

TASK ASSIGNMENT AND SCHEDULING IN UAV MISSION PLANNING  
WITH MULTIPLE CONSTRAINTS

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WITH MULTIPLE CONSTRAINTS**

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# ABSTRACT

## TASK ASSIGNMENT AND SCHEDULING IN UAV MISSION PLANNING WITH MULTIPLE CONSTRAINTS

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In the recent years, unmanned aerial vehicles (UAVs) have started to be utilized as the first choice for high risk and long duration tasks, because UAVs are cheaper; they are hard to be noticed and they can perform long duration missions. Furthermore, the utilization of UAVs ensures to reduce the risk to the human life. Examples of this kind of missions includes signal collection, surveillance and reconnaissance and combat support missions. It is valuable to develop a fully autonomous UAV fleet to perform these kinds of tasks when it is needed, because this kind of missions usually start at unexpected times. Other problems which is in the set of high risk and long duration tasks are multiple constraint UAV scheduling, and target assignment problem. In this problem, a fleet of UAVs are supposed to traverse a set of target areas within a limited area. The targets are only available within certain time windows and need to be traversed promptly. Moreover, for some large target areas multiple UAVs are needed to perform the task. The objective of this problem is to find a complete scheduling and UAV-target assignment that minimizes the total fuel consumption of the UAVs. This problem is a highly critical real time problem and needs to be solved almost in real-time. Therefore, methods doing exhaustive search are infeasible. Most of the methods in the literature, try to solve this problem by evolutionary approaches. In this thesis, we developed an algorithmic method to solve this problem. This method uses divide and conquer method to solve this problem. In this way, the problem is transformed into a combination of multiple small problems. We designed a method

to convert these small problems into transportation problems. Each transportation problem is solved with simplex algorithm. The method proposed is compared with various methods and has been shown to provide fast, acceptably optimal and reliable results.

**Keywords:** Unmanned aerial vehicle (UAV), algorithmic solution, UAV scheduling and target assignment, divide and conquer approach, transportation problem

## ÖZ

### ÇOKLU KISITLAMALAR İÇEREN İNSANSIZ HAVA ARAÇLARI İLE GÖREV PLANLAMA PROBLEMİNDE ZAMAN PLANLAMASI VE HEDEF GÖREVLENDİRMESİ

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Son yıllarda yüksek risk içeren ve uzun zaman gerektiren görevler için insansız hava araçları (İHA'lar) ilk tercih olarak seçilmeye başlanmıştır. Çünkü İHA'lar ucuz, farkedilmesi zor ve uzun süreli işlere uygun araçlardır. Dahası İHA'ları kullanmak insan hayatının riske atıldığı durumları azaltmaktadır. Bu tip görevlere örnek olarak sinyal toplama, keşif ve gözetleme görevleri ve muharebe destek görevleri gösterilebilir. Bu tip işleri gerektiği zaman yapabilecek tamamen otonom bir İHA filosa sahip olmak değerlidir. Çünkü genellikle bu tip görevler beklenmedik zamanlarda başlamaktadırlar. Yüksek risk içeren ve uzun zaman gerektiren görevlere bir başka örnekte İHA'lar ile zaman planlaması ve hedef görevlendirmesi problemidir. Bu problemde İHA filosu belirli bir alandaki bir dizi hedef alanını katetmelidir. Hedefler sadece belirli bir zaman aralığında uygun olup o zaman aralığı içinde katedilmelidirler. Dahası bazı geniş alana yayılmış hedefleri katetmek için birden fazla İHA gerekmektedir. Bu problemde amaç İHA'lar için tam bir zaman planlaması ve İHA-hedef görevlendirmesi yapmanın yanı sıra yakıt tüketimini de azaltmaktır. Bu problem oldukça kritik ve gerçek zamanlı bir problemdir ve hızlı bir şekilde çözülmesi gerekmektedir. Bu sebepten dolayı tam kapsamlı arama yapmak bu problem için uygun değildir. Literatürdeki çoğu metod, bu problemi evrimsel metodlarla çözmeye çalışmaktadır. Bu tezde biz problemi çözebilmek için algoritmik bir metod üretmiş durumdayız. Bu metod, problemi çözmek

için böl ve yönet yöntemini kullanmaktadır. Böylece, problem bir çok küçük probleminin birleşimine dönüştürülmektedir. Biz oluşan bu alt problemleri, ulaştırma problemlerine çeviren bir dizayn ürettik. Ortaya çıkartılan ulaştırma problemleri simplex algoritması ile çözülmektedir. Önerilen metod bir çok farklı metodla karşılaştırılmış olup hızlı, yeterince optimal ve güvenilir sonuçlar ortaya çıkarttığı görülmüştür.

Anahtar Kelimeler: İnsansız hava araçları (İHA), algoritmik çözüm, İHA'lar ile zaman planlaması ve hedef görevlendirmesi, böl ve yönet yöntemi, ulaştırma problemi

*To my family*

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## LIST OF ABBREVIATIONS

2D	two dimensional
GA	genetic algorithm
IH	intlinprog heuristic
LP	linear programming
NP	non deterministically polynomial
PC	personal computer
SA	simplex heuristic algorithm
TV	television
TW	time window
HGA	hybrid greedy algorithm
GHz	giga hertz
LTS	long term support
UAS	unmanned aerial systems
VRP	vehicle routing problem
MILP	mixed integer linear programming
UASs	unmanned aerial systems
UAVs	unmanned aerial vehicles



# CHAPTER 1

## INTRODUCTION

Combinatorial optimization is one topic covered both in computer science and in applied mathematics. In combinatorial optimization problems, the objective is to determine the best object among the set of a finite object set. Common examples of combinatorial optimization problems are traveling salesman problem, cutting stock problem, packing problems and task assignment problem. In *Task assignment* problem there are a number of agents and there are a number of tasks to be performed. For each target, only one agent can be assigned. For each agent-task pair, the cost of performing that task changes. The objective is to assign agents to tasks that minimizes the total cost. If agents and tasks are thought to be two disjoint sets, then the problem transforms into finding the minimum weight matching in a weighted bipartite graph. In graph theory, the graphs whose vertices can be grouped into two disjoint sets are called bipartite graphs. An example bipartite graph can be seen at Figure 1.1.

Another example to combinatorial optimization problems is *job scheduling* problem. In this problem,  $n$  different jobs are need to be scheduled to  $m$  different machines while trying to minimize the total length of the schedule.

For military missions some areas are critical and need to be observed. Generally starting time of these missions are unpredictable. Thus, the observation times of these areas are also changeable. Because of its unpredictable nature, it is not always possible to find enough soldiers for this job. Furthermore, it is dangerous and requires long hours of routine work. Instead of this, using autonomous vehicles is a wiser idea. On the other hand, UAVs are fast, cheap and easy to manage autonomous vehicles. By using this idea, many of the armies started to make research on using UAVs in these

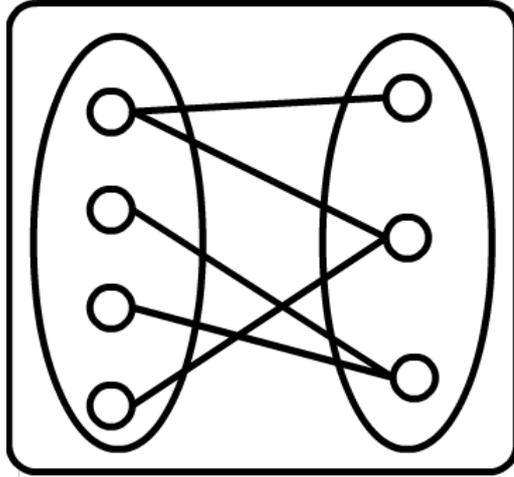


Figure 1.1: An example transportation network

type of missions. It both consists of scheduling and task assignment jobs. Scheduling and assignment of the UAVs, must be done before the mission starts. The solution methods to make planning varies according to the aim of the simulations. This variation is caused by the *constraints* used in the simulations. Constraints are simply the details of the mission. They define physical and operational rules for the simulation. An area should be covered in only a specific time periods. This is an example for the operational constraints for this mission. Physical constrains can be related to environment assumptions, UAVs and targets. Visibility is an example of physical constraint. According to weather conditions visibility of an UAV can change. It affects its decisions and behaviors of the UAV.

Methods that find optimal results for combinatorial optimization problems are infeasible. For example, brute force algorithm takes exponential time to solve this types of problems and for this kind of time critical tasks it is too slow. Hence, finding acceptable solutions in a small amount of time is critical. In this thesis, we developed a method to solve UAV mission planning problem efficiently and find a complete scheduling. It is based on dividing the problem into smaller problem instances and solving the smaller problem instances with an effective method. The proposed method in this thesis, divides the problem according to the time windows. This method does not eliminate the time information. In spite of the fact that transportation problems does not contain any time information, the created sub problems

still have time information. So, we introduced an approach to transform the sub-problems into transportation problems. It transforms the problem into aggregated multiple transportation problems. We solved transformation problems with primal simplex network algorithm. After, we combine the results of transportation problems to find the complete solution. We also developed a greedy and brute force algorithm and two more newly created methods. We compared this solution to the 4 above mentioned methods that we have developed.

The structure of the thesis is as the following. Chapter 2 gives some background information about the problem and previously proposed solutions. In this chapter, the problem is formally defined. Then in Chapter 3, we provide a summary of the works related to our topic. Related works are examined under three major sections. These are, application areas of the problem, problem types and different solution methods used in this area. Next, in Chapter 4, we explain our approach and the other developed approaches for comparison. We explained working structures of these algorithms with examples. Then, in the Chapter 5, we provided the tests performed and then explain our inferences from that tests. Lastly, conclusion and the future work that can be done are explained in Chapter 6.



## **CHAPTER 2**

### **BACKGROUND**

#### **2.1 UAV Mission Planning**

UAV Mission Planning is a very critical task in both military operations and non-military activities. Despite the fact that it has various types, some basic rules are the same for all of the problems.

Uav mission planning task is a combinatorial optimization problem in which the goal is to find an optimal UAV task assignment that minimizes the total cost. Some key features of the problem are explained in the rest of this paragraph. Targets are the pre-defined places in the environment that have to be covered. What to do when reaching a target depends on the problem description, but it has to be traversed. The traversing order of targets is generally not provided, it should be determined by the solution algorithm to minimize costs. UAVs are used to complete the mission. There can be one UAV to complete the mission or there may be more than one UAV cooperating. In case more UAVs needed, the cooperating UAVs are called as UASs. Base station is a place where the UAVs start the mission. Each operation in UAV mission planning starts from a base station and ends in the base station. For cost measurement; generally covered distance, elapsed time or consumed fuel quantities are used. To complete a mission planning, three important goals have to be achieved. The first one is, all of the targets must be traversed. The second one is, utilized UAVs must return to the starting base station coordinates. Lastly, the proposed results must satisfy the constraints defined by the problem. Constraints are the rules defined by the problem to simulate the problem in a more realistic manner. Constraints can be put on UAVs,

targets or the environment. Constraints concerning UAVs generally limit UAV behaviors physically. Target constraints can add new features to targets, such as capacity. Lastly examples of environmental constraints are obstacles, forbidden areas etc.

In the UAV mission planning problem introduced in this thesis, targets have time windows and capacities. We propose a solution technique which divides the problem into many small problems. Then, we solve each of the problems efficiently and combine the results. The division process makes use of target windows. After eliminating time windows, the problem starts to look like a combination of many transportation problems. Our method proposes to solve these small problems with simplex algorithm. In the next section we provide background information about this idea and the other implemented methods.

## **2.2 Background Information About the Solution Methods**

### **2.2.1 Transportation Problem**

Transportation problem is a subset of network flow problems where the aim is to make an optimal transportation from a set of sources to a set of destinations.

In Figure 2.1, a simple transportation problem instance is given. In the problem setting, each source-destination has some predefined capacity which is specified by the integer in the figure. These capacities represent number of products to be transferred from sources to destinations. The arcs represent the cost of going from a specific source to a specific destination. The goal of this problem is to find the maximum possible flow available by taking minimum costs.

This problem is similar to the case of UAV mission planning having targets without time windows. Furthermore, the problem is suitable for UAV mission planning scenarios because it finds integer solutions.

There are many linear programming algorithms that can yield both efficient results and the optimal solution

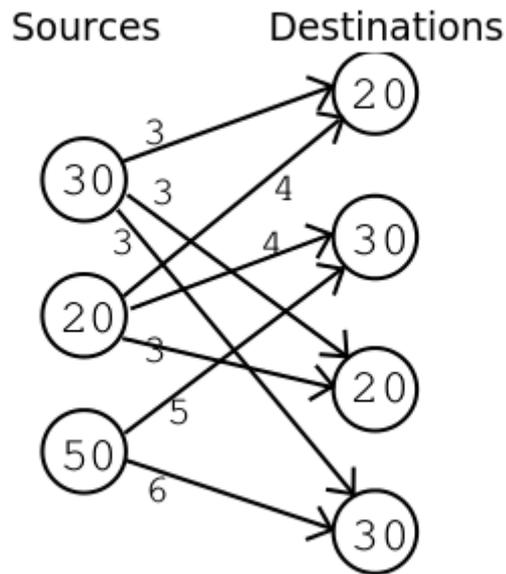


Figure 2.1: An example transportation network

### 2.2.2 Brute Force Algorithm

Brute force algorithm or exhaustive search algorithm is a common programming technique in which optimal solution is guaranteed to be found. It achieves this by simply enumerating all of the possible solutions in the solution space. Exhaustive search algorithm is generally easy to implement and will find the best solution if it exists. The downside of the brute force algorithms are, their processing time grows proportional to number of all possible solutions for that problem. This, causes the algorithm to take too much time for most of the problems. Because of this reason, brute force algorithm is usually chosen for the algorithms which have small solution spaces. Another usage of brute force algorithms are on large problems with limited input sizes. This usage is also common because brute force algorithm is usually used to compare solution qualities of other algorithms on small size inputs.

In spite of its combinatorial nature, brute force algorithm still can be used in large sized solution sets. The main idea is, to somehow reduce the solution space with a known algorithm and then use brute force algorithm to solve that smaller problem

instance. For example, if some of the solution space is known to be not optimal, that part of the solution space may be eliminated by a preprocessing technique. After preprocessing is made, the brute force algorithm will take less amount of time. Also, another usage of brute force algorithms is to first reduce the size of the problem with the help of a heuristic algorithm and then use a brute force algorithm to solve that reduced problem.

### **2.2.3 Greedy Algorithm**

Greedy algorithm is a type of heuristic algorithm which tries to find a solution by choosing the best possible option at each decision point. The nature of the algorithm is fast, because its processing time is not proportional to solution space. It usually can not find optimal solutions for large sized inputs, because choosing the local optimal solutions at each decision point does not guarantee to find the optimal solution.

Despite the disadvantages of optimality, greedy algorithm is still a widely used algorithm technique. For the cases which has a very large solution space, finding optimal solutions is not feasible. For this kind of situations, finding some correct solution in a small amount of time is valuable. For this kind of combinatorial problems where approaching exact algorithms is not possible, greedy based algorithms are commonly used.

### **2.2.4 Linear Programming**

Linear programming is a mathematical optimization method which tries to find best possible outcome. The technique that linear programming uses is to represent the problem as equations. With the help of bounding of the possible solutions and the maximum and minimum possible values of the solutions, its objective function is defined by a convex polyhedron. The solution space is the region involved by this convex polyhedron. The optimal solution is the point where the objective function takes its maximum or its minimum value.

Linear programming is a common method in operations research area. It can be used

to solve planning, production and transportation problems.

### **2.2.5 Mixed Integer Linear Programming**

Integer programming is a special version of linear programming where all of the solution variables are restricted to have integer values. In some of the problems, only a specific part of the solution variables has the restriction of having integer values. For this kind of cases, mixed integer linear programming is used. In most of the scheduling problems, some values are restricted to have integer values. For example the number of UAVs can not be a floating point number. For these kind of situations, mixed integer linear programming method is used.

### **2.2.6 Branch and Bound Method**

This is a paradigm to solve discrete and combinatorial problems. Its aim is, same as the brute force algorithm. Both algorithms wants to find the optimal solution by searching the solution space. Despite the similarities, branch and bound method is a more robust method because it eliminates some parts of the solution space.

The idea of branch and bound method is to see the state space as a rooted tree and it traverses the tree branches to find the optimal solution. Before going into a branch, the algorithm checks the upper and lower estimated bounds to be an optimal solution. If the branch can not provide a better solution than the current solution, then that branch is eliminated from the state space.

This method is generally used for NP-Hard problems. While having better processing times compared brute force algorithm, this algorithm is still too slow to be used in real time problems.

### **2.2.7 Simplex Algorithm**

Simplex algorithm is also a special type of linear programming algorithm. It also makes use of linear equalities, linear inequalities and bounds. The difference of sim-

plex algorithm is that it works much faster than most of the linear programming algorithms. The idea in the simplex algorithm is, to first look at the corners of the polyhedron where the optimal solutions are most likely to be. With this way, it provides faster solutions than the most of the linear programming methods.

### **2.2.8 Matlab Intlinprog Utility**

The main idea of this approach is, to divide the problems into multiple transportation problem instances and find an alternative solution to multiple constraint UAV scheduling problem. Matlab is chosen for implementation because in Matlab there is an optimization toolbox available which provides many tools to solve these kind of planning problems.

Then, the appropriate function in Matlab optimization toolbox should be chosen. To find the most suitable option to solve the problem with Matlab, the problem in hand is analyzed at first.

The purpose in transportation problem is to minimize cost depending on all other expressions as constraints. All the expressions including the objective function, are linear in this problem. Matlab provides a table to choose the suitable function to use for the problem. So, linprog is chosen Figure 2.2.

Though linprog seems really appropriate for the problem setting, it can not be used for the solution. Because in the solution matrix it returns, there can be some non-integer values. This function minimizes the objective function and finds the optimal result but while making this, it assign non-integer values to solution variables. In this problem, the cost is calculated as the summation of the products of the cost to go to a particular target and the number of UAVs sent (the flow). So, finding number of UAVs sent as a non-integer value is a wrong answer for this problem even though it is the optimal answer. This problem needs integer values for the solution variables. There is a function available for achieving this condition. Name of this function is intlinprog. It takes exactly same inputs with the linprog function. The only difference is that it finds optimal result achievable with the integer values. So, intlinprog is used to implement transportation function.

Constraint Type	Objective Type				
	Linear	Quadratic	Least Squares	Smooth nonlinear	Nonsmooth
None	n/a ( $f = \text{const}$ , or $\min = -\infty$ )	quadprog, Theory, Examples	\, lsqcurvefit, lsqnonlin, Theory, Examples	fminsearch, fminunc, Theory, Examples	fminsearch, *
Bound	linprog, Theory, Examples	quadprog, Theory, Examples	lsqcurvefit, lsqlin, lsqnonlin, lsqnonneg, Theory, Examples	fminbnd, fmincon, fseminf, Theory, Examples	*
Linear	linprog, Theory, Examples	quadprog, Theory, Examples	lsqlin, Theory, Examples	fmincon, fseminf, Theory, Examples	*
General smooth	fmincon, Theory, Examples	fmincon, Theory, Examples	fmincon, Theory, Examples	fmincon, fseminf, Theory, Examples	*
Discrete	bintprog, Theory, Example				

Figure 2.2: A table showing Matlab optimization toolbox functions [41].

The signature of the function used is as the following:

`[x,fval]=intlinprog(f,intcon,A,b,Aeq,beq,lb,ub);`

Intlinprog is an integer programming algorithm implementation. So, it only solves the problems given in a specific form. That form is summarized below:

$$\min_x f^T x \quad (2.1)$$

$$A.x \leq b, \quad (2.2)$$

$$Aeq.x = beq \quad (2.3)$$

$$lb \leq x \leq ub \quad (2.4)$$

The Equation 2.1, shows the objective function of the linear programming problem. The Equation 2.2, shows the linear inequalities, Equation 2.3 shows, the linear equalities and finally Equation 2.4 shows, bounds of the linear programming problem.

## **CHAPTER 3**

### **RELATED WORK**

In the last 5 years the number of people using UAVs more than doubled the amount of people using UAVs before. Similarly, the number of producers and developers are also increased. This situation encouraged growing interest in the research involving UAVs. In the recent years, with the technological advances there is a dramatic increase in the number of research made which make use of UAVs.

In this chapter we provide published work on UAV applications, different problem types and solution methods in the literature.

#### **3.1 UAV Applications**

The UAV applications were used to be limited with only military applications before but nowadays the situation has changed. Now, there is a wide variety of applications on the civilian and military areas are produced every other day [52]. The rest of the chapter provides examples of published works about military and non-military applications.

##### **3.1.1 Non-military Applications**

Non-military applications including both civil and commercial applications are usually in the areas of; aerial mapping, oil and gas industry, broadcasting, disaster managing, homeland security etc [6]. UAVs can easily make broadcasting from a place where a man can not reach and also they can change place with the help of controller.

As an illustration to broadcasting usage of the UAVs some of TV channels started to use UAVs in live TV coverage [1]. Not to mention, another usage of UAVs is for monitoring critical places like oil pipelines to avoid unexpected hazardous situations [42]. In the same fashion, another notable usage of UAVs in civilian applications is on the disaster management topic. After natural disasters occurs there is a need to gain information from the disaster area in a fast fashion. UAVs are suitable for this job because they are small, they do not need any human controller in the dangerous area and they are durable. S. M. Adams and C. J. Friedland at al. [2] and T. Chou and M. Yeh and Y. Chen and Y. Chen at al. [12] studied disaster management because of the above mentioned reasons. Environmental protection is an important issue in the nowadays world. For protecting some habitats there is a need to monitor them. Monitoring constantly is important for realizing changes and good for realizing extraordinary urgent situations. Using an unmanned system, for example a flock of UAVs, is suitable for this kind of a long and stable job. R. AI-Tahir and M. Arthur at al. [3] studied on a system to make aerial mapping of a small area to both protect and sense the changes in that environment.

### **3.1.2 Military Applications**

Likewise non-military applications there are numerous military applications that make use of UAVs. The current and possible future applications are explained in the study of D. Glade at al. [28]. According to this study the applications that can use UAVs are the followings; transportation, signals collection, intelligence, surveillance and reconnaissance, combat support missions etc. Most of the military missions includes collecting various kind of data and signals. Mini UAVs are hard to be recognized, durable and fast. Because of these reasons mini UAVs are used for these kind of missions [13]. Surveillance missions are hard missions for most of the pilots because they took long time and in the great majority of the missions the pilots have to wait. They don't have chance to use their hardly won skills. Because of these situations much research is made on autonomous surveillance [25, 40]. Identically for the combat support missions and reconnaissance mission using autonomous agents are preferred [21].

## 3.2 Problem Types

In this part we provide problem variations related to our problem. The problem studied in this thesis is a multiple-constraint UAV scheduling and target assignment problem and it is also a special version of surveillance and reconnaissance mission. Surveillance and reconnaissance missions are usually complex problems that most of the problem instances can not be solved by using a single UAV. Instead of that, cooperation of UAVs is needed to achieve some certain needs of that specific problem [11]. Because of this reason, most of the studies in the literature is on cooperation. However, another notable thing about this research area is the variability of the problem instances. There are numerous different problem instances and for each of them the needs of the problem changes in a different manner. This yields many different solutions for many different problems [24].

For mission planning problems, the major reason of the variability is the nature of constraints. Each different problem has a different nature; so, this causes that each different problem has different constraints. There are roughly two types of constraints. The ones used in the applications which try to simulate a realistic model in terms of the UAV aerodynamics which takes real time constraints into account. The second type of constraints are focusing on scheduling and target assignment. They have some simple assumptions for the aerodynamics of the UAVs but they focus on solving more complex assignment and scheduling problems.

To demonstrate one of the frequently used constraints in realistic models is minimum segment length [17, 7, 37, 53]. This constraint is about the behavior of the UAVs. A UAV must not change any attitude in the flight before traveling to some predetermined distance when this constraint is present. This constraint is generally used in simulations which is used to make a more realistic design. By this constraint an UAV can complete one turn before starting another. Next example constraint is maximum turn angle constraint. This constraint is also associated with the limits of the real life UAVs. UAVs can make turns with some limited angles. The problems having this constraint takes maximum turn angle limit into consideration while finding solutions [33, 32]. Furthermore, another constraint is minimum flight altitude constraint. This constraint can be seen in 3D simulations mostly. All of the aerial vehicles should fly

above a minimum flight altitude [57, 29, 16]. UAVs in real life can not dive or climb with very steep angles like 90 degrees this type of behavior can cause the UAV to fall down. Hence, maximum climbing angle or maximum diving angle are limited. This constraint is again for 3D simulations [58, 60].

The first example of constraints that are frequently used in target assignment and scheduling problems is the capacity constraint. This constraint is generally used in vehicle routing problem VRPs. In these kind of problems every vehicle has a constant capacity and can not serve more than that amount [19, 56, 9]. Moreover another different problem instance is the one with heterogeneous UAVs. In this problem instance, UAVs can have different velocities and fuel capacities in the beginning on the scenario. So, choosing the best suitable UAV for the mission is also an another important issue [35, 22]. Vehicle routing problem with time windows is another problem instance in which targets can only be visited in some predetermined time periods [15, 10, 38]. Next constraint is stochastic one. Here one or more of the components of the problem are random. For example, sources, demands or time windows can be stochastic [43]. Fuel constrained systems is one of the popular problem instances. In this problem instance UAVs have limited fuels. So, while assigning UAVs to targets one should control if the fuel to go from current place of the UAV and go back to base station of the UAV is enough [27].

### **3.3 Solution Methods**

There are numerous problem instances and solution methods for UAV mission planning problems. Solution methods can be classified into two main clusters: Centralized methods and decentralized methods. In centralized methods there should be a control system and there should be agents. First, the control system gathers agent status information and environmental information. Then, it runs the algorithm and finds the flight plan, UAV target assignments and their velocities. After that the control system passes information to UAVs and the UAVs do their job [5, 47]. In decentralized methods, the mechanism is a little different. First of all, there is no control system in the decentralized method. The idea is to divide the main problem into many small problem instances and solve them one by one [36, 31, 45]. While doing this, the biggest

problem is with the coordination of the UAVs because there is no control system that is telling what to do. Each UAV should analyze its status, communicate with the other UAVs and then decide what to do. The advantage of the decentralized approaches is that each sub problem requires a small search space. However for big search spaces and complex problems centralized approaches seem to be more feasible. In the following chapters we provide example studies that uses decentralized and centralized methods.

### **3.3.1 Decentralized Methods**

The first example of the decentralized model is the strategy of having a single leader for the flock and the others follows it [20, 49]. The idea of this method is selecting a leader and position other UAVs according to it. With this way it is possible to find exact places of the whole flock just by finding the place of the leader. Although this looks a good and clear strategy to approach, if the leader fails this strategy loses its effectiveness. Furthermore, the communication between the flock members is not effective in this application. The second example of the decentralized model is virtual structure method. In this method, the whole flock behaves like a single entity [39]. Next, there is a key concept about decentralized model; it is behavior based method. In this method, the team goes within a formation but with the environmental changes they need to adopt to the changing environments. They change formation in order to adopt to environment. Behavior based model is inspired from animal flocks. For example, bird flocks change formation when they are passing through a narrow place. They can even go one by one if the passageway is too narrow. After the environment is big enough they return to starting formation. In behavior based method this types of behavior is aimed. They preserve this formation by communicating with each other [55, 54]. Another method is to changing planning space into the field space. This way, all the obstacles and the places that UAVs should not go are surrounded by repulsion forces and the goal is surrounded by forces of attraction. The collision avoidance and reaching goal by agents are achieved by this method. The method is called artificial potential field method [23, 14]. One of the key decentralized approaches is graph theory based method. In this model a graph containing UAV information and communication links are created. The nodes correspond to UAVs and edges corre-

sponds to relationships between the UAVs. With the help of this graph UAV control is achieved. Furthermore, addition of new nodes and deletion of existing nodes are easy to control [44].

### 3.3.2 Centralized Methods

For the centralized methods a control system exists in the environment. Control system runs the offline algorithm and transmits this information to the UAVs. There are two main cluster of algorithms for the solutions. These clusters are exact algorithms and heuristics. In exact algorithms, the main idea is to find the best solution by searching the whole solution space. Although searching the whole solution space is too slow for even the easy problems, with some elimination from the solution space, algorithms can find faster solutions. The elimination process is usually done before starting the search. These eliminations are usually made on solutions that seem obviously wrong. The variation on exact problems are usually caused by the differences in elimination techniques. Exact algorithms usually find optimal solution but despite the eliminations they are still too slow for large and complex problems. Because of this reason, there are many heuristic approaches in the area. The idea behind the heuristic approaches is finding an acceptable solution in a fast way. They may not find optimal solution for all of the problem instances but they are fast and if finding the optimal solution is not a must and finding an acceptable true solution is enough then using heuristics is a good idea. Heuristics usually search a small but effective part of the solution space. Because of that they find local optimal solutions but in a fast way [46].

Integer programming is a good example of the exact algorithms for the UAV mission planing problems. The problem is encoded into a mathematical model. The constraints are converted into mathematical equalities and inequalities. Then, an objective function is created. With the help of these equalities and inequalities, the optimal values are found with the help of matrix operations. For example, Bellingham at al. [8] worked on control and coordination of UAV flocks with Mixed Integer Linear Programming (MILP). Some of the variables are integers where some others are not. The mixed term comes from the mixture of the integer and floating point values in

the solution variables. Similarly, Schouwenaars, Moor, Feron and How worked UAV mission planning using MILP [48]. Another notable exact algorithm is dynamic programming. In dynamic programming the aim is to break up the problem into smaller instances and solve them. Then, combine the solutions of the smaller problems to reach the optimal solution. Alighanbari and How et al. [4] worked on a cooperative target assignment problem in adversarial environments with dynamic programming.

UAV mission planning algorithms usually have a complex nature. So, exact approaches are infeasible in terms of running times for most of the cases. For this reason, heuristic approaches are more popular. For the heuristic paradigm, there are three types of algorithms. *Probabilistic algorithms*, *genetic algorithms* and *heuristic algorithms*. In probabilistic approaches, the agents make choices randomly to take to the next step. Although probabilistic approaches are not very effective they provide searching through the different areas of the solution space. They are usually used in hybrid approaches [18]. The genetic algorithms or evolutionary algorithms are one of the most popular heuristic methods [26, 50, 30, 51, 34]. Heuristic algorithms are in the same direction with the genetic algorithms. In a certain way, they create a small subset of the solution space and they find the solution using that small subset. The subset is not randomly chosen. They usually build these subsets by making guesses using the domain information that is available to them. One example of heuristic algorithm is greedy algorithm. As the name implies greedy algorithm chooses the best possible option at each decision point. Although this seems like a good idea to reach the optimal solution sometimes the algorithm should not choose the best possible option. The real importance of greedy algorithms is that they are fast. Because, they search only a small part of the solution space [59].



## CHAPTER 4

### PROPOSED WORK

This chapter provides a detailed description of our study of multiple constraint UAV scheduling and target assignment problem. First, the problem structure is defined. Then, the constraints of the problem in hand are defined. After that, the environmental assumptions are given. Then, a brief explanation about how inputs are generated will be made. Lastly, proposed algorithms are presented.

#### 4.1 Problem Description

UAV mission planning is a type of target assignment and UAV scheduling problem. UAV mission planning problem is defined as determining assignment and scheduling of UAVs to targets that minimizes the total cost. First of all, there are some UAVs and targets in this problem. At the beginning of the problem UAVs start from a base station. Then, each of the targets should be visited by some required number of UAVs. All UAVs must return back to the base station, after the traversal. Figure 4.1 contains an example. This example shows a snapshot from a UAV mission planning scenario. In the example, rectangle represents base station and circles represents targets. Arcs connecting the rectangle and the circles represent costs to reach from one of the connected nodes to another. For this example, there is no environment constraints. Environment of the problem defines the physical rules that taken into consideration while solving the problem. When there is no environment constraint, UAVs can use euclidean distance path to go from one target to another. All UAVs and targets have unique ids to distinguish from each other. Again in the figure all the

targets has something like  $tw=[1-5]$ . Tw stands for time window. Targets are only available in these time intervals and they have to be visited at the beginning of the time windows. Furthermore, once a UAV reached a target it can not leave it before departure time of that target. So, if two target's time windows are overlapping, the same UAV can not visit both of them. Some targets have larger areas to be covered than others. So, sometimes one UAV can not finish the mission at that target itself. For this kind of situations, number of UAVs needed must be defined. If a target need is 2, then that target must be traveled by two UAVs and these UAVs can not be the same UAV. Simulation time that is placed in the top left corner shows the time of the mission. Time windows are activated according to that time value.

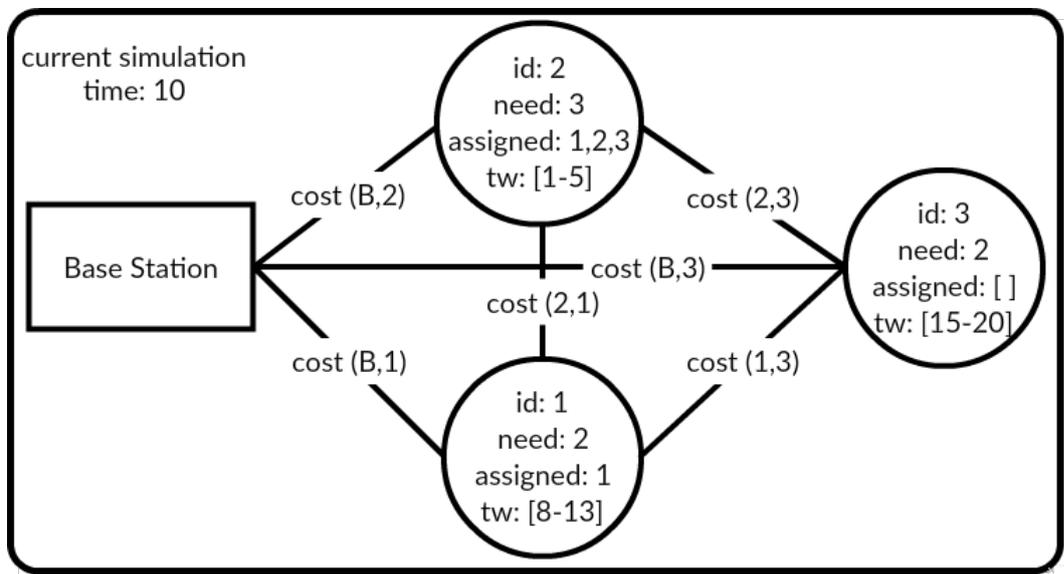


Figure 4.1: A sample drawing of the studied problem

In Figure 4.2, a base station holding 5 UAVs is provided. As can be seen from the figure, each UAV has its own ids and apart from ids, all of the UAVs are identical. Note that, UAVs are homogeneous. When the mission is completed, the current coordinates and the base station coordinates of the UAVs must be the same. Each UAV has a fuel constraint. To be able to achieve a mission, UAVs should have enough fuel in their tanks to return back their own base stations.

The objective is to minimize the cost of traversal of all targets. The total cost is calculated by the summation of the total consumed fuels by all UAVs.

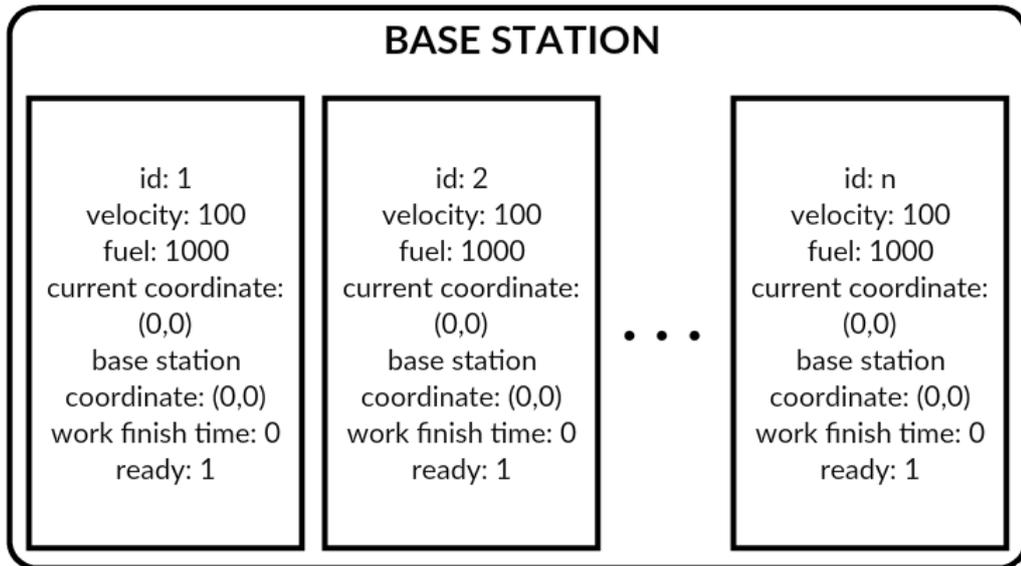


Figure 4.2: UAV structures in this problem

It's assumed that the environment is 2D. This means that the flying altitudes of the UAVs are not important and do not have any effect on the outcomes of the problem. Also, there is no obstacles in the environment, meaning that any UAV can move to a destination without changing direction. There is no constraint on the movements of the UAVs. UAVs can make any movement they want. Once a mission is provided it can not be interrupted and changed. This means that the problem in hand does not have a dynamical environment and offline planning is effective. Moreover, no weather conditions are taken into consideration like wind, rain, lightning etc.

To make a better understanding in Figure 4.3, we provide an example problem. Then, we explain how to determine an optimal solution.

In Figure 4.3, we see the base station and the targets. The nodes in this figure are placed according to their coordinates to see how the targets are positioned. In this example we assume the velocities and the fuel consumptions of the UAVs are 1. This makes easier to calculate the cost. Then, to observe the time window constraints we put the targets on a time line in Figure 4.4. In this figure we see that target-1 and target-2's time windows are overlapping. In this case, we can not send same UAV to both of them. Target-3 is separated from the other two, so there is no constraint for it. Also, the numbers on the lines shows the starting and finishing time of the time

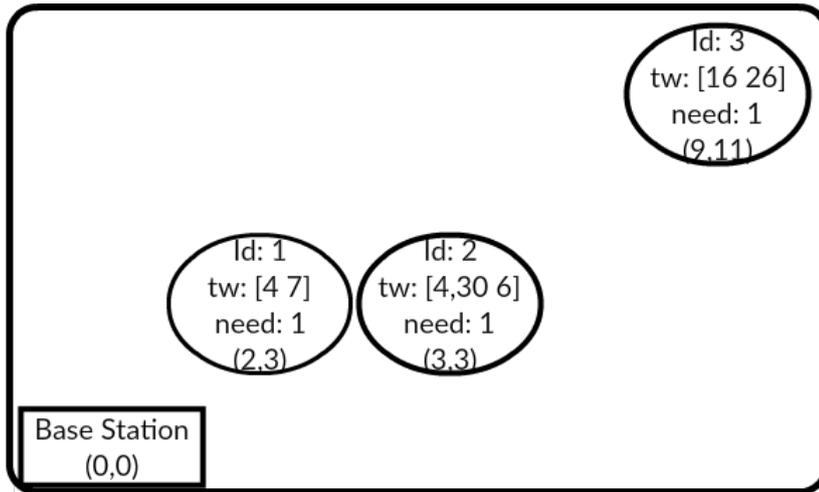


Figure 4.3: An example problem setting, targets are placed according to their coordinates.

windows.

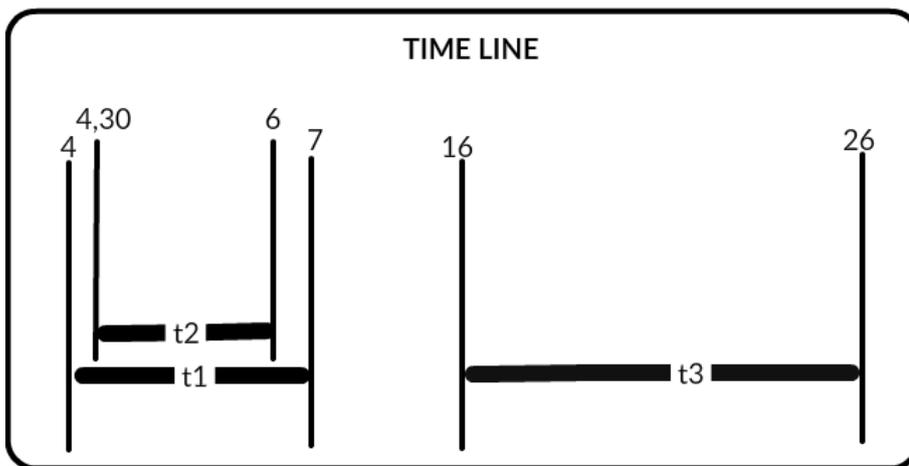


Figure 4.4: An example problem setting, targets are placed according to their time windows.

To be able to decide which choice is the best, we created a cost table that shows the cost to reach from one target to another. In Table 4.1, 0 represents the base station and the other numbers represent the ids of the targets.

The optimal solution for this problem instance is provided at Table 4.2. Targets and assigned UAVs are provided with their costs. While deciding this solution, things to

Table 4.1: Table shows costs to reach from a target to another.

Cost Matrix				
	0	1	2	3
0	0	3,61	4,24	14,21
1	3,61	0	1	10,63
2	4,24	1	0	10
3	14,21	10,63	10	0

consider are the constraints. The first of the current constraints is not sending the same UAV to both target-1 and target-2. The second one is to minimize the fuel. Table 4.2 represents the best possible solution for this problem. On the other hand, in Table 4.3, we provide another possible solution. The difference between these two solutions is, in solution-2 a new UAV is send from the base station. On the contrary, solution-1 sends the UAV which is closer to target-3. With this way it minimizes the fuel consumption

Table 4.2: Optimal solution for this problem is presented in this table.

Solution 1- Total Cost : 17,85		
Target Id	Uav Id	Cost
Target 1	UAV 1	3,61
Target 2	UAV 2	4,24
Target 3	UAV 2	10

Table 4.3: Another possible solution to solve this problem.

Solution 1- Total Cost : 22,06		
Target Id	Uav Id	Cost
Target 1	UAV 1	3,61
Target 2	UAV 2	4,24
Target 3	UAV 3	14,21

## 4.2 Proposed Solution Methods

In this part, algorithms developed to solve UAV mission planning problem are described. There are five algorithms proposed in the scope of this thesis. The two of them are developed to test the proposed algorithms in terms of solution quality and performance. The other three of them are approaches with similar infrastructures.

They are developed in order to solve multiple constraint UAV mission planning problem and assess their performances.

#### 4.2.1 Brute Force Algorithm

This algorithm is developed to compare the results of the other implemented algorithms. The aim of the comparison is to measure the difference between the optimal result and the results of the other algorithms for small sized inputs. The brute force algorithm only generates complete scheduling.

<p><b>Data:</b> <math>T</math>: is the vector of targets</p> <p><b>Data:</b> <math>U</math>: is the vector of UAVs</p> <p><b>Result:</b> <math>bestResult</math>: Optimal UAV-target assignment</p> <pre> 1 <math>n \leftarrow</math> total target needs; 2 <math>permUAVs \leftarrow n</math> permutations U with repetition; 3 <math>permTargets \leftarrow</math> permutations of T; 4 <math>bestCost \leftarrow Inf</math>; 5 <b>for</b> (<math>i \leftarrow 1</math> <b>to</b> <math>lengthPermTargets</math>) <b>do</b> 6     <b>for</b> (<math>j \leftarrow 1</math> <b>to</b> <math>lengthpermUAVs</math>) <b>do</b> 7       Generate the candidate assignment; 8       <math>boolFeasibility \leftarrow</math> Test the feasibility of the candidate assignment; 9       <math>tempCost_n \leftarrow cost(\text{candidate assignment})</math>; 10      <b>if</b> (<math>tempCost &lt; bestCost</math>) <b>and</b> (<math>boolFeasibility == True</math>) <b>then</b> 11          <math>bestCost \leftarrow tempCost</math>; 12          <math>bestResult \leftarrow</math> candidate assignment; 13      <b>else</b> 14          do nothing; 15      <b>end</b> 16    <b>end</b> 17 <b>end</b> </pre>
--

**Algorithm 1:** Brute force algorithm

In Algorithm 1, the general structure of the implemented brute force algorithm is provided. While generating UAV permutations, repetitions also taken into consideration

because the same UAV can go more than one target. For target permutations, we only generated permutations without repetition. By creating these two sets, we are able to search whole solution space. A pre-elimination procedure can be added to this implementation in order to increase the efficiency of this algorithm.

#### **4.2.2 Greedy Algorithm**

This algorithm is also developed to compare the results of the other algorithms to assess test the solution qualities and time consumptions. The greedy algorithm has a faster nature than brute force algorithms. Actually it is one of the fastest approaches to a problem. So, this provides a lower limit on the elapsed time constraint. If an algorithm displays a performance that has a similar speed then that algorithm can be counted as a really effective algorithm. Constructing an algorithm which has a similar speed as greedy algorithm, cost values smaller than greedy algorithm and solution finding ratio larger than greedy algorithm is a really valuable achievement for this problem. To be able to make these kinds of experiments greedy algorithm is

also implemented.

<p><b>Data:</b> <math>T</math>: is the vector of targets</p> <p><b>Data:</b> <math>U</math>: is the vector of UAVs</p> <p><b>Result:</b> <math>Result</math>: Found UAV-target assignment</p> <pre> 1 <math>sortedTargets \leftarrow sort(T)</math> /* sort targets according to their    time windows, in increasing order */ 2 <b>while</b> (<math>isEmpty(sortedTargets) == False</math>) <b>do</b> 3   <math>chosenTar \leftarrow chooseTarget(sortedTargets)</math>; 4   <math>U \leftarrow updateUAVstatus(currentTime)</math>; 5   <b>if</b> <math>anyReadyUAVs(U) == True</math> <b>then</b> 6     <math>chosenUAV \leftarrow chooseLeastCostUAV(chosenTar, U)</math>; 7     <math>(U, sortedTargets) \leftarrow sendUAVtoTarget(chosenUAV, chosenTar)</math>        /* update UAVs and targets */ 8     <math>Result \leftarrow Result + (chosenUAV, chosenTar)</math> 9   <b>else</b> 10    <math>currentTime \leftarrow updateTime()</math>; 11  <b>end</b> 12 <b>end</b> </pre>
---

**Algorithm 2:** Greedy Algorithm

Algorithm 2 shows the steps of greedy the algorithm. The main idea of the greedy algorithm is, to send UAVs to targets with the minimum cost as long as there are UAVs ready. If all the UAVs are on a mission, current time is updated to the time when first UAV will finish its work. Traversing order of targets are defined with their time windows in this algorithm. If start of

### 4.2.3 Divide and Conquer Algorithm

The idea to solve multiple constraint UAV scheduling and target assignment problem is to divide the problem into smaller problem instances and solve the smaller instances. Then, combine the results. As can be seen in Figure 4.5, converting UAV mission planning into multiple transportation algorithms is the key point of our proposal. Hybrid greedy algorithm, intlinprog heuristic algorithm and simplex heuristic

algorithm uses this divide and conquer idea. The only difference is each of them solves the sub problems with a different algorithm. Actually they take their names from the approaches they attack to solve transportation problem. Apart from that infrastructures of all of these three algorithms are the same. They are implemented in order to see which one of them suits best for the problem.

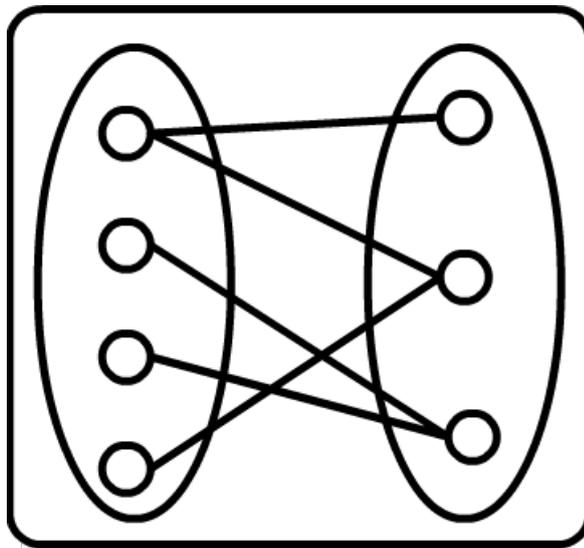


Figure 4.5: UAV structures in this problem

The idea behind our proposal is to cluster the targets according to their time window values. Then from the beginning of the each cluster solve the transportation algorithm and then combine all the results. This algorithm is actually a special version of greedy algorithm but it should have find much more effective results than greedy algorithm because the amount of search space scanned is much more than the greedy algorithm. It is important to realize that when all of the targets are individual clusters this idea works as in the exactly same way with the greedy algorithm but this is only one

extreme case.

```
Data:  $T$ : is the vector of targets
Result:  $Clusters$ : is the vector of clustered targets
1  $sortedTargets \leftarrow sort(T)$  /* sort targets according to their
   time windows, in increasing order */
2 while ( $isEmpty(sortedTargets) == False$ ) do
3    $chosenTar \leftarrow chooseTarget(sortedTargets)$  /* choose target
   with smallest time window start */
4    $newClusterFlag \leftarrow True$ ;
5   for ( $j \leftarrow 1$  to  $length(Clusters)$ ) do
6     if  $overlapWithCurrentCluster(chosenTar) > 0.7$  then
7        $currentCluster \leftarrow currentCluster + chosenTar$ ;
8        $newClusterFlag \leftarrow False$ ;
9       break;
10    else
11      do nothing;
12    end
13  end
14  if  $newClusterFlag == True$  then
15     $currentCluster \leftarrow newCluster$ ;
16     $newCluster \leftarrow chosenTar$ ;
17     $Clusters \leftarrow Clusters + newCluster$ ;
18  else
19    do nothing;
20  end
21 end
```

**Algorithm 3:** Clustering Algorithm

Making an effective clustering is the other problem that should be solved. The clustering algorithm we proposed appears in Algorithm 3. The idea is to build the clusters incrementally. Figure 4.6 helps to understand the clustering algorithm with an example. As one can see, target-1 and target-2 has 70 percent time overlap, so they are put in the same cluster. There is no overlap between target-2 and target-4 so a new cluster

is starts with target-4. After target-4, the next target is the target-5. The overlap ratio between target-4 and target-5 is 40 percent so they are also put in different clusters.

After the clusters are constituted the selected algorithms run on these clusters and after all of them are finished the results are combined and the overall results is found. This approach does not offer to find the optimal solution but it does a offer quickly better than greedy approach.

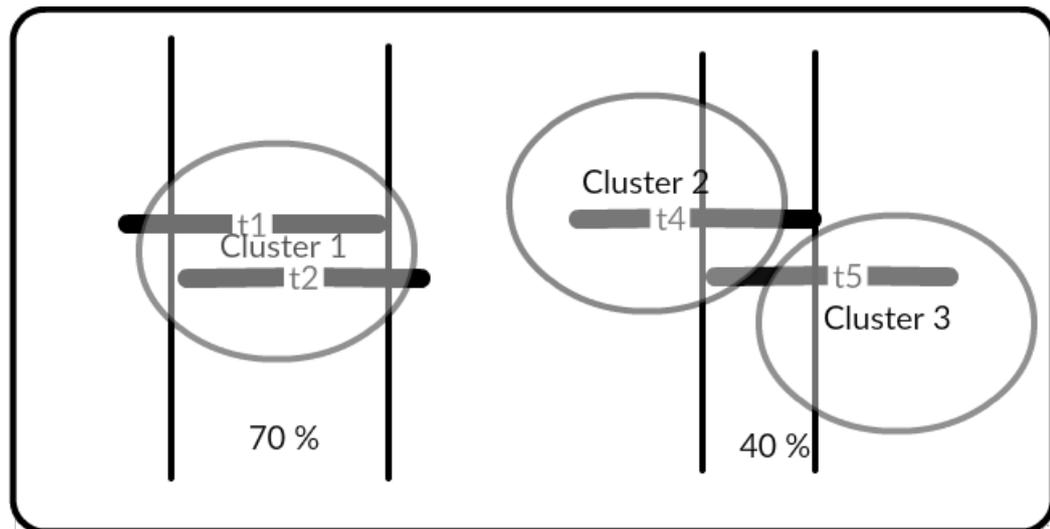


Figure 4.6: UAV structures in this problem

#### 4.2.4 Hybrid Greedy Algorithm

This algorithm uses the above mentioned divide and conquer method. As the solution of the transportation problem it uses a brute force approach. The aim is to find near optimal solutions. Despite its power, the weakness of this algorithm is also similar to brute force algorithm, this algorithm will work slowly. In spite of the fact that the algorithm is not as slow as brute force algorithm it may not be feasible for large sized problems. However, it has the chance to find better solutions from simplex heuristic and intlinprog heuristic for small sized inputs because they only look at possible flow from chosen UAVs to chosen targets.

### 4.2.5 Intlinprog Heuristic Algorithm

This algorithm also uses the above mentioned divide and conquer method. After the clusters are constituted, this algorithm solves transportation problem with a modified linear programming algorithm. To implement this algorithm, the Matlab optimization toolbox's intlinprog function is used. The data is prepared to be suitable for intlinprog's parameters. Then intlinprog is run for each of the transportation problem instances. At the end the found solutions are combined. In Algorithm 4, the steps of Matlab's intlinprog algorithm is presented. Mix integer linear programming and branch and bound algorithms are explained in the background section.

**Data:**  $T$ : is the vector of targets

**Data:**  $U$ : is the vector of UAVs

**Result:** *Result*: Found UAV-target assignment

- 1 it reduces the problem size by applying some linear programming preprocessing;
- 2 solves the initial problem with the help of linear programming;
- 3 performs mixed integer linear programming and cut generation to tighten the LP relaxation;
- 4 uses branch and bound algorithm to reach the optimal solutions;

#### Algorithm 4: Intlinprog Algorithm

In the following parts we explain how we constructed the equations and the objective function to use linear programming. Also, we explain the usage of intlinprog algorithm to solve UAV mission planning problem.

#### 4.2.5.1 Construction of the Linear Programming Equations

In this part the problem is converted into a mathematical model. For linear programming problems linear equalities, linear inequalities, bounds for the solution variables and an objective function need to be generated.

To construct linear equalities, constraints of the problem need to be extracted. There are two constraints available for this problem:

1. Only some of the UAVs are available for each sub-problem. And each UAV can only visit one target at a time. Because of this, for each of the source, the number of items it can send to destination is fixed.
2. For each target, the number of UAVs it needs is fixed.

According to these constraints, the equations are constructed. It is assumed that, the number of sources available is  $n$  and the number of destinations available is  $m$ . For each source and destination, there is exactly one constraint. So, there can be  $n + m$  equations for this problem.

For each of the sources, the number of UAVs it have is shown as:  $S_1, S_2, S_3 \dots S_n$ . This shows how many UAVs that each of the source can send. For each of the destinations, the number of the UAVs they can receive is shown as:  $D_1, D_2, D_3 \dots D_m$ .

Number of UAVs sent from each source to destination is hold in a matrix. This matrix is used to construct the equations. In Table 4.4, we see an example of this matrix. In the matrix, number of UAVs sent from  $S_1$  to  $D_1$  is showed with  $u_{11}$  and UAVs sent from  $S_2$  to  $D_2$  is showed with  $u_{22}$  and so on.

Table 4.4: A matrix showing UAVs send from a source to a destination

	$d_1$	$d_2$	$d_3$	...	$d_m$
$S_1$	$u_{11}$	$u_{12}$	$u_{13}$		$u_{1m}$
$S_2$	$u_{21}$	$u_{22}$	$u_{23}$		$u_{2m}$
$S_3$	$u_{31}$	$u_{32}$	$u_{33}$		$u_{3m}$
...					
$S_n$	$u_{n1}$	$u_{n2}$	$u_{n3}$		$u_{nm}$

With the help of the matrix and the above mentioned constraints, the following  $(n+m)$  number of linear programming equations are constructed:

$$\sum_{i=1}^m u_{1i} = S_1 \tag{4.1}$$

$$\sum_{i=1}^m u_{2i} = S_2 \quad (4.2)$$

$$\sum_{i=1}^m u_{3i} = S_3 \quad (4.3)$$

$$\sum_{i=1}^m u_{ni} = S_n \quad (4.4)$$

$$\sum_{i=1}^n u_{i1} = D_1 \quad (4.5)$$

$$\sum_{i=1}^n u_{i2} = D_2 \quad (4.6)$$

$$\sum_{i=1}^n u_{i3} = D_3 \quad (4.7)$$

$$\sum_{i=1}^n u_{im} = D_m \quad (4.8)$$

For this problem no inequalities exist. Also, no upper bounds exist for the number of UAVs sent. So, upper bounds can be chosen as infinite. It is not possible to send negative number of UAVs to a target. So, lower bound for the solution variables is 0.

The objective function of the problem defines the value that needs to be minimized. In this problem value to be minimized is the total cost. The total cost is the accumulated sum the fuels consumed by all of the UAVs. Let's call the total cost C. Equation 4.9 shows the way C is calculated:

$$C = \sum_{i=1}^m (\sum_{j=1}^k c_{ij}) \quad (4.9)$$

$c_{ij}$  is the cost of going from the current location of the UAV<sub>j</sub> to the target<sub>i</sub>s location. The  $m$  in the formula is the number of targets and the  $k$  in the formula represents the number of UAVs at each of the targets.

#### 4.2.5.2 Usage of Intlinprog Algorithm

In this part usage of Matlab's `intlinprog` function is explained. The signature of the function used is as the following:

$$[x, fval] = \text{intlinprog}(f, \text{intcon}, A, b, Aeq, beq, lb, ub)$$

As the Matlab suggests `x` and `fval` are the outputs of the function. `f`, `intcon`, `A`, `b`, `Aeq`, `beq`, `lb` and `ub` are the input parameters of the function. In `intlinprog`, the inputs represent the various constraint types of the problem and the outputs represents the results. `intlinprog` takes input in the following order: first linear inequalities then linear equalities and finally bounds. According to this, `A` is the matrix of linear inequalities and `b` is a vector holding the right hand side values of the inequalities. Similarly, `Aeq` and `beq` are used for equalities. `lb` is the vector holding lower bound values and `ub` is the vector holding upper bound values.

#### 4.2.6 Simplex Heuristic Algorithm

In this chapter proposed work of this thesis is explained. The idea is to divide scheduling and assignment parts of the problem. Solve each assignment problem individually and combine the results. In this idea, there are two important parts: The first one of them is how to divide this problem. The solution of this part is explained in divide and conquer algorithm section. The second part of this idea is how to construct a transportation problem.

The divided problem has time window constraints and scheduling of the UAVs is needed. On the other hand, transportation problem finds a transportation plan but it does not do scheduling. It only makes an assignment between sources and destinations. It does not do anything related to time. Meaning that, the solutions it produces does not indicate the orders of the assignments. Furthermore, the solutions it produces are optimal solutions.

In the rest of the section, we provide a design to transform the sub-problems into transportation problem. Then, a brief explanation about why simplex algorithm is chosen to solve the generated transportation problem is given. Finally, a pseudo code

explaining the working structure of the primal simplex network algorithm is provided.

#### 4.2.6.1 Problem Design

In this part design to transform sub-problems into transportation problems are explained. The steps of this process is explained in Algorithm 5.

```

Data:  $U$ : is the vector of UAVs
Data:  $T$ : is the vector of targets
Data:  $currentTime$ : is the vector of targets
Result:  $Sources$ : source vector of the transportation problem
Result:  $Destinations$ : destination vector of the transportation problem
1  $AvailableTargets \leftarrow findAvailableTargets(targets,currentTime)$ 
2 for  $i \leftarrow 1$  to  $lengthUAVs$  do
3   for ( $j \leftarrow 1$  to  $lengthAvailableTargets$ ) do
4     if ( $reachTime(UAVs[i]) < target[i].timeWindowStart$ ) then
5        $availableUAVs \leftarrow availableUAVs + UAVs[i]$ ;
6       break;
7     else
8       do nothing;
9     end
10  end
11 end
12  $Sources \leftarrow ClusterSameCoordinateUavs(availableUAVs)$ ;
13  $Destinations \leftarrow availableTargets$ ;
14  $solveTransportationAlgorithm(Sources, Destinations)$ ;

```

**Algorithm 5:** Transportation algorithm design

To explain the algorithm better, an example is prepared. In this example, we examine a mission planning problem. With the help of diving procedure explained above we find first sub-problem. Then, convert it to a transportation problem. In Figure 4.7, a snapshot from a simulation is provided. In this example, at the current time instance there are 10 UAVs in the base station and 5 UAVs are in the targets. The works of the UAVs with ids 1, 2 and 3 are finished at 9th second and the works of the UAVs with

ids 4 and 5 finishes at 13th second.

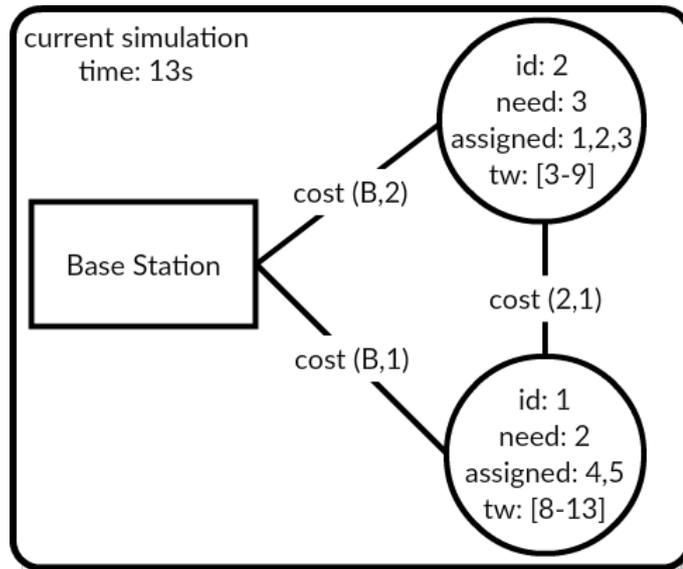


Figure 4.7: An example snapshot from UAV Mission Planning Problem

The next cluster to be visited can be seen in Figure 4.8. In this figure we see three targets to cover. To eliminate scheduling, we determine the ready UAVs. Then, from this set we make an elimination. For each UAV, we calculate the possible reaching times to targets. If reaching time is smaller than or equal to starting time of the target, then that UAV is marked as available. After the eliminations made, the next step is to cluster UAVs according to their coordinates. For this example, there are three clusters of UAVs; base station, the UAVs at target 1 and the UAVs at target 2. Each of these places are taken as sources. Also, the targets are the destinations. Needs of each target is, taken as the capacity on that destination.

In Figure 4.9, the resulting transportation problem is provided. The left circles represent the sources and the right circles represent the destinations. The capacity of each source and destination is written into the center of the circles. The capacities of the sources are 10, 3 and 2 respectively. Because, at base station there are 10 UAVs, at target-1 there are 2 UAVs and at target-2 there are 3 UAVs. After this point, the problem is converted into a transportation problem. The solution of transportation problem yields a minimum cost flow. For our case, results show optimal UAV-target assignments. In Figure 4.9, arcs between sources to destinations shows how many UAVs

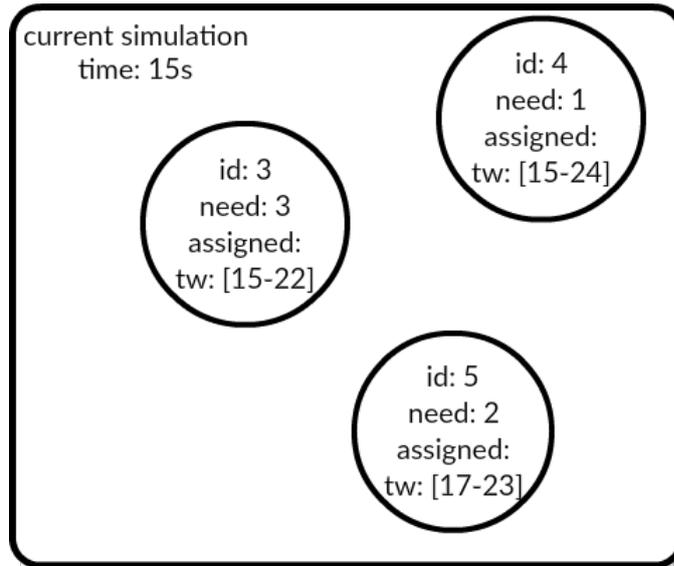


Figure 4.8: An example snapshot from UAV Mission Planning Problem

are send from each source to destination. After solving the transportation problem, we add cost of each UAV to the current time value and calculate the times that UAVs are assigned to targets. With this way time information is added to this design.

Each assignment found from the sub-problems, are placed into a table holding the results. After solving all of the sub-problem a scheduling and assignment table showing the results is produced. An example solution table can be seen in Table 4.5.

Table 4.5: An example table showing the result of an UAV mission planning

Assignments	Target Ids	Uav Ids	Assigned Time	Consumed Fuel
1	1	1	0,8	15
2	1	2	4,30	45
3	1	7	6,35	64
4	2	1	6,45	10
5	3	4	8,32	22,5
6	3	5	10	38

#### 4.2.6.2 Primal Network Simplex Algorithm

For primal simplex algorithm, we used an implemented algorithm provided at Matlab file exchange. It takes a capacity matrix, source matrix and a cost matrix. It returns

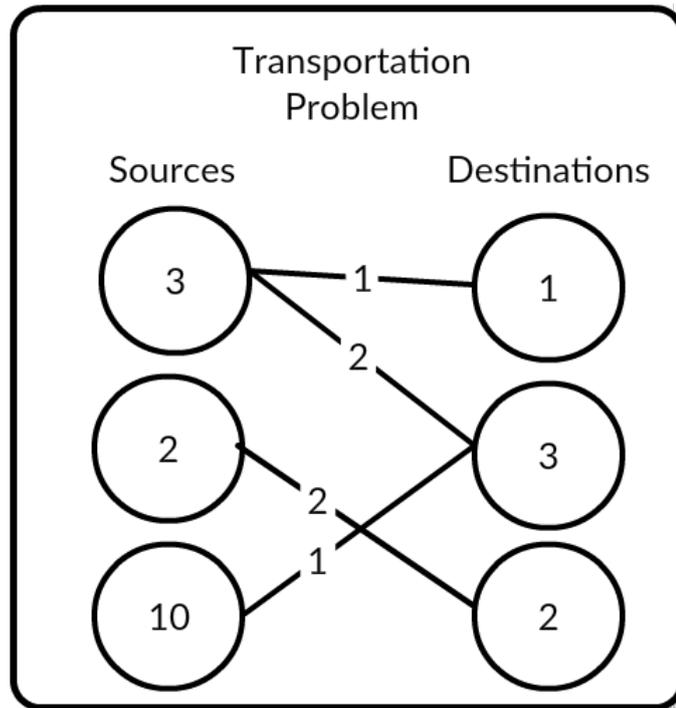


Figure 4.9: An example snapshot from UAV Mission Planning Problem

the minimum cost flow matrix.



## CHAPTER 5

### EVALUATION AND RESULT

In this chapter, performance evaluation of the proposed algorithms presented. We show that simplex heuristic method fits better to this problem compared to the other algorithms. The algorithms are evaluated on their effectiveness and their solution qualities. We created a testing environment. The first thing we do was to examine the effect of the size of the problem on the performance of algorithms. Also, similar problems are grouped together and all the algorithms are tested with characterized input. The aim of doing this is to reveal the vulnerabilities or strengths of the algorithms.

#### 5.1 Test Environment

First of all, Matlab provides a wide range of algorithms readily programmed. Matlab has an optimization toolbox and this toolbox offers a wide range of algorithms to use. This toolbox, comes already installed with the new versions of the Matlab. Furthermore, the structure of the problem is suitable for object based programming. It is also possible to make object oriented programming with Matlab. Also, the transportation algorithm requires a matrix based approach and Matlab fits in all of these necessities nicely. Because of these reasons, Matlab is chosen as the programming language for this study.

The version of the Matlab used in the experiments is Matlab R2014b. The project is developed on a PC which has a 64 bit 3.40 GHz Intel i7 processor. The operating system is Ubuntu 14.04.2 LTS operating system.

## 5.2 Data Sets

Three different input sets are prepared to evaluate the algorithms. All of the algorithms are run on these sets and the solutions are compared. While making comparison average results of the algorithms are used.

The first input set contains problem instances of different sizes. The aim here is to test the effect of problem size on the performances of the algorithms. Data sets inputs are classified in three different clusters; small sized, medium sized and large sized. Small input set contains 3 to 5 targets, medium ones have 7 to 10 targets and large inputs have 20 to 25 targets. For each input type, 10 different inputs are created.

The second input set clusters the input in terms of time window values of targets. The overlap rate of target time windows are calculated for each of the inputs. The inputs are clustered according to their overlap rates. The aim is to observe the behaviors of the algorithms on various overlap rates. Clustered inputs are created for different size of inputs. Each cluster has 10 inputs cases.

The last input set is used to test the algorithms with handcrafted and randomly created inputs. The aim of creating hand crafted inputs is to test some extreme cases. Furthermore, it is possible to provide problem sets that are guaranteed to have complete scheduling solutions with manual input creation. Unfortunately, randomly generated inputs can not give this guarantee. On the other hand, random input generation is needed to increase the variability of the inputs. All small sized, medium sized and large sized inputs are created for both handcrafted input and randomly created input. For each different sizes, an input set having 10 inputs are generated.

We developed a function to create randomly generated inputs. The function takes ratio of overlaps, maximum and minimum time windows values and the coordinate ranges as parameters. Then it generates random inputs satisfying these constraints. Note that for randomly generated inputs, it is not possible to assess whether the created mission can be solved fully or partially.

In the following sub-sections we provide explanations about overlapping ratio.

### 5.2.1 Overlapping Ratio

Overlapping ratio is calculated by the rate of overlaps in the target set. To calculate this ratio, definition of overlapping must be done clearly. In this problem overlap is the case when two targets time windows cover the same time interval. These time intervals do not have to overlap completely, only a small overlap is an enough reason to send two different UAVs to these targets because the UAV must stay at that target until its time window finishes.

Figure 5.1, shows two different problem structures. For both of the problems the overlapping ratio is same, it is  $2/3$ . Overlapping ratio is calculated by dividing the number of targets that overlap to the total number of targets. For this example, target-1 and target-2 are overlapping. So, there are two overlapping targets and the total number of targets is 3. Because of that, the ratio is  $2/3$ . For the second example, first two targets are totally overlapping but this does not change the ratio.

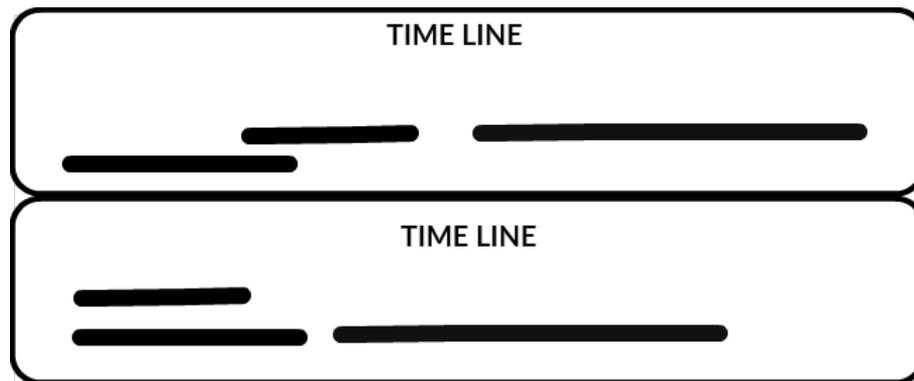


Figure 5.1: Two cases where overlapping ratio is same.

### 5.3 Input Generation

Two types of input are generated. We generated some test cases manually. The second type of input is generated automatically and the inputs are randomized.

For random input the user can specify the number of targets, number of UAVs, maximum and minimum coordinates of targets, maximum and minimum time window

values and the ratio of overlaps. In Algorithm 6, the steps of random input generation

is provided.

```
Data:  $T$ : is the vector of targets  
Data:  $U$ : is the vector of UAVs  
Data:  $OR$ : expected overlapping ratio  
Data:  $CI$ : coordinate interval  
Data:  $V$ : velocity  
Data:  $F$ : fuel  
Data:  $UN$ : UAV numbers needed  
Data:  $NU$ : number of UAVs  
Data:  $NT$ : number of targets  
Result:  $UAVs$ : A randomly created UAV vector  
Result:  $Targets$ : A randomly created Target vector  
1 for  $i \leftarrow 1$  to  $NU$  do  
2   |    $randomCoordinate \leftarrow rand(CI)$ ;  
3   |    $UAVs \leftarrow createRandomUAVs(i, F, V, randomCoordinates)$ ;  
4 end  
5 for  $i \leftarrow 1$  to  $NT$  do  
6   |   if  $overlapRatio(U) < OR$  then  
7   |   |    $randomCoordinate \leftarrow rand(CI)$ ;  
8   |   |    $lenClusters \leftarrow length(Clusters)$ ;  
9   |   |    $selectedCluster \leftarrow randi(0, lenClusters)$ ;  
10  |   |    $tempTarget \leftarrow createRandomTarget(i, randomCoordinates, UN)$ ;  
11  |   |    $Targets \leftarrow tempTarget$ ;  
12  |   |    $Clusters \leftarrow addToCluster(selectedCluster, tempTarget)$ ;  
13  |   |    $overlapRatio \leftarrow updateOverlapRatio(Targets)$ ;  
14  |   |   else  
15  |   |   |    $tempTarget \leftarrow$   
16  |   |   |    $createRandomNonOverlappingTarget(i, Clusters, UN)$ ;  
17  |   |   |    $overlapRatio \leftarrow updateOverlapRatio(Targets)$ ;  
17  |   |   end  
18 end
```

**Algorithm 6:** Random Input Generation

This random input generator makes it very easy to test the algorithms. Furthermore with the help of overlap ratio parameter, inputs with different characteristics can be created. In Figure 5.2, an example explaining the algorithm is provided. At first situation two targets are already generated and current overlap rate is smaller than the expected overlap ratio. In this situation, algorithm adds another target to overlapping targets area. There is only one overlapping region, so the new generated target added into that region. After this situation, current overlap ratio increases and it becomes larger than the expected overlap ratio. Then the next target is added to a randomly chosen area where no overlap occurs to small down the overlap ratio.



Figure 5.2: Random input generation example

#### 5.4 Algorithms Used in the Tests

In this part we present the algorithms developed to solve UAV mission planning. From this point algorithms are referred with the abbreviations provided in the Table 5.1.

Table 5.1: Table showing compared algorithms

Algorithm Name	Abbreviation
Brute Force Algorithm	BFA
Greedy Algorithm	GA
Intlinprog Heuristic	IH
Hybrid Greedy Algorithm	HGA
Simplex Algorithm	SA

## 5.5 Performance Evaluation

In this section how the performance of all of the algorithms are evaluated is explained. The main aim of this study is finding good complete solutions by consuming small amount times and finding solutions where brute force algorithms are not feasible. Some performance metrics are defined for this purpose. Algorithms are evaluated with these performance metrics in different sized inputs.

Performance metrics for this evaluation phase is fuel consumed, time elapsed and the percentage of complete scheduling.

- **Fuel Consumed:** Total fuel consumed by all the UAVs is the cost measure of this problem.
- **Complete Scheduling:** The second performance metric is the percentage of complete scheduling. The multiple constraint UAV scheduling and target assignment problem is a NP-Hard problem. It is not feasible to find optimal solution in a short period of time. Because of this reason, heuristics and approximation techniques are mostly used. To test the success rates of these kinds of approaches we calculate complete scheduling rates of these algorithms in different data sets and situations. The complete scheduling is defined as visiting all the targets and fulfilling their needs. Furthermore, all of the UAVs have to return back to their own base stations to make a complete scheduling.
- **Time Elapsed:** The last performance metric is the time elapsed. Time constraint is important because the use of this problem is usually on military areas and fast solution mechanism is a must for these problems.

### 5.5.1 Hand Crafted Test Cases

In this part first we compare all of the algorithms with brute force algorithm to see the quality of the results in terms of fuel consumption. Then they are compared with each other with variant size of inputs.

#### 5.5.1.1 Comparison with brute force algorithm

The comparison with brute force algorithm is made separately because it is not feasible for the large sized inputs. Figure 5.3 shows time needed to solve problem as its size gets larger for brute force algorithm. This curve shows that brute force algorithm is infeasible for larger sized inputs. It is impossible to find optimal solutions for large size inputs. However, we can compare the fuel consumption rates of the algorithms in small sized inputs.

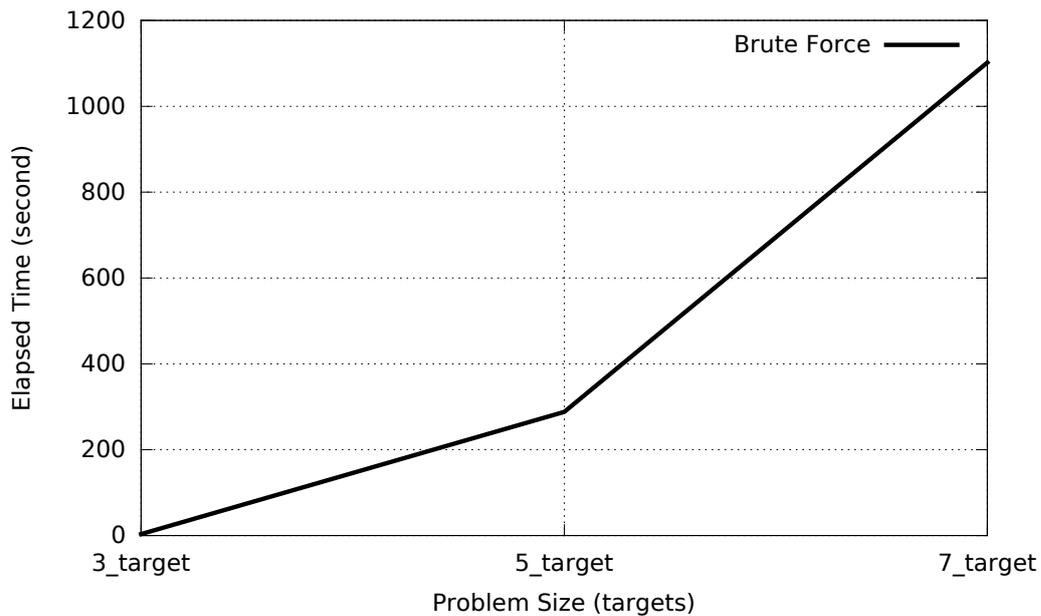


Figure 5.3: Brute force algorithm time consumption in different size of missions

In Table 5.2, all of the algorithms are compared for different test cases having 3 targets. BFA takes more time than others. Although the other four algorithms require acceptable times, HGA seems to be slower. For this data set GA could not find a complete scheduling for 3 of the cases. Here, targets are separate from each other.

When the time windows of targets are separate and there is some time between them the greedy choice made by the algorithm can end up with incomplete scheduling. For the other algorithms, finding a complete scheduling for this input set is possible. To analyze the fuel consumptions, the characteristic of the data set should be known at first. For 3 targets case, there is only limited scenarios: All the targets are separated from each other or they overlap. For two inputs target time windows fully overlapped. For these two scenarios SA and IH find optimal solution. For the separated cases, SA and IH acted like GA which was expected. HGA finds good solutions for this small sized input as expected. Because it covers most of the solutions space. Another key point to realize is SA and IH provide the same results. The elapsed times are also similar. The elapsed time differences between GA, SA and IH will be clearer with the larger inputs.

Table 5.2: All algorithms are compared on 3 target hand crafted inputs

Cases	BFA		GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
3_1	3,53	14,12	0,00	-	0,00	16,55	1,00	16,55	0,00	16,55
3_2	3,36	17,18	0,00	17,18	0,00	17,18	1,00	17,18	0,00	17,18
3_3	3,31	15,84	0,00	22,78	0,00	22,78	1,00	15,84	0,00	15,84
3_4	3,46	13,28	0,00	-	0,00	27,06	1,01	13,28	0,00	13,28
3_5	3,44	16,11	0,00	-	0,00	16,11	1,00	16,11	0,00	16,11

In Table 5.3, the algorithms are compared for test cases with 5 targets and 5 UAVs. Time consumption difference really starts to be realized. BFA terminates in nearly 5 minutes. Where the GA, SA and IH worked in 0,001-0,002 seconds interval. HGA is clearly slower. Solution qualities are similar to 3 target case. SA and IH gave the same results and some of the results are closer to brute force algorithm compared to GA. These are cases where the number of overlaps are higher. For the cases having targets in mostly separated situations the GA, SA and IH show similar behaviors. The HGA is again able to find closer solutions compared to BFA except one case. For that case, HGA failed to make a complete solution.

Table 5.3: All algorithms are compared on 5 target hand crafted inputs

Cases	BFA		GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
5_1	288,55	16,07	0,00	30,06	0,00	24,63	5,38	-	0,00	24,63
5_2	285,36	20,18	0,00	20,18	0,00	20,18	5,36	20,18	0,00	20,18
5_3	295,50	18,84	0,00	25,78	0,00	25,78	5,45	18,84	0,00	25,78
5_4	290,00	18,88	0,00	30,06	0,00	31,87	5,40	20,52	0,00	31,87
5_5	287,46	18,84	0,00	30,02	0,00	18,84	5,39	18,84	0,00	18,84

### 5.5.1.2 Comparison for larger sized inputs

In this subsection solutions of the algorithms compared with larger inputs. BFA solutions are not included in this part. To see the behaviors of the algorithms in a clearer way, graphics will be used to explain the solutions. The graphics will show the results for all problem sizes. The results of fuel consumption, elapsed time and completion ratio are analyzed separately.

**Elapsed Time:** In Figure 5.4, elapsed times for each algorithms for different input sizes is provided. This time data for each problem size is the average values provided by algorithms in each of the test cases with different problem sizes. Despite the good solutions it gave, HGA looks like too slow to be used for real time mission planning scenarios. However, the other three solutions looks fast enough to be used for mission planning scenarios. Another key point that can be observed from this output is, the time difference between SA and IH. They always find the same solutions but SA finds this in a faster fashion. The difference was not clear for small sized inputs but for the larger input sizes the difference is obvious. Speed of SA is similar to GA. The GA is slightly faster but it is not a noticeable difference for these sizes of inputs.

**Fuel Consumed:** For the small sized inputs HGA shown to find better solutions than the others. SA and IH find the same results and these results were better than GA for some cases. For the other cases, GA, SA and IH generate similar results. Figure 5.5 shows the fuel consumption results for bigger size of data. This graphic is created with the help of average fuel consumptions of each algorithm on each of the different sized test cases. For the bigger size data, HGA still finds smaller cost

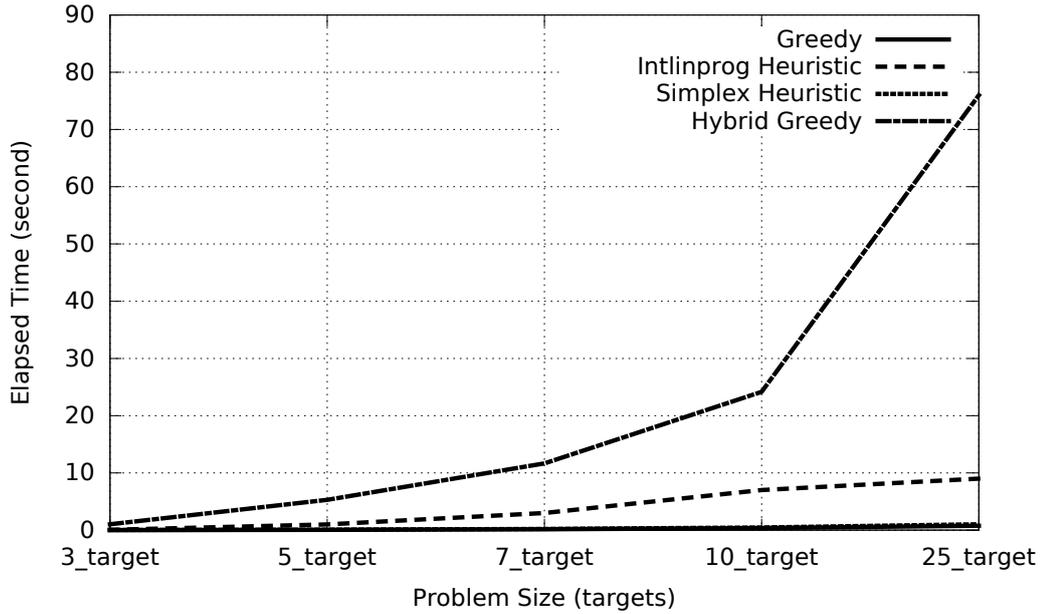


Figure 5.4: Time consumptions of the algorithms for different size of hand crafted inputs

results than others. The difference between GA and SA becomes more noticeable as the input size gets larger. The results of the SA is clearly better than GA's solutions for large input sizes. The consumptions of IH and SA are continues to be same as expected.

**Scheduling Completion Ratio:** Figure 5.6 shows the percentages of completions of the algorithms for each of the different sized test cases. The percentages are calculated according to the success rates in specific problem sizes. For the three target case the success rate of GA is lower than the other algorithms. The reason of this is explained in the previous sections. For the other input sizes, all of the algorithms have high percentages. We can realize that HGA and GA overall percentages are lower than SA and IH. These two heuristics seem to find more reliable solutions than the other two algorithms. Actually, one of the objectives was to find complete solutions for large sized problems. SA and IH seems to satisfy this need.

Finally, Table 5.4 shows some results generated by four algorithms on a data set with 20 targets and 5 UAVs. In terms of fuel consumption, the best results came by HGA.

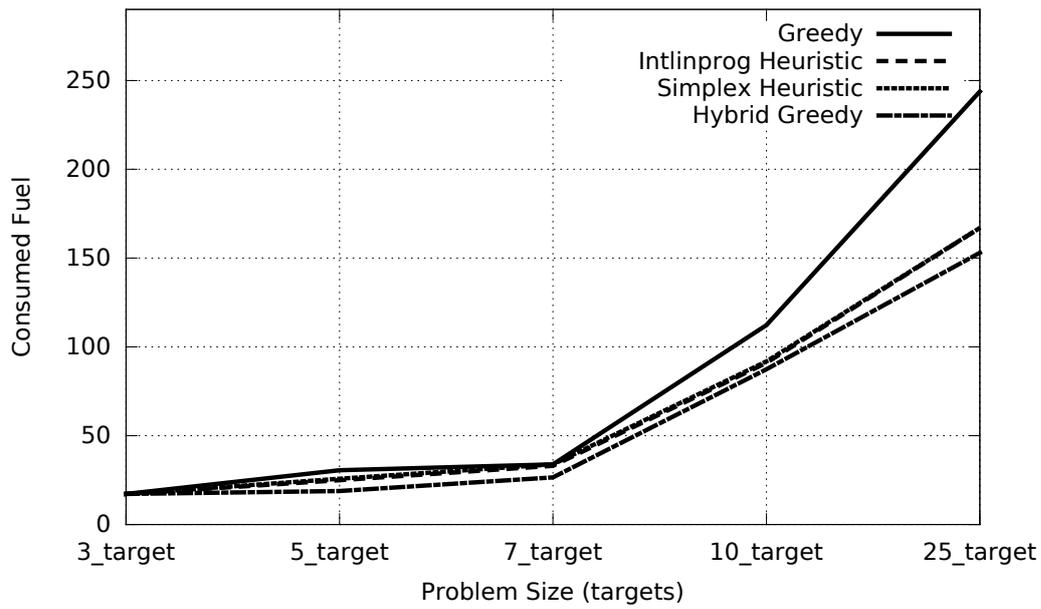


Figure 5.5: Fuel consumptions of the algorithms for different size of hand crafted inputs

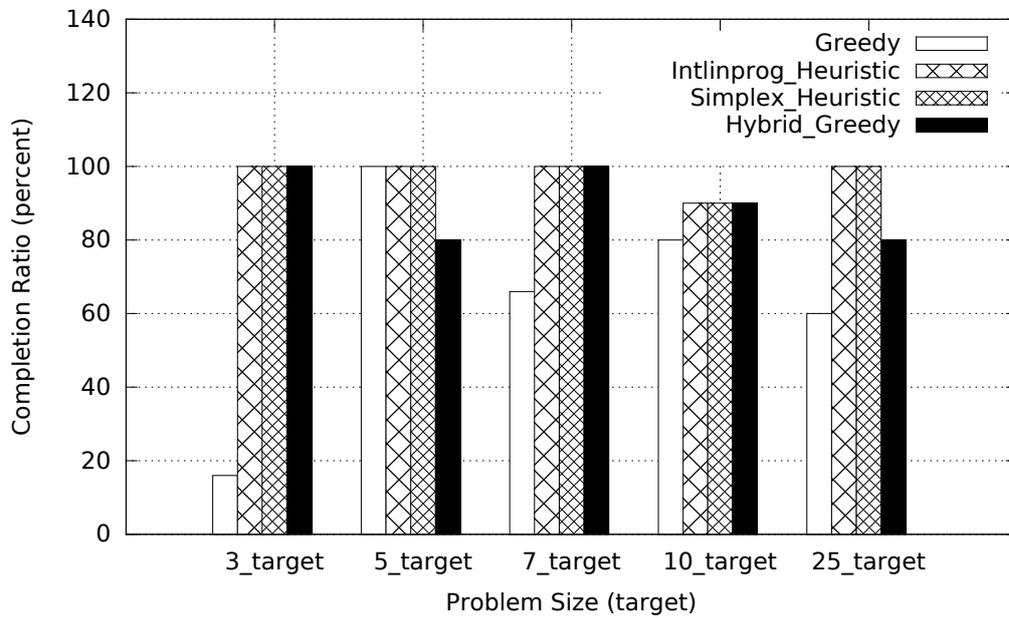


Figure 5.6: Scheduling completion ratios of the algorithms for different size of hand crafted inputs

SA and IH follow HGA. As the problem size gets larger greedy algorithm produces results having larger cost values than the others. The least elapsed times is due to SA and GA. IH is significantly slower than the other two algorithms. HGA starts to be infeasible after this point because of its combinatorial nature. For first two inputs, none of the algorithms could find a complete solution but it may be the situation that there are no complete solutions for that case. For the large inputs, we can not know whether there can be a complete solution available or not.

Table 5.4: Detailed comparison of algorithms in a large sized database for hand crafted inputs

Cases	GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
20_1	0,03	-	9,11	-	77,01	66,56	0,04	-
20_2	0,03	-	9,12	-	88,00	81,76	0,02	-
20_3	0,01	50,95	4,32	50,41	31,28	53,41	0,02	50,41
20_4	0,01	71,88	3,98	56,91	28,15	53,19	0,02	56,91
20_5	0,02	69,02	4,55	69,01	36,10	69,02	0,02	69,01

## 5.5.2 Randomly Generated Test Cases

Randomly generated test cases are used to be able to enlarge the solution space that is tested by the algorithms. User can specify the minimum and maximum time window to determine the size of the simulation time. User can also specify the maximum and minimum coordinates of targets to determine if the targets are close to each other or they are sparse in the area. Lastly, user can specify overlap ratio of the time windows to choose the type of the mission.

All algorithms are run on the different inputs and the results are analyzed in 2 parts. In the first part, we compared the results of the algorithms with the BFA. Then we compare the four of the algorithms with each other.

### 5.5.2.1 Comparison with brute force algorithm

In this part we compare algorithms with BFA on small sized inputs. Also we compare this results with the results obtained at hand crafted inputs part.

In Table 5.5, we see the results gained from 3 target size randomly created inputs. First thing the mention is, processing times of the algorithms. BFA is clearly slower than others with its results near to 5 seconds. On the other hand, HGA is also slower than SA, GA and IH. Compared to BFA, HGA produces faster solutions. The other three algorithms, provided similar results which are approximately 0,0001 seconds for each of them. Compared to hand crafted test cases, in these test cases all of the algorithms are worked a little bit slower. Main reason of this situation is most likely to be the sparse nature of the random inputs. In terms of fuel consumption, there is not much difference between the algorithms. HGA is able to find same results with the BFA. SA and IH algorithms find the same results as expected. Apart from one test case they always found the optimal solutions. GA could not find a complete scheduling for most of the test cases. For the only case, that GA worked completely, it managed to find the optimal solution. Finally, scheduling completion ratios are compared. For this input set, GA can only find one of the inputs. For the others it failed to find a complete scheduling. The input size is small and targets are sparse in the environment. For this kind of a situation, when GA made a wrong choice this causes it to fail immediately. On the contrary to other algorithms, have a wider scope of choices at decision points. This saved them from making incomplete scheduling.

Table 5.5: All algorithms are compared on 3 target randomly created inputs

Cases	BFA		GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
R_3_1	4,56	14,97	0,00	-	0,00	15,34	1,11	14,97	0,00	15,34
R_3_2	4,78	18,34	0,00	18,34	0,00	18,34	1,12	18,34	0,00	18,34
R_3_3	4,32	16,35	0,00	-	0,00	16,35	1,11	16,35	0,00	16,35
R_3_4	4,39	17,49	0,00	-	0,00	17,49	1,12	17,49	0,00	17,49
R_3_5	4,45	16,20	0,00	-	0,00	17,12	1,12	16,20	0,00	17,12

In Table 5.6, results of all algorithms for 5 target input size is provided. In terms of time, BFA starts to be infeasible at this point. Furthermore, HGA is slower than the other three algorithms with the processing time of nearly 6-7 seconds. The SA, IH and GA are still works near to 0,001 , 0,002 seconds which can be counted fast. Despite the processing times are really small, IH is slower than other two algorithms. To compare with hand crafted inputs, again all of the algorithms are processing slower. The next performance metric is the fuel consumption. Ranking in the fuel consump-

tions are similar to the hand crafted input results. Again the optimal results came from BFA. HGA follows BFA in terms of consumed fuel levels. Then SA and IH follows the other two. Finally, the most fuel consumption is made by GA. Despite the similar rankings with the hand crafted input case a change is visible in the fuel consumptions. For the randomly created inputs, the performance of SA and IH are increased. The fuel consumption results are really close to HGA results. The last performance metric to compare the algorithms is completed scheduling ratio. For 5 target problem size the mission completion ratio of the GA is again below the others. Furthermore, HGA also failed to make complete scheduling for two of the inputs. The SA and IH again produced reliable results by finishing in the all of the inputs.

Table 5.6: All algorithms are compared on 5 target randomly created inputs

Cases	BFA		GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
R_5_1	334,58	25,79	0,00	49,43	0,00	26,67	6,35	-	0,00	15,34
R_5_2	361,45	34,52	0,00	17,28	0,00	34,98	6,57	34,98	0,00	18,34
R_5_3	355,32	28,00	0,00	17,54	0,00	28,00	6,45	28,00	0,00	16,35
R_5_4	338,16	26,78	0,00	23,63	0,00	34,03	6,50	-	0,00	17,49
R_5_5	345,8	27,45	0,00	37,77	0,00	41,77	6,43	-	0,00	17,12

### 5.5.2.2 Comparison for larger sized inputs

**Elapsed time:** Figure 5.7 shows the elapsed times for four algorithms in test cases with 3, 5, 7, 10 and 25 targets. Although the results, are similar to the handcrafted inputs case, there are some changes. As a difference from handcrafted case, IH running time increased faster. This is probably because the sparse nature of the random inputs. Apart from that the results are similar to the handcrafted input case. The time curve of HGA grows much faster than others. The SA and GA are close to each other in terms of elapsed times.

**Fuel consumed:** Figure 5.8 shows the fuel consumption levels. For the randomly created input case, GA produces worse results than the hand crafted inputs. However, for the SA, the results are closer to HGA. We can say that SA and IH perform better for the random input cases. On the other hand, fuel consumption values of the HGA

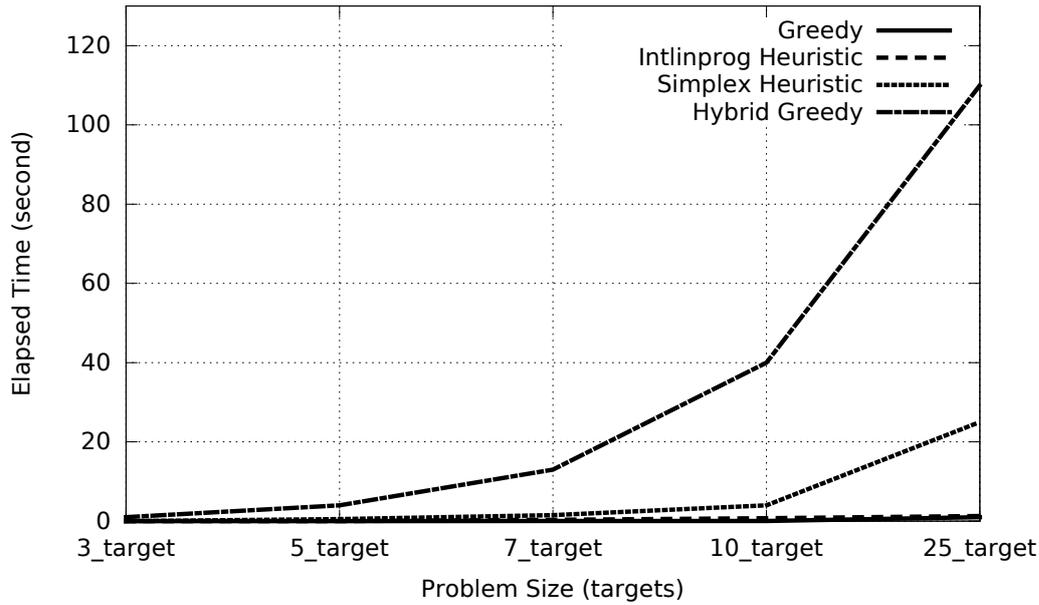


Figure 5.7: Time consumptions of the algorithms for different size of randomly created inputs

are still a little bit better than SA and IH.

**Scheduling Completion Ratio:** Lastly, in Figure 5.9 scheduling completion rates of the algorithms are provided. GA performs badly for the missions which have time windows that are sparse. Both GA and HGAs success ratios are falling down in the larger sized inputs. SA and IH algorithms again seem more reliable for larger sized inputs. Another interesting point to discuss about this results is for the 25 target case. The algorithms completion rates are under 40 percent. No complete scheduling would be possible due to the randomly created inputs but unfortunately there is no optimal algorithm to test this situation in a this size of a big input.

Lastly, we present a detailed results table for large sized inputs for randomly created inputs case in the table 5.7. In this table, we compared 4 of the algorithms with a 20 target size large input set. As the input size gets larger, all of the differences between the result of the algorithms becomes recognizable. HGA took more than 2 minutes to complete the mission planning. This is far too much for a real time job. Furthermore, this consumption can grow further with the larger problem instances. IH provided

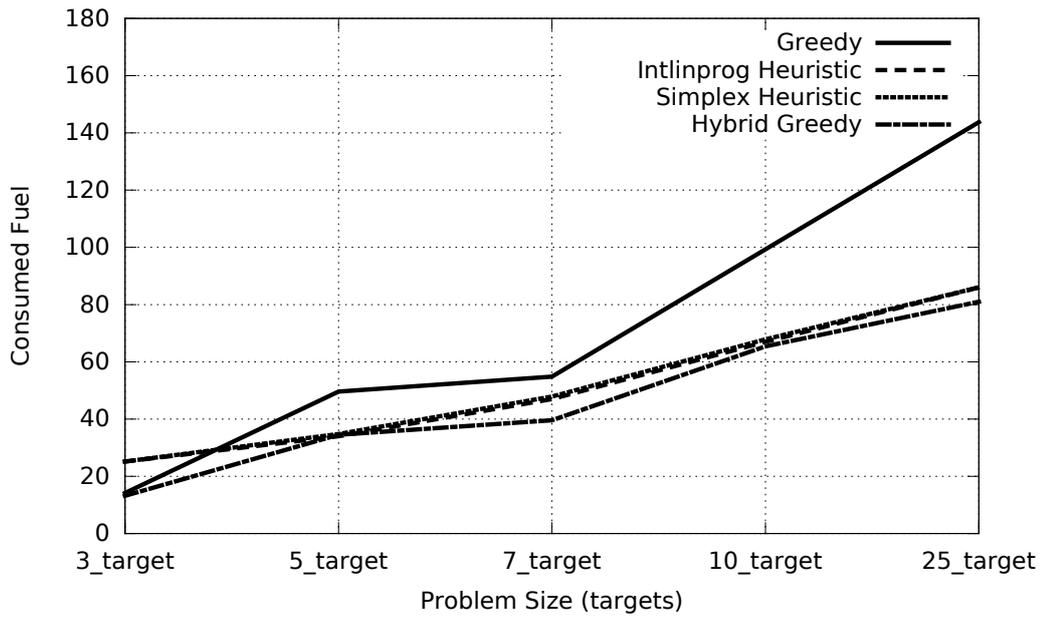


Figure 5.8: Fuel consumptions of the algorithms for different size of hand crafted inputs

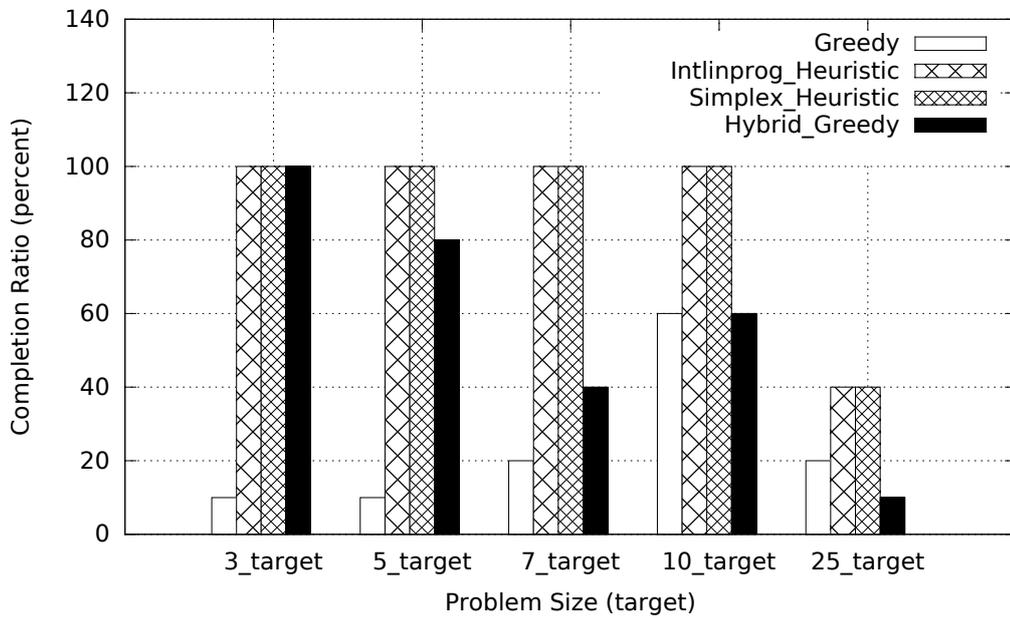


Figure 5.9: Fuel consumptions of the algorithms for different size of hand crafted inputs

also slow responses for this kind of a real time job. On the other hand, the velocities of GA and SA are similar and acceptable for real time missions. Completion rates of the methods are also interesting. For the first two inputs, none of the algorithms are able to provide complete solutions. This situation may be caused by the structure of the input, but it is impossible to determine this for this size of a input. For the reminder of the inputs, GA gave two results over three and HGA provide one complete solution over three inputs. On the other hand SA and IH provided complete solutions for all of the inputs. HGA can only find one complete solution but the fuel consumption level of it was the best among the other four of the algorithms, then SA and the IH comes. For each of the completed schedulings, they were by far better than the greedy algorithms in terms of fuel consumption.

Table 5.7: Detailed comparison of algorithms in a large sized database for randomly created inputs

Cases	GA		IH		HGA		SA	
	Time	Fuel	Time	Fuel	Time	Fuel	Time	Fuel
R_20_1	0,03	-	9,11	-	5,13	-	0,04	-
R_20_2	0,03	-	21,18	-	128,19	-	0,02	-
R_20_3	0,01	114,05	4,17	75,59	84,42	66,56	0,02	75,59
R_20_4	0,01	-	7,18	83,18	-	-	0,02	83,18
R_20_5	0,02	157,17	34,17	90,31	37,81	-	0,02	90,31

### 5.5.3 Comparison on Different Problem Structures

In this part we observe the behaviors of the algorithms on differently structured test cases. The structure of a test case, is determined by the overlapping rates of the time windows of the targets. We created 3 different data sets. The first data set, includes test cases that consists targets that have low overlapping ratios. In the second data set, we grouped the test cases which have medium overlapping ratios. Finally, in the last data set we have test cases having higher overlapping ratios. In the low overlapping ratio data set, test cases have overlap rates between 0 and 30 percent. Medium overlap data set have inputs having overlapping ratios between 30 and 70 percent. And the rest of the tests cases placed in the high overlap data set. For this part, both the hand crafted test cases and the random test cases are mixed and placed inside the data sets. In this part, our aim is to see the effect of problem structure

on the algorithms. Size of the input can change the performance of the algorithms drastically. To make a healthy measurement, we choose a fixed problem size and make the measurements in this size. While deciding the size, we examined the results for hand crafted and randomly created inputs. We choose a size in which there is enough complexity of the problem and most of the algorithms have high mission completion rates. Because of this reason, we choose a 10 target problem size. In the following sub sections, we examined the effect of problem structure with the help of elapsed time, fuel consumption and completion rate values. In the results, we used average values produced by the algorithms.

### **5.5.3.1 The effect of target formation on time**

In Figure 5.10, we can see an overview of the elapsed times of the algorithms for 10 target problem size. In the following paragraphs, we commented on the time behaviors of each different algorithms on the different problem structures.

**Hybrid Greedy Algorithm:** According to the figure, HGA spends most of its time on the problems having low number of overlaps. When the problem have low number of overlaps, the number of decision points increases. At each decision point, making a brute force search is costly. Because of that, this algorithm works slowly for low overlap rates. Another issue to mention is, the difference between medium overlap cases and high overlap cases. In the medium overlap cases, the time elapsed for HGA is low but for the high overlap cases the elapsed times increases compared to medium overlap cases. For the medium overlap cases, the situation is optimal for HGA. The number of decision points are low and the number of UAVs and targets that will exist in the transportation problem are likely to be low. This leads this algorithm to operate in a fast way. For the high overlap cases, number of decision points are even less. On the contrary, in each transportation problem the number of targets and UAVs are likely to be high which adds complexity to the transportation problems. For other algorithms, this kind of a complexity does not change the processing speed of the algorithm. Nonetheless, for the brute force algorithm it causes a recognizable speed reduction.

**Greedy Algorithm:** GA worked fast for all of the input types. The structure of the algorithm does not seem to effect it in terms of time.

**Intlinprog Heuristic Algorithm:** IH worked fast for the low overlap and medium overlap rates. For the cases having high overlap rates, the efficiency of the algorithm is lower compared to other two. In the cases consisting high level of overlaps, there is a possibility to have low number of decision points. Low decision points yields low number of and fairly large sized transportation problems. IH solves transportation problem but it is not a very fast approach. Hence, large sized transportation problems seems to be effecting the speed of IH.

**Simplex Heuristic Algorithm:** Similar to GA, SA did not effected much by the structure of the algorithm. For the high overlap cases, the size of the transportation algorithm increases. SA solves transportation algorithm fast. So, this increase did not effect the overall spent time much. Still, it is possible to recognize a small increase of time in the high overlap cases.

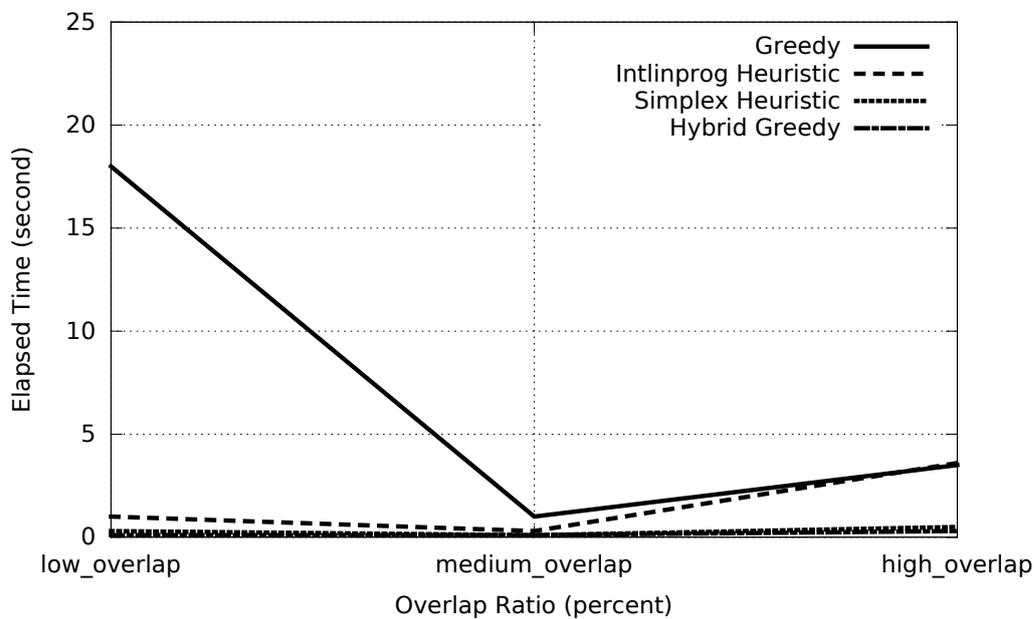


Figure 5.10: Figure showing a diagram showing time elapsed in seconds for each of the different problem structures for algorithms.

### 5.5.3.2 The effect of target formation on fuel consumption

In the Figure 5.11, fuel consumption levels of the algorithms are provided. In the figure, we can recognize that all of the algorithms are getting close to each other in the high overlap case. The reason of this is, the similar infrastructures of the algorithms designed. All of the algorithms in this thesis are implemented with similar infrastructures. Because of this, for the high overlap case they give similar responses. They can find different solutions to transportation problem but the number of transportation problems are low for this problem formation. This, causes them to find similar results. Apart from that, algorithms fuel consumption levels does not seem to be effected by the target formations.

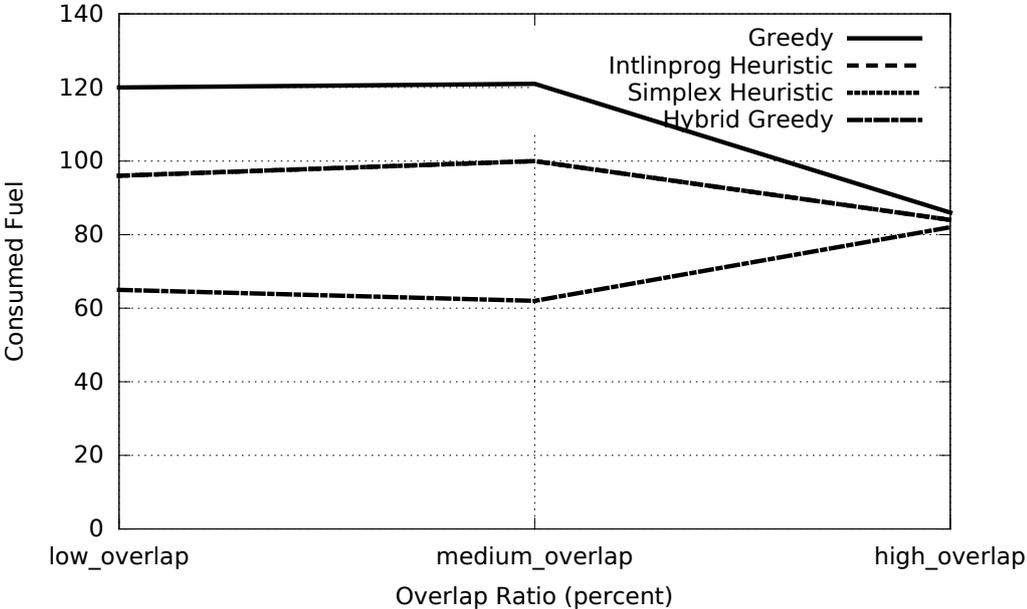


Figure 5.11: Figure showing a diagram showing consumed fuel levels of the algorithms on various different problem structures

### 5.5.3.3 The effect of target formation on mission completion rates

In Figure 5.12, scheduling completion rates of the algorithms on different target structures are provided. The results shows that, all of the algorithms are effected from the low overlap ratio. GA is effected more than the others. The reason is already ex-

plained before. When the targets are sparse and there are time differences between the time windows of the targets, the wrong choices of the GA ends up with incomplete scheduling. The reason of providing bad results for the low overlap rates is same with GA for all the other algorithms. Because, all the other algorithms implemented have greedy natures. After the dividing process, best results for the transportation problems are searched. This makes other algorithms a special version of the GA. So, sometimes they effected from the problem structure in the same way as GA.

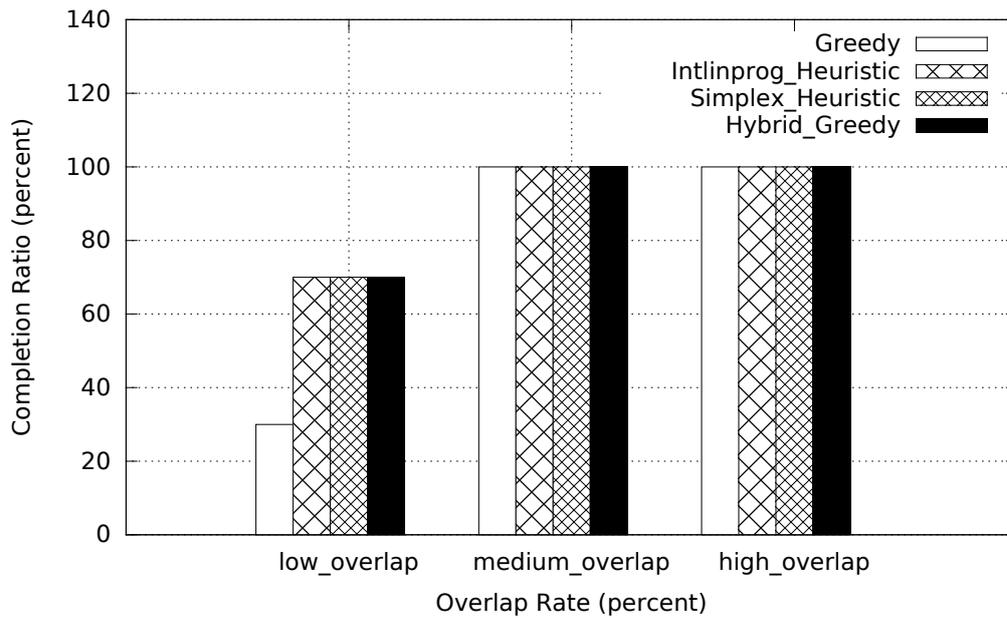


Figure 5.12: Figure showing a diagram showing scheduling completion rates of the algorithms on various different problem structures

The results shows that the best approach for this problem is SA. For small sized inputs, HGA finds results closer to BFA but for larger inputs, working time of the algorithm is too much. Furthermore, for large size inputs the ratio of finding a complete scheduling is low for HGA. GA is a faster choice but the fuel consumption values for the large size inputs is worse than the SA values. IH always finds the same results with SA but it is significantly slower than SA for large size inputs. SA, is the best alternative in terms of reliability efficiency and effectiveness.

## CHAPTER 6

### CONCLUSION AND FUTURE WORK

#### 6.1 Conclusions

In this thesis we worked on multiple UAV scheduling and target assignment problem. The main idea was to approach a divide and conquer type of idea. We wanted to divide this complex problem into multiple small problems which we know how to solve. So, the main consideration after this point was to how to divide the problem and then how to solve the sub problems. The constraint that effects the solutions most is the time windows for this problem. Targets' time windows defines a time period for each target and intersection amounts of the time windows affects' the possible complete schedulings for the mission. So, dividing procedure to group the targets according to their time windows is developed.

After making a division, according to time windows, the small problems created are started to look like transportation problems. For each of the clusters created, transportation problem is solved. In order to solve the transportation problem, 3 different approaches were created. The first one is the linear programming method. The second approach is solving the small problems with the help of simplex algorithm. Last approach is solving the small problem instances with brute force algorithm. Furthermore, to see the effectiveness and the solution quality of these 3 different methods a brute force and a greedy algorithm is also developed.

The five algorithms developed are tested with various different data. There were two main test beds. First one was the hand crafted inputs and the second class of test cases were randomly created inputs. The algorithms are tested with the inputs from

3 targets size to 25 targets size and evaluated according to 3 different performance metrics. These performance metrics are elapsed time, fuel consumption as a cost measurement and lastly the rate of complete scheduling to see the reliability of the algorithms.

Despite the fact that brute force algorithm always finds the optimal solutions it is too slow to be feasible. To be able to find the optimal results it tries to search a big part of the solution space. For this kind of a NP-Hard problem this yields a too complex solution. This problem is usually needed for real time applications and for a real time application this much running time is not suitable.

Greedy algorithm provides fast results independent from the size of the problem set. Although being fast is important for real time problems, the failure rate of the greedy algorithm is too much for a critical mission. Furthermore, for the large input sizes the quality of the solutions created by the greedy algorithm was lower than the other tried approaches.

Intlinprog heuristic is a similar method to our solution. The only difference from our algorithm was the solution method of the small transportation problems. This approach solves the transportation problems with linear programming. Linear programming is fast enough for solving one transportation problem but it is slower than simplex algorithm. As the number of small transportation problems increases this small difference between the working times gets larger.

After comparing the all of the implemented approaches with the data sets, we can say that the most suitable algorithm from this 5 algorithms for UAV mission planning problem is the simplex heuristic method. It provides fast, reliable and acceptable solutions in a fast fashion.

## **6.2 Future Work**

In this thesis an UAV mission planning problem with multiple constraints is solved. The main constraints were the fuel capacity of UAVs, capacity of targets, time windows of the targets and minimum cost. Some more constraints can be added to this

problem to make it more realistic.

In this thesis all of the UAVs were identical and there were no constraints for UAVs that limits their movement. This kind of constraints can be added to achieve more realistic UAV design. For instance, minimum segment length constraint, minimum turning angle constraint, maximum diving angle constraint and heterogeneous UAVs [33] are different ideas that can be added to achieve a more realistic UAV design.

Another idea can be to simulate a more realistic environment. For this study there were no constraints in the environment and the environment was 2D. In the environment there can be some obstacles or no flight zones or some critical regions. Furthermore a 3D environment design can be developed. These constraints can also be added to achieve a realistic environment design.



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