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SIXTH GRADE STUDENTS' ACHIEVEMENT LEVELS, ERRORS, AND UNDERLYING REASONS OF THE ERRORS REGARDING COMPREHENSION AND ORDERING OF INTEGERS

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ABSTRACT

SIXTH GRADE STUDENTS' ACHIEVEMENT LEVELS, ERRORS, AND UNDERLYING REASONS OF THE ERRORS REGARDING COMPREHENSION AND ORDERING OF INTEGERS

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Purposes of the study are three-fold. The first purpose is to investigate middle school sixth grade students' achievement levels regarding comprehension and ordering of integers. The second purpose is to investigate errors made by middle school sixth grade students regarding comprehension and ordering of integer. The third purpose is to explore underlying reasons of the students' errors regarding comprehension and ordering of integers. A mixed-method design was utilized to reach these purposes.

Partipants were 262 sixth grade students from one public middle school in Etimesgut district of Ankara. Data were collected via Integer Achievement Test (IAT) including 8 open ended questions during 2013-2014 spring semester. Individual interviews were conducted 8 participants to amplify their answers to the problems.

Findings indicated that achivement levels of the participants in comprehension questions were considerably high and in ordering questions were modarate. Findings revealed that students made errors regarding comprehension and ordering of integers.

For the comprehension and ordering questions, identified errors were applying incomplete solution strategy, not justifying symbol manipulation, misusing positive and negative signs, ordering as inverse sequence, ordering as arbitrary sequence, taking incorrect reference point, ignoring the given information and making incorrect alignment. Reasons of students' errors were also explored. The reasons for the students' errors regarding comprehension and ordering of integers were misunderstanding of magnitude of numbers on number line, reading the question carelessly, supposing integers with same signs are closer to each other than they are to integers with opposite sign and overgeneralizing properties of natural numbers to integers.

Key Words: Integer, comprehension, ordering, achievement levels, errors

ÖZ

ALTINCI SINIF ÖĞRENCİLERİNİN TAM SAYILARI KAVRAMA VE SIRALAMA KAVRAMLARINDAKİ BAŞARI DÜZEYLERİ, YAPTIKLARI HATALAR VE BU HATALARIN NEDENLERİ

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Çalışmanın amaçları üç kısımdan oluşmaktadır. Çalışmanın birinci amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama kavramlarındaki başarı düzeylerini incelemektir. Çalışmanın ikinci amacı, bu öğrencilerin tam sayıları kavrama ve sıralama kavramlarındaki yaptıkları hataların belirlenmesidir. Çalışmanın üçüncü amacı, öğrencilerin yaptıkları hataların nedenlerinin araştırılmasıdır. Bu amaçlara ulaşmak için karma bir araştırma yöntemi kullanılmıştır.

Çalışmaya Ankara'nın Etimesgut ilçesinden 262 altıncı sınıf devlet ortaokulu öğrencisi katılmıştır. Veriler, 8 açık uçlu soru içeren Tam Sayı Başarı Testi aracılığıyla 2013-2014 öğretim yılı bahar döneminde toplanmıştır. Ek olarak, toplam

8 katılımcı ile katılımcıların testteki cevaplarını açıklamaları amacıyla bireysel görüşmeler yapılmıştır.

Çalışmanın bulguları katılımcıların kavrama sorularındaki başarılarının yüksek, sıralama sorularında orta seviyede olduğunu göstermiştir. Ayrıca, bulgular öğrencilerin tam sayıları kavrama ve sıralama konularında hatalar yaptığını göstermiştir. Kavrama soruları için belirlenen hatalar; eksik çözüm stratejisi uygulama, yanlış sembol manipülasyonu, pozitif ve negatif işaretlerin yanlış kullanımı, verilen bilgi ihmali ve yanlış hizalamadır. Sıralama soruları için belirlenen hatalar; ters sıralama, rastgele sıralama, yanlış referans noktası alma, yanlış sembol manipülasyonu, pozitif ve negatif işaretlerin yanlış kullanımı, verilen bilgi ihmali ve yanlış hizalamadır.

Öğrencilerin yaptıkları hataların sebepleri de incelenmiştir. Öğrencilerin yaptıkları hataların sebepleri; sayı doğrusu üzerindeki sayıların büyüklüğünü yanlış anlama, soruyu dikkatsiz okuma, aynı işaretli tam sayıların farklı işaretli tam sayılara göre daha yakın olduğunu varsayma ve son olarak doğal sayıların özelliklerini tam sayılara genellemedir.

Anahtar Kelimeler: Tam sayı, kavrama, sıralama, başarı düzeyi, hata

To My Husband and My Family

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LIST OF ABBREVIATION

MoNE	Ministry of National Education
METU	Middle East Technical University
NCTM	National Council of Teachers of Mathematics
IAT	Integer Achievement Test

CHAPTER 1

INTRODUCTION

The fundamental goal of mathematics education is to enable all students to perform at the highest level within their mathematical learning process. However, very few students are able to attain this level; the majority of them experience difficulties in learning (Tall & Razali, 1993). In addition, Hiebert and Carpenter (1992) stated that it is crucial to understand mathematics in order to learn it effectively. Understanding mathematics does not only require acquiring some basic skills or concepts, but also establishing relationships between them. Only when appropriate ideas exist and new relationships and connections are established then understanding the highly complex process of mathematics can be possible (Lehrer, 1999). Therefore, it is significant to learning mathematics by understanding it because mathematics is a tool by which people create opportunities and options for their future (NCTM, 2000).

It can be claimed that students investigate answers to questions about mathematics itself as well as the purposes in learning mathematics by considering students' long educational lives. For instance, Lappan et al. (1989) stated that students might think that making computations, solving mathematical problems, or playing with numbers are major goals of learning mathematics. In addition, they may consider mathematics to be the accumulation of disconnected rules (Lappan & Even, 1989). This perception of middle school students regarding mathematics may continue in this way throughout the following years of their educational lives. The standards of the National Council of Teachers of Mathematics (NCTM, 2000) in the USA and the curriculum documents of the Ministry of National Education (MoNE, 2009) in Turkey involve widely accepted guidelines on these issues. In Turkey, the fundamental aims of the middle school curricula involve helping students to acquire skills of critical thinking, creativity, communication, investigation, problem solving,

using information technologies, and entrepreneurship. Additionally, the mathematics curriculum particularly focuses on reasoning, and relationship among concepts. These skills allow students to deal with more meaningful learning of mathematics (MoNE, 2009). Similarly, the last revised mathematics curriculum concentrates on conceptual learning, relationship among mathematics concepts, being fluent in operations and problem solving skills. Moreover, the mathematics curriculum emphasizes that students should value mathematics (MoNE, 2013). Firstly, by considering the importance of meaningful learning of mathematics, students' achievement levels in domain of integers were investigated in this study.

Integers have a crucial role in learning mathematics by understanding it because results of many studies investigating the integer conception revealed that integer is both complex and requires great effort to learn (Dereli, 2008; Janvier, 1983; Kilhamn, 2008; Mc Corkle, 2001). Since there are strong prerequisite relationships among integers and other issues, a student who already has learning difficulties in integers will find it difficult to succeed in the following subjects such as algebra (Lamb et al., 2012; Vlassis, 2004). On the other hand, while students are learning mathematical concepts, they may make some errors and have difficulties in relation to mathematical concepts. The integer conception is one of the important mathematical concepts which students tend to have errors, difficulties in while learning the mathematical concept (Janvier, 1983). Even though students might have many opportunities to experience basic integer concepts before they begin school, such as balancing a checkbook, understanding temperatures, and keeping the score when playing some games, these situations do not prevent making errors and experiencing difficulties in learning integers. In order to be successful in teaching such a significant and problematic subject, knowing the errors and difficulties of students has an important role (Yetkin, 2003). In this regard, students' errors regarding comprehension and ordering of integers while solving questions related to the concepts would be valuable to investigate.

As mentioned above, developing an understanding of mathematics is a difficult goal, and the subject of integers, which is a part of the mathematical curriculum in middle schools, is one of its fundamental components (Kilhamn, 2008). Furthermore, to be more successful in mathematics, a comprehensive understanding of integers

associated with misconceptions about integers which cause difficulties is needed (Akyüz, Stephan, & Dixon, 2012; Vlassis, 2004). Hence, one of the purposes of the present study is to investigate underlying reasons of students' errors regarding comprehension and ordering of integers.

1.1 Purpose of the Study

Even though integers have been an important part of the mathematics curriculum in middle schools in Turkey, students still make errors and have difficulties in understanding integer concepts (İşgüden, 2008). The findings of the study conducted by İşgüden (2008) revealed that students' difficulties are listed as follows: whether zero is an integer or not, the place of negative numbers on the number line, placing integers on a number line, ordering two or more negative integers and relating integers with other number sets such as whole numbers. In addition to these, the meaning of integers also poses difficulty for middle school students (Kilhamn, 2008).

The results of these studies demonstrate that learners need help to understand and construct concepts of integers because they make errors in the integer concept such as perceiving integers as natural numbers. Students should not overgeneralize natural number properties to integers. To overcome this overgeneralization made by middle school students, there is a need for more information associated with how students think about integers and how their thinking process changes by learning the integer concept, which can be obtained by identifying students' errors and the underlying reasons of the errors. Analyzing how Turkish middle school students perceive integers might help the students and also teachers to deal with integers in a more meaningful way.

In this study, middle school 6^{th} grade students' achievement levels, the errors they make and the reasons underlying those errors regarding comprehension and ordering of integers were investigated. Based on these aims, the study adopted the following research questions:

1. What are the achievement levels of middle school sixth grade students regarding comprehension of integers and ordering of integers?
2. What are the errors that middle school sixth grade students make regarding comprehension of integers and ordering of integers?

3. What are the reasons underlying middle school sixth grade students' errors regarding comprehension of integers and ordering of integers?

1.2 Definitions of the Important Terms

Operational and constitutive definitions that are related to the research questions are presented below:

Integer: The whole numbers 0, 1, 2, 3, 4… , together with the negatives of the whole numbers - 1, -2, -3, -4,…. are called integers (Bennett & Nelson, 2001).

Comprehension of Integers: It refers to what integers are, how they are represented, how they are related to each other and to whole numbers. It also refers to both the direction and the magnitude of integers (MoNE, 2009).

In this study, comprehension of integers includes: recognizing integers, reading and writing integers, integer representations, interpretation of integers, the meaning of integers.

Ordering of Integers: It refers to the fact that integers are sequenced and ordered (e.g., -7 is the number greater than -8 and less than -6); however, there is no need to establish a relation to a countable amount or quantity (Clements & Sarama, 2007).

In comparing elements, different criteria are used for different ordering relations. One can compare every pair of elements in some order, like the familiar less-than ordering of natural numbers (Ponce, 2007). In this study, ordering integers includes making comparisons which are less than or greater than between integers.

Error: "An error is a mistake, slip, blunder, or inaccuracy, and a deviation from accuracy" (Luneta & Makonye, 2010, p.36). In this study, it is defined as the wrong answers made as a result of negligence, or having deficiency in knowledge.

In this study the nature of error as it was used, is explained in terms of student's responses to a question in IAT and students' explanations in the interview.

1.3 Significance of the Study

Students who experience learning difficulties in any topic are bound to experience difficulties in achieving success in the subsequent topics (Dikici and İşleyen, 2004).

Failure to gain a prerequisite concept relevant to the mathematic topic will cause difficulties in the learning process because math topics have a strong sequential structure (Altun, 2008). Issues that students have difficulty in must be determined and it is necessary to identify these challenges (Yudariah ve Roselainy, 2001). In addition to these, it is necessary for teachers to foreknow the difficulties students face when they learn certain issues and concepts in order to achieve effective teaching, and so that they can discuss frequent errors with their students and can plan their teaching process based on these errors (Fischbein, 1987; Janvier, 1985; Julie 2013; Ponce, 2007; Spang, 2009). Integers are one of the mathematics topics in the mathematics curriculum as of 6^{th} grade, and the topic of integers is quite important because it is functional in the other following topics. The role of integers in the development of higher level mathematical concepts, such as algebra, makes it one of the most important and essential conceptual subjects in the middle school mathematics curriculum (Christou & Vosniadou, 2012, Vlassis, 2004). When students get to second term of sixth year, they begin to learn algebra that is based on integers. Furthermore, teaching and learning rational numbers and exponential numbers that are based on integers begin in seventh grade and continue in eighth grade. This situation indicates that the topic of integers should be handled and studied differently at different levels. Therefore, to provide a high level of readiness in these domains, the identification of errors middle school sixth grade students' make and the difficulties they encounter come into prominence regarding integers.

Awareness of errors and difficulties faced by students is important for studies focusing on learning (Rasmussen, 1998). Besides, studies of many researchers revealed that students have some difficulties in understanding integers at any class level (İşgüden, 2008; Körükçü, 2008). Hence, students' entire educational life in mathematics is affected by the difficulties that are experienced and unresolved with regard to integers. In addition, the literature review on learning and teaching integers in middle schools in Turkey demonstrates many unanswered questions about students' errors, the difficulties they encounter. In order to help the students eliminate their errors by means of correct knowledge, teachers should have an understanding of the underlying reasons of students' errors regarding integers. Moreover, teachers need this information to establish an effective teaching environment to increase their students' achievement levels regarding the integer

concept. Searching the underlying reasons of errors may reveal some misconceptions related to the concept. If teachers have an understanding of students' misconceptions and underlying reasons of errors, they can help the students replace their misconceptions with scientifically correct knowledge and they can teach how to conceptually deal with integers. Hence, analyzing underlying reasons of 6^{th} grade middle school students' errors regarding comprehension and ordering of integers is essential. Information obtained from this study may assist teachers in detecting and correcting common errors that students make while dealing with integers.

Furthermore, when students learn integers, they need to reason with numbers which cannot be modeled physically. For students, integers are the first number set, which includes numbers that cannot be modeled physically. Many studies have attempted to decide which model and real-world contexts would be most useful to enhance students' understanding of integers (Akyüz & Stephan, 2012). The literature review reveals that the more suitable activities and examples of integers are not clear in terms of learning negative numbers and constructing concepts of negative numbers. On the other hand, Işıksal (2009) stated that there is no consensus among researchers regarding the right time of teaching the integer concept and operations. In addition to these problematic sides of the integer concept, the literature review also reveals that there are few studies about how students make sense of integers when compared to the number of studies about natural numbers and fractions (Işıksal, 2009). Moreover, there are limited studies done in Turkey in the domain of integers (Dereli, 2008; Ercan, 2010; Işıksal, 2009; İşgüden, 2008; Keşan & Kaya 2007; Köroğlu&Yeşildere, 2004; Körükçü, 2008). Hence, it is significant to conduct a study which shows students' achievement levels, errors and underlying reasons of the errors regarding integers. The results of such a study would give valuable information related to comprehension and ordering of integers.

1.4 Organization of the Study

In this chapter, the purpose of the study, the significance of the study and the definitions of important terms have been explained. The second chapter is devoted to the literature review, which presents definitions of important terms, such as error, misconception and integer. Additionally, related studies integer, common errors, difficulties and misconceptions regarding the concept of integers. The third chapter

includes information about the design of the study, the population and sample, data collection instruments, data collection procedure, analysis of data, assumptions and limitations. The fourth chapter presents findings of the study with respect to the achievement levels of middle school sixth grade students in the questions of the Integer Achievement Test (IAT), and the errors they made regarding comprehension and ordering of integers and underlying reasons of the student' errors. The last chapter involves the discussion and implications of the study and presents recommendations for further studies.

CHAPTER 2

LITERATURE REVIEW

The purpose of the current study was two-fold. First, it aimed to investigate middle school 6th grade students' achievement levels regarding comprehension and ordering of integers, and the errors they made regarding this topic. Secondly, it aimed to examine the underlying reasons of those errors.

In accordance with the purposes of the study, the literature was reviewed and summarized under four categories, namely mathematical errors and misconceptions, the definition of integer, common errors, difficulties and misconceptions regarding the concept of integers and lastly studies on integers.

2.1. Mathematical Errors and Misconceptions

Although the terms *error* and *misconception* are generally used together or interchangeably, error and misconception do not have the same meanings in mathematical ideas and procedures (Luneta & Makonye, 2010). In order to explain these terms, the definitions of error and misconception, and the relationship between them are explained in this section.

In many of the research studies in the related literature, there are various definitions of the term *error*. Luneta and Makanye (2010, p. 35) define *error* as "… a mistake, slip, blunder, or inaccuracy and a deviation from accuracy". According to Rouche (1988), there is a connection between creativity in a new situation and imagination with an error which reveals inadequacy of knowledge. A lack in mastery of basic facts, concepts and skills lead to the error. Students make errors that are different from algorithmically based errors (Olivier, 1989). Additionally, three types of errors defined, which are unsystematic, systematic or random (Green, Piel, & Flowers,

2008). Unsystematic errors are unintended, non-repeated and trivial incorrect answers. Learners can correct unsystematic errors when they recognize them (Khazanov, 2008). Contrary to unsystematic errors, repeated wrong responses are systematic errors and these wrong answers are regularly recreated across time and space. Moreover, learners cannot realize systematic errors, which are based on symptoms of a faulty hypothesis referred to as misconceptions (Green, et. al, 2008; Nesher 1987). Lastly, random errors may or may not be repeated. Similar to Green et al. (2008), Cox (1975) categorized errors into two groups as systematic errors and random errors. Repeated wrong answers were defined as systematic errors, which is in agreement with the definition of Green et al. (2008). These types of wrong answers were detected in specific algorithmic calculations. Conversely, random errors, which are similar to the random error definition of Green et al. (2008), do not give any evidence, associated with the repeated wrong thinking process.

As for the reasons why students make errors, it can be inferred that although teachers have an important role in transferring the currently accepted disciplinary concepts to students' fresh minds (Yağbasan & Gülçiçek, 2003), students do not come to schools with fresh minds (Resnick, 1983). They prepossess numerous ideas, knowledge and conceptions expressing some of the mathematical and scientific phenomena that are related to the concepts presented in the classroom (Smith, diSessa, & Roschelle, 1993). Sometimes these students' conceptions are inconsistent with expert concepts, which lead to the creation of a gap between students' conceptions and the corresponding expert concepts. Therefore, misconceptions may be the underlying reasons of the errors made by students.

At this point, then, it is essential to define the concept of *misconception*. According to Yağbasan and Gülçiçek (2003), misconception occurs when an individual attributes meaning to a concept that is fundamentally different from its scientific meaning. It is seen that the term "conception" comprises the origin of all these terms; thus, the term "conception" is important in understanding the term "misconception" (Hammer, 1996), which is used to address students' conceptions that build a systematic pattern of errors (Smith, diSessa, & Roschelle, 1993). Similarly, Yağbasan and Gülçiçek stated that misconception is a deviation among the internalized form of a concept and its scientific meaning (2003). According to Swan

(2001), a ‘misconception’ is not wrong thinking; however, it is a concept in embryo or a local generalization that the student has made. Basically, it may be a natural process of development. These definitions emphasize the fact that it is a conception deficient in quality to be considered as a scientific conception. Considering how misconceptions occur, it can be said that misconceptions are often caused when new information is added to an inadaptable knowledge base, producing consecutive, synthetic models (Behr, Harel, Post & Lesh, 1992). Behr et al. (1992) claimed that some misconceptions may derive from new concepts not being vigorously connected with the student’s previous concepts. On the other hand, some other misconceptions may stem from the lack of some essential details of the knowledge-scheme which has been oversighted in the design of the teaching material. Furthermore, misconceptions can be stable and difficult to change (Garfield & Ahlgren, 1988).

Even though the terms *error* and *misconception* are not synonymous, they are highly interrelated. These terms are used interchangeably because they are generally confused by many people. Whereas misconception is the perception of people’s concepts different from their scientifically accepted counterparts (Keşan & Kaya, 2007), error is the consequence of the misconception. Casey, Ernest and Koshy (2000) defined the term error as “a wrong idea or wrong action that often is the result of a misconception, but not always so” (p.172). Because of the relationship between error and misconception, as mentioned in the previous definitions of Keşan et al. (2007) and Casey et al. (2000), analysis of errors helps to understand the reasons underlying the errors, which may be misconceptions, and, thus, to focus on the possible misconceptions held by students. In other words, all misconceptions should be defined as an error; however, not all the errors can be regarded as a misconception (Eryılmaz & Sürmeli, 2002). To sum up, according to Nesher (1987), in the relationship between error and misconception, error is the image on the surface and there is a misconception that causes and controls the formation of that image.

Thus, it can be maintained that students who make errors in many of the mathematics topics, one of which is the topic of integers, have misconceptions as underlying reasons of the error related to those topics. Having defined error and misconception, the following section will be devoted to the definition of integer.

2.2. Definition of integer

Teacher and students may describe integer concepts by using written symbols, spoken language, concrete materials, and real world examples (Herstein, 1996). However, there is no doubt an account for much of the difficulty of teaching and learning about integers (Davidson, 1987). In addition, there is no agreement on a single definition for integers used in the literature (Musser et al., 2003). In consideration of the purposes of the study, it is essential to review the definitions of integers that emerge in the related literature.

Integers are defined as follows: "A set which comprises of positive integers, negative integers, and zero; and the set are shown with the notation of Z" in the middle school mathematics curriculum (TTKB, 2005, p.132). Another similar definition is expressed indicating that "The set of integers, I, consists of the positive integers, the negative integers, and zero. I= {…,-4, -3, -2, -1, 0, 1, 2, 3, 4 …}" (O' Daffer, Charles, Cooney, Dossey, and Schielack, 2008, p.249). One often notices that the terms "integer", "directed number", "positive or negative number", "signed number" and "number opposite" are used interchangeably at the middle school level to refer to the same mathematical entity, namely the set of numbers …-3, -2, -1, 0, +1, +2, +3… (Rising, 1974).

Musser, Burger, and Peterson (2003) give a detailed explanation of positive and negative integers: "A set in which 1, 2, 3 … are called positive integers, and "the numbers -1, -2, -3… are called negative integers". Furthermore, zero does not have a sign and it is "neither a positive nor a negative integer" (p.319). Dyke, Rogers, and Adams (2009) state that "positive numbers, zero and negative numbers are called signed numbers" (p.674). However, signed numbers may not be considered as integers by middle school students. Hence, teachers should use the term "integers" rather than the signed numbers terminology when they explain the subject of integers (Adams, 2009).

In a study by Cohen and Hativa (1995), students expressed negative numbers in several ways: as a number below zero, smaller than zero, small than all positive numbers, numbers to the left of zero; zero minus something, or subtracting a large number from a small number. Moreover, several students expressed negative

numbers by writing out an example, rather than a verbal definition, which resulted in a negative number (e.g., 5 - 8), or by drawing a number to the left of zero on a number line. Furthermore, some students gave examples based on daily experience, e.g., a thermometer with numbers below zero, or a place that is geographically below sea level.

For the specific topic of negative numbers, Piaget (1960) affirms:

> Regarding the spontaneous numerical operations, we all understood them at all times, since their application to the economical exchanges or to the paths traveled, which when buying more than what has been paid, a debt is acquired and when going back more than what has been advanced, a backwards progress is carried out, which duly constitutes a use in the action of the negative number. (p.109)

In the explanation of Piaget, many daily life examples are given in order to understand the action of negative numbers. For instance, when a student experiences a situation where he/she buys more than what has been paid for, the student can understand the concept of integers and its necessity.

Now that the concepts of error, misconception and integer have been defined, the next section will highlight common errors, difficulties and misconceptions regarding integers.

2.3 Common Errors, Difficulties and Misconceptions Regarding Integers

In primary education, students experience difficulties in understanding and learning mathematical concepts (Bulut et. al, 2001), particularly 'integers', which is one of the most problematic topics (Dereli, 2008; Hayes & Stacey, 1990; Kilhamn, 2008; Körükcü, 2008). Janvier (1983) states that students experience difficulties in understanding concepts regarding negative numbers and in performing procedures with negative numbers. The literature review on students' understanding of integers shows that students typically have difficulties and misconceptions with regard to integers. Thus, literature was particularly reviewed in terms of students' errors, difficulties and misconceptions with regard to integers.

According to Schindler et al. (2013) both the handling of real-world situations, such as comparing temperatures below zero, and the handling of inner mathematical situations, such as solving the equation $x+4=1$, are associated with the concept of

negative integer. Fraenkel (1995) states that negative integers are regarded as a rather basic topic and, thus, can be learned in middle schools. In fact, according to Peled, Mukhopadhyay, and Resnick (1988), children build up internal representations of negative numbers before they receive formal school instruction on negative integers. Thus, the introduction of negative integers does not create great difficulties for students, except for the multiplication of a negative number by another negative number.

Several studies show low success rates of students solving computations involving negative numbers (Human & Murray 1987, Murray 1985), which indicate deeply-rooted and widely-held misconceptions. These difficulties are not limited to the above mentioned multiplication operation, in which students do actually experience great difficulty; yet the difficulties experienced also stem from other mathematical operations, such as addition, subtraction and ordering of integers.

Another study, conducted by Pratiwi et al. (2013), aimed to investigate the students' understanding of the notion of negative number through number line activities. The study was conducted with nineteen students of 9-10 ages. The research results revealed that students tried to interpret negative numbers based on their actual knowledge on natural numbers. In more detail, the students admitted that negative numbers were the inverse of whole numbers. In other words, students considered that all properties of negative numbers were the same with the properties of positive numbers. In fact, students became aware of the fact that the sign before the numbers attributed meaning to them, so their previous knowledge regarding natural numbers not only fostered their understanding of integers but also confused them.

Similarly, a study by Bruno et al., (1999) concentrated on students' difficulties related to negative numbers. The study revealed that the students encountered difficulties when they used their existing conceptions about natural numbers to make sense of negative numbers. It was found that the students could regard negative numbers as the inverse of positive numbers; however, this idea was deficient. Similarly, a study by Ponce (2007) found that many students experienced difficulties in making the transition from working with whole numbers to working with integers. Consequently, many students become confused about different types of numbers and do not understand that all different types of numbers are part of the system of real

numbers (Bruno, et. al., 1999). The results of the study indicated that in order to prepare children to understand the wider numerical system, the notion of negative numbers must be well-understood.

In another study, Linchevski and Williams (1999) also state that students have difficulties in enhancing the concept of negative numbers. Linchevski and Williams (1999) found that the pre-assumed structure of natural numbers that exists in students' minds can serve as an auxiliary element when students learn the positive numbers. On the other hand, in terms of negative numbers, students cannot reach informal information by observing and experiencing the environment because there are no non-positive objects or groups of objects in the physical world (Davidson, 1992; McCorkle, 2001). Actually, students cannot observe negative numbers in the environment concretely so they have difficulties in learning integers (Davidson, 1992; Mc Corkle, 2001).

A study was conducted by Bishop et al. (2014), which concentrated on obstacles and affordances of learners within the domain of integers and on the similarities and differences between children's conceptions and the historical acceptance of negative numbers. The researchers stated that adults, as opposed to children, often operate with numbers proficiently, especially with negative numbers without deep thought. The various metaphors, contexts, and understandings that they bring with problems facilitate them to think of and use numbers easily in multiple ways. For instance, considering the number -5, one can interpret -5 as any of the following: an action of removing 5 from a set, the integer between -6 and -4, an action of moving 5 units left or down, the number yielded when 5 is added to 0, the location on a number line (coordinate plane, etc.) 5 units to the left of, or below 0, and a representation of a $5 debt. Moreover, Henley (1999) stated that negative numbers can be considered as indicators of a process that should be carried out of subtraction in spite to a mathematical object. In history, mathematicians considered subtractive numbers as conceptions for negative numbers (Henley, 1999). According to Gallardo (2002) the minus sign in -3 showed the intention of subtracting 3 from a certain number, but the subtracted number was not given; thus subtractive numbers were considered as "quantities to be subtracted" (Henley, 1999, p. 647).

In addition, Lamb et al. (2012) claimed that students' interpretation and usage of the minus sign are facets of symbol sense, which Arcavi (1994) described as "making friends with symbols" (p. 25). "Making friends with symbols" contains an understanding of symbols, usage process and reading symbols. In more detail, students can distinguish the minus sign which represents an operation from the minus sign which belongs to a negative integer without difficulty. The results of this research indicated that many students in middle school and even high school do not have fully improved sense of symbol associated with the minus sign. It was found that this limited view of the minus sign impedes with students' abilities to truly understand some concepts related to integers, such as the process of solving equations, and to make sense of variables. In the same study, Lamb et al. (2012) believed that encouraging students' sense making of the three meanings of the minus sign and the students' ability to identify when each meaning might be appropriately used are significant. The researchers suggested that students needed time and attention to learn the different meanings of the minus sign, to recognize and distinguish the appropriate meaning in a problem, to understand when the meaning shifts during problem solving, and to easily move among the meanings.

One other study carried out by Gallardo (2002), explored students' efficiency in operations where the solution of equations were related to the domain of integers in which spontaneous answers in problems conveyed negative solutions. This study was conducted with thirty-five students. According to the results of the study, students considered numbers as they would do in counting objects. The results also revealed that when students work with negative numbers in the form of symbols, students are forced to free themselves of the concrete meaning of words that are included in real life problems. For instance, a student can consider ten feet below sea level or ten steps backward easily; however, (– 20) has little or no meaning for students; it is an isolated bit of information.

As for Bishop et al. (2013), they postulate that students, during their school experiences, experience and develop three underlying understandings of numbers: an ordinal, a cardinal, and a formal understanding of numbers (Bishop et al., 2013; Baroody & Wilkins, 1999; Clements & Sarama, 2007; Fuson, 1992; Lakoff & Núñez, 2000). Students need each of these understandings to strongly comprehend

integers and they utilize more than one understanding of number to reason about a single integer problem (Bishop et al., 2013). Bishop et al. (2013) stated that the idea of order is a basic principle of our number system. When students learn counting and reasoning about smaller and greater numbers, they initially engage in ordering. Bishop et al. (2013) claimed that an ordinal (or positional) view of number is associated with the idea of ordering relations. In this view, numbers are sequenced and ordered such as −4 is a number greater than −5 and less than −3; however, it has no relation with countability or quantity (Clements & Sarama, 2007). Bishop et al. (2013) designated that a cardinal view of number, which is a countable quantity, is the second understanding of a number. A number's cardinality is the number of objects it represents; indeed, this way of understanding number is related to numeration, counting, and magnitude. Bishop et al. (2013) recognized that a relationship is present between counting and the cardinality of a set of objects within the realm of whole numbers. For example, counting like 1, 2, 3, 4 can be considered as the existence of four pencils. Cardinal and ordinal meanings of number are not meanings that are independent of each other for children. When children's mathematics extends to the entire set of integers, one must pay attention to new issues and questions that emerge with respect to cardinal and ordinal meanings of number. Bishop et al. (2013) claimed that a formal entity is the third view of number. Students generalize from what they already know to be true about whole numbers and operations with them. According to Bishop et al. (2013), in this understanding, numbers can be treated abstractly. When children's understandings of numbers extend to new domains, such as from whole numbers to integers or from whole numbers to rational numbers, students have opportunities to search for and make use of underlying structures. For instance, a child might use a formal understanding of a number to reason about the statement −4−−4 by extending principles and rules he or she has discovered about whole numbers to negative numbers such as any number subtracted from itself is zero. This type of generalization is an example of a formal approach to number (Bishop et al., 2013).

In addition, Bishop et al. (2014) state that the idea of order is a fundamental principle of our number system. Similarly, Lakoff and Núñez (2000) designated order as a foundational component of mathematical cognition and arithmetic as "motion along a path" as one of their four "grounding metaphors" (p.21). By using the idea of

symmetry on the number line, they asserted that negative numbers are constructed as point locations within this motion metaphor. To illustrate, when some young students solved problems containing integers, they emphasized the idea of order, not its formal form. Especially these students constrained some kind of ordering on **Z** and then used the ordinal, or positional, nature of numbers in their strategies. "Ordering negative numbers is complex because there are two possible orderings that are supported by thinking about common contexts-the standard ordering and ordering by magnitude (absolute value)" (Widjaja, Stacey &Steinle 2011, 81). Students confused these two possible orderings so they encountered some difficulties when they ordered integers. More specifically, students did not know when they should order integers according to common context-the standard ordering or ordering by magnitude. That's why, students made some errors and had difficulties regarding ordering integers.

Bishop et al. (2014) also observed the underlying mathematical ideas of order in some children's ways of reasoning. According to the results of the study, 43% (20 of 47) of the students in the study by Bishop et al. used the ordinal and sequential nature of integers at least once in solutions that included counting or the use of the number line, the context of motion and movement. For instance, many students who correctly solved the problem 3 – 5 = c used a counting-back strategy. Generally, students counted backward beginning with 3, like "3, 2 (put down one finger), 1 (put down second finger), zero (put down third finger), -1 (put up fourth finger), -2 (put up fifth finger). The answer is -2." In this strategy, -2 is a position; it is the place one finds when beginning with 3 and counting back from 5; it is the number before -1 and after -3. The results of the study also showed that young children's conceptions about the mathematical idea of order were used successfully to solve new and novel problems including integers although they were not as formalized as the modern mathematical definition of ordering. Hence, Bishop et al. (2014) stated that an order-based understanding of negative numbers is a cognitive affordance for children's integer reasoning. It was found that ordered numbers are locations on a number line or elements in an ordered sequence. This view of ordered numbers is also supported by both mathematicians and children to engage with integer tasks using number lines and counting sequences. Schindler and Hussman (2013) suggested that previous knowledge of students regarding the ordering of integers should be considered while digging into basics of negative integers.

In another study, Gallardo (1995) documented that less than 40% of the 12-to-13-year-old children she interviewed were able to subtract integers. Similarly, Kloosterman (2012) implied that more than one fourth of all 13-year-old children were unable to add a positive number to a negative number, and further, about half were unable to divide integers, correctly.

Difficulties experienced by students in relation to integers were also investigated in Turkey, even though, the number of studies were limited. One example of a study examining the difficulties experienced in integers by middle school students was conducted by Köroğlu and Yeşildere (2004). They studied how the unit of integers was handled by middle school 7th grade students. The study was conducted with 78 students, thirty-nine of which comprised the experimental group and the other half of constituted the control group of the research study. Research results revealed that middle school students had difficulties in expressing sets of integers and held misconceptions regarding signs. According to the results of the study, middle school students were not able to decide which number was an integer in the given set of integers which included only negative numbers. Köroğlu and Yeşildere (2004) stated that the relations between number sets were the main points of consideration in mathematics. Not being able to determine the relations between integers and natural numbers, students will establish new misconceptions in mathematics in the following years. The results of the study also revealed that students could not clearly determine whether the positive or negative signs of integers (+, or -) indicated the signs of the integers or the operation of the integers. Actually, one of the misconceptions encountered in adding or subtracting the integers was that students confused the sign in front of the integer in that they could not identify whether it belonged to the operation or the integer. It was found that students were also confused about how to make operations with integers when integers had different signs. In fact, students were confused while adding negative and positive integers. Likewise, Spang (2009) stated that middle school students were not able to distinguish between the subtraction sign and the sign of a negative number, and nor could they distinguish between the plus sign and the sign of a positive number. Moreover, it was found that students experienced difficulties with exponential integers and with dividing zero by an integer. Köroğlu and Yeşildere (2004) underlined that students do not understand logically but memorize the fact that the result of dividing integers by zero is

undefined. Therefore, students confuse the division of zero by an integer and the division of integers by zero.

Similar to Köroğlu and Yeşildere (2004), Ercan (2010) conducted a study with 628 middle school 7^{th} grade students who were selected randomly. Students were administered a test including examples and non- examples of integers. Research results indicated that the sign of a number had an important role in deciding whether a number was an integer or not, and students were not sure when integers were written in a decimal or a rational number format. This study also required students to justify their decisions as to why some items in the test were examples of integers and some were not. The results of the study showed that students were undecided about the written form of integers through different number sets (e.g. whether 3.0 is an integer and a decimal number) and many different causes were articulated to verify why some items were integers and others were not. The justifications provided by these middle school students pointed out that the concept of integer is perceived differently and hence they have different integer descriptions.

Another study by İşgüden (2008) examined the difficulties that 7th and 8th grade students experienced in integers with respect to writing a set of integers, and placing positive and negative integers on a number line. The study showed that students experienced difficulties in the following areas: deciding whether zero is an integer or not, where the negative integers are on the number line, how negative integers are ordered. The first difficulty was experienced in deciding whether zero was an integer or not. For students, zero meant "having nothing" (Lytle, 1992; Levenson, Tsamir & Tirosh, 2007). Students considered positive integers as counting numbers and they regarded zero in the same way, so positive integers and zero did not pose any problems (Steiner, 2009). According to Steiner (2009), in order to be successful with integers, students need to understand that zero can be considered in many different ways. Zero might be added to any number without changing the number's value because zero is the identity element for the operation of addition. Zero stands for the same number of positives as negatives and so long as the number of positives added is the same as the number of negatives added to a number, the value of the result does not change. Zero can be considered in many ways, such as two neutral pairs, which is a different meaning from having nothing, and students can be confused by

this thought. Until the time a student experiences negative integers, zero always means the same as "nothing". The second difficulty is related to the places of negative integers on the number line. The results of İşgüden's study indicated that students were confused about the places of negative integers on the number line. In fact, some students could not be sure as to whether negative integers were to the left of zero and positive integers to the right of zero on the number line. The third difficulty was experienced in the ordering of integers. It was found that students could not order integers correctly. Likewise, according to Julie et al. (2013) many students still make errors when they compare two negative integers. Students still think that the way to compare two negative numbers is the same as the way to compare two positive integers. For instance, when students were asked to compare -25 and -20, some students believed that -25 is greater than -20 because 25 is greater than 20. When they ordered the positive integers, they started from the smallest; however, when they ordered negative integers, they started to order them from the largest. More specifically, students were asked to order the following numbers from the smallest to the largest: -23, -29, 34, 27, 13, -22, -16, and 31. Many of the students' answers were wrong as they provided a response, such as -16, -22, -23, -29, 13, 27, 31, 34.

By considering these difficulties that students encountered, one can infer that teaching negative numbers is also crucial in order to decrease the level of difficulties faced by students. In teaching negative numbers, some contexts had already been used by researchers such as the context of the temperature, abacus and the dice (Linchevski & Williams, 1999), assets and debts (Stephan & Akyuz, 2012), and the use of the number line to represent the operation in negative numbers (Heefer, 2011). As for a study conducted by Julie et al. (2013), it was indicated that establishing a context to introduce negative integer was difficult for teachers. According to the results of the study, at some points, some of the contexts mentioned above could help the learner to better understand integers. However, some of them were appropriate for higher grades as they included some abstract calculations. For example, in Indonesia, students do not know the condition in sub-zero temperatures, so the construction process of negative integers with the temperature context is difficult for them (Pratiwi et al., 2013). More specifically, these students experience great difficulty in constructing negative integers in their mind when the teacher uses the

temperature to introduce negative integers because they do not experience sub-zero temperatures in their real life context.

As previously mentioned, the research conducted by Pratiwi et al. (2013) implied that the local culture is an important factor to consider when the teacher chooses a context to teach a mathematical concept. Results of the study demonstrated that some of the contexts mentioned above could help the learner to better understand negative numbers. However, some of those included abstract calculations so they were appropriate for higher grades, and probably not useful for lower grades. The results of the study also showed that the students wanted to know why negative numbers emerged in their life. They need to experience negative numbers in a particular situation in their life. The first conception of students about negative numbers may emerge when they are doing an operation of subtraction of numbers without signs, yielding a result with signed numbers. According to Pratiwi et al. (2013), students' initial phase of comprehension regarding negative numbers should be reinforced by means of a meaningful way of learning. Students might conceive negative numbers if they learn negative numbers through a familiar context that trigger their common sense and intuition.

In addition to the importance of teaching integers mentioned above, the number line also has a crucial role in not only teaching but also learning the concept of integer. Resnick (1983) claimed that even before school entry, most children constitute a mental number line for the positive numbers. Primarily, children use the number line to compare the relative sizes of numbers and after the first years of schooling they progressively connect the number-line representation to the operations of addition and subtraction. Furthermore, the usage of the number line to make comparisons among negative numbers was recommended for grade levels between five and eight in the curriculum of the National Council of Teachers of Mathematics (1989).

Van de Walle (2004) claimed that in order to have a better understanding of integers and operations with integers, students need experience in both the number line model and the neutralization model. Wilkins (1996) indicated that the number represents a given number of objects, which is the cardinal conception of number, while a given number represents a position relative to other numbers, which is the ordinal conception of numbers. The number line model accentuates the ordinal concept of

number, while the neutralization model accentuates the cardinal concept of number. The cardinal concept of number is learned before the ordinal concept of number because children usually learn to count numbers of objects before understanding their order or position (Wilkins, 1996). However, both concepts of ordinal and cardinal understandings have great significance for students in order to understand integers completely (Davidson, 1987).

In addition, the number line is used more than the device itself and indicates an underlying way of reasoning the idea of order. (Wallis, 1685). Two different types of number line conceptions are studied by Peled (1991). The first type contained a continuous number line where numbers were ordered from lesser to greater. The second type of number line contained a divided number line that was disjointed at zero. Actions were considered either toward or away from zero. The student would determine how much was needed to obtain zero and then continue from there. In another study, Janvier (1983) notes two types of models which are equilibrium models and number line models. Subway stations associated with direction and weather problems associated with temperature are some applications of the number line model. These models provide opportunity to implement more concrete meaning to the numbers on the number line since students are familiar with the idea of opposites such as “east” and “west” or “above” and “below”.

A research was conducted by Widjaja et al. (2011) to explore the misconceptions of the number line, which are revealed when pre-service primary teachers locate negative decimals on a number line. Data were collected from 94 pre-service primary teachers. The results of the study indicated that many pre-service teachers also underline the need to provide them with strong visual models of negative numbers and their ordering in the distorted geography on the number lines. In addition, some standard models of negative numbers (e.g., lending and borrowing money) are unhelpful in ordering negative numbers.

Consequently, in order to eliminate misconceptions and minimize errors regarding negative integers, some activities are suggested in various sources. NCTM (2000) recommended that by extending the number line and through familiar applications, all students in grades 3 to 5 ought to investigate numbers less than 0. NCTM (2000) stated that simple games that include losing and gaining points or experiences with

debt can enable very young children to grasp the idea that it is possible to have less than nothing.

For better understanding of negative integers, NCTM, in their publication, *Principles and Standards for School Mathematics* (NCTM, 2000) recommends the inclusion of three activities in the fifth through eighth grade. The first activity entails problem situations in which students learn that negative numbers are rational extensions of positive numbers. In this context, students should solve problems of the form x – y, where x is less than y and both x and y are positive integers. The second type of activity enables students to compare and contrast two negative integers in order to understand the similarities and the differences between negative and positive integers. These types of activities help students to learn order within the negative integers. In this way, students can realize the importance of the magnitude of integers. The third type of activity gives students the opportunity to become aware of the fact that operations with negative integers are just extensions of the properties and operations with counting numbers.

Thus far, some of the common errors students made, difficulties they encountered and some misconceptions they held regarding integers have been highlighted. However, it is also crucial to review various studies conducted on integers, which are summarized and constitute the topic of the following section.

2.4 Studies on Integers

According to Galbraith (1974) the necessity for negative numbers emerges in the area of measurement both everyday life and in the mathematical realm of numbers. Thus, it is natural that the concept of integers is dealt with at all three levels of education: at elementary, secondary, and tertiary levels. Students encounter the concept of integers throughout their entire educational life in various contexts. To illustrate, at middle school level, integers are seen in the measurement of temperature, credit and bills, sea level, or blood groups, while at higher levels of education, they are confronted in some theorems, such as Euclid's Algorithm or prime numbers (Herstein, 1996).

In the Australian curriculum (Australian Curriculum Assessment and Reporting Authority (ACARA) (2011), the continuation of number patterns is mentioned as of

Year 1 and could result in the discovery of negative numbers; however, they are not specifically mentioned until Year 6. In the years 7 and 8, students need to construct on their intuitive understandings to use negative and positive integers to represent and compare quantities and extend the number properties developed with positive integers to negative integers as well (ACARA, 2011).

A large part of a student's school life is spent learning the numerical system. Bruno et al. (1999) designated that there are several extensions of the numerical system to assimilate. Students begin to study numerical systems using natural numbers and finish off studying with real numbers. In addition, they have noted that many students complete their school life feeling confused about the different kinds of numbers and cannot recognize all kinds of numbers. Neither can these students realize that all sorts of numbers are part of a single numerical system, which is the system of real numbers. Bruno and Martinon (1999) claimed that the long process of number learning is the main reason for such confusion. Furthermore, number learning often occurs without one single idea on numbers. Similarly, there is no single unifying perspective regarding how to go about teaching numbers. Therefore, students normally study each of the numerical systems in an isolated way (Bruno & Martinon, 1999).

Integers are related to other mathematical concepts. To illustrate, Steiner (2009) stated that integers are seen not only in algebra, but also in such daily activities as balancing a checkbook, understanding temperatures, and keeping scores when playing some games. The results of a study conducted by Steiner (2009) indicated that the lack of understanding integers is an important reason underlying the difficulties many students experience in beginning level algebra. "The extension of the numerical domain from natural numbers to integers during the process by which twelve to thirteen-year-old students acquire algebraic language constitutes an essential element for achieving algebraic competence in the resolution of problems and equations" (Gallardo, 2002, p. 171). Students are interested in negative numbers as coefficients, constants, and as solutions to equations in algebra.

Likewise, students also experience difficulties in solving algebraic equations (Vlassis, 2002), simplifying algebraic expressions (Christou & Vosniadou, 2012; Lamb et al., 2012), and comparing quantities (Vlassis, 2004) although, they receive

formal school instruction on negative integers in last years. For instance, when students try to add -2x + 7x, students often separate, or ignore, the negative sign in the expression -2x, then they add 2x and 7x to obtain 9x, and then they reattach the negative sign to the expression to get -9x. Students have the tendency to either assign only natural numbers to literal symbols or treat expressions such as -x as though they represent exclusively negative quantities (Christou & Vosniadou, 2012; Lamb et al., 2012). As a result of this study, it was found that the knowledge of negative integers is of importance in the proficiency of solving algebraic equations and simplifying algebraic expressions.

The concept of integer is more abstract than natural numbers for middle school students. In fact, "Non-positive integers are not representable concretely as manipulable objects" (Davidson 1987, p.431) and they are not physically appreciable on their own. Hence, representations have a crucial role in the teaching and learning processes of negative integers. There are most particularly four types of representation, which appeared to be meaningful in getting to know the negatives (Bruno, 1997). The first representation is on the number line or other ordinal arrangements regarding the order of integers (like ... -3 -2 -1 0 1 ...). The second one is a quantity representation, which students mostly learn in relation to natural numbers (e.g. "4 means four pencils"). The third one is the representation within a real-world context (e.g. temperatures, debts and assets). Finally, the last representation is the symbolic representation (e.g. -5 or "minus five"). In addition, physical models give students opportunities to make abstract ideas more concrete, to relate new knowledge to previous concepts. Physical models encourage insight into new ideas, and increase student achievement and motivation (Fennema, 1972; Parham, 1983; Sowell, 1989; Suydam & Higgins, 1977). According to Steiner (2009) models have been used to help students obtain a better understanding of integers and operations with integers. Models should be selected with the intention to motivate students and help them to connect mathematics to the world in which the learner lives (Steiner, 2009). In this way, students can transform abstract ideas to concrete forms which students can relate. Moreover, Russ and Kurtz (1993) stated that models help teachers to equip students with different learning styles and provide teachers with more effective tools of assessment and, thus, help them meet individual needs of students more effectively.

In another research, although students are first introduced to integers with the general model of number line, most children and teachers use, it seems that the neutralization model is more suitable for the comprehension of integers and operations with integers (Davidson, 1987). Children learn and understand cardinality, which is associated with the amount or number of objects, before ordinality, which is associated with position (Davidson, 1987). It would seem, then, that more researchers should search the neutralization model. Janvier (1983) stated that in recent years many other studies in which the neutralization model was benefitted from have been conducted. Some practices of the neutralization method are voting, dancing, attitude, and hot air balloons, all of which students are familiar with. This familiarization gives them the opportunity to transfer their previous knowledge to a new topic.

There are some views related to the concept of integer. Order, magnitude, logical necessity/formal, and computational ways of reasoning are stated as four cognitive affordances in the domain of integers (Bishop et al., 2014). According to Henley (1999) the development of this type of understanding is the main challenge for mathematicians. That negative integers are the quantities to the left of zero on the number line is one view of negative numbers as opposed to the view that negative integers are less than nothing. Their ordering meant that the expression $-2 < -1$ could be interpreted as -2 comes before -1. It does not inhibit to the comment that -2 is smaller in magnitude than -1. Today, students can attain a representation with an alternative view of numbers as ordinal. Therefore, for children to approach negative numbers as an instructional goal, one should consider using ordering relations, magnitude, and logical necessity/formal views of number, and there should be flexibility among these ways of reasoning.

2.5. Summary of the Literature Review

In this chapter, the literature reviewed related to the purposes of the study was presented. In accordance with the purposes of the current study, first of all, definitions of error and misconception and the relationship between them were stated. Subsequently, definitions of the concept of integer were included. In addition, studies on students' common errors, difficulties and misconceptions from related literature were presented. Lastly, some studies on integers were reviewed.

In brief, literature indicated that students made errors and held misconceptions in relation to integers, and that it was not easy to identify all of them; nevertheless, these issues were highlighted as much as possible. Integers are, indeed, complex for learners, so they make errors and establish misconceptions related to integers (Janvier, 1983). To illustrate, middle school students have difficulties in expressing a set of integers and misconceptions regarding signs (Köroğlu & Yeşildere, 2004), in understanding concepts of negative numbers and in performing procedures with negative numbers (Janvier, 1983). Furthermore, many studies have documented students' various difficulties in ordering integers (Bishop, 2014; Julie et al. 2013; etc.). Whether zero is an integer or not, where the negative integers are on the number line, and how negative integers are ordered are some examples difficulties, which are related to ordering integers. Moreover, they have difficulties in solving computations involving negative numbers and operating with integers (Human & Murray 1987, Murray 1985). This review indicated that integers is an abstract concept students learn (Davidson, 1992; Mc Corkle, 2001), and provides a basis for middle school students regarding the upcoming subjects like solving algebraic equations, simplifying algebraic expressions, magnitude; therefore, it has an important role in middle school mathematics (Christou & Vosniadou, 2012; Steiner, 2009; Vlassis, 2002).

As indicated in literature the topic of integers is rarely studied specifically, and the situation is no different in Turkey, where the number of studies on integers with middle school sixth grade students is limited (Dereli, 2008; Ercan, 2010; İşgüden, 2008). Some difficulties students experience in relation to integers are stated in literature. However, there is a gap in the literature in connection to research focusing on the achievement level of middle school sixth grade students, errors made by the students regarding comprehension and ordering of integers and the underlying reasons of these errors. All in all, the literature review supports this study, the purpose of which was to investigate the achievement levels of middle school sixth grade students, the errors they made and the underlying reasons of the errors regarding comprehension and ordering of integers. Results of such a research study would be a significant contribution to the literature in order to gain awareness of middle school 6^{th} grade students' errors and the reasons underlying these errors regarding comprehension and ordering of integers.

CHAPTER 3

METHODOLOGY

The focus of this chapter is the methodology used to conduct this study. More specifically, this chapter will give information about the research design, the participants, the data collection methods, data analysis procedures, the internal and external validity of the study, and assumptions and limitations.

3.1 Design of the Study

The purpose of this study was to investigate middle school 6th-grade students' achievement levels, errors regarding comprehension and ordering of integers and underlying reasons of those errors. The study was conducted in the spring semester of the 2013-2014 academic year in Ankara.

The research design of the study could be considered as a mixed method design, which includes the cross-sectional survey design and semi-structured interviews, since a single methodology would not have been sufficient to collect the essential data required to answer the research questions. Creswell and Plano Clark (2007) defined mixed method as:

> Mixed research is a research design with assumptions as well as methods of inquiry. As a methodology, it involves the assumptions that guide the direction of the collection and analysis of data and the mixture of qualitative and quantitative approaches in many phases in the research process. As a method, it focuses on collecting, analyzing, and mixing both quantitative and qualitative data in a single study or series of studies. (p. 5)

More specifically, the cross-sectional survey design was used in order to identify students' errors. Fraenkel and Wallen (2006) stated that a cross-sectional survey design includes collecting data at a single point in time from a sample that represents

the population. The group could be considered as the middle school 6th-grade students from 10 different classes of a single school.

Particularly, in order to identify underlying reasons of sixth grade students' errors regarding comprehension and ordering of integers, an in-depth study was required; thus semi-structured interviews were conducted. To sum up, first, quantitative data were collected through the achievement test and then qualitative data were obtained through the interviews conducted with selected students.

3.2 Participants of the Study

In this study, all sixth grade students of public middle schools in Ankara were identified as the target population. The accessible population of this study was determined as all sixth grade students of public middle schools in Etimesgut, to which the results of the study will be generalized. Thus, the sampling method used in the first phase was convenience sampling, which involved collecting data from the individuals who were readily available (Fraenkel & Wallen, 2006). The study was conducted in the middle school where the researcher was working as a math teacher. In the sample selection procedure, the research was carried out with 262 students who were sixth grade students in one of the public middle schools in the Etimesgut district, Ankara. The 10 6th grade classes in the middle school comprised the sample of the study. The average age of these students was 11 and the boy to girl ratio was almost equal with a slight male dominance, as can be seen in Table 3.1 Detailed demographics regarding gender, age and grade level of the participants of the study are presented in Table 3.1 below.

Table 3.1 Participants' Demographic Information

Class	Sample size (n)	Age (years)	Gender	
		Average	Boys	Girls
6-A	25	11	13(52%)	12(48%)
6-B	26	11.12	13(50%)	13(50%)
6-C	27	11.11	13(48.15%)	14(51.85%)
6-D	24	10.85	12(50%)	12(50%)
6-E	27	11	14(51.85%)	13(48.15%)
6-F	26	10.79	13(50%)	13(50%)
6-G	27	11	13(48.15%)	14(51.85%)
6-H	26	10.85	14(53.85%)	12(46.14%)
6-I	29	11.14	14(48.28%)	15(51.72%)
6-J	25	10.96	13(52%)	12(48%)
Total (N)	**262**	**10.98**	**132(50.37%)**	**130(49.63%)**

In the second phase of the study, semi-structured interviews were conducted to reach data regarding the underline reasons of students' errors. In order to select the students to be interviewed, participants were categorized into three groups in terms of their ability levels, measured in accordance with their performance in the achievement test: The first group was defined as "high", meaning they correctly answered most of the items and their scores were higher than 60 according to rubric; the second group was defined as "medium", meaning their scores were between 40 and 60; and the third group was defined as "low", meaning their scores were lower than 40. Low and medium levels of students were separately asked whether they were voluntary to participate in semi-structured interviews. 8 students from low and medium levels (approximately 3% of all participants) were willing to participate in the interview. The aim of categorizing students and asking each level of students separately for voluntariness was to provide participation in interviews from low and medium performance levels. Students were selected from low and medium performance levels in order to identify the underlying reasons of sixth grade students' errors regarding comprehension and ordering of integers. The sampling procedure and participants of the study is summarized in Figure 3.1.

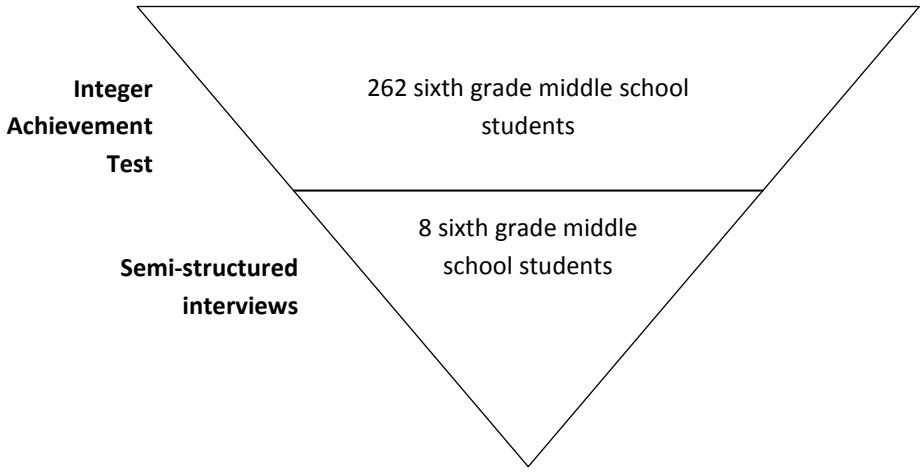


Figure 3.1 Sampling procedure and participants of the study

3.3 Data Collection Methods

The data for this study were collected through the Integers Achievement Test (IAT), which was designed and adapted by the researcher based on the objectives of the 6th grade curriculum of the Ministry of National Education. The interviews were conducted with the students who were selected based on their responses to the questions in the achievement test. The data collection methods, instruments, and procedures are explained below in detail.

3.3.1 Achievement Test

The achievement test (IAT) was developed specifically for this study to identify middle school sixth grade students' errors related to comprehension and ordering of integers. Firstly, the 6th grade objectives of Turkish National Middle School Mathematics Education Curriculum were examined. In the 6th-grade curriculum, there were two objectives specific to the topic of integer. The first objective was that students be able to explain the concept of integers. The second objective was that students be able to order integers. Subsequently, the literature on the errors and difficulties of students related to integers was reviewed. Then, taking into consideration the curriculum objectives, the researcher developed the items of the Integers Achievement Test (IAT), some of which were adapted from the related literature. In the process of developing the items of IAT, the researcher matched each question with the objectives in order to ensure that there was at least one item which

measured an objective stated in the sixth grade mathematics curriculum. IAT, which was composed of 8 items, was developed using this information. Explanations and details of each item in IAT are given below.

The first item which was developed by the researcher in IAT measures students' knowledge of expressing the given statement as an integer. Some statements, such as "below zero", "left of zero on the number line", "loss" and "depth", indicate negative integers. On the other hand, some statements, such as "above sea level" and "money in the wallet", indicate positive integers. To solve this item, students needed to have an understanding of daily life examples of negative and positive integers. The 1st question is presented in Figure 3.2 below:

Q1:

Write each following statement as an integer.

a) 7^0C below zero ……

b) 500 m above sea level …….

c) 12 units left of zero on the number line ……..

d) Loss of $25 ……..

e) $10 in your wallet …….

f) A depth of 1200 meters …….

Figure 3.2 The first item of IAT

The second item in IAT measures the students' knowledge of ordering integers. In the question, three girls' comparisons of their hair lengths were given and these comparisons included negative integers. Students were expected to explain the ordering of these integers, which represented girls' hair lengths. The second item was adapted from the study of Littell (2008). The 2nd question is presented in Figure 3.2 below:

Q2:

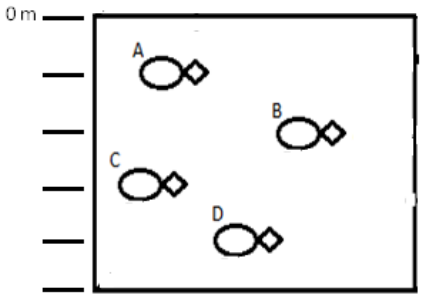
Simge and Rüya are comparing their hair length to their friend Yağmur's hair length. Simge states that her hair is +4 cm compared to Yağmur's hair and Rüya states that her hair is3 cm compared to Yağmur's hair. Who has the shortest hair? Write the girls' names in order of their length from the shortest to the longest.

Figure 3.3 The second item of IAT

Similar to the first item, the third item, which was developed by the researcher in IAT, measures the students' knowledge of comprehending integers. Moreover, the third item evaluated the students' knowledge of identifying a location as an integer. Locations of four fish in an aquarium, whose surface is 0 meter, were shown in the question. Students were expected to express locations of fish as integers by considering the surface of the aquarium to be 0 meter. The 3rd question is presented in Figure 3.4 below:

Q3:

The locations of four fish in an aquarium are shown. Aquarium is completely filled with water and the water surface is 0 meter. Between each of the lines to the left of the aquarium is 1 meter. Write the integers corresponding to the points A, B, C, D, which shows the locations of the fish in the aquarium.



A:

B:

C:

D:

Figure 3.4 The third item of IAT

The fourth item in IAT, which was developed by the researcher with the help of an experienced mathematics teacher, measures the students' knowledge of ordering

integers. For this purpose, all buttons of an elevator were given in the figure and ground floor was represented as zero. Students were expected to identify the numbers of the elevator buttons, which were pressed by Ayşe and the nurse. The fourth question is presented in Figure 3.5 below:

Q4:

When Ayşe took the elevator from the ground floor in a hospital, she pressed the wrong elevator button. She went to the radiology service instead of the blood collection service. With the aid of a nurse in the elevator, the upper floor was pressed. Identify the number of buttons which were pressed by Ayşe and the nurse. Please explain your answer.

Ayşe:

……

……

……

Nurse:……

……

……

……...

Figure 3.5 The fourth item of IAT

The fifth item in IAT also measures the students' knowledge of comprehending integers. In this item, four pairs of integers and the reference, which is -2, were given. Students were expected to identify which integer in the pair was closer to the reference integer by using a number line. This fifth item was adapted from Lappan and Glenda (2006). The 5th question is presented in Figure 3.6 below:

Q5:

For each pair of temperatures, identify which temperature is further away from-2 ^{0}C. Please explain your answers.

a. 6^0C or -6 ^{0}C?

Why? ……

……..

b. -7^0C or 3 ^{0}C?

Why? …….....

.....……

c. 0 ^{0}C or -5 ^{0} C?

Why? ……

……..

d. -10 ^{0}C or 7 ^{0}C?

Why?

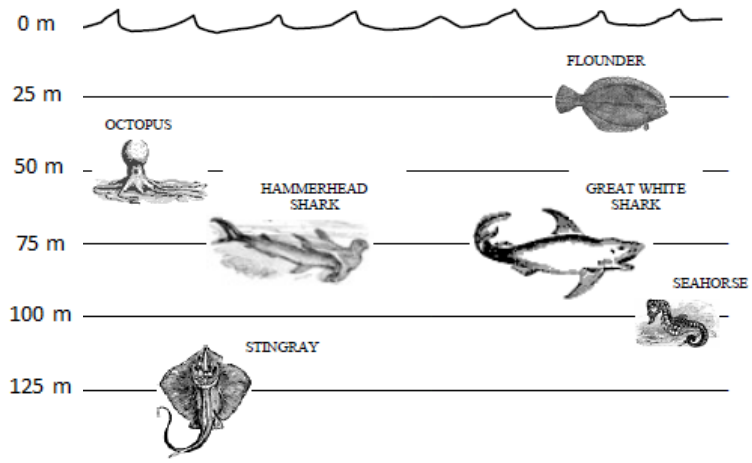
.……

Figure 3.6 The fifth item of IAT

The sixth item in IAT, which measures students' knowledge of comprehending and ordering integers, has six sub-questions. In the first part, students were expected to find the deepest point at which a sea creature lived and students were expected to provide a detailed explanation for this sub-question. In the second part, students were expected to identify the sea creature which lived at a depth of 100 m. In the third part, students were expected to identify the depth of stingray. In the fourth part, students were expected to decide whether the sea horse or the great white shark lived closer to sea level. Moreover, students were expected to explain their answer in detail for this sub-question. In the fifth part, students were expected to write the depth of each sea creature as an integer. In the last part, students were expected to order the depth of each sea creature. The sixth item was adapted from the study of Littell, (2008). The 6th question is presented in Figure 3.7 below:

Q6:

Living beings as fish and octopus live in the sea. Factors such as dissolved oxygen and pressure etc., which are necessary for the existence of these living beings, can differ with respect to depth. Therefore, these living beings exist at different depths. In the picture below, some examples of underwater living beings are given. Answer the questions according to the pictures below.



a. Which sea creature exists at the deepest point? Explain your answer.

b. Which sea creature exists at the 100 m depth? …….

c. At which depth does stingray exist? ..……

d. Which of the sea horse and great white shark lives closer to sea level? Explain your answer…….

e. Please write depth of each sea creatures as an integer.

……

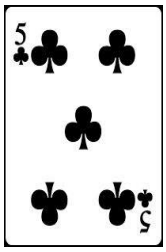
f. Please order integers which were found at part e from smallest to largest.

Figure 3.7 The sixth item of IAT

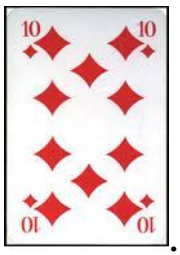
The seventh item of IAT, which was developed by the researcher, measures students' knowledge of comprehending integers. In this item, there is a game in which black cards indicate gain; red cards indicate damage as much as the number written on the card. Some cards were given and students were expected to write integers for each card. The 7th question is presented in Figure 3.8 below:

Q7:

Ahmet and Batuhan developed a game by using a deck of cards. Cards with pictures and A are removed from the deck of cards. They decide that each red card indicates damage to the extent of the number on the card and each black card means gain to the extent of the number on the card. For example, a card which represents +5 is shown below.



Batuhan's cards are shown in the figures below. Please, write each card as an integer.



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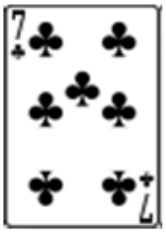

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Figure 3.8 The seventh item of IAT

The eighth item in the IAT measures students' knowledge of ordering integers by using a number line. For this purpose, a rule is developed in that a positive integer indicates increasing temperature, while a negative integer indicates decreasing temperature. Students were expected to find the new temperature according to the result of change in the thermometer by using the number line. The eighth item was adapted from Lappan and Glenda (2006). The 8th question is presented in Figure 3.9 below:

Q8:

In this problem, positive integer means increasing temperature and negative integer means decreasing temperature. Initially, the thermometer showed 25 ^{0}C. After changes in temperature in the thermometer, which degree does it show? (Show your work on the number line.) (For each part of the question, the thermometer showed 25 ^{0}C, initially.)

a. +10^{0}C ……

……...……

…….....

b. -2^{0}C ……

……

……..

…….

c. +30^{0}C ……

……...……

……..

Figure 3.9 The eighth item of IAT

3.3.2 Interview Protocol

One of the purposes of this study was to investigate the underlying reasons of middle school sixth grade students' errors regarding comprehension and ordering of integers. In order to reach this purpose, eight participants were selected for interviews that provided a better understanding of students' approaches to the IAT items, and an in-depth exploration of the errors they made and their underlying reasons.

In this phase, eight of the students were interviewed by the researcher individually in their own classrooms in order to provide a comfortable environment. They were asked to clarify their written answers, and explain the strategies behind their solutions. Moreover, they were expected to give explanations for the items they had left blank or their answers that were unclear. Participants' initial responses to interview questions were probed by the interviewer with such questions as "Why do

you think like this?, How did you get this answer?, What strategy did you use?". These kinds of questions were asked for each of the problems. All the interviews were audio-taped and transcribed. In addition, interviews helped stimulate the thought processes of students and they enabled the researcher to describe how students perceived integers. The semi-structured interview protocol is provided in Appendix D.

3.4 Pilot Study

The pilot study of IAT was conducted in a middle school in the Etimesgut district, Ankara at the beginning of the 2013-2014 spring semester. The purposes of the pilot study were to determine the duration of the implementation of the achievement test, to reveal the points which could cause problems in the actual administration, and to check the validity and reliability of IAT.

The participants of the pilot study were comprised of 46 middle school sixth grade students who were administered an eight-item test in their mathematics class (eighteen sub items) (See appendix C). In the pilot study, students were expected to complete the given achievement test in thirty minutes; however, they experienced some difficulties. At the end of the thirty minutes, they either submitted their papers with some blank answers or requested more time. Thus, the time duration allowed to complete the test was extended in the actual administration from thirty minutes to forty minutes. Furthermore, students experienced some problems in the language of the eighth question. The initial version of the 8^{th} problem is shown in Figure 3.10 below:

Q8:

In this problem, positive integer means increasing temperature and negative integer means decreasing temperature. Initially, thermometer showed 25 ^{0}C. After changes in temperature in thermometer, which degree does it show?

a. +10^{0}C
……..

b. -2^{0}C
……..

c. +30^{0}C
…….

Figure 3.10 Initial version of the 8th question

As difficulties were faced in understanding this question the pilot study, changes were made in the language of the question. The statements "Show your work on the number line" and "For each part of the question, the thermometer showed 25 ^{0}C, initially" were added to question for clarification. The final version of 8th question is presented in the Figure 3.11 below:

Q8:

In this problem, positive integer means increasing temperature and negative integer means decreasing temperature. Initially, the thermometer showed 25 ^{0}C. After changes in temperature in the thermometer, which degree does it show? (Show your work on the number line.) (For each part of the question, the thermometer showed 25 ^{0}C, initially.)

a. +10^{0}C……...…….

b. -2^{0}C …….

c. +30^{0}C ……

Figure 3.11 The final version of the 8th question

After the pilot study and these revisions, the final version of the questionnaire was obtained and IAT was administered to 262 middle school sixth grade students. The last form of IAT is provided in Appendix C.

3.5 Validity and Reliability

Validity refers to the meaningfulness, appropriateness and correctness of the conclusions that the researcher draws from the data collected (Fraenkel & Wallen, 2005). In the study, to check the content validity of achievement test, the table of specification was prepared based on the objectives in the middle school mathematics curriculum. The instrument was submitted to experts in mathematics education for content validation. Before the pilot study, three mathematics educators in the Middle School Mathematics Education program of two different universities had evaluated the items of the instrument in terms of appropriateness of the items in relation to the objectives and the purposes of the study, the table of specification, the usage of mathematical terms, and the clarity of the statements. The table of specification of IAT is presented in Table 3.2 below:

Table 3.2 Table of Specification for the IAT Items

	Objectives	
Integers	Students are able to comprehend integers	1a, 1b, 1c, 1d, 1e, 1f, 3, 5, 6b, 6c, 6e, 7
Integers	Students are able to order integers	2, 4, 8, 6a, 6d, 6f

Reliability is the consistency of the results at a different point in time, location, and situation. The more consistent the scores are between different raters and occasions, the more reliable the assessment is thought to be (Moskal & Leydens, 2000). To check the reliability of the instrument employed in the current study, two types of rubrics were prepared by the researcher and the scoring observer agreement method was used. The data were assessed by a researcher and a second rater, who was a graduate student in mathematics education, by using these rubrics. The inter-rater reliability was calculated and a 0.98 correlation was found to exist between the two ratings.

3.6 Data Collection Procedure

Before the administration of IAT, the official permissions were taken from the Middle East Technical University Human Subjects Ethics Committee. Subsequently, the researcher obtained permission from the head of the school at which the researcher worked and where the data were to be collected. The purpose and the procedure of the study were explained to six mathematics teachers of the middle school. 262 middle school sixth grade students were administered the questionnaire at the beginning of the 2014 spring semester, in March, after the necessary permissions were taken. Data were gathered when all topics related to integers were covered. In total, 40 minutes was given to the students for completing the achievement test, which was administered in the participants' own classrooms, which were similar in terms of environment. Then the interviews, which were mentioned in detail in the Interview Protocol section, were conducted and audio-recorded. A schedule indicating the order of data collection is given in Table 3.3.

Table 3.3 Time Schedule for Data Collection

Date	***Events***
October-February 2013-2014	Development of the measuring tool
March 2014	Pilot study-last revision of the measuring tool
March 2014	Implementation of IAT
March- April 2014	Conducting interviews
April- August 2014	Analysis of the data

3.7 Data Analysis

In order to investigate the research questions, an item-based analysis was conducted using the SPSS PASW program. As the achievement test had two objectives, two rubrics were developed by the researcher for each objective to evaluate the achievement levels of the participants. In addition to these, the categorization of errors were developed to rate the variety of different types of errors that participants made and the answers of the participants were evaluated quantitatively with respect to their accuracy using these error categorizations. The rubrics and the categorization of errors were developed according to the answers of the students in the pilot study

and also by considering the objectives of the questions. In the first phase, the answers of items were analyzed with respect to rubrics and categorized by type and frequency. Table 3.4 and Table 3.5 shows the details of the rubrics.

Table 3.4 Rubric for comprehension questions

Scores	Answer Types
0	No answer/ Had no mathematical understanding
1	Identified the integers incorrectly
2	Identified some integers correctly but some integers incorrectly
3	Identified the integers correctly but some integers were not evaluated
4	Identified the integers correctly

Table 3.5 Rubric for ordering questions

Scores	Answer Types
0	No answer/ Had no mathematical understanding
1	Ordered the integers incorrectly
2	Ordered some integers correctly but some integers incorrectly
3	Ordered the integers correctly but some integers were not evaluated
4	Ordered the integers correctly but without an explanation or with an inappropriate explanation
5	Ordered the integers correctly but had limited mathematical knowledge
6	Ordered the integers correctly with an acceptable explanation

In the second phase, errors that were revealed from the responses of the participants, whose scores were 1, 2 and 3 based on the rubrics for the first and second objectives, were categorized under two main categories for a better analysis of the students' errors. All of the answers were separated into categories. More specifically, the error classification system developed by Tirosh (2000) was adapted as a basis for this data analysis. She refers to the work of other researchers (Ashlock, 1990; Fischbein, Deri, Nello, & Marino, 1985; Graeber, Tirosh, & Glover, 1989), who have studied elements of each of these categories. She classified student error according to the following three categories: Algorithmically based errors, intuitively based errors and errors based on formal knowledge. In this research *errors based on formal*

knowledge of integers was observed as the first main category. For comprehension questions, errors based on formal knowledge have three sub-categories as: *applying incomplete solution strategy, not justifying symbol manipulation* and *misusing positive and negative signs*. The second main category, which did not have a good match in the literature, but originated from the data was "other errors". Other errors have two sub-categories as: *ignoring the given information* and *making incorrect alignment*. For ordering questions, errors based on formal knowledge have five sub-categories as: *ordering as inverse sequence, ordering as arbitrary sequence, taking an incorrect reference point, not justifying symbol manipulation* and *misusing positive and negative signs*. Other errors have two sub-categories as: *ignoring the given information,* and *making incorrect alignment*. In conclusion, a total of two main categories and eight sub-categories were determined, and were checked by a second coder.

In this study, after sub-categories were prepared, they were reread, reexamined, and categorized by the researcher first and then by the second coders in order not to miss any detail. During the process, new sub-categories were added; some of them were replaced with more appropriate ones. After the necessary changes, the categories were created as mentioned above.

The third phase of the analysis dealt with reasons underlying errors regarding comprehension and ordering of integers. In phase two, data obtained from eight interviews were read and transcribed. Namely, audiotapes of interviews were listened to and questionnaires of these eight students were checked simultaneously and the answers were categorized. The reasons that were revealed from the responses of the eight interviewee and they were categorized based on participants' responses by the researcher. The responses were evaluated according to the categories of reasons for a better in-depth analysis of the underlying reasons of students' errors. In this research reasons were classified as the following four categories: *misunderstanding of the magnitude of numbers on the number line, reading the question carelessly, supposing that integers with the same signs are closer to each other than they are to integers with the opposite sign and, lastly, overgeneralizing the properties of natural numbers to integers.*

3.8 Assumptions and Limitations

The main assumptions and limitations about the study are discussed in this section. Firstly, it was assumed that there were no differences among students regarding their age, intelligence, and socioeconomic background. Furthermore, it was assumed that all the students answered all the questions in IAT and the interview honestly, seriously, and carefully.

Secondly, the sampling procedure could be regarded as a limitation of the study since the participants were selected by applying the non-random sampling method. Moreover, the sample includes only sixth grade public middle school students in Ankara. Hence, the generalizability of the results of this study to a larger population would be limited.

Thirdly, data were derived from the achievement test which was designed and adapted by the researcher. If different questions were asked in the achievement test, results could be different from the actual results. Furthermore, the findings of the present study were limited to the participants' ability of self-expression since items of the IAT required expression of their problem solving process or their reasoning for a given answer.

3.9 Internal Validity (Credibility) and External Validity (Transferability)

In order to trust research results, research studies need to be determined by validity and reliability of the design. "Regardless of the type of research, validity and reliability are concerns that can be approached through careful attention to a study's conceptualization and the way in which the data are collected, analyzed, and interpreted, and the way in which the findings are presented" (Merriam, 2009). As standards of rigor in research studies differ in quantitative and qualitative studies, credibility, transferability, dependability and confirmability are discussed in qualitative researches instead of the terminology of internal validity, external validity, reliability, and objectivity (Lincoln & Guba, 1985).

3.9.1 Internal validity or credibility

Internal validity of the study refers to the observed differences on the dependent variables affected by the independent variable (Fraenkel & Wallen, 2006). In other

words, observed results are not related to the dependent variable itself, but to some unintended variables. For every research design, different internal validity threats can be cited. Fraenkel and Wallen (2006) stated that there are three main threats to internal validity in survey research; namely, mortality (loss of subjects), instrumentation and location.

Mortality: The threat for the present study might have been mortality which was also called as loss of subjects. Some of the subjects might have been absent during the administration time (Fraenkel & Wallen, 2006). In other words, loss of participants may affect the results of the study since their data may cause a difference in the findings. Mortality was not a threat for the current study since cross-sectional survey, in which data were collected at one point of time, was conducted.

Instrumentation: Instrumentation threat is related to usage of instruments (Fraenkel & Wallen, 2006). Instrumentation threats involve instrument decay, data collector characteristics and data collector bias. Scoring procedure and changes in instrumentation over time means instrument decay (Fraenkel & Wallen, 2006). In order to control instrument decay, the answers of the students in the current study were evaluated by two scorers based on the rubrics and scorers agreed on scoring. Hence, instrument decay was taken under control. Fraenkel and Wallen (2006) state that if data are collected by different people, data collector characteristics can be a threat for the study. In this study, in order to control the threat, the data collection procedure was standardized and then five data collectors were informed by the researcher about the implementation procedure of the instrument. When the data collector affects the outcomes of the data, data collector bias can be a threat for the study (Fraenkel & Wallen, 2006). In the current study, there was no interaction between data collectors and students during the administration of the achievement test. Besides, a detailed answer key was prepared and used while scoring the answers. Since the researcher remained unbiased and non-directive during the data collection and data analysis, data collector bias did not become a threat for the present study.

Location: If the location where the data are collected has an effect on the outcomes of the study, location threat may occur (Fraenkel &Wallen, 2006). Location was not

a threat for the current study because data were collected from students in their classrooms.

To overcome probable problems regarding credibility, Merriam (2009) offers five strategies: triangulation, member checks, adequate engagement in data collection, researcher's position, and peer examination. In this study, researcher's position, and peer examination were used to increase credibility of the study.

Researcher's position

Researchers are the primary instrument for gathering and analyzing data for all qualitative studies (Merriam, 1998). All observations and analyses are decided by researchers' worldview, values, and perspectives. In order to gain and derive meaningful information, the researchers can make arrangements in their data. Researchers may overlook some particular situations, make mistakes, or biases stand out; even though, researchers take into consideration for magnifying possibilities for collecting and producing meaningful information.

"Rather than trying to eliminate these biases or subjectivities, it is important to identify them and monitor them as to how they may be shaping the collection and interpretation of data" (Merriam, 2009, p.15). Explanation of the biases uncovers researchers' expectations and perspectives which influence research studies conducted (Fraenkel & Wallen, 2006). In this study, to reduce biases, the researcher conducted a pilot study for the open-ended questions. What's more, she described in detail how the research setting was created and what the research findings were.

Peer examination

Peer examination means making critic on research findings together with colleagues (Merriam, 1998). "But such an examination or review can also be conducted by a colleague either familiar with the research or one new to the topic" (Merriam, 2009, p.220).

In this study, the researcher reexamined findings with the help of one graduate student, who is in-service mathematics teacher in a public middle school, from the mathematics education department. In addition, the researcher has also worked with

her supervisor. The researcher studied with the in-service mathematics teacher to scan the data and assess whether the findings were convenient with the data.

3.9.2 External validity or Transferability

The external validity "is concerned with the extent to which the findings of one study can be applied to other situations" (Merriam, 1998). The sample should be representative of the population in terms of nature and environmental issues to generalize the results of the study to the population. The results of this study could not be generalized to a larger population because the sampling method was convenience sampling and only one public school was used to collect data. Nevertheless, at certain conditions, the results of this study could be generalized to a population.

Fraenkel and Wallen (2006) defined this type of generalizability as ecological generalizability which refers to "the extent to which the results of a study can be generalized to conditions or settings other than those that prevailed in particular study" (p. 108). There may be schools with students who have the same academic and social characteristics in other districts. Thus, the results of this study may be generalized to middle school sixth grade students under the same conditions with the participants of the current study.

"In qualitative research, a single case or small, purposeful sample is selected precisely because the researcher wishes to understand the particular in depth, not to find out what is generally true of the many" (Merriam, 2009, p.224). In order to do to make sure external validity (transferability) in qualitative studies is increased by rich and thick description for research situation. Moreover, to be able to transfer research findings to natural situations, transferability is enhanced by rich and detailed description for research situation (Merriam, 1998). In this study, research findings are transferred to cases which are similar to the properties of this study. Hence, results of the study can be transferred to middle school sixth grade students who have similar experiences with the participants of the interview on integers.

CHAPTER 4

RESULTS

The purpose of the study was to investigate middle school 6th grade students' achievement levels regarding comprehension and ordering of integers of integers, errors that they make regarding this topic and the underlying reasons of those errors.

In this chapter, the results of the data are presented regarding two different objectives of the study, namely comprehension of integers and ordering of integers. More specifically, the results addressing the first question of the study are presented under the following headings: analyses of comprehension questions and analyses of ordering questions. The results addressing the second research question of the study are presented under the title errors regarding comprehension and ordering of integers gathered from the integer achievement test (IAT), which was conducted with the aim of revealing the errors of 6^{th} graders regarding the comprehension and ordering of integers. The results addressing the last research question of the study are presented under the title of underlying reasons of errors regarding comprehension and ordering of integers, gathered from the interviews.

4.1 Analysis of the Comprehension Questions

The Integer Achievement Test includes four questions and three sub-items of the fifth question related to comprehension of integers. Comprehension indicates identifying negative and positive integers in different cases. In this section, the results of the analysis of the data collected through the comprehension questions are presented. Middle school 6^{th} grade students' achievement levels and errors regarding comprehension of integers were analyzed benefiting from the rubrics prepared by the researcher. To this end, their wrong answers and wrong explanations were coded and categorized under related themes.

4.1.1 Comprehension Question 1

The first comprehension question is as follows.

Write each following statement as an integer.

g) 7^0C below zero ……

h) 500m above sea level …….

i) 12 units left of zero on the number line ……..

j) Loss of $25 ……..

k) $10 in your wallet …….

l) A depth of 1200 meters …….

Figure 4.1 Comprehension Question 1

As can be seen in the question, students were asked to represent the given statement as an integer. While "7^0C below zero", "12 units left of zero on the number line", "Loss of $25" and "A depth of 1200 meters" indicated negative integers such as -7, -12, -25, -1200, respectively; "500m above sea level" and "$10 in your wallet " indicated positive integers such as +500 and +10, respectively. 6^{th} grade students' answers were evaluated according to the rubric presented below.

Table 4.1 Rubric for Comprehension Question 1

Scores	Answer Types
0	No answer/ Had no mathematical understanding
1	Identified the integers incorrectly
2	Identified some integers correctly but some integers incorrectly
3	Identified the integers correctly but some integers were not evaluated
4	Identified the integers correctly

Based on the rubric, students' answers were coded as 0 if they did not provide an answer to the question or if they had no mathematical understanding. In particular, responses that were not relevant to integers as they indicated no mathematical understanding or were left totally blank, students' responses were coded as 0. Namely, students failed to demonstrate any cognitive evidence of identifying an integer. Their answers were coded 1 if they identified the integers incorrectly. Their answers were coded as 2 if they identified some integers correctly but some integers incorrectly. Students' answers were evaluated as partially or completely correct. In

more detail, answers were coded as 3 if students identified the integers correctly, but some integers were not evaluated. Students' answers were coded as 4 if all integers were identified correctly. To summarize, students' answers were coded as 1 and 2 if their answers were wrong and their answers were coded as 3 and 4 if their answers were correct.

The results of the analysis of 262 6^{th} grade students' answers to comprehension question 1 are presented in Table 4.2.

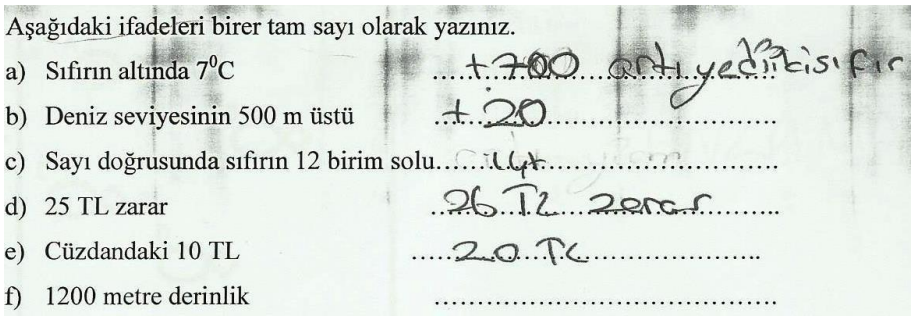
Table 4.2 Frequency of the Answers for Comprehension Question 1

Codes	0	5 (1.9%)
	1	2 (0.8%)
	2	51 (19.5%)
	3	13 (5.0%)
	4	191 (72.9%)
Total		262 (100.0%)

As can be seen in Table 4.2, 5 students (1.9%) among 262 students did not provide an answer or had no mathematical understanding. In particular, these students' responses were not relevant to the integers as they indicated no mathematical understanding or were left totally blank.

To illustrate, the incorrect answer of Participant 21, which is an example of "had no mathematical understanding" is presented below:

The response of Participant 21 is as follows:


Aşağıdaki ifadeleri birer tam sayı olarak yazınız.

a) Sıfırın altında 7^0C ….+700 artı yedinin sıfır

b) Deniz seviyesinin 500 m üstü ….+20……

c) Sayı doğrusunda sıfırın 12 birim solu……14……

d) 25 TL zarar ….25 TL zarar……..

e) Cüzdandaki 10 TL ……20 TL……..

f) 1200 metre derinlik ……

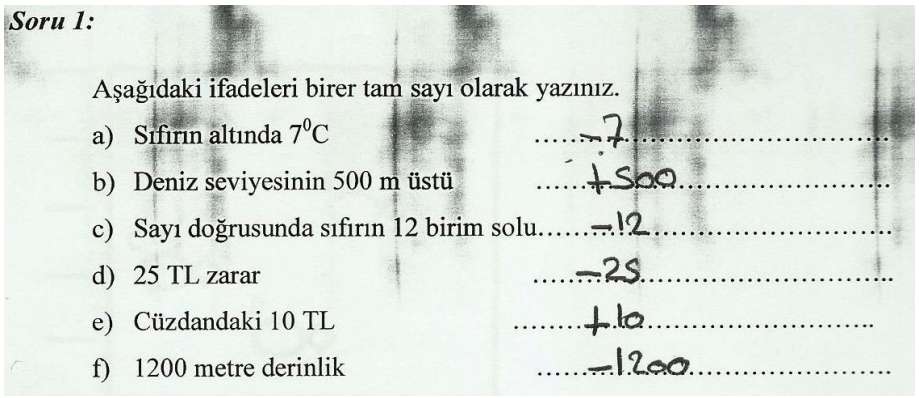
Figure 4.2 Answer of Participant 21 to Item 1

As seen from the participant's response, "+700, +20, 14+, 26 TL loss, 20 TL" were not relevant to the correct answer of item 1.

Two students (0.8%) could not represent the given statements with correct integers. Fifty-one students (19.5%) could represent some of the given statements with correct integers but they could not represent the rest of the statements with correct integers. When the correct answers of the students were analyzed, it was seen that 13 students (5.0%) could represent some of the given statements with correct integers, but the rest of the statements were not evaluated. The remaining 191 students (72.9%) could represent all statements with correct integers. To summarize, 53 (20.3%) students' answers were coded as 1 and 2 because their answers were wrong, and 204 (77.9%) students' answers were coded as 3 and 4 because their answers were correct.

To illustrate, the correct answer of Participant 1 is presented below:

Participant 1:


Soru 1:

Aşağıdaki ifadeleri birer tam sayı olarak yazınız.

a) Sıfırın altında 7^0C ……-7……

b) Deniz seviyesinin 500 m üstü ……+500……

c) Sayı doğrusunda sıfırın 12 birim solu……-12……

d) 25 TL zarar ……-25……

e) Cüzdandaki 10 TL ……+10……

f) 1200 metre derinlik ……-1200……

Figure 4.3 Answer of Participant 1 to Item 1

As can be seen in the participant's answer, "7^0C below zero", "12 units left of zero on the number line", "loss of $25" and "A depth of 1200 meters" were identified as -7, -12, -25, -1200, respectively; "500m above sea level" and "$10 in your wallet " were identified as +500 and +10, respectively.

For item 1, the errors made by the students are presented under related themes.

4.1.2 Comprehension Question 2

The second comprehension question is given below.

The locations of four fish in an aquarium are shown. Aquarium is completely filled with water and the water surface is 0 meter. Between each of the lines to the left of the aquarium is 1 meter. Write the integers corresponding to the points A, B, C, D, which shows the locations of the fish in the aquarium.

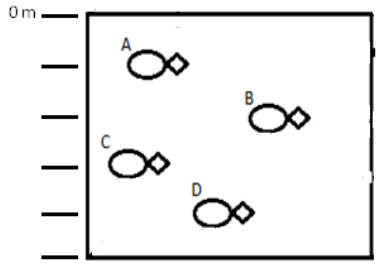


Figure 4.4 Comprehension Question 2

As can be seen in the question, students were asked to identify integers for each location of the fish. The locations of A, B, C and D indicated -1, -2,-3 and -4, respectively. 6th grade students' answers were evaluated according to the rubric presented below.

Table 4.3 Rubric for Comprehension Question 2

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Identified the locations of fishes incorrectly
2	Identified some locations of fishes correctly but some locations of fishes incorrectly
3	Identified the integers correctly but some integers were not evaluated
4	Identified the integers correctly

The answers of 262 6th grade students were analyzed and the results are presented in Table 4.4.

Table 4.4 Frequency of the Answers for Comprehension Question 2

Codes	0	19 (7.3%)
	1	62 (23.7%)
	2	7 (2.7%)
	3	0 (0.0%)
	4	174 (66.4%)
Total		262 (100.0%)

As Table 4.4 illustrates, nineteen students (7.3%) among 262 students did not provide an answer or had no mathematical understanding. In particular, these students' responses were not relevant to the integers as they indicated no mathematical understanding or were left totally blank. As can be observed in Table 4.4, 62 students (23.7%) could not identify locations of the fish. Furthermore, 7 students (2.7%) could not identify some of the levels of the fish, but they could identify the rest of the levels where the fish existed. When the correct answers of the students were analyzed, it was seen that 174 students (66.4%) could identify all the levels where the fish existed. However, there were no responses that were coded as 3. In more detail, all of the answers were coded as either right or wrong. To summarize, 69 (26.4%) students' answers were coded as 1 and 2 because their answers were wrong, and 174 (66.4%) students' answers were coded as 3 and 4 because their answers were correct.

To illustrate, the correct answer of Participant 1 is presented below:

Participant 1:

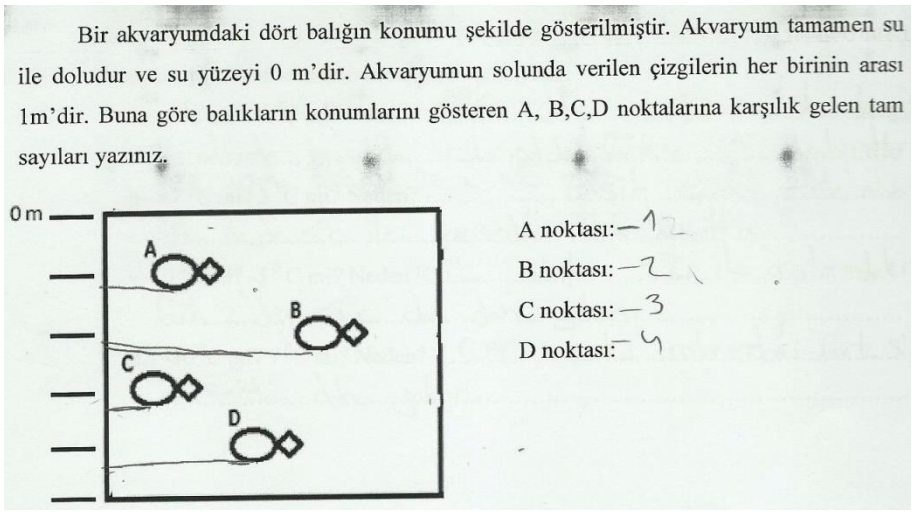


Figure 4.5 Answer of Participant 1 to Item 3

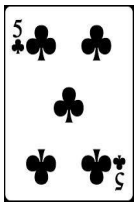
As can be seen in the participant's answer, the location of the fish of A, B, C and D were identified as -1, -2,-3 and -4, respectively.

For item 2, the errors made by the students are presented under related themes.

4.1.3 Comprehension Question 3

The third comprehension question is presented below.

Ahmet and Batuhan developed a game by using a deck of cards. Cards with pictures and A are removed from the deck of cards. They decide that each red card indicates damage to the extent of the number on the card and each black card indicates gain to the extent of the number on the card. For example, a card which represents +5 is shown below.



Batuhan's cards are shown in the figures below. Please, write each card as an integer.

.....

.....

.....

.....

Figure 4.6 Comprehension Question 3

As can be seen in the question, there was a game in which black cards indicated gain; red cards indicated damage to the extent of the number written on the card. Some cards were given and students were expected to write integers for each card. -10, -5, +3 and +7 were the answers of the question. 6th grade students' answers were evaluated according to the rubric presented below.

Table 4.5 Rubric for Comprehension Question 3

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Identified cards' numbers incorrectly
2	Identified some cards' numbers correctly but some cards' numbers incorrectly
3	Identified the integers correctly but some integers were not evaluated
4	Identified the integers correctly

The answers of 262 6th grade students were analyzed and the results are presented in Table 4.6.

Table 4.6 Frequency of the Answers for Comprehension Question 3

Codes	0	25 (9.5%)
	1	10 (3.8%)
	2	45 (17.2%)
	3	0 (0.0%)
	4	182 (69.5%)
Total		262 (100.0%)

As can be observed, 25 students (9.5%) among 262 students did not answer the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Ten students (3.8%) could not identify integers for each game card. As can be observed in Table 4.6, 45 students (17.2%) could not identify some integers for some cards, but they could identify integers for the rest of the cards. When correct answers of the students were analyzed, it was seen that 182 students (69.5%) could identify all integers for each card. However, there was no answer which was coded as 3. In more detail, all of the students answered the question either correctly or incorrectly. To summarize, 55 (21.0%) students' answers were coded as 1 and 2 because their answers were wrong and 182 (69.5%) students' answers were coded as 4 because their answers were correct.

As an example, the correct answer of Participant 4 is presented below:

Participant 4:

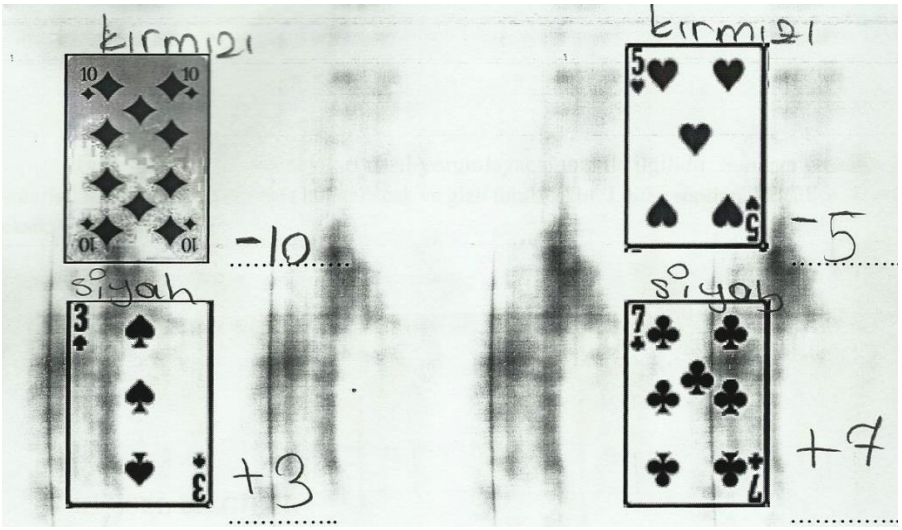


Figure 4.7 Answer of Participant 4 to Item 7

As can be seen in the participant's answer, -10, -5, +3 and +7 were the answers of the question.

For item 3, the errors made by students are presented under related themes.

4.1.4 Comprehension Question 4

The fourth comprehension question is given below.

For each pair of temperatures, identify which temperature is further away from -2 ^{0}C. Please explain your answers.

e. 6^{0}C or -6 ^{0}C?
Why? …….

f. -7^{0}C or 3 ^{0}C?
Why? ……..……

g. 0 ^{0}C or -5 ^{0} C?
Why? ……

h. -10 ^{0}C or 7 ^{0}C?
Why? ……

Figure 4.8 Comprehension Question 4

As can be seen from the question, four pairs of integers and the reference -2 were given in the question. Students were expected to identify which integer in the pair

was further away from the reference integer by using the number line. 6, none of -7 and 3, 0 and -10 are the answers of the question, respectively. 6th grade students' answers were evaluated according to the rubric presented below.

Table 4.7 Rubric for Comprehension Question 4

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Decided further integers correctly
2	Decided some further integers correctly but some further integers incorrectly
3	Decided some further integers correctly but some further integers were not evaluated
4	Identified the integers correctly

The answers of 262 6th grade students were analyzed and the results are presented in Table 4.8.

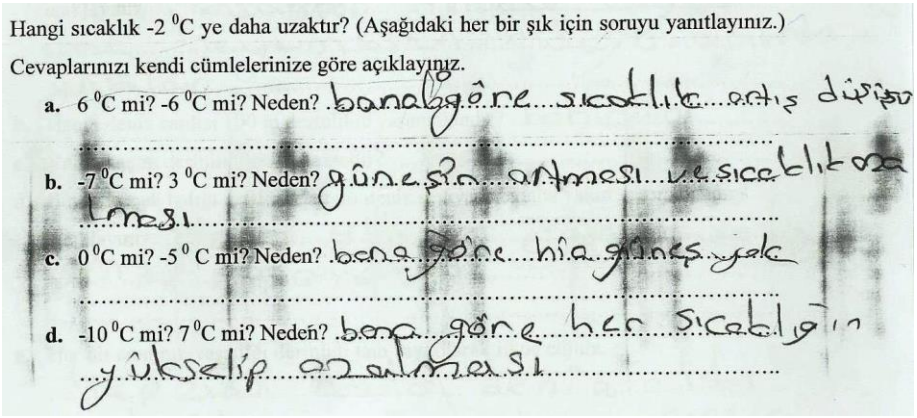
Table 4.8 Frequency of the Answers for Comprehension Question 4

Codes	0	24	(9.2%)
	1	15	(5.7%)
	2	127	(48.5%)
	3	7	(2.7%)
	4	89	(34.0%)
Total		262	(100.0%)

Table 4.8 shows the assessment of the answers given to comprehension question 4. As can be seen, 24 students (9.2%) among 262 students did not provide an answer to the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank.

To illustrate, the incorrect answer of Participant 21, which is an example of "had no mathematical understanding", is presented below:

The response of Participant 21 is as follows:


Hangi sıcaklık -2 ^{0}C ye daha uzaktır? (Aşağıdaki her bir şık için soruyu yanıtlayınız.)
Cevaplarınızı kendi cümlelerinize göre açıklayınız.

a. 6 ^{0}C mi? -6 ^{0}C mi? Neden? bana göre sıcaklık artış düşü

b. -7 ^{0}C mi? 3 ^{0}C mi? Neden? güneşin artması ve sıcaklığın artması

c. 0 ^{0}C mi? -5 ^{0} C mi? Neden? bana göre hiç güneş yok

d. -10 ^{0}C mi? 7 ^{0}C mi? Neden? bana göre her sıcaklığın yükselip azalması

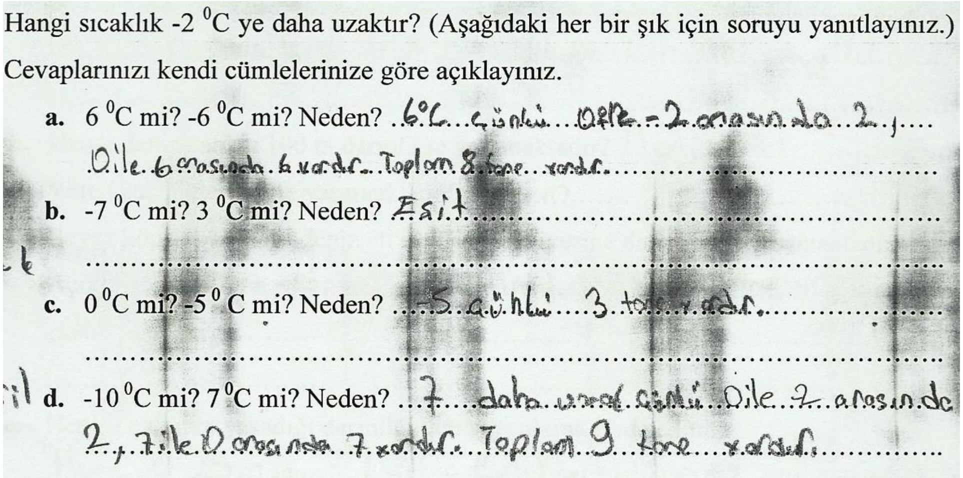
Figure 4.9 Answer of Participant 21 to Item 5

As can be seen in the participant's response, there were not relevant answers to the correct answer of item 5. Student wrote irrelevant things as a response.

Fifteen students (5.7%) could not identify integers further away from -2 for each choice of the question. As can be observed in Table 4.8, 127 students (48.5%) could not identify further away integers for some choices of the question, but they could identify further away integers for the rest of the choices of the question. When the correct answers of the students were analyzed, 7 students (2.7%) could identify further away integers for some choices of the question, but the rest of choices were not evaluated. It was seen that 89 students (34.0%) could identify all further away integers for each card. In more detail, all of the students answered the question either correctly or incorrectly. To summarize, 142 (54.2%) students' answers were coded as 1 and 2 because their answers were wrong and 96 (36.7%) students' answers were coded as 4 because their answers were correct.

As an example, the correct answer of Participant 80 is presented below:

Participant 80:


Hangi sıcaklık -2 ^{0}C ye daha uzaktır? (Aşağıdaki her bir şık için soruyu yanıtlayınız.)
Cevaplarınızı kendi cümlelerinize göre açıklayınız.

a. 6 ^{0}C mi? -6 ^{0}C mi? Neden? 6°C çünkü 0'la -2 arasında 2, 0 ile 6 arasında 6 vardır. Toplam 8 tane vardır.

b. -7 ^{0}C mi? 3 ^{0}C mi? Neden? Eşit

c. 0 ^{0}C mi? -5 ^{0} C mi? Neden? -5 çünkü 3 tane vardır.

d. -10 ^{0}C mi? 7 ^{0}C mi? Neden? 7 daha uzak çünkü 0 ile 2 arasında 2, 7 ile 0 arasında 7 vardır. Toplam 9 tane vardır.

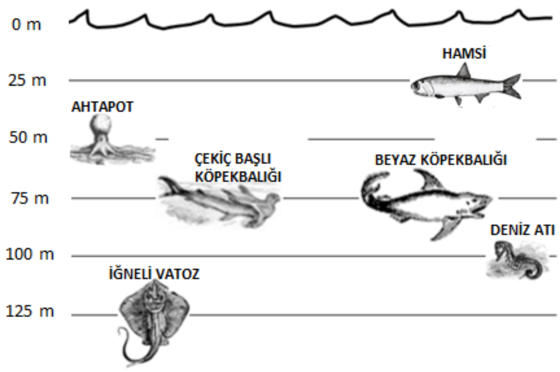
Figure 4.10 Answer of Participant 80 to Item 5

As can be observed in the participant's answer, Participant 80 identified that 6, none of -7 and 3, 0 and -10 were further away from the reference integer, respectively.

4.1.4 Comprehension Question 5

The fifth comprehension question is presented below:

Living beings as fish and octopus live in the sea. Factors such as dissolved oxygen and pressure etc., necessary for the existence of these living beings can differ with respect to depth. Therefore, these living beings exist at different depths. In the picture below, some examples of underwater living beings are given. Answer the questions according to the pictures below.



b. Which sea creature exists at a depth of 100 m? ………………………………..
c. At which depth does stingray exist?..………………………………………….
e. Please write depth of each sea creatures as an integer.

Figure 4.11 Comprehension Question 5

Three sub-items (item b, item c and item e) of the fifth question were related to comprehension of integers. As can be seen in the question, students were asked to

identify the depth of stingray as -125 in item c and from top to bottom the depths of each creatures as -25, -50, -75, -100 and -125, respectively in item e. In item b, students were asked to identify the sea horse, which exists at a depth of 100 m. In item c, students were asked to identify depth of stingray. In item e, students were asked to identify each depth of sea creatures as an integer. 6th grade students' answers were evaluated according to the rubrics presented below.

Table 4.9 Rubric for Comprehension Question 5 (item b)

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Identified sea creature exists at a depth of 100 m incorrectly
2	Identified some sea creature exists at a depth of 100 m incorrectly but some incorrectly
3	Identified some sea creature exists at a depth of 100 m correctly but some were not evaluated
4	Identified sea creature exists at a depth of 100 m correctly

The answers of 262 6th grade students were analyzed and the results are presented in Table 4.10, 4.12 and 4.14.

Table 4.10 Frequency of the Answers for Comprehension Question 5 (item b)

Codes	0	8	(3.1%)
	1	3	(1.1%)
	2	0	(0.0%)
	3	0	(0.0%)
	4	251	(95.8%)
Total		262	(100.0%)

Table 4.10 shows the assessment results of the answers given to comprehension question 5 item b. As can be seen, 8 students (3.1%) among 262 students did not provide an answer to the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Three students (1.1%) could not identify the sea creature that existed at a depth of 100m. As can be observed in

Table 4.10, there was no answer which was coded as 2. When the correct answers of the students were analyzed, it was seen that 251 students (95.8%) could identify the sea creature that existed at a depth of 100m. However, there was no answer that was coded as 3. To sum up, 3 (1.1%) students' answers were coded as 1 because their answers were wrong and 251 (95.8%) students' answers were coded as 4 because their answers were correct.

Table 4.11 Rubric for Comprehension Question 5 (item c)

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Identified depth of stingray incorrectly
2	Identified some depth of stingray incorrectly, some of them correctly
3	Identified some depth of stingray correctly, some of them were not evaluated
4	Identified depth of stingray correctly

Table 4.12. Frequency of the Answers for Comprehension Question 5 (item c)

Codes	0	10 (3.8%)
	1	3 (1.1%)
	2	0 (0.0%)
	3	0 (0.0%)
	4	249 (95.0%)
Total		262 (100.0%)

Table 4.12 shows the assessment results of the answers given to comprehension question 5 item c. As can be seen, 10 students (3.8%) among 262 students did not provide an answer the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Three students (1.1%) could not identify the depth of the stingray. As can be observed in Table 4.12, there was no answer that was coded as 2 or 3. When the correct answers of the students were analyzed, it was seen that 249 students (95.0%) could identify the depth of the stingray. In more detail, all of the students answered the question either correctly or

incorrectly. To sum up, 3 (1.1%) students' answers were coded as 1 because their answers were wrong and 249 (95.0%) students' answers were coded as 4 because their answers were correct.

Table 4.13 Rubric for Comprehension Question 5 (item e)

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Identified depth of each sea creatures incorrectly
2	Identified some depth of sea creatures incorrectly, some of them correctly
3	Identified some depth of sea creatures correctly, some of them were not evaluated
4	Identified depth of each sea creatures correctly

Table 4.14. Frequency of the Answers for Comprehension Question 5 (item e)

Codes	0	39	(14.9%)
	1	61	(23.3%)
	2	0	(0.0%)
	3	0	(0.0%)
	4	162	(61.8%)
Total		262	(100.0%)

Table 4.14 shows the assessment results of the answers given to comprehension question 5 item e. As can be seen, 39 students (14.9%) among 262 students did not provide an answer to the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Sixty-one students (23.3%) could not identify the depths of the sea creatures. As can be observed in Table 4.14, there was no answer that was coded as 2 or 3. When the correct answers of the students were analyzed, it was seen that 162 students (61.8%) identified all the depths of the sea creatures. In more detail, all of the students answered the question either correctly or incorrectly. To sum up, 61 (23.3%) students' answers were coded as 1 because their answers were wrong and 162 (61.8%) students' answers were coded as 4 because their answers were correct.

To illustrate, the correct answer of Participant 80 is presented below:

Participant 80:

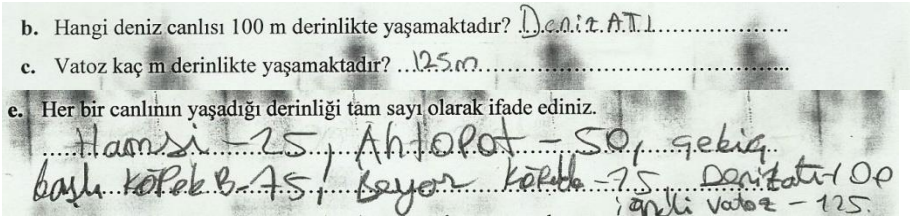


Figure 4.12 Answer of Participant 80 to Item 6-b, 6-c and 6-e

As can be seen in the participant's answer, Participant 80 identified the depth of the stingray as -125 in item c and from top to bottom the depths of each creatures as -25, -50, -75, -100 and -125, respectively in item e. In item b, students identified the sea horse which existed at a depth of 100m.

For item 5, errors of students are presented under related themes.

4.2. Analysis of the Ordering Questions

The Integer Achievement Test includes three questions and three sub-items of the fourth question related to ordering of integers. Ordering indicates arranging integers according to their magnitude from largest to smallest or vice versa. In this section, the results of the analysis of the data collected through the ordering questions are presented. Middle school 6th grade students' achievement levels and errors regarding ordering of integers were analyzed benefiting from the rubrics prepared by the researcher. To this end, their wrong answers and wrong explanations were coded and categorized under related themes.

4.2.1 Ordering Question 1

The first ordering question is given below.

Simge and Rüya are comparing their hair length to their friend Yağmur's hair length. Simge states that her hair is +4 cm compared to Yağmur's hair and Rüya states that her hair is-3 cm compared to Yağmur's hair. Who has the shortest hair? Write the girls' names in order of their hair length from the shortest to the longest.

Figure 4.13 Ordering Question 1

As can be observed in the question, students were asked to order girls' hair lengths from the shortest to the longest. Also, they were asked to express their reasons. The correct answer is Rüya < Yağmur < Simge. 6^{th} grade students' answers were analyzed according to the rubric below:

Table 4.15 Rubric for Ordering Question 1

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Ordered the girls' hair lengths incorrectly
2	Ordered some girls' hair lengths correctly but some incorrectly
3	Ordered some girls' hair lengths correctly but some were not evaluated
4	Ordered girls' hair lengths correctly but without explanations or with inappropriate explanations
5	Ordered girls' hair lengths correctly but had limited mathematical knowledge
6	Ordered girls' hair lengths correctly with an acceptable explanation

According to rubric, students' responses were coded as 0 if students did not provide an answer to the question or students had no mathematical understanding. In particular, responses that were not relevant to integers as they indicated no mathematical understanding or were left totally blank, students' responses were coded as 0. Namely, students failed to demonstrate any cognitive evidence of ordering two or more integers. Their answers were coded as 1 if they ordered the integers incorrectly; their answers were coded as 2 if they ordered some integers correctly but some integers incorrectly. Students' correct answers were evaluated as partially or completely correct. In more detail, answers were coded as 3 if students ordered the integers correctly, but some integers were not evaluated. Students' answers were coded as 4 if all the integers were ordered correctly without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as an inappropriate explanation. Students' answers were coded as 5 if all the integers were ordered but had limited mathematical knowledge. In particular, responses in which students gave correct answers but explained their process only in accordance with the algorithms they had memorized were coded as 5. Lastly, students' answers were coded as 6 if all the integers were ordered correctly with an acceptable explanation. In more detail, clear

and understandable explanations were described as acceptable explanations. To summarize, students' answers were coded as 1 and 2 if their answers were wrong and their answers were coded as 3, 4, 5 and 6 if their answers were correct.

The analysis results of 262 6th grade students' answers are presented in Table 4.16 below:

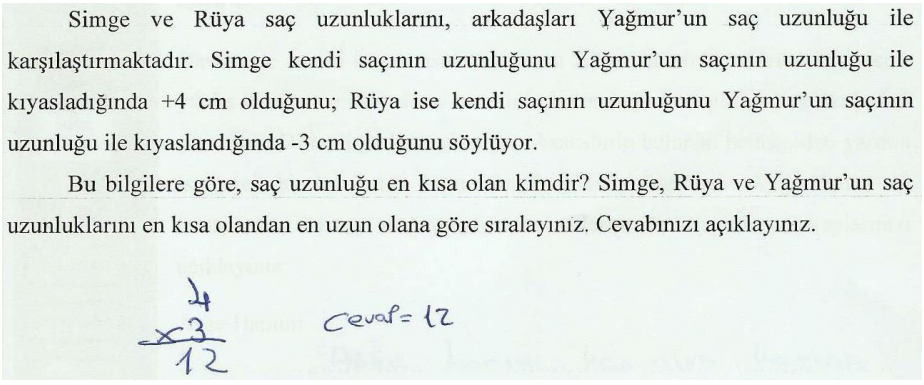
Table 4.16 Frequency of the Answers for Ordering Question 1

Codes	0	30	(11.5%)
	1	51	(19.5%)
	2	9	(3.4%)
	3	37	(14.1%)
	4	58	(22.1%)
	5	31	(11.8%)
	6	46	(17.6%)
Total		262	(100.0%)

According to Table 4.16, 30 students (11.5%) among 262 students could not provide an answer or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank.

To illustrate, the incorrect answer of Participant 24, which is an example of "had no mathematical understanding", is presented below:

The response of Participant 24 is as follows:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

4
x3
12

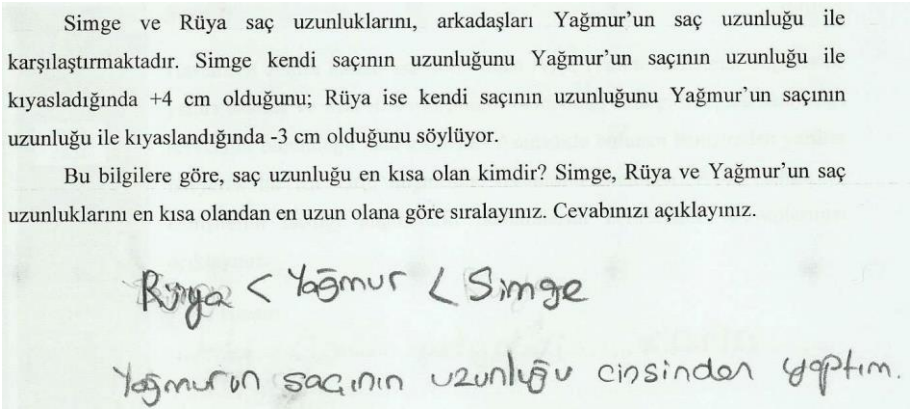
Cevap = 12

Figure 4.14 Answer of Participant 24 to Item 2

As observed in the participant's response, "4x12=12" was not relevant to the correct answer of item 2.

Fifty-one students (19.5%) could not order girls' hair lengths from the shortest to the longest. Nine students (3.4%) could order some girls' hair lengths, but they could not order the rest of girls' hair lengths. When the correct answers of the students were investigated, it was seen that 37 students (14.1%) could order some girls' hair lengths, but the rest of girls' hair lengths were not evaluated. Fifty-eight students (22.1%) could order the girls' hair lengths from the shortest to the longest, but without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as inappropriate.

To illustrate, the correct answer of Participant 127, which is an example of "inappropriate explanation", is presented below:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Rüya < Yağmur < Simge

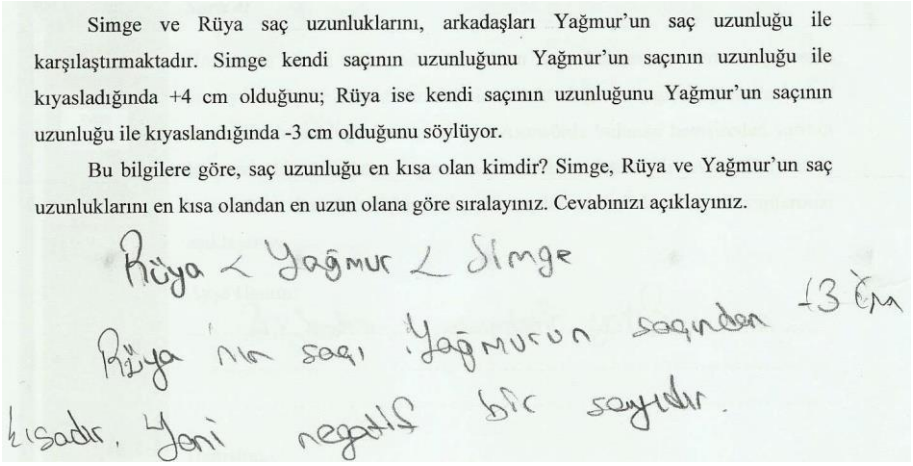
Yağmurun saçının uzunluğu cinsinden yaptım.

Figure 4.15 Answer of Participant 127 to Item 2

As can be seen in the participant's response, the explanation of the participant was not sufficient to understand the participant's thoughts and the ways by which the participant solved item 2. The explanation of the participant was meaningless, thereby inhibiting the comprehension of the reasons provided in the answer.

Thirty-one students (11.8%) could order the girls' hair lengths but they had limited mathematical knowledge. Particularly, these students' responses were correct but they explained their process only in accordance with the algorithms they had memorized.

To illustrate, the correct answer of Participant 54, which is an example of "had limited mathematical knowledge", is presented below:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Rüya < Yağmur < Simge

Rüya nın saçı Yağmurun saçından 13 cm kısadır. Yani negatif bir sayıdır.

Figure 4.16 Answer of Participant 54 to Item 2

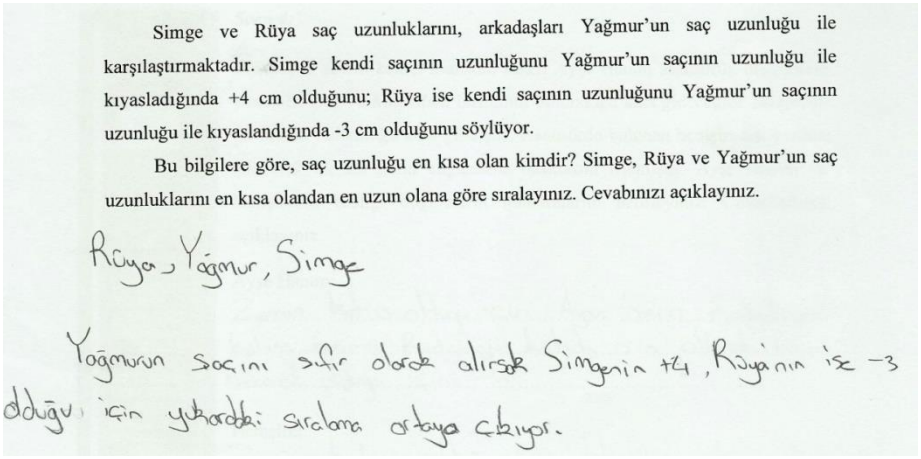
As can be observed in the participant's response, "Rüya's hair length is shorter than Yağmur's hair length because -3 is a negative integer" was not a complete explanation of the correct answer of item 2. The participant's response was correct but Participant 54 explained his/her solution process only in accordance with the algorithms s/he had memorized.

As seen in Table 4.16, only 46 students (17.6%) could order the girls' hair lengths with an acceptable explanation. In more detail, clear and comprehensible explanations were described as acceptable explanations.

To summarize, 60 (22.9%) students' answers were coded as 1 and 2 because their answers were wrong, and 172 (68.5%) students' answers were coded as 3, 4, 5 and 6 because their answers were correct.

To illustrate, the correct answer of Participant 92 is presented below:

Participant 92:



Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Rüya, Yağmur, Simge

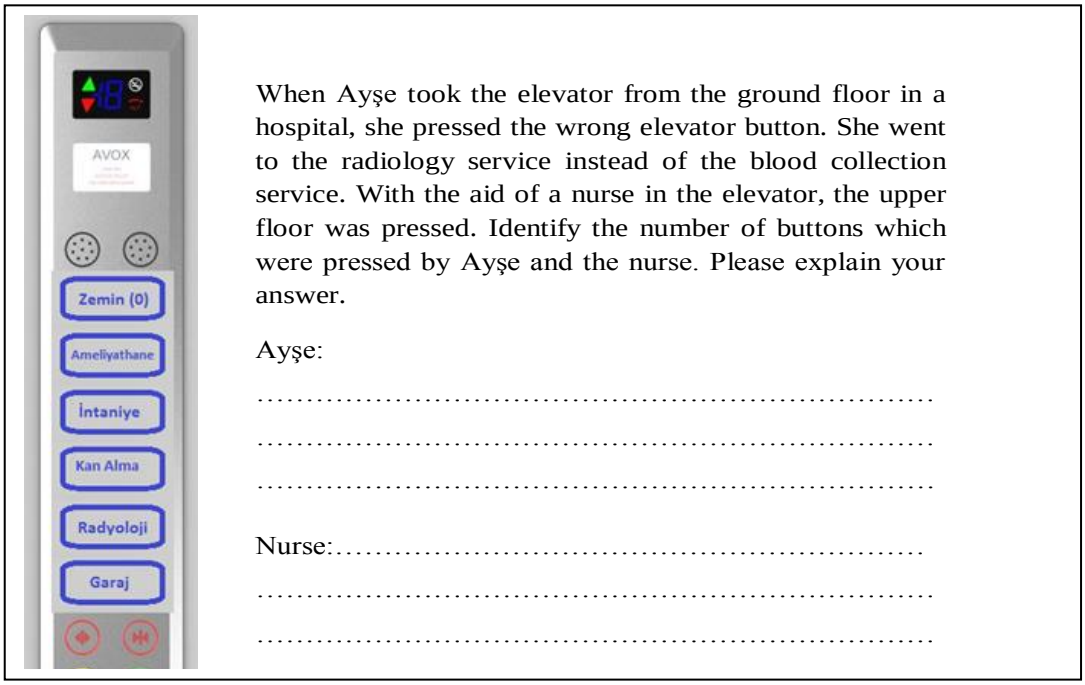
Yağmurun saçını sıfır olarak alırsak Simgenin +4, Rüyanın ise −3 olduğu için yukarıdaki sıralama ortaya çıkıyor.

Figure 4.17 Answer of Participant 92 to Item 2

As can be seen in the participant's answer, Participant 92 ordered the girls' hair lengths from the shortest to the longest correctly. Rüya's hair length is the shortest; Simge's hair length is the longest according to the student's answer.

4.2.2 Ordering Question 2

The second ordering question is given below.



When Ayşe took the elevator from the ground floor in a hospital, she pressed the wrong elevator button. She went to the radiology service instead of the blood collection service. With the aid of a nurse in the elevator, the upper floor was pressed. Identify the number of buttons which were pressed by Ayşe and the nurse. Please explain your answer.

Ayşe:

……

……

……

Nurse:……

……

……

Figure 4.18 Ordering Question 2

As can be observed in the question, students were asked to express the numbers of the elevator buttons which were pressed by Ayşe and the nurse. In the question, all

the buttons of an elevator were given in the figure and the ground floor was represented as zero. Ayşe pressed the wrong elevator button, and then she asked for help from the nurse in the elevator in order to go to the right service. Ayşe pressed -4 and the nurse pressed -3, which are the correct answers to this question. Students' answers were analyzed according to the rubric presented below.

Table 4.17 Rubric for Ordering Question 2

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Ordered buttons incorrectly
2	Ordered some buttons correctly but some incorrectly
3	Ordered some buttons correctly but some were not evaluated
4	Ordered buttons correctly but without explanations or with inappropriate explanations
5	Ordered buttons correctly but had limited mathematical knowledge
6	Ordered buttons correctly with an acceptable explanation

The results obtained from the analyses of the answers of 262 6th grade students are presented in Table 4.18.

Table 4.18 Frequency of the Answers for Ordering Question 2

Codes	0	47	(17.9%)
	1	83	(31.7%)
	2	11	(4.2%)
	3	6	(2.3%)
	4	45	(17.2%)
	5	5	(1.9%)
	6	65	(24.8%)
Total		262	(100.0%)

Table 4.18 displays the descriptive results obtained from the assessment of the responses to ordering question 2. Forty-seven students (17.9%) among 262 students did not provide an answer to the question or had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Eighty-three students (31.7%)

could not identify the buttons which were pressed by the nurse and Ayşe. As can be seen in Table 4.18, 11 students (4.2%) could identify one of the buttons which were pressed by nurse and Ayşe. The results revealed that 6 students (2.3%) could identify one of the buttons which were pressed by the nurse and Ayşe, but the other button was not evaluated by the students. Moreover, 45 students (17.2%) could identify the buttons which were pressed by the nurse and Ayşe without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as an inappropriate explanation. Five students (1.9%) could identify the buttons which were pressed by the nurse and Ayşe, but they had limited mathematical knowledge. Particularly, these students' responses were correct but they explained their process only in accordance with the algorithms they had memorized. The remaining 65 students (24.8%) could identify the buttons which were pressed by the nurse and Ayşe with an acceptable explanation. In more detail, clear and comprehensible explanations were described acceptable explanations.

To summarize, 94 (35.9%) students' answers were coded as 1 and 2 because their answers were wrong, and 121 (46.2%) students' answers were coded as 3, 4, 5 and 6 because their answers were correct.

To illustrate, the correct answer of Participant 3 for item 2 is presented below:

Participant 3:

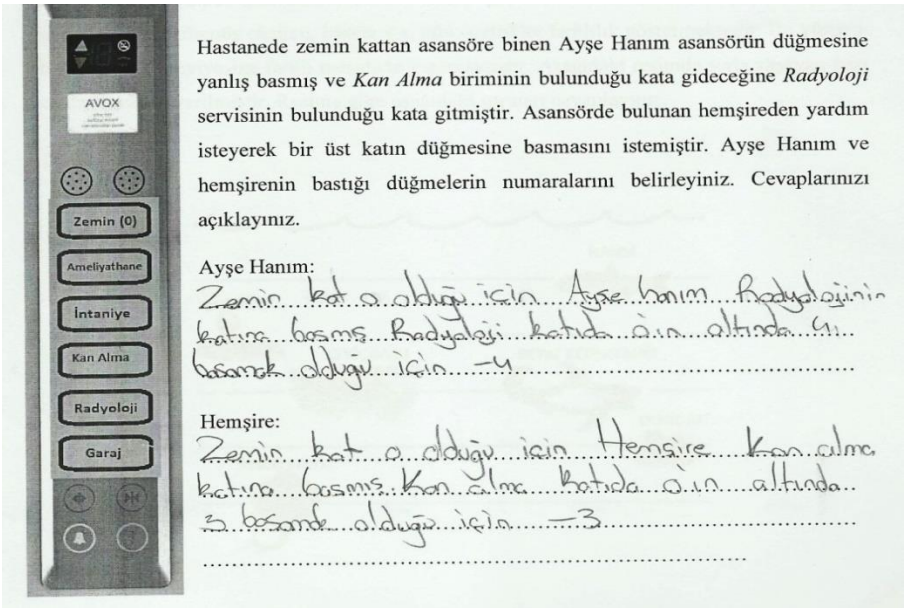


Figure 4.19 Answer of Participant 3 to Item 4

As can be seen in the participant's answer, Participant 3 expressed the numbers of the elevator buttons as Ayşe pressing -4 and the nurse pressing -3.

4.2.3 Ordering Question 3

The third ordering question is below.

In this problem, positive integer means increasing temperature and negative integer means decreasing temperature. Initially, the thermometer showed 25 ^{0}C. After changes in the temperature in the thermometer, which degree does it show? (Show your work on the number line.) (For each part of the question, the thermometer showed 25 ^{0}C, initially.)

d. +10^{0}C ……
……...……

e. -2^{0}C ……
……

f. +30^{0}C ……

Figure 4.20 Ordering Question 3

As can be observed in the question, students were asked to find the new temperature according to the result of the change in the thermometer by using the number line for each sub item. +35, -23, +55 are the answers of the question, respectively. Students' answers were analyzed according to the rubric presented below.

Table 4.19 Rubric for Ordering Question 3

Scores	Answer Types
0	No answer/ Had no mathematical understanding
1	Stated temperatures incorrectly
2	Stated some temperatures correctly but some incorrectly
3	Stated some temperatures correctly but some were not evaluated
4	Stated temperatures correctly but without explanations or with inappropriate explanations
5	Stated temperatures correctly but had limited mathematical knowledge
6	Stated temperatures correctly with an acceptable explanation

Table 4.20 Frequency of the Answers for Ordering Question 3

Codes	0	83	(31.7%)
	1	34	(13.0%)
	2	34	(13.0%)
	3	4	(1.5%)
	4	31	(11.8%)
	5	1	(0.4%)
	6	75	(28.6%)
Total		262	(100.0%)

According to Table 4.20, 83 students (31.7%) among 262 students did not provide an answer to the question or they had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Thirty-four students (13.0%) could not find the new temperature according to the result of the change in the thermometer. As can be seen in Table 4.20, 34 (13.0%) students could not find the new temperature according to the result of the change in the thermometer for some sub-questions. The results revealed that 4 students (1.5%) could find the new temperature according to the result of the change in the thermometer for some sub-questions, but some sub-questions were not evaluated. Moreover, 31 students (11.8%) could find the new temperature according to the result of the change in the thermometer without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as inappropriate explanations. One student (0.4%) could find the new temperature according to the result of the change in the thermometer but the student had limited mathematical knowledge. Particularly, the student's response was correct but she explained the process only in accordance with the algorithms she had memorized. The remaining 75 students (28.6%) could find the new temperature according to the result of the change in the thermometer with an acceptable explanation. In more detail, clear and comprehensible explanations were described acceptable explanations.

To sum up, 68 (26.0%) students' answers were coded as 1 and 2 because their answers were wrong and 111 (42.3%) students' answers were coded as 3, 4, 5 and 6 because their answers were correct.

To illustrate, the correct answer of Participant 4 for item 3 is presented below:

Participant 4:

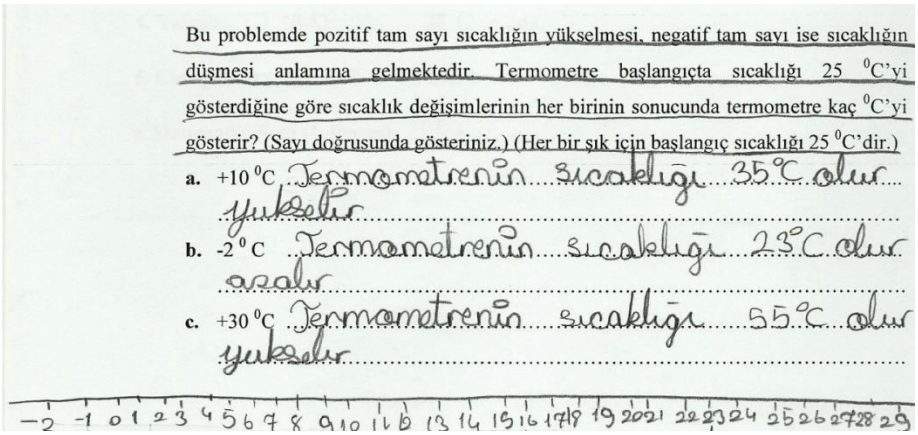


Figure 4.21 Answer of Participant 4 to Item 8

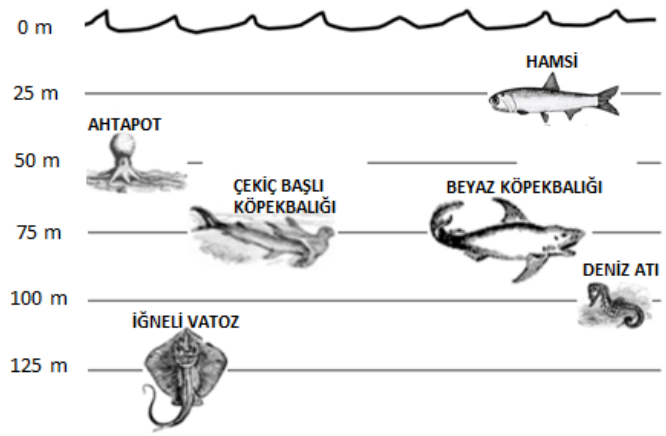
As observed in the participant's answer, the student found the new temperatures as +35, -23, and 55 according to the result of the change in the thermometer, respectively.

4.2.4 Ordering Question 4

The fourth ordering question is given below.

Q6:

Living beings as fish and octopus live in the sea. Factors as such dissolved oxygen and pressure etc., which are necessary for the existence of these living beings can differ with respect to depth. Therefore, these living beings exist at different depths. In the picture below, some examples of underwater living beings are given. Answer the questions according to picture below.



a. Which sea creature exists at the deepest point? Explain your answer.

……

d. Which of the sea horse and great white shark lives closer to sea level?

Explain your answer…….

f. Please order integers which you find at part e from smallest to largest.

Figure 4.22 Ordering Question 4

Three sub-items (item a, item d and item f) of the fourth question were related to ordering of integers. As can be seen in the question, students were asked to identify the creature existing at the deepest point in item a and whether the great white shark or the sea horse lived closer to sea level item d. In item f, students were asked to order the integers which were found in part e. 6th grade students' answers were evaluated according to the rubrics presented below.

The answers of 262 6th grade students were analyzed and the results are presented in Tables 4.21, 4.23 and 4.25.

Table 4.21 Rubric for Ordering Question 4 item a

Scores	Answer Types
0	No answer/ Had no mathematical understanding
1	Stated sea creature exists at the deepest point incorrectly
2	Stated some sea creature exist at the deepest point correctly but some incorrectly
3	Stated some sea creature exist at the deepest point correctly but some were not evaluated
4	Stated sea creature exist at the deepest point correctly but without explanations or with inappropriate explanations
5	Stated sea creature exist at the deepest point correctly but had limited mathematical knowledge
6	Stated sea creature exist at the deepest point correctly with an acceptable explanation

Table 4.22 Frequency of the Answers for Ordering Question 4 item a

Codes	0	14	(5.3%)
	1	7	(2.7%)
	2	0	(0.0%)
	3	0	(0.0%)
	4	142	(54.2%)
	5	25	(9.5%)
	6	74	(28.2%)
Total		262	(100.0%)

Table 4.22 shows the assessment results of the answers given to ordering question 4 item a. Fourteen students (5.3%) among 262 students did not provide an answer to the question or they had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Seven students (2.7%) could not find the creature existing at the deepest point in the sea. As can be seen in Table 4.22, there was no student who had given a partial answer to the question. Moreover, 142 students (54.2%) could find which creature, which was the stingray, existed at the deepest point in the sea without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as inappropriate explanations. Twenty-five students (9.5%) could find the creature, which was the stingray, existing at the deepest point in the sea but they had limited mathematical

knowledge. Particularly, these students' responses were correct but they explained their process only in accordance with the algorithms they had memorized. The remaining 74 students (28.2%) could find the creature that existed at the deepest point in the sea with an acceptable explanation. In more detail, clear and comprehensible explanations were described acceptable explanations.

To summarize, 7 (2.7%) students' answers were coded as 1 because their answers were wrong and 241 (91.9%) students' answers were coded as 4, 5 and 6 because their answers were correct.

Table 4.23 Rubric for Ordering Question 4 item d

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Determined sea creature lives closer to sea level incorrectly
2	Determined some sea creature lives closer to sea level correctly but some incorrectly
3	Determined some sea creature lives closer to sea level correctly but some were not evaluated
4	Determined sea creature lives closer to sea level correctly but without explanations or with inappropriate explanations
5	Determined sea creature lives closer to sea level correctly but had limited mathematical knowledge
6	Determined sea creature lives closer to sea level correctly with an acceptable explanation

Table 4.24 Frequency of the Answers for Ordering Question 4 item d

Codes	0	9 (3.4%)
	1	35 (13.4%)
	2	0 (0.0%)
	3	0 (0.0%)
	4	83 (31.7%)
	5	15 (5.7%)
	6	120 (45.8%)
Total		262 (100.0%)

Table 4.24 displays the descriptive results obtained from the assessment of ordering question 4 item d. Nine students (3.4%) among 262 students did not provide an

answer to the question or they had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Thirty-five students (13.4%) could not decide whether the great white shark lived closer to sea level than the sea horse or vice versa. As can be seen in Table 4.24, there was no student who had given a partially correct answer to the question. Moreover, 83 students (31.7%) could indicate that the great white shark lived closer to the sea level than the sea horse in the sea, but without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as inappropriate explanations. Fifteen students (5.7%) could indicate that the great white shark lived closer to the sea level than the sea horse in the sea but they had limited mathematical knowledge. Particularly, these students' responses were correct but they explained their process only in accordance with the algorithms they had memorized. The remaining 120 students (45.8%) could indicate that the great white shark lived closer to the sea level than the sea horse in the sea with an acceptable explanation. In more detail, clear and comprehensible explanations were described as acceptable explanations.

To conclude, 35 (13.4%) students' answers were coded as 1 because their answers were wrong and 218 (83.2%) students' answers were coded as 4, 5 and 6 because their answers were correct.

Table 4.25 Rubric for Ordering Question 4 item f

Scores	**Answer Types**
0	No answer/ Had no mathematical understanding
1	Ordered each sea creatures as integers incorrectly
2	Ordered some sea creatures as integers correctly but some incorrectly
3	Ordered some sea creatures as integers correctly but some were not evaluated
4	Ordered sea creatures correctly but without explanations or with inappropriate explanations
5	Ordered sea creatures correctly but had limited mathematical knowledge
6	Ordered sea creatures correctly with an acceptable explanation

Table 4.26 Frequency of the Answers for Ordering Question 4 item f

Codes	0	108	(41.2%)
	1	54	(20.6%)
	2	0	(0.0%)
	3	0	(0.0%)
	4	26	(9.9%)
	5	0	(0.0%)
	6	74	(28.2%)
Total		262	(100.0%)

Table 4.26 presents the results of the analysis of the answers given to ordering question 4 item f. According to Table 4.26, 108 students (41.2%) among 262 students did not provide an answer to the question or they had no mathematical understanding. In particular, these students' responses were not relevant to integers as they indicated no mathematical understanding or were left totally blank. Fifty-four students (20.6%) could not order the integers which were found in part e. There was no student who had given a partially correct answer to the question. When the correct answers of the students were investigated, it was seen that 26 students (9.9%) could order the integers which were found in part e but without an explanation or with an inappropriate explanation. Particularly, irrelevant or meaningless explanations were described as inappropriate explanations. Also, there was no student who could order the integers which were found in part e with limited mathematical knowledge. Particularly, no student's response was correct but one explained one's process only in accordance with the algorithms one had memorized. As seen in Table 4.26, only 74 students (28.2%) could order the integers which were found in part e with an acceptable explanation. In more detail, clear and comprehensible explanations were described as acceptable explanations.

To summarize, 54 (20.6%) students' answers were coded as 1 because their answers were wrong and 100 (38.1%) students' answers were coded as 4 and 6 because their answers were correct.

To illustrate, the correct answer of Participant 80 for item 4 is presented below:

Participant 80:

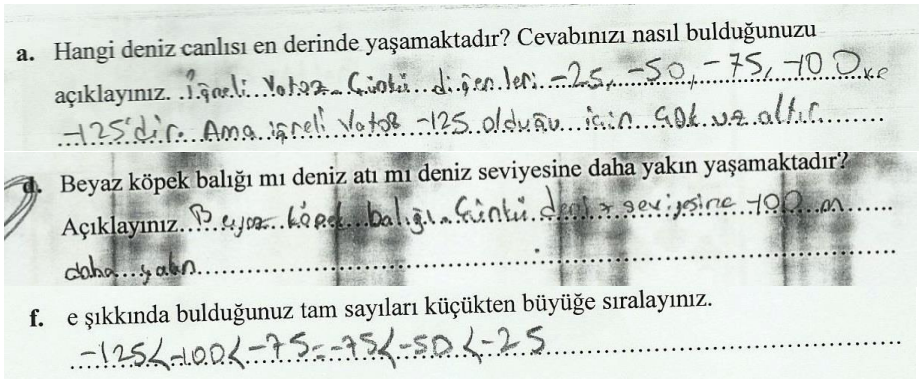


Figure 4.23 Answer of Participant 80 to Item 6-a, 6-d and 6-f

As observed in the participant's answer, Participant 80 found the creature at the deepest point to be the stingray in item a. Participant 80 indicated that the great white shark lived closer to sea level than the sea horse in item d. Lastly, in item f, the student ordered the integers found in part e.

4.3 Errors Regarding Comprehension and Ordering of Integers

The second purpose of this study was to investigate middle school sixth grade students' errors regarding comprehension and ordering of integers. In accordance with this purpose, this section deals with errors that emerged in questions regarding comprehension and ordering of integers. Errors regarding comprehension and ordering of integers are presented in the following parts.

4.3.1 Errors Regarding Comprehension of Integers

One of the purposes of this study was to investigate errors made by middle school sixth grade students regarding comprehension of integers. The other purpose was to investigate the underlying reasons of those errors. Before identifying the underlying reasons of the participants' errors, the information about errors regarding comprehension of integers was analyzed in IAT, which the students had completed. The findings related to errors regarding comprehension of integers are presented in this section.

Errors were categorized under two main categories as: *errors based on formal knowledge of integers* and *other errors.* Errors based on formal knowledge of

integers have three sub-categories as: *applying incomplete solution strategy, not justifying symbol manipulation* and *misusing positive and negative signs*. Other errors have two sub-categories as: *ignoring the given information* and *making incorrect alignment*. In accordance with the purpose, Table 4.27 presents frequencies and percentages of errors of the participants, regarding each item on a categorical basis.

Table.4.27 Frequencies (and percentages) of students' errors regarding comprehension questions

	ERRORS BASED ON FORMAL KNOWLEDGE			***OTHER ERRORS***		
Categories / *Items*	*Applying Incomplete Solution Strategy*	*Not justifying Symbol Manipulation*	*Misusing Positive and Negative Signs*	*Ignoring the Given Information*	*Making Incorrect Alignment*	*Total*
Item 1-a	3 (27.3%)		8 (72.7%)	-	-	11
Item 1-b	3 (37.5%)	3 (37.5%)	2 (25.0%)	-	-	8
Item 1-c	3 (8.6%)	5 (14.3%)	27 (77.1%)	-	-	35
Item 1-d	3 (42.9%)		4 (57.1%)	-	-	7
Item 1-e	3 (37.5%)	3 (37.5%)	2 (25.0%)	-	-	8
Item 1-f	3 (15.0%)	4 (20.0%)	13 (65.0%)	-	-	20
Item 3	-	6 (6.9%)	56 (64.4%)	10 (11.5%)	15 (17.2%)	87
Item 5			124 (98.4%)	2(1.6%)		126
Item 6-b	-	-	5 (100%)	-	-	5
Item 6-c	-	-	4 (44.4%)	5 (56.6%)	-	9
Item 6-e	-	7 (10.5%)	54 (80.1%)	3 (9.4%)	-	67
Item 7	5 (7.6%)	4 (5.3%)	30 (45.5%)	15 (22.7%)	12 (18.9%)	66
Total	23	32	205	33	27	

As it can be seen in Table 4.26, students made many errors regarding comprehension of integer questions. In other words, many students were not able to show correct expressions for the comprehension of integers. According to Table 4.27, the most popular error that students made was errors based on formal knowledge of integers. *Misusing positive and negative signs* was the most popular error in the sub-categories of errors based on formal knowledge of integers. Although they are not too frequent, students made the error of *not justifying symbol manipulation* in questions that required identifying integers in problem situations. *Applying incomplete solution strategy* was the least popular error that students made. *Ignoring the given information* was the most popular error in the sub-categories of other errors. *Making incorrect alignment,* which is a sub-category of other errors, showed an increment in items related to determine an integer for a location in given question. Examples of students' responses are given below under the category headings given in Table 4.27.

4.3.1.1 Errors Based on Formal Knowledge

Errors based on formal knowledge refer to axioms, definitions, theorems, and proofs (Fischbein, 1994). There are three sub-categories of this error type, namely *applying incomplete solution strategy, not justifying symbol manipulation* and *misusing positive and negative signs.*

Applying Incomplete Solution Strategy: *Applying incomplete solution strategy* is the first sub-category in errors based on formal knowledge. *Applying incomplete solution strategy* appeared when students knew the general method of solution; however, they could not apply the method to the question. For example, a student knew that the statement of under sea level implies a negative integer; however, the student could not express a depth as a negative integer. The student wrote his/her solution plan as an answer instead of applying the plan and could not find final answer. In more detail, a student could not express the statement of "10 meter under sea level" as "-10". As it is seen in Table 4.27, errors under this sub-category were the most frequently made errors in items 1-b and 1-d. In item 1-b, 3 students (37.5%) among 8 students and in item 1-d, 3 students (42.9%) among 7 students made errors which were *applying incomplete solution strategy*. In item 1-a, 3 students (27.3%) among 11 students, in item 1-c, 3 students (8.6%) among 35 students, in item 1-e, 3 students (37.5%) among 8 students made errors which were

applying incomplete solution strategy. Lastly, in item 1-f, 3 students (15%) among 20 students and in item 7, 5 students (7.6%) among 66 students made errors which were *applying incomplete solution strategy*. To illustrate, the error of *applying incomplete solution strategy* made by Participant 25 in item 1 is presented below:

Participant 25:

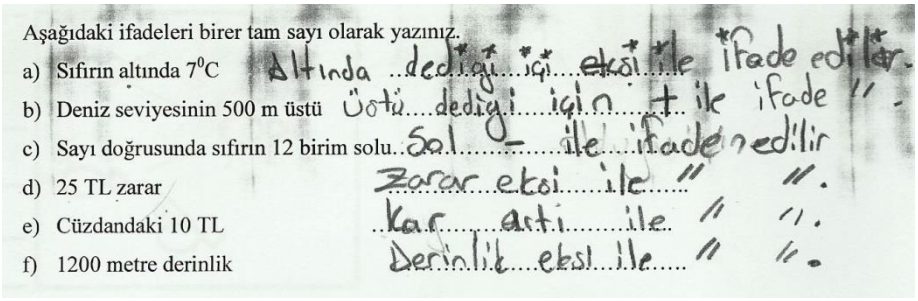


Figure 4.24 Answer of Participant 25 to Item 1

Participant 25 expressed that "7^0C below zero", "12 units left of zero on the number line", "loss of $25" and "A depth of 1200 meters" indicated negative meanings; however, she did not identify these negative meanings as negative integers such as "-7","-12", "-25", "-1200", respectively. Similarly, Participant 25 expressed that "500 m above sea level" and "$10 in your wallet " indicated positive meanings; however, she did not identify these positive meanings as positive integers such as "+500" and "+10". Hence, she made the error of *applying incomplete solution strategy*.

For example, it was seen that Participant 26 made the error of *applying incomplete solution strategy* in item 7 as presented below:

Participant 26:

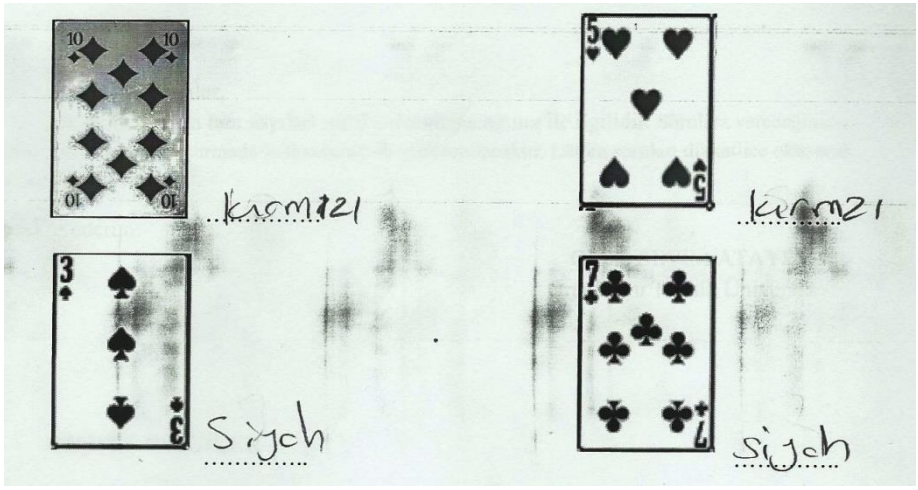


Figure 4.25 Answer of Participant 26 to Item 7

Participant 26 expressed the colors of each card as red and black. However, he had to express red cars as negative integers, such as "-10" and "-5" and he had to express the black cards as positive integers, such as "+3" and "+7". He could not express each colored card as an integer, so he made the error of *applying incomplete solution strategy.*

Not justifying Symbol Manipulation: Table 4.27 shows that *not justifying symbol manipulation* is the second sub-category in errors based on formal knowledge. *Not justifying symbol manipulation* meant that the student just picked some numbers or words from the task or wrote some numbers arbitrary and worked with them in ways irrelevant to the context of the question or tried to identify an integer with irrelevant numbers. For example, the student expressed incorrect representations instead of a negative integer in the context of the question. The student wrote "0.12" instead of "-12". This decimal number was irrelevant to the context of the problem. Another example is a student writing "%25" instead of "-25". This percentage was also irrelevant to the context of the problem Errors under this sub-category were the most frequently made errors in items 1-e and 1-b. In item 1-e, 3 students (37.5%) among 8 students; in item 1-b, 3 students (37.5%) among 8 students; in item 1-c, 5 students (14.3%) among 35 students; in item 1-f, 4 students (20.0%) among 20 students made errors of *not justifying symbol manipulation*. In item 3, 6 students (6.9%) among 87 students; in item 6-e, 7 students (10.5%) among 67 students made errors of *not justifying symbol manipulation*. Lastly, 4 students (5.3%) among 66 students and 2 students (1.6%) among 126 students made errors of

not justifying symbol manipulation in item 7. For instance, it was seen that Participant 1 made the error of *not justifying symbol manipulation* in item 3 as presented below:

Participant 1:

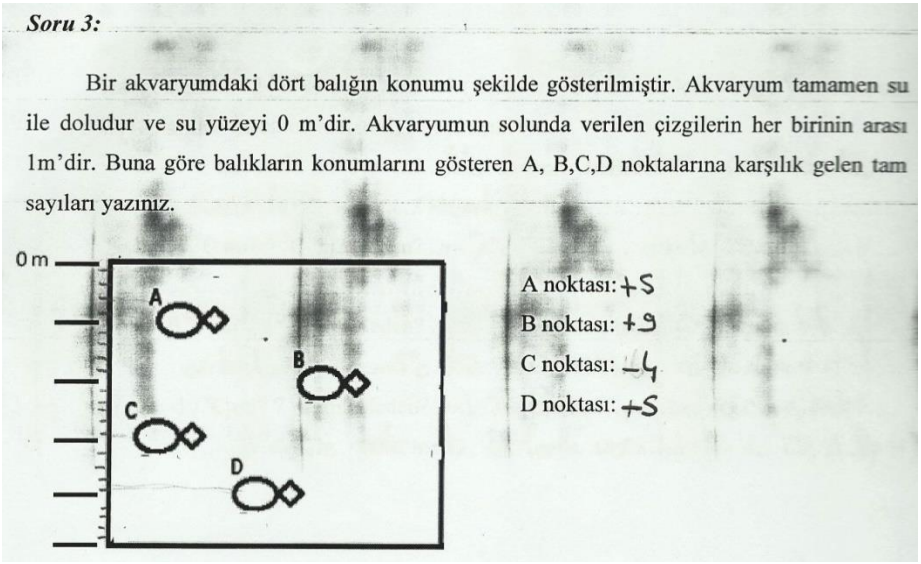


Figure 4.26 Answer of Participant 1 to Item 3

Participant 1 had to express each level of the fish in the aquarium. "-1", "-2", "-3" and "-4" were the levels of A, B, C and D, respectively. However, she did not identify the levels of the fish with correct integers. She stated that "+5", "+9", "-4" and "+5" were the levels of the fish, respectively. These integers were not relevant with the actual levels of the fish. Therefore, she made the error of not *justifying symbol manipulation.*

To illustrate, the error of *not justifying symbol manipulation* made by Participant 27 for item 6-e as presented below:

Participant 27:

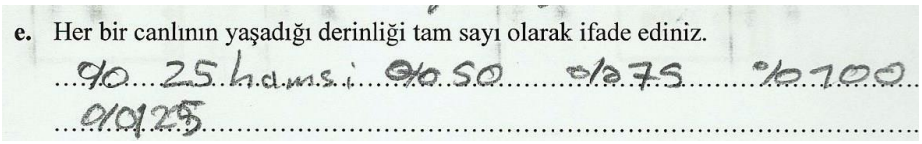


Figure 4.27 Answer of Participant 27 to Item 6-e

Participant 27 had to express the depth of each sea creature as integers such as "-25" for the anchovy, "-50" for the octopus, "-75" for the sharks, "-100" for the sea horse and "-125" for the stingray. However, he expressed the depth of each sea creature in percentages such as "25%", "50%", "75%", "100%" and "125%". These percentages were irrelevant to the context of the question. So he made the error of *not justifying symbol manipulation*.

Misusing Positive and Negative Signs: It was observed that *misusing positive and negative signs* is the last sub-category in errors based on formal knowledge. This error type was related to students' limited conceptions about integers. In this error type, students could not determine when they needed to use integers. Students could use positive integers instead of negative integers or vice versa. As presented in Table 4.27, *misusing positive and negative signs* was the most frequently made error in items 1-a, 1-c, 1-d, 1-f, 3, 5, 6-b, 6-e, 7. In item 6-e, 54 students (80.1%) among 67 students; in item 1-a, 27 students (77.1%) among 35 students; in item 1-c, 8 students (72.7%) among 11 students; in item 1-f, 13 students (65%) among 20 students made the error of *misusing positive and negative signs*. In item 3, 56 students (64.4%) among 87 students; in item 1-d, 4 students (57.1%) among 7 students made the error of *misusing positive and negative signs*. Moreover, in item 7, 30 students (45.5%) among 66 students; in item 6-c, 4 students (44.4%) among 9 students; in item 1-b and 1-e, 2 students (25.0%) among 8 students and 2 students (25.0%) among 8 students made the error of *misusing positive and negative signs*. In item 5, 124 students (98.4%) among 126 students made the error of *misusing positive and negative signs*. Lastly, 5 students (100%), who were all those students who made some errors in item 6-b, made the error of *misusing positive and negative signs* in item 6-b. To illustrate, the error of *misusing positive and negative signs* made by Participant 27 for item 1-c is presented below:

Participant27:

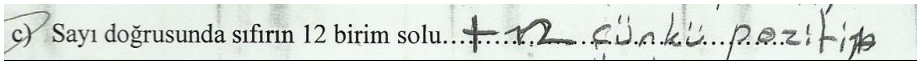


Figure 4.28 Answer of Participant 27 to Item 1-c

Participant 27 could not express the statement of "12 units left of zero on the number line" as a negative integer such as "-12". He wrote a positive integer as "+12" instead of a negative integer as "-12". Hence, he made the error of *misusing positive and negative signs*.

For example, it was seen that Participant 27 made the error of *misusing positive and negative signs* in item 3 as presented below:

Participant 27:

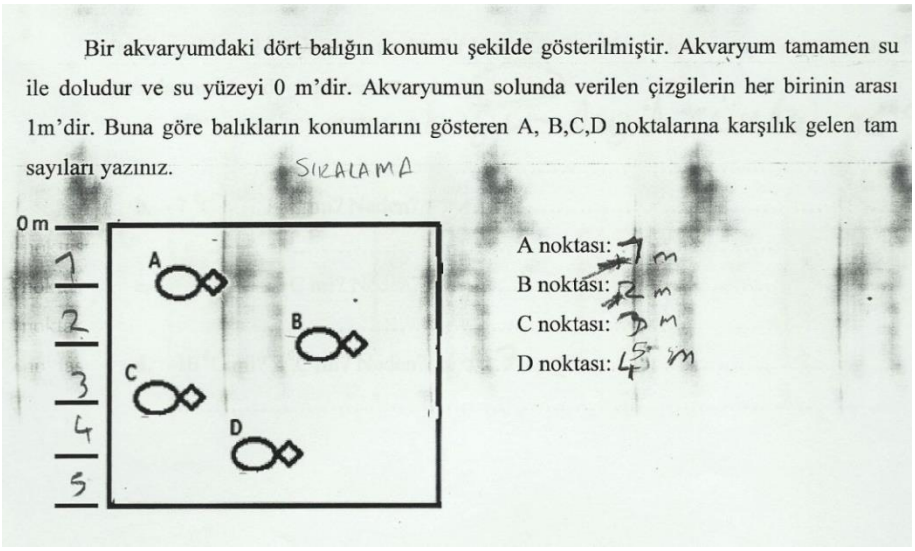


Figure 4.29 Answer of Participant 27 to Item 3

Participant **27** did not identify each level of the fish as negative integers such as "-1", "-2", "-3" and "-4". Instead, he wrote positive integers such as "1", "2", "3" and "4" for each level of the fish. Therefore, he made the error of *misusing positive and negative signs.*

4.3.1.2 Other Errors

In this study, the category of *other errors* includes the students' errors other than those in the first category of errors based on formal knowledge. There are two sub-categories of this error type, namely *ignoring the given information* and *making incorrect alignment.*

Ignoring the Given Information: *Ignoring the given information* is the first sub-category of other errors. This error was related to what was being given and what

was being asked for in the questions. Students ignored some information when they tried to solve the question. In other words, students did not take into consideration all the information given in the questions. In this type of error, the solution methods used by the students were correct; however, their final answers were incorrect.

As can be observed in Table 4.27, *ignoring the given information* was the most frequently made error in item 6-c. It was seen that 5 students (56.6%) among 9 students made the error of *ignoring the given information* in item 6-c. In item 7, 15 students (22.7%) among 66 students; in item 3, 10 students (11.5%) among 87 students; in item 6-e, 3 students (9.4%) among 67 students made the error of *ignoring the given information*. To illustrate, the error of *ignoring the given information* made by Participant 175 for item 6-c is presented below:

Participant 175:

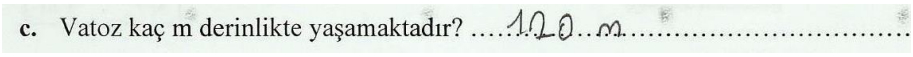


Figure 4.30 Answer of Participant 175 to Item 6-c

In the question, Participant 175 wrote "120 m" instead of "125 m". He ignored the information given on the picture in the question. However, he needed to be careful about the given information in order to give a complete answer to question 6-c. Hence, he made the error of *ignoring the given information.*

For example, it was seen that Participant 24 had made the error of *ignoring the given information* for the problem in item 6-e as presented below:

Participant 24:

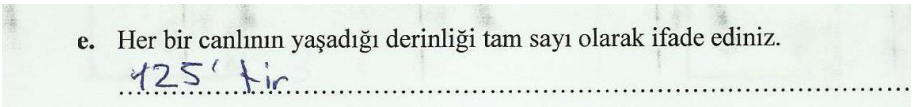


Figure 4.31 Answer of Participant 24 to Item 6-e

Participant 24 did not express the depth of each sea creature as an integer. She only expressed the depth of one sea creature as "125". However, she needed to write the depths of each sea creature in the question. The depth of one sea creature was not

enough to answer this question because there should have been six depths in the answer, totally. Thus, she made the error of *ignoring the given information.*

Making Incorrect Alignment: It was seen that *making incorrect alignment* is the second sub-category of other errors. In this error, students could not align the correct integers to correct locations which were given in the question. Moreover, students could identify the sign of each integer for each card; however, they could not write the correct number for all the cards. As can be observed in Table 4.27, 27 students made the error of *making incorrect alignment* in item 3 and 7. This error was the most frequently made error in item 3. In item 3, 15 students (17.2%) among 87 students and in item 7, 12 students (18.9%) among 66 students made the error of *making incorrect alignment*. The answer of Participant 9 can be presented as the first example for this type of error.

Participant 9:

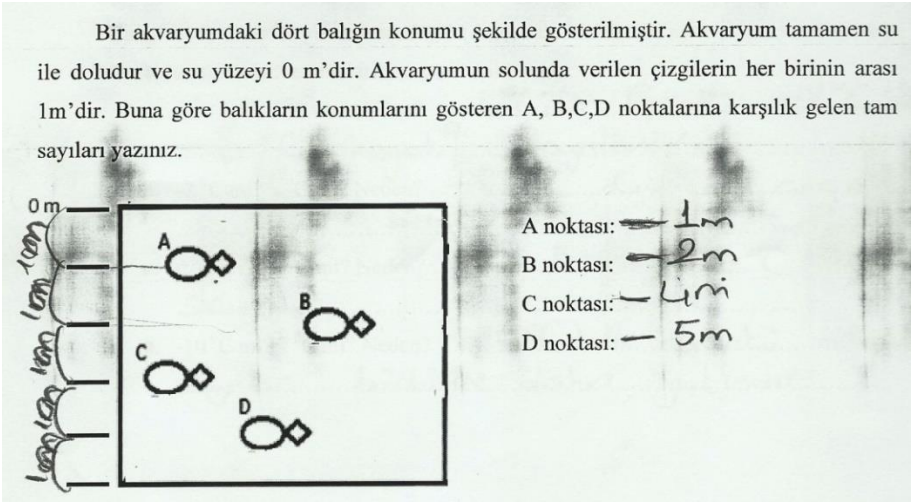


Figure 4.32 Answer of Participant 9 to Item 3

Participant 9 did not identify all the locations where the fish existed. For the locations of the points C and D, she wrote "-4" and "-5" instead of "-3" and "-4", respectively. She did not align the locations of points C and D according to the lines given on the left of the aquarium. Therefore, she made the error of *making incorrect alignment.*

For example, it was seen that Participant 1 made the error of *making incorrect alignment* in item 7 as presented below:

Participant 1:

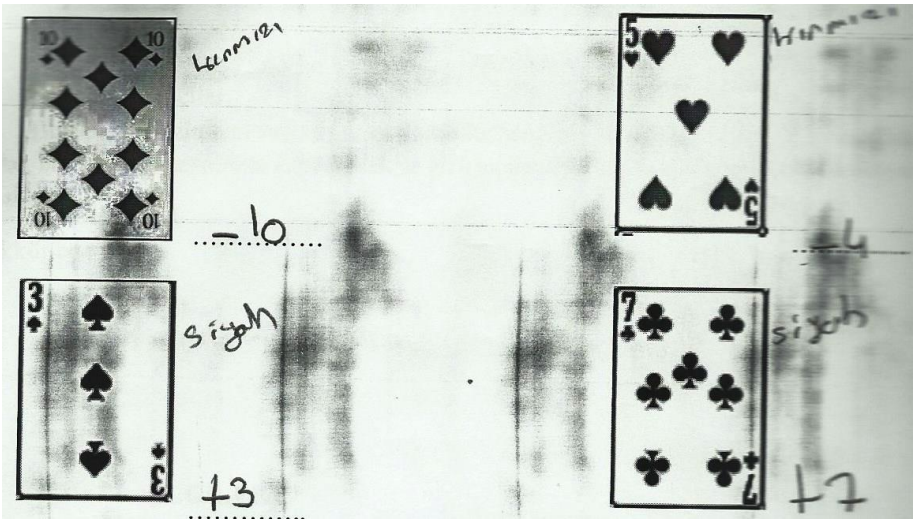


Figure 4.33 Answer of Participant 1 to Item 7

Participant 1 did not identify one of the cards with a correct integer. She identified the correct signs of all cards; however, she did not write the correct integer for the card, heart 5. She wrote "-4" instead of "-5" for heart 5. This is an example of the error of *making incorrect alignment*.

As a consequence, this study showed that sixth grade students made errors regarding comprehension of integers. The other purpose of this study was to investigate sixth graders' errors regarding ordering integers which is explained in the following heading.

4.3.2 Errors Regarding Ordering of Integers

One of the purposes of this study was to investigate errors made by middle school sixth grade students regarding ordering of integers. The other purpose was to investigate underlying reasons of those errors. Before identifying the underlying reasons of participants' errors, the information about errors regarding the ordering of integers was analyzed in IAT, which the students had completed. The findings related to errors regarding the ordering of integers are presented in this section.

Errors were categorized under two main categories as: *errors based on formal knowledge on integers* and *other errors.* Errors based on formal knowledge have five sub-categories as: *ordering as inverse sequence, ordering as arbitrary sequence,*

taking incorrect reference point, *not justifying symbol manipulation* and *misusing positive and negative signs*. Other errors have two sub-categories as: *ignoring the given question* and *making incorrect alignment*. In accordance with the purpose, Table 4.28 represents frequencies and percentages of errors of the participants, regarding each item on a categorical basis.

Table.4.28 Frequencies (and percentages) of students' errors regarding ordering questions

	ERRORS BASED ON FORMAL KNOWLEDGE					***OTHER ERRORS***		
Categories / *Items*	*Ordering as Inverse Sequence*	*Ordering as Arbitrary Sequence*	*Taking Incorrect Reference Point*	*Not Justifying Symbol Manipulation*	*Misusing Positive and Negative Signs*	*Ignoring the Given Information*	*Making Incorrect Alignment*	*Total*
Item 2	8 (13.6%)	41 (69.5%)				10 (16.9%)		59
Item 4	22 (23.2%)		14 (14.7%)		43 (45.3%)		16 (16.8%)	95
Item 6-a	4 (57.1%)	3 (42.9%)						7
Item 6-d	32 (94.0%)			1 (3.0%)			1 (3.0%)	34
Item 6-f	54 (100.0%)							54
Item 8			16 (57.1%)	1 (3.6%)		11 (39.3%)		28
Total	120	44	30	2	43	21	17	

As it is seen in Table 4.28, students made errors regarding ordering of the integer questions. In other words, many students were not able to show the correct expressions for the ordering of integers. According to Table 4.28, the most popular error is errors based on formal knowledge of integers that students made. *Ordering as inverse sequence* is the most popular error in the sub-categories of errors based on formal knowledge of integers. *Ordering as arbitrary sequence* is the second popular error and *misusing positive and negative signs* is the third popular error in the sub-categories of errors based on formal knowledge of integers. Although they are not too frequent, students made the error of *taking incorrect reference point*. *Not justifying symbol manipulation* is the least popular error that students made. In the

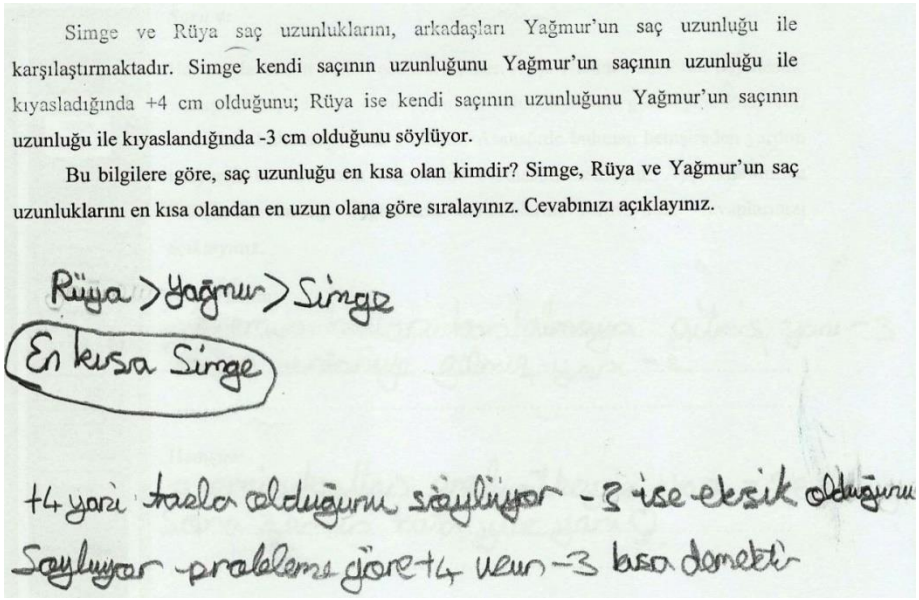
sub-categories of other errors, *making incorrect alignment* is the most popular error. I*gnoring the given information* is the second popular error in the sub-categories of other errors. Examples of students' responses are given under the category headings given in Table 4.28.

4.3.2.1 Errors Based on Formal Knowledge

Errors under this category have been mentioned in the previous section which was about errors of comprehension of integers. There are five sub-categories of this error type, namely *ordering as inverse sequence, ordering as arbitrary sequence, taking incorrect reference point, not justifying symbol manipulation and misusing positive and negative signs.*

Ordering as Inverse Sequence: It was observed that *ordering as inverse sequence* is the first sub-category of errors based on formal knowledge regarding ordering of integers. *Ordering as inverse sequence* appeared when students could not order integers correctly. They wrote not only the largest integer as the smallest integer, but also the smallest integer as the largest integer. For instance, students wrote "-3>-2>-1" instead of "-1>-2>-3" when they tried to order integers from the largest to the smallest. As seen in Table 4.27, errors under this sub-category were the most frequently made errors in item 6-d and 6-f. In item 6-d, 32 students (84.3%) among 38 students and in item 6-f, 54 students (100.0%) among 100 students made errors of *ordering as inverse sequence*. Moreover, in item 6-f, there was only one error type, which was *ordering as inverse sequence*. In item 4, 22 students (23.2%) among 95 students; in item 2, 8 students (13.6%) among 59 students and in item 6-a, 4 students (57.1%) among 7 students made the error of *ordering as inverse sequence*. To illustrate, the error of *ordering as inverse sequence* made by Participant 137 for item 2 is presented below:

Participant 137:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Rüya > Yağmur > Simge

En kısa Simge

+4 yani fazla olduğunu söylüyor -3 ise eksik olduğunu

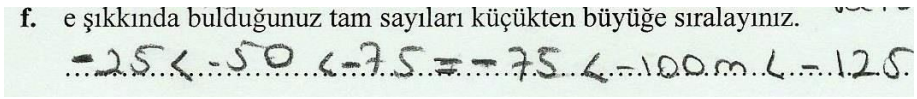
Soylüyor problems göre +4 veya -3 kısa denebilir

Figure 4.34 Answer of Participant 137 to Item 2

Participant 137 did not order the girls' hair lengths from the shortest to the longest. First, he had to indicate each hair lengths as an integer. Then he had to order these integers from the smallest to the biggest. He tried to explain the implications of "+4" and "-3" given in the question; however, he did not use these implications when he tried to order the girls' hair lengths. Hence, *ordering as inverse sequence* emerged as an error in the response of Participant 137.

For example, it was seen that Participant 256 made the error of *ordering as inverse sequence* in item 6-f as presented below:

Participant 256:


f. e şıkkında bulduğunuz tam sayıları küçükten büyüğe sıralayınız.

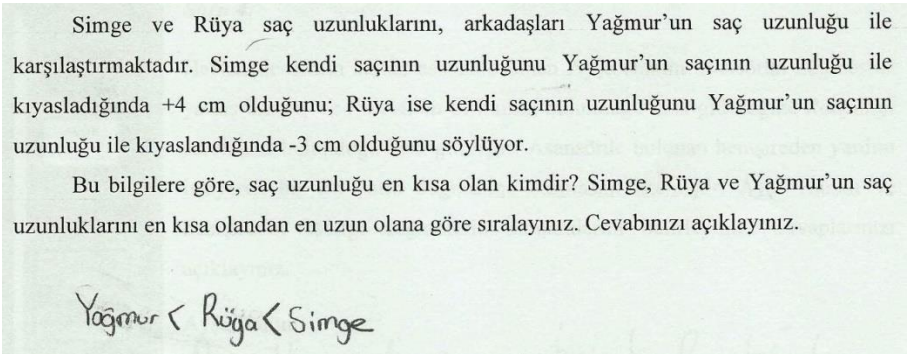
-25 < -50 < -75 = -75 < -100 m < -125

Figure 4.35 Answer of Participant 256 to Item 6-f

Participant 256 did not order the integers found in part e from the smallest to the biggest. She expressed the biggest integer as the smallest integer and vice versa. If she had changed the sign of the bigger integer instead of the sign of the lower integer, the ordering would have been right. Thus, she made the error of *ordering as inverse sequence.*

Ordering as Arbitrary Sequence: Table 4.28 shows that *ordering as arbitrary sequence* is the second error type in the categories of errors based on formal knowledge. In this error, students ordered the integers randomly, so the answers were incorrect. Students' answers were not based on any criterion. For example, they wrote "-3>-1>-2" instead of "-1>-2>-3" when they tried to order the integers from the largest to the smallest. The error within this sub-category was the most frequently made error in item 2. In item 2, 41 students (69.5%) among 59 students and in item 6-a, 3 students (42.9%) among 7 students made the error of *ordering as arbitrary sequence*. For instance, it was seen that Participant 198 made the error of *ordering as arbitrary sequence* in item 2 as presented below:

Participant 198:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Yağmur < Rüya < Simge

Figure 4.36 Answer of Participant 198 to Item 2

Participant 198 did not order the girls' hair lengths from the shortest to the longest. He did not explain the reasons of his answer. He tried to order the girls' hair lengths arbitrarily without any explanation. Therefore, *ordering as arbitrary sequence* emerged as error in the response of Participant 198.

To illustrate, the error of *ordering as arbitrary sequence* made by Participant 20 for item 6-a is presented below:

Participant 20:

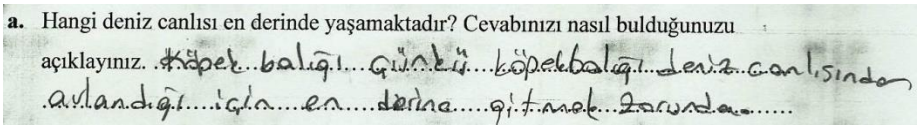

a. Hangi deniz canlısı en derinde yaşamaktadır? Cevabınızı nasıl bulduğunuzu açıklayınız. köpek balığı... çünkü... köpekbalığı deniz canlısında avlandığı için en derine gitmek zorunda.....

Figure 4.37 Answer of Participant 20 to Item 6-a

Participant 20 did not find the stingray to be existing at the deepest point in the sea. He said that the shark exists at the deepest point in the sea. The stingray existed at "-125 m"; however, the shark existed at "-75" m in the sea. He did not identify depths of each sea creature as integers, so he could not find the sea creature which existed at the deepest point in the sea by ordering the integers. Hence, he made the error of *ordering as arbitrary sequence.*

Taking Incorrect Reference Point: It was observed that *taking incorrect reference point* is the third error type in categories of errors based on formal knowledge. In this error, a reference point was given to order integers in the questions; however, students did not take into account the reference point given in the questions. They tried to give base their answer on another reference point found by the students themselves. For example, ground floor was given as "0" (zero) in the question; students did not take into account ground floor as a reference point when they tried to solve the questions. As seen in Table 4.28, in item 4, 14 students (14.7%) among 95 students and in item 8, 16 students (57.1%) among 28 students made errors which were *taking incorrect reference point*. To illustrate, the error of *taking incorrect reference point* made by Participant 200 for item 4 is presented below:

Participant 200:

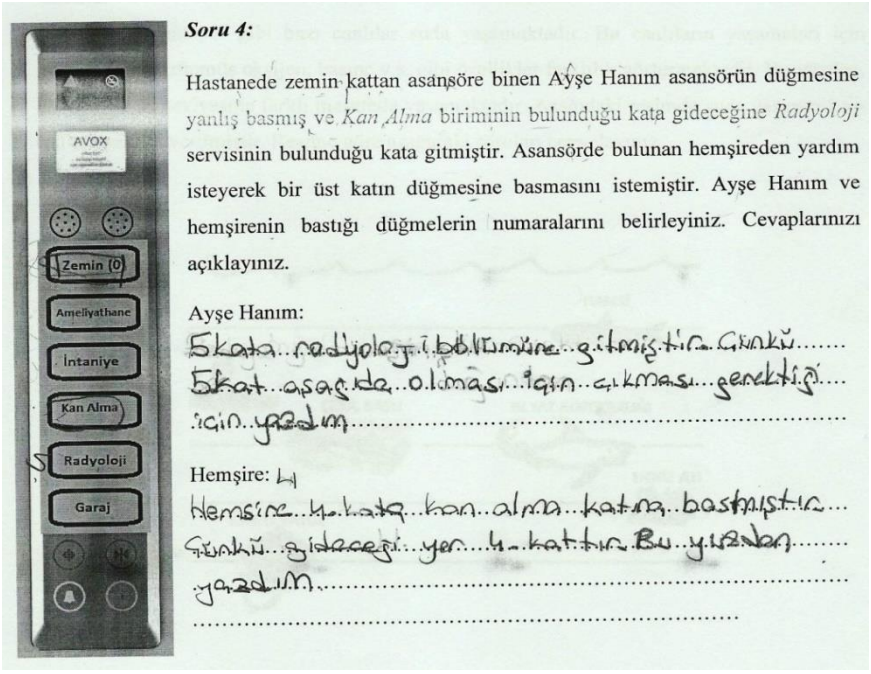
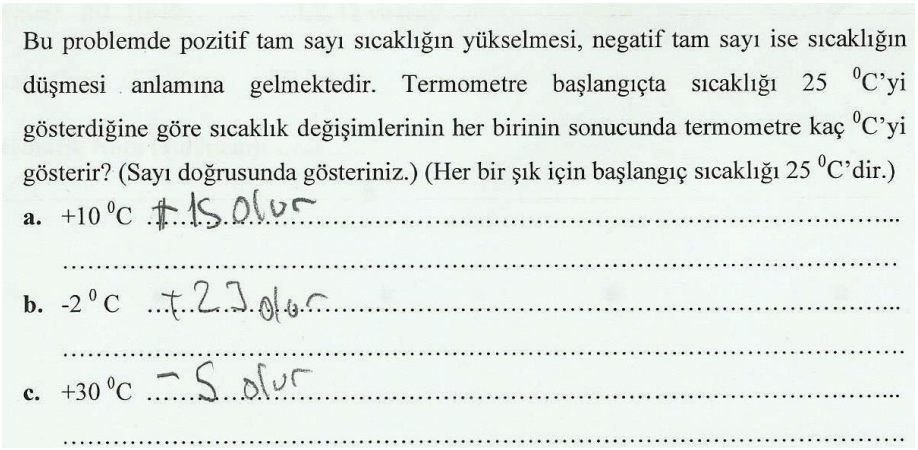


Figure 4.38 Answer of Participant 200 to Item 4

Participant 200 did not identify the buttons of the elevator as integers. In the question, ground floor was represented as zero. She had to identify the other buttons according to ground floor. Based on ground floor being represented as "0" (zero), the other buttons from surgery to garage were represented as "-1", "-2", "-3", "-4" and -5 respectively. While she was solving the question, she ignored the reference point which was the ground floor. Consequently, she did not order the buttons with correct integers. Therefore, *taking incorrect reference point* emerged as an error in the answer of Participant 200.

The answer of Participant 75 in item 8 can be presented as an example for this type of error.

Participant 75:


Bu problemde pozitif tam sayı sıcaklığın yükselmesi, negatif tam sayı ise sıcaklığın düşmesi anlamına gelmektedir. Termometre başlangıçta sıcaklığı 25 ^{0}C'yi gösterdiğine göre sıcaklık değişimlerinin her birinin sonucunda termometre kaç ^{0}C'yi gösterir? (Sayı doğrusunda gösteriniz.) (Her bir şık için başlangıç sıcaklığı 25 ^{0}C'dir.)

a. +10 ^{0}C +15 olur

b. -2 ^{0}C +23 olur

c. +30 ^{0}C -5 olur

Figure 4.39 Answer of Participant 75 to Item 8

Participant 75 did not use the given reference point, which was 25 ^{0}C when she tried to solve the question. As a result, she did not find the new degrees after changes in temperatures. She tried to solve all parts of question with different initial temperatures such as "+10", "-2", "+30", respectively. Hence, *taking incorrect reference point* was seen as an error in this solution.

Not Justifying Symbol Manipulation: It was seen that *not justifying symbol manipulation* is the fourth sub-category of errors based on formal knowledge. As mentioned in 4.3.1, this error was related to irrelevant methods of solution or numbers and numbers within the context of the problem. As can be seen in Table 4.28, it was the least seen error in the answers of the participants. In item 6-d, 1

student (5.2%) among 38 students and in item 8, 1 student (3.6%) among 28 students made the error of *not justifying symbol manipulation*. The answer of Participant 204 in item 6-d can be presented as an example for this type of error.

Participant 204:

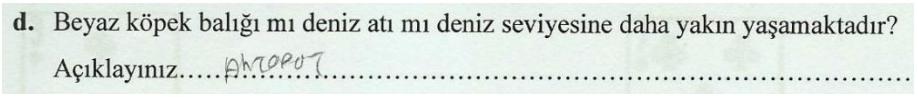


Figure 4.40 Answer of Participant 204 to Item 6-d

Participant 204 had to choose between the sea horse and the great white shark to answer the question. Octopus was given as an answer to the question; however, octopus was irrelevant to the question. Hence, this error type was *not justifying symbol manipulation*.

To illustrate, the error made by Participant 113 was *not justifying symbol manipulation* for item 8 as presented below:

Participant 113:

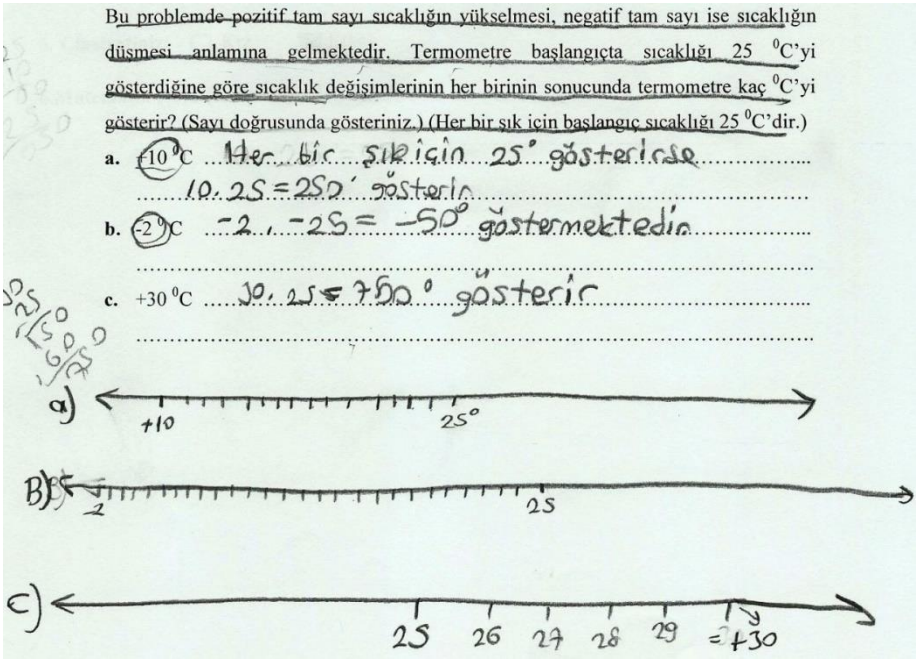


Figure 4.41 Answer of Participant 113 to Item 8

Participant 113 could not find the correct solutions in the question. He picked some numbers from the task and worked with them in irrelevant ways to the context of the

problem. In more detail, he multiplied each choice with 25 ^{0}C. These multiplications were irrelevant with the solution of the problem. Hence, *not justifying symbol manipulation* was seen as an error in the answer of Participant 113.

Misusing Positive and Negative Signs: It was seen that *misusing positive and negative signs* is the last sub-category of errors based on formal knowledge. As mentioned in 4.3.1, this error was related to students' limited conceptions about integers. As can be observed, in item 4, 43 students (45.3%) among 95 students and in item 6-d, 4 students (10.5%) among 38 students made the error of *misusing positive and negative signs*. As an example, the answer of Participant 13 is given below.

Participant 13:

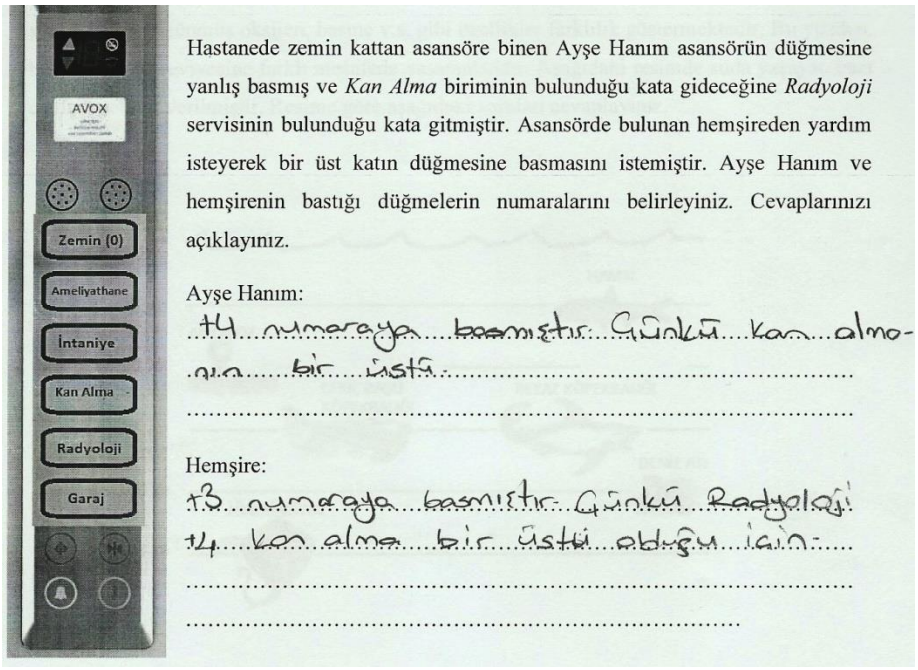


Figure 4.42 Answer of Participant 13 to Item 4

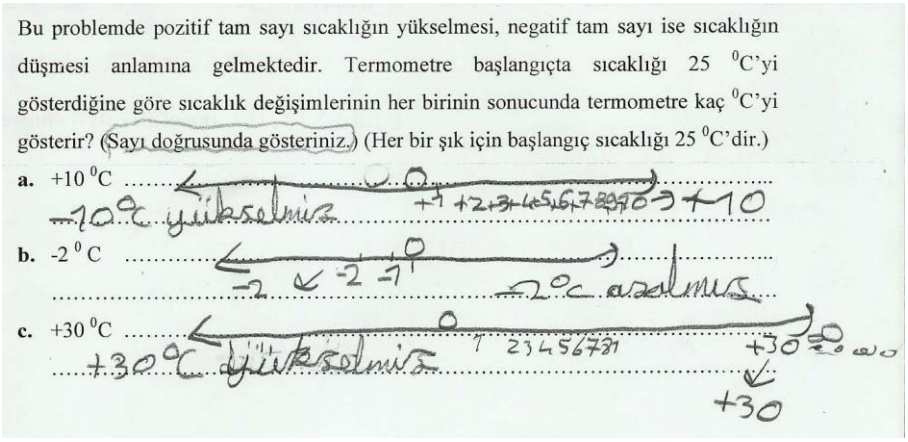
Participant 13 did not use negative integers in her solution. She had to order the buttons of the elevator as negative integers because all the buttons were under ground floor. However, she expressed that "+4" and "+3" represented radiology and blood service, respectively. Hence, *misusing positive and negative signs* emerged as an error in the response of Participant 13.

4.3.2.2 Other Errors

Errors under this category have been mentioned in the previous section which was about errors of comprehension of integers. There are two sub-categories of this error type, namely *ignoring the given information and making incorrect alignment.*

Ignoring the Given Information: It was observed that *ignoring the given information* is the first error type within the category of other errors. As mentioned in 4.3.1, students' procedures were correct; however, their final answers were incorrect in this error. In this error, they did not take into account some of the given information. Only in item 8, 11 students (39.3%) among 28 students; in item 2, 10 students (16.9%) among 59 students made the error of *ignoring the given information.* For example, it was seen that Participant 27 made the error of *ignoring the given information* in item 8 as presented below:

Participant 27:


Bu problemde pozitif tam sayı sıcaklığın yükselmesi, negatif tam sayı ise sıcaklığın düşmesi anlamına gelmektedir. Termometre başlangıçta sıcaklığı 25 ^{0}C'yi gösterdiğine göre sıcaklık değişimlerinin her birinin sonucunda termometre kaç ^{0}C'yi gösterir? (Sayı doğrusunda gösteriniz.) (Her bir şık için başlangıç sıcaklığı 25 ^{0}C'dir.)

a. +10 ^{0}C ..... 10°C yükselmiş

b. -2 ^{0}C ..... 2°C azalmış

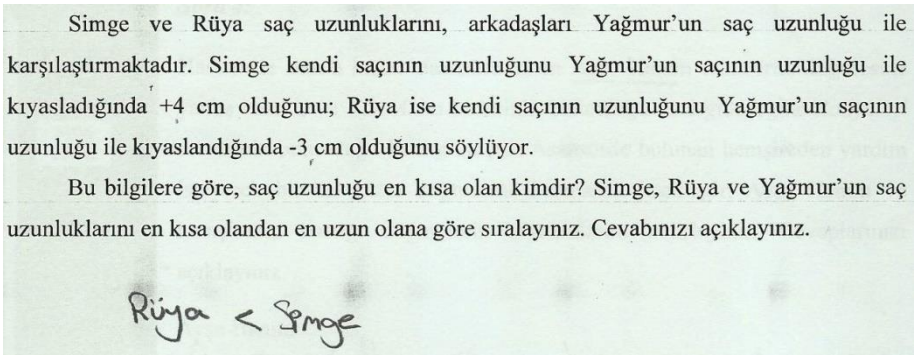
c. +30 ^{0}C ..... +30°C yükselmiş

Figure 4.43 Answer of Participant 27 to Item 8

In his solution, the student drew number lines to represent changes in temperature. In the question, the initial temperature was given as 25^0C for all parts of the question. However, he ignored the given information when he solved the question. He tried to solve the question by assuming that the initial temperature was not 25^0C. Hence, *ignoring the given information* was seen as an error in this solution.

To illustrate, the error of *ignoring the given information* made by Participant 35 for item 2 is presented below:

Participant 35:


Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Rüya < Simge

Figure 4.44 Answer of Participant 35 to Item 2

Participant 35 did not order all the girls' hair lengths. She only ordered two of the girls' hair lengths. She ignored Yağmur's hair length when ordering hair lengths. In more detail, one of the hair lengths was missing in the student's solution. Thus, *ignoring the given information* was seen as an error in the answer of Participant 35.

Making Incorrect Alignment: As can be observed in Table 4.28, *making incorrect alignment* is the second error type in the sub-category of other errors. As mentioned in 4.3.1, students could not align the correct integers to the correct locations, which were given in the question. In item 4, 16 students (16.8%) among 95 and in item 6-d, 1 (3.0%) student among 34 students made the error of *making incorrect alignment*. For example, it was seen that Participant 177 made the error of *making incorrect alignment* in item 6-d as presented below:

Participant 177:

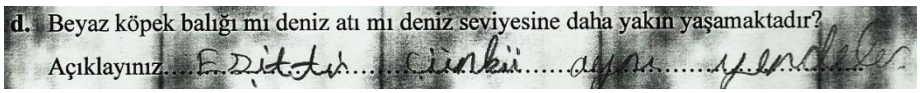

d. Beyaz köpek balığı mı deniz atı mı deniz seviyesine daha yakın yaşamaktadır?
Açıklayınız... Eşittir... Çünkü... aynı... yerdeler...

Figure 4.45 Answer of Participant 177 to Item 6-d

Participant 177 did not align the level of the sea horse and the level of the great white shark according to the sea level. Their levels were different; however, he said that their levels were the same. In other words, the student said that the great white shark and the sea horse existed at the same level. Therefore, he made the error of *making incorrect alignment.*

4.4 Underlying Reasons of Errors regarding Comprehension and Ordering of Integers

The purpose of this study was to investigate middle school 6th grade students' achievement levels, and the errors that they made regarding comprehension and ordering of integers. The other purpose was to investigate the underlying reasons of those errors. Before identifying the underlying reasons of the participants' errors, information about errors regarding comprehension and ordering integers was analyzed in IAT, which students had completed before the interviews and were presented in the previous section.

In this section, after the in-depth analysis of the responses collected from the interviews for which the number of participants was reduced, made for the purpose of identifying the underlying reasons of students' errors, the reasons revealed from the responses and explanations of the eight interview participants were categorized under four headings.

Students' responses in the interviews showed that the reasons behind their errors could be grouped under four categories as: *misunderstanding of the magnitude of numbers on the number line, reading the question carelessly, supposing that integers with the same signs are closer to each other than they are to integers with the opposite sign and, lastly, overgeneralizing properties of natural numbers to integers.* In accordance with the purpose, in this section, the researcher described possible underlying reasons of errors students made during the learning process of comprehending and ordering integers. Table 4.28 represents frequencies of reasons behind errors of the students who participated in the interviews, regarding each item on a categorical basis.

Table.4.29 Frequencies of reasons behind students' errors regarding comprehension and ordering questions

	REASONS			
Categories / *Items*	*Misunderstanding of the magnitude of numbers on the number line*	*Reading the question carelessly*	*Supposing integers with the same signs are closer to each other than they are to integers with the opposite sign*	*Overgeneralizing properties of natural numbers to integers*
Item 1-c	3	-	-	-
Item 1-f	2	-	-	-
Item 2	-	1	-	-
Item 3	4	-	-	-
Item 4	4	5	-	-
Item 5	-	-	4	-
Item 6-c	2	-	-	-
Item 6-e	4	-	-	-
Item 6-f	-	-	-	2
Item 7	-	2	-	-
Item 8	-	6	-	-
Total	19	14	4	2

According to Table 4.28, the most popular reason was *misunderstanding of the magnitude of numbers on the number line. Reading the question carelessly* was the second popular reason. Although they were not too frequent, *supposing integers with the same sign are closer to each other than they are to integers with the opposite sign* and, *overgeneralizing properties of natural numbers to integers* were other reasons behind students' errors. Examples of students' responses and explanations to interview questions are given below under the category headings given in Table 4.28.

Misunderstanding of the Magnitude of Numbers on the Number Line

It was seen that *misunderstanding of the magnitude of numbers on the number line* is the first and the most popular underlying reason of students' errors regarding comprehension and ordering of integers. This underlying reason appeared when students could not identify or order integers on the number line correctly. As it was seen in Table 4.29, *this reason* was observed in item 1-c, item 1-f, item 3, item 4, item 6-c and item 6-e. In item 1-c, 3 students; in item 1-f, 2 students; in item 3, 4 students; in item 4, 4 students; in item 6-c, 2 students and in item 6-e, 4 students made errors owing to this underlying reason. In addition to these, there were two different versions of the reason of *misunderstanding of the magnitude of numbers on the number line.*

The first version of the reason was observed in item 1-c. Findings revealed that students had difficulties in understanding the some of the properties of numbers on the number line. In more detail, they considered that the left of zero on the number line represented positive integers, whereas the right of zero on the number line represented negative integers. The conception of the number line was not learned completely by the students. Students were confused about the places of the positive and negative integers in reference to zero on the number line. Participant 80 was one of the students who made an error which is misusing negative and positive signs in item 1-c because of the misunderstanding of the magnitude of numbers on the number line and she explained her method of solution as follows:

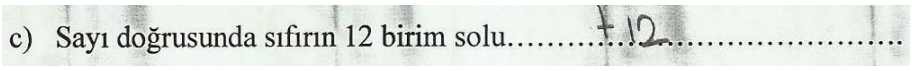


Figure 4.46 Answer of Participant 80 to item 1-c

Participant 80: "The answer is +12 because when you go to the right of the number line, numbers decrease. When you go to the left of the number line, numbers increase."

As can be clearly seen in the explanation of the student, Participant 80 considered the left of the zero on the number line to represent positive integers. She also considered

the right of the zero on the number line to represent negative integers. She confused the places of positive and negative integers in reference to zero on the number line.

The second version of this reason was observed in item 1-f, item 3, item 4, item 6-c and item 6-e. Students do not comprehend the actual meaning of "depth" in reference to sea level or water surface. Actually, the meaning of depth includes under sea level or under 0. However, students ignored the actual meaning of "depth" when they tried to solve integer questions. Thus, students could not determine the locations under water by using negative integers. Furthermore, they considered that when the depth increased, the number which represented the depth also increased.

Participant 94 was one of the students who made an error which is misusing negative and positive signs in item 1-f because of misunderstanding of the magnitude of numbers on the number line, and she explained her method of solution as follows:

Figure 4.47 Answer of Participant 94 to item 1-f

Participant 94: "The answer is +1200. If the depth increases, the number will increase. The deeper the location is, the bigger the number which represents the depth is."

Although her response suggested that she viewed her explanation meaningful, she did not successfully answer the question, in which the student was expected to indicate the 1200 meter depth as -1200. According to Participant 94, when the depth increased, the integer also increased. So she considered that the depth should be represented with a positive integer instead of a negative integer.

Participant 94 was one of the students who made an error which is misusing negative and positive signs in item 3 because of misunderstanding of the magnitude of numbers on the number line and she explained her solution as follows:

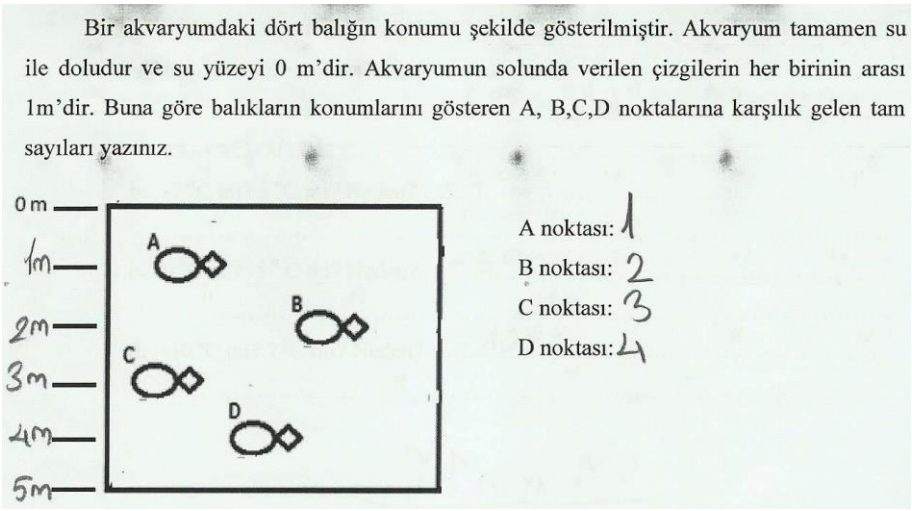


Figure 4.48 Answer of Participant 94 to item 3

Participant 94: "The aquarium shows which fish is at the deepest point or the shallowest point. Which fish is at the deepest point is given in the question as information. If the sea level is represented with 0, the location of fish A is -1. Sorry, +1. If we go one unit below sea level, the location of fish A will be +1."

Analysis of the explanation provided by Participant 94 shows that she could not successfully answer the question that required the student to identify the location of fish A as -1 meter. Participant 94 had difficulty seeing the relationship between the depth and negative integers. So she considered that the depth should be represented with a positive integer instead of a negative integer.

Participant 95 was one of the students who made an error which is misusing negative and positive signs in item 3 because of misunderstanding of the magnitude of numbers on number line and he explained his method of solution as follows:

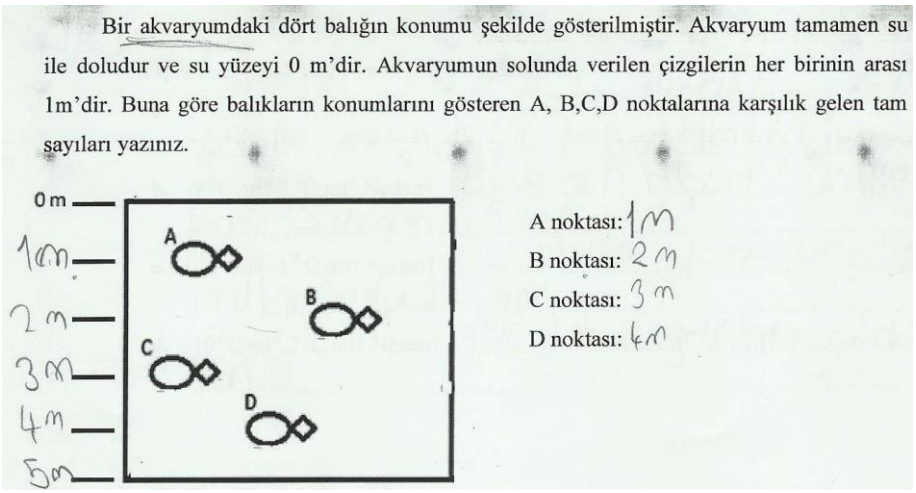


Figure 4.49 Answer of Participant 95 to item 3

Participant 95: "The locations of fish A, B, C and D are +1, +2, +3 and +4, respectively because the fish are under water level in the aquarium."

As can be clearly seen in the explanation of Participant 95, he did not need to use negative integers to represent the locations of the fish under the 0 level in the aquarium. He considered that the inside of the aquarium should be represented with positive integers instead of negative integers. He confused the places of positive and negative integers according to 0 level in an aquarium.

Reading the Question Carelessly

It was observed that *reading the question carelessly* is the second underlying reason of students' errors regarding comprehension and ordering of integers. This underlying reason appeared when students could not understand the question completely. Moreover, students could not comprehend what was asked in the question and some of the statements given in some questions. Students understood questions; however, their understandings missed a piece of information from the question. These incomplete understandings ended up with incorrect answers in students' solutions.

As it was seen in Table 4.29, this reason was observed in item 2, item 4, item 7 and item 8. In item 2, 1 student; in item 4, 5 students; in item 7, 2 students; and also in item 8, 6 students made errors owing to this underlying reason.

Participant 75 was one of the students who made an error which is ignoring the given information in item 6-e because of reading the question carelessly and he explained his solution as follows:

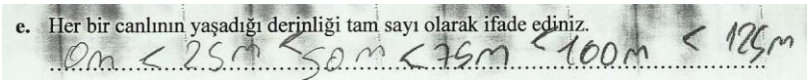


Figure 4.50 Answer of Participant 75 to item 6-e

Participant 75: "I ordered the depths of each sea creatures from the smallest to the biggest. I notice what is asked in the question, now. The ordering was not asked in the question. However, I ordered the integers in item 6-e. I confused the questions of 6-e and 6-f.

As can be clearly seen in the explanation of Participant 75, he did not understand the given information in the question correctly. He did not understand what was asked in the question. Hence, he could not give the correct answer for this question.

Participant 95 was one of the students who made an error which is ignoring the given information in item 4 because of reading the question carelessly and he explained his solution as follows:

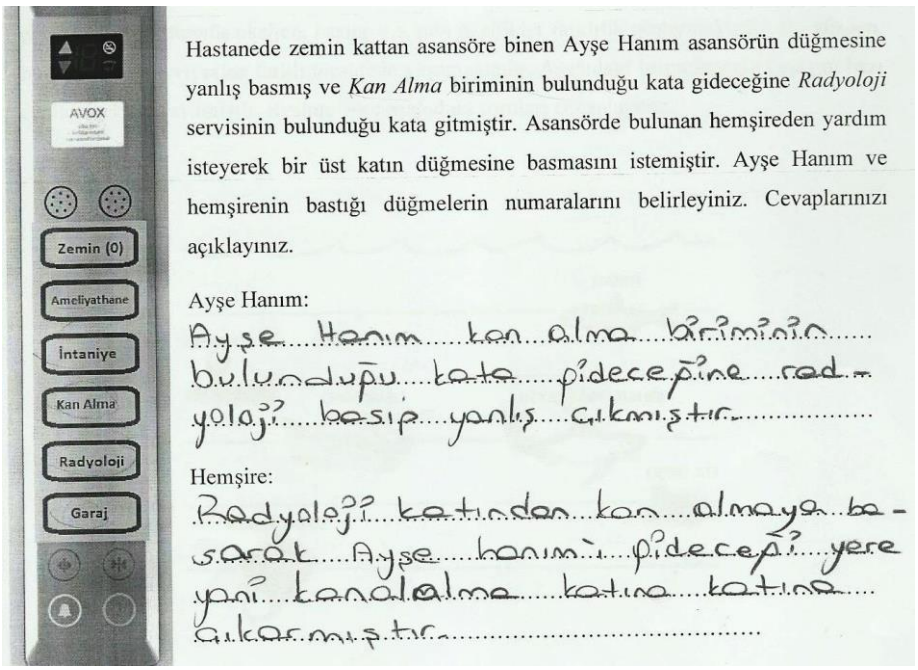


Figure 4.51 Answer of Participant 95 to item 4

Participant 95: “I did not notice that the numbers of the buttons should be identified as integers. So I did not use an integer when I explained the answers. Ayşe went to radiology. The nurse went to the blood selection service. Later Ayşe wanted help from the nurse in order to go to the blood selection service. However, I noticed that I needed to give my answer by using integers after I read the question again. The button which was pressed by Ayşe is -4. The button which was pressed by the nurse is -3.”

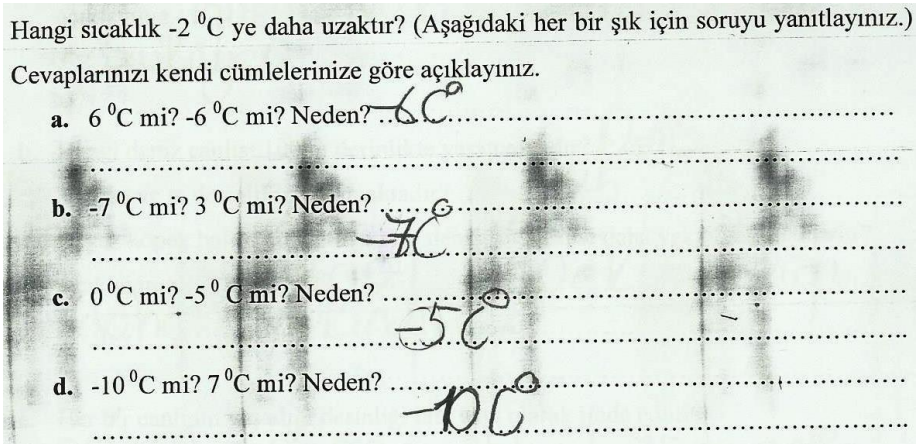
Although his response suggested that he viewed his explanation meaningful, he did not successfully answer the question that required him to identify the buttons which were pressed by the nurse and Ayşe. He could not understand all the given information in the question completely. So he did not use the numbers when he gave his answers to item 4.

Supposing integers with the same sign are closer to each other than they are to integers with the opposite sign

Table 4.29 demonstrates *supposing integers with the same sign are closer to each other than they are to integers with the opposite sign* is the third underlying reason of students’ errors regarding comprehension and ordering of integers. This underlying reason appeared when students considered that a negative integer is always closer to another negative integer than a positive integer. Meanwhile, a positive integer is always closer to another positive integer than a negative integer. Furthermore, students ignored the distance between two integers when they tried to find the furthest one between a negative and a positive integer according to another negative integer. They considered that a positive integer is always further away to a negative integer than another negative integer without any calculation of the distance between integers.

As it was seen in Table 4.29, *this reason* was observed only in item 5. In item 5, 4 students made an error owing to this underlying reason.

Participant 88 was one of the students who made an error which is misusing negative and positive signs in item 5 because of supposing integers with the same sign are closer to each other than they are to integers with the opposite sign, and he explained his solution as follows:


Hangi sıcaklık -2 ^{0}C ye daha uzaktır? (Aşağıdaki her bir şık için soruyu yanıtlayınız.)
Cevaplarınızı kendi cümlelerinize göre açıklayınız.

a. 6 ^{0}C mi? -6 ^{0}C mi? Neden? -6°

b. -7 ^{0}C mi? 3 ^{0}C mi? Neden? -7°

c. 0 ^{0}C mi? -5 ^{0} C mi? Neden? -5°

d. -10 ^{0}C mi? 7 ^{0}C mi? Neden? -10°

Figure 4.52 Answer of Participant 88 to item 5

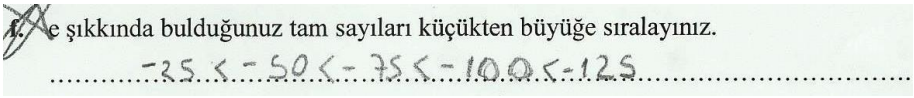
Participant 88: "A negative integer is closer than a positive integer to another negative integer. -6 and -2 are negative integers, so -6 and -2 are closer to each other than +6. "

Participant 88 considered that a negative integer is always closer to another negative integer. This idea caused him to provide an incorrect answer.

Overgeneralizing properties of natural numbers to integers

It was observed that *not realizing differences between integers and natural numbers* is the fourth underlying reason of students' errors regarding comprehension and ordering of integers. When students did not realize differences between integers and natural numbers, this reason of making an error emerged. Students overgeneralized properties and rules of natural numbers to integers, so they gave incorrect answers to integer questions. For instance, students considered that -1 is the smallest negative integer in all negative integers. As it was seen in Table 4.29, this underlying reason was observed only in item 6-f. In item 6-f, 2 students made errors owing to this underlying reason.

Participant 75 was one of the students who made an error which is ordering as inverse sequence in item 6-f because of overgeneralizing properties of natural numbers to integers, and he explained his solution as follows:


f. şıkkında bulduğunuz tam sayıları küçükten büyüğe sıralayınız.

-25 < -50 < -75 < -100 < -125

Figure 4.53 Answer of Participant 75 to item 6-f

Participant 75: "The answer is -25< -50< -75< -100< -125 because 25 is normally smaller than the other numbers."

The analysis of the explanation provided by Participant 75 shows that he overgeneralized the rules for natural numbers to integers, especially in ordering questions. Thus, his answer was incorrect.

As a consequence, this study showed middle school sixth grade students' achievement levels, the errors they made regarding comprehension and ordering of integers. In addition, there were various reasons underlying those errors.

4.5 Summary of Findings

The aims of this study were three-fold. The first purpose of this study was to investigate middle school sixth grade students' achievement levels while solving questions regarding comprehension and ordering of integers. According to the findings of the study, achievement level of students was higher in comprehension questions; also achievement level of students was medium in ordering questions. As a matter of fact, students' achievement level was higher in comprehension questions than ordering questions.

The second purpose of the present study was to investigate errors made by middle school sixth grade students related to comprehension and ordering of integers. According to the results, it was found that the students made errors while solving the questions related to comprehension and ordering of integers. Their errors were categorized into two main categories as: *errors based on formal knowledge of integers* and *other errors*. Errors based on formal knowledge of integers had three sub-categories regarding comprehension questions as: *applying incomplete solution strategy, not justifying symbol manipulation and misusing positive and negative signs.* The most common error was *misusing positive and negative signs* in the sub-categories of errors based on formal knowledge regarding comprehension questions. Other errors have two sub-categories regarding comprehension questions as: *ignoring the given information* and *making incorrect alignment*. Moreover, nearly an equal number of students made these errors. On the other hand, while students tried

to solve the questions regarding ordering questions, they also made some errors from each main category. Errors based on formal knowledge of integers had five sub-categories regarding ordering questions as: *ordering as inverse sequence, ordering as arbitrary sequence, taking incorrect reference point, not justifying symbol manipulation and misusing positive and negative signs.* The most common error was *ordering as inverse sequence* in the sub-category of errors based on formal knowledge regarding ordering questions. Other errors have two sub-categories which are same with sub-categories of other errors regarding comprehension questions. Furthermore, nearly an equal number of students made these errors.

The third purpose was to investigate underlying reasons of errors made by middle school sixth grade students regarding comprehension and ordering of integers. According to the results, four reasons were found behind students' errors regarding comprehension and ordering of integers; namely, *misunderstanding of magnitude of numbers on the number line, reading the question carelessly, supposing integers with same signs are closer to each other than they are to integers with opposite sign and lastly, overgeneralizing properties of natural numbers to integers.* The most frequently found reason was *misunderstanding of magnitude of numbers on the number line.*

CHAPTER 5

DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

The motivation for this study was to investigate middle school sixth grade students' achievement levels and to examine errors made by the students regarding comprehension and ordering of integers. The study also aimed at revealing underlying reasons of the errors made by the students. In Chapter I, the need for analyzing students' achievement levels, errors and underlying reasons of the errors they made while solving questions regarding integers was established. In Chapter II, definitions of error, misconception and integer were provided. Furthermore, the results of several studies related to integers, errors made by students' errors and difficulties encountered by students related to integers were mentioned. Next, Chapter III dwelled on the development of the achievement test in addition to research design and methodology. Both quantitative and qualitative findings of the study were presented in parallel with the research questions in Chapter IV. This final chapter concentrates on the research questions in light of the quantitative and qualitative findings in Chapter IV. Moreover, some implications for educational practices are suggested and some recommendations are made for future studies.

5.1 Discussion of the Results

The purposes of this study were to investigate middle school sixth grade students' achievement levels and errors they made regarding comprehension and ordering of integers. This study also aimed at examining the underlying reasons of errors.

This chapter is organized in such a way that each section refers to the research questions in order. To be more specific, in the first section the achievement levels of the participants in comprehension and ordering questions are discussed. Next, students' errors are discussed with an emphasis on their frequencies. Lastly, the

underlying reasons of students' errors are discussed. The results are also compared and contrasted with previous research studies in the literature.

5.1.1 Discussion of Achievement Level

To address the first research question, the achievement levels of students in comprehension and ordering of questions were investigated. The results of middle school sixth grade students' answers to the five comprehension questions showed that the majority of them answered the questions correctly. In other words, in most of the comprehension questions, nearly two third of the students gave correct answers. More specifically, 204 students among 269 students answered the first comprehension question correctly; 174 students among 269 students answered the second comprehension question correctly; 182 students among 269 students answered the third comprehension question correctly; and 96 students among 269 students answered the fourth comprehension question correctly. Lastly, 251 students, 249 students and 162 students among 269 students correctly answered the fifth comprehension questions parts b, c and e, respectively. This high achievement level of students might be due to their experiences in the three understandings of number; namely, an ordinal, a cardinal, and a formal understanding of number (Bishop et al., 2013; Baroody & Wilkins, 1999; Clements & Sarama, 2007; Fuson, 1992; Lakoff & Núñez, 2000). In more details, students have experience related to cardinality of numbers so this experience might help students to identify integers. Moreover, each of these understandings contribute highly to the comprehension of integers, and students utilize more than one understanding of number to reason when they solve comprehension questions in the IAT (Bishop et al., 2013). Another reason of the high achievement level of students might be due to their internal representations regarding negative numbers before they receive formal school instruction on negative integers (Peled, Mukhopadhyay & Resnick, 1988). In other words, before students learn the concept of integer in the school, they hold some information and experience related to negative numbers.

More specifically, the achievement of students in comprehension question 1 may derive from the fact that students' teachers might generally have used the statements asked in the question to introduce the integer topic. For example, teachers use statements such as "500 m above sea level", which was asked in the question, as real

life examples of the integer topic. Moreover, students' textbooks include similar statements which consist of daily life examples related to integers. The achievement level of nearly two third of students in comprehension questions 1, 2, 3 and 5 (item b, item c and item e) was found to be high. This might be due to the fact that these four questions involve models such as the aquarium, deck of cards, number line, and a picture from under sea level. Models have been used to help students obtain a better understanding of integers and should help to connect mathematics to the world in which the learner lives. In this way, students can convert abstract ideas to concrete form which students can relate (Steiner, 2009). Thus, models included in the comprehension questions help students understand the questions more easily and concretely, thus enabling students to obtain high achievement levels in comprehension questions 1, 2, 3 and 5 (item b, item c and item e). When the achievement levels of students in comprehension question 4 were taken into consideration in comparison to those of other comprehension questions, the findings revealed that there was a decrease in the achievement level of students. More specifically, only 96 students among 269 students answered comprehension question 4 correctly. In the question, students had to decide upon the closer integer to the given integer which was different from zero. In the textbooks, there are such questions asking for the integer that is closer to zero. However, there is no question that asks for the integer that is closer to the given integer which is different from zero. In fact, the questions in the textbook require students to find the closer integer according to only zero. Therefore, the lower achievement levels in this question might be due to the fact that the question is different from the questions given in the textbooks of the students in the integer topic. This reason is also valid for the moderate level of achievement of students regarding ordering question 3, in which students were asked to find the final temperatures according to the changes given in the question by using a number line. Moreover, the moderate level of student achievement in ordering question 3 might have resulted from the fact that students could not use the number line efficiently. Consequently, it can be said that students were not familiar with the type of questions asked in comprehension question 4 and ordering question 3 in terms of the context used in the question, so there was a decrease in the achievement levels in comprehension question 4 and ordering question 3.

It was observed that comprehension question 5 (item b) was the most correctly answered comprehension question, whereas comprehension question 4 was the least correctly answered one. When the highest rate of correct answers in comprehension question 5 (item b) is to be considered, it might be the case that students can see the depths of each sea creature from the picture given in the question directly. In more detail, students do not need to interpret any information given in the question to give a correct answer to comprehension question 5 (item b).

According to the results of middle school sixth grade students' answers to the ordering questions, it was found that nearly all students correctly answered ordering question 4 (item a), nearly two third of them correctly answered ordering questions 1, 4 (item d) and nearly half of them correctly answered ordering questions 2, 3 and 4 (item f). More specifically, the number of students who answered ordering question 1 correctly was 172 out of 269. For ordering question 2, 121 students among 269 students answered the question correctly. In ordering question 3, 111 students among 269 students answered it correctly. The number of students who answered ordering question 4 correctly was 241 out of 269, 218 out of 269 and 100 out of 269 for 6a, 6d and 6f, respectively. Students' achievements in the ordering questions were found to be at moderate level. The reason of this finding may be the case that some ordering questions involve a number line. Students used to use number lines when they ordered numbers. Resnick (1983) claimed that even before school entry, most children constitute a mental number line for the positive numbers. Primarily, children use the number line to compare the relative sizes of numbers, and after the first years of school they progressively connect to the number-line representation to the operations of addition and subtraction (Resnick, 1983). Number lines made ordering integers easy for students. In other words, students understood the ordering questions and they ordered integers easily because number lines helped them to order integers. Thus, students' familiarity with number lines facilitated ordering integers, enabling students' to obtain moderate levels of achievement in ordering questions.

Another reason of this finding may be the case that students learn counting and reasoning about smaller and greater, children experienced ordering, initially (Bishop et al., 2013). In other words, they used to order reasoning about smaller and greater. However, reasoning about greater and smaller with negative numbers is difficult for

students when the findings of İşgüden (2008) are taken into account. Students still think that the way to compare two negative numbers is the same as the way to compare two positive integers (Julie et al., 2013). This erroroneous way of thinking might be the reason why the achievement level of students in the ordering questions was not high. To be more specific, the moderate achievement level of students in ordering question 1 may derive from students' reasoning about smaller and greater because many of the students solved these questions using the strategy of comparing instead of representing each hair length as an integer and then ordering these integers. As mentioned previously, students learn to reason about smaller and greater in early grades (Bishop, 2013 et al., 2013). The reason underlying the moderate achievement level of students in ordering question 2 might be students' challenges in understanding gradual parts of the question. In more detail, students have to answer the first part of the question before they answer the second part of the question. The lowest achievement level of students was observed to be in ordering question 4, which includes part f. This might have resulted from students' generalizations about whole numbers and integers. In the question, students were expected to order five negative numbers from the smallest to the biggest. However, some of them ordered the negative numbers by considering them as positive numbers. For example, students thought that -25 is smaller than -50. In fact, students considered negative numbers as whole numbers. They generalized what they already knew to be true about whole numbers to operations in integers (Bishop et al., 2013). Hence, the reason why students obtained the lowest achievement level in ordering questions 4, which includes part f, can be attributed to students' generalization of the rule of ordering whole numbers to ordering negative numbers.

To sum up, the findings indicated that students' achievement level in comprehension questions is higher than their achievement level in ordering questions. In fact, the results revealed that students obtained high achievement levels in more of the comprehension questions than they did in the ordering questions.

5.1.2 Discussion of Students' Errors Regarding Comprehension and Ordering of Integers

The students' errors were investigated to address the second research question of the present study. The results of the study revealed an overall lack of experience of

students with integer concepts. The students made errors in the items that assessed their knowledge regarding comprehension and ordering of integers. Considering the students' errors based on IAT items, two main groups of errors based on formal knowledge and other errors were identified regarding comprehension and ordering of integers. The category of errors based on formal knowledge have six sub-categories, namely applying incomplete solution strategy, not justifying symbol manipulation, misusing positive and negative signs, ordering as inverse sequence, ordering as arbitrary sequence and taking an incorrect reference point. The second category called other errors has two sub-categories, namely ignoring the given information and making incorrect alignment.

Errors based on formal knowledge revealed information regarding the students' conceptual knowledge. The most common error based on formal knowledge was the misusing of positive and negative signs. In this error, students used positive integers instead of negative integers. This finding was also consistent with the findings of previous research studies in which students' errors and difficulties regarding integers were reported (Dereli, 2008; Hayes and Stacey, 1990; Janvier, 1983; Kilhamn, 2008; Körükcü, 2008). The reason of this error might be students' lack of conceptual knowledge regarding integers since if they had conceptualized the integers as a different number set from whole numbers, they might not have made this error. In other words, the students who made this error might not have known the difference between negative and positive numbers. Students who made the error of misusing positive and negative signs did not represent the depths given in the questions as negative integers. Another example is that students who made this error did not represent the left of the number line with a negative integer. This might be due to the fact that students might not have known the relationship between integers and natural numbers, which is in line with the findings of Köroğlu and Yeşildere (2004). The second common error based on formal knowledge was ordering as an inverse sequence, which was made while ordering integers given in the questions. This error might also have occurred due to the lack of conceptual knowledge regarding integers since most probably they ordered the integers by considering them as natural numbers. To state it differently, students assumed that the properties of integers and natural numbers were the same, so they could not order the given integers correctly, which is in line with the findings of Julie et al. (2013). In other words, many students

still make errors when they compare two negative integers because they think that the way to compare two negative numbers is the same as the way to compare two positive integers (Julie et al., 2013). The third common error based on formal knowledge was ordering as arbitrary sequence. The students who made this error ordered integers randomly. This error might also have occurred due to lack of conceptual knowledge regarding integers and the concept of ordering since it is highly likely that they ordered integers randomly. In fact, students' lack of conceptual knowledge regarding the concept of ordering was the main reason of ordering as arbitrary sequence. These findings concur with the findings of Bishop et al. (2014) because in their study, they stated that ordering relations or an order-based understanding of negative numbers is a cognitive affordance for children's integer reasoning. Order-based understanding of negative numbers poses difficulties for the students so they made errors in ordering as arbitrary sequence. Hence, overgeneralizing properties of natural numbers to integers might be the main reason underlying the misusing of positive and negative signs, ordering as inverse sequence and ordering as arbitrary sequence.

Taking the incorrect reference point was another error based on formal knowledge. The students who made the error of taking the incorrect reference point did not understand the right reference points in the questions. In more detail, this error was observed when the students did not understand the given reference points in the questions and, thus, based their answers on incorrect reference points. In other words, they tried to solve the questions by ignoring the given reference points. For this reason, taking the incorrect reference point was observed as a sub-category of errors based on formal knowledge. This might be due to the fact that students may be unfamiliar with these types of questions which required taking the reference point as zero and then identifying other integers in the questions. In parallel with the findings of Steiner (2009), students need to understand that zero as a reference point can be considered in many different ways in order to overcome the error of taking the incorrect reference point.

Applying an incomplete solution strategy was another error based on formal knowledge. This sub-category of error based on formal knowledge was observed when students knew the general method of solution in a question; however, they

could not apply the method. The reason of this error might be students' incompetence of applying their solution plan to the questions. In fact, although students are conscious about the ways of correct solutions, they did not know how they could implement their plan in order to solve the questions correctly. The last sub-category of error based on formal knowledge was not justifying symbol manipulation. This error emerged when students just picked some numbers or words from the task and worked with them in ways irrelevant to the context of the question or tried to identify an integer with irrelevant numbers. This error was also observed in some other studies (Janvier, 1983; Linchevski & Williams, 1999). Besides, students might not have understood the given information in the questions, causing them to work with irrelevant numbers instead of the given integers. In fact, students worked on methods of solutions that were inappropriate instead of correct methods of solution, so the error of not justifying symbol manipulation was observed. The reason of this error might be students' lack of general mathematical knowledge. To be more specific, the students' lack of mathematical knowledge related not only to the integer concept but also to other mathematical topics.

On the other hand, there were errors other than the errors based on formal knowledge, which was mentioned under the category of other errors. The most common type of errors within this category was ignoring the given information. This error was observed when students did not take into consideration all the information given in the questions. This may have resulted from making a minor error while reading the questions carelessly since most of the students who made this error read one or two pieces of information incorrectly. Another reason underlying the other errors sub-category might be the lack of students' understanding of what was being given and what was being asked for in the questions. The last sub-category of other errors was making incorrect alignment. In this error, students could not align the correct integers to the correct locations which were given in the question. In other words, the students were confused about the location of the integers given in the picture or model in the questions, thus causing this error to emerge. Some common explanations might be given for these findings. The common reason underlying these errors might be reading items carelessly since the given answers were not appropriate. In other words, the reason underlying them might be the insufficient attention devoted to the reading of the questions.

5.1.3 Discussion of Underlying Reasons of Students' Errors Regarding Comprehension and Ordering of Integers

To address the third research question, the reasons underlying students' errors were investigated. The findings related to this research question were discussed in the following parts in order of most to least frequently observe. Analyses revealed four reasons that were common for students' errors regarding comprehension and ordering questions. These were misunderstanding of the magnitude of the numbers on the number line, reading the question carelessly, supposing that integers with the same signs are closer to each other than they are to integers with the opposite sign and, lastly, overgeneralizing the properties of natural numbers to integers. Besides, the most common reason was found: misunderstanding of the magnitude of the numbers on the number line.

To begin with, misunderstanding of the magnitude of the numbers on the number line seemed to be a problem that was hindering students' from giving the correct answers in the questions. Similarly, low achievement of students was documented in previous studies (Bishop et al., 2013; Widjaja et al., 2011). In line with the results of these studies, the findings of the present study highlighted students' lack of knowledge regarding the number line and the magnitude of numbers. For instance, some students considered that the left of zero on the number line represented positive integers, whereas the right of zero on the number line represented negative integers. This inadequate and incorrect knowledge constituted a handicap mostly in comprehension questions 1 (item c, item f), 2, 5 (item c, item e) and ordering question 2. This finding of the current study is consistent with that of Widjaja et al., (2011), who explored misconceptions regarding the number line. This finding was parallel with the results of the study of Baturo (2000). In the study, it is suggested that the number line comprises a difficult model for students to manipulate (Baturo, 2000). The reason for this finding might be related to the fact that while teaching the number line, the teacher may not be laying sufficient emphasis on all its properties and he/she may not be giving students the opportunity to gain practice in examining all the properties of the number line. In fact, students' experiences related to the number line are not sufficient to solve integer questions by using the number line. In

addition, students' teachers might not have used sufficient examples with the number line related to integers in their classroom instruction.

Another example is that some students did not infer that the meaning of below zero or left of the number line could form the word "depth". In more detail, they considered that when the depth increased, the number which represented the depth also increased because students had misunderstandings about the magnitude of numbers on the number line. To illustrate, students represent a depth, which shows the place of sea creatures under sea with a positive number instead of a negative number. The findings of the present study revealed that students do not need to use negative integers to represent a depth. It might be inferred that students were also unfamiliar with the examples which involved "depth". If teachers use vertical number lines instead of usual number lines, students might use negative integers to represent a depth.

As a second common underlying reason of students' errors regarding comprehension and ordering of integers was reading the question carelessly. Generally, students' procedures were correct; however, their final answers were incorrect because of lack of attention. In fact, students could not comprehend exactly what was asked in the question and some of the statements given in some questions. To be more exact, students actually understood the questions; however, their understandings missed a piece of information incorrectly. These understandings ended up with incorrect answers in their methods of solution. This finding is in agreement with the findings of Ryan and William (2007), which showed these reasons do not stem from students' conceptual development, but from students' lack of adequacy in mathematical language (Yurtsever, 2012).

Data analysis also demonstrates that supposing that integers with the same signs are closer to each other than they are to integers with the opposite sign was the third reason, which is also a misconception. This was observed in only ordering question 2. Students ignored the distance between two integers when they tried to find the furthest one between a negative and a positive integer according to another negative integer. They considered that a positive integer is always further away to a negative integer than another negative integer without any calculation of the distance between. For example, this might be due to the fact that students were confused about the

places of negative numbers on the number line (İşgüden, 2008). The students misapplied the rules for ordering natural numbers to integers. In more detail, the students were unable to combine the information of negative numbers and the number line; therefore, they could not make a correct judgment of ordering integers.

As a fourth and last reason underlying students' errors regarding comprehension and ordering of integers was overgeneralizing properties of natural numbers to integers, which is also a misconception. This reason emerged only in ordering question 5 (item f). Literature review revealed that students have a tendency to make errors because of overgeneralizing properties of natural numbers to integers (Bruno et al., 1999; Linchevski & Williams, 1999; Ponce, 2007; Pratiwi et al., 2013). In line with the findings of previous research, some students experienced difficulties in the question in such a way that they ordered negative numbers by assuming them like natural numbers. In other words, students try to interpret negative numbers based on their actual knowledge on whole numbers. However, the number of students who made errors because of overgeneralizing properties of natural numbers to integers was found to be low despite the great emphasis on this reason, which is also a misconception as stated in the related literature. That is to say, the students could deal with the ordering questions that required ordering negative and positive numbers, and a few students experienced challenges. This finding might have resulted from the fact that the ordering questions in the current study, such as item item f, were conventional ordering questions that appear in the initial stages of formal learning of ordering positive and negative numbers. In other words, students might have recognized the question structure as usual questions that they deal with in their school lessons. Considering the analysis results of the interview data, it can be inferred that students focus solely on the sentences and numbers in the problem. Therefore, while interpreting the results of the current study it is important to note that the high or moderate level of achievement in comprehension and ordering questions should not be understood as if students' understanding of integer is conceptual. In fact, students might not have sufficient conceptual understanding related to integers, which can be inferred from analysis results of the data obtained in interviews.

All in all, it was seen that the results of this study not only confirmed the findings of previous studies but also moved the discussion one step ahead. The present study not only examined students' achievement level in comprehension and ordering questions but also dealt with errors of students and why they might have solved the questions incorrectly. Therefore, this study took a deeper look at students' development of integer reasoning, particularly comprehending and ordering integers, and hence obtained the elements of the bigger picture. Thus, it is highly likely that the findings of this study will provide some implications for educational practices and recommendations for further studies. Therefore, the following two sections seek to shed light into the practical and research-based issues in line with the findings of the present study accompanied with the findings of previous studies.

5.2 Implications for Educational Practices

Findings of the present study revealed that students' achievement level in ordering questions were at moderate level. However, despite this moderate level of achievement in ordering questions, it was observed that students also made errors regarding ordering of integers. In fact, a moderate level of achievement in ordering questions does not indicate that students might not have made errors in ordering questions. It was discussed that a moderate level of achievement of students in ordering questions might be resulting from the fact that students learn counting and reasoning about smaller and greater. In fact, children experienced ordering, initially (Bishop et al., 2013). In other words, they used to order reasoning about smaller and greater in their early years of education. Therefore, decreasing or preventing students' errors in ordering questions, should be promoted in order to develop students' conceptual understanding of ordering integers. Students' errors increase particularly in non-routine questions in the IAT. Hence, non-routine questions, such as ordering question 1, should be asked to students when the teacher covers the ordering of integers in order to develop students' understanding regarding ordering of integers.

The findings of this study provides mathematics teachers, curriculum developers, textbook writers and teacher educators with essential information related to students' achievement level in comprehension and ordering questions, students' errors regarding comprehension and ordering of integers and the underlying reasons of

students' errors. The findings of this study revealed that sixth grade students made six different errors regarding comprehension and ordering of integers. Mathematics teachers, curriculum developers, textbook writers and teacher educators should take the students' errors into consideration to establish an effective teaching environment and prepare effective learning materials while teaching the integer concept.

More specifically, teachers could benefit from findings of the study regarding middle school sixth grade students' errors and underlying reasons of these errors regarding comprehension and ordering of integers. Firstly, teachers could benefit from middle school sixth grade students' errors and reasons of the errors regarding comprehension and ordering of integers. Seminars or in-service programs to provide awareness may be organized for teachers in order to prepare suitable teaching plans which could eliminate the errors. For example, teachers can solve additional questions regarding comprehension and ordering integers in order to develop middle school students' conceptual understandings of integers. Hence, their errors can be eliminated. Moreover, some of the students' errors might be due to their misconceptions. Hence, the teachers can become aware of students' errors and in this way they can investigate the reasons underlying students' errors regarding comprehension and ordering of integers. Some of the reasons underlying students' errors might be misconceptions regarding the concepts. Thus, teachers can find appropriate solutions and overcome those errors and misconceptions.

Furthermore, teacher educators can also benefit from the findings of the study. More specifically, pre-service middle school mathematics teachers can be informed about middle school students' errors and the underlying reasons of errors regarding comprehension and ordering of integers. Since pre-service teachers will be aware of the defined errors and the reasons underlying them, they can prepare suitable teaching plans to prevent these errors when they are in-service teachers. In addition, discussions related to how pre-service teachers make their prospective students understand the comprehension and ordering of integers might improve their pedagogical content knowledge related to integers. These discussions could be beneficial for improving pre-service teachers' affects in teaching the concept of integers conceptually.

5.3 Recommendations for Further Research Studies

The participants of the current study were selected by means of convenience sampling from the accessible population which consisted of public schools in the Etimesgut District of Ankara. Since the sampling method is not one of the random sampling methods, the findings of the study could not be generalized to a large population. Some recommendations might be made considering the sample of the study. In order to generalize the findings of the study to a population, the same study could be replicated with a sample randomly selected from nationwide schools in such a way that the sample would be representative of all sixth grade students in Turkey.

As mentioned above, the concept of integer is complex and cannot be understood and learned all at once by children; it has to be acquired through a long process of sequential development through carefully organized sequences of teaching. There is still much scope in this area for further research. The researcher investigated the errors and possible reasons of sixth graders' errors through the application of IAT and an interview to analyze the answers of the students in-depth. Further research could be designed as interventions to find out how misconceptions which emerged as the reasons of errors can be remedied. In other words, in further work, an experimental research could be done by designing a method of instruction to examine the effect of the treatment.

The research in this study may prove useful in the areas of both professional development of teachers and practice. The identification of these specific errors related to integers along with their reasons can be a powerful tool to pinpoint the weak areas in instruction that need to be developed further.

This study was designed as a survey method supported with individual interviews; hence, some changes might be done in the research methodology of the present study. In order to see the changes in students' achievement levels, errors and reasons of the errors, a longitudinal study beginning with students in sixth grade and observing the same students' development of comprehension and ordering of integers throughout their middle school education might be conducted. In other words, further studies could be conducted with a smaller group that can be observed for the changes over a few years.

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APPENDICES

APPENDIX A. PERMISSION OBTAINED FROM METU APPLIED ETHICS RESEARCH CENTER

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
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20.02.2014

Gönderilen : Doç. Dr. Mine IŞIKSAL BOSTAN
İlköğretim Fen ve Matematik Eğitimi

Gönderen : Prof. Dr. Canan Özgen
IAK Başkanı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Fen ve Matematik Eğitimi Bölümü öğrencisi Gizem Sevim Atayev'in "6. Sınıf Öğrencilerinin Tam Sayıları Yorumlama Ve Sıralama Konusu İle İlgili Hataları, Zorlukları Ve Kavram Yanılgıları Üzerine Bir Çalışma "isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

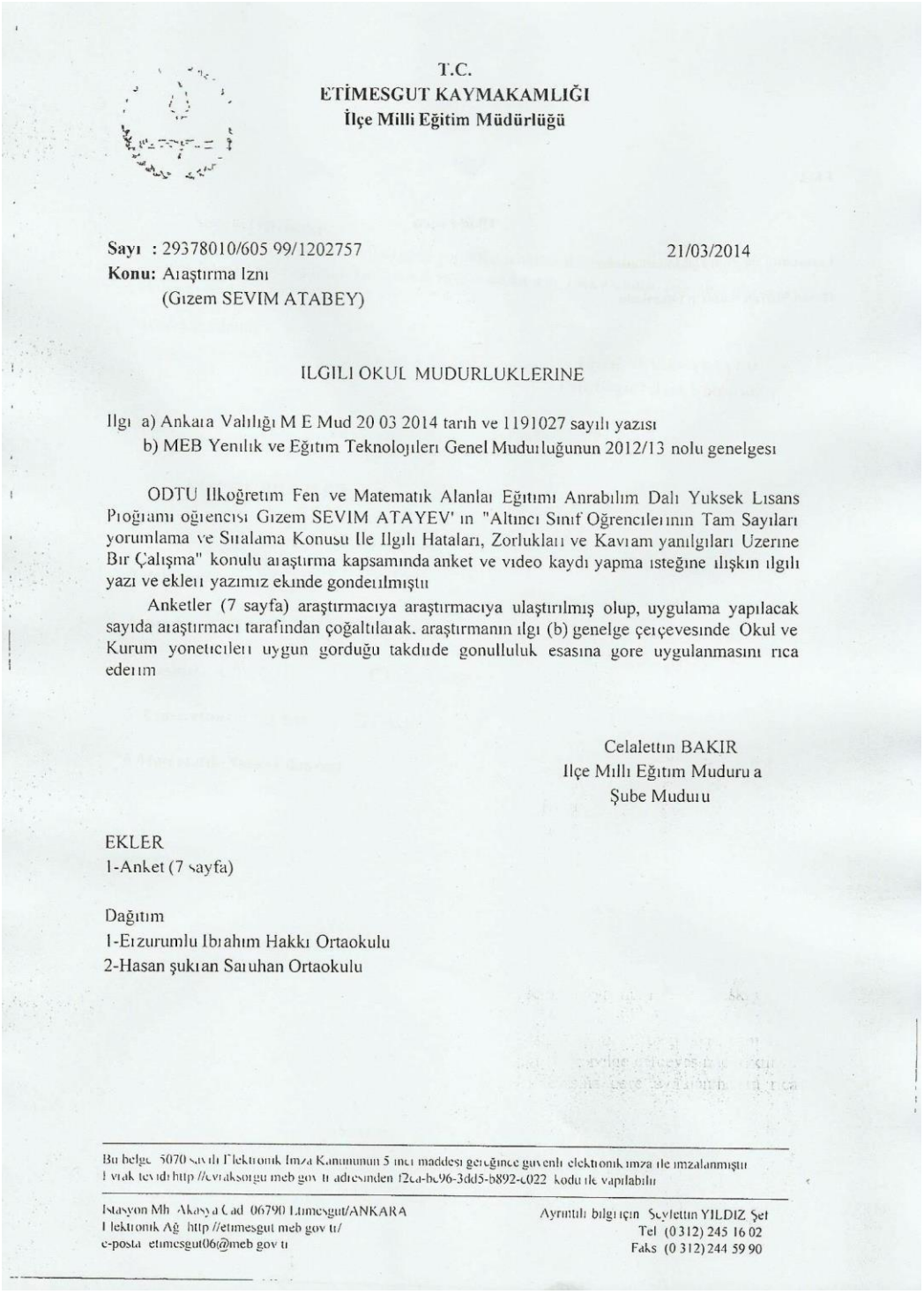
Etik Komite Onayı

Uygundur

20/02/2014

Prof.Dr. Canan Özgen
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

APPENDIX B. PERMISSION OBTAINED FROM MINISTRY OF EDUCATION



T.C.
ETİMESGUT KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Sayı : 29378010/605 99/1202757 21/03/2014
Konu: Araştırma Izni
(Gizem SEVIM ATABEY)

ILGILI OKUL MUDURLUKLERINE

Ilgı a) Ankara Valılığı M E Mud 20 03 2014 tarıh ve 1191027 sayılı yazısı
b) MEB Yenılık ve Eğıtım Teknolojıleri Genel Muduluğunun 2012/13 nolu genelgesi

ODTU Ilkoğretım Fen ve Matematık Alanlaı Eğıtımı Anrabılım Dalı Yuksek Lısans Pıoğıamı oğıencısı Gızem SEVIM ATAYEV' ın "Altıncı Sınıf Oğrencıleı ının Tam Sayıları yorumlama ve Sıralama Konusu Ile Ilgılı Hataları, Zorlukları ve Kavıam yanılgıları Uzerıne Bır Çalışma" konulu aıaştırma kapsamında anket ve vıdeo kaydı yapma ısteğıne ılışkın ılgılı yazı ve ekleı i yazımız ekınde gondeılmıştı

Anketler (7 sayfa) araştırmacıya araştırmacıya ulaştırılmış olup, uygulama yapılacak sayıda aıaştırmacı tarafından çoğaltılaıak, araştırmanın ılgı (b) genelge çeıçevesınde Okul ve Kurum yoneticileı ı uygun gorduğu takdıde gonulluluk esasına gore uygulanmasını rıca edeı ım

Celalettın BAKIR
Ilçe Mıllı Eğıtım Muduru a
Şube Muduıu

EKLER
1-Anket (7 sayfa)

Dağıtım
1-Erzurumlu Ibrahım Hakkı Ortaokulu
2-Hasan şukıan Saıuhan Ortaokulu

Bu belge 5070 sayılı Elektronik Imza Kanununun 5 inci maddesi gerçeğince guvenlı elektronik ımza ile ımzalanmıştı
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APPENDIX C. INTEGER ACHIEVEMENT TEST

Sevgili Öğrenciler,
Bu çalışma sizin tam sayıları nasıl yorumlayacağınız ile ilgilidir. Sorulara vereceğiniz yanıtlar, bilimsel bir araştırmada kullanılacak ve gizli tutulacaktır. Lütfen soruları dikkatlice okuyarak eksiksiz yanıtlayınız.

Teşekkür ederim.

Gizem SEVİM
ATAYEV Orta Doğu
Teknik Üniversitesi

KİŞİSEL BİLGİLER

1.Adınız: …….

2. Okulunuz: ……

3. Sınıfınız: ☐ 7. Sınıf ☐ 6. Sınıf

4. Yaşınız: ☐ 10-12 ☐ 12 ve üstü

5. Cinsiyetiniz: ☐ Kız ☐ Erkek

6.Matematik Notu (1.dönem): ……..

TAM SAYILAR BAŞARI TESTİ

Bu test tam sayılarla ilgili 8 soru içermektedir. Bazı sorular bir ya da birkaç alt sorudan oluşmaktadır. **Lütfen tüm soruları cevaplamaya çalışınız.**

Soru 1:

Aşağıdaki ifadeleri birer tam sayı olarak yazınız.

a) Sıfırın altında 7^0C …….
b) Deniz seviyesinin 500 m üstü …….
c) Sayı doğrusunda sıfırın 12 birim solu…….
d) 25 TL zarar ……..
e) Cüzdandaki 10 TL ……..
f) 1200 metre derinlik …….

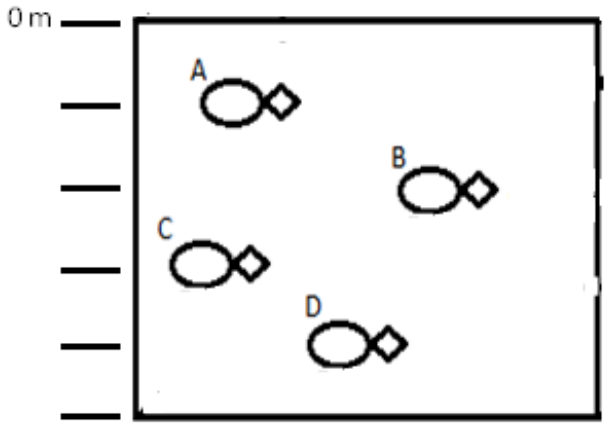
Soru 2:

Simge ve Rüya saç uzunluklarını, arkadaşları Yağmur'un saç uzunluğu ile karşılaştırmaktadır. Simge kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyasladığında +4 cm olduğunu; Rüya ise kendi saçının uzunluğunu Yağmur'un saçının uzunluğu ile kıyaslandığında -3 cm olduğunu söylüyor.

Bu bilgilere göre, saç uzunluğu en kısa olan kimdir? Simge, Rüya ve Yağmur'un saç uzunluklarını en kısa olandan en uzun olana göre sıralayınız. Cevabınızı açıklayınız.

Soru 3:

Bir akvaryumdaki dört balığın konumu şekilde gösterilmiştir. Akvaryum tamamen su ile doludur ve su yüzeyi 0 m'dir. Akvaryumun solunda verilen çizgilerin her birinin arası 1m'dir. Buna göre balıkların konumlarını gösteren A, B,C,D noktalarına karşılık gelen tam sayıları yazınız.



A noktası:

B noktası:

C noktası:

D noktası:

Soru 4:

Hastanede zemin kattan asansöre binen Ayşe Hanım asansörün düğmesine yanlış basmış ve *Kan Alma* biriminin bulunduğu kata gideceğine *Radyoloji* servisinin bulunduğu kata gitmiştir. Asansörde bulunan hemşireden yardım isteyerek bir üst katın düğmesine basmasını istemiştir. Ayşe Hanım ve hemşirenin bastığı düğmelerin numaralarını belirleyiniz. Cevaplarınızı açıklayınız.

Ayşe Hanım:

……

……

……

……

Hemşire: ……

……

……

……

Soru 5:

Hangi sıcaklık -2 ^{0}C ye daha uzaktır? (Aşağıdaki her bir şık için soruyu yanıtlayınız.) Cevaplarınızı açıklayınız.

a. 6 ^{0}C mi? -6 ^{0}C mi?

Neden? ……

……

b. -7 ^{0}C mi? 3 ^{0}C mi?

Neden? …….

……

c. 0 ^{0}C mi? -5 ^{0} C mi?

Neden? …….

……

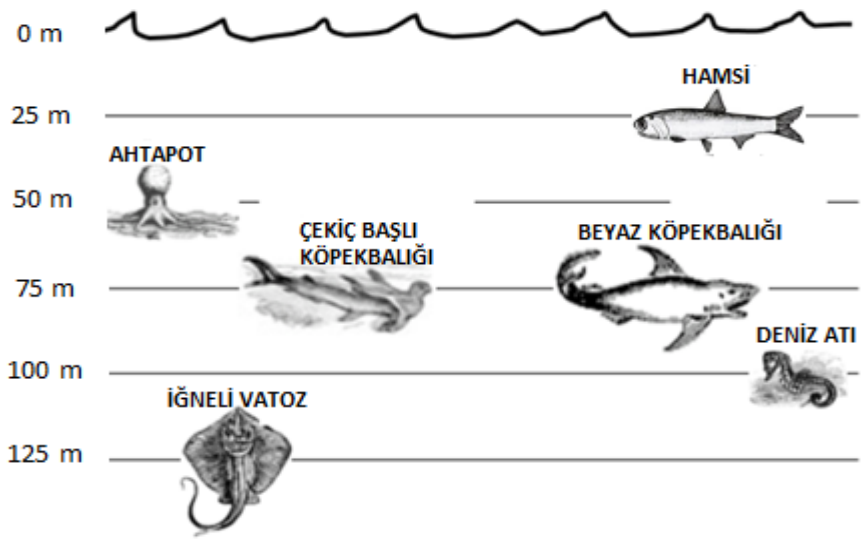
d. -10 ^{0}C mi? 7 ^{0}C mi?

Neden? ……

……

Soru 6:

Balık, ahtapot gibi bazı canlılar suda yaşamaktadır. Bu canlıların yaşamaları için gerekli olan çözünmüş oksijen, basınç v.s. gibi özellikler farklılık göstermektedir. Bu yüzden, bu canlılar su seviyesine farklı mesafede yaşamaktadır. Aşağıdaki resimde suda yaşayan bazı canlı örnekleri verilmiştir. Resime göre aşağıdaki soruları cevaplayınız.



a. Hangi deniz canlısı en derinde yaşamaktadır? Cevabınızı nasıl bulduğunuzu açıklayınız. …….....
.....

b. Hangi deniz canlısı 100 m derinlikte yaşamaktadır? …….

c. Vatoz kaç m derinlikte yaşamaktadır? ……...

d. Beyaz köpek balığı mı deniz atı mı deniz seviyesine daha yakın yaşamaktadır? Açıklayınız……
……

e. Her bir canlının yaşadığı derinliği tam sayı olarak ifade ediniz.

……

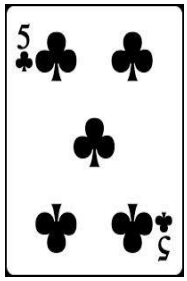
f. e şıkkında bulduğunuz tam sayıları küçükten büyüğe sıralayınız.

……

Soru 7:

Ahmet ve Batuhan iskambil kâğıtlarını kullanarak bir oyun geliştirmiştir. 52 kartlık bir iskambil kâğıdı destesi içinden resimli kartları ve "A" harfinin olduğu kartları çıkarmışlardır.

Her kırmızı kartın üzerinde yazan sayı kadar zarar, her siyah kartın ise üzerinde yazan rakam kadar kazanç olduğuna karar verilmiştir. Örneğin, aşağıdaki kart +5 tam sayısına karşılık gelmektedir.

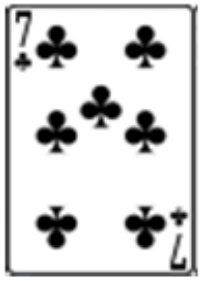


Aşağıda Batuhan'ın çektiği kartlar gösterilmiştir. Bu kartların hangi tam sayılara karşılık geldiğini yazınız.

……

……..

……..



……..

Soru 8:

Bu problemde pozitif tam sayı sıcaklığın yükselmesi, negatif tam sayı ise sıcaklığın düşmesi anlamına gelmektedir. Termometre başlangıçta sıcaklığı 25 ^{0}C'yi gösterdiğine göre sıcaklık değişimlerinin her birinin sonucunda termometre kaç ^{0}C'yi gösterir? (Sayı doğrusunda gösteriniz.) (Her bir şık için başlangıç sıcaklığı 25 ^{0}C'dir.)

a. +10 ^{0}C

……

……

b. -2 ^{0} C

……

……

c. +30 ^{0}C

……

……

APPENDIX D. INTERVIEW QUESTIONS

Görüşme Soruları

1) Yaptığın çözüm yolunu anlatır mısın?
2) Neden bu şekilde düşündün?
3) Neden bu çözüm yolunu tercih ettin?
4) Problemden ne anladın?
5) Problemi çözerken verilen bilgileri inceledin mi?

APPENDIX E: TURKISH VERSION OF EIGHT INTERVIEWEES' EXPLANATION

80 kodlu öğrencinin soru 1-c için açıklaması:

[+12. Çünkü sayı doğrusunda sağa gittikçe sayılar küçülüyor. Sola gittikçe sayılar büyüyor.]

94 kodlu öğrencinin soru 1-f için açıklaması:

[+1200 çünkü derinlik arttıkça aşağıya doğru sayı da büyüyecek. Yani, indikçe daha da çoğalacak.]

94 kodlu öğrencinin soru 3 için açıklaması:

[Akvaryum bana hangisinin en derinde olup veya yüksekte olup olmadığını gösteriyor. Balıkların hangisinin daha derinde olup daha da yüksek pozitif, negatif olup tam sayı olarak belirttiğini soru bana bilgi olarak vermiş. Su yüzeyi 0 ise, A balığının konumu -1 pardon +1 olur. Bir aşağıya inersek +1 olur.]

95 kodlu öğrencinin soru 3 için açıklaması:

[A balığı +1, B balığı +2, C balığı +3 ve D balığı +4 çünkü balıklar akvaryumun aşağı tarafında.]

75 kodlu öğrencinin soru 6-e için açıklaması:

[Her bir deniz canlısının yaşadığı derinliği küçükten büyüğe sıraladım. Şimdi, sorunun ne sorduğunu fark ettim. Soruda sıralama sorulmamış ama ben tam sayıları sıralamışım. 6-e ile 6-f sorularını karıştırmışım.]

95 kodlu öğrencinin soru 4 için açıklaması:

[Düğmelerin numaralarını belirlemem gerektiğini fark etmemişim. Bu yüzden tam sayı kullanmadan açıklama yaptım. Ayşe Hanım radyolojiye çıkmış. Hemşire ise Ayşe hanımın isteği üzerine kan alma birimine çıkmış. Ama şimdi soruyu tekrar okuyunca cevabımı sayı olarak vermem gerektiğini fark ettim. Ayşe Hanım'ın bastığı düğme -4'dür. Hemşirenin bastığı düğme ise -3'dür.]

88 kodlu öğrencinin soru 5 için açıklaması:

[a. Eksi hani eksiye daha yakındır. -6 da eksi var. -2 de – var. İkisi de negatif olduğu için birbirine daha yakındır.]

75 kodlu öğrencinin soru 6-f için açıklaması:

[-25< -50< -75< -100< -125. Çünkü 25 normalde de rakamlarda öbür rakamlara göre küçük olduğu için.]

APPENDIX F. TURKISH SUMMARY

Giriş

Matematik eğitimin öncelikli hedeflerinden biri tüm öğrencilerin matematik öğrenme sürecinde en yüksek performanslarını göstermelerini sağlamaktır. Ama çok az öğrenci bu seviyeye ulaşabilmektedir ve çoğu öğrenci, matematik öğrenmede zorluklar yaşamaktadır (Tall & Razzali, 1993). Ayrıca, matematiği öğrenmek için onu anlamak şarttır (Hiebert & Carpenter, 1992). Matematiği anlamak, bazı temel yeteneklerin yanı sıra bu temel yetenekler arasındaki ilişkileri öğrenmeyi de içerir. Matematiği anlayarak öğrenmek çok önemlidir çünkü matematik insanların gelecekleri için fırsatlar yaratabildikleri önemli bir araçtır (NCTM, 2000).

Tam sayılar matematik eğitiminde önemli bir role sahiptir çünkü tam sayılar üzerine yapılan birçok araştırmanın sonucu, tam sayılar konusunun karmaşık ve öğrenmek için ciddi emek gerektiren bir konu olduğunu göstermiştir (Dereli, 2008; Janvier, 1983; Kilhamn, 2008; Mc Corkle, 2001). Tam sayılar konusu cebir gibi diğer temel matematik konuları için ön koşul durumunda olduğundan, tam sayılar konusunda zorluk yaşayan öğrenciler, bağlantılı konularda da zorluklar yaşamaktadır (Lamb et al., 2012; Vlassis, 2002). Tam sayılar, öğrencilerin öğrenirken hata yapmaya eğilimli olduğu matematik konularından biridir (Janvier, 1983). Tam sayılar gibi önemli ve sorunlu bir matematik konusunun öğretiminde başarılı olmak için, öğrencilerin yaptıkları hataları ve yaşadıkları zorlukları bilmek, bahsedilen başarıya ulaşmakta önemli bir role sahiptir (Yetkin, 2003). Sonuç olarak, öğrencilerin tam sayıları kavrama ve sıralama konularında yaptıkları hataları araştırmak önemli bir başlık oluşturmaktadır. Ayrıca, bu hataların kaynaklarını yani sebeplerini bilmek, bu hataları önlemekte etkili olacağı için öğrencilerin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataların nedenleri de araştırmak için önemli bir başlık oluşturmaktadır.

Çalışmanın Amaçları

Bu çalışmanın amacı üç kısımdan oluşmaktadır: Çalışmanın birinci amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki başarı seviyelerini tespit etmektir. Çalışmanın ikinci amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataları incelemektir. Çalışmanın üçüncü amacı ise, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataların arkasındaki sebepleri incelemektir.

Çalışmanın Önemi

Herhangi bir matematik konusunda zorluklar yaşayan öğrenciler, zorluk yaşanılan konuyu takip eden diğer matematik konularında, istenilen başarıya ulaşmakta da problem yaşamaktadırlar (Dikici & İşleyen, 2004). Önkoşul durumundaki herhangi bir matematik konusunda istenilen başarı seviyesine ulaşılmaması, bu konuyla bağlantılı diğer matematik konularında da zorluklara sebep olacaktır çünkü matematik konuları güçlü sıralı ve bağlantılı bir yapıya sahiptir (Altun, 2008). Öğrencilerin zorluk yaşadığı, hata yaptığı konuların belirlenmesi ve bu hataların tanımlanması gereklidir (Yudariah ve Roselainy, 2001). Bunlara ek olarak, etkili bir öğretim yapabilmek için öğretmenlerin, öğrencilerin hangi konularda hangi zorlukları yaşadıklarını bilmeleri bir gerekliliktir. Öğretmenler sıklıkla yapılan hataları öğrencileri ile tartışmalı, ders planlarını bu hataları göz önünde bulundurarak hazırlamalıdır (Fischbein, 1987; Janvier, 1985; Julie 2013; Ponce, 2007; Spang, 2009).

Tam sayılar, 6.sınıf matematik müfredatındaki konulardan biridir. Tam sayılar konusu önemli bir konudur çünkü kendisinden sonraki konularla ilişkisi güçlüdür. Cebir gibi ileri seviye matematik kavramlarının gelişiminde tam sayıların ciddi bir rolü vardır ve bu durum tam sayılar konusunu ortaokul matematik müfredatı içerisinde daha önemli ve kritik bir hale getirmektedir (Christou & Vosniadou, 2012, Vlassis, 2004). Öğrenciler 6.sınıfın ikinci döneminde tam sayılarla ilişkili olan cebir konularını öğrenmeye başlamaktadırlar. Ayrıca, tam sayılar konusu ile ilişkili olan rasyonel sayıların ve üslü sayıların öğretimi ve öğrenimi de 7.sınıf seviyesinde başlamakta ve 8. Sınıf seviyesinde de devam etmektedir. Bu durum, tam sayılar konusunun farklı sınıf seviyelerinde ele alınmasının ve çalışılmasının gerekliliğini işaret etmektedir. Sonuç olarak, bu alanlarda yüksek seviyede hazırbulunuşluk

sağlamak için, altıncı sınıf öğrencilerinin tam sayılar konusundaki hatalarının tanımlanması öncelikli hale gelmektedir.

Öğrencilerin yaptıkları hatalardan ve karşılaştıkları zorluklardan haberdar olmak öğrenme odaklı çalışmalar için çok önemlidir (Rasmussen, 1998). Bununla birlikte, bazı araştırmalar, öğrencilerin tam sayılar konusunda her sınıf seviyesinde zorluklar yaşadıklarını göstermiştir (İşgüden, 2008; Körükçü, 2008). Sonuç olarak, öğrencilerin tüm matematik eğitim hayatı, bu bahsedilen zorluklardan ve bunların çözülmemesinden etkilenmektedir. Ayrıca, tam sayıların öğrenimi ve öğretimi üzerine yapılan alan yazın taraması, Türkiye' deki ortaokul öğrencilerinin hataları ve yaşadıkları zorluklar üzerine birçok cevapsız soru olduğunu göstermektedir. Öğrencilere tam sayılar konusundaki hatalarını, doğru bilgiyle değiştirmelerinde yardımcı olabilmek için, öğretmenlerin bu hataların arkasındaki sebeplere hakkında da bilgilerinin olması gerekmektedir.

Diğer taraftan, öğrencilerin tam sayılar konusundaki başarı seviyelerini arttırmak amaçlı etkili bir öğretim ortamı oluşturabilmeleri için, öğretmenlerin hataların arkasındaki sebepler bilgisine ihtiyacı bulunmaktadır. Hataların arkasındaki sebeplerin bazıları ilişkili konuyla alakalı kavram yanılgıları olabilmektedir. Eğer öğretmenler öğrencilerin hatalarının nedeni olan kavram yanılgılarını bilirlerse, öğrencilerine kavram yanılgılarını, bilimsel bilgi ile değiştirmelerinde yardımcı olabilirler. Sonuç olarak, 6. sınıf öğrencilerinin tam sayılar konusundaki yaptıkları hataların sebeplerinin analiz edilmesi önemlidir. Bu çalışmanın sonuçları tam sayılar konusundaki yapılan hataları bulma ve düzeltmede öğretmenlere destek olacaktır.

Diğer yandan, araştırmacılar arasında tam sayılar ve tam sayılarla işlemler konularının öğretim zamanı hakkında ortak bir fikir yoktur (Işıksal, 2009). Tam sayıların bu sorunlu yönünün yanında, alan yazın taraması doğal sayılar ve kesirler konuları ile kıyaslandığında tam sayılar konusu ile ilgili az sayıda çalışma bulunduğunu açığa çıkarmıştır (Işıksal, 2009). Ek olarak, Türkiye' de sınırlı sayıda tam sayı konusu üzerine araştırma bulunmaktadır (Dereli, 2008; Ercan, 2010; Işıksal, 2009; İşgüden, 2008; Keşan & Kaya 2007; Köroğlu&Yeşildere, 2004; Körükçü, 2008). Sonuç olarak öğrencilerin tam sayılar kavrama ve sıralama konularındaki başarı seviyeleri, yaptıkları hataları ve bu hataların arkasındaki sebepler araştırma yapılmasına ihtiyaç bulunan konulardır.

Önemli Terimlerin Tanımları

Tam sayı: Doğal sayılar 0, 1, 2, 3, 4… , ile birlikte bu doğal sayıların negatifleri -1, -2, -3, -4… tam sayılar olarak adlandırılır (Bennett & Nelson, 2001).

Tam sayıları kavrama: Tam sayılar nelerdir, nasıl temsil edilir, doğal sayılarla nasıl bağlantılıdır ifadeleri tam sayıları kavrama tanımını açıklar (MoNE, 2009).

Tam sayıları sıralama: Bu tanım tam sayıların sıralı olduğunu (örneğin, -7, -8' den büyüktür ve -6' dan küçüktür), bu sıralamanın miktar ifadesi ile bir ilişkisinin olmadığını ifade eder (Clements & Sarama, 2007).

Hata: Bir hata, kayma, yanlışlık ya da kesinlikten sapmadır (Luneta & Makonye, 2010).

Yöntem

Evren ve Örneklem

Bu çalışmanın örneklemini Ankara'nın Etimesgut ilçesindeki devlet okullarından birine devam eden 262 altıncı sınıf öğrencisi oluşturmaktadır. Bu öğrenciler Ankara'nın Etimesgut ilçesindeki 1 devlet okulundan uygun örnekleme yöntemiyle seçilmiştir. Bu öğrencilerin temel özellikleri aşağıda Tablo 1'de verilmiştir.

Tablo 1 Çalışmanın Katılımcıları ve Temel Karakteristikleri

Sınıflar	Sayı	Yaş	Cinsiyet	
		Ortalama	Erkek	Kız
6-A	25	11	13(52%)	12(48%)
6-B	26	11.12	13(50%)	13(50%)
6-C	27	11.11	13(48.15%)	14(51.85%)
6-D	24	10.85	12(50%)	12(50%)
6-E	27	11	14(51.85%)	13(48.15%)
6-F	26	10.79	13(50%)	13(50%)
6-G	27	11	13(48.15%)	14(51.85%)
6-H	26	10.85	14(53.85%)	12(46.14%)
6-I	29	11.14	14(48.28%)	15(51.72%)
6-J	25	10.96	13(52%)	12(48%)
Toplam	**262**	**10.98**	**132(50.37%)**	**130(49.63%)**

Araştırma Soruları

Bu çalışmanın üç tane araştırma sorusu vardır.

1. Altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki başarı seviyeleri nelerdir?

2. Altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hatalar nelerdir?

3. Altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataların sebepleri nelerdir?

Araştırma Yöntemi

Araştırmada nitel ve nicel araştırma yöntemlerini birleştiren karma bir araştırma yöntemi kullanılmıştır.

Veri Toplama Araçları

Çalışmanın verileri katılımcıların başarı testine verdikleri cevaplar ve bireysel görüşmeler aracılığıyla toplanmıştır.

Tam Sayı Başarı Testi

Katılımcıların tam sayıları kavrama ve sıralama konularındaki başarı seviyelerini, yaptıkları hataları incelemek için bir başarı testi hazırlanmıştır. Bu başarı testi 8 adet açık uçlu sorudan oluşmaktadır. Bu sorulardan dördü alan yazınından adapte edilmiştir. Geriye kalan sorular ise araştırmacı tarafından geliştirilmiştir. Bu tam sayı başarı testinin hazırlanma sürecinde altıncı sınıf ortaokul matematik programında yer alan kazanımlar göz önüne alınmış ve belirtke tablosu hazırlanmıştır. Hazırlanan belirtke tablosu aşağıda Tablo 2'de verilmiştir.

Tablo 2: Tam Sayı Başarı Testindeki Sorular ile İlgili İçerik Tablosu

	Kazanımlar	
Tam Sayılar	Öğrenciler tam sayıları açıklar.	1a, 1b, 1c, 1d, 1e, 1f, 3, 5, 6b, 6c, 6e, 7
Tam Sayılar	Öğrenciler tam sayıları sıralar.	2, 4, 8, 6a, 6d, 6f

Başarı testi belirtke tablosuna göre hazırlandıktan sonra üç uzman görüşü alınmıştır ve 40 altıncı sınıf öğrencisi ile pilot çalışma yapılmıştır. Bu pilot çalışma sonucuna göre, verilen 30 dakika sürenin yetersiz olduğu sonucuna varılmış ve testin tamamlanması için uygun olan sürenin 40 dakika olmasına karar verilmiştir. Bunlara ek olarak, 40 yedinci sınıf öğrencisinin cevapları matematik eğitimi yüksek lisans öğrencisi olan ikinci kişi tarafından değerlendirilmiştir. Araştırmacı ve bu kişinin verdiği skorlar arasındaki korelasyon %98'dir.

Bireysel Görüşmeler

Başarı testi uygulandıktan sonra katılımcıların teste verdikleri cevaplar derinlemesine incelenmiş, başarı seviyelerini sınıflandırmak için kodlar oluşturulmuştur. Bireysel görüşmeler için katılımcılar bu kodlara göre seçilmişlerdir. Görüşme sorularının araştırma sorularına yönelik olup olmadığını belirlemek için bir uzmandan görüş istenmiş ve bir öğrenci ile pilot görüşme yapılmıştır.

Görüşmeler yaklaşık olarak 30 dakika sürmüştür ve bu süre içerisinde katılımcılardan başarı testine verdikleri cevapların açıklanması istenmiştir.

Öğrencilerin açıklamaları “Bu sonuca nasıl ulaştın?”, “ Neden böyle düşündün?”, “Nasıl bir strateji kullandın?” gibi açık uçlu sorularla detaylandırılmıştır.

Görüşmeler yapıldıktan sonra bu görüşmelerin deşifreleri yapılmıştır. İlköğretim bölümündeki bir yüksek lisans öğrencisi ile birlikte ortaya çıkan temalarla ilgili çalışmalar yapılmıştır. Ayrıca, öğrencilerin başarı testindeki cevaplarının yaklaşık %20’si ortak kodlayıcı ile birlikte incelenmiştir.

Veri Toplama Süreci

Çalışmanın verileri 2013-2014 eğitim öğretim yılının bahar döneminde toplanmışlar. Veriler toplanmadan önce gerekli etik izinler alınmıştır. Mart ayında pilot çalışma yapılmış ve yine Mart ayında çalışmanın asıl verileri toplanmıştır. Asıl verilerin toplanma aşamasında, Tam Sayı Başarı Testi tek okulda aynı zamanlarda yapılmıştır. Sınıflardan birinde araştırmacının kendisi uygulama yapmıştır. Diğer sınıflara da farklı öğretmenler uygulama yapmıştır. Diğer sınıflarda uygulama yapacak öğretmenler, uygulama öncesi araştırma ile ilgili bilgilendirilmişlerdir. Başarı testinin uygulamasından birkaç hafta sonra seçilen öğrencilerle bireysel görüşmeler yapılmıştır.

Veri Analizi

Çalışmanın amaçlarına ulaşması için iki farklı veri çeşidi analiz edilmiştir. Bunlar öğrencilerin başarı testindeki cevapları ve bireysel görüşmelerin yazılı kopyalarıdır. Öncelikle öğrencilerin başarı düzeylerinin, hatalarının belirlenmesi için başarı testindeki cevaplar incelenmiştir. Daha sonra öğrencilerin yaptıkları hataların nedenlerinin belirlenmesi için görüşmelerin transkriptleri analiz edilmiştir.

Öncelikli olarak öğrencilerin başarı testindeki cevapları belirlenen dereceli puanlama anahtarında, cevaplar 0 ile 5 arası kodlanmıştır. Genel olarak, kabul edilebilir açıklama ile verilen doğru cevaplar 5; eksik açıklamalarla verilen doğru cevaplar 4; yanlış ya da açıklamasız verilen doğru cevaplar 3; kısmi doğru verilen cevaplar 2; yanlış cevaplar 1; alakasız cevaplar veya boş bırakılan sorular ise 0 olarak kodlanmıştır. Daha sonra kavrama ve sıralama sorularındaki bu cevapların sıklıkları ve yüzdeleri ayrı ayrı hesaplanarak başarı düzeyleri belirlenmiştir. Tam sayıları kavrama ve sıralama konularıyla ilgili tüm sorularda yapılan hataları belirlemek için her soruda 1 ve 2 kodu verilen katılımcı sayısı belirlenmiş ve bu katılımcıların

cevapları derinlemesine incelenerek, tam sayıları kavrama ve sıralama konularındaki yapılan hatalar belirlenmiştir.

Katılımcıların hatalarının arkasında yatan nedenlerinin belirlenmesi için öğrencilerin testteki cevapları ve görüşme transkriptleri derinlemesine incelenmiştir. Bunun yanı sıra alan yazındaki ilgili çalışmalardan kategoriler taranmıştır. Tüm bu süreçte diğer bir ilköğretim bölümü yüksek lisans öğrencisi ile çalışılmış ve uzlaşma şartı aranmıştır.

Bulgular ve Tartışma

Bu çalışmanın üç amacı bulunmaktadır. Çalışmanın birinci amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki başarı seviyelerini tespit etmektir. Çalışmanın ikinci amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataları incelemektir. Çalışmanın üçüncü amacı, altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataların arkasındaki sebepleri incelemektir.

Çalışmanın bulguları öğrencilerin tam sayıları kavrama sorularında yüksek başarı seviyesine sahip olduğunu göstermiştir. Bu yüksek başarının sebeplerinden biri öğrencilerin negatif sayılarla ilgili formal bir eğitim almadan önce, tam sayılara dair zihinsel temsil biçimleri geliştirmeleri olabilir (Peled, Mukhopadhyay & Resnick, 1988). Diğer bir ifadeyle, okula başlamadan önce öğrencilerin negatif sayılarla ilgili bazı deneyimlerinin olması ve bu deneyimlere bağlı olarak bilgilerinin olması, kavrama sorularındaki yüksek başarının sebeplerinden biri olabilir. Diğer yandan, bazı kavrama sorularının benzerlerinin, öğrencilerin ders kitaplarında bulunması da sebeplerden biri olabilir. Bulgular, öğrencilerin tam sayıları sıralama sorularında orta seviyede başarılı olduklarını göstermiştir. Bu başarı seviyesinin sebebi sıralama sorularından bazılarının sayı doğrusu içermesi olabilir. Öğrenciler, sıralama yaparken sayı doğrusu kullanmaya alışkındır. Birçok çocuk zihinlerinde okula başlamadan önce pozitif sayılar için sayı doğrusunu oluşturmaktadır (Resnick, 1983). Sonuç olarak, sayı doğrusu öğrencilerin sıralama sorularını anlamalarını kolaylaştırmış olabilir. Diğer yandan, sıralama sorularının bir kısmının öğrencilerin alışkın olduğu soru tiplerinden olmaması bu orta seviyedeki başarının sebeplerinden biri olabilir.

Bu çalışmada altıncı sınıf öğrencilerin tam sayıları kavrama ve sıralama konularında yaptıkları hatalar da incelenmiştir. Bu hatalar kavrama soruları için ve sıralama soruları için ayrı ayrı incelenmiştir. Katılımcıların yaptıkları hatalar iki ana başlık altında toplanmıştır. Bu ana başlıklar: kavramsal bilgiye dayanan hatalar ve diğer hatalardır. Bulgular göstermiştir ki, kavrama soruları için yapılan hatalar; eksik çözüm stratejisi uygulama, yanlış sembol manipülasyonu, pozitif ve negatif işaretlerin yanlış kullanımı, verilen bilgi ihmali ve yanlış hizalamadır. Katılımcılar tarafından en çok yapılan hata pozitif ve negatif işaretlerin yanlış kullanımıdır. Bu bulgu literatürdeki diğer çalışmalarla da tutarlıdır (Dereli, 2008; Hayes and Stacey, 1990; Janvier, 1983; Kilhamn, 2008; Körükcü, 2008). Bu hatanın sebebi öğrencilerin tam sayıları kavrama ve sıralama ile ilgili kavramsal bilgilerindeki eksiklikler olabilir. Ayrıca Tam Sayı Başarı Testi'ndeki bazı soruların katılımcıların alışık olmadığı soru tipinde olduğu için bu hata yapılmış olabilir.

Katılımcılar tarafından kavrama sorularında ikinci sıklıkla yapılan hata verilen bilgi ihmalidir. Bu hatayı yapan öğrenciler, soruda verilen bir ya da birkaç veriyi, soruyu çözmeye çalışırken kullanmamaktadır. Bu hatanın temel sebebi, öğrencilerin soruları okurken dikkatsiz bir şekilde okuyup, sorunun çözümünde kullanılması gereken bazı verileri kaçırmaları olabilir.

Diğer yandan, öğrencilerin sıralama sorularında da hata yaptığı gözlemlenmiştir. Sıralama soruları için belirlenen hatalar; ters sıralama, rastgele sıralama, yanlış referans noktası alma, yanlış sembol manipülasyonu, pozitif ve negatif işaretlerin yanlış kullanımı, verilen bilgi ihmali ve yanlış hizalamadır. Yani öğrenciler, nerede negatif nerede pozitif sayıları kullanmaları gerektiğine doğru karar vermekte zorluk çekmektedirler. Bu hatalar içince en çok gözlemlenen hata ters sıralamadır. Bu hatanın sebebi, öğrencilerin negatif sayıları sıralama konusundaki kavramsal bilgi eksikliği olabilir. Daha detaylı söylemek gerekirse, öğrenciler iki veya daha fazla negatif sayıyı küçükten büyüğe ya da büyükten küçüğe doğru sıralarken, negatif sayıları pozitif sayılar gibi düşünüp sıralama yapmaktadır (Julie et al., 2013). Bu hatanın temel sebebi, öğrencilerin doğal sayıların özelliklerini tam sayılara genellemesi olabilir.

Katılımcılar tarafından sıralama sorularındaki ikinci sıklıkla yapılan hata rastgele sıralamadır. Bu hatanın sebebi de öğrencilerin negatif sayıları sıralama konusundaki

kavramsal bilgi eksikliği olabilir. Bishop ve arkadaşlarının (2014) çalışmasına göre negatif sayıları sıralama konusu öğrenciler için bilişsel yani kavramsal bir zorluk içermektedir. Bu çalışma ile paralel olarak, öğrenciler negatif sayıları sıralamaya çalışırken zorluk yaşadıkları için hata yapmaktadır. Bu hatalardan biri de yanlış referans noktası almadır. Yanlış referans noktası alma hatası, öğrencilerin soruda verilen referans noktasını anlamaması ya da ihmal ederek rastgele bir referans noktası belirleyip soruyu çözmeye çalışması sonucu oluşmuştur. Bu hatanın üstesinden gelebilme için, öğrencilerin sıfırı referans noktası olarak düşünebilmesi gerekmektedir (Steiner, 2009).

Bu çalışmada altıncı sınıf öğrencilerinin tam sayıları kavrama ve sıralama konularındaki yaptıkları hataların arkasında yatan sebepler de incelenmiştir. Toplam da dört ana sebep bulunmuştur. Bu sebepler: sayı doğrusu üzerindeki sayıların büyüklüğünü yanlış anlama, soruyu dikkatsiz okuma, aynı işaretli tam sayıların farklı işaretli tam sayılara göre daha yakın olduğunu varsayma ve son olarak doğal sayıların özelliklerini tam sayılara genellemedir. Bu sebeplerden en yaygın görüleni sayı doğrusu üzerindeki sayıların büyüklüğünü yanlış anlamadır. Hatalarının arkasında bu sebep yatan öğrenciler, sayı doğrusunun solunun pozitif sayıları, sağının negatif sayıları temsil ettiğini düşünmektedir. Diğer bir deyişle, öğrenciler pozitif ve negatif sayıların yerini sayı doğrusunda karıştırmaktadır. Eğer öğretmenler tarafından derste sayı doğrusunun tüm özellikleri üzerinde yeteri vurgu yapılırsa veya öğrencilere sayı doğrusunun tüm özelliklerini inceleme, tecrübe etme imkânı sağlanırsa hatalara arkasında yatan bu sebep kısmen de olsa azaltılabilir. Dolayısıyla bu sebeple bağlantılı olan hataların görülme sayısı da azalabilir.

İkinci sıklıkla görülen hata sebebi soruyu dikkatsiz okumadır. Hatalarının arkasında bu sebep olan öğrenciler, soruyu okurken bazı bilgileri dikkatsizlik sonucu kaçırmışlardır. Eksik bilgi ile soruları çözmeye çalışmışlardır. Bu bulgular Ryan ve William (2007) ile paralel olarak, bu sebepten kaynaklı hatalar, öğrencilerin kavramsal gelişimleriyle alakalı değildir.

Öğrencilerin hatalarının arkasındaki üçüncü sebep ise aynı işaretli tam sayıların farklı işaretli tam sayılara göre daha yakın oldukları varsayımıdır. Bu sebep aynı zamanda bir kavram yanılgısıdır. Bu sebepten dolayı hata yapan öğrenciler, herhangi bir negatif sayının başka bir negatif sayıya, bir pozitif sayıya göre her zaman daha

yakın olduğunu düşünmektedir ya da herhangi bir pozitif sayının başka bir pozitif sayıya, bir negatif sayıya göre daha yakın olduğunu düşünmektedir. Örnek olarak, öğrencilerin " -7, -1' e +1' den daha yakındır. Çünkü -7 ve -1 aynı işaretli sayılardır" gibi açıklaması verilebilir

Öğrencilerin hatalarının arkasında yatan sebeplerden sonuncusu ise doğal sayıların özelliklerini tam sayılara genellemedir.

Doğurgular

Bu çalışmanın sonuçları matematik öğretmenleri, öğretmen eğiticileri, program geliştiriciler ve ders kitabı yazarları için önemli bilgiler sunmaktadır.

Bu çalışmanın bulguları göstermiştir ki; ortaokul altıncı sınıf öğrencileri tam sayıları kavrama ve sıralama konularında sırasıyla yüksek ve ortalama başarı seviyelerine sahiptir. Yüksek ve orta seviyeli başarı göstermelerine rağmen, bu öğrenciler tam sayıları kavrama ve sıralama sorularını çözerken hatalar yapmışlardır. Ayrıca bu hataların başlıca nedenleri vardır. Öğretmenler, öğretmen eğiticileri ve ders kitabı yazarlarına bu öğrencilerin yaptıkları hataları ve bu hataların nedenlerini dikkate alarak daha etkili öğrenme ortamı hazırlamaları önerilmektedir.

Öğrencilerin bu kavramlarla ilgili yaptıkları hatalar ve bu hataların nedenleri ile ilgili bilgilendirme yapmak için okullardaki öğretmenlere ve öğretmen adaylarına yönelik seminerler organize edilebilir. Böylece öğretmenler öğrencilerin hatalarını ve bu hataların nedenlerini fark ederek uygun ders planı hazırlayabilir ve hataların sebepleri bilgisinden faydalanarak, öğrencilerin hatalarını ortadan kaldırabilirler. Aslında, öğrencilerin yaptıkları hataların en büyük sebebi kavramsal bilgilerindeki eksikliktir. Öğrencilere belli kalıplarda sorular sormak da öğrencilerin kavramsal gelişiminin eksik kalmasına sebep olabilir. Bu nedenle, öğretmenlere öğrencilerin bu kavramlarla ilgili çok yönlü düşünmelerini sağlayacak sorular sorması önerilmektedir.

Araştırmanın Varsayımları ve Sınırlılıkları

Araştırmanın ilk varsayımı öğrencilerin tam sayıları kavrama ve sıralama konularındaki başarılarının geliştirilen test aracılığıyla ölçülebileceğidir. Ayrıca, araştırmanın ilk varsayımı öğrencilerin Tam Sayı Başarı Testi'ni cevaplarken

dikkatli, içten ve açık yürekli olduğudur. Ek olarak, öğrencilerin yaşlarının, zekâ düzeylerinin ve sosyoekonomik geçmişinin benzer olduğu varsayılmıştır.

Çalışmanın katılımcılarının bulabildiğini örnekleme yoluyla seçilmesi, sonuçların daha geniş bir popülasyona genellenmesini sınırlandırmaktadır. Ayrıca görüşmeler için seçilen öğrencilerin amaca yönelik seçilmesinden dolayı görüşmelerden elde edilen veriler bu katılımcılarla sınırlı olabilir.

Ayrıca, elde edilen sonuçlar Tam Sayı Başarı Testi'ndeki sorularla sınırlıdır çünkü farklı sorular için farklı bulgular elde edilebilir. Bunlara ek olarak, bulguların elde edilmesinde öğrencilerin soruların çözümleriyle ilgili yaptığı açıklamalar önemli bir yere sahip olduğu için, bulgular öğrencilerin kendini ifade edebilme yeteneğiyle sınırlıdır.

Öneriler

Aynı çalışma olasılıklı örnekleme yöntemlerinden biriyle seçilerek tekrarlanabilir. Aynı konular üzerindeki hataların sebebi olarak çıkan kavram yanılgılarının nasıl üstesinden gelineceğini araştırmak için deneysel bir çalışma yapılabilir. Bu çalışmada uygulanan iki farklı öğretim metodundan hangisinin kavram yanılgılarını giderebileceği araştırılabilir.

Çalışmanın yönteminde değişiklik yapılarak da farklı çalışmalar yapılabilir. İlk olarak, boylamsal bir çalışma yürütülebilir. İkinci olarak, tam sayıları kavrama ve sırlama ilgili testler geliştirilerek, bu çalışmayla aynı amacı taşıyan başka bir çalışma yürütülebilir.

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APPENDIX G

TEZ FOTOKOPİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☒

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : SEVİM ATAYEV
Adı : Gizem
Bölümü : İlköğretim Fen ve Matematik Eğitimi

TEZİN ADI (İngilizce) : Sixth Grade Students' Achievement Levels, Errors and Underlying Reasons of the Errors Regarding Comprehension and Ordering of Integers

TEZİN TÜRÜ : Yüksek Lisans ☒ Doktora ☐

1. Tezimin tamamı dünya çapında erişime açılsın ve kaynak gösterilmek şartıyla tezimin bir kısmı veya tamamının fotokopisi alınsın. ☐

2. Tezimin tamamı yalnızca Orta Doğu Teknik Üniversitesi kullanıcılarının erişimine açılsın. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☐

3. Tezim bir (1) yıl süreyle erişime kapalı olsun. (Bu seçenekle tezinizin fotokopisi ya da elektronik kopyası Kütüphane aracılığı ile ODTÜ dışına dağıtılmayacaktır.) ☒