TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) DEVELOPMENT OF PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS IN STATISTICS TEACHING: A MICROTEACHING LESSON STUDY

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ABSTRACT

TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) DEVELOPMENT OF PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS IN STATISTICS TEACHING:
A MICROTEACHING LESSON STUDY

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The current study examined the development of technological pedagogical content knowledge (TPACK) of preservice mathematics teachers in statistics teaching with virtual manipulatives in the context of a microteaching lesson study (MLS). Since "technology integration is described by how and why it is used rather its amount or type", integrating technology into education necessitates investigation of instructional uses of digital tools and resources (Earle, 2002, p. 8). TPACK framework suggested by Koehler and Mishra (2009) for technology-integrated teaching in which teaching is assumed as "an interaction between what teachers know and how they apply what they know" in different contexts lies as a fundamental idea behind this study (p. 62).

Qualitative efforts were used in order to address research questions through two MLS groups. At the beginning, 9 participants volunteered for the interviews in order to investigate their views towards statistics teaching and technology integration into it. Distinguishing Teacher Assessment in Statistics (DTAS) Test were applied in order to analyze their content and pedagogical content knowledge. Before MLS began, a workshop was conducted to introduce virtual manipulatives which can be used in statistics teaching while recalling their statistical content knowledge. Through MLS period, each group prepared one lesson plan including a virtual manipulative for the instruction of a specific statistical concept. Data collected through MLS for each group contained three consecutive group discussions.

Based on the findings, it was argued that preservice mathematics teachers' TPACK changed and developed through MLS. They were observed to have significant developments regarding TPACK knowledge domains, especially in statistical content knowledge, statistical pedagogical knowledge and technological content knowledge.

Keywords: Technological Pedagogical Content Knowledge (TPACK), Microteaching Lesson Study (MLS), Statistics Teaching, Virtual Manipulatives.

ÖZ

İLKÖĞRETİM MATEMATİK ÖĞRETMENLİĞİ ADAYLARININ TEKNOLOJİK PEDAGOJİK ALAN BİLGİSİ GELİŞİMLERİ: BİR MİKRO ÖĞRETİM DERS ARAŞTIRMASI

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Bu çalışma, bir mikro öğretim ders araştırması (MÖDA) kapsamında matematik öğretmenliği adaylarının teknolojik pedagojik alan bilgisinin (TPAB) gelişimini sanal manipulatifler ile istatistik öğretimi açısından incelemiştir. "Teknoloji entegrasyonu, onun miktarından ve türünden çok nasıl ve neden kullanıldığıyla açıklanması" gerektiği için eğitimde teknoloji entegrasyonu, dijital araçların ve kaynakların öğretim ve kullanımları soruşturma gerektirmektedir (Earle, 2002, s. 8). Farklı öğretim bağlamlarında, "öğretmenlerin neyi bildikleri ve onları nasıl uyguladıkları arasındaki etkileşim" olarak kabul edilen teknoloji entegre edilmiş öğretim için Koehler ve Mishra (2009) tarafından önerilen TPAB çerçevesi bu çalışmanın arkasında temel fikir olarak yatıyor (p. 62).

Araştırma sorularına cevaben, iki MÖDA grubu için nitel araştırma yöntemleri kullanılmıştır. Başlangıçta, 9 katılımcı istatistik öğretimi ve onların teknoloji

entegrasyonu konusunda bakış açılarını almak üzere görüşmeler için gönüllü oldu. Ayırt edici İstatistik Öğretmen Değerlendirmesi (AİÖD) katılımcıların alan ve pedagojik alan bilgilerini analiz etmek amacıyla uygulandı. MÖDA başlamadan önce, bir atölye çalışması ile istatistiksel alan bilgileri hatırlatılarak istatistik öğretiminde kullanılabilecek sanal manipulatifler tanıtıldı. MÖDA boyunca, her grup, belirli bir istatistik kavramı için sanal manipulatifleri eklemek üzere bir ders planı hazırladı. MÖDA aracılığıyla her grup için toplanan veri bu ders araştırma gruplarının üçer ardışık grup tartışmalarını içermektedir.

Bulgulara dayanarak, MÖDA boyunca matematik öğretmen adaylarının TPAB’lerinin değiştiği ve geliştiği iddia edilmiştir. Katılımcıların özellikle istatistik alan bilgisi, istatistiksel pedagojik bilgi ve teknolojik alan bilgileri başta olmak üzere, TPAB bilgi boyutları açısından önemli gelişmeler gösterdikleri gözlenmiştir.

Anahtar Kelimeler: Teknolojik Pedagojik Alan Bilgisi (TPAB), Mikro Öğretim Ders Araştırması, İstatistik Öğretimi, Sanal Manipulatifler.

To my son, *Poyraz*

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Last words are for Poyraz, my son: You are the unique person who make my life sense and I am still dreaming of the times when we will drink our coffee cheek by jowl while talking maybe about these days.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MLS	: Microteaching Lesson Study
TPACK	: Technological Pedagogical Content Knowledge
CK	: Content Knowledge
PK	: Pedagogical Knowledge
TK	: Technological Knowledge
PCK	: Pedagogical Content Knowledge
TCK	: Technological Content Knowledge
TPK	: Technological Pedagogical Knowledge
TPSK	: Technological Pedagogical Statistical Knowledge
DTAS	: Distinguishing Teacher Assessment in Statistics
TI-STW	: Technology-Integrated Statistics Teaching Workshop
GD(s)	: Group Discussion(s)

CHAPTER 1

INTRODUCTION

With the emergence of educational technology, teaching preservice teachers how to teach with technology has recently become an important concern of teacher educators. Initial expectations of teacher education programs were that preservice teachers should have a deep and broad content knowledge (Niess, 2005). Then, these were affected by the pedagogical content knowledge (PCK) notion (Shulman, 1987), since PCK suggested that there are interrelations between the facts, concepts, principles with the students' learning. Therefore, preservice teachers were expected to organize their teaching according to content and student learning, which is an intersection of content knowledge and pedagogical knowledge and it was named as PCK. However, preservice teachers currently need to develop a technology-PCK (TPCK) notion (Niess, 2005), which was later defined as technological pedagogical content knowledge (TPACK) by Koehler and Mishra (2008). While characterizing teaching as a *wicked problem* (Rittel, & Weber, 1973), TPACK was introduced in order to deal with its complex nature as interrelations among content, pedagogy and technology. TPACK is the interactions between and among these three core knowledge dimensions (Koehler, & Mishra, 2008).

Using the fractals as metaphor, it was explained that "topics such as statistics, graphing, coordinategeometry, matrices, probability, combinatorics and many other mathematics-related topics are in an ongoing state of change and evolution (Heid, 2005 as cited in Grandgenett, 2008, p. 149). Consequently, the teacher education programs should not underestimate the role of technology, which fits best with the interweaving content with pedagogy in order to create effective instruction. However, teacher education programs has still been focusing too much on 'what' technologies to use and there was not enough imaginative thinking on 'how' these technologies could be embedded in order to have an effective instruction (Grandgenett, 2008). TPACK framework provides a sound background for generating such effective implementations. While teacher education programs traditionally dealt with mostly on learning about technology, currently they were integrating pedagogical approaches to teach with technology, particularly in teaching methods courses. However, little research about how these efforts support the development of TPACK of preservice teachers was made (Niess, 2005).

This study is a content and technology based approach which Niess (2011) offered for the categorization of research directions for TPACK development, since she categorized the collaborative lesson studies (Groth, Spiekler, Bergner, & Bardzell, 2009) as a content and technology based approach. Groth, et al. (2009) explained two contrasting paradigms in order to assess teachers' knowledge. One is basically focused on quantitative efforts which tries to measure the effect of mathematics teacher education programs with a psychometric approach. The other is based on qualitative efforts and considers the case descriptions of "teachers' *accounts of practices* which can portray the complex interrelationships among different aspects of teachers' knowledge and their relationships to teaching" (Simon, & Tzur, 1999, p. 263, as cited in Groth, et al., 2009). From the similar point of view, lesson study was offered as a professional development process, as well (Stiegler, & Hiebert, 1999; Lewis, 2009).

Harris, Grandgenett and Hofer (2010) claimed that there are three types of data for assessing TPACK of teachers and these data types fit very well with the data collected through a lesson study, as well. These are "self-report (via interviews, surveys, or other generated documents, such as reflexive journal entries), observed behavior, and teaching artifacts, such as lesson plans. Since teachers' knowledge is typically reflected through actions, statements, and artifacts. . ." (p. 3834). They also explained the process of triangulation among these types of data. They claimed that teachers' knowledge could be recognized through actions, statements and artifacts instead of being observable.

Therefore, techniques or instruments which are used for assessing TPACK of teachers should support distinction and identification of knowledge dimensions and a scope of teachers' TPACK in systematic, reliable and valid ways. Consequently, the triangulation between data types would help in order to better understand the nature of TPACK by inference (Harris, et al., 2010).

Grandgenett (2008) claimed that mathematics teachers currently had a responsibility of "imagining technology connections, determining the benefit of related instructional strategies, and putting it altogether for an effective mathematics lesson" (p. 156). Consequently, teacher preparation programs should be organized in a continuously-structured manner due to technology. Niess (2008) emphasized teaching methods courses at most and she claimed that methods courses could present ways for integrating content, pedagogy and technology to the preservice mathematics teachers. While engaging with the design of lessons, preservice teachers had to change their mindsets and behaviors which were formed through their own learning experiences (p. 226). Grandgenett (2008) also offered some characteristics which teacher education programs should have while assuming TPACK as a base for structuring them: (1) a successful program should encourage an imaginative openness regarding use of technology, (2) it would not treat content, pedagogy and technology as separately. This means that the courses offered should include use of technology. (3) Methods courses should present carefully selected TPACK-related examples. (4) TPACK experiences should be reflected as culturally relevant. (5) A successful program should present the alternative use of technology rather than in content or pedagogy, such as use of technology for classroom management. Lastly, (6) students, as individuals, should never be underestimated as technology is becoming more ubiquitous and pervasive.

1.1 Technology integration in education and mathematics education

In the past, teacher education was focused only on content knowledge and it was assumed that knowing the content area was enough to teach the subject effectively (Doering, Veletsianos, Scharber, & Miller, 2009). However, for the last two decades, it has been acknowledged that how teacher will teach is as important as what and why teachers teach (Shulman, 1987). In addition, teacher education has also been affected by the ongoing developments in technology and technological resources for education. Consequently, in a similar manner, how teachers could enhance their teaching via technological resources has been recognized to be more important than what teachers know about technology and technological resources (Koehler, & Mishra, 2009). In this sense, the notion that how mathematics teachers would enhance their technology integration skills for teaching was the initial concern of this study.

Historically, the use of technology in teaching was effected with the arousal of PCK notion in the late 80s and 90s. Since teachers' beliefs about how to teach mathematics were related with how they learned mathematics, their pedagogical knowledge degraded the use of technology to demonstration, verification and drill and practice, as being the initial attempts of technology integration (Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, Özgün-Koca, & Kersaint, 2009). Therefore, it was eventually argued that "the major limitations of computer use in the coming decades are likely to be less a result of technological limitations than a result of limited human imagination and the constraints of old habits and social constructs" because of the lack of effective examples of technology integration (Kaput, 1992, p. 515). Then the arguments about technology integration were observed to change in nature through the past decades. The effectiveness of technology integration in teaching has been debated for decades; how technology integration is taking place, what the consequences of technology use in teaching and how technology integration into mathematics teaching could be improved and enhanced were begun to be discussed (Kaput, 1992). Earle (2002) explained the foundation of this claim later that "integrating technology is not about technology – it is primarily about content and instructional practices. Integration is defined not by the amount or type of technology used, but by how and why it is used" (p. 8).

In early 2000s, it was recognized that teachers need a professional development for integrating technology in teaching, since they were not capable enough to design technologically-rich lessons. Through the following decade, it was also claimed that strategies for technology integration into teaching could not be developed as fast as the technology (Niess, et al., 2009).

Recently, technology integration has been mostly approached from the aspects of technological tools while underestimating their emphasis upon content and pedagogy. Therefore, aroused technology-based learning activities were generated as independent from content or context, supposing that they could be applicable in any grade level, in any classroom, in any content area (Harris, Mishra, & Koehler, 2009). Another important fact which was claimed as one of the impacts on technology integration is the following: Today's teachers are assumed to be *digital immigrants* while their students are in fact *digital natives* with apparently different experiences with digital technologies (Prensky, 2001; as cited in Niess, 2012), which was the unique problem of this era of teacher education in terms of technology integration. Therefore, teachers would be in the future like as described below:

> Tomorrow's teachers must be prepared to rethink, unlearn and relearn, change, revise and adapt. All preservice teachers are faced with learning to incorporate goals and outcomes beyond their own content areas – guiding student learning about the new and emerging information and communication technologies while students are also learning the content with these technologies (Niess, 2008, p. 225).

1.2 Difference between Mathematical and Statistical Knowledge and Technology integration in statistics teaching

Statistics and mathematics recently were viewed as quite different subjects. Through the lens of liberal arts, Moore (1998) re-defined statistics as:

> Statistics is a general intellectual method that applies wherever data, variation and chance appear. It is a fundamental method, because data, variation and chance are omnipresent in modern life. It is an independent discipline with its own core ideas rather than, for example, a branch of mathematics (p. 1254).

Based on this definition, it can definitely be claimed that mathematical thinking differs from statistical thinking. While mathematics suppresses the context, statistical concepts depends context as obligatorily. Mathematics, in fact, is based on abstract patterns ultimately. However, statistics, especially in data analysis is based on "how the threads of those patterns interweave with the complementary threads of the story line" (Cobb, & Moore, 1997, p. 803). Therefore, it can be argued that statistics need context in order to have a meaning. Based on this view, "statistical thinking offers simple but non-intuitive mental tools for trimming the mass of information, ordering the disorder, separating sense from nonsense, and selecting relevant from the irrelevant many" (Ben-Zvi, 2000, p. 129).

The comprehension of the difference between mathematical and statistical thinking eventually lead to a different approach to teach statistics, which made use of educational technology. While statistics was traditionally taught with a focus on computations, formulas, procedures, etc., "the current emphasis on statistics teaching was based on statistical reasoning and the ability to interpret, evaluate, and flexibly apply statistical ideas" (Ben-Zvi, 2000, p. 130). Consequently, Ben-Zvi (2000) concluded that exploratory data analysis (EDA) has been the inevitable result of this approach to teaching statistics with the emergence of technology. He defined EDA as "the discipline of organizing, describing, representing, and analyzing data, with a heavy reliance on visual displays as analytical tools and, in many cases, on technology" (p. 130). There are four major mottos which were being associated with EDA: (1) *look at the data* which is for initial analysis of data, (2) *look between the data* which is for making comparisons, (3) *look beyond the data* which is for making informal inference, and (4) *look behind the data* which is evaluating it in the context (Curcio, 1989, as cited in Ben-Zvi, 2000; Shaughnessy, Garfield, & Greer, 1996).

The EDA approach of statistics teaching leaves discussion and decision making process under uncertainty (Shaughnessy, et al., 1996) and hence, technological tools were seen as an ideal teaching tool in data handling since using visualizations and simulations in order to understand statistical concepts and methods is easy through the use of technology (Ben-Zvi, 2000).

As an addition to Pea's (1987) *amplifier metaphor* for educational technology, Ben-Zvi (2000) evaluated it with a *reorganization metaphor*. Rather than seeing technology as only amplifying human capabilities, author suggested that technology presents opportunities to develop students cognitively and socio-culturally. Ben-Zvi claimed that technology shifts the activity to a higher cognitive level, it changes the objects of the activity, it has a focus on transforming and analyzing the representations, it supports the situated cognition modes of thinking and problem solving, statistical conceptions can be accessed through the use of graphics, and it constructs meaning by conceptions by the use of the representative ambiguity (Ben-Zvi, 2000, pp. 140-143).

While seeing technology as a powerful tool for cognitive development in statistics and as an indispensable action to statistical thinking, Ben-Zvi (2000) categorized the several technological tools to teach statistics as "statistical packages (tools), microworlds, tutorials, resources (including Internet resources), and teachers' metatools" (p. 144). Statistical packages are simply the software which are based on spreadsheet use in order to deal with data. MEDASS Light, Fathom, TinkerPlots are some examples for them. Even MS Excel can be considered under statistical packages category. Second one is microworlds which consist demonstration of statistical concepts, visualizations, interactive experiments and simulations. Through the microworlds, one can have the option to manipulate the graphs, parameters, and methods. Prob Sim and Sampling Distributions, which are mostly used in high school and university level statistics, are the examples of this category. Tutorials, which is the third category, are some programs which tutor students on specific statistical skills and test their statistical knowledge. ActivStats, ConStats, The Authentic Statistics Stack are the examples for tutorials. Resources are the fourth category for technological tools in statistics teaching, which is in fact the consequence of creation of Internet. Ben-Zvi (2000) claimed that the development of Web-based tools would become more sophisticated and useful resources, since access to Internet is more common and easy today. The fundamental resources are online course materials, online texts, Java demonstrations, electronic journals and newsletters, electronic discussion lists, data (many data sources), and general links. Last category is teachers' metatools, which presents an interface that enables teachers to adapt software to their specific audience and educational goals. Virtual manipulatives, which are the heart of this study, simply falls into the Ben-Zvi's (2000) resources category of teaching tools in teaching statistics.

1.3 Purpose of the Study

Considering the existing literature on knowledge needed to teach mathematics and different uses of technological resources for statistics teaching, the current study aimed to teach how to use of virtual manipulatives to preservice mathematics teachers via performing a microteaching lesson study, as an initial attempt. Since the participants were preservice mathematics teachers, the study conducted here is a microteaching lesson study. Consequently, it was aimed that preservice teachers would experience some developments in their knowledge levels regarding the knowledge dimensions aroused from TPACK framework while learning how to teach statistics via virtual manipulatives. In particular, a workshop was designed to teach the use of virtual manipulatives in statistics teaching as a part of a 4th year undergraduate course for preservice mathematics teachers. Moreover, two groups of microteaching lesson study were formed with the participants of the study. Because of the nature of lesson study, it was aimed that preservice teachers will increase their knowledge levels in terms of knowledge dimensions in TPACK framework, but in particular how this development has taken place in which knowledge dimensions were examined.

1.4 Research Questions

As Kaput (1992) was emphasized that an educational decision to implement a new technology in a classroom is “bounded to the available physical resources, but more importantly, by the decision makers’ vision and expectations regarding the likely contribution of the new technology to the achievement of their educational objectives” (p. 548). Regarding preservice mathematics teachers as the *decision makers* in their future professional lives, the major focus of this study becomes their visions and expectations since they participated in a lesson study. While considering the major research question as ‘how preservice teachers will develop their knowledge levels through a microteaching lesson study regarding the use of virtual manipulatives in statistics teaching?’ the study address the following research questions:

a. What views do preservice middle school mathematics teachers have towards (a) statistics teaching, and (b) towards technology integration into statistics teaching (virtual manipulatives, simulations, and the like) at the beginning of the microteaching lesson study?

b. What content knowledge and pedagogical content knowledge did preservice middle school mathematics teachers have regarding statistics and statistics teaching at the beginning of the microteaching lesson study?

c. How do preservice middle school mathematics teachers’ technological pedagogical content knowledge change through microteaching lesson study?

i. What technological pedagogical content knowledge do preservice mathematics teachers have while teaching statistics via virtual manipulatives?

ii. To what extent does the microteaching lesson study contribute to participants' technological pedagogical content knowledge?

While addressing these research questions, group-case study dynamics (Yin, 2014; Leavy, 2014; Fernández, 2005) were used in order to explore the participants' professional development through a microteaching lesson study. This qualitative approach allowed the researcher to investigate and explore the preservice mathematics teachers' TPACK development and to what extent these developments occurred during microteaching lesson study, while enhancing their statistics teaching with the integration of virtual manipulatives.

1.5 Significance of the Study

Revealing the relationship between technology integration as an important part of today's teachers' knowledge needed for teaching for both mathematics and statistics; and understanding that microteaching lesson study is one of the considerable professional development of preservice mathematics teachers, it was suggested to present professional development sessions for preservice mathematics teachers. Particularly, preservice mathematics teachers' knowledge needed for statistics teaching might offer teacher educators as another way for their professional development because of the changing views towards statistics teaching.

In our revised school curriculum, which started to be instructed in middle-level schools in Turkey in September, 2013, statistics was the subject which paid higher attention rather previous curricula (MONE, 2013). It was considered as a separate learning area named as data handling and it was included in all grades from 5th grade through 8th grade. However, the density of probability was reduced compared to previous curriculum, and its instruction is placed into the 8th grade level only with a basic understanding of probability. Moore (1997, as cited in Biehler, Ben-Zvi, Bakker, & Makar, 2012) recommended some changes from the statistical point of view, in that of *content* (more key concepts, and data analysis, and less probability), *pedagogy* (fewer lectures, more active learning) and *technology* (for data analysis and simulations). Thus, the new curriculum could be identified as a well-reflection of Moore's recommendation that it enhances more statistics and less probability while leaving the deeper conceptual knowledge to the high-school level. As a consequence of this approach with the emergence of technology integration into teaching, TPACK was aroused as a total knowledge package for mathematics teachers (Niess, et al., 2009).

What earlier studies showed that preservice mathematics teachers had a less comprehension of statistics and probability compared with the other learning areas of curriculum, that is, they found probability and statistics subjects difficult to teach especially because of their lack of content knowledge related with them (Quinn, 1997; Stohl, 2005). Contemporary efforts are addressing the same issue as well so that teacher education should be enhanced while giving an attention to statistics teaching for mathematics teachers (Stohl, 2005). Ponte, Oliveira and Varandas (2002) claimed also that "preservice teachers need to acquire some skills related with applications such as word processing, spreadsheets, statistics software, electronic mail, subject-specific educational software, and the Internet in terms of information, search and production" They emphasized especially the inadequacy of preservice teachers' technological knowledge regarding statistics teaching, Internet navigation and use of electronic mail (p. 95).

Because of the universality and accessibility of the Internet, virtual manipulatives as a technological tool which is one of the categories listed by Ben-Zvi (2000) for statistics teaching, were integrated into this study in order to develop their statistics teaching capabilities via technology. In the literature, there has been no study that explicitly lesson study for integrating virtual manipulatives while emphasizing the development of statistics teaching of preservice mathematics teachers. Virtual manipulatives were studied with mostly the concepts regarding fractions, patterns, and geometry (Moyer-Packenham, Salkind & Bolyard, 2008). For instance, there is not any specific study which was conducted for teachers' use of virtual manipulatives regarding statistics or their mathematical or statistical understanding and thinking regarding the use of virtual manipulatives.

Preservice teachers needed to learn and develop their TPACK according to what Niess (2008) suggested for teacher educators and the current study also presents an alternative response to her question of how preservice teacher education could be arranged to support knowing and thinking involved in TPACK. Niess (2008) also called attention to methods courses in such a way that preservice teachers should learn how to teach TPACK ways of thinking before they are ready to teach through methods courses since she claimed that methods courses provides practical experiences to preservice teachers (pp. 227-228).

Although professional development efforts through classroom practices was agreed to foster effective teacher education, it is unclear that how such effective teacher education will be designed and which tasks should be included in order to enhance teachers' knowledge needed for teaching mathematics and improve their mathematical thinking as well as statistical thinking (Fernandez, 2002). Therefore, lesson study was currently emphasized as a form of professional development while responding the above questions (Fernandez, 2002). The current study is significant that it represents a way of professional development of preservice mathematics teachers through lesson study.

Therefore, on the whole, this study was expected to have some positive effects on mathematics teacher education programs, especially in methods courses. Since the study used lesson study as a professional development process regarding TPACK; lesson plans, video-cases, and such similar teaching artifacts could be seen as an important experience in order for developments of prospective teachers in terms of their teaching and learning skills. While considering how teachers integrate technology into content and pedagogy as intertwiningly rather than considering what teachers know about technology, presented lesson study here could be an effective way in order to develop prospective teachers' TPACK through statistics teaching. As placing an important part of the school curriculum, statistics and statistical reasoning needed to be revised in terms of teacher education programs as well. Considering that many teachers' previous experiences were only based on descriptive statistics, recent approaches for statistics teaching and learning should be introduced to them with the help of technological improvements (Pfannkuch, & Ben-Zvi, 2011). As a result, teacher education programs needed to rethink the opportunities regarding the nature, role and purpose of statistics from which prospective teachers benefited. The lesson study conducted here is important as well, since it presents a learning experience for prospective teachers in order to develop their statistical thinking.

1.6 Definitions of Important Terms

The definitions of important terms used through the study were presented below:

Virtual manipulatives: A virtual manipulative is defined as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (Moyer, Bolyard & Spikell, 2002, p. 373). Further they were described as "dynamic visual/pictorial replicas of physical manipulatives (such as pattern blocks, base-10 blocks, geometric solids, tangrams, or geoboards)" (Moyer-Packenham, et al., 2008, p. 2). They further explained the characteristics of virtual manipulatives as in the following: "They are placed on the Internet as *applets*, or smaller stand-alone versions of application programs. Users move the computer mouse to manipulate these dynamic, visual objects. The ability to manipulate virtual manipulatives makes them particularly useful in teaching mathematics interactively" (Moyer-Packenham, et al., 2008, p. 2). Through the study, reader could meet with some examples of virtual manipulatives, such as the JAVA demonstrations in the National

Council of Teachers of Mathematics (NCTM) Illuminations Web site, Shodor Education Foundation Curriculum Materials Web site, and the like.

Knowledge dimensions: The phrases of knowledge components or domains or dimensions referred in this study to the components or bodies of knowledge which aroused from the TPACK framework (Koehler, & Mishra, 2009). Namely, the three core components of teachers' knowledge are content knowledge (CK), pedagogical knowledge (PK) and technological knowledge (TK). As having equal importance with the former ones, the interactions among and between these three core knowledge dimensions were represented with pedagogical content knowledge (PCK), technological content knowledge (TCK), technological pedagogical knowledge (TPK) and technological pedagogical content knowledge (TPACK).

Lesson study: As having its origins from Japanese teacher education, lesson study referred in this study could simply be described as a form of professional teacher education in which a group of teachers work collaboratively in order to design a lesson as emphasizing a specific subject (Lewis, 2009).

Microteaching lesson study: Microteaching lesson study (MLS) referred to a specific form of lesson study throughout the study and introduced in Fernández's (2005) work at first. MLS is a lesson study in which studying teachers are prospective teachers. The cycle of lesson study is performed as in the same iterations in an MLS. During MLS, microteaching is applied as a scaled-down lesson by a prospective teacher to his/her peers. Through MLS, groups of prospective teachers study on designing a lesson while fostering collaboration and reflection among them. MLS groups discuss their progress, implement their lesson plan and watch the video-taped lesson in order to discuss it further and decide on stopping the cycle of lesson study.

CHAPTER 2

REVIEW OF RELATED LITERATURE

This study investigates the development of TPACK of preservice elementary mathematics teachers in statistics through the virtual manipulatives. Therefore, the review of related literature here summarizes the key issues in order to prepare the ground which research questions will be sought. First, TPACK framework was mentioned with its dimensions through the studies which cover TPACK development of teachers, assessment of TPACK, etc. Then, statistics education was mentioned as stressing on the necessities of its inclusion into elementary mathematics curriculum mainly. Third, the studies which focus TPACK development and assessment in relation to statistics education were mentioned and some enthusiasms from them were explained. Lastly, a brief summary ended the review of related literature highlighted with the significance of this study.

2.1 TPACK Framework and its Dimensions

The approach suggested by Koehler and Mishra (2009) for teaching with technology integration in which teaching is assumed as "an interaction between what teachers know and how they apply what they know" in different contexts lies as a fundamental idea behind this study (p. 62). In order to define what technological pedagogical content knowledge (TPACK) is, they assume that technology, pedagogy and content should lie at the heart of good teaching and it was claimed that these three core knowledge bases form the TPACK framework (Koehler & Mishra, 2009, Niess, 2005; 2011, Zhao, 2003). More specifically, TPACK is assumed as the interconnection and intersection of content, pedagogy and technology (Margerum-Leys & Marx, 2004; Mishra & Koehler, 2006; Niess, 2005; Pierson, 2001; Zhao, 2003 as cited in Niess, 2011). Moore (1997) also suggested that there is a synergy between content, pedagogy and technology in between. He claimed that "the most effective teachers will have a substantial knowledge of pedagogy and technology, as well as comprehensive knowledge about and experience applying the content they present" (Moore, 1997, p. 134).

As it was described that the development of TPACK framework shows similarities with the emergence of pedagogical content knowledge (PCK) historically (Koehler & Mishra, 2008; Koehler & Mishra, 2009; Niess, 2011), TPACK framework shows resemblance to the successive development attempts of the "understanding and application of the parent construct – PCK" (Niess, 2011, p.300).

Since teaching would be described as an "ill-structured, complex domain" by Koehler and Mishra (2008, p. 3), integrating technology to it seems to be a "wicked problem" (Rittel & Webber, 1973, p.10, as cited in Koehler & Mishra, 2008). Based on the explanation of what a wicked problem is, it is argued that "there is no definitive solution to a technology integration problem" (Koehler & Mishra, 2008, p. 11). However, the ill-structured and wicked nature of teaching necessitates "understanding a variety of complex concepts (and their contextually defined interactions), and that these concepts interact in patterns that are not consistent across cases" (Koehler & Mishra, 2008, p. 11).

Previously, content knowledge has taken more consideration than other knowledge bases of teaching (Shulman, 1986; Veal & MaKinster, 1999, as cited in Mishra & Koehler, 2006) and generally pedagogy and content knowledge have been perceived as distinct domains in research (Shulman, 1987, as cited in Mishra & Koehler, 2006). Later, there is a shift towards to pedagogy concentrating on general pedagogical strategies independent of and beyond the content knowledge (Ball & McDiarmid, 1990, as cited in Mishra & Koehler, 2006). Another consideration what Shulman (1997) claimed that there is a dichotomy between teacher education programs as being either content or pedagogy oriented. As a result, Shulman (1986) offers a new knowledge dimension, namely PCK and it was defined as knowledge needed for teaching the content while considering the different pedagogical contexts, including “the ways of representing and formulating the subject that make it comprehensible for others” (p. 9). Therefore, according to Shulman (1986), teachers should also have a notion of PCK besides with knowledge of content and pedagogy. These efforts related with knowledge bases for teaching emphasized the work of developing a framework for TPACK currently as described below in detail.

TPACK framework was first offered as an adaptation of four components of PCK in order to explain the technology-integrated PCK (TPCK) (Niess, 2005). She uses the Grossman’s (1989, 1990) four central components of PCK while changing them to fit with technology integration as components of TPCK framework as follows: “(1) an overarching conception of what it means to teach a particular subject integrating technology in the learning; (2) knowledge of instructional strategies and representations for teaching particular topics with technology; (3) knowledge of students’ understandings, thinking and learning with technology in a particular subject; (4) knowledge of curriculum and curriculum materials that integrate technology with learning in the subject area” (Borko & Putnam, 1996, p. 690, as cited in Niess, 2005, p. 511, Niess, 2006, p.197). In her study, she tries to investigate TPCK of five cases which are preservice teachers from both mathematics and science through a one-year multi-dimensional science and mathematics teacher preparation program. On the conclusion, Niess (2005) figured out that the adapted components of PCK in order to provide evidence for the framework which she offered for TPCK was a “beginning” effort (Niess, 2005, p. 522).

Then, the TPACK framework proposed by Mishra and Koehler (2006) and including seven elements interacting with each other as in the following figure, while generating two new knowledge dimensions as technological pedagogical knowledge (TPK) and technological content knowledge (TCK), which is shown in Figure 2.1 below.

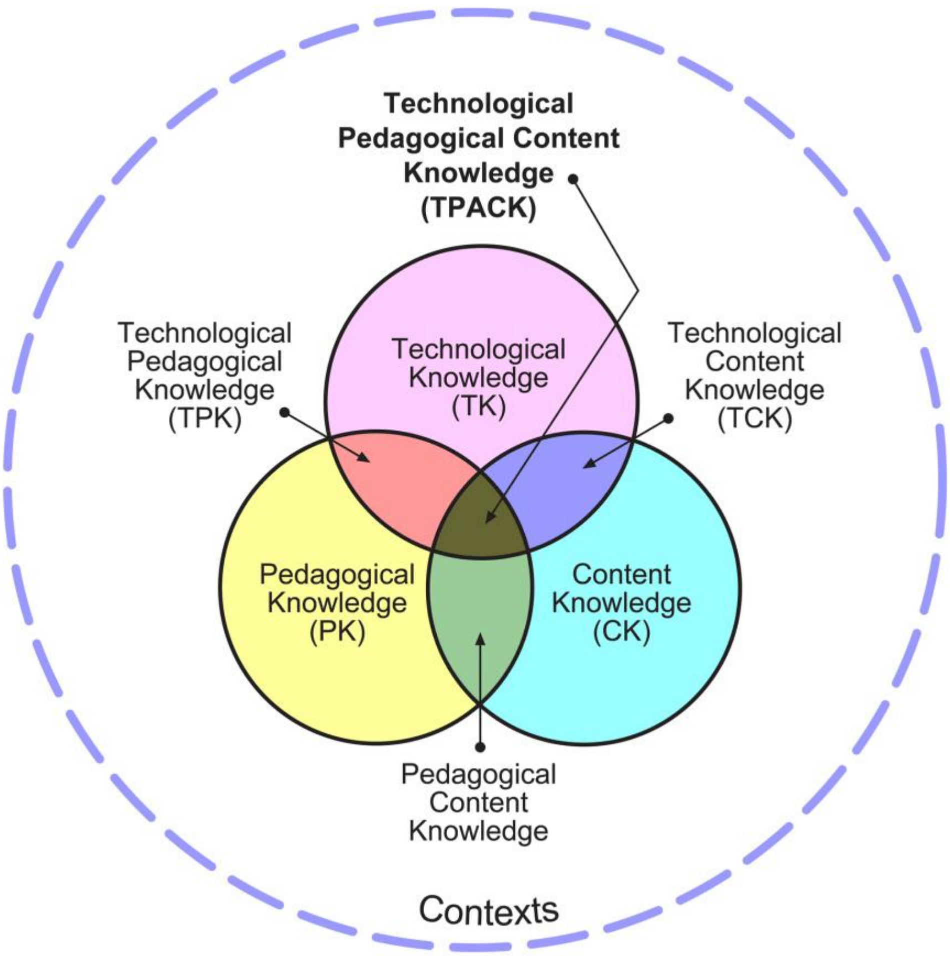

Figure 0-1 TPACK Framework and its knowledge components (Koehler & Mishra, 2009, p. 63).

Content knowledge (C or CK) is defined as teachers' subject matter knowledge to be taught or learned. Knowledge needed for teaching mathematics is different from that for teaching physics and the knowledge needed to teach in elementary level is different from that in university level as well. Content knowledge basically includes the "knowledge of concepts and theories, ideas, organizational frameworks, knowledge of evidence and proof as well as established practices and approaches towards developing such knowledge" (Shulman, 1986, as cited in Koehler & Mishra, 2009). Besides this definition of CK, Ball, Hill and Bass (2005) suggested a definition for mathematical content knowledge as well, they argued that it has two main elements: "*common* knowledge of mathematics that any well-educated adult should have and mathematical knowledge that is *specialized* to the work of teaching and that only teachers need to know" (p. 22). Although specialized content knowledge shows similarities to PCK (Shulman, 1986), they did not include knowledge of students or teaching into their mathematical knowledge package for teaching and both of these elements are mathematical knowledge (Hill, Ball & Schilling, 2008).

Pedagogical knowledge (P or PK) is defined as "deep knowledge about the processes and practices or methods of teaching and learning and how it encompasses, among other things, overall educational purposes, values, and aims" (Mishra & Koehler, 2006, p. 1026). PK includes understanding of how students learn in which levels, how to manage classrooms and time, how to plan a lesson and how to make assessments. 'Teaching and learning' can be used instead of pedagogy, since pedagogy, the term, do not clearly emphasize the 'multiplicity of inputs to teaching and learning.' "Teaching and learning also incorporate the knowledge of curriculum, knowledge of learners, and schools along with pedagogy" (Niess, 2011, p. 301). A deep PK implies comprehensive understanding of how students learn and build knowledge networks (Koehler & Mishra, 2009).

PCK is the knowledge required to be able to decide on what teaching approaches, methods or techniques matches with content and ability to arrange the elements of content for better teaching (Mishra & Koehler, 2006) and which is not a different explanation than that of Shulman's (1986). Moreover, PCK includes knowing about common misconceptions and strategies to prevent those misconceptions, knowing and using alternative teaching methods to present the content while being aware of students' prior knowledge. Briefly, "PCK is concerned with the representation and formulation of concepts, pedagogical techniques, knowledge of what makes concepts difficult or easy to learn, knowledge of students' prior knowledge and theories of epistemology" (Mishra & Koehler, 2006, p. 1027).

Its rapidly changing structure makes defining technology knowledge (T or TK) difficult. In order to avoid being outdated, explanation about TK could be done using the definition of FITness (Fluency of Information Technology) which was offered by

Committee of Information Technology Literacy of the National Research Council (NRC, 1999, as cited in Koehler & Mishra, 2009). Specifically, they claim that FITness is more than computer literacy requiring a deeper understanding information technology than that of in computer literacy. Therefore, from this point of view, TK develops the ability of one person to manage different complex tasks using information technology.

TCK, technological content knowledge, has actually a historical background since technology has been used in order to comprehend the related content deeply, such as in medicine (using X-rays), in archeology (Carbon-14 technique for dating) or in physics. Therefore, technology and content could be linked through education and the definition of TCK becomes an “understanding of the manner in which technology and content influence and constrain one another” (Koehler & Mishra, 2009, p. 63). Having a deep TCK, a teacher must accomplish the subject matter they teach with the application of different technologies and s/he could have the ability to choose the best technology which is applicable to subject matter, or vice versa (Koehler & Mishra, 2009).

TPK, technological pedagogical knowledge, is the understanding of how technology can be adapted to various teaching and learning contexts. Therefore, teacher should have the ability to choose one from a range of technological tools and applications being aware of their constraints and affordances so as to match with pedagogical strategies or designs (Koehler & Mishra, 2009). For instance, TPK includes the knowledge of using technology for everyday teacher works such as grading, taking attendances, class records, or knowledge of technologically practical ideas for special teaching purposes such as using chat rooms, blogs, virtual manipulatives, discussion boards or forums (Mishra & Koehler, 2006).

Technological pedagogical content knowledge (TPACK) is a new knowledge base which combines technology, pedagogy and content knowledge, which are three ‘core’ dimensions, together. Thus, TPACK is defined as “it is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technology; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones" (Mishra & Koehler, 2006, p. 66). Niess (2005, 2006) suggested also an explanation for TPACK by referring back to the knowledge domains and their overlaps and integrations with each other. She considered important that teachers need to understand the mathematics (content), teaching and learning (pedagogy) and technology deeply as well as the integrated knowledge of these domains and their overlap and integration of these domains. What Niess (2005) puts forward is that "TPCK for teaching with technology means that as teachers think about particular mathematics concepts, they are concurrently considering how they might teach the important ideas embodied in the mathematical concepts in such a way that the technology places the concept in a form understandable by their students" (Niess, 2005, p. 196).

Hofer and Grandgenett (2012) summarized the studies regarding the popularity of TPACK through the literature after its introduction, as in the following:

> Early work in TPACK focused primarily on understanding the construct (e.g., Archambault & Barnett, 2010; Koehler & Mishra, 2009; Mishra, Koehler, & Henriksen, 2011) and how TPACK is operationalized in teacher planning (e.g., Harris, Mishra, & Koehler, 2009; Mouza & Wong, 2009) and practice (e.g., Cox & Graham, 2009; Hofer & Swan, 2008). More recently, researchers have begun to focus on specific approaches to helping preservice and inservice teachers develop their TPACK (e.g., Cavin, 2008; C. R. Graham et al., 2009) and on developing, validating, and applying instruments to measure TPACK in a variety of ways (e.g., Hofer & Harris, 2010; Schmidt et al., 2009) (p. 86).

Considering preservice teacher education, three fundamental effortsin order to develop preservice teachers' TPACK were listed as educational technology courses, content-specific teaching methods courses and longitudinal and integrated coursework studies (Hofer & Grandgenett, 2012). In the first category, there was a study which applied pre- and post-tests of the adapted version of Survey of Preservice Teachers' Knowledge of

Teaching and Technology through an educational technology course in order to specify the increase in understanding TPACK (Koh, Chai & Tsai, 2010; as cited in Hofer & Grandgenett, 2012). Cavin (2008) conducted a microteaching lesson study with a group of preservice mathematics teachers in order to develop their TPACK through a course. Another effort in order to assess TPACK is through content-specific teaching methods courses and field experiences courses. For instance, 20 preservice mathematics teachers were studied through a mathematics teaching methods course by using a pre/posttest design with the help of an attitude survey and it was concluded that the idea of integrating technology into lessons was captured as a tool both for support and develop students' conceptual understanding (Özgün-Koca, Meagher & Edwards, 2010; as cited in Hofer & Grandgenett, 2012). There is another study which used multiple data in order to develop TPACK of four preservice teachers through their practice teaching course. Participants were requested to design and implement a series of technology integrated lessons and researchers observed their implementations and gave feedback (Figg & Jaipal, 2009; as cited in Hofer & Grandgenett, 2012). Therefore, the importance of lesson planning and implementation efforts aroused. Third type of effort in order to develop TPACK is the longitudinal and integrated coursework studies. For instance, two preservice mathematics teachers were studied through an educational technology course and a teaching mathematics methods course with the help of several types of data: interviews, lesson plans, notes, videotapes, and their microteaching sessions, and hence, it was concluded that their TPACK was significantly developed (Akkoç, 2011). Niess (2005) investigated the preservice mathematics and science teachers together regarding TPACK development through one year of their teacher preparation program. There were microteaching experiences and full-time student teaching through the study. Niess (2005) concluded that some of the participants could grasp the interconnection between technology and the subject although it was emphasized through their preparation program.

There are also some other approaches used for TPACK development summarized in the literature. Initial attempts were based on self-assessment (Robyler, & Doering, 2010); Mishra, Koehler, Shin, Wolf and DeSchryver (2010) suggested a learning-by-design approach. There are collaborative lesson studies offered, for instance the work by Groth, Spiekler, Bergzer, & Bardzell's (2009), as well as the metacognitive exploration of TPACK emerging from curricula and technological shifts were studied (Hughes, & Scharber, 2008). Alternatively, there are TPACK-based case and action studies have been conducted (Dawson, 2007; Pierson, 2008; Mouza, & Wong, 2009). There are also technology-mapping and peer assessment studies in the literature (Angeli, & Valenides, 2009, as cited in Niess, 2011). Angeli and Valenides (2009) used TPACK framework as a theoretical basis for their introduction of ICT-TPACK.

2.2 Statistical Knowledge and Statistics Education

Statistics teaching could be considered as important and different than mathematics teaching because of already established differentiation between the disciplines of statistics and mathematics. Groth (2007) developed the definition of statistical knowledge by Moore (1988) in his article and he emphasized that knowledge needed for teaching mathematics and knowledge needed for teaching statistics have the similar structure since statistics uses mathematics, and thus he used the knowledge needed for teaching mathematics (Hill, et al., 2008) as the starting point for describing the knowledge for teaching statistics. Groth (2007) used the Guidelines for Assessment and Instruction in Statistics Education, GAISE (Franklin, et al., 2007) framework in order to differentiate teaching statistics and teaching mathematics from each other as explaining them for two types of content knowledge proposed by Hill, et al. (2008) specifically common content knowledge and specialized content knowledge. Therefore, "GAISE framework presents four components of statistical investigation, these are formulating questions, collecting data, analyzing data and interpreting the results" (Groth, 2007, p. 429). He then exemplified each component of statistical investigation for both common and specialized content knowledge. The proposed framework of statistical knowledge for teaching by Groth (2007) can be seen in the following Figure 2.2.

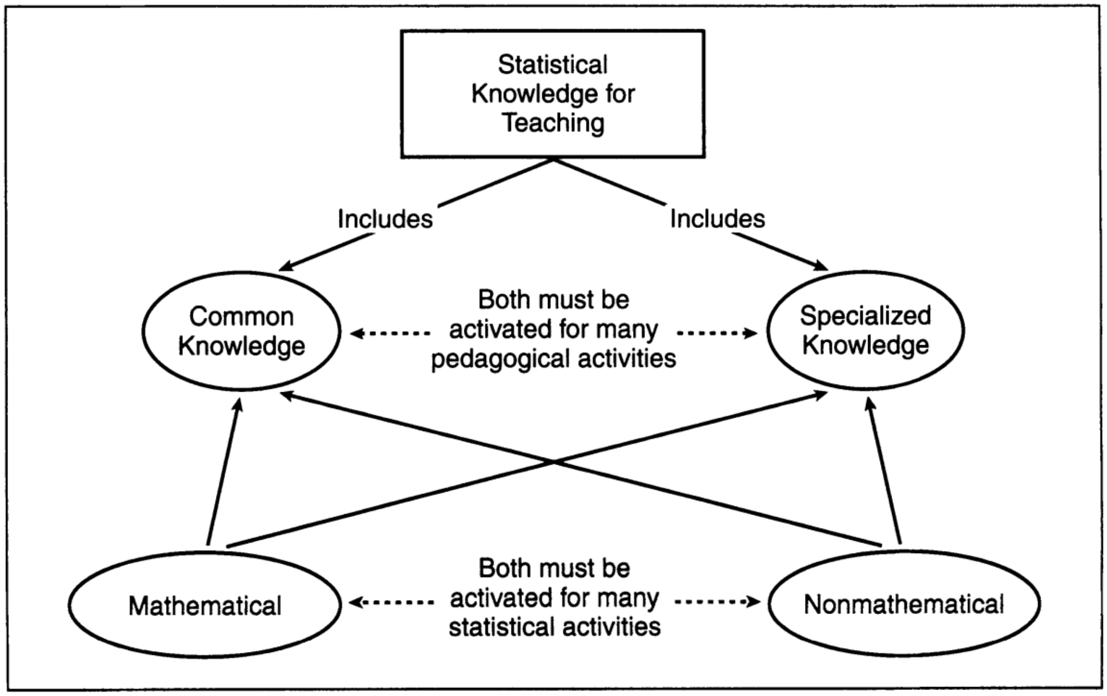

Figure 0-2 Hypothesized structure of statistical knowledge for teaching (Groth, 2007, p. 429).

Groth (2007) summarized also the studies which investigated teachers' knowledge needed for statistics teaching regarding both common and specialized content knowledges. Based on the findings of these studies, it was concluded that there is no need to evaluate common and specialized knowledge separately because of the contextual nature of teaching (Confrey & Makar, 2001; Groth, 2006; Putnam & Borko, 2000, as cited in Groth, 2007).

There was also a need to analyze the statistical thinking and to explain explicitly how practitioners' thought processes occur during data-based enquiry. Wild and Pfannkuch (1999) proposed a 4-dimensional framework for understanding statistical thinking in empirical enquiry in a broader sense. These dimensions are (1) the investigative cycle, (2) types of thinking, (3) the interrogative cycle and (4) dispositions. The suggested model of Wild and Pfannkuch's (1999) was in the following Figure 2.3.

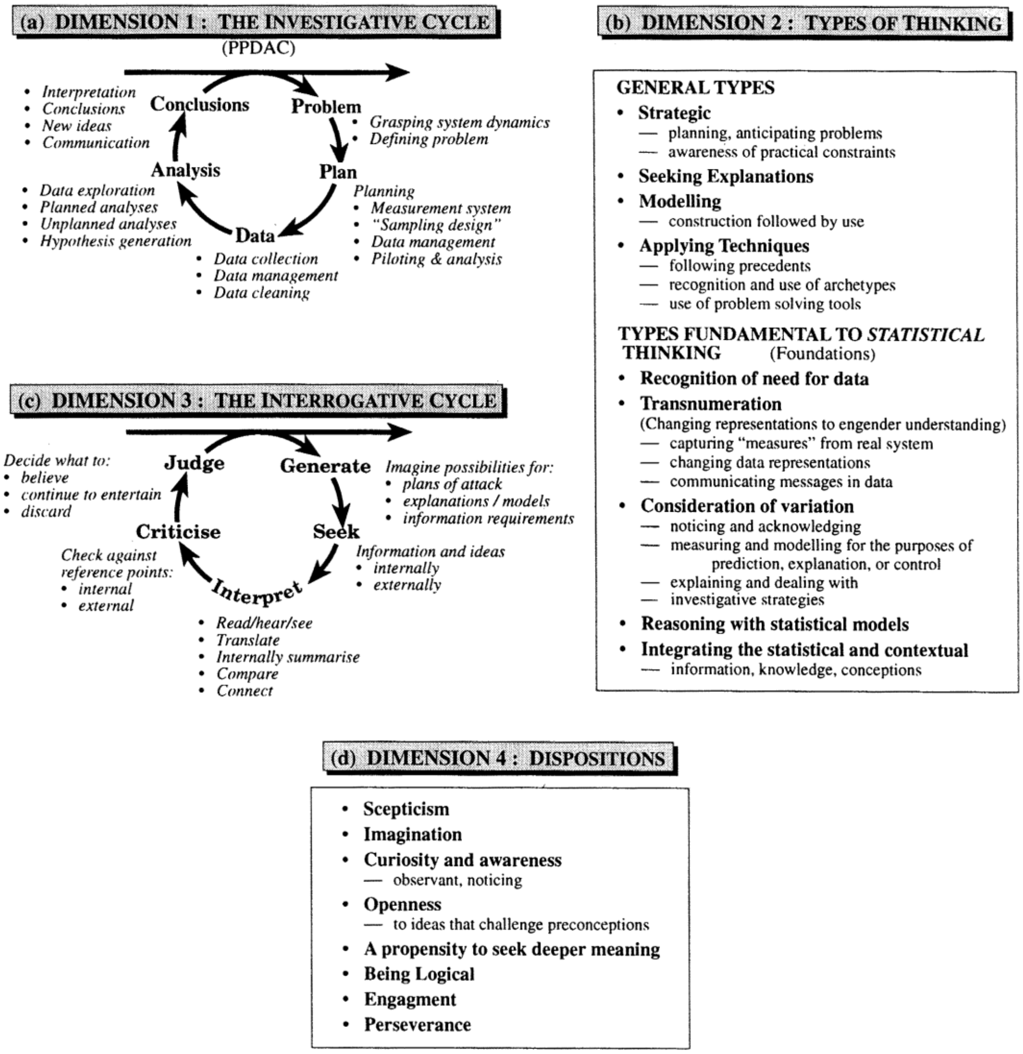

Figure 0-3 Wild & Pfannkuch's (1999) 4-dimensional framework for statistical thinking in empirical enquiry (p. 226).

First dimension, the investigative cycle, deals with the one's actions and thoughts about during a course of statistical investigation and mostly known as PPDAC cycle (first letters of problem, plan, data, analysis and conclusions). Secondarily, types of thinking are several types of thinking strategies. While some of them are specific to statistical thinking, some others are for general thinking, such as strategic, seeking explanations, modelling and applying techniques. Types fundamental to statistical thinking are the recognition of data, transnumeration, variation, reasoning with statistical models and integrating the statistical with the contextual. *Transnumeration* was defined as the ability to represent the data in a variety of different data displays in order to raise the understanding and to interpret the data better for real settings (Wild & Pfannkuch, 1999). Explanation of *variation* could simply be based on the aspect that statistical thinking is dependent on learning and decisions making under uncertainty. It "starts from noticing variation in a real situation, and then influences the strategies [and] continues in the analysis and conclusion stages through determining how one acts in the presence of variation, which may be to either ignore, plan for, or control variation" (Pfannkuch & Wild, 2004, p. 18). For another type of thinking, reasoning with models was proposed with an aggregate-based reasoning rather than an individual-based one. Aggregate-based reasoning focuses on patterns and relationships in a set of data (Pfannkuch & Wild, 2004). Lastly, a synthesis of statistical thinking and contextual thinking could be done in order to interpret the data in its real setting (Wild & Pfannkuch, 1999). Third dimension is the interrogative cycle which is an "interrogative and evaluative process that requires effort to make sense of the problem and the data with the aim of eventually coming to some resolutions about the problem and data during that dialogue" (Pfannkuch & Wild, 2004, p. 41). The dispositions as the last dimension deals with the abilities of the thinker in order to connect relations with other perspectives listed in the Figure 2.3.

The investigative cycle (or known as PPDAC cycle) is mostly studied by statisticians and statistical educators (MacGillivray & Pereira-Mendoza, 2011). First phase, problem-posing, was considered as the most important phase in the cycle since it was concluded that teachers need to learn "to use the driving question to orchestrate a project" (Marx et al., 1994, p. 535, as cited in Makar & Fielding-Wells, 2011). There were studies to teach teachers how to generate effective problems (Allmond & Makar, 2010, as cited in Makar & Fielding-Wells, 2011). Another study showed that teachers could develop the planning phase when their statistical content knowledge increased as well (Fielding-Wells, 2010). Regarding data analysis phase, it was concluded that giving opportunities to create their own data representations to students aroused effective statistical thinking (Cobb, 1999, as cited in Makar & Fielding-Wells, 2011). For the last phase, conclusions, teachers are expected to be capable of using questioning strategy effectively in order to fulfill students' reasoning (Makar & Fielding-Wells, 2011).

Pfannkuch and Wild's (2004) ideas about statistical thinking later developed in order to build a model for teachers' statistical pedagogical knowledge by Burgess (2008) while being emphasized the model offered by Ball, Thames and Phelps (2005) and Hill, Schilling and Ball (2004) in their studies for teachers' needed knowledge for teaching mathematics. The suggested model for teachers' knowledge needed for teaching statistics is in the following Figure 2.4.

<table>
<tr><td colspan="2" rowspan="3"></td><td colspan="4">Statistical knowledge for teaching</td></tr>
<tr><td colspan="2">Content knowledge</td><td colspan="2">Pedagogical content knowledge</td></tr>
<tr><td>Common knowledge of content (CKC)</td><td>Specialized knowledge of content (SKC)</td><td>Knowledge of content and students (KCS)</td><td>Knowledge of content and teaching (KCT)</td></tr>
<tr><td rowspan="4">Thinking</td><td>Transnumeration</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Variation</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Reasoning with models</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Integration of statistical and contextual</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Investigative cycle</td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Interrogative cycle</td><td></td><td></td><td></td><td></td></tr>
</table>

Figure 0-4 Framework suggested by Burgess (2011) for teachers' knowledge needed for teaching statistics (p. 264).

Burgess' (2011) framework was useful to profile teachers' knowledge and to describe the challenges that teachers face during the teaching of investigations since it relates all components of Wild and Pfannkuch's (1999) statistical thinking framework (p. 267). Burgess (2008) characterized this model while emphasizing the differences between two teachers when they were given a task to be benefited considering their classroom practices (Eichler, 2011). In another work, the researcher used this model in order to investigate four teachers' statistical content knowledge needed for teaching and it was concluded that each cell appeared in the model as a different type of knowledge were descrbied as well as which one is needed and used and which one is needed but not used were identified (Burgess, 2007, as cited in Burgess, 2011).

Based on above two emerging frameworks for statistical thinking and teachers' knowledge needed for teaching statistics, the need for modelling statistical pedagogical content knowledge or statistical pedagogical knowledge for teachers aroused. Godino, Batanero, Roa and Wilhelmi (2008) described the five components of pedagogical content knowledge for statistics: (a) *epistemology*, which is the epistemological reflection of the different meanings of concepts, (b) *cognition*, which is the prediction of students' learning difficulties, errors, obstacles, and strategies, (c) *teaching resources and techniques*, which are the effective teaching experiences examples, didactic tools; and critical capacity to analyze some curricular documents, (d) *affect*, which is the ability to deal with students' interest, beliefs, and attitudes, and (e) *interaction*, which is the ability to create good communication in the classroom and to use assessment as a way to guide instruction (p. 1).

Since statistical pedagogical knowledge is being relatively a new issue for statistics education, there is only a study to measure the statistical pedagogical knowledge of 45 very experienced teachers through the Rasch Measurement (Bond & Fox, as cited in Callingham & Watson, 2011). It was concluded that as teachers continued to progress through the scale, they showed increased statistical understanding both for themselves and for their students while their responses were getting more inter-connected and complex (Callingham & Watson, 2011). Statistics education need further investigation of statistical pedagogical knowledge of teachers.

2.2.1 Technological Pedagogical Statistical Knowledge (TPSK)

Rather than describing via different types of knowledge domains and their overlap domains as it was done for TPACK; Lee and Hollebrands (2011) presents TPSK as nested circles as in the Figure 2.5 below:

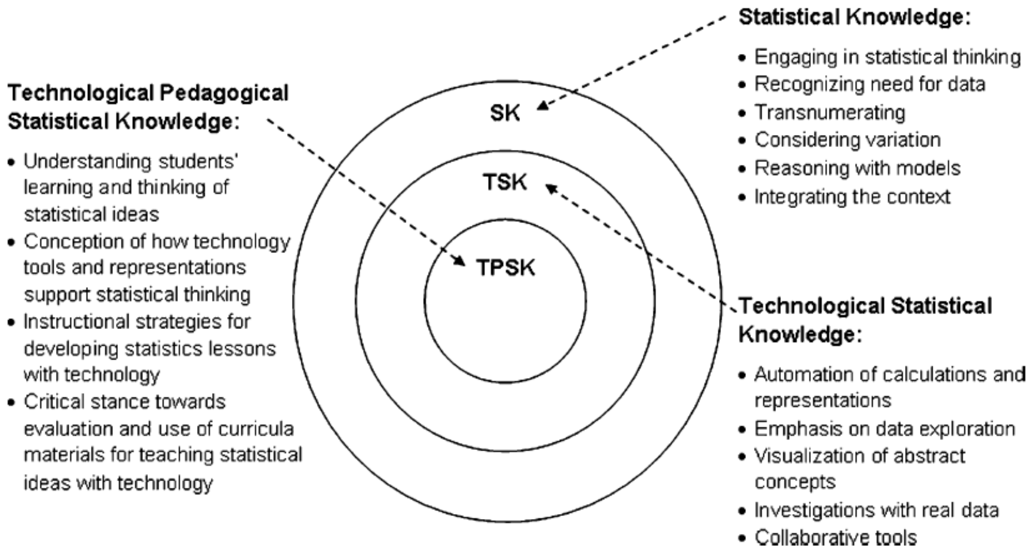

Figure 0-5 TPSK Framework and its knowledge components (Lee & Hollebrands, 2011, p. 362).

The outer circle is statistical knowledge which is the knowledge needed for a teacher to engage in statistical thinking. Their claim is that a teacher should firstly be able to capable of statistical knowledge and thinking abilities before dealing with pedagogy and technology in teaching statistics. Thus, the innermost circle is the TPSK and “founded on and developed with teachers’ knowledge in the outer two sets of technological statistical knowledge (TSK) and statistical knowledge (SK)” (Lee & Hollebrands, 2011, p. 361).

According to many of the researchers, statistical thinking necessitates a different process than mathematical thinking (delMas, 2004; Pfannkuch & Ben-Zvi, 2011; Rossman, et al., 2006, as cited in Lee & Hollebrands, 2011). From the point of view of TPSK framework, in order “to engage in statistical thinking, teachers should recognize the need for properly collected data to examine situations and make decisions, rather than relying on personal experiences or anecdotal evidence. Teachers should then be able to engage in transnumeration (Wild & Pfannkuch, 1999) as a process of transforming a representation between a real system and a statistical system with the intention of engendering understanding (Pfannkuch & Wild, 2004). Thus, teachers should be able to collect measures, represent them meaningfully with graphs and computed statistical measures, and translate their interpretations back to the context" (Lee & Hollebrands, 2011, p. 362).

There are several different technological tools available for dealing with data such as graphing calculators, spreadsheets, statistical packages like SPSS, SAS, Minitab. Some other software like Fathom, Tinkerplots, and Probability Explorer allow users to deal with data dynamically. Internet or online resources such as Java Applets, interactive multimedia systems can be added to this set of technological tools for a statistics lesson (Ben-Zvi, 2000). Ben-Zvi (2000) categorized the technological tools available today as statistical packages (tools), microworlds, tutorials, resources (online course materials, online texts, Java demonstrations, electronic journals and newsletters, electronic discussion lists, data, general links) and teachers' metatools.

Thus, this variability in technological advances necessitates that teachers should have a specialized knowledge about technology particular to statistics. Lee and Hollebrands (2011) used the work of Pea (1987) and Ben-Zvi (2000) in order to provide an explanation for TSK. They appreciate that "technological tools are used in two different ways: to amplify our abilities to solve problems or to reorganize the way we think about problems and their solutions" (Pea, 1987, as cited in Ben-Zvi, 2000, p. 15-16). The five aspects of TSK shown is the TPSK diagram above were adapted from Chance, et al. (2007, as cited in Lee & Hollebrands, 2011).

What Lee & Hollebrands (2011) offered as TPSK is that it is the specialized subset of knowledge for teachers of statistics which encompasses TSK and SK (p. 364). Although pedagogical issues seems underestimated in TSK or SK, they come forward in TPSK.

“TPSK can allow teachers to consider how students think and reason about statistics with and without technology. This implies that they have the specialized content knowledge that Groth (2007, as cited in Lee & Hollebrands, 2011) hypothesized as particular to statistics teachers, and that they are familiar with common ways that students may approach statistical tasks… They should also know how technology can promote different reasoning that may facilitate a transition to aggregate-based thinking” (Lee & Hollebrands, 2011).

In Prodromou’s (2015) study, the researcher revealed the relationships between knowledge dimensions in TPSK framework through their professional development programs about TinkerPlots. It was concluded that teachers needed to be aware of students’ statistical thinking. Besides, Prodromou (2015) claimed that their study presented a way to develop teachers’ TPSK about samples and sampling through the use of TinkerPlots. Another study investigated TPSK framework and TPACK framework in order to better investigate the knowledge components of knowledge needed for statistics teaching through an analysis of course design, namely EarlyStatistics, and resulted in a statistics technological pedagogical content knowledge (STPACK) framework (Serradó-Bayés, Meletiou-Mavrotheris & Paparistodemou, 2014). Through another design-based approach, it was concluded that teachers had the chance to develop and face with the challenges regarding TPSK when they were involved in a technologically-rich environment about the statistical subject of comparing distributions (Madden, 2014). Lee, Kersaint, Harper, Driskell & Leatham (2012) also revealed how 62 prospective teachers use their SK and TSK through problem solving with the help of Fathom and TinkerPlots. The researchers concluded that teachers who have a deeper SK and TSK can plan more effective statistics lessons and generate more beneficial learning environments for the sake of students’ statistical problem solving.

From the TPACK point of view, TPSK seems like the figure below and coincides with the foundations of TPACK and they emphasized the foundations of TPSK lies under the TPACK framework (Lee & Hollebrands, 2011). Instead of content knowledge, statistical knowledge was shown; that is, the content is statistics. Instead of TCK, TSK was specified as technological statistical knowledge. They also insistently did not mention pedagogy since they claimed that every aspect of this framework includes the knowledge needed for teaching and learning and thus pedagogy is mixed in each component of the framework (Lee & Hollebrands, 2011, p. 361). Since, TPACK is designed for technological pedagogical content knowledge for a particular content (Koehler & Mishra, 2005; Niess, 2006), TPACK turns out to be TPSK from the point of view of statistics, obviously, which is very consistent with the TPACK framework, as shown in the Figure 2.6 below.

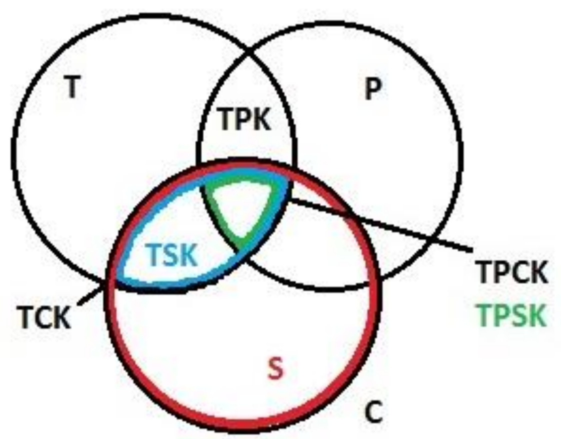

Figure 0-6 Consistency between the frameworks TPACK and TPSK.

2.3 Virtual Manipulatives

From the statistics teaching point of view, virtual manipulatives could be referred to what Ben-Zvi (2009) offered as a technological tool for statistics teaching. He described them as “interactive programs which can be accessed through the Web, based on JAVA programming language, and are simple, self-run demonstrations of statistical concepts”

(p. 148). User could easily enter the data set in order to see the desired demonstration of the previously programmed statistical concept while being connected to Internet.

Virtual manipulatives could be regarded as cognitive technological tools, as well according to Zbiek, Heid, Blume and Dick (2007) since students work with the representations of the objects and the their resulting actions as visually. Moyer-Packenham, et al. (2008) listed some examples of virtual manipulatives as NCTM's Illuminations Web site, Shodor Web site, and the like. The fundamental characteristics of virtual manipulatives summarized by them as "these characteristics include applets that present dynamic electronic: (a) pictorial images only, (b) combined pictorial and numeric images, (c) simulations, and (d) concept tutorials, which include pictorial and numeric images with directions and feedback" (Moyer-Packenham, et al., 2008, p. 2).

Several studies also showed that virtual manipulatives have a significant effect on students' mathematics achievement either they were used alone or with a combination with physical manipulatives (Bolyard, 2006; Moyer, Niezgoda & Stanley, 2005; Reimer & Moyer, 2005; Suh, 2005; Suh & Moyer, 2007; Suh, Moyer, & Heo, 2005, as cited in Moyer-Packenham, et al., 2008). These studies conducted with kindergarten students regarding understanding of the patterns, second grade students regarding their conceptual understanding of regrouping process and third grade students regarding conceptual understanding of fractions. Therefore, it could be concluded that virtual manipulatives are one of the considerable parts of mathematics teaching and learning. Regarding teachers' use of virtual manipulatives in their K-8 grade lessons, Moyer-Packenham, et al. (2008) concluded that teachers preferred to implement virtual manipulatives "when they were central to the *lesson* and to the learning and development of the *mathematics* in the lesson" (p. 14).

2.4 Lesson Study

Originated from Japanese teacher development, lesson study (*jugyou kenkyuu* in Japanese) is an instructional development process where teachers study collaboratively and comment continually on their "research lessons" (*kenkyuu jugyou* in Japanese), where a research lesson is the lesson that teachers designed, planned, discussed and revised by themselves (Lewis, 2000). Lesson study gained a worldwide attention in 1999 and it has been suggested as an effective way "to build professional knowledge base for teaching and to improve teaching and learning" (Stigler and Hiebert 1999; Yoshida 1999, as cited in Lewis, 2009, p. 95).

Lewis (2000) described five characteristics of a lesson study as in the following: First, research lessons are observed by other teachers. This means that observation can be made by the faculty members, or a wider group, even it can be open to all over in Japan. Second, research lessons are planned collaboratively during a long time. Participated teachers work on a specific topic and they try to design an approach of how to teach that topic. Third, research lessons are designed to fulfill a goal of education or achieve a vision of education. This characteristic of research lesson means that the participants in the lesson study group has a wider goal of education or vision beyond the specific subject mentioned in the previous characteristics of a research lesson. These can be, for instance, 'being active problem-solvers' or 'developing individuality' as they are also a part of Japanese national education debate. When the duration of lesson study has been regarded, this property does not seem weird to the participant teachers but in fact they find themselves in developing scientific thinking opportunities. Fourth one is that research lessons are recorded. The recordings can be done audio-taped, video-taped, field notes, observational notes or students' work. Even, other teachers can ask for collecting particular types of data in the school which lesson study conducted. The last one is that research lessons are discussed. All research lessons are planned collaboratively within a group. Then the group members, sometimes with a faculty member or an outside researcher comments on their work together, share their ideas with each other (Lewis, 2000, p. 5-6).

A lesson study process is composed of 4 steps in a cyclic way, as shown in figure 3.1 below: (1) studying curriculum and goals, (2) planning the research lesson collaboratively, (3) implementing research lesson, and (4) reflecting on research lesson.

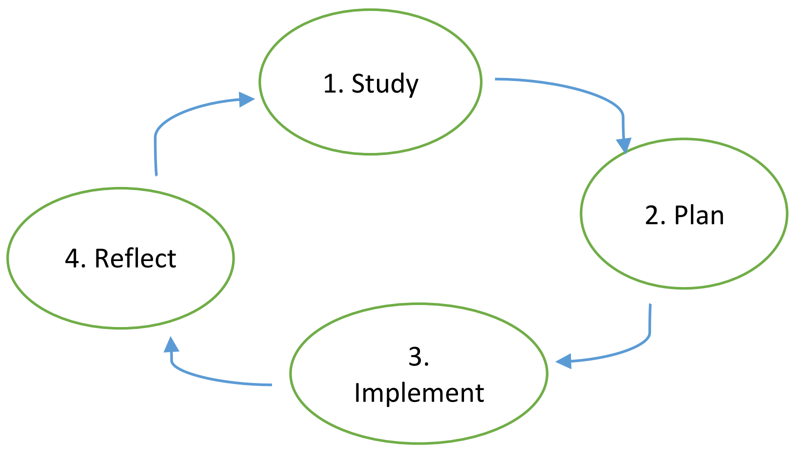

Figure 0-7 Lesson study cycle.

In the *study* phase, participants study on curriculum and objectives and they choose the long-term goals for the learning development of their students. In the *plan* phase, group members plan collaboratively their research lessons including the anticipated students' responses. The *implement* phase means the implementation of research lesson to a group of students, and the last phase includes the reflection on implemented research lesson, the teachers should decide whether on revising the research lesson and implement the revised one. This cycle of research lesson turns till the decision of no more revisions is needed is taken (Lewis, 2009; Fernandez & Yoshida, 2004)

There is an increasing body of work which conducted lesson studies in order to explore teachers' knowledge development and lesson studies were suggested to use in order to enhance teachers' practices (Lewis & Tsuchida, 1997; Stiegler & Hiebert, 1999; Yoshida, 1999, as cited in Fernandez, Cannon & Chokshi, 2003). Apart from these approaches, it was also known that traditional lesson study did not focus originally on technology. However, some researchers included lesson study in their studies in order to explore knowledge development through the integration of educational technology (Cavin, 2007; Chai, Koh, & Tsai, 2010; Groth, Spickler, Bergner, & Bardzell, 2009, as cited in Jones, 2012). The research lessons designed through these studies had a technology component in order to explore teachers' knowledge dimensions regarding technology integration.

2.4.1 Microteaching Lesson Study (MLS)

Although traditional lesson study is actually originated from in-service teacher education, it has also been used in preservice teacher education contexts (Murata and Pothen, 2011; Rock, 2003, as cited in Leavy, 2014). Fernández (2005) defined this type of lesson study as microteaching lesson study (MLS) which is conducted with prospective teachers and implementation takes place as microteaching. MLS is different from usual microteaching, since it enhances collaboration among prospective teachers in order to plan a lesson, revise it and implement it as it was the case for lesson study.

MLS provides preservice teachers with the knowledge development. Leavy (2014) describes the reasoning behind the lesson study with preservice teachers as concerning on content and pedagogical content knowledge, presenting different instructional practice ways, aiming student learning such as procedural and conceptual knowledge. Therefore, lesson study conducted in this study with preservice teachers could be argued as development of knowledge in statistics teaching was aimed and integrated with teachers' practices in order to generate inter-connections among knowledge domains (Leavy, 2014, p. 5). Since the lesson study described here provides the preservice elementary mathematics teachers with the opportunity of development of planning and teaching skills in statistics teaching within the knowledge domains aroused by TPACK framework, the research was conducted through MLS.

MLS efforts was done in order to investigate preservice teachers' PCK initially (Fernández, 2005; Fernandez & Robinson, 2007, as cited in Cavin, 2007). Cavin (2007) conducted MLS in order to evaluate the development of preservice mathematics teachers regarding TPACK and the researcher concluded that teachers developed an awareness of how their lesson planning actions were changing in a student-centered learning environment and when technology was included. However, there was no research to date concerning MLS to explore TPACK of preservice teachers for spefically statistics teaching with technology. That is the main focus of the current study.

CHAPTER 3

METHODOLOGY

3.1 Research Context

This study was conducted with students of Elementary mathematics education program. This program has nearly 40 graduates every year. The education took 4 years (Middle East Technical University, 2014). The program is being part of an education faculty in a university in Ankara, Turkey. This university has a big campus which all other departments of the university are located in it and the language of education is English.

There are nine mathematics courses, courses related with education, courses specific to mathematics education, and Turkish, English and history, namely *Principles of Kemal Atatürk I and II*, courses in this program of elementary mathematics education. In the last year of education, students attend to two practice teaching courses in each semester, as well (Middle East Technical University, 2014).

In the second year of education, students of department of elementary mathematics education took two statistics courses, namely *introduction to statistics and probability 1 and 2,* from department of statistics, which are STAT 201 and STAT 202 courses. Students attend these courses in each semester in their second year, respectively. These courses are the two separate parts of ‘introduction to statistics and probability’ The subtopics which were included in STAT 201 are descriptive statistics, probability, combinatorial methods, conditional probability, random variables, univariate, bivariate distributions, expectation, variance, covariance, correlation, some useful distributions, central limit theorem and estimation. The subtopics which were included in STAT 202 are mainly related with inferential statistics; hypothesis testing, analysis of variance, joint probabilities (several random variables), goodness of fit tests, nonparametric tests and some basics of linear models (Middle East Technical University, 2014).

Another course which is worth to emphasize here is the ELE 329, namely instructional technology and material development course. Student of Elementary Mathematics Education Program attend to this course in their third year of undergraduate education. This course includes characteristics of various instructional technologies, the place and use of technologies in instructional process, development of teaching materials through instructional technologies (worksheets, transparencies, slides, videotapes, computer-based course materials, etc.), and assessment of the qualities of various teaching materials (Middle East Technical University, 2014). At the end of the course, students are basically expected to develop a sense of instructional material development specific to mathematics education.

3.2 Method of Inquiry

The research design of the study was based on the dynamics of the case study approach, which Yin (2014) defined as "an empirical inquiry that investigates a contemporary phenomenon (the 'case') in depth and within its real world context, especially when the boundaries between phenomenon and context may not be clearly evident" (p. 16). Main components of the case study design are apparent in this study which Yin (2014) outlined them as case study's questions; its propositions, if any; its unit(s) of analysis -'case'-; the logic linking the data to the propositions; and, the criteria for interpreting the findings.

Each lesson study group is considered to be a case; since there are two lesson study groups, this study could be evaluated as an *explanatory multiple case-embedded design*. It is not holistic design since the units of analysis used in this study shows variability such as using instruments, interviews and group discussions (Yin, 2014). The data analysis technique used in this study is the logic model which is specified as "it stipulates and operationalizes a complex chain of occurrences or events over an extended period of time [and they are used] to examine a theory of change (that is, a presumed sequence of events as in a neighborhood revitalization process) or to assess an intervention" (Yin, 2014, p. 155).

The analysis was configured through two lesson study groups and each lesson study group was considered as a distinct case of a contemporary phenomenon, which is teaching statistics in elementary education through the use of virtual manipulatives within their real-life contexts.

3.2.1 Microteaching Lesson Study Groups

A major goal of MLS was to help preservice teachers to integrate technology into teaching of statistics concepts. First, groups were informed about the lesson study process and how their artifacts (lesson plans) would be at the end of lesson study. Participants were also said to select an objective from school curriculum regarding statistics and to design their lesson plan according to it. After they were introduced the online resources which can be used in statistics teaching through the workshop (TI-STW), each group started to design their lesson plan. Then, groups separately discussed their decision-making processes throughout their lesson plans with the researcher. This was the first group discussion for each group.

According to their decisions and feedbacks from the researcher, they decided to revise their lesson plans and worked on it on their own. Then, researcher and each group met with each other in order to discuss and critique about lesson plan for a second time. This was the second group discussion. Based on discussion notes and feedbacks, they decided to make revision on their lesson plans for a second time before the implementation.

After revising their lesson plans, they implemented them to a group of students who were their class-mates in about onehour. The implementation was a scaled-down lesson.

In other words, it was like a compressed lesson into one-hour lesson period since both of their lesson plans were two-lesson hours long. Researcher video-taped each implementation in order to comment and critique on them later.

During third group discussions, researcher and members of each group met together lastly, they watched the video-taped lesson implementation and they discussed the flow of the lesson. Each group made self-criticism about their actions, questions, responses to students' questions, activities, and so on. At the end of third group discussions, each group was asked to reach a decision about whether their lesson plan needed further revision or whether they want to re-implement them. Since both groups concluded that their lesson plans needed only minor revisions, they didn't want to implement them again and this was the end of the process.

Basically, MLS was completed in one cycle with one implementation for both groups based on their decisions. Before the TI-STW, each participant was informed about the lesson study process, how they would be involved in and how they would work on their lesson plans. Members of each group met with each other once before first and second group discussions in order to prepare and revise their lesson plans. During the group discussions, they reflected on their self-developments, they criticized themselves, they searched for the online resources, they decided on how they will organize the lesson plan. Thirdly, they implemented their lesson plans as a microteaching and it was video-taped in order to watch it later. Each group determined a member to be the teacher for their lesson plan and researcher arranged a technologically-suitable classroom which their lesson plan necessitated. As the last step, the cycle of lesson study ended with their reflections, critiques and decisions for further revision and implementation through third group discussion which took place with the researcher. Therefore, MLS described mainly above is a good example of lesson study and it could further be labeled as a microteaching lesson study since the participants are prospective mathematics teachers.

3.3 Participants and Groups

There were nine preservice elementary mathematics teachers enrolled in this study. All of them were in their fourth year of undergraduate education. They were from an elementary mathematics education program in one of the universities in Ankara. Two of the preservice teachers were male and the rest were female.

Participants enrolled voluntarily throughout the data collection procedures for this study. At the beginning of the course of Nature of Mathematical Knowledge for Teaching, students, who were all fourth year of their undergraduate education in the program were announced about the current study. More than 10 students became volunteer for the interview as the first step of data collection procedure. Then, only nine of them were interviewed, rest of them were not able to participate to the interview since the researcher and the volunteered students couldn’t reach a conclusion about the time and place in order to conduct the interview. Therefore, 9 participants were involved in all data collection procedures.

Participants’ age were in between 22 and 24, which is normal age for fourth year of undergraduate education. They have some type of teaching experiences before the study, which was summarized in the following Table 3.1 with their ages. Nearly half of the participants had a teaching experience as a community service voluntarily. Almost all of the participants had a tutoring experience which is private teaching to a single student or a group of 2 or 3 students at most. Two of the participants had a teaching experience which could be accepted as a real teaching experience, since they worked for a dersane which is like a semi-structured informal school where students enrolled in order to develop their academic skills for examinations.

Table 3-1 Characteristics of participants regarding age and teaching experience.

Group Name	Name of the participant as abbreviated	Age	Teaching Experience
Group A	Banu	23	teaching voluntarily as a community service for 3 years
	Alp	22	tutoring
	Ezgi	23	-teaching voluntarily as a community service for 4 months -tutoring
	Esen	23	-teaching voluntarily as a community service for 2 years -tutoring
	Gizem	23	-teaching voluntarily as a community service for 2 years -tutoring -practice teaching (as intern teacher in a practice school)
Group B	Emel	22	tutoring
	Serhat	24	tutoring
	Zehra	23	teaching in a real classroom (dersane) for one year
	Şenil	23	teaching in a real classroom (dersane) for 2 months

Regarding the high school type, which participants graduated from, 8 of 9 participants were graduated from an Anatolian teacher high school which is a high school in order to give a preliminary teacher education in high school level, i.e., it can be claimed that this type of school is a preparation phase of enrolling a teacher education program in university level. One participant was graduated from a regular high school in abroad, in a different country.

Five of the participants preferred to become a mathematics teacher deliberately. However, the rest unintentionally started the teacher education program, that is; their first preference for the choice of department in the university entrance examination was not being a teacher or a mathematics teacher at all.

During MLS, all of the participants formed two groups voluntarily, one consisted 4 members, and the other 5. They formed the groups according to the friendship relations among them since they were expected to study together for the weeks for MLS.By the beginning of TI-STW, they started to work with their groups since they would have been planning a lesson at the end of the workshop.

There was a kind relationship between researcher and each participant. The researcher tried to make sure that they can freely ask a question or comment on anything to her. Throughout data collection period and MLS, they were easily connected with each other in any time, for instance, via e-mail, instant messaging services, phone, etc. When they asked a question to the researcher, she tried to answer them as instantly as possible.

3.4 Data Collection Procedures and Tools

Data collection started with the interviews with the participants. At the end of the interview, each participant was asked to respond to the Distinguishing Teacher Assessment in Statistics (DTAS) test. Then, all participants were enrolled into the TI-STW workshop with their fourth year classmates for four weeks. This workshop was integrated into a fourth year undergraduate course named “Nature of Mathematical Knowledge for Teaching”. Although the workshop was not a data collection tool, MLS has been started in its last week regarding the lesson plan preparation and it was audio- and video-recorded. MLS was conducted through lesson study cycle described above through group discussions and lesson plans. Therefore, there are 3 steps in data collection process, namely; interviews, DTAS and MLS. Table 3.2 describes these steps with the dates in detail.

Table 3-2 Schedule for Data Collection Period.

Schedule for Data Collection Period			
Groups and # of participants		Group A (including 5 participants) Group B (including 4 participants)	Date and Duration
Interview		All participants were interviewed before workshop.	October 27th –28th, 2014. Interviews last one week (approximately 1 hour each) and audio-recorded, *transcribed and analyzed*.
Instrument: Distinguishing Teacher Assessment in Statistics (DTAS)		Instrument was administered to all participants during interviews.	DTAS was conducted as a part of interview and *analyzed*.
Workshop: TI-STW		All participants attended workshops together with the class.	October 21st – November 11th, 2014. Workshop last four weeks as a part of one of their undergraduate course in 4th year. It has been both video- and audio-recorded.
Microteaching Lesson Study	Lesson Plans	1 lesson plan was prepared. It started in the fourth week of workshop.	Lesson plans were *analyzed*.
	First Group Discussions (GDs)	Their first draft of lesson plan was discussed.	November 22nd- 27th, 2014. First GDs were audio-recorded, transcribed verbatim, *coded and analyzed*.
	Second Group Discussions (GDs)	Their second draft of lesson plan was discussed.	November 28th - December 9th, 2014. Second GDs were audio-recorded, transcribed verbatim, *coded and analyzed*.
	Implementation of Lesson Plan	They implemented their third draft of lesson plan as a microteaching.	December 9th – 16th, 2014. Implementations were video-recorded.
	Third Group Discussions (GDs)	Their implementation has been criticized and they decided if there is a need for further revision or secondary implementation.	December 12th – 19th, 2014. Third GDs were audio-recorded, transcribed verbatim, *coded and analyzed*.

3.4.1 The Interview

Interview was the initial data collection tool for the current study. All of the participants attended to a one-to-one semi-structured interview. Before conducting the interview, the interview protocol including the questions was prepared by the researcher (Appendix A). The interview for each participant took about 50-60 minutes and all of them were audio-recorded and later transcribed verbatim. Interview was conducted in the second week of the TI-STW workshop, after all of them had already learned about what TPACK is and what its knowledge dimensions are. Each interview composed of two parts, former one inludes the questions which were asked to understand each participant's view towards statistics and statistics teaching overall. In the latter part of the interview, each participant was requested to respond to DTAS, which was mentioned in the next part.

During the first part of the interview, each participant was asked questions about his/her educational background at first. Then, s/he was expected to respond about his/her past teaching experiences and preferences of being an elementary mathematics teacher while they were entering to the university. His/her views towards technology integration into mathematics classes were asked as well. What his/her opinions/views were for teaching via technology, being a teacher in a technologically-rich mathematics classroom, etc. His/her knowledge of technological tools/resources were also asked to the participant. For instance, each participant was asked what technological resources/tools they could use in teaching a statistical subject.

In sum, the interview helped researcher in order to understand what participants' academic background is, what their views towards integrating and using technology in the mathematics classes are. In order to analyze their development in terms of knowledge dimensions aroused from TPACK framework, interview played an important role.

3.4.1.1 Distinguishing Teacher Assessment in Statistics (DTAS)

Distinguishing Teacher Assessment in Statistics (DTAS) test was administered to the participants at the end of the interview (Appendix B). Each participant was expected to respond the questions in interview format since his/her thoughts were understood while s/he was responding to the questions. His/her responses were also audio-recorded and later transcribed verbatim.

DTAS was composed of 8 multiple-choice and 5 open-ended questions in order to assess the content knowledge and pedagogical content knowledge of the preservice elementary mathematics teachers regarding statistics. Twelve of the items in the test were prepared by Bush, Ronau, McGatha and Thompson (2002) and used with their permission. The main aims of their test were: "(1) to describe the breadth and depth of mathematics content knowledge so that researchers and evaluators can determine teacher knowledge growth over time, the effects of particular experiences (courses, professional development) on teachers' knowledge, or relationships among teacher content knowledge, teaching practice, and student performance and (2) to describe elementary school teachers' strengths and weaknesses in mathematics knowledge so that teachers can make appropriate decisions with regard to courses or further professional development" (Center for Research in Mathematics and Science Teacher Development, 2008, para. 1). Only the items related with statistics were included into DTAS from the two tests they sent to the researcher since the original tests included both statistics and probability subjects. The researcher met first with these tests in the Jacobbe's (2007) dissertation. Jacobbe (2007) also generated another item and used it in his test. Then Jacobbe's (2007) item was also included as the last item of DTAS. This item was like an overview of the subjects considered through the previous items.

All of the items were translated to Turkish language, but the English version was also carried with in each interview for the participants. Since all of the participants had a fluent English comprehension because of the language of education in the university, each participant could easily refer back to the English version of the test when they couldn't understand the translated phrases.

Items in DTAS covered the main statistical subjects, namely, (a) measures of center (mean, median, mode), (b) fundamental data displays (bar graph, stem and leaf display, scatterplot, box and whiskers plot, pie graph, line graph, frequency table) and (c) measures of spread (range). The following Table 3.3 shows the statistical subject(s) covered through each item in DTAS.

Table 3-3 Statistical subjects covered through the items in DTAS.

Item Numbers	Item Type	Statistical Subjects Covered through the item
Item 1	Multiple-choice	Measures of center (median)
Item 2	Multiple-choice	Data displays (stem and leaf plot)
Item 3	Multiple-choice	Measures of center (mode, median, mean)
Item 4	Multiple-choice	Data displays (scatterplot, line graph, box and whiskers plot, pie graph)
Item 5	Multiple-choice	Data displays (scatterplot)
Item 6	Open-ended	Data displays (box and whiskers plot)
Item 7	Open-ended	Data displays (line graph, biased graphic displays)
Item 8	Open-ended	Data displays (pie graph)
Item 9	Multiple-choice	Data displays (line graph)
Item 10	Multiple-choice	Data displays (line graph)
Item 11	Multiple-choice	Data displays (frequency table)
Item 12	Open-ended	Data displays (bar graph)
Item 13	Open-ended	Measures of spread (range)

Responding to DTAS by each participant took approximately 20-30 minutes as a part of each interview. Since application of DTAS was the secondpart of the interview, participants were also asked about the subjects covered through the items at the end of the test. Participants' responses to DTAS were considered in planning the workshop later in the study, since the same concepts were covered through TI-STW, but in a more detailed way. Therefore, on the whole, the responses to the items in DTAS were helped to understand what content knowledge and pedagogical content knowledge each participant had regarding statistics.

3.4.2 Technology-Integrated Statistics Teaching Workshop (TI-STW)

To recall participants' background knowledge about statistics and statistics teaching as well as to show the integration of virtual manipulatives as an alternative statistics teaching technique were the aims of the workshop. Participants were not much sure about their statistical content knowledge in order to teach the statistical concepts and they were not aware of the usage of virtual manipulatives as an alternative tool for statistics teaching. Therefore, implementing the workshop in order to achieve to these objectives was valuable for the sake of their statistics teaching capabilities as well as the aim of this study. Since workshop has been planned as a part of a course, not only participants of this study were enrolled but also the other student teachers benefited from it.

Workshop which took 4 weeks (12 hours) was integrated into the course named as 'ELE 465 – Nature of Mathematical Knowledge for Teaching.' This course offered in 4th year to students of the elementary mathematics education program and its main focus was the issues regarding the nature of mathematical knowledge needed in mathematics teaching. Through the course, students were expected to explore the mathematical content knowledge, pedagogical content knowledge and technological pedagogical content knowledge; and to make relations between these knowledge types. As another outcome of the course, preservice teachers should be able to describe the misconceptions that students might encounter and to generate the ways of overcoming these misconceptions. They were also expected to predict probable misconceptions regarding the subjects through the elementary mathematics curriculum. Besides, students develop their procedural understanding as well as their conceptual understanding while making reasoning (Middle East Technical University, 2014).

Regarding the overview of course objectives summarized above, TI-STW started according to the course continuum in 3rd and lastly implemented in 6th week. In the previous weeks, the concepts of ‘knowledge for teaching mathematics’ was covered. Instructor analyzed the mathematical knowledge framework presented by Ball, Thames and Phelps (2008) through their paper. The rest of the course was planned according to subjects covered in curriculum and statistics and probability was selected as the first subject because of the workshop. (The syllabus of this course was attached as Appendix C at the end.) While considering the objectives and aims of this course, integration of the workshop into it was suitable. Throughout the course, students had the opportunity to be involved with TPACK as well as alternative teaching strategies for statistics, which are the essential focus of this study.

In the first week of TI-STW, TPACK was introduced to the students and discussed altogether. At the end of this lesson, a group activity, which asks for examples of new knowledge dimensions aroused from TPACK, was implemented to the class. For instance, students were expected to give an example for a reflection of TPK with its definition. When these questions have been finished, TPACK game (http://www.matt-koehler.com/tpack-game-is-back/) was played in each group after its video-demonstration to the class. Through the game, students were expected to think of the missing content, pedagogical approach or technological approach when the two of them were randomly selected. They started to list the specific subjects, pedagogical approaches and technological approaches, wrote them onto little papers and put them in C (content), P (pedagogy) and T (technology) cups, respectively. Then, they picked, for instance one paper from C-cup and one from P-cup, then tried to find a suitable technological approach to their selected approaches. They also filled the cups with their new findings for C, P and T approaches. The implementation of the activity took one hour and students seemed to enjoy the game.It also helped them to figure out what TPACK framework offers for mathematics teaching and to understand how it distinguishes the knowledge dimensions from each other.

Second week passed with the presentation of the students regarding the main students’ misconceptions and mistakes related with statistics. Management of the lesson was mostly by the instructor and the role of researcher was the observer. Consequently, this lesson might be evaluated as a preliminary step for the next two weeks of workshop.

During the 3rd and 4th weeks of workshops, activities related to statistical concepts were implemented to the students. All of the activities and related instructor sheets were prepared by the researcher according to the fundamental statistical subjects, which were listed in Table 3.4, as well as adapted from other resources (Albert & Rossman, 2001; Rossman & Chance, 2013; Moore & Notz, 2009; Moore, McCabe & Craig, 2009)

Table 3-4 Activity sheets used in TI-STW regarding statistical subjects.

	Number of activity sheet	*Topics covered in each activity sheet*
3rd Week	Activity Sheet 1	Data and Variables
	Activity Sheet 2	Frequency Tables and Bar Graphs
	Activity Sheet 3	Displaying and Describing Distributions
4th Week	Activity Sheet 4	Measures of Center
	Activity Sheet 5	Measures of Spread
	Activity Sheet 6	Comparing Distributions
	Activity Sheet 7	Graphical Displays of Association

By the beginning of the 3rd week, students were sent an online form (named as ‘mini survey for workshop’) to collect data in order to response to the questions asked in activity sheets. Therefore, students used their own set of data in order to follow the activity sheets both in 3rd and 4th week activities during the workshop. These activity sheets and mini survey form is presented in Appendix D and E.

Activity sheet 1 focused on data and variables, specifically student teachers recalled their background knowledge about data, data set and variable types. Both categorical and continuous (measurement) variable types were analyzed through some questions using the data collected through mini survey. Distribution of the variable or variability has been also stressed in the activity sheet 1.

In the second activity sheet, student teachers dealt with the frequency tables, bar graphs and dotplots. The difference of the variable used in a bar graph or in a dotplot was also emphasized and their representations through virtual manipulatives were introduced to them. Student teachers observed how to use some examples of virtual manipulatives in order to draw a bar graph and a dotplot. At the end of this activity sheet, they were also directed to discuss possible pedagogical strategies/ teaching methods while integrating virtual manipulatives. Besides, students discussed the advantages and disadvantages of virtual manipulatives.

Activity sheet 3 focused on the issue of distribution of data. Basic features of data distribution, which are center of the distribution, spread of data and shape of the distribution, have been stressed. Peaks, clusters and outliers have been mentioned. Through the given questions, student teachers were expected to discuss the shape, distribution and variability of the data sets while comparing their dot plots. Stemplots (stem and leaf displays) and histograms were also recalled and related virtual manipulatives were introduced to the class. Using the data collected through mini survey, they drew histogram and stemplot, they saw the relation between a stemplot and a histogram while dealing with a virtual manipulative which shows the transition from stemplot to histogram visually. At the end of the activity sheet there was a discussion related with the differences between a histogram and a bar graph, which recalled their knowledge about variable types and specific characteristics of histogram and bar graph. Besides, they again discussed the possible pedagogical approaches, teaching methods in statistics teaching while implementing these virtual manipulatives introduced through activity sheet 2.

Activity sheet 3 was also the end of the 3rd week of workshop. At the end of each activity sheet there were some homework in order to make them to be involved in virtual manipulatives and to be familiar their usage more. Therefore, student teachers were provided with more activities after 3rd week of the workshop.

Last week was started with the activity sheet 4 which deals with the measures of center. Mean, median and mode were the main issues in this activity sheet. Student teachers dealt with how to find the measures of center of a data set and saw it also using a virtual manipulative. They also distinguished their differences from each other. They discussed in which situations they are applicable with their purposes.

Activity sheet 5 dealt with the measures of spread. They recalled the concepts of quartiles, range, and five-number summary. Nearly all of the student teachers stressed that they have firstly heard about five-number summary, for instance. They recalled the related knowledge by comparing two dotplots of data sets whose mean are the same. Then, students were introduced with the box-and-whiskers plot. Most of the students have seen it for the first time, as they stated during the workshop. They also dealt with the drawing of boxplot while using a virtual manipulative.

Activity sheet 6 was related with comparing distributions. While asking some questions, student teachers were directed to think deeply about the data distribution. They were also introduced with the side-by-side stemplot which shows the two sets of data in one stemplot. They also compared the two boxplots which they drew through virtual manipulative for the two sets of data (see the activity sheet 6 in Appendix D). The goal of this activity sheet was to make prospective teachers to be able to discuss the comparison of distributions while using virtual manipulatives.

In the last activity sheet, graphical displays of association were mentioned, specifically, the concept of scatterplot was recalled. The related virtual manipulative example for drawing a scatterplot was also given and questions asked through activity sheet were responded accordingly.

Therefore, as to summarize all of the activity sheets, the background knowledge of student teachers about fundamental statistical concepts were reminded through these 7 activity sheets prepared by the researcher while synthesizing them with the virtual manipulatives as a new teaching technique for them. At the end of each activity sheet there were small discussions about how the idea of integrating a virtual manipulative to a statistics lesson for the elementary grade level. Student teachers generated ideas and showed their views on a lesson plan which I asked at the end of 4^{th} week.

Each group of prospective teachers (4-5 of them) prepared a lesson plan during the last hour of workshop and they continued to work with them for one week. The two groups who are the participants of this study worked with their lesson plans more, which formed another group of data for this study.

TI-STW has been applied in the same classroom where the course took place. By the third week, researcher organized all students' seats since they were expected to group work. Because of the large number of students (43 students) enrolled in this course and the necessity of computers connected to internet, students were requested to use their own computers (laptops) throughout the third and fourth weeks of TI-STW. Eventually, each group worked with one or two computers.

3.4.3 Lesson Plans

At the end of TI-STW, groups of 4-5 student teachers were asked to prepare a lesson plan which shows an integration of a virtual manipulative for a statistical concept. They could have the opportunity to use the examples of virtual manipulatives shown in TI-STW or they could search the Web for a new one. Through this lesson plan, it was aimed to observe their capabilities for the integration of virtual manipulatives into statistics teaching. These lesson plans were also specified to be one of the course requirements. Therefore, involvement of the classroom was over by the end of TI-STW and first draft of lesson plans.

The two groups (Group A and Group B) which are composed of the participants of this study continued to work on their lesson plans till the end of the study. This was the beginning of MLS centered on this study.

Researcher gave a lesson plan format to the participants and they were asked to follow the path determined before. The lesson plan format is shown in Table 3.5:

Table 3-5 Lesson Plan Format.

Please use the following lesson plan format in order to design a lesson which includes any **statistics** subjects through grades 5-8 while integrating technologies which especially you learned through this workshop in order to teach them.

Lesson Plan Format

Steps	Main Learning Activities	Students' Anticipated Responses	Remarks on Teaching

Explanations / Definitions for Column Headings:

1. "Steps" Column: Give short, general descriptors for each segment of the lesson (e.g., group work, whole-class discussion, etc.)
2. "Main learning activities" column: Describe each segment in more detail, including items like: the problems students will be asked to solve, as well as the activities students are to do during each segment.
3. "Students' anticipated responses" column: Describe how you expect students to react to each of the main learning activities. What will they find easy? What will they find difficult? What will they find interesting? Etc.
4. "Remarks on teaching" column: Provide notes about teacher actions that must be carried out in order to help students succeed with each segment of the lesson. Include things like special instructions given to students, questions you plan to ask, or aspects of the lesson upon which you especially wish to focus students' attention.

This four-column lesson plan format was chosen in order "to draw teachers' attention toward matching instructional activities with students' perceived learning needs and assessing students' progress toward learning goals" (Groth, et al., 2009 p. 395.). A specific lesson plan format provided also with the evaluation of lesson plans in an easier way for both the researcher and the instructors of the course in which TI-STW was included.

Each group prepared a lesson plan in the expected format and revised it two times before the implementation, based on their decisions made through group discussions.

3.4.4 Group Discussions (GDs)

When the last week of TI-STW was completed, each MLS group started to study as separate from the course which TI-STW was included. Therefore, the researcher and each group made an appointment for each group discussion (GD) beforehand.

GDs were a part of MLS as a necessity of conducting a lesson study. Because of having two lesson study groups in this study, there are three consecutive GDs for each group. First two GDs were made according to their lesson plans, but the last one were conducted through the implementation of their lesson plans. The continuum of the GDs for each group was presented above in the Table 3.2 which shows the data collection period.

Each GD lasted 1 to 1.5 hours and each was audio-recorded and later transcribed verbatim. GDs were conducted in order to understand the participants' decisions during planning and implementing the lesson. Therefore, the structure of these GDs was dependent to their lesson plans and their implementations. In each GD, the role of the researcher was the moderator of the discussion which took place and the guide for the aroused questions. Researcher directed questions to the participants in order to determine their aims/goals in their lesson plans.

Because of the nature of the lesson study, researcher also played a guide role for the group members during the GDs. Group members sometimes requested directions from the researcher throughout planning their lessons. Researcher took field notes during the group discussions as well regarding their further actions or changing decisions about the lesson plans.

Based on the results of our discussions and their changing design strategies in lesson planning they decided to revise their lesson plans twice before its implementation. Since the study is a MLS and implementation would take place as microteaching, they didn't want to implement their lesson plans twice to the same audience. Therefore, researcher and the group members met with each other for another GD for a second time before the implementation. Consequently, each group revised their lesson plans twice and discussed them twice before the implementation.

Third GDs passed as dependent to the implementation. During third discussions, researcher and the group members were also watching the video-record of the implementation. When the researcher had a question about a moment of the lesson, video was stopped, group responded to that question, and then continued to watch again. This process lasted till the end of the video-record of the implementation. After they watched the video, researcher asked the question of 'do you want to revise your lesson plan, and do you wish to re-implement it after this revision?' Both groups claimed that there was no need to any further major revision, therefore there was no need to re-implement it. Eventually, this decision has stopped the lesson study cycle and generated the last GD for each group.

The transcriptions of all GDs were coded using the TPACK codebook and they constituted the main part of the data with the lesson plans.

3.5 Data Analysis

Data analysis of the study was performed qualitatively according to the nature of case study regarding the lesson study requirements. Throughout the data collection period; interviews, group discussions, lesson plans and the results of DTAS were analyzed, together in order for the validating credibility of the study.

For the data collected through interviews, codes were derived from the initial categories aroused through the interview questions. These themes were high school type, preference of being a mathematics teacher, teaching experience, views towards technology and technology integration into teaching, views towards statistics and statistics teaching. All codes were formed according to all possible answers which participants could give during the interviews and organized under these categories. Generated form of themes and codes for the interview can be seen in the Appendix F.

Secondarily, TPACK codebook (Hughes, 2012) was realized as suitable in terms of the theoretical background framed in this study (attached as Appendix G). Since TPACK codebook was derived from the TPACK framework considering all knowledge dimensions, all codes were used to analyze the data collected for this study without further revision. Each knowledge dimension was treated as a separate theme and the corresponding codes were determined through the explanatory specifications regarding each theme. For instance, the code of 'knowledge about the ways in which content and technology reciprocally related to one another' was specified as a determinant for the TCK theme. Therefore, the data gathered through group discussions in MLS was analyzed in a similar way. During the coding phase, researcher took the most emphasized theme into account. For instance, if group members mentioned that they preferred to use questioning strategy as a pedagogical approach, then such phrase was coded as belonging to PK theme. However, if they stated that preferring questioning strategy is a better approach while dealing with virtual manipulatives, then such phrase was coded as belonging to TPK theme. During coding continuous long discussions, sometimes it was hard to determine the themes to which chunks of phrases belong; but it was decided based on the negotiations among the researcher and the second coder simultaneously that, the most emphasized knowledge dimension would be coded.

After the consistency has been reached among the researcher and second coder, coding phase was finished by both of them and they compared their codes whether they referred to the same phrases of the participants. This effort was made for all of the GDs in the chronological order.

Since TPACK codebook presents an organized structure to analyze the TPACK framework considering each knowledge dimension separately, findings were summarized according to this structure throughout the findings chapter.

3.6 Quality of the Study

Being consciously aware of the philosophical approaches underlying the qualitative inquiry, some alternative judging criteria summarized by Patton (2002) were selected through the use of *angles of vision* (Peshkin, 2001) in order to address the quality of this study.

Objectivity of the inquirer is one of the validating techniques of data collected from the participants through interviews and group discussions (Patton, 2002). Researcher asked the questions directly to the participants. She didn't direct them to any desired or expected response or she never judged their responses. Participants were felt to be relaxed and comfortable for both physically and mentally. Therefore, they could tell their opinions and ideas without feeling under pressure in all meetings. At the beginning of each discussion and interview, participants were told to request for an interruption if they desired, as well. Before the interviews, participants were also presented a consent form (attached as Appendix H) if they accept to be a participant of this study. This consent form included also the main aims of the study, what the expectations would be from the participants. All of the participants were informed as written in the consent form and as orally by the researcher that audio-records will never be shared with anyone other than the research team.

Another effort used in this study in order for validating data is known as *peer review* (Creswell, 2007). After data was coded by the researcher, a second coder coded the data. However, this effort was applied only into the data analysis phase of the study. Second researcher belonged to the research team of this study compensated the previous effort through *peer debriefing sessions* (Lincoln & Guba, 1985). He monitored the researcher all the time during data collection, data analysis and interpreting the findings of the study. They always were in a close negotiation between each other in terms of experiences regarding collecting the qualitative data, organization of the findings and the design of the study.

While transcribing the audio-taped data, researcher tried to write also the participants' gestures such as hesitations, stops, smiles, and so on, into words before the coding phase. This effort provided both the researcher and the second coder with a data as objective as possible while coding.

Second coder was a faculty member with a doctoral degree in mathematics education working at another university. She could be accepted as quite experienced in qualitative inquiry and she is both a researcher and an instructor in her department. Before she coded the data herself, researcher and the second coder communicated with each other in order to evaluate their understandings from TPACK codebook and to maintain a similar infusion from a small part of data. Then, they coded the rest of the data separately. They used the same codebook for all data, and consequently the objectivity was assumed to be provided through the coded transcriptions. At the end of this effort, both the researcher and the second coder were observed to derive the similar decisions throughout the data coding. Therefore, on the whole, data collection and analysis of this study serves objectivity in a certain level.

Researcher's effort to collect data about the participants' academic background knowledge, their demographic information and other information provided a *thick description* in order to present an objective research context in order for validating the data collected. At the beginning of the interviews, each participant were asked about their preferences/views towards becoming a teacher/mathematics teacher or asked about their hesitations regarding teaching mathematical or statistical subjects. Main aim in these questions was to establish a clear understanding about their perceptions and also to have a close relationship in order to maintain the nature of case study. Describing the

MLS period in detail strengthens the quality of the study in terms of lesson study necessities, which can be considered a part of thick description as well.

Through the all data collection phases (interviews, group discussions, DTAS, lesson plans), researcher tried her best to maintain the *reflexivity* while being conscious about all alternating views of participants towards the issues considered in this study (Patton, 2002). Researcher also appreciated those different views of participants and tried to be as fair as possible in terms of relationships with each participant. Considering having two different cases (two different MLS groups), researcher also tried to be fair towards both of them. Since reflexivity also necessitates having an awareness of 'biases, values, experiences' which the researcher brought into the study, she tried to reflect all of these distinguishing views of participants while interpreting the findings (Creswell, 2007, p. 243).

Triangulation of data sources used in order to maintain the *trustworthiness* of the study. Data collected through interviews, group discussions, DTAS and lesson plans were analyzed together in order to maintain the consistency. What the participants said during the interview was observed in the results of DTAS; and those results were also captured through the GDs during MLS groups (Patton, 2002; Creswell, 2007).

Expert audit review was another effort in order to establish the *credibility* of the study since the second researcher, who was a member of research team, reviewed the study and directed the researcher in data collection and data analysis phases (Guba, 1981; Patton, 2002).

Therefore, on the whole, it could be claimed that the study laid on strong grounds regarding the credibility issues specified in detail above.

3.7 Limitations of the Study

Because of the nature of lesson study or microteaching lesson study, a unique setting was studied as it was described above. Therefore, the transferability of the findings could have some limitations.

Throughout the steps of lesson study cycle, preservice teachers who participated in this study have a direct relationship with the researcher, i.e., they could directly got feedbacks, comments and similar response from the researcher during GDs. Therefore, one might think that the increase in their understanding of TPACK regarding technology integration could be a result of pleasing the researcher for their progress. Besides, it might be thought that participants could be encouraged to show an increase in their instructional capabilities during the lesson study groups. Although the researcher explained clearly to them that they should use their knowledge for teaching a statistical subject, participants were free to choose which suggestions or comments they will consider most while they were planning their lessons.

In addition, since the findings of the study were based on group performance, one might think that the findings could not be inferred directly as their individual improvement. However, their research lesson was their collaborative product, and members of each group have always worked together in order to design and plan it.

Since participants were all told to study on a lesson plan in order to teach a specific statistical subject, one might also feel that they behaved more cautious since they were aware of their weaknesses about teaching statistics, especially integrating technology. On the other hand, the implementation of DTAS was before lesson study period began and researcher have already an idea of what their statistical understanding was according to DTAS results.

Dealing with only virtual manipulatives as a way for integrating technology in statistics teaching could lead one to think it as a limitation for the study. Basically, the participants described their inadequacies about several technological tools which can be used in statistics teaching during their interviews. Excel was the only offered technology for statistics teaching, since they were not aware of any specific technological tools to statistics teaching, such as software, virtual manipulatives, etc. Therefore, virtual manipulatives became not only a simplistic but also a productive start for the increase in their capabilities in the use of technology in statistics teaching.

CHAPTER 4

FINDINGS

This chapter presents the findings of the study. The findings are organized as two case studies and each case is a lesson study group. Findings were organized according to data collection period in chronological order which were interviews initially, DTAS results secondarily and, the group discussions aroused from MLS. Then, each part first describes the findings of Group A which included 5 preservice mathematics teachers and secondly Group B which was composed of 4 of them, regarding the following research questions respectively:

a. What views do preservice middle school mathematics teachers have towards (a) statistics teaching, and (b) towards technology integration into statistics teaching (virtual manipulatives, simulations, and the like) at the beginning of the microteaching lesson study?
b. What content knowledge and pedagogical content knowledge did preservice middle school mathematics teachers have regarding statistics and statistics teaching at the beginning of the microteaching lesson study?
c. How do preservice middle school mathematics teachers' knowledge dimensions change regarding TPACK framework through microteaching lesson study?
    i. What TPACK do preservice middle school mathematics teachers have while teaching statistics via virtual manipulatives?

ii. To what extent does the microteaching lesson study affect these changes in knowledge dimensions aroused from TPACK framework of preservice middle school mathematics teachers?

All of the nine participants have been interviewed at the beginning of the study and they formed two groups for lesson study process (5 participants were in Group A and 4 in Group B). Both groups (groups A and B) completed the planned lesson study process, they prepared their lesson plans, discussed with the researcher, revised it, discussed it secondarily as a group again, then implemented it, and lastly, discussed their lesson plans with the researcher in order identify if there is a need for any further revision and secondary implementation of it. Therefore, on the whole, three GDs and three drafts of lesson plan for each group have been obtained during the lesson study process.

4.1 Findings based on the interview including DTAS

This part summarizes the findings based on interviews of each participant respectively regarding the first research question. The views of each preservice middle school mathematics teacher towards statistics teaching and towards technology and use of technology in teaching/statistics teaching were outlined.

Nine participants were interviewed in the second week of the workshop, and there were two periods in each interview. In the first part, they were asked questions related mainly with the reason of their choice of being mathematics teacher, how much teaching experience they had, views towards technology, statistics and statistics teaching, and technology use in statistics teaching. They also wanted to make a self-critique for themselves regarding content knowledge, pedagogical knowledge and technology knowledge during the interviews. Second part of the interview was passed with the DTAS regarding their content knowledge and pedagogical content knowledge. Below, the main findings based on interviews are presented with the results of DTAS for each participant. In order to make the chapter more comprehensible, interviews and DTAS results were organized with respect to the lesson study groups which they studied later through.

4.1.1 Findings based on the interview

Eight of nine participants were graduated from an Anatolian teacher high school before the university and one participant was graduated from a high school which was abroad. Five of the participants preferred to become a mathematics teacher deliberately. However, the rest unintentionally started the teacher education program, that is; their first preference was not being a teacher or mathematics teacher. Except one, all participants had previously practiced teaching in their university education: 5 of them did tutoring, 4 of them had teaching practice voluntarily as a community service, and 3 of the participants also mentioned that they had a chance to practice teaching in a real class (private school or dersane). Most of them have been experiencing their teaching practice for the last one or two years. Therefore, it could be claimed that almost all of the participants had a chance to practice teaching either in a small group of students or in a real classroom.

Six of the participants had mentioned that they could have some inadequacies in statistics, especially. Two participants were thinking that they were capable enough to handle statistical concepts very well. Almost all of the participants specified that they didn’t have a chance to teach a statistics subject in their teaching experiences, or they said that they were not asked about statistics at all from their students. Only one participant has stressed that she has already prepared a lesson plan which was a requirement of a lesson as a part of university education. However, she said that she had no chance to implement it. Most of the participants claimed also that there was small amount of consideration for statistics teaching or statistics teaching methods in the courses which they took in the past, particularly the methods courses. However, they claimed that it was not enough to learn well.

Concerning their views towards technology or technology integration to teaching of mathematics, there were different responses from participants in their interviews. More than half of the participants (5 of 9) had a wish to integrate technology in their future teaching. They stated also that they would try to use as much technology as possible if they had a chance to integrate, i.e., the infrastructure of classrooms, etc. They (5 of 9) also claimed that they would try to develop themselves in technology integration into their lessons. Some of them (3 of 9) also claimed that technology gives alternative visual representation opportunities in teaching. One participant mentioned about advantages of using technology in saving time on certain classroom tasks such as using a smartboard instead of regular blackboard in a lesson saves time. Another one claimed that technology makes teaching easier. However, there were also comments that were critical about technology integration. Two participants especially stated that technology is needed but, not as important as content or pedagogy. In other words, they claimed that technological capabilities of a teacher are necessary, but these capabilities are not as important as content or pedagogical skills. One of these two participants were hesitant about the advantages of technology use. She claimed that technology can be terrifying for herself especially when considering social media. She thinks that widespread use of tablets, smart phones, etc. of the teenagers gives harm to the students. She argued that those students let themselves get carried away by tablet or pc games, and as a result, technology is becoming an obstacle to develop themselves and they couldn’t learn how to produce something new. Finally, she argued that technology, in fact, should allow students to be more productive. Her responses towards technology was quite different than other participants.

Above findings were the most prominent ones aroused from interviews concerning the research questions of the study. In the following part, DTAS results were outlined.

4.1.2 Findings based on DTAS

DTAS results of the groups A and B regarding each participant were investigated through the summary showed in the following tables, Table 4.1 and Table 4.2, respectively.

Table 0-1 DTAS results of Group A members in terms of items.

Group A/ Item Numbers	Ezgi	Banu	Alp	Esen	Gizem
Item1 Median	+	+	+	+	+
Item2 Stem and leaf display	-	+	+	+	+
Item3 Mode, median, mean	-	+	+	+	+
Item4 Data displays	-	-	-	-	-
Item5 Scatterplot	-	-	+	-	-
Item6 Box and whiskers plot	+	+	+	+	Partial
Item7 Biased graphic displays	Partial	Partial	Partial	Partial	+
Item8 Pie graph	+	+	+	+	+
Item9 Line graph definition	-	-	+	-	-
Item10 Line graph	+	+	+	+	+
Item11 Frequency Table	+	+	+	+	+
Item12 Bar graph	-	-	+	+	+
Item13 Range	+	Partial	+	+	Partial

'+' means correct response, '-' means wrong response, 'Partial' means partially correct response to the open-ended items. 'no response' means participant had no response for the item at all. Items 6, 7, 8, 12 and 13 are open-ended items, the rest are multiple-choice items.

Based on the results of these two tables, it could be claimed that participants were successful in responding to the multiple-choice items in DTAS, in general. The most problematic item was fourth item which almost all participants (8 of 9) answered as wrong. The item was asking the best data display type for the distribution of achievement test scores of fourth grade students in a school. They couldn't choose box-and-whiskers plot as best data display among other alternatives, since they stated that they couldn't remember it very well. Second problematic item was asking the definition of a line graph and 3 of 9 participants could respond as correct. The correct response expected from the participats was ‘a graph with a vertical and horizontal axis that is primarily used to show changes over time’ Next item was also about line graphs. It was asking the average difference of the salaries given to two different groups of people for a period of time showed in line graphs, and all participants responded correctly. There were also some incorrect responses to second item, which was related with stem-and-leaf-plots. It could be claimed that preservice teachers in this studyhad lack of knowledge related with stem-and-leaf-plots. Thirdly, the scatterplot item was again responded correct by nearly half of the participants (4 of 9). They mostly couldn’t understand the item since they analyzed only the showed values in the scatterplot, although they were expected to predict a non-given value in order to find the correct response.

Table 0-2 DTAS results of Group B members in terms of items.

Group B/ Item Numbers	Emel	Serhat	Şenil	Zehra
Item1 Median	+	+	+	+
Item2 Stem and leaf display	-	+	-	+
Item3 Mode, median, mean	+	+	+	+
Item4 Data displays	-	-	-	+
Item5 Scatterplot	-	-	+	-
Item6 Box and whiskers plot	+	No response	+	+
Item7 Biased graphic displays	+	Partial	Partial	+
Item8 Pie graph	+	Partial	+	+
Item9 Line graph definition	-	+	-	+
Item10 Line graph	+	+	+	+
Item11 Frequency Table	+	+	+	+
Item12 Bar graph	+	-	-	+
Item13 Range	+	+	+	+

‘+’ means correct response, ‘-‘ means wrong response, ‘Partial’ means partially correct response to the open-ended items. ‘no response’ means participant had no response for the item at all. Items 6, 7, 8, 12 and 13 are open-ended items, the rest are multiple-choice items.

The following Table 4.3 and Table 4.4 summarizes the responses of the participants to each open-ended item in detail. Most problematic one was related with bar graph display. The item shows a line graph of a data set of categorical variable (transportation way of students in a school, categories are by bus, by car or by walking) and participants were asked how teacher should give feedback to the student which drew such a graph for this data set. Nearly half of the participants (4 of 9) responded that student brought such a homework was right in selecting the line graph, even one participant claimed that student should select histogram instead of line graph for such a data set.

Table 0-3 Responses of Group A participants to open-ended items in DTAS in detail.

Group A/ Item numbers	Item6 Box and whiskers plot	Item7 Biased graphic displays	Item8 Pie graph	Item12 Bar graph	Item13 Range
Ezgi	Correct response	Partial response (understood the difference, but said that graph 1 is more biased)	Correct response	Wrong response (said that line graph is true but histogram is more suitable)	Correct response
Banu	Correct response	Partial response (understood the difference, but said that graph 1 is more biased)	Correct response	Wrong response (said that line graph is true)	Partial response (selected 140 for x and couldn’t explain its reason in a logical way)
Alp	Correct response	Partial response (understood the difference but couldn’t say which is more biased, then selected graph 1 as more biased)	Correct response	Correct response	Correct response
Esen	Correct response	Partial response (understood the difference, but said that graph 1 is more biased)	Correct response	Correct response	Correct response
Gizem	Partial response (said that the most successful class is class 2 and couldn’t read the graph)	Correct response	Correct response	Correct response	Partial response (selected 140 for x and couldn’t explain its reason in a logical way, or said so because of not understanding the target line)

*wrong response: completely irrelevant response
*no response: participant does not answer
*correct response: completely correct response
*partial response: participant’s answer is partially correct, and gave also wrong explanations.

Another problematic item was the seventh item which 6 of 9 participants responded partial. This item was about biased graphic displays: there were two line graphs showing the same set of data. The increasing trend was seen as more sharply in one of them than the one in the other graph since the intervals were different in these two graphs. Then, the questions were: (a) How are the graphs different? (b) How could Graph 1 be used in an argument? (c) How could Graph 2 be used in an argument? The participants who responded as partial understood the difference between the two line graphs. However, they chose the first one (which uses larger interval in y-axes) as biased. They explained further that differences among values in the data set couldn't be recognized because of the large interval, so they couldn't determine each value. Therefore, it could be claimed that they had some inadequacies about what can be seen/expected/understood from a data display. In other words, they couldn't read the graph.

Table 0-4 Responses of Group B participants to open-ended items in DTAS in detail.

Group B / Item numbers	**Item6 Box and whiskers plot**	**Item7 Biased graphic displays**	**Item8 Pie graph**	**Item12 Bar graph**	**Item13 Range**
Emel	Correct response	Correct response	Correct response	Correct response	Correct response
Serhat	No response	Partial response (understood the difference, but said that graph 1 is more biased)	Partial response (not enough explained how he would teach)	Wrong response (said that the student gave exactly the correct response)	Correct response
Şenil	Correct response	Partial response (understood the difference, but said that graph 1 is more biased)	Correct response	Wrong response (said that line graph is correct)	Correct response
Zehra	Correct response	Correct response (understood the difference, but instead of selecting one of them, said that they should be evaluated in different contexts)	Correct response	Correct response	Correct response (mixing about x a little, but could be accepted as correct)

*wrong response: completely irrelevant response
*no response: participant does not answer
*correct response: completely correct response
*partial response: participant's answer is partially correct, and gave also wrong explanations.

The results for sixth item is also worth to analyze that participants seemed successful in responding such a question related with box-and-whiskers plot. Above it was claimed that participants couldn’t remember the necessary knowledge about boxplot. When they saw the boxplot showed in this question, they tried to activate their background knowledge and responded mostly as correct. Most of them didn’t remember the important points in a boxplot. Even some of them thought the numbers written below as the number of students.

The last item in DTAS was mainly about range concept and only two of the participants provided partially accurate response. They couldn’t make a reasonable explanation for the right answer although they chose it as the predicted value for the distance.

Therefore, on the whole, it could be claimed that participants lacked necessary subject matter knowledge and pedagogical content knowledge regarding especially data displays, which are bar graph, line graph and boxplot. They needed to refresh their prior knowledge about data displays, i.e., they needed to revise how to deal with a data display, what should be expected to see from a data display, what information can be grasped through a data display, and so on.

4.2 Findings based on group discussions and lesson plans

This part presents the group discussions of each group as a case and they comprise the main data of this study with the lesson plans. The lesson study process is maintained here and three group discussions for each group (two of them before the implementation and the one is after implementation) is analyzed here regarding the second research question herewith the lesson plans which each group prepared, discussed and commented onto. Sequentially, first, second and third group discussion of each group is summarized herewith the discussed lesson plan below.

4.2.1 Findings based on initial group discussions (GD)

This part will summarize the findings based on first group discussion regarding the knowledge domains which characterizes the TPACK framework, namely, content knowledge (CK), pedagogical knowledge (PK), technological knowledge (TK), pedagogical content knowledge (PCK), technological pedagogical knowledge (TPK), and technological content knowledge (TCK) for each group respectively herewith the first draft of lesson plan they prepared.

4.2.1.1 Initial GD of Group A (including 5 preservice elementary mathematics teachers)

In this part, the main findings of the initial group discussion of Group A regarding the lesson plan they prepared as first draft (attached as Appendix I) were summarized. Their lesson plan were about teaching the histogram concept. Their plans began by activating prior knowledge of students about bar graph. At first, they present bar graph with all of its characteristics and later they pass onto stem and leaf display and histogram, respectively. For all data displays, they used the same set of data.

4.2.1.1.1 Content Knowledge (CK)

The main findings from the initial GD of Group A were: (a) initial GD presented a effective negotiations which implies that their content knowledge about the *type of variables* used in data displays (especially in bar graph) is weak; (b) they could not show enough understanding of data displays regarding their fundemantal differences, especially *differences between a bar graph and a dotplot*; (c) they could not choose the *right example for a bar graph*. Some parts of conversations are presented as examples for these findings as below.

Group A tried to teach the histogram concept while beginning with the bar graph concept, they firstly planned to give a set of data to the students in order to draw a bar graph. The data was the scores of 16 students taken from a course as 22, 24, 15, 10, 17, 30, 29, 15, 10, 30, 15, 22, 17, 24, 11, and 15. They wanted from their students to draw a bar graph for this data set. They claim that the scores of the students in this data set is both a categorical and a quantitative variable at the same time, then they suppose that this data can be drawn as a bar graph or a stem and leaf display which resulted as the first major finding which their inadequate knowledge about the types of variables used in data displays. The following conversation exemplifies the conversation about type of variable through initial GD of Group A:

> Researcher: […] Will he be able to draw this with the bar graph? In this situation?
> Esen: We think he would, considering he already has a knowlage about it, in advance.
> Ezgi: We hoped he would draw it.
> Researcher: What my point is; are these notes categorical data or continuous data? We thought we'd say something different to 22, 19, 24 which finally will lead to 1 1 1, I mean we thought about calling it apple and pear, fruits that everyone loves…
> Banu: It means numbers are the categories.
> Gizem: Then we shouldn't have connected apple and pear with stem and leaf display on the next transition. It should always be seperated. It would also take too long and kids would be in both. We choose a data which is both continuous and categorical. Also with this way children won't get tired.
> Ezgi: Yes, we meant to use both.

In their plans, after showing the data in the bar graph, they intended to show the same set of data as a stem and leaf display to their students as the second part of their research lesson. Their aim was to make a transition from bar graph to stem and leaf display, later a second transition from stem and leaf display to histogram. However, during the conversation, they realized that the transition from bar graph to stem and leaf display seemed different than they thought, because of their lack of knowledge about the variable type used in bar graphs, again. Gizem realized that if a transition from bar graph to stem and leaf display is possible, then the reverse could be possible, too, as can be seen from below conversation:

Gizem: I'm confused when the stem and leaf display is turned clockwise. It doesn't create a bar graph?
Researcher: No, it doesn't.
Gizem: I mean, I wouldn't expect to create this when I turn this. There's space between them.

As Moore (2009) suggested in his book, bar graphs, pie charts or pictograms are suitable alternatives for the distribution of a categorical variable. Although Group A members seemed to be aware of this information, they claimed that the scores can be assumed as categories. Then, I offered another alternative which their students might actually think. The suggestion was shown in Figure 4.1 that individuals in the data set, where A, B … P are the students, could be assumed as categories as well, as in the following figure:

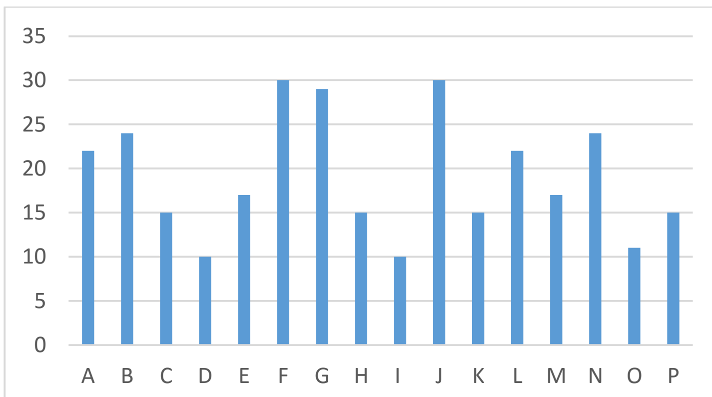

Figure 0-1 Alternative bar graph when students in data set were assumed as categories.

Their drawing for the bar graph of this data set is shown in the Figure 4.2 below, categorical variable is set at students' scores. Since they used boxes instead of dots, they consider this dotplot as a bar graph.

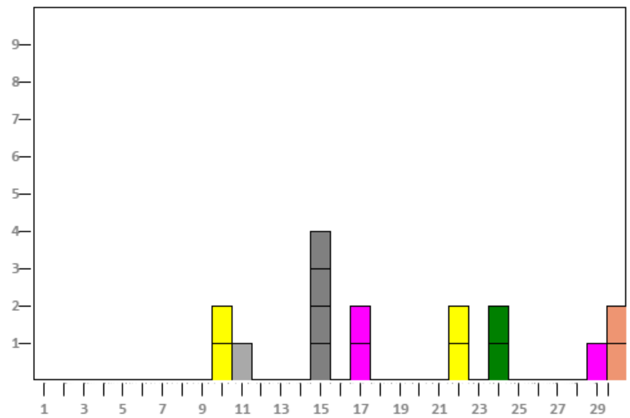

Figure 0-2 Group A drawing for the bar graph for students' scores.

The following conversation demonstrates the comparison of two above alternatives for the bar graph of the data set they used and concludes with that the group actually couldn't differentiate a bar graph from a dotplot, which implies our second major finding as lack of understanding about the differences between a bar graph and a dot plot:

> Researcher: The bar graph will be something like this, then. For instance; B is the student, C; up to how much, there're students up to P. It will be something like; Student A got 22, Student B got 24. It should be something like this. It has to be something like this, doesn't it? There needs to be one column for each.
> Gizem: Actually, it is not like that.
> Banu: We drew it on shodor. We didn't draw it like this. Is there?
> Researcher: There's more.
> Banu: We did something like this. [Shows the drawing above.]
> Gizem: Yes, notes, number of people.
> Researcher: Actually, that wasn't the bar graph; that was the dotplot.

While they were preparing the lesson plan, the group did not considered that their students could offer such a bar graph demonstration regarding the example I suggested. After they realized the difference between a bar graph and a dot plot, they continued the discussion as in the following:

> Gizem: This is what I was talking about. Do you understand me now?

Researcher: Yes, indeed. I didn't think of this one, I thought you were having a problem. I felt that way.
Gizem: Yes, indeed. I couldn't remember its name. I even told you that this graph wasn't a bar graph, it was something else.
Esen: I directly assumed it was a bar graph.
Gizem: I didn't but I couldn't remember the name.
Researcher: Remember I even told you about . . .
Ezgi: We call it a dotplot because we accumulated?
Gizem: I couldn't remember it. No, no, I even told you about it while discussing but you said it was a bar graph. You said it was drawn dot by dot. And then I assumed it was something else.
Banu: I don't remember saying something like that.

Another important finding from initial GD of Group A is that they could not came up with an accurate example for introducing bar graph to their students. After above conversation, they noticed the differences between a bar graph and a dotplot. Then, they started to think about the reasons for their choice of example. They tried to make a transition between bar graph, stem and leaf display and histogram in order not to get their students' minds mixed. In all parts of their lesson plan, they tried to use the same example in order for providing a unity among all three types of data displays. Then, they started to think about why they chose it and replace it with a different one with my feedback and comments. I highlighted that bar graphs are used for distribution of a categorical variable. Their discussion about the reasons of using that example is as below:

Banu: We want him to see the categorization.
Gizem: Yes, indeed.
Esen: How can he categorize it? It's already there.
Banu: We couldn't agree on it. Should we ask a fruit question?
Gizem: Yes, it is there, we discussed it a lot.
Banu: We even searched for bar graph questions online, to see what kind of questions there are, again there were apple questions and grading questions. Then, we wanted to use this website, we decided to use grading to use this tecnology, because we wouldn't be able to use it if were apple and peas.
Gizem: We adapted it. […]
Ezgi: At some point we thought that maybe we should use grading system up to 5, since it is 1, 2, 3, and 5 in an elementry school.
Banu: Only because we wouldn't be able to use stem and leaf display.

> Ezgi: Indeed. That's why we drew it.
> (Everyone says yes.)
> Esen: Just to make transition easier.
> Gizem: Just to use this one and that one on the next. Yes, indeed.
> (Everyone says yes.)
> Gizem: Or it was apple and peas. We seriously created everything from the beginning.
> [Second Example]
> Esen: Our aim here was to make things easier for the teacher.
> Gizem: Indeed.
> Esen: We connect them so children can understand the relations between them little more and to make the transition easier. We thought by this, teacher wouldn't have to deal with it.

The discussion about the example used follows with thinking about other possible alternatives for a bar graph as below:

> Gizem: We actually thought about asking the class about which fruit they like better and write it on the board. And make it a bar graph. Everybody makes as individually. But then we couldn't connect it to stem and leaf display.
> Esen: Just to use that table.
> Gizem: Then we thought about using shoe sizes […]
> Ezgi: Numbers of relatives.

As can be seen from above discussion about the purpose of usage of the same example in all of the displays they used. Since, they disregard the main variable type used in a bar graph, the discussion resulted with a conclusion of changing the example used in the lesson plan. Afterwards, they even thought about the omitting the bar graph-stem and leaf display transition from the lesson plan, as in the following conversation:

> Banu: Actually it would be okay if we just skipped the bar graph and took the histogram of the stem and leaf display and asked about the histogram. But we said let's compare bar graph with histogram.
> Gizem: When the objective is different.
> Banu: How will we make a transition from the bar graph after that?
> Researcher: Does the objective necessarily say to compare bar graph and histogram?
> (Everyone says yes)
> Gizem: Yes.
> Researcher: Okay, so is there an objective before that, something like [student] understands and comprehends the histogram?

> (Everyone says yes.)
> Gizem: Of course there is.
> (Everyone says yes)
> Researcher: So you could have skipped bar graph and passed on that, as well.
> Banu: Yes, we could have.

The discussion about the starting point of the lesson among group members did not reach a consensus, then they left it for second draft of lesson plan. Most prominent moments of the group discussion referring to content knowledge of preservice elementary mathematics teachers were presented above. Next part summarizes the initial GD of Group A from the pedagogical knowledge view.

4.2.1.1.2 Pedagogical Knowledge (PK)

As can be seen from the TPACK codebook (Hughes, 2013), knowledge of general teaching methods and strategies, checking for understanding, knowledge of learners and their background, knowledge of general assessment strategies (e.g., tests, oral, project-oriented tasks), classroom management techniques and lesson planning activities and preparation refer to the pedagogical knowledge of teachers. Based on the analysis of data regarding the pedagogical knowledge, Group A concerns about *knowledge of general teaching methods and strategies at most*, their secondarily important concern for pedagogical knowledge is *knowledge of learners and their background*. Another important consideration of the group members were the *lesson planning and preparation*. However, they did not refer directly to the assessment strategies or classroom management strategies at all during initial GD.

The topmost finding regarding pedagogical knowledge domain is that the participants had enough *knowledge of general teaching methods and strategies*. In their research lesson, Group A uses several pedagogical strategies such as direct instruction, questioning and think-pair-share method. Moreover, they explained how these strategies are used throughout the lesson. It can be resulted that they are capable of explaining these strategies and how they are applied to a lesson.

Through the initial GD, Group A explained their aim for choosing direct instruction method for the beginning of the lesson so as to activate the prior knowledge of their learners. They explained that they didn't choose any group work for students, they expected students to work individually throughout their lesson plan.

The other pedagogical strategy they used was the questioning technique. They prepared ten discussion questions for every part of the lesson. Some questions were as follows: 'What do you think when viewing this relationship?', 'How can we determine the axes of this graph?', 'What categories of information would you place in the graph of this data?' All of the questions were open-ended, which could lead students to discuss about the issue and require reasoning. Moreover, group members were aware of the sub-questions which could emerge from the main question asked, they also thought about the possible further questions and students' responses throughout the lesson. Group A also mentioned about the questioning strategy they used in many parts of their lesson plans for the purpose of guiding the students. They thought that learners could reach the desired outcome at the end of the questioning phases, therefore they described their questions as guide, as in the following conversation:

> Gizem: Our questions actually guide [students] to it, I mean, it fits in the way we made. We wrote in there, something like I'm in a group.
> […]
> Gizem: It's like it's guiding [students] to what we want to do.

Then, they summarized the reasons for deciding on all of the pedagogical strategies they used in the lesson plan. Their top considerations in making decisions about the teaching methods were the objective of the lesson, the duration, technology usage. Group A also considered the flow of the lesson as important, they desired a fluent lesson without allowing to lose the motivations of students. The related part of the discussion is as follows:

> Researcher: You said you do something like direct instruction because you remember all the information and it goes faster this way, by that do you mean you make all these choices based on this?
> (Everybody says yes)

> Researcher: What was your first criterion? What was it based on? Because there's something like, let's not choose this one but this one.
> Gizem: Like I said, to use the tecnology, and the time was limited and we wanted to show them all.
> Banu: Yes, we thought about that.
> Gizem: Those were the things that guided us. We have a website which we will use, we have two hours, that's our objective, what we can do to go through it without boring the children or without making them lose interest on the topic, so we went towards this.

The second-most important finding regarding pedagogical knowledge is the *knowledge of learners and their background* throughout the GD1 by Group A. Nearly for all parts of the lesson plan, they haven't disregarded background knowledge of learners, and they always referred back to the objective of the lesson plan and the unit plans of all grades in elementary mathematics curriculum. They have already learned about the prior knowledge of the students before teaching the histogram concept to them. For instance, they assumed that the students had been taught about bar graph in grade 6, since they wanted to activate their prior knowledge about the bar graph before teaching histogram concept. Below conversation shows this discussion:

> Researcher: I mean to teach students about the bar graph, students have already learnt it last week. And they also learnt that it has to be used with a certain type of variable. You're just summarizing it to them.
> Gizem: Just like you thought here, we …
> Esen: Whereas they don't it either.
> Researcher: But you've already taught them this last week. As a teacher. (Everyone says yes)
> Gizem: No, not last week, it was even last year. That's why we've put a reminder. In case they forgot.
> Researcher: Ah, at the 7th grade.
> Gizem: Especially.
> Esen: That's why we gave 20 minutes.
> Ezgi: We opened and looked at the curriculum, yes indeed it was at the 6th grade.
> Banu: At the 6th grade.
> Esen: I remember it was the 7th grade.

Group A also searched the all unit plans for grades 6, 7 and 8 in order to decide how to present stem and leaf display in their lesson plan. They realized that stem and leaf display is being taught to the students without naming it especially. Their discussion regarding the learners' background knowledge of stem and leaf display is below:

> Researcher: […] And at this point, you assume they know about the stem and leaf display, right?
> Esen: Yes, I think it was at 6th grade.
> (Everyone says yes)
> Gizem: They don't know its name but they know it as a display.
> Ezgi: They only know it as a display.
> Researcher: They don't know it is a stem and leaf display but they know the display.
> (Everyone says yes.)

Another concern of Group A is the motivation of the learners regarding the knowledge of learners. They concerned about the needs of their students and they were aware of these needs. Group members expressed it relating the choice of using the same example for all transitions in the lesson plan, as can be seen from the following conversation:

> Alp: We meant to lead the class through one example to make things look more connected.
> Gizem: Yes. And not to waste any time. Children lose their attentions so easily, they move from website to another, one question to another.

Third finding based on the initial GD of Group A is that they are capable of how to prepare a lesson plan and they know what a lesson plan requires. Throughout the GD1, participants in the Group A referred to *lesson planning and preparation* as well. They mentioned about the lesson plan format which I asked for them, the steps or parts of the lesson, the columns of the lesson plan format, the sequences of the parts of the lesson and the duration of the lesson.

Since they found that lesson plan format which I asked for them is different than they are used to prepare, they stressed that they had some difficulties in understanding it. While explaining that they chose an objective from the 8^{th} grade elementary mathematics curriculum, they also explained these difficulties. Since they were not familiar with the format I gave, they stressed that they had some difficulties in order to understand how they arrange the lesson periods, at most.

The participants also explained why they chose the objective related with histogram from the curriculum. They mentioned that they started to think about the objective during the workshop and decided on histogram, as in the following conversation:

> Researcher: You especially chose histogram. Why did you?
> Banu: Why we chose histogram, actually…
> Gizem: Actually, histogram was very suitable to use for the websites you gave. There were stem and leaf display, bar graph, histogram, we could have used all of them in this. And we would have a lot of sources.
> Banu: I guess we were impressed at the first week of the workshop. We don't have anything from the second week. Only the first week. (Giggles.)
> Gizem: Because you said it in the first week, I mean we already had thought about it. There was a draft.
> Researcher: I told you to prepare a lesson plan at first week.
> (Everyone says yes)
> Gizem: We had already planned it at the end of that week. We prepared a simple draft.

Then, they stressed out the steps they specified during the lesson plan while specifying duration for each step. Steps constitute the phases followed by one another during 80 minutes which makes two classes. There were 4 four steps in their lesson plan as specified below:

> Researcher: Let's assume there are 4 steps. And you put a duration for each one of them. And only the last one doesn't have a duration. In here, you said it would make a think-pair-share.
> Gizem: This was end of the last class.
> Esen: We thought; at the end of the class, since it alredy lasts 40 minutes, there's 20 minutes at the beginning and another 20 minutes at the beginning of the second hour, that's 20 minutes too.
> Gizem: Yes, It's 2 hours for each. 20 minutes for each.
> Researcher: I'm assuming it has to be 80 minutes.
> (Everyone says yes)

The main issue regarding the lesson plan preparation in determining the starting point of the lesson. One point was activating the prior knowledge about bar graph first, then introducing a new concept which is histogram and comparing it with bar graph at the end. In order to decide on the starting point of the lesson, they went back to their instructional objective and Banu reminded the other members as that our objective is about mostly comparing the bar graph with histogram. The related part of the discussion is as follows:

> Banu: Actually our objective is to show the comparison between the histogram and bar graph. In order to do that, first…
> Researcher: There is such a thing, right? (Objective)
> Banu: Yes, there is. There's histogram at the 8th grade.
> Researcher: What does it say exactly? Does it say student knows the difference between a histogram and a bar graph?
> Gizem: Knows the difference. Yes.
> Banu: Yes, it's like that. That's why we decided to remind the bar graph. And then to tell about the histogram and ask a comparison question at the end. We went with this way. Our purpose was to compare those two.

The other idea for starting point of the lesson was introducing histogram while making transition from stem and leaf display to it, then making a comparison between histogram and bar graph at the end which was their objective. Activating prior knowledge about bar graph is then placed into the comparison part of the lesson. The related discussion is in the following:

> Banu: We also could have assumed students already knew about the bar graph and then told them about the histogram and compare these two by saying they've already learnt about it.
> Ezgi: Yes, we could have had them compare.

This discussion about choosing these two different sequences of the steps of the lesson promoted a good discussion that they considered nearly all pedagogical issues very well onto this conflict. They also offered another solution that was to increase the duration of the lesson. However, they saw that they could not reach a consensus during GD, they left all questions aroused from this conflict as unanswered. The following conversation regarding this clash:

> Researcher: Okay, then what is the last decision we agree on? Do we skip the bar graph?
> Ezgi: Yes, we do.
> Gizem: I don't know. I think we should make a decision later.
> Ezgi: Will it work in this way?
> Banu: In this way, the operation…
> Researcher: To me, it feels like in this way there are a lot of operations and it's difficult to control.

Alp: What if we increase the duration?

Including an activity sheet into their lesson plan is also a part of lesson plan preparation. Considering the use of activity sheet in their lesson, they thought about their aim for to use it in the lesson, what kind or what type of questions/problems will be included in it. They also decided on in which step of the lesson, activity sheet will be distributed. One part of this decision making process about using and preparing an activity sheet is in the following:

> Researcher: If you want to use an activity sheet, what that activity sheet is and what it contents. . .
> Gizem: First, the activity sheet is here and there will be some problems.
> Researcher: Okay.
> Gizem: […] And by giving a space, just like we said about stem and leaf display, we can go back to the problem in case we need to, something different for this.

As can be noticed from above, it could be claimed that members of Group A have enough knowledge about lesson planning and preparation. They are capable of considering all of the elements which a lesson plan includes and they promoted a good negotiation with each other about these issues while suggesting many other alternatives, as well.

The most important parts of the discussion regarding pedagogical knowledge of preservice elementary mathematics teachers were listed above. It could be argued that Group A has pedagogical knowledge. Since they could take all possible concerns into account in their lesson plan, their understandings about their learners and their background, general teaching methods and strategies, and lesson planning and preparation were well.. Next part describes the technological knowledge of the participants in Group A.

4.2.1.1.3 Technological Knowledge (TK)

According to TPACK codebook (Hughes, 2013), the determinants of technological knowledge for teachers are operating computer hardware, using standard software tools (e.g. Ms Word etc.) for non-educational use, installing and removing peripheral devices (e.g., USB drives, microphones, etc.), troubleshooting equipment, using appropriate vocabulary and knowledge of current and emergent technologies in society. These were not coded in first GD of Group A but observed through workshop, group discussions and lesson plan implementation periods. Thus, it can be said based on these observations that all of the participants have those abilities which refers back that they have technological knowledge very well.

4.2.1.1.4 Pedagogical Content Knowledge (PCK)

TPACK codebook (Hughes, 2013) specifies that pedagogical content knowledge has three marks: *knowledge of teaching/representing subject matter to students* (e.g., techniques, representations, and analogies), *identifying and addressing student subject-specific misconceptions or mistakes* and content-specific assessment strategies. Although participants haven't referred to content-specific assessment strategies, they discussed how they represent the subject to the students and how teaching will take place at most. There were some concerns for preventing for possible misconceptions or mistakes of the students, as well.

They considered the possible ways of representing the bar graph, stem and leaf display and histogram which are subjects of their lesson throughout the GD. *The visual elements of the lesson, the representation of bar graph, the aims of questioning strategy, representation of example used (example used for all data displays)* were most concerned issues during GD and these are accepted as descriptors of PCK. Therefore, it could be argued that their pedagogical content knowledge is enough to discuss these issues and they are summarized below with most prominent examples of conversations.

The discussion regarding pedagogical content knowledge started with the use of visuals in teaching, after I asked the question as "In the future, do you prefer teaching like the way I showed to you in the workshop?" During workshop, I presented them to teach statistics with the help of online resources as an alternative way, and my aim in asking this question is whether they were impressed by this teaching strategy or not. It seemed that they were impressed by that and use of such visual elements in a lesson was an effective way to teach a subject to the students. They also proposed an alternative way to my method in case of a deficiency in technological infrastructure of the classroom. What they offered is that they could follow the teacher while she is presenting the visuals from her computer instead of every student follow the lesson on their own computer. That means, Group A emphasizes the importance of using such visual elements in order to present a subject so as to take the attentions of their learners.

They discussed the reasons and the teachers' remarks on choosing the first example they used for bar graph. Their aim in using those data for drawing a bar graph was to make students discover grouping into categories and they also stressed that they would never mention about the variable types as categorical or quantitative variable since they were aware that curriculum do not allow to teach variable types. However, they believe that students can realize that grouping ability of bar graph although the variable type won't be mentioned. The following conversation shows this issue:

> Banu: [Students] won't say it's continous or not continuous, or if it's categorical or continuous, because the result will be openly eligible.
> Gizem: Like the way we show stem and leaf display, they understand the difference, they also can understand the difference here, grading apple and peas, more of a continuous data…
> Esen: Understanding the difference between data.
> Gizem: They can understand but it's a continuous data, we are not sayig it's not a discontinuous data.
> Ezgi: Also, I think we focused more on the categorization in that. I mean the way to categorize it.
> Gizem: Yes, there's also in this part.
> Ezgi: It can lead to categorical but then it'll get in the subconscious.
> Gizem: We don't give that.
> (Everyone says yes)

When the above discussion and their questioning strategy used for the bar graph are combined, it is understood that they tried to make students realize the preference of categorical variable in bar graphs rather than quantitative variables. Then, they thought about their example once more regarding the aim of grouping and confused for a while since they realized that those data do not serve for this aim, as in the following:

> Researcher: And there's something else, in here it says, what do you think when viewing this data, what do you expect children to see here? When we come to the main point, to which type of graph…
> Banu: We want them to see the categorization.
> Gizem: Yes, indeed.
> Esen: How can he categorize, I mean it's obviously there.
> Banu: We couldn't come to turn of agreement there, actually, if we should as a fruit question or not.
> Gizem: Yes, we discussed it a lot.

Related with the discussion about this example, they also mentioned about the other alternatives presenting the bar graph to the students, they referred to methods of mathematics teaching course and explained a physical manipulative used to form a bar graph with it, as below:

> Gizem: There was something we saw at the methods [course], Was it latch, or was it throwing something on, the one we counted the bar graph by one by?
> Banu: Yes, indeed it was a latch.
> Esen: What was it like? I don't remember that one.
> Gizem: It wasn't long. I mean that. Think of sticks, there's one stick for each category, like one stick of each grade right now, I attach one latch, and at the end student gathers these latches as a column as high as their height.
> Banu: Makes a column by gathering them.
> Gizem: Actually the idea behind it is still the same in here.

Last three comments indicate that the group members were aware of representing bar graphs through different alternatives. They could negotiate about the possible outcomes of the representations they used. From this point of view, it could be claimed that, to some degree they had an understanding of representing bar graphs in different ways.

They discussed also about questioning strategy they used in lesson plan and the reasons for asking and choosing the questions they would ask to students. Moreover, they discussed the possible questions which students could ask to teacher. At the moment they realized the change in the categories when they change the axes regarding the data set for bar graph, they considered the anticipated responses from the students. They expected that students couldn't easily understand the difference in both bar graphs offered by me and by them. Another consideration was, one of the students might offer the bar graph which I offered, then there could happen an unintended discussion about bar graphs among students. In order to handle with these two different advisable bar graph displays, they were planning to take some precautions while using questioning strategy. Some part of the conversation is as follows:

> Researcher: Now, you ask something like, ‘what do you expect to see from this data’, I get something out of it, right?
> (Everyone says yes) […]
> Researcher: Now, Let’s assume [a student] said ‘I want to see the grades that other students got’ then, he might be thinking of [the graph that I offered to the group members], we should take it in the right way.
> Gizem: Would it be okay if we say, I just thought of it, while guiding at first, these are the grades, and the teacher of the class wants these grades to be easily seen and read. Would it be easier to comment on if it was separate or if it was higher than other grades, how many people got the highest grade.
> Esen: So, you mean we should make a change on the question.
> Gizem: While guiding, or maybe not on the question but while guiding. With which way I can see it easily, it’s basically the same with this one here. I can’t make a comment through this though, but in other case, for example, there’re 5 students who got the highest grade. Then it’s a hardworking class, 5 of them got 30.
> Banu: Okay, what if, I mean this graph is correct, the child can lead to this. But also there’s the opposite sitution, I mean just like what Gü said, the other one also can use it, in order to see the frequency.
> Researcher: By changing the axes.

Together with my feedback on the expressions of questions/problems used in lesson plan (e.g., bar graph example), they offered to shift them into story-like problems in order to refer to real-life conditions. It could be said that they could grasp what I meant here easily and responded in the expected way, as in the following:

> Researcher: I also have a suggestion for you, let’s think more real life setting here. The grade scale of a student is actually something between 0 and 30, right?
> Banu: It’s something like a quiz.
> Gizem: Up to 30.
> Researcher: Starting from 10, I guess, 10 is the lowest and 30 is the highest, you must have took 20 as the range.
> Gizem: We did, so it won’t be difficult.
> Banu: It’s like a quiz which worths 30 points.

> Researcher: So then, make it something better. Something with a background story.
> Gizem: So, we change the question into a story and make it a story problem.
> Ezgi: Something like, they had a quiz in the math class etc…
> Gizem: Yes, yes. Then, just like what I said, the teacher will see it easily and we can connect it to the story more.
> (Everyone says yes)

Secodarily most important finding regarding PCK is that Group A *showed much effort on identifying and addressing student-specific misconceptions or mistakes*. All of the participants in the group concerned about the possible misconceptions which students could experience and the mistakes of the students. For instance, they realized that the *data set used for drawing bar graph can lead students to misconceptions*. Since they used it as the beginning of the lesson and to activate the background knowledge about bar graphs, they suggested that it could be a bad idea to start with, as in the following:

> Ezgi: But then, there's something I like to ask. Okay, we thought about it this way, but, will it be a problem for the student? I mean we are making a new introduction activity, would it confuse his mind even, before entering in his head?
> Researcher: Now, yes, when we talk about it this way and when children try to do it in that what…
> Ezgi: In that way, it wouldn't be a little. It's more like being cumbersome.
> Researcher: In that situation, you will need to control it very well. It can be a problem then.

Related with the same possible misconception, group members discussed students' anticipated responsed and how they would overcome this misconception. They offered again further questioning in order to make them discover the difference between the two bar graphs above. The discussion among group members is as follows:

> Researcher: But again there's something like, what you actually want to see is the frequency, because it gives out the frequency. I mean when we draw the bar graph like this. I mean when we draw the axes like this. (I'm referring to their drawing.)
> (Everyone says yes)
> Researcher: If we don't choose it like this, (I'm referring to my drawing). But choose like this, we're able to see the category directly.

> (Everyone says yes)
> Esen: For example, if the student says something like, ‘teacher, I chose it like this’, how are we supposed to change him?
> Researcher: That’s it!
> Gizem: I can’t say he’s wrong, I say it’s also correct. ‘You know this one, but let’s also draw with in this way’
> Banu: So, basically we are asking him to determine the axes, by asking if we can switch the axes. We can also ask it by asking if we can show it in different ways.
> Gizem: Exactly. How this graph will be if the axes switch?
> Banu: I mean it’s already a very probable answer.

It could be argued from the above two parts of conversation that the group members could take some precautions in order to handle the possible misconceptions which can occur during a lesson. In their case, it was related *with drawing a bar graph and which axes refer which variable in the data set*. Thus, they thought that they should direct students with some questions in order to reach the desired answer. It could be claimed that Group A has enough knowledge about the possible misconceptions related with their objective used in their lesson although they said that they have never searched for possible misconceptions about statistics, especially for bar graph, stem and leaf display or histogram. As it has been presented above, the participants’ content knowledge about statistical concepts was weak.

Group A mentioned about a second example for students’ misconceptions regarding the transitions from bar graph to stem and leaf display and from stem and leaf display to histogram. They realized that their drawing of bar graph and the resulting histogram would have the same properties, therefore they could not find a reason to make students compare them. Since they couldn’t describe their own drawing as a dotplot before GD, they also disregarded the misconceptions which could arouse. After they understood that it was a dotplot which is very far from a bar graph, they started to think about possible consequences of this misinformation and realized that it could actually happen to be a misconception in students. The following part of the discussion shows this issue clearly:

> Gizem: I mean, after that, when a comparison is given, we also draw some attention to those spaces between and the use of a different data

> while comparing them at the 4th step. So, I believe it will be a misconception for the children to create something in here. In order to call it a histogram, he definitely needs to create a different graph.
> Researcher: Then.
> Gizem: I think, if he creates something similar to what you showed, (referring to my example), it'll be wrong.

The discussion about misconceptions and mistakes of students continued with my feedback about the misconception related with the characteristics of bar graph and histogram. I stated that it is a misconception that students generally cannot specify the differences between bar graph and histogram since they seem very similar to each other. Students cannot see the differences between a bar graph and a histogram because of their similar bar-like displays in such a way that bars are distinct in a bar graph referring to distinct categories. However, bars represent groups of individuals continuously in a histogram. Then, I offered Group A that they should be conscious about these differences and take their students' attention into them in an efficient way. Therefore, it can be claimed that they need to improve their PCK regarding identifying and addressing students' misconceptions/mistakes.

Summarizing, the participants in Group A showed descriptors of PCK at most in identifying and addressing student-specific misconceptions or mistakes and in the knowledge of teaching and representing subject matter to the learners. They have never mentioned about the content-specific assessments during GD as they didn't mention about assessment as a part of PK in the above part. It could be claimed that their PCK is enough to describe or think about students' possible misconceptions and mistakes and to produce precautions in order to overcome with them. They also considered the possible representation ways of the concept they covered in lesson plan very well.

4.2.1.1.5 Technological Content Knowledge (TCK)

The three specifications of TCK are knowing about the existence of a variety of content tools for particular content tasks; especially tools that experts in this field might use, operating/knowledge of content-based technologies in which content learning is foregrounded and knowledge about the ways in which content and technology reciprocally related to one another, according to Hughes' (2013) TPACK codebook. The participants of Group A referred nearly all of these codes and it could be sait that they have a limited understanding in order to express their TCK regarding those codes. It could be further claimed that they learned much about the varieties of technological tools to represent statistical concepts through the workshop which was held before. Regarding the online resources presented for statistical concepts in the workshop, they remembered some other online resources for other mathematical concepts such as algebra. For instance, one website which I demonstrated to them during workshop was NCTM's illuminations' website, they mentioned that they were aware of it, but they haven't further searched the website for some other resources for statistics. Because of the newness of online resources for them, Group A could be claimed that they started to develop their skills in TCK with the help of workshop and the group discussions.

While addressing that their choice of online resources in their lesson plan, participants explained the other alternatives which are suitable for the objective of the lesson plan. Gizem expressed that they sometimes used MsExcel for drawing data displays and she claims that Excel could also be used for data displays. Then, they compared the data displays they drew with the data displays in the online resources which I offered as an alternative in the workshop. The following conversation shows this expression:

> Gizem: When I write reflection, and if you read from the objectives also, I looked at the ones directly. Some of the questions from the activity sheets, were the ones which we took out of the websites and translated into Turkish, were very suitable with the objectives. But I guess some of them were not suitable for those that we made. I mean, they weren't in the objectives. Apart from them, there was something my mind got caught up on; mod median, and the mean, we found them directly on that Excel you gave us. For example, Alp and I, I don't know, I mean you stayed on the other side but, we've never used that website. I mean, if the child knows how to use Excel, then he doesn't need to use those websites like ours. We checked afterwards.
> Banu: We checked.
> Gizem: Exactly. Ours happened that way, we found it and said what it is. That maybe.

Although participants stressed that they were not aware of existing any online resource which could be used in teaching statistics during pre-interviews, they realized through workshop that they disregarded some websites since they haven't seen them in teaching statistics before. This supports their limited understanding of TCK regarding teaching statistics. Before workshop, even they haven't searched the web for any online resource to be used in teaching statistics, as can be understood from their pre-interviews. Their comments were as follows:

> Researcher: [In the interview] you said there was nothing specific you know about statistics and said that you didn't know about the virtual manipulatives.
> (Everyone says yes)
> Banu: Actually, it is like this; we knew about the NCTM, but we probably didn't think about the statistics part of the NCTM. Actually it was what we used.
> Gizem: Yes, I only knew the thing. And we even used that graph.
> Researcher: By NCTM, you mean stuff on the NCTM website…
> Banu: Those in the illuminations. Boxplot and etc.
> Gizem: There was one you put, that was familiar with me. But that other one called shodor…
> Banu: Yes, we weren't familiar with the shodor. We've used NCTM.
> Gizem: I didn't know about it at all. There was another website, the first one you gave, we didn't know about that one either.
> Banu: It was something like learning activity or academic learning. It was chapter by chapter.
> Esen: It had A, B, C chapters.

When they realized that the bar graph which they draw on their choice of virtual manipulative (http://www.shodor.org/interactivate/activities/PlopIt/) was in fact a dotplot, they started to think about other alternatives which a bar graph can be drawn within it. They tried to remember the other websites which I demonstrated them during workshop rather than searching the web for it like I did for workshop. The discussion is in the following:

> Researcher: Yes, of course, now I don't think of it as a dotplot but as as a bar graph. Then, we need to do this like, you probably need to choose something else. Maybe a program that really draws bar graph. That would be more beautiful.

> Banu: We thought about using the one from NTCM.
> Ezgi: Yes.
> Gizem: There wasn't a bar graph in NTCM.
> Esen: No, there weren't.
> Gizem: I think there was another bar graph thing. Yes.
> Researcher: Yes, there was. It goes as a bar graph in several places, other places.

While group members were sharing their ideas about the alternative starting points for the lesson, such as starting with stem and leaf display. It seems that they were trying to remember the other online resources which were mentioned during workshop. They were not intending to search for the web, for instance. The related conversation can be seen below:

> Banu: I thought of something right now. Let's take this. For example, if we take this, I take this from here, I take bar graph from here, I started with stem and leaf display and histogram, even here, and we use the stem and leaf-histogram part on NTCM website.
> Ezgi: Yes.
> Banu: After that, after telling about the histogram, we give a brief bar graph example, and briefly summarize bar graph…
> Researcher: And you remind.

As can be seen from above discussions, all TCK references have gone through the bar graph example they used in the lesson plan. They mostly considered about the alternative online resources mentioned through workshop rather than searching the web, which shows their limited understanding of TCK.

4.2.1.1.6 Technological Pedagogical Knowledge (TPK)

TPACK codebook (Hughes, 2013) treats TPK with many specifications, they are: Motivating students through technology; differentiating instruction when technology is used; ability to organize collaborative work with technology; holding students accountable for equipment used; developing strategies for assessing student work with technology; knowing about the existence of a variety of technological tools for particular general pedagogical tasks; ability to repurpose commercial software for general teaching; knowing about the time required to teach with particular technologies (prediction may be said); ability to envision potential student problems with particular technologies and plan relevant activities to support those students; generating alternatives in the event of technological failures; ability to explain a computer procedure to students (e.g. through modelling); using technology for lesson plan preparation; using technology for general assessment; and, knowledge of infrastructure at school site.

The participants discussed mostly on the *differentiation in instruction when technology is used*, *holding students accountable for used equipment*, *generating alternatives in the event of technological failures*, *knowing the variety of technological tools* and *knowing the time required for teaching with technology* throughout the discussion. The main finding for TPK of the participants that they have a little understanding of TPK since their pedagogical understanding is relatively higher than the rest of the other knowledge domains but not as much as TCK. Since TPK is accepted as an intersection of TK and PK, this finding can be asserted as reasonable.

Their first attention was about needed time for technology usage in teaching. Since they experienced a loss of time through the use of online resources during workshop, they stressed that their students could live the same. They had to make transitions between statistical concepts and related online resources quickly during the workshop because of its intensive nature. Therefore, they specified that they sometimes could not use their time efficiently while making these transitions. Gizem expressed her worries about the time management as "You introduced different websites to us. While we were dealing with them, we lost some time at first. We thought that loss of time can happen when students are navigating between the websites"

Similarly, they experienced a difficulty while using a data set given during workshop. They stated that they learned some lessons from this experience regarding a technical failure. They expressed that they should consider some precautions according to the technological tool or online resource which they will use in their future teaching. In one of the virtual manipulatives used in their lesson plan, the max value of the data set could be 30 at most because of a requirement. During workshop, they experienced a failure with it since there were some values more than 30 in the data set. They stressed that they thought about this issue and organized their example of data set accordingly as having values less than 30. This is one of the things they learned through workshop and it could be claimed that workshop had an effect in order to increase their awareness on TPK. The discussion related with this issue is as follows:

> Researcher: […In the workshop] there were also some things that even I didn't think of, there were errors, for example that 30, as the highest, I realized it afterwards.
> (Everyone says yes)
> Researcher: Because I tried some of them, while preparing the workshop, but some of them I did not. And one of them came across. One from the data. We had a problem there.
> Gizem: But we got passed it quickly.
> Researcher: But also, I hadn't examine the data completely. Maybe I should have left out those outliers. Maybe, I should have left out that one said 65. Completely. But since I didn't leave it out, it's like, I gave it to you without going throught with the cleaning part. I wish I have cleaned the data.
> Ezgi: But, it's okay. We all realized it and maybe later while using it, we can move by considering this isssue.
> Gizem: Yes.
> Researcher: Yes, you took a lesson out of this for yourselves.

While discussing about the advantages and the possible benefits of using technology in teaching, the group members made a comparison according to differentiation of instruction when technology is used. They compared the usage of online resource by each student with the usage of it via watching from the board. They emphasized that there is a big difference between them and they supported their claim as saying that they chose individual work just for this reason. Then, it could be said that they can differentiate instruction via use of technology as long as they have the knowledge of varieties of technological tools which can be used for a specific subject. Therefore, it could be claimed that their development in TPK is dependent to their knowledge about varieties of technological tools in teaching a subject, which is TCK in fact. The following conversation shows this argumentation clearly:

> Researcher: And here, I'm interrupting right away. In here, does using a virtual manipulative [for drawing histogram] make things easier for you, for the teacher, and do you believe it will also increase the achievement of the students?
> Esen: Now, if the student does it by himself, it definitely will increase, but there is a difference between making it by himself and seeing it on the board, a big difference I believe.
> Gizem: Yes, I think so, too.
> Researcher: Then, students themselves…
> Gizem: That's why we called it an individual work, actually. Each student will open it and do it individually.
> Alp: Just like we did in the class [during the workshop].
> Ezgi: Again, it's just like thought here.
> (Everyone says yes)

They also stressed about the time required for the technogical activity while students were holding accountable for their individual work on computers. They stated that students will do computer work as individually and they will be responsible on their own work. Therefore, Group A thought that they prevented loss of time and provided an efficient time management in their lesson. This can be accepted as an indication of development of TPK for the participants in the group and the issue can be seen in the below conversation:

> Gizem: Also I don't bother drawing them one by one, there's a graph, only by putting dots, I mean I both do it by myself and don't waste any time. Because I'm using tecnology.

At the end of their GD, Group A was in a conflict whether some changes in the sequence of the steps of the lesson plan and in the pedagogical strategies they used are needed or not. They also had negotiations about students' anticipated responses and taking precautions for those in order not to be resulted with misconceptions. In all aspects of the above discussion points, there are attributions for all knowledge domains of TPACK framework.

So, on the whole, it could be claimed that Group A has lack of CK in statistics especially in the type of variables, differences between a bar graph and a dotplot and choosing the example for a bar graph. However, their PK is far beyond their CK since they considered mostly their learners' background knowledge, the teaching methods and strategies they would choose in every aspect of the lesson planning and preparation phases. Regarding TK, the participants have all of the basic technological abilities such as operating a computer hardware, working with some software, installing and removing some devices, and so on. These capabilities were observed through workshop, group discussions and their lesson plan implementation period. The participants have enough understanding of PCK since they could discuss the varieties of teaching or representing the subject to their students and their possible misconceptions and mistakes which are specific to statistics concepts. However, they showed a limited comprehension regarding TCK because they mostly learned the content-specific technological tools via workshop and they expressed their comprehension based on the information served only on workshop without further searching about other technological varieties. Lastly, the participants showed a better understanding in TPK rather than TCK since their high capabilities in PK. However, their TPK was not as high as PK which could be argued based on their above conversations and arguments.

4.2.1.2 Initial GD of Group B (including 4 preservice elementary mathematics teachers)

The main findings of initial GD of Group B were listed in this part. This initial GD was done through their lesson plan presented as first draft to the researcher as it was done for Group A above. The lesson plan is attached as Appendix J at the end.

In their lesson plan, Group B tried to teach the fundamental characteristics of a bar graph and a histogram while activating the prior knowledge of learners about bar graph and histogram. They have two objectives: (a) to form and interpret a histogram according to a data group, (b) to show the data related to research questions in circle, bar, line or histogram when they are appropriate and make transformations among these graphs. In order to accomplish these objectives, they started first by activating prior knowledge about bar graph. They asked their students to form a bar graph by using a physical manipulative which they prepared on their own. There are 5 or 6 sticks which refer to each job and rings which put through the stick in this manipulative. Then, they showed their students a transition from a dotplot to a frequency bar graph using an online resource which was mentioned during the workshop. The students also follow an activity sheet for above steps. Then the second part of the lesson begins with drawing the histogram. They asked students for listing the data set to the board and for forming a histogram. Then, the lesson ends with a comparison of bar graph and histogram concepts. Throughout the lesson, students use a data set in all steps of the lesson. Before the lesson, the students are requested to bring their data about occupations of their fathers and how many years they have worked in those occupations. In the bar graph part, the occupations of the fathers are used; in the second part of the lesson, the duration how many years their fathers have been working was used for the histogram.

The initial GD of the Group B was summarized regarding the knowledge domains in TPACK framework like for Group A above.

4.2.1.2.1 Content Knowledge (CK)

There are three important findings emerging from initial GD of group B regarding content knowledge: (a) The participants in Group B seemed to lack of statistical content knowledge in bar graph. It could also be specified as a *misconception related with data distribution*. (b) They could not explain *why intervals are used in drawing a histogram*. (c) *They did not know the pictograph as a data display*.

While Group B asks students for putting their rings to the physical manipulative which they prepared to construct the bar graph, they let half of the class to put their rings first and asks some question to the class: “What can you say about the other half? Will the appearance of tool be the same with this or not? Why?” When I asked the reason of asking those questions to them, group members explained that their aim is to show the spread of data is not always the same as they explained in the following:

> Researcher: […] Why do you arrange the timing by dividing students into two groups? I mean you want first half to be filled at first,
> Şenil: We, half of them fill up this, then you look, and tell the student, will the other half be filled exactly like this, or will there be differences? Actually, that data doesn't spread the same, but with the number of people changing.
> Serhat: That is, it can't make generalization.
> Şenil: For example, maybe the student makes the first half and assumes the other half is also the same. In order to prevent that.
> Emel: There were many. For example there were a lot of teachers, is the second part the same?
> Şenil: If 5 out of 10 got teachers, will other 5 out of then get teachers also?
> Emel: Will he move according to a ratio?
> Şenil: Actually we think of it as mean, median, mode, our thing here is to show that data doesn't always spread the same.
> Researcher: So, you are a little focused on the data distribution, right?
> Şenil: Well, yes.
> Emel: It means we are asking a question about it.
> Şenil: We little bit tried to do something with the mean they had learnt before, but…
> Zehra: And after that we ask if we can generalize it to school, it's like a preparation for that.

As can be realized from the above part of discussion, they claim that they are trying to show students cannot generalize the data. They even expressed that they make a connection between measures of central tendency while dealing with the physical manipulative. A distribution of a group of data has some charateristics: the *center* of the distribution, the *variability* of the distribution and the *shape* of the distribution. The center of the distribution means where data are; the variability of data means how data spread out; and, the shape of the distribution provides a general idea about how values are distributed (Albert & Rossman, 2001; Moore & Notz, 2009; Moore, McCaib & Craig, 2009). In the conversation above, Group B tries to make a relation between the number of values in a group of data with the distribution of that data. However, the distribution of a group of data is not related with the number of the values, as can be seen from the above explanation. Thus, it is understood that Group B lacked necessary knowledge about the relation between data distribution and the sampling of a population.

In the second part of the lesson plan which Group B prepared, they tried to make an introduction to histogram and used a second set of data which was related with the first set of data. While they were asking jobs of fathers whose students, they are now asking the years passed for the job of the fathers. They tried to make a connection between first part of the lesson and the second part of the lesson in this way. While they were dealing with categorical data for the bar graph, they were dealing with continuous data for histogram. Using such kind of data, they were making a link in these two groups of data throughout the lesson. Second part of the lesson begins with writing the years which fathers spent for their jobs to the board by the students. Then, there is a question which teacher asks to the students as "Can we apply this procedure for the whole school? Will it be logical?" When I asked the aim of this question to the group members, there is a discussion related with the intervals used for histogram starts among us, as in the following parts:

> Zehra: Our purpose here is, I mean we can face with numbers from 1 to 20 and up to 30 here. We don't know how high the number can get, and at school, there'll be a lot of numbers. Won't it be hard to write them down one by one? How we make it easier, we are asking students to divide them into pieces, actually.
> […]
> Zehra: But the reason why we ask that question there is that the student gives us a feedback with interval. I mean that's not the part we pass on to interval, this is the part we pass on to interval […] In here, after this question, the child is searching for the answer of the question, which is; yes this is difficult, and what else can we apply for this? […] the answer we expect there is interval.
> Researcher: Yes, I see that, you expect the child to give the interval as the answer.
> Şenil: For example; if he says something like 'Can we express the 2-4 interval with a group of data?', that's something we want them to discover, since we assumed they have already knew about the histogram, we thought, if something like this happens, they would think of it this way.
> Researcher: But is that something related to the amount of data or with, well what if he said something like, let's say you're the range of data differs between 17 and 25 years, there are data of 30 students.
> Emel: Hmmm.

> Şenil: Well.
> Researcher: What I'm trying to say here is that, then the question will be useless for you.

As can be seen above, they tried to make a false connection between the number of values in the sample with the range. I understand they think that range increases when the sample increases. As Moore, McCabe and Craig (2009) suggested in their book, a histogram "breaks the range of values of a variable into classes and displays only the count or percent of the observations that fall into each class" (p. 12). There is no relation between the number of observations or cases in sample with drawing its histogram. Thus, Group B thinks that histogram is more reasonable way to look at large sets of data and they use intervals just for this reason while making a histogram. Next conversation related with this issue explains the conflict in which the group fell, as below:

> Zehra: […] Then what if we change the question?
> Şenil: What if it said different varieties, what if we received different data, we can as something like; would it be more logical for us to use bar graph?
> Zehra: I wonder if it would make things easier for us.
> Şenil: That also becomes something like range.
> Zehra: We should take the certain information from the students in advance. I mean, we shouldn't have the data that moment, in such a case, if the range is too much, we can use this question, but if the range is not too much, the teacher can write down some extra himself. For example; in here, how we would use this data, if there were ones and twenty-fives and other things. Then it'd be that easy for us.
> Şenil: See didn't work in their hands, then how they can pass on to histogram, we say if they could, but they couldn't.
> Zehra: Well.
> Researcher: So, how can I put this, we need to think about what is the use in using interval. There needs to be a question that questions that. What is the aim of using interval, let's start with that. What do you think its aim is? Why do we divide histogram with intervals? While making the histogram, why do we divide the data into intervals? […]
> Serhat: To make it look easier, I mean visually, otherwise it might be longer and more complicated.

From above part, it is understood that they try to keep an eye on data set since they form the related data instantly during the class. In case of having a large range at the end, they try to change the action of the teacher with some questions. However, asking such a question which aim is to make students discover why intervals are used in a histogram as they claim, is not related with the number of observations in the data set. As to summarize, using a procedure of applying to whole school does not have any relation with the intervals used in histogram.

From this conversation, it is found that the group members do not have necessary knowledge about why intervals used in drawing a histogram of a set of data and the value of range is independent from the number of observations in the data set.

Third important finding is related with the lack of knowledge about pictograph. When I offered as a different way of displaying the first group of data (categorical data of fathers' jobs), they said that they heard pictograph as the first time.

So, on the whole, Group B lacks of some basic necessary statistical content knowledge in bar graph, histogram and pictograph, which are also fundamental data displays. Further, it can be claimed that they have some misconceptions related with these data displays. They believed that there is relation between the distribution of the data with the number of observations or cases in the data set. Besides, they haven't learned the pictogragh type of data display before, as understood from above conversations.

4.2.1.2.2 Pedagogical Knowledge (PK)

Considering the analysis according to TPACK codebook (Hughes, 2012), Group B refers to *lesson planning activities and preparation* at most. Secondarily most coded item is *knowledge of general teaching methods and strategies* in their first GD. Thirdly, it was seen that *knowledge of learners and their background and checking for understanding* were concerned by Group B. They never mentioned about general assessment strategies or classroom management strategies at all in their first GD.

Lesson planning activities and preparation is the most common indicator of pedagogical knowledge of Group B in their first GD. Group B proceeded through all the steps of preparing a lesson plan starting with choosing an objective at first. They wanted to compare bar graph and histogram in their lesson. They first make students remember bar graph concept in the first part of the lesson and in the second part they give histogram concept with a comparison with bar graph at the end. They discussed about histogram whether they will introduce it for the first time or they will make students remember it after the previous lesson about histogram. They thought that presentation of the lesson depends on this, as in the following conversation:

> Researcher: […] By reviewing the bar graph, you also review the histogram.
> Emel: Yes.
> Researcher: Let's state something here; I mean we've already said it but […] that's how I understood it. I felt like they are also learning the histogram from the beginning. […]
> Zehra: But isn't [our lesson plan] a little insufficient, for this?
> Researcher: Insufficient? No, I don't think so.
> Şenil: Since we think it is insufficient, maybe we act like we are teaching the histogtam directly, what we did actually is that can we show this data group with a bar graph? We thought if we can't show it, if we have a large data group, and we show it with bar graph, will it be useful, we even wrote it. But then we thought about it and if we give it to the students directly, will something appear in their heads about this.
> Zehra: Because before that about the histogram […] we won't be seeing a histogram.
> Şenil: We're conflicted thinking that it may be ineffective.

This conversation is an example of choosing among objectives to implement in a lesson plan and Members of group B disagreed with each other after my comments and left it to solve in the second GD. From here, it could be said that they have an understanding that an instructional objective does have an important effect on the organization of the lesson.

They also discussed how to design the physical manipulative which they used for the bar graph and in which sequence of the lesson they will use it during first GD. They explained the main trick of this physical manipulative as all of the beads should be equal in size and as big as students can see from their seats. They also thought that they didn't want to name the columns of this physical bar graph manipulative since they will instantly collect the data. The related conversation can be seen below:

> Şenil: We created this manipulative and already thought about it a little, we put, following one another, each student puts in the category in whichever his father belongs in, with rings and beads in his hand […]
> Emel: Yes, yes.
> Researcher: We need to write it there though.
> Şenil: Yes, sure.
> Researcher: You can even arrange it in advance. I mean like, here is for an officer, here is for a teacher.
> Zehra: We've already thought we can do it with something like sytropor.
> Şenil: Since we don't really know which job it is, we didn't write them directly. We didn't want to write them when we have no data.
> Zehra: There's no point writing a teacher if there isn't a teacher at all.
> Researcher: Yes. But there has to be an officer. There has to be a self-employed, so maybe you can write these there, arranging empty stickers.
> Emel: Yes.

Related with designing such a physical manipulative, Group B also considers that columns can be arranged as if they can be shifted with each other, as well.

Group B explained why they collect the data instantly and use it for drawing bar graph and histogram, as in the following part of the discussion:

> Researcher: So, you teach histogram in the previous lesson.
> Şenil: Yes. As in the application way.
> Zehra: How can we pass on to real life setting?
> Researcher: In here, it's more like a intensifying about how to show it in real life setting, more like a comparison with bar graph.
> Emel: Its relationship with bar graph…
> Şenil: And since it's a reasearch on their own levels, and they can learn more, we will do it as something extra. It will just intensify his learning.

They thought that collecting data from their students is a nice real-life application which can be used in bar graph and histogram concepts. Thus, it can be claimed that Group B considers real-life application opportunities during lesson plan preparation.

Group B also prepared an activity sheet for their students. Activity sheet is used generally a transition method from class activity to individual activity. After students follow the steps on the board, they fill the related tables in the activity sheet. Then, teacher wants from students to draw the bar graph and histogram. There are also some supplementary questions related with the bar graph and histogram concepts.

Using complimentary examples for data collection procedures in the lesson (jobs of fathers and the years worked by fathers in that job) is also an indicator for lesson planning activities and preparation. They explained that using such an example will make the transition from bar graph to histogram easier and comprehensible and it works for unity of the lesson.

Another discussion which is worth mentioning was about the data collection procedure in the lesson plan. Since they want to show their preparedness, they discussed about possible ways of data collection such as before the lesson or during the lesson. We first discussed possible distribution of the data collected, such as the range of data, min and max values of data, grouping jobs rather than specifying each different job (jobs like greengrocer, butcher, etc. can be grouped as shopkeeper). At the end, I commented that they should mention all of these possibilities and necessary precautions on "remarks on teaching" part of the lesson plan. They agreed that teacher should have an idea about the data before it is collected. Later, one member of Group B offered to collect the data before the lesson since she claimed that making intervals is very important for the histogram part of lesson, so the range is. She thinks that it is difficult to show the intervals while drawing histogram if data collected during the lesson has a small range. The related conversation is as follows:

> Zehra: We better take this data from the students in advance. So, we won't have the information in our hands that exact second, in this case, if the range is too much, we use this question, but if it's not, teacher may write down a few questions himself. For example; if there were some 1s and 25s, how could we use this data? Then, it would be a lot easier for us to do.
> Şenil: If it's not in their hands, then how can they pass on to histogram? We say what if it could, but it didn't.
> Zehra: Yes.

These are the most prominent moments in the first GD of Group B related with the lesson planning activities and preparation. On the whole, it can be resulted that Group B mostly have an understanding of how lessons are planned and how activities are organized during a lesson. They also consider transitions during the lesson as important.

Secondarily, Group B refers much to *knowledge of general teaching methods and strategies* during their first GD. They use several teaching methods and teaching materials in their research lesson such as questioning technique, discussion technique, group working, activity sheets, manipulatives, and virtual manipulatives. Since they prepare an activity sheet for the students to work with indivually, Group B planned to discuss the issues covered on activity sheet with some discussion questions. Thus, it could be said that their major teaching strategy is questioning and discussion methods during their research lesson. Throughout first GD, we discussed their questions, how they ask those questions to students and what they expect from students to discover with them.

In the first part of the lesson which was related with bar graph, Group B included a question as “do all data represent the same quantity, if yes, what do they represent, if no, why, what can you say about the other half?” Then, our discussion starts with what their expectation is from such a question, as in the following:

> Şenil: So, when you put a bead for each student, for example; they put 5 beads to teacher, and 5 to doctor, but one student came along and he put a single bead to the teacher, does this one bead have the same value as the other 5, or after first 5, does it look like he made it look bigger when he out the 6th? That was what we were trying to explain.
> Researcher: Yes.
> Emel: What we are saying is each ring represents the number of jobs.
> Şenil: Yes.
> Researcher: So, each ring stands for a job, sorry represents a person?
> (Everyone says yes)
> Researcher: It represents a person, doesn’t it?
> Şenil: Yes, one person.
> Researcher: Are you trying to say this?
> (Everyone says yes)

Group B stressed out that they will make students to realize each bead refers to one father. Since I thought that their aim is to specify the frequencies of the jobs, I tried to analyze their expectation indeed. At the end of the discussion about the question, they agreed to change the statement of the question in order to make it comprehensible.

One moment of first GD was again related with the question about bar graph display, which came to their minds during the discussion. Below shows the related conversation and the conseqeunces:

> Şenil: Now something came to my mind, can we ask something like this; let's say father of one of the students in the class is a teacher instead of a doctor. How the graph would change according to this?
> Researcher: Well, then wouldn't this part be good to apply, here, for that question?
> Emel: Yes, I think it can be understood.
> Şenil: He can do it both seeing it, and then draw the graph on his activity sheet.
> […]
> Researcher: Would columns changing their places corrupt the meaning of the graph, would it or not, I don't know, I couldn't express it very well. I couldn't write the sentence that well, you can edit it after.
> Emel: Yes.
> […]
> Şenil: And we also said, will it also change according to someone or two poeple?
> Researcher: How does it effect the graph if one or two of them change their places?
> Şenil: Yes, it's better.

At the end, they decided to include these questions into lesson and they left them for second draft of lesson plan to think about the writing statements of them.

Thirdly, Group B discussed *knowledge of learners and their background*. In their first GD, group members seemed to consider their students' background much and they seemed they planned the lesson according to those issues. For example, group B discusses about the duration of the lesson whether it is suitable for objectives and their background, as in the following parts:

> Şenil: I think 2 hours of lessons will be just enough.
> Researcher: Not more, right? That's also what I guessed.
> Zehra: Maybe it could have been a little less.
> Şenil: 2 hours won't be insufficient, will it?

> Zehra: But okay.
> Şenil: I think it won't. Because they are not that young.
> Zehra: Okay but, level of the class is not that low.
> Researcher: 8th grade, right? So the duration is 2 hours. Or let's say 80 minutes.
> (Everyone says yes)
> Researcher: Now here, a graph, one second, I want to ask something, do you teach bar graph to students all over again or do you just remind them?
> Şenil: We remind them.
> Zehra: We are done and now we remind them.
> Emel: They've already seen it at 5th grade.
> Researcher: Okay because that's how I understood it. There's a part for reminding bar graph.

As can be seen from above conversations, they paid attention to students' background and their grade level while planning the lesson regarding the bar graph section. Group B also discusses the adequacy of the lesson regarding the histogram section, as below:

> Researcher: […] okay, actually you review the bar graph, and it means you also review the histogram as well.
> Emel: Yes.
> Researcher: So, let's express that, I mean we've already said it. I mean, on the previous one, that's how I understood it. On the other hand I also felt like they were learning the histogram all from the beginning. […]
> Zehra: Isn't our [lesson plan] that a little insufficient for that?
> Researcher: Insufficient? No, I don't see it that way.
> Şenil: Actually we, we should't act like we teach the histogtram directly, since we think it's insufficient, we actually did something like, can we also show this data with bar graph? If we can't, I mean if we have a large data set, and we show it with a bar graph, will it be helpful? We've already written it. But then we thought if we give it directly, maybe it won't liven up in students' mind.

Group B discussed that whether their lesson is enough to introduce the histogram concept or not during the first GD. Therefore, they left this issue for the next GD to decide. This issue was much concerned issues regarding knowledge of learners and their background through pedagogical knowledge. Based on these conversations, it could be claimed that Group B has an understanding of their students' background knowledge and they tried to take this into account while planning their research lesson.

Therefore, on the whole, much of the conversation regarding pedagogical knowledge was related with three issues: lesson planning activities and preparation, knowledge of general teaching methods and strategies and knowledge of learners and their background. There are also some conversations regarding checking for understanding, however they have already mentioned through the above quotations, so it is not worth to mention them specifically. Group B didn't refer to classroom management techniques or knowledge of general assessment strategies at all during first GD. Nevertheless, Group B could be said to have pedagogical knowledge. They tried to think various teaching alternatives, teaching methods which accord with their objectives and their students' background knowledge.

4.2.1.2.3 Technological Knowledge (TK)

TPACK codebook (Hughes, 2013) claims that operating computer hardware, using standard software tools (e.g. Ms Word etc.) for non-educational use, installing and removing peripheral devices (e.g., USB drives, microphones, etc.), troubleshooting equipment, using appropriate vocabulary and knowledge of current and emergent technologies in society are the indicators of TK. These were not coded in first GD of group B as was for group A. However, these codes were observed through workshop, group discussions and lesson plan implementation periods. Thus, it can be said based on these observations that all of the participants have those abilities which refers back that they have technological knowledge very well.

4.2.1.2.4 Pedagogical Content Knowledge (PCK)

According to TPACK codebook (Hughes, 2013), first GD of group B have two indicators of PCK mainly: *knowledge of teaching/representing subject matter to students* and *identifying and addressing student subject specific misconceptions or mistakes*. Group B has rarely mentioned about the content-specific assessment strategies.

As mentioned before, in the lesson plan, Group B planned to divide the class into two halves while dealing with the physical manipulative (putting a ring for each job of father in order to make a bar graph showing jobs of fathers of the class). They claimed that they did it because they wanted to show that data cannot necessarily continue with the same ratio, i.e. first half of the data should not need to look like the second half of the data, (which was investigated in the CK part of Group B). The relevance of this conversation with PCK is that they also think that this is a way of representing or forming a bar graph to the students with its features. For example, they even further claimed that they could compare with the whole school in order to make a generalization of the data, as can be seen below:

> Researcher: […] So do, you think is [dividing students into two groups and filling the graph after] necessary? I'd like to get your opinion on that.
> Zehra: It's not about dividing the class into two groups, but right now I thought maybe it can be compared to a school. Dividing students into two groups can be too much but can we generalize it to school?
> Şenil: We didn't actually divide the students into two groups.
> Zehra: What I mean by divide is actually first half and second half.
> Şenil: Half of them do it first and the other half does it after. For example; first 10 students go did it and 10 other students are left, after that?

The problem here emerged from the lack of content knowledge about the data distribution and they tried to make it connect with sampling of a population which is false. Therefore, lack of content knowledge here affects their way of thinking representing/teaching the subject of drawing bar graph or discussing on bar graph, i.e., PCK of preservice teachers of Group B.

In order to teach the concept of bar graph, Group B uses physical manipulatives, activity sheets and virtual manipulatives, respectively. While dealing with physical manipulative, Group B aimed to focus on collecting data together from whole class (since they collect the data instantly) and try to show the distribution of the data (which is mentioned above). Secondarily, they want from students to write data in table format as below in Table 4.5 in order to help to draw the bar graph in the next step:

Table 0-5 Table offered by Group B as collecting data from their students.

Occupation	# of fathers

Thirdly, this table will help students to form the bar graph while dealing with the virtual manipulatives. In the physical manipulative part, Group B neither preferred to specify that the resulting view of physical manipulative is a bar graph demonstration, nor they preferred to specify that the table is a frequency table. As a last step, students are told that the outcome of the virtual manipulative is the bar graph display of this data set. This was their way of representation of forming a bar graph for a data set. At this point, it could be claimed that their PCK was enough to generate such an expression of bar graph concept with its all features, however they haven't thought of such a table could also be represented as a frequency table.

Group B discusses the representation of virtual manipulatives part, as in the following:

> Researcher: And then you want children to enter this website.
> Şenil: One minute.
> Researcher: learner.org. For dot plot.
> […]
> Researcher: Yes, you didn't get them to make bar graph.
> Şenil: No, we didn't. They enter the website from there, and do the bar graph according to given information on the site, and after that, it showed how to get to the bar graph through dots, those dots, and those dots showed how much of a space each dot presented. After that we get them make bar graph.
> Researcher: While making bar graph using this tool and the information you see in the site. Oh, with this one.
> Emel: He will continue uniting those two.
> Zehra: Actually that was also our point of putting dots, at the graph, it will benefit from the website.
> Şenil: Those dots are also in this physical manipulative. Each ring in our physical manipulative is also in there.

At first, Group B cannot find a virtual manipulative which directly forms a bar graph with data you entered. Then, they used learner.org in order to show drawing a bar graph (learner.org shows forming a bar graph while making transition from line plot for a continuous data set, which will be mentioned in TCK part). At the end of the conversation, I offered them to search for a virtual manipulative which accords with their initial aim and they accepted. It is worth to emphasize here also that they did not realize that learner.org uses continuous data set to form bar graph while they are dealing with categorical variable (jobs of fathers). Besides, learner.org turns the line plot to form bar graph of raisins per box (virtual manipulative mentioned here can be reached through the link http://www.learner.org/courses/learningmath/data/session2 /part_e/index.html) which Group B specified that they liked it very much because of this turning operation. Consequently, their lack of TCK here affects their PCK in a negative way. The related conversation can be seen below:

> Researcher: [In the workshop] I especially gave them in scatterplot part. […] It's actually only thing I found for scatterplot, I mean I liked it a lot, then I looked and it had everything including scatterplot and bar graph.
> Şenil: Okay, We'll search it, we haven't realized them.
> Zehra: We really liked that turning thing, we actually focused on that directly.
> Researcher: I mean, you can also give that directly, but like I said, there's something, you pass on to continuous while going on categorical, there's something.
> Emel: Okay.
> Şenil: Then we take it out and put one of them.

For the histogram part of the lesson plan, Group B started with a class activity that students come to the board and writes how many years their fathers worked in that job. If the same number occurs, the student writes next to the first one. When I asked the reason of this activity to the group members, they explained that they tried to make students to realize the necessity of using classes in drawing histogram. The related conversation can be seen below:

> Şenil: For example; Ali's father has been working for 6 years, Ali goes and writes down 6. Let's say it was the first time someone works for 6 years in the class. […] And then for example; other students write down, one of them writes 10, one who writes 10, writes it next to 10, one of them writes 5 […] And then other students stands out and says my father

has been working for 6 years, what he will do, he will write 6, next to 6 again. At first we thought about putting a tick mark next to 6, but then they'd think there's one from 6, since there's a tick, we said let's write 6 and that way 6 and 6 will be equal. So that they mean the same thing.
Zehra: It's what I was going to say.
Researcher: So, you don't go into the thing, the interval.
Şenil: No, no we don't. We want them to notice it.

While doing this activity, Group B brings all data together on the board without specifying classes and they asked some questions to introduce the classes for drawing histogram to the students. For example, they ask the question of "what about analyzing these data by grouping instead of one by one?" and, then the question of "can we make again a bar graph using these new data? If yes, how? If no, which graph can we make?" Group B aims that students think about the display type of such data set and discovery the use of intervals for histogram. This type of representing histogram concept shows that Group B tries to make an introduction to histogram concept. Therefore, it can be claimed that they have an understanding of PCK in order to teach histogram concept, but it is not enough to make students to consider how classes are determined.

As understood from the above parts of the discussion, we discussed a lot about aim of using clasess in drawing a histogram. The most prominent cause of this is their lack of understanding about using classes in histogram. Consequently, they could not ask the purpose of using classes to their students. Another reason is that Group B makes a false connection between using classes and the range of data (while trying to connect it generalization to whole school). They have a misconception that if a data set includes more values, then its range is large. Because of this misconception, they tried to make a comparison with whole class or whole school, and they make students to think about this. So, on the whole, they have a lack of understanding about using classes (which I mentioned before in CK part above) and these misunderstandings or misconceptions directs Group B to represent the class concept in a wrong way and they could not find the right questions to ask to the students as well. When I mentioned about this, Group B realized that their misconception and left the revision of the questions for the second draft of the lesson plan. As a result, their PCK seems inadequate for addressing the knowledge of teaching/representing subject matter to students.

Secondarily, Group B referred to identifying and addressing student subject-specific misconceptions or mistakes a lot in their initial GD. During bar graph part of the lesson, Group B asks the question of "do all data represent the same quantity?" to the students, and they expect that students will realize the y-axis is actually a number line. However, their question do not refer to their expectation in a clear way, as can be seen from the following part of discussion:

> Researcher: […] What I mean is, is frequency what you are trying to ask here, the answer to this question, 'what does do all data represent the same quantity mean'?
> Zehra: It doesn't show that all datas are the same for example in here, it's between 3 and 4, the difference between 4 and 5 is 1, and actually the difference between those two is the same. In order to show the unit thing, for example we can make this gap 5, 10, 15 or 2-3 but it's something like we are holding the intervals equal. It's actually what I was trying to explain a little bit […]
> Researcher: Then why are you trying to explain this? Why are you trying to stress it especially?
> Zehra: Because, while drawing the bar graph, students may not draw it equally. In order to prevent that.
> Researcher: What do you mean by equal? Does he need to pay attention to these intervals here?
> Şenil: Yes, for example; when they're drawing the fifth, they can draw a little bit on 4, while those intervals need to be equal. The interval 3 to 4 and the one 4 to 5, need to be the same.
> Researcher: Well, okay.
> Zehra: Actually it has the same logic in fractions. It needs to be equal.
> Researcher: I see. That's the number line thing, I see, here is not an actual number line, there's a categorical situation, but you expect them to see there as a number line.
> Şenil: That those intervals are all equals.

Their aim is to show students that the intervals on y-axis should be drawn equally and all bars (categories in this case) should be arranged according to y-axis clearly in order to prevent any misconception among students. They believe that this can be resulted as a misconception which teacher should be precautious about it. Related with this concept again, Group B was precautious again during physical manipulative period. They were aware of using the unique rings to form the columns for each job since one ring is equivalent to the interval on the y-axis; otherwise it can cause a misconception in the students' minds.

While using the virtual manipulative for bar graph display (learner.org), Group B was not aware of continuous variables used in it since the virtual manipulative uses the transition from line plot to bar graph (raisins per box). When I explained this to the group members as a possible reason to emerge a misconception for students, they realized it immediately and they wanted to think about a different virtual manipulative to use it in that period of the lesson.

Consequently, Group B eventually were convinced that asking questions about generalizations to whole class or whole school can cause misconceptions in the students' ways of thinking. Therefore, they left the revisions of these questions (both for bar graph and histogram parts of lesson plan). They understood that their questions were not related with the aim of understanding the use of intervals in histogram. They realized that their lack of CK caused to think wrongly and to direct wrong questions which in fact altogether effects their PCK.

4.2.1.2.5 Technological Content Knowledge (TCK)

While searching for the virtual manipulatives for bar graph, they specified that they liked the transition from line plot to bar graph in learner.org, they wanted to use it in drawing bar graph. However, they misinterpret it since it uses continuous data which is not suitable way of relating with their categorical data set. They also claimed that they couldn't find any bar graph demonstration which uses the data entered, they found virtual manipulatives which shows the bar graph display with its own data set. After I mentioned my worries about the variable differences and said that there are available virtual manipulatives which uses data entered, they realized that they do not know the existence of a variety of content tools. For example, they neither know the existence of virtual manipulatives for pictograph since they do not know it at all (mentioned in CK part).

Considering the selection of virtual manipulative for histogram, they also claimed that they couldn't find any one of them which shows the transition from frequency table to histogram using the data entered. They only found virtual manipulatives which show the transition from stem and leaf display to histogram which they didn't prefer at all. Because of this, they neglected the use of virtual manipulative for drawing histogram and used only the Table 4.6 shown below they give in activity sheet. However, they left further search about such a virtual manipulative for the second draft of lesson plan, when I persuaded them that they can find one.

Table 0-6 Table used by Group B in order to form histogram.

1-5	6-10	11-15	16-20	21-25	25+

Students were expected to draw this table onto their sheets. For this moment of the lesson, I proposed to use an applet instead of drawing it onto the sheets. Then, the group members specified that they couldn't find a website which shows the data as in the table above, the related conversation is in the following:

> Researcher: Now here, I want to say something else, an important point, I thought why you weren't doing that in here?
> Şenil: Where?
> Researcher: I guess you are making them draw with hand?

> Emel: We couldn’t find that either.
> Researcher: It’s the same, you could have used that again.
> Emel: We haven’t gotten into that website yet.
> Şenil: And since we haven’t gotten into that.
> Zehra: So that’s because of it.
> Emel: We thought a transition from frequency […] to histogram, and I thought we’d have it with the website. But we couldn’t find a website that would first make the frequency and then show histogram after.
> Zehra: And also…
> Şenil: There’s but, It uses only its own data.
> Zehra: The [transition] from the stem and leaf display, the ones that are already done, but this actually…
> Researcher: You don’t get into stem and leaf display.
> Zehra: It’s actually without it, it’s something that gives stem and leaf display subtly. It doesn’t stand out but it’s there.
> Researcher: Okay, you didn’t need to get into that. There’s no such a thing.
> Zehra: That’s why it looked like it’d be easier to turn. Student will see through it.

Another comment of mine was about pictograph: After I mentioned what pictograph is, one member said that she will search for it and can include to lesson plan.

As a result, Group B didn’t make enough search for the variety of virtual manipulatives, therefore, they could not choose the best available one for their objectives. It can be claimed that they have a lack of knowledge of existence of variety of tools which can be used for particular subjects.

Besides to above result, they seemed that they could not make a connection between content and technology reciprocally related to one another throughout the initial GD. For example, a part of conversation related with selection of virtual manipulative for bar graph (using its own data set in learner.org in order to show the bar graph display) is as follows:

> Researcher: Now, I have only one thing, it feels like there’s some kind of a misconnection, I have only worry. How can we prevent this misconnection?
> Zehra: We’ve actully discussed it before but couldn’t come up with a solution.

> Researcher: Now, you already collect a data here […] and instead of just drawing a bar graph, in here students goes in again, and there's something, what was it, a rice?
> Zehra: We had to use the websites, we had to.
> Researcher: Okay, you need to use the websites, but there were some other websites that are for drawing a bar graph.

Moreover, they explained that another reason for choosing learner.org is showing the transition from line plot to bar graph. However, they misregarded it since it doesn't fit their data set. While their data set is categorical, virtual manipulative uses continuous data set, which they never realized before our discussion. Hence, it can be claimed again that members of group B have lack of understanding of the relation between content and technology reciprocally. This can be a result of lack of CK again.

As looking at their lesson plan in general from the TCK point of view, Group B seemed to have tried to integrate the lesson with technology as technology should be integrated in some way. However, it was not in the expected manner which should be in the favor of developing the students' thinking ways or showing students that there are other ways of thinking about the concept.

4.2.1.2.6 Technological Pedagogical Knowledge (TPK)

According to Hughes' (2013) TPACK codebook, there are lots of codes referring to TPK. However, the initial GD of Group B slightly refers to them. There are only two coded sections of discussion related with codes of *knowing about the existence of a variety of technological tools for particular pedagogical tasks* and *differentiating instruction when technology is used*. Therefore, it can be claimed that initial GD of Group B did not show enough indicators of TPK since group members did not have a developed TPK in order to teach a subject while integrating technology regarding pedagogical tasks together.

When Group B could not find any virtual manipulative using data entered for drawing bar graph, they said that they chose the learner.org. Since learner.org used its own data set, they planned to show only forming the bar graph although it uses continuous data.

Hence, they differentiated their instruction according to available technology they found. The related conversation is as follows:

> Şenil: But we thought we would do it this way since we were able to write down our own data directly. Otherwise it'd be awesome if we could write our own data.
> Zehra: We had already searched for it, it was our first goal.

Another indicator of TPK is the code of knowing the existence of a variety of technological tools for particular pedagogical tasks. The conversation again is related with the learner.org which shows the transition from line plot to bar graph using continuous data set. Group members stressed that they chose this virtual manipulative in order to show this transition to the students since it will be beneficial for them. The related section can be seen below:

> Zehra: Because we really liked that transition.
> Şenil: It'll be awesome if we can find it.
> Zehra: It'll be better to see the transition.
> Researcher: Yes, I've liked it as well.
> Zehra: We also prepared the physical manipulative something like that so that the students would see it directly.

So, on the whole, Group B do not show much understanding of TPK necessarily. They rarely referred to the codes of TPACK codebook. It could be claimed that they have not enough understanding to relate technology knowledge with pedagogical tasks which in fact TPK according to TPACK framework.

As a summary of the initial GD of Group B, their lesson plan has little indicators of some components of TPACK framework. Pedagogical knowledge is the most prominent knowledge dimension of group members or it can be said that their pedagogical knowledge has been developed more compared with other knowledge dimensions regarding the lesson plan they prepared. However, their lack of content knowledge in the mentioned subjects above effects very much their PCK and TCK, since their way of represention the subject matter pedagogically or technologically depends on their CK. Their TPK is the least developed knowledge dimension compared to others. Last, but not least, their initial GD seemed as a decision making process for their lesson plan since they could not reach a consensus for most of the parts of the lesson, then they wanted to discuss with me and even they stressed that I chose the discussion topics which there was a conflict among them. Therefore, most of the issues investigated in this discussion are left for the second GD in order to decide on them.

4.2.2 Findings based on second group discussions

This part will summarize the second GDs of both groups of A and B based on the second drafts of their lesson plans. These discussions are also coded according to TPACK codebook (Hughes, 2013), then the format used for initial GD was used again. To begin with, second draft of lesson plan of each group was given with a comparison with the initial versions. At the end of each part, a summary was added.

4.2.2.1 Second GD of Group A

In this part, second GD of Group A will be investigated through the second draft of their lesson plan (can be seen in Appendix K). Second GD is outlined like the initial one according to the knowledge domains in TPACK framework below. In their lesson plan, group members started lesson with an introduction to stem and leaf display and they are making a transition to histogram from stem and leaf display by rotating it. In the second part of the lesson, they will activate the prior knowledge of their students about bar graph with a comparison with histogram. Therefore, it could be said that overall lesson plan have seen to be changed, especially the sequence of the lesson and the examples used for the displays. The detailed analysis of the findings regarding second GD is in the following as organized with the knowledge domains.

4.2.2.1.1 Content Knowledge (CK)

There are three most important findings regarding CK of group members in Group A. During the histogram part of the lesson, they plan to mention about the formula of histogram (how class width is found, how number of classes is calculated, etc.) and the steps of drawing a histogram since Turkish elementary mathematics curriculum includes them as objectives for histogram concept. However, group members has *not enough understanding about the guideline for histogram* and they do *not know exactly the terminology used for histogram concept* (both in Turkish and English languages). Third major finding is in fact a misconception related with stem and leaf display and histogram concept together: Group members has an understanding that each *classes (or stems) will be halved when the class width is halved*. These findings refer again to lack of content knowledge in the concepts of stem and leaf display and histogram. It can be realized that they are different than the initial GD's findings since they have seemed to complete their inadequacies in variable type, differences between a bar graph and a dotplot and choosing the right example for a bar graph.

Firstly, group members did not have enough understanding about how the formula of histogram is applied to a set of data. Since elementary mathematics curriculum includes a guideline for finding the number of classes in a histogram and finding the class width, they wanted to include to their lesson plan as well. In the curriculum, there was an explanation stating "while forming a histogram, range is divided by the chosen number of classes and class width is determined as the minimum natural number according to the following inequality:

$$\frac{range}{number\ of\ classes} < class\ width\text{"}$$

which was also their objective in the lesson plan. However, they were not sure about the calculations. They have a misunderstanding that class width is mostly be chosen as 10 dealing with the formula. Besides, they have another inaccurate information that class width should be n-1 which n refers to the number of values in data. The related conversation is as follows:

> Researcher: How is the formula exactly? I don't remember it exactly.
> Alp: In general they say 10, if they don't, was the number of data – 1, no that's…
> Ezgi: Number of data -1, yes.
> […]
> Researcher: For example let's look for this one then, what happens, what is the smallest range?
> Ezgi: Let's see. For example 0 came up 4 times.
> Ezgi: Yes.
> Researcher: And isn't 21 is the smallest?
> (Everyone says yes)
> Researcher: Is divided by that?
> Alp: They usually take the below as 10.
> Researcher: They usually take the below as 10?
> Ezgi: Actually today we saw that standart deviation thing, and [our statistics teacher] said as number of data – 1.
> Esen: No, but it wasn't that.
> Researcher: You're talking about the one you saw in the statistics lessons.
> (Everyone says yes)
> Alp: [Our statistics teacher] told us to take it as 10 in general, I think she said if not take it as 10, it was n – 1. If the question doesn't specifically said 10.
> Banu: I remember like it was n - 1.
> Researcher: Do you also remember it like n - 1?
> Banu: No, I remember like number of the data was n - 1.
> Researcher: Okay, let's calculate from here, what happens, while the child is writing.
> Alp: 28/14 which equals to 2. That came up really well.
> Gizem: But, it doesn't come up in our data. The thing you say doesn't apply here.
> Researcher: Let's check that one. 56 divided by…
> Gizem: 24 - 1, 23.
> Researcher: What happens, 51 divided by 23 equals to something like 2.
> Ezgi: Yes, yes.
> Researcher: But then if the class width is 2, it'll be too much, but won't it be a nonsense histogram?
> Gizem: Yes.

Alp: That’s why they usuall take it as 10, when we make it 10, something like 5 comes up. I don’t know why but usually 5 comes up anyway.

Later during the second GD, one of the group members searched for the formula of histogram group size specified as in the curriculum and found the following:

Esen: For the formula they say, it says you find the class width for another data set, it explained the value of class width, while finding it, it asks to take the smallest number out of the biggest number, and then to determine the number of classes, as many as we want, it says we can pick up the number of classes less then 10, when we determine it ourselves.
Alp: But the number of the classes…
Researcher: I see, it determines the number of the classes, and after that, it determines the class width according to that. I see.
(Everyone says yes)

Therefore, it could be claimed that they have some inadequacies related with objectives and formula of histogram although they included them in their lesson plan. They have some knowledge about the formula from their statistics courses during second year of education or from their mentor teachers during practice teaching periods.

Secondarily important finding is that group members in Group A often used the wrong terminology either in Turkish or English language and they mixed the ‘range’, ‘class width’ with each other most of the time. A related part of the discussion can be seen below:

Gizem: While I was writing this, I thought about what we thought, why we are drawing now, why we are expressing here again, and we find range, and that range is actually, don’t we do the meaning of finding, our aim is to find the range.
Banu: Yes. To express it.
Researcher: Not range. The class width.
Gizem: Yes, yes.
Banu: Yes.
Gizem: Actually…
Researcher: I suggest you don’t use it in a wrong way in the class. Because range is something diffirent and class width is something different.
Banu: Yes. Because range refers to the interval.
Gizem: Yes. I’m sorry.
Alp: One of them is the difference between the biggest and the smallest.

> Researcher: When we talk about the class width, for each column, this (I'm showing), not that one, right?
> (Everyone says yes)
> Researcher: But range is the difference between max and min.
> (Everyone says yes)
> Researcher: So there's a point that we should be careful about it.
> Gizem: I always thought those two as the same…
> Alp: What is the Turkish of range then?
> Researcher: "Açıklık". Range only means "açıklık", but in here it's class width.
> Banu: For example, is it something like interval, but when we call it "aralık", it sounds something like interval.
> Researcher: In statistics the interval is not used, but the range.
> (Everyone says yes)
> Researcher: And with that it didn't say "aralık" but said "açıklık" in Turkish. So they used a different terminology.

Instead of using 'class width', they mostly used 'range', 'group range' or 'interval' as terms. They used 'range' since they mixed the calculation of class width as subtracting minimum value from the maximum value of data set. They used 'group range' as class width since the Turkish terminology for class width is 'grup açıklığı' as the exact translation. They used also 'interval' since the Turkish translation for width is similar to it. Although we used Turkish language while discussing, they used mostly the English words for them, but the terminology they used were not accurate. Besides, they mostly used inaccurate Turkish terminology such as 'aralık' for class width. So, on the whole, it could be claimed that they have lack of terminology usage in histogram concept, which is problematic during instruction for the students.

Last prominent finding was related with a misconception in both stem-and-leaf display and histogram. While discussing about how their data set is distributed through histogram or stem and leaf display, one of the group members explained that each class is halved when the class width is decreased to its half. She explained that halving class width effects the distribution of the data and students could realize the difference of the distribution while working with virtual manipulative. The related part of conversation can be seen in the following:

> Ezgi: For example; we gave it 10 here, but can we also, when we reduce it to 5, we actually doubly increase these data here, I mean with goes like this with 0 and 0.5, it becomes 4 here, and we reduce this, I mean, can't we just show this like this? For example, we take it, with 0 and 0.2 here, the one we showed as lower and upper…
> Researcher: Yes, the thing here, you mean we do it here.
> Ezgi: No, not here, I am not saying we should do it here, we can do it there, but I mean, wouldn't it be more clear if we explained it like this? Because I mean, now this will stay here, that one will fall aside, this one will stay, this one will fall aside, this one will stay, will fall aside, will increase 2, and I feel like it's more about setting down the logic.
> Researcher: As the class width decrease, you say it changes based on the display.
> Ezgi: Yes. We reduced it to half.

During the workshop, I introduced that stem and leaf display can be drawn using splitting stems into two parts having class widths as 5 in order to increase the number of classes, which is newer to the preservice elementary mathematics teachers (Moore, McCabe & Craig, 2009, p. 11). However, they disregarded that values in a class is not necessarily equal in size for each halves of the class. Therefore, it can be claimed that there was a misconception or misunderstanding related with the class widths while drawing the stem and leaf display or histogram.

As to summarize, the content knowledge of group members has some inadequacies related with histogram concept, its drawing and its terminology use, which are different misunderstandings or misconceptions aroused in the initial GD of them.

4.2.2.1.2 Pedagogical Knowledge (PK)

The second GD of Group A regarding PK has passed through the *lesson planning and preparation* as the most, *knowledge of learners and their background*, *checking for understanding of their learners*, *knowledge of general teaching methods and strategies* and *classroom management strategies* in respectively. Compared with the initial GD, Group A discussed less about lesson planning and preparation phases. They have learned the lesson plan format as well. There is a conversation related with this claim is as follows:

> Ezgi: Can we forget about the first one? There is no first one actually.
> Esen: Actually there is something like, we couldn't focus ourselves on the first one, and didn't know what to do. We had a problem like that. And when you explained it we were able to catch it in a more clear way.
> Researcher: Yes, I also felt something like that. Because you told me you had some problems with the format.

Compared with first draft of lesson plan, second version of their lesson plan has a completely different sequence of steps of the lesson. Instead of beginning with activating prior knowledge about bar graph as in the first draft, they placed it after histogram concept and started with stem and leaf display to their second draft of lesson plan.

They also changed all of the examples throughout the lesson compared with the initial one. For example, since they started with stem and leaf display, they used a continuous data set which is according to this display type. This part forms the activating prior knowledge step of lesson as well. They addressed that they will ask all of the questions can be asked about stem and leaf display at this period in order to make a transition to histogram concept. Specifically, they are planning to ask about the numbers of cases showed in stem and leaf display, the mode of the data set (most repeated value in the data set) and applying to background knowledge in order to handle the misconceptions about stem and leaf display. They also planned that this part should be given through direct instruction and questioning methods. Their related conversation is in the following:

> Researcher: And you decided this lesson will be better with this part, with the best questionning, in an individual way, without the group work, etc. Because . . . what is the reason? You should say.
> Gizem: Group work would just confuse their heads.
> Alp: And it's also a waste of time.
> Gizem: For a question, for example you know but I don't, there needs to be only one correct answer, do it as you know, I don't…
> Researcher: So you say you are doing it like this because you also reminded it.
> Gizem: Indeed.
> (Everyone says yes)
> Banu: No, since we activated the previous data, I don't think, I also think 20 minutes activate their prior knowledge.

While activating their prior knowledge, group members starts with the activity sheet they prepared for their students. Above part mentioned about the first item in the activity sheet which included a stem-and-leaf display already prepared. Second part of the lesson starts with drawing another stem-and-leaf display using a different set of data through a virtual manipulative (http://illuminations.nctm.org/Activity.aspx?id=3476). Students were expected to form a stem and leaf display using the activated prior knowledge during the first step of the lesson. This part includes also the transition to histogram concept. They plan to rotate the stem and leaf display prepared here so as to name it as histogram. The related part of conversation as in the following:

> Gizem: It's not on the website, we will rotate it ourselves.
> Banu: We want them to draw it in their notebooks.
> Ezgi: We will get them rotate it on their notebooks.
> Gizem: For example we gave them the data.
> Researcher: Or they can draw it here.
> Banu: Yes. We actually said activity sheet.
> Gizem: Child sees the data, and draws it on the website.
> Alp: Then we said it here.
> Banu: We already called activity sheet there. Yes.
> Researcher: Then they will make stem and leaf display over here. And make histogram there.
> Banu: No, not like this.
> Gizem: No.
> Banu: They will create the stem and leaf display on NCTM website by giving this data.
> Researcher: Oh, they create stem and leaf display on there.
> Banu: We want them to draw the stem and leaf display to the activity sheet in which they created in their notebooks.
> Gizem: So that they can have it under their hands.
> Banu: And we will use the smartboard with them, we told them that we would draw it there. While we rotate the stem and leaf display on the board, they rotate in on their activity sheets.

This second step of the lesson continues with studying with virtual manipulative. The virtual manipulative on the Illuminations Web site has an option that a data set can be shown with different types of displays, such as stem and leaf display and histogram. Group members uses this property of virtual manipulative and asks from their learners to preview the histogram of their data set which they already displayed with stemplot. The transition to histogram concept takes place during this period and they will explain this as below:

> Gizem: […] And then, I say, for example; I draw it like this, at the same time, children have already entered the data that are given, and when they enter, they will have this in their hands, it will be on the board, and also they will open those websites and see the histogtam.
> Researcher: They will click on the histogram button.
> Gizem: Yes, they will click on the histogram button, they will also see that. […]
> Gizem: We've said it creates a different graph. A different display, and we call it histogram.
> Esen: We call it histogram.
> Banu: Yes, then we explain how to draw histogram.
> Gizem: And, range, class width, how to determine the range, I will explain how to write it one by one. How to gather the columns.

There is also a discussion related with data set used for this part of the lesson among the group members. Since they will make the students change class width while studying with virtual manipulative, they will expect from students to answer questions related with the distribution of data set through different class widths. Then, my suggestion was that their data set should show this difference and they should revise the data set according to it, as in the following:

> Researcher: In order to make that spread look better, we can take those numbers…
> Gizem: To look better? Then both the highest and the lowest…
> Researcher: I mean, sharper things to the lowest-highest parts, and maybe we can change here according to that.
> (Everyone says yes)
> Researcher: Think about it. For example one makes a peak so much in one place. For example in here, like this one can be a little lower, I don't know.
> Gizem: We can switch it with this one.
> Ezgi: Because for example; there will be 3, 3 again, it wil look like 3 took its place, I wonder if there are only 3 and 4…
> Researcher: It can make a peak in somewhere, I don't know, there can be 9 terms. The shape may look better this way.
> (Evereyone says yes)

Then, lesson continues with the comparison of the concepts of bar graph and histogram as the last step of the lesson. They turned back to activity sheet again in order to show a data including categorical variable and its bar graph together. Students will first analyze the data from the frequency table and see its bar graph. They chose the discussion technique in order to activate the prior knowledge about bar graph since their students have learned bar graph concept formerly. The related conversation is as follows:

> Researcher: In here, there is a categorical data, right?
> (Everyone says yes)
> Researcher: And the one below its bar graph version.
> (Everyone says yes)
> Researcher: After that you drew this and showed.
> Gizem: No, we don't draw it at that point.
> Banu: He will look at it, as a visual, we give it that way.
> Researcher: For the lesson plan you mean. You prepared this and drew and showed, and gave to the children, and they from here, what do you ask about this later?
> Banu: What do we ask?
> Esen: We ask its most obvious feature.
> Gizem: Yes, well first we ask about things that remind bar graph. To remind.
> Banu: Yes we didn't mean to do the thing.
> Researcher: But you say bar graph.
> Banu: Yes we do.
> Gizem: Yes we do.
> Researcher: You especially express that it's a bar graph.
> Gizem: Not to teach but to remind, but its name is bar graph.
> Banu: We say that it's a bar graph, that's why here is which types of movies are most liked, the ones that liked the most and the least.

They do not want to mention about bar graph more since they thought that students knew the concept for the years and activating prior knowledge is enough. They only asked a question which makes students to do comparison with histogram, as in the following:

> Researcher: Okay, and then about the bar graph which is the most liked movie, and which is the least one, you say drama or for example romance. And after that, can we show the data…
> Banu: We ask if we can show it in histogram.
> Researcher: So, is this the only question you ask about bar graph?

> (Everyone says yes)
> Esen: We didn't get deep inside it.
> Gizem: Because they've known it from, I don't know which grade, and will be seeing. Now, they are subtly hidden inside the questions.
> Banu: They'll have be seen this at 6th grade.

Their lesson ends with the comparison of bar graph and histogram concepts and Group A thinks that a summary as a last step is not necessary. Since their comparison step includes think-pair-share method with some discussion questions, it is enough to summarize the whole lesson as well. The related part of the conversation is as follows:

> Researcher: Do you summarize it after you explain all the differences between bar graph and histogram?
> Banu: No, we haven't written that.
> Alp: We didn't think it would be necessary.
> Researcher: There's no such a thing.
> Alp: We thought think-pair-share already would be something like a summary.
> Researcher: I mean, yes. It look a bit like summary, you both remind bar graph and histogram and compare them. Like that yes. Nice. I think this, there is actually a difference between first and second lesson plan, and this is way better, I found it really professional. Thank you for your effort.

In general, Group A revised their lesson plan in a more consistent and an organized way with respect to its initial version. The steps of the lesson and their expectations from students were much clearer than the previous one. Their lesson plan can be claimed that the teacher knows how to follow the lesson in a more understandable and clearer way than the previous one. Therefore, it can be said that their pedagogical knowledge regarding lesson planning and preparation seems to be more developed than the previous lesson planning efforts of Group A. This was the first finding regarding PK of Group A.

Another finding was related with knowledge of learners and their background. Based on coding of second GD, members of Group A referred more onto this issue compared with initial GD. They revised their lesson plan without ignoring the knowledge of learners and their prior knowledge. The new sequence of the steps of the lesson accords with it as well. They haven't disregarded the background knowledge of students about stem and leaf display and bar graph concepts. Besides, they mentioned that the measures of center can be reminded onto the stem and leaf display in order to investigate it completely during first step of the lesson, however; they realized that these can be too much with respect to their lesson objective. The conversation is as follows:

> Gizem: I may ask median here.
> Researcher: Can we ask mode, or questions like can we find the median?
> Gizem: Since he already seen it at 7th grade.
> Alp: Actually it can be useful in the median as well.
> Researcher: I mean they will remember stem and leaf display, we can ask anything which we want students to realize or think about.
> Ezgi: Okay.
> Researcher: Anything we want them to brainstorm about.
> Gizem: Can I ask something? For example; we ask mode median here, and explained that, but then I don't know it feels disconnected. I mean we will tell about histogram, but I tell about this and I tell about that. . .
> Researcher: So you mean, you better don't get inside it? I see.
> Gizem: I talk about mode median, and stem and leaf display, and later histogram, and bar graph, it's too much for an hour. We say, student sees this, and we turn it from here. I don't want there to have too much objectives. My aim is not its characteristics.

While dealing with possible misconceptions about stemplot, they also offered to mention about 'placevalue' concept or 'distributive law' which students already knew before. One of the members suggested them as in the following:

> Gizem: I understand, if he thinks something like that, then I pass onto placevalue, and basically I separate the places.
> […]
> Gizem: For example I can say, since I started with ones and tens. For example; they know about distributive law. I say; I thought about that; for example from here, it's like I'm separating 2, it becomes 21 and 25. And then, there is no need to separate them one by one.

Therefore, an important finding can be resulted as Group A dealt with students' background knowledge more as they integrated with lesson plan more deeply. The lesson plan preparation supported the preservice teachers to form more relationships among knowledge dimensions in TPACK framework and made them more conscious about PK.

Thirdly, Group A considered checking for understanding more in their revised lesson plan. Throughout the lesson, they checked for students' understanding using questioning technique. For example, in the first step of the lesson, they expected from students to write the data set related with the stemplot given to students in the activity sheet. The related conversation is as follows:

> Researcher: […] You give [stemplot] to students.
> (Everyone says yes)
> Researcher: You give it as it is. And then you check if the student understands what it is or not.
> Alp: Yes. Yes expect them to write down all the data.

In the histogram step of the lesson plan, Group A planned to change the class width used in drawing histogram onto the virtual manipulative. They have taken some precautions while dealing with the changing class width through virtual manipulative for both drawing stemplot and histogram in order to check for students' understanding. Their aim is to hear the same response that changing class width in both stemplot and histogram makes the same effect on distribution of data on both displays. The related parts of conversation are as follows:

> Researcher: So, we all agree at this point, it already shows 10 by 10, he will pass onto histogram, then he will see how histogram changes when he reduces the class width to 5. And then maybe show how a stem and leaf display can be, I mean you may not have them make it on the website but, you can ask students how it would change.
> (Everyone says yes)
> Researcher: Because you were already making a transition by drawing.
> Gizem: Yes.
> Researcher: They are supposed to give the same answer there, something like: 'It would come up as the same, teacher.'
> […]
> Gizem: […] For example when we take [class width] as 5, 6 as the number of [classes] increase, the columns also increase, this time their heights gets smaller, what it means.
> Researcher: Yes, an expanding data like this. . .
> Gizem: Yes, exactly.
> Researcher: I mean, maybe we should also expect children to give this answer.
> Gizem: Yes, we can think one by one.

> Researcher: When we say what is going on here, they can say something like, this data is expanding. Maybe something like this.
> (Everyone says yes)

Therefore, it can be claimed that members of group A paid more attention to students' understanding in their revision process of lesson plan and they planned secondarily while considering students' anticipated responses more than the previous lesson plan. This finding can be specified as prominent regarding that it makes me to think about a more developed PK or more conscious PK of Group A.

Group A referred to general teaching methods and strategies which they used during revision of lesson plan in their second GD. They mentioned almost the same teaching methods as direct instruction, questioning, think-pair-share method as a discussion technique, using activity sheet and individual work. The teacher again controls the steps of the lesson based on the activity sheet like in the previous lesson plan Group A presented. They mostly didn't change their initial decisions and planned to go through with the questions they ask in the lesson.

Like in the initial lesson plan, Group A specifically did not pay attention to general assessment strategies in the revised one. However, they are observed to consider about classroom management skills which was not seen in the initial GD. The reason might be determining the classroom which the implementation of lesson plan will take place. Since the classroom will be a computer laboratory, members of group A planned not to include group work among students since the students' desks are would not be suitable to move. The related conversation is in the following:

> Researcher: You can also arrange the class setting for yourselves.
> Gizem: Should I change it when I enter the class? I will definitely change the seat of Alp…
> Researcher: We can make a group work, they can sit individually. Or in twos.
> Ezgi: But, how can we make them as a circle in the computer lab?
> Researcher: Yes, yes.
> Gizem: I can only change their sittings.
> Researcher: But at this point the computer lab will be available.
> Banu: And, do we have a group work?

> Gizem: No, we have class discussion.
> Alp: There was think-pair-share.
> Gizem: That too, as discussion, there's no need to change sits.

It is understood from here that members of group A considered classroom management strategies more important compared with first lesson plan and they used them as a factor to decide on selecting teaching methods and strategies.

Hence, to summarize, Group A demonstrated higher effort to develop their knowledge about pedagogical strategies and issues in each factor of PK mentioned in TPACK codebook (Hughes, 2013). It can be claimed that their PK seemed to be developed during second GD and through revised lesson plan compared with one in initial GD through first lesson plan.

4.2.2.1.3 Technological Knowledge (TK)

As stressed in the TK part of initial GD for Group A, technological knowledge factors mentioned in TPACK codebook (Hughes, 2013) still observed during second GD. Besides, group members seemed to capable enough using smartboard placed in classrooms and planned to integrate it into their revised lesson plan. They had necessary knowledge of how to control smartboard, how to write onto smartboard, how to rotate a picture in smartboard and so on. The related part of the discussion is as follows:

> Researcher: What do you do after?
> Banu: After that, I think we need to guide. At the rotation part. On our smartboard.
> Researcher: After, now, how is that thing you call rotation?
> Ezgi: We hold it like this directly.
> Researcher: You hold it like this (by holding the paper in the hand)
> Ezgi: Yes, exactly.
> Gizem: Yes, exactly. I will draw in on the board but the children will rotate it.
> Ezgi: Yes. And also we are more comfortable doing it on the board. They can rotate it right away.
> Researcher: Do they? Okay, awesome.
> Gizem: And then those, I drew them on the board and they rotated it like this, we explain it like that, for example like; right now I assume there are some stuff there.

> Researcher: Can you draw on the smartboard with a pen?
> Gizem: With a special pen. We can also do it in colours. One with red, one with blue and one with yellow.
> Alp: It can even be written by a finger.
> Researcher: Okay then.
> Gizem: It needs first to be activated.

From here, it can be claimed that Group A started to think to include technology and technological tools/items into their lesson more as compared with the initial lesson plan. Therefore, members of group A said to have developed TK which is also a part of TPACK framework.

4.2.2.1.4 Pedagogical Content Knowledge (PCK)

Regarding PCK factors specified in TPACK Codebook (Hughes, 2013), second GD of members of group A has been coded nearly twice as high as the initial GD. This means that Group A dealt with representation of subject matter to the students and identifying/addressing students' misconceptions or mistakes more than the previous one.

Specifically, members of group A often discussed *how to ask questions during the lesson and in which order they will be asked; sequence of the lesson activities and which steps of lesson are included; teaching formula of histogram, steps of drawing a histogram and how class width is analyzed by students; and, implementation of teaching methods especially questioning and discussion (think-pair-share) regarding teaching/representing techniques/methods/strategies.* These were different issues than the ones covered in initial GD. Therefore, Group A revised their lesson plans based on previous issues and there was nothing brought to discuss to the second GD.

First conversation specifying PCK is related with the very first question asked during the stemplot period of the lesson: 'how many supermarket are there?' Then, they realized that they have another alternative unless their students came up with the right answer to above question. Two group members discussed it as in the following:

> Gizem: No, I will first ask what I was going to, how many there will be, how many stores, how many cases…
> Banu: You directly ask how many stores there are.

> Gizem: Yes. For example; my first question […]
> Gizem: Then he says 20 of them, then he will say…
> Ezgi: Yes, me too.
> Gizem: […] This was my sequence I thought of.
> Banu: Personally, I wouldn't start with there are stores, I would start directly with reading it, I mean I'd introduce like how they give a meaning to it, and how we write it.
> Ezgi: But would you give tens and ones?
> Banu: I'd get into the number of stores after actually. Tens and ones would depend on the children's situation, no need if they know about it.
> Gizem: So, there's no need.
> Banu: But if they are wrong, you get into tens and ones, and then into explanation, and then to maximum and minumum.
> Gizem: Then, after explaining all of these, since he will say 21, 22, 23, we will need to make an explanation and tell them that it will be enough to count just here…
> Banu: Yes, they will already be seen there, and then I think we should write down the number of hats and toys on the bottom. We've changed the question.

Here, group members discussed that students can have a misconception about reading the stemplot if they can't remember the related background knowledge. Group A discussed how to continue with the lesson whether students don't remember about stem and leaf display or not. This was a finding that Group A is conscious about different alternatives for representing or asking about the stemplot according to their students' background knowledge.

Another important discussion made about representing way of formula of histogram and steps of drawing a histogram in the second GD. As stated earlier, members of group A wanted to mention about the formula and the steps of drawing histogram since the elementary mathematics curriculum included them as well. Once they made the transition from stemplot to histogram on the board and students make it with their activity sheets, teacher teaches that there is a formula for finding class width in histogram and there are steps to draw a histogram. In my opinion, this conflicts with their previous transition activity since their aim was to demonstrate that histogram looks like stemplot very much which we do not have steps or formula for forming a stemplot. Therefore, they discriminated these two displays while giving a formula and steps for one of them and not giving for the other. As I understood, group members wanted to show histogram in a meaningful way where there is a logic behind the drawing it especially through the use of virtual manipulatives; however, they seemed to make their work easy to handle when they want to include formula or steps of drawing histogram. Hence, it could be claimed that preservice teachers cannot defend their initial aims when they feel the pressure of corresponding to the curriculum since they have been told that their lesson plan will be for today's teachers and should be applicable in a math class. The related conversation is as follows:

> Banu: We said it because it's written on the objective.
> Gizem: Exactly.
> Alp: But I think we should find this class width.
> Gizem: I think I will give too.
> Esen: We said we'd say it, in the steps.
> Ezgi: I mean maybe we can take it out and then give.
> Researcher: I'm opposed to this idea, I mean feeling like we need to solve every problem no matter what it is with a formula.
> Banu: It already doesn't stick to the mind.
> Researcher: Personally; I'm not a fan of getting children to memorize all this. I don't know if you see this necessery, is it necessery to give it this way? I mean even if you never mention it, when the group is explained at the stem and leaf display, it will mention class width there.
> Gizem: Yes, he will see that it's tens by tens.
> Ezgi: Maybe we can explain its logic. I mean, it's like this, the child will understand that he needs to devide it to find… Because we reach to a small piece, from the total I mean, we can say that.
> Researcher: Then, you, was it on this page, so you basically talk about this right? You talk about the formula?
> Alp: Yes.
> Banu: Yes, like we said, steps of the histogram, we said the steps of drawing the histogram. […]

As can be seen from above conversation, Ezgi, for example, explained the idea of making the transition from stem and leaf display to histogram in order to make students to discover how to find class width and number of classes. Despite this, they insist on mentioning about the formula and the steps of drawing histogram.

Besides, they explained the reasons of teaching formula to find the class width in histogram as students will meet with such questions about histogram concept, as in the following conversation:

> Gizem: Okay then we do it like we do it in the class, as it's on the curriculum.
> Researcher: I assume you especially want to give this.
> Ezgi: No, no. Because it's on the curriculum.
> Gizem: I mean it's already on the curriculum.
> Gizem: They will be held responsible from these.
> Alp: Class width won't appear to children 10 by 10 as it's on stem and leaf display.
> Gizem: Yes, it will actually appear like this, like it will give this first and then have them draw the histogram.
> Researcher: I mean you don't have to give it just because it's on the curriculum, there's no such a thing.
> Banu: I thought, while giving the steps about histogram, rather saying range equals bla bla in the 3th step, we should determine the class width while drawing the histogram, maybe while talking about it we can say 'there's also a formula like this'. Of course with explaining it first.
> Researcher: Then you, yes, actually. . .
> Banu: I mean, so that it won't something like 3th step equals to etc.
> Gizem: Actually, also while drawing stem and leaf display, there is a range while saying tens by tens, 0, 1, 2, 3, I increase 10 by 10, then I need to determine class width while drawing the histogram, we can pass on from this to that by saying how much I need to increase this.
> Ezgi: Yes, exactly. For example; at this point, it's a class width like this, I think most of the students can not explain exactly what a class width means. For example; let's say the class width included 5 in there, they don't know exactly what that means, and we can give this and that only because they go with memorizing all of them.

Although I reminded that they might not use the way of representing histogram concept as in the curriculum, they tried to convince me about questions which students encounter during their exams. It can be claimed that members of Group A thought that they feel that they should rely on curriculum strictly since their students will encounter the subjects as are covered in curriculum in the national or class exams. (These findings related with curriculum pressure will be evaluated in discussion part later.)

There is a question which Group A asked to students: ‘What is the purpose of following steps of drawing histogram?’ When I asked the reason for the asking of this question, the conversation is as below:

> Researcher: Okay yes, what is the purpose of that question?
> Ezgi: Actually, it’s stem and leaf display, our goal of drawing this, can we say this in a simple way? The reason why we draw and rotate it.
> Banu: What did we think of this while drawing it?
> […]
> Gizem: While writing this I thought we were thinking about, why we are drawing right now, what we are expressing, we find the range, but do we actually do the meaning of finding the range, our goal there is to find the range.
> Banu: Yes. To express this.

As can be seen, group members cannot explain the aim of directing such a question clearly, which emerges as another finding that members of group A are claimed to disregard their initial attempt of discovery of finding class width through transition from stemplot and they overstated the usage of steps while drawing histogram.

Another important discussion made about the last period of the lesson which bar graph and histogram are compared with each other. Group A planned to make the representation of bar graph with a new data set including categorical variable in the activity sheets of students. There is a frequency table for the data set used and there is a bar graph for this data, already. Then, they discussed about how they will continue to this period and the questions they will ask at this moment, as in the following:

> Researcher: […] You prepared this, […] you also give it to children, then what do you ask about it?
> Esen: We ask the most obvious quality.
> Gizem: Yes, well first we ask about things that remind bar graph. To remind.
> Banu: Yes we didn’t mean to do the thing.
> Researcher: But you say bar graph.
> Banu: Yes we do.
> Gizem: Yes we do.
> Researcher: You especially express that it’s a bar graph.
> Banu: We say that it’s a bar graph, that’s why we didn’t want to get deeper in bar graph since we recall it only.

> Gizem: It is not to teach, but to recall.
> Banu: We say it is a bar graph and here we ask 'which types of movies are most liked, the ones that liked the most and the least'.
> Gizem: How it would be if we didn't give its name?
> Ezgi: I also thought about it, maybe we shouldn't have use the bar graph name, maybe we could make a transition by saying it's a different than the previous graph.
> Gizem: Well, it is bar graph, he should also know it's bar graph, what difference there is between them, he will make an assumption.
> Gizem: He will, I think it won't matter if he knows its name or not.
> Ezgi: Okay.
> […] Banu: I think, let's not create a confusion there.
> Ezgi: Okay, I don't know, I thought it would make a transition to histogram…
> Banu: But our goal here is to raise an awareness.

They also discussed the questions in this period and why they chose these questions and how they will ask them, as in the following conversation:

> Researcher: Okay, and then about the bar graph which is the most liked movie, and which is the least type, you say drama or for example romance. And after that, can we show the data. . .
> Banu: We ask if we can show it in histogram.
> Researcher: So, is this the only question you ask about bar graph? (Everyone says yes)
> Esen: We didn't get deep inside it.
> Gizem: Because they've known it from, I don't know which grade, and will be seeing. Now, they are subtly hidden inside the questions.
> Banu: They'll be seeing this at 6th grade.
> Gizem: They already know about this. From there we pass onto this, to the second question.
> Researcher: And then using histogram, he'll say, 'yes teacher there's a student', what will you do when he says it can be drawn?
> Gizem: What was our steps?
> Researcher: Which steps, steps while drawing the histogram?
> Gizem: Yes.

As can be realized from above conversation that members of group A plans that they will explain the differences of bar graph and histogram by reminding students about the steps of drawing histogram. This is another prominent finding related with use of formula of histogram and steps of drawing it, related with above finding. It can be claimed that Group A seems to manage students' questions easily with the help of such representing way of histogram without making explanations about whether categories are used in or not, for instance.

Last important finding is related with a decision whether they include another step of the lesson as summary of all subjects covered through it. Their conversation showed that there is no need to any more steps for a summary.

On the whole, members of group A can be claimed to be conscious about their teaching/representing techniques/strategies as much as in their initial GD. Although we discussed through almost two different lesson plans (initial and revised ones), they were still precautious about asking questions, sequence of questions, in which activity these questions will be asked, whether there is a need for summary part additionally to the lesson, and so on. Besides, it is resulted that they feel pressure for relying on curriculum onto their decisions for representing/teaching the subject matter.

Secondarily, members of group A was observed to think about alternative solutions whether their students have a misconception or make a mistake. They have predicted these misconceptions and they tried to make new relations based on students' background knowledge especially for *reading stemplot and finding the class width while drawing histogram*. Without searching literature about existence of such a misconception, Group A thinks that students might multiply the rows in stemplot with its first digit. The related discussion is as follows:

> Gizem: I mean, we said child may think as 2x1. I mean, he may have done it wrong.
> Banu: We thought if he learnt it wrong or don't remember correctly…
> Researcher: Is there really a misconception like that?
> Esen: It can be done. We thought it.
> Gizem: They can […]
> Researcher: […] Is there a real misconceotion in the literature or is it just your opinion?
> Banu: No, no we didn't write it based on the literature. Yes, it's our opinion.
> Esen: We've never checked the literature. It's our opinion.

> Ezgi: Here we actually thought, the multiplication table is usually given in a square shape. It's whether, to multiply or to add, I mean they multiply there, I wonder if it can be the same thing here?
> Researcher: It feels like they are distributing.
> Ezgi: Yes, yes.
> Gizem: They multiply this with this, maybe there isn't, and then he will multiply this with that and get to the result.
> Ezgi: That is what we thought. I mean it came up little bit from that.
> Gizem: Because their shapes are alike.
> Researcher: So you make a comment saying there can be a misconcepiton like this.
> (Everyone says yes)
> Esen: We didn't checked the literature.

They claimed also that students could misinterpret the rows with no digits as supermarkets without having toys in the example they used for stemplot. When I asked them whether they have some overcoming methods for these misconceptions, they proposed relations with two subjects: place value concept (tens, ones, etc. as digit names) and distributive law. The related conversation is below:

> Gizem: We said, for example; the child says first market and the second market, we wrote it here, maybe I can say, let's think of here as the placevalue, here is tens digit and here is ones digit. Or maybe I can ask something like, how many stores there are, I mean the child has to check first, and say there're 5 stores, I wonder if we are sure.
> Alp: Or we can say what zero stands for.
> Gizem: Yes. We thought we can take it further with these kind of questions.
> Researcher: So in this case you ask everything there is to be asked about stem and leaf display?
> (Everyone says yes)
> Gizem: But depends on the children and the answers that will come.
> Researcher: Okay.
> Gizem: We may never get into this, if they know.

They suggested that the digit on the left could be said as tens and the other digits on the right of line could be said as ones. This solution seems nice unless students can think of a rational number such as 2,1455578 for the third row of their example. When I reminded them about this, they said that their statistics instructor has also mentioned about this in their statistics courses during their second year of education. The related discussion of misconceptions aroused because of drawing style of stemplot since it shows that group A thought about possible misconceptions and overcoming methods seriously, is as in the following:

> Researcher: So, we can even say there are imaginary lines like that.
> Banu: Yes, it can be said that way.
> Researcher: Those are some series like this.
> Gizem: I mean if we ask how many stores there are, each one of them here, 1, 2, 3, 4, 5, 6, 7, 8, 9 it goes there anyway, the question of how many stores goes there also. More than 5.
> Ezgi: There's also an imaginary line, because I thought otherwise it'd take one to this.
> Gizem: Yes, that's also what I thought. Because we split them digit by digit.
> Researcher: Then…
> Ezgi: Actually, the student is aware that each of them is in different places, but to there, I mean, he only thinks this line as the main line.
> Gizem: I agree.
> Researcher: Yes, if you draw this on the board, I mean they always draw those lines really close which can lead them to think that. That happens.
> (Everyone says yes)

They also generated another sequence in order to overcome the misconception related with the number of cases observed in a stem-and-leaf plot as in the following:

> Gizem: Okay I want to ask something. I was asking about how many cases there were, the child collected here and said 15, (ones right side of the line), here (left side of the line) and said 5, which equals to 20.
> Researcher: So, you are saying he also counted those.
> Ezgi: Yes, this is also a misconception.
> Gizem: Now, I thought what I would say then, I mean I didn't draw this line for nothing. (Giggles)
> Ezgi: By the way, another question came up.
> Gizem: For example I can say, since I started with ones and tens. For example; they know about distributive law. I say; I thought about that; for example from here, it's like I'm distributing 2, it becomes 21 and 25. And then, there is no need to separate them one by one.
> Ezgi: I have to say something, if you say distributing it may refer to multiplying. Don't do that.
> Banu: Yes.
> Ezgi: Since we said 2x1, I'd think it was a multiplication.
> Researcher: Then, the child may think he is adding them together.

> Ezgi: Yes, yes, multiplying is just like adding, anything can come up.
> Banu: I also want to say something. I said here, we will explain this relationship by giving open form, but I think when you give the form, once again, you actually block it.

They thought about the above possible misconception which is making addition of all digits seen in the plot while finding the number of cases and they suggested 'distributive law' as a method for overcoming it. Then they discussed other issues which can be resulted because of the idea of multiplication and addition in distributive law. This finding shows that they were capable enough to be able to discuss the possible misconceptions and they can make relations with the previous knowledge of students.

Another possible misconception offered by Group A regarding class width of histogram is that student can have difficulty in finding or determining the class width of a given histogram. Their possible suggestion to overcome it with the help of virtual manipulatives is as follows:

> Gizem: Actually, also while drawing stem and leaf display, there is a range while saying tens by tens, 0, 1, 2, 3, I increase 10 by 10, then I need to determine a class width while drawing the histogram, we can pass on from this to that by saying how much I need to increase this.
> Ezgi: Yes, exactly. For example; at this point, it's a class width like this, I think most of the students can not explain exactly what a class width means. For example; let's say the class width included 5 in there, they don't know exactly what that means, and we can give this and that only because they go with memorizing all of them.
> Researcher: Yes, that also bothers me, we force children to memorize all these formulas, so that they remember it, we may never get into that, I don't remember who it was who said it Ezgi or Banu, first you can tell what it is, how you find it or even draw some different histograms 10 by 10.
> Banu: Yes.
> Ezgi: Yes.
> Alp: You can have them draw again.
> Researcher: So that child can also see that it is different. At the end, we are integrating technology for such aims.
> Gizem: We make it 5 and it'll be a new histogram again.
> Ezgi: Yes.

They also specified that this misconception related with finding class width in histogram can be overcome while using the formula mentioned in curriculum although I addressed that there is another way rather than memorizing steps of drawing histogram. It is interesting that although group members were aware that students were forced to memorize the concept while giving such a formula, they tried to justify their way of representation on the basis of curriculum.

So, on the whole, it can be claimed that members of group A could think about the misconceptions which can possibly arouse in their students' minds and see the mistakes students might experience. Further, they had the ability to generate some alternative representing or teaching strategies whether they are reasonable or not in order to manage them.

4.2.2.1.5 Technological Content Knowledge (TCK)

Concerning TCK of group A members, it can be claimed that they were observed to have a developed understanding for the issues which are knowing the existence of a variety of content tools, operating content-based technologies and alternative ways in order for relating content and technology.

Regarding TCK, we discussed mostly on the website of NCTM's illuminations page (http://illuminations.nctm.org/Activity.aspx?id=3476) and the usage of smart boards existing in the classroom. This virtual manipulative offers to students and teachers to show the data in five different displays which are box plotter, bubble, scatterplot, histogram and stem and leaf display. For each display, only one set of data can be used. Group members have already learned it and explained as in the following:

> Gizem: There's something about this website and it's that you enter one data and it will be enough.
> Researcher: I checked again and there were different buttons.
> Banu: Yes.
> Esen: Exactly.
> Gizem: We don't need to enter the data twice, then. After creating the stem and leaf display, student should also draw it on here, because when

> he gets home, he won't have anythnig in his hands about it. We said, if he draws on here, then he will hav…
> Researcher: You mean they will keep the [activity sheets].
> Gizem: Yes.
> Gizem: He will rotate it, and when he clicks on the histogram button, he can directly see it by previewing it. So they will both check the website and use it, and we thought it would be a lot easier. In order them to see.
> Researcher: So, what is your main goal of using tecnology here?
> Gizem: So that it would make it easier to enter the data. Because, while entering the data child doesn't do it, I mean he needs to enter 5, 6, 7 in this order, we give them this as ordered, but even if we didn't, stem and leaf display is created automatically on there.

Based on above conversation, it can be understood that they also learned that values of data is seen as ordered although they were entered as mixed in stem and leaf display. They thought that students could realize stem-and-leaf plot sequences the values of data because of this property of virtual manipulative (can be seen from below pictures showed in Figure 4.3). This was the finding showing that group members could relate content and technology reciprocally with each other.

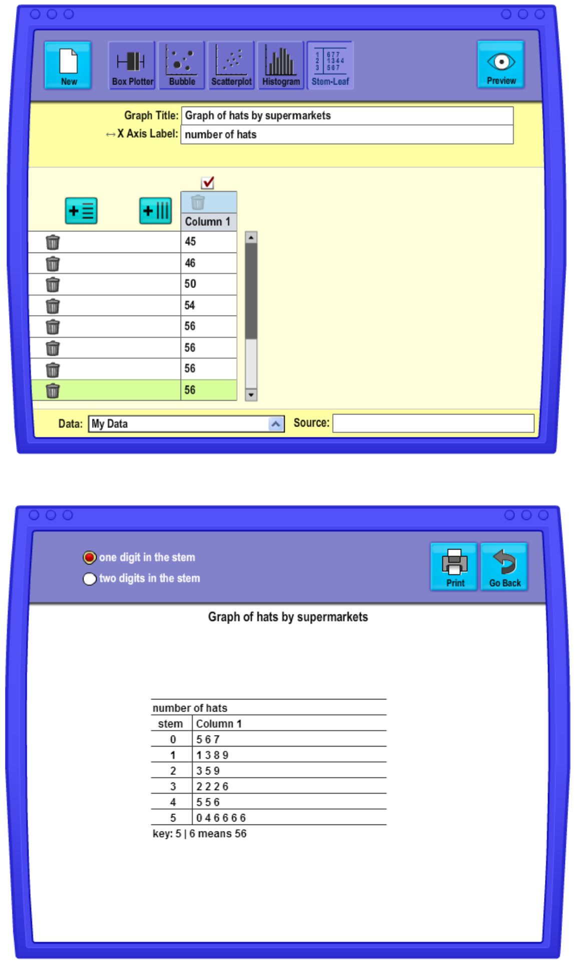

Figure 0-3 Screenshots of virtual manipulative used for Group A.

While working with the virtual manipulative; on the other hand, some of the displays need two columns (as two categories, groups, attributes, etc.) such as scatterplot since it matches two attributes of the same case. In order to draw a scatterplot, user should just add more columns which shown as in the Figure 4.4 below:

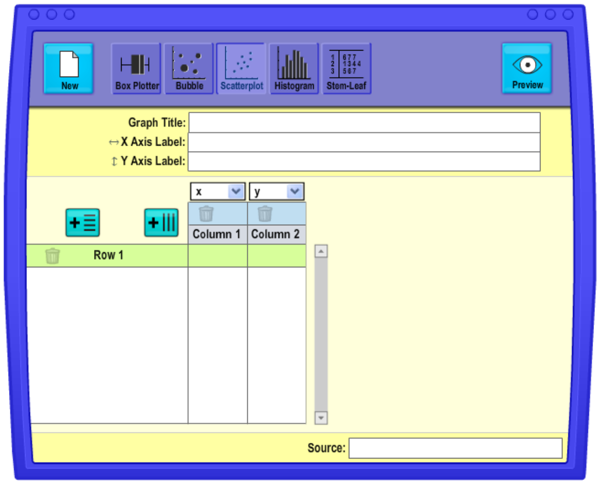

Figure 0-4 Screenshot of the virtual manipulative used for Group A for drawing scatterplot.

Therefore, group members thought that students in their implementation period of the lesson plan could experience some difficulties in understanding the number of columns since they experienced the same as well. The related conversation is as follows:

> Banu: We were also surprised about that.
> Esen: We were confused at first, we even checked when we entered.
> […]
> Banu: Now, I'm thinking that, microteaching can be confusing also, will there be two columns? I think help will be needed again. […] even we were confused about which column to write.

Since I already introduced this virtual manipulative during workshop and their classmates were included in it, I pacified group members while making them to remember this fact.

The virtual manipulative offered them to show the same set of data for both histogram and stem and leaf displays, they decided to make the transition from stemplot to histogram while using it. Then, their expectation is to make their students to realize histogram as a new type of display which was different from bar graph from the past. The related conversation is in the following:

> Gizem: […] and after that, I say, for example, I draw it like this, at the same time children have already entered data from there, they will come

> back a step and have this in their hands, this will be on the board, and they will also open the website and see histogram on there.
> Researcher: They click on the histogram button.
> Gizem: Yes. They click on the histogram button and will histogram also on there. Later on, this is the question I will ask: what does it look like? since they've already seen bar graph, they will say 'it's a graphic, there're columns, and bars', after I will say 'no, it is not. It's a histogram.'

It can be claimed that group members could make relations among content and technology considering them as alternative tools to introduce new subjects for their students. Since they dealt with the virtual manipulative sufficiently, they could see this relation as a transition between contents.

Another idea of the members of group A was to show the option of changing the class width (it was specified as interval size in the virtual manipulative) to the students. Using this option, they were planning to justify the relation of number of classes with the class width. The related conversation is as follows:

> Researcher: [...] Then you go back to the illuminations.
> (Everyone says yes)
> Banu: He will click on the histogram button.
> Researcher: This time, you see he doesn't click on the histogram but on the stem and leaf display.
> Banu: He will both see the histogram and that.
> Researcher: Okay, he will see the histogram.
> Esen: Banu, you should, we will make that [class width] something like 10.
> Ezgi: Yes, we'll make it 10.
> Gizem: Yes, I was going to say that. For example, now we showed it, and he cliked, class width can be seen there, a kid, someone who doesn't know anything about it, will say what a class width is. Maybe after this, we can drive their attention to this again. It shows 10, it's over here is actually 10. We can also change it.
> Alp: What can we say about the difference?
> Gizem: For example; if I change the class width to 5, this one here will also change, the columns will change.
> Alp: Maybe we can say, while stem and leaf display looks like this, why does histogram look much bigger?
> Researcher: What is bigger about it?

> Alp: For example; while there're 3 columns over there, on the previous question, maybe on the thing, histogram will be seen on more columns. We can ask what the reason is. I mean, it's because of the class width.

Therefore, it can be claimed very well that they were quite capable of using this virtual manipulative that they could generate different teaching strategies or subject matters in order to reach their objectives.

On the whole, they seemed to have made sufficient research about the virtual manipulative and they used their subject matter knowledge to connect with technology very well, in revised lesson plan. Based on these findings exemplified with above quotations from second GD, Group A can be claimed to have a developed understanding of TCK.

4.2.2.1.6 Technological Pedagogical Knowledge (TPK)

Members of group A were expected to increase their understanding of TPK based on the above findings regarding TCK. It can be claimed that group members improved their techno-pedagogical strategies/experiences. Below part shows some conversation which supports this major finding.

Firstly, they knew the infrastructure of classroom which the implementation will take place and planned to include using smart boards, for example, since classroom/laboratory has one. The related conversation can be visited in TK part since it was also showing that their ability to use the smart board. Besides, they have a prediction about how much time will be needed to do the activities requiring technology in their lesson. While dealing with entering data into virtual manipulative, they realized that students do not have to sequence the values as in order and that prevents time-consuming, which is showed in the following conversation:

> Gizem: [Data] is created with a single column, there's that, it's not always a waste of time. The student will just enter in there.

Another important finding was that group members tried to relate the method used by virtual manipulative in order to calculate class width with the formula of histogram. However, they could not make an explanation for this process of virtual manipulative since they have lack of knowledge about the formula of histogram. The related conversation (we were calculating the class width of data set given in stemplot in their activity sheet) has been given before in the CK part.

Group members realized that students might have a problem while dealing with class width in the histogram which is drawn with virtual manipulative. One of the members explained this as in the following:

> Alp: Since we find it through the class width formula, that formula. 10 may not come up according to the data we gave. I say, they may have a problem if it’s not 10.

Since Alp thinks that there will be a difference with the class width found by the formula and the one found by virtual manipulative, students might have a problem while understanding. This finding shows that they are disregarding the preparation of a desired data set for their lesson plan.

The most important finding regarding TPK is that group members could generate alternative instruction techniques since they know the existence of alternative technological tools which they can use in their lesson plan in terms of pedagogical strategies. For example, Banu imagined the implementation of their lesson plan in a real elementary mathematics classroom, and realized that there probably no computers for students, but one computer for teachers in general. Then she proposed an instruction way as in the following:

> Banu: Yes, I mean, if it was in a real class with children, and the teacher were to reflect it, then we would open the website, say and show that there is one, you enter the data here.

Since we mostly talked about theoretically best structured classrooms having lots of technological tools etc., she also thought the real-life situation and could suggest a way to overcome such a problem.

While discussing about the transition part of the lesson, they evaluated the property of virtual manipulative (using the same data set in stemplot and histogram) as a transition between the steps of their lesson plan, i.e., they planned the lesson according to it. This also supports the above finding that group members could differentiate their pedagogical strategies when technology is used.

Group members are also aware that using virtual manipulative provides them to show histograms with different class widths for the same data set and to their students and make them to compare with each other in order to comprehend the subject better. This supports the major finding that Group A knew the existence of variety of technological tools (in this case, the properties of virtual manipulative) for their particular pedagogical tasks.

As to summarize, members of group A can be said to have a developed comprehension of TPK with respect to their initial GD, except for the negative effects of deficiencies in their CK onto TPK.

The second GD of Group A was investigated above in terms of knowledge domains in TPACK framework and it can be claimed that there is a significant difference between their views observed in the initial GD and the ones in second GD. That is, they seemed to have worked hard while revising their lesson plan and taking the results of initial GD seriously. These findings further will be discussed in detail in Chapter 5.

4.2.2.2 Second GD of Group B

This part summarizes the second GD of Group B in which their references to knowledge dimensions stated in TPACK framework are investigated based on second draft of their lesson plan. In their revised lesson plan (can be seen in Appendix L), they haven't changed the general idea of starting with bar graph as activating prior knowledge in the first part, and explaining and drawing histogram which was introduced before with a comparison of bar graph and histogram in the second part of the lesson. However, they changed the virtual manipulative which they used for bar graph demonstration, they planned to use a pictograph which shows jobs of fathers of their students. For the histogram part, they also aimed to show it in a virtual manipulative which is NCTM's illuminations' page. Besides, they revised problematic questions which we decided as they can arouse misconceptions or difficult to understand because of flow of sentence. We discussed mostly on forming the data set since they plan to collect the data instantly and also much about sequence of the lesson. Analysis of second GD of Group B can be seen below through knowledge dimensions in TPACK framework.

4.2.2.2.1 Content Knowledge (CK)

Group B observed to not have sufficient comprehension about *why intervals are used in histogram* or *why we need histogram for continuous data sets*. This was the major finding related with CK of group members. One of them still claimed that their set of data, which was years of students' fathers worked, can be drawn by a bar graph and histogram is needed only if we increased the number of values in that data set. She explained her opinions as in the following:

> Zehra: It's not wrong, I mean the one student makes is not wrong either. At this point, would it be a generalization? I mean it happens good or bad for around 20 people.
> Emel: That's what we said last week.
> Zehra: Yes, we also said it last week.
> Şenil: No, it wouldn't. We thought about that too.
> Zehra: Last week.
> Emel: About how it would be if we made a generalization with the school.
> Zehra: It was already in the example last week.

She thinks that the graph can be drawn and it can be said as a bar graph which is not wrong for the distribution of a continuous data set (shown below):

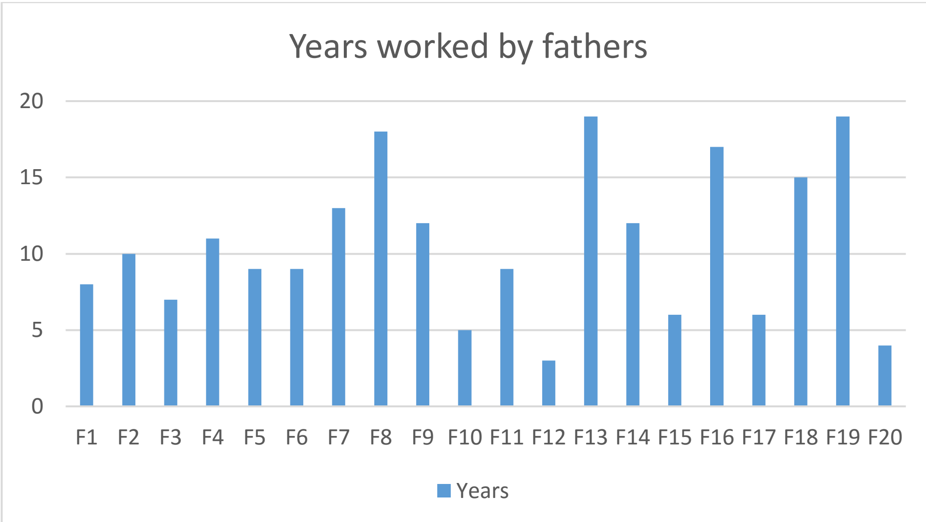

Figure 0-5 Bar graph offered by a member of Group B for the years worked by fathers.

Then, I directed some questions for them to discuss about the meaning of such a graph or what this graph tells us about. They explained as in the following:

> Researcher: Now, let's say: Ali got 8, Mehmet got 10, and Can got 25, and there're 30 students in the class, if I make 30 columns, one is for 8, one is for 9, 10, 12, etc.
> Zehra: We can find the part where it becomes frequent.
> Researcher: Yes, and then, it won't even matter if the columns change their places. No matter what data will not lose its value. It won't be wrong.
> (Everyone says yes)
> Researcher: But what will I understand from this graph then?
> Şenil: Nothing!
> Researcher: What I can understand, in the best case; I can see the highest column and the lowest one.
> Emel: The highest and the lowest…
> Zehra: Or if there is one that repeated.
> Researcher: So, yes, I mean, I only see the columns.
> Emel: That's it.
> Researcher: I can't see anything else but the columns. What is our aim in histogram? Is it to see distribution?
> Zehra: To see the frequency.
> Researcher: Excuse me?

> Zehra: Isn't is little bit about showing the interval where it becomes frequent? I mean, in which part it gets closer and in which part it gets the least closer.

They said that histogram is used to see the distribution of data, which in fact all data displays used for the same purpose. Then, one of them claimed that students can think of a question of which father worked in how many years cannot be seen. That is, histogram cannot show the individual differences because of using classes. However, they claimed that above graph can be used to see the individual differences among data set, and hence, if there is need to see these individual differences, it can be used thoroughly. Then the discussion continues with my feedback about this issue, as in the following conversation:

> Şenil: Will they do, I mean, I want to get a good information, I create the histogram, okay, but, in here I'm not getting a good information.
> Researcher: How?
> Şenil: For example; I wonder who the person who worked for 15 years is. But I can not learn it from there, does this graph make any sense to you? Wouldn't there be someone who says that?
> Researcher: About here?
> Şenil: Yes.
> Researcher: Why wouldn't there be.
> Şenil: And I am saying what if then?
> Zehra: But, that's why we did bar graph first and then moved on to histogram, the reason why we are doing histogram.
> Şenil: But we didn't do how many years it was on bar graph.
> Zehra: He can see it on the bar graph. The one who worked the longest and the shortest. While doing that, for this example, I'm sorry, he does bar graph before histogram.
> Şenil: I mean you should connect it with student's idea, I mean you don't say it's wrong but instead you, like we said about histogram, you say it'll be better if we use this one rather that one.
> Zehra: But they both have different goals. The goal here is different and the goal over there is different.
> Şenil: I'm saying what we say in case the student says something like that to us.
> Zehra: We can; if he is focusing on the result you can go with that but the goal we are focusing in here is to see in what interval it becomes frequent.

> Researcher: Okay, but then what can be our goal here which [I drew a graph top], let's say that too.
> Zehra: I mean, we are able to see it one by one here.
> Şenil: To get an exact data.
> Researcher: In here, Ali got 8, on our graph.
> Şenil: We know whose father is worked how much, and we know whose father worked the longest worked.
> Zehra: And the one who worked the shortest.
> Şenil: Actually…
> Researcher: But one second, in here, it's not wrong, but I mean, how can I say this, it's not wrong but it also has no meaning.
> Şenil: Yes.
> Researcher: I mean that's the thing we should say there actually. I mean, this graph is not wrong but, I mean the data we have which is about durations and years, I'm actually trying to say it's continuous, I am not using that word but, we are trying to make the data…
> Şenil: A bar graph.
> Researcher: Actually it doesn't have a meaning when we express it with this way with a bar graph.

This finding shows that group members still have a misunderstanding about histogram and they clearly claimed a useless graph as a graph can be used to see individual differences. Therefore, it can be claimed that Group B still has some lack of content knowledge about histogram and data distribution on histogram.

4.2.2.2.2 Pedagogical Knowledge (PK)

As it was observed during the initial GD of Group B, they have mostly discussed how lesson planning and preparation of the revised lesson plan will be done in their second GD in terms of PK. They mentioned about alternative teaching methods and strategies which they can use in the steps of the lesson as well as they touched on knowledge of learners and their background and checking for understanding issues in their second GD. However, they haven't included any specific assessment method or classroom management technique into lesson plan, and hence they never made a discussion about them as they did in their initial GD.

The topic whether they will give an introduction to histogram or they will revise histogram in their lesson plan had been left to decide later. In the second GD, it was seen that they decided to make a revision of histogram concept otherwise duration of lesson could be longer and time management could be very difficult, as they claimed so.

Firstly, they discussed much about the sequence of steps and questions asked in the first part of the lesson which is activating prior knowledge of students about bar graph. During this part, group members started with bar graph demonstration using physical manipulative which they constructed with the beads. After students finish this activity, they will ask some discussion questions about bar graph such as ‘if we change the place of columns, does the meaning of graph change?’ Later, they will make students to enter the websites given to use the virtual manipulative in order to draw pictograph and bar graph for the data of fathers’ jobs, at the end of this part of the lesson. While they decided to order the sequence like this, they thought about an alternative and decided that this new one is more meaningful, as in the following:

> Şenil: Something just came up to my mind, after doing this, what if I directly move to graphs and after the graphs ask these 3, and I mean the 7th question will be like their answers. Yes, I mean, in pictograph, when I add a doctor and take away one teacher, it will come to it again, it will have the same shape as the bar graph. It makes more sense to me. Putting the 4th question below the bar graphs, and asking the 7th question after that, I mean, it’s like wanting them to give the answer as it’s the result of these.
> Emel: It can be.

As they stressed that this new method is more meaningful since discussion technique with the questions after the graphic demonstrations through physical manipulative and virtual manipulative will be more efficient since it will effect as supplemental activity, which also shows that Group B could offer alternative teaching methods or strategies. That is, it can be claimed that they have the ability to generate alternative teaching styles for their objectives.

Besides, how activity sheet is used by the students was also discussed. Activity sheet is used for the whole lesson including both bar graph and histogram parts of the lesson and prepared according to the steps of the lesson plan. There was a drawing activity which students would draw the histogram which they saw on the board after painting activity.

They seemed to be not clear about including it to the sheet as can be seen from below conversation:

> Researcher: One second, is it also on the activity sheet?
> Şenil: Yes. The thing they will shade, they will actually enter it there.
> Researcher: There.
> Şenil: Yes. They will enter this like that. So that they can have a histogram in their hands. They will also enter and do that interval. But you don't think it is too unneceserry, do you?
> Researcher: I mean, it's like, those who want to do it can do it, and it can be something optional, because I think he does enough over there.
> Şenil: Yes, there's nothing about histogram.
> Researcher: In here, you already say what we did on top. Right?
> Şenil: Yes, exactly.

Most prominent discussion regarding lesson planning and preparation is about the data collection procedure they used during the lesson. Since students will collect the data instantly, they will be unsure about the distribution of data for both bar graph and histogram demonstrations. For this reason, one of the group members offered an idea of collecting the data, as in the following conversation:

> Şenil: Now, I think, it's highly possible that we have a problem with the data. I mean, we talked to the teacher. At the same age and etc. I mean, our fathers are also at the same age, okay, but you said first child and the second child. I mean, at the end, our range will probably be between 10 or 25 years. And I think, in general, there won't be any changes in this interval. Do you think we should change the question, as the duration of your father's last job?
> Zehra: We've already talked about it last week. We will take both of the data, and we will use which one comes handy.
> Şenil: How?
> Zehra: I remember we talked about it last week. I remember it even was teacher's suggestion.
> Researcher: Yes, I guess. I remember we talked about something like that. I guess.
> Zehra: If there is a problem with the year, look at the last job. And move accordingly. But now, there can also be problems with the last job, that's way we take all the data, and use the one which one is handy in the class.
> Emel: What do you mean when you say all the data?
> Şenil: I mean, do you think it'd be enough just knowing the last job?
> Zehra: I think we should know both, all jobs and the last.

Şenil: Will the last job be enough to gather all of this?
Zehra: There might be a problem with the last job.
Şenil: I mean, what will we do, we collected the data one day earlier, and data that we have in hands, they're all around 10 years. How will we apply this? I mean, I'm saying this so we think what we will do a day after. We need to have something in advance in our hands. Because if not, then we might have some problems.

They eventually offered an alternative of using number of years when fathers' worked in their last jobs, and decided to use the data which is practical and suitable. Although data sets used in the classroom seem unimportant or they are not paid enough attention, as Group B discussed and thought that deciding on which data set is used for which lesson objective is highly important step in lesson planning. For this reason, it can be claimed that Group B has a developed understanding regarding lesson planning and preparation.

Then they realized that they haven't got the whole data set since they collected the data instantly. Because they asked students to paint the table reflected onto the board according to the classes of years, Neither Group B will know the exact value for each student's father nor students. However, these values will be needed for the next step of the lesson since they will enter data for virtual manipulative. The related part of the discussion is below:

Researcher: Then, one second, now it just came to my mind, while entering there…
Şenil: We give a value, an exact value.
Researcher: We give an exact value, but then how will it be? I mean, here, let's say there're 3 children, between 1 and 5, we don't know how much are those, and then how will it work?
Şenil: Now I will say something, how will we…
Researcher: I mean, now, you say to children, whose father that worked between 1 and 5 years, colour this part, those who worked between 6 – 10 colour that part, but you actually don't know if the student coloured for 1 year of work or 3, or 5. Right?
(Everyone says yes)
Researcher: As a result of that, if the student passes on to this part after colouring this, and pass on to that part after colouring this,
Şenil: Weren't we supposed to collect these before we come to the class?
(Everyone says yes)
Zehra: That's how we determine the class width.

Şenil: We improve the tool according to that.
Researcher: Okay. Okay then.
Zehra: These are things that we already know.
Researcher: Then you have to give this data to the children.
Şenil: Yes.
Emel: Yes, it's true.

During the second part of the lesson, Group B prepared an activity which students are asked to paint the cells of a table which is drawn according to the classes like in the histogram. For example, one student will come to the board and paint the cell belongs to class of 6-10 if his/her father has worked between this intervals. At the end of this activity, a histogram (having a class width of 5) will eventually be constructed. The related discussion is left undecided in second GD and later they offered to set some values for each student in the classroom, and hence they will have the whole data set, and they will know everything about the data such as its distribution, its range, etc. Later in the second GD, they generated a way to overcome this situation as in the following:

Şenil: But actually, after having them do this, maybe we should reveal the real data, I mean you worked so hard but…
Researcher: Before that, for example, before the 9th step, maybe we can also reflect that data to the board.
Şenil: So, we can say, these are the data we collected from you, we turned them into a graph.
Researcher: Data can stand next to that graph. Then, kids can still come and colour here, so it'll be interactive.
Zehra: His own number.
Researcher: You can even control it from there.
Şenil: Yes. I mean, also draws his own number.
Researcher: So, he draws his own number. As a result of that, he can also have this in his hands, because it will be needed for there.
Şenil: Right, right. We couldn't do that thing.
(Everyone says yes)
Zehra: Then I add it.
Researcher: To the 9th step.
Şenil: Say that we will reveal the data itself on the board before the 9th board.
Researcher: Or you can reflect it with this, together.
Şenil: At the same time.
Researcher: Yes, at the same time.

Based on above last conversations, it can be claimed that Group B is unaware of which step requires what and it can be named a weakness for them that they could not build the lesson in terms of its steps and requirements.

On the whole, Group B showed their performance on knowledge of general teaching methods and strategies and lesson planning and preparation very much. There was not much difference in their selected teaching methods for this lesson plan in which direct instruction and questioning technique are the leading. They planned to perform the implementation based mostly on the activities in the activity sheet as in the first draft of lesson plan. Besides, most important discussions were for data collection and data set itself. Since their questions are directly related with data set and the distribution of data, they are aware that their aim is to make students to think about their aim of asking those questions since they expect some responses. This effort of Group B is considered as important and hence, it can generally be claimed that members of group B had enough understanding of PK in order to be able to plan such a lesson. However, they lacked attention about the transitions between the steps of the lesson in order to construct a smooth flow.

4.2.2.2.3 Technological Knowledge (TK)

As it was specified in the initial GD of Group B, all of the codes were observed during the second GD as well. Therefore, there is no change in the major finding regarding TK of group members.

4.2.2.2.4 Pedagogical Content Knowledge (PCK)

Group B referred to PCK attributions more in the second GD than in their initial one. They mostly discussed about techniques or strategies to teach/represent the subject matter to students. The never mentioned about the content-specific assessment strategies here. Their aims for questions they asked to students were stressed out and in which sequence they will ask which question was also rearranged during second GD. They also mentioned about their aims in selecting the activities through the steps of the lesson. The way of filling the table on the board for drawing the histogram and the possible questions in the activity with virtual manipulative for histogram were also discussed. There are the most important parts of the second GD regarding PCK.

Firstly, group members discussed the aims of questioning technique used in the bar graph part of the lesson. What their expectations from students while responding the questions are also addressed. The related part of the discussion is in the following:

> Researcher: How does the graph be effected from that? Then how do you expect children to respond this here?
> Şenil: Yes, we came to this part.
> Emel: The meaning we take out of the graph will change. For example, if I put one to teacher from the doctor, the doctor will be one less, and teacher one more. I mean, like this.
> Şenil: Maybe the doctor was the biggest one, and when we put one to the teacher, the teacher becomes the biggest. Like that.
> Researcher: But maybe then, we may need to reconsider it according to the graph that comes up. Let's say, the doctor is the biggest, when we take one from the doctor…
> Emel: Like, we put one from the doctor to there.
> Researcher: Something like that, you need to coordinate it according to the situation of your class.
> Şenil: It can be, since we will also organize it according to the class.
> Researcher: Then your goal is to ask everything about frequency there's to be asked.
> (Everyone says yes)

They described their fundamental aim in the questioning strategy used in first part of the lesson as that students should remember all of the properties regarding bar graph, the variable types used in bar graph, the frequency concept for each category, etc. From this point of view, their aim is comprehensible since they wanted to activate students' prior knowledge about bar graph.

How they asked questions in this questioning period was also described with their aim. They also explained the relation of this activity with the activity sheet. The related conversation is in the following:

> Şenil: There're 3 questions over there, we ask them. And they answer them here. Actually, it's like we ask them directly here, but after answering them in the class, there'll be something like a discussion.
> Researcher: So, they will write the answers first, and you will give them a time to write those answers.
> Şenil: Yes.
> Researcher: And after that, you combine the answers of the class.
> Şenil: Yes. Or we can ask a question, I don't know, but when we go question by question there can be a little bit of confusion. A discussion at the end, after giving all the 3 questions can be better.

Before discussing the answers among students, they first expect from students to answer the questions in their activity sheets. At the end, they will combine their answers in order to form a discussion environment, which was a strategy to discuss and talk about all ideas emerged in a classroom.

Then, they explained the reason for using two other demonstrations for bar graph after forming it in the physical manipulative: virtual manipulative for pictograph and another virtual manipulative for bar graph. At first, I thought that students will choose one of them, however, they said that they will implement both of them. The conversation is as follows:

> Şenil: No, I want them to do both, what was its name, the one we did in the class when you sent it, a bar graph with dots. Actually, also in here, each picture can represents a dot, like that abacus over there, each ring in there, represents a picture over there.
> Researcher: Yes. Yes.
> Şenil: We actually wanted them to pass as each one in that picture represents each unit and piece of the bar graph.
> Researcher: And in order to do that, they will start with the pictograph first and move on to bar graph after.
> Şenil: Exactly.
> Researcher: Okay.
> Şenil: In an order.

Since they want to make a transition from physical manipulative to pictograph, then another transition from pictograph to bar graph; they planned to add all three demonstrations for bar graph, which are also suitable for their data set. Although this seems too much for activating bar graph knowledge, it can be accepted regarding their rationale for transitions between steps.

At the end of the conversation above, one of the members realized that it is better to use questioning after these three demonstrations of bar graph, as in the following:

> Şenil: Actually, something came to my mind. Now, we make them do this, and we said what if we change it there, but what if we put these, under the graph as a visual, how would the visuals be, the changes? Or let's put it this way, we asked these questions here. They did it here, and later on, 3 in the 4th, in the questions we asked.

Then, the rest of the group convinced about this idea as below:

> Şenil: He can do 5, and later 6, and then we ask 4, and then he can pass on to 7.
> Zehra: So, you are saying the order as 5, 6, 4.
> Şenil: Yes.
> Zehra: It can be. It makes more sense.
> Şenil: It just came to my mind, I think it'd be nice. Because, I mean, here, I feel like there will be a little disconnection here. Because, okay, you asked the student like you put that ring here, what happened, etc., aren't we trying to improve student's bar graph ability, if I put the ring, and the teacher increases, then there's no point.
> Zehra: So, you are saying that he sees it as a visual directly.
> Şenil: Yes. I mean that's my opinion, I don't know what you think.
> Zehra: Makes more sense.
> Emel: Okay, let's do it that way. We can also do it that way, we'll see.
> Researcher: Then you changed this sequence [of asking the questions].
> Şenil: Yes.

The two conversations above are examples of decision-making process of selecting teaching strategies in order to represent a subject matter at best. Then, it can be claimed that members of group B were capable of discussing among different possible teaching/representing strategies which also shows their understanding of PCK.

In the second part of the lesson, they also used the questioning for histogram concept and they explained what their aims in asking the questions here, as in the following:

> Researcher: You ask what the names of those columns in the histogram are.

> Şenil: What it can be, I mean the bar graph, if anyone in the class wants to create a bar graph, if we were to create a bar graph, like before columns had names like doctor and teacher, we ask what names can be given here, if he again says doctor, what he will determine as the duration, for example, there's a doctor who worked for 4 years, how I will draw someone who worked for 6 years on top of that? He won't be able to do that.

This strategy was meaningful regarding that it tried to show the logic behind the histogram concept from the reverse.

Then, they discussed that students could offer a 'bar graph' which can be drawn as x-axis shows each father and y axis shows the years which was showed before in CK part. Regarding PCK, I offered to Group B that they can draw this bar graph onto the board in order to show its incoherence as a strategy like above.

In order to save time for the flow of lesson, they discussed the possible alternatives for filling the table which will be shown on board in order to draw histogram. They offered raising students' hands, or drawing checkmarks, as in the following:

> Zehra: What if we say, students whose father has been working for 1, 2, 3, 4, 5 years, rise their hands, they rise their hands, then, those between 6 and 10 years rise their hands, and then it continues. I mean, what if we use the hand rising method.
> Şenil: To be it's there, I mean it's there. I don't know.
> Zehra: Instead of they come to the board. Because they come to the board often.
> Şenil: But children may not be able to understand if it falls or not in that interval, but they may also understand it.
> Zehra: If we especially say the first one 1, 2, 3, 4, 5, between 1 and 5, I mean if we determine it as who worked between 1- 5 years, it wouldn't be difficult for them to understand.
> Şenil: I don't know, I think it might be, there can be a complication.
> Serhat: What if we do all the questions one by one and mark each student?
> Emel: There's that too.
> Researcher: As far as I understood, you actually want students to come up to the board because you want it to be interactive so that students can participate. Right?
> Zehra: But we make them come to the board very often. I mean the student will come up to the board all the time according to this.

Şenil: He can, I mean, will he just sit for 2 lessons?
[…]
Şenil: We said paint here, but, I think, we can also say thick also.
Zehra: We called it paint so it will be easy to see directly that it's a histogram.
Şenil: Okay, but then how will we colour it on the board?
Researcher: You can shade it like this on the board.
Şenil: We can shade it, it will be okay, right?
Researcher: Maybe he can not colour it fully. But he creates a histogram here, so yes.
(Everyone says yes)

These are alternatives for drawing histogram onto the board interactively. While they are representing the subject matter, they also make students to be motivated during this activity.

For the last part of the lesson, I offered an alternative to investigate the class width of histogram. After they make an activity for drawing the histogram on the board, they will make students to draw the histogram through a virtual manipulative. During the activity with virtual manipulative, I offered to make students to realize the changes when class width is changed. The related part of conversation is as follows:

Researcher: After that, they already create a histogram here, and after that, they wil create one also here, and compare them.
(Everyone says yes)
Researcher: But, if you make them do this by hand at first, and then give this afterwords, for example, you can also make them do it with a different class width here. They are able to enter the class width on the website right?
Şenil: First it determines it by itself but then we can change the class width.
Researcher: You can change it. Then you can also ask questions about the shape and etc.
Emel: What happens if we change it, we can ask about the shape.
Researcher: Yes, you can. How is the shape, it'll probably be something ample.

Based on two conversation above, it can be claimed that they are capable of producing some alternatives for representing a subject matter. However, their understanding can be said to as not enough since their CK effects their way of thinking, so does PCK.

Therefore, on the whole, it is observed during second GD of Group B that their PCK is not developed as compared to the observations in the initial one. They need to improve themselves regarding PCK, especially in terms of knowledge of representing/teaching strategies more, so do their CK since CK was observed as importantly related with PCK among their first and second GDs.

4.2.2.2.5 Technological Content Knowledge (TCK)

During second GD, members of group B talked about the issues regarding TCK as they were in their initial GD. They mostly discussed the introduction of pictograph which was not included in the first draft of their lesson plan. Another discussion regarding TCK went around the implementation of virtual manipulative for drawing histogram. They were observed to be involved much about the issues of how virtual manipulatives will be implemented and what their relations with the content are. Therefore, it can be claimed that they seemed to have developed their understanding of TCK although they rarely criticize the effects of usage of virtual manipulatives.

Firstly, group members talked about virtual manipulative for pictograph which I introduced them before in the previous GD as another way of demonstrating categorical variable like bar graph. They seemed that they liked this idea and planned to include it into their first part of the lesson. They also searched for a virtual manipulative which can draw a pictograph and found a visual one (http://primaryschoolict.com/pictograph/). The following conversations show that they explained to me how it is used and how it can be implemented:

> Şenil: We use it. It can be changed, for example you type doctor, and you open it, if you can.
> Emel: Which one?
> Researcher: One second, I guessed I entered that.
> Şenil: I examined it, it can be entered, and it's very nice. For example, this one, you click on this.
> Emel: Yes, it changed on there.
> Şenil: By deleting it.
> Researcher: It does something by researching the Internet.

> Şenil: For example, type doctor.
> Researcher: For example, I typed sports, it even found a visual for that.
> […]
> Researcher: And also, it does something like this on pictograph, right, I mean it can this one out of there, take it from here?
> Şenil: Yes, it can. It deletes one of this.
> Emel: After putting this, it can take it from there and put it over there.

Above conversation is related with a question which they decided to ask after all demonstrations of bar graph: 'if we put one ring to another column, how does the graph be affected from that?' One member of Group B explains that students can easily change the values in the virtual manipulative for pictograph. It can be claimed that group members not only searched for a virtual manipulative but they also thought about how students can work with it.

However, one of the group members criticized that students could fall in a mistake since changes which will be done on pictograph cannot be realized as well as they can on physical manipulative. She offered that there can be a problem since pictograph will use different pictures for each column which can be difficult to compare. The related conversation is as follows:

> Zehra: But there might be a problem now at the thing, since they are both different from one and other, those pictures are on pictograph, and there might be a problem. For example, in here, ring is always the same ring, so we exactly know they are all the same size. So I think it makes more sense to take one ring and put it on the other.
> Şenil: Okay, the picture… okay, it also makes sense, but it's not important to put the ring, it's to understand the bar graph.

Then, another group member explained the issue with an understanding of why virtual manipulatives is included in a lesson. She addressed that their aim is to make students to understand bar graph concept although there are pictures used in pictograph. This finding is important since it shows that they began to develop their understanding of TCK while they are concentrating on the relations of technology and content in order to achieve lesson objectives.

In their revised lesson plan, Group B changed the virtual manipulative for histogram. After they make the board activity of drawing histogram, they start to activity with virtual manipulative, but they were observed to haven't enough worked with it. Then I again gave feedback about it, as in the following:

> Researcher: […] but since you are doing something like this, and there's something beautiful about the histogram, the class width thing, that's something that can be changed, it even gives its formula on the objectives.
> Şenil: Well, we never actually got into that.
> Researcher: So, it means it's something important. I mean, you don't get into the formula anyway.
> (Everyone says yes)
> Researcher: Okay, then don't get it. Since you think he can draw histogram in many different ways, because one of the children can ask something like, why you categorized with 1 and 5.
> Zehra: So you would do it this way.
> Researcher: A question like that might come up there, since you say they're very smart.
> Şenil: Yes, exactly.

Since their activity of filling/painting a table in order to construct the histogram takes class width as 5 units, I suggested them to specify that class width can be changed and take different values. While changing the class width in virtual manipulative, students can see the effects of this change instantly on the graph. They liked this idea and left to add it for the next revision of lesson plan.

So, on the whole, members of group B showed a developed performance regarding the relations of content and technology and implementation of technology into a lesson as compared with their initial GD.

4.2.2.2.6 Technological Pedagogical Knowledge (TPK)

Group B members referred rarely to the issues related with TPK (Hughes, 2013) as they did in their initial GD. However, they tried to consider students' motivation while instructing with technology in the second draft of lesson plan which is different from the previous one. For example, they investigated the virtual manipulative which they searched for pictograph while they were predicting that students will like it very much. Since virtual manipulative allows to change the columns, pictures, etc., each pictograph drawn by a student will be unique and this provides also the varieties of demonstration of the same set of data. The following conversation is as follows:

> Şenil: Some nice doctor pictures come up.
> Emel: You can even change the pictures.
> Researcher: I understand.
> Şenil: It can be taken down, student can use anyone he wants.
> Researcher: So, we only change this question then.
> Şenil: Yes, they all can be changed.
> Researcher: This is a very interesting website actually. With pictures and stuff.
> Şenil: I searched for some stuff. Nothing good came up. Then the thing you said came to my mind. Called pictograph, and I thought I'd give it a shot, and I did. It was one of the below ones, I found it after searching and searching.
> […]
> Şenil: It's something fun.
> Zehra: It's something very cute.

As can be seen from above conversation, Group B members might even be said to be excited to show this virtual manipulative in the lesson. They liked it very much and they want to share this with their students. This finding is important since it shows a slightly developed view towards their understanding of TPK.

As an addition to above finding, they plan to make students to paint the cells of the table for histogram since the table will be reflected to the board. Then, each student can come and paint the related cell. Hence, they think that this activity will also increase their students' motivation for the lesson.

Another important finding aroused in second GD of Group B is related with knowledge of classroom infrastructure. Since they are familiar with the classroom in which they will implement the lesson plan, they knew the available technological and physical equipment of the classroom. They said that there is also a computer for teacher and teacher can reflect the table mentioned above. The related conversation is as follows:

> Şenil: Yes. I mean, I don't know which one it will be, drawing or reflecting.
> Researcher: I suggest you reflect it, don't bother drawing.
> Şenil: Yes, we'll do it in the lab anyway.
> Researcher: Yes. You'll also have a computer, and they will have one too. Wait a second, does the teacher have a computer?
> Şenil: Yes, there's one.
> Zehra: To control.
> Şenil: But in which lab? The one on the central corner?
> Researcher: No, in the one on the corner.
> Şenil: There's one in there.

This finding also shows that they have a developing comprehension of TPK and their views towards technology integration into mathematics classes are changing.

As a summary of the second GD of Group B, they can be claimed to have slightly developed understanding of knowledge dimensions of TPACK framework when compared with their initial GD. Besides, they were observed that they have changing views towards using virtual manipulatives as a technological tool for mathematics teaching regarding that they increased the usage of virtual manipulatives in their revised lesson plan.

4.2.3 Findings based on third GDs

This part will summarize the major findings of groups A and B regarding third GDs after their implementations take place. They revised their lesson plans once more before the discussion period. Then, they implemented them and implementations were video-recorded. During third GDs, each group discussed their implementation while watching their video-records while investigating the moments of lessons with their pros and cons. Third GDs end with my question of whether they need to revise their lesson plan again and whether they need to implement after revision. The following sections describe each group regarding their third GD based on knowledge dimensions of TPACK framework. Their secondarily revised lesson plans will also be summarized.

4.2.3.1 Third GD of Group A and video-record of implementation

In this part, third GD of Group A will be analyzed based on the video-record of implementation of lesson plan which was secondarily revised (third draft of lesson plan can be seen in Appendix M at the end). In their second revision, members of group A haven't made many changes except data set they used for drawing histogram. They again started with stem and leaf display to activate students' prior knowledge about it and make a transition to histogram while rotating it and make students to draw a histogram with the help of a virtual manipulative. Second part of lesson began with activating knowledge about bar graph and compared it with histogram at the end.

During third GD, they discussed the implementation of their lesson plan overall at first. Then, we started to watch its video-record. We stopped the video when there was a need to discuss or they want to explain something. Then, we started back to watch again. This continued till the end of the video. At the end, after discussion about implementation is completed, I asked whether they want to revise the lesson plan once more and whether they want to implement it again. They responded to me that there is no need to reimplementation since there is not any important or radical change needed to lesson plan.

During the lesson, the teacher who was one group member, followed their lesson plan completely. She was careful about time management and applied each step as indicated in the lesson with its determined teaching style. For instance, she was careful about following the questions in order to direct to students. The only problem they faced during implementation was technological failures such as smartboard sometimes stopped working and website of virtual manipulative (NCTM's illuminations page) didn't work. However, they have taken some precautions for them; for instance, they got screenshots of virtual manipulative before and they showed them to students.

4.2.3.1.1 Content Knowledge (CK)

Gizem, who actually taught the lesson, was observed to respond satisfactorily to each question which was asked by students. Some of these questions throughout the lesson were listed: “don’t we write the 0’s in [stemplot, while listing the data]?”, “My teacher don’t do [y axes] show the number of hats?”, “My teacher, our column numbers are increasing, then, is the number of markets increasing?”, “My teacher, how did we define the class width?”, etc. When student asked the question related with increase on number of classes as class width decreases, the conversation between Gizem and student is in the following:

> Ezgi: I will ask something. Our column numbers are increasing. Then, are the number of stores also increasing?
> Gizem: Yes, your friend asked a good question. Now, I am changing it, there the columns are increasing or decreasing.
> (came to the board, tried to show it from her computer but smartboard stopped working. And, she showed the screenshots which they prepared before)
> Gizem: Now, is it this one which we obtained first? When we changed the class width to 5, I obtained this. Here, there are more columns. What about then, are the number of stores or toys are increasing or decreasing too?
> Class: No.
> Gizem: What is the decreaing or increasing thing then?
> Class: Class width.
> Gizem: Class width decreases. What happens when it decreases?
> Class: It makes more classes.
> Gizem: Yes, the number of classes are increasing. In fact, the number of toys or stores are not changing. You can think of like this: you have 50 apples and you grouped them in 3. The remaining is 1 apple. Or, you grouped them as 5. In the former one, you have more groups. In the latter one, you have less groups. There is no change in the number. Then, when we turned to our example, we grouped them with a class width of 9,8 and then 10. In fact, the number didn’t change again. But, the number of classes decreased. Did we agree on that? Do not close this right now. Let’s keep the histogram when you did the class width as 5.
> (time as 34:53, from the video-record of implementation).

She responded to her while giving the apple example as grouping 50 apple as 3 or 5 apples does not make any difference since the total apple number does not change.

Students seemed convincing after this response and this is a good example of claiming that Gizem has a developed CK regarding their lesson objectives since we never discussed such an example during our first and second GDs. Although the example was not completely related with a display of data set, it gives the idea of grouping and class width to the students and makes their minds clear. Since CK was not referred by Group A as much as it was in their initial and second GDs, it can be claimed that CK of Group A seemed to be improved throughout lesson study, regarding third GD.

4.2.3.1.2 Pedagogical Knowledge (PK)

During third GD, Group A concerned general teaching methods and strategies and lesson planning activities at most regarding PK.

The implementation of the lesson plan have taken place according to their lesson plan very strictly. Gizem applied the teaching strategies that they determined before and tried to apply them as much as possible. For example, she applied questioning technique while directing further questions to the students. However, she sometimes was not careful enough for the waiting time after a question asked. Its reason was mostly a natural result of microteaching since their students were their friends in fact. An example of questioning period of the lesson which was related with bar graph is as follows:

> Gizem: Now, can you say while looking at there? Which type of movie is the most preffered?
> Class: romance.
> Gizem: You said romance. What about the least preffered?
> Class: drama
> Gizem: Okay, what else can we say about this bar graph?
> Student 1: Science fiction and comics are preferred as the same amount.
> Gizem: Yes, is that right? Both of them has 4. What else?
> Banu: Why is there a gap between the columns?
> Gizem: Why there is then? The gap between? Is there anybody who can answer to this question?
> Student 2: Doesn’t it look like the one which we saw a little earlier?

> Gizem: Does it? Now, you have the bar graph on your sheets. You have histogram on your computer screen. Look at them both. Do they look like each other?
> Class: Yes, they do.
> Student 3: In my opinion, they do not a little since there is a class width issue here for example, but there is not such thing there.
> Class: But, their looks are similar.
> Gizem: But, you said they look like each other in terms of their shapes. Then, what If I ask such a question then, can I show this data of type of movies with a histogram?
> (From the video-record of implementation)

As it can be seen above, Gizem applied questioning technique while continuing with further questions. Since students know already the subject, she responded questions instantly and wait time for the questions couldn't be applied.

Based on the above part of the implementation, members of group A often criticized their questioning skills and the questions they asked during the lesson. They were also aware that questioning technique couldn't be applied as it should be because of microteaching, however, they claimed that there is not any better technique suitable to their objectives other than questioning. They also complained that they have limited time for waiting responses. Teacher, as the implementer of their lesson plan, explained this as in the following:

> Gizem: Now, I think, while using the questioning, someone even took a saying, for everything I asked, one or two, or maybe one of them answered. But of course it's a result of the short duration but while doing the questioning a longer duration should be given, so that more answers can be taken from everyone.
> Researcher: Of course. It's, what was it called?
> Gizem: It's based on our short duration and the variety of the topics we tell.
> Researcher: Yes, yes.
> Gizem: Because the duration was really short.
> Researcher: If it was an 8th grade, it'd look more natural.
> Gizem: Yes, exactly.
> Researcher: I can't think of a group work like this one, it'd get interrupted often.
> Alp: I think so too, it's so much better this way.

> Gizem: Because we couldn't even do it here, thinking it it's okay or not. I mean it's so,

As it can be seen from above, group members agreed that questioning is better strategy than group working. Besides, they claimed that Gizem applied questioning very well according to their lesson objectives.

Members of Group A discussed the starting point of the lesson as well. Gizem started with an introduction of stem and leaf display to the lesson and she said 'we will see a new data display today' without saying histogram. During discussion, other group members complained about it and suggested saying 'we will go into histogram subject today' as the introduction to the lesson. Then Gizem defended herself as in the following conversation:

> Gizem: I'm thinking that; I wouldn't say I was going to explain histogram at first.
> Esen: Because we start with stem and leaf display.
> Gizem: Because yes, since I'll be moving on from there, and will connect histogram and bar graph at the end, and make a comparison.
> Researcher: So it would be more like a surprise?
> Gizem: Yes, to make it a surprise. But after that I said I'd only say we were going to do something about statistics and data, which you had already learned, we would do something about this, saying we were still on the statistics subject. I wouldn't say I was going to explain histogram.

This discussion shows their hesitations about lesson planning effects onto students' motivation, which is also a finding regarding PK.

When activity sheets were distributed to the students, Gizem announced one student to read first question aloud in class. This surprised other group members since they don't expect it during the implementation. Gizem explained it as below:

> Researcher: Why do you do this by the way?
> Gizem: Because I normally make them read, children never read.
> Alp: They can understand it better.
> Gizem: Normally, when I lecture, at 7, and when I say let's read something, everyody usually starts talking with the one next to them, and no one reads anything. […] When I say let's read, everybody listens to me. Mean while, they listen to their friends. That's why I made them read while I lecture. It's a habbit.

From her previous experiences (tutoring, voluntarily teaching sessions, etc.) Gizem observed that reading aloud makes students to be more involved into lesson and she thinks that it is an effective practice during the lessons and also makes classroom management easier.

During third GD, Gizem explained that Banu behaved like an assistant for teacher since she helped teacher in warning about steps of lesson and time management as well. Banu helped Gizem in troubleshooting smartboard as well. I offered also that one member could behave like a secondary teacher who help students during computer work. The related conversation and Gizem's response is in the following:

> Gizem: Banu was already like that, I mean, she put a tick to everything I said and didn't say. She had a lesson plan in her hand.
> Researcher: So, Banu did a little bit of assistance then.
> Gizem: Yes, yes. I had already told Banu, to be my assistant and to keep the time. And a track of what I said and did no say, and to follow the lesson plan.
> Researcher: Actually, Banu could have, I mean, like there is a problem with the computer there, and someone…
> Gizem: Someone came. We made eye contact with Banu.
> Researcher: No, no, with the students' computer, you could have walked among the students.
> Esen: I actually thought about that.
> Ezgi: Because at some point I stand up.
> Gizem: Yes, yes.
> Ezgi: Then I said I bothered, since I gave her…
> Gizem: Because, I was supposed to be alone. In a normal class, based on the Turkish standards, but I request Banu to just follow and see if I have any missing points.

They were aware that one teacher could implement a lesson in a standard public school, and hence they haven't thought about such an option of having two or more teachers, one is following the lesson steps and the other(s) deal with students, and so on. This finding shows that group A is conscious about procedures of implementing a lesson in a public school regarding planning it.

Further, they even thought that Banu's assistance provided a better time management since microteaching had to be in a less amount of time than planned. Although they have limited time, group A succeeded to go with the steps of lesson continually without passing one and they finished the lesson on time. The related conversation is as follows:

> Researcher: Apart from that, how was the transitions between the tasks, I mean, there're 3 steps in your lesson, how was those transitions?
> Gizem: I mean, it was fast, but since we were in a constant eye contact with Banu…
> Banu: I checked the duration. And did things in there.
> Gizem: You checked the time. Like, now get a little fast or slow down. I think it is better that way, it ended just in time. It is duration was just.
> Researcher: Yes, I really liked the time management. […]
> Gizem: Yes.

As it can be understood from above, not only they planned their lesson in detail but also they were well-prepared for the lesson and they took some precautions against for possible problems, which makes it an important finding regarding PK.

Gizem dealt with students while they were studying with virtual manipulatives especially. When a student faced with a problem, she got close to him/her and tried to solve the problem. One member of Group A evaluated it as low class size allowed to this. She thinks that it could be difficult when class size is much more, as in the following:

> Ezgi: […] We were really a few people. Even it was a real middle school student, the management might have been a little more relaxed, but it's very difficult in a crowded class. For example, even in that problem you need to go through a few people. To see if they did it or not.
> Gizem: Yes, I went through a lot.
> Ezgi: In 18 people.

Group A claimed that classroom management could be difficult in a big class. However, they said that they planned to implement this lesson in two hours in a real classroom environment. Therefore, it can be said that there is no problem is their lesson plan in order to implement it in an elementary mathematics classroom.

As a summary regarding PK dimension of their lesson, Group A showed an effective performance concerning lesson planning activities and preparation and classroom management issues at most. They also considered students' background knowledge during the lesson. However, they still do not pay special attention to assessment of lesson objectives. Questioning strategy might be thought as an attempt to check for understanding of students.

4.2.3.1.3 Technological Knowledge (TK)

In the third GD, group members mentioned about troubleshooting of smart board regarding TK. During their implementation, they experienced a problem while using smart board: It was not responding to their actions, and it locked up for some time. They explained this moment as in the following:

> Gizem: I think there's a problem with the smartboard. I mean, in there, yes. […] I used it but it did not work. I mean. I didn't reflect it like that, but I think it just didn't reflect. And it was seen as a teacher, I also think there was a problem with the smartboard. I mean, it wasn't clicking, then we made an eye contact with Banu and she went over the computer, but she couldn't do either.
> Researcher: So you mean it was stucked.
> Esen: On the top part, click on the smart ink button and from there you need to choose the mouse not the pen, I saw it at the end of the class.
> Gizem: But no, no, no, it didn't click on the mouse, and before that, did you do that to choose mouse?
> Esen: Yes. I closed the pages to go on to the mouse.
> Gizem: At first I tried to choose the mouse on that computer over there, I never could. I couldn't even open the file, couldn't open the link, I couldn't open the website of NCTM, because it wouldn't open. It got stucked at some point.
> Esen: It got stucked.

Based on above conversation, group members can be claimed to have necessary technical information for basic operations in order to run a smart board. Gizem, as the implementer, was said before by other group members as an experienced user of smart board for lesson activities. She said she like to use smart board in class and she is good at using it, as well. Therefore, it can easily be said that they have necessary knowledge to use smart board, and they can also be said to have some solutions for troubleshooting smart board, which together shows their understanding of TK as developed.

4.2.3.1.4 Pedagogical Content Knowledge (PCK)

PCK was the most referred knowledge dimension during third GD. Members of Group A discussed more about alternatives of teaching/representing subject matter to students and some about identifying student mistakes and misconceptions.

Members of Group A tried to differentiate their responses towards the students' questions emerged in the lesson. A student asked the question of "don't we write the 0's?" in the stem-and-leaf session while writing the data set shown in stemplot, and teacher responded to that student as in the following:

> Gizem: Yes, I was saying that. Now, we think of like tens. But, we think of here one by one as ones. Then, what happens to my numbers? What happens then?
> Class: 42, 49…
> Gizem: Let me write here. That is, here is tens, here is ones. Let's write from the beginning.
> Ezgi: Don't we write the numbers include 0s?
> Gizem: We write the numbers including 0s. Your friend gave an example. There is no number between 0 and 1. There in no one in the ones. If there is, I would write 0. There is not even. Then, I am skipping it. I skipped 1. I came to 2. Tell me and write together.
> Class: 21, 24, 25, 25, 25, 27, 28, 30, 34, 36, 36, 40, 42, 49.
> (From the video-record of implementation)

Besides, there is another question related with number of cases in data set. This second question refers also to the similar consequence with the previous one. The related part of the lesson with the response of the teacher is below:

> Student 1: What if, is there any store which do not include any toys? It sells, but its store is empty. Can we say 15 as the least?
> Student 2: But, then, don't we show it close to the 0?
> Gizem: For example, if it would like that you said: you see the 0 here, right, isn't it our tens? If I were write 0 here, then I would have a value for that. Then, I would say there is a store which has no toys at all. Then, since there is 0 here, your claim is not correct. Did we agree on this?
> (From the video-record of implementation)

During third GD, one of the group members criticized these responses, and she offered an alternative. The related parts of the discussion are as follows:

Banu: […] I think we didn't give a satisfying answer there. He said, as far as I remember, we'd take it if it was also next to digit one, I don't think it's a satisfying answer for the child. I mean, then why 0 and 1 are there. I don't think we couldn't do much over there, and it stayed there.
[Second part…]
Gizem: And I said, we already called them tens and ones, but there isn't even a 0 in the digit one, then I mean nothing, depending on what I said, I mean because it'd be wrong to talk about stem and leaf display again. I mean it's means I'm explaining the diagram. They already know it from connecting it, and for whose you don't know it there isn't even a 0 in the ones, and then I can't write it, so I skip it, I made a fast transition saying it's not at 1, not next to 1, then I skip to 2, I mean I don't know what else can be said. Because it'd be like explaining stem and leaf display again. Like explaining how to draw it. Then I thought it would be way too long, it'd be like little bit of that, little bit of this.
Alp: I agree.
Gizem: Because my goal is histogram.
Esen: What if it said, it was 8 toys, right?
Gizem: Yes, this one is toy.
Esen: If there were 8 toys, we could have taken it as 08, bu we don't have that kind of data.
Gizem: Isn't it like saying that? I don't have anything on the ones, not even 0. I remember I said, if it said 0, it would mean one of them has no toy.
Alp: Indeed.
Gizem: I remember giving 0 as an example.
Alp: It explaind it clearly.
Esen: I feel like [student 2] said that.
Gizem: After [student 2] said it, I added the 0. If I wrote 0 there, it would make it 0 and 0, which would mean there isn't any toys in one of my stores, I remember giving an example like that. But I didn't give any examples at 8. I don't know how it would be if I did, but then it'd be like I was explaining diagram.
Alp: I think it's enough.
[Third part…]
Banu: Instead of saying there's no digit one, we can say if there were also numbers next to it, I mean, it would be more open if an explanation was made such as; we don't have a toy between 0 and 10 or between 10 and 20. Frankly because, there weren't single numbers or numbers with tens. I mean not like 11, 12. It would be more obvious if we say it's not like 8 or 9. If I were a student, I'd get confused when you said something like this.
Esen: A number like that could have been given, not 13, not 15.

> Banu: If it was here, how would it be? It'd be 8 or 9, I mean it means 5, 6, with one number, there isn't a store with single number toys. And also it can be said that there's not a store with toys in tens either.
> Researcher: So, it can be also said that there isn't any store that has no toys.
> Gizem: Another example apart from the 00 could have been given.
> Banu: Yes. It could have been right.

As it can be seen from above conversations, Group A considered the teacher's response very important and discussed about the mistakes that implementer teacher made during the lesson. They offered many alternatives and gave reasons for those.

There was a similar discussion which was about teacher responses to a student's question during the lesson. The question was related with the axes of the histogram and how to name them. Student asked that "do these show the number of hats?" implying x-axes. Then, Gizem answered while directing her to the y-axes, and explained the relation between number of hats and columns in the histogram. Group members claimed that her explanation was too detailed and it can be cumbersome for students to understand. One member of Group A offered Gizem to use breaks after her questions if this was her only solution to response to that student. Group members criticized the implementer especially for these reasons and they explained their alternatives.

Group A continued to discuss the responses of implementer teacher for the second part of the lesson as well. This part of the lesson goes completely with the questioning technique and teacher seemed not to control questions and students' responses enough. In fact, it started well at first and then teacher lost her control because of time management as she explained during group discussion. Members of Group A made some critiques about Gizem's responses for students' questions aroused during questioning and I gave some feedback about one possible question which can be directed to students in order to make them realize the differences between histogram and bar graph more. The question was "what happens when we don't use gaps between columns in the bar graph?" and I think that it can help to distinguish the difference of bar graph from histogram very well. One student asked a similar question to it and Gizem referred back to steps of drawing a histogram while asking the question “then, how can we find the class width of such a graph?” It can be claimed that Gizem couldn’t direct the class very well with her questions to the objected outcome and Group members including Gizem discussed about alternatives which implementer teacher would react at that moment.

Group A also offered a solution to the problem which students faced with during the use of virtual manipulative. Students had some difficulties to enter data set into the website, to preview the data display, etc. They proposed preparing a leaflet like a user manual which could be applicable in this case, as in the following:

> Gizem: […] And it could have also been. There could have been some instructions to use these websites. Something like a little preivew, I mean we click on the eye button, like does it appear when we click on the eye button, I mean they realized it own their own or I told them.

As a summary, third group discussion regarding PCK mostly about teacher’s reactions and responses during questioning parts of the lesson. Members of Group A criticized these as well as their questions written on the lesson plan very much. They offered some precautions for the implementer of this lesson plan in order to direct questions carefully, control the responses of students more efficiently and manage the discussion well. They offered some changes about the statements of questions and addition of one or two questions as a result of the discussion. Group A also thought that using a leaflet or manual for the virtual manipulative would be beneficial for the students and saves time for the future implementations. There was not a specific conversation about students’ misconceptions or mistakes during third group discussion.

4.2.3.1.5 Technological Content Knowledge (TCK)

After PCK and PK, TCK was the most referred knowledge dimension during third group discussion of Group A. They discussed about the implementation of the virtual manipulative which they used for drawing histogram and stem and leaf display in the lesson.

During the implementation of the lesson plan, Gizem managed the task of virtual manipulative well. She directed the students about how to enter data set, how to preview data and how to use the tabs appeared as histogram, stemplot, scatterplot, box plot, etc. in the virtual manipulative. She also explained how to react in case of a mistake which students probably make. She walked around the classroom and dealt with almost every student during this task in order to check their data sets, data displays, etc. Related part of video transcription of the lesson is as follows:

> Gizem: Everybody opened that website, there are columns, we don't need to add any extra columns, and there is no need even to order the data. We will write one by one. (She is walking around the class, showing the students)
> Gizem: We are writing them one by one. We are writing while adding. (She is walking around the class, helping the students who encountered a problem, explaining on the computer, from the video-record of implementation)

Group members discussed about Gizem's actions during the task of virtual manipulative and they mostly evaluated her implementation as efficient. While taking preview of a data display in the virtual manipulative, the program gives an error as 'too much value' and Gizem explained how she solved this error and how she reacted during the lesson in order to prevent it, as in the following conversation:

> Gizem: […] I also solved that problem, it was based on, now they made those columns directly putting plus buttons, and after that I announced it in the class. Even they leave it empty after doing it, it looks full on the website, and even we don't enter data. And in that case, I again announced it to everyone, to delete them saying it looks like there's data even if there isn't any.

It can be claimed that group members took some precautions in order to make students not to make a mistake during the virtual manipulative part of the lesson. Group members seemed to master the virtual manipulative very well.

During third GD, group members also discussed about the benefits of including a virtual manipulative into lesson plan as a response to my questions of "Was the task of virtual manipulative beneficial in teaching histogram?" and "Was it a tool for the objective or did it become the objective of the lesson?" Related part of the discussion is in the following:

> Banu: I don't think so, the website became a tool in this case. And it became really handy, let's assume it's not a website, I mean if it was a teacher lecturing, it'd take a very long time to give a single data, and it'd draw a histogram based on that, but they went from stem and leaf display to histogram directly, and saw it.
> Gizem: And we rotated it, I think it was really easy. They saw it directly there. When they click on the histogram.
> Ezgi: Gizem showed it very quickly, she called the columns in a very fast way.

It can be claimed that group members agreed to include the virtual manipulative as a helping tool to reach their lesson objective. They emphasized that they used it not to teach itself but to teach histogram and stemplot concepts. Hence, it can be said that group members realized the aim of using technology in their lessons.

During the implementation of the lesson while dealing with the change of class width in the histogram, virtual manipulative computes its class width as 9.8 at first. Then, Gizem asked students to change it to 4.9, 5 or 10 and to discuss their effects onto histogram. Group members implied also that these non-natural numbers for class width are expected since their mentor teachers computed in a similar way (using the formula of histogram) during practice sessions and rounded to the nearest natural number in order to make computation easier. Therefore, they didn't want to arrange the data set in order to obtain a class width which is natural number.

There was also a similar discussion regarding changing the class width in histogram. During the implementation, there was a moment when Gizem asked students the min and max values for class width in histogram. I proposed them to include the same question after changing the class width in order to strengthen the instruction and they were agree with me.

As a result, group members referred to TCK often in their third GD and they seemed to understand the underlying concepts behind it. They discussed the task of virtual manipulative for teaching histogram with its alternative ways. They discussed their aim of using it into the lesson. Besides, they discussed its relations to content while generating new possible questions which can be added to the lesson. Therefore, it can be claimed that members of group A obtained an enough understanding of relations between technology and content reciprocally and operating a virtual manipulative which content learning is foregrounded.

4.2.3.1.6 Technological Pedagogical Knowledge (TPK)

TPK was the least referred knowledge dimension during third GD of members of Group A after CK. They mentioned about their precautions in case of students' mistakes or a technological failure.

They explained that they took the screenshots of tasks which they do in their lesson plan before the implementation. When they had some problems to activate smartboard during the lesson, Gizem immediately used the screenshots which they intended to show via smartboard at that moment and this saved the continuum of the lesson and time management. They mentioned about these failures of smartboard and said that students haven't realized the screenshots.

Moreover, Gizem's reactions towards the mistakes which students made during working with virtual manipulative were discussed. Group members explained that Gizem recognized the reason of this mistake instantly and managed the students with her right directions in order to overcome it. These reactions of implementer teacher discussed as positively and it shows the relation of technology and pedagogy very well.

As a result, it can be claimed that members of group A has an understanding of TPK and they are aware of their proficiencies regarding TPK.

As a summary for third GD of Group A, it can be considered as a significant step in developing knowledge dimensions regarding TPACK framework since the findings showed that their outcomes in third GD were getting different and richer than the ones of first and second GDs. They started to refer almost each knowledge dimension more often and they overcome their deficiencies related with CK and they started to discuss more about PCK, TCK and TPK in their GDs respectively. These comparative arguments will be mentioned later deeply at the end of this chapter.

4.2.3.2 Third GD of Group B and video-record of implementation

Third GD of Group B will be analyzed here based on the video-record of implementation of secondarily revised lesson plan (third draft of lesson plan can be seen in Appendix N at the end). In their second revision, members of Group B haven't made many changes on lesson plan except that they reordered the questions in the first part of the lesson (bar graph part) and there is an inlusion of data set in the second part of the lesson (histogram part). Before making students draw the histogram on virtual manipulative, they gave the data set while reflecting it to the board.

As in the case of Group A, I and Group B started to discuss the implementation of lesson plan overall at the beginning, then turned to watch its video-record. We had conversations between the breaks of watching the video. At the end, I asked group members whether they want to revise their lesson plan once more or whether they want to re-implement it. Group B responded that there is no need to reimplement it again as their lesson plan doesn't need to an extended revision. Therefore, their eventual decision was that there is no need to one more implementation. However, one of the group members, Serhat, has commented such an unexpected manner that he wouldn't implement their lesson plan at all with its final version since he thought that he couldn't control this lesson plan. The related conversation can be seen below:

> Serhat: Frankly, I don't think they will be able to keep their focus on it, according to me. From my perspective.
> Researcher: Then, which part would you change?
> […] Serhat: For example; there was a bar graph, while explaining this, I'd show, after that when I move on to histogram, maybe a few, maybe if it gets into it, I'd show the things of histogram, maybe I think I could have done the thing.

> […] Researcher: But then you'd be changing the whole lesson plan, I understand it from this, am I understanding your answer wrong?
> Serhat: Yes, little bit.

Serhat commented that he couldn't control the activities including virtual manipulative since he thinks that they take very much time. Hence, Serhat proposes to make the lesson shorten, and to exclude some of the virtual manipulative activities from the lesson plan. However, he couldn't explain his thoughts clearly; he mentioned about his feelings indeed. Another group member Zehra also claimed that time management can be difficult in their lesson plan, then she said she would rather neglect some of virtual manipulatives, for example, the virtual manipulative used for histogram is useless. Her explanations can be seen below:

> Zehra: It came to me that histogram is more effective, on the part when students coloured rather than going through on the writing. […] I mean through the website, that if it's necessery to see it through on the website or not, because it was a waste of time and…
> Şenil: It was so good when they coloured it, it turned out to be the histogram.
> Zehra: Let's assume he entered a wrong data and wrong data came up for each student.
> Researcher: I mean, you may not make them draw histogram on the second website?
> Zehra: Website once or twice in the class is enough, third time would be a little bit challenging, in my opinion. […] we have 3 of them. One after another. I think, in order to draw students' attention, it's too much.

In fact, Zehra claimed that there are so much virtual manipulative activity in the lesson plan that the time and classroom management during their phase would be very difficult for the implementation in a real class. That is, her concerns are very similar to Serhat's ones and they both actually propose to make the lesson plan shortened while excluding some virtual manipulative. However, their groupmates, Şenil and Emel were not agree with this idea and they defended their lesson plan that the continuum of the lesson is provided with the inclusion of virtual manipulatives so exclusion of them doesn't work for the sake of lesson plan. At the end of this conversation, Zehra seemed to be convinced by her group mates and Serhat explained later that he thought he would not implement it by himself in a real class. As a result, they responded summarizingly that another implementation wouldn't be different much since lesson plan do not need a radical change in its structure.

During the implementation of lesson plan, Group B haven't experienced much problem. Teacher tried her best to follow the lesson plan, that is; she applied all of the steps in the lesson, asked all of the questions, and made the all of the activities of virtual manipulative. However, she couldn't control the activity sheets enough which she distributed to the students at the beginning: she only warned them twice to do the task specified on activity sheet and couldn't check if students did or not. Besides, there is a problem related with smartboard like in the implementation of Group A before. However, they overcame it with the help of screenshots again. They had also a problem with the projector connected to the computer, they couldn't reflect a table to the board. Instead of reflecting to the board, they used smartboard to fill the table when smartboard started to work again. Except for these, implementation of lesson plan of Group B was an example of microteaching, even students feel themselves as real eighth grade students and a real classroom environment was felt till the end.

Third GD of Group B was analyzed according to the video-record and its transcription of lesson plan implementation. Analysis of the findings was done according to the knowledge dimensions appeared in TPACK framework, as it was the case for previous GDs of each group.

4.2.3.2.1 Content Knowledge (CK)

CK was the only knowledge dimension in the third GD of Group B to which they didn't directly refer. They haven't mentioned about the facts, rules, concepts or principals regarding their lesson objective. Therefore, members of group B can be claimed to have developed their CK regarding the topics they covered in their lesson plan, which was an expected finding.

4.2.3.2.2 Pedagogical Knowledge (PK)

First, the issue of lesson planning activities and preparation will be analyzed. Group B planned to collect the data which is used for bar graph and histogram before the lesson. However, they decided to prepare their own data set and arrange it to the students since they hesitated to work with a large range or too many categories. They wanted to take a precaution in order to avoid such a data set by distributing a small paper on which a job and a year is written. Before the lesson begins, they sticked these papers onto each computer and announced to them as "your father's job and how many years he has worked are written in the papers on the computers." During third GD, group members discussed this as a last-minute action before the lesson since they said that they couldn't collect the data from all of the students. Consequently, they emphasized that this precaution was useful.

During the implementation, Emel as the teacher forgot to follow the activity sheets as they specified in the lesson plan. Through the lesson, she announced to students to work with activity sheets two or three times; however, there were questions to be answered by the students left at the end of the lesson. It might be because of her excitement or the flow of discussion part of the lesson. She criticized herself as in the following conversation:

> Emel: And then I planned something. I mean, I'll bring it with the activity sheets, and have them make it on the activity sheets also, I told it like three times.
> Şenil: It didn't do…
> Emel: I mean I didn't tell them to write down the questions on there, but I didn't…

Later through discussion, group members also claimed that following activity sheets would cause time loss if they integrate it into the lesson as they intended. They also thought that students haven't followed the activity sheets even it was announced to work with them. The following part of the conversation shows this discussion:

> Zehra: […] I mean there's always an activity sheet, you turn to the computer, and then to the activity sheet, and to the computer again.

> Şenil: By the way, the activity sheet hasn't being made yet.
> Zehra: Yes.
> Şenil: What would have happened if we did that too?
> Researcher: What do you mean it hasn't being made?
> Şenil: It's never been coloured.
> Researcher: I think some of them coloured it.
> Şenil: A little but no one really paid attention. […]
> Emel: But like I said, it didn't do a lot. For example; we could have said that these questions were also on the sheets, and let's fill them from there, but I mean, mine is good.

Although they thought that activity sheets can be time-consuming, Emel suggested that she could make students to answer the questions by writing on activity sheets during discussion part of the lesson since discussion questions were written on them. Eventually, they were aware that activity sheets were not included as they planned before and they shouldn't necessarily be included in their lesson.

During the questioning in the second part of the lesson (histogram), Emel asked lastly a question as "can you give examples of histogram or bar graph with a data set?" and I emphasized that this was a powerful question to finish the lesson. Group members also specified that the given examples were more than they expected. The related conversation is as follows:

> Researcher: It's good that you asked for these examples here, it made them give examples.
> Şenil: Yes.
> Researcher: I mean, it really made them think while reviwing it.
> Emel: Yes, they thought a lot. I wasn't expecting it. Like, okay let's find some at home, they answered a lot.
> Şenil: There was nothing left to do at home.

Summarizingly, group members showed their understanding of lesson planning and preparation abilities enough through third GD, which is an important finding of the study.

Group members also discussed the knowledge of learners and their background often during third GD. During the physical manipulative and filling the table parts, Şenil was the first student to do the task. For example, she said loudly "my father's job is worker and I put the ring here” and they expected to make each student to say his/her action aloud to the class. However, the rest of the class haven’t said such a sentence. Şenil explained that this would be probably like they expected since eighth grade students found such an action as joyful. Members of Group B also realized that students found the virtual manipulative for pictograph as very enjoyful. I also agree with them since the virtual manipulative was dealing with the pictures directly searched through the web according to the written job either in English or in Turkish. The selected pictures were very attractive for the class and students enjoyed much during the implementation. Group members realized such a reaction of students and it showed their ability of understanding their learners.

There was also a discussion among us about the question related with mean during the questioning period of histogram. Emel asked the question “can we predict the mean according to the histogram formed here?” We discussed the suitability of such a question in this lesson and whether they know about the mean of a data set. Group members claimed that eighth grade students have already learned about the mean, and consequently such a question can be asked. The related conversation can be seen below:

> Researcher: You go to mean from the questions about the histogram, okay these are mean, in the curriculum, and do they see it before the thing or after?
> Zehra: They see it before.
> Şenil: There’s mean at 6th grade, mode and median at 7th grade.
> Researcher: Nice. Then you’ll also be reminding that.

This shows also their awareness of background knowledge of the learners, which is a finding for the study.

Members of Group B also mentioned about their classroom management skills in our discussion. For example, they were aware that left side of the class was more interested in the lesson and they were dealing with the tasks more as compared with the right side of the class. Besides, group members specified that the control of the physical manipulative tool could be difficult and takes more time in a real class although they found very enjoyful while watching the video-record of the implementation. Some members claimed that classroom management could be difficult in a real class since they have lots of tasks included in the lesson and they took more time in reality. We determined that this lesson was a very interactive one since it provides much participation of students at last. Consequently, they concluded that teacher should be aware of this fact and should use his/her classroom management skills very well in a real class. Because of these reasons, they also thought that an implementation in a real class could take more than 2 hours, so they had some time management hesitations as well.

To summarize, group members showed their pedagogical abilities very much during third GD since they referred to the issues regarding PK much in our discussion. Hence, it can be claimed in general that members of group B have a developed understanding of PK and used it through implementation and lesson planning phases of the lesson plan.

4.2.3.2.3 Technological Knowledge (TK)

We discussed only the troubleshooting equipment issue regarding TK during third GD. As it was specified before, group members have a developed TK. During the implementation of the lesson plan, teacher couldn't use the projector to reflect the histogram in order to make students to fill it, then she used smartboard instead of projector as an alternative way. Another problem happened during the virtual manipulative for histogram, that is; web page has not opened because of flash requirements. However, Emel immediately showed the screenshots of histogram which they already took before the lesson. This is an example that shows their ability to solve the problems regarding technological equipment during the lesson.

4.2.3.2.4 Pedagogical Content Knowledge (PCK)

PCK was the secondarily most referred knowledge dimension by Group B during third GD. Regarding PCK, alternative ways of teaching/representing subject matter to students and identifying and addressing student misconceptions or mistakes were analyzed. Group B never mentioned about the assessment strategies for their lesson as it was the case in previous GDs.

Group B mentioned about the projector problem they experienced during the implementation. What they planned was before the lesson is to reflect the table in Table 4.6 to the board in order to form histogram by filling the cells. After students form the histogram, teacher was going to show the virtual manipulative for histogram reflected by smartboard. Then, students was going to see both and compare them when class width is changed through the virtual manipulative. When projector didn't work to reflect, Group B couldn't use it in their implementation, they only used smartboard for both reflecting and using it to fill the table. Therefore, they couldn't make students to analyze the comparison issued by class width as they intended. Emel explained their previous idea as in the following:

> Şenil: It was going to be reflected by the projector and be shaded there.
> Researcher: He was going to shade it with the pen, right, I remember talking like that?
> Şenil: Yes.
> Emel: I say, it stays on the side.
> Şenil: We wanted them to compare those two.
> Emel: The [class width] over there didn't change but, the [class width] on the thing would change, I mean in the website.
> Researcher: Yes.
> Emel: I mean, he would see those, and compare with those questions, and we were going to do the thing […]

The above conversation was an example since it shows the ability of Group B for generating alternative representation methods to analyze the class width of a histogram. Their intent to include to lesson plan was never mentioned in our previous GDs. Mentioning about such issue shows their capability to evaluate different representation ways, as well.

It is also worth to emphasize here that there was a similar discussion related with first part of the lesson. They discussed the alternative ways of ordering the tasks included in bar graph part. They started with physical manipulative tool, continued with pictograph and virtual manipulative for bar graph and finished with a questioning about bar graph concepts. That is, there were three different representations of bar graph respectively and a discussion at the end. We discussed together with the transitions between these tasks and the order of them. I found necessary to begin with physical manipulative since it also showed the data set. Because they distributed the values to each student before the lesson, students didn't have the data set at their hands. They also emphasized that physical manipulative task was interesting to start with the lesson since students easily concentrated on the topic. Besides, the order of the tasks was discussed. They specified that some of the questions asked at the end should be moved between pictograph and bar graph virtual manipulatives. They thought that the transition from pictograph to bar graph was not nice and it should be improved while asking some questions before the virtual manipulative of bar graph. Related conversation can be seen below:

> Zehra: Yes, it would have been nice, if one or two questions came in. While moving on to bar graph from pictograph.
> Emel: I mean it looks like we moved on to bar graph suddenly. I mean, they won't see it quickly, or I think they may see the bar graph there.
> Şenil: Yes.

This finding shows the lesson planning abilities of group members. Group members could see the consequences of different implementation way of tasks in a lesson.

Most of the discussion regarding PCK was around the questioning parts of the implementation. End of the bar graph part, beginning and end of histogram parts were gone through questioning. Following conversation shows the beginning of histogram part from the lesson:

> Emel: Now, friends, I wanted two different data from you. The second one was related with the years worked by father. Now, we use this second one. We will make again a graph. Which graph can we make?
> Student 1: It would again a bar graph.
> Emel: Then, what if when we do a bar graph, [student 1]?
> Student 1: My father is retired, we write 15 near to retired.
> Emel: How will we do this? Let's think of here (showing the physical manipulative) for example, we are using the second one, the one which is for the duration of fathers worked.
> Student 1: Only the years?
> Student 2: Will we think only the years?
> Emel: We will think only the years, ah, he thought that can we do something while thinkin both the job and the duration…

Student 1: No, how many years did our father work in general? Here, it is not written like that. Then, I skipped the retired ones. I will write 10 years instead of writing doctor, for example, 20 years instead of teacher, 15 years instead of officers. Let's do the same again. My father worked for 15 years. I would put a ring to the place of 15.
Emel: You say you write years instead of jobs. I understood.
Student 1: Yes.
Emel: Is that right according to you?
Şenil: Don't we need too much writing then?
Student 3: For example, if we write 5, 10, 15, and the like, then what if for 6, what will we do then?
Emel: [Student 1], now [student 3] asked a question, try to answer it.
Student 3: Or, if we would write all one by one, then if 6 appers only once, or the others appear only once, then, there happens lots of things.
Student 1: I made up the numbers since I saw 15 in mine, and saw 30 in others, I thought that they are multiples of 5.
Student 4: My father worked 11 years, for example.
Şenil: Then, wouldn't be it so long then?
Student 1: I didn't see you.
Emel: Then, should we write them all?
Student 5: Let's make it histogram, instead of writing them all one by one.
Student 6: Let's make a histogram.
Emel: Now then, what is the point in making a histogram? Why did you think of making a histogram?
Student 2: There will be a lot of data.
Student 5: Since years are passing, we could have the chance to see the gaps better.
Emel: Now, yes, for this reason, according to the opinions of your friends, histogram will be more beneficial for us. Since, if we do the all one by one, I would make like [student 1] offered. However, when I would do that, what will I understand from such a graph? We are making interpretations from graphs, right?
Student 2: We would say only 15 for the retired.
Student 1: We could understand the number of fathers maybe.
Emel: Then, if we do the histogram, then the data will provide more benefit to us. Then, we will make a histogram.
(From the video-record of implementation)

Questioning method seems well-applied here in this part except for some points, and we discussed them together. I criticized them presenting histogram as the only solution to draw the second group of data set (years what fathers worked) since Emel started questioning with the question of “which graph would you prefer to draw this data?” At the end, she couldn’t connect the ideas generated by students together and her reaction was like there was no other alternative to histogram, but stem and leaf display, for instance. Then she grasped the idea and specified as in the following:

> Researcher: Maybe here, you said, why we draw histogram, we draw histogram because we can’t draw bar graph, but what if a student comes along as asks why they can’t draw stem and leaf display, what would you say ? How would you answer?
> Zehra: It’s not in the curriculum.
> Researcher: It could be, the child may know it.
> Zehra: We don’t spesifically say it because there is not.
> Emel: I’d say let’s do it, I’d say okay let’s do. […]
> Şenil: But it would take 2 hours before they understand it, stem and leaf display is not such an easy subject.
> Researcher: It would, and also, how can I put this, I think it’s not very nice to say to the student that there isn’t an alternative way, and that’s way we draw histogram, and bar graph is no good.
> Emel: We could have said histogram was one of the graph we were drawing.

It can be claimed that Emel showed a developed understanding of PCK, so as the other group members since she understood my hesitation about her response and generated the right reaction as an alternative.

Another conversation related with questioning again was about a question of “does data show a big difference, or are values close to each other, what can we say according to the histogram?” I asked them to explain their aim in asking such questions. They responded to me that they tried to make students to think about distribution of data. Although they weren’t expecting to use the word ‘distribution’ by students, they aimed to hear about some sentences about data referring to its distribution. The related conversation is below:

> Researcher: I don’t understand why you were asking this question.
> Emel: Which question?
> Researcher: I mean at that side, one second, let’s listen to that part again.

> Zehra: You want to see what they get out of histogram, that's purpose of the question. (Emel is asking, it might show varieties, their values are close, and what can we say here?)
> Researcher: How do you expect student to answer this question?
> Emel: I mean, in here, is the standart deviation less or more, to see if we can understand the distribution of the data before getting into that too much.
> Şenil: She is trying to do it before pronouncing the name distribution.
> […]
> Emel: Even she doesn't say distribution, the majority is so different like this, there're some sharp values, or some that gathered in the middle, or like that makes peak.
> Researcher: Gathered in the middle, making peak. Okay.

It can be claimed based on above conversation that group members used the questioning technique with respect to its fundamental aim, however they couldn't rephrase their question in a right form of sentence. That is, they were aware of the objectives of integrating questioning technique into their lesson, which shows their understanding of PCK, but shows a necessity to practice more in application of questioning method.

Members of group B discussed also the students' misconceptions or mistakes during third GD a little. There was a problem which students experienced during the virtual manipulative for histogram. Students entered data set which they saw it as reflected to the board. Emel prepared the data set as a table: first column shows the numbers of values and second column shows the value. During implementation, I realized that some students understood the first column as some data and tried to enter as 1, 2, 3, 4, … without noticing them as numbers of values. When I explained this problem to group members, they said that they never realized it during the lesson and they never thought that students would misunderstand the table. We immediately generated another way of presenting data set as can be seen from below conversation:

> Researcher: Let's continue, now that you have this graph, a lot of problems came up due to it. […] now that you wrote students in there. I mean, you wrote the number.
> Şenil: I wish we wrote a, b, c.
> Emel: It never came to my mind.

> Researcher: Or you might have left it empty. Not write something at all. You could have just given it like a list. You could have given it like a series by putting commas between. It made it confusing when you gave it, if people at your age is confused by this, middle school students' would be easily confused, because I really really helped.
> Emel: I never thought of a, b, c. But I wrote on it as student 1, 2, 3, 4, 5 and and the years.
> Şenil: We thought it wouldn't confuse, right?
> Researcher: Since it came up more than I thought, now here, there was a problem with this display.
> Şenil: Let's make it a, b, c.
> Researcher: I wish, you shouldn't have entered into a b c thing. Because this time, someone might have tried to write it also. Because of the row thing. It's the best to give it in listing or it could have been a simple, straight list.
> Emel: I also thought it would be simpler and would look more beautiful that way.

Therefore, it can be claimed that Group B need to observe their learners' more during an implementation and could see their mistakes or misconceptions earlier in order to take a reaction at the time.

Consequently, it could be claimed that Group B need to develop their abilities to observe students' possible mistakes or misconceptions more, however, they have an understanding of representing a subject matter in different ways.

4.2.3.2.5 Technological Content Knowledge (TCK)

TCK was the least referred knowledge dimension during third GD of Group B members. An obvious discussion regarding TCK was about questioning included in histogram part of the lesson. Emel directed the questions related with changing class width of histogram and possible effects of these changes on histogram to the students. The related part of the lesson was given below:

> Emel: What if I asked while looking at histogram, how many students are there whose father worked for two years? Can you answer?
> Student 1: 1
> Student 2: 2
> Emel: Is there one father? [Student 1] says so.

> Zehra: But, he changed the class width to 1. For example, I did 4, and I couldn't find the answer.
> Student 1: I was about to say that. For example, if class width is changed to 1, we can see how many people worked in how many years easily.
> Emel: In fact, doesn't it look like the display we discussed at the beginning? For example, there is one father for 2 years, two fathers for 3 years. Isn't it a long bar graph at the end?
> Student 1: That is, it seems so.
> Emel: Then, can I understand anything from such a display?
> Student 1: But, we can change it through this program.
> Emel: In order to find how many people exactly?
> Student 1: Yes.
> Emel: If we have such kind of technology, then we could. But, what if we couldn't change the class width, can we say the same?
> Class: We can't say something.
> Student 3: I see some fathers between 2 and 7 years, but I can't say about exactly how many years they worked, such as is it 2, or 3, or 4?
> (From the video-record of implementation)

Emel was addressing here that classes are more important than values in a histogram, that is; individual values could not be realized through a histogram. She was not expecting such a response from students. Then, she tried to explain what happens to a histogram when class width is changed to 1 to students. Hence, she made a clever action while referring back to a previous discussion between [Student 3] and herself about bar graph or histogram choice for the second group of data set. However, she couldn't show its effect on histogram since virtual manipulative had a problem in her computer as discussed before and she preferred to explain the issue as orally without drawing manually to the board.

This is a finding that members of Group B have an understanding of how technology and content are related reciprocally with each other, that is; their understanding of TCK.

4.2.3.2.6 Technological Pedagogical Knowledge (TPK)

TPK was thirdly-most referred knowledge dimension during third GD of Group B. They analyzed their abilities to generate suitable activities to students' possible mistakes/misconceptions, to explain a computer procedure to students and to generate alternatives in the event of technological failures. They had also some conversations for technological infrastructure of classroom.

During pictograph task, teacher made it with the student. That is, she showed how to enter data set to draw the pictograph to the students. Consequently, students had a chance to watch how to draw it and it was like an example for students. Members of group B realized that and explained it as in the following:

> Emel: I've done those too, but then it changed after. That's why I entered them again by myself.
> Researcher: Okay, it's better. That you enter.
> Emel: I was going to do it again, but I took a printscreen for comparison in case I didn't do it again.
> Zehra: It's really good that Emel says what she does here.
> Researcher: Yes, it's nice. It's also good that we show while doing it. Like I said it's good that she didn't take a printscreen. Yes, a problem came up with the histogram, okay, but while doing the others, if students get lost, or not able to do it, at least copy the teacher.
> Şenil: Yes.
> Emel: Yes.

While drawing histogram through filling the table on smartboard, they also realized that it could be longer in a real class. They explained that students could have difficulties while filling on smartboard since only one student could use the smartboard because its non-multifunctional property. Besides, students will not be carefull enough that smartboard feels every touch on itself, that is; some false drawings might occur because of its sensitivity. They also realized that the lines of table disappeared because of filling. Then, Emel explained that she had to redraw those at the end of this activity. The related conversation is in the following:

> Şenil: Students, of course will do it slower.
> Researcher: They might do it slower in the real class.
> Emel: And also those gaps grasps those…
> Researcher: It can be really risky doing it on a smartboard in the class, because student's arm will touch it, hit it, or something, there can be a lot of shading. Like the time Gizem's arm touched it. But our main aim was to do it on the board. But, I mean we had to do it on the board.

Emel: For example, since those gaps closed suddenly, and edge lines can also be coloured…
Researcher: Yes, you've already passed over it.
Emel: Yes, later on I passed over it. Because it wasn't obvious.
Researcher: It's nice.
Şenil: Someone told you.
Emel: I wouldn't have thought of it if someone didn't. It's better or else they wouldn't understand.

Based on above two conversations, it can be claimed that members of Group B have an understanding of how to react with their students as learners during a technology-integrated activity, that is, they started to improve their ability to plan relevant activities for students who experience possible technological problems.

Their precautions in the event of a technological failure were also discussed regarding TPK. They prepared screenshots of their online tasks in case of a problem, eventually, they needed to use some of them. Hence, it could be evaluated as an example of their understanding of TPK.

Group B was also aware of technological infrastructure of the classroom. Emel mentioned it that she couldn't have the opportunity to observe students very well because of existing paravanes between lines of students' desks. She said that there was not any program inside the teachers' computer which is connected to students' computers, so she couldn't observe their monitors either. The related conversation is below:

Emel: And now that you can't see the student here, those things, screens.
Researcher: You can't see it, it's that. And the thing.
Emel: And you can't see what they are doing on the computers.
Researcher: As far as I know, on the system, the teacher can connect to all the students' computers, can click and open it on the screen. And do the intervention over there.
Şenil: It's good.
Emel: It can also be done here. We could have done it here.
Researcher: Is there that kind of a connection in this computer lab?
Emel: I didn't do anything on here. But in our highschool, computer of the teacher would see everybody.
Researcher: Yes.

Giving an example from her high school years, she showed she paid an attention to know infrastructure of classroom. Hence, it can be claimed that Group B has an awareness of TPK regarding knowledge about infrastructure.

As a result, third GD of Group B referred showed that they have an understanding of TPK regarding generating relevant activities in case of a technological problem and technological infrastructure of classroom.

Next chapter discussed highlighted findings throughout the findings chapter.

CHAPTER 5

DISCUSSION

This chapter discusses the major findings through the related literature. The organization of this chapter was made according to the knowledge dimensions, which are CK, PK, TK, PCK, TCK, and TPK, aroused in TPACK framework for the groups A and B. Lastly, all of these knowledge dimensions were discussed together in order to investigate their development in TPACK, as the seventh knowledge dimension. The findings of MLS conducted with members of both groups A and B and the DTAS results of them were discussed together in the following parts, according to the knowledge dimensions. Each part presented below with an in-depth summary of the related findings discussed them through the related literature.

5.1 Content Knowledge (CK)

In their first draft of lesson plan, Group A was starting with bar graph concept, and making a transition to stem-and-leaf display, then finally resulting with histogram. They wanted to use the same set of data in order to achieve transitions between the steps of the lesson. However, in first GD; Group A was observed as: (a) Their knowledge of type of variables used in data displays (especially in bar graph) was weak; (b) They did not have enough understanding of data displays regarding their fundamental differences, especially differences between a bar graph and a dotplot; and (c) they couldn’t choose the right example for a bar graph. Because of these problems, they realized that they needed to reconsider the example they used for bar graph and histogram for the second draft of their lesson plan. Since their aim was to use the same data set as an example, they realized that they couldn't use the same data set for both data displays. They understood that *bar graph and histogram need different types of variables*. When DTAS results considered regarding variable type, 2 of 5 members of Group A had also difficulty in understanding the differentiation of variable in a bar graph, regarding item 12. They could not catch the variable type used in line graph demonstration. As Jacobbe and Horton (2010) concluded similarly in their study that preservice teachers' understanding of data displays was weak, regarding the selection and construction of data displays. In order to investigate the understanding of the participants in terms of data displays, researcher used the taxonomy offered by Friel, Curcio and Bright (2001) in their instrument, whose levels are reading the data, computations, comparisons, trends, selection and construction of data displays. They have added the fifth level to the Friel et al.'s (2001) taxonomy in order to investigate the type of variables more closely. The major result was that their participants had a low comprehension of data displays. However, their achievement in first three levels were higher than the last two levels of the taxonomy. Therefore, it could be concluded that selecting type of variables are problematic for prospective teachers, although they are capable in understanding reading the data in a graph.

In their second draft of lesson plan, they decided to make a change in their lesson plan: they omitted the bar graph part of their lesson; that is, they planned to start with stem-and-leaf display, and then to make a transition to histogram while using the same set of data. At the end of these steps, they activated the students' prior knowledge about bar graph while making them to compare histogram with bar graph. They explained that they liked the transition from stemplot to histogram, and hence they haven't thought to omit stemplot although it was not included in the eighth grade curriculum. Bright and Friel (1996, 1998) have outlined the benefits of using transitions among data displays in order to emphasize the structural relationships between graphs. They mentioned also the transition from stemplot to histogram which Group A did in their lesson plan. Authors suggested that stem-and-leaf plots provides a natural transitional device in order to highlight the equal-width intervals in histogram. Since ‘both histograms and stemplots group data in equal intervals, for instance by 10s, it is theoretically easy to rotate a stemplot 90^0 counterclockwise and visualizing a histogram drawn over the stems’ (Bright, & Friel, 1998, pp. 79-80). Virtual manipulative which Group A included into their lesson plan shows exactly this transition.

However, Group A was observed again as having some problems regarding CK in their second GD. Their problems were: (a) they did not have enough understanding about guideline of histogram; (b) they were not capable enough using right terminology for histogram (in both English and Turkish languages); and (c) they had a misconception that each class (or stem) is halved when class width is halved. Regarding these findings, the misconception has originated from lack of knowledge in grouping the data set, while histograms or box plots use grouped data; bar graphs or picture graphs use original data (Friel, Bright, Frierson, & Kader, 1997). Because of using grouped data, histograms, in fact, ‘bury’ the individual data points (Bright, & Friel, 1998), where some misconceptions occur as a result of this fact.

Based on above findings of initial and second GDs of Group A, it can be claimed that they overcome their inadequacies about CK of type of variables and they realized that dotplot and bar graph are different data displays, so, they need different types of variables. Further, it can be claimed that they had a better understanding of these concepts and this was reflected on their lesson plan. However, they had some problems and misconceptions in histogram concept as it was observed during second GD. Through our second GD, they tried to understand their inadequacies about the terminology used in histogram and the class width issue of histogram and stemplot both. We discussed the right terminology needed and tried to overcome their class width misconception together. Second GD of Group A finished while leaving less revision than previous revision regarding CK.

Third GD of Group A has passed with some critiques of group members on themselves. However, these critiques were quite positive that preservice teachers' examples used to respond students were quite appropriate and convincing. That is; Group A overcome their weaknesses regarding CK in their third draft of lesson plan. They prepared right questions while having enough knowledge to make satisfied and clear responses without making students to direct misconceptions. Therefore, third GD was different from our previous ones since we haven't discussed the inadequacies, but we discussed these examples. Besides, they were even observed to discover some new facts about dotplots and histogram. Dot plots are actually special histograms which ungrouped data is used and dotplots can be used for small data sets (delMas, Garfield, & Ooms, 2005). However, it was stressed that a dotplot cannot be effective while dealing with large sets. In our discussions, members of Group A discovered this fact, as well. They even concluded that a dotplot is a histogram whose class width is 1. Since they lacked knowledge about the difference between a dotplot and a bar graph at first, they were observed to have improvements in their 'graph sense' (Friel et al., 2001).

When the findings of Group B regarding CK were investigated it was revealed that there is a decreasing trend in CK codes. Group B was observed to have a lack of statistical content knowledge in bar graph in all three GDs of Group B. For instance, they did not intend to explain why intervals are used in drawing a histogram; and, they didn't know the pictograph as an alternative data display for categorical variable in their initial GD. While discussing about the data distribution in a bar graph, members of Group B were asking some questions like "is there any effect of one half of data set on its other half while drawing bar graph?" and they were planning to make students to think about that the spread of data is not always the same. Eventually, they were trying to make students to make generalizations about data set, which such an issue was never discussed with members of Group A. This was claimed as a serious misconception before since it was understood that they were lack of necessary CK about data distribution and sampling concepts. The same misconception was also observed in the histogram part of their lesson plan through the similar questions they directed to students. This showed that they were making an incorrect relation *between number of values in a data set with its range*, then they used this relation in order to explain the necessity of preferring a histogram for large data sets. Therefore, it was claimed that they have also serious problems in understanding the usage of intervals in histogram. Besides, they were observed to not know the pictograph as alternative for bar graph display. At the end, it was clearly claimed that they have problems in understanding some basic statistical concepts in the initial GD and we together tried to discuss their misconceptions via group discussion through my questions about the concepts.

The distribution concept has been considered as important, since there are some "depending concepts on distribution and being depended on by others" (Reading & Canada, 2011, p. 224). Distribution is defined as "the arrangement of values of a variable along a scale of measurement resulting in a representation of the observed or theoretical frequency of an event" (Leavy, 2006, p. 90), and this concept is considered as fundamental to statistical thinking (Pfannkuch & Wild, 2004). Nine concepts were identified as dependent to distribution: center, variability, shape, density, skewness, relative frequency, probability, proportionality and causality. Sampling distribution, statistical confidence and statistical significance have been accepted as depending on the concept of distribution (Reading & Canada, 2011, p. 225-226). Bakker and Gravemeijer (2004) described three levels of understanding the concept of distribution, where levels goes from least sophisticated level to the more one. First, a distribution of a data set is simply viewed as a set of individual values. Second level implies that a distribution is expressed with its underlying characteristics, such as center, spread and skewness. Third level as the final improvement level for distribution concept is that a distribution is viewed as an aggregate. Based on this framework, members of Group B demonstrated only the first level of understanding since they could not deal with the data set as an aggregate. They treated data set as halving it into two, and tried to observe how the spread of data will be. They had such a hesitation that after first half of their students formed the bar graph, the other half of them could think of the spread would be the same. Consequently, they couldn't relate the distribution with spread, as one of the characteristics of distribution, which is also the second level of the framework for understanding distribution (Bakker & Gravemeijer, 2004).

In the second GD, Group B seemed to have the same misunderstandings/misconceptions about data distribution in a data display. They claimed that histogram is the only data display to see the distribution of a set of data, therefore, they stated that there is no problem in trying to draw a bar graph for a data set containing continuous variables. In fact, there was a disagreement between group members about this claim. Then, I tried to make them to think about the aim of drawing a histogram through some questions during the discussion. At the end, they seemed to be convinced that a bar graph for continuous variable (Figure 4.5) is in fact a useless or pointless effort since it doesn't give an idea for the distribution of data set. Lack of CK of members of Group B could be claimed still not in the third level of understanding distribution. Suggesting such a bar graph actually proved their weakness in understanding that data sets should be evaluated as aggregate.

However, as another finding from the second GD, their effort to learn more about pictograph could be accepted as a minor improvement in their understanding of statistical concepts. So, on the whole, it can be claimed that their serious problems in statistical CK was observed again although they seemed to overcome their misconceptions regarding making a generalization for a sample to the population while *reading the data*, which Friel et al. (2000) categorized in their task taxonomy. Together with the results of DTAS conducted to participants before MLS, it can be claimed that their understanding of data displays are weak. The reasons for using data displays are described as in two ways: analysis and communication. Former was expressed as "a data display forces us to notice what we never expected to see" (Tukey, 1977, p. vi, as cited in Friel et al., 2001). Second was explained as "a good graph forces us to see the information the designer wanted to convey" (Kossyln, 1994, p. 271, as cited in Friel et al., 2001). Based on the results of item 7 in DTAS, more than half of the participants (6 of 9) responded partially to it. It asked which line graph formed for the salaries of workers for 4-months period could be used in an argument. Partial responders to this item used the small graph scale in favor of seeing the individual values. In other words, they claimed that individual values could be distinguished when a small scale is used. Consequently, they could not see the biased characteristic of this line graph. Graph scale was expressed as having effect on reading the frequency of values (Friel, et al., 2001). What Leinhardt, Zaslavsky, and Stein (1990) found that "the shape of a graph changes depending on the scale necessitates a conceptual demand" (p. 17, as cited in Friel et al., 2001) was the weakness of the participants regarding item 7.

Codes implying CK was only coded once in the third GD of Group B. This was an expected finding that they implemented their lesson as they seemed to overcome the misconceptions mentioned above. That is, there was not any inaccurate observations regarding CK. Group B emphasized that bar graphs are only for categorical variables and histograms are only for continuous variables, for instance. Even, there was a question at the end of their lesson plan which they made students to think of an example for a bar graph and a histogram displays separately while attracting students' attention on variable type. They also gave up halving data set in order to make students to think of generalization for the population. They seemed that they realized that they were only dealing with the data distribution, not the sampling of a population. Their questions were not problematic regarding statistical concepts without making students to fall in misconceptions or misunderstandings. On the whole, it can be claimed that they overcome their misconceptions about data distribution, they understood the difference between the so-called dotplot (since their dotplot was not ordering the values) and the bar graph. Even, they were observed that they understood how to comment on data distribution in a data display. Hence, it can clearly be claimed that they had a significant improvement in their statistical CK throughout their MLS progress, as it was expected.

Below Table 5.1 summarized the findings of both groups of A and B regarding their CK through initial and second GDs. Since CK was not coded in their third GDs, there is no row specified for third GD in the table.

Table 0-1 Findings through initial and second GDs of Groups A and B regarding CK

Groups/ Findings through GDs regarding CK	Group A	Group B
Initial GD	Weak knowledge of type of variables used in data displays (especially in bar graph) Not enough understanding of data displays regarding their fundamental differences especially differences between a bar graph and a dotplot Not able to choose the right example for a bar graph	Having a misconception related with data distribution Not able to explain why intervals are used in drawing a histogram Not knowing the pictograph as a type of data display
Second GD	Not enough understanding about guideline of histogram Not capable of using right terminology for histogram (in both English and Turkish languages) Having a misconception that each class (or stem) is halved when class width is halved	Lack of understanding why intervals are used in histogram Lack of understanding why continuous data sets are needed for histogram

As Group A did in their lesson plan, Group B used the same instructional objective regarding histogram. At the beginning, both groups started with bar graph concept, while Group A tried to make a transition from bar graph to stemplot (inaccurate) and then another transition to histogram, Group B also started with bar graph and continued with histogram. Group B never mentioned stemplot in their lesson plan. Unlike Group A, Group B used their own data sets, one is for their students' fathers' jobs and the other is for the years worked by the fathers. Besides, Group B explained that they wanted to use relational data sets in order to provide a unity through their lesson. In their second GDs, while Group A made a major change in the flow of their lesson plan, Group B changed only the bar graph part of their lesson plan. Group A omitted to start with bar graph since they understood that they couldn't make a transition from bar graph to stemplot or histogram as they realized that these require different types of variables. Since Group B learned about pictograph, they included it as a third type of bar graph representation. Regarding transitions, Group A used the transition from stemplot to histogram, while Group B used only the transitions between different types of bar graph representations. Both approaches were discussed above and it was claimed that they had an increase in their CK regarding different data displays. That is, both groups had an improved graph sense what Friel et al. (2001) described.

In their third GDs, neither Group A nor Group B referred to CK in terms of any adequacy since they all overcome their weaknesses through GDs and reflected this improvement both in their lesson plans and their implementations. Therefore, it was concluded that Group A and Group B developed their CK through MLS especially regarding data displays. Further, it could be claimed that they improved their graph sense since they all overcome their problems regarding all data displays which they included in their lesson plans.

5.2 Pedagogical Knowledge (PK)

In the initial GD of Group A, they seemed to deal with knowledge of general teaching methods and strategies at most, knowledge of learners and their background as secondarily and lesson planning and preparation at least, regarding PK. However, they have never referred to checking for understanding, classroom management or assessment strategies throughout initial GD. Members of Group A discussed several teaching methods and strategies (direct instruction, questioning, think-pair-share method) during initial GD and their appropriateness to lesson objectives. They were aware of how to integrate these teaching methods into their lesson. Group A also showed their consideration for their learners' background during GD, that is; they have already learned what their students' prior knowledge are from the unit plans. For instance, they knew that stemplot was not directly included in the school curriculum, i.e., it was touched on without specifically naming it. For this reason, they stated that they never thought to exclude the stemplot part of the lesson. Besides, they thought that using the same set of data as the only example for bar graph, stemplot, histogram parts of the lesson would be a strong motivator since transitions would be easier. This was also a suggested approach in order to develop the comprehension about data representations (Friel, et al., 1997). Authors offered that transitions from line plots to bar graphs, or transitions from stemplots to histogram are some strategies to improve graph comprehension. Considering this strategy as important, Group A tried to shape their lesson planning according to it. Besides, members of Group A learned this fact through the TI-STW before their lesson study began. Therefore, it can be concluded that their improved CK affected their pedagogical approaches which they used in their lesson plan. Then, they offered two different sequences of steps which are included in their lesson and they discussed which one is suitable more to their objective. This part of discussion was important since it showed their capabilities towards lesson planning activities and preparation very well and they discussed the issue with its all aspects.

In their second GD, Group A was observed as they reached a consensus for starting point of the lesson: they decided to begin with stem-and-leaf display, to continue with histogram concept and then to finish with activating prior knowledge about bar graph while making comparison with histogram at the end. They also changed their examples (data sets) used for these parts of the lesson. From this point of view, it can be said that their first and second draft of lesson plans were quite different, besides to this, group members specified that they spent more effort on the second draft. Therefore, lesson planning activities and preparation issue was less-referred issue during second GD compared with previous one. Secondly, the issue of knowledge of learners and their background was more referred in the second GD of Group A as compared with initial one. For example, members of Group A thought about background knowledge needed for stemplot or histogram and how to activate them while directing questions in a detailed way. This was the effect of their improved CK onto PK, again.

As compared with initial GD, Group A discussed checking for understanding issue regarding PK in the second GD. That is, they were more conscious about their expectations from students since they explained their aims of choosing among teaching strategies in the favor of students' understandings. They were more aware of students' possible actions and reactions during lesson before its implementation. Since they confided in their second draft of lesson plan more, they were more aware of what their expectations would be from their students. This is an expected consequence that their PK was also improved as they improved their CK. This can be claimed a significant difference between initial and second GDs of Group A, so as first and second draft of lesson plans.

During both initial and second GDs, Group A never mentioned about assessment strategies, however, in their second GD, Group A pointed their concern to classroom management issue since they realized that they implement the lesson plan in a computer laboratory and group work cannot be a clever idea because of classroom management. They also stated that they considered classroom management strategies as a factor to decide on selecting teaching methods and strategies.

During third GD, general teaching methods and strategies and lesson planning activities and preparation issues were discussed at most. They especially discussed the questioning strategy they implemented and they criticized the teachers' reactions during questioning. Although there were some limitations such as implementing questioning in a microteaching and time limitations, they concluded that any better technique than questioning couldn't be found according to their lesson objectives. This finding is consistent with Leavy's (2014) findings that she performed 14 lesson studies in three years with preservice teachers. Her participants' most visited pedagogical approach in their lesson plans was *questioning* strategy. While revising their lesson plans, participants dealt more with the construction and wording of open-ended questions, they aimed also that these questions should provide inferential reasoning (Leavy, 2014). The required knowledge for teachers to focus on statistical thinking was emphasized under the classification of PK, as well, or what Niess (2005) called as teaching and learning in terms of pedagogy.

Third GD continued with some comments on Gizem’s reactions or announcements which were more specific considerations compared with previous GDs. That is; while their hesitations about issues regarding PK were more general in previous GDs, they discussed the moments of implementation of the lesson in a detailed way. From this aspect, it can be claimed that GDs are going from general to specific, which is an expected and important finding. Although members of Group A considered PK issues in nearly the same percentage in all of their GDs, there is a qualitative difference between them.

There were also some discussions for the issue of classroom management in third GD of Group A. After implementation, Group A realized that classroom management issue should be more considered in a real classroom environment. Based on this finding, it can also be claimed that Group A seemed to have underestimated classroom management and they comprehended its importance after implementation. This might be the cause of the nature of microteaching which is the technique they used to implement their lesson plan. Since their students were their classmates, they haven’t paid enough attention to classroom management issues.

As an interesting finding, Group A never mentioned about assessment strategies included in their lesson plan through all of GDs at all. Although they haven’t specifically referred to it, some techniques or strategies (questioning, activity sheets, etc.) could be evaluated as assessment methods. Group members were expected to point to this issue.

Overall, Group A demonstrated more satisfying performance regarding PK issues as going through their GDs in respectively. That is, PK issues were discussed superficially during first and second GDs, they negotiated with each other in a profound manner.

Regarding the findings of Group B, they mentioned about mostly the lesson planning activities and their preparation, general teaching methods and strategies and learners' background knowledge and their understanding during the initial GD. They have never mentioned about classroom management or general assessment strategies at all in the initial GD. They considered choosing the objective as a necessity for the flow of lesson. They also emphasized that they collected the data they will use for the bar graph and histogram as instantly since it made students to be more involved into lesson and consequently designed the physical manipulative for bar graph display as a result of this reason. Their use of complimentary data (jobs of father and how many years they worked for it) were thought as beneficial to make transition between the steps of the lesson. However, they had some hesitations that collected data could have a small range than forming histogram based on that data set cannot give a nice example of histogram for the students. Group B also preferred to include an activity sheet so as to make students work individually.

Dealing with the general teaching methods/strategies, Group B was observed to include many teaching methods such as questioning, group working, activity sheets, manipulatives, and virtual manipulatives. They planned to use activity sheets in order to direct discussion among class. (The tendency to use the technique of questioning has been already discussed in Group A's part above.) Group B also discussed the time required for those activities mentioned above while regarding learners' background knowledge and their grade level. At the end of discussion, they couldn't make a decision on whether the lesson is suitable to introduce histogram or the lesson is only enough to revise it. So, on the whole, it was claimed that members of group B have an understanding of PK since they could think of suitable teaching strategies, techniques, materials, etc. according to lesson objectives and their consciousness about students' background knowledge based on initial GD.

In the second GD, Group B discussed the same issues as in the initial GD: They considered lesson planning activities and preparation as the most so were in previous

GD. Their second draft of lesson plan was more prepared than its previous draft and they seemed to be more organized here. For example, they reached to the decision that histogram concept should be given like a subject revision since they claimed that time management could be difficult when making an introduction. First part of their lesson plan was more organized that they now used three different representations of bar graph display with some questions at the end. They could generate some other alternative representations for the bar graph part of the lesson since they produced many ideas regarding the sequence of the tasks. Members of group B discussed about data set again which they will use for bar graph and histogram during the second GD. They stated that they will produce two different data sets (years worked in total or years worked in last job) in order to increase the chance of having more applicable data to draw histogram (to see the classes better). Tendency of Group B towards using real data, which were collected from their students, could be explained their effort in making students to engage with real settings more (Hall, 2011). However, it was explained that teachers need to be very careful while working with real data since all real data cannot be suitable for students' interests. When 'students cannot establish a relation between data and the problem which is investigated, the expected engagement cannot be formed' (Hall, 2011, p. 337). However, it was emphasized that, as it was the case for Group B, if students were working with their own data, then they were observed to be more involved with the subject (Turner, 2006; Wong, 2006; Catley, 2007, as cited in Hall, 2011). Therefore, this effort of Group B could be argued as an improvement of PCK.

As a weakness of Group B, they disregarded the necessity of presenting data set in order to draw the histogram through virtual manipulative. Since they made students to fill the table (to form histogram) on the board, students will never know each value in the data so they couldn't enter the data to the virtual manipulative. Therefore, on the whole, most of the discussion regarding PK was about steps of the lesson and data set collected from students. It can be claimed that they considered the lesson planning more important than their first draft of lesson plan and they thought more in detail and more productive in generating alternative teaching strategies in their second GD. Including physical manipulative as well as pictograph and the bar graph for the same set of data in order for comprehension of bar graph were a well-reflection of PK regarding the representation ways of a specific concept. As being aware of the available technological resources, members of Group B could diversify the representation of bar graph concept as in three different types.

In third GD of Group B, PK was the most referred knowledge dimension by group members, which was unexpected. It could be explained as the causes of their final decision whether they want to reimplement it or not, where group members divided into two, on the idea of whether a radical change is needed or not. They have discussed about the lesson planning activities and preparation of their lesson plan. They also discussed knowledge of learners and their background and classroom management techniques a little which is different from their previous GDs. Besides, they never mentioned about general assessment strategies at all like in their previous GDs. They discussed about their data set. Also, they said that they couldn’t use activity sheets as they planned before, however, neglecting them was beneficial for the sake of time management. They criticized their questions and teachers’ questioning skills in the questioning periods of the lesson and their effects on students’ thinking. They also stressed that they thought of some details in order to make students more motivated during their lesson. For the first time, classroom management strategies were discussed in a GD and it was understood that they were aware that their tasks included in lesson plan could be difficult to manage in a real class. Since their lesson was very interactive as they claimed so, classroom management would be more important. This finding could be explained as an effect of microteaching lesson study since they had a chance to implement their lesson during MLS and consequently, their understanding towards PK was differentiated.

In summary, Group B discussed all of pedagogical elements of their lesson plan and its implementation very well, except for assessment issues in their third GD, and it includes the most satisfying conclusions compared with previous two GDs. Hence, it could be claimed that Group B has a developed understanding of PK especially after implementation of their lesson plan, as it was expected.

Based on above discussion of both groups of A and B, below Table 5.2 showed the main discussed issues together. As an interesting finding that both Group A and Group B never discussed the assessment issue through their GDs. Its reason might arouse from microteaching since their students, i.e., their classmates, had already knew the subject and all members of groups were aware of this fact.

Table 0-2 Findings through GDs of Groups A and B regarding PK

Groups/ Findings through GDs regarding PK	Group A	Group B
Initial GD	Several teaching methods including direct instruction, questioning, think-pair-share method Awareness of students' background knowledge Using the same set of data for the transition from bar graph to stemplot and then to histogram as a motivator Offered two different sequence of steps of lesson	Collecting data instantly from their students during the lesson as a motivator Using the relational data sets for the sake of flow of the lesson Several pedagogical strategies including questioning, group working, activity sheets, physical manipulatives, virtual manipulatives Time required for the tasks included in the lesson
Second GD	Made a decision to start with the transition from stemplot to histogram as the first part of the lesson Including bar graph-histogram comparison at the end Changing data sets for histogram and bar graph displays	Recalling the background knowledge for histogram rather than introducing it Three different representations of bar graph: physical manipulative, virtual manipulatives for pictograph and bar graph Discussion about data set collected from students as for engaging in a real setting
Third GD	More emphasis on questioning strategy they used in their lesson plan Discussion of classroom management in the implementation	Data set collected during the lesson Time management issue for not being able to pass to activity sheets Questions asked through questioning strategy Classroom management issue

Both Group A and Group B used several teaching methods in their lesson plans in order to diversify the representation of the concepts which they included. Their lack of knowledge of statistical subjects mentioned in above part directed both groups to use transitions as well as to use the data sets in an inaccurate manner.

Unlike Group A, Group B thought about time required for the tasks which they included in their lesson plan more especially after they implemented it. This was the main factor which they discussed in order to decide on revising their lesson plan and re-implementing it once more in third GD. However, both groups discussed especially classroom management issue after their implementation more since they reached an awareness of classroom management issue since they practiced.

As going through from initial to third GDs, both groups discussed the PK issues in a more qualitative manner. That is, their discussions became more specific and they dealt with them in a more detailed way. For instance, when they were discussing about the questioning strategy they used, they later discussed the questions they asked, the sequence of the questions, possible responses of teacher and possible questions from students.

5.3 Technological Knowledge (TK)

Although it was specified in the findings chapter that TK was not coded since members of Group A were observed to have expected capabilities regarding TK according to codebook as it was the case for Group B (Hughes, 2013). However, smartboard interactions of members of both groups are worth to mention here as accepted to investigate it under TK. During second and third GDs, it was observed that almost all members of Group A were not capable enough to use smartboard effectively. Only Gizem, the implementer of the lesson plan, was very familiar with using it, and the rest was not much capable enough to use. After implementation, Group A discussed the problems they experienced in using smartboard in third GD. From this aspect, it can be claimed that all members of Group A benefited in using smartboard and learned to implement smartboard in a lesson without making it as the aim of the lesson, but as the tool. Overall, members of group A have more than enough understanding of TK and they were open to learn new technology and implement them in their lessons as well.

However, Group B was seemed not to be capable enough using smartboard since they needed some help from their friends while working with it especially in filling the table on smartboard. This could be claimed not only as a weakness regarding TK, but also as an indicator of being inexperienced in a real setting. Their attitudes towards including technology in their lesson as another factor for this finding. Recalling the findings from the interview, it was claimed that one participant from Group B thought that integrating technology is not as necessary as CK, for instance. She was the only participant having such an approach towards technology integration and it might affected their improvement regarding TK.

5.4 Pedagogical Content Knowledge (PCK)

In the initial GD, Group A discussed knowledge of teaching/representing subject matter to students and identifying and addressing student subject-specific misconceptions or mistakes covered in first draft of their lesson plan. Regarding teaching/representing subject matter issue, they mentioned that visual elements are good for increasing motivation of students. Besides, they explained that using the same set of data provides a lesson flow since making transitions while using the same example would be easier. They also discovered that the example was not suitable for bar graph as discussion continues. They realized that teacher couldn't sufficiently respond to the students throughout the questioning strategy since they wanted to make students to differentiate the variable type in bar graph and histogram. Then, they gave an example of a physical manipulative which can be applicable for demonstrating a bar graph. These moments also made group members to think deeply for the implementation of questioning strategy as well. What they aimed at the beginning of questioning was not going to result in their desired way, as they realized.

Overall, it was concluded before that members of Group A were capable enough to think about the teaching/representing strategies according to their content and objectives.

Therefore, it can be argued that their inadequacies in statistical knowledge in terms of selecting the type of variables, or ‘graph sense’ affected their pedagogical approaches. That is, they felt that they need to change their lesson plan completely since they wanted to use the same set of data for both bar graph and histogram displays. Pfannkuch (2008) summarized what teachers should experience in order to enhance their statistical knowledge and PCK. He mentioned that teachers who teach statistics need to learn the ‘game of statistics’ first, and need to build statistical concepts as a secondary dimension, which are also the two dimensions of Pfannkuch and Wild’s (2004) 4-dimensional statistical thinking framework and he claimed that these two are specific for teachers’ learning. From pedagogical content knowledge point of view, Pfannkuch (2008) suggested that teachers who developed statistical knowledge regarding the above dimensions would have improved their PCK as well. Moreover, his arguments about how teacher learning should take place is parallel with lesson study requirements as a professional development process for teachers, as in the following:

> Effective teacher learning includes teachers participating in activities, as students, in simulated classroom settings, reflection on and studying the theoretical basis or rational for the teaching method from a learner and teacher perspective, observing demonstrations by experts, the teacher educators, and having the time to learn in and and from practice within a professional learning community (Moore, Cobb, Garfield & Meeker, 1995; Ball & Cohen, 1999, as cited in Pfannkuch, 2008, p. 251).

In the initial GD, members of Group A evaluated also that using the same set of data can lead to a misconception for students. Besides, they emphasized that their inadequacy in differentiating a bar graph from a dotplot can lead to a misconception. As stated before, since they have already some misconceptions and inadequacies in CK, they couldn’t easily determine the possible misconceptions which students might experience. However, they tried to generate some precautions after my feedbacks regarding misconceptions or mistakes during initial GD. As a result, Group A seemed to have enough understanding about teaching/representing ways of subject matter to students while they need to improve their PCK regarding determining possible misconceptions or mistakes.

PCK was the most referred knowledge dimension in the second GD of Group A. They discussed how to ask questions and in which sequence, and the way they teach the formula of histogram and class width to the students additionally to the issues covered in the initial GD. Their discussion about questions asked in the lesson and the sequence of them was satisfying enough to conclude that they were conscious about representing the subject matter to students. However, discussion mostly continues among the curriculum needs afterwards that it was claimed that they felt obligated to include formula of histogram into their lesson plan. Since they did not have sufficient CK about construction of histogram, they couldn't provide reasonable explanations in response to my questions during second GD. That was the reason in concluding there was a curriculum requirement. This is, to some degree, conflicting with their tendency to include a transition from stem and leaf display to histogram, although stem and leaf displays are not included in school curriculum. Therefore, it can be argued that their PCK had stronger influence on their decisions, than their PK or CK regarding the guideline of histogram.

However, they were more productive in negotiating about addressing/identifying students' misconceptions or mistakes compared with their initial GD. They thought that there might be some misconceptions regarding stemplot and histogram. After overcoming their inadequacies in bar graph-dotplot concepts in their first lesson plan, they started to think deeply about histogram and stemplot concepts which they included in their lesson plan. Their possible solutions in order to overcome these misconceptions were also discussed during second GD. Overall, it could be claimed that Group A experienced an improvement about identifying and addressing students' possible misconceptions or errors as compared with their initial GD. In general group members were observed as more involved regarding the issues of PCK compared with first lesson plan they prepared.

As it was the case for the second GD, PCK was the most apparent knowledge dimension in the third GD of Group A. The discussion for the issue of teaching/representing subject matter to students was mostly about the questioning strategy. They discussed and criticized almost all aspects of questioning technique they implemented during the lesson, for instance, the questions student teacher asked, the responses of teacher, the mistakes that teacher made while responding towards students' questions, and the like. They generated many other alternatives for responses made or questions asked during the implementation. Besides, Group A offered a user manual for students which explains basic properties of virtual manipulative they used in their lesson. In addition, Group A did not specifically mention about students' misconceptions or mistakes as they did in their previous GDs. Overall, they criticized themselves regarding to the issues of PCK through almost every moment of lesson and generated many alternatives in order to teach subject matter to students. Therefore, it can be claimed that members of group A has improved their understanding of PCK and seemed to be deeply involved with the issues of PCK in their third GD as compared with their first and second GDs. They explained the teaching methods or representing techniques in a more detailed way while specifying their possible effects on misconceptions or mistakes of students. They were more conscious about the existence of students in the context of teaching and they started to consider expected students' responses more important since they realized that those responses could affect the flow of the lesson.

Overall, members of Group A were very much involved with the issues regarding PCK in their three GDs. Especially after they experienced the implementation of their lesson plan, this much of involvement was expected since they had the chance of getting into a context of teaching and learning through implementation. Based, on the framework suggested by Godino, Batanero, Roa and Wilhelmi (2008) for the statistical pedagogical knowledge, members of Group A demonstrated some performance in some components, especially in cognition and teaching resources and techniques as observed through their GDs during their MLS effort. They could be claimed that they improved their statistical pedagogical knowledge when GDs were compared chronologically. Their discussions about selection of teaching methods and resources which were included into the lesson could be established well in terms of above components of statistical pedagogical knowledge.

However, since teachers' statistical pedagogical knowledge is currently being a new issue, it is still a challenge for teacher educators and researchers in order to find ways to enhance the teachers' learning of statistics as well as to develop teachers' statistical pedagogical knowledge (Godino, Ortiz, Roa & Wilhelmi, 2011) regarding the differences in mathematical and statistical thinking. Therefore, findings of this study reflected in both TPACK framework which is applicable to any subject matter and proposed models of knowledge needed for teachers in order to teach statistics.

Regarding the main findings from Group B, they discussed teaching/representing subject matter to students and identifying and addressing subject-specific misconceptions or mistakes mostly in their initial GD. Their lack of CK in data distribution in a data display caused that they thought to represent bar graph features while halving data set to make generalizations on the population. Therefore, it was claimed before that their lack of CK affected their understanding of PCK here since they preferred to present bar graph because of their misconception here. From this point of view, it could be argued that there is an obvious relation between CK and PCK. Then, Group B also planned to use physical manipulatives, activity sheets and virtual manipulatives in order to represent bar graph, although they didn't stress that the table on activity sheet is in fact a frequency table and their choice of virtual manipulative was in fact for a dotplot display, which supports again the same relationship between CK and PCK here. In the histogram part of their lesson plan, Group B tried to make students to focus on the classes aroused while filling the table on the board. However, later in the questioning period of histogram part of the lesson, they were directing students in an incorrect way in order to represent class width concept because of their misconception about it. So, on the whole it was seen clearly that there is a direct relationship between CK and PCK in terms of representing subject matter to students. They had also some precautions in order not to make students to fall in a misconception in understanding the bar graph concept. They were aware that the intervals (on the axes, i.e., number lines) should be drawn carefully and the rings used in physical manipulative should be selected as exactly same with each other. Therefore, it can be claimed in general that Group B has not enough understanding of PCK since they were lack of necessary statistical CK regarding their lesson plan, i.e., their PCK was mostly affected by their inadequate CK which was resulted as poor.

In the second GD, Group B discussed again representation strategies of subject matter more regarding PCK. Since they were more organized and knew more about what to do in the bar graph part of the lesson, they discussed mostly the sequence of the tasks and the content and the aims of the questions mostly. They knew that what they expect from students and what students' reactions might be more in second GD. Therefore, they considered the sequence of the tasks and the one for questions here as important. Their discussions among themselves for deciding on to choose the sequences was an example of their developed PCK, as it was claimed before. They also discussed the possible alternatives for filling the table in the histogram part such as raising hands, coming to boards, etc. Then, it was claimed that they were aware of generating such alternatives in favor of time management and balancing students' motivation. Since they included a virtual manipulative for drawing histogram, I also made them to think about changing the class width while working it, so that; their students could realize the changes on histogram immediately. Then, it was concluded before that their way of representing class width to students was affected by their lack of CK regarding class width in histogram. So, on the whole, it could be claimed that their discussions regarding PCK was more satisfying as compared with their initial GD since they presented and discussed many representation or organization ways of the tasks included in their lesson plan. However, it was seen the effect of CK again in a negative way because of their weak CK. This finding could be explained by the Pfannkuch and Wild's (2004) 4-

dimensional statistical thinking framework again that teachers need to learn *transnumeration* in order to enhance their statistical thinking. Transnumeration is defined simply by Burgess (2011) as 'the ability to represent the data in various ways with making more sense of the data' (p. 261).

In third GD, Group B focused on the same issues regarding PCK and the content-specific assessment strategies were not mentioned again as in the previous GDs. Group B's alternative representation of class width because of technical failure in projector was a finding which shows their developed PCK. They again discussed the sequence of tasks related with bar graph (physical manipulative, virtual manipulative-pictograph, virtual manipulative-bar graph, questioning; respectively) and didn't like the transition from pictograph to bar graph during the implementation and offered to start questioning here and finish it at the end. This discussion shows their improved understanding of PCK. They also made critiques on the way of application of questioning, the questions included, and teachers' responses during third GD, which was also a satisfying discussion in favor of PCK. Their implemented questioning technique was also very systematic and teacher's guided questions were effective. They only couldn't realize the mistake which students met within the implementation because of the table of data set of the years worked by fathers. Hence, it was claimed that they needed to observe their students more during implementation and should react instantly to prevent it.

Therefore, on the whole, it could be claimed that Group B's understanding of PCK was mainly affected by their lack of CK in some important statistical concepts during initial and second GDs. Although they could generate alternative ways of representing the subject matter, their weak CK directed them in an incorrect way. On the contrary, they were observed to have improved their PCK as well as CK in third GD, which is an expected finding. Besides, improvement members of Group B could be explained according to framework for statistical pedagogical knowledge described above (Godino, et al., 2011). Members of Group B showed especially increased improvement in its two specific components, which are *teaching resources and techniques* and *affect.* Their thought of different representation ways of the subject while taking students' motivation and interest into consideration were well-described with the help of these two components.

When we compared the main findings as can be seen in the Table 5.3 below, most of the discussions regarding PCK for both groups were about representing the subject matter to the students in different ways and the sequences of the tasks and the questions which they asked through questioning strategy. It was already stressed that they both never pointed on content-specific assessment strategies at all. It might the reason of microteaching as it was discussed above.

Table 0-3 Findings through GDs of Groups A and B regarding PCK

Groups/ Findings through GDs regarding PCK	Group A	Group B
Initial GD	The visual elements of the lesson as motivator for students Representation of bar graph with an emphasis on categorization through bar graph Aims of using questioning strategy Example used for all data displays A misconception aroused from using the data set for drawing bar graph Awareness of the misconception might be aroused from the bar graph, which was in fact a dotplot	Representation of bar graph concept as halving the class in order to take responses of students as a result of their weak CK about distribution Physical manipulative which they designed Not realized that the table used for bar graph is in fact a frequency table Having the misconception related with class width in histogram
Second GD	How to ask questions during the lesson and in which order they will be asked Sequence of the lesson activities and which steps of lesson are included Teaching formula of histogram, steps of drawing a histogram and how class width is analyzed by students Implementation of questioning and think-pair-share method	Representation of bar graph in three different ways and the sequence of them Questions during bar graph part of the lesson Not enough dealing with data through histogram regarding class width
Third GD	Intensive discussion about questioning: Questions student teacher asked, the responses of teacher, the mistakes that teacher made while responding towards students' questions	Sequence of tasks related with bar graph (physical manipulative, pictograph, bar graph, questioning) Teachers' responses during questioning Being precautious about students' mistakes

Both GDs of Group A and Group B continued from general to specific, they differed and varied as qualitatively. For instance the conversation of both groups regarding questioning strategy could be an example which might show the depth of the discussions regarding PCK. Probably the most important effect of weaknesses of groups regarding CK was observed through PCK, which was already concluded that there is a direct relation between them. Without having deep CK, members of groups couldn't showed sufficient PCK especially at the beginning of MLS.

5.5 Technological Content Knowledge (TCK)

Understanding TCK has some indicators according to TPACK codebook such as knowing about the existence of a variety of content tools for particular content tasks, operating/knowledge of content-based technologies which content learning is foregrounded and knowledge about the ways in which content and technology reciprocally related to each other. TCK demonstrated an increasing trend in the initial, second and third GDs of Group A respectively. In their initial GD, Group A was claimed as they started to develop their TCK skills, since they just met with the virtual manipulatives, which was the unique concern of TI-STW and the current study. They haven't searched for another examples of virtual manipulatives which can be used for statistics teaching. Even their selection for bar graph display was not right, since it was a dotplot display in fact. Although dotplots were covered in the workshop, they misinterpreted it and planned to use it for bar graph. Therefore, it was claimed easily that Group A was not capable enough to discuss the issues regarding TCK during their initial GD. Obviously, their lack of CK had an effect on their TCK which was observed through their initial GD.

In their second GD, it can generally be observed that Group A has expanded control over virtual manipulatives they use in lesson plan. They knew them better and they have checked their properties already in order to relate them with their subject matter. That is, they learned the virtual manipulative (NCTM's illuminations page) very well after omitting the virtual manipulative for dotplot since NCTM's illuminations page is the only virtual manipulative which they planned to use in their second draft of lesson plan.

During second GD, it can be argued that, group members investigated the virtual manipulative critically, i.e., they have worked with carefully and spent some time on it to deal with data, contrary to their initial GD findings. They also realized that they could make the transition from stemplot to histogram while using this virtual manipulative since it provides students with using same set of data in order to investigate the both displays. Besides, they realized that they could represent the class width issue in histogram concept with the help of virtual manipulative. Therefore, on the whole, members of Group A were observed to have developed understanding of TCK as compared with their performance during initial GD since they weren't capable enough to establish a relationship between technology and content they teach in their first lesson plan. When Technological Pedagogical Statistical Knowledge (TPSK) framework was considered, as Lee & Hollebrands (2011) stated in their article that knowing technological tools for teaching statistics provides teachers with in-depth conceptual understanding, effective engagement in exploratory data analysis, visualization of abstract concepts, and knowing how to deal with data sets according to different statistical concepts. From this point of view, members of Group A realized that virtual manipulatives for statistics were working well for the above issues and they started to learn them better in order to deal with statistical concepts well.

In third GD, members of Group A discussed the reactions of the student teacher regarding TCK and they seemed to be able to take some precautions for students as they dealt with the virtual manipulative more when compared with their previous lesson plans. For instance, Gizem was explaining the tricky parts of virtual manipulative which students could easily make a mistake in, such as, entering the data set. They also stressed that they warned the class not to form more than needed rows (i.e., the rows needed for data input while working with virtual manipulative) just because of this consciousness. Overall, they were quite aware of all aspects of virtual manipulative they used in their lesson, such as possible mistakes students can make, nature of class width used in histogram, etc. Consequently, it could be concluded that their use of virtual manipulative served both as an *amplifier* and a *reorganizer* regarding technology integration into their lesson (Pea, 1987; Ben-Zvi, 2000). Their use of virtual manipulatives changed their understanding of histogram and dotplot as well, since they discovered that dotplot is a special histogram whose class width is 1, as stressed above in CK part. Therefore, this finding certified what TPSK Framework claimed as technological statistical knowledge can be founded and improved while basing on a powerful statistical knowledge (Lee & Hollebrands, 2011). Since they used TPSK as nested circles which means that the inner circle is the subset of the outer circle, it could be derived that technological statistical knowledge cannot be developed without statistical knowledge, as it was found in this study.

Therefore, it can clearly be claimed that Group A showed an increasing concern in their initial, second and third GDs respectively. They started to relate content and technology reciprocally with each other, they applied such established relations into their lesson planning, lesson revisions, and finally into their implemented microteaching. That is, it could be claimed that they have improved their TCK capabilities and understanding.

Regarding the main findings through GDs of Group B, TCK was one of the least referred knowledge dimensions during the initial GD of Group B. They were lack of relating the content in their lesson plan with the technology reciprocally. Besides, their knowledge of existing technological tools for particular content tasks was weak. They chose the incorrect virtual manipulative for displaying bar graph, for instance. Their lack of CK in dotplot and in variable type used in bar graph had an effect on this selection. They stated that they couldn't find a virtual manipulative for bar graph. They also didn't know the pictograph as an alternative data display for categorical variable. Therefore, they decided to search for it for the second draft of their lesson plan. For the histogram part of their lesson plan, they also specified that they preferred to use virtual manipulative showing the transition from line plot to histogram since they didn't want to make a transition from stemplot to histogram. Their lack of CK was affected in this preference again. Therefore, it was claimed that their understanding of TCK was weak in order to make effective relations between content and technology and in order to select suitable virtual manipulative for their content tasks.

In the second GD, Group B was observed to be more involved in the issues regarding TCK. They, for instance, searched for the web in order to find a virtual manipulative for pictograph and included into their lesson plan, which I was impressed much. They also worked with it enough to learn its properties such that changing the names affects the pictures appeared in the display, and so on. Their discussion about the different pictures used for different bars in pictograph was also satisfying enough that they tried to make a relation between technology and content. Besides, they changed their previous virtual manipulative for histogram and they planned to use it while comparing the histogram formed through table. However, Group B disregarded its property of changing class width immediately. After my feedbacks, they wanted to decide on later. As a result, it can be claimed that Group B has showed an improved performance regarding TCK since they started to make some connections between content and technology which wasn't observed in their initial GD. However, they needed to spend more time in working with virtual manipulative to generate representation ways of content tasks with it.

In third GD, TCK was the least referred knowledge dimension by Group B members. On the contrary to that, teacher's reaction during the questioning period of histogram part of the lesson was very satisfying enough to claim that their understanding of TCK has been improved and they were observed to have worked with virtual manipulatives to learn its properties very well. The teacher was consciously aware that changing class width to 1 in the histogram turns it to a dotplot, and eventually every value of data set can be seen. There was a discussion about this dotplot in the first part of the lesson after the question of "which data display should we use in order to draw years of fathers worked?" Then, teacher made a nice recall to this moment while making the class width change in the second part. This shows their improved understanding of TCK.

Therefore, on the whole, although its percentage was less in third GD than in previous ones, TCK of Group B was observed as developing enough to be able to relate technology with content reciprocally. While their understanding of TCK was weak because of their lack of CK in their initial GD, they started to think about virtual manipulatives more as how to integrate their lesson plan and how to be effectively benefited from it (Lee & Hollebrands, 2011).

When we compared the main findings of groups A and B through their all GDs, the Table 5.4 given below will help in order to investigate their development in TCK overall.

Table 0-4 Findings through GDs of Groups A and B regarding TCK

Groups/ Findings through GDs regarding TCK	Group A	Group B
Initial GD	Not searching for different virtual manipulatives for bar graph and using the one for dotplot inaccurately	Choosing incorrect virtual manipulative for bar graph Not knowing pictograph as a different representation of categorical data Incorrect preference of using the transition from line plot to histogram
Second GD	Expanded control over using virtual manipulatives they used in the lesson and dealing with data Awareness of making the transition from stemplot to histogram using virtual manipulative Awareness of making an emphasis on class width via virtual manipulative for histogram	Searching about pictograph and dealing with its features in detail Disregarding the property of class width issue in virtual manipulative selected for histogram
Third GD	Realizing the mistakes which students could experience when working with virtual manipulative Awareness of all aspects of virtual manipulative regarding the students' mistakes and class width issue Discovery that a dotplot is in fact a histogram whose class width is 1	Having knowledge of all aspects of virtual manipulatives they included in the lesson Discovery that a dotplot is in fact a histogram whose class width is 1

Both Group A and Group B were observed to gain more knowledge about virtual manipulatives they used in their lesson plans through their GDs in chronological order. Since they improved their TCK skills as they dealt more with the virtual manipulatives and they knew their features and characteristics more, they discovered both the fact that a dotplot is in fact a histogram whose class width is 1. This significant finding showed that groups improved their TCK in terms of relating content with technology reciprocally each other, as they improved their CK through their GDs, which was the inevitable consequence of the relation between CK and TCK.

5.6 Technological Pedagogical Knowledge (TPK)

According to TPACK codebook, there are many specifications of TPK (see Appendix G). In their initial GD, members of Group A especially considered the time management of integrating technological resources in their lesson plan and they also considered the technical failures which students might live through virtual manipulatives. However, they were aware that technological tools such as virtual manipulatives could be very helpful for students to learn the subject better. Therefore, it could be claimed that they understood the necessity of technology for the instruction not only demonstrating to the students on the board but also making them to work with technology closer. Because of being inexperienced in implementing technology in statistics teaching, they only treated it with its prominent disadvantages at first. So, on the whole, it could be claimed that members of Group A realized that there is a need to use technology in mathematics instruction but they didn't know how to integrate it. Hence, members of Group A could be said that they started to develop not only their views through technology integration but also their understanding towards TPK, that is; they started to improve their capabilities of TPK in their initial GD. Moreover, based on the definition of TPK (Koehler & Mishra, 2009), members of Group A recognized that there is a need to know the specific pedagogical approaches while dealing with a technological resource/tool in teaching. Therefore, it was expected in the initial GD that they needed first to be very familiar with virtual manipulatives with their all specifications before using them for pedagogical purposes, as Koehler and Mishra (2009) stated.

In their second GD, they were firstly aware of the classroom infrastructure which they will implement later, and hence they planned to include the usage of smartboard into their lesson plan as well. Since they made radical changes in their lesson plan, they also investigated the requirements of virtual manipulative for histogram and stemplot very well. For example, they emphasized that students do not have to enter the values in data set as ordered and consequently it saves time, as TPK was described as knowing the affordances and limitations of technological tools/resources in order to establish the best relation with the pedagogical approaches (Koehler & Mishra, 2009). Besides, they stressed also that they chose the NCTM's illuminations page since it provides the transition from stemplot to histogram while using the same set of data, which also favors the fluency of the lesson. Consequently, it can be claimed that group members could think of different pedagogical strategies as they are more involved in how to work with virtual manipulatives in a lesson. However, group members had a difficulty in explaining how to deal with class width issue both through virtual manipulative and formula of histogram, since different class width values could arouse through them. Then, one member of Group A stressed that there is a suitable data set needed for their lesson, at the end. This can be explained that there is a relation between TPK and PK since they offered to change their data set in favor of students' understanding of the class width concept. Since they were aware that different class width values could be taken from the virtual manipulative, they suggested to think more about data set in order to prevent students' misconceptions, which shows also the relation with PCK. Besides, it could be claimed that this finding corresponds to the explanations by Zhao (2003) that 'what technologies teachers should know should interface directly with what teachers do in their teaching' (p. 7). Further, Zhao (2003) described his understanding of technology-pedagogy integration as four dimensions: (a) technology for classroom management, (b) technology for instruction, (c) technology for teachers to know more about their students, and (d) differentiating technologies for different subject matters (p. 8); and it could be argued that these four function as foundation for TPK. So, on the whole, it could be claimed that Group A demonstrated an improvement in their understanding of TPK and they could think technological tools which they could use in their lesson plan more pedagogically, which they couldn't do in their initial GD.

In their third GD, TPK was the least referred knowledge dimension among all. However, the conversations related with TPK were satisfying that they could criticized teacher's responses very well and they generated alternatives for the problems they experienced during the lesson. That is, the qualitative dimension is satisfying although they seemed to discuss the issue less. Their preparation of taking screenshots in case of a technical failure during the lesson showed their improved preparedness very well. Besides, teacher recognized the students' mistakes while working with virtual manipulative (addition of unnecessary rows, etc.) and directed students to overcome it immediately. This example can be claimed as an indicator of their improved understanding of TPK, as well. Therefore, Group A showed an effective progress in developing their understanding of TPK in their first and second GDs, they prepared themselves according to the issues they realized before, and implemented their TPK abilities into their lesson plan.

Regarding the main findings from Group B, TPK was the least referred knowledge dimension in the initial GD of Group B. However, it has an increasing trend through the GDs respectively. In the initial GD, they slightly referred to elements of TPK listed in TPACK codebook (Hughes, 2013) except that they chose the virtual manipulative for histogram because they couldn't find any other one which supports entering new data. This shows that they didn't make enough search through web in order to find one even they couldn't realize that the one which I showed in the workshop is suitable for their aim. Group B also explained their reason to select learner.org in order to show the transition from line plot to histogram since they were treating dotplot as a bar graph in their initial GD. Therefore, it was claimed before that Group B need to develop their understanding towards TPK. Their views towards technology and technology integration were related with their understanding of TPK at the beginning, according to the findings of interviews with the participants. Since the mentioned two participants were in this group and they were not reluctant to integrate technology in teaching, or have not seen technology as an essential part of teaching, it was claimed that their approaches towards TPK was affected. However, such beliefs were claimed that "they have not been taught with technology in their middle or high school years, consequently, their beliefs, attitudes and thinking towards technology should mature" (Niess, 2005, p. 511).

In second GD, TPK was again the least referred knowledge dimension like in the initial one. However, Group B showed a developing understanding towards it. Their search for pictograph made them to be excited concerning an effect on students' motivation. It was observed that they even had fun while working on the virtual manipulative for pictograph. Their view towards painting cells to form the histogram was also such an indicator that they started to think of pedagogical elements of technological tools included in a lesson. They were also conscious about classroom infrastructure since they knew where they will implement their lesson plan and they were familiar with that classroom with its technological equipment inside. Therefore, it can be claimed that they had a slightly improved understanding of TPK as observed through second GD.

TPK was one of the knowledge dimensions which has an increasing trend in third GD of Group B. Group B explained that watching the teacher could be motivated for students in order to be able to work with the virtual manipulative and they claimed that this action of teacher was planned deliberately. They also realized that especially after the implementation, using smartboard interactively with students (filling table in smartboard) could cause a mess resulting to not being able to control time and classroom management during the lesson because of its non-multifunctional property. Hence, it can be claimed that Group B started to think of advantages/disadvantages or negative/positive sides of technological tools included into lesson concerning pedagogical strategies. In other words, they slightly improved their understanding on how to apply pedagogical strategies during a technology-integrated lesson. Before the implementation, they already prepared some screenshots of virtual manipulatives in case of having a technical failure. Besides, they were conscious about that having a program in main computer which can control every other computer in a class could be beneficial for lessons including virtual manipulatives like theirs. So, on the whole, it could be claimed that they experienced some improvements slightly in their understanding towards TPK since they didn't refer more than half of the codes listed in codebook at all.

Below Table 5.5 which shows the main findings through all GDs of Group A and Group B regarding TPK could help in order to better investigate their understandings towards TPK.

Table 0-5 Findings through GDs of Groups A and B regarding TPK

Groups/ Findings through GDs regarding TPK	Group A	Group B
Initial GD	Time management of integrating virtual manipulatives into their lesson Technical failures which students might live through virtual manipulatives Awareness of benefits of using virtual manipulatives regarding students' learning Hesitations to use virtual manipulatives in real class settings Becoming aware that there is a need to know pedagogical approaches when dealing with technology	Not enough search the Internet for suitable virtual manipulative for histogram Effect of their attitudes towards technology integration onto their lesson plan
Second GD	Classroom infrastructure Deciding on using smartboard in their lesson Knowing the affordance of virtual manipulative selected for histogram Selecting virtual manipulative for histogram as it permitted the transition from stemplot Awareness of need to a suitable data set in order to emphasize the class width issue well	Searching the Internet for pictograph and finding one example and including it into the lesson Painting cells of the table in order to form the histogram regarding pedagogical issues Classroom infrastructure
Third GD	Preparedness regarding possible technical failures Recognizing students' mistakes while working with virtual manipulative in the implementation	Watching the teacher who worked with virtual manipulative as a motivator for students Not knowing the technical features of smartboard and not being able to control it effectively Need a computer program which control every computer in classroom

Their understanding of TPK especially differed after they implemented their lesson plans. It might be claimed as the consequence of being inexperienced in practicing especially in practicing such a technology-integrated lesson before. For instance, both groups realized that there could be technical failures which micght effect the flow of the lesson after implementation. Group A started to think about them a little earlier than Group B as it can be seen Table 5.5 above. Since it was discussed before, being inexperienced in teaching practice effected their TPK comprehension as well as their PK comprehension. Therefore, it could be claimed that there is a significant relation between PK and TPK. However, it was concluded that they still needed to improve their TPK since both groups didn't referred much of the codes regarding TPK even in their third GD.

The implementation phase of MLS helped to investigate the findings regarding both PK and TPK since it presented a before-implementation and an after-implementation period at the end. Effect of classroom practice which Leavy (2014) emphasized in her study as presenting the "complex and interconnected knowledge demands of mathematics teaching" (p. 3). As Laborde and Perrin-Glorian (2005, p. 1, as cited in Leavy, 2014) described:

> The classroom is a place where knowledge is transmitted through various processes, in particular through situations that contextualize knowledge and through interactions about this knowledge amongst people (teacher and students) who act within and on these situations. Thus situated at an intermediate position between the global educational system and the microlevel of individual learning processes, the classroom teaching situation constitutes a pertinent unit of analysis for didactic research in mathematics, that is, research into the ternary didactic relationship which binds teachers, students and mathematical knowledge.

Dealing with classroom practices of preservice teachers on behalf of TPK and PK could be best captured through MLS since it presented implementation and reflection phases in order to decide on pedagogical approaches continually to select as well as other issues.

5.7 Technological Pedagogical Content Knowledge (TPACK)

This part discussed the overall improvement of TPACK of Groups A and B regarding above body of discussion of each knowledge dimension aroused in TPACK framework. Group A and Group B both experienced significant improvements in TPACK knowledge dimensions especially in CK, TCK and PCK regarding statistics teaching via integrating virtual manipulatives. Further it could be claimed that the findings regarding CK, PCK and TCK showed that there is an interconnected way of relation among them. Their improvement which groups experienced through PK and TPK could be claimed to be a slightly less than their improvements in CK-PCK-TCK. It was already established that there is an effect of being inexperienced in classroom practice on this finding. However, it was claimed that they started to be more aware and conscious about the pedagogical issues when they include technology in their lessons. When it was stressed that the CK issued in this study is in fact SK of preservice mathematics teachers, these relations were established before Lee et al. (2012) as teachers having a deeper SK and statistical thinking assisted their improvements towards TSK as well and this development could be observable in their planning lessons regarding TPSK framework.

It was already emphasized that there is an evolving approach for presenting different courses which demonstrate to preservice teachers how to deal with technology in their lessons and therefore, aiming to increase their capabilities of integrating technology by educational researchers (Hofer & Grandgenett, 2012). Therefore, this study which conducted two MLS helped in order to understand the interconnected nature of knowledge needed for teaching mathematics as well as statistics as it was stressed before that there was a need to understand the knowledge development of preservice teachers in their teacher education period (Hofer & Grandgenett, 2012).

As another conclusion, Group A and Group B differed in their attitutes towards integrating technology in their lesson plans, and it was resulted that their way of dealing with the virtual manipulatives showed a slight difference between them.

Although participants enrolled two statistics courses in their second year of undergraduate education, especially with a deeper involvement in descriptive statistics as well inferential statistics, it was not observed as a development in their statistical thinking especially in their initial GDs, as it was concluded above. This issue was explained before by Groth (2007):

> Requiring teachers to take more statistics courses that are mathematically structured does not appear to be the answer, because such courses do not highlight nonmathematical aspects of common statistical knowledge (Cobb & Moore, 1997). Even courses structured around the investigation of data are not entirely adequate, because they may not explicitly attend to specialized knowledge for teaching. Just as teachers need new kinds of mathematics courses rather than just more courses (Kilpatrick, Swafford, & Findell, 2001), they need new kinds of statistics courses that help develop common and specialized mathematical and nonmathematical knowledge (p. 434).

Therefore, on the whole, MLS conducted in the current study helped in order to investigate the development process of preservice mathematics teachers regarding all of the knowledge dimensions in TPACK framework as well as the knowledge dimensions in TPSK framework ans showed the inter-connected nature of knowledges needed for teaching mathematics, and consequently teaching statistical subjects included in school curriculum.

5.8 Implications of the Study

The findings of the current study could have considerable implications for mathematics teacher education regarding statistics teaching and could suggest alternative ways to teacher educators in order to develop preservice teachers' TPACK as they participate in microteaching lesson study. These are outlined in the following paragraphs.

Based on the findings which revealed the inter-connected structure of knowledge dimensions from this study and as technology integration in mathematics classes develops, it is important to place an emphasis on needed knowledge dimensions for teaching mathematics. Preservice teachers should be aware of those knowledge dimensions as well as regarding the differences between mathematical and statistical knowledge they need for their classes. Moreover, preservice mathematics teachers should be more aware of the requirements of new school curriculum in which statistics was emphasized more compared to previous curricula. Further, they could have learning opportunities of technology integration in statistics teaching more throughout their teacher education programs.

Since being *digital immigrants* of this unique era, preservice teachers should be consciously aware that their students are *digital natives* and develop their understandings of TPACK knowledge dimensions while technology, and consequently technology integration is evolving and differentiating (Prensky, 2001). It was already concluded from the findings regarding the professional development of preservice mathematics teachers through this study, in order to reveal this consciousness, teacher education programs need also constantly rearrange their courses while following the educational technology enhancements. The findings of this study presents microteaching lesson study as an alternative of professional development of preservice mathematics teachers. Preparing lesson plans, watching the video-tapes of implementations of them as well as taking feedbacks from their teacher educators provide preservice teachers to investigate their development in terms of their teaching skills.

Based on the findings of the current study, TPACK framework could also be considered as a web of relations between knowledge dimensions, CK, PK, TK, PCK, TCK and TPK. Since the relations between and among these knowledge types needed for technology integration into mathematics classes are apparent in the findings, the TPACK framework could be interpreted in a different diagram, shown in the Figure 5.1 below. This diagram offers also opportunity to consider which knowledge dimension has a more prominent effect on the others.

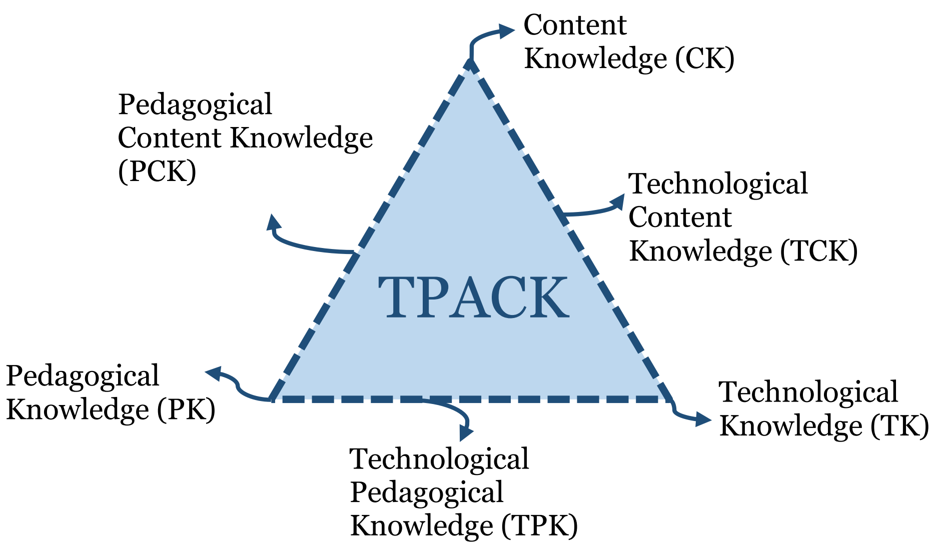

Figure 0-1 New interpretation of TPACK Diagram

In this new interpretation of TPACK, the vertices of triangle show the three core knowledge dimensions needed for teaching, which are knowledge of content, pedagogy and technology. These vertices can be moveable in such a way that more emphasized or privileged knowledge type or types become more apparent, and effects the shape of the triangle. The three edges of the triangle show the three other knowledge dimensions which shows the interrelations between any two of three core knowledge dimensions, specifically they are PCK, TCK and TPK. When the vertices are moved, these edges become shorter or longer, which demonstrates the highlighted importance of related knowledge dimension. Since as it was indicated in the TPACK framework offered by Koehler and Mishra (2009) that the interpretation of these knowledge dimensions are dependent on the teaching context, they were demonstrated as dashed lines. Eventually, the shaded area of triangle refers to technological pedagogical content knowledge (TPACK) since all of the other six knowledge dimensions have an effect on TPACK. More specifically, TPACK understanding was generated from the six knowledge dimensions together.

According to the findings of the current study supports this new interpretation of TPACK since findings regarding each knowledge domain form an interconnected web of relations between and among six knowledge dimensions, i.e., which knowledge dimension is perceived more attention than other, and how they are related with each other. Therefore, new TPACK diagram fits with the findings and have probably effect on future interpretations of knowledge dimensions needed for teaching mathematics.

5.9 Recommendations for Further Research

There are some recommendations for future research as a consequence of the findings of the current study. Since lesson study and MLS are served as alternative professional development ways for both preservice and in-service teachers, more research should be conducted in different contexts. Research also needed to investigate the professional development of both preservice and in-service teachers' use of different technological tools in mathematics and statistics teaching. From the technology-integration point of view, there is a need to develop teachers' teaching skills, they need to be more involved with technologically-rich environments.

Since lesson study was originated from Japan, it has becoming popular throughout the world and becoming familiar in Turkey nowadays. Therefore, more inquiry for lesson study and MLS could be done in the context of Turkish mathematics teacher education, as well. When considering especially the change in school curriculum regarding statistics, more research should be designed in order to investigate statistical thinking and understanding of mathematics teachers.

With a TPACK perspective, there should be more inquiry about how development of TPACK is taking place in different contexts, or in specific contexts. The interrelations between and among the knowledge dimensions aroused from TPACK framework should be researched more in order to analyze the relations between knowledge dimensions more, for instance in the subject of data displays or measures of spread, etc.

Further, preservice teachers' beliefs and attitudes regarding technology use in teaching, or especially in statistics teaching should also be studied especially before their professional development period begin.

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APPENDICES

APPENDIX A – Interview Protocol

Kendi çalışmamla ilgili kısa bir giriş ve öğrencinin geçmiş çalışmalarıyla ilgili önbilgiler alındıktan sonra aşağıdaki sorular yöneltilecektir.

1. Bir matematik dersinde pedagoji sizin için ne ifade etmektedir?
   a. Örnek verebilir misiniz?
   b. Ne kadar pedagojik bilginiz olduğunu düşünüyorsunuz? Kendinize 1'den 5'e kadar puan vermeniz gerekse kaç puan olur? (1: Zayıf, 5: Güçlü) Neden bu puanı verdiniz?
   c. Pedagojik bilginizi geliştirmek için neler yapıyorsunuz? Ya da neler düşünüyorsunuz?

2. Bir matematik dersinde alan bilgisi (content) sizin için ne ifade etmektedir?
   a. Örnek verebilir misiniz?
   b. Ne kadar alan bilgisine sahip olduğunuzu düşünüyorsunuz? Kendinize 1'den 5'e kadar puan vermeniz gerekse kaç puan olur? (1: Zayıf, 5: Güçlü) Neden bu puanı verdiniz?
   c. Alan bilginizi geliştirmek için neler yapıyorsunuz? Ya da neler düşünüyorsunuz?

3. Bir matematik dersinde teknoloji sizin için ne ifade etmektedir?
   a. Örnek verebilir misiniz?
   b. Ne kadar teknolojik bilginiz olduğunu düşünüyorsunuz? Kendinize 1'den 5'e kadar puan vermeniz gerekse kaç puan olur? (1: Zayıf, 5: Güçlü) Neden bu puanı verdiniz?
   c. Teknolojik bilginizin gelişmesi için neler yapıyorsunuz? Ya da neler düşünüyorsunuz?

4. Geçtiğimiz hafta derste öğrendiğimiz TPACK çerçevesi kapsamında öğrendiğiniz diğer bilgi boyutlarıyla (TCK, PCK, TPK) ilgili neler düşünüyorsunuz? Bir matematik öğretmeninin bu bilgilerde yeterli olması sizce gerekli midir? Neden?
   a. TPACK çerçevesinin diyagramını baz alırsak kendinizi hangi bilgi boyutlarında yeterli hissediyorsunuz? Hangilerinde yetersiz hissediyorsunuz?
   b. Özellikle güçlü olduğunuzu düşündüğünüz bilgi boyutu ya da boyutları hangileridir?
   c. Özellikle güçsüz olduğunuzu düşündüğünüz bilgi boyutu ya da boyutları hangileridir?
   d. Kendinizi hangi bilgi boyutlarında geliştirmeyi isterdiniz? Ya da planladığınız çalışmalar var mı?
   e. Kendi matematik öğreniminizi düşünürsek, teknolojiyle öğretmek nasıl olacak?

5. Yukarıda verdiğiniz cevapları baz alarak TPACK'i nasıl tanımlarsınız? Bu bilgi boyutunun matematik öğretmenliği boyutu için önemini tartışır mısınız?

APPENDIX B – DTAS

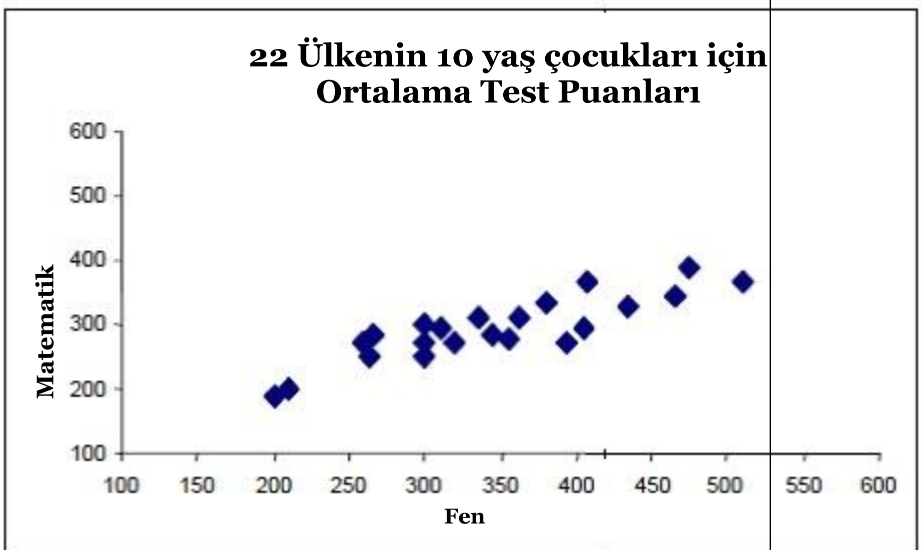
Matematik için Tanısal Öğretmen Değerlendirme (DTAS)
(Louisville Üniversitesi'nin izniyle alınmış iki testten **istatistik** sorularının çekilmesiyle hazırlanmış ve Türkçe'ye çevrilmiştir.)

İsim ______________ Tarih ______________

Açıklama: Aşağıdaki testte istatistikle (merkezi eğilim ölçüleri ve ilgili sorular bulunmaktadır. Sorulara mümkün olduğunca açık cevaplar veriniz ve gerekirse cevaplarınıza kısa açıklamalar ekleyiniz. Zaman ayırdığınız için çok teşekkür ederim.

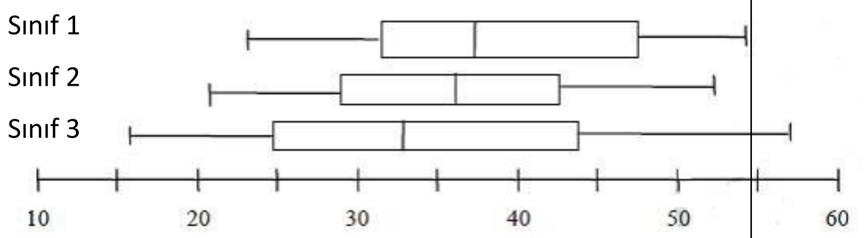
Soru	Cevap
1. Aşağıdaki veri, bir sınıfta öğrencilere dağıtılan şekerleme paketlerinden çıkan kırmızı şekerlerin sayısını göstermektedir. Bir grup katılımcının şeker paketinden çıkan kırmızı şekerler aşağıdadır: 3, 6, 11, 5, 6, 4, 15, 13, 6, 11. Aşağıdakilerden hangisi bu veri kümesinin ortanca değerini gösterir? a. 6 b. 5 c. 11 d. 8	
2. Bir sınıftaki 6. sınıf öğrencilerinin, okulun jimnastik dersinde bir koşuyu ne kadar sürede (saniye cinsinden) tamamlayabilecekleri araştırıldı. Elde edilen süreler gövde-yaprak gösterimiyle aşağıdaki gibidir. Buna göre hangisi doğrudur? 2 | 4 8 3 | 1 3 4 5 6 7 8 4 | 1 2 3 7 7 8 6 | 0 1 4 8 9 a. En kısa süre 28 saniyedir. b. Öğrencilerin yarısı 41 saniyenin üstünde süreye sahiptir. c. En uzun süre 60 saniyedir. d. Öğrencilerin %50'si 38 saniyenin altında süreye sahiptir.	

3. Aşağıdaki veri kümelerinden hangisinin tepe değeri 7, ortalaması 9 ve ortancası 8'dir? a. 8, 10, 7, 7, 16 b. 12, 7, 11, 7, 8 c. 8, 4, 8, 7, 5 d. 19, 7, 8, 7, 7	
4. Aşağıdaki grafik ya da gösterimlerden hangisi bir okuldaki 4. Sınıf öğrencilerinin başarı test puanlarının dağılımını en iyi gösterir? a. Serpme gösterimi b. Çizgi grafiği c. İki çubuk gösterimi (kutu gösterimi) d. Daire grafiği	

5. Aşağıdaki serpme gösterimi, 22 ülkenin 10 yaş çocuklarına uygulanan, biri matematik biri fen olmak üzere, iki testteki ortalama puanlarını göstermektedir. Bir ülkedeki fen testinde ortalama 550 puan alan çocukların matematik testinde alacağı puanı aşağıdakilerden hangi şık en iyi tahmin eder?  a. 380-410 arası b. 490-520 arası c. 570-600 arası d. 320-350 arası	

6. Aşağıdaki iki çubuk gösterimi 3 farklı sınıfın aynı testten aldıkları test puanlarıdır.

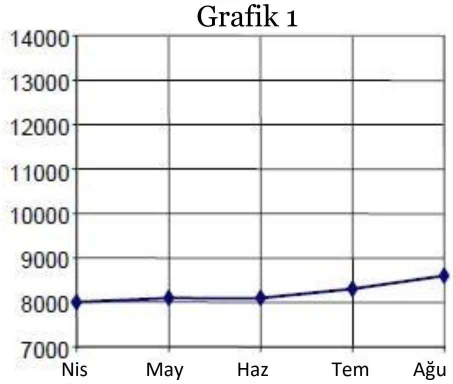
a. Hangi sınıf testte en başarılı olmuştur, hangi sınıf en başarısız olmuştur?
b. Seçimlerinizi, verilen iki çubuk gösterimlerine dayanarak yorumlayınız.



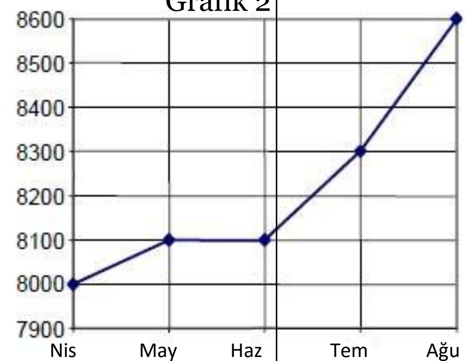
7. Aynı veri, bazen yanlı (yanıltıcı) olabilecek şekilde, birçok yolla gösterilebilir. Aşağıdaki iki grafik de bir eğlence parkında çalışan işçilere Nisan-Ağustos aylarında ödenen günlük ortalama ücretleri göstermektedir.

a. Grafikler ne açıdan farklıdır?
b. Grafik 1 nasıl tartışılabilir, hangi durumlar için avantajlıdır?
c. Grafik 2 nasıl tartışılabilir, hangi durumlar için avantajlıdır?
(Cevaplarınızı açıklayınız.)

Grafik 1



Grafik 2



<table>
<tr><td>

8. Bir öğrenci, sınıf arkadaşları ile yaptığı bir araştırmada, onların bir haftada video oyunlarına ayırdıkları süreyi incelemiştir. Bu öğrenci, araştırmanın verilerini aşağıdaki tabloda göstermiştir:

Süre (saat)	1	3	5	7	7 den fazla
Öğrenci sayısı	3	6	10	5	6

Öğrenci, bu veri için daire grafiğini kullanacağını ve bu grafiğin her diliminin açılarını bulması gerektiğini biliyordu. Öğrenci sayısıyla 10'u çarptı ve 30^o, 60^o, 100^o, 50^o ve 60^o elde etti. Daire grafiğini çizdiğinde ise boşluk kaldığını gördü.

a. Bu öğrenci nasıl bir hata yapmaktadır?
b. Bu öğrenciye nasıl yardım ederdiniz?

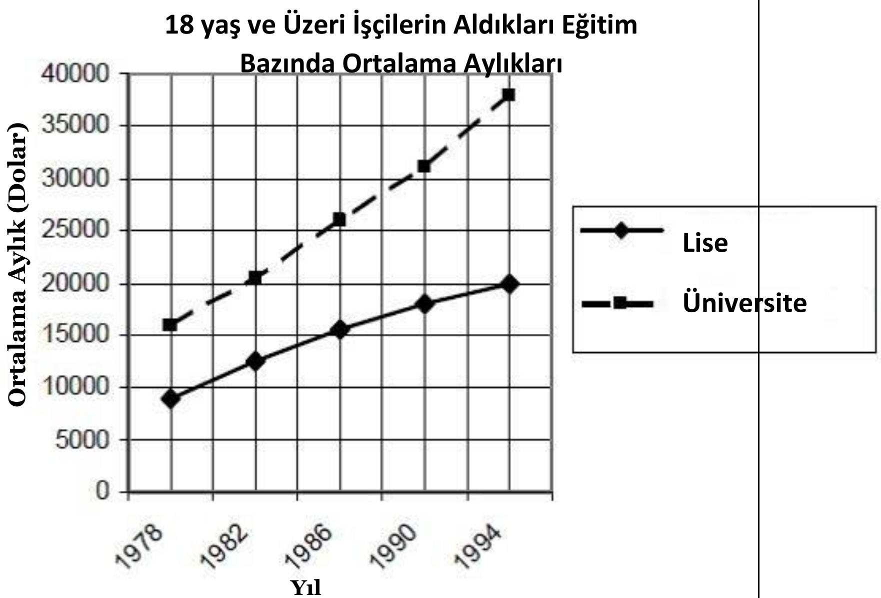
</td><td></td></tr>
<tr><td>

9. Aşağıdakilerden hangisi çizgi grafiğini en iyi tarif eder?

a. Medyanı, çeyrekleri ve veri kümesinin en küçük ve en büyük değerini gösteren grafiktir.
b. Verilerin X'le ifade edilerek yatay bir sayı doğrusu üzerinde gösterildiği grafiktir.
c. Ölçülen her maddeyi noktalarla gösteren grafiktir. Bir noktanın iki koordinatı, o maddeye ait iki özelliği gösterir.
d. Yatay ve düşey eksenlerden oluşan ve öncelikle zaman içindeki değişiklikleri gösteren grafiktir.

</td><td></td></tr>
</table>

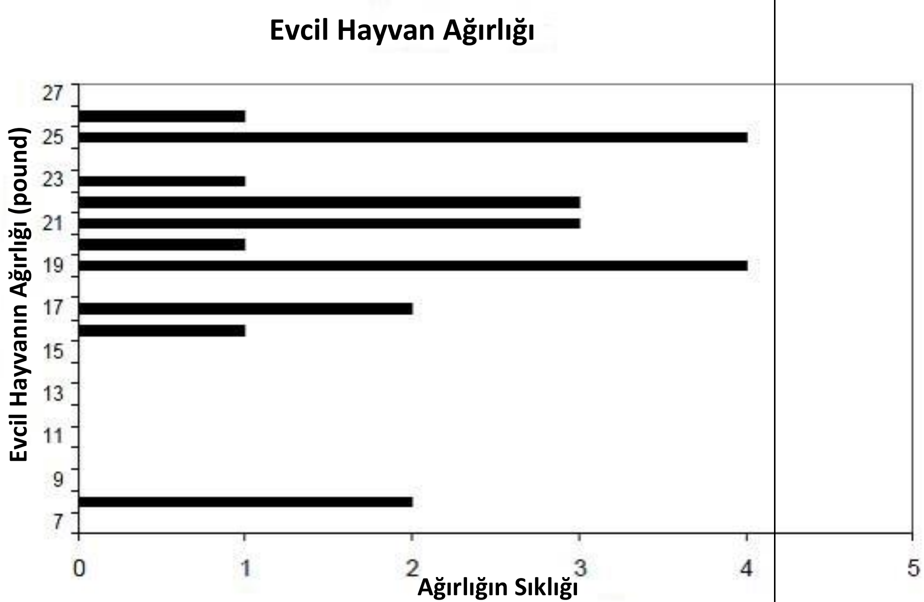
10. Aşağıdaki grafik 18 yaş ve üzeri işçilerin ortalama aylıklarını aldıkları eğitim bazında göstermektedir. Aşağıdakilerden hangisi doğrudur?



Bu iki grubun ortalama aylık ücret farkı...

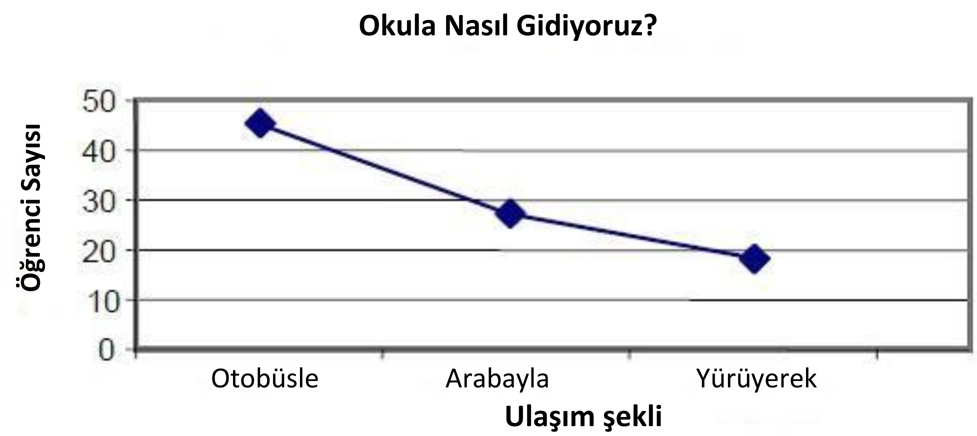
a. Aynı kalmaktadır.
b. Zamanla artmaktadır.
c. Zamanla azalmaktadır.
d. Zamanla iki katına çıkmaktadır.

11. Aşağıdaki cümlelerden hangisi aşağıda gösterilen veriyi tam olarak ifade <u>etmemektedir</u>?	

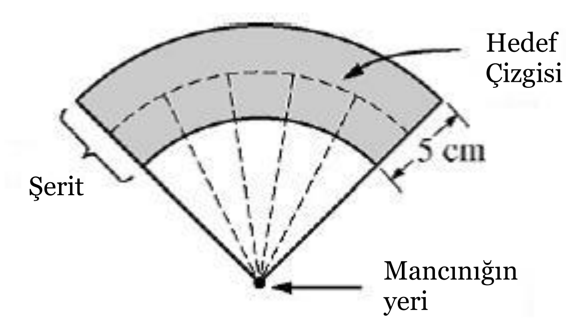


a. 19 ve 25 en sık görülen ağırlıklardır.
b. 8 bir aykırı değerdir.
c. Verilerde 9 ile 15 arasında bir boşluk vardır.
d. En az sıklıktaki ağırlık 8'dir.

12. Bir öğrenci 4. Sınıfa devam eden 90 öğrenciden okula nasıl ulaştıkları hakkında veri toplamıştır. Öğrenci bu veriyi aşağıdaki grafikte göstermiştir. Bu öğrenciye nasıl dönüt verirdiniz?

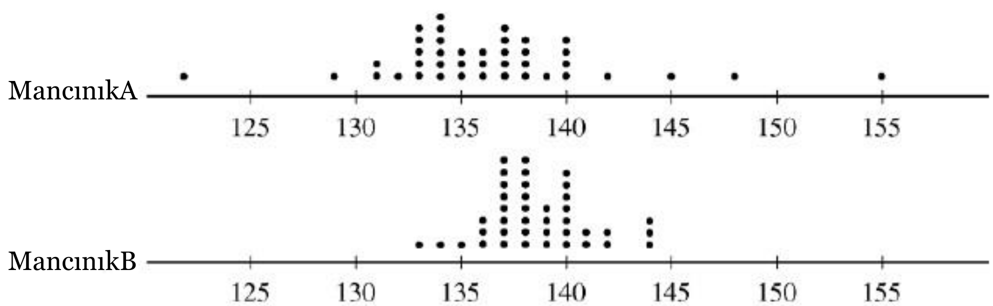


13. İki aile bir ilköğretim okulunun festivali için oyuncak bir mancınık tasarlıyor. Bu oyunu oynamak için, öğrenciler mancınığı kullanarak pinpon toplarını fırlatıyorlar ve topları önceden belirlenmiş 5cm genişliğindeki bir şeride isabet ettirmeye çalışıyorlar. Aşağıdaki şekilde görüldüğü gibi bir hedef çizgisi bu bandın ortasından çiziliyor. Bu hedef çizgisi üzerindeki noktalar mancınıktan eşit mesafededir.



Eğer bir top taralı alana isabet ederse, öğrenci bir ödül kazanacaktır.

Bu iki aile birbirlerinden biraz farkla iki mancınık tasarladılar. Daha fazla mancınık üretmeden önce bu ikisini test ettiler. Benzer koşullar altında iki mancınıktan da 40ar pinpon topu atışı yaptılar ve topların kat ettiği mesafeyi ölçtüler. Aşağıda bu atışların mesafelerini iki mancınık için de santimetre cinsinden gösteren noktasal grafikleri görebilirsiniz.



a. Durumu kendi cümlelerinizle anlatınız.
b. Bu, hangi tür grafik gösterimine girer?
c. Bu gösterime dayanarak hangi bilgilere ulaşabilirsiniz (örneğin, ortalama, ortanca, tepe değer, açıklık)?
d. Eğer, bu mancınıkları tasarlayan aileler pinpon toplarının belirlenen şeride isabet etme olasılığını artırmak isterlerse, hangi mancınık, mancınık A ya da mancınık B, daha uygun olur? (Mancınık A ya da B 5cm genişliğindeki şeride isabet ettirilecek şekilde istenildiği yere konulabilir.) Cevabınızı ispatlayınız.
e. D şıkkında seçtiğiniz mancınığı hedef çizgisinden kaç cm uzağa yerleştirmelisiniz? Bu mesafeyi neden seçtiğinizi açıklayınız.

APPENDIX C – Syllabus of the Course ELE 465

ELE 465

Nature of Mathematical Knowledge for Teaching

Fall 2014

Instructor:

Assoc. Prof. Dr. Bxxxxx Çxxxxxx

Office: EF-xx, Telephone: 210 xxxx, e-mail: xxxx@metu.edu.tr

Teaching Assistants:

Mxxxxx Axxxx

Office: EF-xx, Telephone: 210 xxxx, e-mail: xxxx@metu.edu.tr

Gamze Kurt

Office: Xxxxx , Phone: 210 xxxx, e-mail: gkurt@metu.edu.tr

Time: Tuesdays, 13:40 – 16:30

Place: EF-xx

Office Hours: Mondays, 10:40-11:30, or by appointment

Course Description

Focus on the questions regarding the nature of mathematical knowledge needed in mathematics teaching. Exploring mathematical content knowledge and its relationship to pedagogical content knowledge and pedagogical knowledge. Investigation of nature of knowledge types through theoretical foundations and practical implications.

Attendance and Participation

Attendance is mandatory and missing 3 blocks will result in a CC as your grade. I strongly recommend you attend the classes from the beginning and participate in discussions.

Required Texts

Readings assigned by the instructor mostly from: Ryan, J. & Williams, J. (2007). *Children's Mathematics 4-15. Learning from Errors and Misconceptions*. Berkshire: McGraw-Hill.

Assignments

I. Participation (5/100)

Participate in all the classroom activities and be prepared to discuss the issues related to these activities. You are expected to join the group works, and contribute to the group's studies and class discussions by speaking, listening, observing, sharing ideas and reflecting on the assigned readings and related materials. Lists of readings and resources will be provided on all topics via the METU Class. It is expected that the reading assignments are completed prior to the assigned lecture.

II. Discussion Questions (5/100)

Prepare <u>at least</u> 1 discussion question for each reading assigned for the week. Post it on METU Class by 10 a.m. on Tuesday.

III. Project (25/100)

You will be presented one or more misconception(s) that students are likely to do in the class and be asked to plan a lesson to re-teach the related concept in order to eliminate the misconception(s). The project should have the following components:

a. **Description of the misconception(s):** What might be the misconception(s) that the students had? What might be missing in the students' understanding of the concept?
b. **A lesson plan:** Develop a lesson plan that can be used in eliminating the misconception(s).
c. **A reflection paper**: At most two-page reflection on the types of knowledge that a teacher need in constructing and implementing this lesson plan.

You are expected to be realistic about the lesson plan you provide. Please consider timing, curriculum, and students' level of understanding. You will have one week to complete this assignment.

IV. Classroom Observation (15/100)

You will be required to observe a series of mathematics classroom in an elementary school and respond to the following questions/issues depending on your observations:

a. Description of the class (Physical setting, number of the students, available materials for the teacher, and etc.)
b. Description of the lesson (The concept of the day, how the class started, continued, and finished, the questions asked, students' responses to the questions and the classroom tasks, and assignments)
c. Description of teacher's knowledge used during the lesson (kinds of knowledge used, evidences for the knowledge types from teacher's actions in the classroom, the relationship between the types of knowledge used in the classroom)

Being careful about the questions that the teacher and the students ask and respond, and the tasks completed in the classroom will help you during your activity. It will be better if you pay considerable attention to these issues during your observations.

V. Lesson Plan (10/100)

You will be required to prepare a lesson plan regarding the instruction given on the 7^{th} and 8^{th} weeks. A lesson plan template will be given to you by the 7^{th} week. You will prepare this lesson plan during the course and revise and re-submit it based on the feedbacks. After revising the lesson plan, you will implement this lesson via microteaching.

VI. Research Paper Presentations (Group) (20/100)

You will be required to present at least two research papers on misconceptions in the selected learning area. Each group has to choose at least one learning area we will cover in the course. For each learning area, you will carefully read the provided research papers and discuss with your group friends, prepare a presentation including at most 10 slides including the short introduction, the methodology used in the research (number of participants, instruments, data analysis), the findings, and the conclusions, be ready for the questions that the class might ask as a group, and present the paper within 10 minutes and respond to the questions for 5 minutes. One research paper in the field of history of mathematics. In your presentation, discuss why and how the history of mathematics can be used in mathematics education. This assignment will be carried out as a group and thus all group members should be ready for the presentations and the possible questions.

VII. Reflection Papers (Group) (20/100)

i. You will be required to write a 4 pages paper reflecting your ideas about the study of original historical texts for your mathematical knowledge for teaching and for your beliefs about mathematics and mathematics teaching. In your paper, please make sure you provide justifications for the ideas you presented. The essay papers will be evaluated based on the degree of details and examples provided.

ii. You will be required to write a reflection paper (at most 2 pages) regarding the technological knowledge to teach mathematics. The paper has to cover your thoughts and ideas about the use of virtual manipulatives in teaching and learning statistics based on the instruction given in 7th and 8th weeks and the lesson plan you prepared. Use the following sample questions in developing your paper: What is your overall opinion about the instruction/lesson plan you prepared?, What are the strengths/weaknesses of this implementation?, What changes and revisions would you make regarding the implementation?

Tentative Schedule

Weeks	***Content***
Week 1 **23/09**	**Overview and organization of the course** Reflection on teaching activity
Week 2 **30/09**	**Knowledge for teaching mathematics** Ball, D., Thames, M, & Phelps, G. (2008). Content knowledge for teaching: What makes it special*? Journal of Teacher Education*, *59*(5), 389-407.
Week 3 **07/10**	**Religious Holiday** **No Class**
Week 4 **14/10**	**Teachers' subject matter knowledge and technological pedagogical content knowledge** Ma, L. (1999). Teachers' subject matter knowledge: Profound understanding of fundamental mathematics. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahvah: Lawrence Erlbaum. Koehler, M. J., & Mishra, P. (2009). What is technological pedagogical content knowledge? *Contemporary Issues in Technology and Teacher Education*, *9*(1), 60-70.
Week 5 **21/10**	**Statistics and Probability** Ryan, J. & Williams, J. (2007). Data-handling, graphicacy, probability and statistics. Research paper presentation
Week 6 **28/10**	**National Holiday** **No Class**
Week 7 **04/11**	**Integrating virtual manipulatives into statistics teaching** Introduction to teaching statistics via 'Virtual manipulatives'
Week 8 **11/11**	**Integrating virtual manipulatives into statistics teaching** Designing a lesson in statistics
Week 9 **18/11**	**Number** Ryan, J. & Williams, J. (2007). Developing number. Research paper presentation
Week **10** **25/11**	**Algebra <u>(Classroom observation due)</u>** Ryan, J. & Williams, J. (2007). From number to algebra. Research paper presentation

Weeks	*Content*
Week 11 02/12	**Geometry – Measurement** Ryan, J. & Williams, J. (2007). Shape, space and measurement Research paper presentation - Geometry
Week 12 9/12	**Measurement** Ryan, J. & Williams, J. (2007). Shape, space and measurement Research paper presentation - Measurement
Week 13 16/12	**History and Pedagogy of Mathematics** Module
Week 14 23/12	**History and Pedagogy of Mathematics <u>(Reflective essay due)</u>** Research Paper Presentation and Discussion
Week 15 30/12	**History and Pedagogy of Mathematics** Designing historical teaching activities Presenting and discussing historical teaching activities

Activity Sheet 1: Data and Variables

1. Please identify the type of each variable for items numbered as 2, 3, 4, 5, 6, 9, 10, 11, 13, 15 in the "Mini Survey for Workshop" with your group mates.
2. Suppose that instead of recording the number of words for 6th item, you had been asked to classify each sentence according to following criteria:

   1-4 words: small sentence
   5-7 letters: medium sentence
   8-10 letters: long sentence
   11 or more letters: very long sentence

   In this case, what type of variable is the size of sentence? Why?
3. Suppose that instead of recording whether or not you have been to Cappadocia for the 13th item, you had been asked to report the number of times that you have been to Cappadocia. What type of variable would this have been? Why?

Activity Sheet 2: Frequency (count) tables and bar graphs

1. Use the question 11 in the data set and fill the frequency (count) table below:

   Then, enter the website: http://www.shodor.org/interactivate/activities/BarGraph/ , and draw the bar graph for this frequency table.

   Another website: http://www.harcourtschool.com/activity/elab2004/gr3/26.html

2. Use the question 10 in the data set to create a dotplot. Enter the website: http://www.shodor.org/interactivate/activities/PlopIt/
3. Go to this website and see the relation between dotplots and bar graphs http://www.learner.org/courses/learningmath/data/session2/part_a/index.html
4. Please discuss the pedagogical strategies / teaching methods / possible learning outcomes / advantages / disadvantages which are highlighted during the use of online resources like above with your group mates.

Homework 1: Women Employed……………………………………………………..

The table below lists the number of men and women employed in the United States in 1995 for different occupations. The numbers listed are in thousands, so the number 163 in the Architect row means that there were 163,000 architects employed in the U.S. in 1995.

Occupation	Male	Female	Total	Percentage
Architect	131	32	163	
Engineer	1772	162	1934	
Math/Computer Science	813	382	1195	
Natural Science	377	142	519	
Physician	524	169	693	
Registered Nurse	136	1841	1977	
Teacher - PK, K	9	489	498	
Teacher - Elem.	276	1462	1738	
Teacher - Second.	530	702	1232	
Lawyer	658	236	894	
Musician	101	60	161	
Photographer	99	37	136	
Barber	73	14	87	
Hairdresser	60	690	750	
Social Worker	233	494	727	
Librarian	31	164	195	

(a) Compute the percentage of women employed in each occupation. Put your percentages in the table above.

(b) Identify the three occupations with the highest percentages of women employed and the three occupations with the lowest percentages of women employed. Make comparisons between your choices asked in the "mini survey" with the results here.

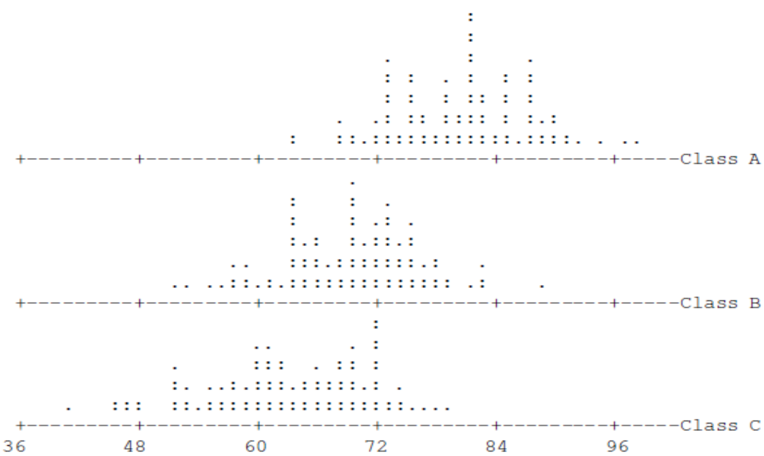
(c) Construct a dotplot of the distribution of percentage women by using online resources that you learned through workshop and take a screenshot of your plot. Based on examining this dotplot, write a brief paragraph describing the key features of this distribution of percentages.

End of Homework 1..

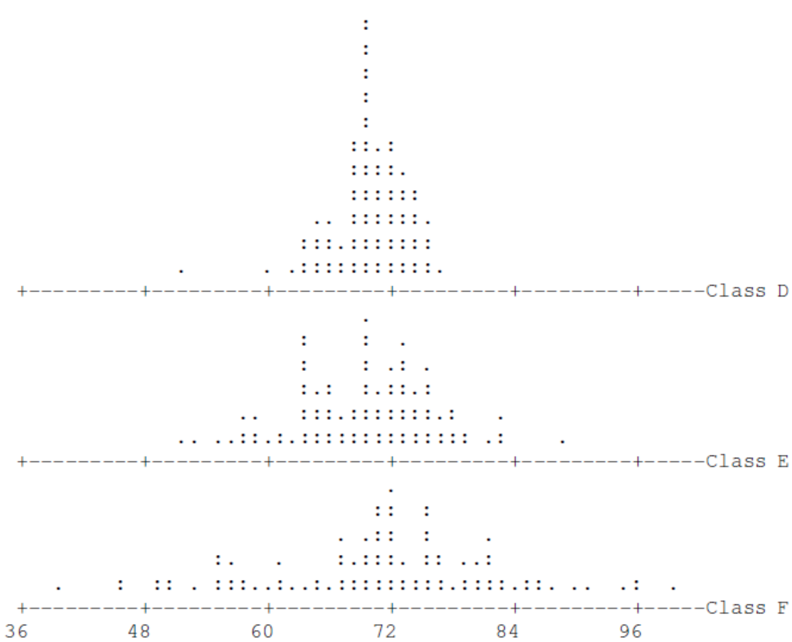
Activity Sheet 3: Displaying and Describing Distributions

1. Let's discuss first the following dotplots for exam scores of three classes which took the same exam.

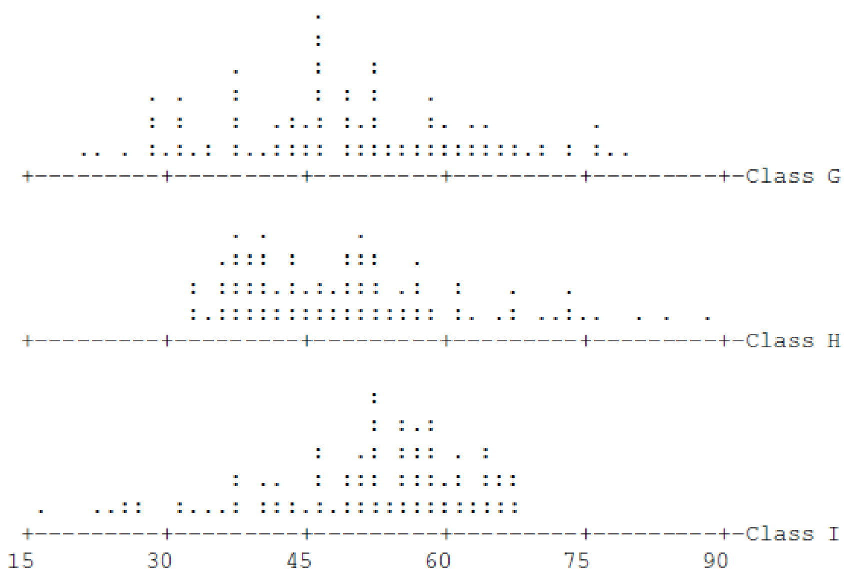
   a. What do you think is the most distinctive difference among the distributions of exam scores in classes A, B, and C?



   b. What is the most distinctive difference among the distributions of scores in classes D, E, and F?



c. What is the most distinctive difference among the distributions of scores in classes G, H, and I?



2. Use the below data set which shows the length of reigns of the Ottoman rulers of Ottoman Empire beginning with Osman Gazi in 1300. Create a stemplot or stem-and-leaf display for this data set using the website: http://www.shodor.org/interactivate/activities/StemAndLeafPlotter/

Ruler	Reign	Ruler	Reign	Ruler	Reign
I. Osman	24	III. Mehmet	8	III. Osman	4
I. Orhan	36	I. Ahmet	14	III. Mustafa	17
I. Murat	29	I. Mustafa	2	I. Abdülhamit	15
I. Beyazıt	13	II. Osman	5	III. Selim	8
I. Mehmet	8	IV. Murat	17	IV. Mustafa	1
II. Murat	28	I. İbrahim	8	II. Mahmut	31
II. Mehmet	32	IV. Mehmet	39	Abdülmecit	22
II. Beyazıt	31	II. Süleyman	4	Abdülaziz	10
I. Selim	8	II. Ahmet	4	V. Murat	0
I. Süleyman	46	II. Mustafa	8	II. Abdülhamit	33
II. Selim	8	III. Ahmet	27	V. Mehmet Reşat	9
III. Murat	21	I. Mahmut	24	VI. Vahdettin	4

3. Use the question 10 in the data set to create the histogram by using the website: http://www.shodor.org/interactivate/activities/Histogram/ or the website: http://www.flashandmath.com/mathlets/statistics/interhisto/BasicHistogram.html
4. Go to the website below and see the relation between a stemplot and a histogram.

   http://www.learner.org/courses/learningmath/data/session3/part_a/interpreting.html
5. Please discuss and identify the differences between a **bar graph** and a **histogram**. In which conditions bar graphs are used, and in which conditions histograms are used?
6. Please discuss the pedagogical strategies / teaching methods / possible learning outcomes / advantages / disadvantages which are highlighted during the use of online resources like above with your group mates.

Homework 2: Marriage Ages…………………………………………………..

Listed below are the ages of a sample of 24 couples taken from marriage licenses filed in Cumberland in June and July of 1993.

couple	husband	wife	couple	husband	wife	couple	husband	wife
1	25	22	9	31	30	17	26	27
2	25	32	10	54	44	18	31	36
3	51	50	11	23	23	19	26	24
4	25	25	12	34	39	20	62	60
5	38	33	13	25	24	21	29	26
6	30	27	14	23	22	22	31	23
7	60	45	15	19	16	23	29	28
8	54	47	16	71	73	24	35	36

(a) Select either the husbands' ages or the wives' ages, and construct a stemplot of their distribution using an online resource which you learned through workshop and take a screenshot of the plot you drew. (Indicate which spouse you are analyzing.)

(b) Construct a histogram for the same group of data you chose by using an online resource that you learned in the workshop and take a screenshot of the graph.

(c) Write a short paragraph describing the distribution of marriage ages for whichever spouse you chose.

End of Homework 2…………………………………………………………….

Activity Sheet 4: Measures of Center

1. Please find the mean of the question 9 (percentages of women lawyers) in the data set by entering the website (http://www.shodor.org/interactivate/activities/PlopIt/) to find the mean interactively.
2. Please find the median of the question 10 (number of cities visited in Turkey by the students) in the data set by entering the website (http://www.shodor.org/interactivate/activities/PlopIt/) interactively.
3. Are the mean and the median be different? Discuss with your group mates. At the end of the discussion, express the differentiations between mean and median by using the websites http://www.shodor.org/interactivate/activities/Measures/ or http://www.shodor.org/interactivate/activities/PlopIt/ again in order to compare the mean and the median together by adding the values to a group of data which you enter.
4. In which conditions mean/median is appropriate? Discuss with your group mates. Could you give examples?
5. For the question 12 (number of countries visited by the students) which measure is more appropriate to identify the center: mean or median? Discuss with your group mates.
6. Please find the mode of the question 15 (number of pets students had) in the data set by using the website (http://www.shodor.org/interactivate/activities/PlopIt/) interactively.

Homework 1: Supreme Court Service……………………………………………..

Use the computer to display the distribution of years of service for all Supreme Court justices who preceded the ones listed in Activity 3-1. These data are listed below:

5	31	32	23	21	21	19	22	6	6	23
1	4	14	6	9	20	3	5	34	13	31
20	34	32	8	14	15	4	8	19	7	3
8	30	28	23	34	10	5	15	23	18	4
6	16	4	18	6	2	26	16	36	7	24
9	18	28	28	7	16	5	7	9	16	17
1	33	15	14	20	13	10	16	5	16	24
13	22	19	34	11	26	10	11	1	33	15
15	20	27	8	5	29	26	15	12	5	14
4	2	5	10							

a) Describe the general shape of the distribution.
b) Based on this shape, do you expect the mean to be larger than the median, smaller than the median, or about the same as the median?
c) Have the computer calculate the mean and median of these years of service. Report these values and comment on your expectation in (b).

End of homework

1………………………………………………………………………..

Activity Sheet 5: Measures of Spread

1. Please find the five-number summary of the question 6 (number of words in statistics) in the data set and draw its box-and-whiskers plot (boxplot) by entering the website http://www.shodor.org/interactivate/activities/BoxPlot/ interactively.
2. Please investigate the website http://illuminations.nctm.org/Activity.aspx?id=3576 in order for making comparisons of boxplots while adding data into a set.

Activity Sheet 6: Comparing Distributions

1. Consider the assertion that men are taller than women. Does this mean that every man is taller than every woman? Discuss with your group mates. If not, write one sentence indicating what you think the assertion does mean.
2. Please use the question 11 (number of cities visited by students) according to gender (question 2) in the data set in order to draw two boxplots for each gender category and compare the plots. Enter the website below http://illuminations.nctm.org/Activity.aspx?id=3476.

Homework 2: Beatles' two albums…………………………………………………

The following table gives the times (in seconds) of the music tracks of "Sergeant Pepper" and "The White Album".

Sergeant Pepper					The White Album				
119	166	205	167	153	165	240	130	190	62
204	156	303	158	163	185	286	167	148	121
155	80	303			140	124	213	232	102
					106	177	160	241	166
					145	195	270	188	253
					162	175	191	495	194

Please draw two boxplots showing the distribution of music tracks of each album together (it is called parallel boxplots). Then, compare the distributions.

End of Homework 2…………………………………………………………………

Activity Sheet 7: Graphical Displays of Association

1. Please use the question 11 (number of cities visited) and the question 16 (number of relatives) in order to show whether there is an association between them. Enter the website http://illuminations.nctm.org/Activity.aspx?id=3476 in order to draw the scatterplot of this association.

Homework 3:
Temperatures……………………………………………………….

For a number of days throughout the year, we recorded the high temperatures in Van and İzmir. Part of the data is given in the table below. Ten days are listed and, for each day, the maximum temperatures for the two cities are listed.

Day	Van Temperature	İzmir Temperature
January 18	11	18
February 3	0	8
October 23	15	21
December 8	8	13
December 26	7	12
January 19	3	9
February 23	7	13
November 7	9	16
December 10	5	11
December 28	7	17

Using the website http://illuminations.nctm.org/Activity.aspx?id=3476 again, draw the scatterplot of the temperature distributions of these two cities. If we knew the temperature in Van for a particular day, could we accurately predict the temperature in İzmir?

End of Homework
3…………………………………………………………………

APPENDIX E – Mini Survey for TI-STW

Please answer the following questions. Your responses will be used as data in our workshop. * Required

1. Write a sentence describing what the word "statistics" means to you. (Here and throughout the text, please write in complete, well-constructed, grammatically correct sentences.) *

2. What is your gender? *

- ○ Female
- ○ Male

3. Which one of the following season best fits you: summer, winter and spring/fall? *

4. Do you think that Turkey should retain or abolish "kuruş" as a coin of currency? *

- ☐ Retain
- ☐ Abolish

5. Rank your opinion of the value of statistics in society on a numerical scale of 1 (completely useless) to 9 (incredibly important). *

	1	2	3	4	5	6	7	8	9	
Completely useless	○	○	○	○	○	○	○	○	○	Incredibly important

6. How many words are in the sentence that you wrote in response to question 1? *

Please write in digits.

7. For each of the following pair of sports, identify the one that you consider more hazardous to its participants: *

- ☐ bicycle riding and football
- ☐ soccer and ice hockey
- ☐ swimming and skateboarding

8. Among the occupations listed below, label with a "W" an occupation where you would expect to find a high proportion of women. Label with an "M" an occupation where you would expect to find a high proportion of men. *

	W	M
Architect	○	○
Nurse	○	○
Primary School Teacher	○	○
Pilot	○	○
Physician	○	○
Musician	○	○
Photographer	○	○
Lawyer	○	○

9. Take a guess as to the percentage of lawyers who are women *

(in percentages)

10. How many cities have you visited in Turkey? *

11. Which city do you think that most visited city in Turkey by your classmates? *

(regarding this class)

12. How many countries have you visited so far? *

13. Have you ever been to Capadoccia in Nevşehir? *

- ☐ Yes
- ☐ No

14. Have you ever been to Europe? *

- ☐ Yes
- ☐ No

15. How many pets did you have so far?

APPENDIX F – Themes and Codes for Interview

Themes and Codes for the Interview	
Themes	Codes
Theme_g: high school type	Code_g1: Anatolian teacher high school Code_g2: high school in abroad
Theme_h: choosing elementary mathematics teacher department	Code_h1: deliberately Code_h2: by coincidence Code_h3: unintentionally Code_h4: forced
Theme_i: teaching experience	Code_i1: teaching in a real classroom Code_i2: microteaching (teaching with their classmates) Code_i3: practice teaching (as intern teacher in a practice school) Code_i4: private teaching (with one or group of students (groups of 2-3 students) (özel ders) Code_i5: teaching voluntarily as a community service
Theme_j: views towards technology	Code_j1: using technology in future teaching Code_j2: wish to develop themselves in technological tools Code_j3: not using technology much Code_j4: technology gives visual opportunities in teaching Code_j5: technology needs continuous improvement Code_j6: technology allows saving time Code_j7: technology is not so necessary like content or pedagogy Code_j8: technology makes teaching easier Code_j9: technology makes no difference in teaching Code_j10: technology is terrifying Code_j11: technology gives harm to students Code_j12: technology should allow students to produce something Code_j13: technology in teaching provides a better learning

Theme_k: views towards statistics and statistics teaching	Code_k1: statistics is always terrifying subject Code_k2: statistics is always joyful Code_k3: statistics teaching needs a hard preparation Code_k4: understanding statistics is different than understanding mathematics

APPENDIX G – TPACK Codebook

TPACK Codebook	
Themes	Codes
Theme_a: Content knowledge (CK-subject matter knowledge)	Code_a1: knowledge of concepts, principles, and relationships in a curricular domain Code_a2: knowledge of the rules of evidence and proof
Theme_b: Pedagogical knowledge (PK)	Code_b1: knowledge of general teaching methods and strategies Code_b2: checking for understanding Code_b3: knowledge of learners and their background Code_b4: knowledge of general assessment strategies (e.g., tests, oral, project-oriented tasks) Code_b5: classroom management techniques Code_b6: lesson planning activities and preparation
Theme_c: Technological knowledge (TK)	Code_c1: Operating computer hardware Code_c2: Using standard software tools (e.g. Ms Word etc.) for non-educational use Code_c3: Installing and removing peripheral devices (e.g., USB drives, microphones, etc.) Code_c4: troubleshooting equipment Code_c5: using appropriate vocabulary Code_c6: knowledge of current and emergent technologies in society
Theme_d: Pedagogical Content knowledge (PCK)	Code_d1: knowledge of teaching/representing subject matter to students (e.g., techniques, representations, analogies) Code_d2: identifying and addressing student subject-specific misconceptions or mistakes Code_d3: content-specific assessment strategies
Theme_e: Technological content knowledge (TCK)	Code_e1: knowing about the existence of a variety of content tools for particular content tasks; especially tools that experts in this field might use Code_e2: operating/knowledge of content-based technologies in which content learning is foregrounded Code_e3: knowledge about the ways in which content and technology reciprocally related to one another

Theme_f: Technological pedagogical knowledge (TPK)	Code_f1: Motivating students through technology Code_f2: differentiating instruction when technology is used Code_f3: ability to organize collaborative work with technology Code_f4: holding students accountable for equipment used Code_f5: developing strategies for assessing student work with technology Code_f6: knowing about the existence of a variety of technological tools for particular general pedagogical tasks Code_f7: ability to repurpose commercial software for general teaching Code_f8: knowing about the time required to teach with particular technologies (prediction may be said) Code_f9: ability to envision potential student problems with particular technologies and plan relevant activities to support those students Code_f10: Generating alternatives in the event of technological failures Code_f11: ability to explain a computer procedure to students (e.g. through modelling) Code_f12: using technology for lesson plan preparation Code_f13: using technology for general assessment Code_f14: knowledge of infrastructure at school site

APPENDIX H – Consent Form

Merhaba,

Ben Gamze KURT. Orta Doğu Teknik Üniversitesi Eğitim Fakültesi İlköğretim Bölümü'nde araştırma görevlisi olarak çalışıyorum. Aynı zamanda İlköğretim Eğitimi Anabilim Dalı'nda devam ettiğim doktora eğitimimde tez aşamasına gelmiş bulunuyorum.

Bu dönem doktora eğitimim kapsamında hazırladığım tez gereği olarak yaptığım çalışmada ilköğretim matematik öğretmeni adaylarının istatistik bilgilerinin teknolojik pedagojik alan bilgisi (TPACK) çerçevesi kapsamında gelişimini incelemeyi hedefliyorum. Bu sayede matematik öğretmen eğitiminin bir parçası olan öğretim yöntem ve teknikleri gibi derslerin de geliştirilmesine ve matematik öğretmen adaylarının istatistik bilgilerinin teknolojik boyutta gelişmesinde katkı sağlayacağını umuyorum.

Bilgi toplamak için planlanan bu birebir görüşmeye katılımınız, sizin tecrübelerinizden yararlanabilmemiz ve şimdiye kadar yaşadığınız bu deneyimlerin ilerleyen dönemde üniversite eğitiminde yaşanacak gelişmeleri etkileyebileceğini gösterebilmemiz için çok değerlidir. Bu görüşmelerde size yöneltilecek sorular çalışmamızın amaçlarıyla doğrudan örtüşmektedir. Kısaca, istatistik için geçmiş alan bilginiz ölçülecek ve TPACK çerçevesinde ortaya çıkan bilgi boyutları bazında bilgi düzeyleriniz belirlenecektir. Konuyla yakından ilgili bu soruları cevaplamanız, katılımcı olarak size herhangi bir zarar vermeyecektir.

Bu noktada, sizden beklenen, sorulara mümkün olduğunca ayrıntılı cevaplar vermenizdir. Söyleyeceğiniz her cümle ilköğretim matematik öğretmeni eğitimine ışık tutar nitelikte olduğundan çalışmamıza çok anlamlı katkısı olacaktır. Birebir yapılacak bu görüşmenin tahminen 30 – 40 dakika arasında süreceği hesaplanmaktadır. Fakat sorulara istediğiniz uzunlukta ve ayrıntıda cevap vermek tamamen size bırakılmıştır, bu anlamda görüşmemiz sizin belirleyeceğiniz şekilde ilerleyecektir.

Görüşme sırasında aynı anda ses kaydı alınması da planlanmaktadır. Görüşme süresince katılımcının vereceği bilgilerin daha sonra özenli bir biçimde analizinin yapılmasını kolaylaştıracak ve sağlamlaştıracak bu işlemden, katılımcı olarak sizin uygun bulmamanız halinde vazgeçilebilir ya da istenildiği anda kayıt durdurulabilir veya yeniden başlatılabilir. Ses kaydını kesinlikle istemediğiniz takdirde görüşme notları tutulacaktır. Görüşmeye katılımınız kesinlikle zorunlu değildir. Katılmamanız veya herhangi bir sebepten ötürü katılmaktan vazgeçmeniz durumunda olumsuz herhangi bir sonuçla karşılaşmanız muhtemel değildir. Başladıktan sonra dahi görüşmeyi durdurabilirsiniz.

Görüşmemiz sırasında edinilen ve kayıt altına alınan bütün bu bilgilerin güvenliği araştırma ekibinin sorumluluğundadır. Herhangi bir şekilde görüşmenin herhangi bir kanalla ibrazı söz konusu değildir. Elde ettiğimiz ses kayıtları ve görüşme notlarına sadece araştırma ekibinin erişimi vardır. Bu kayıtların kimliğinizi açığa çıkaracak şekilde eğitim aldığınız kuruma ya da bir başka kuruma verilmesi söz konusu değildir. Araştırma sona erdikten belli bir süre sonra kayıtlar ve görüşme notları imha edilecektir.

Araştırmamıza yönelik sorularınız olması durumunda benimle ve/veya tez danışmanımla iletişime geçebileceğiniz bilgiler aşağıdaki gibidir:

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Eğer bu çalışma için ayrıntıları yukarda açıklanmış olan birebir görüşmelerde gönüllü olmak istiyorsanız, lütfen aşağıda belirtilen yere isminizi ve tarihi yazarak imzalayınız.

Teşekkür ederim.

İsim: İmza:

Tarih:

APPENDIX I – First Draft of Lesson Plan of Group A

Steps	Main Learning Activities	Students' Anticipated Responses	Remarks on Teaching
1)Activating Prior Knowledge (individual work) (Duration:20 min.)	Giving a problem including categorical data (problem1) and asking stu. that which graph can be used to show this categorical data, then expecting to form a bar graph.	Expecting that selecting bar graph to show this data and this problem should not be difficult for them because of their prior knowledge.	"What do you think when viewing this data? "What categories of information would you place in the graph of this data?" "How can we determine the axes of this graph?" "You should use this website to form bar graph http://www.shodor.org /interactivate/activities/PlopIt/"
2)Activating prior knowledge (individual work) (Duration:20 min.)	Taking attention to the stem and leaf display which is formed in the right side of the bar graph in the website.	Expecting that stu. may remember the stem and leaf display without knowing the name of it.	After taking attention to the stem and leaf display, we will ask them questions like that "How many students took … point?" If students can not remember their prior knowledge, we will give another example on the board.

3)Connecting stem and leaf display knowledge to the histogram and giving information about histogram (individual work) (Duration:20 min.)	Asking to rotate stem and leaf display in the given website and writing it in their notebooks. Then asking them to form a graph from this display in their notebooks. After that we will give steps of drawing histogram.	Expecting to notice relationship between histogram and stem and leaf display and notice the kind of data that used in forming histogram.	"What do you think when viewing this relationship? "What kind of information would you place in the histogram?" "How can we determine the axes of this graph?" "What is the purpose of following these steps?"
3)Comparing bar graph and histogram (think,pair,share)	Expecting that notice and explain the difference between histogram and bar graph by giving an example (problem2).	Expecting that they may think both of the graphs look very similar. They may question the categories of	"Can we show the data of first graph by using histogram ?" "Why?" "What are the differences of these graphs and used data?"

		informations used in these graphs. They may confuse about determining range of data in histogram.	

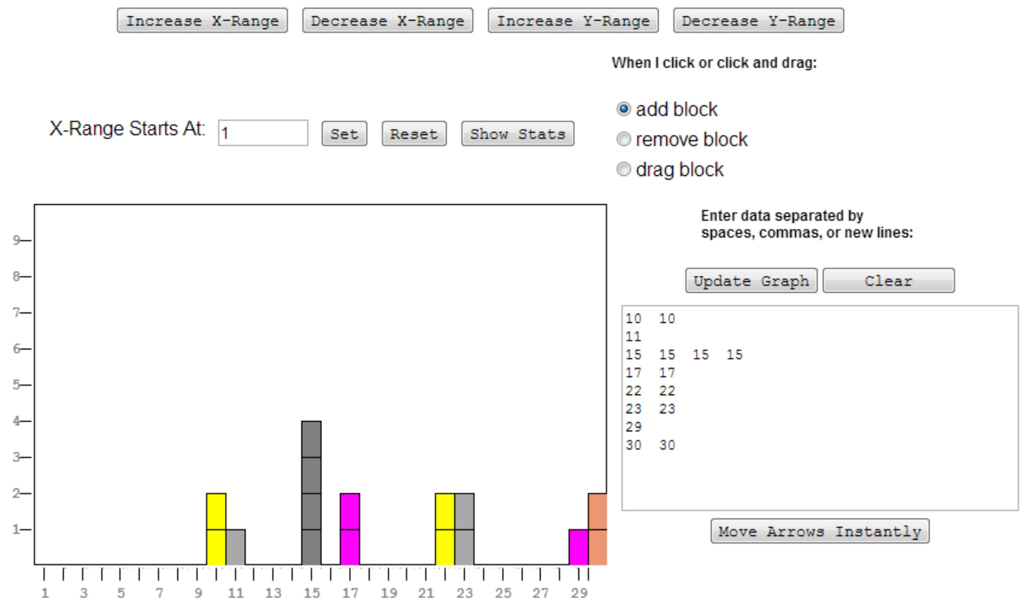
Explanations / Definitions for Column Headings:

1. "Steps" Column: Give short, general descriptors for each segment of the lesson (e.g., group work, whole-class discussion, etc.)
2. "Main learning activities" column: Describe each segment in more detail, including items like: the problems students will be asked to solve, as well as the activities students are to do during each segment.
3. "Students' anticipated responses" column: Describe how you expect students to react to each of the main learning activities. What will they find easy? What will they find difficult? What will they find interesting? Etc.
4. "Remarks on teaching" column: Provide notes about teacher actions that must be carried out in order to help students succeed with each segment of the lesson. Include things like special instructions given to students, questions you plan to ask, or aspects of the lesson upon which you especially wish to focus students' attention.

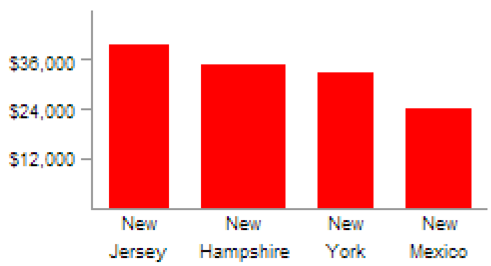
Problem 1: In a recent geometry test, these students got the following grades:

Grade:	A	B	C	D	E	F	G	H
Students:	22	24	15	10	17	30	29	15
Grade:	I	J	K	L	M	N	O	p

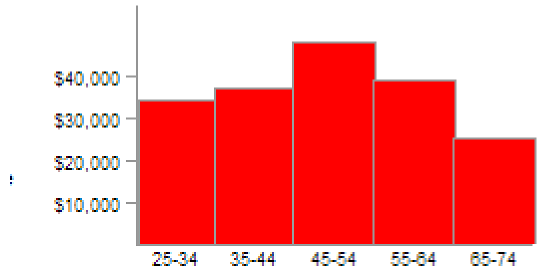
Students: 10 30 15 22 17 24 11 15



Problem 2: The bar chart below shows average income for the four cities ; New Jersey, New York, New Hampshire, and New Mexico.



The histogram below shows income for five age groups.



$40,000
$30,000
$20,000
$10,000
25-34
35-44
45-54
55-64
65-74

APPENDIX J – First Draft of Lesson Plan of Group B

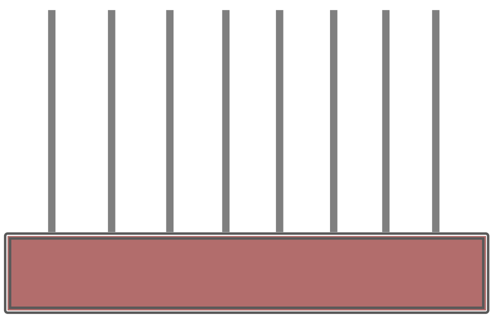
Grade Level: 8

Objectives: Form and interpret a histogram according to a data group.

Show the data related to research questions in circle, bar, line or histogram when they are appropriate and make transformations among these graphs.

Steps:

1) Students are wanted to bring their data about occupations of their fathers and how many years they have worked in those occupations. Also, teacher will bring this tool to the class.



2) Before beginning the activity, ask students what they remember about the bar graph. Want them to give examples of some data which they can make bar graph with it.

3) Then, say students that they will put a ring to the column in the tool in which their fathers' job exists respectively. Half of the class will put first and then the other half. While they do this, distribute activity sheets to students.

4) After the first half of students put their rings, ask the following questions to class which exist also in the sheets:

Do all data represent the same quantity? If yes, what do they represent? If no, why?

What can you say about the other half? Will the appearance of tool be the same with this or not? Why?

5) Want from the other half to put their rings. Then, ask the following questions:

Which occupation is most/least common among fathers?

Can we generalize this situation to our school?

If students do not understand the second question and for example doctor is the most common job, ask “Will the doctor be a most common job in the school?” If students say yes, direct them to the second question. Ask what the most/least common were when the half of students put their rings?

6) Want from students to fill the table in the sheet. This table will help them while making bar graph.

7) Want from students to enter the site http://www.learner.org/courses/learningmath/data/session2/part_a/index.html to see how bar graph is formed using dot plot.

8) Then, ask “How can you make a bar graph using this tool and the information you see in the site?” After students draw their graphs, want one of them to show their graph and explain how they did it. Then, ask the class whether they found the same thing or not.

9) Now, want from students to write the second data they bring i.e. how many years their fathers worked, to the board respectively. If the same number occurs, students will write it next to the other. After students finished, ask

Can we apply this procedure for the whole school? Will it be logical? What can you suggest to avoid this situation?

We expect students to say using interval. If they do not, ask “What about analyzing these data by grouping instead of one by one?” Then, ask “Can we make again a bar graph using these new data? (????) If yes, how? If no, which graph can we make?

We expect students to say histogram.

10) Then, draw this table to the board:

1-5	6-10	11-15	16-20	21-25	25+

Want each student to paint a cell in which their data exist from bottom to top. After each student painted, histogram will emerge automatically. Students will also draw this table to their sheets and make the same procedures.

11) After histogram is formed, ask the following questions:

According to this graph, what can you say about the number of fathers who worked two years?

Can we find maximum and minimum years like in the bar graph?

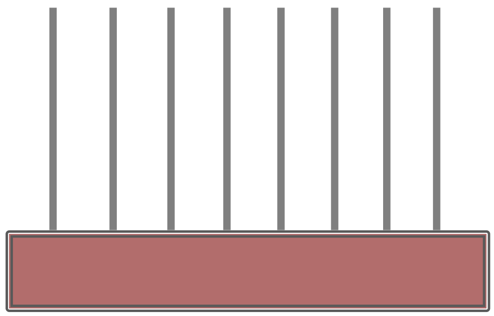
Which types of data are used in bar graph and histogram?

What other differences exist between bar graph and histogram?

12) At the end of the lesson, ask students to bring data types which they can use in bar graph and histogram.

ACTIVITY SHEET

1) Draw rings like in the tool.



2) Answer the following questions:

a) Do all data represent the same quantity? If yes, what they do represent? If no , why?

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b) What can you say about the other half? Will the appearance of tool be the same with this or not? Why?

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c)Which occupation is most/least common among fathers?

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d) Can we generalize this situation to our school?

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3) Write the number of fathers for each occupation.

Occupation	# of fathers

4) Make a bar graph according to the tool and information in the site. Explain your thinking.

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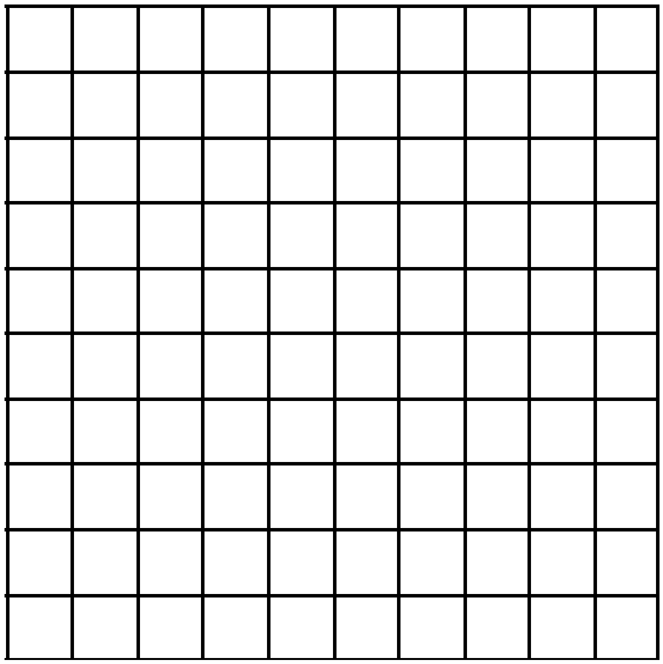
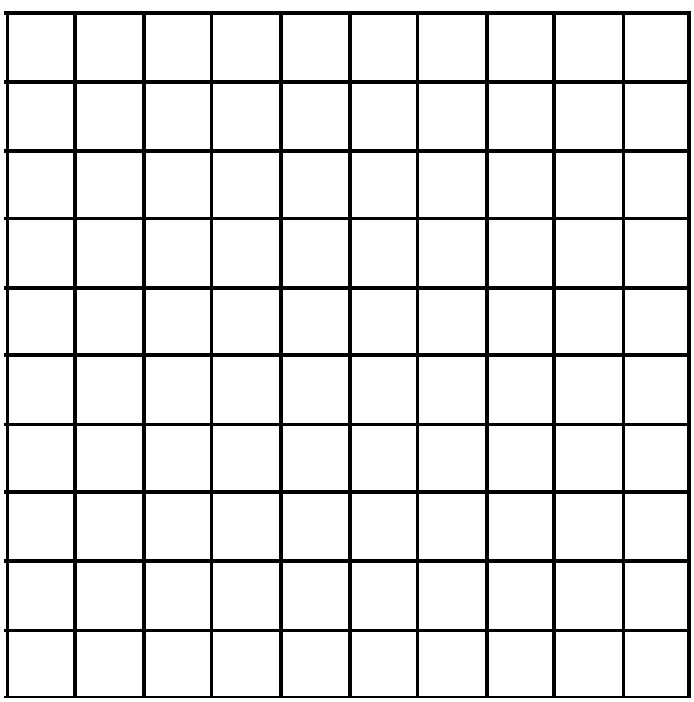


5) Construct the graph by using different colored pencils. Then, answer the questions.



a) According to this graph, what can you say about the number of fathers who worked 2 years.

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b) Can we find the max and min years like in the bar graph?

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c) Say other differences between bar graph and histogram.

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d) Which types of data are used in bar graph and histogram?

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APPENDIX K – Second Draft of Lesson Plan of Group A

Topic: Histogram
Grade Level: 8
Duration: 80 min.
Objectives:

- Bir veri grubuna ilişkin histogram oluşturur ve yorumlar.
- Araştırma sorularına ilişkin verileri uygunluğuna göre sütun grafiği histogramla gösterir ve bu gösterimler arasında dönüşümler yapar.

Lesson Plan Format

Steps	Main Learning Activities	Students' Anticipated Responses	Remarks on Teaching
1)Activating prior knowledge (individual work) (Duration: 20 min.) Direct instruction and questioning method.	Firstly, we will ask the problem 1a and we will remind the stem and leaf display without using its name. We will take attention to the relationship between the numbers in stem and leaf display and the numbers of toys. Then we will explain this relationship by giving the open form of stem and leaf display diagram.	Expecting that some students may remember the stem and leaf display without knowing the name of it. Furthermore, we will explain the stem and leaf display diagram by using problem 1 for students who did not remind it. Also, some students may have misconceptions that stem and	After taking attention to the stem and leaf display, we will ask them questions like that "Ok, what is the relationship between these numbers (in stem and leaf display) and the numbers of toys? "How many super markets are there?"

	(example of open form: 21 21 25 25 25 27 28)	leaf multiply with each other (2x1). Some students also may think that the numbers in the stem part shows the name of the super market. (In the first super market, there is no toy. Therefore, the numbers in the leaf part shows the number of toys.)	"What is the least number of toys? (and the most)" Furthermore, we will give a hint that stem part shows the tens and the leaf part shows the ones.
2)Connecting stem and leaf display knowledge to the histogram and giving information about histogram (individual work) (Duration: 20 min.) Direct instruction	Asking to form stem and leaf display diagram of the problem 1b by using website of NCTM. Then asking them to draw the diagram that they formed in website in their activity sheets. After that, we will rotate the stem and leaf display	Expecting to notice relationship between histogram and stem and leaf display. We will take attention to columns and by using columns they will reach the number of hats in each column. Students may trouble with the	"What do you think about this relationship?" "What kind of information would you place in the histogram?" "How can we determine the axes of histogram?"

	diagram by drawing it on the board. (They also will rotate diagram in their activity sheets.) And, we will construct histogram by covering the around of data with a line. We will indicate the histogram and we will give steps of drawing a histogram. (range=grup açıklığı,objective) Then, we will ask to use the histogram button in NCTM, so they will analyze the histogram.	determining name of the axes of histogram and range of it. Also, they will have difficulties to notice the meaning of the axes.	"What is the purpose of following steps of drawing histogram?"
3)Activating prior knowledge of bar graph and then comparing bar graph and histogram Discussion method	We will activate the knowledge of bar graph by showing a bar graph. For this, we will use visuals (visual1). By using visuals,	Expecting that they may think both of the graphs look very similar. They may question the categories of information used in these graphs.	"Which types of movie are most liked, and which are least liked?" "Can we show the data of

(think, pair, share) Duration: 10+30 min.	we will remind the properties of bar graph (type of data). (10 min.) After that, we will take attention to the histogram formed in problem 1b. Then, we will expect to notice and discuss the difference between histogram and bar graph.	(We will not mention the continuous and categorical data; we solely expect to realize the difference of data to form these graphs.) Also, they may confuse about determining range of data in histogram.	visual1 by using histogram? Why?" "What does the meaning of spaces between columns in bar graph?" "What are the differences between data of bar graph and histogram?"

Explanations / Definitions for Column Headings:

5. "Steps" Column: Give short, general descriptors for each segment of the lesson (e.g., group work, whole-class discussion, etc.)
6. "Main learning activities" column: Describe each segment in more detail, including items like: the problems students will be asked to solve, as well as the activities students are to do during each segment.
7. "Students' anticipated responses" column: Describe how you expect students to react to each of the main learning activities. What will they find easy? What will they find difficult? What will they find interesting? Etc.
8. "Remarks on teaching" column: Provide notes about teacher actions that must be carried out in order to help students succeed with each segment of the lesson. Include things like special instructions given to students, questions you plan to ask, or aspects of the lesson upon which you especially wish to focus students' attention.

ACTIVITY SHEET

Problem 1: a) A super market purchase specialist formed a stem and leaf display diagram to show the number of toys in each super market.

0	
1	
2	1 4 5 5 5 7 8
3	0 0 4 6 6
4	0 2 9

b) A super market purchase specialist recorded the number of hats in below:

5,6,7,10,11,13,18,19,23,25,29,32,32,32,36,45,45,46,50,54,56,56,56,56

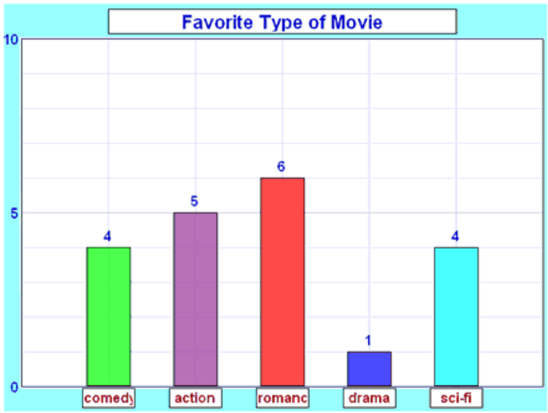
By using website of NCTM (http://illuminations.nctm.org/Activity.aspx?id=3476), can you help her to form stem and leaf display diagram of the numbers of hats in these super markets?

Visual1:

Imagine you just did a survey of your friends to find which kind of movie they liked best:

Table: Favorite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**SciFi**
4	5	6	1	4

We can show that on a bar graph like this:



APPENDIX L – Second Draft of Lesson Plan of Group B

Grade Level: 8

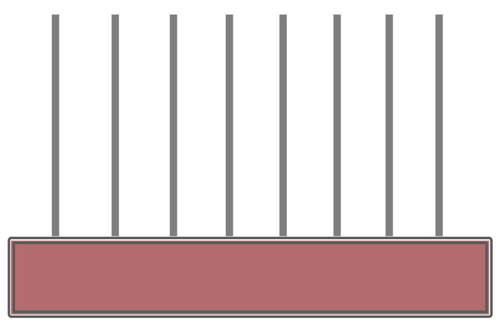
Duration: 80 min.

Objectives:

Show the data related to research questions in circle, bar, line or histogram when they are appropriate and make transformations among these graphs.

Steps:

1) Students are wanted to bring their data about occupations of their fathers and how many years they have worked in those occupations. Also, teacher will bring this tool to the class.



2) Before beginning the activity, ask students what they remember about the bar graph. Want them to give examples of some data which they can make bar graph with it.

3) Then, say students that they will put a ring to the column in the tool in which their fathers' job exists respectively. Half of the class will put first and then the other half. While they do this, distribute activity sheets to students.

4) Then, ask the following questions:

-Which occupation is most/least common among fathers?

-If we change the place of columns, does the meaning of graph change ?

-If we put one of rings another column, how does the graph be affected from that?

5) Want from students to fill the table in the sheet. This table will help them while making bar graph.

6) Want from students to enter the sites and write their datas .
http://primaryschoolict.com/pictograph/
http://www.shodor.org/interactivate/activities/BarGraph/

7) Then, ask "How can you explain the relationship between this tool and the graphs you see in the sites?"

8) Let's think about second data you bring. We will use one of the graph we learned before. Which can be used for this data? (We expect the students to say histogram)
After students answers, we will ask their reasoning. If students say we can make a bar graph, we ask what the column names will be in bar graph?

9) Then, draw this table to the board:

1-5	6-10	11-15	16-20	21-25	25+

Want each student to paint a cell in which their data exist from bottom to top. After each student painted, histogram will emerge automatically. Students will also draw this table to their sheets and make the same procedures.

10) Construct a histogram by using http://illuminations.nctm.org/activity.aspx?id=3476

11) After histogram is formed, ask the following questions:

According to this graph, what can you say about the number of fathers who worked two years?

Can we find maximum and minimum years like in the bar graph?

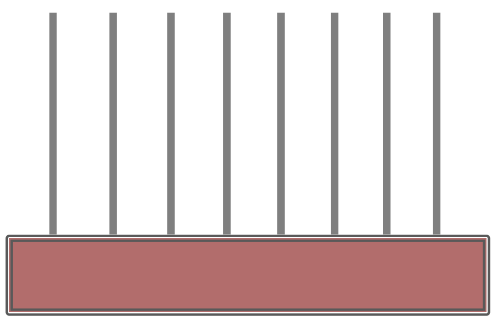
Which types of data are used in bar graph and histogram?

What other differences exist between bar graph and histogram?

12) At the end of the lesson, ask students to bring data types which they can use in bar graph and histogram.

ACTIVITY SHEET

1) Draw rings like in the tool.



2) Answer the following questions:

a)Which occupation is most/least common among fathers?

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-If we change the place of columns, does the meaning of graph change ?

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-If we put one of rings another column, how does the graph be affected from that?

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3) Write the number of fathers for each occupation.

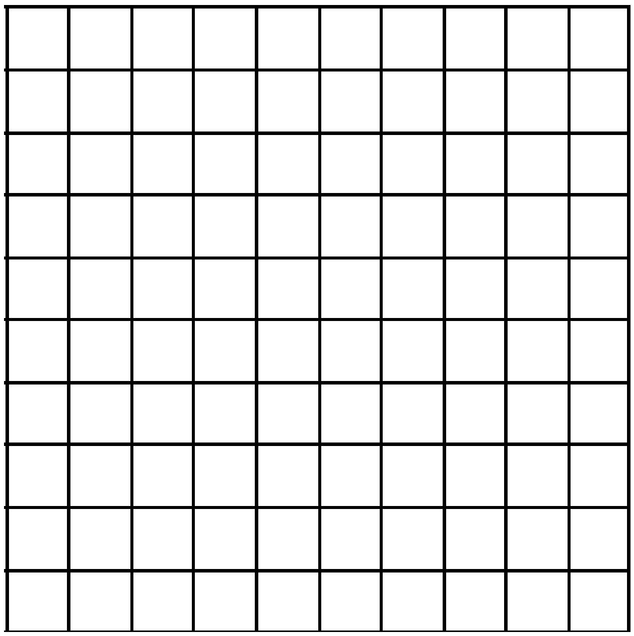
Occupations	# of fathers

4) Then, ask "How can you explain the relationship between this tool and the graphs you see in the sites?"

http://primaryschoolict.com/pictograph/

http://www.shodor.org/interactivate/activities/BarGraph/

5) Construct the graph by using different colored pencils. Then, answer the questions.



a) According to this graph, what can you say about the number of fathers who worked …. years.

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b) Can we find the max and min years like in the bar graph?

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c) Say other differences between bar graph and histogram.

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d) Which types of data are used in bar graph and histogram?

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http://primaryschoolict.com/pictograph/

http://www.shodor.org/interactivate/activities/BarGraph/

eğlenceli bir oyun [bar graph için çok uygun]

http://www.softschools.com/math/data_analysis/pictograph/games/

http://mrnussbaum.com/smartpoll2/

histogram için

http://illuminations.nctm.org/activity.aspx?id=3476

APPENDIX M – Third Draft of Lesson Plan of Group A

Topic: Histogram
Grade Level: 8
Duration: 80 min.
Objectives:

- Bir veri grubuna ilişkin histogram oluşturur ve yorumlar.
- Araştırma sorularına ilişkin verileri uygunluğuna göre daire grafiği, sıklıktablosu, sütun grafiği, çizgi grafiği veya histogramla gösterir ve bu gösterimler arasında dönüşümler yapar.

Lesson Plan Format

Steps	Main Learning Activities	Students' Anticipated Responses	Remarks on Teaching
1)Activating prior knowledge (individual work) (Duration: 20 min.) Direct instruction and questioning method.	Firstly, we will ask the problem 1a and we will remind the stem and leaf display without using its name. We will take attention to the relationship between the numbers in stem and leaf display and the numbers of toys. Then we will explain this relationship by giving the open form of stem and leaf display diagram.	Expecting that some students may remember the stem and leaf display without knowing the name of it. Furthermore, we will explain the stem and leaf display diagram by using problem 1 for students who did not remind it. Also, some students may have misconceptions	After taking attention to the stem and leaf display, we will ask them questions like that "Ok, what is the relationship between these numbers (in stem and leaf display) and the numbers of toys? "How many super markets are there?"

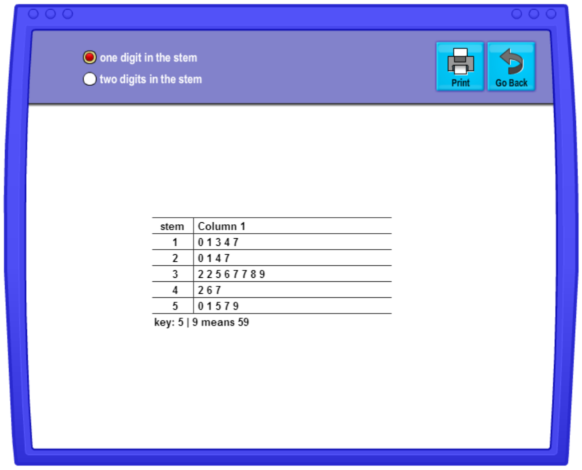
	(example of open form: 21 24 25 25 25 27 28)	that stem and leaf multiply with each other (2x1). Some students also may think that the numbers in the stem part shows the name of the super market. (In the first super market, there is no toy. Therefore, the numbers in the leaf part shows the number of toys.)	"What is the least number of toys? (and the most)" Furthermore, we will give a hint that stem part shows the tens and the leaf part shows the ones.
2)Connecting stem and leaf display knowledge to the histogram and giving information about histogram (individual work) (Duration: 20 min.) Direct instruction	Asking to form stem and leaf display diagram of the problem 1b by using website of NCTM. Then asking them to draw the diagram that they formed in website in their activity sheets. After that, we will rotate the stem and leaf display diagram by	Expecting to notice relationship between histogram and stem and leaf display. We will take attention to columns and by using columns they will reach the number of hats in each column.	"What do you think about this relationship?" "What kind of information would you place in the histogram?" "How can we determine the axes of histogram?"

	drawing it on the board. (They also will rotate diagram in their activity sheets.) And, we will construct histogram by covering the around of data with a line. We will indicate the histogram and we will give steps of drawing a histogram. (range=grup açıklığı,objective) Then, we will ask to use the histogram button in NCTM, so they will analyze the histogram.	Students may trouble with the determining name of the axes of histogram and range of it. Also, they will have difficulties to notice the meaning of the axes.	"What is the purpose of following steps of drawing histogram?"
3)Activating prior knowledge of bar graph and then comparing bar graph and histogram	We will activate the knowledge of bar graph by showing a bar graph. For this, we will use visuals (visual1). By using visuals, we will remind the properties of	Expecting that they may think both of the graphs look very similar. They may question the categories of information used in these graphs.	"Which types of movie are most liked, and which are least liked?" "Can we show the data of visual1 by using histogram?

Discussion and questioning method (think, pair, share) Duration: 10+30 min.	bar graph (type of data). (10 min.) After that, we will take attention to the histogram formed in problem 1b. Then, we will expect to notice and discuss the difference between histogram and bar graph.	(We will not mention the continuous and categorical data; we solely expect to realize the difference of data to form these graphs.) Also, they may confuse about determining range of data in histogram.	Why?" "What does the meaning of spaces between columns in bar graph?" "What are the differences between data of bar graph and histogram?"

Explanations / Definitions for Column Headings:

1. "Steps" Column: Give short, general descriptors for each segment of the lesson (e.g., group work, whole-class discussion, etc.)
2. "Main learning activities" column: Describe each segment in more detail, including items like: the problems students will be asked to solve, as well as the activities students are to do during each segment.
3. "Students' anticipated responses" column: Describe how you expect students to react to each of the main learning activities. What will they find easy? What will they find difficult? What will they find interesting? Etc.
4. "Remarks on teaching" column: Provide notes about teacher actions that must be carried out in order to help students succeed with each segment of the lesson. Include things like special instructions given to students, questions you plan to ask, or aspects of the lesson upon which you especially wish to focus students' attention.


one digit in the stem
two digits in the stem
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stem | Column 1
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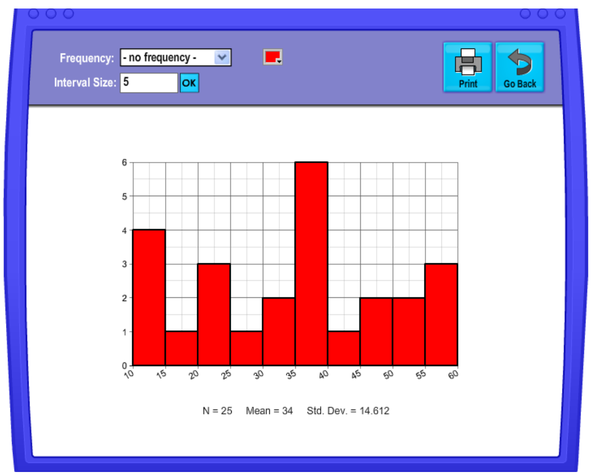
2 | 0 1 4 7
3 | 2 2 5 6 7 7 8 9
4 | 2 6 7
5 | 0 1 5 7 9
key: 5 | 9 means 59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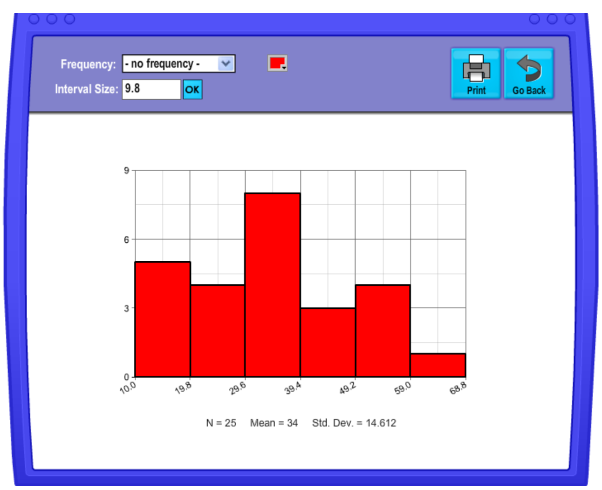
N = 25 Mean = 34 Std. Dev. = 14.612


Frequency: - no frequency -
Interval Size: 9.8
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Print
Go Back
N = 25 Mean = 34 Std. Dev. = 14.612

ACTIVITY SHEET

Problem 1: a) A super market purchase specialist formed a data display diagram to show the number of toys in each super market.

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2	1 4 5 5 5 7 8
3	0 0 4 6 6
4	0 2 9

b) A super market purchase specialist recorded the number of hats in below:

10 11 13 14 17 20 21 24 32 32 35 37 37 36 38 27 39 46

42 47 50 51 55 57 59

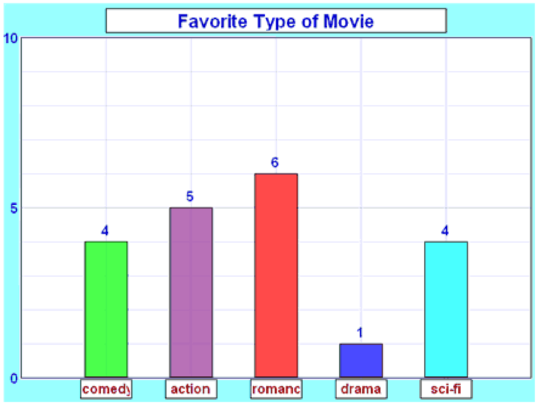
By using website of NCTM (http://illuminations.nctm.org/Activity.aspx?id=3476), can you help her to form a diagram of the numbers of hats in these super markets?

Visual1:

Imagine you just did a survey of your friends to find which kind of movie they liked best:

Table: Favorite Type of Movie				
Comedy	**Action**	**Romance**	**Drama**	**SciFi**
4	5	6	1	4

We can show that on a bar graph like this:



APPENDIX N – Third Draft of Lesson Plan of Group B

Grade Level: 8

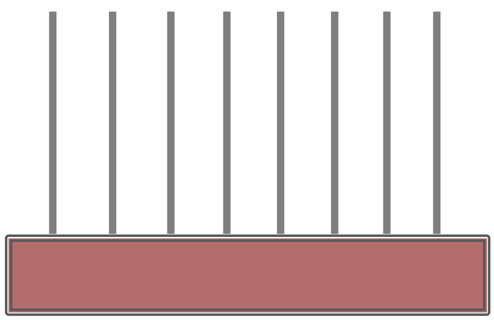
Duration: 80 min.

Objectives:

Show the data related to research questions in circle, bar, line or histogram when they are appropriate and make transformations among these graphs.

Steps:

13) Students are wanted to bring their data about occupations of their fathers and how many years they have worked in those occupations. Also, teacher will bring this tool to the class.



14) Before beginning the activity, ask students what they remember about the bar graph. Want them to give examples of some data which they can make bar graph with it.

15) Then, say students that they will put a ring to the column in the tool in which their fathers' job exists respectively. While they do this, distribute activity sheets to students.

16) Want from students to fill the table in the sheet. This table will help them while making bar graph.

17) Want from students to enter the sites and write their datas .

http://primaryschoolict.com/pictograph/

http://www.shodor.org/interactivate/activities/BarGraph/

18) Then, ask "How can you explain the relationship between this tool and the graphs you see in the sites?"

19) Then, ask the following questions:

-Which occupation is most/least common among fathers?

-If we change the place of columns, does the meaning of graph change?

-If we put one of rings another column, how does the graph be affected from that?

20) Let's think about second data you bring. We will use one of the graph we learned before. Which can be used for this data? (We expect the students to say histogram)

After students' answers, we will ask their reasoning. If students say we can make a bar graph, we ask what the column names will be in bar graph? Then ask;

- What benefits can we take from this graph?
- What information can we infer from it?
- After these questions, say students histogram can be more useful and beneficial.

21) Then, draw this table to the board:

1-5	6-10	11-15	16-20	21-25	25+

Want each student to paint a cell in which their data exist from bottom to top. After each student painted, histogram will emerge automatically. Students will also make the same procedures in their sheets.

22) Want from students to construct a histogram by using http://illuminations.nctm.org/activity.aspx?id=3476. Then, ask

- Looking the graph, can you estimate average years of fathers' working?
- Are years of fathers' working closed to each other? Do they show much difference? Can you find that difference approximately?
- If we change interval, how does histogram change?
- What can you say about the number of fathers who worked two years?

23) After histogram is formed, ask the following questions:

Can we find maximum and minimum years like in the bar graph?

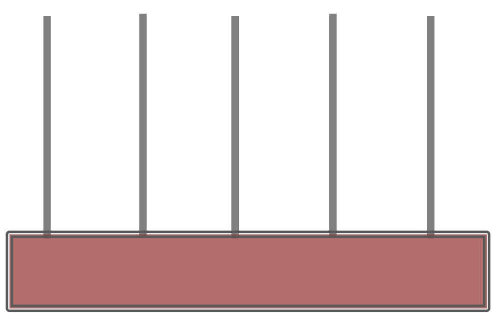
Which types of data are used in bar graph and histogram?

What other differences exist between bar graph and histogram?

24) At the end of the lesson, ask students to bring data types which they can use in bar graph and histogram.

ACTIVITY SHEET

1) Draw rings like in the tool.



2) Write the number of fathers for each occupation.

Occupations	# of fathers

3) Enter these sites respectively.

http://primaryschoolict.com/pictograph/

http://www.shodor.org/interactivate/activities/BarGraph/

4) How can you explain the relationship between this tool and the graphs you see in the sites?

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5) Answer the following questions:

a) Which occupation is most/least common among fathers?

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b) If we change the place of columns, does the meaning of graph change ?

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c) If we put one of rings another column, how does the graph be affected from that?

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6) Construct the graph by using different colored pencils. Then, answer the questions.

1-5	6-10	11-15	16-20	21-25	25+

http://illuminations.nctm.org/activity.aspx?id=3476

a) Looking the graph, can you estimate average years of fathers' working?

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b) Are years of fathers' working closed to each other? Do they show much difference? Can you find that difference approximately?

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c) If we change interval, how does histogram change?

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d) What can you say about the number of fathers who worked two years?

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7) Answer the followings.

a) Can we find the max and min years like in the bar graph?

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b) Say other differences between bar graph and histogram.

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c) Which types of data are used in bar graph and histogram?

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APPENDIX O – Turkish Summary of the Study

1 GİRİŞ

Eğitim teknolojilerinin ortaya çıkmasıyla beraber, öğretmen adaylarının teknolojiyi derslerine nasıl entegre edeceklerini öğretmek öğretmen eğitimcileri için önemli bir ilgi alanı haline gelmiştir. Öğretmen eğitimi programlarının ilk beklentileri öğretmen adaylarının derin ve geniş bir alan bilgisine sahip olması gerektiği idi (Niess, 2005). Sonra, bu beklentiler pedagojik alan bilgisi (PAB) kavramıyla etkilenmiştir (Shulman, 1987). PAB öğrencilerin öğrenmesi ile gerçekler, kavramlar, ilkeler arasında ilişkiler olduğunu önermektedir. Bu nedenle, öğretmen adaylarından öğretimlerini alan bilgisi ve öğrencilerin nasıl öğrendiklerine göre düzenlemeleri beklenmektedir. Ancak, öğretmen adayları şu anda teknolojik-PAB (Niess, 2005) olarak tanıtılan ve daha sonra Koehler ve Mishra (2008)'nın teknolojik pedagojik alan bilgisi (TPAB) olarak tanımladığı, bilgilerini geliştirmeleri gerekmektedir. Öğretimin *sorunlu* olarak karakterize edilmesiyle beraber (Rittel, & Weber, 1973), TPAB alan, pedagoji ve teknoloji bilgileri arasındaki ilişkilerin karmaşık doğası ile başa çıkmak için kullanılmaya başlandı. TPAB bu üç temel bilgi boyutunun arasındaki etkileşimleridir (Koehler & Mishra, 2008).

Bu çalışma, TPAB gelişimi için araştırma yollarının sınıflandırılması için Niess (2011)'in alan ve teknoloji tabanlı yaklaşım olarak önerdiği yaklaşımı kullanmaktadır. İşbirlikçi ders araştırmaları (Groth, Spiekler, Bergner, & Bardzell, 2009) bu yaklaşıma bir örnektir. Groth et al. (2009) öğretmenlerin bilgisinin değerlendirilmesi için birbiriyle zıt olarak iki paradigma önermiştir. Birincisi, temelde psikometrik yaklaşımla, matematik öğretmeni eğitimi programlarının etkilerini ölçmeye yönelik nicel çabalara dayanmaktadır. Diğeri ise nitel çabalara odaklanmaktadır ve temelde "öğretmenlerin bilgilerinin farklı yönlerini ve aralarındaki karmaşık ilişkileri açığa çıkaran *öğretmen uygulamaları*nın ve bunların onların öğretimiyle olan ilişkilerine bakması" fikrini kullanmaktadır (Simon, & Tzur, 1999, s. 263). Benzer bakış açısıyla, ders araştırmaları da profesyonel gelişim süreci olarak görülebilir (Stiegler, & Hiebert, 1999; Lewis, 2009).

Harris, Grandgenett ve Hofer (2010) TPAB değerlendirmesine uygun olan üç çeşit veri olduğunu iddia etti ve bu veri tipleri bir ders araştırması sürecinde toplanan verilerle iyi uyum sağlamaktadır. Bunlar "öz-rapor (röportajlar, anketler ya da diğer üretilen belgelerin, bu tür dönüşlü günlük girişleri olarak aracılığıyla), gözlenen davranış ve ders planı gibi öğretmen eserleridir. Çünkü öğretmenlerin bilgisi onların eylemlerine, cümlelerine ve eserlerine rahatlıkla yansır (s. 3834). Araştırmacılar, bu veri türleri arasındaki üçgenleme sürecini de açıklamışlardır. Bu nedenle, TPAB değerlendirmesi için kullanılan bu teknikler ve araçlar, öğretmenlerin TPAB'nin kapsamıyla, bilgi boyutlarının ayrımı ve belirlenmesini sistematik, geçerli ve güvenilir bir şekilde desteklemelidir. Sonuç olarak, veri türleri arasındaki üçgenleme, TPAB doğasını daha iyi anlamak için yardımcı olabilir (Harris ve diğerleri, 2010).

1.1 Matematik Eğitiminde Teknoloji Entegrasyonu

Geçmişte, öğretmen eğitimi sadece alan bilgisi üzerine odaklanmaktaydı ve alan bilgisine sahip olmanın konuyu etkili bir şekilde öğretmek için yeterli olduğu kabul edilmiştir (Doering, Veletsianos, Scharber, & Miller, 2009). Ancak son 20 yıldır, öğretmenlerin nasıl ve neden öğrettikleri de ne bildikleri kadar önemli görülmeye başlanmıştır (Shulman, 1987). Buna ek olarak, öğretmen eğitimi eğitimde teknoloji entegrasyonun devam eden gelişmeleri ile beraber teknolojik kaynaklar tarafından etkilenmiştir. Sonuç olarak, benzer bir şekilde, öğretmenlerin teknolojik kaynaklar aracılığıyla öğretimlerini nasıl geliştirmeleri gerektiği, onların teknoloji ve teknolojik kaynaklar hakkında ne bildiklerinden daha önemli kabul edilmiştir (Koehler & Mishra, 2009). Bu anlamda, matematik öğretmenlerinin öğretimlerini kendi teknoloji entegrasyonu becerilerini nasıl geliştirecekleri düşüncesi bu çalışmanın birincil amacı oldu.

Tarihsel olarak, öğretimde teknoloji kullanımı 80'lerin ve 90'ların PCK kavramının ortaya çıkmasıyla gerçekleşti. Matematik öğretmek konusunda öğretmenlerin inançları nasıl matematik öğrendikleriyle ilişkili olduğundan, teknoloji kullanımının ilk girişimleri onların pedagojik bilgilerini gösterim, doğrulama ve uygulama çeşitlerine indirgemiştir (Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, Özgün-Koca, & Kersaint, 2009). Bu nedenle, teknoloji uygulamalarının etkili örneklerinin eksikliği sebebiyle, "bilgisayar kullanımın sınırlılığı önümüzde yıllarda teknolojik sınırlamaların bir sonucu olmasından daha çok insan hayal gücünün sınırlarından ve eski alışkanlıkların ve sosyal yapıların sonucu olarak ortaya çıkmasının muhtemel olduğu" iddia edilmiştir (Kaput, 1992, s. 515). Sonra, geçmiş yıllar boyunca teknoloji entegrasyonu ile ilgili tartışmalar onun doğasının değiştirilmesiyle ilgili olarak şekillenmiştir. Öğretimde teknoloji entegrasyonunun etkinliği yıllardır tartışma konusu olmuştur ve nasıl yapıldığı, sonuçlarının öğretimde nasıl etkili olduğu ve matematik eğitimine entegrasyonun nasıl geliştirileceği ve etkinleştirileceği tartışılmaya başlanmıştır (Kaput, 1992). Earle (2002) bu iddianın temelini "teknoloji entegrasyonu teknoloji hakkında değil – alandaki ve öğretimdeki uygulamalarıyla ilgilidir, entegrasyon kullanılan teknolojinin miktarı ve çeşidiyle değil, nasıl ve neden kullanıldığıyla tanımlanmalıdır" diyerek açıklamıştır (s. 8).

1.2 Matematiksel ve İstatistiksel Düşünüşler arasındaki Fark ve İstatistik Eğitiminde Teknoloji Entegrasyonu

İstatistik ve matematiğin son yıllarda oldukça farklı konular olduğu kabul görmüştür. Liberal sanatlar merceğinden, Moore (1998) istatistiği yeniden tanımlamıştır:

> İstatistik veriler, varyasyon ve şans söz konusu olduğunda geçerli olan entelektüel bir genel yöntemdir. Veri, varyasyon ve şans modern hayatta her an var olduğu için temel bir yöntemdir. Kendi çekirdek fikirleri ile başlı başına, örneğin matematik dalından, bağımsız bir disiplindir (s. 1254).

Bu tanımı temel alarak, matematiksel ve istatistiksel düşünmenin farklı olduğu kesinlikle iddia edilebilir. Matematik bağlamı göz ardı ederken, istatistik zorunlu olarak bağlama dayalıdır. Sonuç olarak, matematik soyut örüntülere dayanır. Ancak özellikle veri analizinde istatistik, "bu örüntülerin ipliklerinin hikayenin tamamlayıcı iplikleriyle nasıl dokunduğuna" dayanmaktadır (Cobb, & Moore, 1997, s. 803). Buna bağlı olarak, istatistiğin anlam kazanması için bağlama ihtiyacı olduğu söylenebilir. Bu görüş temel alındığında, "istatistiksel düşünme bilgi kütlesini kırpma, saçma gelenle saçma olmayanı ayırma, düzensizliği düzenleme ve alakasız birçok şeyden alakalı olanı ayırmak için basit ama sezgisel olmayan zihinsel araçlar sunar" (Ben-Zvi, 2000, s. 129).

Matematiksel ve istatistiksel düşünme arasındaki farkın anlaşılması, eğitim teknolojisi kullanımını öne çıkararak istatistiği öğretmek için farklı bir yaklaşıma yol açar. İstatistik, geleneksel olarak hesaplamalara, formüllere ve izleklere odaklanarak öğretiliyor iken, "istatistik öğretimine verilen mevcut önem, istatistiksel muhakeme ve istatistiksel fikirleri, yorumlama, değerlendirme ve esnek bir şekilde uygulama yeteneklerine dayalıdır" (Ben-Zvi, 2000, s. 130).

1.3 Çalışmanın Amacı

Matematik öğretimi için gereken bilgi ve istatistik öğretiminde kullanılan farklı teknolojik kaynaklarla ilgili mevcut alanyazın göz önüne alındığında, mevcut çalışma temelde matematik öğretmen adaylarına sanal manipulatiflerin kullanımını bir mikro öğretim ders araştırması yoluyla öğretme amacını taşımaktadır. Katılımcılar matematik öğretmen adayları olduğu için uygulanan çalışma aslında bir mikro öğretim ders araştırmasıdır. Sonuç olarak, sanal manipulatifleri istatistik öğretiminde kullanmayı öğrenerek öğretmen adaylarının TPAB çerçevesinde ortaya çıkan bilgi boyutlarında gelişim göstereceklerі beklenmiştir.

Özel olarak belirtilmesi gerekirse, öğretmen adaylarının lisans eğitimlerinin dördüncü yılında sunulan bir derse entegre edilmek üzere, istatistik öğretiminde sanal manipulatifleri nasıl kullanacaklarını anlatan bir atölye çalışması tasarlanmıştır. Dahası, çalışmanın katılımcılarıyla iki mikro öğretim ders araştırması grubu oluşturulmuştur. Ders araştırması modelinin doğası gereği, öğretmen adaylarının TPAB çerçevesinde ortaya çıkan bilgi boyutlarında gelişim göstermeleri hedeflenmiştir. Özel olarak, bu gelişimin hangi bilgi boyutlarında ve nasıl gerçekleştiği incelenmiştir.

1.4 Araştırma Soruları

Kazanımlara ulaşmada yeni teknolojilerin muhtemel katkılarını ele alarak, Kaput (1992) sınıfta yeni bir teknoloji kullanımı kararının, "hali hazırda var olan fiziksel kaynaklardan çok, *karar verici*lerin vizyonu ve beklentileriyle sınırlı olduğunu" vurgulamaktadır. Katılımcıların öğretmen adayları olduğu gerçeğiyle, bu çalışma onların karar verme süreçlerini, vizyonlarını ve beklentilerini ders araştırması yöntemiyle incelemektedir. Çalışmanın ana araştırma soruları aşağıdaki gibidir:

a. Mikro öğretim ders araştırması başlangıcında, ilköğretim matematik öğretmeni adaylarının (a) istatistik öğretimine ve (b) istatistik öğretiminde teknoloji entegrasyonuna (sanal manipulatifler, simülasyonlar, vb.) bakış açıları nelerdir?

b. Mikro öğretimin ders araştırması başlangıcında, istatistik alanını ve istatistik öğretimini göz önüne alarak, ilköğretim matematik öğretmeni adaylarının alan bilgileri ve pedagojik alan bilgileri nedir?

c. Mikro öğretim ders araştırması aracılığıyla ilköğretim matematik öğretmeni adaylarının teknolojik pedagojik alan bilgileri (TPAB) nasıl gelişmiştir?

    i. Öğretmen adaylarının sanal manipulatifleri kullanarak istatistik öğretirken sahip oldukları TPAB'leri nedir?

ii. Mikro öğretim ders araştırması öğretmen adaylarının TPAB'lerine hangi ölçüde katkı sağlar?

Bu araştırma sorularına cevaben, mikro öğretim ders araştırması yoluyla katılımcıların profesyonel gelişimleri, grup-olay incelemesi dinamikleri kullanılarak incelenmiştir (Yin, 2014; Leavy, 2014; Fernandez, 2005). Bu nitel araştırma yaklaşımı, araştırmacıya matematik öğretmeni adaylarının TPAB gelişimlerini ve mikro öğretim ders araştırmasının sanal manipulatiflerin kullanarak istatistik öğretimlerinin iyileştirilmesi ile birlikte bu gelişimin ne ölçüde olduğunu inceleme açısından imkân sağlamıştır.

1.5 Çalışmanın Önemi

Eylül 2013 itibariyle uygulamaya geçirilen, revize edilmiş ortaokul matematik programımızda, önceki programa nazaran, istatistik en çok dikkat edilmiş alandır (MEB, 2013). İstatistik, veri işleme adıyla ayrı bir öğrenme alanı olarak 5. sınıf düzeyinden 8. sınıf düzeyine kadar işlenmek üzere ayrı bir öğrenme alanı olarak değerlendirilmiştir. Bununla birlikte, önceki programdan farklı olarak, olasılık konusunun yoğunluğu sadece temel olasılık kavramları ele alınarak azaltılmış ve onun öğretimi sadece 8. sınıf matematik programına dâhil edilmiştir. İstatistiki bakış açısıyla, Moore (1997, akt. Biehler, Ben-Zvi, Bakker, & Makar, 2012) *alan* (daha çok kavram ve veri analizi ve daha az olasılık), *pedagoji* (daha az konu anlatımı ve daha fazla aktif öğrenme) ve *teknoloji* (veri analizi ve simülasyonlar için) açısından bazı önerilerde bulunmuştur. Dolayısıyla, yeni matematik programı, daha çok istatistik ve daha az olasılık vurgulaması ve derin kavramsal öğrenme süreçlerini de lise düzeyine bırakmasıyla Moore'un önerilerinin iyi yansımış bir hali olarak betimlenebilir. Bu yaklaşımın öğretimde teknoloji entegrasyonuyla beraber bir sonucu olarak, TPAB matematik öğretmenlerinin gereksinimleri olan bütün bilgi boyutlarını sunmasıyla ortaya çıkmıştır (Niess ve diğerleri, 2009).

Önceki çalışmalar, matematik öğretmeni adaylarının matematik programındaki diğer öğrenme alanlarına kıyasla istatistik ve olasılık konularında daha az kavrayışa sahip olduklarını göstermektedir; şöyle ki, öğretmen adaylarının istatistik konularında yetersiz alan bilgisine sahip olmaları onların bu konuları öğretmeyi zor bulduklarına sebep olmaktadır (Quinn, 1997; Stohl, 2005). Günümüze ait çalışmalar, matematik öğretmen eğitimin onların istatistik öğretimine önem vererek aynı soruna işaret etmektedir (Stohl, 2005). Ponte, Oliveira and Varandas (2002) "öğretmen adaylarının kelime işleme, elektronik tablolama, istatistik yazılımları, elektronik posta, konuya özgü eğitim yazılımları ve internet gibi uygulamalarla ilgili yeteneklerinin bilgi, araştırma ve üretim açısından gelişmesi gerektiğini" iddia etmişledir (s. 95). Araştırmacılar özellikle öğretmen adaylarının istatistik öğretimiyle ilgili teknolojik bilgilerinin, internet araştırması ve elektronik posta hakkındaki bilgilerinin yetersizliğini vurgulamışlardır.

İnternetin evrenselliği ve erişilebilirliği sayesinde, Ben-Zvi (2000)'nin istatistik öğretiminde kullanılabilecek teknolojik kaynaklardan biri olarak kullanılmak üzere sunduğu sanal manipulatifler, öğretmen adaylarının teknoloji entegre ederek istatistik öğretimlerini geliştirmeye yönelik olarak bu çalışmaya dahil edilmiştir. Alanyazında, öğretmen adaylarının sanal manipulatiflerle istatistik öğretiminde teknoloji entegrasyonlarını geliştiren özel bir çalışma yoktur. Sanal manipulatifler, çoğunlukla kesirler, örüntüler ve geometri konularında çalışılmıştır (Moyer-Packenham, Salkind & Bolyard, 2008). Örneğin, öğretmenlerin ya da öğretmen adaylarının istatistik konularında sanal manipulatif kullanımlarını veya onların istatistiki ve matematiksel düşünme biçimlerini ve kavramalarını inceleyen özel bir çalışma bulunmamaktadır.

Sınıf uygulamalarıyla profesyonel gelişim çabalarının etkili öğretmen eğitimini sağladığı bilinmesine rağmen, öğretmenlerin matematik öğretmesi için gereken bilgilerin geliştirilmesi ve onların matematiksel düşünüşlerinin olduğu kadar istatistiksel düşünüşlerinin de iyileştirilmesi için etkili bir öğretmen eğitiminin nasıl tasarlanacağı ve hangi görevleri içereceği kesin değildir (Fernandez, 2002). Bu nedenle, ders araştırması yukarıdaki sorulara cevaben bir profesyonel gelişim yöntemi olarak vurgulanmıştır

(Fernandez, 2002). Bu çalışma, ders araştırması yöntemiyle matematik öğretmeni adaylarına özel bir profesyonel gelişim yolu sunması açısından önemlidir.

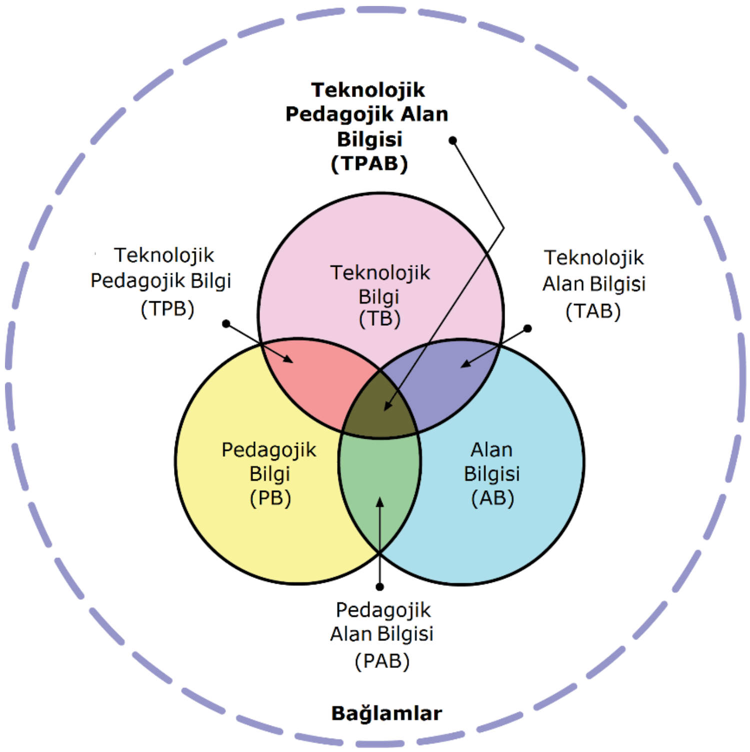
Sonuç olarak, bu çalışmanın, öğretim yöntemleri dersleri öncelikli olarak, matematik öğretmenliği eğitimi programlarında olumlu etkilerinin olacağı beklenmektedir. Okul matematiği programında önemli bir yer edinerek, istatistik ve istatistiksel muhakemenin öğretmen eğitimi programlarında yeniden ele alınmasını ve gözden geçirilmesi gerekecektir. Birçok öğretmenin önceki tecrübelerinin sadece betimsel istatistik alanında olduğu göz önüne alınırsa, istatistik öğretimine teknolojik gelişmelerinin de yardımıyla için güncel eğilimlerle yaklaşılmalıdır (Pfannkuch, & Ben-Zvi, 2011). Bu nedenle, matematik öğretmenliği programları istatistiğin doğası, rolü ve amacı açısından aday öğretmenlerin yararlanabileceği fırsatlar sunarak yeniden tasarlanmalıdır. Bu çalışmada yürütülen ders araştırması da, öğretmen adaylarına istatistik öğretimini öğrenme tecrübeleri sunması ve bu yönde onları geliştirmesi açısından önemlidir.

2 ALANYAZIN ÇALIŞMALARININ İNCELENMESİ

2.1 TPAB Çerçevesi ve TPAB Bilgi Boyutları

Koehler ve Mishra (2009) tarafından, farklı eğitim ortamlarında "öğretmenlerin ne bildiğiyle, bildiklerini nasıl uyguladıkları arasındaki etkileşim" olarak tanımladıkları öğretimde teknolojinin entegrasyonu yaklaşımı bu çalışmanın temelinde yatan fikirdir (s. 62). Teknolojik pedagojik alan bilgisini (TPAB) tanımlamak için, teknoloji, pedagoji ve alan bilgisinin iyi bir öğretimin kalbinde yatan bilgiler olduğunu varsaymışlardır ve bu üç öz bilgi boyutu TPAB çerçevesini oluşturur (Koehler & Mishra, 2009, Niess, 2005; 2011, Zhao, 2003). Daha özel bir ifadeyle, TPAB teknoloji, pedagoji ve alan bilgilerinin arasındaki kesişimler ve bağıntılar olarak kabul edilmiştir (Margerum-Leys & Marx, 2004; Mishra & Koehler, 2006; Niess, 2005; Pierson, 2001; Zhao, 2003 akt. Niess, 2011). Moore (1997) alan bilgisi, teknoloji ve pedagoji arasında bir sinerji olduğunu önermiştir. Araştırmacı aynı zamanda, "en etkili öğretmenlerin sadece alan bilgisi ve uygulamadaki tecrübeleriyle değil teknoloji ve pedagoji alanında da oldukça zengin bilgilerinin olması gerektiğini" ileri sürmüştür (Moore, 1997, s. 134).

Mishra ve Koehler (2006)'in önerdiği TPAB çerçevesi ve içerdiği yedi bilgi boyutu aralarındaki ilişkilerle birlikte aşağıdaki Şekil 2.1'de gösterilmiştir. Bu yaklaşımla araştırmacılar teknolojik pedagojik bilgi (TPB) ve teknolojik alan bilgisi (TAB) olmak üzere iki yeni bilgi boyutunu da tanımlamışlardır.



Şekil 2.1 TPAB Çerçevesi ve içerdiği bilgi boyutları (Koehler & Mishra, 2009, s. 63)

Alan bilgisi (A ya da AB) öğretmenlerin öğretmek ya da öğrenmek için kazanmaları gereken konu bilgisidir. Matematik öğretimi için gereken bilgi örneğin fizik öğretimi için gerekli olan bilgiden farklıdır ve bunu ortaokulda öğretmek için gereken bilgi üniversite düzeyinde öğretmek için gereken bilgiden de farklıdır. Alan bilgisi, esasen, kavramları, teorileri, fikirleri, örgütsel çerçeveleri, bulgu ve kanıt bilgilerini olduğu kadar bunların gelişimine yönelik yaklaşımlarda var olan uygulamaları içeren bilgi boyutudur (Shulman, 1986, akt. Koehler & Mishra, 2009).

Pedagojik bilgi (P ya da PB), "öğretim ve öğrenimin yöntemleri ve uygulamaları ve süreçleri ve bunların bütün eğitimsel amaçlar, değerler ve hedeflerle beraber neyi kapsadığı hakkındaki derin bilgidir" (Mishra & Koehler, 2006, p. 1026). PB öğrencilerin hangi düzeylerde nasıl öğrendiklerini, sınıf yönetiminin nasıl yapılacağını, ders planının nasıl hazırlanacağını ve ders değerlendirmesinin nasıl yapılacağını içerir. Derin bir PB, öğrencilerin nasıl öğrendiğine ve nasıl bilgi ağları oluşturduklarına yönelik bilgiye işaret eder (Koehler & Mishra, 2009).

Pedagojik alan bilgisi (PAB) hangi öğretim yöntem ve tekniklerinin hangi konuda seçileceği kararının verilebilmesi ve iyi bir öğretim ögelerinin yer değiştirilebilmesi becerilerini gerektirir (Mishra & Koehler, 2006) ve bu anlamda Shulman (1986)'ın tanımına uymaktadır. Dahası, PAB yaygın kavram yanılgılarını ve bunları önlemek için gereken stratejileri, öğrencilerin geçmiş bilgilerinin farkında olarak alternatif öğretim yöntemlerini bilmeyi ve kullanmayı içerir. Kısaca, "PAB, kavramların sunumları, pedagojik teknikler, bir kavramı öğrenmeyi zorlaştıran veya kolaylaştıran şeyleri bilme ve öğrencilerin geçmiş bilgilerini bilme ve bilgi-bilim teorileriyle ilişkilendirilmiştir" (Mishra & Koehler, 2006, p. 1027).

Hızla değişen yapısıyla, teknolojik bilgiyi (T ya da TB) tanımlamak zorlaşmaktadır. Geçersiz olmasını engelleyecek bir biçimde, TB hakkında açıklama Ulusal Araştırma Konseyi tarafından FITness (Bilgi teknolojilerinin akışı)'nın tanımı kullanılarak yapılabilir (NRC, 1999, akt. Koehler & Mishra, 2009). Özel olarak, araştırmacılar FITness'ı bilgisayar okuryazarlığından daha çok daha derin bir bilgi teknolojisi anlayışının gerektirdiğini savunurlar. Bu nedenle, bu bakış açısıyla, TB herhangi bir bireyin farklı karmaşık görevlerde teknoloji kullanımını yönetebilme becerisini geliştirir.

Teknolojik alan bilgisi (TAB) ilişkili olduğu alanı daha iyi anlamak için kullanılması açısında yıllar öncesine dayanır. Örneğin, tıpta (x ışınlarının kullanımı), arkeolojide (tarihleme için Karbon-14 metodu) veya fizikte kullanılagelmiştir. Bu nedende, teknoloji ve alan bilgisi eğitim için de ilişkilendirilebilir ve tanımı "teknoloji ve alan bilgisinin birbirini ne yönde etkilediği ve sınırladığının anlaşılması" olarak yapılabilir (Koehler & Mishra, 2009, p. 63). Derin bir TAB'ye sahip olarak, bir öğretmen konuyu farklı teknolojileri uygulayarak öğretebilmeli ve konuya en uygun teknolojiyi seçebilme ya da bunun tersi becerisine sahip olmalıdır (Koehler & Mishra, 2009).

Teknolojik pedagojik bilgi (TPB) farklı öğretim ve öğrenme bağlamlarında teknolojinin nasıl uyumlaştırılacağını anlamadır. Bu nedenle, bir öğretmen, pedagojik stratejiler ve yaklaşımların sunduğu fırsatları ve yaptığı sınırlamalarının farkında olarak geniş bir teknolojik araçlar listesinden farklı öğretim ve öğrenme bağlamlarında teknolojiyi nasıl kullanacağını bilmelidir (Koehler & Mishra, 2009). Örneğin, TPB öğretmenin günlük işleri olan notlandırma, yoklama alma, sınıf kayıtları için kullanmasını, çevrimiçi yazışma odaları, bloglar, sanal manipulatifler, tartışma tahtaları veya forumlar gibi teknolojik olarak pratik fikirleri özel öğretim amaçları için kullanabilme bilgisini içerir (Mishra & Koehler, 2006).

Teknolojik pedagojik alan bilgisi (TPAB) teknoloji, pedagoji ve alan bilgisini birleştiren yeni bir bilgi boyutudur ve TPAB'nin üç öz bilgi boyutunu temsil eder. Bu sayede, TPAB, "teknoloji kullanarak kavramların sunumlarının anlaşılmasını gerektiren etkili öğrenmenin, konuyu pedagojik yaklaşımlarla yapılandırıcı biçimlerde öğretmenin, öğrenilecek konuyu neyin zor veya kolay yaptığını ve öğrencilerin karşılaştığı sorunları teknolojiyle çözmeyi bilmenin, öğrencilerin geçmiş bilgilerinin farkında olmayı bilmeyi ve eski bilgileri güçlendirerek teknoloji sayesinde yeni bilgileri inşa etmenin nasıl olacağını bilmenin temeli" olarak tanımlanır (Mishra & Koehler, 2006, p. 66).

Hofer ve Grandgenett (2012) TPAB'nin tanıtıldığı günden beri alanyazında popüler olmasına dikkat çekerek başlıca çalışmaları aşağıdaki gibi sıralamıştır:

> "TPAB ile ilk çalışmalar onun yapıtaşlarını anlamaya (örneğin, Archambault & Barnett, 2010; Koehler & Mishra, 2009; Mishra, Koehler, & Henriksen, 2011), TPAB'nin öğretim planlamasını nasıl bir işleme tabi tuttuğuna (örneğin, Harris, Mishra, & Koehler, 2009; Mouza & Wong, 2009) ve uygulamasına (örneğin, Cox & Graham, 2009; Hofer & Swan, 2008) odaklıydı. Bugünlerde, araştırmacılar öğretmenlerin ve öğretmen adaylarının TPAB'lerinin geliştirilmesine yönelik özel yaklaşımlara (örneğin, Cavin, 2008; C. R. Graham et al., 2009) ve farklı yollarla TPAB'nin değerlendirilmesi için testlerin geliştirilmesi, bunların geçerliliği ve uygulanmasına (örneğin, Hofer & Harris, 2010; Schmidt et al., 2009) odaklandılar." (s. 86).

2.2 İstatistiki Bilgi ve İstatistik Eğitimi

İstatistik ve matematiğin farklı disiplinler olarak ele alınmasıyla birlikte, istatistik öğretimi de matematik öğretiminden farklı olarak önemle ele alınabilir. Groth (2007) Moore (1988)'un istatistik için verdiği tanımı geliştirdi ve aslında istatistik matematiği *kullandığı* için, istatistik öğretmek için gereken bilgilerin matematik öğretmek için gereken bilgilerle yapısal olarak benzediğini vurgulamıştır. Dolayısıyla, araştırmacı, istatistik öğretmek için gereken bilgileri belirtmek için başlangıç noktası olarak matematik öğretmek için gereken bilgileri (Hill, Ball & Schilling, 2008) kullanmıştır. Groth (2007) İstatistik Öğretimi ve Değerlendirmesi için Kuralları (Franklin, ve diğerleri, 2007) çerçevesini kullanmıştır ve istatistik ve matematik öğretimini birbirinden ayırarak bunları Hill ve diğerlerinin (2008) yaygın alan bilgisi ve özelleştirilmiş alan bilgisi kavramlarını kullanarak açıklamıştır.

2.2.1 Teknolojik Pedagojik İstatistik Bilgisi (TPİB)

TPAB'de olduğu gibi farklı bilgi boyutlarını ve bunların kesişimi ile oluşan bilgi boyutlarını belirtmek yerine, Lee ve Hollebrands (2011) teknolojik pedagojik istatistik bilgisi (TPİB) çerçevesini içiçe daireler olarak tanımlamıştır. En dıştaki daire bir öğretmenin istatistiksel düşünebilmesi için gereken istatistiki bilgisini gösterir. Araştırmacılar, bir öğretmenin istatistik öğretirken pedagoji ve teknolojiyle ilgilenmeden önce ilk olarak istatistik bilgisine ve istatistiksel düşünme becerisine sahip olmaları gerektiğini iddia etmektedirler. Dolayısıyla, en içteki daire de TPİB olarak belirtilmiş ve "dıştaki teknolojik istatistik bilgisi (TİB) ve istatistik bilgisi (İB) dairelerinin içinde alt kümesi olarak geliştirilmiştir" (Lee & Hollebrands, 2011, s. 361).

Birçok araştırmacıya göre, istatistiksel düşünme matematiksel düşünmeden farklı bir süreç gerektirir (delMas, 2004; Pfannkuch & Ben-Zvi, 2011; Rossman, et al., 2006, akt. Lee & Hollebrands, 2011). TPİB çerçevesinden yaklaşarak, istatistiksel düşünebilmeleri için öğretmenler, durumları incelemek için kendi kişisel tecrübelerine ya da hikâye tarzındaki bulgulara değil düzgün bir şekilde toplanmış verileri analiz edebilme ve karar verebilme becerisine sahip olmalıdır. Öğretmenler, aynı zamanda gerçek bir sistemle istatistiki bir sistem arasındaki durumların geçiş sürecini anlamak için karşı-numaralama (transnumeration) (Wild & Pfannkuch, 1999) konusunda beceriye sahip olmalıdır (Pfannkuch & Wild, 2004). Dolayısıyla, öğretmenler ölçütleri toplayabilme, grafiklerle ve istatistiksel ölçümlerle anlamlandırma ve kendi yorumlarını bağlam içinde değerlendirebilmelidir" (Lee & Hollebrands, 2011, s. 362).

TPAB bakış açısıyla, TPİB uyum göstermektedir ve TPAB'nin ve TPİB'nin temelinde benzer eğilimler bulunmaktadır (Lee & Hollebrands, 2011). Alan bilgisi yerine istatistik bilgisi; TAB yerine ise TİB kullanılmıştır. TPİB çerçevesi her yönden pedagoji her bir bilgi boyutunda bulunduğu için, araştırmacılar, özel olarak pedagojik bilgiden ayrıca bahsetmemişlerdir (Lee & Hollebrands, 2011, s. 361). TPAB herhangi özel bir konu için teknolojik pedagojik alan bilgisi olarak tasarlandığı için (Koehler & Mishra, 2005; Niess, 2006), TPAB bu durumda TPİB'ye dönüşür. Dolayısıyla, TPAB çerçevesiyle uyum içindedir.

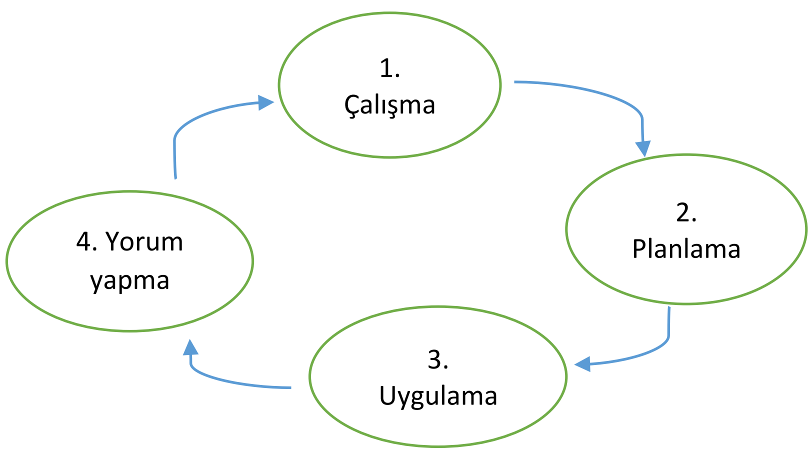
2.2.2 Sanal Manipulatifler

Sanal manipulatifler, "matematiksel bilgiyi inşa etme fırsatları sunarak bir dinamik nesnenin etkileşimli, internet bazlı görsel sunumları" olarak tanımlanmıştır (Moyer, Bolyard & Spikell, 2002, s. 373). Dahası, araştırmacılar, " örüntü blokları, onluk taban blokları, geometrik şekiller, tangram, geometri tahtaları gibi fiziksel manipulatiflerin dinamik görsel/resimsel kopyaları" olduklarını" belirtmişlerdir (Moyer-Packenham, et al., 2008, s. 2). Ek olarak, sanal manipulatiflerin özellikleri aşağıdaki gibi aktarılmıştır: "internetten erişilebilen applet, ya da bağımsız uygulama programlarıdır. Kullanıcılar, bilgisayar faresini kullanarak dinamik görsek nesneyi hareket ettirebilir. Sanal manipulatifleri kullanma becerisi matematiği etkileşimli öğretebilme konusunda yarar sağlar" (Moyer-Packenham, et al., 2008, s. 2). Çalışma boyunca, okuyucu sanal manipulatiflerin bazı örnekleriyle karşılaşacaktır, bunlar: NCTM'in illuminations Web sitesinde bulunan JAVA gösterimleri, Shodor Eğitim Vakfı'nın Program materyalleri Web sitesi ve benzeri.

2.2.3 Ders Araştırması ve Mikro Öğretim Ders Araştırması (MÖDA)

Menşei, Japon öğretmen gelişimini olan ders araştırması (Japonca'da *jugyou kenkyuu*) öğretmenlerin işbirliği içinde çalıştıkları ve sürekli olarak araştırma derslerine (Japonca'da *kenkyuu jugyou)* yorum yaptıkları gelişim sürecidir. Araştırma dersleri, öğretmenlerin kendilerinin hazırladığı, planladığı, tartıştığı ve revize ettikleri derslerdir (Lewis, 2000). Ders araştırması 1999 yılında dünya çapında ilgi çekti ve "öğretim ve öğrenimi geliştirmek ve profesyonel bir bilgi temeli inşa etmesi" açısından etkili bir yöntem olarak sunuldu (Stigler and Hiebert 1999; Yoshida 1999, akt. Lewis, 2009, p. 95)

Bir ders araştırması süreci aşağıdaki Şekil 2.2'de görüldüğü gibi döngüsel biçimde 4 adımdan oluşur:



Şekil 2.2 Ders araştırması döngüsü

Geleneksel olarak, ders araştırması hizmet-içi öğretmenlerle yürütülmesine rağmen, öğretmen adaylarının bağlamlarında da kullanılagelmiştir (Murata and Pothen, 2011; Rock, 2003, akt. Leavy, 2014). Fernández (2005) bu tip ders araştırmasını mikro öğretim ders araştırması (MÖDA) olarak tanımlamıştır ve uygulamanın mikro öğretim biçiminde öğretmen adayları tarafından yapıldığını açıklamıştır. MÖDA, ders planı hazırlama, revize etme ve uygulama yaparken öğretmen adayları arasındaki işbirliğini geliştirmesi açısından genel ders araştırmasından farklıdır.

MÖDA çalışmaları, ilk olarak öğretmen adaylarının PAB'lerini incelemek üzere yürütülmüştür (Fernández, 2005; Fernandez & Robinson, 2007, akt. Cavin, 2007). Cavin (2007) MÖDA uygulamasını öğretmen adaylarının TPAB gelişimlerini değerlendirmek ve onları teknoloji entegre edildiğinde öğrenci merkezli öğrenme ortamlarında ders planı hazırlama etkinliklerinde bilinçlendirmek için yürütmüştür. Bununla birlikte, öğretmen adaylarının istatistik öğretimine özel TPAB'lerinin incelenmesini önemseyen herhangi bir çalışma bugüne kadar yapılmamıştır. Bu da bu çalışmanın ana odak noktasıdır.

3 ÇALIŞMANIN YÖNTEMİ

3.1 Çalışmanın Deseni: Durum Çalışması

Bu çalışmanın deseni durum çalışması yaklaşımının dinamiklerini temel almaktadır. Yin (2014) durum çalışmasını "özellikle görüngü ve bağlamın sınırlarının açıkça ifade edilemediği durumlarda, çağdaş bir görüngüyü (durumu) kendi gerçek bağlamında derinlemesine inceleyen, gözleme dayalı araştırma" biçimi olarak tanımlamıştır (s. 16). Bu çalışma desenin ana bileşenleri Yin (2014)'in belirttiği biçimde bu çalışmada da görülmektedir: inceleme birimi – durum –, veri ile önermeleri ilişkilendiren mantık; ve bulguları yorumlama kriterleri. Her bir ders araştırması grubu bir durum olarak ele alınmıştır ve iki ders araştırma grubu olduğu için açıklayıcı gömülü çoklu-durum deseni olarak değerlendirilebilir. Bütünsel bir desen olarak düşünülemez çünkü bu çalışmada inceleme birimi test uygulanması, görüşmeler yapılması ve grup tartışmaları uygulanması ile çeşitlilik göstermektedir (Yin, 2014).

3.2 Mikro Öğretim Ders Araştırma (MÖDA) Grupları

MÖDA'nın en önemli hedefi öğretmen adaylarının istatistik kavramlarının öğretiminde teknolojiyi entegre etmesine yardımcı olmasıdır. İlk olarak, gruplara MÖDA sürecinde ve ders araştırması sonunda nasıl eserler (ders planları) üretecekleri hakkında bilgi verildi. Katılımcılara ayrıca istatistikle ile ilgili okul programından bir kazanım seçmek ve ona göre kendi ders planlarını tasarlayacakları söylendi. Atölye (TB-İÖA) yoluyla istatistik öğretiminde kullanılabilir sanal manipulatifler tanıtıldı, sonra, her grup kendi ders planını tasarlamaya başladı. Ardından, araştırmacı ile gruplar ayrı ayrı bir araya gelerek bu ders planları boyunca karar verme süreçleri hakkında grup tartışmaları yapıldı. Bu, her grup için ilk grup tartışması oldu.

Kendi kararlarına ve araştırmacının geri bildirimlerine göre, ders planlarını revize etmeye karar verdiler ve üzerinde çalıştılar. Ardından, araştırmacı ve her grup tartışmak ve ders planının ikinci versiyonu hakkında eleştiri yapmak için bir araya geldi. Bu ikinci grup tartışması oldu. Tartışma notları ve geri bildirimler doğrultusunda, ders planının uygulanmasından önce ikinci kez kendi ders planları üzerinde revizyon yapmaya karar verdiler.

Ders planlarını revize ettikten sonra, yaklaşık bir saat süreyle kendi sınıf arkadaşları olan bir grup öğrenciye uyguladılar. Uygulama küçük ölçekli bir ders oldu. Başka bir deyişle, her iki grubun iki-ders saati olarak planladıkları ders planları, bir ders saatine sıkıştırılmış bir ders gibiydi. Araştırmacı daha sonra onların yorum ve eleştirilerini almak amacıyla her uygulamayı video olarak kaydetti.

Üçüncü grup tartışmaları boyunca, araştırmacı ve her grubun üyeleri son olarak bir araya geldiler, onlar video kaydı alınmış ders uygulamasını izlediler ve dersin akışını tartıştılar. Her grup öğrencilerin soruları, etkinlikler, kendi eylemleri, soruları, yanıtları hakkında özeleştiri yaptı. Üçüncü grup tartışmaları sonunda, her gruptan kendi ders planlarını bir kez daha revize etmek ya da bu revizyon doğrultusunda yeniden uygulamak isteyip istemedikleri hakkında bir karara varmaları istendi. Her iki grup da ders planları sadece küçük revizyonlar gerektiği sonucuna varmıştır, dolayısıyla yeniden uygulama ihtiyacı görmediler ve dolayısıyla bu, ders araştırması sürecinin sonuydu. Kısaca, ders araştırması, yalnızca bir döngüde tamamlanmış oldu.

3.3 Katılımcılar ve Gruplar

Ankara'da bir üniversitede, ilköğretim matematik öğretmenliği programının 4. sınıfında öğrenim gören 9 öğretmen adayı bu çalışmaya gönüllü olarak katılmıştır. Katılımcıların ikisi erkek, yedisi ise kadındır. Katılımcıların yaş aralığı 22 ila 24'tür. Katılımcılardan 8'i lise öğrenimini bir Anadolu öğretmen lisesinde tamamlamıştır, diğeri yurtdışında tamamlamış yabancı uyruklu bir öğrencidir. Katılımcılardan 5'i matematik öğretmeni olmayı özellikle istemiştir, diğerleri Öğrenci Seçme Sınavı'nda aldıkları sonuca göre ilk tercihlerinin bu olmadığını belirtmişlerdir.

MÖDA boyunca bütün katılımcıların gönüllü olarak istedikleri şekilde iki grup oluşturmaları istendi, kendi aralarındaki arkadaşlık ilişkileri ve grup tartışmaları haricinde beraber çalışabilme olanaklarını da değerlendirerek iki grup oluşturdular. Atölyenin başlamasıyla beraber kendi gruplarında çalışmaya başladılar.

3.4 Veri Toplama Süreci ve Araçları

Veri toplama süreci katılımcılarla bireysel görüşme yapılarak başlamıştır. Görüşme sonunda, her katılımcıya görüşme sonunda İstatistikte Ayrıştırıcı Öğretmen Değerlendirmesi (İAÖD) testi uygulanmıştır. Bütün katılımcılar, 4 hafta boyunca Teknoloji Bazlı-İstatistik Öğretimi Atölyesi (TB-İÖA)'ne katılmışlardır. Bu atölye onların 4. Sınıfta aldıkları bir ders olan 'öğretim için gereken matematiksel bilginin doğası' dersine entegre edilmiştir. Bu atölye bir veri toplama amacı olmamasına rağmen MÖDA sürecinin başlangıcı, ders planlarını hazırlamaya başladıkları hafta olan atölyenin son haftasında gerçekleşmiştir ve video kaydı alınmıştır. Dolayısıyla, bu çalışmada 3 adımda veri toplanmıştır: görüşmeler, İAÖD testi ve MÖDA. Aşağıdaki Tablo 3.1'de bu süreç ayrıntılı olarak görülebilir.

3.5 Görüşme ve İstatistikte Ayrıştırıcı Öğretmen Değerlendirmesi (İAÖD)

Bütün katılımcılar yaklaşık 50-60 dakika süren birebir ve yarı-yapılandırılmış bir görüşmeye (Appendix A'da görülebilir) katıldılar. Görüşme boyunca, onların geçmiş eğitim hayatları, istatistik alanına, istatistik öğretimine ve istatistik öğretiminde teknoloji kullanımına ilişkin bakış açıları hakkında sorular soruldu. Görüşme sonunda her katılımcıya İAÖD uygulandı, bu test Bush, Ronau, McGatha ve Thompson (2002)'ın izinleri ile alınan ve matematik öğretmenlerinin istatistik ve olasılık konularındaki alan ve pedagojik alan bilgilerini ölçmeye yönelik bir testtir. Ortaokul matematik programındaki bütün istatistik kavramlarını ve konuşlarını ele alan bu testin içerdiği sorulardan 5 tanesi açık uçlu olmak üzere, diğerleri çoktan seçmeli sorulardır. Testin son sorusu, Jacobbe (2007)'nin doktora tezinden alınmış ve diğer bütün soruların ilgili olduğu konuları gözden geçirmesi sebebiyle teste eklenmiştir.

3.6 Teknoloji Bazlı-İstatistik Öğretimi Atölyesi (TB-İÖA)

Görüşme ve testlerin uygulanmasından sonra, katılımcılar ve diğer öğrenciler, TB-İÖA'nın entegre edildiği derste atölyeye başlanmıştır. Atölye, toplamda 12 saatlik süreyle 4 haftada tamamlanmıştır. Atölye boyunca, toplam yedi etkinlikle ortaokul matematik programında ele alınan bütün istatistik konularına değinilmiştir.

Tablo 3.1 Veri Toplama Süreci ve Zamanlaması

Veri Toplama Süreci ve Zamanlaması				
		A Grubu	B Grubu	Tarih ve Süre
Öğretmen Sayısı		5	4	
Görüşme		Bütün katılımcılarla atölyeden önce görüşme yapıldı.	Bütün katılımcılarla atölyeden önce görüşme yapıldı.	27 –28 Ekim, 2014. Görüşme yaklaşık bir saat sürdü, ses kaydı alındı, kodlandı ve analiz edildi.
Test: İstatistikte Ayrıştırıcı Öğretmen Değerlendirmesi (İAÖD) testi		Görüşmenin bitiminde her katılımcıya uygulandı.	Görüşmenin bitiminde her katılımcıya uygulandı.	Analiz edildi.
Atölye: Teknoloji Bazlı-İstatistik Öğretimi Atölyesi (TB-İÖA)		Bütün katılımcılar sınıfla beraber atölyeye katıldı.	Bütün katılımcılar sınıfla beraber atölyeye katıldı.	21 Ekim – 11 Kasım 2014. Atölye 4 hafta sürdü, video ve ses kaydı yapıldı.
Mikro Öğretim Ders Araştırması	Ders planları	Atölyenin son haftasında 1 ders planı hazırlandı.	Atölyenin son haftasında 1 ders planı hazırlandı.	Analiz edildi.
	İlk Grup Tartışması	Ders planının birinci versiyonu tartışıldı.	Ders planının birinci versiyonu tartışıldı.	22 -27 Kasım 2014. Ses kaydı alındı, birebir yazıya geçirildi, kodlandı ve analiz edildi.
	İkinci Grup Tartışması	Ders planının ikinci versiyonu tartışıldı.	Ders planının ikinci versiyonu tartışıldı.	28 Kasım – 9 Aralık 2014. Ses kaydı alındı, birebir yazıya geçirildi, kodlandı ve analiz edildi.
	Ders planının uygulanması	Mikro öğretim yoluyla, ders planının 3. Versiyonunu uyguladılar.	Mikro öğretim yoluyla, ders planının 3. Versiyonunu uyguladılar.	9 – 16 Aralık 2014. Video kaydı yapıldı.
	Üçüncü Grup Tartışması	Uygulamaları kritik edildi, yeni bir revizyona ve yeniden uygulamaya ihtiyacın olup olmadığı konusunda karar verdiler.	Uygulamaları kritik edildi, yeni bir revizyona ve yeniden uygulamaya ihtiyaç konusunda karar verdiler.	12 – 19 Aralık 2014. Ses kaydı alındı, birebir yazıya geçirildi, kodlandı ve analiz edildi.

Atölye boyunca kullanılan etkinliklerde adı geçen istatistik kavramları şunlardır: Veri ve değişkenler; sıklık tablosu ve sütün grafiği; dağılımı gösterme ve belirtme; merkezi eğilim ölçüleri; merkezi dağılım; dağılımların karşılaştırılması; ve ilişkilendiren grafik gösterimleri. Bu etkinliklerde, konuyla ilgili sanal manipulatifler tanıtılmış ve istatistik öğretiminde derste nasıl kullanılabileceği konusunda tartışılmıştır.

3.7 Ders Planları

Atölyenin sonunda her gruptan öğrendikleri sanal manipulatifleri ya da internet araştırması sonucu bulabilecekleri başka bir sanal manipulatifi kullandıran istatistik öğretimine yönelik bir ders planı hazırlamaları istenmiştir. Bu aşamadan sonra, sınıfın katılımı sona ermiş ve MÖDA süreci başlamıştır. Araştırmacı, katılımcılara bir ders planı formatı vermiş ve ders planlarını buna göre hazırlamalarını istemiştir (Table 3.5'te görülebilir.)

3.8 Grup Tartışmaları

Her grupla üçer grup tartışması yaparak, toplamda altı grup tartışması yapılmıştır. Her biri yaklaşık 1 ya da 1,5 saat sürmüş, ses kaydı alınmış, birebir yazıya aktarılmış, kodlanmış ve analiz edilmiştir. Grup tartışmaları boyunca, grup üyelerinin ders planı hazırlama süreci boyunca karar verme mekanizmaları, kullandıkları pedagojik stratejiler ve teknolojik kaynaklar (sanal manipulatifler) hakkında tartışılmıştır. Ders araştırması yönteminin doğası gereği, araştırmacı yönlendirici rolde bulunmuştur ve tartışma süresince grup üyelerinin soruları olması durumunda cevaplayarak onları yönlendirmiştir.

Her grup, uygulamadan önce ders planlarını iki kere revize ettiler, bu revizyonlarla ilgili de iki kere grup tartışması yapılmış oldu. Ders planlarının uygulanmasından sonra, uygulamalarına ve yeniden revizyona veya yeniden uygulama yapmaya gerek olup olmadığı konusunda kararlarını almak üzere bir kere daha grup tartışması yapılmış ve kararları doğrultusunda MÖDA döngüsü bir seferde tamamlanmıştır.

3.9 Veri Analizi

Nitel araştırma yaklaşımlarına uyan veri analizi süreci durum araştırmasının doğasına uygun olarak yürütülmüştür. Veri toplama süreci boyunca elde edilen görüşme, test sonuçları, ders planı ve grup tartışması verileri üçgenleme yoluyla çalışmanın güvenilirliğini sağlamak amacıyla birlikte analiz edilmiştir.

Görüşme yoluyla elde edilen verilerin analizi için, görüşme sorularının ilişkili oldukları temalar önceden belirlenmiş, kodlar oluşturulmuş ve ikinci kodlayıcı ile birlikte gözden geçirilmiştir. Bu kodlar Appendix F'de görülebilir. İkinci olarak, grup tartışması için TPAB Kod Elkitabı (Hughes, 2012) (Appendix G'de görülebilir) kullanılmıştır. Bu elkitabı TPAB çerçevesinden elde edildiği için grup tartışması yoluyla elde edilen verilere uygun olduğunu farkedilmiş ve ikinci kodlayıcıyla da fikir birliği sağlanmıştır.

3.10 Çalışmanın Niteliği

Nitel araştırmaların temelinde yatan felsefi yaklaşımların bilinçli olarak farkında olarak, Patton (2002)'ın özetlediği bazı alternatif yargılama kriterleri bu çalışmanın kalitesini ele almak üzere (Peshkin, 2001)'in görme açıları seçilerek kullanılmıştır.

Araştırmacının tarafsızlığı, hakem denetimi, hakem sorgulama seansları, ikinci kodlayıcı kullanılması, derin betimleme, düşünümsellik, veri kaynaklarının üçgenlemesi ve uzman denetim görüşü stratejileri bu çalışmanın geçerliğini ve güvenirliğini sağlamak amacıyla kullanılmıştır.

4 BULGULAR

4.1 Görüşme ve İAÖD testinden elde edilen bulgular

Görüşmede elde edilen bulgulara göre, 6 katılımcı özellikle istatistik alanında kendilerini yetersiz bulduklarını belirtti. Katılımcılardan ikisi istatistik kavramlarıyla başa çıkabilecek beceride olduklarını belirtti. Neredeyse bütün katılımcılar geçmişte herhangi bir istatistik konusunun öğretimiyle ilgili bir tecrübelerinin olmadığını vurguladılar. Biri, aslında bir ders kapsamında bir istatistik konusuyla ilgili bir ders planı hazırladığını, ancak uygulama fırsatı bulamadığını belirtmiştir.

Teknoloji ve eğitimde teknoloji entegrasyonuna ilişkin bakış açılarını ele alırsak, farklı yanıtlarla karşılaşılmıştır. Beş katılımcı ileride öğretimlerinde, imkânlar elverdiği ölçüde, teknoloji kullanmayı istediklerini belirtti. Aynı katılımcılar teknoloji entegrasyonu konusunda kendilerini geliştirmek istediklerini de açıkladılar. Üç katılımcı teknolojinin öğretimi görsel anlamda çeşitlendirdiğini düşünüyorlardı. Biri, teknolojinin örneğin akıllı tahta kullanımı gibi bazı sınıf aktivitelerinde zaman kazandırması açısından avantajlı olduğunu söyledi. Bunlarla birlikte, bazı katılımcılar ise olumsuz görüş bildirdiler, 2 katılımcı teknolojinin gerekli olduğunu ama alan bilgisi ya da pedagojik bilgi kadar önemli olmadığını belirtti. Bunlardan biri, öğretimde teknoloji kullanımı konusunda endişelerini aktardı: Özellikle sosyal medyayı düşündüğünde, teknolojinin korkutucu/ürkütücü bir tarafının olduğunu, ve bu anlamda gençlere zarar verebileceğini belirtti. Aynı katılımcı, gençlerin her zaman erişebildikleri telefon ve tabletler sayesinde tüketime yönelik hareket ettiklerini ve kendilerini geliştiremediklerini iddia etti. Kısaca, teknolojinin üretken bireyler olmaya özendirme konusunda avantajlı olabileceğini, ancak bu amaçla uygulandığında faydalı olabileceğini iddia etti.

4.1.1 İAÖD Testinden Elde Edilen Bulgular

Bu testin sonuçları, her iki grubu da kendi içinde değerlendirmenin daha kolay olacağı fikriyle ayrı olarak hazırlanmıştır. Aşağıda bulunan Tablo 4.1 ve Tablo 4.2'de sonuçlar görülebilir.

Tablo 4.1 A Grubu'na uygulanan İAÖD testinden soru bazında elde edilen bulgular

A Grubu/Soru numaraları	Ezgi	Banu	Alp	Esen	Gizem
Soru1 Medyan	+	+	+	+	+
Soru2 Dal yaprak gösterimi	-	+	+	+	+
Soru3 Mod, medyan ve ortalama	-	+	+	+	+
Soru4 Veri gösterimleri	-	-	-	-	-
Soru5 Serpme gösterimi	-	-	+	-	-
Soru6 Kutu gösterimi	+	+	+	+	Eksik
Soru7 Yanıltıcı grafik gösterimleri	Eksik	Eksik	Eksik	Eksik	+
Soru8 Daire grafiği	+	+	+	+	+
Soru9 Çizgi grafiği tanımı	-	-	+	-	-
Soru10 Çizgi grafiği	+	+	+	+	+
Soru11 Sıklık tablosu	+	+	+	+	+
Soru12 Sütun grafiği	-	-	+	+	+
Soru13 Açıklık	+	Eksik	+	+	Eksik

'+' doğru ve tam olarak verilen cevap, '-' yanlış verilen cevap, 'eksik' tam olarak doğru verilmeyen cevap anlamına gelir. 'cevap yok' katılımcının cevaplamadığı ya da cevaplayamadığı sorudur. 6, 7, 8, 12 and 13 numaraları sorular açık uçlu olup, diğerleri çoktan seçmeli sorulardır.

En problemli soru sütun grafiğiyle ilgili olan sorudur. Bu soru, kategorik değişkenler gösterilmiş bir veri grubunun (bir okuldaki öğrencilerin okula nasıl geldikleri sorulmuş, yürüyerek, otobüsle ve arabayla şeklinde cevaplar alınmış) çizgi grafiğini göstermektedir. Öğretmen adaylarına, bu şekilde ödev hazırlayan bir öğrencinize ne yönde dönüt verecekleri sorulmuştur. Katılımcıların neredeyse yarısı (9'da 4'ü) öğrencinin çizgi grafiği seçiminin yanlış olmadığını söylemiştir. Hatta, bir katılımcı çizgi grafiği yerine histogramla göstermeliydi şeklinde dönüt vereceğini belirtmiştir.

Tablo 4.2 B Grubu'na uygulanan İAÖD testinden soru bazında elde edilen bulgular

B Grubu/ Soru Numaraları	Emel	Serhat	Şenil	Zehra
Soru1 Medyan	+	+	+	+
Soru2 Dal yaprak gösterimi	-	+	-	+
Soru3 Mod, medyan ve ortalama	+	+	+	+
Soru4 Veri gösterimleri	-	-	-	+
Soru5 Serpme gösterimi	-	-	+	-
Soru6 Kutu gösterimi	+	Cevap yok	+	+
Soru7 Yanıltıcı grafik gösterimleri	+	Eksik	Eksik	+
Soru8 Daire grafiği	+	Eksik	+	+
Soru9 Çizgi grafiği tanımı	-	+	-	+
Soru10 Çizgi grafiği	+	+	+	+
Soru11 Sıklık tablosu	+	+	+	+
Soru12 Sütun grafiği	+	-	-	+
Soru13 Açıklık	+	+	+	+

'+' doğru ve tam olarak verilen cevap, '-' yanlış verilen cevap, 'eksik' tam olarak doğru verilmeyen cevap anlamına gelir. 'cevap yok' katılımcının cevaplamadığı ya da cevaplayamadığı sorudur. 6, 7, 8, 12 and 13 numaraları sorular açık uçlu olup, diğerleri çoktan seçmeli sorulardır.

4.1.2 Grup Tartışmaları ve Ders Planlarından Elde Edilen Bulgular

Burada sunulan bulgular, çalışmanın ana bulgularını oluşturmaktadır. Bu ana bulgular, her bir bilgi boyutu bazında her iki grup için bir arada olmak üzere tablolar halinde aşağıda sunulmuştur.

4.1.2.1 Alan Bilgisi

İlk olarak, A Grubu'nun alan bilgisiyle ilişkili olan bulguları listelenmiştir. A Grubu üyeleri ders planlarının ilk versiyonunda, sütun grafiği kavramıyla derslerine başlıyorlardı, ikinci kısımda buradan dal yaprak gösterimine geçiş yaparak, histogram kavramıyla derslerini bitiriyorlardı. Bütün bu farklı gösterimlerde aynı veri grubunu kullanmaya çalıştılar. Ders planlarının ikinci versiyonunda, sütun grafiğiyle başlamaktan vazgeçtiler ve dal yaprak gösteriminden histograma geçiş yaparak, dersin son kısmında sütun grafiğini histogramla karşılaştırarak öğrencilerin geçmiş bilgilerini hatırlatmak istediler. Dolayısıyla, ilk ve ikinci grup tartışmaları alan bilgisi açısından bu geçişlerde yaptıkları istatistik bilgisi hatalarını tartışmakla geçmiştir. Üçüncü grup tartışmasında ise, dersin uygulanması süresince alan bilgisi anlamında bir hata yapmadıkları için alan bilgisi konusundan bahsedilmemiştir.

B Grubu ise ilk ders planlarında A Grubu'nun tercihi gibi sütun grafiğiyle başladılar, konuyu hatırlattıktan sonra kazanımları olan histogram konusuyla devam ettiler. Burada, özellikle değişken tipi ve veri dağılımı konusunda alan bilgisi açısından eksiklikleri olduğu gözlemlendi. İkinci ders planlarında, veri dağılımı ile ilgili yaptıkları hatalar devam ediyordu, bu nedenden yeniden revize ederek uygulamış oldular. Üçüncü grup tartışmasında herhangi bir alan bilgisi eksikliği gözlenmediğinden kodlanmamıştır.

A ve B gruplarının ilk ve ikinci grup tartışmaları boyunca alan bilgisi açısından en belirgin bulguları aşağıdaki Tablo 4.3'te görülebilir.

Tablo 4.3 Grupların alan bilgisi bazında grup tartışmalarından elde edilen bulguları

Gruplar/ Grup Tartışmaları	A Grubu	B Grubu
İlk Grup Tartışması	Veri gösterimlerinde (özellikle sütun grafiği) değişken tipi konusunda eksiklik Veri gösterimlerinin temel farklarını anlamada yetersizlik, özellikle sütun grafiği ve noktasal gösterim Sütun grafiği için uygun örneği seçememe	Veri dağılımıyla ilgili bir kavram yanılgısı Histogramda grup genişliğinin neden kullanıldığını açıklayamama Resimçizitin bir veri gösterimi olduğunu bilmeme
İkinci Grup Tartışması	Histogramın formülüyle ilgili yetersiz anlayış Hem İngilizce hem de Türkçe' de histogramla ilgili terimleri iyi bilmeme Her grup yarılandığında, grup genişliğinin de yarılandığı şeklinde bir kavram yanılgısı	Histogramda grup genişliğinin neden kullanıldığını açıklayamama Histogramda sürekli veri gruplarının neden kullanıldığını açıklayamama

4.1.2.2 Pedagojik Bilgi

Her iki grup da ders planı hazırlama süreçlerinde ve bütün grup tartışmalarında pedagojik stratejilerin çeşitliliği, öğrencilerin geçmiş bilgileri ve ders planı hazırlanması konularından sıkça söz etmişlerdir. Bunun yanı sıra, ne A ne de B Grubu, öğrencilerin değerlendirmesinden söz etmişlerdir. Her iki grubun da bütün grup tartışmaları boyunca pedagojik bilgi açısından ele aldıkları konular aşağıdaki Tablo 4.4'te görülebilir.

Tablo 4.4 Grupların pedagojik bilgi bazında grup tartışmalarından elde edilen bulguları

Gruplar/ Grup Tartışmaları	A Grubu	B Grubu
İlk Grup Tartışması	Birçok öğretim yöntemini ekleme (doğrudan öğretim, soru sorma, düşün-tartış-paylaş yöntemi, sanal manipulatifler) Öğrencilerin geçmiş bilgilerinin farkında olma Dersin geçişlerinde aynı veri grubunu motive edici unsur olarak kullanma Dersin akışı için iki farklı sıralama önerme	Kendi öğrencilerinden elde edecekleri gerçek veriyi motive edici unsur olarak kullanma Dersin akışı için ilişkili veri grupları kullanma Birçok farklı pedagojik strateji kullanma: soru sorma, grup çalışması, etkinlik kâğıdı, fiziksel ve sanal manipulatifler, Dersin kısımları ile ilgili zamana gösterdikleri önem
İkinci Grup Tartışması	Dal yaprak gösteriminden histograma geçiş yaparak derse başlamaya karar verme Sütun grafiği ve histogram karşılaştırmasını dersin sonuna bırakma Histogram ve sütun grafiği için farklı veri gruplarını kullanma	Histogramı ilk kez öğretme yerine hatırlatma yapılması Sütun grafiği için 3 farklı gösterim: fiziksel manipulatif, resimçizit ve sanal manipulatif Gerçek veri kullanımı hakkında tartışma
Üçüncü Grup Tartışması	Ders planında kullandıkları soru sorma yöntemini vurgulama Uygulamadan sonra sınıf yönetimiyle ilgili tartışma	Ders esnasında toplanan gerçek veri grubu Etkinlik kâğıtlarını kullanmadıkları için zamanlama konusunda tartışma Soru sorma aşamasında sordukları sorular Sınıf yönetimi

4.1.2.3 Teknolojik Bilgi

Her iki grup da grup tartışmaları ve atölye çalışmaları süresince TPAB Elkitabında bahsedildiği şekilde teknolojik bilgiye sahip oldukları gözlenmiştir. Özel olarak, katılımcıların akıllı tahta kullanımından bahsedilebilir. Özellikle, uygulamadan sonra grup üyeleri akıllı tahta kullanımını tecrübe etmeleri gerektiğini belirtmişlerdir.

4.1.2.4 Pedagojik Alan Bilgisi

Her iki grup da pedagojik alan bilgisi açısından konuyu farklı türlerde sunma ve aktarma biçimleri hakkında ve öğrencilerin konuya özgü kavram yanılgısı ve hatalarından sıkça söz etmişlerdir. Bununla birlikte, ne A Grubu ne de B grubu öğrencilerin konuya özgü değerlendirmelerinden bahsetmişlerdir. Her iki grubun üçer grup tartışmaları boyunca pedagojik alan bilgisi açısından ele aldıkları konular aşağıdaki Tablo 4.5'te görülebilir.

Tablo 4.5 Grupların pedagojik alan bilgisi bazında grup tartışmalarından elde edilen bulguları

Gruplar/ Grup Tartışmaları	A Grubu	B Grubu
İlk Grup Tartışması	Dersin görsel ögelerini motive edici unsur olarak ele alma Sütun grafiğinin gruplama açısından vurgulanarak gösterilmesi Soru sorma tekniğinin amaçları Bütün veri gösterimlerinde seçilen örnek Sütun grafiğini çizerken kullandıkları veri grubuyla ilgili bir kavram yanılgısı Sütun grafiğinin işlenişinin öğrencilerde kavram yanılgısına sebep olabileceğini fark etmeleri (aslında bir noktasal gösterim olduğunu anlama)	Dağılım konusundaki eksik bilgileri nedeniyle, sütun grafiği için verilerin yarım yarım doldurulması Kendi tasarladıkları fiziksel manipulatif Sütun grafiği için kullandıkları tablonun aslında sıklık tablosu olduğunu fark etmemeleri Histogramda grup genişliği ile ilgili kavram yanılgısına sahip olmaları
İkinci Grup Tartışması	Soru sorma tekniğinde soruların nasıl ve hangi sırayla sorulacağı Ders etkinliklerinin ve izlenen adımların sıralanması Histogram formülünün öğretilmesi, histogram çizme adımları, öğrencilerin grup genişliğini nasıl betimledikleri Soru sorma ve düşün-tartış-paylaş tekniklerinin uygulanması	Sütun grafiği için kullanacakları 3 farklı gösterim ve onların sıralanması Sütun grafiği kısmında yapacakları soru sorma tekniği Grup genişliğine dikkat ederek histogram üzerinde veriyle nasıl ilgileneceklerini bilmemeleri
Üçüncü Grup Tartışması	Soru sorma tekniği hakkında yoğun bir tartışma: öğretmenin sorduğu sorular ve yanıtları, öğrencilerin yanıtlarına karşılık öğretmenin verdiği yanlış dönütler	Sütun grafiği gösterimlerinin sıralanması Soru sorma tekniği sırasında öğretmenin verdiği cevaplar Öğrencilerin yaptıkları hataların farkında olma

4.1.2.5 Teknolojik Alan Bilgisi

Her iki grup da, bütün grup tartışmaları boyunca TPAB elkitabında belirtildiği şekilde teknolojik alan bilgisi bağlamında, konuya özgü teknolojik araçların ve kaynakların çeşitliliği ve bu konuya özgü teknolojileri alan bilgisi ön planda tutarak ve alan bilgisi ve teknolojik bilgiyi sürekli ilişkilendirerek işleme konularından sıkça bahsetmişlerdir. Aşağıdaki Tablo 4.6'da grupların teknolojik alan bilgisi bazında temel bulguları listelenmiştir.

Tablo 4.6 Grupların teknolojik alan bilgisi bazında grup tartışmalarından elde edilen bulguları

Gruplar/ Grup Tartışmaları	A Grubu	B Grubu
İlk Grup Tartışması	Sütun grafiği için sanal manipulatif için yeterince araştırma yapmama, noktasal gösterim için olanı kullanmaya çalışma	Sütun grafiği için uygun olmayan sanal manipulatif seçimi Resimçizitin kategorik değişkenlerin gösterimi için kullanıldığı bilmeme Çizgisel gösterimden histograma geçiş için yanlış manipulatif seçimi
İkinci Grup Tartışması	Derste kullandıkları sanal manipulatifler hakkında yeterince bilgi ve kontrol sahibi olma ve veriyle ilgilenebilme Dal yaprak gösteriminden histogram geçiş yapabilecekleri sanal manipulatifler hakkında farkındalık Histogram için kullandıkları sanal manipulatifte grup genişliğine vurgu yapabiliyor olmalarını fark etme	Resimçiziti araştırma ve onun bütün özelliklerini öğrenme Histogram için seçilen sanal manipulatifte grup genişliğiyle ilgilenebiliyor olmayı göz ardı etme
Üçüncü Grup Tartışması	Sanal manipulatiflerle çalışırken öğrencilerin yapabileceği hataların farkında olma Sanal manipulatifi öğrencilerin düştükleri hatalar ve grup genişliği konuları açısından iyi öğrenme Noktasal gösterimin aslında grup genişliği 1 olan özel bir histogram olduğunu keşfetme	Derste kullandıkları sanal manipulatifleri bütün yönleriyle iyi öğrenme Noktasal gösterimin aslında grup genişliği 1 olan özel bir histogram olduğunu keşfetme

4.1.2.6 Teknolojik Pedagojik Bilgi

TPAB Elkitabına göre teknolojik pedagojik bilginin varlığını gösteren birçok özellik belirtilmiştir. Hem A Grubu hem de B Grubu bu konulardan sık olmasa da bahsetmişlerdir. Özellikle B grubu görüşmeden elde edilen bulgulara göre teknolojinin eğitimde entegrasyonu konusunda birtakım endişelere sahip olsa da, onların bu endişeleri teknolojik pedagojik bilgilerinin gelişimini etkilememiş göründüğü ortaya çıkarılmıştır. Her iki grubun teknolojik pedagojik bilgi boyutundan ele aldıkları temel konular aşağıdaki Tablo 4.7'de ayrıntılı bir şekilde görülebilir.

Tablo 4.7 Grupların teknolojik pedagojik bilgisi bazında grup tartışmalarından elde edilen bulguları

Gruplar/ Grup Tartışmaları	A Grubu	B Grubu
İlk Grup Tartışması	Ders planlarına ekledikleri sanal manipulatiflerin zaman yönetimi Sanal manipulatiflerle çalışırken öğrencilerin yaşayabilecekleri teknik sorunlar Öğrencilerin öğrenmesi açısından sanal manipulatif kullanımının faydaları hakkında farkındalık Gerçek sınıf ortamında sanal manipulatif kullanımı konusunda endişeleri Teknoloji entegrasyonuyla birlikte kullanılabilecek pedagojik stratejileri bilme gereğini fark etme	Histogram için uygun sanal manipulatif için yeterince araştırma yapmama Teknoloji entegrasyonu konusunda kendi tutumlarının ders planlarına olan etkileri
İkinci Grup Tartışması	Sınıfın teknolojik altyapısı Akıllı tahta kullanmaya karar verme Histogram için seçilen sanal manipulatifin olanaklarını bilme Dal yaprak gösteriminden histograma geçişe izin veren sanal manipulatif seçme Grup genişliğini iyi vurgulamak için uygun bir veri grubuna ihtiyaç olduğunu fark etme	İnterneti resimçizit için tarama, bir örneği bulma ve derse ekleme Histogram oluşturmak için tabloyu tarama Sınıfın teknolojik altyapısı
Üçüncü Grup Tartışması	Olası teknik aksaklıklar için hazır olma Uygulama sırasında öğrencilerin yaptığı teknik hataları fark etme	Sanal manipulatifi kullanan öğretmeni öğrencilerin izlemesinin motive edici unsur olması Akıllı tahtaya yeterince hakim olmama ve özelliklerini etkili bir biçimde kullanamama Sınıftaki bütün bilgisayarları kontrol eden bir ana programa olan ihtiyaç

5 YORUM VE TARTIŞMA

Bu bölüm TPAB çerçevesinde ortaya çıkan her bilgi boyutunun birlikte tartışılması sonucu A Grubu ve B Grubunun TPAB gelişimlerinin yorumlanmasını ve tartışmasını içermektedir. A ve B grupları sanal manipulatiflerin istatistik öğretimine entegre edilmesini öğrenmesi açısından özellikle AB, TAB ve PAB boyutlarından önemli gelişmeler göstermişlerdir. Bundan başka, AB, PAB ve TAB ile ilgili bulgulara dayanarak, bu bilgi boyutlarının ilişkili ve bağlantılı oldukları açıkça iddia edilebilir. Grupların PB ve TPB ile ilgili yaşadıkları deneyimlere göre bu bilgi boyutlarındaki gelişimleri için, AB-PAB-TAB gelişimlerinden daha az olduğu söylenebilir. Bu bulguya gerçek sınıfta öğretim konusunda deneyimsiz olmalarının bir etkisinin olduğu iddia edilebilir. Ancak, kendi derslerinde teknolojiyi kullanacaklarında pedagojik konular hakkındaki farkındalıklarının ve bilinçlerinin gelişmeye başladığı iddia edilebilir. Bu çalışmada bahsi geçen AB aslında matematik öğretmeni adaylarının istatistik bilgileri olduğu vurguladığında, bu ilişkiler Lee ve diğerleri (2012) tarafından daha önce daha derin bir istatistik bilgiye ve istatistiksel düşünmeye sahip olan öğretmenlerin teknolojik istatistik bilgilerinin gelişimine yardımcı olduğu ortaya çıkarılmıştı. Bu gelişim aynı zamanda teknolojik pedagojik istatistik bilgisi çerçevesine ilişkin ders hazırlama çabalarında da gözlenebilir olduğu iddia edilmiştir.

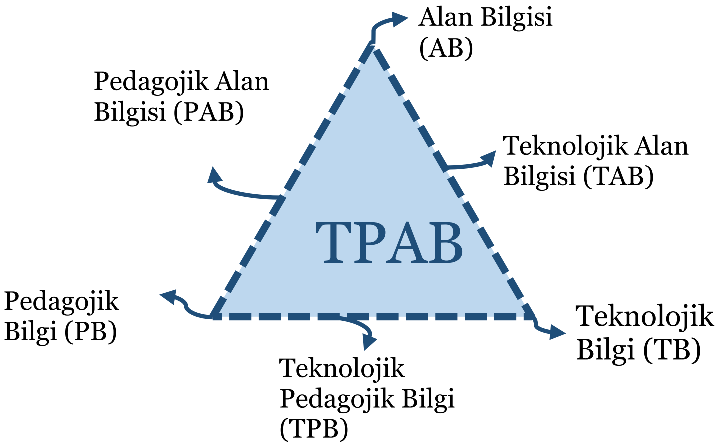
Üniversite öğretiminin bir parçası olarak, öğretim üyeleri tarafından öğretmen adaylarına teknolojiyi derslerine nasıl entegre edeceklerini gösteren ve onların eğitim teknolojileri hakkında yeteneklerini artırmaya yönelik farklı derslerin sunumunun giderek daha yaygınlaştığı daha önce vurgulanmıştı (Hofer ve Grandgenett, 2012). Bu nedenle, iki MÖDA yürütülen bu çalışma matematik için olduğu kadar istatistik öğretimi için de gereken bilgilerin birbiriyle ilişkili yapısını açıklamaktadır. Daha önce vurgulandığı üzere, öğretmen adaylarının öğretmen eğitimleri boyunca bilgilerinin nasıl geliştiğini anlamaya yönelik bir ihtiyaç olduğu açıktır (Hofer & Grandgenett, 2012).

Başka bir sonuç olarak, A ve B Grubu ders planlarında teknoloji entegrasyonu bağlamında farklı tutum içinde oldukları için onların sanal manipulatifleri ders planı hazırlarken ele alış biçimleri küçük farklılıklarla sonuçlanmıştır.

Öğretmen eğitimlerinin ikinci yılında betimsel istatistiğe olduğu kadar çıkarımsal istatistiği de ele alan iki farklı istatistik dersine katılmış olmalarına rağmen, özellikle ilk grup tartışmalarında istatistiksel düşünüşlerinin gelişimi olarak gözlenmemiştir. Bu sorun daha önce Groth (2007, s. 434) tarafından aşağıdaki gibi açıklanmıştır:

> "Öğretmenlerin matematiksel olarak yapılandırılmış daha çok istatistik dersi almasını gerektirmek bir çözüm olarak görünmemektedir, çünkü bu dersler yaygın istatistik bilgisinin matematiksel olmayan yönlerini öne çıkarmaz (Cobb & Moore, 1997). Hatta veri incelemesine odaklanan derslerin yapılandırılması tamamıyla yeterli değildir. Çünkü öğretmeye özgü bilgiye özel olarak katkıda bulunmaz. Öğretmenlerin daha çok dersten ziyade daha farklı matematik derslerine ihtiyacı olduğu kadar (Kilpatrick, Swafford, & Findell, 2001), yaygın istatistik ve istatistiğe özgü matematiksel ve matematiksel olmayan bilgileri içeren daha farklı istatistik derslerine de ihtiyaçları vardır" (s. 434).

Bu çalışmalarının bulgularına dayanarak, öğretmenlerin teknoloji entegrasyonu için bilmeleri gereken bilgi boyutları arasındaki ilişkiler de bağlantıların daha iyi gösterilebileceği yeni bir TPAB çerçevesi önerilmektedir. Aşağıda Şekil 5.1'de bu yeni gösterim incelenebilir.



Şekil 5.1 TPAB çerçevesinin yeni yorumu

Bu yeni çerçeveye göre, TPAB Çerçevesinde üç temel bilgi boyutu olarak tanımlanan alan bilgisi, pedagojik ve teknolojik bilgiler bu üçgenin köşelerini oluşturmaktadır. Üçgenin kenarları bu üç temel bilginin ikişerli etkileşimleriyle ortaya çıkan bilgi boyutlarıdır. Üçgenin köşelerinin öğretmenin içinde bulunduğu öğrenme bağlamında farklı yerlerde bulunabilir, dolayısıyla üçgenin şekli değişebilir. Bu sayede öne çıkan bilgi boyutlarını anlamak ve yorumlamak daha etkili biçimde gerçekleşmiş olur. Matematik öğretmeninin teknoloji entegrasyonu için bilmesi gereken TPAB, bütün bu altı bilgi boyutunun birlikte değerlendirilip ele alınmasıyla elde edilen bilgi boyutu olduğu iddia edilmektedir. Dolayısıyla derin bir TPAB için, matematik öğretmeni diğer bütün bilgi boyutlarına hâkim olmalı ve aralarındaki ilişkilerin ve bağlantıların farkında olmalıdır.

APPENDIX P – Curriculum Vitae

PERSONAL INFORMATION

Surname, Name: Kurt, Gamze
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Marital Status: Married
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EDUCATION

Degree	Institution	Year of Graduation
MS	METU Elementary Mathematics and Science Education	2009
BS	METU Elementary Mathematics Education	2003
High School	Cumhuriyet Lisesi, Konya	1998

WORK EXPERIENCE

Year	Place	Enrollment
2007- Present	METU Department of Elementary Education	Research Assistant
1997-2003	MONE	Elementary Mathematics Teacher

FOREIGN LANGUAGES

Advanced English, Middle level German

PUBLICATIONS

1. Kurt, G. (2010). *Teacher Educators' Perspectives in Turkish Teacher Education Context: Changes in 1982, 1998 and 2006*. Lap Lambert Academic Publishing.

2. Kurt, G. (2015. The investigation of understanding of preservice elementary mathematics teachers about data displays. In *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education.* (pp. 651-657). Prague.

HOBBIES

Classical Turkish Music, Chorist, Japan food, Movies, Theatres, Travellling, Different Cultures

APPENDIX R – Thesis Photocopy Permission Form

ENSTİTÜ

Fen Bilimleri Enstitüsü	☐
Sosyal Bilimler Enstitüsü	X
Uygulamalı Matematik Enstitüsü	☐
Enformatik Enstitüsü	☐
Deniz Bilimleri Enstitüsü	☐

YAZARIN

Soyadı : KURT
Adı : GAMZE
Bölümü : ELE

TEZİN ADI (İngilizce) : TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE (TPACK) DEVELOPMENT OF PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS IN STATISTICS TEACHING: A MICROTEACHING LESSON STUDY

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora X

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. X

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: