MODELING TEMPERATURE AND PRICING WEATHER DERIVATIVES BASED ON TEMPERATURE

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ABSTRACT

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Weather Derivatives are financial contracts prepared to reduce weather risks faced by economic actors to regulate cash flows and protect earnings. The weather derivatives may be in the forms of options, futures, swaps, and bonds whose payout are dependent on some weather indices. The firms in the sectors like energy, insurance, agriculture, construction use weather derivatives mostly. Weather derivatives are different than the traditional financial derivatives on several occasions. Traditional financial derivatives are based on some assets like stocks, bonds, foreign exchange, interest rate etc. that are traded on the market. Besides, weather derivatives are based on a weather index, which is not traded. Also financial derivatives are generally used to hedge price risk, while weather derivatives are used to hedge volume risk. Because of different nature of the weather derivatives its pricing is different than the pricing of other financial derivatives. In addition, although it is possible to write a derivative that uses any weather index like temperature, humidity, and wind speed etc. most of the weather derivatives that are traded on market are based on temperature. Within this context, in this thesis, models for temperature and pricing issues of the weather derivatives based on temperature will be evaluated. Moreover, the applicability of the weather derivatives to Turkey will be investigated.

Keywords : weather derivatives, temperature-based derivatives, temperature modeling, temperature risk, jump processes, option valuation in incomplete markets

ÖZ

SICAKLIĞIN MODELLENMESİ VE SICAKLIĞA DAYALI İKLİM TÜREVLERİNİN FİYATLANDIRILMASI

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İklim türevleri ekonomik aktörler tarafından, nakit akımını düzenlemek ve karlılığı korumak amacıyla, karşı karşıya kaldıkları iklim risklerini azaltmak için düzenledikleri finansal sözleşmelerdir. İklim türevleri, getirileri belli hava endekslerine bağlı olan, opsiyon, futures, swap ve tahvil şeklinde olabilirler. Çoğunlukla enerji, sigorta, tarım, inşaat gibi sektörlere dahil firmalar iklim türevlerini kullanırlar. İklim türevleri, geleneksel finansal türevlerden bir kaç noktada ayrılırlar. Geleneksel finansal türevler pazarda ticareti yapılan hisse senedi, tahvil, döviz kuru, faiz oranı gibi varlıklara dayanırlar. Fakat iklim türevleri ticareti söz konusu olmayan bir hava endeksine bağlıdır. Ayrıca finansal türevler genellikle fiyat riskine karşı önlem amacıyla yapılırken iklim türevleri hacim riskine karşı yapılır. Farklı doğası nedeniyle iklim türevlerinin fiyatlandırılması diğer finansal türevlerin fiyatlandırılmasından farklıdır. Ek olarak, sıcaklık, nem, rüzgar hızı gibi herhangi bir hava endeksine bağlı olarak bir türev ürün yazmak mümkün olmakla birlikte, piyasada ticareti yapılan iklim türevlerinin büyük kısmı sıcaklık üzerinedir. Bu bağlamda, bu tez çalışmasında sıcaklık modelleri ve sıcaklığa dayalı iklim türevlerinin fiyatlandırılması değerlendirilecektir. Ayrıca, iklim türevlerinin Türkiye'de uygulanabilirliği araştırılacaktır.

Anahtar Kelimeler: iklim türevleri, sıcaklığa dayalı türevler, sıcaklık modellemesi, sıcaklık riski, sıçrama süreçleri, eksik piyasada opsiyon değerlemesi

To My Wife Sevinç and My Daughter Özde

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ACF	Autocorrelation Function
ARCH	Autoregressive Conditional Heteroskedasticity
ARIMA	Autoregressive Integrated Moving Average
BM	Brownian Motion
BSM	Black-Scholes Model
CPP	Compound Poisson Process
CAR	Continuous Time Autoregressive
CDD	Cooling Degree Days
CCDD	Cumulative Cooling Degree Days
CHDD	Cumulative Heating Degree Days
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
HDD	Heating Degree Days
OU	Ornstein-Uhlenbeck
PACF	Partial Autocorrelation Function
TR	Temperature Risk
TTR	Total Temperature Risk
WD	Weather Derivatives

CHAPTER 1

INTRODUCTION

1.1 General Information

Weather affects businesses from agriculture to tourism[35].Estimates showed that a big portion of the business activity are weather sensitive [43]. The impact of weather on business can be in the form of a reduction on profits to a total disaster in case of a heavy storm [28].

Story about weather derivatives (WD) began in 1990s with climate extremes and major storms that caused financial losses. The response of the financial markets was to present instruments called weather derivatives to be used for transferring or reducing the risk caused by weather [35, 43].

Weather derivatives are tools where companies use against non-catastrophic weather events. These may include warmer or colder than the usual periods, rainy or dry periods etc. These unusual periods are frequent and can cause significant decrease in profits that depend to the weather. The stability of profits is an important topic such that weather derivatives are desirable tools in case of existence of sensitivity of business to weather conditions. Benefits of stable profits are listed as [28]

• low volatility in profits can reduce cost of borrowed money

• when a company is open to public low volatility in profits results with a high value for the company

• bankruptcy risk is reduced by low volatility in profits

In the literature it is seen that particularly the energy and power sectors use tools for hedging weather risks [39, 38]. But weather derivatives can be used by many different companies from many different sectors.

First appearance of the weather derivatives was in the US energy industry in 1997. While there exists a trade on contracts based on electricity and gas prices it was realized that this trade can be extended to the contracts based on the weather that may hedge weather risk. The market grew fast as other companies realized the benefits of these contracts. Later, the market was extended to Europe and Japan[28, 12].

1.1.1 What is a Weather Derivative?

Weather derivatives are contingent claims written on some weather indices whose values are obtained from weather data. Some of these weather indices include daily average temperature, cumulative annual temperature, heating degree days, cooling degree days, precipitation, snowfall, wind [35].

1.1.2 Examples of Weather Hedging

In the following, examples were given to reveal effect of weather on different businesses. In most of the time, volume of sales is affected [28].

• a natural gas supply company may sell less gas in a winter season that is warmer than the usual

• a ski resort attracting less visitors in case of little snow

• a clothes retailing company may have problems with sales in summer clothes in case of a colder than the usual summer

All these risks could be hedged using WD [28].

1.1.3 Why Weather Derivatives Exist?

There are four effects that discussed in the literature as the cause of emerging of weather derivatives:

• Climate change and weather variability: Climate change accepted as a fact for a majority of people. This also resulted with rising concerns about its economic, social, political effects. Financial impacts of climate change may be hedged by WD[43].

• Deregulation of the US energy sector: This is perhaps the most important key factor in development of WD. By losing monopoly power on prices, deregulated companies focused on profits more[43, 39, 38, 12].

• Convergence: Increased awareness about hedging and protection against risks led capital and insurance markets come closer. WD can be considered as an extension in this process[2].

• Commoditization of weather and climate: Developments in weather observations through better equipment and better processing capacities of computers led production of accurate and valuable weather data. This also resulted with commoditization of weather forecasting [43].

1.1.4 Differences Between Weather and Ordinary Derivatives

Several items make WD different than classical derivatives:

• The most important one is that the weather is not traded. In other words, the underlying is not a traded asset [39, 7].

• Another fundamental difference is that financial derivatives are used for price hedging. On the other hand, WD are useful for quantity hedging [7].

• The weather derivative markets are much less liquid than traditional commodity markets. This is mainly due to the fact that weather is a location-specific issue and as a result it is not a standard commodity [7].

1.1.5 Differences Between Insurance and Weather Derivatives

Although many similarities exist between insurance policies and WD contracts, there are some important differences regarding coverage and payouts. Some of the important differences may be listed as following [43, 28, 39, 2]:

• For standard insurance contracts, it is needed for a proof of loss and an interest to be insured. WD differs from these kinds of insurance contracts because they have neither of these two requirements.

• The moral risk removed since the weather indices are out of control of the parties.

• There is a minor difference between the loss and the payout in an insurance contract. In WD, on the other hand, the returns from the contract may not match the risk faced by the buyer.

• Derivative positions must be re-evaluated as time passes, but this is generally not the case in insurance contracts.

• Tax liabilities may be different.

• The accounting treatment and contractual structure may be different.

• A WD can be used to produce profit from the weather in addition to hedging.

• One important difference is that insurance contracts are designed for high risk – low probability events. On the other hand, WD are designed for low risk – high probability events.

• In WD, two parties having counter effects from the weather can come together and hedge each other's risk.

1.1.6 Weather Forecasts

One question may be asked about usage of weather forecasts instead of WD. Nevertheless, the main obstacle against weather forecasts is in their forecasting horizon. A company with long term plans that cover several years cannot use weather forecasts. On the other hand, WD can be used for extended time periods.

1.1.7 Weather Derivatives Market

The first weather derivative was issued by US energy firm Enron on over-the-counter market in USA in 1997 [18]. Today, there are two main markets that offer standard products to be automatically traded:

• Chicago Mercantile Exchange (CME)

• London International Financial Futures and Options Exchange (LIFFE)

1.1.7.1 Weather Contracts

Weather contracts may be in the form of swaps, futures, and call/put options based on weather indices [35]. Following parameters are used in weather contracts[35, 28]

• The contract type

• The contract period (e.g. February 2016)

• The underlying index: Specifies one of the indices that discussed below.

• An official weather station where weather data will be obtained

• The strike level

• The tick size: This is the monetary amount to be paid or received for each index value

• The maximum payoff: Some contracts may contain a maximum monetary value to be paid or received for the contract.

Some indices can be listed as following:

• Based on Temperature: These types of contracts mainly based on Heating Degree Day (HDD) and Cooling Degree Day (CDD). A degree day corresponds to the measure of deviation temperature from 65"F (or equivalently 18"C). The idea is that as temperature deviates from 65"F, more energy will be needed for heating and cooling. As a result, these type of contracts offer companies to hedge against unexpectedly cold or warm periods. In practice, HDDs are used for winter periods and CDDs are used for summer periods. Other variables may include the monthly or daily average temperature in addition to monthly and yearly cumulative temperatures.

• Rainfall: Total rainfall on a given area is measured to be based on rainfall contracts. Nevertheless, these type of contracts attracted less interest compared to temperature based contracts because of difficulties in modeling of rainfall [35].

• Wind speed: As electricity production through wind mills increased, special attention was given to wind speed contracts that are based wind power indices [35].

Some contract types can be listed as following:

• Options: Mostly used options are HDD/CDD calls and puts as well as some combination strategies.

• Bonds: In these types of contracts, payments about interest and nominal values are made contingent to an index [39].

• Swaps: Based on a weather index, two parties agree to exchange a variable and a fixed amount on a given date [2].

• CME Futures: These are agreements to buy or sell an index at a specific future date [35].

1.2 Problem Statement

As stated in the excellent book by [40] derivative pricing is different from asset pricing; basic securities can be used to determine an arbitrage-free price of a security without taken into consideration other assets or markets. This is done with some formulas obtained from equivalent martingale measures and partial differential equations. Alternatively, the essence of arbitrage pricing of derivatives lies in the assumption that the market is complete [58]. When a market is complete, one can create risk-free portfolios that mimic the behavior of an asset. Converting asset behavior into a martingale by changing measures eliminates the need to consider an individual's risk preferences. However, in the case of weather derivatives based on temperature, none of the above-mentioned methods can be used. When underlying is not a traded asset market will be incomplete. As a result, the problem of pricing temperature-based derivatives is basically the problem of pricing in incomplete markets. Related with this, a second issue will be to define risk attitudes of the economic entities. When market is incomplete there are basically two ways that can be followed: First one requires certain techniques to change the market to a complete one. Second method continues to consider the market as incomplete. Besides, in the second case, some utility functions have to be used to reveal risk preferences. Some of the important studies about incomplete markets were mentioned in the literature review part.

Current study offers a third way to deal with pricing in incomplete markets. As a first step, a temperature model will be defined. By using the model, index calculations will be done. With these index values, a new setup will be defined for pricing. In this setup, the market will continue to be incomplete as this situation represents a more realistic approach. Another motivation behind continuation of incompleteness of the market will be obtained from usage of risk-neutral probabilities. In fact, it will be shown that usage of risk-neutral probabilities ends up with super-hedging that limits its usage in weather derivatives. As a result, after defining risk of the temperature, the differences between the risk of an ordinary asset and the risk of the temperature will be revealed. This is a necessary step because temperature affects businesses in different ways. In other words, temperature affects different entities in a personalized way. For example, a warmer than the usual winter season affects a retail gas selling company different than a beverage company. The personalized risk of the temperature will be, then, used to define a personalized price for a candidate company. Then by using some functions called as objectives, which are set by the entity itself, trading behavior of the candidate company will be revealed. In the study, profit maximization will be considered as the objective of a candidate company. With the aim of profit maximization, the trading behavior of the mentioned company will be investigated. Moreover, it will be shown that the discussion regarding market price of risk of temperature is inconclusive supporting the need for a new setup for pricing.

The difference of this study from the existing studies appears on two grounds: In the first, current study defines risk in a different way than the literature that results development of a personalized price of an option. In the second, current study uses objective functions instead of utility functions. By this way, a more realistic approach for the trading behavior of the candidate company will be revealed.

1.3 Related Literature

The literature about weather derivatives can be divided into several groups: first group is about modeling weather events; second is about pricing issues related with WD; and third one is about potential uses of WD. It is normal that some studies fall into two or three groups while others study just one of the above. As the focus of this study is about modeling temperature and pricing temperature based derivatives, in the literature review part, mainly temperature-based studies were covered.

In the literature, the main tool that was used to model temperature was mean-reverting processes, namely Orstein-Uhlenbeck (OU) processes. In one of the mostly cited study, [2] develops an OU process to represent temperature. By using equivalent martingale measures approach, the authors determines the price of an option by taking market price of risk as a constant and by considering temperature is a traded asset. [4] models temperature as a continuous time autoregressive process for Stockholm. They report a clear seasonal variation in regression residuals. Their proposed model is a higher-order continuous time autoregressive process, driven by a Wiener process with seasonal standard deviation. While pricing futures and options they consider Gaussian structure of temperature dynamics. [38] develop a stochastic volatility model to represent evolution of temperature. They work in an environment of equilibrium previously defined by [2]. Vasicek model was used in describing stochastic model for temperature. [59] propose a model that uses wavelet neural networks to model an OU temperature process with seasonality in the level and time-varying speed of mean reversion. They argue that wavelet networks can model the temperature successfully. [58] use neural networks to model seasonal component of the residual variance of an OU temperature process, with seasonality in the level and volatility. Authors also use wavelet analysis to identify seasonality component in temperature process as well as in the temperature anomalies. They suggest that this approach can be used in pricing weather derivatives by performing Monte Carlo simulations. [1] use an OU process driven by a Levy noise to model daily average temperatures. Model also includes a seasonally adjusted asymmetric ARCH process for volatility. Author uses Normal Inverse Gaussian and Variance gamma distributions to model disturbances. Then prices of out of the money call and put options compared. [25] extends the model proposed by [2] with a consideration to ARCH/GARCH effects to reflect clustering of volatility in temperature. They report that HDD/CDD call price is higher under ARCH-effects variance than under fixed variance, while put price is lower. Further they declare that despite weather options have different pricing methods than traditional financial derivatives, the effect of mean and standard deviation is same in both of the cases.

Instead of dynamic models, some authors offered time-series models to represent temperature. [7] apply a time series approach in modeling temperature. Time series modeling shows conditional mean dynamics and strong conditional variance dynamics in daily average temperature. Their model included a trend, seasonality represented by a low-ordered Fourier series, and cyclical patterns represented by autoregressive lags. For conditional variance dynamics the contributions are coming from seasonal and cyclical components. Authors used Fourier series and GARCH process to represent seasonal volatility components and cyclical volatility components respectively. [8] use an equilibrium model that is a generalization of Lucas model of 1978 [36] to include weather as another source of uncertainty. Temperature in their case was modeled with seasonal cycles and uneven variations throughout the year. Temperature was related to aggregate dividend or output. Finally they suggest that market price of weather risk is significant for temperature derivatives. They also add the only time for temperature derivative to be discounted with risk free rate is when correlation between aggregate dividend and temperature is low and/or investor's risk aversion is low. They said that these were not supported by empirical evidence. [18] focus on estimation of average temperature as an analysis of extreme values that is used to find a model for temperature maxima and minima. Author states that AR-GARCH model, which is the model offered by [7] and regression model, which is actually the model of [4] yield superior point estimates for temperature but extreme value model outperform these models in density forecasting. [54] uses marginally normalized time series where original data of the temperatures are standardized using the mean values and variances of the estimated deterministic seasonal cycles. Standardization is done by subtracting mean values of seasonality data from the original data and then dividing these by the corresponding standard deviations. A non-stationary AR model was used to quantify anomalies by applying normalized data. They report that this model fits better than an ordinary AR model for the normalized temperature data sets and exhibits a significant seasonal structure in their autocorrelation. [21] extends the study by [8] such that instead of using a given risk aversion coefficients, authors used generalized method of moments and simulated method of moments to estimate it. [11] extends the study of [8] by employing time series model of [7] and using extended power utility function instead of constant proportional risk aversion utility function.

Complying with the aim of the study, two more articles were found within literature that uses comparison of existing models. The article by [41] compares six different models. They argue that models that rely on auto-regressive moving average processes offer better fit than models that use Monte Carlo simulations. But this situation is not valid when predictions considered. Second article by [48] uses four models to compare and suggests that all models perform better when predicting Heating Degree Days than Cooling Degree Days. Continuously they argue that all models underestimate the variance of errors.

Several models offered for pricing, which is the most problematic issue of weather derivatives. In this case, it was seen that pricing of WD is mainly based on two approaches: First one is the dynamic valuation; second one is about equilibrium asset pricing. In his famous book, [26] uses a valuation framework for temperature derivatives where probability distribution estimation of the temperature index was used as expected payoff of the option. After assigning a tick value, option price was calculated by discounting by the risk free rate. In another famous book by [6], temperature derivatives are priced according to a given benchmark derivative with the idea that derivatives are related to each other and that there are certain rules for derivatives to follow in these relationships to prevent arbitrage possibilities. [56] focuses on indifference pricing that aims to bridge financial and actuarial approaches for the valuation of financial assets. Indifference pricing does not attempt to predict a market price but rather calculates price boundaries. Indifference pricing is about as described by the authors that the amount of money where a potential buyer (or seller) of weather insurance is indifferent in terms of expected utility between buying (or selling) and not buying (selling), constitutes an upper (lower) limit for the contract price. It does not require assumption of continuous trading. It is actually based on utility maximization. Another study about pricing issue is revealed by [46]. Authors use Lucas model [36] to find an equilibrium pricing model that was emphasized as superior to other models. The authors suggest that a temperature series for Fresno follows a mean-reverting BM with discrete jumps and autoregressive conditional heteroscedastic errors. They use this model to price CDD options. Burn-rate method, Black-Scholes and Merton approximation and equilibrium Monte Carlo simulations were developed to compare prices of options. They argue that these prices developed by three different methods showed differences. Since underlying is not traded it is not possible to define arbitrage free pricing but this can be addressed by designing an appropriate equilibrium pricing model, which is established by calculating prices for CDD put and call options in a representative production region. An equilibrium pricing model in a multi commodity setting is offered by [33]. Authors define a model where agents optimize their hedging portfolios that include weather derivatives. Supply and demand for hedging activities were combined in an equilibrium pricing model. Summer day options, which are popular in Japan were priced by good deal bounds by [30]. [31] finds price of weather options based on [2] and [4]. In addition, by using Korea Composite Stock Price Index, they calculates market price of risk.

Some of the studies that focus on different aspects of modeling temperature and pricing issues can be listed as following: An analysis of weather derivatives and market is given by [45], by adopting a cultural economy approach. [57] discusses weather risk hedging in three European countries by the weather derivatives traded at CME. [47]

incorporates meteorological forecast into pricing of weather derivatives. Authors state that inclusion of meteorological forecasts accurately explains market prices of temperature futures traded in CME. [16] state that, by analyzing observed prices of US temperature futures, an index modeling approach without de-trending captures the prices well and weather forecasts influence prices up to 11 days ahead. Without de-trending temperature futures yield biased valuations by overpricing winter contracts and underpricing summer contracts. Use of the principal component analysis in generation of daily time series to be used in weather derivatives market was discussed in [17]. [19] develops four different regime-switching models of temperature to be used in pricing temperature based derivatives. They revealed that a two-state model; governed by a mean-reverting process as the first state and by a Brownian motion as the second state, was superior than the others. [49] argue that weather ensemble predictions consist of multiple future scenarios for a weather variable. They can be used to forecast density of the payoff from a weather derivative. Mean of the density is the fair price of derivative and distribution of the mean is important for a couple of factors like value at risk models. In their paper authors use 10-day-ahead temperature ensemble predictions to forecast mean and quantiles of density of the payoff from 10-day HDD put option. They also argue that ensemble based forecasts compare favorably with those based on a univariate time series GARCH model. Focusing on the temperature indices density, [9] proposed a generalization of ARFIMA-GARCH model with time-varying memory coefficients. Usage of weather forecasts in pricing of weather derivatives is discussed in [29]. Authors presented two methods for strong seasonality in probability distributions and auto-correlation structure of temperature anomalies. For the first case, they offer a new transform that allows seasonality varying non-normal temperature anomaly distributions to be cast into normal distributions. For the second case, they present a new parametric time series model that captures both the seasonality and the slow decay of the autocorrelation structure of observed temperature anomalies. Their model was supposed to be valid in case of slowly varying seasonality. In addition, they offered a simple method that was valid in all cases including extreme non normality and rapidly varying seasonality.

Pricing in incomplete markets has been discussed by many researchers. The theory of incomplete markets was discussed in [37]. In another study [10] tries to connect standard arbitrage pricing with expected utility maximization. [55] uses partial hedging where hedging portfolio formed by minimizing convex measure of risk. [24] uses effect of risk aversion on investment timing and value of the option to define a parameter region where investment signals were given. [15] use marginal substitution value approach for pricing in incomplete markets.

1.4 Scope and the Structure of the Thesis

1.4.1 Scope

Although weather derivatives are written on highly varied weather indices this thesis focuses on weather derivatives based on temperature as a big portion of the weather contracts are written on temperature. On the other hand, the ideas proposed in this thesis can be extended to other forms of contracts especially in the areas of definition of weather risk and pricing.

1.4.2 Structure of the Study

This thesis mainly composed of 4 parts:

• Introduction contains general information about weather derivatives and problem statement

• Second part contains information about temperature itself; its properties, existing models for temperature, comparison of existing models in predicting temperature, and finally a new model for temperature will be proposed. In addition, an approximated distribution for the temperature and value of an HDD will be defined.

• Third part is about defining temperature risk and pricing

• Fourth part contains conclusions.

CHAPTER 2

MODELING TEMPERATURE

2.1 Preliminaries

As this thesis is mainly about temperature-based derivatives, some basic terminology is defined in the following:

• Daily temperature:

$$T_i = \frac{T_i^{max} + T_i^{min}}{2} \tag{2.1}$$

where i represents a certain day, and T_i^{max} and T_i^{min} are maximum and minimum temperatures of the given day.

• HDD for a given day:

$$HDD_i = max(0, Base - T_i) \tag{2.2}$$

where $Base$ is a pre-determined temperature level, T_i is the average temperature calculated as in Equation (2.1) for a given day i. As a standard, $Base$ is equal to 65 Fahrenheit or 18 Celsius degrees.

• Cumulative HDD (CHDD):

$$CHDD = \sum_{i=1}^{N} HDD_i \tag{2.3}$$

where HDD_i is calculated as in Equation (2.2), and N is the time horizon, which is generally a month or a season.

• CDD for a given day:

$$CDD_i = max(0, T_i - Base) \tag{2.4}$$

where $Base$ is a pre-determined temperature level, T_i is the average temperature calculated as in Equation (2.1) for a given day i. As a standard, $Base$ is equal to 65 Fahrenheit or 18 Celsius degrees.

• Cumulative CDD (CCDD):

$$CCDD = \sum_{i=1}^{N} CDD_i \tag{2.5}$$

where CDD_i is calculated as in Equation (2.4), and N is the time horizon, which is generally a month or a season.

• Payoff of a call option on HDD:

$$V_T = max(0, CHDD - K) * tick \tag{2.6}$$

where V_T is the value of the call option at time T, K is the exercise value, and $tick$ is a monetary value that provides conversion from degrees to money.

Pricing of a WD based on temperature usually starts with modeling of the underlying temperature indices. As stated by [7], temperature modeling is important for both supply and demand side. On demand side, one needs to know the risk that will be caused by weather to determine the hedging. On supply side, weather forecasting is required to define derivative prices since other methods do not work.

In this chapter, comparison of the existing models will be discussed first; then, the properties of the temperature will be revealed; in the light of all available information, a model for temperature and resulting indices that are based on temperature will be given; finally, calculations regarding index values will be revealed.

2.2 Comparison of Existing Temperature Models

2.2.1 Methodology

As mentioned earlier in the study, although a number of models exist for temperature, five models have the biggest emphasis in the literature whether as a base model to be further developed or as models to be compared with the newly developed models. Simulations based on these models will be developed and compared to find the model with highest efficiency and predict the data well. These models are:

• Model based on Historical Burn Analysis

• Model based on [8], which will be called as Cao Model

• Model based on [7], which will be called as Campbell Model

• Model based on [2], which will be called as Alaton Model

• Model based on [4], which will be called as Benth Model

Calculations and analysis were done by the R statistical software. In the remaining parts of the section above models will be introduced in addition to the data used.

2.2.2 Data

In this part of the study, mean temperature data of Chicago from O'Hare International airport and Ankara from Esenboga airport in Fahrenheit will be used. Chicago daily temperature data starts from 1/1/1974 to 12/31/2010 and contains 13502 values. Ankara daily temperature data includes same time span and contains 13480 values. Because of the limited number, missing values were completed by linear interpolation. As a result 13505 values were obtained for each of the cities. For the ease of calculations Feb 29s were excluded from series. Temperature data was obtained from National Climatic Data Center. Mean temperature data was obtained by Equation (2.1). Data is partially shown in Figure 2.1.

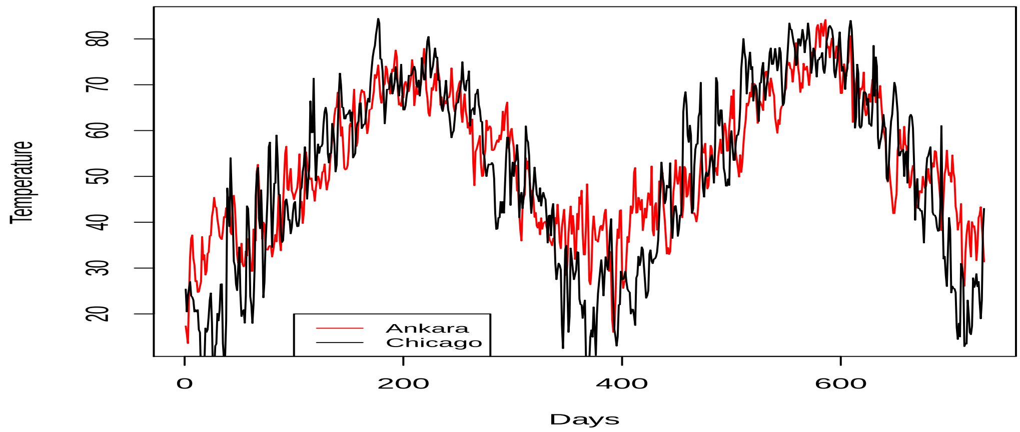


Figure 2.1: Temperatures of Ankara and Chicago between 2009-2010

In Table 2.1, some statistics about temperature data of two cities are given:

Table 2.1: Temperature Statistics

	Ankara	Chicago
Mean	49.86	50.67
Median	50.45	52.4
Standard Deviation	15.74	19.85
Maximum	84.75	91.95
Minimum	-10.05	-20.05
Skewness	-0.2438	-0.3258
Kurtosis	-0.656	-0.747

Figure 2.2 and Figure 2.3 show the bimodal structure of the data as shown on the histograms although bimodality is more apparent in Ankara.

In Figure 2.4 and Figure 2.5 yearly mean temperatures and yearly standard deviations are given for both of the cities respectively. Simple regressions for yearly mean temperatures of the two cities revealed an upward trend being higher in Ankara. The coefficients of trend variables are 0.081445 for Ankara and 0.0345 for Chicago. But only

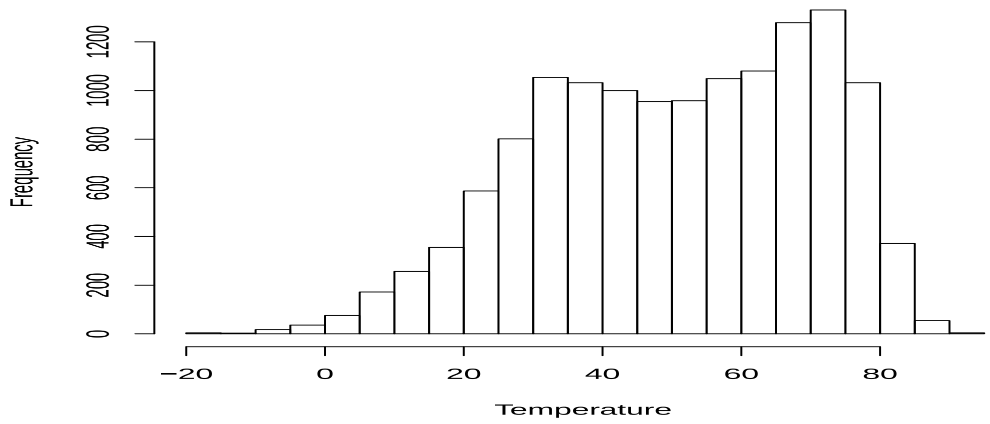


Figure 2.2: Chicago Temperature Histogram

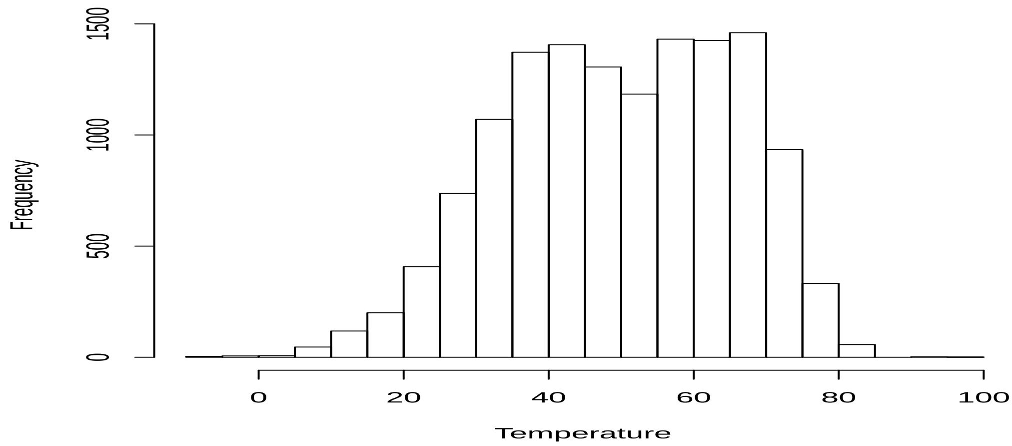


Figure 2.3: Ankara Temperature Histogram

the coefficient of Ankara is statistically significant. On the other hand, yearly standard deviations show a negative trend for Chicago and a positive one for the Ankara. Nevertheless the results are not statistically significant. The coefficients of independent variables are 0.02243435 for the Ankara and -0.04215 for the Chicago.

When averages and standard deviations of the means and standard deviations of monthly mean temperatures are calculated, it is shown that highest average of the monthly average temperatures belongs to June and the lowest average temperature belongs to January for Chicago. In addition, the lowest average of standard deviation belongs to August and the highest to March. For Ankara, the highest average of the monthly average temperatures belongs to August and the lowest average temperature belongs to January while the lowest average of standard deviation belongs to August and the highest to February. When monthly mean temperatures and standard deviations are

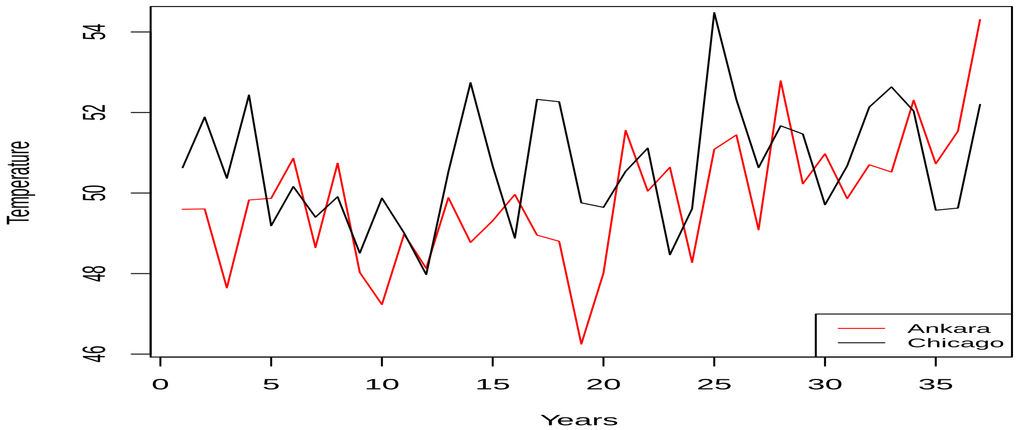


Figure 2.4: Yearly Mean Temperatures

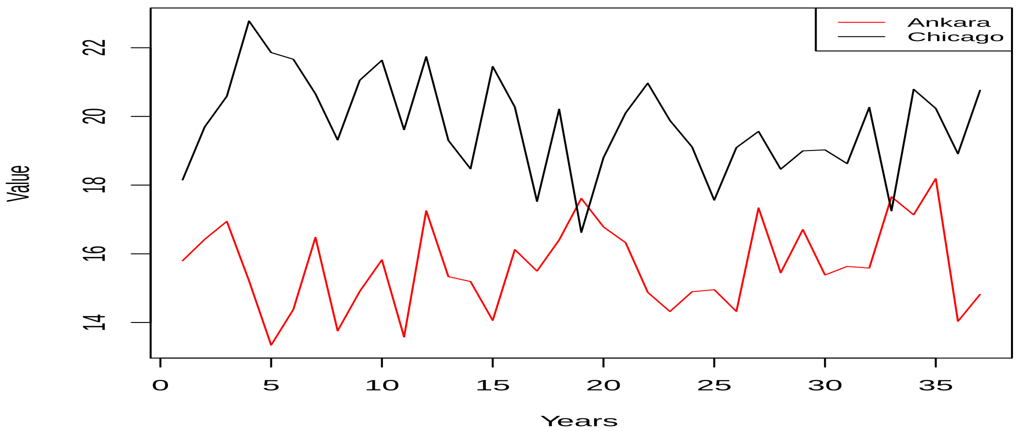


Figure 2.5: Yearly Standard Deviations

investigated for possibility of existing trends, for Chicago, only standard deviations of June and November showed negative trends with statistically significant coefficients, whereas in Ankara, there are positive trends from June to September monthly mean temperatures and none in the standard deviations.

Figure 2.6 shows averages of daily temperatures for both of the cities. To constitute this figure, first temperatures of each day of a year were found and then their averages were calculated. Figure 2.7 shows standard deviations of temperatures of each day. As shown in the figures, there is a seasonal pattern in both of the cities. Averages increase and variations decreases in summer months and vice versa.

In the following part, the models that were selected for comparison purposes will be introduced.

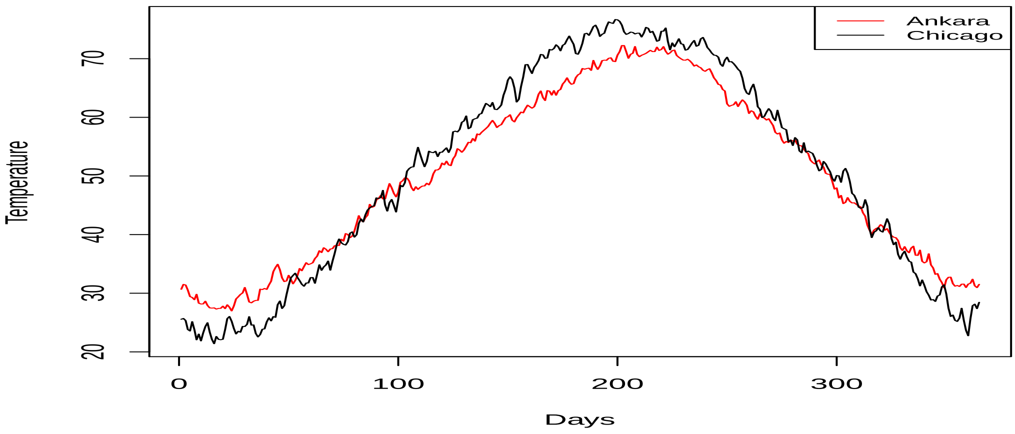


Figure 2.6: Daily Temperatures

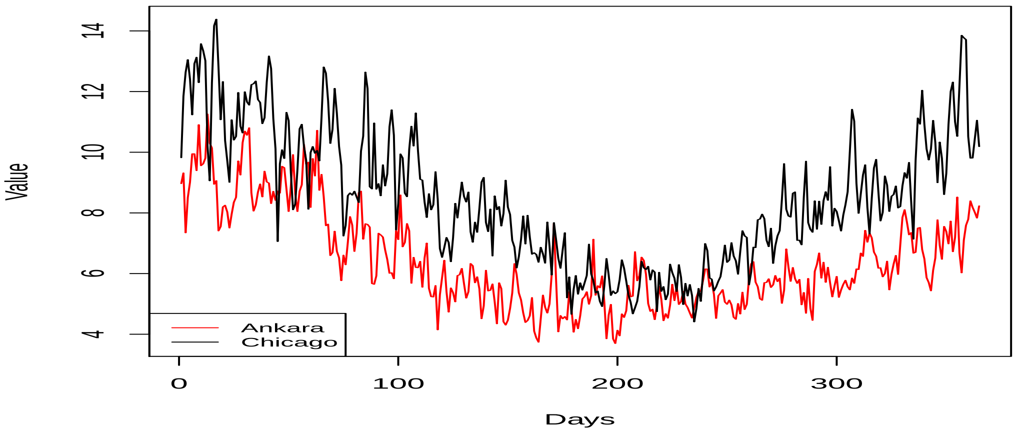


Figure 2.7: Daily Standard Deviations

2.2.3 Models

2.2.3.1 Historical Burn Analysis

This approach simply considers past data to calculate price of a weather derivatives. In this study, HDDs and CDDs of the two cities calculated for 37 years and their averages were found.

2.2.3.2 Cao Model

[8] stacked daily temperature observations in a vector Y_t for $t = 1, 2, 3, \ldots, T$ and corresponding historical average temperature for each day is stacked in vector $\bar{Y}_t$. After removing mean and trend, the residual daily temperature is expressed as

$$U_t = Y_t - (\frac{\beta}{365}(t - \frac{T}{2}) + \bar{Y}_t) \tag{2.7}$$

where β is global warming trend parameter and $t = 1, 2, \ldots, T$. Authors assume that U_t, daily temperature residual, follows a k-lag auto-correlation process as following:

$$U_t = \sum_{i=1}^{k} \rho_t U_{t-i} + \sigma_t \xi_t \tag{2.8}$$

where $\sigma_t = \sigma_0 - \sigma_1 \left| sin(\frac{\pi t}{365} + \phi) \right|$ and $\xi_t \sim i.i.d.N(0, 1)$

In this setup, ξ_t represents randomness in the temperature changes. Volatility specification using the sine wave reflects the requirement of high volatility in the winter and lower in the summer. ϕ captures starting point of sine wave. Autocorrelation setup captures autoregressive nature of temperature. Seasonal variation is captured by $\bar{Y}_t$ and since $\bar{Y}_t$ represents daily historical average temperature within the sample, half of the sample must be over historical average and other half must be lower than it.

2.2.3.3 Campbell Model

[7] define a model of temperature that is composed of a trend, a seasonal effect, and a cycle effect. The first effect is defined by a polynomial deterministic trend function. A Fourier series is used to define seasonality. The cycle effect is defined by autoregressive lags. Authors allow conditional variance where contributions come from seasonal and cyclical components. Seasonal volatility component approximated by a Fourier series and cyclical volatility by a generalized autoregressive conditional heteroscedasticity (GARCH) model. Then, model for temperature is shown by the following formulas:

$$T_t = Trend_t + Seasonal_t + \sum_{l=1}^{L} \rho_{t-1} T_{t-l} + \sigma_t \epsilon_r \tag{2.9}$$

$$Trend_t = \sum_{m=0}^{M} \beta_m t^m \text{ and } Seasonal_t = \sum_{p=1}^{P} (\sigma_{c,p} \cos(2\pi p \frac{d(t)}{365}) + \sigma_{s,p} \sin(2\pi p \frac{d(t)}{365})) \tag{2.10}$$

$$\sigma_t^2 = \sum_{q=1}^{Q}(\gamma_{c,q}\cos(2\pi q\tfrac{d(t)}{365}) + \gamma_{s,q}\sin(2\pi q\tfrac{d(t)}{365})) + \sum_{r=1}^{R}\alpha_r(\sigma_{t-r}\epsilon_{t-r})^2 + \sum_{s=1}^{S}\beta_s\sigma_{t-s}^2 \tag{2.11}$$

where $\epsilon_t \sim i.i.d(0,1)$ and $d(t)$ is a repeating function that cycles $1...365$.

2.2.3.4 Alaton Model

[2] suggests a mean reverting process with the variation term that differs between months but constant within each month. Then model becomes:

$$dT_t = \tfrac{dT_t^m}{dt} + a(T_t^m - T_t)dt + \sigma_t dW_t \tag{2.12}$$

$$T_t^m = A + Bt + C\sin(wt + \varphi) \tag{2.13}$$

where $w = \frac{2\pi}{365}$ and φ is the phase angle.

2.2.3.5 Benth Model

[4] define a general continuous time autoregressive model, which is continuous time analogue of an AR(p) time series. This class of models are called CAR(p). The model is defined by following Ornstein- Uhlenbeck process $X(t)$ in $\mathbb{R}^p$ for $p \geqslant 1$:

$$dX(t) = AX(t) + e_p\sigma(t)B(t) \tag{2.14}$$

where $B(t)$ is a Brownian motion, e_k is the kth unit vector in $\mathbb{R}^p$, $k = 1,..,p$. In addition, $\sigma_t > o$ is a real-valued and square integrable function and A is pxp matrix of the form:

$$[A = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ . & . & . & ... & . \\ 0 & 0 & 0 & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & -\alpha_{p-2} & ... & -\alpha_1 \end{bmatrix}] \tag{2.15}$$

where α_k, $k = 1..p$ are constants.

Following CAR (p) model, the temperature dynamics was introduced as:

$$T(t) = \Lambda(t) + X_1(t) \tag{2.16}$$

where $\Lambda(t)$ is a deterministic seasonal function representing average temperature. Stochastic process $X(t)$ can be represented explicitly as:

$$X(S) = exp(A(s-t))x + \int_t^s exp(A(s-u))e_p\sigma(u)dB(u) \tag{2.17}$$

where $s \geqslant t \geqslant 0$ and $X(t) = x \in \mathbb{R}^p$.

2.2.4 Results and Discussion for Comparison of Existing Models

When structures of the temperature models studied it is seen that they are mainly composed of two parts: A seasonal + trend parts, and a variation part. This may be analogous to pricing conventional derivatives by BSM method, which considers asset returns in two parts: Deterministic contribution part and a stochastic contribution part [27]. The seasonal + trend parts can be considered as a body that is used to determine a mean structure of the temperature data. The variation part is what is left after removing seasonal or the mean part. It is the discrepancy over the long term mean. In this part of the study seasonal + trend, and variation parts will be examined separately first and then their power to represent actual data will be considered in terms of fitting the data. In addition, model's prediction powers will be evaluated. To do this, models applied to data that covers 1974 to 2010. After finding necessary coefficients, models run one year ahead to evaluate how models will fit to the year 2011. Fits to existing data was evaluated in two parts: First, revealed model's seasonality + trend parts and data's correlation coefficient was calculated. Then all model parameters including variation were used to find correlation coefficient again. This process was repeated for year 2011.

Table 2.2 summarizes models according to their approaches for the seasonality + trend and variation, how the models fit the data, and how they predict one year ahead.

Table 2.2: Model's fit to Past and Predicted Data

	Seasonality + Trend				Seasonality+Trend+Variation			
Model	R^2 (1974-2010)		R^2 (2011)		R^2 (1974-2010)		R^2 (2011)	
	Ankara	Chicago	Ankara	Chicago	Ankara	Chicago	Ankara	Chicago
Cao	0.911	0.900	0.932	0.918	0.970	0.960	0.931	0.917
Campbell	0.909	0.897	0.927	0.918	0.970	0.959	0.882	0.913
Alaton	0.905	0.894	0.931	0.917	0.968	0.944	0.930	0.918
Benth	0.905	0.894	0.930	0.919	0.905	0.894	0.930	0.919
Average	0.908	0.896	0.930	0.918	0.953	0.939	0.918	0.917

A general look at the table first indicates that all models have values close to each other. There is no clear model that fits better than the others. This is especially true for predictions part that tried to measure fit of the models for year 2011.

In all cases seasonality and trend explains 90% percent and over of the existing temperature data. When variation part included all models increase their fit up to 97%.

This is, on the other hand, not true when prediction fits considered. Addition of variation part does not increase the fit. This suggests that prediction is mainly done by seasonality + trend parts of the models. This is supported by the fact that prediction fits outperform fits of the years 1974 to 2010.

Table 2.3 to 2.6 was formed to represent predictive powers of the models. After running simulations for each model, the temperature of the year 2011 was predicted and HDD and CDD values for the months of January, February, March, November, December, May, June, July, August, and September were calculated according to the predictions. These HDD and CDD values then compared with actual values of 2011. A positive percentage in the tables represents over prediction and a negative percentage means the opposite.

Table 2.3: HDDs of Ankara

		Cao		Campbell		Alaton		Benth		HBA	
	Actual	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.
January	993	1125	132	1122	129	1041	48	1036	43	1124	131
February	856	923	67	1011	155	889	33	892	36	922	66
March	818	783	-35	945	127	794	-24	794	-24	783	-35
November	926	711	-215	569	-357	649	-277	649	-277	712	-214
December	942	985	43	934	-8	918	-24	919	-23	987	45
Total	4535	4527	492	4581	776	4291	406	4290	403	4528	491

*Pre.: Predicted **Dif.:Difference

Table 2.4: HDDs of Chicago

		Cao		Campbell		Alaton		Benth		HBA	
	Actual	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.
January	1353	1253	-100	1268	-85	1180	-173	1192	-161	1280	-73
February	1065	1037	-28	1114	49	1011	-54	1011	-54	1037	-28
March	874	820	-54	975	101	868	-6	868	-6	820	-54
November	559	707	148	558	-1	733	174	734	175	708	149
December	916	1130	214	1025	109	1061	145	1060	144	1131	215
Total	4767	4947	544	4940	345	4853	552	4865	540	4976	519

Table 2.5: CDDs of Ankara

		Cao		Campbell		Alaton		Benth		HBA	
	Actual	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.
May	0	0	0	0	0	0	0	0	0	5	5
June	16	8	-8	0	-16	73	57	71	55	42	26
July	227	148	-79	47	-180	191	-36	190	-37	163	-64
August	166	159	-7	108	-58	143	-23	143	-23	174	8
September	17	4	-13	20	3	8	-9	9	-8	30	13
Total	426	319	107	175	257	415	125	413	123	414	116

When the tables investigated, it is seen that:

- Although winter months have higher variation HDD calculations were more accurate

Table 2.6: CDDs of Chicago

		Cao		Campbell		Alaton		Benth		HBA	
	Actual	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.	Pre.	Dif.
May	66	3	-63	0	-66	9	-57	9	-57	52	-14
June	194	145	-49	55	-139	199	5	200	6	177	-17
July	444	299	-145	252	-192	342	-102	343	-101	301	-143
August	273	249	-24	253	-20	270	-3	270	-3	255	-18
September	78	59	-19	88	10	46	-32	45	-33	101	23
Total	1055	755	300	648	427	866	199	867	200	886	215

than CDD calculations for all methods. This is consistent with the literature.

• Related with the above result best predictions came to Chicago HDD values where Chicago winter months have the higher variation.

• Best predictions for each group achieved by different models, which is considered as best model for temperature depends on geography and time.

• Interestingly, the HBA, which is only based on average of CDD and HDD values performed well in all cases. It has been thought that this maybe because of having moderate levels of variation of variations. So that averages have a comparable prediction power. This is consistent with the data fitting results of Table 2 where seasonality + trend parts of the models fitted data mainly without an additional increase from variation parts.

• A big portion of calculations underestimated both CDD and HDD values. This situation becomes more valid in case of CDD calculations. This suggests that models underestimate the variation part. This is again consistent with the literature.

• Benth and Alaton Models produced highly close values. Within this context, when absolute value of all error values summed the lowest value belongs to Benth Model with a value of 1266. Second one is Alaton Models with a value of 1282. Others are listed as; HBA = 1341, Cao Model= 1443, and Campbell Model= 1805.

2.2.5 Conclusions for Comparison of Existing Models

The calculations showed that the best model to predict temperature for temperature based derivatives changes according to time and geography. So that all models must be examined for their fit to data and then simulations must be run to select appropriate one for specific location and timing. Although this makes pricing of temperature based derivatives a complex one, it is still possible to find local prices by using existing models. One advantage of this situation is that location specific entities would develop or use products more suitable to their needs. On the other hand, a disadvantage would be since products will become more location specific the illiquidity for the weather derivatives will continue if not worsen.

2.3 Properties of Temperature

After the comparison of the some of the existing models and having a non-conclusive result, it has been made an extensive research on the literature to find other models that may show better results than the compared models. Within this context, a wide range of volatility models and more general models have been studied ranging from parametric to non-parametric, from discrete-time to continuous-time models. These research and additionally, investigation on the existing data revealed that temperature has some important properties to consider. In the following, these properties are listed.

• Trend: There is a slowly moving upward trend in temperature. In almost all of the cities a positive trend was captured as shown in Figure 2.8.

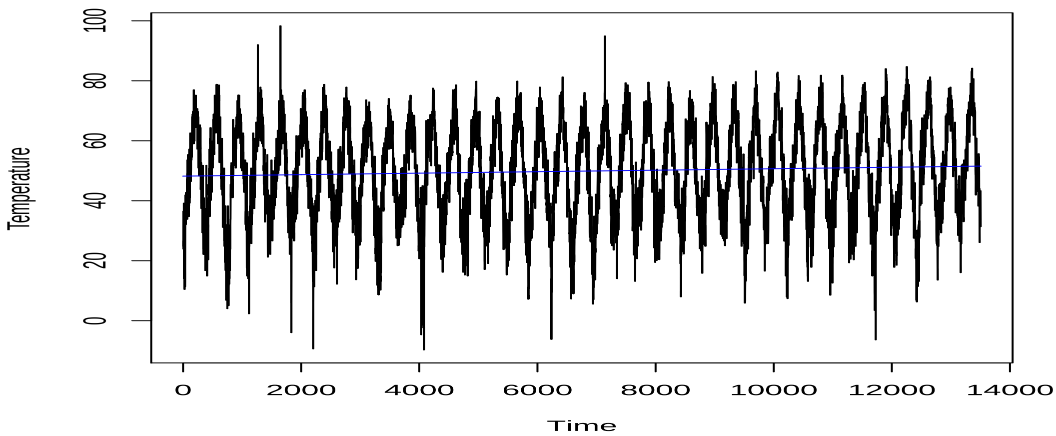


Figure 2.8: Trend in Temperature of Ankara

• Seasonality: There is seasonality in temperature. Figure 2.9 shows existing seasonality in temperature of Ankara.

• Mean reversion: It is not possible for daily temperature to deviate from mean temperature for long terms. Again, a closer look at Figure 2.8 clearly shows existence of a mean reversion property in temperature data.

• Auto-correlation: A day's temperature is not independent than previous day's temperature. Also, it means that short-term behavior will differ from the long-term behavior. A closer look at Figure 2.10 clearly indicates existence of autoregressive nature of the data. This is also shown in Figure 2.11.

• Higher variation in winter and lower in summer: Another important feature of the temperature data is that winter months have higher variation than summer months. This is shown in Figure 2.7. In other words, variance of temperature rises in the winters and declines in the summers.

• GARCH like disturbances: When mean temperature subtracted from the temperature

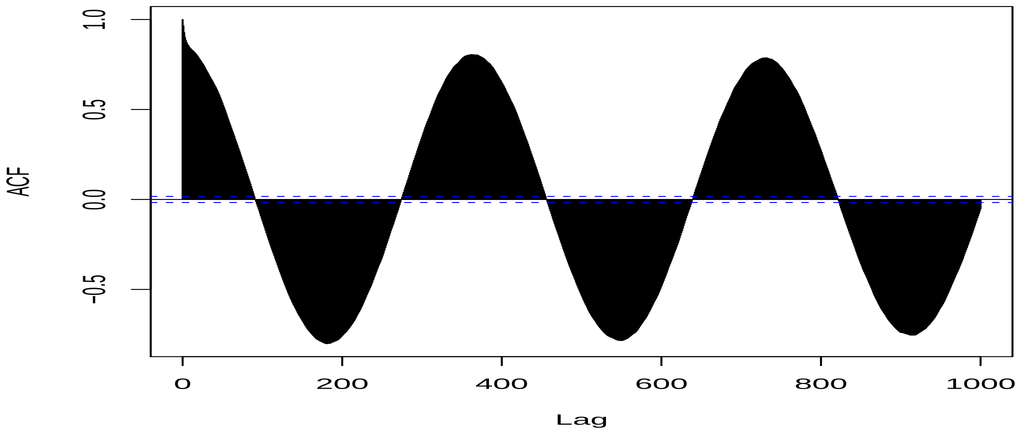


Figure 2.9: Auto-correlation Function of Ankara with lag=1000 to Represent Seasonality

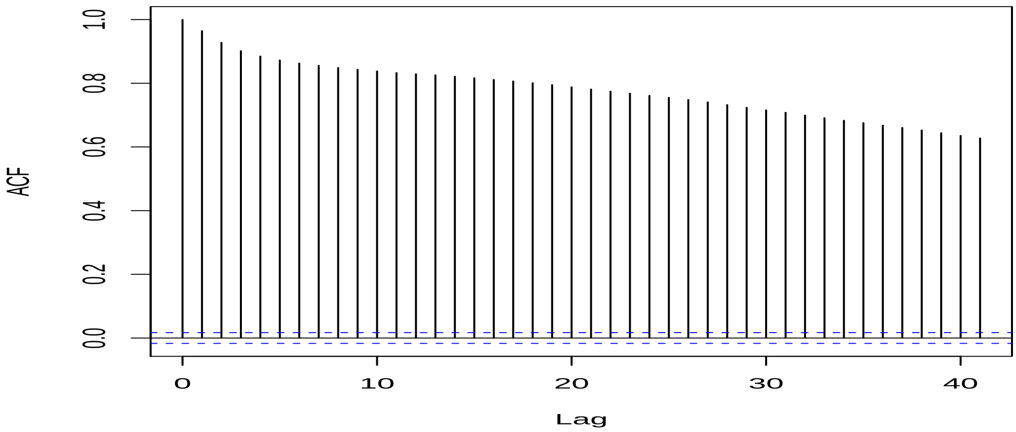


Figure 2.10: Auto-correlation Function of Ankara

data volatility of temperature was obtained. An investigation on this volatility indicates that there exist volatility clusters similar to asset returns. Secondly volatility evolves over time in a continuous manner. Third volatility does not converge to infinity. Lastly, the response of the volatility to big positive and big negative changes is different [51].

• Estimation limits of models: This one is actually not a property of temperature. It is a property of the models that try to explain temperature such that all the models have some limitations in explaining existing data and in predictions. This means it does not matter whatever model is used, some of temperature data will not be included or predicted by models.

• Jumps: Since there are limits in explaining the data and making predictions by the

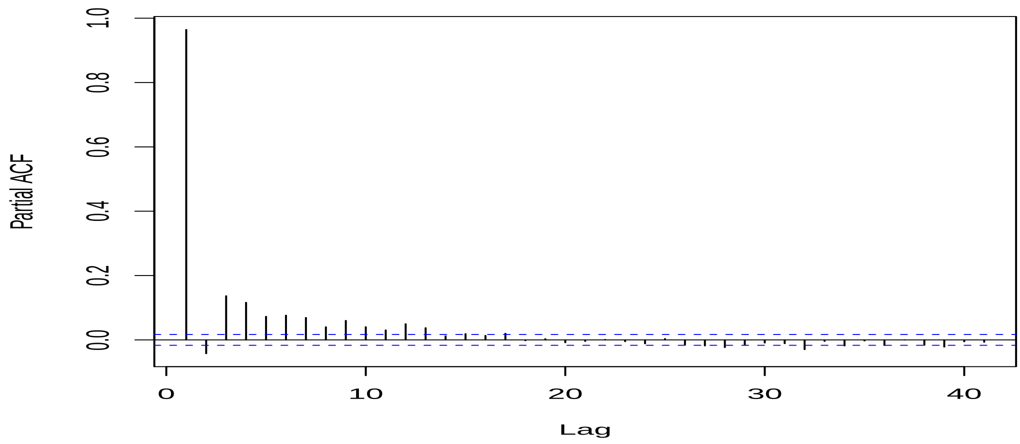


Figure 2.11: Partial Auto-correlation Function of Ankara

existing models it was found that there are out-of-the-bound values that were called as jumps. Existing models fails to cover these jumps and it was found that prediction error were mainly caused by these values.

• Locality: Temperature behaves locally that requires caution in making generalizations.

• Smoothness: Depending on the selection of the time span of the temperature data, parameters tends to be differ. As time span gets longer parameters start to be smoother.

2.4 Proposal of a Temperature Model

Keeping all these information in mind, a model for temperature was offered as following: To represent trend, seasonality, and mean reversion properties, a mean reverting process was offered by using a periodic function to represent mean process similar to [2]. To represent conditional structure of volatility ARCH process were selected after careful examination of wide range of models such as stochastic volatility models, ARIMA etc. The problem for all these models were their incapacity to create volatility clusters. By choosing an ARCH-type model, this problem was solved.

In addition, models with a Brownian motion suffer from the fact that the model will not be able to represent data if there are values above a threshold. This is a known fact as defined in theorem of "modulus of continuity". In the literature, especially in econometrics, there are methods to exclude these above-the-threshold values namely jumps. But in the context of weather derivatives one must do the reverse. It means these jumps must be included in the calculations. The reason for this is that the aim of a temperature model for weather derivatives is to find some index values like HDD, CDD. Since these index values are a summation that collects departures from some base value, excluding jumps will result underestimation of the index. To overcome this problem an additional entity to represent jumps must be added. In addition, observations showed that there are actually two types of jumps: slowly and fast mean reverting jumps. As a result, two different types of jumps were included into the model.

As a note, observations on temperature data revealed an interesting property. In some simulations, it was found that long term value of Hurst component, which is a tool from fractional Brownian motion, in temperature is equal to 0,5. This value validates usage of BM. But when short term Hurst component is considered, this value changes in a band of 0,36 – 0,66. This really affects the success of the model by simply changing the jump behavior. When Hurst component moves downward or upward from 0.5, number of jumps increases. This is an important factor in considering how many data to include in finding parameters of any model. This is due to the fact that having a long time span for the data will result with smoothness such that finding lower values for the parameters of any model.

2.4.1 The Temperature Model

The temperature model is a OU-process driven by a Levy process that contains a Brownian motion (BM), and two mean reverting compound Poisson processes (CPP). Volatility of Brownian motion is a process whose coefficients derived from ARCH disturbances. The model is represented as following:

$$dT_t = \{\tfrac{dT_t}{dt} + b(T_t^m - T_t)\}dt + dL_t \tag{2.18}$$

T_t^m is a cyclical process of temperature and represented in Equation 2.19.

$$T_t^m = A + Bt + C\sin(wt + \varphi) \tag{2.19}$$

where $w = \dfrac{2\pi}{365}$ and φ is the phase angle.

Differential of the driving Levy process dL_t is defined as following:

$$dL_t = \sigma_t dW_t + dY_t + dZ_t \tag{2.20}$$

Brownian component of L_t will be approximated by the ARCH (1) model. To represent the temperature's different jump structures, dY_t and dZ_t are defined as quick and slow mean-reverting OU processes driven by compound Poisson processes with intensities of λ_Y and λ_Z, respectively. A similar usage of a mean-reverting jump process combination was found in the modeling of spot electricity prices in [23]. All the components of the driving Levy process are assumed independent.

$$dY_t = -\alpha Y_t dt + dQ_t \tag{2.21}$$

where $Q_t = \sum_{i=1}^{N_t^Y} U_i$, U_i are i.i.d. random variables and $U_i \sim N(\mu_Y, \delta_Y^2)$.

and

$$dZ_t = -\beta Z_t dt + dR_t \tag{2.22}$$

where $R_t = \sum_{i=1}^{N_t^Z} V_i$, V_i are i.i.d. random variables and $V_i \sim N(\mu_Z, \delta_Z^2)$.

The solution of these non-Gaussian processes are [14]:

$$Y_t = y_0 e^{-\alpha t} + \int_0^t e^{\alpha(s-t)} dQ_s \tag{2.23}$$

and

$$Z_t = z_0 e^{-\beta t} + \int_0^t e^{\beta(s-t)} dR_s \tag{2.24}$$

Solution of the Equation 2.18 is given as follows:

$d(T_t - T_t^m) = b(T_t^m - Tt)dt + dL_t$

Let $K_t = T_t - T_t^m$. Then;

$dK_t = -bK_t dt + dL_t$ where $dL_t = \sigma_t dW_t + dY_t + dZ_t$.

$K_t = e^{-bt} M_t$

$M_t = e^{bt} K_t$

$dM_t = be^{bt} K_t dt + e^{bt} dK_t = be^{bt} K_t dt + e^{bt}(-bK_t dt + dL_t)$

$dM_t = e^{bt} dL_t$

$M_t = M_0 + \int_0^t e^{bu} dLu$

$K_t = e^{-bt} M_t$

$M_0 = K_0$

$K_t = e^{-bt} K_0 + e^{-bt} \int_0^t e^{bu} dL_u$

$T_t - T_t^m = e^{-bt}(T_0 - T_0^m) + e^{-bt} \int_0^t e^{bu} dL_u$

$$T_t = T_t^m + e^{-bt}(T_0 - T_0^m) + e^{-bt} \int_0^t e^{bu} dL_u \tag{2.25}$$

To find value of a temperature-based derivative, one needs distribution of the underlying temperature defined in Equation 2.25. Nevertheless, there is no closed form solution for the distribution of temperature. One way to address this problem is to use characteristic functions. Then by applying inversion techniques, it may be possible to find required information.

In the following, methods to derive characteristic function of the temperature and application of inversion techniques will be represented.

2.5 The Characteristic Function of the Temperature

To find characteristic function of the Equation 2.25, [14] will be followed. A summary of the method offered by [14] can be found in the Appendix. As a first step, it is needed to find characteristic exponent of the driving Levy process defined in Equation 2.20. Characteristic exponent $\psi(u)$ is given by $Ee^{iuL} = e^{\psi(u)}$. The characteristic exponent of the Equation 2.20 will be obtained through the solution of the L_1. A closer look to the Equation 2.20 reveals that it is composed of three independent processes. In addition, dY_t and dZ_t terms in Equation 2.20 have exactly same structure as the processes defined in Equation A.1. In other words, the Equation A.1 represents solutions of processes Y_t and Z_t. As a result the solution can be written directly by using the Equations A.1, A.2, and A.3.

$$L_1 = \int_0^1 \sigma_u dW_u + y_0 e^{-\alpha} + \int_0^1 e^{\alpha(s-1)} dQ_s + z_0 e^{-\beta} + \int_0^1 e^{\beta(s-1)} dR_s \tag{2.26}$$

Now, the Equation 2.26 will be investigated part by part to find their characteristic exponents.

2.5.1 Characteristic Exponent of Brownian Motion

Characteristic exponent of Brownian part will be:

$$\psi_{BM}(u) = -\frac{1}{2}u^2 C \tag{2.27}$$

where $C = E(\int_0^1 \sqrt{\sigma_t^2} dW_u)^2 = \int_0^1 \sigma_u^2 du$

2.5.2 Characteristic Exponents of Jump Processes

In the current model, jump processes dY_t and dZ_t have exactly the same structure with the Equation A.1. Levy part of the Equation A.1 will correspond to Compound Poisson processes (CPP) of the current model. Then, by the Equation A.3 and A.4, characteristic exponent of CPP will be found. Consider following:

Let ϕ_f be characteristic function of jump size distributions.

$\phi_f(w) = \int_{-\infty}^{\infty} e^{iwz} f(x)dx = E[e^{iwx}]$

The characteristic exponent of CPP will be:

$\psi_f(w) = \int_{-\infty}^{\infty} \{e^{iwx} - 1\}\lambda f(x)dx = \lambda \int_{-\infty}^{\infty} \{f(x)e^{iwx} - f(x)\}dx$

$\psi_f(w) = \lambda\{\int_{-\infty}^{\infty} f(x)e^{iwx}dx - \int_{-\infty}^{\infty} f(x)dx\} = \lambda\{\phi_f(w) - 1\}$

Now, by the Equation A.4, characteristic exponent and characteristic function of jump processes Y_t and Z_t will be as follows:

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \int_0^1 \lambda_Y\{\phi_{fY}(ue^{\alpha(r-1)} - 1\}dr$

$E\{e^{iuZ_1}\} = \exp\{iuz_0e^{-\beta} + \int_0^1 \lambda_Z\{\phi_{fZ}(ue^{\beta(r-1)} - 1\}dr$

Because jump size distributions are normal, these expressions can be written more explicitly as following:

$$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y \int_0^1 \{e^{iu\mu_y e^{-\alpha(r-1)} - \frac{1}{2}u^2\delta_Y^2 e^{2\alpha(r-1)}\}} - 1\}dr \tag{2.28}$$

$$E\{e^{iuZ_1}\} = \exp\{iuz_0e^{-\beta} + \lambda_Z \int_0^1 \{e^{iu\mu_Z e^{-\beta(r-1)} - \frac{1}{2}u^2\delta_Z^2 e^{2\beta(r-1)}\}} - 1\}dr \tag{2.29}$$

The integrals in the Equations 2.28 and 2.29 are hard to be evaluated, if not impossible. As a result, an approximation method was developed as following:

Let $A = iu\mu_Y e^{-\alpha}$ and $B = \dfrac{1}{2}u^2\delta_Y^2 e^{-2\alpha}$. Let $e^{\alpha r} = g(r)$. Then;

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y \int_0^1 \{e^{Ag(r) - Bg^2(r)} - 1\}dr\}$

Further, let $D(r) = Ag(r) - Bg^2(r)$ where $e^{D(r)} = 1 + D(r) + \dfrac{D^2(r)}{2!} + ...$

It is known that, being $-\infty < x < \infty, e^x = \sum_{n=0}^{\infty} \dfrac{x^n}{n!} < \infty$, such that $|x| < \infty$. As a result, $|D(r)| < \infty$. Then, by using linear approximation,

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y \int_0^1 \{1 + D(r) - 1\}dr\}$

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y \int_0^1 \{Ag(r) - Bg^2(r)\}dr\}$

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y A \int_0^1 e^{\alpha r}dr - \lambda_Y B \int_0^1 e^{2\alpha r}dr\}$

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y iu\mu_Y e^{-\alpha}(\dfrac{e^{\alpha} - 1}{\alpha}) - \lambda_Y \dfrac{1}{2}u^2\delta_Y^2 e^{2\alpha}(\dfrac{e^{2\alpha} - 1}{2\alpha})\}$

$E\{e^{iuY_1}\} = \exp\{iuy_0e^{-\alpha} + \lambda_Y iu\mu_Y(\frac{1-e^{-\alpha}}{\alpha}) - \lambda_Y \frac{1}{2}u^2\delta_Y^2(\frac{1-e^{-2\alpha}}{2\alpha})\}$

Similarly, the Equation 2.29 was changed to:

$E\{e^{iuZ_1}\} = \exp\{iuz_0e^{-\beta} + \lambda_Z iu\mu_Z(\frac{1-e^{-\beta}}{\beta}) - \lambda_Z \frac{1}{2}u^2\delta_Z^2(\frac{1-e^{-2\beta}}{2\beta})\}$

Information regarding error of this approximation is given in the appendix.

The characteristic exponents of the jump processes will be:

$$\psi_Y(u) = iuy_0e^{-\alpha} + \lambda_Y iu\mu_Y(\frac{1-e^{-\alpha}}{\alpha}) - \lambda_Y \frac{1}{2}u^2\delta_Y^2(\frac{1-e^{-2\alpha}}{2\alpha}) \quad (2.30)$$

$$\psi_Z(u) = iuz_0e^{-\beta} + \lambda_Z iu\mu_Z(\frac{1-e^{-\beta}}{\beta}) - \lambda_Z \frac{1}{2}u^2\delta_Z^2(\frac{1-e^{-2\beta}}{2\beta}) \quad (2.31)$$

2.5.3 Characteristic Function of the Temperature

Finally, the characteristic function of the temperature model is ready to be written explicitly. By the Equation A.4,

$E\{e^{iuT_t}\} = \exp\{iu(T_t^m + e^{-bt}(T_0 - T_0^m)) + \int_0^1 \psi_T(ue^{b(s-t)})ds\}$

where

$\int_0^t \psi_T(ue^{b(s-t)})ds = -\int_0^t \frac{1}{2}u^2e^{2b(s-t)}Cds + \int_0^t iue^{b(s-t)}y_0e^{-\alpha}ds +$
$\int_0^t \lambda_Y iue^{b(s-t)}\mu_Y(\frac{1-e^{-\alpha}}{\alpha})ds - \int_0^t \lambda_Y \frac{1}{2}u^2e^{2b(s-t)}\delta_Y^2(\frac{1-e^{-2\alpha}}{2\alpha})ds +$
$\int_0^t iue^{b(s-t)}z_0e^{-\beta}ds + \int_0^t \lambda_Z iue^{b(s-t)}\mu_Z(\frac{1-e^{-\beta}}{\beta})ds - \int_0^t \lambda_Z \frac{1}{2}u^2e^{2b(s-t)}\delta_Z^2(\frac{1-e^{-2\beta}}{2\beta})ds$

$\int_0^t \psi_T(ue^{b(s-t)})ds = -\frac{1}{2}u^2C(\frac{1-e^{-2bt}}{2b}) + iuy_0e^{-\alpha}(\frac{1-e^{-bt}}{b}) + \lambda_Y iu\mu_Y(\frac{1-e^{-\alpha}}{\alpha})(\frac{1-e^{-bt}}{b}) -$
$\lambda_Y(\frac{1}{2})u^2\delta_Y^2(\frac{1-e^{-2\alpha}}{2\alpha})(\frac{1-e^{-2bt}}{2b}) + iuz_0e^{-\beta}(\frac{1-e^{-bt}}{b}) + \lambda_Z iu\mu_Z(\frac{1-e^{-\beta}}{\beta})(\frac{1-e^{-bt}}{b}) -$
$\lambda_Z(\frac{1}{2})u^2\delta_Z^2(\frac{1-e^{-2\beta}}{2\beta})(\frac{1-e^{-2bt}}{2b})$

More explicitly,

$E\{e^{iuT_t}\} = \exp\{iu(T_t^m + e^{-bt}(T_0 - T_0^m)) - \frac{1}{2}u^2C(\frac{1-e^{-2bt}}{2b}) + iuy_0e^{-\alpha}(\frac{1-e^{-bt}}{b}) +$
$\lambda_Y iu\mu_Y(\frac{1-e^{-\alpha}}{\alpha})(\frac{1-e^{-bt}}{b}) - \lambda_Y \frac{1}{2}u^2\delta_Y^2(\frac{1-e^{-2\alpha}}{2\alpha})(\frac{1-e^{-2bt}}{2b}) + iuz_0e^{-\beta}(\frac{1-e^{-bt}}{b}) +$
$\lambda_Z iu\mu_Z(\frac{1-e^{-\beta}}{\beta})(\frac{1-e^{-bt}}{b}) - \lambda_Z \frac{1}{2}u^2\delta_Z^2(\frac{1-e^{-2\beta}}{2\beta})(\frac{1-e^{-2bt}}{2b})\}$

or,

$$\begin{aligned} E\{e^{iuT_t}\} = \phi_T(u) = \exp\{iu(T_t^m + e^{-bt}(T_0 - T_0^m)) +& \\ iu(\tfrac{1-e^{-bt}}{b})\{y_0e^{-\alpha} + \lambda_Y\mu_Y(\tfrac{1-e^{-\alpha}}{\alpha}) + z_0e^{-\beta} + \lambda_Z\mu_Z(\tfrac{1-e^{-\beta}}{\beta})\}& \\ - \tfrac{1}{2}u^2(\tfrac{1-e^{-2bt}}{2b})\{C + \lambda_Y\delta_Y^2(\tfrac{1-e^{-2\alpha}}{2\alpha}) + \lambda_Z\delta_Z^2(\tfrac{1-e^{-2\beta}}{2\beta})\}\}& \quad (2.32) \end{aligned}$$

2.6 Measuring a HDD by Using Characteristic Function

In the rest of the study, the focus will be on measuring HDDs. It is easy to transport the calculations into other index types. In the current case, inversion techniques will be used to find value of a HDD and its distribution. Thereon, CHDD values will be obtained.

2.6.1 Approximating Density Function of the Temperature

First inversion formula will be applied to characteristic function of the temperature defined in Equation 2.32. Before applying inversion formula, following shortcuts that derived from the Equation 2.32 will be defined and used. Let $f(x)$ and $\phi(z)$ be density function and characteristic function of temperature, respectively.

$$T^* = T_t^m + e^{-bt}(T_0 - T_0^m) \tag{2.33}$$

$$M = (\tfrac{1-e^{-bt}}{b})\{y_0 e^{-\alpha} + \lambda_Y \mu_Y (\tfrac{1-e^{-\alpha}}{\alpha}) + z_0 e^{-\beta} + \lambda_Z \mu_Z (\tfrac{1-e^{-\beta}}{\beta})\} \tag{2.34}$$

$$M^* = T^* + M \tag{2.35}$$

$$V = (\tfrac{1-e^{-2bt}}{2b})\{C + \lambda_Y \delta_Y^2 (\tfrac{1-e^{-2\alpha}}{2\alpha}) + \lambda_Z \delta_Z^2 (\tfrac{1-e^{-2\beta}}{2\beta})\} \tag{2.36}$$

Then, by inversion formula:

$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-izx}\phi(z)dz$

$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-izx}e^{izM^* - \frac{1}{2}z^2V}dz$

$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iz(M^*-x) - \frac{1}{2}z^2V}dz$

$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2V + iz(M^*-x)}dz$

By completing the square;

$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{\frac{-V}{2}(z - \frac{i(M^*-x)}{2V})^2 - \frac{(M^*-x)^2}{4(\frac{V}{2})}} dz$

Let $u = \frac{i(M^*-x)}{2V}$.

$f(x) = \frac{e^{-\frac{(M^*-x)^2}{4(\frac{V}{2})}}}{2\pi}\int_{-\infty}^{\infty} e^{-\frac{V}{2}u^2}dz$

$f(x) = \frac{e^{-\frac{(M^*-x)^2}{4(\frac{V}{2})}}}{2\pi}\sqrt{\frac{2\pi}{V}}$, by using the identity $\int_{-\infty}^{\infty} e^{-ax^2}dx = \sqrt{\frac{\pi}{a}}$

$$f(x) = \frac{1}{\sqrt{2\pi V}}e^{-\frac{(x-M^*)^2}{2V}} \tag{2.37}$$

2.6.2 Inversion Formula Applied to HDD

The main motivation behind option valuation is to find expected value of a contingent claim. In other words, value of a contingent claim is its expected value obtained from payoff function of the claim. When the HDDs are considered it is clear that they are contingent claims on how temperature deviates from a base temperature. As a result, the aim of this part is to find expected value of a HDD to be based in calculations regarding pricing. One way to do this is obtained by, first, finding Fourier transform of the HDD. Then, inverse Fourier transform will be applied to both Fourier transform of a HDD and characteristic function of the temperature [34].

2.6.2.1 Fourier Transform of a HDD

For a given day, HDD is calculated by Equation 2.2. Fourier transform of this expression can be found as following:

Let $x = T_t$, $w(x)$ is HDD's payoff function given in the Equation 2.2, $Base = B$, and $\widehat{w}(z) = \mathcal{F}[w(x)]$, $z \in \mathbb{C}$ be its generalized Fourier transform. Then,

$\widehat{w}(z) = \int_{-\infty}^{\infty} \exp(izx)w(x)dx$

$\widehat{w}(z) = \int_{-\infty}^{B} Be^{izx}dx - \int_{-\infty}^{B} xe^{izx}dx$

By using integration by parts,

$\widehat{w}(z) = \frac{Be^{izx}}{iz}\Big|_{-\infty}^{B} - \frac{xe^{izx}}{iz}\Big|_{-\infty}^{B} - \frac{e^{izx}}{z^2}\Big|_{-\infty}^{B}$

For the convergence, it must be $\mathcal{I}m\,(z) < 0$.

$$\widehat{w}(z) = -\frac{e^{izB}}{z^2}, \mathcal{I}m\,(z) < 0 \tag{2.38}$$

2.6.2.2 Inversion Applied to the Equation 2.38 and Characteristic Function of the Temperature

In this case, inversion will be applied to $\widehat{w}(z)\phi_T(-z)$, where $\widehat{w}(z)$ is defined in the Equation 2.38 and ϕ_T is the characteristic function defined in the Equation 2.32.

Let temperature in Equation 2.25 defined in shorthand notation as $T_t = T^* + \Lambda_t$, where T^* is defined as in Equation 2.33 and $\Lambda_t = e^{-bt}\int_0^t e^{bu} dL_u$, which is derived from Equation 2.25. Characteristic function of the Λ_t can be obtained from Equation 2.32 and be written as;

$$\phi_\Lambda(u) = \exp\big(iu(\tfrac{1-e^{-bt}}{b})\{y_0 e^{-\alpha}\lambda_Y \mu_Y(\tfrac{1-e^{-\alpha}}{\alpha}) + z_0 e^{-\beta} + \lambda_Z \mu_Z(\tfrac{1-e^{-\beta}}{\beta})\}$$
$$-\frac{1}{2}u^2(\tfrac{1-e^{-2bt}}{2b})\{C + \lambda_Y \delta_Y^2(\tfrac{1-e^{-2\alpha}}{2\alpha}) + \lambda_Z \delta_Z^2(\tfrac{1-e^{-2\beta}}{2\beta})\}\big)$$

Then,

$E[HDD] = E[\frac{1}{2\pi}\int_{iv-\infty}^{iv+\infty} e^{-izT_t}\widehat{w}(z)dz]$ where E represents expectations.

$$E[HDD] = \frac{1}{2\pi}E[\int_{iv-\infty}^{iv+\infty} e^{-iz(T^*+\Lambda_t)}\widehat{w}(z)dz]$$

$$E[HDD] = \frac{1}{2\pi}E[\int_{iv-\infty}^{iv+\infty} e^{-izT^*}e^{-iz\Lambda_t}\widehat{w}(z)dz]$$

By Fubini theorem, interchanging the order of integration will give;

$$E[HDD] = \frac{1}{2\pi}E[\int_{iv-\infty}^{iv+\infty} e^{-izT^*}\phi_\Lambda(-z)\widehat{w}(z)dz]$$

To find an appropriate contour, consider following;

According to Cauchy's Theorem, if $f(z)$ is analytic everywhere within a simply-connected region, then; $\oint_C f(z)dz = 0$ for every simple closed path C lying in the region of analyticity. Consider rectangular contour in complex plane as shown in Figure 2.12. Then,

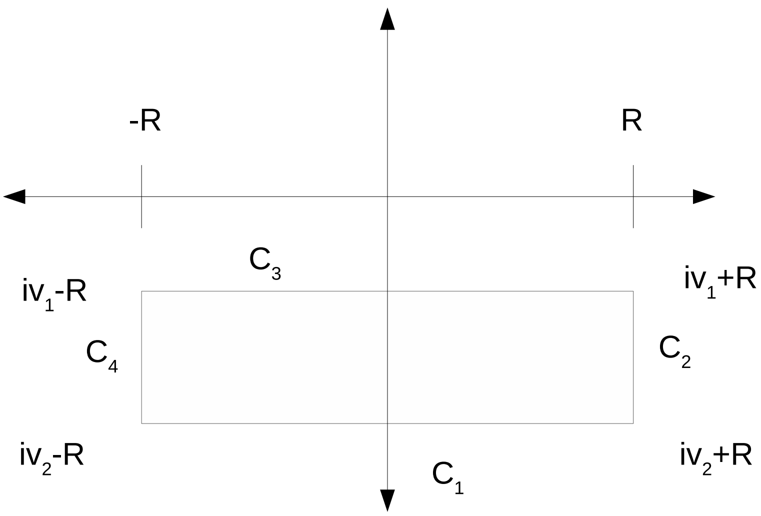


Figure 2.12: Cauchy's Theorem

$$\oint_{C_1} f(z)dz + \oint_{C_2} f(z)dz + \oint_{C_3} f(z)dz + \oint_{C_4} f(z)dz = 0$$

It can be shown that, $\mathbb{R} \to \infty$, $\oint_{C_2} f(z)dz = 0$ and $\oint_{C_4} f(z)dz = 0$. It can be said that;

$$\int_{iv_2-\infty}^{iv_2+\infty} f(z)dz = -\int_{iv_1+\infty}^{iv_1-\infty} f(z)dz = \int_{iv_1-\infty}^{iv_1+\infty} f(z)dz$$

By Equation 2.38, $\mathcal{I}m\,(z) < 0$. In addition, by Cauchy's Theorem, it is possible to select any line parallel to the x-axis. Let $z = u + iv$. For ease, it is selected that $v = -1$. Now, the problem becomes, by using the contour shown in Figure 2.13, to evaluate following integral:

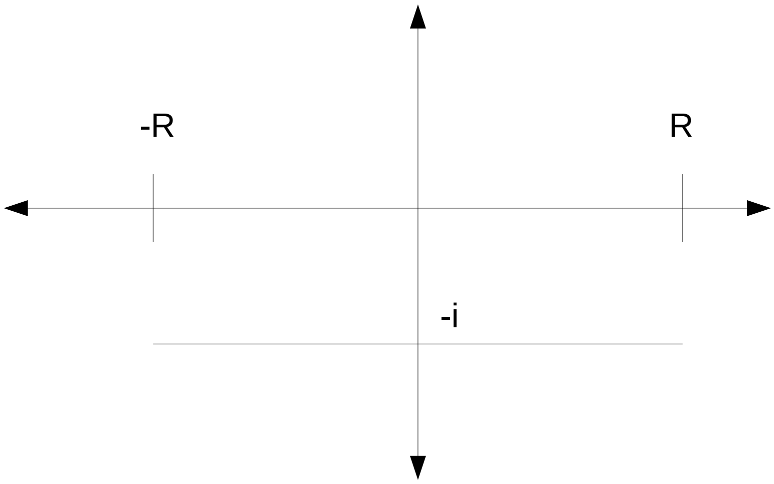


Figure 2.13: Selected contour

$E[HDD] = \frac{1}{2\pi}\int_{-i-\infty}^{-i+\infty} e^{-izT^*}\phi_\Lambda(-z)\widehat{w}(z)dz$ where $z = u - i$.

$E[HDD] = \frac{1}{2\pi}\int_R e^{(-1-iu)T^*}\phi_\Lambda(-u+i)\widehat{w}(u-i)du$

$$E[HDD] = \frac{e^{-T^*}}{2\pi}\int_R e^{-iuT^*}\phi_\Lambda(-u+i)\widehat{w}(u-i)du \tag{2.39}$$

Nevertheless, it was not possible to evaluate this integral analytically. As a result, the elliptic package of R statistical software [22] was applied to evaluate the integral numerically. The results indicated that the integral is equal to the $B - M - T^*$. So, it was concluded that;

$$E[HDD] = B - M - T^* \tag{2.40}$$

2.7 Finding CHDD

Previous result was about HDD of a single day. Temperature options are generally written on for a month or a season. To find cumulative HDD values for a period, independence of HDDs for each day will be considered. It was found that, with linear approximation, HDDs follow normal distribution with mean M^* and variance V. It is known that sum of independent random variables with a normal probability distribution corresponds to a new random variable that follows again normal distribution with a new mean that equals to sum of means and sum of variances. As a result, for P days, the mean of the CHDD will be $PB - PM - PT^*$ and the approximated distribution will be;

$$CHDD \sim N(PB - PM - PT^*, PV) \tag{2.41}$$

2.8 Numerical Estimates

As part of the study, the success of the temperature model offered has been tested in terms of representing the data and forecasting. Within this respect following steps were conducted.

Data: Temperature data is available for 12 cities covering a time span of 38 years starting from 1974 to 2011. In the first part of the estimates 37 years of data were used to estimate parameters. The temperature data of year 2011 was used to make one-year-ahead predictions.

Design: Simulation was designed to be run in two dimensions. First one is to be run on different cities. Second dimension is designed to capture changes in the parameters through time for each city. Within this respect, simulations were started to be run by using 5 years initially. Then they continued by including one more year for each turn. In each turn error estimates, HDD and CDD estimates were calculated. Results were compared with actual HDD and CDD values. A turn, on the other hand, consisted of 10000 runs. At the end of runs average value of 10000 runs were calculated. In other words, we have started with 5 years of data. By using this data parameters were estimated. Then, 10000 runs were conducted and their average was calculated. The whole process were repeated by adding one more year of data reaching 6,7,8,... years of data. Finally, parameters of the year that offered best estimates of the HDDs and CDDs were chosen to be used in one-year-ahead predictions.

Discretization: Discretization was done by using Euler approximation [27, 52, 20] as following:

$$T_{t+1} = T_t + T^m_{t+1} + T^m_t + b(T^m_t - T_t) + H_{t+1} + Y_t - \alpha Y_t + (Q_{t+1} - Q_t) + Z_t - \beta Z_t + (R_{t+1} - R_t) \tag{2.42}$$

where $H_{t+1} = \sqrt{\gamma_0 + \gamma_1 H_t}\epsilon_t$, γ_0 and γ_1 are ARCH parameters and $\epsilon \sim N(0,1)$

Parameter Estimation: Parameters of T^m_t were found by least squares method. To make this, $Y_t = a_1 + a_2 t + a_3 sin(wt) + a_4 cos(wt)$ was fitted to temperature data [2]. Then, parameters were obtained by $A = a_1$, $B = a_2$, $C = \sqrt{a_3^2 + a_4^2}$, and $\varphi = arctan(\frac{a_4}{a_3}) - \pi$.

Mean reversion parameter b were estimated as $\widehat{b} = -log\Big(\frac{\sum_{i=1}^n \frac{[(T_{i-1} - T^m_{i-1})(T_i - T^m_i)]}{\sigma^2_{i-1}}}{\sum_{i=1}^n \frac{[(T_{i-1} - T^m_{i-1})^2]}{\sigma^2_{i-1}}}\Big)$ by [5].

Mean reversion parameters of jump parts were estimated as $\widehat{\alpha} = 1$ and $\widehat{\beta} = -log\left(\frac{\sum_{i=1}^{n} x_{i-1}x_i}{\sum_{i=1}^{n}[X_{i-1}^2]}\right)$ [27].

Simulation of jumps: Having a different structure, jumps were simulated separately and then the results were added to discretized model. For this aim, first, jumps were detected. To do this, after removing the mean from the actual data, the values above two standard deviations were selected. These jumps then, separated into two categories as single jumps and more than one jump to constitute fast and slow mean reverting jumps respectively. Sample means and sample standard deviations were found to represent jump sizes. In addition, intensities were found by $\widehat{\lambda} = \frac{no.of jumps}{no.of observations}$. Then, 10000 runs were realized. In each run, first, jump times were found by using intensities. Then, on each jump time, random draws were realized by using means and standard deviations obtained from data. Finally, discretized jumps were added into discretized jump model.

Parameter estimates for 12 cities were shown in the Appendix. Moreover, R scripts that were written to find parameter estimates were included in the Appendix. Using these parameter estimates, one-year-ahead simulations were conducted to find prediction power of the current model along with three other models. In addition, HDD predictions were calculated based on the Equation 2.40. Following results were obtained and shown in Table 2.7 and Table 2.8.

Table 2.7: One-year ahead prediction and error values for HDDs

City	Actual	The Model	Error	Campbell	Error	Benth	Error	HBA	Error	Eq. 40	Error
Ankara	6047	4980	-17.65%	6203	2.58%	5377	-11.08%	5940	-1.77%	6398	5.81%
Beijing	5880	5705	-2.98%	5571	-5.26%	5039	-14.30%	5319	-9.54%	6218	5.75%
Cairo	573	322	-43.81%	746	30.19%	353	-38.39%	701	22.34%	718	25.31%
Chicago	5980	5814	-2.78%	6098	1.97%	5849	-2.19%	6337	5.97%	3560	-40.67%
Dallas	2151	1777	-17.39%	2202	2.37%	1767	-17.85%	2176	1.16%	1118	-48.02%
Istanbul	3607	2872	-20.38%	3808	5.57%	2972	-17.61%	3485	-3.38%	4520	25.31%
Los Angeles	1441	1250	-13.26%	1127	-21.79%	1119	-22.35%	1244	-13.67%	1608	11.59%
New York	4378	4391	0.30%	4699	7.33%	4516	3.15%	4683	6.97%	4949	13.04%
Paris	3940	4988	26.60%	5006	27.06%	4190	6.35%	4781	21.35%	4902	24.42%
Sydney	1277	1000	-21.69%	1381	8.14%	1002	-21.54%	1343	5.17%	1086	-14.96%
Tokyo	2925	2648	-9.47%	3238	10.70%	2671	-8.68%	764	-73.88%	3309	13.13%
Washington	3572	3758	5.21%	3751	5.01%	3724	4.26%	3851	7.81%	3749	4.96%

Table 2.8: One-year ahead prediction and error values for CDDs

City	Actual	The Model	Error	Campbell	Error	Benth	Error	HBA	Error
Ankara	426	742	74.18%	175	-58.92%	412	-3.29%	417	-2.11%
Beijing	1576	1542	-2.16%	1214	-22.97%	1647	4.51%	1447	-8.19%
Cairo	3221	4243	31.73%	2763	-14.22%	3427	6.40%	3218	-0.09%
Chicago	1071	771	-28.01%	647	-39.59%	866	-19.14%	909	-15.13%
Dallas	3585	2556	-28.70%	2353	-34.37%	2903	-19.02%	2792	-22.12%
Istanbul	1428	1297	-9.17%	792	-44.54%	1224	-14.29%	1089	-23.74%
Los Angeles	426	490	15.02%	569	33.57%	495	16.20%	681	59.86%
New York	1297	1104	-14.88%	906	-30.15%	988	-23.82%	1046	-19.35%
Paris	264	118	-55.30%	63	-76.14%	189	-28.41%	314	18.94%
Sydney	1308	1251	-4.36%	840	-35.78%	1206	-7.80%	1157	-11.54%
Tokyo	1832	1608	-12.23%	1282	-30.02%	1716	-6.33%	434	-76.31%
Washington	1927	1739	-9.76%	1451	-24.70%	1503	-22.00%	1609	-16.50%

Analysis of numerical estimates:

• Best estimates of HDDs and CDDs were obtained for different time periods as shown in the Appendix. This is mainly a characteristic of the temperature as it changes in its long-term behavior. It can be said that it is not a good way to use all the existing data for a city. Instead, every location must be scanned and evaluated for different time periods to obtain best prediction results.

• The current model is equally successful in HDD and CDD predictions.

• Having a good estimate of HDD and CDD values does not necessarily correspond to best fit of the model to temperature data. Main motivation behind this result might be that inclusion of jumps is ending up a better estimation of index values while deteriorating the fit of the model to the data.

• Current model showed its capacity in changing conditions of temperature data. For example, in Chicago both of the jump types were statistically significant and the model predicted HDDs accurately for Chicago. On the other hand, Tokyo did not have any jumps during entire data span and the current model was still able to make accurate predictions for HDDs in Tokyo.

• Interestingly, Historical Burn Analysis that contains long term HDD and CDD averages were successful in predictions. This is mainly due to the fact that temperature does not have change in large especially for certain locations.

• As expected, approximated HDD calculations obtained from Equation 2.40 were less accurate than simulations. Nevertheless, predictions based on Equation (40) were still successful. Estimated HDDs of Los Angeles and Washington were better than any other model.

• Final comment: There are 125.000 weather stations around the world. In this comparison only 12 stations were compared. As a result, it is impossible to say a model is better than all others. However, it was concluded that the current model and Equation 2.40 are successful in certain locations and for certain time periods and have a value to be evaluated.

CHAPTER 3

PRICING

Classical option valuation is based on the idea of risk-neutral valuation. The process is that, instead of usage of real probabilities to obtain asset price averages, risk-neutral probabilities that are also called as Q-probabilities are found that eliminates the need to investigate risk-aversion behavior of the market participants. This permits to define a unique price for the derivative based on this asset. If these Q-probabilities were not defined it might not be possible to define a unique price because each market participant has a unique risk aversion behavior that leads different price perceptions for the asset price that also lead different price perceptions for the derivative written on this asset. After defining Q-probabilities, a hedge portfolio is composed of a riskless and a risky asset that, for each time increment, will mimic the behavior of the derivative.

The setup with Q-probabilities and the hedge portfolio cannot be realized without the concept of complete market. In practical terms, this means that, for each possible value of the asset within the described time horizon, there exist potential buyers and sellers such that any amount of the asset and resultantly any hedge portfolio can be realized.

In the current case of pricing temperature-based derivatives, there are crucial differences from the mentioned pricing method. These differences may be concentrated into two questions:

• What will happen when the underlying, temperature, is not traded?

• Then, what will be the market price of risk?

These two differences make WD market incomplete [18]. When the market is incomplete one cannot apply no-arbitrage pricing since there is no way to replicate the portfolio (or payoff of a portfolio) by portfolio of basic securities [49]. As a result, there is no generally accepted pricing formula. This led some ad hoc solutions to be used in pricing. Some of the current pricing methods can be listed as following:

• Historical Burn analysis: To estimate a fair value, historical burn analysis calculates the average of realized payoffs. For example, to find the value of a put option written on CHDD for January, method considers last 10-20 years of past data for realized CHDDs.

• Modeling and Simulating Weather Events: Focusing on a certain weather events, models and simulations are run to find expected values of the event. First step is to develop a model for the process. After specifying the model, expectation of discounted future payoff is calculated to be used to price contingent claims. But this process is path-dependent which means one needs Monte Carlo simulations since it is not possible to find closed form solutions. Then a large number of simulations are run to determine possible average simulated payoffs that are further discounted for the time value of money. As easily seen, the success of this process is dependent on temperature process.

• Dynamic Valuation Methods: These are mainly attempts to apply risk-neutral valuation into temperature-based derivatives. An example can be found in [2].

Apart from current pricing methods, current study offers a different approach based on an analysis of "Who needs a temperature-based derivative?" and "Under what conditions an economic entity trades a temperature-based derivative?" By answering these questions, it will also be possible to find some solutions for the cases of non-traded underlying and related market price of risk. The approach is explained and presented later in the chapter as follows:

• First step contains an answer to the question "Who needs a temperature-based derivative?" Within this concept, temperature risk will be defined. It will be shown that certain businesses are affected from temperature in an adverse way and need hedging.

• In second step, the temperature risk and classical asset risk will be compared to reveal differences between them. In addition, it will be presented that the discussion regarding market price of temperature risk is inconclusive.

• A fair value for a temperature-based derivative will be presented with real probabilities to be used for hedging purposes.

• It will be shown that the value of a temperature-based derivative ends up with superhedging if risk-neutral probabilities were used.

• By considering inconclusive results regarding market price of temperature risk and inappropriateness of usage of risk-neutral probabilities in valuation of temperature-based derivatives, a new approach for pricing was offered. The new approach is based on the idea that the temperature risk is dependent on the business type such that it is personal. In return, this personal risk will be reflected in a personal price of the derivative. Within this respect, a company with an objective of profit maximization is considered to form the derivative price. The company is evaluated to reveal "Under what conditions an economic entity trades a temperature-based derivative?"

3.1 Measuring Temperature Risk

First step is to define temperature risk. There seems to be a general consensus on the definition of temperature risk. There is an adverse effect of temperature on business, and this effect reveals itself in volume. In simple terms, temperature risk is volume risk; it affects sales [8].

Once having identified temperature risk, the problem arises of how to measure it. Change in expected sales is a good candidate to begin with. The main problem with measuring change in expected sales is that many other factors may cause a change in expected sales. In other words, temperature risk needs to be isolated. In the domain of derivatives based on temperature, this problem may be the least discussed. In any case, a multivariate linear regression method to isolate the effect of temperature on sales can be used [44]. Another study that deals with measuring weather risk can be found in [50]. Next is to consider how a change in sales will be reflected in profits of the company.

The next idea is not an observation but rather a requirement for any company to survive. It will be assumed that each economic entity is able to measure its exposure to temperature.

At this point, the focus will be on a single company with an obvious exposure to temperature to reveal the relationship between temperature index and sales of the company. As a candidate company, consider a retail gas seller where the company is concerned about sales and profit of the next January. For this type of a company, it is expected that a relationship between sales and temperature will exist. Some examples for existence of this relationship can be found in [44, 42]. In summary, for the mentioned gas company, it is expected a relationship between its sales and temperature as shown in following figure:

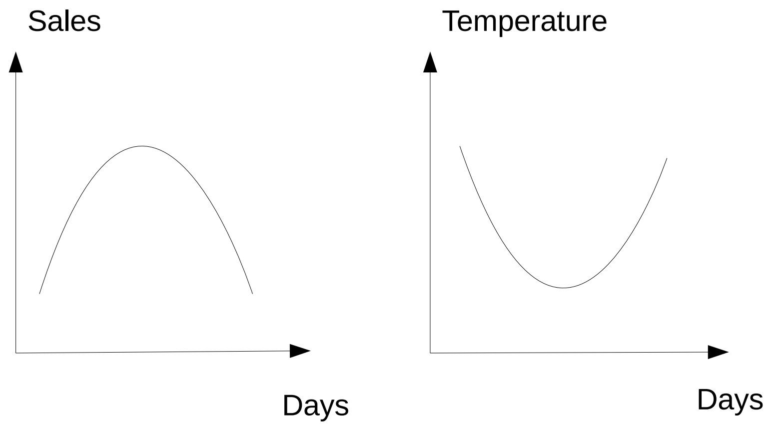


Figure 3.1: Relationship Between Sales and Temperature

By using Equation 2.2, a new form of Figure 3.1 will be established in terms HDDs derived from temperature as shown in Figure 3.2.

For this company, it is now possible to reveal the relationship between sales and HDDs by using regression analysis such that, for each additional HDD, there will be an increase in sales. With the additional assumption that this relationship will continue in the near future, another relationship can be constructed between expected sales (ES) and CHDD as shown in the following equation:

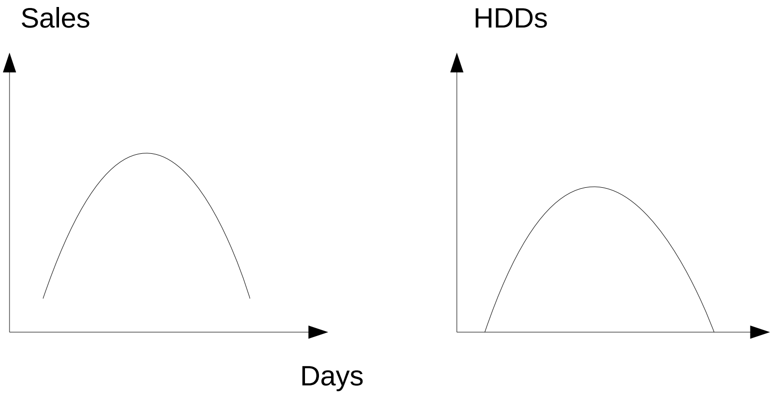


Figure 3.2: Relationship Between Sales and HDDs

$$ES = a + bCHDD \tag{3.1}$$

where ES is expected sales.

An important note for the the Equation 3.1 is that CHDD is a stochastic value in reality, but in this case a corresponding value is assumed for each possible value of CHDD. This actually shows a payoff function where for each CHDD there is a certain amount of sales.

The above setup is shown in Figure 3.3 where the Equation 3.1 corresponds to ES Line in the figure.

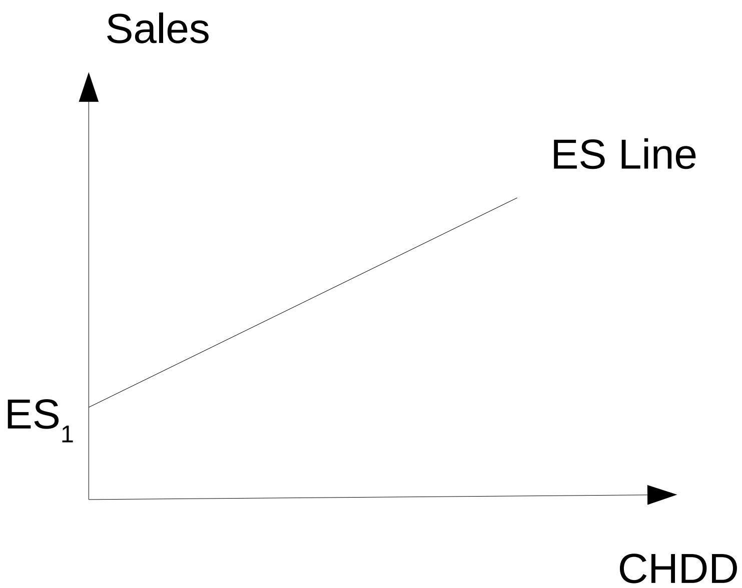


Figure 3.3: Relationship Between Sales and CHDD

To ascertain the magnitude of the effect of temperature, some additional assumptions are made regarding cost and revenue functions of this company. For simplicity, linear cost and revenue functions will be used as in the following equations.

$$ExpectedCost = C = \Theta + ExpectedSales \tag{3.2}$$

where Θ is a constant.

$$ExpectedRevenue = R = Price * ExpectedSales \tag{3.3}$$

where $Price$ is the constant price of the commodity sold by the company and $Price > 1$.

Therefore, profits of the company can be written as the following equation:

$$ExpectedProfit = P = R - C = Price * ES - \Theta - ES = ES(Price - 1) - \Theta \tag{3.4}$$

The aim is to construct a relationship between CHDD and profit functions to ascertain magnitude of effect of temperature on the company. This relationship is shown Figure 3.4 with the assumption that $\Theta > a * Price$.

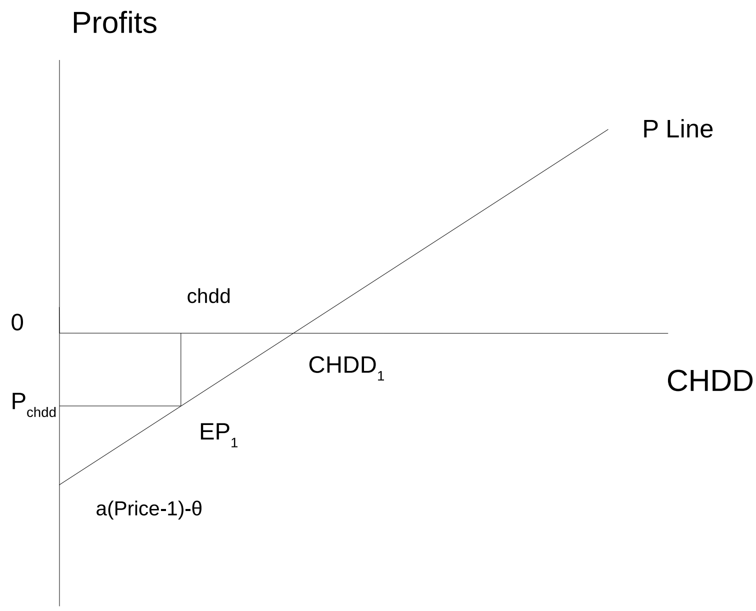


Figure 3.4: Relationship Between Expected Profit and CHDD

What is achieved by Figure 3.4 was that it is constructed a relation between each CHDD and profit of the company such that every CHDD value now represents a monetary value in terms of positive and negative profits. In the figure, there is a deterministic relation between each level of CHDD and a certain level of profit. What is unknown is that what will be the CHDD value for the proposed period.

Now, the risk of the company can be defined as having a low value of CHDD for a certain period that leads to a loss. In other words, the risk of the company will fall left of $CHDD_1$ and will be covered by P Line. This possible loss will be referred to as temperature risk (TR) and is equal to the area covered by $0CHDD_1a(Price - 1) - \Theta$. This is actually the total TR (TTR) and can be realized if CHDD for the next January becomes zero. In reality, the CHDD for the next January is not known because it is a stochastic value. Therefore, TR can be written as:

$$TR = TTR - \int_0^{chdd} PdCHDD = \int_{chdd}^{CHDD_1} PdCHDD \tag{3.5}$$

where $chdd$ is unknown value of CHDD for a certain period.

After defining temperature risk, the focus can be shifted to differences between classical asset risk and temperature risk. The market price of temperature risk will also be evaluated.

3.2 Temperature risk vs. classical asset risk

To reveal the differences between temperature-based derivatives and classical derivatives, consider the underlying of an ordinary derivative namely stock and the underlying of a temperature-based derivative namely CHDD. Assume that they both have a value of 100 at present time. It is calculated that they both have equal probabilities for an up or down movement after a certain period.

At the first sight, both of the underlying items are similar, at least in their values and in their expected values. Besides, there are some fundamental differences:

• First, stock is about having something. On the other hand, temperature is about something is exposed to. Besides, to continue, assume that they represent the same thing.

• For the owner of the stock, a decrease in the price will mean a loss in wealth of the owner. In case of CHDD, a decrease in index value does not necessarily mean a decrease in the wealth. For example, for a gas company, a decrease in the index value means a decrease in the sales. But, at the same time, this decrease may result an increase in the sales of a beverage company.

• In addition, a decrease of the price of the stock from \$100 to \$50 means a decrease that equals to \$50 for the owner. Besides, a decrease in the index value from 100 to 50 may result an unknown amount of loss for the mentioned gas company. This fact also requires considering the well-known concept of 'risk aversion in the small' and 'risk aversion in the large'. These concepts emphasize the fact that people show different risk aversion behavior when there is a change in the amount of risk exposed.

• One share of Intel Inc. means the same thing for anybody who owns it in every part of the world. On the other hand, index value of CHDD will have a meaning only for the periphery of the weather station where temperature is measured.

These differences, simply, imply that the amount of temperature risk is related with business type. It is something personal for each of the economic entities. In fact, this conclusion will lead to a personalized price as described at the end of this chapter.

On the other hand, although different in nature, the temperature risk still be investigated in terms of market price of risk. It was found that there are studies in the literature that investigate risk premiums for weather derivatives with inconclusive results. For example, [13] examines the efficiency of weather futures on CME in HDD and CDD futures. They stated that risk premiums varied from negative to positive values across cities.

One interesting perspective for the market price of risk about temperature-based derivatives comes from Hull [26] and Turvey [53]. Hull [26] states that weather derivatives have no systemic risk. Consequently, the payoffs of these claims can be calculated with real probabilities. Turvey [53] also developed and supported this idea. He suggested using CAPM to estimate the market price of risk.

These different approaches regarding market price of temperature risk supported the idea that considers trading behavior of a company that is exposed to temperature risk instead of considering risk attitudes of the same company in valuation of a temperature-based derivative.

Next section offers a value for an option written on CHDD with real probabilities that can be used in hedging the temperature risks.

3.3 An Approximated Fair Price of Temperature Based Put Option

Under linear approximation, it was found that temperature distributed normally with mean M^* and variance V as defined in the Equations 2.35 and 2.36, respectively. Consider following:

$CHDD = HDD$ of Day 1 $+ HDD$ of Day 2 $+ .. + HDD$ of Day P

In addition, the idea of independence of HDDs will be followed. Then the distribution function will be $CHDD \sim N(PB - PM^*, PV)$. Let K, which may be set to $CHDD_1$ from the Figure 3.4, is the strike value, $x = CHDD$, and $f(x)$ is the probability density function of CHDD. Then, the value of a put option on CHDD will be equal to:

$$E[max(K - x, 0)] = \int_{-\infty}^{K} (K - x) f(x) dx$$

$$E[max(K - x, 0)] = \frac{K}{\sqrt{2\pi PV}} \int_{-\infty}^{K} e^{-\frac{(x - PB + PM^*)^2}{2PV}} dx - \frac{1}{\sqrt{2\pi PV}} \int_{-\infty}^{K} x e^{-\frac{(x - PB + PM^*)^2}{2PV}} dx$$

Let $u = x - PB + PM^*$, then;

$$E[max(K - x, 0)] = \frac{K - PB + PM^*}{\sqrt{2\pi PV}} \int_{-\infty}^{K - PB + PM^*} e^{-\frac{u^2}{2PV}} du$$
$$- \frac{1}{\sqrt{2\pi PV}} \int_{-\infty}^{K - PB + PM^*} u e^{-\frac{u^2}{2PV}} du$$

Let $N(x) = \int_{-\infty}^{x} f(x)dx = P(X \leq x)$. In other words $N(x)$ is the probability that a variable with a mean of$PB - PM^*$ and a variance of PV is less than x, the first integral after the equality is:

$$(K - PB + PM^*)N(K - PB + PM^*)$$

The second integral after equality is equal to $\frac{\sqrt{PV}}{\sqrt{2\pi}}e^{\frac{-A^2}{2PV}}$ where $A = K - PB + PM^*$. The solution will be

$$E[max(K - x, 0)] = (K - PB + PM^*)N(K - PB + PM^*) + \frac{\sqrt{PV}}{\sqrt{2\pi}}e^{\frac{-A^2}{2PV}}$$

Now, when this result is multiplied with tick value, which is a pre-agreed monetary value that converts each CHDD into money and discount the result with risk-free rate, price of the option will be obtained.

$$E[max(K-x, 0)] = e^{-r(T-t)}\left[(K-PB+PM^*)N(K-PB+PM^*)+\frac{\sqrt{PV}}{\sqrt{2\pi}}e^{\frac{-A^2}{2PV}}\right]*tick \tag{3.6}$$

What is found with the Equation 3.6 is a fair price based on real probabilities. It may be also called as actuarial price since it is based on expected values.

Next section will show that option value will be super-hedging value if risk-neutral probabilities were used.

3.4 Risk-Neutral Pricing

In this section, a put option written on CHDD with a strike value of K will be priced. Besides, CHDD is composed of HDDs. A closer look at HDDs reveals that realizations of HDDs can be represented by binomial model such that if HDD is realized there will be an up movement and otherwise there will be 0. As a result, binomial model for option pricing can be used for this case. The calculations were done by the book of Bjork [6]. For the up movement, the best candidate for HDD will be the mean value of an HDD, which was found in the Equation 2.40. Let U = Equation 2.40. Then, following representation can be defined.

For each day, there will be either an up movement that equal to mean HDD or a down movement that adds nothing to the cumulative index. The probability for the up movement, p_u is found by $\int_{-\infty}^{B} f(x)dx$, while the probability of down movement is equal to $p_d = 1 - p_u$. Let risk-free rate $R = 0$. The model satisfies the condition of being arbitrage-free in the form of $d \leq 1 + R \leq u$ by definition. In addition, martingale measure for the current model is defined as $CHDD_t = E^Q[CHDD_{t+1}]$. The problem arises when it is calculated the risk neutral probabilities. Because the CHDD is either up or remain same, the only possibility to satisfy the condition $CHDD_t = E^Q[CHDD_{t+1}]$ is to set risk neutral probability for up to 0 and risk neutral probability for down to 1 i.e. $q_u = 0$, $q_d = 1$ if $R = 0$. In addition, the value for the up movement

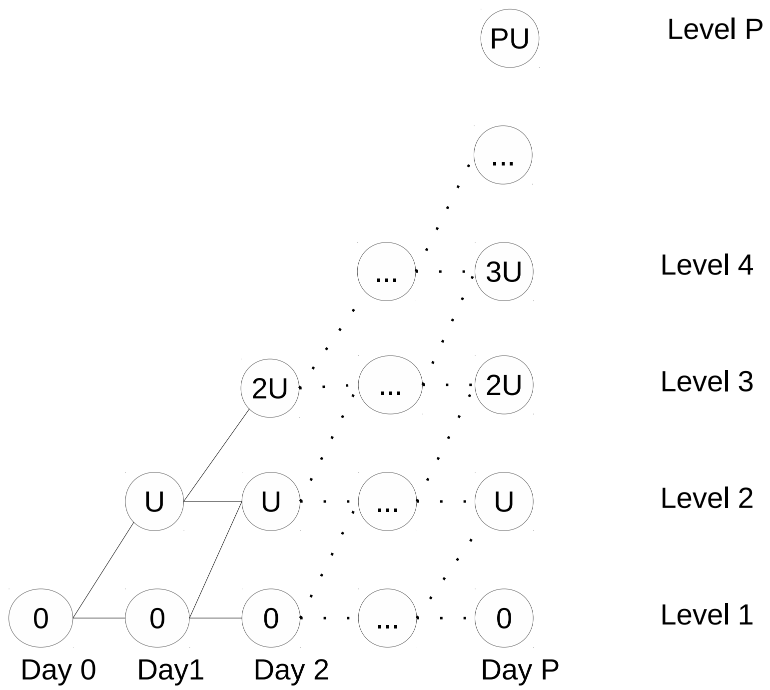


Figure 3.5: Construction of CHDD Tree

represented by u must be changed for each node of the binomial tree and the d that represents down movement must be equal to 1 for each node in the tree. If it is set that $K = PU$, under the given setup, the value of the option will be equal to K itself.

The reasons behind the above result are due to the fact that CHDD is a summation process and certainly a sub-martingale compared to underlying of an ordinary option. The only possibility to obtain martingale form of the underlying process CHDD is then to consider the whole summation process and reflect it as a constant.

One interesting implication of this result can be found with re-investigation of Figure 3.4. Figure 3.6 is constructed with the combination of Figure 3.4 and Figure 3.5.

In the figure, each level of TR is connected to up movements created by HDDs. This time TR becomes a super-martingale and when TR is converted into a martingale it will be equal to TTR. These ideas supported with following propositions.

Proposition 3.1. *Temperature Risk (TR) is a super-martingale.*

Proof. Assume $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space. $\Omega = [0, CHDD_1]$, $\mathcal{F} = \sigma(CHDD)$, $CHDD = 0 \ldots CHDD_1$, TR is adapted to $\mathcal{F}$, $\mathcal{P}$ is Gaussian with (μ, σ^2). It is obvious that CHDD values obtained at time horizon do not fit perfectly into a Gaussian distribution. However, they were assumed as Gaussian by Equation 2.37. Let c_t be any CHDD value between 0 and $CHDD_1$, and $s < t < CHDD_1$. $P_(c_t)$ is profit function evaluated at c_t. Then $E[TR_t] = \left[\frac{(CDDD_1 - E(c_t))E[\mathcal{P}_{c_t}]}{2}\right]$ and $E[TR_s]$ can be written similarly. Here, $E[P_{c_t}] = Price * a + Price * b * E(c_t) - \Theta - a - b * E(c_t)$. As seen from the equations, the differences come from $E[c_t]$ and $E[c_s]$. Because these pairs are

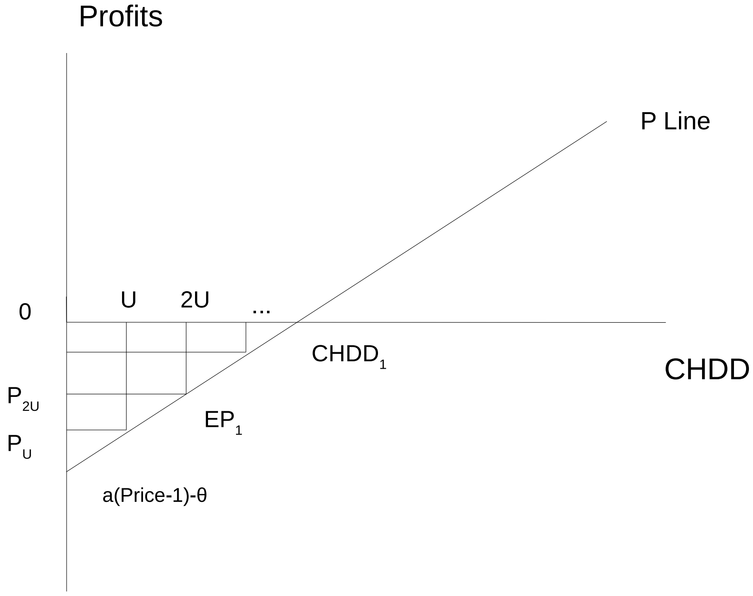


Figure 3.6: Evolution of TR

obtained from a cumulative process, and for each time step, expected value of HDD is positive, $E[c_t] > E[c_s]$ at $\mathcal{F}_0$. Additionally, $E[P_{c_t}] > E[P_{c_s}]$ (corresponding to a lower level of loss), and $c_t > c_s$. Then, $E[TR_t] < E[TR_s]$. Given $\mathcal{F}_s$, $E[\frac{c_t}{\mathcal{F}_s}] > c_s$ and $E[\frac{TR_t}{\mathcal{F}_s}] < TR_s$. $\square$

Proposition 3.2. *Total Temperature Risk (TTR) is a martingale.*

Proof. The same probability space defined in Proposition 3.1 is still valid. TR value is calculated as the difference between areas under revenue and cost functions subtracted from TTR such that, for any CHDD value, $c_t < CHDD_1$, and $E[TR_t] = \frac{CHDD_1(a(Price-1)-\Theta)}{2} - \frac{((a(Price-1)-\Theta)+E[P_{c_t}])E(c_t)}{2}$. By Proposition 2.3.1 of Lamberton [32] also referred to as Doob decomposition, super-martingales can be written as $SM_t = M_t - A_t$, where SM_t is a super-martingale process, M_t is a martingale, and A_t is an non-decreasing sequence. The above equation exhibits the same structure: $\frac{CHDD_1(a(Price-1)-\Theta)}{2}$ is a constant being a martingale by definition, $\frac{((a(Price-1)-\Theta)+E[P_{c_t}])E(c_t)}{2}$ is an increasing function. Similarly, $E[TR_s]$ can be written for any $c_s < c_t < CHDD_1$ and $|\frac{((a(Price-1)-\Theta)+E[P_{c_t}])E(c_t)}{2}| > |\frac{((a(Price-1)-\Theta)+E[P_{c_t}])E(c_t)}{2}|$ $\square$

The importance of the propositions lies in the fact that a company who wants a hedge for a possible risk with a certain amount will be willing to pay for the hedge an amount related with the magnitude of the risk. Nevertheless, in the current case with risk-neutral probabilities, it is not the actual risk but the total risk is considered for the hedge. For example, assume that the actual risk with real probabilities are calculated as the area covered by $2UP_{2U}CHDD_1$. What is expected in this case is that if company enters into a transaction for a put option written on CHDD with a strike value of $CHDD_1$ it will be willing to pay an amount related with the mentioned area. But instead, the risk neutral valuation forced the company to consider the total risk instead of the mentioned area. This means that the company will use total risk in pricing the put option. This example shows the importance of usage of real probabilities in risk measuring.

The situation described in previous paragraph is a known subject called as super-hedging. It means in simple words that hedging the total risk. In a pricing manner, the price of the hedge will be equal to total risk. Using the total risk instead of actual risk will definitely prevent any form of transaction both in terms of hedge demanders and suppliers. As a result using risk-neutral probabilities are not ideal for pricing a temperature based derivative.

The discussion in this section up to now can be summarized as follows:

• The risk of temperature has different structure than underlying of an ordinary derivative like stocks. In case of stocks, the risk has emerged from themselves in the form of a decrease in their prices. On the other hand, the risk of temperature emerged from its effect on businesses. In this case the type of the business becomes important.

• The concept of market price of risk is inconclusive. This is observed both in theoretical and empirical studies.

• Risk-neutral pricing of temperature based derivatives ends with super-hedging that prevents any form of trade.

Under these circumstances, only option remained for pricing a temperature based derivative is to use utility functions. This brings additional problems. For example, what will be the utility function of a company? Is it the utility of owners or managers? To overcome this problem, researchers assign utility functions like exponential, power etc. Besides, this brings another problem. Depending on the choice of the utility function, the price that is calculated will change.

By considering all above obstacles, a different perspective was developed and presented in the next section.

3.5 A New Setup for Pricing

In this setup, utility functions are replaced by some objective functions. As a result, instead of maximizing a utility function, any economic entity either being an individual or a company will try to achieve an objective. In the current case a natural objective for the mentioned company is to maximize its expected profit given in Equation 3.4. Then the question becomes under what conditions this company will maximize its profits. The answer of this question will reveal the trading behavior of the company. Consider following theorem:

Theorem 3.3. *The mentioned company will buy a put option if following condition is* *satisfied:*

$$E[max(K - CHDD, 0)]N(K - PB + PM^*) \geq C \tag{3.7}$$

where $E[max(K - CHDD, 0)]$ and $N(K - PB + PM^*)$ is given in Equation 3.6. C is the cost of the put option.

Proof. The Equation 3.4 is the expected profit in case where there is no trade for options. If there is a chance to trade an option, the mentioned company will prefer a put option with a strike value equal to chdd from the Figure 3.4. Assume that company buys an amount of the put option equal to ϵ with a cost of C. Let $tick$ value equal to \$1. There will be two states at the end of the determined period depending on the payout of the option. The setup is given in the following:

Let x to represent CHDD. Then;

State 1:

$\big((a+bE[x])(Price-1)-\Theta-\epsilon C+\epsilon E[max(K-x,0)]\big)$ where E represents expectation

State 2:

$\big((a + bE[x])(Price - 1) - \Theta - \epsilon C\big)$

The probability $N(K - PB + PM^*)$ also defines the probability of State 1. Then, probability of State 2 will be $1 - N(K - PB + PM^*)$. Under these probabilities the expected value of the two states will be:

$$\begin{split}\big((a + bE[x])(Price - 1) - \Theta - \epsilon C + \epsilon E[max(K - x, 0)]\big)N(K - PB + PM^*)+\\ \big((a + bE[x])(Price - 1) - \Theta - \epsilon C\big)(1 - N(K - PB + PM^*))\end{split} \tag{3.8}$$

The mentioned company will enter a trade for a put option if the Equation 3.8 is greater or equal to no trade case such that

$\big((a+bE[x])(Price-1)-\Theta-\epsilon C+\epsilon E[max(K-x,0)]\big)N(K-PB+PM^*)+\big((a+bE[x])(Price-1)-\Theta-\epsilon C\big)1-N(K-PB+PM^*) \geqslant (a+bE[x])(Price-1)-\Theta$

Besides, the profit function with no trade case is also reflected in the left hand side of the equation with a probability of 1. Consequently, subtracting the right hand side from the left hand side will result;

$-\epsilon C + \epsilon E[max(K - x, 0)]N(K - PB + PM^*) \geq 0$

$E[max(K - x, 0)]N(K - PB + PM^*) \geq C$ □

Although above calculations are given for a candidate company, since profit function dropped out and tick value is equal to \$1, the Equation 3.7 defines a general case valid for any company dealing with a put option with $K = chhd$ where chdd obtained from Figure 3.4. As a result, following definition is given for the case $E[max(K - CHDD, 0)]N(K - PB + PM^*) = C$:

Definition 3.1. When $E[max(K - CHDD, 0)]N(K - PB + PM^*) = C$, the value C is called the general price valid for any economic entity.

The above setup can be extended by an idea called as shadow price, which gives the effects of the resources in a production process on profit. In the current case, the profit function is designed as a deterministic function such that it is a payoff function of the index value CHDD. As a result, CHDD can be seen as the resource that produces the profit. It is possible to measure the effect of one unit of change in CHDD on profit and use it as the price of one unit of CHDD. Again assuming $CHDD = x$, consider following:

$$\frac{dProfit(x)}{dx} = b(Price - 1) \tag{3.9}$$

The value given in Equation 3.9 is a good candidate for the tick value mentioned in Equation 3.6. Now, by using the Equations 3.6 and 3.9, a new definition can be given:

Definition 3.2. A personalized price for a specific company of the put option $E[max(K - CHDD, 0)]$ is given by

$$E[max(K - CHDD, 0)]N(K - PB + PM^*)b(Price - 1) = C \tag{3.10}$$

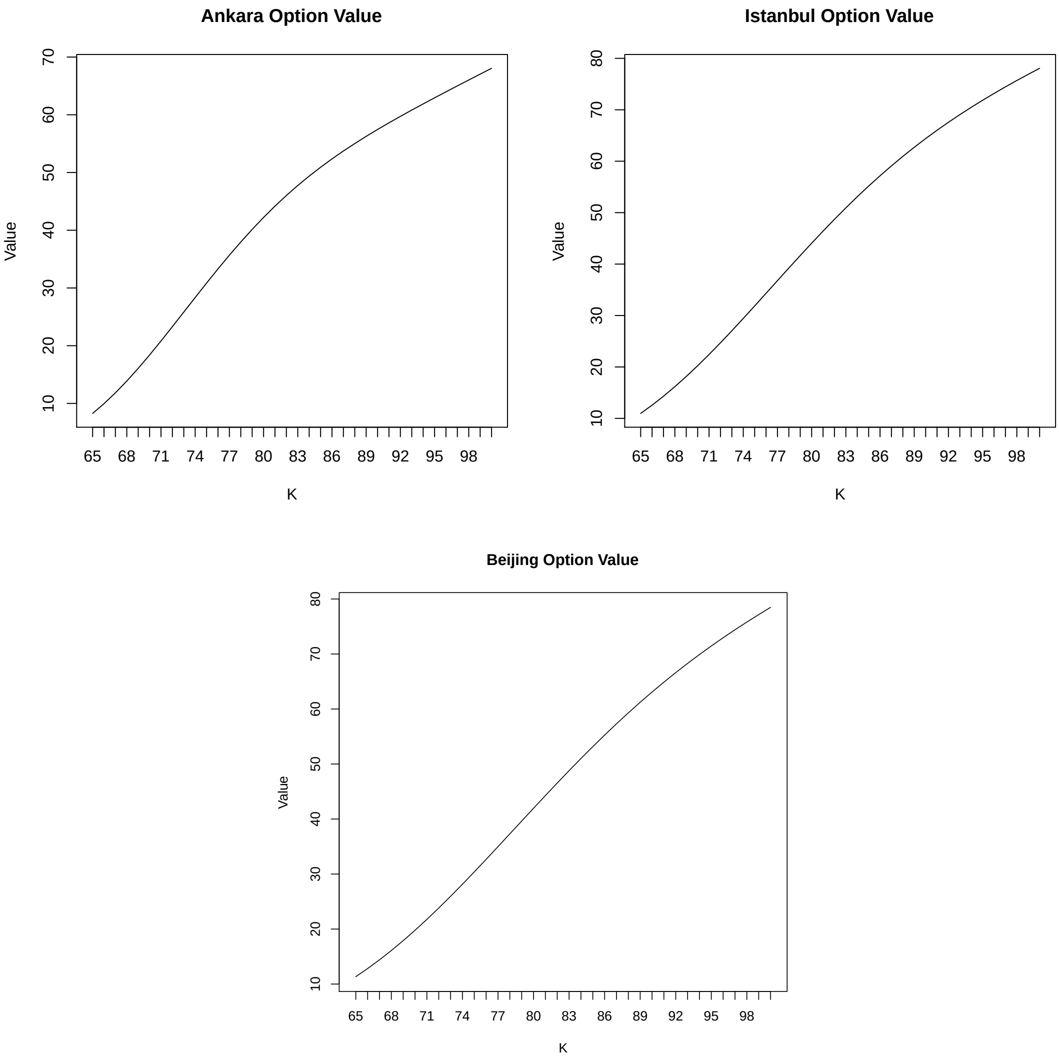
Discussion for A New Setup for Pricing

Equation 3.7 defines the profitable conditions for the company. Three conditions can be revealed from the Equation 3.7. When $E[max(K-x, 0)]N(K-PB+PM^*) \geq C$, the company will buy the option. If $E[max(K - x, 0)]N(K - PB + PM^*) = C$, the company will be indifferent between buying the option or doing nothing. As a final case, when $E[max(K - x, 0)]N(K - PB + PM^*) \leq C$, it is the best interest of the company to sell the option as it maximizes the profit. The value C, when $E[max(K - x, 0)]N(K - PB + PM^*) = C$, can be defined as the fair price as it does not result with a positive profit.

From here a connection between the current approach and the utility approaches can be established. Since the current setup is based on the idea of profit maximization, it coincides with the utility approach based on wealth maximization. The gain is that this statement is true for any utility function choices.

By using Equations 3.7 and 3.6 a numerical estimate for the price of a HDD for several cities was developed. For this aim, one day ahead estimate of temperature was found. Mean and standard deviation of the approximated distribution were calculated according to the Equations 2.33 to 2.36. The value of C in Equation 2.36 was approximated by conditional variance of the ARCH model. Tick value was taken as \$1. Strike value K was taken as an interval from 65 to 100. The estimated values are shown in Figure 3.7.

Figure 3.7: Estimated Values of an HDD



As mentioned earlier, Equation 3.7 defines a general price and trading behaviors for any company as profit function dropped out from the equation. Besides, this generality does not give much insights about what actually a put option with strike value of K means for a specific company. This deficiency was corrected with replacement of tick value with Equation 3.9. This replacement was rationalized with the idea that, unlike to ordinary assets, temperature affects economic entities on different scales. As a result, a personalized price must apply for each of the economic entities. Moreover, Equation 3.10 and Definition 3.2 state that the mentioned company will enter a trade for the option if there is a possibility for arbitrage. If the fair price available, the company will be indifferent to enter a trade or do nothing. In the presented case, the expected profit will always be at maximum. As a result, the utility of the company will always be at maximum. The case of risk aversion will prevent this kind of maximums for profit and utility. It is believed that having a profit at maximum and resulting maximum at utility will direct the company to follow the presented approach not the sub-optimal risk aversion behavior.

Final words can be said for potential uses of temperature-based derivatives in Turkey. It is clear that, like the rest of the world, some Turkish companies are exposed to temperature risk. With the models and methods presented in this study, Turkish companies may have hedge their risk. As a starting point in this process, HDD and CDD futures and options can be offered within Borsa Istanbul for major cities of Turkey.

CHAPTER 4

Conclusions

Temperature based derivatives are the result of business act that seeks hedging mechanisms for adverse effect of temperature on business. The aim of the study was to cover the topic of temperature derivatives from defined properties of temperature to describe how pricing can be done. Keeping applicability of proposed models and methods, the study started with defining a temperature model based on its properties. After that, inversion methods applied on to the characteristic function of the temperature to obtain expected value of one type of temperature based index namely HDD. In addition, inversion method led finding an approximated distribution for the temperature and HDDs. The, expected values of the HDDs were combined to obtain cumulative HDDs, which are base for temperature based derivatives.

In pricing part of the study, after defining temperature risk, it was found that the discussion about market price of temperature risk was inconclusive. In addition, it was shown that risk-neutral pricing of a temperature-based derivative will result with superhedging. Moreover, the temperature risk was shown to be different than classical asset risk in the sense that temperature risk is related with the business type i.e. it is personal. Summing all these information led to development of a personalized price based on personalized temperature risk.

In summary, following conclusions and contributions to the literature were derived:

• Derivatives written on temperature are based on index values obtained from temperature data. These indices are basically measured as deviations of temperature from a threshold value. This makes measuring deviations from a base temperature in the form of jumps important for any temperature model. In the current study, different kind of jumps were included into the temperature model and handled by using different techniques.

• Simulation results showed that each location must be evaluated for different time periods during parameters estimation to obtain best predictions of temperature index values.

• Unlike to existing models that consider temperature risk as the result of the temperature itself like in stocks, current model shows that financial risk caused by temperature is different from classical asset risk. This risk is dependent on the business type. It is in fact a personal risk. For example, in case of a rise in the temperature levels in January, we expect a rise in the profits of a beverage firm while we expect a decline in the profits of a retail gas seller. In our study we showed a way to measure this temperature risk.

• Almost all of the existing pricing methods are based on risk-neutral valuation. This study showed that risk-neutral valuation in temperature based derivatives ends with super-hedging.

• Current study offers a pricing scheme that is different than the classical pricing approaches that are based on risk-neutrality or risk-aversion concepts. Instead of utility functions, current study employs more realistic and practical approach in terms of objective functions that are set by the firm itself. In return, a personalized price was obtained based on personalized temperature risk as a result of realization of an objective.

REFERENCES

[1] A. Ahčan, Statistical analysis of model risk concerning temperature residuals and its impact on pricing weather derivatives, Insurance: Mathematics and Economics, 50(1), pp. 131–138, 2012.

[2] P. Alaton, B. Djehiche, and D. Stillberger, On modelling and pricing weather derivatives, Applied Mathematical Finance, 9(1), pp. 1–20, 2002.

[3] R. G. Bartle, *The elements of real analysis*, volume 2, Wiley New York, 1964.

[4] F. E. Benth and J. š. Benth, The volatility of temperature and pricing of weather derivatives, Quantitative Finance, 7(5), pp. 553–561, 2007.

[5] B. M. Bibby and M. Sørensen, Martingale estimation functions for discretely observed diffusion processes, Bernoulli, pp. 17–39, 1995.

[6] T. Björk, *Arbitrage theory in continuous time*, Oxford university press, 1998.

[7] S. D. Campbell and F. X. Diebold, Weather forecasting for weather derivatives, Journal of the American Statistical Association, 100(469), pp. 6–16, 2005.

[8] M. Cao and J. Wei, Weather derivatives valuation and market price of weather risk, Journal of Futures Markets, 24(11), pp. 1065–1089, 2004.

[9] M. Caporin and J. Preś, Forecasting temperature indices density with time-varying long-memory models, Journal of Forecasting, 32(4), pp. 339–352, 2013.

[10] P. Carr, H. Geman, and D. B. Madan, Pricing and hedging in incomplete markets, Journal of financial economics, 62(1), pp. 131–167, 2001.

[11] C.-C. Chang, J.-B. Lin, and W.-M. Shen, Pricing weather derivatives using a predicting power time series process, Asia-Pacific Journal of Financial Studies, 38(6), pp. 863–890, 2009.

[12] S. A. Changnon, New risk assessment products for dealing with financial exposure to weather hazards, Natural Hazards, 43(3), pp. 295–301, 2007.

[13] L. Chincarini, No chills or burns from temperature surprises: An empirical analysis of the weather derivatives market, Journal of Futures Markets, 31(1), pp. 1–33, 2011.

[14] R. Cont and P. Tankov, *Financial modelling with jump processes*, Chapman & Hall, 2004.

[15] M. Davis, Pricing weather derivatives by marginal value, Quantitative Finance, 1(3), pp. 305–308, 2001.

[16] G. Dorfleitner and M. Wimmer, The pricing of temperature futures at the chicago mercantile exchange, Journal of Banking & Finance, 34(6), pp. 1360–1370, 2010.

[17] C. Dreveton and Y. Guillou, Use of a principal components analysis for the generation of daily time series, Journal of applied meteorology, 43(7), pp. 984–996, 2004.

[18] D. J. Dupuis, Forecasting temperature to price cme temperature derivatives, International Journal of Forecasting, 27(2), pp. 602–618, 2011.

[19] R. Elias, M. Wahab, and L. Fang, A comparison of regime-switching temperature modeling approaches for applications in weather derivatives, European Journal of Operational Research, 232(3), pp. 549–560, 2014.

[20] C. Gouriéroux, *ARCH models and financial applications*, Springer-Verlag New York, Inc, 1997.

[21] H. Hamisultane, Utility-based pricing of weather derivatives, The European Journal of Finance, 16(6), pp. 503–525, 2010.

[22] R. K. Hankin et al., Introducing elliptic, an r package for elliptic and modular functions, Journal of Statistical Software, 15(7), pp. 1–22, 2006.

[23] A. Hayfavi and I. Talasli, Stochastic multifactor modeling of spot electricity prices, Journal of Computational and Applied Mathematics, 259, Part B, pp. 434–442, 2014.

[24] V. Henderson, Valuing the option to invest in an incomplete market, Mathematics and Financial Economics, 1(2), pp. 103–128, 2007.

[25] H.-H. Huang, Y.-M. Shiu, and P.-S. Lin, Hdd and cdd option pricing with market price of weather risk for taiwan, Journal of Futures Markets, 28(8), pp. 790–814, 2008.

[26] J. C. Hull, *Options, futures, and other derivatives*, Pearson Education Inc, 2012.

[27] S. M. Iacus, *Option Pricing and Estimation of Financial Models with R*, John Wiley & Sons Ltd, 2011.

[28] S. Jewson and A. Brix, *Weather derivative valuation: the meteorological, statistical, financial and mathematical foundations*, Cambridge University Press, 2005.

[29] S. Jewson and R. Caballero, Seasonality in the statistics of surface air temperature and the pricing of weather derivatives, Meteorological Applications, 10(04), pp. 367–376, 2003.

[30] T. Kanamura and K. Ōhashi, Pricing summer day options by good-deal bounds, Energy Economics, 31(2), pp. 289–297, 2009.

[31] J. Kim, D. Sheen, and S. Shin, Option pricing of weather derivatives for seoul, East Asian Journal on Applied Mathematics, 2(04), pp. 309–325, 2012.

[32] D. Lamberton and B. Lapeyre, *Introduction to stochastic calculus applied to finance*, CRC press, 1996.

[33] Y. Lee and S. S. Oren, An equilibrium pricing model for weather derivatives in a multi-commodity setting, Energy Economics, 31(5), pp. 702–713, 2009.

[34] A. Lewis, *Option Valuation Under Stochastic Volatility*, Finance Press: Newport Beach, CA, 2000.

[35] J. London, *Modeling derivatives applications in Matlab, C++, and Excel*, FT Press, 2007.

[36] R. E. Lucas Jr, Asset prices in an exchange economy, Econometrica: Journal of the Econometric Society, pp. 1429–1445, 1978.

[37] M. Magill and M. Quinzii, *Theory of incomplete markets*, volume 1, Mit press, 2002.

[38] M. Mraoua and D. Bari, Temperature stochastic modeling and weather derivatives pricing: empirical study with moroccan data, Afrika Statistika, 2(1), 2007.

[39] A. Müller and M. Grandi, Weather derivatives: a risk management tool for weather-sensitive industries, The Geneva Papers on Risk and Insurance. Issues and Practice, 25(2), pp. 273–287, 2000.

[40] S. N. Neftci, *An Introduction to the Mathematics of Financial Derivatives*, Academic Press, USA, 2000.

[41] T. Oetomo and M. Stevenson, Hot or cold? a comparison of different approaches to the pricing of weather derivatives, Journal of Emerging Market Finance, 4(2), pp. 101–133, 2005.

[42] A. Pardo, V. Meneu, and E. Valor, Temperature and seasonality influences on spanish electricity load, Energy Economics, 24(1), pp. 55–70, 2002.

[43] J. S. Pollard, J. Oldfield, S. Randalls, and J. E. Thornes, Firm finances, weather derivatives and geography, Geoforum, 39(2), pp. 616–624, 2008.

[44] J. Pres, Measuring non-catastrophic weather risks for businesses, The Geneva Papers on Risk and Insurance-Issues and Practice, 34(3), pp. 425–439, 2009.

[45] M. Pryke, Geomoney: An option on frost, going long on clouds, Geoforum, 38(3), pp. 576–588, 2007.

[46] T. J. Richards, M. R. Manfredo, and D. R. Sanders, Pricing weather derivatives, American Journal of Agricultural Economics, 86(4), pp. 1005–1017, 2004.

[47] M. Ritter, O. Musshoff, and M. Odening, Meteorological forecasts and the pricing of temperature futures, Journal of Derivatives, 19(2), p. 45, 2011.

[48] F. Schiller, G. Seidler, and M. Wimmer, Temperature models for pricing weather derivatives, Quantitative Finance, 12(3), pp. 489–500, 2012.

[49] J. W. Taylor and R. Buizza, Density forecasting for weather derivative pricing, International Journal of Forecasting, 22(1), pp. 29–42, 2006.

[50] C. Toeglhofer, R. Mestel, and F. Prettenthaler, Weather value at risk: on the measurement of noncatastrophic weather risk, Weather, Climate, and Society, 4(3), pp. 190–199, 2012.

[51] R. S. Tsay, *Analysis of financial time series*, John Wiley & Sons, 2010.

[52] R. S. Tsay, *An introduction to analysis of financial data with R*, John Wiley & Sons, 2014.

[53] C. Turvey, Weather derivatives for event-specifier risk, American Journal of Agricultural Economics, 81(5), 1999.

[54] M. Wakaura and Y. Ogata, A time series analysis on the seasonality of air temperature anomalies, Meteorological Applications, 14(4), pp. 425–434, 2007.

[55] M. Xu, Risk measure pricing and hedging in incomplete markets, Annals of Finance, 2(1), pp. 51–71, 2006.

[56] W. Xu, M. Odening, and O. Musshoff, Indifference pricing of weather derivatives, American Journal of Agricultural Economics, 90(4), pp. 979–993, 2008.

[57] C. C. Yang, L. S. Li, and M.-M. Wen, Weather risk hedging in the european markets and international investment diversification, The Geneva Risk and Insurance Review, 36(1), pp. 74–94, 2011.

[58] A. Zapranis and A. Alexandridis, Weather derivatives pricing: Modeling the seasonal residual variance of an ornstein–uhlenbeck temperature process with neural networks, Neurocomputing, 73(1), pp. 37–48, 2009.

[59] A. Zapranis and A. Alexandridis, Modeling and forecasting cumulative average temperature and heating degree day indices for weather derivative pricing, Neural Computing and Applications, 20(6), pp. 787–801, 2011.

APPENDIX A

Summary of [14] for non-Gaussian OU processes

Current model can be considered as a non-Gaussian OU process. These type of processes were studied by [14]. According to the authors, given a Levy process (Z_t),$(y_t)_{t\geqslant 0}$ is defined as;

$$y_t = y_0 e^{-\lambda t} + \int_0^t e^{\lambda(s-t)} dZ_s \tag{A.1}$$

The process (y_t) verifies the SDE:

$$y_t = y_0 - \lambda \int_0^t y_s ds + Z_t \tag{A.2}$$

Its local behavior id defined as;

$dy_t = -\lambda y_t dt + dZ_t$

To find the characteristic function of the Equation A.1, following Lemma is defined:

Lemma 15.1 from [14]:

Let $f : [0, T] \to R$ be left-continuous and $(Z_t)_{t\geqslant 0}$ be a Levy process. Then;

$$E\{exp(i \int_0^T f(t) dZ_t)\} = exp\{\int_0^T \psi(f(t))dt \tag{A.3}$$

where $\psi(u)$ is the characteristic exponent of of Z.

Then characteristic function of y_t is found by applying Equation A.3 to Equation A.1:

$$E\{e^{iuy_t}\} = exp\{iuy_0 e^{-\lambda t} + \int_0^t \psi(ue^{\lambda(s-t)})ds\} \tag{A.4}$$

APPENDIX B

Error of Approximation

The error of the linear approximation is equal to [3]:

$$e^x - 1 - x = \frac{x^2}{2!} + \frac{x^3}{3!} + ... = E_1(x) = \int_0^x (x-t)e^t dt$$

Interestingly, the evolution of the characteristic functions shows that inclusion of an element from the approximation series affects the characteristic function through corresponding moment. For example, inclusion of the third element from the series starts to affect the characteristic function from the third moment of the function. This is, actually, the result of the structure of the approximation. Approximation requires inclusion of powers of D. The u parameter within the D, then, will have powers of D at least equal to the power value itself. As a result, it can be concluded that linear approximation finds the first moment correctly, while approximating the variance. Other moments are found to be zero. Likewise, second degree approximation fixes the variance and brings in approximations of the third and fourth moments. Continuously, third degree approximation does not change first two moments, fixes the third moment, updates the fourth moment, and brings in fifth and sixth moments and so on.

APPENDIX C

Parameter Estimates for 12 Cities

Table C.1: Parameter Estimation for HDD Calculations

City	Year	A	B	C	phase	b	γ_0	γ_1	Fast_Intensity	Fast_Mean	Fast_SD	Slow_Intensity	Slow_Mean	Slow_SD	beta
Ankara	5	50.19	0.001854	21.32	-2.77	0.19	95.95	0.964	0.0016438	-0.5543834	0.4924429	0.0054795	-6.1314104	5.1345993	0.38
Beijing	5	56.73	-0.00176	27.73	-2.89	0.37	66.18	0.980	0	0	0	0.00109589	-3.161959	2.420366	0.48
Cairo	29	70.23	0.000344	13.69	-2.73	0.35	239.81	0.955	0.0042513	-0.4124696	1.585721	0.00302315	-1.690717	2.075651	0.70
Chicago	15	50.57	0.00014	24.71	-2.79	0.27	117.25	0.961	0.004414	-3.119119	2.524564	0.005632	-6.375580	5.517740	0.34
Dallas	34	65.32	0.000232	20.00	-2.83	0.30	294.61	0.936	0.0045931	-2.498749	2.391683	0.00604351	-6.1932845	4.966445	0.29
Istanbul	9	58.54	0.00081	18.12	-2.67	0.24	184.55	0.948	0.004262	-0.958549	2.047743	0.002740	-3.133173	1.813471	0.56
Los Angeles	21	63.83	-0.0001	6.65	-2.47	0.23	338.76	0.917	0.0062622	0.3575345	1.352837	0.01108937	0.81062689	2.798516	0.46
New York	7	54.64	0.000548	21.78	-2.72	0.29	165.02	0.945	0.0015656	-1.6825421	2.360227	0.00508806	-3.8112152	3.30503	0.68
Paris	6	54.58	-0.00122	15.45	-2.83	0.21	147.12	0.949	0.0054795	-0.0954452	1.895953	0.00547945	-1.67898	3.505224	0.55
Sydney	37	63.45	0.000154	9.5	-2.81	0.64	527.0	0.876	0.0146612	1.5064313	2.606529	0.0033321	1.5601199	3.226385	0.91
Tokyo	8	61.22	0.000375	18.38	-2.62	0.53	180.28	0.954	0	0	0	0	0	0	0
Washington	34	58.62	0.000030	21.75	-2.79	0.28	173.30	0.952	0.0045125	-1.980577	1.755655	0.00354553	-4.9014241	4.06658	0.39

Year represents amount of data in years that produced best estimation results.

SD represents standard deviation.

Table C.2: Parameter Estimation for CDD Calculations

City	Year	A	B	C	phase	b	γ_0	γ_1	Fast_Intensity	Fast_Mean	Fast_SD	Slow_Intensity	Slow_Mean	Slow_SD	beta
Ankara	5	50.19	0.001854	21.32	-2.77	0.19	95.95	0.964	0.0016348	-0.5543834	0.4924429	0.0054795	-6.1314104	5.1345993	0.38
Beijing	19	54.97	0.000003	27.15	-2.92	0.35	63.31	0.981	0.000721	-1.5276715	1.531992	0.00100937	-2.9315846	2.019894	0.39
Cairo	5	71.94	0.002198	13.39	-2.74	0.37	104.84	0.983	0.0021918	0.3088935	1.644226	0.00273973	-1.7313942	0.945926	0.54
Chicago	7	51.70	-0.00034	25.18	-2.80	0.25	108.27	0.964	0.003131	-3.281461	3.025694	0.005871	-5.240072	4.234712	0.39
Dallas	5	69.86	-0.00172	20.02	-2.83	0.27	314.93	0.934	0.0032877	-3.2390974	1.069774	0.00657534	-3.3012769	3.345035	0.44
Istanbul	5	59.32	0.00134	18.10	-2.69	0.26	183.29	0.950	0.002192	0.067349	0.601966	0.002740	-2.406968	1.833246	0.56
Los Angeles	32	63.81	-0.000056	6.70	-2.47	0.21	312.83	0.923	0.0058219	0.3913695	1.425177	0.00984589	1.02398907	3.358289	0.40
New York	7	54.64	0.000548	21.78	-2.72	0.29	165.02	0.945	0.00155656	-1.6825421	2.360227	0.00508806	-3.8112152	3.30503	0.69
Paris	8	54.66	-0.00085	15.60	-2.84	0.22	150.30	0.949	0.005137	-0.4207048	1.581089	0.00513699	-0.1355112	4.770146	0.26
Sydney	9	65.07	0.000171	9.54	-2.86	0.54	426.97	0.903	0.0152207	1.6436984	1.81751	0.00182648	2.58516471	2.615411	0.77
Tokyo	18	61.60	0.000031	18.53	-2.63	0.50	214.39	0.945	0	0	0	0	0	0	0
Washington	8	57.78	0.000801	22.01	-2.78	0.28	169.08	0.953	0.0041096	-1.6365612	1.493845	0.00376712	-3.2540612	2.309414	0.56

Year represents amount of data in years that produced best estimation results.

SD represents standard deviation.

Table C.3: P-values of Parameters for HDD Calculations

City	a_1	a_2	a_3	a_4	γ_0	γ_1	Fast_Mean	Slow_Mean
Ankara	0	0	0	0	NS	0.00263	0.1609	0
Beijing	0	0	0	0	NS	0.0082	0.3174	0.02142
Cairo	0	0	0	0	NS	NS	0.0879	0
Chicago	0	NS	0	0	0	0	0	0
Dallas	0	0	0	0	NS	0	0	0
Istanbul	0	0	0	0	NS	NS	0.1021	0
Los Angeles	0	0	0	0	NS	NS	0.07382	0
New York	0	<0.001	0	0	NS	<0.001	0.2038	0
Paris	0	0	0	0	NS	<0.01	0.8557	0.01075
Sydney	0	0	0	0	NS	<0.05	0	0
Tokyo	0	0	0	0	NS	NS	NA	NA
Washington	0	<0.05	0	0	NS	0	0	0

NS:Not statistically significant
NA:Not available
a_1 to a_4 are regression parameters as explained in Section 2.8

Table C.4: P-values of Parameters for CDD Calculations

City	a_1	a_2	a_3	a_4	γ_0	γ_1	Fast_Mean	Slow_Mean
Ankara	0	0	0	0	NS	0.00263	0.1609	0
Beijing	0	NS	0	0	NS	0	0.096	0
Cairo	0	0	0	0	NS	NS	0.6717	0.001395
Chicago	0	NS	0	0	<0.01	0	0.03218	0
Dallas	0	0	0	0	NS	NS	0.01897	0
Istanbul	0	0	0	0	NS	NS	0.7978	0
Los Angeles	0	0	0	0	NS	NS	0.02789	0
New York	0	<0.001	0	0	NS	<0.001	0.2038	0
Paris	0	0	0	0	NS	<0.01	0.3038	0.8347
Sydney	0	<0.01	0	0	NS	NS	0	0.0056
Tokyo	0	<0.01	0	0	NS	<0.01	NA	NA
Washington	0	0	0	0	NS	<0.001	0.009	0

NS:Not statistically significant
NA:Not available
a_1 to a_4 are regression parameters as explained in Section 2.8

APPENDIX D

R Codes for Parameter Estimation

• Main function:

```
rm(list=ls(all=TRUE))
source("Path\\find_jumps.R")
source("Path\\find_arch.R")
source("Path\\Arch_Sim.R")
source("Path\\Jump_Sim.R")
source("Path\\Mean_Rev.R")
source("Path\\find_beta.R")

Temperature=read.csv(Path)
Temperature=Temperature$temp

Year=5;End=13505
Results=array(0:0,c(33,18))
for(kk in 1:33){
Beginning=End-Year*365+1
Dump_Temp=Temperature[Beginning:End]

Size=Year*365;time=1:Size
temperature.lm=lm(Dump_Temp~time+sin(2*pi*time/365)
+cos(2*pi*time/365))
Parameters=coef(temperature.lm)
A=Parameters[1]
B=Parameters[2]
C=sqrt(Parameters[3]^2+Parameters[4]^2)
phase=atan(Parameters[3]/Parameters[4])-pi
Resid=residuals(temperature.lm)
Fit=fitted.values(temperature.lm)

Jump_Parameters=find_jumps(Size,Dump_Temp)

Arch_Parameters=find_arch(Dump_Temp)
```

```
b=Mean_Rev(Dump_Temp,Fit,Size)
TA=Arch_Sim((Dump_Temp[1]-Fit[1]),Arch_Parameters,Size)

alpha=1;
TY=Jump_Sim(Jump_Parameters[1:3],Size,alpha)

beta=find_beta(Jump_Parameters[7:(Size+6)],Size)
TZ=Jump_Sim(Jump_Parameters[4:6],Size,beta)

T=array(0:0,Size);T[1]=Dump_Temp[1]
HDD=0;CDD=0
if(T[1]>65)
CDD=T[1]-65
if(T[1]<65)
HDD=65-T[1]

for(j in 2:Size){
T[j]=T[j-1]+Fit[j]-Fit[j-1]+
b*(Fit[j-1]-T[j-1])+TA[j]+TY[j]+TZ[j]
if(T[j]>65)
CDD=CDD+(T[j]-65)
if(T[j]<65)
HDD=HDD+(65-T[j])
}

HDD_exact=0;CDD_exact=0
for(i in 1:Size){
if(Dump_Temp[i]>65)
CDD_exact=CDD_exact+(Dump_Temp[i]-65)
if(Dump_Temp[i]<65)
HDD_exact=HDD_exact+(65-Dump_Temp[i])
}
Difference=sum((T-Dump_Temp)^2)

Results[kk,1]=A;Results[kk,2]=B;Results[kk,3]=C;
Results[kk,4]=phase
Results[kk,5]=Arch_Parameters[1];
Results[kk,6]=Arch_Parameters[2];
Results[kk,7]=Jump_Parameters[1];
Results[kk,8]=Jump_Parameters[2]
Results[kk,9]=Jump_Parameters[3];
Results[kk,10]=Jump_Parameters[4]
Results[kk,11]=Jump_Parameters[5];
Results[kk,12]=Jump_Parameters[6]
Results[kk,13]=beta;Results[kk,14]=HDD;
Results[kk,15]=HDD_exact
Results[kk,16]=CDD;Results[kk,17]=CDD_exact;
```

```
Results[kk,18]=b

Year=Year+1
print(kk)
}
write.table(Results,Path)
```

- Script that find jumps

```
find_jumps=function(S,DT){
jump_array=array(0:0,S);
jump_array_slow=array(0:0,S);
jump_array_fast=array(0:0,S)
SD=sd(DT);MEAN=mean(DT);DT=DT-MEAN
for(i in 1:S){
if(DT[i]>=0){
if(DT[i]>2*SD)
jump_array[i]=DT[i]-2*SD
else
jump_array[i]=0
}
else{
DT1=DT[i]*(-1)
if(DT1>2*SD)
jump_array[i]=DT[i]+2*SD
else
jump_array[i]=0
}
}
if(jump_array[1]==0){
jump_array_slow[1]=0
jump_array_fast[1]=0
}
else{
if(jump_array[2]==0)
jump_array_fast[1]=jump_array[1]
else
jump_array_slow[1]=jump_array[1]
}
if(jump_array[S]==0){
jump_array_slow[S]=0
jump_array_fast[S]=0
}
else{
if(jump_array[(S-1)]==0)
jump_array_fast[S]=jump_array[S]
```

```
else
jump_array_slow[S]=jump_array[S]
}
for(j in 2:(S-1)){
if(jump_array[j]!=0){
if(jump_array[j-1]==0&&jump_array[j+1]==0)
jump_array_fast[j]=jump_array[j]
else
jump_array_slow[j]=jump_array[j]
}
else{
jump_array_fast[j]=0
jump_array_slow[j]=0
}
}
Dump_Fast=array(0:0,sum(jump_array_fast!=0))
Dump_Slow=array(0:0,sum(jump_array_slow!=0))
CF=1;CS=1
for(k in 1:S){
if(jump_array_fast[k]!=0){
Dump_Fast[CF]=jump_array_fast[k]
CF=CF+1}
if(jump_array_slow[k]!=0){
Dump_Slow[CS]=jump_array_slow[k]
CS=CS+1}
}
Mean_Fast=mean(Dump_Fast);SD_Fast=sd(Dump_Fast)
Mean_Slow=mean(Dump_Slow);SD_Slow=sd(Dump_Slow)

Slow_Counter=0
for(i in 1:(S-1))
if(jump_array_slow[i]!=0&&jump_array_slow[i+1]==0)
Slow_Counter=Slow_Counter+1

Fast_Intensity=(S-sum(jump_array_fast==0))/S
Slow_Intensity=Slow_Counter/S
if((S-sum(jump_array_fast==0))<2){
Fast_Intensity=0;Mean_Fast=0;SD_Fast=0}
if((S-sum(jump_array_slow==0))<2){
Slow_Intensity=0;Mean_Slow=0;SD_Slow=0}
return(c(Fast_Intensity,Mean_Fast,SD_Fast,
Slow_Intensity,Mean_Slow,SD_Slow,
jump_array_slow,jump_array_fast))
}
```

- Script to find Arch Parameters

```
library(tseries)
find_arch=function(DT){
Coef_Arch=coef(garch(DT,order=c(0,1),
grad='numerical',trace=FALSE))
return(Coef_Arch)
}
```

- Script to find beta

```
find_beta=function(J,S){
Sum1=0;Sum2=0;
for(i in 2:S){
Sum1=Sum1+(J[i]*J[i-1])
Sum2=Sum2+(J[i-1])^2
}
beta=(-1)*log(Sum1/Sum2)
if(!is.finite(beta))beta=0
return(beta)
}
```

- Script to find mean reversion parameter

```
Mean_Rev=function(T,F,S){

Sum1=array(0:0,11680)
Sum2=array(0:0,11680)

for(i in 2:S){

Sum1[i]=((T[i-1]-F[i-1])*(T[i]-F[i]))/sd(T[1:i])
Sum2[i]=(T[i]-F[i])^2/sd(T[1:i])
}

Sum1[!is.finite(Sum1)]=0
Sum2[!is.finite(Sum2)]=0

b=(-1)*log(sum(Sum1)/sum(Sum2))
return(b)
}
```

- Script for Arch based simulation

```
Arch_Sim=function(Init,AP,S){
```

```
S1=10000
AS_Dump=array(0:0,c(S1,S))
AS_Dump[,1]=Init
AS=array(0:0,S)
for(i in 1:S1){
for(j in 2:S){
AS_Dump[i,j]=sqrt((AP[1]+AP[2]*AS_Dump[i,j-1]))*rnorm(1)
if(!is.finite(AS_Dump[i,j]))AS_Dump[i,j]=0
}
}

for(k in 1:S)
AS[k]=mean(AS_Dump[,k])

return(AS)
}
```

- Script for jump simulation

```
Jump_Sim=function(JP,S,P){
Sim_Size=10000
Y=array(0:0,dim=c(Sim_Size,S));J=array(0:0,S);Y[,1]=0

for(k in 1:Sim_Size){
t=0;t_count=array(1);t_count[1]=0;lambda=JP[1]

for (i in 1:S){
u=runif(1)
t=t+(-log(u)/lambda)
if(t>0&&t<S){
t_count=append(t_count,t)
}
else{
i=S}
}
t_count=round(t_count)

Q=array(0:0,S)
Q[1]=0
Dump=array(1)

for(i in 2:length(t_count)){
q=rnorm(1,JP[2],JP[3])
Dump=append(Dump,q)
x=t_count[i]
Q[x]=Q[x]+q
```

```
}


for(i in 2:S)
Y[k,i]=Y[k,i-1]-P*Y[k,i-1]+Q[i]
}
for(j in 1:S)
J[j]=mean(Y[,j])
return(J)
}
```

CURRICULUM VITAE

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EDUCATION

Degree	Institution	Year of Graduation
M.S.	Atılım University Computer Engineering	2004
B.S.	METU Department of Economics	1993
High School	Merzifon Lisesi	1987

PROFESSIONAL EXPERIENCE

Year	Place	Enrollment
1995-2002	Different Banks	Different Positions
2004-2009	Atılım University	Instructor
2013-Present	Different Companies	Programmer & Analyst

PUBLICATIONS