A GENETIC ALGORITHM FOR HEALTHCARE FACILITY LOCATION PROBLEM

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ABSTRACT

A GENETIC ALGORITHM FOR HEALTHCARE FACILITY LOCATION PROBLEM

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In this study, we consider the problem of locating emergency healthcare facilities in urban areas. Upon emergency occurrence, patients are directed to any one of the emergency centers with a likelihood that depends on the travel time. Moreover, the survival, that represents the severity of the consequences of the emergency situation, is also probabilistic and is a function of the travel time. A mathematical model is constructed under the objective of maximizing expected number of survivors while determining the location of predetermined number of facilities. Characteristics of this model under certain situations, such as when a concave or convex survival function is used, or when the facility is located on a line or a network, are investigated. After presenting the analytical findings, we propose a Genetic Algorithm based solution approach to solve the model for locating healthcare facilities on a network. Lastly, we present its performance and findings of the numerical study.

Keywords: Facility Location Models, Gravity Models, Gradual Coverage, Healthcare, Genetic Algorithm

ÖZ

SAĞLIK KURULUŞU KONUMLANDIRMA PROBLEMİ İÇİN BİR GENETİK ALGORİTMA

İŞBİLİR, MELİKE
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Bu çalışmada, kentsel bir alanda acil durum sağlık kuruluşlarının konumlandırılması problemi düşünülmüştür. Acil durumun gerçekleşmesi üzerine, hastalar acil müdahele merkezlerinden her birine, ulaşım sürelerine bağlı olasılıklarla yönlendirilebilirler. Bunun yanı sıra, hastanın durumunun ağırlığını temsil eden hayatta kalma olasılığı da, hastaneye ulaşım süresine bağlı olan bir fonksiyondur. Bu kapsamda, belli sayıda tesisi beklenen hayatta kalan hasta sayısını ençoklayacak şekilde konumlandırmak üzere bir matematiksel model kurulmuştur. Modelin matematiksel özellikleri, konveks ya da konkav hayatta kalma olasılığı fonksiyonu kullanılması, tesislerin bir çizgi ya da ulaşım ağı üzerine konumlandırılması gibi farklı durumlar için incelenmiştir. Analitik bulguların sunumundan sonra, bir ulaşım ağı üzerinde tesislerin konumlandırılmasına yönelik modelin çözümü için Genetik Algoritma (GA) tabanlı bir çözüm yöntemi önerilmiştir. Son olarak, önerilen yöntemin performansı değerlendirilmiştir ve sayısal analizlerden elde edilen bulgular sunulmuştur.

Anahtar Kelimeler: Tesis Yer Seçimi Modelleri, Yerçekimi Modelleri, Kademeli Kapsama, Sağlık Hizmeti, Genetik Algoritma

To my dear family

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CHAPTER 1

INTRODUCTION

Facility location models are used extensively for locating public facilities where people's well-being is one of the main concerns. Models developed so far are utilized as a part of significant efforts to improve healthcare services as well, and new approaches continue to emerge in this area. As there are several problems in healthcare that require use of operations research techniques, we are particularly interested in locating a given number of healthcare facilities in this thesis.

There is an ongoing effort to increase the applicability of location models and make better decisions by incorporating new features that represent real life situations. In this study, we will mainly consider two of these. To begin with, coverage idea is an old and important concept in facility location models, which mainly represents the view that a demand region with a distance to the facility beyond a given threshold value can not be served. Despite its extensive usage in many fields including healthcare as well, binary coverage idea is later criticized and gradual and partial coverage models are proposed. Secondly, while it is assumed that customers are served by the closest facility in the traditional models, this is later questioned and behaviors of customers are analyzed. Consequently, several gravity models, which consider factors such as distance to facilities, and size, attractiveness of the facilities, are proposed for reflecting customer choice.

In this study, we approach the problem of locating healthcare facilities considering those two mentioned features. Here, we particularly consider emergency medical situations. Instead of assuming a patient is covered if he/she is within the service border of an hospital, we take into account the time it takes to reach that hospital and evaluate his/her survival probability based on this time. Additionally, as gravity models suggest, we can not know for sure if the patient will be treated at the closest hospital. Therefore, a likelihood function, which represents the probability that this patient will choose a particular hospital is taken into account.

The problem environment is considered to be a city, where each demand region is represented by a node. Demand amounts, which is the number of patients having a medical emergency, are considered to be in accordance with the population of regions. A weighted complete graph is used to represent the city and a given number of identical, noncapacitated healthcare facilities are located in the scope of this problem.

The proposed mathematical model for this problem has a nonlinear objective function. Moreover, as the number of demand regions and the number of facilities to be located increases, complexity increases significantly. Therefore, a Genetic Algorithm based solution approach is presented in this study.

The remainder of this thesis is organized as follows. In Chapter 2, literature survey on coverage models and gravity theory as well as their usage in healthcare area are presented. Later, in Chapter 3, proposed problem is given and the problem environment is explained in detail. In Chapter 4, characteristic of the problem is investigated under certain conditions; such as when concave and convex survival probability functions are used, and when single or multiple facilities are located. Analytical findings are presented in this section. In Chapter 5, a Genetic Algorithm based solution approach is presented for the given problem. Later in Chapter 6, computational study is conducted, results are presented and discussed. Lastly, evaluation of the study and future research directions are given in Chapter 7.

CHAPTER 2

LITERATURE REVIEW

As average lifetime and consequently population of the world increases, difficulties in planning and management of healthcare systems have arisen. Similar to many other systems, considerable efforts have been observed to improve healthcare systems in the last few decades by utilizing Operations Research (OR). A survey by Rais and Viana [20] reveals that the main academic studies on healthcare optimization issues include demand forecasting, capacity planning, patient scheduling, resource scheduling, logistics and location selection. As for the location selection, it is stated that there are two main areas; namely, healthcare centers and emergency vehicle locations. This thesis is about the problem of locating emergency centers. Daskin and Dean [10] classify facility location models used in healthcare applications as the set covering model, the maximal covering model, and the *P-median* model. Nature of our problem led us to a special case of covering models. For this reason, first covering models are introduced in Section 2.1, later the special case, gradual covering models and their applications in the literature are reviewed in Section 2.2. Lastly, attractiveness models in facility location are introduced and a few studies considering attractiveness concept in healthcare are reviewed in Section 2.3.

2.1 Covering Problems in Facility Location

Before starting to give details about different covering models developed in the literature, coverage idea should be explained simply. A customer is "covered" if (s)he is served by a facility that is located within a prespecified distance from (her)him. This idea was firstly introduced by Hakimi [14]. In this study, the objective of the model is determining the minimum number of policeman in highway network such that no one is farther from a policeman than a predetermined distance S. He formulates the problem in a weighted graph denoted by G, and $d(x, y)$ denotes the distance between points x and y on G. Let X_p be a subset of vertices consisting of p nodes on the graph and let

$$d(v_i, X_p) = (min[d(v_i, x_1), d(v_i, x_2), ..., d(v_i, x_p)]) \tag{2.1}$$

All the nodes in G are covered by the subset X_p if:

$$d(v_i, X_p) \leq S \tag{2.2}$$

Subset X_p is said to be an optimum set if there is no other set X_r that covers G, where $r < p$.

Two traditional covering models, set covering problems and maximal covering location problems are presented in Section 2.1.1 and Section 2.1.2, respectively.

2.1.1 Set Covering Problem

While Hakimi [14] was the first one coming up with the idea of coverage, first mathematical model in covering problems was developed by Toregas et al [26]. Objective of this model is to minimize the total number of located facilities while satisfying coverage constraints for all demand nodes. Toregas et al [26] state that the problem can be most applicable to location of emergency services, as well as other services such as schools, libraries etc. For this reason, S is defined as the response distance, even though it may be considered as coverage distance or time for other types of problems.

The notation used for the Set Covering Problem (SCP) is summarized in Table 2.1.

Table 2.1: The Notation Used for the Set Covering Problem

Indices:	
i	Index for demand points
j	Index for alternative facility locations
Sets:	
N	Set of demand nodes, i=1,2, ... , $\|N\|$
M	Set of alternative facility locations, j=1,2, ... , $\|M\|$
N_i	Set of facility locations that covers node i=1,2, ... , $\|N\|$
Decision variables:	
x_j	Equals 1 if facility is established at point j, 0 otherwise
Parameters:	
d_{ij}	Distance or response time from location j to demand point i
S	Coverage distance

Formulation of the set covering problem is as follows:

$$\text{Min} \quad \sum_{j \in M} x_j \tag{2.3}$$

$$\text{s. to} \quad \sum_{j \in N_i} x_j \geq 1 \qquad \forall i \in N \tag{2.4}$$

$$x_j \in \{0, 1\} \qquad \forall j \in M \tag{2.5}$$

The objective function (2.3) minimizes the number of facilities located. Constraint (2.4) ensures that each demand is being covered, and (2.5) is the integrality constraint. Toregas et al [26] utilize linear programming supplemented by the addition of a single cut constraint. Suppose that p^0 is the optimal solution of the linear programming relaxation and it is a fractional value. This means, in any feasible integer solution, at least $\lfloor p^0 \rfloor$ +1 servers must be located. It is claimed that addition of this single cut constraint always resulted in integer solutions, therefore the proposed model solves the set covering problem to optimality.

The set covering problem was extended in several ways later. *Weighted Set Cover-*

ing Problem (WSCP) attaches costs to facilities and minimizes the cost of locating facilities while ensuring all demand nodes are covered. In the set covering problem, when facilities are assumed to be noncapacitated, demand of different customers are not needed to be represented on the model. In order to make the problem more adaptable to real-world situations, Current and Storbeck [8] present a formulation for the capacitated set covering problem including demand explicitly in the model. Additionally, Murray et al [19] use spatial representation of demand areas and proposes two models; namely, *Location Set Covering Problem (LSCP)-Implicit* and *LSCP-Explicit*. Both of these models minimize the number of facilities located while allowing a demand area being covered by multiple facilities. Under implicit covering problem, coverage levels which represent the maximum number of facilities a coverage combination can include are defined and there is a given minimum acceptable coverage percentage for each level. The facilities are partitioned in discrete sets for each demand node depending on whether they provide coverage at the acceptable percentage for each level. Eventually, each demand area must be completely covered by some combination of facilities at some level k. Explicit covering, on the other hand, creates sets of facility combinations for each demand area that completely cover that area, which enables one to keep track of facilities serving to each demand region.

Due to the probabilistic nature of emergency situations, the set covering model was extended later considering some probabilistic factors. ReVelle and Hogan [22] estimate the busy fractions of servers in the zones around each demand region and aim to minimize number of servers while ensuring each demand is served with a reliability of at least α. ReVelle and Hogan [24] present the derivations of this probabilistic set covering problem; namely, *"α-reliable P-center problem"* and *"maximum reliability allocation problem"*. Both models locate P facilities by taking busy probabilities of the service vehicles into account. The fist one minimizes the maximum time or distance within which service is available with α reliability. The solution is obtained by solving the original probabilistic model successively for smaller values of S. When a decrease in S causes an increase in the number of facilities to be located, the previous S value gives the smallest time or distance within which service can be provided with α reliability. The second problem aims to maximize the minimum reliability of service. The solution is obtained by solving the original probabilistic model successively for larger values of α. If an increase in α causes an increase in the number of facilities to be located, the previous α value gives the largest minimum reliability provided by P facilities.

2.1.2 Maximal Covering Location Problem

As mentioned earlier, another covering concept utilized extensively in locating healthcare facilities is *Maximal Covering Location Problem (MCLP)*. MCLP is based on the idea of maximal service distance, that is used to label a demand region as covered or not. When location decisions are to be made with limited budget, covering all demand areas without changing S may not be feasible. In this case, a cost-effectiveness curve obtained by SCP is examined to observe the effect of changing S on total cost, i.e., minimum number of facilities to be located. The model proposed by Church and ReVelle [5] seeks to cover as many customers as possible within S using the limited resources. Sets, parameters and decision variables needed to designate MCLP can be seen in Table 2.2.

MCLP is formulated as follows:

$$\text{Max} \quad \sum_{i \in N} a_i y_i \tag{2.6}$$

$$\text{s. to} \quad \sum_{j \in N_i} x_j \geq y_i \qquad \forall i \in N \tag{2.7}$$

$$\sum_{j \in M} x_j = P \tag{2.8}$$

$$x_j \in \{0, 1\} \qquad \forall j \in M \tag{2.9}$$

In the model, (2.6) maximizes the total demand covered while (2.7) ensures that y_i will be equal to 1 only when demand point i is covered by one or more facilities. The number of facilities to be located is limited to P as stated in (2.8) and (2.9) is the integrality constraint. Church and ReVelle [5] state that objectives of public sector location problems rather than the ones in private sector are taken into consideration in MCLP better, which makes it an extensively used model in healthcare facility location

Table 2.2: Notation Used for Maximal Covering Location Problem

Indices:	
i	Index for demand regions
j	Index for alternative facility locations
Sets:	
N	Set of demand nodes, i=1,2, ... , $\|N\|$
M	Set of alternative facility locations j=1,2, ... , $\|M\|$
N_i	Set of facility locations that covers node i=1,2, ... , $\|N\|$
Decision variables:	
x_j	Equals 1 if facility is established at point j, 0 otherwise
y_i	Equals 1 if node i is covered by at least one facility, 0 otherwise
Parameters:	
d_{ij}	Distance or travel time from location j to demand point i
S	Coverage distance
P	Number of facilities to be located
a_i	Demand amount at node i

problems. Two greedy heuristic solution approaches and two different techniques for resolving the fractional solutions obtained by the linear programming model are proposed for solving the problem.

Later, various extensions of the model proposed by Daskin and Dean [10] are studied. As in SCP, Murray et al [19] present the implicit and explicit versions of the problem. Church [6] solves the model on a continuous plane without defining discrete facility locations. Like it was applied to SCP, Current and Storbeck [8] consider capacitated facilities and propose the *capacitated version* of MCLP.

Besides these extensions, probabilistic version of MCLP that was presented by ReVelle and Hogan [23] is an important problem in healthcare facility location problems. This model locates P servers while considering that a server may not always be available to serve demand points. The problem is known as the *the Maximum Availability*

Location Problem (MALP). The objective of the model is to maximize the population covered with a predefined reliability. There are two versions of the model that differ in computation of the fraction of time that the servers are busy. The first version, *MALP I*, assumes that busy fractions of all servers in the system is the same. Whereas, *MALP II*, relaxes this assumption and computes site-specific busy fractions rather than system-wide measures.

Another probabilistic extension of MCLP is presented by Daskin and Mark [9]. The model, which is known as *Maximum Expected Covering Location Model (MEXCLP)*, is based on the idea that not all located facilities will be able to respond to demands at all times. It is assumed that the state of a facility may be *"broken down"* with a known probability r. This probability is the same for all facilities, and the problem aims to maximize expected covered population under these assumptions. Besides an integer programming model, a heuristic approach that aims to find solutions for all P values is presented. Results of the heuristic approach tested on a network for different ranges of r are given as well.

As mentioned before, set covering and maximal covering location problems are the two traditional classes in covering problems. Main principles and objectives of both models are explained as well as some extensions of them in this section. These models and their extensions are being used extensively to solve healthcare facility location problems. However, most of them are constituted based on the traditional coverage idea, which labels demand points as covered or not. Our problem definition considers all demand regions as being covered to some extent, which is represented by a survival probability. This situation requires understanding partial and gradual coverage concepts, which are explained in Section 2.2.

2.2 Partial Coverage and Gradual Coverage Problems

Defining a coverage distance specific to a service means that locating a facility is purposeful for a customer only if the distance between them is within the predefined critical value. When such a critical value is known, objectives such as minimizing total distance or cost may not be meaningful and coverage models that aim to minimize the number of facilities located or maximize covered population are used instead. Despite this advantage, binary coverage concept, which means dividing points as "covered" or "uncovered" has been criticized later on.

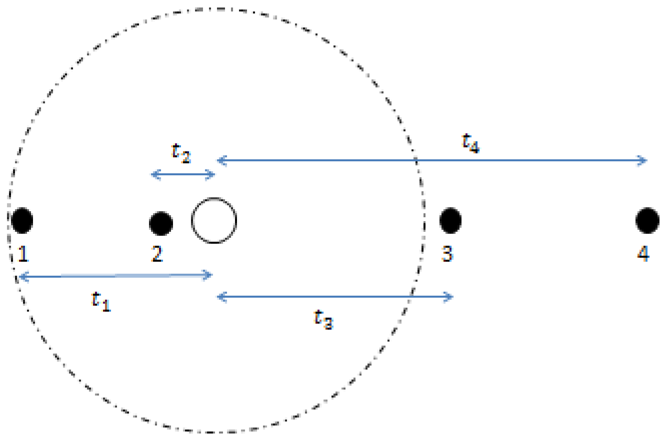

Figure 2.1: Facility and Demand Nodes

In Figure 2.1, there exists one facility and four demand nodes around it. t_i values represent the travel time between each demand region i and the facility. Demand nodes beyond the dashed lines are not served since they are not within a distance S of the facility. It can be seen that although relative distances of customer 1 and customer 3 to the facility are close to each other, node 1 is considered as covered while node 3 is considered not covered according to the binary coverage principle. Moreover, there is not a way to differentiate states of nodes 1, 2, and nodes 3, 4 even though their distances to the facility are quite different. In order to reflect the effect of distances or travel times between demand regions and facilities to their state of coverage in a more realistic approach, partial and gradual coverage concepts have been proposed.

2.2.1 Partial Coverage

The Generalized Maximal Covering Location Problem (GMCLP) presented by Berman and Krass [1] is an extension of the MCLP. This study states that binary coverage assumption may be unrealistic in many applications. From the retailers' point of view, the situation is expressed as follows: customers within 1-2 miles of the facility are considered as fully covered since they are believed to constitute primary trading area of the facility. Beyond that distance, customers are only partially covered, which cannot be represented by binary coverage approach. In order to incorporate decreasing levels of coverage into location models, they define a decreasing stepwise function of the distance. Consider a network $G = (N, E)$, where N and E represents the nodes and edges, respectively. There is a weight assigned to each node i, w_i, which can for example represent the population of node i. Distance between any two point i,j $\in G$ is denoted by $d(i, j)$. For a given set of locations X_p on G, the distance between node i and its closest facility in the set is defined as

$$d(X_p, i) = \min_{j \in X_p} d(j, i) \tag{2.10}$$

As explained earlier, it is assumed that a customer can be covered at different levels depending on its distance to the closest open facility. Coverage level decreases as distance increases, and partial coverage uses a step-function to represent this situation. Predefined critical levels on distances constitute ranges, and there exists a corresponding coverage level for each distance range. For each node i, k coverage radii are defined such as $r_i^0 < r_i^1 < r_i^2 < \ldots < r_i^k = \infty$ with associate coverage levels $a_i^1 = 1 > a_i^2 > \ldots > a_i^k \geq 0$. For a given set of locations X_p and level l≤k, let

$$N(X_p, l) = \left\{ i \in N | r_i^{l-1} \leq d(X_p, i) < r_i^l \right\} \tag{2.11}$$

Equation (2.11) is used to define the set of all nodes whose shortest distance to X_p is in the range [r_i^{l-1},r_i^l). The defined set is discrete for all values of l and each i belongs to exactly one such set. Depending on the range of the set i belongs to, a coverage level a_i^l is attributed to node i. If M is the set of potential locations and P is the number of facilities to be located, the GMCLP can be written as

$$\max_{X_p \subseteq M, |X_p| = p} \sum_{l=1}^{k} \sum_{i \in N(X_p, l)} w_i a_i^l \tag{2.12}$$

It is proven by Berman and Krass [1] that an optimal set of solutions exist on nodes of the network. This study also presents several integer programming formulations of the problem. Two solution approaches based on LP-relaxation and a greedy heuristic are proposed. IP model and the proposed solution approaches are tested on 226 instances and the computational results are presented in the study.

2.2.2 Gradual Coverage

As mentioned earlier, Berman and Krass [1] find binary coverage assumption unrealistic. Similarly, Berman et al [2] state that the assumption of "abrupt" termination of coverage in MCLP may be unrealistic in many potential applications. Therefore, an alternative approach to coverage, which can be considered as an extension of GM-CLP, is presented. Single coverage distance is replaced with two coverage radii l and u. A point is assumed to be fully covered if it is within distance l of it's closest facility and is not covered if the distance to the closest facility is beyond distance u. When the distance to facility takes a value between two coverage radii, the node is assumed to be partially covered, which is represented by a "coverage decay function".

They consider the problem of locating p facilities on a network $G = (N, E)$ where N and E are the set of nodes and the links, respectively. Demands are assumed to be originated from nodes. Characteristic of the problem has been examined for particular situations. When the gradual decay function is taken to be a decreasing convex function, it is proven that an optimal set of locations exists in a discrete subset of G consisting of N and certain breakpoints on the edges. These breakpoints are the points on the network which are at a distance l_i or u_i from each node $i \in N$. However, a similar conclusion could not be obtained for the gradual decay functions with a structure other than convex. They also present a mathematical formulation, and the notation used can be seen in Table 2.3.

(2.13) maximizes the total "value". (2.14) ensures that P facilities will be located. A node can be covered from i only if there is a facility at i (2.15). Each demand can be covered only once (2.16). (2.17) is the integrality constraint.

Table 2.3: Notation Used for the Gradual Covering Decay Location Problem on a Network

Indices:	
i	Index for demand regions
j	Index for alternative facility locations
Sets:	
N	Set of demand nodes, i=1,2, ... , $\|N\|$
M	Set of alternative facility locations, j=1,2, ... , $\|M\|$
$M_i(1)$	Set of facility locations that fully covers node i, $j \in M$; $0 \leq d_i(j) \leq l_i$
$M_i(2)$	Set of facility locations that partially covers node i, $j \in M$; $l_i < d_i(j) \leq u_i$
Decision variables:	
y_j	Equals 1 if a facility is established at point $j \in M$, 0 otherwise
g_{ij}^x	Equals 1 if a facility is located at j, 0 otherwise, ∀j $\in M_i(x)$, x=1,2
g_{ij}	Equals g_{ij}^1 if $j \in M_i(1)$, g_{ij}^2 if $j \in M_i(2)$, 0 otherwise
c_{ij}	Equals w_i if $j \in M_i(1)$, $w_i f_i(d_{ij})$ if $j \in M_i(2)$, 0 otherwise
Parameters:	
d_{ij}	Distance or response time from any node j to any node i
w_i	Demand weight associated with node $i \in N$
P	Number of facilities to be located
d_{ij}	Distance from node i to facility j
$f_i(d)$	Gradual decay function of distance d

$$\text{Max} \quad \sum_{i \in N} \sum_{x \in M} g_{ij} c_{ij} \tag{2.13}$$

$$\text{s. to} \quad \sum_{j \in M} y_j = P \tag{2.14}$$

$$y_j \geq g_{ij} \qquad \forall i \in N, j \in M \tag{2.15}$$

$$\sum_{j \in M} g_{ij} \leq 1 \qquad \forall i \in N \tag{2.16}$$

$$y_j, g_{ij} \in \{0, 1\} \qquad i \in N, j \in M \tag{2.17}$$

Since the gradual covering decay location model is a special case of uncapacitated facility location problem, Berman et al [2] state that the same solution approaches; namely, the greedy heuristic and LP-relaxation, are applicable to this problem as well.

Drezner et al [12] solve the gradual covering problem on 2-dimensional plane. Objective function of the problem is formulated as minimizing noncoverage rather than maximizing coverage. In this way, the objective function becomes similar to the Weber problem except that the cost function is not linear in the distance. Two coverage radii, l_i and u_i are used as in [1], as well. The problem is to find the best location of a facility while the total cost for all demand points is minimized. Notation used to formulate the problem is given in Table 2.4.

Table 2.4: Notation Used for the Gradual Covering Problem

Indices:	
i	Index for demand regions
Sets:	
N	Set of demand nodes, i=1,2, ... , n
Decision variables:	
X	Unknown location of the new facility
$d_i(X)$	Distance between point i and the new facility
Parameters:	
w_i	Demand weight associated with node $i \in N$

Cost function used is defined as:

$$c_i(d) = \begin{cases} 0 & \text{d} \leq l_i & (2.18) \\ w_i(d - l_i) & \text{l}_i \leq d \leq u_i & (2.19) \\ w_i(u_i - l_i) & \text{d} \geq u_i & (2.20) \end{cases}$$

Objective function is:

$$\text{Min} \quad F(X) = \sum_{i \in N} c_i[d_i(X)] \tag{2.21}$$

When Euclidean distance is used as the distance metric, the solution to the problem is proven to be in the convex hull of the demand points. A branch and bound solution procedure is proposed to solve the problem in finite steps within an acceptable accuracy. They also discuss the potential areas that gradual covering problem may be applied to. One of these applications is medical facility location. It is stated that, survival rate decreases with the time it takes to reach the patient. Given model can be applied to represent this situation by taking survival rate constant until a critical point and severity of the disease starts after that point, l_i. Condition of the patient no longer worsens after the second critical point, u_i.

A modified version the mathematical model in [12] is presented by Karasakal and Karasakal [16]. Here, demand points are identical in terms of their weights. There exists two critical points, l_i and u_i with the same applications in [12]. Coverage function, on the other hand, may be of any type; continuous or discrete; linear or nonlinear. It is stated that large size problems can not be solved using mathematical programming packages and heuristic approaches do not guarantee reaching an optimal solution. Therefore, a lagrangean relaxation based solution procedure is proposed and it is claimed to be effective based on computational results.

Applications of Gradual Coverage in Healthcare

As mentioned by Drezner et al [12], gradual coverage can be used to represent survival rate in healthcare related problems. So far, there is a limited number of studies incorporating gradual coverage into healthcare problems. Among these, one of them is closely related to the subject of this thesis and must be explained in detail.

Gradual coverage was first introduced in emergency medical service models by Erkut et al [13]. This study focuses on ambulance location problem and aims to maximize the expected number of survivors from patients having a cardiac arrest. Four survival probability functions are examined which may take different parameters such as time spent until cardiopulmonary resuscitation (CPR), defibrillation and advanced cardiac life support (ACLS) into account. Among these, the survival function presented by Valenzuala et al [28] is used and simplified by making some assumptions on time related parameters. Survival probability depends on response time, which is represented as a function of distance, $R(d)$, and some explanatory variables $\mathbf{O}$, such as qualification of ambulance staff, existence of a witness, receiving CPR from a bystander or not. Assuming that probability distributions of $R(d)$ and $\mathbf{O}$ are known, Monte Carlo simulation is used to derive survival probability as a function of distance. However, uncertainty in the response time was ignored and survival probability is taken as $E[S(E[R(d)], \mathbf{O})]$ in the firstly presented model. Notation used is given in Table 2.5.

Table 2.5: Notation Used for Ambulance Location Problem

Indices:	
i	Index for demand regions
j	Index for alternative facility locations
Sets:	
N	Set of demand nodes, i=1,2, ... , $\|N\|$
M	Set of alternative facility locations, j=1,2, ... , $\|M\|$
Decision variables:	
y_j	Equals 1 if facility is established at point j, 0 otherwise
x_{ij}	Equals 1 if demand node i is served by the facility established at point j, 0 otherwise
Parameters:	
w_i	Demand weight associated with node $i \in N$
t_{ji}	Travel time from candidate location j to demand node i
t_d	Pretravel delay
$s(t)$	Survival rate as a function of response time
P	Number of facilities to be located

The mathematical model based on MCLP aims to maximize expected number of patients who survive. (2.23) ensures that demand point i can be served by j only if an EMS vehicle is located there. Each demand point is served by a single facility (2.24). No more than P facilities can be located (2.25). (2.26) is the integrality constraint.

$$
\begin{aligned}
\text{Max} \quad & \sum_{i \in N} \sum_{j \in M} w_i s(t_{ji} + t_d) x_{ij} && && (2.22)\\
\text{s. to} \quad & \sum_{i \in N} x_{ij} \leq n y_j && \forall j \in M && (2.23)\\
& \sum_{j \in M} x_{ij} = 1 && \forall i \in N && (2.24)\\
& \sum_{j \in M} x_j \leq P && && (2.25)\\
& y_j, x_{ij} \in \{0, 1\} && \forall i \in N, j \in M && (2.26)
\end{aligned}
$$

After presenting the main model, they also give some extensions in order to make the model more realistic. First one of them, referred as the maximal expected survival location problem (MEXSLP), is based on MEXCLP. This model incorporates busy probabilities and allows allocation of more than one EMS vehicle to each demand point. The second one, the maximal survival location problem with probabilistic response time (MSLP+PR), only modifies the objective function of the base model. This extension uses precomputed survival rates for each $i - j$ pair, considering the uncertainty in response times. The last one considers both the uncertainty in response time and the busy probabilities of vehicles, and is referred as the maximal expected survival location problem with probabilistic response time (MEXSLP+PR). Results of the optimization models tested on real life data are also presented in the study.

Similarly, Knight et al [17] focuses on locating EMS vehicles by incorporating survival functions as well. While the study by Erkut et al [13] was restricted to cardiac arrest patients, they consider multiple-classes of patients and define category based survival probabilities. Another contribution of the study is taking into account stochastic utilization of ambulances and congestion at each ambulance station. While doing this, an iterative approach is used to calculate busy probabilities based on queuing theory.

Providing equity is an important issue in emergency management, and Chanta et al [4] aim to locate EMS vehicles while minimizing inequity of the system. The problem considers both survival rates and availability of EMS vehicles. Survival function given by Valenzuala et al [27] is rewritten based on some assumptions made in this study. These assumptions are related to times spent until a call is placed to the emergency center, the start of CPR and defibrillation. Eventually, a survival function that only depends on response time is obtained and busy probabilities are estimated based on hypercube queuing model. Objective of the study is minimizing total envy, where envy is defined as the difference between satisfaction levels of customers. Satisfaction levels are associated with survival probabilities rather than distance to facilities. An integer programming formulation is presented, and results for varying P values are given as well.

2.3 Attractiveness Models in Facility Location Problem

Most of the coverage models explained in the previous sections use the assumption of single allocation, that is each demand point is served by its closest facility. In spite of that, attractiveness models are constituted based on the idea that customers are not necessarily served by the closest facility. This idea is firstly put forward by Reilly [21] and supported by an empirical national study. The model proposed in that study is later criticized by Huff [15] and an alternative model is proposed.

History of development of attractiveness models starts with the pilot study of Reilly [21], in which data collected from retail merchants and by door-to-door questionnaires are analyzed. Main purpose is to find out the interaction between smaller towns and large cities; on the other hand, field survey results bring him to the conclusion that only size of the residual areas is not enough to explore this interaction. A third dimension, population of the cities, must be considered as a predictive factor as well. Following that, a nationwide survey is conducted in the United States. Reilly's inference based on this data is that population and size of the cities are enough to identify breakpoints between two cities. As a result, he presents the following equation:

$$\frac{B_a}{B_b} = (\frac{P_a}{P_b})^N * (\frac{D_b}{D_a})^n \qquad (2.27)$$

where

B_a: the business which City A draws from any intermediate given town,

B_b: the business which City B draws from any intermediate given town,

P_a: population of city A,

P_b: population of city B,

D_a: distance of city A from the intermediate town,

D_b: distance of city B from the intermediate town,

N: Increase rate of the outside trade as population increases,

n: Decrease rate of the outside trade as distance from city increases.

Three decades after this first attractiveness model was presented, a more extensive field survey is conducted and an alternative model is proposed by Huff [15]. Data collected by interviewing households and firms are analyzed and the results show that the proportion of customers patronizing a given shopping center varied with distance, size of the merchandise, product type and proximity of the shopping area to its competitors. In addition to these findings, Huff states that Reilly's model had some limitations. Firstly, the model assumes that an area may fall into coverage zone of only one store, whereas there may exist gradual declines of sales potential as distances increased. Secondly, while Huff's field survey reveals that customers may be willing to travel further distances for certain product types, Reilly's model does not differentiate product types in terms of their effect on trading areas. In order to overcome these limitations, a new model is proposed which represents a theoretical abstraction of consumer spatial behavior. Notation used is given in Table 2.6 and a formal expression of the probability that a customer at point i traveling to shopping center at point j is as follows:

$$P_{ij} = \frac{\frac{S_j}{{t_{ij}}^{\lambda}}}{\sum_{j \in M} \frac{S_j}{{t_{ij}}^{\lambda}}} \tag{2.28}$$

Also, the expected number of customers in place i that shop at area j can be estimated by

$$E_{ij} = P_{ij} C_i \tag{2.29}$$

where C_i represents the number of customers at i.

Table 2.6: Notation Used for the Gravity Model

Indices:	
i	Index for demand regions
j	Index for shopping centers
Sets:	
N	Set of demand nodes, i=1,2, ... , $\lvert N \rvert$
M	Set of shopping centers, j=1,2, ... , $\lvert M \rvert$
Parameters:	
t_{ij}	Travel time from demand node i to shopping center j
λ	Parameter reflecting the effect of travel time on various kinds of shopping trips
S_j	Size of shopping area j
P_{ij}	Probability of a consumer at i travelling to shopping center j

The idea of relaxing proximity assumption is later used in several traditional facility location models. Among them, "The gravity p-median model" presented by Drezner and Drezner [11] is closely related to our study, since we aim to locate a given number of healthcare facilities by incorporating the attractiveness principle. Their problem is to locate p facilities on nodes of a network by minimizing the total distance traveled by all customers to their selected facilities. The probability of a customer selecting a particular facility is proportional to the attractiveness of that facility and inversely proportional to the distance between them. They also prove that optimal locations are not always on nodes of the network, but restrict facilities to be located on nodes. Two heuristic approaches, namely, the steepest descent and tabu search, are proposed in the study and claimed to give impressive results.

Even though attractiveness models are initially constituted to define trading areas, their applications are extended to various location problems, including public sector facility locations as well. In the next section, we will focus on attractiveness models applications in healthcare problems.

Applications of Gravity Models in Healthcare

Bucklin [3] studies the applicability of attractiveness models in healthcare. This study reviews all attractiveness models, and comments on interpretations and possible values of parameters. The main purpose is to examine the relation between the value of exponent $-\lambda$ and distance. In the process of exploring this relationship, patient data for a group of hospitals are analyzed. As a result, a curve representing the hypothesized logistic relationship between λ and distance is obtained. This curve reveals that small exponents close to zero are suitable for relatively short distances. A rapid increase in λ occurs as distance increases until some point. Later, it continues to increase with decreasing slope and reaches a maximum value. However, it is stated by Bucklin [3] that this hypothesis is very tentative and requires further evaluation.

The study by Lowe and Sen [18] uses the attractiveness model given by Sen and Sööt [25] in order to estimate the effects of certain changes in healthcare system in an urban hospital market. These include the changes in hospital payment policy and hospital closures. In this study, diseases are also categorized and the effects are examined for each category. It is also shown that people will travel further distances for more specialized health services, as in the case of shopping for certain type of products.

Congdon [7] analyzes the impact of reconfigurations of emergency hospital services on patient flows from homes to hospitals. These configurations may include strategic decisions such as opening new facilities or closure of existing ones, expanding capacities of hospitals in terms of number of beds. Flows are modeled based on attractiveness models. Simulation based Bayesian methods are used for estimation.

CHAPTER 3

PROBLEM DEFINITION

In this section, the problem studied will be explained. The idea behind splitting demand to different facilities and benefits of using survival functions in healthcare facility location problems are explained, and the assumptions made are described.

As mentioned in Section 2.3, attractiveness models brought the idea that customers are not necessarily served by the closest facility. It is seen in cases such as capacitated facilities are considered, or servers are not always available to serve customers, that allocating customers to different facilities may be allowed. Otherwise, when proximity of demand nodes and facilities brings improvement to the objective function, customers are considered to be served by the closest facility.

Considering an emergency situation, it is obvious that time spent until first medical intervention is of vital importance. This time depends on factors such as the distance between incident location and the hospital chosen to go, traffic on the selected route, availability of necessary medical staff and the equipment. As it is of high importance to reach medical service as quick as possible upon occurrence of emergency, it is naturally thought that the patient would prefer to be treated at the closest facility. However, this may not always be the case observed in real life situations. Upon occurrence of a medical emergency situation, patients are either transported to a nearby hospital by an emergency service vehicle or this transportation is conducted by a witness or the patient himself/herself. In the first case, EMS vehicle's destination may be predetermined for certain incident types and locations, or EMS staff may be directed to a certain hospital after arrival of the call. In an environment where all the hospitals are identical and uncapacitated, the optimal choice would be transferring the patient to the closest hospital. In the latter case, patient or a bystander makes the decision about which treatment centre to go. This decision may be affected by many factors some of which originates from the fact that hospitals are not identical in people's perception. These factors can include their past experiences, reputation of the nearby hospitals, suggestions from friends as well as the distances from the patient's location to the hospitals under consideration.

As human mind is a complex system, a perfect mathematical representation of this choice would require all of these factors and more. In order to reach a simpler representation, the idea behind attractiveness models is used. Even though gravity models first emerged from the observations from travels to shopping centers, its usage spread to other areas including healthcare as explained in Section 2.3. Among these limited number of studies, one of the factors whose effect on patient's behavior is studied is distance to the hospitals, which can also be considered as the travel time. Since one of the leading factors affecting patients' choice in emergency situation is travel time, it is found acceptable to use a likelihood representation function of travel time in this thesis. In this way, we not only reflect the importance of travel time on this decision fairly, but also relax the assumption of always choosing the closest hospital with a more realistic one.

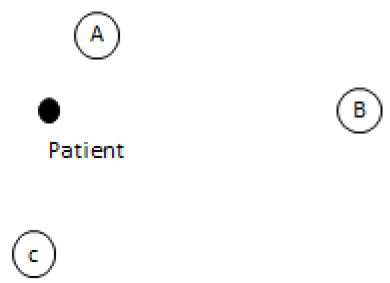

Figure 3.1: Facilities and the Demand Node

The likelihood function proposed gives the probabilities of a patient to travel each one of the open facilities. Consider the small example given in Figure 3.1. There are three hospitals surrounding the patient and travel times between the patient and them are t_A, t_B and t_C. When the likelihood of this patient traveling to any of these hospitals is represented by $P_{facility}$, the value corresponding to visiting facility A is as follows;

$$P_A = \frac{\frac{1}{t_A}}{\frac{1}{t_A} + \frac{1}{t_B} + \frac{1}{t_C}} \tag{3.1}$$

Equation 3.1 is a special form of equation (2.28), where S_j values are all equal to one, which can be considered as all the facilities are identical. λ value reflecting the effect of travel time is taken to be one as well. General form of the function will be presented in Section 4.2.

As mentioned in Section 2.2, partial and gradual coverage models emerged as a result of binary coverage models being insufficient to represent real life situations. This was demonstrated with the help of a small example in Figure 2.1.

Now let's expand the example in order to see the effects of using binary coverage and gradual coverage. Consider the example in Figure 3.2, where t_{ij} values represent the travel times between two points. Suppose that there are 10 patients on each node, and S value is equal to 10, the nodes are located on a line and the facility can be located anywhere on this line. A continuous maximal covering approach would result in coverage of three nodes with a single facility. This can be achieved by locating the facility anywhere between the points A and B in Figure 3.2. Point A is at a distance 6 minutes away from node 2 and 10 minutes away from node 4. Node B is located at a distance 10 minutes away from node 2 and 6 minutes away from node 4. Any of these alternate solutions covers 30 patients.

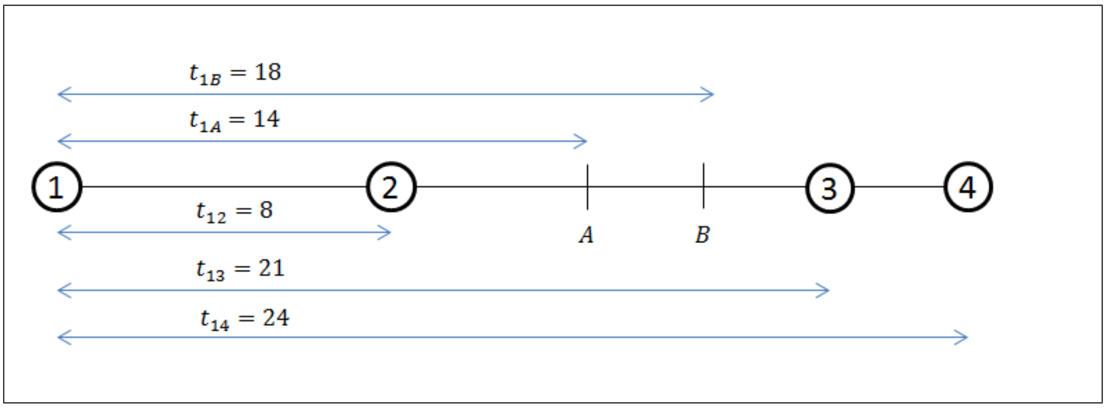

Figure 3.2: Locating a Single Facility on a Line

Now, let's examine the effect of using a gradual coverage function instead of binary coverage. Survival probability at point x equals to $e^{-t_x/4}$, where t_x stands for the travel time between the facility located and the patient at point x. Figure 3.3 shows expected number of survivors for each node separately and all the patients on the network, when facility is located at any point from A through B.

In the first situation, where maximal coverage is aimed, node 1 is not considered as covered, however changing the location of the facility between A and B has a small effect on the expected number of survivors at this demand point. Similarly, while anywhere on the line [AB] is optimal for the first situation, it is seen that expected number of survivors change remarkably for nodes 2, 3 and 4 as facility is moved from A to B. In total, there are approximately 2.5 more survivors that can be gained by moving facility from point A to point B, whereas this gain could not be noticed by maximizing the total demand covered.

This was the case where all demand points has 10 patients. Consider that there are 10 patients on nodes 1, 2 and 3 while there are 50 on node 4. In this case, moving the facility from A to B would increase the expected number of survivors from 12.85 to 22.17. It is seen that the gain obtained by using a survival probability function and maximizing the expected number of survivors can improve the decision substantionally depending on the number of patients, which are partly neglected while using the maximal coverage approach.

As mentioned before, this study locates emergency hospitals by using an objective function which considers survival probabilities. There are many studies in the literature examining the relationship between incident type dependent factors and the survival probability. Most of them focus on cardiac arrest survivals, and only a few among them provides survival functions of response time. In this study, different hypothetical functions are considered. In this way, it is aimed to include emergency situations that may differ in terms of degree of urgency. Our model considers a single-class of patients which means only one type of survival function is used at a time.

As both the survival probability and likelihood functions are dependent on response time, computation of this value plays an important role on the decision. Travel time in an urban area is known to be traffic dependent; however, in this study, stochasticity

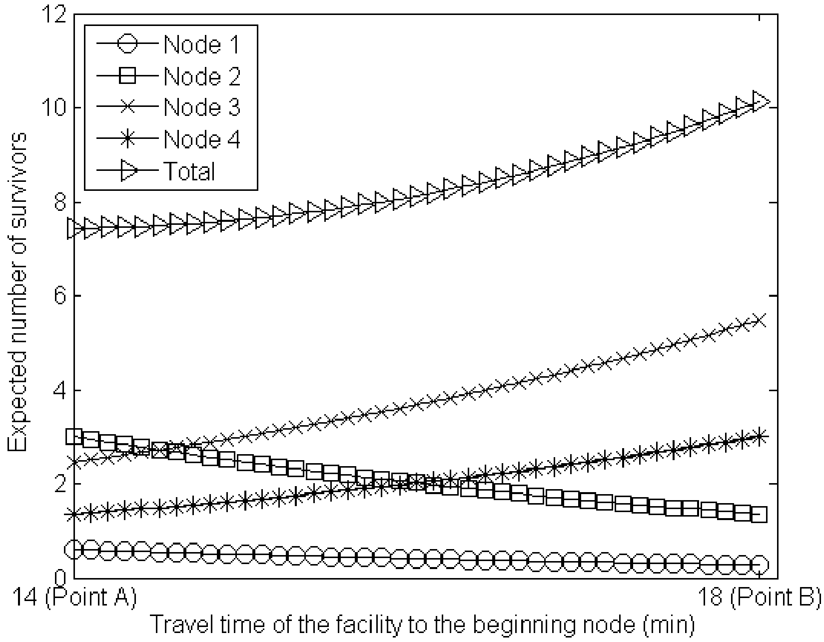

Figure 3.3: Change in the Expected Number of Survivors as the Facility Moves from A to B

in travel time is ignored. The assumption given in the study by Chanta et al [4] is used and it is assumed that 2 minutes are required to travel 1 mile.

Since the budget constraint should be considered as well, a specified number of facilities are to be located within the scope of this problem. These facilities are assumed to be identical and noncapacitated, which means that survival probability is not affected by the facility type and it is not required to consider number of patients a hospital can serve, while calculating likelihood values.

CHAPTER 4

MATHEMATICAL MODEL AND ANALYTICAL RESULTS

In this section, mathematical models for single facility and multiple facilities location problem are given and some analytical findings about the optimal locations when different survival functions are in use are explained.

4.1 Locating a Single Facility

4.1.1 Locating a Single Facility on a Line

Consider the problem of locating a single facility on a line, [AB]. This line has n vertices in set $V = \{v_1, v_2, ..., v_n\}$, consisting of the beginning and ending points as well and having a distance l_i from the beginning of the line. Patients are located on these nodes and are allowed to travel in both directions to reach the hospital. There are h_i patients on each node i whose time to the hospital is $t_i(x)$, and their survival probabilities are represented by $\pi(t_i(x))$. The facility can be located anywhere on the line, and x represents the travel time from facility to the beginning node. Under these circumstances, the problem is designed as a continuous location problem, and the following formulation maximizes the expected number of survivors on this line:

$\mathbb{P}$

$$\text{Max} \quad \sum_{i=1}^{n} h_i * \pi(t_i(x)) \tag{4.1}$$

$$\text{s. to} \quad t_i(x) \geq x - l_i \qquad \forall i \in N \tag{4.2}$$

$$t_i(x) \geq l_i - x \qquad \forall i \in N \tag{4.3}$$

$$x \in [AB] \tag{4.4}$$

Now, let's analyze the properties of this formulation under two cases: (1) π is a convex function decreasing in travel time, (2) π is a concave function decreasing in travel time.

Theorem 1. The optimal solution to problem $\mathbb{P}$ occurs at a demand node when a single facility is located on a line under case (1).

Proof

Consider the first case and let k, $k+1$ represent two adjacent demand nodes on the line. If the single facility is located on $s^* \in$ [k,$k+1$], a θ function for defining the relative location of s^* is expressed as follows;

$$\theta = \frac{t_{ks^*}}{t_{k,k+1}} \tag{4.5}$$

Equation 4.5 implies that s^* moves from k to $k+1$ through the line as θ changes from 0 to 1, where t values define travel times between any two points. In this case, travel time from any demand node i to point s^* can be defined as a function of θ.

$$\delta_i(\theta) = min\left\{t_{ik} + \theta t_{k,k+1}; t_{i,k+1} + (1-\theta)t_{k,k+1}\right\} \tag{4.6}$$

By utilizing Equation 4.6, the objective function of the problem is rewritten as follows;

$$\text{Max} \sum_{i=1}^{n} h_i * \pi(\delta_i(\theta)) \tag{4.7}$$

Since $\delta_i(\theta)$ is a linear function of θ and π is convex decreasing in t, $\pi(\delta_i(\theta))$ is found to be a convex function of θ. Consequently, the objective function is convex between k and $k+1$ which implies that moving the facility from anywhere on the line [k,$k+1$] to k or $k+1$ cannot cause the objective function value worsen. This shows that optimal solution always exists on one of the demand nodes when a convex decreasing survival function is used.

Existence of the optimal solution on the nodes in every case enables us to replace continuous formulation by a discrete location problem mathematical formulation. Additional indices, parameters, variables and the formulation are as follows:

Indices

$j = 1, ..., n \in V$ Candidate facility locations

Parameters

t_{ij} The travel time between point i and point j

$\pi(t_{ij})$ Survival probability of a patient at point i who travels to the hospital at point j

Decision Variables

$$y_j = \begin{cases} 1 & \text{if a facility is located at point } j \\ 0 & \text{otherwise} \end{cases} .$$

$$\text{Max} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} h_i * \pi(t_{ij}) * y_j \tag{4.8}$$

$$\text{s. to} \quad \sum_{j=1}^{m} y_j = 1 \tag{4.9}$$

$$y_j \epsilon \{0, 1\} \qquad \forall j \in V \tag{4.10}$$

Objective function (4.8) maximizes the expected number of survivors on the network. Constraint set (4.9) ensures that only one hospital is opened and set (4.10) specifies the integrality constraints.

Now, let's consider case (2). Under this case, it is not possible to show that optimal solution to $\mathbb{P}$ occurs at a demand node when a single facility is located on a line.

If the survival function is concave decreasing, $\pi(\delta_i(\theta))$ is found to be concave in θ between two adjacent points as well, whereas this does not give a clue about the optimal location of the facility. Therefore, the characteristic of the objective function between the beginning and ending point should be examined. For this purpose, let 1 and N represent the first and last demand nodes on the line, respectively. θ and the travel time function are modified as follows:

$$\theta = \frac{t_{1,s^*}}{t_{1,N}} \tag{4.11}$$

$$\delta_i(\theta) = max\left\{\theta t_{1,N} - t_{1,i}; t_{1,i} - \theta t_{1,N}\right\} \tag{4.12}$$

Since $\delta_i(\theta)$ is convex in θ, when survival function is concave decreasing in travel time, $\pi(\delta_i(\theta))$ becomes a concave function. This proposes that the optimal solution can exist anywhere on the line, not necessarily on demand points. The θ value that equates first derivative of the objective function to 0 will give the optimal result.

4.1.2 Locating a Single Facility on a Network

As our problem definition is to locate hospitals in an urban area, a network is required to represent demand regions and the transportation paths between them. Let *N(V,E)* be this connected and undirected network where the node set V represents demand points and E represents the links connecting them. Consider the two cases that convex and concave survival functions are used as in Section 4.1.1.

Theorem 2. The optimal solution to problem $\mathbb{P}$ occurs at a demand node when a single facility is located on a network under case (1).

Proof

Let π be a convex decreasing function of the travel time, and k, k +1 represent the source and sink nodes of any arc $e \in E$. When the single facility is located on s^* which exists on the arc e, θ and δ functions in Equations (4.5) and (4.6) are applicable to this case as well. However, shape of the δ function will be concave in this instance. Consequently, $\pi(\delta_i(\theta))$ is found to be convex, and so is the objective function given in Equation (4.7). This corresponds to optimal solution being on the nodes, which enable one to consider demand points as the candidate facility locations and use the discrete location problem formulation given by Equations (4.8), (4.9) and (4.10).

When a single facility is located on a network under case (2), which means π is taken to be a concave decreasing function of the travel time, characteristic of the objective function between two adjacent demand points is found to be neither convex nor concave. Therefore, an inference about the optimum location of the facility could not be made.

4.2 Locating Multiple Facilities

When more than one facility are to be located on a given network or line, the objective function includes one more term besides demand values and the survival probabilities. A likelihood function is used to represent the probability of a patient at a particular node selecting a specific hospital. When this probability is incorporated into the model, any conclusion about the optimum location of facilities on a line or network, under the use of concave or convex function could not been obtained. When the candidate set of facilities are defined as the demand nodes and certain breakpoints on the arcs of a network, it is observed computationally by complete enumeration that the optimal solution is not always on the demand nodes when a concave function is used. However, a similar counter example could not be found when a convex survival probability function is considered, which means that optimal solutions always appeared on the demand nodes.

Berman et. al [2] present a theorem that optimal locations exist on nodes when a convex non-increasing decay function is used with the objective of maximizing total demand weight covered. Suppose S^* is an optimal set of locations, which contains a point s^* that is an interior point on a link. Set of nodes on the network are partitioned as N_{s^*} and $\overline{N}_{s^*}$, representing the nodes that are closer to s^* and nodes that are closer to some other location on the network, respectively. The resulting objective function utilizing these set definitions is as follows:

$$\text{Max} \sum_{i \in N_{s^*}} h_i * \pi(\delta_i(\theta)) + \sum_{i \in \overline{N}_{s^*}} h_i * \pi(t_i(S^* - s^*)) \tag{4.13}$$

While the first term in Equation (4.13) is a function of θ, the second term is a constant. Since $\pi(\delta_i(\theta))$ is a convex function with respect to θ as shown in Section 4.1.1, setting θ value to 0 or 1 cannot decrease the objective function value. While it is assumed that each demand point is served by their closest facility in this study, our model considers that demand points are served by all facilities with varying probabilities. As these probabilities are calculated based on the travel times between facilities and demand nodes, changing θ value between 0 and 1 would change the travel times, consequently the probabilities. Under the condition that, likelihood values would not change as θ changes, the proof in [2] applies to our problem as well.

In this study, candidate facility locations are restricted to the set of demand points and the mathematical formulation constructed for the single facility location problem is modified as follows for the multiple facility location problem:

Decision Variable:

ES_i: Expected survival probability of the patient at node i

$\mathbb{P}1$

$$\text{Max} \quad Z1 = \sum_{i \in V} h_i * ES_i \tag{4.14}$$

$$\text{s. to} \quad \sum_{j \in V} y_j = P \tag{4.15}$$

$$ES_i = \left(\frac{\sum_{j \in V} y_j * \frac{1}{t_{ij}} * \pi(t_{ij})}{\sum_{k \in V} y_k * \frac{1}{t_{ik}}}\right) * (1 - y_i) + \pi(t_{ii}) * y_i \qquad \forall i \in V \tag{4.16}$$

$$y_j \epsilon \{0, 1\} \qquad \forall j \in V \tag{4.17}$$

In the context of this problem, it is also assumed that a patient will be treated at the hospital that is located on its region if such a facility exists. This is ensured by constraint (4.16).

As we deal with a healthcare facility location problem, providing equity in service to patients should be taken into account as well. For this purpose, two more objective functions are defined for this problem. The first one given in Equation (4.18) aims to maximize the minimum survival rate of patients on the network. The corresponding problem, $\mathbb{P}2$, considers each patient separately: that is, number of patients on nodes are not taken into account. The gap between the survival rates is minimized only by increasing the lowest value. On the other hand, the second function given in Equation (4.19) has rather a system-wide approach. $\mathbb{P}3$ aims to minimize the total weighted envy in the system, where envy is defined as the difference between survival probabilities of nodes in the network. If survival probability of a node A is less than another one, objective function value is increased by the multiplication of the difference between these two values and number of patients at node A.

$\mathbb{P}2$

$$\text{Max} \quad Z2 = \underset{i \in V}{\text{Min}} \{ES_i\} \tag{4.18}$$

s. to 4.15- 4.17

$\mathbb{P}3$

$$\text{Min} \quad Z3 = \sum_{i \in V} \sum_{l \in V, l \neq i} h_i * Max\{0, ES_l - ES_i\} \tag{4.19}$$

s. to 4.15- 4.17

In addition to these, the problem that aims to maximize expected number of survivors where each patient is served by the closest facility and survival rate depends on the response time is presented. This version of the problem will be called $\mathbb{P}4$ from now on.

Decision Variable

$$x_{ij} = \begin{cases} 1 & \text{if demand node } i \text{ is served by the facility } j \\ 0 & \text{otherwise} \end{cases}.$$

$\mathbb{P}4$

$$\text{Max} \quad Z1 = \sum_{i \in V} h_i * \pi(t_{ij}) * x_{ij} \tag{4.20}$$

$$\text{s. to} \quad \sum_{j \in V} y_j = P \tag{4.21}$$

$$\sum_{j \in V} x_{ij} = 1 \qquad \forall i \in V \tag{4.22}$$

$$x_{ij} \leq y_j \qquad \forall i, j \in V \tag{4.23}$$

$$y_j, x_{ij} \epsilon \{0, 1\} \qquad \forall j \in V \tag{4.24}$$

Drezner and Drezner [11] show in their study that the objective function value of standard p-median problem is not higher than the objective function value of gravity p-median, where the objective is minimizing the total distance traveled. A similar proof can be done for our problem as well.

Theorem 3. Let P define a set of facility locations. In this case, it can be claimed that the optimal objective function value of model $\mathbb{P}1$ is not higher than the objective function value of $\mathbb{P}4$.

Proof

$$\sum_{i \in V} h_i * \frac{\sum_{j \in V} \pi(t_{ij}) * y_j * \frac{1}{t_{ij}}}{\sum_{k \in V} (y_k * \frac{1}{t_{ik}})} \tag{4.25}$$

$$= \sum_{i \in V} h_i * \frac{\sum_{j \in P} \pi(t_{ij}) * \frac{1}{t_{ij}}}{\sum_{k \in P} \frac{1}{t_{ik}}} \tag{4.26}$$

$$\leq \sum_{i \in V} h_i * \frac{\sum_{j \in P} \operatorname*{Max}_{j \in P} \{\pi(t_{ij})\} * \frac{1}{t_{ij}}}{\sum_{k \in P} \frac{1}{t_{ik}}} \tag{4.27}$$

$$= \sum_{i \in V} h_i * \operatorname*{Max}_{j \in P} \{\pi(t_{ij})\} \tag{4.28}$$

CHAPTER 5

SOLUTION METHODOLOGY

The mathematical programming model presented in Section 4.2 is nonlinear, therefore it is not guaranteed to reach an optimal solution using commercial optimization solvers. Moreover, computation time increases enormously as number of nodes and facilities to be located increases. For this reason, a Genetic Algorithm based solution approach is developed which successfully solves large instances in reasonable computation times. Structure of the algorithm is as follows;

- **Representation Scheme**

  Our problem has two sets of decision variables and the ones defining likelihood values are nonlinear functions of location decisions variables. Therefore, defining a representation scheme only for y_j variables would be enough.

  We represent the facility location with a $1 \times P$ vector where P is the number of facilities to be located. Considering there are n candidate sites, each element of the array can take any value ranging from 1 to n as long as they are different. In this representation, i^{th} element shows which candidate site is opened as the i^{th} one, where i is defined in the set $\{1, \cdots, P\}$. An example with P=4 and n=90 candidate locations is

10	28	35	79

- **Fitness Function**

  Z1 is used as the fitness function. As we aim to maximize the expected number of survivors in $\mathbb{P}1$, higher values stand for higher fitness.

- **Initial Population Generation**

  Pop_size individuals are generated randomly.

- **Parent Selection**

  In any generation, all individuals in the population are allowed to be selected as parents. Two parents are selected randomly to generate offspring and removed from the population. This process is repeated for *pop_size/2* times to select all parents for reproduction.

- **Crossover Operators**

  Crossover is applied to all parents with a determined probability p_c. Crossover operator merges the arrays representing two parents. If the same gene appears in both individuals, it is copied to both offspring. Later, an array consisting of remaining genes is constructed and shifted in order to ensure diversity. First and second halves of the newly constructed array are transmitted to first and second offspring, respectively. An example of crossover process is

Parent 1:	10	28	35	79
Parent 2:	10	32	46	83

  As 10 appears in both parents, it becomes the first gene of both offspring. Mergence of remaning genes and newly consructed array by reordering remaining genes are as follows;

1:	28	35	79	32	46	83
2:	79	46	35	28	83	32

  Resulting offspring;

Offspring 1:	10	79	46	35
Offspring 2:	10	28	83	32

- **Nearest Neighbor Search**

  Since *pop_size* can not be large enough to capture all candidate locations in the initial population, ensuring the generation of each location in the further steps is desired. In order to manage this wisely rather than generating random genes, a nearest neighbor search algorithm has been proposed which is applied to all children. A random gene is selected in each individual and two more offspring are generated by altering the subject gene with two closest nodes. An example of this this process is as follows:

  Offspring 1:

10	79	46	35

  A number between 1 and p is generated randomly.

  Say p=3. Let the closest nodes to node 46 be 51 and 65. In this case, nearest neighbor search algorithm generates the following offspring.

  Offspring 2:

10	79	51	35

  Offspring 3:

10	79	65	35

  The best of these three individuals in terms of fitness function value is selected as the offspring and other two are disposed.

- **Mutation Operators**

  Mutation is applied to each individual bit of each offspring with a predetermined probability p_m. If a gene is to be mutated, it is replaced with a location chosen randomly from the candidate set as long as it does not cause same genes taking place in the subject individual. Otherwise, random selection process continues until a feasible solution is obtained.

- **Fitness Function Evaluation and New Population Generation**

  The population generated in the previous iteration and children generated by crossover and succeeding local search, mutation operations are gathered in a pool. After fitness functions are calculated, individuals are sorted in decreasing fitness values. In order to ensure spreading good features, best *pop_size* individuals are selected to form the new population.

- **Stopping Condition**

  When 95% of the individuals in the population reach to the same objective function value, the algorithm stops.

CHAPTER 6

COMPUTATIONAL STUDY

In this section, first, computational study on the assessment of the proposed solution approach is presented. In more detail, in Section 6.1 problem instances and survival probability functions used for testing the model are given. In Section 6.2, the parameters of GA are fine tuned. Section 6.3 gives the results of the proposed solution approach, and evaluates them by comparing with the optimal values. Here, we also statistically analyze the effects of factors such as city structure and problem size on the algorithm performance. While the first three parts are mainly about constructing the computational setting and a successful solution procedure, in the rest of this section, characteristics of the presented problems are analyzed using computational results and effects of some parameter changes on decisions are investigated. In Section 6.4, we aim to observe the differences brought by the gravity idea on location decisions. Location decisions of three problems presented in the previous section are given and compared in Section 6.5. The effect of number of facilities on the optimal objective function values is analyzed, and some properties worth discussing are explained with the help of a small example in Section 6.6. Finally, decisions obtained by alternative objective functions are evaluated using the performance measures in Section 6.7.

6.1 Computational Setting

This section presents the specifications of two important points of the computational setting, namely, city structure and survival function. Demand regions are randomly generated in a way to represent two different city structures. In the first one, demand

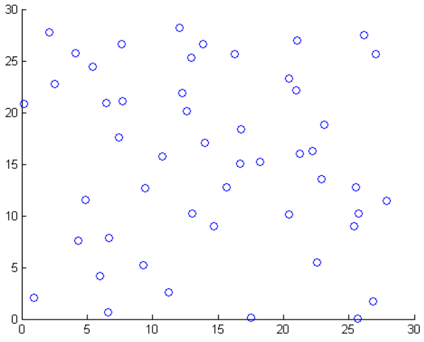

Figure 6.1: Uniformly Distributed Demand Regions

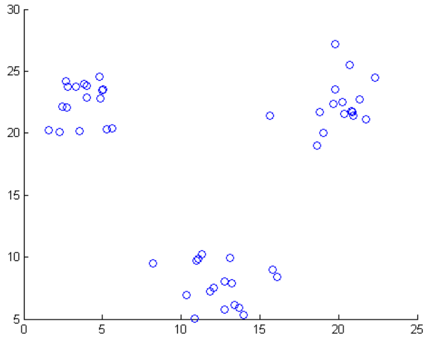

Figure 6.2: Clustered Demand Regions with 3 Centers

Table 6.1: Problem Instances

N	City Structure	Number of Centers	p
20	Uniform	-	3,4,5,6,7
30	Uniform	-	3,4,5,6,7
40	Uniform	-	3,4,5,6,7
50	Uniform	-	3,4,5,6
100	Uniform	-	2,3,4
20	Clustered	2,3	3,4,5,6,7
30	Clustered	2,3,4	3,4,5,6,7
40	Clustered	2,3,4	3,4,5,6,7
50	Clustered	2,3,4,5	3,4,5,6
100	Clustered	2,4,6,8,10	2,3,4

regions are uniformly distributed in a 30x30 miles city. Second structure consists of normally distributed centers and normally distributed demand regions around those centers. They can be seen in Figures 6.1 and 6.2, respectively. It is assumed that all regions are connected, therefore Euclidean distances are used when travel times between any nodes are calculated. Problem instances used for testing the performance of the proposed solution approach are given at Table 6.1.

For simplicity, instances are named based on the following rule:

City Structure (C for Clustered, U for Uniform)_k(Number of clusters)_(Number of demand regions)_p(Number of facilities to be located)

The second point needs to be determined for computational study is the survival probability. Two survival functions used in the literature for reflecting survival chance from a cardiac arrest situation are mentioned in Section 2.2. These are decreasing convex functions of the response time, which lead to death in minutes. In order to include other emergency situations except cardiac arrests and observe the effect of using different survival functions, two hypothetical functions are generated, which can be seen in Figure 6.3. The convex function is given by Equation (6.1), and the concave one is given by Equations (6.2a) and (6.2b). In order to prevent negative survival rates, it is assumed that survival probability is 0 for travel times higher than or equal to 74. Representing the emergency situation with a concave function instead of the convex one obviously reflects a decrease in the level of urgency. Since the survival functions used for a similar purpose in the previous studies are convex decreasing, parameter setting and performance evaluation of the algorithm is conducted using the hypothetical convex function generated. The concave function is used for the comparison made in Section 6.4.

$$scvx(t) = (e^{0.262t} + 0.1)^{-0.15} \tag{6.1}$$

$$scnv(t) = \begin{cases} 0 & \text{if} \geq 74 \quad (6.2a) \\ 0.99 - (\dfrac{t^{1.8} * 3}{84^2}) & \text{otherwise} \quad (6.2b) \end{cases}$$

It should also be noted that we mainly focus on maximizing expected number of survivors on the network in this study, which is presented by $\mathbb{P}1$. Therefore, the proposed solution approach is developed particularly to solve $\mathbb{P}1$ successfully. However, it is also used to solve $\mathbb{P}2$ and $\mathbb{P}3$ by only altering the objective function equation in the algorithm. In the next section, fine tuning of GA parameters using this computational setting is explained.

6.2 Calibration of Genetic Algorithm Parameters

As initial runs of the genetic algorithm based solution approach gives satisfying results, no more attempt have been made to try a new approach or add new features to

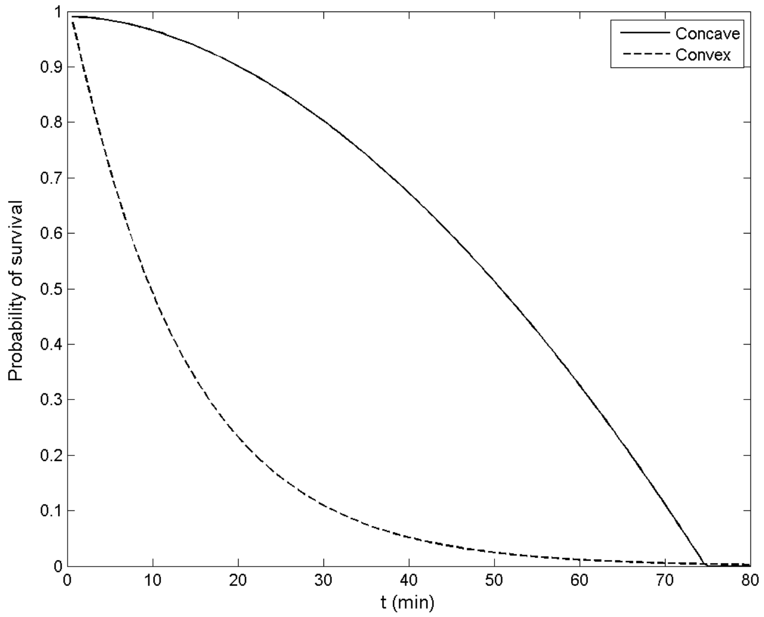

Figure 6.3: Survival Functions

the existing one. Alternatively, it is aimed to improve performance of the solution approach by tuning the algorithm parameters. Trial experiments led us to use the values given in Table 6.2 for parameters and conduct a full factorial design with two levels and three factors.

Table 6.2: Levels of Parameters

Factors	**Levels**	
	-	+
Population size	50 (30)	100 (50)
Crossover probability	0.6	1
Mutation probability	0.01	0.05

The number of feasible solutions in our instances range between 1140 and 18,643,560. As the population size constitutes only a small proportion of the search space, increasing it further than 100 individuals is not approved. However, for small instances, working with 100 individuals may unnecessarily increase the computation time. Therefore, for problems with less than 20,000 solutions, levels of *pop_size* are defined as 30 and 50, for the low and high levels, respectively. Two problem instances, which have uniformly distributed demand regions, are selected to represent the small and large instances in the experimental design;

- $Ins1$: *N*=50, *p*=3

- $Ins2$: *N*=100, *p*=4

Objective function values for *Ins1* and *Ins2* are compared with the optimal values and percentage errors are taken into account for an experimental design. Minitab outputs consisting of main effects and interaction plots of the percentage errors for $Ins1$ can be seen at Figure 6.4 and 6.5. Figure 6.4 shows that increasing population size and crossover probability has a decreasing effect on the percentage error, while mutation probability does not change it remarkably. However, Figure 6.5 suggests that there is an interaction between crossover probability and mutation probability, so this should be analyzed in order to decide which level of mutation probability should be selected for higher level of crossover probability. When crossover probability is equal to 1, increasing mutation probability from 0.01 to 0.05 decreases percentage error as well. Therefore, for small instances, parameter values are selected as 50, 1 and 0.05 for pop_size, p_c and p_m, respectively.

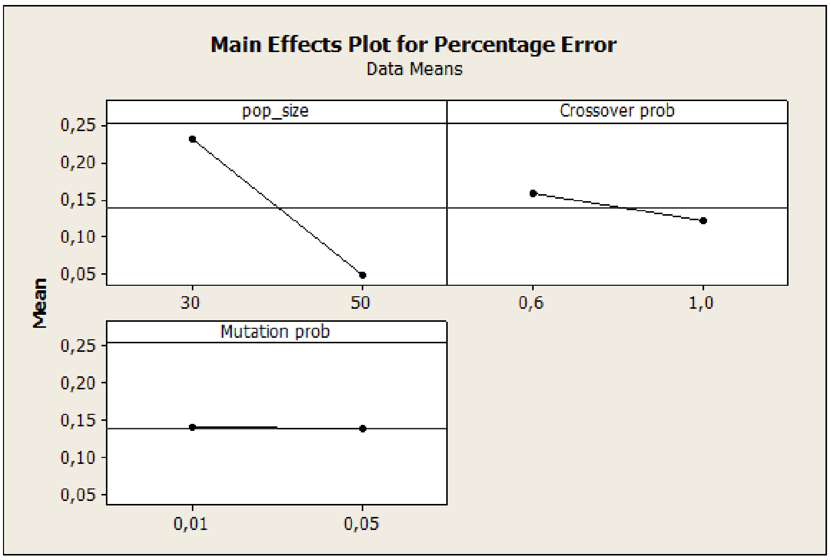

Figure 6.4: Main Effects Plot for Percentage Error for *Ins1*

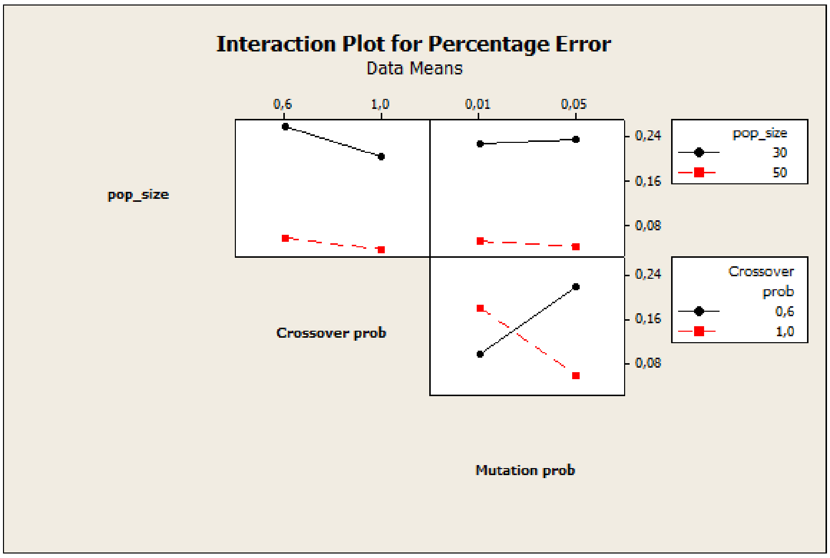

Figure 6.5: Interaction Plot for Percentage Error for *Ins1*

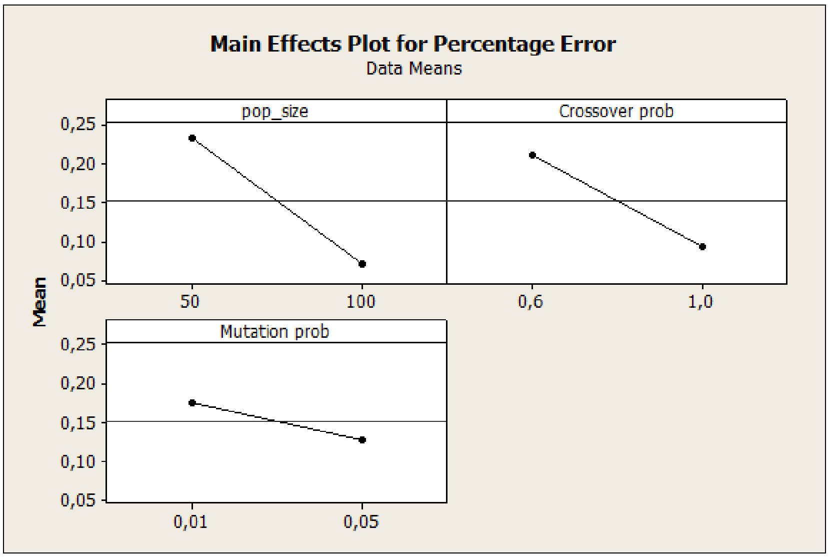

Figure 6.6: Main Effects Plot for Percentage Error for *Ins2*

Minitab outputs consisting of main effects and interaction plots of the percentage errors for $Ins2$ is can be seen at Figures 6.6 and 6.7. These two figures illustrate that the algorithm gives better results for higher values of all three parameters and that there is no interaction between them. Therefore, for large instances, parameter values are selected as 100, 1 and 0.05 for pop_size, p_c and p_m, respectively.

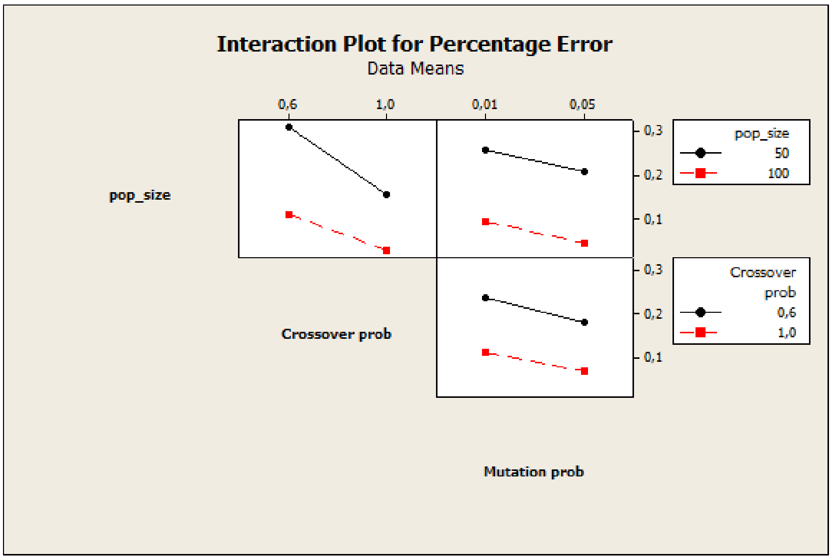

Figure 6.7: Interaction Plot for Percentage Error for *Ins2*

6.3 Performance Evaluation of the Solution Approach

Four sets of instances given in Table 6.1 are generated randomly for testing the model. Optimal solutions of these instances are obtained by total enumeration, and the proposed solution approach is run for 100 replicates. Table A.1 summarizes the average values reached by comparing total enumeration and genetic algorithm results of four sets. These experiments were run on a 3.10 GHz Intel Core i7-4770S computer with 16.0 GB of RAM.

Table A.1 presents the performance of the proposed solution approach for each problem instance. Percent deviation stands for the gap between GA solution and optimal value. Minimum, maximum deviations in 100 replications and average of them are given under this measure. Quality gives the number of replications that optimal value was reached in 100 replications, which can be considered as the probability of finding the optimal solution using our solution approach. A summary of these results are as follows;

- Average quality in 93 instances are 96.33%
- Maximum of the maximum deviations from optimal is 0.438% observed for

instance C_k2_100_p2.

- Minimum quality value is 76.25% observed for instance C_k8_100_p2. Maximum deviation from optimal for this instance is 0.322%.
- It takes in total 863.96 seconds to solve 93 instances using GA, while 273279.86 seconds are necessary when total enumeration is used.

As mentioned before, instances consist of differently structured cities and different number of clusters, demand regions and facilities to be located. This enables us to analyze the possible factors that may affect performance of our algorithm. We are particularly interested in how the algorithm performs in different city structures, with different problem sizes and with different number of clusters of demand regions. Quality and average deviation values are considered for these tests.

Observation 1 *The proposed solution approach performs in clustered cities at least as good as it does in uniformly distributed demand regions.*

Table 6.1 shows 22 instances have uniform structures and 71 instances have clustered structures. Considering all instances for a comparison would mean including different number of instances for different N values in clustered and uniform sets. For a fair comparison, three paired tests are applied and performance in uniform structures are compared with clustered ones with 2, 3 and 4 centers. As normality assumption is not satisfied for both the quality and average deviation values, Wilcoxon Signed Rank Test is utilized. Notation used is as follows;

$Difference\_q$ = Quality for uniform structure - Quality for clustered structure
$Difference\_d$ = Maximum deviation for uniform structure - Maximum deviation for clustered structure

1) Comparison for uniform and 2 centered clustered structures

H_0= $Md_{difference\_q}$= 0
H_1= $Md_{difference\_q} \neq 0$

$H_0 = Md_{difference\_d} = 0$

$H_1 = Md_{difference\_d} \neq 0$

Table 6.3: Test Results for Uniform and 2 Centered Clustered Structures

	N	Wilcoxin Statistics	p	Estimated Median
Difference_q	22	174.0	0.127	1.750
Difference_d	22	142.0	0.626	0.0002

P values for both quality and average deviation reveal that algorithm performance does not differ in these structures at a 0.05 significance level.

2) Comparison for uniform and 3 centered clustered structures

For this case, hypothesis testing for the inequality of performances shows that algorithm success changes between two structures. Therefore, the following tests are applied in order to understand for which one the performance improves;

$H_0 = Md_{difference\_q} = 0$

$H_1 = Md_{difference\_q} \leq 0$

$H_0 = Md_{difference\_d} = 0$

$H_1 = Md_{difference\_d} \geq 0$

Table 6.4: Test Results for Uniform and 3 Centered Clustered Structures

	N	Wilcoxin Statistics	p	Estimated Median
Difference_q	19	32.5	0.011	-1.313
Difference_d	19	149.0	0.003	0.0015

P values reveal that average deviation is significantly lower and quality is significantly higher for 3 centered clustered structures.

3)Comparison for uniform and 4 centered clustered structures

H_0 = $Md_{difference\_q}$= 0
H_1 = $Md_{difference\_q} \neq 0$

H_0 = $Md_{difference\_d}$=0
H_1 = $Md_{difference\_d} \neq 0$

Table 6.5: Test Results for Uniform and 4 Centered Clustered Structures

	N	Wilcoxin Statistics	p	Estimated Median
Difference_q	19	53.5	0.287	-1.000
Difference_d	19	107.0	0.156	0.0016

P values for both quality and average deviation reveal that algorithm performance does not differ in these structures at a 0.05 significance level.

To sum up, three tests show that algorithm performance either improves or does not change significantly when it is used in a clustered structure instead of a uniform one.

Observation 2 *Problem size does not affect algorithm performance significantly.*
In this part, it is aimed to observe how the algorithm performs for different problem sizes. Since all solutions containing p different nodes are feasible for $\mathbb{P}1$, number of all p-combinations in set N defines the problem size. Considering the break points of number solutions in our instances, five classes for problem sizes are determined. Due to violation of normality assumption, a Kruskal-Wallis Test is applied to these samples. Results are presented in Tables 6.6 and 6.7.

Table 6.6: Test Results of Five Classes for Average Deviation Values

Class	Number of solutions	N	Median	Ave. rank	Z
1	1000-10000	20	0.0026	58.1	2.08
2	10000-100000	23	0.0005	37.1	-2.02
3	100000-1000000	22	0.0006	44.3	-0.53
4	1000000-10000000	19	0.0009	45.5	-0.27
5	10000000-20000000	9	0.0017	57.2	1.20
DF=4 p=0.091					

Table 6.7: Test Results of Five Classes for Quality Values

Class	Number of solutions	N	Median	Ave. rank	Z
1	1000-10000	20	97.88	44.6	-0.45
2	10000-100000	23	99.25	59.2	2.49
3	100000-1000000	22	98.38	48.6	0.32
4	1000000-10000000	19	97.75	40.5	-1.17
5	10000000-20000000	9	96.25	31.1	-1.86
DF=4 p=0.058					

There are two main results obtained by these analyses. First, it is seen that algorithm performance does not differ significantly between these five classes at a 0.05 significance level. Second, there is not a decreasing trend on the performance as problem size increases. On the contrary, it is seen that algorithm performance is at lowest values for classes 1 and 5, which are the smallest and largest instances.

Observation 3 *A trend in algorithm performance with increasing number of clusters in cities is not observed.*

We are interested in how our algorithm performs for different number of clustered regions in a city where number of nodes on the network remain constant. Since sample sizes for these observations are rather small, hypothesis testing is not utilized. Instead, we take averages of two performance measures over valid p values for cities having 100, 50 and 40 demand regions. Results are presented in Figures 6.8 and 6.9.

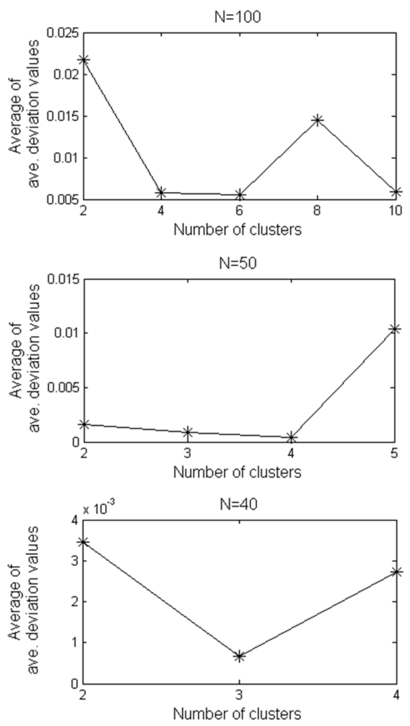

Figure 6.8: Change in the Average Deviation with Increasing Number of Clusters

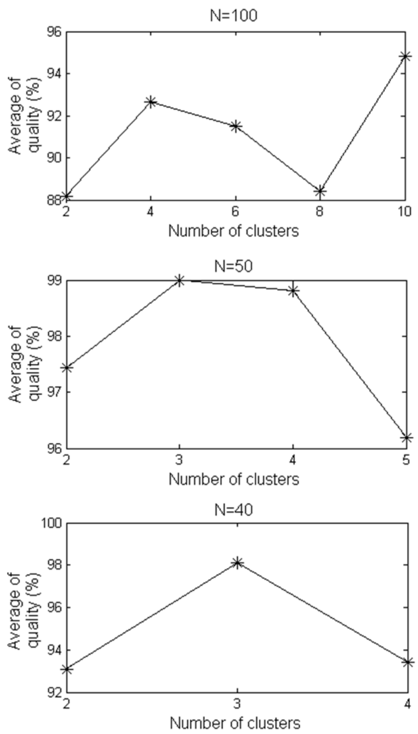

Figure 6.9: Change in the Quality with Increasing Number of Clusters

It is seen that both measures are most of the time in accordance with each other and neither of them shows a trend in performance. So, it is not possible to claim that algorithm performs better for a specified number of clusters.

To sum up, based on given analyses, we claim that the proposed solution approach is robust to problem size, city structure and number of centers in clustered cities.

6.4 Analyzing the Effects of Gravity Idea on the Location Decisions

Our study combines gradual coverage and gravity models in order to approach the problem of locating healthcare facilities in a more realistic way and make better decisions for the sake of all. Examples given in Section 2.2 and Chapter 3 reveals the effect of using a survival function instead of partitioning regions as covered or not on location decisions. Similarly, contribution of incorporating the gravity idea when locating healthcare facilities is questioned using the computational results in this sec-

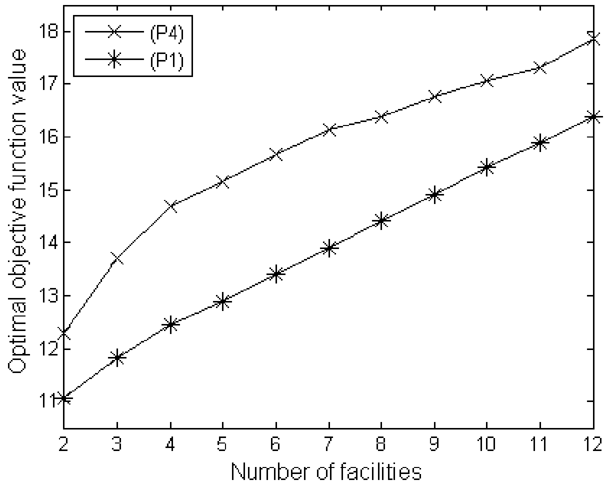

Figure 6.10: Z1 Values Under the Optimal Solution to $\mathbb{P}1$ and $\mathbb{P}4$ for N=20

tion. Therefore, results of $\mathbb{P}1$ are compared with results $\mathbb{P}4$. Figure 6.10 presents the optimal objective function values of $\mathbb{P}1$ and $\mathbb{P}4$ which are solved using the same network and same survival probability function.

As the figure illustrates, Z1 values obtained under $\mathbb{P}4$ are higher than Z1 values obtained under $\mathbb{P}1$ for all p values, which is an expected result as proposed in Section 4.2. Secondly, locating first 2 to 4 facilities makes the greatest contribution both when $\mathbb{P}1$ and $\mathbb{P}4$ are considered. Later, as p value increases, marginal increase in Z1 under $\mathbb{P}1$ does not change remarkably, which means the shape of the function is close to linear. However, there is a rapid decrease in the marginal increase of Z1 under $\mathbb{P}4$ when 5 facilities are located instead of 4. This is caused by the fact that locating one more hospital changes the state of less regions as p increases since each node is served by the closest facility, where state stands for the expected survival probability. However, when $\mathbb{P}1$ is solved, locating one more facility is beneficial for some regions while it worsens the state of other regions whose expected distance to the existing facilities were shorter than distance to the the newly located facility. Reasons of this situation will be explained in detail and simplified by an example in Section 6.6. Here, we analyze how the marginal increase in objectives of two models should be interpreted for practical reasons. In facility location problems, it may be aimed to select the best locations for p facilities or decide the number of facilities to be located due to the

Table 6.8: Comparison of $\mathbb{P}1$ and $\mathbb{P}4$

Demand	Convex Function		Concave Function	
	CinL(%)	CinES(%)	CinL(%)	CinES(%)
Unit	98.9	5.81	98.9	1.65
Random values between [1,10]	98.9	4.52	100	1.15

budget constraint. When p is a decision variable, the gain on the objective function value obtained by locating $p + 1$ facilities instead of p may be evaluated in order to decide if the gain justifies additional expense. At this point, we see from the figure that marginal increases and changes in the slope differ remarkably under $\mathbb{P}1$ and $\mathbb{P}4$. This suggests that decisions about the optimal number of locations may differ when proximity assumption in $\mathbb{P}4$ is relaxed. That is, locating 8 facilities instead of 7 may be found unnecessary when the gain and costs are evaluated under the assumption of each patient being treated at the closest hospital. However, when this assumptions is replaced with a more realistic one, $\mathbb{P}1$ is obtained and the marginal increase of Z1 may suggest that locating more hospital is affordable considering the resulting increase in wellness of people.

With the intention of observing to what degree decisions change when proximity assumption is relaxed, $\mathbb{P}1$ and $\mathbb{P}4$ are solved on the instances given at Table 6.1. Instances are solved with both convex and concave survival functions in order to see whether the level of urgency affects the percentage change of solutions obtained by $\mathbb{P}1$ and $\mathbb{P}4$. For similar purposes, it is firstly assumed that there is a single patient at each demand point. Then, number of patients on each node are given random values values between [1,10]. For a fair comparison, total expected number of survivors obtained by solutions of $\mathbb{P}4$ are calculated by using the objective function and the likelihood function of $\mathbb{P}1$.

Observation 4 *Relaxing proximity assumption changes solutions almost always regardless of the survival function shape and demand quantities.*

Results obtained by comparing solutions of 93 problem states are presented in Table 6.8. CinL shows the percent of problem instances for which the optimal locations under $\mathbb{P}1$ and $\mathbb{P}4$ change. If the optimal location sets are not exactly the same, then they are assumed to be different. CinES presents the percent change in total expected number of survivors when $\mathbb{P}1$ is used instead of $\mathbb{P}4$. CinL values suggest that optimal locations change almost always when gravity idea is incorporated into the decision process. Having varying demand over regions do not alter CinL values remarkably. However, the decrease on CinES values for both convex and concave functions may suggest that optimal locations tend to be on nodes having higher demand amounts in both $\mathbb{P}1$ and $\mathbb{P}4$, which results in having more nodes in the intersection of optimal location sets of these problems.

6.5 Comparison of Alternative Objective Functions

It is mentioned before that providing equity is an important issue in healthcare problems, which led us to consider two more objective functions for location decisions of healthcare facilities. Among these, $\mathbb{P}2$ has an individual basis approach and it aims to increase the minimum expected survival probability on the network as much as possible. On the contrary, $\mathbb{P}3$ has rather a system-wide approach and aims to minimize the total difference between survival levels, which is defined as "envy". With the purpose of observing the difference on locations decision each problem leads to, $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ are solved on the same network having 50 regions and one patient in each region.

Empty circles in Figure 6.11 represent demand regions, while filled squares represent demand regions with a facility located on it. When the figure is analyzed, it is seen that $\mathbb{P}1$ locates facilities either close to center of city or around the regions between center and corners. Facilities are not located exactly on nodes lying closest to the corners since proximity to as much regions as possible is desired. Due to a similar reason, optimal locations exist near the center of clusters consisting of close neighboring regions. $\mathbb{P}2$ divides the city in four (equal to p value) regions and locates hospitals close to the centers of them. In this way, a big proportion of the patients' expected distance to hospitals become neither too short nor too long. By pursuing proximity of hospitals to every single region, minimum survival rate in the system is maximized. As mentioned before, $\mathbb{P}3$ tries to minimize the total envy system by keeping survival probabilities of regions close to each other. At this point, one may concern about that if this situation could lead to keeping probabilities at low levels without pursuing high number of survivors in the system. However, since there have to be p facilities located, we already know that it is inevitable to have 4 regions with maximum possible survival rate. Therefore, the model tries to bring other regions' survival probabilities to levels which are not far distant from each other and also close to the maximum survival rate as much as possible. In a way, it could be stated that variance in the system is minimized by this problem when there are equal number of patients at each region.

Later, it is assumed that 16 regions taking place on the upper left side of the city could have more than one patient and randomly generated values between [1,10] are assigned to them as demand amounts. There are still one patient at the each one of other regions. It is aimed to observe the effect of demand concentrating over a region on decisions of all three problems. Results of $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ with new demand amounts are presented in Figure 6.12. As $\mathbb{P}1$ aims to maximize expected number of survivors in the system, proximity to more patients is taken into account primarily. Therefore, hospitals are located on the upper left part of the city, where most of the patients take place. $\mathbb{P}2$ considers each individual's survival probability separately, so demand amounts on nodes do not alter location decisions as seen. $\mathbb{P}3$ minimizes total weighted envy, where weights are associated with number of patients on nodes. Therefore, we see that it gives priority to service of higher demand nodes when compared to the solution in Figure 6.11; however, proximity to furthest areas is not neglected completely, even if there is a single patient there.

6.6 The Effect of Number of Facilities on the Optimized Objective Functions

In this section, we will analyze how Z1, Z2 and Z3 values change as number of facilities to be located increases. All $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ are solved on a network consisting of uniformly distributed 20 demand regions with one patient at each one.

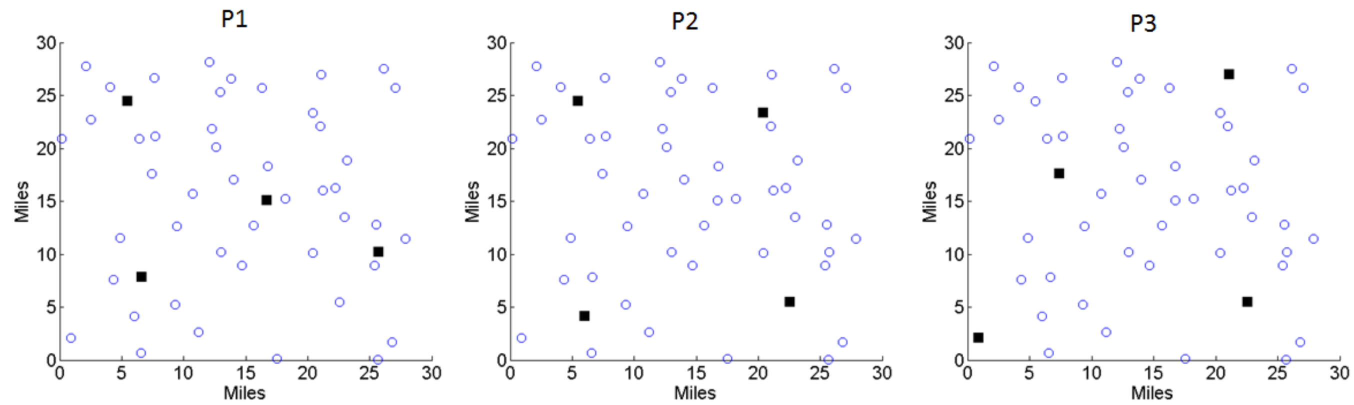

Figure 6.11: Single Patient at Each Demand Node

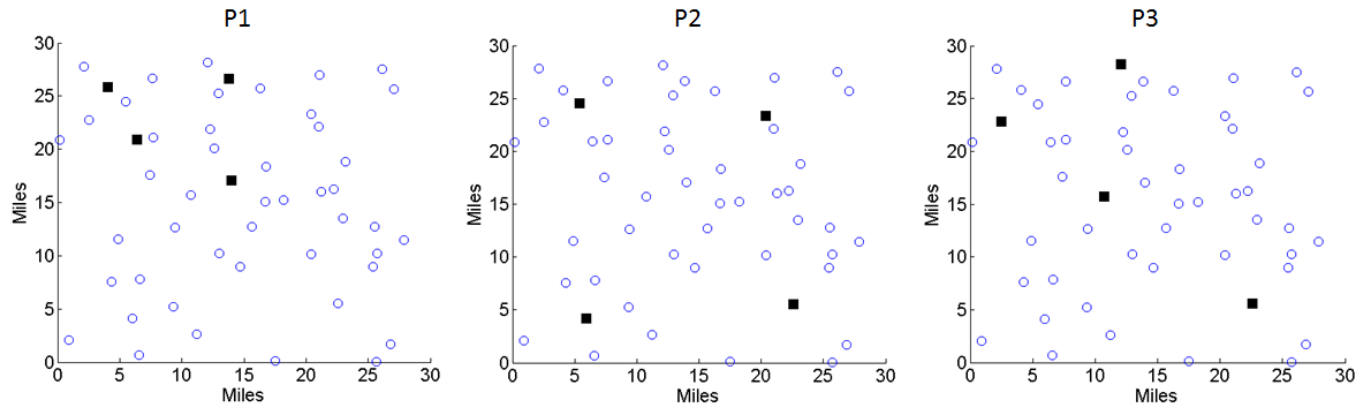

Figure 6.12: Demand is Concentrated Over the Upper Left Corner of the City

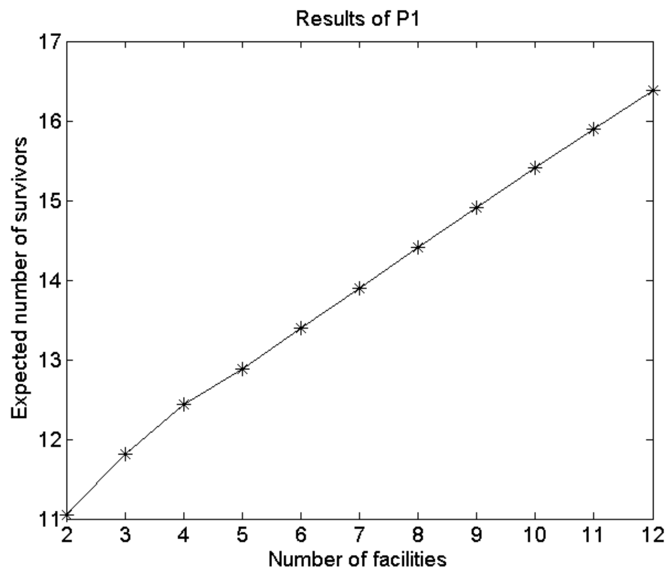

Figure 6.13: Results of $\mathbb{P}1$

Results of $\mathbb{P}1$ are shown in Figure 6.13. As discussed before, optimized expected number of survivors (ENS) on the network strictly increases as p value increases. The curve has a linear-like shape which means marginal increases have close values.

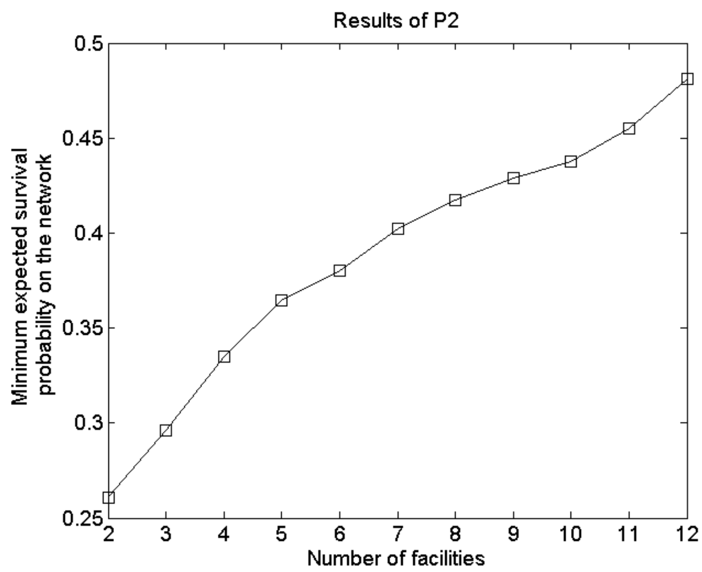

Figure 6.14: Results of $\mathbb{P}2$

Results of $\mathbb{P}2$ are presented in Figure 6.14. We see that Z2 values monotonously increase as more facilities are located. Increasing p until 5 brings a remarkable improvement to minimum expected survival probability (MESP) on the network. The concave-like shape of the objective function curve until p=10 indicates that marginal increases tend to lower as more facilities are located.

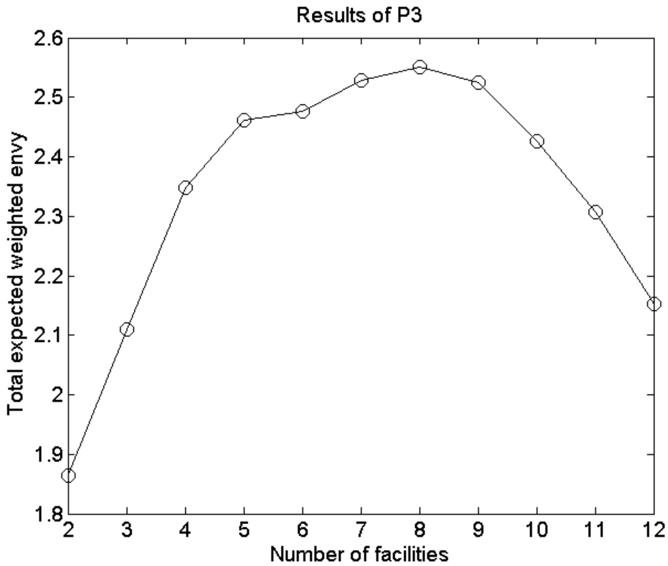

Figure 6.15: Results of $\mathbb{P}3$

Results of $\mathbb{P}3$ are presented in Figure 6.15. As mentioned before, $\mathbb{P}3$ aims to minimize the total expected weighted envy (TEWE) on the network. However, we see that TEWE strictly increases, that is Z3 value worsens, as number of located facilities increases up to some point. After that critical p value, Z3 has a monotonously decreasing behavior.

Observation 5 *Increasing the number of facilities located sometimes causes a worsening in Z2 and Z3.*

Even though it has not been observed on the objective function value of $\mathbb{P}2$ in Figure 6.14, locating a new facility causes a decrease in MESP for some problem instances. An example of this situation is seen when the model is solved on a network of uniformly distributed 30 demand regions and it is presented in Figure 6.16. Reasons of why $\mathbb{P}2$ and $\mathbb{P}3$ behave in this manner are explained with the following simple example.

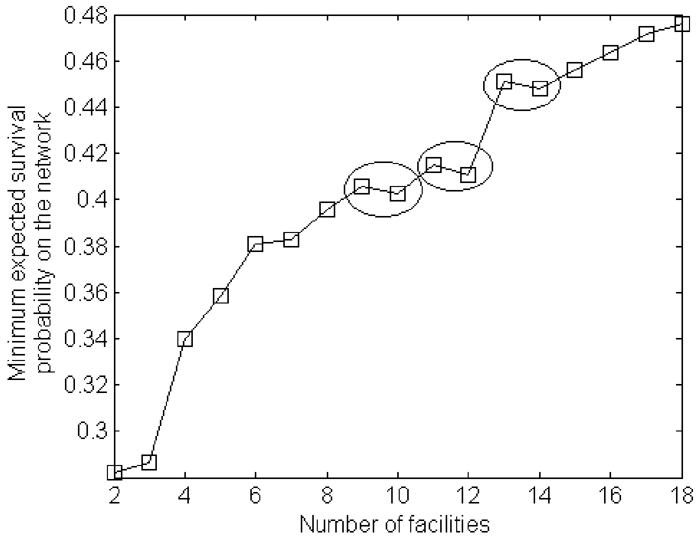

Figure 6.16: Results of $\mathbb{P}2$ for N=30

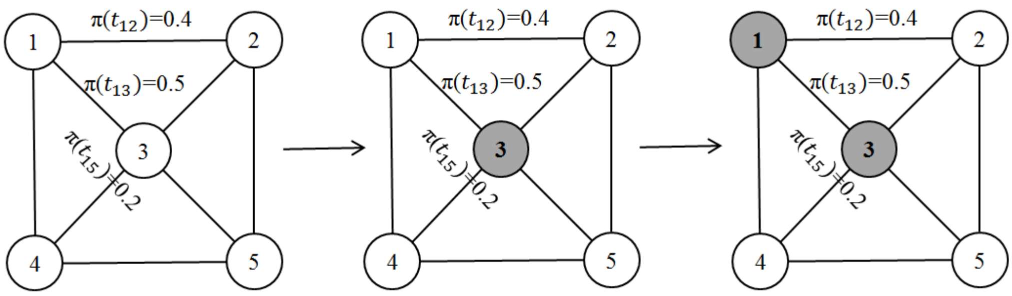

Figure 6.17: Solutions of $\mathbb{P}2$ and $\mathbb{P}3$ Separately

Consider the network given in Figure 6.17 consisting of 5 demand regions and equal number of patients on each of them. For simplicity, exact values of travel times between nodes are not given, instead, survival probabilities obtained by a hypothetical survival function are presented in the figure. Other probabilities are as follows;

- $\pi(t_{12})=\pi(t_{14})=\pi(t_{45})=\pi(t_{25})$
- $\pi(t_{13})=\pi(t_{23})=\pi(t_{34})=\pi(t_{35})$

- $\pi(t_{15})$=$\pi(t_{24})$
- $\pi(t_{11})$=$\pi(t_{22})$=$\pi(t_{33})$=$\pi(t_{44})$=$\pi(t_{55})$=1

Additionally, likelihood values are calculated using the t_{12}=$\sqrt{2}t_{13}$ equation and other equalities in distances implied by the items given above.

Let's consider $\mathbb{P}2$ first and locate a single facility with the aim of maximizing MESP on this network. Locating it on one of 1, 2, 4 and 5 would results in MESP value being equal to 0.2. When it is located at the center instead, that is node 3, this value becomes 0.5. Now let's examine what happens when p=2 for this problem. Alternatives and resulting survival expected probabilities at each node are given at Table 6.9.

Table 6.9: Resulting Survival Probabilities When Two Facilities are Located

Locations	Node 1	Node 2	Node 3	Node 4	Node 5	Minimum expected survival probability in the system
1&2	1	1	($\frac{1}{2}$*0.5) +($\frac{1}{2}$*0.5) =0.5	($\frac{0.5}{0.85}$*0.4) +($\frac{0.35}{0.85}$*0.2) =0.31	0.31	0.31
1&3	1	($\frac{0.5}{1.2}$*0.4) +($\frac{0.7}{1.2}$*0.5) =0.46	1	0.46	($\frac{2}{3}$*0.5) +($\frac{1}{3}$*0.2) =0.4	0.4
1&5	1	($\frac{1}{2}$*0.4) +($\frac{1}{2}$*0.4) =0.4	($\frac{1}{2}$*0.5) +($\frac{1}{2}$*0.5) =0.5	0.4	1	0.4

At it is seen from Table 6.9, maximum value of MESP is obtained by locating facilities either on 1 & 3 or 1 & 5, and this value is equal to 0.4. When two facilities are located instead of one, we assume that patients could go to any of them with the given probabilities. For some regions, that are closer to 3 than 1, this means traveling to the further hospital from time to time, which causes a worsening in their state of wellness. If it was assumed that each node is covered by the closest region as in traditional models, then locating the second facility on node 1 would only change state of node 1 and not cause any worsening in terms of the objective function value.

Table 6.10: Results of the Instances in Figure 6.17

	p=1 (Facility is located at node 3)	p=2 (Facilities are located at nodes 1&3
	Expected Survival Probability	Expected Survival Probability
Node 1	0.5	1
Node 2	0.5	0.46
Node 3	1	1
Node 4	0.5	0.46
Node 5	0.5	0.4
Min. expected survival probability in the system	0.5	0.4
Total expected weighted envy in the system	0.5+0.5+0+0.5+0.5 =2	0+(0.54+0.54)+(0.54 +0.54)+(0.6+0.6 +0.06+0.06)=3.48

Now, let's consider $\mathbb{P}3$ and minimize total expected weighted envy when a single facility is located. Alternatives and the resulting TEWE are as follows;

- Located at node 1: 0+(0.6+0.1)+(0.5)+(0.6+0.1)+(0.8+0.2+0.3+0.2)=3.4
- Located at node 3: (0.5)+(0.5)+0+(0.5)+(0.5)=2

A single facility is located on node 3 considering the envy values above. When two facilities are to be located, survival probabilities given in Table 6.9 are considered and envy amounts for alternatives are calculated as follows;

- Located at node 1&2: 0+ 0+ (0.5+0.5)+ (0.69+0.69+0.19)+ (0.69+0.69+0.19) =4.14

- Located at node 1&3: 0+ (0.54+0.54)+ (0.54+0.54)+ (0.06+0.06+0.6+0.6) +0 = 3.48

- Located at node 1&5: 0+ (0.6+0.1+0.6) + (0.5+0.5)+ (0.6+0.1+0.6) +0 =3.6

Optimal locations for two facilities are determined as nodes 1 & 3 considering these values. When the second facility is located, survival probability of one more demand node becomes 1, and this results in other patients feeling envy of one more region. Consequently, TEWE increases from 2 to 3.48. This increase would continue until a critical number of facilities are located on the network.

This example demonstrates that even though increasing number of hospitals in a city would be beneficial for public on any ground, gain obtained may not be explicitly observed by evaluating MESP and TEWE values. Therefore, regardless of the fact that $\mathbb{P}2$ and $\mathbb{P}3$ performs better compared to $\mathbb{P}1$ when it comes to provide equity for public, these two problems can not be used by themselves when discussing how many hospitals to establish. In this case, evaluating other performance measures, such as ENS becomes necessary.

6.7 Evaluation of Decisions Under Alternative Objective Functions

In this section, we will evaluate the performances of problems $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ using ENS, MESP and TEWE values. A city consisting of 20 uniformly distributed demand regions with a single patient on each region is considered for this analysis. As we mainly present the problem of locating facilities while maximizing expected number of survivors in this thesis, how the corresponding model, $\mathbb{P}1$, performs in terms of equity measures should be investigated. Similarly, we want to analyze the sacrifice made from ENS values while aiming equity in the system by $\mathbb{P}2$ and $\mathbb{P}3$. It is also aimed to examine results of $\mathbb{P}2$ and $\mathbb{P}3$, in order to see if they have similar behaviors in terms of MESP and TEWE measures since both seeks equity with different objective functions.

Z1 is evaluated under the solutions obtained by $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$, and resulting ENS values are presented in Figure 6.18. It is seen that, when problems $\mathbb{P}2$ and $\mathbb{P}3$ are

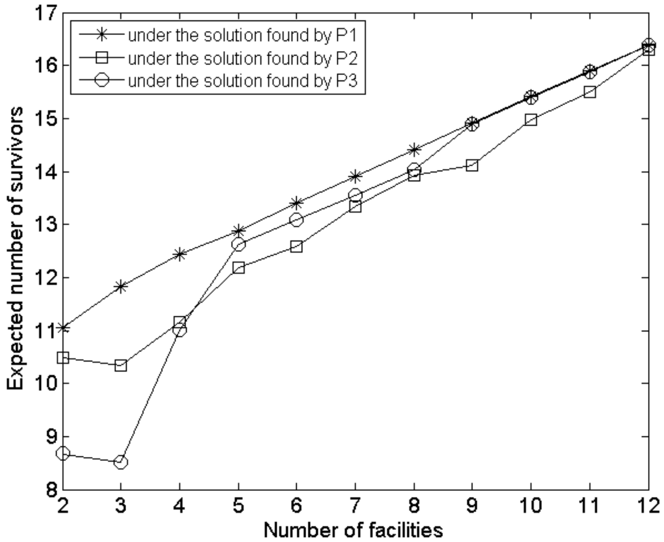

Figure 6.18: Evaluation of Solutions Under $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ According to ENS Values

solved, ENS in the system may decrease even when number of located facilities on the network is increased. However, this situation is observed for smallest p values and after a certain number of facilities, ENS values strictly increase under the solutions of all models. It is also seen that $\mathbb{P}1$ and $\mathbb{P}3$ gives exactly the same results when more than 8 facilities are located.

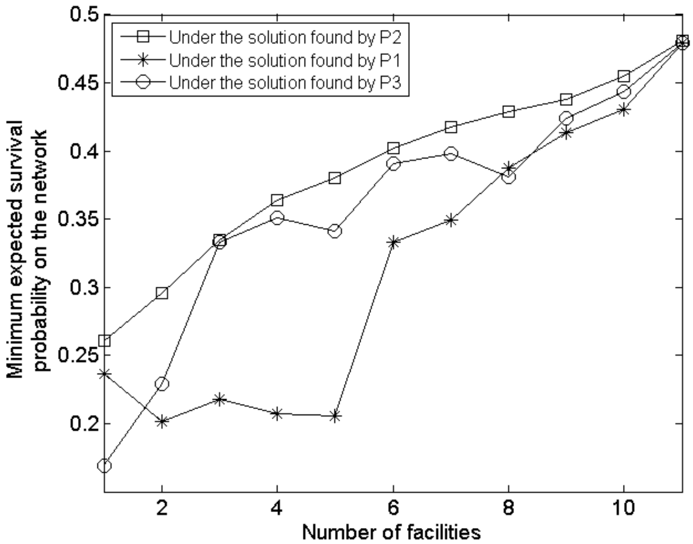

Figure 6.19: Evaluation of Solutions Under $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ According to MESP Values

Values obtained by evaluating Z2 under the solutions of all three problems are presented in Figure 6.19. Here, it is seen that outmost demand regions are neglected when $\mathbb{P}1$ is solved since it aims to locate facilities close to high demand areas. The gap between MESP values obtained by $\mathbb{P}1$ and $\mathbb{P}2$ is highly likely to get bigger when demand nodes have different number of patients. We also see that when number of facilities are increased from 6 through 12, MESP under the solution found by $\mathbb{P}1$ monotonously increases and reaches the optimal level at the end. However, the same is not true for results of $\mathbb{P}3$ since their MESP values fluctuate as p increases. Still, it can be claimed that $\mathbb{P}3$ has a better performance than $\mathbb{P}1$ for this performance measure since its MESP values are most of the time closer to optimal. This is an expected result as both problems pursue equity in the system.

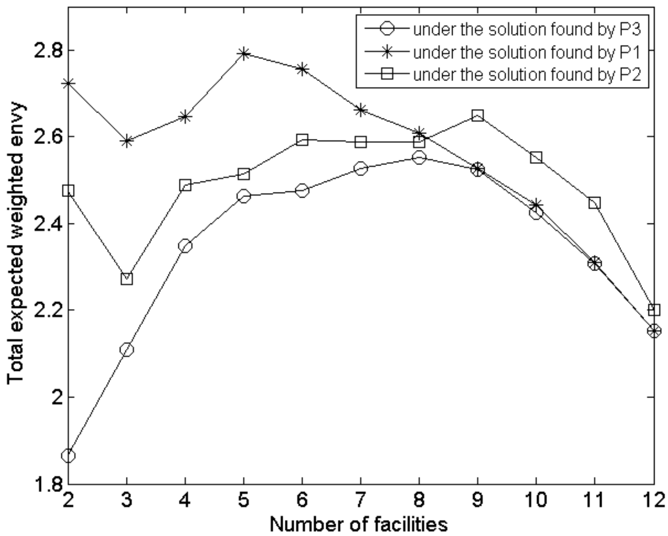

Figure 6.20: Evaluation of Solutions Under $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ According to TEWE Values

Z3 is evaluated under the solutions obtained by $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$, and resulting TEWE values are presented in Figure 6.20. Similarity of behaviors of $\mathbb{P}2$ and $\mathbb{P}3$ are observed in this figure as well. TEWE values under both of them have a concave-like shape and marginal increase-decrease amounts are close to each other in both curves. On the other hand, since $\mathbb{P}1$ favors high demand areas for locating facilities, further regions are ignored and it creates an increase in TEWE for small number of facilities. Due to the nature of Z3, locating more facilities decreases TEWE after some p value. This explains the improvement in this measure of $\mathbb{P}1$ after the 5^{th} facility.

All in all, three important observations are obtained from these three figures.

Observation 6 *For p values that are higher than a certain value, problems $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ give close results in terms of the three performance measures proposed.*

Observation 7 *When small number of facilities are to be located, it is seen that $\mathbb{P}1$ has an inferior performance in terms of providing equity in the system. Similarly, it is observed that for the same levels of p, ENS values obtained by the results of $\mathbb{P}2$ and $\mathbb{P}3$ are quite different from the optimal value. Therefore, when limited budget allows locating only a small number of facilities, neither of these alternative objective functions gives satisfying results and multi-objective decision making approaches may need to be utilized.*

Observation 8 *That Z3 value worsens until locating a certain number of facilities makes the objective function of this problem open to criticism. However, it should also be noted that the decisions obtained by this problem gives the closest values to optimal ones in terms of ENS and MESP measures. This could indicate that the model gives meaningful results in terms of providing balance between MESP and ENS values.*

CHAPTER 7

DISCUSSION AND CONCLUSION

In this thesis, we studied the problem of locating healthcare facilities where well-being of patients having a medical emergency is aimed by three different objective functions. Firstly, literature survey of location models used in this area are reviewed by focusing on two particular subjects; namely, gradual coverage models and gravity theory. Studies considering these models in healthcare related problems are reviewed as well. Later, gradual coverage idea is utilized since we consider that survival probability of a patient is represented by a decreasing function of response time. Additionally, by relaxing the assumption of proximity, it is considered that a patient could visit any one of the hospitals with some probability. A likelihood function is obtained by modifying a gravity model in the literature and is used to calculate the mentioned probabilities. In this context, we define our problem as locating a given number of healthcare facilities with the aim of maximizing expected number of patients. Where a weighted complete graph is used to represent the city that we would like to locate facilities, demand regions are denoted by nodes and weight of the nodes give the number of patients in that region. Under the given objective, characteristic of the problem in which facilities can be located anywhere on the network is analyzed by examining certain conditions. First, it is shown that the optimal location would always be on a node when a single facility is located on a line with the use of a convex decreasing survival probability function. Later, it is shown that under the same conditions, optimal facility location would not necessarily be on a node when a concave decreasing function is considered. Similarly, it is proven that optimal location of a single facility on a network will exist on one of the ones when a convex decreasing function is used. An optimality condition for the case of locating multiple facilities on network could not be reached for either of survival functions. However, there exists a proof in the literature that optimal location of facilities exist on nodes under the use of a convex decreasing function and when each region is served by the closest facility. It is shown that under certain conditions, this proof applies to our problem as well. In addition to these, when a discretized model in which some points on links are denoted as candidate facility location is solved using a convex function, obtained optimal locations were always on demand regions. In contrast, locating facilities on links are encountered when a concave function is used. Consequently, demand regions are considered as candidate facility locations in our problem definition. Since the proposed model is nonlinear and computation time increases significantly as the number of demand regions and facilities increases, a heuristic solution approach is developed which gives satisfying results in reasonable time.

As providing equity is an important issue in public facility location problems, we consider two more additional objective functions with the same setting and constraints. One of them aims to maximize the minimum expected survival rate on the network, while the other one aims to minimize total envy where envy emerges from the inequities between expected survival probabilities of patients. In the computational study, location decisions of three problems are presented and compared. It is seen that as p value increases, Z1 value always improves while Z2 and Z3 may worsen from time to time. Behavior of objective function of $\mathbb{P}2$ is important in the sense that it shows effects of opening one more facility on an individual basis. It suggests that state of a particular individual is improved remarkably only when the facility is located nearby. Consider the facilities could be any type, not necessarily a hospital. In this case, the convex decreasing function we use would represent customers' utility as a function of the travel time. In this general case of the problem, results of $\mathbb{P}2$ show the expected utility of a particular customer may worsen when a new facility is located far away. As an illustration, with the view of a customer, locating a new shopping center in addition to a certain number of centers will decrease his/her expected utility as long as it is not located very close to that customer. In the computational study, it is also showed that relaxing the proximity assumption when $\mathbb{P}1$ is under consideration changes location decisions in almost all instances regardless of the survival function shape and number of patients on regions.

A weakness of the model proposed is the assumption of identical and noncapacitated facilities. As a future study, different sized facilities can be incorporated into the problem definition. This would bring two features: first, likelihood function is to be modified since different sizes may mean different attractiveness, second, patients may need to travel from one hospital to another due to full capacity which changes the response time. Another weak point of the thesis is about the travel times between regions. In this thesis, it is assumed that travel time is an linear function of the distance; however, it has a stochastic feature especially in a city environment. As travel time affects both the survival probability and likelihood values, expected survival probability of a patient on a particular node may change depending on the time of day. For instance, probability of choosing a facility may be higher at night then it is at daytime. Therefore, one focus can be towards considering stochastic travel times in this model as a future study. A final future study can be made by considering that patients may exist not only on nodes but also on paths connecting them. In this way, emergency situations such as traffic accidents can be taken into account as well. As demand amounts on links would be in accordance with road traffic intensity, considering this with stochastic travel times together can be an interesting extension for the problem presented.

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APPENDIX A

Table A.1: Performance of the Solution Approach

Instance Name	Optimal objective function value	Percent Deviation			Quality (%)	Total Time	
		Min	Max	Ave.		GA	Enum
U_k0_20_p3	11.356	0.000	0.059	0.002	99.250	1.657	0.647
U_k0_20_p4	12.107	0.000	0.000	0.000	100.000	1.936	2.562
U_k0_20_p5	12.734	0.000	0.092	0.006	98.250	2.451	8.061
U_k0_20_p6	13.340	0.000	0.000	0.000	100.000	5.655	20.093
U_k0_20_p7	13.829	0.000	0.020	0.000	99.750	7.185	40.630
U_k0_30_p3	16.598	0.000	0.189	0.011	95.750	2.224	2.948
U_k0_30_p4	17.402	0.000	0.011	0.000	99.500	5.718	20.268
U_k0_30_p5	18.135	0.000	0.000	0.000	100.000	7.574	106.611
U_k0_30_p6	18.753	0.000	0.022	0.000	90.250	9.293	448.971
U_k0_30_p7	19.333	0.000	0.015	0.001	98.000	10.900	1559.452
U_k0_40_p3	21.379	0.000	0.012	0.002	96.000	2.743	9.481
U_k0_40_p4	22.226	0.000	0.103	0.001	99.750	7.637	88.718
U_k0_40_p5	23.000	0.000	0.049	0.009	94.500	10.230	644.569
U_k0_40_p6	23.653	0.000	0.091	0.002	99.000	11.200	3821.755
U_k0_40_p7	24.207	0.000	0.016	0.002	97.250	14.328	18767.868
U_k0_50_p3	26.002	0.000	0.213	0.005	97.250	3.685	23.275
U_k0_50_p4	26.848	0.000	0.019	0.004	90.250	10.863	276.009
U_k0_50_p5	27.574	0.000	0.087	0.002	99.000	12.807	2570.721
U_k0_50_p6	28.198	0.000	0.085	0.008	87.500	17.819	19605.948
U_k0_100_p2	54.977	0.000	0.287	0.012	95.750	4.738	11.493
U_k0_100_p3	57.001	0.000	0.006	0.001	96.750	16.212	379.047
U_k0_100_p4	58.644	0.000	0.036	0.001	98.250	20.148	9277.828
C_k2_20_p3	16.108	0.000	0.066	0.001	99.000	1.567	0.566

Table A.1 (Continued)

Instance Name	Optimal objective function value	Percent Deviation			Quality (%)	Total Time	
		Min	Max	Ave.		GA	Enum
C_k2_20_p4	16.490	0.000	0.006	0.000	98.500	1.899	2.426
C_k2_20_p5	16.689	0.000	0.033	0.001	95.500	2.319	7.929
C_k2_20_p6	16.932	0.000	0.003	0.000	93.750	6.889	20.105
C_k2_20_p7	17.153	0.000	0.004	0.000	99.000	7.846	40.607
C_k2_30_p3	23.715	0.000	0.059	0.001	97.750	2.179	2.948
C_k2_30_p4	24.122	0.000	0.011	0.000	98.750	6.457	20.148
C_k2_30_p5	24.347	0.000	0.008	0.000	98.750	7.764	106.213
C_k2_30_p6	24.595	0.000	0.000	0.000	100.000	9.265	449.314
C_k2_30_p7	24.807	0.000	0.025	0.000	94.750	11.486	1559.070
C_k2_40_p3	31.253	0.000	0.140	0.013	88.250	3.355	9.477
C_k2_40_p4	31.862	0.000	0.037	0.001	99.000	8.496	88.546
C_k2_40_p5	32.001	0.000	0.019	0.001	98.500	10.755	644.725
C_k2_40_p6	32.333	0.000	0.025	0.000	95.750	13.246	3814.926
C_k2_40_p7	32.527	0.000	0.047	0.003	84.000	16.521	18744.140
C_k2_50_p3	38.016	0.000	0.157	0.004	96.500	3.725	23.334
C_k2_50_p4	38.735	0.000	0.050	0.001	99.250	10.232	276.695
C_k2_50_p5	38.969	0.000	0.017	0.001	97.750	13.738	2570.378
C_k2_50_p6	39.345	0.000	0.044	0.001	96.250	17.148	19631.275
C_k2_100_p2	76.161	0.000	0.438	0.051	84.000	5.011	11.509
C_k2_100_p3	75.999	0.000	0.096	0.009	90.750	19.145	379.149
C_k2_100_p4	77.305	0.000	0.045	0.004	89.750	23.096	9277.493
C_k3_20_p3	16.003	0.000	0.000	0.000	100.000	1.346	0.562
C_k3_20_p4	15.014	0.000	0.085	0.005	98.500	2.004	2.445
C_k3_20_p5	15.338	0.000	0.080	0.002	99.500	2.320	7.905
C_k3_20_p6	15.653	0.000	0.000	0.000	100.000	5.971	20.183
C_k3_20_p7	16.259	0.000	0.055	0.001	99.250	8.100	40.790
C_k3_30_p3	22.406	0.000	0.030	0.001	99.500	1.918	2.968
C_k3_30_p4	22.596	0.000	0.000	0.000	100.000	6.076	20.132
C_k3_30_p5	22.970	0.000	0.069	0.001	99.750	8.385	106.299
C_k3_30_p6	23.420	0.000	0.000	0.000	100.000	9.428	450.394
C_k3_30_p7	23.690	0.000	0.012	0.000	99.750	10.998	1563.972
C_k3_40_p3	29.179	0.000	0.000	0.000	100.000	2.556	9.465
C_k3_40_p4	29.313	0.000	0.013	0.000	99.500	7.905	88.605
C_k3_40_p5	29.600	0.000	0.007	0.001	95.250	10.140	644.850
C_k3_40_p6	29.993	0.000	0.023	0.001	96.250	13.524	3824.376

Table A.1 (Continued)

Instance Name	Optimal objective function value	Percent Deviation			Quality (%)	Total Time	
		Min	Max	Ave.		GA	Enum
C_k3_40_p7	30.235	0.000	0.020	0.000	99.500	14.490	18766.912
C_k3_50_p3	36.791	0.000	0.027	0.000	99.250	3.328	23.330
C_k3_50_p4	36.997	0.000	0.053	0.002	98.250	11.027	276.793
C_k3_50_p5	37.461	0.000	0.038	0.001	98.750	13.276	2580.299
C_k3_50_p6	38.031	0.000	0.051	0.001	99.750	16.132	19698.706
C_k4_30_p3	21.098	0.000	0.000	0.000	100.000	1.826	2.960
C_k4_30_p4	21.868	0.000	0.000	0.000	100.000	5.456	20.175
C_k4_30_p5	22.199	0.000	0.030	0.000	99.750	7.384	106.521
C_k4_30_p6	22.504	0.000	0.030	0.000	99.750	9.279	453.475
C_k4_30_p7	22.860	0.000	0.000	0.000	100.000	10.969	1573.734
C_k4_40_p3	27.172	0.000	0.140	0.006	96.250	2.928	9.469
C_k4_40_p4	28.436	0.000	0.055	0.001	99.250	8.157	88.585
C_k4_40_p5	28.708	0.000	0.026	0.001	97.750	10.274	646.831
C_k4_40_p6	29.103	0.000	0.012	0.001	91.250	13.531	3844.067
C_k4_40_p7	29.515	0.000	0.073	0.005	82.500	17.895	18945.896
C_k4_50_p3	33.713	0.000	0.111	0.001	98.500	3.447	23.322
C_k4_50_p4	35.606	0.000	0.000	0.000	100.000	9.860	276.839
C_k4_50_p5	35.915	0.000	0.004	0.000	99.750	12.918	2580.865
C_k4_50_p6	36.383	0.000	0.017	0.000	97.000	15.007	19762.870
C_k4_100_p2	62.272	0.000	0.143	0.003	95.500	4.709	11.493
C_k4_100_p3	67.417	0.000	0.127	0.001	99.500	14.050	378.330
C_k4_100_p4	70.387	0.000	0.085	0.013	83.000	19.381	9292.032
C_k5_50_p3	31.683	0.000	0.149	0.024	91.500	3.630	23.353
C_k5_50_p4	33.263	0.000	0.000	0.000	100.000	9.703	277.284
C_k5_50_p5	34.914	0.000	0.000	0.000	100.000	12.432	2581.512
C_k5_50_p6	34.550	0.000	0.124	0.018	93.250	16.464	19780.482
C_k6_100_p2	60.945	0.000	0.200	0.009	94.000	4.596	11.505
C_k6_100_p3	63.854	0.000	0.045	0.003	96.000	15.207	378.997
C_k6_100_p4	66.094	0.000	0.042	0.004	84.500	20.777	9299.633
C_k8_100_p2	57.990	0.000	0.322	0.036	76.250	4.627	11.556
C_k8_100_p3	60.855	0.000	0.076	0.003	95.250	14.160	380.135
C_k8_100_p4	62.567	0.000	0.177	0.005	93.750	19.972	9327.405
C_k10_100_p2	57.657	0.000	0.120	0.004	98.000	4.416	11.540
C_k10_100_p3	61.082	0.000	0.040	0.012	92.250	13.059	380.385
C_k10_100_p4	63.028	0.000	0.044	0.001	94.250	19.776	9301.681