DEVELOPING PROSPECTIVE MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING QUADRILATERALS THROUGH A VIDEO CASE-BASED LEARNING ENVIRONMENT

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ABSTRACT

DEVELOPING PROSPECTIVE MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING QUADRILATERALS THROUGH A VIDEO CASE-BASED LEARNING ENVIRONMENT

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The aim of this study was to examine the developments in prospective middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge about quadrilaterals as they attended to a teaching experiment that was designed in a video case-based learning environment. Data was collected from eight prospective teachers during the fall semester of 2014-2015 in the scope of an elective course. In data collection process, multiple data sources were utilized such as clinical individual pre- and post-interviews, initial and revised lesson plans, teaching experiment sessions, reflection papers, group discussions and field notes. Data was analyzed by using qualitative methods. Clinical pre-interviews and initial lesson plans indicated that prospective teachers had various difficulties and inadequacies in definitions, constructions, classifications, and properties of quadrilaterals. However, in the progress of teaching experiment sessions requiring analyzing and discussing student's mathematical thinking in micro-case videos, considerable improvements were observed mostly in prospective teachers' pedagogical content knowledge about quadrilaterals. Thus, they developed awareness about what students' possible conceptions, misconceptions, difficulties and their possible reasons can be. Furthermore, they enriched their instructional strategies to overcome problematic situations in students' mathematical thinking regarding quadrilaterals. On the other hand, post-interviews revealed that there were also great developments in subject matter knowledge about quadrilaterals in addition to pedagogical content knowledge about quadrilaterals. In this sense, they corrected their errors in pre-interviews and they expanded their knowledge about definitions, constructions, classifications, and properties of quadrilaterals.

Keywords: Video Case-Based Learning Environment, Micro-case Videos, Prospective Teacher Education, Knowledge for Teaching Quadrilaterals, Teaching Experiment

ÖZ

MATEMATİK ÖĞRETMEN ADAYLARININ VİDEO DURUM TEMELLİ ÖĞRENME ORTAMINDA DÖRTGENLERLE İLGİLİ ÖĞRETİMSEL BİLGİLERİNİN GELİŞİMİ

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Bu çalışmanın amacı, ilköğretim matematik öğretmen adaylarının video durum temelli öğrenme ortamında tasarlanmış bir öğretim deneyine katılımları sürecinde gerçekleşen dörtgenlerle ilgili konu alan bilgilerindeki ve pedagojik alan bilgilerindeki gelişimleri incelemektir. Veriler 2014-2015 sonbahar dönemi boyunca sekiz ilköğretim matematik öğretmen adayından seçmeli bir ders kapsamında toplanmıştır. Veri toplama sürecinde bireysel klinik ön ve son görüşmeler, ders planları ve revize edilmiş ders planları, öğretimsel deney oturumları, yansıtıcı düşünce raporları ve grup tartışmaları gibi çoklu veri kaynaklarından yararlanılmıştır. Klinik ön görüşmeler ve ilk ders planları öğretmen adaylarının dörtgenlerin tanımı, çizimi, sınıflaması ve özellikleriyle ilgili çeşitli sıkıntılara ve yetersizliklere sahip olduklarını göstermiştir. Fakat video durumlarındaki öğrencinin matematiksel düşünüşünü analiz etmeyi ve tartışmayı gerektiren öğretimsel deney oturumlarında ise çoğunlukla öğretmen adaylarının dörtgenlerle ilgili pedagojik alan bilgilerinde kayda değer ilerlemelerin olduğu gözlemlenmiştir. Bu sayede, öğretmen adayları öğrencilerin muhtemel kavrayışları, kavram yanılgıları ve zorluklarının ve öğrenci düşünüşündeki problemli durumların nedenlerinin neler olabileceği ile ilgili bir farkındalık geliştirmişlerdir. Ayrıca, öğretmen adayları öğrencilerin dörtgenlerle ilgili problemli durumlarını gidermeye yönelik öğretimsel stratejilerini zenginleştirmişlerdir. Diğer taraftan, son görüşmeler öğretmen adaylarının dörtgenlerle ilgili pedagojik bilgilerinin yanında konu alan bilgilerinde de büyük gelişmeler olduğunu ortaya çıkarmıştır. Bu bağlamda, öğretmen adayları ön görüşmelerdeki hatalarını düzelterek dörtgenlerin tanımı, çizimi, sınıflaması ve özellikleriyle ilgili bilgilerini genişletmişlerdir.

Anahtar Kelimeler: Video Durum Temelli Öğrenme Ortamı, Mikro Durum Videoları, Hizmet Öncesi Öğretmen Eğitimi, Dörtgen Öğretim Bilgisi, Öğretim Deneyi

To memory of my lovely father Ramazan Bayık

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LIST OF ABBREVIATIONS

MCVCs	Micro-Case Video Clips
NCTM	National Council of Teachers of Mathematics
MoNE	Ministry of National Education
EME	Elementary Mathematics Education
PSTs	Prospective Teachers
SMK	Subject Matter Knowledge
PCK	Pedagogical Content Knowledge
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
SnNC	Sufficient but not Necessary Conditions
NnSC	Necessary but not Sufficient Conditions
NSC	Necessary and Sufficient Conditions
nNnS	Neither Necessary nor Sufficient Conditions
ADRP	After Discussion Reflection Paper
BDRP	Before Discussion Reflection Paper
PT	Prototypical Examples
NPT	Non-Prototypical Examples
PPT	Partial-Prototypical Examples
NH	Non-Hierarchical Examples
PH	Partial-Hierarchical Examples
H	Hierarchical Examples

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CHAPTER I

INTRODUCTION

Teaching mathematics is complex in nature because it requires well-developed content knowledge, an understanding about how students reason and learn mathematical concepts; and knowledge on different instructional strategies (e.g. Ball & McDiarmid, 1990; Fauskanger, 2015; Harrington, 1999). For instance, in many cases, teachers should be able to answer students' questions from the conceptual aspect rather than instrumental aspect (Borko & Putnam, 1996; Tchoshanov, 2011) because the conceptual knowledge involves knowing the meaning of mathematical representations, explaining the reason why certain algorithms and procedures work in particular situations and establishing strong connections between mathematical concepts instead of knowing only facts and procedures. On the other hand, having sufficient subject matter knowledge alone is not enough to provide an effective teaching to students (Shulman, 1986). In this regard, teachers need to understand the relationship between what they need to know and how they teach (Davis & Simmt, 2006; Mason & Davis, 2013). Furthermore, they also should know the concepts or topics which students have difficulties and misconceptions and the strategies to overcome these misconceptions because their knowledge affects students' conceptions (Tirosh, 2000). For this reason, understanding the nature of teachers' knowledge becomes an important issue among teacher educators (Ball, Thames, & Phelps, 2008; Hill, Sleep, Lewis, & Ball, 2007). After the increasing attention to teacher knowledge, many studies which were conducted on different mathematical concepts indicate that not only students but also teachers do not have adequate knowledge to teach a mathematical concept for elementary level students even it does not matter what the subject or the concept is (e.g. Ball, 1990a, 1990b; Even, 1993; Işıksal & Çakıroğlu, 2011; Hines, & McMahon, 2005; Ma, 1999; Tirosh, 2000; Toluk-Uçar, 2009).

Among the learning domains of mathematics, geometry is an important component for every curriculum in all countries (Common Core State Standards Initiative [CCSSI], 2010; Ministry of National Education [MoNE], 2013; National Council of Mathematics [NCTM], 2000) because geometry is a key element to understand and to facilitate students' visualization and reasoning abilities (Clements & Battista, 1992; Mammana & Villani, 1998). Specifically, NCTM (2000) and MoNE (2013) imply the importance of analyzing characteristics and properties of two- and three-dimensional geometric shapes and developing mathematical arguments about geometric relationships. In this regard, one of the basic topics of geometry is quadrilaterals which include the concepts of rectangle, square, rhombus, parallelogram, kite and trapezoid. Comprehending the attributes and properties of these shapes is crucial to construct the inclusive relationship between them (e.g. every square is a rectangle). The inclusive relation of quadrilaterals contributes the development of geometrical thinking and mathematical argumentation, deductive reasoning and proof (Fujita, 2012; Fujita & Jones, 2007). To reason these relationships, learners imagine shapes of geometric figures and examine their properties conceptually by using the attributes of shapes such as angles, sides, or diagonals. However, even learners know definitions of shapes, but related studies indicate that they generally do not recognize the relationship between the definition and the image of related mathematical concept. In a similar vein, a number of local and international studies about quadrilaterals have shown that many of students and prospective and inservice teachers have various difficulties in the issue of how they correctly and formally define and classify quadrilaterals (Akuysal, 2007; Currie & Pegg, 1998; De Villers, 1994; Doğan, Özkan, Karlı-Çakır, Baysal & Gün, 2012; Erez & Yerushalmy, 2006; Monaghan, 2000; Okazaki & Fujita, 2007). In general, the results of the studies indicate that teachers are not equipped with necessary content and pedagogical content knowledge about geometrical concepts (Chinnappan, Nason, & Lawson, 1996; Fuys, Geddes, & Tischler, 1988; Hershkowitz & Vinner, 1984; Leikin, Berman, & Zaslavsky, 2000; Mayberry, 1983; Swafford, Jones, & Thornton, 1997). This situation might be related to the current structure of the professional development programs used in undergraduate level at universities.

Teacher education programs at universities have crucial role to adequately meet the needs of prospective teachers before they actively enter to the professional occupations in the schools. However, traditional professional development programs utilized for training prospective teachers are generally fragmented structure (Guskey, 2002; Hawley & Valli, 1999). Traditional approaches in teacher education accept teachers as a "conscious decision maker" who can precisely transfer theoretical knowledge into practical situations (Clark, 1986; Özçınar & Deryakulu, 2011). However, obtaining enough theoretical knowledge does not guarantee that it could be transferred directly to the practical situations (Cole & Knowles, 1993; Goodlad, 1990; Veenman, 1984; Zeichner & Tabachnick, 1981). In this regard, it seems that traditional approaches are not enough to establish a strong bridge between theory and practice in teacher education. Furthermore, it can be deduced that as educators continue to utilize current traditional teacher training programs at the universities, it is difficult to make practical, methodological or theoretical contributions to prospective teachers' existing knowledge.

Prospective mathematics teachers take the courses on "*teaching of mathematics methods*" and "*field experience*" during their undergraduate education. For example, they may have ideas about how to teach a mathematical concept by means of teaching of mathematics methods course. Furthermore, they have opportunities to observe experienced teachers in complex classroom environment within the scope of "*field experience*". However, there are some arguments about the complexity of field experiences (e.g. Santagata, Zannoni & Stigler, 2007). For instance, prospective teachers are generally prone to concentrate on superficial or irrelevant features of classroom environment such as students talking each other, the sound of their voice, and the gestures they used, in the absence of necessary guidance on how to conduct observations (Fuller & Manning, 1973). Another problematic issue is that field experience can expose prospective teachers to a limited instructional strategies and student groups in isolation (Little, 1993) with little time and the lack of access to their colleagues' work (Sherin, 2004). Under this limitation, they may see limited numbers of strategies, which they observed, are suitable to teach a concept without thinking alternative ones. From these perspectives, traditional approaches in teacher education programs are severely being criticized in recent years (e.g. Abell & Cennamo, 2004; L. Shulman, 1992) because prospective teachers should be equipped in all these knowledge types before graduating the teacher education programs at the universities. As a result, how the gap between theory and practice could be reduced by the alternative approaches become the primary concern of teacher educators. At this point, case-based professional development in teacher education have been seen as an alternative approach to establish robust connections among theory and practice at least for over past two decades (Butler, Lee, & Tippins, 2006; Hammerness, Darling-Hammond, & Shulman, 2002; Lundeberg, Bergland, Klyczek, Mogen, Johnson & Harmes, 1999; Merseth, 1991; Shulman L., 1992) because of the many of potential benefits such as promoting critical and reflective thinking, developing SMK and PCK, or examining the complex nature of the practice.

In 1990s, the researchers concentrated on text-based (narrative) cases like photocopies of student work in classroom situations (Barnett, 1991; Merseth & Lacey, 1993; Shulman, 1992; Stein, Smith, Henningsen, & Silver, 2000). The immense improvements in the field of technology, researchers begin to focus on video-cases instead of narrative cases. Especially after the late of 1990s, researchers tended to use video cases for both prospective teacher education (e.g. Frederiksen, Sipusic, Sherin, & Wolfe, 1998; Seago, 2004; Sherin, 2003b, 2004) and inservice teacher education (e.g. Copeland & Decker, 1996; Daniel, 1996; Friel & Carboni, 2000; Goldman & Barron, 1990) since video-cases have been seen as "a window into the classroom that conveys the complexity and subtlety of classroom teaching as it occur in real time" (Brophy, 2004, p.287).

After video cases have been highlighted as a powerful tool for teacher professional development, educators suggested the use of video cases in order to facilitate especially prospective teachers' SMK and PCK (Ball, 2000; Ball & Cohen, 1999; Hiebert, Gallimore, & Stigler, 2002; Lampert, Heaton, & Ball, 1994). However, researchers generally used classroom videos that involve information about various dimensions of classrooms such as the students, the teacher, management, climate, pedagogy, mathematical thinking (e.g. Sherin, Jacobs, & Philipp, 2011; van Es & Sherin, 2008). In this sense, prospective teachers encounter a multi-dimensional structure of the classroom in the video-cases. Because of this complex structure, prospective teachers cannot always directly focus on student mathematical thinking when analyzing the classroom videos (Chamberlain, 2005; Ding & Dominguez, 2015; Freese, 2006; Kagan, 1992; Olkun, Altun & Deryakulu, 2009; Shapiro, 1991). Instead, they tend to notice various issues such as classroom management, the teacher's reactions or classroom climate when first examining a classroom video case. However, in recent years, there have been an increasing close attention to the use of videos involving students' mathematical thinking instead of complex classroom situations (e.g. Jacobs, Lamb, & Philipp, 2010; Sherin, 2007; van-Es, 2011). They generally produced video cases by cutting the events in which a student or students solve problem on the board in the mathematics classroom in order to serve the clips to the prospective teachers. In a classroom environment, a student's mathematical thinking on a particular concept depends on many factors such as the teacher's questions, teaching environment, students' characteristic features, or time limitation in the lessons. For example, students on the board could not explain their mathematical ideas in more detail because of time limitation, or timidity from the teacher or their friends in the classroom. For this reason, some details related to a student's mathematical thinking may be missed in the classroom. Aforementioned limitations of classroom situations reveal the necessity of production and usage of specially-designed educational videos that purely and directly concentrate on the students' mathematical thinking. Accordingly, as a strong proposal in the current study, I thought that producing and using "*micro-case videos*" that reflect students' mathematical thinking can be used an alternative effective approach to promote prospective mathematics teachers content related knowledge and pedagogical content knowledge. In other words, the use of "*micro-case videos"* in this manner serves purely a student-centered perspective instead of focusing on multi-dimensions of complex classroom learning.

In the study, I defined "*micro-case video*" as a specially-designed educational video that involves a collection of significant events related to an individual's mathematical thinking process on particular mathematical concepts or problem situations when the learner works on structured content-related tasks in an isolated non-classroom learning environment (*Note*: Further details about micro-case videos are explained in section 2.5). Just like "*microscopes*", micro-case videos allow zooming in students' particular ideas about a mathematical concept or problem situation. In other words, micro-case videos can provide opportunities to understand how different children understand mathematical concepts in different ways (Friel & Carboni, 1997; Jacob et al., 2010). In this way, micro-case videos might afford the opportunity to receive various students' thinking, and to compare and contrast different thinking processes. Similar to the arguments that are related to the affordances of case-based pedagogy, prospective teachers will be able to more easily identify student misconceptions (Hill & Collopy, 2003); to improve and increase their reasoning about student thinking and development (Harrington, 1999; Lundeberg, 1999) and decision making abilities (Grossman, 1992; Jay, 2004; Merseth, 1992) as well as their subject, pedagogical and professional knowledge (Manouchehri, 2002; Mayo, 2002) in a more efficient way by analyzing and discussing micro-case videos. Considering these strong arguments, in the current study, micro-case video clips are utilized to examine the developments in prospective teachers' SMK and PCK on quadrilaterals by integrating a video-case based professional development program.

Although there are many studies about quadrilaterals in the literature, this subject was chosen to investigate in this study because it is known to be difficult for students in all grade levels and prospective teachers and in-service teachers. Quadrilaterals are central concepts of geometry in all grade levels. Figures and properties of them have crucial role in understanding other geometric concepts such as solids, area, and perimeter. As a result, a comprehensive investigation is needed in order to assert how prospective teachers' subject matter knowledge and pedagogical content knowledge develop in the process of video case analyses and group discussions. Prospective middle school mathematics teachers are chosen as main participants, because the results of this study might give future implications to policy makers and scholars in terms of organizing textbooks, designing their lessons and teacher education programs. In order to increase teachers' content and pedagogical content knowledge, researchers focused on preservice teacher training programs because prospective teachers will become in-service teachers in the future. For these reasons, in the current study, it is aimed to examine the nature and developments of prospective middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge about quadrilaterals as they attend to a teaching experiment designed in a video case-based learning environment that requires analyzing, interpreting, reflecting, and discussing of micro-case video clips. More specifically, it is aimed to answer following research questions in this study:

1.1 Research Questions

1) What is the nature of prospective middle school mathematics teachers' existing subject matter knowledge and pedagogical content knowledge on quadrilaterals before attending to a teaching experiment designed within video case-based learning environment?
    - What do they know about definitions, constructions, classifications, and properties of quadrilaterals before attending to the teaching experiment?
    - What do they know about students' ways of mathematical thinking about students' mis/conceptions, difficulties, and confusions as well as their reasons related to definitions, constructions, classifications, and properties of quadrilaterals before attending to the teaching experiment?
    - What are prospective teachers' instructional approaches for teaching quadrilaterals before attending to the teaching experiment?

2) How do prospective middle school mathematics teachers develop or change their knowledge about quadrilaterals as they attended to the teaching experiment designed within video case-based learning environment?
3) What is the nature of prospective middle school mathematics teachers' subject matter knowledge and pedagogical content knowledge on quadrilaterals after they attended to the teaching experiment?

1.2 Significance of the Study

This study aimed to investigate the nature and development of prospective middle school mathematics teachers' knowledge about quadrilaterals throughout a teaching experiment designed within video case-based learning environment. The most significant aspects of this research are explained in the following paragraphs.

First of all, it might be questioned that why do you conduct this research while there are many of studies about quadrilaterals? The answer of this question can make clear the importance of this study. In recent years, there have been a huge number of local and international studies that were centered to understand students' and pre-service teachers' or in-service teachers' conceptions about quadrilaterals. Many of them concentrated on why understanding definitions, classifications and properties of quadrilaterals is difficult for both learners in all grade levels and inservice/preservice teachers (Fischbein, 1993; Fujita & Jones, 2007; Hershkowitz, 1990; Nakahara, 1995; Tall & Vinner, 1981; Vinner & Hershkowitz; 1980; Walcott, Mohr & Kastberk, 2009). These theoretical and empirical studies commonly investigated how learners recognize critical properties of the figures, how they define the concepts considering necessary and sufficient conditions, or how they classify quadrilaterals according to inclusive or exclusive relations. Their findings generally bear many of similarities even participants of the studies were different grade levels or ages. The tendency of these studies was to reveal learners' conceptions, misconceptions, and difficulties about definitions and classifications of quadrilaterals. Although there have been many details about students' understanding about quadrilaterals in related literature, researchers does not utilize the results of these studies in prospective teacher education programs and undergraduate courses. Furthermore, the findings of research on teachers' knowledge about students' mathematical thinking indicated that there are substantial gaps between learners' actual conceptions, misconceptions, and difficulties and teachers' predictions about them. However, teacher educators emphasize that teachers should be able to anticipate, attend to, and comprehend students' ways of various thinking (Ball et al., 2008) as a significant component of teacher knowledge since teachers' knowledge about students' thinking influences on their instructional strategies and decisions.

It is necessary to find some alternative ways in which teacher educators should be are able to find effective ways to enhance teachers' knowledge instead of only describing the problematic situations in teachers' knowledge. From this point of view, related literature indicates case-based research (Harrington & Garrison, 1992; Mayo, 2004) and noticing theory (e.g. Sherin & van Es, 2005; van Es & Sherin, 2002) are an efficient ways to train prospective teachers for the different teaching environments. In recent years, researchers have been focusing on case-based approach that involves classroom videos and prospective or inservice teachers' noticing abilities in order to improve teachers' knowledge on students' mathematical thinking. However, enhancing teachers' "*professional noticing of students' thinking*" is a complex and challenging issue by using classroom video cases due to the complex nature of classrooms teaching environment. Yet, it should be clarified that how teachers improve their knowledge and noticing abilities on students' thinking through their examination of students' mathematical thinking process in (specially-designed) videos (Sherin et al., 2011). At this point, I propose that prospective teachers need professional development experiences to improve their skills and knowledge about a mathematical concept by collectively analyzing and discussing specially-designed micro-case videos that involve students' mathematical thinking instead of all classroom settings. From this point, the current study can provide prospective teachers with insights when they become teachers with the responsibility to teach mathematical concepts to their students by considering students' actual conceptions, misconceptions, and errors. Thus they can have a chance to expand and enrich their anticipations and instructional decisions. In this regard, the results of current study are important in terms of giving ideas about the effectiveness of using micro-case videos to promote prospective teacher SMK and PCK in a video-based learning environment.

Having adequate theoretical knowledge on an issue does not guarantee adequacy of practical knowledge on the issue. Prospective teacher training programs generally try to equip prospective teachers with theoretical knowledge instead of practical knowledge. However, educators haven't found this tendency adequate in terms of gaining all necessary knowledge and abilities for being a teacher. For example, in traditional teacher training programs, prospective teacher have no opportunity to directly analyzing and discussing all details of students' reactions, responses, and conceptions. They generally graduate from universities by getting limited knowledge about students' understanding in their school experience and mathematics teaching methods courses. Doubtlessly, the most prominent contribution of current research is to give an alternative approach to the researchers in terms of how a robust link between prospective teachers' theoretical and practical knowledge of mathematics can be established (Butler et. al, 2006; Masingila & Doerr, 2002). In this study, pre-service teachers able to analyze students' different thinking processes via micro-case video clips, to discuss their ideas with their colleagues in a social learning environment and to monitor the changes of their own knowledge of quadrilaterals. On the other hand, prospective teachers have opportunity to stop and replay videos flexibly to analyze and reflect on student thinking and to develop ways to facilitate student learning (Masingila & Doerr, 2002). This situation gives opportunities them to change their instructional plans by considering students' mathematical thinking in the video-cases (van Es & Sherin, 2010). As result, they are able to propose new alternative and multiple instructional methods (Stockero, 2008). In this sense, examining micro-case videos may facilitate and support prospective teachers' critical thinking abilities as they interpret and make inferences about critical situations in the video cases. Furthermore, the effect of analyzing video cases on the prospective teachers' knowledge about seventh grade students' conceptions might be manifested by the pre-interviews and post-interviews in the current study. By this means, pre-service teachers have opportunity to recognize the changes in their knowledge world, which can make them a reflective teacher for their future instructions.

Another important issue is to use of micro-case videos which is an emerging point in the current study. In general, researchers focused on video cases conducted in the classroom environment. However, a single student mathematical thinking is a focal point of a micro-case video in this study. In other words, a single student's thinking process about quadrilaterals was monitored and recorded in each video case instead of monitoring and recording whole classroom. How does the role of using micro-case videos explain from the point of the significance of the study? The use of micro-case videos in this manner serves a more student-centered perspective to the teachers because students' mathematical thinking processes and misconceptions may be missed in classroom video recordings. To become a good in-service teacher, prospective teachers gain knowledge and experience about students' conceptions in addition to their misconceptions and difficulties. With this regard, seventh grade students having different concept images about quadrilaterals are selected for the videotaping of their processing in this research. From this perspective, the results of this study might give ideas to the mathematics educators in terms of the importance of understanding of thinking processes of students who are at any geometric understanding level, which provides them to examine rich, diverse learning situations via specially-designed educational video clips.

In the current study, it is also significant in order to inform policy makers whether they should make revision or improvement for teacher education programs taking into consideration of students' and pre-service teachers' needs because this study makes use of micro-case videos as a professional development tool. Additionally, the results of this research also may be used to develop and revise the presentation of quadrilaterals in the elementary mathematics curriculum because this research gives information about both students' and teachers' understanding related to the concepts of quadrilaterals and their properties. As a last word, it can be claimed that this study also may contribute not only to the international literature but also to the Turkish literature on video-case based pedagogy because there is limited number of studies interested in the uses of video cases for teacher professional development in Turkey (e.g. Olkun et al., 2009; Osmanoğlu, Işıksal, & Koç, 2012; Osmanoğlu, Koç, & Işıksal, 2013).

1.3 Definitions of the Important Terms

Considering the purpose and the research questions in the study, there are some crucial technical terms related to "case-based approaches in teacher education", "types of teacher knowledge", and "quadrilaterals". Since it is necessary to clarify the meaning of essential terms, all the terms utilized within the current study are constitutively and operationally described in the following.

Case-based pedagogy: According to L. Shulman (1986), a case in classroom teaching context is ―… a piece of controllable reality, more vivid and contextual than a textbook discussion, yet more disciplined and manageable than observing or doing work in the world itself (J. Shulman, 1992, p.xiv)". Case-based pedagogy involves the using cases to help teachers broaden the knowledge and qualifications that are necessary to respond to the complexity and authenticity of real classrooms (Merseth, 1991, 2003; J. Shulman, 1992; Sykes & Bird, 1992). In the current study, case-based pedagogy refers to a way in which prospective teachers reflect their ideas and make discussion with their peers about specially-designed video cases involving student's mathematical thinking process in a social constructivist learning environment.

Micro-case videos: Video cases are one of the typical case examples among lengthy narrative cases, short narrative cases, multimedia cases, and hyper-media cases. In the literature, Richardson and Kyle (1999) describe video cases as ―…multimedia presentations of classroom actions and analyses that include moving pictures (usually on videocassette) of classroom action (p.122). In the current study, video cases refer to the specially-designed video clips that are called "*micro-case videos*" and each of them involves a student's mathematical thinking instead of a classroom situation. "*Micro-case video*" is defined as a specially-designed educational video that involves a collection of significant events related to an individual's mathematical thinking process on particular mathematical concepts or problem situations when the learner works on structured content-related tasks in an isolated non-classroom learning environment. The main characteristics features that are involved by micro-case video clips can be listed as follows: (i) a collection of specially-designed selected-edited events, (ii) a learner's thinking process, (iii) structured content-related tasks or problem situations, and (iv) isolated non-classroom learning environment.

Subject matter knowledge: Shulman (1986) defined subject matter knowledge (or it is called content knowledge) as "the amount and organization of knowledge per se in the mind of teacher" (p.9). It does not mean that subject matter knowledge does not much differ from knowing facts. In this sense, Shulman (1986) emphasized the crucial role of content knowledge as providing explanations and definitions for students. This knowledge type also includes the important points about "why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions, both within the discipline and without, both in theory and practice" (p.9). In this research, subject matter knowledge refers to prospective middle school mathematics teachers' knowledge about definitions, constructions, classifications, and properties of quadrilaterals. More specifically, prospective teachers are supposed to write definitions and hierarchical relationship of quadrilaterals by using their properties with regard to sides, angles, diagonals. In addition, their knowledge about the reasons of these hierarchical relationships of quadrilaterals is involved to the scope of the subject matter knowledge.

Pedagogical content knowledge: Shulman (1986) defined pedagogical content knowledge as:

> …the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations — in a

> word, the most useful ways of representing and formulating the subject that makes it comprehensible to others…. Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (Shulman, 1986, p.7).

In general meaning, pedagogical content knowledge involves teachers' knowledge of students' possible conceptions, misconceptions, and difficulties; knowledge of the possible sources of them; and knowledge of how these problematic situations can be solved (Ball & Bass, 2000; Hill, Rowan, & Ball, 2005). Moreover, prospective teachers' suggestions to overcome the students' misconceptions and difficulties are included the scope of pedagogical content knowledge. In the current study, it is concentrated on two important dimensions of pedagogical content knowledge as knowledge of content and students (KCS) and knowledge of content and teaching (KCT) (Ball et al., 2008). In the scope of the current study, while KCS involves the proficiency on anticipating students' possible conceptions, errors and difficulties and determining the task that students can find challenging, interesting or motivating, KCT requires the proficiency on selecting suitable examples for different purposes; assessing the advantages and disadvantages of representations used in teaching process; and identifying affordable methods and strategies to teach a concept.

Concept image and concept definition: Vinner (1991) assumed the existence of two different cells in one's cognitive structure for the image and definition of the concept based on their previous research (Tall & Vinner, 1981; Vinner, 1983; Vinner & Hershkowitz, 1980). Concept image is the set of all the mental representations associated in the students' mind with the concept name (Tall &Vinner, 1981; Vinner, 1983). The image might be nonverbal and implicit, that is, it evokes in learners' mind. On the other hand, concept definition constitutes a form of words which are used to specify the concept (Tall & Vinner, 1981; Vinner, 1983). In this research, the theory of concept image and concept definition is used to select seventh grade students for videotaping their thinking process while they are engaging tasks related to quadrilaterals. Furthermore, the theory is utilized when explaining the existing situations and developments in prospective teachers' knowledge on quadrilaterals.

Inclusive and exclusive definition: Usiskin and Griffin (2008) mention two types of definitions of the concepts belong to special quadrilaterals family: "*exclusive definition*" and "*inclusive definition*". For instance, there are two different definitions of trapezoid in geometry textbooks. While one is that a quadrilateral with exactly one pair of parallel sides, another is that a quadrilateral with at least one pair of parallel sides. As the former one is an example of exclusive definitions (e.g. parallelograms are not also trapezoids), latter one is a type of inclusive definitions (e.g. parallelograms are also trapezoids).

Prototypical and non-prototypical example*:* The prototype examples were usually the subset of examples that had the "longest" list of attributes all the critical attributes of the concept and those specific (noncritical) attributes that had strong visual characteristics" (Hershkowitz, 1990, p.82). Learners often see the figures in a static way rather than in the dynamic way that would be necessary to understand the inclusion relations of the geometrical figures (de Villiers, 1994). For instance, students receive square is not a rectangle because of their misconception about the length of the opposite sides of rectangle. For this study, prototype examples are used as a reflective tool of both seventh grade students' and prospective middle school mathematics teachers' conceptions and misconceptions of quadrilaterals. For instance, they are accustomed to engage squares like in Figure 1-a, they do not recognize Figure 1-b as being an example of square.

Figure 1. (a) A prototypical example of square (b) a non-prototypical example of square

Classifications of quadrilaterals: In the literature, there are three different classification types of quadrilaterals such as hierarchical classification, partition classification, and non-hierarchical classification. Hierarchical classification is defined as "the classification of a set of concepts in such a manner that the more particular concepts form subsets of the more general concepts" (De Villiers, 1994, p.11). The researcher also defined partition classification as the classification where "the various subsets of concepts are considered to be disjoint from one another" (p.11). To be more precise, an example situation is given to express the operational meaning of each classification type in the following. In this study, if a learner treats all parallelograms, rhombuses, rectangles, and squares as the examples of the set of trapezoid, it is evaluated that this learner is able to make hierarchical classification in terms of trapezoid. If a learner treats only parallelograms as a trapezoid example, it is evaluated that this learner can make partition classification. Finally, if a learner does not consider all parallelograms, rhombuses, rectangles, and squares as an example of trapezoid, this learner makes non-hierarchical classification in terms of trapezoid.

Overgeneralization and undergeneralization errors: Two types of common errors that are exhibited by students have been described in the literature as undergeneralization and overgeneralization (Klausmeier & Allen, 1978). Undergeneralization occurs when examples of a concept are encountered but are not identified as examples. It results when the examples provided in instruction are not sufficiently different from one another in the variable attributes (Klausmeier & Allen, 1978; p.217). In the context of this study, for example, a student who has experienced only right trapezoids having exactly one pair of parallel sides may not identify trapezoids not having right angle even it has exactly one pair of parallel sides. On the other hand, overgeneralization occurs when examples of other concepts treated as members of target concept (Klausmeier & Allen, 1978; p.217). In the current context, a quadrilateral having no parallel sides and non-equal length of sides or a polygon having more than four sides may be treated as an example of trapezoid, which indicates the presence of overgeneralization error.

CHAPTER II

LITERATURE REVIEW

This study aimed to understand the nature and development of middle school mathematics teachers' knowledge about quadrilaterals throughout a teaching experiment designed within video case-based learning environment. For this purpose, relevant literature was divided into seven parts. The first part gives information about types and components of teacher knowledge in the light of different frameworks. In the second part, geometric knowledge about quadrilaterals and international and local studies investigating teachers' knowledge about quadrilaterals are presented and discussed in terms of their differing methodological approaches, and findings, and theoretical and practical implications for researchers and teacher educators. In the following part, the importance and different usages of cases for teachers' professional development are mentioned. Some empirical studies on the uses of video cases in teacher education were mentioned in the fourth part. The the need for micro-case videos in teacher education are specifically emphasized in the fifth part.Then, social constructivist theory and other theories used in case-based teacher education were summarized in the last two parts of literature review.

2.1 Frameworks for Teacher Knowledge

It is an undeniable fact that what teacher knows has a crucial effect on their organization of lessons and students' knowledge. Especially, teachers' knowledge of students' thinking might have important influence on their teaching practice (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Fennema, Carpenter & Peterson, 1989). Related literature indicates that the nature and types of teachers' knowledge have been studied by different researchers for many years (Ball et al., 2008; Cochran, DeRuither, & King, 1993; Fennema & Franke, 1992; Grossman, 1995; Peterson, 1988; Shulman, 1986). Some prominent frameworks on teacher knowledge are discussed in a chronological order.

Shulman (1986) proposed a theoretical framework that having different categories of teacher knowledge: "*subject matter knowledge*", "*pedagogical content knowledge*" and "*curricular knowledge*". He defined teachers' subject matter knowledge as "the amount and organization of the knowledge per se in mind of the teacher" (p.9). According to Shulman (1986), it is not enough to know only the mathematical structures, rules and principles to become a good teacher. At the same time, they should know the reasons and underlying factors of them. On the other hand, he defines pedagogical content knowledge as a kind of content knowledge. Pedagogical content knowledge involves "the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations, the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p.9). It also contain "an understanding of what makes the learning of specific topics easy or difficult, the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons" (p.9). Moreover, knowledge about students' thinking processes is related to this type of knowledge. Finally, curricular knowledge comprises of the scope and sequence of a subject and materials that utilized while teaching.

Similarly, Peterson (1988) proposed a framework by building on Shulman's framework. She grouped teachers' knowledge into three categories: how students think in content areas, how to facilitate growth in students' learning and self-awareness of their own cognitive processes. Unlike Shulman, curricular knowledge is not placed in the Peterson's framework. There are also some overlapping points between Shulman's and Peterson's frameworks. For instance, the understanding of how students learn in specific domain from the first category of Peterson's framework is covered by pedagogical content knowledge from Shulman.

Different from these researchers, Grossman (1995) asserted a more comprehensive framework for teacher education than above frameworks. It includes six types of knowledge: "*knowledge of content*", "*knowledge of learning*", "*knowledge of general pedagogy*", "*knowledge of curriculum*", "*knowledge of context*", and "*knowledge of self*". According to Grossman, content knowledge both involves subject matter knowledge and pedagogical knowledge of the subject matter. As a result, pedagogical content knowledge stated by Shulman is included under this category. Knowledge of learners is nearly the same as Peterson's categories of how students think in specific subject. It includes students' potential conceptions, misconceptions and difficulties of a particular topic Knowledge of curriculum is the same as Shulman's category. However, the last two knowledge types in Grossman's framework are obviously not mentioned in the frameworks discussed above. For instance, knowledge of self refers teachers' knowledge of their personal values and educational philosophy, dispositions, strengths, and weaknesses (Grossman, 1995).

In another framework which shapes subject matter knowledge and pedagogical content knowledge, Ball, Thames, and Phelps (2008) proposed a refinement to Shulman's categories because they found the definition of teacher knowledge in Shulman's model is not clear to conduct the empirical studies about teacher education. Therefore, they introduced "*Mathematical Knowledge for Teaching* (MKT)".

They divided subject matter knowledge into two sub-knowledge domains as "*common content knowledge*" (CCK) and "*specialized content knowledge*" (SCK) (see Figure 2). In this division, CCK means mathematical knowledge being not specific to teaching. On the other hand, SCK refers to mathematical knowledge that is necessary to teach mathematics. Furthermore, they mentioned "horizon content knowledge" as a component of subject matter knowledge and they defined it as "awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p.403). In addition to the dimensions of subject matter knowledge, Ball and her colleagues (2008) also divided pedagogical content knowledge into three subcategories: "*knowledge of content and students*" (KCS), "*knowledge of content and teaching*" (KCT), and "*knowledge of content and curriculum*" (see Figure 2).

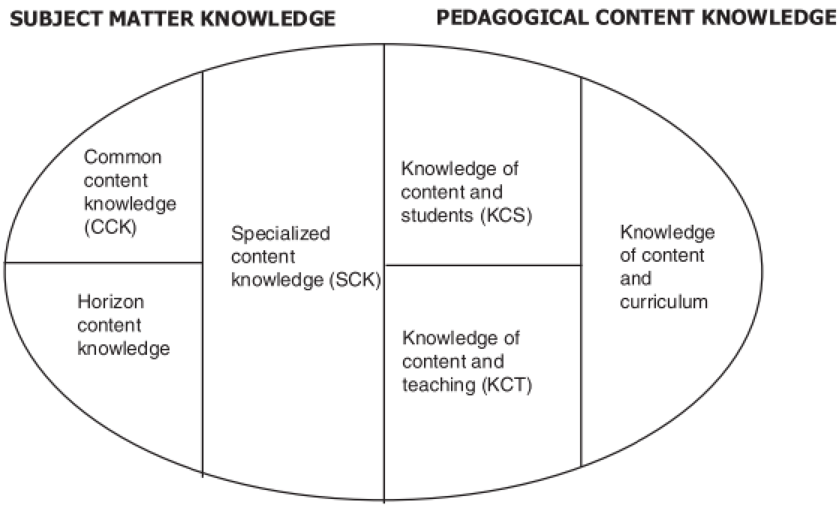


Figure 2. Domains of mathematical knowledge for teaching (Ball et al., 2008, p.403)

In this categorization, KCS can be defined as the combination of knowing about students and knowing about mathematics. Teachers having this knowledge type must predict students' possible conceptions, misconceptions, difficulties and errors. Teachers also need to anticipate what students can find challenging, interesting or motivating when deciding to use an example or mathematical activities. In sum, it is clearly seen that KCS requires the presence of strong interactions between particular mathematical subject or concepts and students' understanding in related subject or concepts. Another domain named knowledge of content and teaching (KCT) combines knowing about teaching and knowing about mathematics, which requires a mathematical knowledge related to instructional design of the mathematical tasks. Some examples of KCT includes selecting suitable examples for different purposes; assessing the advantages and disadvantages of representations used in teaching process; and identifying affordable methods and strategies to teach a concept. In other words, KCT is related to the knowledge involving how to teach mathematical concepts and procedures. As a final component of pedagogical content knowledge, Ball and her colleagues (2008) mentioned knowledge of curriculum which has similarity with Shulman's curricular knowledge.

In the literature, it is usually assumed that subject matter knowledge and pedagogical content knowledge are interrelated (Ball, 1991; Shulman, 1986, 1987). However, the number of research is limited to support and illustrate this relationship. Subject matter knowledge, pedagogical content knowledge, and general pedagogy are more overlapping than discrete (Marks, 1990). Moreover, teachers' knowledge of subject matter also affects how they represent the nature of knowing within a content area to their students (Grossman, 1995). Ball (1991) finds that teachers with weak conceptual understanding of mathematics are likely to represent the nature of mathematical knowing as rule bound. The lack of conceptual understanding may lead to the misuse of instructional strategies. In this sense, teachers' knowledge of subject matter has its contribution to classroom instruction. Teachers' subject matter knowledge may also contribute both to their selection of curricula and to their critiques of specific curriculum materials (Grossman, 1990).

In the current study, it was aimed to study prospective teachers' subject matter knowledge and pedagogical content knowledge on quadrilaterals by using seventh grade students' micro-case video clips as a tool. In order to analyze prospective teachers' subject matter knowledge, Shulman's, Ball's and Grossman's frameworks are combined. Prospective teachers' personal definitions, constructions, and classifications of quadrilaterals were analyzed for in order to determine their subject matter knowledge. In terms of pedagogical content knowledge, Shulman's (1986) and Ball and her colleagues' (2008) definitions were used. In this regard, pre-service teachers' existing and developing knowledge on (i) common conceptions and misconceptions held by the elementary school students; (ii) the possible sources of these conceptions and misconceptions, and (iii) the strategies that pre-service teachers used to overcome these misconceptions, the representations that prospective teachers used to reason their understanding, and the strategies that pre-service teachers used to explain the concepts of quadrilaterals and definitions and properties of them will be investigated in order to understand PSTs pedagogical content knowledge.

After this review of the general characteristics of various frameworks related to subject matter knowledge and pedagogical content knowledge, the details of the studies about teachers' knowledge on quadrilaterals is mentioned in the following section.

2.2 Teacher Knowledge about Quadrilaterals

In recent years, researchers have proposed several theories and frameworks regarding the teaching and learning of geometry, including concept image and concept definition (Tall & Vinner, 1981; Vinner, 1991); common cognitive paths (Vinner & Hershkowitz, 1980); prototypical phenomenon (Hershkowitz, 1990); figural concepts (Fischbein, 1993; Mariotti & Fischbein, 1997); personal and formal figural concepts (Fujita, 2012; Fujita & Jones, 2007); and dynamic figural concepts (Walcott et al., 2009). Prospective teachers' subject matter knowledge and pedagogical content knowledge in terms of understanding students' conceptions about quadrilaterals have been examined as the focus of many of studies by utilizing aforementioned theories and frameworks. In the current study, it is not aimed to identify teachers' conceptions, misconceptions, difficulties and predictions about students' possible conceptions, misconceptions, difficulties on quadrilaterals in order to reflect PSTs' SMK and PCK. Instead, this study focused on the developmental processes in PSTs' SMK and PCK on quadrilaterals throughout a teaching experiment designed in a video case-based learning environment. At this point, it is important to give a summary about how teachers' conceptions about quadrilaterals were asserted in the literature. This review sheds on light how researchers identified and determined teachers' conceptions, misconceptions and inadequateness on quadrilaterals by using various theoretical perspectives in the teaching of geometry. As a result, interpreting the results of the current study will be meaningful when explaining the developmental process in PSTs' SMK and PCK on quadrilaterals. For this reason, in this part of the literature review, both international and local studies on teachers' knowledge related to quadrilaterals are summarized by focusing on critical points having direct relation with the purpose of the study.

2.2.1 The importance of concept image and concept definition in geometry

The terms of "*concept image*" and "*concept definition*" were firstly proposed by Vinner and Hershkowitz in 1980 as a theoretical framework. This framework serves significant contributions to the literature in terms of explaining how concept images influence students' conceptions and learning processes by emphasizing the role of learners' previous learning and pre-conceptions about mathematical concepts. In the current study, the aim is not to examine prospective teachers' concept images and concept definitions about quadrilaterals. However, it is necessary and useful to explain this theoretical framework that involves the components of concept image and concept definition to understand how PSTs develop their knowledge about quadrilaterals because this framework has a potential in terms of developing substantial ideas about learners' mathematical thinking (Bingölbali, 2016). From this aspect, concept image and concept definition components and the results of some studies that were conducted based on this framework were mentioned to provide a philosophical and theoretical background on how learners comprehend geometric concepts.

Vinner and Hershkowitz (1980) proposed "*common cognitive paths*" that refer a statistical method for identifying a path that learners follow to select or realize similar concepts. The basic idea is as follows:

> Denote by a, b, c, d respectively the subgroups of people that answered correctly the items that test aspects A, B, C, D. Suppose, finally, that it was found that $a \supset b \supset c \supset d$. We may claim then that $A \rightarrow B \rightarrow C \rightarrow D$ is a common cognitive path for this group (in the sense that nobody in the group can know D without knowing also A, B, C and so on) (Vinner and Hershkowitz, 1980, p.182-183).

In this sense, Nakahara (1995) investigated Japanese primary school children's common cognitive paths related to quadrilaterals and reached that students had a path such as parallelogram → rhombus → trapezoid. Likewise, Okazaki and Fujita (2007) conducted a study with 263 Japanese and Scottish trainee elementary school teachers to reveal prototypical phenomenon and common cognitive paths in teachers' understanding of the quadrilaterals. They reported that while Japanese prospective teachers' path can be square/rhombus, rectangle/parallelogram and finally square/rectangle, Scottish prospective teachers' path was more likely to be rectangle/parallelogram, square/rectangle and square/rhombus. The results of such kind of studies are important to understand students' and prospective teachers' conceptions about quadrilaterals at the international level.

In order to assert learners' cognitive structures, Vinner (1991) assumed the existence of two different cells in one's cognitive structure for the image and definition of the concept based on their previous research (Tall & Vinner, 1981; Vinner, 1983; Vinner & Hershkowitz, 1980). In the framework, concept image is defined as following:

> […] the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures (Tall & Vinner, 1981, p.152)

The image might be nonverbal and implicit. On the other hand, concept definition constitutes a form of words which are used to specify the concept (Vinner, 1991). According to this framework, suitable and robust interactions between concept definition and concept image might guarantee the conceptual learning rather than instrumental ones.

In many of studies on triangles and quadrilaterals, researchers utilized Tall and Vinner's (1981) and Vinner's (1991) studies in order to examine the nature of the relationship between learners' concept image and concept definition for all grade levels (e.g. Gutierrez & Jaime, 1999; Hershkowitz, 1989; Hershkowitz & Vinner, 1984). Unfortunately, learners do not make sense to link between the two elements because there might be irrelevant properties about the concept evoking in the learners' mind specifically. For example, Gutierrez and Jaime (1999) conducted a study with 190 prospective primary teachers by using Vinner's (1991) framework in order to examine their concept images, difficulties, and errors about the concept of altitude of a triangle in a written task. They concluded that the presence of formal definition and previous classroom activities on the altitude of triangle influence PSTs' performances on the task. In order to reflect PSTs' concept images, they grouped participants' errors related to the altitude of triangle into five main categories: altitude vs median; altitude vs. perpendicular bisector; limitation to internal altitudes; disregard of length; fixation on side; and marked base as distracter. Considering these errors, they concluded that pre-service teachers have incomplete concept images and ill-connections between concept image and concept definition related to the altitude of triangle. Similarly, the results of some studies indicate that many of students at different grade levels have a concept image of equilateral triangle having a right angle or slanted sides of equal length (Burger & Shaughnessy, 1986; Clements & Battista, 1992). Consequently, the results of many of studies indicated that pre-service teachers, in-service teachers, and students have same misconceptions when asked them to respond same task. Some researchers concluded that prospective teachers' and inservice teachers' concept images on quadrilaterals were slightly better than those of the students (e.g. Hershkowitz, 1989; Hershkowitz & Vinner, 1984).

Fujita and Jones (2006a) investigated whether there is a relationship between pre-service primary teachers' concept images and concept definitions. For this purpose, they selected 158 pre-service primary school teachers at first year of teacher education program. They asked teachers to identify and construct some quadrilaterals. In addition, they asked some questions in order to understand how pre-service primary school teachers make relationship between quadrilaterals. Results of this research show that teacher did not comprehend the hierarchical relationship between quadrilaterals. Moreover, disconnectedness was found among their concept images and concept definitions.

Additionally, same researchers focused on pre-service teachers' concept images about only one quadrilateral such as parallelogram (Fujita & Jones, 2006b) and square (Fujita & Jones, 2007). The results of these studies presented that most of pre-service teachers have not got complete concept image. Instead, they had only prototype examples of quadrilaterals in their minds. In a similar way, Pickreign (2007) investigated fourteen pre-service teachers' perceptions about relationship between parallelograms. The results of the research present teachers have incomplete or incorrect definitions of rhombus and square. They classify parallelograms taking into consideration of their appearances rather than their properties.

Considering the discrepancy between learners' concept images and concept definitions, Hershkowitz (1989) offered that if learners are encountered limited examples having common figural features of a geometric concept in school or other contexts, these examples lead to prototypes phenomenon by focusing on possible influences of prototypical examples on the learners' cognitive structures on quadrilaterals. The meaning of prototypical phenomenon is explained in the following section.

2.2.2 Prototypical understandings related to quadrilaterals

Prototypes can be defined as a first or early example that is used as a model for what comes later. Related literature indicates that "*prototypicality*" influences on the interpretations of geometric constructions (Noirfalaise, 1991) because geometric figures can be illustrated in different versions. For example, a right triangle can be constructed in different orientations and sizes. They are all visual images of a geometric concept. The main focus of the current study is not to reveal prospective teachers' prototypical understandings, but prototypical phenomenon is a significant theoretical perspective to illustrate PSTs' knowledge involving limited concept images about quadrilaterals and developments in these images. Furthermore, prototypical phenomenon is also helpful and necessary when interpreting the influences of prototypical examples on PSTs' and students' constructions and classification of quadrilaterals. For this reason, in this part, I mentioned about what prototypical understanding means, how it influences learners' conceptions in the light of some theoretical and empirical studies.

Hershkowitz (1990) defined the "*prototype examples*" as the subset of examples that had the "longest" list of attributes all the critical attributes of the concept and those specific (non-critical) attributes that had strong visual characteristics (p.82)". Learners often see figures in a static way rather than in the dynamic way that would be necessary to understand the inclusion relations of the geometrical figures (de Villiers, 1994). For instance, students receive square is not a rectangle because of their misconception about the length of the opposite sides of rectangle. In the literature, there are various studies focusing on students and teachers' prototypical understanding about quadrilaterals (e.g. Fujita, 2012; Monaghan, 2000; Nakahara, 1995; Okazaki, 1995; Okazaki & Fujita, 2007; Vinner & Hershkowitz, 1980). In these studies, researchers utilized and extended Hershkowitz's prototype phenomenon of geometrical figures in order to understand the role of prototypical figures on learners' conceptions of quadrilaterals. The common results of the studies generally reported that learners could not recognize quadrilaterals (e.g. square and rectangle) in different orientations due to the influence of prototypical figures although they were able to define the concepts correctly. For example, it is reported that although most of students consider rhombus as a parallelogram example, they did not treated square and rectangle as being an example of parallelogram due to the influence of prototypical concept images.

As another perspective, some researchers developed an idea in which geometrical concepts are characterized as having double nature by two aspects: "*figural*" and "*conceptual*" (Fischbein, 1993; Mariotti & Fischbein, 1997) similar to the concept image and concept definition (Vinner, 1991), respectively. While figural aspect involves spatial properties like shape, position, and magnitude; conceptual aspect involves abstract and theoretical nature as ideality, abstractness, generality and perfection. According to Fischbein (1993), figural aspect is generally more dominant than conceptual one. For example, parallelograms do not look like a trapezoid, but they are formally trapezoids considering the formal exclusive definition of trapezoid in our context. Based on these ideas, Fujita and Jones (2007) proposed the ideas of "*personal and formal figural concepts*". "*Formal figural concepts*" involve formal concept images and definitions in Euclidian geometry. However, "*personal figural concepts*" are constituted through individuals' own geometry learning experiences about geometric shapes. For instance, "rectangle is a parallelogram with four right angles" is a formal figural concept definition. Besides, the expression of "a rectangle is a quadrilateral with only opposite sides congruent and four 90° angles" reflects a learner's personal figural concept.

Taking account aforementioned theories and frameworks, contradictions between concept images and concept definitions may elicit misconceptions in students' mind when classifying quadrilaterals. Furthermore, many researchers claimed that prototypical examples can lead misconceptions and create inconsistencies between the definitions and hierarchical relations of quadrilaterals (Fujita, 2012; Fujita & Jones, 2006; Hershkowitz, 1990; Pratt & Davison, 2003). In this regard, the studies investigating teachers' conceptions about relations of quadrilaterals are expressed in the following part of the literature review after giving important relevant theoretical perspectives.

2.2.3 Learners' understandings on the definitions and classification of quadrilaterals

Definitions and relations among quadrilaterals are emphasized in The National Council of Teachers of Mathematics [NCTM] standards for grades 6-8 as the following:

> All students should precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties (NCTM, 2000, p.232).

From this point of view, prospective teachers are also expected to have adequate knowledge about definitions and classifications of quadrilaterals. Consequently, because knowledge about definitions and classifications of quadrilaterals is an essential part of PSTs' SMK and PCK about quadrilaterals, I reviewed the literature on definitions and classifications of quadrilaterals.

Researchers in literature considered that definitions and classifications of quadrilaterals are closely related to each other because differences in definitions lead different classification of quadrilaterals. The presence of a close relationship between definitions and classification was emphasized by Poincaré (1952) who is a well-known French mathematician:

> The aim of each part of the statement of a definition is to distinguish the object to be defined from a class of other neighboring objects. The definition will not be understood until you have shown not only the object defined, but the neighboring objects from which it has to be distinguished, until you have made it possible to grasp the difference, and have added explicitly your reason for saying this or that in stating the definition (p.133).

From the above expressions, it can be inferred that critical properties of geometric figures are used to define a concept. By this way, definitions allow us to involve a concept into a suitable class of objects which have related critical properties. In this regard, quadrilaterals are seen as the best subject to examine the intertwined nature of definitions of the concepts and their classifications. Consequently, several researchers focused on the close relationship between definitions and hierarchical structures of quadrilaterals in their studies (e.g. De Villiers, 1994; Fujita, 2012; Fujita & Jones, 2007; Schwarz & Hershkowitz, 1999; Usiskin & Griffin, 2008).

Among these researchers, De Villers (1994) proposed two different classifications types for quadrilaterals as "*hierarchical classifications*" and "*partition classification*". While hierarchical definition refers the "the classification of a set of concepts in such a manner that the more particular concepts form subsets of the more general concepts" (p.11), partition classification is defined as the classification where “the various subsets of concepts are considered to be disjoint from one another” (De Villers, 1994, p.11). The accurateness of the classification does not depend on the types of them. Instead, it depends on the purposes and personal preferences. However, many of researchers prefer to use hierarchical classifications of quadrilaterals because they believed that using hierarchical classifications give opportunities to the learners in order to establish relationship between more general and specific concepts, to make deductions the properties of the concepts, and to produce alternative definitions for a concept (De Villers, 1994; Fujita, 2012; Fujita & Jones, 2007). Researchers also emphasize that learners improve their ability to comprehend the transitivity (e.g. if a square is a rectangle and a rectangle is an isosceles trapezoid then a square is an isosceles trapezoid), asymmetry (e.g. a rectangle is a parallelogram but a parallelogram is not a rectangle), and opposite asymmetry of relations between geometric shapes (e.g. a square is a rectangle and a rectangle is not a square; but while all properties of a rectangle are valid for a square, all properties of a square are not valid for a rectangle) by virtue of the functionality of hierarchical classifications (Fujita & Jones, 2007; Schwarz & Hershkowitz, 1999). Moreover, it is stated that the inclusive relation of quadrilaterals contributes the development of geometrical thinking and mathematical argumentation, deductive reasoning and proof (Fujita, 2012; Fujita & Jones, 2007).

Fujita and Jones (2007) investigated trainee elementary school teachers’ understanding of definitions and their knowledge of inclusive relations between quadrilaterals in Scotland. The researchers reached a result in which they proposed the presence of a gap between learners’ personal figural concepts and formal figural concepts. Their results showed that although teachers were able to correctly draw geometric figures, they had difficulties in defining and classifying them. Based on Fujita and Jones’s (2007) study, Fujita (2012) examined trainee teachers and lower secondary school students’ understanding of inclusive relations of quadrilaterals. As a result, they offered a theoretical model and method to identify learners’ cognitive development on inclusive relations of quadrilaterals by synthesizing past and current theories such as van Hiele’s model, figural concepts, prototype phenomenon, etc.

They resulted that most of learners recognize quadrilaterals by prototypical examples, which makes difficult for students to understand hierarchical relations between quadrilaterals.

As another remarkable study, Usiskin and Griffin (2008) conducted a study about classifications of quadrilaterals by analyzing various textbooks from the year 1838 to 2008 in order to examine change in definitions through years and provide equivalent definitions of the concepts excluding trapezoid. They proposed two groups of definitions such as "*exclusive definitions*" and "*inclusive definitions*". They explained that when "one definition purposely excludes what the other definition includes; we call the one definition an exclusive definition and the other definition an inclusive definition" (p.4). For example, if a trapezoid is defined exclusively as "a quadrilateral with exactly one pair of parallel sides" (p.27), then rectangles and trapezoids would be grouped as disjoint subgroups. In contrast, if the trapezoid is defined inclusively as "a quadrilateral with at least one pair of parallel sides" (p.27), then all rectangles would be a taken as a subgroup of trapezoids. Therefore, it is clearly seen that while inclusive definitions are related to hierarchical classifications, exclusive definitions lead to partition or exclusive classifications of quadrilaterals similar to the De Viller's (1994) categorization. In other words, types of classifications changes on the basis of the choice of exclusive or inclusive definitions.

Therefore, these important empirical and theoretical studies showed the connections between prototypical examples, definitions and classifications of quadrilaterals. Moreover, classification of quadrilaterals is a crucial mathematical ability because it enables students' better understanding in terms of differentiating similarities and differences of figures (Welter, 2001). However, complex nature of the relationships between the concepts causes difficulties in terms of understanding inclusive definitions and corresponding hierarchical classifications (Fujita & Jones, 2007; Schwarz & Hershkowitz, 1999). In this sense, some researchers especially focused on learners' defining abilities on quadrilaterals. Because teachers' knowledge on definitions of quadrilaterals is an important part of the current study, some theoretical studies on mathematical definitions and empirical studies related to especially teachers' defining abilities are mentioned in the following subsection by summarizing and synthesizing their crucial results.

2.2.4 Teachers' use of mathematical definitions of quadrilaterals

Definitions were taken in hand both mathematical point of view and pedagogical point of view in related literature and the standards of curriculum. For example, NCTM (2000) articulated that giving opportunities experience with definitions enables students to appreciate the power of precise mathematical language. Besides, in terms of helping the developments of students' appropriate concept images and concept definitions, teachers' role was offered by Poincaré (1952) as the following:

> They [students] should be made to see they do not understand what they think they understand, and brought to realize the roughness of their primitive concept, and to be anxious themselves that it should be purified and refined (p.123).

From this point of view, knowing appropriate definitions and selecting pedagogically suitable definitions in classroom teaching are accepted as significant components of teachers' knowledge for teaching mathematics (Ball & Bass, 2003; Ball, Bass, & Hill, 2004). More specifically, Ball and Bass (2003) stated that teachers have responsibilities to select appropriate definitions by taking account their students' needs and levels instead of using definitions given in textbooks. Similarly, Winicki-Landman and Leikin (2000) considered the selection and use of definitions in the classroom teaching is a fundamental component of a teacher's pedagogical content knowledge. To sum, considering the importance and necessity of definitions in PSTs' both SMK and PCK related to quadrilaterals, I needed to mention some details about the role of definitions on concept acquisition of quadrilaterals and the results of previous studies that were generally conducted with prospective teachers in this part.

Definitions play a crucial role as an important language form in teaching and learning of mathematics. When considering the fundamental roles of mathematical definitions in problem solving, argumentation and proof, identifying mathematical concepts (De Villiers, 1998; Silfverberg, 2003), making relationship among concepts (Mariotti & Fischbein, 1997), and ensuring oral and written communication for mathematics teaching and learning (Thompson & Rubenstein, 2000), utilizing definitions effectively in the instructional processes is a crucial and necessary component of teachers' subject matter knowledge and pedagogical content knowledge (Ball, Bass, & Hill, 2004).

Both some mathematicians and mathematics educators have widely preferred *inclusive definitions* involving hierarchical relations among concepts since they functionally and economically allows to establish an inclusion between more particular concepts and more general concept (De Villers, 1994; de Villiers, Govender, & Patterson, 2009; Heinze, 2002; Kaur, 2015; Shir & Zavlavsky, 2002; Usiskin & Griffin, 2008). In this regard, some researchers (e.g. Solow, 1984; Vinner, 1991; Winicki-Landman & Leikin, 2000) outlined logical principles that should be fulfilled in defining a mathematical concept, which include defining as giving a name, establishing necessary and sufficient conditions for the concept, using only previously defined concepts, minimality, and arbitrariness. To mention but a few, there are a variety of statements for every mathematical concept, which constitutes necessary conditions-the concept properties-or sufficient conditions that is indication of the concept (Winicki-Landman & Leikin, 2000, p.17). For instance, having four sides is necessary, but not sufficient for a quadrilateral to be a square. However, with the provision of necessary and sufficient conditions, a class of equivalent definitions occurs and each definition in equivalence class becomes mathematically correct. In this regard, it may be critical for teachers to select a definition amongst a number of equivalent definitions while teaching a mathematical concept. In order to do an effective selection, it is important that a mathematical definition should also be considered from didactical perspective, because providing all requirements that fulfils all logical/mathematical principles is not sufficient to put a didactically sound definition. Namely, a definition must be both mathematically correct and didactically suitable when teaching mathematical concepts. In this regard, Leikin & Winicki-Landman (2001) stated that when determining to utilize equivalent mathematical definitions, it should be assessed not only from the *epistemological* aspects but also from the *cognitive* (What definition is the most suitable within a given project for teaching?), *instructional* (What definition is the most suitable within a given project for teaching?), and *didactical (pedagogical)* (What relationship is established between the personal meaning learnt and the institutional meaning intended?) aspects. In parallel this idea, didactically suitable definition for the instructional processes was explained based on some conceptions (Winicki-Landman & Leikin, 2000) such as relying on previously learned concepts (Edwards & Ward, 2003), learners' intellectual development, zone of proximal development of the learners (ZPD), intuitiveness (Fischbein, 1987; Mariotti & Fischbein, 1997), and elegance (Vinner 1991; Van Dormolen & Zaslavsky, 2003).

Mathematical and pedagogical considerations are mutually complementary components for providing both correct and suitable definitions of mathematical concepts in schools. While prospective teachers' personal definitions are a fundamental part of their "*subject matter knowledge*", instructional definitions intended to be shared with students can be regarded as an essential indicator of their related "*pedagogical content knowledge*". In addition, teachers' knowledge of mathematical definitions affects their instructional preferences and pedagogical strategies when teaching mathematical a mathematical concept (Leikin & Zazkis, 2010; Zazkis & Leikin 2008). If teachers have sufficient pedagogical content knowledge on definitions, they can select and utilize suitable definitions considering their students' cognitive abilities and ages. From this point of view, some researchers aimed to examine teachers' defining abilities on quadrilaterals (De Villiers & Govender, 2002; Pickreign, 2007; Vinner, 1991; Zazkis & Leikin, 2008; Zaslavsky & Shir, 2005). Details about some related studies are mentioned in the following.

For example, Shir and Zaslavsky (2001) addressed secondary schools' mathematics teachers' conceptions of mathematical definitions of square. They gave a questionnaire involving eight equivalent statements to the teachers and asked them to decide whether to accept or reject each statement as a definition of a square and to provide their reasoning for the decision. After teachers working individually and in a group having 3-5 persons, they made a whole class discussion. The results of the study showed that teachers disagreed when deciding to accept a statement as a definition of square. Similarly, Zazkis and Leikin (2008) conducted a study to examine 40 prospective secondary mathematics teachers' understanding of the definition of a square. At the beginning of the task, teachers were asked to write as many definitions as they can for a square. However, only five prospective teachers listed appropriate definitions including necessary and sufficient conditions in addition to accurate mathematical terminology and 26 out of 40 teachers could write at least one appropriate definition. They concluded that prospective teachers disagreed when deciding the validity of a definition in terms of providing necessary and sufficient conditions as in the case of Shir and Zaslavsky's (2001) findings. In another research, Pickreign (2007) conducted a descriptive study to investigate 40 prospective elementary mathematics teachers' understanding of the properties and relationships among parallelograms. In data collection process, he asked teachers to give written definitions of rhombus and rectangle Results of the study indicate that only nine prospective teachers articulated an adequate definition of rectangle and only one of them provided an adequate description of rhombus.

Consequently, the results of studies that examine prospective mathematics teachers' definitions of a quadrilateral revealed that the preservice teachers tended to define prototypical figures under the influence of visual characteristics of prototypical figures. This situation showed unsuitable connections between their personal and formal figural concepts. Furthermore, they generally provided inappropriate mathematical language usages in their definitions. On the other hand, from pedagogical perspective, they had difficulties to determine which definition is more suitable when teaching the concept to the students in a specific grade level or age group. In sum, related literature generally shed light on the inadequate nature of prospective teachers' subject matter knowledge and pedagogical content knowledge relate to definitions of quadrilaterals.

2.2.5 National studies about teachers' knowledge on quadrilaterals

In recent years, several studies related to quadrilaterals and basic geometric concepts have been conducted in Turkey. These studies can be grouped as the studies that aimed to examine middle school students' (grade 4-8) conceptions about quadrilaterals (Aktaş & Cansız-Aktaş, 2012; Akuysal, 2007; Doğan, Özkan, Karlı-Çakır, Baysal & Gün, 2012; Duatepe-Paksu & Ubuz, 2009; Erbaş & Aydoğan-Yenmez, 2011; Ergün, 2010; Özerem, 2012; Türnüklü, 2014a; Ubuz & Üstün, 2004; Ulusoy, 2015); secondary school students' (grade 9-12) conceptions (Cansız-Aktaş & Aktaş, 2012; Ubuz, 1999; Yılmaz, Durgut & Alyeşil-Kabakçı, 2008); prospective primary and middle school mathematics teachers' knowledge about quadrilaterals (Aslan-Tutak & Adams, 2015; Aslan-Tutak, 2009; Cantürk-Günhan, 2014; Cantürk-Günhan & Çetingöz, 2013; Çetin & Dane, 2004; Çontay & Duatepe-Paksu, 2012; Duatepe-Paksu, İymen & Pakmak, 2012; Duatepe-Paksu, Pakmak & İymen, 2012; Erşen & Karakuş, 2013; Koç & Bozkurt, 2011; Olkun & Toluk, 2004; Erdoğan & Dur, 2014; Öztoprakçı, 2014; Türnüklü, 2014a; Türnüklü, 2014b; Türnüklü, Gündoğdu-Alaylı & Akkaş, 2013); and inservice teachers knowledge about quadrilaterals (Akkaş & Türnüklü, 2014, 2015). Numerous studies in Turkish context indicate increasing interest of Turkish researchers on the subject of quadrilaterals in terms of understanding especially middle school students' and prospective middle school mathematics teachers' knowledge in recent years. In the following, considering the purpose of the current study, the details of teacher-centered studies instead of student-centered studies are mentioned by comparing and contrasting their important results.

Aslan-Tutak (2009) carried out a study to investigate three preservice teachers' geometry learning and their geometry content knowledge for the case of quadrilaterals by both qualitative and quantitative methods. In qualitative part of the study, she focused on three pre-service teachers' geometry knowledge and their usages of effective instructional ways for students' learning. Based on the results of qualitative part of the study, pre-service teachers' geometry content knowledge was limited and they had difficulties in classifying quadrilaterals. In the quantitative part of the study, she compared geometric content knowledge of control (n=48) and treatment (n=54) groups of the pre-service teachers and to specify the increase of geometry knowledge of pre-service teachers in the experimental group. The results revealed that both the treatment group participants' geometry knowledge significantly increased and the control group participants' geometry knowledge also improved. Although the knowledge increase of the participants in treatment group was greater than the increase in the control group participants, the difference was not statistically significance.

In a descriptive study, Duatepe-Paksu, İymen and Pakmak (2012) investigated 45 preservice middle school mathematics teachers' geometrical content knowledge about classification of parallelogram. They concluded that these preservice teachers could not establish class inclusion among trapezoid and parallelogram.

Differently, Akkaş and Türnüklü (2014; 2015) examined middle school mathematics teachers' pedagogical content knowledge regarding student knowledge and teaching strategies about quadrilaterals by interviewing with 30 in-service teachers working in 12 different schools in Turkey. Their results indicated that teachers considered their students' previous learning when teaching the concepts. Researchers' another conclusion was that teachers thought that students' mistakes can be grouped into three main categories as mistakes regarding defining quadrilaterals, mistakes regarding visual property, classification of quadrilaterals, and family relation within quadrilaterals. Moreover, they mentioned what kinds of strategies teachers used when teaching quadrilaterals to their students (e.g. using formal definition, informal personal definitions or listing properties of the figure; using daily life example materials or drawing figures).

Furthermore, from the pedagogical aspect, Türnüklü (2014a) investigated 68 prospective middle school mathematics teachers' perceptions about special quadrilaterals and their inclusive relations in order to reveal their common cognitive paths. PSTs' lesson plans were used as the main data source. The results of the study showed that PSTs had some misconceptions about the inclusive relations among quadrilaterals. She also identified prospective teachers' common cognitive paths as parallelogram/rhombus, square/rectangle and square/rhombus association. Considering related international studies, it is noted that there was a similarity between Scottish prospective teachers' common cognitive paths and that of Turkish prospective teachers (Okazaki & Fujita, 2007).

Instead of solely focusing on teachers' subject matter knowledge or pedagogical content knowledge, Cantürk-Günhan and Çetingöz (2013) focused on preschool teachers' subject matter knowledge and pedagogical content knowledge on basic geometric concepts (e.g. triangle, square) in the classroom environment through a case study design. According to the results, both prospective teachers could not use appropriate mathematical language when describing geometric concepts. Furthermore, the researchers reported that prospective teachers generally used real life examples and employed activities involving visual examples to provide a better understanding for the children. Another important result of the study emphasized the inadequateness of prospective teachers' knowledge about students' conceptions, misconceptions and errors on basic geometric concepts. In a similar vein, Cantürk-Günhan (2014) conducted a case study to understand five pre-service teachers' subject matter knowledge and pedagogical content knowledge about quadrilaterals. She found that prospective teachers' subject matter knowledge and pedagogical content knowledge was not sufficient for interpreting the characteristic features of quadrilaterals.

In recent years, national studies have frequently been conducted to examine prospective middle school mathematics teachers' conceptions about quadrilaterals by utilizing different theoretical frameworks such as Vinner's (1991) concept image and concept definition (Erşen & Karakuş, 2013; Türnüklü, Gündoğdu-Alaylı & Akkaş, 2013; Türnüklü, 2014a; Erdoğan & Dur, 2014). For instance, Türnüklü Gündoğdu-Alaylı and Akkaş (2013) carried out a qualitative study with 36 prospective middle school mathematics teachers in order to show how they define, image, and classify quadrilaterals throughout semi-structured interviews. They concluded that some PSTs had incomplete connections between concept image and concept definition. In the results, they stated that PSTs could not recognize the difference between rhombus and square and they had difficulties in drawing of trapezoid. Further, the results indicated that PSTs tended to prefer partition classification instead of focusing on inclusive relations among quadrilaterals. Similarly, in another study, Türnüklü (2014b) examined middle school students' and prospective teachers' concept images regarding trapezoid in a qualitative study. She reached that both students and preservice teachers used non-critical properties in non-formal and incorrect definitions and they made overgeneralizations.

Very similar to Türnüklü et al.'s study (2013), Erdoğan and Dur (2014) also executed a study in which they tried to understand 57 preservice mathematics teachers' personal figural concepts and their classification of quadrilaterals by administering a questionnaire. They reported the dominant nature of prototypical figures on preservice mathematics teachers' personal concept images. According to the researchers, preservice teachers could not completely establish hierarchical relations among quadrilaterals under the negative influence of prototypical concept images. They also concluded that even though the preservice teachers could provide formal definitions of quadrilaterals, their prototypical images influenced their personal figural concepts.

Taking account of indispensability and importance of definitions in mathematics, some researchers solely focused on the prospective teachers' defining abilities on special quadrilateral concepts (e.g. Aytekin & Toluk-Uçar, 2011; Duatepe-Paksu, Pakmak & İymen, 2012; Erşen & Karakuş, 2013; Koç & Bozkurt, 2011; Öztoprakçı, 2014). Among them, Aytekin and Toluk-Uçar (2011) investigated 36 practicing teachers' understanding of square, rectangle, trapezoid, and parallelogram as reflected by the definitions they generate by a written questionnaire.

Teachers generated 357 statements for the concepts. The results revealed that one third of teachers produced inappropriate definitions for each concept. Likewise, Koç and Bozkurt (2011) focused on pre-service teachers' definitions of major geometric concepts. For this purpose, they examine how 162 first year mathematics pre-service teachers define and draw two and three dimensional geometric shapes. Results indicate that prospective teachers have more difficulties on defining than on drawing. Furthermore, Duatepe-Paksu, Pakmak and İymen (2012) specifically concentrated 45 preservice teachers' descriptions on a rhombus task in terms of providing necessary and sufficient conditions in individual interviews sessions. Their results indicated that some of preservice teachers generally could not see the relationship properties of rhombus and its definition in order to establish a definition involving necessary and sufficient conditions.

As a noteworthy study, Öztoprakçı (2014) examined five preservice teachers' cognitive processes in constructing and assessing definitions and classification of quadrilaterals under the support of the Geometer's Sketchpad learning activities in her doctoral thesis by a qualitative case study. She conducted one-to-one clinical interviews with each preservice teacher in the Human-Computer Interaction Laboratory. The findings showed that using the Geometer's Sketchpad in learning environment was found effective to improve preservice teachers' cognitive processes of identifying critical properties for the definitions of quadrilaterals, assessing mathematical importance of a definition, understanding the relations among quadrilaterals to establish hierarchical classifications.

2.2.6 Summary of literature review about quadrilaterals

In literature, there are a lot of comprehensive international theoretical and empirical studies about why understanding definitions, classifications and properties of quadrilaterals is difficult for both the learners in all grade levels and inservice/preservice teachers. Some of these theoretical frameworks are Vinner's concept image and concept definition (Tall & Vinner, 1981; Vinner & Hershkowitz;

1980, 1983), Fischbein's figural concepts (Fischbein, 1993), personal and formal figural concepts (Fujita & Jones, 2007), dynamic figural concepts (Walcott, Mohr & Kastberk, 2009) and prototype phenomenon of geometric figures (Hershkowitz, 1990; Nakahara, 1995; Okazaki, 1995). These theoretical and empirical studies commonly investigated how learners recognize critical properties of the figures, how they define the concepts considering necessary and sufficient conditions, or how they classify quadrilaterals according to inclusive, exclusive relations. Their findings generally bear many similarities even participants of the studies were different grade levels or ages. Some crucial and common results of teachers-based studies can be summarized in order to show what kinds of conceptions, misconceptions, and difficulties the learners had as in the following:

- Memorization of concept definition independent of concept image or developing concept images apart from concept definition
- Insufficiency in using both mathematically and grammatically correct language in the definitions
- Inability to choose mathematically, instructionally and pedagogically suitable definitions of the concepts
- Barriers to think quadrilaterals in a flexible/dynamic way due to the influences of prototypical examples that learners encountered in previous learning environments
- Classifying quadrilaterals partially or exclusively instead of considering all hierarchical relationships between quadrilaterals by adopting an inclusive way.

On the other hand, quadrilaterals have also been a popular subject in Turkey because many of researchers have carried out their research to examine students' and teachers' conceptions about quadrilaterals for last ten years. However, it can be clearly seen that almost all teacher-based national studies focused on understanding the existing situation of pre-service teachers' or inservice teachers' knowledge on quadrilaterals by virtue of replication studies rather than trying to develop their knowledge by using various theoretical, instructional or methodological approaches.

Their results generally pointed out the incomplete and inadequate nature of pre-service teachers or in-service teachers' subject matter knowledge and pedagogical content knowledge on quadrilaterals.

To sum up, many studies indicated that both teachers and students have difficulty to comprehend definitions, constructions, and classifications of quadrilaterals. In addition to these studies, some researchers focused on traininig studies instead of descriptive ones. In training studies, researchers generally used dynamic geometry applications to enhance teachers' knowledge (de Viller & Govender, 2002; Öztoprakçı, 2014) or mostly students' knowledge (Erez & Yerushalmy, 2007; Furinghetti & Paola, 2002; Jones, 2000; Özçakır, 2013) about geometric concepts. They used dynamic geometry softwares such as GeoGebra, Cabri, Geometer Sketchpad, and Shape Maker. They generally concluded that if learners are actively engaged in defining, constructing, and classifying activities in a dynamic geometry learning environment, they can establish appropriate mental models of geometric figures and conceptual understanding of their properties. However, these studies mostly aimed to develop primary and middle school students' conceptions. On the other hand, although there are some studies that were conducted to enhance teachers' conceptions on quadrilaterals they focused on teachers' subject matter knowledge on a specific area as defining and classifiying of quadrilaterals. Therefore, it is necessary to find some possible ways in which researchers are able to effectively enhance teachers' subject matter knowledge and pedagogical content knowledge instead of only describing the problematic situations in teachers' knowledge. In this regard, mathematics educators and the educators in other learning domains offered case-based pedagogy for teachers' professional development as an alternative effective approach.

In the following part of the literature review, the most necessary and important information about the place of case-based pedagogy in teacher education, the uses of video cases, the necessity of micro-case videos, and the theoretical approaches utilized in case-based teacher education were given respectively by considering the purpose and the research questions of the current study.

2.3 Case-based Pedagogy for Teachers' Professional Development

Training well-qualified and knowledgeable teachers is substantially challenging task for teacher educators (Harrington, 1999). From this point of view, it is necessary to design effective teacher preparation programs aiming in order to prepare prospective teachers for the realities of classrooms (Shulman, J., 1992) by developing their subject matter and pedagogical knowledge. In this regard, traditional teacher training programs at universities like lecture-based instructional methods have been criticized for being deprived of establishing strong connections among theory and practice (Abell & Cennamo, 2004; L. Shulman, 1992). In the absence of the necessary connections among theory and practice, many prospective teachers have difficulty when transferring their theoretical knowledge to practical learning environments when they become inservice teachers at schools (Ball, 2000; Doyle, 1986; L. Shulman, 1992; Merseth, 1999). For this reason, teachers and educators have begun to explore new effective methods. At this point, case-based professional development in teacher education have gained more attention and become increasingly popular among educators at least for over past two decades. Thus, researchers see the uses of case-based instructional approaches as both a helper and a solvent in order to minimize the problems in teacher education (Lundeberg, 1999; Lundeberg et al., 1999; Masingila & Doerr, 2002; Merseth, 1991; L. Shulman, 1992; Van Den Berg & Visscher-Voerman, 2000).

There are various definitions of cases based on their purposes and uses (Merseth, 1996). For example, Bruner (1986, 1990) considers cases as a way of knowing (as cited in L. Shulman, 1992). According to J. Shulman (1992) a case in classroom context is "…a piece of controllable reality, more vivid and contextual than a textbook discussion, yet more disciplined and manageable than observing or doing work in the world itself (p.xiv)". Case-based instruction is defined as an instructional design method in which learners analyze and solve cases through observation, discussion, reflection, and discussion (Ertmer & Stepich, 1999).

At this point, it is useful to explain why researchers have increasingly preferred to use case-based pedagogy in teacher education at least for over past two decades. In other words, what are the potential benefits of case-based pedagogy in teacher education? The intention in preparation and utilization a case is to help teachers to broaden knowledge and qualifications that are necessary to respond to complexity and authenticity of real classrooms (Merseth, 1991, 2003; Richardson, 1996; J. Shulman, 1992; L. Shulman, 1986; Sykes & Bird, 1992). Thus, prospective teachers can get the opportunity to learn "*to think like a teacher*" through analyzing, discussing, and reflecting on authentic cases. More specifically, the reasons why educators intensively prefer to utilize cases for teachers' professional development are also related to potential benefits of case-based approaches. In the light of the related literature, the notable benefits of using cases in teacher education are summarized as follows: (a) promoting teachers' critical and reflective thinking and decision making abilities (Butler, Lee, & Tippins, 2006; Grossman, 1992; Jay, 2004; Mayo, 2004; Merseth, 1992), (b) developing content and pedagogical knowledge in subject-specific context (Fernandez, 2005; Manouchehri, 2002; Mayo, 2002); (c) providing a means of understanding theoretical principles, (d) giving opportunity to teachers to examine effectively about the complex structures of practice, (e) overcoming the potential limitations of field experiences (Masingila & Doerr, 2002), and (f) providing opportunity to cope with ambiguities and dilemmas of schooling like determining the way of appropriate instruction; (g) learning in a community through analysis, reflection and discussion of cases in social interactional environment (Arellano, Barcenal, Bilbao, Castellano, Nichols, & Tippins, 2001; Shulman, J., 1992). Considering the benefits of the cases in teacher education, it might be useful to mention some details about types of cases in literature.

In general, there are several types of cases such as text-based cases, video-based cases, and multi-media cases. In 1990s, most of the studies that investigate the uses of cases in teacher education have focused on mostly text-based cases of classroom situations, including written or printed documents such as diaries, photocopies of student work, observer's notes, and so on (Barnett, 1991; Merseth &

Lacey, 1993; Shulman, 1992; Stein, Smith, Henningsen, & Silver, 2000). Nowadays, there are different kinds of cases that have been used for prospective and/or inservice mathematics teacher education such as *lengthy narrative cases* (e.g. Hillen & Hughes, 2008), *short narrative cases* (e.g. Schifter, Bastable, & Russell, 2008) and *video cases* (e.g. Goldsmith & Seago, 2008; Seago, Mumme & Branca, 2004; Van Zoest & Stockero, 2008). In order to clarify the effect of different presentations of cases, Moreno and Valdez (2007) conducted a study in which they used a classroom case in both text and video format to examine students' learning and ability to transfer educational psychology principles to novel classroom situations. Their results indicated that video group had higher transfer and retention scores than the other groups. Similarly, since video cases begin to be seen as a more powerful and authentic tool than narratives cases in teacher education (Valmont, 1995; Wetzel, Radtke, & Stern, 1994), there has been an interest in using video cases in order to improve prospective teachers' subject matter knowledge and pedagogical content knowledge (Ball & Cohen, 1999; Ball, 2000; Barnett, 1991; Hiebert, Gallimore, & Stigler, 2002; Lampert, Heaton, & Ball, 1994).

2.3.1 Video-cases in teacher education

By the innovations of technological equipment such as portable video, people can easily reach any of multimedia tools. Therefore, teacher educators have been using video recordings for microteaching sessions since the late of 1960s by summarizing teachers' learning via watching brief clips of classroom instruction (Allen, 1966; Allen & Clark, 1967; Allen & Ryan, 1969; Limbacher, 1971; Ward, 1970). After the late of 1990s, researchers began to use video for in-service teacher education (Frederiksen, Sipusic, Sherin, & Wolfe, 1998; Gwyn- Paquette, 2001; Seago, 2004; Sherin, 2003a, 2004; Tochon, 1999) and for methods courses in pre-service teacher education (Copeland & Decker, 1996; Daniel, 1996; Friel& Carboni, 2000; Goldman & Barron, 1990). With the tendency of using video cases for teachers' professional development, researchers focused on teacher progress in identifying crucial moments and examining student thinking to deepening their pedagogical content knowledge and reflective thinking (Brophy, 2004; Jacob, Lamb, Philipp, Schappelle, & Burke, 2007; Santagata, Zannoni, & Stigler, 2007).

The reason why video have become popular and crucial as the means of instruction and evaluation in teacher education can lurk in its considerable affordances although it has some drawbacks at the same time. In this regard, Sherin (2004) categorized the affordances and drawbacks of using video-cases in teacher education. She offered three drawbacks of using videos before explaining its affordances. In this purpose, she specified all drawbacks as in the following:

i) *Passive role of the person*: Persons views the video without any opportunity to interact the persons in the video clips. Unlike the teacher or observer, viewer has no chance to ask a student in the video clip to make a detailed explanation or to elaborate an idea.

ii) *Limited information*: Classroom is a complex learning environment. The data obtained from video cases are limited only the information captured by video cameras. For example, because video cameras look only one direction at a time, viewers cannot know all students engaged the activity or what students are doing. Furthermore, viewers cannot turn her/his head to look around whenever they want to examine something in the classroom.

iii) *Lack of capturing wide-variety of contextual information*: Video gives information at a time. However, it involves no information about what happened in earlier days and weeks. Similarly, it cannot be gained information about the broader atmosphere of school or the students educating other classrooms.

Sherin (2004) also focused on particular three affordances of video. She offered first two of them based on the work of Latour (1990). These affordances are summarized in the following:

(i) *Video is a lasting record*: A video record can be paused and rewound in any time and can be watched again and again to examine students' statements, constructions, and specific conversations. One may watch and listen a student's specific drawing and explanations in multiple times as a live observation. At the same time, a researcher can get more information about the details of interactional processes between teacher and students or students in a group via external microphones. Additionally, when someone communicates a group of students, video camera get all interactional process of remaining groups of students.

(ii) *Video can be collected, edited, and recombined*: Although video involves chronologically a period of time of a classroom interaction, it can be divided into segments by using video editing opportunities. Furthermore, these rearranged videos can be involved in a video library that develops for teacher education (Frederiksen, 1992). Such types of libraries could include the excerpts of video collections about specific themes such as teachers' actions to manage classroom, students' mathematical understanding about a particular topic, and interactional process between students and teacher. Furthermore, by the help of current developing advances in technology, particular video clips can be electronically linked to curriculum materials.

(iii) *Video sustains a set of practices that are very different from teaching*: The permanent and editable nature of videos allows educators to organize different set of practices to promote teacher professional development and to recognize the new ways for teaching and learning (Putnam & Borko, 2000; Sherin, 2002).when examining a pedagogical issue in video case, prospective or inservice teachers have a luxury of time to think and reflect their ideas on the issue. They can spend time to explore the ways of alternative interpretations and instructional strategies. In addition, they get opportunity to see different colleagues' classrooms and various teaching ways by comparing and contrasting these alternative ways. As a result, teachers can determine the effectiveness of a particular pedagogical strategy. On the other hand, because the complex nature of classroom practice, video clips enable that teachers can spend extended amount of time analyzing a small particular event in classroom practice.

Considering related literature, it seems that it is not enough to group the affordances of videos under only aforementioned three headings. From this point of view, uses of video cases offer many opportunities to teachers and teacher educators: video (a) allows one to enter into the complex nature of classrooms (e.g. Richardson & Kyle, 1999) (b) provides an collaborative discussion and reflection environment (e.g. Lundeberg, Levin, & Harrington, 2000); (c) gives opportunity to examine alternative pedagogical approaches by comparing and contrasting different instructional strategies (e.g. Sherin, 2004); (d) enables to examine interaction analysis among students and teacher (e) supports teachers' noticing interpretive and evaluative stance rather than descriptive ones (e.g. Sherin & Han, 2004) and so on.

2.4 Studies on the Uses of Video Cases in Teacher Education

There are several studies in which researchers utilized case-based pedagogy in order to enhance teachers' professional development. While some of these studies made use of narrative cases, some others employed video or multimedia cases. At this point, it may be useful to recall that the current study is aimed to understand the developments in prospective middle school mathematics teachers' knowledge throughout analyzing and discussing of micro-case video clips involving seventh grade students' mathematical thinking process on quadrilaterals. Taking account the purpose of the current study, it is solely focused on video case-based studies that were conducted with preservice or inservice (especially mathematics) teachers who teach primary (grade 1-4), elementary (grade 5-8) or secondary (grade 9-12) school students. In this regard, some vital and related international and national studies are presented in this part of the literate review.

In general manner, most of studies using video case-based training programs in both inservice and preservice teacher education focused on teachers' noticing of students' mathematical thinking in a general way without making emphasis on particular mathematical concepts and domains. However, exploring the development of teachers' knowledge or teachers' professional noticing abilities in particular mathematical domains might be crucial to enhance teachers' professional development (Garet, Porter, & Desimone, 2001; Kennedy, 1998; Walkoe, 2014) considering the influences of teachers' developing knowledge on the students' achievement and conceptual understanding (e.g. Kennedy, 1998). In this regard, researchers have begun to focus on the studies addressing both inservice teachers and mostly preservice teachers' knowledge or professional noticing of students' mathematical thinking on particular mathematical concepts by video-based instructional ways in very recent years (e.g. Ding & Dominguez, 2015; Huang, Kulm, Li, Smith, & Bao, 2011; Ingram, 2014; Jacobs, Lamb, and Philipp, 2010; Olkun et al., 2009; McDuffie, Foote, Bolson, Turner, Aguirre, Bartell, Drake, & Land, 2014; Santagata, 2009; Schack, Fisher, Thomas, Eisenhardt, Tassell, & Yoder, 2013; Sleep & Boerst, 2012; Taylan, 2015; Walkoe, 2014). Consequently, it is necessary to give the details of video case-based studies that concentrate on the development of mathematics teachers' professional noticing of students' mathematical thinking about particular mathematical concepts in order to see their theoretical and methodological characteristics and contributions to the literature. Thus, it is possible to picturize how international and local studies addressed case-based pedagogy in terms of theoretical, methodological, and didactical aspects and how they made contributions to mathematics teacher education.

For instance, Santagata (2009) implemented a video-based professional development program to sixth grade mathematics teachers during two consecutive years in five low-performing schools. At the end of the first year, the researcher determined teachers' problems related to basic understanding about ratio-proportion, knowledge about students' mathematical conceptions, their abilities when analyzing students' works beyond evaluating them as only right or wrong. In the second year implementation, she addressed four changes/modifications in the modules: "(a) increased specificity of content-related questions, (b) focus on common students' misconceptions, (c) refinement of facilitators' planning and variation in professional development discourse structure, and (d) increased guidance in the analysis of student thinking." (p.48).

Schack et al. (2013) carried out a comprehensive study in order to improve the professional noticing abilities of prospective elementary school mathematics teachers in the context of early numeracy thinking. They collected their data from 94 preservice teachers in mathematics method courses at three different universities by virtue of five-session module developed by researchers considering three components skills (attending, interpreting, and deciding) of professional noticing framework. The results of pre and post-assessment indicated that prospective teachers significantly improved their professional noticing skills.

Similarly, Ding and Dominguez (2015) investigated six Chinese lower secondary mathematics prospective teachers' knowledge and beliefs regarding teaching pedagogies and students' mathematics knowledge when they analyze video clips involving students' procedural errors consisting of exchanging the order of Cartesian coordinates in the process of applying the distance formula. More specifically, they tried to describe how prospective teachers attended, interpreted, and responded to a video case where a student exchanged the order of the coordinates when applying the distance formula. They emphasized the inconsistencies between prospective teachers' responses in three tasks. The results of the study also indicated that teachers' noticing was influenced by their prior experiences, knowledge and beliefs.

In the video club context, Walkoe (2014) aimed to develop seven USA preservice teachers' noticing of students' algebraic thinking in terms of broadening their perspectives to algebra concepts, taking account of different students' algebraic thinking, and reasoning about students' conceptions in more detail. Over an eight week period, preservice teachers watched and discussed classroom video clips taken form teachers' algebra classes. She analyzed preservice teachers' knowledge-based reasoning by determining the depth of group discussions about students' algebraic thinking in order to understand how they examine students thinking either by evaluating a general sense or looking at the details in students' conceptions. The results of the study revealed that participating in a video club not only provided teachers articulated substantively conceptual aspects of students' algebraic thinking, but also gave opportunities them to reason about the thing they noticed in deeper ways. At the end of the study, the researcher suggested that researchers can be examined how preservice teachers' professional noticing abilities develop in different mathematical subjects such as spatial abilities in geometry or generalization in statistics for the future studies.

On the other hand, Huang et al. (2011) conducted an exploratory study in China with sixteen in-service primary school mathematics teachers having teaching experience from two years to twenty years via a 5-day video-case based training program about fractions and decimals. They examined their data by utilizing the framework in terms of mathematical content knowledge, general pedagogy, and pedagogical content knowledge to capture how these teachers' approaches change when evaluating the videos. Based on the data analysis, they found that teachers shifted their perspectives from general pedagogical issues to mathematical and pedagogical content knowledge on fractions and divisions instead of focusing solely generic classroom management issues. Consequently, they concluded that video-based training program based on a specific mathematical concept such as fractions positively influenced on the development of teachers' understanding of related mathematical content and pedagogy and their reflective thinking abilities.

Another study was carried out in order to assess the effectiveness of an experimental elementary mathematics field experience course by proposing a prediction assessment rubric for evaluating prospective teachers' knowledge of children's mathematical thinking about part-whole relationship in fractions (Norton, McCloskey, & Hudson, 2011). By establishing and using a model of child's mathematics, prospective teachers predicted how the child answers to a mathematical task. In the findings, researchers made emphasis on the effectiveness of the instrument that they developed. They concluded that their implementation indicates moderate to high degrees of interrater reliability in using the rubric to make assessment about prospective teachers' models and predictions. Finally, they suggest that prediction assessments effectively assess the prospective teachers' pedagogical content knowledge.

In addition to the international studies, there are also some video case-based studies that focused on teacher noticing abilities in online video discussion environments or teachers' knowledge (Koç, 2011; Osmanoğlu, Koç, & Işıksal, 2013; Osmanoğlu, Işıksal & Koç, 2012; Osmanoğlu, Işıksal & Koç, 2015) or professional noticing abilities of students' mathematical understanding in specific mathematical concepts such as numbers and arithmetic (Olkun et al., 2009), multiplication and division (Taylan, 2015), problems in modeling perspective (Didiş, 2014; Didiş, Erbaş, Çetinkaya, Çakıroğlu & Alacacı, 2015). Parallel with the aim of the current study, it is mentioned about the details of national studies that focus on teachers' knowledge and professional noticing of students mathematical understanding in particular concepts instead of mentioning about the studies related to teachers' noticing in a general manner independent from the mathematical subject or concepts.

For example, according to case-based instructional design, Olkun, Altun, and Deryakulu (2009) developed the developmental process of a digital learning tool (Learning Tool for Elementary School Teachers (L-TEST)) involving children's mathematical thinking for the ages of 4–11 years on the subjects of numbers, arithmetic, and geometric shapes as an teacher training project. They aimed to help prospective and inservice teachers get to know students thinking by the virtue of video cases. Finally, they conducted a usability test for the developed learning tool. The results revealed that teachers provided two main benefits of such kind of learning tool: supporting and enhancing their knowledge about children's basic mathematical concepts, and getting new information about children's geometric thinking and their strategies.

In a very recent study, Taylan (2015) examined third grade teachers' professional noticing of students' mathematical thinking on multiplication and division. As the data sources, she used video records of three consecutive mathematics classes, students' written documents, field notes, and videotaped interviews conducted with the teachers. She examined how teachers notice particular events of classroom instruction by using following components: (i) student thinking (e.g. student strategies, student understanding, student difficulty, making connections, and providing explanations), classroom norms (e.g. checking work, partnership providing feedback for peers admitting/learning from mistakes), and (iii) students characteristics (e.i. students' personality or attributes). As an important result, she found that teachers' noticing might influence their instructional approaches and their students' learning in better ways.

In her doctoral thesis, Didiş (2014) examined twenty five prospective secondary school teachers' knowledge of students' mathematical thinking within an undergraduate course context about mathematical modelling. She collected data within four two-week cycles. Prospective teachers firstly examined a non-routine mathematical task. Then, they examined and discussed a group of high school students' solutions to the task by analyzing students' written documents and video episodes. The results revealed that prospective teachers' predictions were inconsistent with on students' actual mathematical thinking ways at the beginning of the course. However, she found that great portion of prospective teachers' predictions become more consistent over the course as they analyzed and discussed students' thinking ways.

Based on Didiş's (2014) doctoral thesis, Didiş, Erbaş, Çetinkaya, Çakıroğlu and Alacacı (2015) published an article in order to reflect secondary school teachers' views about the role of analyzing students' mathematical work in comprehending students' ways of thinking. They reached that prospective mathematics teachers found examining the students work useful in terms of being aware of, understanding, interpreting students' ways of thinking. They emphasized the positive influences of using students' work from real classroom settings on the development of teachers' pedagogical content knowledge.

In sum, based on the literature review, it is evident that there are limited studies on the use of cases and the influences of cases-based pedagogy on teacher education in Turkey. Thus, it was necessary to conduct a study on what prospective teachers gain from the case-based professional development programs especially designed in particular mathematics domains. Based on this necessity, I proposed the use of micro-case videos in prospective teacher education. In the following, I provide explanations about what micro-case video is, why it is necessary, and how researchers produce such video cases to develop prospective teachers' knowledge on specific mathematical concepts or domain.

2.5 The Need for Micro-Case Videos in Teacher Education

In the literature, there are different uses of videos in teacher education such as microteaching, interaction analysis, modeling expert teaching, video-based cases, hypermedia programs and field recording (Sherin, 2004). In general, many of video-based studies that were conducted to enhance learning to notice for teachers professional development involves the noticing of classroom features, such as student-teacher interaction and communication in classroom environment (e.g. Brophy, 2004; Goldman, Pea, Barron, & Derry, 2007; Star & Strickland, 2008; van Es, 2011).

Classroom videos involve information about various dimensions that can be noticed when analyzing video clips such as *actor* (the students, the teacher, and others), *topic* (management, climate, pedagogy, mathematical thinking), and *stance* (describe, evaluate and interpret) (Sherin, Jacobs, & Philipp, 2011; van Es & Sherin, 2008). These dimensions indicate the complex and multidimensional structure of classroom environment in terms of prospective mathematics teachers. Among these dimensions, understanding students' mathematical thinking has particularly crucial role to promote the effectiveness of prospective teachers' professional development. Especially, reform-based approach for teacher education encourages teachers to get detailed information about students' misconceptions, difficulties, errors and thinking processes on a mathematical concept by adopting a flexible approach to instruction (Ball & Cohen, 1999; National Council of Teachers of Mathematics [NCTM], 2000; Ministry of National Education [MoNE], 2013). However, teachers who began to analyze classroom videos may not have a focused and direct attention on students' mathematical thinking due to other factors in classroom teaching (Chamberlain, 2005; Ding & Dominguez, 2015; Freese, 2006; Kagan, 1992; Olkun et al., 2009;

Shapiro, 1991). Instead, they tend to notice different things such as teachers' explanation, students' conversations, or classroom environment when first examining a classroom video case.

In the classroom videos, the noticeable features of students' mathematical understanding are hidden in the complex nature of the classroom. At this point, "*professional noticing*" enables teacher more focused noticing on students' mathematical ideas (Jacobs, Lamb, & Philipp, 2010; Sherin et al., 2011; Star & Strickland 2008; van Es 2011) because mathematics educators make emphasis on the significance of interpreting and eliciting the meaning of students' mathematical works as a primary goal to ensure effective mathematics teaching. To achieve this, teachers must also have knowledgeable on students' possible conceptions, misconceptions, difficulties, and errors in any subject domain in order to access students' thinking and get opportunity to devise their own instructional decisions (Ding & Dominguez, 2015; Jenkins, 2010). The results of several studies indicate that prospective teachers experience a number of difficulties at the beginning of their teaching careers in terms of interpreting their observations (Jacobs et al., 2010; Star & Strickland 2008; van Es & Sherin, 2008; van Es & Sherin, 2002) that are especially related to interpreting students' mathematical thinking. Because of the difficulties, they generally tend to describe mathematical situations in the learning environment rather than interpreting the meaning of a situation or finding solutions for problematic events (e.g., Santagata et al., 2007; Roth-McDuffie, Foote, Bolson, Turner, Aguirre, Bartell, Drake, & Land, 2013).

When considering the courses given in prospective teacher education programs, observing classroom videos are doubtlessly helpful in constructing a bridge between university learning to classroom practice in terms of prospective teachers. Furthermore, though classroom videos, prospective and inservice teachers can broaden their knowledge on different instructional strategies, and classroom culture in terms of interactional process and classroom management, and curricula (Sherin, 2004, p.14). However, we should question that whether the examination of only classroom videos or some interesting moments of classroom videos can be enough to promote prospective mathematics teachers' professional development in terms of understanding students' mathematical thinking.

In order to attend, interpret, and reflect students' mathematical thinking thoroughly, it is crucial to detect and edit suitable moments as the cases in the classroom videos. For the selection of video excerpts from classroom environment, Linsenmeier and Sherin (2009) identified three types of video clips as *What*, *Wow*, and *Hmm* clips that are useful to increase productivity of discussion on student mathematical thinking. These clips refers to "what just happened?", "I never thought of that!", and "there is something interesting in here", respectively. Additionally, Sherin, Linsenmeier and van Es (2009) characterized classroom video clips of student mathematical thinking according to three dimensions such as *Window*, *Clarity*, and *Depth* by rating twenty six video clips as being low, middle and high on each dimension (see Table 1).

Table 1. Three dimensions of classroom video clips of student thinking (Sherin et al., 2009, p.216)

Dimensions	Critical question	Levels of each dimension		
		Low	Medium	High
Window	Is there evidence of student thinking in the video clip?	Little evidence of student thinking from any source	One or more sources of information exist, but little detail provided	Detailed information from one or more sources
Depth	Are students exploring substantive mathematical ideas?	Task is routine for student; calls for memorization or recall on part of student	Some sense-making applied to routine task	Student engages in math sense-making, works on task at conceptual level
Clarity	How easy is it to understand the student thinking shown in the video?	Student thinking not transparent	Much of student thinking transparent, though some ideas may be unclear	Student thinking transparent; viewer sense-making not called for or single interpretation obvious

They examined the relationship between the video clips and productivity of the discussions of the clips. Based upon the findings, they proposed that familiarity with these dimensions leads teacher educators who desire to characterize productive video clips to use with teachers. In these studies, video clips that involve students' mathematical thinking are selected and identified through cutting and editing classroom videos (e.g. small video excerpts that display clear students' explanations how s/he solved the problem on the board) rather than focusing on a single student's thinking process on a mathematical concept thinking process within a specific video production process. Absolutely, the way used by Sherin et al. (2009) is helpful to determine which part of a classroom video can be used for a productive discussion of students' mathematical thinking. However, conducting videos in complex classroom environment in which teachers make natural instruction requires extra time and effort. Furthermore, in classroom videos, it is almost impossible to examine students' mathematical thinking in both natural and isolated manner. Accordingly, I think that producing and using "*micro-case videos*" that reflect student mathematical thinking can be an effective way to promote prospective mathematics teachers' content related knowledge and pedagogical content knowledge in terms of understanding students' mathematical thinking and developing alternative instructional strategies to problematic situation in students' conceptions. Thus, the use of *micro-case videos* in this manner serves purely a student-centered perspective to the teachers instead of focusing on multi-dimensions of complex classroom learning. The uses of micro-case videos in prospective teacher education programs can be effective and helpful to fill the gap in video case-based professional development context. In the following, I provide information about definition and main characteristics of micro-case videos.

2.5.1 A proposal for definition and main characteristics of micro-case videos

In the scope of this research, it is necessary to answer following questions: What are the definition and the main characteristics of micro-case video clips? What is the difference of micro-case videos from classroom or other types of video cases? In the current study, I defined "*micro-case video*" as a specially-designed educational video for prospective mathematics teachers that involve a collection of significant events related to an individual's mathematical thinking process on particular mathematical concepts or problem situations when the learner works on structured content-related tasks in an isolated non-classroom learning environment. Based on the definition, I listed four main features that are associated with micro-case video clips as follows: (i) isolated non-classroom learning environment, (ii) an individual's thinking process, (iii) structured content-related tasks or problem situations, and (iv) a collection of specially-designed selected-edited events. The details of main characteristic features are expressed in the following:

2.5.1.1 Isolated non-classroom learning environment

Micro-case video clips involved a learner's thinking process in an isolated non-classroom environment. In a classroom environment, there are many of student-related, teacher-related, and classroom-related factors that can affect student learning and thinking process (Grubaugh & Hauston, 1990). Students' prior knowledge, level of participation in class, classroom pacing, time limitation, class climate (e.g. teacher-centered or student-centered), supportive or non-supportive learning environment, teachers attitudes about teaching, learning, and students can be given as only some example factors that might have effect on the level of students' self-expression in the classroom. For example, because of having stage fright on the board under the influence of math anxiety, a student may not adequately express his/her mathematical ideas about the issue discussed in the classroom (Jackson & Leffingwell, 1999; Lyons, 1989; Malkoç & Kaya, 2015). Another crucial point is that some teachers can inappropriately use classrooms as a bully pulpit, which might create traditional teacher-centered learning environment instead of student-centered learning environment. In such a case, students cannot actively take a role in the center of the learning process to explain their ideas and responses (Berry & Sharp, 1999; Çubukçu, 2012; Sharma, Millar & Seth, 1999). For these reasons, isolated non-classroom environments can be more suitable than the classrooms in order to foster learners' mathematical thinking in more depth. Especially, well-structured clinical interviews can be used as an alternative way to deepen learners' mathematical conceptions in the current study to produce fruitful video-cases.

2.5.1.2 An individual's thinking process

In this research, type of video case is named by using the prefix of "*micro*" because it involves only a student's mathematical thinking on a particular mathematical subject. Such kinds of clips can be called "*micro-case videos*" and the classroom videos can be thought as the examples of "*macro-case videos*". As mentioned before, related literature indicates the complexity and multidimensional structure of the classroom cases. In classroom video cases, researchers generally focused on many of students' mathematical thinking at the same time due to the nature of classroom context. Students in the classroom cannot be thought separate from their peers and the teacher in the classroom environment. For the similar reasons, it is almost impossible to reach the details related to how a student reason about particular mathematical concepts. From the point of prospective teachers, examining a learner's thinking in more detail is a great opportunity to enhance their knowledge about student mathematical thinking before graduating teacher education program. However, they have almost no chance to examine different achievement level student's conceptions and thinking processes in the courses such as method courses, field practice or school experience at universities. At this point, as a strong argument, it is proposed that micro-case videos involving a single student's thinking process open the door to the world of student's ideas.

2.5.1.3 Structured content-related tasks or problem situations

Sykes and Bird (1992) mentioned two types of cases as "*content-specific cases*" and "*context-specific cases*" and they gave some examples to explain these cases. According to Sykes and Bird (1992), while case related to multiplication of fraction (Barnett, 1991) is an example of content-specific cases, case on teaching Alaskan communities (Kleinfeld, 1992) is an example of context-specific cases. In this regard, micro-case videos in the present study can be thought as an example of content-specific cases because micro-case videos involve middle school students' mathematical thinking processes on content-related tasks or problem situations about quadrilaterals. The reasons why I preferred structured content-related tasks or problem situations in micro-cases were explained in the following.

Teachers' proficiency on conceptual knowledge is emphasized as a prerequisite component of their subject matter knowledge. As a new perspective, Tchoshanov (2011) documents three types of teacher content knowledge: type 1: knowledge of facts and procedures; type 2: knowledge of concepts and connections; and type 3: knowledge of models and generalizations. He reports that type 2 knowledge has a potential role to predict of teaching that will positively effect on students' achievement. In this sense, structured content-related tasks are utilized in order to foster students' knowledge of concepts and connections among the concepts by considering mathematics educators' suggestions. Mathematics educators have stressed the importance of developing students' conceptual understanding, the ways of reasoning, and higher level of problem solving competencies rather than focusing on procedural or short-cut heuristic algorithmic processes (Davis, Maher& Noddings, 1990; Goldin, 1997; von Glasersfeld, 1991) because these tasks give opportunity to "*enter the students' mind from the conceptual aspect*" considering individual difference and the diversity of their mathematical understandings (Davis, 1984; Hazzan & Zazkis, 1999). By this way, researchers may draw inferences about the changing knowledge structure and cognitive processes and the possible meanings of learners' verbal or written statements.

2.5.1.4 A collection of specially-designed selected-edited events

Sherin (2004) stated that "video can be collected, edited and reorganized into a format that differs from its original presentation" (p.12). Similarly, micro-case video clips require a careful and structured video production process. An educational video production process involves different steps such as planning, videotaping, archiving, selecting and editing as similar stages in the case of cinematography. Why editing is necessary for the micro-case videos before sharing the videos to the prospective teachers? Raw videos may involve many of unnecessary data that are unrelated to the researcher's purpose. Furthermore, researchers may want to chance the flow of events in the raw video in order to catch a more effective situation and strengthen the integrity of the clip. Thus, editing process permits both composition and decomposition of pieces of a video in a specific manner. In conclusion, micro-case video clips constitute a collection of selected-edited significant events with regard to the aim of the researcher and some other criteria. (*Important note*: all details of the criteria that I utilized when producing micro-case video clips are explained in method section.) In micro-case video production, preparing video cases that are completely suitable with researchers' purposes can be more possible and easier than other video preparation process. As a result, researchers have opportunities to select necessary video cases among a rich collection of specially-designed selected-edited events related to students' mathematical thinking.

2.6 Social Constructivist Theory

Constructivism makes emphasis on the idea that learners construct their own learning via engaging mathematical practices mostly by the way of social interaction (Cobb, Yackel, & Wood, 1992). According to this theory learners have active roles in any learning environment and they build mathematical knowledge based on their existing knowledge, experiences, and beliefs. Thus, the main idea in constructivism may be related to the storage of knowledge in learners' mind because learners do not store given information as separate pieces. Instead, they try to understand knowledge by developing arguments and establishing a connection between them in order to internalize obtained knowledge (Perkins, 1991).

In the literature, two types of constructivism were mentioned as radical constructivism and social constructivism (Karagigorgi & Symeou, 2005). Radical constructivists claim that the process of knowledge construction is dependent on the individual's interpretations as being isolated from social context. On the other hand, social constructivists see construction of knowledge is not solely depend on individuals' subjective interpretations. This knowledge construction is also socially situated and it grows out social interaction with others (Tobin & Tippins, 1993). In the current study, because I adapted social constructivism assumptions while producing a video case-based learning environment, I found necessary and useful to mention some details about social constructivist theory in the following.

Social constructivist theory was emerged based on Vygotsky's social and cultural perspective and Piaget's cognitive constructivist perspective (see Piaget & Inhelder, 1969). While Piaget see knowledge as the mental organization of the learner's individual experience, Vygotsky (1978) considered knowledge as a social and cultural entity. More specifically, Piaget also acknowledged the role of social interaction on learning. In this regard, he stated "…individual would not come to organize his operations in a coherent whole if he did not engage in thought exchanges and cooperation with others…" (p.174). In this regard, both Vygotsky and Piaget emphasized the role of social interaction on learners' cognitive change and intellectual development. However, there are some differences between Piaget's and Vygotsky's view in terms of the role of social exchange on learner's cognition. While Vygotsky focused on social interaction between more capable peer and learners, Piaget (1965) mainly see social relationship between equal peers. This difference made contributions to the current study because I combined two perspectives when examining prospective teachers' knowledge development in both individual process and in group discussion process between participants having different perspective on a mathematical issue.

As a strong argument, Vygotsky (1978) proposed that "learning awakens a variety of internal development process that are able to operate only when the …[learner] is interacting with people in the environment and with his peers" (p.90). In addition to the importance of social interaction on learning, considering the role of individual's own experience, ability, and knowledge on learning, Vygotsky (1978) proposed the construct of zone of proximal development (ZPD). Accordingly, he stated that there are two developmental levels in ZPD. This argument if often supported by the following explanations:

> Any function of the child's development appears twice, or on two planes. First it appears on the social plane and then on the psychological plane. First it appears between people as an inter-psychological category and then within the child as an intra-psychological category (Vygotsky, 1978, p.63).

While one is related to what a learner individually can perform, the second level identifies what this learner can do by the help of support, which indicated that there is a zone between these two developmental levels. In this sense, Vygotsky (1978) describes ZPD as the following:

> The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers (p.86).

From this point of view, ZPD provides learners interaction to support their capacity in order to reconstruct mathematical concepts through modifications in a constructivist learning environment (Steffe, 1991). Thus, social constructivist theory emphasizes that learning is defined not to be solely individual process, but also a social construct produced in social discourse (e.g. Pitsoe, 2007) because learners have a chance to actively participate to the learning environment with the teacher and their peers by using their existing knowledge in order to construct new knowledge. In the classrooms or small groups, learners can bring their own perspectives to the learning context. Thus, social interaction allows the presence of multiple perspectives on the content (Duffy & Cunningham, 1996; Schreiber & Valle, 2013). On the other hand, from the mathematics education perspective, Ernest sees learning as the construction of knowledge through socially situated conversation. Moreover, he puts emphasis on the necessity of the knowledge construction through active participation and learners' interactions. In this sense, Ernest's philosophy provides an approach to mathematics education in terms of social constructivist perspective. In summary, learning is active, contextual, and social in social constructivist theory. In another mathematics education perspective, Stephan, Bowers, Cobb, & Gravemeijer (2003) also emphasize that learning is a process involving both individual and social aspects. In this perspective, there is no primacy over individual process or social process. They called social constructivism as emergent perspective. In this regard, they strongly argued that there is a strong connection among individual and social learning processes and this processes cannot be thought as a separate components from learning development because their existences depends on the existence of each other. As a result of this assumption, researchers assumed that the emergent perspective takes account of learners' individual mathematical development as they participate in practices that are carried out in a social and cultural environment such as classrooms (Cobb, 2000; Yackel & Cobb, 1995). In the light of related literature, critical features of a social constructivist approach were formulated in many studies (e.g. Beck & Kosnik, 2006; Hang, Meijer, Bulte, & Pilot, 2015) as in the following: learning is social; knowledge is experience-based and constructed by learners; all aspects (e.g. attitudes, emotions) of a learner are connected; and learning communities should be inclusive and equitable.

As compatible with the nature of social constructivist theory, case-based instruction also provides an environment where the learners actively participate to class or group discussion (Mayo, 2002). By this way, in such kinds of environments, not only the learner get opportunity to construct knowledge individually via own experiences, but also to enhance his/her learning through social interaction and reflection while interpreting and discussing cases (Mayo, 2002). As a result, it is suggested that case-study method can be utilized to facilitate the development of reflective thinking and deepening learners' conceptual understanding. In this regard, the findings of Mayo's (2004) study indicated that learners were able to find solutions to problems together and reconstruct their existing knowledge by the help of social interaction in case-based settings.

In the current study, social-constructivist approach was adapted due to some reasons. To be clarify and justify the reasons why I utilized social-constructivist theory when prospective teachers attended to a teaching experiment designed within video case-based learning environment, it is necessary to mention other theoretical perspectives used commonly in case-based teacher education. In this regard, I summarized some notable characteristics of "*situated perspective of learning theory*", and "*noticing theory*". Then, I explained the reasons why social constructivist theoretical approach was preferred in the current study at the end of following part.

2.7 Other Theoretical Approaches Used in Case-Based Teacher Education

2.7.1 Situated perspective on cognition and learning

Situated learning was defined as: 'the notion of learning knowledge and skills in contexts that reflect the way the knowledge will be useful in real life' (Collins, 1988, p.2). Furthermore, apprentice observing community of practice is admitted as a critical characteristic feature of situated learning (Herrington & Oliver, 2000). Lave and Wenger (1991) proposed situated perspective in which learning occurs through interaction and participation in a particular community of practice situated in authentic learning environments such as teachers' own classrooms (Putnam & Borko, 2000). According to this theory, social relationship prepares an environment in which learning occurs (Greeno, 1997) because the learners move from the periphery with the role of observer to the center of the community with the role of fully participant. This social participation is named as “legitimate peripheral participation” (Lave & Wenger, 1991). Legitimate peripheral participation enables the participant to become a member in the culture of a group via a community of practice. In this sense, Lave and Wenger (1991) mentioned that ‘to be able to participate in a legitimately peripheral way entails that newcomers have broad access to arenas of mature practice’ (p.110). By the help of situated perspective, teachers are able to adapt their knowledge to the different situational contexts from the situation they are currently learned (Shulman, J., 1992), which give opportunity them to think flexibly (Lundeberg et al., 1999) and to explore the context domain from alternative points (Merseth, 1996; Van den Berg & Visscher- Voerman, 2000) within case-based instructional studies. As a result, using a Situative perspective in educational settings contributes to the emergence of strong professional learning communities that can foster the development of professional knowledge and improvement of practice (Little, 2002).

There are several studies utilizing situated learninig theory in case-based pedagogy of teacher education (Abell, & Cennamo, 2004; Doerr & Thompson, 2004; Herrington & Oliver, 2000; Leinhardt, 1990; Putnam & Borko, 2000). Among them, Herrington and Oliver (2000) asserted critical elements of situated learning as in the following for the researchers who want to design a learning environment in a multimedia program based on situated learning theory and to explore students’ perceptions of learning environment in more depth. These critical elements are: providing authentic context reflecting the way the knowledge will be used in real-life; providing access to expert performances and the modelling of processes; providing multiple roles and perspectives; supporting collaborative construction of knowledge; promoting reflection and articulation; providing coaching and scaffolding; and providing for authentic assessment of learning within the tasks. Their case-based study conducted in multimedia learning environment revealed that prospective teachers collaboratively learned to teach through group discussions and reflective thinking (Herrington & Oliver, 2000).

2.7.2 Noticing theory

Noticing theory provides information about how teachers notice classroom interactions. In this theory, the development of teacher noticing is examined by serving classroom situations like “cases” to teachers (van Es & Sherin, 2002). In other words, noticing framework expects teachers to be able to establish connections between teachers’ knowledge to broader principles of teaching and learning by transferring their knowledge to different situations, which is compatible with the use of cases in teacher education. In this sense, researchers found meaningful to combine case-based pedagogy and noticing theory in order to develop teachers’ professional vision. From this point of view, the nature and functions of noticing theory in teacher education were mentioned in the following.

Noticing is a natural part of everyday life. However, noticing in professional or intentional meaning is different from everyday noticing (Mason, 2002) because professional noticing enables people in a profession to realize complex situations in particular ways (Jacobs, Lamb, & Philipp, 2010). In this regard, many of researchers who are studying in mathematics teacher education have focused on “teacher noticing” as a new theoretical construct in order to train well-qualified teachers having necessary skills to manage complex classroom environment and to increase students’ learning (Borko, Jacobs, Eiteljorg, & Pittman, 2008; Borko, Virmani, Khachatryan & Mangram, 2015; Goldsmith & Seago, 2011; Jacobs, Lamb, & Philipp, 2010; Sherin & van Es, 2009; Star & Strickland, 2008; van Es & Sherin, 2002; van Es, 2012a; 2012b). After researchers concentrated on noticing in teacher education, different conceptualizations about teacher noticing have been proposed in the literature. Among them, van Es and Sherin (2002) developed learning to notice framework to describe how teachers notice classroom interactions in video-cases by using a software program. They offered three key aspects of teacher noticing that are;

i. identifying what is important or noteworthy about a classroom situation,
ii. making connections between the specific events and the broader principles of teaching and learning,

iii. using what one knows about the context to reason about classroom interactions (van Es & Sherin, 2002, p.573).

The first aspect of the above conceptualization focused on how teacher identify noteworthy event in a particular situation of complex classroom environment. It is difficult to attend all aspect of a teaching situation in a video clip such as interaction among student or students-teacher, classroom management, student's mathematical understanding, and teacher's strategies or instructional ways. Instead, teachers must select what they will attend or respond to throughout the lesson (van Es & Sherin, 2002, p.573). The second feature of noticing theory emphasizes the ability of making connection between specific events and broader principles rather than solely describing a situation. The last characteristic of noticing is related to what one knows about the context in order to reason and interpret noteworthy events. It was evident form the research (e.g. Chi, Glaser, & Farr, 1988) that as individual become more experienced in a particular domain or context, they began more adaptable at interpreting the situations they encounter in the particular domain.

Sherin (2007) identified professional vision based on the Goodwin's words (1994) that professional vision involves "ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p.606). In line with this description, Sherin (2007) identified professional vision as the combination of two processes of "selective attention" and "knowledge-based reasoning". Selective attention is related to how the teacher makes decision about where to pay attention on a given moment because classroom context are very complex. In this sense, she grouped selection attention into two main categories as Actor (e.g. teacher, student, and other) and Topic (e.g. management, climate, pedagogy and math thinking). On the other hand, she proposed two main dimensions for the component of knowledge-based reasoning. These two main dimensions are Stance (e.g. describe, evaluate, and interpret) and Strategy used to explore student math thinking (e.g. restating student ideas, investigating meaning of student idea, generalizing and synthesizing across student ideas). According to her, selective attention and knowledge-based reasoning interact in a dynamic manner.

Many of researchers concentrated on knowledge-based reasoning proposed by Sherin (2007) in their studies in order to examine the nature of teachers' noticing related to knowledge-based reasoning (e.g. Baş, 2013; Borko et al., 2015; van Es & Sherin, 2006, 2008a, 2008b). For instance, van Es and Sherin (2006) considered Stance dimension; involving describe, evaluate and interpret; that teachers utilize to examine practice in classroom videos. According to the framework, *Describe* refers to statements that recounted the events that unfolded in the clip. *Evaluate* refers to statements that were judgmental in nature, in which the teachers commented on what was good or bad or could or should have been done differently. *Interpret* refers to statements in which the teachers made inferences about what they noticed, with the intent of explaining what happened and why. They distinguished the types of comments related to student math thinking as *Level 1*: identify statements made by students, *Level 2*: Analyze the meaning of student ideas, and *Level 3*: generalization & synthesis of student ideas.

As another prominent study, Jacobs, Lamb, and Philipp (2010) focused on teacher noticing from a more specific aspect. Namely, they concentrated on "*professional noticing of children's mathematical thinking*" in the context of whole-number operations. Instead of focusing on various dimensions of noticing, they particularly searched how teachers notice mathematical ideas that students raise. In a cross-sectional study, they collected their data from 131 prospective elementary school teachers and experienced K-3 teachers differing in experience years by using two video clips involving children's strategies in problem solving processes as the main data collection tool. In the light of the obtained results, they provided three interrelated skills for professional noticing of children's mathematical thinking such as "(i) attending to children's strategies; (ii) interpreting children's understandings; and (iii) deciding how to respond on the basis of children's understandings" (p.173). The first skill is related to how teachers pat attention to mathematically noteworthy things in details of children's strategies. Second skill refers how teachers construct an understanding on children's mathematical thinking. The final skill is mainly related to teachers reasoning ways when responding to children's mathematical understandings (e.g. kinds of potential instructional strategies and responses). Their results revealed that teachers had difficulties in all three interrelated skills of professional noticing of children's mathematical thinking. As a result, they concluded that teachers' professional noticing is a complex and challenging issue as being parallel with the complexity of students' ideas.

Another important framework was developed by van Es (2011) in order to examine how and in which degree seven fourth and fifth grade elementary school teachers learn to notice student mathematical thinking by using a video-cases involving the excerpts of classroom. She proposed a framework in which she mentioned the degree of noticing in terms of four levels as follows: baseline noticing, mixed noticing, focused noticing, and extended noticing. Considering these levels, she also divided noticing two central dimensions such as "*What teachers notice"* that is similar "*selective attention*" component in Sherin's study (2007) and *"How teachers notice"* that resembles with "*knowledge-based reasoning*" in Sherin's study (2007). She explained all dimensions of the framework for learning to notice student mathematical thinking as in Table 2.

In a similar vein, by modifying van Es (2011)'s learning to notice student thinking framework, Borko, Virmani, Khachatryan and Mangram (2015) used a framework when analyzing teachers' video discussions. They analyzed teachers' conversations in two dimensions of noticing as what teachers notice and how teachers notice. In this framework, "*What teachers notice*" refers to the topics and subjects the teachers attended to when discussing the video. On the other hand, "*How teacher notice*" refers to the ways in which teachers reasoned and analyzed what they observed (Borko et al., 2015, p.98). Specifically, they used a similar way that van Es (2011) did in order to capture the depth of teachers' analysis in video discussion process. They used the following framework: Level 1: Conversations in which teachers described or evaluated events in the video with little evidence to support analysis (Code: describe/evaluate); Level 2: Conversations in which teachers made interpretive and analytic comments about the events in the video clip (code: interpret/analyze); Level 3: Conversations in which teachers either generalized events to principles of teaching and learning or proposed alternative pedagogical solutions (code: generalize/propose alternatives).

Table 2. Framework for learning to notice student mathematical thinking (van Es, 2011, p.139)

	Level 1 Baseline	Level 2 Mixed	Level 3 Focused	Level 4 Extended
What teachers notice	Attend to whole class environment, behavior, and learning, and to teacher pedagogy	Primarily attend to teacher pedagogy Begin to attend to particular students' mathematical thinking and behaviors	Attend to particular students' mathematical thinking	Attend to the relationship between particular students' mathematical thinking and between teaching strategies and student mathematical thinking
How teachers notice	From general impression of what occurred Provide descriptive and evaluative comments Provide little or no evidence to support analysis	From general impression and highlight noteworthy events Provide primarily evaluative with some interpretive comments Begin to refer to specific events and interactions as evidence	Highlight noteworthy events Provide interpretive comments Refer to specific events and interactions as evidence Elaborate on events and interactions	Refer to specific events and interactions as evidence Elaborate on events and interactions Make connections between events and principles of teaching and learning On the basis of interpretations, propose alternative pedagogical solutions

2.7.3 Theoretical perspective utilized in the current video case-based study

In this part, the information about which theoretical perspective used in the current study will be given with the reasons. In this study, it was utilized micro-case video clips involving a student's mathematical thinking in a particular concept rather than classroom videos involving a complex classroom environment. From this perspective, aforementioned frameworks and studies based on situated learning theory and noticing theory were prepared to assess how teachers analyze "classroom situations”. For this reason, noticing theory is not completely consistent with the structure of micro-case videos. Classroom video-cases includes multi-dimensional structure such as pedagogy, climate, management, mathematical thinking. Yet, micro-case videos, special production video clips, involve single student’s mathematical thinking and a researcher who is within observer and questioner role. By this way, micro-case video clips enables teacher to start to analyze videos with a focused attention because they need to concentrate only student’s actions, responses, drawings in the clips rather than interaction between student and teacher, management issues.

This study involves a group of prospective middle school students who are responsible for reflect, discuss, and share their ideas about students’ mathematical thinking in a social learning environment by the guidance of researcher. Moreover, the main focus is to understand the developmental process of prospective teachers’ subject matter knowledge and pedagogical content knowledge about a particular mathematic subject (quadrilaterals). In other words, it is vital to understand how they construct or reconstruct mathematical concepts or students’ conceptions. Thus, there is constructivist and concept-based approach in the current study. From this perspective, utilizing social constructivist theory was found more suitable and reasonable than other theories.

CHAPTER III

METHODOLOGY

The purpose this study is to understand the nature and development of middle school mathematics teachers' knowledge for teaching of quadrilaterals throughout a classroom teaching experiment designed within video case-based learning environment. In accordance with this purpose, I firstly mentioned design of the current study, reasons why I preferred classroom teaching experiment methodology. Throughout this chapter, I also gave information about the context and participants, data sources, planning procedures of teaching experiment, implementation procedures of teaching experiment, data analysis procedures, trustworthiness of the study, and (de)limitations of the study.

3.1 Research Design: Teaching Experiment Methodology

In this study, classroom teaching experiment methodology (Cobb, 2000) was utilized in order to examine the nature and development of middle school mathematics teachers' knowledge about quadrilaterals. From this point of view, brief information about the nature and characteristics of teaching experiment was given because this information was necessary to understand the reasons why this method was preferred in the current study.

The primary aim of constructivist teaching experiments for researchers is to provide explanations of students' mathematical conceptions, reasoning processes and changes in them (Steffe & Thompson, 2000). Similarly, Yackel, Gravemeijer, and Sfard (2011) also stated that "the primary goal when conducting a constructivist teaching experiment is to gain insight into the development of students' mathematical reasoning" (p.12). In other words, teaching experiment study not only aims to identify the beginning and ending situation of the learners' conceptions or knowledge, but also it examines how learners progress throughout the experiment by indicating the ways learners use to restructure, change and organize their existing knowledge (Steffe & Thompson, 2000). Thus, throughout the teaching experiment process it is possible to observe learners when they work on mathematical tasks and to make inferences about how they restructure specific mathematical concepts in terms of investigators (von Glasersfeld, 1995). On the other hand, classroom teaching experiment is a natural extension of constructivist (one-to-one) teaching experiment methodology. In the case of classroom teaching experiment, learners restructure their knowledge by interacting with the teacher and their peers rather than with only the teacher in a social context. More specifically, researchers also emphasize that learning is a process involving both individual and social aspects (Cobb, 2000; Stephan, Bowers, Cobb, & Gravemeijer, 2003; Yackel & Cobb, 1995). In this perspective, there is no primacy over individual process or social process. From this point of view, classroom teaching experiment methodology was found suitable with the aim of the current research because the aim of this study is to examine how prospective middle school mathematics teachers develop their knowledge about quadrilaterals throughout a video-based professional development program in a social constructivist environment. Consequently, a classroom teaching experiment methodology was carried out in order to examine PSTs' knowledge development processes.

A teaching experiment consists of a sequence of teaching episodes (Steffe, 1983). More specifically, a teaching episode involves following elements: a teacher/researcher, one or more students, a witness of teaching episodes, and a method of recording what transpires during the episodes (Steffe & Thompson, 2000). Furthermore, before conducting a teaching experiment, the researcher should fulfill some requirements such as identifying (i) a learning objective for the participants, (ii) existing research on the related mathematical topic, and (iii) participants' readiness. In this regard, I primarily examined existing research on students' and teachers' conceptions on quadrilaterals. This examination gave me opportunities to understand some problematic issues related to both students' and teachers' knowledge and it enabled me to determine learning objectives and the general structure of teaching episodes in this study. Finally, I determined participants' readiness before conducting a teaching experiment (*Note*: Further details were explained in preparation procedures of teaching experiment part).

In a teaching experiment "the researcher acts as a teacher" (Steffe, 1991, p.177). The researcher/teacher put aside her conceptions and did not insist that prospective teachers learn what she knows (Norton & D'Ambrosio, 2008; Steffe, 1983) in order to explore learners' knowledge development on related mathematical subject. The primary goal of teaching episodes is to understand how the learners [re]construct knowledge and produce ways to make explicit their processes in a social interaction process. In this sense, the researcher/teacher adopts two crucial roles such as (a) asking critical essential questions and providing situations in which learners can actively participate and learn, and (b) analyzing how learning occurs in teaching episodes (Steffe, 1991). From this point of view, I was both the teacher and researcher in this study by adopting aforementioned two crucial roles.

Another important characteristic of teaching experiment is that it requires long-term interaction from 6-weeks to 2 years with the learners (Yackel, Gravemeijer, & Sfard, 2011). Moreover, it includes a dynamic passage from one state of knowledge to another. In other words, it gives information about both what students do and how they do. Considering objectives in this study and the nature of teaching experiment, I interacted with the participants about eight weeks. Furthermore, I continued to communicate them until the semester ended.

Another characteristic is that qualitative data is generally obtained in teaching experiment rather than quantitative data due to the nature of teaching experiment including a huge data set coming from the sequences of teaching episodes and clinical interviews (Cobb & Steffe, 2011). Concordantly, I fully obtained qualitative data by using multiple data sources such as individual clinical pre- and post-interviews, group discussions, initial and revised lesson plans, reflection papers, field notes.

3.2 Context of the Study

From the broader perspective, this study is related to prospective middle school mathematics teacher education program. For this reason, I firstly explained broad context of the study in this part. For this study, I and my supervisor opened an elective course for fourth year prospective teachers considering the purposes and research questions of the current study. In the following, I provided information about context and participants of the study.

3.2.1 Broad context of the study

The context of the study is the undergraduate middle school mathematics teacher education program, which is a four-year undergraduate program. This is one of the major teacher education programs in Ankara, Turkey. In this education program, the means of instruction is English. Graduates of The Elementary Mathematics Teacher Education (EME) program are qualified to teach mathematics in middle schools, grades from 5 to 8 (ages 10–14) in Turkey. The program offers content (mathematics, physics, and statistics) courses, education sciences courses, and elementary mathematics education courses. Prospective teachers mostly take mathematics courses in the first 2 years. In the following semesters, they began to take courses such as methods of teaching mathematics, school experience, and practice teaching. The undergraduate curriculum for the program is represented in Appendix 9.

Prospective teachers have opportunities to learn how they can effectively design the teaching and learning process of mathematics during their mathematics teaching methods courses and practice teaching. The mathematics teaching methods courses are offered in their third year. Each of mathematics topics were covered in 5 class hours according to the course book by Van de Walle, Karp, Karp, & Bay-William (2013) in mathematics teaching methods course in order to guide prospective teachers in their thinking process. Related with the topic of this study, quadrilaterals, specifically geometric concepts for middle school grade levels is one of the mathematical contents that the method course entails. When dealing with each mathematical topic in the course book, including quadrilaterals, instructor of the course supports prospective teachers within the context of discussing content knowledge and pedagogical content knowledge related to mathematical topics. Moreover, prospective take two practice teaching courses in their last year. In the first school experience course, prospective teachers solely observe students and teacher in natural classroom environment without making active teaching for 14 weeks. On the other hand, they have opportunities both observe and make teaching practices in the second school experience and teaching practice courses.

In sum, there is no course in the context of the undergraduate middle school mathematics teacher education program to directly and closely observe and examine middle school students' mathematical thinking. This absence in the current prospective teacher education program creates the necessity of courses in which prospective teachers closely examine students' mathematical thinking on specific mathematical concepts. Considering this absence and purposes of the current study, I and my supervisor decided to open an elective course named "Projects in elementary science and mathematics education" at the fall semester of the 2014-2015 academic years in an undergraduate mathematics teacher education program of a state university in Ankara, Turkey. I provided details of the general structure of this elective course and participants who took the elective course in the following.

3.2.2 The context of "Projects in elementary science and mathematics education" course

For this study, an undergraduate course as an elective course with the name of "Projects in elementary science and mathematics education" was offered to fourth year prospective teachers in the fall semester of the 2014-2015 academic years. In the catalogue description, the course of "Projects in elementary science and mathematics education" is explained as a project-based course designed to help prospective teachers to work on a theoretical or practical needs related to elementary (science or) mathematics education by investigating of current research studies in elementary (science and) mathematics education and their applications in classroom settings. To the extent practical, students are expected to develop projects related to their own primary area of study and give a project report. As a specific course description, this course provides a unique chance to students to develop a research project in the area of mathematics education and gain an experience on designing and conducting a research to understand students' understanding of mathematical concepts in the middle schools.

Considering the general catalogue description and specific course description, we prepared the structure of the course including two main phases. In the first phase of the course, prospective teachers analyzed middle school students' conceptual knowledge and thinking styles about quadrilaterals through the medium of micro-case video clips that were prepared by the researchers. Furthermore, prospective teachers prepared their lesson plans in this direction.

In the second phase of the course, prospective teachers had conducted an independent study towards understanding students' conceptual structures in-depth by approaching a mathematical concept or subject that they chose within the scope of their research questions. More specifically, following steps were carried out in second phase of the course: (1) Selection of the concept/subjects on which to be studied, sharing them in the classroom and taking feedbacks from their peers and researchers, (2) preparation of the questions to be asked to the students and preparation of substructure of the research, sharing them in the lesson and taking feedbacks from their peers and researchers, (3) conducting interviews with the students in middle schools and sharing preliminary perceptions in the classroom, (4) writing the reports and sharing data analysis processes and the results of their studies in the classroom.

Consequently, in the first phase of the course, I aimed to examine how senior class prospective middle school (grade 5-8) mathematics teachers can develop their SMK and PCK about quadrilaterals within a video case-based approach by using the teaching experiment methodology. For this purpose, I used the data obtained in the first phase of the course in order to investigate developments in PSTs' knowledge about quadrilaterals. More specifically, I gave all details about data collection tools and the structure of the first phase of the course after I introduced information about the participants of the study.

3.3 Participants of the Study

As the students to the course, we preferred to admit fourth year prospective teachers who completed pure mathematical courses some of their required educational courses that are Methods of Teaching Mathematics I, Methods of Teaching Mathematics II, and School Experience. As a result, eight senior female prospective middle school mathematics teachers took the elective course. Thus, the study was carried out by eight senior students attending Elementary Mathematics Teacher Education program in a public university in Ankara, Turkey.

Because quadrilaterals is the subject chosen to examine prospective teachers' knowledge development, it was found useful to give participants' grades of some educational courses and pure mathematical courses that mostly involve content about learning and teaching of geometry. In this regard, the information about participants' academic background is asserted in Table 3. According to the table, prospective teachers completed the required courses of Analytic Geometry, Elementary Geometry, Method of Teaching Mathematics I, and Method of Teaching Mathematics II. Furthermore, some participants (Beril, Zehra, Ece, and Emel) took also Teaching of Geometric Concept as an elective course.

Their calculated cumulative grade points (Cum-GPA) indicated that their points were between 2.58 and 3.88. More specifically, two participants' Cum-GPAs were above 3.50 out of 4.00 three PSTs' Cum-GPAs were between 3.00 and 3.50, and three PSTs' Cum-GPAs were between 2.00 and 2.50. The distribution of PSTs' Cum-GPAs indicates that the participants in the teaching experiment consisted of a variety of PSTs having different academic achievement level.

Table 3. Participants' academic backgrounds

Pseudonyms of the participants	Courses involving learning and teaching of geometric concepts					
	Analytic geometry	Elementary geometry	Method of teach math I	Method of teach math II	Teach of geo concepts*	Cum-GPA**
Aslı	DD	CC	CC	BB	X	2,58
Deniz	CC	CC	CC	CB	X	2,81
Beril	DD	CB	BB	AA	BB	2,90
Oya	CB	CC	CB	BA	X	3,08
Zehra	BB	BA	DC	BA	BB	3,01
Ece	BB	BB	BB	AA	BB	3,18
Maya	AA	BA	AA	BA	X	3,63
Emel	AA	AA	AA	AA	AA	3,88

*X means that the student did not take the course of "*teaching of geometric concepts*".
** Cum GPA indicates cumulative grade points of all taken courses in all semesters out of 4. The coefficient of the grades corresponds to DD-1, DC-1.5, CC-2; CB-2.5, BB-3, BA-3.5, and AA-4.

3.4 Data Collection Tools

The main data sources in the current study were individual clinical pre- and post-interviews, PSTs' initial and revised lesson plans, reflection papers, group discussions, and field notes. The functions and involvement of each data source were explained in the following. I mentioned all details how I used each data collection tool in the sections of preparation and implementation procedures of teaching experiment (see section 3.5 and section 3.6).

3.4.1 Individual clinical pre- and post-interviews and tasks

Clinical interview has been used as technique in the teaching experiment studies after Jean Piaget's studies on child knowledge development in 1975s because he proposed that observation and standardized tests are not enough to obtain detailed information about a child's cognitive processes (Ginsburg, 1997; Opper, 1977). Furthermore, clinical interviews give opportunity to "enter the students' mind" considering individual difference and their mathematical understanding (Clement, 2000; Hazzan & Zazkis, 1999; Koichu & Harel, 2007; Newel & Simon, 1972). In this study, individual clinical pre- and post-interviews were conducted two purposes as (i) to prepare of micro-case videos, and (ii) to understand prospective teachers' initial and final state of subject matter knowledge and pedagogical content knowledge related to quadrilaterals. In this regard, I prepared a data collection tool involving various tasks covered with fully open-ended questions. I utilized same tasks in both pre-interviews and post-interviews. I explained preparation of tasks and purposes of each task in clinical interviews in the following.

3.4.1.1 Preparation of individual clinical interview tasks

In order to prepare individual clinical pre- and post-interview tasks, I examined some questionnaires about quadrilaterals in the literature (Fujita, 2012; Nakahara, 1995; Okazaki, 1995; Öztoprakçı, 2014) and geometry standards in instructional programs (Common Core State Standards Initiative [CCSSI], 2010; Ministry of National Education [MoNE], 2013; National Council of Teachers of Mathematics [NCTM], 2000). In this regard, I decided to prepare questions considering five main themes: *definitions*, *constructions*, *identifications*, *properties*, and *classifications* of quadrilaterals as seen in Table 4. I determined these five main themes considering 6-8 grades in geometry standards in specifically NCTM (2000) and MoNE (2013). In NCTM (2000), instructional programs from prekindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. More specifically, in geometry learning domain, grade 6-8 expectations are illustrated as following: In grades 6–8 all students should

- precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties;
- understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects;

- create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship (NCTM, 2000, p.232).

Above standards indicates that having knowledge on definitions, classification, and properties of quadrilaterals are very crucial to understand quadrilaterals. Considering these important themes, I prepared "*definition questions*", "*properties questions*" and "*classification questions*" (see Table 4). Furthermore, examination of relationship between learners' concept images and concept definitions are seen important to understand their acquisition of geometric concepts. From this point of view, I also used "*constructions questions*". Finally, I prepared "*identification questions*" to understand PSTs' identification of geometric figures since many of researchers have prepared specific mathematical tasks involving both examples and non-examples in order to examine how learners identify examples of geometric concepts such as triangles (Burger & Shaughnessy, 1986; Tsamir et al., 2008), the altitude of triangles (Gutiérrez & Jaime, 1999), quadrilaterals (Clements & Battista, 1991), square (Razel & Eylon, 1991; Zazkis & Leikin, 2008), parallelogram (Fujita, 2012; Petty & Jansson, 1987), trapezoid (Ulusoy, 2015), and circles and prism (Razel & Eylon, 1991; Tsamir et al., 2015). The results of these studies indicate that asking students to identify examples of a concept among a set of examples and non-examples can give information about students' reasoning about specific mathematical concepts.

In general manner, Table 4 explains what kinds of questions in tasks (see Appendix 1) I utilized in order to learn PSTs' initial personal, instructional and anticipative knowledge on definitions, constructions, selections, properties, and classifications of quadrilaterals. Because I want to examine PSTs' existing knowledge about quadrilaterals, it was important to prepare the tasks that clearly reveal PSTs' knowledge about middle school students' possible conceptions, misconceptions, and difficulties in addition to their SMK. I explained further details about questions in the following.

Table 4. Types of questions in the tasks of individual clinical pre-/post-interviews

Question types*	# of questions	Types of questions	Related questions
Definition questions	7	Open-ended questions about PSTs' personal, instructional definitions of quadrilaterals and their anticipations about students' possible definitions/ descriptions.	1-2-3-4-5-6-7
Construction questions	9	Open-ended questions about PSTs' personal, instructional constructions of quadrilaterals and their anticipations about students' possible constructions.	8-9-10-15-16-17-22-23-24
Identification questions	6	Open-ended questions about PSTs' personal selections of quadrilaterals and their anticipations about students' possible selections.	11-12-18-19-25-26
Properties questions	7	Open-ended questions about PSTs' personal and instructional knowledge about the properties of quadrilaterals and their anticipations related to students' possible conceptions about properties of quadrilaterals.	13-14-20-21-27-28-29
Classification questions	2	Questions asking PSTs to represent hierarchical relations among quadrilaterals by a diagram and to present the ways how they teach hierarchical relations to their students.	30-31

* For details of questions, you can examine Appendix 1.

In all tasks, I utilized grid paper because usage of grid/dot paper is strongly emphasized and suggested in the objectives related to the constructions and identifications of two dimensional geometric figures (Ministry of National Education [MoNE], 2013) in revised Turkish curriculum. Furthermore, it is thought that using grid paper can be useful to observe participants' reasoning about critical and non-critical attributes of any geometric figure considering the unit squares in grid paper. Taking into account this recommendation, I prepared all examples and non-examples in the present study by using grid paper.

3.4.1.1.1 Definition questions

There are seven questions related to the definition of the quadrilaterals in the tasks (see Task 1 in Appendix 1). In the three defining questions (questions 1-3-5), participants were asked to personally define parallelogram, rhombus, and trapezoid, respectively. These questions aimed to evaluate PSTs' SMK about definitions of the concepts because the main focus was whether PSTs are defining a geometric concept by listing many redundant properties or by using both necessary and sufficient properties; and whether they were aware of the inclusive relations between geometric shapes were examined. On the other hand, four questions (2-4-6-7) were prepared to evaluate PSTs' PCK about the definitions of quadrilaterals. More specifically, three questions of them were organized to learn how they give instructional definitions of parallelogram, rhombus, and trapezoid. Remaining one question was added in order to get information about PSTs' predictions about students' possible descriptions of quadrilaterals and reasons of possible problems in students' definitions/descriptions of quadrilaterals.

3.4.1.1.2 Construction questions

There are nine questions related to constructions of quadrilaterals (see Task 2, Task 3, Task 4 in Appendix 1). The questions of 8-15-22 in the tasks were prepared to understand what kinds of drawings PSTs will construct when asking them to draw more than three different examples of parallelogram, rhombus, and trapezoid. These questions gave information about PSTs' SMK on their examples spaces about constructions of quadrilaterals. Furthermore, tasks involved the questions of 9-16-23 that aimed to understand PSTs' anticipations about students' possible correct or incorrect constructions of parallelogram, rhombus, and trapezoid, respectively. By these questions, it can be possible to understand whether PSTs are aware of students' overgeneralization and undergeneralization errors, constructional difficulties, or the errors arising from the inadequate knowledge on basic geometric concepts. Finally, I added the questions of 10-17-24 into the tasks in order to understand PSTs' instructional preferences about the constructions of quadrilaterals. These questions were important to understand PSTs' instructional example spaces. Moreover, in all constructions, I aimed to obtain information about PSTs' knowledge in terms of prototypicality and inclusive relations among quadrilaterals.

3.4.1.1.3 Identification questions

In the pre-/post-interviews, identification tasks consisted of six questions (see 11-12-18-19-25-26 in Appendix 1). Question 18 was prepared to ask PSTs to select parallelogram among given different polygons. On the other hand, in question 19, I asked them to say and write what students' possible parallelogram identifications can be. In this regard, I aimed to explore what kinds of figures PSTs admit as an example of parallelogram among different polygons, which gives idea about PSTs' SMK related to the image of parallelogram in their minds. Similar questions were prepared for the concepts of rhombus and trapezoid.

To be clearer, I explained the preparation and involvement of "*parallelogram identification task*" in detail. At the beginning of preparation of parallelogram identification task, I examined all studies in which researchers have prepared specific mathematical tasks involving both examples and non-examples in order to examine how learners identify examples of geometric concepts such as triangles (Burger & Shaughnessy, 1986; Tsamir et al., 2008), the altitude of triangles (Gutiérrez & Jaime, 1999), quadrilaterals (Clements & Battista, 1991; Öztoprakçı, 2014), square (Razel & Eylon, 1991; Zazkis & Leikin, 2008), parallelogram (Fujita, 2012; Petty & Jansson, 1987), trapezoid (Ulusoy, 2015), and circles and prism (Razel & Eylon, 1991; Tsamir et al., 2015). In these studies, researchers generally used prototypical/non-prototypical examples, hierarchical/non-hierarchical examples, and non-examples in identification tasks. In this sense, parallelogram identification task involved 14 quadrilaterals as in Figure 3. More specifically, the task included 10 examples (1-2-4-5-7-9-10-11-13-14) and 4 non-examples (3-6-8-12) of parallelogram.

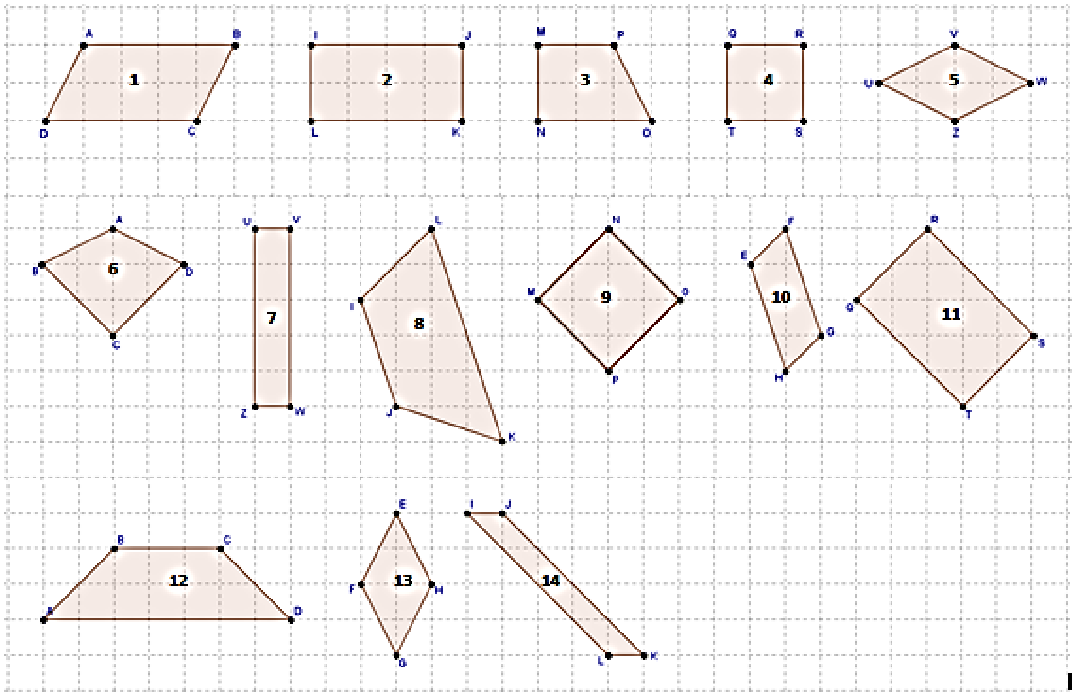


Figure 3. Parallelogram identification task

3.4.1.1.4 Properties questions

There are also seven questions by which I wanted to understand what PSTs know about side, angle, and diagonal properties of parallelogram, rhombus, and trapezoid. While three questions (13-20-27) were prepared to evaluate PSTs' SMK on the properties of quadrilaterals, another three questions (14-21-28) were involved to the task to obtain knowledge what PSTs' predictions about students' possible conceptions related to the side, angle, and diagonal properties of quadrilaterals. On the other hand, question 29 was prepared to learn PSTs' instructional strategies that they can use when teaching the properties of quadrilaterals in their future lessons. Details of items can be examined by visiting the end of the Task 2, Task 3, and Task 4 in Appendix 1.

3.4.1.1.5 Classification questions

At the end of the all questions related to definition, construction, identification, and properties, there were two questions for the classification of quadrilaterals (see Task 5 in Appendix 1). Question 30 was prepared to obtain data how PSTs' represent hierarchical relations of quadrilaterals in a diagrammatic representation. Thus, it can possible to PSTs' ability to transfer their knowledge from one representational type to another one. Question 31 was added to get information about PSTs' instructional preferences that they decide to use either inclusive relations or exclusive relation or partial-inclusive relations of quadrilaterals in their future instructional plans.

In order to construct credibility of all items, independent experts from the Faculty of Education were asked to match the questionnaire items with the related five sections. Moreover, experts checked the format of the instrument in terms of clarity of the language and directions, irrelevant information and physical appearance of the paper. Furthermore, I piloted pre-interview questions with three prospective teachers who were not participants to the current study.

3.4.2 Initial and revised lesson plans

Many of studies conducted with beginning teacher and pre-service teachers indicated that they have inadequate knowledge about curriculum materials and teaching strategies (Ball & Feiman-Nemser, 1988; Grossman & Thompson, 2004; Nicol & Crespo, 2006) because choosing a material or teaching technique requires making critique of their effectiveness and considering the appropriateness to the students' needs. In this regard, lesson plans have a great importance to see PSTs' pedagogical considerations on teaching of quadrilaterals in more realistic and detailed way.

In this study, examination of PSTs' initial lesson plans was considered as an important step in the teaching experiment preparation and implementation processes. In the preparation of teaching experiment, data obtained from PSTs' initial lesson plans were used to select and organize micro-case video clips for teaching experiment sessions. Further, initial lesson plans were also used to get information about PSTs' existing pedagogical knowledge about instructional strategies. On the other hand, in the implementation process of teaching experiment, I used PSTs' lesson plan revisions as an important data source because these revisions and PSTs' reflective notes about the reasons why they needed to make revisions in their lesson plans were crucial to understand the changing and developing SMK and PCK about quadrilaterals throughout the teaching experiment. Specifically, it gave more ideas about prospective teachers' changing pedagogical decisions and approaches to the problematic situations in students' mathematical thinking about quadrilaterals.

3.4.3 Reflection papers

In individual video analysis process, participants were asked to write a reflection paper for each video clip. For this reason, individual reflection papers have crucial importance to understand PSTs' individual perspectives and knowledge related to students' mathematical thinking about quadrilaterals in micro-case video clips. For this purpose, I prepared a guidance involving the general structure of individual video analysis reflection paper. In the reflection paper, prospective teachers were asked to explain what they found interesting in video clip and what their idea about the topic in the video was before analyzing the video case. In the second question of reflection paper requested PSTs to answer following main question and its sub-questions:

What did you notice while watching videos individually?

- Explain student's thinking process in the video case (e.g. procedural/conceptual, misconception/ difficulty/ misunderstanding)
- If students made incorrect answers in her/his explanations/ constructions/ selections what can the reasons of their difficulty/misconceptions be?

- Do the correct explanations/ constructions/ selections of students show that they certainly have complete knowledge about selected mathematical concepts?
- What are the other points that you noticed in the video clips?

Above questions generally focused on how PSTs comprehend students' mathematical thinking in video clip, how they identify problematic situation in student thinking in more detail. They wrote their reflection paper while individually watching each video clip. Some of them preferred to write after finishing individual video analysis. After they completed their individual reflection paper, the researcher collected them and initiated a group discussion after a short break.

PSTs also requested a reflection paper that I called them as "*after discussion reflection papers*" (ADRP) at the end of the two sessions of video analysis and group discussions in each teaching experiment week. More specifically, these reflection papers involved following questions:

- Explain if there was any change in your thinking after the group discussion process? How did discussion environment influence your thinking? (Link between previous knowledge or give some example speeches between you and your friends)
- Propose some recommendations for classroom applications/teaching methods to develop student's mathematical thinking and to overcome their misconceptions/ misunderstandings in the video clips. (*Think as you teach these concepts*…)

In these questions, first question aimed to obtain information about PSTs' changing SMK and PCK after the group discussions of video clips. Second question was prepared to get more information about how PSTs develop pedagogical decisions to overcome students' misconceptions and to enhance their conceptions. PSTs wrote this reflection paper at the end of each teaching session. Consequently, these reflection papers provide information on how PSTs develop their knowledge after the influences of group discussions and interactions. In line with social constructivist approach, I had opportunities to examine the influence of social interactions on PSTs' knowledge developments.

3.4.4 Group discussions

As mentioned before, social-constructivist approach was used in the current study. In this approach, social aspects of learning gains importance in addition to individualistic view of learning. From this point of view, group discussions were used to broaden the prospective teachers' perspectives on noteworthy events in micro-case video clips. As the facilitator of the group discussion, I used framework for the facilitation of video-based discussion that was developed by van Es, Tunney, Goldsmith, and Seago (2014). I mentioned all details related to the facilitation of group discussion in section 3.6.2.

3.4.5 Field notes

Fraenkel and Wallen (2006) described field notes as "the researchers' written account of what they hear, see, experience, and think in the course of collecting and reflecting on their data" (p.516). They also proposed two types of field notes: descriptive and reflective. Descriptive field notes involves everything in the setting such as participants' behaviors and facial expressions, materials and physical appearance of the settings, and particular events during the study, etc. (Bogdan & Biklen, as cited in Fraenkel & Wallen, 2006). On the other hand, reflective field notes involves information about the researcher's personal ideas and comments about what is being observed, such as the problems related to the analysis or design of the study; possible factors that might affect the study; or any kind of conflicts or concerns, etc. (Bogdan & Biklen, as cited in Fraenkel & Wallen, 2006). In the current study, I utilized both descriptive and reflective field notes as data sources in order to provide an assessment and critique on research process. I prepared a paper involving two columns. In the first column, I generally wrote descriptive notes such as date, names of participant(s), participants' gestures and facial expressions in both clinical interviews and group discussions. On the other hand, in the second column, I generally wrote reflective notes such as what participant might think, why they saw an event in video as a noteworthy event etc.

3.5 Planning Procedures of Teaching Experiment

In the planning of teaching experiment, I followed three steps as in Figure 4. I explained all details about each step in the following subsections.

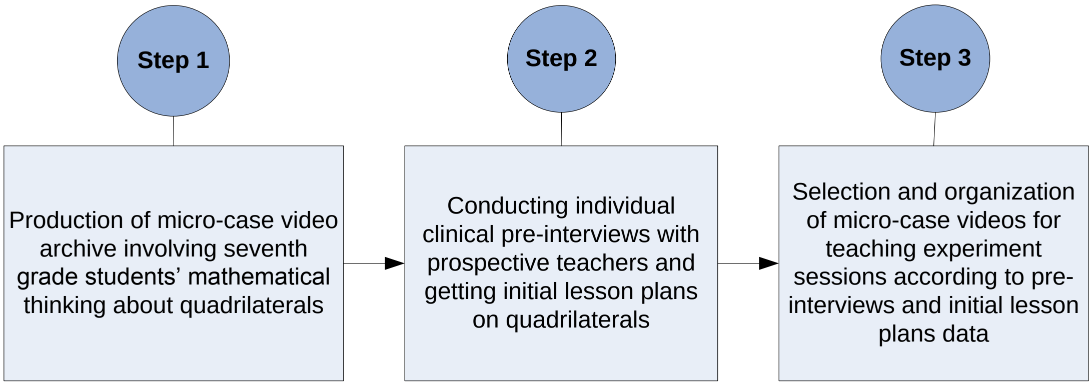


Figure 4. Steps in planning of teaching experiment

3.5.1 Production of micro-case videos

As mentioned before, I defined "*micro-case video*" as a specially-designed educational video that involves a collection of significant events related to an individual's mathematical thinking process on particular mathematical concepts or problem situations when the learner works on structured content-related tasks in an isolated non-classroom learning environment. The main characteristics that are involved by micro-case video clips can be listed as follows: (i) a collection of specially-designed selected-edited events, (ii) a learner's thinking process, (iii) structured content-related tasks or problem situations, and (iv) isolated non-classroom learning environment. In the following, I mentioned about case production process.

3.5.1.1 Case production interviews with seventh grade students

For the production of micro-case video clips, I initially selected a middle school located in the capital city of Turkey considering the easy accessibility to me. I preferred to select seventh grade students in order to conduct "*case production interviews*" because objectives about basic geometric concepts and quadrilaterals are placed mostly in seventh grade geometry learning area in national curriculum (Ministry of National Education [MoNE], 2013). In the school, there were two seventh grade classes with 47 students (21 in 7-A and 26 in 7-B) in total. At the beginning, I had an informal interview with mathematics teachers of the both classes to get information about students' mathematics grades and personal characteristics (e.g. talkative). Furthermore, each class was observed for four hours in order to monitor students' behaviors and each teacher's mathematics instructions. At the end of the observations, I asked students and their families to participation to the study via a consent form (see Appendix 2). 16 seventh grade students in both classes decided to participate to the study. I conducted "*case production interviews*" with 16 seventh grade students aged twelve or thirteen who were enumerated from S1 to S16 as in Table 5.

Their achievement levels were categorized according to their average math grades belonging to the first and second semester. Semester grades were categorized as 5-5 was *high*; 5-4, 4-5 4-4, 3-4 and 4-3 were *moderate*; and 3-3 and lower ones were *low* math achievement. According to semester grades, three students' grades were selected among low level, six were moderate level and seven were high level. Thus, I aimed to obtain a rich video clip bank involving students' mathematical thinking at different achievement levels. I think that the diversity of students' achievement level might give ideas to prospective teachers in terms of comparing students' different thinking styles and conceptions related to same mathematical concept when they examine micro-case video clips in teaching sessions.

Table 5. Demographic information of seventh grade students

Number	Gender	Class	Average note in grade 7		Achievement level
			Math I	Math II	
S1	F	7-B	4	5	Moderate
S2	F	7-B	5	5	High
S3	F	7-A	5	5	High
S4	F	7-A	2	3	Low
S5	F	7-A	5	5	High
S6	F	7-A	4	4	Moderate
S7	F	7-A	4	4	Moderate
S8	F	7-B	5	5	High
S9	F	7-B	3	4	Low
S10	F	7-B	4	4	Moderate
S11	M	7-A	5	5	High
S12	M	7-B	5	5	High
S13	M	7-A	2	3	Low
S14	M	7-A	5	5	High
S15	M	7-B	4	4	Moderate
S16	M	7-A	3	2	Moderate

I conducted all "*case production interviews*" in a suitable room of the school, which is approximately 20 m^2 (see Figure 5).

Figure 5. Case production interview room

In order to videotape student's mathematical thinking, I utilized *single-camera video production* technique and outlined the process of working with one video camera from beginning to end. Using *multiple-camera settings* is more suitable than using *single-camera settings* to produce classroom videos due to the situations in which simultaneous angles might be required to examine multiple aspects of classroom interactions. However, I think that single-camera settings are sufficient to produce micro-case videos because students' drawings and writings may be clearer in single-camera video production throughout close-up shots.

In order to produce video cases, I conducted *case production interviews*. For the interviews, I prepared tasks that are similar to the tasks I implemented to prospective teachers in pre-and post-interviews. Then, I conducted interviews into two sessions with each student. The reason why I conducted two sessions instead of one session depended on time and middle school student's attention. First, I piloted all questions in case production interviews with different achievement level students. After this piloting, I observed that asking all questions in one session might be very long and tiring for middle school students.

In the tasks, there are different questions about definitions, constructions, selections, and properties of quadrilaterals. Moreover, I asked students to construction questions related to basic geometric concepts (e.g. construction of equal length of line segments). In detail, geometric task about prerequisite knowledge on geometry (e.g. construction of two parallel line segments) and parallelogram were asked to the student in the first interview. On the other hand, the task related to rhombus and trapezoid was handled in the second interview. Specifically, information about questions in the tasks (*Note*: see Appendix 3 which involves tasks and questions used in the second case production interview). Questions types and structure of each case production interview are given in subsequent sections.

3.5.1.1.1 Question types in case production interviews

Mathematics educators have stressed the importance of developing students' conceptual understanding, the ways of reasoning, and higher level of problem solving competencies rather than focusing on procedural or short-cut heuristic algorithmic processes (Davis, Maher & Noddings, 1990; Goldin, 1997; von Glasersfeld, 1991). From this point of view, I prepared tasks and questions in case production interviews based on conceptual aspects instead of instrumental aspect. In this sense, in the interviews, I asked different types of questions to seventh grade students as in Table 6: "*performance questions*", "*unexpected why questions*", "*twist questions*", "*construction tasks*" that I called construction question, "*give an example task*" (I called it as "*exemplification questions*") and "*reflection questions*" (Hazzan & Zazkis, 1999).

Table 6. Questions types in case production interviews

Question Types	Example questions
Performance	-Could you define rhombus in your own words? -Can you calculate the measurement of other angles of a parallelogram if the measurement of one angle of this parallelogram is given as 70˚.
Unexpected why	-Why do you think a parallelogram can have more than four sides? -Why do you think that the sum of interior angle of any quadrilateral can be both 180˚ and 360˚?
Twist	-You draw this figure. Now, can you show the diagonals of this figure? -What can you say about the length of diagonals of this figure?
Construction	-Can any parallelogram in which the length of all sides are equal be?
Exemplification	-Could you draw three different parallelogram examples?
Reflection	-Alara thinks that square is also a rhombus. However, Fatih does not think square as an example of rhombus. In this situation, Do you agree Alara or Fatih? How do you convince the student who you disagree?

I asked *performance questions* to reveal students' understanding of a specific concept in quadrilaterals. The main interest was not to evaluate their performance by focusing what participants are doing. Instead, I was interested in how and why they are explaining the concepts rather than their performance. I asked *why questions* in order to reveal or clarify students' mathematical thinking in unexpected places, which allows to me to go beyond understanding the successfully applied algorithms or memorized rules. I utilized *construction tasks* to obtain data how students built mathematical objects which satisfy certain properties. Moreover, I also asked exemplification questions to observe students' concept images about quadrilaterals. Furthermore, I used reflection questions how students provide arguments to justify her/his own thinking and to convince someone.

3.5.1.1.2 The structure of the first case production interview

Prerequisites of quadrilaterals involve the construction of congruent angles, parallel/ perpendicular/ equal length line segments, and knowledge on diagonal. Lack of knowledge on prerequisite concepts may influence on middle school students' conceptions related to quadrilaterals. In order to produce information about how students conceptualize basic prerequisites of quadrilaterals, they were asked to construct equal length line segments, parallel/perpendicular line segments, and congruent angles. For this reason, some constructions were made students in the first case production interviews to get information about their prerequisite knowledge of basic geometric concepts before asking the questions about parallelograms. The structure of the first interview was given in Figure 6.

In the first case production interviews, students used grid paper, ruler, geoboard and colored pencils whenever they wanted when constructing shapes. All interviews were recorded via HD camera and audio recorder. Each interview took approximately 50 minutes. In the first interviews, students firstly were asked to construct equal length of line segments in the grid paper. After they sketched the figures, they explained the reason why their constructions were equal length and how they understand their equality in terms of length. Similarly, they constructed two parallel line segments in the grid paper. The other constructions were made by using similar ways.

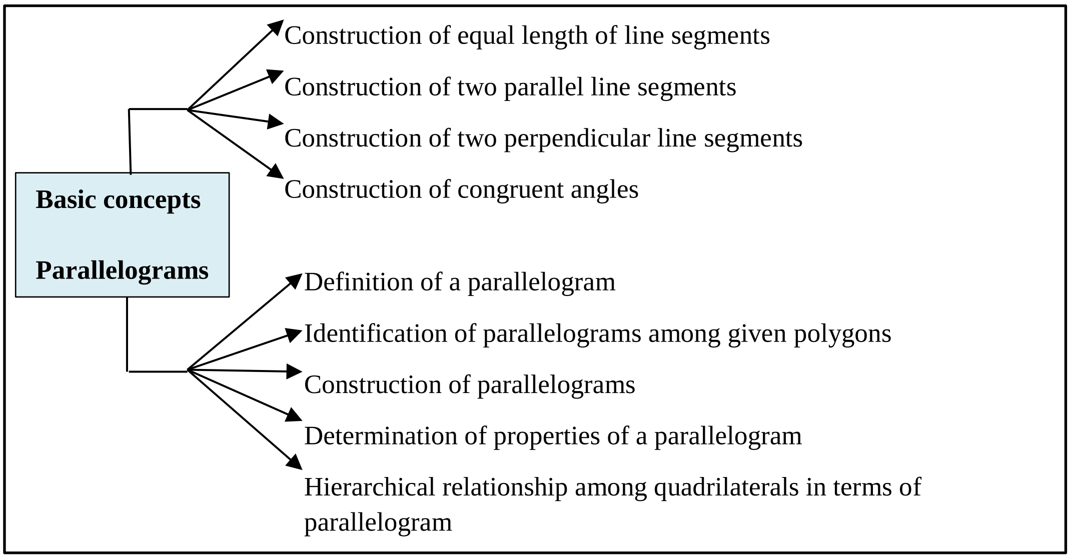


Figure 6. The structure of the first case production interview

3.5.1.1.3 The structure of the second case production interview

Second case production interviews were conducted with same sixteen students, displayed in Table 5, in a different time from the time of first interview. Similarly, students used grid paper, ruler, geoboard and colored pencils whenever they wanted while constructing shapes. Each interview took approximately 30 minutes. Specifically, in these interviews, students were asked to answer the questions about rhombus and trapezoid. The organization of the questions was demonstrated in Figure 7.

In this task, definition/description of rhombus was firstly asked to the students. After describing orally, students wrote down their definition/description of rhombus. Then, I asked them to identify rhombuses among given quadrilaterals. After identification part, they constructed a rhombus in the grid paper by using ruler and determined the properties of rhombus according to sides, angles and diagonals.

Finally, I asked some conditions to understand their ideas about hierarchical relationship among quadrilaterals in terms of rhombus. Trapezoid questions were asked to the students in similar way.

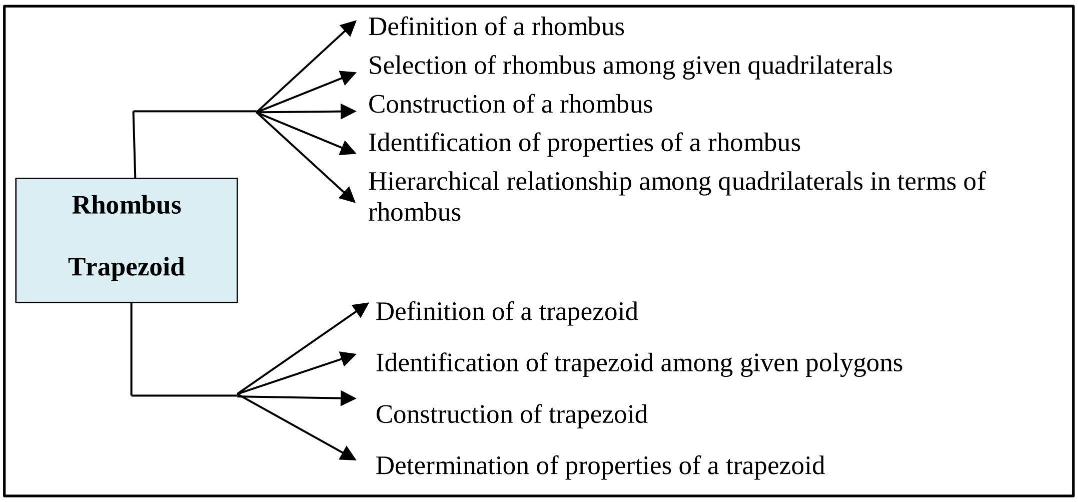


Figure 7. The structure of the second task-based clinical interview

In conclusion, I produced approximately 1000 minutes of raw video data set involving seventh grade students' mathematical thinking about basic geometric concepts and quadrilaterals. At this point, it is necessary to prepare an archive because archiving allows the researchers to determine which segments of the video to examine and to begin to see patterns within and across segments (Barron & Engle, 1998). Moreover, it is useful to decide which segments should be transcribed at what detail level. In the following, I gave information about how I archived case production interview data that were conducted with seventh grade students.

3.5.1.2 Achieving micro-case video clips

I think that transcription of 1000 minutes raw video data is not feasible and reasonable because transcribing all qualitative data set can be both time-consuming and unnecessary. For this reason, archiving/segmentation of raw videos is useful and necessary to cover raw video into meaningful small pieces without completely transcribing case videos in the planning process of a teaching experiment. Thus, these archives function as a "*video database*" that can be used when selecting and organizing appropriate micro-case videos for teaching experiment sessions. I mentioned archiving criteria and strategies in the following sections.

3.5.1.2.1 Archiving criteria of micro-case videos

When archiving videos, I also took into two criteria: (i) length of videos, and (ii) windows, depth, and clarity.

3.5.1.2.1.1 Length of MCVCs

Determining the length of videos is important issue because someone who is not accustomed to examine educational videos may find video cases as slow and boring (Jaworski, 1990). She also mentioned that "it is rarely possible to show more than ten minutes of real time of a lesson before people fidget, or start to exchange comments" (Jaworski, 1990, p.64). Accordingly, in the literature, researchers generally do not use video cases more than ten minutes (e.g. Seago, 2004; Sherin, Linsenmeier & van Es, 2009) because they found long videos ineffective in terms of providing a productive video discussion. More specifically, Seago and Mumme (2002) did not utilize more than six minutes, Colestock and Sherin (2009) used 3-8 minutes video segments, Sherin (2001) utilized average 5 minutes video clips, and van Es (2011) used 7 minutes video segments in her study. Considering recommendation in literature, I generally prepared video segments between 0-10 minutes in the archives.

3.5.1.2.1.2 Windows, Depth, and Clarity

Raw videos involve many of useful and useless events. Therefore, it is necessary to find a reasonable way for preparing suitable video clips for the teaching experiment sessions. As mentioned before, Sherin, Linsenmeier and van Es (2009) hypothesized that there are three dimensions such as windows, clarity and depth (see Table 7) in order to establish a video clip that promotes teachers' group discussions of students' mathematical thinking.

Sherin et al.'s (2009) conclusion was that watching a clip that is high in depth does not always guarantee productive conversations. Instead, the clips that are low in depth lead productive discussions. Moreover, they did not find clarity as a sole deterministic factor when choosing productive video clips. Consequently, in order to providing a productive discussion environment to the PSTs, I considered following three types in terms of "*windows-clarity-depth*", respectively: "*high-high-low*"; "*high-low-low*"; "*high-low-high*" and "*high-high-high*" by considering the Sherin et al.'s (2009) proposal about selection of productive video clips.

Table 7. Three dimensions of student thinking (Linsenmeier & Sherin, 2009, p.421)

Dimension	Description	Questions to consider
Windows	Evidence of students' mathematics thinking	– Is student written work visible? – Do students explain their ideas verbally? – Do we see students' gestures or facial expressions?
Clarity	Ease of understanding students' ideas	– Am I confused about what students are doing or saying? – Do I understand the students' ideas or methods?
Depth	Nature of students' mathematics thinking	– Are students involved in routine tasks based on memorization and rote recall? – Are students engaged in mathematical reasoning and problem solving?

3.5.1.2.2 Archiving strategies of micro-case videos

I utilized two strategies in the construction of charts after previewing the video segments again. First strategy was based on the identification and summarization of all events in a chronological order for each seventh grade student's video data. In the first strategy, I initially previewed raw videos. After that, I took time-indexed notes known as *content logs*, for the events in the video. Content logs enabled me to develop a quick sense of the corpus of the data and to save time the selection of segments for the further detailed analysis. When dividing raw video into events, I wrote both descriptive and reflective notes about student's mathematical thinking in separate columns in a table. Appendix 4 illustrates an example video segmentation of S13's mathematical thinking in the case production interview. As in Table 26 (see Appendix 4), video production interview took approximately 60 minutes with S13. I divided this 60 minutes raw video data into 11 significant events. I placed in and out of each segment in raw video. Thus, I think that S13's thinking process can be analyzed chronologically as a whole. At the same time, this segmentation may allow me to select a piece of video in production of a micro-case video for teaching sessions. This table was useful to detect "*unpredictable/noteworthy events*" in one student's thinking. However, this table did not allow comparing how different students think about same subject, topic or concept at first glance. It was hard to compare different students' thinking by using many of tables. At this point, I utilized second approach in order to archive video segments based on different students' mathematical thinking for a specific concept or situation like in Table 27 (see Appendix 5). This table involved information about 16 seventh grade students' definitions of parallelogram. It gave opportunity to detect easily students' different *mathematical thinking* related to same mathematical concept. In this sense, I identified each student's mathematical thinking about same part of the relevant task. Thus, it gave opportunity to detect easily students' different *mathematical thinking* related to same mathematical concept.

3.5.2 Conducting individual clinical pre-interviews with prospective teachers and acquisition of initial lesson plan

Conducting individual clinical pre-interviews with prospective teachers who are participants of the study was found important before starting teaching experiment sessions in terms of gaining insights and knowledge about their initial SMK and PCK levels on quadrilaterals. Thus, the developments in PSTs' knowledge could be observed throughout the teaching experiment process. Furthermore, this interview data was valuable for the selection and organization of micro-case videos that were used in teaching experiment sessions. For these reasons, I conducted pre-interviews before preparing teaching experiment sessions. There was not any time restriction in the implementation process of pre-interviews. Each interview took approximately one and a half hour for each participant in a silent room at the Faculty of Education. Each prospective teacher individually participated to the pre-interview (*Note*: I had explained the involvement of interviews in section 3.4.1).

In line with the tendency in the literature about the use of clinical interviews in educational studies, partially standardized version of clinical interviews was preferred in the current study because it makes possible to compare a participant's responses with one another (Opper, 1977). For this reason, the tasks and questions in the study are standardized, but still permit the interviewer freedom to introduce some additional probing questions when the interviewer finds inconsistency between learner's responses or doubts whether the responses reflect completely the learner's real thinking (Opper, 1977).

By adopting partially standardized version of clinical interviews, five tasks were sequentially implemented to the PSTs. The arrangement of the tasks in pre-interviews is displayed in Figure 8. In the first task of the pre-interview, PSTs asked to make personal, instructional definitions of parallelogram, rhombus, and trapezoid and to explain their anticipations about how any seventh grade student can describe/define these concepts. Furthermore, following prompting questions in were asked to understand participants' mathematical and pedagogical considerations about definitions of quadrilaterals: What are your personal definitions for the concepts of parallelogram, rhombus, and trapezoid? Which definitions of those concepts do you prefer to use for your future instruction? Why do you prefer such a definition in any teaching situation? Which instructional approaches do you prefer when utilizing the definitions of the concepts in your future lessons (e.g. teacher-centered approach or student-centered approach)? How can selection of a definition influence your instructional process?

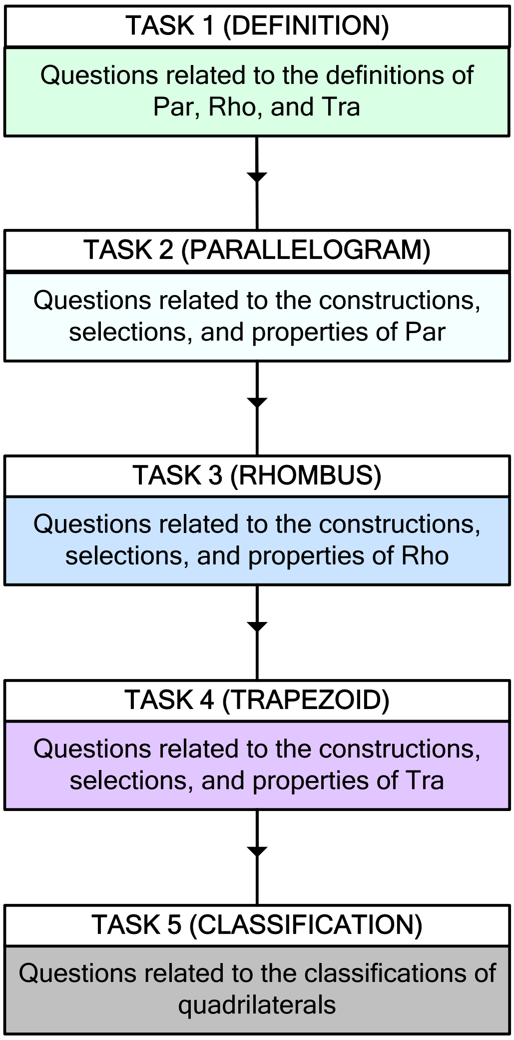


Figure 8. The arrangement and involvement of the tasks in pre/post interviews

After the participant completed the first task, I implemented the remaining task to the participant as seen in Figure 8. In the second task, participant asked to give information about the questions related to the constructions, identifications, and properties of parallelogram. In the following, Task 3 and Task 4 were implemented to examine PSTs' SMK and PCK related to the constructions, selections, and properties of rhombus and trapezoid. Finally, questions related to classification of quadrilaterals were asked to the participants in the fifth task. At those tasks, some additional prompting questions were asked in order to assert participants' personal, predictive, and instructional drawings of parallelogram, rhombus, and trapezoid. For example, in construction items, can you draw at least three different drawings for each concept (e.g. parallelogram, rhombus, trapezoid)? What do you think about students' possible drawings for those concepts? Why do you think that students might produce drawings that you predicted? What kinds of drawings do you prefer to utilize for your future instruction? Which instructional approaches do you prefer when utilizing the drawings of the concepts in your future lessons (e.g. from definition to drawings, from drawings to definitions or using both of them at the same time).

At the end of pre-interviews, I asked each participant to prepare an example lesson plan in 7-10 days as if to teach quadrilaterals the seventh grade students. Lesson plans have a great importance to see PSTs' pedagogical considerations on teaching of quadrilaterals. From this point of view, I considered that examination of PSTs' initial lesson plans is an important step when selection of micro-case videos for teaching experiment sessions.

In the following section, I provided details how I used the data involving PSTs' existing knowledge about quadrilaterals that were obtained via individual clinical pre-interviews and PSTs' initial lesson plans in the process of selection and organization of teaching experiment sessions.

3.5.3 Selection and organization procedures of micro-case video clips

Selection and organization procedures of micro-case videos for teaching sessions were executed as in Figure 9. As seen in Figure 9, before starting the teaching experiment, I initially examined literature and video clip bank produced in planning process of teaching experiment. Thus, I prepared a tentative MCVC list based on previous literature on quadrilaterals and video databases that the researcher archived. I used not only literature but also PSTs' initial SMK and PCK on quadrilaterals were used as a significant determinative factor when choosing and organizing MCVCs in the raw video data of seventh grade students' mathematical thinking. After I analyzed PSTs' pre interviews data, I updated this tentative MCVC list that involved 35 video segments. When updating, I added some new MCVCs and I moved some MCVCs from the list. Then, I planned to prepare possible micro-case video clips for teaching experiment sessions considering pre-interview results. According to this plan, PSTs required to analyze two MCVCs in each teaching experiment week. In the following, I explained how I prepared and revised MCVCs throughout the teaching experiment process.

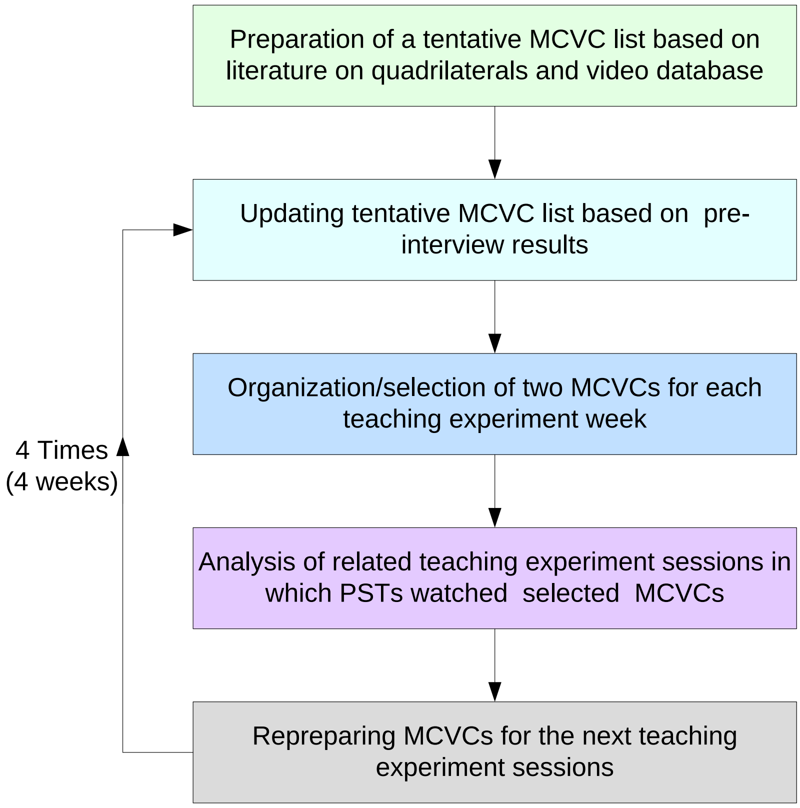


Figure 9. MCVC preparation and organization process

The result of previous studies on quadrilaterals, the data obtained from seventh grade students in video production interviews, and pre-interviews data that conducted with PSTs helped to the determination of objectives of the experiment and organization of MCVCs. Literature and obtained data provided a clarification for

PSTs’ and students’ conceptions about quadrilaterals. For example, most of PSTs generally could not provide necessary and sufficient conditions in their personal and instructional definitions. Moreover, they could not anticipate students’ possible definitional errors and difficulties. For these reasons, all MCVCs involved students’ definitions/descriptions related concept. In the following, I explained all details about organization and revision of MCVCs for each teaching experiment week. Final version of MCVCs was displayed in Table 9 in order to show crucial characteristics of each MCVC. The duration of clips ranged from 4.23 to 10.05 (minutes/seconds).

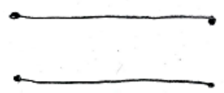
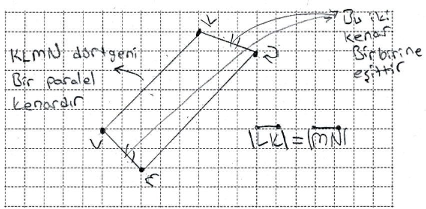
3.5.3.1 Preparation of MCVCs in the first week of teaching experiment

At the beginning of the teaching experiment, I decided to prepare two MCVCs that involved students’ mathematical thinking about parallelogram. This decision was reasonable because parallelogram is a concept that PSTs and students had more difficulties in understanding of parallelogram than rhombus or trapezoid. Furthermore, the set of parallelogram involve rhombus, rectangle and square according to the inclusive relations of quadrilaterals. Based on these arguments, MCVC1 and MCVC2 in Table 9 were prepared for the first week of the teaching experiment.

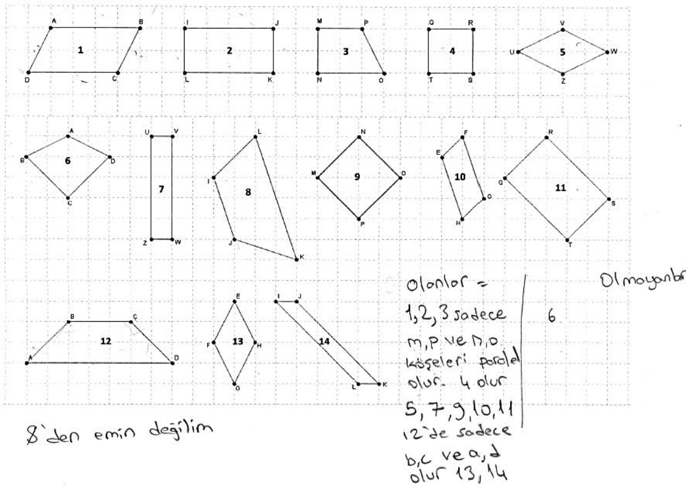
To be more evident, I explained first micro-case video (MCVC1) in more detail. MCVC1 was a 5.02 minutes video clip that involves video segments related to a moderate mathematical achievement level student’s parallelogram definition, constructions, and selections. Transcription of MCVC1 was given in Table 8.

Based on PSTs’ lack of knowledge about students’ overgeneralization errors in definitions and constructions, I prepared such a video because pre-interviews indicated that PSTs had limited knowledge about students’ overgeneralization errors on quadrilaterals. In this regard, MCVC1 involves many noteworthy unexpected events for the PSTs. In MCVC1, the student could not provide a correct description and construction for parallelogram. In this sense, I think that this clip is useful to illustrate the connection among student's concept image and concept definition because student thought that two parallel line segments are an example of parallelogram although she stated there are four sides of parallelogram at the beginning of the interview. In the following, she drew two parallel line segments to exemplify parallelogram. In conclusion, this clip clearly showed how a student describes, classifies, and understands relationships among quadrilaterals in terms of parallelogram.

Table 8. Conversation between researcher and student in MCVC1

Time interval	Content	Conversation between researcher and student
00:00-00:45	Parallelogram definition	R: How do you orally describe parallelogram? S: Parallelogram is two line segments in same proportion. R: How many sides do have a parallelogram? S: Four. R: Can any parallelogram have more than four sides? S: No. R: How do you write definition of parallelogram? S: Definition: Parallelogram is expansion of two line segments with same proportion through a point starting from that point[1]. Two parallel line segments can be given as an example of parallelogram.
00:45-3:03	Parallelogram construction	R: How do you draw a parallelogram? S: After I determine two points, I merge these two points.  R: Can you construct another parallelogram example in this grid paper? S: I can draw (*she drew initially [LK] and [MN]*) I named them as [KL] and [MN]. 

[1] Turkish version: Paralelkenar iki doğru parçasının bir noktadan başlayıp o nokta boyunca aynı orantıda ilerlemesidir.

		R: At the beginning of the interview, you said that a parallelogram has four sides. However, you drew [KL] and [MN]. Please explain the reason why you drew such a figure? S: It is necessary four sides to be a parallelogram. R: How many sides does this figure have? S: I think this [figure] may not be a parallelogram. R: Is it necessary that parallelogram must be a quadrilateral? S: No. R: If I ask you to draw a parallelogram having four sides, how do you draw it? S: I will complete [KL] and [MN]. I added other sides. R: Is the quadrilateral of LMNK an example of parallelogram? S: Hmm… [LM] and [KN] seem differently inclined. However, it can be related to my construction. I think LMNK quadrilateral is a parallelogram.
3:03-5:02	Parallelogram identification	R: Which figures do you identify a parallelogram?  S: 1 and 2 are parallelogram In third figure, only [MP] and [NO] are parallelogram. 4 and 5 are parallelogram examples. 6 is not a parallelogram. 7 is also parallelogram. In eighth figure, only [IJ] and [LK] are parallelogram. However, I am not sure whether eighth figure is a parallelogram or not. R: If you are not sure you can write the number of figure to indicate your indetermination. S: I am not sure about figure 8. 9 is also a parallelogram. R: Is there a specific name of figure 9? S: It is square. 10, 11, 13, and 14 are also parallelogram. However, in figure 12, [BC] and [AD] are parallelogram.

MCVC2 was 7.26 minutes video clip that involves a high mathematics achievement level seventh student's video segments about parallelogram definition, constructions, and selections. Some students treated hexagon as an example of parallelogram in video production interviews. One of student's related overgeneralization error involved MCVC2 as an unexpected situation. The student had prototypical and nonhierarchical concept images about parallelogram in MCVC2. At the beginning of the clip, the student described parallelogram as "*a distorted figure like a pushed down form of rectangle or square*". Furthermore, in the parallelogram selection part of the clip, she did not consider a rotated square and a rotated rectangle as a square and rectangle, respectively. This might be an expected situation for PSTs. However, in MCVC2, the student changed her decisions about relations among quadrilaterals through the process. This is interesting because PSTs were unaware of unstability of students' decisions. MCVC1 and MCVC2 were prepared "*high-high-high*" in terms of "*windows-clarity-depth*" by considering the Sherin et al.'s (2009) proposal.

3.5.3.2 Preparation of MCVCs in the second week of teaching experiment

Before conducting pre-interviews, I had planned to prepare video clips about students' mathematical thinking on rhombus for the second week of teaching experiment. More interestingly, I did not expect PSTs had difficulty in angle and diagonal properties of quadrilaterals. However, pre-interviews indicated that they had inadequate knowledge especially on diagonal properties of parallelogram and rhombus. For this reason, MCVC3 and MCVC4 were reorganized in order to develop their knowledge related to the properties of quadrilaterals. As seen in Table 9, MCVC3 was 4.27 minutes video clip that involves a low mathematics achievement level seventh student's video segments about parallelogram definition, constructions, and angle and diagonal properties. Besides, MCVC4 was 6.15 minutes video clip that involves a high mathematics achievement level seventh student's video segments about parallelogram definition, constructions, angle and diagonal properties. Prospective teacher generally thought that if students correctly define and construct a geometric figure, they also know properties of this figure. In MCVC3 and MCVC4, although students correctly constructed parallelogram, they made many mistakes about diagonal and angle properties of parallelogram. These micro-case videos helped to broaden PSTs' limited perceptions.

In the first week experiment sessions and pre-interviews results, I recognized that PSTs did not consider the influence of lack of knowledge related to prerequisite knowledge about quadrilaterals on students' conceptions of quadrilaterals. Considering these insufficiency, I also designed a special video segment about angle concept in order to show PSTs in group discusion process of MCVC3. On the other hand, the student in MCVC3 could not differentiate between diagonal and corner. To provide a different perspective, I reorganized MCVC4 that involves students' misconceptions about diagonal properties of parallelogram.

3.5.3.3 Preparation of MCVCs in the third week of teaching experiment

At the beginning of the teaching experiment, I decided to use two MCVCs that include video segments on students' prototypical and non-hierarchical understanding about rhombus. .In that situation, pre-interview results indicated that the involvement of these video clips comprised of expected situations in terms of prospective teachers. Furthermore, after I recognized PSTs had inadequate knowledge about properties of quadrilaterals and students' lack of knowledge on basic geometric concepts such as perpendicularity and parallelism. For these reasons, I decided to reorganize MCVC5 and MCVC6. In this regard, I added new video segments involving student's thinking on diagonal properties of rhombus in addition to rhombus definition, constructions video segments to MCVC5. By this way, PSTs could recognize students' misconceptions about basic geometric concepts in teaching experiment sessions. Consequently, MCVC5 was 7.13 minutes video clip that involve a moderate mathematics achievement level student's mathematical thinking about rhombus. On the other hand, I added a new video segment in which a low mathematics achievement level seventh grade student's inconsistencies between rhombus and square to the student's rhombus definition and constructions in MCVC6. Thus, MCVC6 was a 4.23 minutes video clip that included a low mathematics achievement level seventh grade student's conceptions about rhombus.

3.5.3.4 Preparation of MCVCs in the fourth week of teaching experiment

In the last week of the teaching experiment, I prepared MCVC7 and MCVC8 that involve students' mathematical thinking on trapezoid after controlling tentative video list. When I analyzed 1000 minutes video production interview data, I recognized that seventh grade students generally had difficulties and misconceptions on trapezoid due to ordinary usage of "*yamuk*" in Turkish language, prototypical understanding and tendency on non-hierarchical relations among quadrilaterals.

After analyzing tentative video list, pre-interviews data, and previous weeks' sessions data, I prepared MCVC7 that was 10.05 minutes video clip in which there were a high mathematics achievement level seventh grade student's trapezoid definition, constructions, and selections video segments. More specifically, the student defined trapezoid based on prototypical trapezoid figure and constructed five-sided polygon as an example of trapezoid in MCVC7. Furthermore, the student provided unstable decisions about relations among quadrilaterals in terms of trapezoid. On the other hand, MCVC8 involved 4.54 minutes video segments in which there were a seventh grade student's definition, constructions, and angle properties of trapezoid. The segment related to properties of trapezoid was added after the analysis of pre-interviews results and previous sessions data.

Table 9. General characteristic of selected micro-case video clips in teaching sessions

Week	Sessions	MCVCs	Length (min.)	Student	Level of student	Mathematical topic in the video**	Windows	Clarity	Depth
W1	Session 1	MCVC1	5.02	S6	Middle (3-4)	D-C-S of Par.	High	High	High
	Session 2	MCVC2	7.26	S8	High (5-5)	D-C-S of Par.	High	High	High
W2	Session 3	MCVC3	4.27	S13	Low (2-3)	D-C-Prop (Ang-Dia) of Par	High	Low	Low
	Session 4	MCVC4	6.15	S1	High (5-4)	D-C-Prop (Ang-Dia) of Par	High	Low	High
W3	Session 5	MCVC5	7.13	S6	Middle (3-4)	D-C-S & Dia of Rho	High	Low	Low
	Session 6	MCVC6	4.23	S4	Low (2-3)	D-C-S of Rho	High	High	High
W4	Session 7	MCVC7	10.05	S5	High (5-5)	D-C-S of Tra	High	Low	High
	Session 8	MCVC8	4.54	S4	Low (2-3)	D-C-S & Side-Ang of Tra	High	High	High

* Abbreviations mean Tra-Trapezoid, Par-Parallelogram, Rho-Rhombus, D-definition, S-Selection, C-Construction, Prop-Properties, Ang-Angle, and Dia-Diagonal

3.6 Implementation Procedures of Teaching Experiment

The general structure of implementation phase of the teaching experiment was illustrated in Figure 10. As seen in Figure 10, after I conducted pre-interviews and the participants prepared their initial lesson plans, I began to conduct micro-case video-based teaching experiment sessions. There were two 90 minutes teaching experiment sessions in each week. All teaching experiment sessions were completed into four consecutive weeks. Teaching sessions were conducted in a seminar room at the Faculty of Education. Participants placed the around of the table and each participant has a personal laptop. Moreover, there was a plasma-screen TV connected with the researcher's laptop.

3.6.1 Individual analyses of micro-case videos

Firstly, prospective teachers individually examined a video case in their personal laptops and wrote a reflection paper in order to answer following questions:

- Explain student's thinking process in the video case (e.g. procedural/conceptual, misconception/ difficulty/ misunderstanding)
- If students made incorrect answers in her/his explanations/ constructions/ selections what can the reasons of their difficulty/misconceptions be?
- Do the correct explanations/ constructions/ selections of students show that they certainly have complete knowledge about selected mathematical concepts?
- What are the other points that you noticed in the video clips?

They wrote their reflection papers while individually watching each video clip. Some of them preferred to write after finishing individual video analysis. After they completed their individual reflection paper, I collected them and initiated a group discussion after a short break.

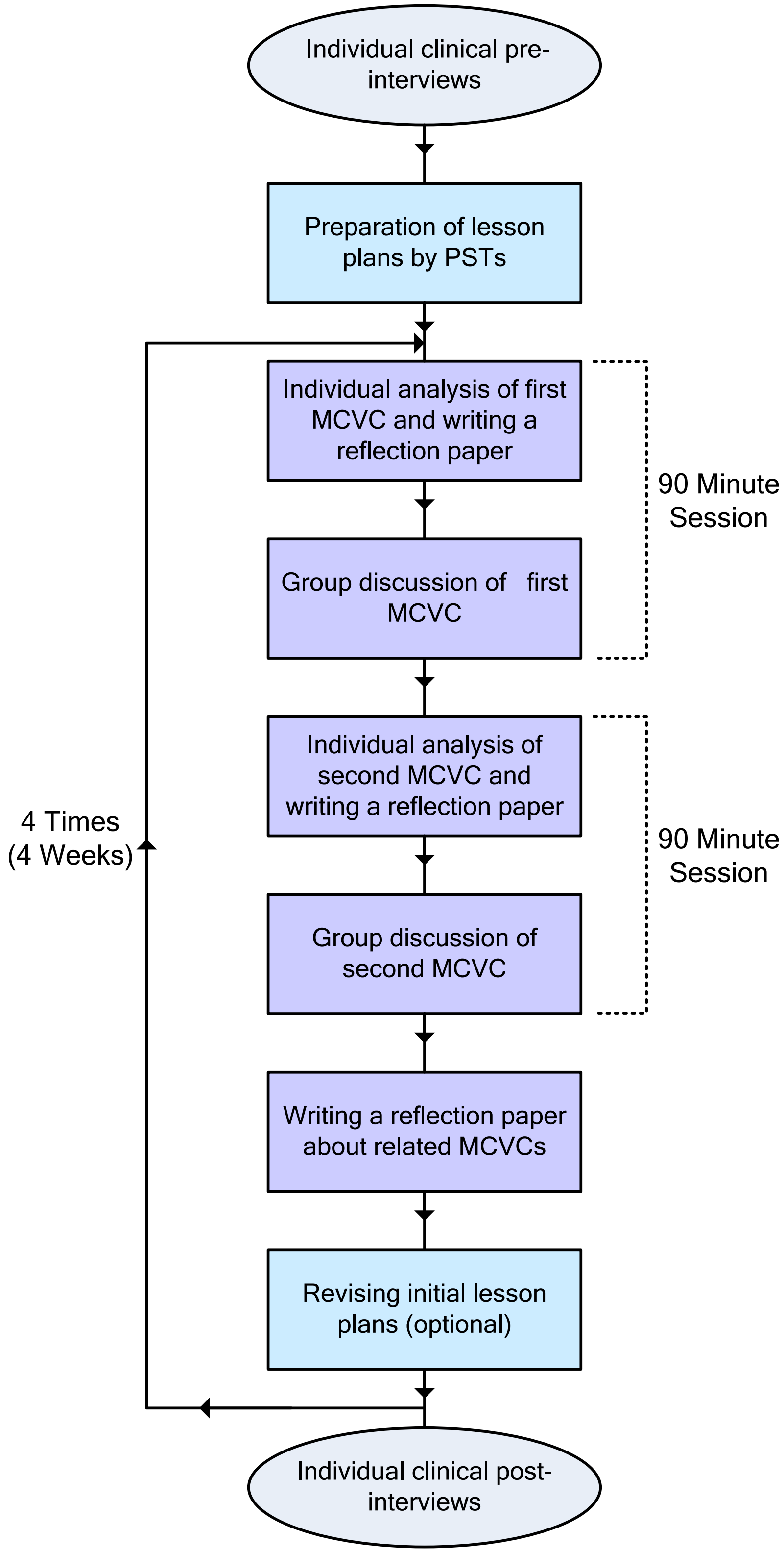


Figure 10. Structure of the teaching experiment

3.6.2 Group discussions about micro-case videos

To be more precise and concrete, how I conducted a teaching experiment session was explained in more detail. In the first week, I gave a flash memory involving MCVC1 and MCVC2 to each participant. They transferred MCVCs to their personal laptops. After they completed the transfer of MCVCs, I asked them to individually analyze MCVC1 and to write a reflection paper about related MCVC. Their individual video analyses and reflection paper writing process took approximately 35 minutes. After they completed individual video analysis and writing reflection paper about MCVC1, I initiated a group discussion in order to elaborate their conceptions and perceptions.

In all group discussions, I utilized framework for the facilitation of video-based discussion that was developed by van Es, Tunney, Goldsmith, and Seago (2014). They proposed four dimensions in the framework for utilizing video in more productive ways: (i) orienting the group to the video analysis task, (ii) sustaining an inquiry stance, (iii) maintaining a focus on the video and the mathematics, and (iv) supporting group collaboration. Details of these dimensions and the facilitator's some example moves were given in the following.

Orienting the group to the video analysis task. This practice is associated to two moves in this dimension such as "*contextualizing*" and "*launching*". I, as a facilitator, used "*contextualizing*" before PSTs individually examine the clip in order to inform the involvement of the each MCVC such as student's gender, concept, and duration of clip. For example, I said that "this five minutes-clip (MCVC1) involves a female seventh grade student's conceptions about definition, construction, and selections of parallelogram." I used "*launching*" at the beginning of the group discussion of each MCVC in order to elicit PSTs' ideas by posing general prompts. For instance, I asked following typical example prompting questions: What did you noticed when examining the video clip? "What did you find interesting in the video?" These prompting questions helped to initiate the group discussion by focusing on some noteworthy events in the video clips.

Sustaining an inquiry stance. In order to maintain the productivity of group discussion, I focused on six particular moves: highlighting and lifting up, pressing and clarifying, and offering an explanation and countering. I used "*highlighting*" when drawing attention to some particular events that prospective teachers did not mention on their own. For example, in group discussion of MCVC1, prospective teacher did not mention anything about the inconsistency between student's definition and construction of parallelogram. At this point, I asked them the following question: "Yes, you said that this student constructed a wrong parallelogram example, but, if you examine the definition and construction as a whole, what can you say the connection among them?" In a similar vein, I utilized some prompts related to "*lifting up*" that "refers to the facilitator taking up noteworthy participant ideas and making them the object of discussion (van Es et al., 2014, p.7)". For example, in group discussion of MCVC3, although one participant proposed that the student do not know the angle relation of parallelogram, another participant claimed that because the student see two variables in parallelogram, he divided 360˚ by 2. At this point, since I found the second participant's idea was interesting and important, I raised this idea for further discussion. Furthermore, I used two moves as "*pressing*" and "*clarifying*" in order to elaborate participants' ideas. Pressing enable PSTs to expand on an idea and to provide further explanation about their reasoning. For this purpose, following from of questions were utilized: "can you tell me more about that?" and "I understood your idea. Can you give some details what you exactly mean?". "*Clarifying*" was used when encouraging the group to get further information on their thinking by rephrasing a participant's idea. Finally, remaining two moves as "*offering an explanation*" and "*countering*" were used "to introduce a way of interpreting or making sense of what was happening in the video (van Es et al., 2014, p.8)".

Maintaining a focus on the video and the mathematics. I utilized three strategies such as "*redirecting*", "*pointing to evidence*", and "*connecting ideas*". Redirecting occurred when the participants focused on an unrelated issue from the involvement of the video case. At this point, I deflected the discussion into mathematical issue. When I want to learn some evidence about the PSTs' proposed ideas, she chanced the direction of the discussion by using some prompts. In order to establish relationship between PSTs' different ideas and issues related to mathematics, I used following example prompts: "This idea is similar to what Ece was doing ?" and" what your strategies are if your students would do same mistakes in your class?".

Supporting group collaboration. By using three strategies, I facilitated the group collaboration: "*standing back*", "*distributing participation*", and "*validating participant ideas*". Sometimes, I gave the group member time to explore an idea and did not involve the discussion. In order to invite different PSTs, I also tried to distributed participation of group discussion. Validation of ideas was utilized in following ways: "it is very interesting. I had not thought that before" or "that could be". Consequently, group discussion of each video case was implemented in similar ways.

3.6.3 Writing after discussion reflection papers (ADRP)

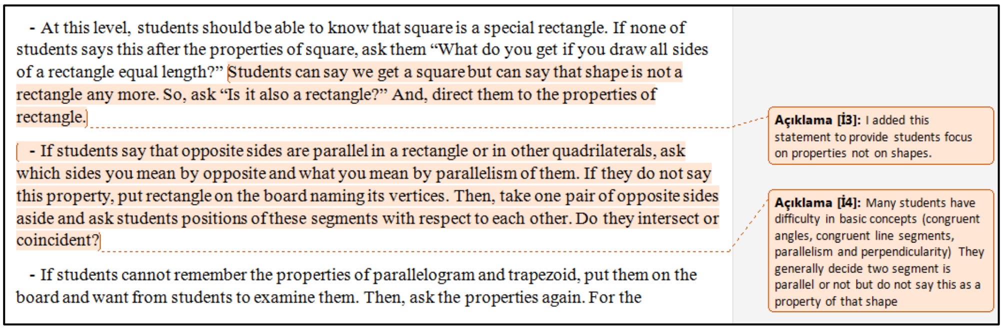
I requested PSTs to write a "*after discussion reflection paper* (ADRP)" at the end of the two sessions of video analysis and group discussions in each teaching experiment week. In these reflection papers, I asked PSTs to respond to the following questions.

- Explain if there was any change in your thinking after the group discussion process? How did discussion environment influence your thinking? (Link between previous knowledge or give some example speeches between you and your friends)
- Propose some recommendations for classroom applications/teaching methods to develop student's mathematical thinking and to overcome their misconceptions/ misunderstandings in the video clips. (*Think as you teach these concepts*…)

They individually wrote these reflection papers in either class or their home. Then, they supplied their ADRP to me as a hardcopy. I preferred to get hardcopy version because they were faster when writing in a paper than writing in computer. Furthermore, they found easier to draw a figure in paper than computer.

3.6.4 Making revisions in lesson plans

As seen in Figure 10, making revisions in the lesson plans was optional for the prospective teachers at the end of each week in teaching experiment. These revisions and PSTs' reflective notes about the reasons why they needed to make revisions in their lesson plans were crucial to understand the changing and developing SMK and PCK about quadrilaterals throughout the teaching experiment. In this sense, before starting a new teaching session (e.g. two days ago), I asked participants to send revised versions of lesson plans involving the reasons why they needed to make revisions. They wrote all reflective comments in lesson plans by following the steps as opening their initial lesson plans in "Microsoft Word Program", opening "review" section, and adding a "new comment". More specifically, one participant's some revisions and reflective comments on the reasons why she changed her initial lesson plan were illustrated in Figure 11.



- At this level, students should be able to know that square is a special rectangle. If none of students says this after the properties of square, ask them "What do you get if you draw all sides of a rectangle equal length?" Students can say we get a square but can say that shape is not a rectangle any more. So, ask "Is it also a rectangle?" And, direct them to the properties of rectangle.

- If students say that opposite sides are parallel in a rectangle or in other quadrilaterals, ask which sides you mean by opposite and what you mean by parallelism of them. If they do not say this property, put rectangle on the board naming its vertices. Then, take one pair of opposite sides aside and ask students positions of these segments with respect to each other. Do they intersect or coincident?

- If students cannot remember the properties of parallelogram and trapezoid, put them on the board and want from students to examine them. Then, ask the properties again. For the

Açıklama [İ3]: I added this statement to provide students focus on properties not on shapes.

Açıklama [İ4]: Many students have difficulty in basic concepts (congruent angles, congruent line segments, parallelism and perpendicularity) They generally decide two segment is parallel or not but do not say this as a property of that shape

Figure 11. An example from Emel's lesson plan revisions

3.6.5 Conducting individual clinical post-interviews

Post-interviews were conducted by using the same tasks after all teaching experiment sessions were completed in order to increase comparability of the results. In the interviews, I asked the main question of "why did you change your initial idea on this question?" in order to understand the changes and developments in PSTs' SMK and PCK on quadrilaterals by considering their initial responses in pre-interviews. Similar to pre-interviewing process, there was not also any time restriction in the implementation process of post-interviews. Post-interviews took between 35-45 minutes.

3.7 Data Analysis

3.7.1 Analysis of pre/post-interviews

In the analysis of pre- and post-interview data, I utilized thematic coding to identify, analyze and report the themes in the data. For this purpose, all data were examined by taking account the phases of familiarization with data, generating initial codes, searching for themes among codes, reviewing themes, defining and naming themes, and producing the final report (Braun & Clarke, 2006). Before familiarization of the data, I examined literature about geometrical theories (e.g. concept image-concept definition, prototypical phenomenon, figural concepts, van-Hiele geometric thinking), and conducted national and international studies about quadrilaterals. In the beginning, I transcribed all interview sessions. Next, in the light of the literature, I generated initial codes and themes. I then turned to the transcribed video data and examined each participant's responses to the items related to quadrilaterals. From this analysis, I defined the characteristics of each themes and corresponding codes for the items on each task. Then, I grouped each PST's responses that are related to the task such as definitions, constructions, selections, properties, and classification of quadrilaterals. I code PSTs' responses in an Excel document to compare all PSTs' responses for each task. Details of coding process were explained in the following.

3.7.1.1 Coding of PSTs' knowledge about definitions of quadrilaterals in pre- and post- interviews

A good mathematical definition is characterized according to some logical principles by different researchers (Edwards & Ward, 2003; Khinchin, 1968; Solow, 1984; Van Dormolen & Zaslavsky, 2003; Vinner, 1991). Among the principles, establishing hierarchical structures between general and specific concepts and providing necessary and sufficient conditions were taken as necessary features of a definition by all researchers. In the current study, I examined prospective middle school mathematics teachers' personal definitions in pre- and post-interviews in terms of (i) establishing necessary and sufficient conditions, and (ii) providing hierarchy between general and specific concepts. The details how I named the themes and codes in the PSTs' personal definitions were given in Table 10.

More specifically, Table 10 shows the codes of participants' personal definitions in terms of establishing necessary and sufficient conditions and providing inclusivity. I generated Theme 1 with four codes in order to examine PSTs' definitions in terms of establishing necessary and sufficient conditions.

For example, PDef-Code1 is related to PSTs' definitions involving sufficient but not necessary defining conditions. More specifically, if a participant defines a concept by listing all known or many of redundant properties, I grouped such kinds of definitions into PDef-Code1 because these definitions were correct uneconomical definitions. More specifically, some students can define parallelogram as "*a quadrilateral having two parallel opposite sides*" or "*a figure having two opposite parallel sides*". In the first definition, it was evident that "*two opposite parallel sides*" is not enough to define parallelogram since trapezoid even can be evaluated a parallelogram example according to this definition. Similarly, second definition does not involve information about whether parallelogram is a "closed" figure or not. As an example of PDef-Code2, if a student define rhombus as "*a figure having four equal length sides*". This definition shows necessary but not sufficient conditions because the definition involved the term of "figure" instead of a "closed-figure". On the other hand, PDef-Code3 indicates a definition that involves both necessary and sufficient defining conditions. Finally, an example definition of trapezoid can be given for the PDef-Code4 as following: "*a rectangular region having the lower base and the upper base*". In this situation, a four-sided figure having no parallel sides can be an alternative representational interpretation of this definition.

Table 10. Coding of PSTs' personal definitions according to themes

	Themes	Name of themes	Codes*	Coding
Personal Definitions	Theme 1	Establishing necessary and sufficient conditions	PDef-Code 1	Sufficient but not necessary
			PDef-Code 2	Necessary but not sufficient
			PDef-Code 3	Necessary and sufficient
			PDef-Code 4	Incorrect
	Theme 2	Providing hierarchy	PDef-Code5	Inclusive definition
			PDef-Code6	Partial inclusive definition
			PDef-Code7	Exclusive definition
			PDef-Code8	Incorrect

On the other hand, Theme 2 was prepared to examine PSTs' definitions in term of providing hierarchy. There are four codes in Theme 2 as seen in Table 10. Usiskin and Griffin (2008) state that when "one definition purposely excludes what the other definition includes; we call the one definition an exclusive definition and the other definition an inclusive definition" (p.4). It indicates non-hierarchical relations of quadrilaterals. For example, if a trapezoid is defined exclusively as "a quadrilateral with exactly one pair of parallel sides" (p.27), then rectangles and trapezoids would be grouped into disjoint subgroups of quadrilaterals. In contrast, if the trapezoid is defined inclusively as "a quadrilateral with at least one pair of parallel sides" (p.27), then all rectangles would be a taken as a subgroup of trapezoids. Consequently, while inclusive definitions are related to hierarchical classifications, exclusive definitions lead to partition or exclusive classifications of quadrilaterals. To sum, I grouped prospective teachers' personal definitions by considering these themes and categories.

PSTs' instructional definitions in pre- and post-interviews were examined considering the studies about definitions (see Table 11). Accordingly, some researchers (e.g. Winicki-Landman & Leikin, 2000) proposed criteria for didactical (pedagogical) suitability of a definition, based on some conceptions such as relying on previously learned concepts, learners' intellectual development, zone of proximal development of the learners, intuitiveness, and elegance.

Table 11. Coding of PSTs' instructional definitions

Definition types	Possible reasons of the instructional preference
In terms of hierarchy Inclusive/Partial inclusive/Exclusive *In terms of economic structure* Economical/Non-economical	– Enable mathematical generalizations and deductive reasoning – Matching students' knowledge and needs (e.g. Zone of proximal development) – Convenience for applying to problem solving – Ease to understand – Intuitiveness (Fischbein, 1987) – Clarity to the students – Building knowledge on the basis of known concepts (Edwards & Ward, 2003; Leikin & Winicki-Landman, 2001; Winicki-Landman & Leikin, 2000) – No reason

In order to understand PSTs' didactical considerations for a definition, I examined how they select appropriate statements for a definition which factors influence on their selections, and their thinking about how the selection of a definition can influence their instructional process (Leikin & Winicki-Landman, 2001; Winicki-Landman & Leikin, 2000). More specifically, I firstly examined the types of PSTs' definitions in terms of hierarchical and economic structure. Then, I coded participants' pedagogical considerations that reflect the reasons why they want to utilize a definition in their future instruction as in Table 11.

Finally, I examined PSTs' predictions about students' possible definitions/descriptions of quadrilaterals. At this point, I made a detailed analysis how PSTs anticipate students correct but incomplete descriptions or incorrect descriptions and their possible reasons in order to understand their pedagogical content knowledge.

3.7.1.2 *Coding of PSTs' knowledge about constructions of quadrilaterals in pre- and post- interviews*

I evaluated prospective teachers' personal drawings for quadrilaterals in terms of whether or not they are drawn hierarchical /partial hierarchical /non-hierarchical and prototypical /partial-prototypical /non-prototypical. Based on the methodological and cognitive approach used by Tsamir, Tirosh and Levenson (2008) in order to understand children' intuitive examples and non-examples about triangle, I grouped prospective teachers' approaches on determining students' possible constructions for parallelogram, rhombus and trapezoid as in the following:

a) *Correct constructions* (involving correct intuitive and non-intuitive examples such as hierarchical and non-prototypical concept images)
b) *Correct but incomplete constructions* (involving only intuitive examples as prototypical concept images that is the result of undergeneralization error)
c) *Incorrect constructions* (involving also intuitive non-examples as a result of overgeneralization error)
d) *Difficulties in constructions* (grid paper usage, determination of parallelism etc.)

I also produced some possible reasons of students' constructional problems that might be proposed by prospective teachers as in the following: learners' intuitions, lack of previous knowledge about basic geometric concepts, disconnection among concept definition and concept images, prototypical reasoning, and the examples given in the textbooks and math lessons. Finally, PSTs' instructional constructions were grouped according to examples types that they planned to use in their future instructions such as prototypical/non-prototypical examples, hierarchical/partial hierarchical/non-hierarchical examples, intuitive examples, and non-examples by examining their didactical considerations when to utilize a specific example/non-examples. Moreover, I coded their teaching approach (e.g. teacher-based, student-based) and the materials (e.g. concrete materials, dynamic geometry applications, exemplification only on board) that they plan to use in teaching of the constructions of quadrilaterals.

3.7.1.3 Coding of PSTs' knowledge on properties and classification of quadrilaterals in pre- and post-interviews

PSTs' verbal and written responses to the items related to properties of quadrilaterals were examined in terms of the correctness. Again, participants' knowledge about classifications of quadrilaterals was examined by analyzing their written and verbal responses to selections and classifications item in the interview tasks. I coded PSTs' responses according to inclusive relations of quadrilaterals.

In the coding process of PSTs' written and verbal responses to all items, I utilized inter and intro reliability approach proposed by Miles and Huberman (1994). In this regard, I firstly determined the number of the agreements among the coders. In the following, I divided the number of agreements by the total number of agreements and disagreements. In this approach, the result that is equal or higher than 70% indicates the presence enough reliability. When two separate researchers coded the responses, there were three disagreements in pre-interview data coding and four disagreements in post-interview data coding. As a result, while the inter-rater reliability of pre-test coding was calculated 3/33= 0.09, 0.09x100=90, the inter-rater reliability of pre-test coding was calculated as 4/33= 0.87, 0.87x100= 87. I reached a consensus about the items that produced disagreements by making an effective discussion with the coders. Furthermore, as a researcher, I coded each interview data thrice three month apart. The intra-rater reliability was found as 98%.

As a conclusion, the verbal and written responses in both interviews were utilized to provide evidences for PSTs' SMK and PCK about quadrilaterals and how they are progressed and restructured by comparing their responses.

3.7.2 Analysis of teaching experiment sessions

In the current study, I determined an analytic three-level analysis approach in order to examine teaching experiment sessions data. In the low-level preliminary analysis, I made a chronological order for teaching experiment session data that consisted of prospective teachers' individual video analysis reflection papers, group discussions, reflection papers that were written after group discussions, and revised lesson plans. After chronological order, I read all reflection papers and group discussions in each week of teaching experiment. In the following, I took specific field notes about PSTs' developments in knowledge about quadrilaterals. Moreover, first viewing of group discussion videos provided an overview of each PST's knowledge development.

In the mid-level analysis process, I transferred all hardcopy reflection papers to the digital form in computer and I transcribed all group discussions videos. I began to identify and divide data set into "idea units" in order to code the PSTs' written statements and verbal explanations. Idea unit is defined as "a distinct shift in focus or change in topic" (Jacobs, Yoshida, Fernandez, & Stigler, 1997, p.13). In this regard, I determined four main idea units that refer PSTs' knowledge about the topics of definitions, constructions, classifications, and properties of quadrilaterals. While any idea unit in a reflection paper sometimes involves one statement, it sometimes involves a paragraph of comments. However, any idea unit in a group discussion data consisted of a conversation about a specific issue such a student's defining abilities. Consequently, the length of an idea unit generally depends on the content (e.g. definition, construction, classification, and properties of quadrilaterals) and the types of data source (e.g. reflection paper, group discussions, and revised lesson plans). After I completed to divide all teaching experiment data into idea units according to content and the types of the data, I passed advanced level analysis.

In the advanced level analysis, I focused on how the developments occurred in PSTs' knowledge about quadrilaterals throughout the teaching experiment process. In this sense, I prepared a "*knowledge development sheet*" in order to examine each participant and the group's knowledge development in each teaching experiment session.

In order to make how I examined participants' knowledge developments throughout teaching experiment process more understandable, I gave the details of analysis and coding structure for the developments of PSTs' knowledge about definitions of quadrilaterals. As mentioned before, PSTs analyzed and discussed two video clips and wrote reflection papers about the clips before and after group discussions in each teaching session. Moreover, they optionally made revisions in their initial lesson plans. In knowledge development sheet, I firstly noted each PST's initial SMK and PCK about definitions of quadrilaterals. Then, I read each PST's written statements about the definitions of quadrilaterals in individual reflection papers that they wrote when individually examining the video clip. Then, I examined each participant's explanations about the definitions in group discussions and I highlighted whether each statement shows development/change in knowledge about definitions of quadrilaterals in terms of SMK and/or PCK or not. After that, I remarked all comments about definitions in reflection papers that they wrote after group discussions. Finally, I examined the changing nature of revised lesson plans in terms of definitions of quadrilaterals. Therefore, I reached a big data that showed how each participant developed their knowledge about definitions of quadrilaterals throughout the teaching experiment process. I used same procedure to determine each participant's knowledge developments in constructions, classifications, and properties of quadrilaterals. To be more evident, I gave an example coding scheme for the developments of PSTs' SMK, and PCK about definitions of quadrilaterals (see Table 12).

Table 12. An example coding of developments in shifts PSTs' knowledge on definitions of quadrilaterals

	Common developments in PSTs' knowledge about definitions From	to
SMK	Exclusive or partial inclusive definitions	Inclusive definitions
	Inability to establish necessary and sufficient conditions for a definition	Establishing necessary and sufficient conditions for a definition
	Inadequate mathematical language usage	Precise mathematical language usage
KCS	Ignoring didactical considerations	Preferring definitions according to didactical considerations
	Focusing only student's correct or incomplete definitions/descriptions	Focusing on students' definitional errors and their possible reasons
	Not realizing the relationship between students' concept image and concept definition	Establishing a strong relationship between students' concept definition and concept image
KCT	Preferring teacher-centered teaching way for the definitions	Adopting student-centered teaching ways
	Giving only definitions of quadrilaterals	Defining and exemplifying also basic prerequisite geometric concepts
	Preferring their personal definitions as instructional definitions	Differentiating personal definitions and instructional definitions

3.8 Trustworthiness of the Study

Validity and reliability are two essential issues that any researcher should consider while conducting a study, analyzing the results and mentioning the quality of study (Patton, 2002; Yıldırım & Şimşek, 2006). For qualitative research, validity and reliability concepts are defined and named different from the validity and reliability concepts in quantitate research. At this point, it is meaningful to mention about Lincoln and Guba's (1985) identification of reliability and validity issues in term of qualitative research. They defined credibility, transferability, dependability, and confirmability terms; referring to internal validity, external validity, reliability and objectivity respectively. According to Lincoln and Guba (1985), these terms may be seen as the indicators of trustworthiness which is the term to be used to show reliability and validity for a qualitative research. How I utilized these strategies in the current study is explained in the following.

3.8.1 Credibility and transferability

Lincoln and Guba (1985) suggested that ensuring *credibility* referring internal validity is one of the most important factors in establishing trustworthiness of a qualitative study. Internal validity deals with the questions of "how research findings match reality?" (Merriam, 1998, p.213). Merriam (1998) explains that six basic strategies to enhance internal validity: Triangulation-using multiple sources, multiple investigators, or multiple methods, member checks, long-term observation, peer examination, participatory or collaborative modes of research and research's biases. In the current study, I utilized most of these methods to provide and increase credibility.

Firstly, I used triangulation. "Triangulation has been generally considered as a process of using multiple perceptions to clarify meaning, verify the repeatability of an observation or interpretation" (Stake, 2000, p.443). In literature, there are four types of triangulation: *data triangulation* (the use of a variety of data sources in a study), *investigator triangulation* (the use of several different researchers or evaluators), *theory triangulation* (the use of multiple perspectives to interpret a single set of data, and *methodological triangulation* (the use of multiple methods to study a single problem or program) (Denzin, 1978; Patton, 1987, 2002). I used multiple data sources such as pre- and post-interviews, reflection papers, group discussions, prospective teachers' initial and revised lesson plans, field notes, observation of individual video analysis processes, and self-development reports. Thus, I used data triangulation in the current study. The investigator triangulation was also utilized because the first teaching session is observed by a professor in mathematics educators. This researcher observed prospective teachers' individual video analysis processes and group discussion of related video clip. The professor gave valuable ideas about the methodological issues of teaching experiment process and what kinds of things he observed interesting in prospective teachers' ways of reasoning about the video clip. I also used member-checking to ensure the credibility of the study. At the end of the task-based post-interviews, I gave their written responses to pre-interviews and post-interviews and asked them whether they agreed with these written ideas or not. Furthermore, I obtained information about the ideas they provided in reflection papers and group discussions. For the methodological triangulation, I used different methodological approach such as observations, interviews, group discussions, and written document analysis. Finally, when interpreting the data, I utilized theory triangulation because I examined prospective teachers' SMK and PCK by considering the theories of prototypical phenomenon, concept image/concept definition, definitions in mathematics, exemplification, and figural concepts. Furthermore, I used different perspectives that focused on teacher' professional noticing in video-based professional development programs.

Another strategy, I used to increase the credibility of the study was prolonged engagement with the participants throughout a semester. Thus, I got opportunities in terms of building thrust with the participants during the period of data collection.

The second criteria to establish trustworthiness in a qualitative study is *transferability* referring to external validity. External validity is related to the question of "How generalizable are the result of the study?" (Merriam, 1998, p.223). However, it is not possible to mention about generalizability for qualitative research because in qualitative research, a single case or small nonrandom sample is selected in order to understand the context in depth not to find what was true across the population. Making statistical generalizations is not to the major aim of the investigator. Transferability was used by ensuring sufficient information about implementation processes in qualitative research. Thus, I provided thick description of the study so that the reader understands it and compare to their own studies. For this research, the context of the study, the selection criteria of the participants, the number of participants, the purpose and context of any instrument to be used in the study, the number of the length of the data collection sessions, and the time period of the study will be explained in detail for ensuring the transferability.

3.8.2 Dependability and confirmability

The third criteria to establish trustworthiness is *dependability* referring to reliability. Reliability is defined as "…to the extent to which research findings can be replicated" (Merriam, 1998, p.220). It means that whether the results of study are dependable and consistent with the data (Merriam, 1998). According to Shenton (2004), how to ensure the dependability of a study depends on the explanations about how research design was implemented, how the data was gathered, and what was done to describe the field in the data. In this study, it is aimed that the replication of the study will be afforded by providing detailed information on the processes of the study. In this sense, I explained all operational and theoretical details of data gathering process, the nature of teaching sessions, and the structure of reflection papers. Furthermore, the coding categories were explained in detail during the coding processes by helping the second coder in order to ensure the dependability. After organizing the codes, researcher and the other coder came together and discuss the codes until they reach an agreement on the categories. Then, main themes were determined. After coding all the data individually with the final version of the codes, randomly selected transcription of teaching sessions and reflection papers were given to the second coder to ensure dependability of the study. The last criteria to establish trustworthiness in a qualitative research is *confirmability* referring the objectivity. Shenton (2004) explains how to ensure confirmability as using triangulation to reduce the researcher bias, providing detailed methodological information. In a similar vein, the confirmability was tried to ensure through triangulation, peer debriefing, detailed description on the methodology of the study, and presence of second coder in this study.

3.9 Ethical issues

In any study, some ethical problems may occur during the processes of data collection and data analysis (Merriam, 1998). For the ethical consideration in this study, it was taken permission from the Ethical Committee at METU (see Appendix 6). After this permission, for the video-taping and the participation of the study, other permissions will be taken from Ministry of National Education (MoNE) (see Appendix 7) and administrators and teacher of schools by using consent form. Additionally, in order to produce video cases that were necessary for the preparation of vide-based professional program, I took permission seventh grade students and their parents to participate in the study via consent form because students' age is under the eighteen years old (see Appendix 8). To ensure honesty in the present study, seventh grade students who willingly was selected to examine conceptions in quadrilaterals through task-based clinical interviewing processes. When producing video cases, I did not involve students' faces and names in the clips. Moreover, I did not give information about the specific name of the school in which I produced video cases involving seventh grade students' mathematical thinking. All the answers to the questionnaire and the following interviews were confidential and no one without the researcher and prospective teachers in the study access to the data. Furthermore, the video cases served to prospective teachers by giving no extra information about the seventh grade student in order to eliminate the bias.

On the other hand, some ethical considerations took into account after video producing part of the study. As mentioned before, an elective course was opened in the department. Some junior and senior class prospective middle school mathematics teachers decided to select this course. In the first meeting, these prospective teachers were informed about the data collection process. In this regard, I gave detailed information at the beginning of the study about the usage of video-camera in all process, pre-and post-interviews, individual video analysis and group discussions about video clips, the general structure of the videos, lesson plans, and reflection papers. As a result, eight senior class prospective middle school mathematics teachers decided to enroll to the course. I did not use the names of prospective teachers in anywhere. Instead, I utilized pseudonyms in reporting the data.

3.10 Limitations and Delimitations of the Study

There are some limitations in this study in terms of: (i) the number of participants, and (ii) the medium of the study, and (iii) the role of researcher. All these limitations and how I tried to handle all of them were explained in the following.

The number of the participants is a limitation of this study because the study was executed by eight senior students who attending Elementary Mathematics Teacher Education program in a public university in Ankara, Turkey**.** The undergraduate course that subjects to the current study was opened to senior class prospective teachers as an elective course with the name of "Projects in elementary science and mathematics education". As a result, the number of participants was limited to eight prospective middle school mathematics teachers. However, the aim is not make a generalization. Instead, focal point in the study is revealing the developmental process of teachers' knowledge in micro-case video clips analysis and discussions. In general, video case-based studies within the video club context involve 1-8 participants in the literature (e.g. Baş, 2013; Sherin, Linsenmeier, & van Es, 2009; Taylan, 2015).

Medium (micro-case video clips) of the study might be cause a limitation in terms of due to the research questions and research purpose. Micro-case video clips involve only students' mathematical thinking process in contrast to the classroom videos. Namely, there is only one dimension as student mathematical thinking in the micro-case video clips. In the literature, Sherin (2007) identified professional vision as the combination of two processes of "*selective attention*" and "*knowledge-based* *reasoning*". Considering and using classroom videos, she grouped selection attention into two main categories as "*Actor*" (e.g. teacher, student, and other) and "*Topic*" (e.g. management, climate, pedagogy and math thinking). However, the current study involves only one component in terms of the dimensions of "*Actor*" (only student) and "*Topic*" (only math thinking) because of the nature of micro-case video clips.

In this teaching experiment, I was the teacher-researcher, which can be a limitation for the study. Teaching experiments aim to observe the learners in their settings and report them to the audience. Combining participation and observation is the main challenge in such kinds of qualitative research (Patton, 2002). For this reason, establishing a balance between being a participant and being a researcher-teacher is crucial to provide correct and unbiased results. In the literature related to the use of video-based professional development programs, researchers emphasizes the importance of the relationship between a group of participating teachers and a facilitator in terms of providing fruitful teacher learning environment (Borko et al., 2015; Borko et al., 2008; van Es, 2012a). Furthermore, Van Es (2012a) mentions the role of facilitator in video-based professional development programs that need "to be created to help members become comfortable making their practice public and analyzing each other's teaching" (van Es 2012a, p.184). As a result, a framework for the facilitation of teachers' analysis of video (van Es, Tunney, Goldsmith, & Seago, 2014) utilized in order to increase the productivity of group discussions and prospective teachers' learning in the current study. In conclusion, researcher bias may be a risky factor for the study (Lincoln & Cuba, 1985). However, making the aim of study to make clear for the participants, studying with voluntary participants, assuring confidentiality, trying to make the participants comfortable during the data collection process, and making a check the interpretation of researcher with the participants were used to reduce the research bias in the current study.

There are also delimitations that were intentionally set as the boundaries by the researcher considering the related literature, the purposes of the study. The delimitations of the study consisted of (i) preferring only senior students in mathematics teacher education program (ii) the involvement of micro-case video clips, (iii) the number of micro-case video clips, and (iv) time period of the teaching experiment. All of them are elaborated in the following.

Preferring only senior students in mathematics teacher education program to the course is a delimitation of the study. As the participants to the course, it was preferred to admit fourth year prospective teachers who completed educational courses and pure mathematical courses involving information about learning and teaching of geometry. As a result, eight fourth year prospective middle school mathematics teachers took the elective course.

One of the delimitations of the study is related to the involvement of micro-case video clips since they involve only seventh grade students' mathematical thinking processes about quadrilaterals (e.g. definitions, constructions, classifications, and properties of quadrilaterals). However, this situation was intentionally decided in consultation with the experts in mathematics education. Furthermore, quadrilaterals are intensively covered in seventh grade teaching programs. Besides, concentrating only quadrilaterals in the teaching experiment process may be admitted as another limitation of the study. At this point, I think that related literature indicated that there is limited number of video-case based studies concentrating a particular mathematical domain (e.g. Didiş, 2014; Taylan, 2015, Walkoe, 2014). Conversely, the structure of the current study can be thought as an emergent perspective for video-based professional development just like a video-based curriculum proposed by Stockero (2008). Consequently, focusing on only quadrilaterals to understand the developments in teacher knowledge might not be evaluated as a limitation.

The number of the micro-case video clips can also be evaluated as the delimitation because this study is limited with the analyses and discussions around the eight micro-case video clips selected and watched during the four weeks of the course. The delimitation in the number of the video clips is substantially related to several factors: (i) the concepts in quadrilaterals taken the scope of the study, (ii) the variety of seventh grade students' conceptions in micro-case production process, (iii) prospective teachers' pre-interview results. These factors acted very crucial role in determination of sufficient, effective, and productive micro-case video preparations and selections for the teaching experiment. Moreover, it gives idea to the researcher about how much time period can be enough video analysis and discussion process.

CHAPTER IV

FINDINGS

This chapter summarized the findings of the current study in three main sections and related subsections. In the first section, prospective middle school mathematics teachers' existing subject matter knowledge (SMK) and pedagogical content knowledge (PCK) about quadrilaterals was analyzed by using their written and verbal responses in individual clinical pre-interviews and initial lesson plans that were conducted before starting teaching experiment sessions. In the second section, I examined how prospective teachers' developed their mathematical knowledge for teaching quadrilaterals throughout the teaching experiment in more detail. In the last section, I summarized the final state of prospective teachers' SMK and PCK about quadrilaterals was summarized by using the data taken from individual post-interviews and revised lesson plans.

4.1 Prospective Teachers' Existing Subject Matter Knowledge and Pedagogical Content Knowledge about Quadrilaterals

This section provides the findings of prospective middle school mathematics teachers' existing subject matter knowledge (SMK) and pedagogical content knowledge (PCK) about (i) *definitions of quadrilaterals*, (ii) *constructions of quadrilaterals*, (iii) *hierarchical relations among quadrilaterals,* and (iv) *properties of quadrilaterals* by the help of constant-comparative method of data analysis. To reveal these findings, I used the data obtained from PST's written and verbal responses in the individual pre-interviews before starting the teaching experiment as well as the initial form of their lesson plans in which they plan to teach quadrilaterals to seventh grade students.

4.1.1 Prospective teachers' existing knowledge about definitions of quadrilaterals

In this part, prospective middle school mathematics teachers' personal and instructional definitions for the concepts of parallelogram, rhombus and trapezoid and their predictions about middle school students' possible definitions/descriptions for these concepts were consecutively presented in order to reflect an essential part of their SMK and PCK about quadrilaterals.

4.1.1.1 Prospective teachers' personal definitions of quadrilaterals

Prospective teachers' personal definitions were examined in terms of providing some logical principles that allow a definition to be mathematically correct; i) *establishing necessary and sufficient conditions*, and ii) *providing hierarchy between general and specific concepts.* The characterization of PSTs' personal definitions of quadrilaterals was asserted in Table 13 in terms of providing inclusivity and establishing necessary and sufficient conditions.

Table 13. Characterization of PSTs' personal definitions in terms of providing inclusivity and establishing necessary and sufficient conditions

PSTs	Parallelogram		Rhombus		Trapezoid	
Aslı	SnNC	Inclusive	NnSC	Exclusive	nNnS	Exclusive
Deniz	SnNC	Inclusive	SnNC	Inclusive	NSC	Exclusive
Beril	SnNC	Inclusive	NSC	Inclusive	NSC	Exclusive
Oya	NnSC	Inclusive	NnSC	Inclusive	NnSC	Exclusive
Ece	SnNC	Inclusive	SnNC	Inclusive	NSC	Exclusive
Zehra	NSC	Inclusive	NSC	Inclusive	NSC	Inclusive
Maya	NSC	Inclusive	NSC	Inclusive	NSC	Inclusive
Emel	NnSC	Inclusive	NSC	Inclusive	NSC	Inclusive

*SnNC: Sufficient but not necessary conditions; NnSC: Necessary but not sufficient conditions; NSC: Necessary and sufficient conditions; nNnS: neither necessary nor sufficient conditions

The analysis of prospective teachers' personal definitions in terms of establishing necessary and sufficient conditions for the parallelogram concept indicated that only Zehra's and Maya's parallelogram definitions involved all necessary and sufficient conditions with accurate mathematical terminological usages (see Table 13). However, Deniz, Ece, Aslı and Beril defined the concept by listing all known properties such as the equality of length of sides or diagonals, which were examples of correct uneconomical definition of parallelogram. Such types of parallelogram definitions were grouped as involving sufficient but not necessary conditions. On the other hand, while Emel defined parallelogram as "*a quadrilateral having two parallel opposite sides*[2]", Oya identified it as "*a figure having opposite parallel sides*[3]". In Emel's definition, it was evident that "*two opposite parallel sides*" is not enough to define parallelogram since trapezoid even can be evaluated a parallelogram example according to this definition. Similarly, Oya used inadequate expressions because she did not mention whether parallelogram is a "*closed*" figure or not. Consequently, Emel and Oya provided necessary but not sufficient conditions when defining parallelogram.

For the rhombus concept, four prospective teachers made correct economical definitions by providing both necessary and sufficient conditions. However, similar to the parallelogram definition, Oya defined rhombus as "*a figure having four equal length sides*[4]". This is a definition including necessary but not sufficient conditions because the definition again involved the term of "*figure*" instead of a "*closed-figure*". On the other hand, Deniz and Ece defined rhombus by mentioning about the angle and diagonal properties in addition to the congruency of all sides of rhombus, which showed that they used sufficient but not necessary conditions in their definitions. They thought that a good definition must involve all known properties of

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[2] Turkish version: Paralelkenar karşılıklı iki kenarı paralel olan dörtgendir.

[3] Turkish version: Paralelkenar karşılıklı kenarları paralel olan bir şekildir.

[4] Turkish version: Eşkenar dörtgen eşit uzunlukta dört kenarı olan bir şekildir.

the defined concepts, which is a very common thinking among learners (De Villers, 1998). Finally, Aslı defined rhombus as "*a figure that all angles are congruent and all sides have equal length*[5]". Because she gave an extra property about the equality of the angles without mentioning about the number of the sides and closeness, this definition might also represent a square or a regular hexagon rather than representing all rhombuses. However, Aslı was unaware of the situation.

Different from parallelogram and rhombus definitions, six prospective teachers interestingly provided necessary and sufficient conditions in their trapezoid definitions. However, five of them provided their definitions based on exclusive relations among trapezoid instead of inclusive relations. On the other hand, Aslı defined trapezoid as "*a rectangular region having a lower base and an upper base*[6]". In this situation, a four-sided figure having no parallel sides can be an alternative representational interpretation of Aslı's incorrect trapezoid definition. Furthermore, Oya's trapezoid definition again involved necessary but not sufficient conditions due to the lack of information about "*closeness*" and "*the number of sides*". At that situation, an unclosed five-sided figure having two opposite parallel sides might be treated as a trapezoid, which causes overgeneralization error. However, Oya was unaware of the consequences of her inadequate personal definitions. Consequently, when considering PSTs' personal definitions, it was evident that prospective teachers were unaware how they can define the concepts of quadrilaterals with minimal conditions in addition to necessary and sufficient ones because they made their definition either by using inadequate expressions or giving extra information (Leikin & Winiki-Landman, 2000; Vinner, 1991; Zazkis & Leikin, 2008).

Although participants generally could not establish necessary and sufficient conditions with a suitable mathematical terminology for the definitions of the concepts, they generally did not have difficulty in providing inclusive descriptions of

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[5] Turkish version: Eşkenar dörtgen tüm açıları eş ve tüm kenar uzunlukları birbirine eşit olan şekildir.

[6] Turkish version: Yamuk alt ve üst tabanı olan dikdörtgensel bölgedir.

the concepts. Interestingly, while they could present inclusive descriptions/definitions of parallelogram and rhombus; they generally preferred exclusive definition of trapezoid. Among them, only Aslı put forth different definitions in terms of inclusivity. For example, while she provided partially inclusive definition for parallelogram because her definition followed extra information such as "*only opposite angles must be congruent in parallelogram*[7]." Moreover, she incorrectly defined rhombus "*a figure that all angles are congruent and all sides have equal length*[8]". From this, it was clearly seen that she treated rhombus as if all angles of it are congruent. Furthermore, she identified trapezoid by an exclusive definition (remember Aslı's definition: "*a rectangular region having a lower base and an upper base*[9]".). The difference among the Aslı's descriptions in terms of inclusivity might be related to her inadequate geometrical content knowledge because her expressions and drawings in individual pre-interviewing process also indicated that she was unaware of the hierarchical relations among quadrilaterals (e.g. rectangles ⊂ parallelograms and squares ⊂ rhombuses).

4.1.1.2 Prospective teachers' instructional definitions of quadrilaterals and predictions about students' possible definitions/descriptions of quadrilaterals

PSTs' instructional definitions are examined in terms of not only mathematical correctness but also pedagogical suitability. More specifically, instructional definitions were analyzed considering some pedagogical considerations such as i) *providing hierarchy*; ii) *establishing necessary and sufficient conditions*; iii) *clarity to the students or ease of understanding*; iv) *intuitiveness*; v) *matching students'*

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[7] Turkish version: Paralelkenarda sadece karşılıklı açılar eştir.

[8] Turkish version: Tüm kenarları eşit uzunlukta olan ve tüm açılaı eş olan bir figürdür.

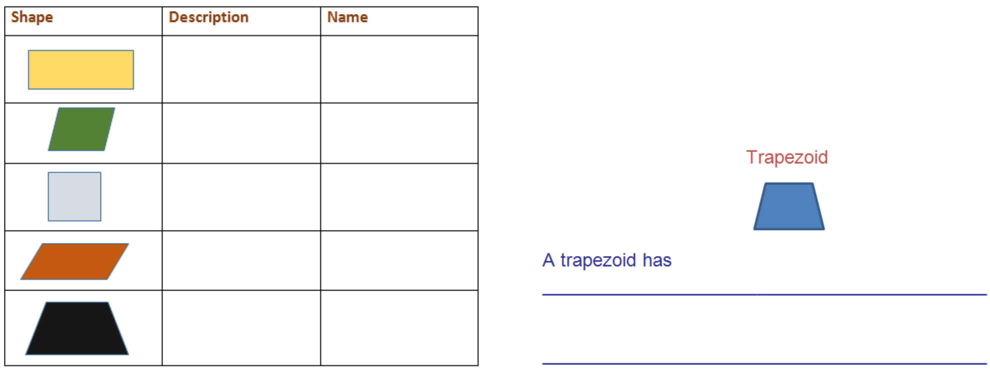
[9] Turkish version: Alt ve üst tabanı olan dikdörtgensel bölgedir.

knowledge and needs vi) *enabling deductive reasoning*. Thus, it was expected to provide a comprehensive and comparable data for prospective teachers' mathematical and pedagogical considerations about definitions of various concepts in quadrilaterals family. Furthermore, PSTs' predictions about how students can describe/define aforementioned concepts were presented at the end of this part.

When explaining their preferences of definitions for any future instructional process, they generally decided to utilize their personal definitions with the limited number of didactical considerations such as "*enabling deductive reasoning*" or "*clarity to the students*". For example, Aslı and Emel generally found economical definitions more effective for learning environment because they claimed that economical definitions might be easy to understand for the students and they can give opportunity to make deductive reasoning. According to them, if they use economical definitions, student may deduce other properties of the concept such as congruency of opposite sides of parallelogram from the given definition. However, some prospective teachers (e.g. Ece, Beril and Deniz) found uneconomical definitions more suitable than economical ones to teach the concepts by drawing attention to their "*clarity*" and "*intuitiveness*". Further, they offered that uneconomical definitions might also provide a complete understanding for the concepts. On the other hand, although they decided to use inclusive definitions for parallelogram and rhombus, most of them preferred to use non-hierarchical definition of the trapezoid by considering students' previous learning and intuitions. For example, Ece made no reasonable explanation why she preferred to use exclusive definition of trapezoid; however, Oya explained pedagogical considerations when selecting the definition of trapezoid in teaching process. She preferred to use exclusive definition of the concept providing the reason that students might have difficulty to understand hierarchical relations among quadrilaterals if the definition of trapezoid is given by considering the inclusive relations of quadrilaterals. However, her explanations indicated that she was unsure which definition type is more suitable in which conditions. In contrast, Emel, Maya and Zehra chose to give inclusive definition when to teach the concept by proposing the benefits of giving relations among quadrilaterals in order to promote the students' conceptual understanding and reasoning abilities. Furthermore, hierarchical definitions were found suitable due to enabling mathematical generalization and deductive reasoning while non-hierarchical definitions were found instructionally suitable by proposing their easy understandable nature and suitability with students' previous learning and intuitions. Considering prospective teachers' instructional definitions from the didactical perspective, it can be concluded that prospective teachers were unaware of the role and influence of a definition on the construction of robust relationship between learners' concept images and concepts definitions because many of them put forth no reasonable idea on why teaching a concept by selecting a specific definition is useful for the students' learning.

On the other hand, in the individual pre-interview process, PSTs provided only a few ideas or predictions about what seventh grade students' possible improper and incorrect definitions/descriptions of quadrilaterals can be. They generally predicted that students may not provide formal definitions of the concepts because of inadequate knowledge about mathematical terminology. Especially, Zehra and Maya who provided all necessary and sufficient conditions for the definitions of the concepts in their personal definitions proposed that students can describe the concept instead of defining. According to them, students might provide correct descriptions of the concepts by listing many redundant properties. Moreover, only Oya and Emel predicted that students might define trapezoid incorrectly because of the usage of the word of "*yamuk*" in Turkish language for "*trapezoid*" by emphasizing on the "*irregular*" meaning of "*yamuk*" in ordinary language. However, any of them did not mention something about how students can use mathematical terms incorrectly or improperly for the concepts of parallelogram or rhombus and they provided limited predictions on the connections between students' inappropriate descriptions and students' conceptions about quadrilaterals. These findings showed that PSTs had inadequate knowledge about determining of students' possible conceptions, misconceptions and difficulties on definitions of quadrilaterals might be.

As a final point, most of PSTs preferred teacher-centered approach for teaching the concepts of parallelogram, rhombus, and trapezoid definitions instead of a student-centered approach. According to teacher-centered approach, they preferred to give definitions or constructions of quadrilaterals on the board or in a Venn diagram in the beginning of their instructional plans. For example, in the lesson plans, while Deniz decided to use the diagram in Figure 12-a, Beril provided a prototypical example for the concepts as in Figure 12-b in order to pass definitions of the concepts.



(a) (b)

Figure 12. (a) Deniz's visual-based strategy to teach the definitions of quadrilaterals. (b) Beril's instructional approach to teach the definitions of quadrilaterals

Similarly, Oya, Maya, Aslı and Ece considered that giving visual representations of the concepts before the definitions is more useful for supporting students' conceptions about quadrilaterals. Their explanations were as following:

> I initially introduce concepts by drawing figures. In any case they can find the definition themselves, because figure is always learnt more easily [Aslı, Initial lesson plan].

> It is hard to see the relationship between quadrilaterals by examining their definitions. However, we can easily teach that square is also a parallelogram by drawing their figures [Ece, Initial lesson plan].

However, Emel and Zehra did not prefer to use definitions in their lesson plans. Instead, they generally focused on properties of quadrilaterals.

4.1.2 Prospective teachers' existing knowledge about constructions of quadrilaterals

In this part, PSTs' personal constructions, predictions about students' possible drawings, and instructional preferences to teach the concepts to middle school students were addressed for the three concepts of parallelogram, rhombus, and trapezoid, respectively. For all categories of personal drawings, predictions about students' drawings, and instructional drawings, prospective middle school mathematics teachers were asked to draw three of more different parallelograms and asked why they thought their drawings were different from each other.

4.1.2.1 Prospective teachers' personal constructions of quadrilaterals

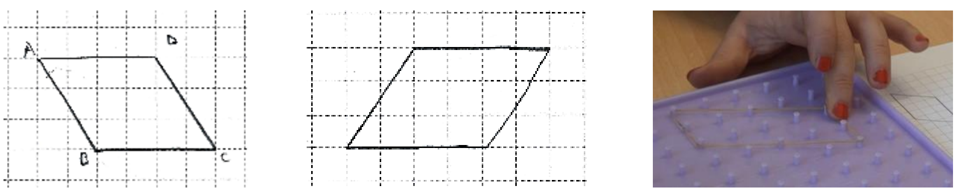
The nature of prospective teachers' personal constructions of quadrilaterals was summarized in terms of prototypicality (e.g. *prototypical*/*partial-prototypical*/*non-prototypical*) and hierarchical structure (e.g. *hierarchical*/*partial hierarchical*/*non-hierarchical*) in Table 14.

Table 14. Characterization of PSTs' personal constructions of quadrilaterals

	Prototypicality and hierarchical structure		
PSTs	Parallelogram	Rhombus	Trapezoid
Aslı	PT & NH	PT & NH	PT & NH
Deniz	PT & PH	PT & NH	NPT & NH
Beril	PT & PH	PPT & H	PT & PH
Oya	PT & PH	PPT & H	PT & PH
Ece	PT & NH	NPT & NH	NPT & NH
Zehra	PT & H	NPT & H	NPT & H
Maya	PT & H	PPT & H	NPT & H
Emel	PT & PH	PPT & H	PT & NH

PT: prototypical examples, PPT: partial-prototypical examples, NPT: non-prototypical examples; H: hierarchical examples, PH: partial-hierarchical examples, NH: non-hierarchical examples

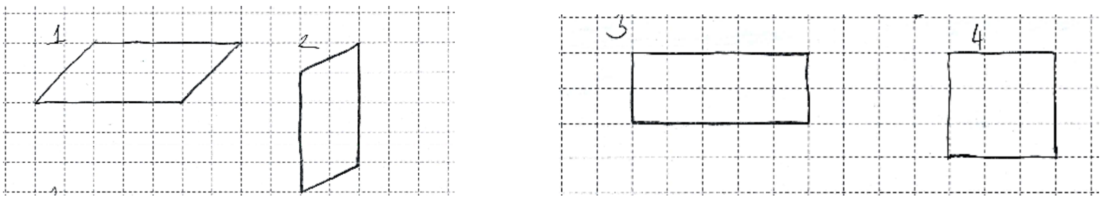
For parallelogram concept, Aslı and Ece constructed only prototypical and non-hierarchical parallelogram examples like first two parallelograms in Figure 13-a. These two figures could be accepted as an indicator for the presence of prototypical and non-hierarchical concept images of parallelogram in their minds. To be sure, I asked the reasons why they thought these two figures were different examples for a parallelogram. Aslı's explanations were as following: "*Paralllеogram rotated 180° can also be considered as parallelogram. Furthermore, we can rotate the page and look the figures. Their size can change also (she indicated* Figure 13-c*).* Aslı's explanations showed that she made differentiation when drawing parallelogram according to only vertical or horizontal orientation of the figures rather than family relations among quadrilaterals. She then used geoboard to produce different parallelograms. However, she only changed the length of the sides instead of producing a figure as rectangle (see Figure 13-b). This situation asserted the influence of the strong visual characteristics of prototypical images on the drawings and constructions of parallelogram as an inclined shape similar to the results of some studies (Fujita 2012; Hershkowitz, 1990). Another essential point in above explanations was that she determined to draw her figures considering the influence of her drawings on students' concept images. This approach was interesting to present participant's didactical considerations when even making personal drawings of parallelogram.



(a) (b)

Figure 13. Aslı's parallelogram constructions (a) in grid paper (b) in geoboard

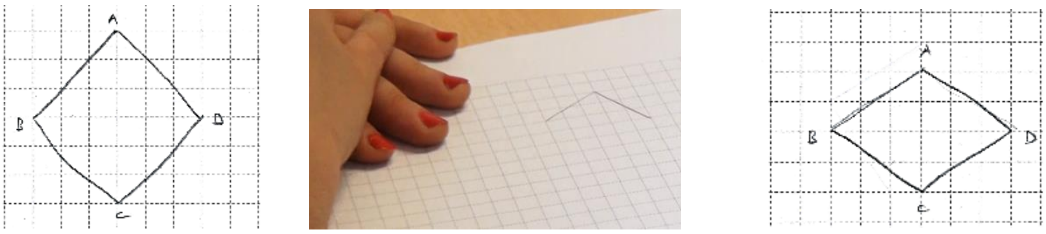
On the other hand, as seen in Table 14, four prospective teachers' (Deniz, Beril, Oya and Emel) personal constructions of parallelogram were grouped into partial-hierarchical and partial-prototypical structure. Inıtially, they drew parallelograms like in Emel's first and second examples in Figure 14-a. These figures showed that they just perceived "difference" as the differentiation of orientation of figures either vertically or horizontally. After thinking for a short time, they elaborated their thinking and produced third and fourth shapes in Figure 14-b. At that time, they partially focused on inclusive relations among quadrilaterals in terms of parallelogram by drawing of square and rectangle, but the image of rhombus was not evoked in their minds. When I asked the question of why you did not draw rhombus they made the similar explanations such as "*it did not come my mind because I thought rhombus is a very special quadrilateral*" (Oya). Finally, as seen in Table 14, Maya and Zehra provided prototypical and hierarchical constructions for parallelogram concept because they added prototypical rhombus into their constructions in addition to prototypical rectangles and squares.



(a) (b)

Figure 14. (a) Emel's parallelogram constructions (b) Emel's parallelogram examples to imply hierarchical relations among quadrilaterals

For rhombus concept, Table 14 illustrated that prospective teachers constructed different types of drawings in terms of prototypicality and hierarchical structure. For example, Aslı and Deniz constructed rhombuses within prototypical and non-hierarchical structure. To exemplify, Aslı's constructions were given in Figure 15. There was an important situation in Aslı's expressions in individual pre-interview because she couldn't distinguish the differences and similarities between rhombus and square. Thus, she was undecided about the correctness of her drawing (see Figure 15-b) with regard to whether a square is also a rhombus or not. However, her inappropriate rhombus definition (only squares could be considered as rhombus according to her definition) and current drawings showed lack of suitable and robust interactions between her concept definition and concept image, which leads instrumental learning rather than conceptual ones (Vinner, 1991). Otherwise, inability to distinguish rhombus from square also revealed her inadequate subject matter knowledge about related concept and its characteristic properties. In order to get a deeper understanding about the participant's thinking process; I gave an opportunity to the participant by asking her to draw again a rhombus figure on the grid paper.



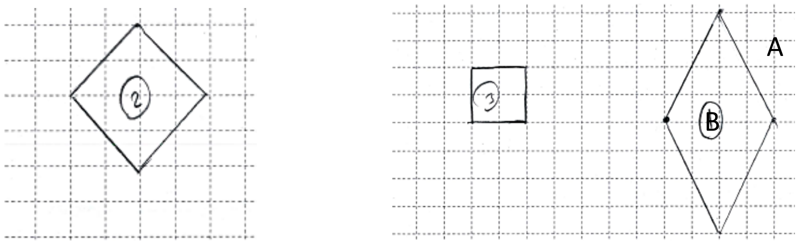
(a) (b) (c)

Figure 15. (a) Aslı's first rhombus construction (b) Aslı's trial-error construction (c) Aslı's final prototypical rhombus construction

Researcher	Could you please draw the figures coming to your mind when said rhombus?
Aslı	(She drew Figure 15-a but she thought it is wrong) I think it is wrong.
Researcher	Why do you think it is wrong?
Aslı	This figure resembled a square. Just because, for a moment I thought that if square is also a rhombus. In my opinion square is also a rhombus.
Researcher	This figure confused you, well now, what do you draw once again.
Aslı	I mean there will show up a figure like this, so I could not be sure if I have to draw or not. Specifying corner points were difficult for me. (She is erasing Figure 15-b)
Researcher	I would say try to draw once again.
Aslı	(She is drawing Figure 15-c.)
Researcher	Are you sure that this figure will be a rhombus?
Aslı	Not really but I suppose I am sure.

Although she attempted to draw a prototypical rhombus she again constructed a non-prototypical square and then she decided to erase the figure (see Figure 15-b). She explained her difficulty to determine suitable points as the corner points of a prototypical rhombus in the grid paper. By the help of researcher, the participant produced a prototypical rhombus in Figure 15-c. This situation asserted participant's inability in using grid paper by ensuring all required properties of a geometric figure.

Table 14 also indicated that four participants (Beril, Oya, Maya, and Zehra) drew partial prototypical rhombus examples considering hierarchical relations among rhombus and square (see the example construction in Figure 16). As a detailed example, in their construction process, I realized that although Oya wanted to draw a prototypical rhombus she drew a non-prototypical square. After thinking for a time, she constructed a new figure, but she also drew a rotated square rather than a prototypical rhombus as in Figure 16-a. In the interviewing process, by the researcher's guidance, she could construct a prototypical rhombus (diamond) in Figure 16-c. She then drew a prototypical square (see Figure 16-b). Her difficulty in construction process might be related to being unaccustomed to using grid paper when drawing a geometric shape by ensuring all necessary and sufficient properties.



(a) (b) (c)

Figure 16. Oya's personal constructions of rhombus

Differently, Zehra and Ece constructed non-prototypical rhombus examples; however, their construction differed in terms of hierarchical aspect (see Table 14). More specifically, as Zehra drew square examples in addition to rhombus examples, Ece only constructed rhombus examples with the rotated ones.

When I examined participants' interview data related to personal trapezoid constructions and explanations in terms of prototypicality, I noticed that four of them (Aslı, Beril, Oya, and Emel) visualized trapezoid considering only prototypical examples even if they provided their figures according to either exclusive relations or inclusive relations among quadrilaterals (remember Table 14). Among them, Aslı and Emel solely focused on the special types of trapezoid in her drawings such as right, scalene, and isosceles trapezoid, respectively in Figure 17.

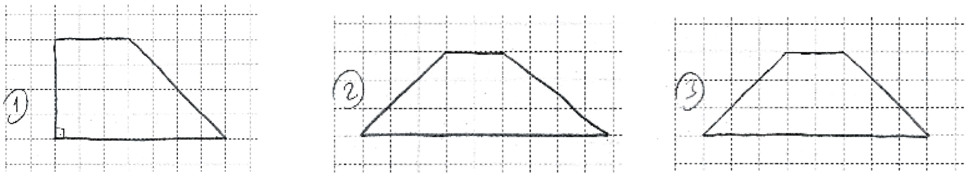


Figure 17. Examples of participants' prototypical trapezoid constructions

However, Beril and Oya added rectangle, parallelogram or square figures to their trapezoid examples. In other words, their constructions were categorized in prototypical and partial-hierarchical constructions as in Figure 18. They started her drawings with prototypical trapezoid types like isosceles and right trapezoid, respectively. Then, they drew third non-prototypical trapezoid figure. This drawing indicated their image related to non-prototypical trapezoid shape as such 180° degree-rotated version of a prototypical trapezoid. Furthermore, the fourth and fifth drawings in Figure 18 reflected the participants' hierarchical understanding about quadrilaterals regarding trapezoid. This understanding was also supported by the following explanations "*there are many of trapezoid examples such as rectangle and square due to the definition of trapezoid*."(Oya).

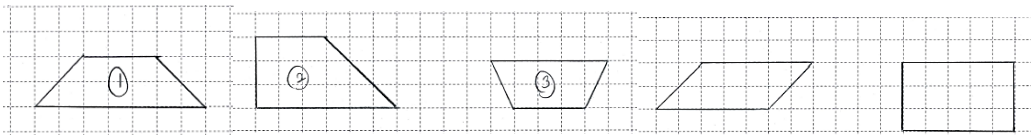


Figure 18. Oya's personal prototypical and partial-hierarchical trapezoid constructions

As seen in Table 14, remaining four participants (Deniz, Ece, Zehra, and Maya) visualized trapezoid considering non-prototypical examples even if they provided their figures according to either exclusive relations or inclusive relations among quadrilaterals. For example, Deniz and Ece constructed their figures according to exclusive relations among quadrilaterals; however, Zehra and Maya could also draw prototypical examples of rectangle, square, rhombus, and parallelogram considering the inclusive relations among quadrilaterals (see examples in Figure 19). For instance, in the following, Zehra expressed that she visualized all rotated versions of the all figures in her mind.

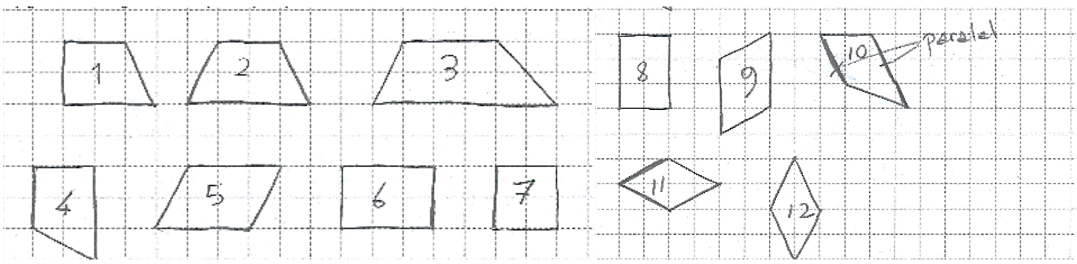


Figure 19. Zehra's personal trapezoid constructions

4.1.2.2 *Prospective teachers' predictions on students' possible constructions of quadrilaterals*

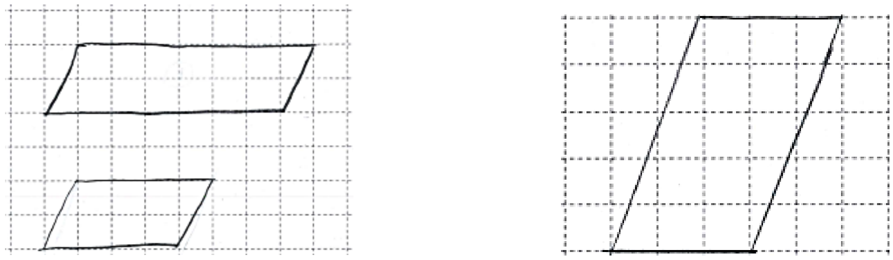
Prospective teachers' predictions about students' possible conceptions, misconceptions, and errors about a mathematical concept was essential to determine their pedagogical content knowledge with respect to understanding of student mathematical thinking. In this regard, Prospective teachers' approaches used to determine students' possible drawings for parallelogram, rhombus and trapezoid were grouped according to the methodological and cognitive approach used by Tsamir, Tirosh and Levenson (2008) as in the following: how they predicted middle school students' possible i) *example drawings;* ii) *non-example drawings*; and iii) *difficulties in drawings.* Furthermore, their knowledge about possible reasons of students' problematic drawings (e.g. learners' intuitions, lack of previous knowledge about basic geometric concepts, disconnection among concept definition and concept images, prototypical reasoning, the examples given in the textbooks and math lessons etc.) was examined according to their inferential ideas. Table 15 summarized prospective teachers' predictions about students' possible constructions involving their examples in terms of prototypicality and hierarchical structure, and non-examples.

Table 15. Participants' predictions on students' constructions of quadrilaterals

PSTs	Parallelogram*	Rhombus	Trapezoid
Aslı	PT & NH	PT & H	PT & NH
Deniz	PT & NH	PT & NH	PT & NH
Beril	PT & NH	PT & H	PT & NH + Non-exp.
Oya	PT & NH	PT & NH + Non-exp.	PT & NH + Non-exp.
Ece	PT & NH	PT & NH	PT & NH
Zehra	PT & NH + Non-exp.	PT & H	PT & NH
Maya	PT & NH	PT & NH	PT & NH
Emel	PT & NH	PT & NH	PT & NH+ Non-exp.

*PT: prototypical examples, PPT: partial-prototypical examples, NPT: non-prototypical examples; H: hierarchical examples, PH: partial-hierarchical examples, NH: non-hierarchical examples; Non-exp.: Non-examples

As seen in Table 15, all prospective teachers generally could predict students' possible correct parallelogram examples rather than considering the drawings showing students' possible non-examples involving their contradictions, misconceptions, and errors. According to them, almost all students are able to draw at least a prototypical parallelogram figure considering exclusive relations among quadrilaterals. For this reason, they generally drew similar parallelograms as in Figure 20-a. Many of them claimed that some students are able to only shorten or extend the length of the sides without any manipulations on the figure such as rotation/orientation changes with an angle different from 90° and its positive integer multipliers. They proposed that limited number of students can draw the rotated shape in Figure 20-b as a non-prototypical parallelogram example. This situation revealed that PSTs couldn't recognize that it was already a prototypical figure.



(a) (b)

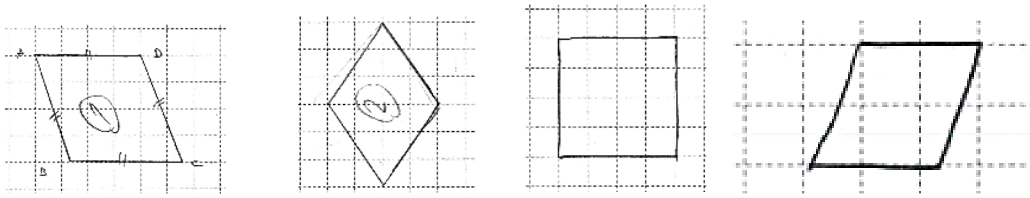
Figure 20. Oya's predictions about students' possible (a) prototypical parallelogram constructions (b) non-prototypical parallelogram construction

After my critical question about why they thought students cannot draw rectangle or square to exemplify a parallelogram figure, they explained their expectations intuitively based on the students' difficulties in comprehending hierarchical relations among quadrilaterals. Yet, the result of previous studies conducted with middle school students in order to understand their conceptions about quadrilaterals revealed that some students can construct partial hierarchical relations. Consequently, prospective teachers' pedagogical content knowledge related to understanding of students' possible mathematical conceptions were limited to only students' correct prototypical and non-hierarchical concept images of parallelogram. In this regard, Shulman (1987) mentioned that understanding students' possible conceptions involves the knowledge about their prior knowledge, learning difficulties, errors, the reasons of the difficulties and misconceptions. From this perspective, only Zehra predicted that students may draw a non-example such as trapezoid just supposing it as a parallelogram since they couldn't pay attention the properties of grid paper. However, she proposed that students can unconsciously draw a trapezoid.

In sum, participants generally predicted both students' drawings involving their correct prototypical concept images of parallelogram with regard to exclusive relations. However, they (excluding Zehra) couldn't construct any drawing that shows students' possible errors such as an overgeneralized situation (e.g. treating trapezoid as an example of parallelogram).

In Table 15, prospective teachers' predictions showed that they only made their predictions in terms of prototypicality and hierarchical relation among rhombus and square. As a result, they only concentrated on what students' possible correct drawings can be rather than focusing on how they may have difficulty or how they may make incorrect drawings such as non-examples. More specifically, four of them (Deniz, Ece, Maya, and Emel) have predicted that students solely draw prototypical rhombus examples according to exclusive relations among quadrilaterals. In this regard, they claimed that students can suppose a rotated square as a prototypical rhombus example. According to them, because students cannot comprehend the difference between a rotated square and prototypical square they can easily think that a square is not a rhombus due to being a special quadrilateral.

Differently, three prospective teachers (Aslı, Beril, and Zehra) predictions could be evaluated were examples of hierarchical and non-prototypical structure. In this sense, they thought that students could construct a square as a rhombus example because all sides of a square have equal length (see example in *Figure 21*). Interestingly, in the interview process, I observed that since Aslı was aware of her inadequate knowledge about the hierarchical relation between rhombuses and squares while making personal drawings for rhombus, she predicted that students also may have same difficulty. Following explanations clearly illustrates the situation: "*In my opinion, they can draw square too. Because they may be confused like me when said "equal length, I will draw a square too"* (Aslı). As a result, she added square into students' possible drawings in Figure 21. Thus, this situation indicated how participant's subject matter knowledge might influence her pedagogical content knowledge about understanding of students' thinking.



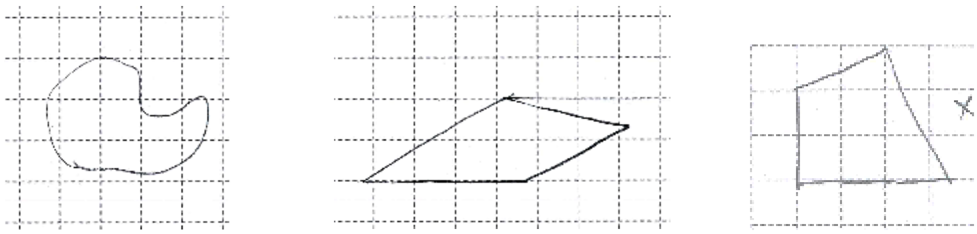
(a) (b)

Figure 21. (a) Oya's predictions (b)Aslı's predictions about students' possible drawings for rhombus

Another crucial point is that only Oya made prediction about students' possible incorrect drawings rather than focusing only on their correct prototypical concept images such as the first shape in Figure 21. She constructed this figure by proposing that students can draw this figure as rhombus even if it is not because they think that there are two square units for each edge. It was interesting that although Oya had no idea about students' incorrect drawings for parallelogram concept, she could give an example of students' possible incorrect drawing for rhombus. This situation indicated that prospective teachers' pedagogical content knowledge can differ as the concepts, phenomenon, etc. differ.

Participants generally made predictions about students' possible trapezoid drawings by not focusing on their possible contradictions and incorrect drawings (see Table 15). They mostly concentrated on students' possible correct but prototypical trapezoid drawings. They considered that students never make a relationship between a trapezoid and rectangle, square, rhombus and parallelogram. According to them, understanding the hierarchical relations especially in terms of trapezoid concept is very difficult for the primary and middle school students. On the other hand, participants' drawings and explanations revealed their inadequate knowledge about students' possible trapezoid drawings reflecting students' difficulties, misconceptions and errors. In this regard, only three PSTs (Beril, Oya, and Emel) provided some predictions on students' possible incorrect drawing and the points in which students can have difficulty in addition to students' correct drawings for trapezoid. For example, they proposed that some students can draw a four-sided irregular quadrilateral having no parallel opposite sides (see Figure 22-b and Figure

22-c) or an irregular figure (see Figure 22-a) because of the nomination of the trapezoid called "*yamuk*" with the meaning of "*irregular*" in Turkish language.



(a) (b) (c)

Figure 22. (a) Emel's prediction (b) Beril's prediction (c) Oya's prediction on students' incorrect drawings of trapezoid

4.1.2.3 Prospective teachers' instructional constructions of quadrilaterals

Prospective teachers' personal drawings reflected their subject matter knowledge about drawing of quadrilaterals. Besides, their knowledge about how any middle school student can draw various quadrilaterals and which type of drawings PSTs prefer to use in their instructional plans about quadrilaterals presented their pedagogical content knowledge. For this reason, prospective teachers' instructional drawings also were examined in terms of prototypicality and hierarchical structure to illustrate their existing pedagogical strategies in this part.

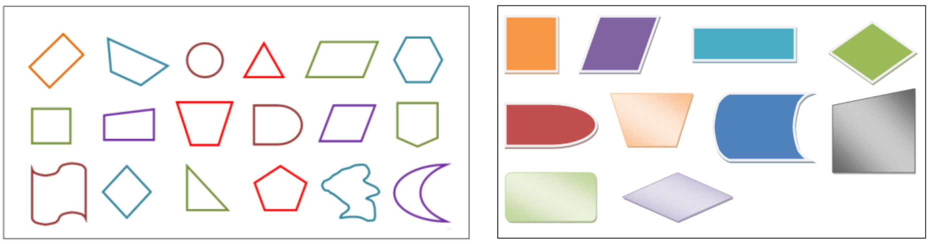
Their example instructional constructions were grouped according to inclusive relations of quadrilaterals and prototypicality in Table 16. When making a comparison between Table 14 and Table 16, I noticed that participants' instructional preferences for the constructions of quadrilaterals were almost same with their personal constructions in terms of prototypicality and hierarchical nature.

Table 16. PSTs' instructional preferences for the constructions of quadrilaterals

PSTs	Parallelogram*	Rhombus	Trapezoid
Aslı	PT & NH	PT & NH	PT & NH
Deniz	PT & PH	PT & NH	NPT & NH
Beril	PT & PH	PPT & H	PT & PH
Oya	PT & PH	PPT & H	NPT & NH
Ece	PT & NH	NPT & H	NPT & NH
Zehra	PT & H	NPT & H	NPT & H
Maya	PT & H+counter-exp.	PPT& H+counter-exp.	PT & H+counter-exp.
Emel	PT & PH	PPT & H	PT & NH

*PT: prototypical examples, PPT: partial-prototypical examples, NPT: non-prototypical examples; H: hierarchical examples, PH: partial-hierarchical examples, NH: non-hierarchical examples; counter-exp.: counter examples

In terms of hierarchical structure, it was evident that only Zehra and Maya planned to give their instructional constructions for quadrilaterals by taking into account inclusive relations among quadrilaterals. The constructions in their lesson plans also supported their preferences as in Figure 23 since they preferred to use prototypical and hierarchical constructions in order to teach quadrilaterals and their properties. Furthermore, only Maya planned to give counter-examples (e.g. implying critical properties of trapezoid by showing a parallelogram example) in her instructional plans in order to make emphasis on critical attributes of the concepts by comparing properties of examples and counter-examples.



(a) (b)

Figure 23. (a) Maya's and (b) Zehra's instructional constructions of quadrilaterals in the initial lesson plans

The participants who had partial-hierarchical or hierarchical constructions proposed two-stage approach in order to teach the concept of parallelogram. In the following, an example excerpt taken from individual pre-interview excerpt illustrated why they proposed to utilize such kind of teaching approach in their instructional plans.

Researcher	Which parallelogram figures do you draw when you teach in your lesson?
Oya	First of all I introduce 1st and 2nd figures and show that two opposite sides are parallel [in Figure 18].
Researcher	Why do you not introduce square and rectangle initially?
Oya	I do not introduce them, because firstly they have to understand what parallelogram means exactly. Their minds do not have to confuse. They do not have to say "Are not square and rectangle different, are they parallelogram now?" Thus introducing them later and discussing is more logical.

In the light of above excerpt, it was evident that Oya believed that after students comprehend all required properties by encountering prototypical parallelogram figures they can be more confident in making relations among quadrilaterals. For this reason, she found useful to firstly utilize prototypical parallelograms when to teach parallelogram. Following that, she preferred to draw a non-prototypical rectangle and a prototypical square. Thus, although she put emphasis on the relations among quadrilaterals superficially, she did not mention any point related to what she will plan to prevent students' possible misconceptions and errors, or which plans, tasks, and activities she will develop to enhance students' conceptions about parallelogram because she was unaware of students' all types of conceptions.

4.1.3 Prospective teachers' existing knowledge about hierarchical relations among quadrilaterals

4.1.3.1 Prospective teachers' SMK about hierarchical relations among quadrilaterals

Prospective middle school mathematics teachers' existing subject matter knowledge about hierarchical relations among quadrilaterals was summarized in Table 17 by considering their identifications of related figures among different polygons (e.g. parallelogram identification task in Figure 24), as well as their oral expressions that reflect how they classify quadrilaterals. Their responses grouped into four categories in terms of hierarchical structure: *hierarchical classification*; *partial-hierarchical classification*, *non-hierarchical classification*; and *overgeneralized classification.* In hierarchical classification included identifications of figures based on inclusive relations between quadrilaterals. Partial hierarchical classifications were based on partial inclusive relations (e.g. considering rhombus as an example of parallelogram, but considering rectangle and square as non-examples of parallelogram). On the other hand, non-hierarchical classification involved exclusive relations between quadrilaterals. Finally, overgeneralized classification involved prospective teachers' overgeneralization errors in identification (selection) of quadrilaterals (e.g. treating trapezoid as an example of parallelogram).

Table 17. Participants' SMK about hierarchical relations among quadrilaterals

PSTs	Parallelogram	Rhombus	Trapezoid
Aslı	Partial-hierarchical	Hierarchical	Non-hierarchical
Deniz	Hierarchical	Hierarchical	Hierarchical
Beril	Overgeneralized classification	Hierarchical	Overgeneralized classification
Oya	Hierarchical	Hierarchical	Hierarchical
Ece	Hierarchical	Hierarchical	Hierarchical
Zehra	Hierarchical	Hierarchical	Hierarchical
Maya	Hierarchical	Hierarchical	Hierarchical
Emel	Hierarchical	Hierarchical	Hierarchical

As seen in Table 17, almost all prospective teachers had knowledge about inclusive relations among quadrilaterals because they provided adequate information about hierarchical relations of quadrilaterals. However, Aslı's and Beril's responses involved some problematic aspects regarding their SMK. In order to provide detailed information about Aslı's difficulties and doubts in personal parallelogram selection process, individual pre-interview excerpt is presented below:

Researcher	How do you make your choices? Which figures are parallelograms?
Aslı	Quadrilaterals are subset of parallelogram also. I could not decide if I should choose them. 1, 10, 11 and 14 are definitely parallelogram. (she is examining the other shapes in Figure 24). If I choose 5, I have to choose 11 and 13.
Researcher	You have already chosen 11. Make up your mind to select the others.
Aslı	Yes. I will select them. 1, 5, 10, 11, 13, 14 are parallelogram.
Researcher	Are these your final decision? Are you saying you are not choosing the others?
Aslı	Yes. Exactly.
Researcher	Why did you decide to choose these figures? For example why did not you choose 4 and 9 as an example of parallelogram?
Aslı	I have chosen them because there are two pairs of opposite parallel sides in each figure. Differently, all sides are equal length in 4 and 9; on the other hand only opposite sides are equal in parallelogram. So actually quadrilaterals are subset of parallelogram.
Researcher	You thought a lot while choosing figure 5. What did you think about then?
Aslı	I thought like this during that time: 5 and 9 are similar.
Researcher	How do you define figure 5?
Aslı	Figure 9 is rhombus and it is almost the rotated form of 4. We can say figure 9 is square.
Researcher	You were undecided when selecting 5 as a parallelogram. I wonder about the reason of it. For example, you have chosen 1 and 14 immediately.
Aslı	Because 1 and 14 were very similar to the figures we saw in our lessons. For figure 5, I thought that if I have to take also rhombus as a subset of parallelogram because its all sides are equal lenght.
Researcher	Did we take it now? We said that rhombus is a parallelogram.
Aslı	Yes, we took it.
Researcher	If you have doubts you can specify there.
Aslı	I'm really not sure about for 9 whether it is a parallelogram or not. I'm not sure for 4 and 9.

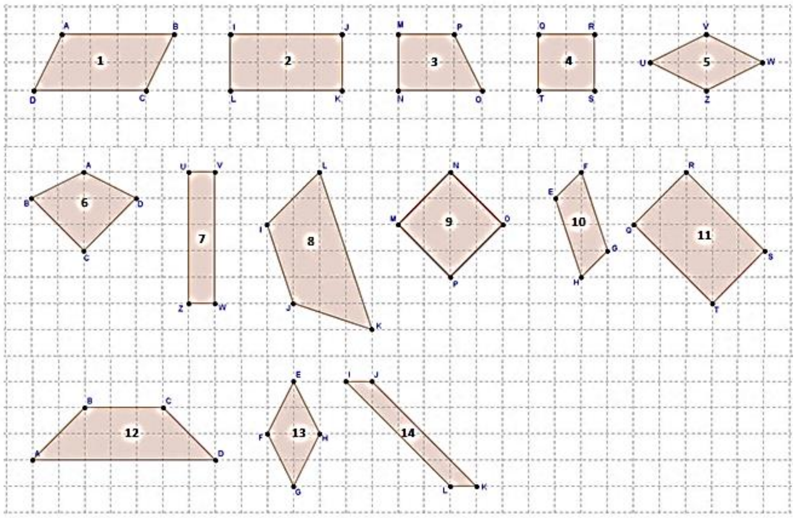


Figure 24. Parallelogram identification task

Aslı's explanations indicated that she firstly focused on prototypical parallelogram figures as 1, 10, and 14. However, although she then selected non-prototypical rectangle (11 in Figure 24) as a parallelogram, she did not consider prototypical rectangle as a parallelogram (2 in Figure 24), which revealed her prototypical concept images about parallelogram in her mind. Afterwards, she was undecided about whether she selects rhombuses (5 and 13 in Figure 24) as a parallelogram example or not. Nevertheless, she added rhombus figures in her parallelogram selections. After researcher asked why she did not consider a square as a parallelogram type, her limited concept images again appeared because she considered parallelogram as a quadrilateral having two shorter and two longer parallel sides. Moreover, she thought a rotated square as an example of prototypical rhombus from a first impression. When I asked the reason why she had difficulty in deciding whether rhombus is a parallelogram or not Aslı's inadequate subject matter knowledge about hierarchical relations among quadrilaterals reemerged because she focused only appearance of a prototypical example of the figure rather than focusing the critical attributes in the inclusive definition of rhombus. Furthermore, researcher's probing question that aimed to understand her quandary about the relationship between square and parallelogram revealed that she thought square as a special type of quadrilaterals due to its equal length sides.

Differently, as Beril considered trapezoid as a parallelogram example, she made overgeneralization error when classifying all given shapes in Figure 24. For example, she also treated trapezoids (e.g. 3, 8, and 12 in Figure 24) as examples of parallelogram in addition to all square, rectangle, parallelogram and rhombuses examples. Furthermore, Beril took five-sided polygon in Figure 25 as a trapezoid example because she only concentrated on the parallelism of two opposite sides rather than focusing on the number of sides.

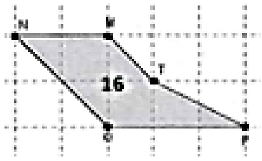


Figure 25. The figure Beril considered as an example of trapezoid

4.1.3.2 Prospective teachers' PCK about hierarchical relations among quadrilaterals

In this part, it was examined what prospective teachers know about students' possible conceptions in terms of hierarchical aspect among quadrilaterals in order to reflect their PCK related to understanding students' mathematical thinking. Furthermore, the information about how they plan to give hierarchical relations among quadrilaterals in their lesson plans were given in order to reveal PSTs' existing PCK. Prospective teachers could successfully predict students' possible prototypical and hierarchical approaches on the selection of parallelograms and rhombuses. However, all of them proposed that students may select trapezoid examples only by considering exclusive relations of quadrilaterals.

To be more precise, how prospective teachers made predictions on students' possible conceptions on hierarchical relations considering on their responses involving predictive ideas on students' possible selections of parallelogram in among polygons in Figure 24 was asserted in Table 18.

Table 18. Participants' classifications for students' possible parallelogram selections

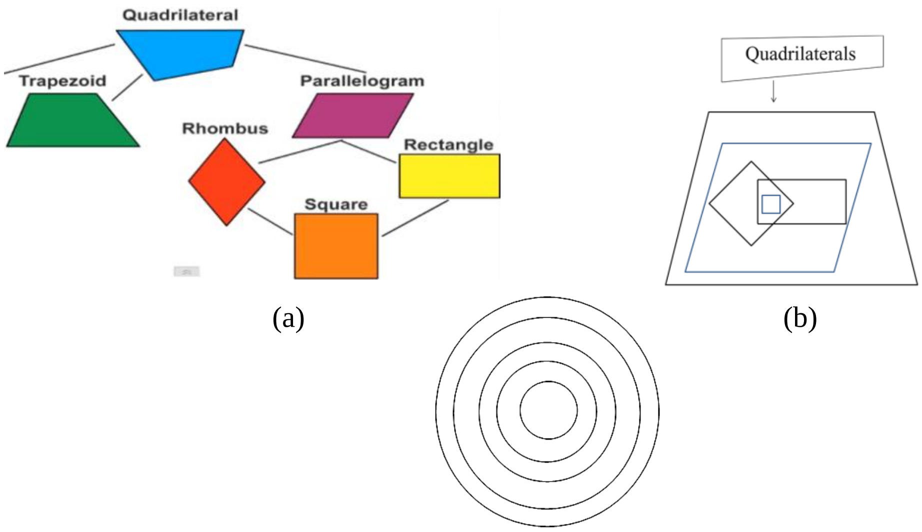
PSTs	Predictions for students' possible parallelogram selections	Meaning of the predictive selections*
Aslı	1-10-14-11	Par + npRec
	1-10-14-11-5-9-13	Par + npRec + Rho
	1-10-14-11-5-9-13-2-7	Par + Rho + Rec
	1-10-14-11-5-9-13-2-7-4	Par + Rho + Rec+ Squ
Oya	1-10-14-5-13	Par + Rho
	1-10-14-5-13-14-2-4-7-9-11	Par + Rho + Rec + Squ
	Selecting figure 12 as a Par	Overgeneralized situation
	Selecting fig 6 having no parallel sides as a Par	Overgeneralized situation
Emel	1-10-14-11	Par + npRec
	1-10-14-5-9-13	Par + Rho
	1-10-14-5-9-13-2-4-7	Par + Rho + pRec + pSqu

*Abbreviations in the column means that Par-Parallelogram, Rho-Rhombus, Rec-Rectangle, Squ-Square, np-nonprototypical, p-prototypical

For example, Aslı made four categories that involve either students' correct selections or correct/incomplete selections in order to provide her predictions on students' possible conceptions about quadrilaterals. For instance, first one presents her prediction on students' prototypical concept images because she pointed not only prototypical parallelogram but also non-prototypical rectangle (11 in Figure 24) because of its visual similarity with a prototypical parallelogram. She continued her predictions about students' understanding involving partial hierarchical relations of quadrilaterals by adding rhombus and rectangle figures. She finalized her predictive selections by proposing that some students can construct all hierarchical relations in terms of parallelogram concept. Differently, Oya focused on students' possible incorrect selections in addition to hierarchical or partial hierarchical selections. To be more precise, she had some prediction on students' possible selection involving only rhombuses and parallelograms. According to her, students can decide whether a figure is a parallelogram considering the presence of inclined opposite parallel sides. For instance, a group of students couldn't select square and rectangle as a parallelogram due to perpendicular sides. Moreover, she proposed a predictive category for high achieved students' possible selections involving complete and correct hierarchical relations among quadrilaterals if they use the meaning of parallelogram definition. Upon my question that aimed to increase her concentration and to elaborate her thinking process, Oya proposed that if students think two parallel opposite sides were enough for a parallelogram they can consider that trapezoid is also a parallelogram. Moreover she elaborated her thinking with the idea of a quadrilateral having no parallel sides (6 in Figure 24) can be taken as a parallelogram if students pass over the absence of parallelism. At this regard, these predictions indicated that she considered both students' correct and incorrect selections.

Emel's predictions had similarities and differences with Aslı's and Emel's predictive selections. For instance, Emel could not predict any students' possible incorrect parallelogram selections. Instead, she focused on students' partial hierarchical and non-hierarchical selections in the light of prototype phenomenon. To be more precise, she proposed a group of students are only able to select parallelogram and non-prototypical rectangle (11 in Figure 24) due to its visual similarity with parallelogram based on exclusive relations among quadrilaterals. Additionally, she claimed that a group of students can make partial relation between quadrilaterals like rhombus is also a parallelogram because of visual similarity between them. Finally, she suggested that a few number of students may comprehend the hierarchical relations completely and correctly. However, Emel's predictions also showed that she has no idea about students' incorrect selections although she had attended high achievement participant category. That is, even if a participant who was well at subject matter knowledge about parallelogram it did not guarantee having a well-structured pedagogical content knowledge on related concept.

As instructional approach to teach hierarchical relations of quadrilaterals, six participants explained their preferences on teaching of quadrilaterals by taking account of inclusivity of quadrilaterals in their lesson plans. They planned to give relations among the quadrilaterals by using Venn diagrams or concept maps in their plans. Five of them utilized to use Venn diagram. Some participants' diagrammatic representations were presented in Figure 26. However among these diagram, Maya's representation was found incorrect as in Figure 26-c because she could not correctly visualize the relations of square, rhombus, and rectangle.



(c)

Figure 26. PSTs' representations for hierarchical relations of quadrilaterals in their lesson plans (a) Deniz' scheme and (b) Oya's scheme (c) Maya' scheme

On the other hand, Zehra and Aslı did not prefer to use such kind of diagrams in their lesson plans. Instead, they generally concentrated on the properties of quadrilaterals or calculations of area of quadrilaterals.

4.1.4 Prospective teachers' existing knowledge about properties of quadrilaterals

4.1.4.1 Prospective teachers' existing SMK about properties of quadrilaterals

In order to reflect problematic situations in PSTs' SMK about the properties of quadrilaterals, their incorrect responses were illustrated in Table 19.

Table 19. PSTs' incorrect responses about properties of quadrilaterals

PSTs	Parallelogram	Rhombus	Trapezoid
Aslı	-Diagonal is angle bisector -Diagonals have equal length	If diagonals are equal length they become perpendicular.	X
Deniz	X	X	X
Beril	-Diagonals are angle bisector -Diagonals have equal length	-Diagonals have equal length.	X
Oya	-Diagonals have equal length	X	X
Ece	-Diagonals are perpendicular. -Diagonal is angle bisector. -Diagonals have equal length.	X	X
Zehra	X	X	X
Maya	-Diagonals are angle bisector.	X	X
Emel	-Diagonals have equal length	-Diagonals have equal length	X

X means that there is no misconception.

Participants provided correct information about side and angle properties of trapezoid. This might be related to the nature of trapezoid having only two characteristic features as (i) at least two opposite sides are parallel, and (ii) adjacent angles along the sides are supplementary. In conclusion, PST's SMK was insufficient when considering their responses about especially properties of parallelogram and rhombus because they could not justify and clarify the reasons why they proposed the properties in Table 19. This table indicated that only Deniz and Zehra provided correct responses for the properties of quadrilaterals. Although remaining prospective teachers provided adequate information about angle, side and diagonal properties of the quadrilaterals, they generally made mistakes when determining diagonal properties of parallelogram and rhombus. For parallelogram concept, Aslı, Beril, Ece and Maya thought that diagonals of any parallelogram are always angle bisectors. Furthermore, Aslı, Beril, Oya, Ece, and Emel claimed that diagonals of any parallelogram are always equal length. Similarly, Aslı, Beril and Emel proposed that diagonals of any rhombus also have equal length. Additionally, Ece proposed that diagonals of parallelogram are perpendicular. In the following, Ece’s interview excerpt illustrates how she had difficulties on diagonal properties of parallelogram in more detail:

Researcher	What can you say about the side, angle and diagonal properties of parallelogram?
Ece1	Yes. Are diagonals intersecting perpendicularly? I am not good at diagonals. Hmm… Intersecting perpendicularly. Why are they intersecting perpendicularly? It should be taught like proof. … I am thinking that diagonals are intersecting in the middle. I have not a clear idea about the issue of diagonals are angle bisector. I remember that diagonals are intersecting perpendicularly.
Researcher	Are diagonals intersecting each other equally?
Ece2	Good question. Yes. For example, if this equals to this and intersect perpendicularly it will not be. Why? We learnt such a thing. (*she is drawing the parallelogram below*). Now, I said they are intersecting perpendicularly. If it is true, if they are intersecting perpendicularly these two would be equal and these should be angle bisector, aren’t they? I would only say its diagonals intersect perpendicularly. I think diagonals are not dividing each other equally, but I cannot remember. If I draw like this it seems that they could not be intersect perpendicularly. For example, if we draw a quite different parallelogram. Are diagonals perpendicular here in your opinion? I think they are not. Normally, it seems perpendicular. But, as figure changes it could not be.
Researcher	Are you giving up your idea?
Ece3	I am confused. For example, we can try on parallelogram (*she meant second figure below*). (*It is recommended to use set square. She is drawing diagonals with the aid of set square. Set square is being used*.) It is obvious that they are not perpendicular in this case. (*She is erasing what she wrote*) Should I write “they are not intersecting perpendicularly”?

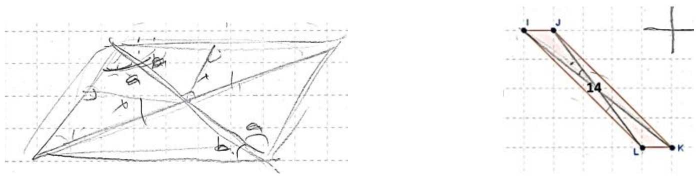


Figure 27. Ece’s constructions of parallelogram to examine angle-diagonal properties

At the beginning of the above excerpt, I prompted a question to Ece in order to understand what she knows about angle, side and diagonal properties of quadrilaterals. Hereon, Ece immediately focused on diagonal properties. She mentioned her inadequate knowledge on diagonal and its properties. Moreover, she claimed that diagonals of a parallelogram are perpendicular without providing any reasonable expression about the reason of their perpendicularity. Besides, her following explanations indicated that she was unsure about whether diagonals of parallelogram are also angle bisectors or not. At that point, I desired to obtain more information about what she knows about intersection of diagonals. To response, Ece drew the first parallelogram shape in Figure 27. However, her statements (see Ece2) revealed that she had difficulty to correctly determine how diagonals of parallelogram intersect each other in terms of perpendicularity. In this regard, she decided to examine diagonals in a different parallelogram example in Figure 27. She saw that diagonals are not always perpendicular in a parallelogram. Then, I asked a new question related to whether the diagonals of parallelogram bisect each other or not.

Researcher	Are diagonals intersecting each other equally?
Ece4	I think they intersect each other equally as they do not intersect perpendicularly.
Researcher	Are you deciding according to appearance of figure? Are you sure?
Ece5	Yes. I am deciding according to appearance of it. I think diagonals intersect each other equally.
Researcher	Well. Are diagonals become angle bisector?
Ece6	Now. Here would be “a” and also here would be a. Then here would be 180˚- 2a [in Figure 27]. Actually it fits nice. What would be if not? This would be b this would also b, this would be a, it would be 180˚- (a + b). Yes it also fits. If this is angle bisector, branches of angle bisector, perpendicular bisectors are equal each other in triangle. Now if we try to find the area of here, triangle, if this would be x and if this would be x… But these are different. But we are looking different triangles now.
Researcher	You said diagonals intersect each other equally. You can continue from that point.
Ece7	I am not sure if they are intersecting each other equally. (Whereupon she stated that she don’t have a clear idea about whether diagonal is angle bisector or not.)

Ece's ideas were interesting because she proposed that if the diagonals are not perpendicular to each other, they can bisect each other. Hereon, I asked her whether she decided according to visual appearance of diagonals or not. In response, she considered the influence of visual appearance of the shapes on her decision making process. In the following, I shifted participant's attention on the relationship between diagonal and angle bisector. In this sense, Ece, made some calculations. However, she could not reach a definite conclusion. In conclusion, such kinds of explanations strongly supported participants' inadequate SMK about diagonal properties of quadrilaterals.

4.1.4.2 Prospective teachers' existing PCK about properties of quadrilaterals

Findings of the study indicated that most of prospective teachers focused on limited numbers of 'students' possible errors related to properties of quadrilaterals. To illustrate, some participants' predictive ideas about students' errors on properties of quadrilaterals were given in the following. Zehra and Maya proposed that students can suppose that all angle measures are same in rhombuses due to their inability to comprehend the differences between square and rhombus. Zehra's explanations show clearly the situation: "*Actually, each square is a rhombus but students may misunderstand it. In my opinion, they may think that the measures of all angles of rhombus should always be equal because its sides are equal*."

Findings also indicated that most of PSTs claimed that students may not have difficulty to determine side and angle properties of quadrilaterals. However, they claimed that students might not provide correctly diagonal properties of quadrilaterals by focusing on different reasons. Some noteworthy examples were given in the following.

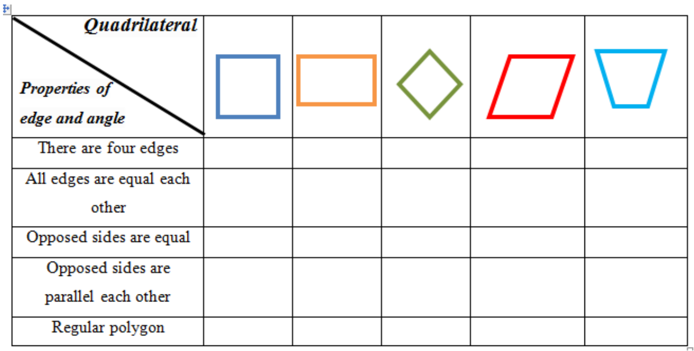
> I do not think that students would not confuse angle and side properties of quadrilaterals because they are easier than diagonal properties. However, I cannot say the same thing for diagonals. For example, students may assume diagonals in equal length in rhombus or they may think diagonals are intersecting perpendicularly in trapezoid [Emel, Pre-interview].

> Even I do not know the diagonal properties completely. I think students cannot know diagonal properties [Ece, Pre-interview].

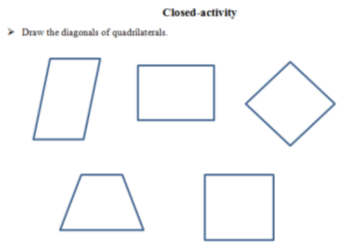
Emel thought that student may found angle and side properties of quadrilaterals easier than diagonal properties. As a result, she proposed that students can consider the lengths of diagonals of any rhombus iare equal or the diagonals of any trapezoid are perpendicular. On the other hand, Ece provided general inferences about students' possible errors rather than specifically identifying what kinds of errors students might have by making emphasis on her own inadequate SMK about diagonal properties. From these statements, we can see how PSTs' SMK might influence their PCK related to understanding of students' mathematical thinking.

It is also important to imply that they could not consider the possible influence of the lack of students' knowledge about basic sub-geometric concepts such as diagonal, parallel and perpendicular line segments on their conceptions related to properties of quadrilaterals.

In order to teach properties of quadrilaterals, PSTs developed different strategies in terms of teaching style, using representations, and question types in their instructional plans. The nature of their strategies was summarized in this part. In the lesson plans, Aslı and Ece utilized a mathematical task involving visual and verbal representations in which they asked students to match the given properties to suitable quadrilateral type. Aslı's activity was asserted as an example in Figure 28. In the activity, Aslı desired to ask students to fill the table in Figure 28-a by using side and angle properties of quadrilaterals. She planned to focus the diagonals of quadrilaterals in her closing activity (see Figure 28-b) where she only asked students to draw the diagonals of given quadrilaterals. Besides, Ece utilized diagonal properties in her matching questions (e.g. for which figures, diagonals bisect each other? and for which figures, intersecting diagonals are perpendicular?) in addition to angle and side properties in her activity. However, they did not prefer to activities that allow exploration of the ways and the reasons why the sum of interior angles in any quadrilateral equals to 180˚ or why the opposite angles of any parallelogram are congruent. Instead, they addressed an activity involving only matching properties and the names of suitable quadrilaterals.



Quadrilateral / Properties of edge and angle					
There are four edges					
All edges are equal each other					
Opposed sides are equal					
Opposed sides are parallel each other					
Regular polygon					



(a) (b)

Figure 28. Aslı’s activity for the properties of quadrilaterals (b) Aslı’s closing activity involving diagonal properties of quadrilaterals

Differently, Maya decided to use a more explorative way because she prepared a lesson plan in which she adopted a student-centered approach within a group study by using technology-supported activity that allows students to discover what types of quadrilaterals can be formed when the diagonals meet various ways (see Figure 29-a). On the other hand, Emel and Deniz utilized some diagrammatic representations and table in order to summarize properties of quadrilaterals at the end of their instruction plans. They adopted a teacher-based approach by giving some verbal expressions about side and angle properties of quadrilaterals. Deniz’s diagram was presented in Figure 29-b as a representative example. Although Deniz rarely focused on diagonal properties of quadrilaterals, Emel fully considered diagonal properties for all types of quadrilaterals. However, they give a misinformation about diagonal properties such as “diagonals of rectangle intersect at right angle.” (Deniz) and “lengths of diagonals are equal for any parallelogram” (Emel).

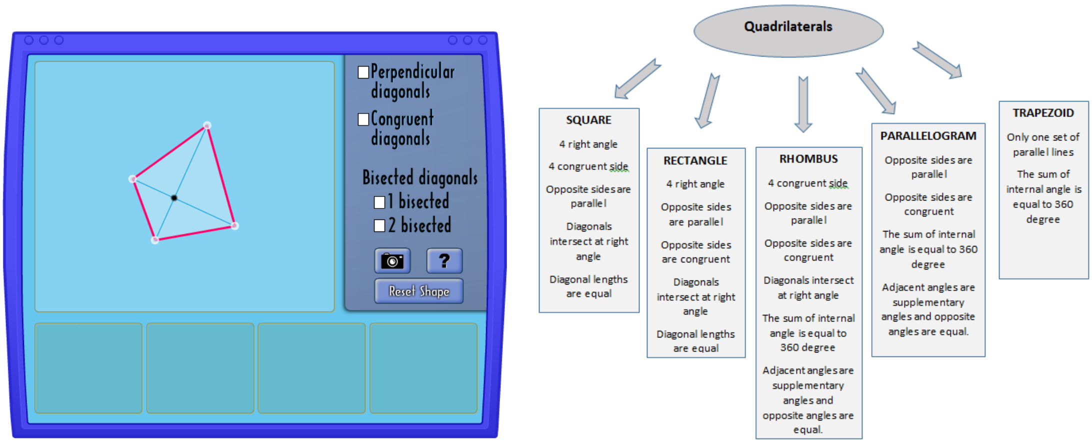


Figure 29. (a) Maya's activity for diagonal property of quadrilateral (b) Deniz' diagram for the properties of quadrilaterals

Different from other participants, Deniz also provided some additional activities in which she focused on angle properties of quadrilaterals. More specifically, she showed the sum of the measurements of interior angles of quadrilaterals and the sum of consecutive angles of parallelogram by referencing the angle properties of a triangle (see Figure 30).

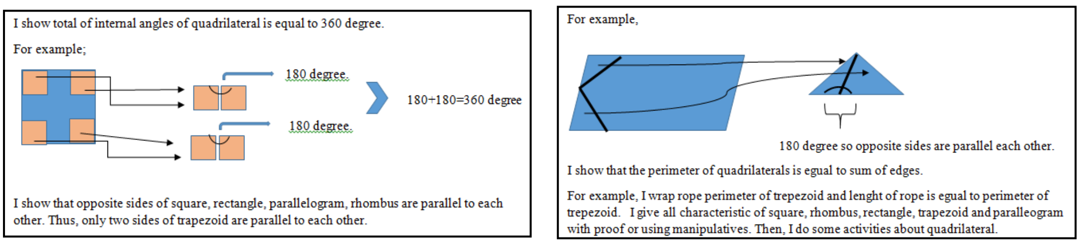


Figure 30. Deniz's additional activities on angle properties of quadrilaterals

As a final point, among participants, Beril's, Zehra's and Oya' instructional plans did not involve any activity or explanations about properties of quadrilaterals. Instead they generally concentrated on hierarchical relations of quadrilaterals.

4.2 Developments in Prospective Teachers' Knowledge about Quadrilaterals in Teaching Experiment Sessions

Developments in prospective middle school mathematics teachers' knowledge about quadrilaterals were presented into four main sections: developments in prospective teachers' knowledge about (i) descriptions/definitions of quadrilaterals, (ii) constructions of quadrilaterals, (iii) prototypical and/or nonhierarchical concept images of quadrilaterals, and (iv) properties of quadrilaterals. According to the corresponding sections, developmental processes in prospective teachers' knowledge were presented by giving some example written statements from reflection papers and illustrative episodes from group discussions that were conducted during the teaching experiment process. Moreover, PSTs' revised lesson plans were utilized to show and make emphasis on the developmental points in more detail.

4.2.1 Developments in prospective teachers' knowledge about definitions of quadrilaterals

In the individual pre-interviewing process, prospective teachers provided only a few ideas or predictions about seventh grade students' possible improper and incorrect definitions/descriptions of quadrilaterals. In this regard, some participants generally predicted that students may not provide formal definitions of the concepts because of inadequate knowledge about mathematical terminology. However, when they began to analyze and discuss the students' descriptions of quadrilaterals in video cases throughout the teaching experiment sessions, all participants could realize students' various improper and incorrect descriptions of quadrilaterals as well as their possible reasons. Furthermore, teachers' knowledge about content and teaching (KCT) was developed in the teaching sessions because they built connections between problems in students' descriptions and alternative instructional strategies in the teaching experiment sessions. All developments in teachers' pedagogical content knowledge about definitions of quadrilaterals were explained in the following by referencing seventh grade students' descriptions that PSTs considered as noteworthy events. Interestingly, throughout the teaching experiment sessions, there was no explicit evidences that showed the developments in PSTs' subject matter knowledge about definitions of quadrilaterals.

4.2.1.1 Developments in PSTs' knowledge while reasoning about students' overgeneralization errors in definitions

Individual pre-interviews data indicated that prospective teachers provided a few ideas about students' possible overgeneralization errors. For example, only Oya and Emel predicted that students might define trapezoid incorrectly and treat irregular polygons as an example of trapezoid because of the usage of the word of "*yamuk*" in Turkish language for "*trapezoid*" by emphasizing on the "*irregular*" meaning of "*yamuk*" in ordinary language. On the other hand, prior to the study, none of the participants mentioned anything about how students could use mathematical terms incorrectly or improperly for the concepts of parallelogram or rhombus and they provided limited predictions on the connections between students' inappropriate descriptions and students' overgeneralization errors about definitions of quadrilaterals. Throughout the teaching experiment sessions, as they analyzed seventh grade students' definitions/descriptions about quadrilaterals in MCVC1, MCVC4, and MCVC8 (see Table 20) they began to realize the relations among students' overgeneralization errors in descriptions of quadrilaterals and their possible reasons. In this regard, how prospective teachers realized, interpreted and discussed the noteworthy events in MCVC1, MCVC4 and MCVC8 including middle school students' descriptions of quadrilaterals were explained in the following.

Table 20. Students' overgeneralization errors in descriptions as noteworthy events

MCVCs	Concept	Students' definitions in MCVCs that PSTs determined as noteworthy events
MCVC1	Parallelogram	Verbal explanation: Parallelogram is two vertical line segments in same proportion[10]. (She provided written definition as following: parallelogram is expansion of two line segments with same proportion through a point starting from that point[11].) Two parallel line segments can be given as an example of parallelogram.
MCVC4	Parallelogram	A geometric figure composed of equal length opposite sides that extend in the same direction[12]. The number of sides can be more than four. A regular hexagon is an example of parallelogram.
MCVC8	Trapezoid	Trapezoids are figures of which all sides are not equal. They can have more than four sides. (e.g. any convex or concave polygon)

4.2.1.1.1 Noteworthy event in MCVC1

As seen in Table 20, a seventh grade student described the concept of parallelogram as "*two vertical line segments in same proportion*" in a moment of MCVC1. The student's parallelogram description in MCVC1 was an unpredictable situation for the PSTs in the current study because they provided no prediction involving that any middle school student can identify parallelogram as "*two vertical line segments in same proportion*" by constructing only two parallel line segments in the individual pre-interviews.

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[10] Turkish version: Paralelkenar aynı orantıda dik iki doğru parçasıdır.

[11] Turkish version: Paralelkenar iki doğru parçasının bir noktadan başlayıp o nokta boyunca aynı orantıda ilerlemesidir.

[12] Turkish version: Paralelkenar aynı doğrultuda uzanan eşit kenar uzunluklarından oluşan geometrik bir şekildir.

4.2.1.1.1.1 *Recognizing and interpreting student's definitional error*

The comments in the individual video analysis reflection papers for MCVC1 revealed that as soon as PSTs analyzed the clip, all of them noticed the student's incorrect description of parallelogram. In that process, some PSTs (e.g. Aslı and Beril) did not focus on the reasons why the student described parallelogram incorrectly and by which ways it can be corrected. More specifically, they firstly explained that the student's descriptions of parallelogram were unexpected. After restating the student's descriptions, they only evaluated the correctness of the student's description of parallelogram in terms of correctness. One example of such comments in reflection paper (BDRP) included:

> Before watching the video, I thought that a student in 7th grade would know the meaning of parallelogram. However, the student may have misconceptions about this, because he defined parallelogram incorrectly and different from my thought [Aslı, BDRP-MCVC1&2].

Besides, Oya, Deniz and Ece not only focused on the correctness of the student's parallelogram description in their reflection papers written in individual video analysis process but also pointed its mathematical meaning by focusing on the relationship between the student's parallelogram description and conceptions. For example, Oya pointed out the lack of sufficient mathematical terminologies such as "*the parallelism of opposite sides*" in the student's description. Deniz and Ece explained their expectations about how seventh grade students can define parallelogram. Furthermore, they inferred that the student's incorrect description and misconceptions about parallelogram influenced her parallelogram constructions and identifications (selections) in the following of the clip. Consequently, they both noticed student's incorrect description of parallelogram and reasoned how an incorrect definition influenced the student's conception of parallelogram after they individually examined MCVC1. On the other hand, Emel, Maya, and Zehra also concentrated on the reasons of why the student made such an incorrect description in their written statements. Their comments indicated that they linked the student's definitional error with various reasons. For instance, Emel claimed that the student had a misconception about the definition of "*quadrilateral*" rather than the definition of "*parallelogram*". Differently, Maya developed an idea in which she thought the student concentrated on the word meaning of parallelogram in Turkish language instead of its conceptual meaning. Further, Zehra proposed that the student could not differentiate among "*corner points*" and "*sides of parallelogram*" and also the student was not aware of "*closeness*" property of quadrilaterals by drawing attention on the inconsistency between student's verbal and written descriptions in MCVC1. Therefore, by virtue of individual analysis of the student's description in MCVC1, they developed new ideas about students' errors in the description of parallelogram and some of them also had opportunities to reason possible reasons of the errors. However, they made no suggestions in order to overcome the student's definitional errors in their written statements.

4.2.1.1.1.2 Elaborating knowledge about reasons of student's definitional error

At the beginning of the group discussion of MCVC1, I sought prospective teachers' general impressions about the student's mathematical thinking in the clip. By the help of my prompting question, the group again concentrated on the student's mathematical work (see Episode 1).

Researcher	What do you think about student thinking generally after watching this video?
Zehra	The first thing drew my attention [in video] is student's [parallelogram] definition. She doesn't think quadrilateral as a closed figure. She mostly finds it sufficient having two parallel line segments to being a parallelogram. Student is not counting quadrilateral as a closed figure as we saw in her definition and she is making her [parallelogram] choices [in oncoming parts of video] according to her definition.
Aslı	She is defining wrongly. She is directly counting corner points [of two line segments] as sides of parallelogram.
Emel	I am also thinking that student has misconception about quadrilateral definition. In my opinion, it is sufficient for a figure to have 4 corners

	instead of having 4 sides according to student.
Deniz	Yes, she is not saying quadrilateral or so in his definition. She is saying [two] lines directly.
Beril	Yes, it is obvious from her definition that this student is mixing parallelism of two lines and parallelogram.
Maya	Alternatively, I think that student may focus on word meaning of parallelogram instead of its definition. For this reason child is treating two parallel line segments as parallelogram and she is making his definition in this way.

Episode 1 taken from MCVC1 discussion

Zehra firstly responded to my question by focusing on student's parallelogram description (Remember pre-interview data in which Zehra have made mathematically correct parallelogram definition in the individual pre-interview.). Parallel with her ideas in individual video analysis reflection paper, she proposed that the student did not think parallelogram as a "*closed"* and *"four-sided*" figure. She continued her explanations with the interpretation of how student perceived parallelogram. At this point, Aslı provided additional information in order to elaborate the issue about why the student made such a definition and developed such kind of mathematical understanding about parallelogram. For this, she claimed that the student treated the endpoints of two parallel line segments as the sides of the parallelogram. Hereon, Emel participated to the discussion by supporting Aslı's interpretation. In the following, Deniz put emphasis on some details in the student's parallelogram description. She mentioned that the description was made considering two parallel line segments does not involve the term of "*quadrilateral*". In response, Beril offered a claim in which she explained student's inability to distinguish the differences between parallel line segments and parallelogram. After Beril's interpretations, Maya proposed an alternative perspective about the reason of the student's error in the description of parallelogram. She suggested that when making the description of parallelogram, the student might reference the meaning of "*paralelkenar*" in Turkish ordinary language instead of the conceptual meaning of parallelogram. In Turkish language, "*paralelkenar*" corresponds with "*parallelogram*" and it is formed by combinations of the words of "*paralel-parallel*" and "*kenar-the edge*". Because of the combinations of words in "*paralelkenar*", Maya concluded that the student conceptualized parallelogram considering linguistics structure of Turkish language instead of developing reasoning based on its conceptual properties. She continued her explanation by claiming that the student treated two parallel line segments as an example of parallelogram. From the discussion, it is obvious that prospective teachers shared different interpretations with their peers. As a result, they had a chance to elaborate their knowledge about the problems in student's parallelogram definition and its possible reasons.

4.2.1.1.1.3 Building connections between student's definitional error and instructional strategies

Towards the end of the group discussion, Aslı shared an idea in which she proposed an alternative way to teach definitions of quadrilaterals to the students. This proposal initiated a discussion involving how they should give instructional definition of parallelogram in mathematic lesson. Episode 2 illustrated how PSTs' enhance their knowledge from the pedagogical perspective on the issue.

Aslı	We can provide students to discover definition also.
Researcher	How do you make your current definition when you think your own previous definition? For example, some of you thought that it would not be enough to give only the parallelism of opposite sides for parallelogram definition.
Beril	Normally we can show that opposite sides are in equal length as a result of parallelism after showing the parallelism. In addition, they can measure individually by ruler.
Deniz	For example, including angles and diagonals [to definition of parallelogram] is exaggeration, because there are students cannot imagine even the figure of parallelogram. Because student cannot imagine, it is hard [for students] to draw that diagonals or so and saying diagonals are intersecting each other equally. Student has to understand parallelogram firstly.
Emel	For this reason, I think we have to focus on quadrilateral definition. I would initially ask student if this is a quadrilateral in your opinion. I would ask what is required for being quadrilateral. If student understand the base conditions in definition she can find other properties herself in any way.

Episode 2 taken from group discussion of MCVC1

When Aslı claimed that it can be given opportunities to the students for exploring definitions themselves, I lead PSTs to evaluate their instructional definitions of parallelogram that they made in individual pre-interviews. For example, in the pre-interview, although Beril had claimed that giving uneconomical definitions of parallelogram in teaching is more useful for the students, she changed her mind in discussion process and she found giving only parallelism of opposite sides when defining parallelogram enough. Similarly, while Deniz preferred uneconomical definitions involving diagonal and angle properties of concepts as instructional definitions in pre-interview, she just claimed that mentioning angle and diagonal properties in definitions of the concepts might be too complicated for the students by referencing the student's incorrect concept image and concept definition of parallelogram in MCVC1. Thus, she developed a pedagogical view on how a suitable instructional definition should be selected. As a result, she argued economical definitions are more useful than uneconomical ones in the teaching process by focusing on their easily understandable nature. Deniz comments were followed by Emel's suggestions. Emel made emphasis on the sub-concepts in geometry that are necessary to understand and make the definition of parallelogram concept. For this purpose, she explained her instructional strategy to teach the definition of parallelogram. To this, she thought that teaching the definition of "*quadrilateral*" is crucial to pass the definition of "*parallelogram*". In her suggestion, she adopted a student-centered approach rather than teacher-centered approach. Thus, they began to build connections among students' thinking and alternative instructional approaches.

In overall, after they discussed student's parallelogram description with the peers, they had opportunities to see alternative interpretations how students might describe parallelogram and what their possible reasons and solutions can be. In this regard, prospective teachers' comments in the reflection papers written after group discussion process also clearly showed the presence of the developments in prospective teachers' pedagogical content knowledge. To illustrate, the developments on prospective teachers' understanding about the relation among the student's ill-conception of parallelogram and the incorrect mathematical language usages were clearly exemplified in the following chosen excerpts taken from after discussion reflection papers (ADRP). For instance, at the beginning of the teaching experiment, Aslı and Beril only had evaluated the correctness of the student's parallelogram description. However, example comments in the after discussion reflection papers indicated that they realized that how the student improperly and informally defined the concept of parallelogram. Additionally, they realized the influence of mathematically incorrect definitions on students' misconceptions as well as instructional strategies involving what possible solutions can be for the student's definitional error. A few example statements follow:

> I want to give each definition at the beginning of lesson because the student in the video clip did not adequately know the definition of the parallelogram. I think it is the base reason for their misconceptions and misunderstandings. So, I think using correct mathematical terms [in the definitions] is very important [Beril, ADRP-MCVC1&2]
>
> I think the terms in definition have to be taught completely [Aslı, ADRP-MCVC1&2].

On the other hand, although Ece evaluated the correctness of student's definition in the MCVC1 when individually examining the clip, she also realized the importance of remembering basic geometric concepts such as "*corner*" and "*angle*" in terms of providing mathematically correct definitions after discussing the student's mathematical thinking in the clips. Her statements were:

> Furthermore it will be beneficial to remind the concepts in definition again while defining; because after discussion I have noticed that student has errors about the [basic geometric] concepts like corner and side in video 1 [Ece, ADRP-MCVC1&2].

Finally, while Zehra, Emel and Maya, as high achiever prospective teachers, provided limited predictions about the relation among students' incorrect descriptions and the conceptions of quadrilaterals in their individual pre-interviews, they had a chance to develop and elaborate their understanding how students define incorrectly and why they made errors when describing the concepts after they analyzed and shared their ideas with their peers in the group discussion. For example, Maya provided following explanations:

> I thought that the student in video were focusing on word meaning [of parallelogram] and didn't know the parallelogram at all. However, it quite drew my interest that, the point my friends caught, student does not know that quadrilateral is a closed figure. I have understood that the words used in definition are very important [Maya, ADRP-MCVC1&2].

From these explanations, it was evident that Maya reflected her own developments in understanding student's mathematical thinking and she also realized the importance of using suitable mathematical terminological usage in the definitions in the group discussion process. In conclusion, the ideas asserted in Episode 2 gave opportunity to revisit their instructional definitions by evaluating a didactical viewpoint in addition to producing some useful suggestions to the student's definitional errors.

4.2.1.1.2 Noteworthy event in MCVC4

As seen in Table 20, a seventh grade student in MCVC4 described parallelogram as "*a geometric figure composed of equal length opposite sides that extend in the same direction*[13]". Moreover, the student said that parallelogram can have more than four sides. Accordingly, the student drew a regular hexagon as an example of parallelogram. In conclusion, these descriptions indicated that the student perceived parallelogram as a closed geometric figure that might have more than four sides.

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[13] Turkish version: Paralelkenar aynı doğrultuda uzanan eşit kenar uzunluklarından oluşan geometrik bir şekildir.

4.2.1.1.2.1 Recognizing and interpreting student's definitional error

The student's description is interesting for PSTs because none of prospective teachers predicted that any student might define parallelogram as a closed figure having at least two pairs of parallel sides in the individual pre-interviews. In this regard, once PSTs individually analyzed the clip, they were generally surprised to see the student's both unpredictable and incorrect parallelogram description. Their comments in the reflection papers of individual video analysis indicated that six of them noticed student's inappropriate description of parallelogram. However, Aslı and Deniz thought that the student defined parallelogram in mathematically acceptable way. For example, Deniz's comments in her reflection paper showed that she was unaware of student's definitional errors. "*The student knows the concept of parallelogram roughly. The definition [made by student] is acceptable and parallelogram figures drawn are correct.*" (Deniz). On the other hand, Maya provided additional interpretations in her reflection paper by focusing on why the student interestingly could not think parallelogram as a four-sided figure. In this regard, she claimed that the student considered the word meaning of "*paralelkenar*" might cause problems in student's conception of parallelogram.

4.2.1.1.2.2 Elaborating ideas about student's definitional error

After they continued to write their reflection papers of individual video analysis, I launched the discussion by posing general prompts to elicit prospective teachers' thinking about student's parallelogram description (see Episode 3).

Researcher	What are your opinions about the student's parallelogram definition?
Deniz	I think her drawings and definition are mathematically appropriate and acceptable.
Emel	But her definition is not correct completely in my opinion, because she said that number of sides didn't influence being a parallelogram [in her definition].
Zehra	She has already chosen pentagon [as a parallelogram in a moment of the video].

Deniz	You are right. Even eight-sided figure is a parallelogram according to the student's definition.
Aslı	I had thought that student has enough information about parallelogram when I heard the definition of student [in video]. But I have just noticed that she said "number of sides didn't influence" [in her definition] showing that she described parallelogram wrongly.
Maya	I think this student [like the student in MCVC1] also focusing on the name of parallelogram. The difference here is, it is obvious from his drawings, this student knows parallelogram as a closed figure.
Emel	I think so. Student is focusing the words "parallel" and "sides" (word by word Turkish translation of parallelogram is "parallelsides") while describing [parallelogram].
Aslı	I have never thought in this point of view.

Episode 3 taken from group discussion of MCVC4

As seen in the Episode 3, Deniz immediately evaluated the student's description in terms of the correctness. However, Emel challenged with the idea of Deniz and she claimed that the student did not define the concept correctly by giving evidence from the clip. As a connecting idea, Zehra provided a detail that was directly related to the issue under discussion. She mentioned that the student treated pentagon as a parallelogram in the clip. After Emel and Zehra evaluated the correctness of the student's description, Deniz and Aslı realized student's definitional error. Up to this point of the discussion, prospective teachers debated the mathematical correctness of student' parallelogram description. However, Maya shifted the discussion in progress by focusing on why the student described parallelogram improperly. She offered a claim about the student's incorrect description which she linked to student's usage of the meanings of the words of "*paralelkenar*" in Turkish language instead of conceptual meaning of parallelogram. Supporting Maya's interpretation, Emel proposed that the student identified the concept considering the words of "*parallel-paralel*" and "*the edge-kenar*" in Turkish language. In response, Aslı stated that she never thought from this perspective. Therefore, although Aslı and Deniz could not realize the errors in the student's description of parallelogram in the process of individual analysis of the video clip, they had opportunities to develop their knowledge about the relation between the mathematical correctness of definition and students' language-based reasoning about quadrilaterals at the end of the group discussion process. Consequently, in the analysis and discussion processes of MCVC4, prospective teachers had opportunities to develop their pedagogical content knowledge about students' definitions of quadrilaterals.

4.2.1.1.3 Noteworthy event in MCVC8

In the clip, a seventh grade student described trapezoid as an irregular figure having non-equal sides and the student constructed a five-sided convex polygon as an example of trapezoid (see Table 20). Before starting the teaching experiment, only Emel and Oya predicted that students might identify trapezoid by referencing the meaning of "*yamuk*" in Turkish ordinary language. However, remaining prospective teachers only predicted that a few students may provide exclusive definition of trapezoid instead of inclusive definition of the concept. In Turkish language, the word of "*yamuk*" is used for the English term "*trapezoid*" in all textbooks and teachers' instruction. However, "*yamuk*" is synonym and also means "*irregular*" in Turkish ordinary language. As a result, students may imagine trapezoid as a figure having more than four sides. In other words, they treat some non-examples as examples by extending their knowledge to another context in an inappropriate way.

4.2.1.1.3.1 Recognizing and interpreting student's definitional error

When PSTs individually analyzed the clip, all of them were surprised to see the student's inappropriate trapezoid description and construction. Furthermore, seven of them not only noticed student's definitional errors but also offered possible reasons of the errors. However, they suggest any solution way to overcome such kind of definitional error. Instead, in the reflection papers, they offered that the student might focus on the word meaning of "*yamuk*" in ordinary language. One example comment included:

> It is hard to predict the definition of student being like this. I would say the student was influenced a lot from the name of trapezoid and said that all sides of figure are not equal. Generally this  figure comes to student's minds. Possibly she drew a figure with five sides I mean she mixed because the word trapezoid does not contain the number of sides as in quadrilateral [Ece, ADRP-MCVC8].

However, Deniz solely could describe how student defined and constructed trapezoid in her reflection paper for individual analysis of MCVC8 instead of focusing on why the student made such kind of incorrect description.

4.2.1.1.3.2 Elaborating ideas about student's definitional error

After the individual video clip analysis, I asked them to explain their thinking about the student's conception of trapezoid in order to elaborate their pedagogical content knowledge related to understanding student's mathematical thinking. As a result, they began to discuss the student's description of trapezoid as in Episode 4.

Researcher	What did you notice about [student's] thinking related with trapezoid when you watched the student in video?
Maya	I think she doesn't know [trapezoid concept].
Oya	This student constructed a trapezoid definition in her way and considered the figures appropriate to her definition [as trapezoid].
Aslı	A superficial definition.
Deniz	I wondered actually how the student found this definition, when I watched the video.
Ece	In my opinion, she thought the meaning in ordinary language when said trapezoid.
Deniz	Is this because the synonym of trapezoid in Turkish?
Beril	The trapezoid concept in her mind corresponds to irregular shape like used in Turkish language. She thinks that trapezoid shape need not be a regular one. She thinks that if all sides are equal in a shape it could not be a trapezoid.
Oya	The things said by student are not definition already. A figure having unequal sides is enough to say trapezoid for her. As a matter of fact she doesn't know the concept.
Aslı	So she is not choosing square and rhombus [as trapezoid] but she is choosing the rest of figures [as trapezoid].
Deniz	For instance, why is she choosing the parallelogram as trapezoid?
Oya	Because all sides are not equal.

Deniz Hmm…

Episode 4 taken from group discussion of MCVC8

Maya provided an inference about student' lack of knowledge on trapezoid. While Oya concentrated on student's informal description of trapezoid, Aslı made an evaluative comment for the student's description as superfluous and vague. At this time, Deniz needed to prompt a question to the peers how a student might produce such kind of description for the concept of trapezoid. In response, Ece claimed an idea involving student's possible approach when defining trapezoid based on the word meaning of "*yamuk*" in Turkish language. Hereon, Deniz asked a new question for a validation. In order to give evidence, Beril tried to explain the details of student's mathematical thinking and description about trapezoid. She showed the student expression of "*trapezoid having non-equal sides*" as an evidence for the errors in student's trapezoid conception. Aslı connected to Beril's and Oya's interpretation by giving a noteworthy event in the clip (e.g. the student did not consider rhombus and square as an example of trapezoid). Then, Deniz wondered why the student thought parallelogram as an example of trapezoid. In response, Oya immediately presented the reason of because the lengths of all sides are not equal in any prototypical parallelogram. Overall, at the end of the group discussion of MCVC8, Deniz had opportunities to develop her knowledge about why the student thought trapezoid as an irregular figure and realized possible influences of ordinary language on the student's conception of trapezoid by referencing student's incorrect trapezoid selections.

> I have noticed that student considers an irregular shape when said trapezoid because of the meaning of ordinary Turkish language, and this thought affects the trapezoid selection [Deniz, Group discussion of MCVC8].

The explanations provided by Deniz clearly indicated her knowledge development related to the possible reasons of student's incorrect trapezoid description.

4.2.1.1.3.3 Building connections between student's definitional error and instructional strategies

PSTs provided no suggestive ideas involving how they overcome the student's incorrect trapezoid description in group discussion process of MCVC8. Interestingly, four PSTs (Aslı, Ece, Deniz, and Beril) proposed some strategies to prevent students' definitional error originated from incorrect language usages in their reflection papers that they wrote at the end of the group discussions of the MCVC8. They thought that these strategies might be helpful to prevent the formation of incorrect concept definition of trapezoid. To be more precise, Beril and Ece claimed that it is important and necessary to give a warning to emphasize the mathematical meaning of trapezoid rather than the meaning in ordinary language before starting the lesson. Some example statements are:

> Some students may have misconceptions like "trapezoid need to have irregular sides" because of the meaning of trapezoid in ordinary Turkish language. Students can particularly be warned in the beginning of the course to prevent this misconception [Beril, ADRP-MCVC8].

Moreover, Aslı argued that determining what students understand from trapezoid is useful to prevent the development of possible similar misconceptions before giving the definition of the concept. As an alternative way, Deniz preferred to make emphasis on the point that trapezoid is a quadrilateral in her instructional plans.

In sum, before participating in the teaching experiment, most of prospective teachers' predictions about students' possible parallelogram descriptions did not involve students' overgeneralization errors in definitions originating from incorrect mathematical terminological usages. From this point of view, they developed their knowledge about how students might perceive parallelogram or trapezoid differently considering the student's descriptions in the video clips (e.g. MCVC1, MCVC4, and MCVC8). Thus, they realized the relation among students' overgeneralization errors in mathematical definitions of quadrilaterals and the influences of linguistic factors and language-based conceptions of quadrilaterals in students' mind. As a result, they had a chance to develop their pedagogical content knowledge in terms of understanding students' mathematical thinking by virtue of analyzing and discussing special designed video cases. Furthermore, interactional process among prospective teachers and me in the group discussion processes helped them to elaborate their interpretations and inferences about student's conceptions rather than only describing and evaluating what student identified the concepts in the video clips. As result, at the end of the teaching experiment process, prospective teachers realized students' incorrect descriptions/definitions as well as they had opportunities to find the reasons of such kinds of errors in the definitions and the possible influences of these errors in students' conceptions about quadrilaterals or the influences of students' ill-conceptions on their descriptions of quadrilaterals by analyzing and discussing the video cases. As a final crucial point, they even offered some specific instructional strategies to overcome students' overgeneralization errors in the definitions of quadrilaterals rather than offering superficial and general instructional strategies.

4.2.1.2 Developments in PSTs' knowledge while reasoning about students' undergeneralization errors in definitions

In the individual pre-interviews, prospective teachers predicted that students can describe the concepts mathematically correct. However, after they examined and discussed students' descriptions in MCVC2 and MCVC7 (see Table 21) involving undergeneralization errors, they realized the possible influence of making only visual reasoning on students' incorrect descriptions of quadrilaterals.

How prospective teachers realized, interpreted and discussed the noteworthy events in MCVC2 and MCVC7 including seventh grade students' undergeneralization errors in descriptions of quadrilaterals were given with all details in the following paragraphs.

Table 21. The nature of students' undergeneralization errors in descriptions as noteworthy events

MCVCs	Concept	Students' definitions in MCVCs that PSTs determined as noteworthy events
MCVC2	Parallelogram	A kind of distorted figure that is obtained by pushing down on rectangle or square[14]. [*The student drew prototypical parallelogram*]
MCVC7	Trapezoid	A figure formed by putting a triangle next to a square or rectangle[15] [*The student drew prototypical right trapezoid*].

4.2.1.2.1 Noteworthy event in MCVC2

As seen in Table 21, a student in MCVC2 described parallelogram as "a kind of distorted figure that is obtained by pushing down on rectangle or square".

4.2.1.2.1.1 Recognizing and interpreting student's definitional error

Comments of prospective teachers' individual analyses of MCVC2 indicated that all of them initially realized the student's improper mathematical terminological usages in the parallelogram description. More specifically, Aslı stated that she did not understand what the student meant with the term of "*pushed down figure*" in her parallelogram description. However, remaining prospective teachers focused on what the student meant with "*pushed down figure*" and the possible reasons why the student used such different terminology in her parallelogram description. In the statements in the reflection papers, Emel, Zehra and Maya generally argued that the

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[14] Turkish version: Dikdörtgenin veya karenin uçlarından bastırılarak yamulmuş bir şeklidir.

[15] Turkish version: Yamuk karenin ya da dikdörtgenin yanına gelen üçgen ile oluşan şekildir.

student made such kind of inappropriate description due to her visual reasoning rather than conceptual ones. To illustrate, Zehra's comments were given in the following:

> This student is very close to my student model that I had predicted. She is making her parallelogram definition over image. Sides of figure should be parallel and there should be short and long sides as well for figure to be a parallelogram. On one hand figure should not be straight as square or rectangle. She is making her definition according to her prototype image rather than understanding the concept [Zehra, BDRP-MCVC2].

From these statements, it was apparent that they considered the student solely focused on visual properties of (a prototypical) parallelogram figure rather than focusing on critical attributes in order to establish formal definition of the concept. Furthermore, three of prospective teachers (Beril, Ece, and Deniz) provided some comments in their individual video analysis reflection papers in order to explain the reasons of student's visual-based approach in the parallelogram description in addition to the meaning of the student's rather intuitive description. They provided an inference in which they claimed that student's math teacher might have defined parallelogram after giving basic (prototypical) constructions of parallelogram on the board.

4.2.1.2.1.2 Elaborating knowledge on student's definitional error

After individual video analysis process, in the group discussion of MCVC2, PSTs elaborated their understanding about the student's mathematical thinking by referencing student's parallelogram description and construction as in Episode 5.

Researcher	Is it enough for you listening the explanation of student related with parallelogram? She said like things "pushed down figure".
Ece	The student said "pushed down figure" but she even did not say pushed down in the same proportion.

Aslı	Expressing it like "pushed down figure" is a very rough definition. I cannot comprehend at the moment how a pushed down figure looks like. Because when you push down a figure, a warp [not containing parallelism] is formed.
Deniz	I think she does not know anything about the definition and properties of parallelogram. She only saw its figure once. She is trying to define parallelogram based on that figure. It seems she does not know anything. She is trying to understand the other figures by comparing to [prototypical] figure.
Zehra	On the other hand, she does not have error about pushed down figure because when she pushed down the figure she obtains a parallelogram in any way. It is not possible to obtain any other figure because opposite sides are equal. She has at least a definition in her mind in her own way.
Maya	I think she learnt something based on memorization. Moreover, she is consistent in herself actually. She can link to parallelogram figure but she is defining [mathematically] incompletely and making relations of quadrilaterals incompletely because of her incorrect or incomplete knowledge.
Aslı	Yes, for example she said parallelogram to hexagon and then she changed her mind by looking other figures, supporting this also.

Episode 5 taken from group discussion of MCVC2

Researcher initiated the discussion in order to get detailed information about prospective teachers' interpretations about student's parallelogram description and conception. Ece commented on the lack of information in the student's description. She claimed that the student did not mention about pushing down on figure in same proportion in the parallelogram description. Hereon, Aslı evaluated the description as a rough definition and she found the student's description meaningless. Similarly, Deniz provided an interpretation about why student made such a description in the clip. She inferred that student did not know anything about definition and properties of parallelogram. According to Deniz, student made visual reasoning by giving student's prototypical parallelogram construction in the clip as evidence. Here, Zehra offered an alternative perspective on the correctness of student's parallelogram description. She pointed that if they pushed down on rectangle, square, or rhombus, the figures always turn into parallelogram. By referencing this situation, Zehra thought that student's description might be evaluated as a reasonable informal parallelogram definition. At this point, Maya connected the ideas proposed by Deniz and Zehra. For Maya, the student constructed some relations among quadrilaterals by the help of rote learning, but she had mathematically inadequate definition of parallelogram due to her insufficient conceptual knowledge about quadrilaterals. Following that, Aslı supported to Maya's interpretation by referencing student's inconsistent responses in the determination of whether hexagon is also a parallelogram or not in the video clip.

Considering Aslı's previous comments written in individual video analysis process, she only stated that she did not understand what the student meant with the term of "*pushed down figure*" in her parallelogram description. However, after the group discussion, she provided following explanation in her reflection paper:

> I understood after video discussion that visual thinking and the teacher's handling way of the issue can be very important in student's perception of subject. In other words, more effective ways should be used instead of using same examples and memorization all the time in lessons [Aslı, ADRP-MCVC2].

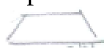
These explanations showed that Aslı had to restructure and elaborate her knowledge about the student's description after recognizing the student's incorrect description of parallelogram by virtue of both individual and group discussion process. Another crucial development was observed in Emel's and Maya's written comments involving their updated knowledge about student thinking. When they individually analyzed the clip, they only make connection between visual reasoning and the student's description. However, after the discussion, they concentrated on the teaching style as a possible reason of the development of visual reasoning instead of conceptual ones.

4.2.1.2.2 Noteworthy event in MCVC7

The student in MCVC7 described trapezoid as “*a figure formed by putting a triangle next to a square or rectangle*[16]” and constructed a prototypical right trapezoid in order to give an example according to the description, which indicated the presence of undergeneralization error in student’s definition. Prospective teachers had a chance to develop ideas about how visual reasoning might influence on students’ descriptions of quadrilaterals during individual video analysis and group discussion by analyzing MCVC7 at the last week of the teaching experiment.

4.2.1.2.2.1 Making inferences about reasons of student’s definitional error

Normally, prospective teachers could not predict that student might identify trapezoid as limited to a specific form of trapezoid (e.g. right trapezoid). In this regard, PSTs’ individual reflective comments indicated that all PTs were surprised to see the student provided an informal description for trapezoid as a right trapezoid. After they realized and interpreted student’s description formed by the visual appearance of the right trapezoid, they commented on why the student identified trapezoid in such a way. They inferred that the student did not consider properties of a trapezoid when defining the concept. According to them, the student just informally described the right trapezoid that she imagined in her mind because of considering the visual characteristics of right trapezoid rather than considering conceptual properties of all trapezoids. One example involved:

> Student thinks that trapezoid is a figure formed by putting a triangle next to a square or rectangle. I mean she is identifying trapezoid with right trapezoid. I did not expect student’s description in this way. I thought that this figure  comes into their head when said trapezoid. But right trapezoid is coming into her head. Again this student cares about appearance actually [Emel, BDRP-MCVC7].

[16] Turkish version: Yamuk karenin ya da dikdörtgenin yanına gelen üçgen ile oluşan şekildir.

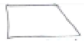
However, Ece and Aslı provided additional judgmental comments in their reflection papers. Aslı offered a claim in her reflection paper of individual video analysis to explain the reason of student's incorrect description by pointing out the possible teaching approach in the student's class. She thought that the teacher might teach the concept based on the limited number of trapezoid examples rather than considering the formal definition of trapezoid. On the other hand, Ece made a prediction about the student's definitional error in MCVC7 by underlying the organization of mathematics textbooks filled with same kind of (prototypical) figures of quadrilaterals.

4.2.1.2.2.2 Elaborating ideas about reasons of student's definitional error

In the group discussion process, I asked to the participants how they had interpreted student's description of trapezoid in order to elaborate their knowledge about student's mathematical thinking. As a result, the group focused on the student's description of trapezoid (see Episode 6).

Researcher	What was your comment when you heard the student's description?
Ece	I thought that students' trapezoid description would be for a normal one instead of a right trapezoid. Like there should be two triangles on each side of a square. But student thought right trapezoid directly and defined it.
Emel	Yes I had also thought like that.
Beril	Student perceives the trapezoid as only a right trapezoid according to [her personal] definition.
Researcher	To be honest I wondered that why did student define right trapezoid instead of other types of trapezoid?
Emel	Because she is caring about appearance [of figure].
Aslı	Yes, appearance.
Oya	She is focusing on appearance. But student can be unfamiliar to other trapezoid types.
Researcher	You may be right but almost half of the students I have interviewed drew right trapezoid and the other half drew isosceles trapezoid.
Aslı	I think their teacher may overemphasize on right triangle in lessons.
The group	(*The group agreed by nodding their head*.)

Episode 6 taken from group discussion of MCVC7

Ece mentioned her expectation that students can define trapezoid by considering two triangles and one square (e.g. ) rather than one triangle and one square (e.g. ). At this point, Emel and Beril supported Ece’s interpretation. I invited the participants to think and to explore the reasons why student focused on right trapezoid in the description instead of other trapezoid types. This was an important moment for PSTs as they had to consider the possible reasons of the student’s incorrect description. Therefore, Emel, Aslı and Oya responded to my question at the same time. They offered that the student took account into the appearance of right trapezoid in her description. Furthermore, Oya made an additional inference in which she claimed that the student might not know the types of trapezoids. After this point, I offered an alternative viewpoint to the issue under discussion. In order to provide evidences about the presence of students’ knowledge about trapezoid types, I gave additional information from the interviews conducted with the seventh grade students. Hereon, Aslı proposed an idea that only she had wrote earlier in her reflection paper during individual analysis of the video clip. Her claim was that because teacher might have overemphasized right trapezoid rather than focusing on definitional properties of the trapezoid in the teaching process, the student provided such a description.

At the end of the Episode 6, all group members accepted Aslı’s and Oya’s proposals as reasonable ideas to clarify the reasons of the problematic situation in the student’s description of parallelogram. In other words, different perspectives especially provided by Aslı and Oya had positive contributions on the developments of other prospective teachers’ pedagogical content knowledge involving understanding students’ description of trapezoid. Following this, I prompted the group to elaborate and to explain all the things that they think about the reasons why the teacher might overemphasize right trapezoid like in Episode 7.

Researcher Why could their teacher overemphasize these typical trapezoid examples?

Maya	Maybe teacher focused on these because students are mixing hierarchical relations of quadrilaterals.
Oya	Maybe there are questions including mostly these types of trapezoids in exams. Like “We are drawing perpendicular and sides are equal in isosceles trapezoids”. I think for that reason I mean.
Maya	May be it could be like this. Teacher may want students to use triangle, square and rectangle during area and circumference calculations because of their familiarity to these concepts. It is easier to calculate area in right trapezoid.
Aslı	It makes sense.

Episode 7 taken from group discussion of MCVC7

This particular prompt challenged prospective teachers own knowledge and encouraged them to think on some possible reasons of the overemphasized situation. Thus, by the help of detailed examination of the student’s thinking in the video, Maya and Oya generated new ideas about why the student described trapezoid according to the visual appearance of right trapezoid. For example, Maya proposed that the teacher may focus on right and isosceles trapezoids because of students’ inability to differentiate the relation among quadrilaterals. Differently, Oya thought that there might be an influence of the involvement of exam questions in the school on the teacher’s examples of trapezoid. Maya extended these ideas by giving the details such as the teacher might desire to give such examples because the calculation of perimeter and area of trapezoid can be easy in a right trapezoid. In sum, at the end of the group discussion of MCVC7, prospective teachers generated new ideas from different perspectives to explain the student’s description of trapezoid. As a result, they developed their knowledge about student’s definitional errors as well as their possible reasons. Emel’s written statements in after discussion reflection paper were given as an example:

> The ideas that I could not predict before are emerged in this week’s group discussion again. For instance, first student had identified trapezoid with right trapezoid. I did not think much that why she was thinking like this. The ideas coming from my friends were like this: Student’s thinking like this may be caused by her teacher’s overemphasizing of special types of trapezoids. The reason why teacher was overemphasizing special types of trapezoid (Student drew right trapezoid when

> requested to draw a trapezoid.) was these types of trapezoids are frequently used in exam questions and area and circumference calculation examples. These ideas broadened my mind [Emel, ADRP-MCVC7&8].

Such kinds of written statements were strong indicators to present the developments in PSTs' knowledge related to understanding students' mathematical thinking.

4.2.1.2.2.3 Building connections between student's definitional error and instructional strategies

By referencing students' students' definitional errors and difficulties in MCVCs, PSTs proposed various instructional approaches/strategies in their after discussion reflection papers. In this sense, Emel, Zehra, and Aslı offered an alternative teaching method involving teaching the concepts based on the definitions rather than focusing on their visual appearances in their reflection papers. Emel's statements were given in the following:

> The students are deciding or describing by focusing on appearance of figure. Furthermore for student it is enough to say trapezoid for a figure to resemble a trapezoid in some way. The properties of figure are shown in one side and the [prototypical] examples on the other side just at the beginning of the section in course books. Actually, we are causing student's thinking like this. Generally if it is considered that visual things draw more attention, it is inevitable for students to think like this. For this reason, in my opinion, figures firstly should not be shown to students when these concepts are taught [Emel, ADRP-MCVC7&8].

It was clearly seen in Emel's written statements that they argued that utilizing and adopting such a teaching method might provide a solution to the negative influences of students' restricted concept images that develops with the effect of visual characteristics of prototypical geometric figures on the concept definitions. Consequently, at the end of the teaching sessions, PSTs established connections between students' inappropriate definitions and students' visual reasoning. Accordingly, they concluded that identifying geometric concepts according to their visual characteristic instead of necessary and sufficient conditions might lead some problematic situations in the establishing mathematically correct definitions of the concepts.

4.2.1.3 Developments in PSTs' knowledge while reasoning about necessary and sufficient conditions in students' definitions

In the video cases excluding MCVC6, students generally provided incorrect informal descriptions of quadrilaterals instead of formal definitions. For this reason, prospective teachers generally focused on students' incorrect descriptions of quadrilaterals and their possible reasons such as mathematical or ordinary language usages or students' approaches in describing the figures visually instead of conceptually in the analyses and discussion of these clips. Consequently, they could not realize the absence of necessary and sufficient conditions in the video cases excluding MCVC6.

4.2.1.3.1 Noteworthy event in MCVC6

In the sixth video case, a seventh grade student provided the definition of "*rhombus is a figure with four sides of equal length*[17]*"*. This definition seems mathematically correct but the student used the term of "*figure*" rather than "*a closed figure*" in the definition. According to this definition, a non-closed figure with four sides of equal length also becomes an example of rhombus despite of being a non-example.

4.2.1.3.1.1 Interpreting the absence of necessary and sufficient conditions in the student's definition

Most probably, because the student's definitions involved small errors, participants could not recognize them when they were individually analyzing the student's

[17] Turkish version: Eşkenar dörtgen eşit uzunlukta dört kenarı olan bir şekildir.

mathematical thinking in the video clip. However, they had opportunity to notice the absence necessary and sufficient conditions in the student's definition after my critical questions in the group discussion process of MCVC6 (see Episode 8).

Aslı	When I saw the definition and such of student I thought that she is a successful student. She had defined correctly.
Researcher	Is there anyone who has a different opinion than Aslı? Is there any missing part in student's definition in your opinion?
Maya	She did not state length [of sides].
Ece	The student mentioned about [equality of] the length of all sides, but she did not mention anything about parallelism of opposite sides [of rhombus].
Oya	The parallelism of opposite sides is already provided when it is stated that the length of opposite sides are equal.
Zehra	Parallelism is already a result of length equality of opposite sides.
Ece	Yes you are right. Okay, I understood the problem [in my thought].
Researcher	Why did you give up the idea of the way of definition as you said?
Ece	It is not a problem but in that situation there are additional statements [in definition].
Oya	The students we watched have never stated the parallelism of sides.

Episode 8 taken from group discussion of MCVC6

At the beginning of the group discussion in Episode 8, Aslı's comments showed that she evaluated the student's definition of rhombus as mathematically correct. At this point, I redirected the discussion to understand how remaining prospective teachers evaluated the correctness of the student rhombus definition in terms of necessary and sufficient conditions. In response, Maya stated that student did not mention "*the equality of the length*" in her definition. In the following, Ece challenged Maya's proposition by pointing the absence of "*the parallelism of opposite sides*" in the student's definition of rhombus. Oya and Zehra participated to the discussion by emphasizing that "*the parallelism of opposite sides*" in a rhombus can be easily deducted from "*the equality of the length"*. After they emphasized "*the parallelism of opposite sides*" as an unnecessary condition for the definition of rhombus, Ece changed her previous idea and agreed with her friends. When I asked to Ece the reason of the change in her thinking, she provided a comment in which she found the condition of "*the parallelism of opposite sides*" as an extra property for the definition of rhombus.

4.2.1.3.1.2 Elaborating ideas about necessary and sufficient conditions

After they discussed the necessity of "*the parallelism of opposite sides*" in the rhombus definition, I asked the question about the presence of other points that they found unnecessary or insufficient in the student's definition of rhombus. This question initiated a new discussion on the presence of necessary and sufficient conditions in the student's definition (see Episode 9).

Researcher	Are there other points that you found unnecessary and insufficient in the student's definition of rhombus?
Emel	I think there are not.
Ece	I think she defined pretty well for a student.
Maya	But she did not stated "identical length" instead of "equal length".
Ece	Yes you are right. That is a lack.
Oya	I think it is hard to know that much detail for a student. Even we learnt the difference between "identical" and "equal" in university.
Researcher	Well, when you think about the student's definition, can this figure [having non-closed sides] be drawn according to her definition.
Oya	Mmm, I have never thought like that.
Ece	It has never drawn my attention.
Emel	Hmm, I have never noticed this situation.
Oya	I had mentioned such a situation in my lesson plan. There is quadrilateral, I had designed an activity to question whether that quadrilateral is rhombus or not. But it did not come into my mind that the statement in definition here is insufficient.
Ece	The importance of the word "closed" is seemed again.
Oya	If he said it was quadrilateral, it would not be sufficient, would it?
Ece	Quadrilateral is defined as closed in the end. Isn't it sufficient?
Oya	Yes it is sufficient then.

Episode 9 taken from group discussion of MCVC6

Ece and Emel immediately provided an explanation in which they found the definition mathematically correct. Differently, Maya pointed the presence of an unsuitable mathematical term usage in the definition considering the difference between the terms of "*equal*" and "*congruent*". While Ece supported Maya's viewpoint, Oya criticized the idea because Oya thought that it was difficult to provide a definition involving fully correct mathematical terminological usages for the students. In order to elaborate participants' interpretations on the student's definition of rhombus, I constructed a figure on the board as an alternative visual interpretation of the student's definition (e.g. a non-closed figure with four sides of equal length). Thereon, they were surprised with such alternative interpretation of the definition. Ece, Oya, and Emel verbally explained the reason why they were amazed. Moreover, Oya provided further information that indicated her awareness in noticing the absence of necessary conditions in the definition. Ece put emphasis on the lack of "*closeness*" in the student's definition. Oya prompted a question to understand if the definition involves the term of "*quadrilateral*" whether it can become correct or not. Following Oya's question, Ece explained the meaning of quadrilateral. After Ece's explanations, Oya understood how the student's definition can be corrected.

In summary, prospective teachers realized the absence of necessary and sufficient conditions at the end of the group discussion. For example, they understood a property involving "*the parallelism of opposite sides*" is an extra property in the rhombus definition, the lack of the term of "*closeness*" or "*quadrilateral*" is an insufficient characteristic in the student definition. Therefore, by virtue of new ideas that were generated in collaborative peer discussion of the clip, prospective teachers had a chance to elaborate and develop their knowledge about students' definitions from the angle of understanding the role of necessary and sufficient conditions for providing mathematically correct definitions. This conclusion was supported with the comments written after group discussion of video clip 6. Some salient statements taken from reflection papers written after group discussion were given below.

> My knowledge about the students' possible definitions was improved after discussion. Student in video did not emphasize the necessity of closeness for figure while defining rhombus. She said that it is only a figure. I saw that student has serious problems in definition of figure [Maya, ADRP-MCVC6].

> There are so many changes in my mind before and after discussion. The most important one of them is that I need to understand students' aim in their definition very well. I have noticed in question "Is it sufficient to say "a figure" and not to say "closed" when defining quadrilaterals" that: What degree will any student be consistent when we draw figures suitable to this student's [personal] definition? [Zehra, ADRP-MCVC6].

Above statements indicated that even though prospective teachers defined all concepts establishing necessary and sufficient conditions in the individual pre-interviews they could not evaluate students' definition in terms of necessary and sufficient conditions before the teaching experiment. However, group discussion enabled them to realize unnecessary conditions or extra properties in the student's rhombus definition.

The summary of the common developments in prospective teachers' pedagogical content knowledge about definitions of quadrilaterals throughout teaching sessions was given in Figure 31.

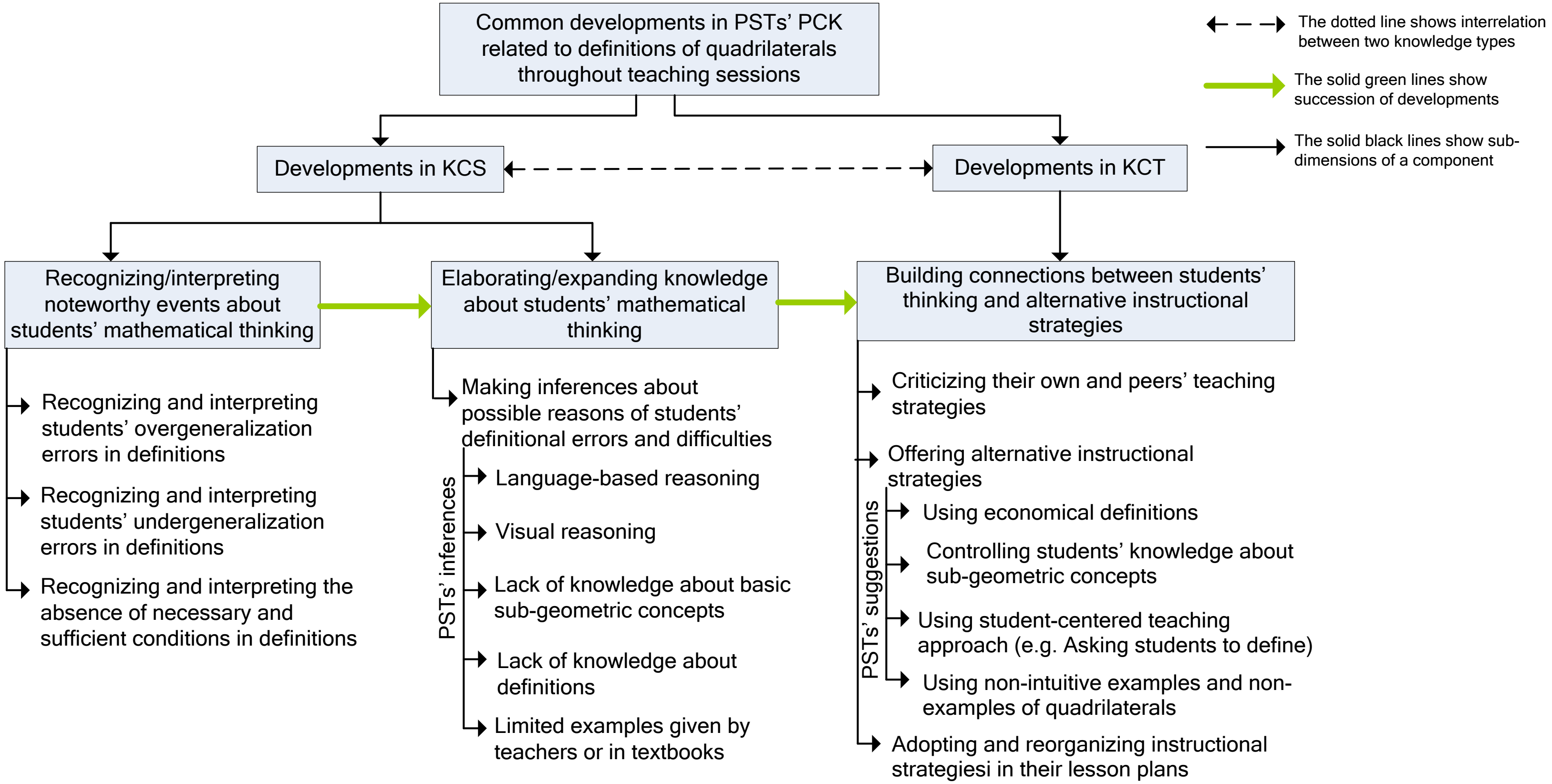


Figure 31. Summary of PSTs' developments in PSTs' PCK related to definitions of quadrilaterals in teaching session

4.2.2 Developments in prospective teachers' knowledge about constructions of quadrilaterals

In the individual pre-interviews, prospective teachers generally did not provide detailed predictions about students' possible "*errors*" and "*difficulties*" when drawing quadrilaterals. They only anticipated the presence of students' possible prototypical and nonhierarchical constructions of quadrilaterals. However, they began to notice students' errors and difficulties in the constructions of quadrilaterals though analyzing and discussing video cases. In this regard, they found students' construction processes in some video cases such as MCVC1, MCVC3, MCVC7, and MCVC8 as noteworthy events in the teaching experiment process. These noteworthy events were grouped into three categories as in Table 22.

Table 22. Students' constructions PSTs determined as noteworthy events in MCVCs

MCVCs	Concept	Students' constructions in MCVCs	Errors/Difficulties
MCVC1	Parallelogram	Two parallel line segments	Overgeneralization error
MCVC1	Parallelogram	A prototypical trapezoid	
MCVC8	Trapezoid	A five-sided convex polygon	
MCVC7	Trapezoid	A prototypical right trapezoid	Undergeneralization error
MCVC7	Trapezoid	A five-sided convex polygon	
MCVC3	Parallelogram	Prototypical and hierarchical parallelogram examples	Difficulty in construction of non-prototypical figures

In the following, how prospective teachers developed their knowledge about constructions of quadrilaterals by recognizing, interpreting, and discussing (i) overgeneralization errors, (ii) undergeneralization errors, and (iii) difficulties in students' construction processes was mentioned in the teaching experiment sessions by highlighting the noteworthy events.

4.2.2.1 *Developments in PSTs' knowledge while reasoning about students' overgeneralization errors in constructions*

4.2.2.1.1 *Noteworthy events in MCVC1*

As seen in Table 22, a seventh grade student firstly constructed two parallel line segments (e.g. [LK] and [MN] in Figure 32) as a parallelogram example. After I asked the student to construct a four-sided figure, the student incorrectly drew two line segments with non-equal length ([LM] and [KN]) and claimed that these two line segments were equal length in spite of being different length.

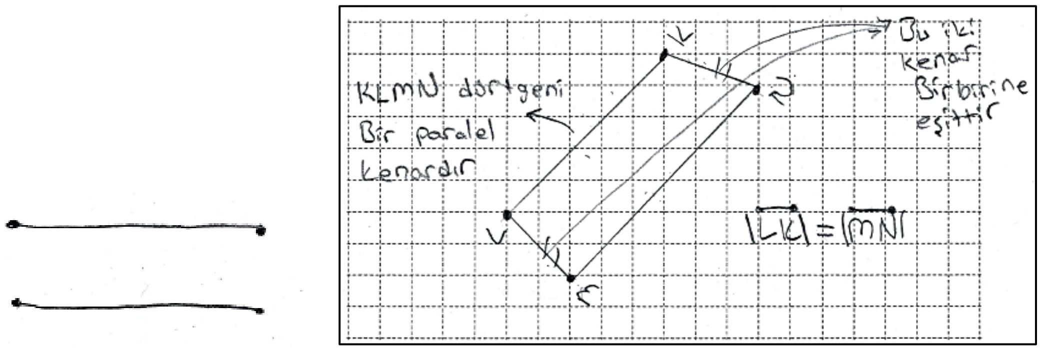


Figure 32. Student's parallelogram constructions in MCVC1

4.2.2.1.1.1 *Recognizing and interpreting student's constructional error*

When prospective teachers individually examined MCVC1, they realized that the student made an incorrect construction of parallelogram. Furthermore, they found this situation as a noteworthy event to comment in their reflection paper because they were surprised to see the student's construction of parallel line segments as an example of parallelogram. While some of them (e.g. Aslı and Deniz) only described how the student constructed parallelogram in the clip; others also provided interpretations about the possible reasons why the student drew an incorrect figure as an example of parallelogram in the clip. Three prospective teachers interpreted the meaning of student's construction of incorrect parallelogram figure in their reflection papers based on student's description. Some notable comments taken from prospective teachers' individual video analysis reflection papers were presented and interpreted in the following:

> The student thought only the parallelism of sides because she did not know parallelogram is also a quadrilateral. For this reason, she considered her construction as parallelogram although it involves non-equal opposite sides and only one pair of parallel opposite sides. I think that the reason of the student's misconceptions is lack of knowledge about definition of parallelogram. Because of that she had difficulty when constructing [a parallelogram][Beril, BDRP-MCVC1].

> The student in video thought that a quadrilateral is parallelogram if it has at least one pair of parallel opposite sides. Furthermore, she thought parallelogram as only two parallel line segments due to the Turkish meaning of "paralelkenar". This student could misunderstand the definition of parallelogram in their math lesson [Ece, BDRP-MCVC1].

In these statements, Beril and Ece inferred that because the student did not know the definition of parallelogram, its construction became incorrect. Furthermore, Ece provided an additional claim about the reason of incorrect construction by focusing on the semantic and syntactic structure of "*parallelogram*" in Turkish language. (Remember that in Turkish language, "*paralelkenar*" used instead of "parallelogram". "*Paralel-kenar*" is a word with the combinations of "*parallel-parallel*" and "*kenar-the edge*".). On the other hand, two prospective teachers (Zehra and Emel) provided an additional comment to explain the reason of incorrect construction by pointing a noteworthy event in the clip. According to them, the student in the video clip treated four corner points as the sides of parallelogram. Finally, Maya focused on the inconsistency between student's expressions and constructions about parallelogram. More specifically, she commented on the student drew LMNK quadrilateral in Figure 32 as an example of parallelogram although the figure involves non-parallel opposite sides of [LM] and [KN].

4.2.2.1.1.2 *Making inferences about reasons of student's constructional error*

After prospective teachers individually examined the video clip, I asked a question in order to understand their expectations about the student's incorrect parallelogram construction in the group discussion process (see Episode 10)

Researcher	Actually, I wondered that have you ever expected such kinds of constructions when you think the student's definition.
Deniz	I had never expected that any student could draw two parallel line segments [as an example of parallelogram].
Beril	I also did not expect.
Zehra	I expected that the student at least would know parallelogram as a closed figure.
Ece	Considering the student's definition, I supposed that she would draw a rectangle.
Researcher	In a moment of the video, the student said parallelogram must have four sides when I asked her how many sides a parallelogram has.
Emel	In my opinion, the student could not differentiate between corner and side. She said there are four sides but, she counted corners as the sides of parallelogram.
Maya	When the researcher asks her to complete figure to being a [four-sided] parallelogram, the student drew two additional sides as [LK] and [MN] [in Figure 32]. Although KLMN is a trapezoid she treated it as a parallelogram. Also, she said this figure seems not regular.
Deniz	The student mentioned about the inclination of [LK] and [MN]. She intuitively understood these line segments are not parallel.
Zehra	After that, the student also said that these line segments are equal length. If student knew the parallelism, there is also a problem because she identified all trapezoids as the examples of parallelogram [in identification task]. I could not understand what the logic of identifying trapezoids as parallelogram was.
Deniz	I think it is clear because the student treated all figures having at least one parallel opposite sides as a parallelogram.
Beril	I think that the student was confused about hierarchical relations between parallelogram and trapezoid. She inversely interpreted this relationship.
Researcher	Ok, why did the student misinterpret this relationship?
Oya	It can be related to side properties of trapezoid since it involves one pair of parallel opposite sides.
Aslı	I think that the student solely focused on the parallelism of sides and she did not know other sub-geometric concepts.
Zehra	I also think that student did not know the closeness and the

	[mathematical] meaning of quadrilateral.
Maya	Alternatively, I think that she defined and drew parallelogram by focusing on word meaning of "paralelkenar" in ordinary [Turkish] language.

Episode 10 taken from group discussion of MCVC1

As a response to my question, Zehra explained her expectation about students' possible construction of parallelogram at least as a closed figure while Ece expected that the student would construct a rectangle considering the student description of parallelogram (Remember the student's description as *parallelogram is two line segments in same proportion*.). After obtaining information about prospective teachers' expectancies about student's parallelogram construction, I raised the discussion on student' parallelogram perception as a four-sided figure by giving the student's some inconsistent explanations from the clip as an evidence. This prompt guided group members to seek the reasons why student provided such inconsistent explanations. At this point, they offered a few details. For example, Emel claimed that the student did not differentiate between corner points (L, K, M and N) of two parallel line segments that she constructed and the number of sides in any parallelogram figure. In the following they began to search some evidences from the clip to make inference about student's mathematical thinking in video. Hereon, Maya changed the direction of discussion by putting emphasis on the non-parallel sides in the student's four-sided parallelogram example. Deniz made a prediction on the reason why the student treated trapezoid as a parallelogram. She proposed that the student thought the figure as a trapezoid after she saw non-parallel sides. As a connecting idea, Beril built her proposal that the student in video clip was confused with hierarchical relation among trapezoid and parallelogram. At this point, I elaborated to the discussion by asking possible reasons of student's confusion about hierarchical relations of quadrilaterals. Here, Oya firstly proposed that the student focused on the presence of one parallel opposite sides in any geometric figure. Aslı supported Oya's interpretation and offered an explanation that the student did not know basic geometric concepts. Zehra provided details about the reasons of student's incorrect construction of parallelogram such as lack of knowledge about closeness of parallelogram and the definition of quadrilateral. Hereon, Maya came to challenge with Oya’s and Zehra’s ideas and then offered an alternative perspective for the reason of student’s incorrect construction. She inferred that the student constructed her figure considering the meanings of the words of “*paralel*” and “*kenar*” in “*paralel-kenar*” that is used for “parallelogram” in Turkish language. In summary, this episode taken from group discussion of MCVC1 indicated that prospective teachers had opportunities to develop their initial perspectives and knowledge about student’s incorrect parallelogram constructions and possible reasons of incorrectness in the constructions by virtue of sharing their ideas in a social learning environment.

4.2.2.1.1.3 Building connections between student’s constructional error and instructional strategies

Up until this point, the group had focused on the student’s constructional error and the possible reasons of the error. However, my question that aims to learn PSTs’ instructional strategies to overcome students’ similar incorrect constructional errors moved the discussion towards a new point in which they focused on some suggestive ideas on the issue as in Episode 11. They now reached a point in the discussion where they need to begin to unpack the pedagogical content knowledge further in order to claim alternative solution strategies.

Researcher	As a teacher, you have ten or fifteen students who had similar conceptions. What will you plan to overcome problem in their conceptions?
Oya	It is clearly seen from the student’s construction that there is a misconception here.
Ece	We should overcome.
Beril	I suggest special quadrilaterals must be taught beginning from trapezoid because students were confused when differentiating between parallelogram and trapezoid. I will make the explanation of every parallelogram is a trapezoid, but every trapezoid is not a parallelogram. Thus, they can understand the relationship between parallelogram and trapezoid. In addition, they are able to understand the necessity of two pair of opposite sides must be parallel in a parallelogram.
Aslı	I think that I can prepare an activity involving grid paper. In the video, I

	realized that the student had difficulties when constructing figures. In my activity, I can ask them to construct the examples of parallelogram or I can use geoboard for same purpose. After I ask them to construct figures, I can observe what they do.
Zehra	I can draw a Venn diagram on the board. Then, I can draw examples of figures in diagram. Thus, students are able to see the difference between parallelogram and trapezoid.
Researcher	Is there anyone who disagrees with or challenges the Zehra's comment?
Emel	I disagree with Zehra because students already have difficulty when drawing a figure. Instead of giving all figures on the board, we can ask them to construct. We understood from the videos that student even treated two parallel line segments as a parallelogram. Furthermore, she could not draw a [prototypical] parallelogram. As I were a teacher, I would ask students is this figure [two parallel line segments] a quadrilateral? Then, I ask the question of what do we need to say this figure a quadrilateral? I prefer a student-centered approach instead of giving all things.

Episode 11 taken from group discussion of MCVC1

At the beginning of the episode, Oya emphasized the presence of the student's misconception by considering the student's incorrect parallelogram construction. Following that, Ece implied the necessity of overcoming such kinds of misconceptions. Hereon, Beril proposed a teaching strategy in which she proposed that starting trapezoid when teaching quadrilaterals can be useful to avoid students' inabilities to differentiate parallelogram and trapezoid. Differently, Aslı explained her future instructional plan by mentioning about which material she wants to use for which purpose. According to her, using grid paper or geoboard might be helpful to prevent students' constructional difficulties. At this point, Zehra suggested an alternative approach in which she aimed to draw a Venn diagram with the examples of quadrilaterals in order to make emphasis on the main differences between parallelogram and trapezoid. Up to this point, they generally focused on instructional materials and representations. However, the question of "*is there anyone who disagrees with or challenges the Zehra's comment?*" activated Emel to explain her disagreement. By referencing the student's parallelogram construction in MCVC1, Emel proposed that instead of drawing all figures on the board in the lesson, it should be asked the students to construct related quadrilaterals. By indicating student's two parallel line segments construction as an example of parallelogram, she claimed that questioning students' knowledge about basic geometric concepts such as the construction and the meaning of "*quadrilateral*" is necessary before starting to construct a parallelogram figure. By the end of the teaching sessions, Emel's interpretation was taken under serious consideration as a milestone by all group's members. As a result, they suggested some different solution ways to overcome student's constructional errors in their after discussion reflection papers and revised lesson plans. For instance, Maya, Ece, and Aslı added grid papers to their lesson plans for the constructions of quadrilaterals. Furthermore, Zehra, Ece, Oya, and Aslı adopted a student-centered approach instead of direct teaching approach in their revised lesson plans. While they had preferred to give all examples of quadrilaterals in a paper before lesson plan revisions, they decided to add grid papers by asking students to draw example figures. Finally, Zehra and Ece added some explanations about the meaning of quadrilaterals in their lesson plans by taking account of Emel's suggestion in Episode 11.

4.2.2.1.2 Noteworthy event in MCVC8

Analyzing and discussing the student's mathematical thinking in MCVC8 also had a contribution to prospective teachers' pedagogical content knowledge related to understanding student's constructional errors and difficulties related to quadrilaterals. At the beginning of the clip, the student described trapezoids as the irregular polygons. Then, she initially drew a five-sided polygon as an example of trapezoid (see Figure 33-a) although she said that she could not remember how to have a shape of trapezoid. In the following of the clip, although the student stated that trapezoids have no parallel sides, she constructed an additional example of trapezoid as ABCD quadrilateral having parallel opposite sides of [AB] and [DC] in the grid paper (see Figure 33-b).

(a) (b)

Figure 33. (a) Student's five-sided construction as an example of trapezoid (b) student's trapezoid construction that she proposed it has no parallel sides

Prospective teachers' individual pre-interview data revealed that only Emel and Oya predicted that students can draw a quadrilateral having no parallel sides as an example of trapezoid. Furthermore, Emel provided an additional prediction on students' possible trapezoid constructions by proposing that students can draw irregular figures, which are not polygon, in order to exemplify trapezoid figure in the grid paper. However, all prospective teachers could not predict students' trapezoid constructions having more than four sides or the inconsistencies among the constructions of trapezoid as seen in Figure 33.

4.2.2.1.2.1 Recognizing and interpreting student's constructional error

When they individually examined the student's thinking about trapezoid in MCVC8, they noticed that student constructed both a five-sided polygon and a four-sided quadrilateral as the examples of trapezoid. In the individual video analysis reflection papers, while Maya only described the figure the student drew, the others made inferences about why the student drew a four-sided quadrilateral having one pair of parallel sides in spite of mentioning the lack of parallelism of the sides of trapezoid or why the student drew five-sided figure. Ece and Aslı focused on the reason of student's five-sided construction of trapezoid. They offered that the student made the constructions considering the word meaning of trapezoid in Turkish language. Ece's statements included:

> Student drew a rotated trapezoid in grid paper without awareness because she said that there is no parallel opposite sides in trapezoid. This was an unpredictable situation for me. The student drew a five-sided figure. There is no word such as “dörtgen” in “yamuk” as in “dikdörtgen” [“dörtgen” corresponds to “quadrilateral” in Turkish language]. Thus, language can be a possible reason of the student’s confusion [Ece, BDRP-MCVC8].

Differently, remaining prospective teachers concentrated on student’s inconsistent responses and second construction of trapezoid in Figure 33. Three of them (Emel, Deniz, and Oya) specifically focused on the student’s inconsistent responses and second construction of trapezoid. They thought that the student might not know the parallelism of line segments because the student drew a non-prototypical trapezoid according to exclusive relations among quadrilaterals in the grid paper although the student previously stated that there were no parallel sides in a trapezoid figure. Finally, Zehra provided a comment in her reflection paper in which she argued that the student drew a five-sided polygon due to the influence of visual appearance of an exclusive trapezoid figure on the student’s concept images about trapezoid. Consequently, individual video analysis data revealed the diversity of prospective teachers’ interpretations and inferences about the meaning and the reasons of student’s constructional error.

4.2.2.1.2.2 Making inferences about reasons of student’s constructional error

In the discussion process that was conducted after prospective teachers finished to individually examine the clip and to write their reflection papers about student’s mathematical thinking about trapezoid, I asked a question in order to understand their expectations about students’ possible trapezoid constructions before participating the teaching experiment (see Episode 15).

Researcher	Do you remember what have you predicted about students’ possible trapezoid constructions in our pre-interviews?
Emel	I expected that student draw an irregular figure. However, I did not expect that she would not draw a polygon because trapezoid is involved

	in quadrilaterals set.
Beril	I also thought that students do not draw a figure having more than four sides as an example of trapezoid.
Oya	I thought that students can treat quadrilaterals having non-parallel sides as the examples of trapezoid.
Researcher	What did you think when you saw the student's trapezoid constructions in video?
Beril	I did not expect the student might draw a five-sided polygon because students learn trapezoid in the context of quadrilaterals.
Ece	I agree with you. Moreover, she constructed a rotated trapezoid [as in Figure 33-b]. Students generally could not draw such a figure.
Oya	However, she said that there is no parallel opposite sides in the figure although there is one pair of opposite sides in Figure 33-b.
Ece	She was unaware about whether her construction involved parallel sides or not.
Aslı	In my opinion, she did not know parallelism concept.
Emel	I think she knew neither parallelism nor trapezoid.
Deniz	Interesting, she drew correctly [a non-prototypical] trapezoid, but she did not know what parallelism means. Also, she said there are no parallel sides.
Emel	I think they coincided.
Aslı	I agree. I think that she did not also know the meaning of quadrilateral well because she firstly drew a five-sided shape in Figure 33-a. In the following of the video clip, she identified parallelogram and rectangles as trapezoids. In my opinion, student made a messy classification in terms of trapezoid.
Ece	When someone asks what trapezoid is, students did not consider the properties of sides. According to students, the presence of on-equal sides in a polygon is enough to identify a figure as a trapezoid.

Episode 12 taken from group discussion of MCVC8

Emel began to express her expectations about students' possible constructions such as an irregular shape. Additionally, she mentioned that she did not expect students could draw a polygon having more than four sides as an example of trapezoid. Here, Beril supported Emel's expectations. Differently, Oya provided an explanation involving an expectation of students' possible quadrilateral construction having no parallel sides as an example of trapezoid. After that, I shifted the discussion on the constructions which the student drew in MCVC8 to understand how they interpreted in more detail. At this point, Beril stated that I did not expect the student might draw a five-sided polygon because students learn trapezoid in the context of quadrilaterals. Ece provided additional information to develop an idea. Her explanations showed that she could not discount any student can draw a non-prototypical trapezoid example. In the following, Oya directly focused on student inconsistent responses about the presence of parallelism of sides in a trapezoid by giving evidence from the clip. In response, Ece suggested that the student was not sure the parallelism of opposite sides in Figure 33. As an alternative viewpoint, Aslı offered the lack of student's knowledge about parallelism concept. Emel added an idea in which she claimed that the student have knowledge about neither trapezoid nor parallelism. Deniz summarized the situation under the discussion by focusing on how the student arranged the line segments as being parallel in the grid paper. After challenging Deniz's claim, Emel explained her interpretation that the student drew parallel line segments in the figure by chance rather than consciously arranging them. After supporting all ideas in the discussion episode, Aslı summarized student's mathematical thinking by giving some evidences from the video clip. As a final point, Ece made an inference about students' trapezoid image in their minds based on the length of the sides of any trapezoid figure. In sum, this episode is significant prospective teachers' justifications and interpretations clearly showed how their knowledge on student's mathematical thinking reemerged in group discussion process. Thus, they reached new conclusions to explain why the student developed such thinking when constructing a trapezoid by sharing their ideas with the peers.

Moreover, after group discussion process, prospective teachers excluding Emel, Deniz, and Oya explained her own development in their after discussion reflection paper. To illustrate, Ece's statements were given in the following:

> I had thought that the student solely focused on the word meaning of trapezoid in Turkish language when individually analyzing the clip. However, I found my friends' inference because they supposed that the student could not know the meaning of parallelism. Basic sub-geometric concepts are very important. Even if students know the word meaning of a concept, they cannot draw the figure if they do not know parallelism. Group discussion enabled me to receive this issue [Ece, ADRP-MCVC8].

As seen in the example statements, prospective teachers noticed the importance of basic geometric concepts in addition to the influence of linguistic factors on the student's trapezoid constructions. To conclude, throughout the teaching experiment process, analyzing and discussing the student's constructions of quadrilaterals in the micro case video clips such as MCVC1 and MCVC8 had many contributions to the prospective teachers' pedagogical content knowledge in terms of understanding middle school students' mathematical work since they noticed both students' overgeneralization errors and questioned their reasons in social learning environment.

4.2.2.1.2.3 Building connections between student's constructional error and instructional strategies

Another crucial development in PSTs pedagogical content knowledge occurred after the group discussion of MCVC8 because they needed to develop different solution strategies in their reflection papers for overcoming students' constructional errors. For example, Zehra and Aslı suggested that it is useful to use interactive geometry programs such as Geogebra for overcoming students' constructional errors of trapezoid in their written statements. Furthermore, four PSTs (Ece, Beril, Emel, and Oya) mentioned the effectiveness of using grid papers in constructions of quadrilaterals. Some example written statements were given in the following:

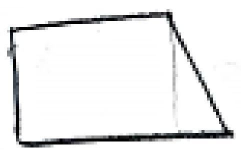
> We should teach how parallel line segments can be constructed by using a student-centered teaching strategy before teaching quadrilateral to the students in order to overcome misconceptions about parallelism that we saw in video clips [Ece, ADRP-MCVC8].
>
> I noticed that the student could not pay attention to whether line segments are linear or not. For example, she tried to construct a square and a triangle to produce a trapezoid when completing a figure having only two sides of trapezoid. In order to overcome such problem, we can ask them to construct figures in grid paper in our lessons. Specifically, we firstly show parallel line segments. In the following, we can ask them to construct various parallel line segments in grid paper [Beril, ADRP-MCVC8].

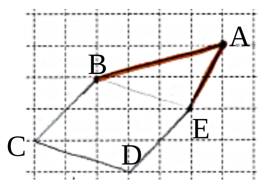
In these proposals, Ece and Beril focused on the importance of determining whether students know basic geometric concept such as parallelism or not before teaching a new concept such as trapezoid. Furthermore, according to them, utilizing grid papers for the constructions of quadrilaterals was found reasonable and efficient with a student-center teaching method. Differently, Maya and Zehra concentrated on the possible positive influence of using non-examples in teaching process for preventing students' overgeneralization errors in which students treat some non-examples as examples by extending their knowledge to another context in an inappropriate way. Consequently, such kinds of solution strategies clearly indicates the inevitable effect of analyzing and discussing MCVCs on PSTs' PCK involving instructional strategies.

4.2.2.2 Developments in PSTs' knowledge while reasoning about students' undergeneralization errors in constructions

4.2.2.2.1 Noteworthy event in MCVC7

As mentioned before, the student in MCVC7 defined trapezoid as the combinations of a square and a triangle at the beginning of the clip. After defining, the student constructed Figure 34-a as an example of trapezoid. I asked her to continue the partial construction as being a trapezoid in the Figure 34-b in which only the sides of [AB] and [AE] were given. At this point, the student constructed a five-sided figure instead of a four-sided figure.





(a) (b)

Figure 34. (a) Student's first trapezoid construction; (b) second trapezoid construction

4.2.2.2.1.1 Recognizing and evaluating student's constructional error

In the individual video analysis process, prospective teachers' reflective comments showed that they were surprised with the student's five-sided construction of trapezoid because the student said that trapezoid has only four sides at the beginning of the clip. In their reflection papers, five of them only focused on the incorrectness of the construction due to having five sides rather than focusing on how the student thought when drawing a five-sided figure and why the student drew five-sided figure. However, Emel and Ece not only evaluated the correctness of the student's construction of five-sided figure in terms of being a trapezoid example but also they proposed some claims involving the possible reasons of incorrect trapezoid construction. Some notable statements taken from individual video analysis reflection papers were exemplified in the following:

> Moreover, the student tried to combine a square and a triangle by looking at her personal definition in figure completion process. However, she could not recognize her trapezoid construction having five sides instead of four sides [Ece, BDRP-MCVC7].

As seen in these example statements, they offered a claim in which they proposed that the student tried to construct a figure considering the student's personal description of trapezoid. In the following, they specifically concentrated on the student's awareness about the incorrectness of the construction of trapezoid. On the other hand, in the individual video analysis reflection paper, Beril drew attention to the inconsistencies between the student previous statement about the number of sides of trapezoid and the student's five-sided construction of trapezoid. Next, she continued her reflections by suggesting a reason why the student constructed five-sided figure as an example of trapezoid. Her comments indicated that she thought that the student did not carefully construct by considering the properties of grid paper or the student might not know all critical features of the trapezoid. In summary, individual video analysis reflections showed that many of participants only described how student constructed trapezoid in the clip without providing interpretive comments.

4.2.2.2.1.2 *Interpreting student's constructional error*

In the discussion process of the clip, I asked a question about student's trapezoid construction in order to understand how prospective teachers will interpret the student's trapezoid constructions within a social learning environment. As seen in Episode 13, this question initiated the discussion between group members and me.

Researcher	What did you notice about the student's constructions of trapezoid?
Ece	When completing the shape in Figure 34-b, the student tried to construct trapezoid considering her personal definition. As a result, she firstly tried to construct a square. Then, she put a triangle to the next of square.
Maya	I think the most interesting thing in video was the student's figure completion process. I never predict she produced such kind of trapezoid.
Emel	Although she said this figure have four sides she constructed five-sided figure.
Ece	In my opinion, she could not pay attention to the non-linearity of the combination of [DE] and [EA].
Deniz	Really, I did not notice [when individually analyzing the video]
Ece	Absolutely, she firstly tried to draw a square as in her definition.

Episode 13 taken from group discussion of MCVC7

After my prompting question, Ece explained the way student constructed trapezoid. She claimed that the student tried to apply her definition on the construction of trapezoid. More specifically, she considered that the student tried to combine a square and a right triangle in the given incomplete figure. Then, Maya explained her inexpectations on a five-sided figure construction. In the following, Emel specifically focused on the inconsistency between student's explanations about the number of any trapezoid and five-sided construction as a trapezoid. At this point, Ece offered a claim for the reason of the student's five-sided trapezoid construction. She claimed that the student did not realize nonlinearity between line segments of [DE] and [EA]. Here, Deniz developed an understanding about the reason why student constructed a five-sided figure as a trapezoid example. In conclusion, as we see it from the episode, mathematical arguments that provided PSTs shifted from descriptive stance to evaluative and interpretive stance because they provided new and alternative comments to explain the reason why this student drew a five-sided figure as an example of trapezoid.

4.2.2.2.1.3 Making inferences about reasons of student's constructional error

In order to elaborate the issue under the discussion, I posited a new question that enabled the elaboration of the discussion like in Episode 14 for thinking additional reasons of the student's incorrect trapezoid construction.

Researcher	Why did the student draw a five-sided polygon as an example of trapezoid?
Emel	If the researcher gave the line segments of [AB] and [AE] as a linear position, the student could draw four-sided figure as trapezoid?
Maya	Because the researcher gave half of the figure in grid paper the student had difficulty to construct remaining part of trapezoid.
Ece	She could not provide linearity of [DE] and [EA]. She envisioned CDEB as a square by considering her definition of trapezoid.
Beril	She could not correctly complete the figure.
Aslı	The student might think the line segments of [DE] and [EA] as linear because they seems linear.
Maya	I agree with you. She could not recognize non-linearity.

Episode 14 taken from group discussion of MCVC7

For instance, Emel made a prediction that if I gave the incomplete figure that can be completed as a prototypical trapezoid, student can easily completed the figure. Based on the Emel's interpretation, Ece again built her idea in which she claimed that the student did not construct linearity between line segments of [DE] and [EA] in the grid paper. Moreover, Beril supported Ece's idea. Then, Aslı provided a new perspective by claiming the student could not realize whether line segments of [DE] and [EA] are linear or not. Hereon, Maya supported her. The statements at the end of the episode was clearly an indicator of the development in Aslı's and Maya's pedagogical content knowledge about students' mathematical thinking because she only described how the student constructed trapezoid in her individual reflection paper.

Moreover, PSTs' reflection papers written after the group discussion also showed up the developments in prospective teachers' pedagogical knowledge about understanding students' constructional errors related to quadrilaterals and their possible reasons. Their comments reflected that they interpreted student's construction beyond the description or evaluation of the noteworthy events in the clip because they commented on how the student reasoned geometrical figures or why the student constructed such a figure. Some developments in their knowledge were explicitly asserted in the following excerpts.

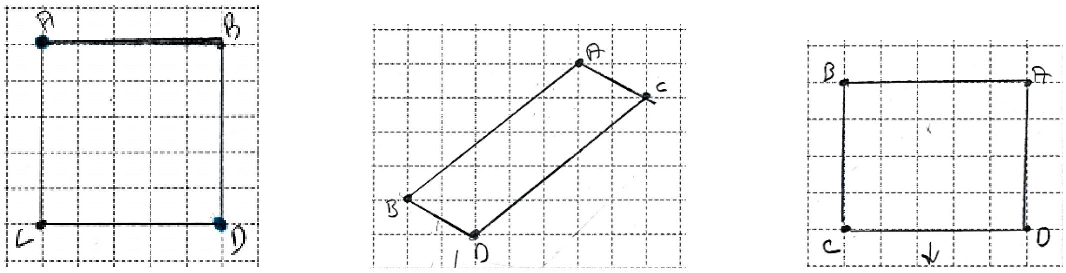
> In the group discussion, by the help of my friends' ideas, I noticed that the student could not recognize whether her construction is a quadrilateral or pentagon [Deniz, ADRP-MCVC7].
>
> In the group interaction, an interpretation about how the student responded the questions was very useful for me. In the group discussion, for example, I recognized that although the student wanted to obtain a quadrilateral when drawing, she could not adjust the points on a linear line in grid paper and she produced a pentagon [Aslı, ADRP-MCVC7].
>
> I had thought that the student in VC7 did not pay attention to the parallelism of opposite sides in the following figure , but I had never thought she tried to construct trapezoid as the combination of a square/rectangle and a triangle before group discussion [Maya, ADRP-MCVC7].

It was evident that Deniz explained that after the group discussion, she noticed the student could not realize whether Figure 34-b is a quadrilateral or pentagon. Furthermore, in individual video analysis, Aslı's comments indicated that she only identified what student draw in the clip. However, after discussion reflection paper, she commented on the development in her knowledge about student's mathematical thinking due to realizing the relation between student's description and constructions of trapezoid. On the other hand, Emel's comments showed that she developed her perspective about the reason why the student drew five-sided figure as a trapezoid example. Before group discussion, she offered a claim that the student tried to construct a figure considering her description of trapezoid. However, after the group discussion, she developed a new viewpoint about the reason of five-sided construction because the group proposed that the student might perceive the line segments of [AB] and [CD] as linear. In a similar way, Maya developed her knowledge about the reason of the student's incorrect drawing since she noticed the relationship between student's description and drawing as the combinations of a square and a right triangle.

4.2.2.3 Developments in PSTs' knowledge about students' difficulties in non-prototypical figure constructions

In the pre-interviews, although prospective teachers predicted students' tendency in construction of prototypical figures, they did not consider students' possible difficulties in construction of non-prototypical quadrilaterals. However, as they analysed and discussed video cases in teaching sessions, they realized that students had difficulties in construction process of geometric concepts. A noteworthy example was given in the following.

The student in MCVC3 firstly constructed a square as an example of parallelogram. After I asked him to construct two more examples of parallelogram, he constructed two additional examples in Figure 35-b and Figure 35-c. Although the student drew a parallelogram in Figure 35-b and a rectangle in Figure 35-c, he inappropriately named the figures as "*parallel-rectangle*" and "*parallel-square*" respectively.



(a) (b) (c)

Figure 35. **(**a) Student's first parallelogram construction as ACDB square; (b) BDCA named as "*parallel-rectangle*"; (c) BCDA rectangle named "*parallel-square*"

Most of prospective teachers' reflection papers written in individual video analysis process of the clip indicated that they did not consider the student's different parallelogram constructions and denominations of these constructions as a noteworthy event when individually analyzing the events in the MCVC3. Instead, they generally focused on the correctness of student's constructions or other significant events in the clip instead of the reason why the student always needed to construct a rectangle or square to provide parallelogram examples. However, after I asked a question about the types of student's parallelogram construction to the group in the discussion process of student's mathematical thinking about parallelogram, they wondered why the student always tried to construct a rectangle or square instead of typical parallelogram examples. Corresponding episode taken from group discussion of MCVC3 was given and interpreted in below.

Researcher	What kinds of figures did the student when I ask him to construct a parallelogram?
All group	Square and rectangle!
Deniz	However I found interesting something. For example, he named quadrilaterals in Figure 35 as parallel-rectangle and parallel-square. I never predict such kind of thinking before watching this video.
Aslı	Such constructions in Figure 35 indicate that he made a personal classification for parallelogram. It is really interesting.
Beril	However, parallel-square named by the student is not actually a square and he was unaware. I think that this student could not differentiate between square, rectangle, and parallelogram.
Deniz	I expected that the student immediately drew a well-known

	parallelogram, but he always drew a rectangle and square. I wonder why did he think that?
Emel	In my opinion, he could not draw other types of parallelogram.
Researcher	Why could not he draw?
Ece	He found easy to construct square in the grid paper.
Zehra	Additionally, he said that I would draw this figure when the researcher drew a prototypical parallelogram in the video.
Ece	He even explained why he could not construct. He believed that he could not construct equal length sides in the grid paper.
Deniz	Interesting! I missed this point.
Maya	I could not pay attention this point when individually analyzing video.

Episode 15 taken from group discussion of MCVC3

I oriented the discussion on the student's constructions of parallelogram. As soon as they examined the constructions, they have realized that the student drew square or rectangle whenever I asked him to draw an example of parallelogram in the clip. At this point, Deniz focused on student's naming style of the quadrilaterals (remember that the student named Figure 35-b and Figure 35-c "parallel-rectangle and parallel-square respectively). Here, Aslı offered a new proposal that the student interestingly asserted a personal classification for parallelogram. However, Beril pointed out the incorrect naming of the parallelograms such as parallel-square. She explained that although the student constructed a rectangle in Figure 35-c, he treated the figure as a square. Furthermore, she claimed that the student did not know the differences between rectangle, square, and parallelogram. Up to this point, they discussed the types of student's parallelogram construction. However, after Deniz wondered the reason why the student always tried to draw a rectangle or square instead of a prototypical parallelogram figure they began to produce new ideas about the possible reasons why the student did not prototypical parallelograms. In response to Deniz, Emel claimed that the student could not draw other kinds of parallelograms. Oya elaborated Emel's idea by proposing that the student found easy to draw square in a grid paper. After I prompted the discussion with a question seeking the information about the reason why the student had difficulty to draw parallelogram figure, Zehra and Ece provided some evidences from the clip in order to explain student's difficulty. They put emphasis on the student's self-explanations about the difficulty in drawing a (prototypical) parallelogram. By the help of these salient evidences, Maya and Deniz realized that they could not carefully pay attention student's explanations in the clip. As a result, they realized some details about student's errors and difficulties in the construction of parallelogram.

The summary of the common developments in prospective teachers' pedagogical content knowledge about constructions of quadrilaterals throughout teaching sessions was given in Figure ***36***.

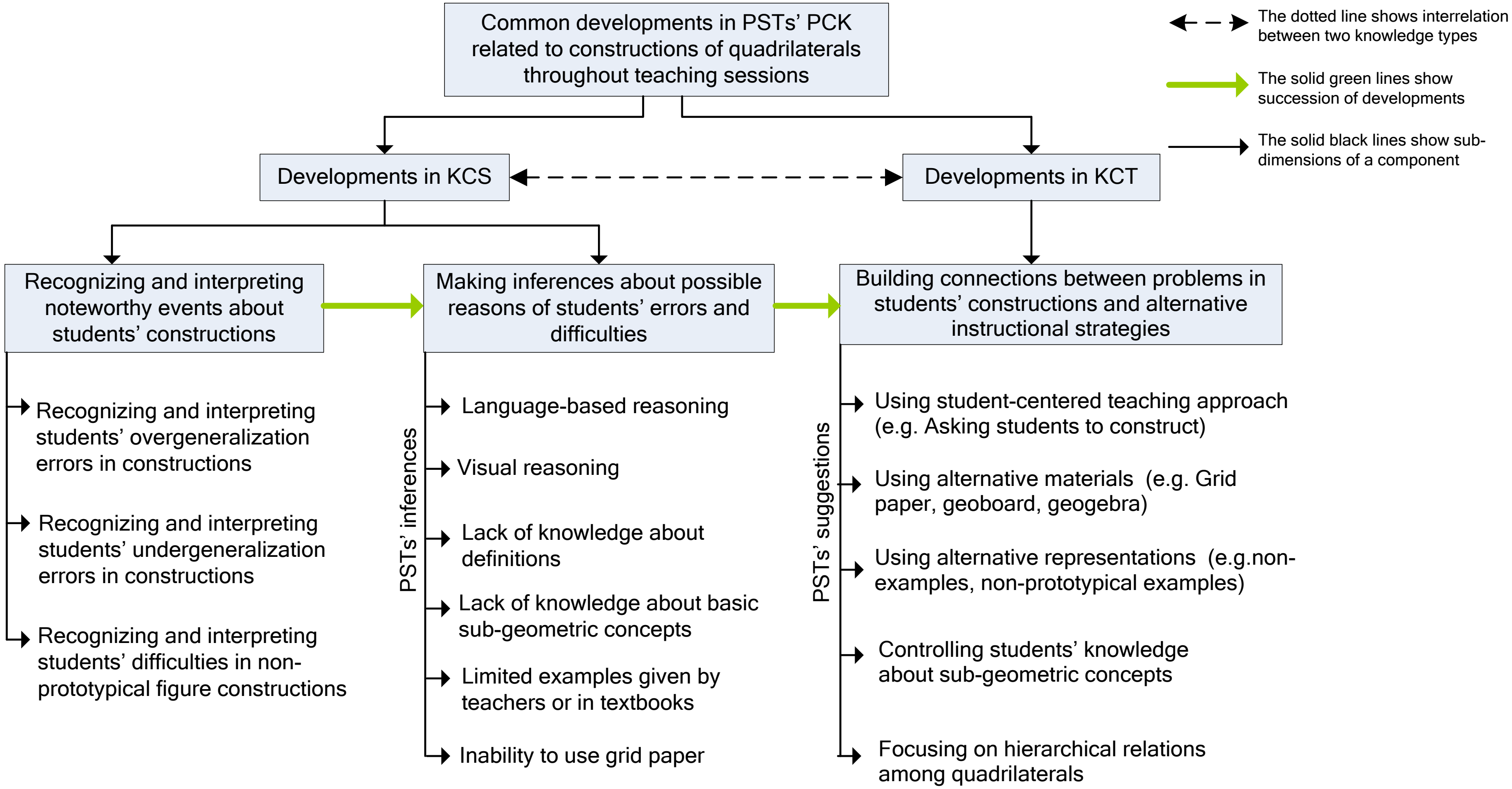
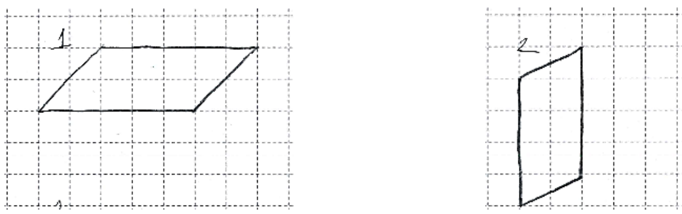


Figure 36. Summary of the developments in PSTs' PCK related to constructions of quadrilaterals in teaching sessions

4.2.3 Developments in prospective teachers' knowledge about students' nonhierarchical/prototypical concept images of quadrilaterals

In the individual pre-interviews that were conducted before the teaching experiment, all prospective teachers predicted that students might have prototypical and nonhierarchical concept images about quadrilaterals. For example, most of them thought that students would not consider rectangle as an example of parallelogram or trapezoid. As another example, they proposed that giving parallelogram example in Figure 37-b to the students when teaching the concept certainly prevents the formation of prototypical concept images about parallelogram like in Figure 37-a.



(a) (b)

Figure 37. (a) PSTs' prototypical parallelogram examples (b) PSTs' non-prototypical parallelogram examples

On the other hand, throughout teaching sessions, they began to recognize that some students' concept images sometimes can be inflexible or unstable (*Note*: I mean that inflexible concept images are the images that are resistance to change. Such images generally develop over a prototypical example. For example, if a student has inflexible concept image about square s/he cannot inflexibly imagine square rotated by 45˚ as an example of square. Instead s/he treats it as an example of rhombus. On the other hand, unstable concept images do not mean a flexible change. I used the term of unstable to reflect changing images inconsistently in a learner's mind. For instance, a student define trapezoid as a figure having no parallel opposite sides, but s/he also can draw a figure having parallel opposite sides as an example of trapezoid or s/he can be confused and change her or his mind while studying on a task.). All details how they realized and interpreted middle school students' unstable or inflexible concept images about quadrilaterals were mentioned in the following paragraphs according to the noteworthy events in the video clips such as MCVC2 and MCVC6.

4.2.3.1 Developing ideas about students' prototypical concept images

4.2.3.1.1 Noteworthy events in MCVC2

The student had prototypical and nonhierarchical concept images about parallelogram in MCVC2. At the beginning of the clip, the student described parallelogram as "*a distorted figure like a pushed down form of rectangle or square*". In the parallelogram selection part of the clip, she selected a rotated rectangle (see 11 in Figure 38) as an example of parallelogram and a rotated square (see 9 in Figure 38) as an example of rhombus. In other words, she did not consider a rotated square and a rotated rectangle as a square and rectangle, respectively. Instead, she said that a rotated square and a rotated rectangle become a rhombus and a parallelogram, respectively. Furthermore, the student considered that a typical square and rectangle cannot be an example of a rhombus and a parallelogram, respectively. This situation showed inflexible nature of the student's conceptions about parallelogram.

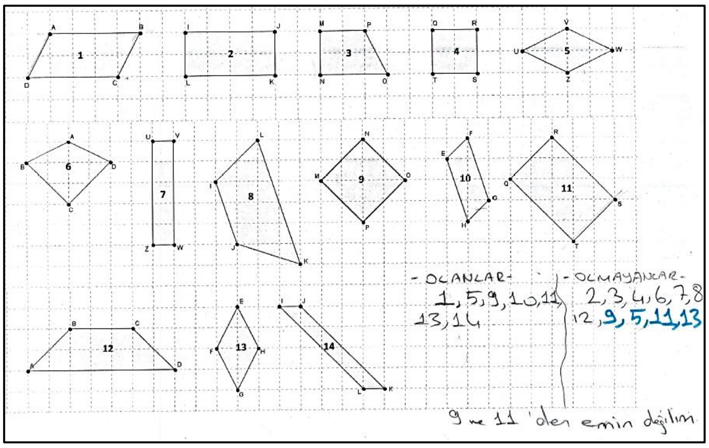


Figure 38. Student's parallelogram selections in MCVC2

Another crucial point in the clip was that the student firstly treated rhombuses (see 5 and 13 in Figure 38) as an example of parallelogram. However, she decided to exclude all rhombuses from her parallelogram selections after discussing whether a square is a parallelogram or not with me. At the end of the clip, the student changed her mind and provided nonhierarchical selections for parallelogram considering the equal sides in rhombus by stating that only opposite sides must be equal in a parallelogram, which showed unstable nature of the student's conceptions.

4.2.3.1.1.1 Interpreting the student's inflexible and unstable concept image

PSTs' comments in individual video analysis reflection papers indicated that they provided some interpretive comments about the student's changing ideas about the relations of quadrilaterals to find possible reasons and possible solutions to them. Some example illustrative comments were as following:

> In my opinion, quadrilaterals were always given in same orientation by mathematics teachers in lessons. As a result, students could not identify figures in rotated forms such as 9 and 11 (see Figure 38). Furthermore, the student had difficulty to consider whether a rhombus is also a parallelogram or not. Firstly, she looked the equality of opposite sides. Then, she changed her decision by focusing the length of all sides in figures. I concluded that this student memorized something without developing an understanding [Aslı, BDRP-MCVC2].

These comments showed that she made some descriptions and interpretations about student's prototypical and nonhierarchical concept images about quadrilaterals. She mentioned that the student did not realize the figures when they turned. Then, she commented on the student's changing ideas about the relation between rhombus and parallelogram by describing the situation in the clip. At the end of the descriptions, she made inference about possible reason of the student's prototypical nonhierarchical concept images by pointing the possibility of student's rote learning in math lessons. Nevertheless, she could not provide detailed interpretations and inferences on how the student reasoned parallelogram and the relations among quadrilaterals or why the student changed her mind about the relations among rhombus and parallelogram throughout the clip. Another example was given in the following:

> The student knew what parallelogram is. However, the appearance of [prototypical] parallelogram was always imagined in her mind. According to student, a figure must be pushed down to be a parallelogram. As a result, rectangle and square are not [examples of] parallelogram. The student used both the equality of the length of sides and parallelism of opposite sides when identifying a figure as parallelogram. She knew the necessity of the equality of the length of sides in parallelogram. However, she thought a figure must involve two [equal length] short sides and two [equal length] long sides to be a parallelogram. In other words, she thought that the length of all sides of parallelogram is not equal. As a result, she did not consider square and rectangle as an example of parallelogram. Firstly, she considered rhombus as a parallelogram because rhombus has parallel opposite sides. However, she changed her identification because she focused on the length of sides. The student could not establish a relationship between rectangle, square, and rhombus due to the lack of understanding about inclusive relations. She paid attention on the visual properties of [prototypical] figure. Although all properties of a figure belong to parallelogram, she gave incorrect responses based n visual properties [Emel, BDRP-MCVC2].

Emel's written comments in her reflection paper of individual video analysis indicated that she initially interpreted student's parallelogram perception based on the student's description and constructions of parallelogram. By referencing her first comment, she inferred that student did not consider rectangle and square as a parallelogram. Then, she interpreted how the student selected parallelogram among different polygon figures. Furthermore, she explained the reason why the student did not consider square and rectangle as an example of parallelogram in detail. According to her, the student firstly focused on the parallelism of opposite sides in a rhombus and considered it as a parallelogram example. However, because all sides of rhombus are equal length the student did not consider rhombus as a parallelogram. As a final point, she inferred that the student always focused on the visual appearance of prototypical parallelogram rather than focusing on critical properties of parallelogram. In conclusion, when prospective teachers individually examined student's mathematical thinking in MCVC2, they realized the unstable nature in the student's selections of parallelogram due to the influence of imperfect concept images developed under prototypicality and exclusivity of quadrilaterals.

4.2.3.1.1.2 Establishing a relation between student's concept image and concept definition

In the group discussion process of MCVC2, I asked a question about how student in the clip reasoned about the relations among quadrilaterals by drawing attention to the student's description of parallelogram (see Episode 16).

Researcher	I expected that the student would consider rectangle and square as parallelogram because she defined parallelogram over rectangle and square. Did you think the same thing?
Beril	When I read the student's [parallelogram] definition, I thought that she was able to establish a relationship between parallelogram and square/rectangle. I had expected that she did not think properties of a figure do not change when pushing down the figure.
Emel	I thought the same thing.
Ece	I thought differently.
Deniz	I also thought differently because the student directly identified square and rectangle as non-examples of parallelogram [in identification task]. She saw the pushed down form of rectangle and square as parallelogram.
Ece	She looked inclined position of figures when saying pushed down.
Aslı	In my opinion, the student could not comprehend the figures when changing its orientation. The reason of this situation can be related to mathematics teacher's teaching styles. Their teacher can draw only prototypical figures in her/his lessons. Furthermore, the teacher may not give information about [invariant] properties of a figure and its rotated form.

Episode 16 taken frrom group discussion of MCVC2

I supposed that the student might select square and rectangle as a parallelogram due to the involvement of the student's description of parallelogram in MCVC2. At this point, Beril and Emel supported my idea. Furthermore, Beril explained that after listening student's description she expected that the student can consider a rhombus as an example of parallelogram because if square is distorted it becomes a rhombus. However, Ece and Deniz disagreed with me and their peers' comments. For example, Deniz proposed that the student thought suppressed form of rectangle or square as a parallelogram, but not typical form of rectangle and square. In the following, Ece offered additional information that is directly related to the issue under the discussion. She offered an observation of an event in the video segment in which the student looked for an inclined quadrilateral because she treated parallelogram as a figure suppressed from cross corners. Hereon, Aslı inferred that the student selected parallelogram under the influence of her rote learning. According to Aslı, math teacher's instructional approach can lead prototypical concept images in the student's mind because giving only prototypical parallelogram example and not giving any rotated quadrilaterals in the lessons. Consequently, by virtue of discussing the events in the video segments, prospective teachers elaborated their knowledge about student's prototypical concept images by considering the student's selections of rotated figures (e.g. 9 and 11 in Figure 38). For instance, while Aslı provided superfluous interpretations about student' prototypical concept images in her individual video analysis reflection paper, she made detailed inferences about the reasons why the student had prototypical concept images about parallelogram in group discussion process.

4.2.3.1.1.3 Building a connection between student's changing concept image and instructional strategies

Up until this point, prospective teachers realized student's changing ideas about prototypical and nonhierarchical concept images of parallelogram by examining and discussing event segments in MCVC2. Furthermore, they had opportunities to develop their knowledge on what can the reasons of prototypical and nonhierarchical concept images of parallelogram can be by the help of group discussion of video clip. After PSTs understood the student's difficulties about hierarchical relation among quadrilaterals and rotated figures, I prompted a new question how they provide an instructional strategy to overcome the student's such kinds of difficulties and errors. This question enabled to see PSTs' potential solution strategies to the problematic situations in the student's prototypical concept images (see Episode 17).

Researcher	If you were a teacher how would you overcome such type of student's misconceptions and errors?
Ece	I rotate figures by different angles. Student must see all kinds of examples because student in video supposed a figure becomes a different

	figure when rotated.
Zehra	I firstly give the definition of concept. Then I ask students to try different constrıctions considering the definition in geoboard.
Maya	For example, I also draw a square and then I ask students what this figure becomes after I rotate it. In school, our teachers generally say if you see a property of geometric figures you can look the figure by rotating. We can show various examples of a specific concept.
Oya	I will do the same.

Episode 17 taken from group discussion of MCVC2

In response to my question, Ece claimed that it is necessary to give rotated figures when teaching quadrilaterals by indicating the student's responses for rotated figures in MCVC2. In the following, Zehra offered using geoboard. She expressed her instructional preferences in which she aimed to use geoboard to ask students to produce figures according to given definitions. Oya supported Zehra's suggestion. This episode indicated how they developed pedagogical decisions throughout group discussion although they were unaware about such a student's mathematical thinking before analyzing the video.

4.2.3.1.1.4 Criticizing and reorganizing instructional strategies in lesson plans

In the following of the group discussion, the group reached a point in the discussion where they needed to think their initial instructional approaches in order to evaluate the form of how they had focused on hierarchical relations of quadrilaterals in the lesson plans as in Episode 18.

Researcher	Ok, how do you teach relationship between quadrilaterals?
Oya	I do not teach them separately. Instead, I will focus on relations between them. If they learn hierarchical relations, their relational understanding develops.
Beril	As I said previous lesson, I will start trapezoid concept to teach quadrilaterals. Thus, students learn concepts from general to specific by comprehending relations among them.
Researcher	Do you agree? Is there anyone who thinks differently?
Emel	For example, I prepared my lesson plan as following: Firstly, I asked students to find properties of figures. At this point, I did not mention

	anything about relation among them. However, I prepared my final two questions to understand students' knowledge about hierarchical relations.
Deniz	I used similar strategies in my lesson plan. Actually, I did not give any rotated figure.
Beril	Yes, I also recognized it in my plan after watching videos.
Maya	I also asked critical questions in my lesson plan. For example, Is every square also a rectangle? However, I did not mentioned about rotated figures. I generally used well-known [prototypical] figures. For this reason, I can add an activity involving rotated figures.
Aslı	I mentioned the relationship between quadrilaterals in my lesson plan. However, I did not give any rotated figure.

Episode 18 taken from group discussion of MCVC2

Oya considered that teaching inclusive relations of quadrilaterals helps the development of students' relational understanding. In the following, Beril expressed her instructional strategy in which she offered that teachers should start to teach quadrilaterals from more general concept (e.g. trapezoid) to more specific concept (e.g. square). Considering the presence of PSTs' different possible viewpoints on inclusive relations of quadrilaterals, I wondered other PSTs' ideas about inclusivity. Emel explained how she placed relations of quadrilaterals into her initial lesson plan. By supporting Emel's explanations, Deniz began to evaluate her own lesson plan. She emphasized on the lack of rotated figures in her lesson plan. Deniz's expression acted other group members in terms of evaluating their lesson plans whether they are involving rotated figures or not. As a result, Beril, Maya, and Aslı also made emphasis on the lack of rotated figures in their lesson plans. Moreover, Maya decided to revise her initial lesson plan by adding an activity involving rotated from of quadrilateral. Because almost every prospective teacher expressed the involvement of lesson plans in terms of hierarchical relations of quadrilaterals and prototypicality, I asked a question in order to understand whether they need a revision on their lesson plans or not. This is an important moment for the group members as they again had to consider the possible solutions to prevent students' prototypical concept images (see Episode 19).

Researcher	Do you need to make any revision in your initial lesson plans?
Ece & Deniz	Definitely yes.

Researcher	What do you plan?
Emel	I think that definitions are very important. We should ask students to define concepts themselves. Teacher should provide guidance them in that process. Then, it is crucial to reinforce students understanding about hierarchical relations [among quadrilaterals].
Aslı	Alternatively, after giving the definition, teachers can ask her/his students to construct figures without making any construction. In that situation, students can draw rotated figures.
Researcher	Are there any different opinion?
Zehra	Instead of drawing figures in paper or constructing figures in geoboard, we can ask students to cut papers. It might be more useful because when they cut figures they can easily rotate them. They can recognize properties of rectangle do not change in rotation.

Episode 19 taken from group discussion of MCVC2

As seen in Episode 19, Ece and Deniz explained that they need to revise their lesson plans. Again, I asked how they will revise their initial lesson plans. This question leaded them to think how they revise their initial lesson plans. As a response, Emel focused on the importance of definitions. She believed that if students know a common definition for each concept in quadrilaterals they can easily make relations among the types of quadrilaterals. As another alternative approach, Aslı suggested that students should construct the figures instead of teachers by taking account of the given definition. After I asked whether there is another alternative viewpoint or not, Zehra proposed cutting out quadrilaterals instead of only constructing on geoboard or grid papers. According to her, rotating figures can be comprehended more effectively by the help of the activities involving cutting paper. In conclusion, before teaching experiment, PSTs only predicted students' possible prototypical concept images. However, throughout group discussion of MCVC2, PSTs increased their attention and awareness on students' inflexible prototypical and non-hierarchical concept images about quadrilaterals. Furthermore, they began to develop some suggestive ideas to overcome students' inflexible prototypical instances at the end of the group discussion.

4.2.3.1.2 Noteworthy event in MCVC6

Analyzing and discussing video clip 6 also contributed to prospective teachers' knowledge about understanding students' unstable or inflexible concept images of quadrilaterals and they got opportunity to develop their knowledge about alternative instructional strategies for the problematic situations in the student's prototypical conceptions about rhombus. In the clip, a seventh grade student selected all rhombuses (e.g. 3, 6, and 9 in Figure 39) and a rotated square (e.g. 2 in Figure 39) as examples of rhombus.

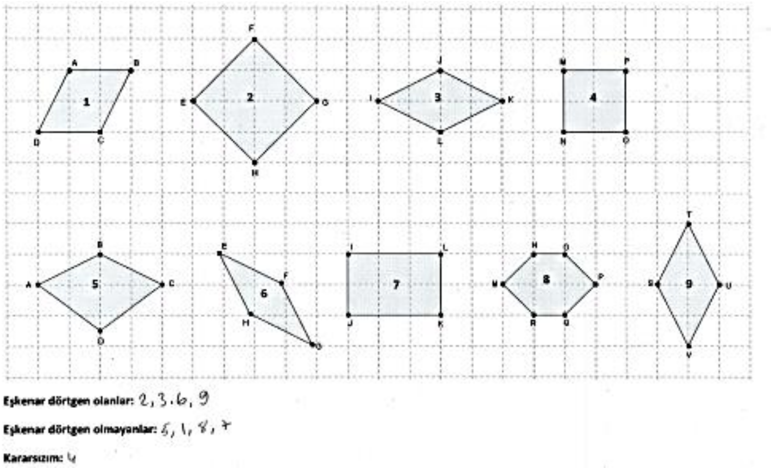
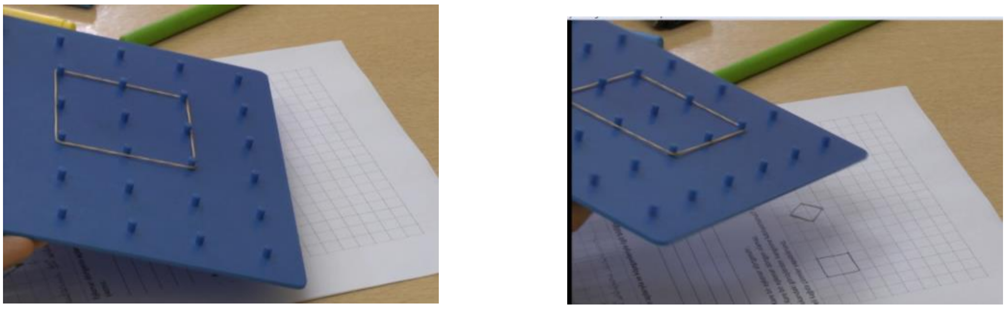


Figure 39. Student's rhombus selections in MCVC6

Furthermore, the student was undecided whether a prototypical square (e.g. 4 in Figure 39) is also a rhombus or not. After the student continued to select rhombuses among the given polygons, I asked the student to explain the relation among square and rhombus in order to elaborate and deepen student's mathematical thinking. At this point, the student used geoboard to show the examples of square and rhombus. When she turned the square like in Figure 40-a, she claimed that it becomes a rhombus like in Figure 40-b. Moreover, the student made interesting explanations such as *"If we do not turn square it is only a square. However, if we turn the square it becomes a rhombus. Hereafter, it is not a square*."



(a) (b)

Figure 40. Student's (a) square example and (b) rhombus example in geoboard

4.2.3.1.2.1 Recognizing and interpreting student's unstable concept image

In the individual pre-interviews, prospective teachers correctly and easily could predict that students generally think a rotated square as an example of rhombus although they do not consider a prototypical square as a rhombus example. However, when prospective teachers began to individually analyze the student's thinking about the relation between rhombus and square in MCVC6, they were surprised to see student's unstable decision making process on whether a square is also rhombus or not. Before starting the teaching experiment, according to prospective teachers, if a student does not consider square as an example of rhombus, rotating square in a geoboard can be the best solution to explain relations among square and rhombus. Yet, they realized that rotating square in a geoboard is not an effective solution because the student thought square as a different figure when making a rotation by using a geoboard in MCVC6. PSTs' some example explanations taken from individual video analysis reflection papers were presented in the following to show how they generally reasoned student's unstable thinking about the relationship between square and rhombus.

In this regard, three participants (Aslı, Deniz, and Ece) provided descriptive and interpretive comments in her reflection paper. Aslı's comments showed that she could not find the reasons of student's unstable responses about the relationship between rhombus and square. Additionally, she had difficulty to find a solution to overcome the instability of student's responses. However, Deniz reached a conclusion that the student could not recognize figures when they are rotated and did not know invariant properties of the figures under the rotation.

> The student said that square and rhombus are two different forms of a figure although we constructed them in geoboard. In other words, she thought the figure changes when making rotation. I am undecided how I can find a solution this confusion because the student could not differentiate between rotated square and prototypical square in geoboard. I think that her responses were very interesting [Aslı, BDRP-MCVC6].

> Finally, we saw that while the student called a rotated square as rhombus, she called a [prototypical] square as not a rhombus. This situation clearly showed that she could not correctly identify figures if they are rotated and she did not know properties of quadrilaterals do not change under rotation [Deniz, BDRP-MCVC6].

Other three participants (Emel, Zehra, and Beril) provided similar written comments about student's concept images of rhombus. They also concentrated on the possible reason of student's unstable thinking in video clip. According to them, the student had difficulty to differentiate square and rhombus due to the rote learning and visual-based reasoning. In other words, they offered that the student always focused on visual properties of a prototypical rhombus when deciding whether a square is also a rhombus or not (see Emel's following comments).

> Another interesting point is that the student did not consider square as a rhombus but, she treated rotated square as a rhombus. I think that she is not aware about rotated square is also a square. When the researcher asked the reason why she selected [a rotated square as a rhombus] student made following explanation: when said rhombus, this figure comes my mind and this figure have equal length sides." Thus, we can see the student pays attention to the visual properties of figure. Rhombus is always imagined in her mind as the following figure  [Emel, BDRP-MCVC6].

Remaining prospective teachers (Ece and Maya) developed different perspectives to express the possible reasons of student's unstable concept images of rhombus. For example, Maya focused on other possible reasons such as not using grid papers and not constructing rotated figures in the lessons. According to Maya, for these reasons, the student did not establish the relations between quadrilateral types. In sum, in the individual video analysis of MCVC6, PSTs understood student's unstable concept images of rhombus and its possible reasons in individual video analysis process.

4.2.3.1.2.2 Building connections between student's prototypical concept image and instructional strategies

In the group discussion of MCVC6, participants concentrated on what possible solution strategies might be in order to prevent such kind of students' prototypical concept images (see Episode 20).

Ece	Normally, we prove the invariant properties of a figure by rotating it. However, the student thought that the figure changes when rotating. We must use an alternative way to show [invariance].
Oya	What will we do?
Ece	I think students do not encounter different examples [of a concept] in their lessons.
Zehra	It is necessary to show different examples when teaching the concepts.
Oya	I think so.
Maya	In my opinion, there is nothing to do at this point. In my lesson plan, I wrote that if the student do not convince we can draw a figure again and rotate it. I see that this strategy even did not work.
Oya	We mentioned previously about a teaching strategy in which we propose to teach quadrilaterals from trapezoid to square. If we say every square is also a rhombus, students can understand [hierarchical relation among square and rhombus].
Emel	I agree with you. We need to teach quadrilaterals from general to specific. Student made visual reasoning [instead of attribute reasoning]. For this reason, we should focus on [critical] properties of concepts.
Zehra	I wonder if we give definition and then ask them to draw figures according to definition. If we initially ask them to construct figure they probably draw [proto] typical examples.
Oya	In my opinion, if students learn the concept based on visual properties, they may not draw different examples. However, we can try to understand their conceptions without giving information about which definition belong to which concept. In that situation, students can draw various examples [instead of drawing only prototypical examples].
Zehra	I mean same thing. We can give a general definition and they try to understand definition. Or, we can show common properties [of rhombus and square] in a scheme.

Episode 20 taken from group disccusion of MCVC6

In Episode 20, Ece made emphasis on the necessity of alternative solution way in order to prove that rotation does not change the properties and the names of geometric figures. At this point, she indicated the ineffectiveness of using only rotating figures on the paper to show invariant properties of the figure. Hereon, Oya prompted a question in order to learn peers' alternative solution strategies. In response, Ece thought that the student's prototypical concept images might be related to the limited example spaces given in the math lessons. As a corroborating idea, Zehra proposed that utilizing figures having different orientation and size at the beginning of the lesson is crucial and necessary for preventing students' prototypical concept images. While Oya considered Zehra's suggestion as a useful way, Maya claimed that there is no other effective solution different from "*rotating figures*" to overcome students' prototypical concept images. After a five seconds silence, Oya offered that starting to teach quadrilaterals from general to specific may be beneficial to point inclusive relations of quadrilaterals. Furthermore, she believed that if a student knows the critical properties of geometric figures, s/he also must establish connection among the figures having hierarchical relations. Emel expressed that she agreed with Oya in terms of suggestive ideas. In spite of peers' different solution strategies, Zehra continued to search about additional instructional approaches that make possible to produce complete concept images in students' minds. She desired to learn her peers' idea about utilizing a definition-based teaching way instead of a teaching way based on only visual characteristics of the figures. Oya challenged with Zehra's proposal because she thought that if a student previously learn the concept according to visual properties, the student could not construct non-prototypical figures. Alternatively, Oya giving definition without the concept name might be more effective than giving definition with the concept name in order to guide students to construct different rhombus examples. Zehra approved Oya's alternative approach. Consequently, above conversation clearly indicated that PSTs elaborated their knowledge on content and teaching about quadrilaterals. Thus, they began to choose which quadrilateral examples to start with and which examples to utilize to take learners deeper into the concept or content. Furthermore, PSTs evaluated the instructional advantages and disadvantages of any example or teaching way to explain a specific concept.

4.2.3.1.2.3 Criticizing instructional strategies in their initial lesson plans

In the following of the group discussion of MCVC6, participants deepened their evaluations about alternative solution methods. For this purpose, they took some notes for the new instructional approaches that proposed in the group discussion for their lesson plan revisions. After they finished their note-taking, I asked whether they need to revise their lesson plans after video analyses and discussion processes. My prompt enabled them to revisit their initial lesson plans and to deepen on the ways how they can enrich their plans as in Episode 21.

Researcher	When you analyzed student's thinking in video, do you need to make a revision on your instructional approaches? If so, why?
Beril	I never expected that students could confuse a figure and its rotated form. For this reason, I did not put rotated figures in my initial lesson plan. After I watched the videos in the lessons, I observe that some students have a difficulty in realizing that the rotated shapes are the same shapes. Considering students' difficulties, I added rotated figures to my lesson plan. Furthermore, when I teach the quadrilaterals, I will show the various rotated versions of the same shape as possible as and I will add them to activities to determine whether they can realize the different versions of the same shapes.
Oya	I can change my main activity involving hierarchical relations among quadrilaterals in this week because I organize quadrilaterals exclusively. In the activity, I asked students to find properties of quadrilaterals one by one. I think such kind of organization can be superficial. In order to emphasize hierarchical relation among quadrilaterals, I can use a teaching approach from trapezoid to square. I think I will change the organization of my initial lesson plan.
Zehra	I also need to revise the appearance of figure. For example, I generally use prototypical figures instead of their rotated forms in my initial lesson plan.
Maya	I also need to change the orientation of my figures in the lesson plan.
Deniz	You are right. I also need to change my figures.

Episode 21 taken from group discussion of MCVC6

Beril explained the influence of video analysis and discussion process on her revised lesson plan in terms of prototypicality. On the other hand, Oya was undecided to change the structure of her initial lesson plan in terms of inclusive relations of quadrilaterals. She developed an idea in which she claimed that starting more general concepts such as trapezoid when teaching quadrilaterals might be more effective than other teaching ways. At the end of the Episode 21, Zehra, Maya, and Deniz again focused on their own instructional strategy that they used in their lesson plans involving only prototypical quadrilateral examples. Zehra, Maya and Deniz decided to change the orientation and size of the figures in their lesson plans in order to avoid producing prototypical concept images. Thus, analyzing and discussing student's mathematical thinking in videos enabled them to criticize and evaluate their initial instructional strategies according to students' needs and conceptions.

4.2.3.1.2.4 Reorganizing instructional strategies in lesson plans

After the teaching experiment of week 3 continued, they individually worked on lesson plans to whatever they want to change. Some noteworthy changes in the lesson plans were exemplified by referencing some PSTs' written statements and constructions in revised lesson plans. For instance, Oya added an activity to her plan in order to teach the relationship between square and rhombus by a property-based approach as in Figure 41 that was heavily suggested as an effective method in the Episode 21. In this activity, she wrote following statements "*I prepared activities related to different quadrilaterals [in my initial lesson plan]. After watching videos, I decided to focus on invariant properties of quadrilaterals in rotation by preparing a new activity*". For the hierarchical relations of quadrilaterals, Oya changed her teaching way in which she planned to mention quadrilaterals from trapezoid to square in her lesson plan.

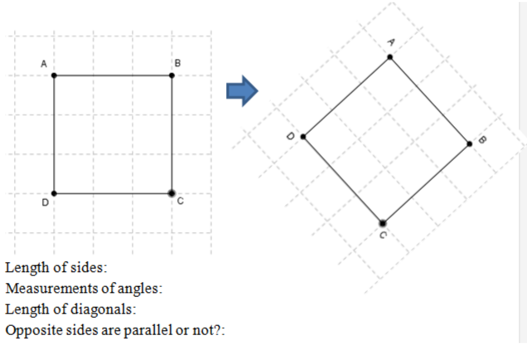
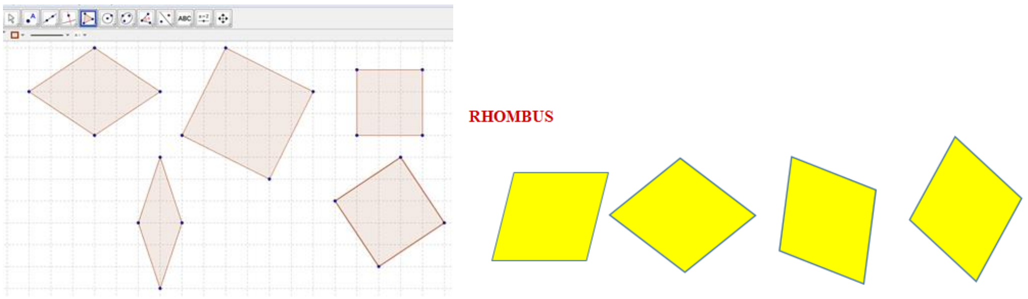


Figure 41. Oya’s activity about square-rhombus relationship in revised lesson plan

Normally, Zehra was aware of the importance of rotated figures in pre-interview conducted before teaching experiment. However, she did not place any non-prototypical example to her initial lesson plan. Parallel with her decisions in the Episode 21, Zehra made revisions on the plans by adding non-prototypical examples for each concept in quadrilaterals. Her new constructions for rhombus drawn in Geogebra were illustrated in Figure 42-a. However, Deniz changed figures only in terms of orientation instead of size and hierarchical relations like in Figure 42-b by writing following explanations:

> From the videos that we watched in the lesson, I recognized that students were not aware of invariant properties of a figure and its rotated form. For this reason, I added many of rotated form of each figure, which influences positively students’ conceptions about a geometric figure [Deniz, Lesson plan reflection].



(a) (b)

Figure 42. (a) Zehra’s rhombus examples in revised lesson plan (b) Deniz’s rhombus examples in revised lesson plan

As a conclusion, such kinds of revisions indicated how analyzing and discussing of video clips influenced and unpacked PSTs' pedagogical content knowledge about both understanding students' mathematical thinking and developing instructional ways. The common developments in prospective teachers' pedagogical content knowledge about students' non-hierarchical and non-prototypical concept images were summarized in Figure 13.

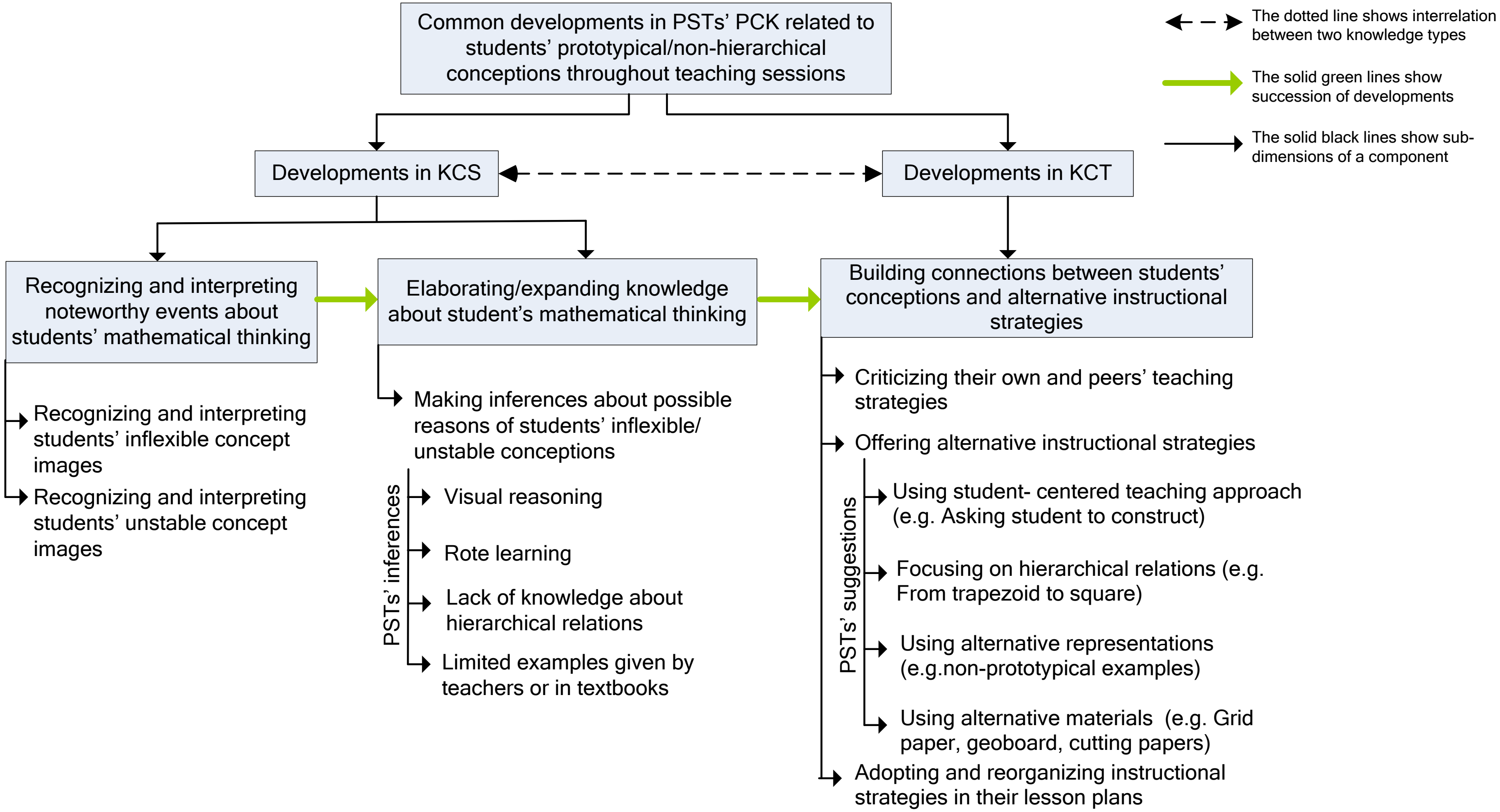


Figure 43. Summary of the developments in PSTs' PCK related to students' prototypical and non-hierarchical conceptions

4.2.4 Developments in prospective teachers' knowledge about properties of quadrilaterals

Individual pre-interviews had revealed that even though the prospective teachers' pedagogical content knowledge related to understanding student's mathematical thinking about properties of quadrilaterals was weaker than their subject matter knowledge about properties of quadrilaterals. According to PSTs' predictions, students easily know side and angle properties of quadrilaterals but they could have difficulty with diagonal properties. However, clinical interviews conducted with many of seventh grade students revealed that the students had various misconceptions about angle properties of quadrilaterals in addition to diagonal properties. For instance, some students could not tell the congruency of opposite angles of any parallelogram or the sum of interior angles of any quadrilateral as 360˚. To illustrate this, the prospective teachers' reasoning process about student's mathematical thinking related to angle properties of quadrilaterals was presented with some example written statements in reflection papers and episodes from group discussions and examples from revised lesson plans.

4.2.4.1 Developments in prospective teachers' knowledge about angle properties of quadrilaterals

4.2.4.1.1 The noteworthy event in MCVC3

A seventh grade student in MCVC3 initially drew ACDB square (see the first construction in Figure 44) to show congruent angles of parallelogram. The student then claimed that only one pair of opposite angles [by marking the angles of A and D] is congruent for any square. Further, I asked the student to draw a prototypical parallelogram example and to show angle properties in the figure in order to understand how the student decided which angles should be congruent in a parallelogram. In response, by the help of the researcher, the student could construct MHTR parallelogram as in Figure 44. The student again claimed that only the angles of M and T are congruent for MHRT parallelogram without offering any explanation why these two angles are congruent.

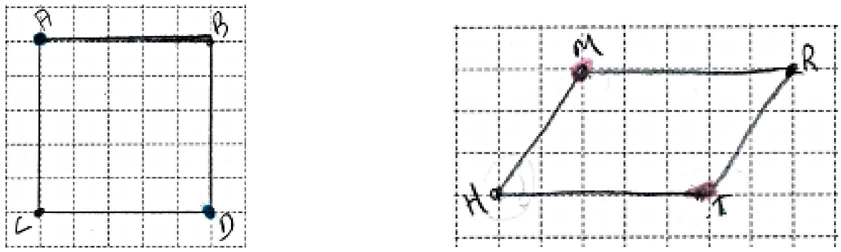


Figure 44. Student's determination of congruent angles of parallelogram in MCVC3

4.2.4.1.1.1 Noticing student's misconception about congruent angles of parallelogram

The comments in prospective teachers' individual video analysis reflection papers indicated that they were very surprised when analyzing the student's reasoning about properties of quadrilaterals in the clip because they could not think any student might consider only congruence of one pair of opposite angles of a parallelogram until analyzing the video clip. Four of them (Emel, Oya, Deniz and Aslı) identified and evaluated how the student decided the angle properties of parallelogram in the clip. In the individual video analysis process, most of prospective teachers tended to only evaluate the student's responses into two piles: correct/knowledgeable or incorrect/not knowledgeable. However, Maya provided some interpretive comments in addition to evaluative and descriptive comments in her individual video analysis reflection paper by drawing attention on the possible reasons of the student's understanding about angle property of parallelogram. Maya claimed that the student could not know the angle properties of parallelogram because the student's mathematics teacher might not adequately put emphasis on the definitions and discuss hierarchical relations of quadrilaterals with the students in the lessons.

4.2.4.1.1.2 Indicating teaching style as a possible reason of the student's misconception

In the group discussion process of MCVC3, I introduced a question that "how did the student examine angles of parallelograms in Figure 44 in order to understand how prospective teachers interpreted student's approach for determining the congruency of angles in any parallelogram. This question initiated an episode of pedagogical reasoning that shifted from the expressions of what the student knows and does not know to the expressions of why the student developed such a perception about angles of parallelogram (see Episode 22).

Researcher	How did the student examine angles of parallelogram in video clip?
Zehra	The student claimed that only one pair opposite angles [of parallelogram] is congruent.
Emel	He did not consider another pair opposite angles.
Researcher	Why did the student develop such kind of thinking?
Ece	It can be related to their mathematics teacher's explanations in lesson. Generally, teachers say opposite angles are congruent in parallelogram and indicates one pair of angles to show congruency between angles. Students can misinterpret teachers' explanations and examples in lessons.
Emel	The student could not reason about why the angles are congruent.
Beril	Yes, student mentioned something based on his rote learning.
Deniz	I wonder that why did he think only the angles of A and Das the congruent angles?
Ece	She thought one pair of angles is enough to say congruency.
Beril	Their teacher might show only one specific pair of opposite angles to explain congruent angles. As a result, students might misinterpret the property.
Oya & Deniz	Probably.

Episode 22 taken from group discussion of MCVC3

As one can see from the episode, Zehra and Emel attempted to describe how the student mentioned congruency of angles of any parallelogram. At this point, I asked the possible reasons why the student thought only one pair of opposite sides were congruent in any parallelogram. Through the question, it became necessary for them to take out their pedagogical knowledge in order to find possible reasons why the student developed such kind of mathematical thinking about angle property of parallelogram rather than describing and evaluating noteworthy events in the clip. For this, Ece suggested that because mathematics teachers generally say that "*opposite angles of parallelograms are congruent*" without emphasizing on "*two pairs of opposite angles*", the student might misinterpret the angle property of parallelogram by focusing on the congruency of only one pair opposite angles instead of two pairs of opposite angles. Following this, Emel and Beril provided supporting ideas by claiming the lack of mathematical reasoning in the student's responses. Hereon, since Deniz was not convinced with her peers' ideas, she immediately prompted a question to the group. She asked the reason why the student concentrated only one pair opposite angles even if he memorized something without reasoning. Here, they reached a point where they needed to spend time to think further in order to offer what other possible reasons of the student's understanding might be. In this regard, Ece provided a proposal in which she claimed that the student found the congruency of one pair of opposite angles enough to indicate angle properties of parallelogram. On the other hand, Beril claimed that the teacher may show to the students only two angles of parallelogram when teaching the angle properties of parallelogram in the instructional process. In the following, Oya and Deniz supported her peer's idea. To conclude, this episode was important because prospective teachers' comments suggested mathematics teachers' limited examples and explanations as a possible reason for the student's conception about angle property of parallelogram.

4.2.4.1.1.3 Relating student's misconception to the student's lack of knowledge about angle concept

In the following of the discussion, Aslı pointed a new noteworthy event as an alternative perspective and changed the direction of the discussion (see Episode 23). Aslı commented on the lack of student's knowledge about angle concept as the reason of the student’s incorrect responses for the congruency of opposite angles of any parallelogram. It was an important moment for the group because they accepted Aslı’s proposal on the student’s misconception about angle concept as a significant event to pursue (see the student’s responses in Figure 44). Normally, in the prospective teachers’ individual video analysis reflection papers, there was no statement emphasizing a relationship between the student’s misconception about angle concept and the student’s inadequate knowledge about the congruency of opposite angles of parallelogram. Instead, they only concentrated on the student’s misconceptions about angle concept and some of them only identified how the student perceived congruent and non-congruent angles in the clip. However, after Aslı’s proposal, the group discussion moved toward analyzing the student’s understanding about angle concept and the relationship between student’s conception of angle and the congruency of angles of any parallelogram in more depth. Following episode illustrates how the prospective teachers evaluated and considered Aslı’s proposal.

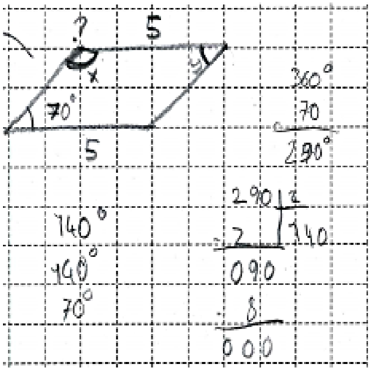
Aslı	I think that the student lack knowledge about angle concept because he mentioned the equality of the rays of [AK and [KL. Furthermore, he indicated them as two different angles. For this reason, the student could not know angle properties of parallelogram.
Emel& Deniz	You are right, he treated rays as angles.
Aslı	However, he actually treated the corner points of B and D in Figure 44 as angles.
Ece	Moreover, the student incorrectly named the angle such as <*MST*.
Zehra	Student also made visual reasoning when determining the congruency of angles.
Emel	Yes, he focused on the length of rays.
Zehra	Probably he tried to provide a similarity between <MKL and <AKL and incorrectly constructed them.
Maya	The student could not know angle concept how do we expect he knew the angle properties of parallelogram?
The group	You are so right.

Episode 23 taken from group discussion of MCVC3

When Aslı individually analyzed the clip, she primarily interpreted student’s conceptions about angles in her individual video analysis reflection paper. Now in this episode, not only she focused on how the student treated the rays of AK and KL as angle by trying to measure the length of the rays but also she established a relationship between the student's angle conception and the student's misconceptions of angle properties of parallelogram. Emel and Deniz supported Aslı's interpretations. Following this, Aslı focused on the inconsistency among the student's perceptions of angles. At this process, the group needed to elaborate their analysis on student's conceptions about congruency of two angles. Here, while Ece evaluated how the student inappropriately named the angles, Zehra pointed that the student's conceptions about congruent angles in Figure 44. After the prospective teachers had concentrated student's various misconceptions and difficulties about congruent angles, Maya restated the relationship between the student's angle conception and the student's misconceptions of angle properties of parallelogram. At the end of the episode, they had agreed that the student's lack of knowledge about angle concept seemed potentially linked to the student's misconception about the congruency of angle of parallelogram.

4.2.4.1.1.4 Elaborating knowledge about the student's approach when determining angle measurements in parallelogram

The discussion process in this way probably leaded Aslı to propose a new interpretation about how the student's lack of knowledge about angle concept can influence the approach the student used when determining the measures of the angles in parallelogram. In more detail, Aslı's and her peers' interpretations were illustrated in Episode 24 in order to show the prospective teachers' developmental process on pedagogical content knowledge in terms of understanding students' mathematical thinking.



The noteworthy event in MCVC3

When interviewing with a seventh grade student in video clip 3, the researcher gave 70˚ in the parallelogram and then asked the student to find the measurements of x and y in the given parallelogram. The student firstly subtracted 70˚ from 360˚. In the following, 290˚ divided by 2. After the division, the student reached 140˚. As a final point, the student offered that x and y had same measurement and stated that
x= 140˚ and y= 140˚.

Figure 45. Student's miscalculations for the angles of parallelogram in MCVC3

Aslı	Additionally, the student could not find the measurement of other angles in parallelogram because he did not know what angle means.
Deniz	He correctly mentioned about the sum of interior angles of parallelogram as 360˚. Then, he subtracted 70˚ from 360˚. Until this point, the student made reasonable calculations. However, I could not understand why did he divide 290˚ by 2? Dividing 290˚ by 3 is understandable because there were three unknown angles in parallelogram.
Researcher	Why did the student divide 290˚ by 2? What are your opinions?
The group	(*Silence and revisiting the noteworthy event in the clip*.)
Ece:	Because there are two variables such as "a" and "b" in parallelogram, student might divide 290˚ by 2. I think that if there were also one additional variable, he might divide 290˚ by 3. I understood that if there is no variable, the student did not need to consider the presence of any angle in parallelogram.
Deniz	Hmm, you are right.
The group	(*All group found Ece's explanations reasonable by nodding their heads.)*

Episode 24 taken from group discussion of MCVC3

As seen in the Episode 24, she claimed that the student might not correctly find the angle measurements of the parallelogram in *Figure 45* due to the student's lack of knowledge about angle concept. In the following, Deniz described how the student found angles. Then, she said that she did not understand why the student divided 290 by 2. Shortly after, I jumped in with an elaboration of the problem about the student's approach. Here, prospective teachers had to reconsider the reasons why the student in the clip correctly could not found other angle measurements of the parallelogram in case one angle measurement was given 70˚ (see Figure 45). For this, some of them examined the student's written responses in the paper and others revisited related part of the video clip in order to analyze the student's expressions. Ece provided an interpretive comment in which she proposed that the student divided 290˚ by 2 due to the presence of the variables such as x and y in Figure 45. Moreover, she suggested that if there was one more variable such as z, the student probably divided 290˚ by 3. At this point, Ece's proposal has been accepted by the discussion group as a possible reason that can give opportunity to clarify the problem in the student's mathematical approach for determining the measurements of the interior angles of the parallelogram. However, Oya still needed to understand the reason why the student did not use his knowledge about the congruency of angles of parallelogram although he previously stated that only one pair of opposite angles of parallelogram is congruent. Oya's interest was taken into consideration by the group members and the discussion moved toward analyzing this problem in more depth (see Episode 25).

Oya:	Interesting, the student said that one pair of opposite angles are congruent. By using this information, I expected that he can understand y is equal to 70˚. Why did not he use this information?
Emel:	I thought the same thing.
Zehra:	However, the student considered only another pair of opposite angles is congruent. 70˚ was given other side.
Ece:	(*She showed the angles of x and y in the paper involving the student's written responses to Oya*.)
Zehra:	There is "x" variable in this situation. Understanding variable is difficult for students. Furthermore, he thinks that y and 70˚ are not equal.
Researcher:	So, Do the presence of variables in Figure 45 influence on student's calculations?
Emel & Maya:	Probably.
Oya:	It is reasonable! I never thought about it before.

Episode 25 taken from group discussion of MCVC3

While Emel only agreed with Oya's thinking, Zehra reminded that the student treated only A and D angles in ACBD square and M and T angles in MHTK parallelogram in Figure 44. Zehra offered this situation as the possible reason of the student's difficulty to find interior angles of parallelogram and she put emphasis on the corner points that the student took congruent angles. When Ece showed x and y variables in Figure 45 to Oya Zehra continued her previous comment. By combining the first comment in the above episode, Zehra put the presence of variables in the parallelogram figure as a problematic situation for the student in calculating interior angles of parallelogram. Just at that moment, I needed to be sure what the other group members think about the situation under the discussion. While Oya, Emel and Maya accepted Zehra's suggested proposal verbally, remaining group members communicated by nodding to show that they supported to Zehra.

4.2.4.1.1.5 Building connections between student's error in angle concept and alternative instructional strategies

After PSTs concentrated and comprehended the student's insufficiencies and misconceptions about angle properties of quadrilaterals and its possible reasons, I asked a question in order to understand how they develop solution strategies to the problematic situations in the student's mathematical thinking. This question initiated a new discussion in Episode 26.

Researcher	If you were a teacher what do you pay attention when teaching angle and diagonal properties of quadrilaterals to such students?
Zehra	For angle concept, I distribute straws in different length to the students. Then, I ask them to construct congruent angles because the student has serious problems about angle concept. Firstly, the student should learn how congruent angles are constructed.
Oya	In my initial lesson plan, I thought to remind polygons as basic geometric concepts. However, it is necessary to remind angle concept.
Zehra	Or, it can be mentioned about what parallelism and equal length mean.
Oya	However, if student have already misconceptions it is difficult to teach by only reminding these concepts.
Beril	It seems that it is necessary to take on from the top.
Researcher	Do you have different opinion?
Oya	We can ask student to cut angles and to superimpose them. Thus, students do not see angles as a corner point and they can understand congruent angles.
Maya	We can also use protractor to teach [angles]. For example, I prepared an activity that similar to mathematics textbook in Ministry of Education in

	my lesson plan. I aimed to teach angle to the students by using protractor in my activity.
Emel	We can teach the meaning of angle between lines because the student did not know. We can say how the sum of interior angles of quadrilaterals can get.
Zehra	I wonder we can distribute various quadrilaterals to the students. Then we ask them to cut its interior angles. In the following, we can ask the question of what is the degree of the sum of angles when you combine all of them together.
Ece	Or student knew the sum of interior angles of triangle. If we draw one diagonal of quadrilateral, two triangles occur in quadrilateral. The sum of interior angles of a triangle is 180˚. By using this information, students can find the sum of interior angles of quadrilaterals adds up 360˚.
Zehra	But, such type of activity can be difficult for middle school students. Students can inappropriately draw two diagonal and find four triangles instead of two triangles.
Maya	In Zehra's example, students can mix angles that they cut if the angles have no name.
Zehra	We can paint and name the angles.
Group	(*The group found Zehra's idea reasonable*.)
Researcher	After you watched and discussed the student's thinking in video, is there any point that you want to change in your lesson plan?
Zehra	In my opinion, we must pay attention to control students' knowledge about basic sub-geometric concepts.
Aslı	I agree with my friend.

Episode 26 taken from group discussion of MCVC3

In the Episode 26, by putting emphasis on the necessity of teaching constructing congruent angles, Zehra explained that she preferred to use "straws" by cutting one of them from its top point in order to show congruent angles to the student because the student did not consider the angles in Figure 46 as congruent angles in a moment of MCVC3.

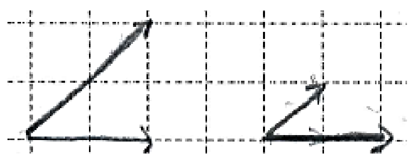


Figure 46. Student's two non-congruent angles construction in MCVC3

By following Zehra's suggestive ideas, Oya again emphasized the necessity of controlling students' knowledge basic geometric concepts such as angle instead of polygon. Zehra provided additional information to Oya's comment by focusing on other basic geometric concepts such as parallelism and equal length line segments. However, Oya challenged with the idea of focusing on basic geometric concepts. She believed that if a student have misconceptions that comes his/her previous learning experiences, focusing on basic sub-geometric concepts in the instructional process of quadrilaterals might not work as expected. To response, Beril suggested that it is necessary to teach the concept again from the beginning. The group did not make any comment on this suggestion. To continue the discussion, I asked the question of “is there any other suggestion about the issue under the discussion? After immediately, Oya provided an alternative approach to overcome the student’ misconception about angle concept. She proposed utilizing cutting paper activities involving congruent angles in Figure 46. According to her, if an angle was superimposed on another congruent angle the student can understand angles constitutes from rays rather than seeing angle as a corner point. Another alternative solution way was offered by Maya. She focused on teaching of using protractor and measuring the angles with the protractor to the students as an effective strategy. Emel approached the issue under the discussion from a different perspective. She concentrated on the teaching of the properties of angles between parallel line segments by indicating the student’s lack of knowledge on related angles in MCVC3. She also suggested that using such an activity might be useful to teach the sum of the measurement of interior angles of quadrilaterals. This suggestion acted an idea in Zehra’s mind because she proposed that giving the angles of a quadrilateral that is cut with scissors to the students and asking them to combine these angles might enable them to see the sum of interior angle measurements of any quadrilateral. Similarly, with the influence of Zehra’s proposal, Ece suggested that showing the presence of two triangles inside of any parallelogram can be an effective way to teach the relationship between the sum of interior angles of triangle and quadrilateral. However Zehra disagree with Ece’s suggestion in terms of the effectiveness. She believed that Ece’s proposal can be difficult for middle school students due to the possibility of drawing unsuitable triangles into parallelogram. Hereon, Maya criticized Zehra’s activity involving cutting the angles of quadrilaterals with s scissors by the students can become complicated if students distinguish the angles after cutting. In response, Zehra suggested that angles can be colored or named before cutting. All group members found this idea reasonable. After that, I wondered what they think about their lesson plans in terms of angle properties of quadrilaterals. Consequently, Zehra and Eda expressed that they need to add information about the prerequisite knowledge involving basic geometric concepts to their lesson plans.

In summary, all these episodes and the comments in the prospective teachers' reflection papers written after the group discussion of the MCVC3 illustrated how their knowledge about understanding student's mathematical thinking related to angle properties of parallelogram has developed and changed. Before starting of the teaching experiment, they provided very few predictions about seventh grade students' difficulties and misconceptions related to the angle properties of quadrilaterals. However, when they individually examined the MCVC3, they firstly noticed that the student misinterpreted the congruency of opposite angles of parallelogram. The comments in their individual video analysis reflection papers revealed that all of them (excluding Ece) described their inexpectations about student's approach in the determination of congruent angles of parallelogram in the clip rather than making inferences or finding solutions to the problems in student's understanding. Instead, in the group discussion process, they had opportunities to share different ideas with the peers. As a result, they needed to focus on the possible reasons of the problems in student's understanding about angle properties rather than identifying only the problematic situations in the clip. For instance, at the end of the first episode, they agreed with "*teaching style*" as a possible reason of the student's misconception about angle property of parallelogram. In the second episode, they related student's misconceptions about angle property of parallelogram to the student's lack of knowledge about basic "*sub-geometric concepts*" such as angle. Towards the end of the group discussion, they tried to "*elaborate*" their knowledge to interpret the student's approach in the determination of angle measurements of parallelogram in case one angle measurement is given 70˚ in more depth. Consequently, when interpreting the student's misconceptions about angle properties of parallelogram in a social learning environment, they heavily concentrated on different possible reasons such as teaching style, the lack of student's knowledge on basic sub-geometric concepts and variables. Thus, video discussion process created a need for prospective teachers to develop new pedagogical content knowledge as they engage with problems of students' understanding. Moreover, after they understood the possible reasons of the student's errors, they began to produce instructional approaches to prevent or correct problems in student's conceptions about angle and angle properties of quadrilaterals. They offered following alternative ways: utilizing manipulatives such as using "drinking straws", cutting paper activities, using protractor to measure angles of quadrilaterals; making emphasize on basic sub-geometric concepts (e.g. parallelism); using the relationship between the sum of interior angles of triangle and quadrilateral within an activity in the group discussion as well as in the after discussion reflection papers by written comments.

4.2.4.1.2 The noteworthy event in MCVC4

A seventh grade student's misconceptions about the congruent angles of parallelogram were involved in MCVC4 (see Figure 47). In the clip, I asked the student to construct a parallelogram and to explain the congruent angles if there are in the figure. In response, the student initially constructed DASK parallelogram and claimed that the angles of D and S are congruent and also the angles of A and K are congruent. After I desired to learn what the student knows about the relationship between the angles of A and D or K and S. The student changed her idea and stated that all angles of parallelogram are congruent instead of focusing on the congruency of opposite angles.

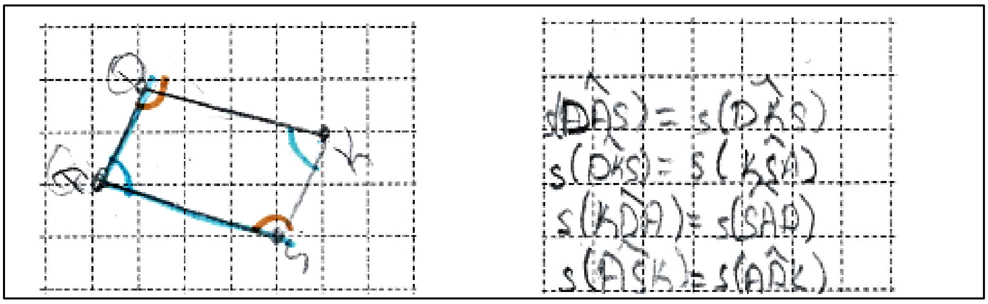


Figure 47. Student's representations of congruent angles of Par in MCVC4

4.2.4.1.2.1 Early interpretations about the student's misconception related to the congruent angles of parallelogram

In the pre-interviews that were conducted before teaching experiment, prospective teachers provided no prediction about what kinds of misconceptions students could have related to the angle properties of parallelogram. However, the comments in prospective teachers' individual video analysis reflection papers showed that they noticed the student had a misconception about angle property of parallelogram because the student treated all angles in a prototypical parallelogram as if they are always congruent. After they noticed the misconception they further provided different interpretations when in individually analyzing student's mathematical work. For instance, Ece and Emel made a connection between the student's misconception and the lack of student's knowledge about angle concept. Besides, Oya and Deniz proposed that the student had such a misconception because the student constructed a rectangle instead of parallelogram. On the other hand, Aslı and Zehra claimed that the student could not distinguish the properties of rectangle and parallelogram in her reflection papers of individual analysis. In sum, reflective comments provided in the individual video analysis indicated that prospective teachers concentrated on three possible reasons to explain the student's misconception such as (a)"the lack of student's knowledge about angle concept", (b) "student's inability to distinguish rectangle and parallelogram" and (c)"constructing rectangle instead of parallelogram".

Surprisingly, Beril and Maya concentrated on solely the incorrectness of the student's answer when individually examining the clip. Examples of such comments involved:

> I'm very surprised because I never predict that a student can think all angles of any parallelogram are congruent before examining the clip. The student did not correctly know the angle property of parallelogram [Beril, BDRP-MCVC4].
>
> I did not understand why the student changed her idea about the congruent angles of parallelogram. In addition, the student could not give any reason about her changing idea [Maya, BDRP-MCVC4).

Noteworthy in Beril's and Maya's comments in reflection papers that they did not understand why the student thinks all angles of any parallelogram are always congruent because they seemed genuinely puzzled by the approach the student used when deciding the congruency of angles of parallelogram. Consequently, Beril and Maya tended to focus on the wrongness of the student's answers about angle properties of parallelogram.

4.2.4.1.2.2 Elaborations in interpretations about the student's misconception

In the group discussion process, I posed a general prompt to elicit and elaborate prospective teachers' ideas about the student's thinking related to congruent angles of parallelogram in MCVC4. The group started to share their ideas with the peers (see Episode 27).

Researcher	When you analyzed the video what did you notice about student thinking related to interior angles of parallelogram?
Zehra	The student was aware of the meaning of angle concept.
Beril	The student said that opposite angles [of parallelogram] are congruent, but she concluded all angles are congruent [at the end of the video].
Researcher	Beril is right. Actually, at the beginning, the student considered that only opposite angles are congruent. Why did the student change her mind later?
Oya	She thought that if line segments are same direction, the angles between the line segments are same. Thus, the student interpreted the situation in

	her own way.
Beril	Student thought that one line segment is common for two angles (*she indicated [AD] in Figure 47*) and other line segments are parallel (*she indicated [DK] and [AS] in Figure 47*).
Emel	In my opinion, the student knew the congruency of opposite angles [in parallelogram]. However, I am not sure about whether the student knew the reason why opposite angles are congruent or not. For this reason, she claimed that the angles of "S" and "D" are congruent. After that, the student supposed that the angles of A and D are also congruent based on the parallelism of [AS] and [DK]. Such kind of response results from the student's misconception about angle concept. This student saw the angle as two line segments instead of the area between two intersecting rays. Furthermore, the student believed that in case the line segments are equal length angles becomes congruent.
Oya	Actually right. We saw in the first week video discussions that students generally focus on the length of line segments when examining angles. They misinterpreted angle and provided incorrect descriptions about angle. The student in MCVC4 also did not know corresponding angles. If the student knew she might provide correct response.
Zehra	But, the student knew alternate internal and exterior angles.

Episode 27 taken from group discussion of MCVC4

In the above episode, while Zehra evaluated whether the student know the angle concept or not, Beril described what she noticed in the clip. Here, I prompted to the participants to explain how they reasoned student's work rather than describing or evaluating its correctness. At this point, they tried to comment on the meaning of the student's responses. Oya and Beril offered an explanation as to how the student considered that all angles of parallelogram are congruent instead of mentioning the congruency of opposite angles. They interpreted the student perception about congruency of angles based on the position of the rays (e.g. DA and DK in Figure 47) and parallel sides (e.g. DK ad AS in Figure 47) in the parallelogram. Emel participated to the discussion by supporting her peers' comments. Furthermore, similar to her individual video analysis comments in the reflection paper, Emel concluded that the student has misconceptions about angle concept after identifying the student's conception of angle.

At the end of the group discussion, Oya and Zehra reached a point in the discussion where they need to begin to unpack their reasoning about student's angle conception. Through this discussion, Zehra and Oya had agreed that-differently from their individual video analysis comments in the reflection paper-the misconception also might be related to the lack of student's knowledge about angle concept. Nevertheless, Zehra would like to review the video before making a new claim. Then, she proposed that since the student misinterpreted the congruency of angles in the parallelogram due to the construction being similar to rectangle in Figure 47 (see episode 28). This claim is significant since it lead to the discussion for the prospective teachers to concentrate on other alternative reason in order to explain the student's misconception.

Zehra	(*After watching a part of MCVC4*) I noticed that the student might establish a similarity between rectangle and her construction in Figure 47 because the student used properties of rectangle when examining properties of parallelogram.
Oya & Deniz	You are right, I think so too.
Researcher	Is Figure 47 a rectangle?
Deniz	In my opinion, it seems a rectangle.
Zehra	(*After carefully examining the student's construction*) this figure is not a rectangle.
Aslı	However, the student used solely properties of rectangle. In my opinion, the student could not differentiate between properties of rectangle and parallelogram.
Researcher	So, do you mean that students' difficulty is related to her constructions?
Oya	Yes, I think that student reasoned by focusing on visual properties of her construction because her construction seems a rectangle. As evidence, many of students decide angle properties by focusing on appearance of figures rather than controlling [critical] properties.
Aslı	I think that student's thinking is not related to her construction. Probably, the student was confused the properties of rectangle and parallelogram.
Beril	In my opinion, the student made an overgeneralization because the diagonals are equal length in squares and rectangles.

Episode 28 taken from group discussion of MCVC4

As seen in the Episode 28, Oya and Deniz supported Zehra's claim. In order to ensure all prospective teachers have evaluated the student's construction as a rectangle instead of an example of parallelogram, I joined to the discussion. My question revealed that Deniz perceived the student's construction as a rectangle instead of a parallelogram. At this point, Zehra challenged with Deniz's explanation. At this point, Aslı proposed that the student could not separate the differences between a rectangle and a prototypical parallelogram based on the properties the student used. To be sure, I asked whether the student's misconception is related to her parallelogram construction or not. Hereon, Oya showed the student's parallelogram construction as the most important reason that lies in the origin of students' misconception, Aslı concentrated on the student's confusion on the rectangle-parallelogram differences. Beril agreed with Aslı and she provided some additional comments to elaborate the issue under the discussion by giving the properties of square and rectangle. In conclusion, they had opportunities to develop awareness about various alternative interpretations for the reasons of the student's misconceptions by sharing their ideas in a social learning environment.

In the following of the discussion, I shifted prospective teachers' attention to what other possible reasons of the student's misconception about congruent angles of parallelogram can be (see Episode 29).

Researcher	Are there any different view?
Maya	An idea currently comes to my mind. I wonder that whether the teacher overemphasize rectangle in the lessons or not.
Beril	Or their mathematics teacher may not adequately emphasize the properties of parallelogram.
Researcher	What do you think about your peers' ideas?
Oya	I make a relation between student error and construction. However, I found my friends' ideas quite reasonable.
Maya	I could not make sense why the student said all angles of parallelogram are congruent, but I understood there were many of reasons to explain student's mathematical thinking in group discussion.

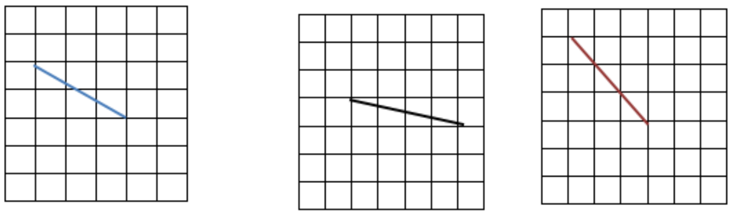
Episode 29 taken from group discussion of MCVC4

After my question, Maya bring a new perspective to the discussion. She suggested that the teacher might overemphasize the properties of rectangle rather than that of parallelogram, which can affect the student's interpretation of congruent angles of parallelogram. It is remarkable development in Maya's pedagogical reasoning about student's thinking because she only evaluated the correctness of the student's responses in her individual video analysis reflection papers. Likewise, Beril made an additional explanation in which she claimed that the teacher might not make enough emphasis on the properties of parallelogram. After the explanations made in the discussion, Oya understood that the teaching methods may have a role in the student's misconception, although she only focused on the student's construction as a possible reason of the misconception when individually analyzing the clip. Furthermore, Maya's final comments indicated that she developed many of ideas about the reasons of the student's misconceptions while she could not provide any reasonable explanation why the student thought parallelogram as a figure having four congruent angles in individual video analysis process.

4.2.4.1.2.3 Building connections between student's error and alternative instructional strategies

After prospective teachers discussed MCVC3 and MCVC4 in the third week of the teaching experiment process, they wrote reflection papers involving alternative solutions ways to the students' errors, difficulties and misconceptions in the clips. Moreover, they made revisions on their lesson plans if they found necessary. Considering PSTs' suggestive ideas and revised lesson plans, it was clearly seen that PST's developments in knowledge about understanding students' mathematical thinking about angle properties of quadrilaterals helped them to propose different solution strategies in order to overcome problems in students' conceptions about angle and angle properties. For example, all PSTs agreed that controlling of students' knowledge on basic sub-geometric concept such as parallelism, and angle is necessary and crucial before teaching parallelogram or rhombus. In addition, they added some activities involving basic geometric concepts. One of them was illustrated in Figure 48. More specifically, Oya added following statements to her lesson plan when adding below activity: "*Before the first activity I may ask students the meaning of some concepts such as edge, side, diagonal or angle to check whether they have essential basic knowledge about the topic since there are some students do not know them*". These explanations indicated how Oya unpacked her pedagogical content knowledge.

b) Draw 2 different parallel lines to the given lines.



This week I add b part in this activity, because in the 7th video, I realize that some students cannot know linearity concept and parallelism adequately. Maybe, thanks to this part of the activity, I can detect the students who have a difficulty about it and I can focus on these concepts before starting quadrilateral.

Figure 48. Oya’s activity about basic geometric concept in revised lesson plan

Similar crucial developments are observed in all prospective teachers’ pedagogical content knowledge about quadrilaterals. Another striking point is that although Maya and Beril only detected students’ errors when individually analyzing the clips, they made explanations about the contributions of the social learning environment to their pedagogical content knowledge in after group discussion reflection paper. More specifically, Beril’s written statements were asserted in the following:

> Before starting to teach quadrilaterals, I think that it is necessary to remind prerequisite basic geometric concepts to the students. It is obvious that inadequate knowledge about corner, diagonal, and angle concepts leads some problems. They have both difficulties in higher concepts [e.g. diagonal and angle properties] and various misconceptions [Beril, ADRP-MCVC4].

4.2.4.1.3 Noteworthy event in MCVC8

We know that adjacent angles of any prototypical trapezoid are supplementary because of the parallel sides. This means that their measures add up to 180˚. However, in MCVC8, the student claimed that if one of adjacent angle is given as 70˚, other one must be 360˚-70˚=290˚. Furthermore, the student made following explanations: “there is no parallel sides in the trapezoid”. However, she also made the construction involving one pair of opposite sides as in Figure 49.

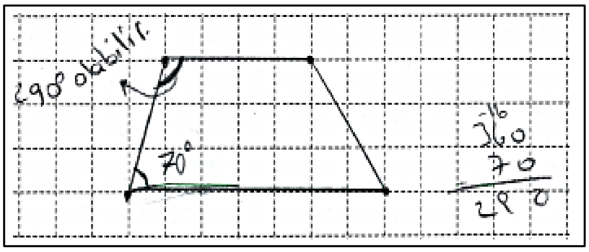


Figure 49. Student’s conception of the sum of adjacent angles of trapezoids in video

Student’s such kind of perception about angles of trapezoid is an unpredictable situation because prospective teachers’ prior knowledge about quadrilaterals that was obtained from individual pre-interviews data revealed that they were unaware about what types of difficulties or misconceptions middle school students might have about angle properties of trapezoid.

4.2.4.1.3.1 Recognizing and interpreting student’s conception about angle property

When prospective teachers individually examined the clip, they were very amazed to see the student’s misconception about the sum of the measurements of adjacent angles of a trapezoid. More specifically, Maya, Aslı and Oya only explained that they had found the student's calculation is different and meaningless without providing any interpretation on the situation in their reflection papers they wrote individual video analysis process. On the other hand, Ece made emphasis on the student’s inability to think the presence of another two angles in her reflection paper. Emel and Beril made a connection between student’s misconception and the student’s lack of knowledge about parallelism concept. Besides, Zehra and Damla proposed that the student misremembered the measures of adjacent angles as adding up to 360˚ instead of 180˚. To conclude, prospective teachers’ comments about the student’s mathematical thinking that they provided in individual video analysis revealed that how they interpreted differently the possible reasons of problematic situations in the student's conceptions.

1.1.1.1.1.1 Elaborations in interpretations about the student's misconception

In the discussion process of MCVC8, my prompting question in Episode 30 extended to the discussion on the student's conception about adjacent angles of trapezoid.

Researcher	As a new point, is there any point that you noticed about the student' thinking related to angle concept?
Beril	Because the student did not know parallelism, she could not correctly find the measure of angle in Figure 49.
Ece	Student focused on two angles and she immediately decided to subtract 70˚ from 360˚. She did not even consider remaining two interior angles.
Emel	The student used 360˚ in subtraction instead of 180˚ by misremembering something. It shows the student's rote learning.
Deniz	Another reason can be related to student's mathematics teacher. The student misremembered the teacher's solution of a similar question in any lesson. As a result, she focused on 360˚ instead of 180˚.
Aslı & Beril	It is reasonable.
Maya	Ezbere gitti yani.
Ece	Parallelism is very crucial to comprehend properties of quadrilaterals. Students should know parallelism before reasoning about properties.
Researcher	Although students know parallelism concept, they could not even reason about angle properties of quadrilaterals.
Ece	Students do not also know angle concept. We saw such situations in previous videos. While students did not interpret angle concept how we will expect they reason about angle properties of quadrilaterals.

Episode 30 taken from group discussion of MCVC8

In the group discussion process, Beril, Ece and Emel repeated their comments written in reflective reports when individually examining the clip involving the student's misconception of the sum of adjacent angles of a trapezoid. To put in more detail, each of them linked the student's misconception with different reasons. For instance, as a possible reason, Beril emphasized that the student lacks necessary knowledge about parallelism concept, Ece and Emel commented on the student's approach by proposing that it was nourished from rote learning based on the student's memorized knowledge capacity. After listening different reasons, the group concentrated on Deniz's interpretation. Deniz reiterated her statements that she wrote in individual video analysis process. She proposed that the student might misremember the rule as if it involves 360˚ instead of 180˚. Furthermore, as a possible reason, she pointed on the influence of the math teacher's possible examples given in the lessons on the student's misconception. At this point, Deniz's comments were taken under serious consideration by some participants (e.g. Aslı and Beril) as a sensible reason of the student's misconception about adjacent angles of trapezoid. Contrary to her peers, Ece again turned the lack of the student's knowledge about parallelism that Beril had suggested at the beginning of Episode 30. Ece put emphasis on the importance of understanding parallelism concept for establishing the connection between angles of trapezoid. Hereon, I needed to give additional information to push Ece to think in more dept. As a result, Ece spent time interpreting the student's mathematical work and she offered the lack of students' knowledge about angle concept as a reason of the misconception considering the noteworthy events in all video clips that the group examined throughout the teaching experiment process.

In the statements of reflection papers written after the group discussion of MCVC7 and MCVC8, the prospective teachers' changing and emerging ideas about understanding students' mathematical mis/conceptions were observed more saliently and explicitly. More specifically, the correctness of the student's responses was a prevalent focus of some PTs' attention (e.g. Aslı, Oya and Maya) when individually analyzing the student's work related to adjacent angles in MCVC8. Moreover, these PTs stayed silent or only participated to the discussion as a supporter role. Following statements taken from the reflection papers clearly pointed that how they reflected the knowledge development that they see in themselves. One example is in Maya's comments:

> Before group discussion, I concluded that the student made such kind of error due to rote learning. However, I recognized that student could misinterpret the property of the sum of interior angles and the sum of consecutive angles of parallelogram. I never

predict such kind of error before watching the video. It is really interesting [Maya, ADRP-MCVC8].

These expressions shows that her focus was not only on the correctness but also on the possible reasons of the of the student's incorrect responses. On the other hand, before the group discussion, Beril and Emel only had proposed the lack of student's knowledge about parallelism concept as a reason lead the misconception. However, Beril, for example, wrote following statements in her reflection paper after the discussion:

> I understood the possible reason why the student subtracted 70° from 180° instead of 360° in the group discussion process. Student previously encountered the sum of interior angles of a quadrilateral. However she might do this calculation by forgetting 360° as the sum of all interior angles [Beril, ADRP-MCVC8]

In general meaning, aforementioned statements was evaluated as an indicator of the participants' updating and deepening pedagogical knowledge in terms of understanding student's mathematical thinking compared to their initial comments. Overall, it was observed that PSTs later statements involved more detailed interpretive comments from different point of views on the student's misconception about adjacent angles of trapezoid. From this aspect, the collection of alterative perspectives coming together in the discussion process was found very reasonable and stimulating by the group members in order to explain why the student thought the sum of the measurement of adjacent angles of trapezoid as 360˚ instead of 180˚. Consequently, sharing the ideas in group discussion provided an opportunity to PSTs to enhance their pedagogical content knowledge in terms o understanding the student's mathematical understanding.

4.2.4.2 Developments in prospective teachers' knowledge about diagonal properties of quadrilaterals

4.2.4.2.1 Noteworthy event in MCVC3

In MCVC3, I asked a seventh grade student to draw the diagonals of HMRT parallelogram (see Figure 50). At this point, the student said that I could not remember what the diagonal is. Hereon, I asked the student to show the corner points of the parallelogram. In the following, the student showed the corner point of R as an example. After that, I again put a question in order to reveal the student's conception about the meaning of diagonal. The student tried to describe the diagonal concept by providing the following statements:

> It did not come back to my memory. I am not sure; it can be the place between two angles constructed inside of the parallelogram. No, I backed down because I could not remember what and how the diagonal is [The student in MCVC3].

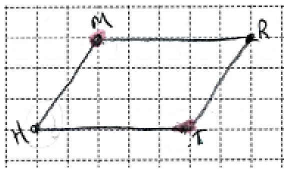


Figure 50. HMRT parallelogram where student treated corners as diagonals

4.2.4.2.1.1 Recognizing student's inadequate knowledge about diagonal

In the individual pre-interviews, all of PSTs thought that any seventh grade student can easily draw the diagonals of quadrilaterals because it is a basic geometric concept taught in the previous grade levels. Furthermore, according to the PSTs, students intuitively can deduce the meaning of "diagonal" if they consider the syntactic and semantic structure of the word of "*köşegen* (diagonal)" in Turkish language (Note: "*köşegen*" forms by the combinations of the words of "*köşe* (corner)" and "*-gen*" in Turkish language.

After PTs individually analyzed the student's mathematical work on diagonals in MCVC3, five of them expressed their surprise to see the student's lack of knowledge about diagonal in their reflection papers. Examples of comments involved:

> I think that any student can know the diagonal concept before watching the video. Interestingly, although the student knew the corner, he could not know diagonal [Beril, BDRP-MCVC3]
>
> The student don't know the diagonal concept because he could not express and draw diagonal in the video. However, I expected that the student can remember what the diagonal is by referencing the word structure of diagonal in Turkish language [Emel, BDRP-MCVC3].

From these comments, it was evident that PSTs tended to evaluate whether the student knew or could not know the diagonal concept. They made no specific inferences about why the student could not remember the drawing and meaning of diagonal in their individual video analysis processes.

In a part of the discussion process of the group discussion of MCVC3 (see episode 31), I asked what they noticed about the student's diagonal conception.

The researcher	What did you notice about student's interpretation related to diagonal concept?
The group	The student did not know the meaning of diagonal.
Ece	She knew what the corner is at least.
Maya	At a moment in video, the student described diagonal as a thing inside of the figure.
Deniz	However, the student was not sure although she made such description.
Emel	Actually, I expected the student could make a deduction considering the meaning of the Word "köşegen-diagonal" in Turkish because it involves the word of "köşe-corner". However, the student could not.

Episode 31 taken from MCVC3 group discussion

In that process, similar to the individual video analysis process, the group again evaluated the student knowledge about diagonal concept. They generally described the situation in the clip or evaluated what the student knew or could not know. However, they did not need to question the reasons why the student did not know anything about diagonal or they did not focus on what the possible influences of lack of knowledge about diagonal on students' conceptions about quadrilaterals can be. Nevertheless, it was concluded that PSTs had opportunity to restructure their knowledge about understanding student's mathematical thinking by realizing a middle school student’s lack of knowledge about basic geometric concepts such as diagonal.

4.2.4.2.2 Noteworthy event in MCVC4

There was a great opportunity for prospective teachers to develop their PCK related to understanding students’ mathematical understanding by virtue of the individual and group analysis of MCVC4. In the clip, a seventh grade student’ representations (see Figure 51) about diagonal properties of parallelogram were given in the following.

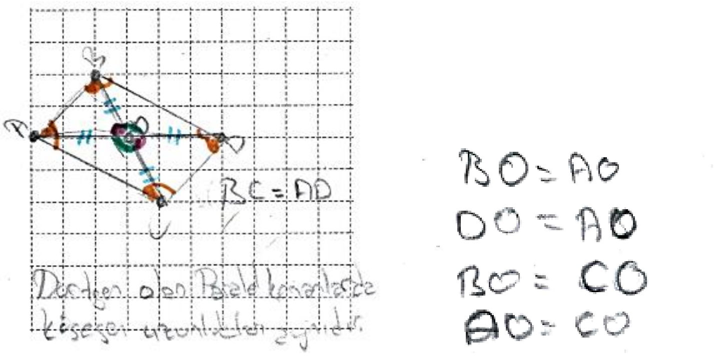


Figure 51. Student’s responses on diagonal properties of parallelogram in MCVC4

After I asked what she knows about diagonal properties of parallelogram, she said that the length of the diagonal of parallelogram must be equal because the diagonals come from the corner points that they have same angle measurements. At this point, I prompted a question to understand what the student thinks about the equality of diagonals for all parallelogram types. Hereon, the student drew a pentagon and analyzed its diagonals. At the end of her examination, she made following comments: “*The diagonals must be equal length for four-sided parallelogram, not for more than four-sided parallelograms.”* In the following, the student continued her ideas about diagonal properties by saying that *the diagonals of any parallelogram are also angle bisectors. For example, the angle of BAD and the angle of DAC have same measurements.* Student’s such kinds of expressions were unpredictable situations for prospective teachers because PSTs generally could not predict the student's conceptions about diagonal properties of parallelogram in the individual pre-interviews. The possible reason why they did not predict the student's misconception about diagonal properties might be associated to the inadequate SMK on related issue.

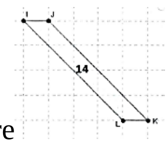
4.2.4.2.2.1 Early comments about student' ways of thinking

Prospective teachers' reflection papers that they wrote in individual video analysis process indicated that Maya, Beril, Ece, Oya, and Emel evaluated the correctness of the student's responses about the length of diagonals of parallelogram. They concluded that the student's conception was correct, reasonable and acceptable. However, Deniz, Aslı and Zehra provided some comments in which they evaluated the student's explanations as the indicators of the student's incorrect conceptions about corresponding diagonal property of parallelogram. Furthermore, they proposed different views about the reasons why the student had such misconceptions in their individual video analysis reflection papers. For instance, Deniz found the student's parallelogram construction as deceptive point in terms of mentioning on the diagonal properties because of the similarity between the student's parallelogram construction and rectangle figure. In addition, Aslı and Zehra argued that the student could not differentiate parallelogram from rectangle.

4.2.4.2.2.2 Recognizing and interpreting their own error and student's error about the length of diagonals of parallelogram

At a point in the group discussion process, they firstly focused on the adequacy of the student's knowledge about the concept of diagonal. As seen in Episode 32, Aslı shifted the discussion from the student's understanding of diagonal concept to the student's misconception about the length of diagonals of parallelogram.

Aslı	The student said that the length of diagonals is equal. This situation is related to being rectangle.

Beril	For example, if the student examined the diagonals of rhombus, she might understand diagonals of parallelogram are not always equal length.
Zehra	You are right, the student have such a misconception in her mind.
Researcher	Do you think the diagonals are equal length in parallelograms?
Emel & Maya	I thought diagonals are always equal length too.
Oya	I also, but I gave up this thinking yet.
Researcher	Why did you change your mind?
Beril	I also thought diagonals are always equal length. Normally, if we think diagonals of squares and rectangles it seems that diagonals are always equal length. However, when I consider the diagonals of rhombus, I recognized my mistake.
Aslı	For example, if we draw the following figure  it is easy to see non-equal length diagonals.
Researcher	In that case, why do students generally suppose that diagonals are always equal length?
Aslı	In my opinion, the student started to analyze diagonals of square and rectangle. Then, she might overgeneralize the equality of length of diagonals to all parallelograms. Furthermore, she said that all angles in her Figure are congruent. By referencing this congruency, the student supposed the length of diagonals is equal [length].
Maya	In my opinion, the student considered well-known [prototypical] figures. As a result, she incorrectly interpreted diagonal property [of parallelogram].

Episode 32 taken from group discussion of MCVC4

At the beginning of Episode 32, Aslı suggested that the student incorrectly took the diagonals as having equal length like that of rectangle. At this point, Beril gave additional example situations to elaborate Aslı's idea. She claimed that the student did not think the rhombus examples when examining the diagonal properties of parallelogram. Zehra explained her agreement with Beril by making emphasis of the possible influence of examining limited parallelogram examples on the student's misconception about the equality of diagonals of parallelogram. Following this, I wanted to learn which prospective teachers think parallelogram has equal length diagonals before the group discussion process. This prompting question revealed that Emel, Maya, Oya and Beril understood their mistakes about the length of diagonals of parallelogram by expressing their previous conception to the group. I pressed PSTs for why they changed their minds by asking a new question. Here, Beril explained that when she considered the diagonals of rhombus in addition to square and rectangle she realized that the diagonals do not have to equal length for every parallelogram. Similarly, Aslı expressed how someone easily can realize the diagonal properties if s/he consider non-prototypical parallelogram examples. I then asked the group to seek a possible reason why the student developed such kind of conception related to diagonals of parallelogram. Aslı resumed her explanation by proposing that the student only concentrated on the diagonals of square and rectangle instead of considering rhombus or other parallelogram figures. Moreover, she claimed that since the student supposed all angles of parallelogram have equal measurements the student believed that diagonals of parallelogram are also equal length. Besides, Maya's comments also supported her peer's comments.

4.2.4.2.2.3 Elaborations in interpretations about the student's misconception

Up until this point, the group had focused on the student's misconception about the length of diagonals of parallelogram and its possible reasons. In that process, some group members realized their own incorrect knowledge about diagonal properties of quadrilaterals. With this awareness, for example, Ece moved the discussion on another noteworthy event involving the student's misconception about the relationship between diagonals and angle bisectors (see Episode 33). This attempting behavior can be evaluated as an indicator to see how Ece developed her knowledge on student's mathematical thinking. Consequently, because of the influence of group discussion in Episode 33, she might adopt an approach which is less conclusive, more detailed and more exploratory.

Ece	Furthermore, the student thought that diagonals are always angle bisectors [in parallelograms].
Researcher	Do you think diagonals of parallelogram as angle bisectors?
Ece & Oya	Sometimes, it can be.
Zehra	Not always. For example, diagonals of rectangle are not angle bisectors.
Emel	In squares, diagonals are also angle bisectors.

Researcher	Students generally treated diagonals as angle bisectors. What can possible reasons of this situation be?
Zehra	The student might not understand parallelism of line segments and congruent angles between parallel line segments.
Researcher	Inadequate knowledge about parallelism is an important factor. What else?
Zehra	Another possible reason can be overexposure of isosceles triangle in mathematics lessons and questions. In ısosceles triangle, diagonal is angle bisector and median. In that situation, Might the student think median as diagonals?
Maya	Diagonal separate parallelogram two similar triangles. In such situation, the student could think that if triangles are same, the measure of angles [constituted by diagonals] in the triangles are also same. Thus, she had a misconception about angle properties of parallelograms.
Oya	Alternatively, students generally have difficulty to measure something in grid paper. İt seems that because they could not measure exactly the angles in grid paper, they might suppose diagonals divide the angle two equal parts.

Episode 33 taken from group discussion of MCVC4

As seen in Episode 33, after Ece described what the student thought about the angles that are separated by the diagonals of parallelogram, I asked a question to the group in order to understand whether they think that diagonals of parallelogram are angle bisectors or not. In response, Ece, Oya, and Zehra asserted that the diagonals of parallelogram are not always angle bisectors. Following this, I deflected the discussion into searching the reasons of the student's misconception of diagonals of parallelogram are also angle bisectors. This question was taken under serious consideration by different participants as they had to consider the possible factors that lead aforementioned misconception in the student's mind. Normally, although Zehra provided no inference to clarify why the student had such kind of misconception in her individual video analysis process, she indicated the possibility of the student's lack of knowledge about parallelism as a possible reason of the student's confusion between diagonals of parallelogram and angle bisectors. At this point, the group members evaluated Zehra's expressions as a quite reasonable inference. However, I posed a question to the group in order to understand what other PSTs think the possible reasons of the student's misconception. Again Zehra asserted that to be asked the questions requiring the usage of diagonal properties of isosceles triangles in the exams might be a challenging point for the students because the diagonals of isosceles triangle are also angle bisectors. As an alternative perspective, Maya proposed that the student might consider two congruent triangles inside of her parallelogram. According to Maya, the student can unsuitably suppose diagonals of parallelogram as angle bisectors by focusing on two congruent triangles. While the group members were concentrating on Maya's proposal, Oya brought a new perspective for identifying the possible reason of the student's related misconception. Oya claimed that since the student could not measure the length of diagonals in grid papers, the misconception might occur. It is important to see that final three comments in Episode 34 clearly showed that how prospective teachers elaborate their knowledge about understanding student's mathematical thinking especially their misconceptions with the possible reasons. Furthermore, reflective comments indicating the developments in PSTs' pedagogical content knowledge also were observed in their reflection papers written after the group discussion of MCVC4. Overall, rather than making a quick evaluation, all PSTs considered alternative reasons that could explain the student's misconception about the diagonals of parallelogram and proposed alternative solutions to the problematic situations in the student's work.

The summary of the common developments in prospective teachers' knowledge about properties of quadrilaterals throughout teaching sessions was given in Figure 52.

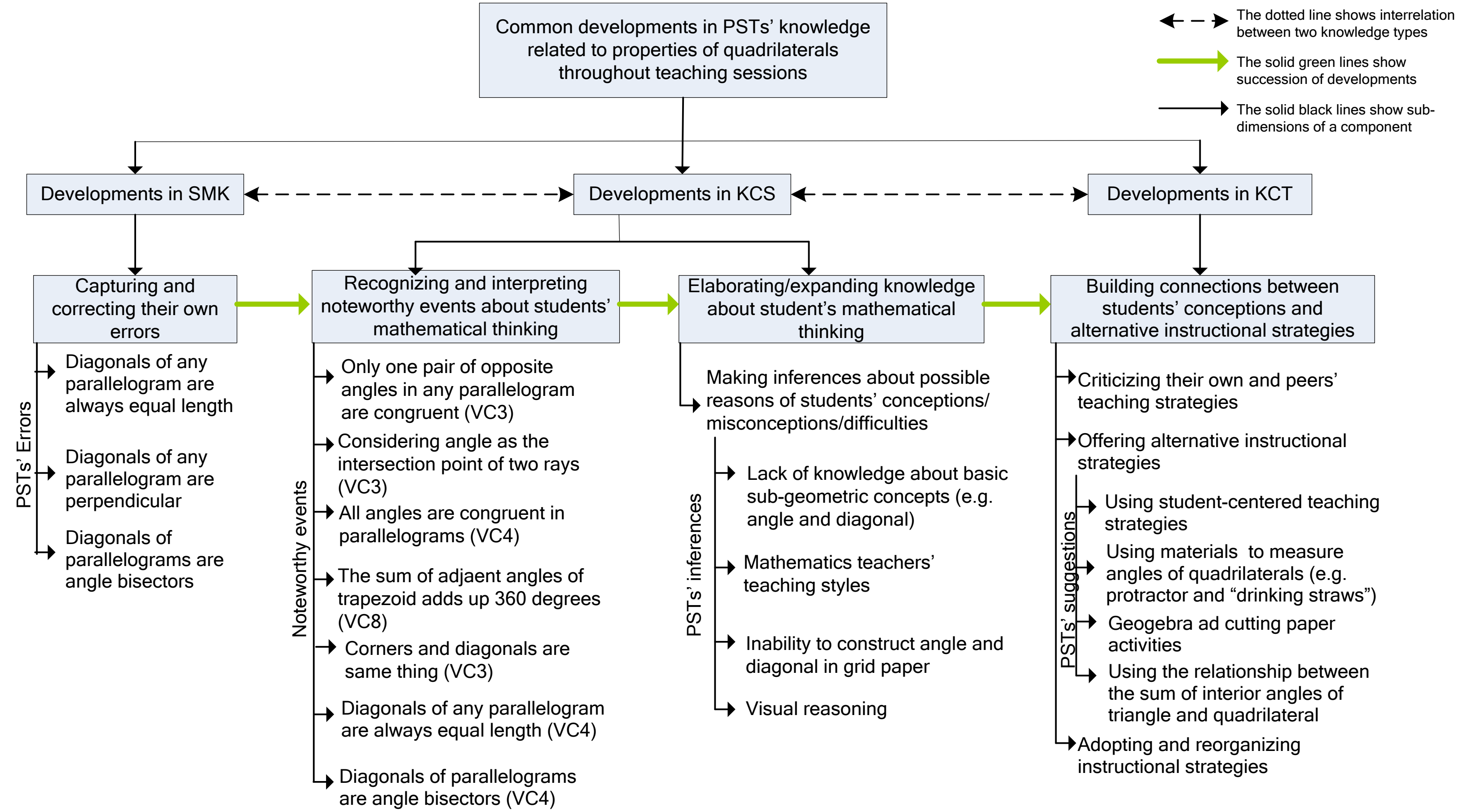


Figure 52. Summary of the developments in PSTs' knowledge related to properties of quadrilaterals in teaching sessions

4.3 The Nature of Prospective Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge about Quadrilaterals after Attending the Teaching Experiment

This part of the results summarized the final situation of prospective middle school mathematics teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) about quadrilaterals at the end of the teaching experiment process. As the data sources, PSTs' written and verbal responses to the post-interview tasks that were same with the pre-interview tasks and revised lesson plans involving their instructional planning to teach quadrilaterals to the seventh grade students. PSTs' responses at the beginning and at the end of the teaching experiment were comparatively presented in the following sub-sections.

4.3.1 The nature of prospective teachers' subject matter knowledge and pedagogical content knowledge about definitions of quadrilaterals

4.3.1.1 Updating personal definitions to establish necessary and sufficient conditions and to provide inclusive relations among quadrilaterals

Comparison of initial and final forms of PSTs' personal definitions of quadrilaterals was presented in Table 23 in terms of providing some logical principles that allow a definition to be mathematically correct; i) establishing *necessary and sufficient conditions*, and ii) *providing inclusivity among quadrilaterals concepts*.

For parallelogram concept, all prospective teachers continued to make their definitions considering inclusive relations among quadrilaterals. As seen in Table 23, they changed their personal definitions in terms of establishing necessary and sufficient conditions.

Table 23. Comparison of PSTs' personal definitions of quadrilaterals in terms of establishing necessary and sufficient conditions and providing inclusivity

PSTs	Parallelogram		Rhombus		Trapezoid	
	Pre-Int.	Post-Int.	Pre-Int.	Post-Int.	Pre-Int.	Post-Int.
Aslı	SnNC Inclusive	NSC Inclusive	NnSC Exclusive	NnSC Inclusive	nNnS Exclusive	NSC Inclusive
Deniz	SnNC Inclusive	NSC Inclusive	SnNC Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive
Beril	SnNC Inclusive	SnNC Inclusive	NSC Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive
Oya	NnSC Inclusive	NSC Inclusive	NnSC Inclusive	NSC Inclusive	NnSC Exclusive	NSC Inclusive
Ece	SnNC Inclusive	NSC Inclusive	SnNC. Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive
Zehra	NSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive
Maya	NSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive
Emel	NnSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Inclusive	NSC Exclusive	NSC Inclusive

*SnNC: Sufficient but not necessary conditions; NnSC: Necessary but not sufficient conditions; NSC: Necessary and sufficient conditions; nNnS: neither necessary nor sufficient conditions.

In the pre-interviews, although only Zehra and Maya provided all necessary and sufficient conditions in their definitions of quadrilaterals, all PSTs (excluding Beril) revised their parallelogram definitions considering necessary and sufficient conditions in the post-interviews. For instance, while Emel had defined parallelogram as "*a quadrilateral having two parallel opposite sides*[18]", Oya had identified it as "*a figure having opposite parallel sides*[19]". After teaching experiment, Emel realized that "*two opposite parallel sides*" is not enough to define

[18] Turkish version: Paralelkenar karşılıklı iki kenarı paralel olan dörtgendir.

[19] Turkish version: Paralelkenar karşılıklı kenarları paralel olan bir şekildir.

parallelogram since trapezoid even can be evaluated a parallelogram example according to this definition. Similarly, Oya recognized inadequate expressions in her previous definition because she did not mention whether parallelogram is a "closed" figure or not. As a result, she changed her definition as "*quadrilaterals having opposite parallel sides*[20]*.*" Consequently, Emel and Oya provided necessary and sufficient conditions when defining parallelogram. On the other hand, while three prospective teachers (Deniz, Ece, and Aslı) had defined parallelogram by listing all known properties such as the equality of length of sides or diagonals in the pre-interviews, they focused only critical properties of parallelogram in their revised definitions. Their revised definition was "*quadrilaterals with opposite sides parallel*[21]". This definition indicated that they adopted economical definitions instead of uneconomical ones.

For rhombus concept, all PSTs (excluding Aslı) provided economical correct inclusive definitions in the post-interviews because they considered all necessary and sufficient conditions for the definition. However, Aslı did not make any change on her previous rhombus definition (remember that she defined rhombus as "*a figure that all angles are congruent and all sides have equal length*[22]"). In the definition, although she gave an extra property about the equality of the angles without mentioning about the number of the sides and closeness, she could not recognize that this definition represents a square or a regular hexagon rather than representing all rhombuses.

For trapezoid concept, in the pre-interviews, six prospective teachers interestingly provided necessary and sufficient conditions based on exclusive relations among trapezoid instead of inclusive relations in Table 23. However, they preferred to define trapezoid according to inclusive relations of quadrilaterals after

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[20] Turkish version: Paralelkenar karşılıklı paralel kenarlara sahip olan dörtgenlerdir.

[21] Turkish version: Paralelkenar karşılıklı kenarları paralel olan dörtgenlerdir.

[22] Turkish version: Tüm açıları eş ve tüm kenar uzunlukları birbirine eşit olan şekildir.

the teaching experiment. Additionally, Aslı corrected mistakes in her previous definition as "*a rectangular region having the lower base and the upper base*[23]". Oya realized that her definition involved necessary but not sufficient conditions due to the lack of information about "*closeness*" and the number of sides.

In sum, at the end of the teaching experiment, PSTs could detect and correct the errors or inadequateness in their personal definitions and they focused on necessary and sufficient conditions to provide economical inclusive definitions of quadrilaterals. PSTs reactions about the student's rhombus definition in the analysis and discussion of MCVC6 can be given as evidence in order to show possible influence of video analysis and discussion process on PSTs' updated SMK. As a conclusion, it can be inferred that teaching experiment process contributed the development of their subject matter knowledge in addition to pedagogical content knowledge. This situation strongly showed the interrelation among SMK and PCK (Ball, 1991; Even, 1993; Shulman, 1986).

4.3.1.2 Developing an awareness about students' definitional errors of quadrilaterals and their possible reasons

In the individual pre-interview process, prospective teachers had provided only a few ideas or predictions about what seventh grade students' possible improper and incorrect definitions/descriptions of quadrilaterals can be. Some of them (Maya and Zehra) predicted that some students can only describe the concepts instead of formally defining them. However, in the post-individual interviews, their responses about middle school students' possible definitions of quadrilaterals differed from the responses in the pre-interviews. For example, they not only focused on the students' correct or incomplete descriptions but also developed awareness about the students' incorrect descriptions of quadrilaterals. More specifically, all PSTs took the students' descriptions especially in MCVC1 and MCVC4 as a referential point in order to

[23] Turkish version: Yamuk alt ve üst tabanı olan dikdörtgensel bölgedir.

mention about students' possible definitional errors related to parallelogram. They concentrated students' overgeneralization errors (e.g. defining two parallel line segments as a parallelogram or treating a regular hexagon as a parallelogram) and its possible reasons in related clips. PSTs' some comments taken from post-interview data was given in the following in order to assert how they developed an idea about students' definitional errors at the end of the teaching experiment.

> I understood that students did not know basic geometric concepts. For example, the student in MCVC1 treated two parallel line segments as an example of parallelogram. Moreover, the student treated beginning and end points of line segments as the sides of parallelogram. In the group discussion, we concluded that the student might focus on the meaning of parallelogram in Turkish language. As a result, we considered language as a reason for the student's misconception [about parallelogram]. Maybe, the student focused on the words of "parallel-paralel" and "edge-kenar" instead of considering "paralelkenar" mathematically. I was not aware of such kind of student's mathematical thinking before watching the video [Ece- Post-interview].

> I never predicted a student can have such kind of misconception [about parallelogram]. While a student could say two parallel line segments is an example of parallelogram, another student could say a regular hexagon is also an example of parallelogram. In this regard, students' conceptual knowledge is very poor. Students do not even know a quadrilateral as a closed figure [Zehra, Post-interview].

For trapezoid concept, in the pre-interviews, only Oya and Emel predicted that students might define trapezoid incorrectly because of the usage of the word of "*yamuk*" in Turkish language for "*trapezoid*" by emphasizing on the "*irregular*" meaning of "*yamuk*" in ordinary language. However, in the post-interviews, all PSTs considered the influence of the meaning of trapezoid in Turkish ordinary language on students' conceptions about trapezoid by referencing the students' incorrect definition and construction in MCVC8. Post-interview data also revealed that PSTs realized the influence of visual aspects of shapes on students' definitions of the concepts. As an example, Aslı provided following explanations:

> In lessons, giving always prototypical examples causes limited conceptions in students' mind. As a result, students generally describe prototypical figures without

> considering [critical] properties. For example, in MCVC7, the student described trapezoid as "*a figure formed by putting a triangle next to a square or rectangle* [24]". For this reason, we should firstly give definitions to the students instead of presenting only prototypical examples in the lessons [Aslı, Post-interview].

Aslı's comments in individual post-interview indicated that she recognized that students could not pay attention to the conceptual properties of the concepts by considering the students' mathematical work in MCVC7. Furthermore, she tried to develop some teaching strategies to prevent the effects of visual properties on students' restricted definitions/descriptions.

4.3.1.3 Focusing on both didactical suitability of an instructional definition and its mathematical correctness by proposing new teaching strategies

At the beginning of the teaching experiment, prospective teachers generally decided to utilize their personal definitions as instructional definitions with the limited number of didactical considerations such as "enabling deductive reasoning" or "clarity to the students". However, at the end of the teaching experiment, post-interview data indicated that they concentrated on didactical suitability of a chosen definition for the learning process in addition to its mathematical correctness. They thought that using well-known concepts by the students in the instructional definitions are crucial to fulfil students' needs. For example, Ece, Emel, Zehra, Maya, Oya made emphasis on selecting known concepts that are familiar for the learners. By this way, they believed that students can build connections between the concepts. Some example statements were given in the following:

> At the beginning [in pre-interviews], I thought a definition that involves all properties about sides, angles and diagonals of a geometric concepts might be useful for the students. However, I recognized that students did not even know the meaning of diagonal and angle in the videos. For this reason, I think that using a clear basic [economical and minimal] definition is more meaningful than a definition involving

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[24] Turkish version: Yamuk karenin ya da dikdörtgenin yanına gelen üçgen ile oluşan şekildir.

> all properties [of the concept]. For example, I will focus on the parallelism of opposite sides when defining parallelogram. Similarly, I will focus on the equal length of sides when defining a rhombus [Ece, Post-interview].

Above statements revealed that PSTs decided to select well-known concepts by the students when considering the students' lack of knowledge and intuitions in the video clips that they analysed an discussed throughout teaching experiment process.

On the other hand, Aslı, Deniz and Beril commented on the lack of necessity of giving extra information in the definitions. In the below statements clearly showed that they took account of economical definitions in post-interviews because economical definitions allow deductive reasoning.

> After watching the videos, I recognized that every student had difficulty about definitions of geometric concepts. Giving all [critical and non-critical] properties in a definition can limit students' minds. Furthermore, it can hinder to deductive reasoning. If we define parallelogram as a quadrilateral having opposite parallel sides, students can easily deduce parallelograms also have opposite equal length sides themselves. For this reasons, I made some changes in my initial definitions [considering minimality and economical definitions] [Aslı, Post-interview].

Another crucial change occurred for PSTs' instructional definition of trapezoid in terms of inclusive relations. In the pre-interviews, while they mostly decided to utilize inclusive definitions of parallelogram and rhombus they preferred to use exclusive definition of trapezoid due to students' intuitions. However, they changed their preference for the instructional definition of trapezoid in the post-interviews. They said that using inclusive definition is necessary to make emphasis on hierarchical relations among quadrilaterals because inclusive definition gives opportunity to the students in order to make relational thinking. When making such kinds of expressions, they always took the students mathematical thinking in the video cases as a referential point. For this reason, it can be inferred that PSTs' instructional preferences were changed after they analysed and discussed the students' descriptions of quadrilaterals in the video cases.

As a final point, it was observed that after they developed an understanding about the possible reasons of students' incorrect or inadequate descriptions, they started to device pedagogically powerful teaching strategies. Prospective teachers' common instructional strategies in pre- and post-interviews were summarized in Figure 53.

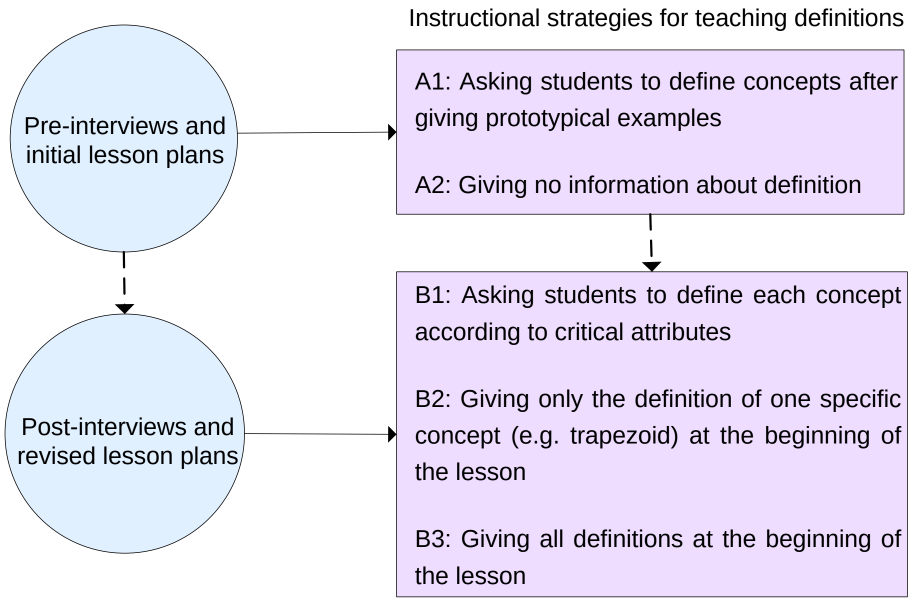


Figure 53. A summary about PSTs' instructional ways to teach definitions

By using information in Figure 53, the shift in each prospective teacher's instructional strategy was illustrated in Figure 54. For example, all participants (excluding Zehra and Emel) preferred asking definitions of parallelogram, rhombus, and trapezoid by giving examples of prototypical examples in the pre-interviews and initial lesson plans. Besides, in the post-interviews and revised lesson plans, three participants (Oya, Zehra, and Deniz) again preferred to give definitions in the lesson by a teacher-centered approach in post interviews and revised lesson plans because they believed that students must know the basic (economical) definition for each concept. However, among them, Oya decided to ask the definition of quadrilateral to the students since she thought that controlling students' previous knowledge about prerequisite knowledge is important to produce correct concept images and concept definitions.

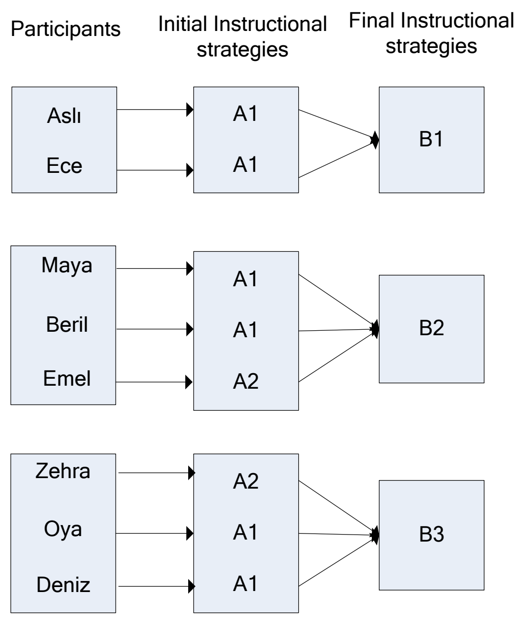


Figure 54. The comparison of PSTs' instructional strategies for teaching definitions

Differently, Beril, Maya, and Emel developed an idea in which they proposed that it is useful to give only the definition of one concept at the beginning of the lesson. Thus, they believed that students can make deductive reasoning to find the definitions of other concepts of quadrilaterals. To be clearer, Beril's explanations taken from revised lesson plan were given in the following:

> Firstly I give the trapezoid definition and properties. In the 8th video, the student thinks all sides in a trapezoid do not equal to each other because of the meaning of the trapezoid in Turkish. Thus, for students who can make the same mistake, I emphasize that it has different meaning in the geometry in the beginning of trapezoid concept [Beril, Revised lesson plan].

Beril's explanations showed the influence of the students' mathematical understanding in MCVC8 on her instructional ways. Alternatively, Maya decided to start the definition of parallelogram by making following explanations in revised lesson plan:

> Use the relation between definitions of the concepts when giving the definitions of each quadrilateral but not give the whole definition at the same time, give them gradually by doing deductions. After the learning of definition of parallelogram say

that a rectangle which has opposite sides and these sides are parallel and same length is a parallelogram [Beril, Revised lesson plan].

On the other hand, Aslı and Ece believed that giving visual representations of the concepts before the definitions is more useful for supporting students' conceptions about quadrilaterals in the pre-interviews and lesson plans. However, they changed their approach at the end of the teaching experiment. They suggested student-centered and definition-based approach instead of using teacher-based and figure-based approach. For example, Ece added following additional directions to teach definitions of the concept to the students (see Figure 55). In her directions, it was clear that she devised a discussion environment in which students can get opportunity to produce the definition of each concept considering critical properties of the concept.

- Ask students to what are their definitions of each quadrilateral according to their properties. Students may have difficulties about definition of trapezoid.
- Discuss the definitions of students.
- Give them definitions by completing the missing parts of their definitions.
- Try to use their words so students may be motivated
- Give importance to definition of trapezoid because of <u>at least</u> one parallelism. Students may get confused. Explain their relations.
- To relate all quadrilaterals, use Venn diagram.

Figure 55. Ece's directions about the teaching of quadrilaterals in revised lesson plan

4.3.2 The nature of prospective teachers' subject matter knowledge and pedagogical content knowledge about constructions of quadrilaterals

4.3.2.1 Shifting from prototypical examples to non-prototypical examples

Prospective teachers' personal constructions of quadrilaterals in post-interviews were compared with the constructions in pre-interviews. According to Table 24 all PSTs tended to draw non-prototypical figures for each concept by considering the hierarchical relations among quadrilaterals at the end of the teaching experiment.

Table 24. Comparison of PSTs' personal constructions of quadrilaterals in terms of prototypicality and hierarchical structure

Participants	Parallelogram*		Rhombus		Trapezoid	
	Pre-Int.	Post-Int.	Pre-Int.	Post-Int.	Pre-Int.	Post-Int.
Aslı	PT-NH	NPT-H	PT-NH	NPT-H	PT-NH	NPT-H
Deniz	PT-PH	NPT-H	PT-NH	NPT-H	NPT-NH	NPT-H
Beril	PT-PH	NPT-H	PPT-H	NPT-H	PT-PH	NPT-H
Oya	PT-PH	NPT-H	PPT-H	NPT-H	PT-PH	NPT-H
Ece	PT-NH	NPT-H	NPT-NH	NPT-H	NPT-NH	NPT-H
Zehra	PT-H	NPT-H	NPT-H	NPT-H	NPT-H	NPT-H
Maya	PT-H	PPT-H	PPT-H	NPT-H	NPT-H	NPT-H
Emel	PT-PH	NPT-H	PPT-H	NPT-H	PT-NH	NPT-H

*PT: prototypical, PPT: partial-prototypical, NPT: non-prototypical; H: hierarchical, PH: partial-hierarchical, NH: non-hierarchical

Moreover, they generally added rotated figures to their lesson plans by using a geometry application. In this regard, I asked the reason why they changed their initial constructions in lesson plans. In response, they generally said that students had difficulties to distinguish square and rhombus or rectangle and parallelogram in video clips. For example, Deniz wrote following statements in her revised lesson plan:

> From the videos that we analyzed and dıscussed, I realized that students could not comprehend rotate form (e.g. 45˚) of a figure. For this reason, I think that rotating figures in different angles can positively influence students' understanding [Deniz, Revised lesson plan].

Thus, they needed to add many of rotated figures to their future instructional plans. From this, we can concluded that prospective teachers both unpacked their subject matter knowledge and enhanced their pedagogical content knowledge in order to devise new instructional approaches by the help of the video clips in teaching experiment.

4.3.2.2 Developing an awareness about students' possible difficulties and errors in construction of quadrilaterals

In the pre-interviews, prospective teachers generally predicted students' correct prototypical and non-hierarchical drawings (remember Table 15). Only Zehra presumed that students may draw a non-example such as trapezoid just supposing it as a parallelogram since they couldn't pay attention the properties of grid paper. Interestingly, all PSTs provided additional constructions of parallelogram such as two parallel line segments, trapezoid, hexagon or octagon in order to show students' overgeneralization errors in the constructions of parallelogram. For rhombus concept, they also give examples of students' possible incorrect drawings in the post-interviews. They drew hexagon, parallelogram, four equal-sided figures that are not closed. While adding the figures to the task in post-interviews, all PSTs made similar explanations in below:

> I never predicted that students made such kinds of errors and difficulties about quadrilaterals. After analyzing the videos, I know that students can treat non-closed figures as parallelogram or they can treat a ten-sided regular polygon as a parallelogram [Oya, Post-interview].
>
> The name of "eşkenar dörtgen"in Turkish language is clear because "eşkenar" corresponds to "equal length sides" and "dörtgen" corresponds to "quadrilateral". However, students can even treat a regular hexagon as a rhombus without focusing on the word of "dörtgen-quadrilateral". Furthermore, I think that a student can see a non-closed figure as a rhombus because students in videos thought non-closed figures as the examples of parallelogram [Aslı, Post-interview].

It was evident from above explanations that PSTs took into account of the situations in video cases that they analyzed individually and discussed with their peers. Finally, all of them constructed non-closed shapes, irregular shapes, and polygons having more than five sides in order to indicate students' possible incorrect drawings of trapezoid, which can be considered as a crucial development in PSTs' PCK when considering their initial responses in pre-interviews.

4.3.2.3 *Developing a student-centered instructional way to teach the constructions of quadrilaterals*

Final form of participants' lesson plans indicated that most of them focused on a student-centered instructional way to teach the constructions of quadrilaterals to the middle school students. According to student-centered instructional plans, they generally kept students in an active role to construct quadrilaterals. In this sense, Ece and Beril added only some comments involving the necessity of giving rotated figures in teaching of quadrilaterals. However, they did not mention any detail about how they show rotated figures to the students.

On the other hand, Emel, Maya and Oya proposed that if students select the asked shapes among different kinds of polygons like in Figure 56, they can understand prototypicality and hierarchical relations among quadrilaterals by the help of teacher's guidance. They devised all polygons in their lesson plans by using Geogebra.

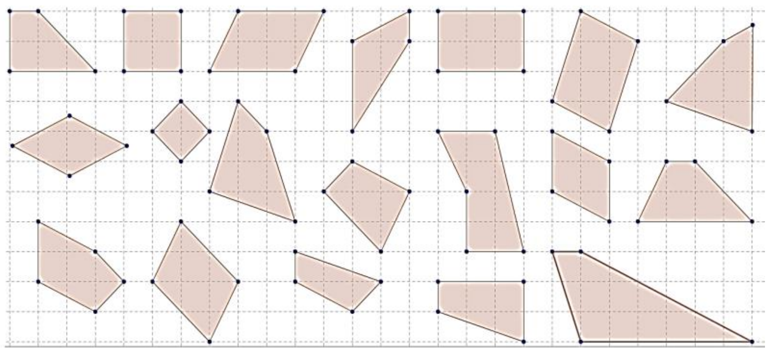


Figure 56. The selection task that Maya prepared to ask students in her revised plan

When I asked the reason why they changed the form of the figures in their lesson plans, they offered almost same explanations. One of them was given in the following:

> I added some figures having different orientations and sizes in my revised lesson plan. I added all figures in grid paper by using Geogebra. Thus, I expect that students can

easily measure equal length sides, parallelism and perpendicularity in grid paper [Oya, Revised lesson plan].

As can be understood from the above explanations, they decided to change their figures with the figures drawn in Geogebra and grid paper in order to make emphasis on the properties involving parallelism and perpendicularity of line segments in quadrilaterals. Alternatively, Zehra, Aslı and Deniz offered the usage of different kinds of materials such as paper-clippers and geoboard when teaching non-prototypical constructions of quadrilaterals. For example, Zehra planned to separate the class into two groups in her final form of lesson plan. She aimed to give papers and clippers to one group and to give geoboards to another groups. She also added each group of students' anticipated answers for the constructions of quadrilaterals according to given definitions (see Figure 57). She finalized her teaching for the constructions of quadrilaterals by using a task similar to the task in Figure 56 in order to reinforce their conceptions and prevent the formation of misconceptions originating from prototypical concept images about quadrilaterals.

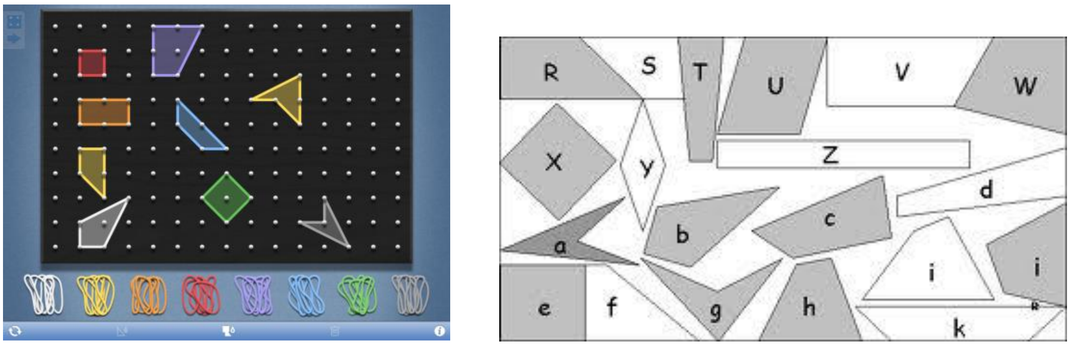
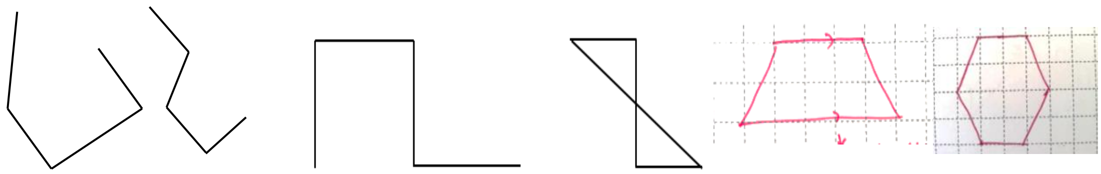


Figure 57. Zehra's predictions about students' anticipated answers for the constructions of quadrilaterals

Aslı proposed another alternative way to teach the constructions of quadrilaterals by aiming to prevent the formation of possible misconceptions or limited concept images in students' mind. By using student-centered teaching approach, Aslı planned to ask students to construct rectangle in geoboard and to cut figures from the paper. She made following comments about her teaching way:

> Students can cut papers their own ways, which enables them to strengthen their understanding about properties of quadrilaterals. Additionally, student can think a figure differently when it is rotated. If we show the figures in different positions, they can comprehend figures and their rotated forms have same properties [Aslı, Revised lesson plan].

These comments indicated that she aimed to teach that rotation does not change the properties and names of the figures. The other strategy proposed by the four participants (Maya, Zehra, Oya, and Ece) is about using the constructions of counter-examples in their instructional plans as an alternative way to teach the constructions of quadrilaterals. In the pre-interviews, only Maya proposed that giving the constructions of counter-examples might be useful to make emphasis on the critical properties of related quadrilateral (Remember that she had offered to use trapezoid, parallelogram, and an irregular figure to emphasize critical properties of parallelogram, rhombus and trapezoid, respectively.) At the end of the teaching experiment, for instance, Zehra and Ece also added some non-closed figures in the polygon tasks (see Figure 58-a and Figure 58-b).



(a) (b) (c)

Figure 58. (a) Zehra's counter examples (b) Ece's counter examples to show quadrilaterals in revised lesson plans (c) Oya's counter-examples to emphasize critical properties of parallelogram in post-interview

Zehra provided following explanations in order to explain why she needed to add such kinds of figures on the task:

> I added these figures because we discussed the effectiveness of giving counter-examples to imply the critical properties of the figures in the video clips we examined

> in the third week. After that, I developed an idea such as "why don't I use counter-examples? Then, I thought to make students have more critical thinking there is need [Zehra, Revised lesson plan].

Differently, Oya aimed to use counter-examples in Figure 58-c to ask students whether trapezoid and hexagon are also parallelogram examples or not. She said that because students sometimes make overgeneralizations as we saw in the video clips, it is meaningful to give counter-examples for an effective teaching of quadrilaterals.

In sum, in the beginning of the teaching experiment, prospective teachers generally preferred teacher-centered strategy; however, they tended to develop student-centered instructional way for teaching of the constructions of quadrilaterals at the end of the teaching experiment. In this process, they suggested using different kinds of materials or representations such as paper-clipper, geoboard, geometric applications (e.g. Geogebra), card activities, and counter-examples.

4.3.2.4 Final State of Prospective Teachers' Subject Matter Knowledge and Pedagogical Content Knowledge about Properties of Quadrilaterals

4.3.2.5 Detecting and correcting errors in their personal knowledge about diagonal properties of quadrilateral

As mentioned before, in the pre-interviews, although participants provided correct information about side, diagonal, and angle properties of trapezoid. PST's SMK was insufficient when considering their responses about especially properties of parallelogram and rhombus as seen in Table 25).

For instance, only Deniz and Zehra had provided correct responses for the properties of quadrilaterals. Remaining prospective teachers generally made mistakes when determining diagonal properties of parallelogram and rhombus in the pre interviews. When considering their responses in pre-interviews, it was evident that Aslı, Beril, Ece and Maya supposed that diagonals of any parallelogram are always angle bisectors. Furthermore, Aslı, Beril, Oya, Ece, and Emel claimed that diagonals of any parallelogram are always equal length. Similarly, Aslı, Beril and Emel proposed that diagonals of any rhombus also have equal length. Additionally, Ece believed that diagonals of parallelogram are perpendicular.

Table 25. Comparison of PSTs' misconceptions about properties of quadrilaterals

Concepts	Misconceptions	Participants having related misconception	
		Pre-int.	Post-int.
Parallelogram	Diagonals have equal length.	Aslı, Beril, Oya, Ece, Emel	X
	Diagonals are angle bisectors.	Aslı, Ece, Beril, Maya	X
	Diagonals are perpendicular.	Ece	X
Rhombus	Diagonals have equal length.	Aslı, Beril, Emel	X
	Diagonals are not always perpendicular.	Aslı	X
Trapezoid	X	X	X

Post interviews' data revealed that they recognized their errors about the angle and diagonal properties of quadrilaterals that they made in the pre interviews and they corrected all of them. They indicated the positive influences of analyzing and especially discussing students' mathematical works in MCVCs on their subject matter knowledge in addition to pedagogical content knowledge development. For instance,

> I was confused about diagonal properties of quadrilaterals in pre-interviews. At that time, I recognized the inadequacy in my knowledge about diagonal properties. However, when my friends provided different ideas and examples in group discussion I understood the reasons of my mistakes. For example, I concluded that the diagonals of parallelogram are always equal length in pre-interview. However, when my friends showed rhombus and its diagonals in the group discussion of MCVC4, I realized that I always focused on prototypical parallelogram examples instead of other examples [Ece, Post-interview].
>
> In pre-interview, after I constructed a classical [prototypical] parallelogram and I said that diagonals are also angle bisectors. However, it was necessary to think other examples [of the concept]. I noticed my mistake in group discussion of videos [Beril, Post-interview].

4.3.2.6 Recognizing the importance of evaluating students' existing knowledge about basic geometric concepts

Pre-interviews data indicated that most of PSTs generally focused on limited numbers of 'students' possible errors. It is also important to imply that they could not consider the possible influence of the lack of students' knowledge about pre-geometric concepts such as diagonal, parallel and perpendicular line segments on their conceptions of quadrilaterals. I mentioned about participants' predictive ideas about students' errors on properties of quadrilaterals were already given in the "section 4.1.4.2". Furthermore, I gave detailed information about how develop an awareness about students' possible misconceptions and difficulties with the possible reasons on properties of quadrilaterals in the "section 4.2.4". For this reason, in this part, I focused on what and how prospective teachers pedagogically concentrated on properties of quadrilaterals in the post-interviews.

In the post interviews, all PSTs made emphasis on the importance of evaluating students' knowledge about basic geometric concepts such as perpendicularity, parallelism, equal length line segments, corner, diagonal etc. In the interviewing process, they claimed that students' many of misconceptions might be related to their lack of knowledge about basic geometric concepts. According to them, before teaching of definitions and properties of quadrilaterals, students' current knowledge about pre-geometric concepts should be controlled and evaluated by using different assessment techniques and activities. They generally concentrated on diagonal concept as well as parallelism and perpendicularity concepts by indicating the students' understanding in video cases (especially in MCVC3 and MCVC4).

For instance, Aslı, Beril, Oya, Maya, and Ece added many of activities and tasks to determine students' existing knowledge about pre-geometric concepts by using similar strategies. For this purpose, Beril, Oya, and Ece preferred to draw some prototypical examples of quadrilaterals and ask students to construct or define what diagonal, angle, edge mean. More specifically, Beril's example activity was asserted in Figure 59. To explain the reason why Beril added such an activity to her lesson plan, she said that I added the activity to detect whether students' basic knowledge is enough to learn quadrilaterals and their properties.

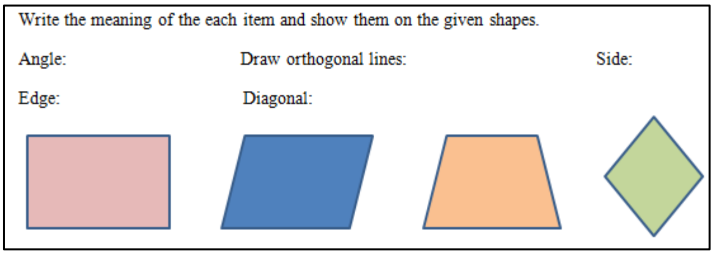
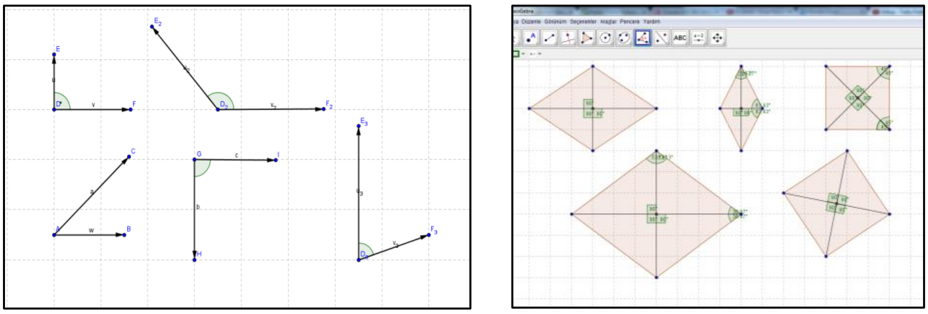


Figure 59. Beril's strategy to determine students' existing knowledge about pre-geometric concepts

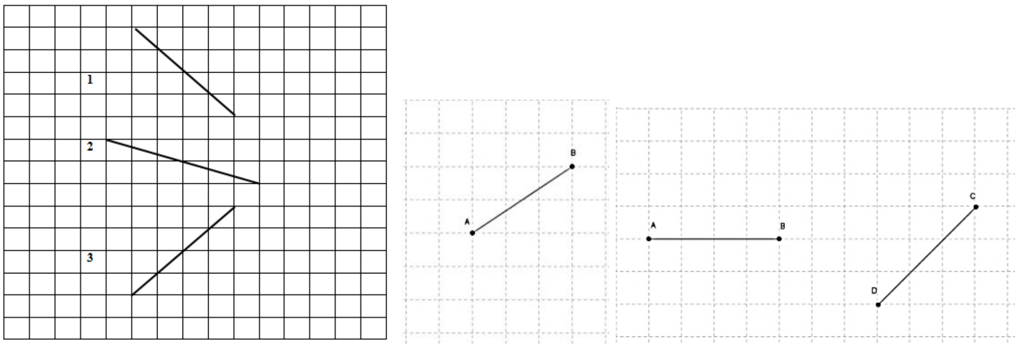
Aslı, Zehra, Oya, Beril, and Maya also focused on the pre-activities enabling to detect students' existing knowledge about the concepts of perpendicularity and parallelism. In these activities, they planned to use student-centered approach. In other words, they asked students to construct both prototypical and non-prototypical parallel line segments and perpendicular line segments. Some specific examples were given in Figure 60. In these activities, while Aslı planned to give protractor to the students in order to measure the angles and to find perpendicular angles in Figure 60-a, Zehra devised to use Geogebra to construct figures as in Figure 60-b and to measure angles of them by adopting a student-centered teaching way .



(a) (b)

Figure 60. (a) Aslı's activity to remind perpendiculartiy concept (b) Zehra's activity to teach intercestion of diagonals of rhombus and perpendicularity concept

On the other hand, Oya, Beril, and Maya utilized almost same instructional way to remind pre-geometric concepts such as perpendicularity and parallelism. They used grid paper and planned to ask students to construct a parallel or perpendicular line segment to the given line segment in the paper. Figure 61 indicated that they generally concentrated on non-prototypical constructions.



(a) (b)

Figure 61. (a) Maya's activity (b) Oya's activity to remind the constructions of parallel and perpendicular line segments

Deniz made little changes on her lesson plans in terms of teaching of quadrilaterals because her initial leasson plan involved many of activities aiminig to teach the angle and diagonal properties of quadrilateras. However, Emel made all revisions in her lesson plan considering the students' specific errors and difficulties in video cases. For example, she added following explanations to the final version of her lesson plan in order to prevent students' possible errors originating from the lack of knowledge about basic geometric concepts and also added an activity involving strips to examine angles between diagonals. She said that many students have difficulties in basic concepts such as congruent angles, congruent line segments, parallelism and perpendicularity.

> If students say that opposite sides are parallel in a rectangle or in other quadrilaterals, ask which sides you mean by opposite and what you mean by parallelism of them. If they do not say this property, put rectangle on the board naming its vertices. Then, take one pair of opposite sides aside and ask students positions of these segments with respect to each other. Do they intersect or coincident? [Emel, Revised lesson plan].

Furthermore, she provided following expressions by considering students' possible errors in the properties of parallelogram and trapezoid. In the revised lesson plan, she said that I added this statement because students can have focused on only one pair of opposite angles in video clip 3.

> If students cannot remember the properties of parallelogram and trapezoid, put them on the board and want from students to examine them. Then, ask the properties again. For the parallelogram, they can say the properties of sides (length of opposite sides) but they cannot be sure about angles. If such a case happens, cut the parallelogram in half from its diagonal and want from students to compare angles of two triangles. If students say that opposite angles are congruent in a parallelogram, ask them to show the angles that they mean by opposite [Emel, Revised lesson plan].

Moreover, many of PSTs again focused on the properties in the closing part of their revised lesson plans. They aimed to evaluate whether students learn side, angle and diagonal properties of quadrilaterals or not. In this regard, Zehra,Aslı, Maya and Oya utilized a table to compare and contrast the common characteristics of the quadrilaterals. Emel preffered to prepare an exit card to ask students following example questions for each quadrilateral: Are the lenghts of diagonals equal? Are diagonals also angle bisectors? etc. Additionally, she planned to ask true-falso questions such as "if I am a parallelogram, then my diagonals are always perpendicular."

In revised lesson plans, they also explained the reason why they needed such kinds of additional activities and why they found these activities are pedagogically appropriate when teaching quadrilaterals to middle school students. Some example comments they provided in the final form of lesson plans were presented in the following:

> I add this activity involving the constructions of parallel and perpendicular line segment to the given line segments, because I realize that some students cannot adequately know the concepts of linearity and parallelism in MCVC7. Maybe, thanks to this part of the activity, I can detect the students who have a difficulty about it and I can focus on these concepts before starting quadrilateral [Beril, Revised lesson plan].

> In new activities that I prepared after group discussions, I aimed to remind definitions, properties of quadrilaterals and basic geometric concepts. I think that the lack of knowledge about basic geometric concepts lead problems in students' understanding for further concepts in geometry. We understood how the lack of knowledge about basic geometric concepts influenced students' conceptions [about quadrilaterals] in videos. In this regard, I took account of students' incorrect responses in MCVC3 and MCVC4 when preparing my new activities [Maya, Revised lesson plan].

In conclusion, these revisions in lesson plans and above explanations indicated that prospective teachers had opportunities to develop their instructional approaches for teaching quadrilaterals by the help of analyzing and discussing video cases that involves seventh grade students' mathematical representations and explanations about definitions, constructions, selections, and properties of quadrilaterals.

CHAPTER V

CONCLUSIONS AND DISCUSSION

The purpose of the current study is to understand the nature and development of middle school mathematics teachers' knowledge about quadrilaterals throughout a teaching experiment designed within video case-based learning environment. In accordance with this purpose, this chapter addressed the conclusions of the research findings and the discussion of the major and critical evaluations, interpretations, judgments, and justifications about the obtained results of the current study by referencing previous studies in the literature. In this regard, this chapter divided into two parts. The first part of the conclusion and discussion section is related to common developments in prospective teachers' knowledge throughout the teaching experiment. The first part is remarkable because I proposed usage of micro-case video clips in undergraduate teacher education as an emerging and new issue in the current study. In the second part of the chapter, the content-specific developments of prospective middle school mathematics teachers' knowledge related to quadrilaterals in video case-based learning environment is discussed in order to present the details of developmental process in prospective teachers' SMK and PCK about quadrilaterals.

5.1 Common developments in prospective teachers' knowledge in micro-case video-based professional development context

In this section, I explained common developments in prospective teachers' mathematical knowledge for teaching of quadrilaterals through micro-case video-based learning environment. When discussing common developments in teachers' knowledge in the current study, I utilized results of some crucial studies that focused on developmental process of teachers' noticing abilities and knowledge development (Jacob et al., 2010; Sherin, 2007; Sherin & van Es, 2005; Stockero, 2008; Taylan, 2015; Tirosh, Tsamir, Levenson, Barkai, & Tabach, 2014; van Es, 2012a, 2012b; van Es & Sherin, 2002, 2006, 2008a, 2008b, 2010; Walkoe, 2014). Developments in PSTs' PCK throughout teaching experiment process were illustrated in *Figure 62*.

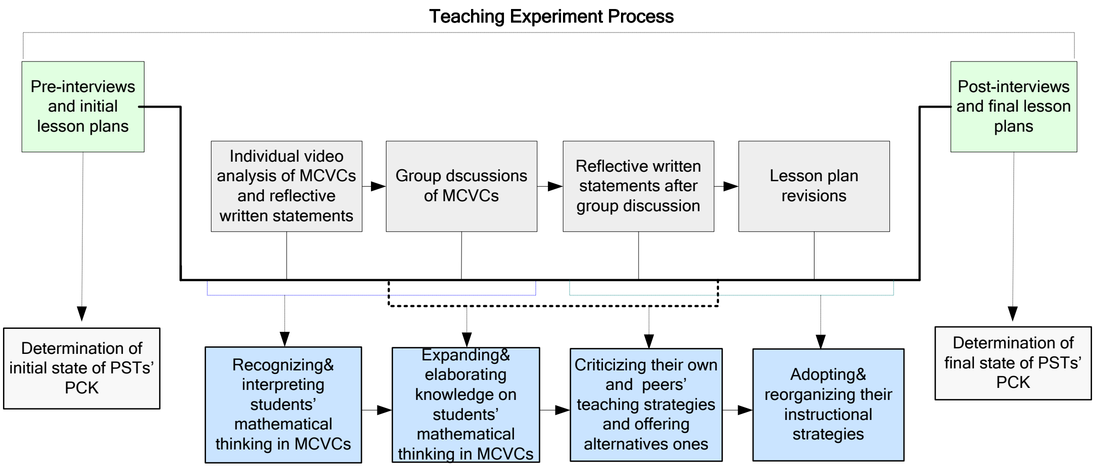


Figure 62. Common developments in PSTs' PCK

In the literature, it is stated that prospective teachers can lack PCK required for sophisticated analyses of teaching and learning (e.g. Hammerness, Darling-Hammond, & Bransford, 2005; Star & Strickland, 2008). In this regard, I used the results of pre-interviews and initial lesson plans when determining PSTs' initial state of knowledge on quadrilaterals and preparing micro-case video clips for teaching experiment sessions. For example, prospective teachers had inadequate knowledge about students' possible limitations and errors about quadrilaterals in pre-interviews. As seen in Figure 62, when prospective teachers individually analyzed micro-case video clips they generally began to recognize and interpret noteworthy events about students' mathematical thinking. By this way, they generally provided mathematically substantial descriptions instead of providing superficial descriptions of students' thinking even in the early meetings of teaching experiment. This result is interesting because the results of relevant prior works showed that in early video club meetings, teachers paid attention a range of issues involving climate, management, teacher-student relationship, and interactional processes (e.g. Sherin & van Es, 2009; Star and Strickland 2008; van Es & Sherin, 2008a, 2010; van Es, 2011) than directly focusing on students' mathematical thinking. In these studies, teacher noticing moved from general issues to student thinking after a series of video club meetings. The difference between other studies on noticing theory and this study can be related to the nature of type of video case. Researchers generally used classroom videos when examining teachers' noticing abilities. Such video cases involve information about complex learning environments as classrooms. Instead, micro-case videos focus on a learner's mathematical understanding as a "*microscope*". Thus, the structure of video might enable prospective teachers to directly attend to students' mathematical thinking and interpreting students' strategies in even early meetings. These results bear some similarities with the results of Jacob et al.' study (2010). They used video cases involving students' mathematical process in whole number operations to examine teachers' "*professional noticing of children's mathematical thinking*". They proposed three skills for professional noticing of children's mathematical thinking as "(i) attending to children's strategies; (ii) interpreting children's understandings; and (iii) deciding how to respond on the basis of children's understandings" (p. 173). Consequently, individual analysis of micro-case videos provided more detailed and focused noticing on students' mathematical ideas in the current study. Thus, prospective teachers began to think about students' defining, construction, and classification abilities on quadrilaterals in more nuanced ways throughout the teaching experiment process. This situation revealed the importance of concentrating a specific concept/subject in mathematics on teachers' knowledge development in terms of understanding students' mathematical thinking. Similarly, researchers concluded that video analysis involving students' mathematical thinking in a particular mathematics concept may help teachers more deeply about student thinking in the related mathematics domain such as multiplication and division (Taylan, 2015), algebraic concepts (Walkoe, 2014), and modelling perspective (Baş, 2013; Didiş, 2014).

In the group discussions of MCVCs, as seen in Figure 62, they generally had opportunities to elaborate/expand their knowledge on students' mathematical thinking with the influences of their peers' ideas. By this way, the results indicated that they began to (i) search possible reasons of students' errors and misconceptions rather than concentrating solely on errors in students' mathematical understanding; (ii) propose specific suggestions considering students' errors in video clips instead of making too general instructional recommendations. Most importantly, even the prospective teachers who provided interpretive and suggestive ideas in individual video analysis process expanded their knowledge about student mathematical thinking in terms of (i) recognizing new details about students' mathematical thinking in micro-case video clip; (ii) elaborating diverse ideas about possible reasons of problematic situations in students' understanding; and (iii) proposing alternative instructional strategies in order to overcome problematic situations in students' understanding in the group discussion processes. These developments in the prospective teachers' knowledge could be related to the nature of social constructivist learning environment in the study. In the group discussion process, prospective teachers had a chance to share ideas with their peers, which lead to deeper discussions away participants. Furthermore, in the following of group discussions, they began to criticize their peers' ideas and offering alternatives. Thus, they were able to conjecture alternative instructional strategies in order to overcome problematic situations in students' understanding in video clips and develop ideas how they can revise their future instructional plans considering peers' ideas and student thinking styles. Such developments may be interpreted based on the Vygotsky (1978)'s study in which it can be concluded two developmental levels of Zone of Proximal Development (ZPD). First level identifies what a learner can do or perform individually and independently. The second level describes what this learner can do in a social learning environment with guidance. From this point, in social interactional process, the role of facilitator and peer learning opportunities in group discussion influenced on the prospective teachers' developments in especially PCK related to quadrilaterals in this research. As a result, prospective teachers had opportunities to share, compare and discuss their ideas with the peers and they adopted different perspectives and conjectures on the related mathematical issue (Cunningham, Duffy, & Perry, 1992). Similar to Palinscar's (1998) idea, individual video-case analysis and group discussion process in this study created disequilibrium between prospective teachers' existing knowledge and newly encountered knowledge in the teaching experiment process. In conclusion, it is observed that sharing knowledge in social constructivist environment helped the prospective teachers to exchange ideas, to restructure their own knowledge and conceptions on the content, to construct pedagogical solutions, to deeply comprehend students' mathematical thinking, to receive feedback, and to support from their colleagues (Hiebert, Morris, & Glass, 2003) throughout the teaching experiment. In more general manner, the results of this study supported the idea that high quality of professional development involves teachers working with peers to examine problematic educational situations related to teaching and learning over sustained periods of time (Guskey, 2003; Hawley & Valli, 1999; Wilson & Berne, 1999; van Es, 2012a).

The results of the study also revealed that remarkable developments occurred in all prospective teachers' knowledge on the strategies they used to overcome students' misconceptions, the representations used to reason their understanding, and the strategies they used to explain the concepts of quadrilaterals and definitions and properties of them. Many of growth indicators were seen in prospective teachers' revised lesson plans that they updated after each week in teaching experiment process. In their lesson plans, they adopted some teaching strategies that were discussed by the group and they then reorganized their initial lesson plans considering peers' ideas and student thinking styles in micro-case video clips. In this regard, their revised lesson plans provided a great contribution in terms of understanding how they enhanced their knowledge with the influence of video analysis and group discussion. In every group discussion, they reconsidered the nature and involvement of their initial lesson plans about teaching of quadrilaterals to seventh grade students and they determined whether they need to revise their lesson plans in terms of changing teaching style, representations, examples, and designing/selecting appropriate new definitional activities, constructional activities etc. Thus, they approached to their initial lesson plans in more didactic and specific ways rather than thinking in utopic and more general ways. As a result, as they keep revising their lesson plans they began to (i) prepare/select student-centered and high-level tasks, (ii) adopt more detailed instructional approaches, (iii) build robust connections between students' errors and appropriate instructional approaches, and (iv) recognize the importance of asking suitable critical questions to the students.

I also illustrated common developments in PSTs' SMK on quadrilaterals in Figure 63 by focusing on a comparison of the results of pre-interviews and post-interviews because explicit evidences that show developments in SMK have rarely seen in individual video analysis and group discussions processes. Instead, the post-interviews results indicated that prospective teachers SMK related to quadrilaterals also enhanced at the end of the teaching experiment.

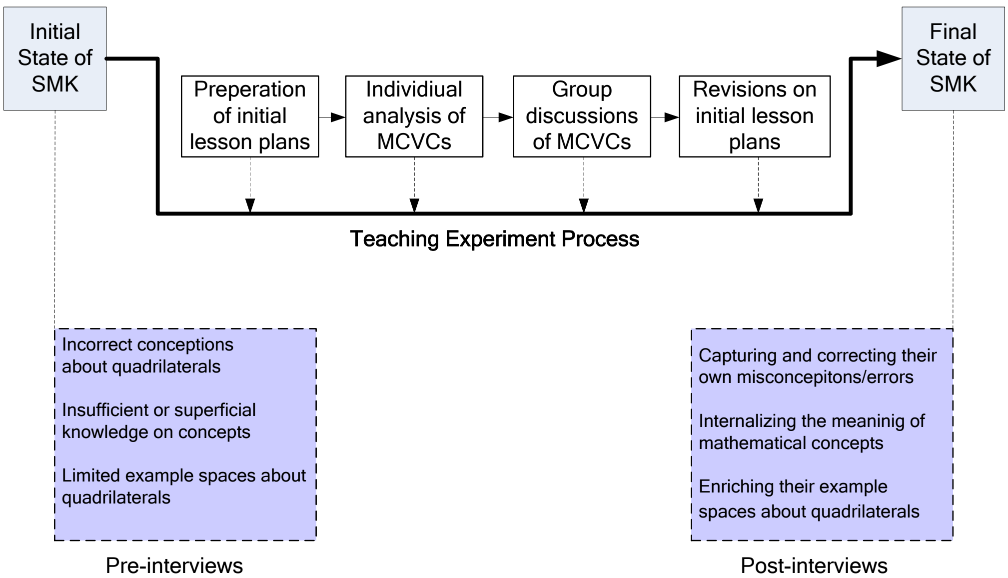


Figure 63. Common developments in PSTs' SMK

As in Figure 63, pre-interviews revealed that PSTs had incorrect conceptions, insufficient or superficial knowledge, and limited example spaces about quadrilaterals. For example, they generally thought only prototypical examples of quadrilaterals. However, in the post-interviews, they enriched their example spaces because they gave both prototypical and non-prototypical examples of quadrilaterals. At this point, it is meaningful to ask the following question: Why did explicit evidences of PSTs' developments in SMK occur in post interviews? This situation is probably related to the involvement of post-interview tasks in which there are many critical questions focusing on their subject matter knowledge (e.g. How do you personally classify quadrilaterals?, How do you represent hierarchical relations of quadrilaterals with a diagram?, Could you select only trapezoids among these polygons? etc.). Similar developments in teachers' classification abilities of quadrilaterals were detected in Öztoprakçı's (2014) study in which she examined prospective middle school mathematics teachers' cognitive processes under the support of the Geometer's Sketchpad learning activities because she asked some questions in the pre and post-interview sessions to the participants in order to examine their subject matter knowledge instead of pedagogical content knowledge.

Other possible reason may be related to the nature of micro-case video clips because these clips involve a collection of significant events related to a learner's mathematical thinking process on particular mathematical concepts or problem situations. In this regard, micro-case video clips can give more opportunities to the learners for directly attending to the students' mathematical thinking instead of focusing on content. Consequently, the results of post-interviews indicated that prospective teachers captured their own mathematical errors and misconceptions and corrected them. In more detail, they recognized their existing misconceptions and errors in the pre-interviews, corrected these errors and misconceptions, encountered new mathematical strategies, and internalized the meaning of the mathematical concepts or ideas at the end of the teaching experiment. Moreover, they emphasized the help and effective influences of analyzing and discussing students thinking in micro-case video clips on the development of their own conceptions.

5.2 The Content-Specific Developments of Prospective Teachers' Knowledge about Quadrilaterals

In this study, it is important to discuss content-specific developments in PSTs' knowledge in addition to the common developments. In this regard, I divided this section into two parts. In the first part, I discussed the nature and developments of PSTs' SMK on quadrilaterals. Then, I discussed the nature and developments of PSTs' PCK on quadrilaterals.

5.2.1 Nature and developments of teachers' subject matter knowledge on quadrilaterals

I discussed the nature and developments in PSTs' SMK on quadrilaterals by focusing on their personal knowledge on definitions, constructions, classifications, and properties of quadrilaterals, respectively in the following. The common developments in PSTs' SMK were presented in Figure 64.

The results of the pre-interviews indicated that many of prospective middle school mathematics teachers generally attempted to construct an *inclusive definition* or *partial inclusive definition* for the concepts. Especially, one prospective teacher (Aslı), having low-academic performance compared to other participants, made *partially inclusive definition*, *inclusive definition* and *exclusive definition* for parallelogram, rhombus and trapezoid respectively. In terms of subject matter knowledge, the difference between each definition revealed the presence of partial conceptions about hierarchical relations among quadrilaterals similar to the results of previous studies conducted with primary school teachers or middle school mathematics teachers (Erdoğan & Dur, 2014; Fujita, 2012; Türnüklü, 2014). This result was quite expected because making definitions based on hierarchical relations requires more sophisticated reasoning (e.g. logical and deductive reasoning) in company with deep mathematical knowledge on related concepts (Burger & Shaughnessy, 1986; Jones, 2000).

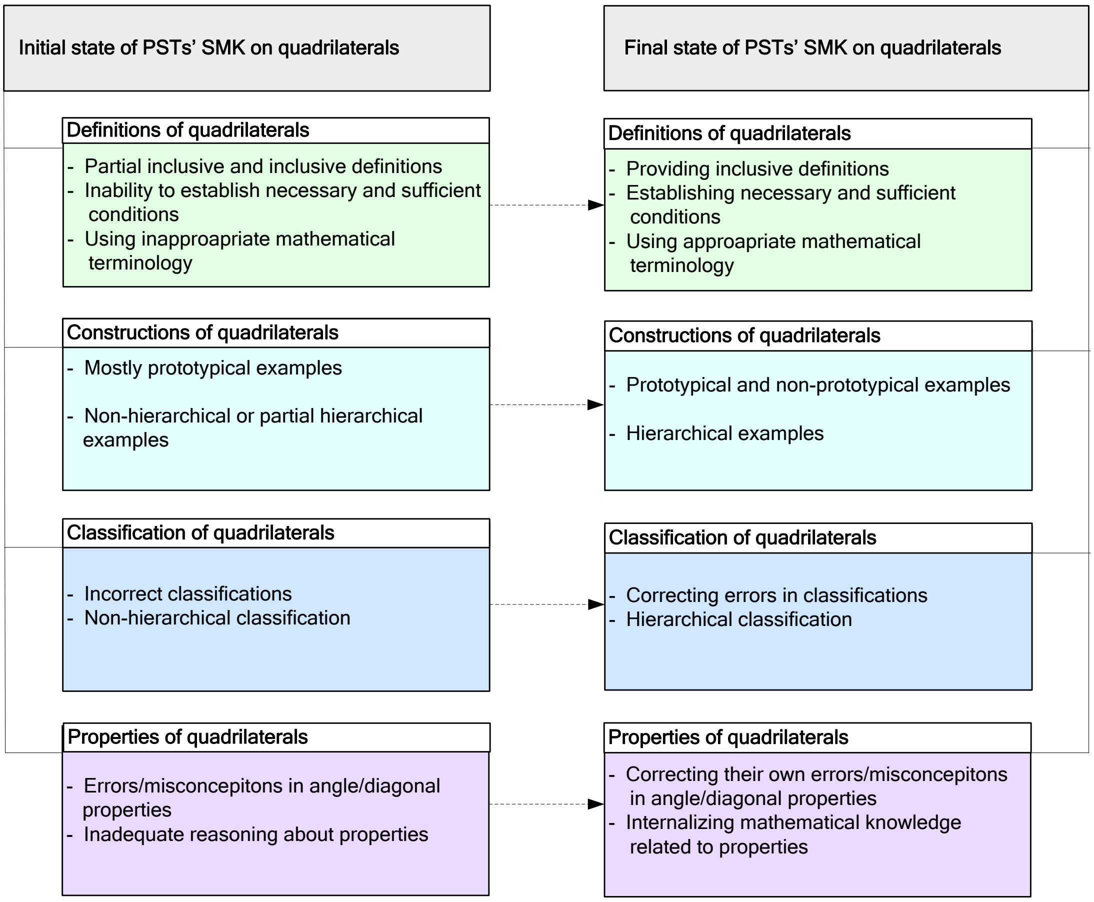


Figure 64. Shifts in PSTs' SMK on quadrilaterals

In addition to the hierarchical aspects of prospective teachers' personal definitions, another crucial result of pre-interviews was that prospective middle school mathematics teachers generally had difficulty to define the concepts of quadrilaterals considering fundamental characteristics of a definition involving *necessary* and *sufficient* conditions. More specifically, they generally either exposed *non-critical attributes* or omitted *critical-attributes* when defining the concepts. As a result, they provided either sufficient but not necessary conditions or necessary but not sufficient conditions in especially parallelogram and trapezoid definitions. Some prospective teachers were also not aware of the need to use precise language, and used the term of "figure" rather than "a closed figure" in definitions. To sum, instead of ensuring all mathematical requirements for definitions of the concepts formally or axiomatically, they defined the concepts personally and intuitively as mentioned Fischbein (1994), namely by trying to remembering memorized definitions which they learned previously in high schools or at university courses. Yet, definitions involving *necessary* and *sufficient* conditions allow learners to make deductive reasoning for determining critical attributes of a concept based on others (De Villiers, Govender, and Patterson, 2009; Winicki-Landman & Leikin, 2000) or producing an equivalent definition for a concept in quadrilaterals.

In the process of analysis and discussion of micro-case video clips involving students' conceptions about quadrilaterals, there were no clear and explicit evidences that show development in prospective teachers' personal definitions of quadrilaterals. Instead, they generally concentrated on student's conceptions, misconceptions, difficulties, or errors their possible reasons regarding the definitions/descriptions of quadrilaterals in video analysis and discussion process. As mentioned before, possible reason of this situation might be related to the involvement of micro-case video clips. Other possible reasons can be related to the nature of selected mathematical subject and learners' previous experiences. However, different form the pre-interview process, important developments in prospective teachers' personal definitions of quadrilaterals were observed in the post-interviews. For example, they updated their initial personal definitions in order to provide mathematically correct economical definitions and to establish necessary and sufficient conditions by making emphasis on inclusive relations among quadrilaterals in the post-interviews. More specifically, while only four prospective teachers provided inclusive definition of trapezoid in pre-interviews, all prospective teachers preferred to give inclusive definition of trapezoid in post-interview. This changing situation can be evaluated as a positive development in prospective teachers' personal definitions because both some mathematicians and mathematics educators and curriculums (e.g. MoNE, 2013) have widely preferred *inclusive definitions* since they functionally and economically allow to establish an inclusion between more particular concepts and more general concept (De Villers, 1994; De Villiers, Govender, & Patterson, 2009; Heinze, 2002; Kaur, 2015; Shir & Zaslavsky, 2002; Usiskin & Griffin, 2008).

Consequently, it is concluded that prospective teachers had a chance to develop their subject matter knowledge related to definitions of quadrilaterals after participating to the micro-case video-based teaching experiment process. As a conclusion, it can be inferred that teaching experiment process contributed the development of their subject matter knowledge in addition to pedagogical content knowledge because PSTs always mentioned the influence of analysis and discussion of MCVCs on their knowledge in post-interviews. This situation strongly showed the interrelation among SMK and PCK (Ball, 1991; Even, 1993; Shulman, 1986). For example, Carpenter, Fennema, and Franke (1996) claimed that if teachers comprehend students' understanding, this would give opportunities to enhance their pedagogical and content knowledge.

In terms of "*constructions of quadrilaterals*", the results of individual clinical pre-interviews indicated that even prospective teachers made inclusive definition for a concept; they generally exemplified the concept within non-hierarchical structure by constructing commonly prototypical examples. For instance, a prospective teacher made non-hierarchical personal drawings for the concepts of parallelogram, rhombus, and trapezoid even though she made inclusive, partial-inclusive and exclusive definitions for the aforementioned concepts respectively. Another striking example was that although a prospective teacher (Emel) made her personal definitions inclusively for the concepts of parallelogram, rhombus and trapezoid; she represented these concepts within partial-hierarchical, hierarchical, and non-hierarchical structure, respectively. These situations can be evaluated as an indicator of the influence of memorized concept definition on their concept images (Vinner, 1991) and the strong influence of intuitive representations on formal conception (Fischbein 1987). At that point, formal definitions can become meaningless if they are not given with all associated examples by teachers or textbooks (Fischbein, 1993). Consequently, it is necessary to reinforce the communication between verbal and pictorial information in order to construct robust relationship among concept image and concept definition (Fischbein, 1993; Vinner, 1991; Vinner & Hershkowitz, 1980, 1983). On the other hand, pre-interview results also revealed that prospective teachers' personal drawings were influenced by prototypical figures (Fujita, 2012). For example, their parallelogram drawings showed their prototypical concept images in their minds because they only shortened or extended the length of the sides without any other types of manipulations on the figure such as rotation/orientation changes with an angle different from 90°. Moreover, prospective teachers made their personal drawing for trapezoid with the influence of prototype images of the concept. These situations clearly can be assessed an indicator for the dominant role of figural aspects of geometric concepts (Fischbein, 1993) because they stated that they always encounters such figures in the textbooks and instructional processes of geometrical concepts. With the influence of these limited concept images (Hasegawa, 1997; Hershkowitz, 1989, 1990), it was an expected situation that prospective teachers generally used prototypical examples of quadrilaterals for their personal drawings of quadrilaterals in the pre-interviews. In the teaching experiment process, prospective teachers did not explicitly mention about what they changed in their minds throughout analyzing and discussing of micro-case video clips. Instead, they concentrated on students' thinking process and instructional approaches to students' difficulties and errors. However, post-interview results showed that they changed or updated their initial personal constructions of quadrilaterals. More specifically, they shifted from prototypical examples to non-prototypical examples of quadrilaterals considering inclusive relations among quadrilaterals.

In terms of "*classifications of quadrilaterals"*, the results indicated that prospective teachers tended to classify quadrilaterals by considering inclusive relations among quadrilaterals. From this point, results met the expectation that learners (especially prospective mathematics teachers) at university level must construct hierarchical relations among quadrilaterals. Obtained results had similarity with the results of studies conducted with elementary school students (De Villiers, 1994; Monaghan, 2000; Erdoğan & Dur, 2014; Türnüklü et al., 2013; Türnüklü, 2014). Furthermore, the idea of preservice and inservice teachers' concept images only slightly better than the students (Hershkowitz, 1989; Hershkowitz & Vinner,

1984) was supported by the results pre-interviews in the current study. Interestingly, the clear evidences showing the developments in prospective subject matter knowledge related to classification of quadrilaterals did not occurred frequently in the video analysis and discussion process although some of them made wrong classifications in the pre-interviews (e.g. Beril took trapezoid as a parallelogram example and Aslı treated square not to be an example of rhombus). Based on the previous research, it is more difficult to determine developmental paths in subject matter knowledge than the developments in pedagogical content knowledge on quadrilaterals when they examine students' works (Aslan-Tutak, 2009). In such situations, prospective teachers generally concentrated on students' conceptions and difficulties instead of explicitly revisiting their own conceptions. However, in the post-interviews, some developments were observed in prospective personal classifications of quadrilaterals. As mentioned before, this situation is probably related to the involvement of post-interview tasks in which there are many critical questions focusing on their subject matter knowledge.

In terms of "*properties of quadrilaterals"*, knowing critical properties of quadrilaterals is crucial to make correct economical definitions, flexible constructions, and inclusive classification of quadrilaterals because it is necessary to know critical properties of the concept in order to fulfill mentioned abilities (De Villers, Govender, & Patterson, 2009; Graumann, 2005; Mason, 2010; Öztoprakçı, 2014; Usiskin & Griffin, 2008). For example, knowing properties of quadrilaterals are important to determine critical properties of geometric figures (De Villers, Govender, & Patterson, 2009; Zazkis & Leikin, 2008). However, related literature indicated that researchers generally focused on learners' defining, construction, and classification abilities instead of directly examining teachers' knowledge and conceptions related to properties of quadrilaterals. In the current study, pre-interviews data revealed that many of prospective teachers incorrectly remembered the diagonal properties of rhombus and parallelogram because they stated axioms and theorems related to diagonals of quadrilaterals instead of reasoning how to work or to think mathematically. More specifically, half of prospective teachers had following misconceptions in the pre-interviews: (i) diagonals of any parallelogram are always equal length, (ii) diagonals of any parallelogram are perpendicular, and (iii) diagonals of parallelograms are angle bisectors. From the pre-interview data, it can be argued that the main reasons of these misconceptions may be related to prospective teachers' tendency to examine prototypical examples of the figures (Fujita, 2012; Gutierrez & Jaime, 1999) and inadequate proof, deductive reasoning and argumentation abilities (Clausen-May, Jones, McLean & Rowlands, 2000; Leung, 2008; Weber, 2001).

On the other hand, video analysis and group discussion processes revealed that prospective teachers recognized their own errors related to diagonal properties of quadrilaterals when encountering their peers' responses and suggestions. They specifically mentioned about the contributions of sharing ideas in a social learning environment to enhancing and unpacking their own conceptions in the post interviews.

5.2.2 Nature and developments of teachers' pedagogical content knowledge on quadrilaterals

I discussed the nature and developments in PSTs' PCK on quadrilaterals by focusing on definitions, constructions, classifications, and properties of quadrilaterals, respectively in the following.

PCK on definitions/descriptions of quadrilaterals. In the individual pre-interview process, prospective teachers provided only a few ideas or predictions about what seventh grade students' possible improper and incorrect definitions/descriptions of quadrilaterals can be. They generally predicted that students may not provide formal definitions of the concepts because of inadequate knowledge about mathematical terminology. As a prominent example, only two prospective teachers (Oya and Emel) predicted that students might define trapezoid incorrectly because of the usage of the word of "*yamuk*" in Turkish language for "*trapezoid*" by emphasizing on the "*irregular*" meaning of "*yamuk*" in ordinary language. Although this is a common overgeneralization error that Turkish middle school students make (Erşen & Karakuş, 2013; Türnüklü, 2014; Ulusoy, 2015) the participants were not aware of it. In the pre-interviews, it is observed that they were unaware of overgeneralization errors in students' descriptions of quadrilaterals (e.g. "parallelogram consisted of two perpendicular line segments in same proportion"; "parallelogram can have more than four sides" or "trapezoid is an irregular figure having non-equal sides") and their possible reasons. However, throughout the teaching experiment process, as they analyzed seventh grade students' conceptions about quadrilaterals in micro-case video clips they began to recognize the relations among students' incorrect descriptions of quadrilaterals and their possible reasons like inappropriate mathematical and ordinary language usages. All prospective teachers understood the possible influences of the meaning of concepts in Turkish ordinary language on students' conceptions. Furthermore, they recognized students' tendency to focus solely visual properties of a prototypical geometric figure rather than focusing on critical attributes for establishing formal definition of the related concept. In the group discussion process, not only they understood students' definitional errors, they began to develop conjectures for their possible reasons such as (i) language-based reasoning, (ii) visual reasoning, (iii) lack of students' knowledge about definitions, (iv) lack of knowledge on basic sub-geometric concepts, (v) the prototypical figures given in the textbooks, and (vi) limited prototypical examples teachers used in instructional process. In conclusion, as they mostly recognized students' errors, misconceptions and their possible reasons in the individual video analysis process they unpacked their pedagogical content knowledge regarding understanding the possible factors of students' definitional errors and misconceptions in especially group discussion process.

When considering the fundamental roles of mathematical definitions in problem solving, argumentation and proof, identifying mathematical concepts (De Villiers, 1998; Silfverberg, 2003), making relationship among concepts (Mariotti & Fischbein, 1997), and ensuring oral and written communication for mathematics teaching and learning (Thompson & Rubenstein, 2000), utilizing definitions effectively in the instructional processes is a crucial and necessary component of teachers' subject matter knowledge and pedagogical content knowledge (Ball, Bass, & Hill, 2004). For this reason, Leikin & Winicki-Landman (2001) stated that when determining to utilize equivalent mathematical definitions, it should be assessed not only from the *epistemological* aspects but also from the *cognitive*, *instructional*, and *didactical* aspects. Despite its importance and necessity of definitions in mathematics learning and teaching, results of the pre-interview data revealed that most of prospective teachers could not consider both *mathematical correctness* and *didactical suitability* of a definition when expressing their instructional preferences for a mathematical definition of quadrilaterals. Furthermore, they were not adequately aware of didactical considerations when preferring a definition for the instructional processes.

In the literature, didactically suitable definition for the instructional processes was explained based on some conceptions (Winicki-Landman & Leikin, 2000) such as relying on previously learned concepts, learners' intellectual development, zone of proximal development of the learners (ZPD), intuitiveness (Fischbein, 1987; Mariotti & Fischbein, 1997), and elegance (Vinner 1991; Van Dormolen & Zaslavsky, 2003). When viewed from this perspective, prospective middle school mathematics teachers in the current study did not generally have didactical considerations when preferring a definition for instructional processes at the beginning of the teaching experiment. More specifically, some prospective teachers only mentioned the importance of controlling students' prior knowledge about parallelism before giving the definition of parallelogram. As an illustrative example, it is useful to remember that Oya suggested her didactical consideration only for trapezoid concept. She personally defined trapezoid inclusively; however, she preferred to use exclusive definition when o teach the concept to the students by the reason of difficult nature of inclusive definition for the students. In other words, when deciding which definition of trapezoid to use for the instruction, she paid attention to the learners' development built on intuitive meaning of trapezoid concept by limited concept images. On the other hand, some of them (e.g. Emel) paid attention to selection of her instructional definitions in terms of ensuring *elegance* and *necessary* and *sufficient* conditions according to inclusive relations among quadrilaterals. Similar to the suggestion of de Villiers, Govender, and Patterson (2009), they believed that if students encounter firstly an elegant and minimal definition of the concept, they can deduce other non-critical properties from the given definition. However, other prospective teachers did not think *intuitiveness* and *minimality* of a mathematical definition as a critical characteristic of the definitions when presenting their instructional preferences. Based on the pre-interview data and prospective teachers' initial lesson plans, it was concluded that that prospective teachers' knowledge about determining didactically suitable definitions for instructional processes was formed by a narrow and inadequate perspective in terms of teachers' professional development before they participated to the video case-based teaching experiment process.

In the teaching experiment process, prospective teachers began to develop their initial instructional decisions and proposed new strategies that they plan to use when teaching definitions of quadrilaterals. In this regard, they concentrated on both didactical suitability and mathematical correctness of an instructional definition in the group discussion process. For instance, because the student's definition involves a small error in MCVC6, prospective teachers could not recognize when individually analyze the student's mathematical thinking in the video clip. However, they had opportunity to notice the absence necessary and sufficient conditions in the student's definition after my critical questions in the group discussion process (remember Episode 8). More interestingly, even high-achiever prospective teachers defined all concepts establishing necessary and sufficient conditions in the individual pre-interviews they could not evaluate students' definition in terms of necessary and sufficient conditions before the teaching experiment. However, group discussion enabled all prospective teachers to realize unnecessary conditions or extra properties in the student's rhombus definition. In this sense, the result of the current study is parallel with the research involving peer interactions, and group working opportunities to the learners in various mathematical contexts (Eizenberg & Zaslavsky, 2003; Leikin, 2004). For instance, a group discussion environment that provides group learning for teachers and the need to share ideas during their attempts are emphasized for mathematics teachers' professional development in terms of their mathematical knowledge, pedagogical content knowledge, and curricular content knowledge (Leikin, 2004).

Another important development in prospective teachers' teaching strategies that after they developed an understanding about the possible reasons of students' incorrect or inadequate descriptions by virtue of individual analysis and group discussions of micro-case video clips, they started to think pedagogically powerful teaching strategies. For example, in the pre-interviews and initial lesson plans, they generally preferred to utilize teacher-centered approach to teach the definitions of the concepts by giving the definitions on the board or Venn diagram. Instead, in post-interviews and revised lesson plans, they developed a teaching strategy in which they adopted student-centered ways such as asking learners to construct or define quadrilaterals according to given critical features. They emphasized the crucial influences of watching different students' various descriptions in the videos and the ideas proposed by their peers in the group discussions in terms of enhancing and unpacking their pedagogical strategies for teaching of quadrilaterals. Thus, analysing and discussing specifically selected and designed video cases can provide efficient and effective learning opportunities. Therefore, teachers can be supported in understanding how different learners understand mathematical concepts in various ways; adopting a student-centred approach instead of teacher-centred perspective (Friel & Carboni, 2000); enhancing pedagogical strategies and their ability to identify a problematic situation with multiple perspectives (Carboni & Friel 2005; Lin, 2005).

PCK on constructions of quadrilaterals. The ability to anticipate and interpret students' responses is crucial component of prospective teachers' pedagogical content knowledge because such type of knowledge provides an understanding on students' mathematical thinking. However, according to pre-interview data, prospective teachers' knowledge about students' possible drawings for the concept of parallelogram, rhombus and trapezoid showed that they had limited pedagogical concept knowledge about students' possible drawings since they generally focused on what can be students' possible correct drawings rather than the incorrect and contradicted ones. For example, pre-interview data indicated that prospective teachers could predict only students' non-hierarchical prototypical drawings in the current study. Unfortunately, they had inadequate knowledge about students' possible partial hierarchical relational thinking and common cognitive paths used when recognizing inclusive relation between similar concepts such as rhombus and parallelogram (Okazaki & Fujita, 2007; Vinner & Hershkowitz, 1980). Besides, prospective teachers could predict students' possible incorrect rhombus drawings in addition to the correct ones. This situation was an expected result because parallelogram/rhombus relations might be grasped more easily than other types of relations among quadrilaterals, which was parallel with the results of the studies conducted with students (Fischbein & Nachieli, 1998; Okazaki & Fujita, 2007) and preservice teachers (Duatepe-Paksu et al., 2012; Fujita & Jones, 2007; Türnüklü, 2014a). On the other hand, in the pre-interviews, only three prospective teachers could predict what can be students' possible incorrect drawings for the trapezoid concept. For example, two of them easily thought that a student might fail to determine parallelism of opposite sides. Furthermore, only one of them could argue that students can treat an oblique shape because of the naming of the concept in Turkish language. Consequently, when evaluating prospective teachers' knowledge about students' possible quadrilaterals drawings, it was clearly seen that their lack of awareness about the points students have difficulties when drawing and their incorrect drawings that developed under the influence of intuitive and visual perceptions (e.g. overgeneralization error like treating two parallel line segments as an example of parallelogram) before participating to video-based teaching experiment.

As a prominent development, when analyzing and discussing of micro-case video clips, prospective teachers enhanced their knowledge and awareness on students' possible incorrect constructions and difficulties in the constructional process of quadrilaterals rather than focusing solely students' correct prototypical and non-hierarchical drawings. The results showed that they realized students' insufficient knowledge about basic sub-geometric concepts such as closeness of quadrilaterals and parallelism that is necessary to determine parallel opposite sides. Furthermore, they noticed students' inability to construct a quadrilateral in grid paper as an interesting and unexpected situation. Finally, prospective teachers saw the influence of incomplete relationship between concept image and concept definition in students' conceptions. These developments can lead an ambiguous teaching in prospective teachers' future instructions because ambiguous teaching lies at the intersection of mathematical content and students' mathematical reasoning (Philipp, 2014).

The ability of planning an effective instructional process by selecting suitable examples and non-examples is necessary to fulfil different cognitive level of students' needs and to activate the interactional processes between concept image and concept definition of each quadrilateral type. As a crucial component of pedagogical content knowledge, the examples or non-examples teachers utilized for their instructional process might support or limit students learning (Leinhardt, 2001; Rowland, Thwaites, & Huckstep, 2003; Zaslavsky & Zodik, 2007, 2014) and reflects their knowledge capacity (Zaslavsky, Harel, & Manaster, 2006; Zodik & Zaslavsky, 2009). In this respect, prospective teachers' instructional drawings that they proposed for their future instructions showed some similarities and differences in the pre-interviews and initial lesson plans. A similar point was that they did not consider utilizing non-examples in order to imply the role of critical and non-critical attributes of a concept for the construction of appropriate and various concept images and example spaces (Vinner, 1983; Zaslavsky & Peled, 1996). Yet, NCTM (2000) emphasizes the necessity of non-examples of the concepts in addition to the many examples of same concept to allow of concept attainment (Petty & Jannson, 1987). However, prospective teachers in the study preferred to construct only non-hierarchical prototypical examples of quadrilaterals as instructional constructions without any didactical considerations (e.g. students' prior knowledge, intellectual level etc.) although they defined parallelogram, rhombus and trapezoid as partial-inclusively or inclusively. Inconsistency between PSTs' concept definitions and concept images in terms of hierarchical structure might be related to intuitive responses rather than formalized mathematical knowledge about related concepts (Vinner, 1991). Furthermore, prospective teachers' instructional constructions were substantially similar to their personal constructions of quadrilaterals. In the literature, it is stated that non-hierarchical and prototypical drawings cannot promote students' understanding in terms of inclusive relations of quadrilaterals and cannot give opportunity students to construct a robust and flexible interaction between concept image and concept definition (Fujita, 2012; Hasegawa, 1997; Okazaki, 1995). As a result, it is not surprising that students have to develop inflexible (static) mental images about quadrilaterals by referencing such types of teacher-generated drawings without reasoning the meaning of the definitions (Fujita, 2012; Monaghan, 2000). As a final remark, it is believed that prospective teachers should use powerful and appropriate instructional examples/non-examples (Ball et al., 2005; Ball et al., 2008) and provide awareness about the affordances and limitations of experiencing with a specific type of example or non-example (Watson & Mason, 2005) before becoming inservice teachers because it is necessary for the development of flexible mental images of concepts in students' mind.

The results revealed that in the video analysis and discussion process and revised lesson plans, prospective teachers adopted student-oriented instructional ways to teach the constructions of quadrilaterals by utilizing different materials and representations. More specifically, they commonly preferred following instructional strategies for constructions of quadrilaterals: (i) asking students to construct quadrilaterals, (ii) a teaching way by beginning trapezoid instead of starting with square, (iii) controlling students' conceptions related to basic sub-geometric concepts, (iv) utilizing different materials and representations such as grid or dot paper, paper-clipper, geoboard and Geogebra, (v) using non-examples, and (vi) asking critical questions to assert students thinking deeply. All these crucial results indicated developments of prospective teachers' knowledge on knowing mathematics curriculum and alternative instructional materials as an important dimension of pedagogical content knowledge (Grossman, 1990). The ideas proposed in group discussions gave opportunities to prospective teachers to build connections between problems in students' constructions and alternative instructional strategies. From this point, the results revealed how development of teachers' knowledge on students thinking influenced their instructional plans and decisions (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Fennema, Franke, Carpenter, & Carey, 1993; Mason, 2002; Nathan & Koedinger, 2000a; 2000b).

PCK on classifications of quadrilaterals. Results of the pre-interviews unexpectedly revealed that even if a prospective teacher is well at subject matter knowledge about classification of quadrilaterals it does not guarantee having a well-structured pedagogical content knowledge on related concept. For instance, while prospective teacher having inadequate SMK about classification of quadrilaterals focused on students' incorrect selections of quadrilaterals originated from overgeneralization errors (Klausmeier & Allen, 1978), another prospective teacher having enough SMK on classification of quadrilaterals considered only students' possible correct and/or incomplete selections of quadrilaterals. On the other hand, they generally could predict partial hierarchical classification majority of students make for especially parallelogram and rhombus (Erez & Yerushalmy, 2006; Monaghan, 2000; Fujita & Jones, 2006) rather than focusing on contradicted situations in which students made a correct inclusive trapezoid definition. Furthermore, they were aware of students' possible difficulties to differentiate between non-prototypical square and rhombus. From such type of examples, it can be concluded that they partially realized the influence of orientation of figures evoking prototypical images in students' minds despite being a non-critical or irrelevant attribute. This difficulty has been heavily emphasized in the studies involving middle school students (Aktaş & Aktaş, 2012; Monaghan, 2000). This result is not compatible with the results conducted with preservice teachers in which they reached teachers' inability to detect students' prototypical images (Akkaş & Türnüklü, 2014; Hannibal, 1999; Türnüklü, Alaylı & Akkaş, 2013). Most probably, the main reason of this difference may be related to giving different orientations in the selection tasks used in pre-interviews, which might lead them enacting to concentrate on prototypical phenomenon.

The most crucial development in prospective teachers' knowledge in content and students throughout teaching experiment process was that they recognized the influence of students' inflexible prototypical concept images and exclusive definitions/constructions of quadrilaterals on the students' classification abilities. As a result, in their lesson plans and group discussions, they decided to ask students to construct figures in different size and orientations by emphasizing hierarchical relations among quadrilaterals in order to from a strong basis for the hierarchical classifications of quadrilaterals.

PCK on properties of quadrilaterals. The results of pre-interviews revealed that prospective teachers' knowledge on common conceptions and difficulties that elementary school students might have was insufficient in terms of anticipating students' possible errors and difficulties regarding properties of quadrilaterals and their possible reasons. However, some of them (n=2) predicted that students can know side and angle properties of quadrilaterals better than diagonal properties. In this regard, it can be concluded that prospective teachers' insufficient subject matter knowledge on especially diagonal properties or inadequate experience with middle school students can be evaluated as a barrier to their pedagogical content knowledge in terms of anticipating students' possible conceptions related to properties of quadrilaterals. It is also important to imply that many of studies revealed that learners struggle with in many of basic geometric concepts such as perpendicularity (Clements, Swaminathan, Hannibal, & Sarama, 1999), altitude of triangle (Gutierrez & Jaime, 1999), angle (Duatepe-Paksu, 2004; Matos, 1999; Mitchelmore & White; 2000; Prescott, Mitchelmore, & White, 2002; Scally, 1991; Ubuz, 1999). Unfortunately, prospective teachers in the current study could not consider the possible influence of the lack of students' knowledge about basic sub-geometric concepts such as diagonal, parallel and perpendicular line segments on their conceptions of quadrilaterals in the pre-interviews and initial lesson plans.

In the analysis and discussion process of micro-case video clips, prospective teachers found the events reflecting students' knowledge of properties of quadrilaterals in the clips as unexpected situations because they never predicted and encountered with such kinds of students' conception before participating to the teaching experiment in the current study. As a result, in the first glance, they noticed student's misconceptions about properties (e.g. student in MCVC3 proposed the congruence of only one pair of opposite angles of a parallelogram). Results indicated that they began to seek possible reasons of student's misconception about angle or diagonal properties of quadrilaterals such as (i) mathematics teachers' teaching style, (ii) lack of students' knowledge about angle, diagonal, perpendicularity, and parallelism concepts, and (iii) insufficient abilities of angle and diagonal constructions. As a result, it is clearly seen that analyzing students' video clips is an effective method to enhance teachers' skills and knowledge for comprehending and interpreting students' various thinking styles and their possible reasons (Ball, 1997).

Pre-interview results also showed that prospective teachers used various strategies in terms of teaching style, using representations, and question types in order to teach or to summarize properties of quadrilaterals at the end of their initial lesson plans. However, they generally listed all properties of a concept rather than mentioning and emphasizing critical properties in their instructional plans. This situation is probably related to their awareness of the importance of necessary and sufficient conditions in establishing mathematically correct definitions. As a crucial point, some prospective teachers' (n=3) initial instructional plans did not include any activity or explanations about properties of quadrilaterals. Instead, they generally concentrated on hierarchical relations of quadrilaterals. Besides, in the initial lesson plans, some misinformation about diagonal properties was detected in some initial lesson plans that were prepared by prospective teachers having misconceptions about diagonal properties. On the other hand, pre interviews and initial lesson plans also indicated that prospective teachers did not give any information aiming to assess students' prior knowledge on prerequisite geometric concepts such as angle, perpendicularity/ parallelism of line segments, constructing congruent angles etc. that are necessary to understand properties of quadrilaterals. This situation can be evaluated as an indicator of the robust relationship between teachers' knowledge on students' thinking and knowledge on instructional approaches. In other words, knowledge of content and student (KCS) and knowledge of content and teaching (KCT) can be seen as an amalgam that shapes pedagogical content knowledge in teachers professional knowledge context (Ball et al., 2008; Tsamir, Tirosh, Levenson, Tabach, & Barkai, 2014).

Results also indicated that after prospective teachers analyzed students' thinking in micro case video clips and discussed the possible reasons of students' misconceptions and errors, they began to propose following alternative solution ways to overcome the student's errors: (i) utilizing manipulatives such as using "drinking straws", (ii) cutting paper activities, using protractor to measure angles of quadrilaterals; and (iii) making emphasize on pre-geometric concepts (e.g. parallelism); using the relationship between the sum of interior angles of triangle and quadrilateral during and after group discussions. In this regard, their revised lesson plans also revealed that they added new objectives and activities focusing on pre-geometric concepts by using grid paper, Geogebra or geoboard. These results have similarities with the results of some video case-based studies conducted with prospective mathematics teachers' development in pedagogical content knowledge regarding particular mathematical concepts (Ding & Dominguez, 2015; Sherin & van Es, 2009). In other words, teachers' learning to interpret students' thinking in a video-based professional development program might extend to their teaching strategies. From this perspective, learning how to use student mathematical understanding helped mathematics teachers to change their instructional decisions in a way that aim to enhance students understanding and to prevent possible misconceptions (Franke, Carpenter, Levi, & Fennema, 2001; Goldsmith & Seago, 2011; Jacobs et al., 2010; Kazemi & Franke 2004; van Es, 2011).

5.3 Implications and Suggestions

In the light of the obtained results, I proposed some implications and suggestions for mathematics teacher educators, curriculum developers, and researchers who want to conduct a study involving micro-case video-based professional development programs in both prospective and in-service teacher education. In this sense, these implications and suggestions might shed light on educators' perspective in terms of filling gaps in the field of mathematics teacher education program in order to support teachers' professional development.

First implication of the findings is related to the use of micro-case video clips in professional development programs. The results of many studies indicated that preservice teachers do not have enough knowledge about students' mathematical thinking, the possible reasons of students' errors, and generating various mathematical strategies and justifications to the problems (Ball et al., 2008; Philipp, 2008; Ubuz & Yayan, 2010; Zembat, 2007). This situation creates a necessity to search ways in which teachers have opportunities to develop their SMK and PCK. To respond this need, the uses video-based professional development programs in prospective teacher education have been increased in recent years. Accordingly, in the current study, it has been shown that using micro-case video clips rather than focusing on classroom videos are very useful and effective way to enhance prospective teachers' professional vision before they become in-service teachers. For this reason, the findings of the study have implications on how to prepare prospective teachers. The courses given in the universities typically provide prospective teachers with immediate access to the student mathematical thinking in more detail (Lowery, 2002; Philipp, 2008; Philipp et al., 2007). Within the design of this video-based study, prospective teachers had many of opportunities to possess a body of rich knowledge about student mathematical thinking via analyzing and discussing micro-case videos (MCVCs). By the help of analysis and discussion of MCVCs and their peers' ideas, they were able to compare and contrast their own conceptions and students' mathematical conceptions. Consequently, it is suggested that designing a teaching environment that concentrates on student mathematical thinking can be used an effective alternative way in the courses at the universities in order to promote PSTs' knowledge related to various mathematics concepts.

In the pre-interviews, I realized that prospective teachers underestimate students' mathematics and they supposed that the most of students can correctly answer the questions related to quadrilaterals. At the beginning of teaching experiment, they evaluated correct answers as evidence for conceptual understanding and incorrect answers as the lack of conceptual understanding as mentioned in Clements and Sarama's (2014) study. However, in reality, studies revealed that students have many of misconceptions, difficulties in quadrilaterals when defining, constructing, and making classifications of quadrilaterals. As they restructure their knowledge on student thinking they became more realistic, analytic and reflective when interpreting student thinking and developing alternative instructional solutions. As an implication, efficient use of micro-case video clips in teacher education help prospective teachers to set realistic mathematical learning goals for their students (Clements & Sarama, 2014) because participating such video-based teacher education program give opportunities them to avoid judgmental discourse before becoming an in-service teacher (Ball & Chazan, 1994; Philipp, 2008).

Another implication is related to the crucial role of socially constructed learning environment on providing fruitful learning opportunity to prospective teachers. Sharing knowledge with the peers elaborated their knowledge by developing subject-related knowledge, obtaining alternative perspectives, understanding the reasons of students' errors, anticipating students' another incorrect answers, and suggesting possible solutions strategies. In this regard, I suggest that researchers can utilize socially constructed learning environment in teacher education programs because learning in a social environment entails enabling learners to develop, contrast and compare, and discuss different perspectives, arguments, and conjectures on the issue (Bednar, Cunningham, Duffy, & Perry, 1992) and they reached a shared understanding (Cobb,1994). For example, at the beginning of each week of mathematics teaching course, prospective teachers can be asked to individually examine selected MCVCs by taking notes. Then, they can start to discuss the video within a group. After that, they developed an instructional way to develop students' mathematical thinking in the video. Thus, developments in prospective teachers' knowledge on students' mathematical thinking can be analyzed throughout a semester.

In the study, I observed that revising lesson plans after each video analysis and discussions give chance them reconsider and modify their instructional decisions in the initial lesson plans. Prospective teachers came to understand the importance of student mathematical thinking in the lesson plan revisions and they enhanced their knowledge about different teaching approaches, representations, and materials considering students' needs and conceptions. In this regard, it might be useful to combine video-based approaches and lesson study approaches in order to facilitate teachers' professional development considering the influence of lesson study activities in PSTs' learning (Murata & Pothen, 2011).

I want to share one another implication for the researchers who want to make large-scale projects about the use of video case-based pedagogy in prospective teacher education. It is evident from the results that using micro-case video clips is useful in terms of supporting teacher professional development. In this sense, I think that a big research team can set "*online case libraries*" involving both classroom videos and specially designed micro-case video clips for each mathematics domain. Thus, all researchers and teachers can utilize valid and effective micro-case video clips to enhance teachers' knowledge in their course designs whenever they want. Furthermore, I propose the necessity of "*video case-based curriculum*" in teacher education as another implication of the current study. For instance, Stockero (2008) reported on the effectiveness of using a video-case curriculum where PSTs in middle school mathematics method course viewed, analyzed, and discussed video clips of students solving mathematical tasks. In the current study, utilized video-based instructional model can be seen as an example of "learning and teaching quadrilaterals curriculum (LTQC)". By this way, video-based curriculums can be used as a learning tool in method courses of universities teacher education programs because they help the development of reflective stance of prospective teachers.

Finally, as an implication, I propose a methodological strategy to the researchers who plan to conduct a study in micro-case video-based learning environment with prospective teachers or inservice teachers. As an important point, conducting pre-interviews with prospective teachers before preparing micro-case video clips were a helpful and useful when selecting "*mathematically unexpected situations*" related to students' mathematical thinking in raw video data. Why is the selection of "*mathematically unexpected situations*" important for the developments of PSTs' SMK and PCK? Researchers stated that exploiting unexpected situations for teachers' professional development may be helpful to support teachers in organizing mathematics lessons in ways that enable them to providing flexible and productive responses to the unexpected situation in future instructions (Chick & Stacey, 2013; Foster, 2014; Rowland & Zazkis, 2003; Sawyer, 2004). Accordingly, Brookfield (2006, pp. xi–xii) believed that teaching is "full of unexpected events, un-looked-for surprises, and unanticipated twists and turns". Furthermore, Rowland and Zazkis (2013) grouped the inservice teacher's response to unexpected situations from students into three categories: to ignore, to acknowledge but put aside, and to acknowledge and incorporate. From this perspective, reaching a high degree of pedagogical content knowledge is so important because prospective teachers' developing knowledge can provide teachers both acknowledgment and incorporation in terms of responding students' mathematically unexpected queries in their future vocational practices (Sawyer, 2004). Consequently, in this research, detecting prospective teachers' existing knowledge on particular related mathematics concepts before they encountered the micro-case video clips can be considered as an effective approach both to understand in which degree each learner's knowledge develops throughout a case-based professional development program and to select/organize micro-case video clips considering PSTs' existing knowledge and needs.

5.3.1 Suggestions for future studies

In the current study, the main concern is the development of prospective teachers' knowledge on quadrilaterals in video case-based professional development context. However, I suggest mathematics educators to examine prospective teachers' developmental process on different mathematical subjects. Quadrilaterals have been examined by various international and local studies. This situation provided a great convenience in terms of exploiting students' conceptions, errors, and misconceptions about quadrilaterals in order to produce effective micro-case video clips. Examining teachers' knowledge development in video case-based professional development programs also gives opportunities to the researchers to understand how they developed their SMK when analyzing student mathematical thinking. In this study, explicit evidences that show developments in SMK have rarely seen in individual video analysis and group discussions processes. Conversely, post-interview results indicated prospective teachers SMK related to quadrilaterals also enhanced at the end of the teaching experiment process. In other context such as statistical reasoning, or covariational reasoning, developments in SMK can be detected more easily due to prospective teachers' mathematical difficulties in concepts related to statistics and covariation. In this regard, I recommend that different groups of prospective teachers' SMK development can be investigated and compared in different mathematical conceptual domains via micro-case video-based development programs.

In this study, I examined eight prospective middle school mathematics teachers developmental process in an elective course. Similar studies can be conducted in classroom environment with all prospective teachers (e.g. Ding & Dominguez, 2015; Jacob et al., 2010). At this situation, it is possible to observe the nature of all prospective teachers' professional noticing abilities in a more realistic environment. The results might indicate how micro-case video clip usages support prospective teachers' SMK and PCK throughout a semester in classroom environment. Another important issue is that I suggested that researchers must examine teachers' knowledge development in different ways by using experimental methods, teaching experiment method or aptitude treatment interaction in order to determine the best way for their professional development. For example, while students' written papers can be given one group, micro-case video clips can be given another group. Furthermore, their results can be compared the results of control group.

Instead of focusing only prospective teachers' knowledge development, researchers may investigate teachers' professional development and noticing abilities in a video club context involving both prospective teachers and inservice teachers. This situation might different contributions to prospective teachers' perspectives in the following ways: (i) understanding expert teachers' perspectives when examining student's understanding, (ii) expanding alternative pedagogical strategies in terms of effectiveness and cognitive aspect, (iii) developing a realistic view to the teaching of a concept rather than approaching in an utopic way. In a study conducted by Hammerness, Darling-Hammond and Bransfor (2005), it is determined that the prospective teachers, intensively participating research activities in teacher training programs are feeling more prepared and being evaluated positively by their employers. Additionally, inservice teachers can be utilized social learning environment involving also prospective teachers in different ways. For example, if researchers select specially designed micro-case videos that reflect students' mathematical thinking in inservice teachers' classroom, inservice teachers can also notice many of noteworthy events related to their own students' mathematical thinking in more detail when analyzing the videos and discussing them with the prospective teachers. Consequently, it is suggested that inservice teachers and prospective teachers' interactional processes should be examined in order to determine influences of analyzing micro-case video clips on their knowledge and beliefs in a video club context.

Another suggestion for the future studies is related to micro-case video production. In the current study, I produced micro-case video clips involving students' mathematical thinking about quadrilaterals considering many of criteria. In the literature, the productivity of video clips in discussion process was examined and proposed selection criteria to determine productivity of clips (Linsenmeier & Sherin, 2009; Sherin, Linsenmeier & van Es, 2009). However, videos used in the literature generally involve classroom situations instead of involving a single student's mathematical thinking in a particular mathematics concept. In this regard, it can be useful finding well-structured effective ways to determine and to select efficiently the productivity of a micro-case video clip without consuming much time.

On the other hand, prospective teachers can be asked for producing micro-case video clips during a semester within the context of school experience. They also asked to determine noteworthy events in the clips that they captured. Thus, researchers have opportunity to observe how prospective teachers try to produce micro-case video clips, how they foster student thinking during video shooting, how they notice and determine noteworthy events in students' thinking in the clips, and how they interpret the critical points related to the student's conceptions throughout a semester. Such kind of research enables us to learn the developmental process of prospective teachers' abilities in task design that is able to foster student mathematical thinking, knowledge related to understanding students' thinking because it is quite important and beneficial for teachers to conduct research in vocational subjects/topics in terms of their professional development (Cochran-Smith, 2003). Accordingly, incorporating prospective teachers in inquiry-based learning processes is a remarkable issue in terms of their occupational improvement. The necessity of giving place to research based education mainly in undergraduate level is a subject emphasized from time to time in international level (Boyer Commisssion, 1998; Brew, 2010).

In my final future suggestion, I recommended to the use of the combination of learning trajectories (LTs) and teachers' noticing in video-based teacher education (Philipp, 2014) because it may give opportunity to PSTs in terms of understanding developmental paths in students' knowledge related to particular mathematics concepts.

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APPENDICES

Appendix 1

PRE-/POST-INTERVIEW TASKS

TASK 1 (DEFINITIONS OF QUADRILATERALS)

Aşağıda isimleri verilen dörtgenleri önce bildiğiniz biçimde, ardından yedinci sınıfta öğrenim gören bir öğrenciye ifade edecek biçimde tanımlayınız.

1) Paralelkenar : Kişisel tanım

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2) Paralelkenar: Öğretimsel tanım

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3) Eşkenar dörtgen: Kişisel tanım

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4) Eşkenar dörtgen: Öğretimsel tanım

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5) Yamuk: Kişisel tanım

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6) Yamuk: Öğretimsel tanım

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7) Sizce herhangi bir yedinci sınıf öğrencisi bu kavramları nasıl tarif edebilir? (Eğer hata ve zorluklara yönelik ifadeler gelirse nedeni nedir?)

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TASK 2

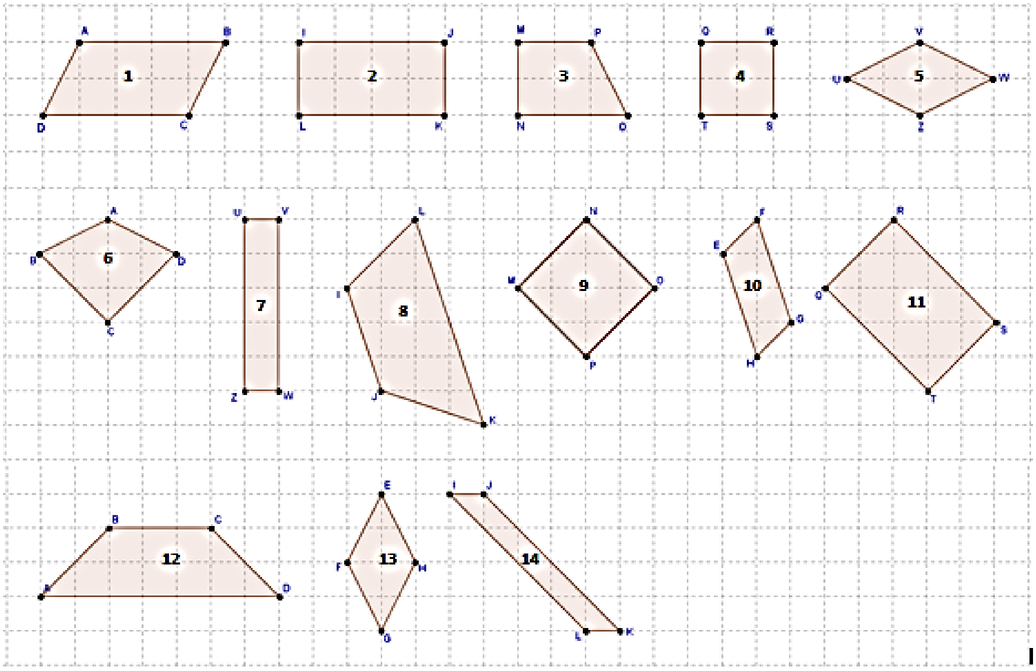
(PARALLELOGRAM TASK)

8) Aşağıdaki kareli kağıda en az üç farklı paralelkenar şekli çiziniz

9) Sizce bir yedinci sınıf öğrencisi paralelkenar şekline dair nasıl çizimler yapabilir? (Varsa öğrencilerin çizimlerindeki hata ve zorluklarının nedeni nedir?)

10) Siz öğrencilerinize paralelkenarı anlatırken nasıl çizimler yaparsınız?

11) Aşağıda paralelkenar olduğunu düşündüğünüz şekiller nelerdir?



Paralelkenar olanlar:

Paralelkenar olmayanlar:

12) Sizce bir yedinci sınıf öğrencisinin yukarıda verilen şekillerle ilgili yapacağı muhtemel seçimler nasıl olur? Neden bu tip seçimler yapmış olabilirler? (Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?)

13) Paralelkenarın bildiğiniz özelliklerini (kenar, açı, köşegen, simetri doğrularına göre) sıralayınız.

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14) Bir yedinci sınıf öğrencisinin paralelkenarın açı, kenar ve köşegen özellikleriyle ilgili kavrayışları nasıl olabilir? (Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?)

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TASK 3

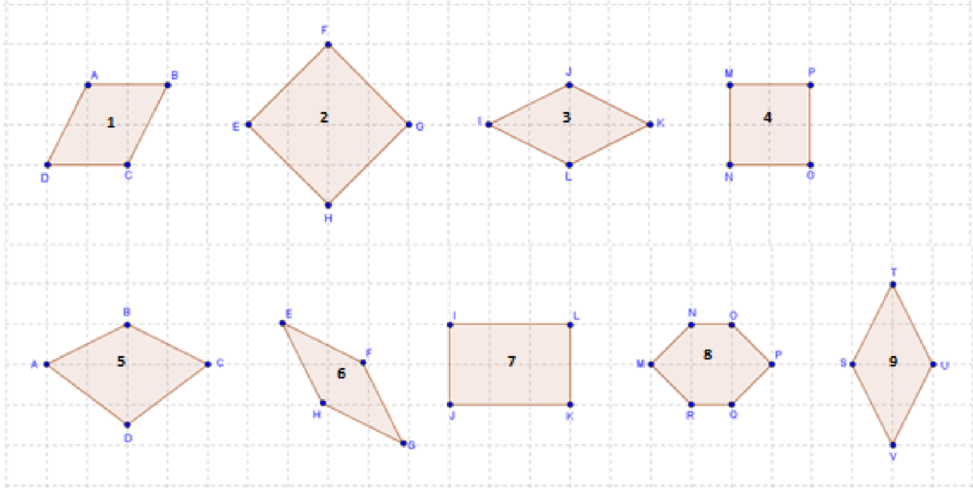
(RHOMBUS TASK)

15) Aşağıdaki kareli kağıda en az üç farklı eşkenar dörtgen şekli çiziniz.

16) Sizce bir yedinci sınıf öğrencisi eşkenar dörtgen şekline dair nasıl çizimler yapabilir? (Varsa öğrencilerin çizimlerindeki hata ve zorluklarının nedeni nedir?)

17) Siz öğrencilerinize eşkenar dörtgeni anlatırken nasıl çizimler yaparsınız?

18) Aşağıda eşkenar dörtgen olduğunu düşündüğünüz şekiller nelerdir?



Eşkenar dörtgen olanlar:
Eşkenar dörtgen olmayanlar:
Kararsızım:

Eşkenar dörtgen olanlar:

Eşkenar dörtgen olmayanlar:

19) Sizce bir yedinci sınıf öğrencisinin yukarıda verilen şekillerle ilgili yapacağı muhtemel seçimler nasıl olur? Neden bu tip seçimler yapmış olabilirler? Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?

20) Eşkenar dörtgenin bildiğiniz özelliklerini (kenar, açı, köşegen, simetri doğrularına göre) sıralayınız.

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21) Bir yedinci sınıf öğrencisinin eşkenar dörtgenin açı, kenar ve köşegen özellikleriyle ilgili kavrayışları nasıl olabilir? (Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?)

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TASK 4

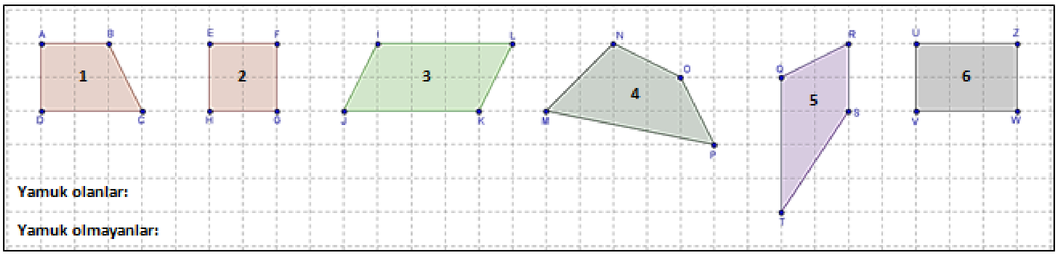
(TRAPEZOID TASK)

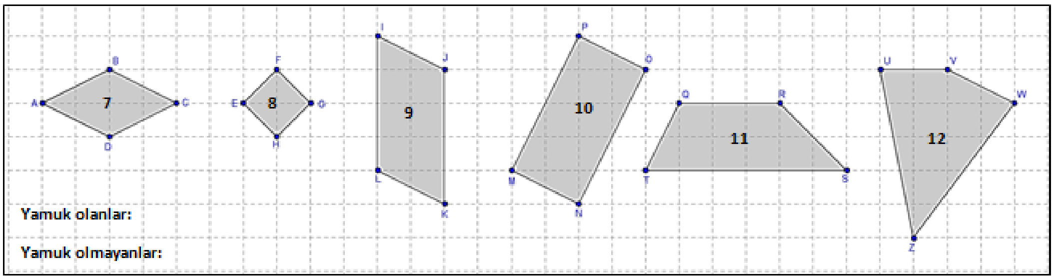
22) Aşağıdaki kareli kağıda en az üç farklı yamuk şekli çiziniz.

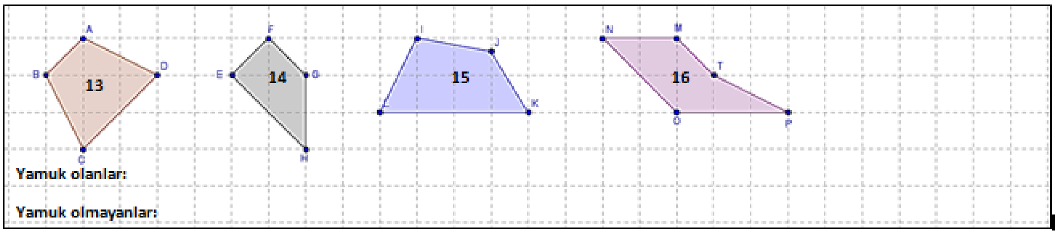
23) Sizce bir yedinci sınıf öğrencisi yamuk şekline dair nasıl çizimler yapabilir? (Varsa öğrencilerin çizimlerindeki hata ve zorluklarının nedeni nedir?)

24) Siz öğrencilerinize yamuğu anlatırken nasıl çizimler yaparsınız?

25) Aşağıda yamuk olduğunu düşündüğünüz şekiller nelerdir?







Yamuk olanlar:

Yamuk olmayanlar:

26) Sizce bir yedinci sınıf öğrencisinin yukarıda verilen şekillerle ilgili yapacağı muhtemel seçimler nasıl olur? Neden bu tip seçimler yapmış olabilirler? (Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?)

27) Yamuğun bildiğiniz özelliklerini (kenar, açı, köşegen, simetri doğrularına göre) sıralayınız.

28) Bir yedinci sınıf öğrencisinin yamuğun açı, kenar ve köşegen özellikleriyle ilgili kavrayışları nasıl olabilir? (Eğer öğrenciler hata ve zorluklar yaşarsa nedenleri nedir?)

29) Öğrencilerine dörtgenlerin özelliklerini nasıl öğretirsiniz ve öğretirken nelere dikkat etmeye çalışırsınız?

TASK 5
(CLASSIFICATION OF QUADRILATERALS)

30) Dörtgenleri Venn diyagramı kullanarak nasıl sınıflarsınız?

31) Dörtgenler arasındaki ilişikleri öğrencileriniz nasıl öğretmek istersiniz? Neden?

Appendix 2

SAMPLE INFORMED CONSENT FORM

GÖNÜLLÜ KATILIM FORMU

Sevgili katılımcı,

Ben Fadime ULUSOY. Orta Doğu Teknik Üniversitesi Eğitim Fakültesi, İlköğretim Bölümü'nde araştırma görevlisi olarak çalışıyorum. Aynı zamanda İlköğretim Fen ve Matematik Eğitimi Anabilim Dalı'nda devam ettiğim doktora eğitimimde tez aşamasına gelmiş bulunuyorum. Bu çalışmada, tez danışmanım Prof. Dr. Erdinç ÇAKIROĞLU ile birlikte yedinci sınıf öğrencilerinin video durumlarını kullanarak ilköğretim matematik öğretmen adaylarının dörtgenlerle ilgili konu alan bilgilerini ve pedagojik içerik bilgilerini araştırılmayı amaçlıyoruz.

Araştırmada verilerin elde edilmesi iki aşamadan oluşmaktadır. İlk olarak, dörtgenler ve özelliklerini içeren ve 25 sorudan oluşan Van-Hiele Geometrik Testi eksiksiz bir biçimde tamamlamaları için …. Okulunun yedinci sınıfında öğrenim gören öğrencilere verilecektir. Testin tamamlanması yaklaşık 35 dakika almaktadır. Testteki soruların cevaplanmasından sonra yedinci sınıfta öğrenim gören bu öğrenciler arasından seçilen kişilerle dörtgenler ve dörtgenlerin özellikleriyle ilgili sorular çözecektir. Bu soru çözümleri öğrencilerle sınıf ortamı dışında okulun uygun olan bir yerinde bireysel olarak ve video kaydı alınarak gerçekleştirilecektir. Bu bakımdan, derslere müdahale edilmesi söz konusu değildir. Araştırmaya katılım gönüllü olup, katılımcıların sonradan vazgeçmesi halinde herhangi olumsuz bir sonuç oluşmayacaktır. Araştırma sırasında toplanan veriler sadece araştırmacının bilgisi dâhilinde olup gerek diğer katılımcılar gerekse başka şahıslar tarafından bilinmeyecektir. Araştırma raporunda okul, katılımcı öğrenci ve öğretmenlerin ismi hiçbir şekilde aynen geçmeyecek, isim kullanılması gerekirse takma isim kullanılacaktır.

Araştırmamıza yönelik sorularınız olması durumunda benimle ve/veya tez danışmanımla iletişime geçebileceğiniz bilgiler aşağıdaki gibidir:

Araş. Gör. Fadime ULUSOY, Adres: ODTÜ, Eğitim Fakültesi, İlköğretim Bölümü, Oda No: EFA-39, ODTÜ/ ANKARA 06531; Telefon: +90 312 210 75 07, e-posta: bfadime@metu.edu.tr

Prof. Dr. Erdinç ÇAKIROĞLU, Adres: ODTÜ, Eğitim Fakültesi, İlköğretim Bölümü, Oda No: 113, ODTÜ/ ANKARA 06531; Telefon: +90 312 210 40 90, e-posta: erdinc@metu.edu.tr

Bu çalışmaya gönüllü olarak katılmayı kabul ediyorsanız, lütfen aşağıda belirtilen yere isminizi ve tarihi yazarak imzalayınız.

Katılımınız için teşekkür ederim.

Ad-Soyad:____________________________ **İmza:**

Tarih:

Appendix 3

QUESTIONS IN CASE PRODUCTION INTERVIEW-2

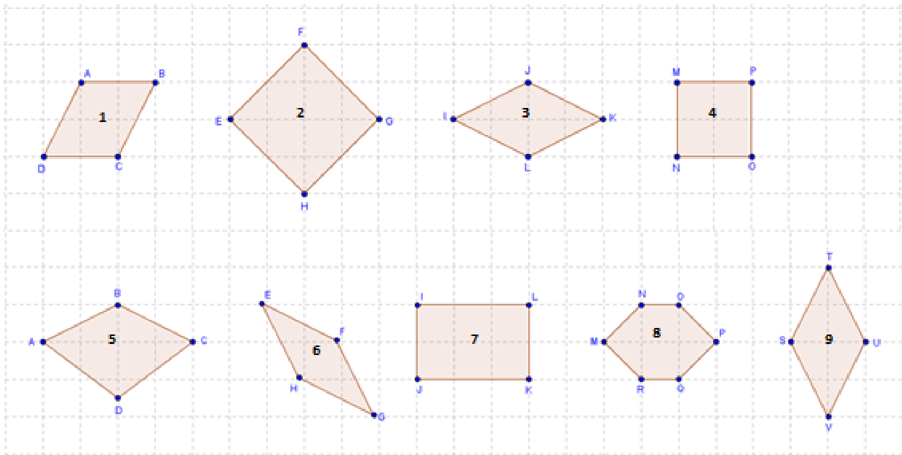
RHOMBUS QUESTIONS

- Eşkenar dörtgen denilince aklına ilk ne geliyor? Bir arkadaşınıza eşkenar dörtgeni nasıl tarif edersiniz?

- Aşağıdaki kareli kağıda eşkenar dörtgen şekilleri çiziniz.

- Eşkenar dörtgenin **kenarlarıyla ilgili bildiğiniz özellikleri** aşağıya yazınız.

- Aşağıdaki şekillerin hangilerinin bir eşkenar dörtgen örneği olup olmadığını belirleyiniz.



Eşkenar dörtgen olanlar:
Eşkenar dörtgen olmayanlar:
Kararsızım:

- Aşağıda verilen kareli kağıda bir eşkenar dörtgen ve bu eşkenar dörtgene ait köşegenleri çiziniz.

- Eşkenar dörtgenin **açılarıyla ve köşegenleriyle ilgili bildiğiniz tüm özellikleri** aşağıya yazınız.

- **Alara**: Kare bir eşkenar dörtgendir.
- **Fatih**: Kare bir eşkenar dörtgen olamaz.

  Siz yukarıdaki görüşlerden hangisine katılıyorsunuz? Düşüncenizi açıklamak için aşağıdaki kareli kağıda çizimler yapabilirsiniz.

- **Zeynep**: Eşkenar dörtgende köşegenlerin birbiriyle dik kesişmesine gerek yoktur.
- **Hasan**: Köşegenleri dik kesişmeyen bir dörtgen asla eşkenar dörtgen olmaz.

  Siz yukarıdaki görüşlerden hangisine katılıyorsunuz? Düşüncenizi açıklamak için aşağıdaki kareli kağıda çizimler yapabilirsiniz.

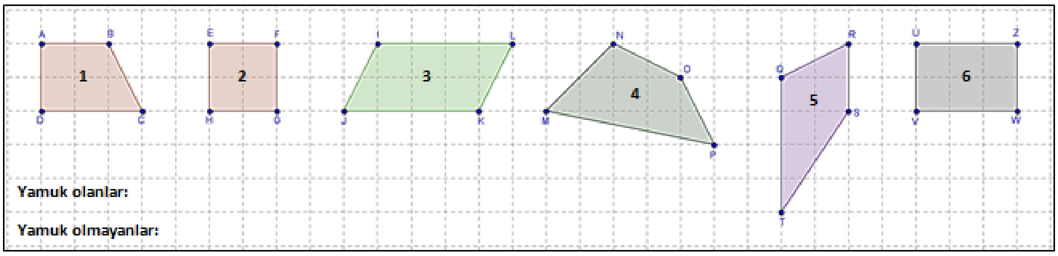
TRAPEZOID QUESTIONS

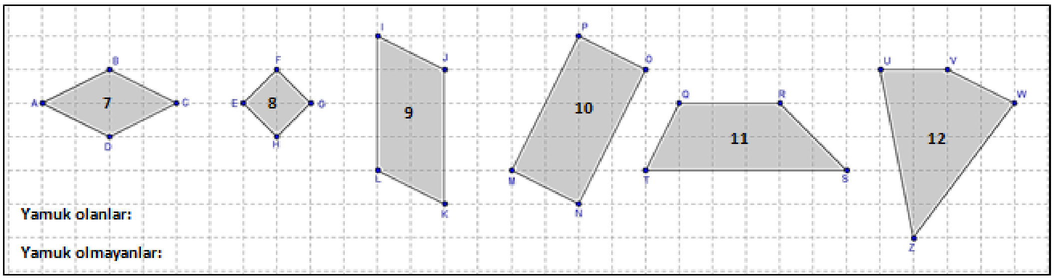
- Yamuk denilince aklına ilk ne geliyor? Bir arkadaşınıza yamuğu nasıl tarif edersiniz?

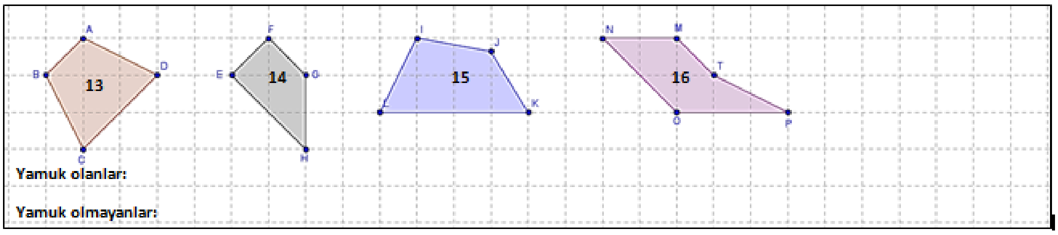
- Aşağıdaki kareli kağıda yamuk şekilleri çiziniz.

- Yamuğun **kenarlarıyla ilgili bildiğiniz özellikleri** aşağıya yazınız.

- Aşağıdaki şekillerin hangilerinin bir yamuk örneği olup olmadığını belirleyiniz.







- Aşağıda verilen kareli kağıda bir yamuk ve bu yamuğa ait köşegenleri çiziniz.

- Yamuğun **açılarıyla ve köşegenleriyle ilgili bildiğiniz tüm özellikleri** aşağıya yazınız.

Appendix 4

AN EXAMPLE OF FIRST ARCHIVING APPROACH

Table 26. The archive of S13's first case production interview

In	Out	Topic	Descriptive notes on student thinking	Reflective notes on student thinking
00:00	03:41	Construction of equal length LSs	Incorrect construction	Difficulty in equal length non-prototypical LSs
03:41	07:28	Construction of parallel LSs	Incorrect and limited constructions	Inability to construct non-prototypical parallel LSs Treating non equal length parallel LSs as non-examples
07:28	10:30	Definition of Par	Incorrect description	Improper mathematical language
10:30	17:45	Construction of Par	Rectangles examples and hexagon	Difficulty in construction of inclined sides of prototypical Par Lack of knowledge about the number of sides of a Par
17:45	22:05	Identification (selection) of Par	Correct selections of parallelograms	Student easily identify parallelograms among various polygons
22:05	22:39	Side properties of Par	Correct responses	Student knew parallelism and equality of length of sides in a parallelogram
22:39	28:36	Construction of perpendicular LSs	Both correct and incorrect constructions	Student had difficulty to draw non-prototypical perpendicular LSs examples
28:36	36:03	Construction of congruent angles	Incorrect constructions	Inability to construct congruent angles in non-prototypical position Student thought rays of an angle must have equal length
36:03	42:11	Construction of diagonals of Par	Lack of knowledge about diagonals	Student thought that diagonal and corner are same thing in a Par.
42:11	53:36	Angle and diagonal properties of Par	Inadequate knowledge on angle property	Student claimed that only one pair of opposite is congruent for any Par. Rote learning on the sum of interior angles of Par Calculations showed that student had inadequate knowledge on decimals
53:36	59:47	Hierarchical relations of quadrilaterals	Correct responses	Student was able to construct relations among square, rhombus and Par

*Abbreviations means that LSs: Line segments; Par: Parallelogram

Appendix 5

AN EXAMPLE OF SECOND ARCHIVING APPROACH

Table 27. The archive of students' parallelogram definition data in 1st interview

In	Out	Student	Student's written description of parallelogram (Turkish version)	Reflective notes on students' thinking
8:27	10:54	S1	Uzunlukları ve doğrultuları birbirine eşit olan karşılıklı kenarlardan oluşan geometrik şekildir. Kenar sayısı önemli değildir.	Incorrect Overgeneralization error
7:25	9:00	S2	Karşılıklı uzunlukları birbirine eşit, karşılıklı kenarları birbirine paralel, açıları dik değil karşılıklı olanları birbirine eşittir.	Listing of properties Undergeneralization error
12:30	15:47	S3	Paralelkenar birbirine paralel iki kenarın oluşturduğu bir şekildir. Örneğin bir dikdörtgen	Inadequate description Hierarchical understanding
8:20	11:10	S4	Paralel birbirine eşit uzunlukta doğru parçalarının birbirini alt ve üst biçimde gelmesiyle oluşur.	Incorrect Inadequate description Overgeneralization error
5:33	7:20	S5	Dikdörtgene benzeyen bir şekildir. Aynı uzaklıkta ve açıları aynı olan bir şekildir.	Visual thinking when defining Undergeneralization error
6:28	8:20	S6	Paralelkenar aynı orantıda iki doğru parçasının bir nokta belirlenip o nokta üzerinde ilerlemesidir.	Incorrect description Inappropriate language usage
7:03	8:14	S7	Karşılıklı kenarları birbirine paralel ve eşit olan dört kenarlı bir şekildir.	Inadequate description. PSTs predicted such a definition
6:26	7:25	S8	Benim aklıma kare veya dikdörtgenin iki çapraz uçlarından bastırılmış bir şekil geliyor.	Visual thinking when defining Undergeneralization error
5:45	7:00	S9	Karşı karşıya denk gelen çizgilerdir.	Incorrect description Overgeneralization error
6:00	6:40	S10	Kare aynı yönde doğru parçaları	Inappropriate language usage and incorrect description
6:40	8:35	S11	Paralelkenar bir dörtgendir karşılıklı kenarları birbirine eşittir.	Inadequate description. Student focused on equality of length instead of parallelism
5:17	7:40	S12	Dörtkenarı bulunan karşılıklı iki kenarı birbiriyle kesişmeyen çokgen türüdür.	Inappropriate description Overgeneralization error
7:28	10:30	S13	Paralelkenarın boyutlarının ve uzunluklarının eşit olması ve aynı hizada olması aklıma gelir.	Incorrect description Inappropriate mathematical language usage
6:10	9:02	S14	Paralelkenar iki doğru parçasının aynı doğrultuda olan doğru parçalarının kenarlarının birleşimidir	Incorrect and inappropriate description
6:14	7:50	S15	Paralelkenar kenarları birbiriyle kesişmeyen bir şekildir ve bu şekil dört kenarlıdır	Inadequate description
7:34	8:27	S16	No description	Student constructed figure

Appendix 6

APPROVAL OF THE ETHICS COMMITE OF METU RESEARCH CENTER FOR APPLIED ETHICS

UYGULAMALI ETİK ARAŞTIRMA MERKEZİ
APPLIED ETHICS RESEARCH CENTER

ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

DUMLUPINAR BULVARI 06800
ÇANKAYA ANKARA/TURKEY
T: +90 312 210 22 91
F: +90 312 210 79 59
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www.ueam.metu.edu.tr

Sayı: 28620816/75 - 226

06 Mart 2013

Gönderilen: Doç.Dr.Erdinç Çakıroğlu
İlköğretim Bölümü

Gönderen : Prof. Dr. Canan Özgen
IAK Başkan Yardımcısı

İlgi : Etik Onayı

Danışmanlığını yapmış olduğunuz İlköğretim Bölümü Doktora öğrencisi Fadime Ulusoy'un "Yedinci ve Sekizinci Sınıf öğrencilerinin Video Durumlarını Kullanarak İlköğretim Matematik Öğretmen Adaylarının Dörtgenlerle İlgili Konu Alan Bilgilerinin ve Pedagojik İçerik Bilgilerinin Araştırılması" isimli araştırması "İnsan Araştırmaları Komitesi" tarafından uygun görülerek gerekli onay verilmiştir.

Bilgilerinize saygılarımla sunarım.

Etik Komite Onayı

Uygundur

06/03/2013

Prof.Dr. Canan ÖZGEN
Uygulamalı Etik Araştırma Merkezi
(UEAM) Başkanı
ODTÜ 06531 ANKARA

Appendix 7

T.C.
ETİMESGUT KAYMAKAMLIĞI
İlçe Milli Eğitim Müdürlüğü

Şubesi :Strateji
Sayı :29378010.605.99- 680 ...../04/2013
Konu :Araştırma İzni
(Fadime ULUSOY)

İLGİLİ OKUL MÜDÜRLÜKLERİNE

İlgi :a)Ankara Valiliği İl M.E.Müd.nün 12/04/2013 tarih ve 605.99-583601 sayılı yazısı
b)Meb Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu genelgesi

ODTÜ İlköğretim Ana Bilim Dalı Doktora Programı Öğrencisi Fadime ULUSOY'un "7. ve 8. Sınıf Öğrencilerinin Video Durumlarını Kullanarak İlköğretim Matematik Öğretmen Adaylarının Dörtgenlerle İlgili Konu Alan Bilgilerinin Araştırılması" konulu tez önerisi kapsamında video kaydı ve uygulama yapma isteğinin uygun görüldüğüne ilişkin ilgi yazı ve ekinde alınan liste yazımız ekinde gönderilmiştir.

Anketler (13 sayfa) araştırmacıya ulaştırılmış olup, uygulama yapılacak sayıda araştırmacı tarafından çoğaltılarak araştırmanın ilgi (b) genelge çerçevesinde, okul ve kurum yöneticileri uygun gördüğü takdirde gönüllülük esasına göre uygulanmasını önemle rica ederim.

Ünsal ÇOLAK
İlçe Milli Eğitim Müdürü a.
Şube Müdürü

Ekler:1-Yazı ve Liste

Dağıtım:
-Toki Göksu Ortaokulu
-Kooperatifler Birliği Ortaokulu
-Hasan Ali Yücel Ortaokulu
-Şehit Rıfat Çelik İlkokulu
-Şehit Abdülkadir Yüzbaşıoğlu Ortaokulu
-Türkkonut Emel Önal İlkokulu

T.C.
ANKARA VALİLİĞİ
Milli Eğitim Müdürlüğü

Sayı : 14588481/605.99/583601 — 12/04/2013
Konu: Araştırma İzni
(Fadime ULUSOY)

Etimesgut İLÇE MİLLİ EĞİTİM MÜDÜRLÜĞÜNE

İlgi : a) Meb Yenilik ve Eğitim Teknolojileri Genel Müdürlüğünün 2012/13 nolu genelgesi
b) ODTÜ'nün 01/04/2013 tarih ve 1721 sayılı yazısı.

ODTÜ İlköğretim Ana Bilim Dalı Doktora Programı öğrencisi Fadime ULUSOY'un "7. ve 8. Sınıf Öğrencilerinin Video Durumlarını Kullanarak İlköğretim Matematik Öğretmen Adaylarının Dörtgenlerle İlgili Konu Alan Bilgilerinin Araştırılması" konulu tez önerisi kapsamında ilçeniz okullarında video kayda ve uygulama yapma isteği Müdürlüğümüz Değerlendirme Komisyonunca uygun görülmüştür.

Anketler (13 sayfa) araştırmacıya ulaştırılmış olup, uygulama yapılacak sayıda araştırmacı tarafından çoğaltılarak, araştırmanın ilgi (a) genelge çerçevesinde, okul ve kurum yöneticileri uygun gördüğü takdirde gönüllülük esasına göre uygulanmasını rica ederim.

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Şube Müdürü

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Okul Listesi (1 sayfa)

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Appendix 8

PARENT APPROVAL LETTER

(VELİ ONAY MEKTUBU)

Sayın Veliler,

Orta Doğu Teknik Üniversitesi İlköğretim Bölümünde doktora tezi kapsamında "yedinci sınıf öğrencilerin video durumlarının kullanarak matematik öğretmen adaylarının dörtgenlerin tanımları ve özellikleriyle ilgili matematik konu alan bilgilerini ve pedagojik konu alan bilgilerini inceleme" isimli çalışmayı yürütmekteyiz. Araştırmamızın amacı öncelikle yedinci sınıf öğrencilerinin dörtgenler konusundaki bilgilerini ölçmektir. Bu amacı gerçekleştirebilmek için çocuklarınızın bazı sorulara cevap vermesine ihtiyaç duymaktayız.

Katılmasına izin verdiğiniz takdirde çocuğunuz soruları okulda ders saati dışında cevaplayacaktır. Çocuğunuzun cevaplayacağı soruların onun psikolojik gelişimine olumsuz etkisi olmayacağından emin olabilirsiniz. Çocuğunuzun dolduracağı anketlerde cevaplarınız kesinlikle gizli tutulacak ve bu cevaplar sadece bilimsel araştırma amacıyla kullanılacaktır. Bu formu imzaladıktan sonra çocuğunuz araştırmaya katılmaktan ayrılma hakkına sahiptir. Araştırma sonuçlarının özeti tarafımızdan okula ulaştırılacaktır. Araştırmayla ilgili sorularınızı aşağıdaki e-posta adresini veya telefon numarasını kullanarak bize yöneltebilirsiniz.

Saygılarımızla,

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Lütfen bu araştırmaya çocuğunuzun katılım durumunu aşağıdaki seçeneklerden size <u>en uygun gelenin</u> altına imzanızı atarak belirtiniz ve bu formu <u>çocuğunuzla okula geri gönderiniz</u>.

A) Bu araştırmaya tamamen gönüllü olarak çocuğum ....'nın katılımcı olmasına izin veriyorum. Çalışmayı istediğim zaman yarıda kesip bırakabileceğimi biliyorum ve verdiğim bilgilerin bilimsel amaçlı olarak kullanılmasını kabul ediyorum.

Velinin Adı-Soyadı: ....

İmza: ....

B) Bu çalışmada çocuğum ....'nın katılımcı olmasına izin vermiyorum.

Velinin Adı-Soyadı: ....

İmza: ....

Appendix 9

UNDERGRADUATE CURRICULUM FOR ELEMENTARY MATHEMATICS EDUCATION PROGRAM

First Year

First Semester	Second Semester
MATH111 FUNDAMENTALS OF MATHEMATICS	MATH112 DISCRETE MATHEMATICS
MATH115 ANALYTIC GEOMETRY	MATH116 BASIC ALGEBRAIC STRUCTURES
MATH117 CALCULUS I	MATH118 CALCULUS II
EDS200 INTRODUCTION TO EDUCATION	CEIT100 COMPUTER APPLICATIONS IN EDUCATION
ENG101 ENGLISH FOR ACADEMIC PURPOSES I	ENG102 ENGLISH FOR ACADEMIC PURPOSES II
IS100 INTRODUCTION TO INFORMATION TECHNOLOGIES AND APPLICATIONS	

Second Year

Third Semester	Fourth Semester
PHYS181 BASIC PHYSICS I	PHYS182 BASIC PHYSICS II
MATH219 INTRODUCTION TO DIFFERENTIAL EQUATIONS	MATH201 ELEMENTARY GEOMETRY
STAT201 INTRODUCTION TO PROBABILITY & STATISTICS I	STAT202 INTRODUCTION TO PROBABILITY &STATISTICS II
ELE221 INSTRUCTIONAL PRINCIPLES AND METHODS	ELE225 MEASUREMENT AND ASSESSMENT
EDS220 EDUCATIONAL PSYCHOLOGY	ENG211 ACADEMIC ORAL PRESENTATION SKILLS
HIST2201 PRINCIPLES OF KEMAL ATATURK I	HIST2202 PRINCIPLES OF KEMAL ATATÜRK II
HIST2205 HISTORY OF THE TURKISH REVOLUTION I	HIST2206 HISTORY OF THE TURKISH REVOLUTION II

Third Year

Fifth Semester	Sixth Semester
MATH260 BASIC LINEAR ALGEBRA	ELE310 COMMUNITY SERVICE
ELE341 METHODS OF TEACHING MATHEMATICS I	ELE329 INSTRUCTIONAL TECHNOLOGY AND MATERIAL DEVELOPMENT
TURK201 ELEMENTARY TURKISH	ELE342 METHODS OF TEACHING MATHEMATICS II
TURK305 ORAL COMMUNICATION	EDS304 CLASSROOM MANAGEMENT
ELECTIVE	TURK202 INTERMEDIATE TURKISH
ELECTIVE	TURK306 WRITTEN EXPRESSION
	ELECTIVE

Fourth Year

Seventh Semester	Eight Semester
ELE301 RESEARCH METHODS	ELE420 PRACTICE TEACHING IN ELEMENTARY EDUCATION
ELE435 SCHOOL EXPERIENCE	EDS416 TURKISH EDUCATIONAL SYSTEM AND SCHOOL MANAGEMENT
ELE465 NATURE OF MATHEMATICAL KNOWLEDGE FOR TEACHING	EDS424 GUIDANCE
ELECTIVE	ELECTIVE
ELECTIVE	

Appendix 10

TURKISH SUMMARY

1 Giriş ve Gerekçe

Matematik öğretimi, çok iyi yapılandırılmış konu alan bilgisi, öğrencilerin matematiği nasıl öğrendikleriyle ilgili derinlemesine bir bilgi birikimi ve bir kavramın öğrencilerin ihtiyaçlarına cevap verecek şekilde öğretimine yönelik pedagojik yaklaşımlarla ilgili zengin bir bilgi yeterliği gerektirmektedir (Ball ve McDiarmid 1990; Fauskanger, 2015; Harrington, 1999). Bu nedenle matematik öğretimi, donanımlı öğretmenler yetiştirebilmek için üzerinde çok çalışılması gereken oldukça karmaşık ve çok boyutlu bir alandır. Bu bağlamda, ilgili alan yazın öğretmen yeterliklerinden biri olarak öğrenci sorularına işlemsel açıdan ziyade kavramsal açıdan cevap verilmesinin önemi üzerinde durmaktadır (Borko ve Putnam, 1996; Tchoshanov, 2011). Çünkü kavramsal bilgi, matematiksel temsillerin anlamını bilme ve bir algoritmanın neden belli bir şekilde çalıştığı gibi daha karmaşık alanlarda bilgi sahibi olmayı gerektirmektedir. Fakat yeterli kavramsal bilgiye sahip olmak öğrencilere iyi bir öğretim sunabilmenin garantisini tek başına veremez (Shulman, 1986). Bu noktada, öğretmenlerden bir konuda neyin bilinmesi gerektiği ile o konunun nasıl öğretilmesi gerektiği arasında sağlam bir ilişki kurmaları beklenmektedir (Davis ve Simmt, 2006; Mason ve Davis, 2013). Sonuç olarak, öğretmen bilgisinin çok yönlü yapısı, eğitimcileri öğretmenlerin sahip olduğu ya da sahip olmaları gereken bilginin doğasını anlama yönünde çalışmaya iten bir güç olmuştur (Ball ve diğ., 2008; Hill ve diğ., 2007). Öğretmen bilgisini anlamaya yönelik ilginin artmasıyla, farklı matematiksel kavramlarla ilgili öğretmenlerin sahip oldukları bilgi düzeyini inceleyen ulusal ve uluslararası platformda birçok çalışma yapılmıştır (e.g. Ball, 1990a, 1990b; Even, 1993; Işıksal ve Çakıroğlu, 2011; Hines ve McMahon, 2005; Ma, 1999; Tirosh, 2000; Toluk-Uçar, 2009). Bu çalışmaların sonuçları, öğretmenlerin hem matematiksel konu ile ilgili hem de o konuyu nasıl öğretecekleriyle ilgili yetersiz bilgiye sahip olduklarını ortaya çıkarmıştır. Bu bilgi eksikliklerin özellikle geometri kavramlarında daha fazla olduğu dikkat çekmektedir.

Matematik öğrenme alanları içinde geometri tüm ülkelerin öğretim programında önemli bir yere sahiptir (Common Core State Standards Initiative [CCSSI], 2010; Milli Eğitim Bakanlığı [MEB], 2013; National Council of Mathematics [NCTM], 2000). Çünkü geometri öğrenme alanı, görselleştirme ve üç boyutlu düşünme gibi önemli becerilerinin öğrencilere kazandırılması ve bu becerilerin geliştirilmesi bakımından kilit bir role sahiptir (Clements ve Battista, 1992; Mammana ve Villani, 1998). Bu bağlamda, iki boyutlu ve üç boyutlu şekillerin karakteristik özelliklerinin tanınması ve incelenmesi ile geometrik ilişkilerle ilgili matematiksel muhakemenin geliştirilmesinin taşıdığı öneme vurgu yapılmaktadır (MEB, 2013; NCTM, 2000). Yapılan bu vurgu göz önünde bulundurulduğunda, geometri öğrenme alanı içinde dörtgenler okul öncesi eğitimden ortaöğretim düzeyine kadar her kademede temel bir geometri konusu olarak kendini göstermektedir. Dörtgenlerin özelliklerini ve kritik bileşenlerini anlamak dörtgenler arasında hiyerarşik ilişkilerin kurulması gibi noktalarda kritik role sahiptir. Çünkü bu hiyerarşik ilişkilerin kurulması geometrik düşünmenin gelişimi, matematiksel argümantasyon yapabilme, çıkarım yapma ve ispatlama becerileri açısından gerekli ve önemlidir (Fujita, 2012; Fujita ve Jones, 2007). Konunun taşıdığı çok yönlü öneme dayanarak yapılan ulusal ve uluslararası çalışmalar maalesef birçok öğretmenin ve öğretmen adayının dörtgenleri doğru ve tam manada tanımlayamadıklarını ve sınıflayamadıkları göstermektedir (Akuysal, 2007; Currie ve Pegg, 1998; De Viller, 1994; Doğan ve diğ., 2012; Erez ve Yerushalmy, 2006; Monaghan, 2000; Okazaki ve Fujita, 2007). Özet olarak, yapılan çalışmaların sonuçları öğretmenlerin dörtgenlerle ilgili gereken konu olan bilgisine ve pedagojik bilgiye yeterince sahip olamadıklarını işaret etmektedir (Chinnappan ve diğ., 1996;

Fuys ve diğ., 1988; Hershkowitz ve Vinner, 1984; Leikin ve diğ., 2000; Mayberry, 1983; Swafford ve diğ., 1997).

Öğretmenlerin konu ile ilgili sahip oldukları bilgi eksiklerin temelinde üniversitelerde verilen mesleki gelişim programlarının geleneksel yapısından kaynaklanan yetersizlikler yatabilir. Çünkü üniversitelerde verilen geleneksel hizmet öncesi öğretmen eğitimi programları teori ile uygulama arasında yeterince güçlü bağlantılar kuramadığı yönünde eğitimciler tarafından uzun zamanlardan beri eleştirilmektedir (Abell ve Cennamo, 2004; L. Shulman, 1992). Yapılan çalışmalar, teori ve pratik arasında kurulan zayıf bağlantıların öğretmen adaylarının mesleki yaşamlarına başladıklarında teorik bilgilerini öğretim ortamlarına aktarırken çeşitli zorluklar yaşadıklarını göstermektedir (Ball, 2000; Doyle, 1986; L. Shulman, 1992; Merseth, 1999). Bu zorlukların önlenmesinde, geçmiş yirmi yıldan beri durum temelli öğretim yaklaşımının hizmet öncesi öğretmen eğitiminde alternatif bir yaklaşım olarak kullanılmasına yönelik bir eğilim ortaya çıkmıştır (Butler ve diğ. 2006; Hammerness ve diğ., 2002; Lundeberg ve diğ., 1999; Merseth, 1991; L. Shulman, 1992). Bu eğilimin ortaya çıkmasında, araştırmacılar durum temelli öğretimsel yaklaşımın sağladığı potansiyel faydaları göz önünde bulundurmuşlardır. Bu faydalar alan yazında genel olarak şu şekilde ifade edilmektedir: (i) Öğretmenlerin kritik düşünme, yansıtıcı düşünme ve karar verme becerilerini geliştirme, (ii) Teorik prensipleri anlamada bir araç görevi görme, (iii) Öğretmenlere öğretmenlik mesleğinin karmaşık yapısını etkili bir şekilde analiz etme imkanı sunma, (iv) Öğretmenlik deneyimi dersinin potansiyel kısıtlıklarının önüne geçme ve (v) Öğretmenlerin konu alan bilgilerini ve pedagojik alan bilgilerini geliştirme.

Sıralanan potansiyel faydaları göz önünde bulunduran araştırmacılar 1990'lı yıllarda öğrencinin sınıf içindeki çalışma fotokopilerini içeren metin-esaslı durumlara odaklanmışlardır (Barnett, 1991; Merseth ve Lacey, 1993; Shulman, 1992; Stein ve diğ., 2000). Fakat özellikle 1990'ların sonuna doğru teknolojinin karşı konulamaz gelişimiyle birlikte hem hizmet öncesi öğretmen eğitiminde (Frederiksen ve diğ., 1998; Seago, 2004; Sherin, 2003b, 2004) hem de hizmet içi öğretmen eğitiminde (Copeland ve Decker, 1996; Daniel, 1996; Friel ve Carboni, 2000;

Goldman ve Barron, 1990) metin-esaslı durumlar yerine video-temelli durumlarının kullanımı popülerlik kazanmaya başlamıştır.

Video-temelli durumlarının öğretmenlerin mesleki gelişimleri açısından güçlü bir araç olarak kabul görmesiyle, eğitimciler video-temelli durumların kullanımını öğretmen adaylarının konu alan bilgilerini ve pedagojik bilgilerini gelişimi açısından önermeye başlamıştır (Ball ve Cohen, 1999; Hiebert ve diğ., 2002; Lampert ve diğ., 1994). Bu bağlamda, özellikle son zamanlarda öğrencilerin matematiksel düşünüşünü içeren video durumlarına odaklanan çalışmalar yapılmaya başlamıştır (ör. Jacobs, Lamb, ve Philipp, 2010; Sherin, 2007; van-Es, 2011). Fakat detaylı bir alan yazın incelemesi yapıldığında yapılan çalışmalarda genel olarak sınıf video durumlarının kullanıldığı ortaya çıkmaktadır. Sınıf ortamını içeren video durumlarının kullanılmasının sağlayacağı muhtemel faydalar kesinlikle göz ardı edilemez. Fakat sınıf ortamı içeriğinde öğrenciler, öğretmen, sınıfın sosyal, fiziki ve pedagojik yapısı gibi çeşitli boyutlarda bilgiler muhteva etmektedir (Sherin, Jacobs, ve Philipp, 2011; van Es ve Sherin, 2008). Bu bakımdan, sınıf video durumları sınıf ortamının karmaşık ve çok boyutlu yapısını içerir. Sınıf video durumlarını analiz ederken, bu karmaşık yapıyla karşı karşıya gelen bir öğretmen adayının doğrudan öğrencinin matematiksel düşünüşüne odaklanması mümkün olmayabilir (Chamberlain, 2005; Ding ve Dominguez, 2015; Freese, 2006; Kagan, 1992; Olkun ve diğ., 2009; Shapiro, 1991). Öğretmen adayları öğrencinin matematiksel düşünüşüne odaklanmak yerine, dikkatlerini öğretmenin sınıf yönetimi, öğrencilerin kendi aralarında yaptıkları konuşmalar ve sınıfın fiziksel yapısı gibi faktörlere yöneltebilirler.

Alan yazındaki video durum temelli çalışmalarda, öğrencilerin matematiksel düşünüşünü içeren video durumları genel olarak sınıf ortamında öğrencinin tahtada soru çözdüğü veya öğretmeniyle konuştuğu zaman dilimlerinin ham video veri setinde kesilip düzenlenmesiyle üretilmeye çalışılmıştır. Fakat sınıf ortamında bir öğrencinin matematiksel düşünüşünü yalın haliyle yansıtması öğretmenin sorduğu sorular, öğretim ortamı, öğrencinin karakteristik özellikleri, öğrencinin arkadaşlarıyla olan sosyal ilişkisi, dersteki zaman sınırı, sınıf atmosferi gibi birçok iç ve dış etkenlere bağlı olabilir. Örneğin, bir öğrenci tahtaya kaldırıldığında matematiksel düşünüşünü öğretmeninden ve arkadaşlarından utandığı için ya da dersin sonunda dar bir zamana denk geldiği için olduğu gibi aktaramayabilir. Diğer taraftan, sınıfta öğrenci odaklı öğretim biçimi yerine öğretmen-odaklı bir öğretim biçimi benimsenmişse ve kavramsal bilgiler yerine soru çözme gibi pratik uygulamalar yoğunluktaysa öğrencilerin bir matematik kavramıyla ilgili sahip olduğu kavrayışların detayına ulaşmak mümkün olmayabilir. Bu ve benzeri durumlar sonucunda da sınıf ortamında öğrencinin matematiksel düşünüşündeki detaylar yok olabilir ya da gizli kalabilir. Sınıf ortamının bahsi geçen sınırlılıkları yalnız bir öğrencinin matematiksel düşünüşünün detaylarını doğrudan içeren özel tasarlanmış video durumlarının üretimine ve kullanımına olan ihtiyacı ortaya çıkarmaktadır. Bu nedenle, bu çalışmada, öğretmen adaylarına Jacobs, Lamb ve Philipp'in (2010) çalışmasındakine benzer şekilde tek bir öğrencinin dörtgenlerle ilgili matematiksel düşünüşünü içeren sınıf dışı bir ortamda klinik görüşmeler yoluyla elde edilen video durumları hazırlanmıştır. Bu tip videolar "*mikro durum videoları*" olarak isimlendirilmiştir. Öğrenci düşünüşüne odaklanan bu videolar bir mikroskop gibi öğrencilerin matematiksel düşünüşündeki detayları yakınlaştıracağı ve detaylandıracağı beklenmektedir. Diğer bir deyişle, bu videoların değişik matematiksel başarı düzeyindeki birçok öğrencinin aynı matematiksel kavramı farklı açılardan nasıl düşünebileceğini gösterme adına faydalı olabileceği düşünülmüştür (Friel ve Carboni, 1997; Jacob ve diğ., 2010). Böylece öğretmen adaylarının öğrencilerin kavram yanılgılarını daha kolay bir şekilde tespit ederek (Hill ve Collopy, 2003), muhakeme yapma becerilerini (Harrington, 1999; Lundeberg, 1999) ve karar verme becerilerini (Grossman, 1992; Jay, 2004; Merseth, 1992) geliştirebilirler. Bu sayede öğretmen adayları konu alan bilgilerini ve pedagojik alan bilgilerini iyileştirerek zenginleştirme fırsatı yakalayabilir (Manouchehri, 2002; Mayo, 2002). Tüm bu güçlü argümanlara dayanarak, bu çalışmada öğrenci düşünüşü içerikli mikro durum videoları bir video durum-temelli mesleki gelişim programına entegre edilerek ilköğretim matematik öğretmen adaylarının dörtgenlerle ilgili konu alan bilgisi ve pedagojik alan bilgilerindeki gelişimi incelemek amacıyla kullanılmıştır.

Daha önce de bahsedildiği gibi alan yazında dörtgenlerle alakalı ulusal ve uluslararası oldukça fazla sayıda çalışma bulunmaktadır. Fakat bu çalışmaların sadece birkaç tanesi dışında (ör. Aslan-Tutak, 2009; Duatepe-Paksu ve Ubuz, 2009; Öztoprakçı, 2014) geneli öğrencilerin ve öğretmen adaylarının yaşadığı zorlukları ve bilgi eksiklerini tarif etmekten öteye gidememiştir. Tüm öğrenme düzeylerinde bu kadar zorluk yaşandığı tespit edilen bir konuda asıl önemli olan nokta, bu zorlukların aşılması için alternatif yaklaşımlar ortaya koymak ve öğretmenlere bilgi birikimlerini geliştirecekleri fırsatlar sunmaktır. Buradan yola çıkarak, bu çalışma kapsamında öğretmen adaylarının dörtgenlerle ilgili sahip oldukları konu alan bilgilerinin ve pedagojik alan bilgilerinin doğasını tespit etmek ve bu bilgileri geliştirmek ve zenginleştirmek amacıyla öğrenci düşünüşü içerikli mikro durum videolarının izlenmesi ve tartışılmasını gerektiren bir öğretim deneyi hazırlanmıştır. Yedinci sınıf öğrencilerin dörtgenlerle ilgili matematiksel düşünüşünü içeren video durumlarının incelenmesini ve tartışılmasını içeren deney tasarımında, ilköğretim matematik öğretmenlerinin dörtgenlerin tanımı, çizimi, sınıflaması ve özellikleriyle ilgili bilgilerindeki gelişimlerin/değişimlerinin incelenmesi amaçlanmıştır. Bu amaç doğrultusunda özel olarak aşağıdaki araştırma sorularına cevap aranmıştır.

1) İlköğretim matematik öğretmen adaylarının öğrenci düşünüşü içerikli video durum temelli bir öğretim deneyine katılmadan önce dörtgenlerle ilgili konu alan bilgileri ve pedagojik alan bilgileri nedir?
2) İlköğretim matematik öğretmen adayları öğrenci düşünüşü içerikli video durum temelli bir öğretim deneyine katılımları sürecindeki dörtgenlerle ilgili konu olan bilgileri ve pedagojik alan bilgilerini nasıl geliştirmişlerdir veya değiştirmişlerdir?
3) İlköğretim matematik öğretmen adaylarının öğrenci düşünüşü içerikli video durum temelli bir öğretim deneyine katıldıktan sonra dörtgenlerle ilgili konu alan bilgileri ve pedagojik alan bilgilerinin doğası nedir?

2 Yöntem

Öğretim deneylerinin ana amacı öğrencilerin ilk elden nasıl matematik öğrendiklerini ve akıl yürüttüklerini anlamak (Thompson, 2000) ve öğretim kararlarını buna göre yönlendirerek öğrencilere daha iyi bir öğrenme ortamı sunmaktır (Cobb, Confrey, diSessa, Lehrer ve Schauble, 2003). Bu yönleriyle öğretim deneyi yöntemi, deneysel çalışmadan ve klinik görüşmelerden ayrılan özelliklere sahiptir. Daha detaylı açıklamak gerekirse, deneysel çalışmalar öğrencilerin kavrayışlarının başlangıç ve son durumlarıyla ilgilenirken, klinik görüşmeler de öğrencilerin hali hazırdaki kavrayışlarının anlaşılması durumuyla ilgilenir. Öğretim deneyleri öğrencilerin matematiksel etkinliklerinin ve davranışlarının modelini ortaya çıkarmada etkilin bir yoldur (Steffe ve Thompson, 2000). Yani öğretim deneyleri sadece öğrencilerin kavrayışlarının başlangıç ve sondaki durumlarını değil süreç içinde var olan bilgilerini nasıl yapılandırdıkları ve geliştirdikleriyle de ilgilenir (Steffe ve Thompson, 2000; Steffe ve Ulrich, 2014). Bu nedenle bu çalışmada ilköğretim matematik öğretmen adaylarının dörtgenlerle ilgili konu alan bilgisi ve pedagojik alan bilgilerindeki gelişimlerin neler olduğu öğretimsel deney yöntemi kullanılarak incelenmiştir.

2.1 Bağlam ve katılımcılar

Bu araştırmanın bağlamını dört yıllık bir öğretmen yetiştirme programının bünyesinde bulunan İlköğretim Matematik Öğretmenliği programı oluşturmuştur. Özel olarak, araştırmanın yürütülmesi amacıyla, 2014-2015 sonbahar döneminde Ankara'da bir devlet üniversitesinin İlköğretim Matematik Öğretmenliği Bölümü'nün son sınıfında öğrenim gören öğrencilere yönelik "İlköğretim fen ve matematik eğitiminde projeler" isimli seçmeli bir ders açılmıştır. Ders içeriğinin tanıtıldığı ilk buluşma sonrasında sekiz sonuncu sınıf öğretmen adayı dersi seçmeye karar vermiştir. Böylece araştırmanın katılımcıları Ankara'da bir devlet üniversitesinin İlköğretim Matematik Öğretmenliği Bölümü'nün son sınıfında öğrenim gören sekiz öğretmen adayından oluşmaktadır. Katılımcı seçiminde amaçlı örneklem seçme tekniğinden yararlanılmıştır. Çünkü seçmeli ders sadece dördüncü sınıflara yönelik açılmıştır. Bu dersin sadece dördüncü sınıflara açılmasının en önemli nedenleri onların Matematik Öğretim Yöntemleri, Okul Deneyimi derslerini almış olmaları ve Araştırma Metotları dersini de hali hazırda alıyor olmalarıdır.

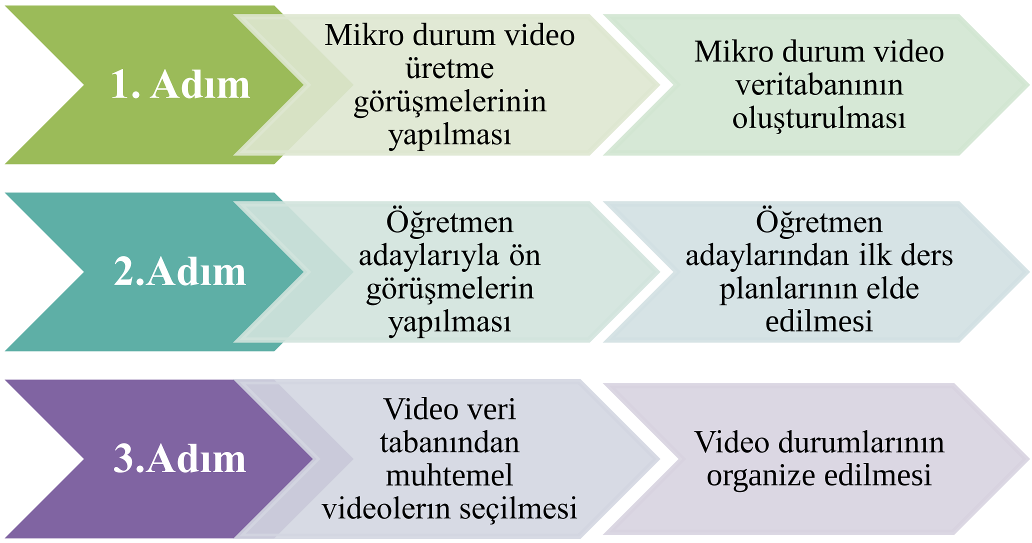
2.2 Verilerin Toplanması

Çalışmada öğretim deneyi sürecindeki detaylar veri toplama sürecini ve veri kaynaklarını anlama adına büyük bir önem taşımaktadır. Bu çalışmada öğretim deneyi iki aşamada gerçekleşmiştir. Birinci aşama öğretim deneyinin hazırlanmasından, ikinci aşama ise öğretim deneyinin uygulanmasından oluşmaktadır.

2.2.1 Öğretim deneyin hazırlanması

Öğretim deneyinin hazırlanma aşaması bireysel klinik ön görüşmelerin ve son görüşmelerin hazırlanması ve dörtgenlerle ilgili ders planı formatının geliştirilmesi ve öğrenci düşünüşü içerikli mikro durum videolarından oluşan bir video bankasının hazırlanması adımlarını içermiştir. Şekil 1 öğretim deneyinin hazırlanma adımlarını göstermektedir.

Katılımcıların düşünce dünyasına girme ve onların var olan bilgi düzeylerini anlamada klinik görüşmelerin etkililiği alan yazında vurgulanan bir noktadır (Clement, 2000; Hazzan ve Zazkis, 1999; Koichu ve Harel, 2007; Newel ve Simon, 1972). Bu çalışmada, klinik ön-görüşmeler her bir katılımcı ile bireysel olarak yaklaşık 60 dakika sürecek şekilde gerçekleştirilmiştir. Görüşme süreci 33 maddeden oluşan beş görevden oluşmuştur: dörtgen tanımlama görevi, paralelkenar görevi, eşkenar dörtgen görevi, yamuk görevi ve dörtgen sınıflama. Görüşmede bir görevin tamamlanmasının ardından ikinci görev devreye girmiştir. Ön görüşmelerdeki bu görevler dörtgenlerle ilgili alan yazın tarandıktan sonra araştırmacı ve alan uzmanı bir matematik eğitimcisi tarafından geliştirilmiştir. Bu maddelerin bir kısmı öğretmen adaylarının o konudaki konu alan bilgisini ölçmeye çalışırken, bir kısmı da öğrenci düşünüşüyle ilgili bilgileri ve öğretimsel yaklaşımlarını anlamaya yönelik tasarlanmıştır. Örneğin, üç madde öğretmen adaylarının kişisel dörtgen çizimlerini anlamaya çalışırken, üç madde öğrencilerin muhtemel çizimleriyle ilgili görüşlerini anlamaya hedeflemiş ve bir diğer üç madde de onların öğretimsel çizimlerini anlamayı hedefleyecek içerikte hazırlanmıştır. Bu maddeler oluşturulduktan sonra alan uzmanlarına ve matematik öğretmenlerine danışılarak maddelerin çalışmadaki katılımcılara olan uygunluğu sağlanmaya çalışılmıştır.



Şekil 1. Öğretim deneyinin hazırlanma adımları

Ön-görüşmeler tamamlandıktan sonra katılımcılardan dörtgenlerin öğretimine yönelik bir ders planı tasarlanmaları istenmiştir. Bu bağlamda, öğretmen adaylarının öğretimsel yaklaşımlarının daha detaylı anlaşılmasını sağlayacak ders planlarının bir formatı hazırlanmıştır. Bu formatta öğretmen adaylarından yedinci sınıf öğrencilere dörtgenleri nasıl öğreteceklerinin detaylarını barındıran bir ders planı hazırlamalarını gerektirecek bir içerik oluşturulmuştur. Bu ders planını katılımcılar yaklaşık 7-10 gün içinde hazırlayıp araştırmacıya teslim etmiştir.

Son ve önemli hazırlık aşaması öğrencinin matematiksel düşünüşünün detaylarını içeren mikro durum videolarının yer aldığı bir video bankasının oluşturulması olmuştur. Bu bağlamda, öğretmen adaylarına uygulanan klinik görüşmelerde yer alan ölçme aracındaki maddelere benzer özellikle maddeler içeren görüşme formları öğrenciler için oluşturulmuştur. Daha sonra, yedinci sınıf öğrencilerle yapılacak görüşmeler için araştırmacıya yakın bir ilköğretim okulu belirlenmiştir. Okulda yer alan iki yedinci sınıfta bulunan 47 öğrenci arasından önceki dönem matematik not ortalamalarına göre belirlenen ve farklı başarı düzeylerinde bulunan 16 kişi video üretme görüşmelerine (video production interviews) katılmaya gönüllü olmuştur. Okulda rehberlik birimi tarafından kullanılan bir odada her bir öğrenci ile iki görüşme yapılmıştır. Birinci görüşme yaklaşık 50 dakika sürerken, ikinci görüşme 30 dakika sürmüştür. Birinci klinik görüşmede katılımcılara paralel/dik doğru parçalarının inşası, eş açıların inşası, eşit uzunlukla doğru parçalarının inşası gibi temel kavramlar ile paralelkenar tanımı, çizimi, seçimi ve özelliklerine yönelik maddeler yöneltilmiştir. İkinci video üretme görüşmesinde ise öğrencilere eşkenar dörtgen ve yamuk kavramlarının tanımı, çizimi, seçimleri ve özellikleriyle ilgili maddeler sorulmuştur. Sonuç olarak toplamda yaklaşık 1000 dakikalık öğrencilerin matematiksel düşünüşünü içeren ham video veri seti elde edilmiştir. Daha sonra bu veri seti nitel araştırma yöntemleri kullanılarak iki tip arşivleme sürecine tabii tutulmuştur. Birinci tip arşivleme sürecinde tanımlama, çizim, seçimler ve dörtgen özelliklerine göre tüm katılımcıların cevapları gruplanmıştır (bkz. Ek 4). İkinci arşivleme tipinde ise her bir katılımcının tüm görevlerdeki durumu dakika aralıklarının da yer aldığı tabloya aktarılmıştır (bkz. Ek 5). Ayrıca yapılan iki tip arşivlemede de iki kriter göz önünde bulundurulmuştur. Birinci kriterde ilgili alan yazındaki öneriler göz önünde bulundurularak mikro durum videolarının süreleri 10 dakikayı geçmeyecek şekilde belirlenmiştir. İkinci olarak, video durumlarının grup tartışmalarındaki verimliliği artturması için Sherin ve diğerleri (2009) tarafından önerilen üç bileşene göre mikro durum videoları tasarlanmıştır. Bu üç bileşen; görünüm, açıklık ve derinlik olarak isimlendirilmiştir. Bu üç bileşene ve bileşenleri ortaya çıkaran araştırmacıların önerilerine göre, öğretim deneyinde kullanılmak üzere "*görünüm-açıklık-derinlik*" bileşenleri "*yüksek-yüksek-düşük*", "*yüksek-düşük-düşük*" veya "*yüksek-yüksek-yüksek*" düzeylerde olan klipler öğretim deneyi oturumları için izletilme ihtimali olan klipler kategorisine alınmıştır. Video kesme ve birleştirme işlemleri profesyonel bir video düzenleme programı olan Adobe Premier Pro CS5.5 kullanılarak yapılmıştır.

2.2.2 Öğretim deneyinin uygulanması

Öğretim deneyinin uygula aşaması, mikro durum videolarının belirlenmesi, öğretim deneyi oturumlarının gerçekleştirilmesi, ders planlarının revize edilmesi ve son olarak görev-temelli klinik son görüşmelerin yapılması adımlarından oluşmuştur.

2.2.2.1 Öğretim deneyi oturumlarında kullanılacak video durumlarının düzenlenmesi

Yapılan tüm görev temelli klinik görüşmelerin ve ders planlarının incelenmesinin hemen ardından öğretim deneyi oturumlarında izletilecek video bankasında yer alan öğrenci düşünüşü içerikli video durumları seçilmeye ve düzenlenmeye başlanmıştır. Bu video durumları hazırlanırken belli ölçütler göz önünde bulundurulmuştur. Bu ölçütler şu şekilde sıralanabilir: (i) video durumunun süresi, (ii) öğrenci düşünüşündeki çeşitlilik, (iii) öğretmen adaylarının dörtgenlerle ilgili sahip oldukları bilgi durumları ve (iv) öğrenci düşünüşü boyutları (Linsenmeier ve Sherin, 2009). Bu ölçütler ve içerikleri kısaca aşağıda açıklanmıştır.

Daha öncede belirtildiği gibi videoların izleyen kişinin dikkatini dağıtmaması ve video olan odağı arttırma adına öncelikle alan yazında önerilen video süresi ve yapılan çalışmalarda yer alan videoların taşıdığı süreler incelenmiştir. Çünkü eğitim videosu izlemek eğer uzun olursa izleyen kişiye hem sıkıcı hem de yavaş gelebilir (Jaworski, 1990). Yapılan çalışmalarda ortalama dakika ile dakika arasında videoların sıklıkla kullanıldığı görülmüştür. Bu çalışmada da 4.23 ile 10.05 dakika/saniye aralığında sekiz video durumu tasarlanmıştır (bkz. Table 9).

Videoların her birinde bir öğrencinin dörtgenlerle ilgili düşünsel sürecine yer verilmiştir. Böylece farklı kavrayışlara sahip birçok öğrencinin matematiksel düşünüşünü içeren zengin bir video durumu seti elde edilmiştir. Videoların hazırlanmasında en kritik ölçüt öğretmen adaylarından ön görüşmelerde ve ilk ders planlarında elde edilen veriler olmuştur. Çünkü öğretmen adaylarının bilgilerinde hatalı ve yetersiz olan noktaların tespit edilmesi hazırlanacak olan videoların içeriğini belirleyen en önemli faktör olmuştur. Örneğin, ön görüşmelerde öğretmen adaylarının genel olarak paralelkenarın köşegen ve açı özelliklerinde yanılgılara sahip oldukları tespit edildiği için ikinci oturumda dörtgen özelliklerine yönelik iki video durumunun hazırlanmasına karar verilmiştir.

2.2.2.2 Öğretim deneyi oturumlarının gerçekleştirilmesi

Öğretim deneyi oturumları dört hafta sürmüştür. Öğretmen adayları her hafta iki video durumunu bireysel olarak incelemiş ve grupça tartışmıştır. Her videonun bireysel izlenme sürecinde öğretmen adaylarından videodaki konu ile ilgili kendi düşüncelerini, videodaki öğrencinin düşünüşünde neleri ilginç buldukları ve nelerin farkına vardıklarını, varsa videodaki öğrenci yanılgılarını ve nedenlerini yansıtıcı bir düşünce raporu olarak yazmaları beklenmiştir. Bireysel video analizi ve yansıtıcı düşünce raporlarının tamamlanmasının ardından her bir video sonunda grup tartışması yapılmıştır. Grup tartışmasının verimliliğini arttırmak adına van Es, Tunney, Goldsmith ve Seago'nun (2014) önerdiği teorik çerçeve kullanılmıştır. Bu teorik çerçeveye göre grup tartışması yöneticisi dört temel görev üstlenmiştir: grubu video analizine yönlendirme, grubun video durumunu sorgulamasını sağlama, video ve matematik ile ilgili odağın devamlılığını sağlama ve grup işbirliğini destekleme. İki videonun da bireysel incelenmesi ve grup tartışmalarının gerçekleştirilmesinden sonra öğretmen adaylarından dörtgenlerle ilgili konu alan bilgisi ve pedagojik alan bilgilerinde meydana gelen gelişmeleri/değişimleri ifade etmelerini gerektiren yansıtıcı bir düşünce raporu yazmaları ve araştırmacıya teslim etmeleri istenmiştir. Katılımcılar bu yansıtıcı düşünce raporlarını oturum sonunda veya evlerinde yazarak araştırmacıya teslim etmişlerdir. Son olarak da katılımcılardan her hafta gerçekleştirilen iki öğretim deneyi oturumunun bitiminde ders planları üzerinde gerekli gördükleri değişiklikleri yapmaları ve bu değişiklikleri neden yaptıklarını yansıtıcı düşüncelerle plan üzerine notlar yazarak açıklamaları istenmiştir. Revizyon yapılan planlar bir diğer öğretim deneyi oturumuna başlamadan birkaç gün önce araştırmacıya mail yoluyla ulaştırılmıştır. Diğer üç öğretim deneyi oturumu da benzer şekilde gerçekleştirilmiştir.

Her hafta yapılan oturumlardan elde edilen bireysel video analizi yansıtıcı düşünce raporları, grup tartışmaları verileri, grup tartışması sonrası yansıtıcı düşünce raporları ve revize edilmiş ders planları incelenerek bir sonraki hafta öğretim deneyi oturumunda kullanılacak videoların tasarlanması veya modifikasyonu sağlanmıştır.

2.2.2.3 Son görüşmelerin yapılması ve revize edilen ders planlarının elde dilmesi

Tüm oturumların bitmesiyle öğretmen adaylarının dörtgenlerle ilgili konu alan bilgisi ve pedagojik alan bilgisindeki son durumun ne olduğunu anlamak adına her bir katılımcı ile klinik bireysel son görüşmeler yapılmıştır. Her bir görüşme ortalama 35-45 dakika sürmüştür. Bu görüşmelerde katılımcılara ön görüşmelerde sorulan dörtgenlerle ilgili maddelerin tümü sorulmuştur. Katılımcıların bu maddelere ve araştırmacının sorduğu ek sonda sorulara verdikleri cevaplar incelenerek onların dörtgenlerle ilgili sahip oldukları bilgilerdeki değişimler/gelişimler incelenmiştir.

2.3 Verilerin Analizi

2.3.1 Bireysel klinik ön-görüşmelerin ve son-görüşmelerin analizi

Bireysel klinik ön görüşmelerin ve son görüşmelerin analizinde temaların ve kodların oluşturulması amacıyla tematik kodlama kullanılmıştır. Bu amaçla, tüm görüşme verileri belirtilen aşamalara göre incelenmiştir: verinin tanınması, ilk kodların oluşturulması, kodlar arasından temaların elde edilmesi, temaların gözden geçirilmesi, temaların tanımlanması ve isimlendirilmesi, tüm tema ve kodları içeren bir son raporun oluşturulması (Braun ve Clarke, 2006). Verilerin tanınması adımı öncesinde, geometri teorileri (ör. Kavram imajı-kavram tanımı, figural kavramlar, prototip fenomeni, van-Hiele geometric düşünce düzeyleri) ve bu teorileri temel alan ve dörtgenlerle ilgili yapılan çalışmalar detaylı bir şekilde incelenmiş ve arşivlenmiştir. Bu arşivleme, alan yazından gelebilecek muhtemel kodların elde edilmesinde büyük bir role sahip olmuştur. Ardından, tüm görüşme videoları yazılı döküm haline getirilmiştir. Daha sonra da veriler içinde dörtgen tanımları, çizimleri, sınıflamaları ve dörtgen özellikleriyle ilgili kısımlar ana temalar olarak ayrılmıştır. Bu temaların altında öğretmen adaylarının konu alan bilgilerine yönelik kısımlar, öğrenci düşünüşünü anlamaya yönelik bilgileri ve öğretimsel stratejilerle ilgili bilgileri alt temalar olarak belirlenmiştir. Tema ve alt temaların belli olmasının ardından her bir tema ve alt temada yer alan muhtemel kodlar ortaya çıkarılmıştır.

Tüm kodların belirlenmesinin ardından kodlayıcılar arasındaki güvenirlik ve araştırmacının kendi içinde sağladığı güvenirlik katsayıları Miles ve Huberman (1994) önerdiği metot kullanılarak hesaplanmıştır. Kodlayıcılar birbirinden bağımsız olarak iki katılımcının ön ve son görüşmelerini incelemişler ve kodlamışlardır. Kodlama sonunda ön-görüşme ve son görüşme veri setindeki anlaşma sağlanan kod sayısı ile anlaşma sağlanamayan kod sayıları belirlenmiştir. Bunun sonucunda da kodlayıcılar arasındaki güvenirlik ön görüşme için 90% bulunurken, son görüşme için 87% bulunmuştur. Daha sonra kodlayıcı ile yüz yüze yapılan bir görüşmeyle kodlamada çıkan anlaşmazlık noktalarında fikir birliğine varılmıştır. Araştırmacı kendi içinde bir güvenirlik sağlama adına aynı veri setini üç ay içinde üç kez kodlamıştır ve sonuçta araştırmacının kendi içindeki güvenirliğinin ortalaması 98% bulunmuştur.

2.3.2 Öğretim deneyi oturumunda elde edilen verilerin analizi

Öğretim deneyi oturumlarında elde dilen verilerin incelenmesinde üç aşamalı analitik bir veri analizi yaklaşımı kullanılmıştır. Birinci aşamada öğretim deneyi oturumlarında elde edilen veriler kronolojik bir sıra ile genel bir incelemeye tabii tutulmuştur. Bu kronolojik sıralamada incelenen veriler sırasıyla şu şekilde olmuştur: bireysel video analizi yansıtıcı düşünce raporları, grup tartışmaları, grup tartışması sonrası yazılan yansıtıcı düşünce raporları, revize edilmiş ders planları. Daha sonra her bir veri setinin incelenmesi sürecinde sürece ve katılımcıların bilgilerine ilişkin hatırlatıcı kısa notlar alınmıştır.

İkinci düzey veri analizinde ise yazılı formatta olan yansıtıcı düşünce raporlarının tümü bilgisayar ortamına aktarılmış ve videolardaki grup tartışmaları yazılı olarak döküm haline getirilmiştir. Tüm veri setinin yazılı bir form almasının ardından, veri setlerindeki "*birim fikirler*" belirlenmeye başlanmıştır (Jacobs, Yoshida, Fernandez ve Stigler, 1997). Birim fikirler öğretmen adaylarının dörtgenlerin tanımı, çizimi, sınıflanması ve özellikleriyle ilgili konu alan bilgileri ve pedagojik alan bilgilerini yansıtan yazılı veya sözlü söylemler olarak belirlenmiştir. Bu söylemler bazen bir cümleden oluşurken bazen bir paragraftan oluşurken bazı durumlarda bir paragraf veya tartışma diliminden oluşmuştur.

Birim fikirlerin belirlenmesi ve kodlanmasının ardından üçüncü düzey veri analizine geçilmiştir. Bu düzeyde, her bir katılımcı için bir "*kişisel bilgi gelişim dokümanı*" hazırlanmıştır. Bu dokümanda ilk olarak her bir katılımcının öğretimsel deney öncesi dörtgenlerle ilgili konu alan bilgisi ve pedagojik bilgilerinin durumu belirtilmiştir. Bu analizin nasıl gerçekleştiğini daha detaylı aktarabilmek adına katılımcıların dörtgen tanımlarıyla ilgili bilgilerinin süreç boyunca ele alınışı detaylı bir şekilde bir sonraki paragrafta açıklanmıştır.

İlk olarak katılımcıların dörtgenlerin tanımlarına yönelik bilgileri kişisel bilgi gelişim dokümanına not edilmiştir. Ardından, bireysel video analizi sırasında yazdıkları yansıtıcı düşünce raporlarında dörtgenlerin tanımına yönelik bilgilerini yansıtan söylemleri dokümana aktarılmıştır. Bunun akabinde, yine her bir katılımcının dörtgenlerin tanımıyla ilgili grup tartışmalarında söylediği ifadeler dokümana eklenmiştir. Son olarak, yine dörtgen tanımıyla ilgili bilgilerini yansıtacak tartışma sonrası yansıtıcı düşünce raporlarındaki söylemleri ve ders planlarındaki değişiklikler ve bu değişikliklerin gerekçeleri "kişisel bilgi gelişim dokümanı" içeriğine eklenmiştir. Benzer şekilde katılımcıların dörtgen çizimleri, dörtgen sınıflaması ve özellikleriyle ilgili bilgilerindeki süreçler de kişisel bilgi gelişim dokumanı içeriğine aktarılmıştır. Bu kişisel bilgi gelişim dokümanları her bir katılımcının dörtgenlerin tanımı, çizimi, sınıflaması ve özellikleriyle ilgili konu alan bilgileri, öğrenci düşünüşünü anlamaya yönelik bilgileri ve öğretimsel stratejilerindeki gelişimleri/değişimleri anlamayı kolaylaştırmıştır. Daha açık olması amacıyla Tablo 1 öğretmen adaylarının dörtgen tanımlarıyla ilgili bilgilerindeki gelişimin kodlanmasını özetle örneklemiştir.

Tablo 1. Öğretmen adaylarının dörtgen tanımlarıyla ilgili bilgilerindeki gelişimin kodlanmasına yönelik örnek

Bilgi türü	Dörtgen tanımlarıyla ilgili bilgilerdeki gelişimler -den/dan	 -ye/ya
Konu alan bilgisi	Hariç tutan ya da kısmi kapsayıcı tanımlar	Kapsayıcı tanımlar
	Bir tanım için gerek ve yeter koşulları sağlayamama	Tanım için gereken gerek ve yeter koşulları sağlama
	Yetersiz matematiksel dil kullanımı	Düzgün matematiksel dil kullanımı
Öğrenci düşünüşünü anlama bilgisi	Öğrencilerin doğru veya kısmi özellikteki tanımlarını kestirme	Öğrencileri hatalı tanımları ve bunların muhtemel nedenlerine odaklanma
	Öğrencilerin kavram imajları ile kavram tanımları arasındaki ilişkinin farkında olmama	Öğrencilerin kavram imajları ile kavram tanımları arasındaki sıkı bir ilişki kurma
Öğretimsel yaklaşımlarla ilgili bilgi	Tanımların öğretiminde öğretmen-merkezli bir öğretim yaklaşımı benimseme	Tanımların öğretiminde öğrenci-merkezli bir öğretim yaklaşımı benimseme
	Sadece ilgili dörtgenin tanımını verme	Dörtgenlerdeki kavramlar için ön koşul olan kavramları da örnekleme ve tanımlama
	Kişisel tanımları öğretimsel tanım olarak kullanma	Kişisel tanımlarını öğretimsel tanımlarından didaktif bir yaklaşımla ayırt etme

3 Bulgular ve Sonuçlar

Araştırma bulgu ve sonuçları araştırma sorularına paralel olarak üç başlık altında verilmiştir. Birinci başlıkta öğretim deneyi öncesi öğretmen adaylarının dörtgenlerle ilgili bilgi durumlarının nasıl olduğuna dair bilgilerin onların ön-görüşme verilerine ve ilk ders planlarına göre yorumlanmıştır. İkinci başlıkta ise öğretmenlerin bilgilerinde öğretim deneyi sürecinde nasıl gelişimler/değişimler yaşandığıyla ilgili özet sonuçlara yer verilmiştir. Son olarak, son görüşme verilerine dayanarak öğretmen adaylarının bilgi durumlarının öğretim deneyi sonunda nasıl bir hal aldığına yönelik sonuçlar özetlenmiştir.

3.1 Klinik Ön-Görüşmelerin Sonuçları

3.1.1 Öğretmen adaylarının dörtgen tanımları ile ilgili konu alan bilgileri ve pedagojik alan bilgileri

Klinik ön-görüşmeler öğretmen adaylarının dörtgenlerin tanımlarıyla ilgili çeşitli eksikliklerinin olduğunu ortaya koymuştur. Örneğin, sekiz öğretmen adayından altısı yaptıkları tanımlarda bir tanımda olması gereken yeter ve gerek koşullara yer verememiştir. Bu yönüyle kişisel tanımlarının matematiksel olarak doğru olup olmadığına çok fazla dikkat etmemişlerdir. Diğer taraftan, öğretmen adayları genel olarak paralelkenar ve eşkenar dörtgen için kapsayıcı tanımları kullanırken, yamuk kavramı için hariç tutan tanımları kullanmışlardır.

Öğretmen adayları öğrencilerin tanımlarda yapacağı muhtemel yanılgılar, hatalar ve zorluklar ile ilgili kısıtlı bilgi sunmuşlardır. Örneğin, öğrencilerin aşırı genelleme hataları yüzünden ortaya çıkabilecek yanlış tarif ve tanımları konusunda nerdeyse hiç kimse fikir yürütememiştir. Birkaç öğretmen adayı genelde öğrencilerin dörtgen tanımlarında özellikle yamuk tanımı için yamuğun günlük dil kullanımından kaynaklanan bazı yanılgılar yaşayabileceklerini savunmuştur. Diğer taraftan, kullanacakları öğretimsel tanımları belirlerken ise didaktik bir yaklaşım sergilemek yerine verdikleri kişisel tanımın aynısını öğretimsel tanım olarak da kullanabileceklerini belirtmişlerdir. Ayrıca öğretmen adaylarının ders planları onların genel olarak tanımları verirken öğretmen- merkezli bir öğretim şeklini tercih ettiğini ortaya koymuştur. Ek olarak, ders planlarında ilkel (prototip) bir şekil vererek öğrencilerden tanım yapmasını isteyen türde etkinliklere yer verdikleri ortaya çıkmıştır.

3.1.2 Öğretmen adaylarının dörtgen çizimleriyle ilgili konu alan bilgileri ve pedagojik alan bilgileri

Öğretmen adaylarının neredeyse hepsi birbirinden farklı en az üç paralelkenar, eşkenar dörtgen ve yamuk örneği çizmeleri istenen maddelerde prototip şekiller çizmeyi tercih etmişlerdir. Onlar için şeklin 180 derecelik dönmüş versiyonu prototip olmayan bir şekil örneği olmuştur. Diğer önemli bir nokta da paralelkenar için çoğunlukla kısmi hiyerarşik çizimler sunarken eşkenar dörtgen için genelde hiyerarşik çizimler sunmuşlardır. Fakat yamuk şekli için çoğunluğu hiyerarşik olmayan çizimler gerçekleştirmiştir.

Öğrencilerin yapacağı muhtemel çizimlerin neler olacağı sorulduğunda ise genel olarak öğrencilerin prototip şekilleri çizeceklerini belirtmişlerdir. Ayrıca öğrencilerin dörtgenler arasındaki ilişkilere dair bilgi birikimlerini ortaya koyacak şekilleri çizmekte zorluk yaşayacaklarını ifade etmişlerdir. Fakat öğrencilerin muhtemel hatalarına ve zorluklarına yönelik tahminler oldukça sınırlı sayıda olmuştur. Örneğin, hiçbir katılımcı öğrencilerin paralel iki doğru parçasını veya düzgün altıgeni bir paralelkenar örneği olarak alabileceğini tahmin edememiştir. Diğer bir nokta da yine hiçbir katılımcı öğrencilerin kareli kağıt kullanımında paralellik belirlemede zorluklar yaşayacaklarını veya benzer başka çizimlerde yaşanacak zorluklar üzerine fikir yürütmemiştir.

Son olarak, öğretmen adaylarının öğretimsel çizimleri kontrol edildiğinde kişisel örnek uzaylarında yer alan prototip örnekleri ve kısmi ya da hiyerarşik ilişkide örnekleri kullanmayı tercih ettikleri gözlemlenmiştir. Sadece bir katılımcı her bir dörtgen çeşidinin kritik özelliklerini vurgulama adına örnek teşkil etmeyen çizimlerden faydalanacağını belirtmiştir.

3.1.3 Öğretmen adaylarının dörtgenlerin sınıflanması ve özellikleriyle ilgili konu alan bilgileri ve pedagojik alan bilgileri

Öğretmen adaylarının altısı dörtgen çeşitlerinin seçilmesini ve sınıflanmasını gerektiren maddelerde dörtgenler arasındaki hiyerarşik ilişkileri göz önünde bulundurmuştur. Katılımcılardan Eda ise paralelkenar için kısmi hiyerarşik sınıflama yaparken, eşkenar dörtgen için hiyerarşik, yamuk için ise hiyerarşik olmayan bir sınıflama yapmıştır. Beril ise yamuk ile paralelkenarın sınıflama ilişkisini karıştırarak aşırı genelleme hataları yapmıştır. Öğrencilerin yapacakları seçimleri tahmin ederken genelde prototip şekiller ve hiyerarşik olmayan ilişkilere odaklanmadan çıkabilecek dörtgen sınıflamasına yönelik problemleri kestirebildikleri halde öğrencilerin yapacağı muhtemel hatalı seçimleri ve nedenlerini kestiremedikleri ortaya çıkmıştır.

Son olarak, tüm katılımcılar paralelkenar ve eşkenar dörtgen için hiyerarşik ilişkileri göz önünde bulundurarak öğretim yapmayı planladıklarını belirtmişlerdir. Ders planlarında genellikle dörtgen ilişkilerini öğretmeye yönelik Venn diyagramını tercih etmişlerdir. Fakat iki öğretmen adayı ders planlarında hiyerarşik ilişkilerin öğretimine yönelik bir etkinlik sunmamıştır.

Öğretmen adaylarının dörtgen özellikleriyle ilgili konu alan bilgileri incelendiğinde ise dörtgenlerin kenar özelliklerin hiçbir sıkıntı yaşamadıkları halde açı ve özellikle köşegen özelliklerinde problemler yaşadıkları ve bilgilerinin yeterli olmadığı ortaya çıkmıştır. Öğrencilerin dörtgen özellikleriyle ilgili yapacakları yanılgılar konusunda ise oldukça kısıtlı bir bilgi birikimine sahip oldukları görülmüştür. Çünkü öğrencilerin köşegen özelliklerinde ne gibi zorluklar yaşayacakları ve bu zorlukların muhtemel nedenlerinin ne olacağıyla ilgili herhangi bir veriye rastlanmamıştır. Bu durumun temel nedeni, öğretmen adaylarının konu alan bilgilerindeki yetersizliklerle ilgili olabilir. Ders planlarında ve klinik görüşmelerde dörtgen özelliklerinin öğretimine yönelik sundukları öğretimsel yaklaşımlar incelendiğinde genel olarak prototip bir şekil verip tablolarda açı, kenar ve köşegen özelliklerine değinme veya öğrencilere inceletme eğiliminde oldukları görülmüştür.

3.2 Öğretim Deneyi Oturumları Sürecinde Öğretmenlerin Bilgi Gelişimleri

3.2.1 Öğretmen adaylarının dörtgenlerin tanımlanmasına yönelik bilgilerindeki gelişimler

Öğretim deneyi oturumları sürecinde elde edilen verilen öğretmen adaylarının video durumlarını izledikçe dörtgenlerin tanımlarına yönelik bilgilerinde önemli gelişimler olduğunu ortaya çıkarmıştır. Bu gelişmelerin en belirgin olanları şu şekilde sıralanabilir: (i) öğrencilerin hatalı tanımları ile matematiksel ve günlük dil kullanımları arasındaki ilişkiyi odaklanma, (ii) öğrencilerin prototip şekillerin görsel özelliklerine odaklanarak tanım yaptıklarını fark etme, (iii) öğrencilerin tanımlarındaki gerek ve yeter koşulların sağlanmamış olduğunu bu nedenle de matematiksel olarak doğru tanım sunmadıklarını fark etme (iv) öğrencilerin yaptıkları tanımlardaki hatalı durumların nedenlerini grup tartışmaları sürecinde sorgulama ve (v) bu hatalı durumların üstesinden gelmek için alternatif yollar geliştirme.

3.2.2 Öğretmen adaylarının dörtgenlerin çizimleriyle ilgili bilgilerindeki gelişimler

Öğretim deneyi oturumları sürecinde elde edilen veriler, öğretmen adaylarının öğrencilerin hatalı dörtgen çizimlerini ve dörtgen çizimi sürecinde yaşadıkları sıkıntıları fark ederek bu problemli durumların nedenlerini sorgulamalarına imkan vermiştir. Örneğin, öğrencinin paralelkenarı neden iki paralel doğru parçası olarak çizdiğini fark etmiş bu durumun birçok muhtemel nedeni olabileceğini grup tartışmalarındaki fikir paylaşımlarında anlamışlardır. Diğer taraftan, dörtgen çizimlerinin öğretilmesi adına birçok alternatif yaklaşım geliştirerek pedagojik alan bilgilerini genişletme fırsatı yakalamışlardır. Bazı strateji örnekleri şu şekilde verilebilir: (i) şekillerin hazır çizimlerini vermek yerine öğrencilerden çizmelerini isteme, (ii) yamuktan kareye doğru giden bir anlatım ve çizim yolu benimseme, (iii) öğrencilerin paralellik, diklik, açı inşası gibi temel geometrik kavramlarla ilgili bilgilerini kontrol etme, (iv) tahtada düz bir anlatım yöntemi benimsemek yerine farklı materyal ve temsil biçimlerinden faydalanma ve (v) öğrencilerin düşüncelerini açıklamalarına fırsat verecek kritik sorular sorma.

3.2.3 Öğretmen adaylarının dörtgenlerin sınıflaması ve özellikleriyle ilgili bilgilerindeki gelişimler

Öğretmen adayların dörtgenlerin sınıflanmasına yönelik bilgilerindeki en önemli gelişim öğrencilerin muhtemel hatalı kavrayışlarının dörtgen seçme ve gruplama süreçlerine olan etkilerini anlama olmuştur. Diğer taraftan, öğretim deneyi oturumları esnasında en önemli gelişmeler öğretmen adaylarının dörtgenlerin özelliklerine yönelik sahip oldukları konu alan bilgileri ve pedagojik bilgilerinde gözlemlenmiştir. Öğretmen adaylarının tümü grup tartışmaları esnasında dörtgenlerin köşegenlerine yönelik kendi kavram yanılgıları fark etmişlerdir. Bu sayede, paralelkenarın köşegenlerini her zaman eşit olduğunu düşünme ve birbirine dik olduğunu düşünme ve köşegenlerin her zaman açıortay olduğuna dair yanılgılarını gidererek konu alan bilgilerini zenginleştirmişlerdir. Diğer yandan, öğretmen adayları öğrencilerin yanılgı ve hatalarını fark ederek bunların muhtemel nedenleri olarak matematik öğretmenin öğretim biçimini, öğrencilerin temel geometri konularındaki bilgi eksikliklerini ve öğrencilerin açı ve köşegen inşasını kareli kağıtta yapmadaki yetersizliklerini göstermişlerdir. Öğrencilerin dörtgenlerin özelliklerine yönelik yaşadıkları sıkıntıları gidermek ve hatalı kavrayışlarını düzeltmek adına öğretim deneyinde grup tartışmaları esnasında ve sonunda farklı öğretimsel yaklaşımlar geliştirmişlerdir. Bunlar şu şekilde özetlenebilir: pipetlerle açı ve özelliklerinin aktarılması, kağıt kesme etkinliklerinin kullanılması, açıölçer ile oluşturulan şekillerin açılarının ölçülmesi, üçgenin iç açıları toplamı ile dörtgenin iç açılar toplamının ilişkilendirilmesi.

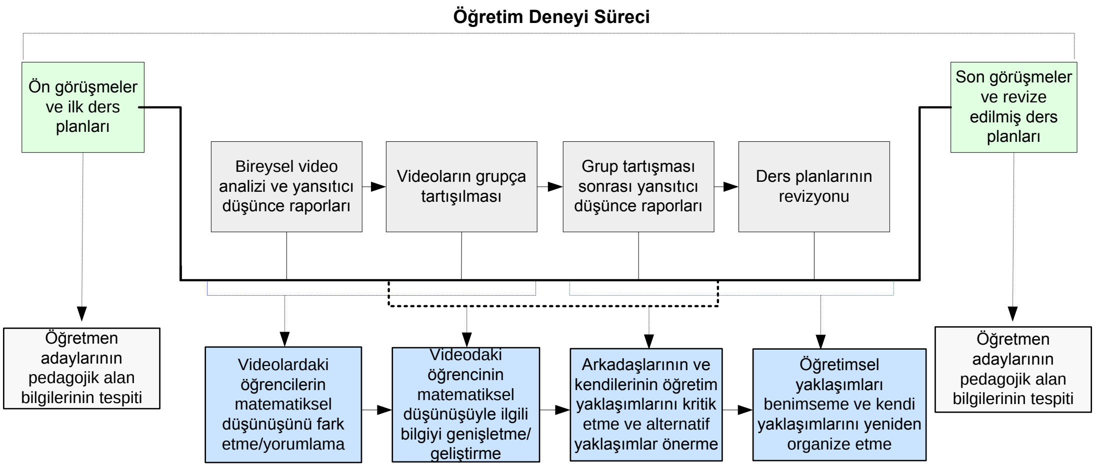
3.3 Klinik Son-Görüşmelerin Sonuçları

Klinik son görüşmeler öğretmen adaylarının dörtgenlerle ilgili bilgilerinin son durumunu görmek adına ilk görüşmede yöneltilen maddelerin tekrar sorulmasıyla elde edilen verilerden oluşmaktadır. Öğretmen adaylarının öğretim deneyi oturumlarında konu alan bilgisinden ziyade her ne kadar pedagojik bilgilerinde gelişme olduğu ortaya çıksa da son görüşmeler katılımcıların konu alan bilgilerinde de büyük gelişimler olduğunu ortaya çıkarmıştır. Örneğin, son görüşmelerde öğretmen adayları ön görüşmelerdeki tanımlarının matematiksel olarak doğru olup olmadığı ve gerek ve yeter koşulları sağlayıp sağlamadığını kontrol ederek düzeltmeler yapmışlardır. Ayrıca yamuk için verdikleri dışlayıcı tanımı kapsayıcı tanımla dörtgenler arasındaki ilişkilerin anlaşılması açısından önemli bularak değiştirmişlerdir. Öğrencilerin yanılgılarına yönelik geliştirdikleri farkındalıkla ek olarak öğretimsel tanımlarını didaktik bir bakış açısıyla revize etmişlerdir.

Son görüşmelerde ortaya çıkan diğer önemli bir sonuç da öğretmen adaylarının prototip ve yarı-hiyerarşik ya da hiyerarşik olmayan dörtgen örneklerine ek olarak prototip olmayan ve hiyerarşik özellikte dörtgen örneklerine de çizimlerinde yer vermeleri olmuştur. Ayrıca son görüşmelerde öğretim deneyi oturumlarında nasıl öğrenci düşünüşündeki detayları anladıklarına ve alternatif pedagojik yaklaşımlar öğrendiklerine dair söylemlerde bulunmuşlardır. Son olarak, tüm öğretmen adayları dörtgen özelliklerine yönelik yanılgılarını gidermişlerdir. Bu durumun nedeni olarak da yapılan grup tartışmalarının önemli bir etken olduğunu belirtmişlerdir. Ayrıca öğrencilerin temel geometrik kavramları bildiklerini varsaymanın büyük bir eksiklik olduğu sonucuna varmışlardır. Böylece dörtgen özellikleriyle ilgili olarak da pedagojik alan bilgilerini zenginleştirdikleri görülmüştür.

4 Tartışma ve Öneriler

Bu çalışmada alan yazında belirtildiği gibi durum-temelli öğretmen yetiştirmenin öğretmen adaylarının bilgilerini nasıl geliştirdiği ve desteklediği ortaya çıkarılmıştır. Özellikle video gibi gelişime açık ve öğrencilerin düşünüşünün okul dışı ortamlarda tüm netliği ile incelenebilmesi öğretmen bilgisini arttırma adına önemli bir araç görevi üstlenmiştir. Öğretmen adaylarının öğretim deneyi oturumlarında pedagojik alan bilgilerindeki genel gelişimler Şekil 2'de özetlenmiştir.

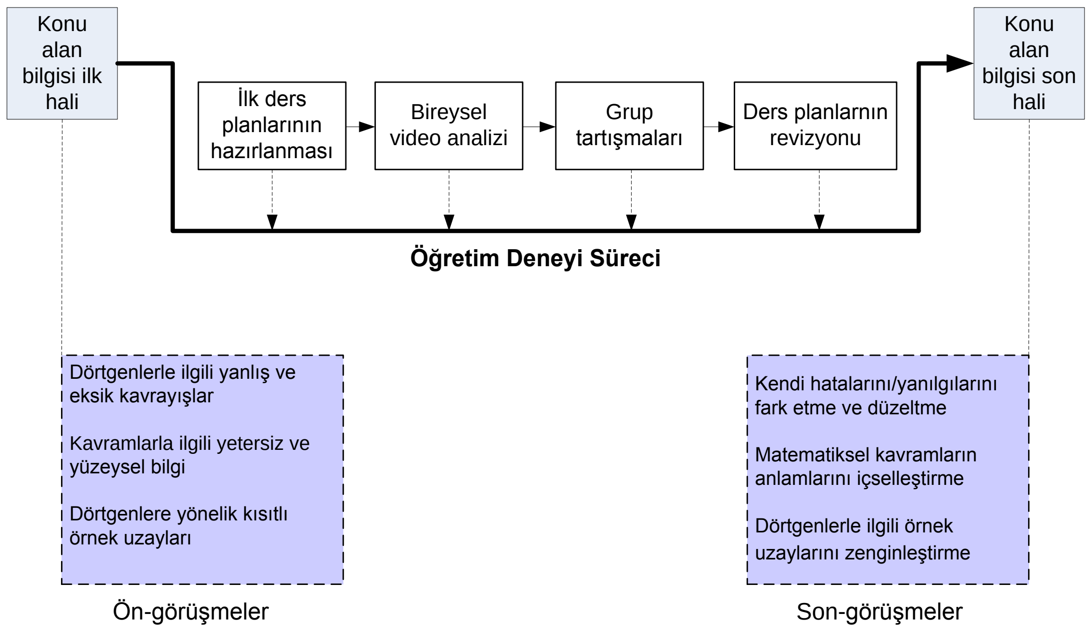


Şekil 2. Öğretmen adaylarının öğretim deneyi oturumlarında pedagojik alan bilgilerindeki genel gelişimler

Şekil 2'de görüldüğü gibi öğretmen adayları mikro durum videolarını bireysel olarak inceledikleri süreçte genel olarak videodaki öğrencilerin matematiksel düşünüşünü fark ederek yorumlama başlamışlardır. Bu süreçte bazı katılımcılar öğrenci düşünüşünü yorumlayıp çıkarımlar sunabilirken bazı katılımcılar sadece öğrencinin verdiği cevabın doğru veya yanlış olup olmadığını değerlendirmiştir. Fakat videodaki öğrenci düşünüşünün grup ile birlikte tartışıldığı süreçte tüm katılımcılar öğrencilerin matematiksel düşünüyle ilgili sahip oldukları bilgiyi genişletme ve geliştirme imkanı bulmuşlardır. Örneğin, bireysel video analizinde sadece öğrencinin yaptığı hatayı tespit eden öğretmen adayları grup tartışmaları esnasında bu hatanın kaynağına ve nasıl giderebileceği gibi hususlara odaklanmaya başlamışlardır. Grup tartışmalarının sonlarına doğru geçen süreç ve grup tartışmaları sonrası yazdıkları yansıtıcı düşünce raporları ise öğretmen adaylarının arkadaşlarının ve kendilerinin öğretimsel yaklaşımlarını kritik ettiğini ve alternatif yaklaşımlar öne sürdüklerini göstermiştir. Son olarak, ders planlarını revize ettikleri öğretim deneyi oturumları sonrası ise öretmen adayları kendi ders planlarındaki öğretimsel yaklaşımları grup tartışmalarında ve video analizi sürecinde edindikleri perspektifle yeniden organize etmişlerdir. Sonuç olarak, bu çalışmanın sonuçları öğretmenlerin mesleki yeterliklerinin gelişimi açısından video-durum temelli çalışmaların etkinliğini vurgulayan çalışmaları destekler nitelikte olmuştur.

Bu çalışmada, bazı çalışmalarda varılan sonuçlardan farklı sonuçlar da elde edilmiştir. Örneğin, alan yazındaki çalışmalar öğretmen adaylarının veya öğretmenlerinin video kulüp buluşmalarının son görüşmelerine doğru yorumlama ve çıkarımda bulunma gibi becerilerini geliştirerek öğrenci düşünüşüne daha çok odaklanmaya başladıklarını göstermiştir (ör. Sherin, 2007; Sherin ve van Es, 2005; van Es ve Sherin, 2008). Fakat bu çalışmada öğretmen adayları öğrencilerin düşünüşünü daha ilk haftaki öğretim deneyi oturumunda yorumlamış ve bu düşünüşe dair çıkarımlarda bulunmuşlardır. Bu durumun temel nedeni belirtilen çalışmalarda kullanılan videoların sınıf videosu olmasıyla ilgili olabilir. Sınıf videolarında sınıf ortamının karmaşık yapısı gereği öğretmenler dikkatlerini sonradan öğrenci düşünüşüne yönlendirebilirler. Çalışma sonuçlarının ortaya çıkardığı önemli bir sonuç da öğretmen adaylarının grup ortamında fikir paylaşımında bulunmalarının onların bilgi gelişimlerini olumlu yönde etkilediğidir. Çünkü bireysel video analizlerinde öğrencilerin yanılgılarını fark eden bir öğretmen adayı, arkadaşlarıyla fikir paylaşımı yaptığı bir grup tartışması sonrasında öğrencinin yanılgısının muhtemel nedenleri üzerinde birçok düşünce geliştirmiş ve alternatif öğretimsel yaklaşım önerilerinde bulunmuştur (Guskey, 2003; Hawley ve Valli, 1999; Wilson ve Berne, 1999; van Es, 2012a, 2012b).

Bu çalışmada öğretmen adaylarının konu alan bilgilerindeki gelişim genel olarak öğretim deneyi oturumlarından ziyade son görüşme verilerinde net bir şekilde görülmüştür. Bu durum videoların sadece öğrenci düşünüşü içerikli olmasıyla ilgili olabilir. Ayrıca son görüşmelerde sorulan sorular, direkt olarak öğretmen adaylarının konu alan bilgileri ölçmeye çalıştığı için konu alan bilgilerindeki gelişimin son görüşmelerde daha net ortaya çıkmasına neden olmuş olabilir. Öğretmen adaylarının konu alan bilgilerindeki genel gelişimler Şekil 3'te özetlenmiştir.



Şekil 3. Öğretmen adaylarının konu alan bilgilerindeki genel gelişimler

Appendix 11

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Ulusoy, Fadime

Nationality: Turkish

Date and Place of Birth: 1 September 1984, Ankara

Marital Status: Married

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EDUCATION

Degree	**Institution**	**Year**
Doctor of Philosophy	Elementary Mathematics Education, Middle East Technical University, Ankara, Turkey	2016
Master of science	Secondary Science and Mathematics Education, Gazi University, Ankara, Turkey	2010
Bachelor of Science	Secondary Science and Mathematics Education, Gazi University, Ankara, Turkey	2007

WORK EXPERIENCE

Years	**Place**	**Enrollment**
June 2010-August 2010	Department of Elementary Education, Kastamonu University, Kastamonu, Turkey	Research Assistant
September 2010-March 2016	Department of Elementary Education, Middle East Technical University, Ankara, Turkey	Research Assistant

PUBLICATIONS

Journal Papers

Yemen-Karpuzcu, S., **Ulusoy, F.**, & Işıksal, M. (2015). Prospective middle school mathematics teachers' covariational reasoning for interpreting dynamic events during peer interactions. *International Journal of Science and Mathematics Education.* Doi: 10.1007/s10763-015-9668-8

Ulusoy, F., & Çakıroğlu, E. (2013). İlköğretim matematik öğretmenlerinin histogram kavramı ile ilgili kavrayışları ve bu kavramın öğretim süreçlerinde yaşadıkları sorunlar [In-service elementary mathematics teachers' conceptions of histogram and difficulties about its teaching process]. *İlköğretim Online, 12*(4), 1141-1156.

Ulusoy, F. (2013). An investigation of the concept of variable in Turkish elementary mathematics teachers' guidebooks. *Journal of Educational and Instructional Studies in the World, 3*(1), 139-149.

International Conference Papers

Ulusoy, F. (2016). The role of learners' example spaces in example generation and determination of two parallel and perpendicular line segments**.** Paper accepted at the meeting of *40th Conference of the International Group for the Psychology of Mathematics Education* (PME 40), Szeged, Hungary.

Ulusoy, F., & Çakıroğlu, E. (2016). Prospective teachers' personal and instructional definitions for quadrilaterals as a lens of their mathematical and didactical considerations. Paper accepted at the meeting of the 13th International Congress on Mathematical Education (ICME-13), Hamburg, Germany.

Ulusoy, **F**. (2015). A meta-classification for students' selections of quadrilaterals: the case of trapezoid. In K. Krainer, & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 598-607). Charles University in Prague: Faculty of Education and ERME.

International Conference Presentations

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Ulusoy, F. (2013, April). An investigation of the concept of variable in Turkish elementary mathematics teachers' guidebooks. Paper presented at *4th*

International Conference on New Trends in Education and Their Implications, Antalya- Turkey. Abstract retrieved from http://iconte.org/FileUpload/ks59689/File/7._iconte_bildiri_ozetleriii.pdf

Yeniterzi, B., & **Ulusoy, F.** (2013, September). Investigation on pre-service mathematics teachers' knowledge about eight grade students' possible errors in exponents. Paper presented at *The European Conference on Educational Research* (ECER), Bahçeşehir University, İstanbul. Abstract retrieved from http://www.eera-ecer.de/ecer-programmes/conference/8/contribution/21092/

Ulusoy, F. (2012, September). Using history of mathematics in learning and teaching of the concept of variable. Paper presented at *The European Conference on Educational Research* (ECER), Cadiz, Spain. Abstract retrieved from http://www.eera-ecer.de/ecer-programmes/conference/6/contribution/16735/

Ulusoy, F. (2012, September). 8th grade students' algebraic thinking processes based on different meanings of variable. Paper presented at *The European Conference on Educational Research* (ECER), Cadiz, Spain. Abstract retrieved from http ://www.eera-ecer.de/ ecer-programmes/ conference/6 /contribution/16732/.

Bayık (Ulusoy), F., & Argün, Z. (2011, July).The effects of eleventh grade students' geometric reasoning processes on their representations. In B. Ubuz (Ed.). *Proceedings of the 35 the Conference of the International Group for the Psychology of Mathematics Education* (PME 35) (Vol. 2, pp.259), Ankara, Turkey: PME.

National Conference Papers & Presentations

Ulusoy, **F**. & Çakıroğlu, E. (2016, September). Ortaokul öğrencilerinin paralelkenarı ayırt ederken kullandıkları yaklaşımlar. Accepted at the meeting of *XII. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (XII. UFBMEK)*. Trabzon, Turkey.

Çakıroğlu, E. & **Ulusoy, F.** (2015, Mayıs). Öğretmen adaylarının küçük ölçekli araştırma projeleri yoluyla öğrenci düşünüşlerini incelemesi: örnek bir ders uygulaması. Paper presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

Ulusoy, F. (2014, Eylül). Ortaokul matematiğinde paralellik ve diklik kavramları: öğrencilerin sahip olduğu imgeler ve yaşadığı yanılgılar. In P. Fettahlıoğlu vd. (Ed.) *XI. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (XI. UFBMEK) Bildiri Özetleri Kitapçığı*. (s.1121-1123). Adana, Türkiye.

Ulusoy, F. & Yeniterzi, B. (2013, Eylül). Sekizinci sınıf öğrencilerinin tam sayıların üslü ifadelerindeki işlemlerle ilgili hataları. *22. Ulusal Eğitim Bilimleri Kurultayı Bildiri Özetleri*. (s. 250), Eskişehir, Türkiye: Pegem Akademi.

Ulusoy, F. (2013, Mayıs). Altıncı sınıf öğrencilerinin iki doğrunun birbirine paralel veya dik olma durumlarına ilişkin sahip oldukları kavrayışlar. *12. Matematik Sempozyumu, Sergi ve Şenlikleri Kitapçığı*, (s. 55-58). Ankara, Türkiye: Milli Eğitim Bakanlığı.

Ulusoy, F. (2012, Eylül). 11. sınıf öğrencilerin farklı temsil türlerinde örüntü genelleme becerileri. *Uygulamalı Eğitim Kongresi*, (s.182-183). Ankara, Turkey: Milli Eğitim Bakanlığı.

Yemen-Karpuzcu, S., **Ulusoy, F.** & Işıksal, M. (2012, Haziran). İlköğretim matematik öğretmen adaylarının geometrik cisimler ile yükseklik-hacim grafiklerini yorumlamaları üzerine bir çalışma. In M. Şahin vd. (Ed.) *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (X. UFBMEK) Bildiri Özetleri*. (s.342). Niğde, Türkiye: Pegem Akademi.

Ulusoy, F., & Argün, Z. (2012, Haziran). 11. sınıf öğrencilerinin çokgenlerle ilgili muhakeme süreçlerinde ürettikleri temsiller arasında kurdukları etkileşimler. In M. Şahin vd. (Ed.) *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (X. UFBMEK) Bildiri Özetleri*. (s.469). Niğde, Türkiye: Pegem Akademi.

Ulusoy, F., & Çakıroğlu, E. (2012, Haziran). İlköğretim Matematiğinde Histogram ve Öğretimi: Öğretmenlerin Algıları, Kavrayışları Ve Sorunları. In M. Şahin vd. (Ed.) *X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi (X. UFBMEK) Bildiri Özetleri*. (s.416). Niğde, Türkiye: Pegem Akademi.

National Poster Presentations

Ayhan, Z., **Ulusoy, F.** & Çakıroğlu, E. (2015, Mayıs). Çarpan-kat kavramından ebob-ekok kavramlarına giden yolda ortaokul öğrencilerinin sahip olduğu bilgiler ve yanılgılar. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

Başer, E. E., **Ulusoy, F.** & Çakıroğlu, E. (2015, Mayıs). Üslü sayılarda kavramsal ve işlemsel bilgi arasındaki kopukluklar: bir durum çalışması. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye. [**Best Poster Award**]

Çakıroğlu E. & **Ulusoy, F**. (2015, Mayıs). İlköğretim matematik eğitiminde işlem akıcılığı. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

Çatman, E., **Ulusoy, F.** & Çakıroğlu, E. (2015, Mayıs). 6. sınıf öğrencilerinin ondalık sayılar hakkındaki düşünüşleri. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

Gülden, B., **Ulusoy, F.** & Çakıroğlu, E. (2015, Mayıs). 7. sınıf öğrencilerinin simetri kavramı hakkındaki bilgileri. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

İnan, M., **Ulusoy, F.** & Çakıroğlu, E. (2015, Mayıs). Ortaokul öğrencilerinin üçgende yükseklik ile ilgili sahip oldukları kavram imajları. Poster presented at *Türk Bilgisayar ve Matematik Eğitimi Sempozyumu-2 (TURKBİLMAT 2).* Adıyaman Üniversitesi, Türkiye.

SCHOLARSHIPS AND GRANTS

- Scholarship of the Scientific and Technological Research Council of Turkey (TUBITAK 2210) for Master Degree, Turkey (2007-2009)
- Scholarship of the Scientific and Technological Research Council of Turkey (TUBITAK 2210) for PhD Degree, Turkey (2010-2015)
- TÜBİTAK 2224-B Yurt İçi Bilimsel Etkinliklere Katılım Desteği (2015, May)
- Financial Support for CERME-9 from ERME Graham Litter Fund (2015, February)
- Financial Support for PME40 from Skemp Fund (2016, August)

HOBBIES

Writing poetry, painting, cooking

Appendix 12

TEZ FOTOKOPİSİ İZİN FORMU

ENSTİTÜ

Fen Bilimleri Enstitüsü ☐

Sosyal Bilimler Enstitüsü ☒

Uygulamalı Matematik Enstitüsü ☐

Enformatik Enstitüsü ☐

Deniz Bilimleri Enstitüsü ☐

YAZARIN

Soyadı : ULUSOY

Adı : FADİME

Bölümü : ELEMENTARY MATHEMATICS EDUCATION

TEZİN ADI (İngilizce) : DEVELOPING PROSPECTIVE MATHEMATICS TEACHERS' KNOWLEDGE FOR TEACHING QUADRILATERALS THROUGH A VIDEO CASE-BASED LEARNING ENVIRONMENT

TEZİN TÜRÜ : Yüksek Lisans ☐ Doktora ☒

1. Tezimin tamamından kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
2. Tezimin içindekiler sayfası, özet, indeks sayfalarından ve/veya bir bölümünden kaynak gösterilmek şartıyla fotokopi alınabilir. ☐
3. Tezimden bir (1) yıl süreyle fotokopi alınamaz. ☒

TEZİN KÜTÜPHANEYE TESLİM TARİHİ: