PROBABILISTIC SLOPE STABILITY ANALYSIS USING LIMIT EQUILIBRIUM, FINITE ELEMENT AND RANDOM FINITE ELEMENT METHODS

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PROBABILISTIC SLOPE STABILITY ANALYSIS USING LIMIT EQUILIBRIUM, FINITE ELEMENT AND RANDOM FINITE ELEMENT METHODS

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ABSTRACT

PROBABILISTIC SLOPE STABILITY ANALYSIS USING LIMIT EQUILIBRIUM, FINITE ELEMENT AND RANDOM FINITE ELEMENT METHODS

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In recent years, geotechnical engineers are moving more towards decision-making processes based on reliability assessment, since accounting for soil variability and carrying out probabilistic analyses result in significant savings in designs, and possible prediction of failure events. Objectives of this study are to investigate and compare different methodologies for probabilistic slope stability analyses as well as with deterministic methodologies, in terms of factor of safety, the probability of failure, and the critical failure surface. For this purpose probabilistic limit equilibrium tools (Slide and Slope/W) and probabilistic finite element tools (Phase 2 and PLAXIS2D) are utilized. Furthermore spatial variability of shear strength of soils is considered by Rslope2D software and newly-developed PLAXIS2D-Python scripting in this study. The effect of coefficient of variation and spatial correlation length is also studied. The most significant findings of this study are: (1) a deterministic FS value greater than 1.00 does not always mean that the slope is “safe” in traditional-sense, examples are shown where factor of safety of 1.50 can have a probability of failure of 30%. (2) As the coefficient of variation (COV) value increases, the probability of failure (PF) increases. (3) The results of probabilistic slope stability analyses are significantly influenced by the spatial correlation length. The results of this study are believed to be useful for further understanding of the probabilistic slope stability concept and the effects of soil heterogeneity on slope stability evaluations with the aim of better geotechnical risk management and communication.

Keywords: probabilistic slope stability, limit equilibrium, random finite element method, spatial correlation length and spatial variability

ÖZ

LİMİT DENGE, SONLU ELEMANLAR VE RASSAL SONLU ELEMANLAR YÖNTEMLERİ KULLANILARAK OLASILIKSAL ŞEV STABİLİTESİ ANALİZİ

Akbaş, Burak
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Zemin değişkenliğini hesaba katmak ve olasılıksal analizler gerçekleştirmek, dizayn açısından olası yenilmelerin öngörüsü açısından önemli kazanımlar ortaya koyduğu için, geoteknik mühendisleri son zamanlarda güvenilirliğe dayalı karar verme değerlendirmelerine yönelmektedir. Bu çalışmanın amaçları şunlardır: olasılıksal ve deterministik şev stabilitesi analiz metotlarının, güvenlik katsayısı, yenilme olasılığı ve kritik yenilme yüzeyi açısından incelenmesi ve kıyaslanması. Bu amaç doğrultusunda, olasılıksal limit denge araçları (Slide ve Slope/W) ve olasılıksal sonlu elemanlar araçları (Phase 2 and PLAXIS2D) kullanılmıştır. Ayrıca, Rslope2D ve bu çalışmada yeni geliştirilmiş olan PLAXIS2D-Python komut yazımı kullanılarak, zeminlerin kayma dayanımının mekansal değişkenliği göz önünde bulundurulmuştur. Varyasyon katsatısının ve mekansal korelasyon mesafesinin etkileri de çalışılmıştır. Bu çalışmanın en kayda değer bulguları şunlardır: (1) 1.00 dan büyük, deterministik güvenlik katsayısı, şevin geleneksel anlamda "güvenli" olduğu anlamına gelmez; örnekler göstermektedir ki 1.50 güvenlik katsayısına sahip bir şev, % 30 yenilme olasılığına sahip olabilmektedir. (2) Varyasyon katsayısı (COV) arttıkça yenilme olasılığı da (PF) da artmaktadır. (3) Olasılıksal şev stabilitesi analiz sonuçları mekansal korelasyon mesafesinden önemli ölçüde etkilenmektedir. Bu çalışmanın sonuçlarının, daha iyi geoteknik risk yönetimi ve iletişimi amacıyla birlikte, olasılıksal şev stabilitesi konseptinin ve zemin heterojenliğinin şev stabilitesi değerlendirmeleri üzerindeki etkilerinin daha iyi anlaşılması konularında yararlı olacağına inanılmaktadır.

Anahtar Kelimeler: olasılıksal şev stabilitesi, limit denge, rassal sonlu elemanlar yöntemi, mekansal korelasyon mesafesi ve mekansal değişkenlik

To my well-beloved family

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Even if I put all elegant words in here, I cannot sufficiently express my gratitude towards my advisor, Asst. Prof. Dr. Nejan Huvaj Sarıhan. I would like to thank her for her endless guidance, support and understanding. Her beyond-measure encouragement and faith for me made everything possible. It is well worth saying that such an opportunity to work with her is privilege.

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I would like to express my special gratitude my mother Ayşe Akbaş, my father Hüseyin Akbaş, my sister Betül Akbaş and my brother Samet Akbaş for their continuous support, encouragement and friendship. I am exclusively grateful for their patience.

Finally, there are lots of names that I cannot put all here but their continuous support and existence are deeply appreciated.

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LIST OF ABBREVIATIONS

CIUC	Consolidated Isotropic Undrained Triaxial Compression Test
COV	Coefficient of Variability
FEM	Finite Element Method
FORM	First Order Reliability Method
FOSM	First Order Second Moment
FS	Factor of Safety
GM	Global Minimum
LEM	Limit Equilibrium Method
LH	Latin-Hypercube
MC	Monte-Carlo
OS	Overall Slope
PF	Probability of Failure
PI	Plasticity Index
RFEM	Random Finite Element Method
RI	Reliability Index
SRF	Strength Reduction Method
TC	Triaxial Compression Test
UC	Unconfined Compression Test
UU	Unconsolidated-undrained Compression Test
VST	Vane Shear Test
PES	Probabilistic Engineered Slopes (deWolfe et al., 2010)
Phase2	Finite Element Software for Soil and Rock Application (Rocscience)

Plaxis 2D	Finite Element Software for Geotechnical Problems (Plaxis bv)
Python	Programming Language (The Python Software Foundation)
Rslope2D	RFEM Slope Stability Software (G.A. Fenton and D.V. Griffiths 1992)
Slide	Limit Equilibrium Software for Slope Stability Analysis (Rocscience)
Slope/W	Limit Equilibrium Software for Slope Stability Analysis (Geo-Slope)

CHAPTER 1

INTRODUCTION

Risk and safety assessment of dikes, earth dams, open pit mines, tailing dams, landfills and natural slopes are becoming more and more important for proper management and mitigation of natural hazards. However, estimating the safety level and performance of the slopes, either man-made or natural (Figure 1.1), is a formidable task since there are quite a bit of uncertainties in relevant material properties (Figure 1.2 and Figure 1.3). Considering the heterogeneity and uncertainty in material properties, together with changes and variability in environmental factors, a probabilistic evaluation of slope stability is necessary. There are several deterministic and probabilistic approaches available in the literature but a thorough understanding based on comparison and synthesis is missing. In this study, different probabilistic approaches are utilized for a number of slope geometries and their results are compared in terms of failure surface, probability of failure and factor of safety. In addition to that, a concept called “Random Finite Element Method” is used to demonstrate the effect and the importance of spatial correlation distance on the probability of failure. In this manner, randomness and heterogeneity of the field is also taken into account. Within this context, several sets of slope stability analyses are carried out.

Figure 1.1 a) Instability of a man-made municipal solid waste slope in the Philippines, 2000 (source: casehistories.geoengineer.org) b) Instability of a natural slope in USA, 1995 (source: pubs.usgs.gov)

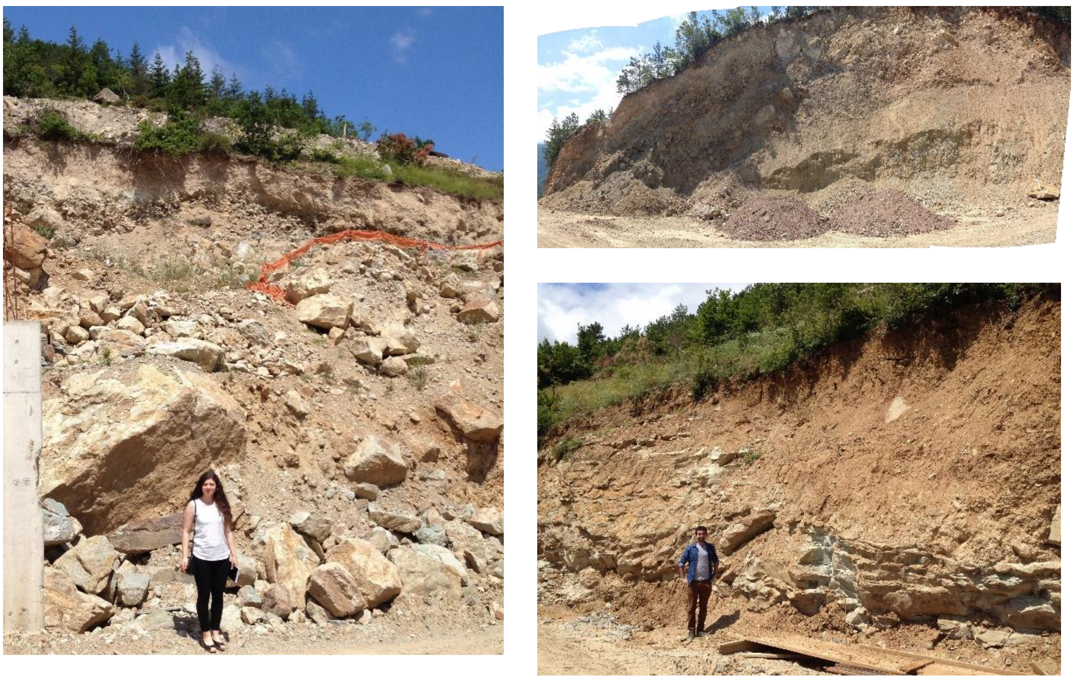


Figure 1.2 An extreme example of heterogeneity and variability in a colluvium (slope debris) in a slope in Artvin, Turkey. Colluvium is described as “gravelly sandy silty slope debris with large rock blocks” (photos courtesy of Dr. Nejan Huvaj)

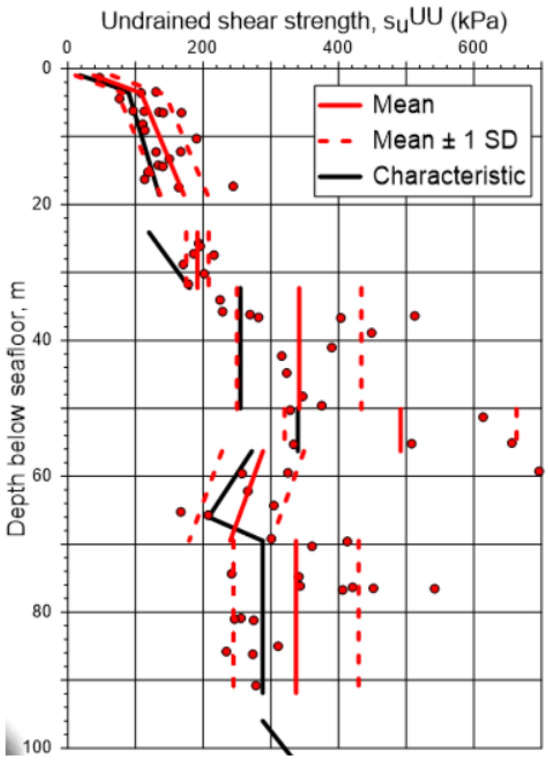


Figure 1.3 Variability in undrained shear strength of a natural clay at an offshore pile foundation site (2015 Rankine lecture presentation by Dr. Suzanne Lacasse, unpublished)

1.1 Problem Statement

Limit Equilibrium Method (LEM) and Finite Element Method (FEM) are widely-used approaches for evaluating the stability of slopes. An extension of the FEM, Random Finite Elements Method (RFEM) can estimate the probability of failure of slopes while accounting for the spatial variability of material properties through spatial correlation length. In the literature, there are many studies about the comparison of different deterministic LEM and FEM solutions (such as Duncan, 1996; Cheng et al., 2007; Hammouri et al., 2008; Babu et al., 2012 etc. among others); as well as some probabilistic studies (e.g. Bhattacharya et al., 2003; Griffiths and Fenton, 2004; El-Ramly et al., 2005; Hammah et al., 2009; Cho, 2009 etc.). Some of these studies are carried out for the first-time slides (i.e. a slope that is experiencing failure for the first-time, with no previous shear movements in the history of the slope), whereas others are for reactivated slides (which have a previous history of shear movement and an existing shear surface). However, there is a lack of comparative studies that emphasize the necessity of the probabilistic analysis and factors affecting the probability of failure. Defining how these factors affect the results and in which trend it does is the key to understand the mechanism and can change how we approach to the specific problems.

1.2 Research Objectives

The "factor of safety" is a widely-used concept in geotechnical engineering since the birth of the soil mechanics discipline almost a hundred years ago. It is, however, a single deterministic number to evaluate the safety in geotechnical engineering. Since it is used for so long, engineers worldwide are used to factor of safety terminology in their daily engineering practice. More and more, in recent years, engineers are moving towards using probabilistic approaches in all their engineering analyses and designs in order to account for the variability in material properties and in environmental/external conditions. In fact, in her 2015 Rankine lecture, Dr. Suzanne Lacasse expressed this trend in her "Evolution of geotechnical practice" graph (Figure 1.4) saying that decision-making based on reliability assessment will be taking a bigger role in geotechnical engineering designs. Accounting for soil variability and carrying out probabilistic analyses result in significant savings in designs, or possible prediction of failures or unwanted events.

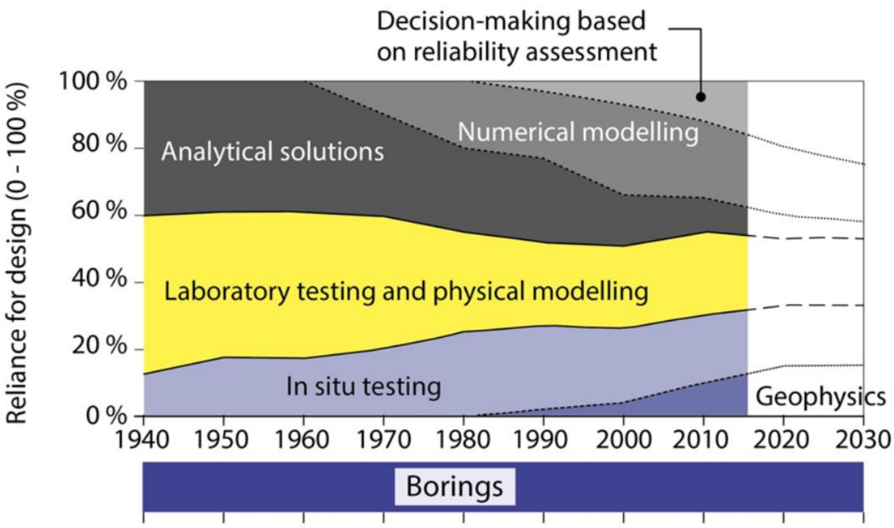


Figure 1.4 "Evolution of geotechnical practice", by Dr. Suzanne Lacasse (2015 Rankine lecture presentation by Dr. Suzanne Lacasse, unpublished)

The broad objective of this study is to demonstrate and show to the geotechnical engineering community the importance of using probabilistic analyses, and to give examples in slope stability for its use. Further specific objectives of this thesis are (i) to investigate and compare different methodologies for probabilistic slope stability analyses as well as with deterministic methodologies; (ii) to study the factors that influence the results of probabilistic slope stability analyses; (iii) to provide guidance and suggestions to practicing geotechnical engineers for evaluating the safety of slopes probabilistically. This study seeks to develop an understanding, or answer, the following questions and conditions:

(1) Can a slope with a deterministic factor of safety larger than 1.0 have a probability of failure larger than 0 %? If so, does that mean giving a single F.S. value is not sufficient for evaluating the condition of the slope? Does the deterministic analysis overestimate or underestimate the safety of a slope? If it does, in which cases or when it overestimates the safety level?

(2) Would deterministic and probabilistic analyses give different results in terms of the most critical failure surface? If so, how different they are, and what are the factors that influence the difference?

(3) Are the above questions influenced by different factors such as the dry vs. wet case, high coefficient of variability vs. low coefficient of variability, different soil types (e.g. undrained clays, mixed soils and drained cohesionless soils)?

(4) Should we consider the "spatial correlation" of material properties in a probabilistic analyses? Does the consideration of spatial correlation give different results than an analysis without any spatial correlation? If so, how different the results are and which parameters affect the results?

(5) What are the effects of considering the spatial correlation length and anisotropy of spatial correlation length in RFEM?

The results of this study are believed to be useful for further understanding of the probabilistic slope stability concept and the effects of soil heterogeneity on slope stability evaluations with the aim of better geotechnical risk management and communication.

1.3 Scope

This study investigates the slope stability under different approaches. In Chapter 2, a literature review is provided. In Chapter 3, the methodology of different analysis types is provided. In Chapter 4, the first three questions of interest are studied. For this purpose, several case studies are analyzed via LEM (Rocscience Slide v6, Slope/W of GeoStudio 2012) and FEM (Rocscience Phase2 v8). In Chapter 5, the effects of spatial correlation are investigated and remaining questions are answered. To do that, a random finite element program called "Rslope2D" created by G.A. Fenton and D.V. Griffiths in 1992 is used. Since this software has some limitations, remote scripting of PLAXIS 2D 2015 is carried out with the help of Python 3.4 in order to apply random finite element method for different cases. In Chapter 6, real-life case studies taken from the literature are analyzed. Finally, in Chapter 7, outcomes of the study are highlighted and topics for further studies are suggested.

CHAPTER 2

LITERATURE REVIEW

Stability of slopes is one of the oldest and arguably the most-studied and the least-understood topic of geotechnical engineering. In their 2014 book entitled Soil Strength and Slope Stability, Duncan, Wright and Brandon says “Evaluating the stability of slopes in soil is an important, interesting, and challenging aspect of civil engineering” (Duncan et al. 2014). There are numerous works in this field in terms of theoretical, analytical, experimental, statistical and numerical approaches, perhaps starting with Terzaghi’s 1950 work entitled “Mechanism of Landslides” (Terzaghi, 1950), and they continue till today Since the solution of slope stability problems requires the understanding of analytical methods and their application, the knowledge of available methods and their limitations becomes a crucial topic. Only after a sound understanding and analyses of a slope stability problem, effective remedial solutions (e.g. stabilization measurements) can be applied if necessary.

2.1 Finite Element and Limit Equilibrium Methods for Slope Stability

Methods for slope stability works can be broadly categorized into two; namely Limit Equilibrium Methods and Numerical Methods. In LEM concept, an investigation of the force equilibrium in a soil mass in a sloping ground which tends to slide down under the effect of gravity (or loading) are carried out. The method is based on the comparison between resisting and driving forces, moments or stresses and the outcome is the “Factor of Safety” of the slope. FS can be calculated in three ways. These are namely limit, force and moment equilibriums (Figure 2.1). In the first definition, FS ratio is between available shear strength and mobilized shear stress. It can be either in terms of effective or total stresses according to field conditions. The shear strength is fully mobilized at the time of failure (e.g. the FS ratio is 1). Available shear strength and mobilized shear strength are defined as in Equation 2.1a and 2.1b, respectively.

$$\tau_f = c + \sigma * \tan\phi \qquad (2.1a)$$

$$\tau = \tau_f / FS \qquad (2.1b)$$

Where; c is cohesion, Φ is internal friction angle, σ is normal stress, τ_f is available shear strength and τ is mobilized shear strength.

Throughout the history, development of LEM concept has been continued and several LEM's are proposed. All methods are based on some assumptions for the shape of the slip surface, the force and moment equilibriums (Table 2.1). According to the specific problem, these assumptions should be considered carefully.

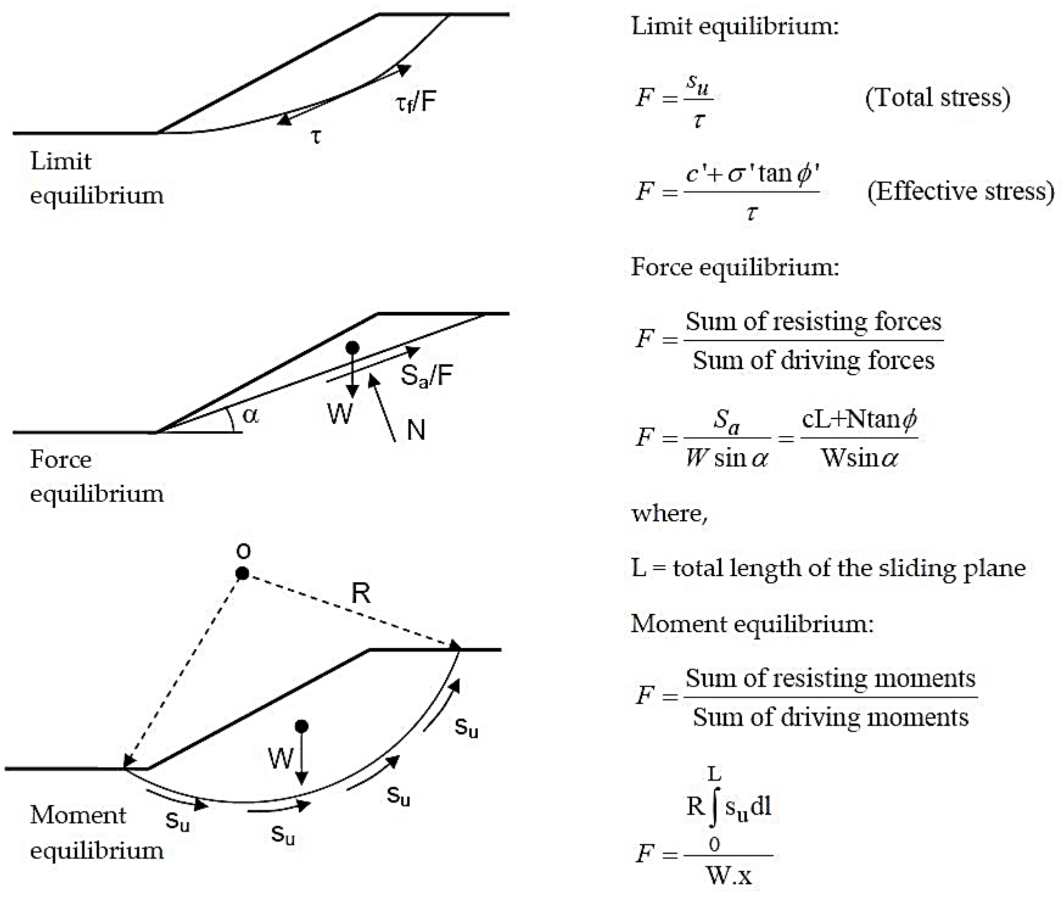


Figure 2.1 Three definitions of factor of safety (Aryal 2006)

Table 2.1 Summary of Limit Equilibrium Methods (Pockoski and Duncan 2000)

Method	**Slip surface shape**	$\sum M_{overall}$	$\sum M_{individual\text{-}slice}$	$\sum H$	$\sum V$
Swedish Circle	Circular	Yes	No	No	No
Ordinary Method of Slices (Fellenius 1927)	Circular	Yes	No	No	No
Bishop's Modified Method (Bishop 1955)	Circular	Yes	No	No	Yes
Morgenstren and Price's Method (Morgenstren and Price 1965)	Any shape	Yes	Yes	Yes	Yes
Spencer's Method (Spencer 1967)	Any shape	Yes	Yes	Yes	Yes
Corps of Eng. Modified Swedish (1970)	Any shape	No	No	Yes	Yes
Lowe and Karafiath (1960)	Any shape	No	No	Yes	Yes
Janbu Simplified (Janbu 1954)	Any shape	No	No	Yes	Yes

Where; $\sum M_{overall}$ is overall moment equilibrium, $\sum M_{individual\text{-}slice}$ is moment equilibrium for individual slice, $\sum H$ is horizontal force equilibrium, $\sum V$ is vertical force equilibrium.

Among the many different numerical methods (finite difference, finite element, discrete element, material point method etc.), perhaps, the most widely-used one is the Finite Element Method. In this method, the whole domain is divided into sub-domains (e.g. finite elements) and these elements are connected to each other through their nodes. Forces and stresses are calculated at these nodal points according to specified material constitutive model. The assembly of all the elements are usually called a "mesh" or a "grid". An example of generated mesh and details of mesh elements for a simple slope geometry is provided in Figure 2.2 and Figure 2.3, respectively. In finite element analyses, the type and size of elements play an important role in order to achieve accurate results. There is a simple analogy in which one can approximate a circle by connecting many small lines along its circumference, and as the length of the lines decreases a better approximation would be achieved. Therefore, the size of the elements should be chosen carefully. If they are too big (e.g. a too coarse mesh), the results may not be accurate enough. If they are too small (e.g. a too fine mesh), required computational time will increase and the results will not get any better after a certain fineness. In finite element analyses of slope stability problems, to obtain a factor of safety, "strength reduction method" is commonly used. In this method, strength parameters (namely cohesion, c and the friction angle, tan Φ) are simultaneously reduced (by the same number) in small increments until the failure occurs. The resulting "strength reduction factor, SRF" is comparable with the commonly used "factor of safety, FS" and defined as in Equation 2.2.

$$SRF = FS = \frac{c}{c_{reduced}} = \frac{\tan\phi}{\tan\phi_{reduced}} \tag{2.2}$$

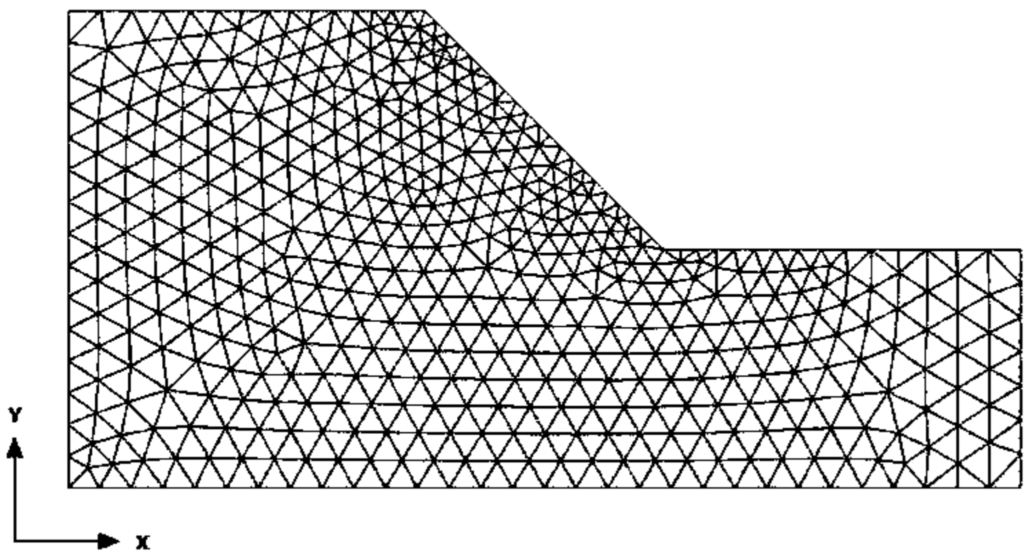


Figure 2.2 Finite element mesh view of a simple slope geometry (generated by PLAXIS 2D 2015)

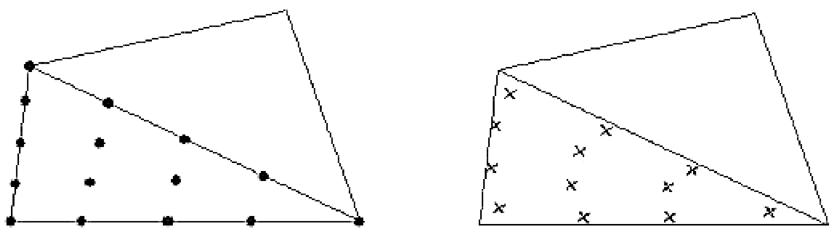


Figure 2.3 15-noded, triangular mesh elements; a) nodes b) stress points (PLAXIS 2D Reference Manual 2015)

The main difference between FEM and LEM is that LEM utilizes the statics of force and moment equilibrium whereas FEM uses stress-strain relationship (e.g. constitutive law). In FEM, stress redistributions can be computed since it is based on stress-strain relationship. Additionally, due to its meshing process, compatibility between structural members and soil media can be done without much problem. However, FEM is computationally more time consuming as compared to LEM. On the other hand, LEM analyses are well established for many decades and their simplicity and relatively good results with well accuracy makes it more widely-used in geotechnical engineering practice. However, when slope fails with a complex mechanism (such as progressive failure etc.), LEM simulations may become inadequate.

Literature studies on LEM and FEM slope stability analyses are extensively available. The studies that present a comparison between these approaches are discussed in the remaining part of this section.

Yu et al. (1998) compares the results obtained from conventional limit equilibrium method to the results of upper and lower bound solutions which are numerical procedures depending on finite element analysis and details of the approach can be found in the mentioned study. According to the study, Tresca yield criterion is used for undrained case whereas Mohr-Coulomb criterion is used for drained condition. In this work, two types of analyses are carried out for a simple slope geometry whose slope inclination and slope height are changed systematically in order to carry out a parametric study. In the first type, undrained slope stability analyses are carried for both constant undrained shear strength and shear strength increasing with depth. In the second one, slopes having cohesive-frictional (c, ϕ) soil are analyzed with constant cohesion and friction angle values. For the second type of analysis, effects of seepage are not included. After their analyses, they are concluded that, among other findings, LEM analyses give reasonable results for homogeneous slopes whereas they underestimate the stability for inhomogeneous slopes with low slope inclination.

Cheng et al. (2007) focuses on the comparison of FS values and slip surfaces obtained from LEM and FEM analyses. For this purpose, two general slope cases are considered. In the first one, homogeneous slope having 6 m height and inclination of 45 degrees is used. All possible combinations of cohesion values (2 kPa, 5 kPa, 10 kPa and 20 kPa) and friction angles (0°, 5°, 15°, 25°, 35° and 45°) are analyzed. Unit weight, elastic modulus and Poisson's ratio are kept constant. For FEM analyses, both associated and non-associated flow analyses are done and obtained slip surfaces and FS/SRF values are compared. Several important conclusions are obtained from these analyses. First, with only a few exemption, SRF obtained from FEM analyses are slightly larger than FS of LEM analyses and associated flow cases have slightly bigger SRF than non-associated cases. Second, for a given small cohesion value, the difference between FS and SRF is greater for higher friction angle. Third, failure surfaces are similar in most cases but associated FEM cases are closer to LEM results than that of non-associated cases. Additionally, for small cohesive values, distinction is very hard to do whereas noticeable difference occurs when cohesive strength gets bigger. Fourth, as the friction angle of the slope is increased, crest side of the slip surface moves closer towards the crest. Fifth, for a given small friction angle, failure surface difference between associated and non-associated cases are greatest for smaller cohesion values. Lastly, failed volume (or area in 2D analysis) gets smaller for increasing friction angle but increases with increasing cohesion values. For the illustration purposes, one of the comparisons is provided below. In that figure, SRM1 and SRM2 means non-associated and associated flow strength reduction methods, respectively. In the second type of analysis, slope having a thin, soft band between two cohesive layers are considered. This layer has zero cohesion and considerably small friction angle compared to the other layers. In order to investigate the effects of the domain size and mesh sizes of FEM analyses, several subcases are studied and the final case is selected for not having any effects from these inputs. The authors are utilized 4 different commercial software for FEM analyses and the results are surprisingly different. Even though the location of the failure surfaces are similar, obtained SRF values are different. For the second case, effects of dilation angle, elastic modulus and 4 other layer configurations are also studied.

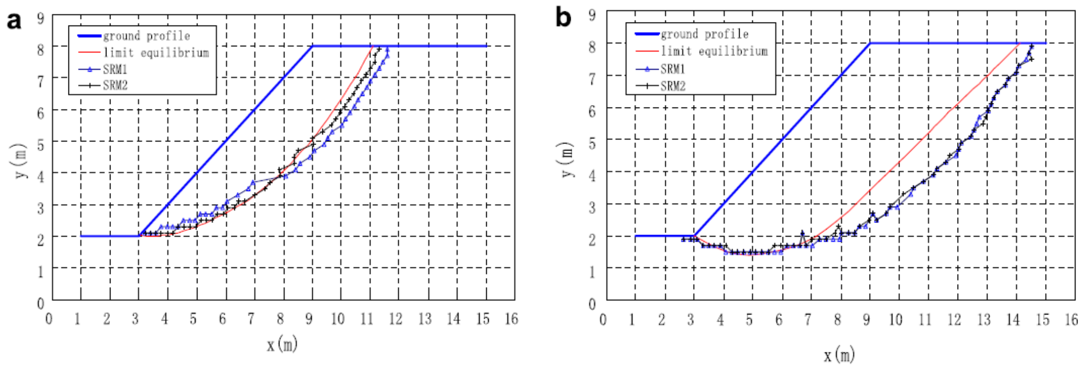


Figure 2.4 Failure surface comparison with increasing cohesion values for $\Phi' = 5°$
a) $c' = 2$ kPa b) $c' = 20$ kPa (Cheng et al. 2007)

At the end of the study by Cheng et al. (2007), several conclusions are presented. First, it is seen that effects of dilation angle, elastic modulus and domain size on slip surface location and SRF are small. Second, for the layered cases with soft band, the outcomes of FEM are very sensitive to number of iteration, element size, nonlinear solution algorithm and flow rules. It is advised that LEM analysis should be used to check the FEM results. Third, during LEM computation, occurrence of local minima is very likely, unlike in FEM computations. Therefore, this is one of the limitations of FEM analyses and LEM and FEM analyses should be used together. Fourth, when soft band is very thin, FEM analysis requires a large number of elements, therefore, a large computational time. This is also one of the limitations and LEM analysis may probably be chosen for this special case. Fifth, when both cohesion and friction angle are very small, numerical problems occurs for FEM analyses and large domain is required for an accurate analysis. Finally, in spite of its drawbacks, FEM analyses have a big advantage which is automatic positioning of failure surface without any assumption for the slip surface shape and trial-error search. However, FEM suffers from being sensitive to nonlinear solution and flow rule while LEM suffers from interslice force assumptions.

Hammouri et al. (2008) compares FEM and LEM in terms of safety factor and location of the obtained slip surfaces. For that purpose, the study utilized PLAXIS 2D v.8 and SAS-MCT v.4 for FEM and LEM analyses, respectively. Analyses are done for both homogeneous and inhomogeneous slopes with one slope inclination and rapid drawdown, presence of tension crack and undrained clayey soil are taken into consideration. Effects of having layered subsoil condition are also investigated for undrained and tension crack cases. A number of analyses are carried out and tendency between drawdown percent and FS, undrained shear strength and FS, tension crack location and FS are investigated. The study also uses a range of internal friction angle and cohesion values so that it can show different safety levels. As long as the given conditions are considered, the study provides an extensive analyses. However, it suffers from lack of the probabilistic concept. Nevertheless, the shape and location of slip surfaces obtained for all cases show a reasonable agreement. However, provided safety factor values are slightly different from each other. The rapid drawdown case with an average of 11 % difference in FS/SRF values between FEM and LEM is the situation with more safety factor difference. On the other hand, tension crack and undrained slope cases both have an average of 4-5 % safety factor difference. However, writers concluded that engineers should carry the both approaches for the critical failure surface analyses.

Mbarka et al. (2010) proposes a combined reliability method with various approaches. In the study, probabilistic strategies such as Monte-Carlo Simulation, First and Second Order Reliability Methods, Mean Values First Order Second Moment Method and Quadrature method are combined with three mechanical models (e.g. Caquot-Taylor, Bishop LEM and FEM). Only the results related to FEM and LEM are briefly presented in here since the others are out of the scope. For this study, a homogeneous test slope having a height of 8 m, inclination of 2V:3H and cohesive-frictional soil is used. For the probabilistic analyses, only the shear strength parameters are used as variables. In the study, the mean values of cohesion and friction angle are used as 9 kPa and 27° whereas COV values are 25 % and 15 %, respectively. Statistical distribution of variables are taken both normal and lognormal for comparison purposes. At the end of the study, they confirmed that deterministic slope stability analyses are insufficient since deterministic failure surface with minimum FS value is not always the most critical slip surface when the reliability analysis is considered. The study also states that critical slip surface depends on the methods used and statistical characteristics of the variables. However, this effect is small for a homogeneous slope and need further investigation. One example is provided below. The writers also state that spatial correlation should be included for the future studies.

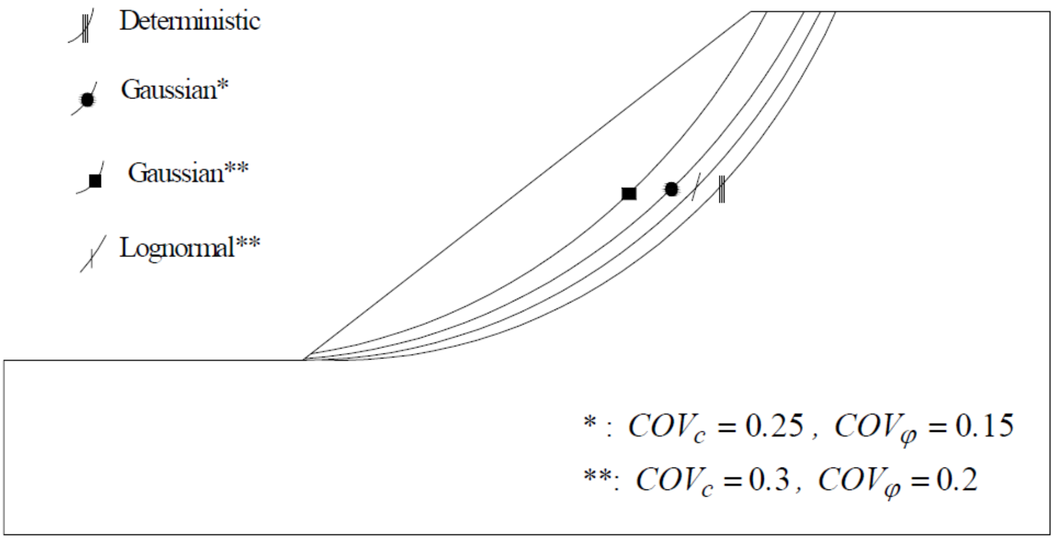


Figure 2.5 Deterministic and probabilistic slip surfaces obtained from Monte-Carlo Simulation based on Bishop LEM for both Normal distribution and Lognormal distribution (Mbarka et al. 2010)

Belczyk et al. (2012) carries out landfill cap stability analyses by using limit equilibrium, finite element and 2D computational limit analysis. Although the last method is out of the scope of this thesis, results related for the first two methods can be briefly summarized here in order to exemplify the concept for different geotechnical structures. As explained by the writers, landfills usually consist of a thin veneer of soil layer on top of a geosynthetic layer and this may cause a stability problem in terms of sliding of the cover and tension failure of the geosynthetic material. For that purpose, the study investigates an appropriate design method. The study makes several conclusions. First, the results of FEM and LEM are similar (less than 1 %) for a uniform cover layer. Second, when scenarios having a seepage parallel to the slope is considered, FS values obtained from LEM analyses are about 4 % bigger than SRF results. Third, for buttressed problems, FS values are about 6 % bigger in LEM analyses. Lastly, the writers concluded that LEM is an easy to use method with a relatively good first estimation for simple problems, however, FEM should be used in conjunction with LEM analyses.

Alemdag et al. (2015) studies the stability of a slope debris in Gümüşhane, Turkey. In the area, survey lines, 14 boreholes and 4 trial pits are drilled and undisturbed samples are obtained in order to determine the soil profile and shear strength parameters. After the necessary laboratory tests, it is determined that soil profile consists of clayey sand, silty sand and low plasticity clay with a peak cohesion values between 2.63 and 16.35 kPa and peak friction angles between 20° and 27° for 4 different cross-sections. In addition to strength parameters, other index parameters are also determined and slope stability analyses are carried out for 4 different cross-sections by deterministic LEM and deterministic FEM. The study also considers the ground acceleration in the field. In their study, Slide v.5 and Phase2 v.6 are used for LEM and FEM analyses, respectively and analyses are done for the worst soil conditions. Among other results, it is concluded that FS values obtained from LEM analyses are slightly greater than that of FEM analyses. Although the original study does not make any comparison between the obtained failure surfaces, slip surfaces are quite different except cross-section 2 of the study. Obtained FS/SRF values as well as one cross-section example are provided for the demonstration purpose.

Table 2.2 Obtained FS/SRF factors for cross-sections (Alemdag et al. 2015)

	Cross-Sections			
Analysis Type	**No: 1**	**No: 2**	**No: 3**	**No: 4**
LEM	1.44	1.80	1.96	1.72
FEM	1.39	1.72	1.59	1.58

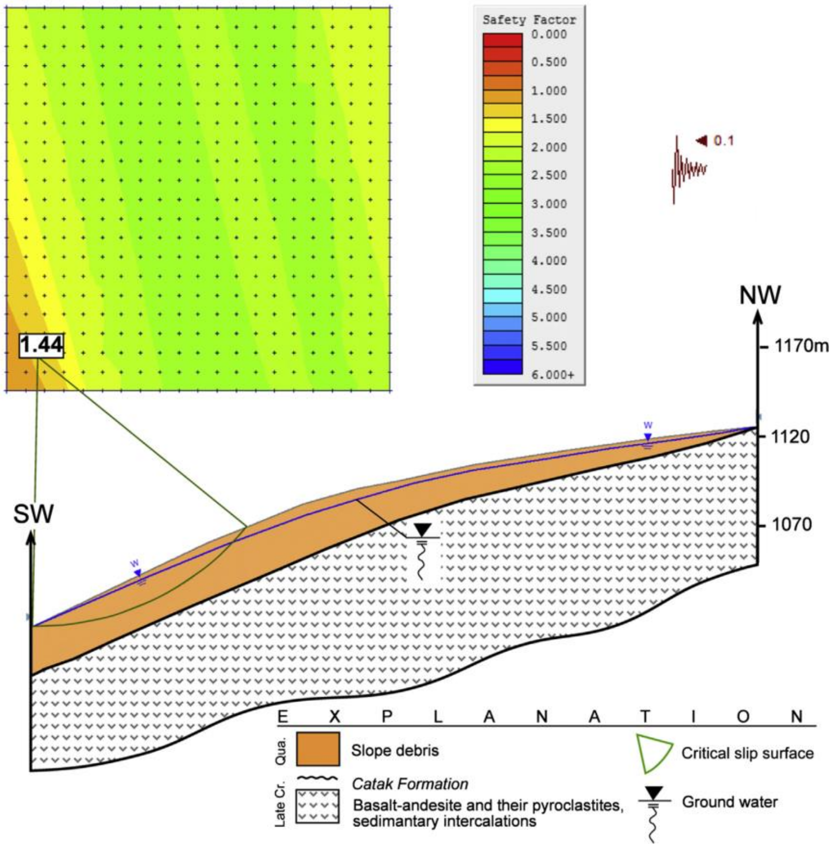


Figure 2.6 LEM analysis result of cross-section 1 (Alemdag et al. 2015)

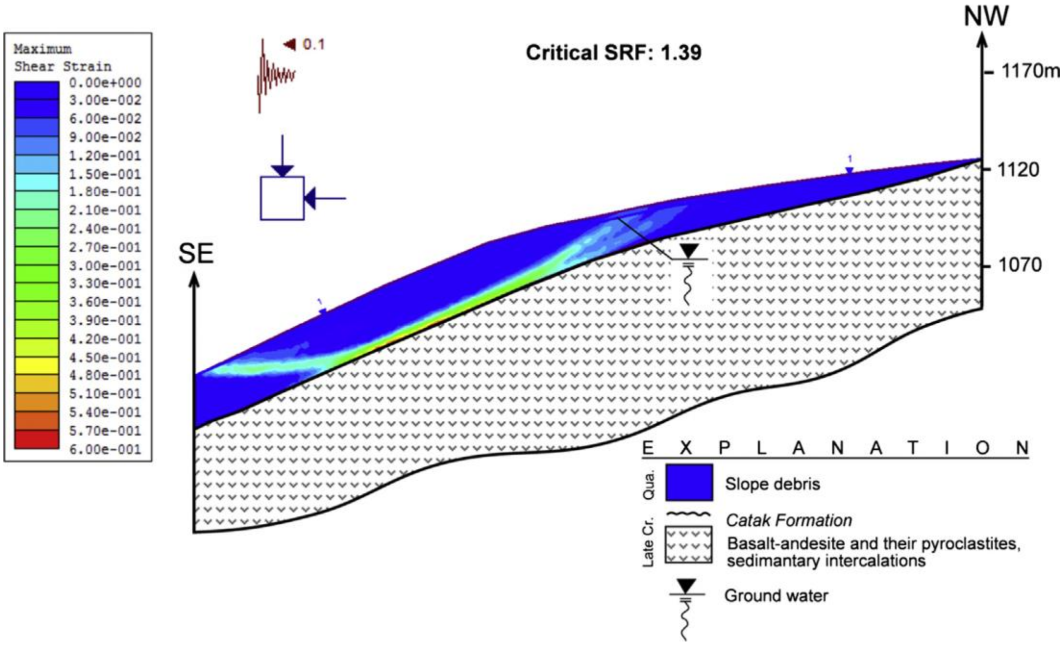


Figure 2.7 FEM analysis result of cross-section 1 (Alemdag et al. 2015)

Shivamanth et al. (2015) investigates the stability analysis of a dyke using LEM and FEM approaches. For this purpose, analyses are done using Slope/W and PLAXIS 2D for different construction and operation conditions. These are stability of dyke just after the construction, stability after bottom ash deposition and rapid drawdown conditions. For the dyke dimensions, earth dam having 14 m height, 6 m crest with 1V:2H slope on both sides are analyzed. For the first 2 m of soil profile, there is soft disintegrated rock and, afterwards, there is hard rock formation for 8 m. After FEM analyses of three cases, it is found out that the most critical condition in terms of stability is the rapid drawdown case. For the comparison purposes, this case is also analyzed with Morgenstern-Price method of Slope/W. FS result obtained from LEM is 1.75 whereas SRF value of PLAXIS 2D is 1.57. That means that LEM gives about 12 % bigger safety factor than FEM approach. Although the original study does not compare the obtained failure surfaces, it is seen that slip surface of LEM is slightly deeper and includes wider range than that of FEM. Both results are provided below.

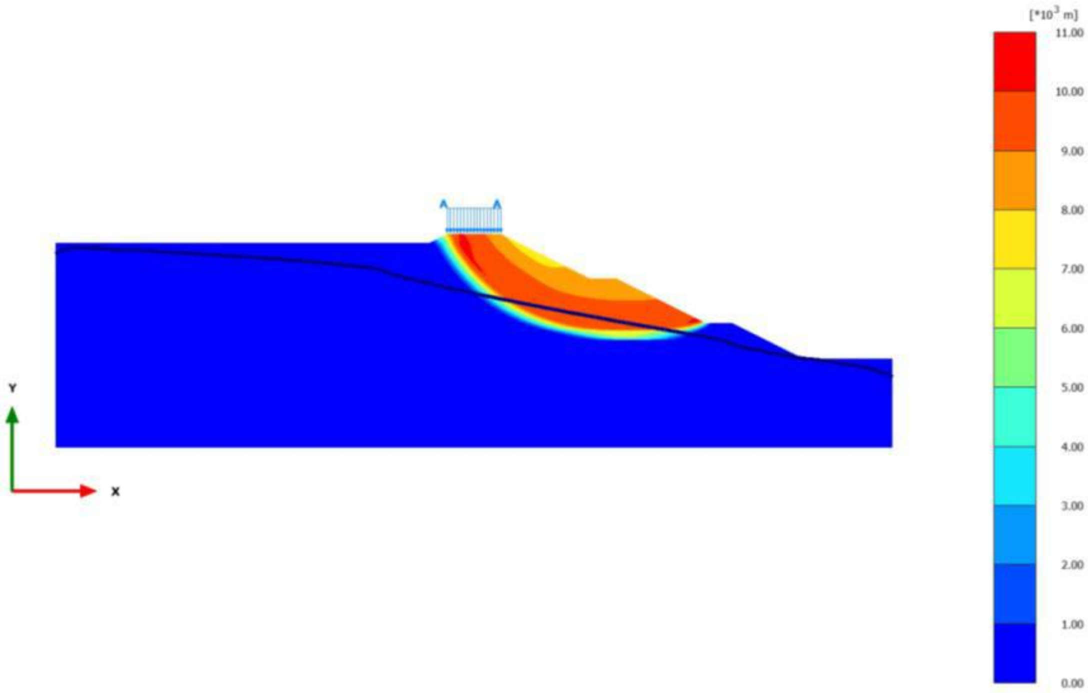


Figure 2.8 Obtained slip surface via FEM (Shivamanth et al. 2015)

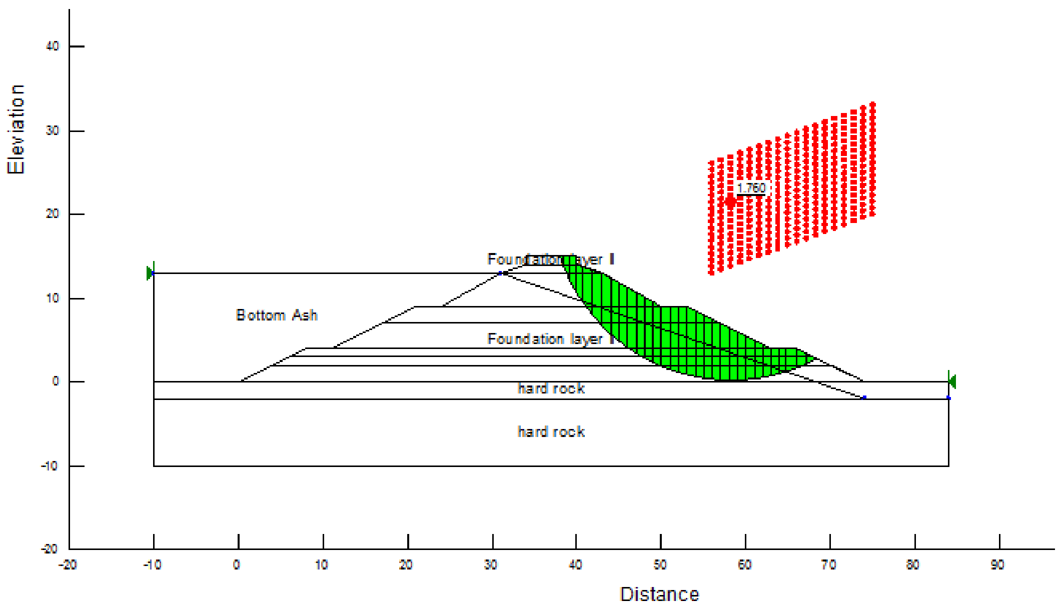


Figure 2.9 Obtained slip surface via LEM (Shivamanth et al. 2015)

Ozbay et al. (2015) investigates two landslides that occurred in Çöllolar lignite mine located in Kahramanmaraş, Turkey. The image of the area after the landslides is provided below. In this study, limit equilibrium and finite element methods are used in order to determine the factors that led to the failures. Additionally, outcomes of FEM and LEM analyses are compared with the results of the other researchers who have studied these slopes previously. For FEM analyses, PLAXIS 2D v.8 is used whereas STB, which uses Bishop's simplified method, is utilized for LEM analyses. The study makes slope stability calculations for 14 cross-sections with three different ground water table (GWT) configurations each. The study states that FEM and LEM results for the southwest and the northwest permanent slopes are similar when GWT at the ground surface. However, for the conveyor region slopes, LEM analyses are resulted in smaller safety factors than FEM analyses. This difference gets bigger as the GWT is lowered. For the failure surfaces, LEM analyses omits the week thin clay layer and result in circular slip surfaces whereas FEM results show a transitional slide onto this region. For the demonstration purpose, FEM and LEM results of the most critical section of permanent slopes are provided below.

Figure 2.10 Landslide area after both failures (Ozbay et al. 2015)

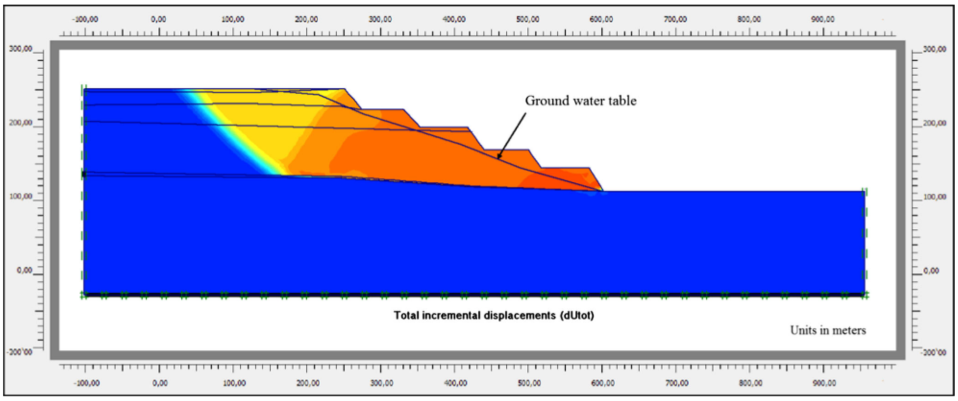


Figure 2.11 FEM slip surface of most critical permanent slope (Ozbay et al. 2015)

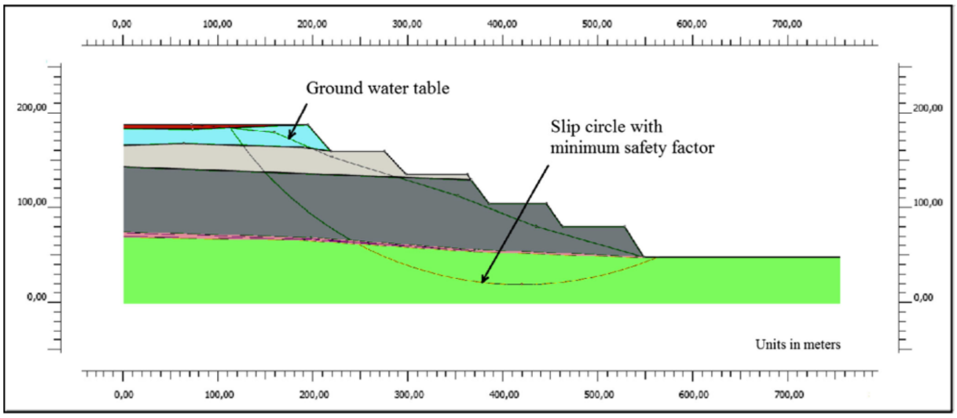


Figure 2.12 LEM slip surface of most critical permanent slope (Ozbay et al. 2015)

Liu et al. (2015) compares the outcomes of LEM analyses with two FEM analyses. These FEM's are namely strength reduction method and enhanced limit equilibrium method. Second FEM method is an optimization type of search method but details, which can be found in the research paper, are out of scope of the current study. In order to compare the obtained failure surfaces and factor of safeties, 4 representative slopes are chosen and analyzed. For the first case, homogeneous slope having 1V:2H slope inclination and undrained soil condition are analyzed. Geometry and geotechnical parameters are taken from Griffiths and Lane (1999). For LEM analyses, both circular and optimized slip surfaces are determined by using Slope/W. It is seen that both failure surfaces and obtained FS/SRF values are in good agreement except for optimized LEM case. Additionally, circular surface LEM case has larger FS than rest of the methods. Effect of Poisson's ratio is also studied and no effect is observed. Obtained failure surfaces are provided below.

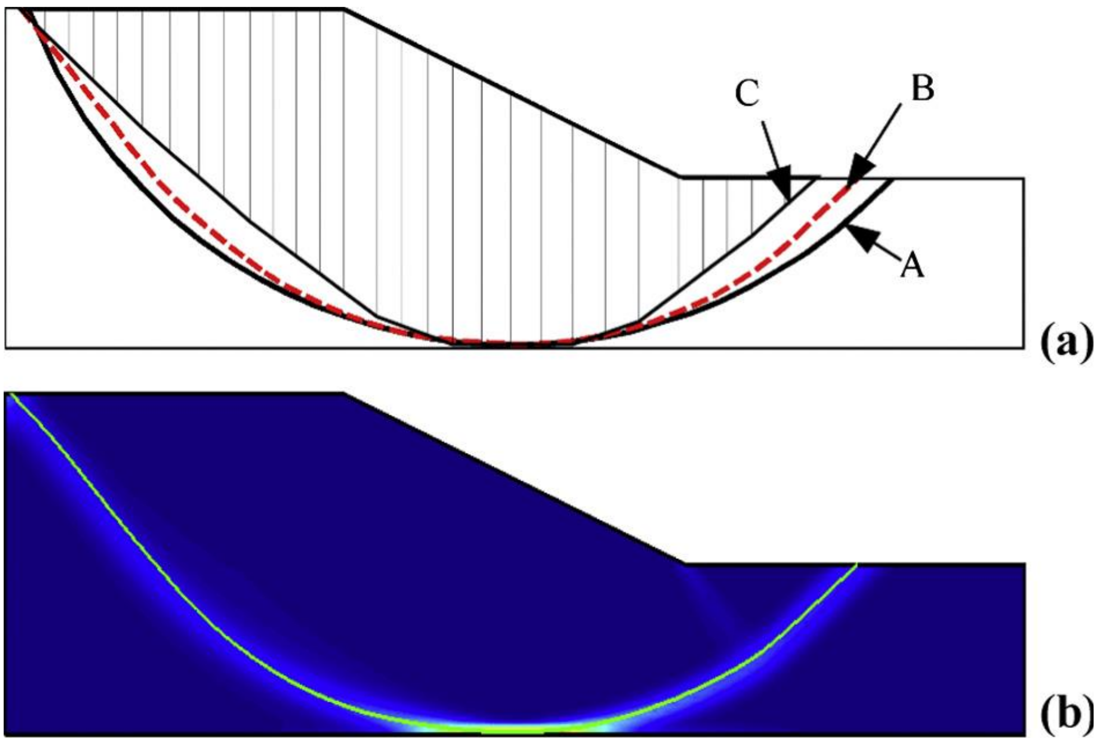
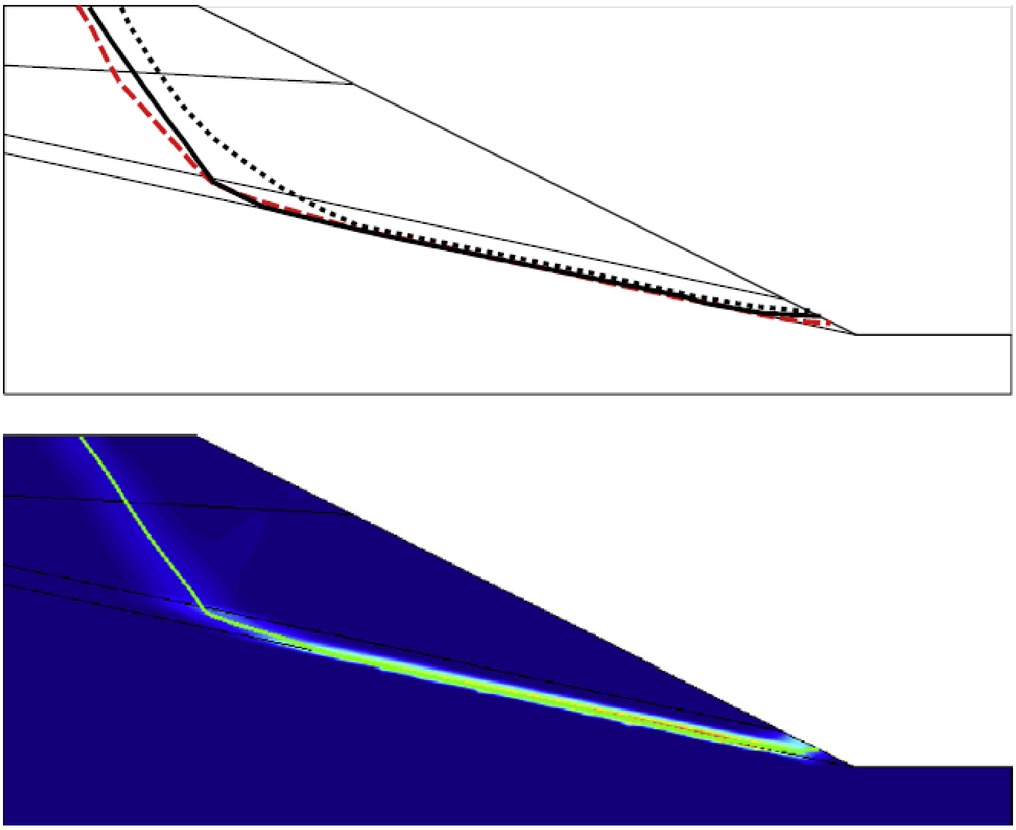


Figure 2.13 Critical slip surfaces a) enhanced LEM surface (B), LEM surfaces (solid lines, A & C) b) FEM slip surface (Liu et al. 2015)

For the second case, homogeneous slope having 1V:2H inclination and cohesion-frictional soil is analyzed. Geometry and geotechnical parameters are taken from Griffiths and Lane (1999). For FEM analyses, both non-associated and associated flow case is studied. For both non-associated and associated cases, it is seen that shape and location of FEM and LEM analyses are essentially same, however, FS of LEM is slightly smaller. This difference is about 1.7 % so it is negligible. For the third case, layered slope having a thin, weak intermediate layer with relatively small cohesion and friction angle is analyzed. It is concluded that both slip surfaces and FS/SRF values are similar. It is also compared with the results of Zolfaghari et al. (2005) study which also analyzed the same slope with LEM and seen that FS obtained in Zolfaghari et al. (2005) study is about 13 % bigger than their study. Failure surfaces of all analyses are provided below for the comparison purposes.



(a)

(b)

Figure 2.14 Critical slip surfaces a) enhanced LEM surface (dashed line), LEM surface (solid lines), Zolfaghari et al. 2005 study (dotted line) b) FEM slip surface (Liu et al. 2015)

For the last case, analyses are done for a multi-stage slope geometry taken from Cheng et al. (2007). For FEM analysis, in addition to obtained SRF value, a sudden displacement increase on the nodes are also assumed as failure criterion. After LEM, non-associated and associated FEM analyses, it is seen that FS/SRF values are quite similar but there are local minima for LEM analysis which have slightly bigger FS values. For FEM analysis, however, local slip surfaces cannot be obtained. For the comparison purposes, all of the slip surfaces are provided below.

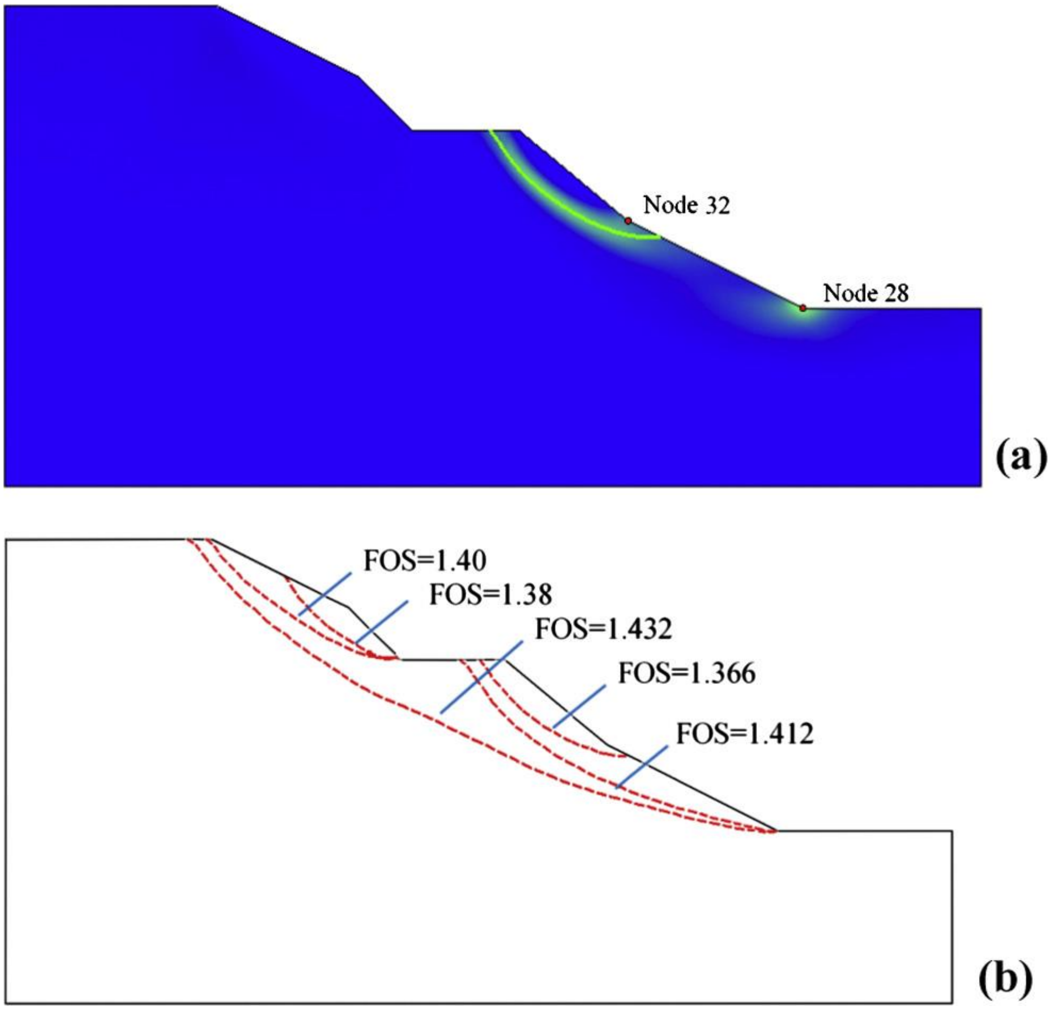


Figure 2.15 Comparison of slip surfaces a) FEM surface b) global and local slip surfaces of LEM (Liu et al. 2015)

In summary, although comparative slope stability studies between limit equilibrium and finite element methods are available in the literature, they still do not lead to solid universal conclusions. Many of the studies found similar safety factor (FS, or SRF) values with LEM and FEM (within 5-10% differences). However, under which conditions LEM gives bigger or smaller FS as compared to FEM is not clear. Many of the studies found significant differences in the critical failure surfaces obtained by LEM and FEM, however not many studies exist in the literature which compares the real measured failure surface with the failure surfaces obtained via LEM and FEM, therefore it is difficult to say which of the LEM or FEM gives more accurate failure surfaces. Most of the studies reviewed in the preceding section were deterministic in nature and they did not discuss any probabilistic approaches and variability in material properties, which is the topic of this thesis.

2.2 Variability of Geotechnical Material Properties

As it is very well known, soil profiles in nature are commonly heterogeneous not just vertically but also horizontally (i.e. multi-layered) and they contain some spatial variability in material properties even within one layer (one material type). Geotechnical engineers frequently carry out site investigations, field testing (SPT, CPT etc), soil sampling and laboratory testing to determine material properties of soils. Most of the times, extrapolations and interpretations are made based on a limited number of boreholes and limited number of soil samples taken from the field. Consequently, the designers should consider the uncertainty and variability in material properties when they carry out analyses and design. In the ideal situation, soil data variability should be accounted for site-specific. However, in most of the cases, site-specific data are either not available or too limited. In that case, reported literature values, maybe in the form of coefficient of variation (COV), are needed as a guidelines. Unfortunately, available statistical values are not feasible for this kind of a general usage since they are derived from total variability analyses assuming a uniform source of uncertainty (Phoon and Kulhawy 1999). The three main sources of geotechnical variability come from aleatoric variability, epistemic variability and transformation uncertainty. First source (also known as inherent uncertainty) is related to historical development of the soil and comes from natural geological process of the field. Since soils are formed by factors, such as weathering, erosion etc., they are exposed to different loadings, physical and chemical progresses. This cycle modifies the geotechnical parameters in a continuous manner. Second source mainly comes from in-situ and laboratory testing errors, soil sampling errors and equipment errors. Effects of these errors on overall uncertainty can be reduced by having more data (e.g. taking more samples). The last source comes into action when determined geotechnical parameters, by means of either in-situ or laboratory measurements, are converted to design values using correlations. There are other errors which mainly comes from human actions but they are mostly neglected (Baecher and Christian 2003). Considering the structural, loading and modelling uncertainties, visual illustration of all uncertainties is provided in Figure 2.16.

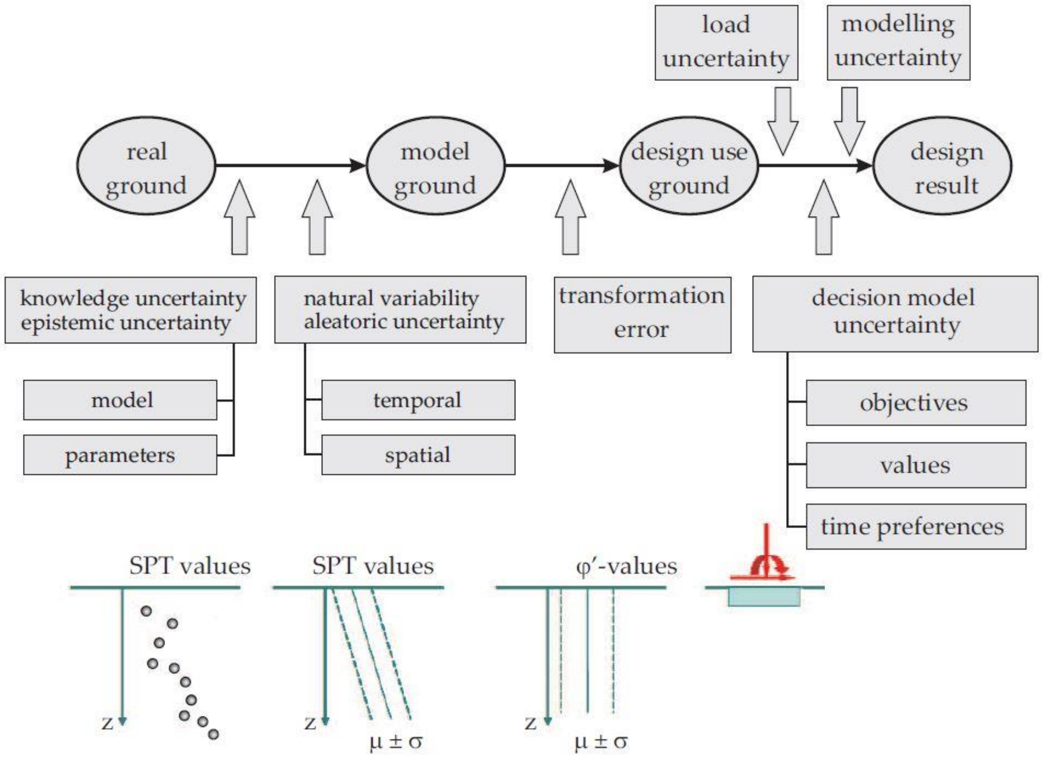


Figure 2.16 Summary of uncertainties in geotechnical design process (Huber 2013)

Although they are limited, it is possible to find collection of reported values in the literature for coefficient of variation of geotechnical properties and in-situ test results. One of the examples is Duncan (2000) study which summarizes the available literature at that time and adds some of his own data into the literature. Within the content of the present thesis study, only the relevant parameters and their COV values are provided in Table 2.3. For details and other values, the reader is referred to original paper and its references. Since the testing procedure and condition of the sampling is not reported in most of the sources, these values should be treated as rough guidelines (Duncan 2000).

Table 2.3 COV's for some geotechnical properties (Duncan 2000)

Geotechnical Property	COV (%)	Source
Unit Weight (γ)	3-7	Harr (1984), Kulhawy (1992)
Buoyant Unit Weight (γ_b)	0-10	Lacasse and Nadim (1997), Duncan (2000)
Effective Stress Friction Angle (Φ')	2-13	Harr (1984), Kulhawy (1992)
Undrained Shear Strength (c_u)	13-40	Harr (1984), Kulhawy (1992), Lacasse and Nadim (1997), Duncan (2000)

Another study is conducted in companion papers of two-part series by Phoon and Kulhawy (1999) which has extensive information about inherit uncertainty, measurement errors and transformation errors. Transformation errors are investigated for Standard Penetration Test, Dilatometer, Vane Shear Test, Plasticity Index and Cone Penetration Test correlations. When available, all the effecting information, such as type of testing, sampling method and soil type, are provided in the study. Additionally, range of mean values of the soil properties are also provided since reported COV values are applicable only for this range. With this information, in the case of lack of data, those values can be extrapolated for an existing design project and can be utilized for a general geotechnical usage for a certain field information. Summary of the study is provided in the Table 2.4 as an approximate design guideline. Within the content of the present study, only the relevant parameters are provided. For other details such as individual COV values of inherent uncertainty, measurement and transformation errors, the reader is referred to Phoon et al. (1995) and Phoon et al. (1999).

Table 2.4 COV's for geotechnical properties (Phoon et al. 1999)

Property	Test	Point COV (%)	Spatial Avg. COV (%)
c_u (UC)	Direct (Lab)	20-55	10-40
c_u (UU)	Direct (Lab)	10-35	7-25
c_u (CIUC)	Direct (Lab)	20-45	10-30
c_u (field)	VST	15-50	15-50
c_u (UU)	q_T	30-40	30-35
c_u (CIUC)	q_T	35-50	35-40
c_u (U)	SPT-N	40-60	40-55
c_u	K_D	30-55	30-55
c_u (field)	PI	30-55	-
Φ'	Direct (Lab)	7-20	6-20
Φ' (TC)	q_T	10-15	10
Φ'_{cv}	PI	15-20	15-20

Where; UC is unconfined compression test, UU is unconsolidated-undrained triaxial compression test, CIUC is isotropically-consolidated undrained triaxial compression test, TC is triaxial compression test, VST is vane shear test, Φ'_{cv} is constant-volume friction angle, q_T is cone tip resistance, K_D is dilatometer horizontal stress index and PI is plasticity index. Additionally, spatial averaging are done over 5 m in the study.

Considering the fact that reported values are valid under certain conditions, designers should be cautious and use these values including engineering judgments.

In addition to COVs, statistical distribution of the parameters is also necessary in order to carry out probabilistic analysis (e.g. uncertainty analysis). This can also be called frequency distribution. Ideally, adequate amount of samples should be repeatedly taken from field and sampling distribution of the parameters should be defined after necessary tests. Unfortunately, this is not always possible. Baecher and Christian (2003) states that sampling distributions of the mean values of soil properties are approximately "Normal". Another comment on the distribution from Duncan (2000) says that "three-sigma rule", which uses normally distributed parameters, described by Dai and Wang (1992) can be used for when having limited or no data. This simple method covers the 99.73% of all values and it covers nearly all possible values. Although this is true for other distributions (Harr 1987) and three-sigma rule is not directly connected to a certain distribution, Harr (1989) stated that one can assume normal (Gaussian) distribution in case of having only expected value (mean) and standard deviation (σ) since it is the least biased method. Baecher and Christian (2003) also states that most of measures of the soil strength can be modelled by normal distribution and says that *"from a random process model view, soil strength is an averaging process, and we should expect the results to display a tendency towards Normal distribution"*. According to Baecher and Christian (2003), in "Central Limit Theorem" developed by Laplace in 1738, distributional summation of large number of random variables converge to normal distribution. Baecher and Christian (2003) also provided probability of a failure of various distribution (Figure 2.17), in which, it can be seen that for most of the ranges, normal distribution is conservative.

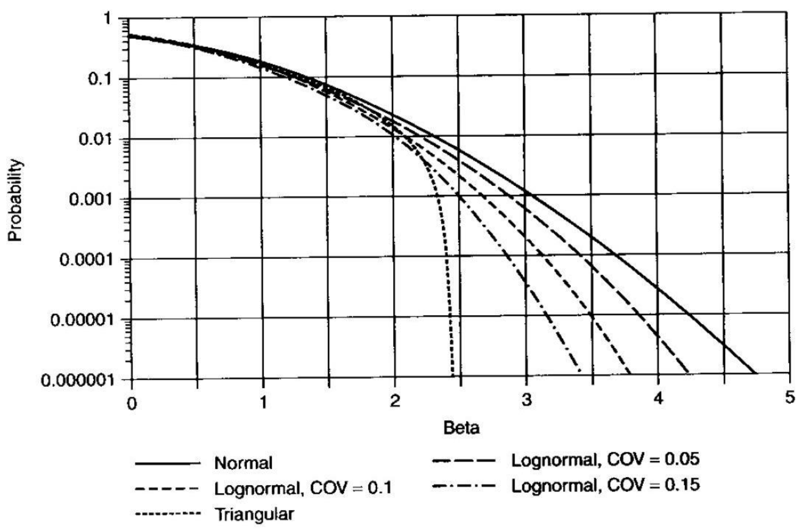


Figure 2.17 Probability of failure (PF) vs Reliability index (RI) for Normal, Lognormal and Triangular distributions (Baecher et al. 2003)

It is also shown by several other studies (JCSS 2001, Lumb 1966, Schultze 1971) that soil properties suit to the normal distribution. However, Jiang et al. (2014) used lognormal random field because the lognormal random variable is a continuous variable and strictly nonnegative, which complies with the physical meaning of most geotechnical parameters (e.g., cu, c, and f). The lognormal random field is frequently used to model the inherent spatial variability of geotechnical parameters and has been shown to perform well in the geotechnical literature (Griffiths et al. 2002; Griffiths and Fenton 2004; Cho 2010; Tabarroki et al. 2013).

2.3 Probabilistic Slope Stability Analyses

Probabilistic slope analysis is developed in order to consider the uncertainties and variability in material properties. It is essentially the evaluation of the performance function. In the case of slope stability issues, this function is the distribution of the factor of safety and it occurs from the statistical characteristics of the input parameters and chosen method. As it is stated and illustrated in the previous section, geotechnical parameters are quite variable inherently. Therefore, there are uncertainties and engineers should adapt a probabilistic analysis approach into their designs. In some of the other disciplines the terms “safety margin” is defined by the difference between capacity (resistance) and applied load. However, geotechnical engineers are mostly familiar with the form of ratio between resistance and load. Although the applied load has also uncertainties, geotechnical engineers are mostly interested with the probabilistic sampling of input material properties (e.g. capacity).

In order to carry out a reliability analysis (probabilistic analysis), there are several methods available and some of them are briefly summarized in the following subsections.

2.3.1 First Order Second Moment (FOSM) Method

This method depends on the first order Taylor’s Series expansion and neglect the higher order terms. Therefore, the method uses the first order approximation of the mean, standard deviation and variance of the performance function. In the slope stability works, this function is the factor of safety which is calculated by the method of slices. General calculation steps of reliability index are as follows:

(1) Determination of all variables that affect the performance function.

(2) Determination of the mean values of each variable and calculation of the best estimate of performance function (mean value of the function).

(3) Choosing the appropriate uncertainty for each variable including spatial variability, estimation errors, data scatters etc. Then, variance of each variable is calculated.

(4) Finding the effect of each variable to the performance function. This is done by taking partial derivatives of the performance function with respect to each variable separately.

(5) Computing the variance of the performance function.

(6) Calculating the reliability index from performance function by assuming its distribution as normal.

One of the great advantages of this method is that it can show the relative contributions of each random variable to the performance function. This can be useful since the designers can focus on the most effecting variable. This is not provided in many of the other methods. On the other hand, main disadvantage of the method is to deal with the partial derivative of the performance function, especially if it has a complicated form.

2.3.2 Point Estimate Method

This method is established by Rosenblueth in 1975 and it is formerly known as Rosenblueth's Method. The owner, however, called the procedure as the "point estimate method". It's an easy to use and straightforward method where the designers are not required to know much of a probability theory. The method approximates the moments of the performance function that contains random variables. As it is, first and second moments of a function are mean and variance, respectively and the square root of the variance is the standard deviation. General procedure steps are as follows:

(1) Determination of all variables that affect the performance function.

(2) Determination of the mean and standard deviation of all variables.

(3) Calculation of the performance function by using all possible combinations of the random variable in the range of plus and minus 1 standard deviation. In other words, if there are 2 random variables (say c and Φ), there will be 4 calculations for performance function. This is calculated by 2^n where n is the number of random variables.

(4) Assigning a weight for each combination by using the formulae$(1 \mp \rho)/4$ where ρ is the correlation value between random variables. It is 1 if two variable is perfectly correlated and -1 if they are inversely correlated. In case of uncorrelated variables, it is zero and weight is 0.125 and constant. Additionally, in the formulae, it is plus when both correlated variables are on the same side of their mean values (e.g. 1 standard deviation below or above) and minus when opposite.

(5) Calculation of mean and variance, therefore standard deviation, of the performance function.

(6) Calculation of the reliability index by using this mean and standard deviation and assumed distribution of the performance function.

(7) Calculation of the probability of failure after assuming a distribution to the performance function.

At first glance, it is seen as over approximate approach. However, its satisfactory accuracy is shown by Baecher et al. (2003) by several numerical case studies, including a slope stability analysis, and it is shown that the method exactly agrees with the theoretical values for a range of practical geotechnical problems. Aside from its simplicity, it can also incorporate the correlation between variables. Additionally, variables can have different statistical distributions. However, when the number of random variables increases, number of the computations will also increase in the form of 2^n. Therefore, the method is well-suited for the problems that have few variables (2 to 6). In general, the method is accurate for the performance function which can be represented by a third-order polynomial or less.

2.3.3 First Order Reliability Method (FORM)

In both of the previous methods, there are two main assumptions which is not usually valid. First one is that having only mean and variance of the random variables and their linear combination is an accurate estimation for a performance function and its moments. Second assumption is to know the form of distribution of performance function so that probability of failure of performance function can be obtained from reliability index information. To overcome these assumptions, Hasofer and Lind

(1974) proposed another method which is also known as "First Order Reliability Method". Baecher et al. (2003) showed for a simple vertical cut problem in cohesive soil that FOSM gives different results depending on how the performance function is constructed. Their proposed method was a geometrical interpretation in which the reliability index is the closest distance failure criterion and the point defined by the mean values of the variables. In the method, they defined a dimensionless variable using the mean and standard deviation of each variable and new performance function by using these dimensionless variables as shown in Equation 2.3 and Equation 2.4. Then, they minimized this new performance function by spreadsheet, iteration etc.

$$x_i' = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \tag{2.3}$$

$$x' = \{x_1', x_2', x_3' \dots \dots \dots \dots \dots x_n'\} \quad and\ g(x') = 0 \tag{2.4}$$

Where; x_i are variables, g is performance function μ is mean and σ is standard deviation of variable.

2.3.4 Monte Carlo Simulation Method

This method is based on randomized input in which variables affecting the performance function is randomly selected from a region of interest and results on the performance function is evaluated. Generalized steps can be defined as follows:

(1) Obtaining geometry of the slope and the most likely (mean) values of the required soil parameters.

(2) Choosing the shear strength model (Mohr-Coulomb, Hoek-Brown etc.)

(3) Determination of the randomly treated input parameters (unit weight, cohesion, internal friction angle, pore water pressure coefficient etc.)

(4) For each random variable, choosing the relevant COV from the literature unless information on the variability of site-specific soil is sufficient.

(5) Choosing the statistical distribution (normal, lognormal etc.) of the random parameter.

(6) Choosing a number of required analysis (N) and sampling method.

(7) Choosing the shape of a slip surface (circular, non-circular etc.) if necessary.

(8) Carrying the slope stability analysis N-times.

This procedure will result in; (i) a statistically distributed, N-times factor of safeties, (ii) probability of failure, (iii) critical probabilistic failure surface and (iv) reliability index depending on the FS distribution. Among the results, (ii) and (iv) should be calculated from the factor of safeties. They can be identified as follows:

(1) Probability of failure (PF) is, in general terms, ratio of the number of analyses that end up with factor of safety smaller than 1.0 to the number of total analyses.

(2) Reliability index (RI) is the representation of the standard deviations between mean value of FS's and critical FS which is 1.0. According to Baecher et al. (2003), RI should be at least around 3.0 to assure the safety of the slope. However, of course, this depends on the desired safety level in the specific project. RI can be calculated as:

i. If distribution of the resultant FS's is normal: $\beta = \frac{\mu - 1}{\sigma}$

ii. If the distribution is lognormal: $\beta_{LN} = \frac{\ln[\frac{\mu}{\sqrt{1+V^2}}]}{\sqrt{1+V^2}}$

Where; β is reliability index, μ is the mean of FS's, σ is standard deviation of the FS's and V is the COV of the FS's (σ / μ).

In order to carry out this method, random numbers have to be generated from a given statistical distribution of the input variables. As stated by Baecher et al. (2003) most of the random number generators use a concept called "linear congruential algorithm" which applies modulo operation (Equation 2.5).

$$Z_{i+1} = a * Z_i + b \ (mod\ m) \tag{2.5}$$

Modulo operation simply gives the remainder of the division of the linear equation by m. The first values is generally called "seed" and can be determined by the user. This can be useful if another user wants to create the same set of numbers by simple using the same initial seed value. Advantages of modulo operation are:

(1) The longest, non-repeating series of number cannot exceed the number m.

(2) Newly generated numbers depend only on the previous one.

(3) It is repeatable.

Drawback of this method is that generated number may not cover the necessary region of interest depending on the values a, b, m, seed and the number of realization. In addition to that, depending on the probability of failure of the performance function, necessary random number realization can be too much. There are several other alternative methods for this sampling purpose. Some of them are namely: importance sampling, antithetic sampling, correlated sampling, control variates, stratified sampling and Latin Hypercube. However, the details of these are not within the scope of this thesis. Interested readers are referred to Baecher and Christian (2003) among other literature.

2.4 Random Finite Element Method

As explained in the introduction section, RFEM is an extension to FEM. The method is the combination of FEM and random field theory (Fenton and Vanmarcke 1990). In this method, random fields of material properties are generated and mapped onto the finite element mesh. The spatial variation of material properties can be correlated to each other by using "spatial correlation length", which is sometimes referred to, in the literature, as the "scale of fluctuation (or autocorrelation length)". This parameter describes the distance over which spatially random variables will tend to be significantly correlated (Griffiths and Fenton 2004). Therefore, large values of spatial correlations length means smoothly varying (more uniform) field. Theoretically, the value of infinity would mean a homogeneous field. This value roughly means that soil samples taken close to each other will be more likely to have similar material properties than that of faraway samples. One of the distinct features of this method, while accounting spatial variability of soil, is that it can seek-out the weakest, most critical path and it does not have to be a certain shape such as a circular surface (Griffiths and Fenton 2004) by using main advantage of FEM.

There is also "anisotropic spatial correlation" in which soil is likely to have longer spatial correlation lengths in the horizontal direction as compared to vertical direction since most soils are deposited vertically (Griffiths and Fenton 2004), (i.e. soils are more uniform in horizontal direction as compared to vertical direction). Nonetheless, in practice, spatial correlation is usually assumed to be isotropic and this is called “stationarity” (Baecher et al. 2003).

In order to carry out the RFEM analyses, G.A. Fenton and D.V. Griffiths created a series of software in 1992. Among those, software named Rslope2D is for 2D finite element analyses of slope stability that considers spatial correlation. It is an open-source coded, publically available, and free of charge software. The software takes into account of the mean, standard deviation and spatial correlation length of the input parameters as well as local averaging of the properties over the finite elements (Fenton and Vanmarcke 1990). The software applies the Monte-Carlo framework.

2.4.1 Spatial Correlation Length

As stated in the previous section, information about scale of fluctuation (spatial correlation length) is scarce. However, it is possible to find a number of studies about the subject. One of them is carried by Phoon et al (1999) and partially provided in Table 2.5. It is a summary of the available literature at that time and also includes number of the studies per geotechnical parameters and the soil types. Lastly, since both of the available studies and data is quite insufficient, these values should be used with caution.

Table 2.5 Spatial correlation length of some geotechnical properties (Phoon et al. 1999)

Property	No. of study	Scale of fluctuation (m)	
		Range	**Mean**
Vertical Fluctuation			
c_u	5	0.8-6.1	2.5
c_u (VST)	6	2.0-6.2	3.8
SPT-N	1	-	2.4
γ_b	1	-	1.6
γ	2	2.4-7.9	5.2
Horizontal Fluctuation			
c_u (VST)	3	46.0-60.0	50.7

Another study is conducted by Huber (2013) as a PhD Thesis at University of Stuttgart. In the study, the author investigated a wide variety of literature and created three databases. These are for rocks, cohesive and frictional soils. Most of the data are based on CPT measurements and do not consider sampling, measurements and statistical uncertainties. The database consists of parameters such as permeability, porosity, modulus of elasticity, pre-consolidation pressure, void ratio, sand fraction, and water content. Most of the reported parameters are out of interest of the current study, related ones are provided in Table 2.6.

Table 2.6 Spatial correlation length of some geotechnical properties (Huber 2013)

Property	Soil Type	$\theta_{vertical}$ (m)	$\theta_{horizontal}$ (m)	Source
$\Phi' + c'$	Sand	5.1	242	Suchomel et al. 2010
Bulk Density	Medium gravel	0.2	-	Tillmann et al. 2008
	Coarse sand, fine gravel	0.3	-	Tillmann et al. 2008
	Medium gravel	0.4	-	Tillmann et al. 2008
Density	Sand	3	-	Ouellet et al. 1987
	Compacted Clay	5	4-5	Baecher et al. 1980
	Compacted Clay	-	10-15	Alber et al. 1986
Modulus of Elasticity	Clay	2-5	-	Jaksa et al. 1999
	Clay	6	298.5	Jaksa et al. 1997
	Clay	-	400	Marache et al. 2009
Φ'	Clay	-	800	Raspa et al. 2008
Unit Weight	Soft Silty Loam	0-3	17-22	Alonso et al. 1975
Unit Weight	Soft Clay	1.2	-	Vanmarcke et al. 1975

Where; θ is correlation distance.

Although developing trend of spatial correlation works are increasing, the literature about spatial correlation length is still quite insufficient and not well documented. In the case of an insufficient data, the approximate spatial correlation length can be used as 0.1 to 0.25 of the size of the problem geometry in each direction (Griffiths and Fenton 2004).

Following paragraphs are devoted to summarize the literature studies which consider the outcome of spatial correlation. The content is restricted to only slope stability work but other works related to geotechnical problems also widely available in the literature. A book written by Griffiths and Fenton (2007) has a variety of spatial variability application in other engineering problems such as seepage problems, settlement analyses, mine pillar stability works and bearing capacity calculations.

Griffiths et al. (2004) makes comparison between RFEM and simple approach that does not consider the spatial correlation. For this purpose, test slope having 1V:2H inclination and undrained subsoil condition is used. Only undrained shear strength is treated as spatially random variable. First, deterministic analysis is carried and, then, simple probabilistic analysis with single random variable is considered. After these, spatial variation is included into the analysis. After observing dependency between PF and COV level, several other analyses are carried out for different COV levels. At the end of the study, it is found out that ignoring the spatial variability will overestimate PF when COV is relatively small, whereas it will underestimate the PF when COV is relatively high. One of the outcome is provided below for the sake of comparison. In the graph below, correlation length values are normalized by slope height which is used as 10 m.

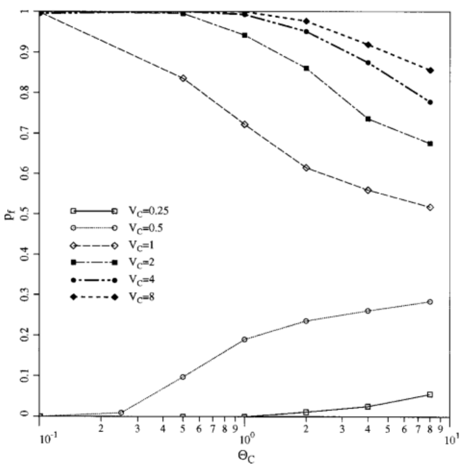


Figure 2.18 PF vs spatial correlation length via RFEM with mean c_u = 50 kPa (Griffiths et al. 2004)

Schweiger et al. (2005) proposes a “random set method” in order to account for the spatial variability. This method is not exactly same as RFEM developed by Griffiths and Fenton, but it is worth to mention. In this study, homogeneous slope having 1V:2H inclination and undrained soil conditions is analyzed. In the analysis, only undrained shear strength is used as a spatially random parameter and others are kept deterministic. Different COV levels and spatial correlation lengths are used in order to see the correlation between these two and probability of failure. No comparison between conventional methods that do not consider the spatial variability are made. However, results are compared with the Griffiths and Fenton (2000) study that uses random field approach and analyzes the same slope. It is seen that PF is affected by both COV and spatial correlation value. One of the results is provided below for the demonstration purposes.

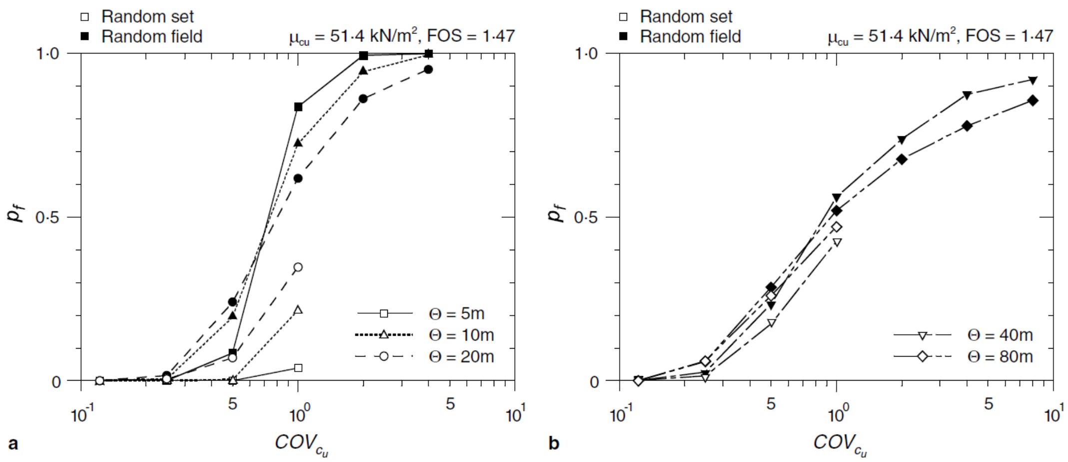


Figure 2.19 Influence of spatial correlation and COV on PF a) small correlation lengths b) large correlation length (Schweiger et al. 2005)

Cho (2007) study focuses on the stability of slopes with Monte-Carlo simulations that considers the spatial variability. For that purpose, two example analyses are carried out. In the first one, two-layered slope having 1V:2H inclination, frictionless upper layer and cohesive-frictional bottom layer condition is analyzed. Effects of correlation length and anisotropic spatial correlation are studied for a fixed COV level. All cohesions, friction angles and unit weights are used as variable parameter. It is seen that PF decreases with decreasing correlation length and isotropic field assumption yields conservative results. Obtained results are provided below.

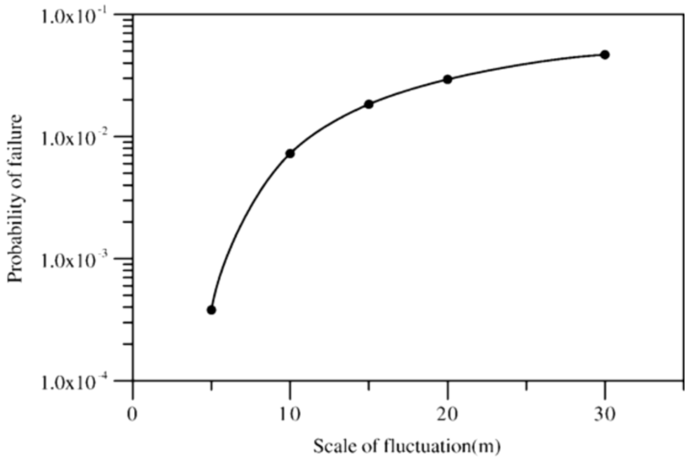


Figure 2.20 Scale of fluctuation vs. PF for isotropic case (Cho 2007)

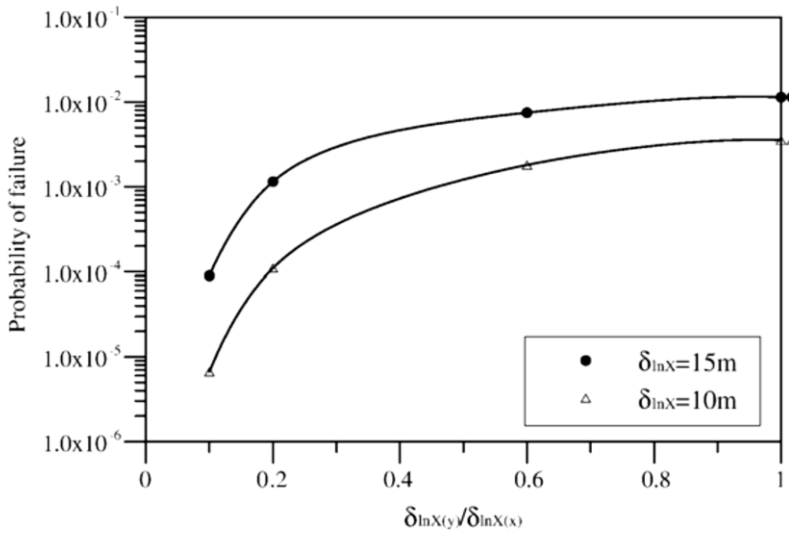


Figure 2.21 Scale of fluctuation vs. PF for anisotropic cases (Cho 2007)

In the second example, stability of the Sugar Creek embankment reported in White et al. (2005) is used along with its geotechnical parameter. Cross-section of the slope is provided below. Although deterministic and probabilistic failure surfaces are slightly different, they both passes through relatively weak layer and have similar safety levels.

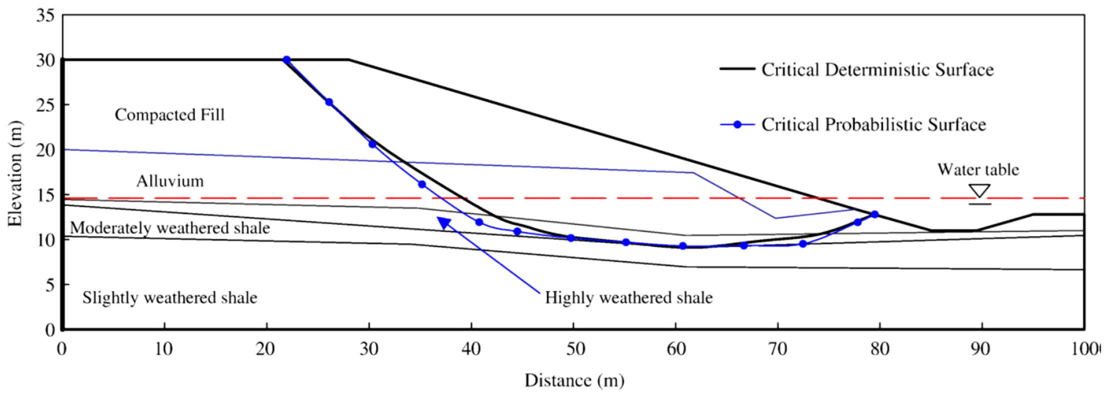


Figure 2.22 Cross-section and slip surfaces of Sugar Creek embankment (Cho 2007)

For this case, scale of fluctuation is not known. Therefore, author uses three correlation length values from the literature as an assumption. Since it is found conservative, isotropic spatial correlation is adopted for all analyses. It is found that PF values for 20 m, 40 m and infinity correlation lengths are 0.12 %, 0.88 % and 1.18 %, respectively.

Griffiths et al. (2009) studies the effects of spatial correlation length on PF by using undrained and cohesion-frictional cases with three different slope angles. Additionally, Monte-Carlo simulation without spatial correlation is carried out in order to make comparison. For undrained case, test slope used in Griffiths et al. (2004) is used with same geotechnical parameters for 1V:1H, 1V:2H and 1V:3H slope inclinations. It is seen that not considering spatial correlation underestimates PF at lower COV values for steeper slopes than flatter slopes. Additionally, for flatter slopes, it is seen that failure surfaces are usually deep and passes through the foundation layer, however, for steeper slopes, slip surface may pass through the toe or pass through deeper levels with higher PF. On the other hand, when spatial correlation is considered, PF of steeper slopes are higher than that of flatter slopes. For the second case, cohesion-frictional slope, similar analysis are carried out. It is observed that inclination has negligible influence on PF and it is also concluded that not accounting for spatial correlation overestimates the PF for relatively low COV whereas underestimates the PF for relatively high COV levels. Exact same result is obtained by Griffiths et al. (2004) study for undrained case. Obtained PF values of drained slope of 1V:3H are provided below.

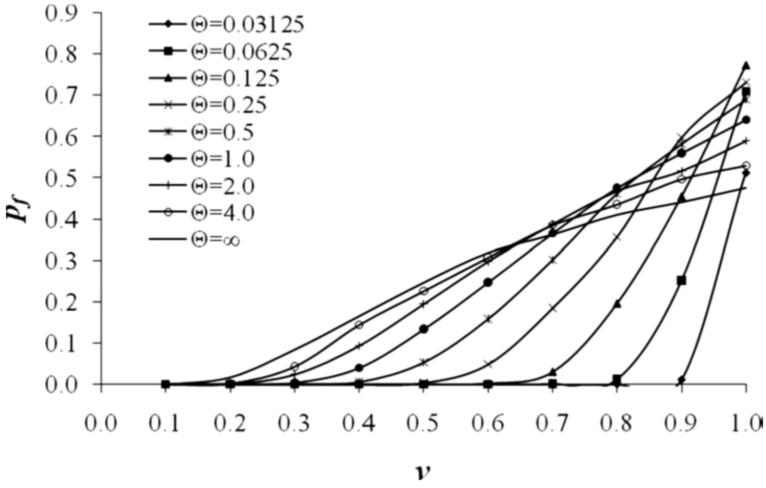


Figure 2.23 PF vs spatial correlation length via RFEM for drained slope (Griffiths et al. 2009)

Cho (2010) study extends the traditional LEM analysis to a probabilistic approach that considers spatial correlation of geotechnical parameters. For that purpose, slope stability routine is created via FORTRAN and random fields are generated via MATLAB. Analyses are done for two cases, namely undrained and cohesion-frictional cases. For the first case, slope having 1V:2H inclination and 5 m height is analyzed. Two types of Monte-Carlo simulations are considered. In the first one, failure surface search is done for every generated random field whereas second type makes probabilistic analysis on previously determined critical failure surface based on mean values. Results of the analyses show that overall PF of random LEM is much bigger than case for fixed critical surface. The reason for that random LEM can search the most critical surface throughout the heterogeneous soil region created by random field.

For the second case, cohesion-frictional slope having 1V:1H inclination and 10 m slope height is analyzed. In addition to the previous analysis, cross-correlation between cohesion and friction angle is also included into the analyses. Obtained results are also compared with FORM analyses. PF vs cross-correlation coefficient results are provided below. It is seen that difference between fixed slip surface and searched slip surface are not that much in terms of PF. However, FORM analyses which use single random variable overestimates the PF.

deWolfe et al. (2010) study analyzes the Fruitgrowers Dam located in Delta Country, Colorado. The study uses a program called PES (Probabilistic Engineering Slopes) coded in FORTRAN 95. According to the authors, it's a 2D probabilistic FEM slope stability analysis software which uses Mohr-Coulomb failure criterion. The study also utilizes Slope/W in order to compare PES performance. Slope/W software can perform 1D soil property correlation along the failure surface whereas PES uses RFEM concept and 2D correlation. Analyses are done for both lower and higher COV levels and it is seen that Slope/W overestimates the PF values for both high and low COV level.

Babu et al. (2012) study focuses on the stability of municipal solid waste landfill. It compares the conventional method where geotechnical parameters are constant to the method that considers the spatial variation. For that purpose landfill with 30 m height and 1V:3H inclination is analyzed. Detailed cross-section and properties of each layer can be found in the paper. In general, it is found that safety level is lower when spatial correlation is considered under the given condition of the study.

Jiang et al. (2014) study developes a Monte-Carlo simulation based approach for evaluation of PF. For that purpose, instead of using a large number of potential slip surfaces, such as in conventional LEM analyses, the study uses what is called representative slip surfaces (RSS). According to the study, this RSS approach is used by many other researchers and found to be an efficient method. After determining the necessary number of RSS, multiple stochastic response surface for each RSS is constructed. Details of these two procedure can be found the original paper. However, in general, a number of random fields are generated by Latin-Hypercube sampling method. Then, for every field, deterministic slope stability analysis is performed by LEM and a number of RSS are found. Then, for every RSS, stochastic response surface is constructed and Monte-Carlo simulation is performed for every response surface to. Then, minimum factor of safety is calculated for every realization and, therefore, PF of the system is determined. The study uses two illustrative example. In the first one, undrained slope having height of 5m and 1V:2H inclination used by Cho (2010) and its geotechnical parameters are used. For this example, as stated in Cho (2010), mean undrained shear strength and COV value are 23 kPa and 30 %, respectively. Only undrained strength is treated as spatially random and vertical and horizontal correlation length used are 2 m and 20 m, respectively. These values are directly taken from Cho (2010). After LEM analyses with 1000 Latin-Hypercube sampling, 71 RSS are found including the critical deterministic slip surface (CDSS) which comes from the analysis using mean values. After the analyses, the study compares the found FS to that of Cho (2010) findings and concludes that proposed method is in good agreement with conventional Monte-Carlo simulation approach using 100000 samples. When the outcome of only CDSS is compared with the results of Cho (2010), it is again seen that the proposed method works well. In addition to that, it is confirmed that probability of failure of an undrained slope is underestimated when only CDSS are used for the Monte-Carlo simulations by giving a 2 times less PF value. After observing the efficiency of the method, several other analyses are carried by using different spatial correlation distances in order to see the sensitivity of the results with respect to autocorrelation length. These results are provided below. In the second example, cohesion-frictional slope of Cho (2010) is analyzed with the same proposed method. Geotechnical parameters are directly taken from Cho (2010). For this particular example, effect of cross-correlation between cohesion and friction is investigated. A good agreement between Cho (2010)'s method and the current method is seen and it is observed that using only CDSS is again underestimates the PF. These results are also provided below. Additionally, correlation between cross-correlation and number of RSS obtained is also studied but those results are not provided since they are out of the scope. As a conclusion, the author states that these results are solely depend on the utilized LEM which is Bishop's method and can be different for other analysis approaches.

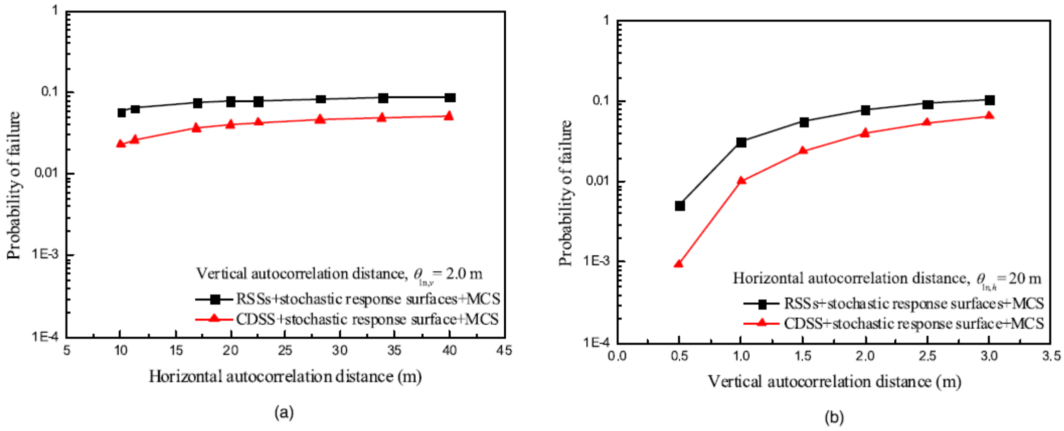


Figure 2.24 PF vs. correlation length for c_u soils a) fixed vertical correlation b) fixed horizontal correlation (Jiang et al. 2014)

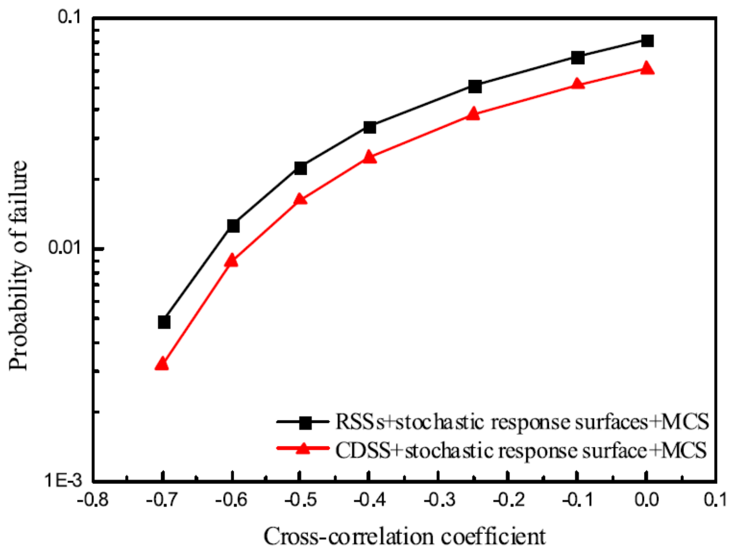


Figure 2.25 Cross-correlation vs. PF for c-phi soils (Jiang et al. 2014)

Le et al. (2014) study focuses on the influence of the spatial variability on the stability of the c-phi soils. Within this purpose, a series of analyses are done in order to determine the effects of autocorrelation length, COV level and cross-correlation between strength parameters. Effects of correlation length are also studied for different deterministic factor of safety levels. Therefore, this study is seen a very important literature source by the author of the current thesis. Slope geometry and geotechnical parameters are provided below. Monitoring points on the slope are placed so that sudden increase in displacement can be captured and counted as failure.

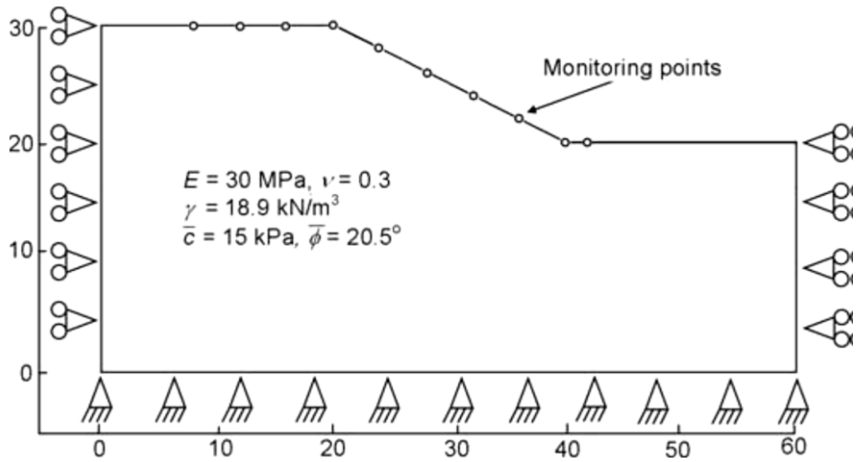


Figure 2.26 Slope geometry and mean geotechnical parameters. Dimensions are in meters (Le et al. 2014)

For the purpose mentioned above, 4 different scenarios are studied. These are namely: 1) random friction and uniform cohesion, 2) random cohesion and uniform friction, 3) random friction and cohesion with no cross-correlation and 4) random friction and cohesion with perfect cross-correlation. Then, for all 4 scenarios, probabilistic FEM analyses are performed under different COV and correlation length values. Only the strength parameters are taken variable and others are kept constant. COV of cohesion and friction are taken equal to each other when both are random. Result of COV vs. mean factor of safety and COV vs. probability of failure are provided below. For a selected case of spatial correlation value, deterministic slope stability analysis results after one random field realization. For these analyses, horizontal correlation length is taken 2 times the vertical correlation lengths.

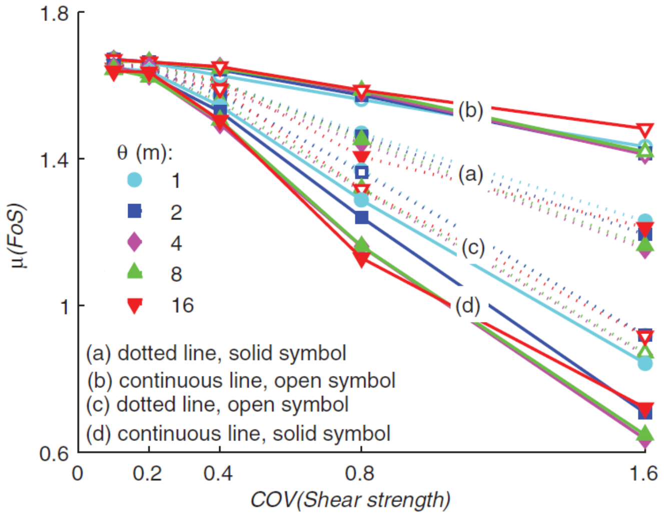


Figure 2.27 COV vs. mean FS for different correlation lengths a) scenario 1, b) scenario 2, c) scenario 3 and d) scenario 4 (Le et al. 2014)

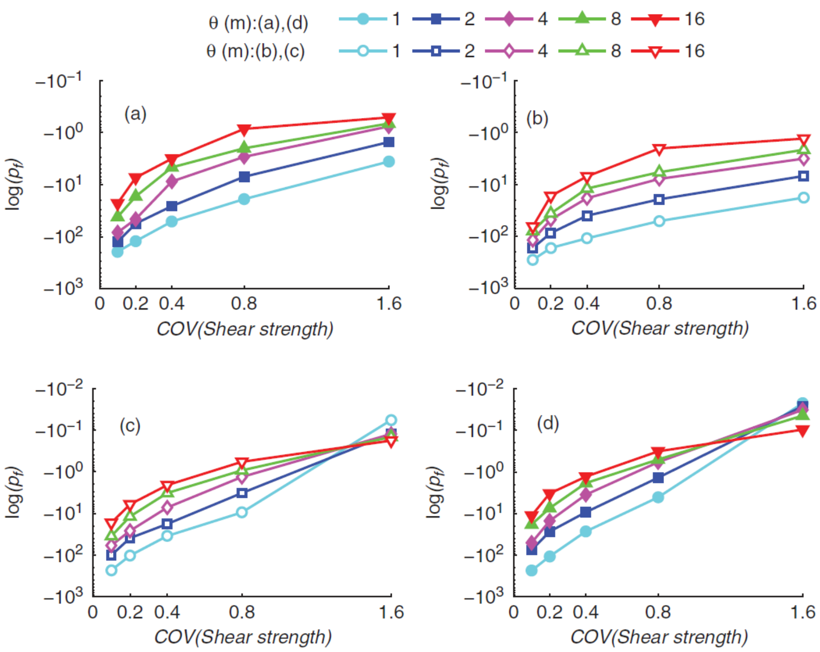


Figure 2.28 COV vs PF for different correlation lengths a) scenario 1, b) scenario 2, c) scenario 3 and d) scenario 4 (Le et al. 2014)

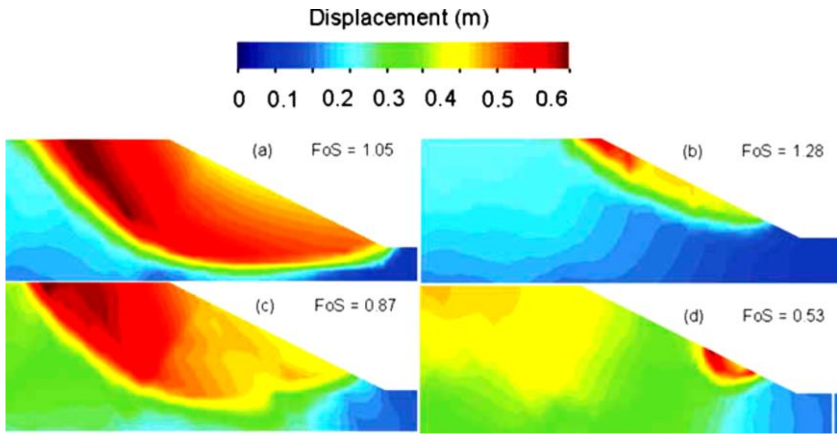


Figure 2.29 Slip surfaces for COV = 1.6 and $\theta_{hor.} = 2*\theta_{vert.} = 4$ m. a) scenario 1, b) scenario 2, c) scenario 3 and d) scenario 4 (Le et al. 2014)

As an additional purpose, the study rearranges the obtained results with respect to dimensionless correlation length by dividing with slope height (θ/H). After that, the authors try to fit a curve to each set of data on θ/H vs. PF graph and obtain a polynomial functions. Coefficients of these functions, of course, depend on COV level and the geotechnical parameters. The study repeats this procedure for other results obtained from the literature. These are Griffiths et al. (2004), Babu et al. (2004) and Hick et al. (2002). For illustration purposes, outcome of one of these fitted curve work is provided below.

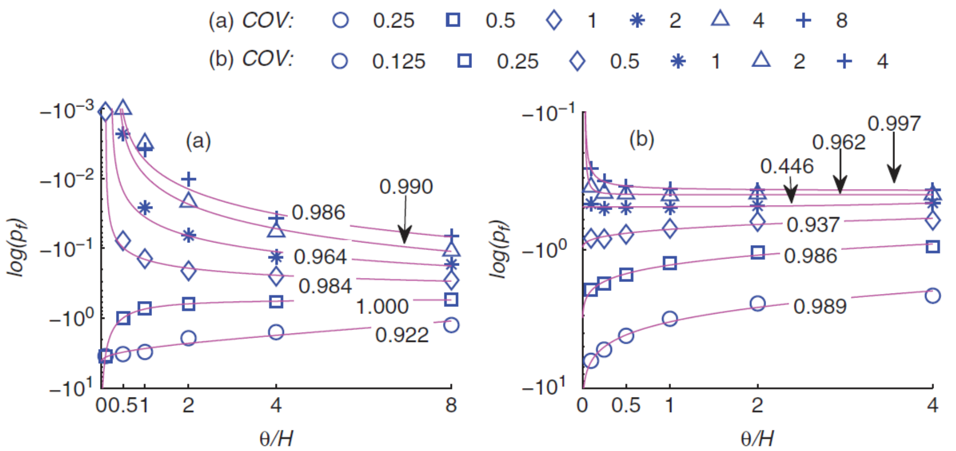


Figure 2.30 θ/H vs. PF fitted curves for different COV values a) Griffiths et al. (2004) data and b) Babu et al. (2004) data (Le et al. 2014)

Numbers on each fitted curve means how well the curve fits the data set (e.g. R^2, coefficient of determination). Although obtained coefficients are not provided here, constructed equations are in the form given below.

$$\log(PF) = A * \left(\frac{\theta}{H}\right)^{B} + C \; where\; A, B\; and\; C\; are\; coefficients \tag{2.6}$$

At the end of the study, several useful conclusions are made. First of all, relative to the scenarios where only 1 strength parameter is used as random, scenarios taking both of them as random increase the PF even for without cross-correlation. Second, as the COV increases, mean FS decreases whereas PF increases. Third, contribution of friction angle on slope stability has more significance than that of cohesion. Fourth, compared to no correlation case, positive correlation increases the probability of failure. Fifth, as the COV increases, PF of the scenarios 1 and 2 increases and after certain points it shows a trend towards constant behavior. However, this changes for scenarios 3 and 4 after certain COV level. Sixth, for scenarios 3 and 4, PF increases faster as COV increases for smaller correlation lengths. Therefore, as far as the analyzed cases considered, autocorrelation distance, COV levels and cross-correlation play significant role on obtained PF values. However, as a result of side analyses, preferred autocorrelation function (Gaussian and Markov functions) makes only negligible effect.

Le et al. (2015) study investigates the influence of spatial variability of the void ratio on unsaturated slope stability subjected to rainfall infiltration in terms of factor of safety and the sliding mass. For that purpose, slope having a 10 m height and 1V:2H inclination is analyzed by using FEM concept. Slope geometry including GWT and meshes are given below. Before applying the gravity loading, spatial field for void ratio is produced and mapped onto the slope geometry. After that, Monte-Carlo simulation is carried with two steps. First one is an infiltration of rainfall for a chosen time period in order to create the stresses, strain and pore water pressure inside the slope. Second step is the strength reduction method so that factor of safety and sliding mass area can be obtained. For the analysis, software named CODE_BRIGHT is used. Details for hydraulic and mechanical properties and their behavior can be found in the original research paper. Besides from FEM computation, same slope is also analyzed with Seep/W and Slope/W. Water pressures are created from Seep/W and used in Slope/W for LEM slope stability analysis for the same rainfall and geotechnical parameters. Resulting curve shows that both method gives similar results although LEM FS values are slightly higher than that of FEM.

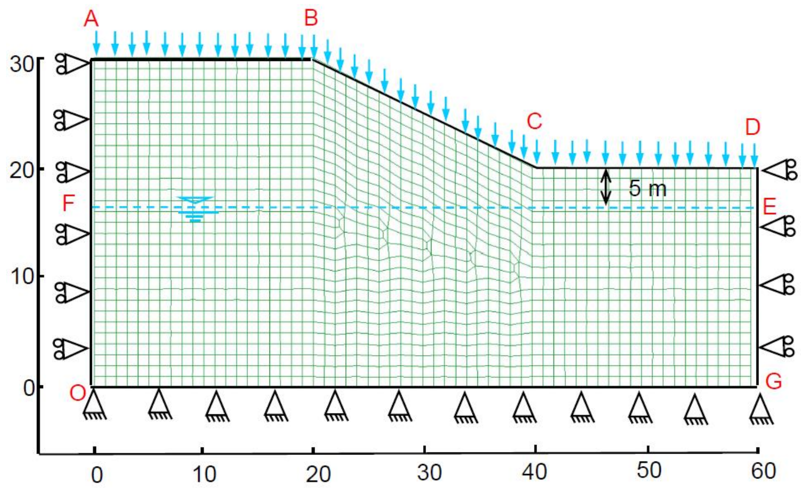


Figure 2.31 Slope geometry with meshes. Units are in m. (Le et al. 2015)

For the first set of analyses, the paper investigates the correlation between COV of void ratio vs. PF and autocorrelation length vs. PF. For this set of analyses, spatial correlation fields are taken as isotropic. In other words, vertical and horizontal correlations are equal to each other. Following resulting curves are obtained. Units of correlation distance in the graph is meter.

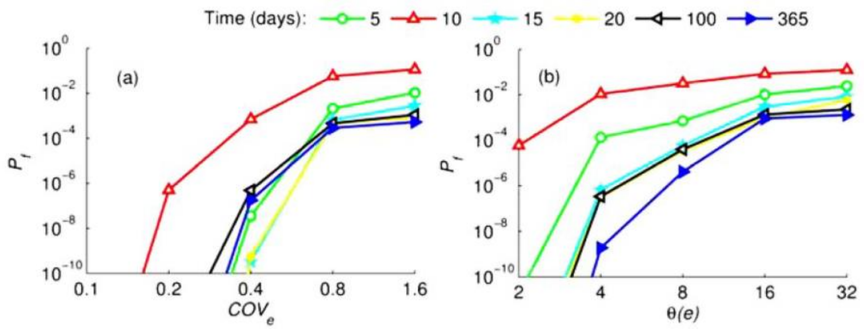


Figure 2.32 COV vs. PF and autocorrelation vs. PF for different times under rainfall (Le et al. 2015)

After the first set of analyses, effects of COV and anisotropic correlation are studied. From these analyses, sensitivity of mean FS and COV of FS are obtained with respect to different COV, autocorrelation and anisotropy of void ratio. These results are provided below with the necessary legends in the graphs. For all curves, e is void ratio, θ is correlation length (in meters), α is anisotropy ratio (θ_h / θ_v) and μ is mean value of the property. At the end of the study, several conclusions are made. First, mean FS decreases as the rainfall water infiltrates and it becomes minimum at the 10 days which is just before rainfall stops. Second, as the COV_e increases, mean FS decreases till 100 days and opposite occurs afterwards. Third, increase in θ results in increase in COV_e. This is probably because of the fact that a single slip surface can pass through less region with different parameters as the correlation length increases. Fourth, changing both α and θ have bigger effects on the results. Fifth, trends of PF are similar for both COV_e and θ_e.

As a last set of analyses, sensitivity of mean sliding mass area and COV of sliding mass area are investigated with respect to different COV, autocorrelation and anisotropy of void ratio. These results are also provided below. As conclusion, first, trend of mean and COV of sliding mass area are similar for different COV_e values. Second, as COV_e increases, variation of the sliding mass area also increases. Third, increase in θ or decrease in α results in bigger variation in size of the sliding mass. In addition to these, similar conclusions and trend like for FS are observed.

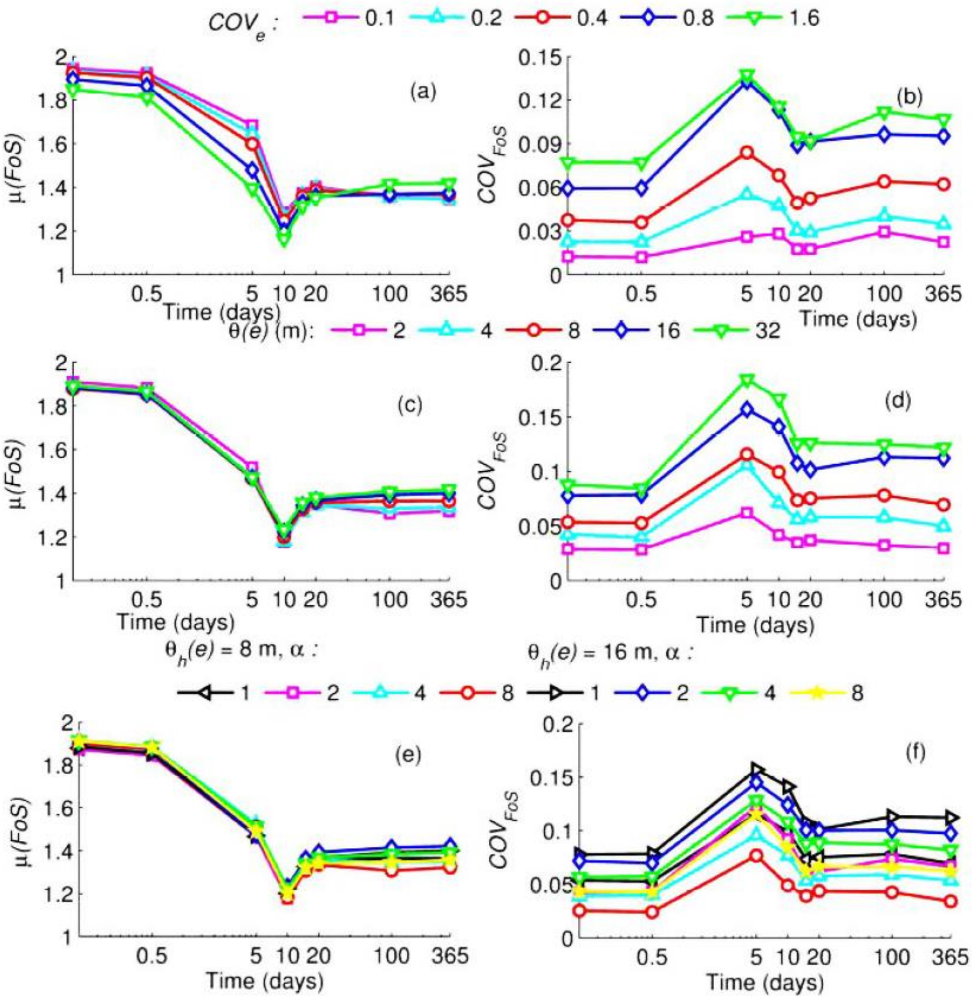


Figure 2.33 Sensitivity of mean FS and COV of FS under different a, b) COV (μ_e = 0.5, θ_e = 8 m.) c, d) θ (μ_e = 0.5, COV_e = 0.8) e, f) α (μ_e = 0.5, COV_e = 0.8) (Le et al. 2015)

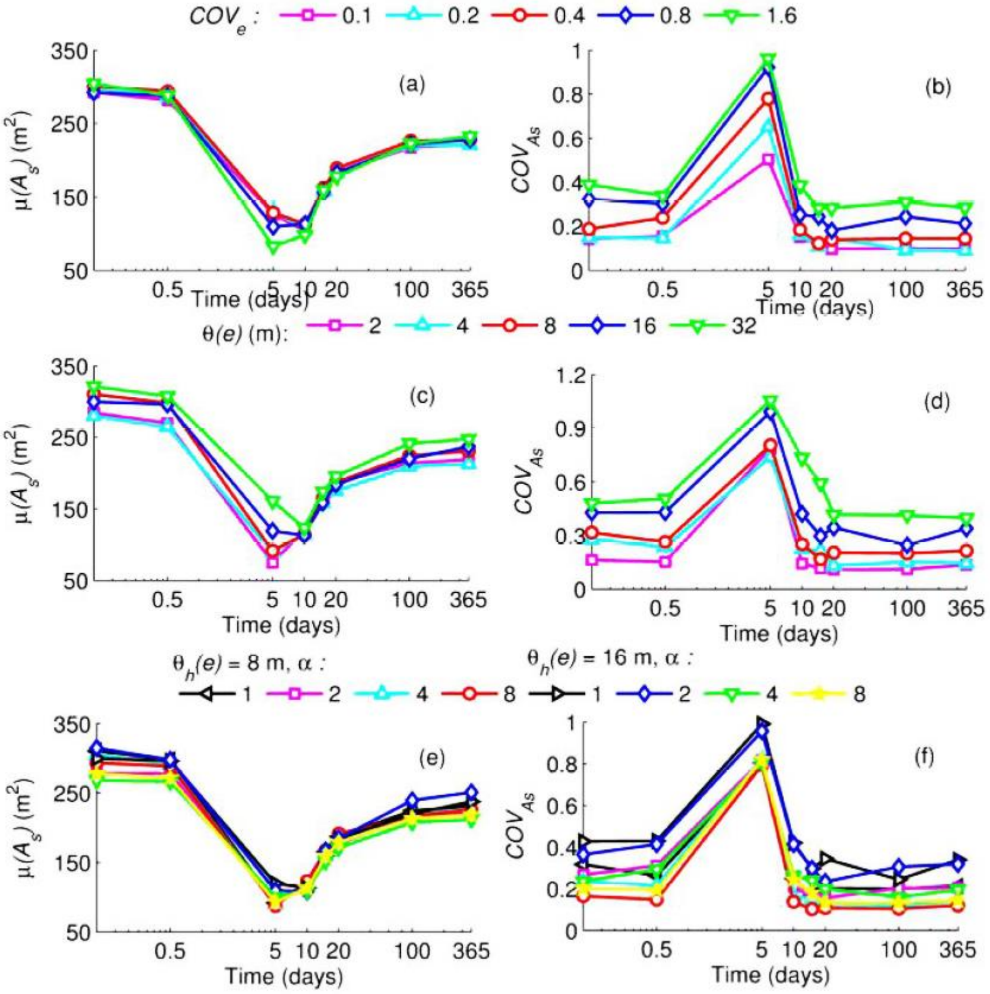


Figure 2.34 Sensitivity of mean sliding area and COV of sliding area under different different a, b) COV (μ_e = 0.5, θ_e = 8 m.) c, d) θ (μ_e = 0.5, COV_e = 0.8) e, f) α (μ_e = 0.5, COV_e = 0.8) (Le et al. 2015)

In summary, as can be seen from the literature studies, there are extensive amount of researches performed on spatial variability of soil parameters and probabilistic slope stability. To determine the safety level and the most critical failure surface in a slope, many of the studies in the literature agree that deterministic slope stability analyses, giving a single value of FS, are insufficient. The probability of failure and critical failure surface depend on the methods used and statistical characteristics of the variables (statistical distribution, COV value, whether spatial correlation length is considered or not and if considered its value etc), however the suggested conclusions are not general enough and universal. It is mostly agreed that as COV increases, the probability of failure increases. For a given COV value, as spatial correlation length increases, PF increases. However there are a few studies that conclude the opposite. Isotropic random field assumption yields conservative results (i.e. higher probability of failure). This thesis study focuses on comparing different approached of probabilistic slope stability analyses, and the influence of the spatial variability on the probability of failure and the critical failure surface in undrained clays, c-phi soils and cohesionless soils.

CHAPTER 3

METHODOLOGY

This chapter gives information about the methodology used in this study, such as the technical details of the calculation methods and software that are utilized for analyses. Throughout the upcoming chapters, several geotechnical tools are used for both LEM, FEM and RFEM. As stated in the introduction chapter, these are namely Rocscience Slide, Slope/W of GeoStudio, Rocscience Phase2, PLAXIS 2D and Rslope2D.

3.1 Limit Equilibrium Analyses: Slide and Slope/W

Rocscience Slide v6 is one of the 2D limit equilibrium slope stability analysis tools. It assumes a failure surface shape; divides the soil mass inside this failure surface into a number of vertical slices and considers force and moment limit equilibrium. If desired, it also has finite element seepage analysis option as a built-in function to consider the pore pressures obtained from seepage analysis. It has several built-in options and can handle complex geometries easily. It mainly consists of three modules. These are Slide Model, Slide Compute and Slide Interpret.

In the model part, users can define an external boundary, material boundaries, ground water table, external loading, supports etc. Soil materials can be created with a desired strength type (default is Mohr-Coulomb) and assigned to a region. The software is also capable of doing analysis for other methods (Spencer, Bishop, Janbu etc.). However, as stated before, the method should be chosen carefully (Table 2.1). The user can also increase the number of slices along with other advanced options to increase the accuracy of the calculations. The analysis can be made for user-defined failure surface or via the grid search option including circular and non-circular surfaces. In addition to deterministic slope stability capacity, Slide can also carry out sensitivity and probabilistic analyses. For the probabilistic analysis, there are two available sampling methods and two analysis types. As for the sampling, “Monte-Carlo (MC)” and “Latin-Hypercube (LH)” sampling methods with desired number of samples (e.g. number of analysis carried) are available. For the analysis types, there are “Global Minimum (GM)” and “Overall Slope (OS)” presented. The user can also define the statistical distribution of the variables. Available distributions are normal, uniform, triangular, beta, exponential, lognormal and gamma. In all of the probabilistic analyses, for variable parameters, interval of mean ± 3 standard deviation is used. Interval is truncated from zero when necessary.

After compute module is used, the outcomes of the analysis can be obtained from interpret module. In this module, a number of results including FS, probability of failure (PF), reliability index (RI) and the critical failure surface(s) can be obtained. This module has lots of useful tools for organizing the desired data. It can export the raw data, insert contours etc.

For the probabilistic analyses in Rocscience Slide, as stated above, one can use both Global Minimum Method and Overall Slope Method together with Monte-Carlo and Latin Hypercube sampling methods as a combination. GM is one of the probabilistic analysis type in which deterministic slope stability analysis is carried out using the mean values of all parameters and one critical slip surface is found. Then, using the generated samples of material properties as random variables, probabilistic analysis is carried out only for this slip surface. In the end, one slip surface with FS, PF and RI values is obtained. Unlike GM, however, in the OS method, entire new critical surface search is repeated N times where N is the number of random samples generated for the OS type probabilistic analysis. It results in several slip surfaces and it can also generate the most critical failure surface. That means, if a user-defined slip surface is used for both methods, instead of searching for the most critical failure surface, both methods naturally give the same results.

For this particular thesis study, OS type of analyses with both MC and LH are carried out since it is more likely to obtain different slip surfaces for different random c-ϕ pairs. For some of the analyses, GM is also used additionally. Among the many available limit equilibrium method of slices, such as Bishop, Swedish, Janbu and

Morgenstern etc., Spencer's method is preferred since it satisfies all equilibrium conditions (e.g. overall moment, individual slice moment, horizontal and vertical force equilibriums) and it is suitable for slip surface of any shape (Pockoski and Duncan, 2000). Additionally, Mohr-Coulomb strength type is used in the analyses. Normal distribution is used in the absence of statistical information about variables.

Despite having some differences, Slope/W 2012 is also working with the same principle (e.g. LEM) as Slide software and they are practically the same. For examples, Slope/W does not have LH sampling options and does analysis comparable to GM type of analysis. However, as opposed to Slide, Slope/W has a slip surface search option by defining the possible entry and exit locations of the surface. In some of the cases, this may be useful.

3.2 Finite Element Analyses: PLAXIS 2D and Phase2

PLAXIS 2D 2015 is one of the 2D finite element geotechnical analysis tools. Besides its plastic deformation analysis and other analyses options, it can also carry out slope stability analysis with strength reduction method. It consists of mainly two parts. These are input and output programs. Input program has 5 subsections, namely soils, structures, mesh, flow conditions and staged construction. Units, dimensions of working place, model type (plane strain or axisymmetric) and finite element type (6 or 15-noded) can be defined in the project properties part. Model geometry can be defined by either inserting borehole or drawing soil polygons in soil or structures subsections, respectively. Materials can be created and assigned into the soil regions by suing the material sets options. In there, soil parameters, type of drainage, constitutive material model, seepage and groundwater conditions can be defined. In the mesh section, geometry is divided into finite elements (forming a mesh) with a desired level of fineness. The software has 5 options for mesh dimensions. These are very fine, fine, medium, coarse and very coarse. Additionally, local fineness can also be added into the meshes. After that, ground water level (e.g. phreatic level) can be added in the flow conditions section. After finalizing the input geometry, desired level of calculation stages (e.g. phases) can be created in accordance with the purpose of the analysis. In slope stability analysis, there should be at least two sages. First one creates the initial field conditions which can be either K_0 loading or gravity loading. For non-horizontal layers, gravity loading should be used in order to create initial stresses. Second phase performs the strength reduction analysis by reducing the cohesion and tangent of the friction angle with the same number until failure occurs, giving the strength reduction factor (SRF). If there are any loading or structural elements, they can be activated or deactivated within these stages. There are also other advanced options available for the stages. In the output, several analysis results, including deformations and stresses, can be viewed for individual stages. After going to the strength reduction phase results, one can obtain the failure surface zone from "Incremental Cartesian Strain" option viewing the incremental shear strains, and reached SRF value can be seen from the "Calculation Information".

Although PLAXIS is a powerful and useful geotechnical tool, due to lack of probabilistic built-in option, not every aspects of the problem can be considered. Since 2014, however, PLAXIS 2D user interface has changed with anniversary edition version and the designers can interrupt the software with PLAXIS VIP license and Python coding. This is explained in one of the later sections. In addition to that, latest update of PLAXIS 2D 2015 is capable of doing a sensitivity analysis but it is very limited.

Despite having lots of common properties, Phase2 v8 has several advantages compared to PLAXIS 2D in terms of probability analysis. It can carry out probabilistic finite element analysis with Rosenblueth's Point Estimate Method. It can easily import from or export into Slide software so both LEM and FEM analyses can be carried out, if desired. After the probabilistic analysis, it gives failure zone, factor of safety and reliability index information.

3.3 Random Finite Element Analyses: Rslope2D

As stated before, G.A. Fenton and D.V. Griffiths created a series of software in 1992. Rslope2D is one of them. It is a 2D finite element slope stability analysis software that considers the spatial variation and correlation of the soil properties. The software takes into account of the mean, standard deviation and spatial correlation length of the input parameters as well as local averaging of the properties over the finite elements (Fenton and Vanmarcke 1990). The software applies the Monte-Carlo method for the probabilistic analysis. It has a user interface in which input parameters and several other options can be specified. Vertical and horizontal correlation distance, statistical distribution of the random material and the number of analyses can be specified as well. Additionally, soil material properties can also be correlated to one another. After the analysis, the software provides a strength reduction value and deformed mesh view which can show the failure surface location.

Despite its advantage of spatial correlation consideration, Rslope2D has some limitations, therefore, disadvantages. One of them is that the software cannot incorporate the existence of the ground water level, i.e. it can only deal with dry cases. Another is that it can only work for simple slope geometry and single layered soils. In other words, geometries having more than one inclined slope surface and layers cannot be analyzed with this software. However, as stated before, it is an open-source coded software so it can be improved if desired.

Rslope2D uses initial generator seed value for the random number generation process. It can either be positive integer or zero. If desired, exact same result (e.g. same sequence of random numbers) can be reproduced by starting the same seed number and input parameters. This is especially useful when investigating the effects of the input parameters on the results (e.g. parametric studies). If the initial seed value is given as 0, the software starts the initial seed value from the computer. Depending on the operating system, it uses either the process ID or system time. If more than one analyses are carried out, second seed number is the previous seed plus one and this goes like that. Therefore, if one wants to regenerate a certain analysis step (say n^{th} analysis), it can be done by giving the seed number of initial seed plus n-1.

3.4 Random Finite Element Analyses: PLAXIS Remote Scripting with Python

As explained before, PLAXIS 2D 2014 or newer versions allows for interrupting the flow of software with remote scripting via Python which is one of the available programming languages. Both inputs as well as outputs can be created, called, stored or altered. For this purpose, aside from the aforementioned software, PLAXIS VIP license and internet connection are required. In the PLAXIS, there are several commands for every specific action and they can be viewed from PLAXIS Command Reference manual which comes with the installation of the software. Along with this, required amount of Python syntax knowledge is necessary. Python 3 is utilized for the scripting purpose.

Remote scripting can also be made partially for a specific purpose. For example, after manually creating inputs and carrying out the analysis, certain output value can be called and stored into an array object via scripting. Even this provides convenience in terms of human effort and time. However, remote scripting is more efficient when it is applied from the start till the end. For example, if a remote scripting file for a slope stability analysis is created with a general, always valid expressions, it can be reused in the future without any further effort. Remote scripting works can also be used for optimization problems where the designer should change almost every input parameters till the desired optimal result is reached. Since doing this procedure manually is quite time consuming, scripting provides convenience for the user.

As explained before, Rslope2D has three main limitations. In order to resolve these limitations, remote scripting is done for PLAXIS 2D that enables the software to carry out multiple slope stability analyses with a randomly assigned material properties. This way complex layered soil geometries with a ground water level can be analyzed with the powerful computational capacity of PLAXIS. At the same time, multiple analyses with a randomly generated and assigned material properties can be carried out with a desired number of realization for example for Monte Carlo simulation. Scripting is also done so that several soil regions are created with different soil strength properties. This way heterogeneous and anisotropic soil media can be created and analyzed.

For the current study, coding is developed so that all analysis steps are automatized. Created Python script is provided as an appendix at the end. General working principles of the coding along with the procedure steps are as follows:

(1) Remote Scripting Server: Under the “expert” tab of PLAXIS 2D, the users can configure and start the server (Figure 3.1). In here, availability of a valid license (should be VIP), connection to remote service website and local connection is shown. This is the first step of remote scripting and it is done before running the Python coding file. As it is stated before, coding is done so that the user does not have to do anything in the PLAXIS 2D environment after this step. The scripting itself connects and carries every step in the PLAXIS 2D.

(2) Running the Python coding file created in this thesis study: First, script sets up a connection between PLAXIS and Python. Then, it askes a number of input parameters to the users via the Python interface.

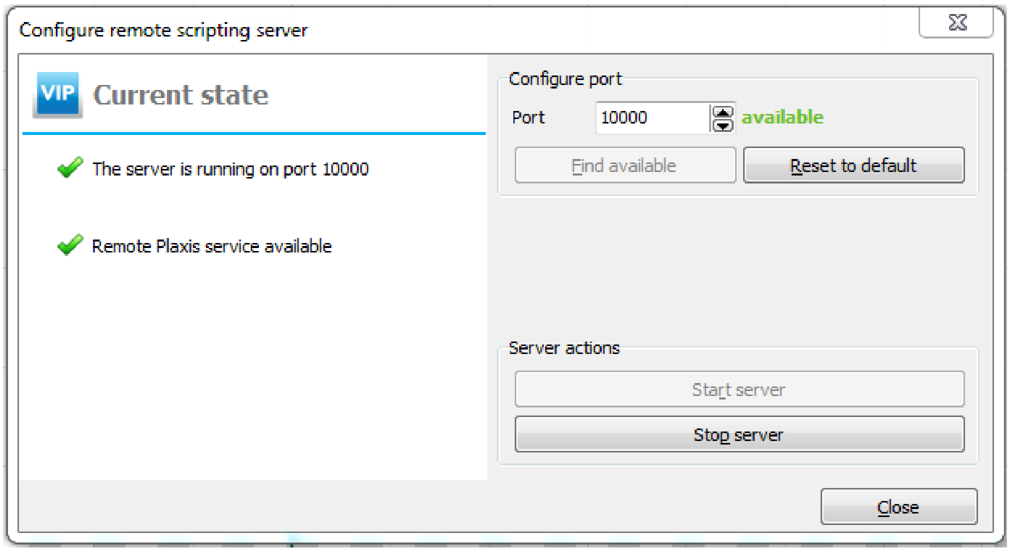


Figure 3.1 Configuration of remote scripting server in PLAXIS 2D

(3) Inputs: The created Python code asks information about the external geometry, number of layers and their boundaries, coordinates of the ground water level, finite element mesh dimension and material properties (e.g. unit weight, Young's modulus, cohesion etc.) with their COV values. The script also asks the horizontal and vertical spacing so that it can divide the geometry in order to create soil regions for heterogeneous and anisotropic analysis. Finally, before starting the analysis, it asks the number of analyses to be performed (each with a different random value for the variables) and desired saving location for the files.

(4) The script, first, sets up the working space in accordance with the dimension of external geometry. Then, it makes PLAXIS to draw the external geometry, layer boundaries and to create the sub regions. After this step, soil materials are created from the given inputs. If chosen random, each material is created

and assigned randomly different from its mean value. Finally, finite element meshing, phreatic level and calculation stages are established.

(5) After strength reduction analysis is finished, PLAXIS file is saved and calculated SRF value is written and stored on a file. Then, all inputs are cleared out in order to be ready for the next analysis.

(6) The analysis is repeated specified run number times.

(7) After all of the analyses are finished, the script calls the file which contains all SRF values and performs a number of statistical computation. In here, probability of failure and reliability index are calculated. Then, they are also written on the existing file.

(8) PLAXIS can sometimes give errors due to numerical calculation problems etc. In order to compensate for this, one try-catch loop is inserted into the code so that if a previously specified error occurs, PLAXIS can skip that analysis and continue with the next one. This is also considered in PF and RI computations.

CHAPTER 4

PROBABILISTIC VERSUS DETERMINISTIC SLOPE STABILITY

In this chapter, several deterministic and probabilistic slope stability analyses are carried out in order to demonstrate the necessity of the probabilistic analysis. Additionally, behavioral trend between factor of safety, probability of failure and coefficient of variability is investigated. Within this purpose, both Limit Equilibrium Method and Finite Element Method are utilized with the help of available geotechnical tools.

This chapter mainly consists of five parts. In the first three of them, comparison between deterministic and probabilistic analyses are inspected under different soil conditions. In the last two parts, aforementioned trend is studied for a selected soil condition. Young's Modulus and Poisson's ratio values are conservatively assumed since they are not affecting both SRF and the location of failure surface when they are not reported. Mesh options are kept as their default values for FEM analyses in Phase2 and PLAXIS 2D software. Outcomes of these five parts are mainly discussed in the later chapters and briefly mentioned in here. In all of the tables, figures and main paragraphs, several abbreviations are used. Their long names and meanings are not re-explained since this is done in the Chapter 3 and in the list of abbreviations.

4.1 Deterministic vs. Probabilistic Analyses: C_u Soils

For this section, soils having an undrained condition are studied. Analyses are carried out for two different safety levels. First analyses types are all having deterministic FS around 1.1 which is close to failure. Second types are all having deterministic FS around 1.6 which is relatively safer. Slope geometry and its geotechnical values are inspired from the generic test slope in Griffiths et al. (2004). However, to create different safety levels, some of the values are modified (Table 4.1).

4.1.1 Slopes Having Near-Failure Factor of Safety

Geotechnical parameters and their statistical values are provided in Table 4.1 along with the geometrical information. Only c_u value is slightly different than its reported value by Griffiths et al. (2004). In Table 4.2, technical details used in the different computation modes with respect to each software are provided.

Table 4.1 Geotechnical parameters and statistical distributions of the generic test slope by Griffiths et al. (2004)

Property	**Mean Value**	**COV (%)**	**Distribution**
C_u	40 kPa	50	Normal
γ	20 kN/m^3	none	none
GWT	none	none	none
Slope Height	10 m	none	none
Inclination	1V:2H	none	none

As it can be seen from Table 4.1, only strength parameter is chosen as probabilistic variable and others are kept deterministic and constant.

Table 4.2 Computational details of the methods

Software	**Explanation**
Slide	OS Analysis with 1000 MC Samples OS Analysis with 1000 LH Samples Method: Spencer
Slope/W	Analysis with 1000 MC Samples
PLAXIS 2D	1 Deterministic FEM Analysis
Phase2	2^1=2 Rosenblueth's Point Estimate Analyses

After all analyses, obtained results are provided in Table 4.3 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.3 Comparison of results

Method	FS or SRF	PF (%)
Slide-Deterministic	1.14	None
Slide-MC	1.03	44.5
Slide-LH	0.94	50.5
Slope/W	1.14	45.4
PLAXIS 2D-Deterministic	1.10	None
Phase2	1.09	45.6

Some of the observations based on these results are: (1) As it can be seen from Table 4.3, a uniform clayey slope having a deterministic FS value greater than 1.0 can have a PF as high as 50%. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM gave similar PF and FS values. (4) All probabilistic LEM methods give similar PF values in the range of 44.5-50.5%. MC and LH sampling methods give slightly different PF values (6% difference). (5) Two LEM software, with both of which 1000 MC probabilistic analyses are carried out, gave very similar PF values, however moderately different FS values (1.03 and 1.14), and Slide MC probabilistic analysis gives more critical FS value compared to Slope/W MC. (6) LEM probabilistic analyses give a lower FS value then deterministic FS, with Slide software.

Failure surfaces of these analyses are provided in the Figure 4.1 to Figure 4.6 with necessary explanations in their caption. As it can be seen from these figures that the critical failure surfaces are different not only between the deterministic and probabilistic analyses but also among the probabilistic methods. The failure surfaces are all different in terms of their maximum depth, their width and the radius of the critical surfaces. It is difficult to reach to a general conclusion about the differences in the critical failure surfaces.

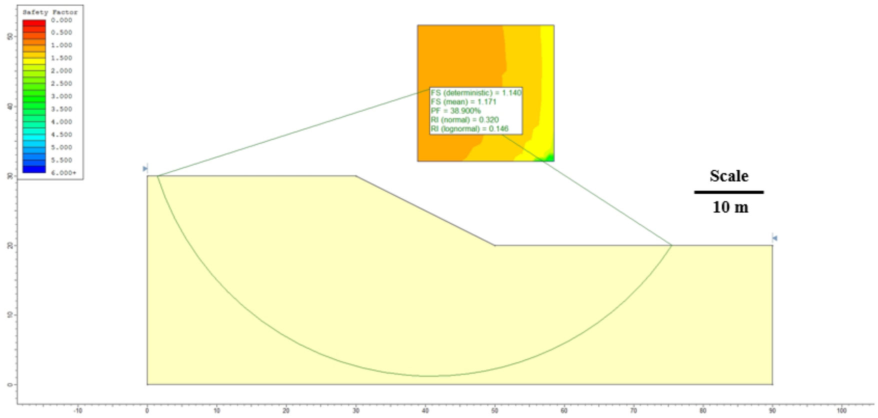


Figure 4.1 Deterministic failure surface from Slide analysis (C_u Soil)

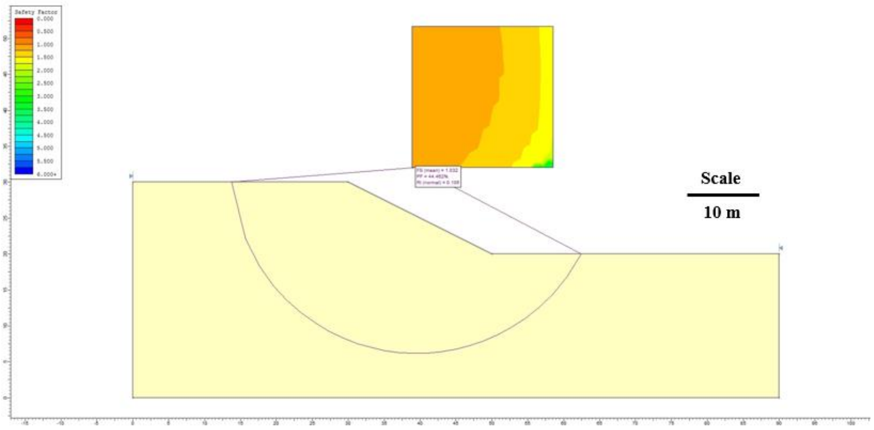


Figure 4.2 The most critical failure surface from Slide-MC analysis (C_u Soil)

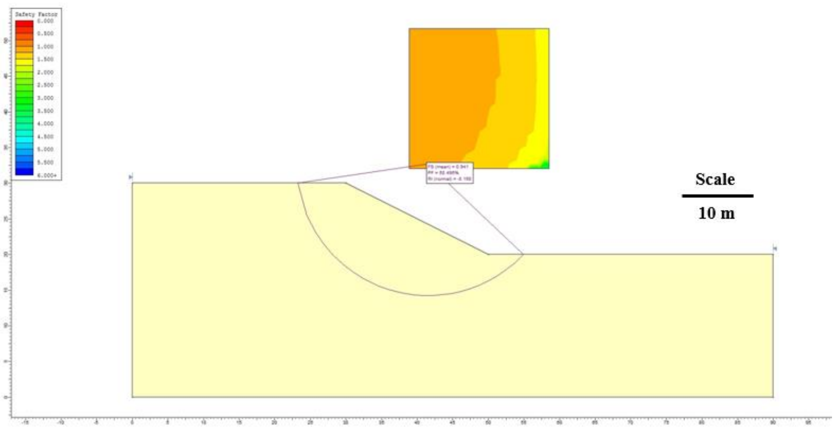


Figure 4.3 The most critical failure surface from Slide-LH analysis (C_u Soil)

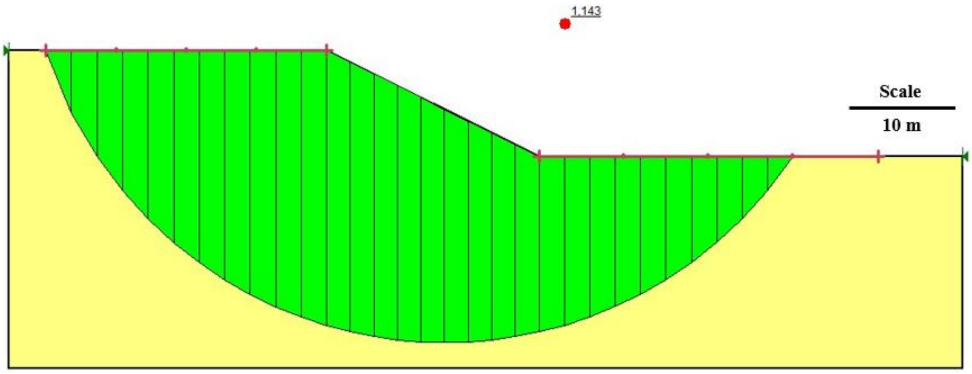


Figure 4.4 Failure surface from Slope/W analysis (C_u Soil)

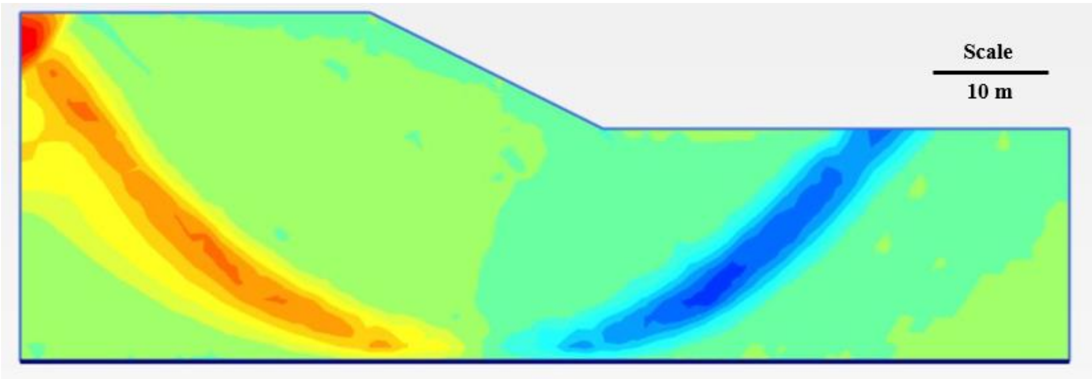


Figure 4.5 Failure zone from PLAXIS 2D analysis (C_u Soil) (red color shows the highest incremental shear strain)

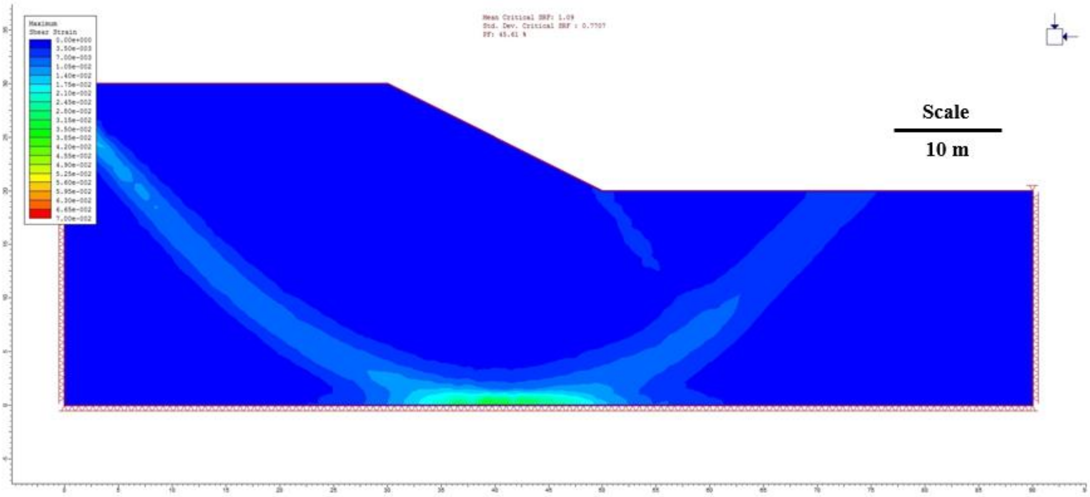


Figure 4.6 Failure zone from Phase2 analysis (C_u Soil)

4.1.2 Slopes Having Relatively Safer Factor of Safety

Similar analyses are repeated with different slope height. Geotechnical parameters and their statistical values are the same as provided in Table 4.1 along with the geometrical information with the only difference being the slope height is 7 m, keeping the same slope angle. Technical details used in the different computation modes with respect to each software are the same as in Table 4.2. Only strength parameter is chosen as probabilistic variable and others are kept deterministic and constant.

After all analyses, obtained results are provided in Table 4.4 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.4 Comparison of results

Method	FS or SRF	PF (%)
Slide-Deterministic	1.61	None
Slide-MC	1.29	28.8
Slide-LH	1.31	29.3
Slope/W	1.67	18.6
PLAXIS 2D-Deterministic	1.57	None
Phase2	1.53	31.1

Some of the observations based on these results are: (1) As it can be seen from Table 4.4, a uniform clayey slope having a deterministic FS value greater than 1.5 can have a PF as high as 30%. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM do not give similar PF and FS values. (4) Among the probabilistic LEM methods, (with both of which 1000 MC probabilistic analyses are carried out), Slide gives higher PF and lower probabilistic FS as compared to Slide. (5) LEM probabilistic analyses give a lower FS value then deterministic FS, with Slide software.

Failure surface of these analyses are provided in the Figure 4.7 to Figure 4.12 with necessary explanations in their caption. As it can be seen from these figures that failure surfaces are different not only between probabilistic and deterministic analyses but also among the probabilistic methods. The failure surfaces are all different in terms of their maximum depth, their width and the radius of the critical surfaces. It is difficult to reach to a general conclusion about the differences in the critical failure surfaces.

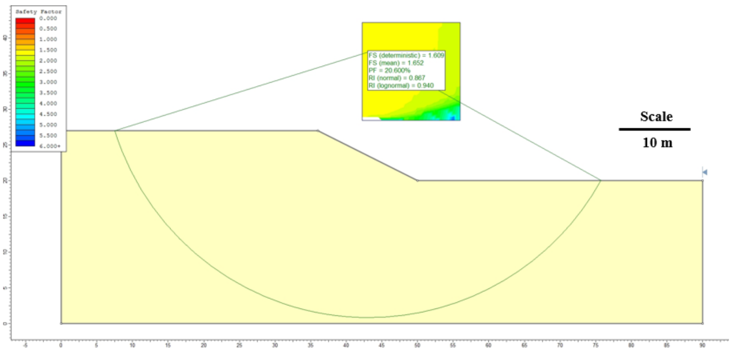


Figure 4.7 Deterministic failure surface from Slide analysis (C_u Soil)

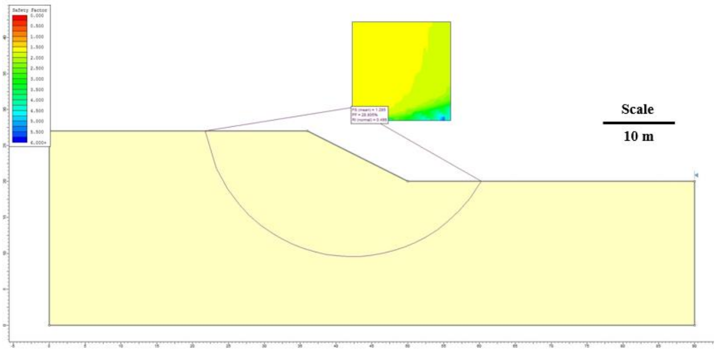


Figure 4.8 The most critical failure surface from Slide-MC analysis (C_u Soil)

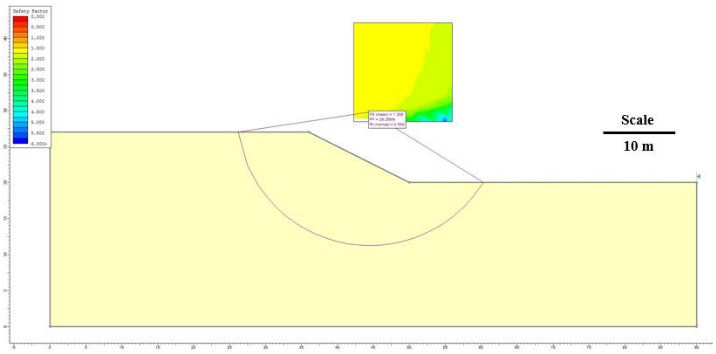


Figure 4.9 The most critical failure surface from Slide-LH analysis (C_u Soil)

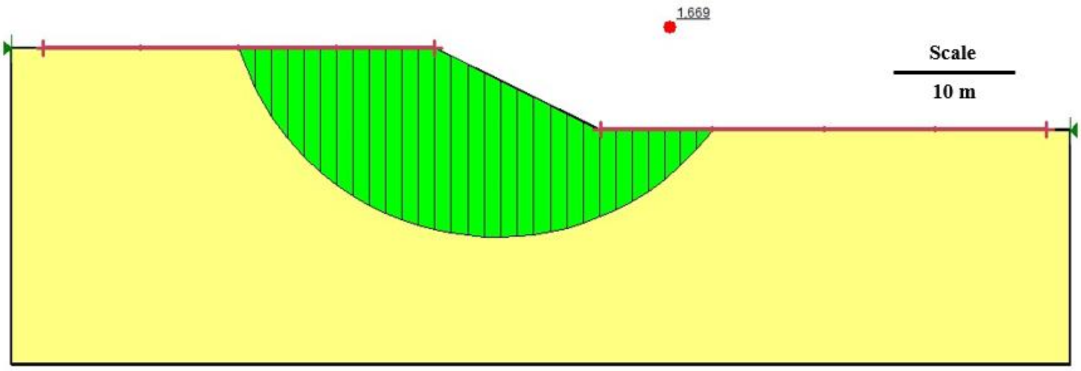


Figure 4.10 Failure surface from Slope/W analysis (C_u Soil)

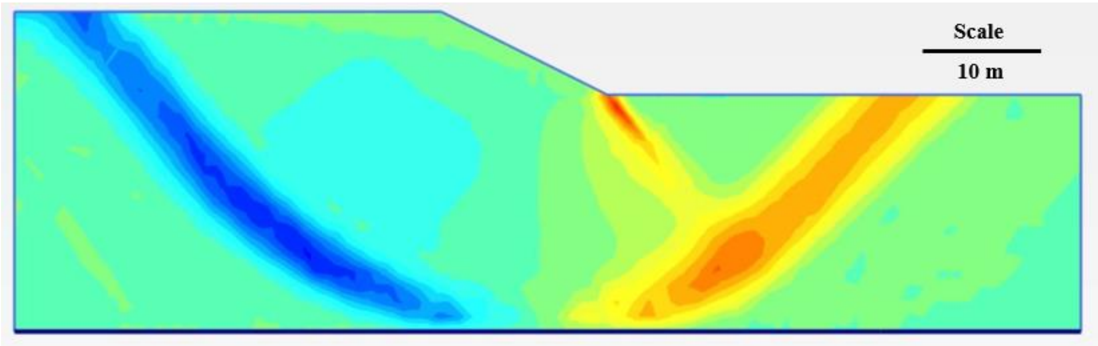


Figure 4.11 Failure zone from PLAXIS 2D analysis (C_u Soil) (red color shows the highest incremental shear strain)

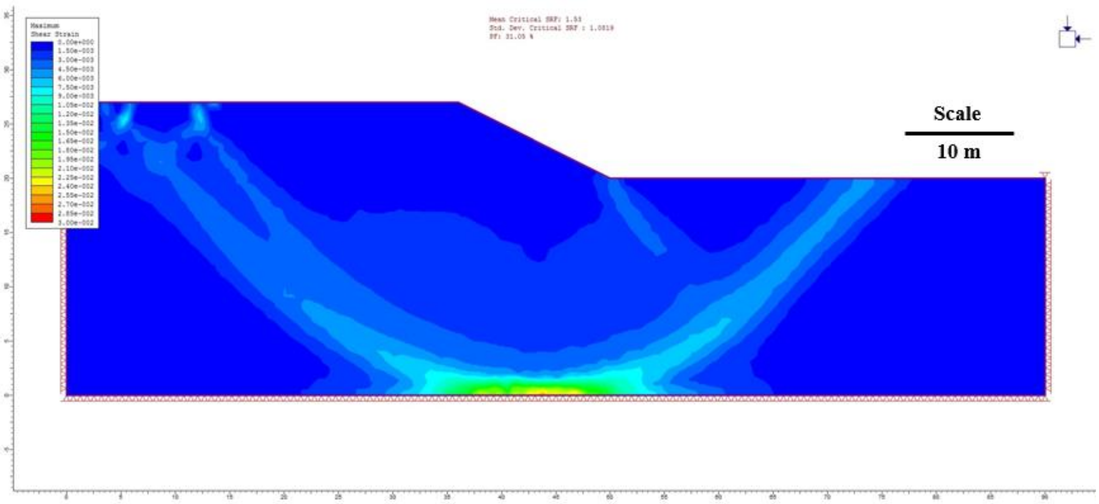


Figure 4.12 Failure zone from Phase2 analysis (C_u Soil)

4.2 Deterministic vs. Probabilistic Analyses: c-phi Soils

For this section, soils having a drained condition and non-zero cohesion values are studied (e.g. a clayey sand). Analyses are carried out for two different safety levels. First analyses types are all having deterministic FS around 1.1 which is close to failure. Second types are all having deterministic FS around 1.6 which is relatively safer. Generic slope in Bhattacharya et al. (2003) is used along with its reported parameters.

4.2.1 Slopes Having Near-Failure Factor of Safety

Geotechnical parameters and their statistical values are provided in Table 4.5 along with the geometrical information. In Table 4.6, technical details used in the different computation modes with respect to each software are provided.

Table 4.5 Geotechnical parameters and statistical distributions

Property	Mean Value	COV (%)	Distribution
c'	18 kPa	20	Normal
Φ'	30°	10	Normal
γ	18 kN/m^3	none	none
GWT	none	none	none
Slope Height	10 m	none	none
Inclination	2V:1H	none	none

As it can be seen from Table 4.5, only strength parameter is chosen as probabilistic variable and others are kept deterministic and constant. Geotechnical values used in this table are exactly same as their reported values. However, slope inclination and GWT are changed so that factor of safety level of 1.1 can be achieved.

Table 4.6 Computational details of the methods

Software	Explanation
Slide	OS Analysis with 1000 MC Samples OS Analysis with 1000 LH Samples Method: Spencer

Table 4.6 Continued

Slope/W	Analysis with 1000 MC Samples
PLAXIS 2D	1 Deterministic FEM Analysis
Phase2	2^2=4 Rosenblueth's Point Estimate Analyses

After all analyses, obtained results are provided in Table 4.7 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.7 Comparison of results

Method	**FS or SRF**	**PF (%)**
Slide-Deterministic	1.19	None
Slide-MC	1.16	12.4
Slide-LH	1.15	12.8
Slope/W	1.27	4.3
PLAXIS 2D-Deterministic	1.11	None
Phase2	1.21	8.7

Some of the observations based on these results are: (1) As it can be seen from Table 4.7, a c-phi type soil slope having a deterministic FS value greater than 1.0 can have a PF as high as 13%, which is a considerable value. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM do not give similar PF and FS values. (4) Among the probabilistic LEM methods, (with both of which 1000 MC probabilistic analyses are carried out), Slide gives higher PF and lower probabilistic FS as compared to Slope/W. (5) LEM probabilistic analyses give a slightly lower FS (0.03 - 0.04 difference) value then deterministic FS, with Slide software.

Failure surface of these analyses are provided in the Figure 4.13 to Figure 4.18 with necessary explanations in their caption. The failure surfaces in all methods seem to be very similar in terms of their maximum depth, their width and the radius of the critical surfaces.

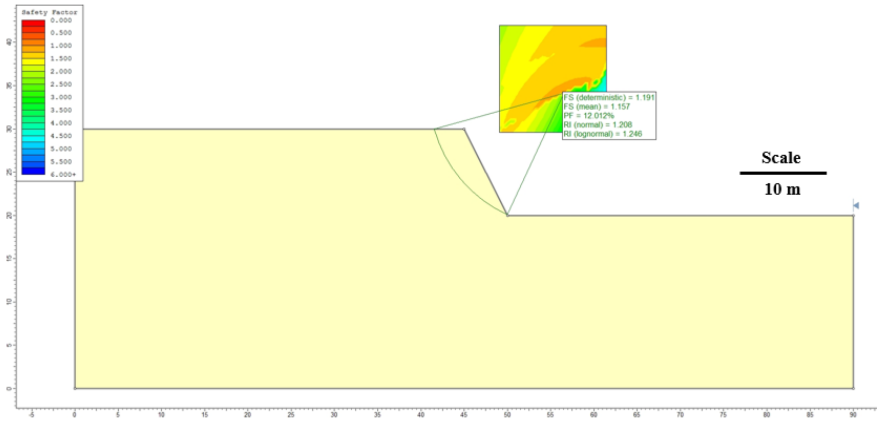


Figure 4.13 Deterministic failure surface from Slide analysis (c-phi Soils)

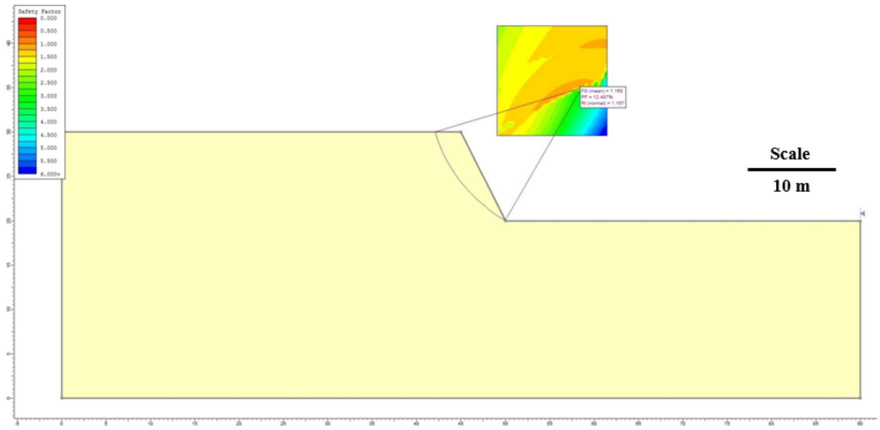


Figure 4.14 The most critical failure surface from Slide-MC analysis (c-phi Soils)

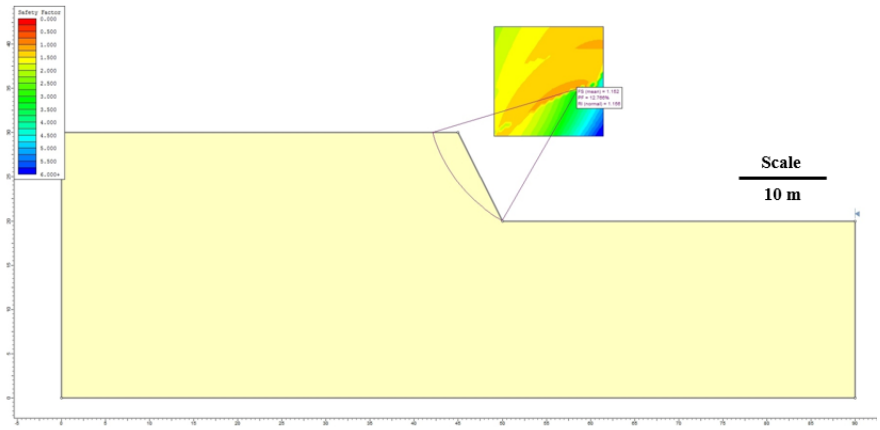


Figure 4.15 The most critical failure surface from Slide-LH analysis (c-phi Soils)

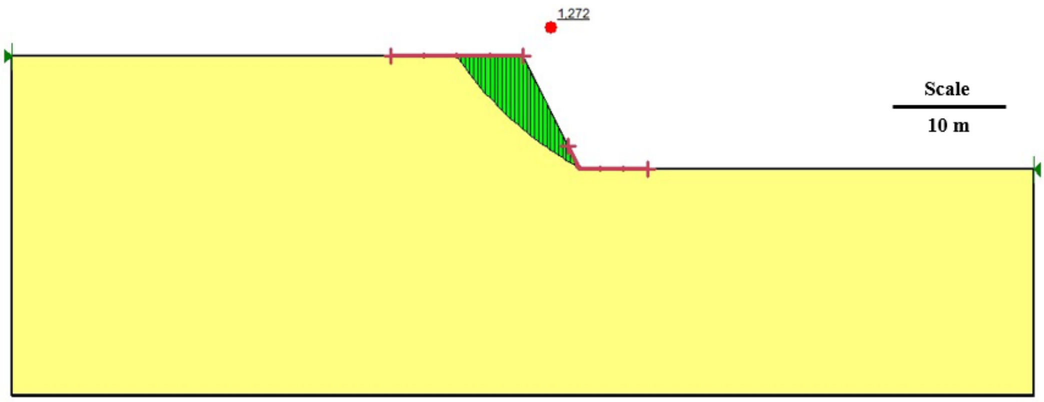


Figure 4.16 Failure surface from Slope/W analysis (c-phi Soils)

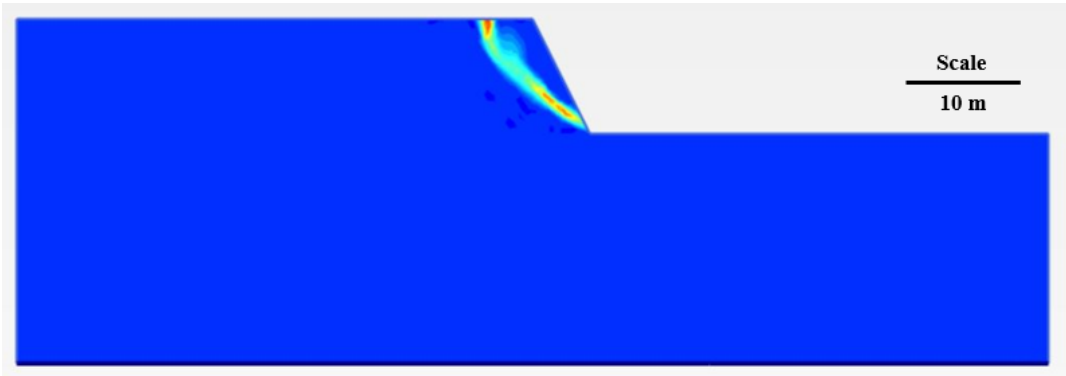


Figure 4.17 Failure zone from PLAXIS 2D analysis (c-phi Soils) (red color shows the highest incremental shear strain)

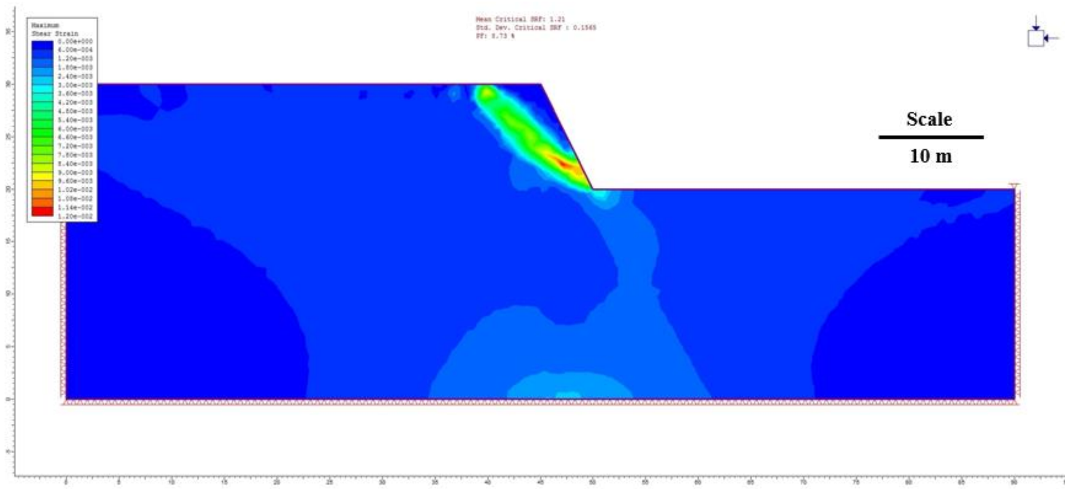


Figure 4.18 Failure zone from Phase2 analysis (c-phi Soils)

4.2.2 Slopes Having Relatively Safer Factor of Safety

Similar analyses are repeated with different slope inclination which is the original reported value. Geotechnical parameters and their statistical values are the same as provided in Table 4.5 along with the geometrical information, with the only difference being the slope angle is now 1V:1H, keeping the same slope height. In Table 4.6, technical details used in the different computation modes with respect to each software are provided.

Only strength parameter is chosen as probabilistic variable and others are kept deterministic and constant. Geotechnical values used in this table are exactly same as their reported values. However, GWT is changed so that factor of safety level of 1.6 can be achieved.

After all analyses, obtained results are provided in Table 4.8 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.8 Comparison of results

Method	**FS or SRF**	**PF (%)**
Slide-Deterministic	1.62	None
Slide-MC	1.66	0
Slide-LH	1.66	0
Slope/W	1.63	0
PLAXIS 2D-Deterministic	1.54	None
Phase2	1.61	0.12

Some of the observations based on these results are: (1) As it can be seen from Table 4.8, this slope does not have any probability of failure. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM gave similar PF and FS values. (4) All probabilistic LEM methods give similar PF values.

Failure surface of these analyses are provided in the Figure 4.19 to Figure 4.24 with necessary explanations in their caption. The failure surfaces in all methods seem to be very similar in terms of their maximum depth, their width and the radius of the critical surfaces.

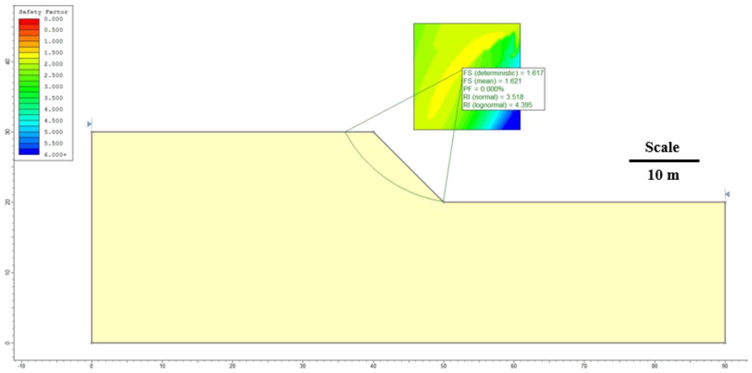


Figure 4.19 Deterministic failure surface from Slide analysis (c-phi Soils)

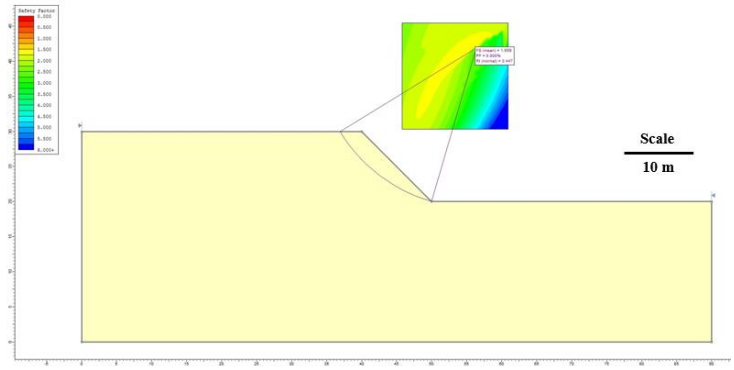


Figure 4.20 The most critical failure surface from Slide-MC analysis (c-phi Soils)

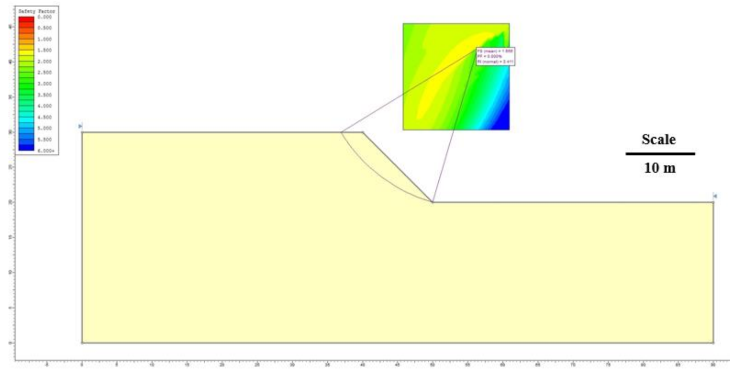


Figure 4.21 The most critical failure surface from Slide-LH analysis (c-phi Soils)

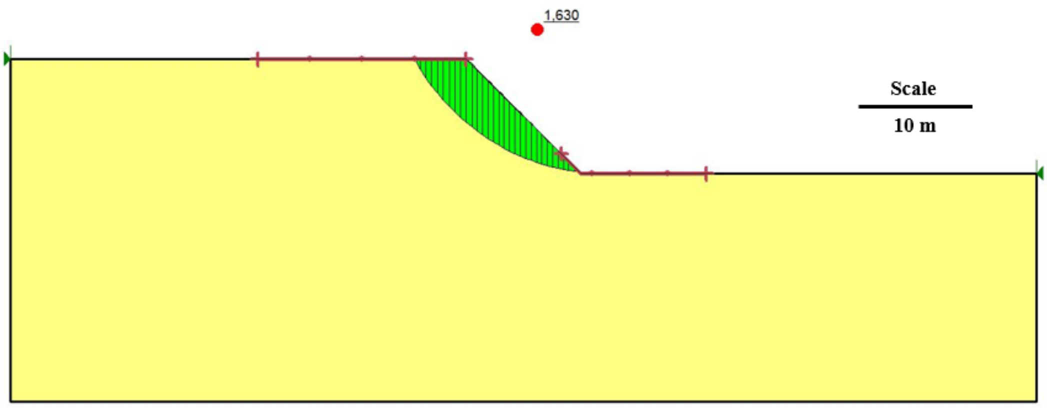


Figure 4.22 Failure surface from Slope/W analysis (c-phi Soils)

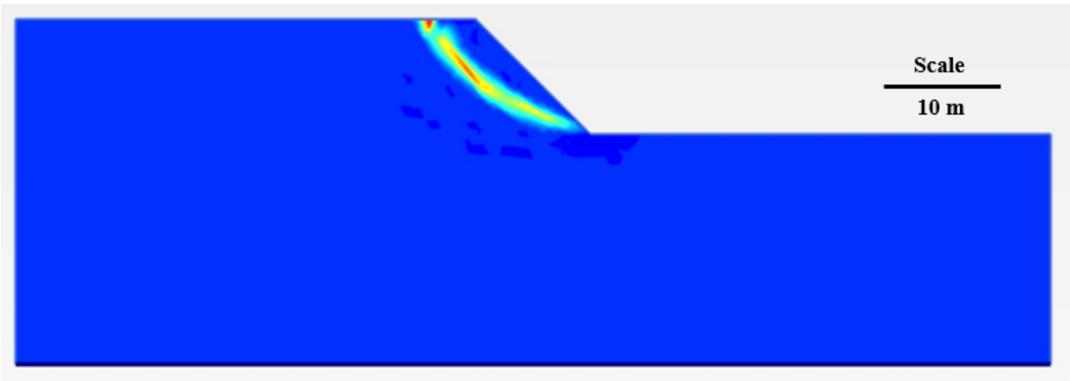


Figure 4.23 Failure zone from PLAXIS 2D analysis (c-phi Soils) (red color shows the highest incremental shear strain)

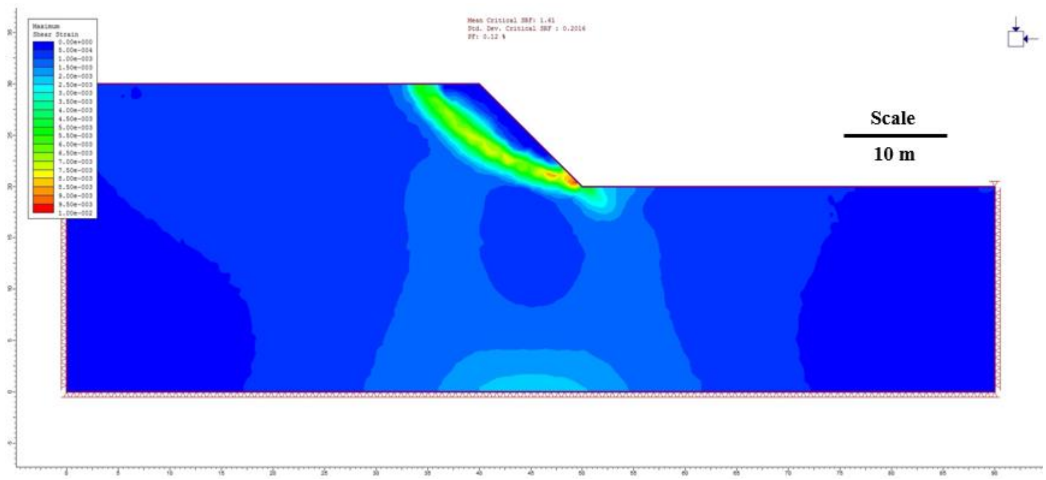


Figure 4.24 Failure zone from Phase2 analysis (c-phi Soils)

4.3 Deterministic vs. Probabilistic Analyses: Cohesionless Soils

For this section, soils having a drained condition and zero cohesion value are studied (e.g. a clean sand). Analyses are carried out for two different safety levels. First analyses types are all having deterministic FS around 1.1 which is close to failure. Second types are all having deterministic FS around 1.5 which is relatively safer. A generic slope geometry is created and geotechnical parameters are assumed from the literature values.

4.3.1 Slopes Having Near-Failure Factor of Safety

Geotechnical parameters and their statistical values are provided in Table 4.9 along with the geometrical information. In Table 4.10, technical details used in the different computation modes with respect to each software are provided.

Table 4.9 Geotechnical parameters and statistical distributions

Property	Mean Value	COV (%)	Distribution
Φ'	30°	10	Normal
γ	18 kN/m^3	none	none
GWT	none	none	none
Slope Height	10 m	none	none
Inclination	1V:2H	none	none

As it can be seen from Table 4.9, only strength parameter is chosen as probabilistic variable and others are kept deterministic and constant. COV value of internal friction angle is assumed from the Duncan (2000) study as an approximate mean value.

One of the problems when reporting the analysis results of cohesionless soils is that geotechnical software show the shallowest surficial slip surface as the most critical one. To overcome this problem, and to be able to see deeper failure surfaces, all failure surfaces with FS value 1.2 or smaller are provided.

Table 4.10 Computational details of the methods

Software	**Explanation**
Slide	OS Analysis with 1000 MC Samples
	OS Analysis with 1000 LH Samples
	Method: Spencer
Slope/W	Analysis with 1000 MC Samples
PLAXIS 2D	1 Deterministic FEM Analysis
Phase2	$2^1=2$ Rosenblueth's Point Estimate Analyses

After all analyses, obtained results are provided in Table 4.11 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.11 Comparison of the results

Method	**FS or SRF**	**PF (%)**
Slide-Deterministic	1.16	None
Slide-MC	1.16	11.7
Slide-LH	1.16	12.4
Slope/W	1.16	11.3
PLAXIS 2D-Deterministic	1.14	None
Phase2	1.21	15.9

Some of the observations based on these results are: (1) As it can be seen from Table 4.11, a cohesionless soil slope having a deterministic FS value greater than 1.0 can have a PF as high as 12%. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM gave similar PF and FS values. (4) All probabilistic LEM methods give similar PF values in the range of 11.3-12.4%. MC and LH sampling methods give slightly different PF values (7% difference). (5) Two LEM software, with both of which 1000 MC probabilistic analyses are carried out, gave very similar PF and probabilistic FS values. (6) LEM probabilistic analyses give same FS as deterministic FS, with Slide software.

Failure surface of these analyses are provided in the Figure 4.25 to Figure 4.28 with necessary explanations in their caption. All failure surfaces are similar to each other, in that they are small and shallow with varying maximum depths.

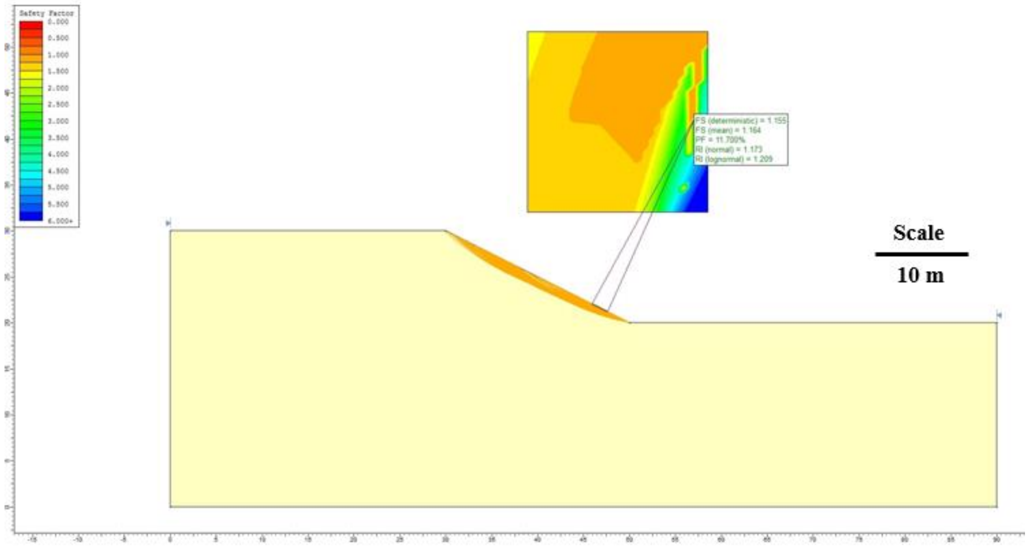


Figure 4.25 Failure surfaces with FS 1.2 or smaller from Slide analysis

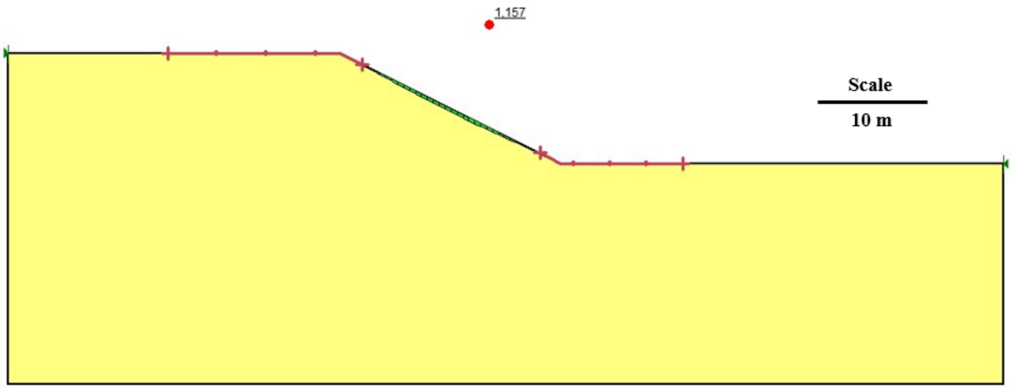


Figure 4.26 Failure surface from Slope/W analysis

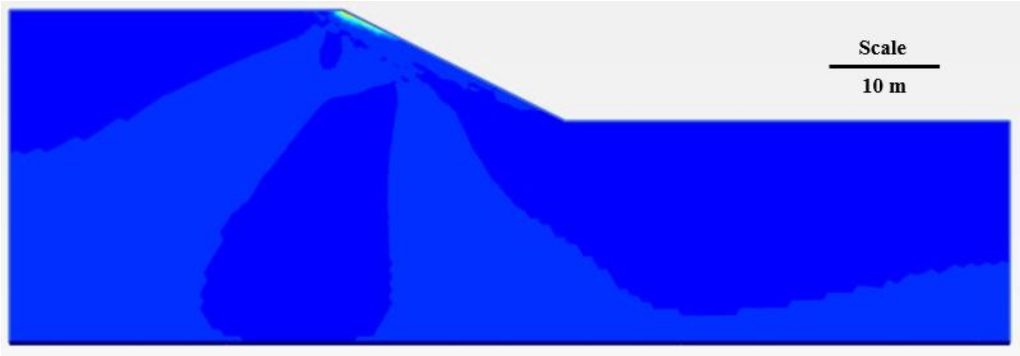


Figure 4.27 Failure zone from PLAXIS 2D analysis

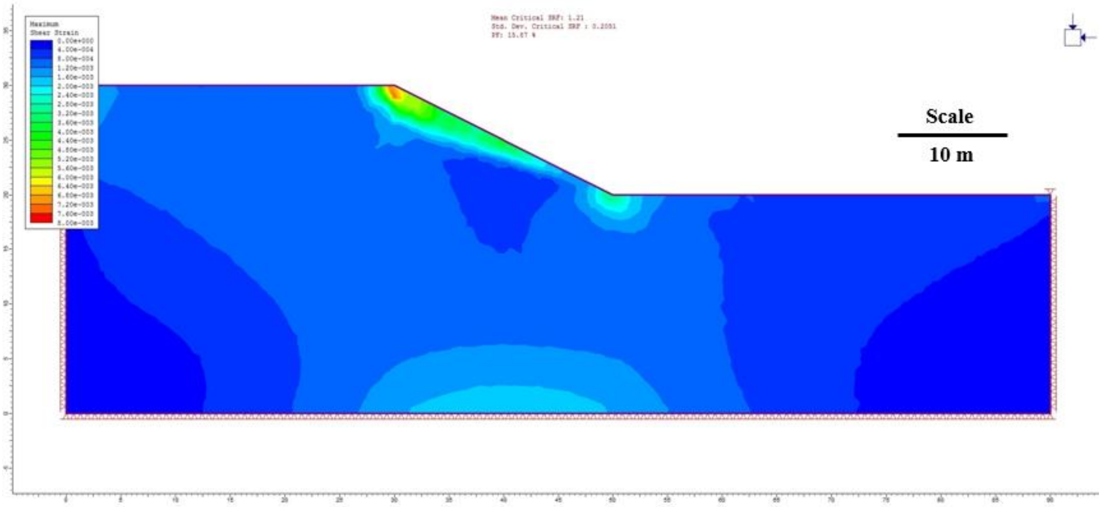


Figure 4.28 Failure zone from Phase2 analysis

4.3.2 Slopes Having Relatively Safer Factor of Safety

Similar analyses are repeated with different slope inclination. Geotechnical parameters and their statistical values are provided as in Table 4.9 along with the geometrical information, with the only difference being the slope angle is now 1V:2.5H. In Table 4.10, technical details used in the different computation modes with respect to each software are provided. Only strength parameter is chosen as probabilistic variable and others are kept constant.

After all analyses, obtained results are provided in Table 4.12 for the sake of comparison. In this table both deterministic and probabilistic results are provided.

Table 4.12 Comparison of the results

Method	FS or SRF	PF (%)
Slide-Deterministic	1.44	None
Slide-MC	1.46	0.2
Slide-LH	1.45	0.2
Slope/W	1.45	0.2
PLAXIS 2D-Deterministic	1.46	None
Phase2	1.49	2.71

Some of the observations based on these results are: (1) As it can be seen from Table 4.12, this slope has negligible probability of failure. (2) The deterministic FEM and LEM gives similar FS values. (3) Probabilistic LEM and FEM gave similar FS values, probabilistic FEM gives slightly larger PF (2.5 % difference). (4) All probabilistic LEM methods give similar PF values. (5) LEM probabilistic analyses give same FS as deterministic FS, with Slide software.

Failure surface of these analyses are provided in the Figure 4.29 to Figure 4.32 with necessary explanations in their caption. For this case, failure surfaces with a FS value 1.5 or smaller are reported. All failure surfaces are similar to each other, in that they are small and shallow with varying maximum depths.

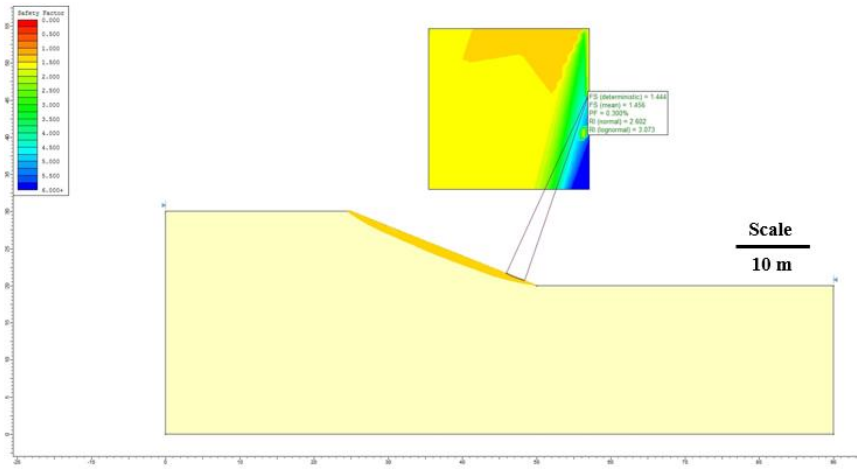


Figure 4.29 Failure surfaces with FS 1.5 or smaller from Slide analysis

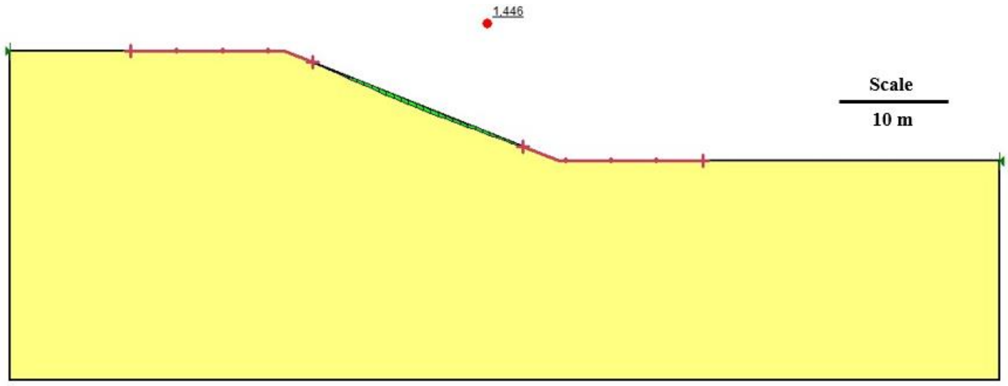


Figure 4.30 Failure surface from Slope/W analysis

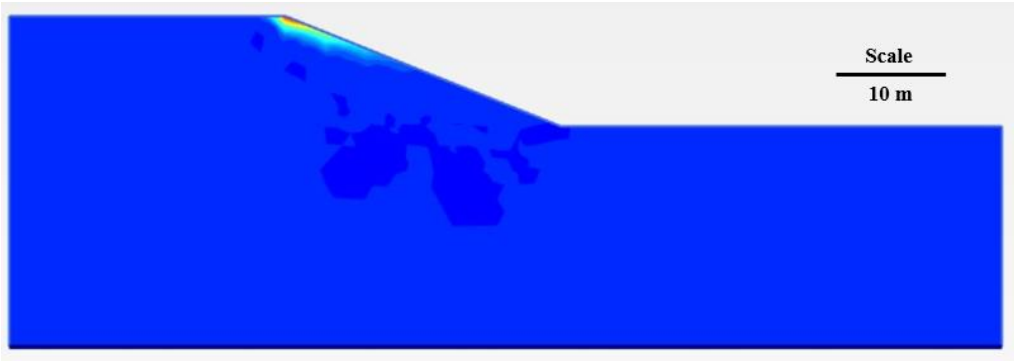


Figure 4.31 Failure zone from PLAXIS 2D analysis

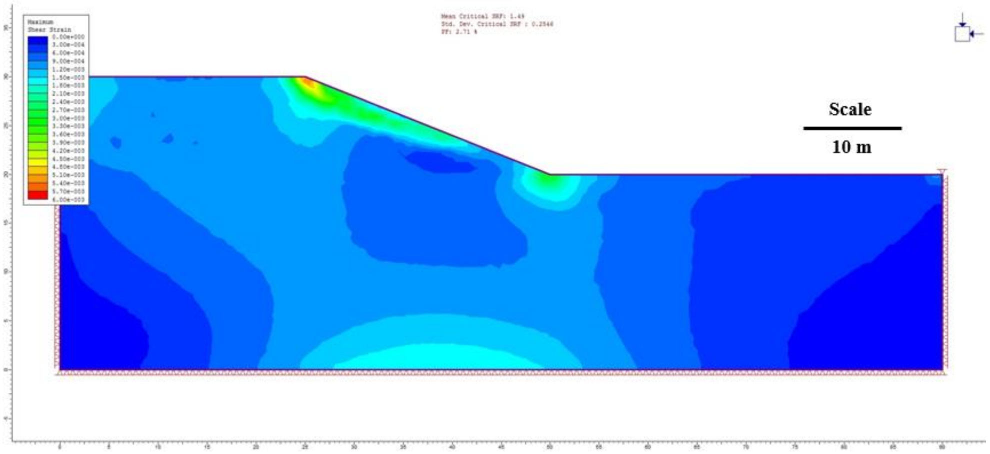


Figure 4.32 Failure zone from Phase2 analysis

4.4 Relation between Factor of Safety and Probability of Failure

In this section, behavioral trend between FS and PF is investigated. For this purpose, generic slope in Bhattacharya et al. (2003) is selected and two probabilistic LEM tools (Slide and Slope/W) and one probabilistic FEM tool (Phase2) are utilized. For Slide software, OS type of computations are carried out for both MC and LH sampling options. In order to create a variety of safety levels, (therefore, different probability of failure values), slope inclination is changed systematically. Since the reported slope has a GWT, for the sake of completeness, analyses are done for both dry and wet slopes. COV values are used as they are reported in Bhattacharya et al. (2003). Analyses details and geotechnical properties are provided in Table 4.13 and Table 4.14, respectively. Dry and saturated unit weights are taken to be the same.

Table 4.13 Analyses details and inclination variation

Property	Value
Slope Inclination (°)	45 to 75 (with 2.5 increments)
Range of FS levels (Dry)	1.08 - 1.66
Range of FS levels (R_u)	0.8 - 1.3

Table 4.14 Geotechnical parameters and statistical distributions

Property	Mean Value	COV (%)	Distribution
c'	18 kPa	20	Normal
Φ'	30°	10	Normal
γ	18 kN/m^3	none	none
R_u (GWT)	0.2	none	none
Slope Height	10 m	none	none

After the analyses, obtained FS and PF values are tabulated and FS vs. PF graphs are obtained for each case. Figure 4.33 and Figure 4.34 are provided for dry and wet case, respectively. Semi-log scale relation is given in Figure 4.35 and Figure 4.36.

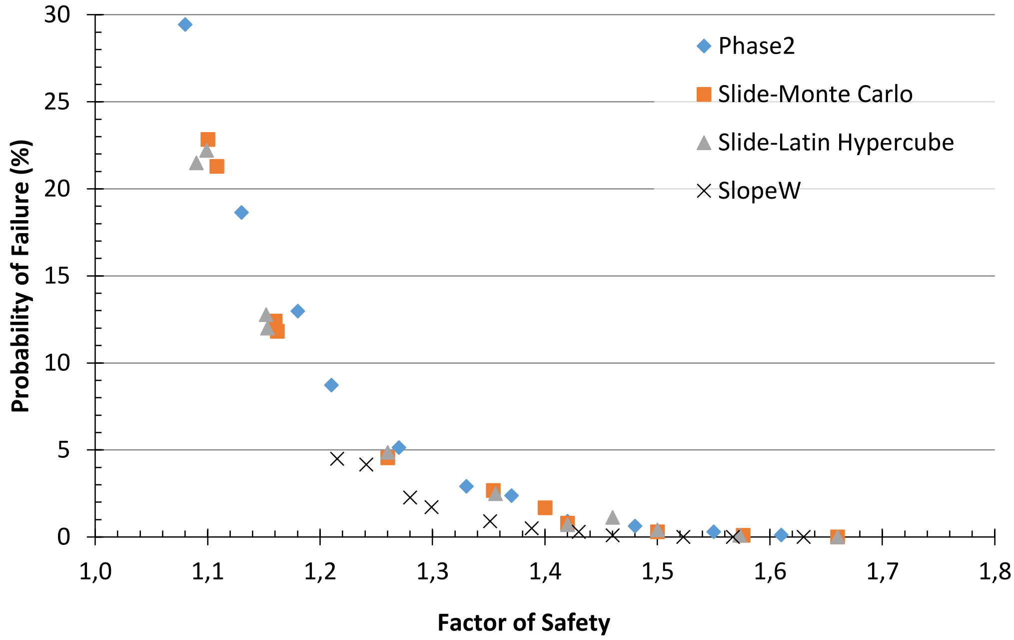


Figure 4.33 Trend between FS and PF (dry case)

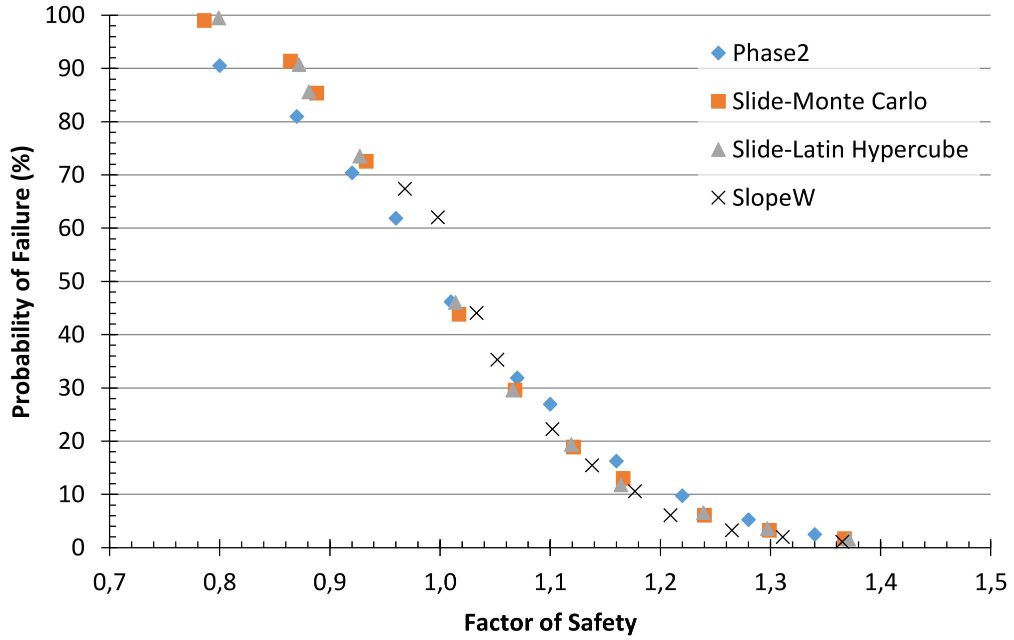


Figure 4.34 Trend between FS and PF (R_u = 0.2 case)

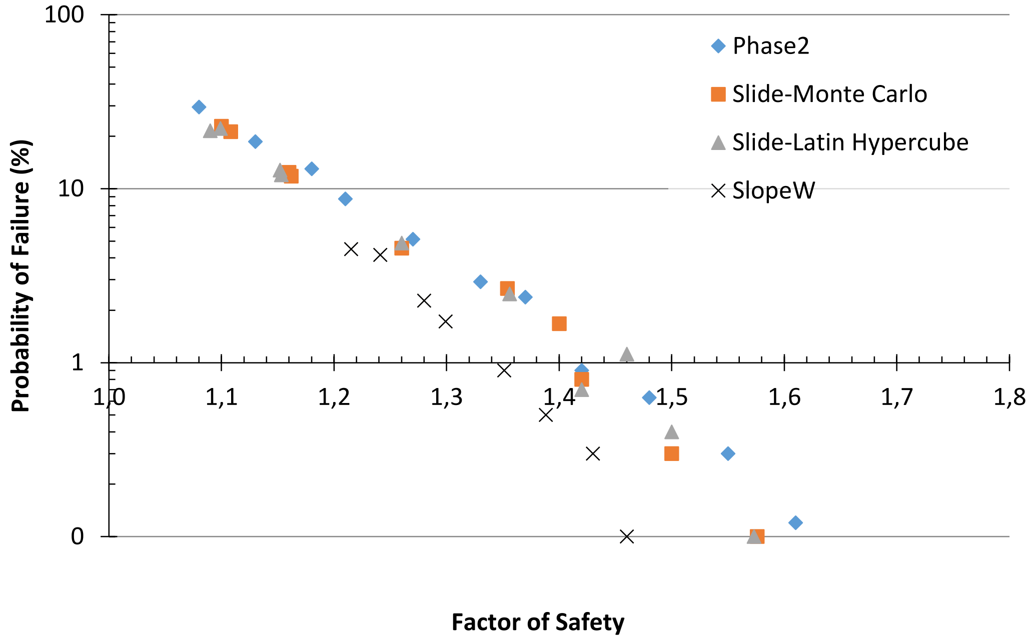


Figure 4.35 Trend between FS and PF in semi-log scale (dry case)

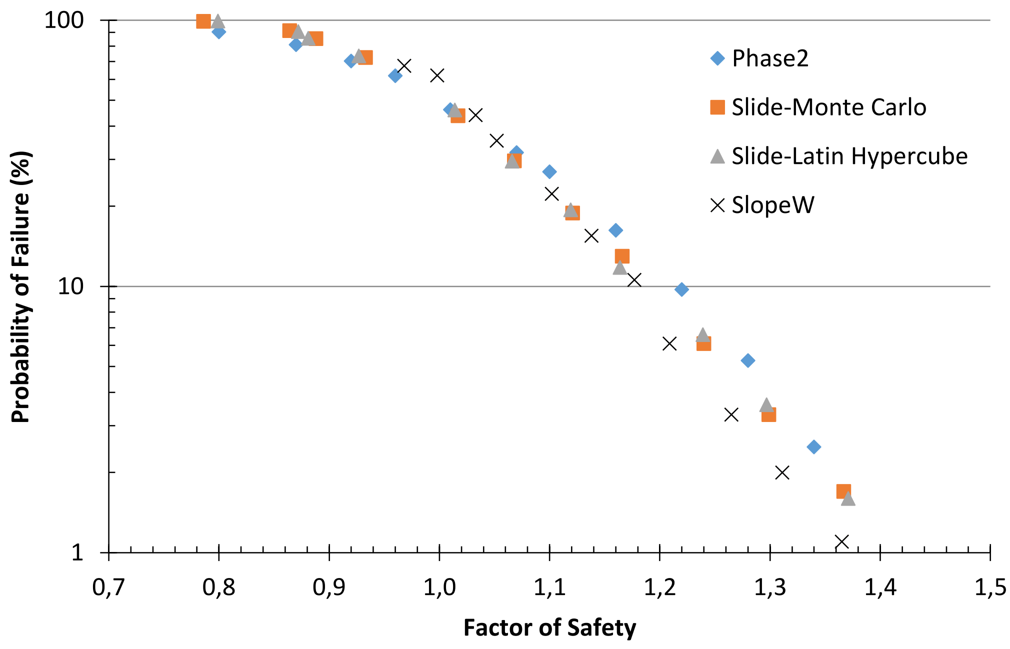


Figure 4.36 Trend between FS and PF in semi-log scale (R_u = 0.2 case)

Obtained FS and PF values for different analysis methods are provided in Table 4.15 and Table 4.16 for dry and wet case, respectively. Some of the observations based on the figures above are: (1) the FS=1.10 corresponds to a probability of failure of 21-23% for the dry slope case, and 19-27% for the wet slope case. Therefore the misconception in geotechnical practice that "any slope with FS larger than 1.00-1.05 is 'safe' (i.e. it will be stable for sure)" is proven to be not correct. Slopes with a FS value 1.1 or larger do not have probability of failure of zero. (2) For probability of failure of less than or equal to 1%, FS value is greater than about 1.35 in both the dry and the wet case. (3) The owner or the designer of a slope can decide about the allowable or "acceptable" probability of failure for that slope and relate it to FS value, which is easier to understand for non-technical people or for traditionally-oriented geotechnical engineers. It should be noted that the specific numbers and values in these conclusions are valid only for the given COV values of 20% for cohesion and 10% for friction angle. As it will be demonstrated in the following section, the COV will influence the PF.

Table 4.15 Obtained FS and PF values (dry case)

	Methods							
	Phase2		**Slide-MC**		**Slide-LH**		**Slope/W**	
Inclination (°)	**FS**	**PF (%)**	**FS**	**PF (%)**	**FS**	**PF (%)**	**FS**	**PF (%)**
70	1.08	29.44	1.10	22.83	1.09	21.49	1.22	4.49
67.5	1.13	18.63	1.11	21.29	1.10	22.20	1.24	4.15
65	1.18	12.98	1.16	11.81	1.15	11.99	1.28	2.27
63	1.21	8.73	1.16	12.41	1.15	12.77	1.30	1.72
60	1.27	5.13	1.26	4.55	1.26	4.86	1.35	0.90
57.5	1.33	2.91	1.35	2.66	1.36	2.48	1.39	0.50
55	1.37	2.38	1.40	1.67	1.46	1.12	1.43	0.30
53	1.42	0.90	1.42	0.80	1.42	0.70	1.46	0.10
50	1.48	0.63	1.50	0.30	1.50	0.40	1.52	0.00
47.5	1.55	0.30	1.58	0.10	1.57	0.10	1.57	0.00
45	1.61	0.12	1.66	0.00	1.66	0.00	1.63	0.00

Table 4.16 Obtained FS and PF values (Ru = 0.2 case)

	Methods							
	Phase2		Slide-MC		Slide-LH		Slope/W	
Inclination (°)	**FS**	**PF (%)**	**FS**	**PF (%)**	**FS**	**PF (%)**	**FS**	**PF (%)**
70	0.80	90.54	0.79	99.03	0.80	99.51	0.97	67.42
67.5	0.87	81.01	0.86	91.41	0.87	90.72	1.00	62.09
65	0.92	70.43	0.89	85.37	0.88	85.62	1.03	44.12
63	0.96	61.91	0.93	72.56	0.93	73.57	1.05	35.32
60	1.01	46.20	1.02	43.88	1.01	46.07	1.10	22.29
57.5	1.07	31.88	1.07	29.63	1.07	29.60	1.14	15.50
55	1.10	26.97	1.12	18.90	1.12	19.40	1.18	10.60
53	1.16	16.26	1.17	13.00	1.16	11.80	1.21	6.10
50	1.22	9.76	1.24	6.10	1.24	6.60	1.27	3.30
47.5	1.28	5.27	1.30	3.30	1.30	3.60	1.31	2.00
45	1.34	2.50	1.37	1.70	1.37	1.60	1.37	1.10

4.5 Relation between Coefficient of Variation and Probability of Failure

In this section, generic slope in Bhattacharya et al. (2003) is used and the effects of the geotechnical variability levels on PF are studied. For this purpose, COV values are systematically changed for both dry and wet cases. To see the partial effects of the strength parameters, analyses are done for only phi random cases, only c random cases and both c-phi random cases (without any correlation between c and phi). Slope inclinations of 45° and 60° are chosen for the demonstration purposes. Slide software is utilized for the computations. Since the main purpose is to see the effects of COV on PF, GM type of analyses with MC sampling are used in Slide software. By this way, failure surfaces are kept constant and only the PF values are changed. The details of the analyses are provided in the Table 4.17. In all of the analyses, statistical distribution of the random variables are used as normal.

Table 4.17 Details of the analyses

	Value	
Property	**Dry Case**	**R_u = 0.2 Case**
c'	18 kPa	18 kPa
Φ'	30°	30°
γ	18 kN/m^3	18 kN/m^3
COV (%)	2.5, 5, 10, 15, 20, 25, 30	2.5, 5, 10, 15, 20, 25, 30
Inclination (°)	45 and 60	45 and 60
Analysis Type	Slide MC (GM)	Slide MC (GM)

After the analyses, resulting PF vs. COV curves are provided in Figure 4.37 to Figure 4.48 with the necessary explanations in their captions.

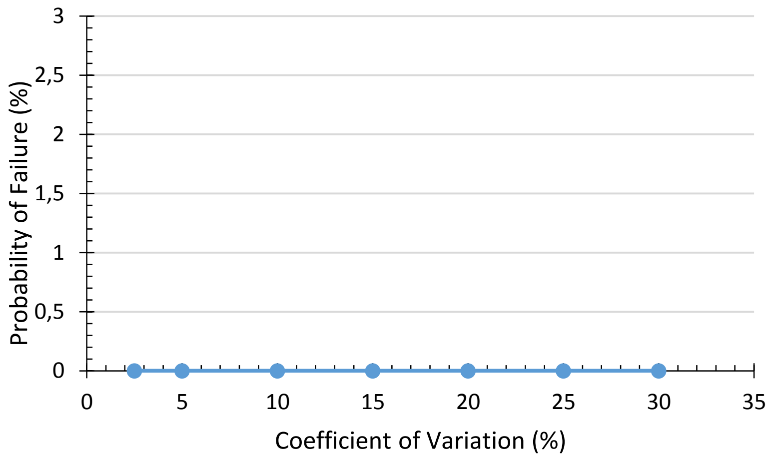


Figure 4.37 COV vs. PF curve for GM-MC (45°, dry, only c random case)

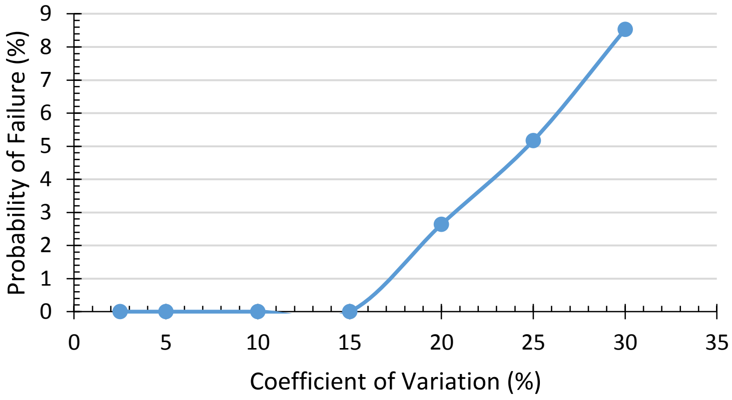


Figure 4.38 COV vs. PF curve for GM-MC (60°, dry, only c random case)

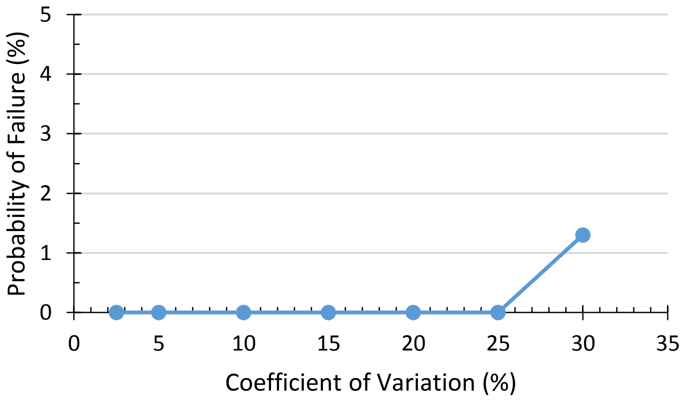


Figure 4.39 COV vs. PF curve for GM-MC (45°, dry, only phi random case)

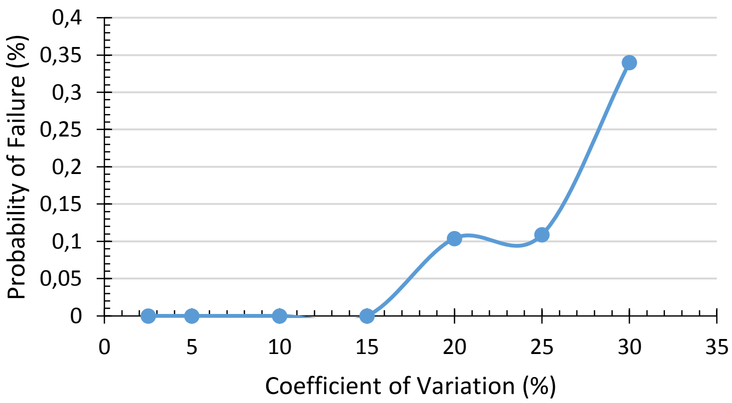


Figure 4.40 COV vs. PF curve for GM-MC (60°, dry, only phi random case)

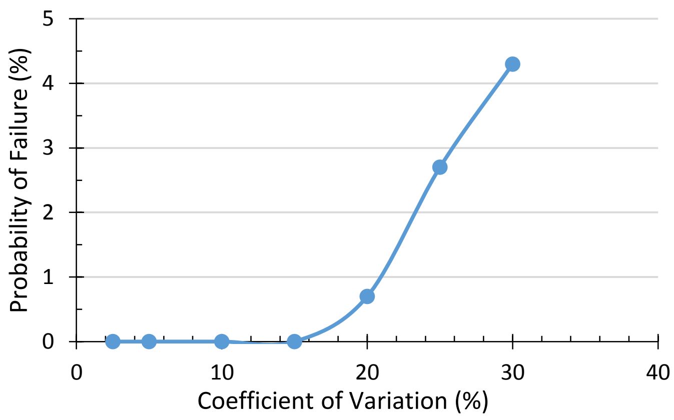


Figure 4.41 COV vs. PF curve for GM-MC (45°, dry, c-phi random case)

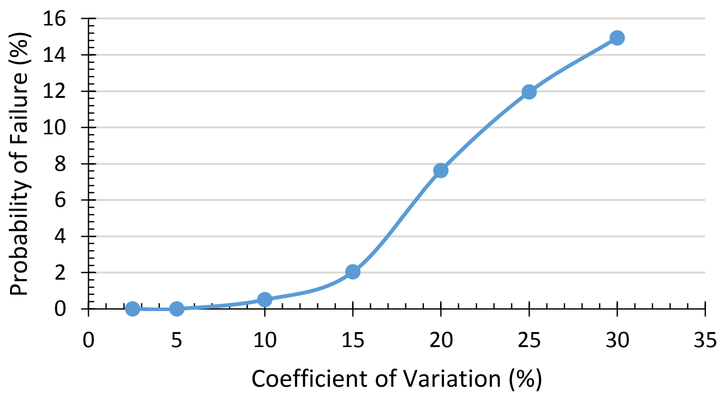


Figure 4.42 COV vs. PF curve for GM-MC (60°, dry, c-phi random case)

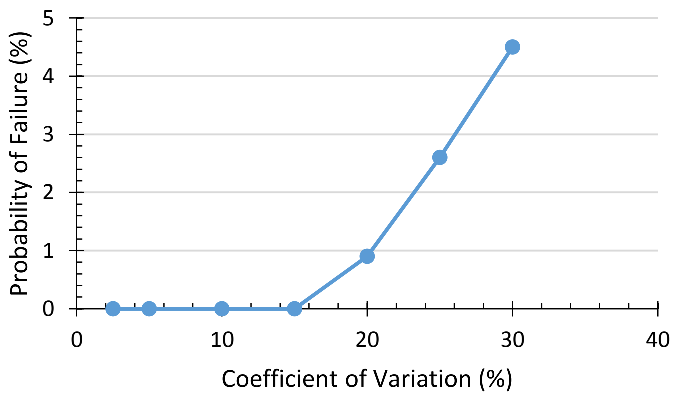


Figure 4.43 COV vs. PF curve for GM-MC (45°, R_u = 0.2, only c random case)

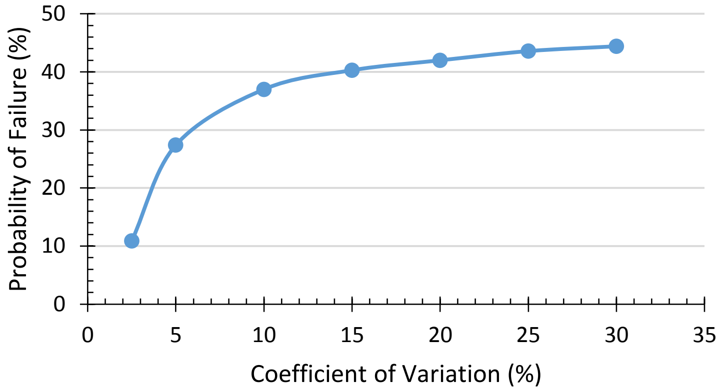


Figure 4.44 COV vs. PF curve for GM-MC (60°, R_u = 0.2, only c random case)

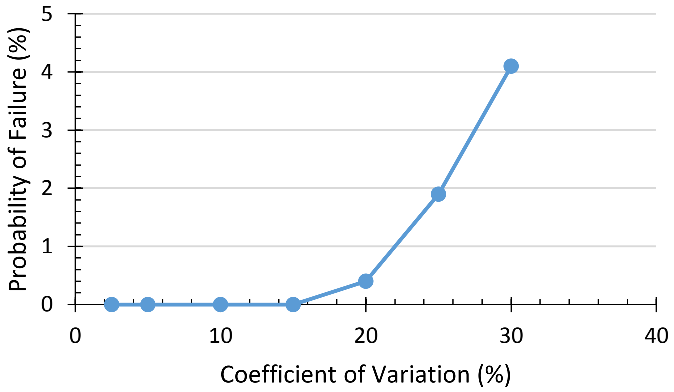


Figure 4.45 COV vs. PF curve for GM-MC (45°, R_u = 0.2, only phi random case)

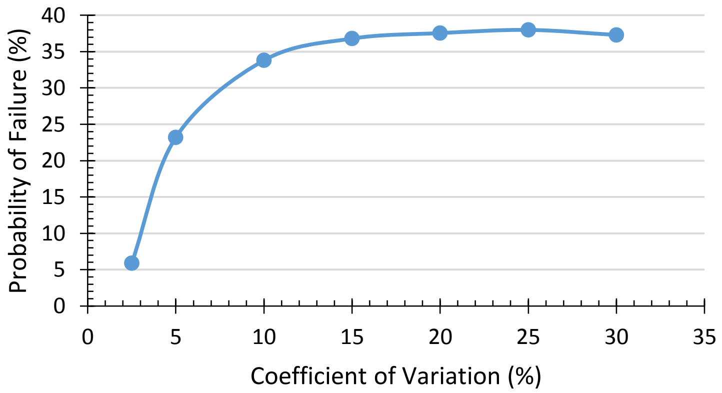


Figure 4.46 COV vs. PF curve for GM-MC (60°, R_u = 0.2, only phi random case)

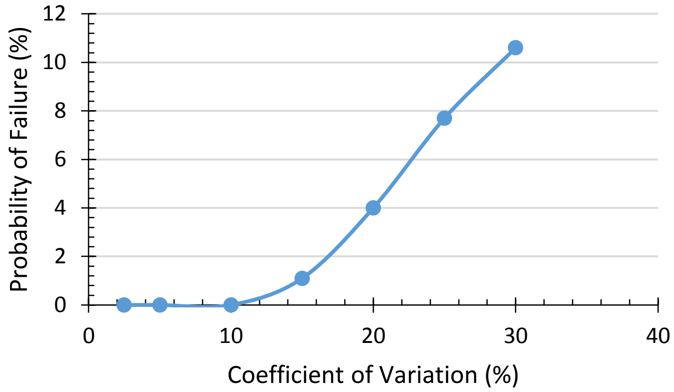


Figure 4.47 COV vs. PF curve for GM-MC (45°, R_u = 0.2, c-phi random case)

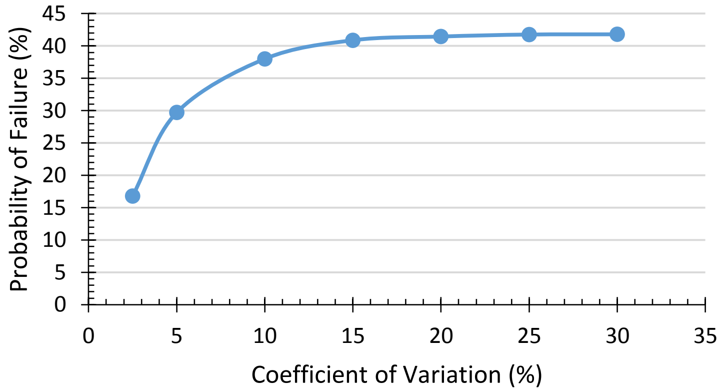


Figure 4.48 COV vs. PF curve for GM-MC (60°, R_u = 0.2, c-phi random case)

As it can be seen from the above figures the probability of failure increases with an increasing COV value for all of the cases. Since PF values of the remaining cases are all zero, conclusion about their behaviors cannot be explained. Additionally, in a few cases (60°, R_u = 0.2 cases), asymptotical behavior is observed as the COV increases.

CHAPTER 5

RANDOM FINITE ELEMENT METHOD

Consideration of spatial variability of soil properties may either overestimate or underestimate the probability of the failure depending upon the case. Hence, in this chapter, the effects of the spatial variability and its depending factors are further investigated.

This chapter mainly consists of three sections. In the first one, analyses carried out in the first three subsections of Chapter 4 are reproduced with exact geotechnical and geometrical information via random finite element code Rslope2D (Fenton and Griffiths 1992). In the second part, effects of the correlation distance, correlation ratio in x-y directions and COV levels on PF results are studied. For this purpose, a selected case is analyzed with Rslope2D. In the final part, PLAXIS-Python remote scripting is demonstrated and the limitations of Rslope2D are eliminated.

5.1 Considering the Spatial Variability: Does it matter?

For this section, as it is stated before, analyses done in the first three subsections of the Chapter 4 are reproduced via Rslope2D. However, for all of the cases, spatial correlation lengths are not presented in the literature. Therefore, it is assumed as the one-tenth of the geometric dimension throughout the all cases. As stated in the literature review chapter, it is a fairly tolerable assumption stated by Griffiths and Fenton (2004). In addition to that finite element mesh size is selected as 1 m by 1 m and initial seed value for random number generation process is given as 1. Obtained random finite element results are provided in Table 5.1 with names consistent with the case names of Chapter 4 (e.g. titles of the subsections). In the last column of the Table

5.1, comments on the obtained results are stated making a comparison between the results provided in Chapter 4.

Table 5.1 Results of RFEM analyses by Rslope2D

Name of the analysis	SRF	PF (%)	Comment
C_u Soils (Near-Failure)	1.1	100	Overestimates PF
C_u Soils (Relatively Safer)	1.6	100	Overestimates PF
c-phi Soils (Near-Failure)	1.2	1	Underestimates PF
c-phi Soils (Relatively Safer)	1.6	0	Same PF
Cohesionless Soils (Near-Failure)	1.2	1.7	Underestimates PF
Cohesionless Soils (Relatively Safer)	1.5	0	Almost same PF

In the Table 5.1, overestimating the PF means that analysis that considers the spatial correlation length results in a higher probability of failure whereas underestimation means that finding a lower PF value. In the last analysis, other methods have either zero or negligible PF values in the order of 1 to 2 %. Besides from having a different probability of failures, factor of safeties and strength reduction factors are in the same orders.

Although the above results might show the necessity of consideration for spatial variability, they may be misleading, since the correlation distances, which are 9 m in x-direction and 3 m in y-direction, are blindly assumed. Another reason might be the COV level as some of the literature studies stated. Hence, these two factors need to be carefully investigated which is the topic of the next section.

In order to detect the failure surfaces, displaced finite element meshes obtained from Rslope2D are provided in the Figure 5.1 to Figure 5.6 with necessary identification information in their captions. Some of the results do not have displaced meshes, therefore, slip surfaces for those analyses cannot be identified in Rslope2D.

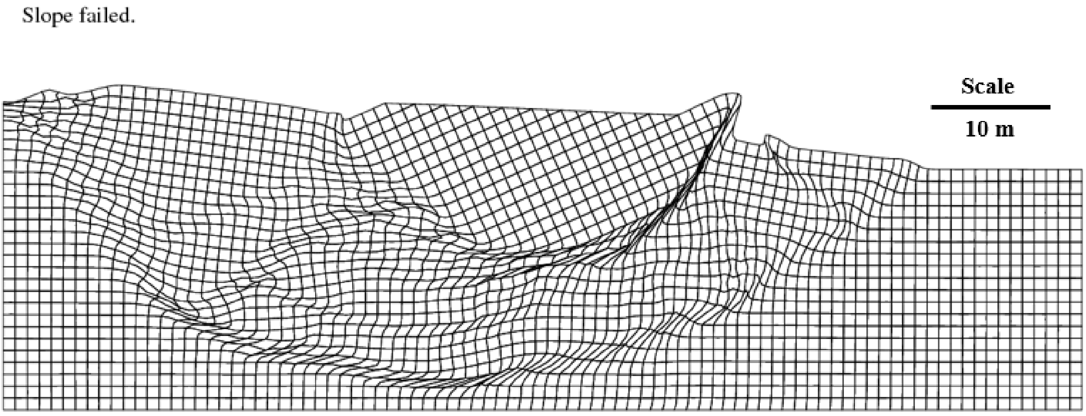


Figure 5.1 Displaced mesh view (C_u Soils, Near-Failure)

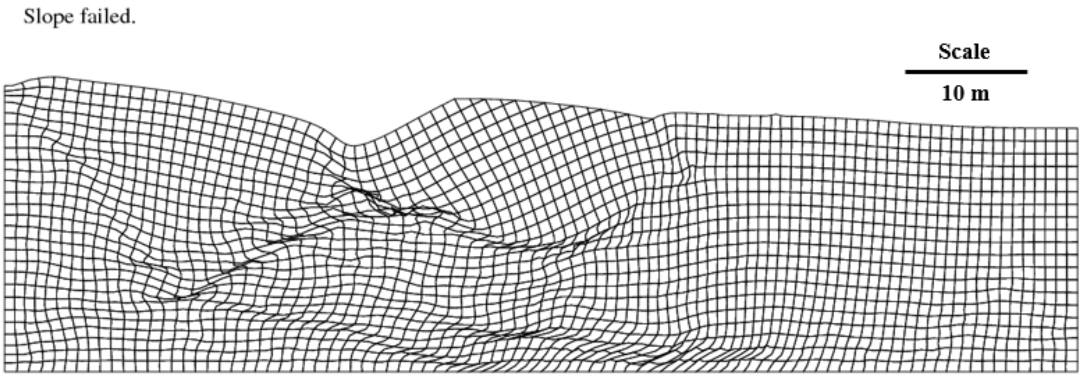


Figure 5.2 Displaced mesh view (C_u Soils, Relatively Safer)

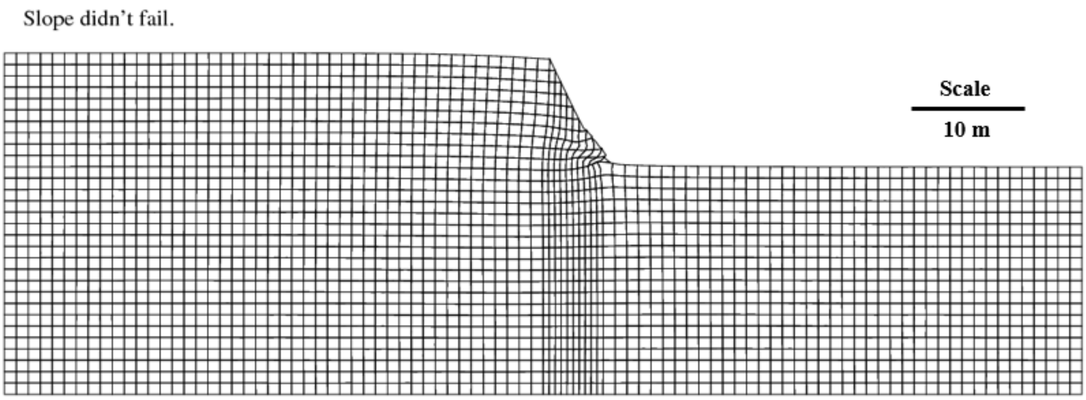


Figure 5.3 Displaced mesh view (c-phi Soils, Near-Failure)

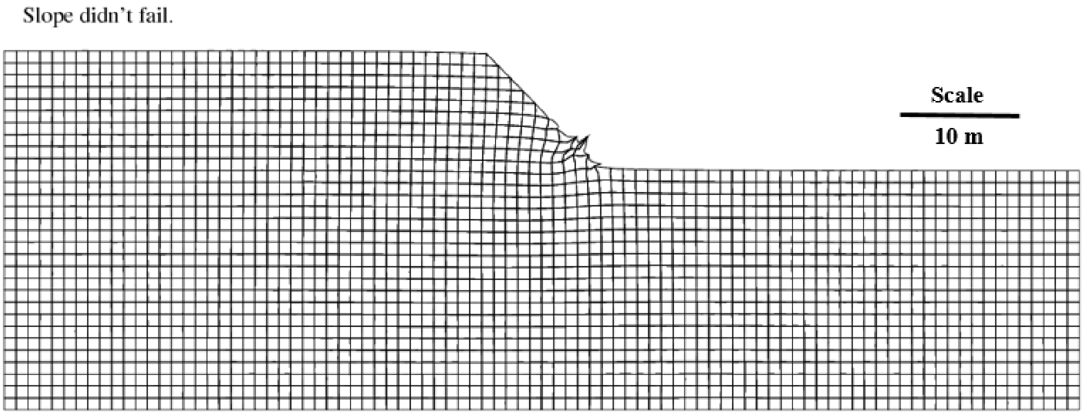


Figure 5.4 Displaced mesh view (c-phi Soils, Relatively Safer)

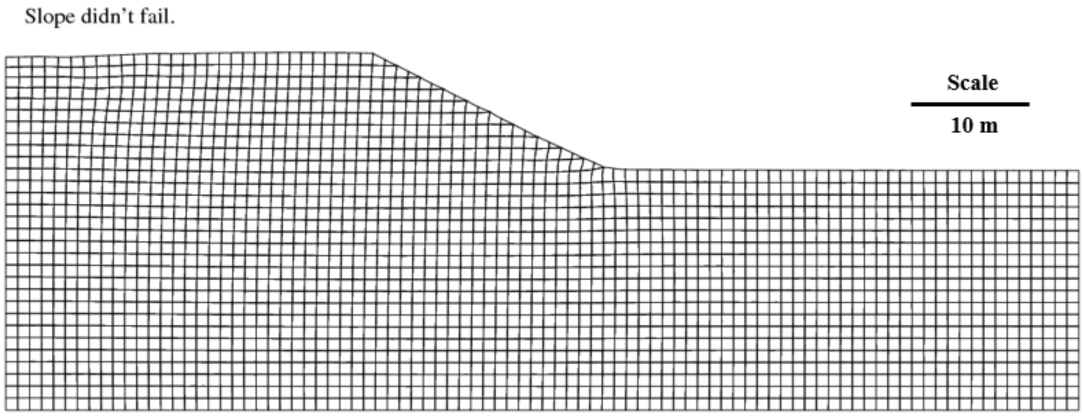


Figure 5.5 Displaced mesh view (Cohesionless Soils, Near-Failure)

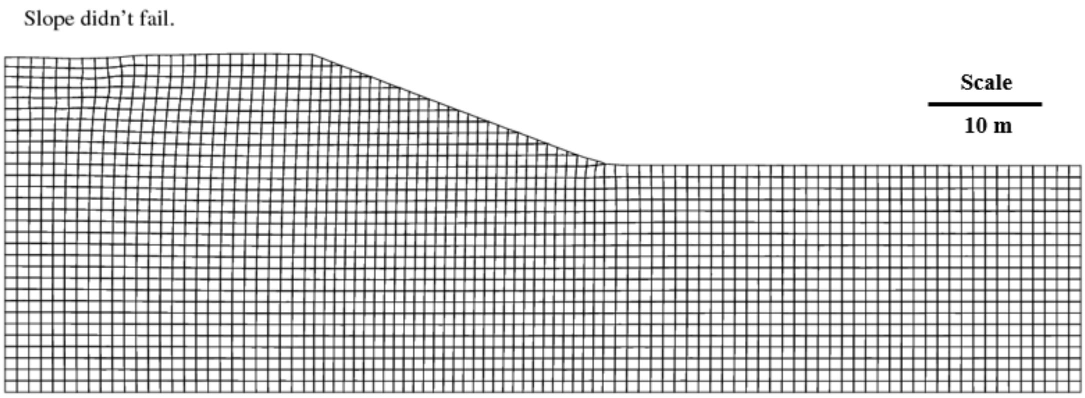


Figure 5.6 Displaced mesh view (Cohesionless Soils, Relatively Safer)

5.2 Relation between Probability of Failure, Spatial Correlation and Coefficient of Variation

In this section, several analyses are carried out in order to investigate the effects of the correlation distance, correlation ratio in x-y direction (anisotropy) and COV levels on PF results. Geotechnical parameters and geometrical details are provided in Table 5.2 and analyses details are given in Table 5.3. For this section, generic test slope in Griffiths et al. (2004) is used. Additionally, in order to investigate the high COV levels, statistical distribution of the random parameters are chosen as lognormal. If it was chosen as normal, three sigma values for cohesion will be zero or lower after COV = 33.3 % which cannot be the case.

Table 5.2 Geotechnical parameters and input values

Property	**Value**
C_u	50 kPa
γ	20 kN/m^3
GWT	none
Slope Height	10 m
Inclination	1V:2H
Element Size	1 m x 1 m
Seed	1
No of Run	1000 MC
FS (deterministic)	1.5

Table 5.3 Details of the analyses

COV (%): 10, 30, 50, 75, 100
Correlation Ratio between x and y directions: 1, 2, 4, 8, 10
Correlation Length in x-direction (m): 6, 8, 10, 12, 14, 16, 18, 20

Correlation ratios in Table 5.3 are defined as follows:

$$Ratio = \frac{Correlation\ Length\ in\ x-direction}{Correlation\ Length\ in\ y-direction} \tag{5.1}$$

In Figure 5.7 to Figure 5.10, for a selected COV, generated random fields of undrained shear strength are provided for different ratios and correlation lengths. Two extreme cases are given in those figure; one is ratios of 1 and 10, the other is, for both ratios, correlation length in x-direction of 6 m and 20 m. As it can be seen from the figure, as the correlation length increases, generated field becomes more homogeneous. However, as the ratio increases, generated field in y-direction becomes more heterogeneous.

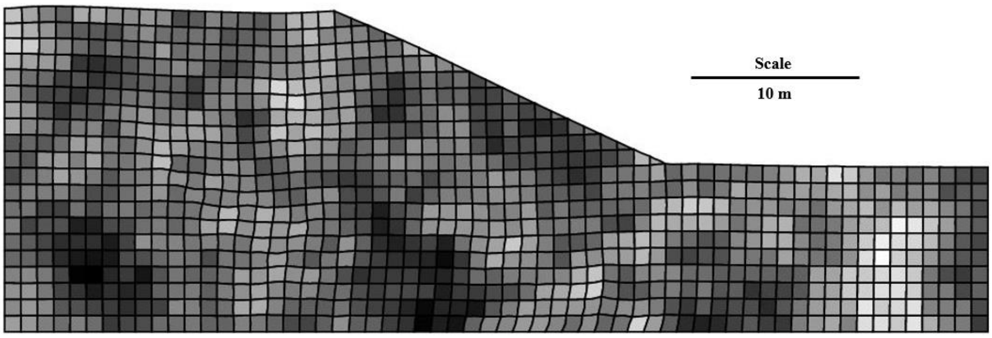


Figure 5.7 Generated random field by Rslope 2D (COV = 50 %, ratio = 1, correlation in x-direction = 6 m)

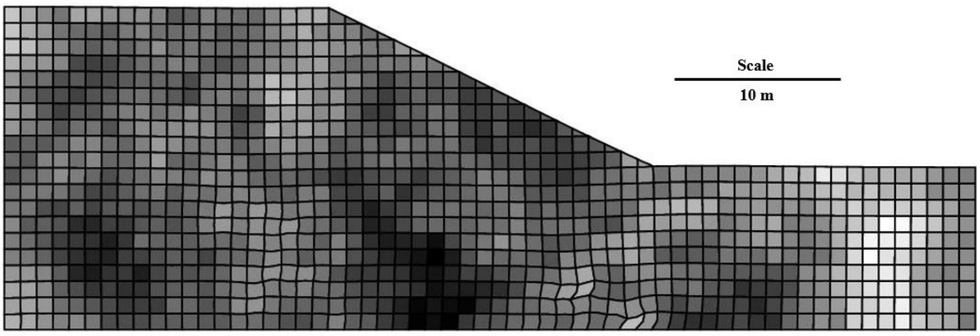


Figure 5.8 Generated random field by Rslope 2D (COV = 50 %, ratio = 1, correlation in x-direction = 20 m)

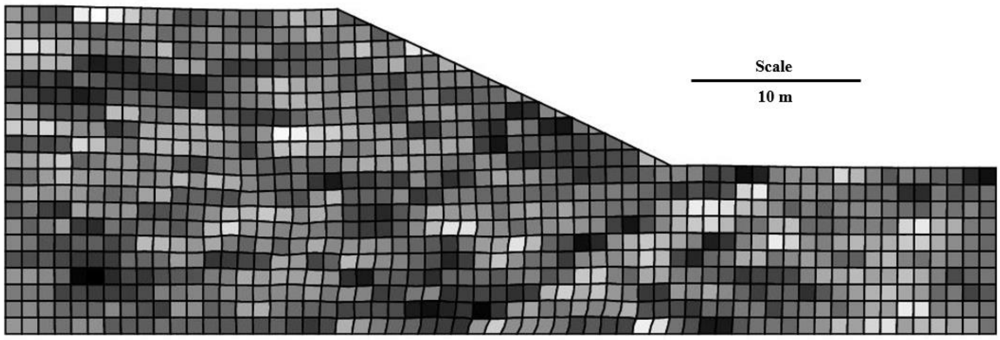


Figure 5.9 Generated random field by Rslope 2D (COV = 50 %, ratio = 10, correlation in x-direction = 6 m)

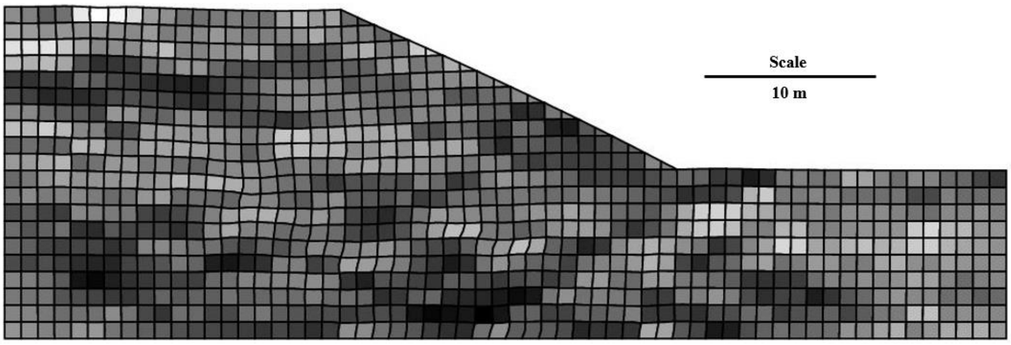


Figure 5.10 Generated random field by Rslope 2D (COV = 50 %, ratio = 10, correlation in x-direction = 20 m)

After all of the analyses, obtained PF values are tabulated and correlation length-PF curves are created with respect to correlation ratios for every COV levels. These graphs are provided in Figure 5.11 to Figure 5.15 with necessary identification information in their captions.

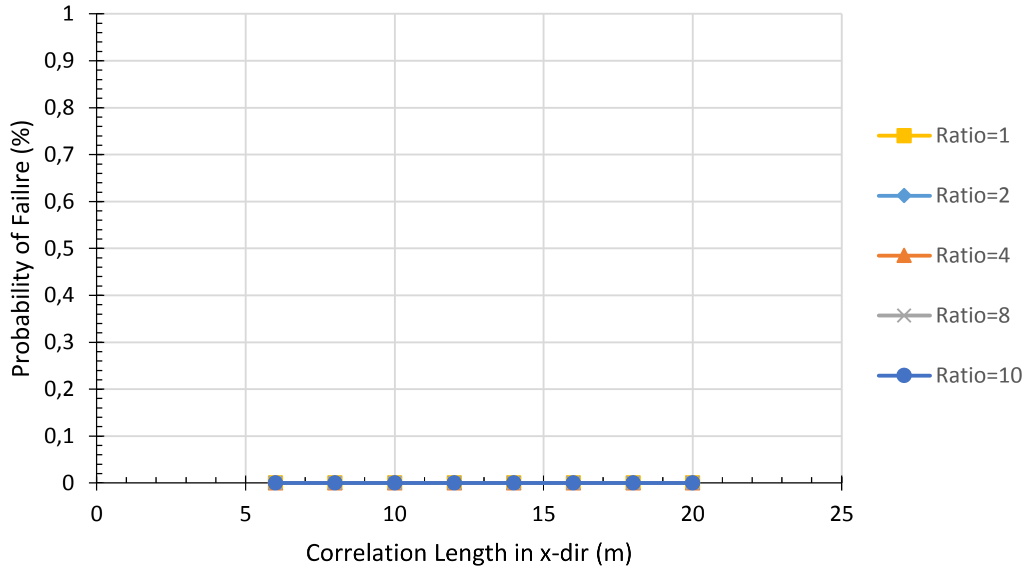


Figure 5.11 Correlation length vs. PF curves (COV = 10%)

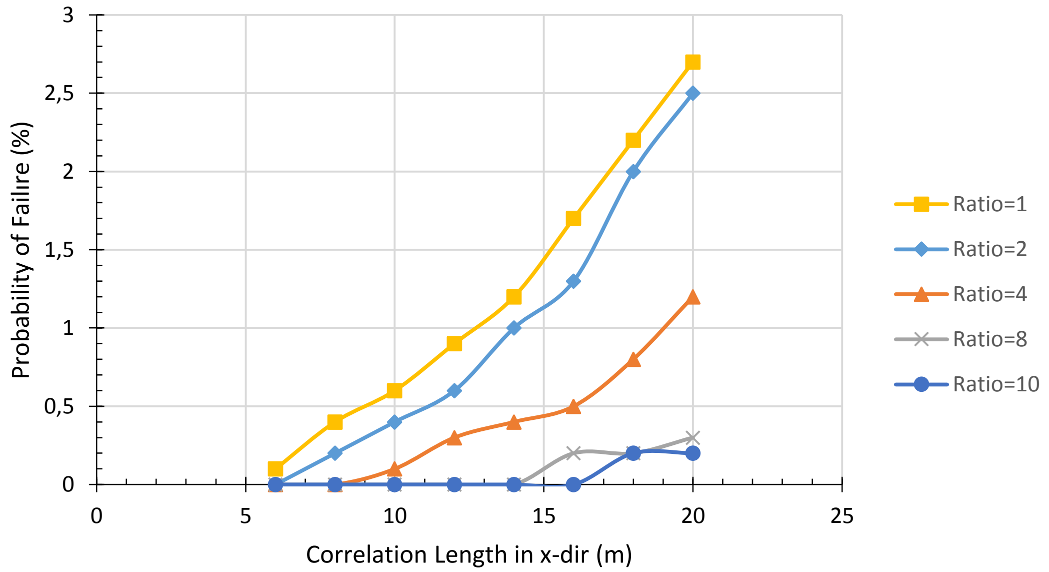


Figure 5.12 Correlation length vs. PF curves (COV = 30%)

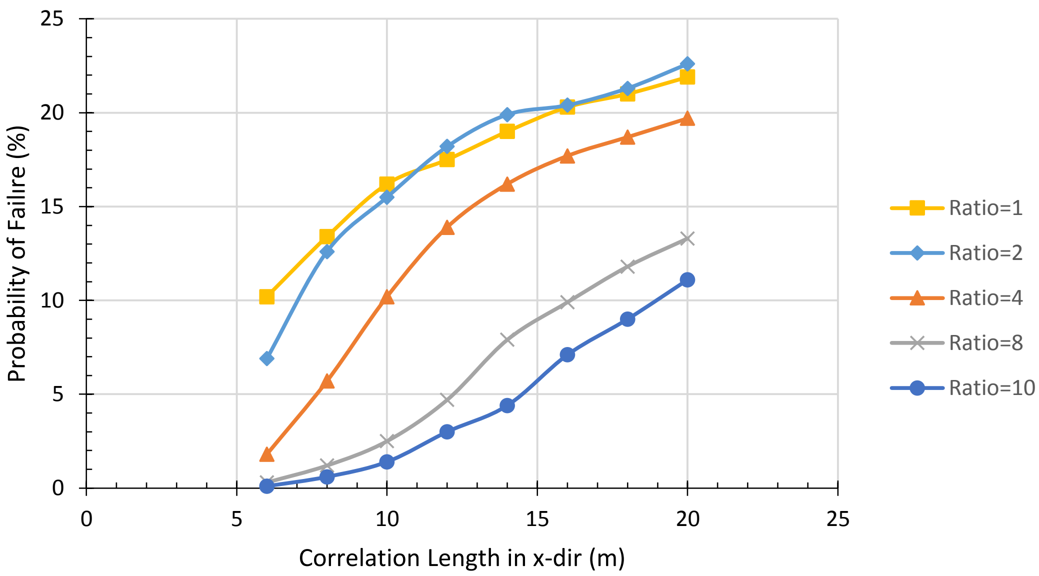


Figure 5.13 Correlation length vs. PF curves (COV = 50%)

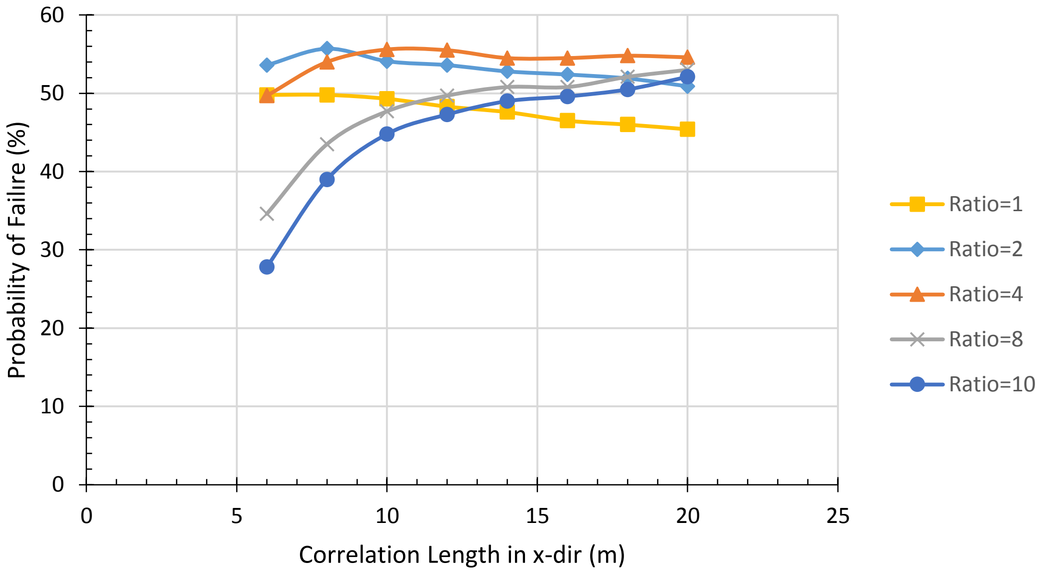


Figure 5.14 Correlation length vs. PF curves (COV = 75%)

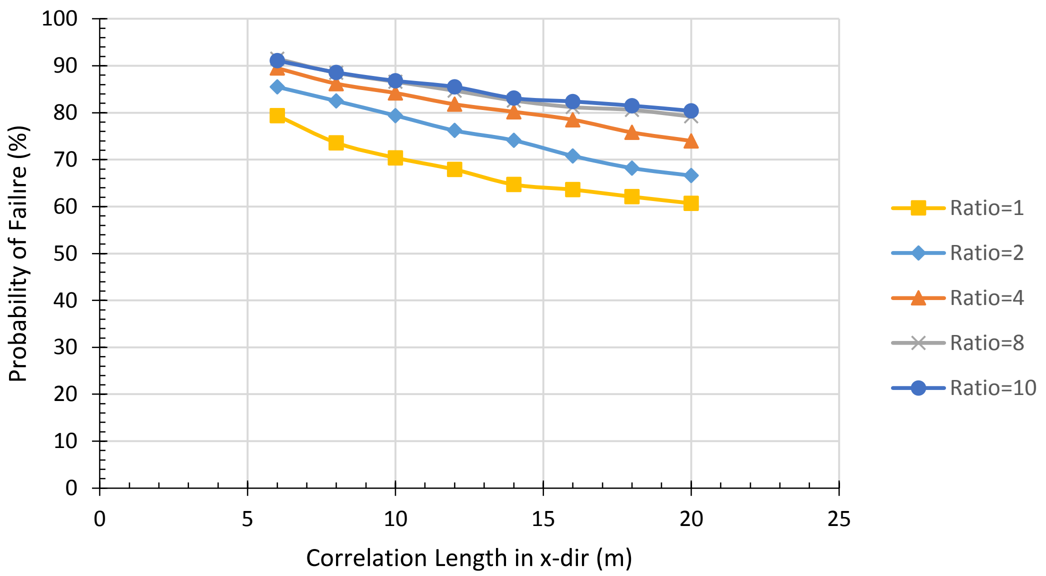


Figure 5.15 Correlation length vs. PF curves (COV = 100%)

Obtained PF values, except for COV = 10% case which is all zero, are tabulated in Table 5.4 to Table 5.7 with respect to COV levels.

Table 5.4 Obtained PF values (COV = 30%)

	PF (%)				
Corr. Length in x-dir. (m)	**Ratio = 1**	**Ratio = 2**	**Ratio = 4**	**Ratio = 8**	**Ratio = 10**
6	0.1	0	0	0	0
8	0.4	0.2	0	0	0
10	0.6	0.4	0.1	0	0
12	0.9	0.6	0.3	0	0
14	1.2	1	0.4	0	0
16	1.7	1.3	0.5	0.2	0
18	2.2	2	0.8	0.2	0.2
20	2.7	2.5	1.2	0.3	0.2

Table 5.5 Obtained PF values (COV = 50%)

	PF (%)				
Corr. Length in x-dir. (m)	**Ratio = 1**	**Ratio = 2**	**Ratio = 4**	**Ratio = 8**	**Ratio = 10**
6	10.2	6.9	1.8	0.3	0.1
8	13.4	12.6	5.7	1.2	0.6
10	16.2	15.5	10.2	2.5	1.4
12	17.5	18.2	13.9	4.7	3
14	19	19.9	16.2	7.9	4.4
16	20.3	20.4	17.7	9.9	7.1
18	21	21.3	18.7	11.8	9
20	21.9	22.6	19.7	13.3	11.1

Table 5.6 Obtained PF values (COV = 75%)

	PF (%)				
Corr. Length in x-dir. (m)	**Ratio = 1**	**Ratio = 2**	**Ratio = 4**	**Ratio = 8**	**Ratio = 10**
6	49.8	53.6	49.7	34.6	27.8
8	49.8	55.7	54	43.5	39
10	49.3	54.1	55.6	47.7	44.8
12	48.3	53.6	55.5	49.7	47.3
14	47.6	52.8	54.5	50.8	49
16	46.5	52.4	54.5	50.8	49.6
18	46	51.9	54.8	52.1	50.5
20	45.4	50.9	54.6	53	52.1

Table 5.7 Obtained PF values (COV = 100%)

Corr. Length in x-dir.(m)	PF (%)				
	Ratio = 1	**Ratio = 2**	**Ratio = 4**	**Ratio = 8**	**Ratio = 10**
6	79.4	85.5	89.5	91.5	91.1
8	73.6	82.5	86.2	88.5	88.6
10	70.4	79.4	84.2	86.6	86.8
12	67.9	76.2	81.8	84.7	85.5
14	64.7	74.1	80.2	82.6	83.1
16	63.6	70.8	78.5	81.2	82.4
18	62.1	68.2	75.8	80.6	81.5
20	60.7	66.6	74	79.2	80.4

It is observed from the figures that as the spatial correlation length increases, PF increases for the COV values of 30 and 50%, whereas the trend shows a mixed behavior (increases & decreases) for the case of COV=75%, and PF decreases with increasing correlation length for the case of COV of 100%.

In the literature, there are some comments about the results of RFEM compared with simple approaches without spatial correlation. Some researchers (El-Ramly et al. 2002; Ji et al. 2013; Griffiths et al. 2004) say that ignoring the spatial variability results in overestimation of PF. In other words, not accounting the spatial variability will results in higher PF and overestimates the risk of failure compared to classical methods without spatial correlation. Some claims the opposite (deWolfe et al. 2010; Griffiths et al. 2004). Griffiths and Fenton (2004) states that ignoring the spatial variability will overestimate PF when COV is relatively small whereas it will underestimates PF when COV is relatively high. The results obtained in this section seem to be confirming Griffiths and Fenton (2004).

Theoretically, when the value of spatial correlation length is infinity it means a homogeneous field. For the case of very large correlation length, and ratio=1 together with a large COV value, one can think of cases, where the material is almost like a uniform material, with mostly high shear strength. On the other hand, for large value of horizontal correlation length, if a weak point is generated during the random field simulation, the nearby points next to it have a greater chance to be weak. The probability of forming a weak zone during random field simulation will influence the PF value.

Another observation is that, for a given horizontal correlation distance, as the ratio increases from 1 to 10, (i.e. the soil is more and more heterogeneous in the vertical direction as compared to horizontal direction) PF decreases for the COV of 30% and 50%. The behavior is mixed (increases & decreases) for the case of COV=75%. And for the case of COV of 100%, as the ratio increases PF increases. When we look into the graphs carefully, we observe that the change is PF value is small for a given ratio; whereas, the change in PF is large for a given horizontal correlation length and changing ratio values. We may conclude that the vertical correlation length is more important issue, since it influences PF more dramatically than the horizontal correlation length does. This is also noted in the literature in that the vertical spatial variability has a much more significant effect on PFs than the horizontal spatial variability (Cho 2007; Ji et al. 2012, Jiang et al. 2014).

When we return to the question posed at the title of this section "Considering the Spatial Variability: Does it matter?" we should answer as "Yes, it matters" and spatial correlation length can influence the PF (most commonly by increasing it for COV less than or equal to 50%).

5.3 PLAXIS Remote Scripting via Python

In this section, series of probabilistic analyses are carried out using PLAXIS 2D with the help of remote scripting via Python. As explained in the methodology chapter (e.g. Chapter 3), coding does not exactly do what Rslope2D does. However, by dividing the layers as equal to the spatial correlation length (or as desired), a number of sub-elements are created and material properties are assigned to them randomly. This is the proposed method by the writer of this thesis and it may be called as "Pseudo-Random Finite Element Method". In order to verify the accuracy of the approach, first three types of analyses in sections 4.1, 4.2 and 4.3 of Chapter 4 are reproduced with Slide, Rslope2D and Python-PLAXIS-MC (1000 Analyses). For the sake of demonstration, only near-failure cases are studied. Spatial correlation length and

PLAXIS-Python division are taken as equal to each other. In Table 5.8, obtained factor of safeties and probability of failures are provided. Last column of the table shows how close the proposed method are to the results of Rslope2D in terms of probability of failure.

Table 5.8 Comparison of three approaches

		Approaches			
		Slide	**Rslope2D**	**Python-PLAXIS-MC**	**Difference (%)**
C_u Soil (Sect. 4.1)	FS	1.20	1.15	1.12	96.4
	PF (%)	41.8	69.3	68.3	
c-phi Soil (Sect. 4.2)	FS	1.19	1.20	1.13	66.1
	PF (%)	13.8	2.0	6.0	
Cohesionless Soil (Sect. 4.3)	FS	1.12	1.2	1.12	21.5
	PF (%)	17.2	4.6	14.5	

From the Table 5.8, it can be said that proposed method can easily be used for first type of soils. Coding also writes the obtained factor of safeties and indicates the analysis having minimum FS value. Therefore, slip surface of this analysis can be used as the most critical failure surface. One of the input slope geometry and layers are given in Figure 5.16 for demonstration purposes. Additionally, slip surfaces are provided in Figure 5.17 to Figure 5.19. In order to reduce the computational time for Python-PLAXIS-MC, dimensions of boundary extensions are shortened but, of course, slope height, inclination etc. are kept as they are in Chapter 4.

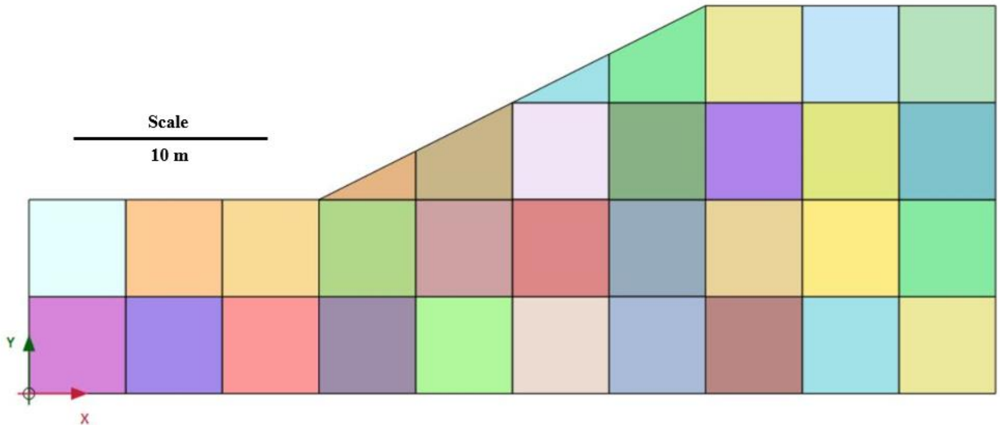


Figure 5.16 Generated field (e.g. layers) for C_u-soil case

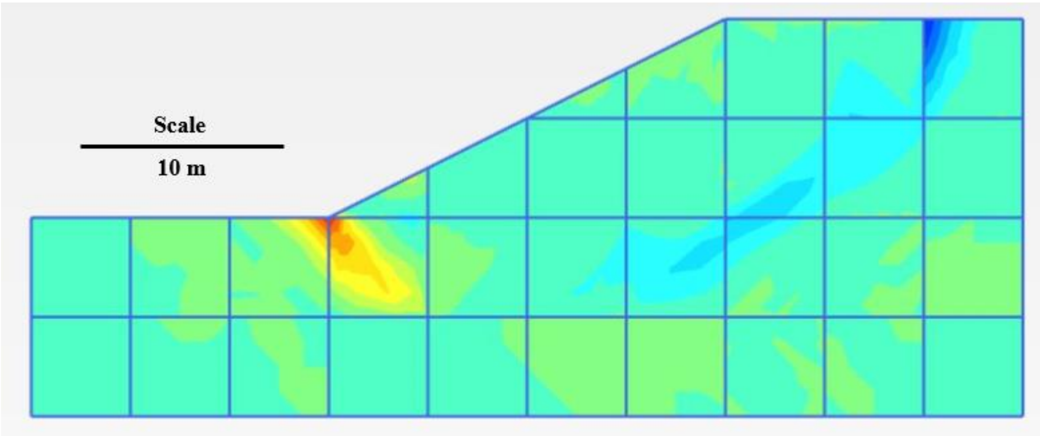


Figure 5.17 Failure surface zone (C_u soils)

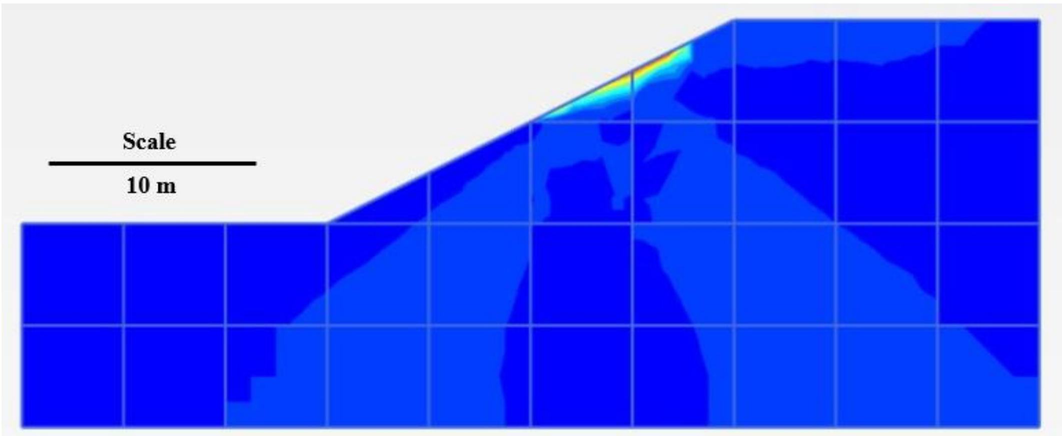


Figure 5.18 Failure surface zone (cohesionless soils)

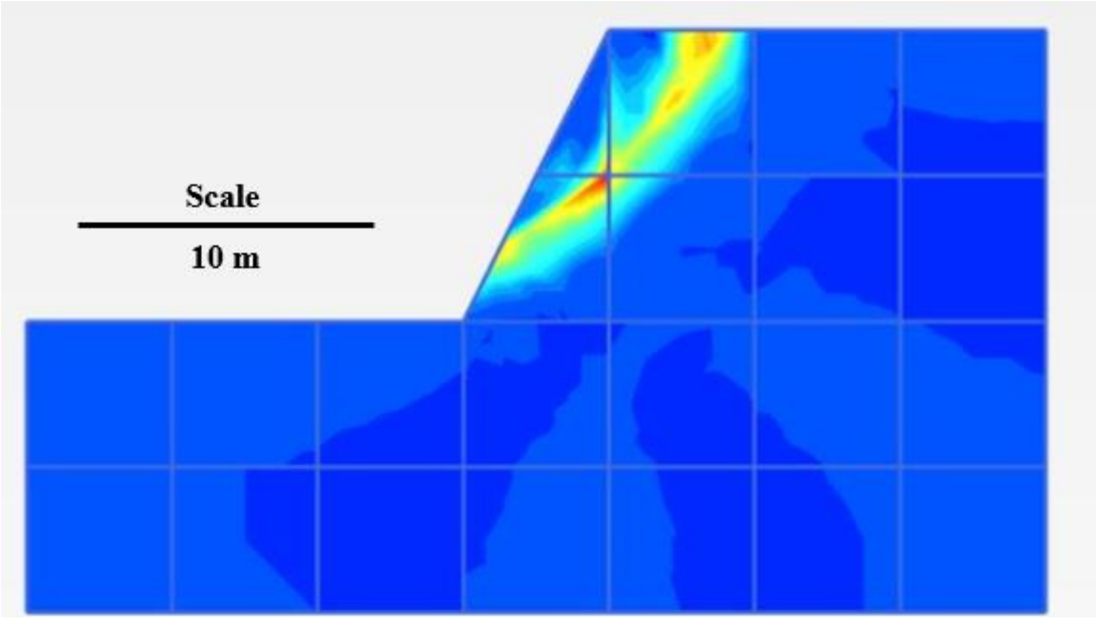


Figure 5.19 Failure surface zone (c-phi soils)

When comparisons are made between previously obtained failure surfaces of Chapter 4 and newly obtained failure zones, it can be said that cohesion-frictional type of soils have almost identical failure surfaces although this type of analyses ranked in second for closeness as shown in Table 5.8. Therefore, it is concluded that proposed approach can also be used for c-phi soils.

5.3.1 Hypothetical Layered Slope

In this section, a hypothetical slope with 2 subsoil layers, 3 slope inclinations and GWT is created. Analyses are done for 1000 Monte-Carlo runs for Slide and 1000 runs for Python-PLAXIS with a subdivision of layers. Geotechnical parameters and analyses details are provided in Table 5.9 whereas comparison of approaches are given in Table 5.10. Resultant slip surfaces are provided in Figure 5.20 and Figure 5.21.

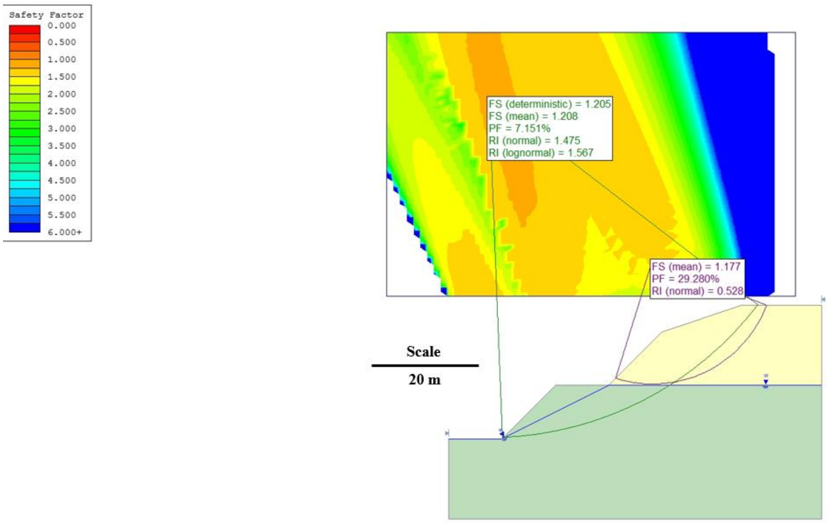


Figure 5.20 Deterministic and the most critical failure surfaces (Slide analysis)

Table 5.9 Geotechnical input parameters

Layer	Property	Mean Value	COV (%)	Distribution
Upper Layer	Φ'	0°	none	none
	c_u	50 kPa	30	Normal
	γ	18 kN/m^3	none	none
Lower Layer	Φ'	30°	10	Normal
	c'	10 kPa	10	Normal
	γ	20 kN/m^3	none	none

Table 5.10 Comparison of results

Method	FS or SRF	PF (%)
Slide-Deterministic	1.21	-
Slide-MC	1.18	29.3
Python-PLAXIS-MC	0.83	42.8

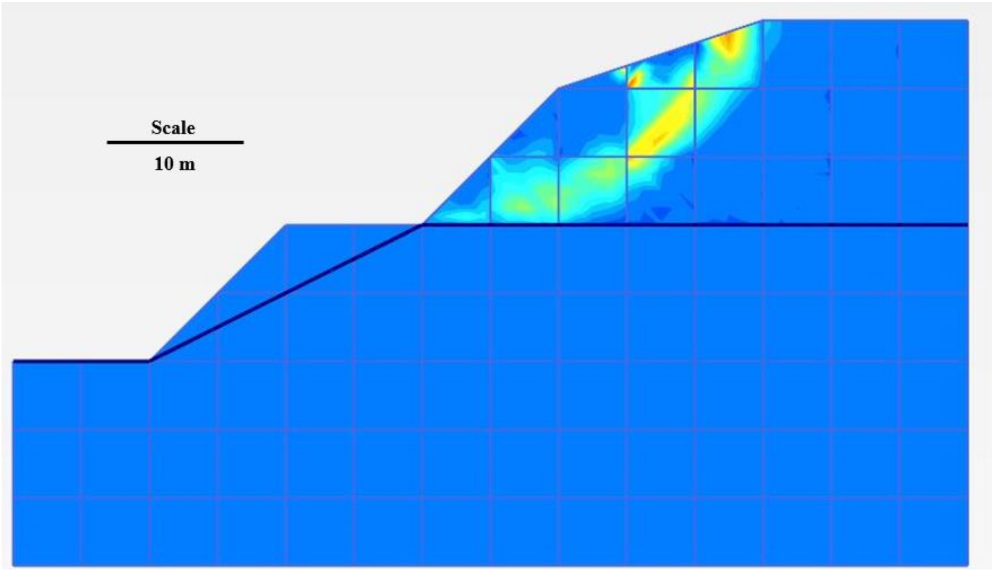


Figure 5.21 Failure surface zone with minimum FS (Python-PLAXIS-MC analysis)

5.3.2 Slope Having Thin and Weak Layer

In this section, one of the example slope geometry studied by Liu et al. (20015) is analyzed by probabilistic LEM (Slide), probabilistic FEM (Phase 2) that utilizes point estimate method and another probabilistic FEM (Python-PLAXIS-MC, 1000 runs) approaches. Geotechnical parameters and obtained results are provide in Table 5.11 and Table 5.12, respectively. Although the original research does only deterministic analyses, failure surface obtained from the study also provided in Figure 5.22. Additionally, failure surfaces obtained from the new analyses are given in Figure 5.23 to Figure 5.25.

Table 5.11 Geotechnical input parameters

Layer (From top to bottom)	Property	Mean Value	COV (%)	Distribution
1	Φ'	20°	10	Normal
	c'	15 kPa	20	Normal
	γ	18.62 kN/m^3	none	none

Table 5.11 Continued

2	Φ'	21°	10	Normal
	c'	17 kPa	20	Normal
	γ	18.62 kN/m^3	none	none
3	Φ'	10°	10	Normal
	c'	5 kPa	20	Normal
	γ	18.62 kN/m^3	none	none
4	Φ'	28°	10	Normal
	c'	35 kPa	20	Normal
	γ	18.62 kN/m^3	none	none

Table 5.12 Comparison of the results

Method	**FS or SRF**	**PF (%)**
Slide-MC	1.48	0
Phase 2	1.12	8.8
Python-PLAXIS-MC	0.73	29.2

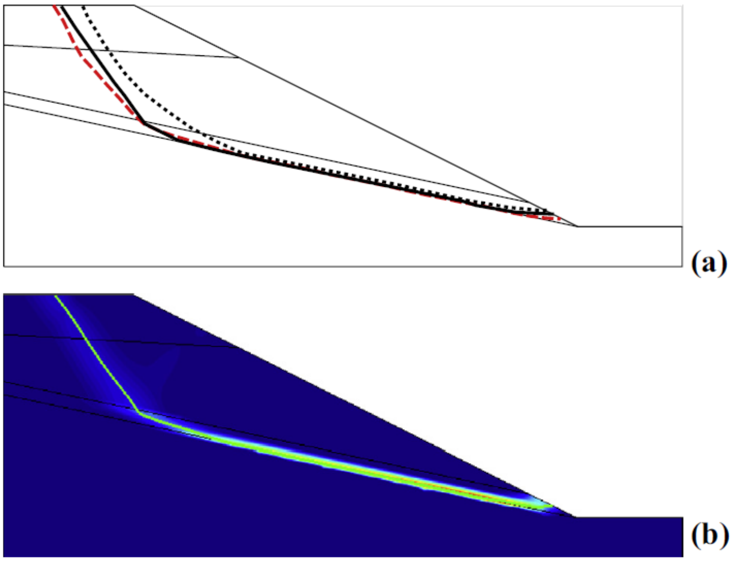


Figure 5.22 Deterministic failure surfaces obtained by Liu et al. (2015)

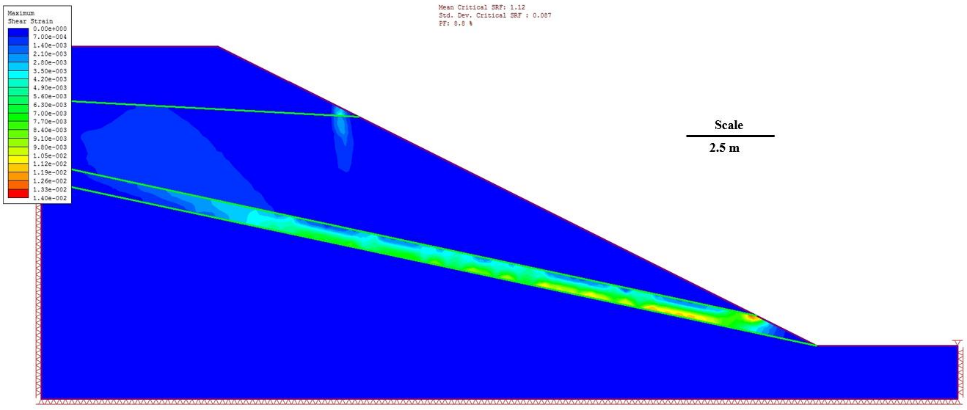


Figure 5.23 Failure surface zone (Phase 2 analysis)

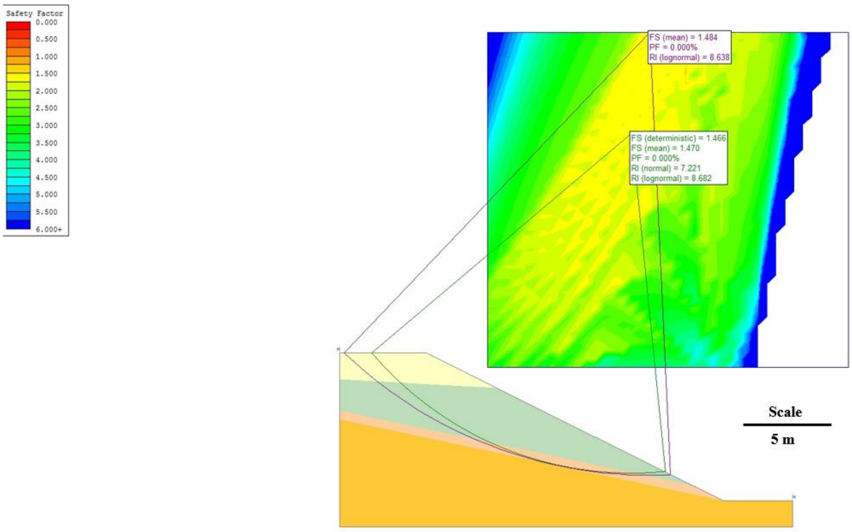


Figure 5.24 Deterministic and the most critical failure surfaces (Slide analysis)

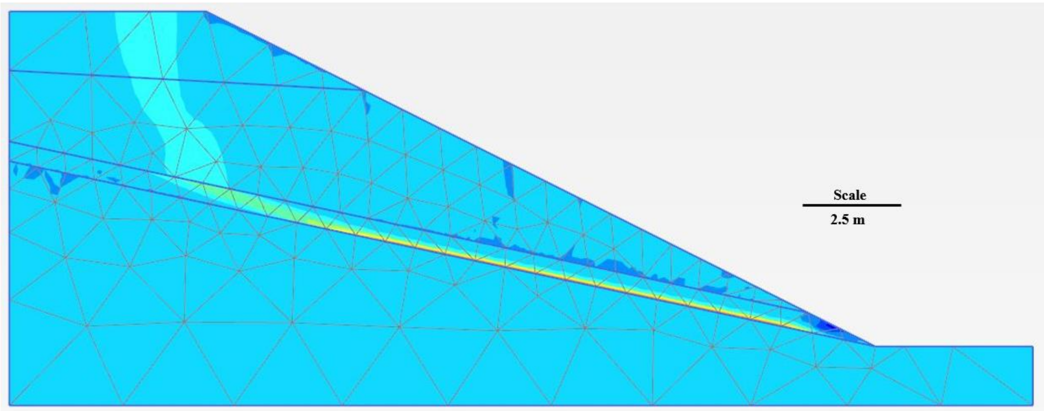


Figure 5.25 Failure surface zone with minimum FS (Python-PLAXIS analysis)

CHAPTER 6

CASE STUDIES

In this chapter, the analyses that are done in Chapter 4 are reproduced for real-life landslides which are taken from the literature. The analyses are done using one LEM tool (Slide) and one FEM tool (Phase2) so that probabilistic limit equilibrium and finite element methods can be compared in terms of failure surface, probability of failure and factor of safety. As it is seen both from the literature studies and the results of Chapter 4, Monte-Carlo and Latin-Hypercube sampling methods give results that are sufficiently close to each other. Therefore, all Slide analyses carried in this chapter are done only for Overall Slope analysis type with 1000 Monte-Carlo sampling. On the other hand, 2^n Phase2 analyses are done for each case, n being the number of random parameters.

6.1 Slope Debris in Gümüşhane, Turkey

Stability of a landslide in Gümüşhane, Turkey is investigated by Alemdag et al. (2015) with a deterministic limit equilibrium and finite element methods. For that purpose, survey lines, trial pits and boreholes are drilled so that 4 cross-sections of the landslide and the geotechnical parameters can be determined by necessary laboratory tests. In accordance with the findings of Alemdag et al. (2015), cross-section number 1 is determined as the most critical one. In this section, this cross-section is analyzed by the aforementioned methods. Geotechnical parameters of slope debris used in the analyses are taken from the original research and COV values are taken from the literature since such information is not provided in the original work. All input parameters are given in Table 6.1 whereas analyses results are provided in Table 6.2. Failure surfaces obtained from the analyses are given in Figure 6.1 and Figure 6.2.

Table 6.1 Geotechnical input parameters (after Alemdag et al. 2015)

Layer	Property	Mean Value	COV (%)	Distribution
Slope Debris	$\Phi^{'}$	20°	10	Normal
	$c^{'}$	2.63 kPa	20	Normal
	γ	20.4 kN/m^3	none	none
Strong Layer (Basalt-Andesite)	$\Phi^{'}$	35°	none	none
	$c^{'}$	200 kPa	none	none
	γ	20 kN/m^3	none	none

Table 6.2 Comparison of the results

Method	FS or SRF	PF (%)
Slide-MC	0.53	100
Phase2	0.48	100

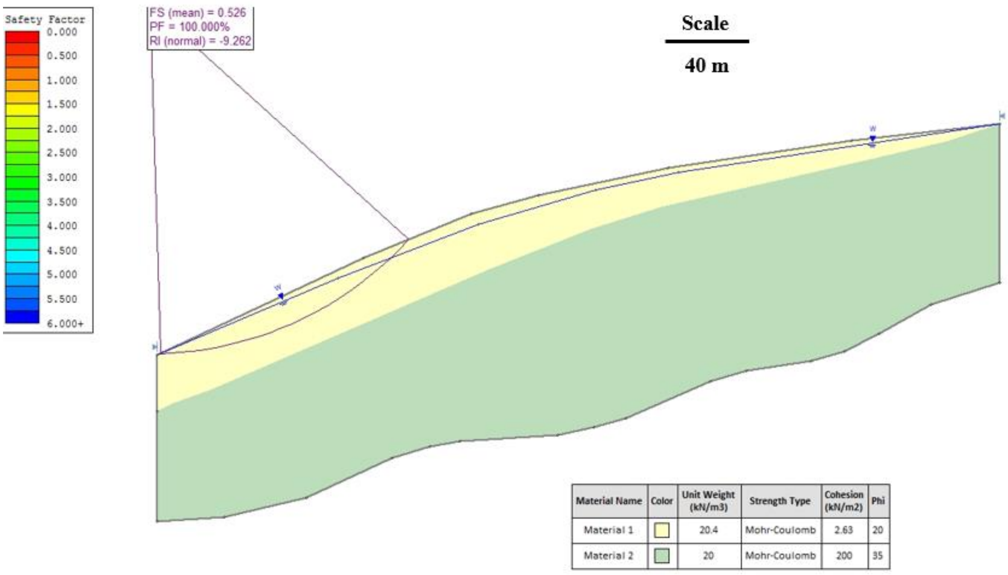


Figure 6.1 Failure surface obtained from Slide

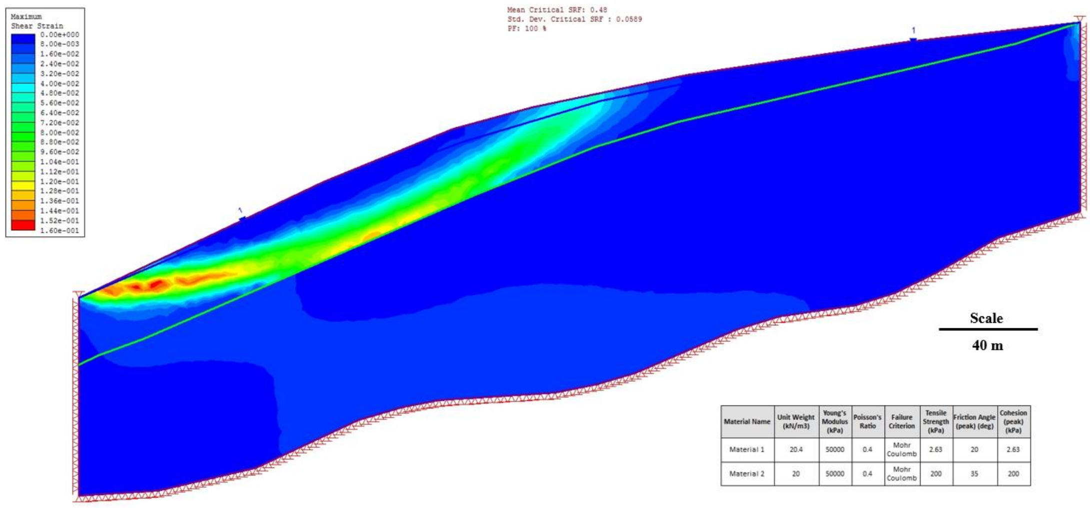


Figure 6.2 Failure zone obtained from Phase 2

Although obtained factor of safeties and probability of failure values are almost the same, failure surfaces of two approaches are slightly different. Finite element analysis has wider and deeper failure zone compared to the one obtained from limit equilibrium method.

6.2 Sugar Creek Embankment in Iowa, U.S.A.

This embankment fill slope is studied by Cho (2007) in order to see the effect of spatial correlation on probability of failure. This study is also included in the literature review chapter with slope geometry and subsoil conditions so it is not provided again in this section. Both the mean values of geotechnical parameters and their COV values are taken from the original work, thus, provided in Table 6.3. Since the deterministic and probabilistic slip surfaces provided in the original research do not pass through the last two layers, only the first three layers' parameters are treated as random in order to shorten the analysis time. Obtained results are given in Table 6.4 whereas resultant slip surfaces are provided in Figure 6.3 and Figure 6.4. This selected case also analyzed with Python-PLAXIS-MC with 1000 runs and the results are provided in Table 6.4. Obtained failure surface is provided in Figure 6.5. For the sake of completeness, the results obtained by Cho (2007) are also included in Table 6.4 and obtained failure surface are given in Figure 6.6.

Table 6.3 Geotechnical input parameters (Cho 2007)

Layer	Property	Mean Value	COV (%)	Distribution
Compacted Fill	Φ'	12°	20	Lognormal
	c'	29 kPa	22	Lognormal
	γ	20.4 kN/m^3	none	none
Alluvium	Φ'	16.5°	21	Lognormal
	c'	33 kPa	62	Lognormal
	γ	19 kN/m^3	none	none
Highly Weathered Shale	Φ'	12.8°	38	Lognormal
	c'	33.2 kPa	60	Lognormal
	γ	20 kN/m^3	none	none
Moderately Weathered Shale	Φ'	21.6°	none	none
	c'	97 kPa	none	none
	γ	20 kN/m^3	none	none
Slightly Weathered Shale	Φ'	23.3°	none	none
	c'	675 kPa	none	none
	γ	21 kN/m^3	none	none

Table 6.4 Comparison of the results

Method	FS or SRF	PF (%)
Slide-MC	1.59	0.20
Phase2	1.42	10.57
Python-PLAXIS-MC	0.79	3.90
Cho 2007 Results		
FORM	1.62	2.01
MC (20 m corr. length in x-dir.)	1.63	0.12
MC (40 m corr. length in x-dir.)	1.63	0.88
MC (∞ corr. length in x-dir.)	1.63	1.18

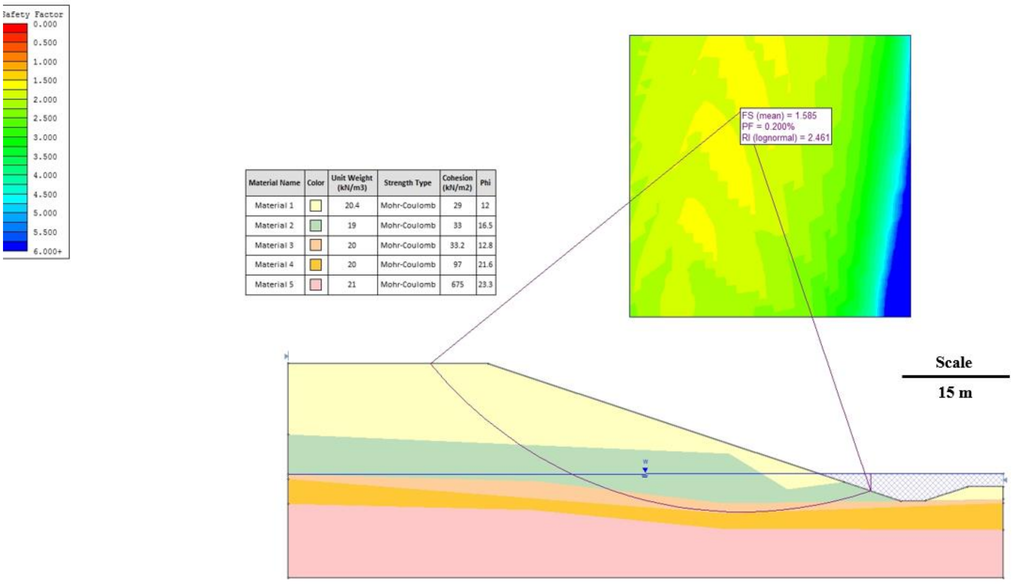


Figure 6.3 Failure surface obtained from Slide

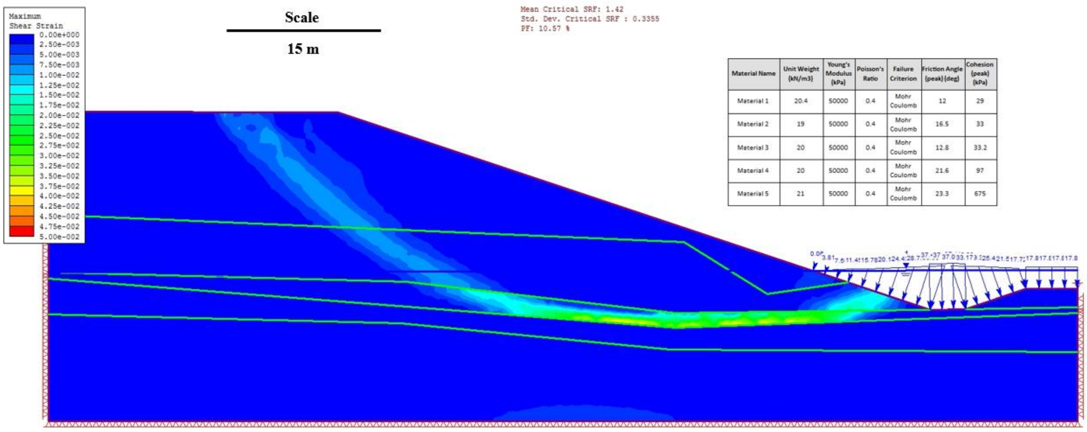


Figure 6.4 Failure zone obtained from Phase 2

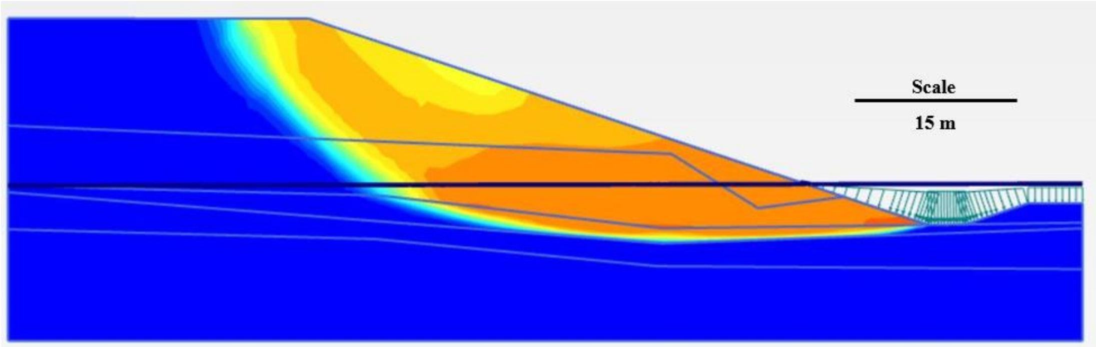


Figure 6.5 Failure surface by Total Displacement contours (Python-PLAXIS-MC analysis)

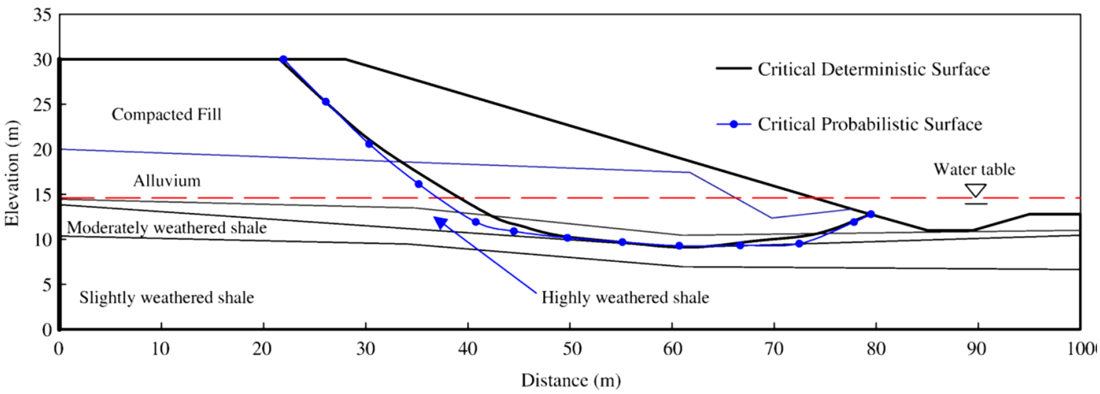


Figure 6.6 Cross-section and slip surfaces of Sugar Creek embankment (Cho 2007)

The FS values from probabilistic LEM (Slide) and FEM (Phase 2) are almost the same, however PF values are different, and FEM gives larger PF. As for the failure zone, FEM gives slightly deeper and wider failure surface. However, when Python-PLAXIS analysis are compared, FS and PF values are different from the other two methods whereas failure surface of Python-PLAXIS is almost same as the other two methods.

6.3 Fruitgrowers Dam in Colorado, U.S.A.

Stability of this dam is analyzed by several authors as well as deWolfe et al. (2010) for pre- and post-liquefaction conditions by using LEM approach. Slope geometry, mean geotechnical values for pre-liquefaction condition and COV values are taken from deWolfe et al. (2010), thus, provided in Table 6.5. The original research study uses two COV levels obtained from the literature. Mean values of these COV levels are used in the current study. Obtained results and failure surfaces are given in Table 6.6, Figure 6.7 and Figure 6.8. For the sake of completeness, the results obtained by deWolfe et al. (2010) are also included in Table 6.4 and obtained failure surface are given in Figure 6.6. Although deWolfe et al. (2010) does not have any probabilistic analyses for pre-liquefaction conditions, deterministic analyses are carried with pre-liquefaction parameters. As it is stated before, this study utilizes Slope/W and program called PES (Probabilistic Engineered Slopes). Obtained FS values and failure surfaces are provided in Table 6.6, Figure 6.9 and Figure 6.10.

Table 6.5 Geotechnical input parameters (deWolfe et al. 2010)

Layer	Property	Mean Value	COV (%)	Distribution
Embankment	Φ'	32°	15	Normal
	c'	20.68 kPa	15	Normal
	γ	20.1 kN/m^3	none	none
Quaternary Alluvium	Φ'	30°	30	Normal
	c'	0.05 kPa	30	Normal
	γ	20.42 kN/m^3	none	none
Foundation	Φ'	30°	30	Normal
	c'	0.05 kPa	30	Normal
	γ	20.42 kN/m^3	none	none

Table 6.6 Comparison of the results

Method	FS or SRF	PF (%)
Slide-MC	1.92	4.82
Phase2	1.47	8.48
deWolfe et al. (2010) results		
Slope/W	1.75	-
PES	1.66	-

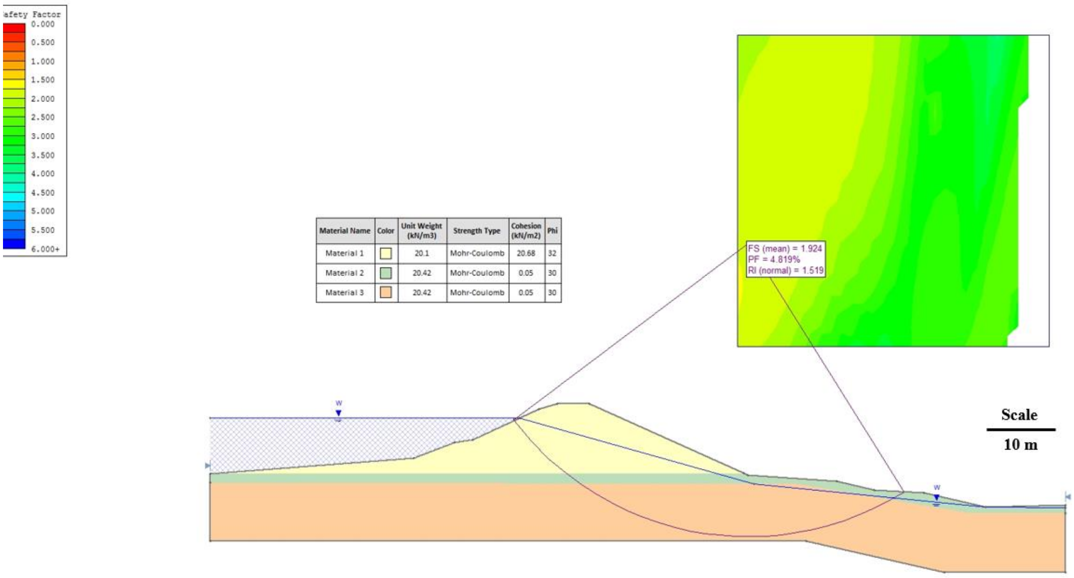


Figure 6.7 Failure surface obtained from Slide

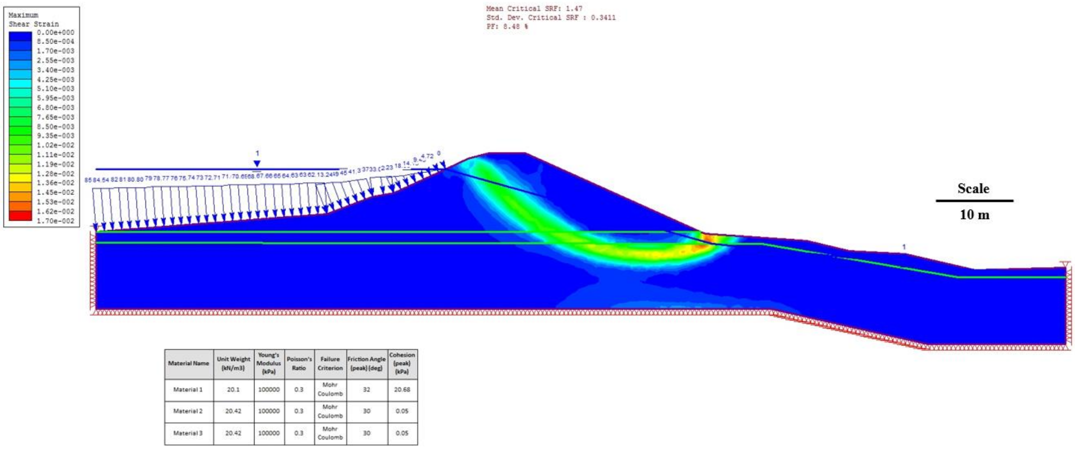


Figure 6.8 Failure zone obtained from Phase 2

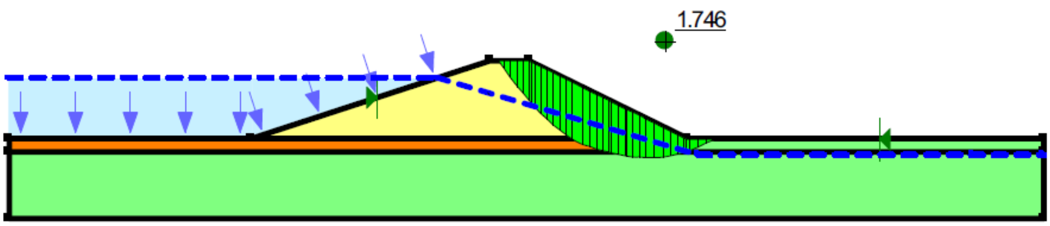


Figure 6.9 Failure surface obtained from Slope/W (deWolfe et al. 2010)

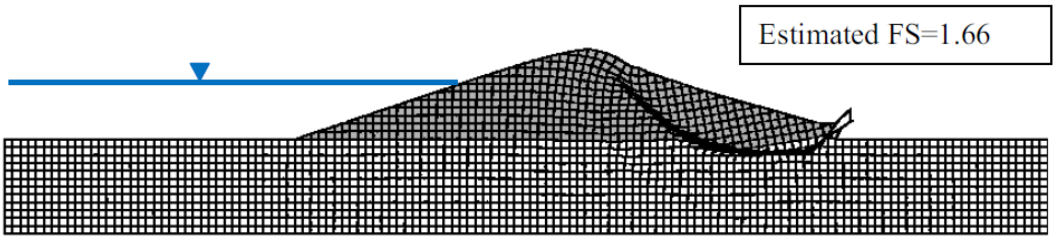


Figure 6.10 Failure surface obtained from PES (deWolfe et al. 2010)

When the results are evaluated, it can be said that probabilistic FEM gives lower FS and higher PF, and a slightly shallower failure surface as compared to probabilistic LEM.

6.4 Çöllolar Lignite Mine in Kahramanmaraş, Turkey

This case is studied by Ozbay et al. (2015) with deterministic FEM and LEM approaches. Series of numerical analyses are performed for total of 14 cross-sections. 8 of them are permanent sections and the rest is temporary (production) slopes. Additionally, three GWT configurations are used in the original research paper. Geotechnical parameters are taken from Ozbay et al. (2015) and COV levels are determined from the literature studies. One from each type of slopes are chosen for this section and one GWT configuration is used for the analyses. Geotechnical parameters and results are provided in Table 6.7 and Table 6.8, respectively. Failure surfaces after probabilistic analyses are given in Figure 6.11, Figure 6.12, Figure 6.13 and Figure 6.14. According to the results of Ozbay et al. (2015), most of the failure surface passes through first four layers. Since there is also the fact that the first layer is quite thin, only the parameters of three layers are treated as random to decrease the computation time. If all layers and their parameters were to be treated as random, estimated computational time for Phase 2 would be 800 hours with a computer of i7 (3.40 GHz) processor and 8 GB RAM.

Table 6.7 Geotechnical input parameters (after Ozbay et al. 2015)

Layer	Property	Mean Value	COV (%)	Distribution
Loam	Φ'	11°	none	none
	c'	48 kPa	none	none
	γ_{unsat}	13.72 kN/m^3	none	none
	γ_{sat}	18.53 kN/m^3	none	none
Blue Clay	Φ'	18°	10	Normal
	c'	13 kPa	20	Normal
	γ_{unsat}	13.79 kN/m^3	none	none
	γ_{sat}	18.41 kN/m^3	none	none
Gyttja	Φ'	32°	10	Normal
	c'	54 kPa	20	Normal
	γ_{unsat}	8.46 kN/m^3	none	none
	γ_{sat}	14.84 kN/m^3	none	none
Lignite	Φ'	32°	10	Normal
	c'	54 kPa	20	Normal
	γ_{unsat}	6.47 kN/m^3	none	none
	γ_{sat}	13.31 kN/m^3	none	none
Black Clay	Φ'	9°	none	none
	c'	8 kPa	none	none
	γ_{unsat}	10.5 kN/m^3	none	none
	γ_{sat}	16.59 kN/m^3	none	none
Green Clay	Φ'	23°	none	none
	c'	33 kPa	none	none
	γ_{unsat}	11.39 kN/m^3	none	none
	γ_{sat}	16.63 kN/m^3	none	none

Table 6.8 Comparison of the results

Section	Method	FS or SRF	PF (%)
Temporary Slope (Cross-section no.5 of Ozbay et al. 2015)	Slide-MC	0.80	98.8
	Phase2	0.56	100
Permanent Slope (Cross-section no.1 of Ozbay et al. 2015)	Slide-MC	0.79	100
	Phase2	0.31	100

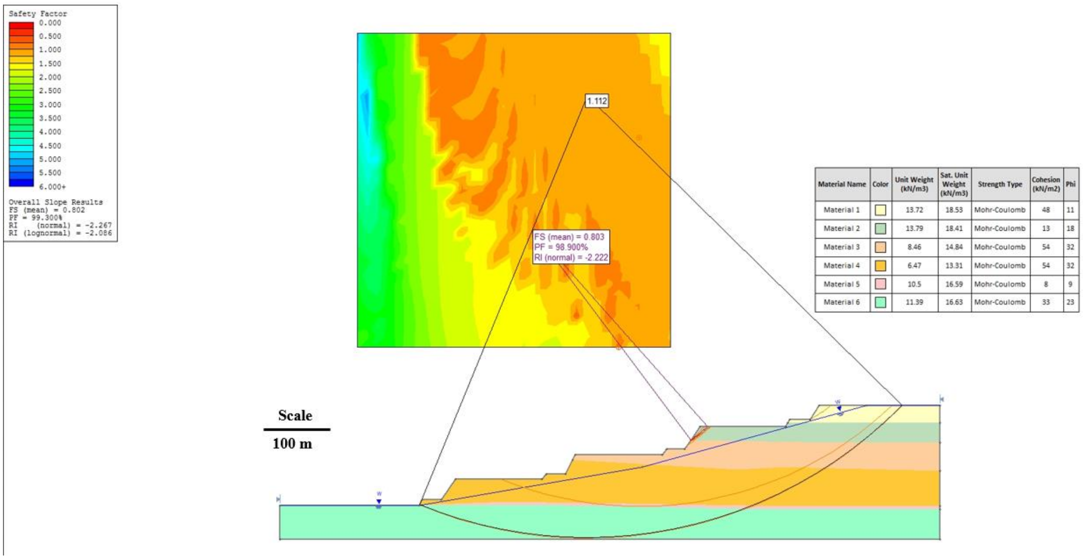


Figure 6.11 Failure surface obtained from Slide (Temporary Slope)

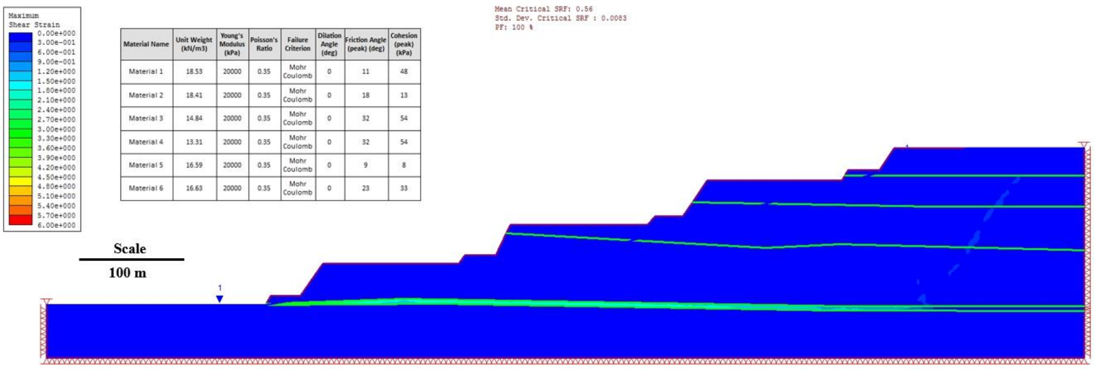


Figure 6.12 Failure zone obtained from Phase 2 (Temporary Slope)

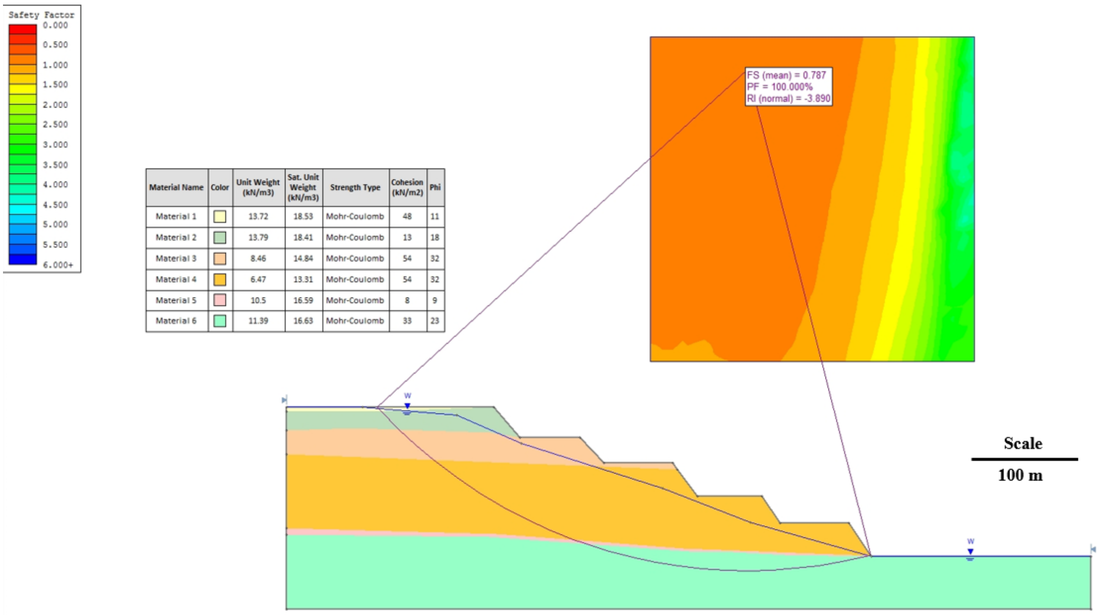


Figure 6.13 Failure surface obtained from Slide (Permanent Slope)

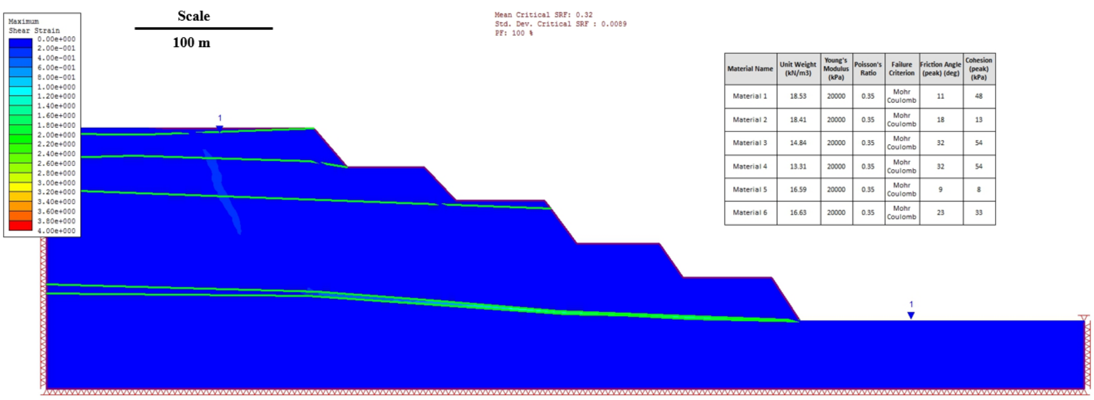


Figure 6.14 Failure zone obtained from Phase 2 (Permanent Slope)

When the results are evaluated, it can be said that probabilistic FEM gives lower FS and the same of slightly higher PF (1 % difference). The failure surface obtained by two methods are quite different. Since only circular failure surface search is conducted, the LEM cannot capture the noncircular failure surface, as for FEM, since a pre-defined shape of failure surface is not imposed, the strain localizations indicate that the failure will take place along the thin, weak, sub-horizontal layers in the slope.

6.5 Lodalen Landslide in Oslo, Norway

This landslide is studied by Suchomel et al. (2010) with three approaches. One of them is basic FOSM approach that does not consider the spatial correlation. Second method is RFEM approach and the third method is an extended FOSM that can consider spatial variability. Slope geometry, geotechnical parameters and COV values are taken from the original work. Input soil parameters and obtained results are given in Table 6.9 and Table 6.10, respectively. Failure surfaces are provided in Figure 6.15 and Figure 6.16.

Table 6.9 Geotechnical input parameters (after Suchomel et al. 2010)

Property	Mean Value	COV (%)	Distribution
Φ'	27.1°	6	Normal
c'	10 kPa	21	Normal
γ	18.6 kN/m^3	none	none

Table 6.10 Comparison of the results

Method	FS or SRF	PF (%)
Slide-MC	1.25	0.5
Phase2	1.22	2.03

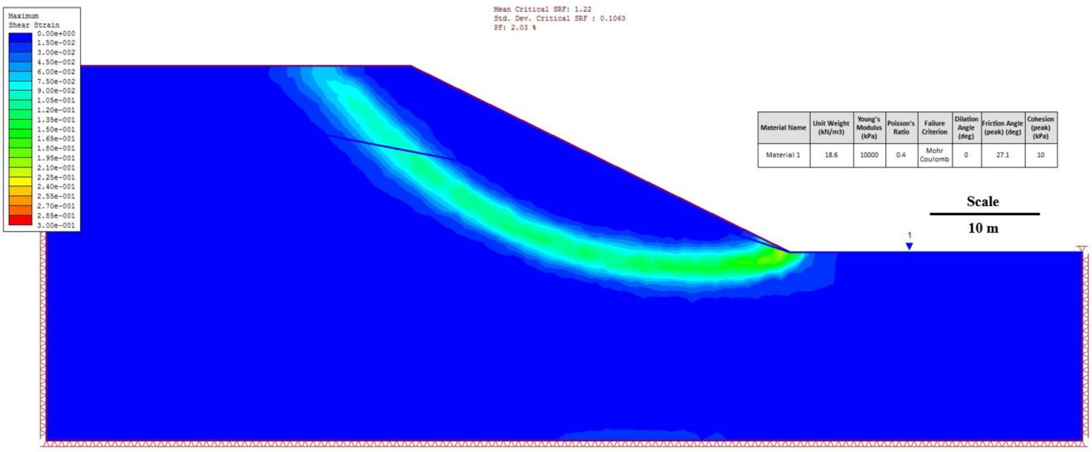


Figure 6.15 Failure zone obtained from Phase 2

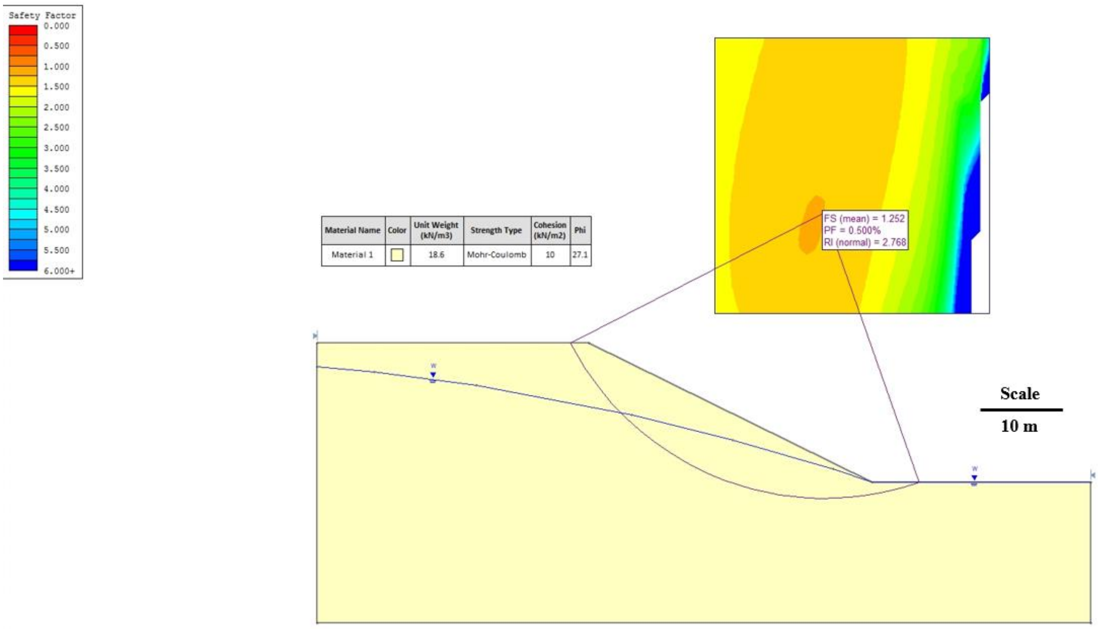


Figure 6.16 Failure surface obtained from Slide

The FS values from probabilistic LEM and FEM are almost the same, however PF values are different, and FEM gives larger PF. As for the failure zone, FEM gives slightly wider failure surface but very similar to LEM critical failure surface.

6.6 Discussion of Results

In this chapter, five real-life landslides are analyzed with probabilistic LEM (Slide-Overall Slope-1000 Monte Carlo) and simple probabilistic FEM (Phase2-point estimate method). The goal is to compare the FS and PF values, together with the critical failure surfaces. Five case histories include all c-phi type soils, with cohesion and friction angle as random variable having COV values mostly in the range of 6-30%, (except with only one embankment case where the cohesion of materials had COV of 60%). For these cases, probabilistic FEM gave either equal or less FS, and equal or greater PF as compared to probabilistic LEM. Failure surfaces are most of the time very similar, with the condition that probabilistic FEM tends to give a slightly deeper and wider failure surface as compared to probabilistic LEM.

CHAPTER 7

CONCLUSIONS

7.1 Summary

In recent years, geotechnical engineers are moving more towards decision-making based on reliability assessment, since accounting for soil variability and carrying out probabilistic analyses result in significant savings in designs, or possible prediction of failure events. The broad objective of this study was to demonstrate the importance of using probabilistic analyses, in terms of examples in slope instability, and to discuss the factors affecting the probability of failure.

There are several deterministic and probabilistic approaches available for slope stability analysis in the literature, however a thorough understanding based on comparison and synthesis is missing. In this study, different probabilistic approaches are utilized and their results are compared in terms of the critical failure surface, probability of failure and probabilistic factor of safety. Within this context, several sets of slope stability analyses are carried out considering:

- Hypothetical slopes and real natural landslides
- Homogeneous and heterogeneous (layered) soil slopes
- Slopes having different traditionally-defined “safety” levels (for example, slopes with different slope inclinations i.e. different FS values)
- For different material behaviors, e.g. for undrained clays (c_u), for mixed (c-phi) soils, and for cohesionless (c=0) soils
- With or without water level

- First-time and reactivated slopes (with peak and residual shear strengths respectively)
- Different COV levels
- Different spatial correlation lengths

The random variables in this thesis were the shear strength parameters cohesion and friction angle. For the statistical distribution of material properties normal distribution is used. Although log-normal distribution is also applicable (e.g. Jiang et al. 2014, Griffiths and Fenton 2004, Cho 2010 etc.), the goal of this study was not focusing on the most accurate statistical distribution representing soil parameters, therefore normal distribution is assumed to be sufficient for this study (JCSS 2001, Lumb 1966, Schultze 1971). Baecher and Christian (2003) also mentioned that using normal distribution is conservative.

In the analyses, the cohesion and friction angle are treated as independent (not-correlated) variables. In reality they are negatively cross-correlated, i.e. the increase of c often corresponds to the decrease of phi. Although it is known that the cross-correlation between cohesion and friction angle affects PF, treating them without cross-correlation gives a larger PF (i.e. more conservative) as noted by Le et al. (2014) and Jiang et al. (2014).

Different methodologies for probabilistic slope stability analyses are compared among themselves, as well as with deterministic methodologies. Limit equilibrium method using method of slices (Spencer, 1967) is used via Rocscience Slide v6, and Slope/W of GeoStudio 2012 softwares. For finite element method, Phase2 v8 of Rocscience and PLAXIS 2D is used. In order to investigate the effects of spatial correlation of material properties a random finite element program called “Rslope2D” created by G.A. Fenton and D.V. Griffiths in 1992 is used. Since this software has some limitations, remote scripting of PLAXIS 2D 2015 is carried out with the help of Python 3.4 in order to apply random finite element method for different cases. The results are compared in terms of the probabilistic FS, PF and the critical failure surface.

7.2 Conclusions

Some of the conclusions reached at the end of this study are:

(1) Slopes having a deterministic FS value greater than 1.00 should not mean that this slope is “safe” in its traditional-sense. Slopes with FS values greater than 1.00 can have a significant probability of failure (for all undrained, c-phi and cohesionless soils). For example an undrained clay slope with a COV value of 50% with a deterministic FS=1.14 can have as high as 50% PF; and the same slope with a deterministic FS of greater than 1.50 can have PF as high as 30%. However, for a cohesionless soil slope having a COV of 10%, a slope having a deterministic FS=1.16 can have PF of 12%. The COV value influences the PF value significantly.

(2) The inverse relation between FS and PF is nonlinear (it is approximately linear in semi-log axes scale). This relation can be obtained and plotted for a given slope geometry and materials, for considering different COV levels, and the owner or the designer of a slope can decide about the allowable or “acceptable” probability of failure for that slope and relate it to FS value.

(3) The probability of failure increases with an increasing COV value for all of the cases studied.

(4) Deterministic LEM and FEM gives very similar FS values, for all undrained, c-phi and cohesionless soils, for near-failure and relatively safer slopes. However there is a slight trend for FEM to give somewhat lower FS values as compared to LEM. (Many of the studies in the literature found similar safety factor (FS, or SRF) values with LEM and FEM (within 5-10% differences)).

(5) Based on the analyses of hypothetical slopes and 5 real landslide cases, probabilistic LEM and FEM do not give similar PF and FS values for most of the cases; probabilistic FEM gives slightly larger PF. For the five real-life landslide cases, all with c-phi type soils, with COV mostly in the range of 6-30%, (except with only one embankment case where the cohesion of materials had COV of 60%), probabilistic FEM gave either equal or less FS (and equal or greater PF) as compared to probabilistic LEM. Failure surfaces are most of

the time very similar, with the condition that probabilistic FEM tends to give a slightly deeper and wider failure surface as compared to probabilistic LEM.

(6) LEM probabilistic gives a slightly lower FS value then LEM deterministic FS, with for undrained clay type and c-phi type soils.

(7) Among the probabilistic LEM methods, (with both of which 1000 MC probabilistic analyses are carried out), although they mostly give similar values, Slide gives higher PF and lower probabilistic FS as compared to Slope/W for near-failure slopes.

(8) The failure surfaces in all deterministic and probabilistic methods seem to be very similar in terms of their maximum depth, their width and the radius of the critical surfaces, for undrained clay type soils. However for other soil types, failure surfaces can be quite different.

(9) The results of probabilistic slope stability analyses (the PF, probabilistic FS and the critical failure surface) are significantly influenced by the COV value and the spatial correlation length.

(10) For COV values less than 50%, as the spatial correlation length increases (i.e. more uniform soil), PF increases. For very large values of COV (such as 100%) PF decreases with increasing correlation length. Griffiths et al. (2004) noted that ignoring the spatial variability (i.e. infinitely large spatial correlation length) will overestimate PF when COV is relatively small, whereas it will underestimate the PF when COV is relatively high.

(11) For COV values less than 50%, for a given horizontal correlation distance, as the ratio increases from 1 to 10, (i.e. the soil is more and more heterogeneous in the vertical direction as compared to horizontal direction) PF decreases. For large values of COV (such as 100%) as the ratio increases PF increases. It is noted that isotropic spatial correlation length assumption yields conservative results (i.e. higher probability of failure) (Cho, 2007).

(12) It is noted that the change is PF value is small for a given ratio and changing horizontal correlation lengths; whereas, the change in PF is large for a given horizontal correlation length and changing ratio values. We may

conclude that the vertical correlation length is a more important issue, since it influences PF more dramatically than the horizontal correlation length does.

(13) The question: “Considering the Spatial Variability: Does it matter?” should be answered as “Yes, it matters” and spatial correlation length can influence the PF.

(14) Deterministic slope stability analyses resulting in a single FS value is no longer sufficient to evaluate the safety of a slope in geotechnical engineering. The deterministic failure surface with minimum FS value is not always the most critical slip surface when the reliability analysis is considered (as was also suggested by Mbarka et al. 2010).

(15) In deterministic slope stability evaluations, there is already a consensus in the literature suggesting to use both LEM and FEM. In order to provide guidance and suggestions to practicing geotechnical engineers, at the end of this study, the author recommends for evaluating the safety of slopes probabilistically, to carry out both LEM Slide OS-MC and FEM Plaxis 2D-Python-MC analyses to reach to a deeper understanding of slope safety and critical failure surface.

(16) Although, in this thesis, only isotropic spatial correlation length is incorporated, it is demonstrated, for the first-time in the literature (to the author's knowledge), that PLAXIS2D-Python couple can be used to develop RFEM to account for spatial correlation length of material properties.

The results of this study are believed to be useful for further understanding of the probabilistic slope stability concept and the effects of soil heterogeneity on slope stability evaluations with the aim of better geotechnical risk management and communication. For example, for a slope with deterministic FS value of 1.50, PF can be decreased from 10^{-1} to 10^{-6} or to any desired level with increasing level of engineering (extensive site investigation, laboratory testing, state-of-the-art analyses, monitoring etc.) (Figure 7.1).

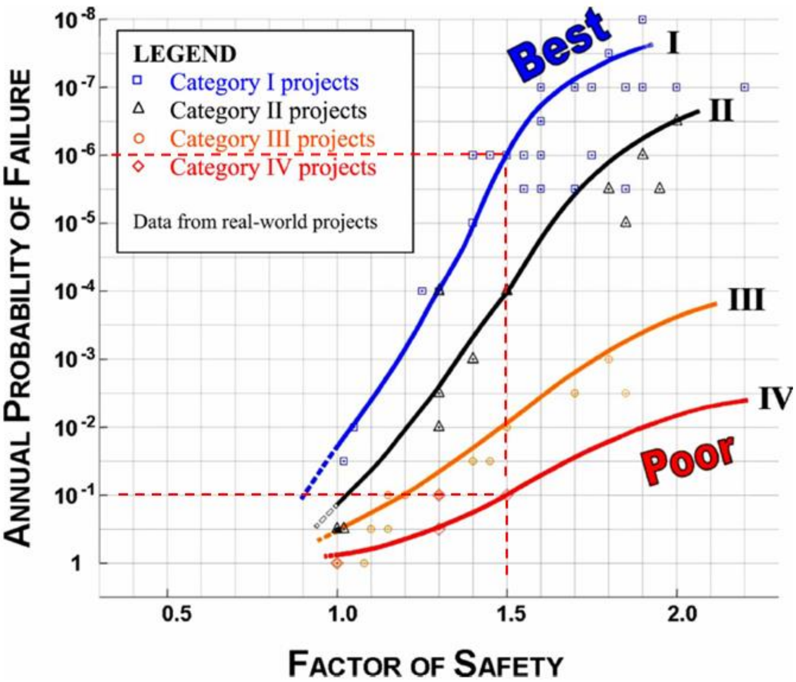


Figure 7.1 Relation between level of engineering and probability of failure (Silva et al. 2008)

7.3 Recommendations for Future Studies

Following topics are recommended for future studies:

- Log normal distribution for material properties
- Not only shear strength parameters, but also Young's modulus, unit weight etc. can be considered as random variables especially when slope deformations are needed from FEM study.
- Cohesion and friction angle can be considered as cross-correlated variables (their correlation coefficient can be typically between -0.20 to -0.70).
- Anisotropy in spatial correlation length in Plaxis2D-Python RFEM
- Collection of data and/or carrying out lab and field studies to determine the COV values and horizontal and vertical spatial correlation distances of material properties

- Layer boundaries, ground water table and loading conditions can also be treated as random variables
- "What is acceptable PF?" needs further study. However, it depends on consequences of failure (Figure 7.2)

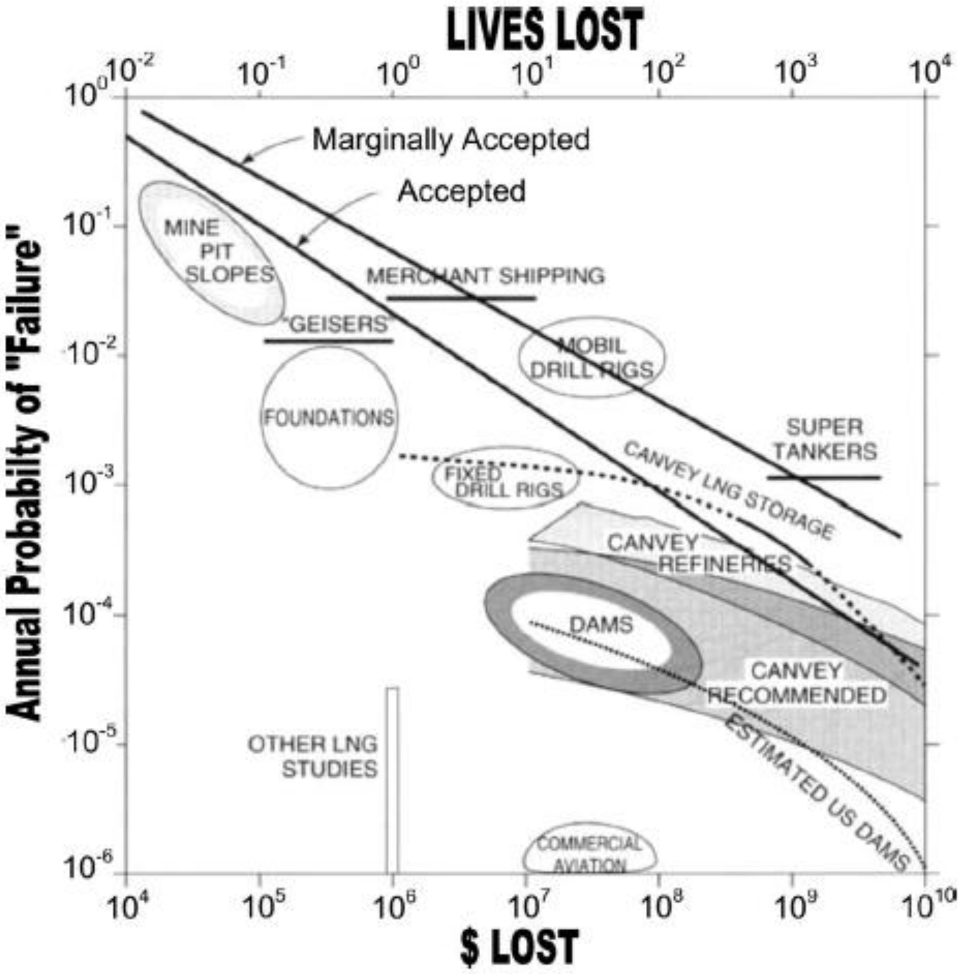


Figure 7.2 Relation between annual probability of failure and consequences of failure (Silva et al. 2008)

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APPENDIX A

PLAXIS-PYTHON REMOTE SCRIPTING CODE

In this section, several codes that are written in Python v. 3 are provided. These codes are used for interrupting the flow of PLAXIS 2D, e.g. remote scripting.

A.1 Main Body of the Script

```
#Setting the connection between Plaxis 2D and Python
localhostport_input = 10000
localhostport_output = 10001
plaxis_path =input("Enter installation location of Plaxis 2D without the final
backslash: ")
import imp
found_module = imp.find_module('plxscripting', [plaxis_path])
plxscripting = imp.load_module('plxscripting', *found_module)
from plxscripting.easy import *
s_input, g_input = new_server('localhost', localhostport_input)
s_output, g_output = new_server('localhost', localhostport_output)
#Asking the coordinates of outer geometry of the slope
Corner_no=int(input("Enter the number of corners that the outer geometry has: "))
print("WARNING:Starting from (0,0) coordinate, enter coordinates clockwise or
counterclockwise")
a=1
geometry=[]
while a<=Corner_no:
    x=float(input("Enter the x coordinate of the corner: "))
    y=float(input("Enter the y coordinate of the corner: "))
    geometry.append(x)
```

```
    geometry.append(y)
    a=a+1
#Asking the coordinates of the Other Layers
Layer_No=int(input("How many horizontal layers?: "))
if Layer_No>1:
    print("WARNING:Starting from top to bottom, please enter the corner coordinates of the layer boundaries (left to right)")
f=1
Layers=[]
while f<Layer_No:
    x1=float(input("Enter the x1 coordinate of the corner: "))
    y1=float(input("Enter the y1 coordinate of the corner: "))
    x2=float(input("Enter the x2 coordinate of the corner: "))
    y2=float(input("Enter the y2 coordinate of the corner: "))
    Layers.append(x1)
    Layers.append(y1)
    Layers.append(x2)
    Layers.append(y2)
    f=f+1
#Asking the coordinates of GWT
Water_Corner_no=int(input("How many points you will use to define the GWT?: "))
if Water_Corner_no>0:
    print("WARNING: Enter the coordinates in directional order(left to right or right to left)")
c=1
GWT=[]
while c<=Water_Corner_no:
    x=float(input("Enter the x coordinate of the corner: "))
    y=float(input("Enter the y coordinate of the corner: "))
    GWT.append(x)
    GWT.append(y)
    c=c+1
#Asking input values to the user
sh=float(input("Enter the horizontal division spacing of the geometry(meter): "))
```

```
sv=float(input("Enter the vertical division spacing of the geometry(meter): "))
FEM_mesh_dim=float(input("Enter the FEM Mesh dimension(meter): "))
g=1
GU=[]
GS=[]
nu=[]
e=[]
cov_e=[]
cohesion=[]
cov_c=[]
phi=[]
cov_phi=[]
drain_type=[]
print("WARNING:Enter the parameters of the Layers from top to bottom")
while g<=Layer_No:
    GU.append(float(input("Enter the unsaturated unit weight of the soil(kN/m^3): ")))
    GS.append(float(input("Enter the saturated unit weight of the soil(kN/m^3): ")))
    nu.append(float(input("Enter the Poissons' Ratio of the soil: ")))
    e.append(float(input("Enter the Young's Modulus of the soil(kPa): ")))
    cov_e.append(float(input("Enter the COV of Young's Modulus in %(e.g. 30)(0, if deterministic): ")))
    cohesion.append(float(input("Enter the cohesion of the soil(kPa): ")))
    cov_c.append(float(input("Enter the COV of cohesion in %(e.g. 30)(0, if deterministic): ")))
    phi.append(float(input("Enter the internal friction angle of the soil(degree): ")))
    cov_phi.append(float(input("Enter the COV of internal friction angle in %(e.g. 30)(0, if deterministic): ")))
    drain_type.append(int(input("Enter Drainage Type Number (0:Drained,\n1: Undrained(Type A:Stiffness and Strength are defined by effective parameters),\n2: Undrained(Type B:Stiffness defined effective, Strength defined undrained),\n3: Undrained(Type C:Stiffness and Strength are defined by undrained parameters)): ")))
    g=g+1
Run_Number=float(input("How many run do you want to do?: "))
save_path=input("Enter your desired saving path for the files: ")
#Setting up the work space and external geometry
```

```
g_input.SoilContour.initializerectangular
(min(geometry[::2]),min(geometry[1::2]),max(geometry[::2]),max(geometry[1::2]))
import random
counter=1
FS=[]
while counter<=Run_Number:
    try:#First, tries the statements. Then, if specified error occurs, does the statements
after "except"
        print('Run No: ', counter)
        g_input.polygon
(geometry[0],geometry[1],geometry[2],geometry[3],geometry[4],geometry[5])
        b=6
        while b<len(geometry):
            g_input.Polygon_1.addpoint (geometry[b], geometry[b+1])
            b=b+2
        h=1
        n=1
        while n<Layer_No:
            g_input.cutpoly (Layers[h-1],Layers[h],Layers[h+1],Layers[h+2])
            h=h+4
            n=n+1
        m=1
        while m<=Layer_No:
            material = g_input.soilmat()
            material.setproperties(
                "SoilModel", 2,
                "DrainageType", drain_type[m-1],
                "gammaUnsat", GU[m-1],
                "gammaSat", GS[m-1],
                "nu", nu[m-1],
                "Gref", random.normalvariate(e[m-1],e[m-1]*cov_e[m-
1]/100)*0.5/(1+nu[m-1]),
                "cref", random.normalvariate(cohesion[m-1],cohesion[m-1]*cov_c[m-
1]/100),
                "phi", random.normalvariate(phi[m-1],phi[m-1]*cov_phi[m-1]/100))
```

```
    g_input.setmaterial (g_input.Soils[m-1],material)
    m=m+1
j=1
i=Layer_No
k=1
while j<=Layer_No:
    if Layer_No>1:
        if j==1:
            #Dividig the geometry horizontally
            x1=max(geometry[::2])-sh
            y1=max(geometry[1::2])
            x2=x1
            y2=Layers[k]
            while x1>Layers[k-1]:
                g_input.cutpoly (x1,y1,x2,y2)
                x1=x1-sh
                x2=x2-sh
            #Dividig the geometry vertically
            x1=Layers[k-1]
            y1=Layers[k]+sv
            x2=max(geometry[::2])
            y2=y1
            while y1<max(geometry[1::2]):
                g_input.cutpoly (x1,y1,x2,y2)
                y1=y1+sv
                y2=y2+sv
        elif j==Layer_No:
            #Dividig the geometry horizontally
            x1=max(geometry[::2])-sh
            y1=Layers[k-4]
            x2=x1
            y2=min(geometry[1::2])
            while x1>min(geometry[::2]):
```

```
            g_input.cutpoly (x1,y1,x2,y2)
            x1=x1-sh
            x2=x2-sh
        #Dividig the geometry vertically
        x1=min(geometry[::2])
        y1=Layers[k-4]-sv
        x2=max(geometry[::2])
        y2=y1
        while y1>min(geometry[1::2]):
            g_input.cutpoly (x1,y1,x2,y2)
            y1=y1-sv
            y2=y2-sv
    else:
        #Dividig the geometry horizontally
        x1=max(geometry[::2])-sh
        y1=Layers[k-4]
        x2=x1
        y2=Layers[k]
        while x1>Layers[k-1]:
            g_input.cutpoly (x1,y1,x2,y2)
            x1=x1-sh
            x2=x2-sh
        #Dividig the geometry vertically
        x1=Layers[k-1]
        y1=Layers[k]+sv
        x2=max(geometry[::2])
        y2=y1
        while y1<Layers[k-4]:
            g_input.cutpoly (x1,y1,x2,y2)
            y1=y1+sv
            y2=y2+sv
else:
    #Dividig the geometry horizontally
```

```
        x1=min(geometry[::2])+sh
        y1=max(geometry[1::2])
        x2=x1
        y2=min(geometry[1::2])
        while x1<max(geometry[::2]):
            g_input.cutpoly (x1,y1,x2,y2)
            x1=x1+sh
            x2=x2+sh
        #Dividig the geometry vertically
        x1=min(geometry[::2])
        y1=max(geometry[1::2])-sv
        x2=max(geometry[::2])
        y2=y1
        while y1>min(geometry[1::2]):
            g_input.cutpoly (x1,y1,x2,y2)
            y1=y1-sv
            y2=y2-sv
    #Creating and setting the material properties
    while i<len(g_input.polygons):
        material = g_input.soilmat()
        material.setproperties(
            "SoilModel", 2,
            "DrainageType", drain_type[j-1],
            "gammaUnsat", GU[j-1],
            "gammaSat", GS[j-1],
            "nu", nu[j-1],
            "Gref", random.normalvariate(e[j-1],e[j-1]*cov_e[j-1]/100)*0.5/(1+nu[j-
1]),
            "cref", random.normalvariate(cohesion[j-1],cohesion[j-1]*cov_c[j-
1]/100),
            "phi", random.normalvariate(phi[j-1],phi[j-1]*cov_phi[j-1]/100))
        g_input.setmaterial (g_input.Soils[i],material)
        i=i+1
    j=j+1
```

```
        k=k+4
    print('Geometry has been successful divided')
    print('Total number of polygon is:', len(g_input.polygons))
    print ('All polygons/soils are successful assigned a randomly generated material properties')
    #FEM Meshing
    g_input.gotomesh ()
    g_input.mesh (FEM_mesh_dim)
    #GWT properties
    if len(GWT)>3:
        g_input.gotoflow()
        if len(GWT)>4:
            g_input.waterlevel (GWT[0],GWT[1],GWT[2],GWT[3])
            d=4
            while d<len(GWT):
                g_input.UserWaterLevel_1.addpoint (GWT[d], GWT[d+1])
                d=d+2
        else:
            g_input.waterlevel (GWT[0],GWT[1],GWT[2],GWT[3])
    #Setting up the calculation stages, calculation, saving
    g_input.gotostages()
    g_input.set (g_input.InitialPhase.DeformCalcType, "Gravity loading")
    g_input.phase (g_input.InitialPhase)
    g_input.setcurrentphase (g_input.Phase_1)
    g_input.set (g_input.Phase_1.DeformCalcType, "Safety")
    if drain_type.count(1)>=1:
        g_input.set (g_input.Phase_1.Deform.IgnoreUndrainedBehaviour, True)
    elif drain_type.count(2)>=1:
        g_input.set (g_input.Phase_1.Deform.IgnoreUndrainedBehaviour, True)
    g_input.calculate()
    if g_input.Phase_1.Reached.ReachedMsf.value>0:
        g_input.save (save_path + r'\Run_no'+str(counter))
        #Saves and writes the calculated FS values (Strength Reduction Values,SRF)
        FS.append(g_input.Phase_1.Reached.ReachedMsf.value)
```

```
        else:
            g_input.set (g_input.InitialPhase.Deform.UseDefaultIterationParams, False)
            g_input.set (g_input.InitialPhase.Deform.ToleratedError, 0.5)
            g_input.set (g_input.InitialPhase.ShouldCalculate, True)
            g_input.calculate()
            if g_input.Phase_1.Reached.ReachedMsf.value>0:
                g_input.save (save_path + r'\Run_no'+str(counter))
                #Saves and writes the calculated FS values (Strength Reduction
Values,SRF)
                FS.append(g_input.Phase_1.Reached.ReachedMsf.value)
            else:
                g_input.save (save_path + r'\Run_no'+str(counter))
                FS.append(g_input.InitialPhase.Reached.ReachedMStage.value)
        with open(save_path+r'\FS.txt','a') as file:
            file.writelines(["{}\t{}\n".format('Run_no_'+str(counter),FS[counter-1])])
        if counter!=Run_Number:
            g_input.clear()
        counter=counter+1
    except plxscripting.plx_scripting_exceptions.PlxScriptingError:#Specified error
        FS.append('NA-Error')
        with open(save_path+r'\FS.txt','a') as file:
            file.writelines(["{}\n".format('NA-Error Occured')])
        if counter!=Run_Number:
            g_input.clear()
        counter=counter+1
print('All Calculations was successful')
#Calculation of Probability of Failure and Reliability Index
if Run_Number>1:
    new_FS=[]
    FS_1=[]
    for value in FS:
        if isinstance(value, float):
            new_FS.append(value)
        elif isinstance(value, int):
```

```
        new_FS.append(value)
    for values in new_FS:
        if values<1:
            FS_1.append(values)
    PF=len(FS_1)/len(new_FS)
    print("Probability of failure is: ", round(PF,3))
    from statistics import mean
    from statistics import stdev
    RI_normal=(mean(new_FS)-1)/stdev(new_FS)
    print("Reliability index (RI) is: ", round(RI_normal,3))
    from math import log
    from math import sqrt
    from math import pow
RI_lognormal=log(mean(new_FS)/sqrt(1+pow(stdev(new_FS)/mean(new_FS),2)))/s
qrt(log(1+pow(stdev(new_FS)/mean(new_FS),2)))
    print("If FS lognormally dist. RI is: ", round(RI_lognormal,3))
    with open(save_path+r'\FS.txt','a') as newfile:
        newfile.writelines(["{}\t{}\n".format('Run with min FS:
',FS.index(min(new_FS))+1)])
        newfile.writelines(["{}\t{}\n".format('Prop. of Failure: ',round(PF,3))])
        newfile.writelines(["{}\t{}\n".format('Normal Reliability Index:
',round(RI_normal,3))])
        newfile.writelines(["{}\t{}\n".format('Lognormal Reliability Index:
',round(RI_lognormal,3))])
        newfile.writelines(["{}\t{}\n".format('No of Analyses with error is: ',len(FS)-
len(new_FS))])
```

A.2 Parameter Conversion for Lognormal Distribution

If lognormal distribution is needed, mean and standard deviation of the geotechnical parameter should be modified so that natural logarithm of the generated distribution will be normally distributed. In this case, following addition should be made into the script where necessary. Example coding is given for cohesion of the soil.

```
from math import log
from math import sqrt
from math import pow
random.lognormvariate(log(cohesion[m-1])-0.5*pow(sqrt(log(1+pow(cov_c[m-
1]/100,2))),2),sqrt(log(1+pow(cov_c[m-1]/100,2))))
```