999) stated that PCK is the understanding of teachers related to students' understanding of a specific subject and organization, representation, and adaptation of particular subject matter topics, problems, and issues with respect to interest and abilities of learners. Also, Niess (2005) defined PCK as “the intersection of knowledge of the subject with knowledge of teaching and learning” (p. 510). As definitions of PCK vary, models of PCK have different structures across different studies. According to study of Kind (2009), categories of PCK and their placement in the model present diversity as in Figure 2.3. However, representations and instructional strategies and subject specific learning difficulties of students are generally common components for pedagogical content knowledge.

| Authors | Representations and instructional strategies | Students' subject specific learning difficulties | Purposes/ orientations/ nature of science | Curricular knowledge | Subject matter knowledge | Context for learning | General pedagogy/ classroom management | Assessment | Socio-cultural issues | School knowledge |
|---|---|---|---|---|---|---|---|---|---|---|
| Shulman (1987) | P | P | K | K | K | K | K | 0 | 0 | 0 |
| Grossman (1990) | P | P | P | P | K | K | K | 0 | 0 | 0 |
| Magnusson, Krajcik and Borko (1999) | P | P | P | P | K | K | K | P | 0 | 0 |
| Marks (1990) | P | P | 0 | P | P | 0 | 0 | 0 | 0 | 0 |
| Fernández-Balboa and Stiehl (1995) | P | P | P | 0 | P | P | 0 | 0 | 0 | 0 |
| Koballa, Gräber, Coleman and Kemp (1999) | 0 | P | 0 | P | P | P | P | 0 | 0 | 0 |
| Cochran, deRuiter and King (1993) | 0 | P | 0 | 0 | P | P | P | 0 | 0 | 0 |
| Veal and MaKinster (1999) | P | P | P | P | P | P | P | P | P | 0 |
| Banks, Leach and Moon (2005) | 0 | 0 | 0 | 0 | P | 0 | P | 0 | 0 | P |

Notes: ‘P’ shows components believed to comprise PCK; ‘K’ denotes a component in a teacher’s knowledge base; ‘0’ shows components not discussed explicitly.

Figure 2. 4. PCK models of some researchers (Kind, 2009, p. 175)

Initially, Shulman (1986b, 1987) suggested that PCK includes two main components, namely the knowledge of representations and instructional strategies and the knowledge of students’ subject specific learning difficulties. He stated that teachers should use instructional strategies such as “the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations” (p. 9). Moreover, subject specific learning difficulties of students include knowledge about misconceptions, naive ideas gained through previous learning, and potential barriers to learning the content. According to Figure, models of PCK represent variety regarding the relation of SMK and PCK and the inclusion of components within PCK.

Grossman (1990) follows the PCK model of Shulman highlighting that SMK and PCK are distinct categories. Also, the researcher expanded the model of Shulman by adding two more components namely, purposes -an inclusive conception of teaching a specific subject- and curricular knowledge. Grossman (1990) defined purposes as “the overarching conceptions of teaching a subject [that] are reflected in teachers' goals for teaching particular subject matter" (p. 8). The researcher included this component in PCK after observing different teaching goals of teachers while teaching the same concept. The researcher proposed that different purposes yield with different instructional strategies. For this reason, purpose of teacher should be included in PCK as a component.

Magnusson et al. (1999) also proposed a PCK model including four components similar with the model of Grossman (1990). They attached orientations to purposes component of PCK. According to researchers, orientations - discovery, conceptual change, process, didactic, and inquiry - affect decisions of teachers related to preference of instructional strategies. Therefore, orientations should be included in PCK. Grossman (1990) and Magnusson et al. (1990) included curricular knowledge in PCK also. According to Magnusson et al. (1990), curricular knowledge is one of the dimensions of teacher knowledge which "distinguishes the content specialist from the pedagogue – a hallmark of pedagogical content knowledge" (p. 103). In addition, Magnusson et al. (1999) added knowledge of assessment within the PCK model advocating that assessment methods must be used in teaching in order to understand whether students have learned the subject matter or not.

Cochran, DeRuiter, and King (1993) argued that the notion of Shulman's PCK reflects a teacher directed model of teaching and Shulman's notion of PCK concentrated on only 'transformation of subject matter'. They advocated that PCK should be transformed into a dynamic form since development of knowledge is a continuous process. For this reason, they constructed Pedagogical Content Knowing (PCKg) by modifying PCK so that it was transformed into a dynamic form which comes from a constructivist view of learning. They defined PCKg as "teacher's integrated understanding of four components of pedagogy, subject matter content, student characteristics and the environmental context of learning" (p. 266). The researchers included additional components in PCK model when compared with the model of Shulman (see Figure 2.3). Additionally, they included SMK, context for learning, and general pedagogy within PCK.

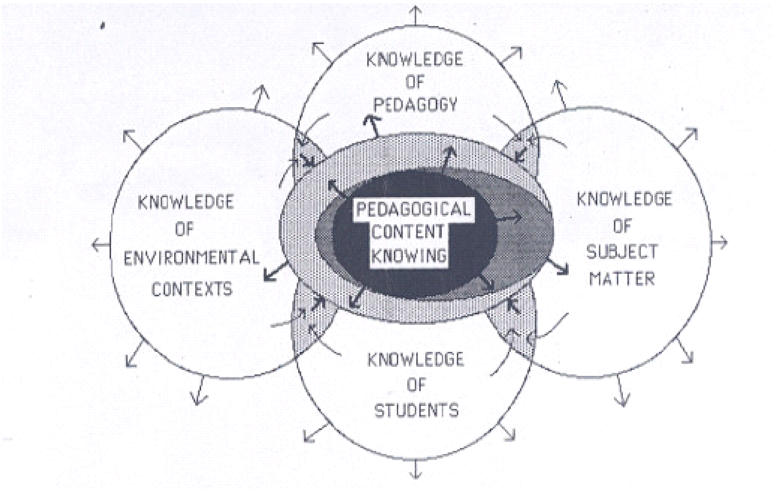



Figure 2. 5. PCK model of Cochran et al. (1993, p. 268)

The other research group, Ball et al. (2008), proposed a new PCK model in which SMK and PCK were treated as separate elements of teacher knowledge. According to this model, PCK includes three components, namely knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum as seen at right side of the oval. Firstly, KCS is defined as the knowledge of teachers related to thinking and learning of students such as deciding on which decimals are mostly struggling for students. It was a primary component of Shulman's (1986) PCK. According to Hill et al. (2008), this component contributes a critical foundation to PCK by taking into account thinking and ideas of students as it is mentioned in Shulman (1986). Secondly, KCT is defined as teachers' knowledge on teaching and mathematics such as making a decision on what to do about difficulties of students regarding particular concepts. And lastly, knowledge of curriculum is related to decision of the teacher about which content should be participated in curriculum. To sum up, this model categorized the knowledge of teaching and knowledge of students as separate components similar to Marks (1990) and Cochran et al. (1993). But, it has modifications related to components of PCK when we compare it with the initial notion of Shulman's PCK. The model proposed by the researchers is presented in Figure 2.2.

Sosa (2011, cited in Carreno et al., 2013) proposed a similar pedagogical content knowledge (PCK) model consisting of knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of curriculum

(KC). The researcher described knowledge of content and teaching (KCT) as the knowledge of how to introduce a new concept in mathematics and the knowledge of different methods while teaching mathematical concepts. Secondly, the researcher categorized the knowledge of content and students (KCS) as general pedagogical knowledge and knowledge about interaction of students with mathematics. The first one is defined as being aware of students' difficulties and requirements related to mathematical topics and the second one is probable misunderstandings that might arise in minds of students while learning mathematics. Lastly, knowledge of curriculum (KC) is described as organization of content in textbooks and the relation of previous and forthcoming mathematical topics.

In MTSK (Carillo et al., 2013), remaining three sub-categories are related to pedagogical content knowledge. The first one is knowledge of features of learning mathematics (KFLM). KFLM is closely related to the understanding of the teachers related to students' thinking when they face with mathematical activities and tasks, similar with KCS in the model of Ball et al. (2008). Additionally, this knowledge includes knowledge of theories and models related to how students learn mathematics and contribution of these theories and models on learning process of students. In MKT model of Ball et al. (2008), KCS refers to the knowledge of both mathematics and learning of students whereas KFLM is dominantly related to how mathematics is learnt by students and mathematics background of a teacher in order to understand the facts regarding students' learning of mathematics. The second sub-category of PCK is knowledge of mathematics teaching (KMT). This knowledge allows teachers to make complex series of decisions related to mathematics teaching, such as choosing an appropriate teaching method, preferring a convenient textbook, and choosing representations or materials for particular concepts of mathematics. The third sub-category is knowledge of mathematics learning standards (KCMLS). According to researchers, KCMLS is similar with KCC including the knowledge of curricular specifications, progression through consecutive years, and minimum standards similar with KC in the teacher knowledge model of Ball et al. (2008). KCMLS additionally includes learning objectives, standards, and measures of performance developed by external examining boards, professional associations, and researchers (Carrillo et al., 2013).

To sum up, there are various knowledge models for knowledge base of mathematics teachers in the literature. In this study, teacher knowledge model of Ball et al. (2008) and Carrillo et al. (2013) was taken into account in which SMK and PCK were treated as separate categories of teacher knowledge. The other important point is assessment of professional knowledge of mathematics teachers that researchers have little agreement on (Wilson, 2007).

### 2.3. Assessment of Knowledge of Mathematics Teachers

In recent years, teacher education has been one of the most popular topics for policy makers. The reason is knowledge of teachers being accepted as one of the key factors for quality of learning (Tatto et al., 2008). After teacher knowledge was identified as an important element for effective teaching, researchers have posed several questions related to teacher knowledge. What teachers should know and what they need to know are most frequently discussed topics for several years in teacher education (Ball & Bass, 2003; Ball et al., 2008; Hill et al., 2007). In the late 1980s, Shulman addressed that current teacher education courses have little effect on the improvement of teaching and learning. As Blömeke and Delaney (2012) stated whether and how teacher training affects the knowledge of teachers is a question that should be asked. For this reason, professional assessment systems have been required in order to understand effects of teacher education on knowledge of teachers. There are mainly three contemporary pressures which make teacher assessment a favorite topic in teacher education. The first one is the attempts for graduating "highly qualified" teachers in order to present better instruction in schools. The second one is observing whether or how teacher training affects the development of teacher capacity, knowledge, and skills. And the third one is related to identification of a domain related to professional knowledge and skills of teachers (Blömeke & Delaney, 2012; Hill et al., 2007).

As Shulman (1986) stated, the roots of teacher assessment were much older. He asserted that he got copies of tests licensing candidates at the county level which belongs to one century ago at that time. All those tests were in the same manner; that is ninety to ninety-five percent of those tests were related to subject matter knowledge. There were few items questioning the pedagogical aspect of the teacher knowledge. He mentioned examinations conducted to teacher candidates in 1980s and stated that those examinations were testing basic abilities such as reading, writing, spelling, calculating, and solving arithmetic problems. Therefore, those tests were just prerequisites in order to be a teacher other than being a professionalized examination for teaching. Shulman argued that evaluation of teachers should be conducted in a way, so that candidates became teachers with respect to their *capacity* to teach. He called the examination process of teacher candidates as assessing the capacities of a professional. He argued that these exams should be prepared by the member of teaching profession rather than legislators and laypersons. Also, Shulman (1986) stated that these examinations should cover both the content and teaching process required by teaching professionals. In addition, he proposed that these examinations should assess the knowledge of learning of students and their backgrounds, principles related to organization of school, finance and management, historical, social, and cultural foundations of education. According to Hill et al. (2007), how teacher knowledge is assessed varies across the approaches of the researchers. Some can be identical to a test which could be given to students and some can include tasks particular to teaching profession. As Hill et al. (2007) stated, "What is measured on tests of teachers' mathematical knowledge? What should be measured? How should it be measured?" are frequently asked questions related to assessment of teacher knowledge. In the following topics, what should be measured and how it should be measured in teacher assessment examinations will be discussed.

### 2.3.1. Aspects of Knowledge for Teaching Mathematics as Assessed in the Examinations

Shulman (1986) stated that there had been no consistency related to the balance of teacher knowledge included in teacher assessment. Sometimes pedagogical knowledge was ignored whereas sometimes subject matter knowledge was overlooked in teacher assessment. He stated that content and pedagogy are components of an indistinguishable body of knowledge. A century ago, pedagogical accomplishment was equal to knowledge of content since pedagogical knowledge was not used as its recent meaning today. Also, there was no distinction between the knowledge of pedagogy and content since it has been a recent tradition. He stated that researchers were generally interested in problems regarding learning of students; however, researchers should also study on problems related to teaching and fundamentals of teacher knowledge, namely sources of teacher knowledge, how the new knowledge is acquired, what a teacher knows and when the teacher comes to know. As Shulman (1986) stated, observations and interviews were better in order to assess teacher knowledge adequately rather than achievement tests. For this reason, he measured content knowledge of teachers by conducting regular interviews related to their teaching and observations of their instructions rather than use of achievement tests to evaluate knowledge of teachers as it is typically used in research literature. He stated that teacher assessment should not be composed of purely content knowledge; on the contrary, teacher assessment should also include aspects regarding teaching in order to discriminate content specialist from the teacher. Shulman also argued that teacher assessment should be conducted by professionals rather than legislators or layperson. Teacher assessment should include knowledge of content, knowledge of teaching process, and curricular knowledge. Also, a well-organized teacher assessment examination should include "knowledge of general pedagogy, knowledge of learners and their backgrounds, principles of school organization, finance and management, and the historical, social, and cultural foundations of education" (Shulman, 1986, p. 14). Also it is stated that content of the assessment should be prepared in a well-organized way in order to choose well-prepared teachers as professionals of teaching.

According to Hill et al. (2007), "...assessment of teachers is hotly contested terrain" (p. 112). They argued that there was not a concrete construct for mathematics knowledge of teachers. Therefore, although all teacher assessment examinations are concentrated on measuring mathematics knowledge of teachers, they measured different aspects of teacher knowledge with different methods. There are several methods to assess knowledge of teachers, namely interviews, observations, given tasks, and portfolios. To illustrate, some examinations measure the ability of teachers with mathematical problems at middle school level (e.g., California Basic Educational Skills Tests), some examinations assess ability of teachers to construct mathematical questions and tasks for students (e.g., Exam for the Certification of Educators in Texas), and some others assess ability of teachers to conceptualize and use mathematical content in teaching (e.g., Massachusetts Tests for Educator Licensure). There has been a disagreement on what mathematics teachers need to know in order to teach. Some argue that general mathematics knowledge is the most important qualification of a mathematics teacher whereas others argue that this knowledge should be complemented by other components of teacher knowledge, namely knowledge of students' thinking regarding a mathematical content or mathematical activities related to teaching of mathematics (Hill et al., 2004). According to study of Hill et al. (2004), teachers' mathematics knowledge should be composed of general mathematics content knowledge and knowledge of mathematics in a more specific way such as using definitions of mathematical content in teaching. That is, mathematics teachers should have mathematics knowledge more than any well-educated adult. As they stated, the purpose of the evaluation of teachers should be how mathematics knowledge is used in teaching – whether the teacher can use mathematical knowledge to construct representations or to correct misconceptions of students – rather than merely examining how much mathematics knowledge is held by the teacher. Hill et al. (2004) stated that basic mathematical knowledge must be acquired by a teacher; however, additional knowledge should be acquired also, such as knowing why mathematical statements are true, multiple representations of mathematical content, features of an accurate definition, and evaluation of mathematical methods, solutions, and representations. Furthermore, review of assessment of elementary mathematics teachers showed that much of the examinations include items simply asking to compute mathematical problems rather than tasks including use of mathematical knowledge for mathematics teaching in classroom (Hill et al., 2004).

The other study was Teacher Education and Development Study in Mathematics (TEDS-M) which investigated achievements, commonalities and differences of prospective teachers from Eastern and Western countries (Kaiser & Blömeke, 2013). In this study, data is gathered in terms of outcomes, institutions and programs, and national policy of participating countries. First, outcomes referred the knowledge of prospective teachers who were participated in the study from different countries. Second, institutions referred the fundamental characteristics of the teacher education programs of different countries. Lastly, national policy referred the context of policy in terms of teacher education across different countries (Tatto et al., 2008). TEDS-M was used to assess competencies of prospective mathematics teachers in their final year of teacher education. Professional knowledge of a teacher is structured by the components of mathematics content knowledge (MCK), mathematics pedagogical knowledge (MPCK) including curricular knowledge, and general pedagogical knowledge (GPK) in the study. The model of professional knowledge in TEDS-M is presented in detail as:

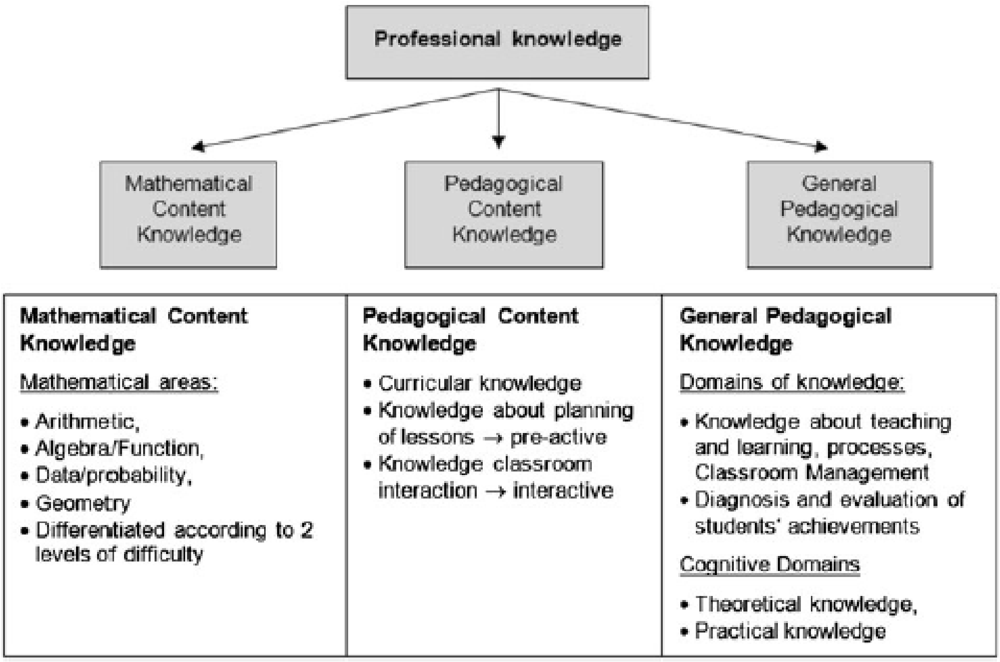



Figure 2. 6. Professional knowledge of teachers (Kaiser & Blömeke, 2013, p. 6)

As a result, subject matter knowledge, pedagogical content knowledge, and general pedagogical knowledge are the three main components of teacher knowledge taken into the consideration in the studies assessing the knowledge of mathematics teachers.

### 2.3.2. Types of Teacher Assessment

Although reliable and valid methods for assessment of teacher knowledge are required, teacher assessment systems which measure categories of teacher knowledge effectively have been seen rarely (Krauss et al., 2008). As assessment of teacher knowledge has become commonplace and number of them has increased over the past 30 years, there have been several teacher assessment techniques regarding their purposes, content, and methods (Hill et al., 2007). Firstly, Hill et al. (2004) conducted a study in order to develop a scale for developing measures of teachers' mathematics knowledge for teaching. In order to measure the content knowledge of mathematics teachers they prepared a test including multiple choice items, complex multiple choice items, and open construct items. In another study, TEDS-M (2008), beliefs, opportunity to learn, subject matter knowledge, and pedagogical content knowledge of pre-service teachers from 17 countries were tapped. Within the study interview and questionnaire techniques were utilized. In order to tap knowledge of pre-service mathematics teachers, questionnaires were used including multiple choice items, complex multiple choice items, and open construct items as similar with Hill et al. (2004). In addition, OECD (2005) reported that "[e]xaminations may include observation of the candidate’s teaching, in-depth interviews or consideration of portfolios with records of achievement and work experience” (p. 22).

Items with multiple choice or short-answer response cannot completely represent knowledge of mathematics teachers since they can measure limited aspects of knowledge of mathematics teaching which has a complex nature (Hill et al., 2007). For this reason, open construct items, interviews, and observations should be utilized additionally in order to tap teacher knowledge (OECD, 2005; Tatto et al., 2008). Furthermore, Hill et al. (2008) stated that expert teachers can be discriminated from non-experts by their detailed interpretations of students' problems. For this reason, measurement of teacher knowledge with multiple choice items may be inadequate because it will be difficult to investigate explanations of teachers related to students' errors since teachers generally choose any plausible answer with multiple choice formats. Ultimately, Hill et al. (2008) directed a question: "Can teachers' KCS be measured in multiple choice formats?" (p. 391). According to the results of the study, multiple choice items can measure KCS if they are prepared well although it is reasonably difficult for large scale assessments. Also, they stated that preparation of multiple choice items is an important process. In multiple choice items, wrong answers often seem absurd so that even little knowledgeable students can correctly choose the expected answer. Therefore, it is difficult to prepare large scale items to assess teacher knowledge in multiple choice formats. Figure 2.7 is an example to multiple choice items from Hill et al. (2008).

Mrs. Jackson is getting ready for the state assessment, and is planning mini-lessons for students focused on particular difficulties that they are having with adding columns of numbers. To target her instruction more effectively, she wants to work with groups of students who are making the same kind of error, so she looks at a recent quiz to see what they tend to do. She sees the following three student mistakes:

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 38 | 45 | 32 |
| 49 | 37 | 14 |
| + 65 | + 29 | + 19 |
| 142 | 101 | 64 |
| (1) | (II) | (III) |

Which have the same kind of error? (Mark ONE answer.)

a. I and II
b. I and III
c. II and III
d. I, II, and III

Figure 2. 7. Sample multiple choice item from Hill et al. (2008, p. 400)

Figure 2.7 is an example to multiple choice items and Figure 2.8 is an example to open-ended items from the study of Hill et al. (2008). Examinations conducted to assess teachers may vary based on their conceptions including knowledge and skills clarified by educational committees which are critical for doing teaching profession. Different examinations assess different aspects of teacher knowledge such as some measure basic skills or liberal arts knowledge while others assess subject matter knowledge and still others assess teaching methods (Mitchell et al., 2001). Hill et al. (2008) conducted a study to investigate whether teachers' knowledge of mathematics and students affects learning of students. In the study, they prepare items in order to measure KCS of teachers. They stated that KCS is an amalgam including both SMK and knowledge of students. Therefore, measures that assess knowledge of students should not solely be related to one of these elements of teacher knowledge. Items should include mathematical knowledge or mathematical reasoning in order to anticipate thinking of students related to particular topics. In addition, they identified that items fell into four categories of KCS, namely common students errors, students' understanding of content (e.g., deciding on which productions of students present better understanding of students), developmental sequences of students (e.g., identifying which concepts are leant by students easier or more difficult, knowing what sixth graders most probably might be able to do), and common student computational strategies.

Consider Jill's response to a subtraction problem. How might she have gotten an answer like this?

$$\begin{array}{r} 51 \\ -\underline{18} \\ 47 \end{array}$$

Figure 2. 8. Sample open-ended item from Hill et al. (2008, p. 400)

In the study of Krauss et al. (2008), they established a test to measure SMK and PCK of secondary mathematics teachers, to identify the level of students, and to determine connectedness with respect to these categories. They prepared two tests within the study, namely PCK and SMK tests. PCK test was composed of knowledge of mathematical tasks (with four open-ended items), knowledge of misconceptions and difficulties of students (assessed by seven scenarios), and knowledge of mathematics-specific instructional strategies (with 10 items for teachers to explain mathematical situations) (see Figure 2.9 for a sample PCK item). Also, CK test (30 items) included mathematical knowledge from secondary level mathematics curriculum. In the test, 34 items were open ended items (see Figure 2.10 for a sample item). According to results of the study, they concluded that the constructed test is reliable and empirically valid regarding both knowledge categories of teachers, corresponding structural model, and different educational backgrounds of teachers.

| | | |
|---|---|---|
| PCK Instruction | A student says: I don't understand why $(-1)\cdot(-1)=1$<br><br>Please outline as many different ways as possible of explaining this mathematical fact to your student. | The "permanence principle," although it does not prove the statement, can be used to illustrate the logic behind the multiplication of two negative numbers and thus foster conceptual understanding:<br><br>$3 \cdot (-1) = -3$<br>$2 \cdot (-1) = -2$<br>$1 \cdot (-1) = -1$<br>$0 \cdot (-1) = 0$<br>$(-1) \cdot (-1) = 1$<br>$(-2) \cdot (-1) = 2$<br><br>-1 +1 |

Figure 2. 9. Sample PCK item from Krauss et al. (2008, p. 720)

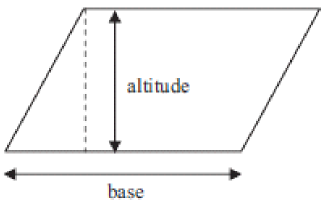
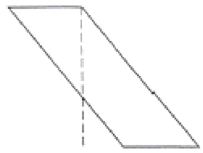
| | | |
|---|---|---|
| PCK Student | The area of a parallelogram can be calculated by multiplying the length of its base by its altitude.<br><br>Please sketch an example of a parallelogram to which students might fail to apply this formula. | <br>Note: The crucial aspect to be covered in this teacher response is that students might run into problems if the foot of the altitude is outside a given parallelogram. |

Figure 2. 10. Sample PCK item from Krauss et al. (2008, p. 720)

The other method for assessing knowledge of teachers is observation and interview techniques. Lee, Brown, Luft, and Roehrig (2007) conducted a study in order to assess beginning science teachers' PCK based on two categories, namely knowledge of student learning and knowledge of instructional strategies. In the study, researchers employed two methods for measuring PCK of science teachers, observation of their classroom practice and interview. At the end of the study, they concluded that assessing PCK of science teachers might be difficult even it is measured by observation and interview methods because of complex nature of the construct PCK.

In addition to studies measuring teachers' knowledge in the literature, there are several teacher assessment examinations in the world. Teacher assessment in different countries will be discussed in following topic.

### 2.3.3. Teacher Education and Knowledge Assessment for Job Readiness in Turkey and Other Countries

By the help of improvements in teacher assessment, teacher knowledge assessment systems have become commonplace for several countries in the world (Tatto et al., 2008). Even if all teacher knowledge assessment examinations measure knowledge of teachers, they might differ in terms of their content and methods (Hill et al., 2007). Firstly, in Turkey, pre-service teachers are selected by a national examination, public personnel selection examination. After they enter the teacher education program, they are educated for four or five years period. In teacher education period, they generally take courses related to subject matter knowledge, pedagogical knowledge, pedagogical content knowledge, and other elective courses. After they complete teacher education program, they enter an examination for teacher assessment in order to be a teacher in governmental schools. This examination consists of multiple choice items from three main areas, basic knowledge and skills, educational sciences, and teacher content knowledge. Basic knowledge and skills examination includes general knowledge of social sciences and mathematics, educational sciences examination includes knowledge related to general education such as educational psychology, developmental psychology, guidance, and measurement and assessment. And lastly, teacher content knowledge examination is composed of subject matter knowledge and pedagogical content knowledge. Teachers are merely evaluated with respect to their achievement on these tests and employed as teachers in governmental schools (ÖSYM, 2013).

In the United States, each state is responsible for its teacher certification and teacher licensing system. As Cronin (1983) stated "Certification is the process of deciding that an individual meets the minimum standards of competence in a profession. Licensing is the legal process of permitting a person to practice a trade or profession once he or she has met certification standards" (p. 175). In the United States, each state has its own standards and competencies for teacher education;

however, there are several communities which identify national standards for teachers such as the Interstate New Teacher Assessment and Support Consortium (INTASC), the National Board for Professional Teaching Standards (NBPTS), the National Council for Accreditation of Teacher Education (NCATE), and National Council of Teachers of Mathematics (NCTM, 2000). In United States, professional licensing of teachers have a long history from today to the late 1600s in order to ensure at least a minimum level of teacher quality (Roth & Swail, 2000). Although each state has its own certification and licensing system, some points are generally valid for all states. To illustrate, in order to be a teacher, teacher candidates have to have at least a bachelor's degree or complete an approved teacher education program. Teacher candidates should have a major or minor in education for elementary school and have a major in their area for middle or high school teaching, have an efficient liberal-arts knowledge, and finally pass teacher assessment examination (Roth & Swail, 2000). In some states, there are tests for admission to teacher education programs whereas in other states testing is conducted for initial licensure (Mitchell, Robinson, Plake, and Knowles, 2001). Also, teacher candidates can get teacher certification by alternative teacher certification, not going through undergraduate teacher education. After teacher candidates have a bachelor degree from an accredited college or university or complete an approved educator preparation program, they can enter teacher certification examinations. For each subject matter area, there exists a test which assesses pedagogical knowledge, pedagogical content knowledge, and subject matter knowledge.

Singapore is one of the countries that performed world-beating success in mathematics (Gonzales et al., 2008; Mullis et al., 2005). In Singapore, admission into teaching profession is done selectively. According to Singapore educators this performance is an indicator of using a coherent curriculum in every school by highly qualified teachers. They stated that highly-qualified teachers are not simply raised by the chance or the cultural respect on teaching. It happens as a result of accurate policy preferences. That is, use of a deliberate teacher education system including selection, compensating, and developing teachers. In Singapore, pre-service teachers from the top one third of the secondary school graduating class are selected cautiously by the Ministry of National Education. Pre-service teachers can enter teacher education programs after completing secondary school, after completing a university degree, or with a decision to change their carrier. Then, teachers are educated at a centralized institution, National Institute of Education (NIE) at Nanyang Technological University. Courses taken by teachers are generally composed of subject matter knowledge and pedagogical knowledge. Also, teachers improve their teaching carrier by learning new practices of teaching and new uses of technology in education, going abroad to observe teaching in other countries, and sharing their experiences with other teachers in Singapore Teachers' Center. In the process of teacher education, pre-service teachers receive a stipend which is equivalent to 60% of a teacher salary provided that they commit to teaching for at least three years. Pre-service teachers are entitled to teacher evaluation after being an in-service teacher. These evaluations are held with respect to the specific performance goals, competencies, training and development plans for the following year and with respect to the reviews and comments of a teacher and a supervisor based on the work performance and the competencies of teachers. Adaptation of teachers to teaching profession is enhanced by internship programs and mid-career entry to the profession. Also, teachers are recruited locally and overseas. For each year, teachers are subjected to 100 hours of professional development. Beginning teachers are mentored by experienced teachers for several years. After being an in-service teacher, teachers are provided awards, scholarships, and sponsorships in order to improve their capacities in teaching profession. Furthermore, beginning and experienced teachers are supported through Induction and Enhanced Mentoring Programs. Evaluation of teachers is conducted by the Enhanced Performance Management System (EPMS), a competency based tool. Like every other professions in Singapore, teachers are reinforced with performance appraisal with respect to their success in teaching, relationship with parents and other related person, and contribution to the school. Teachers are evaluated with respect to competencies such as observable characteristics, subject matter knowledge, classroom management, teaching skills. Competencies of teachers are composed of learning and development of students, contribution of teachers to community of school, cooperation with parents, and professional development (Steiner, 2010). Teacher evaluation is conducted in order to evaluate, direct, and promote teachers. In Singapore, there is a strong connection among the Ministry, the National Institute of Education and the schools which lead to a continuous improvement and high-consistency in education. Teachers are annually assessed in order to investigate whether they have potential to continue their carrier as master teacher, specialist in curriculum and research, or school leader. If they have potential for one of these professions, they are prepared for their new roles by taking special training. By thinking this system, it can be understood that success of Singaporean teachers are provided by the connection among highly qualified teaching, influential student learning, and effective organization of schools.

By the help of high performance of students in Programme for International Student Assessment (PISA), Finland has received an international attention (Hendrickson, 2012). In Finland, teaching profession is one of the most respected jobs (Opetusalan Ammattijärjestö (OAJ), 2008). Teacher education is conducted in universities and polytechniques. Pre-service teachers should firstly have a Bachelor's degree, then a Master's degree. In their major subject they take basic, high school, and advanced studies and basic and high school studies in their minor subjects which proceed for 5 years. However, students who want to be a kindergarten teacher should have a 3-years education only. Teacher education includes both pedagogical studies and subject studies. Primary school teachers (1-6) have to apply a teacher education program in order to be a teacher whereas secondary school teachers can follow two paths in order to be a teacher. Firstly, they may have a MA degree and apply separately teacher education or they may directly apply teacher education programs. In Finland, successfully completing a teacher education program is adequate in order to be a teacher (OAJ, 2008). In conclusion, assessment of teacher knowledge differs in different countries. In some countries, it is sufficient to complete a teacher education program while in some other countries prospective teachers are required to take an examination in order to legitimate their competency. The following section will be the summary of the literature.

### 2.4. Summary of the Literature

Teacher competency is a central concern in teacher education. It refers to the knowledge, skills, abilities, and beliefs of the teachers for the achievement of future K-12 students. There have been several institutions who prepare teacher standards in order to have competent teachers. These standards mainly have two components. The first one is related to beliefs, motivation, and self-regulation of teachers. The second one is related to professional knowledge of teachers. Professional knowledge of teachers is composed of three fundamental categories, namely pedagogical knowledge, subject matter knowledge, and pedagogical content knowledge. Pedagogical content knowledge is related to knowledge of teachers regarding teaching of the subject and learning of it by students whereas content knowledge refers to sufficient knowledge of teachers related to the subject they will teach. In addition, pedagogical knowledge can be defined as knowledge of educational principles and aspects such as guidance, classroom management, measurement and assessment, and educational psychology. That is, it is a type of knowledge which includes general aspects of education.

When literature is reviewed, pedagogical content knowledge can be divided into three categories, namely knowledge of teaching, knowledge of learning of students, and knowledge of curriculum. Also, content knowledge can be separated into three categories, common content knowledge, content knowledge specific to teachers, and knowledge of structure of mathematics. In literature, these concepts were discussed in detail. Since teacher knowledge is an important concern for teacher quality, assessment of teacher knowledge has become a hotly discussed topic among researchers. How teachers should be assessed, which components of teacher knowledge should be measured, and which kind of questions should be used have been mostly discussed topics in teacher assessment. There have been several arguments related to teacher assessment; however, there has not been an agreement on these questions yet. Therefore, the results of these studies have showed that further studies are needed in order to identify a concrete teacher knowledge assessment system and in order to improve content and structure of these systems.

# CHAPTER 3

# METHODOLOGY

The first purpose of this study was to investigate the structure of mathematics teacher content knowledge examinations in Turkey and Texas based on the curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics for teaching. The second purpose of this study was to explore how similar and different the mathematics teacher knowledge examinations in Turkey and Texas were with respect to the curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics for teaching. In the study, following questions were researched:

What are the structures of mathematics teacher content knowledge examinations of Turkey and Texas in terms of curricular levels, fields of mathematics, cognitive domain, and domains of the mathematics knowledge for teaching?

What are the similarities and differences between mathematics teacher content knowledge examinations in Turkey and those in Texas in terms of curricular levels, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching?

This chapter outlines the methodology of the study including research design, selection of the sample, data analysis of the study, and reliability.

## 3.1. Research Design

This is a qualitative study in which content analysis techniques were used so as to analyze the data to investigate the distribution of the items with respect to the related categories in mathematics teacher content knowledge examinations. Using qualitative methods permits the researcher to collect and evaluate the data in a detailed and effective manner (Patton, 1990). In this study, content analysis technique was used. Content analysis is the technique in which communications of human-like textbooks, essays, novels, and articles are investigated rather than directly studying on human behavior (Fraenkel & Wallen, 2005). Within the study, elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas were analyzed. The frequency and percentage of the items were investigated based on the curricular levels, fields of mathematics, cognitive domain, and sub-categories of mathematics teacher knowledge in order to have a better image of the examinations.

### 3.2. Data Sources

Mathematics teacher content knowledge examinations in Turkey and Texas were introduced in the following topics. The examinations were analyzed within this study due to the convenience of resource. In Table 3.1, frequencies of items for each examination were given.

Table 3. 1. Total number of items for each examination

| | **Elementary Mathematics Teacher Content Knowledge Examination** | **Secondary Mathematics Teacher Content Knowledge Examination** |
|---|---|---|
| **Turkey** | 50 | 50 |
| **Texas** | 80 | 80 |

#### 3.2.1. Mathematics Teacher Content Knowledge Examinations in Turkey

In Turkey, teachers have been selected to public institutions by a centralized examination since 2000s (ÖSYM, 2014). Kamu Personeli Seçme Sınavı (KPSS), the examination for the selection of public personnel, is conducted by a public institution, the Center for Assessment, Selection and Placement (ÖSYM), in order to select teachers and other public personnel (ÖSYM, 2014).

KPSS was composed of two main sections, namely the general ability-general knowledge test and the educational sciences test until the KPSS implemented in

2013. General ability test includes mathematics and Turkish language items and general knowledge test includes the items from the history, law, geography, and culture. Also, the educational sciences test includes items related to teaching principals and techniques, counseling, educational psychology, and developmental psychology. Although there were a section related to the educational sciences, KPSS had not assessed the knowledge of elementary and secondary mathematics teachers based on the knowledge of mathematics which is specific to the mathematics teaching profession. For the last two years, a third section, mathematics teacher content knowledge examination (ÖABT), has been included in the KPSS. In ÖABTs, there were two main sections, subject matter knowledge and pedagogical content knowledge. The purpose of the KPSS is the admission of civil personnel to the public institutions in Turkey. Therefore, the purpose of general culture-general knowledge, educational sciences, and ultimately ÖABTs, as parts of KPSS, are for the admission of teachers to the public schools in Turkey (ÖSYM, 2014). The passing criterions of ÖABTs indicate that Turkey has a norm-referenced examination. Therefore, teachers are ranked with respect to the achievement of their counterparts in ÖABT. That is, prospective teachers are selected for the public schools with respect to their achievements in comparison to the achievement of other teacher candidates.

Since 2013 ÖABT was the first examination in assessing the knowledge of mathematics specific to the teaching profession, in Turkey, it was the only examination which evaluates the knowledge of mathematics teachers when this study was conducted. In addition, after the ÖABT which was implemented in 2013, ÖSYM began to release only 10% of the items in the examinations publicly rather than releasing all of these items (ÖSYM, 2013). Therefore, the ÖABT implemented in 2013 was chosen to have an insight of how the assessment of the knowledge of mathematics teachers was done in Turkey. In this study, elementary and secondary mathematics teacher content knowledge examinations in Turkey will be called as ÖABT (5-8) and ÖABT (9-12).

### 3.2.2. Mathematics Teacher Content Knowledge Examinations in Texas

Examinations for the assessment of teacher knowledge have been conducted for more than a hundred years in the United States of America. Even though each state has its own teacher certification examination, it can be said that assessment of teacher knowledge has a diverse history (Hill et al., 2007). Since examinations for teacher assessment have been conducted for several years, there have been several studies related to the assessment of the knowledge of mathematics teachers in the United States. In addition, the United States of America was one of the few countries (England and the United States of America) implementing examinations after teacher education programs (Wang, Coleman, Coley, & Phelps, 2003). For this reason, mathematics teacher content knowledge examinations of United States of America were preferred within the study. In this study, some states were requested for the mathematics teacher certification examinations in these states; however, only the state of Texas gave the opportunity to use the mathematics (4-8) and mathematics (8-12) representative forms of the state's teacher certification examination. The representative forms of the state's teacher certification examination were developed by the Educational Testing Service and published in 2006. Also, conversations with the Texas Education Agency and the other states were given in Appendix B.

Texas Educator Certification Examination includes two parts in general. Individuals from Texas, other states, or another country are required to pass the examinations including the Texas Higher Education (THEA) and Texas Examination for Educator Standards (TExES) or hold a teacher certification in order to be a teacher in Texas (TEA, 2014). The first part is the THEA which includes multiple choice items from reading, writing, and mathematics areas in general. And, the second part is the TExES which includes multiple choice items related to the knowledge of mathematics and the knowledge of teaching of mathematics. These examinations are prepared by Educational Testing Service (ETS). These are criterion-referenced examinations in which teacher candidates are examined in order to investigate whether they achieve identified learning standards, criterions, or objectives (ETS, 2014a). Therefore, teacher candidates have to get the scaled score of 240 as the minimum passing score in the range of 100-300 in TExES in order to be a teacher in Texas (THEA, 2014; ETS, 2014b). Therefore, teacher candidates have to present the minimum passing score to represent the minimum level of competency required to be an entry-level educator in Texas (ETS, 2014b).

In order to attain the representative forms of TExES, the researcher sent an e-mail to TEA. After two months, the Non Disclosure Agreement (NDA) among TEA, Educational Testing Service, and the researcher was signed after the correspondences with TEA. Then, the hardcopy of representative tests were received by the researcher within two months after the non disclosure agreement was approved. Under non disclosure agreement, the researcher have agreed to notify Educational Testing Service and provide a copy of the derived materials at least 60 days prior to any final deadline of the study and the researcher have also agreed to return the representative tests to TEA within 30 days of the completion of the academic research.

TEA provided the representative forms of the elementary (4-8) and secondary (8-12) mathematics teacher content knowledge examinations, TExES (4-8) with code 115 and TExES (8-12) with code 135 respectively. TEA stated that if the examinations are publically disclosed, neither the teacher certification examinations nor any of the items will be administered again (see the conversations with TEA in Appendix B). Therefore, TEA provided the representative forms of TExES (4-8) and TExES (8-12) for the study. These examinations were prepared by Educational Testing Service (ETS) and similar in every material with respect to the administered tests but do not contain the items that have been or will be used in an administered test. Also, representative tests are designed closely parallel to the administered tests and normally they are sold only to an authorized educator preparation program for limited and controlled use in preparing their candidates. However, since representative forms will be used for academic purposes in this study, TEA provided them to the researcher with no charge (For further correspondences with TEA, see Appendix B.) In this study, representative forms of elementary and secondary mathematics teacher certification examinations will be called as TExES (4-8) and TExES (8-12).

## 3.3. Content Analysis of Mathematics Teacher Content Knowledge Examinations

The investigation of the frameworks of the examinations in Turkey and Texas indicated that there is not a publically released framework including competencies and standards for mathematics teaching related to the preparation of examinations.

Conversely, there exists a publically released framework while preparing the examinations to assess the competencies of teachers in consideration of detailed teacher certification standards in Texas (TEA, 2006). Since examinations in Turkey and Texas are not prepared with respect to the same framework, the framework of TExES could not be used for the analyses of examinations in both Turkey and Texas. Therefore, the analyses of the examinations were conducted with respect to the curricular levels and cognitive domain frameworks used in the TEDS-M study (Tatto et al., 2008), fields of mathematics framework and mathematics knowledge for teaching framework. Categorization of sample items with respect to these four variables was presented in Appendix A.

### 3.3.1. Analyses of Examinations based on the Curricular Levels and Fields of Mathematics

Curricular levels and fields of mathematics were two aspects in which mathematics teacher content knowledge examinations were analyzed within this study. The examinations were investigated based on these two aspects in order to present how items are distributed in the examinations as it is done in the study of Tatto et al. (2008). In this study, the curricular levels were specified as in Table 3.2. The framework in Table 3.2 was adapted from the study of Tatto et al. (2008). For both elementary and secondary mathematics teacher content knowledge examinations, Table 3.2 was used in order to investigate the examinations in terms of curricular levels. Since $4^{th}$ grade is included in elementary school in elementary mathematics curriculum of Texas, the scope of the elementary school was accepted as $4^{th}$-$8^{th}$ grades in the framework (TEA, n.d.). In addition, in order to separate elementary and high school categories of the curricular levels, the scope of the high school was accepted as $9^{th}$-$12^{th}$ grades for both Texas and Turkey although the scope of the high school includes $8^{th}$-$12^{th}$ grades in high school mathematics curriculum of Texas (TEA, n.d.). Moreover, the mathematics content beyond the level of high school was accepted as the advanced level as it is presented in Table 3.2.

Table 3. 2. Framework for curricular levels (adapted from Tatto et al., 2008)

| | |
|---|---|
| **Elementary** | Mathematics content that is typically thought at elementary school level ($4^{th}$-$8^{th}$ grade) |
| **High School** | Mathematics content that is typically thought at high school level ($9^{th}$-$12^{th}$ grade) |
| **Advanced** | Mathematics content that is typically thought beyond high school |

Categorization of items in terms of curricular levels was separately conducted based on the elementary school and high school mathematics curricula of Turkey and Texas (MONE, 2009; MONE, 2011; TEA, n.d.). Furthermore, items were examined based on the fields of mathematics. Number, algebra, geometry and measurement, and data analysis and probability are fields of mathematics specified by educational studies (MONE, 2009; MONE, 2011; NCTM, 2000; ÖSYM, 2013; Tatto et al., 2008; TEA, n.d.). In this study, mathematics content was divided into four domains: algebra, calculus, applied mathematics, and geometry. Since items included mathematics content from the elementary school to advanced level, number field was analyzed under algebra field and calculus is identified as a separate field. While coding items, each item was classified into one of these fields with respect to the mathematics content of the related item. Moreover, classification of mathematical topics and concepts with respect to these four fields was done based on the framework prepared by the researcher in consideration of the mathematics handbooks prepared by Pearson (1990) and Rainbolt and Gallian (2010).

While coding the items, they were categorized into sub-categories of curricular levels and fields of mathematics in a dependent way. Therefore, sample items were presented with respect to the frameworks of both curricular levels and field of mathematics as presented in Table 3.2 and Table 3.3. Categorization of items was presented based on curricular levels and fields of mathematics in the following three tables by the use of sample items.

Table 3. 3. The framework of the fields of mathematics with respect to the curricular levels (Pearson, 1990; Rainbolt and Gallian, 2010)

| | Elementary | High School | Advanced |
|---|---|---|---|
| **Algebra** | Number<br>Sets<br>Operations<br>Equations<br>Algebraic expressions<br>Inequalities | Types of proofs<br>Cartesian product quadratic and other nonlinear equations and inequalities<br>Complex numbers<br>Matrices and determinant<br>Linear equation systems | Groups, ideals, rings, and fields<br>Matrices and determinants<br>Vector Spaces and linear transformation<br>Elementary number theory<br>Set Theory<br>Function series<br>Fourier series |
| **Calculus** | -------- | Functions<br>Polynomials<br>Limit and continuity<br>Derivative<br>Integral<br>Trigonometry | Functions<br>Differentiation<br>Integral calculus |
| **Applied Mathematics** | Probability and statistics<br>Statistical terms<br>Permutation and combination<br>Table and graphics | Permutation, combination<br>Binomial expansion<br>Probability and statistics | Differential Equations Power Series Solutions<br>The Laplace transformation<br>Combinatorics probability<br>Statistics calculus of errors |
| **Geometry** | Line, line segment, and ray<br>Angles<br>Polygons<br>Similarity<br>Circle<br>Three dimensional figures<br>Transformational geometry<br>Pattern and tessellations<br>Area and volume | Coordinate geometry<br>Polygons and coating in plane<br>Right prisms and pyramids<br>Circle<br>Right circular cylinder, Right circular cone, and sphere<br>Lines and triangles<br>Transformations<br>Conics<br>Vectors in space<br>Line and plane in space<br>Solid geometric objects<br>Tessellations, rotation, and perspective in space | Plane geometry<br>Plane trigonometry<br>Spherical trigonometry<br>Vector algebra<br>Analytical geometry<br>Differential geometry |

The first sample item was from TExES (4-8) representative form (TEA, 2010a). The item was categorized as an intermediate algebra item since the item includes mathematics knowledge from the intermediate level based on the high school mathematics curriculum of Texas (TEA, n.d.). Moreover, the item was categorized into the algebra field since it included mathematics knowledge related to the properties of mathematical operations (see Figure 3.1).

| | |
|---|---|
| **Curricular levels:** | Intermediate |
| **Field of mathematics:** | ALGEBRA |

3. **Use the addition problem below to answer the question that follows.**

$$\begin{array}{r} 12 \\ 29 \\ 88 \\ \underline{+\ 11} \end{array}$$

When given the addition problem above, a student quickly said "140." When asked how she solved the problem, the student replied, "I added 88 and 12 to get 100, and 29 and 11 to get 40. Then I added these two numbers together." Which of the following two properties of addition did the student use in solving this problem?

A. Associative and commutative

B. Associative and additive identity

C. Commutative and additive identity

D. Distributive and additive inverse

Figure 3. 1. A sample item from TExES preparation manual for mathematics (4-8) (TEA, 2010a)

The second item was from ÖABT (5-8) which was coded as an intermediate item since the item includes mathematics knowledge from the intermediate level based on the curriculum of high school mathematics in Turkey (MONE, 2011). Moreover, the item was categorized into the calculus field since it included mathematics knowledge related to the trigonometric functions (see Figure 3.2).

| | |
|---|---|
| **Curricular levels:** | Intermediate |
| **Field of mathematics:** | CALCULUS |

**2.** $\tan 2x = u$

olarak veriliyor.

**Buna göre, $\sin 4x$ 'in $u$ türünden ifadesi aşağıdakilerden hangisidir?**

A) $\dfrac{1-u^2}{1+u^2}$  B) $\dfrac{u^2-1}{u^2+1}$  C) $\dfrac{2u}{1+u^2}$

D) $\dfrac{u}{1-u^2}$  E) $\dfrac{1+u^2}{u^2-1}$

Figure 3. 2. A sample item from ÖABT (5-8) (ÖSYM, 2013a, p. 1)

| | |
|---|---|
| **Curricular levels:** | Advance |
| **Field of mathematics:** | ALGEBRA |

**14. Sayılabilir kümeler için aşağıda verilen ifadelerden hangisi <u>yanlıştır</u>?**

A) $K$ sayılabilir bir küme ise her $T \subset K$ için $T$ kümesi de sayılabilirdir.

B) $L$ ve $M$ sayılabilir kümeler ise $L \cup M$ kümesi de sayılabilirdir.

C) $L_1, L_2, \dots, L_n$ küme ailesi sayılabilir ise $\bigcup_{i=1}^{n} L_i$ kümesi de sayılabilirdir.

D) $L$ sayılabilir bir küme ve $M \subset L$ ise $L \setminus M$ kümesi de sayılabilirdir.

E) $L$ sayılabilir bir küme ve $L \subset T$ ise $T$ kümesi de sayılabilirdir.

Figure 3. 3. Sample item from ÖABT (5-8) (ÖSYM, 2013b, p. 4)

The last sample item for the curricular levels and fields of mathematics was an advanced level algebra item as it was presented in Figure 3.3. The level of the item was advance since mathematics knowledge included in this item was beyond the high school level mathematics according to the high school mathematics curriculum in Turkey (MONE, 2011). The field of mathematics of the item was algebra since mathematics knowledge included in the item was related to the set theory.

In the categorization of items with respect to the curricular levels and the fields of mathematics, some items could not be categorized into the curricular levels and the fields of mathematics since there was not a mathematics content included in the item. Therefore, these items were classified into the uncategorized section based on the curricular levels and fields of mathematics while presenting the results.

### 3.3.2. Analyses of Examinations based on the Cognitive Domain

In this study, cognitive domain was one of the variables which were used to analyze mathematics teacher content knowledge examinations. Items of these examinations were investigated in order to present how the distribution and balance of items were established based on levels of cognitive domain in similar with the study of Tatto et al. (2008). Data analyses of four examinations can give insight about the structure of examinations based on the levels of the cognitive domain. In Tatto el al. (2008), the study assessed and evaluated mathematics teachers in 17 countries and the TIMMS 2007 assessment frameworks was used for the investigation of the cognitive domain of items.(Mullis et al., 2007; Tatto et al., 2008). Similarly, the framework of the cognitive domain was gathered from two studies in this study, TIMMS 2015 and TIMMS Advanced. Although the cognitive domain frameworks in these three studies were similar in content, the cognitive domain frameworks of TIMMS 2015 and TIMMS Advanced were preferred since the categories of the cognitive domain in these studies were more convenient for this study. To illustrate, the compute sub-category of knowing category referred to basic computations related to derivatives, polynomial functions, and equations while in the same sub-category of TIMMS 2007 participants were required to carry out routine algorithmic procedures such as addition, multiplication, and division with whole numbers, fractions, decimals and integers. Therefore, this framework was more convenient for the items in the examinations in Turkey and Texas which included high level of mathematics knowledge and skills beyond elementary and high school mathematics (Garden et al., 2006; Gronmo et al., n.d). This study analyzed the cognitive domain of items in examinations under three levels, namely knowing, applying, and reasoning. In addition, there were sub-categories of these levels as presented with their explanations in Table 3.4.

In the categorization of items with respect to the cognitive domain, some items could not be categorized into the components of the cognitive domain since there was not a mathematics content included in the item. Therefore, these items were classified into the uncategorized section based on the cognitive domain while presenting the results.

Table 3. 4. Framework of the cognitive domain (adapted from Garden et al., 2006; Grønmo et al., n.d.)

| | |
|---|---|
| **KNOWING** | **Knowing and understanding basic language of mathematics, essential mathematical concepts and properties, and computational tools and procedures; making extensions beyond existing knowledge; judging the validity of mathematical statements and methods.** |
| **Recall (K.1)** | Recall definitions, terminology, number properties, geometric properties, mathematical conventions, and notation. |
| **Recognize (K.2)** | Recognize entities that are mathematically equivalent (e.g., different representations of the same function or relation). |
| **Classify/Order (K.3)** | Classify numbers, expressions, quantities, and shapes by common properties. |
| **Compute (K.4)** | Carry out algorithmic procedures (e.g., determining derivatives of polynomial functions, solving a simple equation). |
| **Retrieve (K.5)** | Retrieve information from graphs, tables, texts, or other sources. |
| **Measure (K.6)** | Use measuring instruments; and choose appropriate units of measurement. |
| **APPLYING** | **Use of knowledge and conceptual understanding in *routine (familiar) problems* and learned procedures; use of skills, facts, procedures, tools, and mathematical understanding of concepts to create mathematical representations.** |
| **Determine (A.1)** | Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution. |
| **Represent / Model (A.2)** | Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship. |
| **Implement (A.3)** | Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures (For example, differentiate a polynomial function, use geometric properties to solve problems). |
| **REASONING** | **Use of logical and systematic thinking; use of intuitive and inductive reasoning; in unfamiliar and novel situations, complex contexts, multi-step problems, and situations including knowledge of different areas which needs higher level cognitive demand over routine problems; observing and making conjectures; use of logical deductions based on specific assumptions, rules, and justifying results.** |
| **Analyze (R.1)** | Investigate given information, and select the mathematical facts necessary to solve a particular problem. Determine, describe, or use relationships among numbers, expressions, quantities, shapes and objects in mathematical situations. |
| **Synthesize/ Integrate(R.2)** | Link different elements of knowledge, related representations, and various mathematical procedures to solve problems. |

Table 3.4 (continued)

| | |
|---|---|
| **Evaluate (R.3)** | Evaluate alternative problem solving strategies and solutions. |
| **Draw Conclusions (R.4)** | Make valid inferences on the basis of information and evidence. |
| **Generalize (R.5)** | Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms. |
| **Justify (R.6)** | Provide mathematical arguments to support a strategy or solution. |

In Figure 3.4, a sample item from ÖABT (9-12) was presented as an example for the knowing category of the cognitive domain. The item was categorized into the SMK category since it included the mathematics knowledge of teachers only. Since this item required the recall of the mathematics knowledge, it was categorized into the recall sub-category of knowing.

| | |
|---|---|
| **Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Level of Cognitive Domain:** | KNOWING (recall) |

**21. Cisimler ile ilgili olarak verilen**

I. Bir cismin sıfır ve kendisinden başka ideali yoktur.

II. Her cisim kendi üzerinde bir vektör uzayıdır.

III. Her tamlık bölgesi bir cisimdir.

IV. Her mertebeden cisim vardır.

**ifadelerden hangileri doğrudur?**

A) I ve II B) II ve III C) III ve IV

D) Yalnız III E) Yalnız IV

Figure 3. 4. A sample item from ÖABT (9-12) (ÖSYM, 2013b, p. 6)

In Figure 3.5, a sample item was given from ÖABT (5-8) which was categorized into the applying category of the cognitive domain. In order to solve this item, the identification and implementation of appropriate methods was required. Therefore, this item was categorized into the applying category. In addition, it should be clarified that the items which required the implementation of appropriate procedures were coded as the implement although they belong to both the determine and implement sub-categories. Also, items which required determination of procedures only were categorized as determine.

| | |
|---|---|
| **Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Level of Cognitive Domain:** | APPLYING (implement) |

**8.** $$\int_{1}^{e} \ln x \, dx$$

**integralinin değeri kaçtır?**

A) $-2$ B) 0 C) 1 D) 2 E) 3

Figure 3. 5. A sample item from ÖABT (5-8) (ÖSYM, 2013a, p. 2)

### 3.3.3. Analyses of the Examinations based on Mathematics Teacher Knowledge Model

Mathematics teacher knowledge model in mathematics teacher content knowledge examinations in Turkey and Texas were investigated within this study in order to get an image of the structure of these two examinations based on the teacher knowledge model taken from the literature. As it is presented in Table 3.5, mathematics teacher knowledge model used in this study is constructed in consideration of related studies in the literature (Aslan-Tutak & Ertas, 2013; Ball et al., 2008; Climent et al., 2013; Flores, Escudero, & Carrillo, 2013; Jakobsen, Thames, & Riberio, 2013; Hill, Sleep, Lewis, & Ball, 2007; NCTM, 2000; Shulman, 1986; Shulman 1987; Sosa, 2011, as cited in Carreno et al., 2013; Zaskis & Mamolo, 2011). Although this framework is guided by several studies in the literature, it is generally structured on the mathematics knowledge for teaching (MKT) model of Ball et al. (2008). The MKT model is powerful and convenient for this study because of being able to describe the mathematics teacher knowledge within both mathematics and educational aspects (Carillo et al., 2013).

Table 3. 5. Mathematics teacher knowledge model

| Mathematics Knowledge for Teaching (MKT) | |
|---|---|
| **Subject Matter Knowledge (SMK)** | **Pedagogical Content Knowledge (PCK)** |
| Common content knowledge (CCK) | Knowledge of Features of Learning Mathematics (KFLM) |
| Specialized mathematics knowledge (SMK) | Knowledge of Teaching Mathematics (KTM) |
| Horizon content knowledge (HCK) | Knowledge of curriculum and mathematics learning standards (KCMLS) |

Based on the model in Table 3.5, mathematics teacher knowledge consisted of two main categories: subject matter knowledge (SMK) and pedagogical content knowledge (PCK). SMK refers to the mathematics knowledge of mathematics teacher which is required for teaching and which is beyond the knowledge required for teaching. SMK was analyzed under three sub-categories: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). Firstly, CCK refers to mathematical knowledge which is used in a wide variety of settings in which knowledge of teaching and knowledge of students' learning are not included. The term, common content knowledge, does not mean that everyone has it; however, it means that adults from other professions also might have it since it is not specific to the teaching profession. It is the knowledge related to the use of the correct pronunciation and definitions of terms, using formulas, rules, properties, and theorems, doing calculations, and solving problems correctly. Teachers must have CCK in order to do the work which they assign their students. Therefore, CCK can be considered as the pure mathematics knowledge which is not specific to mathematics teachers, but must be acquired by all mathematics teachers. In this study, CCK included of five components as presented below:

- **CCK1:** The knowledge to simply calculate an answer or solve a question in a correct way
- **CCK2:** The knowledge of definitions, rules, properties, theorems, formulas, and axioms (or postulates) related to a specific topic
- **CCK3:** The knowledge to use mathematical notation to understand and solve the mathematical question (Items were coded as CCK3 only when the mathematical notation was directly asked in the item rather than when the mathematical notation was needed while solving question in the item.)
- **CCK4:** The knowledge to recognize a wrong answer or inaccurate definition (Items were coded as CCK4 when the prospective teachers are required to identify the wrong choices and choose the correct one by eliminating the wrong ones.)

- **CCK5:** The knowledge of representations of mathematical content including tables, figures, geometric shapes, materials, and models (Items were coded as CCK5 only when the item was required to construct a geometric figure or a graphic while solving the question in the item rather than understanding a geometric figure or graphic given in the item.)

Secondly, the other sub-category of SMK is specialized content knowledge (SCK) which refers to the mathematical knowledge requiring deep understanding in mathematics. It is the knowledge which is purely mathematical and specific to mathematics teaching profession which is generally not required or necessary in other contexts or professions. It is the knowledge related to responding students' "why" questions since only teachers have to explain "why" when multiplying a number with 10 results with adding a 0 at the right hand side of the number. Moreover, it is the knowledge of theoretical meaning behind procedures, critiquing and developing definitions, and history of mathematical concepts. Within this study, components of SCK were framed as:

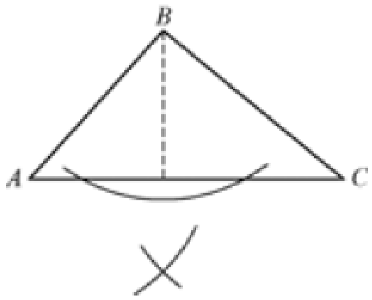
- **SCK1:** The knowledge of mathematical explanations to explain and justify mathematical procedures and unseen steps behind procedures (e.g. why you invert and multiply to divide fractions)

- **SCK2:** The knowledge of arguing, generalizing, and exploring
- **SCK3:** The knowledge to interpret the root of students' mathematical errors, especially unfamiliar ones
- **SCK4:** The knowledge for the discussion of alternate definitions
- **SCK5:** The knowledge of history of mathematical concepts

A SMK item from the preparation manual for mathematics (8-12) was presented in Figure 3.6. Preparation manuals are for the preparation of teacher candidates to TExES. Items could not be published in this study because the required permission could not provided by TEA; therefore, similar items from preparation manuals with the items in representative forms are presented in this study. In Figure 3.6, the item was classified into the SMK category since it requires knowledge of mathematics only. In addition, since mathematics knowledge behind the procedures of geometric construction was needed (SCK1), this item was categorized into the SCK.

| | |
|---|---|
| **Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Sub-domain of Mathematics Knowledge for Teaching** | Specialized Content Knowledge (SCK) |

23. **Use the figure below to answer the question that follows.**




The figure above shows the markings for constructing an altitude of a given triangle, $ABC$. This is based on constructing

A. a ray that bisects one of the angles of $\triangle ABC$.

B. the perpendicular bisector of one of the sides of $\triangle ABC$.

C. a point equidistant from the vertices of $\triangle ABC$.

D. a line through one of the vertices and perpendicular to one of the sides of $\triangle ABC$.

Figure 3. 6. A sample item from TExES preparation manual for mathematics (8-12) (TEA, 2010b, p. 52)

The last sub-category of SMK was horizon content knowledge (HCK) which is described as the mathematics knowledge which is related to having a sense of larger mathematics environment. It is the knowledge the learner might or might not have in the future, but should have if he/she becomes a mathematics teacher. Also, HCK is the knowledge which guides making judgments about mathematical importance of a topic, emphasizing key points, understanding connections among mathematical concepts, being aware of the mathematical opportunities, reasoning and making a proof to establish validity of ideas in mathematics, and being aware of the interdisciplinary applications. It is defined as the advanced mathematical knowledge on elementary mathematics in terms of concepts, connections among concepts, and major disciplinary ideas and structures. It is not only the awareness of the connections among mathematical concepts (i.e. knowing the relationship between matrix algebra and geometry), but also the global knowledge of evolution of mathematical content. For the purposes of this study, HCK was categorized into three components:

- **HCK1:** The knowledge of how the content being taught is situated in and connected to the broader disciplinary territory that students may or may not meet in the future
- **HCK2:** The knowledge of reasoning and proof and knowledge of where ideas come from and how the truth or validity is established in the discipline
- **HCK3:** The knowledge for the awareness of interdisciplinary applications

Figure 3.7 presents a sample item from ÖABT (9-12). This item was classified into SMK since only mathematics knowledge was needed. Moreover, this item was categorized as HCK since the types of proof and the process of making proofs were included in the item (HCK2).

| | |
|---|---|
| **Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Sub-domain of Mathematics Knowledge for Teaching** | Horizon Content Knowledge (HCK) |

**45.** Bir matematik öğretmeni, öğrencilerinden iki tek sayının toplamının çift sayı olduğunu ispatlamalarını istemiştir. Üç öğrencinin yapmış olduğu ispat aşağıda verilmiştir.

Ali

$\forall\, n \in Z$ olmak üzere,
$n+1$ tek sayıdır.
$n+3$ tek sayıdır.
$n+1+n+3=2n+4$
$2(n+2)$ çift sayıdır.

Burcu

Herhangi iki tek sayı alalım ve toplamlarını inceleyelim.
$1+3=4$ çift sayıdır.
$3+5=8$ çift sayıdır.
$5+7=12$ çift sayıdır.
Buna göre, iki tek sayının toplamı çifttir.

Ceyda

$\forall\, k,n \in Z$ olmak üzere
$2k+1$ tek sayıdır.
$2n+1$ tek sayıdır.
$2k+1+2n+1=2(k+n+1)$
$2(k+n+1)$ bir çift sayıdır.

**Buna göre, ispatı doğru yapan öğrenci ve kullandığı yöntem aşağıdakilerin hangisinde birlikte verilmiştir?**

| | Öğrenci | İspat Yöntemi |
|---|---|---|
| A) | Ali | Tümevarım |
| B) | Burcu | Olmayana ergi |
| C) | Ceyda | Doğrudan ispat |
| D) | Ali ve Burcu | Tümevarım |
| E) | Ali ve Ceyda | Doğrudan ispat |

Figure 3. 7. A sample item of ÖABT (9-12) (ÖSYM, 2013b, p. 13)

The other category of teacher knowledge was pedagogical content knowledge (PCK). In mathematics teacher knowledge model, PCK of a mathematics teacher is explained as the knowledge and ability of a mathematics teacher in relation to the teaching of mathematics and learning of mathematics of students. PCK is also investigated under three sub-categories: knowledge of features of learning mathematics (KFLM), knowledge of teaching mathematics (KTM), and knowledge of curriculum and mathematics learning standards (KCMLS). Firstly, KFLM stands for the knowledge which includes the combination of mathematics knowledge and mathematical conceptions of students. It includes common conceptions, misconceptions, and common errors of students related to a specific topic, such as common misinterpretations of equivalence sign. Also, it includes common difficulties and requirements of students in mathematics education. It is the pedagogical knowledge which is totally related to students' learning of mathematics. The components of KFLM can be presented as:

- **KFLM1:** The knowledge to decide which of several errors are most likely to be made by students
- **KFLM2:** The knowledge to anticipate what students are likely to think
- **KFLM3:** The knowledge of students' learning difficulties, misconceptions, and incomplete thoughts related to a concept
- **KFLM4:** The knowledge of theories and models of how students learn mathematics (Van Hiele geometric thinking levels etc.) and the knowledge of strategies while students do mathematics
- **KFLM5:** The knowledge to anticipate what students will find interesting and motivating

A sample item was presented in Figure 3.8. Since the item was related to the learning of students, it was classified into PCK. In addition, the items was categorized into KFLM since it required knowledge related to incomplete thinking of students or misconceptions of students (KFLM3). This item was not categorized into the knowledge of teaching mathematics (KTM) since prospective teachers were not possessed an answer related to methods, strategies, or procedures of teaching in a specific case in this item. Rather, the knowledge of students' misconceptions and incomplete thinking were required.

| | |
|---|---|
| **Knowledge Domain:** | Pedagogical Content Knowledge |
| **Sub-domain of Mathematics Knowledge for Teaching** | Knowledge of Features of Mathematics (KFLM3) |

**45.** $\sqrt{4^2+3^2}=7$ **olduğunu söyleyen bir öğrencinin bu cevabı vermesinin nedeni aşağıdaki düşüncelerden hangisi olamaz?**

A) $a^2+b^2$ ile $(a+b)^2$ birbirine eşittir.

B) $a^2+b^2$ nin karekökü $(a-b)(a+b)$ 'ye eşittir.

C) $a^2+b^2$ nin karekökü $a+b$ 'ye eşittir.

D) $a^2+b^2$ ile $(a-b)^2+2ab$ birbirine eşittir.

E) $\sqrt{a+b}$ ile $\sqrt{a}+\sqrt{b}$ birbirine eşittir.

Figure 3. 8. A sample item of ÖABT (5-8) (ÖSYM, 2013a, p. 13)

The second sub-category of PCK was knowledge of teaching mathematics (KTM) which referred to the knowledge including the combination of mathematics knowledge and knowledge of methods for teaching. It is the knowledge which includes teachers' knowledge of the evaluation and use of representations for teaching of mathematical concepts, methods and procedures which are used in mathematics education, and making appropriate instructional decisions while teaching mathematics.

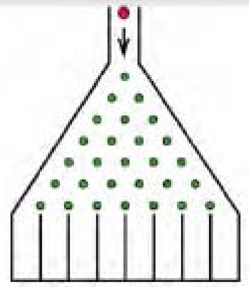
- **KTM1:** The knowledge to choose which examples to start with and which examples to use to take students deeper into the context and to correct students' misconceptions.
- **KTM2:** The knowledge to evaluate the instructional advantages or disadvantages of representations while teaching a specific content
- **KTM3:** The knowledge to choose and use of an appropriate representation or certain material for learning a concept or mathematical procedure
- **KTM4:** The knowledge to take complex series of decisions in order to perform the task of teaching, make the choice of an appropriate textbook, select a representation for a specific concept, or get a specific resource material related to a particular topic

- **KTM5:** The knowledge to make a decision about appropriate methods and procedures and use them for particular cases of teaching mathematics

In Figure 3.9, a sample item from ÖABT (9-12) was presented. Since the knowledge for teaching mathematics was needed to answer this item, it was classified into PCK. In addition, the knowledge to evaluate the instructional advantages and/or disadvantages of representations while teaching a specific content (KTM2) and the knowledge to choose and use of an appropriate representation or certain material for learning a concept or mathematical procedure (KTM3) were required in this item. Therefore, the item was coded as KTM2 and KTM3.

| | |
|---|---|
| **Knowledge Domain:** | Pedagogical Content Knowledge |
| **Sub-domain of Mathematics Knowledge for Teaching** | Knowledge of Teaching Mathematics (KTM2 & KTM3) |

**49.**



Şekildeki olasılık makinesinde üstten atılan bir topun engellere çarptıktan sonra engelin sağından gitme olasılığı ile solundan gitme olasılığı birbirine eşittir.

**Bu makineyi matematik dersinde kullanmak isteyen bir öğretmenin amacı aşağıdakilerden hangisi olamaz?**

A) Topun verilen bir yolu takip etme olasılığının kaç olduğunu göstermek

B) Eş olasılı örneklem uzayında gerçekleşen olayların olasılığını göstermek

C) Deneysel olasılık ile teorik olasılık değeri arasındaki ilişkiyi fark ettirmek

D) Olasılık değerleri ile Pascal üçgeni arasındaki ilişkiyi fark ettirmek

Figure 3. 9. A sample item of ÖABT (9-12) (ÖSYM, 2013b, p. 15)

The third sub-category of pedagogical content knowledge was knowledge of curriculum and mathematics learning standards (KCMLS). KCMLS was the knowledge including the knowledge of mathematics curriculum, mathematics learning standards, types of assessment techniques, and appropriate assessment techniques in mathematics education. The knowledge of mathematics curriculum is also means the specifications in the curriculum and connections among mathematical topics within the mathematics curriculum. That is, it is not the knowledge of the relationship of mathematical topics in terms of pure mathematics; on the contrary, it is the knowledge of relations of mathematics topics in the curriculum. The components of KCMLS are presented as:

- **KCMLS1:** The knowledge of curricular specifications and connections among current topics and previous and forthcoming ones, the progression from one year to the next
- **KCMLS2:** The knowledge of learning objectives and standards
- **KCMLS3:** The knowledge of the curriculum that students are learning in other subject areas
- **KCMLS4:** The knowledge of mathematical process standards, such as problem solving, reasoning and proof, connections, communication, and mathematical representations and modeling
- **KCMLS5:** The knowledge of types of assessment techniques
- **KCMLS6:** The knowledge of appropriate assessment techniques to guide instruction and evaluate the progress of learners

In Figure 3.10, a sample item was presented from ÖABT (9-12). Since the item was included the knowledge related to teaching of mathematics, it was classified into PCK. Moreover, the item was categorized into knowledge of curriculum and mathematics learning standards since it included the knowledge of curriculum in mathematics education (KCMLS2).

| | |
|---|---|
| **Knowledge Domain:** | Pedagogical Content Knowledge |
| **Sub-domain of Mathematics Knowledge for Teaching** | Knowledge of Curriculum and Mathematics Learning Standards (KCMLS2) |

**43.**

I. $f: R \rightarrow R$, $f(x) = -x^2 + 4x + 1$ fonksiyonunun alacağı en büyük değeri bulunuz.

II. $f: R \rightarrow R$, $f(x) = 2x + 1$ fonksiyonu birebir ve örten midir?

III. $f: R \rightarrow R^+, f(x) = 3^x$ fonksiyonunun grafiğini çiziniz.

**Yukarıdaki soruları çözmek için gerekli kazanımlar uygulanmakta olan Ortaöğretim Matematik Dersi (9, 10, 11 ve 12. Sınıflar) Öğretim Programı'nda <u>ilk kez</u> kaçıncı sınıf düzeyinde ele alınmaktadır?**

| | I. | II. | III. |
|---|---|---|---|
| A) | 12. | 10. | 12. |
| B) | 10. | 9. | 12. |
| C) | 10. | 9. | 11. |
| D) | 12. | 9. | 11. |
| E) | 10. | 10. | 12. |

Figure 3. 10. A sample item from ÖABT (9-12) (ÖSYM, 2013b, p. 12)

### 3.4. Reliability

In a qualitative study, the agreement between two or more coders is one of the methods for checking reliability (Fraenkel & Wallen, 2005). In this study, coding of items with respect to the cognitive domain and mathematics teacher knowledge model was conducted by three researchers who were graduate students in mathematics education. For the categorization of the items, researchers were provided a detailed explanation of frameworks before they began the coding of the items. Then, three researchers came together to analyze some of the items which were selected randomly from each examination. In this process, categorizers discussed on the categorization of items in relation to explanations in the frameworks when they had not an agreement on categorizations or when they had wanted to make changes on these categorizations. Then, categorizers coded some of the items individually and came together again in order to compare their categorizations. When the researchers agreed on the categorization of the items, they began the categorizations individually. After coding of items had been completed, researchers came together again and discussed for the items which were coded differently by one or more categorizer(s). If a disagreement was occurred, then researchers discussed this item until they reached an agreement related to the coding of this item. Discussions on the categorization of the items had proceeded until all the items in the examinations were coded the same by the three categorizers.

Cognitive domain and mathematics teacher knowledge variables were coded by the three categorizers for the items whereas curricular levels and fields of mathematics variables were coded by the researcher individually. Curricular levels of the items are classified with respect to the elementary and secondary mathematics curricula of Turkey and Texas. While categorizing items, no interpretations were required since classification of topics was identified with respect to the related curricula. For this reason, items were not coded by the other categorizers. In addition, the framework for fields of mathematics was also prepared with respect to the handbooks of mathematics (Pearson, 1990; Rainbolt & Gallian, 2010). Therefore, no interpretation was needed while coding the items for the fields of mathematics. For this reason, items were categorized with respect to the fields of mathematics by the researcher only.

# CHAPTER 4

# RESULTS

The first purpose of this study was to investigate the structure of the mathematics teacher content knowledge examinations in Turkey and Texas based on curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics teachers. The second purpose of this study was to investigate how similar and different the mathematics teacher knowledge examinations of Turkey and Texas were with respect to curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics teachers. In this study, following questions were researched:

What are the structures of teacher content knowledge examinations of Turkey and Texas in terms of curricular levels, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching?

What are the similarities and differences between teacher content knowledge examinations of Turkey and Texas in terms of curricular levels, fields of mathematics, cognitive domain, and domains of mathematics knowledge for teaching?

In this chapter, results are presented based on four main sections, namely curricular levels, fields of mathematics, cognitive domain, and knowledge for teaching mathematics. Analysis of items in examinations was conducted with respect to the related frameworks of four variables. For each variable, distributions of items in ÖABT (9-12), ÖABT (9-12), TExES (4-8), and TExES (8-12) were given respectively. In each section, results were reported with tables consisting of frequencies and proportions of items in the examinations on the basis of related categories and sub-categories.

### 4.1. The Structure of Mathematics Teacher Content Knowledge Examinations in terms of Curricular levels and Fields of Mathematics

Curricular levels were one of the variables in which items were analyzed. In terms of curricular levels, items were classified into three sub-categories, elementary, high school, and advanced. For the items in both ÖABTs and TExESs, elementary level refers to mathematics content that is typically thought at elementary school ($4^{th}$-$8^{th}$ grade), high school level refers to mathematics content that is typically thought at high school ($9^{th}$-$12^{th}$ grade) and advance level refers to mathematics content that is typically thought beyond the highest level of high school. Analyses of items were reported in this section below the titles of four examinations, respectively.

#### 4.1.1. The Distribution of the Items for Elementary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

Table 4.1 presents the distribution of the SMK and PCK items of ÖABT (5-8) and TExES (4-8) with respect to the curricular levels. It can be inferred from Table 4.1 that mathematics content that is typically thought at the elementary school ($4^{th}$-$8^{th}$ grade) was not included in the SMK items in ÖABT (5-8), whereas more than quarter of the items in TExES (4-8) were categorized into the elementary level. In Table 4.1, most of the SMK items in ÖABT (5-8) were classified into high school and advance levels. That is, most of the SMK items in ÖABT (5-8) included the mathematics content which is typically thought at high school ($9^{th}$-$12^{th}$ grade) and beyond the highest level of high school, whereas items in TExES (4-8) were proportionally distributed across all three categories of curricular levels. Moreover, it can be inferred from the Table 4.1 that, in both ÖABT (5-8) and TExES (4-8), PCK items were mostly categorized into the levels that the prospective teacher will teach. In addition, 4 of the PCK items were labeled as uncategorized since these PCK items did not include mathematics content. Therefore, they could not be categorized with respect to curricular levels regarding the mathematics curricula of Turkey and Texas.

Table 4. 1. Frequency of SMK items with respect to curricular levels in ÖABT (5-8) and TExES (4-8)

| Curricular levels | ÖABT (5-8) | | TExES (4-8) | |
|---|---|---|---|---|
| | SMK f(%) | PCK f(%) | SMK f(%) | PCK f(%) |
| Elementary | 0 (0) | 7 (70) | 26 (37.1) | 6 (60) |
| High school | 26 (65) | 2 (20) | 33 (47.1) | 1 (10) |
| Advance | 14 (35) | 0 (0) | 11 (15.7) | 0 (0) |
| Uncategorized | 0 (0) | 1 (10) | 0 (0) | 3 (30) |
| Total | 40 (100) | 10 (100) | 70 (100) | 10 (100) |

### 4.1.2. The Distribution of the Items with respect to Curricular levels and Fields of Mathematics for Elementary Mathematics Teacher Content Knowledge Examination in Turkey and Texas

Table 4.2 presents the analysis of SMK items with respect to the curricular levels and fields of mathematics in ÖABT (5-8) and TExES (4-8). According to the Table 4.2, the SMK items were mostly classified into calculus and geometry fields in ÖABT (5-8) while SMK items in TExES (4-8) were mostly categorized into algebra field. Most of the algebra items in TExES (4-8) were categorized into elementary school and high school levels while the SMK items in ÖABT (5-8) were mostly categorized into advanced level. There were fewer items categorized into the applied mathematics field when compared with other three fields in ÖABT (5-8). Almost all of the items classified into calculus and geometry sub-categories were at the high school level and items which were classified into applied mathematics were dominantly at the advanced level. Also, applied mathematics items were mostly categorized into advanced level in ÖABT (5-8) while they were mostly categorized into elementary and advanced level in TExES (4-8). There were no SMK items in ÖABT (5-8) from the elementary level in the algebra field. That is, prospective teachers were not questioned from the level they would teach in the future from algebra and calculus fields in ÖABT (5-8). In addition, especially items in ÖABT (5-8) were generally prepared with mathematics content which is beyond the highest grade the prospective teacher would teach for each field of mathematics.

Table 4. 2. Frequency of SMK items in ÖABT (5-8) and TExES (4-8) with respect to curricular levels and fields of mathematics

| | Algebra f (%) | | Calculus f (%) | | Applied Mathematics f (%) | | Geometry f (%) | |
|---|---|---|---|---|---|---|---|---|
| | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES |
| Elementary | 0 (0) | 15 (21.4) | 0 (0) | 0 (0) | 0 (0) | 5 (7.1) | 0 (0) | 6 (8.6) |
| High school | 4 (10) | 11 (15.7) | 10 (25) | 12 (17.1) | 2 (5) | 2 (2.9) | 11 (27.5) | 8 (11.4) |
| Advanced | 7 (17.5) | 1 (1.4) | 2 (5) | 5 (7.1) | 5 (12.5) | 5 (7.1) | 0 (0) | 0 (0) |
| Total | 11 (27.5) | 27 (38.5) | 12 (30) | 17 (24.2) | 7 (17.5) | 12 (17.1) | 11 (27.5) | 14 (19.6) |

Table 4. 3. Frequency of PCK items with respect to curricular levels and fields of mathematics in ÖABT (5-8) and TExES (4-8)

| | Algebra f (%) | | Calculus f (%) | | Applied Mathematics f (%) | | Geometry f (%) | |
|---|---|---|---|---|---|---|---|---|
| | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES |
| Elementary | 5 (50) | 5 (50) | 0 (0) | 0 (0) | 1 (10) | 0 (0) | 1 (10) | 1 (10) |
| High school | 0 (0) | 0 (0) | 1 (10) | 1 (10) | 0 (0) | 0 (0) | 1 (10) | 0 (0) |
| Total | 5 (50) | 5 (50) | 1 (10) | 1 (10) | 1 (10) | 0 (0) | 2 (20) | 1 (10) |

*Since PCK items in ÖABT (5-8) and TExES (4-8) were comprised of the mathematics knowledge from the elementary and high school, the advance category did not presented in the table.

Table 4.3 presents the distribution of PCK items in ÖABT (5-8) and TExES (4-8) based on curricular levels and fields of mathematics. It can be concluded that most of the PCK items were categorized into the elementary level and algebra field. It can be inferred that nearly all of the items includes pedagogical content knowledge from the grades the prospective teacher will teach, namely from the elementary school level in both ÖABT (5-8) and TExES (4-8).

### 4.1.3. The Distribution of the Items for Secondary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

Table 4.4 presents the distribution of the SMK and PCK items with respect to curricular levels. Most of the SMK items in ÖABT (9-12) were almost equally distributed on the high school and the advanced levels while SMK items in TExES (4-8) were mostly categorized into the high school level. Also, it is remarkable that quarter of all SMK items the in TExES (8-12) were from the elementary level, whereas there was no test item from the elementary level in the ÖABT (9-12). In Table 4.4, PCK items in TExES (8-12) were distributed equally through elementary and high school levels while most of the PCK items in ÖABT (9-12) were categorized into the high school level, namely at the level the prospective teacher would teach. Lastly, 1 of the PCK items was labeled as uncategorized. The reason was the SMK item was not categorized in relation to its mathematics content since only SMK items including mathematics knowledge could be categorized based on curricular levels.

Table 4. 4. Frequency of SMK items with respect to curricular levels in ÖABT (9-12) and TExES (8-12)

| Curricular levels | ÖABT (9-12) | | TExES (8-12) | |
|---|---|---|---|---|
| | SMK<br>f (%) | PCK<br>f (%) | SMK<br>f (%) | PCK<br>f (%) |
| Elementary | 0 (0) | 1 (12.5) | 18 (24.7) | 3 (42.9) |
| High school | 22 (52.4) | 7 (87.5) | 40 (54.8) | 3 (42.9) |
| Advanced | 20 (47.6) | 0 (0) | 15 (20.6) | 0 (0) |
| Uncategorized | 0 (0) | 0 (0) | 0 (0) | 1 (14.2) |
| Total | 42 (100) | 8 (100) | 73 (100) | 7 (100) |

### 4.1.4. The Distribution of the Items with respect to Curricular levels and Fields of Mathematics for Secondary Mathematics Teacher Content Knowledge Examination in Turkey and Texas

Table 4.5 presents the distribution of SMK items in ÖABT (9-12) and TExES (8-12) regarding the curricular levels and the fields of mathematics. In ÖABT (9-12) and in TExES (8-12), SMK items were distributed proportionally across fields of mathematics. However, the SMK items were mostly categorized into the algebra and geometry fields. According to Table 4.5, most of the calculus and geometry items were from high school level in ÖABT (9-12) and in TExES (8-12); in other words, from the grades the prospective teachers would teach. For algebra and applied mathematics categories, the SMK items were dominantly classified into advanced level in ÖABT (9-12) while they were proportionally distributed across the three curricular levels in TExES (8-12). That is, the prospective secondary mathematics teachers in Turkey were dominantly responsible for applied mathematics at the advanced level. In addition, since no items were categorized into the elementary level in ÖABT (9-12); it can be inferred that prospective elementary mathematics teachers were not questioned by the SMK items from the level they would teach in the future in Turkey.

Table 4.6 presents the distribution of PCK items based on curricular levels and fields of mathematics. In Table 4.6, PCK items were distributed across the four fields of mathematics proportionally in ÖABT (9-12) and in TExES (8-12). In addition, most of the items were from the algebra and calculus fields in ÖABT (9-12).

Table 4. 5. Frequency of SMK items with respect to curricular levels and fields of mathematics in ÖABT (9-12) and TExES (8-12)

| | Algebra f (%) | | Calculus f (%) | | Applied Mathematics f (%) | | Geometry f (%) | |
|---|---|---|---|---|---|---|---|---|
| | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES |
| Elementary | 0 (0) | 6 (8.2) | 0 (0) | 0 (0) | 0 (0) | 6 (8.4) | 0 (0) | 6 (8.4) |
| High school | 3 (7.1) | 8 (11.2) | 9 (21.6) | 11 (15.0) | 1 (2.4) | 4 (5.6) | 9 (21.6) | 18 (25.2) |
| Advanced | 6 (14.3) | 6 (8.2) | 4 (9.6) | 5 (7.0) | 8 19.2) | 4 (5.6) | 2 (4.8) | 0 (0) |
| Total | 9 (21.4) | 20 (27.6) | 13 (31.2) | 16 (22.0) | 9 (21.6) | 16 (19.6) | 11 (26.4) | 24 (33.6) |

Table 4. 6. Frequency of PCK items with respect to curricular levels and fields of mathematics in ÖABT (9-12) and TExES (8-12)

| | Algebra f (%) | | Calculus f (%) | | Applied Mathematics f (%) | | Geometry f (%) | |
|---|---|---|---|---|---|---|---|---|
| | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES | ÖABT | TExES |
| Elementary | 1 (12.5) | 0 (0) | 0 (0) | 0 (0) | 0 (0) | 1 (14.3) | 0 (0) | 1 (14.3) |
| High school | 1 (12.5) | 2 (28.6) | 3 (37.5) | 0 (0) | 1 (12.5) | 0 (0) | 1 (12.5) | 1 (14.3) |
| Total | 2 (25) | 2 (28.6) | 3 (37.5) | 0 (0) | 1 (12.5) | 1 (14.3) | 1 (12.5) | 2 (28.6) |

*Since PCK items in ÖABT (9-12) and TExES (8-12) were comprised of elementary and high school level mathematics knowledge, the advanced sub-category did not presented in the table.

To sum up, some conclusions can be drawn related to the distribution of items regarding curricular levels and fields of mathematics. It can be yielded that SMK items were completely belong to high school and advance levels in ÖABT (5-8) and ÖABT (9-12). That is, there was no SMK item from elementary school level in the ÖABTs in Turkey. However, the results indicated that SMK items in TExES (4-8) and TExES (8-12) were distributed across the curricular levels although they were mostly categorized into the high school level. Results presented that there was not an explicit difference in terms of the PCK items among four examinations with respect to fields of mathematics. In the examinations, PCK items mostly included the mathematics knowledge from the grades the future teacher will teach. However, it is remarkable that half of the PCK items in TExES (8-12) were categorized into the elementary level and the remaining ones were categorized into the high school level.

### 4.2. The Structure of Mathematics Teacher Content Knowledge Examinations in terms of the Cognitive Domain

In this section, the results of the analyses of SMK items were presented based on the cognitive domain. Mathematics teacher content knowledge examinations were analyzed with respect to the categories and sub-categories of the cognitive domain framework (Garden et al., 2006; Grønmo et al., 2014).

#### 4.2.1. The Distribution of Elementary and Secondary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

In Table 4.7, the distribution of SMK items in ÖABT (5-8) and TExES (4-8) were presented. As Table 4.7 presented, SMK items in ÖABT (5-8) were mostly categorized into the applying category. That is, most of the SMK items included the routine (familiar) questions which require the identification and implementation of known procedures and the use of representations in routine mathematical situations while the SMK items in TExES (4-8) were mostly classified into knowing category. Therefore, most of the SMK items required the knowledge of mathematical rules, properties, concepts, and definitions or use of basic computations in mathematics. Moreover, there was no SMK item coded as reasoning in both examinations. Therefore, there were no items including the logical thinking and reasoning in unfamiliar situations or complex contexts which require a higher level cognitive demand over routine problems. Rather, items included recognition of mathematical rules, procedures, and concepts or knowledge of determination or implementation of known procedures in routine mathematical questions.

Table 4. 7. The frequency of SMK items with respect to the categories of the cognitive domain in ÖABT (5-8) and TExES (4-8)

| Cognitive Domain | ÖABT (5-8) f (%) | TExES (4-8) f (%) |
|---|---|---|
| Knowing | 7 (17.5) | 48 (68.6) |
| Applying | 33 (82.5) | 22 (31.4) |
| Reasoning | 0 (0) | 0 (0) |
| Total | 40 (100) | 70 (100) |

According to the results, the SMK items in ÖABT (5-8) which were categorized into the knowing category were mostly classified as the recall (4 items among 7 SMK items) and the compute (2 items of 7 SMK terms) sub-categories. That is, the SMK items categorized as the knowing category were mostly related to knowing mathematical concepts, procedures, and rules or related to carrying out basic computations in mathematics. Items which were classified into the applying category were mostly coded as the represent (11 items among 33 SMK items) and the implement (all of the items coded as applying) sub-categories. There were no items classified into the determine sub-category since the SMK items including the implementation of a mathematical procedure included the identification of the mathematical procedure comprised at the same time. For this reason, most of the SMK items were classified into the implement sub-category although these items included the characteristics of the determine sub-category.

Results indicated that most of the SMK items coded as the knowing were classified into the compute (37 items among 48 SMK items) sub-category in TExES (4-8). Moreover, items coded as the applying were mostly classified into the implement (16 items among 22 SMK items) sub-category. In ÖABT (5-8), items mostly included knowledge for implementing appropriate methods and solutions in familiar problems and situations whereas in TExES (4-8) most of the items included recall and use of known procedures in mathematics. In addition, there were no items classified into the reasoning category in both examinations.

### 4.2.2. The Distribution of Elementary and Secondary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

The distribution of the SMK items in ÖABT (9-12) and TExES (8-12) were presented in terms of the cognitive domain in Table 4.8. According to results, most of the SMK items in ÖABT (9-12) and TExES (8-12) were classified into the applying category and the SMK remaining items were categorized into the knowing category. More specifically, the SMK items in TExES (8-12) which were coded as the knowing were classified into the recall (9 items among 26 items), the compute (10 items among 26 items), and the retrieve (6 items among 26 items) sub-categories. Also, the SMK items in the applying category were dominantly categorized into the implement (37 items among 46 items) sub-category. It could be inferred that most of the SMK items in ÖABT (9-12) which were coded as the knowing were classified into the recall (8 items among 11 items) sub-category. Moreover, most of the SMK items coded as the applying were mostly classified into the implement (30 items among 31 items) sub-category. In both examinations, there were no items categorized into the reasoning category. In addition, 1 of the SMK item was coded as uncategorized since it did not include a mathematical content; therefore, it cannot be categorized into one of the categories of the cognitive domain

Table 4. 8. The distribution of SMK items with respect to levels of the cognitive domain in ÖABT (9-12) and TExES (8-12)

| Cognitive Domain | ÖABT (9-12) f (%) | TExES (8-12) f (%) |
|---|---|---|
| Knowing | 11 (26.2) | 26 (35.6) |
| Applying | 31 (73.8) | 46 (63.0) |
| Reasoning | 0 (0) | 0 (0) |
| Uncategorized | 0 (0) | 1 (1.4) |
| Total | 42 (100) | 73 (100) |

In conclusion, it can be inferred that most of the items were classified as applying for all four examinations, except for TExES (8-12). Therefore, majority of the SMK items included knowledge of identification and implementation of mathematical procedures in familiar situations and contexts in the examinations.

### 4.3. The Structure of Mathematics Teacher Content Knowledge Examinations in terms of the Mathematics Knowledge of Teachers

In this study, mathematics knowledge for teaching (MKT) was one of the variables in which items were analyzed. MKT was divided into two categories, subject matter knowledge (SMK) and pedagogical content knowledge (PCK). SMK was divided into three sub-domains: common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). Moreover, pedagogical content knowledge (PCK) was also divided into three sub-domains: knowledge of futures of learning mathematics (KFLM), knowledge of teaching mathematics (KTM), and knowledge of curriculum and mathematics learning standards (KCMLS).

Items were initially categorized based on the domains of mathematics knowledge for teaching, subject matter knowledge and pedagogical content knowledge. Then, they were categorized with respect to sub-domains of knowledge for teaching mathematics. The results of the analyses of items were presented for the mathematics teacher content knowledge examinations in Turkey and Texas in the following topics.

#### 4.3.1. The Distribution of Items for Elementary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

Table 4.9 presented the distribution of items in ÖABT (5-8) and TExES (4-8) with respect to the domains of knowledge for teaching mathematics. According to the results, all of the SMK items were coded as the CCK in ÖABT (5-8) and TExES (4-8). There were no items categorized into the SCK and HCK sub-categories in both examinations. That is, all of the items consisted of the mathematics knowledge that was not unique to mathematics teachers; in other words, the mathematics knowledge which can also be known by adults from other professions. In addition, there were more PCK items in ÖABT (5-8) in comparison to the proportion of the PCK items in TExES (4-8). Moreover, PCK items were distributed proportionally through three sub-categories of the PCK in both examinations. However, most of the PCK items were categorized into the KFLM sub-category in ÖABT (5-8) and most of the PCK items in TExES (4-8) were categorized into the KTM sub-category.

Table 4. 9. The frequency of the items with respect to the domains of knowledge for teaching mathematics in ÖABT (5-8) and TExES (4-8)

| Mathematics Knowledge of Teacher (MKT) | ÖABT (5-8) f (%) | TExES (4-8) f (%) |
|---|---|---|
| Subject matter knowledge (SMK) | | |
| Common content knowledge (CCK) | 40 (80) | 70 (87.5) |
| Specialized content knowledge (SCK) | 0 (0) | 0 (0) |
| Horizon content knowledge (HCK) | 0 (0) | 0 (0) |
| Total SMK items | 40 (80) | 70 (87.5) |
| Pedagogical content knowledge (PCK) | | |
| Knowledge of features of learning mathematics (KFLM) | 7 (14) | 4 (5) |
| Knowledge of teaching mathematics (KTM) | 3 (6) | 7 (5) |
| Knowledge of curriculum and mathematics learning standards (KCMLS) | 3 (6) | 3 (3.75) |
| Total PCK items | 10 (20)* | 10 (12.5)* |

*Since an item could be classified into more than one sub-category, total number of the PCK items is seen as if it should be more than 10.

The results showed that the SMK items were dominantly included CCK1 (the knowledge of simply calculating an answer or solving a question) and CCK2 (the knowledge of definitions, rules, properties, theorems, formulas, and axioms related to a specific topic). The results presented that half of the PCK items included KFLM2 (anticipating how students think) and KFLM3 (being aware of students' difficulties on mathematical concepts) in both ÖABT (5-8) and TExES (4-8). It can be concluded that there were nearly no items coded as KFLM1 (the knowledge to decide which of several errors are most likely made by students) and KFLM5 (the knowledge to anticipate what students will find interesting and motivating) in the examinations. Also, in ÖABT (5-8), the items including the KTM were mostly coded as KTM1 (deciding on appropriate methods, examples for teaching) and KTM5 (procedures for particular cases of teaching mathematics) while the items in TExES (4-8) were mostly coded as KTM3 (use of an appropriate representation or certain material while teaching a mathematical concept) and KTM5 (the knowledge to use appropriate methods and procedures for particular cases of mathematics education).

Lastly, 1 of the PCK items were coded as KCMLS1 (the knowledge of curricular specifications and connections among topics, the progression from one year to the next ones) and 1 of them was coded as KCMLS2 (the knowledge of learning objectives and standards) while there were no items coded as including KCMLS1 and KCMLS2 in TExES (4-8). There were no PCK items which were consisting of KCMLS3 (the knowledge of curriculums of other subject areas) in both ÖABT (5-8) and TExES (4-8). In TExES (4-8), there were 3 items including the knowledge of assessment techniques while there were no items in ÖABT (5-8) including the knowledge of assessment techniques.

### 4.3.2. The Distribution of Items for Secondary Mathematics Teacher Content Knowledge Examinations in Turkey and Texas

Table 4.10 presented the results of analyses in ÖABT (9-12) and TExES (8-12) with respect to domains of knowledge for teaching mathematics. In Table 4.10, most of the SMK items were coded as CCK in both ÖABT (9-12) and TExES (8-12). In addition, there were more PCK items in ÖABT (9-12) in comparison to the proportion of the PCK items in TExES (8-12). Also, PCK items were distributed across the sub-categories of PCK proportionally in both examinations.

Table 4. 10. Frequency of items with respect to mathematics knowledge of teachers ÖABT (9-12) and TExES (8-12)

| Mathematics Knowledge of Teacher (MKT) | ÖABT (9-12) f (%) | TExES (8-12) f (%) |
|---|---|---|
| Subject matter knowledge (SMK) | | |
| Common content knowledge (CCK) | 41 (82) | 69 (86.3) |
| Specialized content knowledge (SCK) | 0 (0) | 4 (5) |
| Horizon content knowledge (HCK) | 1 (2) | 5 (6.3) |
| Total SMK items | 42 (84) | 73 (91.3)** |
| Pedagogical Content Knowledge (PCK) | | |
| Knowledge of features of learning mathematics (KFLM) | 5 (10) | 3 (3.8) |
| Knowledge of teaching mathematics (KTM) | 3 (6) | 2 (2.5) |
| Knowledge of curriculum and mathematics learning standards (KCMLS) | 3 (6) | 3 (3.8) |
| Total PCK items | 8 (16)* | 7 (8.8)*** |

*Since an item could be classified into more than one sub-category, total number of PCK items is seen as if it should be more than 8.

**Since an item could be classified into more than one sub-category, total number of SMK items was seen as if it should be more than 73.

***Since an item could be classified into more than one sub-category, total number of PCK items is seen as if it should be more than 7.

According to the results, CCK items in both ÖABT (9-12) and TExES (8-12) mostly included CCK1 (the knowledge to simply calculate an answer or solve a question) and CCK2 (the knowledge of definitions, rules, properties, theorems, formulas, and axioms related to a specific topic) in similar with the results of the analyses in ÖABT (5-8) and TExES (4-8). That is, most of the SMK items included common knowledge of mathematics to define mathematical concepts and to use mathematical procedures. There were fewer items requiring CCK4 (the knowledge to recognize a wrong answer) and CCK5 (the knowledge of representations of mathematical concepts) in both examinations.

In ÖABT (9-12), there were no test items categorized into the SCK sub-category and there was only 1 item categorized into the HCK sub-category. That is, almost all of the items in ÖABT (9-12) consisted of mathematics content which is not unique to mathematics teachers; in other words, mathematics content which can be possessed by adults from other professions. However, in TExES (8-12), there were 4 items categorized into the SCK sub-category. In more detail, 2 items were coded as SCK1 (the knowledge of mathematical explanations to clarify and justify mathematical procedures and unseen steps behind procedures) and 2 items were coded as SCK2 (the knowledge of arguing, generalizing, and exploring). Also, there were 5 items categorized into the HCK sub-category in TExES (8-12). More specifically, 1 item was coded as HCK1 (the knowledge of relationship of mathematical content in its broader disciplinary territory) and 4 items were coded as HCK3 (the knowledge for the awareness of interdisciplinary applications of mathematics). There was no item related to HCK2 (the knowledge of reasoning and proof and knowledge of where ideas come from and how truth or validity is established in mathematics) in both examinations.

The results indicated that 4 of the PCK items classified into both KFLM2 and KFLM3 (anticipating how students think and being aware of students' difficulties on mathematical concepts) in ÖABT (9-12). Also, 2 items were categorized as KFLM4 (theories and models on how students learn mathematics). Similarly, 3 of the PCK items in TExES (8-12) were coded as KFLM3 (the knowledge of learning difficulties and misconceptions of students). It can be concluded that there were no test items based on KFLM1 (the knowledge to decide which of several errors are most likely made by students) and KFLM5 (the knowledge to anticipate what students will find interesting and motivating) in both ÖABT (9-12) and TExES (8-12). The second sub-category of PCK was Knowledge of Teaching Mathematics (KTM). There were 2 items coded as KTM1 (deciding on appropriate methods, examples, and procedures for particular cases of teaching mathematics) and 1 item was coded as KTM3 (the use of an appropriate representation or certain material while learning a mathematical concept) in ÖABT (9-12) whereas 2 of the PCK items which were categorized into the KTM sub-category were coded as KTM3, the knowledge to choose and use of an appropriate representation or material for learning of a mathematical concept, in TExES (8-12).

Lastly, 1 of the PCK items consisted of KCMLS1 (the knowledge of curricular specifications and connections among topics, the progression from one year to the next) and 1 of the PCK items included KCMLS2 (the knowledge of learning objectives and standards) in ÖABT (9-12). Moreover, 1 item was coded as KCMLS4 (the knowledge of mathematical process standards, problem solving, reasoning and proof, connection, communication, and mathematical representations and modeling) in ÖABT (9-12). In TExES (8-12), only 3 items were categorized into the KCMLS, 2 of the PCK items were coded as KCMLS1 (the knowledge of curricular specifications and connections among topics, the progression from one year to the next ones). Although 2 of the PCK items was coded as KCMLS5 (the knowledge of types of assessment techniques), and KCMLS6 (the knowledge of appropriate assessment techniques to guide instruction and evaluate progress of learner) in TExES (8-12), there were no items including the knowledge of prospective teachers related to the type of assessment techniques or the preference of appropriate assessment techniques in ÖABT (9-12).

It can be inferred that, regarding the SMK items, prospective teachers were mostly responsible for basic mathematics knowledge which was not unique to mathematics teachers in both ÖABT (9-12) and TExES (8-12). The distribution of the PCK items in ÖABT (9-12) indicated that the PCK items were dominantly comprised of the knowledge including difficulties and misconceptions of students whereas there were no items related to the assessment techniques in teaching. In contrast, there were some items related to the knowledge of assessment techniques of teachers in TExES (8-12) in addition to the PCK items including knowledge of students' learning and knowledge of teaching mathematics.

### 4.4. Summary of Results

The first purpose of the study was to investigate the distribution of items in elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas with respect to the curricular levels, fields of mathematics, cognitive domain, and domains of knowledge for teaching mathematics. The results indicated that the SMK items in the examinations generally included mathematics content from the high school and advance levels; in other words, from the high school mathematics and beyond. In addition, there were no SMK items from the elementary level in both ÖABT (5-8) and ÖABT (9-12) whereas the SMK items in TExES (4-8) and TExES (8-12) were distributed across the three curricular levels in a more balanced way. Moreover, the PCK items in the examinations generally included mathematics content from the level at which the future teacher would teach. In addition, it can be concluded that there were fewer items from the applied mathematics field in comparison to the other three fields of mathematics. Also, there were more SMK items categorized into the algebra field in the examinations in Texas in comparison with the examinations in Turkey while there were more SMK items categorized into the calculus field in the examinations in Turkey in comparison with the examinations in Texas.

In this study, the cognitive domain was divided into the three categories, namely the knowing, applying, and reasoning. When the results were examined, most of the SMK items were from the applying category in ÖABT (5-8), ÖABT (9-12), and TExES (8-12). That is, most of the SMK items included use of the knowledge and conceptual understanding in familiar problems and learned procedures. Only in TExES (4-8), the SMK items were mostly categorized into the knowing category, the knowledge of basic mathematical concepts, properties, and computations. Also, in the examinations, there were no test items from the reasoning category which required the higher level cognitive demand over routine problems and included the use of logical and systematic thinking and the use of intuitive and inductive reasoning.

Lastly, the distribution of SMK and PCK items in elementary and secondary mathematics teacher content knowledge examinations were investigated on the basis of the domains of knowledge for mathematics teaching. According to the results, most of the items were SMK items in the examinations since the there were fewer PCK items in comparison to the proportion of the SMK items. Moreover, almost all of the SMK items were categorized into the CCK, namely mathematical knowledge which was used in a wide variety of settings. In other words, mathematics knowledge which is not unique to teaching was mostly included in elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas. As results presented, there were few items including knowledge of mathematical notation in the examinations. Although most of the items included the knowledge of mathematical notation, few of them were directly asking the notation of a mathematical concept. Therefore, there were few items coded as CCK3 (the knowledge of mathematical notation). The SMK items categorized into the SCK (the knowledge unique to mathematics teachers) sub-category was slightly included in TExES (8-12) and they were not included in the other examinations. Furthermore, the PCK items including the knowledge of learning of students were mostly included in ÖABT (5-8) and ÖABT (9-12) whereas a balanced distribution was observed in TExES (4-8) and TExES (8-12) based on the knowledge of students' learning, the knowledge of teaching, and the knowledge of curriculum and assessment.

# CHAPTER 5

# DISCUSSION

The first purpose of this study was to investigate mathematics teacher content knowledge examinations in Turkey and Texas based on curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics teachers. The second purpose of this study was to investigate how similar and different the mathematics teacher knowledge examinations in Turkey and Texas were with respect to curricular levels, fields of mathematics, cognitive domain, and knowledge of mathematics teachers.

In this chapter, elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas are compared and discussed based on curricular levels and fields of mathematics, cognitive domain, and domains knowledge for teaching mathematics.

## 5.1. Discussion of the Results

The results of the elementary and secondary mathematics teacher content knowledge examinations were discussed according to the structure of the tests. Therefore, headings in this chapter are determined as discussion of mathematics teacher content knowledge examinations based on curricular levels and fields of mathematics, cognitive domain, and domains of knowledge for teaching mathematics, respectively. The first and second research questions are discussed under following three headings.

### 5.1.1. Discussion of Mathematics Teacher Content Knowledge Examinations based on the Curricular Levels and Fields of Mathematics

In this study, elementary and secondary mathematics teacher content knowledge examinations in Turkey and Texas were separately analyzed with respect to curricular levels and fields of mathematics. The nature and extend of teachers' knowledge is an unknown topic, whether it is comprised of basic skills at the grades the prospective teacher will teach or a specific professional mathematics knowledge such as advanced calculus, linear algebra, abstract algebra, or differential equations has been an unanswered question yet (Ball, Hill, & Bass, 2005). One of the main components of teacher knowledge was subject matter knowledge. Although the professional knowledge of mathematics teachers is accepted as the mathematics knowledge of high school or elementary school mathematics in some studies (Baumert et al., 2010), it includes not only the knowledge of broad mathematics from a more detailed and advanced perspective, but also the knowledge of mathematics at the grade level teachers are assigned to teach (An, Kulm, & Wu, 2004; Baumert et al., 2010; National Mathematics Advisory Panel, 2008). The results of this study showed that most of the SMK items in the examinations in Turkey were from high school or advance level. That is, there were no SMK items from the elementary school level. According to the results, especially prospective elementary mathematics teachers in Turkey need to possess mathematics knowledge beyond the level they would teach. At this point, it can be criticized whether having advanced level of mathematics knowledge guarantees that the teacher has the conceptual understanding of the mathematical concepts from elementary levels. In other words, whether having an advanced level of mathematics knowledge ensure the success of the teacher in mathematics education? Ma (1999) answered this question by stating that the completion of advanced courses successfully does not ensure the understanding of the elementary mathematics. Even if university mathematicians assume advanced mathematics as a refinement and an extension of elementary mathematics, the researcher stated that it is possible to pass advanced courses without conceptualizing how they enlighten the elementary mathematics. To illustrate, according to the comparative study of Ma (1999), the Chinese teachers were better than the American teachers in terms of the mastery of elementary school mathematics although the American teachers seemed to have a superior education than the Chinese teachers since they were all graduated from colleges and several of them had master degree. Therefore, it can be concluded that having an advanced knowledge of mathematics may not refer to having a deep knowledge of elementary mathematics or provide a better instruction in elementary school. For this reason, in order to improve mathematics education, prospective mathematics teachers might also be responsible for mathematics knowledge from elementary school in examinations measuring teacher knowledge. The results of Texas showed that SMK items in the examinations in Texas were proportionally distributed within curricular levels. That is, even if the items were dominantly concentrated on the high school level, they were distributed proportionally across each curricular level. As a result, it can be concluded that the results regarding the SMK items were similar with the argument of Hill et al. (2007). That is, examinations which assessed the SMK of teachers did not work in a dependent way although they were concentrated on the same purpose, assessing knowledge for teaching mathematics. The reason might be that these examinations assessed different components of teacher knowledge by using different methods and by concluding different results (Hill et al., 2007).

According to the results, PCK items were mostly from the elementary school level for the prospective elementary mathematics teachers and from the high school level for the prospective secondary mathematics teachers. In other words, prospective mathematics teachers were mostly posed questions from the level they would teach in the future. Results also indicated that there existed a proportional distribution of PCK items with respect to curricular levels in the examinations in both Turkey and Texas.

Results showed that there was no trend in relation to the distribution of the SMK items in the examinations based on the fields of mathematics, namely algebra, calculus, applied mathematics, and geometry. However, the items which were classified within the applied mathematics field were fewer than in the other fields of mathematics in the examinations used in both Turkey and Texas. In Turkey, there are fewer mathematics courses related to the applied mathematics field as identified by the Council of Higher Education of Turkey. For this reason, it might be concluded that the frequency of the items in categories might be proportional to the number of courses taken in the teacher education program in Turkey. In addition, results indicated that subject matter knowledge items from the applied mathematics field were mostly categorized into the advanced level in the mathematics teacher content knowledge examinations in Turkey while the subject matter knowledge items from the applied mathematics field were distributed proportionally across the components of the curricular levels in the examinations in Texas. In Turkey, mathematical concepts from the applied mathematics field is mostly included in high school and university education while these concepts were included in the mathematics curricula in Texas beginning from the elementary school to high school level and beyond the high school. Therefore, this result also indicated that the frequency of the items might be proportional to the inclusion of mathematical concepts in the mathematics curricula used in Turkey and Texas (MoNE, 2011; TEA, n.d.). In the mathematics teacher content knowledge examinations in Texas, test items were mostly categorized into algebra field. Therefore, it might be concluded that most of the items in the examinations in Texas were from the algebra learning field since there are substantial number of mathematics concepts constituting algebra in the middle school and high school mathematics curricula of Texas (TEA, n.d.).

### 5.1.2. Discussion of Mathematics Teacher Content Knowledge Examinations based on Cognitive Domain

In this section, the analysis of the SMK items based on the cognitive domain will be discussed. According to the results, it was observed that most of the SMK items in the examinations in Turkey were categorized into the applying category. That is, most of the SMK items included the use of mathematical knowledge, skills, facts, and principles in familiar problems and learned procedures in the examinations in Turkey. Also, some of the SMK items were categorized as knowing which includes the knowledge and understanding of basic language of mathematics, essential mathematical concepts, properties, and procedures. As a result, it can be inferred that the SMK items of the examinations in Turkey included the knowledge of mathematical concepts and procedures and the knowledge of implementation of regular procedures.

Upon the analysis of TExES (4-8), it was observed that, most of the SMK items were categorized into the knowing category and the remaining items were categorized into the applying category. Conversely, the results regarding the analysis of TExES (8-12) items indicated that most of the SMK items were categorized into the applying category and the remaining ones were categorized into the knowing category. In conclusion, except for TExES (4-8), SMK items were mostly categorized into the applying category in the examinations of both Turkey and Texas. That is, most of the items were related to the common procedures of the determination or implementation in mathematics questions. Furthermore, there were no SMK items categorized into the reasoning category in the examinations in both Turkey and Texas. The reason might be related to the type of the items. The preparation of the multiple choice items which can be categorized as the reasoning might be difficult because of the complex nature of teacher knowledge. In addition, since measurement of categories of the cognitive domain such as logical thinking and reasoning might be measured better by open-ended items rather than multiple choice items. For this reason, there might be no item categorized as the reasoning.

#### 5.1.3. Discussion of Mathematics Teacher Content Knowledge Examinations based on the Knowledge for Teaching Mathematics

In the present study, items were also investigated with respect to the domains of knowledge for teaching mathematics. Knowledge of teaching mathematics should include conceptual understanding of mathematics, why the method or procedure works and how it is generalized (Ball, Lubienski, & Mewborn, 2001; Kahan et al., 2003). As a result, there was a common point among the four examinations that they measured especially one aspect of teacher knowledge, almost all of the SMK items required common content knowledge (CCK), which means they comprised general mathematics knowledge and skills which were not unique to mathematics teachers. It was the type of knowledge which could also be used or needed in areas other than mathematics. There was hardly any test item including mathematics knowledge and skills of teachers unique to mathematics teaching, specialized content knowledge (SCK) or horizon content knowledge (HCK). That is, SMK items were not specific to the mathematics teaching profession; but they included general mathematics knowledge which could be answered by adults from other professions also. Teachers are required not only to know facts, rules, and procedures, but also to conceptualize why it is so; in other words, they should have conceptual understanding of the concept (Shulman, 1986). Therefore, the other dimensions of teacher knowledge might be included in the examinations assessing teacher knowledge. In addition, examinations in Turkey and Texas had similar structures regarding subject matter knowledge of mathematics teachers. That is, common content knowledge of teachers mostly measured in the examinations in Turkey and Texas.

Results also indicated that there was a balanced distribution among the three components of pedagogical content knowledge, namely knowledge of learning of students, knowledge of teaching mathematics, and knowledge of mathematics curriculum. According to the results, it can be inferred that most of the items in the mathematics teacher content knowledge examinations in Turkey were related to the knowledge of learning of students while there was not a tendency in terms of the distribution of the PCK items in the mathematics teacher content knowledge examinations in Texas. Moreover, results revealed that slightly more PCK items were included in the examinations in Turkey in comparison to the examinations in Texas. Conversely, there were more SMK items in the examinations in Texas in comparison to the examinations in Turkey. Therefore, it might be concluded that PCK items have more importance in the examinations in Turkey when compared with the examinations in Texas. On the contrary, SMK items have more importance in the examinations in Texas in comparison to the examinations in Turkey.

Ma (1999) described subject matter knowledge as deep understanding of fundamental mathematics and stated that mathematics knowledge of teachers does have a vital role in teaching mathematics. Pedagogical content knowledge is also a very important factor since it is related to curriculum, instruction, and learning of students (Ball et al. 2001; Baumert et al., 2010). Therefore, neither just profound subject matter knowledge nor a deep and broad pedagogical knowledge alone is adequate for effective teaching (An et al., 2004; Shulman, 1986). That is, there should be a supportive interaction between pedagogical knowledge and subject matter knowledge (An et al., 2004). Shulman (1986) argued that here had been an inconsistency in the distribution of the domains of teacher knowledge in the examinations assessing teacher knowledge. Sometimes subject matter knowledge was more important and sometimes pedagogical knowledge was favored while assessing teachers. The results of the analyses in this study illustrated that most of the items were related to subject matter knowledge and the remaining items were related to pedagogical content knowledge. Therefore, it might be interpreted as while preparing these examinations, subject matter knowledge was perceived as the most important part of the teacher knowledge.

Knowledge for teaching mathematics is defined as the mathematical knowledge which is required to teach mathematics (Ball et al., 2008; Hill et al., 2004). Therefore, the preliminary question while preparing examinations assessing knowledge of teachers could be worded as: What is the mathematical knowledge required for teaching mathematics? Or, which dimensions of knowledge are required in order to teach mathematics? As Ball et al. (2001) stated, not only which courses teachers have taken or the professional subject matter knowledge they have is, but also whether and how they use this knowledge while teaching mathematics should be an important concern in teacher education. Although subject matter knowledge of teachers includes substantial elements, it does not guarantee that a teacher with mere strong subject matter knowledge can teach mathematics effectively (Ball et al., 2008). There is a lack of studies assessing pedagogical knowledge of teachers apart from the subject matter knowledge (Hill et al., 2008). That is, teacher assessment is mostly considered as an assessment of subject matter knowledge rather than pedagogical content knowledge or other dimensions of teacher knowledge although pedagogical content knowledge is proposed as one of the most important components for accomplishment of effective teaching (NBPTS, 2004; NRC, 1996). Therefore, teachers are required to possess knowledge of teaching a specific content to a group of students rather than just having knowledge of subject matter or knowledge of pedagogy (Park & Oliver, 2008). Therefore, components of teacher knowledge might be included in examinations of teacher assessment in a balanced way.

All in all, the study implies that mathematics teacher content knowledge examinations generally have similar structures except for analyses with respect to cognitive domain and curricular levels. It can be concluded that examinations in both Turkey and Texas have similar structures in consideration of knowledge for teaching mathematics, namely they have mostly included common content knowledge of teachers.

### 5.2. Implications and Recommendations for Further Research Studies

Professional knowledge of teachers is one of the predictors for the achievement of students (Ball, Hill, & Bass, 2005, Ball et al., 2008; Rowan, Chiang, & Miller, 1997). For this reason, the dimensions of teacher knowledge should be studied in order to see which dimension influences the achievement of students the most. Specialized mathematics knowledge of teachers is one of the most important components of subject matter knowledge of teachers in order to predict achievement of students (Ball et al., 2008). The results indicated that mathematics teacher content knowledge examinations contradict with the argument of Ball et al. (2008) since the examinations mostly included common content knowledge of teachers. As a result, future studies might be conducted while assessing the knowledge of mathematics teachers in such a way that specialized content knowledge and the other dimensions of the knowledge of mathematics for teaching might be included in the assessment of the professional knowledge mathematics teachers.

In addition, most of the items in mathematics teacher content knowledge examinations in Turkey and Texas included common content knowledge among the other sub-domains of teacher knowledge. One of the reasons why there was a high number of items coded as common content knowledge might be attributed to the elusive nature of borders among the dimensions of teacher knowledge. Since there have been unclear points among some dimensions of teacher knowledge, categorization of items might be biased in terms of knowledge for teaching mathematics. The distinction among the domains of teacher knowledge is important for the effective assessment of teacher knowledge (Hill et al., 2008). However, there are some difficulties in differentiating the components of teacher knowledge because of these unclear points (Carrillo et al., 2013). Firstly, these difficulties might be appeared between common content knowledge and specialized content knowledge since it is unclear when common content knowledge ends and specialized content knowledge starts for some mathematical situations. In addition, sometimes differentiating the specialized content knowledge from the knowledge regarding the features of students learning might become difficult while categorizing an item. For this reason, in ensure teacher quality, future studies might be conducted in order to identify the unclear points across the components of teacher knowledge. Therefore, assessment of the teacher knowledge might be conducted in a better way by the identification of the structure and dimensions of teacher knowledge.

In this study, the comparison of mathematics teacher content knowledge examinations in Turkey and Texas (ÖABT and TExES) was concluded with several results that might be beneficial for the improvement of ÖABT. However, it should be clarified that TExES is a section in Texas Educator Certification in order to certify the teacher candidates in Texas while ÖABT is a section in KPSS in order to rank and select the teacher candidates in Turkey. Although TExES and ÖABT were similar in context and structure according to the results of this study, they have some distinct characteristics. In ÖABT, prospective teachers have to be successful with respect to the achievement of their counterparts while prospective teachers have to be successful with respect to the predetermined passing standards in TExES. Therefore, similar studies might be conducted with other countries that have similar structure with Turkey within the application of the examination and the interpretation of the scores taken from the examination. Moreover, similar criterions might be included within the requirements of mathematics teacher content knowledge examinations in Turkey in order to have more qualified teachers. Furthermore, TExES has been prepared with respect to the publically available and detailed frameworks while there has been not a publically available and detailed framework related to the preparation of mathematics teacher content knowledge examinations in Turkey (TEA, 2010a; TEA, 2010b).

The items investigated in this study were comprised of multiple choice test items. That is, there were no other types of items in order to assess the knowledge of mathematics teachers. Some researchers claimed that multiple choice items could not measure the professional knowledge of teachers (Ball et al., 2005). On the other hand, some others stated that multiple choice and complex multiple choice items can be used to evaluate all knowledge domains (Hill et al., 2008; Tatto et al., 2008). However, multiple choice items do not permit respondents to provide a detailed explanation, interpretation, or demonstration related to the item as much as open constructed response items do since open constructed items provide respondents with the opportunity to develop answers for a question or to produce representations for mathematical concepts (Tatto et al., 2008). For this reason, knowledge of teachers can be assessed by interviews and open-ended tasks in addition to multiple choice items (Hill et al., 2008). As a result, it can be concluded that examinations for the assessment of teacher knowledge included in this study might be insufficient in order to measure the professional knowledge of teachers. Only multiple choice items were administered in the scope of mathematics teacher content knowledge examinations in Turkey and Texas although teacher knowledge has a complex nature, which should be evaluated in detail (Hill et al., 2007; Tatto et al., 2008). Therefore, additional sections such as open-ended items, interviews, observations, and portfolios might be included in teacher content knowledge examinations to measure the knowledge of teachers in a more detailed and effective way.

There are some limitations for the study. Since there are some unclear points across the dimensions of mathematics knowledge for teaching, this might be a limitation related to the categorization of the items in terms of the mathematics knowledge for teaching. In addition, mathematics teacher content knowledge examinations in Turkey and Texas have some different characteristics related to some points, such as the purpose of the examination, the preparation techniques of the examination, and the interpretation of the scores of prospective teachers although these examinations were similar in content based on the results of this study. Therefore, these different characteristics of the examinations might be a limitation for a comparative study.

Finally, further studies might be conducted in relation to the examinations in other countries. Since this study merely involves mathematics teacher content knowledge examinations in Turkey and Texas, the content of examinations of several countries might expand the scope of the current study. Moreover, similar studies might be conducted for further mathematics teacher content knowledge examinations, especially in Turkey. By means of these kinds of studies, deficiencies and improvements might be determined among the teacher content knowledge examinations.

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# APPENDICES

## A. ITEM SAMPLES

### 1. ÖABT (5-8)

| | |
|---|---|
| **Curricular levels:** | High school |
| **Field of Mathematics** | Calculus |
| **Content Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Sub domain of Mathematics Teacher Knowledge** | CCK1-CCK2 |
| **Level of Cognitive Domain:** | APPLYING (represent-implement) |

**2.** $\tan 2x = u$

olarak veriliyor.

**Buna göre, $\sin 4x$'in $u$ türünden ifadesi aşağıdakilerden hangisidir?**

A) $\dfrac{1-u^2}{1+u^2}$ B) $\dfrac{u^2-1}{u^2+1}$ C) $\dfrac{2u}{1+u^2}$

D) $\dfrac{u}{1-u^2}$ E) $\dfrac{1+u^2}{u^2-1}$

**Source:** ÖSYM (2013a, p. 1)

## 2. ÖABT (9-12)

| | |
|---|---|
| **Curricular levels:** | High school |
| **Field of Mathematics:** | Algebra |
| **Content Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Sub-domain of Mathematics Knowledge of Teacher** | CCK1-CCK2 |
| **Level of Cognitive Domain:** | KNOWING (Compute-Retrieve) |

**13.** Z tam sayılar kümesi olmak üzere,

$$A = \{n^3 \mid n \in Z, -4 \le n \le -1\}$$

$$B = \{(-3)^n \mid n \in Z, 1 \le n \le 4\}$$

kümeleri veriliyor.

**Buna göre, aşağıdakilerden hangisi doğrudur?**

A) $A \times B$ 'nin eleman sayısı 16'dır.

B) $A \cap B$ 'nin eleman sayısı 2'dir.

C) $A \cup B$ 'nin eleman sayısı 8'dir.

D) $A \setminus B$ 'nin eleman sayısı 2'dir.

E) $B \setminus A$ 'nın eleman sayısı 2'dir.

**Source:** ÖSYM (2013b, p. 4)

### 3. ÖABT (5-8)

| | |
|---|---|
| **Curricular levels:** | Elementary |
| **Field of Mathematics** | Geometry |
| **Domain:** | Pedagogical content knowledge (PCK) |
| **Sub-Domain:** | KFLM4 |

**47.** Van Hiele, geometrik düşünmenin gelişiminin aşamalı olarak aşağıda verilen beş düzeyde gerçekleştiğini belirtmektedir.

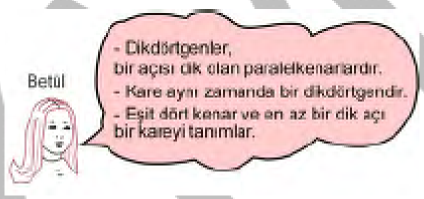
1. Düzey: Öğrenci, şekilleri genel görsel özelliklerine göre tanır ve adlandırır.

2. Düzey: Öğrenci, şekillerin özelliklerini belirtir.

3. Düzey: Öğrenci, geometrik şekiller arasında ilişkiler kurar.

4. Düzey: Öğrenci, bir aksiyomatik yapıyı kullanabilir ve bu yapı içinde ispatlar yapar.

5. Düzey: Öğrenci, farklı aksiyomatik sistemler arasındaki benzerlik ve farklılıkları anlar.




**Bu açıklamayı yapan Betül, <u>en az</u> hangi Van Hiele geometrik düşünme düzeyi içinde yer alır?**

A) 1. Düzey B) 2. Düzey C) 3. Düzey

D) 4. Düzey E) 5. Düzey

**Source:** ÖSYM (2013a, p. 14)

**4. ÖABT (9-12)**

| | |
|---|---|
| **Curricular levels:** | High school |
| **Field of Mathematics** | Algebra |
| **Content Knowledge Domain:** | Pedagogical Content Knowledge (PCK) |
| **Sub-Domain of Mathematics Knowledge of Teacher:** | KCMLS1-KCMLS2 |

**43.**

I. $f: R \rightarrow R$, $f(x) = -x^2 + 4x + 1$ fonksiyonunun alacağı en büyük değeri bulunuz.

II. $f: R \rightarrow R$, $f(x) = 2x + 1$ fonksiyonu birebir ve örten midir?

III. $f: R \rightarrow R^+$, $f(x) = 3^x$ fonksiyonunun grafiğini çiziniz.

**Yukarıdaki soruları çözmek için gerekli kazanımlar uygulanmakta olan Ortaöğretim Matematik Dersi (9, 10, 11 ve 12. Sınıflar) Öğretim Programı'nda ilk kez kaçıncı sınıf düzeyinde ele alınmaktadır?**

| | I. | II. | III. |
|---|---|---|---|
| A) | 12. | 10. | 12. |
| B) | 10. | 9. | 12. |
| C) | 10. | 9. | 11. |
| D) | 12. | 9. | 11. |
| E) | 10. | 10. | 12. |

**Source:** ÖSYM (2013b, p. 12)

5. **ÖABT (9-12)**

| | |
|---|---|
| **Curricular levels:** | Advanced |
| **Field of Mathematics** | Geometry |
| **Content Knowledge Domain:** | Subject Matter Knowledge (SMK) |
| **Sub-Domain of Mathematics Knowledge of Teacher:** | CCK2 |
| **Level of Cognitive Domain:** | KNOWING (Recall) |

**42.** Euclid Geometrisi'nin beş postulatından birine yönelik şüpheler ve bunun üzerine yapılan çalışmalar Euclid dışı geometrilerin ortaya çıkmasına zemin hazırlamıştır.

**Buna göre, bu postulat aşağıdakilerden hangisidir?**

A) Merkezi ile yarıçapı verilen bir çember çizilebilir.

B) Bir doğru parçası sınırsız bir şekilde uzatılabilir.

C) İki noktadan bir ve yalnız bir doğru geçer.

D) Bütün dik açılar eştir.

E) Bir doğruya dışındaki bir noktadan yalnız bir tek paralel doğru çizilir.

**Source:** ÖSYM (2013b, p. 12)

## 6. ÖABT (5-8)

| | |
|---|---|
| **Curricular levels:** | Advanced |
| **Field of mathematics** | Applied Mathematics |
| **Question Type:** | Subject Matter Knowledge (SMK) |
| **Sub domain of Mathematics Teacher Knowledge** | CCK1-CCK2 |
| **Level of Cognitive Domain:** | APPLYING (implement) |

**28.** Bir X rastgele değişkeninin olasılık yoğunluk fonksiyonu

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$$

biçiminde tanımlanıyor.

**Buna göre, $P(1,5 < X < 2)$ olasılığı kaçtır?**

A) $\frac{1}{9}$ B) $\frac{1}{8}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$ E) $\frac{3}{2}$

**Source:** ÖSYM (2013a, p. 8)

# B. CORRESPONDENCES WITH TEXAS EDUCATION AGENCY AND THE EDUCATION AGENCIES OF OTHER STATES IN THE UNITED STATES

## 1. Response of Michigan Department of Education for Release of Teacher Certification Examinations

| | |
|---|---|
| **Konu:** | Re: michigan teacher certification examination |
| **Gönderen:** | "Jones, Edwardeen (MDE)" <JonesE9@michigan.gov> |
| **Tarih:** | 10 Mayıs 2013, Cuma, 10:39 am |
| **Alıcı:** | "nurbanu.yilmaz@metu.edu.tr" <nurbanu.yilmaz@metu.edu.tr> |
| **Öncelik:** | Normal |
| **Seçenekler:** | Tüm Başlıkları Göster \| Yazdırılabilir Şekilde Göster \| Bunu dosya olarak indir |

No. Testing materials can not be given out for research purposes.

Sent from my iPhone

On May 10, 2013, at 2:53 AM, "nurbanu.yilmaz@metu.edu.tr"
<nurbanu.yilmaz@metu.edu.tr> wrote:

## 2. Response of New York State Department of Education for Release of Teacher Certification Examinations

| | |
|---|---|
| **Konu:** | RE: getting previous tests for research purposes cdt |
| **Gönderen:** | "ERM es-east-customersupport" <es-east-customersupport@pearson.com> |
| **Tarih:** | 13 Mayıs 2013, Pazartesi, 10:34 pm |
| **Alıcı:** | nurbanu.yilmaz@metu.edu.tr |
| **Öncelik:** | Normal |
| **Seçenekler:** | Tüm Başlıkları Göster \| Yazdırılabilir Şekilde Göster \| Bunu dosya olarak indir \| HTML olarak göster |

Thank you for contacting Evaluation Systems regarding the New York State Teacher Certification Examinations (NYSTCE).

Evaluation Systems cannot provide you with the data you have requested. In accordance with the policies established by the New York State Education Department (NYSED), Evaluation Systems releases program data only to the New York State Education Department and other parties authorized by NYSED.

We are committed to providing the highest level of customer service, and we hope we have addressed your inquiry. If you have other questions or need further assistance, please contact us again at your convenience. If you prefer, you may call NYSTCE Customer Service at (800) 309-5225 or (413) 256-2882. Customer Service Representatives are available 9:00 a.m.-5:00 p.m. eastern time, Monday-Friday, excluding holidays. The Automated Information System is available 24 hours daily.

Customer Service
Evaluation Systems
Pearson

## 3. Response of the Texas Education Agency and the Educational Testing Service for the Release of the Representative Teacher Certification Examinations

| | |
|---|---|
| **Konu:** | PIR 19730 (Yilmaz) Non Disclosure Agreement for Release of Requested Information |
| **Gönderen:** | "PIR" <pir@tea.state.tx.us> |
| **Tarih:** | 17 Temmuz 2013, Çarşamba, 6:08 pm |
| **Alıcı:** | "nurbanu.yilmaz@metu.edu.tr" <nurbanu.yilmaz@metu.edu.tr> |
| **Öncelik:** | Normal |
| **Alındı bilgisini oku:** | gönderildi |
| **Seçenekler:** | Tüm Başlıkları Göster \| Yazdırılabilir Şekilde Göster \| Bunu dosya olarak indir \| HTML olarak göster |

Mr.  Yilmaz,

Attached is the non disclosure agreement (NDA) between ETS, TEA and yourself so that TEA may provide you with the representative test for your request.  Please note that the agreement will need your signature and a physical address where the test can be mailed, as we anticipate that it is in paper format and bound.

You may send the signed NDA by mail, fax, or e-mail.

Open Records Coordinator
Texas Education Agency
1701 North Congress Avenue
Austin, Texas 78701-1494
FAX: (512) 463-9838 | Email: PIR@tea.state.tx.us<mailto:PIR@tea.state.tx.us>

If you have trouble accessing the document or if you have any questions please contact me.

Sincerely,
Alejandra Gallegos

## 4. The E-mail of Texas Education Agency related to Posting the Representative Mathematics Teacher Certification Examinations

| | |
|---|---|
| **Konu:** | PIR 19730 (Yilmaz) Delivery Status Update |
| **Gönderen:** | "PIR" <pir@tea.state.tx.us> |
| **Tarih:** | 18 Eylül 2013, Çarşamba, 5:03 pm |
| **Alıcı:** | "nurbanu.yilmaz@metu.edu.tr" <nurbanu.yilmaz@metu.edu.tr> |
| **Öncelik:** | Normal |
| **Alındı bilgisini oku:** | gönderildi |
| **Seçenekler:** | Tüm Başlıkları Göster \| Yazdırılabilir Şekilde Göster \| Bunu dosya olarak indir \| HTML olarak göster |

Mr. Yilmaz:

I am writing to check on the delivery of the information that was provided to you in response/agreement for PIR 19730.  The information was sent via mail on August 15, 2013 and the last tracking update was that it left the New York postal service on August 20th.

At this time could you confirm if you have received the information?

Thank you in advance for your time and response.

Sincerely,
Alejandra Gallegos
Public Information Request Coordinator

## 5. The E-mail from Texas Education Agency related to the Actual Teacher Certification Examinations and Representative Tests

| | |
|---|---|
| **Konu:** | RE: texas teacher certification examination: TEA PIR# 19730 |
| **Gönderen:** | "Meitler, Montgomery" <Montgomery.Meitler@tea.state.tx.us> |
| **Tarih:** | 27 Haziran 2013, Perşembe, 7:37 pm |
| **Alıcı:** | "nurbanu.yilmaz@metu.edu.tr" <nurbanu.yilmaz@metu.edu.tr> |
| **Cc:** | "PIR" <pir@tea.state.tx.us> |
| **Öncelik:** | Normal |
| **Seçenekler:** | Tüm Başlıkları Göster \| Yazdırılabilir Şekilde Göster \| Bunu dosya olarak indir |

Mr. Yilmaz,

I am writing in regard to your Public Information Request (TEA PIR# 19730) for teacher certification tests previously administered by the Texas Education Agency (TEA), specifically "mathematics 4-8 (with code 115) and mathematics 8-12 (with code 135)." As I have previously indicated, we are opposed to releasing these tests because (A) if the tests are publically disclosed, neither the tests nor any of the test questions could be administered again, and (B) the tests contain questions that would otherwise be used on future teacher certification tests. Public disclosure of these exams will compromise future examinations and require TEA to develop new exams at a considerable expense to the state of Texas. However, we believe we have developed a proposal that will satisfy both your research interest and our confidentiality concerns, while sparing everyone the 45 business days required for a ruling from the Office of the Attorney General and the uncertainty and expense of the public information request process.

In addition to the tests administered for certification, TEA's testing provider, Educational Testing Service (ETS), develops parallel "Representative Tests" that are similar in every material respect to the administered tests but do not contain questions that have been or will be used in an administered test. These Representative Tests are designed to closely parallel the administered tests and normally are sold only to authorized Educator Preparation Programs for limited and controlled use in preparing their candidates. TEA is willing to provide you with a Representative Test for each of the two tests you requested if you will agree to keep the test contents confidential and withdraw your public information request for the administered tests. Because the Representative Tests are similar in every material respect to the administered tests, we believe they should satisfy your research needs.
Please let us know your response to this proposal by July 3, 2013. If it would be helpful to you, I would be happy to arrange a call with representatives from TEA and/or ETS so that they can answer any questions you may have about the Representative Tests.

Thank you,

W. Montgomery Meitler
Assistant Counsel /
Confidentiality Officer
Office of Legal Services
Texas Education Agency
1701 N. Congress Ave.
Austin, Texas 78701-1494
512-463-7830
Fax: 512-475-3662