ANALYSIS OF STRESS IN THE ELASTIC STATE IN A SOLID CYLINDER SUBJECTED TO PERIODIC BOUNDARY CONDITION

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ABSTRACT

ANALYSIS OF STRESS IN THE ELASTIC STATE IN A SOLID CYLINDER SUBJECTED TO PERIODIC BOUNDARY CONDITION

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An analytical model is developed to investigate the thermoelastic response of a solid cylinder subjected to periodic boundary condition. Time dependent periodic boundary condition for the solid cylinder is treated by the help of Duhamel's theorem. The corresponding thermoelastic equation is obtained in terms of radial displacement by using basic equations of elasticity under generalized plane strain presupposition. Analytical solution of the thermoelastic equation is obtained to determine the distributions of stress, displacement and elastic strains. Different time dependent and periodic functions are used to describe the surface temperature. The corresponding thermoelastic behavior of the solid cylinder is examined and put into view in the form of figures and tables.

Keywords: Solid cylinder, Periodic boundary condition, Transient heat conduction, Duhamel's theorem, Thermoelasticity

ÖZ

PERİYODİK SINIR KOŞULU UYGULANAN İÇİ DOLU SİLİNDİRİN ELASTİK GERİLME DURUMUNDA GERİLİM ANALİZİ

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Yüzey sıcaklığı zamanla ve periyodik olarak değişen içi dolu bir silindirin termoelastik davranışı, bu çalışmada geliştirilen analitik bir model yardımıyla incelenmiştir. Silindirin zamana bağlı periyodik sınır koşulu Duhamel teoremi kullanılarak çözümlenmiştir. Bu koşullar altında silindirin termoelastik denklemi, elastisitenin temel denklemleri ve genelleştirilmiş düzlem şekil değiştirme kabulu altında radyal yer değiştirme cinsinden elde edilmiştir. Termoelastik denklemin analitik çözümü elde edilerek silindirin içerisindeki gerilim, yer değiştirme ve elastik şekil değiştirmeler belirlenmiştir. Silindirin periyodik olarak değişebilen yüzey sıcaklığını ifade edebilmek için farklı, zamana bağlı periyodik fonksiyonlar kullanılmıştır. Silindirin bu değişik fonksiyonlara karşılık gelen termoelastik davranışı araştırılmış ve değişik grafik ve tablolarla okuyucunun incelemesine sunulmuştur.

Anahtar Kelimeler: İçi dolu silindir, Periyodik sınır koşulu, Geçici ısı iletimi, Duhamel teoremi, Termoelastisite

To my family

Yağız Yedekçi, Hatice Çiftci, Fuat Çiftci, Yuşa Çiftci

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CHAPTER 1

INTRODUCTION

The shape of the structural material is changed by applying external loads which are classified as mechanical and thermal loads. This change in shape is called deformation. If the deformation is elastic, the material returns to its initial shape and size when the applied external loads are removed. However, when the load is sufficient to permanently deform the material, it is called plastic deformation. Change in temperature causes thermal stresses which are built up in the structure.

Concepts of thermoelasticity and thermal stresses rose after the birth of elasticity. Shortly after the basic formulation of elasticity, Duhamel studied the formulation of elasticity problems including the effect of temperature variation in 1835, and hereby the first paper on thermoelasticity was published in the Journal de l'Ecole Polytechnique in 1837 [7]. In order to improve the theory, F. Neumann [27], E. Almansi [2], O. Tedone [32] and W. Voigt [35] studied on the formulation of thermoelasticity equations in 1885, 1897, 1906 and 1910, respectively. Thermoelasticity problems were reduced to elasticity problems by creators of the theory of elasticity, B. de Saint-Venant, G. Lame, and P.S. Laplace. With this method, many basic thermoelasticity problems were studied within classic theory of elasticity. In 1873, the work on a solution in integral representation for a sphere subjected to an arbitrarily distributed temperature was studied by C.W. Borchardt [6]. Another important papers that made a significant contribution to thermoelasticity were published by J. Hopkinson [22] of 1874 on thermal stresses in a sphere, by A. Leon of 1905 on a hollow cylinder, and by S. Timoshenko [33] of 1925 on bi-metallic strips.

M.A. Biot [4] investigated the properties of two-dimensional distributions of thermal stresses in 1935 and J.N. Goodier [20] introduced the concept of the thermoelastic potential and considered the effect of non-continuous temperature fields in 1937. Due to the demand of new technologies, research on thermal stresses was accelerated during and after the Second World War. The distribution of temperature in specific situations, finding thermal stresses in parts of complex mechanical systems, assessment of allowed stresses in various materials and in various loading conditions, matters of stability, problems of viscoelasticity, of fatigue, and thermal shock become the main issues of theoretical and experimental investigations [21].

Further progress was made by the publication of books. The first book on thermal stresses was prepared by E. Melan and H. Parkus [26] in 1953 and on thermoelasticity by W. Nowacki [29], in Polish, in 1960. Again in 1960, a very enlightening monograph was written by B.A. Boley and J.H. Weiner [5]. Two years later, W. Nowacki[30] composed a monograph on thermoelasticity in English. Many years later, a book on thermal stresses was published which was specifically written as a textbook by N. Noda, R.B. Hetnarski, and Y. Tanigawa [28]. A detailed information about history of thermal stress analysis can be obtained from the book of Richard B. Hetnarski and M. Reza Eslami, written in 2009 [21] by the interested readers.

Considering the importance mentioned above, the effect of temperature on materials still attracts a great deal of attention of the researchers. Especially, thermoelastic response of fundamental structures like spherical shells, disks, tubes and cylinders is an important issue in engineering design, due to the prediction of failure and improving reliability. Even if there are many studies about analysis of stresses and deformation and determination of temperature distribution of these basic structures in literature, studies in recent years have been mentioned below chronologically. Orcan [31] presented the exact solution of the distribution of stress and deformation in an elastic-perfectly plastic cylindrical rod with uniform internal heat generation under generalized plane strain condition. Orcan and Eraslan [16] studied the transient solution of the coupled thermoelastic-plastic deformation of a heat-generating tube with temperature dependent material properties. Eraslan and Akış [11] analyzed elastic-plastic stress distribution in a nonisothermal rotating annular disk by the use of Tresca and von Mises criteria. Eraslan [9] examined exact solutions for nonisothermal variable thickness rotating disks represented by different thickness profiles. Eraslan and

Argeşo [12] developed a computational to predict the thermal stresses in generalized plane strain axisymmetric structures and afterwards extended this study to add the cylinders and tubes with temperature dependent physical properties [13]. Eraslan and Kartal [15] developed a computational model to predict the thermoelastic response of a cooling fin of variable thickness with or without rotation. Temperature dependency of modulus of elasticity, uniaxial yield limit, coefficient of thermal expansion and thermal conductivity of the fin were examined. Eraslan, Arslan and Mack [14] investigated the distributions of stress, strain and displacement in a linearly strain hardening elastic-plastic hollow shaft subject to a positive temperature gradient and monotonously increasing angular speed. In this study, a remarkable result is that the occurring temperature gradient generally reduces the elastic limit angular speed whereas for moderate values of the temperature difference a slight increase in the fully plastic angular speed can be observed. Eraslan [10] assessed the effects of material nonhomogeneity and nonisothermal conditions on the stress response of pressurized tubes by virtue of a computational model. Arslan, Mack and Eraslan [3] studied the stress distribution in a rotating solid shaft with temperature dependent yield stress subject to a temperature cycle and it was shown that the stress distribution is altered significantly by the temperature cycle. Eraslan and Tokdemir [18] investigated the thermoelastic analytical solution of a variable thickness cooling fin problem. They obtained the temperature distribution in the fin for the given heat and centrifugal loads and the corresponding stress state by means of the analytical solutions of energy and thermoelastic equations.

Although periodic heat loads has a great importance in engineering, very little work has been done on thermoelastic response of such structures. Jahanian [24] investigated the stresses in a long hollow circular cylinder subjected to rapid cooling of the exterior surface, and in his another study [23], the total accumulation of plastic strains of a thick walled tube subjected to rapid heating and cooling of the inside of the cylinder over four cycles was presented. Eraslan [8] obtained analytical solutions for thermally induced axisymmetric elastic and elastic-plastic deformations in nonuniform heat-generating composite tubes with fixed ends. By using four different boundary conditions, thermoelastic solutions were attained. Orcan and Eraslan [17] examined the thermoelastoplastic response of a linearly hardening cylinder subjected to a nonuniform heat source and convective heat transfer condition at the external boundary. Ahmadikia and Rismanian [1] determined heat transfer of a fin that is subjected to a periodic boundary condition analytically. This study showed that a unsteady boundary condition in the base fin caused shocks and the hyperbolic heat conduction equation can violate the second thermodynamic law under some unsteady boundary conditions. Kaya and Eraslan [25] examined the thermoelastic behavior of a long tube subjected to periodic heating analytically. In this study, the temperature distribution is obtained by the solution of heat conduction equation and the boundary condition is treated by Duhamel's theorem.

In the present study, a solid cylinder is subjected to a time dependent periodic boundary condition. The effects of this periodic boundary condition on temperature, stress, strain and displacement distributions in the solid cylinder are investigated. Two different materials and periodic functions are taken into consideration. For this purpose, an analytical model is developed and the results are verified by using a finite element model. Also, results are displayed in the form of figures and tables.

CHAPTER 2

PROBLEM DEFINITION AND SOLUTION

2.1 Temperature Distribution

The temperature distribution in the solid cylinder is governed by one dimensional unsteady heat conduction equation in cylindrical coordinates given by

$$\frac{1}{\alpha_T}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \quad ; \quad 0 \leq r < b \quad , \quad t > 0, \tag{2.1}$$

where α_T represents the thermal diffusivity, $T(r,t)$ the temperature at radial position r at time t and b the outer radius. Initially the solid cylinder is at zero temperature

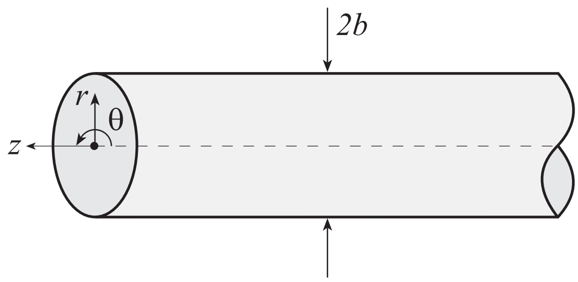

Figure 2.1: Solid cylinder profile

and for times $t > 0$ the surface temperature of the solid cylinder is subjected to periodic boundary condition described by the function $F(t)$. Under these circumstances the boundary and initial conditions can be expressed as

$$\begin{aligned} T(0,t) &= \text{finite}, \ t > 0, \\ T(b,t) &= F(t), \ t > 0, \\ T(r,0) &= 0, 0 \leq r \leq b. \end{aligned} \tag{2.2}$$

To handle time dependent periodic boundary condition Duhamel's Theorem

$$T(r,t) = \int_0^t F(\beta) \frac{\partial}{\partial t} \Phi(r, t-\beta)\, d\beta, \tag{2.3}$$

is used, in which $\Phi(r,t)$ is the solution of the auxiliary problem given by

$$\frac{1}{\alpha_T} \frac{\partial \Phi}{\partial t} = \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial r^2} \quad ; \quad 0 \leq r < b \quad , \quad t > 0, \tag{2.4}$$

subjected to

$$\begin{aligned} \Phi(0,t) &= \text{finite}, \; t > 0, \\ \Phi(b,t) &= C = \text{constant}, \; t > 0, \\ \Phi(r,0) &= 0, 0 \leq r \leq b \end{aligned} \tag{2.5}$$

For simplicity we take $C = 1$. Because of the nonhomogeneous boundary condition at the surface, we propose a solution for $\Phi(r,t)$ of the form

$$\Phi(r,t) = Y(r,t) + Z(r). \tag{2.6}$$

Substituting this solution into $\Phi(r,t)$ gives

$$\frac{1}{\alpha_T} \frac{\partial Y}{\partial t} = \frac{1}{r} \frac{\partial Y}{\partial r} + \frac{\partial^2 Y}{\partial r^2} + \frac{1}{r} \frac{dZ}{dr} + \frac{d^2 Z}{dr^2} \quad ; \quad 0 \leq r < b \quad , \quad t > 0. \tag{2.7}$$

Let

$$\frac{1}{r} \frac{dZ}{dr} + \frac{d^2 Z}{dr^2} = 0, \tag{2.8}$$

so then

$$\frac{1}{\alpha_T} \frac{\partial Y}{\partial t} = \frac{1}{r} \frac{\partial Y}{\partial r} + \frac{\partial^2 Y}{\partial r^2}. \tag{2.9}$$

By doing so, the last equation is divided into two equation, one of partial differential equation, the other one ordinary differential equation. Boundary and initial conditions should also be determined from the original problem. So, by applying first boundary condition we obtain

$$\Phi(0,t) = Y(0,t) + Z(0) = \text{finite}. \tag{2.10}$$

If we let

$$Z(0) = \text{finite}, \tag{2.11}$$

then

$$Y(0,t) = \text{finite}. \tag{2.12}$$

From the other boundary condition we get

$$\Phi(b,t) = Y\,(b,t) + Z(b) = 1, \tag{2.13}$$

and if we let

$$Z(b) = 1, \tag{2.14}$$

then,

$$Y\,(b,t) = 0. \tag{2.15}$$

Hence, the ordinary differential equation with the conditions can be written as

$$\frac{1}{r}\frac{dZ}{dr} + \frac{d^2Z}{dr^2} = 0, \tag{2.16}$$

$$Z\,(0) = \text{finite}, \tag{2.17}$$

$$Z(b) = 1. \tag{2.18}$$

and the partial differential equation becomes homogeneous

$$\frac{1}{\alpha_T}\frac{\partial Y}{\partial t} = \frac{1}{r}\frac{\partial Y}{\partial r} + \frac{\partial^2 Y}{\partial r^2} \tag{2.19}$$

and hence, amenable to an analytical solution with the conditions

$$\begin{aligned} Y\,(0,t) &= finite, \\ Y\,(b,t) &= 0, \\ Y\,(r,0) &= -Z\,(r)\,. \end{aligned} \tag{2.20}$$

The solution for $Z\,(r)$ is

$$Z\,(r) = C_1 \ln r + C_2 \tag{2.21}$$

If we apply the boundary conditions; we require that $C_1 = 0$ as $\ln(0) \to -\infty$ and

$$C_2 = 1, \tag{2.22}$$

is obtained. Then, the solution for $Z\,(r)$ becomes

$$Z\,(r) = 1. \tag{2.23}$$

The partial differential equation for$Y(r,t)$ with the conditions indicated above, can be solved by the application of separation of variables. A solution of the form is assumed.

$$Y\,(r,t) = \theta(t)R(r). \tag{2.24}$$

Differentiating and then substituting into Eq. (2.19) we come up with

$$\frac{1}{\alpha_T}\left(\frac{1}{\theta}\frac{d\theta}{dt}\right) = \frac{1}{R}\left(\frac{1}{r}\frac{dR}{dr} + \frac{d^2R}{dr^2}\right). \tag{2.25}$$

Since the left hand side of this equation depends only on t and the right hand side only on r, they must be equal to a constant. If we take it as $-\lambda^2$ this last equation becomes

$$\frac{1}{\alpha_T}\left(\frac{1}{\theta}\frac{d\theta}{dt}\right) = \frac{1}{R}\left(\frac{1}{r}\frac{dR}{dr} + \frac{d^2R}{dr^2}\right) = -\lambda^2. \tag{2.26}$$

Now, separation is possible:

$$\frac{d\theta}{dt} + \alpha_T\lambda^2\theta = 0, \tag{2.27}$$

$$\frac{1}{r}\frac{dR}{dr} + \frac{d^2R}{dr^2} + \lambda^2 R = 0. \tag{2.28}$$

The solution for $R(r)$ is

$$R(r) = C_3 J_0(\lambda r) + C_4 Y_0(\lambda r) \tag{2.29}$$

in which J_0 and Y_0 are the Bessel functions of the first and second kind. Since $R(r)$ should be finite at the center of the cylinder $C_4 = 0$ as $Y_0(r)$ is not finite at this location.Then, the solution for $R(r)$ takes the form

$$R(r) = C_3 J_0(\lambda r). \tag{2.30}$$

Second boundary condition for $Y(r,t)$ implies

$$R(b) = 0, \tag{2.31}$$

giving

$$J_0(\lambda b) = 0, \tag{2.32}$$

the roots of which provides the eigenvalues λ_n for $n = 1, 2, \ldots \infty$ of the eigenvalue system. The corresponding eigenfunction is

$$R(r) = J_0(\lambda_n r). \tag{2.33}$$

On the other hand, the solution for $\theta(t)$ becomes

$$\theta(t) = A_n\, e^{-\alpha_T\lambda_n^2 t}. \tag{2.34}$$

Hence, one solution for $Y\,(r,t)$ is

$$Y\,(r,t) = \theta(t)R(r) = A_n\, e^{-\alpha_T\lambda_n^2 t} J_0(\lambda_n r) \tag{2.35}$$

and by superposition

$$Y(r,t) = \sum_{n=1}^{\infty} A_n e^{-\alpha_T \lambda_n^2 t} J_0(\lambda_n r), \tag{2.36}$$

is also a solution. To find the constant A_n, we make use of the initial condition $Y(r,0) = -Z(r)$, i.e.

$$-1 = \sum_{n=1}^{\infty} A_n J_0(\lambda_n r) \tag{2.37}$$

Multiplying both sides by $rJ_0(\lambda_m r)$ and integrating from 0 to b we get

$$-\int_0^b rJ_0(\lambda_n r)dr = \sum_{n=1}^{\infty} A_n \int_0^b rJ_0(\lambda_n r)J_0(\lambda_m r)dr, \tag{2.38}$$

which reduce to

$$-\int_0^b rJ_0(\lambda_n r)dr = A_n \int_0^b r\left[J_0(\lambda_n r)\right]^2 dr, \tag{2.39}$$

since

$$\int_0^b rJ_0(\lambda_n r)J_0(\lambda_m r)dr = 0 \ \text{ if } \ m \neq n. \tag{2.40}$$

Then

$$\begin{aligned} A_n &= -\frac{\int_0^b rJ_0(\lambda_n r)dr}{\int_0^b r\left[J_0(\lambda_n r)\right]^2 dr} \\ &= -\frac{2}{\lambda_n b}\frac{1}{J_1(\lambda_n b)}. \end{aligned} \tag{2.41}$$

Substituting into $Y(r,t)$ one obtains

$$Y(r,t) = -\frac{2}{b}\sum_{n=1}^{\infty} e^{-\alpha_T \lambda_n^2 t}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}. \tag{2.42}$$

Hence, the solution for $\Phi(r,t)$ becomes

$$\Phi(r,t) = 1 - \frac{2}{b}\sum_{n=1}^{\infty} e^{-\alpha_T \lambda_n^2 t}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)} \tag{2.43}$$

Finally the temperature distribution of solid cylinder is obtained as

$$\begin{aligned} T(r,t) &= \int_0^t F(\beta)\frac{\partial}{\partial t}\Phi(r,t-\beta)\,d\beta \\ &= \int_0^t F(\beta)\frac{\partial}{\partial t}\left[1 - \frac{2}{b}\sum_{n=1}^{\infty} e^{-\alpha_T \lambda_n^2 (t-\beta)}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\right] d\beta \\ &= \frac{2\alpha_T}{b}\sum_{n=1}^{\infty} e^{-\alpha_T \lambda_n^2 t}\frac{\lambda_n J_0(\lambda_n r)}{J_1(\lambda_n b)}\int_0^t e^{\alpha_T \lambda_n^2 \beta}F(\beta)\,d\beta. \end{aligned} \tag{2.44}$$

Integrating by parts

$$\int_0^t e^{\alpha_T\lambda_n^2\beta}F(\beta)\,d\beta = \frac{1}{\alpha_T\lambda_n^2}\left[e^{\alpha_T\lambda_n^2 t}F(t) - F(0) - \int_0^t e^{\alpha_T\lambda_n^2\beta}\frac{dF(\beta)}{d\beta}d\beta\right] \quad (2.45)$$

Substituting

$$\begin{aligned} T(r,t) &= \frac{2\alpha_T}{b}\sum_{n=1}^{\infty} e^{-\alpha_T\lambda_n^2 t}\frac{\lambda_n J_0(\lambda_n r)}{J_1(\lambda_n b)}\frac{1}{\alpha_T\lambda_n^2}e^{\alpha_T\lambda_n^2 t}F(t) \quad (2.46)\\ &\quad -\frac{2\alpha_T}{b}\sum_{n=1}^{\infty} e^{-\alpha_T\lambda_n^2 t}\frac{\lambda_n J_0(\lambda_n r)}{J_1(\lambda_n b)}\frac{1}{\alpha_T\lambda_n^2}F(0)\\ &\quad -\frac{2\alpha_T}{b}\sum_{n=1}^{\infty} e^{-\alpha_T\lambda_n^2 t}\frac{\lambda_n J_0(\lambda_n r)}{J_1(\lambda_n b)}\frac{1}{\alpha_T\lambda_n^2}\int_0^t e^{\alpha_T\lambda_n^2\beta}\frac{dF(\beta)}{d\beta}d\beta\\ &= \frac{2}{b}F(t)\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\\ &\quad -\frac{2}{b}\sum_{n=1}^{\infty} e^{-\alpha_T\lambda_n^2 t}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\left[F(0) + \int_0^t e^{\alpha_T\lambda_n^2\beta}\frac{dF(\beta)}{d\beta}d\beta\right]. \end{aligned}$$

Note that the solution for $Y(r,t)$ satisfies the condition $Y(r,0) = -1$, hence

$$Y(r,0) = -1 = -\frac{2}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)} \quad (2.47)$$

which gives

$$\frac{2}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)} = 1, \quad (2.48)$$

then,

$$T(r,t) = F(t) - \frac{2}{b}\sum_{n=1}^{\infty} e^{-\alpha_T\lambda_n^2 t}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\left[F(0) + \int_0^t e^{\alpha_T\lambda_n^2\beta}\frac{dF(\beta)}{d\beta}d\beta\right] \quad (2.49)$$

or

$$\begin{aligned} T(r,t) &= F(t) - \frac{2}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}e^{-\alpha_T\lambda_n^2 t}F(0) \quad (2.50)\\ &\quad -\frac{2}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\int_0^t e^{\alpha_T\lambda_n^2(\beta-t)}\frac{dF(\beta)}{d\beta}d\beta. \end{aligned}$$

In the special case if $F(t) = A\sin(t)$, $F(0) = 0$ and $T(r,t)$ becomes

$$T(r,t) = A\sin(t) - \frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\psi_1(\alpha_T,\lambda_n,t) \quad (2.51)$$

where

$$\psi_1(\alpha_T,\lambda_n,t) = \frac{-\alpha_T\lambda_n^2 e^{-\alpha_T\lambda_n^2 t} + \alpha_T\lambda_n^2\cos t + \sin t}{1+\alpha_T^2\lambda_n^4}. \quad (2.52)$$

For this case temperature gradient is derived as

$$\frac{\partial T\left(r,t\right)}{\partial r}=\frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_1(\lambda_n r)}{J_1(\lambda_n b)}\psi_1(\alpha_T,\lambda_n,t). \tag{2.53}$$

If $F(t) = At\cos(t)$, the temperature and its gradient take respectively the forms

$$T\left(r,t\right)=At\cos(t)-\frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t), \tag{2.54}$$

$$\frac{\partial T\left(r,t\right)}{\partial r}=\frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_1(\lambda_n r)}{J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t). \tag{2.55}$$

where

$$\begin{aligned}\psi_2(\alpha_T,\lambda_n,t) \;=\; & \frac{e^{-\alpha_T\lambda_n^2 t}(\alpha_T\lambda_n^2-\alpha_T^3\lambda_n^6)}{\left(1+\alpha_T^2\lambda_n^4\right)^2} \\ & +\frac{\cos t\left(t-\alpha_T\lambda_n^2+t\alpha_T^2\lambda_n^4+\alpha_T^3\lambda_n^6\right)}{\left(1+\alpha_T^2\lambda_n^4\right)^2} \\ & +\frac{\sin t\left(-t\alpha_T\lambda_n^2+2\alpha_T^2\lambda_n^4-t\alpha_T^3\lambda_n^6\right)}{\left(1+\alpha_T^2\lambda_n^4\right)^2}.\end{aligned} \tag{2.56}$$

Two different time dependent functions are selected as the periodic boundary conditions. One of these functions is $A\sin(t)$ and the other is $At\cos(t)$. In Figure 2.2, the variations of the periodic boundary conditions in the time interval, 0-20 hours, are shown. For this figure, A is taken as 1. Dash line refers to the function $A\sin(t)$ and solid line to $At\cos(t)$. It is seen that both functions have a period of 2π, but their amplitudes are different from each other. The amplitude of $\sin(t)$ remains constant while the amplitude of $\cos(t)$ increases considerably with time, i.e.,

$$\lim_{t\to\infty} At\cos(t)=\infty. \tag{2.57}$$

Hence, the function $At\cos(t)$ is not finite as $t \longrightarrow \infty$ and can not be used for very large times. However, $A\sin(t)$ is used for all times and also stresses and displacements show the same behavior in each period of $\sin(t)$.

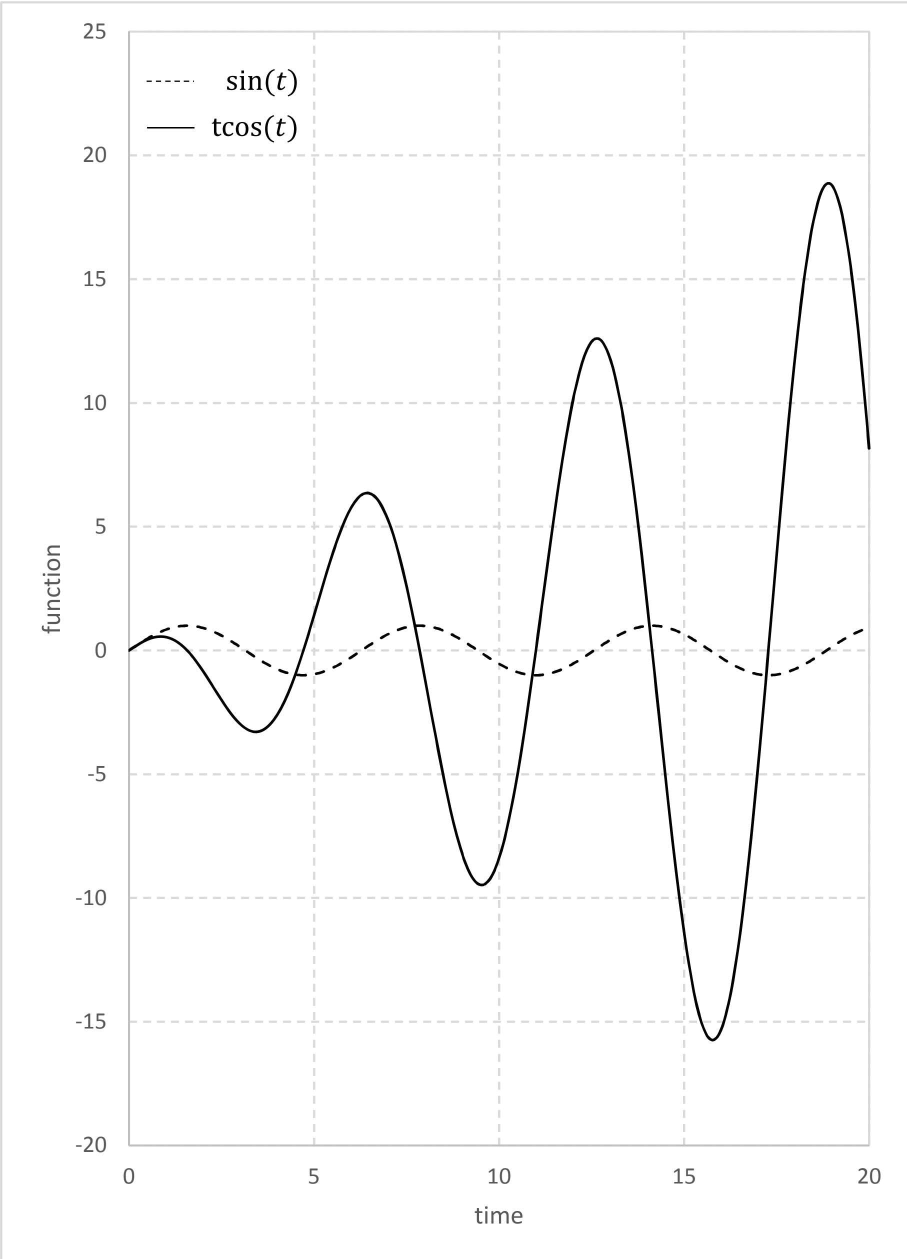

Figure 2.2: Variation of the periodic boundary conditions

Analytical solution is verified in comparison to the computational model. Eraslan and Varlı [19] developed this computational model with Finite Element Method, and it is used by adapting it to this study. Due to the computational model is established in the nondimensional form, some parameters are needed to adjust. For verification, amplitude of the functions, A, is taken as 2, while the thermal diffusivity α_T is set to $1\ m^2/h$ and structural steel is used as material. In figures solid lines refer to the analytical solutions, dots to computational solutions. Figures 2.3 and 2.4 show the verification of temperature distribution and temperature gradient in the solid cylinder subjected to function $F(t) = Asin(t)$ at different time steps, while in Figures 2.5 and 2.6 verification of temperature distribution and temperature gradient in the solid cylinder are presented at different time steps for the case F(t)=Atcos(t). As one can see, analytical results are identical to those from computational model.

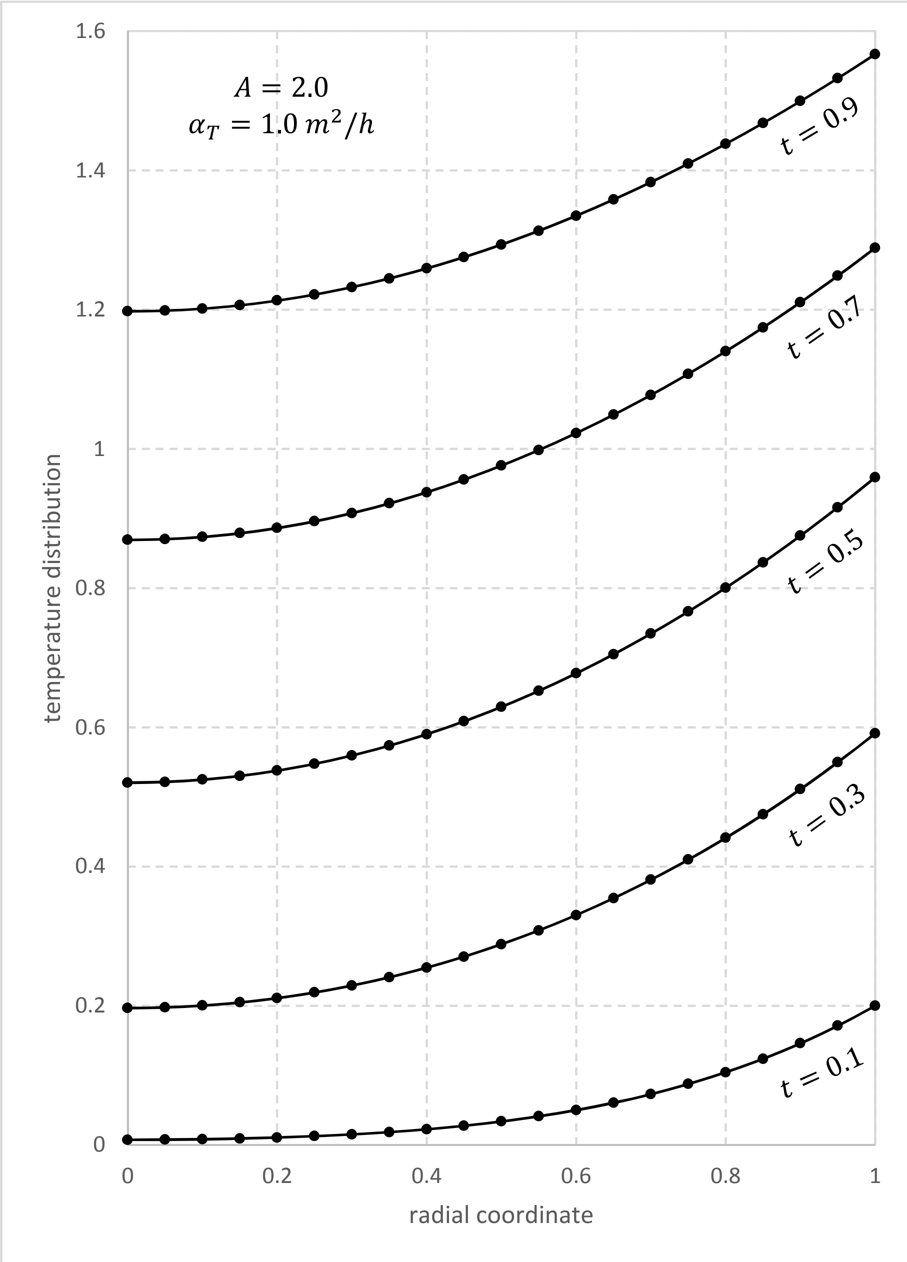

Figure 2.3: Verification of temperature distribution in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

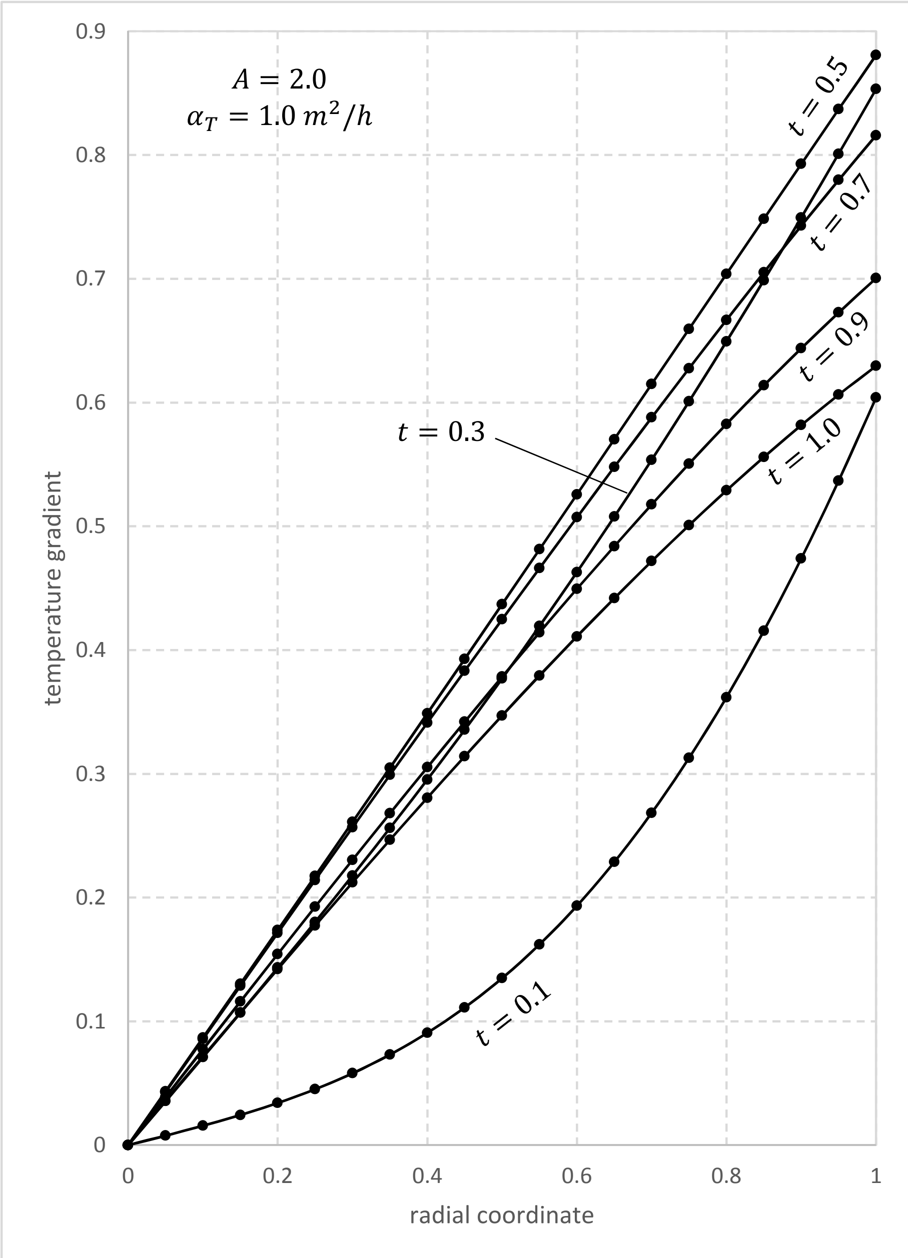

Figure 2.4: Verification of temperature gradient in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

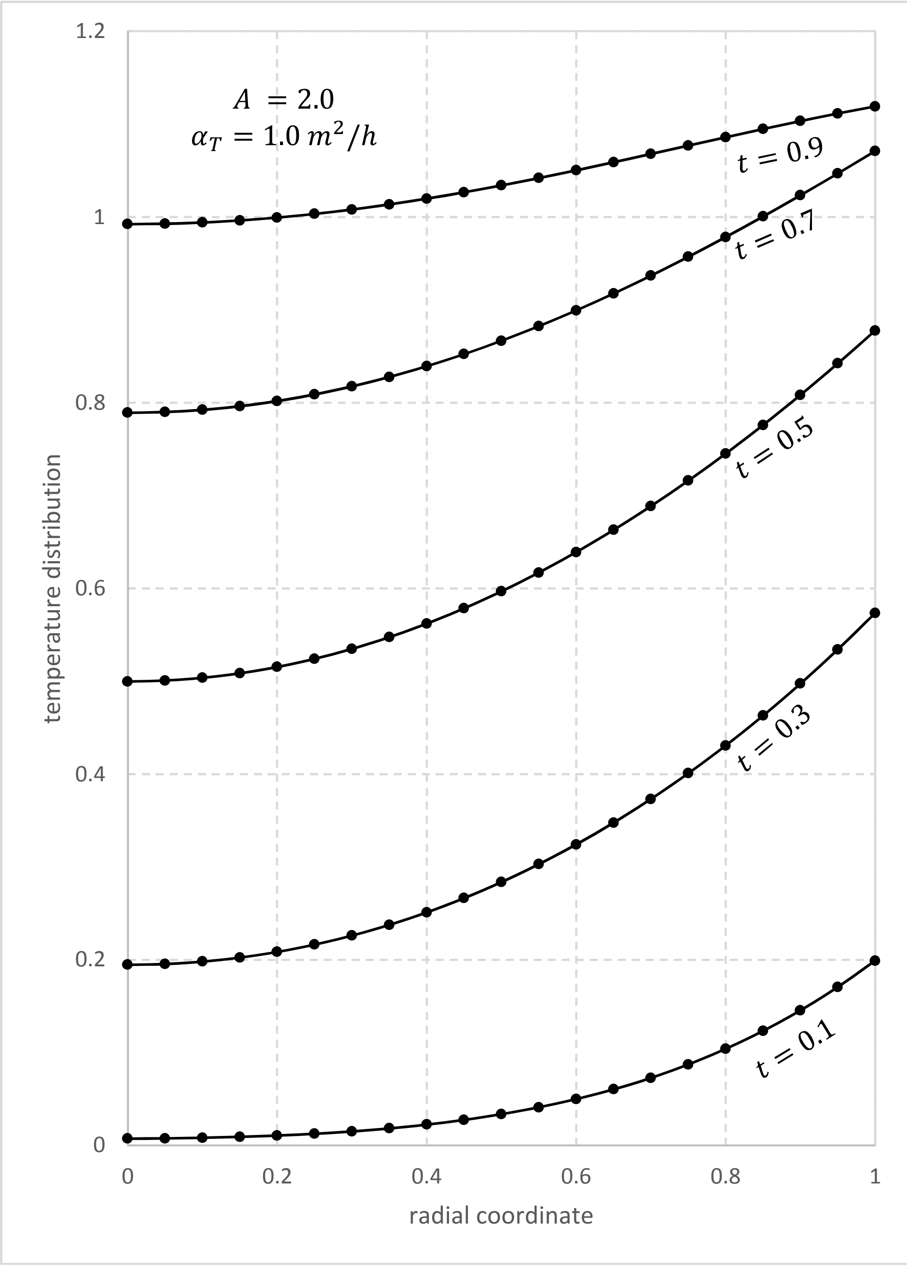

Figure 2.5: Verification of temperature distribution in the solid cylinder made of structural steel at different time steps for the case $F(t) = Atcos(t)$.

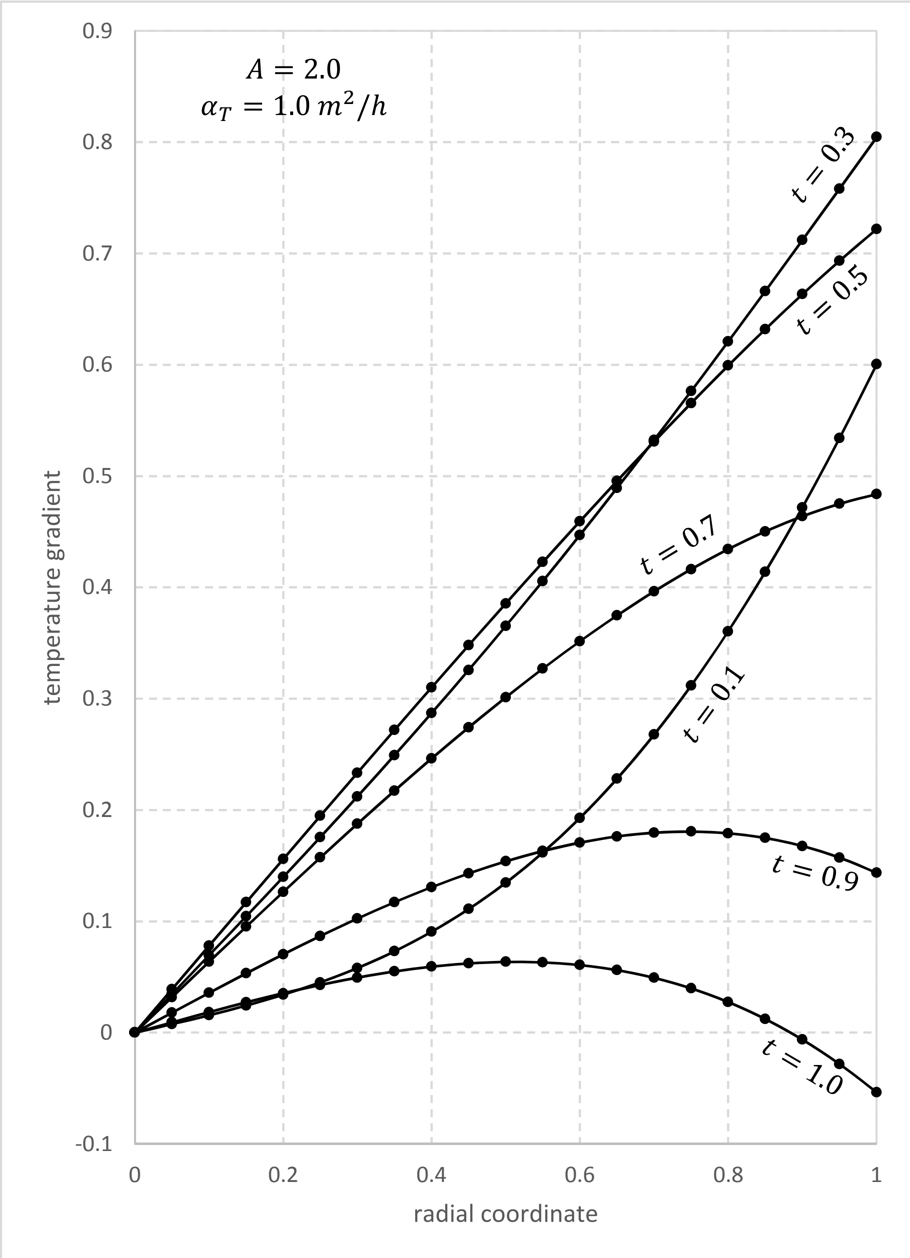

Figure 2.6: Verification of temperature gradient in the solid cylinder made of structural steel at different time steps for the case $F(t) = Atcos(t)$.

2.2 Elastic Solution

In following Timoshenko and Goodier's [34] notation and the basic equations of elasticity as provided therein are used. The equation of equilibrium and strain-displacement relations are

$$\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0, \tag{2.58}$$

$$\epsilon_\theta = \frac{u}{r} \quad ; \quad \epsilon_r = \frac{du}{dr}. \tag{2.59}$$

Using equations of the generalized Hooke's law the total strains ϵ_r, ϵ_θ and ϵ_z in the radial, circumferential and axial directions: (r, θ, z) can be written as

$$\epsilon_r = \frac{1}{E}\left[\sigma_r - \nu\left(\sigma_\theta + \sigma_z\right)\right] + \alpha\Delta T, \tag{2.60}$$

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu\left(\sigma_r + \sigma_z\right)\right] + \alpha\Delta T, \tag{2.61}$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_r + \sigma_\theta\right)\right] + \alpha\Delta T. \tag{2.62}$$

In these equations σ_r, σ_θ and σ_z represent the radial, circumferential and axial stress components, u the displacement in the radial direction r, E the modulus of elasticity, ν the Poisson's ratio, α the coefficient of thermal expansion, and $\Delta T = T(r,t) - T_{ref}$ the temperature gradient at any radial location r, and time t in the cylinder with T_{ref} being a reference temperature. In case of the generalized plane strain, the axial strain $\epsilon_z = \epsilon_0 = constant$, hence from Eq. [2.62] the axial stress is determined in terms of radial and circumferential stresses as

$$\sigma_z = E\epsilon_0 - \alpha E\Delta T + \nu\left(\sigma_r + \sigma_\theta\right). \tag{2.63}$$

The axial stress is eliminated in total strain expressions and the results are substituted in the strain-displacement relations to give

$$\sigma_r = \frac{E\epsilon_0\nu}{(1+\nu)(1-2\nu)} + \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\frac{u}{r} + (1-\nu)u'\right] - \frac{E\alpha\Delta T}{(1-2\nu)}, \tag{2.64}$$

$$\sigma_\theta = \frac{E\epsilon_0\nu}{(1+\nu)(1-2\nu)} + \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{u}{r} + \nu u'\right] - \frac{E\alpha\Delta T}{(1-2\nu)}. \tag{2.65}$$

Inserting the radial and circumferential stresses into the equation of equilibrium, Eq. (2.58) leads to the elastic equation in terms of the radial displacement as

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = \frac{\alpha\left(1+\nu\right)}{(1-\nu)}r^2\frac{dT}{dr}. \tag{2.66}$$

This is a Cauchy-Euler nonhomogeneous type differential equation which assumes the general solution

$$u(r) = A^{*}r + B^{*}\frac{1}{r} + \frac{\alpha\,(1+\nu)}{(1-\nu)}\frac{1}{r}\int_0^r \eta T(\eta,t)d\eta, \tag{2.67}$$

in which A^{*} and B^{*} are arbitrary integration constants. Since u and the stresses are finite at $r = 0$ we require that $B^{*} = 0$. Therefore,

$$u(r) = A^{*}r + \frac{\alpha\,(1+\nu)}{(1-\nu)}\frac{1}{r}\int_0^r \eta T(\eta,t)d\eta. \tag{2.68}$$

It is important to note that the expression

$$\frac{1}{r}\int_0^r \eta T(\eta,t)d\eta, \tag{2.69}$$

is an indeterminate of type $0/0$ and it is determined by using L'Hospital rule:

$$\lim_{r\to 0}\frac{1}{r}\int_0^r \eta T(\eta,t)d\eta = \lim_{r\to 0}\frac{\frac{d}{dr}\int_0^r \eta T(\eta,t)d\eta}{\frac{d}{dr}(r)} = \lim_{r\to 0}\frac{rT(r,t)}{1} = 0. \tag{2.70}$$

If we substitute the displacement into the radial and circumferential stress expressions, Eqs. (2.64) - (2.65), we obtain

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)}(\epsilon_0\nu + A^{*}) - \frac{E\alpha}{(1-\nu)}\frac{1}{r^2}\int_0^r \eta T(\eta,t)d\eta, \tag{2.71}$$

$$\sigma_\theta = \frac{E}{(1+\nu)(1-2\nu)}(\epsilon_0\nu + A^{*}) + \frac{E\alpha}{(1-\nu)}\frac{1}{r^2}\int_0^r \eta T(\eta,t)d\eta - \frac{E\alpha T}{(1-\nu)}. \tag{2.72}$$

Note again that

$$\frac{1}{r^2}\int_0^r \eta T(\eta,t)d\eta = \frac{0}{0},$$

hence, like above its actual value is obtained by using L'Hospital rule again as

$$\lim_{r\to 0}\frac{1}{r^2}\int_0^r \eta T(\eta,t)d\eta = \lim_{r\to 0}\frac{\frac{d}{dr}\int_0^r \eta T(\eta,t)d\eta}{\frac{d}{dr}(r^2)} = \frac{T(0,t)}{2}. \tag{2.73}$$

By substituting σ_r and σ_θ into σ_z, the axial stress now can be written as

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[\epsilon_0(1-\nu) + 2\nu A^{*}\right] - \frac{E\alpha T}{(1-\nu)}. \tag{2.74}$$

At $r = 0$, the radial and circumferential stresses become

$$\sigma_r(0) = \sigma_\theta(0) = \frac{E}{(1-2\nu)(1+\nu)}(\epsilon_0\nu + A^{*}) - \frac{E\alpha T(0,t)}{2(1-\nu)}, \tag{2.75}$$

and the displacement vanishes

$$u(0) = 0. \tag{2.76}$$

In these equations we have 2 unknown constants to be determined. For this purpose boundary condition

$$\sigma_r(b) = 0, \tag{2.77}$$

and

$$F_z = \int \sigma_z dA = 0, \tag{2.78}$$

are used. In these equations F_z represents the axial force. The second condition implies that the cylinder can expand or contract freely in the axial direction as its temperature is increased or decreased. Due to the axial symmetry, the condition for the vanishing axial force can be written as

$$F_z = \int \sigma_z dA = 2\pi \int_0^b r\sigma_z dr = \int_0^b r\sigma_z dr = 0. \tag{2.79}$$

Then, by using these conditions ϵ_0 and A^* constants are determined as

$$\epsilon_0 = \frac{2\alpha}{b^2} \int_0^b rT(r,t)dr, \tag{2.80}$$

$$A^* = \frac{\alpha(1-3\nu)}{b^2(1-\nu)} \int_0^b rT(r,t)dr. \tag{2.81}$$

If we substitute the temperature distribution with a boundary condition $F(t) = A\sin t$, into ϵ_0 and A^* we obtain

$$\begin{aligned} \epsilon_0 &= \frac{2\alpha}{b^2} \int_0^b rT(r,t)dr \\ &= \frac{2\alpha}{b^2} \int_0^b \left(A\sin(t) - \frac{2A}{b} \sum_{n=1}^{\infty} \frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)} \psi_1(\alpha_T, \lambda_n, t) \right) rdr \\ &= \frac{2\alpha}{b^2} \left(\frac{Ab^2}{2} \sin(t) - \frac{2A}{b} \sum_{n=1}^{\infty} \frac{b}{\lambda_n^2} \psi_1(\alpha_T, \lambda_n, t) \right) \\ &= A\alpha \sin(t) - \frac{4A\alpha}{b^2} \sum_{n=1}^{\infty} \frac{1}{\lambda_n^2} \psi_1(\alpha_T, \lambda_n, t) \end{aligned} \tag{2.82}$$

and

$$\begin{aligned}
A^* &= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b rT(r,t)dr \\
&= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b A\sin(t)rdr \\
&\quad -\frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b \frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\psi_1(\alpha_T,\lambda_n,t)rdr \\
&= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\left(\frac{Ab^2}{2}\sin(t)-\frac{2A}{b}\sum_{n=1}^{\infty}\frac{b}{\lambda_n^2}\psi_1(\alpha_T,\lambda_n,t)\right) \\
&= \frac{A\alpha(1-3\nu)}{2(1-\nu)}\sin(t)-\frac{2A\alpha(1-3\nu)}{b^2(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^2}\psi_1(\alpha_T,\lambda_n,t). \qquad (2.83)
\end{aligned}$$

For the case, $F(t) = At\cos t$, ϵ_0 and A^* become

$$\begin{aligned}
\epsilon_0 &= \frac{2\alpha}{b^2}\int_0^b rT(r,t)dr \\
&= \frac{2\alpha}{b^2}\int_0^b r\left(At\cos(t)-\frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t)\right)dr \\
&= \frac{2\alpha}{b^2}\left(\frac{Ab^2}{2}t\cos(t)-\frac{2A}{b}\sum_{n=1}^{\infty}\frac{b}{\lambda_n^2}\psi_2(\alpha_T,\lambda_n,t)\right) \\
&= A\alpha t\cos(t)-\frac{4A\alpha}{b^2}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^2}\psi_2(\alpha_T,\lambda_n,t) \qquad (2.84)
\end{aligned}$$

and

$$\begin{aligned}
A^* &= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b rT(r,t)dr \\
&= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b At\cos(t)rdr \\
&\quad -\frac{\alpha(1-3\nu)}{b^2(1-\nu)}\int_0^b \frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t)rdr \\
&= \frac{\alpha(1-3\nu)}{b^2(1-\nu)}\left(\frac{Ab^2}{2}t\cos(t)-\frac{2A}{b}\sum_{n=1}^{\infty}\frac{b}{\lambda_n^2}\psi_2(\alpha_T,\lambda_n,t)\right) \\
&= \frac{A\alpha(1-3\nu)}{2(1-\nu)}t\cos(t)-\frac{2A\alpha(1-3\nu)}{b^2(1-\nu)}\sum_{n=1}^{\infty}\frac{1}{\lambda_n^2}\psi_2(\alpha_T,\lambda_n,t). \qquad (2.85)
\end{aligned}$$

Here, $\psi_1(\alpha_T, \lambda_n, t)$ and $\psi_2(\alpha_T, \lambda_n, t)$ are the outcomes of Duhamel's integral. The other integral that is used in the above calculations can be derived as

$$\begin{aligned}\int_0^r \eta T(\eta,t)d\eta &= \int_0^r \eta At\cos(t)d\eta \\ &\quad -\frac{2A}{b}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n\eta)}{\lambda_n J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t)\eta d\eta \\ &= \frac{Ar^2}{2}t\cos(t) - \frac{2A}{b}\sum_{n=1}^{\infty}\frac{r}{\lambda_n^2}\frac{J_1(\lambda_n r)}{J_1(\lambda_n b)}\psi_2(\alpha_T,\lambda_n,t). \qquad (2.86)\end{aligned}$$

Convergence of an analytical solution at early times is difficult. As time progresses convergence becomes easier. In this study, all verifications are done in the time interval $0 \leq t \leq 1$ in order to determine the possible errors. However, it is seen that the results obtained from two different models are in perfect agreement. As mentioned before, solid lines belong to the analytical solutions, dots to computational solutions in figures. Figures 2.7 and 2.8 present the verification of radial, circumferential, axial and von-Mises stress components, and the displacement in the radial direction of the cylinder subjected to periodic boundary condition function $F(t) = Asin(t)$ at $t = 0.2$ and $t = 0.5$, respectively. In Figures 2.9 and 2.10 verification of stresses and displacement distributions are demonstrated at $t = 0.2$ and $t = 0.5$ for the case $F(t) = Atcos(t)$.

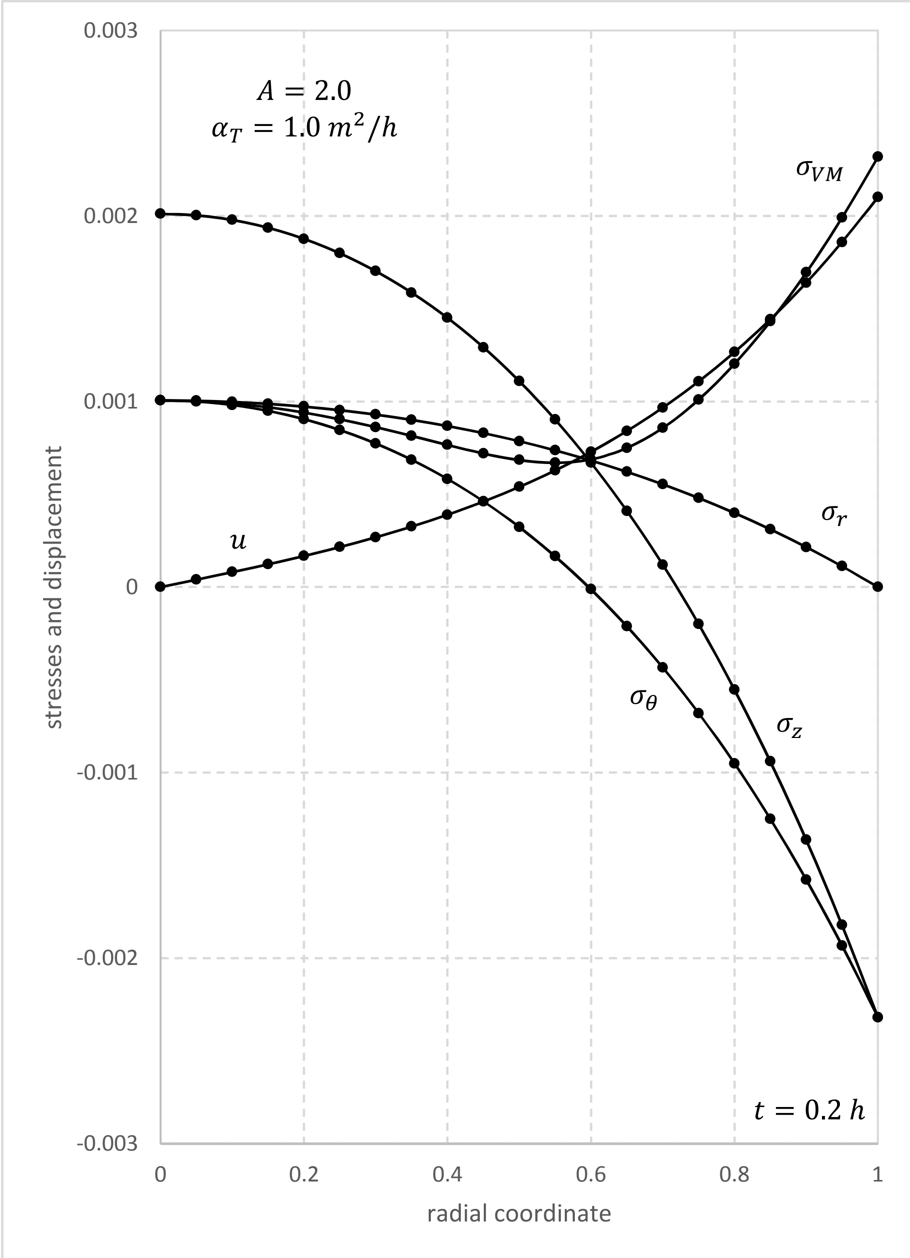

Figure 2.7: Verification of stresses and displacement in the solid cylinder made of structural steel at $t = 0.2$ for the case $F(t) = Asin(t)$.

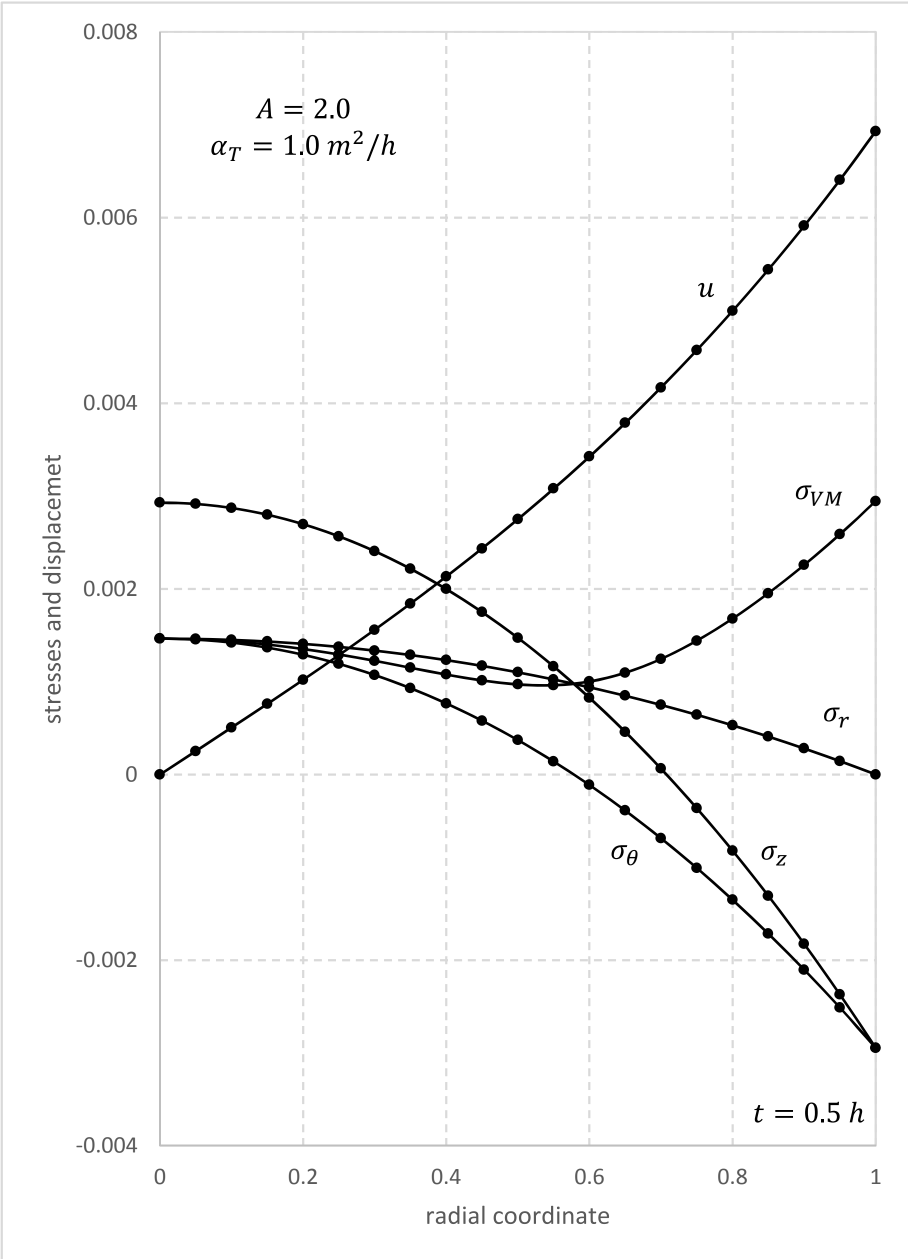

Figure 2.8: Verification of stresses and displacement in the solid cylinder made of structural steel at $t = 0.5$ for the case $F(t) = Asin(t)$.

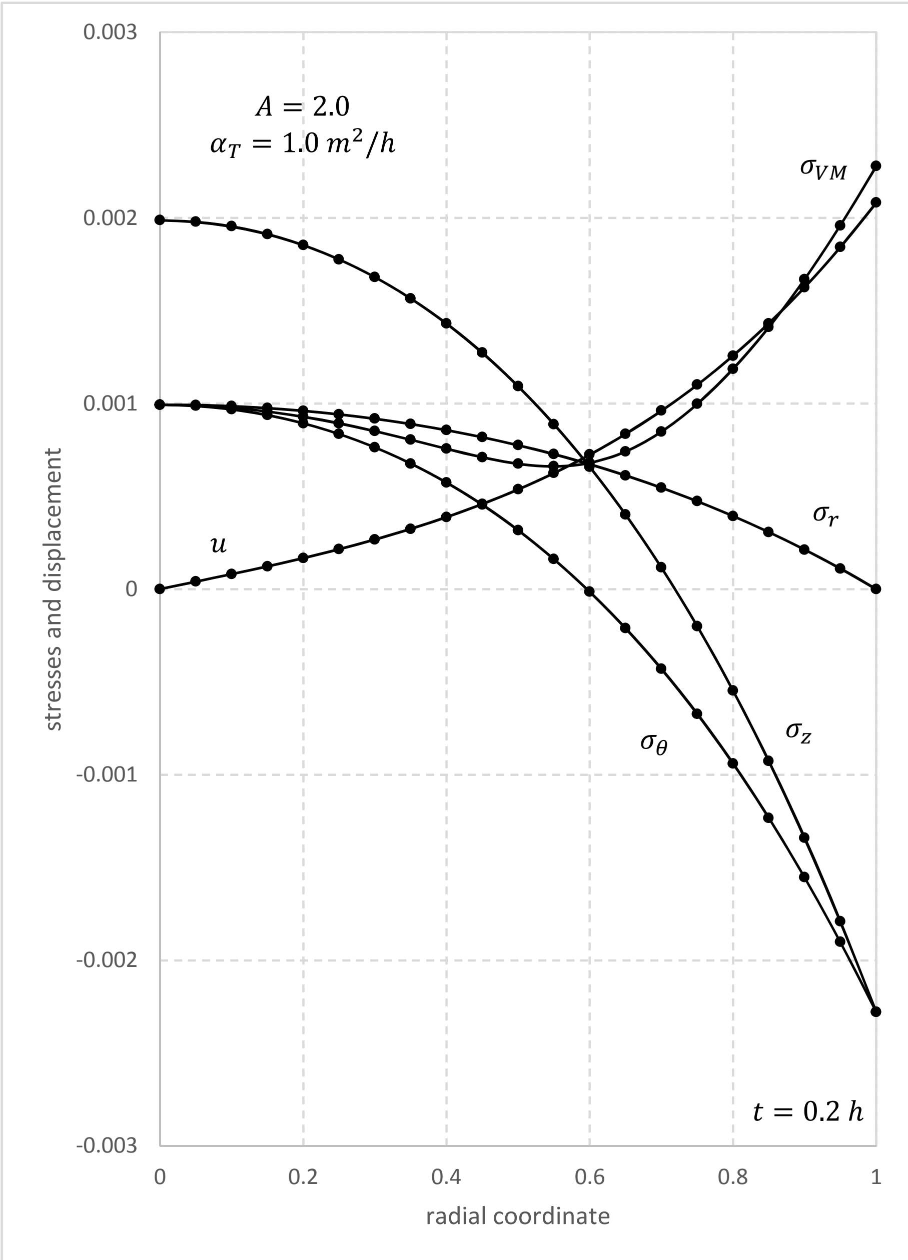

Figure 2.9: Verification of stresses and displacement in the solid cylinder made of structural steel at $t = 0.2$ for the case $F(t) = Atcos(t)$.

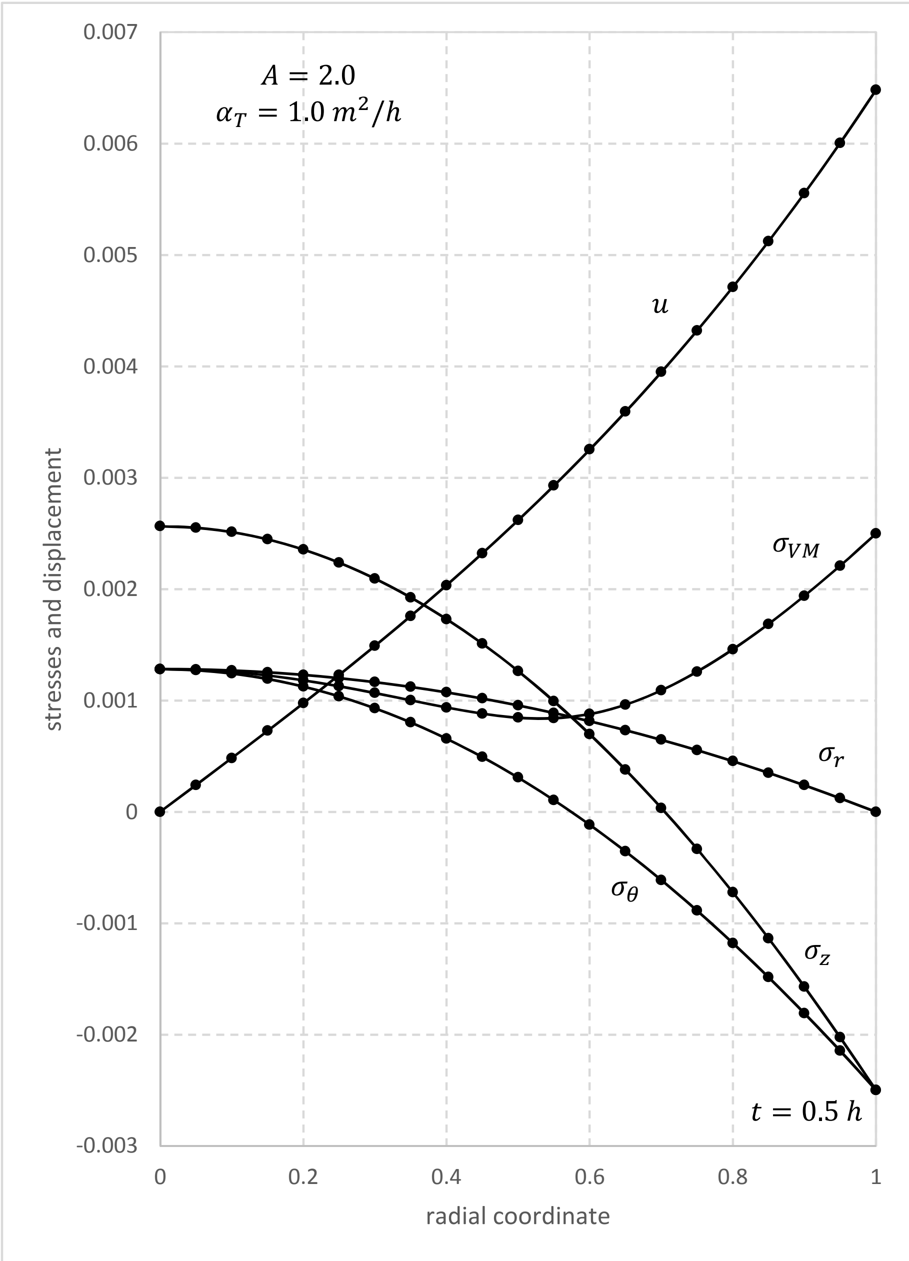

Figure 2.10: Verification of stresses and displacement in the solid cylinder made of structural steel at $t = 0.5$ for the case $F(t) = Atcos(t)$.

CHAPTER 3

RESULTS AND DISCUSSIONS

In the present study, the thermoelastic response of a solid cylinder subjected to time dependent periodic boundary condition is investigated. To analyze the effect of the periodic boundary conditions, two different boundary conditions are taken into consideration. Furthermore, the thermoelastic response of two different materials are examined under the same periodic boundary conditions.

The radius of the solid cylinder is taken as $b = 1$. In Figures 3.1 to 3.12, time dependent periodic boundary condition is selected as $A\sin(t)$. Here, A is the amplitude of the function. From Figure 3.1 to 3.8 structural steel (A36) is used as material, while in Figures 3.9 - 3.11 Aluminum 2014-T6 (UNS A92014) is used. In the calculations, the following numerical values of the parameters are used: For steel, Poisson ratio $\nu = 0.3$, thermal expansion coefficient $\alpha = 11.72 \times 10^{-6}$ K^{-1}, modulus of elasticity $E = 210$ GPa and yield stress $\sigma_Y = 270$ Mpa; For Aluminum, Poisson ratio $\nu = 0.33$, thermal expansion coefficient $\alpha = 22.7 \times 10^{-6}$ K^{-1}, modulus of elasticity $E = 73.1$ GPa and yield stress $\sigma_Y = 414$ Mpa.

Figure 3.1 shows the temperature distribution in the solid cylinder at different time steps. It can be seen that surface temperature at the boundary equals to the value of the function at that time. A is chosen as 97.2218 so that the material does not exceed the elastic limit, and thermal diffusivity α_T is equal to $0.0480744\ m^2/h$ for this material. By taking derivative of the temperature distribution with respect to radial coordinate, temperature gradient is obtained and presented in Figure 3.2. Hereby, the rate of change in temperature in the radial coordinate is seen in this figure. Radial stress inside the solid cylinder is shown in Figure 3.3. In radial direction, there is no stress at the boundary surface, hence $\sigma_r = 0$ at $r = b$, and as seen in Figure 3.3 the boundary condition is verified. For different times, circumferential and axial stresses along the radial coordinate are calculated and plotted in Figures 3.4 and 3.5, respectively. Both stresses follow the same trend. In Figure 3.6, von-Mises stress at different time steps is presented. σ_θ and σ_z take their maximum value at $t = 4.33$ and thus von-Mises stress reaches the elastic limit of the material at that time. When the Figure 3.7 is examined, it is seen that at early times there is almost no displacement, and since the structure is a solid cylinder at the center $u = 0$. As mentioned before, von-Mises stress takes its maximum value at $t = 4.33$, Figure 3.8 shows all stress components and displacement at that time.

The thermoelastic response of the cylinder corresponding to the temperature distribution is depicted in Figure 3.9. A is selected as 372.2782 and thermal diffusivity α_T is equal to 0.221652 m^2/h for this material. Temperature gradient along the radial coordinate can be seen in Figure 3.10. von-Mises stress reaches the elastic limit of the material at $t = 3.77$, radial, circumferential and axial stress components, and the displacement in the radial direction is plotted at that time in Figure 3.11. In addition, variation of the axial strains in the solid cylinder made of structural steel and aluminum with time can be observed in Figure 3.12. It can be seen that the axial strains follow the same trend as the plot of the periodic boundary condition.

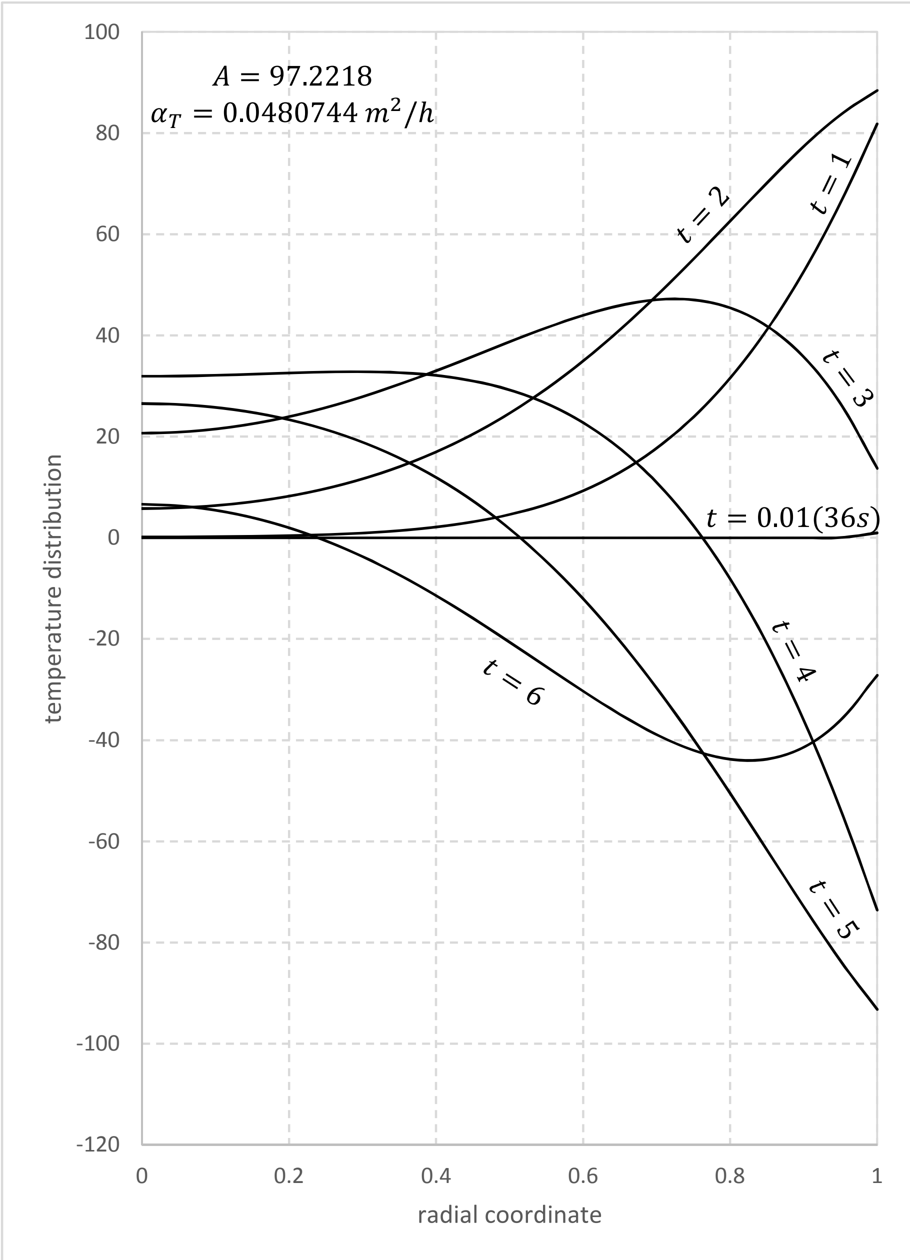

Figure 3.1: Temperature distribution in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

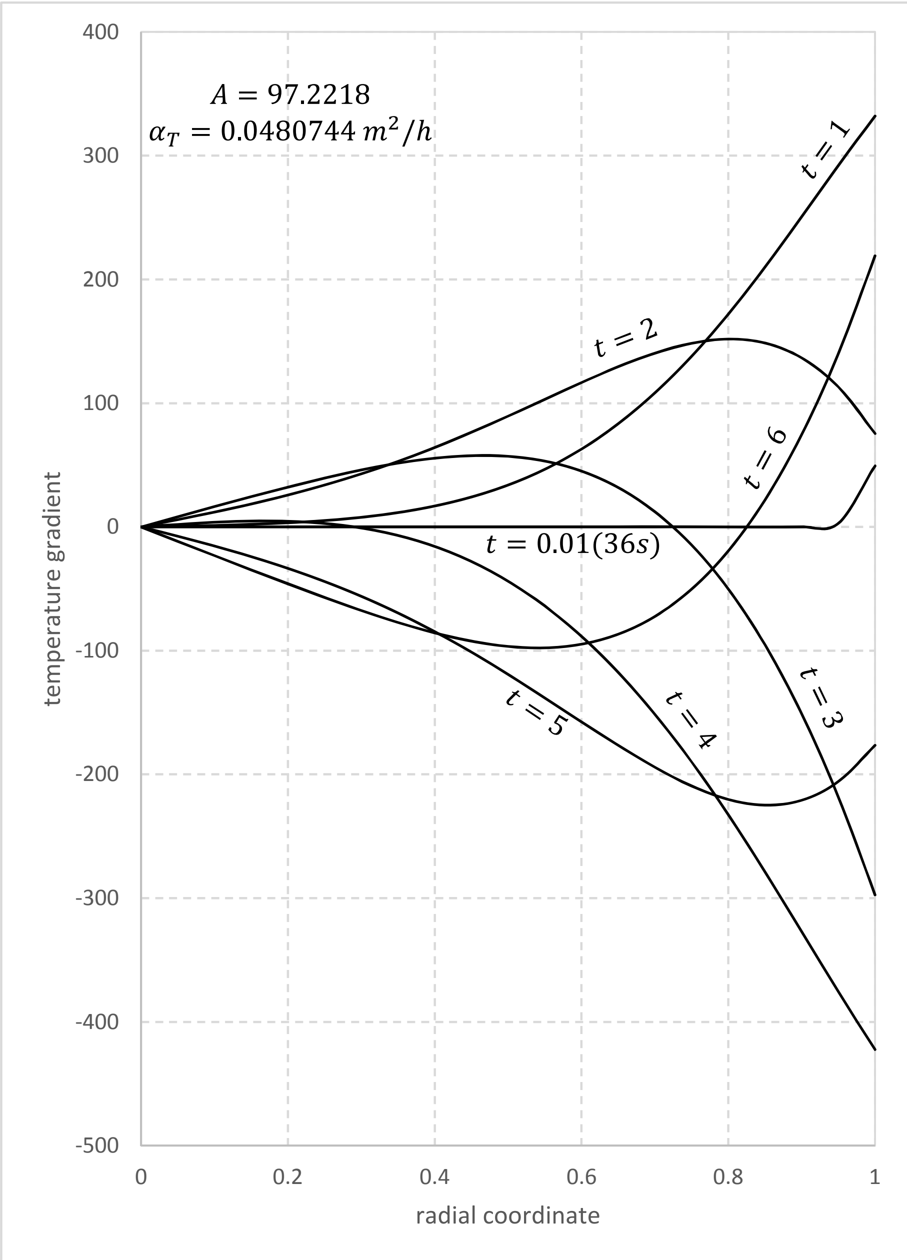

Figure 3.2: Temperature gradient in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

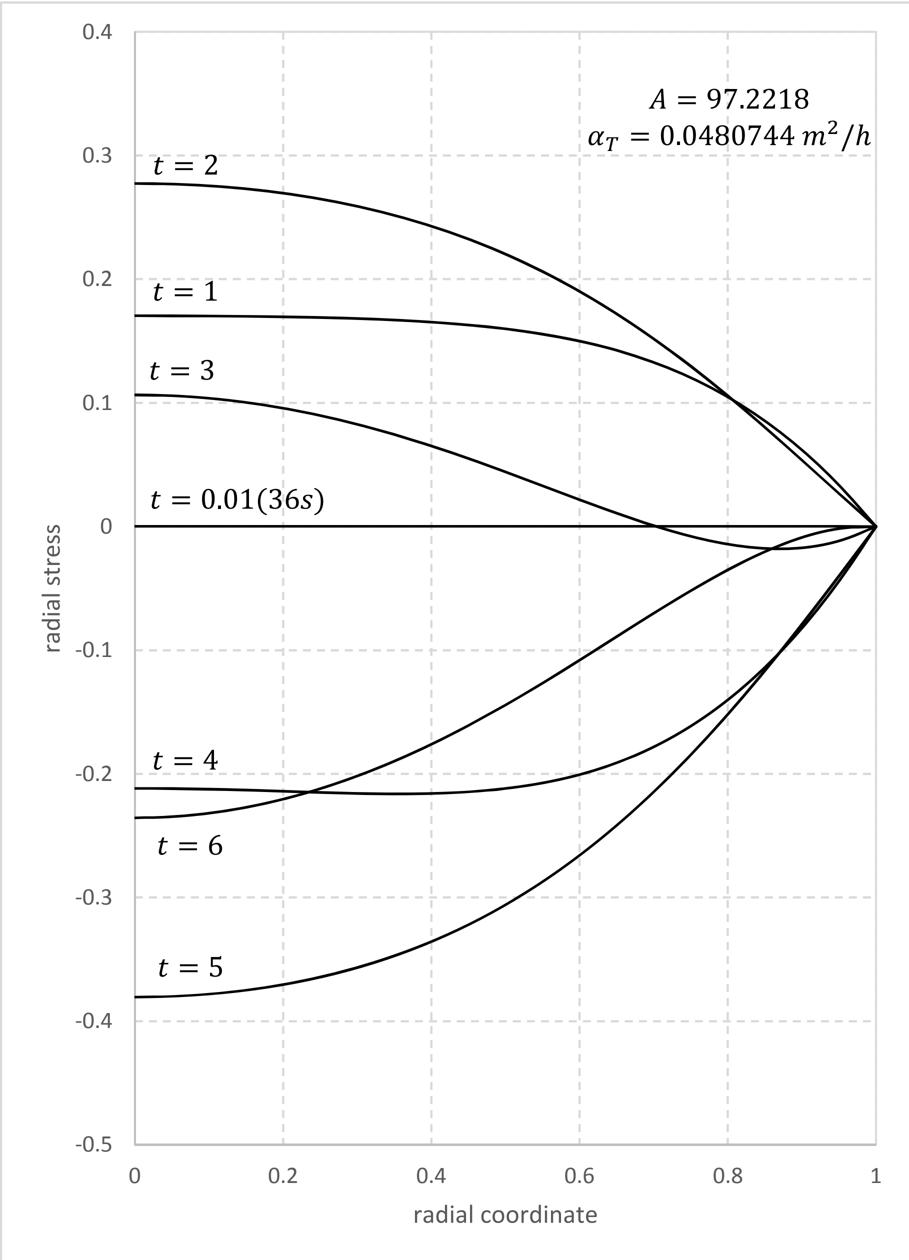

Figure 3.3: Radial stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

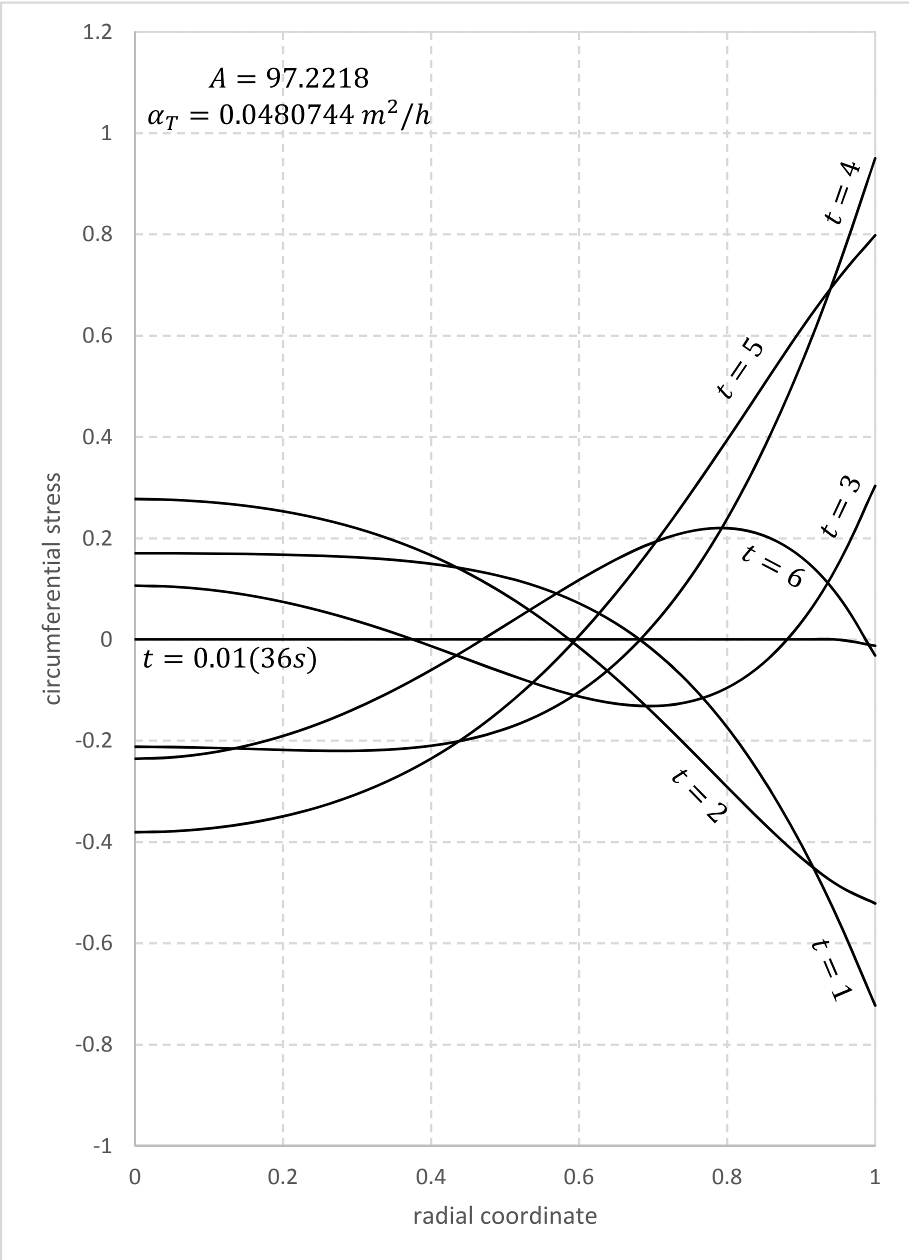

Figure 3.4: Circumferential stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

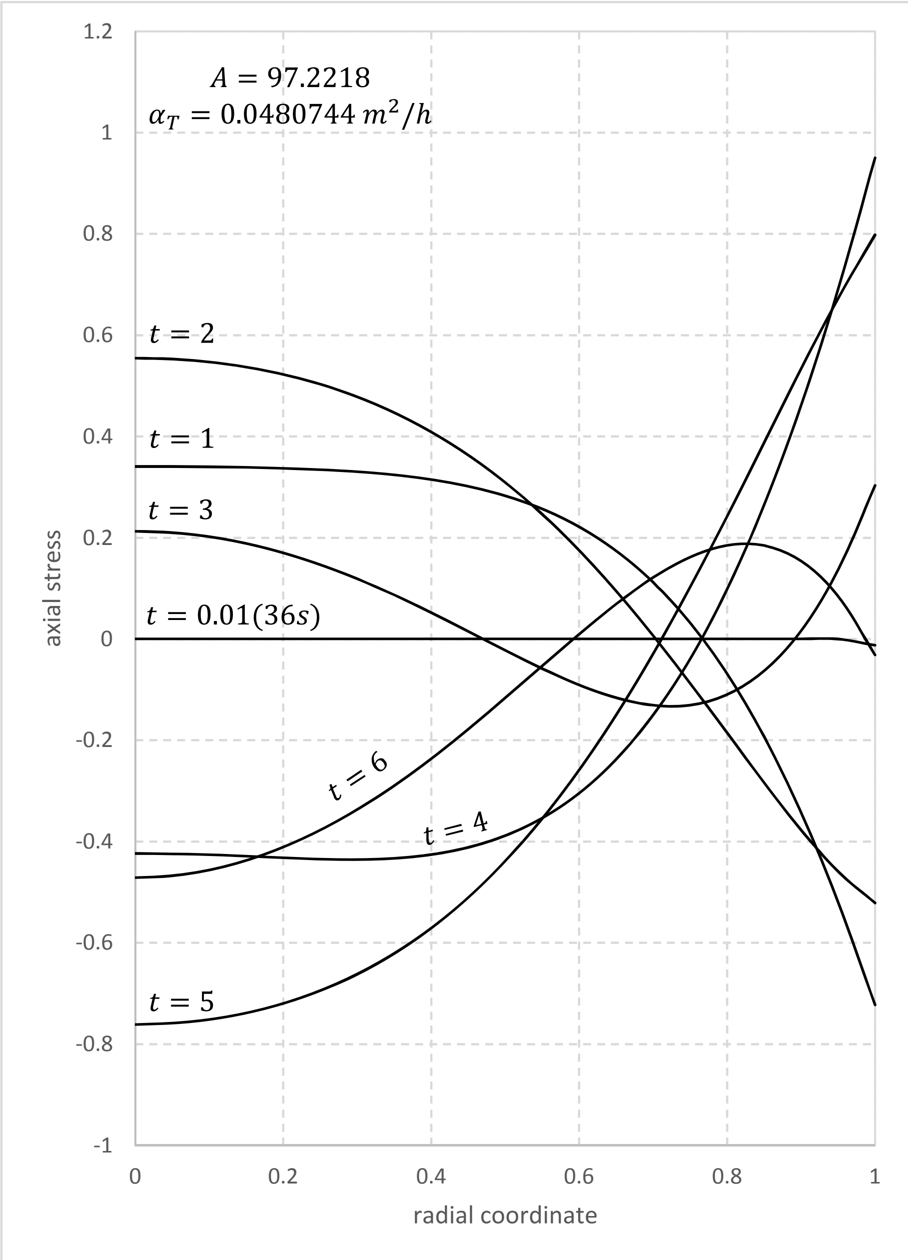

Figure 3.5: Axial stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

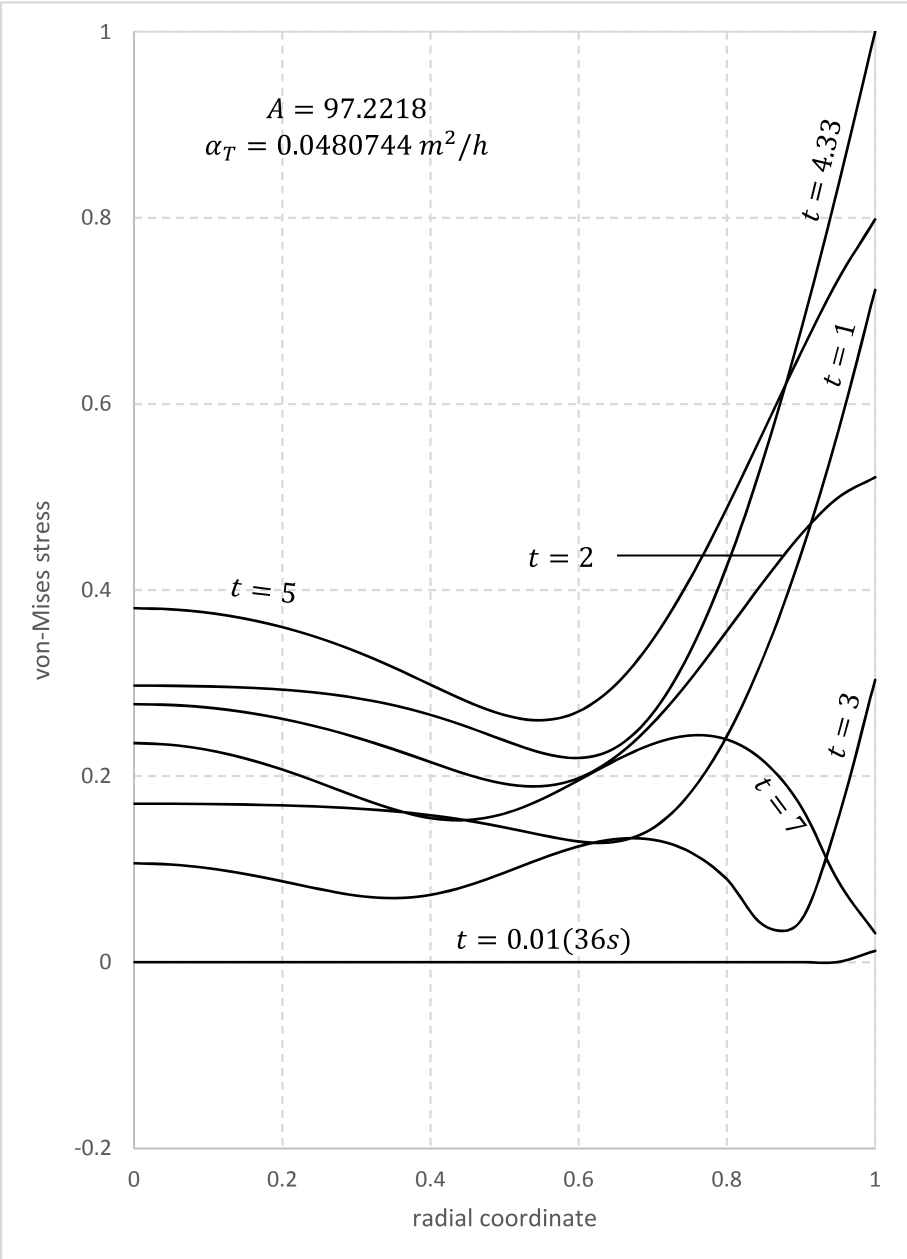

Figure 3.6: von-Mises stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

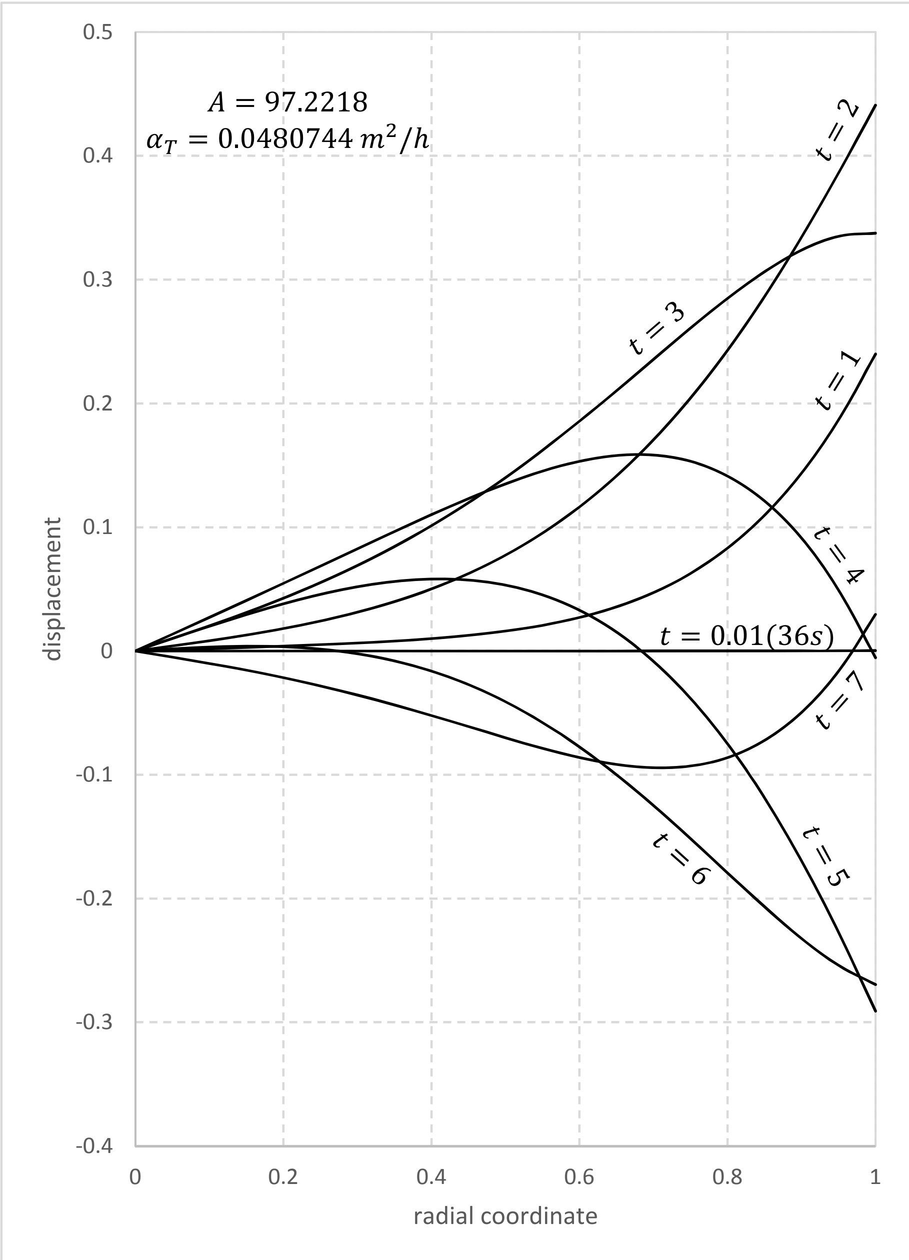

Figure 3.7: Displacement in the solid cylinder made of structural steel at different time steps for the case $F(t) = Asin(t)$.

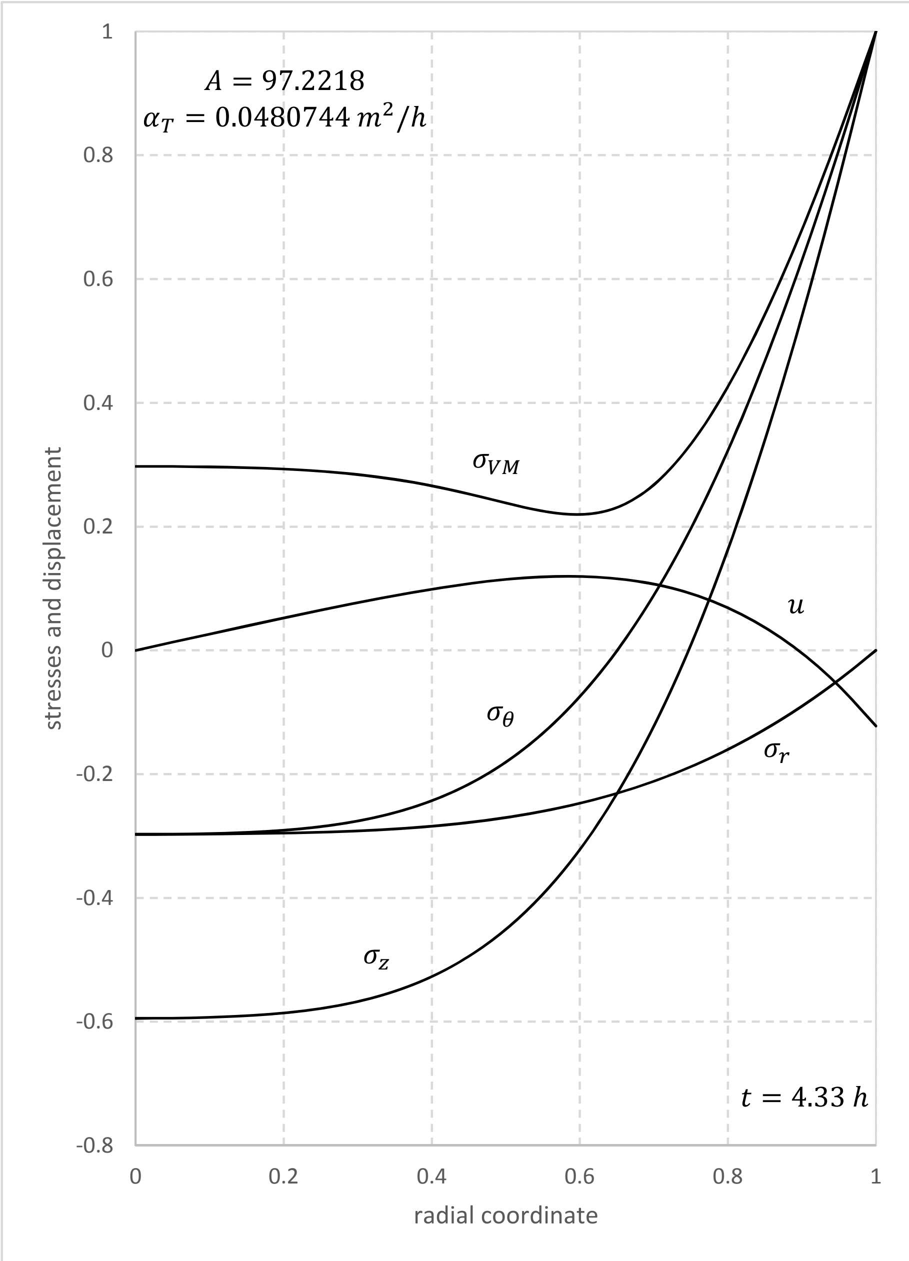

Figure 3.8: Stress and displacement distributions in the solid cylinder made of structural steel at $t = 4.33$ for the case $F(t) = Asin(t)$.

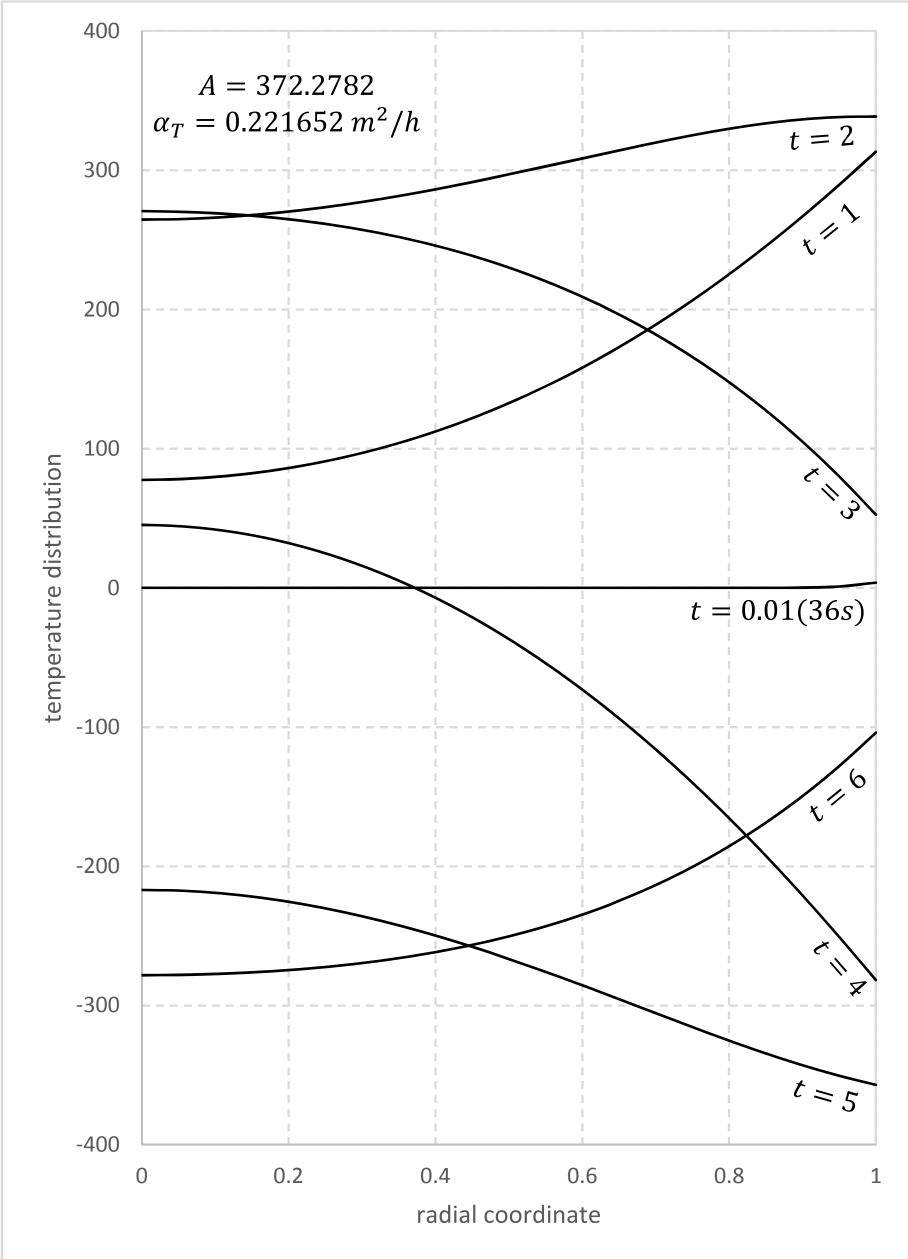

Figure 3.9: Temperature distribution in the solid cylinder made of aluminum at different time steps for the case $F(t) = Asin(t)$.

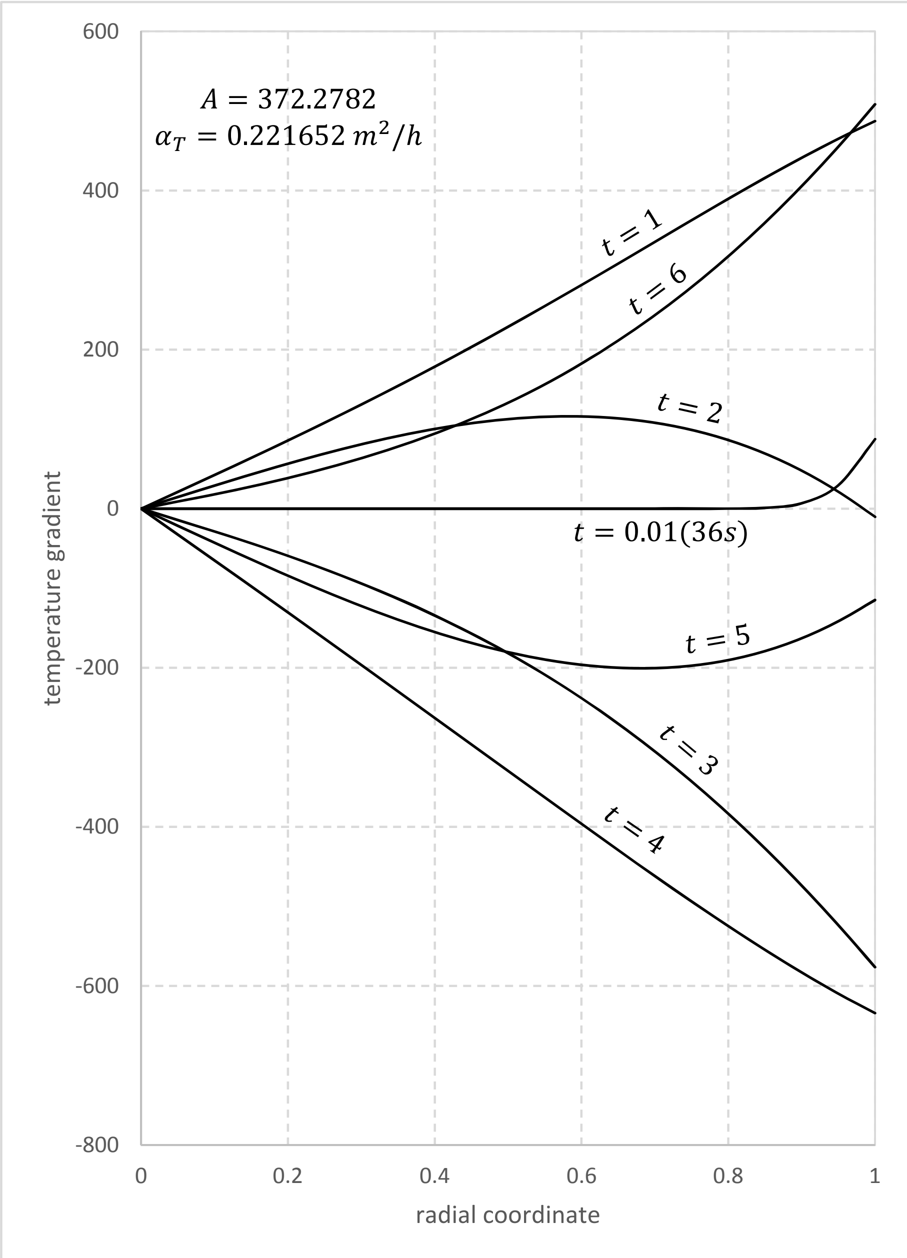

Figure 3.10: Temperature gradient in the solid cylinder made of aluminum at different time steps for the case $F(t) = Asin(t)$.

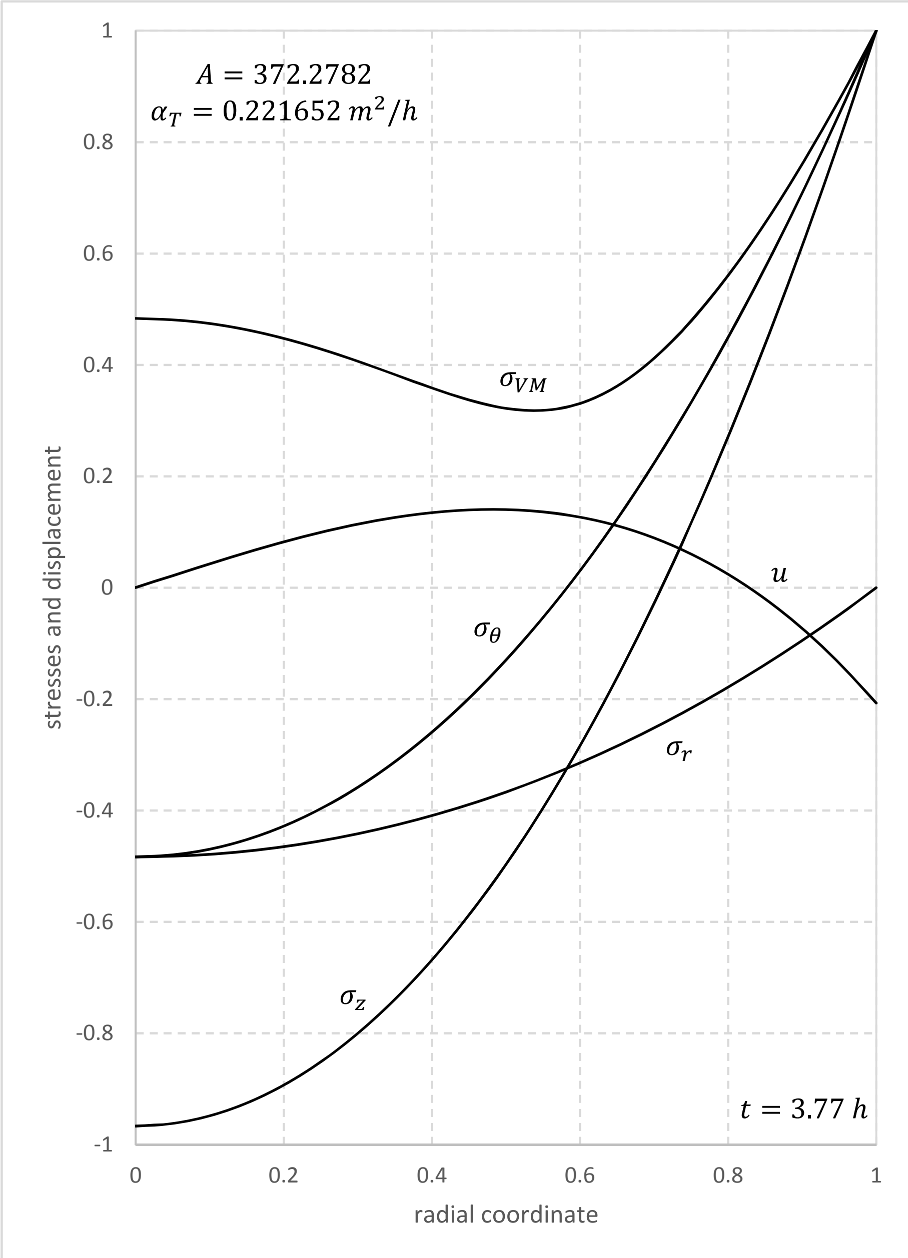

Figure 3.11: Stress and displacement distributions in the solid cylinder made of aluminum at $t = 3.77$ for the case $F(t) = Asin(t)$.

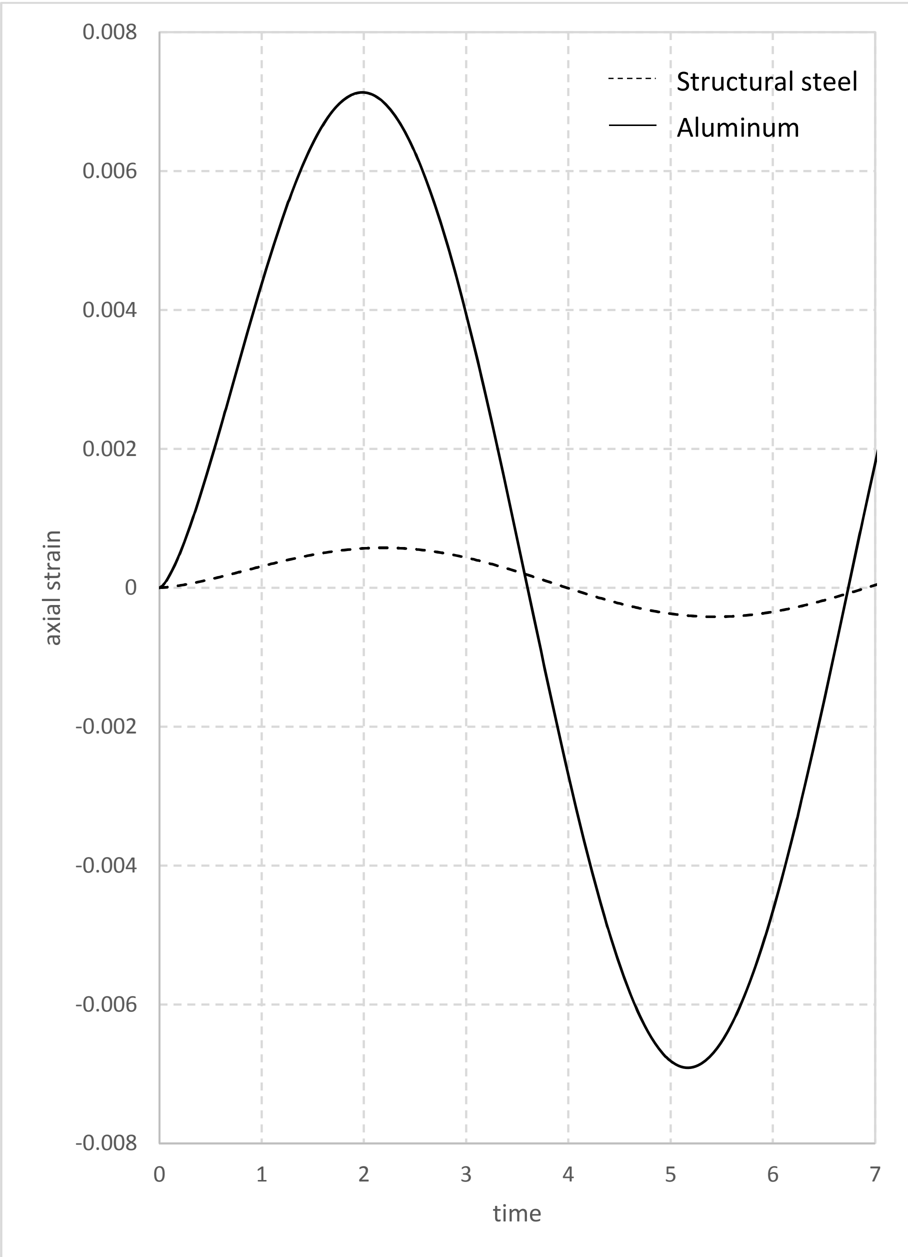

Figure 3.12: Comparison of the axial strains in the solid cylinder made of structural steel and aluminum for the case $F(t) = Asin(t)$.

In Figures 3.13 to 3.24, time dependent periodic boundary condition is selected as $At\cos(t)$. From Figure 3.13 to 3.20 structural steel is used as material, while in Figures 3.21 - 3.23 Aluminum is used. For steel, the amplitude of the function, A, is determined as 11.715 and α_T is equal to $0.0480744\ m^2/h$ again. Temperature distribution and temperature gradient in the solid cylinder at different time steps are shown in Figures 3.13 and 3.14. In Figure 3.15, radial stress, in Figure 3.16 circumferential stress, in Figure 3.17 axial stress and in Figure 3.19 radial displacement distributions are plotted for different time steps. Figure 3.18 presents the variation of von-Mises stress for different times and it is observed that von-Mises stress obtains its maximum value at $t = 9.19$. The distributions of σ_r, σ_θ, σ_z, σ_{VM} and u at $t = 9.19$ are drawn in Figure 3.20.

For Aluminum, A, is found as 45.7191 and $\alpha_T = 0.221652\ m^2/h$. In Figures 3.21 and 3.22, temperature distribution and temperature gradient along the radial direction are indicated. Stresses and displacement at $t = 8.68$, when the von-Mises stress arrives at its maximum value, is plotted in Figure 3.23. Finally, the distributions of the axial strains in the solid cylinder made of structural steel and aluminum are depicted in Figure 3.24.

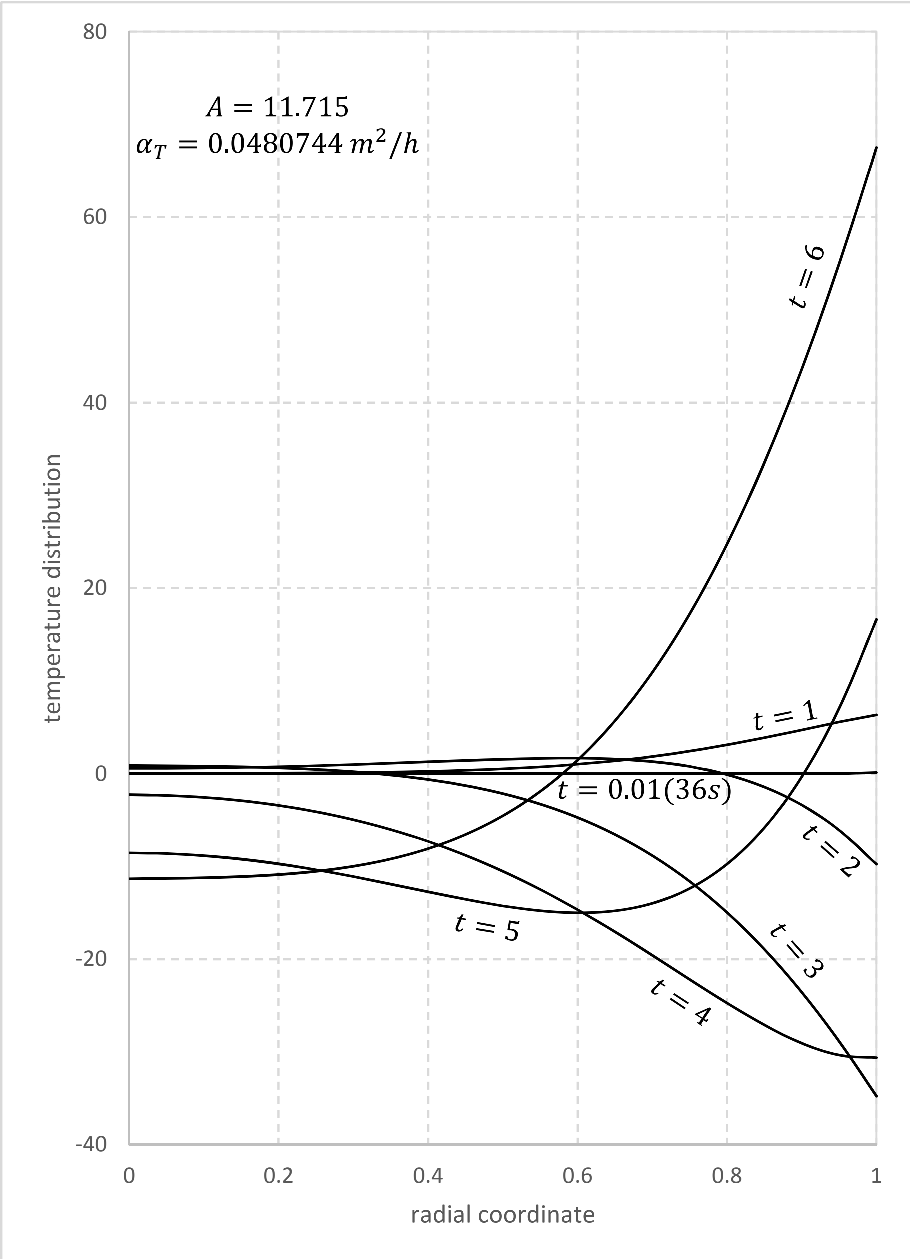

Figure 3.13: Temperature distribution in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

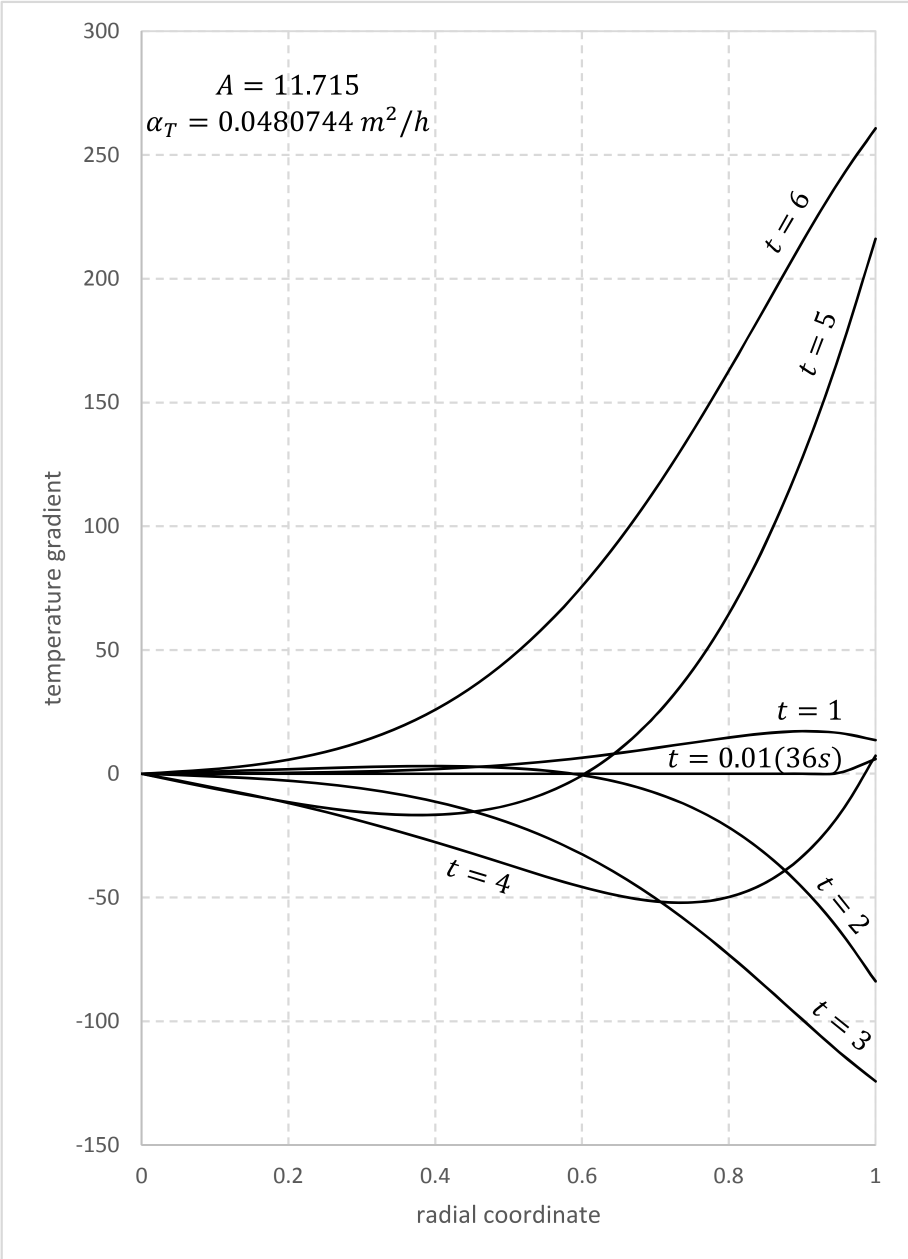

Figure 3.14: Temperature gradient in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

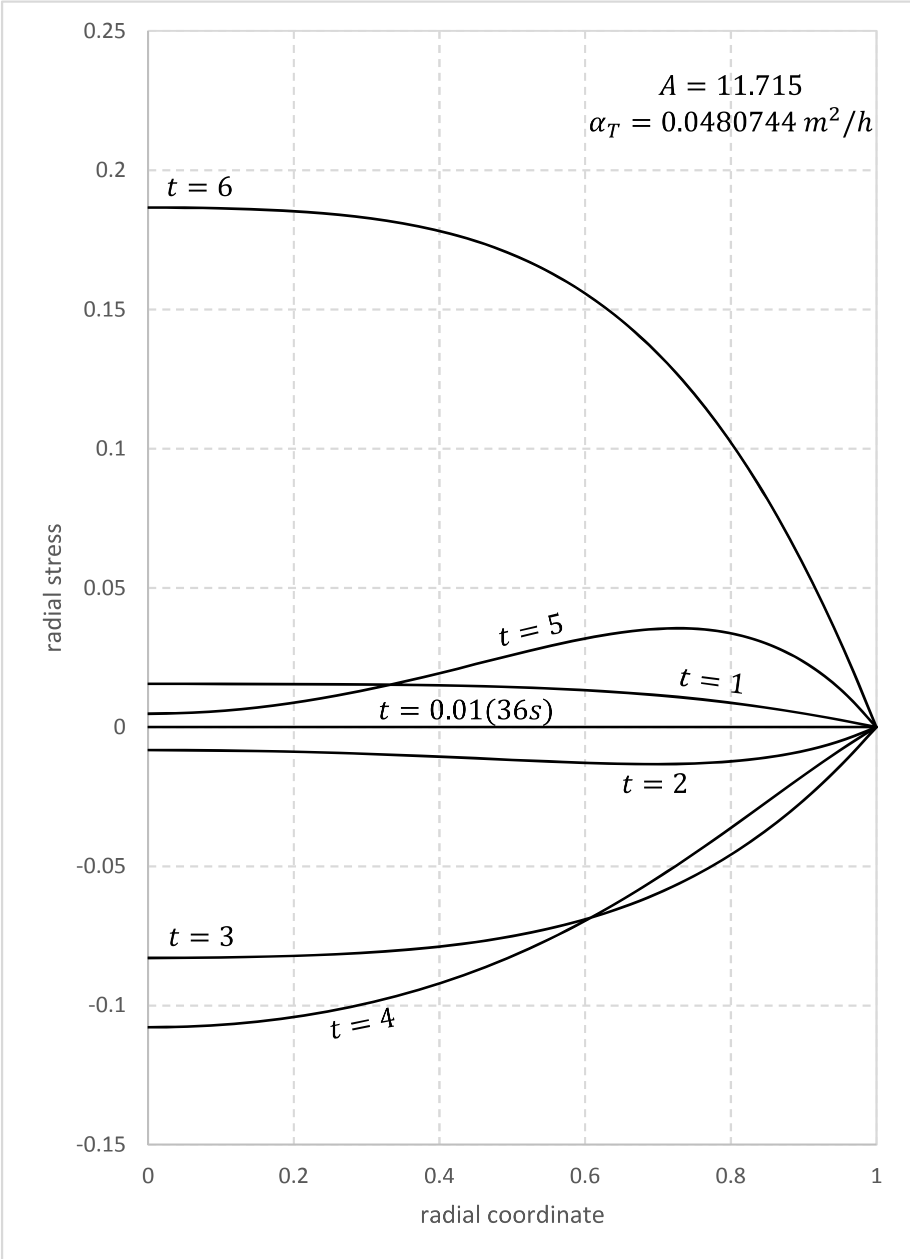

Figure 3.15: Radial stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

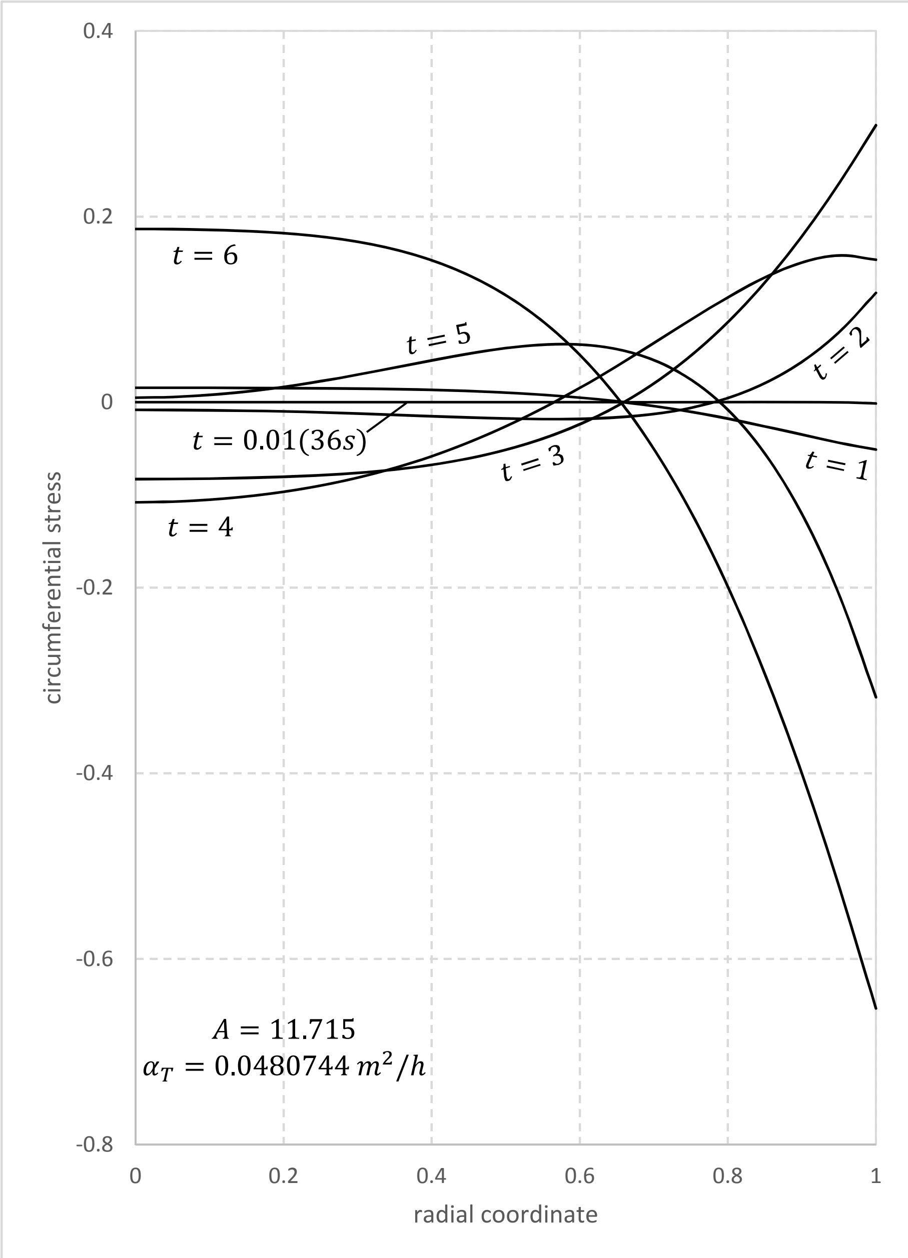

Figure 3.16: Circumferential stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

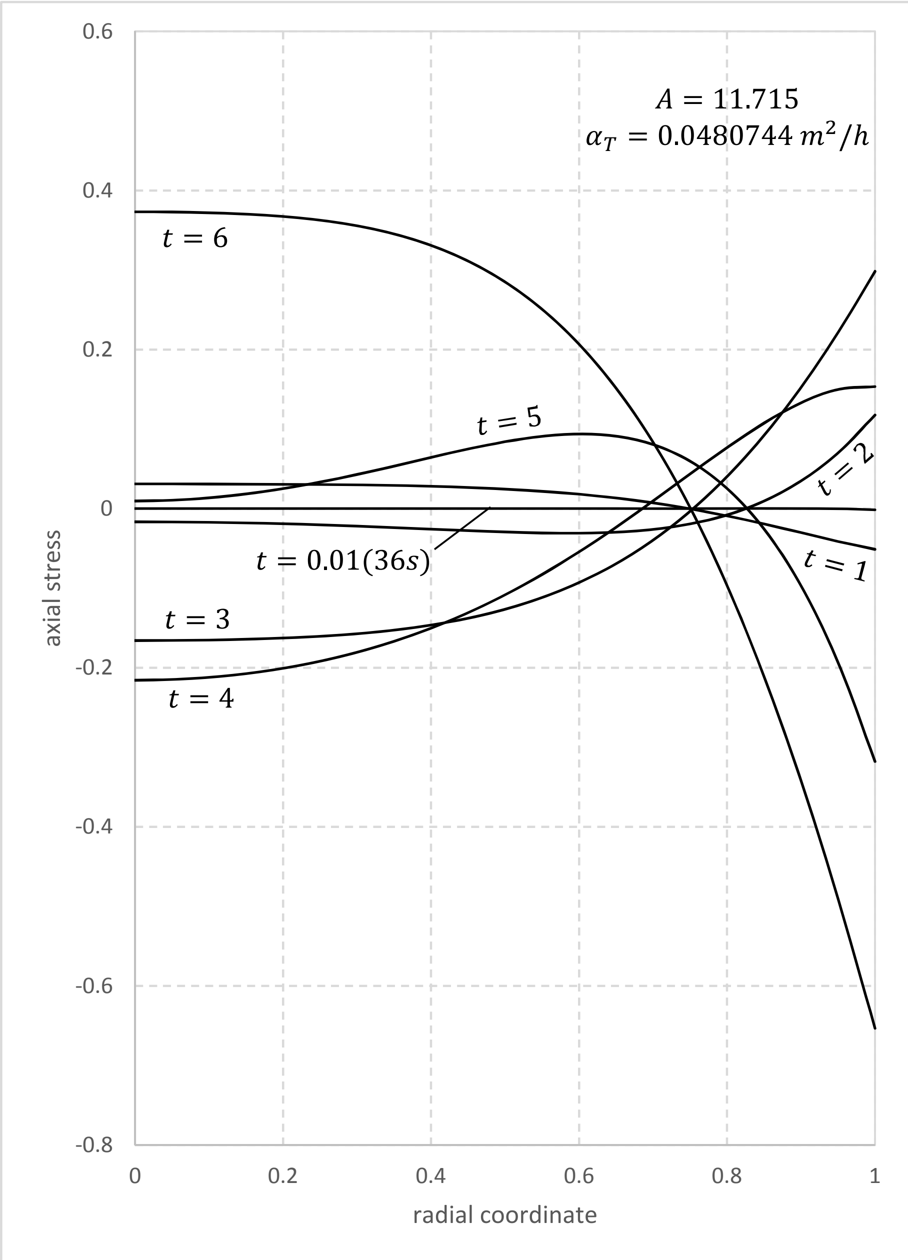

Figure 3.17: Axial stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

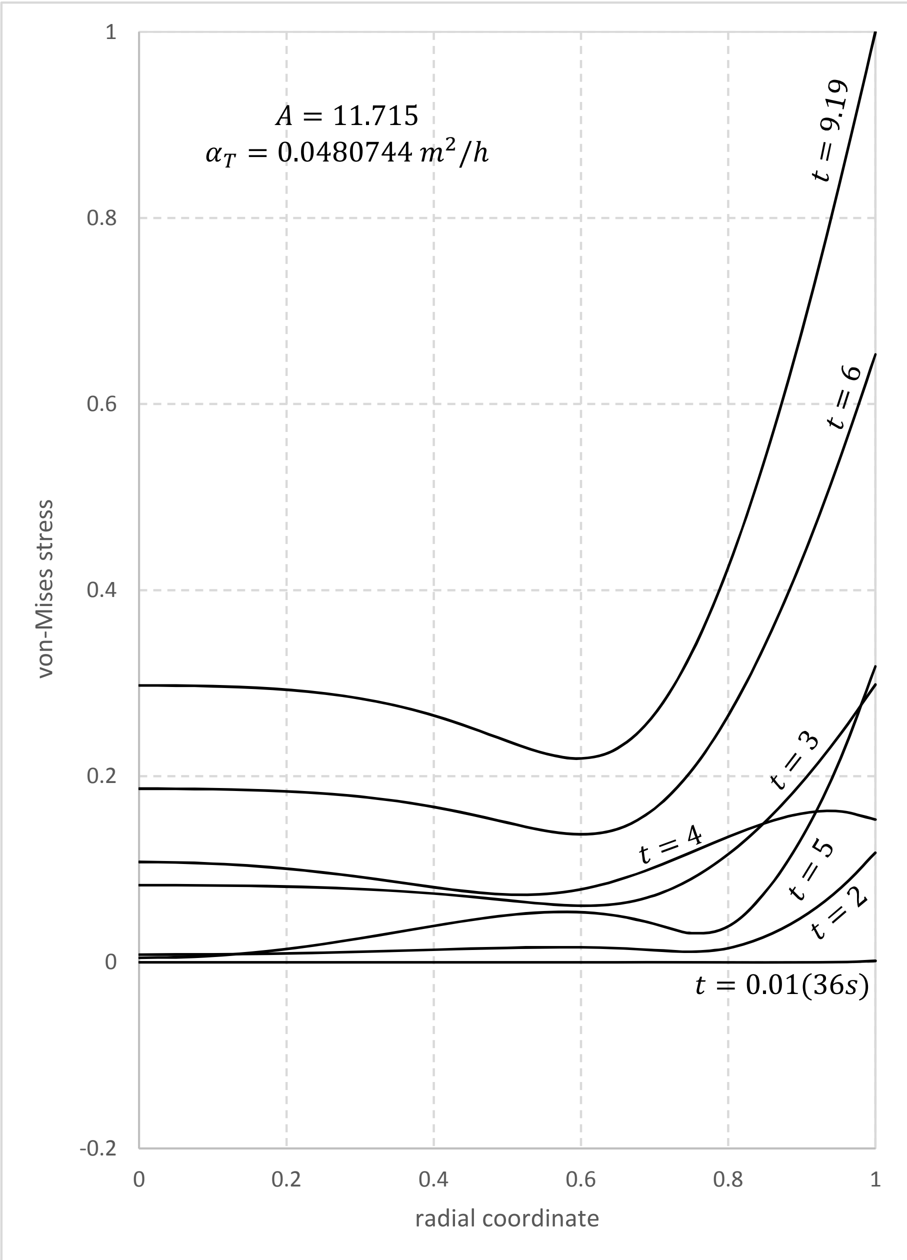

Figure 3.18: Von-Mises stress in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

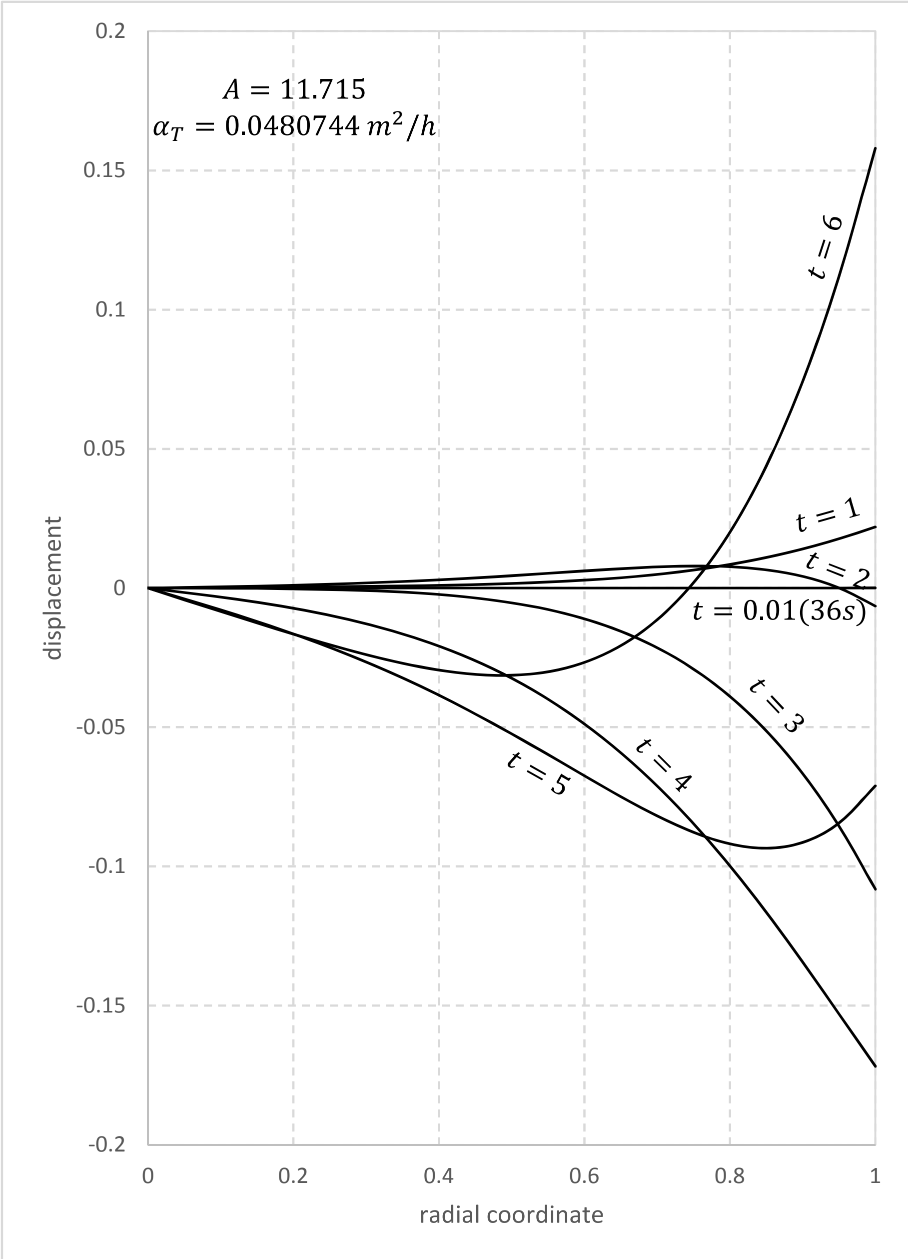

Figure 3.19: Displacement in the solid cylinder made of structural steel at different time steps for the case $F(t) = AtCos(t)$.

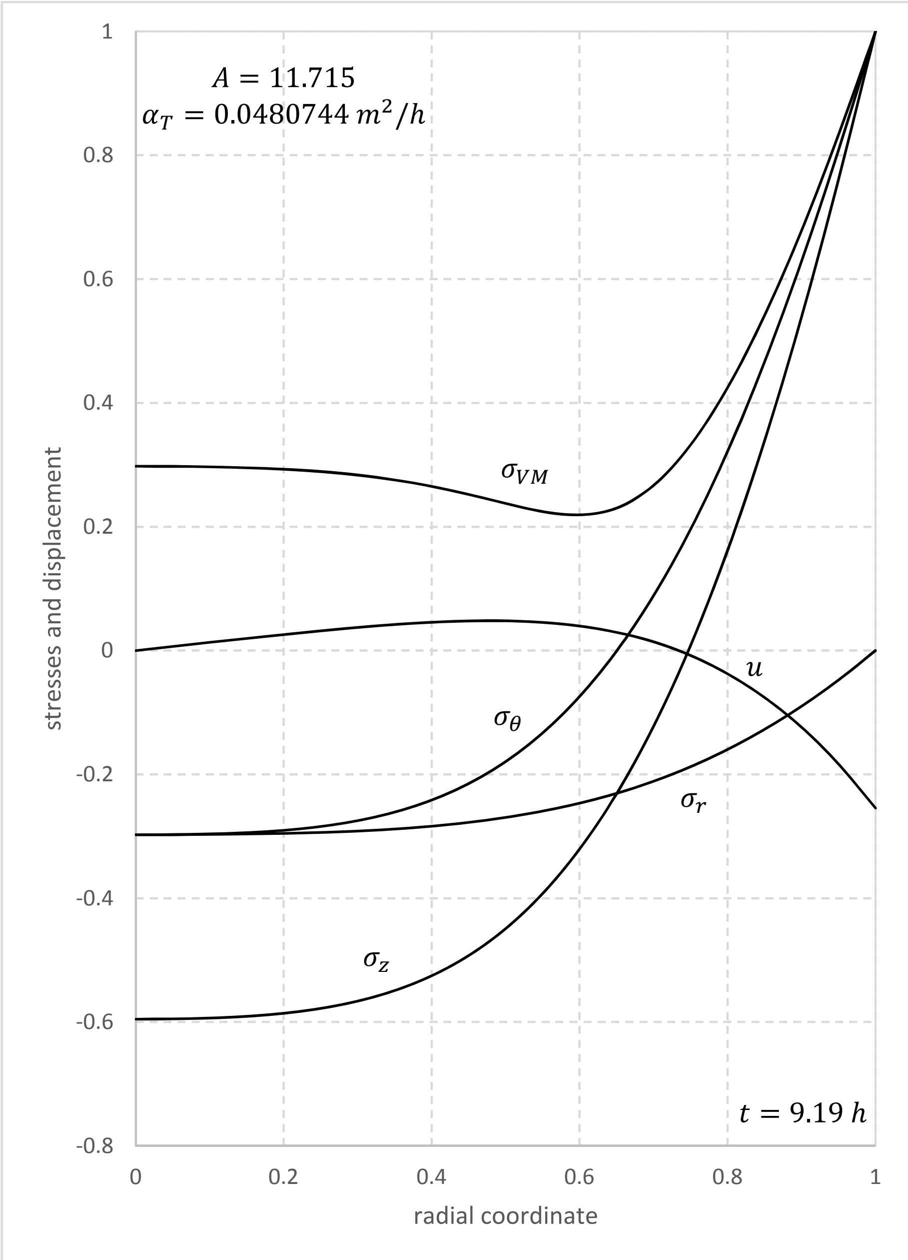

Figure 3.20: Stress and displacement distributions in the solid cylinder made of structural steel at $t = 9.19$ for the case $F(t) = AtCos(t)$.

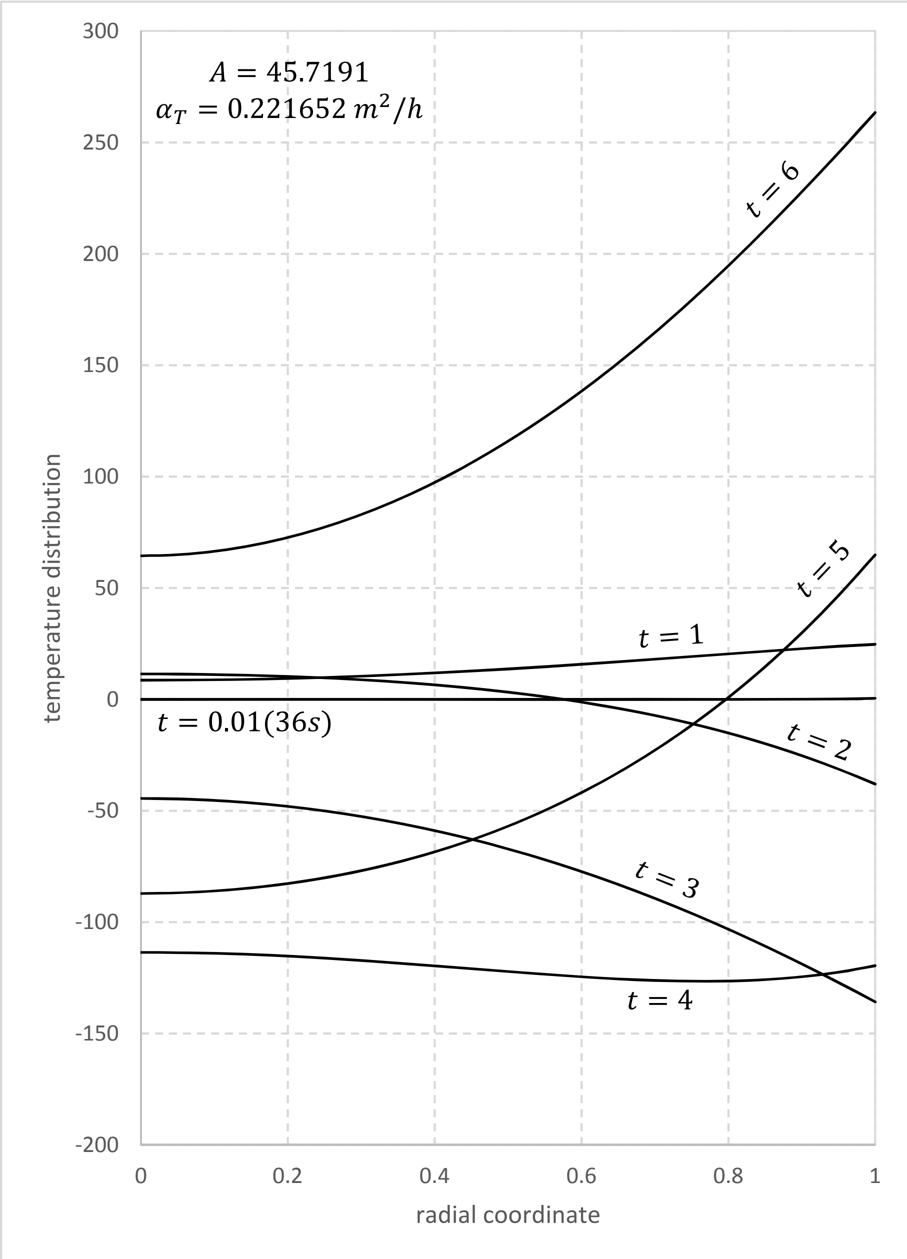

Figure 3.21: Temperature distribution in the solid cylinder made of aluminum at different time steps for the case $F(t) = AtCos(t)$.

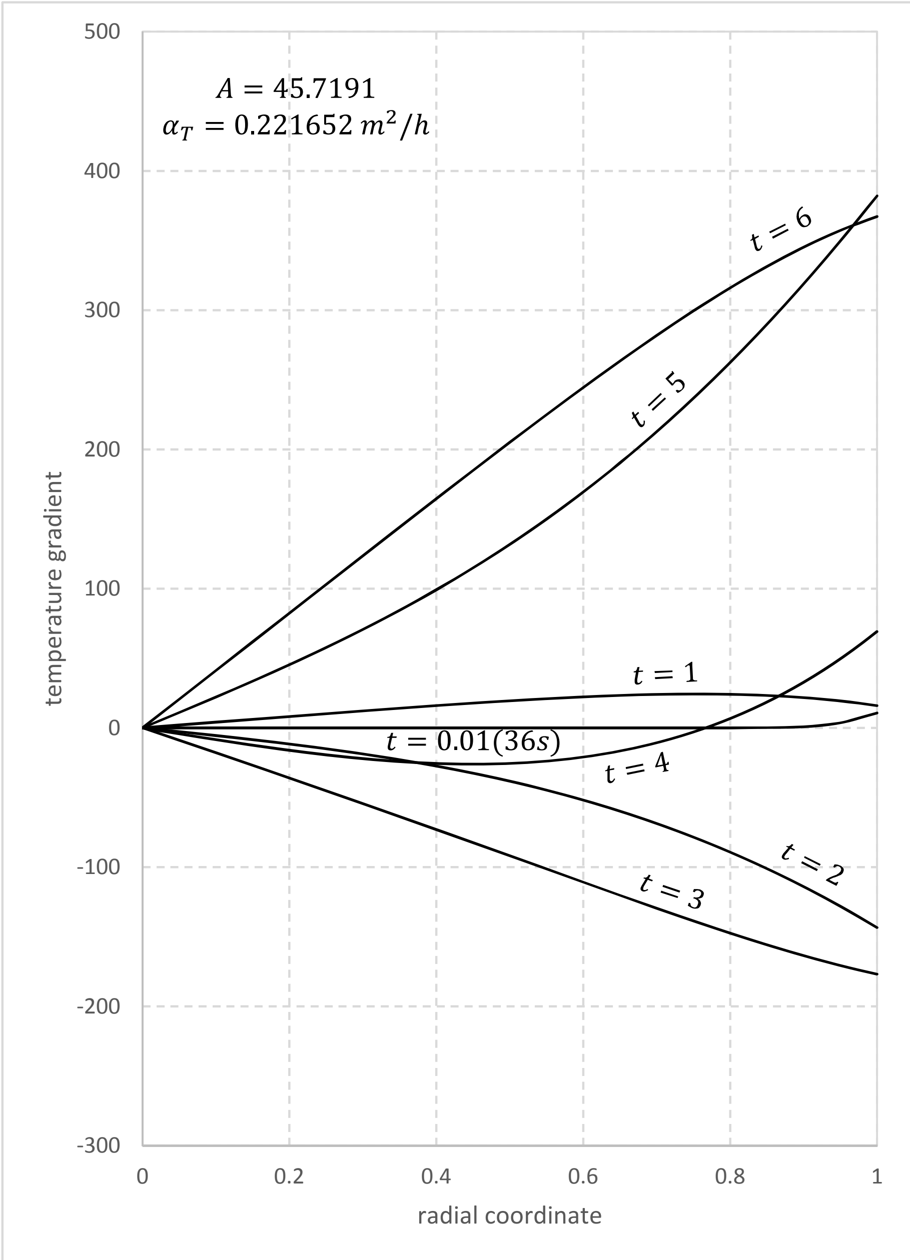

Figure 3.22: Temperature gradient in the solid cylinder made of aluminum at different time steps for the case $F(t) = AtCos(t)$.

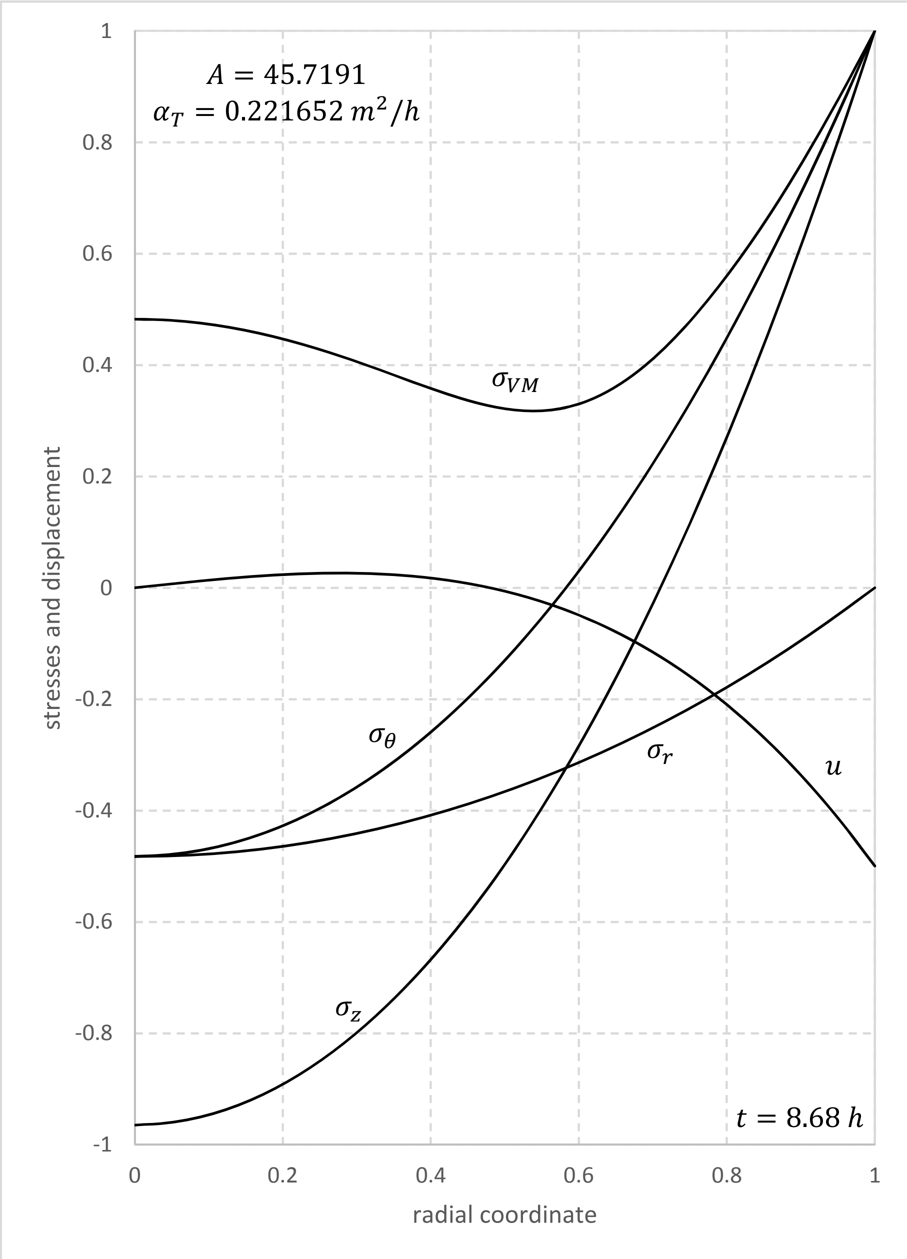

Figure 3.23: Stress and displacement distributions in the solid cylinder made of aluminum at $t = 8.68$ for the case $F(t) = AtCos(t)$.

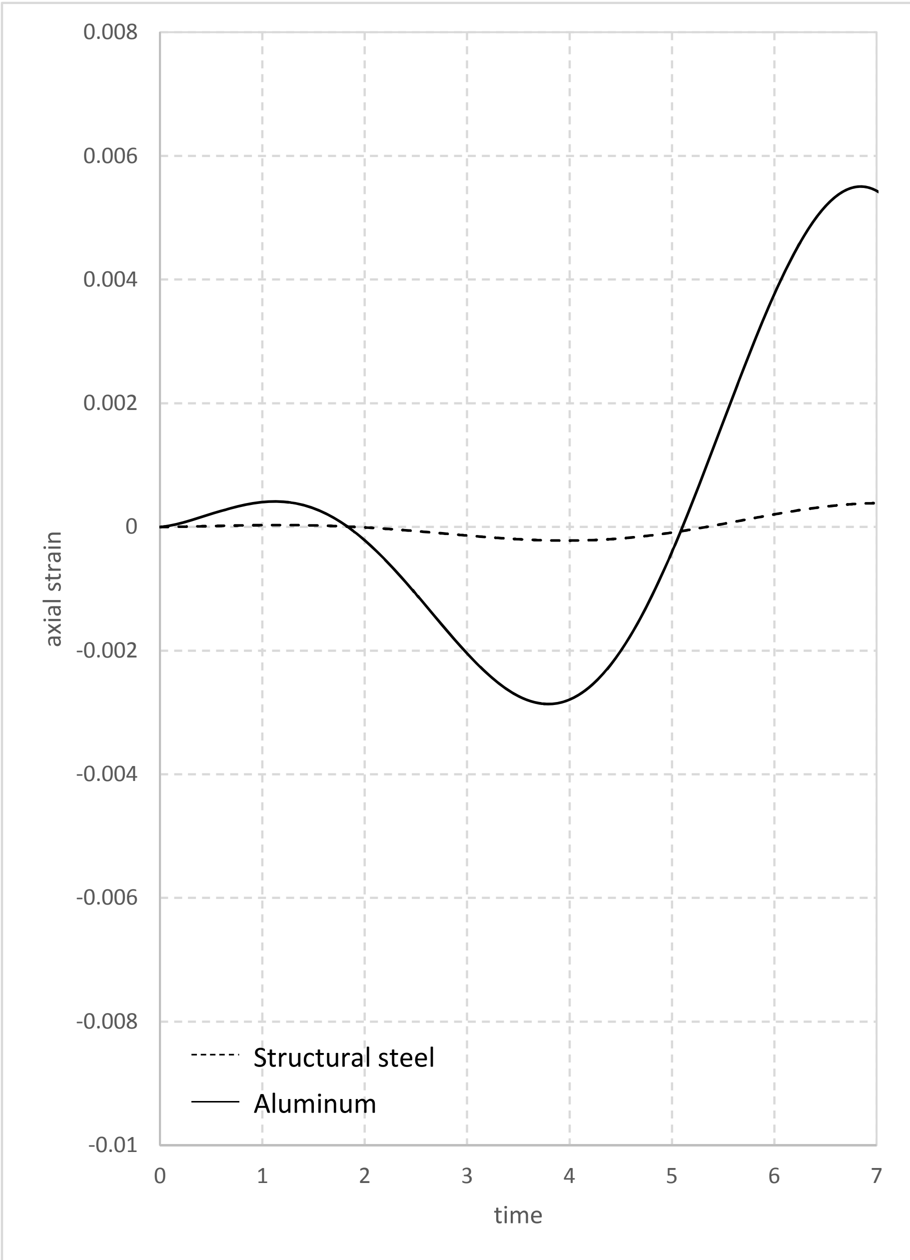

Figure 3.24: Comparison of the axial strains in the solid cylinder made of structural steel and aluminum for the case $F(t) = Atcos(t)$.

CHAPTER 4

CONCLUSIONS

In literature, studies about thermoelastic response of basic structures subjected to periodic boundary conditions exist. Nevertheless, these studies provides specific solution for a specific function of periodic boundary condition by using Fourier series method. This study, on the other hand, enables us to obtain general solution of the problem by the use of Duhamel's theorem. This general solution is applicable to use all kinds of time dependent and periodic functions.

In this study, an elastic solid cylinder problem is examined under time dependent periodic boundary condition. Parametric analyses are performed for two different materials. In addition, two different time dependent functions are used as periodic boundary condition. Firstly, one dimensional unsteady heat conduction equation, whose boundary condition is nonhomogeneous, is solved analytically by the application of Duhamel's theorem and temperature distribution in the cylinder is obtained. Then, by using basic equations of elasticity, radial, circumferential and axial stress components, and the displacement in the radial direction are determined. For verification, temperature and stress distributions are also obtained with Finite Element Model. The results of this solution are compared to those of an analytical solution and it is seen that they are in perfect agreement. Finally, the results of the solutions are presented in figures.

Results demonstrate that the radial stress in the solid cylinder vanishes at $r = b$ and the displacement at the center of the cylinder is equal to zero. It means that boundary conditions are satisfied. It can be seen from the figures that axial strains show the same behavior with the corresponding function which defines the periodic boundary condition. Moreover, maximum difference between stresses and the von-Mises stress occurs at the surface of the solid cylinder, and if the amplitude of the boundary condition functions is increased, cylinder enters the plastic region starting from the surface.

In the future, as the extension of this study, elastic-plastic or plastic analysis can be obtained for the solid cylinder subjected to periodic boundary conditions. Also, the thermoelastic response of tubes and spherical shells can be investigated by using the same method.

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APPENDIX A

MATERIAL PROPERTIES OF SOME METALS

Material	Material Properties							
	Density $\rho(g/cm^3)$	Elastic Modulus $E(GPa)$	Specific Heat $C_p(J/kg.K)$	Tensile Strength $\sigma_t(MPa)$	Yield Strength $\sigma_Y(MPa)$	Thermal Cond. $K(W/m.K)$	Thermal Diff. $\alpha_T(m^2/s)$	Thermal Exp. Coeff., $\alpha(1/K)$
Structural Steel (ASTM 36)	7.8	210	480	480	270	50	13×10^{-6}	11×10^{-6}
Stainless Steel (AISI 442)	7.61	200	460	540	310	21.7	6×10^{-6}	10.2×10^{-6}
Aluminum (1200-H16)	2.69	70	900	150	130	230	95×10^{-6}	23.4×10^{-6}
Aluminum (2014-T6)	2.8	73	870	490	430	150	62×10^{-6}	22.7×10^{-6}
Aluminum (3103-H12)	2.73	71	890	130	90	160	66×10^{-6}	23.1×10^{-6}
85/15 Red Brass (Half-Hard H02)	8.75	120	380	395	340	160	48×10^{-6}	18.7×10^{-6}
Titanium (Grade 1) (R50250)	4.5	105	520	300	220	22.6	10×10^{-6}	8.7×10^{-6}
Zinc-Aluminum 8 (Z35636)	6.28	86	440	240	210	120	43×10^{-6}	23×10^{-6}
Bronze (Grade A) (C51000)	8.86	110	380	325	130	84	25×10^{-6}	17.8×10^{-6}
90/10 Copper-Nickel (Half-Hard H02)	8.94	140	380	470	63	44	13×10^{-6}	17×10^{-6}
Co-Cr-Mo Alloy (UNS R30075)	8.3	210	450	660	450	13	3×10^{-6}	12×10^{-6}

APPENDIX B

FORTRAN CODES

B.1 Program for Root Finding

```
      PROGRAM ROOT FINDING
C     -----------------------
C     PURPOSE OF THE PROGRAM:
C     -----------------------
C       This program calculates the roots of
C     the eigenvalue equation which are obtained
C     for a solid cylinder which is initially at
C     zero temperature, for times>0 the surface
C     temperature of the solid cylinder is determined
C     by the prescribed function of time F(t).
C       First, input parameters are introduced. Then,
C     by using RFALSI subroutine roots of the
C     eigenvalue equation are obtained. Results
C     are printed to the output file, which is
C     titled as 'A-ROOTS.DAT'.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) FUN( LAMBDA ) : Calculates the eigenvalue
C                        function.
```

```
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     A               : Initial guess
C     B               : Final guess
C     DX              : Increment of the radial coordinate
C     NR              : Number of eigenvalues
C     ITMAX           : Maximum iteration number
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     ROOT            : Roots of the eigenvalue equation
C
C
      IMPLICIT NONE
      INTEGER ITMAX, I, N, NJ, NC, NR
      DOUBLE PRECISION A, B, DX, TOL, FUN, P, ADUM, BDUM, ROOT
      PARAMETER ( NR = 2500 )
      DIMENSION ROOT(NR)
      EXTERNAL FUN
C
      OPEN(6,FILE='A-ROOTS.DAT')
      N     = 10000
      A     = 0.0D0
      B     = 8000.0D0
      DX    = (B-A)/N
      NJ    = 0
      NC    = 10
      ITMAX = 50
      DO 10 I = 1, N
        B     = A + DX
```

```
        P      = FUN(A) * FUN(B)
         IF (P .LT. 0.0D0) THEN
            IF ( NJ+1 .GT. NR) GO TO 20
           ADUM     = A
           BDUM     = B
            CALL RFALSI (FUN, ADUM, BDUM, NC, ITMAX)
            NJ       = NJ + 1
            ROOT(NJ) = ADUM
         END IF
         A = B
 10    CONTINUE
 20    CONTINUE
C
       WRITE(6,200) "DATA"
       DO 40 I = 1, NR, 2
       IF (I .EQ. NR) THEN
        WRITE(6,100)  ROOT(I)
       ELSE
         WRITE(6,100) I, ROOT(I), I+1, ROOT(I+1)
        ENDIF
        IF ((MOD((I+1),100).EQ.0).AND.(I+1.LT.NR-100)) THEN
         WRITE(6,200) "DATA"
        ENDIF
 40    CONTINUE
       WRITE (6,200) "END"
       PAUSE
       STOP
 100   FORMAT(5X, '*  R(', I4, ') /', F15.10,'D0 / ,',
      3      '  R(', I4, ') /', F15.10,'D0 /')
 200   FORMAT (8X, A)
       END
C
C
```

```
C     ---------------------------------------
      DOUBLE PRECISION FUNCTION FUN( LAMBDA )
C     ---------------------------------------
     IMPLICIT NONE
     DOUBLE PRECISION LAMBDA, B, DBJ0, DBJ1
      B = 1.0D0
C
      FUN = DBJ0(LAMBDA * B)
C
      RETURN
      END
C
C     =======================================
      SUBROUTINE RFALSI (F, A, B, NC, ITMAX)
C     =======================================
      IMPLICIT DOUBLE PRECISION (A-H , O-Z)
      INTEGER  ITER, ITMAX, NC
      EXTERNAL F
C ----------------------------------------------------------
C   SUBROUTINE RFALSI COMPUTES THE ROOT OF A NONLINEAR
C   EQUATION
C     F(P) = 0
C   USING METHOD OF FALSE POSITIONS (REGULA FALSI).
C
C   PARAMETER LIST :
C   ----------------
C   F     : THE NAME OF THE EXTERNAL FUNCTION THAT DEFINES
C           THE FORM OF THE EQUATION F(P) = 0. THIS
C           FUNCTION SHOULD BE DECLARED EXTERNAL IN
C           THE CALLING PROGRAM.
C   A, B  : END POINTS OF F SUCH THAT F(A) AND F(B)
C           HAVE OPPOSITE SIGNS. IF F(A) AND F(B) HAVE
C           THE SAME SIGN SUBROUTINE RETURNS TO THE CALLER
```

```
C            PRINTING AN ERROR MESSAGE. ON OUTPUT, THE
C            COMPUTED ROOT P IS ASSIGNED TO A.
C   TOL    : ERROR BOUND TO TERMINATE THE REGULA FALSI
C            ITERATIONS. ITERATIONS ARE TERMINATED WHEN
C            ABS (P - P0) .LT. TOL, WHERE P0 IS THE
C            PREVIOUS ITERATION VALUE OF P.
C   ITMAX  : MAXIMUM NUMBER OF ITERATIONS ALLOWED. ON
C            RETURN ITMAX IS SET EQUAL TO THE NUMBER
C            OF ITERATIONS PERFORMED TO HIT THE
C            GIVEN ERROR BOUND.
C
C   AHMET N. ERASLAN
C   3 - 7 - 1993
C -----------------------------------------------------
C... INITIALIZE
      DATA P0 /0.0/
     ES = 0.5D0*10.0**(2 - NC)
      FA = F(A)
      FB = F(B)
      IF ((FA * FB) .GT. 0.0) WRITE(6,200)
      IF ((FA * FB) .GT. 0.0) RETURN
C
C... ITERATION LOOP :
      DO  10  ITER = 1 , ITMAX
      FA = F(A)
      P  = A - FA * (B - A) / (F(B) - FA)
     EA = DABS((P - P0) / P) * 100.0
      IF ( EA .LT. ES )  GOTO 20
      FP = F(P)
      IF ((FA * FP) .LT. 0.0)  A = P
      IF ((FA * FP) .GT. 0.0)  B = P
 10   P0 = P
C
```

```
C... ITERATIONS CONVERGED; SET NUMBER OF ITERATIONS
C    AND RETURN
 20   ITMAX = ITER
      A     = P
      RETURN
 200  FORMAT(///10X,'FROM SUBROUTINE RFALSI :'/
     1          10X,'F(A) AND F(B) HAVE THE SAME SIGN,'
     2          ,' METHOD IS NOT APPLICABLE...')
      END
```

B.2 Program for Temperature Distribution

B.2.1 For the case $F(t) = Asin(t)$

```
      PROGRAM TEMPERATURE DISTRIBUTION
C     -----------------------
C     PURPOSE OF THE PROGRAM:
C     -----------------------
C
C       This program calculates the temperature distribution
C     of a solid cylinder which is initially at zero
C     temperature, for times>0 the surface temperature of
C     the solid cylinder is determined by the prescribed
C     function of time F(t). For this program, function
C     is selected as
C                          F(t) = A sin(t).
C       First, input parameters are introduced. Then, the
C     summation is performed. Finally, the temperature
C     distribution is obtained. Results are printed to the
C     output file, which is titled as 'A-OUT.DAT'.
C
C
```

```
C     ----------------
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which
C                       calculate BesselJ0, BesselJ1,
C                       BesselY0 and BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the eigenvalue
C                       equation.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
C                           Duhamel's theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND          : Number of correct digits
C     ALPHA       : Thermal diffusivity
C     A           : Non-zero constant
C     B           : Boundary surface
C     DR          : Increment of the radial coordinate
C     NPTS        : Number of eigenvalues
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     T           : Temperature distribution
C
C
      IMPLICIT NONE
      INTEGER ND, NPTS, I, J
```

```
      PARAMETER ( NPTS = 2500 )
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, TIME,
     1               R, DHM, A, B, T, DBJ0, DBJ1,
 1               L ,LI, ALPHA, DR
      DIMENSION L(NPTS)
      COMMON /EIG/L
      EXTERNAL DHM
      OPEN(6,FILE='A-OUT.DAT')
C
      ND    = 8
      ALPHA = 0.0480744D0
      A     = 97.2218D0
      B     = 1.0D0
      DR    = B / 20.0D0
C
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      DO 40 TIME = 0.0D0, 10.0D0, 0.01D0
      WRITE(6,*)
      WRITE(6,*) '  TIME = ',TIME
      WRITE(6,*)
      R = 0.0D0
      DO 30 J = 1, 21
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ0(LI*R)/(LI*DBJ1(LI*B))
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, TIME)
       EA  = (SUM - SOLD)/SUM*100.0
       T   = A*DSIN(TIME) - SUM
       SOLD = SUM
```

```
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      WRITE(6,100) R , T, I
      R    = R + DR
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(5X,F5.2,5X,F12.8,5X,I5)
      END
C
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
      IMPLICIT NONE
      DOUBLE PRECISION ALF, L, T, ARG, T1
C
      ARG = -ALF*L**2*T
      IF (DABS(ARG) .GT. 250.0D0) THEN
       T1 = 0.0D0
      ELSE
       T1 = DEXP(ARG)
      ENDIF
C
      DHM = (-T1*ALF*L**2 + ALF*L**2*DCOS(T) + DSIN(T))
     2       / (1.0D0 +  (ALF**2*L**4))
C
      RETURN
      END
C
```

B.2.2 For the case $F(t) = Atcost(t)$

```
      PROGRAM TEMPERATURE DISTRIBUTION
C     -----------------------
C     PURPOSE OF THE PROGRAM:
C     -----------------------
C
C       This program calculates the temperature distribution
C     of a solid cylinder which is initially at zero
C     temperature, for times>0 the surface temperature of
C     the solid cylinder is determined by the prescribed
C     function of time F(t). For this program, function
C     is selected as
C                          F(t) = A t cos(t).
C       First, input parameters are introduced. Then, the
C     summation is performed. Finally, the temperature
C     distribution is obtained. Results are printed to the
C     output file, which is titled as 'A-OUT.DAT'.
C
C
C     ----------------
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which
C                       calculate BesselJ0, BesselJ1,
C                       BesselY0 and BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the eigenvalue
C                       equation.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
```

```
C                            Duhamel’s theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND         : Number of correct digits
C     ALPHA      : Thermal diffusivity
C     A          : Non-zero constant
C     B          : Boundary surface
C     DR         : Increment of the radial coordinate
C     NPTS       : Number of eigenvalues
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     T          : Temperature distribution
C
C
      IMPLICIT NONE
      INTEGER ND, NPTS, I, J
      PARAMETER ( NPTS = 2500 )
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, TIME, R,
     1                 DHM, A, B, T, DBJ0, DBJ1, L,
     1                 LI, ALPHA, DR
      DIMENSION L(NPTS)
      COMMON /EIG/L
      EXTERNAL DHM
      OPEN(6,FILE='A-OUT.DAT')
C
      ND    = 8
      ALPHA = 0.0480744D0
      A     = 11.715D0
      B     = 1.0D0
```

```
      DR     = B / 20.0D0
C
      ES     = 0.5D0*10.0D0**(2 - ND)
C
      DO 40 TIME = 0.0D0, 10.0D0, 0.01D0
      WRITE(6,*)
      WRITE(6,*) '   TIME = ',TIME
      WRITE(6,*)
      R = 0.0D0
      DO 30 J = 1, 21
      SUM    = 0.0D0
      SOLD   = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ0(LI*R)/(LI*DBJ1(LI*B))
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, TIME)
       EA   = (SUM - SOLD)/SUM*100.0
       T    = (A*TIME*DCOS(TIME)) - SUM
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      WRITE(6,100) R , T, I
      R    = R + DR
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(5X,F5.2,5X,F12.8,5X,I5)
      END
C
C
```

```
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
      IMPLICIT NONE
      DOUBLE PRECISION ALF, L, T, ARG, T1, T2, T3
C
      ARG = -ALF*L**2*T
      IF (DABS(ARG) .GT. 250.0D0) THEN
       T1 = 0.0D0
      ELSE
       T1 = DEXP(ARG)
      ENDIF
C
      T2 =  T1*(ALF*L**2 - ALF**3*L**6) + DCOS(T)*
     2       (T - ALF*L**2 + T*ALF**2*L**4
 2       + ALF**3*L**6)
      T3 =  DSIN(T)*(-T*ALF*L**2 + 2*ALF**2*L**4
     2       - T*ALF**3*L**6)
C
      DHM =  (T2 + T3)/(1.0D0 +  (ALF**2*L**4))**2
C
      RETURN
      END
C
```

B.3 Program for Temperature Gradient

B.3.1 For the case $F(t) = Asin(t)$

```
      PROGRAM TEMPERATURE GRADIENT
C     -----------------------
C     PURPOSE OF THE PROGRAM:
```

```
C     -----------------------
C       This program calculates the temperature gradient
C     of a solid cylinder which is initially at zero
C     temperature, for times>0 the surface temperature
C     of the solid cylinder is determined by the prescribed
C     function of time F(t). For this program, function
C     is selected as
C                          F(t) = A sin(t).
C       First, input parameters are introduced. Then, the
C     summation is performed. Finally, the temperature
C     gradient is obtained. Results are printed to
C     the output file, which is titled as 'A-OUT.DAT'.
C
C     ----------------
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which calculate
C                       BesselJ0, BesselJ1, BesselY0 and
C                       BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the eigenvalue
C                       equation.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
C                           Duhamel's theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND         : Number of correct digits
C     ALPHA      : Thermal diffusivity
```

```
C     A          : Non-zero constant
C     B          : Boundary surface
C     DR         : Increment of the radial coordinate
C     NPTS       : Number of eigenvalues
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     DT         : Temperature gradient
C
C
      IMPLICIT NONE
      INTEGER ND, NPTS, I, J
      PARAMETER ( NPTS = 2500 )
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, TIME, R,
     1                 DHM, A, B, DT, DBJ1, L ,LI,
     1                 ALPHA, DR
      DIMENSION L(NPTS)
      COMMON /EIG/L
      EXTERNAL DHM
      OPEN(6,FILE='A-OUT.DAT')
C
      ND    = 8
      ALPHA = 0.0480744D0
      A     = 97.2218D0
      B     = 1.0D0
      DR    = B / 20.0D0
C
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      DO 40 TIME = 0.0D0, 10.0D0, 0.01D0
      WRITE(6,*)
      WRITE(6,*) '  TIME = ',TIME
```

```
      WRITE(6,*)
      R = 0.0D0
      DO 30 J = 1, 21
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ1(LI*R)/DBJ1(LI*B)
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, TIME)
       EA  = (SUM - SOLD)/SUM*100.0
       DT   =  SUM
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      WRITE(6,100) R , DT, I
      R    = R + DR
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(5X,F5.2,5X,F15.8,5X,I5)
      END
C
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
      IMPLICIT NONE
      DOUBLE PRECISION ALF, L, T, ARG, T1
C
      ARG = -ALF*L**2*T
      IF (DABS(ARG) .GT. 250.0D0) THEN
```

```
        T1 = 0.0D0
       ELSE
        T1 = DEXP(ARG)
       ENDIF
C
       DHM = (-T1*ALF*L**2 + ALF*L**2*DCOS(T) + DSIN(T))
     2       / (1.0D0 +  (ALF**2*L**4))
C
       RETURN
       END
C
```

B.3.2 For the case $F(t) = Atcost(t)$

```
       PROGRAM TEMPERATURE GRADIENT
C      -----------------------
C      PURPOSE OF THE PROGRAM:
C      -----------------------
C
C        This program calculates the temperature
C      gradient of a solid cylinder which is initially
C      at zero temperature, for times>0 the surface
C      temperature of the solid cylinder is determined
C      by the prescribed function of time F(t). For
C      this program, function is selected as
C                             F(t) = A t cos(t).
C        First, input parameters are introduced. Then,
C      the summation is performed. Finally, the
C      temperature gradient is obtained. Results are
C      printed to the output file, which is titled
C      as 'A-OUT.DAT'.
C
C
```

```
C     ----------------
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which
C                       calculate BesselJ0, BesselJ1,
C                       BesselY0 and BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the eigenvalue
C                       equation.
C
C     ------------------
C     CONNECTED FUNCTION:
C     ------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
C                           Duhamel’s theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND              : Number of correct digits
C     ALPHA           : Thermal diffusivity
C     A               : Non-zero constant
C     B               : Boundary surface
C     DR              : Increment of the radial coordinate
C     NPTS            : Number of eigenvalues
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     DT              : Temperature gradient
C
C
      IMPLICIT NONE
      INTEGER ND, NPTS, I, J
```

```
      PARAMETER ( NPTS = 2500 )
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, TIME, R,
     1                 DHM, A, B, DT, DBJ1, L ,LI,
     1                 ALPHA, DR
      DIMENSION L(NPTS)
      COMMON /EIG/L
      EXTERNAL DHM
      OPEN(6,FILE='A-OUT.DAT')
C
      ND    = 8
      ALPHA = 0.0480744D0
      A     = 11.715D0
      B     = 1.0D0
      DR    = B / 20.0D0
C
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      DO 40 TIME = 0.0D0, 10.0D0, 0.01D0
      WRITE(6,*)
      WRITE(6,*) '  TIME = ',TIME
      WRITE(6,*)
      R = 0.0D0
      DO 30 J = 1, 21
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ1(LI*R)/DBJ1(LI*B)
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, TIME)
       EA  = (SUM - SOLD)/SUM*100.0
       DT   =  SUM
       SOLD = SUM
```

```
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      WRITE(6,100) R , DT, I
      R    = R + DR
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(5X,F5.2,5X,F15.8,5X,I5)
      END
C
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
     IMPLICIT NONE
     DOUBLE PRECISION ALF, L, T, ARG, T1, T2, T3
C
     ARG = -ALF*L**2*T
      IF (DABS(ARG) .GT. 250.0D0) THEN
       T1 = 0.0D0
      ELSE
       T1 = DEXP(ARG)
      ENDIF
C
      T2 =  T1*(ALF*L**2-ALF**3*L**6) + DCOS(T)*(T-ALF*L**2
     2      + T*ALF**2*L**4 + ALF**3*L**6)
      T3 =  DSIN(T)*(-T*ALF*L**2+2*ALF**2*L**4-T*ALF**3*L**6)
C
      DHM =  (T2 + T3)/(1.0D0 + (ALF**2*L**4))**2
C
      RETURN
      END
```

```
C
```

B.4 Program for Elastic Solution

B.4.1 For the case $F(t) = Asin(t)$

```
      PROGRAM ELASTIC SOLUTION
C     -----------------------
C     PURPOSE OF THE PROGRAM:
C     -----------------------
C       This program calculates the thermoelastic
C     response of a solid cylinder subjected to
C     periodic boundary condition. For this program,
C     periodic boundary condition which is a function
C     of time is selected as
C                     F(t) = A sin(t).
C       First, input parameters are introduced. Then,
C     non-zero integration constant A1 (i.e. A*) and
C     axial strain EPS0 are calculated. By using
C     subroutine 'ELAST', stress components and
C     displacement are obtained. Finally, results
C     are printed to the output file, which is titled
C     as 'A-ELAST.DAT'.
C
C     ----------------
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which
C                       calculate BesselJ0, BesselJ1,
C                       BesselY0 and BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the
C                       eigenvalue equation.
```

```
C
C     ---------------------
C     CONNECTED SUBROUTINE:
C     ---------------------
C     1) ELAST(R, T, A1, EPS0, SIGR, SIGT, SIGZ, U):
C        Calculates the stress components and displacement.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
C                           Duhamel’s theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND              : Number of correct digits
C     ALPHA           : Thermal expansion coefficient
C     ALPHAT          : Thermal diffusivity
C     A               : Non-zero constant
C     B               : Boundary surface
C     NU              : Poisson’s ratio
C     E               : Modulus of elasticity
C     SIGY            : Yield stress
C
C     ------------------
C     OUTPUT PARAMETER :
C     ------------------
C     SIGR            : Radial stress
C     SIGT            : Circumferential stress
C     SIGZ            : Axial stress
C     U               : Radial displacement
C     EPS0            : Axial strain
```

```
C
C
      IMPLICIT NONE
      INTEGER ND
      DOUBLE PRECISION ALPHA, ALPHAT, B, SIGR, SIGT,
     1                 SIGZ, SIGY, VM, NU, E, R, T,
     1                 T1, INTG, A1, EPS0, U, AD, A,
     1                 DHM
      COMMON /CONVER/ ND
      COMMON /PROPS/ ALPHA, ALPHAT, B, E, NU, SIGY, A
      OPEN(6, FILE='A-ELAST.DAT')
C
      ND     = 8
      ALPHA  = 11.72D-06
      ALPHAT = 0.0480744D0
      A      = 97.2218D0
      B      = 1.0D0
      NU     = 0.3D0
      E      = 210.0D09
      SIGY   = 270.0D06
C
      WRITE(6,*) ' ND     = ', ND
      WRITE(6,*) ' ALPHA  =  ', ALPHA
      WRITE(6,*) ' ALPHAT = ', ALPHAT
      WRITE(6,*) ' A      = ', A
      WRITE(6,*) ' B      = ', B
      WRITE(6,*) ' NU     =  ', NU
      WRITE(6,*) ' E      = ', E
      WRITE(6,*) ' SIGY   = ', SIGY
C
      DO 40 T = 0.00D0, 10.0D0, 0.01D0
      T1    = INTG (B, T)
      A1    = ALPHA*(1.0D0 - 3.0*NU)/(B**2*(1.0D0
```

```
     2         - NU)) * T1
      EPS0  = 2.0*ALPHA/B**2*T1
      WRITE(6,*)
      WRITE(6,*) ' TIME = ',T
      WRITE(6,*)
      WRITE(6,*) ' EPS0 = ',EPS0
      WRITE(6,*)
      WRITE(6,*) '       R          SIGR          SIGT
     2        SIGZ         SIGVM         U'
      WRITE(6,*)
C
      DO 30 R = 0.0D0, 1.0D0, 0.05D0
      CALL ELAST (R, T, A1, EPS0, SIGR, SIGT, SIGZ, U )
      VM    = SQRT(0.5*((SIGR-SIGT)**2+(SIGR-SIGZ)**2
3         +(SIGT-SIGZ)**2))
      WRITE(6,100) R , SIGR, SIGT, SIGZ, VM, U
C
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(10(2X,F10.6))
      END
C
C     -------------------------------------------------------
      SUBROUTINE ELAST(R, T, A1, EPS0, SIGR, SIGT, SIGZ, U)
C     -------------------------------------------------------
      IMPLICIT NONE
      DOUBLE PRECISION ALPHA, A, B, E, NU, SIGY, R, T, SIGR,
     1                 SIGT, SIGZ, U, EPS0, A1, TEMP, INTG,
     1                 AD, T1, T2, T3, T4, T5
C
      COMMON /PROPS/ ALPHA, AD, B, E, NU, SIGY, A
```

```
C
      T1  = 1.0D0 - NU
      T2  = 1.0D0 + NU
      T3  = 1.0D0 - 2.0*NU
C
      IF (R .EQ. 0.0D0) THEN
       T4   = TEMP (R, T)
       SIGR = E*(EPS0*NU + A1)/(T3*T2) - E*ALPHA*T4
     1        /(2.0*T1)
       SIGT = SIGR
       SIGZ = E*EPS0 - ALPHA*E*T4 + NU*(SIGR + SIGT)
       U = 0.0D0
      ELSE
       T4   = TEMP (R, T)
       T5   = INTG (R, T)
       SIGR = E*(EPS0*NU + A1)/(T3*T2) - E*ALPHA*T5
     1        /(R**2*T1)
       SIGT = E*(EPS0*NU + A1)/(T3*T2) + E*ALPHA*T5
     1        /(R**2*T1) - E*ALPHA*T4/T1
       SIGZ = E*EPS0 - ALPHA*E*T4 + NU*(SIGR + SIGT)
       U    = A1*R + ALPHA*T2/(T1*R)*T5
      ENDIF
       SIGR = SIGR / SIGY
       SIGT = SIGT / SIGY
       SIGZ = SIGZ / SIGY
       U    = U*E / (SIGY*B)
      RETURN
      END
C
C     -------------------------------------
      DOUBLE PRECISION FUNCTION TEMP (R, T)
C     -------------------------------------
      IMPLICIT NONE
```

```
      INTEGER ND, NPTS, I
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, T2, T3,
     1                 T, R, E, NU, A, B, DBJ0, DBJ1,
     1                 L, ARG ,LI, ALPHA, SIGY, AD, DHM
      DIMENSION L(2500)
      COMMON /EIG/L
      COMMON /CONVER/ ND
      COMMON /PROPS/ AD, ALPHA, B, E, NU, SIGY, A
C
      NPTS  = 2500
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ0(LI*R)/(LI*DBJ1(LI*B))
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, T)
       EA  = (SUM - SOLD)/SUM*100.0
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      TEMP = A*DSIN(T) - SUM
      RETURN
      END
C
C     -------------------------------------
      DOUBLE PRECISION FUNCTION INTG (R, T)
C     -------------------------------------
      IMPLICIT NONE
      INTEGER ND, NPTS, I
```

```
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, T2, T3,
     1                 T, R, E, NU, B, DBJ1, L, ARG,
     1                 LI, ALPHA, SIGY, AD, A, DHM
      DIMENSION L(2500)
      COMMON /EIG/L
      COMMON /CONVER/ ND
      COMMON /PROPS/ AD, ALPHA, B, E, NU, SIGY, A
C
      NPTS  = 2500
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1  = R*DBJ1(LI*R)/(LI**2*DBJ1(LI*B))
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, T)
       EA   = (SUM - SOLD)/SUM*100.0
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      INTG = A*R**2*DSIN(T)/2 - SUM
      RETURN
      END
C
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
      IMPLICIT NONE
      DOUBLE PRECISION ALF, L, T, ARG, T1
C
      ARG = -ALF*L**2*T
```

```
      IF (DABS(ARG) .GT. 250.0D0) THEN
       T1 = 0.0D0
       ELSE
       T1 = DEXP(ARG)
      ENDIF
C
      DHM = (-T1*ALF*L**2 + ALF*L**2*DCOS(T) + DSIN(T))
     2       / (1.0D0 +  (ALF**2*L**4))
C
      RETURN
      END
```

B.4.2 For the case $F(t) = Atcost(t)$

```
      PROGRAM ELASTIC SOLUTION
C     -----------------------
C     PURPOSE OF THE PROGRAM:
C     -----------------------
C       This program calculates the thermoelastic
C     response of a solid cylinder subjected to
C     periodic boundary condition. For this program,
C     periodic boundary condition which is a function
C     of time is selected as
C                     F(t) = A t cos(t).
C       First, input parameters are introduced. Then,
C     non-zero integration constant A1 (i.e. A*) and
C     axial strain EPS0 are calculated. By using
C     subroutine 'ELAST', stress components and
C     displacement are obtained. Finally, results
C     are printed to the output file, which is titled
C     as 'A-ELAST.DAT'.
C
C     ----------------
```

```
C     CONNECTED CODES:
C     ----------------
C     1) BSSLY0Y1J0J1 : Contains functions which
C                       calculate BesselJ0, BesselJ1,
C                       BesselY0 and BesselY1 functions.
C     2) BLOCK DATA   : Contains roots of the eigenvalue
C                       equation.
C
C     ---------------------
C     CONNECTED SUBROUTINE:
C     ---------------------
C     1) ELAST(R, T, A1, EPS0, SIGR, SIGT, SIGZ, U):
C     Calculates the stress components and displacement.
C
C     -------------------
C     CONNECTED FUNCTION:
C     -------------------
C     1) DHM( ALF, L, T ) : Calculates the integral of
C                           Duhamel's theorem.
C
C     ------------------
C     INPUT PARAMETERS :
C     ------------------
C     ND              : Number of correct digits
C     ALPHA           : Thermal expansion coefficient
C     ALPHAT          : Thermal diffusivity
C     A               : Non-zero constant
C     B               : Boundary surface
C     NU              : Poisson's ratio
C     E               : Modulus of elasticity
C     SIGY            : Yield stress
C
C     ------------------
```

```
C     OUTPUT PARAMETER :
C     ------------------
C     SIGR           : Radial stress
C     SIGT           : Circumferential stress
C     SIGZ           : Axial stress
C     U              : Radial displacement
C     EPS0           : Axial strain
C
C
      IMPLICIT NONE
      INTEGER ND
      DOUBLE PRECISION ALPHA, ALPHAT, B, SIGR, SIGT, SIGZ,
     1                 SIGY, VM, NU, E, R, T, T1, INTG, A1,
     1                 EPS0, U, AD, A, DHM
      COMMON /CONVER/ ND
      COMMON /PROPS/ ALPHA, ALPHAT, B, E, NU, SIGY, A
      OPEN(6, FILE='A-ELAST.DAT')
C
      ND     = 8
      ALPHA  = 11.72D-06
      ALPHAT = 0.0480744D0
      A      = 11.7150D0
      B      = 1.0D0
      NU     = 0.3D0
      E      = 210.0D09
      SIGY   = 270.0D06
C
      WRITE(6,*) ' ND     = ', ND
      WRITE(6,*) ' ALPHA  =  ', ALPHA
      WRITE(6,*) ' ALPHAT = ', ALPHAT
      WRITE(6,*) ' A      = ', A
      WRITE(6,*) ' B      = ', B
      WRITE(6,*) ' NU     =  ', NU
```

```
      WRITE(6,*) ' E       = ', E
      WRITE(6,*) ' SIGY    = ', SIGY
C
      DO 40 T = 0.00D0, 10.0D0, 0.01D0
      T1     = INTG (B, T)
      A1     = ALPHA*(1.0D0-3.0*NU)/(B**2*(1.0D0-NU))*T1
      EPS0   = 2.0*ALPHA/B**2*T1
      WRITE(6,*)
      WRITE(6,*) ' TIME = ',T
      WRITE(6,*)
      WRITE(6,*) ' EPS0 = ',EPS0
      WRITE(6,*)
      WRITE(6,*) '        R          SIGR          SIGT
     2       SIGZ          SIGVM         U'
      WRITE(6,*)
C
      DO 30 R = 0.0D0, 1.0D0, 0.05D0
      CALL ELAST (R, T, A1, EPS0, SIGR, SIGT, SIGZ, U )
      VM     = SQRT(0.5*((SIGR-SIGT)**2+(SIGR-SIGZ)**2
     2        +(SIGT-SIGZ)**2))
      WRITE(6,100) R , SIGR, SIGT, SIGZ, VM, U
C
   30 CONTINUE
   40 CONTINUE
      PAUSE
      STOP
  100 FORMAT(10(2X,F10.6))
      END
C
C     -----------------------------------------------------
      SUBROUTINE ELAST(R, T, A1, EPS0, SIGR, SIGT, SIGZ, U)
C     -----------------------------------------------------
      IMPLICIT NONE
```

```
      DOUBLE PRECISION ALPHA, A, B, E, NU, SIGY, R, T,
     1                 SIGR, SIGT, SIGZ, U, EPS0, A1,
     1                 TEMP, INTG, AD, T1, T2, T3, T4, T5
C
      COMMON /PROPS/ ALPHA, AD, B, E, NU, SIGY, A
C
      T1  = 1.0D0 - NU
      T2  = 1.0D0 + NU
      T3  = 1.0D0 - 2.0*NU
C
      IF (R .EQ. 0.0D0) THEN
       T4   = TEMP (R, T)
       SIGR = E*(EPS0*NU + A1)/(T3*T2) - E*ALPHA*T4/(2.0*T1)
       SIGT = SIGR
       SIGZ = E*EPS0 - ALPHA*E*T4 + NU*(SIGR + SIGT)
       U = 0.0D0
      ELSE
       T4   = TEMP (R, T)
       T5   = INTG (R, T)
       SIGR = E*(EPS0*NU + A1)/(T3*T2) - E*ALPHA*T5/(R**2*T1)
       SIGT = E*(EPS0*NU + A1)/(T3*T2) + E*ALPHA*T5/(R**2*T1)
     1        - E*ALPHA*T4/T1
       SIGZ = E*EPS0 - ALPHA*E*T4 + NU*(SIGR + SIGT)
       U    = A1*R + ALPHA*T2/(T1*R)*T5
      ENDIF
       SIGR = SIGR / SIGY
       SIGT = SIGT / SIGY
       SIGZ = SIGZ / SIGY
       U    = U*E / (SIGY*B)
      RETURN
      END
C
C     -------------------------------------
```

```
      DOUBLE PRECISION FUNCTION TEMP (R, T)
C     -------------------------------------
      IMPLICIT NONE
      INTEGER ND, NPTS, I
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, T2, T3,
     1                 T, R, E, NU, A, B, DBJ0, DBJ1,
     1                 L, ARG, LI, ALPHA, SIGY, AD, DHM
      DIMENSION L(2500)
      COMMON /EIG/L
      COMMON /CONVER/ ND
      COMMON /PROPS/ AD, ALPHA, B, E, NU, SIGY, A
C
      NPTS  = 2500
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1 = DBJ0(LI*R)/(LI*DBJ1(LI*B))
C
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, T)
       EA  = (SUM - SOLD)/SUM*100.0
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      TEMP =  (A*T*DCOS(T)) - SUM
      RETURN
      END
C
C     -------------------------------------
      DOUBLE PRECISION FUNCTION INTG (R, T)
```

```
C     -------------------------------------
      IMPLICIT NONE
      INTEGER ND, NPTS, I
      DOUBLE PRECISION ES, SOLD, SUM, EA, T1, T2, T3,
     1                 T, R, E, NU, B, DBJ1, L, ARG,
     1                 LI, ALPHA, SIGY, AD, A, DHM
      DIMENSION L(2500)
      COMMON /EIG/L
      COMMON /CONVER/ ND
      COMMON /PROPS/ AD, ALPHA, B, E, NU, SIGY, A
C
      NPTS  = 2500
      ES    = 0.5D0*10.0D0**(2 - ND)
C
      SUM   = 0.0D0
      SOLD  = SUM
      DO 10 I = 1, NPTS
       LI = L(I)
       T1  = R*DBJ1(LI*R)/(LI**2*DBJ1(LI*B))
       SUM = SUM + 2.0*A/B*T1*DHM(ALPHA, LI, T)
       EA  = (SUM - SOLD)/SUM*100.0
       SOLD = SUM
      IF (DABS(EA).LT. ES) GO TO 20
   10 CONTINUE
   20 CONTINUE
      INTG = A*R**2*T*DCOS(T)/2 - SUM
      RETURN
      END
C
C     ------------------------------------------
      DOUBLE PRECISION FUNCTION DHM( ALF, L, T )
C     ------------------------------------------
     IMPLICIT NONE
```

```
      DOUBLE PRECISION ALF, L, T, ARG, T1, T2, T3
C
      ARG = -ALF*L**2*T
       IF (DABS(ARG) .GT. 250.0D0) THEN
        T1 = 0.0D0
       ELSE
        T1 = DEXP(ARG)
       ENDIF
C
       T2 =  T1*(ALF*L**2 - ALF**3*L**6) + DCOS(T)*
      2       (T - ALF*L**2 + T*ALF**2*L**4 + ALF**3*L**6)
       T3 =  DSIN(T)*(-T*ALF*L**2+2*ALF**2*L**4
      2       - T*ALF**3*L**6)
C
       DHM =  (T2 + T3)/(1.0D0 + (ALF**2*L**4))**2
C
       RETURN
       END
C
```