MULTI-ECHELON DYNAMIC CAPACITATED FACILITY LOCATION PROBLEM FOR THE RECOVERY OF WASTE

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submitted by **İSTENÇ TARHAN** in partial fulfillment of the requirements for the degree of **Master of Science in Industrial Engineering Department, Middle East Technical University** by,

Prof. Dr. Gülbin Dural Ünver ___________
Dean, Graduate School of **Natural and Applied Sciences**

Prof. Dr. Murat Köksalan ___________
Head of Department, **Industrial Engineering**

Assoc. Prof. Dr. Sedef Meral ___________
Supervisor, **Industrial Engineering Dept., METU**

Examining Committee Members:

Assoc. Prof. Dr. Ferda Can Çetinkaya ___________
Industrial Engineering Dept., Çankaya University

Assoc. Prof. Dr. Sedef Meral ___________
Industrial Engineering Dept., METU

Assoc. Prof. Dr. Seçil Savaşaneril ___________
Industrial Engineering Dept., METU

Assoc. Prof. Dr. Canan Sepil ___________
Industrial Engineering Dept., METU

Assist. Prof. Dr. Mustafa Kemal Tural ___________
Industrial Engineering Dept., METU

Date: ___________

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Name, Last name :

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ABSTRACT

MULTI-ECHELON DYNAMIC CAPACITATED FACILITY LOCATION PROBLEM FOR THE RECOVERY OF WASTE

Tarhan, İstenç
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Waste Electrical and Electronic Equipment (WEEE) Directive became a European Union Law in February, 2003. Turkey introduced an akin regulation in May, 2013 to give original manufacturers of electrical and electronic equipment the responsibility of making provisions for the collection and recovery of wastes. Due to the lack of existing infrastructure for recovery, studies related to recovery networks are supposed to increase in Turkey following the relevant regulation. In this study, we formulate a mathematical model for the dynamic capacitated facility location problem with two-echelons consisting of collection, consolidation and disassembly centers. The proposed model determines the locations and opening times of the centers, expansion of capacities as well as the transportation of returns from collection centers to disassembly centers through consolidation centers. Since the proposed model is difficult to be solved in reasonable times, we develop a heuristic approach decomposing the original problem into subproblems by the rolling horizon approach and invoking Lagrangean relaxation and variable neighborhood search in each subproblem. After solving all subproblems deriving from the rolling horizon approach, the original problem with the reduced solution space is solved the solution of which is the final solution of the proposed heuristic. The proposed heuristic approach is tested on a set of problems that we generate. The computational results show that the commercial solvers can be time-consuming even for moderate size problems whereas the proposed heuristic can find solutions for large scale problems with a reasonable optimality gap in much less time than the commercial solvers.

Keywords: Waste recovery network, WEEE, dynamic capacitated facility location, multi-echelon, rolling horizon approach, Lagrangean relaxation, variable neighborhood search

ÖZ

ATIKLARIN GERİ KAZANIMI İÇİN ÇOK SEVİYELİ VE DİNAMİK KAPASİTELİ TESİS YER SEÇİMİ PROBLEMİ

Tarhan, İstenç
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Atık Elektrikli ve Elektronik Eşyalar (AEEE) yönergesi, Şubat 2003'te Avrupa Birliği yasası haline geldi. Üreticilere, elektrikli ve elektronik eşya atıklarının geri kazanımı sorumluluğunu yükleyen benzer bir yasa, Mayıs 2013'te Türkiye'de yürürlüğe girdi. Geri kazanım için gerekli altyapısının yetersiz olmasından dolayı, ilgili yasayı takiben, Türkiye'de geri kazanım şebekesine ilişkin yapılan çalışmaların artması bekleniyor. Bu çalışmada; toplama, konsolide etme ve demontaj merkezlerinden oluşan iki seviyeli, arttırılabilir kapasiteli tesis yer seçimi problemi için bir matematiksel model geliştirilir. Söz konusu model, ilgili merkezlerin nerede ve ne zaman açılacağına, kapasite artırımlarına ve aynı zamanda toplama merkezlerindeki atıkların konsolide etme merkezleri aracılığıyla demontaj merkezlerine taşınmasına karar verir. Geliştirilen modelin makul süre içerisinde çözümünün zor olmasından dolayı, yuvarlanan ufuk yaklaşımı ile esas problemi alt problemlere ayıran ve her bir alt problem için Lagrange gevşetmesini ve değişken komşu esaslı arama metasezgiselini çağıran sezgisel bir yöntem geliştirilir. Yuvarlanan ufuk yaklaşımından doğan alt problemlerin tamamının çözümünden sonra, çözüm alanı daraltılmış orijinal problem çözülür ve önerilen sezgisel yöntemin nihai sonucu elde edilir. Önerilen sezgisel yöntem tarafımızca oluşturulan test problemleri üzerinde test edilir. Ticari bilgisayar yazılımları orta ölçekli problemlerde dahi çok fazla zaman harcıyor iken, önerilen sezgisel yöntemin büyük ölçekli problemler için çok daha kısa süreler içinde tatmin edici sonuçlara ulaştığı görülür.

Anahtar Kelimeler: Geri kazanım şebekesi, AEEE, dinamik kapasiteli tesis yer seçimi, çok seviyeli, yuvarlanan ufuk yaklaşımı, Lagrange gevşetmesi, değişken komşu esaslı arama

To my family

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CHAPTER 1

INTRODUCTION

In modern business management, individual businesses cannot operate as if they are isolated and no longer strive as completely autonomous entities; yet, they compete as supply chains (Douglas et al., 1998). Definition of supply chain can be accomplished in various ways. Christopher (2013) defines supply chain as the network of organizations associated with different processes/activities in connection through upstream and downstream linkages that is developed so as to produce value as product or service for end customers. Supply chain can also be described as the functions within and outside a company constituting a chain generating value in the form of products and services to the customers according to Cox (1995). All definitions of supply chains point to coverage of different activities in relation to produce value. Purchasing, production, marketing and logistics are some of the activities that are involved in most of the supply chains. Consideration of such activities as interacting with each other in a chain requires an integrative philosophy to manage the total flow of a distribution channel from suppliers through end user which can be called as supply chain management, SCM, (Martha et al., 1997).

A traditional supply chain covers the units from suppliers to retailers including plants, warehouses and distribution centers as well as the goods flowing among these units like raw materials, work-in-process inventory and finished goods. Raw materials are obtained from suppliers to be processed so as to produce goods in plants. Warehouses and distribution centers are the intermediate stages between plants and retailers to provide storage service, benefiting from economies of scale etc.

All stages/components along turning raw materials into products and handing them to customers can be included in a supply chain. The scope of supply chains and the interactions among the stages require an efficient SCM. La Londe and Bernard (1997) state that aims of SCM are to provide enhanced customer service and economic value via the consideration of the flows of physical goods and information from source to the consumption simultaneously. Supply chain consists of a broad variety of activities from strategic to operational level. Strategic level decisions have long-lasting effects and are associated with the number, location and type of facilities in the supply chain. Horizon of tactical level decision can be defined in months and includes decisions related to purchasing, production, inventory and transportation. Operational level decisions are regarded on daily or weekly basis such as scheduling and routing. An effective SCM can save a significant portion of the costs associated with all these decision-making levels.

Distribution network design problem (DNDP) is a fundamental part of SCM that aims at locating facilities effectively as well as determining the distribution scheme of the products in the network. DNDP includes both strategic and tactical level decisions and requires rigorous planning due to its long lasting effects.

DNDPs have long been studied in the operations research literature and cover a broad range from single-period, single-echelon deterministic linear models to multi-period, multi-echelon stochastic nonlinear models and continue evolving with new perspectives in the global industry and advancements in operations research. Studies in developing mathematical models and solution techniques for DNDPs date back to 1970s. At the beginning, DNDPs were fine tuning the logistics of products from raw materials to the end customer. Products are obviously still streaming in the direction of the end customer but an increasing flow of products is coming back due to product recovery, goods return, or overstock for a whole range of industries covering WEEEs, pharmaceuticals, beverages and so on (Brito and Dekker, 2003). A substantial part of product returns arises on the purpose of product recovery which bears a growing importance.

Returns of used products can be due to various reasons like defective products, end-of-life products, products not used by their owners anymore, etc. Different products in terms of these reasons and different views of companies bring along diverse product recovery options such as repair, refurbishment, remanufacturing, cannibalization and recycling (Thierry et al., 1995). The purpose of repair is to turn damaged products into "working order" status and it requires limited operations. The quality of repaired product is generally less than the quality of new products. Refurbishment, on the other hand, brings used products up to a specified quality and can be used to upgrade products by replacing out dated parts with superior versions. Unlike repair and refurbishment, remanufacturing brings used products up to some quality standards that are as rigorous as those for new products. Approved parts and products are disassembled, and then repairable products are fixed and assembled into remanufactured products. The purpose of cannibalization is to recover a limited set of reusable parts to supply components to repair, refurbishment and remanufacturing. Recycling, on the other hand, is to reuse materials from used products, and with recycling identity of used products is lost unlike other recovery options. Recycling appears as the last option in general. Used products which are not convenient for recovery are disposed by landfilling or incinerations. It is quite reasonable to believe that used products have still some value within them in many cases when these various recovery options are considered. Schema of product recovery options by Thierry et al. (1995) can be seen in Figure 1.1.

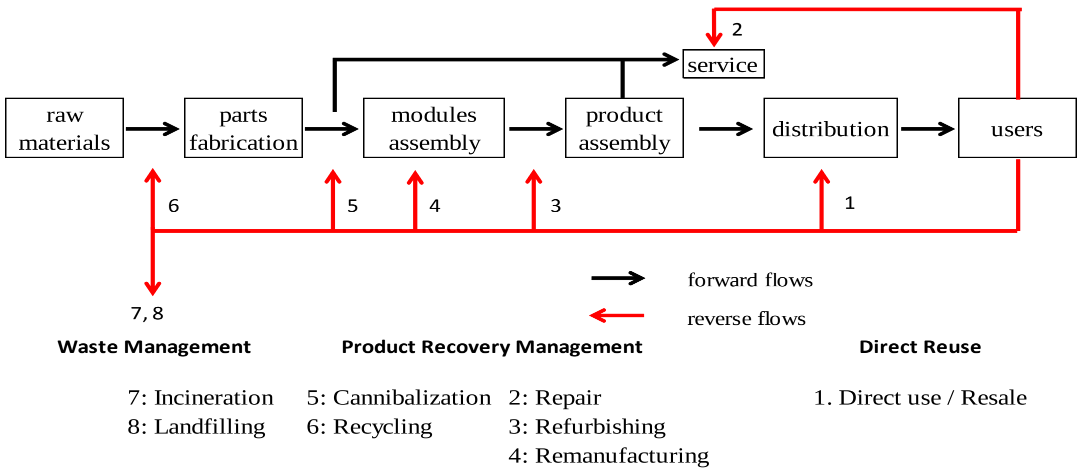

Figure 1.1. Schema of recovery options

Main drivers of the product recovery are stem from commercial reasons which are directly related to environmental concerns of the society, economic lucrativeness and regulative restrictions by governments. Deterioration in the environment has increasing the environmental consciousness of people that tend to prefer companies caring of the green world. In addition to the commercial side, recovery can be economically feasible since product recovery options can be economically rewarding.

Such reasons forced the companies to consider recovery of used products more seriously. Another reason behind the increasing concern for product recovery is regulative restrictions set by governments. One of the most prevalent restrictions is arising from WEEE-Directive of European Union (EU), which became an European Union Law in February, 2003 together with RoHS-Directive (Restriction of Hazardous Substances Directive). Main incentive of the relevant law is to put burden on producers of WEEEs to recycle used products and satisfy recycling targets which are set by EU. Although the origin of WEEE-Directive is EU, non-members of EU also set similar obligations for WEEE producers (Oliveira et al., 2012).

As the efforts for product recovery increase, it brings about a new realm: Product Recovery Management (PRM). PRM encompasses the management of all used and discarded products, components, and materials that fall under the responsibility of a manufacturing company in the first place. The objective of PRM is to recover as much of the economic (and ecological) value as reasonably possible, thereby reducing ultimate quantities of waste. Recovery alternatives like repair, refurbishing, remanufacturing, cannibalization and recycling are assessed in this perspective (Thierry et al., 1995).

Increased attention to product recovery practices has extended the scope of traditional supply chain management by drawing attention to collection, demanufacturing, and remanufacturing operations, i.e. the reverse channel which consists of final-users, collectors, demanufacturers, and remanufacturers (Üster et al., 2009). As a result, PRM has engendered a new kind of logistics called "Reverse Logistics" which is defined as the movement of products or materials in the opposite direction for the purpose of creating or recapturing value, or for proper disposal (Tibben-Lembke et al., 2002). Fleischmann et al. (1977) point out that reverse logistics is not necessarily a systematic picture of forward distribution. In Table 1.1, differences between forward and reverse logistics can be seen (Tibben-Lembke et al., 2002).

As it is seen in Table 1.1, reverse logistics is mostly less determinant, visible, consistent and smooth when compared to forward logistics, and product recovery entails a more complex logistics than traditional downward flow of products which in the end leads to the requirement for an efficient PRM. Thus, researchers and practitioners in this field encounter with non-identical networks established to get over obstacles and fuzziness depending on the recovery context.

Table 1.1. Comparison of forward and reverse logistics

Forward Logistics	**Reverse Logistics**
Relatively straightforward forecasting	More difficult forecasting
One to many transportation	Many to one transportation
Uniform product quality	Non-uniform product quality
Uniform product packaging	Damaged product packaging
Clear destination	Unclear destination
Standardized channel	Exception driven
Clear disposition options	Unclear disposition
Relatively uniform pricing	Dependent pricing on many factors
Importance of speed	Low priority of speed
Easily monitored costs	Less visible costs
Consistent inventory management	Inconsistent inventory management
Manageable product lifecycle	More complex product lifecycle
Negotiation between parties straightforward	Complicated negotiations
Marketing methods well-known	Complicated marketing
Real-time information available for tracking	Less transparent visibility of processes

M. Fleischmann et al. (2000) describe the differences between product recovery networks and traditional production-distribution networks, and the commonalities among recovery networks. The major differences between product recovery networks and traditional networks appear to arise on the supply side. In traditional production-distribution systems, supply is typically an endogenous variable in the sense that timing, quantity, and quality of delivered input can be controlled according to the system's needs. In contrast, supply is largely exogenously determined in product recovery systems and may be difficult to forecast. Hence, supply uncertainty appears to be a major distinguishing factor between product recovery and traditional production-distribution networks. Following the distinguishing factors of recovery networks, authors split variables concerning recovery situations into three categories: (i) product characteristics, (ii) supply chain characteristics and (iii) resource characteristics.

As long as the supply chain characteristics are concerned, recovery networks differentiate in:

- degree of centralization
- number of levels
- links with other networks
- open vs closed loop structure
- degree of branch co-operations

Recovery context drives the density of the network, width of the entire chain, direction of flows and the parties responsible for setting up the network, as network design practices reveal how a recovery network varies depending upon context.

As it is stated above, forward and reverse supply chains have distinct characteristics. Yet, it can be rewarding to integrate forward and reverse supply chains if additional value can be created from returns by recovery options like refurbishment, remanufacturing and reuse. In such integrated supply chains which are called as closed loop supply chains (CLSC), forward supply chain does not end with customers. Returns are collected in CLSC and additional value from returns can be captured by demanufacturing them to material/component level, refurbishing or reusing them (Guide and Van Wassenhove, 2009). Then, valuable part of returns can be reintroduced into the forward supply chain. Unlike the bidirectional flows in CLSC, there is one direction flows in traditional/forward supply chains. Similarly, pure recovery networks include only the collection, hence processes required to recover returns and reintroduction of recovered parts into forward supply chains is excluded. Such one directional supply chains are called as open loop supply chains (OLSC).

In most of the recovery practices, collected returns proceed along traditional open loop recovery networks which consist of collection centers, sorting/consolidation centers and treatment centers (Cahill et al., 2010). Regardless of return collection policies and network managers in charge, collection centers are common in recovery networks, since the volume of wastes is high and product recovery networks spread out to a large spatial scale. Bounds of recovery networks can range from an administrative district to a country. In the case of a large scale network, returns at collection centers can be consolidated at consolidation centers which can also be used for the execution of operations like sorting, inspection and pre-processing (Fleischmann et al., 2000). Consolidation centers are succeeded by disassembly centers where removal of the components and specific materials from products are executed (Sodhi and Reimer, 2001). Hazardous, useless or economically unviable components/materials are sent to disposal areas and the rest of waste is sent to recovery centers.

Invocation of WEEE Directive in European Law in February, 2003 has increased people’s concern for the recovery of WEEEs. As many other non-EU countries, Turkey has introduced an akin regulation in May, 2013 and rendered producers responsible for the recovery of WEEEs. Turkey is lack of an adequate recycling infrastructure and this regulation is supposed to lead to investments in the construction of recovery networks (Kilic et al., 2015). It is believed that uptrend interest in the recovery of WEEEs along with new regulation will lead to an increase in the studies on the design of product recovery networks in Turkey.

In this study, we have regarded the problem of constructing open-loop product recovery network on a national-scale determining the best sites for consolidation centers and disassembly centers, capacity expansion decisions related to disassembly centers along with the allocation of returns from collection centers to disassembly centers through consolidation centers in a long multi-period planning horizon. Then, we can classify the problem as Multi-Echelon Dynamic Capacitated Facility Location Problem (MEDCFLP) which is a special type of Facility Location Problems (FLPs), and a class of Network Design Problems (NDPs). The objective of the relevant problem is to minimize the total cost comprising of the costs of facilities and transportation cost.

In Chapter 2, FLP literature is reviewed and relevant studies in the literature are classified based on the number of echelons and the number of periods.

In Chapter 3, problem environment is discussed and a mathematical model is formulated for the MEDCFLP. Assumptions related to the problem context, decision variables, parameters and constraints are explained in detail.

In Chapter 4, the rolling horizon approach (RHA) nesting lagrangean relaxation (LR) which calls for the variable neighborhood search (VNS) in the end is proposed. Implementation of the proposed solution procedure is described regarding MEDCFLP.

In Chapter 5, the procedure of test instances generation is displayed. Computational results of the proposed solution procedure are evaluated based on the performance measures presented in this chapter.

In the final chapter, Chapter 6, the conclusions are stated along with the suggestions for further research.

CHAPTER 2

LITERATURE REVIEW

Recovery network design practices have been arising since economic incentives, regulative acts and environmental concern boost and enforce recovery. In the meantime, literature associated with recovery networks continue expanding and efforts in this direction make NDP literature, and MEDCFLP literature as well, progress and increment the scope of the problems like classical plant location problems which have been studied extensively for years and a wide literature exists on.

M. Fleishcmann et al. (2001) give examples of prevalent recovery network design practices. One of the prevalent network design practices is conducted by Louwers et al. (1999). The design of a large-scale European recycling network for carpet waste is developed by the cooperation of some chemical companies with the European carpet industry. Recovery opportunities for valuable resources like nylon fibers and the restrictive environmental regulations turned out to be the main drivers for this project. Through the network, used carpets are collected, sorted, and preprocessed in regional recovery centers to create value from their materials. Ammons et al. (1997) set up a similar network with a chemical company concerning the collection of used carpets, processing of collected carpets by separating reusable materials, and a remainder to be landfilled, and end-markets for recycled materials. These two papers examine the conditions under which recovery is economically viable, and recognize the volume of returns as the major factor in this scope.

The electronics industry is one of the most prominent sectors in product recovery. Many original equipment manufacturers (OEM) start taking back and recovering their products. In this context, several copier manufacturers reconsider their logistics networks (Krikke, 1998). Given an existing forward distribution network, logistics structures for reverse channel functions such as collection, inspection, and remanufacturing are investigated. Similar issues arise for computer manufacturers (Berger and Debaillie, 1997). On the other hand, electronics product recovery may also be attractive for the specialized third parties, such as the example of a U. S. cellular telephone remanufacturer (Jayaraman et al., 1999). In this case, a new logistics network is developed including collection, remanufacturing and redistribution activities. Reusable packaging is another important area of product recovery. A logistics service provider in the Netherlands considers a logistics system for reusable plastic containers that are rented as transportation packaging (Kroon and Vrijens, 1995). To this end, the number and locations of depots for storing empty containers need to be determined. In the Netherlands, the design of a sand recycling network is considered by a consortium of construction waste processing companies (Barros et al., 1998). Since sand from processing demolition waste may be polluted, it needs to be inspected and possibly cleaned before being reusable, e.g., for road construction, and authors design a logistics network comprising of both cleaning facilities and storage locations. In the German steel industry, a recycling network for production residuals is discussed on a branch level (Spengler et al., 1997). By-products of steel production need to be recycled in view of the extended environmental regulation and increasing disposal costs. Therefore, processing facilities need to be installed allowing by-products to be reintegrated in the steel production process or sold as secondary materials to other industries. Achillas et al. (2011) study a real world case in Greece to optimize the electronics products' reverse logistics network where scenarios to be tested are generated via collaboration with producer compliance scheme (PCS) and the proposed model is proven to be cost-efficient in the perspective of decision maker of the problem -government and local authorities- when considering historical data related to recovery of WEEEs in Greece. Similar to the study of Achillas et al. (2011), it is a frequent practice to develop a network on the national-scale, since all manufacturers in the nation are exposed to the regulations ruled by the government. In this aspect, Wagner (2009) examines Maine's program, which was the first US state to mandate producer responsibility for recycling household e-waste and finds out that shared recovery cost responsibility among producers, municipalities, and consumers results in a significant reduction in disposal, and also a corresponding increase in the number of environmentally friendly products. Walther and Spengler (2005) study the impact of WEEE-directive in Germany, and a model for the assessment of existing infrastructure in regard to future scenarios of WEEE treatment is developed. Performance of the existing infrastructure for the satisfaction of recovery and recycle targets in different future scenarios is evaluated.

Although recovery network design practices briefly stated above broaden the range of the literature, conventional structure of FLPs remains standing in those practices. As a result, classification schemes for FLPs are still valid and can be applied to recovery network problems as well as other facility location problems. General structure of classification scheme by Hamacher and Nickel (1998) has five positions that represent in order: the number and type of new facilities, decision space type (i.e. discrete or continuous), particularities of specific location problem (i.e. capacity restriction), relation of new and existing facilities (i.e. distance matrix) and description of objective function. Arabani and Farahani (2012) develop a more detailed scheme where static-dynamic discrimination divides the entire literature into two groups each of which is divided into sub-groups based on the objectives of the problems (i.e. p-median, p-center, allocation problems). Min et al. (1998) develop their own taxonomy and classification schemes for Location-Routing Problems (LRP). Their classification scheme is valid for FLPs as well and involves a more comprehensive scheme regarding both the problem characteristics and solution methodologies when compared to other classification schemes for FLPs or LRPs. Authors classify problems with regard to problem perspectives and solution methods. One of the problem perspectives is associated with the number of layers/echelons that the networks consist of. Nature of demand and supply is the second class since it would determine the problem to be either deterministic or stochastic and influence the way problem is handled significantly. Number of facilities as well as layers is used for further categorization. Some classes like size of vehicle fleets, vehicle capacities and time windows can be ignored due to the absence of routing consideration in FLPs. Features of facilities as capacitated/uncapacitated and primary layer/intermediate layer, number of periods in the planning horizon, type of the objective function and model data are used to subcategorize problem perspective classes as well. Solution methods are classified as either exact or heuristic algorithms in a generic form. Although FLPs are NP-hard, some exact procedures are proposed for the solution of FLPs, but these methods are applicable for limited sized problems in general. Nevertheless, exact algorithms -MIPs solved by optimization softwares in particular- are not rare in the literature likewise Lagrangean relaxation, metaheuristics, Benders Decomposition (BD) etc.

As it is defined at the beginning of Chapter 3 in detail, the problem environment in this study is a multi-echelon dynamic capacitated facility location problem. However, the difficulty of matching our study to the studies in the literature exactly leads us to discuss slightly different environments related with them. In this regard, studies reviewed are placed under four sections based on the two attributes below:

1) Number of echelons (single or multiple)
2) Number of periods (single or multiple)

2.1. Single-Echelon, Single-Period FLP

The simplest class of FLP has a single echelon and location decisions are made considering a single period. A further simplification is the assumption of uncapacitated facilities. Although single-echelon, single-period uncapacitated FLP (UFLP) is simpler than the other classes of FLP, there are remarkably many examples of networks belonging to the relevant network class. Similarly, there are many reverse logistics problems that belong to the class of UFLP. UFLP can be viewed as a basis for more complicated facility location problems.

Le Blanc et al. (2004) offer a UFLP model for Auto Recycling Nederland (ARN) which is a branch organization for the collection and recycling of end-of-life vehicles (ELV) in the Netherlands. Researchers used the existing collective scheme of the organization to incorporate transportation side and discussed a redesign of the LPG-tank recycling network. MIP model is used to select depot locations (to degas LPG) and allocation of ELV dismantlers to the depots. Since intervals of collection are known, researchers incorporated vehicle routing problem (VRP) into the model and turned UFLP into LRP. VRP is solved a priori to the model by calculating transportation cost for each ELV dismantler and each depot, that becomes input for the optimization model. In VRP, a heuristic combining nearest neighborhood heuristic and local search techniques examining the movement of a location from one route to another are used as the solution procedure. The network designed by the authors is put into practice in the following months.

Min et al. (2006) propose a mixed integer, nonlinear programming model (MINLP) developed for an e-business company to determine the desirable holding time for consolidation of the returned products into a large shipment which is one of the few studies addressing the time for consolidation of the returned products. Tradeoff in storing products at the initial collection points through using a discount rate according to the shipping volume between the initial collection points and the succeeding layer that is the return centers. An upper bound on the distance between a customer and its nearest collection point is set to ensure return of products. A genetic algorithm (GA) is employed to solve the model (Min et al., 2006). Chromosome with single dimensional array, binary tournament method for parent selection, inversion of each gene with some small probability for mutation and fitness function derived from the objective function with a penalty term due to capacity violation characterize the GA in this paper.

Sahyouni et al. (2007) develop a generic facility location model for CLSC integrating different stages of a product's life cycle. Based on three life cycle stages -introductory, maturity and decline-, researchers propose three alternative network models. In the introductory stage, forward distribution network is dominant since returns are limited at the beginning. There are hybrid and stand-alone forward facilities, and there cannot be any facility dealing with reverse flows solely in the corresponding model. In the maturity stage, both hybrid, forward-related and reverse-related facilities are allowed. In the decline stage, returns have a significant portion of total flow. Therefore, the network model becomes reverse dominant which offers two types of facilities: hybrid and stand-alone reverse facilities. Researchers prefer using Lagrangean relaxation as the solution methodology through relaxing the constraints assigning forward and reverse demand points to facilities. Authors also propose a network similarity metric, basically quantifying the differences between any two network models by capturing the total distance between them, in order to measure to what extent alternative models differ from one another.

Hansen et al. (2007) provide an example of primal-dual algorithm nesting variable neighborhood decomposition search (VNDS) to solve simple plant-location problem (SPLP). Authors use VNDS to find the solution of primal problem. In shaking phase of VNDS; add, drop or exchange move is selected depending on dedicated probabilities to generate a new solution through reduced variable neighborhood search (RVNS). After shaking step using RVNS, algorithm proceeds to local search step; at the beginning of local search, the number of open facilities of incumbent solution that is fixed in local search step is set to l which points the decomposition phase of VNS. Open facilities of incumbent solution to be fixed are determined based on the minimum distances among open facilities. After fixing l open facilities in incumbent solution and assignments to them as well, new facilities are determined to be opened to ensure that all users are assigned to a facility based on a subroutine resting upon closest distances between users and facilities. If the number of facilities in the new solution is less than a certain number, m, local search is achieved by VNS; if it is greater than m but less than n, RVNS is deployed for local search. Otherwise, the number of open facilities from the incumbent solution to be fixed becomes l_{min}. If best solution is improved after local seach or l gets equal to l_{max}, l returns to l_{min}; else, l increases to $l+1$. This procedure continues until a pre-determined time limit is exceeded. To derive a lower bound on the primal problem, restricted dual solution counting on integer-friendliness of strong LP relaxation is solved to find an initial dual solution. Sliding Simplex is run to reach the exact solution of dual primal from the initial dual solution. If best solution is not improved after a certain number of iterations, VNDS proceeds to branch and bound algorithm (B&B) to be able to find optimal solution. The proposed algorithm advances the record of the largest size SPLP optimal solution of which can be found back then.

Aras and Aksen (2008) assume that a drop-off strategy for returns is in place in their model determining locations of collection centers. The proportions of product holders who return products are dependent on both incentive price, which is a decision variable in the model, and distances to collection centers. Amount of possible returns is derived from demand history of householders. Proposed model considers the tradeoff between the positive value of returns, which is assumed to be equal to unit cost saving from a return less the unit variable cost for a return and the incentive price offered. The problem addressed is called as collection center location problem (CCLP) and it is a MINLP. The model is solved by SBB solver in GAMS which performs B&B. Yet, Aras and Aksen also propose a heuristic employing nested Tabu Search (TS) and Fibonacci search methods. Fibonacci search is used to find the best incentive value in the inner loop and TS finds the optimal solution in outer loop.

Rivera and Ertel (2009) analyze different scenarios based on the target collection rates of e-waste products in Mexico, specifically automobiles, although Mexico is suffering from the lack of e-waste recovery regulations which is the driver of the studies like the one by Achillas et al. (2010). All scenarios are tested via UFLP formulation and solved through the facility location software SITATION (Daskin, 2006).

Although UFLP is one of the most studied logistics problems, it can fail in representing the reality in the presence of capacity limitation which eventually leads to the studies for the capacitated facility location problems (CFLP).

Louwers et al. (1999) works with a continuous decision space for the locations of facilities serving to re-use of carpet materials. A heuristic solution is proposed to overcome the complexity since continuous decision space increases problem size excessively. In the first step of the heuristic, estimated number of facilities and their capacities are calculated where facilities with higher capacities are given a higher priority. Locations of facilities are determined in the succeeding step and allocations are made with a greedy method in the final step. Stepwise moves related to capacity and location decisions are employed to derive new solutions from the final solution in each iteration.

Sambola et al. (2007) solve one-echelon capacitated plant location problem with distance constraints. This study is a more-complex version of a simple CFLP. Customers assigned to a particular plant should also be assigned to vehicles associated with the corresponding plant and sum of distances from customers that share the same vehicle to the plant they are assigned to should be under a specific limit; this is the differentiating part of the study from a standard capacitated plant location problem. In the first step of the proposed algorithm, a constructive heuristic is developed that indexes plants with respect to the maximum number of customers can be assigned to relevant plants based on the demand and the distance of the customers, and associated costs with opening those plants. Set of the open plants is determined based on the indices and assignment of customers to the open plants and vehicles are made basically in ascending order of the associated cost with assignments. Following the constructive heuristic, TS is invoked to develop the constructed/initial solution that includes a variety of neighborhoods regarding status of plants, customers' assignments to plants and vehicles. Proposed TS is built in layers; outer layer deals with the status of plants, intermediate layer deals with the assignments of customers to plants and the last layer is related with the assignment of customers to vehicles. The proposed solution procedure achieves an optimality gap varying from 0.05% to 8% in reasonable solution time.

Kara and Onut (2010) proposed a modest network with two layers: customers and recycling centers. Recycling is the only recovery option because of the product on hand that is paper. The decision maker appraises profitability of collecting the returns available based on the amount of paper demand in regard of the investment cost for recycling centers. Uncollected returnable products are penalized in the objective function that maximizes net revenue. Number of recycling centers to be opened, as well as the capacities of them, is limited. Kara and Onut (2010) use both two-stage stochastic programming and max-min robust optimization to include the randomness in the returns' volume. They show that the proposed model is capable of solving a realistic recycling problem in İstanbul, Turkey. Each approach addresses a different perspective. A risk-averse may choose robust optimization to use whereas a risk-seeker may prefer stochastic programming.

Rahmaniania et al. (2012) propose a VNS heuristic to solve a single-echelon single-period CFLP. Robustness constraints are included in the study and the objective is to minimize the worst case cost. Instead of shifting to the next neighborhood when current one fails at improving the best solution, they choose to use acceptance probabilities like in Simulated Annealing (SA). Once a new solution is generated, it can be accepted as the incumbent solution with a specific probability. Likewise SA, cooling function is included in the model which reduces the probability of accepting non-improving solutions as the search proceeds. Computational results show that the proposed algorithm achieves 2.66% gap in 849 seconds, while CPLEX could obtain 4.39% gap in 2297.36 seconds.

2.2. Single-Echelon, Multi-Period FLP

FLP is a strategic-decision oriented problem while tactical and operational decisions can be included too. Thus, it can be worthwhile to anticipate the changes in the future and to regard possible investments to be done on future dates in a strategic perspective. Therefore, it is prevalent to have multi-period planning horizon in FLP.

Shulman (1991) develop a Lagrangean Heuristic for CFLP with discrete expansion sizes. Demand satisfaction constraints are chosen to be relaxed and Lagrangean dual problem is decomposed into subproblems, one for each discrete location. At this point, "discrete location" term should be clarified. The relaxation of constraints ensuring that demands are met is common in the literature since it enables to handle each facility independently. Since Shulman (1991) formulates a model that allows for multiple facilities types of which may vary on a particular discrete location, a discrete location can host more than one facility. Subproblems resulting from the relaxation determine the numbers and types of facilities to be opened as well as the periods to open them for each discrete location. He chooses dynamic programming (DP) as the solution tool for the subproblems. Two different heuristics are proposed to generate a feasible solution from the Lagrangean dual problem. The first heuristic exploits both the location and transportation variables of the subproblems, while the second one takes only the location variables into consideration. The first heuristic, which classifies demand nodes as under-served, served and over-served based on the Lagrangean dual solution and adjusts product flows in a greedy manner built upon the stated classifications, outperforms the second heuristic.

Realff et al. (1999) come with a network designated for carpets with the knowledge of material composition details of carpets, processes required to recycle carpets and their precedence relations. They investigate the optimal structure of the reverse production system where a collection site can be allowed to execute all operations in the reverse production system. Although the proposed model determines the locations of only collection sites, capability options like inspection, treatment, etc. that exist for each candidate site lead to a non-pure and diffuse single echelon network. They work with the maximization type of objective function including end uses of materials.

Canel et al. (2001) propose a heuristic algorithm employing DP, B&B and Delta & Omega Rule (DO) (Akinc and Khumawalat, 1977) to make decisions on opening, reopening and closing facilities in a multi-period horizon. In the first phase of the algorithm, DO determines which facilities should be opened, closed or reopened. In the second phase, B&B produces candidate facility configurations regarding the set of open/close/reopen facilties given from phase 1. Configuration alternatives are evaluated in the third and final phase by DP. Computational efforts show that if there are dominant facilities in the problem, the proposed algorithm is quite efficient since success of the first phase, having a crucial role as the predecessor of the second and the third phase, is highly dependent on such facilities.

Antunes and Peeters (2001) study the school network planning in Portugal and Simulated Annealing (SA) is performed to solve the proposed model, as capacity exchange moves on facilities selected randomly are executed in each iteration. Performance of the proposed model is compared with the add & interchange (ADD) local heuristic method and, as the problem size increases, the proposed model outweighs ADD.

Hosseini and Jenab (2004) use Tabu Search (TS) to solve a multi period two-layer facility location problem with the capacity expansion problem. An array including cells in the number of regions (Layer 1) and centers (Layer 2) is used in the algorithm. In the corresponding array, regions are assigned to the nearest center cell at the right side of them. By this array representation, open centers and allocation of regions to these centers are shown within an array. Neighborhood search is achieved by pairwise exchange moves. In the proposed metaheuristics, infeasible solutions are not accepted or repaired, and the new neighborhood solution is generated when an infeasible solution is obtained due to the capacity constraints. Benders Decomposition (BD) is used to compare the performance of the proposed algorithm and computational results show that solutions of the proposed algorithm deviates from the solutions of BD within a range of 0% to 7%. In addition to this, solution time is very short when compared with Benders' solution time, and for large scale problems, the proposed algorithm outperforms BD, since BD has shortcomings in solving large scale problems.

Velasquez and Melo (2004) address the multi-period single echelon dynamic capacitated facility location problem for which they propose a model allowing for shifting capacities of facilities in the existing infrastructure in a multi-period environment. Open / close facility location decisions are handled by capacity shifts too; capacity can be relocated to a new location which represents opening a new facility, while shifting all capacity of a facility is equivalent to the closure of it. VNS is preferred as the solution technique. Four neighborhoods are defined to execute VNS. In the first one, decision of opening / closing facility is shifted one period forward or backward; both facility and direction of the period shift are selected randomly. In the second neighborhood, a facility is opened / closed in period t where both the facility and period t are chosen randomly as similar to the first neighborhood. In the third neighborhood, an existing facility and a close facility are chosen randomly, as well as period t. Existing facility is shut down in period t and other facility is opened in period t+1. In the fourth and the last neighborhood, several facilities are chosen and their opening / closing decisions are altered at most by one period where all selections are done randomly. On the average, 0.5% optimality gap is achieved by the proposed VNS, while solution time of VNS is less than one fourth of MIP solver's solution time.

Soto (2009) studies the dynamic capacitated fixed charge location problem (DCFLP) with open/reopen/close facility decisions and evaluates BD and LR for the single-echelon DCFLPs. Capacity constraints are relaxed in LR. Based on the values of the decision variables in the subproblems, upper bound is generated by solving a transportation problem for each period. Subgradient procedure is selected to ensure the convergence through optimal solution. BD approach is implemented by fixing variables pertaining to open/reopen/close decisions and generating pareto optimal cuts which are strengthened cuts regarding the degeneracy. BD outperforms LR for large-sized problems, while LR is better in small-sized problems.

Soto and Üster (2011) alter the model formulation by Soto (2009) and include a modified model formulation where reopening and closing facilities are excluded and facilities should be kept open until the end of the planning horizon once they are opened. Computation results show a similar trend as Soto's previous work and BD outweighs LR in the majority of test instances.

Melo et al. (2012) develop a TS heuristic for multi-period, multi-commodity CFLP under a budget constraint. Tabu move is selected to change a status of one facility in a single period. Repair mechanism is proposed to be used in case of infeasible solutions following Tabu moves. Feasible solutions are generated by actions like closing facilities, postponing them or scheduling them to earlier periods. Feasible solutions generated via the repair mechanism may have been visited previously in the search. In this case, status of a facility is altered in the entire horizon. Alterations are guided by status of the corresponding solution in previous solutions. The proposed heuristic is compared against another heuristic authors had developed for a similar problem (Melo et al., 2011) and it is found out that TS performs better than their former heuristic, which is basically a local search based heuristic and rejects infeasible solutions throughout the entire search process, in most of the test instances.

Jena et al. (2014), similar to Antunes and Peeters (2001), solve a FLP focusing on capacity planning as modular capacity expansion/contraction is evaluated in a multi-period environment. They use LR facilitating to divide the original problem into sub-problems as many as the number of facilities. A heuristic procedure is introduced to generate feasible solutions guided by the classification of demands in sub-problem solutions: “exactly met”, “under-served” and “over-served”. Variables that are frequently observed in the feasible solutions obtained in Lagrangean iterations are fixed and a reduced MIP model with the relevant set of fixed variables is run after a certain number of iterations periodically. The proposed model has a very low integrality gap and near optimal solutions are obtained.

2.3. Multi-Echelon, Single-Period FLP

A traditional supply chain covers the units from suppliers to retailers as it is stated previously. Thus, there have always been many researchers dealing with multi-echelon FLPs to capture the characteristics of supply chains as much as possible. Growing importance of the integration of suppliers, manufacturers and customers and advancing communication technologies are increasing the attention and effort in such problems (Selldin and Olhager, 2003).

Tragantalerngsak et al. (1997) evaluate LR alternatives as a solution tool to two-echelon CFLP. In the context of the problem, products follow the path from uncapacitated depots to customers through facilities which are capacitated. They propose six different set of constraints to relax in LR. The constraints that are relaxed are capacity, demand, one-to-one assignments and logical constraints ensuring product flows to the open sites only. Computational results show that to relax the demand and the logical constraints surpass the other relaxation alternatives.

Pirkul and Jayaraman (1998) provide LR with a heuristic method to obtain feasible solutions from the subproblems to locate warehouses and plants in a distribution network. This paper is a modified version of the study by Pirkul and Jayaraman (1996). In their former study, number of warehouses and plants to be opened is specified exactly where there is an upper bound on the maximum number of open warehouses and plants in the latter. LR makes it possible to divide the original problem into two subproblems handling opening decisions of warehouses and plants separately. Authors propose a heuristic algorithm to construct a feasible solution from the Lagrangean solution: customers not assigned to warehouses in the Lagrangean solution are considered and penalty costs for not assigning these customers to the cheapest and second cheapest warehouse are calculated; then the ratio of these two costs is used in the assignments of customers to warehouses. Warehouse-plant assignment is achieved in a similar approach. Computational results prove that the proposed model produces solutions with a negligible gap in a reasonable time compared to MIP model that is solved to optimality.

Barros et al. (1998) propose a network model for processing sieved sand which includes individual problem characteristics. They use specific properties belonging to sand utilized to draw process graphs of each type of sands, and then directions of flows are determined by the corresponding process graphs. As it is observed in many papers in the literature, they are inspired from the regulations of the Dutch government concerning construction waste. Two levels of the proposed network model stand for the candidate regional depots and treatment facilities with capacity alternatives. The proposed model is NP-hard; and hence a heuristic combining LR and problem-specific procedures including valid inequalities to obtain and improve lower and upper bounds on the optimal solution is offered.

Jayaraman et al. (1999) develop a CLSC network model to determine location of remanufacturing/distribution facilities where there is a limit on the maximum number of the facilities. Production and inventory of products and components are included in the model which is very common in supply chain problems. Although the proposed model covers effects of storage, it is a single-period problem and authors use average storage values to embed inventory effect into the model. GAMS optimizer is used to test the model.

Krikke et al. (1999) present one of the most detailed models constructing a reverse logistics network in the literature. Although the proposed model is intended to be generic, it requires advanced knowledge of the technical properties of the products. This study is a chapter of a more comprehensive dissertation. Chapter 2 and Chapter 3 concern the formulation of product recovery and disposal strategy (PRD) for single-product and multi-product cases, respectively. PRD strategies are obtained by evaluating alternative disassembly sequences. Based on the PRD strategy developed, process graph for each product which reflects the sequences of processes needed to implement a PRD strategy is drawn. In the proposed model; reuse, remanufacturing, recycling and disposal are accepted as the recovery options. In Chapter 6, a case study named “Business case Océ: reverse logistics network re-design for copiers” is included. Transformations from products to modules, from modules to components and from components to materials in the order are represented in the constraints. For each process, reductions in flows due to material loss or emissions during the processes are considered which points out the requirement of advanced knowledge of products and processes. Capacity alternatives for each candidate facility location and secondary markets at the levels of products, components and materials are included in the model. Authors believe that echelons are user-defined and problem owner must assign processes to echelons. As for the solution procedure, they come up with a heuristic which exploits the echelon structure by building the network echelon by echelon. They develop alternative echelon configurations (sub-systems) by the two-step procedure. In the first step, processes are assigned to echelons regarding the minimization of the number of reverse arcs and in the following step, multiple processes are clustered in echelons. To cluster and exchange processes among echelons produces alternative echelon configurations containing no information about flows. Flows among facilities are optimized by using linear programming as the last step of the whole heuristic. In the end, a network with a pre-specified number of echelons is constructed. Then, an improvement step is executed by testing the possibility of capacity expansion exchanges followed by the foregoing step that is optimization of the transportation cost.

Unlike the majority of reverse logistics literature which regards recycling or remanufacturing as recovery options, Jayaraman et al. (2003) cover refurbishment since “product recalls” is the focus of the problem they handle. Candidate sites are given for the locations of collections and refurbishment facilities where the number of each facility type to be opened is limited and must be in a specified range. As for the solution method, Jayaraman et al. (2003) use Heuristic Concentration (HC) where sub-problems with a set of decision variables reduced based on the previous iterations in the heuristic are iteratively solved to optimality. It is shown that HC is superior to MIP in respect of both solution quality and computation time.

Beamon and Fernandes (2004) design a network consisting of collection centers and warehouses which are used through forward distribution and reverse flow for remanufacturing. In the problem context, collection centers preceding warehouses have burden of sorting and inspection. Collection centers are not authorized to make inspection in many papers in the literature, since equipment required for inspection is relatively expensive. Likewise, sorting capability is also assured by some special equipment except the case of manual sorting and this extra equipment requires further investments. However, such expenses on this equipments are not reflected in the model. Although planning horizon consists of a single period, present worth of costs is used in the objective function coefficients to capture the measure of investment value.

Listeş and Dekker (2005) extend the model of Barros et al. (1998) to stochastic programming. They intend to use two-stage stochastic programming which is common for non-deterministic FLP. Yet, they decide to use a three-stage stochastic programming method, since two-stage stochastic programming tends to protect scenarios with high return supply and a solution to such scenarios could be too costly. Location decisions are made in the first two stages and allocation decisions are reserved for the third stage in the proposed three-stage stochastic programming. They assume that average supply of scenarios with low and high returns have to be met in the first stage. At the second stage, after the actual return supply is expected to be revealed, additional decisions related to opening location decisions are made if return supply is high. At the third stage, directions of flows are determined. This method provides the possibility of working with diverse scenarios which reflect uncertainty better than the two-stage stochastic programming method.

Hong et al. (2006) propose a max-min robust optimization model for e-scrap reverse production infrastructure to process the used televisions, monitors and CPUs in the state of Georgia, USA. Stakeholders are state government and industrial firms together, where municipal collection sites and non-profit recycling sites are assumed to collect e-scrap. In the literature, uncertainty generally encompasses nondeterministic nature of demands and returns, whereas constraint matrix (i.e. participation rate, reusability percentages, capacity utilization in collection facilities) is assumed to be uncertain in the proposed model. As recovery options; reuse, remanufacturing and recycling are considered. The proposed model determines locations of sites related to collection by regarding the given process sites. Due to large scale of the network, MIP results in poor performance. Therefore, authors prefer using the heuristic proposed by Assavapokee et al. (2006). In the end, it is shown that e-scrap collection may be seen as a positive financial resource in the case of the high reusability of e-scrap.

Cordeau et al. (2006) introduce a flexible model formulation for CFLP which designs a network from scratch. The proposed model gives decisions on the suppliers to collaborate with, locations of warehouses and plants and transportation mode to haul products in the network. Due to the complex structure of the model, authors describe two approaches to solve the problem: BD and B&B. Inefficiency of the traditional BD is overcome by pareto optimal cuts which outweigh other cut alternatives in the presence of degeneracy. For a further improvement on BD, initial cuts are generated by solving linear relaxation of the MIP model iteratively as integrality constraints are introduced for most promising variables in each iteration until a feasible solution is obtained. Computation results show that BD approach performs slightly better than B&B.

Wang and Yang (2007) investigate a MIP model with multi-commodity and single-period characteristics solving an integrated facility location and configuration model for recycling e-waste where the corresponding network includes collection, storage, recycle and final treatment sites. To decrease computational efforts, they propose two heuristics comprising of different combinations of random selection (RS), heuristic concentration (HC), modified heuristic expansion (MHE) and MCC which is a modified version of the heuristic introduced by Jayaraman (2003). RS is used as a benchmark heuristic in order to compare the others. The first heuristic algorithm is a hybrid of RS, HC and MHE. The second one is a hybrid of MHE and MCC. In the end, authors prove that the second algorithm dominates the other heuristic and CPLEX in terms of time performance, whereas the solution quality of the first algorithm is better.

Lu and Bostel (2007) propose a two-echelon CLSC network with three types of facilities which are intermediate centers (working as disassembly center), remanufacturing centers and producers. Authors utilize LR approach for the corresponding discrete location problem. Constraints ensuring demand satisfaction, collection of all available returns and assignments to only open intermediate centers are relaxed. Following the third relaxation, problem is separated into two sub-problems; in the first subproblem, producers and remanufacturing centers are included, while in the second subproblem intermediate centers are considered. Due to the first two relaxations, the first subproblem is decomposed further into sub-problems for each potential site available to producers and remanufacturing facilities. Feasible solution to be an upper bound is generated by fixing the location decisions in the subproblems and solving the assignment problem subsequently. Surrogate constraints are introduced to improve the efficiency of LR yielding the gap below 1% for all test instances while the performance of MIP is worse in overall, especially for large-scale instances.

Salema et al. (2007) propose a MIP for a generic CLSC network with uncertainty on product demands and returns. This generic model is tested on a Spanish company named Iberian. The proposed model ignores the existing infrastructure and determines locations of factories, warehouses and disassembly centers which are the layers of a network with two echelons. While authors prefer using strict return constraints by giving high penalty costs to unsatisfied returns to reflect legislation enforcement, processes on returns are not handled in the proposed model and returns are conceived equivalent to material procurement; in other words, customers returning products are treated as suppliers. Therefore, the proposed model can be easily converted to an OLSC. A scenario-based model is applied to investigate different patterns of demands and returns, and the resulting model is solved to optimality by B&B method.

Chouinard et al. (2008) introduce a CLSC with 5 layers from customers to suppliers. Network design decisions concern the locations of service centers (responsible for the collection of returns and distribution of products to customers), processing centers (where both recovery and manufacturing operations are executed) and warehouses, transportation cost and material acquisition cost from suppliers. Distribution of products to customers is undertaken by the nearest service centers to customers. To reduce the problem size, product families are introduced instead of including each particular product in the model. Product families are defined based on two criteria: process requirements and economic impacts on the network design decisions. Links between product families are represented through bill of materials (defined according to assembly/disassembly trees) and products are categorized into five classes: unknown, new, good, deteriorated and unusable. Stochastic programming is preferred to consider uncertainty in volumes of demand and returns. Yet, proportions of products at a particular state in total number of returns are deterministic. Sample average approximation (SAA) method is utilized to evaluate a large sample of scenarios generated by Monte Carlo sampling method. In the end, a problem specific heuristic is developed based on SAA which solves the model for a subset of scenarios and obtains a network configuration in the first place. Then, this configuration is applied to all scenarios and infeasible ones are discarded and the outcome becomes a lower bound on the optimal solution. In case of a feasible network configuration for all scenarios, the outcome becomes an upper bound on the optimal solution. The heuristic progresses by trying different sets of scenarios to construct a network configuration until the gap between lower bound and upper bound falls below an acceptable threshold.

Wollenweber (2008) solves the two-echelon CFLP. He uses split costs, instead of constant fixed costs, for facilities such that fixed cost of a facility varies depending on its capacity which is selected from the set of discrete capacity alternatives placed as variables in the model. Since the proposed model is NP-hard, he develops a solution approach that generates an initial solution by a construction heuristic and improves the relevant solution via VNS nesting VNDS. The construction heuristic starts by solving linear relaxation of the model; non-binary variable that is the closest one to an integer value is converted to a binary variable and relaxation of the proposed model is run again and so on, until a feasible solution is obtained. Add / drop moves are randomly performed on the final solution of the construction heuristic. VNS follows add / drop moves and generates a new solution in shaking step by changing capacities of facilities selected randomly. A new solution is exposed to VNDS which is employed for local search. VNDS includes neighborhoods of add, drop and swap moves in order. At the beginning of VNDS, facilities of exactly one stage are fixed and relaxation model is solved as bounded with fixed facilities. Then, construction heuristic runs to generate a feasible solution from the solution of the relaxed model. Solution generated by the construction heuristic is exposed to the neighborhood operations of VNDS. The proposed solution method outperforms state-of-art MIP solver in all test instances.

Demirel and Gökçen (2008) come up with a generic model for a CLSC with multiple products. The proposed model inquires how remanufacturing can lead to a profit in forward logistics while determining locations of disassembly centers, distribution centers and collection centers under capacity constraints. Unlike many papers published after WEEE Directive, the proposed model does not deal with recycling. GAMS optimizer is used to test scenarios generated by the estimated parameters.

Lee and Dong (2008) design a network with three layers: original equipment manufacturers -for end-of-lease computer products-, hybrid processing facilities that handle remanufacturing as the sole recovery option and customers. The corresponding model is developed to determine locations of facilities belonging to the first two layers. Instead of capacity restriction on facilities, products' flow on each arc is limited. They develop a two-stage heuristic approach to improve results by the MIP which is computationally intractable. In the first stage, locations of facilities are chosen randomly and network simplex method is applied to determine paths of flows. The second stage of the heuristic is to improve the obtained solution by TS where new solutions are produced by the swap of flow arcs, insertion of nodes in the network and 2-opt exchange of non-adjacent arcs. Computational results demonstrate that the proposed heuristic method works well for small-size test instances, but results in average gap ranging from 10% to %12 in large-size test instances.

Pishvaee et al. (2009) examine uncertainty in CLSCs through stochastic optimization. They develop a MIP model that determines locations of facilities at three layers: production/recovery, distribution/collection and disposal. Scenario-based stochastic approach is used to capture the uncertainty factor in the problem. Scenario-dependent parameters are embedded into MIP model, leading to duplication of each constraint as the number of scenarios and actualization possibilities of scenarios are added to the objective function. MIP model is solved to optimality which is a moderate solution for all scenarios. Although deterministic model using average parameter values gives better solution, stochastic approach is more powerful in terms of deviations among scenarios.

A contemplation of forward and reverse supply chains simultaneously with hybrid facilities active in both forward and reverse direction is handled by Üster and Easwaran (2010). The proposed model determines hybrid centers (HC) assigned as both distribution center and collection center, and hybrid sourcing facilities (HSF) which handle remanufacturing and serve as sourcing locations for new parts. To get over the complexity derived from simultaneous consideration of forward and reverse channels, they present two alternative solution methods: MIP with branch and cut method (B&C) and BD with strengthened multiple dual cuts, respectively. Computational results show that BD outperforms B&C in most of the test instances.

Pishvaee et al. (2010) develop a generic model including given recovery centers capable of different treatment types: recycling, refurbishment and remanufacturing. Returns are segregated into two groups: the ones to be disposed of and those to be the objects of recovery. To achieve this segregation, inspection is executed in capacitated collection centers the locations of which are determined by the proposed model. Since CFLP is NP-hard, specialized heuristics/metaheuristics are appreciated in the time complexity of the problem. Therefore, Pishvaee et al. (2010) adopt SA and auxiliary methods like priority-based encoding, 2-opt and 3-opt neighborhood search are combined and embedded into SA. Efficiency of the proposed methodology in solution quality and time is proven after the comparison with B&B in the view of negligible gaps achieved after a significant decrease in solution time.

Achillas et al. (2010) design a reverse logistics network for the Region of Central Macedonia where there is an existing infrastructure and candidate capacitated storage facilities are evaluated to be opened or not. Since the decision maker is the regulative authorities, not private companies, proposed MIP model is an OLSC regarding only backward flow of end-of-life EEEs and test cases which are moderate in size are solved via mathematical programming language AMPL.

Lieckens and Vandaele (2011) offer a model taking stochastic delays into account due to various processes like collection, production and transportation as well as disturbances due to various sources of variability like uncertain supply and uncertain process times by using queuing theory. The model constructs a CLSC consisting of customers, retailers, remanufacturing/manufacturing plants, evaluation centers, distribution centers, primary and secondary markets where manufacturing plants are decided by the model. The queuing analysis is valid as long as the network is in a steady state condition and all flows within the network are assumed to have known distributions and the proposed model works with expected values. They apply the differential evolution heuristic (DE) to get over the complexity of the problem which is in the class of MINLP and NP-complete.

Piplani and Saraswat (2011) formulate a model to construct a CLSC consisting of repair and refurbishment centers with the existence of uncertainty in demands and returns. Authors reckon that there is no known distribution fitting into non-deterministic demands and returns. Since stochastic programming cannot be applicable to the problems where demands and returns do not have known distributions, they choose min-max robust optimization as the solution method. The proposed model works with modular products and returns of these products are divided into 2 groups: the ones with warranty and the ones out of warranty. The formulated model has to repair the products in the first class and determines whether to repair the products or not in the second class based on profitability. Repair and refurbishment operations are executed at the modular level in-house or outsourced to a repair vendor. The model is tested on a company providing after-sale service for laptops and desktops in the Asia-Pacific Region. The model is solved under the circumstances of the existing network (binary variables are fixed) by linear programming and compared with a new network from scratch.

Amrani et al. (2011) develop a heuristic using TS as nested in VNS to solve three-layer (production/distribution centers, distribution centers and demand zones) CFLP. Different configurations of facilities varying in process capacity and storage limit are offered for each site location in the model and, the proposed model determines capacity and storage size of facilities as well as the locations of them. TS is employed as a local search procedure in VNS and invoked whenever a new neighbor solution is generated by VNS. Six types of neighborhoods structures, which are generated by "add", "drop" and "exchange" moves on production/distribution and distribution layer, are presented in the study. Authors make a distinction between exchange moves executed on distribution centers and other moves, since the former ones have more possibilities to be examined than others. Therefore, all moves except exchange operations on distribution centers are used in a way that standard VNS procedure follows. In shaking step of VNS, a new solution is generated by these neighborhoods except the distinct one that is the neighborhood derived from exchange moves on disassembly centers. Following this step, TS is called to be used as the local search procedure and exchange moves on disassembly centers are used to generate new solutions in TS. The proposed heuristic achieves an optimality gap under 1.5% in all test runs.

Multi-objective models are scarce in reverse logistics literature. Ramezani et al. (2013) prefer working with three objectives, namely: maximizing profit, customer responsiveness and quality of the logistics network. Definition of the objectives are to maximize net profit, to maximize the customer service level in both forward and reverse direction and to minimize the number of defective materials acquired from suppliers -and increase sigma quality level-, respectively. The network is constructed from scratch and locations of plants, collection centers, distribution centers, hybrid processing centers and disposal centers serving for distribution of products and remanufacturing/ disposal of returns are determined by the model. To capture multi-objective nature of the model, authors chose ε-constraint method where one of the objectives is set as the real objective function and rest of them are inserted into the model as constraints with allowable bounds. Since Ramezani et al. (2013) study in a stochastic environment and use two-stage stochastic programming, their proposed model is even complex for a single objective. Therefore, ε-constraint method is quite plausible to be used. At the first stage of stochastic programming, decision variables specifying network configuration are taken as the first stage variables before observing uncertainty. The second stage variables are related to product flows in the networks. Stochastic programming may not give the optimal solution for any scenarios, and for some scenarios, for the good of overall solution, results can be inadequate. To avoid such outcomes, they set a lower bound on profit for each scenario and try different bound values and then acquire a set of pareto optimal solutions in the end.

Although traditional FLP mostly deals with networks associated to production environment, similar problem structures are utilized to design telecommunication networks. Gendron et al. (2013) propose a generic model to be used to design telecommunication networks and develop a heuristic algorithm for it. The proposed model decides where to open uncapacitated facilities (depots and satellites) and which depot-satellite path to be followed by each customer. LR is used to relax the constraint ensuring the link between depots and satellites. Feasible solutions are obtained by the neighborhood operations on the given set of facilities that the sub-problems provide. B&B guided by the reduced costs in the sub-problems is performed to improve lower and upper bounds. Although some test instances could not be solved with acceptable gaps; on the average, the proposed heuristic provides solutions with small gaps in reasonable time.

Firoozi et al. (2013) design a two-echelon distribution network consisting of a supplier, distribution centers and retailers. Locations of distribution centers are decided by the proposed model, whereas the locations of existing supplier and retailers are given beforehand and new suppliers/retailers are not allowed. Distribution centers pull products from the supplier with economic order quantity policy (EOQ). The proposed model minimizes total cost comprising of holding inventory and safety stock cost besides costs relating to the installations of distribution centers and transportation cost. They handle perishable goods and perishability is represented by lifetime of products. Although the model formulation covers a single period, known pattern of EOQ policy allows authors to include inventory side in their model. LR is used as solution tool and constraint associated with assignments between distribution centers and retailers is relaxed which enables subproblems to handle retailer layer and distribution center layer independently. At the end of each iteration, UB is generated with a greedy approach assigning retailers to distribution centers one by one. For small-size problems, optimality is reached for all instances while less than 0.1% gap is obtained on the average for large-size problems.

2.4. Multi-Echelon, Multi-Period FLP

The enlargement in the extent of supply chains is stated at the beginning of Section 2.3. As the size of supply chain extends and the level of investment required to build up them increases, responsiveness of supply chains and anticipation of them to be adaptable to possible changes in the future get more significant. In this respect, multi-period planning horizon is more essential for large scale supply chains like the ones with multi-echelon.

Some authors believe that supply of returns, which is the basis of uncertainty in reverse logistics, can be stabilized by some additional burden on producers. Mansour and Zarei (2008) emphasize that returns can be more predictable if producers have the responsibility of collecting returns which are farther to the closest collection center than a certain distance range. Otherwise, responsibility of dropping the returns off to the collections centers is left to product owners. In the proposed model for CLNP including dismantlers, shredders and metal separators besides material suppliers, collection centers, distribution centers and plants, returns of automobiles are taken into account in a planning horizon consisting of multiple periods. To get over the problem complexity and find the solution in a reasonable time, they develop a heuristic method which starts with multi-start search to obtain initial solutions for the location sub-problem. Allocation sub-problem is solved by a greedy algorithm (Glover and

Kochenberger, 2003). Solutions obtained after the greedy algorithm are improved by problem specific procedures which include both exact and greedy algorithms.

Min and Ko (2008) propose a multi-period model including both forward and reverse channels and consisting of three layers: manufacturers, warehouses and repair centers for forward and reverse flows, respectively and customers. Locations and capacity expansions of warehouses and repair centers, and hybrid option combining warehouses and repair centers are evaluated by the model from the perspective of 3PL service provider. They adapt GA to their model. Roulette wheel method is chosen to select parents, and two-point crossover and random mutation are applied to generate new generations. The calculation of the fitness function requires solving transshipment problem which is converted to a distribution problem by dummy sources and destinations and solved by C++. Near optimal solutions within 2% gap are obtained by the proposed algorithm.

Salema et al. (2009) develop a location-allocation model developed for the simultaneous design of forward and reverse channels. Strategic decisions such as network design are considered together with tactical decisions such as production, storage and distribution planning. It is difficult to integrate strategic and tactical decisions since they occur at different period scales. To overcome this problem, Salema et al. (2009) use two interconnected time scales: macro and micro time. Strategic decisions are made at macro level, while minding tactical decisions at micro level. Inclusion of transportation time which is very rare in the literature is achieved thanks to these two time scales. They consider utilization rates all along the network and set lower bounds on inventory amount, production quantities and flows among facilities where there are also upper bounds on them. Objective function is to maximize net revenue of the system where income arises from the sales to secondary market. The proposed model constructs the network from scratch and determines locations of factories, warehouses and disassembly centers. The proposed model is solved to optimality by B&B. Authors admit that B&B method is not sufficient to overcome the computational burden and stimulate efforts for different solution techniques such as BD.

In practice, it is very common for original product manufacturers to cooperate with 3PLs since reverse logistics may not be economically viable in regard of costly requirements such as sorting/inspection and treatment equipments if each manufacturer runs its own reverse network. As a response to rising interest in 3PLs, Hyun and Gerald (2007) consider 3PLs coming with a challenging property that they must make a sequence of interrelated decisions over time since they operate for a number of different clients. Authors propose a model covering demands from multiple clients and belonging to a class of multi-period, multi-commodity, two-echelon, capacitated location models. Warehouses are transshipment points in the forward channel, whereas repair centers are transited in the reverse channel. The proposed model also allows for hybrid facilities which work as both warehouse and repair center capacities of which are expandable. Facilities can be open in non-successive periods as a result of close and reopen decisions. Yet, cost function in the objective function depends on the order of periods that facilities are opened which leads to a non-linear objective function. Due to the complexity of the problem, authors propose a GA based heuristic. Gene chromosome representation is preferred and a solution has chromosomes as much as the number of periods. Fitness function is the sum of objective function with a penalty cost associated with capacity violation. Combined roulette wheel and elitism method are used for the parent selection method. Two point crossover and random mutation are used to generate new offspring. Computational results are reasonable and encouraging with short computation time.

Alumur et al. (2012) propose a profit maximizing framework for reverse logistics network with multi-period, multi-commodity with the information of bill of materials and possibilities of modular capacities. The model includes all treatment options such as reuse, remanufacturing and recycling. The proposed model determines the locations of inspection centers and remanufacturing plants, return flows, capacity expansion decisions, material purchase decisions and inventory. Authors test the model for the case of washing machines and tumble dryers in 40 collection centers located by the municipalities in the 40th most populated cities in Germany. A single-period model with average values of the parameters over the entire planning horizon is formulated to assess the effect of multi-period horizon, and efficiency of developing a multi-period model is proven.

Lee and Dong (2009) study a CLSC incorporating uncertainty in the number of returns and in demands of customers with known distributions. Stochastic nature increases the inherent complexity of FLPs which are NP-hard. Moreover, objective function includes non-linear parts. Therefore, a solution method is developed by integrating a sample approximation scheme with SA. Initial location solutions are obtained by opening the facilities with minimum operating cost and flows are allocated by a greedy algorithm. Products are shipped to the nearest facility unless there is lack of capacity. Otherwise, the second nearest facility is considered. A one-cut point method on the representation of solutions is applied to generate new solutions. Risk prevention due to the uncertainty in the problem is undertaken by using SAA which uses expected value functions of a random sample approximated by the corresponding sample average function. Test solutions show that the proposed methodology outperforms deterministic optimization approach in large-scale problems and in moderate-size problems; the proposed methodology achieves small gaps within reasonable time.

Hasani et al. (2011) propose a comprehensive model for strategic CLSC under interval data uncertainty for demand and the proposed model in their paper intends to fit to environment of food and high-tech electronics manufacturing industries which are exposed to perishability. Perishability brings forth introduction of “warehouse life-time period” notion into the model. The proposed model incorporates multiple products, multiple periods, and three echelon supply chain including suppliers, manufacturers, warehouses and retailers conveying foods or high-tech products where perishability is represented by warehouse life-time and time-dependent prices, respectively. In the proposed model, uncertain demand and purchasing cost parameters are seen as the key elements which are exposed to dramatic changes. Therefore, different scenarios are generated in a specific uncertainty level range for corresponding parameters, to be used in the interval robust optimization technique. Constraints representing amounts of returns from the warehouse due to expiration of warehouse life-time and products remaining in the warehouse are non-linear. A variable transformation technique is executed for model linearization. Computational analysis is executed via LİNGO software.

Gomes et al. (2011) design a nationwide recovery network for WEEEs. They intend to find best locations for collection and sorting centers by incorporating both strategic and tactical level decisions via two interconnected time scales as defined in Salema et al. (2009). Planning activities such as storage are handled in micro-period whereas location of facilities and volumes to be collected are set in macro-period. Different freight modes, alternative time horizons and network structures are evaluated in case studies.

Assavapokee and Wongthatsanekorn (2012) design a reverse production system infrastructure for the state of Texas, USA and focus on reuse, disassembly and reassembly activities such as repairing, refurbishing and remanufacturing of electronic products. Minimum population coverage constraints limit the search space of the problem where stakeholder is the local government. Although local government's perspective is on the focus of the problem, remanufacturing activities are included in the model and approximate financial support to make the collection of electronic products financially viable is calculated based on the model's outcome. Unlike many papers in the literature, authors consider the number of workers with different capabilities with respect to various tasks which are derived from multi-tasking characteristics of the processes in reverse production system.

Wang et al. (2008) expand the literature of multi-echelon CFLPs in a multi-period planning horizon by the inclusion of customer choice behavior and utilize bi-level programming to encompass customer choice behavior along with the objective to minimize total cost of distribution network construction. The problem of interest decides locations and capacities of central and regional distribution centers as well as the distribution of commodities between distribution centers and stores. GA is preferred as solution tool where at the beginning of each iteration, SUE assignment model is run for each chromosome (candidate solution) since demand is elastic and dependent on network structure. Bi-level programming enables execution of a GA model nesting SUE assignment model because upper level decision maker, who has charge on the network design , knows how lower level decision makers behave. Proposed model gives at most 4% gap in all test instances.

Abbas et al. (2011) develop a hybrid GA with Pattern Search (PS) to solve a multi-period multi-echelon FLP. The proposed model determines locations of manufacturing and warehouses sites as well as supplier selection and distribution paths. GA stage includes tournament selection, intermediate crossover, uniform mutation performed on gene-coded chromosomes. PS is employed on the best solution of each iteration to intensify it. Pattern vectors, direction vectors in the set of {-1,0,1}, are generated at the initialization step of PS and they are multiplied by a specified scalar (delta). As long as the current solution improves, delta is redoubled and solution space is expanded. Otherwise, contraction is rendered by halving delta. Test runs show that optimality gap of the hybrid heuristic ranges from 1% to 8%.

You et al. (2011) use GA approach to solve a FLP regarding opening/reopening collection and treatment centers for waste returns. Assignments between different layers are rarely represented in GA chromosomes. In this study, a gene chromosome encoding is used and decoding operator is executed to turn gene-chromosome into a matrix representation where open/reopen decisions and assignment information are retained. Near-optimal solutions within 0.11% optimality gap are obtained via the proposed GA.

Tang et al. (2013) develop a BD algorithm for CFLP with capacity expansion possibility. BD leads to subproblems with no integer variables as fixing binary variables: facility open decisions and facility-customer assignments. Dual formulations of the subproblems help to generate optimality and feasibility cuts. Authors propose the disaggregation of Benders primal cuts as separating subproblems in two lower level problems dealing with transportation quantities and capacity expansions, respectively. To divide subproblems into smaller ones ends up with multi-generations of cuts which restrict solution space more rapidly than traditional cuts. In the case of degeneracy, pareto optimal cuts are generated to limit solution space as much as possible. For the vast of the modest-size problems, optimal solutions are found; however, the proposed model is not as effective as intended for large-size problems.

Hinojosa et al. (2000) study multi-period two-echelon CFLP. Authors include opening and closing facility decisions in their model, and limits on the minimum number of open facilities for the beginning and end of the problem horizon are introduced. LR is employed to decompose the problem into two subproblems by relaxing the constraint that ensures demands are met. Further decomposition of subproblems is achieved, since subproblems can be solved for each facility independently. A heuristic solution to construct a feasible solution from the subproblems is proposed. Heuristic solution, firstly, provides a set of facilities to be opened, while guaranteeing that total capacity would be enough to meet demand. As it is stated, LR enables the problem to be decomposed into subroblems, each for a particular facility. Therefore, the influence of opening a particular facility on the objective function of Lagrangean dual problem is known for all. They assign indices to the facilities based on their influence on the subproblem objective function and determine the set of open facilities guaranteeing sufficient capacity. In the second step of the algorithm, transportation problem is solved to optimality. An average optimality gap between 0.24% and 5.12% is obtained in test instances.

As the lack of long-established practices related to recovery of waste in Turkey is considered, it is likely that quantity of returns that can be collected will be less than that expected in the first years. As recovery activities become commonplace and consciousness of recovery raises by time, quantity of returns is expected to increase in later years. Thus, it is normal to reckon that recovery network will expand as the quantity of returns increases. Our proposed model takes this fact into account through a multi-period planning horizon and the inclusion of capacity expansion over time. Hence, the problem as we define it belongs to the class of multi-echelon dynamic capacitated facility location problems, MEDCFLP. The taxonomy of the papers summarized in this section can be found in Table A.1 in Appendix A.

CHAPTER 3

PROBLEM FORMULATION

In this chapter, firstly, we define structure of the recovery networks and our problem environment. Then, we discuss the assumptions of the model and introduce a MIP model representing our problem environment and the given assumptions. Notations of the model and requirement of developing a heuristic for the model are discussed in the rest of the chapter.

3.1. Problem Environment

In Chapter 1, a brief comparison between forward distribution networks and product recovery networks is provided. Product recovery encompasses a set of operations from collection of returns to treatment of returns for recovery and distinction between forward and backward networks can be made from the very beginning: collection of returns. As forward distribution networks are driven by the demand of customers at the end of the network, recovery networks start with returns of products and to ensure that recovery network operates as it is supposed to be, collection of product returns from end users must be guaranteed. There are different policies to guarantee collection of returns such as pick-up policy, pay as you throw policy; and the selection of these policies to apply is highly dependent on the characteristics of the society procuring waste (McMillen and Skumatz, 2001). It is stated in Chapter 1 that WEEE Directive – one of the driving factors of product recovery- imposes a burden on producers and as return collection policies vary, managers of product recovery networks vary too. Strict burden for the recovery of waste entails requirement of an outspreaded network to make sure of the collection of returns and recovery of them. Different forms of cooperations between municipalities and producers exist in practice to manage product recovery networks.

In our study, we intend to contribute to the efforts in the design of product recovery network in Turkey and propose a model to construct a network serving to the recycling of WEEEs. We include collection, consolidation and disassembly centers in recovery network structure. While collection and disassembly centers are included in almost all recovery networks, consolidation centers are usually incorporated into large-size networks. In accordance with the efforts of organizations like White Goods Manufacturer Association and Informatics Industry Association in Turkey in planning to construct a nation-wide network for the recovery of WEEEs (Subaşı, 2011 and TÜBİSAD, 2013), we consider a nation-wide recovery network and include consolidation centers for the economies of scale and other advantages of aggregation as well. In Figure 3.1, a straightforward representation of the network in our study is demonstrated.

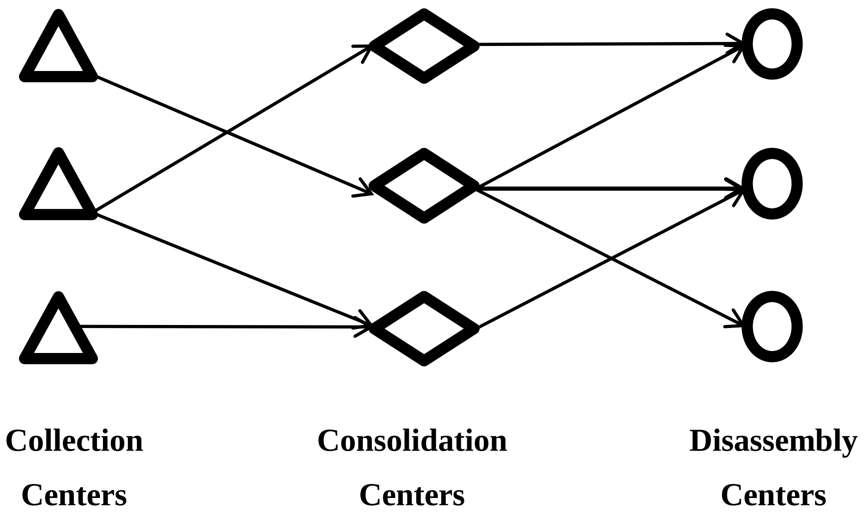

Figure 3.1. Recovery network representation in our problem

A distinctive feature of the proposed model from the existing studies in the literature is the limit imposed on the maximum distance between collection centers and disassembly centers that returns can flow through. There are various incentives for such a limit: policies as Ro-HS Directive limits hazardous substances in products and enforces disposal of such substances in a short time after returns are collected. These enforcements are leading to the requirement of a dense network to get returns to disassembly centers as fast as possible. Moreover, in the case of remanufacturing, recoverable components must be sorted out and processed to be used in manufacturing plants, and arrival of these reusable components to the plants should be on time in order to maintain smooth production.

As it is supposed that the entire network is better to be managed by a specific cooperation, it is possible to make assignments between collection centers and consolidation centers, and between consolidation centers and disassembly centers, when considering the overall efficiency of the network. Many-to-many assignments - multi-sourcing in other words- can be adequate in facility location problems if there is no opposite obligation (i.e. householders drop their returns at the closest collection center in drop-off policy and many to many assignment is not realistic in that case). In our case, many-to-many assignment turns out to be possible. It can be cost-effective as well as being capacity utilization friendly (disassembly centers are capacitated and many to many assignment is appropriate to utilize capacity of disassembly centers effectively, since it is more flexible in assignments in contrast to other assignment schemes).

Our proposed model constructs a recovery network consisting of collection, consolidation and disassembly centers by considering a multi-period planning horizon. Objective function of the model is to minimize the total cost that is comprised of facility -consolidation and disassembly center- installation and operating cost, transportation cost among the facilities and the capacity expansion cost related to disassembly centers. Decisions on the locations and the opening period of facilities, flows of waste/returns and capacity expansions are made with regard to the relevant objective function.

3.2. Model Formulation

In this section, we introduce a MIP model for our MEDCFLP. Assumptions, notations and sets, parameters, decision variables, objective function and constraints of the corresponding model are presented in the following sub-sections.

3.2.1. Assumptions of the Model

Before we formulate our model, we discuss the assumptions we make in the development of the MIP model.

1. The values of following parameters are deterministic and known.

- ❖ Locations of collection centers
- ❖ Alternative location sites for potential consolidation centers and disassembly centers
- ❖ Quantity of returns at collection centers in each period
- ❖ Initial capacity of disassembly centers
- ❖ Capacity expansion module of disassembly centers
- ❖ Capacity expansion cost of disassembly centers
- ❖ Fixed cost of operating consolidation centers and disassembly centers

2. Once a consolidation center is opened, it cannot be closed until the end of the planning horizon.
3. Once a disassembly center is opened, it cannot be closed until the end of the planning horizon.
4. Disassembly centers can only be expanded at the beginning of each period.
5. All consolidation centers have equal fixed costs that are static over time.
6. All disassembly centers have equal fixed costs that are static over time.
7. All disassembly centers have the same capacity at the beginning of the planning horizon.
8. Capacity expansion module of each disassembly center is given. Capacity can be expanded only at the beginnings of periods and expansion must be exactly as one capacity module.
9. All returns at collection centers must be transported to disassembly centers through consolidation centers. Direct transportation from collection centers to disassembly centers is not allowed.
10. It is not allowed to transport returns among centers in the same layer (i.e. there cannot be any flows among collection centers).
11. Returns at collection centers can be transported to more than one consolidation center.
12. Returns at consolidation centers can be transported to more than one disassembly center.
13. If the itinerary distance between a collection center and a disassembly center through a consolidation center is more than the pre-specified distance

threshold, returns cannot be transported from that collection center to that disassembly center through that consolidation center.

14. There is only one type of product to be returned. It can be a real product or an artificial product representing an aggregation of multiple products.
15. Processing rate of the product is the same for all disassembly centers and equal to 1.
16. Unit transportation cost is taken as the Euclidean distance between centers that returns are transported through.

3.2.2. Definitions of the Sets

J : Set of collection centers $J = \{1,2,...,j,...|J|\}$

K : Set of consolidation centers $K = \{1,2,...,k,...,|K|\}$

M : Set of disassembly centers $M = \{1,2,...,m,...,|M|\}$

t : time period index $t = \{1,2,...,t,...,T\}$

3.2.3. Parameters of the Model

s_{jt} : Quantity of returns at collection center j in period t

$f1_k$: Fixed cost of installing and operating consolidation center k in a period

$f2_m$: Fixed cost of installing and operating disassembly center m in a period

I_{m0} : Initial capacity of disassembly center m

ec_{mt} : Capacity expansion cost of disassembly center m in period t

p_m : Amount of possible capacity expansion in each period for disassembly center m

d_{jkm} : Total distance from collection center j to disassembly center m through consolidation center k

TH : Limit on maximum distance that the returns follow from collection centers to disassembly centers through consolidation centers

$$cv_{jkm} : \begin{cases} 1 & \text{if } d_{jkm} \text{ is less than } TH \\ 0 & \text{o/w} \end{cases}$$

3.2.4. Decision Variables of the Model

X_{jkmt}: Percentage of returns sent from collection center j to disassembly center m through consolidation center k in period t over total returns at collection center j in period t

$$Y_{kt}: \begin{cases} 1 & \text{if consolidation center } k \text{ is open in period } t \\ 0 & \text{o/w} \end{cases}$$

$$Z_{mt}: \begin{cases} 1 & \text{if disassembly center } m \text{ is open in period } t \\ 0 & \text{o/w} \end{cases}$$

$$e_{mt}: \begin{cases} 1 & \text{if disassembly center } m \text{ is expanded in period } t \\ 0 & \text{o/w} \end{cases}$$

I_{mt} : Capacity of disassembly center m in period t

3.2.5. The Model

$$\text{P} = \text{Minimize} \sum_{j\in J}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} X_{jkmt}\, s_{jt}\, d_{jkm} + \sum_{k\in K}\sum_{t\in T} Y_{kt}\; f1_k + \sum_{m\in M}\sum_{t\in T} Z_{mt}\; f2_m + \sum_{m\in M}\sum_{t\in T} e_{mt}\; ec_{mt} \tag{1.1}$$

s.to

$$\sum_{k\in K}\sum_{m\in M} X_{jkmt}\, cv_{jkm} = 1 \qquad \forall\, j\in J,\, t\in T \tag{1.2}$$

$$I_{m,t-1} + e_{mt}\; p_m = I_{mt} \qquad \forall\, m\in M,\, t\in T \tag{1.3}$$

$$\sum_{j\in J}\sum_{k\in K} X_{jkmt}\, s_{jt} \le I_{mt} \qquad \forall\, m\in M,\, t\in T \tag{1.4}$$

$$X_{jkmt} \le Y_{kt} \qquad \forall\, j\in J,\, k\in K,\, m\in M,\, t\in T \tag{1.5}$$

$$X_{jkmt} \le Z_{mt} \qquad \forall\, j\in J,\, k\in K,\, m\in M,\, t\in T \tag{1.6}$$

$$Y_{kt} \le Y_{k,t+1} \qquad \forall\, k\in K,\, t\in T \tag{1.7}$$

$$Z_{mt} \le Z_{m,t+1} \qquad \forall\, m\in M,\, t\in T \tag{1.8}$$

$$X_{jkmt} \geq 0,\ Y_{kt} \in \{0,1\},\ Z_{mt} \in \{0,1\},\ e_{mt} \in \{0,1\} \quad \forall\ j \in J, k \in K, m \in M, t \in T \quad (1.9)$$

In the objective function (1.1), the original problem P minimizes the total cost of the network comprising of transportation, facility installation cost in addition to operation cost and capacity expansion cost. Constraint (1.2) ensures that all returns at collection centers are transported to disassembly centers. Constraint (1.3) reckons the capacities of disassembly centers for each period based on the realization of capacity expansions. Constraint (1.4) limits inbound flows to disassembly centers to their capacities. Constraint (1.5) and (1.6) preclude the returns to be transported through unopened consolidation centers and disassembly centers, respectively. Constraint (1.7) and (1.8) ensure that consolidation centers and disassembly centers cannot be closed anymore once they are opened. Constraint (1.9) includes nonnegativity and binary constraints.

Original problem, P, is an extension of CFLP. Mirchandani and Francis (1990) show that two-stage capacitated facility location problem (TSCFLP) is NP-Hard. Our proposed model is polynomially reducible to TSCFLP which means that it is NP-Hard, too. Thus, to solve even the medium-sized problems via commercial optimization tools can be time consuming. Ultimate aim of the proposed model is to make some contribute to the design of a nat ion-wide recovery network which can be a very extensive one. It is unlikely to have optimal solutions for such large problems by using commercial optimization tools. Thus, we need to develop a heuristic method to get optimal or near optimal solutions in reasonable times. Our model is still NP-Hard even if it has a planning horizon of single-period. It is obvious that multi-period property increases the complexity of the problem further. In order to alleviate the complexity of the problem due to its multi-period property, Rolling Horizon Approach (RHA) is employed which is recommended by Archetti and Speranza (2014). RHA which is explained in the next chapter enables the proposed model to be handled through modest sub-problems having shorter planning horizons. As it is specified above, our proposed model is still quite complex with the shorter planning horizons. Hence, LR is applied to sub-problems brought forth by RHA. LR has been successfully implemented to diverse facility location problems by Beasley (1993), Marin and Pelegrin (1999), Guignard and Ryu (1992), Fisher (1981), Klincewicz and Luss (1986), Barcelo and Casanovas (1984), Mazzola and Neebe (1999) pursuing prominent works of Held and Karp (1971) and Geoffrion and McBride (1978). LR provides satisfactory solutions in general while it is very lucrative in the aspect of time spent for computations. Yet, duality gap, which is explained in the following chapters, exists in Lagrangean problem and LR fails at reaching optimal solution in this case. To improve the solution obtained by LR, VNS is employed at the end of LR to search for better solutions in the neighborhood of best solution found by LR. In the end, near-optimal solution to the original problem is attained, since RHA cannot succeed to find the optimal solution in most of the cases. Lastly, a version of the original mixed integer model with the reduced solution space is introduced to improve the solution that RHA provides.

CHAPTER 4

SOLUTION APPROACH

As optimization tools and mathematical solution techniques have been developed, problems in the literature tend to be extended and comprehensive owing to such developments facilitating to solve large-sized problems. Supply chain problems can be counted as an example of this trend. In this kind of multi-dimensional problems, researchers intend to utilize all opportunities of mathematical programming models and heuristic schemes; and they are interested in combining these solution tools to obtain high quality solutions (Archetti and Speranza, 2014). Puchinger and Raidl (2005), Doerner and Schmid (2010), and Ball (2005) propose classification schemes for combinations of heuristics and exact algorithms. Archetti and Speranza (2014) develop their own classification scheme based on the recent contributions back then and propose the classification below:

1- Decomposition approaches

2- Improvement heuristics

3- Branch-and-price / column generation – based approaches

Decomposition approaches are basically built on generating subproblems which are easier to be handled when compared to the original problem. Subproblems are solved independently and a final feasible solution for the original problem is obtained from the solutions of subproblems.

Alternative algorithms to solve large scale optimization problems are presented by Nemhauser and Wolsey (2014). In our study, we use RHA to mitigate the complexity of our problem.

4.1. Rolling Horizon Approach (RHA)

RHA is one of the decomposition approaches which is applied to the problems having multi-period environment. Rationale behind RHA is to solve subproblems, that are generated by temporal decomposition, sequentially and consider the solutions of previous subproblems as fixed/frozen in the solution of each subproblem. Agra et al. (2014), Merce and Fontan (2003), Araujo et al. (2007), Rakke et al. (2011) provide implementations of RHA in lot-sizing, routing and scheduling problems. Bredström and Rönnqvist (2006) provide one of the rare applications of RHA in supply chain problems. RHA is a generic idea for problems in which decisions are made over time, and this leads to the diversity of the fields RHA is applied to. It is also applicable to multi-period facility location problems. A brief explanation of RHA is given in the next section. Computational efficiency of RHA in our problem context is shown in Chapter 5.

4.1.1. Framework of Rolling Horizon Approach

Main idea of RHA is to solve subproblems with shorter horizons iteratively (Rakke et al., 2011). As it is specified above, RHA is a generic approach and applied to various fields like scheduling, lot-sizing and routing. The characterization of the approach dependent on the problem is how to divide the original problem into subproblems, in other words, to determine the length of the horizon of subproblems.

Four notions should be introduced to explain RHA: frozen period, central period, forecasting period and out-of-scope period. These four periods are adjacent to each other and integration of them successively is equal to the planning horizon of the original problem. The order of these four periods can be seen in Figure 4.1.

In each iteration of RHA, we solve a subproblem considering the central period and forecasting period subject to the fixed decisions in the frozen periods, while out-of-scope period is totally ignored. Union of central and forecasting period is the planning horizon of the subproblem. The difference between central and forecasting period is that integrality constraints in the forecasting period are relaxed. Thus, the part of the forecasting period in the subproblem solution can be infeasible with respect to the constraints in the original problem. After solving a particular iteration, central and forecasting periods are shifted to adjacent periods in the next iteration while central period is merged to frozen periods – and the decisions in the central period are fixed-, forecasting period becomes central period and a part of out-of-scope period becomes the new forecasting period. As a result, the length of the central period determines the number of subproblems to be solved. As the length of the central and forecasting period increases, the accuracy of RHA tends to be higher while computation time of the entire approach strictly depends on the length of the horizons. Thus, trade-off between solution quality and computational time must be evaluated by decision makers.

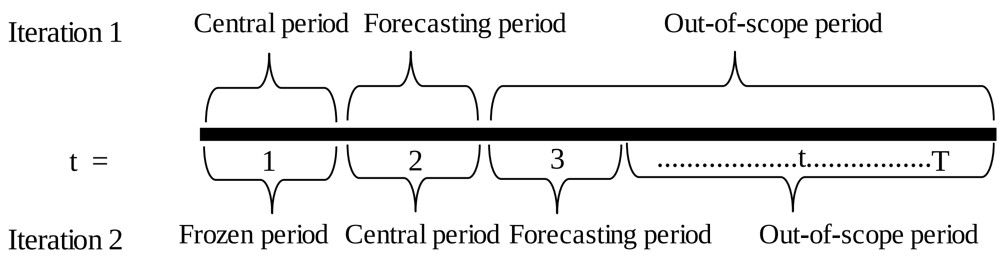

Figure 4.1. Iterative procedure of RHA

In Figure 4.2, standard RHA is presented. Variants of this standard approach can be generated by making minor adjustments. Merce and Fontan (2003) propose to freeze only a certain part of the decision variables in central periods (i.e. freeze all binary decision variables except capacity expansion variables in our model). Another tuning action can be to maintain a certain part of binary constraints in forecasting periods.

The complexity of the original problem derives from the wide extent due to its multi-echelon and multi-period characteristics. Without any mitigating actions in these aspects, the original problem remains laborious. For this purpose, RHA is applied to decompose the original problem into smaller subproblems in temporal dimension.

Original problem is a two-echelon CFLP in disregard of multi-period property which is NP-Hard and difficult to be handled by its own nature. Thus, the main intention of RHA is to decompose the whole model into subproblems as small as possible without a significant loss from solution quality. To this end, length of central periods and forecasting periods are settled as to have one period (l^c=1, l^f=1) while the length of the entire planning horizon has five periods.

In Chapter 5, computational results show that overall loss in solution quality is acceptable under these settlements.

t^{d1}: beginning period of frozen periods, t^{d2}: end period of frozen periods

t^{c1}: beginning period of central periods, t^{c2}: end period of central periods

t^{f1}: beginning period of forecasting periods, t^{f2}: end period of foracasting periods

l^{c} : length of the central periods

l^{f} : length of the forecasting periods

$t^{d1} = \varnothing$, $t^{d2} = \varnothing$

$t^{c1} = 1$, $t^{c2} = t^{c1} + l^{c} - 1$

$t^{f1} = t^{c2} + 1$, $t^{f2} = t^{f1} + l^{f} - 1$

s=1

t=1

ns = number of subproblems to be solved

$ns = \lceil T / l^{c} \rceil$

while $s < ns$

Relax binary constraints of $Y_{kt} \forall t \in [t^{f1}, t^{f2}]$, $Z_{mt} \forall t \in [t^{f1}, t^{f2}]$, $e_{mt} \forall t \in [t^{f1}, t^{f2}]$

Solve P with time periods $[t^{c1}, t^{f2}]$

t^{d1}=1, t^{d2}=t^{c2}

Freeze Y_{kt} $\forall t \in [t^{d1}, t^{d2}]$, Z_{mt} $\forall t \in [t^{d1}, t^{d2}]$, e_{mt} $\forall t \in [t^{d1}, t^{d2}]$

$t^{c1} = t^{c2} + 1$, $t^{c2} = t^{c1} + l^{c} - 1$

$t^{f1} = t^{c2} + 1$, $t^{f2} = \min(t^{f1} + l^{f} - 1, T)$

$s = s+1$

end

Figure 4.2. Standard RHA

4.2. Lagrangean Relaxation (LR)

LR is one of the most-preferred heuristic algorithms due to its computational efficiency and varied applicability. LR enables to relax hard/complicating constraints (or "linking constraints" which ensures the unity of the problems and hinders the partition of the problems into independent subproblems) and allows generating subproblems that can be handled easily compared to the original problems. Although its applications vary based on problem characteristics, general procedure is identical and consists of three parts: generation of lower bound, generation of upper bound and update of the Lagrange multipliers.

The relaxed problem always has an objective function value that is less (higher) than or equal to the optimal solution of the original problem in minimization (maximization) problems, since the problem becomes more flexible due to the relaxations of a set of constraints. Thus, the relaxed problem provides lower bound (upper bound) to the optimal solution. Since our original problem P is a minimization problem, the relaxed problem provides lower bound for it.

Second part of LR is to obtain an upper bound to the original problem. To use LR as nested in other exact or heuristic algorithms is very common in the literature. Most-preferred hybrid algorithm including LR is B&B nesting Lagrangean which has examples by Tragantalerngsak et al. (1997), Gendron et al. (2013) and Fisher (1981). In B&B approach, Lagrangean is used to improve bounds of the optimal solution and accelerate the algorithm. In such cases where Lagrangean is nested in another algorithm that provides an upper bound, upper bound generation can be redundant. However, in a standard LR, upper bound generation is required. Since the lower bound obtained by the relaxation is generally infeasible, a primal heuristic must be developed to produce an upper bound based on the results of the relaxed problem.

The third and last part of the Lagrangean is to update the Lagrange multipliers. To calculate the objective function of the relaxed problem, the problem should be concave nondifferentiable since Lagrange multipliers and decision variables in the relaxed constraints are multiplied in the objective function. Considering the complexity of such problems, it is prevalent to determine Lagrange multipliers separately. Performance of the Lagrangean relaxation is very dependent on the closeness of the Lagrange multipliers to the optimal multiplier values since the lower bound would not converge to the upper bound until appropriate multipliers are obtained which can be difficult due to numerous possibilities. There are various algorithms used to update Lagrange multipliers such as subgradient optimization (Naum, 1985), bundle method (Crainic et al., 2001) and multiple adjustment method (Fisher et al., 1986).

Subgradient optimization is more common than the other update methods since it is easy to implement and it provides satisfying solutions. Thus, proposed LR in this paper uses subgradient optimization too.

4.2.1. Framework of Lagrangean Relaxation

In combinatorial optimization problems, it can be difficult to obtain optimal solutions via commercial optimization tools or exact/heuristic algorithms handling the problem as a whole. Accordingly, decomposition approaches can be useful in such cases. LR is a method of decomposition; constraints are partitioned into two groups: hard constraints and easy constraints. Hard constraints are removed from the original problem and transferred into the objective function by being penalized them with proper weights (Ahuja et al., 1993). The original problem is reduced to problems which are easier to be solved and have structures for which there are satisfactory solution methods. An illustration of LR is as follows:

$$\text{Z} = \text{Minimize } \; c\,x \tag{2.1}$$

s.to

$$A\,x = b \tag{2.2}$$

$$B\,x \le d \tag{2.3}$$

$$x \ge 0 \;\text{ and integer} \tag{2.4}$$

In Problem Z, constraint (2.2) is an equality constraint and can be classified as “hard constraint” whereas inequality constraint (2.3) can be labeled as “easy constraint”. Rationale behind LR is to relax hard constraint which is the considerable reason of the problem complexity. The relaxed constraint is represented in the objective function with a Lagrangean multiplier, $\lambda \ge 0$. The problem after relaxation appears as:

$$\text{Z}_{LR} = \text{Minimize } \; c\,x + \lambda * (b - A\,x) \tag{2.5}$$

s.to

$$B\,x \le d \tag{2.6}$$

$$\lambda \geq 0 \tag{2.7}$$

$$x \geq 0 \text{ and integer} \tag{2.8}$$

The optimal solution of Z_{LR} can be reached by finding optimal Lagrangean multiplier values. Solutions to Z_{LR} serve as lower bound to the original problem since a set of its constraints is relaxed and solution space is enlarged.

4.2.2. Issues of Lagrangean Relaxation

As it is stated in the previous sections, LR is a solution method which is very generic and applicable to various types of problems. Although the main procedure is explicit, there are two major issues to be tackled to reach a proper lower bound in reasonable time (Beasley, 1996):

1- Tactical issue: How will Lagrange multipliers be updated to provide a fast convergence to the optimal solution of the relaxed problem?

2- Strategic issue: Which sets of constraints should be chosen to be relaxed to obtain a proper lower bound?

4.2.2.1. The Tactical Issue

The classical approach of maximizing the objective function would be the "steepest ascent method" if the objective function of the relaxed problem were differentiable. Unfortunately, this method is not valid in general since objective function is not differentiable everywhere, especially at the optimal point (Bertsimas and Tsitsiklis, 1997). Due to the complexity brought forth by nondifferentiability, determination of Lagrange multipliers is detached from the relaxed problem and solved as a separate problem in general.

One of the most-preferred methods to update Lagrange multipliers is the subgradient optimization. It is an extension of the gradient optimization to tackle the nondifferentiability problem. Subgradient optimization and its variants have remained as one of the most-preferred and effective methods to update Lagrange multipliers for decades. Sherali and Myers (1988) stated that subgradient optimization can be quite adequate when the relaxed problem has a well-known problem structure. Since then, many contributions have been made proving efficient convergence properties of the subgradient optimization (Nemhauser and Wolsey, 2014).

One of the most popular drawbacks of the subgradient optimization is its zig-zag pattern in the iterations. Many studies have been conducted to modify classic subgradient optimization to overcome this pattern (Brannlund, 2001) as giving priority to the recent iterations while updating Lagrange multipliers. Yet, classic subgradient optimization is still appealing to the researchers due to its simple implementation and computational efficiency.

4.2.2.2. The Strategic Issue

The structure of the relaxed problem is totally dependent on the set of constraints to be relaxed. Different constraints to be relaxed yield various relaxed problems differentiating from each other in the tightness of lower bounds and computation efforts required. In the illustration of LR in Section 4.2.1, hard constraints appear to be equality constraints and this is not always the case. Equality constraints may not be as decisive and complicating as inequality constraints; or, more than one constraint should be relaxed in some cases. Therefore, the set of constraints to be relaxed is not always clear.

LR is one of the oldest algorithms still used in combinatorial optimization. Since the introduction of LR, studies are conducted to develop modified versions (variants) of traditional Lagrangean. One of these studies belongs to Guignard and Kim (1987) proposing Lagrangean Decomposition (LD). LD assigns different decision variables for different sets of constraints and adds artificial constraints to the relaxed problem ensuring that decision variables assigned will be equal.
With the introduction of decision variables assigned, proposed model becomes:

$$\text{P} = \text{Minimize } c\,x \tag{2.9}$$

s.to

$$A\,x = b \tag{2.10}$$

$$B\,y \leq d \tag{2.11}$$

$$x = y \tag{2.12}$$

$$x \geq 0 \text{ and integer} \tag{2.13}$$

$$y \geq 0 \text{ and integer} \tag{2.14}$$

The relaxation of the constraint (2.12) enables Z_{LR} to be divided into subproblems where easy constraints and hard constraints are handled separately. After the relaxation of the constraints (2.12), Lagrangean model becomes:

$$Z_{LB} = \text{Minimize } cx + \lambda(x - y) \tag{2.15}$$

s.to

$$Ax = b \tag{2.16}$$

$$By \leq d \tag{2.17}$$

$$x \geq 0 \text{ and integer} \tag{2.18}$$

$$y \geq 0 \text{ and integer} \tag{2.19}$$

$$\lambda \text{ unrestricted} \tag{2.20}$$

Beltran (2004) proposed a modified Lagrangean called "semi-Lagrangean relaxation" which majors on equality constraints. Equality constraints are transformed into inequality constraints by adding a "less than or equal to" and a "greater than or equal to" constraint. Then, one of these two new inequality constraints is relaxed.

Employment of semi-Lagrangean relaxation on our illustration model leads to the model below:

$$P = \text{Minimize } cx \tag{2.21}$$

s.to

$$Ax \leq b \tag{2.22}$$

$$Ax \geq b \tag{2.23}$$

$$By \leq d \tag{2.24}$$

$$x \geq 0 \text{ and integer} \tag{2.25}$$

$$y \geq 0 \text{ and integer} \tag{2.26}$$

A partial relaxation of the equality constraint, $Ax=b$, can be achieved by relaxing the constraint (2.23). In this case, Lagrangean model becomes:

$$Z_{LB} = \text{Minimize } cx + \lambda * (b - Ax) \tag{2.27}$$

s.to

$$Ax \leq b \tag{2.28}$$

$$By \leq d \tag{2.29}$$

$$x \geq 0 \text{ and integer} \tag{2.30}$$

$$y \geq 0 \text{ and integer} \tag{2.31}$$

$$\lambda \geq 0 \tag{2.32}$$

Variants of LR increase relaxation alternatives to be evaluated in the aspect of solution quality and computational time. As Geoffrion and McBride (1978) stated, in general, relaxed problems that can be solved easily yield worse lower bounds than the problems requiring more computational efforts. Researchers face the trade-off between solution quality and computational time, and make the decision based on the main objective of the research.

4.2.3. Lagrangean Relaxation in Our Model

There are seven sets of constraints excluding binary and nonnegativity constraints in our model. Even if LR methods like LD and semi-Lagrangean are ignored, traditional LR provides (2^7-1) candidate sets of constraints that can be relaxed. Although many of these sets are not worth to be evaluated since they do not lessen the complexity of the problem significantly, constraints to be relaxed are not clear and candidate sets of constraints should be evaluated.

To relax constraint (1.2) does not change the structure of the model and results in the same problem P except the fact that returns at collection centers do not have to be sent to disassembly centers anymore. Relaxation of constraint (1.3) excludes capacity expansion opportunity and initial capacities cannot change over the planning horizon. After relaxing constraint (1.4) in our model, relaxed problem becomes multi-period two-echelon uncapacitated facility location problem. To relax constraints (1.5) enables the original problem to be partitioned into two subproblems: multi-period single echelon facility location problem with capacity expansion, and an easy problem associated with consolidation layer. Relaxation of constraint (1.7) does not provide any convenience for the solution of the original problem. Constraint (1.8) and (1.9) guarantee that consolidation and disassembly centers cannot be closed once they are opened, respectively. In the absence of these constraints, "reopen/close facility" possibilities appear in the model. All of the relaxations presented except the relaxation of constraint (1.5) do not provide the decomposition of the original problem into smaller subproblems. On the other hand, relaxation of the constraint (1.5) seems promising:

$$\begin{aligned} \mathrm{P}^{LR} = \text{Minimize} & \sum_{j\in J}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} X_{jkmt}\, s_{jt}\, d_{jkm} + \sum_{k\in K}\sum_{t\in T} Y_{kt}\ f1_k \\ & + \sum_{m\in M}\sum_{t\in T} Z_{mt}\ f2_m + \sum_{m\in M}\sum_{t\in T} e_{mt}\ ec_{mt} \\ & + \sum_{j\in J}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} \lambda_{jkmt}\,(X_{jkmt} - Y_{kt}) \end{aligned} \qquad (2.33)$$

s.to

$$\sum_{k\in K}\sum_{m\in M} X_{jkmt}\, cv_{jkm} = 1 \qquad \forall\ j\in J,\ t\in T \qquad (2.34)$$

$$I_{m,t-1} + e_{mt}\ p_m = I_{mt} \qquad \forall\, m\in M,\, t\in T \qquad (2.35)$$

$$\sum_{j\in J}\sum_{k\in K} X_{jkmt}\, s_{jt} \le\ I_{mt} \qquad \forall\, m\in M,\, t\in T \qquad (2.36)$$

$$X_{jkmt} \le Z_{mt} \qquad \forall\ j\in J,\, k\in K,\, m\in M,\, t\in T \qquad (2.37)$$

$$Y_{kt} \le Y_{k,t+1} \qquad \forall\, k\in K,\, t\in T \qquad (2.38)$$

$$Z_{mt} \le Z_{m,t+1} \qquad \forall m \in M, t \in T \qquad (2.39)$$

$$X_{jkmt} \ge 0, Y_{kt} \in \{0,1\}, Z_{mt} \in \{0,1\}, e_{mt} \in \{0,1\} \qquad \forall j \in J, k \in K, m \in M, t \in T \qquad (2.40)$$

$$\lambda_{jkmt} \ge 0 \qquad \forall j \in J, k \in K, m \in M, t \in T \qquad (2.41)$$

In the absence of the constraint (1.5), original problem can be partitioned into two independent problems associated with consolidation centers and disassembly centers, respectively. The problem related to consolidation centers (P^c) is easy to be solved whereas the latter problem (P^d) is a multi-period single-echelon dynamic capacitated FLP.

Subproblems P^d and P^c are as follows:

$$\begin{aligned} \mathrm{P}^d = \text{Minimize} & \sum_{j\in J}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} X_{jkmt}\, s_{jt}\, d_{jkm} + \sum_{m\in M}\sum_{t\in T} Z_{mt}\ f2_m \\ & + \sum_{m\in M}\sum_{t\in T} e_{mt}\ ec_{mt} + \sum_{j\in J}\sum_{k\in K}\sum_{m\in M}\sum_{t\in T} \lambda_{jkmt}\ X_{jkmt} \end{aligned} \qquad (2.42)$$

s.to

$$\sum_{k\in K}\sum_{m\in M} X_{jkmt}\, cv_{jkm} = 1 \qquad \forall j \in J, t \in T \qquad (2.43)$$

$$I_{m,t-1} + e_{mt}\ p_m = I_{mt} \qquad \forall m \in M, t \in T \qquad (2.44)$$

$$\sum_{j\in J}\sum_{k\in K} X_{jkmt}\, s_{jt} \le I_{mt} \qquad \forall m \in M, t \in T \qquad (2.45)$$

$$X_{jkmt} \le Z_{mt} \qquad \forall j \in J, k \in K, m \in M, t \in T \qquad (2.46)$$

$$Z_{mt} \le Z_{m,t+1} \qquad \forall m \in M, t \in T \qquad (2.47)$$

$$X_{jkmt} \ge 0,\ Z_{mt} \in \{0,1\},\ e_{mt} \in \{0,1\} \qquad \forall j \in J, k \in K, m \in M, t \in T \qquad (2.48)$$

$$\lambda_{jkmt} \ge 0 \qquad \forall j \in J, k \in K, m \in M, t \in T \qquad (2.49)$$

$$\mathrm{P}^{c} = \text{Minimize} \sum_{k \in K} \sum_{t \in T} Y_{kt}\ f1_{k} - \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} \lambda_{jkmt}\, Y_{kt} \tag{2.50}$$

$$Y_{kt} \leq Y_{k,t+1} \qquad \forall\, k \in K, t \in T \tag{2.51}$$

$$Y_{kt} \geq 0 \qquad \forall\, k \in K, t \in T \tag{2.52}$$

$$\lambda_{jkmt} \geq 0 \qquad \forall\ j \in J, k \in K, m \in M, t \in T \tag{2.53}$$

P^{c} can be handled easily by decomposing into smaller problems for each consolidation center $k, k \in K$. Yet, P^{d} is still difficult to be solved. A further relaxation of a constraint in addition to constraint (1.5) can be a practical action to be able to tackle P^{d} more comfortably. Tragantalerngsak et al. (1997) compared the relaxation of different sets of constraints belonging to a two-echelon CFLP which is similar to our original problem. Authors showed that to relax demand satisfaction constraint and the constraint linking echelons simultaneously outperforms other relaxation alternatives. Pirkul and Jayaraman (1998) preferred to relax similar constraints and showed the computational efficiency of the relaxation. As other similar examples in the literature, to relax constraint (1.2) and (1.5) in P simultaneously seems quite promising for our model too. Thus, we chose to relax these two constraints. After the relaxation, the relaxed problem P^{d} becomes as follows:

$$\begin{aligned} \mathrm{P}^{d} = \text{Minimize} & \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} X_{jkmt}\, s_{jt}\, d_{jkm} + \sum_{m \in M} \sum_{t \in T} Z_{mt}\ f2_{m} \\ & + \sum_{m \in M} \sum_{t \in T} e_{mt}\ ec_{mt} + \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} \lambda_{jkmt}\ X_{jkmt} \\ & - \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} \mu_{jt}\ X_{jkmt} \end{aligned} \tag{2.54}$$

s.to

$$\sum_{k \in K} \sum_{m \in M} X_{jkmt}\, cv_{jkm} = 1 \qquad \forall\ j \in J,\ t \in T \tag{2.55}$$

$$I_{m,t-1} + e_{mt}\ p_{m} = I_{mt} \qquad \forall\, m \in M,\, t \in T \tag{2.56}$$

$$\sum_{j\in J}\sum_{k\in K} X_{jkmt}\, s_{jt} \le I_{mt} \qquad \forall\, m\in M,\ t\in T \qquad (2.57)$$

$$X_{jkmt} \le Z_{mt} \qquad \forall\, j\in J,\ k\in K,\ m\in M,\ t\in T \quad (2.58)$$

$$Z_{mt} \le Z_{m,t+1} \qquad \forall\, m\in M,\ t\in T \qquad (2.59)$$

$$X_{jkmt} \ge 0,\ \ Z_{mt}\in\{0,1\},\ \ e_{mt}\in\{0,1\} \qquad \forall\, j\in J,\ k\in K,\ m\in M,\ t\in T \quad (2.60)$$

$$\lambda_{jkmt} \ge 0,\ \ \mu_{jt} \ge 0 \qquad \forall\, j\in J,\ k\in K,\ m\in M,\ t\in T \quad (2.61)$$

The relaxation of the constraint (1.5) already enabled the original problem to be divided into two independent subproblems: P^{c} and P^{d}. This structure still holds after the relaxation of the constraints (1.2) while it is also possible to divide P^{d} into $|M|$ subproblems handling P^{d} separately for each disassembly center thanks to this additional relaxation. Then, we can denote subproblem P^{d} solved for disassembly center m as P^{d}_{m}.

P^{d} and P^{c} are solved independently and sum of the objective functions of these two problems is equal to the objective function of P^{LR}. Objective function value of P^{LR} at iteration k with multipliers λ^{k} and μ^{k} is as follows:

$$\mathrm{P}^{LR}\left(\lambda^{k}, \mu^{k}\right) = \mathrm{P}^{d}\left(\lambda^{k}, \mu^{k}\right) + \mathrm{P}^{c}(\lambda^{k}) \quad \text{where} \quad \mathrm{P}^{d} = \sum_{m\in M} \mathrm{P}^{d}_{m}$$

Lagrangean problem is solved in an iterative manner and as it is stated above, Lagrangean multipliers are determined separately from subproblems using subgradient optimization. Therefore, constant parts of the objective functions, i.e., parts that do not consist of any decision variables, can be excluded from the objective functions. As a result, objective function value of P^{LR} becomes:

$$\mathrm{P}^{LR}\left(\lambda^{k}, \mu^{k}\right) = \mathrm{P}^{d}\left(\lambda^{k}, \mu^{k}\right) + \mathrm{P}^{c}(\lambda^{k}) + \sum_{j\in J}\sum_{t\in T} \mu^{k}_{jt}$$

Subgradient optimization iteratively converges through optimal Lagrangean multipliers and lower bound to the original problem is the maximum value achieved by the relaxed problem among all iterations.

$$\text{LB}\left(\lambda^{*}, \mu^{*}\right) = \underset{k}{\text{Max}}\left\{\text{P}^{LR}\left(\lambda^{k}, \mu^{k}\right)\right\}$$

Relaxation of constraints (1.2) and (1.5) provides significant convenience to be able to solve the original problem. Yet, meanwhile, it enlarges the solution space and disregards the relaxed constraints in a sense. Addition of surrogate constraints for these relaxed constraints, which are called as "valid inequalities" in the literature, can undertake the role of disregarded constraints to a certain degree. Surrogate constraints ("valid inequality" term will be used instead of "surrogate constraint" in the rest of the study to avoid misleading since "surrogate constraint" term is used in different manners in the literature) enable to benefit from the relaxations in tighter dual subproblems. The important point in the selection of valid inequalities is to resemble the relaxed constraints as much as possible, while the structure of dual subproblems is not violated.

Nemhauser and Wolsey (2014) cite that to develop strong valid inequalities having a substantial effect on the solution can be compelling. For example, after the relaxation of constraint (1.2), there is no obligation to transport any of the returns at collection centers to disassembly centers and it is possible to determine that none of the consolidation and disassembly centers should be opened. To avoid such a result, valid inequalities ensuring that each collection center must be covered by at least one pair of (open consolidation centers, open disassembly centers) can be inserted into the relaxed model ("Coverage" term is referred to many times in this paper. To clarify this notion, let's say $Cover_{jt} = \left\{d_{jkm} \leq TH \text{ where } Y_{kt} = 1 \text{ and } Z_{mt} = 1\right\}$. If $Cover_{jt} \neq \varnothing$, it means that collection center *j* is covered in period *t*). However, the insertion of such a valid inequality would increase the complexity of subproblems remarkably since subproblems would encompass set covering problem in this case which is difficult to be solved on its own. Heuristic algorithms can be integrated into the algorithm to overcome the complexity of the subproblem after valid inequalities but the objective function of dual subproblems would not necessarily serve as a lower bound in that case (Desaulniers et al., 2006). Preliminary experiments are executed to observe the effect of such a valid inequality by developing a heuristic algorithm and no substantial enhancement is observed. Although it is difficult to guarantee that each collection center is covered by at least one pair of (open consolidation centers, open disassembly centers), there are useful valid inequalities that can be embedded into the Lagrangean model easily.

Coverage of all collection centers may not be satisfied in dual subproblems. Yet, to have at least one open consolidation center and disassembly center can be ensured via the following valid inequalities:

$$\sum_{k \in K} Y_{kt} \geq 1 \tag{2.62}$$

$$\sum_{m \in M} Z_{mt} \geq 1 \tag{2.63}$$

Relaxed constraint (1.2) ensures that all returns at collection centers are transported to disassembly centers. In the absence of this constraint and constraint (1.5) as well, each disassembly center is evaluated independently, while there is no guarantee that constraint (1.2) is satisfied. Therefore, it is possible to have X_{jkmt} equal to 0 for all $j \in J$, $k \in K$, $t \in T$ and a particular $m \in M$. On the other hand, it is also possible to have X_{jkmt} equal to 1 for all $j \in J$, $k \in K$, $t \in T$ and a particular $m \in M$. In the latter case, infeasibility occurs due to over-satisfaction of the constraint (1.2). Since each disassembly center is handled independently, we do not have absolute control on the returns' flows. Yet, a limitation associated with returns' flows can be introduced for each disassembly center as follows:

$$\sum_{k \in K} X_{jkmt} \leq 1 \tag{2.64}$$

Constraint (2.64) avoids returns sent from collection center $j \in J$ to disassembly center $m \in M$ to exceed total returns at corresponding collection center j.

4.2.3.1. Solution Methodology of First Subproblem P^{d}

Subproblem P^{d} determines the flows from collection centers to disassembly centers through consolidation centers along with the decisions related to disassembly centers to be opened. Each disassembly center $m \in M$ has a limited capacity which can be expanded at the beginning of each period and inbound flows to disassembly centers are limited by their capacitites. In the presence of constraint (1.2), total inbound flows to disassembly centers originating from a particular collection center $j \in J$ must be equal to total amount of returns at corresponding collection center j. Yet, after the relaxation of constraint (1.2) and introduction of valid inequality (2.64), this limitation transforms so that inbound flows to a disassembly center $m \in M$ coming from collection center $j \in J$ cannot be greater than total amount of returns at corresponding collection center j. Therefore, inbound flows to a disassembly center $m \in M$ are not dependent on the flows to other disassembly centers and in result, each disassembly center can be handled independently if constraint (1.2), which is a linking constraint between both consolidation and disassembly centers, is ignored. Subproblem P^d related to disassembly center m can be labeled as P_m^d .

P_m^d evaluates whether to open disassembly center m or not. Further evaluation is executed to assess gainings of a capacity expansion if disassembly center m is determined to be opened. In the evaluation of disassembly center $m \in M$, the benefits of opening disassembly center m are compared with the fixed cost of the corresponding disassembly center m. If the benefits compensate for the required cost for them, final decision turns out to be opening disassembly center m.

$$\text{Minimize} \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} X_{jkmt} \, (s_{jt} \, d_{jkm} + \lambda_{jkmt} - \mu_{jt}) + \sum_{m \in M} \sum_{t \in T} Z_{mt} \, f2_m + \sum_{m \in M} \sum_{t \in T} e_{mt} \, ec_{mt} \tag{2.65}$$

Objective function (1.1) can be transformed to (2.65). $(s_{jt} \, d_{jkm} + \lambda_{jkmt} - \mu_{jt})$ is the weight of X_{jkmt} on the objective function which can be denoted as w_{jkmt}. X_{jkmt} values are determined based on the rank of their weights under the limitation of the capacity. The structure of subproblem P_m^d is known as the “knapsack problem” in the literature. Basic knapsack problem can be modeled as:

$$\text{Min } c\,x \tag{2.66}$$

$$wx \leq b \tag{2.67}$$

$$x \in \{0,1\} \tag{2.68}$$

Martello and Toth (1990) study various versions of the knapsack problem like 0-1 Knapsack, 0-1 Multiple Knapsack and Bin-Packing Problem and propose different algorithms to these problems. Versions studied by Martello and Toth (1990) are very similar to subproblem P_m^d with a significant difference: X variable in knapsack problems is constrained to be integer, or binary in general, whereas X variable in subproblem P_m^d is continuous. In the absence of integrality constraint on X variable, knapsack problem is called as "fractional knapsack problem" (Goodrich and Tamassia, 2002). Fractional knapsack problem and greedy algorithm regarding it are very well known. Greedy algorithm for fractional knapsack problem is simply based on the comparison between the weight of X variables on the objective function and the amount that they consume from the total capacity.

Ultimate objective of subproblem P_m^d is of minimization type. Thus, X_{jkmt} should be equal to 0, if w_{jkmt} is greater than or equal to 0, since a positive value of X_{jkmt} would not make any contribution to the objective function. Since we know that X_{jkmt} variable consumes the capacity of disassembly center m in proportion to s_{jt}, marginal benefit of X_{jkmt} (mb_{jkmt}) to the objective function considering the capacity constraint of disassembly center m in period t can be calculated as:

$$mb_{jkmt} = w_{jkmt} \,/\, s_{jt}$$

The ascending order of mb_{jkmt} for $m \in M$ in period $t \in T$ can be shown as below:

$$mb_{jkmt}{}^{(1)} \leq mb_{jkmt}{}^{(2)} \leq mb_{jkmt}{}^{(3)} \leq mb_{jkmt}{}^{(4)} \leq \ldots\ldots \leq mb_{jkmt}{}^{(|J|*|K|)}$$

$mb_{jkmt}{}^{(1)}$ has the biggest marginal benefit on the objective function of subproblem P_m^d since it is a minimization problem. So, primal choice for inbound flow to disassembly center m in period t should be flow from collection center j through consolidation center k where $\left\{(j,k) \mid mb_{jkmt} = mb_{jkmt}{}^{(1)}\right\}$. In this fashion, capacity of disassembly center m in period t should be consumed from $mb_{jkmt}{}^{(1)}$ to $mb_{jkmt}{}^{(|J|*|K|)}$

in order until marginal benefit becomes nonnegative or capacity of disassembly center m in period t is fully utilized.

At this point, it should be noted that constraint (1.1) indicates X_{jkmt} must be equal to 0 if cv_{jkm} is equal to 0. Thus, w_{jkmt} can be revised as $w_{jkmt}\, cv_{jkm}$. In result, weight of X_{jkmt} variable becomes 0, as well as mb_{jkmt}, if cv_{jkm} is equal to 0. As following, there cannot be any flows from collection center j to disassembly center m through consolidation k where $\{(j,k) \mid cv_{jkm} = 0\}$ since it is not feasible.

If we presume that disassembly center $m \in M$ is open in period $t \in T$, then, assignments denoted by X_{jkmt} can be determined as it is shown in Figure 4.3.

FA :

$R_{mt} = I_{m0}$

for $m \leftarrow 1$ to $|M|$

for $t \leftarrow 1$ to 2

Order mb_{jkmt} in ascending order, set $i = 1$

while $R_{mt} > 0$ and $mb_{jkmt} < 0$

$$\left.\begin{aligned} X_{jkmt} &= \min(R_{mt} / s_{jt}, 1) \\ R_{mt} &= R_{mt} - X_{jkmt}\, s_{jt} \end{aligned}\right\} \quad \text{where} \quad \left\{ (j,k) \mid mb_{jkmt} = mb_{jkmt}^{(i)} \right\}$$

$i = i + 1$

end

end

end

Figure 4.3. Determination of assignment values X_{jkmt}

It should be noted that LR is employed in each RHA subproblem where central period and forecasting period are chosen to be one only. Consequently, time horizon of LR becomes equal to two and our proposed solution algorithm for LR explained in the following sections is regarding problem P^d with a reduced time horizon consisting of period $t \in T$ where $|T|=2$.

Then, the contribution derived from opening disassembly center $m \in M$ in period $t \in T$ to the objective function would be as:

$$kd_{mt} = \sum_{j \in J} \sum_{k \in K} \sum_{t}^{2} w_{jkmt} X_{jkmt}$$

kd_{m1} represents how the objective function of the Lagrangean problem P^{LR} would be influenced by opening disassembly center $m \in M$ in the first period (central period), while kd_{m2} shows the effect of opening disassembly center $m \in M$ in the second period (forecasting period) on the objective function of the Lagrangean problem P^{LR}.

kd_{m1} includes the effect of the assignments in the second period ($w_{jkm2} X_{jkm2}$) as well, since constraint (10.8) forces disassembly centers to remain open until the end of the planning horizon once they are open. Thus, to open disassembly center $m \in M$ in the first period means that it has to be open in the second period too. In the end, entire RHA subproblem should be solved in evaluating to open disassembly centers in central period.

Along with the benefits, there is a fixed cost associated with disassembly centers to be operated. Then, net benefits of disassembly centers become:

$$nk_{mt} = kd_{mt} + \sum_{t}^{2} f1_{mt}$$

Disassembly center $m \in M$ is worth opening only if its positive effects outrun the associated cost with it, in other words, if nk_{mt} has a negative value. The relevant decision process OD is presented in Figure 4.4.

nk_{m1} denotes the net benefit/effect of opening disassembly center m in the central period to the objective function of P^{LR}. Since P^{LR} is a minimization problem, nk_{m1} has to be negative to determine disassembly center m to be opened in the central period. Yet, this is not sufficient to make a judgment regarding the central period due to the fact that disassembly centers opened in the central period have to remain open in the forecasting period and decisions associated with the central period have to consider the effects on the forecasting period too. Therefore, $nk_{m1} + nk_{m2}$ should be negative too if disassembly center m would be opened in the central period. In sum, the central period decisions are evaluated as isolated from the other periods as well as by the total effect on the planning horizon of P^{LR} in the decision making scheme OD.

OD :

```
for m ← 1 to |M|
     if       nk_m1 < 0   and   nk_m1 + nk_m2 < 0
              Z_m1 = 1  and  Z_m2 = 1
    elseif    nk_m2 < 0
              Z_m2 = 1
    endif
end
```

Figure 4.4. Evaluation of disassembly centers to be opened

After the decision making scheme OD, disassembly centers to be opened and when to open them are determined. In the next step, capacity expansion opportunity should be evaluated for open disassembly centers.

Procedure FA manages initial capacities of disassembly centers to be utilized in the most effective way, as assignments are determined based on marginal benefits in ascending order until no more gain is achieved by further assignments or capacities are fully utilized. As it is noted in 8th clause in Section 3.2.1 (Assumptions of the Model), disassembly centers can be expanded by the exact pre-specified amount, p_m, at the beginning of each period. For the evaluation of capacity expansions, we presume that all disassembly centers determined to be opened by scheme OD are expanded. Then, expanded capacity is utilized and new assignments are done in a similar fashion with procedure FA. Afterwards, the effects of new assignments are calculated. Capacity expansion decisions rest upon the difference between expansion cost and the benefits derived from it.

In FA, assignments are constrained with the initial capacities whereas capacities to be utilized are equal to capacity expansion modules, p_m, in procedure CA in Figure 4.5.

CA :

$R_{mt} = p_m$

for $t \leftarrow 1$ to 2

for $m \in \{ m \mid Z_{mt} = 1 \}$

Order mb_{jkmt} in ascending order where $j \in \{ j \mid \sum_{k \in K} X_{jkmt} < 1 \}$, set $i = 1$;

while $R_{mt} > 0$ and $mb_{jkmt} < 0$

$$\left.\begin{aligned} X'_{jkmt} &= X_{jkmt} \\ X_{jkmt} &= \min\,(X_{jkmt} + R_{mt} / s_{jt}, 1) \\ X''_{jkmt} &= X_{jkmt} - X'_{jkmt} \\ R_{mt} &= R_{mt} - X_{jkmt}\, s_{jt} \end{aligned}\right\} \quad \text{where } \left\{ (j,k) \mid mb_{jkmt} = mb_{jkmt}{}^{(i)} \right\}$$

$i = i + 1$

end

end

end

Figure 4.5. Utilization of expanded capacity

Other difference between FA and CA is that FA is free in assignments since there are no prior assignments at the beginning of FA (all returns are waiting to be collected at collection centers). Yet, Procedure CA has to omit assignments X_{jkmt} where $\sum_{k \in K} X_{jkmt} = 1$ after FA is executed, since valid inequality constraint (2.64) limits returns' flows. Then, marginal benefits to be included in procedure CA are ordered as below:

$$mb_{jkmt}{}^{(1)} \le mb_{jkmt}{}^{(2)} \le mb_{jkmt}{}^{(3)} \le \ldots\ldots \qquad \text{where } j \in \{ j \mid \sum_{k \in K} X_{jkmt} < 1 \}$$

As it is shown in Figure 4.5, X'_{jkmt} is equal to the assignment variables prior to the capacity expansion and X''_{jkmt} calculates additional assignments deriving from the capacity expansions.

CA procedure determines the additional assignments that can be set if capacity expansions in central period or forecasting period in disassembly centers are realized.

Yet, CA procedure does not show the outcome of the realization of expansions in both central and forecasting period. In CA procedure, we calculate the effect of capacity expansion by p_m for central and forecasting period. Yet, we should consider the effect of capacity expansion by $2\,p_m$ in forecasting period (expansion in central period provides additional p_m capacity in forecasting period and if another expansion in forecasting period follows the expansion in central period, capacity in forecasting period would have a further increment by p_m) when both central and forecasting periods are exposed to capacity expansion. Then, the influence of further expansion p_m in the forecasting period can be calculated by CA2 procedure in Figure 4.6.

CA2 :

$R_{mt} = p_m$, $t = 2$

for $m \in \{ m \mid Z_{mt} = 1 \}$

Order mb_{jkmt} in ascending order where $j \in \{ j \mid \sum_{k \in K} X_{jkmt} < 1 \}$, set $i = 1$;

while $R_{mt} > 0$ and $mb_{jkmt} < 0$

$$\left. \begin{array}{l} \dot{X}_{jkmt} = X_{jkmt} \\ X_{jkmt} = \min(X_{jkmt} + R_{mt} / s_{jt}, 1) \\ \ddot{X}_{jkmt} = X_{jkmt} - \dot{X}_{jkmt} \\ R_{mt} = R_{mt} - X_{jkmt}\, s_{jt} \end{array} \right\} \quad \text{where} \quad \left\{ (j,k) \mid mb_{jkmt} = mb_{jkmt}^{(i)} \right\}$$

$i = i + 1$

end

end

end

Figure 4.6. Utilization of the additional capacity in the forecasting period

If we presume the realization of capacity expansion in the central period, X''_{jkm2} would be the influence of it on the assignment variables in the forecasting period. In the case of additional expansion in the forecasting period, $\ddot{X}_{jkm2}$ would present additional assignments emanating from the corresponding capacity increase. Then, we can present gross benefits of capacity expansions as:

$$ke_{m1} = \sum_{j \in J} \sum_{k \in K} \sum_{t}^{2} w_{jkmt} \, X''_{jkmt}$$

$$ke_{m2} = \sum_{j \in J} \sum_{k \in K} w_{jkm2} \, X''_{jkm2}$$

$$ke_{m3} = ke_{m1} + \sum_{j \in J} \sum_{k \in K} w_{jkm2} \, \ddot{X}_{jkm2}$$

where ke_{m1} represents the gross benefit derived from capacity expansion in the central period, ke_{m2} represents the gross benefit derived from capacity expansion in the forecasting period, and ke_{m3} represents the gross benefit obtained if capacity expansion in the central period is followed by an expansion in the forecasting period.

After all, final capacity expansion decisions can be taken based on the consideration of gross benefits of expansions and associated costs with them. Decision making scheme OC in Figure 4.7 shows the corresponding procedure.

At the beginning of OC scheme, we evaluate the capacity expansion in the central period. To expand capacity in the central period, gross benefit of capacity expansion in central period over the subproblem horizon (central and forecasting periods) must exceed the cost of expansion in the central period; in other terms, condition C1 must be satisfied.

C1: $ke_{m1} + ke_{m2} + ec_{m1} < 0$

Yet, this is not enough for expansion in the central period since gross benefits over the subproblem horizon may especially come from the gross benefit in the forecasting period (ke_{m2}) and it may be wiser to expand capacity in the forecasting period instead of central period. Therefore, we should also ensure that net benefit of expansion in the central period outweighs the cost of capacity expansion in the forecasting period.

C2: $ke_{m1} + (ec_{m1} - ec_{m2}) < 0$

If condition C2 is satisfied too, capacity expansion should be realized in the central

OC :

```
for m ∈ { m | Z_m1 = 1 }
      if        ke_m1 + ke_m2 + ec_m1 < 0  and  ke_m1 + (ec_m1 - ec_m2) < 0
                e_m1 = 1
      elseif    ke_m1 + ke_m2 + ec_m1 < 0
                if   ke_m3 + ec_m1 - ke_m2 < 0
                e_m1 = 1  and  e_m2 = 1
                else
                e_m2 = 1
                end
       elseif   ke_m2 + ec_m2 < 0
                e_m2 = 1
       endif
end
for m ∈ { m | Z_m2 = 1 }
      if        ke_m2 + ec_m2 < 0
                e_m2 = 1
      endif
end
```

Figure 4.7. Determination of capacity expansions for disassembly centers

period. Otherwise, it cannot be said necessarily that capacity expansion in the central period must be disregarded. We should consider the effect of the expansion in the central period on the forecasting period. Capacity expansion by $2\,p_m$ in the forecasting period can only be achieved by the realization of expansions in both periods. In the case of high benefits obtained from capacity expansion by $2\,p_m$ in the forecasting period, we should still consider the capacity expansion in the central period. In such a case, capacity can be expanded in the central period if its contribution over central and forecasting periods is more than the associated cost, in other words, if condition C3 is satisfied.

C3: $ke_{m3} + ec_{m1} - ke_{m2} < 0$

If C1 is satisfied where neither C2 nor C3 is satisfied, it should be deduced that condition C1 is satisfied mostly owing to the benefits of capacity expansion in the forecasting period, ke_{m2}; and hence, the forecasting period should be exposed to capacity expansion.

Dissatisfaction of condition C1 indicates that expansion in the central period does not provide benefits in any case. Then, capacity expansion in the forecasting period should be evaluated.

C4: $ke_{m2} + ec_{m2} < 0$

If condition C4 is satisfied, expansion in the forecasting period must be realized.

At the end of capacity expansion decisions, gross benefits and net benefits of disassembly centers $m \in M$ should be updated as follows:

$$kd_{mt} = \sum_{j \in J} \sum_{k \in K} \sum_{t}^{2} w_{jkmt} X_{jkmt}$$

$$nk_{mt} = kd_{mt} + \sum_{t}^{2} f1_{mt} + \sum_{t=1}^{t=2} e_{mt}$$

After the relaxation of the constraint (1.2), it is not an obligation to transport returns at collection centers to disassembly centers. At the initial iterations of the proposed solution methodology for subproblem P_m^d, none of the centers can be lucrative to be opened and all returns can be left at collection centers since it is difficult to find good Lagrange multipliers at the beginning. To avoid such a case, we added valid inequality constraints (2.62) and (2.63), for consolidation and disassembly centers respectively, to our model as stated in Section 4.2.3. The proposed solution method for subproblem P_m^d does not consider constraint (2.62) and does not guarantee that this constraint would be satisfied. Then, there should be an additional step at the end of the solution process of subproblem P_m^d to ensure that valid inequality constraint (2.62) is met. Procedure E1 in Figure 4.8 is proposed to be followed at the end of the corresponding solution process. It is not necessary to invoke Procedure E1 if at least one disassembly center in the central period is determined to be opened in previous steps.

If there is not any open disassembly center in the central and forecasting periods, Procedure E1 regards nk_{mt} values to find the disassembly center that would result in the minimum increase in the objective function in case of opening and opens disassembly center $m \exists nk_{m1} = \min_{m}(nk_{m1})$. If there is at least one open disassembly center *m* in the forecasting period, while there is none in the central period, effects of opening disassembly center m ϵ M' where $M' = \{m \mid Z_{m2} = 1\}$ in the central period too and opening disassembly center $m \, \epsilon \, M\text{-}M'$ in both central and forecasting period should be evaluated. In the end, opening decision with the least cost is made by procedure E1.

E1 :

if $\sum_{m \in M} Z_{m1} = 1$, do nothing

elseif $\sum_{m \in M} Z_{m1} = 0$ and $\sum_{m \in M} Z_{m2} = 0$

find $m \exists nk_{m1} = \min_{m}(nk_{m1})$ and set $Z_{mt} = 1$ for $t \in \{1, 2\}$

else

calculate $nk'_{m1} = kd_{m1} + f1_{m1}$ for $m \in \{m \mid Z_{m2} = 1\}$

if there is a disassembly center $m' \exists nk'_{m1} = \min_{m}(nk_{m1}, nk'_{m1})$

$Z_{m't} = 1$ for $t \in \{1, 2\}$

elseif there is a disassembly center $m \exists nk_{m1} = \min_{m}(nk_{m1}, nk'_{m1})$

$Z_{mt} = 1$ for $t \in \{1, 2\}$

end

end

Figure 4.8. Ensuring the satisfaction of constraint (2.63)

4.2.3.2. Solution Methodology for the Second Subproblem

Subproblem P^c determines the consolidation centers to be opened and decides in which period they should be opened. There is only one constraint bounding the solution space except binary constraint on Y_{kt} variables; constraint (2.51) enforces consolidation centers not to be closed once they are open. In subproblem P^c, there is no linkage between different consolidation centers and the lack of such a linkage enables to solve P^c for each consolidation center $k \in K$ independently. Subproblem P^c solved for consolidation center $k \in K$ can be denoted as P_k^c and objective function of P^c (2.50) can be transformed to the objective function (2.69).

$$\text{Minimize} \sum_{k \in K} \sum_{t \in T} Y_{kt} * (f1_{kt} - \sum_{j \in J} \sum_{m \in M} \lambda_{jkmt}) \tag{2.69}$$

Weight of Y_{kt} is equal to $(f1_{kt} - \sum_{j \in J} \sum_{m \in M} \lambda_{jkmt})$ which can be denoted as ns_{kt}. Then, based on ns_{kt} values, consolidation center $k \in K$ is evaluated to be opened for each period $t \in T$. If the minimum value of ns_{kt} for a particular consolidation center $k \in K$ is negative, the corresponding consolidation center is determined to be opened in the relevant period. Otherwise, Y_{kt} becomes zero in order to minimize the objective function (2.69). Decision process related to P_k^c is presented in Figure 4.9.

OK :

for $k \leftarrow 1$ to $|K|$

    if $ns_{k1} + ns_{k2} < 0$ and $ns_{k1} < 0$

        $Y_{k1} = 1$ and $Y_{k2} = 1$

    elseif $ns_{k2} < 0$

        $Y_{k2} = 1$

    end

end

Figure 4.9. Evaluation of consolidation centers to be opened

Similar to the execution of procedure E1 after the solution of P_m^d, procedure E2 in Figure 4.10 should be called, if necessary after decision process OK in the same manner as procedure E1.

***E2* :**

`if` $\sum_{k \in K} Y_{k1} = 1$

do nothing

`elseif` $\sum_{k \in K} Y_{k1} = 0$ and $\sum_{k \in K} Y_{k2} = 0$

find $k \ \exists \ ns_{k1} + ns_{k2} = \min_{k}(ns_{k1} + ns_{k2})$ and set $Y_{kt} = 1$ for $t \in \{1, 2\}$

`else`

find $\mathrm{mn} = \min(ns_{k'1}, ns_{k''1} + ns_{k''2})$ where $k' \in \{k \mid Y_{k2} = 1\}$ and $k'' \in \{k \mid Y_{k2} = 0\}$

`if` there is a consolidation center k′ $\exists \ ns_{k'1} = mn$

$Y_{k't} = 1$ for $t \in \{1, 2\}$

`elseif` there is a consolidation center $k'' \ \exists \ ns_{k''1} + ns_{k''2} = mn$

$Y_{k''t} = 1$ for $t \in \{1, 2\}$

`end`

`end`

Figure 4.10. Ensuring the satisfaction of constraint (2.62)

4.3. Primal Heuristic

Union of the solutions belonging to P_m^d and P_k^c constitutes the optimal solution of the Lagrangean dual problem, P^{LR}. Similarly, the sum of the objective functions belonging to P_m^d and P_k^c is equal to the objective function value of the Lagrangean dual problem. Lagrangean dual problem may not be feasible for the original problem. Thus, to obtain a feasible solution to the original problem, which would be an upper bound to it, an alteration algorithm based on the solution of Lagrangean dual problem has to be employed in most of the LR applications. The generation of the feasible solution to the original problem is remarkably significant since it provides practical/valid information applicable to real world problems whereas lower bound is not valid in actuality due to its infeasibility. Yet, it should be noted that lower bound generation can be the most crucial part, if LR is used within another algorithm and serves so as to provide a lower bound to it (Barketau et al., 2013). Barketau et al.

(2013) use LR to strengthen lower bound to B&B algorithm which is very common in the literature.

Drezner and Hamacher (2004) state that greedy algorithms are the most common methods to generate upper bounds in Lagrangean Relaxation. Jena et al. (2014), Lu and Bostel (2007), Pirkul and Jayaraman (1998), Tragantalerngsak et al. (1997) and Firoozi et al. (2013) propose several greedy algorithms designed by considering the characteristics of the problems they study to generate upper bounds to their Lagrangean Relaxation applications. Our problem in this study has a different character due to some of its various features like multi-period planning horizon, multi-echelon network structure, dynamic capacity and coverage limitation. Thus, we choose to develop our own greedy algorithm to generate an upper bound to the original problem P.

4.3.1. Greedy Algorithm

Greedy algorithms work in a local perspective and decide to proceed to the next step having the most benefit in finite next step alternatives as hoping to find the global optimal solution. Greedy algorithm generates a feasible solution step by step and selects best move in incumbent situation while disregarding the effects of the selected move in the long term.

Upper bound generations for LR via greedy algorithms are achieved, in general, by taking Lagrangean dual problem as the initial solution and progressing by greedy moves starting from the initial solution, or alternatively running a construction heuristic exploiting the materials Lagrangean dual problem provides is run in a greedy manner (Pirkul and Jayaraman, 1998). The latter method is used in this study to generate feasible solutions to the original problem P.

In CFLPs solved via LR, upper bound generation can be reduced to an assignment/allocation problem if total capacity in Lagrangean dual subproblem is forced to be greater than or equal to the sum of all demands (Soto and Üster, 2011). Another method in the generation of upper bounds is to alter capacity preferences regarding the minimum capacity sufficient to satisfy demands and to handle transportation problem after the alteration of capacity (Jena et al, 214). Such methods do not fit our model properly, since the limitation of coverage of collection centers obstructs to guarantee reaching an optimal solution if total capacity exceeds total demand. As an example, a solution having capacity more than total demand can be infeasible if none of the pairs of open consolidation center_open disassembly center covers a particular collection center *j*. Since greedy algorithms using Lagrangean dual problem as initial solution generally major on the balance between demand and capacity, it can fail in our problem; and this is the main reason to choose generating upper bound to the original problem P by a construction heuristic in a greedy manner.

4.3.2. Greedy Algorithm as Primal Heuristic

Greedy algorithm resting upon solutions of subproblems P_m^d and P_k^c is proposed to be used as the primal heuristic. Subproblem P_m^d makes decisions related to the path of flows and location & opening time decisions of disassembly centers, while P_k^c regards only location & opening time decisions of consolidation centers. Since there is no relation between subproblems P_m^d and P_k^c, returns' flows in P_m^d can be transported through consolidation center $k \in K$ which is determined to be close in P_k^c. Relaxation of linkage constraint (1.5) in the original problem enables to decompose the problem P spatially and consider disassembly centers and consolidation centers independently. Spatial decomposition is leading to infeasibility of the dual problem along with the relaxation of constraint (1.5).

Upper bound generation is required to get over the infeasibility in the Lagrangean dual subproblem and simultaneous consideration of consolidation centers and disassembly centers is necessitated. In addition to this, returns at collection centers cannot traverse distance more than *TH* until disassembly centers which hinder to handle transportation between layers separately. As a result of this necessity, in our primal heuristic, we choose to evaluate disassembly centers and consolidation centers pair by pair: *(k,m)* pairs where $k \in K$ and $m \in M$ are assessed among themselves and prominent pairs are determined to be opened. Selection process of prominent pairs is explained in the rest of the chapter.

Our construction heuristic in a greedy approach basically consists of six parts executed sequentially where first three parts are dealing with the first (central) period of the problem and last three parts cope with the second (forecasting) period. In the first part, adequate *(k,m)* pairs, $k \in K$ and $m \in M$, are assessed to be opened in the central period based on the information provided by subproblems P_m^d and P_k^c. In the second part, the unused capacity of the disassembly centers determined to be opened in the first part is utilized in an effective way. In the third and last part, new pairs *(k,m)* where $k \in K$ and $m \in M$ are opened in the central period if there are still returns at collection centers that could not be transferred to disassembly centers after the first two parts. Other parts of the primal heuristic are executed in the same fashion as the first three parts, since the forecasting period takes the role of the central period in these parts.

4.3.2.1. Part 1: Evaluation of *(k,m)* Pairs in Central Period

Subproblems P_m^d and P_k^c mostly give infeasible solutions regarding the constraints of the original problem P. Yet, they can provide solid information assisting to make decisions relevant to the original problem and Lagrangean dual subproblems can be exploited to generate feasible solutions (Knudsen et al., 2013).

In subproblem P_m^d, nk_{mt} variables denote the net benefits provided by opening disassembly center $m \in M$ in period $t \in T$ and ns_{kt} variables denote the net effect of opening consolidation center $k \in K$ in period $t \in T$ on the objective function. These two variables indicate contributions / benefits of disassembly centers $m \in M$ and consolidation centers $k \in K$ in the objective function, respectively. As it is stated above, our greedy construction heuristic is based on the evaluations of consolidation center - disassembly center pairs and it is not unreasonable to allege that a *(k,m)* pair, $k \in K$ and $m \in M$, becomes prominent over other pairs if sum of nk_{mt} and ns_{kt} is greater than others'.

$$np_{kmt} = \sum_{t \in T} ns_{kt} + nk_{mt}$$

np_{kmt} represents net benefit of opening both consolidation center k and disassembly center m in period t ,$k \in K$, $m \in M$ and $t \in T$, on the objective function of P^{LR}. Gross benefit of nk_{mt} is coming from inbound flows to disassembly m (kd_{mt}) and if it is greater than the cost of installation of disassembly center m, net benefit becomes negative which is favorable, since the objective function is to be minimized. Thus, nk_{mt}, as well as np_{kmt}, depends on inbound flows (in other words, returns allocated to disassembly centers) . Since np_{kmt} is influential in our upper bound generation, allocations in subproblem P_m^d should not be ignored due to the dependency of np_{kmt} on them. Thus, as we choose prominent pairs based on np_{kmt} values, allocations of returns to chosen pairs in subproblem P_m^d take part in upper bound solution too. Algorithm FU in Figure 4.11 explains the first part of our primal heuristic.

FU :

$Y_{kt}, Z_{mt}, X_{jkmt}, e_{mt} \rightarrow$ variable values in subproblem P_d and P_e

$\hat{Y}_{kt}, \hat{Z}_{mt}, \hat{X}_{jkmt}, \hat{e}_{mt} \rightarrow$ variable values in upper bound

Set $\hat{X}_{jkmt} = 0, \hat{Y}_{kt} = 0, \hat{Z}_{m1} = 0 \quad \forall\ j \in J, k \in K, m \in M, t \in T$

$S_1 = \left\{ j \mid \sum_{k \in K} \sum_{m \in M} \hat{X}_{jkm1} < 1 \right\}$

$S_2 = \left\{ (k,m) \mid \sum_{j \in S_1} X_{jkm1} >= ml \text{ and } (\hat{Y}_{k1} = 0 \text{ or } \hat{Z}_{m1} = 0) \right\}$

$ml = \frac{|J|}{|M|} * ml1$

i=1

while $S_1 \neq \emptyset$ and $S_2 \neq \emptyset$

select $(k,m) \exists\, np_{km1} = np_{km1}^{i}$ where $np_{km1}^{1} = \min_{k,m} np_{km1}$

if $\sum_{j \in S_1} X_{jkm1} >= ml$

$\hat{Z}_{mt}$=1 and $\hat{Y}_{kt}$=1 and $\hat{e}_{m1} = e_{m1}$ where $t \in \{1,2\}$

$\hat{X}_{jkm1} = \min\,(1 - \sum_{k \in K} \hat{X}_{jkm1}, X_{jkm1}) \quad \forall\ j \in J$

endif

$i = i$+1

end

Figure 4.11. First part of the feasible solution construction for the central period

In Algorithm FU, np_{kmt} values are ranked in an ascending order and *(k,m)* pairs are started to be opened in the central period based on np_{kmt} values by row. As *(k,m)* pairs are determined to be opened, allocations to the chosen pair are made based on X_{jkm1} values of the corresponding pair for collection centers that could not have set their whole returns yet.

In this approach, it can be possible to decide a *(k,m)* pair to be opened and assign it only one collection center $j \in J$. Yet, in this case, corresponding pair may not worth opening. To measure such an event, we introduce a threshold "*ml*" such that a *(k,m)* pair cannot be opened if it does not host "*ml*" of collection centers. Threshold "*ml*" is determined to be a product of $\frac{|J|}{|M|}$ by "*ml*" which is a given input to the problem.

"*ml1*" is set to 2 in our solution model which means that the number of collection centers assigned to *(k,m)* pair in subproblem P_m^d should be at least as two times the number of the collection centers for a candidate disassembly center on the average. As we rest on the solutions of subproblems P_m^d and P_k^c by constructing a feasible solution, specified threshold measures to count upon subproblems totally and does not allow opening a *(k,m)* pair if the number of collection centers assigned to that pair is not satisfying.

In the end, algorithm FU terminates if there is no *(k,m)* pair that could host at least ml of the collection centers or all returns at collection centers are sent to disassembly centers in the central period.

4.3.2.2. Part 2: Utilization of the Unused Capacity in the Central Period

In the first part of the primal heuristic, a set of consolidation center – disassembly center is opened and allocations are accomplished based on subproblem P_m^d. Solution constructed by the first part is expected to be infeasible, since allocations in this part are strictly dependent on X_{jkmt} values in P_m^d and due to the relaxation of constraint (1.2), P_m^d does not necessarily satisfy the requirement of transferring all returns at collection centers to disassembly centers. Infeasibility of subproblem P_m^d because of unsatisfied constraint (1.2) directly reflects on the feasibility status of the first part. Therefore, the second part of the primal heuristic succeeds after the first part.

In the first part of the primal heuristic, capacity utilization of disassembly centers to be opened is not concerned with even if unused capacity can serve for uncollected returns at the collection centers.

In the second part, unused/available capacity is tried to be utilized in an effective way by SU algorithm described in Figure 4.12.

SU :

$$KK_m = I_{m0} + \hat{e}_{m1} - \sum_{j\in J}\sum_{k\in K} \hat{X}_{jkm1}\, s_{j1} \quad \forall\; m \in M$$

$$\hat{K} = \{k \mid \hat{Y}_{k1}=1\},\ \hat{M} = \{m \mid \hat{Z}_{m1}=1\} \text{ and } \tilde{\tilde{M}} = \{m \mid KK_m > 0 \text{ and } m \in \hat{M}\}$$

$$U_{km} = \{ j \mid cv_{jkm} > 0\}$$

$$U^1 = \left\{ j \mid j \in S_1 \text{ and } j \subset \bigcup_{k\in\hat{K}, m\in\tilde{\tilde{M}}} U_{km} \right\}$$

while $U^1 \neq \varnothing$

$$close^1_j = \min_{k\in \hat{K},\, m\in\tilde{\tilde{M}}} \left(d_{jkm}\right) \quad \text{and} \quad j^1_{km} = \left\{(k,m) \mid d_{jkm} = close^1_j\right\} \quad \forall\; j \in U^1$$

$$close^2_j = \min_{k\in \hat{K},\, m\in\tilde{\tilde{M}} \text{ and } (k,m)\neq j^1_{km}} \left(d_{jkm}\right) \text{and} \quad j^2_{km} = \left\{(k,m) \mid d_{jkm} = close^2_j\right\} \quad \forall\; j \in U^1$$

Select $(k,m,j) \ni d_{j,\, j^2_{km}} - d_{j,\, j^1_{km}} = \max(close^2_j - close^1_j)$

$$\hat{X}_{jkm1} = \hat{X}_{jkm1} + \min\,(1 - \sum_{k\in K}\hat{X}_{jkm1}, \min(\frac{KK_m}{s_{j1}}, 1))$$

update KK_m, $\hat{M}, \tilde{\tilde{M}}$ and U^1

end

Figure 4.12. Utilization of unused capacity of disassembly centers in central period

$\hat{K}$ is the set of consolidation centers, $k \in K$, determined to be opened in the first part and $\tilde{\tilde{M}}$ is the disassembly centers, $m \in M$, that are opened in the first part and have unused capacity. $close^1_j$ is the smallest one of the distances, d_{jkm}, from collection center j that has unsent returns to disassembly center $m \in \tilde{\tilde{M}}$ through consolidation center $k \in \hat{K}$, where pair *(k,m)* covers collection center *j*.

$$close_j^1 \le close_j^2 \le close_j^3 .... close_j^{|\hat{U}_j|} \quad \forall \; j \in U^1$$

$$\text{where } \hat{U}_j = \left\{ (k,m) | \;\; j \in U_{km} \text{where } k \in \hat{K}, m \in \tilde{\hat{M}} \right\}$$

The second part of the primal heuristic deals with the difference between $close_j^1$ and $close_j^2$. Returns in the collection center *j* that has the maximum difference are allocated to the closest *(k,m)* that covers collection center *j* where disassembly center *m* has unused capacity.

This procedure goes on until there is no capacity left to be utilized or consolidation center - disassembly center pairs that have unused capacity cover none of the collection centers with uncollected returns.

4.3.2.3. Part 3: Handling Uncollected Returns in the Central Period

After the first two parts, there can still be uncollected returns in the central period due to two reasons:

1. Total capacity of disassembly centers determined to be opened in the central period is less than the total returns to be collected at the collection centers.
2. Consolidation center - disassembly center pairs with available capacity to be used cover none of the collection centers with the uncollected returns.

In the existence of one of these two reasons, new disassembly centers or consolidation centers, or possibly both, have to be opened to transfer uncollected returns at collection centers to disassembly centers. As new centers are opened to guarantee that there are no returns left uncollected, it is intended to achieve this purpose with the minimum cost dedicated to new centers. In this manner, we develop a simple algorithm, AP, to assign opening probability for each candidate consolidation center *k* – disassembly center *m* pair where at least one of consolidation center $k \in K$ and disassembly center $m \in M$ is close in the central period. Algorithm AP in Figure 4.13 is founded on the measures like the number of collections centers with uncollected returns that are covered by candidate consolidation center *k* – disassembly center *m* pair and total operating costs of the corresponding pair.

AP :

$$ar = \frac{\sum_{j \in J} s_{j1}}{2|M|}$$

`while` $S_1 \neq \varnothing$

Calculate $p_{km} \;\; \forall \;\; (k,m) \exists \hat{U}_{km} \neq \varnothing$, Select $(k,m) \exists \; p_{km} = \max(p_{\text{km}})$

`if` $\hat{Z}_{m1} = 0$

$\hat{Y}_{k1} = 1, \hat{Y}_{k2} = 1$ and $\hat{Z}_{m1} = 1, \hat{Z}_{m2} = 1$

`elseif` $\hat{Z}_{m1} = 1$ and $KK_m \geq ar$

$\hat{Y}_{k1} = 1, \hat{Y}_{k2} = 1$

`end`

Select $(k,m) \exists \;\; close^1_{km} = \min(close^1_{km})$ where $close^1_{km} = \min\limits_{j \in \bigcup\limits_{k \in \hat{K}} \hat{U}_{km}} (d_{jkm})$

Set $j = j^1_{km}$ where $j^1_{km} = \left\{ j \,|\, d_{jkm} = close^1_{km} \right\}$

$\hat{X}_{jkm1} = \min\,(1 - \sum\limits_{k \in K} \hat{X}_{jkm1}, \min(\dfrac{KK_m}{s_{j1}}, 1))$, update $KK_m, \hat{U}_{km}$ and S_1

`end`

`end`

Figure 4.13. Opening new centers ensuring that all returns are collected in the central period

p_{jkm} denotes the probability of returns at collection center *j* transferred to disassembly center *m* via consolidation center *k* and calculated as:

$$p_{jkm} = (1 - (2\begin{Bmatrix} f1_k & \text{if } k \notin \hat{K} \\ 0 & \text{o/w} \end{Bmatrix} + \; 2\begin{Bmatrix} f2_m & \text{if } m \notin \hat{M} \\ 0 & \text{o/w} \end{Bmatrix}) / tf_j \;) cv_{jkm}$$

$$\forall \; (k \notin \hat{K} \text{ or } m \notin \hat{M}) \text{ and } j \in \hat{U}_{km}$$

$$\text{where } tf_j = \sum_{\hat{U}_{km} \supset j} 2\begin{Bmatrix} f1_k & \text{if } k \notin \hat{K} \\ 0 & \text{o/w} \end{Bmatrix} f1_k \; + \; 2\begin{Bmatrix} f2_m & \text{if } m \notin \hat{M} \\ 0 & \text{o/w} \end{Bmatrix} \;\; \forall \; j \in S_1$$

and $\hat{U}_{km} = \{ j \mid j \in S_1 \text{ and } j \in U_{km} \}$.

As the cost of opening centers in *(k,m)* pair , $j \in \hat{U}_{km}$, becomes lower compared to the costs of other pairs that satisfy $j \in \hat{U}_{km}$, probability of transferring returns at collection center *j* to disassembly center m via consolidation center *k* increases. Sum of the probabilities over collection centers is the main determining factor in the third part of the primal heuristic:

$$p_{km} = \sum_{j \in S_1} p_{jkm}$$

p_{km} shows the priorities of candidate pairs to be selected, and as higher as *(k,m)* pair's probability (p_{km}) is, it becomes more preferable to get consolidation center *k* and disassembly center *m* opened.

The pair with maximum probability is selected to be opened in any cases if disassembly center m associated with the corresponding pair is not already open. Yet, if it is, it should be ensured that unused capacity of disassembly centers should be at least "*ar*" which is equal to half of the average returns per candidate disassembly center in the central period.

This condition serves as a guarantee that consolidation center *k* associated with the corresponding pair is not opened, unless unused capacity of disassembly center *m* in the pair is greater than a prespecified value, "*ar*". Otherwise, uncollected returns cannot be allocated to the selected pair due to insufficient capacity although selected pair covers many of the collection centers with uncollected returns.

In the end, collection centers with the uncollected returns are allocated to open *(k,m)* pairs with available capacity in the order of distance to it, - allocation of the closest collection center *j* is done in the first place and so on- until there are no returns left uncollected or capacity of the disassembly center *m* in selected pair is fully utilized. New pairs of condolidation center-disassembly center are determined to be opened in the same fashion based on the probabilities that are updated after the selection of each new pair. This procedure proceeds until all returns at collection centers are collected and transferred to disassembly centers.

4.3.2.4. Part 4: Evaluation of (*k,m*) Pairs in the Forecasting Period

After the first three parts, it is guaranteed that all returns at collections centers are transferred to disassembly centers. Yet, decisions taken in these parts do not consider the satisfaction of constraints associated with the second (forecasting) period. Last three parts are employed in the same manner as the first ones to make sure that all constraints related to the forecasting period are satisfied.

Firstly, *(k,m)* pairs selected to be opened in the central period are handled and assignments to these open pairs in the forecasting period are determined in this part of the primal heuristic. np_{km2} values are the main determinants in the assignment process as similar to the procedure in the first part. Determination of returns' flows to *(k,m)* pairs opened so far is explained in Figure 4.14.

FU2 :

$S_1^2 = \left\{ j \mid \sum_{k \in K} \sum_{m \in M} \hat{X}_{jkm2} < 1 \right\},\ i = 1$

while $i <= |\hat{K}||\hat{M}|$

select $(k,m) \exists\ np_{km2} = np_{km2}^{i}$ where $np_{km2}^{1} = \min_{k,m} (np_{km2})$ where $k \in \hat{K},\ m \in \hat{M}$

$\hat{X}_{jkm2} = \min\ (1 - \sum_{k \in K} \hat{X}_{jkm2}, X_{jkm2}) \quad \forall\ j \in J$ and $(k,m) \exists\ np_{km2} = np_{km2}^{i}$

$i = i + 1$

end

end

Figure 4.14. Assignments in the forecasting period to the centers opened in the first three parts

In Algorithm FU2, *(k,m)* pairs are evaluated based on np_{km2} values of consolidation and disassembly centers that are opened in the central period and pair with minimum np_{km2} is selected firstly, and assignments are settled as abiding by the assignments in subproblem P_m^d. This procedure continues until there are no uncollected returns in the forecasting period, or consolidation-disassembly center pairs that are opened in the central period cover none of the collection centers with uncollected returns.

$\text{FU2}'$ algorithm below is equivalent to FU except the period dealt with. FU2 decides which *(k,m)* pairs to open in the forecasting period by regarding np_{km2} values. Algorithm $\text{FU2}'$ is described in Figure 4.15.

FU2′ :

$S_2^2 = \left\{ (k,m) \mid \sum_{j \in S_1^2} X_{jkm2} >= ml \text{ and } \hat{Y}_{k2} = 0 \text{ or } \hat{Z}_{m2} = 0 \right\}$ and $i = 1$

while $S_1^2 \neq \varnothing$ and $S_2^2 \neq \varnothing$

select $(k,m) \ni np_{km2} = np_{km2}^i$

if $\sum_{j \in S_1^2} X_{jkm2} >= ml$

$\hat{Y}_{k2} = 1$ and $\hat{Z}_{m2} = 1$ and $\hat{e}_{m2} = e_{m2}$

$\hat{X}_{jkm2} = \min\,(1 - \sum_{k \in K} \hat{X}_{jkm2}, X_{jkm2}) \quad \forall\ j \in J$

endif

$i = i + 1$

end

Figure 4.15. Opening new centers in the forecasting period

4.3.2.5. Part 5: Utilization of Unused Capacity in the Forecasting Period

In the fourth part of the primal heuristic, algorithm $\text{FU2}'$ determines *(k,m)* pairs to be opened in the forecasting period and assignes collection centers to the selected pairs based on subproblem P_m^d. However, final solution at the end of the fourth part can be infeasible since relaxed constraint (1.2) is not satisfied for all returns at collection centers in the forecasting period in subproblem P_m^d and algorithm $\text{FU2}'$ does not make assignments that do not exist in subproblem P_m^d. Therefore, in a similar fashion to algorithm SU in the second part, algorithm SU2 is developed which is evolved from algorithm SU by replacing the central period with the forecasting period. Algorithm SU2 is employed to consume unused capacity of open disassembly centers in the forecasting period effectively. Algorithm SU2 is explained in Figure 4.16.

SU2 :

$$KK_m^2 = I_{m0} + \sum_{t=1}^{t=2} \hat{e}_{mt} - \sum_{j \in J} \sum_{k \in K} \hat{X}_{jkm2}\ s_{j2} \quad \forall\ m \in M$$

$$\hat{K}^2 = \left\{ k \mid \hat{Y}_{k2} = 1 \right\},\ \hat{M}^2 = \left\{ m \mid \hat{Z}_{m2} = 1 \right\} \text{ and } \tilde{\hat{M}}^2 = \left\{ m \mid\ KK_m^2 > 0 \text{ and } m \in \hat{M}^2 \right\}$$

$$U^2 = \left\{ j \mid j \in S_1^2 \text{ and } j \subset \bigcup_{k \in \hat{K}^2, m \in \tilde{\hat{M}}^2} U_{km} \right\}$$

`while` $U^2 \neq \varnothing$

$$close_j^1 = \min_{k \in \hat{K}^2,\ m \in \tilde{\hat{M}}^2} \left(d_{jkm} \right) \text{ and } j_{km}^1 = \left\{ (k,m) \mid d_{jkm} = close_j^1 \right\} \quad \forall\ j \in U^2$$

$$close_j^2 = \min_{k \in \mathrm{P}_m^d,\ m \in \tilde{\hat{M}}^2 \text{ and } (k,m) \neq j_{km}^1} \left(d_{jkm} \right) \text{and } j_{km}^2 = \left\{ (k,m) \mid d_{jkm} = close_j^2 \right\} \quad \forall\ j \in U^2$$

$$\text{Select } (k,m,j) \ni \mathrm{d}_{j,\ j_{km}^2} - \mathrm{d}_{j,\ j_{km}^1} = \max(close_j^2 - close_j^1)$$

$$\hat{X}_{jkm2} = \hat{X}_{jkm2} + \min\ (1 - \sum_{k \in K} \hat{X}_{jkm2}, \min(\frac{KK_m^2}{s_{j2}}, 1))$$

update $\mathrm{KK}_m^2, \hat{M}^2, \tilde{\hat{M}}^2$ and U^2

`end`

Figure 4.16. Utilization of unused capacity of disassembly centers in the forecasting period

4.3.2.6. Part 6: Handling Uncollected Returns in the Forecasting Period

First three parts of the primal heuristic attempts to find a feasible solution satisfying all constraints in the original problem for the central period. These three parts are called in order and part 3 is employed only if previous parts fail in constructing the feasible solution. Sixth part of the primal heuristic has the same role as the third one, and algorithm AP in the third part is converted to algorithm AP2 in the sixth part so as to address the forecasting period instead of the central period.

Algorithm AP2 is explained in Figure 4.17.

AP2 :

$ar^2 = \sum_{j \in J} s_{j2} / (2|M|)$

`while` $S_1^2 \neq \emptyset$

Calculate $p_{km}^2 \ \forall \ (k,m) \ \exists \hat{U}_{km}^2 \neq \emptyset$**,** Select $(k,m) \exists \ p_{km}^2 = \max(p_{km}^2)$

`if` $\hat{Z}_{m2} = 0$**,** $\hat{Y}_{k2} = 1$ and $\hat{Z}_{m2} = 1$

`elseif` $\hat{Z}_{m2} = 1$ and $KK_m^2 \geq ar^2$**,** $\hat{Y}_{k2} = 1$

`endif`

`while` $KK_m^2 > 0$ and $\bigcup_{k \in \hat{K}^2} \hat{U}_{km}^2 \neq \emptyset$

Select $(k,m) \exists \ close_{km}^1 = \min(close_{km}^1)$ where $close_{km}^1 = \min_{j \in \bigcup_{k \in \hat{K}^2} \hat{U}_{km}^2} (d_{jkm})$

Set $j = j_{km}^1$ where $j_{km}^1 = \left\{ j \mid d_{jkm} = close_{km}^1 \right\}$

$\hat{X}_{jkm2} = \min\,(1 - \sum_{k \in K} \hat{X}_{jkm2}, \min(\frac{KK_m^2}{s_{j2}}, 1))$, update KK_m^2 ,$\hat{U}_{km}^2$ and S_1^2

`end`

`end`

Figure 4.17. Opening new centers ensuring that all returns are collected in the forecasting period

p_{jkm}^2 is calculated in a similar way to p_{jkm} as the central period in the latter one turns into the forecasting period in the former one. p_{jkm}^2 is calculated as below:

$$p_{jkm}^2 = (1 - (\begin{Bmatrix} f1_k & \text{if } k \notin \hat{K}^2 \\ 0 & \text{o/w} \end{Bmatrix} + \begin{Bmatrix} f2_m & \text{if } m \notin \hat{M}^2 \\ 0 & \text{o/w} \end{Bmatrix}) / tf_j\ ) cv_{jkm}$$

$$\forall\ (k \notin \hat{K}^2 \text{ or } m \notin \hat{M}^2) \text{ and } j \in \hat{U}_{km}^2$$

$$\text{where } tf_j = \sum_{\hat{U}_{km}^2 \supset j} \begin{Bmatrix} f1_k & \text{if } k \notin \hat{K}^2 \\ 0 & \text{o/w} \end{Bmatrix} f1_k + \begin{Bmatrix} f2_m & \text{if } m \notin \hat{M}^2 \\ 0 & \text{o/w} \end{Bmatrix} \ \forall\ j \in S_1^2$$

and $\hat{U}^2_{km} = \{ j \mid j \in S^2_1 \text{ and } j \in U_{km} \}$

Then, p^2_{km} which is the probability of opening *(k,m)* pair in the forecasting period can be calculated with the following equation:

$$p^2_{km} = \sum_{j \in S^2_1} p^2_{jkm}$$

In this chapter, our approach to construct feasible solutions for original problem P based on the information P^d and P^c provide is explained. At the end of part 6 of the primal heuristic, a feasible solution is obtained for the original problem P the objective function value of which objective function is denoted by P^{UB}.

$$\mathrm{P}^{UB} = \sum_{j \in J} \sum_{k \in K} \sum_{m \in M} \sum_{t \in T} \hat{X}_{jkmt}\, s_{jt}\, d_{jkm} + \sum_{k \in K} \sum_{t \in T} \hat{Y}_{kt}\ f1_k$$
$$+ \sum_{m \in M} \sum_{t \in T} \hat{Z}_{mt}\ f2_m + \sum_{m \in M} \sum_{t \in T} \hat{e}_{mt}\ ec_{mt}$$

Feasible solutions obtained can serve as upper bounds to the problem P which is used in the subgradient optimization explained in the next section. Ultimate target of LR is to construct a feasible solution that is optimal or at least very close to optimal for the problem P.

4.4. Subgradient Optimization

Methods to generate lower and upper bounds to the original problem are explained in previous sections. Proposed methods reckon that Lagrangean multiplers are pre-given at the beginning of each iteration, and at this point, subgradient optimization is employed in our study to update Lagrangean multipliers at the end of each iteration. Subgradient optimization is developed by Poljak (1967) and many authors have been referring to and employing it since then, like Held et al. (1974), Goffin (1977), Sandi (1979) and Nesterov (2012). Beasley (1996) describes subgradient optimization as an iterative method that updates the Lagrangean multipliers so as to drive them to converge through optimal Lagrangean multipliers, and in the end, maximize the lower bound (for minimization problems) that the Lagrangean dual subproblem provides.

Subgradients which are partial differentials of the Lagrangean dual subproblem at a specific point are preferred over gradients when the function is nondifferentiable.

As an example, g is a subgradient of a convex function f at x ϵ **dom** f, where

$$f(y) \geq f(x) + g^T (y - x) \quad \forall y \in \text{dom } f$$

Due to nondifferentiability of the function, subgradient direction cannot be steepest descent or ascent as the gradient direction. Yet, fundamental measure is not the function value; it is the Euclidean distance of the current point to the optimal point (Boyd et al., 2003) and Euclidean distances of solutions to the optimal point decrease in each iteration of the subgradient optimization, thanks to acute angle, $g^T (y - x)$. Thus, subgradient g^T enables to converge to the optimal solution with the selection of proper step sizes.

There are various alternatives to determine the step size in iteration t, α^t, and three alternatives that are widely used in the literature are stated below:

1- fixed step size: $\alpha^t = \alpha$

2- fixed step length: $\alpha^t = s / \| g^{(t-1)} \|_2$ where s is a pre-given step length

3- diminishing step size: $\alpha^t \to 0, \sum_{t=1}^{t=\infty} \alpha^t = \infty$

"Diminishing step size" differentiates from the first two step size update methods since it guarantees to reach optimality with proper tunings while other two methods cannot guarantee the convergence to the optimal solution (Vandenberghe, 2013).

While "diminishing step size" guarantees the convergence to the optimal solution, it assumes that optimal value is accessible as diminishing step size is calculated by:

$$\alpha^t = \frac{s^t (f(x^*) - f(x^t))}{\|g^t\|^2} \quad \text{where } 0 \leq s^t \leq 2$$

Although optimal solution is not known prior to subgradient optimization, a surrogate value (upper bound) that is close to the optimal value can make this step size update method viable. Primal heuristic as introduced in Section 4.3 can be employed to attain such a proper surrogate value in our study.

s^t is the step length in "diminishing step size" method that lies between 0 and 2, and determination of s^t is up to the decision maker of the problem addressed. In general, s^t is initiated by a value that is less than or equal to 2, and it is halved if lower bound does not improve within a pre-specified number of iterations.

After all, Lagrange multipliers ϕ^t can be updated as below:

$$\phi^t = \phi^{t-1} + \alpha^t * d^t$$

There are several alternatives to choose the search direction (d^t). Search direction is generally set to the subgradient (Holmberg and Yuan, 2000). Another common approach in determination of search direction is not to focus on solely last iteration and to use an aggregated search direction of previous iterations with emphasis on recent iterations. Crowder's rule (1976) is the most widely used approach regarding aggregated search directions and determines search direction as below:

$$d^t = d^{t-1}\,\delta + d^{t-2}\,\delta^2 ....+ d^1\,\delta^{t-1} \quad \text{where } \delta \text{ is a constant and } 0 < \delta < 1$$

Since δ is between 0 and 1, Crowder's rule puts more emphasis to latest iterations in comparison to first iterations which are included in the determination of search direction too.

4.4.1. Subgradient Optimization in Our Model

Subgradient optimization with "diminishing step size" is very common in the literature since it is appealing to researchers with its easy adaptation to problems and its capability to converge through optimal solution. Likewise, in our study, we chose to use subgradient optimization with "diminishing step size" approach to update Lagrangean multipliers. There are two sets of constraints decided to be relaxed in the original problem P. Lagrangean multipliers dedicated to each set share same step size as they are updated. Yet, directions differ based on the subgradients and subgradients at iteration *i* are calculated as:

$$MS^i_{jt} = 1 - \sum_{k \in K} \sum_{m \in M} X^i_{jkmt}$$

$$LS^i_{jkmt} = X^i_{jkmt} - Y^i_{kt}$$

MS^i_{jt} is the subgradient belonging to constraint (1.2) in iteration *i* while LS^i_{jkmt} is the subgradient of constraint (1.5) in iteration *i*.

Step size in iteration *i* can be calculated as below:

$$ss^i = \frac{sl^i * (UB * 1.1 - LB^i)}{\left\| MS^i_{jt} \right\|^2 + \left\| LS^i_{jkmt} \right\|^2}$$

The difference between the best feasible solution in all iterations (upper bound) and lower bound at iteration *i* is used at the numerator of step size ratio in a standard step size calculation. Yet, as Galvao et al. (2002) stated, convergence can be very slow as the gap between lower bound and upper bound is substantially small. Therefore, to multiply upper bound with a small number can accelerate converge without misleading.

After all, Lagrangean multipliers can be updated as follows:

$$\mu^i_{jt} = \max\left(0, \mu^{i-1}_{jt} + MS^i_{jt}\ ss^i\right)$$
$$\lambda^i_{jkmt} = \max\left(0, \lambda^{i-1}_{jkmt} + LS^i_{jkmt}\ ss^i\right)$$

Since Lagrangean multipliers have to be nonnegative, they get equal to 0 if subgradients lead them to a negative value.

Step length *sl* is halved if lower bound does not improve within a pre-specified number of iterations, ni_{max}.

$$ni_{max} = \frac{Niter}{100}$$

where *Niter* is the maximum number of iterations in Lagrangean Heuristic. *Niter* is explained in the next section.

4.5. Stopping Criteria of Lagrangean Relaxation

It is expected to converge through optimal solution iteratively via subgradient optimization. In ideal case, LR is terminated after the achievement of the equality between lower bound and upper bound in a reasonable time. Yet, this may not be the case due to two reasons:

1- The lower bound may not reach the upper bound.
2- Execution time of the algorithm until the achievement of equality between lower and upper bound can be tremendous.

Due to the integrality gap explained in Section 4.6, first reason above generally exists in our problem. Thus, another stopping criteria except the equality of lower bound and upper bound must be introduced.

A ratio of the difference between upper bound and lower bound over lower bound can be used as stopping criteria. As upper bound and lower bound approaches to each other, gap becomes smaller and LR is terminated when the gap falls under a certain threshold which is decided to be 0.05% in our problem.

$$\text{Gap} = \frac{\text{UB - LB}}{\text{LB}}$$

To achieve a gap less than 0.05% can be difficult as well as the equality of upper bound and lower bound. In this situation, there should be another criteria ensuring the termination of LR. Maximum number of iterations is used as a stopping criteria too to guarantee the termination of the algorithm and it is set to 2000, *Niter* = 2000.

It should also be stated that probability of improving upper bound decreases as upper bound approaches optimal solution. Thus, step length *sl* is halved after a pre-given number of iterations with no upper bound improvement. Thus, LR stops when step length *sl* gets too small which means that the improvement of upper bound is not likely anymore. After all, sl_{min} is introduced as another stopping criteria; LR terminates when step length *sl* gets smaller then sl_{min} which is decided to be 0.05, sl_{min} = 0.05.

4.6. Integrality Gap

The integrality gap can be described as the ratio of the difference between optimal solution obtained by linear relaxation and integer optimal solution divided by the latter one (Jena et al., 2014).

Formulation of Lagrangean dual problem is independent of the types of the variables in the original problem; in other words, formulation of P^{LR} would not be different if all variables in the original problem P were integer or real numbers. In this fashion, the equality of lower and upper bounds in LR can be achieved only if strong duality holds and there must be zero gap between linear relaxation solution and optimal integer solution for strong duality. Thus, performance of LR depends on how close linear relaxation solution is to integer optimal solution.

In our problem, integrality gap reached up to 20% in preliminary runs of LR and LR does not give satisfying solutions in the instances with high integrality gap, since its performance is dependent on integrality gap. Integrality gap is more dependent on the model formulation and valid inequalities get more significant with regards to the integrality gap. Although proper alterations were tried on the formulation, integrality gap for alternative model formulations continued to vary around the range specified above.

Stated by Martin (1999) , duality gap is the same as integrality gap when the dual of the integer program is the dual of the linear programming relaxation. Boyd and Udell (2014) and Vujanic et al. (2015) propose different methods, which are computationally expensive, to reduce the duality gap. Yet, authors do not guarantee that there would be noteworthy decrease in the duality gap despite the computational effort. Therefore, we decide to use VNS which is easy to model and has been successful in diverse kinds of problems to improve the solution LR generates. All in all, proposed LR in our study can be represented as depicted in Figure 4.18.

START
INIT $\mathrm{LB} = -\infty, \mathrm{UB} = +\infty,\ \lambda_{jkmt} = 0, \mu_{jt} = 0, no\_improve = 0, iter = 1, sl = 2$
while Gap > 0.05 and *iter* <= *Niter* and *sl* > sl_{min}
    SOLVE Lagrangean dual problem

Figure 4.18. Representation of LR in our study

```
SOLVE Subproblem P_m^d
        CALL procedure FA
        CALL scheme OD
        CALL procedure CA and CA2
        if Σ_{m∈M} Z_{m1} > 0, STOP, else
            CALL procedure E1
        endif
SOLVE Subproblem P_k^c
    CALL scheme OK
    if Σ_{k∈K} Y_{k1} > 0, STOP, else
        CALL procedure E2
    endif
UPDATE Lower Bound
    if P^LR(λ_jkmt, μ_jt) > LB, LB = P^LR(λ_jkmt, μ_jt) and no_improve=0
    else    no_improve = no_improve+1
            if no_improve = ni_max, sl = sl / 2 endif
    endif
RUN Primal heuristic
    RUN Primal heuristic for central period
        CALL procedure FU
        if Σ_{k∈K} Σ_{m∈M} X_{jkm1} = 1 ∀ j ∈ J, STOP , else
            CALL procedure SU
            if Σ_{k∈K} Σ_{m∈M} X_{jkm1} = 1 ∀ j ∈ J, STOP, else
                CALL procedure AP
            endif
        endif
    RUN Primal heuristic for forecasting period
        CALL procedure FU2 and FU2′
        if Σ_{k∈K} Σ_{m∈M} X_{jkm2} = 1 ∀ j ∈ J, STOP , else
            CALL procedure SU2
            if Σ_{k∈K} Σ_{m∈M} X_{jkm2} = 1 ∀ j ∈ J, STOP, else
```

Figure 4.18. (cont’d)

```
                                        CALL procedure AP2
                                endif
                        endif
            if P^UB < UB, UB=P^UB endif
    UPDATE Lagrange multipliers and Gap
    iter = iter+1
end
STOP
```

Figure 4.18. (cont'd)

4.7. Variable Neighborhood Search (VNS)

It can be difficult to handle complex combinatorial problems with conventional methods and exact algorithms. In this situation, heuristics can be developed to find an approximate solution in a reasonable time. Heuristics are usually problem-specific, which exploit characteristics of the problem on hand, as well as the generic methods that prove to be successful in certain kinds of problems. It should be noted that knowledge about the problems solved by a heuristic algorithm must hold for the problem on hand to employ the corresponding heuristic (Rothlauf, 2011).

Metaheuristics is another subfield of optimization techniques. Metaheuristics can be labeled as "black box optimization" in a sense, since they represent strategy guidelines to develop a solution methodology regardless of the problem characteristics in a generic form (Luke, 2014). Not to be problem-specific extends the kinds of problems for which metaheuristics are applicable and use of metaheuristics on diverse problems have been increasing due to the wide-applicability of it. Wide-applicability of metaheuristics is achieved by their success in two aspects: intensification and diversification (Blum and Roli, 2003). Intensification is the exploitation of the search experience and diversification refers to exploration of the search space while it also aids not to stick at local optima. Some of the most well known metaheuristics are genetic algorithm (GA), tabu search (TS), simulated annealing (SA) and variable neighborhood search (VNS). Each of these metaheuristics has different intensification/diversification techniques which are not problem-specific and represents a similar perspective.

As it is stated previously, in our study, integrality gap of the proposed model can reach up to 20%. Performance of LR is influenced negatively as the integrality gap increases and this situation leads to the requirement of an additional improvement tool to be called for after LR. In accordance with this purpose, we choose to adapt VNS into our approach.

VNS, proposed by Mladenovic and Hansen (1997), is a metaheuristic exploring different neighborhoods in order enabling to escape from local optima of a particular neighborhood by shifting among the neighborhoods, since local optima of a neighborhood is not necessarily local optima for other neighborhoods.

VNS, as other metaheuristics, has diverse application fields like scheduling, vehicle routing, facility location, fleet management and graph problems. Although VNS are preferred mostly for problems like scheduling and vehicle routing rather than facility location problems (FLP); there are successful applications of VNS on FLP by Amrani et al. (2011), Hansen et al. (2007), Rahmaniania et al (2012) and Wollenweber (2008). Standard VNS is depicted in Figure 4.19.

Determine set of neighborhood structures N_k for $k=1,...,k_{\max}$ and set $k=1$

while termination condition isn't satisfied

$i=1$

while $i < i_{max}$

Shaking: Generate a point x' randomly from the kth neighborhood of x $(x' \in N_k(x))$

Local Search: Apply local search with x' as initial solution; x'' is obtained

by local search

if x'' is better than incumbent solution x, $x \leftarrow x''$ and $i = i_{max}+1$;

else, set $i = i+1$ **endif**

end

if $i = i_{max}+1$, $k=1$, **else** $k = k+1$ **endif**

end

Figure 4.19. General procedure of VNS

Solution obtained at the end of LR in our study will be the initial solution of VNS. Since LR provides valuable solutions to be used as initial solutions to the succeeding improvement algorithm, it would be substantial to adapt an approach after LR that intensifies Lagrangean solution as preventing to get stuck at local optima. In this aspect, we view VNS as a convenient approach to be embedded into the proposed algorithm. One of the other advantages of VNS having influences on the preference of it is that it reduces search space for the local search procedure significantly (Perez et al., 2006) and it can be quite fast with proper initial solutions. Simplicity of VNS and no requirements of additional parameters to be determined are other important advantages of VNS.

There are different variants of VNS like reduced VNS (RVNS), variable neighborhood decomposition search (VNDS), Variable Neighborhood Descent (VND) and Skewed VNS.

Hansen and Mladenovic (2009) state three questions to be asked when a local minimum solution has been reached:

1- In which direction to go?

2- How far?

3- How should one modify moves if they are not successful?

Authors state that question 1 is pertaining to reaching any feasible solution and the simple answer is to choose direction at random. In regard of question 2, it seems natural to explore first the vicinity of the current solution. Yet, if the valley surrounding is broad, to explore the vicinity of the current solution may not be sufficient; in this case, question 3 arises and authors again propose a natural answer which is to go further. These aims are pursued by RVNS which is derived from general VNS procedure as dropping local search step from it to save time. RVNS is presented in Figure 4.20.

In the end, RVNS is determined to be called after LR to improve the solution generated by it in our study.

Determine set of neighborhood structures N_k for $k=1,...,k_{\max}$ and set $k=1$

while termination condition isn't satisfied

$i=1$

while $i < i_{max}$

Shaking: Generate a point x′ randomly from the kth neighborhood of x ($x' \in N_k(x)$)

if x'' is better than incumbent solution x, $x \leftarrow x''$ and $i = i_{max}+1$;

else, set $i = i+1$ **endif**

end

if $i = i_{max}+1$, $k=1$, **else** $k = k+1$ **endif**

end

Figure 4.20. General procedure of RVNS

4.7.1. Multi - Layer Reduced Variable Neighborhood Search in Our Problem

VNS is based on exploring different neighborhoods and selection of neighborhood sets is crucial. Possible neighborhood operations are strictly dependent on the structure of the problems to be solved.

Glover and Kochenberger (2003) state that if search space is pertaining to location variables, neighborhood structures usually contain so-called “Add”, “Drop” and “Exchange” moves; “Add” move changes the status of a closed site as open whereas “Drop” move turns a closed site to an open one. “Exchange” move behaves as a combination of Add & Drop move and opens a site concurrently with the closure of an open site.

Hansen and Mladenovic (2009) propose add, drop and exchange moves, which are very common in local search algorithms in the literature, as introducing a simple version of VNS. Likewise, Daskin and Maass (2015) define these moves as the most general neighborhood operations. Therefore, it is common to use these moves in VNS applications as expected. Dias et al. (2006) develop a hybrid GRASP / VNS algorithm and use add, drop and exchange moves in both GRASP and VNS. Aloise et al. (2003) develop a VNS including nine neighborhood operations composed of add, drop and exchange moves.

Similarly, Wollenweber (2008), Amrani et al. (2011), Hansen et al. (2007) and Rahmaniania et al. (2012) use add, drop and exchange moves, too. In the end, we decided to develop a RVNS based on the neighborhood operations comprised of these moves. Our RVNS algorithm is resting on multilayer VNS (MLVNS) method developed by Gendron et al. (2011). MLVNS partitions neighborhoods into multiple layers. Layers are ordered based on the complexity of neighborhood operations associated with them; the simplest neighborhood operations belong to layer 1 and so on. Rationale behind MLVNS is to invoke simple neighborhood operations more frequently while more complex operations are supposed to be called less often.

In MLVNS that is shown in Figure 4.21, each layer $l \in L$ has a set of neighborhoods from k_1^l to $k_{\max}^l$. At the beginning of MLVNS, Layer 1 is called and N_k^l is operated similarly as basic VNS. Layer 2 is invoked when termination conditions of Layer 1 are satisfied. Layer 2 generates a new solution x' by neighborhood operations within it and algorithm proceeds from the beginning of layer 1 as x' becomes an incumbent solution. Layer 1 progresses as it is supposed to be and terminates after the realization of its termination conditions. Then, layer 2 generates another solution x'' to be an incumbent solution to layer 1.

This procedure continues until all neighborhood operations of layer 2 are examined. Then, layer 3 is invoked to generate a solution to be the incumbent solution to layer 2. Layer 2 generates a new solution and algorithm proceeds from Layer 1 and so on. In the realization of the improvement of the best solution, MLVNS returns the first layer and algorithm starts from the initial neighborhood operation of the current layer.

Gendron et al. (2011) deal with a location-distribution problem for a multi-channel retailing company selling a wide variety of products. To cope with demand fluctuations, guaranteeing service quality and consolidation due to small or medium-size products to be delivered, they design a multi-echelon distribution network. The corresponding multi-echelon network includes central warehouses, cross-docking terminals and small-docking terminals called satellites from where shipments are delivered to customers. They choose to use add, drop and exchange moves to generate neighborhoods and constitute layers based on these moves; drop and exchange moves are regarded easier than add moves and drop and exchange moves affiliated with terminals and satellites are included in layer 1. In layer 2, neighborhoods are generated by adding a new terminal or a new satellite.

MLVNS(x : current solution, l_{max} : number of layers) :

Determine set of layers L for l=1,...,$l_{\max}$

Determine set of neighborhood structures N_k^l for k=1,...,$k_{\max}^l$

for $l \leftarrow 1$ to $l_{\max}$

Order the $k_{\max}^l$ neighorhood structures : $N_{k_1}^l$,.....,$N_{k_{\max}}^l$

repeat

$k \leftarrow 1$

repeat

Order (randomly) the $m_{\max}^k$ sub-neighborhood structures : ${}^1N_k^l$,......, ${}^{m_{\max}^k}N_k^l$

$m \leftarrow 1$

repeat

Generate $x' \in {}^mN_k^l(x)$

MLVNS(x', l-1)

$m \leftarrow m+1$

until $f(x') < f(x)$ or $m > m^{\max}$

if $f(x') < f(x)$

$x \leftarrow x'$ and $k \leftarrow 1$

else

$k \leftarrow k+1$

endif

until $k > k_{\max}^l$

until termination condition is satisfied

end

Figure 4.21. General procedure of MLVNS

It can be practical to call simple neighborhood operations more often in VSN and partition VNS into layers based on the complexity of these operations. Yet, it can also be worthwhile to give priority to promising operations, and layers can be established in this respect. In addition to this, VNS starts from small neighborhoods and proceeds through large neighborhoods (Hansen and Mladenovic, 2001). To invoke more promising and small neighborhood operations primarily is the main motivation of our RVNS model, and this motivation leads to partition our RVNS model into multiple layers; hence, it is called multilayer RVNS (MLRVNS). Original problem P in this study is dealing with a nation-wide recovery network. Thus, size of the problem can be too extensive for a multi-layer VNS. Thus, it is proposed to segment our MLRVNS into two parts in Figure 4.22.

A multi-layered structure nesting all neighborhood operations as Gendron et al. (2011) propose can be computationally ineffective as the problem size increases. As it is explained in the rest of this section, neighborhood operations in our MLRVNS result in a vast number of neighbor solutions due to the capacity expansion decisions and multi-period environment. Thus, to be able to cope with computational difficulty of the multi-layered structure, layers associated with disassembly centers can be invoked firstly, which can be named as MLRVNS-1, next; the best solution obtained in MLRVNS-1 becomes the initial solution of MLRVNS-2 which includes layers associated with consolidation centers. Then, as a cyclic process, the best solution at the end of MLRVNS-2 becomes initial solution for MLRVNS-1 and so on.

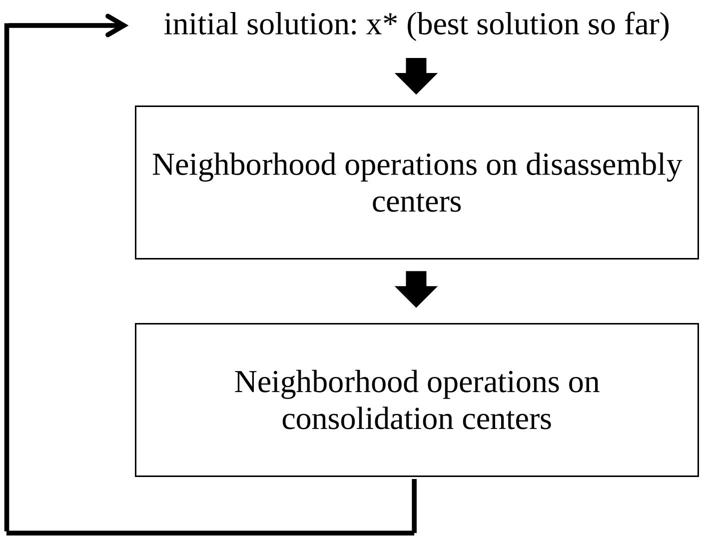

Figure 4.22. Segmentation of MLRVNS

There are two layers in MLRVNS-1 and MLRVNS-2 as stated below:

Layer 1: Drop Capacity Expansion in Disassembly Center $m \in M$

Drop Disassembly Center $m \in M$

Exchange Capacity Expansion in Disassembly Center $m \in M$

Exchange Disassembly Center $m \in M$

MLRVNS 1

Layer 2: Add Disassembly Center $m \in M$

Layer 3: Drop Consolidation Center $k \in K$

Exchange Disassembly Center $k \in K$

Layer 4: Add Consolidation Center $k \in K$

MLRVNS 2

As Gendron et al. (2011) propose to establish layers in regard of complexity, it can be considered to order neighborhood operations within a layer based on their complexity/simplicity; the simplest neighborhood operation can be ranked first to be employed. Complexity of a neighborhood operation can be measured by how difficult it is to solve original problem P after the generation of a new solution by it. In our case, a linear transportation problem follows each neighborhood operation which fixes binary variables and determines how to transport returns under the new configuration of the centers on account of the latest drop / exchange / add move. Transportation problem which is employed after each neighborhood operation is the crucial part in terms of solution time, and it is decided to be solved via commercial linear programming solver which provides fair computational time. Since the running time of the corresponding commercial linear programming solver does not change notably for different network configurations, simplicity of the problem to be solved after neighborhood operations does not vary based on the types of operations. In this case, a new approach, rather than complexity-oriented approach, related to ordering neighborhood operations within layers should be introduced; this can be an order regarding a balance between the number of candidate neighbors associated with a particular neighborhood operation and to what degree neighborhood operations are promising. In this perspective, neighborhood operations related to the central periods are granted the prior rank instead of the ones in the forecasting period, since more rapid improvements can be obtained by the alterations in the central period which have effects on the forecasting period too as the number of candidate neighbors does not vary significantly depending on the period. Moreover, decisions in the central period are more important than the ones in the forecasting period, since they are fixed in subsequent subproblems. After the prioritization of the neighborhood operations in the central period over the ones in the forecasting period, it should be decided in which order drop and exchange moves would be employed. Drop moves are preferred to be employed firstly since the number of candidate drop moves is supposed to be less than the number of candidate exchange moves. First layer of MLRVNS-1 runs as a simple RVNS represented in Figure 4.20. In Figure 4.23, structure of layer 1 is depicted.

Notation:

$X^* \rightarrow$ best solution so far, $O^* \rightarrow$ objective function value of X^* (upper bound)

N_v^l :Neighbors of incumbent solution by executing vth neighborhood operation in layer l

N_1^1 : Drop capacity expansion of disassembly center m in both periods where $e_{m1} = 1$ & $e_{m2} = 1$

N_2^1 :Drop capacity expansion of disassembly center m in central period where $e_{m1} = 1$

N_3^1 :Exchange capacity expansion of disassembly center m with disassembly center m' in central period where $e_{m1} = 1$, $e_{m'1} = 0$ & $Z_{m'1} = 1$

N_4^1 :Drop capacity expansion of disassembly center m in forecasting period where $e_{m2} = 1$

N_5^1 :Exchange capacity expansion of disassembly center m with disassembly center m' in forecasting period where $e_{m2} = 1$, $e_{m'2} = 0$ & $Z_{m'2} = 1$

N_6^1 :Drop disassembly center m in both periods where $Z_{m1} = 1$

N_7^1 :Drop disassembly center m in central period where $Z_{m1} = 1$

N_8^1 :Drop disassembly center m in forecasting period where $Z_{m1} = 0$ & $Z_{m2} = 1$

N_9^1 :Exchange disassembly center m with disassembly center m' in central period where $Z_{m1} = 1$ & $Z_{m'1} = 0$

N_{10}^1 :Exchange disassembly center m with disassembly center m' in forecasting period where $Z_{m1} = 0$, $Z_{m2} = 1$ & $Z_{m'2} = 0$

$nn_{\max}^{lv}(X)$: maximum number of solutions to be generated from solution X via neighborhood v in layer l

Figure 4.23. Representation of layer 1 in MLRVNS

Define function **func**(N_v^1):

$nn = 1$
find $nn_{\max}^{1,v}(X)$
do while $N_v^1(X) \neq \emptyset$ and $nn \leq nn_{\max}^{1,v}(X)$ and $exit$=0
generate a solution $X' \in N_v^1(X)$ with probability $p_v^1(X')$
if $O' \leq O^*$
$O^* = O', X^* = X', X = X', exit = 1$
else
$N_v^1(X) \rightarrow N_v^1(X) - \{X'\}$
update probability p_v^1, $nn = nn+1$
endif
end

MLRVNS - 1 (1 - a) :
$v = 1$
while $v < 11$
$exit = 0$
run func(N_v^l)
if $exit = 1, v = 1$ **else** $v = v+1$ **endif**
end

MLRVNS - 1 (1 - b) :
$V^1 = \{4, 5, 8, 10\}$
$i = 1$
while $i < 5$
$exit = 0$
run func$(N_{V^1(i)}^1)$
if $exit = 1, i = 1$ **else** $i = i+1$ **endif**
end

Figure 4.23. (cont'd)

In layer 1 of the MLRVNS, drop and exchange moves are executed on disassembly centers and capacity expansions in the order stated in Figure 4.23. Neighborhood operations are started to be employed on a particular solution in the search of a better solution and the search returns the beginning of the layer once an improvement in upper bound is achieved; and then the new best solution becomes the incumbent solution. This process continuous until all neighborhood operations in layer 1 are examined and no improvement is achieved. Initial neighborhood operations in layer 1 belong to the alterations in capacity expansion decisions. The reason behind the ordering of capacity expansion neighborhoods prior to neighborhoods related to dropping / exchanging disassembly centers is that neighborhood solution space of the former one is much less than the latter one's.

Although layer 1 covers a wide neighborhood space, it can be impractical / ineffective to evaluate all possible neighbor solutions generated by exchange moves, N_9^1 and N_{10}^1. Moreover, cyclic nature of MLRVNS in Figure 4.22 shows that MLRVNS-1 needs to be called multiple times in MLRVNS. Thus, a limitation on the number of neighbor solutions to be evaluated instead of evaluating all of them can be time-saving. $nn_{max}^{1,v}$ denotes this limitation in Figure 4.23. Number of possible drop and exchange moves associated with capacity expansions is not excessive and they do not need to be limited. Thus, $\mathrm{nn}_{\max}^{1,v}$ is equal to ∞ for $v \in \{1,2,\ldots8\}$; yet, a limitation on N_9^1 and N_{10}^1 is required. $nn_{max}^{1,9}$ and $nn_{max}^{1,10}$ vary based on the size of the neighborhood space. In Section 5.1, different values are assigned to $nn_{max}^{1,9}$ and $nn_{max}^{1,10}$ for different test instances.

Limitation on the number of neighbor solutions to be evaluated means that some of the neighbor solutions are excluded in the search for a better solution. Thus, as setting such limits, neighbor solutions to be evaluated should be chosen in such a way that they comprise good solutions as many as possible. At this point, scoring of variables in Scatter Search (SS) (Laguna, 2002) can be useful. SS is based on generating new solutions from a set of solutions called the reference set. Generation of new solutions is achieved by combining solutions in subsets of the reference set. A new solution after the combination of a particular subset of reference set is constructed based on variables' scores. In default procedure of SS, variables are scored as below:

$$score(i) = \frac{\sum_{j \in S} O^j \; x_j^i}{\sum_{j \in S} O^j}$$

where O^j is objective value of solution j in subset s

$x_j^i = 1$ if variable i takes value of 1 in solution j

Variables are scored based on objective values of solutions to be combined. By using a similar approach, neighbor solutions to be generated in MLRVNS can be determined. LR is run before MLRVNS and at the end of each iteration of LR, a feasible solution is generated. These feasible solutions can be considered a big subset of reference solutions. Then, each variable can be scored as below:

$$score(i) = \textbf{normalized}\left(-\frac{\sum_{j \in PH} P^j \, x_j^i}{\sum_{j \in PH} P^j} \right)$$

where PH is a set of distinct solutions obtained by the primal heuristic,

P^j is the objective value of solution j,

$i \in \left(Z_{mt} \cup Y_{kt} \right)$ and $x_j^i = 1$ if variable i takes value of 1 in solution j.

We would like to represent the favourability of the variables in direct proportion to their scores. Since the original problem P is a minimization problem, the ratio in the default procedure of SS does not satisfy the relevant representation. Thus, negative of the corresponsing ratio is used for scoring variables in this study. Normalization of negative scores is executed in the range [0,1] in order to work with positive score values. A high score for a variable *i* means that value of variable *i* is equal to 1 in many of good solutions obtained in LR phase. On the other hand, a low score indicates that solutions are less likely to be favorable in general when variable *i* is equal to 1. Thus, it is more likely to have positive effects after exchanging disassembly centers having low scores with the ones having high scores. Evaluation of exchange moves with highest $nn_{max}^{1,v}$ scores can be an option; yet, such a procedure relies on solutions in LR's last iterations too much and in addition to this, diversity may not be achieved. Thus, it is decided to assign probabilities to moves to be evaluated in N_9^1 and N_{10}^1 as high as their scores. In this fashion, the diverse solutions can be evaluated in the corresponding neighborhoods while moves with higher scores are more likely to be evaluated. After all, $pr_v^1(m,m')$ is introduced to MLRVNS; $pr_v^1(m,m')$ is the probability of generating a new solution by exchange move i in the neighborhood $v \in \{9, 10\}$ including disassembly centers m and m'.

$$pr_v^1(m,m') = \frac{\mathbf{normalized}(score(m') - score(m))}{\sum_{m \in M^v} \sum_{m' \in M'^v} \mathbf{normalized}(score(m') - score(m))}$$

where $M^9 = \{m \mid Z_{m1} = 1\}$, $M'^9 = \{m \mid Z_{m1} = 0\}$

$$M^{10} = \{m \mid Z_{m1} = 0 \text{ and } Z_{m2} = 1\},\ M'^{10} = \{m \mid Z_{m2} = 0\}$$

$nn_{max}^{1,9}$ and $nn_{max}^{1,10}$ limit the number of solutions to be generated in N_9^1 and N_{10}^1, respectively. Purpose of the introduction of $pr_v^1(m,m')$ is to decrease the probability of excluding the promising solutions of N_9^1 and N_{10}^1. Normalized values of score differences are used to obtain nonnegative probabilities. In the end, in function N_v^1 of layer 1, a new solution X' is generated from incumbent solution X by move i including disassembly centers m and m' with probability $pr_v^1(m,m')$.

Layer 2 (shown in Figure 4.24) which consists of add moves related to disassembly centers follows layer 1. New solution generated in layer 2 becomes incumbent solution to layer 1. If upper bound is updated in layer 1, layer 2 starts from the beginning with the new best solution; otherwise, layer 2 generates a new solution following the order of neighborhoods within it.

Notation:

N_1^2: Add a capacity expansion to disassembly center m in both periods
where $e_{m1} = 0$ & $e_{m2} = 0$ & $Z_{m1} = 1$ & $Z_{m2} = 1$

N_2^2: Add a capacity expansion to disassembly center m in the central period
where $e_{m1} = 0$ & $Z_{m1} = 1$

N_3^2: Add a capacity expansion to disassembly center m in the forecasting period
where $e_{m2} = 0$ & $Z_{m2} = 1$

N_4^2: Add a disassembly center m in both periods where $Z_{m1} = 0$

N_5^2: Add a disassembly center m in the forecasting period where $Z_{m1} = 0$ & $Z_{m2} = 0$

Figure 4.24. Representation of layer 2 in MLRVNS

Define function **func**(N_v^2):

$nn = 1$

find $nn_{max}^{2,v}(X)$

do while $N_v^2(X) \neq \varnothing$ and $nn \leq nn_{max}^{2,v}(X)$ and $exit = 0$

generate a solution $X' \in N_v^2(X)$ with probability $p_v^2(X')$

if $O' \leq O^*$

$O^* = O', X^* = X', X = X', exit = 1$

else

  if $v \in \{1, 2, 4\}$

  run ***MLRVNS - 1 (1 - a)***

  else

  run ***MLRVNS - 1 (1 - b)***

  endif

    if $X^* \neq X'$

    $exit = 1$

    else

    $N_v^2(X) \to N_v^2(X) - \{X'\}$

    update probability p_v^2

    $nn = nn + 1$

    endif

endif

end

MLRVNS - 1 (2) :

while $v < 6$

$exit = 0$

run func(N_v^2)

if $exit = 1, v = 1,$ **else** $v = v + 1$ **endif**

end

Figure 4.24. (cont'd)

Layer 1 is nested into layer 2 and solutions generated by layer 2 are subject to neighborhood operations in layer 1. At the end of this nested structure; drop, exchange, add, add & drop and add & exchange moves on disassembly centers are included in MLRVNS-1. Similar to the limitation on the number of neighbor solutions generated by exchange moves, add moves in layer 2 can be limited too since number of possible add moves can be too many when considering the fact that layer 1 is called after each add move. Add moves on capacity expansions do not need to be limited owing to the small size of their neighborhood solution space. Thus, $nn_{max}^{2,1}$, $nn_{max}^{2,2}$ and $nn_{max}^{2,3}$ are equal to ∞; yet, a limitation on N_4^2 and N_5^2 is required. Values assigned to $nn_{max}^{2,4}$ and $nn_{max}^{2,5}$ for different test instances are given in Section 5.1.

As it is stated before, we would like to disregard unfavorable solutions as much as possible as setting limits, $nn_{max}^{2,4}$ and $nn_{max}^{2,5}$. Then, $pr_v^2(m)$ is introduced to denote the probability of generating a new solution by adding disassembly center m in neighborhood v.

$$pr_v^2(m) = \frac{score(m)}{\sum\limits_{m \in M'^v} score(m)}$$

where $M'^4 = \{m \mid Z_{m1} = 0\}$

$M'^5 = \{m \mid Z_{m2} = 0\}$

As following the end of MLRVNS-1, MLRVNS-2 starts to run by using the best solution in MLRVNS-1 as the initial solution. MLRVNS-2 includes neighborhood operations associated with consolidation centers and the structure of it is very similar to MLRVNS-1. First layer of MLRVNS-2, layer 3, is represented in Figure 4.25.

Notation:

N_1^3 :Drop consolidation center k in both periods where $Y_{k1} = 1$

N_2^3 :Drop consolidation center k in the central period where $Y_{k1} = 1$

N_3^3 :Drop consolidation center k in the forecasting period where $Y_{k1} = 0 \,\&\, Y_{k2} = 1$

N_4^3 : Exchange consolidation center k with center k' in the central period where $Y_{k1} = 1 \,\&\, Y_{k'1} = 0$

N_5^3 : Exchange consolidation center k with center k' in the forecasting period where $Y_{k1} = 0, Y_{k2} = 1 \,\&\, Y_{k'2} = 0$

Figure 4.25. Representation of layer 3 in MLRVNS

Define function **func**(N_v^3):

$nn = 1$

find $nn_{max}^{3,v}(X)$

do while $N_v^3(X) \neq \varnothing$ and $nn \leq nn_{max}^{3,v}(X)$ and $exit = 0$

generate a solution $X' \in N_v^3(X)$ with probability $p_v^3(X')$

if $O' \leq O^*$

$O^* = O', X^* = X', X = X', exit = 1$

else

$N_v^3(X) \rightarrow N_v^3(X) - \{X'\}$

update probability p_v^3

$nn = nn + 1$

endif

end

MLRVNS - 2 (1 - a) :

$v = 1$

while $v < 6$

$exit = 0$

run func(N_v^3)

if $exit = 1, v = 1$ **else** $v = v + 1$ **endif**

end

MLRVNS - 2 (1 - b) :

$V^2 = \{3, 5\}$

$i = 1$

while $i < 3$

$\text{exit} = 0$

run func($N_{V^2(i)}^3$)

if $exit = 1, i = 1$ **else** $i = i + 1$ **endif**

end

Figure 4.25. (cont'd)

Drop moves on consolidation centers are executed in layer 3 as similar to drop moves on disassembly centers in layer 1. Since consolidation centers are uncapacitated, drop moves in layer 3 are limited to closing open consolidation centers of the incumbent solution. Layer 3 progresses until all neighborhood operations within it are executed and no improvement is achieved. Layer 4 follows the end of layer 3. Layer 4 has the same structure with layer 2 except the centers focused on. Layer 4 deals with add moves on consolidation centers instead of disassembly centers in layer 3. In Figure 4.26, representation of layer 4 is shown.

Notation:

N_1^4 : Add a consolidation center k in both periods where $Y_{k1} = 0$

N_2^4 : Add a consolidation center k in the forecasting period where $Y_{k1} = 0 \,\&\, Y_{k2} = 0$

Define function **func**(N_v^4) :

$nn = 1$, find $nn_{max}^{4,v}(X)$

do while $N_v^4(X) \neq \varnothing$ and $nn \leq nn_{max}^{4,v}(X)$ and $exit = 0$

generate a solution $X' \in N_v^4(X)$ with probability $pr_v^4(X')$

if $O' \leq O^*$ **then** $O^* = O', X^* = X', X = X', exit = 1$

else

  if $v = 1$

  run ***MLRVNS - 2 (1 - a)***

  else

  run ***MLRVNS - 2 (1 - b)***

  end

    if $X^* \neq X'$ **then** $exit = 1$

    else

    $N_v^4(X) \rightarrow N_v^4(X) - \{X'\}$, update probability $pr_v^4(X'), nn = nn + 1$

    endif

endif

end

MLRVNS - 2 (2) :

while $v < 3$

$exit = 0$

run func(N_v^4)

if $exit = 1, v = 1,$ **else** $v = v + 1$ **endif**

end

Figure 4.26. Representation of layer 4 in MLRVNS

As similar to layer 1 and layer 2, the number of solutions that can be generated by exchange moves and add moves on consolidation centers can be too many and evaluation of all neighborhoods can be ineffective in terms of the computation time. Thus, $nn_{max}^{3,v}$ and $nn_{max}^{4,v}$ are set to limit the number of solutions to be generated by exchange and add moves on consolidation centers. Again, evaluation of all drop moves is not time consuming and $nn_{max}^{3,1}$, $nn_{max}^{3,2}$ and $nn_{max}^{3,3}$ can be equal to ∞. Values of $nn_{max}^{3,4}$, $nn_{max}^{3,5}$, $nn_{max}^{4,1}$ and $nn_{max}^{4,2}$ are specified in Section 5.1. p_v^3 and p_v^4 are calculated in the same way as p_v^1 and p_v^2.

Main drawback of the represented MRVNS is to split neighborhood operations related to disassembly centers and consolidation centers completely in each iteration. Neighborhood operations on consolidation centers are started to be employed after all operations on disassembly centers are completed. In other words, neighborhood operations associated with consolidation centers and disassembly centers respectively are not placed in a layered structure but in order. The influence of the alterations in consolidation centers on decisions related to the disassembly centers is included in the algorithm iteratively. The preference of the corresponding absolute distinction in each iteration has arisen from satisfactory solutions obtained in test instances within relatively short computation times through the employment of the proposed MLRVNS.

4.7.2. Stopping Criteria of Multi - Layer Reduced Variable Neighborhood Search (MLRVNS)

Termination conditions of MLRVNS are regarding the total running time of the algorithm, the improvement achieved since the beginning of it and the number of successive iterations without improvement of the best solution. We find it convenient to stop the algorithm if gap percentage, which is derived from the difference between the minimum objective function value obtained in MLRVNS and upper bound in LR, exceeds a pre-specified threshold, *pl*:

$$\text{Gap}_{\text{VNS}} = \frac{\text{UB - O}^*}{\text{UB}}$$

where O^* is best objective value obtained in MLRVNS,

UB is upper bound coming from LR

If Gap_{VNS} exceeds threshold *pl*, then MLRVNS stops. Likewise the minimum gap percentage to stop LR, *pl* is set to 5%, $pl = 0.05$.

This threshold may be difficult to be exceeded. Moreover, it can be impossible to exceed it, if optimality gap of Lagrangean solution is small. Therefore, we also use running time of the algorithm as the termination condition, and MLRVNS is not allowed to run after 2000 seconds, *Ntime*=2000. Entire procedure of MLRVNS is represented in Figure 4.27.

START

INIT $VNS_{initial} = UB, VNS^{1}_{best} = UB, no\_improve\_vns = 0, time\_elapsed = 0, iter = 1$

$$gap_{VNS} = \frac{VNS_{initial} - VNS_{best}}{VNS_{initial}}$$

where $VNS_{initial}$ and VNS^{1}_{best} are the best objective value

in LR and iteration 1 of MLRVNS,respectively

do while $gap_{VNS} < pl$ *and* $time\_elapsed < Ntime$ and $no\_improve\_vns < ni\_vns_{max}$

RUN ***MLRVNS - 1 (1)***

if $gap_{VNS} \geq pl$, STOP **endif**

RUN ***MLRVNS - 1 (2)***

if $gap_{VNS} \geq pl$, STOP **endif**

RUN ***MLRVNS - 2 (1)***

if $gap_{VNS} \geq pl$, STOP **endif**

RUN ***MLRVNS - 2 (2)***

if $VNS^{iter}_{best} = VNS^{iter-1}_{best}$ **then** $no\_improve\_vns = no\_improve\_vns + 1$

else $no\_improve\_vns = 0$

endif

$iter = iter + 1$

end

Figure 4.27. Representation of MLRVNS

Finally, it should be noted that best solution does not necessarily improve in each iteration of MLRVNS, since it can stack at local optimum and fail in reaching the global optimum. Therefore, we should end the algorithm after a certain number of iterations without best solution improvement, ni_vns_{max} that is determined to be 3, meaning that MLRVNS stops if three iterations in row fail in improving the best solution, $ni\_vns_{max} = 3$.

4.8. Modifications in Subproblems of the Rolling Horizon Approach

As our solution methodology for the original problem P, we develop an algorithm by RHA which is a nesting of hybrid LR and MLRVNS in subproblems of it. In previous sections of this chapter, our solution methodology is explained in detail. For the simplicity, this explanation is regarding first RHA subproblem of the proposed model where there is no pre-given decisions like opening / expanding a center. However, all RHA subproblems except the first one are influenced by the decisions in the previous subproblems. Therefore, relations and information transfers between RHA subproblems should be clarified.

We stated in Section 4.1.1 that values of all binary variables in the central period of a particular RHA subproblem *s*, $s \in \{1,2..,S\text{-}1\}$, are fixed in the rest of RHA subproblems. Then, opening decisions related to consolidation and disassembly centers are interacted within RHA subproblems as below:

if $\hat{Y}_{k1} = 1$ in subproblem $\bar{s}$ then $Y_{k1} = 1$ in subproblem s where $s \in \{\bar{s}+1,...,S\}$

if $\hat{Z}_{m1} = 1$ in subproblem $\bar{s}$ then $Z_{m1} = 1$ in subproblem s where $s \in \{\bar{s}+1,...,S\}$

Similarly, once a capacity expansion decision is taken in a central period of a RHA subproblem, expanded capacity must be taken into account in the rest of the RHA subproblems.

Then, initial capacity should be updated at the beginning of each RHA subproblem as below:

$$I^{s}_{m0} = \hat{I}^{s\text{-}1}_{m0} + \hat{e}^{s\text{-}1}_{m1} \qquad \forall\, s \in S, m \in M$$

where $\hat{I}^{s}_{m0}$ is the initial capacity of disassembly center m in subproblem s,

$\hat{e}^{s-1}_{m1}$ is capacity expansion variable for disassembly center m

in the central period of subproblem $s-1$.

In the light of interactions among RHA subproblems, our solution method including LR and VNS should be altered.

In the decision process OD for subproblem P^{d}_{m}, net benefits of opening disassembly centers are evaluated for relevant decisions. Yet, if a disassembly center m is determined to be opened in a central period of a RHA subproblem s, it has to remain open in the rest of RHS subproblems regardless of its net benefits. Then, decision process OD in RHS subproblem s, OD^{s}, can be represented as shown in Figure 4.28. Similarly, decision process OK in RHS subproblem s, OK^{s}, is shown in Figure 4.29.

***OD^s* :**

for $m \leftarrow 1$ to $|M|$

    if $(nk_{m1} < 0$ and $nk_{m1} + nk_{m2} < 0)$ or $\hat{Z}^{s-1}_{m1} = 1$

        $Z_{m1} = 1$ and $Z_{m2} = 1$

    elseif $nk_{m2} < 0$

        $Z_{m2} = 1$

    endif

end

Figure 4.28. Decision process OD in RHA subproblem s

***OK^s* :**

for $k \leftarrow 1$ to $|K|$

    if $(ns_{k1} + ns_{k2} < 0$ and $ns_{k1} < 0)$ or $\hat{Y}^{s-1}_{k1} = 1$

        $Y_{k1} = 1$ and $Y_{k2} = 1$

    elseif $ns_{k2} < 0$

        $Y_{k2} = 1$

    endif

end

Figure 4.29. Decision process OK in RHA subproblem s

Our primal heuristic in a particular RHA subproblem must consider the decisions taken in previous RHS subproblems as well. Main objective of primal heuristic is to generate a feasible solution. Thus, it should keep the centers open once they are determined to be opened in previous RHA subproblems. Therefore, at the beginning of the primal heuristic, opening decisions below must be taken:

$$\hat{Y}^{s}_{k1} = 1 \text{ and } \hat{Y}^{s}_{k2} = 1 \text{ if } \hat{Y}^{s-1}_{k1} = 1 \qquad \forall s \in S, k \in K$$

$$\hat{Z}_{m1} = 1 \text{ and } \hat{Z}_{m2} = 1 \text{ if } \hat{Z}^{s-1}_{mt} = 1 \qquad \forall s \in S, m \in M$$

As a feasible solution is generated at the end of each Lagrangean iteration, opening center decisions in the primal heuristic are settled based on np_{kmt} which is the sum of ns_{kt}, net benefit of opening consolidation center *k* on the objective function of P^{LR}, and nk_{mt}, net benefit of opening disassembly center m on the corresponding objective function. Net benefit of a particular center is calculated as extracting the associated cost with opening the corresponding center from gross benefit derived from opening it. As a result, instead of evaluating opening consolidation and disassembly centers separately, they are evaluated simultaneously based on np_{kmt} values in the primal heuristic. Yet, it should be noted that calculation of np_{kmt} should be different if at least one of the consolidation center *k* and disassembly center *m* in a particular pair *(k,m)* is determined to be opened in previous RHA subproblems. As net benefit is calculated for such a pair *(k,m)*, opening cost of the center within relevant pair that is determined to be opened in previous subproblems should be ignored, since it must remain open regardless of its costs and benefits. Thus, ns_{kt} and nk_{mt} values can be altered as below:

$$nk^{s}_{mt} = \begin{cases} kd_{mt} + \sum_{t}^{2} f1_{mt} & \text{if } Z^{s-1}_{m1} = 0 \\ kd_{mt} & \text{o/w} \end{cases} \text{ and } ns^{s}_{kt} = \begin{cases} f1_{kt} + \sum_{j \in J} \sum_{m \in M} \lambda_{jkmt} & \text{if } \mathrm{Y}^{s-1}_{k1} = 0 \\ \sum_{j \in J} \sum_{m \in M} \lambda_{jkmt} & \text{o/w} \end{cases}$$

MLRVNS in a particular RHS subproblem should consider the decisions in previous RHS subproblems and be revised as well. As it is stated above, consolidation and disassembly centers must remain open in subsequent RHA subproblems once they are determined to be opened in a subproblem *s*. Therefore, as we explore the neighborhoods of an incumbent solution in MLRVNS, we exclude neighbor solutions that violate the feasibility and close any of the consolidation and disassembly centers determined to be opened in previous RHA subproblems. As a result, we ignore all solutions in N_v^l that close centers determined to be opened in previous RHA subproblems:

$$\text{if } \hat{Y}_{k1}^{s\text{-}1} = 1,\ Y_{k1}^{s} = 1 \text{ in } N_v^l \qquad \forall s \in S, k \in K, v \in V, l \in L$$

$$\text{if } \hat{Z}_{mt}^{s\text{-}1} = 1,\ Z_{m1}^{s} = 1 \text{ in } N_v^l \qquad \forall s \in S, m \in M, v \in V, l \in L$$

There is no existing infrastructure at the beginning of our proposed model and first RHA subproblem develops a network for the first two periods (central and forecasting period) from scratch. Subsequent RHA subproblem fosters the corresponding network if necessary and so on. As a result, the extent of RHA subproblems decreases as RHA proceeds. This leads to the necessity of differentiating stopping conditions for each subproblem. It is expected that a particular RHA subproblem requires more computation time when compared to subsequent subproblems. Thus, we decide to decrease the stopping criteria, *Niter* and *Ntime*, gradually as RHA iterates. These parameters have already been settled for the first RHA problem in Section 4.5 and Section 4.7.2 as below:

$Niter^1 = 2000$ and $Ntime^1 = 2000$ where $Niter^s$ and $Ntime^s$ are stopping conditions of LR and MLRVNS, respectively, in subproblem $s \in S$

$Niter^s$ and $Ntime^s$ values are decided to be halved in each RHA subproblem as below:

$$Niter^s = \frac{Niter^{s\text{-}1}}{2} \quad \forall\, s \in S\text{-}\{1\} \text{ and } Ntime^s = (5-s)*500 \quad \forall\, s \in S$$

Thanks to halving of stopping criteria parameters, our proposed model allocates more time to initial RHA subproblems while latter subproblems are terminated in shorter time.

Since the half of the step length in LR is dependent on *Niter* that decreases as the proposed algorithm proceeds, condition to halve step length should be modified too.

In Section 4.4.1, ni_{max} is determined to be centesimal of *Niter*. This ratio can work for the first subproblem of RHA; yet, in subsequent subproblems, ni_{max} can be too small and step length can decrease rapidly which hinders to exploit good step sizes. Hence, ni_{max} is determined to be dependent on the order of subproblems in RHA. Step length *sl* is halved in subproblem $s \in S$, if lower bound does not improve within a pre-specified number of iterations, ni^s_{max}:

$$ni^s_{max} = \frac{s\, Niter^s}{100} \qquad \forall\, s \in S$$

4.9. Improvement of the Rolling Horizon Approach

RHA can yield rewarding solutions by separating the original problem P into subproblems with shorter time horizons which enables to decrease solution time of the proposed model. Yet, RHA usually fails in reaching the optimal solution since it considers near future of the concerned central period in each subproblem and it does not cover the entire planning horizon of the problem. Thus, final solution attained by RHA is open to improvement. Efficiency of RHA depends on the accuracy of the decision variable values in the central periods to be fixed. There are two undesirable possibilities which can be regarded as the main drawback of RHA in our problem. These possibilities are:

- To fail in opening prominent/promising centers or expanding disassembly centers in regard to the entire planning horizon
- To open centers or expanding disassembly centers which seem appropriate in a particular sub-horizon whereas it is inefficient in total.

We propose to run algorithm UU (underutilization) in Figure 4.30 to mitigate possible negative outcomes associated with the possibilities stated.

Algorithm UU evaluates the necessity of consolidation and disassembly centers determined to be opened at the end of RHA based on their utilization rates. Opening / expanding decisions related to the centers having higher utilization rate than a pre-given utilization threshold in the best solution are fixed in the original problem P and problem $\overline{\mathrm{P}}$ that is the original problem P bounded with fixed variables is solved to optimality by the commercial solver. Owing to the algorithm UU, centers determined to be opened in the best solution at the end of RHA, but are underutilized, can be excluded from the best solution. Utilization rate of a disassembly center, u_m, is simply calculated by dividing the total inbound flow through the relevant disassembly center over the entire planning horizon to the summation of its capacity in each period of the planning horizon. If $u_m \geq 0.75$, opening /expanding decisions associated with the disassembly center m is fixed in problem $\overline{\mathrm{P}}$. Subsequently, utilization rate of consolidation center k, nu_k, is calculated by dividing total flows to fixed disassembly centers through consolidation center k to total flows visiting it. If $nu_k \geq 0.75$, opening decisions associated with consolidation center k is fixed in problem $\overline{\mathrm{P}}$. Finally, problem $\overline{\mathrm{P}}$ is solved to optimality and final solution of our proposed solution procedure is obtained.

UU :

for $m \leftarrow 1$ to $|M|$

$$u_m = \frac{\sum_{j\in J}\sum_{k\in K}\sum_{t=1}^{t=T} X_{jkmt}\, s_{jt}}{\sum_{t=1}^{t=T}(I_{m0} + \sum_{w=1}^{w=t} e_{mw})}$$

end

$$nu_k = \frac{\sum_{j\in J}\sum_{m\in M^u}\sum_{t=1}^{t=T} X_{jkmt}\, s_{jt}}{\sum_{j\in J}\sum_{m\in M}\sum_{t=1}^{t=T} X_{jkmt}\, s_{jt}} \quad \text{where } M^u = \{m \mid u_m \geq 0.75\}$$

$$\overline{\mathrm{Z}}_{mt} = \mathrm{Z}_{mt},\ \overline{\mathrm{e}}_{mt} = \mathrm{e}_{mt} \quad \forall\, t \in T, m\, \exists \{\mathrm{u}_m \geq 0.75\}$$

$$\overline{\mathrm{Y}}_{kt} = \mathrm{Y}_{kt} \quad \forall\, t \in T, k\, \exists \{\mathrm{nu}_k \geq 0.75\}$$

SOLVE problem $\overline{\mathrm{P}}$ where $\overline{Z}_{mt}, \overline{Y}_{kt}$ and $\overline{e}_{mt}$ are fixed

end

Figure 4.30. Improvement algorithm of RHA

After all, a brief view of entire solution procedure is shown in Figure 4.31.

$t = 1$;

for $s \leftarrow 1$ to $|S|$

central period $= t$ and forecasting period $= t + 1$

SOLVE LR for subproblem s

RUN MLRVNS by using the best solution of LR as initial solution

fix Z_{mt}, Y_{kt}, e_{mt} and update initial capacities of disassembly centers

$t = t + 1$

end

RUN algorithm UU

Figure 4.31. Representation of the entire solution procedure

CHAPTER 5

COMPUTATIONAL STUDY

In this chapter, we evaluate the performance of the proposed approach based on the test instances we generate. Firstly, we explain the design of our experiments and how test instances are generated. Then, performance measures for evaluation are introduced, and lastly, results of the computational study are discussed.

5.1. Design of Experiments

To test the performance of the proposed model, we generate diverse sets of test instances since there is no test instances library for two-echelon dynamic capacitated FLPs. Different test instance generation procedures are proposed by the researchers for the problems belonging to a similar class with the problem in this study. We generate our test instances as referring to the procedure proposed by Thanh et al. (2012) who study two-echelon FLP with modular capacity. We draw upon different papers, which are cited in the rest of this section, to generate test instances.

Demands of customers are derived from Melo et al. (2005). Authors produce demands for the first period with uniform distribution in the interval [0,25] and increase demands by a percentage between 5% and 10% in the following period and so on. Similarly, we develop quantity of returns at collection centers as:

$$s_{j1} = \mathrm{U}[0, 25] \qquad \begin{array}{l} \textbf{for } t \leftarrow 1 \text{ to } T \\ s_{jt} = s_{j,t-1} \, (\mathrm{U}[105, 150])/100 \\ \textbf{end} \end{array}$$

Return quantities increasing monotonically represent the pattern of the customer’s/householder’s preferences. Return quantities are expected to increase as consciousness and awareness of recovery options increase by time. Network representation of the problem is constructed as following the procedure by Cordeau et al. (2006).

For each location of consolidation center and disassembly center, Euclidean coordinates are generated randomly in the unit square [0,1] x [0,1]. Distances, d_{jkm}, are calculated by summing the Euclidean distance between collection center $j \in J$ and consolidation center $k \in K$, and consolidation center $k \in K$ and disassembly center $m \in M$:

$$d_{jkm} = \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2} + \sqrt{(x_m - x_k)^2 + (y_m - y_k)^2}$$

where x_j, x_k, x_m and y_j, y_k, y_m are x and y axis coordinates of collection center $j \in J$, consolidation center $k \in K$ and disassembly center $m \in M$, respectively.

There is no unit transportation cost coefficient in the objective function and distance is directly equal to the unit transportation cost. Therefore, d_{jkm} is updated as below to represent the cost of transportation:

$d_{jkm} = d_{jkm}\ \mathrm{U}[0{,}100]$ where $\mathrm{U}[0{,}100]$ represents transportation cost per unit distance.

Initial capacity of disassembly centers is determined via the procedure by Melkote and Daskin (2001). Proposed procedure is regarding a single-period problem and assigns the same initial capacity to all capacitated facilities based on the following ratio:

$K_{min} = \dfrac{\sum_{i \in I} d_i}{|M|}$, where d_i denotes the demand of customer $i \in I$ and M is the set of capacitated facilities.

Melkote and Daskin (2001) determine the capacities of facilities as:

$K = K_{min} + \alpha$, where α can be determined as a product of average demand of customers.

For example, α can be determined as:

$\alpha = 2\dfrac{\sum_{i \in I} d_i}{|N|}$ where N is the number of customers

Authors' procedure can be applied to our problem while determining the initial capacities. Yet, our original problem P has a multi-period horizon whereas Melkote and Daskin (2001) study a single-period problem. Therefore, we cannot simply use sum of demands in the single period to calculate K_{min}. We choose to use demands in the medium period of the entire planning horizon T which can be denoted as $t_m = \lceil |T|/2 \rceil$ in the calculation of K_{min}. Then, K_{min} can be denoted as:

$$K_{min} = \frac{\sum_{j \in J} s_{j3}}{|M|}$$

As result, all disassembly centers have the same initial capacity in our problem:

$I_{mo} = K_{min} + 25\alpha$, where $\alpha \in \{10, 20\}$ and 25 is the limit on the maximum demand of collection centers.

Two different values for α are introduced to examine the effect of initial capacities on the performance of the proposed solution procedure.

Thanh et al. (2012) refer to the study of Cortinhal and Captivo (2003) to determine fixed costs of the facilities and we use the same approach to assign operating costs to consolidation and disassembly centers as they do.

Cortinhal and Captivo (2003) calculate the fixed cost of a facility as below:

$f_i = \mathrm{U}[0, 90] + \mathrm{U}[100, 110]\sqrt{a_i}$ where a_i is the capacity of facility i.

While fixed cost is calculated as above, authors multiply the distances between facilities with $\mathrm{U}[0, 100]$ to calculate unit transportation cost between these facilities. In other words, transportation cost coefficient is generated from the uniform distribution between 0 and 100. Coefficient of capacity in the calculation of fixed cost above is coming from the corresponding transportation cost distribution. Similar to Cortinhal and Captivo (2003) approach, unit transportation costs are uniformly distributed values between 0 and 100 in our problem. Yet, main difference between the corresponding study and our problem is that our problem has two-echelon structure whereas the reference paper deals with a single-echelon problem. Thus, operating costs of disassembly and consolidation centers cannot be determined as exactly as Cortinhal and Captivo (2003) propose.

As it is stated previously, operations executed within disassembly centers are more complex than the ones in consolidation centers and this causes disassembly centers to have higher costs to be operated. If it is intended to have equivalent operating costs to the ones in the reference paper, we can prefer higher coefficients in the calculation of operating costs of disassembly centers when compared to the parameters Cortinhal and Captivo (2003) use; and contrarily, operating costs of consolidation centers can be calculated by smaller coefficients. In the end, equivalent operating costs can be obtained.

We can determine operating costs of disassembly centers as below:

$$f2_m = \mathrm{U}[\,0\,,180\,] + \mathrm{U}[\,200\,,220\,]\sqrt{I_{m0}}\ .$$

It is seen that operating costs of disassembly centers would be higher than facility fixed costs in the reference problem. Yet, we assume that we would have similar operating costs on the average when costs of consolidation centers and disassembly centers are considered aggregately, since operating costs of consolidation centers are less than the costs of disassembly centers and they can compensate for higher operating costs of disassembly centers. We reckon that average operating cost of consolidation centers can be at most as half of the average operating cost of disassembly centers and we calculate the operating costs of consolidation centers as follows:

$$f1_k = AF\, W_k\,,$$

where W_k is generated by $\mathrm{U}[0.25\,,0.5]$ and $AF = \dfrac{\sum_{m \in M} f2_m}{|M|}$

Our problem environment includes dynamic capacity derived from capacity expansion opportunity at the beginning of each period (at this point, the 8th clause in Section 3.2.1 should be recalled: capacity can be expanded only at the beginnings of periods and expansion must be exactly by one modular capacity). Thus, along with the initial capacities, size of capacity expansion modules and costs associated with them must be determined. Capacity expansion modules are calculated as below:

$$p_m = 0.25 I_{m0}$$

It should be fair to think that cost of unit capacity expansion should be higher than operating cost of a unit initial capacity:

$$\frac{ec_{mt}}{p_m} \geq \frac{f2_m}{I_{m0}}$$

In this sense, ec_{mt} values can be determined as:

$$ec_{mt} = 1.25 \frac{f2_m}{I_{m0}} p_m$$

Thus, unit capacity expansion cost is 0.25 times higher than operating cost of a unit capacity.

Another parameter in the model to be determined is *TH* which is the maximum distance allowed for returns to be moved. *TH* is determined to be 60th percentile of sum of the distances from collection centers to disassembly centers through consolidations centers.

Parameters of our problem are determined as Thanh et al. (2012) propose and the proposed procedure is explained above. Besides problem parameters, we should determine the parameters used in our solution methodology as well.

First of all, it should be noted that planning horizon of our model is determined to be five years where each year corresponds to a period. Although a set of diverse problem instances are generated which are explained in the following parts, planning horizon has remained as five years in all these instances since the length and number of periods seem reasonable to us for a strategic planning and additionally, there are already considerably many parameters to assign a set of values to test the proposed model even if planning horizon does not vary among test instances.

As stated in Section 4.1, the scheme of the decomposing original problem into subproblems by RHA is dependent on some parameters determination of which is left to the user of the approach. Length of the central and forecasting periods, and the selection of the variables to be fixed in subproblems are the key parameters/decisions influencing the performance of the corresponding approach. We decide to make both the length of central and forecasting periods equal to one (l^c=1, l^f=1); in other words, horizon of subproblems derived from RHA is determined to be two years. Mainly, there are two reasons relating to two-year length of subproblems; firstly, small-size subproblems are easy to be handled and they provide significant savings in solution time, especially in regard of location and capacity expansion decisions simultaneously. When the length of both central and forecasting period is equal to 1, subproblem P_m^d basically evaluates whether to open disassembly center or not, and if it is viable to open it, P_m^d determines when to open it (in period 1 or period 2) and evaluates capacity expansions in suitable periods. As the horizon of subproblem P_m^d extends, capacity expansions to be evaluated increase exponentially which leads to higher complexity. The other reason of preferring short subproblem horizon is pertaining to the structure of our solution procedure. In our solution methodology, we do not relax binary variables in the forecasting period unlike conventional RHA since relaxation of binary constraints would not change the structure of either LR or MLRVNS. To retain binary constraints in the forecasting period mitigates the negative effect of short subproblem horizon.

After the designation of RHA, structure of subproblems invoking LR to be solved becomes clear. Our Lagrangean Heuristic uses subgradient optimization for Lagrange multipliers' update and several parameters related to subgradient optimization need to be determined as the initial value of step length in Lagrange multiplier update and initial multiplier values. Although step length value has influence on the convergence towards optimal solution explicitly, there is not any prevalent step length determination procedure in the literature. Nonetheless, Held et al. (1974) cite that to have step length between 0 and 2 ensures geometric convergence towards optimal solution. In the end, we set step length to 2 at the beginning of LR, sl^0=2.

As initialization of Lagrange multipliers, we determine to set them to 0 at the beginning of Lagrangean Heuristic which is more frequently preferred in the literature rather than other alternatives, like initializing values of Lagrange multipliers one at a time and using dual values for the initialization of multipliers (Siamitros et al., 2004).

nn_v^l values are very influential on the performance of MLRVNS. We intend to use different limitation values depending on the size of the problems. nn_v^l values are calculated as below:

$nn_{max}^{l,v} = \frac{|N_v^l|}{r_v^l}$ where r_v^l is a parameter depending on the number of relevant potential centers

In Table 5.1, calculation of r_v^l values is shown.

Table 5.1. r_v^l values to be used in the calculation of $nn_{max}^{l,v}$

	Disassembly center				Consolidation centers			
	Exchange	Exchange	Add	Add	Exchange	Exchange	Add	Add
$\|K\|,\|M\|$	r_9^1	r_{10}^1	r_4^2	r_5^2	r_4^3	r_5^3	r_1^4	r_2^4
40	20	20	5	5	20	20	5	5
25	10	10	4	4	10	10	4	4
15	5	5	3	3	5	5	3	3
10	2	2	2	2	2	2	2	2

Since the number of neighbor solutions increases exponentially with the number of potential centers, r_v^l values are determined in such a way that a larger portion of possible neighbor solutions are disregarded (excluded due to the limitation) as the number of potential candidate centers increases. It should be noted that there is no limitation on exchange and add moves if the number of relevant potential centers is less than 10.

All in all, problem types in test instances are shown in Table 5.2.

Table 5.2. Problem types in test instances

# of collection centers $\mid J \mid$	# of potential consolidation centers $\mid K \mid$	# of potential disassembly centers $\mid M \mid$	Value of α
40	15	5	10
40	15	5	20
80	15	5	10
80	15	5	20
80	25	5	10
80	25	5	20
80	25	10	10
80	25	10	20
80	40	15	10
80	40	15	20
100	25	5	10
100	25	5	20
100	25	10	10
100	25	10	20
100	25	15	10
100	25	15	20
100	40	5	10
100	40	5	20
100	40	10	10
100	40	10	20
100	40	15	10
100	40	15	20
200	40	5	10
200	40	5	20
200	40	10	10
200	40	10	20

The parameters defining the problem type are the number of collection centers $|J|$, the number of potential consolidation centers $|K|$, the number of potential disassembly centers $|M|$ and initial capacities of disassembly centers I_{m0}. As it is stated before, I_{m0} is equal to the sum of $K_{\min}$ and $\alpha * 25$ and $\alpha \in \{10,20\}$. Different values of α enable to analyze the influence of initial capacities on the performance of the proposed solution approach.

As having different numbers of potential centers from 5 to 40 and collection centers from 40 to 200, a wide range of problem types is aimed to be reached. Since generation of test instances includes randomness, 5 instances for each problem type are generated to mitigate the effect of randomness in evaluating the results for test instances. In the end, we have carried out the experiments with 130 test instances.

5.2. Performance Measures

As evaluating the performance of the proposed solution approach, following performance measures are used:

1. The following gap ratios can be used to measure performances of the proposed solution approach.

    a. Ultimate performance measure of the proposed solution approach is the gap between the optimal solution and the best solution found by the proposed approach. Normally, this gap is equal to the ratio of the objective value difference of corresponding solutions to the optimal value. Yet, to find the optimal solution can be so time consuming for large size problems. Therefore, 36,000 central processing unit (CPU) seconds is set as a limit to search for the optimal solution via commercial solver.

$$\text{Gap}=\frac{\text{UB} - \text{P}^*}{\text{P}^*}$$

where UB is the best solution obtained at the end of entire solution procedure, and

P^* is the best solution found by commercial solver in 36,000 CPU seconds.

b. Gap measures the performance of the proposed approach in comparison to the performance of CPLEX in 36,000 CPU seconds. It should be noted that Gap is not necessarily equivalent to optimality gap, since CPLEX can fail in finding optimal solution in 36,000 CPU seconds. Gap_1 shows the ratio of the difference between the best solution found by the proposed approach and the lower bound given by CPLEX solver to the latter value. Gap_1 can be considered as an upper bound on the optimality gap whereas Gap is a lower bound on the optimality gap.

$\mathrm{Gap}_1 = \dfrac{\mathrm{UB} - \mathrm{LB}}{\mathrm{LB}}$, where LB is the lower bound CPLEX solver found.

c. Performance of proposed solution approach is strictly dependent on the performance of RHA. Thus, original problem P is also solved by using RHA via commercial solver that is run to solve subproblems of RHA instead of using LR and MLRVNS.

$$\mathrm{Gap}_{\mathrm{RHA}} = \frac{\mathrm{P}^{\mathrm{RHA}} - \mathrm{P}^*}{\mathrm{P}^*}$$

where $\mathrm{P}^{\mathrm{RHA}}$ is the best solution obtained by using RHA in commercial solver

Maximum running time limit of 36,000 seconds should also be set for RHA problem for fair comparison. Yet, such a limitation cannot be set easily when using RHA, since RHA leads to a number of subproblems to be solved. For example, even a single subproblem may not be solved to optimality in 36,000 seconds for large size problems. Therefore, time limit on each subproblem of RHA rather than a total time limit can be rational. Following time limits are used in the calculation of $\mathrm{P}^{\mathrm{RHA}}$:

$Ntime^{1}_{RHA} = 20000$

if $rt^{1}_{RHA} < 15000$, $Ntime^{2}_{RHA} = 15000$ **else** $Ntime^{2}_{RHA} = 10000$ **endif**

if $rt^{1}_{RHA} + rt^{2}_{RHA} < 20000$, $Ntime^{3}_{RHA} = 10000$ **else** $Ntime^{3}_{RHA} = 7500$ **endif**

if $rt^{1}_{RHA} + rt^{2}_{RHA} + rt^{3}_{RHA} < 30000$, $Ntime^{4}_{RHA} = 6000$ **else** $Ntime^{4}_{RHA} = 2500$ **endif**

where $Ntime_{RHA}^{s}$ is running time limit on subproblem s of RHA, and

rt_{RHA}^{s} is running time of subproblem s of RHA

d. LR and MLRVNS are used to solve subproblems of RHA in the proposed solution approach. To test their performance, solution of the proposed approach should be compared with the solution of RHA where subproblems are solved by commercial solver.

$$\mathrm{Gap}_2 = \frac{\mathrm{UB} - \mathrm{P}^{\mathrm{RHA}}}{\mathrm{P}^{\mathrm{RHA}}}$$

e. How close the lower and upper bound approaches towards each other in LR is an important measure to evaluate the performance of LR.

$$\mathrm{Gap}_{\mathrm{LR}} = \frac{\mathrm{UB}_{\mathrm{LR}} - \mathrm{LB}_{\mathrm{LR}}}{\mathrm{LB}_{\mathrm{LR}}}$$

where $\mathrm{UB}_{\mathrm{LR}}$ and $\mathrm{LB}_{\mathrm{LR}}$ are upper and lower bounds obtained at the end of LR.

In our solution approach, LR is invoked in each subproblem of RHA. Therefore, in each RHA subproblem, particular $\mathrm{Gap}_{\mathrm{LR}}$ values are obtained. Instead of regarding all $\mathrm{Gap}_{\mathrm{LR}}$ values, it is decided to use $\mathrm{Gap}_{\mathrm{LR}}^{\max}$ that is the maximum LR gap among subproblems of an instance.

$\mathrm{Gap}_{\mathrm{LR}}^{\max} = \underset{s \in S}{\mathrm{Max}}\,(\mathrm{Gap}_{\mathrm{LR}}^{s})$, where $\mathrm{Gap}_{\mathrm{LR}}^{s}$ is $\mathrm{Gap}_{\mathrm{LR}}$ in subproblem $s \in S$.

f. Performance of MLRVNS can be measured by observing the improvement achieved by it on the solution obtained by LR. Performance of MLRVNS varies among RHA subproblems of a particular test instance. Maximum improvement of MLRVNS achieved in subproblems is used as the performance indicator of MLRVNS.

$$\mathrm{Gap}_{\mathrm{VNS}}^{s} = \frac{\mathrm{UB}_{\mathrm{LR}}^{s} - \mathrm{UB}_{\mathrm{VNS}}^{s}}{\mathrm{UB}_{\mathrm{LR}}^{s}}$$

$$\mathrm{Gap}_{\mathrm{VNS}}^{\max} = \underset{s \in S}{\mathrm{Max}}\,(\mathrm{Gap}_{\mathrm{VNS}}^{s})$$

where $\mathrm{UB}_{\mathrm{LR}}^{s}$ is the best solution obtained at the end of LR in subproblem $s \in S$, and $\mathrm{UB}_{\mathrm{VNS}}^{s}$ is the best solution obtained at the end of MLRVNS in subproblem $s \in S$.

g. Proposed solution approach ends with the run of algorithm UU which is succeeding the solution termination of the last RHA subproblem in MLRVNS. Performance of algorithm UU can be measured by comparing the best solutions found by it and MLRVNS.

$$\mathrm{Gap}_{\mathrm{UU}} = \frac{\mathrm{UB}_{\mathrm{VNS}} - \mathrm{UB}}{\mathrm{UB}_{\mathrm{VNS}}}$$

2. Solution times in CPU seconds can be used for performance evaluation. Total solution time of proposed approach, rt_{H}, is the sum of solution times of LR, MLRVNS and algorithm UU: $\mathrm{rt}_{\mathrm{LR}}$, $\mathrm{rt}_{\mathrm{VNS}}$ and $\mathrm{rt}_{\mathrm{UU}}$. Solution times of the mentioned sub-algorithms of the proposed solution approach are required to measure the performance of it within itself. To be able to assess the performance of hybrid of LR and MLRVNS in the solution of original problem P by RHA, time required to solve original problem P by RHA via commercial solver, $\mathrm{rt}_{\mathrm{RHA}}$, should be found. Ultimately, performance of the proposed approach should be compared with performance of commercial solver, and $\mathrm{rt}_{\mathrm{cplex}}$ that is solution time of original problem P via commercial solver is required for such a comparison.

5.3. Experimental Results

All steps of the proposed solution approach except algorithm UU is coded in MATLAB R2012b. At the end of MATLAB code, GAMS 23.9.5.is called to solve algorithm UU with CPLEX. Optimal solution of the original problem P (or best solution of it in 36,000 CPU seconds) and best solution to the original problem by using RHA ($\mathrm{P}^{\mathrm{RHA}}$) are obtained via GAMS too. All computational studies are conducted on an Intel Intel® Pentium® i7-2640 M 2.8 GHz processor with 4 GB RAM under Windows 7 operating system.

Summary of test instances' results are given In Table 5.3 and Table 5.4 (results of all test instances are given in Table B.1 in Appendix B). In table 5.3, performance measures of RHA and the proposed solution approachs are presented.

CPLEX is unable to solve 64 problem instances to optimality in 36,000 seconds. In the solution of 14 problem instances by RHA, at least one subproblem has to stop before reaching optimal solution due to running time limit. In Table 5.3, problem types that include at least one instance that could not be solved to optimality are marked with an asterisk sign (*).

Table 5.3. Performance measures of RHA and the proposed solution approach

|J|	|K|	|M|	α	Avg. CPLEX CPU (rt_{cplex})	Avg. RHA CPU (rt_{RHA})	Avg. Heuristic CPU (rt_H)	Avg. Gap (%)	Avg. Gap 1 (%)	Avg. Gap 2 (%)	Avg. Gap RHA (%)
40	15	5	10	285.08	203.43	225.57	1.172	1.172	0.601	0.565
40	15	5	20	228.53	155.23	221.36	0.733	0.733	0.677	0.056
80	15	5	10	2001.74	2259.86	322.27	2.055	2.055	0.896	1.149
80	15	5	20	1203.26	987.81	248.23	1.469	1.469	0.467	0.998
80	25	5	10	2587.16	1500.56	627.28	2.274	2.274	0.925	1.339
80	25	5	20	798.94	646.47	511.78	1.617	1.617	0.603	1.006
80	25	10	10	27185.69*	14122.02	885.58	2.975	3.345	1.345	1.608
80	25	10	20	36,000.00*	23378.36*	883.68	1.506	3.798	1.333	0.186
80	40	15	10	35389.91*	28768.40*	2746.82	1.764	3.716	2.180	-0.396
80	40	15	20	33084.40*	19634.97	2567.39	0.441	3.484	1.111	-0.657
100	25	5	10	3054.43	1492.77	756.38	2.974	2.974	1.490	1.460
100	25	5	20	3552.99	736.30	645.18	2.526	2.526	1.545	0.960
100	25	10	10	29636.44*	18935.76	1239.60	2.329	3.555	1.001	1.322
100	25	10	20	23626.45*	12082.64	1117.24	1.569	3.593	1.670	-0.106
100	25	15	10	29134.73*	14315.26	1668.58	2.291	5.300	2.254	0.062
100	25	15	20	36,000.00*	23131.85	1492.83	-2.380	5.808	1.244	-3.580
100	40	5	10	26999.66*	16326.95	1488.62	1.782	4.407	1.928	-0.135
100	40	5	20	19654.01*	12375.01	1300.46	2.933	3.500	2.666	0.276
100	40	10	10	33143.97*	21223.60	1862.59	-0.616	3.624	1.547	-2.138
100	40	10	20	31890.26*	20772.72	1721.96	2.308	3.794	1.737	0.563
100	40	15	10	36,000.00*	27029.94*	3305.93	-0.643	3.616	0.820	-1.435
100	40	15	20	33869.92*	25740.26*	2568.88	2.900	6.004	2.617	0.294
200	40	5	10	36,000.00*	24614.30	1798.50	-1.667	2.410	0.841	-2.486
200	40	5	20	18780.71*	14189.21	1527.74	2.542	3.830	2.414	0.115
200	40	10	10	36,000.00*	30344.97*	3008.14	-2.943	5.042	1.243	-4.144
200	40	10	20	33230.12*	27411.72*	2531.66	1.854	5.579	2.083	-0.224
Average				21897.63	14706.94	1433.63	1.299	3.432	1.432	-0.129

It should be noted that Gap and Gap_{RHA} can be negative for 64 problem instances could not be solved to optimality. Besides, binary constraints are relaxed in forecasting period as solving original problem P by RHA via GAMS whereas they are retained in the proposed solution approach. Therefore, Gap_2 can be negative as well. Performance indicators of LR, MLRVNS and algorithm UU are given in Table 5.4.

Table 5.4. Performance measures of LR, MLRVNS and algorithm UU

$\|J\|$	$\|K\|$	$M\|$	α	Avg. rt_{LR}	Avg. Gap_{LR}^{max} %	Avg. rt_{VNS}	Avg. Gap_{VNS}^{max} %	Avg. rt_{UU}	Avg. Gap_{UU} %
40	15	5	10	104.07	11.079	116.26	1.798	5.24	0.591
40	15	5	20	122.72	9.259	94.12	1.162	4.52	0.063
80	15	5	10	129.63	13.184	179.04	2.657	13.60	0.646
80	15	5	20	112.52	14.980	127.02	3.842	8.68	0.553
80	25	5	10	164.44	15.660	436.33	2.917	26.51	0.541
80	25	5	20	135.36	11.751	364.51	2.966	11.91	0.489
80	25	10	10	283.74	20.150	566.93	2.756	34.91	0.978
80	25	10	20	313.64	17.777	523.41	3.031	46.63	1.199
80	40	15	10	734.63	14.650	1954.50	3.002	57.68	0.885
80	40	15	20	697.47	13.871	1794.51	2.214	75.41	1.050
100	25	5	10	245.57	16.535	474.71	2.947	36.10	0.700
100	25	5	20	210.93	10.494	408.65	2.765	25.60	0.443
100	25	10	10	445.85	13.223	743.80	2.905	49.95	0.384
100	25	10	20	412.91	14.997	622.83	2.909	81.50	0.824
100	25	15	10	526.68	20.129	1068.67	3.464	73.23	0.984
100	25	15	20	514.18	14.907	885.49	4.518	93.16	0.820
100	40	5	10	399.24	16.896	1033.42	1.926	55.97	0.741
100	40	5	20	399.69	13.866	839.01	3.897	61.76	0.674
100	40	10	10	413.24	16.291	1321.88	1.872	127.46	0.899
100	40	10	20	477.50	17.063	1188.03	3.056	56.42	1.036
100	40	15	10	758.82	17.505	2432.87	3.119	114.24	1.606
100	40	15	20	683.76	18.647	1781.02	3.518	104.11	1.159
200	40	5	10	639.90	14.801	1058.00	2.640	100.60	0.319
200	40	5	20	567.09	14.167	848.18	2.611	112.46	1.102
200	40	10	10	1251.06	21.933	1665.16	3.445	91.92	0.896
200	40	10	20	1052.60	20.129	1368.49	2.188	110.58	1.097
Average				453.73	15.536	919.11	2.851	60.78	0.795

Average values of performance measures for each problem type are given in Table 5.3 and Table 5.4. In Table 5.3, it can be seen that the proposed solution approach gives satisfying solutions in short times when compared to the performance of CPLEX solver. Ultimate performance measure of the proposed approach is Gap_1 that compares lower bounds found by CPLEX solver and the best solutions found by the corresponding approach. In the consideration of the average Gap_1 values for each problem type, maximum Gap_1 is equal to 6.00% and Gap_1 of the proposed approach is equal to 3.43% on overall average; in other words, proposed approach achieves 3.43% gap with the lower bounds of the optimal solutions when considering all test instances lower bounds of which are found (see Table B.1 in Appendix B).

'Gap' compares solutions found by proposed approach and CPLEX, while Gap_1 evaluates proposed approach based on the lower bounds found by CPLEX solver. Maximum Gap is equal to 2.98% and it is achieved for problem type 80x25x10x10 (80 collection centers, 25 consolidation centers, 10 disassembly centers and α =10). Average Gap of the proposed approach is equal to 1.30%. As it is stated previously, 64 of total 130 instances could not be solved to optimality and it should be reminded that Gap is not equivalent to optimality gap; it is a lower bound on the optimality gap. For example, average gap of 5 problems types are negative due to running time limitation as seen in Table 5.3. If relevant 64 instances are ignored and results of particular test instances are considered instead of average results for problems sizes, maximum Gap is 5.15% which is achieved for an instance of problem type 100x40x15x20 and average Gap becomes equal to 1.37%. Although many of the problems that could be solved to optimality are small size problems, maximum and average Gap values are still satisfying for the instances for which we could find the optimal solutions. When regarding average solution time of the proposed solution approach, it is approximately equal to one fifteen of average solution time of CPLEX.

Performance of the proposed heuristic is strictly dependent on the performance of RHA. Although handling binary constraints in the forecasting period differs in the proposed approach and standard RHA and optimal solution achieved by using RHA is not necessarily an upper bound on the proposed approach, optimal RHA solution is supposed to be an upper bound to the proposed approach in most of the instances.

Gap_{RHA} is equal to -0.30% on average of problem types. 13 of total 130 instances could not be solved to optimality by standard RHA due to maximum running time limit on it. Negative value of average Gap_{RHA} is deriving from the fact that average Gap_{RHA} can reach up to -4% for large size problems while it is also satisfying for small size problems; maximum of average Gap_{RHA} values is 1.61%. Average solution time of RHA is almost two thirds of average solution time of CPLEX. RHA can solve test instances within small gaps in shorter times than CPLEX; yet, solution time of it can still be reduced which is the driving reason to use LR and MLRVNS in this study.

Gap_2 shows how close the solution of the proposed approach gets to the solution of standard RHA subproblems of which are solved by CPLEX rather than a hybrid of LR and MLRVNS as in our case. On the average, Gap_2 is varying between 0.47% and 2.67%. Gap_2 values are satisfying especially when considering the fact that the proposed solution approach produces better solutions then standard RHA in 31 of total 130 instances due to retainment of binary constraints in the forecasting period and the time limitation of standard RHA.

On the average, 32% of total running time of the proposed solution approach is spent for LR and 64% of it is spent for MLRVNS, while the rest of the running time belongs to the algorithm UU. In Table 5.4, average values of $\mathrm{Gap}_{\mathrm{LR}}^{\max}$, $\mathrm{Gap}_{\mathrm{VNS}}^{\max}$ and $\mathrm{Gap}_{\mathrm{UU}}$ are given. LR is employed in each subproblem of RHA; in other words, LR provides a lower bound and an upper bound for each RHA subproblem. For a particular test instance, its $\mathrm{Gap}_{\mathrm{LR}}^{\max}$ is equal to the maximum gap between upper of lower bounds among RHA subproblems of the corresponding instance. As considering subproblems of RHA, latter ones are strictly dependent on solutions of first subproblems and LR can give small gaps towards the last subproblems although $\mathrm{Gap}_{\mathrm{LR}}$ is not small in first subproblems. Therefore, $\mathrm{Gap}_{\mathrm{LR}}^{\max}$ is on the interest of this study rather than minimum gap. Minimum value of average $\mathrm{Gap}_{\mathrm{LR}}^{\max}$ is equal to 9.26% which is not rewarding. LR is employed for each subproblem of RHA; in other words, LR provides a lower and upper bound for each subproblem of RHA. Since this study is dealing with the optimal solution of the entire problem instead of optimal solution of each RHA subproblem, optimal solutions of subproblems are not found. Yet, preliminary runs are executed for first RHA subproblems of test instances to observe how close upper bounds of LR are to optimal solutions; gaps between upper bounds obtained from LR and optimal solutions are ranging between 1% - %5 in general. Although the gap between LR upper bound and optimal solutions are within acceptable limits, lower bounds do not approach towards the upper bound as expected.

MLRVNS has the largest portion of total running time of the proposed solution approach. MLRVNS is called after LR in each subproblem of RHA and $\mathrm{Gap}_{\mathrm{VNS}}^{\max}$ for a particular test instance represents the maximum improvement on the upper bound of LR among RHA subproblems of the corresponding instance. Influence of MLRVNS decreases as algorithm proceeds to latter subproblems, since they are dependent on previous subproblems and there are not too many decisions in latter subproblems to make alterations (it is possible to open all centers in the first period and not to make any investments in the following periods. In this case, MLRVNS do not even need to be run for subproblems except the first one). Therefore, as similar to the case of LR, maximum improvement achieved by MLRVNS is concerned instead of minimum improvement of it in this study. $\mathrm{Gap}_{\mathrm{VNS}}^{\max}$ is ranging between 1.16% and 4.52% and MLRVNS achieved improvement in all test instances. Although the lower bound does not approach the upper bound as expected in LR, primal heuristic achieves to generate upper bound within 1% - 5% of optimal solution. More importantly, LR provides a set of diverse solutions including promising solutions as well to MLRVNS, enabling to decrease the size of neighborhoods significantly.

Algorithm UU is executed after solving the original problem using RHA. In 123 of total 130 test instances, best solution is improved by algorithm UU. Maximum improvement achieved by it is 2.73% which is gratifying in regard of the fact that at most 247.25 seconds are spent for the corresponding algorithm in all test instances.

Solution times of CPLEX and the proposed solution approaches have distinct characteristics in regard of how solution times of instances sharing the same problem type vary. Solution time of CPLEX solver shows high standard deviation, while solution times of the proposed approach do not deviate significantly among test instances with the same problem type. Since standard RHA is composed of subproblems that have a similar structure to the original problem, standard deviation in standard RHA is high as well.

Standard deviations of CPLEX and the proposed approach along with its components are shown in Table 5.5.

Table 5.5. Standard deviations of the solution times of CPLEX and the proposed approach with its components

|J|	|K|	|M|	α	Standard deviation (σ)					
				CPLEX	Standard RHA	Proposed heuristic	LR	VNS	algorithm UU
40	15	5	10	207.093	172.214	23.236	21.558	25.534	2.779
40	15	5	20	202.745	141.177	29.075	18.709	33.891	1.459
80	15	5	10	1235.168	1977.885	123.738	26.680	94.528	8.946
80	15	5	20	944.925	917.577	66.560	23.452	64.286	5.932
80	25	5	10	1434.121	1153.069	112.064	32.612	110.903	14.499
80	25	5	20	405.827	631.242	114.334	55.233	96.816	6.770
80	25	10	10	9678.951	10728.459	94.702	82.307	124.581	7.528
80	25	10	20	0.000	10516.060	129.628	54.347	76.229	22.409
80	40	15	10	1364.213	7359.840	239.056	53.166	277.119	11.987
80	40	15	20	4032.951	6466.516	398.001	78.450	344.626	23.447
100	25	5	10	3069.200	1559.450	114.886	20.764	115.011	21.519
100	25	5	20	5126.693	431.453	45.719	42.226	82.409	19.330
100	25	10	10	14229.350	13391.850	127.547	32.473	118.509	19.128
100	25	10	20	17105.319	4406.428	108.585	60.407	67.895	18.921
100	25	15	10	15351.205	8514.169	379.955	78.309	371.433	43.017
100	25	15	20	0.000	6886.554	369.649	26.896	397.106	33.810
100	40	5	10	12646.785	8461.600	92.319	26.772	128.160	39.474
100	40	5	20	16414.094	12227.988	108.950	38.898	95.559	46.396
100	40	10	10	4197.364	6105.236	389.236	40.809	396.938	65.696
100	40	10	20	6013.122	5592.316	266.950	50.969	275.145	22.267
100	40	15	10	0.000	9055.759	775.021	135.609	738.395	51.768
100	40	15	20	3318.054	10020.003	833.780	106.328	713.836	38.705
200	40	5	10	0.000	5649.062	166.404	59.365	104.346	52.460
200	40	5	20	17106.543	13016.014	86.512	32.643	67.366	83.063
200	40	10	10	0.000	3498.555	670.549	162.121	596.117	46.185
200	40	10	20	6193.648	3564.693	369.147	156.969	317.293	37.324
Average				5395.284	5863.276	239.831	58.387	224.386	28.647

Although samples for each problem type are not enough to make credible inferences, they can still show several properties of the relevant algorithms, since there is a significant difference between standard deviations of the proposed solution approach and CPLEX – and standard RHA as well. Most possible reason of high standard deviation in CPLEX is that it has many different background algorithms and performance of each algorithm depends on many factors that can vary among problem instances. Besides, degree of simplicity of generating initial solution can show significant variation among problem instances. In contrast to CPLEX solver, both LR and MLRVNS have a straight operation structure such that they follow the same steps regardless of parameters which lead to the robustness of the proposed solution approach.

In the following section, performance measures on the proposed solution approach are elaborated, and effects of the number of possible consolidation centers, disassembly centers and collection centers are analyzed as well as the effects of initial capacity on the performance of the corresponding approach.

5.4. Analysis of Performance Measure

Effect of the problem types on the performance measures is analyzed in this section. Problem type is determined by the number of collection, consolidation, disassembly centers and their initial capacities. Effect of each determinant factor of problem types is studied in the following sections.

It should be underlined that it maybe mislead to infer a direct relationship between a parameter and Gap values. Even if a specific parameter and Gaps show a similar trend, they do not necessarily have a causal relationship. Gap values may or may not increase with the increase of a specific parameter. Thus, the trend of gap values cannot be estimated regarding the problem sizes. Input parameters of some test instances may be more adequate for subgradient optimization and result in smaller $\text{Gap}_{\text{LR}}^{\text{max}}$ or the opposite case may occur. $\text{Gap}_{\text{VNS}}^{\text{max}}$ is strictly dependent on the solutions of Lagrangean solution, since its purpose is to improve the final solution of it. Not to be able to generalize the relationship between $\text{Gap}_{\text{LR}}^{\text{max}}$ and problem sizes is valid for the relationship between $\text{Gap}_{\text{VNS}}^{\text{max}}$ and problem sizes as well. Algorithm UU is invoked at the end of RHA when all LR and MLRVNS runs are finished. Decision variables of final solution at the end of RHA are fixed based on the utilization rates, and algorithm UU solves the original problem P size of which is reduced due to fixed variables. Performance of UU is dependent on how many variables are fixed; as the number of variables fixed increases, the possibility of improvement by algorithm UU decreases, and there is no causal relationship between the goodness of the solution at the end of RHA and the number of variables to be fixed. Therefore, it is difficult to make a judgment about the performance of algorithm UU at all.

Although relationship between Gaps and problem types cannot be generalized, solution time can have considerable correlation with determinant parameters of problem types, and they are analyzed in the rest of this chapter.

5.4.1. Effects of the Number of Collection Centers on the Performance of the Proposed Solution Approach

Effects of the number of collections centers on the solution times are analyzed in this section. For the purpose of comparison, six pairs of problem types are chosen such that in each pair, there are two problem types which are distinct due to the number of collection centers. In Table 5.6, selected pairs of problem types can be seen. Six pairs of problem types in Table 5.6 show that most of the solution times increase by the number of collection centers as expected. Such an increase is not observed only in $\mathrm{rt}_{\mathrm{UU}}$ in pair 5 and $\mathrm{rt}_{\mathrm{RHA}}$ in pair 2.

As it is stated in previous section, performance of algorithm UU is depending on the utilization rates of the final solution at the end of RHA rather than the goodness / accuracy of it. Thus, a decrease in $\mathrm{rt}_{\mathrm{UU}}$ after $|J|$ arises from 100 to 200 can be possible although it is supposed to be less probable than increase in $\mathrm{rt}_{\mathrm{UU}}$ along with the rise in the number of collection centers. $\mathrm{rt}_{\mathrm{RHA}}$ is supposed to show a similar behavior with $\mathrm{rt}_{\mathrm{cplex}}$ since their structures are alike, but it experiences a decrease as $|J|$ increases in pair 2 while $\mathrm{rt}_{\mathrm{cplex}}$ has a positive interaction with |J| in all pairs. Even though pair 2 can be an outlier, it cannot be said that there is a positive relationship between $\mathrm{rt}_{\mathrm{RHA}}$ and $|J|$.

In all six pairs of problem types, average values of $\mathrm{rt}_{\mathrm{cplex}}$, rt_{H}, $\mathrm{rt}_{\mathrm{LR}}$ and $\mathrm{rt}_{\mathrm{VNS}}$ increase as the number of collection centers increases. Although there seems to be a positive correlation between the number of collection centers and solution times, it cannot be said that there is a strong correlation between them. For example, in pair 1, the number of collection centers doubles whereas there is a gradual increase on it in pair 4, although increase of $\mathrm{rt}_{\mathrm{LR}}$ is higher in the latter pair.

Table 5.6. Effects of the number of collection centers on solution times

①									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
15	5	10	**40**	285.08	203.43	225.57	104.07	116.26	5.24
			80	2001.74	2259.86	322.27	129.63	179.04	13.60
Increase (%)				602.159	1010.868	42.868	24.563	54.000	159.297
②									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
25	5	10	**80**	2587.16	1500.56	627.28	164.44	436.33	26.51
			100	3054.43	1492.77	756.38	245.57	474.71	36.10
Increase (%)				18.061	-0.520	20.580	49.332	8.796	36.183
③									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
25	10	10	**80**	27185.69	14122.02	885.58	283.74	566.93	34.91
			100	29636.44	18935.76	1239.60	445.85	743.80	49.95
Increase (%)				9.015	34.087	39.976	57.131	31.197	43.104
④									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
40	15	10	**80**	29636.44	18935.76	1239.60	445.85	743.80	49.95
			100	36,000.00	27029.94	3305.93	758.82	2432.87	114.24
Increase (%)				21.472	42.745	166.693	70.198	227.085	128.703
⑤									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
40	10	10	**100**	33143.97	21223.60	1862.59	413.24	1321.88	127.46
			200	36,000.00	30344.97	3008.14	1251.06	1665.16	91.92
Increase (%)				8.617	42.978	61.503	202.741	25.969	-27.881
⑥									
$\|K\|$	$\|M\|$	α	$\|J\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
40	5	10	**100**	26999.66	16326.95	1488.62	399.24	1033.42	55.97
			200	36,000.00	24614.30	1798.50	639.90	1058.00	100.60
Increase (%)				33.335	50.759	20.816	60.282	2.378	79.737

Similar cases exist for solution times of other algorithms too. Moreover, solution time increase in the percentage deriving from different number of collection centers exceeds 1000% and 600% for rt_{cplex} and rt_{RHA} in pair 1, respectively, while there is no such a rapid increase in other pairs. This unstable increase can be a sign of a weak interaction between the number of collection centers and solution times.

As it is stated in previous section, performance of algorithm UU is depending on the utilization rates of the final solution at the end of RHA rather than the goodness / accuracy of it. Thus, a decrease in $\mathrm{rt}_{\mathrm{UU}}$ after |*J*| arises from 100 to 200 can be possible although it is supposed to be less probable than increase in $\mathrm{rt}_{\mathrm{UU}}$ along with the rise in the number of collection centers. $\mathrm{rt}_{\mathrm{RHA}}$ is supposed to show a similar behavior with $\mathrm{rt}_{\mathrm{cplex}}$ since their structures are alike, but it experiences a decrease as |*J*| increases in pair 2 while $\mathrm{rt}_{\mathrm{cplex}}$ has a positive interaction with |J| in all pairs. Even though pair 2 can be an outlier, it cannot be said that there is a positive relationship between $\mathrm{rt}_{\mathrm{RHA}}$ and |*J*|.

In all six pairs of problem types, average values of $\mathrm{rt}_{\mathrm{cplex}}$, rt_{H}, $\mathrm{rt}_{\mathrm{LR}}$ and $\mathrm{rt}_{\mathrm{VNS}}$ increase as the number of collection centers increases. Although there seems to be a positive correlation between the number of collection centers and solution times, it cannot be said that there is a strong correlation between them. For example, in pair 1, the number of collection centers doubles whereas there is a gradual increase on it in pair 4, although increase of $\mathrm{rt}_{\mathrm{LR}}$ is higher in the latter pair. Similar cases exist for solution times of other algorithms too. Moreover, solution time increase in the percentage deriving from different number of collection centers exceeds 1000% and 600% for $\mathrm{rt}_{\mathrm{cplex}}$ and $\mathrm{rt}_{\mathrm{RHA}}$ in pair 1, respectively, while there is no such a rapid increase in other pairs. This unstable increase can be a sign of a weak interaction between the number of collection centers and solution times.

Percentage increases of solution times derived from different number of collection centers seem to be more limited in rt_{H} and $\mathrm{rt}_{\mathrm{VNS}}$ than solution times of other algorithms. Solution time of MLRVNS is more robust to the changes in the number of collection centers when compared to other algorithms. Since a large portion of solution time of the proposed approach belongs to MLRVNS, rt_{H} shows a similar behavior as well.

In couples of pairs (2,3) and pairs (5,6), problem types within pairs have the same number of collection centers and consolidation centers, while the numbers of disassembly centers are different. Influence of the increase in |*J*| is getting higher in both rt_{VNS} and rt_{LR}, and rt_{H} as well, as |*K*| increases. Therefore, there can be a positive interaction between the influence of |*J*| on rt_{VNS}, rt_{LR}, rt_{H} and the number of disassembly centers. rt_{cplex} and rt_{RHA} do not show such a behavior against the increase in disassembly centers in the corresponding couples of pairs. Yet, this can result from test instances in the corresponding couples that cannot be solved to optimality and terminated early. For example, in pair 5 and pair 6, all instances of problem type with 200 collection centers fail in finding optimal solutions and they are terminated after 10 hours. If they are allowed to run until optimal solutions are found, behaviors of rt_{cplex} and rt_{RHA} against the changes in |*M*| could be similar. Therefore, exact influence of |*M*| on the correlation of |*J*| with rt_{cplex} and rt_{RHA} cannot be observed in these pairs.

5.4.2. Effects of the Number of Consolidation Centers on the Performance of the Proposed Solution Approach

Effects of the number of consolidation centers on the solution times of algorithms are analyzed in this section. Eight pairs of problem types are chosen such that in each pair, there are two problem types which are distinct due to the number of consolidation centers. In Table 5.7, selected pairs of problem types can be seen.

Table 5.7. Effects of the number of consolidation centers on solution times

①									
|*J*|	|*M*|	α	|*K*|	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	5	10	**15**	2001.74	2259.86	322.27	129.63	179.04	13.60
			25	2587.16	1500.56	627.28	164.44	436.33	26.51
Increase (%)				29.25	-33.60	94.64	26.86	143.70	94.94
②									
|*J*|	|*M*|	α	|*K*|	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	5	10	**25**	3054.43	1492.77	756.38	245.57	474.71	36.10
			40	26999.66	16326.95	1488.62	399.24	1033.42	55.97
Increase (%)				783.95	993.74	96.81	62.58	117.70	55.02

Table 5.7. (cont'd)

③									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	10	10	**25**	29636.44	18935.76	1239.60	445.85	743.80	49.95
			40	33143.97	21223.60	1862.59	413.24	1321.88	127.46
Increase (%)				11.84	12.08	50.26	-7.31	77.72	155.16
④									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	15	10	**25**	29134.73	14315.26	1668.58	526.68	1068.67	73.23
			40	36,000.00	27029.94	3305.93	758.82	2432.87	114.24
Increase (%)				23.56	88.82	98.13	44.08	127.65	56.01
⑤									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	5	20	**15**	1203.26	987.81	248.23	112.52	127.02	8.68
			25	798.9356	646.4742	511.779	135.3558	364.5108	11.9124
Increase (%)				-33.60	-34.55	106.17	20.29	186.96	37.17
⑥									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	5	20	**25**	3552.99	736.30	645.18	210.93	408.65	25.60
			40	19654.01	12375.01	1300.46	399.69	839.01	61.76
Increase (%)				453.17	1580.71	101.57	89.49	105.31	141.23
⑦									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	10	20	**25**	23626.45	12082.64	1117.24	412.91	622.83	81.50
			40	31890.26	20772.72	1721.96	477.50	1188.03	56.42
Increase (%)				34.98	71.92	54.13	15.64	90.75	-30.77
⑧									
$\|J\|$	$\|M\|$	α	$\|K\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	15	20	**25**	36,000.00	23131.85	1492.83	514.18	885.49	93.16
			40	33869.92	25740.26	2568.88	683.76	1781.02	104.11
Increase (%)				-5.92	11.28	72.08	32.98	101.13	11.75

At first glance, it can be seen that rt_{LR} decreases as the number of consolidation centers increases in pair 3, and both rt_{cplex} and rt_{RHA} decrease as $|K|$ increases in two pairs. Reverse effects of the increase in |K| on rt_{cplex} are observed in pair 5 and pair 8 that both have α equal to 20. It can be stated that there seems a positive correlation between |*K*| and rt_{cplex} when α is equal to 10. Yet, this is not the case if α rises to 20. When rt_{RHA} and rt_{LR} are considered, it cannot be stated that there is a positive correlation between corresponding solution times and the number of consolidation centers, although they increase along with the increase in |*K*| in most of the pairs.

In regard of the difference between the number of consolidation centers of problems; in pairs (2,3,4), the number of consolidation centers increases by 15 while corresponding value is 10 in pair 1. It should be cited that solution space of neighborhoods in MLRVNS increases exponentially; thus, a specific increment on the number of consolidation centers leads to the expansion of neighborhoods much more than the increment in the number of consolidation centers. In pairs (2,3,4), both the lowest |*K*| value (25) and the increment on them (15) are higher than the corresponding values in pair 1 (15 and 10). At the end, it is expected that pairs (2,3,4) have higher increase in rt_{VNS} than the corresponding increase in pair 1. Yet, this expectation cannot be observed in Table 5.7. On the contrary, pair 1 has the highest increase in rt_{VNS}. One reason for such a case can be the fact that there is an increase of 66.7% in the number of consolidation centers in the first pair, whereas this percentage is equal to 60% in pairs (2,3,4). Higher percentage increase of |*K*| in the first pair can be a determining factor on having more rapid influence on rt_{VNS}. Besides the higher percentage increase of |*K*| in pair 1, it should also be noted that performance of MLRVNS is remarkably dependent on the ratios given in Table 5.2 that vary based on problem sizes. Given ratios are used to limit the neighborhood solutions that can be generated by exchange & add moves. As the number of centers in a particular layer increases, limitation on the number of neighborhood solutions that can be generated by an exchange or add move on relevant layer increases even more. In other words, the portion of neighborhood solutions that are excluded increases as the number of potential centers increases. This can be the reason behind the failure of expectations about more rapid increase of rt_{VNS} in pairs (2,3,4).

In Table 5.7, pair 1-5, pair 2-6, pair 3-7 and pair 4-8 are identical except the value of α; α is equal to 10 in first four pairs while it is equal to 20 in last four pairs. As α rises from 10 to 20, none of the algorithms experiences one directional influence of α on the effects of |*K*| on solution times. In other words, there are both decreases and increases in the influence of $|K|$ on solution times of all algorithms as α rises from 10 to 20. Therefore, α has no effect on how much solution times are affected by the change in $|K|$.

5.4.3. Effects of the Number of Disassembly Centers on the Performance of the Proposed Solution Approach

Reactions of solution times to changes in the number of disassembly centers are studied in this section. Six groups of problem types that share the same $|J|$, $|K|$, α and differentiate in the number of dissassembly centers, $|M|$- are selected to compare solution times within and between relevant pairs. Six groups to be considered in this section can be seen in Table 5.8.

$\mathrm{rt}_{\mathrm{cplex}}$ seems to increase dramatically as the number of disassembly centers increases from 5 to 10 in pair 1, pair 2, pair 5 and pair 6. In fact, many of the test instances in problem types that have higher $|M|$ in corresponding pairs could not be solved to optimality and terminated after 10 hours without finding the optimal solution. If they were allowed to run until optimal solution is found, it would be possible for percentage increase in $\mathrm{rt}_{\mathrm{cplex}}$ to be even higher than the current precentages which are 870.28%, 950.79%, 4406% and 564.97% for corresponding pairs, respectively. Likewise, most of the test instances of problem types in pair 3 and pair 4 -even the instances of problem types with lower $|M|$- have reached time limitation and terminated early. Therefore, the trend of $\mathrm{rt}_{\mathrm{cplex}}$ in pair 1, 2, 5 and 6 is not caught in pair 3 and pair 4, although a similar trend is expected in these pairs as well. Pair 4 shows the effect of early terminations clearly.

In group 2, an increase in $\mathrm{rt}_{\mathrm{VNS}}$ by 56.67% is observed by the rise of $|M|$ from 5 to 10 while increase ratio reduces to 43.68% when $|M|$ moves from 10 to 15. Contrarily, in group 3, $\mathrm{rt}_{\mathrm{VNS}}$ experiences a higher increase, 84.07%, after the movement of $|M|$ from 10 to 15 than the increase achieved by the rise of $|M|$ from 5 to 10. In a standard VNS, the latter case is expected, since the number of neighborhoods increases as problem size increases as stated before. Yet, since the ratios in Table 5.2 change the disposition of the relationship between problem size and solution time of VNS, the case in group 2 becomes possible too.

Table 5.8. Effects of the number of disassembly centers on solution times

①

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	25	10	**5**	2587.16	1500.56	627.28	164.44	436.33	26.51
			10	27185.69	14122.02	885.58	283.74	566.93	34.91
Increase (%)				950.793	841.115	41.178	72.548	29.933	31.669

②

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	25	10	**5**	3054.43	1492.77	756.38	245.57	474.71	36.10
			10	29636.44	18935.76	1239.60	445.85	743.80	49.95
			15	29134.73	14315.26	1668.58	526.68	1068.67	73.23
Increase-1 (%)				870.278	1168.501	63.887	81.558	56.686	38.360
Increase-2 (%)				-1.693	-24.401	34.606	18.130	43.676	46.597

③

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	40	10	**5**	26999.66	16326.95	1488.62	399.24	1033.42	55.97
			10	33143.97	21223.60	1862.59	413.24	1321.88	127.46
			15	36,000.00	27029.94	3305.93	758.82	2432.87	114.24
Increase-1 (%)				22.757	29.991	25.122	3.508	27.914	127.738
Increase-2 (%)				8.617	27.358	77.492	83.626	84.046	-10.370

④

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
200	40	10	**5**	36,000.00	24614.30	1798.50	639.90	1058.00	100.60
			10	36,000.00	30344.97	3008.14	1251.06	1665.16	91.92
Increase (%)				0.000	23.282	67.259	95.507	57.388	-8.621

⑤

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	25	20	**5**	798.94	646.47	511.78	135.36	364.51	11.91
			10	36,000.00	23378.36	883.68	313.64	523.41	46.63
Increase (%)				4405.995	3516.286	72.668	131.716	43.593	291.427

⑥

$\|J\|$	$\|K\|$	α	$\|M\|$	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	25	20	**5**	3552.99	736.30	645.18	210.93	408.65	25.60
			10	23626.45	12082.64	1117.24	412.91	622.83	81.50
			15	36,000.00	23131.85	1492.83	514.18	885.49	93.16
Increase-1 (%)				564.973	1541.001	73.168	95.758	52.413	218.322
Increase-2 (%)				52.372	91.447	33.618	24.528	42.172	14.301

Therefore, it can be said that there is a positive correlation between the number of disassembly centers and rt_{VNS} but degree of the influence of changing $|M|$ on rt_{VNS} is difficult to be estimated. Likewise the situation in group 2, influence of increments of $|M|$ decreases as $|M|$ gets higher in group 6 which is identical to group 2 except α values. This can be a sign of the fact that initial capacities do not affect the increase of rt_{VNS} deriving from the changes in $|M|$. rt_{LR} behaves similarly as rt_{VNS} in group 2 , group 3 and group 6 and comments for rt_{VNS} are valid for it too. The difference between rt_{LR} and rt_{VNS} appears in the consideration of couple of groups (1,5) and couple of groups (2,6) where groups in each couple are identical with each other except initial capacities. As considering group 1 and group 5, it seems that increase in rt_{VNS} is higher in group 5 that has α equal to 10. rt_{VNS} shows a different attitude as regarding group 2 and group 6. In this couple of groups, higher increase in rt_{VNS} is achieved in group 6 where α is equal to 20. As it is stated above, no effects of initial capacities on the increase of rt_{VNS} deriving from the changes in $|M|$ is observed. Yet, for rt_{LR} , it seems that the increments in rt_{VNS} get higher as initial capacities increase in couple of groups (1,5) and couple of groups (2,6). This can be an indicator of positive relationship between initial capacities and the influence of $|M|$ on rt_{LR} .

It can be seen in group 1 and group 2 that the rise of $|M|$ from 5 to 10 has more influence on both rt_{VNS} and rt_{LR} , as the number of collection centers increases. This relation is valid when considering group 3 and group 4 too. Relevant groups are identical except the number of collection centers and higher increase in rt_{VNS} and rt_{LR} as $|M|$ is moved from 5 to 10 is achieved in group 4 than the increase in group 3 where former group has more collection centers. Therefore, there might be a positive relationship between the number of collection centers and the influence of $|M|$ on rt_{VNS} and rt_{LR} . Conclusions on the behavior of rt_{VNS} against changes in $|M|$ are valid for rt_{H} too.

5.4.4. Effects of Initial Capacities on the Performance of the Proposed Solution Approach

In previous three sections, effects of the number of collection centers, consolidation centers and disassembly centers, which are parameters defining problem sizes, on the solution times are studied. In this section, influence of initial capacities on solution times is analyzed. Eight pairs of problem types selected for comparison can be seen in Table 5.9.

Increases of initial capacities have similar effects on rt_{cplex}, rt_{RHA}, rt_{H} and rt_{VNS} that decrease along with the expansions in initial capacities whereas rt_{LR} increases following the expansion of initial capacity in pair 1, pair 4 and pair 5. In previous sections, increase in average values of rt_{cplex} has reached up to 1000% in several pairs. Obviously, it is not expected to observe such decreases in rt_{cplex} along with the increase of initial capacities; it is impossible indeed to decrease solution times by 1000%. Yet, decreases in solution times along with the expansion in Table 5.9 are equivalent to increases in solution times along with the downsizing of initial capacities and, in this perspective, none of the pairs experiences a change in solution time by such high degrees as 1000%.

Table 5.9. Effects of the initial capacities on the solution times

①									
$\|J\|$	$\|K\|$	$\|M\|$	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
40	15	5	**10**	285.08	203.43	225.57	104.07	116.26	5.24
			20	228.53	155.23	221.36	122.72	94.12	4.52
Increase (%)				-19.838	-23.695	-1.869	17.922	-19.044	-13.835
②									
$\|J\|$	$\|K\|$	$\|M\|$	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	15	5	**10**	164.591	111.655	148.354	81.569	63.779	-1.357
			20	124.427	81.063	122.614	74.070	46.284	-3.557
Increase (%)				-24.402	-27.399	-17.350	-9.194	-27.429	162.177
③									
$\|J\|$	$\|K\|$	$\|M\|$	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
80	25	5	**10**	2587.16	1500.56	627.28	164.44	436.33	26.51
			20	798.94	646.47	511.78	135.36	364.51	11.91
Increase (%)				-69.119	-56.918	-18.414	-17.689	-16.460	-55.067

Table 5.9. (cont'd)

④									
|J|	|K|	|M|	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	40	5	**10**	26999.66	16326.95	1488.62	399.24	1033.42	55.97
			20	19654.01	12375.01	1300.46	399.69	839.01	61.76
Increase (%)				-27.206	-24.205	-12.640	0.113	-18.812	10.353
⑤									
|J|	|K|	|M|	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	40	10	**10**	33143.97	21223.60	1862.59	413.24	1321.88	127.46
			20	31890.26	20772.72	1721.96	477.50	1188.03	56.42
Increase (%)				-3.783	-2.124	-7.550	15.550	-10.125	-55.736
⑥									
|J|	|K|	|M|	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
100	40	15	**10**	36,000.00	27029.94	3305.93	758.82	2432.87	114.24
			20	33869.92	25740.26	2568.88	683.76	1781.02	104.11
Increase (%)				-5.917	-4.771	-22.295	-9.892	-26.794	-8.873
⑦									
|J|	|K|	|M|	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
200	40	5	**10**	36,000.00	24614.30	1798.50	639.90	1058.00	100.60
			20	23288.001	16783.265	1448.362	437.924	937.406	65.277
Increase (%)				-35.311	-31.815	-19.468	-31.564	-11.398	-35.110
⑧									
|J|	|K|	|M|	α	Avg. rt_{cplex}	Avg. rt_{RHA}	Avg. rt_{H}	Avg. rt_{LR}	Avg. rt_{VNS}	Avg. rt_{UU}
200	40	10	**10**	36,000.000	30344.974	3008.142	1251.060	1665.157	91.924
			20	33230.116	27411.716	2531.663	1052.596	1368.485	110.581
Increase (%)				-7.694	-9.666	-15.840	-15.864	-17.816	20.296

At the end, it can be said that there is a negative relationship between initial capacities and rt_{cplex}; yet, effect of initial capacities on rt_{cplex} seems more limited compared with other parameters analyzed in previous sections. Besides; in pair 4, pair 5 and pair 6, number of collection centers and consolidation centers are the same whereas the number of disassembly centers increases from pair 4 to pair 6. It is seen that maximum decrease in rt_{cplex} is achieved in pair 4 and minimum decrease is experienced in pair 5. Therefore, there is no effect of the number of disassembly centers on the influence of initial capacities on rt_{cplex}. In couples of pairs (1,2), (4,7) and (5,8), none of the parameters are different except the number of customers. It seems that influence of capacity expansion increases with the number of collection centers in all of the couples mentioned. Hence, it is reasonable to claim that greater initial capacities become more influential as the number of collection centers increases. All inferences on rt_{cplex} are valid for rt_{RHA} too, since it shows a similar behavior as rt_{cplex} in Table 5.9.

Likewise the behavior of rt_{cplex}, rt_{VNS} has a negative relationship with initial capacities as well. Again, effect of initial capacities on rt_{VNS} is more limited when compared with other parameters analyzed before. Highest influences of initial capacities on rt_{VNS} are observed in pair 2 and pair 6 that have problem sizes 80x15x5 and 100x40x15, respectively. If all test instances in this study are to be classified as small, moderate and large size problems, problems in pair 2 can be regarded as small size problems, whereas size of the problems in pair 6 is high. Then, it can be said that degree of the negative effect of initial capacities on rt_{VNS} is not dependent on problem sizes. In regard of couples of pairs (1,2), (4,7) and (5,8), behavior of rt_{VNS} differs from rt_{cplex}. In couple of pairs (4,7), influence of the increase in initial capacity on rt_{VNS} decreases as the number of collection centers increases, while converse situation is observed in other couples. Therefore, there is no relationship between the influence of initial capacities on rt_{VNS} and $|J|$.

As it is seen in Table 5.9, effect of the increase in initial capacities on rt_{LR} is within a range between -31.56% and 17.92% which has ends in both negative and positive region. Therefore, no effect of initial capacities on rt_{LR} has been observed in test instances. Although reactions of rt_{LR} to the expansion of initial capacities vary, it shares a similar trend with rt_{cplex} and rt_{RHA} such that the influence of initial capacity expansions increases with $|J|$. In previous analysis of solution times, it has been observed that characteristics of rt_{H} is more similar to rt_{VNS} rather than rt_{LR}, since more solution time is spent for MLRVNS when compared to LR. However, there is a positive relationship with the influence of initial capacity expansions on rt_{H} and $|J|$ while there is no such a relation with rt_{VNS} as it is stated.

CHAPTER 6

CONCLUSION AND FURTHER RESEARCH

6.1. Conclusion

In this study, we have dealt with a two-echelon dynamic capacitated facility location problem regarding the construction of a recovery network. Purpose of the problem is to determine the locations of consolidation and disassembly centers, when to open them, timing and locations of capacity expansions as well as transportation of returns from collection centers to disassembly centers with the least cost. Mathematical model is formulated as an MIP and we propose a heuristic approach using LR and MLRVNS along with RHA. RHA allowed us to decompose the original problem into subproblems with smaller planning horizons. LR and MLRVNS are employed in each subproblem of RHA. Subgradient optimization is used in LR to update Lagrange multipliers. Two constraints are relaxed in LR; which allow us to decompose subproblems of RHA even further, and in the end, a subproblem has arise for each disassembly center and consolidation center. Primal heuristic is proposed to generate feasible solutions and it uses the effects of opening consolidation and disassembly centers on the objective function of Lagrangean dual problem as basis. Since best solutions obtained by LR are not satisfying, MLRVNS is developed to be invoked after LR in each RHA subproblem. MLRVNS includes drop / exchange / add moves on centers in a layered structure. Improvement algorithm is developed to be run after all subproblems of RHA are solved. Its principle is to solve a reduced size of the original problem as fixing a set of centers based on their utilization rates in the best solution obtained so far.

To evaluate the performance of proposed solution approach, test instances are generated and best solutions of these instances -that are obtained by CPLEX solver allowed to run at most 10 hours- are used to compare the solutions of the proposed solution approach. Computational results show that there is 1.3% gap on average between best solutions of CPLEX and the solutions found by the proposed solution approach that achieves a relevant gap in less than fifteenth of solution times of CPLEX. Contributions of this study to the supply chain literature are as follows:

- Studies on MEDCFLPs are rare in the literature and none of the studies set limits on the maximum distance that flows can be moved to the best of our knowledge.
- In none of the studies dealing with a similar problem, RHA has been used. We have used RHA to decompose the original problem in a temporal manner. RHA has been effective to reduce solution times and enables us to study with small subproblems. It has been shown that RHA should be considered when dealing with large scale FLPs.
- It is shown that hybrid of LR and MLRVNS can produce good results for multi-echelon multi-period FLPs. To use scoring method in SS can be time-saving in MLRVNS as intensifying the promising regions. Besides, feasible solutions generated in LR can serve as a good reference set for scoring method. In the end, LR can provide both good initial solution and a set of solutions that form an opinion about promising neighborhood operations to MLRVNS. In case of having a convenient set of solutions that includes good solutions as maintaining diversity, MLRVNS can produce satisfying solutions in short times.
- RHA is open to improvement due to its decomposed structure. Drawbacks of considering subproblems instead of the whole problem can be mitigated by fixing favorable parts of the solution RHA provides, and original problem can be solved again as bounded with fixed decisions.

6.2. Further Research

Our study can be extended mainly in two directions: the extension of the proposed heuristic approach and the extension of the problem structure. Proposed heuristic can be extended to be improved; or, alterations can occur in the problem structure to generate a new problem type. Extension alternatives in both kinds are presented in the following sections.

6.2.1. Extensions of the Proposed Heuristic

Possible extensions of the proposed heuristic are as follows:

- One of the biggest drawbacks of the proposed heuristic is to fail in approaching lower bound towards upper bound in LR phase. Our model can be formulated in a different way or stronger valid inequalities can be added to the constraints of the model. In most of CFLPs, a valid inequality ensuring that total supply is greater than or equal to total demand is inserted into the model. Such a constraint can be difficult to be handled due to dynamic capacity in our problem. Besides, it may not be very useful since it does not guarantee feasibility, because of the maximum distance constraints on flows in our problem. Yet, a convenient valid inequality would be promising.
- Lengths of the central and forecasting periods can be increased to decrease the optimality gap of RHA solutions. Instead of fixing all variables in the central periods, more sophisticated selection procedures can be used to measure fixing decisions that would result in inefficiency when their effects are considered on the entire planning horizon.
- Our primal heuristic works as a construction heuristic based on the effects of opening consolidation and disassembly centers on the objective function of Lagrangean dual problem. Instead of such a construction heuristic, repair mechanism on the final solution of Lagrangean dual problem to reach a feasible solution can be processed. Pirkul and Jayaraman (1998) take the final solution of Lagrangean dual solution and open / close facilities based on proximity to the customers. To generate feasible solutions by such an approach that repairs the solution on hand instead of constructing a new one may be a good idea to obtain better upper bounds.
- Performance of MLRVNS is promising while LR does not perform as expected. LR can be replaced by other heuristics that can provide good initial solutions to MLRVNS like a heuristic that uses linear relaxation of the model and increases the number of binary values iteratively until a feasible solution is obtained. In the absence of LR, MLRVNS needs a new method to exclude non-promising solutions since reference set would not be provided by LR in that case and evaluation of all neighborhood solutions would be too costly in time. In such a case, a heuristic that determines promising neighbor solutions can be developed. Study of Amrani et al. (2011) is a good example of the

corresponding case. They use TS to generate only promising neighborhood solutions, since large size of the problem they study makes it impossible to work with a method like VND.

- Velasquez and Melo (2004) study a multi-period CFLP and use VNS as a solution technique. Neighborhoods defined by them are different from the ones in this study such that more randomness is included in the generation of neighbor solutions in their study. For example, in a particular neighborhood, the number of facilities to be added and dropped simultaneously are determined randomly as well as the periods to take relevant actions. Contrarily, in our study, there is a more deterministic structure. Role of randomness can be increased in our MLRVNS. Besides, expanding the number of neighborhoods can be a good idea.
- At the end of RHA, exact algorithms can be run to find optimal solution since it gets more significant to find optimal solution when the problem dealt with concerns strategic decisions.

6.2.2. Extensions of the Problem Structure

Possible extensions of the problem structure are as follows:

- As Fleischmann et al. (2000) state, uncertainty is the major distinguishing factor between forward and reverse logistics. Therefore, uncertainty can be included in the model. Especially, amount of returns can have high degree of uncertainty and inclusion of it into the model would contribute significantly to practices of the solution. Uncertainty can be included in the model through generating different scenarios. In such a case, solution methods like robust optimization and stochastic programming can be useful.
- There is a single commodity or a single group of commodities in our problem. Yet, in practice, recovery networks consist of multiple commodities. Hence, multiple products or product groups can be covered by the problem.
- Returns at collection centers are transported to disassembly centers through consolidation centers in our problem. There is no sorting or inspection process that is considered during the transportation of returns. Hence, recovery options like recycling, reuse etc. are not examined in this study. Recovery of products

can be included in the model in such a way that returns are exposed to different recovery options based on the probabilities associated with each recovery option. In the presence of different recovery options, direction of flows would vary based on the types of recovery.

- As it is stated in Section 3.1, organizations belonging to different sectors like White Goods Manufacturer Association and Informatics Industry Association in Turkey are studying the ways of creating value from returns and remanufacturing can be a viable option in that case. Our problem can be reformulated in the perspective of a group of producers that share the same recovery network and are willing to use valuable parts of returns in their production systems.
- We assume that capacity expansions can occur by an exact amount pre-given. This assumption can be relaxed and amount of capacity expansions can be unbounded.
- Our problem has a planning horizon of five years and time value of money should be included in the model. In addition to this, piecewise linear costs or nonlinear costs can be used to catch the effect of economies of scale in the problem.
- Once a center is opened, it has to remain open until the end of the planning horizon. Since costs of disassembly centers are supposed to be considerably high and closing or relocating them can be costly, this assumption seems appropriate. Yet, for consolidation centers which require less investments, this assumption can be relaxed and closing / reopening consolidation centers can be provided as an option in the model.

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APPENDIX A

TAXONOMY TABLE

Table A.1. Taxonomy of the literature research

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Le Blanc et al. (2004)	local search	single	single	linear	deterministic	open loop	no	capacitated	single	single
Mansour and Zarei (2008)	heuristic	single	single	linear	deterministic	close loop	no	capacitated	multiple	multiple
Min and Ko (2008)	GA	single	single	linear	deterministic	close loop	yes	capacitated	multiple	multiple
Salema et al. (2009)	B&B	single	single	linear	deterministic	close loop	no	capacitated	multiple	multiple
Hyun and Gerald (2007)	GA	single	multiple	nonlinear	deterministic	close loop	no	capacitated	multiple	multiple
Alumur et al. (2012)	commercial solver	single	multiple	linear	deterministic	close loop	yes	capacitated	multiple	multiple
Lee and Dong (2009)	sample approximation scheme with SA	single	single	nonlinear	stochastic	close loop	no	capacitated	multiple	multiple
Hasani et al. (2011)	commercial solver	single	single	linear	stochastic	close loop	no	uncapacitated	multiple	multiple
Gomes et al. (2011)	commercial solver	single	multiple	linear	deterministic	open loop	no	capacitated	multiple	multiple
Assavapokee and Wongthatsanekorn (2012)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	multiple	multiple

Table A.1. (cont'd)

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Wang et al. (2008)	GA nesting SUE	multiple	single	linear	deterministic	open loop	yes	capacitated	multiple	multiple
Abbas et al. (2011)	hybrid of GA and PS	single	single	linear	deterministic	open loop	no	capacitated	multiple	multiple
You et al. (2011)	GA	single	multiple	linear	deterministic	open loop	no	capacitated	multiple	multiple
Tang et al. (2013)	BD	single	single	linear	deterministic	close loop	yes	capacitated	multiple	multiple
Hinojosa et al. (2000)	LR	single	single	linear	deterministic	open loop	no	capacitated	multiple	multiple
Tragantalerngsak et al. (1997)	LR	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Pirkul and Jayaraman (1998)	LR	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Barros et al. (1998)	LR	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Jayaraman et al. (1999)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Krikke et al. (1999)	heuristic	single	multiple	linear	deterministic	open loop	no	capacitated	single	multiple

Table A.1. (cont'd)

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Jayaraman et al. (2003)	HC	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Beamon and Fernandes (2004)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Listeş and Dekker (2005)	two stage stochastic programming	single	single	linear	stochastic	open loop	no	capacitated	single	multiple
Hong et al. (2006)	heuristic for min-max robust optimization	single	single	linear	stochastic	open loop	no	capacitated	single	multiple
Cordeau et al. (2006)	BD	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Wang and Yang (2007)	heuristic	single	multiple	linear	deterministic	open loop	no	capacitated	single	multiple
Lu and Bostel (2007)	LR	single	single	linear	deterministic	close loop	no	uncapacitated	single	multiple
Salema et al. (2007)	B&B for scenario-based model	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Hinojosa et al. (2000)	LR	single	single	linear	deterministic	open loop	no	capacitated	multiple	multiple
Tragantalerngsak et al. (1997)	LR	single	single	linear	deterministic	close loop	no	capacitated	single	multiple

Table A.1. (cont'd)

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Pirkul and Jayaraman (1998)	LR	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Barros et al. (1998)	LR	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Jayaraman et al. (1999)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Krikke et al. (1999)	heuristic	single	multiple	linear	deterministic	open loop	no	capacitated	single	multiple
Jayaraman et al. (2003)	HC	single	single	linear	deterministic	open loop	no	capacitated	single	multiple
Beamon and Fernandes (2004)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Listeş and Dekker (2005)	two stage stochastic programming	single	single	linear	stochastic	open loop	no	capacitated	single	multiple
Hong et al. (2006)	heuristic for min-max robust optimization	single	single	linear	stochastic	open loop	no	capacitated	single	multiple
Cordeau et al. (2006)	BD	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Wang and Yang (2007)	heuristic	single	multiple	linear	deterministic	open loop	no	capacitated	single	multiple

Table A.1. (cont'd)

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Lu and Bostel (2007)	LR	single	single	linear	deterministic	close loop	no	uncapacitated	single	multiple
Salema et al. (2007)	B&B for scenario-based model	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Chouinard et al. (2008)	SAA heuristic for stochastic programming	single	single	linear	stochastic	close loop	no	capacitated	single	multiple
Wollenweber (2008)	VNS	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Demirel and Gökçen (2008)	commercial solver	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Lee and Dong (2008)	heuristic nesting TS	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Pishvaee et al. (2009)	stochastic programming	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Üster and Easwaran (2010)	BD	single	multiple	linear	deterministic	close loop	no	uncapacitated	single	multiple
Pishvaee et al. (2010)	SA nesting local search	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Achillas et al. (2010)	commercial solver	single	multiple	linear	deterministic	open loop	no	capacitated	single	multiple

Table A.1. (cont'd)

Paper	Solution method	# of objectives	# of products	Objective function	Parameters	Supply chain structure	Expandable capacity	Capacity limitation	# of periods	# of echelons
Lieckens and Vandaele (2011)	differential ebolution heuristic for queuing analysis	single	single	nonlinear	stochastic	close loop	no	capacitated	single	multiple
Piplani and Saraswat (2011)	min-max robust optimization	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Amrani et al. (2011)	TS nested in VNS	single	single	linear	deterministic	close loop	no	capacitated	single	multiple
Ramezani et al. (2013)	ε-constraint method with stochastic programming	multiple	single	linear	stochastic	close loop	no	capacitated	single	multiple
Gendron et al. (2013)	B&B nested in LR	single	single	linear	deterministic	close loop	no	uncapacitated	single	multiple
Firoozi et al. (2013)	LR	single	single	linear	deterministic	open loop	no	uncapacitated	single	multiple

APPENDIX B

COMPUTATIONAL RESULTS

Table B.1. Computational results

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
40	15	5	10	220,1	215,5	249,2	128,9	110,9	9,4	32758	32950	32950	32758	0,586	0,586	0,000	6,925	1,769	0,962
40	15	5	10	353,6	121,7	213,5	126,4	85,4	1,8	31902	31902	31902	31902	0,001	0,001	0,000	8,655	1,620	0,000
40	15	5	10	611,4	495,8	251,4	90,1	155,5	5,8	137927	137927	138927	137927	0,725	0,000	0,725	14,114	1,365	0,408
40	15	5	10	149,6	119,7	213,5	88,0	121,3	4,1	32073	32494	33094	32073	3,183	1,313	1,846	9,845	1,675	0,224
40	15	5	10	90,7	64,5	200,3	86,9	108,3	5,0	22757	22967	23067	22757	1,362	0,923	0,435	15,857	2,562	1,360
40	15	5	20	546,9	393,7	210,0	140,1	66,9	3,0	115096	115273	115273	115096	0,154	0,154	0,000	11,696	0,035	0,000
40	15	5	20	241,1	164,1	217,1	139,4	71,1	6,6	52984	52984	53984	52984	1,887	0,000	1,887	10,508	1,489	0,223
40	15	5	20	70,6	62,9	259,8	127,6	127,1	5,1	100855	100855	100855	100855	0,000	0,000	0,000	7,081	1,051	0,019
40	15	5	20	34,8	44,4	182,6	107,5	70,5	4,6	13357	13357	13557	13357	1,497	0,000	1,497	7,366	1,315	0,072
40	15	5	20	249,4	111,0	237,3	98,9	135,1	3,3	25979	26012	26012	25979	0,127	0,127	0,000	9,644	1,922	0,000

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
80	15	5	10	2872,1	4768,8	514,7	167,0	323,8	24,0	163239	165121	167175	163239	2,411	1,153	1,244	12,160	2,454	1,841
80	15	5	10	2693,8	3779,4	329,1	134,9	181,0	13,2	197761	201250	202673	197761	2,484	1,764	0,707	14,794	4,217	0,251
80	15	5	10	3029,6	1876,9	270,8	133,0	116,8	21,1	119043	120001	121762	119043	2,284	0,805	1,467	8,792	3,442	1,128
80	15	5	10	1183,4	793,1	321,2	120,1	197,5	3,6	159234	161349	161669	159234	1,529	1,328	0,198	13,955	1,029	0,000
80	15	5	10	229,7	81,2	175,6	93,2	76,2	6,1	186901	188203	189827	186901	1,566	0,697	0,863	16,220	2,143	0,012
80	15	5	20	2020,7	2320,2	324,5	138,0	177,9	8,7	166994	168888	169769	166994	1,662	1,134	0,522	10,587	2,302	0,201
80	15	5	20	476,1	427,4	237,7	133,3	94,6	9,9	115421	117328	117328	115421	1,652	1,652	0,000	12,609	2,686	1,084
80	15	5	20	2360,9	1572,7	176,1	111,0	47,3	17,8	134031	136983	138011	134031	2,969	2,202	0,750	15,933	2,327	0,749
80	15	5	20	221,7	220,5	308,7	98,0	205,9	4,8	154032	154032	155666	154032	1,061	0,000	1,061	18,566	6,019	0,730
80	15	5	20	936,9	398,3	194,1	82,4	109,5	2,3	98640	98640	98640	98640	0,000	0,000	0,000	17,207	5,874	0,000

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
80	40	15	10	36000,0	33704,5	2558,6	827,2	1659,7	71,7	498133	498133	502887	484589	0,954	0,000	0,954	10,291	2,893	0,742
80	40	15	10	36000,0	35919,4	2584,5	720,8	1816,8	46,8	204668	197295	203107	-	-0,763	-3,603	2,946	15,620	2,228	1,285
80	40	15	10	32949,5	21980,8	2766,2	718,3	2004,2	43,6	249228	255782	258340	249228	3,656	2,630	1,000	14,778	2,594	1,325
80	40	15	10	36000,0	32484,7	2676,3	716,4	1895,3	64,5	278045	275241	280746	-	0,971	-1,009	2,000	12,111	1,892	0,696
80	40	15	10	36000,0	19752,6	3148,6	690,5	2396,5	61,7	210417	210417	218834	-	4,000	0,000	4,000	20,451	5,402	0,376
80	40	15	20	36000,0	14169,0	3174,6	761,5	2327,4	85,7	281346	276465	277400	-	-1,403	-1,735	0,338	9,660	1,599	0,125
80	40	15	20	36000,0	20983,3	2675,1	752,6	1864,6	57,9	198502	195592	194056	-	-2,240	-1,466	-0,785	12,365	1,373	0,122
80	40	15	20	29518,3	29967,8	2550,6	742,7	1740,8	67,1	277014	284244	287087	277014	3,636	2,610	1,000	17,790	2,042	1,323
80	40	15	20	36000,0	18705,2	2150,2	646,2	1392,5	111,4	170070	163267	168166	-	-1,120	-4,000	3,000	14,237	3,063	2,151
80	40	15	20	27903,7	14349,5	2286,5	584,3	1647,3	54,9	348935	353488	360559	348935	3,331	1,305	2,000	15,304	2,992	1,531

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	25	5	10	4403,2	3941,9	908,6	239,7	612,1	56,8	90945	93017	94226	90945	3,608	2,278	1,300	21,830	4,202	1,097
100	25	5	10	1919,6	1845,6	606,7	279,9	302,3	24,5	184421	186140	189274	184421	2,631	0,932	1,684	17,971	3,409	0,641
100	25	5	10	1171,9	274,6	803,6	224,8	534,1	44,6	143014	145984	148953	143014	4,153	2,077	2,034	16,856	3,102	0,627
100	25	5	10	39,3	42,7	688,1	236,7	447,1	4,3	64504	64504	65075	64504	0,885	0,000	0,885	12,138	2,357	0,000
100	25	5	10	7738,2	1359,1	774,9	246,7	477,8	50,3	213226	217519	220886	213226	3,592	2,013	1,548	13,881	1,666	1,133
100	25	5	20	545,0	461,2	593,2	230,4	355,2	7,6	75912	77973	79928	75912	5,290	2,715	2,507	10,353	1,389	1,084
100	25	5	20	12620,9	992,2	599,6	268,0	294,1	37,5	188440	188496	190349	188440	1,013	0,030	0,983	11,134	4,237	0,192
100	25	5	20	882,4	738,3	666,3	216,2	435,8	14,3	77956	78241	79467	77956	1,938	0,366	1,567	10,116	3,547	0,432
100	25	5	20	2558,5	1294,1	693,9	177,6	462,6	53,7	102418	104149	105971	102418	3,469	1,690	1,749	11,601	2,498	0,443
100	25	5	20	1158,1	195,7	672,9	162,4	495,5	15,0	187902	187902	189626	187902	0,917	0,000	0,917	9,264	2,153	0,064

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
80	25	5	10	875,6	248,1	536,5	196,0	331,2	9,3	176238	178021	180547	176238	2,445	1,012	1,419	14,578	4,080	0,472
80	25	5	10	2561,5	1125,0	677,8	188,2	463,3	26,3	128255	128255	130051	128255	1,400	0,000	1,400	13,723	2,962	0,598
80	25	5	10	1523,0	704,3	529,7	170,2	343,9	15,6	86909	89012	89151	86909	2,580	2,420	0,156	10,484	2,174	0,986
80	25	5	10	3640,8	2578,2	797,2	153,8	606,2	37,2	94009	95717	97298	94009	3,499	1,817	1,652	19,961	3,580	0,166
80	25	5	10	4334,8	2847,2	595,2	114,0	437,1	44,1	103495	104992	104992	103495	1,446	1,446	0,000	19,552	1,789	0,482
80	25	5	20	732,9	124,7	547,2	190,5	353,4	3,3	194843	194843	194843	194843	0,000	0,000	0,000	7,880	3,797	0,000
80	25	5	20	1103,1	506,9	600,0	173,8	414,2	12,0	69111	69883	69883	69111	1,117	1,117	0,000	11,613	2,313	0,063
80	25	5	20	1055,5	921,0	444,2	157,0	275,8	11,3	83489	84464	85038	83489	1,855	1,168	0,680	10,766	3,550	0,784
80	25	5	20	119,3	81,0	620,0	94,6	503,1	22,2	65474	66555	67177	65474	2,601	1,651	0,935	15,611	2,940	0,524
80	25	5	20	983,8	1598,8	347,5	60,8	276,0	10,7	78384	79241	80351	78384	2,509	1,093	1,401	12,885	2,231	1,075

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
80	25	10	10	29847,0	10347,1	815,9	390,9	391,0	34,0	135448	137663	140663	135448	3,850	1,635	2,179	18,342	2,945	2,463
80	25	10	10	36000,0	15147,9	1045,6	335,1	679,4	31,1	104045	105348	105874	-	1,758	1,252	0,499	17,203	2,363	0,151
80	25	10	10	18832,4	7019,3	826,0	280,9	502,6	42,5	82208	83952	84792	82208	3,143	2,121	1,001	21,897	1,674	2,133
80	25	10	10	15249,0	5892,7	843,7	226,4	575,3	42,1	130615	132914	134589	130615	3,043	1,760	1,260	14,528	4,741	0,047
80	25	10	10	36000,0	32203,1	896,7	185,4	686,4	24,8	115439	116904	118993	-	3,079	1,269	1,787	28,780	2,059	0,097
80	25	10	20	36000,0	23736,3	1036,0	392,3	561,3	82,5	172017	174465	174814	169073	1,626	1,423	0,200	21,473	2,302	0,906
80	25	10	20	36000,0	25931,0	995,8	339,0	604,0	52,9	129874	125460	129460	-	-0,319	-3,399	3,188	15,933	5,018	1,208
80	25	10	20	36000,0	27061,3	734,0	294,2	402,1	37,7	52679	53638	54282	-	3,043	1,820	1,201	17,828	1,667	0,609
80	25	10	20	36000,0	34230,8	861,1	294,1	542,2	24,8	185032	186940	189885	182231	2,623	1,031	1,575	19,204	3,522	1,905
80	25	10	20	36000,0	5932,3	791,4	248,7	507,4	35,3	134959	135034	135710	-	0,556	0,056	0,501	14,449	2,648	1,368

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	25	10	10	36000,0	14295,7	1248,1	475,9	732,5	39,7	399696	397714	405409	391429	1,429	-0,496	1,935	12,199	3,638	0,857
100	25	10	10	36000,0	32618,0	1181,1	428,1	693,9	59,0	192519	194102	195867	-	1,739	0,822	0,909	11,481	3,232	0,375
100	25	10	10	4182,2	1951,5	1055,3	408,2	589,5	57,5	35657	36881	36919	35657	3,539	3,433	0,103	19,180	3,194	0,143
100	25	10	10	36000,0	32618,4	1349,3	483,7	794,4	71,2	192162	194931	196329	-	2,168	1,441	0,717	10,338	2,776	0,529
100	25	10	10	36000,0	13195,2	1364,3	433,3	908,7	22,3	204851	207743	210526	-	2,770	1,412	1,340	12,919	1,683	0,018
100	25	10	20	36000,0	12394,9	1265,9	475,4	695,3	95,2	175943	173390	175060	-	-0,502	-1,451	0,963	12,598	2,320	0,560
100	25	10	20	1743,3	14935,0	1193,0	419,9	685,0	88,2	39707	40127	41233	39707	3,843	1,058	2,756	22,914	2,848	0,391
100	25	10	20	36000,0	4391,9	1010,0	360,7	549,6	99,7	127859	125430	127019	-	-0,657	-1,900	1,267	13,728	4,188	0,163
100	25	10	20	8388,9	14492,8	1036,7	341,9	624,5	70,3	124080	126143	128228	124080	3,343	1,663	1,653	15,804	2,541	1,757
100	25	10	20	36000,0	14198,5	1080,6	466,6	559,9	54,2	78335	78414	79757	-	1,815	0,101	1,713	9,941	2,649	1,247

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	25	15	10	36000,0	12019,2	1965,0	537,5	1323,3	104,2	174504	172391	177839	-	1,911	-1,211	3,160	22,319	4,449	0,332
100	25	15	10	1673,7	1500,2	1539,0	415,1	1081,5	42,4	72917	74301	75531	72917	3,585	1,898	1,655	20,695	4,144	0,663
100	25	15	10	36000,0	16715,0	1317,5	518,0	785,4	14,1	148802	144912	150709	-	1,282	-2,614	4,000	25,535	3,560	0,855
100	25	15	10	36000,0	24789,0	2168,4	527,0	1527,4	114,0	127942	133030	133632	124873	4,447	3,977	0,453	17,218	2,890	1,163
100	25	15	10	36000,0	16553,0	1352,9	635,7	625,8	91,4	104978	103153	105217	-	0,228	-1,738	2,001	14,879	2,278	1,909
100	25	15	20	36000,0	33210,3	1598,8	491,0	1054,8	53,0	230767	219344	213118	-	-7,648	-4,950	-2,838	19,933	5,229	0,558
100	25	15	20	36000,0	14814,5	1730,8	548,6	1074,5	107,8	170816	172380	177276	165431	3,782	0,916	2,840	12,832	4,641	1,669
100	25	15	20	36000,0	26000,0	1035,9	529,0	418,9	88,0	264507	231936	238895		-9,683	-12,314	3,000	10,187	5,315	0,033
100	25	15	20	36000,0	20776,4	1186,3	518,7	525,6	142,0	153184	153184	156844	150152	2,389	0,000	2,389	14,235	3,301	1,446
100	25	15	20	36000,0	20858,1	1912,3	483,6	1353,6	75,1	186925	184023	185544	-	-0,739	-1,552	0,827	17,348	4,104	0,395

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	40	5	10	36000,0	22990,0	1511,8	409,2	1071,9	30,6	254787	241023	246651	-	-3,193	-5,402	2,335	17,055	3,825	0,057
100	40	5	10	36000,0	20638,3	1605,2	361,1	1198,0	46,1	209836	215193	217325	-	3,569	2,553	0,991	18,145	2,409	0,658
100	40	5	10	36000,0	23370,0	1417,3	421,3	946,7	49,4	72504	70918	72300	-	-0,281	-2,187	1,949	14,035	1,886	0,021
100	40	5	10	17512,8	9715,4	1375,2	382,1	868,4	124,7	149546	152015	156424	149546	4,599	1,651	2,900	18,743	1,471	1,327
100	40	5	10	9485,5	4921,1	1533,6	422,4	1082,1	29,1	115109	118230	119960	115109	4,214	2,711	1,463	16,504	0,038	1,643
100	40	5	20	36000,0	21862,3	1257,2	451,3	760,1	45,8	228190	227042	231402	-	1,408	-0,503	1,920	10,836	4,481	0,488
100	40	5	20	967,8	773,4	1150,8	344,3	781,1	25,3	63474	63474	65937	63474	3,880	0,000	3,880	21,526	4,112	0,681
100	40	5	20	5721,1	2842,2	1321,6	400,9	898,3	22,4	194289	198824	203561	194289	4,772	2,334	2,383	11,162	3,612	1,110
100	40	5	20	19581,2	7835,6	1449,6	388,3	978,8	82,5	53097	53923	54078	53097	1,848	1,556	0,287	14,988	4,908	0,406
100	40	5	20	36000,0	28561,7	1323,0	413,5	776,7	132,8	104682	102582	107568	-	2,757	-2,006	4,861	10,820	2,373	0,686

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	40	10	10	36000,0	15640,7	1298,9	350,9	798,0	150,0	390831	382791	392931	-	0,537	-2,057	2,649	14,937	3,001	0,591
100	40	10	10	36000,0	29920,4	2071,6	434,6	1582,5	54,6	340495	314984	317844	-	-6,652	-7,492	0,908	18,303	2,968	0,669
100	40	10	10	36000,0	24921,0	1635,4	401,5	1009,3	224,6	236698	224010	226731	-	-4,211	-5,360	1,215	17,986	1,355	1,036
100	40	10	10	26704,2	16170,8	2262,4	459,6	1720,2	82,6	354665	361387	366938	354665	3,460	1,895	1,536	14,940	1,230	0,569
100	40	10	10	31015,7	19465,1	2044,6	419,6	1499,4	125,6	193348	197843	200671	193348	3,787	2,325	1,429	15,287	0,808	1,628
100	40	10	20	36000,0	15640,1	1909,1	534,1	1298,5	76,5	295986	294595	297433	-	0,489	-0,470	0,963	17,689	2,789	1,018
100	40	10	20	33391,4	26389,7	1504,1	526,5	947,7	29,9	399146	409233	419052	399146	4,987	2,527	2,399	15,573	4,915	0,848
100	40	10	20	21491,4	14276,7	1862,9	468,3	1344,1	50,5	281960	287934	291090	281960	3,238	2,119	1,096	20,242	2,462	0,495
100	40	10	20	36000,0	25640,5	1369,7	435,3	852,3	82,1	205021	199378	204341	-	-0,332	-2,752	2,489	15,075	1,556	2,071
100	40	10	20	32568,6	21916,7	1963,9	423,4	1497,5	43,0	305235	309489	314869	305235	3,156	1,394	1,738	16,736	3,558	0,748

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
100	40	15	10	36000,0	25240,5	4143,7	661,0	3393,6	89,1	235437	231847	234440	228733	-0,423	-1,525	1,118	27,553	4,697	0,909
100	40	15	10	36000,0	19372,7	3217,3	877,8	2254,6	85,0	193931	187623	191376	-	-1,317	-3,253	2,000	18,193	3,857	1,875
100	40	15	10	36000,0	40000,0	2377,4	605,3	1697,5	74,6	226887	228148	222837	-	-1,785	0,556	-2,328	13,297	2,503	1,358
100	40	15	10	36000,0	32041,8	2757,4	731,5	1824,4	201,5	216936	213051	215835	-	-0,508	-1,791	1,307	15,292	2,412	2,535
100	40	15	10	36000,0	18494,7	4033,8	918,6	2994,2	121,1	149979	148239	151204	144365	0,817	-1,160	2,000	13,191	2,128	1,353
100	40	15	20	32911,8	32875,5	3710,6	829,3	2760,5	120,8	198219	208436	210374	198219	6,132	5,154	0,930	21,914	2,261	1,489
100	40	15	20	36000,0	15048,3	3047,0	741,4	2160,0	145,6	355118	348932	358379	-	0,918	-1,742	2,707	16,908	1,941	2,549
100	40	15	20	36000,0	32912,2	2484,5	675,7	1700,2	108,6	200626	189736	195634	-	-2,488	-5,428	3,109	17,123	2,487	0,190
100	40	15	20	36000,0	33377,2	1624,2	614,8	905,1	104,4	103176	104314	107368	-	4,063	1,103	2,928	23,253	5,048	1,207
100	40	15	20	28437,8	14488,1	1978,1	557,6	1379,3	41,1	115492	118243	122278	115492	5,876	2,382	3,412	14,039	5,855	0,361

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
200	40	5	10	36000,0	28483,1	1961,6	709,0	1181,0	71,5	373834	361404	361947	-	-3,180	-3,325	0,150	12,348	2,817	0,253
200	40	5	10	36000,0	27777,5	1896,2	684,8	1110,6	100,8	655596	629704	640190	625324	-2,350	-3,949	1,665	12,239	4,321	0,423
200	40	5	10	36000,0	14693,7	1895,7	645,5	1095,3	155,0	492814	478317	482901	-	-2,012	-2,942	0,958	13,925	1,400	0,236
200	40	5	10	36000,0	25854,5	1596,5	587,7	979,6	29,2	562481	553107	555108	-	-1,311	-1,666	0,362	20,045	2,627	0,530
200	40	5	10	36000,0	26262,8	1642,5	572,5	923,4	146,5	550290	547273	553137	539945	0,517	-0,548	1,072	15,447	2,037	0,153
200	40	5	20	36000,0	28132,2	1443,5	603,1	778,3	62,2	174004	172784	177968	-	2,278	-0,701	3,000	12,303	2,801	1,162
200	40	5	20	36000,0	25192,9	1652,9	589,1	949,1	114,7	149150	144676	147570	-	-1,059	-3,000	2,001	10,331	1,994	0,052
200	40	5	20	18319,5	16025,3	1532,5	566,4	857,4	108,7	93482	96731	100535	93482	7,545	3,476	3,933	10,945	1,933	0,657
200	40	5	20	1994,7	1071,7	1449,5	558,9	861,2	29,5	88980	88980	90540	88980	1,753	0,000	1,753	13,551	1,053	2,731
200	40	5	20	1589,3	524,0	1560,2	518,0	795,0	247,2	113635	114544	116126	113635	2,192	0,800	1,381	23,703	5,272	0,906

Table B.1. (cont'd)

Problem Type				rt						Objective				Gap(%)					
|J|	|K|	|M|	α	cplex	RHA	H	LR	VNS	UU	cplex	RHA	H	LB	Gap	RHA	2	LR	VNS	UU
200	40	10	10	36000,0	27901,6	3422,7	1443,3	1871,1	108,3	211738	201594	204642	-	-3,351	-4,791	1,512	24,326	4,369	1,261
200	40	10	10	36000,0	33199,4	3403,3	1355,1	1900,1	148,1	521439	489148	490280	-	-5,976	-6,193	0,231	25,659	5,664	0,976
200	40	10	10	36000,0	25638,8	2872,6	1289,0	1468,7	114,8	457234	445952	458410	434369	0,257	-2,467	2,794	18,563	1,003	1,173
200	40	10	10	36000,0	33809,7	1888,5	1090,6	750,7	47,2	476613	464823	470713	450234	-1,238	-2,474	1,267	16,228	0,804	0,339
200	40	10	10	36000,0	31175,3	3453,7	1077,4	2335,2	41,1	687335	654360	657048	-	-4,406	-4,798	0,411	24,890	5,387	0,729
200	40	10	20	36000,0	31993,0	2879,2	1330,4	1422,1	126,7	265632	256780	262187	-	-1,297	-3,332	2,106	20,564	2,626	0,600
200	40	10	20	36000,0	24823,4	2634,6	1021,2	1496,1	117,3	144347	144077	146841	137771	1,728	-0,187	1,918	16,771	2,063	0,030
200	40	10	20	36000,0	23587,8	1903,0	979,3	818,5	105,2	327469	323914	330393	-	0,893	-1,086	2,000	19,633	2,229	1,447
200	40	10	20	22150,6	26477,6	2656,7	971,7	1633,6	51,4	507560	520642	530781	507560	4,575	2,577	1,947	21,295	2,280	1,188
200	40	10	20	36000,0	30176,8	2584,8	960,4	1472,1	152,3	293181	295838	303064	-	3,371	0,906	2,443	22,382	1,744	2,218