THE EFFECT OF METACOGNITIVE INSTRUCTIONAL METHOD ON ELEVENTH GRADE STUDENTS' METACOGNITIVE SKILL AND MATHEMATICAL PROCEDURAL AND CONCEPTUAL KNOWLEDGE

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I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

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ABSTRACT

THE EFFECT OF METACOGNITIVE INSTRUCTIONAL METHOD ON ELEVENTH GRADE STUDENTS' METACOGNITIVE SKILL AND MATHEMATICAL PROCEDURAL AND CONCEPTUAL KNOWLEDGE

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The purpose of the study was to investigate the effect of metacognitive instructional method, compared to traditional instruction on eleventh grade science student's mathematical procedural and conceptual knowledge, and metacognitive skills. Sixty-six eleventh-grade students in a school in Bandung City, Indonesia took part in this study. Matching-only pre-test-post-test control group design was conducted. The classes were randomly assigned to experimental and control group. In the experimental group metacognitive instructional method called IMPROVE (Introduction, Metacognitive Inquiry, Review, Practicing, Obtaining Mastery, Verification, and Enrichment) was applied, whereas in the control group traditional instruction was used to teach composition and inverse function, infinite sequence and series, and line equations topics within 9 weeks. The data collection tools used were Procedural and Conceptual Knowledge Test, Metacognitive Awareness Inventory. Using Multivariate Analysis of Covariance, the main effects of teaching methods, gender, and interaction between them were investigated. Consequently, (1) IMPROVE instructional method was more effective in supporting students' procedural and conceptual knowledge, and regulation of cognition, (2) there was gender difference in students' procedural and conceptual knowledge, and regulation of cognition in favour of female students. No interaction between instructional methods and gender was found.

Another purpose of the study was to explore students' experience with metacognitive instructional method. Therefore, qualitative data was compiled. Fourteen students who had different abilities from experimental group were interviewed semi-structurally after the treatment. The data was transcribed, coded and categorized. Generally, students took benefits from IMPROVE instructional method even though at the beginning they encountered with several challenges.

Keywords: Metacognitive Instructional Method, IMPROVE Instructional Method, Mathematical Procedural Knowledge, Mathematical Conceptual Knowledge, Metacognition.

ÖZ

ÜSTBİLİŞSEL ÖĞRETİM YÖNTEMİNİN ON BİRİNCİ SINIF ÖĞRENCİLERİNİN ÜSTBİLİŞSEL BECERİLERİNE VE İŞLEMSEL VE KAVRAMSAL MATEMATİK BİLGİLERİNE ETKİSİ

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Bu çalışmanın amacı üstbilişsel öğretim yönteminin on birinci sınıf öğrencilerinin işlemsel ve kavramsal matematik bilgilerine ve üstbilişsel becerilerine etkisini incelemekti. Endonezya'da Bandung şehrinde bulunan bir okulda okuyan altmış altı on birinci sınıf öğrencisi bu çalışmada yer aldı. Eşleştirmeli ön test-son test kontrol grup deseni kullanılmıştır. Sınıflardan rastgele seçilen bir sınıf deney, diğer sınıf kontrol grubu olarak berlirlenmiştir. Fonksiyonların bileşkesi ve tersi, sonsuz diziler ve seriler, ve doğru denklemlerini anlatmak için, deney grubunda IMPROVE isimli üstbilişsel öğretim yönteminin (Giriş, Üstbilişsel sorgulama, Gözden geçirme, Pratik yapma, Ustalaşma, Doğrulama ve Zenginleştirme) öğretim yöntemi, kontrol grubunda ise geleneksel yöntem dokuz hafta boyunca uygulanmıştır. Kullanılan veri toplama araçları İşlemsel ve Kavramsal Matematik Bilgi Testi, Üstbiliş Farkındalık Ölçeği, gözlem ve görüşmelerdir. Bu çalışmanın hipotezlerini test etmek için Çok Değişkenli Kovaryans Analizi kullanılmıştır. Elde edilen verilere göre, (1) öğrencilerin işlemsel ve kavramsal matematik bilgisinde ve bilişin düzenlenmesinde IMPROVE öğretim yöntemi daha fazla etkiliydi ve (2) kız öğrencilerin işlemsel ve kavramsal matematik bilgilerinde ve bilişin düzenlenmesinde daha etkin olduğunu göstermiştir. Öğretim yöntemi ile öğrenci cinsiyetinin arasındaki etkileşim bulunmamıştır.

Bunlara ek olarak, çalışmanın bir diğer amacı üştbiliş öğretim yöntemi uygulanan gruptaki öğrencilerin deneyimlerinin incelenmesiydi. Bunun için nitel veriler toplanmıştır. Bunlardan biri olan yarı yapılandırılmış görüşmeler IMPROVE uygulaması sonrasında on dört farklı yeteneklere sahib öğrenci ile yapılmıştır. Görüşmeler birebir olarak yazılmış, kodlamış ve kategoriler elde edilmiştir. Sonuç olarak, öğrencilerin büyük bir çoğunluğu uygulama sırasında bazı zorluklarla karşılaşmalarına rağmen çeşitli kazanımlar da elde etmişlerdir.

Anahtar Kelimeler: Üstbilişsel Öğretim Yöntemi, IMPROVE Öğretim Yöntemi, İşlemsel Matematik Bilgisi, Kavramsal Matematik Bilgisi, Üstbiliş

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LIST OF ABBREVIATIONS

PROC	: Procedural Knowledge
CONC	: Conceptual Knowledge
MAI	: Metacognitive Awareness Inventory
KC	: Knowledge of Cognition
RC	: Regulation of Cognition
MANCOVA	: Multivariate Analysis of Covariance
ANCOVA	: Analysis of Covariance
Pre-PROC	: Pre-test of Procedural Knowledge
Post-PROC	: Post-test of Procedural Knowledge
Pre-CONC	: Pre-test of Conceptual Knowledge
Post-CONC	: Post-test of Conceptual Knowledge
Pre-KC	: Pre-test of Knowledge of Cognition
Post-KC	: Pre-test of Knowledge of Cognition
Pre-RC	: Pre-test of Regulation of Cognition
Post-RC	: Post-test of Regulation of Cognition
IMPROVE	: Introduction, Metacognitive Questions, Practice, Review, Obtain Mastery, Verification, Enrichment and Remedial
PROCA	: Knowledge of mathematical formal language or symbol representation system
PROCB	: Knowledge of collection of formulas and algorithms
CONCA	: Knowledge of general principles
CONCA1	: Explanation of concepts tasks
CONCA2	: Evaluation of examples tasks
CONCB	: Knowledge of the principles underlying procedures
CONCB1	: Application and justification of procedures tasks
CONCB2	: Evaluation of procedures tasks
M	: Mean or Average
SD	: Standard Deviation

Max.	: Maximum
Min.	: Minimum
SKEW.	: Skewness
KURT.	: Kurtosis
DVs	: Dependent Variables
IVs	: Independent Variables
Sig.	: Significance
N	: Sample Size
df	: Degrees of Freedom
SPSS	: Statistical Package for the Social Sciences
HA	: High Achiever Student
MA	: Moderate Achiever Student
LA	: Low Achiever Student

CHAPTER 1

INTRODUCTION

The sine qua non objectives of mathematics instruction in various levels of education around the world are to focus on improvement of students' conceptual understanding of central mathematical ideas and procedural fluency – what are often known as rigorous goals in students' learning (Lampert, 2001). Correspondingly, the NCTM Principles and Standards for School Mathematics (2000) gave special attention to the significance of establishing a strong foundation in the underlying concepts and skills in learning mathematics. The standards then encouraged mathematics teachers to modify their instructional strategies in order to assist students in both mastering procedures and obtaining a deep comprehension of mathematical concepts.

However, the balance of students' ability to perform algorithm successfully and to grasp mathematical ideas properly in mathematics learning in fact seems to be far from what is expected as vast majority of mathematics classrooms tend to put the first over the latter. In most occasions, a plethora of students learn mathematics by dealing with routine problems and manipulative practice. The proverb "practice makes perfect" has become mantra that influences all activities within this school mathematics culture. Learning mathematics then is perceived as process of mastering predetermined knowledge and procedure. Teachers do their job by presenting topics in brief and simple, informing students the proper and efficient algorithm, and subsequently letting students to practice individually on mathematics exercise (Goos, 1996). Students become highly skilled by imitating what teachers have done and doing excessive practice. As a result, mathematics classrooms have been dominated by memorization of rules and blind execution which pay little attention to understanding and consciousness (Kloosterman, 2002). Moreover, this state of affairs might lead students to perceive mathematics as lesson whose concepts or topics are not integrated or connected each other and subsequently they do not learn mathematics in meaningful manner (Richland, Stigler, & Holyoak, 2012).

It is in fact in line with what Richards (2002) described as “school mathematics culture” in which teaching process is a transfer of information. Mathematics lesson has been taught traditionally in which teachers play role as dispenser of knowledge and the students are described as vessel into which knowledge is to be poured (Boulton-Lewis, Smith, McCrindle, Burnett, & Campbell, 2001). Rather than learning how to make sense of the logical structure of mathematics, within this tradition students come to see mathematics as a lesson which consists of set of rules for representing strings of numbers and letters that have to be memorized for passing mathematical tests or examinations (Kloosterman, 2002). Moreover, Binkley et al. (2012) summarized real condition of mathematics classrooms currently. They revealed that by and large teachers and schools were likely to emphasize on what was assessed and evaluated rather than on what the principal ideas, therefore learning objectives were no longer being considered.

This condition also influences students’ achievement in mathematics subject in which they encountered difficulties in learning mathematics and solving various mathematical problems. Errors or mistakes that students make when solving problems as a matter of fact stem from their deficiency of both conceptual knowledge and procedural knowledge (Johari, Nor Hasniza, & Mahani, 2012).

Procedural knowledge and conceptual knowledge should be interwoven and integrated each other (Kilpatrick, Swafford, & Findell, 2001). Procedural knowledge can assist students in solving problem and calculating accurately, effectively, and efficiently (Rittle-Johnson, Star, & Durkin, 2012). While, conceptual knowledge leads to meaningful learning in mathematics lesson (Long, 2011), assist students in promoting flexibility (Crooks & Alibali, 2014), and improves consciousness and regulation of cognitive (Moos & Azevedo, 2008).

Accordingly, teachers have to encourage students to improve both procedural knowledge and conceptual knowledge in learning mathematics through exposing appropriate instructional methods. Augmentation of attention to integrate and combine procedural knowledge and conceptual knowledge, however, does not parallel with its application in actual mathematics classrooms. The reason might be that teachers did not have adequate information with regard to widely appropriate and applicable instructional methods in regular classrooms that aimed at developing procedural knowledge and conceptual knowledge (Ma, 1999).

Baker and Czarnocha (2002) revealed that developing both students' procedural knowledge and conceptual knowledge could be performed by promoting metacognitive skills. Rittle-Johnson et al (2001) argued that metacognitive skill is a bridge for students to attain conceptual knowledge from procedural knowledge through iterative process. Those arguments emphasized the importance of metacognition in developing students' procedural knowledge and conceptual knowledge. Therefore, according to them development of procedural and conceptual knowledge could not be separated from the development of metacognition.

Metacognition which is defined commonly as thinking about thinking allows students to recognize their personal ability, encountered problems or tasks, and strategies will be used (Schraw, 1998). When accomplishing a task, metacognition enables students to regulate their cognitive process by making plan, activating control, and evaluation (Schraw, 1998). To do this, students are required to ask questions to themselves implicitly or explicitly before, during, and after accomplishing tasks (Mevarech & Amrany, 2008). By developing metacognitive skills, learners can learn in meaningful situation, flexible manner, and careful with strong foundation of mathematics concepts, and subsequently they can cope with various tasks successfully (Veenman, Wilhelm, & Beishuizen, 2004) and promote achievement in problem solving (Mevarech & Kramarski, 2014). In this century, metacognition is important indicator of a term called "educated intellect" which leads to effective learning. It assists student in developing independent and high achievement which are necessary in facing new century (Papleontiou-louca, 2003).

Due to its importance, over the past decades the application and research of metacognition has become trends in mathematics education and efforts has been made in order to develop students' metacognitive skills. According to Schraw (1998) metacognitive skill can be trained by means of certain instruction that encourages students to regulate their cognitive process and understand their personal ability, tasks or problem, and strategies. Currently, metacognitive instruction, as an instructional method, has taken respectable attention from several researchers as it serves to aid pupils in monitoring and controlling effectiveness and accuracy of their own understanding of mathematics topics or concepts so that students can develop their knowledge and be aware of their own thought processes (e.g. Askell-Williams, Lawson, & Skrzypiec, 2012; Lee, Yeo, & Hong, 2014; Tok, 2013).

There are several metacognitive instructional methods that have been developed and implemented in various studies such as Polya's heuristic for solving mathematics problem (2014), Schoenfeld's metacognitive instructional model (1985), IMPROVE instructional method (Mevarech & Kramarski, 1997), Verschaffel's model of metacognition instruction (1999) and Singapore model of mathematics problem solving (Fan & Zhu, 2007). All of the methods capitalize on self-directed metacognitive questions but vary in that of details, scope and age range. Polya's and Schoenfeld's metacognitive instructional models are designed to be implemented with under-graduated students and on single complex, unfamiliar and non-routine problems, whereas IMPROVE instructional method, Verschaffel, and the Singapore model can be implemented to younger students and for a set of problems or even an entire curriculum.

IMPROVE is powerful mathematics instructional method as it can be implemented widely in regular mathematics classrooms which consist of heterogeneous ability students (Mevarech & Kramarski, 1997). Mevarech and Kramarski (1997) introduced IMPROVE instructional method which is an acronym of all the instructional procedures which stand for the method: (1) Introducing the novel concept, (2) Metacognitive inquiry, (3) Practicing, (4) Reviewing, (5) Obtaining proficiency on higher and lower cognitive progress, (6) Verification, and (7) Enrichment and remedial. As a matter of fact, the method stems from cognition, social cognition and metacognition, thus it consists of three interdependent elements: metacognitive activities, cooperative setting, and systematics provision of feedback-corrective-enrichment (Mevarech & Kramarski, 1997). The purpose of this method is to promote students' capability of activating metacognitive processes which in turn amplify students' understanding of mathematical concepts (Mevarech & Fridkin, 2006). With respect to metacognitive inquiry, Mevarech and Kramarski (1997) revealed that it involves four facets of self-addressed questions, to wit, comprehension questions, connection questions, strategic questions, and reflection questions. Each question contributes to students' proliferation in promoting metacognitive process and understanding.

Considerable studies of the implementation of IMPROVE instructional method have been conducted including that of effect on mathematics achievement in algebra (Mevarech & Kramarski, 1997), students' mathematical knowledge, mathematical reasoning and metacognition (Mevarech & Fridkin, 2006), algebraic procedural and real-life problems concerning conceptual mathematical explanations (Kramarski, 2008a), authentic and standard tasks (Kramarski, Mevarech, & Arami, 2002), mathematical achievement of students with mathematical learning difficulties (Grizzle-Martin, 2014), and mathematics achievement and regulation of cognition (Mevarech & Amrany, 2008). These studies corroborate that IMPROVE instructional method has many benefits over traditional instruction in terms of cognitive and metacognitive aspects such achievement, metacognitive skills, as well as solving various types of problems. However, according to Mevarech and Fridkin (2006) further study is necessary to investigate the effect of IMPROVE instructional method on high school students' metacognitive skills in different country by considering gender differences. Furthermore, its effect on mathematical procedural and conceptual knowledge is still open to be investigated.

To sum up, procedural knowledge, conceptual knowledge, metacognition (knowledge of cognition and regulation of cognition) are important elements for students to reach academic achievement, and IMPROVE as a metacognitive instructional method has a potential of improving those elements. Besides, according to the literature there are several gaps in studies of the implementation of IMPROVE instructional method, procedural knowledge, conceptual knowledge, and metacognitive skills. Therefore, the present study is aimed to examine the effect of IMPROVE instructional method on students' procedural knowledge, conceptual knowledge, and metacognitive skills. In addition, students' experiences with IMPROVE instructional method are investigated in this study.

The Purpose of the Study

In the light of the above literature, the benefits of IMPROVE instructional method on mathematics education inspired this study to investigate the effect of IMPROVE instructional method over traditional instruction on 11th grade high school students' procedural and conceptual knowledge, knowledge of cognition, and regulation of cognition.

1.1. Significance of the Study

Both procedural and conceptual knowledge are crucial for students especially in mathematics learning (Lauritzen, 2012). Accordingly, presenting effective instruction that interweave the two knowledge is problematic issues for mathematics educators. In a plethora of occasions, mathematics lessons are presented partially in which emphasizing solely procedural knowledge and setting aside conceptual knowledge (Hasenbank, 2006). As a matter of fact, this issue is of ongoing, unresolved, and crucial problem that always occur in many places, particularly in Indonesia, and serious and effective efforts have to be made immediately. This problem also consequently would affect how mathematics are perceived by the students (Kloosterman, 2002). Therefore, at this point instructional method plays inevitable significant role and applying traditional instruction that heavily emphasizes on procedural knowledge evidently is not appropriate solution to address the aforementioned problem.

In addition to procedural and conceptual knowledge, Binkley et al. (2012) listed necessary skills that students have to develop in this century – they called it as twenty-first century skills, one of which is metacognition. Unfortunately, metacognitive skills couldn't be developed in regular teaching that attach great importance to transmission and learning rote. It is important to note that considerable studies revealed that metacognitive skills played notable role in enhancing students' achievement in learning mathematics and problem solving (Ozsoy & Ataman, 2009; Pennequin, Sorel, & Mainguy, 2010; Teong, 2003). Besides, as metacognitive ability could be taught (Martinez, 2006), thus for two last decades researchers have made effort to develop instruction that embed metacognitive skills in teaching and learning process (Kramarski et al., 2002; Mevarech & Amrany, 2008; Mevarech & Kramarski, 1997) .

One of the first metacognitive instructional methods implemented for elementary until high school levels is IMPROVE instructional model which developed by Mevarech and Kramarski (1997). In the literatures, its significant effects have been reported, such as, the improvement of mathematics achievement (Mevarech & Amrany, 2008; Mevarech & Kramarski, 1997), students' mathematical knowledge, mathematical reasoning and metacognition (Mevarech & Fridkin, 2006), algebraic procedural and conceptual mathematical explanations (Kramarski, 2008a), authentic and standard tasks (Kramarski et al., 2002), mathematics achievement of students with mathematical learning difficulties (Grizzle-Martin, 2014), and regulation of cognition (Mevarech & Amrany, 2008). Since it was first developed, studies related to IMPROVE instructional method have proved its validity ecologically in which it effectively could be implemented in regular classroom by common mathematics teachers.

This study develops a new application of IMPROVE instructional method which contributes to the structure knowledge of metacognitive instructional method as it is necessary to examine theoretically and practically the effect of IMPROVE instructional method on students from different countries (Mevarech & Fridkin, 2006). In addition, by considering previous studies, it is mostly found that the use of IMPROVE instructional method only on specific mathematics topics. Moreover, it is necessary to implement IMPROVE instructional method on various topics in mathematics within regular classroom and investigate its effect on procedural knowledge, conceptual knowledge, and metacognitive skills. With respect to the implementation of IMPROVE instructional method in Indonesia particularly, there are very few research studies conducted about the effect of IMPROVE instructional method for improving students' procedural, conceptual knowledge, and metacognitive skills.

What is more, the literature suggests that further research would be necessary to investigate the impact of IMPROVE instructional method on high school students in mixed gender classroom with emphasizing both knowledge of cognition and regulation of cognition (Mevarech & Fridkin, 2006). In addition, this study would address the need to conduct observations and interview under instructional method in order to uncover its effectiveness (Mevarech & Fridkin, 2006).

Based on the objectives of mathematics lesson in the new national mathematics curriculum in Indonesia, procedural knowledge, conceptual knowledge, as well as metacognitive skills have to be attached simultaneously in teaching and learning process. Therefore, mathematics curriculum developer and Indonesian mathematics teachers particularly could gain valuable information about the implementation of IMPROVE instructional method on students' procedural knowledge, conceptual knowledge, and metacognition in mathematics classrooms. In general, the findings of this study will contribute to mathematics education field considering that procedural, conceptual knowledge, and metacognitive skills are important elements for students to reach academic achievement, and IMPROVE as a well-known metacognitive instructional method has a potential to promote those elements.

In addition, the developed lesson plans and teaching materials used in this study might be applied by mathematics teachers in their lessons. The developed lesson plans and teaching materials can be modified by mathematics teachers by considering students' prior knowledge and ways of thinking. Thereunto, the implementation of IMPROVE instructional method can be introduced and attached in mathematics textbooks by authors who are trying to seek for teaching strategy to develop students' metacognitive skills.

The result of this study may be applied as foundation for the policy makers to strengthen their support in formulating and activities that will contribute to the effort of teachers' professional development. Student teachers in educational faculty may be provided comprehensive insight with respect to IMPROVE instructional method, and they could implement the instruction when they are conducting experience field in teaching practice. In addition, IMPROVE instructional method can be trained for in-service teachers in professional development.

With respect to procedural and conceptual knowledge, as there are various conceptions about definition of conceptual knowledge in which it influences the way how to assess it, this study also can subscribe significant information pertaining to valid and reliable instruments for measuring and assessing procedural knowledge and conceptual knowledge by means of new developed framework (Crooks & Alibali, 2014).

Moreover, this study provides extensions and likelihood of future research topics that were elucidated during this current study but weren't its focus.

1.2.Definition of Important Terms

In this section definitions of important terms will be illuminated.

- Metacognitive Knowledge or Knowledge of Cognition: It refers to personal recognition of their and general cognition (Schraw, 1998) or initial contemplation of a task. Metacognitive knowledge considers interaction among knowledge about person, task, and strategies (Flavell, 1979). There are three distinct metacognitive awareness: declarative knowledge, procedural knowledge, and conditional knowledge. Declarative knowledge is defined as knowledge concerning person's own self as a learner and what factors affect ones' performance. Procedural knowledge refers to knowledge about the way how to do things. Conditional knowledge refers to understanding of condition in which declarative and procedural knowledge are used.
- Metacognitive Regulation or Regulation of Cognition: It refers to set of activities to regulate learners' cognition when learning or accomplishing tasks. Metacognition regulation consists of three elements: planning, monitoring, and evaluating (Schraw, 1998). Planning activities refer to task analysis and goal setting by selecting appropriate strategies and providing sources required for accomplishing tasks. Monitoring refers to on-line awareness of understanding and task performance. Evaluating activity refers to judging the result or answer and efficiency of students' learning.
- IMPROVE: IMPROVE is an abbreviation of which stands for sequential teaching steps that construct the method: Introduction, Metacognitive self-directed questions, Practice, Review, Obtaining mastery, Verification, and Enrichment (Mevarech & Kramarski, 1997).
- Traditional Instruction*:* It is an instruction in which students are instructed by means of lecture and emphasizing calculation and manipulation of mathematical symbols, hence students learn mathematics in the absence of meaning.
- Procedural Knowledge: It refers to students' ability to implement, calculate, and execute symbols representation system and algorithms to solve problem

accurately, efficiently, and appropriately (Lauritzen, 2012; Rittle-Johnson, Siegler, & Alibali, 2001).

- Conceptual knowledge: It refers to comprehension of general principles such as mathematics rules, meaning of symbols, and domain of structure (concept) and knowledge of principle underlying procedures by making use of connection (Crooks & Alibali, 2014).
- Mathematics Topics: It refers to three consequent mathematics topics presented for 11^{th} grader in the 1^{st} semester: Composition and Invers Function, Infinite Sequence and Series, and Line Equations.
- High School in Indonesia: It is a school which provides students with part or all of their secondary education from year 10 to year 12. It comes after finishing junior high school (year 7 to year 9) and is followed by higher education. Students are required to attend this level from ages 15 to 17, scheduling between 32 until 36 hours of class a week. In this level, students are categorized in three distinct specializations (i.e. science, social, and language) based on their interest and achievement in junior high school.
- 11^{th} grader in Indonesia: Students in this level are typically 16 or 17 years of age. The eleventh grade is the second year in high school level. The topics of mathematics taught in this level are algebra, function, geometry, statistics, and preliminary calculus.

1.3.The problems

In this section, the main problems, sub-problems and hypotheses of the study are stated.

1.3.1. The Main Problems

1. What is the effect of different teaching methods and gender on 11^{th} grade science major Indonesian students' procedural knowledge, conceptual knowledge, metacognitive skills compared to traditional instruction?
2. How do 11^{th} grade science major Indonesian students experience IMPROVE instructional method?

1.3.2. The Sub-problems

1. What is the main effect of teaching methods (IMPROVE instructional method and traditional instruction) on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition, when all students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled?
2. What is the main effect of gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled?
3. What is the interaction effect between teaching methods and gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled?

1.4.Hypotheses

1. There is no statistically significant main effect of teaching methods (IMPROVE instructional method and traditional instruction) on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled.
2. There is no statistically significant main effect of gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural

knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled.

3. There is no statistically significant main interaction effect between teaching methods and gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled.

CHAPTER 2

LITERATURE REVIEW

In this chapter, a closer look is taken at related literature of the current study. At the beginning, theoretical frameworks of procedural and conceptual knowledge, and detail information concerning IMPROVE instructional method will be discussed. This is followed by presenting prior studies related to variables used in this study. General information with respect to education system in Indonesia will be illuminated before ending the chapter with a section presenting a few concluding remarks of the literature review to highlight important parts of the study.

2.1. Procedural and Conceptual Knowledge

Mathematical knowledge can be explained into two distinct elements: knowledge about 'how' (skill or knowledge) and knowledge about 'why' (understanding) (Lauritzen, 2012). Most researchers have defined knowledge about 'how' as procedural knowledge and knowledge about 'why' as conceptual knowledge. There has been a great deal of debate pertaining to whether the two are categorized as types of knowledge or quality of knowledge. Cognitive psychologists and mathematics educators are those who have opposite views with respect to this categorization. While psychologists argue that the two knowledge are differed in terms of types, mathematics educators argue that it describes quality of students' understanding (Star & Stylianides, 2013).

Star and Stylianides (2013) described knowledge quality as query of how deep and well something is comprehended. It can be at superficial level, deep level, or whatever in the midst of them. To put it differently, students' knowledge can be put in certain point along continuum of understanding. Superficial knowledge is explained as procedural knowledge, whereas deep knowledge is explained as conceptual knowledge. Mathematics educators, by considering this, contend that conceptual knowledge is in some way better than the rest (Maciejewski, Mgombelo, & Savard, 2011). Besides, knowledge type is question about what is known. The terms procedural and conceptual indicate what sort of knowledge being characterized (Star & Stylianides, 2013). Accordingly, at the same time a person can have both or one of them.

Not surprisingly, these controversial arguments lead to divergence relative to its definition and, consequently, subsequent exploration pertaining to assessment in mathematics teaching and learning. While it is categorized as knowledge type, most researchers are not likely to encounter significant problem developing instruments to evaluate students' knowledge, thanks to its distinct and well-defined characteristics. However, developing valid instrument to measure students' knowledge quality is likely to face difficulty (Star & Stylianides, 2013).

The terms procedural knowledge and conceptual knowledge as a matter of fact were introduced initially in 1980s by Hiebert and Lefevre. Various researchers used different terms such as instrumental understanding and relational understanding (Sfard, 1991), syntactic and semantic (Nesher, 1986), operational and structural understanding (Skemp, 1978), fragmented conception and cohesive conception (Crawford, Gordon, Nicholas, & Prosser, 1994) to represent the types of knowledge. Even though they did not use identical terms, no fundamental differences were found and generally they possessed common tenets relatives to what are called as procedural knowledge and conceptual knowledge (Hiebert & Lefevre, 1986)

2.1.1. Procedural Knowledge

Procedural knowledge has been defined in numerous ways by researchers. In straightforward manner, Rittle-Johnson, et al (2001) defined it as the ability to implement formula to solve problem. It heavily relies on computational ability and implementation of procedures (Lauritzen, 2012). Hiebert and Lefevre (1986) provided detail insight about procedural knowledge by focusing on two separate elements, to wit, the knowledge of mathematical formal language or symbol representation system and the knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks.

a. The knowledge of mathematical formal language or symbol representation system encompasses students' familiarity with symbol and consciousness of the syntactic formulas. The familiarity and consciousness refer to students' recognition and judgment whether mathematical ideas are expressed in plausible form. For instance, students who hold procedural knowledge can acknowledge that the expression $3.5 \div \square = 2.71$ is plausible and that the expression $3.5 + = \square 2$ is not plausible configuration syntactically. Lauritzen (2012) enlightened that this knowledge did not consider calculation or that of meaning, but rather being able to distinguish plausible from implausible form in use of writing symbols. He also put symbols as fundamental elements in constructing procedures or algorithms.
b. The knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks is composed of mathematical algorithms, rules, or formulas. To reach intended result, certain steps and procedures have to be accomplished in sequential manner. The sequence of procedures or step-by-step procedure is key feature in executing provided formulas. To put it in another way, students, who hold this element of knowledge, recognize the order of steps that have to be performed firstly, secondly, and so forth. Actually, those steps are linked each other, yet it is in restricted areas. For instance, the expression $f \circ g(x) = f(g(x))$ leads students to think that the first step is dealing with $g(x)$ and substituting that of result to $f(x)$. Even though awareness of sequence is taken into consideration, reasons behind it cannot be presented consciously. In addition, to reach final answer, in a plethora occasions mathematical symbols are substituted with numerical value. Therefore, it is clearly that this element of knowledge is still in superficial level of knowledge.

Claims that the need of procedural knowledge or superficial knowledge in the future of mathematics education is less cannot justify that it should be disregarded in mathematics classroom at all. Several researchers proposed that in order for students to achieve conceptual knowledge, they are in need of mastering procedural knowledge (Lauritzen, 2012). In addition, developing procedural knowledge in flexible manner could assist students in gaining proficiency in problem solving (Rittle-Johnson et al., 2012).

Question about in what way that procedural knowledge can be taught best is subsequent orientation that attract mathematics educators' attention. Miller and Hudson (2007) proposed five guidelines applied to boost procedural knowledge: (1) providing sequential set of steps that direct to the answer of the problem; (2) generalizing the steps of the strategy; (3) encouraging to undertake an extroverted performance, applying a cognitive and metacognitive strategies, or executing a formula; (4) simplifying the word and the number of the steps; and (5) helping students do retrieval the steps of the strategy by means of mnemonic. In this guidelines, Miller and Hudson emphasized the importance of metacognitive strategy that can aid students in advancing their procedural knowledge.

Pertaining to assessment of procedural knowledge, mathematics educators as well as researchers had made effort to develop instruments to evaluate students' procedural knowledge. Definition and categorization of procedural knowledge determine types of instruments used to measure it. There are different arguments about deciding whether certain instruments being used are intended to measure procedural knowledge or conceptual knowledge. On this, Lauritzen (2012) claimed that the two knowledge cannot be assessed by means of one instrument, rather it is necessary to design independent conceptual and procedural task and to investigate students' attainment in such tasks. In addition, Schneider and Stern (2005) found that considerable studies have been conducted to measure procedural knowledge, yet it tended to be unclear that it can be measured with sufficient reliable and valid assessment instrument. Therefore, current research in mathematics education is in need of effort to develop and construct reliable, valid, and independent instruments for measuring procedural knowledge.

Generally speaking, as procedural knowledge emphasizes actions deal with symbols, and algorithms, thus most researchers typically employ routine tasks which stress on computational and executional aspects (Rittle-Johnson et al., 2001). In addition, Star (2000) argued that since procedural knowledge is considered as skill that asks students to perform it correctly, thus, he proposed the use of non-verbal assessment through observing the execution of a rule.

Mathematics teaching and learning which emphasizes on solving routine problem often leads students to make use of certain strategies through application of specified rules without constructing their conceptual knowledge. Frequent execution contributes to develop students' automaticity in solving certain problem. Therefore, students become advance in procedural knowledge through carrying out rehearsal or drill as Hatano and Inagaki (1986) called it as routine experts. However, the automaticity become problem since it gives rise to students' meaninglessness and lack of consciousness in learning mathematics. On this situation, students should develop conceptual knowledge.

2.1.2. Conceptual Knowledge

While procedural knowledge is filled with accounts of symbols, rules, formulas, and algorithms in discrete manner, conceptual knowledge is described most obviously as knowledge which is rich in connections (Hiebert & Lefevre, 1986). With respect to the definition, as a matter of fact there was no common and clear convention among mathematics educators and notably researchers. Diversity in how researchers defined it consequently has made obstacle among new researchers to hold consistently and generally acceptable definition as well as the means of how it is assessed. Crooks and Alibali (2014) conducted meta-analysis and proposed a framework by integrating theory and measurement of conceptual knowledge. They suggested two components of conceptual knowledge that can be applied across mathematical domains: general principle knowledge and knowledge of principles underlying procedures. However, the classification is still novel and in need of subsequent investigation.

a. Knowledge of general principles. According to Crooks and Alibali (2014), general principles could be recognized as something that did not related to certain procedure. Therefore, it could consist of rules, domain structure, or symbols. This knowledge does not relate to knowledge of execution or calculation as they solely talk about principles, ideas, or meaning of certain domain.

   De Jong and Ferguson-Hessler (1996) pointed out conceptual knowledge as static knowledge about facts, concepts, and principles that apply within a certain domain. In addition, conceptual knowledge also can be defined as an integrated and functional grasp of mathematical principles (Kilpatrick et al., 2001). Understanding rules or properties means that students can give reasonable statements about certain rules or properties. In contrast, memorizing rules or properties is an ability possessed by those who possess procedural knowledge. While providing reasonable statements, students can make connection to other rules or properties, thus knowledge of connection also come to play in this point. The connection links all necessary information so that networks are constructed. Therefore, the knowledge focuses on (1) the rules or properties itself and (2) the way it is constructed (Thanheiser, 2012).

Conceptual knowledge emphasizes understanding of principles that regulate a knowledge domain (Rittle-Johnson et al., 2001). As principles can be explained also as regularities within certain domains, knowledge of domain structure can be included in this classification (Crooks & Alibali, 2014). Pertaining to this, Richland, Stigler, and Holyoak (2012) hypothesized a term, namely, conceptual structure of mathematics. For instance, the difference between structure of arithmetic and geometric sequences is arithmetic sequence has a constant difference between terms, whereas geometric sequence has constant ratio between terms. In addition, in order to understand domain structure of certain concept, other concepts might be included or applied, thus knowledge of connection comes into play in this occasion.

Hiebert and Wearne (1986) proposed the definition of conceptual knowledge as semantic knowledge that rich in explanation of connection amid symbols and their meanings. In addition, Ploger and Hecht (2009) used the term of awareness to describe the meaning of mathematical symbols. Awareness of and understanding of the meaning are main differences of knowledge of symbols in procedural knowledge and conceptual knowledge. For instance, $f(x)$ is a mathematical symbol whose mean is a map value of x to $f(x)$. Subsequently, in order to provide comprehensive meaning of certain symbol, students might be necessary to relate it with others symbol or to concepts, thus knowledge of connection comes into play. Lachance and Confrey (2001) also claimed that linking symbols with its meanings is very crucial for improving students' understanding of mathematical concepts. In addition to understanding meaning, exploration of how and why the symbols developed and how it connects to other mathematical symbol system have to be taken into account.

b. Knowledge of the principles underlying procedures. On the one hand knowledge of general principles deals with non-procedural activity, on the other hand knowledge of the principles underlying procedures pervades understanding of how procedures work and the aim of each step in procedure when solving problem (Crooks & Alibali, 2014). Executing calculation merely without understanding the reasons behind it is categorized as procedural knowledge. Therefore, the most crucial parts in this knowledge is to provide reasonable arguments of execution or justification of statement.

   Unpacking reasons to develop meaning of mathematical procedure is in need of powerful knowledge of general principles. Through recognizing general principles, students provide arguments to explain the reasons of why certain steps are consequences of others (Kilpatrick et al., 2001) and when principles are applicable to certain situation (Thanheiser, 2012). To put in different way, each step is sequentially integrated each other as the prior steps can explain the subsequent steps and between them there will be some mathematical explanations. Providing reason by considering rules, properties, or others procedures can be described as ability to make relationship, thus knowledge of connection also is necessary.

   Focusing on goal of problem also has to be taken into consideration. By this, students can select appropriate procedures and monitor their works. Awareness and consciousness take place in this process as they implicitly or explicitly make planning and regulate their cognitive thinking to generate reasonable actions. Eventually, they can demonstrate accuracy over process from beginning to the end and reach intended solution. In addition, they can apply and extend these activities correctly to variety of domain (Kilpatrick et al., 2001).

Just as procedural knowledge has benefit for students in solving problems, conceptual knowledge has too. Since the aim of mathematics instruction is to improve students' understanding, most mathematics educators tend to put conceptual knowledge over procedural knowledge. It is useful to understand the reasons behind such argument held by mathematics educators in favouring conceptual knowledge. Through compelling several previous studies, there are at least four advantages of conceptual knowledge either cognitively, affectively or meta-cognitively.

First, conceptual knowledge leads to meaningfulness in learning mathematics. Meaning, according to Hiebert and Lefevre (1986), is constructed through recognizing and establishing connection among units of information. As conceptual knowledge underlines connection between prior information and new information, students can catch the reason why certain concepts emerge and work. Absence of providing the connection causes rote learning in which students merely memorize the information i.e. facts and propositions and inhibit them in learning mathematics meaningfully. Long (2011) made description as to learning rote that it is habitual repetition and disregarding conceptual knowledge and, it didn't construct skill or knowledge that can be related to other skill or knowledge. Meaningfulness, in addition, might increase the likelihood of retrieving information easily for long time as it served structural and alternative route for recalling.

Second, conceptual knowledge can assist students in promoting flexibility. Cobb (1988) contended that constructed powerful conceptual structures would help students solving problems in various contexts. The structure itself is derived from sequential connection among knowledge and it becomes repertoire for students when dealing with mathematical tasks. In a plethora of case, students are able to tackle easily familiar and routine tasks and conversely getting difficulties when encountering unfamiliar or non-routine tasks. With respect to relation with flexibility in procedural knowledge, conceptual knowledge also provides opportunities for students to choose appropriate procedures in a given situation (Crooks & Alibali, 2014).

Consciousness and regulation of thinking are aspects that gain positive increment when conceptual knowledge are obtained. Nesher (1986) expressed the need for mathematics teachers to teach for understanding as it assisted in elaborating monitoring control systems which was very crucial in executing procedures.

With respect to question about how conceptual knowledge are best taught, it is interesting to pay attention to what the Harvard Graduate School of Education has proposed (Leach & Moon, 2008). They developed a framework that consisted of four concepts: generative topics, understanding goals, understanding performance, and ongoing assessment. Actually, each concept emphasizes metacognition as inseparable part in teaching and learning to improve conceptual knowledge. In addition, it is benefit for students to begin mathematics lesson by activating their prior knowledge and extending it to novel situations. To do this, teachers can propose questions or problems that encourage students to think reasonably and construct connection (Engelbrecht, Harding, & Potgieter, 2005).

It is important to note that definition of conceptual knowledge definitely influences how it is taught. As we base our definition on what Crooks and Alibali (2014) proposed in which conceptual knowledge is combination of knowledge of general principles and knowledge of principles underlying procedures, thus we are necessary to select appropriate teaching method fits with those characteristics. However, since these constructs are novel, it is in need of further investigations.

Definition of conceptual knowledge also determines the way how it is assessed. Nevertheless, the lack of its consistent and general definition seem to be a challenge for mathematics educators as well as researchers to construct valid and reliable instruments. Classification of procedure and non-procedure proposed by Crooks and Alibali (2014) comes into being as novel guidelines for those who interested in further investigation as to measuring conceptual knowledge. They advised other researchers to develop instruments that were specifically constructed to explore each of the two classifications of conceptual knowledge (Crooks & Alibali, 2014).

According to Crooks and Alibali (2014), the knowledge of general principle knowledge could be measured by means of two sorts of tasks, to wit, explanation of concept tasks and evaluation of example tasks. Explanation of concept tasks ask students to give definitions for symbols and elements of domain structure by means of verbal explanation. Meanwhile, evaluation of example tasks demand students to acknowledge examples, definitions, or statements of principles. Besides, the knowledge of principles underlying procedures could be measured by means of two specific tasks, to wit, application and justification of procedures tasks and evaluation of procedures tasks. Application and justification of procedures tasks ask students to address problems or to give verbal explication of their own problem-solving procedures. Finally, evaluations of procedures tasks ask students to evaluate or judge the presented procedures whether it is correct or incorrect.

2.1.3. Relation between procedural knowledge and conceptual knowledge

Even though the two types of knowledge have been considered as distinct entities, there are enough evidences indicated that there are linked in favourable ways. Lauritzen (2012) revealed that both knowledge are very essential need for students to be competence in mathematics. Moreover, the National Council of Teachers of Mathematics (NCTM, 2000) has encouraged teachers to develop teaching methods that enable students to comprehend the way how mathematical concepts interrelate each other to generate a coherent whole. The absent of connection between procedural knowledge and conceptual knowledge may cause meaningless learning as well as a complex task cannot be solved entirely. Just as a coin has two inseparable faces, the use of skill and understanding are indivisible in mathematics lesson.

As a consequence, there is no longer question whether instruction should focus solely on procedural knowledge or conceptual knowledge, yet how the two are developed in balance. Lauritzen (2012) confirmed that discussion pertaining to concepts-first and procedural-first in teaching mathematics concepts should no longer appear as they can develop iteratively. Both knowledge developed iteratively means that increasing in one sort of knowledge can lead to increasing in other sort of knowledge (Rittle-Johnson et al., 2001).

2.2.Metacognitive Instructional Method

2.2.1. Metacognition

The study of metacognition was firstly initiated in 1979 by Flavell (Flavell, 1979). Metacognition is commonly and simply defined as cognition about cognition which refers to second level of cognition, thinking about thinking, knowledge about knowledge, or reflection about actions (Papaleontiou-Louca, 2008). Knowledge as an object refers to knowledge about personal skill, problems, strategy to deal with the problem (Schneider & Artelt, 2010). Metacognition also can be explained as regulation of cognition or personal ability to adjust or monitor their cognitive activity to improve more effective understanding (Baker & Brown, 1984).

According to those conceptualization, metacognition comprises of awareness of multiple knowledge and also the way how these are applied and managed in learning activities to cope with problem or tasks. Metacognition then is not a single concept, rather than it is composed of several elements. Based on work of Flavell (1987) who as the prominent researcher in metacognition, Schraw (1998) distinguished metacognition into two main components, namely, knowledge of cognition and regulation of cognition.

Knowledge of cognition or metacognitive knowledge refers to personal recognition of their own cognition and general cognition (Schraw, 1998). It is categorized as initial contemplation of a task, thus it considers interaction among knowledge about person, task, and strategies (Flavell, 1979). Knowledge of person refers to student's understanding about their personal capability. Knowledge of task refers to student's perception about completion of a task. Knowledge of category refers to analysis and selection of appropriate and correct strategy (Babich, 2010).

By considering the interaction, learners can admit their cognitive strength and weaknesses. The interaction in addition has led researchers to categorize knowledge of cognition into three distinct metacognitive awareness: declarative knowledge, procedural knowledge, and conditional knowledge. Declarative knowledge briefly refers to knowing about things. It is knowledge concerning persons' own self as a learner and what factors affect ones' performance. Procedural knowledge can be explained simply as knowledge about the way how to do things. It highly relates to heuristic and strategies which are taken by learners to accomplish tasks. Finally, conditional knowledge is understanding condition in which declarative and procedural knowledge used is categorized as conditional knowledge. It refers to the query of why and when (Schraw, 1998).

The second element of metacognition is regulation of cognition which refers to set of activities to regulate learners' cognition when learning or accomplishing tasks (Papleontiou-louca, 2003; Schraw, 1998). As it takes place before, during, and after activities, thus educational psychologists applied the term self-regulation which included three regulatory element of metacognition, namely, planning, monitoring, and evaluating (Baker, 2010). Regulating cognitive lead students to make plan and provide learning resources, monitor and control their knowledge and performance, and evaluate their achievement in various occasion such as knowledge acquisition, task accomplishment and personal goal achievement (Schraw, 1998).

Planning activities let in selecting proper strategies and providing sources required for accomplishing tasks. It involves activities before commencing a task such as predicting, ordering, and allocating time and attention (Schraw, 1998). Students can enhance their monitoring and regulation of cognition by taking planning into consideration (Papleontiou-louca, 2003). Monitoring activities take place when students engage in learning or task accomplishment, thus it is on-line awareness of understanding and task performance (Schraw, 1998). Metacognition regulates and monitor students' cognition by allowing them to be aware of thinking process and guiding them in applying effective strategies to accomplish tasks (Shamir, Mevarech, & Gida, 2009). Finally, evaluating activities refer to judging the result or answer and efficiency of students' learning. It also includes re-assessing and checking out conclusion and goals (Schraw, 1998).

In addition, even though it was assumed theoretically that knowledge of cognition and regulation of cognition were mutually related and compensatory (Flavell, 1987) , a number of conflicting findings have occurred with respect to the relation between metacognitive knowledge or knowledge of cognition and metacognitive regulation or regulation of cognitive. One is that both of the components were related in reading abilities, yet the improvement of knowledge of cognition was found moderately over the school year (Meloth, 1990). Meanwhile, Schraw and Dennison (1994) found moderate correlation between the two components, yet the compensatory of the relation was not supported with enough evidence. A second is that regulation of cognition and knowledge of cognition were not related which meant that there was no guarantee that students who develop knowledge of cognition would regulate their cognition (Mevarech & Amrany, 2008). These findings are important since the components of metacognition are separated each other, an agreement with respect to the relation between them cannot be concluded commonly.

Since it has been introduced in 1979 by Flavell (Flavell, 1979), metacognition has attracted many researchers in broad spectrum of disciplines, notably mathematics education. Based on what has been initially investigated in the field of psychology, educational researchers have embarked on developing, investigating, and analysing the role of metacognitions on mathematics education theoretically and empirically (Schneider & Artelt, 2010). Increasing studies on metacognition in this field from the last four decades implies that metacognition definitely plays significant roles for students' learning. Veenman et al. (2004) revealed that metacognitive skills led students to learn in meaningful situation, flexible manner, and careful with strong foundation of mathematics concepts, and subsequently they can cope with various tasks successfully. Carr (2010) added that even though metacognitive knowledge and skills were not the only factors that influence students' learning, devoid of metacognition students might face difficulties in learning. Moreover, metacognition is one important factor to effective learning as it assists students in developing independent and high achievement which are necessary in facing new century (Papleontiou-louca, 2003). In mathematics education, concern with metacognition has developed from the studies on mathematical problem solving as metacognitive skills play important role in improving mathematical problem solving achievement (Mevarech & Kramarski, 2014; Ozsoy & Ataman, 2009; Schneider & Artelt, 2010).

The importance of metacognitive skills in mathematics education has led researchers to develop ways to improve students' metacognitive skills. According to Schraw (1998) metacognitive skills could be trained by means of certain instruction. Large evidences has been shown that teaching metacognitive skill is plausible and it can be employed to improve students' learning (Nietfeld & Schraw, 2002; Thiede, Anderson, & Therriault, 2003). It means that there is a possibility to create certain circumstance that assists in boosting students' metacognition. Veenman, Van Hout-Wolters, and Afflerbach (2006) proposed three crucial principles for successful metacognitive instructional method: a) attaching metacognition in the content of subject matter to ensure connection between the prior knowledge and new knowledge, b) providing information in respect to the benefit of metacognition to students so that they can apply the initial extra enterprise, c) sustained training to warrant the smooth and maintained the use of metacognitive activities.

In addition, several researchers (Goos, Galbraith, & Renshaw, 2002; Kramarski et al., 2002; Kramarski & Mevarech, 2003) reported that metacognitive skills might be developed within environment that provided social interactions. They claimed that students' self-awareness and improvement of cognitive skills were facilitated within social interactions. Kramarski (2004) supported the importance of modification of classroom management into small cohort learning that led students with the chance to build mathematical meaning by engaging them in mathematical discourse through self-metacognitive questioning as a way to develop metacognitive skills.

Students with different learning abilities also might take benefit from instruction that embedded metacognitive skills. Studies conducted by Grizzle-Martin (2014) and Schneider and Artelt (2010) indicated that both normal students and students with mathematics learning difficulties or low performances could improve their mathematics achievement after they were exposed to metacognitive instruction. It has been a consistent finding for more than 30 years that there has been positive relationship between students' successful in certain domain and level of metacognition (Baker, 2010). Therefore, study to develop appropriate instructional interventions that considers diversity in students' ability to enhance metacognition is necessary to be conducted.

2.2.2. IMPROVE Instructional Method

Based on principles proposed by Veenman, Van Hout-Wolters, and Afflerbach (2006), Mevarech and Kramarski (1997) developed metacognitive instructional method, namely, IMPROVE. The method is grounded on theoretical frameworks of self-regulated learning, social cognition, and cognition. It combines those theoretical frameworks to obtain high quality of teaching process and learning outcome. The method also can be applied flexibly in various learning contexts with discrepant types of problem or tasks (Mevarech & Kramarski, 2014). This method is also known as one of the first metacognitive instructional method designed for mathematics lesson for various level of school from primary to college students. Based on combination of those various theoretical frameworks, the method consists of three interdependent elements: metacognitive questioning, cooperative setting, and systematic provision of feedback-corrective-enrichment (Mevarech & Kramarski, 1997).

Metacognitive questions are self-directed questions which are used to increase students' awareness of their own comprehension and assist them in learning to regulate their cognitive activities while and after problem solving (Tzohar-Rozen & Kramarski, 2014). Students are directed to answer questions which focus on understanding mathematical problems, construct relationship between prior and present knowledge, select and apply appropriate strategies to cope with the problem, and reflect on the performed process and obtained result (Schneider & Artelt, 2010). These questions are central to IMPROVE instructional method as it is attached into and adapted to teaching materials at various stages in teaching unit (Eggert, Ostermeyer, Hasselhorn, & Bögeholz, 2013). Research in metacognition and mathematical problem solving has revealed that self-questioning techniques might promote students' achievements (Kramarski & Mevarech, 2003; Veenman et al., 2006). Therefore, presenting metacognitive questions should not be ignored by teachers and it should become inseparable part in teaching and learning process.

Cooperative setting has been emphasized heavily in implementing IMPROVE instructional method. Students may work in small groups which consist of four heterogeneous students in terms of their prior knowledge i.e. one high, two middle, and one low ability student (Mevarech & Kramarski, 1997). Considerable educational researchers had established clear evidences with respect to positive impact of cooperative setting on students' cognitive, social skills, and self-regulation (Hossain & Ahmad, 2013; Kramarski et al., 2002). Students who engage in cooperative setting communicate each other to discuss about the problem and they make effort collectively to come up with common solution. During this process, through activating their prior knowledge each student expresses their thought about the problem from various aspects and each argument is examined its validity. This validation process can enhance students' thinking and the provision of on-line regulation of cognitive sources (Mevarech & Kramarski, 1997).

Feedback-corrective-enrichment in IMPROVE instructional method, according to Mevarech and Kramarski (1997), enables students to achieve mastery on the problems or tasks and extend their mathematical thinking. To do this, formative test which focus on the main objective of presented topic is administered to all students. The tests consist of various types of problems and most of which are high level mathematics problems. Result of these tests is a feedback that can describe students' mastery level in which students who do not achieve mastery will be provided corrective activities, whereas those who achieve mastery will engage in enrichment activities relative to the unit. Teacher, at this session, considers students' level of ability and based on that he or she provides various task or activities.

As a matter of fact, IMPROVE is an abbreviation of which stands for teaching steps that construct the method: Introduction, Metacognitive self-directed questions, Practice, Review, Obtaining mastery, Verification, and Enrichment. In addition, every teaching steps considers the development of the two elements of metacognition i.e. metacognitive knowledge and metacognitive regulation (Mevarech & Fridkin, 2006).

a. Introduction. In the first step, teachers provide introduction of the new topics, concepts, or procedures to the whole class by modelling the metacognitive questioning strategies.
b. Metacognitive self-directed questions. The questions can be proposed in small groups or individualized settings. There are four sorts of metacognitive questions that students are instructed to apply and take turns in asking and answering (Kramarski, 2008b). These four sort of questions are presented in three occasions: prior to, during, after having finished learning activities (Eggert et al., 2013).
    - The comprehension questions are addressed to engage students in understanding topics, concepts, or tasks (Kramarski et al., 2002; Mevarech & Amrany, 2008; Mevarech & Fridkin, 2006). Students, initially, read aloud the topics, concepts, or task, and explain those in their own words to check understanding of what they have read. Students also can discuss the meaning of the concepts with other students. The common questions exposed are "what is the concepts/task all about?", "what is the meaning of this concept?"
    - IMPROVE instructional method provides great opportunity for students to build connection between new knowledge and what they have learned by means of connection questions. Connection can be constructed through focusing the similarities and the differences between two concepts/tasks (Mevarech & Kramarski, 1997). Example of connection question is "Can you come up with similarities and differences between these concepts/tasks

and concepts/tasks you have understood or solved in the past, what and why?"

- Strategic questions lead students to elaborate or select appropriate strategies to accomplish tasks. It is also important for students to be aware of what they are doing by thinking-aloud. Reasoning plays important role in these occasions as students provide reasonable arguments pertaining to strategies they have selected and elaborated (Mevarech & Kramarski, 2014). The instance of question is "What are strategies/principles necessary for accomplishing the task, and why? Please explain your reasons!"
- Reflection questions are proposed in order to assist students in monitoring their progress when they accomplish task, help them making adjustment of strategies used when they encountered difficulties, and lead them to check back and analyse their works so that they can employ it in other tasks (Mevarech & Kramarski, 2014). The examples of question are "Does the result make sense?", "Are there any other strategies I can use to solve this problem?", and "Am I stuck? Why?"

c. Practice. Following introduction to novel concepts or tasks, students practice by exerting and responding metacognitive self-addressed questions (Mevarech & Kramarski, 1997). IMPROVE instructional method enables students to practice processing skills, conceptual knowledge, and assessing mathematical tasks. The practice could be applied in individual or small group setting with teachers' assistance as necessary.
d. Review. IMPROVE instructional method also takes reviewing the new materials or the main ideas into consideration. Teachers review such things by means of self-addressed metacognitive questions. It can be performed in individual, small group, or entire class setting (Mevarech & Kramarski, 1997).
e. Obtaining mastery. Obtaining mastery refers to teachers' effort ensuring that students reach intended higher and lower cognitive process (Mevarech & Kramarski, 1997).
f. Verification. Verification is defined as the validation process of a proposition by empirical ways (A. Stevenson, 2010). Verification has been assumed as a two-fold activity which involves cognitive and metacognitive domain (Artzt & Armour-

thomas, 1992). Therefore, IMPROVE instructional method allows students to verify the acquisition of cognitive and metacognitive skill according to the application of feedback-corrective process (Mevarech & Kramarski, 2014).

g. Enrichment and Remediation. As normal classrooms consist of students which has various abilities, enrichment and remedial are necessary to be integrated in order to fulfil students sundry needs (Freeman, Raffan, & Warwick, 2010). Enrichment actually allows students to experience greater depth, breadth, as well as context in learning, going beyond what the curriculum has constrained (Bandura, 1997). In contrast, remediation is designed to serve to the needs of students who are behind the expected level of achievement and incapable of keeping pace with teaching-learning in a normal classroom (Selvarajan & Vasanthagumar, 2012).

2.3.Review of research studies

In this section, relation among procedural knowledge, conceptual knowledge, metacognitive skills, and IMPROVE instructional method is illuminated by considering conducted studies.

2.3.1. Procedural Knowledge, Conceptual Knowledge, and Metacognitive Skills

In the literatures there are three discrepant models regarding the way how procedural and conceptual knowledge and metacognitive skill are related each other. Firstly, Piaget model claimed that procedural knowledge is a necessity for improving metacognition and conceptual knowledge. Secondly, Vygotsky model argued that developing both procedural knowledge and conceptual knowledge required metacognition (Baker & Czarnocha, 2002). Thirdly, Rittle-Johnson et al (2001) proposed that metacognitive skill is a bridge for students to attain conceptual knowledge from procedural knowledge through iterative process. Understanding the way how the three models develop practically in classroom may lead teachers to select and apply appropriate teaching method.

Metacognitive instruction as a teaching method may support students to understand the way how they learn (Mevarech & Kramarski, 1997). By employing it, students can elaborate proper arrangement during teaching and learning process (Jaafara & Ayubb, 2010) and obtain effective learning and later academic achievement (McCormick, Dimmitt, & Sullivan, 2013). This instruction has been implemented to promote development of students' procedural knowledge and conceptual knowledge in various school levels. Jbeili (2012) conducted study to investigate the effect of cooperative learning which integrated metacognitive scaffolding on elementary students' mathematics procedural knowledge and conceptual knowledge in learning and accomplishing tasks of addition and subtraction of fractions. Working with 240 fifth-grader male students in Jordan, researcher conducted a quasi-experimental research. The result of the study indicated that students who were instructed with metacognitive scaffolding embedded in cooperative learning outperformed students who were instructed by traditional instruction in mathematical procedural knowledge ($p < 0.05$ and $ES = 0.43$) and conceptual knowledge ($p < 0.05$ and $ES = 0.34$).

In middle school level, Sari and Özdemir (2013) conducted study aimed at investigating the effects of a metacognitive instructional method on middle school students' conceptual and procedural knowledge of algebraic expressions and equations. The researchers employed two seventh grade classes from a public school in Turkey. By implementing quasi-experimental design, they acquired significant result in which experimental group was better than control group in conceptual and procedural knowledge scale provided ($p < 0.05$). Unfortunately, the study to investigate the effect of metacognition instruction which involves high school students' procedural knowledge and conceptual knowledge is rare and in need of further investigation.

2.3.2. Research about IMPROVE Instructional Method

IMPROVE instructional method can be implemented by ordinary teachers in regular classroom which consists of large amount of students with various mathematical abilities. This innovative instructional method was firstly introduced in 1997 by Mevarech and Kramarski and has become powerful alternative for teachers to promote students' mathematical achievement in various levels of schools and wide range of topics. The reason lies in the fact that theoretically students are not able to attain mathematical competency devoid of metacognitive thought process which are fundamental for academic success (Kramarski et al., 2002).

The advantages of IMPROVE instructional method had been shown evidently in either cognitive and metacognitive aspect, such as: improving mathematics achievement on standard and authentic tasks (Kramarski et al., 2002), mathematical knowledge, reasoning, and metacognition (Mevarech & Fridkin, 2006), algebraic procedural and conceptual mathematical explanation, and problem solving (Kramarski, 2008b), achievement and regulation of cognition (Mevarech & Amrany, 2008), socio-scientific decision making (Eggert et al., 2013), and critical thinking ability (Anggoro, Bambang Sri Kusumah, Darhim, & Afgani, 2014).

Those advantages might occur due to that IMPROVE instructional method considers cooperative setting. Metacognitive instructional method without attaching cooperative setting could not enhance optimally students' achievement in learning and problem solving. For instance, in 2002, Kramarski et al conducted a study to investigate the effect of cooperative learning with or without metacognitive instruction on students' solution when dealing with mathematical authentic tasks. They worked with 91 seventh graders registered at three different classrooms. By implementing qualitative and quantitative research methods, they came up with the result that students who were instructed with IMPROVE instructional method outperformed their counterparts who were instructed with only cooperative learning in solving authentic and standard tasks.

Conversely, cooperative setting without embedding metacognitive instruction also could not assist students in developing understanding of mathematical concept. For instance, Kramarski (2004) conducted the research aimed to explore the discrepant impact of cooperative learning with or without metacognitive instruction on making sense of graphs. In the study, 196 students who were six eighth-grade classes randomly selected from two junior high schools in Israel whose socioeconomic status were equal participated in the study within two weeks. In the study, Linear Graph was used as a main topic and it was divided into 10 lessons. Results of the study showed that students who were in cooperative learning environment with the metacognitive instruction significantly outperformed their counterparts who were in cooperative learning without the metacognitive instruction on graph interpretation ($F(1,194) = 23.90$; $p < 0.0001$) and graph construction ($F(1,194) = 3.35$; $p < 0.05$). Therefore, metacognitive instruction and cooperative setting cannot be separated in teaching and learning process in elementary and middle school.

However, interestingly IMPROVE instructional method evidently could be applied also within individualized setting in pre-college level. For instance, Mevarech and Fridkin (2006) conducted study to investigate the effects of IMPROVE instructional method on students' mathematical knowledge, mathematical reasoning, and metacognition. Eighty-one students who participated in the study took a pre-college course in mathematics in Israel. The students studied the course mathematical functions which were taught for 12 hours a week during one month (around 50 hours). Results indicated that IMPROVE students significantly outperformed their counterparts on both mathematical knowledge and mathematical reasoning ($F (1, 78) = 10.14$ and 15.45; $p = .002$ and $.001$ respectively). This result led Mevarech and Fridkin (2006) to suggest that IMPROVE instructional method could be applied in either individualized or cooperative setting. However, relation between participants' age and regulation of cognition might be explanation for this case. According to Silvers et al., (2012), age has been correlated with linear increasing in cognitive control tasks, thus little teachers' assistance has been perceived to be adequate for adult students.

In addition to regulation of cognition, theoretically IMPROVE instructional method also take into knowledge of cognition consideration in that of implementation (Mevarech & Fridkin, 2006). However, interesting result has been found in the study of Mevarech and Amrany (2008) who investigated direct and delayed impact of IMPROVE instructional method on regulation of cognition and mathematics achievement. Sixty-one girls-high-school students participated in the study and they took the topic of Growth and Decay within a month for the four-point credit on the matriculation. The results showed that IMPROVE instructional method could enhance students' mathematics achievement ($F (1, 58) = 4.79$, $p = .033$) and regulation of cognition ($F (1, 58) = 4.55$, $p < .05$), yet there was no significant result on knowledge about cognition ($F (1, 58) = 1.823$, $p > .05$). They claimed that though IMPROVE instructional method allowed students to implement different facets of cognitive regulation processes, it didn't serve to improve their knowledge about cognition. Therefore, they argued that knowledge about cognition had not strong correlation to a high level of regulation. It was also due to the fact that the study emphasized heavily on regulation of cognition, rather than knowledge about cognition. As a matter of fact, based on their study, Mevarech and Fridkin (2006) revealed that the effect of

IMPROVE instructional method could be discerned evidently in eight scales of metacognitive skills, to wit, knowledge of cognition (declarative, procedural, conditional) and regulation of cognition (planning, monitoring, debugging, evaluating, and information managing).

IMPROVE instructional method allowed students to turn on metacognitive processes such as making linkage, applying strategies, and evaluating the outcome, it also led students to regulate their cognition and subsequent high academic achievement (Mevarech & Fridkin, 2006). Further research would be necessary to investigate the impact of IMPROVE instructional method on high school students in mixed gender classroom with emphasizing both knowledge of cognition and regulation of cognition. In addition, as most of studies have focused on certain specific mathematics topics, it would be worth to investigate the effect of this instructional methods on different sequential mathematics topics.

2.3.3. Gender differences

Of particular interest for this study is investigation of the role of gender that plays in the relationship between teaching methods and students' procedural, conceptual knowledge, and metacognitive skills. There are differences between the terms of sex and gender (Unger, 1979) in which sex refers to biological characteristic and gender refers to cultural characteristics. Geist and King (2008) claimed that gender differences between males and females are not based on biological reasons, instead the way they learn mathematics. Therefore, researchers have been investigating gender difference in mathematics learning (i.e. Else-Quest, Hyde, & Linn, 2010 and Lindberg, Hyde, Petersen, & Linn, 2010; Voyer & Voyer, 2014).

With respect to gender differences in procedural and conceptual knowledge, previous studies reported inconsistence conclusions. For instances, Hutkemri and Zakaria (2012) conducted a study to investigate the implementation of GeoGebra-enriched mathematics instruction on students' procedural and conceptual knowledge in the topic of function. Working with 284 Indonesian students in two different schools, they revealed that there was no gender difference on students' conceptual and procedural knowledge. This result fit well with study conducted by Mosia (2014). Mosia (2014) conducted a study whose aim was to analyse how gender have an effect on mathematics performance of 10-12 grade students in South Africa. There were 5254 10^{th},11^{th}, and 12^{th} grade students participated in the study and a mathematics questionnaire on various cognitive levels was capitalized on to identify students' difficulties in mathematics. Using one-way analyses of variance and t-test, the result of the study indicated that there was no gender difference in the mean scores of manipulative procedural knowledge, logical reasoning, problem solving, number concept and function concept in grade 10 and 11. Nevertheless, in grade 12 there was a signification gender difference in the mean scores of manipulative procedural knowledge and function concept in favour of male students.

Meta-analysis conducted by Else-Quest et al. (2010) and Lindberg et al. (2010) showed that there was gender difference in mathematics performance and achievement in favour of male students. However, several research suggests that female students had better mathematics achievement than male students (Voyer & Voyer, 2014). In Indonesian context, result of TIMSS assessment in 2011 showed that Indonesian female students were more successful than male students in mathematics achievement (Mullis, Martin, Foy, & Arora, 2012). Chouinard and Roy (2008) revealed that in some cases, high school female students had more positive attitudes toward mathematics than high school male students which become reason to the difference in mathematics achievement between them.

Applying Schraw's (1998) framework of metacognition, the present study also investigated the gender differences in knowledge of cognition and regulation of cognition in a sample of high school students. Previous research indicated that inconsistency findings with respect to the gender differences in metacognitive ability. Several research suggested that there were no gender differences between male and female students in metacognitive ability. For instance, Sperling, Howard, Miller, and Murphy (2002) investigated the gender differences in knowledge of cognition and regulation of cognition of elementary and middle school students. Working with 344 children in third through ninth grades, the result of the study indicated that no gender differences were found for younger students [t (129) = 1.17, p = .24] and for the older students [t (246) = .62, p = .43].

On the other hand, gender differences in metacognitive ability were found in several studies. Ciascai and Lavinia (2011) worked with 91 8th grade students in Romania to investigate the likelihood of gender differences with respect to metacognitive skills. Using the Junior Metacognitive Awareness Inventory, the result showed that that there are significant differences between male and female students on the use of prior knowledge in problem-solving, planning, knowledge about one's own intellectual strengths and weaknesses, the use of various learning strategies and monitoring the learning process. However, the study didn't confirm evidently whether female or male students possessed higher metacognitive skills. Several studies such as Bidjerano (2005) and Wu (2014) suggested that female students had better metacognitive skills than male students. Wu (2014) conducted study that investigated how knowledge of metacognitive strategies and navigation skills mediate the relationship between online reading activities and printed reading assessment and electronic reading assessment. Working with 34104 fifteen-year-old students, the result of the study showed that female students performed better in knowledge of metacognitive strategies, navigation skills and printed reading assessment. The similar result was also uncovered by Bidjerano (2005) in which female students outperformed male students in the use of rehearsal, organization, metacognition, time management skills, elaboration, and effort. These inconclusive results encourage the researcher to analyze gender differences in metacognitive skills in Indonesian context.

Even though there is extensive research on the gender differences in mathematics learning there is little research on role gender has played in the relationship between the implementation of IMPROVE instructional method and student' procedural knowledge and conceptual knowledge, and metacognition. Therefore, the present study investigates the effect of IMPROVE instructional method on students' procedural knowledge and conceptual knowledge, and metacognition by considering gender differences. As a matter of fact, the present study would answer the implication of study conducted by Mevarech and Fridkin (2006) to investigate differential effect of IMPROVE instructional method on boys and girls. In the study, it was assumed that as IMPROVE instructional method tends to be heavily verbalized instructional method, female students would gain more benefit than male students in terms of mathematics achievement and metacognition.

2.3. Education in Indonesia

In this section, general view of education in Indonesia is presented. Subsequently, as the present study concerns about high school students, thus explanation about mathematics in high school level is reviewed.

2.3.1. General View

Children in Indonesia commenced their education at early education or pre-school from the age of three to four for play group and four to six for kindergarten. The goal of early education is to provide opportunity for children to develop and grow both physically and mentally and to assist them in making good preparation for the subsequent level of education. The early education is not compulsory for children, yet the number of children enrolled in this level has been increasing constantly. In Indonesia, basic education covers nine years which is separated into two levels: elementary school and junior high school. It is compulsory for all children without exception. The goal of basic education is to develop attitudes and abilities, to provide basic knowledge and skills required for life in society, and to prepare students for participating in secondary education. Elementary school begin at the age of seven. Elementary school lasts for 6 years (from grade 1 until grade 6). Subsequently, junior high school level lasts for three years (from grade 7 until grade 9). At the end of the academic year, 6^{th} and 9^{th} graders take national exam to evaluate students' learning and performance on certain subjects by considering competency standards. The total score obtained in the exam is used to apply the subsequent school level. In the wake of accomplishing junior high school, students may continue to secondary school level. In this level there are three types of schools: senior high school (general track), senior vocational school, and Islamic senior school (faith-based school). It lasts for three years (from grade 10 until grade 12). The goal of secondary level school is to improve intelligence, knowledge, personality, character, and skills to live independently and to prepare for pursuing higher education. Then, students may continue to pursue higher education after secondary school level. Higher education level provides various opportunities include undergraduate (Sarjana I), diplomas (Diploma I until IV), and postgraduate level (Sarjana 2 or Magister and Sarjana 3 or Doctorate). Summary of education levels in Indonesia is presented in the Figure 2.1.

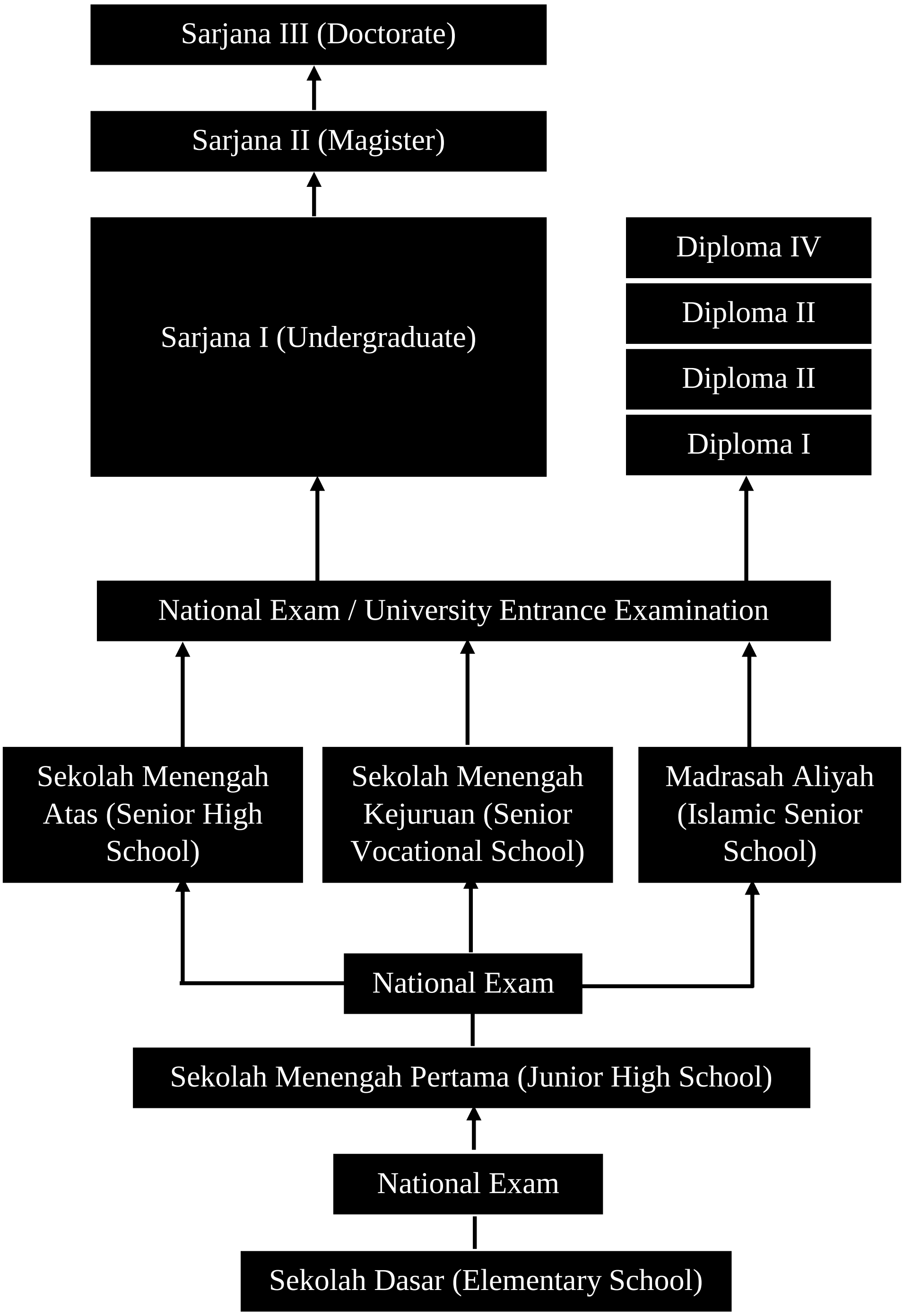


Figure 2.1. Education system in Indonesia

2.3.2. Mathematics in Secondary School Level

In 2013, Indonesian Ministry of Education and Culture has developed new curriculum – the 2013 curriculum. The modification of the curriculum has influenced various aspects, particularly mathematics education in upper secondary or high school level. Compared to the prior curriculum which emphasized heavily on students' understanding, reasoning, problem solving, and communication, the 2013 curriculum enlarged its scopes involving various types of knowledge that students have to be achieved. The new curriculum (BSNP, 2013) for high school mathematics gives insight into the demands for what is referred to as competence in school mathematics, by stating that: "Understanding, applying, and analysing factual, procedural, conceptual knowledge and metacognitive skills based on their curiosity about science, mathematics, technology, art, culture and humanity within insight on humanity, national, state and civilization related to causes of phenomena and events in a specific field of study to solve problems." The new curriculum in substance added metacognitive skills in the objective. Therefore, it emphasizes the importance of dealing with four categories of knowledge: facts, procedures, concepts, and metacognition in learning mathematics. As a matter of fact, it fit with classification of structure of the knowledge dimension suggested by Anderson and Krathwohl (2001) as a revision of the Bloom's taxonomy which merely focused on three categories of knowledge, to wit, facts, procedures, and concepts knowledge. Metacognition was added into structure of the knowledge dimension, in as much as most researchers continue to point out the significance of students' awareness on their learning achievement.

In Indonesia, secondary level is split up into two parts, to wit, junior and senior high school. Secondary education starts with junior level for students who have reached the age of 12 or 13 before the school year starts. In the wake of three-year period (grades 7, 8, and 9) at this level, students continue their education and enrol at senior level which starts from grade 10 to grade 12. Students who have finished their junior high school level are given opportunities to select their favourite high school (most of them are public schools) by considering their scores at national and school examination.

In the senior level, at the beginning of the year, new students are assigned into three divisions of school programs, to wit, science, social, and language and culture programs. The students are placed based on their achievement in junior level, psychological tests, interviews, and placement test which consists of tests of mathematics, English, science and social. Therefore, in this level there are three cohorts of students based on their ability and interests. The aim of these classifications are to provide a large opportunity for pupils to develop their interest in program in accordance with their interest at university level later and also to develop their interest in a certain discipline or a particular skill (BSNP, 2013).

Students in senior high school level are exposed to various mathematics topics which are under broad concepts of algebra, geometry, statistics, probability, discrete mathematics, and calculus. Those concepts are distributed into specific topics such as real numbers; exponential, logarithm, and its inequalities; algebra; geometry and transformations; function and trigonometric equations; limit of algebraic function; matrices; combinatorics; statistics and probability; derivative of algebraic function; linear programming; conic section; space geometry; econometric; growth and decay; vectors; mathematical induction; integration; and logic. In addition, the aforementioned topics are separated into two main categories, namely mandatory topics and specialized topics. Table 2.1 presents detail information relative to categories of mathematics topics across grade and programs for each semester.

Table 2.1. List of high school mathematics topics across categories and grade in each semester

Categories	Semester	10th Grade	11th Grade	12th Grade
Mandatory Topics	1	1. Exponent and Logarithm 2. Equality and Inequality of Absolute Value 3. System of Linear Equations and Inequalities (2 and 3 variables) 4. Matrices 5. Relation and Functions 6. Sequences and Series	1. Linear Programming 2. Matrices 3. Composition and Inverse Function 4. Infinite Sequence and Series 5. Line Equations 6. Sine and Cosine Law	1. Matrices 2. Interest, Growth, and Decay 3. Mathematical Induction 4. Space, Face, and Plane Diagonal 5. Integral
Specialized Topics	1	1. Exponential and Logarithmic Function 2. System of Linear and Quadratics Equation (Two Variables) 3. System of Quadratics Inequalities	1. Polynomial 2. Conics Section 3. Intersection of Two Circles	1. Application of Matrices 2. Vector 3. Financial Mathematics 4. Composition of Geometric Transformation 5. Space Geometry 6. Trigonometry 7. Definite Integral 8. Partial Integration
Mandatory Topics	2	1. Quadratics Equation and Its Function 2. Trigonometry 3. Introduction to Space Geometry 4. Limit of Algebraic Function 5. Statistics 6. Probability	1. Statistics 2. Probability 3. Equation of Circle 4. Transformation 5. Derivatives 6. Integrals	

Table 2.1. Continued

Categories	**Semester**	**10th Grade**	**11th Grade**	**12th Grade**
Specialized Topics	2	1. Absolute, rational, and irrational Inequalities 2. Plane Geometry 3. Trigonometric Equation	1. Statistics 2. Limits of Function 3. Derivative of Trigonometric Function 4. Application of Differentiation	

Besides, time allocation for learning mathematics in classroom in each grade is different. Detail information about time allocation for students in a week is illuminated in Table 2.2. All students regardless their grades and programs are presented with four-hour lesson of mandatory mathematics topics in a week. However, students who are enrolled in science program are provided with more four-hour lesson of specialized mathematics topics. One-hour lesson is equal to 40 minutes. Generally speaking, for all public schools and for most private schools, lesson starts at 7 o'clock in the morning and end at 3 o'clock in the afternoon from Monday to Friday. Students take break twice in a day school, to wit, short break (09.40 – 10.00) and long break (12.00 – 13.00).

Table 2.2. Time allocation in a week for mathematics lesson across grades and programs

Grade	**Program**	**Lesson hour in a week**		
		Mandatory Topics	Specialized Topics	Total
10th	Science	4	3	7
	Non Science	4	-	4
11th	Science	4	4	8
	Non Science	4	-	4
12th	Science	4	4	8
	Non Science	4	-	4

With respect to mathematics text books, the government in fact publishes mathematics textbooks (students' book and teacher's book) and supplies it to all mathematics teachers and students freely. It is also available online in pdf version and it can be downloaded freely (http://bse.kemdikbud.go.id/buku/kurikulum2013). However, the contents of the book are, according to opinion of several mathematics teachers, too difficult for students to be comprehended as most of explanation of the topics provided seems to be directed to university level. Therefore, several teachers decide to capitalize on certain text book produced by private publishers available in market as main source in teaching. In addition to it, they also draw on various mathematics text books along with textbooks published by government as complement of the main source.

Concerning secondary mathematics classroom practices in Indonesia, there are not much research studies that illuminate condition of it. The following is daily mathematics instruction regularly applied by a secondary mathematics teacher. At the beginning of the lesson, teacher checks students' attendance by calling students name loudly. Then the teacher introduces new topic by writing formulas related to the topic and explain the elements of that formula. After explaining the formula, the teacher presents several examples of questions. This is followed by explanation of strategies and solution of the questions. Then the teacher gives several minutes for students to take note what has been written in board. This is followed by exercises for students and the teacher generally capitalizes on common mathematics text book as a source of exercises by selecting several questions that appropriate and similar to what has been presented in the examples. Students strive to solve the problem given individually and several of them who sit next to the other work together. Meanwhile, the teacher walks and comes near to several students for giving guidance. The teacher observes students' activities and evaluates students' activities and students' thinking. In general classroom is dominated by the teacher and it seems that this strategy is very common used in many mathematics class in Indonesia. Therefore, in Indonesia, as is the case in other countries in the world, high school teaching of mathematics tends heavily to focus on procedural knowledge. On the contrary, conceptual knowledge obviously has not been taken into consideration. It has been common contention among Indonesian teachers that students are enough to be equipped with procedural knowledge so that they will be able to pass not only national examination in the last year of their upper secondary level but also university entrance. Despite the national curriculum put emphasis heavily on conceptual understanding and problem solving, this type of instruction appears to prevail in most classrooms.

All secondary school students are expected strongly to pass in the national examination held by National Board of Education Standardization (BSNP – Badan Standar Nasional Pendidikan) – an organization under the Department of Education and Culture. Along with exam administrated by school, the national exam determines students successful in high school level. For those who pass the exam, they may or may not continue to higher education, whereas for those who fail the exam have to take alternative exams or take 1 year more in 12^{th} grade. Therefore, the national exam is categorized as high stake exam and it makes students, teachers, school principals, etc. attach great importance to it. The exam consists of 40 multiple choice questions and most of which evaluate students' procedural knowledge in that students are expected to be able to apply their competence in application of mathematics formulas, algebraic manipulation, and calculation.

Based on the result of national exam conducted at 2015, it was reported that the average of students' mathematics scores was still below the average of scores of all subjects achieved by high school students in the exam nationally. Therefore, mathematics seems to be problem for Indonesian students. In addition, several studies indicated that most Indonesian students showed low attitudes towards mathematics and it is worsened by condition in which most mathematics teachers tend to be rated less favourably than other subjects' teachers (Maulana, Opdenakker, den Brok, & Bosker, 2012).

Research in mathematics education especially in high school level is still in progress. Most of studies are reported in local or national journal and it is very rarely published in international journal. The internationally published studies however are most directed to improvement of classroom practice in elementary school level such as Realistic Mathematics Education (Sembiring, Hadi, & Dolk, 2008; Widjaja & Dolk, 2010) and improvement of teaching profession such as lesson study (Marsigit, 2007; Saito, Harun, Kuboki, & Tachibana, 2006; Saito, Imansyah, Kubok, & Hendayana, 2007).

Concerning high school mathematics teachers, most of them are those who have graduated from a four-year undergraduate program in mathematics education. Mathematics education in university provides various courses which are under three broad categories of courses, to wit, mathematics courses, educational courses, and general courses. In addition, before completing their education in the last year student teachers have to take field experience of teaching practice in appointed schools within one semester. In wake of completing a four-year undergraduate program, according to the new regulation, all student teachers are expected to take 'Professional Teacher Education Program' within 1 year.

After being in-service teachers, professional developments also are provided in various forms. For instance, in-service teachers are expected to attend meeting of local mathematics teachers (*MGMP - Musyawarah Guru Mata Pelajaran or KKG – Kelompok Kerja Guru*) in certain period. The aim of the meeting is to discuss about the implementation of mathematics instruction in their own schools. In addition, range of seminars and workshops are provided and offered by government (educational ministry), universities and non-governmental organizations to support teachers' professional development.

2.4. Summary

The main objective of teaching and learning mathematics essentially is to improve students' mathematical understanding by means of meaningful learning process so that they can deal with various problems in their school or daily life (BSNP, 2013). In fact, however, implemented teaching strategies mostly tend to emphasize development of rote knowledge and overlook deep and meaningful mathematical knowledge (Hasenbank, 2006). Conceptual knowledge plays very crucial role in learning mathematics as it is benefit for meaningfulness (Hiebert & Lefevre, 1986), and flexibility (Crooks & Alibali, 2014) monitoring and regulation (Nesher, 1986).

Conceptual knowledge has been defined by researchers in numerous manners and there has not been accepted general formulation for various domain, nebulous, and poorly operationalized (Crooks & Alibali, 2014). This condition gives rise to difficulty in how it is measured. Therefore, it is necessity for researchers to confirm clear and explicit definition of conceptual knowledge and to choose and apply tasks that fit well with the selected definition (Crooks & Alibali, 2014). The present study considers the new developed definition of conceptual knowledge proposed by Crooks and Alibali (2014) as the framework of the study. The framework is applied due to its novelty and it includes comprehensive and detail delineation of conceptual knowledge based on previous studies.

Both procedural knowledge and conceptual knowledge have to be taken into consideration in teaching and learning mathematics in a balanced way (Lauritzen, 2012). The acquisition and development of the two knowledge in fact could not be provoked successfully without embedding metacognition in instruction (Haapasalo, 2013). Besides, metacognitive skill could be taught in certain environment which lead students to be aware of what they are doing and regulate their cognitive process (Schraw, 1998).

IMPROVE is an powerful metacognitive instructional method which consists of three interdependent elements: metacognitive questions, cooperative setting, and feedback-corrective-enrichment (Mevarech & Kramarski, 1997). The instruction is capitalized on in this study as it could be implemented widely in regular mathematics classrooms which consist of heterogeneous ability students. In addition, previous studies indicated that IMPROVE instructional method could enhance students cognitive and metacognitive aspect, such as: improving mathematics achievement on standard and authentic tasks (Kramarski et al., 2002), mathematical knowledge, reasoning, and metacognition (Mevarech & Fridkin, 2006), algebraic procedural and conceptual mathematical explanation, and problem solving (Kramarski, 2008b), achievement and regulation of cognition (Mevarech & Amrany, 2008), socio-scientific decision making (Eggert et al., 2013), and critical thinking ability (Anggoro, Bambang Sri Kusumah et al., 2014).

The effect of the implementation of IMPROVE instructional method on gender would be worth to be investigated as previous studies do not allude this issue further. What is more, the literature suggests that the implementation of IMPROVE instructional method could be examined theoretically and practically its effectiveness in different contexts or countries (Mevarech & Fridkin, 2006). In addition, it would be necessary to investigate the impact of IMPROVE instructional method on high school students in mixed gender and regular classrooms with emphasizing knowledge of cognition and regulation of cognition. Besides, as most of conducted studies have focused on certain specific mathematics topics, it would be worth to study the effect of this instructional method on various mathematics topics. Consequently, it is valuable to conduct an experimental research which is aimed to examine the effect of metacognitive instructional method on eleventh grade students' procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition.

In the present study, there are six variables that are taken into account: procedural knowledge, conceptual knowledge, IMPROVE instructional method, gender, knowledge of cognition, and regulation of regulation. Definitions, theoretical frameworks, examples as well as previous findings relative to those variables were illuminated above in order to provide clear insight into base of the present study. Those variables were employed in mathematics classrooms to support students in achieving academic success based on the main objective of national curriculum for upper secondary school mathematics. Based on the literature reviews, in this study it was expected that IMPROVE instructional method would enhance students' procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation cognition. It is also expected that IMPROVE instructional method would have greater impact on female students than male students.

CHAPTER 3

METHOD

After related literature on research questions of this study were illuminated in the previous chapters, subsequently in this chapter, research design, participants, variables used in the analysis, instruments applied for assessment, instructional materials, procedure followed, treatments implemented, how treatment fidelity was confirmed, statistical analysis conducted, unit of analysis, and assumptions and limitation, and internal and external validity as well as trustworthiness related to the study are presented.

3.1.Research Design

To address the research questions guiding this study, quantitative research design was utilized and qualitative data was compiled. The purpose of conducting quantitative research is to test hypotheses, and investigate cause and effect relation, whereas compelling qualitative data is aimed at understanding and interpreting social interaction occur within particular context (Lichtman, 2012). With respect to sample size in this study, the number of sample in the two research designs was different due to their distinct objectives. Therefore, for this study two samples were required. The sample of the quantitative part of the study in fact could not be selected randomly from the population and the assignment of the sample to experimental and control group could not be carried out randomly since the groups were classified by the school administration before the study or two intact classes were used. Hence, this study became quasi-experimental design in which matching-only pre-test-post-test control group design was selected (Fraenkel, Wallen, & Hyun, 2015). This quasi-experimental quantitative study seeks to determine the relationship between the implementation of two different teaching methods and students' procedural, conceptual knowledge, and metacognitive skills. Nevertheless, the two intact classes we randomly assigned to two treatments, namely, IMPROVE instructional method and traditional instruction. Table 3.1 summarizes the design of the study.

Table 3.1. Design of the study

Groups	Pre-Tests	Treatments	Post-Tests
Experimental Group	KC	IMPROVE	KC
	RC	Instructional Method	RC
Control Group	PROC	Traditional	PROC
	CONC	Instructional	CONC

Note. KC: Knowledge of Cognition; RC: Regulation of Cognition; PROC: Procedural Knowledge; CONC: Conceptual Knowledge

As shown in Table 3.1, at the beginning of the study, students in both groups were given pre-test. Subsequently, students in experimental group were taught with IMPROVE instructional method whereas students control group were taught with traditional instruction on composition and inverse function, infinite sequence and series, and line equation concepts. In the wake of the treatments, procedural and conceptual knowledge test, and metacognitive awareness inventory were administered to both experimental and control groups. In addition to these tests, semi-constructed interviews were conducted with 14 students in experimental group. Besides, to match students in the experimental and control group, covariates analyses were implemented on pre-test scores (Pre-KC, Pre-RC, Pre-PROC, and Pre-CONC).

3.2.Participants of the Study

The target population of this study was all 11th grade high school science students in Bandung, a city in West Java, Indonesia. The accessible population all 11th grade high school science students from high schools located at the North Region of Bandung City. The sample of the study was chosen from the accessible population by using convenience sampling. In educational field, convenience sampling was applied as it was very difficult to pick up random or systematic non-random sampling (Fraenkel et al., 2015).

There were five high schools in the accessible population and only one of them was convenient to be included in the study. This school is private school whose medium instruction is mainly Indonesian language. There are approximately 704 students with 39 teachers. Gender distribution of the school is almost half and half. There are 4 science classrooms, and by considering students' previous results of mathematics tests, out of four classes, the researcher and the teacher agreed to pick up two classes. The selected classes were randomly assigned to an experimental and a control group. There were 66 students in these classes and they took part in the study. The age of students varies between 15 and 16. Details about the participants of the study were summarized in Table 3.2.

Table 3.2. Number of students in relation to groups and gender

Gender	Number of Students		Total
	Experimental Group	Control Group	
Male	14	11	25 (37.88%)
Female	20	21	41 (62.12%)
Total	34 (51.52%)	32 (62.12%)	66

As seen in Table 3.2, there are 34 students (51.52%) in experimental group which was exposed to IMPROVE instructional method and 32 students (48.48%) in control group which was exposed to traditional instruction. In addition, the sample of the study were 25 male students (37.88%) and 41 female students (62.12%). In detail, there are 14 male students and 20 female students in experimental group; and there are 11 male students and 21 female students in control group.

In addition, when collecting qualitative data, 14 students from experimental group (41% of total students in experimental group) were interviewed after treatment. The students were selected based on their scores on Post-PROC and Post-CONC in which five high achiever students (one male and four female students), four moderate achiever students (two males and two female students), and five low achiever students (five male students) were selected in this interview. Initially, students were listed based on the scores on Post-PROC and Post-CONC in descending order. Five students who were on the top of list were selected as high achiever students, four students were in the middle of the list were chosen as moderate achiever students and five students who were in the bottom of the list were selected as low achiever. Each student was interviewed within 20-30 minutes.

In this study researcher worked with an 8-year experienced mathematics teacher. The researcher work with her as she was young teacher who possessed interest in developing her ability in teaching mathematics. Working with young teacher in research study was very helpful. Her age is 30 years old. The teacher is a graduated from a four-year undergraduate program in department of mathematics education.

3.3.Variables

Independent variables (IVs) of the study were teaching methods and gender. The independent variables were categorical and measured in nominal scale. The teaching methods variable had two levels, to wit, traditional instruction and IMPROVE instructional method. Correspondingly, gender had two levels, namely, male and female.

Dependent variables (DVs) of the study were pre-test scores of procedural knowledge test (Pre-PROC), conceptual knowledge test (Pre-CONC), knowledge of cognition (Pre-KC), and regulation of cognition (Pre-RC), post-test scores of procedural knowledge test (Post-PROC), conceptual knowledge test (Post-CONC), knowledge of cognition (Post-KC), and regulation of cognition (Post-RC). All variables were continuous and measured in nominal scales. In Table 3.3, detail information about the variables used is given.

Table 3.3. Variables used in the study

Variables	Types	Types of Value	Scales
Teaching Method Gender	IV	Categorical	Nominal
Pre-PROC Pre-CONC Pre-KC Pre-RC Post-PROC Post-CONC Post-KC Post-RC	DV	Continuous	Interval

3.4.Data Collection Instruments

The measuring tools applied in this study were procedural knowledge test (PROC), conceptual knowledge test (CONC) and metacognitive awareness inventory which consisted of knowledge of cognition (KC) and regulation of cognition (RC). Detail information with respect to these instruments is discussed in the following sub-sections.

3.4.1. Metacognitive Awareness Inventory

Metacognitive Awareness Inventory (MAI) was developed by Schraw and Dennison (1994). It serves to measure of student's metacognitive skills before and after treatments. Based on the classification of metacognition, that is, metacognitive knowledge and metacognitive regulation, the MAI consists of 52 statements. There are 17 questions related to the knowledge of cognition (KC) factor for a maximum possible score of 85. There are 35 questions related to the regulation of cognition (RC) factor for a maximum possible score of 175. This instrument is scored by means of 5-point Likert–type scale ranging from 1 (always false) to 5 (always true). Higher scores correspond to greater metacognitive knowledge and greater metacognitive regulation. In this instruments there were no negative items so that there was no need to reverse the scores.

The 52 statements were divided into nine elements: *declarative knowledge* (eight statements e.g. I know what kind of information is most important to learn); *procedural knowledge* (four statements e.g. I have a specific purpose for each strategy I use); *conditional knowledge* (five statements e.g. I use different learning strategies depending on the situation); *planning* (five statements e.g. I set specific goals before I begin a task); *information management strategies* (10 statements e.g. I create my own examples to make information more meaningful); *monitoring* (seven statements e.g. I find myself pausing regularly to check my comprehension); *debugging* (five statements e.g. I change strategies when I fail to understand); and *evaluating* (six statements e.g. I ask myself if I learned as much as I could have once I finish a task).

As the original MAI was in English language, thus the instrument was translated into Indonesian language. The process of translation involved two experts in both languages. The instrument, firstly, was translated into Indonesian language by an English teacher. Secondly, the obtained result was translated back to English language by an Indonesian student in English Department. Eventually, the original form and the result of translation were compared and examined in order to converge on single understanding and context-based. The original and translated versions are given in Appendix C and Appendix D.

Besides, confirmatory factor analysis was conducted in order to present evidence about whether the two elements of metacognitive skills (knowledge of cognition and regulation of cognition) were assessed by means of MAI. To begin with, the descriptive statistics and assumptions of confirmatory factor analysis were necessary to be checked and evaluated. According to (Tabachnick & Fidell, 2012), the assumptions are: sample size and missing data; multivariate normality and outliers; linearity; absence of multicollinearity and singularity; and residuals.

In this study, there were 169 participants in pilot study and 35 observed variables for regulation of cognition and 17 observed variables for knowledge of cognition. The ratios were 1:5 for regulation of cognition and 1:10 for knowledge of cognition. These ratios were adequate given that the reliability of the knowledge cognition and regulation of cognition in the pilot study were high (Tabachnick & Fidell, 2012). There were no missing data.

Indication of normality of the observed variables was evaluated by means of skewness and kurtosis values. As the skewness values didn’t exceed between -2 and 2 therefore, all observed variables were normally distributed. Considering Appendix P, all skewness values are negative and it means that all distributions are skewed in the similar direction. Therefore, linearity assumption could be said to be confirmed (Tabachnick & Fidell, 2012). Univariate outliers were evaluated by means of boxplots as explained in Tabachnick and Fidell (2012). Subsequently, multivariate outliers were assessed using Mahalanobis distances as described in Tabachnick and Fidell (2012). For evaluating assumption of multicollinearity, it is necessary to check the correlations among the observed variables. Appendix Q presents the correlation matrix for knowledge of cognition and regulation of cognition separately. All the correlations are significant and the values less than .80. and thus assumption of multicollinearity was confirmed (Tabachnick & Fidell, 2012). As a result, the assumptions of confirmatory factor analysis were not violated.

Confirmatory Factor Analysis was conducted by using Lisrel 8.8 program on our proposed CFA model. In the model, knowledge of cognition and regulation of cognition are the latent variables. Figure 3.1 shows the model that resulted in a good fit for Indonesian Version of MAI. As a result, the model shown in Figure 3.1 indicated that there was a good fit between the model and observed data ($\chi 2$ = 2994.35, p = 0.00, GFI= 0.93; AGFI= 0.92; RMSEA= 0,056; SRMR= 0.047, RMR = 0.59) by comparing to the criterion proposed by Jöreskog and Sörbom (1993). Detailed output is given in Appendix R. Therefore, in order to confirm if the MAI really assesses knowledge of cognition and regulation of cognition, the confirmatory factor analysis was conducted. Items 3, 5, 10, 12, 14, 15, 16, 17, 18, 20, 26, 27, 29, 32, 33, 35, and 46 were included in knowledge of cognition factor, and items 1, 2, 4, 6, 7, 8, 9, 11, 13, 19, 21, 22, 23, 24, 25, 28, 30, 31, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 47, 48, 49, 50, 51, and 52 were included in regulation of cognition factor. As a result, a good fit between the modified model and the data was obtained.

According to Schraw and Dennison (1994), the original version of MAI had the Cronbach alpha reliability coefficient of .88 for knowledge of cognition and .88 for regulation of cognition. Whereas, after it was translated into Indonesian language the Cronbach alpha reliability coefficient of MAI based on pilot study and main study was similar, that is, .81 for knowledge of cognition and .90 for regulation of cognition which meant that the questionnaire has relatively high internal consistency, inasmuch as a reliability coefficient above .70 is considered as acceptable (Kline, 2011). In addition, these values mean that the knowledge of cognition and regulation of cognition scores represent students' metacognitive skills at an acceptable level.

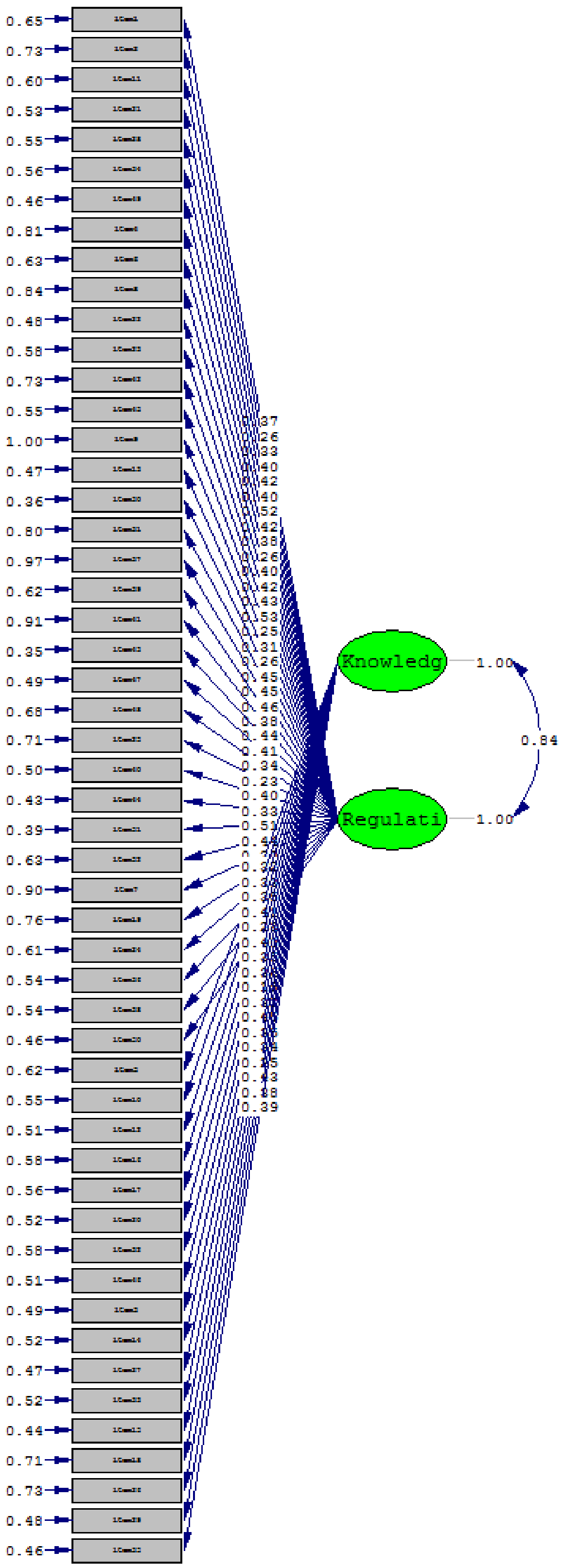


Figure 3.1. The model of Indonesian version of MAI

3.4.2. Procedural and Conceptual Mathematics Test

Procedural knowledge test (PROC) and conceptual knowledge test (CONC) were utilized to gauge students' procedural knowledge and conceptual knowledge of composition and inverse function, infinite sequences and series, and line equations topics. In this study, theoretical framework proposed by Hiebert and Lefevre (1986) was capitalized on for defining and measuring procedural knowledge. According to Hiebert and Lefevre (1986), procedural knowledge consisted of (1) knowledge of mathematical formal language or symbol representation system and (2) knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks. The developed items related to procedural knowledge were considered by this classification. Therewith, this study took theoretical framework revealed by Crooks and Alibali (2014) as foundation for defining and measuring conceptual knowledge. According to Crooks and Alibali (2014), conceptual knowledge consisted of general principle knowledge and knowledge of principles underlying procedures. General principle knowledge was measured using explanation of concepts tasks and evaluation of examples tasks. Meanwhile, knowledge of principles underlying procedures was measured using application and justification of procedures tasks and evaluation of procedures tasks.

In addition, the instruments of procedural and conceptual knowledge were developed by considering the objectives of the topics declared and determined within Indonesian's curriculum of secondary school mathematics. In the curriculum, there were eight learning objectives: four learning objectives for composition and inverse function; two learning objectives for infinite sequence and series; and two learning objectives for line equations. Based on these objectives, 31 questions were formed in which 9 items for procedural knowledge and 22 items for conceptual knowledge. With respect to table specification of PROC and CONC, it is presented in Appendix A. Meanwhile, in the following table, distribution of number of items in relation to topics and type of knowledge is presented.

Table 3.4. Distribution of number of items in relation to topics and type of knowledge

Topics	Number of Items		Total
	PROC	CONC	
Composition and Inverse Function	4	7	11
Infinite Sequences and Series	3	7	10
Line Equation	2	8	10
Total	9	22	31

To construct the instrument, various sources such as mathematics textbooks, internet, and previous research were reviewed in order the questions to satisfy with the curriculum's requirements. There were 26 questions which were developed by researcher, two questions were taken from mathematics textbooks, two questions were taken from previous study, and one question was taken and adapted from internet. Detail information about sources information and the items is given in the following table.

Table 3.5. Sources of questions

Sources of Questions	Items
Researcher	1,2,3,4,6,7,8,9,10,12,13,14,15,16,17,18, 21,22,23,25,26,27,28,29,30,31
Textbooks	11, 24
Other study	5,19
Internet	20

With respect to the type of questions, the researcher utilized essay type of test. The reason lies in the fact that all important concepts presented in the topics could be examined thoroughly and students' both procedural and conceptual knowledge could be assessed properly. Even though essay questions have deficiencies such as scoring problems, inadequate sampling of achievement, and writing problem, it is capable of assessing highest level learning outcomes and emphasizing the integration and application of concepts (Waugh & Gronlund, 2013).

Afterwards, the items were evaluated, reviewed and examined by three professors and two high school mathematics teachers. They evaluated if there were any ambiguity in the sentences within items, if there were any mistakes in writing and answer keys, and if the objectives and the items were assessed its compatibility. Based on the feedback, some suggestions and minor revisions were made in the PROC and CONC. Therefore, content validity and face validity of this instrument were fulfilled. The questions of PROC and CONC were given in Appendix B.

Pilot study was conducted in which the procedural knowledge test (PROC) and conceptual knowledge test (CONC) were administered to 94 students from two different schools in Bandung City (one public school and one private school). As the instruments was applied to measure students' procedural and conceptual knowledge of the aforementioned topics, it would be administered for students who learned the topics, to wit, science major 12^{th} grade classes. The test was piloted in Bandung City as the main study had been planned to be conducted in Bandung City.

The rubric of the instruments was developed by the researcher by making adaptation of rubric developed and used by Oregon Department of Education Mathematics Problem Solving Official Scoring Guide (2011). The researcher used this rubric due to its compatibility with the variable used in the study, that is, procedural and conceptual knowledge. The rubric was adjusted based on the type of knowledge being measured. For example, in scoring items related to knowledge of mathematical formal language or symbol representation system, (1) score 0 was given if students chose incorrect option, (2) score 1 was given if students chose the correct option, but without explanation, (3) score 2 was given if students chose the correct option, but the reason did not make sense, (4) score 3 was given if students chose the correct option and the reason makes sense, but there was a little mistake in the procedure, (5) score 4 was given if students chose the correct option and the reason made sense, but there was a mistake in writing correct mathematical symbol, and (6) score 5 was given if students chose the correct option, the reason made sense, and no mistakes. Detail rubric was given along instruments of procedural and conceptual knowledge in Appendix B. Scoring of procedural knowledge and conceptual knowledge tests was conducted by the researcher and a mathematics teacher. The researcher initially trained the mathematics teacher how to conduct scoring the PROC and CONC by considering the developed rubric. When scoring students' answers, each scorer evaluated students' works on certain question initially. In other words, each response on similar question was evaluated initially in order to notice the differences among them easily and avoid bias in collecting data. Possible maximum scores that students would get if they solved all questions correctly were 45 and 110 for procedural knowledge test and conceptual knowledge test respectively. While their minimum scores were 0 (zero).

Eventually, result of evaluations conducted by the two scorers were compared and correlation between them were obtained. For pilot study correlation between the researcher and the other's scoring was 0.92. Meanwhile, the correlation for pre-test and post-test were 0.94 and 0.90 respectively. The values indicated that the correlations between scorers were obviously high. Therefore, it could indicate the reliabilities of the scoring process made by merely one scorer.

Afterwards, item analysis was conducted to evaluate the items in terms of item difficulty and the Cronbach alpha reliability coefficient. The item difficulty is the proportion or percentage of examinees taking the test who responded the item correctly (Crocker & Algina, 1986). According to Adkins (1974) a higher difficulty index demonstrates an easy item which means that majority students are able to solve problem. Correspondingly, a small difficulty level shows a difficult item. The averages mean of difficulty levels were .36 and .39 for PROC and CONC respectively which showed that it was difficult test for 11th grade level students. With respect to discriminant index, the average was 0.36 which meant that the items were categorized as reasonably good and very good items. The detail of all item difficulties and discriminant index were given in appendix L.

The Cronbach alpha reliability coefficient of PROC and CONC based on pilot study was .407 and .732 respectively. It means that CONC has relatively acceptable internal consistency, whereas PROC has low internal consistency. This low internal consistency led the researcher to conduct revision of several items in PROC. There were six items which were revised, to wit, items 14 and 17 (due to difficult items), and items 24, 27, 29, and 30 (due to low discriminant index). By preserving its objectives, the researcher along with the teacher modified the items such as changing the questions (items 14 and 24), modifying the number used (items 17 and 30), and changing statements (items 27 and 29). As a result, when alpha coefficient calculated with data obtained post administration of PROC and CONC, they were .74 and .82 respectively and these values were as acceptable (Kline, 2011). Therefore, according to results of the pilot study and main study, the PROC and CONC test were appropriate.

3.4.3. Interview

Interviews were conducted in this study to support effort in answering research question with respect to students' experiences with IMPROVE instructional method. With reference to Fraenkel and Wallen (2012), interviewing is conducted with the aim of investigating what people think and how they feel concerning something. There are four facets of interviews, one of which is semi-structured interview or verbal questionnaire in which it is composed of series of questions designed to obtain certain answer from respondents in informal manner (Fraenkel et al., 2015). In this study, this facet of interview was conducted with several students in experimental group after treatment as the interviews are best conducted at the end of the study in order to frame replies to the researcher's opinions of how things are. The aim of the interview was to answer research questions concerning students' experience with metacognitive instructional method in the topics of composition and inverse functions, infinite sequence and series, and line equations. To do this, 14 students from experimental group were interviewed within 20-30 minutes. The students were selected based on their achievement test of Post-PROC and Post-CONC (five high achiever students, four moderate achiever students, and five low achiever students). Knowledge, opinion, and feeling types of questions were asked in this interview. Four basic questions were proposed: (1) the differences from the previous instruction, (2) the obtained benefit from IMPROVE instructional method, (5) useful activities in IMPROVE instructional method, and (4) their encountered challenges or problems during instruction. Detail information with respect to the questions of semi-structured is given in Appendix H. During interview, students also were asked about procedural and conceptual questions relative to the topics based on their answers or responses in post-tests. Interviews were audio taped and transcribed. The transcriptions then were analysed. The analysis process involved two different persons, one of which was researcher and the other was a master student on mathematics field. The process included listening the tape, coding, and categorizing under certain themes.

3.4.4. Classroom Observation

Observation is one tool to obtain first-hand information by observing participants of the study and places at a research site (Creswell, 2012). Classroom observations were conducted in this study to support effort in answering research questions with respect to students' experience with IMPROVE instructional method and to confirm treatment fidelity or treatment verification (see section 3.7 about treatment fidelity and verification). The researcher played role as complete observer and multiple observations were conducted (Fraenkel et al., 2015) within nine-week treatment period. Therefore, the researcher did not interfere with the process of the instruction. During observations, all learning occurrences were recorded by means of two recording devices. The focus of observation was students' posed and responded questions, expressions, and activities during the implementation of teaching methods. All of these were discussed in terms of their development of procedural, conceptual knowledge, and metacognitive skills. Besides, the researcher took notes about important occurrence ensued during the lessons.

3.5.Procedures

The procedure for conducting this study from the beginning to the end of the dissertation writing included several steps. These steps were listed in order with respect to time. The main steps followed during the preparation of this dissertation can be described as follows:

- Determination of research problem
- Identification of keywords
- Literature review and reading the sources obtained from literature review
- Theoretical framework
- Lesson plan and instructional materials
- Development of instruments
- Pilot of the study
- Research permission

- Teacher training and piloting IMPROVE instructional method
- Participants were selected
- Administration of pre-test
- Main study (treatment was given)
- Administration of post-test
- Interviews
- Data analysis
- Writing thesis

Initially, based on the researcher's interests on improvement of students' procedural knowledge, conceptual knowledge, and metacognitive skills, the effectiveness of IMPROVE instructional method were identified as an essence of this study. The researcher obtained enough information and support from previous findings towards the study by means of searching database. Most of literature review was obtained from ERIC (Educational Resources Information Centre), SSCI and International Dissertation Abstract, Academic Search Complete, Social Science Citation Index, JSTOR, Taylor and Francis, Wiley Inter Science, Pro Quest (UMI) Dissertations and Theses, METU Library Theses and Dissertations by using important keywords such as metacognitive instructional method, procedural knowledge and conceptual knowledge, metacognitive skills and gender differences. Obtained literatures from related studies were examined in detail to determine theoretical framework of the study.

Subsequently, lesson plans, instructional materials, and measurement tools were developed in accordance with reviews of professors who were expert in mathematics and mathematics education. Pilot administrations of the measurement tools were conducted in July 2015. 169 students participated in piloting MAI questionnaire and 94 students participated in piloting PROC and CONC. By considering item difficulty, discriminant index, reliability of tests scores, and expert opinion necessary improvements and revisions were performed.

Then, location of the study or school was determined. The school was visited and informed about the study and research permission was obtained from school principal. A volunteer 11^{th} grade mathematics teacher was appointed and trained its implementation in class, and how lesson plans were applied in classrooms. Subsequently, by considering teachers' suggestions and schools' principal's consideration, two intact classes were selected and assigned to experimental and control group randomly. Pre-tests of all measurement tools were administered to both experimental and control group within two-hour lesson or 80 minutes to recognize students' initial procedural knowledge, conceptual knowledge, and metacognitive skills on the topics of composition and inverse function, infinite sequence and series, and line equation. Implementation period lasted for approximately ten weeks (4 class-hours in a week) in the 2015-2016 fall semester. Researcher observed lessons as a non-participant observer and along with other observer rated classroom observation checklists (see Appendix G). Before the implementation of teaching method, the researcher conducted interview with the teacher, classroom observation, teacher training, and piloting study. These activities were accomplished within three weeks. In addition, during implementation the researcher and the teacher came together on Tuesday and Thursday every week to review the previous instruction and prepare for the next lesson so that prior and possible deficiencies could be resolved.

Following the accomplishment of the implementations, post-tests were administered to both groups and semi-structured interviews were conducted with 14 students from experimental group. Data obtained from pre-test and post-test were entered to SPSS to perform necessary analysis. The qualitative data from interviews were transcribed. Descriptive and inferential analyses were done to test the hypotheses of this study and interpret the raw data. The transcribed interviews were coded and the drawings categorized under levels.

3.6.Implementation of the Treatment

The study included 17 sessions within approximately 9-week treatment period (four 40 minute sessions per-week) on the three topics, to wit, composition and inverse function, infinite sequence and series, and line equations. In order to develop teaching material or lesson plan, the researcher considered various references such as previous studies concerning IMPROVE instructional method (Mevarech & Kramarski, 1997), metacognitive awareness inventory questionnaire (Schraw & Dennison, 1994), objective of lesson found in mathematics curriculum for high school (BSNP, 2013) (see Table 3.6 and Table 3.7), definition of procedural knowledge (Hiebert & Lefevre, 1986) and conceptual knowledge (Crooks & Alibali, 2014). The steps of IMPROVE instructional method were introducing the new concepts, metacognitive questioning, practice, review, obtaining mastery, verification, and enrichment and remedial. However, obtaining mastery, verification, and enrichment and remedial steps were performed after all materials explained. Therefore, regularly two-hour lessons were separated into four steps, to wit, introducing the new concepts, metacognitive questioning, practice, and review. In addition, the items existing in the MAI were taken into consideration to be foundation in developing lesson plan or teaching material. When it came to mathematics part, definition of procedural and conceptual knowledge became fundamental consideration. The researcher employ mathematics text book provided by the Indonesia Government and it is available online. The developed teaching material were evaluated by experts in mathematics education and two mathematics teachers. At first stage, revision was conducted by considering theoretical framework of all aforementioned references. Subsequently, at the second stage after it was translated to Indonesian language, revision was made by considering actual condition. The conducted revisions were about the reduction the number of questions in practice part and removing unnecessary parts. The example of lesson plan is presented in Appendix K.

Before the implementation, the teacher, who had over 8 years' experience in teaching mathematics, was trained for implementation of IMPROVE instructional method. Before, training, the teacher was given documents related to procedural knowledge, conceptual knowledge, metacognitive skills, and IMPROVE instructional method and she was requested to read it. The documents were prepared by the researcher. Subsequently, the researcher and the teacher met twice in which each meeting spent approximately 40 minutes to discuss about the documents. The teacher also was given examples of lesson plan and teaching material and she was informed about how she should follow it.

Afterwards, pilot study was conducted in which the teacher was asked to implement IMPROVE instructional method in other two classes within similar school. The topics used were determinant of matrices and invers of matrices and the lesson plans and teaching materials were prepared by the researcher. Based on the result of reflection of the pilot study, the researcher and the teacher made adjustment in terms of number of question in activity sheet, flexibility in proposing metacognitive questions, teachers' moving in classroom, and time limitation.

In addition, before each class session, researcher and the teacher conducted briefing to highlight the important points of lesson plans. The lesson plans were implemented by the teacher during the fall 2015-2016 school term for both experimental and control groups. Instruments were administered during a week right before and after implementation period.

3.6.1. Traditional Instruction

Traditional instruction was implemented by similar teacher for students in control group. The students in control group learned similar topics as the experimental group, yet activities such as metacognitive questioning, cooperative learning, and systematic provision of feedback-corrective-enrichment were not attached in their learning process. The main characteristics of the traditional instruction was that the teacher dominated learning process or teacher-oriented. The teacher sought to transmit knowledge and thought to passive students, limiting their initiated questions, independent thought or interaction between them. The teacher was already capitalizing on traditional instruction in her classes as it was obviously seen when the researcher conducted observation for several sessions. Before, entering control class, the researcher frequently requested the teacher to continue to implement this daily instruction for control group, and to not to apply teaching steps in IMPROVE instructional method. However, the class employed similar tasks that experimental group also used.

The following description delineates the implementation of traditional instruction carried out by the teacher. At the beginning, the teacher checked students' attendance by mentioning students' name loudly. Then, teacher introduced new topics by briefly reminding students about necessary related prior knowledge that would be drew on to help them comprehending the new topics. Subsequently, teacher explained new topics by writing down related formula and demonstrating how to apply it in straightforward example of questions related to the topic.

Table 3.6. Learning objectives of each topic

No	Topics		Learning Objectives	Lesson Hour
1	Composition and inverse function	1.	Students are able to describe the concept of function and apply algebraic operation (addition, subtraction, multiplication, and division) on function	2 x 40’
		2.	Students are able to analyse the concepts and properties of function and perform algebraic manipulation in determining inverse function and inverse of a function	6 x 40’
		3.	Students are able to describe and analyse properties of a function as a result of operation of two or more other function	2 x 40’
		4.	Students are able to describe concepts of composition of function and apply it in daily life context	2 x 40’
2	Infinite sequence and series	1.	Students are able to describe the concept of infinite sequence as a function whose domain is natural numbers	4 x 40’
		2.	Students are able to apply the concept of infinite sequence and series in solving simple problem	4 x 40’
3	Equation of straight lines	1.	Students are able to analyse properties of parallel and perpendicular line and apply it in solving problem	4 x 40’
		2.	Students are able to analyse curves through several points to conclude a straight line, parallel line, or perpendicular line	4 x 40’

Table 3.7. The details of used lesson plans

Topics	Subtopics	Duration	Objectives
Composition and inverse function	Algebraic Operation on Function	2 x 40’	1.1
	Concept of Composite Function	2 x 40’	1.2
	Properties of Composite Function	2 x 40’	1.3
	Concept of Inverse Function	2 x 40’	1.2
	Properties of Inverse Function	2 x 40’	1.2
	App. of Composite and Inverse Function	2 x 40’	1.4
	Remedial and Enrichment	2 x 40’	
Infinite sequence and series	Infinite Sequences	2 x 40’	2.1
	Infinite Geometric Series	2 x 40’	2.1
	Application of Infinite Geometric Series 1	2 x 40’	2.2
	Application of Infinite Geometric Series 2	2 x 40’	2.2
	Remedial and Enrichment	2 x 40’	
Equation of straight lines	Angles with Parallel Lines	2 x 40’	3.1
	Slope of Lines	2 x 40’	3.1
	Parallel and Perpendicular Lines	2 x 40’	3.1
	Line Equation	2 x 40’	3.2
	Remedial and Enrichment	2 x 40’	

The purpose of presenting example of question was to aid students applying the formula in simple situation and student might see the type of question that might appear in the following exercises. The teacher then gave an opportunity for students to take note which she wrote in board. Asking questions to students during the lesson was rarely instead explaining and exercise became dominant in this instruction. Then, teacher gave exercise and requested students to accomplish it individually. Teacher also guided students individually and provided opportunity for student to ask questions. Teacher walked and came near to several students for giving guidance. Several students at the same time who would like to ask about question come near to the teacher. In addition, there was no discussion among the students. In the end of the lesson, the teacher let students to take break without presenting summary of what they learned within two-hour lesson. This type of instruction was consistently carried out by the teacher in the control group during the implementation.

3.6.2. The Implementation of IMPROVE Instructional Method

Students in the experimental group were exposed to IMPROVE instructional method. One week before the implementation, explanation concerning procedural knowledge, conceptual knowledge, metacognition, and metacognitive instructional method were distributed to students in paper form and they were requested to read it at home. One week after the distribution, the teacher checked whether the students read it or not, and most of them read it. The purpose of this were to lead students to be aware of the importance of procedural knowledge, conceptual knowledge, and metacognitive skills in learning mathematics, and to inform students with respect to description of the steps of IMPROVE instructional method.

Students then were divided into groups in which each group consisted of two students. Students' partners were selected by considering students' previous achievement in mathematics exam. According that score, students were listed in descending order, then the researcher paired high achiever students with low achiever students. Working in pairs might help students discussing and sharing their ideas freely without hesitation and each member might have great opportunity to contribute to the process of accomplishing tasks in activity sheet.

The first element of IMPROVE instructional method was introducing to the new concept. There were three important activities conducted in this part: activating prior knowledge, explanation of new concept, and presentation of examples. At this part, in each session the teacher initially asked metacognitive questions to help students reminding the previous concepts and connecting it to the new concepts. For instances, "*What do you remember about functions and relations? Can you tell me how functions differ from relations? What are the domain and range mean?*" Students answered the teachers' questions by expressing their thinking and knowledge related to the previous concepts. The teacher noticed students' answers to recognize whether they possessed sufficient foundation for comprehending the subsequent concepts as lack of prior knowledge impeded students to grasp the idea of new concepts. Then, the teacher explained the topics through questioning, presenting figure, and giving daily life examples in which when teaching she considered students' distinctions on their prior knowledge and levels of thinking. Therefore, she posed questions, presented figures, and gave daily life examples in various ways so that each student in classroom could comprehend the presented topics. The teacher wrote definition of concepts, and explained meaning of each expression in the definition as explaining the definition of certain concept word by word might help students to understand it. When mentioning one mathematical concepts, the teacher sometimes asked students to make visualization it or present example of it. Afterwards, the teacher presented examples. Most examples of concepts were simple problems so that students could deal with it. This is followed by the teacher's metacognitive questions such as: "*What is the problem about? What are the similarities and differences between this problem and the previous problem? What are appropriate strategies for dealing with this problem? Do we perform calculation correctly?*" Subsequently, when solving example questions the teacher modelled aloud strategies for dealing with it and provided reasoning with respect to the use of the strategies. At the end, the teacher checked the process and solution and provided alternative strategies. However, in several occasion, the teacher also asked students if there were students who would like to solve it. Several examples could be solved by several students. Students who solved example questions were asked to give explanation to other students. The teacher proposed questions to verify process and solution of presented problem. When solving example problems, the teacher guided students by means of metacognitive questions. The introduction part took approximately 20-30 minutes.

Second element of IMPROVE instructional method was metacognitive questioning. It was one of the three interdependent components of IMPROVE instructional method (Mevarech & Kramarski, 1997). As a matter of fact, the metacognitive questions were used by both the teacher and students throughout lesson time. In other words, they were inherent in each step of IMPROVE instructional method. When introducing the new concept, the teacher posed comprehension and connection questions to aid students in reminding previous concepts, comprehending new concepts, and building connection among them. Strategic and reflective questions also were asked when she tried to cope with example of questions relative to the topic. During practice, the teacher and students capitalized on all sort of metacognitive questions to help students solve presented problems in activity sheets. In reviewing, the teacher posed heavily comprehension questions to check students' apprehension with respect to material learned. In verification, students presented their works in front of classroom and the teacher posed mostly strategic and reflective questions. Reflective questions were proposed when students solved problems and after they solved it completely. All questions were proposed by the teacher initially, and students also used it when they worked in pairs. In addition, the teacher proposed all sort of the questions to different students so that all of them were given similar opportunity to develop their thinking process. In enrichment and remedial session, the teacher used the metacognitive questions to help low achievers to enhance their understanding and performance in addressing problems, while high achievers used the questions to deal with advance problems by themselves. The metacognitive questions were designed and prepared initially by the researchers for both the teacher and students, yet in the implementation due to unforeseen circumstances and the teacher's way of communication, modification in terms of wording and extension of questions were undertaken. The list of metacognitive questions is presented in Appendix J.

The third element of IMPROVE instructional method was practicing in which students worked in groups to solve problems given in classroom activity sheet. One of three interdependent of IMPROVE instructional method was cooperative setting (Mevarech & Kramarski, 1997). Cooperative setting in IMPROVE instructional method was reflected in this part, namely practicing. Compared to the original version of IMPROVE instructional method in which a group consisted of four students with distinct prior knowledge: one high, two moderate, and one low-achiever, in this study researcher grouped students into two. The reason lies in the fact that if students work in pairs, students would have greater opportunities to develop their thinking process and take important role in solving the problems as well as it would minimize the likelihood of noise due to high numbers of group member. All students also were given two-page of metacognitive questions sheet and they were requested to read it before, during, and after solving problems. With reference to Mevarech and Kramarski (1997) the metacognitive questions used in practicing part were arranged and designed in order to guide students to be aware of the problem solving process and to self-regulate their progress. When working in pairs, every student strived to address mathematics problem presented in the activity sheet and then described his or her reasoning by responding the given questions stated in the two-page of metacognitive questions interchangeably. There are four types of metacognitive questions employed in this teaching stage: comprehension questions; strategic questions; connection questions; and reflective questions. The Table 3.8 illuminates several instances of metacognitive questions stated in the sheet.

With respect to problems or tasks presented in activity sheets, generally within a session students had to deal with three tasks. The central aim of presenting tasks in activity sheet was to develop students procedural and conceptual knowledge, thus constructed tasks or problems were given to assist students in applying learned concepts in various context or type of questions, to guide students to discover general rules or formulas by means of proofing, to lead students to grasp the ideas presented or deepen their comprehension by means of figures or graphics, to help students dealing with non-routine problems, and to assist them in drawing conclusion and expressing by means of their own words. Students were expected to accomplish all tasks in activity sheet within approximately 20 minutes.

When students worked in pairs, the teacher provided guidance by joining at least two pairs for approximately 10 minutes and involved in group discussion. The teacher modelled the use of metacognitive questions in addressing problems in activity sheet by reading aloud and presenting reasoning of each conducted step of the solution.

When joining the group discussion, the teacher also paid attention to what students did in their effort to solve the problems and gave guidance if necessary. In the wake of working in pairs, several students were requested to present their works and write it in board. Subsequently, they explained their works to the whole class by using metacognitive questions. The teacher allowed other pairs to ask questions of reveal and share their alternative ideas relative to the presented problem. The teacher monitored students' process of obtaining correct answers and use of metacognitive questions. If there were any pairs that made mistake in solving problem, the teacher asked the whole class to observe the mistake and perform correction. The teacher also provided guidance by proposing additional metacognitive questions.

Table 3.8. Instances of metacognitive questions in practice part

Type of Questions	Instances of Questions
Comprehension Questions	1. What is the problem about? 2. What is the meaning of this term? 3. What is important information given in question?
Strategic Questions	1. What is the strategy appropriate for dealing with the problem? Why? 2. What is the first step? What is the next step? 3. Are there any other strategies to solve this problem? What and how?
Connection Questions	1. What are differences between the prior problem and the problem you are working on? 2. Have you solved this type of problem successfully before? 3. How do strategies used in the previous problem differ from strategies that you have selected in this problem?
Reflective Questions	1. What are you doing right know? Why? 2. Do we calculate correctly? 3. Does the result make sense? Why?

The fourth element of the IMPROVE instructional method was reviewing. Reviewing was conducted at least in three distinct spots: (1) beginning of lesson, (2) during explanation, and (3) end of lesson. At the beginning of lesson, the teacher reviewed material that learned in previous session. In order to check students' comprehension with respect to what are being learned, teacher reviewed small part of the concepts learned. At the end of lesson, the teacher reviewed the important ideas of the lesson learned in that day with all students or highlighted the topics by emphasizing its summary. In these occasions, the teacher observed students' common difficulties and then provided necessary supplementary explanation to the whole class. Identification of students' difficulties was necessary to be foundation for the teacher to consider the alternative explanation to deal with it. The teacher also conducted reviews by proposing metacognitive questions such as, "What have you learned yesterday/today? Can you repeat what identity function does mean?" besides, the students were expected to answer questions asked or provide explanation of the concept learned by means of their own words.

The fifth element of IMPROVE instructional method was obtaining mastery. The instruction expected all students to reach mastery in which it was checked in formative test conducted after accomplishing certain topic. Formally according to high school mathematics curriculum in Indonesia, a student could be claimed as reaching mastery if he or she is able to reach 75% of all requirements in mathematics classroom (BSNP, 2013). In this occasion, formative test was administered initially. The test consisted of three or four questions and students were requested to solve it individually within 20 to 30 minutes.

The sixth element of IMPROVE instructional method was verifying. The teacher verified students' acquisition of cognitive and metacognitive skills. After administering formative tests, the teacher checked and verified students' answer along with the students. Based on its results, the teacher categorized the students into two distinct categories: high achievers and low achievers.

The seventh element of IMPROVE instructional method is enrichment and remedial. As a matter of fact, this element was derived from one of three interdependent components of IMPROVE instructional method, namely systematic provision of feedback-corrective-enrichment. This systematic provision adjusted the learning time to the needs of every pupil and thus it led them to reach mastery on the tasks or topics and extended their mathematical thinking (Mevarech & Kramarski, 1997). Enrichment and remedial were conducted within two-hour lesson after all material of certain topic were explained or at the end of each topic. Students who didn't achieve mastery (75% correct) on the formative tests were provided corrective activities. The corrective activities consisted of conceptual and procedural guidance by presenting various types of questions. At the end of guidance process, these students were given second formative test whose questions were parallel to the initial formative test. Meanwhile, students who were able to reach 75% correct on the test were given enrichment activities concerning to the topics. The enrichment activities consisted of advance tasks that centred on reasoning and application of the topics. In the wake of solving those tasks, high achiever students could check their answers by looking at answer key given by the teacher. In this session, therefore, the teacher dealt with two types of students, to wit, low achievers and high achievers. Students were no longer working in pairs, rather they are requested to work individually and along with other students if necessary. Enrichment and remedial process were conducted within approximately 30 minutes. In addition, each session students were given homework in which help students to develop their ability in comprehending the topics and the teacher used this homework as foundation for recognizing students' ability and providing feedback if students made mistakes. Students were expected to pay attention to correction given by the teacher.

In addition, students in experimental group were provided with reflective journal three times during the implementation (in the lecturer 6, 11, and 16). Students were asked about their difficulties, the strategy to deal with it, and their performance during lesson. Students wrote reflective journal ten minutes before ending the class session. Detail of questions in reflective journal is given in Appendix I.

3.7.Treatment Fidelity and Verification

Shaver (1983) revealed that verifying the application of independent variables is extremely crucial in teaching methods studies. In this study, the verification was aimed to ensure that whether the students within experimental group were taught with IMPROVE instructional method properly and whether students within control group were instructed with traditional instruction. Therefore, there were no other alternative explanations concerning difference in dependent variables. To do this, there were several methods that were be performed by researcher: defining the IMPROVE instructional method and traditional instruction clearly, developing detailed lesson plans (see Appendix K), observation checklist (see Appendix G), and teacher training.

Related to defining the instructions clearly, literature reviews on IMPROVE instructional method provided framework how the instruction should or should not be implemented. There were several previous studies that explicate stages of IMPROVE instructional method in detail (Kramarski et al., 2002; Mevarech & Fridkin, 2006; Mevarech & Kramarski, 1997). Subsequently according to those studies, lesson plans for the three topics were developed. During development, experts in mathematics education such as supervisor and other lectures provided guidance to the researcher. In line with their suggestions, necessary revisions were conducted on it.

Before implementing IMPROVE instructional method to experimental group, the teacher was trained by researcher. It was conducted within two weeks in which at the first week the training was centred on theoretical aspect and practical aspect in the second week. The teacher initially was presented by explanation regarding metacognition, procedural knowledge, conceptual knowledge, and IMPROVE instructional method. Technical information related to the aforementioned terms were discussed and the teacher expressed her understanding. In addition, in order to verify teachers' understanding of metacognitive instructional method, pilot study was conducted within two sessions. Result of the pilot study indicated that the teacher was able to implement IMPROVE instructional method appropriately even though there were several deficiencies at the outset.

In addition, treatment verification of the treatment was confirmed by rating classroom observation checklists. To do this, observation checklists was developed by researcher in agreement with literature reviews and suggestions from the supervisor. The checklist consisted of steps of IMPROVE instructional method and for experimental group it determined the degree to which the teachers implemented it that was formulated with lesson plans (see Appendix G). Whereas, for the control group, it checked the absence of the steps of IMPROVE instructional method. In the study, the researcher also worked along with other observer who worked as mathematics teacher in other private school to fill the observation checklists. The purpose of involving two observers in this study was to preclude bias of the researcher and hence to acquired more reliable result. Treatment verification of the study was confirmed by evaluating classroom observation checklist throughout 8 sessions or 16 class hours (four of them were in topics of composition and inverse function, two in topics of infinite sequences and series, and two in topics of line equations) implementation period both in experimental and control group. It meant that 57% of all sessions were observed. The checklist indicated the level to which the teacher implemented IMPROVE instructional method which is arranged within lesson plans. Table 3.9 gave insight into descriptive statistics of each item in the checklist. It is obviously evident from the table that items related with the experimental group have higher mean scores than those for the control group.

There were 33 items in the checklist in which each item was coded as ‘1’ for never or no, ‘2’ for partially or occasionally, and ‘3’ for yes or frequently, and ‘0’ for not applicable. Not applicable for certain items means that the items are not applied in each lesson instead it is applied in particular session. When observing lesson time, observers were also counting the occurrence of several countable items, and based on the obtained number it was decided whether it was categorized as occasionally and frequently.

In addition, to decide whether the observed differences between groups in Table 3.9 are statistically significant or not, both parametric and non-parametric tests were used. For this purpose, independent t-test was applied for the parametric test and Kruskal-Wallis test for the non-parametric test while comparing the groups by using these scores. According to Table 3.10 and Table 3.11, there are significant mean differences between groups on the dependent variables.

Table 3.9. Results of classroom observation checklist

Item No	Experimental Group		Control Group	
	Mean	SD	Mean	SD
1	3	0	2	1.07
2	3	0	1	0
3	2.75	0.46	1	0
4	3	0	2.5	0.53
5	3	0	1.75	0.46
6	3	0	2.125	0.64
7	3	0	1.625	0.74
8	3	0	1	0
9	3	0	1.375	0.52
10	3	0	3	0
11	2.375	0.92	1	0
12	3	0	3	0
13	3	0	1.875	0.64
14	3	0	1	0
15	3	0	1	0
16	3	0	2.25	1.03
17	3	0	1	0
18	2.5	0.53	1	0
19	2.625	0.52	1	0
20	2.25	0.89	1.125	0.35
21	3	0	1.375	0.52
22	2.75	0.46	1.25	0.46
23	2.5	0.53	2.5	0.53
24	2	0	1	0
25	2	0	1.375	0.52
26	2.375	0.74	2.375	0.74
27	2	0	1	0
28	3	0	3	0
29	2	0	1	0
30	.375	0.71	0	0
31	0	0	0	0
32	0	0	0	0
33	0	0	0	0

Table 3.10. Independent Samples t-Test (Parametric Test)

	Levene's Test for Equality of Variances		t-test for Equality of Means		
	F	Sig.	t	df	Sig. (2-tailed)
Scores	.000	.988	4.533	64	.000

Table 3.11. Kruskal-Wallis Test

	Scores
Chi-Square	18.406
df	1
Asymp. Sig.	.000

In order to obtain reliable results from the observation checklist, the sessions were observed by two observers and it were compared. Hence, items had two scores for these sessions across groups. In the analyses, item scores for these sessions were capitalized on based on the average of the scores given by the two observers. Table 3.12 gives insight into detail information with respect to the correlation coefficients between these scores given by two observers across group. With reference to the data presented in the table, the correlations of observation checklists between the two observers were calculated and it is found that most of the values were high (greater than .90). Therefore, it could indicate the reliabilities of the observations made by merely one observer.

Table 3.12. Correlations between two observers

Group	Correlations in each session							
	1	2	3	4	5	6	7	8
Control Group	.95	.93	.92	.94	.93	.92	.93	.95
Experimental group	.96	.94	.98	.98	.98	.95	.94	.97

In brief, based on the result of descriptive statistics it showed that treatment verification was supported for IMPROVE instructional method. In addition, to understand the difference between the two teaching methods, independent t-test and Kruskal-Wallis test were conducted and the result indicated that characteristics of IMPROVE instructional method was applied in experimental group and it differed from that traditional instruction. To conclude, the treatment verification is confirmed in this study.

3.8.Data Analysis

In order to answer research problems, quantitative and qualitative study were conducted. Therefore, this study covered quantitative and qualitative data. Pre-test and post-test scores of procedural knowledge (PROC), conceptual knowledge (CONC), knowledge of cognition (KC), and regulation of cognition (RC) were compiled as quantitative data. In addition, students' gender, classrooms, and scores in previous examination were also collected. All obtained data were entered into computer and analysed statistically with IBM Statistical Package for the Social Sciences (SPSS) program.

3.8.1. Descriptive statistics

Descriptive statistical procedures were applied to find the mean, standard deviation, minimum and maximum scores, range, skewness, kurtosis, correlation coefficients of the obtained data for each variables. The obtained result allowed the researcher to make description about sample, thus the possibility of influencing dependent variable could be analysed. In addition, these statistical procedures were used to check assumptions that are required to conduct inferential statistics.

3.8.2. Inferential statistics

After conducting descriptive statistics, inferential statistics were applied on the obtained data as the purpose was to generalize results obtained from the sample to the population. Multivariate analysis of covariance (MANCOVA) was conducted to test the hypotheses of this study. MANCOVA was chose and conducted since the study worked with two intact groups in which one group might be superior to another group in particular condition. Hence, it was necessary to equate the condition at least on one independent variable by means of covariate analysis. There were four variables treated as covariate: Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC. However, before running MANCOVA, there were several assumptions underlying it that had to be checked. Based on data analysis using SPSS, it was observed that all assumptions were met, then the researcher continued to proceed the analysis with MANCOVA. MANCOVA was conducted with four dependent variables: Post-PROC, Post-CONC, Post-KC, and Post-RC. Subsequently, as the test resulted in a significant difference for group membership and gender, follow-up ANCOVAs were performed in order to investigate effects of each independent variable, to wit, teaching methods and gender on each dependent variable separately.

3.8.3. Qualitative Data

Interview data analysis was conducted with the aim of investigating the students' experience with IMPROVE instructional method. In this study, the analysis was the organization and interpretation of the students' responses acquired during the interviews in an attempt to look for similar meanings. Therefore, according to Creswell (2012), an prominent step in the process of analysing interview data is investigating the common sense of the data. At first, the participants' responses which were recorded were labelled as their first letter of their names and related letter of their second names in order to facilitate the author in identifying their responses during analysis and fulfilling ethical considerations. It is important to note that the interviews were conducted using Indonesian language. Four questions posed to the students were of themes.

Then process of coding the data started. Coding as described by Charmaz (2006) was classification of data segments in which a simple label was given to condense and delineate each section of data. However, before breaking into codes, the recorded interviews were played and listened several times by the researcher and a master student in department of mathematics from beginning to end to facilitate a holistic comprehension and get sense of each participant's responses that were related to the interview questions. While listening, notes were taken to keep track of initial thoughts about codes to investigate further by two persons independently. Students statements in Indonesian language were transcribed and printed out. The transcribes were read two times by coders. Subsequently, to begin coding the data, all possible codes extracted from individual interviews were listed in separated Microsoft Excel worksheet. In the worksheet, table was constructed which consisted of students' initial name in the columns and questions of interview in the row. Within the intersections of the cells, each coder wrote possible codes and necessary comments.

At first, there were few possible codes assigned, but as the analysis progressed, more codes emerged. Comparison between constructed codes was carried out and discussed to reach common categories. When the coding was complete and common categories were obtained, then statements from students' interview related to the categories were selected and transcribed. Each transcript was edited by removing any irrelevant statements and personal identifiers. Subsequently, the edit transcript was translated properly in English without modifying its meaning. Inter-rater reliability in coding was confirmed by computing percentage of agreement between two independent raters' coding. At the end, 83% agreement in coding was reached. Even though the value showed high level of inter-rater agreement, there were several disagreed matters, after further discussion full agreement was achieved on codes. The result of coding process is presented in Table 4.20.

With respect to classroom observation, all teaching processes were recorded using video recording. During observation, the researcher and other observer also took field notes to catch important issues. Subsequently, after the implementation of the study, the recordings were played three times by the researcher and important events were noted and transcribed. Along with field notes, these note regarding important events were used to support students' responses of questions asked in the interview.

3.9.Unit of Analysis

In an experimental study, the experimental unit should also be the unit of statistical analysis (Festing & Altman, 2002). The reason is that if both the unit of analysis and the experimental unit are the similar things, then the independence of observation is fulfilled. Nonetheless, in this study, the experimental unit is each intact classroom i.e. control group and experimental group, whereas the unit of analysis is each student. Since the unit of analysis and experimental unit are not similar, thus generation a claim in regard to independence of observation is not possible.

In fact, during the lesson it was inevitable that there were lots of interactions among students. Thus, by considering this reality, independence of observation was difficult to be said when the treatments were applied. Nevertheless, interaction among students when data collection process i.e. pre-test and post-test was not allowed and the teacher ensured that students did their tests individually. As a result of this, independence of observation might be assumed at least during measurement processes.

3.10. Assumptions, Delimitations and Limitations of the Study

There are several assumptions for this study which were given below:

- The participants responded the items on the instruments seriously, consciously, and truthfully.
- Treatments were given according to the lesson plans developed by the researcher.
- The tests were administered according to the regulations for test administration prepared by the researcher.
- Independence of observations was satisfied.
- Characteristics of the teacher who implemented the treatments were not influenced by division of experimental and control group.

In addition, delimitations of the study were as follows:

- The results of this study are delimited to 66 11th grade science students.
- The results are delimited to composition and inverse function, infinite sequences and series, and line equations.
- Implementation period was delimited to 9 weeks.
- The interview was restricted to 14 students in experimental group.
- The quantitative data was delimited from four measurements: procedural knowledge; conceptual knowledge; and metacognitive awareness inventory.

Limitations of the study were:

- The study was not able to provide random sampling.
- Students procedural and conceptual knowledge on the three topics might be influenced by other factors such as affective aspects that are not controlled in this study.

3.11. Internal and External Validity of the Study

In this part, description relative to internal and external validity of the study will be explained below in detail.

3.11.1. Internal Validity of the Study

Internal validity means that there should not be other alternative explanation relative to relationship observed between two or more variables (Fraenkel et al., 2015). So as to the independent variable influences purely dependent variable, several threats to internal validity have to be controlled such as subject characteristics, mortality, location, instrumentation, testing, history, maturation, attitude of subjects, regression, implementation.

Subject characteristics could be a threat to internal validity as relationship observed between variables might be due to difference in individuals such as gender, ability, and age. Even though the study applied convenient sampling, the groups were randomly assigned to the intact classes. In order to address these differences, holding certain variables constant, applying appropriate statistical design and using analysis of covariance was performed.

Mortality could be a threat to internal validity as the participants of the study might withdraw from the study due to unexpected situation and as a result it affects the outcome of the study. In order to address this threat, researchers asked the teacher to remind students the date of pre-test and post-test as well as instructed them to attend those tests. All students from the two groups attended pre and post-treatment tests, thus there were no missing data or losses during the study.

In this study, location that could be alternative explanation of relationship observed between variables was not a threat. The number of students in each class was almost equal (32 students in control group and 34 students in experimental group). In addition, two classrooms that the study took place were almost similar in terms of the size, lighting, and other physical conditions.

Application of instruments in the study could be a threat as changes in the instruments and scoring manners, data collector characteristics, and data collector bias might influence the result of the study. However, in this study, both control and experimental group were given similar set of pre and post-test questions and duration of the administration of those tests involved a long enough period of time (two and half months). In addition, detail rubrics for each instrument were used and data were collected by the teacher.

History could be threat as unexpected events may occur when the study is conducted and it influences the outcome of the study. In this study, there were not unplanned events took place during the study as the researcher and the teacher made timeline of treatment by considering school academic calendar for fall semester 2015-2016. Therefore, history was not a threat for this study.

Maturation could be a threat to internal validity as alteration during treatment might be due to factors related to passing of time. In this study, maturation was not threat since the study was conducted within two and half months and the age of participants were 15-16 years old in average.

Students also might experience the Hawthorne Effect since they might know they were being observed during the study. This might result in special attention and recognition received by students. In this study, in order to eliminate the problem of attitude of subjects, the researcher conducted classroom observations for various classes by video-taping it and it started from three weeks before the implementation. Therefore, students might think that other classes also received similar treatment.

Regression could be a threat since when score difference is investigated in a group whose pre-test scores is extreme low or high. In this study selecting appropriate design, that is, analysis of covariance was applied to cope with this threat.

Implementation could be a threat to internal validity as the experimental group may receive unintended manners that provide them an undue benefit affecting outcome. To overcome this possible threat, both groups were instructed by the same teacher and treatment fidelity and treatment verification were examined.

3.11.2. External Validity of the Study

According to Fraenkel and Wallen (2012), external validity refers to applying obtained result of certain study to novel setting, people, or samples. The result of this study, based on MANCOVA, indicated that there were statistically significant mean differences between experimental and control group on collective dependent variables of Post-PROC, Post-CONC, Post-KC, and Post-RC scores after adjusting for pre-existing difference in students' Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC in favour of experimental group which was exposed to IMPROVE instructional method. Besides, this study involved 66 participants in which most of them are low or medium achievers. They also were not selected randomly from the population. Therefore, the findings of this study might be generalized merely to the sample that possess similar characteristics and contexts. As a result, generalization is limited.

3.12. Trustworthiness in Qualitative Part

Both reliability and validity are requirement for all sorts of research methods. In qualitative research, trustworthiness could be described by means of dependability, transferability, credibility, and confirmability (Guba & Lincoln, 1982).

a. Dependability

The term dependability referred to how interpretation of data should be consistent among researcher and other persons (Koch, 2006). In order to support dependability, in-depth information about all the process of the research was illuminated. Rich and thick description of the contexts and students as well as the interview questions and data collection procedures were described. In this study, interviews were audio recorded so that there was no any losing information.

b. Transferability

According to Hoepfl (1997), transferability referred to the applicability of a working hypotheses to other context. Therefore, the similarity between the original context and the context to which it is transferred should be similar. However, the researcher couldn't ensure the transferability of findings. There were two means to support transferability, to wit, theoretical purposive sampling and thick description (Guba & Lincoln, 1982). In qualitative part, purposive sampling was conducted in which high, moderate, and low achieving students were selected to be interviewed by the researcher. The second way that could be done by the researcher was to provide thick descriptions with respect of all process of the study. Subsequently, other researchers could apply the findings of this study to other context.

c. Credibility

Credibility is also important in qualitative research. There are several means to ensure the credibility: prolonged engagement, persistent observation, peer debriefing, triangulation, referential adequacy materials and member checks (Guba & Lincoln, 1982).

Prolonged engagement referred to the presence of researcher in the location of the study where the instruction was being implemented long enough to develop trust with the students, undergo the broadness of variation and to deal with distortions due to the presence of the researcher in the location. In this study, the researcher was on the location of the study within 3 months and it seemed that it was enough for the researcher to be in there. In addition to prolonged engagement, persistent observation was conducted. It referred to a technique conducted to pay attention to specific phenomena under a study so that important and unimportant focus were determined. In this study, the researcher focused on students' experience with the IMPROVE instruction.

With respect to triangulation, in this study the researcher used multiple methods of data collection such as interview, observation, reflective journal, and literature review. Peer debriefing was evaluated after the completion of transcribing. It involved discussion with a disinterested peer (a master student from mathematics department). He was requested to construct codes and themes from the given transcriptions. At the end of peer debriefing, all coded interviews were compiled and compared to check their consistency. It was found that the number of codes constructed by researcher was more than that of disinterested peer. However, these codes indicated high parallelism and similarity between the two.

Referential adequacy materials mean that document, videotapes, audio recordings, pictures, and other raw materials are compiled during the implementation of the study and archived without analysis. In this study, these materials were collected.

d. Confirmability

Confirmability deals with evaluation of the findings whether they are really generated by means of the inquiry or not. There are three strategies to ensure confirmability, to wit, triangulation, practicing reflexivity, and confirmability audit (Guba & Lincoln, 1982). In this study, reasons for formulating the study in a certain way, decisions during the study, the rationale or meaning behind those decisions, and the quality of those decisions after application were uncovered.

When all aspects were considered, in short, all the process were explained in detail to support trustworthiness. In order to provide trustworthiness, while collecting the data several considerations were taken such as: (1) making clear the objective of the study before conducting the interviews; (2) establishing convenient and flexible environment; (3) keeping away intervention and personal reflections when interviewing; and (4) provide enough time to students to respond.

3.13. Ethical Considerations

In this study, the participants were informed with respect to the purpose of the study and they participated voluntarily. They were not forced in any way and possessed option to quit the study at any time. All obtained data were compiled by the researcher and explored by the researcher. Collected data will not be applied for other purposes. This study maintained confidentiality of all subjects such as school names, students' names, teacher's name, classroom's name and so forth. Unrelated information about students or teacher were not disclosed in this study.

CHAPTER 4

RESULTS

This chapter presented descriptive analysis of pre-tests scores and post-tests scores, determination of covariates, evaluation of assumptions of MANCOVA, result of investigation of the effect of different teaching methods and gender on students' procedural knowledge, conceptual knowledge, and metacognitive skills, students' experiences with IMPROVE instructional method, interview results on students' procedural and conceptual knowledge, and the summary of the results.

4.1.Descriptive Analysis

The descriptive statistics for pre-tests scores and post-test scores are presented for each level of the independent variables and the interactions among them. Since inferential statistics for main and interaction effects of the independent variables on the dependent variables will be carried out, descriptive statistics relative to the main effects and interaction effects are presented for the dependent variables.

4.1.1. Descriptive Statistics for Pre-Test Scores

Descriptive statistics for pre-test of procedural knowledge (Pre-PROC), conceptual knowledge (Pre-CONC), knowledge of cognition (Pre-KC), regulation of cognition (Pre-RC) scores across groups were summarized at Table 4.1.

Table 4.1. Descriptive statistics for all pre-test scores across groups

TEST	GROUPS	N	M	SD	MIN.	MAX.	SKEW.	KURT.
Pre-PROC	Control	32	1.31	1.65	.00	6.00	1.333	1.067
	Experiment	34	1.12	1.75	.00	6.00	1.993	3.421
Pre-CONC	Control	32	2.50	2.65	.00	8.00	.659	-.787
	Experiment	34	2.59	3.09	.00	10.00	1.004	-.149
Pre-KC	Control	32	60.31	6.85	48.00	76.00	.384	-.511
	Experiment	34	61.92	6.79	49.00	78.00	.384	-.102

Table 4.1. Continued

TEST	**GROUPS**	**N**	**M**	**SD**	**MIN.**	**MAX.**	**SKEW.**	**KURT.**
Pre-RC	Control	32	122.00	15.34	96.00	154.00	.193	-1.108
	Experiment	34	125.56	13.43	98.00	162.00	.164	.434

In general, according to Table 4.1, there were no large differences between means of the control and experimental groups on all pre-tests scores. Pre-PROC and Pre-CONC measured students' initial procedural and conceptual knowledge on composition and inverse functions, infinite sequences and series, and line equations topics. The mean score of Pre-PROC for control group was 1.31 (SD = 1.65) and for experimental group was 1.12 (SD = 1.75). The mean score of Pre-CONC for control group and experimental group were 2.50 (SD = 2.65) and 2.59 (SD = 3.09) respectively. As the mean scores of the control and experimental groups were very close to each other, it could be said that the students from the two groups possessed similar level of initial knowledge on these topics before the implementation of the treatments. It could be said also that maximum scores gained by the students was low if compared to the possible maximum scores total of PROC and CONC, to wit, 45 and 110 respectively.

Similar to Pre-PROC and Pre-CONC scores, the mean of pre-test scores of students' metacognitive skills which consisted of knowledge of cognition (Pre-KC) and regulation of cognition (Pre-RC) were very close each other. The mean of pre-test scores of Pre-KC for control group and experimental group were 60.31 (SD = 6.85) and 61.92 (SD = 6.79) respectively out of the possible maximum scores of 85, whereas the means score of Pre-RC for control group and experimental group were 122.00 (SD = 15.34) and 125.56 (SD = 13.43) respectively out of the possible maximum scores of 175. Since there were 17 items and 35 items of KC and RC respectively and using Likert–type scale ranging from 1 (always false) to 5 (always true), it could be said that in average students in control group and experimental group expressed 'true' in metacognitive awareness inventory which means that most students initially possessed moderate to high metacognitive skills.

With respect to the value of skewness and kurtosis of all pre-tests scores, it was obviously seen in the table that most of the scores were normally distributed as their values fell between -2 and +2. However, scores of Pre-PROC for experimental group were not normally distributed based on the value of kurtosis as it exceeded the maximum range of criteria of normal distribution.

In addition, descriptive statistics for all pre-tests scores across gender were summarized at Table 4.2.

Table 4.2. Descriptive statistics for all pre-tests scores across gender

TEST	GENDER	N	M	SD	MIN.	MAX.	SKEW.	KURT.
Pre-PROC	Male	25	.36	.76	.00	3.00	2.397	5.834
	Female	41	1.73	1.89	.00	6.00	1.168	.395
Pre-CONC	Male	25	1.04	2.26	.00	9.00	2.758	7.419
	Female	41	3.46	2.83	.00	10.00	.364	-.712
Pre-KC	Male	25	61.96	6.55	52.00	74.00	.494	-.901
	Female	41	60.63	7.01	48.00	78.00	.353	-.087
Pre-RC	Male	25	121.32	13.32	96.00	143.00	-.041	-.844
	Female	41	125.37	14.95	100.00	162.00	.130	-.496

In general, as reported in Table 4.2. there were slight differences between means of male and female students on pre-tests scores. The mean score of Pre-PROC for male was 0.36 (SD = .76) and for female was 1.73 (SD = 1.89) out of the possible maximum scores of 45. Meanwhile, the mean score of Pre-CONC for male and experimental group were 1.04 (SD = 2.26) and 3.46 (SD = 2.83) respectively out of the possible maximum scores of 110. Based on that mean scores, it seemed that female students' initial procedural and conceptual knowledge were better than that of male students. The maximum scores obtained by male and female students were low compared to possible maximum scores of PROC and CONC.

Regarding metacognitive skills, there were two distinct results in which female students had higher mean score on Pre-RC (M = 121.32, SD = 13.32) than that of male students (M = 125.37, SD = 14.95) out of the possible maximum scores of 85, and male students had higher mean score on Pre-KC (M = 61.96, SD = 6.55) than that of female students (M = 60.63, SD = 7.01) out of the possible maximum scores of 175. It could be said that in average male and female students expressed 'true' in statements of metacognitive awareness inventory which means that most students initially possessed moderate to high metacognitive skills.

With respect to the values of skewness and kurtosis of all pre-tests scores, it was obviously seen in the table that not all the scores were normally distributed as the values of skewness and kurtosis of scores of Pre-PROC and Pre-CONC for male students didn't fall between -2 and +2.

4.1.2. Descriptive Statistics for Post-Test Scores

In the wake of describing descriptive statistics for all pre-tests scores, in this section descriptive statistics for post-tests scores is presented. Descriptive statistics for Post-PROC, Post-CONC, Post-KC, and Post-RC scores of control and experimental groups are summarized at Table 4.3.

Table 4.3. Descriptive statistics for all post-tests scores across groups

TEST	GROUPS	N	M	SD	MIN.	MAX.	SKEW.	KURT.
Post-PROC	control	32	29.81	8.39	11.00	45.00	-.138	-.461
	experiment	34	33.79	6.89	16.00	45.00	-.616	-.018
Post-CONC	control	32	48.84	10.49	32.00	75.00	.758	.206
	experiment	34	62.12	11.90	40.00	93.00	.626	.442
Post-KC	control	32	60.63	7.10	42.00	73.00	-.739	.396
	experiment	34	63.79	6.78	54.00	85.00	1.210	1.784
Post-RC	control	32	121.50	15.03	88.00	150.00	-.430	.346
	experiment	34	131.03	13.18	104.0	175.00	.867	2.848

According to Table 4.3, there were differences between means of the control and experimental groups on post-test scores. Post-PROC and Post-CONC measured students' procedural and conceptual knowledge on composition and inverse function, infinite sequences and series, and line equation concepts after the treatments. The mean score of Post-PROC for control group was 29.81 (SD = 8.39) and for experimental group was 33.79 (SD = 6.89) out of the possible maximum scores of 45. Whereas the mean score of Post-CONC for control group and experimental group were 48.84 (SD =10.49) and 62.12 (SD = 11.90) respectively out of the possible maximum scores of

110. It seemed that there were large differences between mean scores of Post-PROC and Post-CONC across groups. It could be said also that maximum scores gained by the students in post-test was better that of scores in pre-tests as there were several students who gained the maximum scores of total of PROC and CONC.

Similar to Post-PROC and Post-CONC scores, the mean scores of students' metacognitive skills which consisted of knowledge of cognition (Post-KC) and regulation of cognition (Post-RC) were different across groups. The mean score of Post-KC for control group and experimental group were 60.63 (SD = 7.10) and 63.79 (SD = 6.78) out of the possible maximum scores of 85, whereas the means score of Post-RC for control group and experimental group were 121.50 (SD = 15.03) and 131.03 (SD = 13.18) respectively out of the possible maximum scores of 175. It could be said that in average students in experimental and control group expressed 'true' in metacognitive awareness inventory which means that most students possessed moderate to high metacognitive skills.

With respect to the value of skewness and kurtosis of all post-tests scores across groups, it was obviously seen in the table that most of the scores were normally distributed as their values fell between -2 and +2. However, scores of Post-RC for experimental group were not normally distributed based on the value of kurtosis as it exceeded the maximum range of criteria of normal distribution. However, based on Kolmogorov Smirnov test for normality, it showed that scores of Posts-RC for experimental group was normally distributed (see Appendix M).

In addition to descriptive statistics for all post-test scores across groups, descriptive statistics of post-test scores of male and female students were summarized at Table 4.4.

Table 4.4. Descriptive statistics for all post-tests scores across gender

TEST	GENDER	N	M	SD	MIN.	MAX.	SKEW.	KURT.
Post-PROC	Male	25	26.04	6.67	11.00	42.00	.122	.700
	Female	41	35.41	6.29	16.00	45.00	-.943	1.377
Post-CONC	Male	25	48.64	10.32	36.00	81.00	1.384	2.939
	Female	41	59.98	12.69	32.00	93.00	.260	.403
Post-KC	Male	25	60.64	6.26	47.00	74.00	-.096	.114
	Female	41	63.24	7.41	42.00	85.00	.127	2.191
Post-RC	Male	25	121.52	13.27	88.00	144.00	-.955	1.395
	Female	41	129.39	15.03	92.00	175.00	.211	1.557

In line with Table 4.4, there were differences between means of male and female students on all post-tests scores. According to Table 4.4 the mean score of Post-PROC for male was 26.04 (SD = 6.67) and for female was 35.41 (SD = 6.29) out of the possible maximum scores of 45. Whereas, the mean score of Post-CONC for male and experimental group were 48.64 (SD = 10.32) and 59.98 (SD = 12.69) respectively out of the possible maximum scores of 110. Based on that mean scores, it seemed that after the implementation of the treatment female students had better procedural and conceptual knowledge than that of male students.

With respect to metacognitive skills, female students had higher mean score on Post-KC and Post-RC (M = 63.24, SD =7.41, and M = 129.39, SD =15.03, respectively out of the possible maximum scores of 85) that male students (M = 60.64, SD = 6.26) and M = 121.52, SD = 13.27, respectively out of the possible maximum scores of 175). It could be said that in average male and female students expressed ‘true’ in metacognitive awareness inventory which means that most students possessed moderate to high metacognitive skills.

With respect to the values of skewness and kurtosis of all pre-tests scores, it was obviously seen in the table that not all the scores were normally distributed as the values of skewness and kurtosis of scores of Post-CONC for male students didn’t fall between -2 and +2. However, based on Kolmogorov Smirnov test for normality, it showed that scores of Posts-CONC for experimental group was normally distributed (see Appendix M).

4.2. Determination of Covariates

Before conducting MANCOVA, the maximum number of covariates applied in the study was determined using certain formula proposed by Huitema (2011).

$$\left(C+\left(J-1\right)\right)/N<0.10 \tag{4.1}$$

In the formula 4.1, C stands for number of covariates, N is the sample size, and J is the number of groups. As in this study there were 66 participants (N = 66) and two groups (J = 2), thus by considering the formula the maximum number of covariates allowed was 5. In this study, there were four covariates and this value didn't exceed the calculated maximum number of covariates. In addition, independent variables which can be used as covariates are determined by considering correlations among dependent variables and covariates. The process is performed so as to determine desired independent variables that meet to the requirement to be treat as covariates. According to Tabachnick and Fidell (2012), to use independent variables as covariates, the desired independent variables should be significantly correlated at least one of the dependent variables and correlations between possible covariates should be less than .80.
The following table presented correlations among possible covariates and the dependent variables.

Table 4.5. Correlations among possible covariates and the dependent variables

Variables	Post-PROC	Post-CONC	Post-KC	Post-RC
Pre-PROC	.457**	.394**	.282*	.197
Pre-CONC	.468**	.484**	.187	.074
Pre-KC	.050	.091	.527**	.431**
Pre-RC	.206	.196	.498**	.564**
**. Correlation is significant at the 0.01 level (2-tailed).				
*. Correlation is significant at the 0.05 level (2-tailed).				

Based on Table 4.5, since Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC were correlated significantly with at least one dependent variable and the values were less than .80. Therefore, all pre-test scores met the requirement to be treated as covariates.

4.3. Assumptions of MANCOVA

Before conducting Multivariate Analysis of Covariance (MANCOVA), there are eight assumptions: sample size; multivariate normality; absence of outliers; homogeneity of variance-covariance matrices; linearity; homogeneity of regression; and reliability of covariates and absence of multicollinearity and singularity which are necessary to be evaluated (Tabachnick & Fidell, 2012). Detail explication relative to assumption associated to the MANCOVA is as follows:

4.3.1. Sample Sizes

According to Tabachnick and Fidell (2012), when applying MANCOVA, in every cell having more cases than the number of dependent variables is required. In this study, the number of dependent variables is five, and it is clearly evident that the number of cases in every cell was more than five as the number of cases ranged from 11 (the number of male students in control group) until 34 (the number of students in experimental group). Therefore, assumption with respect to the sample size was met.

4.3.2. Multivariate Normality

The multivariate normal distribution become foundation for significance tests for MANCOVA. Multivariate normality indicates that the sampling distributions of means of the numerous dependent variables in every cell and their linear combinations are normally distributed (Tabachnick & Fidell, 2012). The assumption of univariate normality was evaluated by discerning the value of skewness and kurtosis of all dependent variables, to wit, Post-PROC, Post-CONC, Post-KC, and Post-RC. Table 4.1, 4.2, 4.3, and 4.4 showed that the values were in the range of -2 and +2 for most cells. While there were several cells: Post-RC (for experimental group); Post-CONC (for male students); and Post-KC (for female students) whose kurtosis value were greater than +2. However, by means of Kolmogorov and Smirnov test, it was indicated that the aforementioned variables were normally distributed (see Appendix M). Therefore, the assumption of univariate normality was met. Multivariate normality in addition can be evaluated by observing Box's test. Based on Table 4.7, as the p value was greater than 0.05, thus the assumption of multivariate normality was satisfied.

4.3.3. Absence of Outliers

According to Tabachnick and Fidell (2012), sensitivity to outliers is one of crucial limitations of MANOVA as outliers can generate either a Type I or a Type II error. Hence, evaluation of outliers is very important in advance of conducting MANOVA. Univariate outliers can be evaluated by using boxplots as delineated in Tabachnick and Fidell (2012). There were no outliers found in the boxplot for each variable. In addition, multivariate outliers can be tested by inspecting the Mahalanobis distance which are generated by multiple regression program. To recognize which cases are outliers, initially it is necessary to determine the critical chi-square value by considering the number of dependent variables as the degrees of freedom. In this study they were four dependent variables. Thus, by using Tabachnick and Fidell's (2013) guidelines, the critical value for four dependent variables is 18.47. Subsequently, the Mahalanobis was calculated and compared to the critical chi-square value. Table 4.6 indicated the calculated Mahalanobis distance value, to wit, 15.11 and this value didn't exceed the critical chi-square value. Therefore, there were not multivariate outliers in the data set.

Table 4.6. Extreme Values

			Case Number	ID	Value
Mahalanobis Distance	Highest	1	53	53.00	15.11313
		2	8	8.00	12.25682
		3	63	63.00	11.67360
		4	32	32.00	11.05145
		5	9	9.00	10.95668
	Lowest	1	18	18.00	.29358
		2	21	21.00	.40950
		3	40	40.00	.42585
		4	13	13.00	.54903
		5	35	35.00	.74597

4.3.4. Homogeneity of Variance-Covariance Matrices

The assumption homogeneity of variance-covariance matrices is that variance–covariance matrices within every cell of the design are sampled from the similar population variance–covariance matrix and can sensibly be pooled to produce a unique estimate of error (Tabachnick & Fidell, 2012). The assumption was evaluated by means of the Box's M test of equality of covariance matrices. Table 4.7 indicated that non-significant result was obtained [F (45, 5742.365) = 1.285, p = 0.096]. By considering this value, it means that the assumption of homogeneity of variance-covariance matrices is not violated.

Table 4.7. Box's test of equality of covariance matrices

Box's M	69.151
F	1.285
df1	45
df2	5742.365
Sig.	.096

4.3.5. Linearity

In agreement with Tabachnick and Fidell (2012), violations of linearity assumptions can decrease the power of the statistical test due to the reasons that (1) the linear combinations of dependent variables do not maximize the isolation of groups for the independent variables, and (2) covariates do not maximize adjustment for error. To confirm the linearity assumption, a straight-line relationship between each pair of dependent variables is required. Therefore, configuration of scatterplots between them can provide insight with respect to linearity (Tabachnick & Fidell, 2012). In the wake of splitting data by gender and teaching method, scatter plots of each dependent variable and the covariate were evaluated. All of the scatter plots indicated linear relationship between each dependent variables and the covariates as they skewed in the similar direction (see Appendix N). Therefore, in this study, linearity of assumptions was confirmed.

4.3.6. Homogeneity of Regression

Heterogeneity of regression in MANCOVA means that there is interaction between the independent variables and the covariates and that a distinct adjustment of dependent variables for covariates is required in distinct groups. If there is interaction between independent variables and covariates, MANCOVA is unreasonably conducted (Tabachnick & Fidell, 2012). In order to meet this assumption, the interactions are expected to be non-significant. The *p* values of the tests were evaluated across alpha level of .01 to serve robustness. Based on tables presented in Appendix O, it showed that the assumptions are satisfied. There are no significant interactions between the covariates and independent variables for each dependent variable.

4.3.7. Reliability of Covariates

In MANCOVA, if reliability of covariates assumption is not violated, the F test for mean differences is more powerful. However, if the assumption is violated, either increased Type I or Type II errors can come about (Tabachnick & Fidell, 2012). Therefore, this assumption plays an important role on the power of MANCOVA result. It can be checked by observing the Cronbach alpha value of each covariate in which the value should be greater than .70 (Kline, 2011). In this study, the reliability of Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC were .74, .82, .81, .89 respectively which demonstrated that the variables were reliable and could be treat as covariates.

4.3.8. Absence of Multicollinearity and Singularity

This assumption implies that when correlations among dependent variables are high, one dependent variable is a near-linear combination of other dependent variables and the dependent variable also produces information that is unnecessary to the information in one or more of the other dependent variables (Tabachnick & Fidell, 2012). Therefore, it is expected that correlations among dependent variables are moderate in which all correlations are less than .80 (Tabachnick & Fidell, 2012). Table 4.8 presents correlations among dependent variable of this study.

Table 4.8. Correlation among dependent variables

D.V.	Post-PROC	Post-CONC	Post-KC	Post-RC
Post-PROC	1	.775**	.272*	.375**
Post-CONC	.775**	1	.288*	.382**
Post-KC	.272*	.288*	1	.764**
Post-RC	.375**	.382**	.764**	1

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Based on the values showed in Table 4.8, it is obvious that all correlations are less than .80, thus the assumption of the absence of multicollinearity and singularity was met.

4.4. Results of Investigation of the Effect of Different Teaching Methods and Gender on Student's Procedural Knowledge, Conceptual Knowledge, and Metacognitive Skills

As stated, MANCOVA would be conducted to address one of sub-problems, to wit, to examine the effect of teaching methods (IMPROVE instructional method and traditional instruction) and gender on 11th grade science major Indonesian students' procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition compared to traditional instruction. As there were no serious violation of the assumptions underlying MANCOVA, the analysis for testing the null hypotheses could be conducted. The MANCOVA was performed on two independent variables, four covariates, and four dependent variables. The independent variables were teaching methods (IMPROVE instructional method and traditional instruction), and gender (males versus females). The covariates were pre-test scores of the procedural knowledge test (Pre-PROC), conceptual knowledge test (Pre-CONC), knowledge of cognition (Pre-KC), and regulation of cognition (Pre-RC). The dependent variables were the post-test scores of the procedural knowledge test (Post-PROC), conceptual knowledge test (Post-CONC), knowledge of cognition (Post-KC), and regulation of cognition (Post-RC). Statistically significant result would be obtained for any *p* value less than 0.05.

If the obtained MANCOVA results show that there are statistically significant main effect of teaching methods, gender, and interaction on combined dependent variables, then multiple univariate ANCOVAs are necessary to be conducted to observe the specific effect of each independent variable on each dependent variable in detail. According to Tabachnick and Fidell (2012) prior to deciding its significance by observing the p values, a Bonferroni type adjustment to alpha value is required to be conducted initially. The aim of this adjustment is to minimize Type I error in case of separate univariate test. To do this, it is suggested to divide the alpha by the number of dependent variables. As .05 alpha level is set at the outset and there are four dependent variables in this study, hence adjusted alpha is set at .0125 level for the univariate test. Consequently, statistically significant result would be obtained for any p value less than 0.0125. The results of MANCOVA were presented in the Table 4.9 and the results of follow-up ANCOVA were given in the Table 4.10.

4.4.1. Result of the Effect of Teaching Methods on Procedural, Conceptual Knowledge, and Metacognitive Skills

The first null hypothesis was "there is no statistically significant main effect of teaching methods (IMPROVE instructional method and traditional instruction) on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled".

As seen in Table 4.9, the results showed that there are statistically significant mean differences between control and experimental group on the combined dependent variables of Post-PROC, Post-CONC, Post-KC, and Post-RC scores when the covariates (Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC scores) were controlled [F (4, 55) = 9.34, Wilks' Lambda = .595, p =.000, partial eta squared = 0.405]. Hence, the null hypothesis 1 was rejected and this difference can be associated to the different instruction between groups. The value of partial eta squared value is .405 and it implies that approximately 41% of the variance in dependent variables can be explained by teaching methods. In addition, the value of observed power which demonstrates the probability of making correct decision is .999 for the main effect of teaching method at .05 level.

As there was significant result in MANCOVA for teaching methods, thus follow-up ANCOVA for each dependent was analysed. With reference to Table 4.10, there are significant mean differences between control and experimental group in post-test score of procedural knowledge (Post-PROC) (F (1, 58) = 9.767, p = .004, partial eta squared = .144), conceptual knowledge (Post-CONC) (F (1, 58) = 36.357, p = .000, partial eta squared = .385), and regulation of cognition (Post-RC) (F (1, 58) = 8.074, p = .006, partial eta squared = .122). All of the effect sizes are small to medium.

Table 4.9. MANCOVA Result

Effect	Wilks' Lambda	F	Hypothesis df	Error df	Sig.	Eta Squared	Observed Power
Intercept	.578	10.06	4.000	55.000	.000	.422	1.000
Pre-PROC	.880	1.87	4.000	55.000	.128	.120	.532
Pre-CONC	.741	4.79	4.000	55.000	.002	.259	.938
Pre-KC	.854	2.36	4.000	55.000	.065	.146	644
Pre-RC	.822	2.99	4.000	55.000	.027	.178	.761
Gender	.753	4.51	4.000	55.000	.003	.247	.922
Groups	.595	9.34	4.000	55.000	.000	.405	.999
Gender* Groups	.863	2.19	4.000	55.000	.082	.137	.608

Table 4.10. Univariate ANCOVA Results

Source	Dependent Variable	Values	df	F	Sig.	Eta Squared	Observed Power
Gender	Post-PROC	522.651	1	15.615	.000	.212	.973
	Post-CONC	497.443	1	6.786	.012	.105	.726
	Post-KC	74.215	1	2.262	.138	.038	.316
	Post-RC	919.987	1	7.118	.010	.109	.747
Groups	Post-PROC	326.892	1	9.767	.003	.144	.867
	Post-CONC	2665.085	1	36.357	.000	.385	1.000
	Post-KC	60.542	1	1.845	.180	.031	.267
	Post-RC	1043.561	1	8.074	.006	.122	.798
Gender * Groups	Post-PROC	.609	1	.018	.893	.000	.052
	Post-CONC	120.023	1	1.637	.206	.027	.242
	Post-KC	83.324	1	2.539	.116	.042	.347
	Post-RC	.184	1	.001	.970	.000	.050

In contrast, there are no statistically significant mean differences between experimental and control groups with reference to their post-KC (F (1, 58) = 1.845, p = .180, partial eta squares = .031). Even though there are slight differences between groups' post-KC scores in favour of experimental group, the estimated marginal means (see table 4.11) are more close to each other as the mean adjustment is applied in covariate analysis. Detail information with respect to mean difference of all dependent variables across groups is presented in Table 4.11.

Table 4.11. Mean comparisons of for each dependent variables scores across groups

Dependent Variable	(I) Group	(J) Group	Mean Difference (I-J)	Std. Error	Sig	95% Confidence Interval for Difference	
						Lower Bound	Upper Bound
Post-PROC	Control	Exp.	-4.68	1.5	.003	-7.68	-1.68
	Exp.	Control	4.68	1.5	.003	1.68	7.68
Post-CONC	Control	Exp.	-13.37	2.22	.000	-17.81	-8.93
	Exp.	Control	13.37	2.22	.000	8.93	17.81
Post-KC	Control	Exp.	-2.02	1.48	.180	-4.98	.95
	Exp.	Control	2.02	1.48	.180	-.95	4.98
Post-RC	Control	Exp.	-8.37	2.94	.006	-14.26	-2.47
	Exp.	Control	8.37	2.94	.006	2.47	14.26

Table 4.11 indicated adjusted mean difference of post-test scores of each dependent variable and all differences are in favour of experimental group in which IMPROVE instructional method was implemented. As a result, students' procedural knowledge, conceptual knowledge, and regulation of cognition in experimental group higher than that of students in control group.

4.4.2. Students' Answers to Procedural Knowledge Test

This study capitalized on theoretical framework proposed by Hiebert and Lefevre (1986) in which procedural knowledge were composed of two separate elements, to wit, *t*he knowledge of mathematical formal language or symbol representation system and the knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks. The following is delineation of students' answers by considering those classifications.

4.4.2.1.The knowledge of mathematical formal language or symbol representation system (PROCA)

Out of nine questions relative to procedural knowledge (PROC) there were four questions related to this category (PROCA) in which two questions were taken from topic of composition and inverse function, one from infinite sequences and series topic, and one from line equations topic. Table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.12. Distribution of number of students relative to scores, items of PROCA, and groups

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	1	2	4	0	0	8	18
	4	3	7	0	0	13	9
	12	4	12	1	4	0	11
	22	6	2	5	0	1	18
	Total	15	25	6	4	22	56
Experimental	1	0	3	0	2	11	18
	4	4	8	1	0	7	14
	12	3	6	0	1	1	23
	22	2	0	0	3	13	16
	Total	9	17	1	6	32	71

Based on the above table, in control group there were 15 students who obtained score 0 in the items (item 1, 4, 12, and 22). Meanwhile there were nine students in experimental group who gained score 0. Score 0 represented a condition in that students were not able to select correct option or they left it blank without any responses. Then, there were 25 students in control group and 17 students in experimental group who acquired score 1 in these items. Score 1 stood for condition in which students who did not provide any explanation even though they choose the correct option. In these items also, there were six students in control group and one student in experimental group who obtain score 2. Score 2 represented condition in which students selected the correct option, yet they did not present reasonable arguments. Subsequently, four students in control group and six students in experimental group got score 3 in these items. Score 3 illuminated students' ability in which they chose certain option as the correct answers inasmuch as the other options were incorrect. In control group there were 22 students and in experimental group there were 32 students who obtain score 4 in these items. In this case, students chose the correct option and provided reasonable arguments, but there was mistake in writing correct mathematical symbol. Finally, score 5 was achieved successfully by 56 students in control group and 71 students in experimental group. These students seemed to possess well procedural knowledge as they were able to decide the incorrect mathematical formal language or symbol representation system. In addition, they were able to provide revision properly.

From the above description, it is obviously showed that the number of students in experimental group who gained score 3, 4 and 5 exceeded the number of students in control group. However, there were more students in control group that obtained score 0, 1, and 2 than students in experimental group. Therefore, it could be concluded that majority students in experimental group could more provide comprehensive explanations than majority students in control group in items related to knowledge of mathematical formal language or symbol representation system.

4.4.2.2.The knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks (PROCB)

In addition to four items related to knowledge of mathematical formal language or symbol representation system (PROCA), there were five questions related to this category (PROCB) i.e. two questions from composition and inverse function topic, two from infinite sequences and series topic, and one from line equations topic. The following table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.13. Distribution of number of students relative to scores, items of PROCB, and groups

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	7	0	1	1	10	14	6
	9	1	1	5	12	7	6
	14	0	1	3	1	17	10
	15	1	0	3	16	3	9
	23	3	6	10	8	1	4
	Total	5	9	22	47	42	35

Table 4.13. Continued

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Experimental	7	1	2	0	2	20	9
	9	2	1	0	4	9	18
	14	2	3	0	1	11	17
	15	1	2	3	12	0	16
	23	2	1	9	12	7	3
	Total	8	9	12	31	47	63

According to the above table, in control group there were five students and in experimental group there were two students who gained score 0 in these items. Score 0 reflected condition in that students were not able to answer the questions as they left it blank without any responses. Then, there were nine students in both groups who acquired score 1 in these questions. Score 1 stood for condition in which students who were not able to grasp the idea presented in the problem. The student misused principles or translated the problem into inappropriate procedures, thus they applied improper formula or properties. In these items also, there were 22 students in control group and 12 students in experimental group who obtained score 2 that represented condition in which students used principles but were unable to translate the problem into appropriate procedures. Then, 47 students in control group and 31 students in experimental group get score 3 in these questions. Score 3 illuminated condition in which students used principles and translate the problem into appropriate procedures, but the students were unable to carry out a procedure completely or they demonstrated their incorrectness when performing calculation or algebraic manipulation. Subsequently, in control group there were 42 students and in experimental group there were 47 students who obtained score 4 in these items. The score 4 referred to condition in which students used principles, translated the problem into appropriate procedures, carried out a procedure completely, but did not use appropriate mathematical language. Several students were not able to write mathematical expression properly. Eventually, score 5 was achieved successfully by 35 students in control group and 63 students in experimental group. These students used appropriate mathematical terms and strategies. They also used mathematical principles and language precisely. They demonstrated their ability in dealing with the question properly and completely. They paid attention to all requirements such as calculation, algebraic manipulation, as well as mathematical expression. The student solved and verified the problem successfully.

From the above description, it is obviously showed that students in experimental group gained score 4 and score 5 more than students in control group. However, there were more students in control group that obtained score 2 and 3 than students in experimental group. In addition, there were slight difference between students in experimental and control groups who acquired score 1 and score 2. Therefore, it could be concluded that majority students in experimental group could more provided comprehensive explanation (score 4 and score 5) than majority students in control group in items related to knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks.

4.4.3. Students' Answer to Conceptual Knowledge Test

In addition to procedural knowledge, the other purpose of this study was to investigate students' conceptual knowledge. As stated at the beginning, this study took theoretical framework of conceptual knowledge revealed by Crooks and Alibali (2014) into consideration in which two elements of conceptual knowledge could be derived across mathematical domains: general principle knowledge (CONCA) and knowledge of principles underlying procedures (CONCB). The following is delineation of students' answers by considering those classifications.

4.4.3.1. General principle knowledge (CONCA)

According to Crooks and Alibali (2014), the knowledge of general principle could be measured by means two facets of tasks, to wit, explanation of concept tasks (CONCA1) and evaluation of example tasks (CONCA2).

4.4.3.1.1.Explanation of concepts tasks (CONCA1)

There were five questions related to this facet of tasks (CONCA1), to wit, two questions from composition and inverse function topic, two from infinite sequences and series topic, and one from line equations topic. Table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.14. Distribution of number of students relative to scores, items of CONCA1, and groups

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	3	0	10	15	4	2	1
	5	6	6	11	8	1	0
	13	3	13	11	5	0	0
	16	8	15	6	3	0	0
	28	0	9	15	6	2	0
	Total	17	53	58	26	5	1
Experimental	3	7	3	6	8	5	5
	5	1	0	2	2	2	27
	13	2	8	9	12	1	2
	16	6	7	14	5	2	0
	28	3	1	13	12	3	2
	Total	19	19	44	39	13	36

Based on the above table, in control group there were 17 students and in experimental group there were 19 students who obtained score 0 in these items. Score 0 reflected a condition in that the students were not able to provide answers as they left it blank without any responses. Then, there were 53 students in control group and 19 students in experimental group who acquired score 1 in these items. Score 1 stood for condition in which the students were misunderstood the question or the student's solution was not fully related to the question. In these items also, there were 58 students in control group and 44 students in experimental group who obtained score 2. Score 2 represented a condition in which students understood one portion of the question and yet translated it into inappropriate mathematical concepts. They provided incomplete explanation and left it partially. Subsequently, 26 students in control group and 39 students in experimental group got score 3 in these items. Score 3 illuminated a condition in which students understood one portion of the question and translated it into appropriate mathematical concepts. Then, in control group there were five students and in experimental group there are 13 students who obtained score 4 in these questions. Score 4 referred the condition in which students understood the complete questions, but did not translate them into inappropriate mathematical concepts. Finally, score 5 was achieved successfully by one student in control group and 36 students in experimental group. Score 5 referred to condition in which students understood the complete questions, translated it into appropriate mathematical concept and the responses were consistent with what was asked. In addition, they were able to demonstrate their ability to provide reasonable explanation about problem and to connect various concepts to provide comprehensive explanation.

From the above description, it is obviously showed that the number of students in experimental group who obtained score 3, 4, and 5 exceed that of students in control group. Nevertheless, there were more students in control group that obtained score 1 and 2 than students in experimental group. Slight difference was found in the number of students in control and experimental group who obtained score 0 in these items. Therefore, it could be concluded that majority in experimental group more provided comprehensive explanation than majority students in control group in explanation of concept tasks items.

4.4.3.1.2.Evaluation of examples tasks (CONCA2)

In the developed instruments to measure students' conceptual knowledge, there were six questions related to evaluation of example tasks, to wit, two items from composition and inverse function topic, two items from infinite sequences and series, and two items from line equations. The following table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.15. Distribution of number of students relative to scores, items of CONCA2, and groups

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	6	3	8	13	8	0	0
	10	4	6	13	7	2	0
	18	4	14	13	1	0	0
	21	0	10	16	6	0	0
	27	2	10	16	4	0	0
	29	2	7	15	8	0	0
	Total	15	55	86	34	2	0
Experimental	6	5	7	15	1	1	5
	10	3	4	20	3	4	0
	18	3	5	15	7	1	3
	21	2	2	13	5	4	8
	27	0	2	0	18	7	7
	29	0	1	15	14	3	0
	Total	13	21	78	48	20	23

According to the above table, in control group there were 15 students and there were 13 students in experimental group who gained score 0 in these items. Score 0 reflected a condition in that students selected the incorrect option or they left it blank without any responses. Besides, there were 55 students in control group and 21 students in experimental group who acquire score 1 in these items. Score 1 stood for condition in which students chose the correct option, but without providing any explanations. In these items also, there were 86 students in control group and 78 students in experimental group who obtain score 2. Score 2 represented condition in which students selected the correct option, but the presented reasons didn't relate to the statements. Then, 34 students in control group and 48 students in experimental group got score 3 in these items. Score 3 illuminated condition in which students selected the correct option and some of the presented reasons did not make sense. They provided improper explanation as they were in confusion and take unrelated concept to explain problem. Several students also used the options presented in the problems and rewrote it as reason of the problems. In control group there were 2 students and in experimental group there were 20 students who obtain score 4 in these questions. Score 4 represented students' ability in which they selected the correct option and the reason presented make sense, but there is a mistake in using appropriate terminology. Also in this case that students expressed that they chose certain option as the answer as they observed that the other answers were incorrect. Finally, none of students in control group obtained score 5 successfully, whereas 23 students in experimental group were able to answer the items correctly. Score 5 meant that students chose the correct option and provided comprehensive reasons.

From the above description, it is obviously showed that number of students in experimental group who obtained score 3, score 4, and score 5 exceeded the number of students in control group. However, there were more students in control group who obtained score 0, score 1, and score 3 than students in experimental group. Therefore, it could be concluded that majority students in experimental group could more provided comprehensive explanation than students in majority control group in items related to evaluation of example tasks.

4.4.3.2. Knowledge of principles underlying procedures (CONCB)

With reference to Crooks and Alibali (2014), the knowledge of principles underlying procedures could be measured by means of two specific tasks, to wit, application and justification of procedures tasks (CONCB1) and evaluation of procedures tasks (CONCB2).

4.4.3.2.1.Application and justification of procedures tasks (CONCB1)

There were seven questions related to application and justification of procedures tasks, to wit, one question from composition and inverse function topic, two from infinite sequences and series topic, and four from line equations topic. The following table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.16. Distribution of number of students relative to scores, items of CONCB1, and groups

Groups	No Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	2	1	11	17	3	0	0
	17	4	10	9	1	0	8
	19	5	1	9	14	1	2
	24	2	0	7	14	3	6
	25	0	0	1	8	15	8
	26	0	0	6	13	7	6
	31	0	0	10	17	5	0
	Total	12	22	59	70	31	30
Experimental	2	0	0	8	23	2	1
	17	3	0	18	5	5	3
	19	3	1	5	14	5	6
	24	3	9	7	9	5	1
	25	0	0	0	15	5	14
	26	0	0	10	9	7	8
	31	0	4	6	15	5	4
	Total	9	14	54	90	34	37

Based on the above table, in control group there were 12 students and in experimental group there were 9 students who gained score 0 in these items. Score 0 reflected a condition in that students did not provide any responses and left it blank. Besides, there were 22 students in control group and 14 students in experimental group who acquired score 1 in these items. Score 1 represented a condition in which students misunderstood the question or the solutions were not fully related to the question.

Subsequently, in these items, there were 59 students in control group and 54 students in experimental group who obtained score 2. Score 2 referred to a condition in which students understood one portion of the question and yet translated it into inappropriate mathematical concepts. Therefore, they were not able to grasp the idea presented in the problem and applied improper formula or properties. Then, 70 students in control group and 90 students in experimental group acquired score 3 that illuminated a condition in which students understood one portion of the question and translated it into appropriate mathematical concepts. They demonstrated their incorrectness when performing calculation or algebraic manipulation. Then, in control group there were 31 students and in experimental group there were 34 students who obtained score 4 in these items. Score 4 meant that students understood the complete questions, but did not translate it into appropriate mathematical concepts. Therewith, they were not able to write mathematical expression properly. Finally, score 5 was achieved successfully by only 30 students in control group and 37 students in experimental group. Score 5 referred to a condition in which students understood the complete questions, translated it into appropriate mathematical concept and the answer was consistent with the questions. These students demonstrated their ability to deal with the question properly and completely. They paid attention to all requirements such as calculation, algebraic manipulation, as well as mathematical expression.

From the above description, it is obviously showed that the number of students in experimental group who obtained score 3, 4, and 5 than students in control group. However, it also indicated that there were more students in control group that obtained score 1, 2, and 3 than students in experimental group. Therefore, it could be concluded that majority students in experimental group more provided comprehensive explanation than majority students in control group in tasks related to application and justification of procedures tasks.

4.4.3.2.2.Evaluation of procedures tasks (CONCB2)

There were four items related to evaluation of procedures tasks, to wit, two items from composition and inverse function topic, one from infinite sequences and series topic, and one from line equations topic. The following table indicated distribution of number of students relative to scores, the items, and groups.

Table 4.17. Distribution of number of students relative to scores, items of CONCB2, and groups

Group	Items	Number of students					
		Score 0	Score 1	Score 2	Score 3	Score 4	Score 5
Control	8	4	7	5	11	3	2
	11	3	1	16	4	0	8
	20	7	1	1	4	13	6
	30	0	7	11	10	3	1
	Total	14	16	33	29	19	17
Experimental	8	9	0	1	6	8	10
	11	2	1	0	8	6	17
	20	1	3	9	4	13	4
	30	4	5	3	19	2	1
	Total	16	9	13	37	29	32

Based on the above table, in control group there were 14 students and in experimental group there were 16 students who gained score 0 in these items. Score 0 represented a condition in which there were no any responses and students left in blank. Besides, there were 16 students in control group and 9 students in experimental group who acquired score 1 that stood for a condition in which students demonstrated their ability in finding the incorrect procedures but they didn’t provide revision or explanation. In these items also, there were 33 students in control group and 13 students in experimental group who obtained score 2. Score 2 represented a condition in which students found the mistake and provided explanation but in incomplete manner. Subsequently, 29 students in control group and 37 students in experimental group got score 3 in these items. Score 3 illuminated a condition in which students demonstrated their ability in finding the incorrect procedures and provided proper explanation yet they did not take effort to revise it. Then, in control group there were 19 students and in experimental group there were 29 students who obtained score 4 in these items. Score 4 meant that the students demonstrated their ability in finding the incorrect procedures, expressing proper explanation, and trying to revise it yet their revision was not appropriate. They were not able to grasp the idea presented in the problem. Most of them applied improper formula or properties. Eventually, score 5 was achieved successfully by 17 students in control group and 32 students in experimental group. These students demonstrated their ability in finding the incorrect procedures, providing proper explanation, making correction correctly.

From the above description, it is obviously showed that the number of students in experimental group who obtained score 3, 4, and 5 exceeded the number of students in control group. However, it also indicated that there were more students in control group that obtained score 1, 2, and 3 than students in experimental group. Therefore, it could be concluded that majority students in experimental group could more provided comprehensive explanation than majority students in control group in items related to evaluation of procedures tasks.

4.4.4. Result of the Effect of Gender on Procedural, Conceptual Knowledge, and Metacognitive Skills

The second null hypothesis was "there is no statistically significant main effect of gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled".

The table 4.10 also indicated that there were statistically significant mean differences between male and female students on the combined dependent variables of Post-PROC, Post-CONC, Post-KC, and Post-RC when the covariates were controlled [F (4, 55) = 4.51, Wilks' Lambda = .753, p = .003, partial eta squared = .247]. Therefore, the null hypothesis 2 was rejected and the difference can be associated to gender difference. The value of partial eta squared value is .247 and it implies that approximately 25% of the variance in dependent variables can be explained by gender. In addition, the value of observed power which demonstrates the probability of making correct decision is .999 for the main effect of teaching method at .05 level.

The follow-up ANCOVA was conducted to investigate the effect of gender on each dependent variable. Based on Table 4.11, significant difference was found between male and female students' post-test scores on procedural knowledge (Post-PROC) (F (1, 58) = 15.615, p = .000, partial eta squared = .212), conceptual knowledge (Post-CONC) (F (1, 58) = 6.786, p = .012, partial eta squared = .105), and regulation of cognition (Post-RC) (F (1, 58) = 7.118, p = .010, partial eta squared = .109). The effect size is small. In contrast, there were no statistically significant mean differences between male and female students on their post-KC (F (1, 58) = 2.262, p = .138, partial eta squares = .038).

Table 4.18. Mean comparisons of for each dependent variables scores across gender

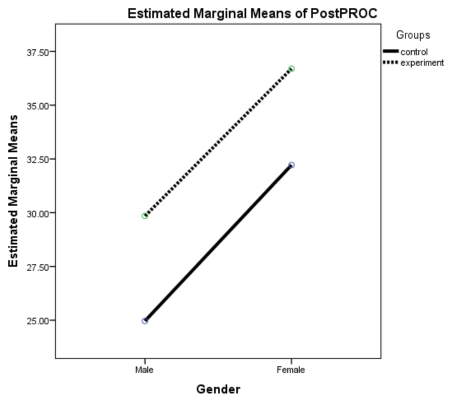
Dependent Variable	(I) gender	(J) gender	Mean Difference (I-J)	Std. Error	Sig	95% Confidence Interval for Difference[b]	
						Lower Bound	Upper Bound
Post-PROC	Male	Female	-7.05	1.78	.000	-10.62	-3.48
	Female	Male	7.05	1.78	.000	3.48	10.62
Post-CONC	Male	Female	-6.88	2.64	.012	-12.16	-1.59
	Female	Male	6.88	2.64	.012	1.59	12.16
Post-KC	Male	Female	-2.66	1.77	.138	-6.19	.88
	Female	Male	2.66	1.77	.138	-.88	6.19
Post-RC	Male	Female	-9.35	3.51	.010	-16.37	-2.34
	Female	Male	9.35	3.51	.010	2.34	16.37

As shown in Table 4.18, the difference is in favour of female students as shown in. Then, it can be concluded that female students regardless of instructed teaching method have higher level of procedural knowledge, conceptual knowledge, and regulation of cognition.

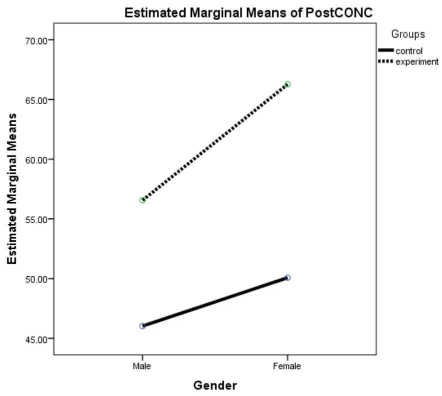
4.4.5. Result of the Interaction Effect of Teaching Methods and Gender on Students' Procedural, Conceptual Knowledge, and Metacognitive Skills

The third null hypothesis was "there is no statistically significant interaction effect between teaching method and gender on the population means of collective dependent variables of 11th grade science major students' post-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition when students' pre-test scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition are controlled".

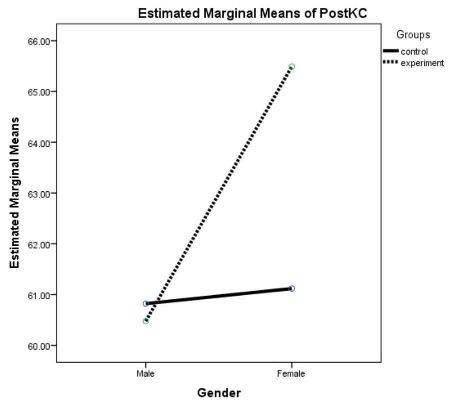
The table 4.10 also indicated that there was no statistically significant interaction of teaching methods and gender on students' combined dependent variables of Post-PROC, Post-CONC, Post-KC, and Post-RC when the covariates were controlled [F (4, 55) = 2.19, Wilks' Lambda = .863, p = .082, partial eta squared = .137]. Therefore, the null hypothesis 3 failed to be rejected. The results of MANCOVA was confirmed with the results of follow-up ANCOVA. With reference to Table 4.11, it was found that groups by teaching method interactions for all of the dependent variables are not statistically significant (F (1, 58) = .018, p = .893, partial eta squared = .000 for Post-PROC; F (1, 58) = 1.637, p = .206, partial eta squared = .027 for Post-CONC; F (1, 58) = 2.539, p = .116, partial eta squared = .042 for Post-KC; and F (1, 58) = .001, p = .970, partial eta squared = .000 for Post-RC). All the effect sizes were small. The interaction plot of gender and teaching methods interactions for all dependent variables are given.



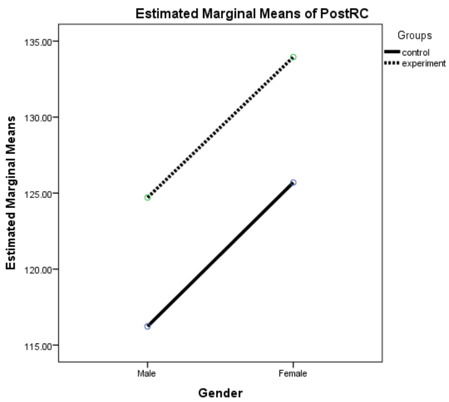
(a)



(b)



(c)



(d)

Figure 4.1. Interaction Plot of Gender and Groups

As seen in the Figure 4.1, there are three figures (a, b, and d) that obviously do not display interaction between class and gender and it is accordance with the statistical calculation. However, there was a figure (c) that displayed slight interaction even though the result of statistical calculation does not support it. By considering 99-

percent confidence intervals in Table 4.19, it is clear that there is slight mean intersection between male and female students' Post-KC scores both in control and experimental group.

Table 4.19. Estimate of mean of each dependent variable across groups and gender

Dependent Variable	Gender	Group	Mean	Std. Error	99% Confidence Interval	
					Lower Bound	Upper Bound
Post-PROC	Male	control	24.96	1.908	21.146	28.786
		experiment	29.85	1.641	26.565	33.134
	Female	control	32.22	1.339	29.537	34.896
		experiment	36.69	1.333	34.028	39.366
Post-CONC	Male	control	46.03	2.824	40.372	51.678
		experiment	56.56	2.428	51.700	61.421
	Female	control	50.07	1.981	46.105	54.036
		experiment	66.27	1.973	62.320	70.219
Post-KC	Male	control	60.82	1.890	57.038	64.603
		experiment	60.48	1.625	57.224	63.728
	Female	control	61.12	1.325	58.465	63.771
		experiment	65.49	1.320	62.849	68.134
Post-RC	Male	control	116.23	3.750	108.724	123.738
		experiment	124.71	3.224	118.253	131.161
	Female	control	125.69	2.630	120.429	130.960
		experiment	133.95	2.620	128.704	139.193

4.5. Students' Experiences with IMPROVE Instructional Method

As mentioned earlier, the second main problems in this study was to investigate students' experiences with IMPROVE instructional method. In order to address this research question, results of semi-structured interviews, classroom observation, and reflective journal were taken into consideration. Based on the questions of the interview, there were four themes defined and students' experiences will be described under these themes. Besides, the results of classroom observation and reflective journal were capitalized on to enrich the explanation. The following table indicates the themes and categories with reference to interview results.

Table 4.20. Themes and categories according to the interview results

A. Comparison to the previous teaching method	***f***
1. More questioning	14
2. Reasons and connections	11
3. Various types of problems	9
B. Strength of the instruction	***f***
1. More confident	12
2. Having better comprehension	10
3. Enjoying lesson	9
4. More awareness	9
C. Useful activities	***f***
1. Discussion and sharing ideas	14
2. Visualization	9
3. Reviewing	7
4. Enrichment and remedial	10
5. Checking answer	12
D. Weakness on the instruction	***f***
1. High level of mathematical problems	9
2. Lack of prior knowledge	8
3. Noise	9
4. Feeling inconvenient at the beginning	12
5. Time limitation	11

4.5.1. Comparison to the previous teaching methods

The first question directed to the students was "What are the differences between the current instruction and the previous instruction?" With respect to it, they revealed that IMPROVE instructional method led to pose and answer more questions (14 students), to reason and construct connection (11 students), and to deal with various types of problems (9 students).

- First Category: More Questions

All students who were interviewed expressed that IMPROVE instructional method led to pose and respond questions.

> RH (HA) said, "I think, in this instruction, there are more questions posed by the teacher to help us understand certain concepts, and we can also pose questions to the teacher freely."
>
> MS (MA) said, "…when we work with our partner, we read and pose questions also to be answered by our partners…"
>
> RS (LA) said, "In this instruction, we are also treated to ask ourselves about the process that we conduct when trying to grasp certain concept and also solving certain mathematical problem".

As stated, metacognitive questioning was one of three fundamental inter-dependent elements of IMPROVE instructional method (Mevarech & Kramarski, 1997). In this instruction there were four types of questions used: comprehension questions, connection questions, strategic questions, and reflective questions. According to the students' statements above, these questions were capitalized on by both the teacher and students in four distinct directions: (1) from the teacher to students, (2) from students to the teacher, (3) from students to their peers, and (4) from students to their self.

Firstly, the teacher posed questions to students throughout the lessons with the aim of providing guidance so that students could activate their cognitive and metacognitive skills. Teacher's guidance was obviously observed in various occasions such as in introduction part. Using metacognitive questions, the teacher assisted students in activating prior knowledge to develop students' conceptual knowledge. Activating prior knowledge was an effort to grasp new knowledge by building connection between them as mathematics concepts were logically interconnected. For instance, when teaching topic of inverse function, the teacher asked students about the concept of bijective function. Another example was that the teacher guided students to comprehend concepts of composition function by presenting figures which clarified the position of functions and its elements such as domain, range, and codomain. The teacher also modelled the use of metacognitive questions during teaching process. The questions such as "What is the goal of the lesson?", "How should we proceed this?", "Why does that answer make sense to you?" were posed so that students were pushed to be aware of what they were doing.

Secondly, students also posed metacognitive questions relative to mathematical concepts to the teacher. According to students' posed questions, the aims were to gain clarification with respect to explanation of certain concept, to ensure that the conducted processes and obtained results were correct, to acquire information about the subsequent processes that should be performed. Table 4.21 showed several examples of questions students posed to the teacher (the data were taken from classroom observation).

Thirdly, students also were requested to pose questions directed to their peers. It was obviously seen in practice part in which students worked with their partners to deal with mathematical tasks in activity sheet provided by the teacher. Before, during, and after solving problems in activity sheet, students read metacognitive questions loudly and started to think deeply about the answer of the questions. As selection of partners was conducted by considering students' achievements in which high achiever and low achiever students worked together, both students could take benefit from asking and answering their peer questions. In practice part, students who faced difficulty in learning mathematics posed any questions to his or her partner who was able to deal with mathematical concepts.

Table 4.21. Examples of questions proposed by several students to the teacher

No	Sub Topics	Example of Questions
1	Properties of Inverse Function	SZ asked, "Will expression of $(f \circ g)^{-1} = g^{-1} \circ f^{-1}(x)$ be valid for all conditions?"
2	Infinite Sequences and Series	i. MA asked, "Why does the formula consist of (n-1)?" ii. KN asked, "What is the meaning of the sentence of the sum of the first n term?" iii. PP asked, "Are there any sequences that are not categorized as arithmetic or geometric sequences?"
3	Geometric Infinite Series	i. MQ asked, "If the value of r equal to -1 or 1, what will happen?" ii. SA asked, "How do we do to make difference between geometric series and infinite geometric series?"
4	Line Equations	i. DC asked, "How can we sure the length of the line similar to that line?" ii. AS asked, "Why do the two corresponding angles have alike measure?" iii. MD asked, "What is the meaning of $\beta - \alpha$?" iv. AS asked, "In what conditions we can use those formula?" v. NH asked, "Why does the statement $x_1 \neq x_2$ has to be written after the expression of $m = \frac{y_2 - y_1}{x_2 - x_1}$?" vi. NH asked, "How does we decide index 1 and 2 when writing m_1 and m_2 ?"

Eventually, IMPROVE instructional method also expected students to pose questions directed to their self. As the teacher always posed reflective questions during and after solving problems, students also got used to ask their self to recognize whether they applied proper procedure and acquired correct result. For instance, reflective questions that were mostly posed by students were "*Am I right*?", "*Is it correct*?", or *"Does it make sense?"* Those questions directed the students to check procedures and concepts which they selected and carried out. These types of questions were obviously seen when several students were given opportunity to present their works in board. However, when they were working in groups or individually, they were also encouraged to use these questions.

> AM (MA) said, "The teacher always asks questions whether the selected strategies and obtained results are correct or not. And we also get used those questions to check our answer and strategies."

- Second Category: Reasons and connections

11 students (two students from low achievers (LA), four students from middle achievers (MA), five students from high achievers (HA)) revealed that in this instruction the students were encouraged to present reasons behind the formula or procedures and connection among existing concepts. The following are instances of students' excerpts with respect to this category.

> AS (HA) said, "We are requested to provide reasons behind strategies that we have chosen and we are conducting"
>
> NA (MA) said, "It is important in this instruction to know the meaning of each process and formula".
>
> RA (LA) said, "…as what our teacher said, that mathematics topics are like a chain. In order to understand one topic, you have to understand previous topics. I think in this instruction we are requested to connect the new topics and the old topics".

> NH (HA) said, “In this instruction, at the beginning of the lesson the teacher always ask us about the old knowledge since it helps us understand the new topics”.

> SI (MA) said, “…mathematics topics or concepts are interrelated each other, and in order to understand certain concept we have to connect it other concepts, and it helps us present reasons”.

According to the students’ statements above, in this instruction they were asked to present reasons and meaning of each process by considering connection among concepts. Comprehension, strategic, and reflective questions are mostly capitalized on to encourage students to equip their action with reasons. As a matter of fact, presenting reasons served as means for students to elaborate procedural and conceptual knowledge. The presented reasons were not independent, instead they were interdependent and connected each other. Thus, so as to understand certain concept, they had to connect to other related concepts. Students’ reasons and connection were obviously reflected in their responses. Students gave responses in two occasions: (1) when answering the teacher’s questions and (2) answering other students’ questions. Students’ responses to the teacher’s questions were obvious frequently than students’ responses to their peers’ questions.

Students responded the teacher’s questions in various teaching steps. For instance, in review part, as students were encouraged to grasp verbal mathematical expression and make representation in their mind, several students were able to express their comprehension by using their own words in proper manner. It was obviously evident when the teacher reviewed the learned material about the difference between convergent and divergent infinite sequences and series, several students explicated their understanding. The following are excerpts taken from classroom observation.

> T asked, “What do you think about the difference between divergent and convergent geometric series?”

> MR said, “Hmm, I think the main difference between them is the ratio, if the ratios fall between -1 and 1 so it is categorized as convergent geometric series.

Convergent geometric series imply that its sum could be defined, whereas divergent geometric series imply that we couldn't determine its sum".

MD said, "Yes, I agree with him, since if the ratio is greater than 1 or less than -1 so we cannot define its sum".

Other examples were that when the teacher asked students about the difference among slope of horizontal and vertical lines.

NH said, "So…, the horizontal line is like this (students put her finger into motion from right to left), which means that the line passes through equal ordinate but different abscissa and by considering the formula of determining slope of line if two points given, we obtain zero in numerator part as two similar numbers are subtracted. So…, we can conclude that slope of a horizontal line is zero".

SA said, "I think, the vertical line …is it like this? (By putting her finger into motion from top to bottom she asks her friend next to her and her friend confirms her question). So, the value of x or abscissa is always similar and ordinates are different, therefore if we use that formula, the denominator is zero as $x_1 = x_2$. So, a number which is divided by zero will equal to undefined number. So, the vertical lines do not possess defined slope".

Students also responded their peers' questions and it was obviously seen in practice part. The following is example of conversation between two students when they worked in group to solve problems related to inverse function (the data were taken from classroom observation).

DF asked, "What should we do first to decide whether $f(x) = \{(-3,3),(-2,2),(0,0),(2,2)\}$ has inverse function or not?"

AS responded, "Based on what the teacher has explained, let us see the definition of inverse function first, it is stated that the function has to bijective. But in the example we see like this. How?"

DF said, “I think we may change it into figure, so that we can easily see the map between domain and codomain.”

AS said, “Good idea, because the first element is domain and the second element is range not codomain. Let us draw the figure.” (AS draws figure and DF pays attention to what AS is doing and sometimes makes comment)

AS said, “Here we see that in the codomain there is one element (2) is used by two elements in domain (-2 and 2), so?

DF said, “So it is not bijective function and automatically it has not inverse function.”

- Third Category: Various Types of Problems

Since in the previous instructions, students tended to be presented with similar type of problems or tasks, 9 students (three students from low achievers (LA), three students from middle achievers (MA), three students from high achievers (HA)) expressed that they had to deal with various types of problems or tasks in this instruction.

AS (HA) said, “In this instruction, there are various types of problems that we have to solve it. The problems are clearly different from what we see in the previous”.

RR (MA) said, “The questions are very different, actually there are similar but in different form. I love questions about application of certain concept”.

MD (LA) said, “I think the problems are different from what we have regularly solved in the previous”.

According to students’ statements, in this instruction they were presented with various types of mathematical problems which were different from problems presented in the previous instructions. During instruction, there were problems or tasks of application of single or multi-concepts, algorithm, proofing the formulas, showing the properties, interpreting properties, drawing figures, and so forth which were given in activity sheets, homework, and remedial and enrichment. In order to solve various types of problems successfully, students were enforced to pay more attention to mathematics concepts devoid of setting aside procedural knowledge.

> AS (HA) said, "As what the teacher has said that in order to solve various mathematical problems successfully, we have to understand the concept initially and practice it frequently".

4.5.2. Strength

The second question directed to the students was "What are advantages that you take in this new instruction?" Students expressed their responses in that within IMPROVE instructional method environment they felt more confident (12 students), to have better comprehension (10 students), to enjoy lesson (9 students), and to have better self-awareness (9 students).

- First Category: More confident

12 students (three students from low achievers (LA), four students from middle achievers (MA), five students from high achievers (HA)) revealed that the instruction made them to be more confident in participating in learning mathematics and solving mathematical problems.

> ML (HA) said, "As we have answered various types of problems successfully, I become more confident to deal with other mathematics problems".

> RR (MA) said, "In this instruction also, we are trained to express our understanding to our partners or other friends, it makes me feel more confident in learning mathematics".

> MR (LA) said, "…I feel more confident to learn mathematics as during learning we solve many problems in activity sheet, homework, quizzes, and so on."

Based on the students' statements, as they were trained to express their thinking and solve diverse mathematical problems in various occasions, they felt more confident in learning mathematics and solving mathematical problems. It was obviously seen that students in this instruction were assigned into groups and given great opportunity to communicate, discuss, and share their ideas freely by using their own words with not only their partners but also other friends and the teacher. In addition, as stated, students were presented frequently with diverse mathematical problems in activity sheets, daily homework, formative assessments, as well as remedial and enrichments, thus they were accustomed to it and students became more confident.

- Second Category: Having better comprehension

10 students (two students from low achievers (LA), three students from middle achievers (MA), five students from high achievers (HA)) expressed that during the implementation of IMPROVE instructional method they obtained better understanding.

> NH (HA) said, “In this instruction, the teacher always starts with asking previous concepts, and I think it helps to understand the new topics.”
>
> MS (MA) said, “I feel like I think and understand about mathematics topics better ...”
>
> MA (LA) said, “…mathematics topics or concepts are interrelated each other, and in order to understand certain concept we have to connect it other concepts, and it helps us present reasons, and thus we can understand what we are learning”.

With reference to students’ arguments, they obtained better understanding in learning mathematics as the teacher encouraged them to connect the new concept to the previous concepts. As a matter of fact, metacognitive questions served as a remarkable tool that played important role to deepen students’ comprehension as it connected previous knowledge to new knowledge. Compared to the traditional instruction which heavily focused on exercises applying given formula to solve mathematical problems, IMPROVE instructional method encouraged students to elaborate their mathematical understanding. It might lead students in experimental group could develop both procedural and conceptual knowledge.

- Third Category: Enjoying lesson

9 students (two students from low achievers (LA), three students from middle achievers (MA), four students from high achievers (HA)) expressed their enjoyment during the implementation.

> AS (HA) said, "Even though at the beginning of the instruction I faced difficulties, but as the time passed, I started to enjoy it, I think it takes time to get used to it".
>
> NA (MA) said, "I enjoy the lesson since we are given opportunities to ask any questions and I understand mathematics topics".
>
> RA (LA) said, "If you understanding mathematics, you can enjoy the lesson, this instruction to be honest contributes to my understanding about mathematics topics. Therefore, I enjoy the lesson".

According to students' statements above, in the instruction they enjoyed learning mathematics and it contributed to improvement of their understanding. The reason lies in the fact that students were provided with greater opportunities to ask any questions, to solve problems by means of various strategies, to express freely, to give responses, to deepen and enrich their knowledge, as well as to be in dynamic environment. After several weeks of the implementation, it was obviously seen that most of the students began to be involved in learning process. They showed their interest and engagement in not only learning process but also accomplishing tasks in activity sheet. They showed their interest in learning mathematics deeper and further and in type of problems presented in activity sheet. Students expressed that they started to think deeply about mathematics concepts. Although, the instruction sometimes confused them, in the wake of presenting clear explanation using metacognitive questions they expressed their pleasure. Several students actively posed questions to the teacher when the teacher explained about certain concept. They also responded the teacher questions in many occasions. The most important improvement in activeness was that students were able to express their own ideas in front of classroom. It is important to note that IMPROVE instructional method provided environment that enhance students to be more active.

- Fourth Category: More awareness

9 students (three students from low achievers (LA), two students from middle achievers (MA), four students from high achievers (HA)) also alluded to improvement of awareness in this instruction.

> RH (HA) said, “In this instruction, it is important to be aware of what you are doing…”
>
> SI (MA) said, “The teacher always warns us to pay attention to the process that we are undertaking”
>
> MD (LA) said, “The questions that the teacher asks always make us to be more awareness when we learn and solve any problems”

According to students’ statements above, the instruction led the students to be more aware of what they were doing as they were requested to pay attention to the process conducted. Metacognitive questions and systematic provision of feedback and corrective environment seemed to influence students’ awareness in learning and solving problems. In many occasions, the teacher posed questions such as “What are we doing right now?”, “Why do we choose this formula to solve this problem?”, “Are we on the right track?”, and so forth. Using these questions students were directed to decide whether they were in right way by considering the reason behind the processes that they were undertaking.

4.5.3. Useful activities

The third question directed to the students was “What are useful activities in this new instruction?” Their responses to this question were discussion and sharing (14 students), visualization (9 students), reviewing (7 students), enrichment and remedial (10 students), as well as checking answer (12 students).

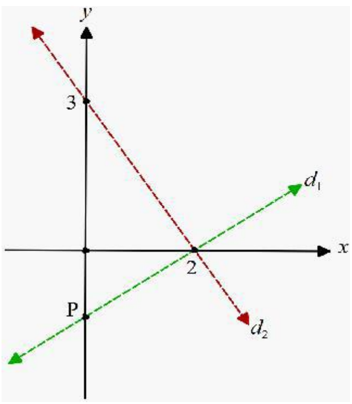
- First Category: Discussion and sharing ideas

All students who were interviewed expressed that discussion and sharing ideas were useful activities in the instruction.

> NH (HA) said, “I think, when work in groups, it is the best time for me to discuss about problems presented in activity sheet”.
>
> RR (MA) said, “Sometimes, when we do not understand of what the teacher has explained, we get better explanation from our partners when discuss it in group working as we can ask questions freely”.

MR (LA) said, "Discussing and sharing our ideas help us solve various mathematics problems".

According to students' arguments above, discussion and sharing were important activities in the instruction as it helped students deal with difficulties in comprehending mathematical concepts and solving mathematical problems. IMPROVE instructional method facilitated students with environment to support sharing and discussion. Sharing and discussion activities were obviously seen when students worked in pairs and presented their works in front of class. In addition, as students were encouraged to solve mathematics problem by means of different strategies, they were requested to share and explain their works in front of class. Besides, other students asked questions related to the procedures and concepts applied as well as proposed alternative strategies to address problem. For instance, the following an example of problem that presented in activity sheet.



Based on the figure line d_1 and d_2 are perpendicular each other. Can you find the way to determine the value of p

Figure 4.2. A problem in activity sheet

The teacher gave an opportunity to group 3 to present their work in board. The Table 4.23 are group's works which combined their writing in board as well as their verbal explanation. The Table 4.22 indicated that group 3 presented their own way to address the problem. However, other group namely group 7 had another strategy to deal with it.

Table 4.22. Examples of groups' works

Group 3's work	Group 7's work
Aim: Determine point p Strategy: Find the line equation of d_1 and as the line passes through the point $(0, p)$, point p can be determined. Procedure: 1. Find slope of d_2, then d_1 $d_1 \perp d_2 \Rightarrow m_1 = -\frac{1}{m_2}$ $\Rightarrow m_2 = -\frac{1}{-\frac{3}{2}} = \frac{2}{3}$ 2. Find line equation of d_1 $d_1 \equiv y = \frac{2}{3}(x-2)$ 3. The line passes through point $(0, p)$ then find point *p*. $p = \frac{2}{3}(0-2)$ $p = -\frac{4}{3}$	Aim: determine point p Strategy: By using property of relation of slope of two perpendicular lines. Procedure: 1. Write the properties $d_1 \perp d_2 \Rightarrow m_1 = -\frac{1}{m_2}$ 2. Determine slope of each line by using the formula of determining slope a line which passes through two points. $m_1 = -\frac{3}{2}$ and $m_2 = \frac{-p}{2}$ 3. Use the property to determine the value of p $m_1 \cdot m_2 = -1 \Rightarrow -\frac{3}{2} \cdot \frac{-p}{2} = -1$ $\Rightarrow p = -\frac{4}{3}$

Subsequently, during presentation, feedback was provided so that if there was a mistakes, the teacher make correction. Therefore, the teacher posed question such as, “How did you determine this to be true?”, “Why can’t we do this?”, and “What information is important for you to consider?”

- Second Category: Visualization

9 students (three students from low achievers (LA), two students from middle achievers (MA), four students from high achievers (HA)) also expressed that figuring out concepts or visualization was useful activities in the instruction.

> RR (HA) said, “The teacher always provides ancillary figure to help us comprehend certain concepts”.
>
> NH (MA) said, “When we cannot grasp the idea of the main concept, we are requested to make figures to lessen our difficulties, and it helps us”.
>
> MD (LA) said, “In this instruction, in various occasion we are trained to make figure related to certain concept, and I think it makes us understand”.

Based on the students’ statements above, visualization of the concepts assisted them in comprehending mathematical concepts so that it might reduce their difficulties when learning mathematics. As a matter of fact, IMPROVE instructional method assisted students in grasping mathematical concepts using figures. Besides, during solving mathematical problems, they were also expected to draw figure so as to recognize the meaning and purpose of the problems. For example, in order to grasp the concept of composition function, the teacher drew a three functions and labelled it. Using this figure students might be able to differ the difference between $f \circ g$ and $g \circ f$. Therefore, students were led to comprehend the meaning of symbol and mathematical expressions which might imply to development of conceptual knowledge. Another example was that when students tried to understand the concept of slope of line. Students were asked to describe the characteristics of lines that had positive, negative, zero, or undefined slopes. By means of figures, students were guided to observe the lines and using formula of determining slope of lines if two points given, characteristics of those lines were uncovered.

- Third Category: Reviewing

7 students (one student from low achiever (LA), two students from middle achievers (MA), four students from high achievers (HA)) also revealed that reviewing was a useful activity in IMPROVE instructional method.

> RR (HA) said, "The teacher always reviews what she has explained to the class, and I think it helps us a lot".
>
> RH (MA) said, "To check whether we understand or not, we are always asked by the teacher and we have to answer it by means of own words."
>
> RA (LA) said, "I think reviewing is very important for us to remind main ideas."

Reviewing also played important role in this instruction. According to students' statements, reviewing helped them check and evaluate their understanding and remind main ideas. Questions posed by the teachers encouraged students to think about what they had learned and students were asked to express using their own words. The example presented in page 115 gave insight into students' responses to the teacher's questions. The teacher also asked various types of questions in this occasion such as: (1) Connection question: "What are the similarities and differences between the concept at hand and the previous ones?"; (2) Comprehension question: "What do you know about the topic so far? Can you explain the concept by your own words?"; (3) Strategic question: "What are the steps to solve the problem? Explain in your own words?"; and (4) reflective questions: "What are your difficulties in this lesson?".

In the questions students were encouraged to connect the topic that they were learning and the previous topics such as the topics divergent geometric series and convergent geometric series. Therefore, students could make clear distinction between them so that they might grasp the concept easily. Besides, questions related to students' recognition of their ability were posed (see examples of comprehension and reflective questions) in order to activate their knowledge of cognition and regulation of cognition.

- Fourth Category: Enrichment and remedial

10 students (four students from low achievers (LA), three students from middle achievers (MA), three students from high achievers (HA)) revealed that enrichment and remedial were useful to gauge how much they have learned.

> AS (HA) said, “It is interesting and very helpful to have one day in which we test our understanding than we enrich or correct our understanding”
>
> MS (MA) said, “I get great benefit from remedial process as the teacher guides us to refine our mistakes”
>
> MA (LA) said, “Even though in that day I was in high tense, but after that I get great chance to refine my understanding and also enrich my knowledge”.

According to the students’ statements above, enrichment and remedial were useful activities in the instruction as it might evaluate their understanding, refine their mistakes made, and enrich their knowledge. Enrichment was a great opportunity for high-achieving students to enrich their knowledge by dealing with high level of mathematics problems. Meanwhile, remedial was provided so that low-achieving students could refine their mistake made and eventually reach mastery intended by the curriculum. Enrichment and remedial was conducted after finishing each topic, thus during implementation, students was facilitated with it three times. In this occasion, students procedural and conceptual knowledge were evaluated by means of mathematical problems given by the teacher. Both high achiever and low achiever students, in fact, took benefit from this activity as they might improve and enriched their knowledge. It facilitated them to recognize their ability after learning process. Misconceptions or mistakes made were refined immediately after the students accomplished the evaluation.

- Five Category: Checking answer

According to the interview, 12 students (three students from low achievers (LA), four students from middle achievers (MA), five students from high achievers (HA)) revealed that after solving problem they were encouraged to check the conducted process and obtained result, and this was useful activity.

> NH (HA) said, “After we solve problems, we are requested to check our answer whether it makes sense or not, and I think it helps us to find mistakes in the process as well as the solution”.
>
> MS (MA) said, “Sometimes, without realizing we make mistake when solving problems, and the teacher always warn us to check our obtained answers”.
>
> MR (LA) said, “As our teacher always requests us to check our answers when we have solved certain problem”.

IMPROVE instructional method let students to perform reflection of what they have conducted. It included the process of checking answer after they have solved certain mathematics problems. As the teacher modelled reflective questions in the introduction part, she got students to question themselves after solving problems whether the intended result was correct or incorrect by checking the conducted process and obtained results. According to students’ statements, checking answer assisted them in looking for mistakes made in the process and solution and made them more confident. The example of reflective questions used to check answers were “Why does that answer make sense to you?”, “Do you check each step of your answer?”, “How can you check your answer?”, and “Did you answer what is asked?”. Checking was obviously seen when students present their works in board in which after they obtained final answer they checked their answer by conducting calculation or algebraic manipulation. Besides, it is important to note that questions posed in activity sheets encourage students to check their answers.

4.5.4. Weaknesses

Finally, the fourth questions directed to the students was “What are the challenges or problems which you encounter when you are taught with the new instruction?” The students’ responses with respect to this question were high level of mathematical problems (9 students), lack of prior knowledge (8 students), noise (9 students), feeling inconvenient at the beginning of the instruction (12 students), and limited time (11 students). Albeit the new instruction gave multiple advantages, inevitably with reference to students answers there were problems that ensued within the implementation.

- First Category: High level of mathematical problems

9 students (5 students from low achievers (LA), 3 students from middle achievers (MA), 1 student from high achiever (HA)) expressed that they faced difficulties with high level of mathematical problems.

> SA (HA) said, “To be honest, most of the presented problems are difficult”.

> SI (MA) said, “When we are given activity sheet and other tasks the problems are mostly difficult”.

> RA (LA) said, “Sometimes I find myself in confusion as the problems are different than normal”.

As stated, the problems presented in this instruction were ranged from straightforward to complex problems. In addition, diverse types of mathematical problems have caused students to encounter hurdles and make strong effort to deal with it. In the previous instruction, students were presented with similar type of problems such as algebraic manipulation and application of formula, yet in this instruction there were various types of problems were given. All low achiever students who were interviewed in this study expressed their difficulty with respect to the high level mathematical problems.

- Second Category: Lack of prior knowledge

8 students (4 students from low achievers (LA), 3 students from middle achievers (MA), 1 student from high achiever (HA)) expressed that they had difficulty in activating prior knowledge. Their deficiency in prior knowledge had impeded their effort to grasp the idea of new concept.

> MS (MA) said, “Our old knowledge helps you understand the new knowledge, and I don’t have it properly”

> NA (MA) said, “…I know that my previous knowledge couldn’t support enough, therefore I face difficulties”.

> MA (LA) said, “…as what our teacher said, that mathematics topics are like a chain. In order to understand one topic, you have to understand previous topics. I think in this instruction we are requested to connect the new topics and the

> old topics, and the most problem that I have is to deal with old knowledge, because I don't remember it".

According to the students' statements above, previous knowledge played important role in effort to comprehend the new concepts as mathematics topics were considered as a chain, thus their deficiency in prior knowledge impeded them to understand the new concepts. When introducing the new concept, the teacher made effort to activate students' prior knowledge using metacognitive questions. The prior knowledge was used as foundation to the subsequent knowledge. For example, the concept of properties of inverse function was given in the wake of concept composition function, bijective function, and function.

- Third Category: Noise

9 students (2 students from low achievers (LA), 3 students from middle achievers (MA), 4 students from high achievers (HA)) expressed that noise was the problem in this instruction.

> MS (HA) said, "When we work in pairs, other friends make noise, and for me it is annoying"
>
> NA (MA) said, "I don't like noise in mathematics classroom since it bothers my concentration"
>
> RS (LA) said, "I couldn't concentrate if the other friends make noise".

According to the students' statements above, when they work in groups several students made noise and consequently several students couldn't concentrate and felt annoyed. In this instruction, classroom environment that allowed students to work in pairs and pose and answer questions to each other inevitably gave rise to noise situation and it occasionally annoyed several students, including some who were interviewed. The teacher and several students in fact warned students who made noise during discussion.

- Fourth Category: Feeling inconvenient at the beginning

12 students (5 students from low achievers (LA), 4 students from middle achievers (MA), 3 students from high achievers (HA)) who were interviewed expressed that in

the beginning of the implementation of the new instruction they felt some inconvenience as it seemed so different from the previous instructions.

> SA (HA) said, "This instruction, to be honest, differed from the previous instructions, thus to get used to it I faced difficulties".
>
> MD (MA) said, "…I think at the beginning of the instruction we faced difficulties as we had to deal with mathematical concepts, and we had to work with new partner".
>
> NH (LA) said, "Even though at the beginning of the instruction I faced difficulties, but as the time passed, I started to enjoy it, I think it takes time to get used to it".

According to the students' statements above, at the beginning of the implementation of IMPROVE instructional method, several students were in difficulties as they worked with new partner and dealt with mathematical concepts heavily. Therefore, classroom situation tended to be in silence and most students didn't engage thoroughly in learning process. As IMPROVE instructional method emphasized heavily verbal explanation, several students expressed their confusion and asked the teacher to present formulas and examples directly. The reason might lay in the fact that in the previous instruction students were accustomed to deal with numerical operation or simple calculation regardless detail delineation concerning concepts and reasoning. For instance, at the outset of the implementation when the teacher explained the concept of domain, AS said, "Miss, I could not catch what you are explaining, can you give us the formula directly?" In addition, students were not accustomed to working with new partners that the teacher and the researcher had assigned. As a result, most of them worked in rigid situation and worked in three and four, albeit they were requested to work in pairs.

- Fifth Category: Time limitation

11 students (5 students from low achievers (LA), 4 students from middle achievers (MA), 3 students from high achievers (HA)) expressed that they worked in limited time.

MS (MA) said, "Sometimes, when we are trying to deal with tasks in activity sheet we are about to solve all questions but unfortunately time is up"

RR (MA) said, "We have to solve three questions in activity sheet within 20 minutes, I think it is not enough".

SA (HA) said, "In this instruction, the teacher always warns us to do something within certain period of time, and sometimes we couldn't catch what we want to do".

According to the students' statement above, at certain occasions several students were unable to meet time limitation due to processing delays. A limited time might impede students to deepen their knowledge or accomplish given problems in activity sheet. IMPROVE instructional method consisted of multiple teaching steps which have to be performed by the teacher within two-hour lesson. Time allocation for each teaching step was organized beforehand, since if one teaching step exceeded the allocated time it might undermine other teaching steps. The teacher and students in particular were expected to adjust their performance by considering time limitation. By and large within a two-hour lesson, time allocation for introducing the new concept was 30 minutes, practicing was 20 minute, presenting was 20 minutes, and reviewing was 10 minutes.

4.6. Interviews on Students' Procedural and Conceptual Knowledge

4.6.1. Students Procedural Knowledge

This study also tried to illuminate students' procedural knowledge of the three topics. By observing students' answers on Post-PROC and interviews conducted to several students in experimental group, information with respect to this was gained. As stated, there were nine items relative to procedural knowledge in used instrument. As described, with reference to Hiebert and Lefevre (1986) procedural knowledge was separated into two distinct elements, to wit, knowledge of mathematical formal language or symbol representation system (PROCA) and knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks (PROCB).

The items related to knowledge of mathematical formal language or symbol representation system expected students to recognize and provide judgment whether mathematical ideas are expressed in plausible form. In the developed instrument, students were presented with five options of mathematical expression and students were requested to choose one of them which was incorrect expression. Students also were asked to provide arguments after selecting their choice. 93% students in experimental group were able to choose the right option. It means that most of the students were familiar with symbols and conscious of the syntactic formulas. However, they obtained various scores on these items based on their given reasons or arguments. With reference to students' explanation, they choose the correct option because of various reasons such as observing that the other options are incorrect options and true comprehension.

> AS said, "I choose this option as I see that the other options are wrong. I don't know this expression. And I am sure that the other options are wrong because I remember it well. I always write it before solving problem."
>
> DH said, "When I learned the topics of composition of function, I didn't find such kind of thing. It is weird."
>
> RH said, "I chose this answer because I am sure that this expression is wrong, it should be like this. I know it because if it is written like this, the meaning will be change"

Student AS and DH were likely to memorize mathematical formal language or learned formulas. Therefore, they were familiar with correct mathematical formal languages. They expressed their unfamiliarity with certain mathematical languages in presented options so they selected it as an incorrect expression. However, they couldn't make any revision to the incorrect one. Besides, student RH observed that there were unfamiliar things in the expressions, and based on her understanding, she made revision correctly. According to her, meaning of mathematical symbol could help her to determine the incorrect one and make revision.

On the contrary, they were several students who didn't answer the items correctly. They revealed that they couldn't make distinction among presented options and they didn't pay attention deeply on formulas presented.

FH said, "I have difficulty in making difference between all options, all of them are correct I think. I didn't remember all of the formulas well."

The items pertaining to knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks (PROCB) expected students to solve routine mathematics problems by applying certain formulas and algorithms. According to Table 4.13, 82% students could provide correct answers. With reference to several students' responses, they revealed that they were able to solve those questions because the questions and the formulas were familiar for them. They added that as they solved numerous problems in various occasions such as classroom activity, homework, formative assessments, and enrichment and remedial, it assisted them in solving these types of problems. The following was an instance of students answer of a question in line equations topics.

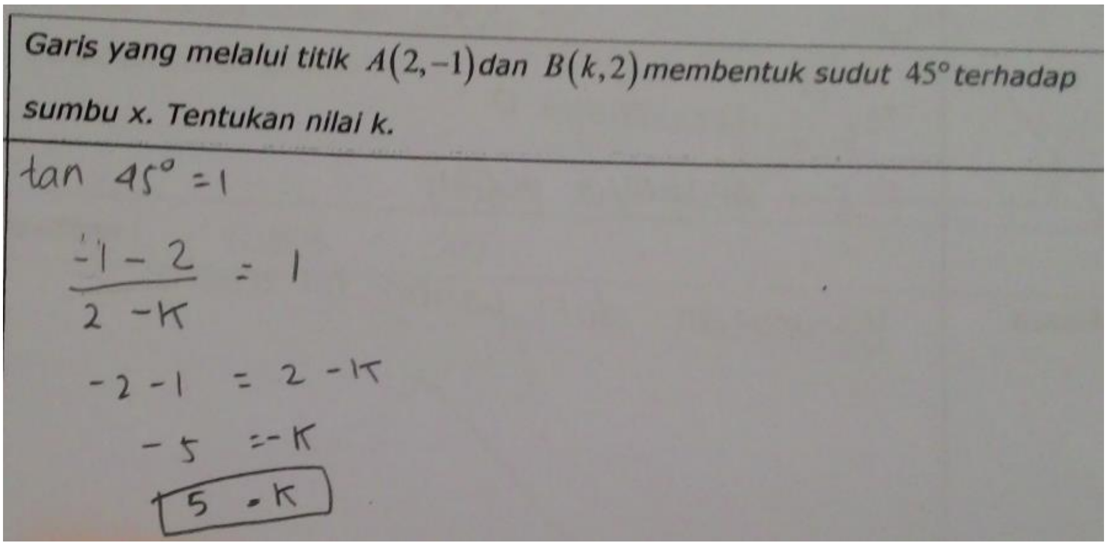


Figure 4.3. A student's answer on item 23

Researcher : Why was this question easy for you?

SD : This question is familiar for me, and I know the formula to solve this question. As you know to find slope of a line if two points are given is this. And the formula of slope of line can also like this if it is known the angle formed by a line and x axis.

RF : This question actually is not unfamiliar to me. I can deal with it. First, I use the formula, and then take important information from question, and put it into the formula. I calculate it, and I can come to this answer."

MD : There are many questions that we have completed in activity sheets, homework, test, and enrichment and remedial. Therefore, we can solve this type of problem."

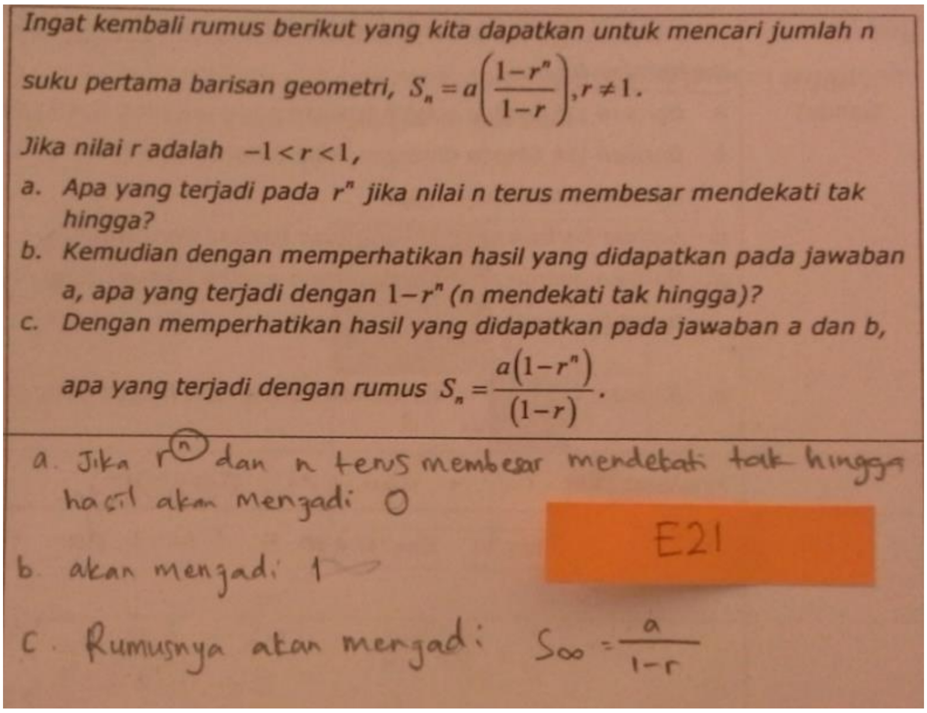
On the contrary, there were several students who couldn't provide correct answer. The reasons that they revealed were that they didn't remember the formulas, they made mistake in calculation or manipulating algebraic expressions, and they misused the concepts.

> DF said, "…actually this question could be solved but I didn't remember the formula."
>
> SA said, "I made mistake since I was careless, I did what I should not"

4.6.2. Students Conceptual Knowledge

In this section, students' conceptual knowledge will be described. Similar to description presented for procedural knowledge, by discerning students' answers on Post-CONC and interview conducted to several students in experimental group, information with respect to this was obtained. With reference to Crooks and Alibali (2014), conceptual knowledge was separated into two elements of knowledge, to wit, knowledge of general principle and knowledge of principles underlying procedures. In addition, each of knowledge could be measured by two facets of tasks, to wit, explanation of concepts tasks and evaluation of examples tasks for knowledge of general principle, and application and justification of procedures tasks and valuation of procedures tasks for knowledge of principles underlying procedures.

Items relative to explanation of concepts tasks expected students to give definitions for symbols and elements of domain structure. To put it differently, the items expected students to provide description with respect to the meaning of symbols or mathematical expressions or concepts. Therefore, the questions were related to explanation of the concepts of composition and inverse of function, infinite sequence and series, and line equations. According to Table 4.14, there were 77% students who could be said to be able to provide answer with respect to these items. In order to be able to answer these items, comprehending the concepts and connecting it to other related concepts were sine qua non. In addition, ability to cope with calculation and algebraic manipulation played important role in answering correctly. The following are examples of students' answers relative to the explanation of discovering formula of geometric infinite series. In this items, students were expected to discover the formula of infinite convergent geometric series through questions presented.



Ingat kembali rumus berikut yang kita dapatkan untuk mencari jumlah n suku pertama barisan geometri, $S_n = a\left(\frac{1-r^n}{1-r}\right), r \neq 1$.

Jika nilai r adalah $-1 < r < 1$,

a. Apa yang terjadi pada r^n jika nilai n terus membesar mendekati tak hingga?
b. Kemudian dengan memperhatikan hasil yang didapatkan pada jawaban a, apa yang terjadi dengan $1-r^n$ (n mendekati tak hingga)?
c. Dengan memperhatikan hasil yang didapatkan pada jawaban a dan b, apa yang terjadi dengan rumus $S_n = \frac{a(1-r^n)}{(1-r)}$.

a. Jika r^n dan n terus membesar mendekati tak hingga hasil akan menjadi 0

E21

b. akan menjadi 1

c. Rumusnya akan menjadi: $S_\infty = \frac{a}{1-r}$

Figure 4.4. A student's answer on item 16

Researcher : What is the meaning of S_n here?

SA : I think that it is the sum of the first n term of geometric sequences.

Researcher : Can you explain to me your statement in detail?

SA : For example, let r equals $\frac{1}{2}$, if the value of n is getting bigger then the value of r^n became 0.

Researcher : Why?

SA : Because if you divide one by a very big number the value approaches zero.

Researcher : You wrote the expression $1-r^n$ became one. Why?

SA : As you know we have $r^n = 0$, therefore $1-r^n = 1$.

Researcher : So what did you get then?

SA :The formula became $S=\frac{a}{1-r}$.

Researcher : What formula is it?

SA : This is formula for infinite geometric series.

Researcher : Convergent or divergent? Why?

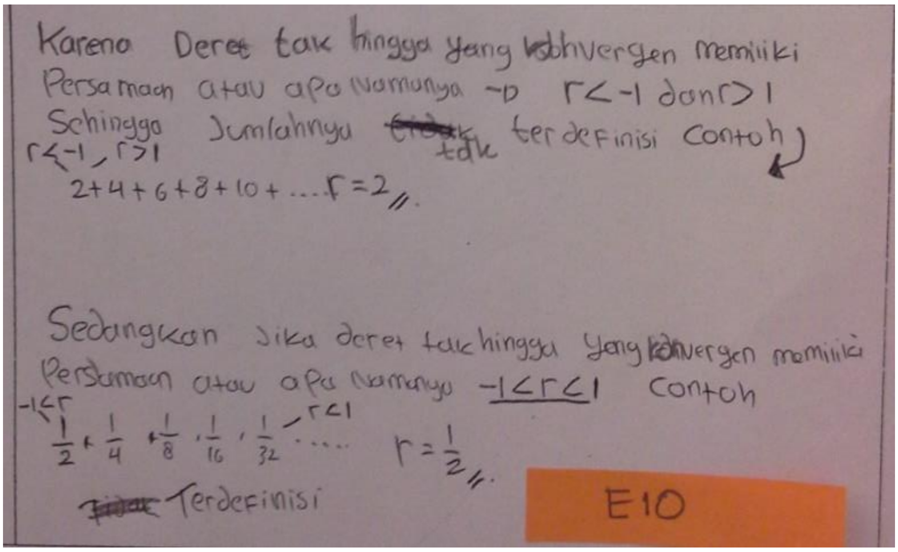
SA : Convergent, because of the ratio.

According to students' excerpts, it was clear that in order to cope with this question, concept of geometric series, calculation, algebraic manipulation, and connection to other concepts such as exponential were required.

On the other hand, there were several students who were not able to provide reasonable arguments. The responses ranged from blank response to taking unrelated concepts. Some of them expressed that they were in confusion and didn't remember concepts and the related concepts. Therefore, their lack of connection to other concept might be reasonable cause as they only comprehended the concept independently.

> SD said, "I actually didn't remember this concept, I am confused."
>
> RF said, "I have learned this and I know this, but don't know how to complete this."

The items relative to evaluation of examples tasks demanded students to acknowledge examples, definitions, or statements of principles. Therefore, the items expected students to choose one incorrect option or one correct option from available options and they were requested to write their arguments. According to Table 4.15, 94% students in experimental group were able to select correct option, yet they obtained different scores based on their presented arguments. Their understanding about mathematical symbol and connection among concepts assisted them to deal with these problems. The following was example of one item which asked students to choose correct statement pertaining to convergent and divergent sequences and series.



Karena Deret tak hingga yang konvergen memiliki Persamaan atau apa namanya → r<-1 dan r>1
Sehingga Jumlahnya ~~tidak~~ tak terdefinisi Contoh
r<-1, r>1
2+4+6+8+10+ ... r=2

Sedangkan jika deret tak hingga yang konvergen memiliki Persamaan atau apa namanya -1<r<1 Contoh
-1<r, r<1
$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ .... $r=\frac{1}{2}$
~~Tidak~~ Terdefinisi

E10

Figure 4.5. A student's answer on item 21

Researcher : Why did you choose this option?

MD : I chose this option because I know that convergent sequence is sequence whose ratio is between -1 and 1 and divergent sequence is sequence whose ratio are greater than 1 and less than -1. And the ratio of this series is $\frac{1}{2}$ which fall between -1 and 1.

However, there were several students who were not able to deal with these tasks. They revealed that they didn't remember the formulas and the meaning of mathematical expression presented in the tasks.

> RA said, "I forget the formulas, therefore I couldn't solve this problem."

> SD said, "I forget the meaning of this expression, therefore it makes me confused."

The items related to application and justification of procedures tasks asked students to address problems or to give verbal explication of their own problem-solving procedures. The items seemed to be similar to items which expected students to solve familiar problems, yet it differed as it was more complicated and needed further reasonable explanation. Therefore, in general the items required students to capitalized on more than two concepts or apply it in different forms of questions. In addition, the students were asked to provide reasonable arguments of execution or justification of statement. According to Table 4.16, 68% students were able to solve it in various responses. The following is an example of item related the application of concept infinite convergent geometric series.

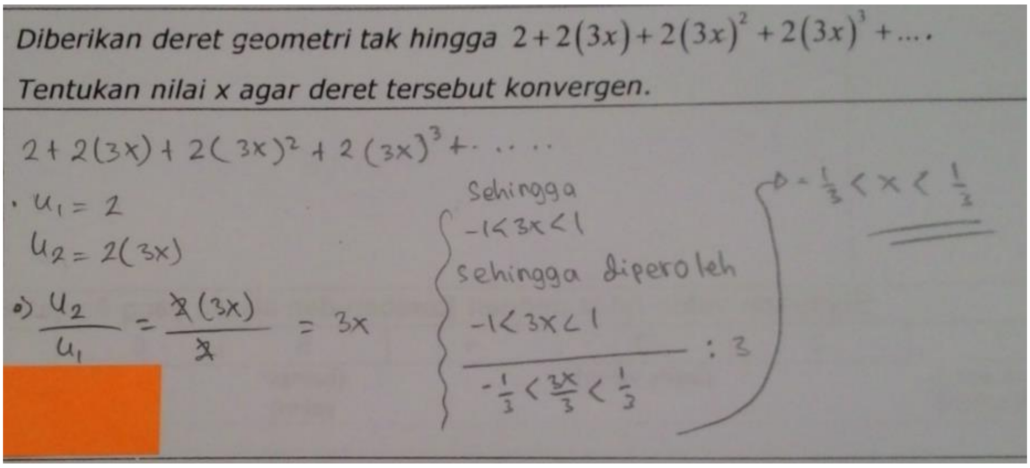


Figure 4.6. A student's answer on item 17

Researcher : What are things that you pay attention after reading this question?

SA : Convergent series, and determining the value of x.

Researcher : Can you explain to me the way how you answered this question?

SA : First since this series is convergent, therefore its ratio has to be fallen between -1 and 1. From this series we obtained its ratio equal to $3x$. We could get it by dividing the second term with the first term. Then we could obtain this result because we could divide the expression by 3. The sign was not change since we divide it by positive number. As a result, the values of x have to be fallen between $-\frac{1}{3}$ and $\frac{1}{3}$.

In order to be able to solve this item, students had to comprehend numerous concepts such as convergence, ratio, division, inequalities, and algebraic manipulation.

However, the rest of the students were not able to cope with these items as they did not provide complete execution, provide reasonable arguments, as well as translate into appropriate procedures. They claimed that they didn't understanding the concept very well, and they didn't check their works in the end.

> AS said, "I know the formula but I don't know the condition of formulas and properties in which it can be applied."

SE said, "I did it carelessly because I didn't check it after solving this."

The items relative to evaluation of procedures tasks ask students to evaluate or judge the presented procedures whether it is correct or incorrect. Therefore, in this items students were presented with one or two procedures and they were asked to choose the correct procedures or to make revision if there were any mistakes. Students were able to deal with these items. According to Table 4.17, 88% students were able to find incorrect procedures. However, their obtained scores depended on their explanation and the presented and correctness of revisions of the incorrect procedures. Several students were able to come up with incorrect procedures as they paid attention to the procedure from the beginning until the end. In addition, their understanding about the concepts, calculation, algebraic manipulation, and procedural knowledge took important role in performing it well.

Researcher : How could you find the incorrect procedures?

SA : First, I read all expressions carefully one by one. For the procedure A, I found that there was a mistake in converting the equation. I thought that the procedure B was true, but I suspected that the procedure B was incorrect. Then similar to what I have done to procedure A. Then I found incorrect procedure here. Then I made revision, and finally after revision both results were similar. Therefore, the similarity of the two expressions were showed.

Researcher : Why did you think that those things were incorrect?

SA : If you move 6x to the left side, then move y to the right side, then it should be like this.

Researcher : How about this?

SA : Similar to the previous I think. It should be like this since if we move this number to the left it should be negative.

According to students' excerpts, in order to deal with this type of item they had to read all procedures from the beginning until the end and pay attention to alteration of mathematical expression in each step.

In contrast, there were several students who were no able to find incorrect procedures. The revealed that they thought that the procedures were correct and did not need to be revised.

> SA said, “at a glance, there is not incorrect procedures and I think there is no need to be revised.”

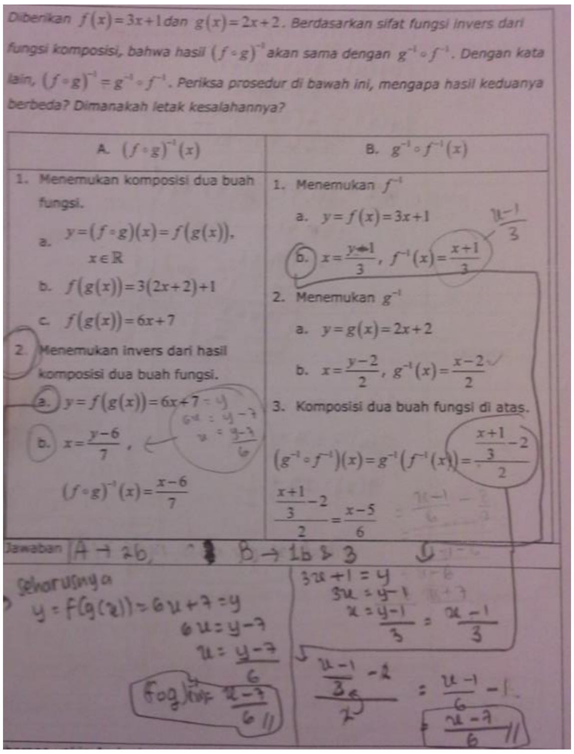


Figure 4.7. A student’s answer on item 8

4.7. Summary of Findings

Results obtained from this study can be summarized as the following.

1. There is statistically significant mean difference of combined scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition of the students in control and experimental group with medium effect size. Subsequently, follow-up ANCOVAs is conducted for each dependent variable, and it is found that control and experimental group significantly differ in procedural knowledge (Post-PROC), conceptual knowledge (Post-CONC), and regulation of cognition (Post-RC) scores with a small to medium effect size. These differences seem to be in favour of experimental group.

2. There is statistically significant mean difference between combined scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition of male and female students with a medium effect size. Then, follow-up ANCOVAs for each dependent variable is conducted, and it is found that male and female students significantly differ in procedural knowledge, conceptual knowledge, and regulation of cognition with a medium effect size. These differences seem to be in favour of female students.
3. There is no statistically significant interaction effect of teaching methods and gender on combined scores of procedural knowledge, conceptual knowledge, knowledge of cognition, and regulation of cognition of male and female students with a small effect size.
4. Based on result of descriptive analysis of students obtained score in all items, it indicated clearly that majority students in experimental group were able to provide more comprehensive arguments in both procedural and conceptual knowledge items than majority students in control group.
5. The results of interview conducted in this study described students' experiences with the implementation of IMPROVE instructional method. Based on interviews and classroom observations, by comparing to the previous instruction they claimed that IMPROVE instructional method lead to pose and answer more questions, to reason and make connection, to deal with various type of problems. Students also expressed that within IMPROVE environment they feel more confident, better in comprehension, better in self-awareness, and enjoy lesson. They also revealed that IMPROVE instructional method provided great opportunity for them to discuss, draw visualization of concepts, review, conduct enrichment and remedial, as well as check answer. In addition to positive responses, they express the challenges or weakness in IMPROVE instructional method such as high level of mathematical problems, lack of prior knowledge, noise, feeling inconvenient at the beginning of the instruction, and limited time.
6. Based on students' interviews of procedural and conceptual knowledge test, in order to solve questions related to procedural knowledge successfully, the requirements were understanding of meaning of mathematical symbols and expressions, frequently working with mathematical formal language, and solving

numerous problems. Meanwhile, in order to solve items related to conceptual knowledge, comprehending certain concept and connecting it to other related concepts, successful in performing calculation, ability in algebraic manipulation, understanding about meaning of mathematical symbols and expressions, and focus on procedures from the beginning until the end were required.

CHAPTER 5

DISCUSSION, CONCLUSSION, IMPLICATION, AND RECOMMENDATION FOR FURTHER RESEARCH

Chapter five consists of four sections. The first section takes a closer look at discussion of the results of the study. This is followed by section of conclusions and implications of the study. As a final remark, recommendations for further research are presented.

5.1. Discussion

As stated, the purposes of the study were to investigate effectiveness of IMPROVE instructional method on 11^{th} grade science students' procedural and conceptual knowledge, and metacognitive skills in topics of composition and inverse function, infinite sequence and series, and line equation, and their experiences in this metacognitive instructional method. The twofold intentions were reflected in the researcher's decision to compile quantitative and qualitative data. In quantitative part, pre-tests and post-tests of students' procedural knowledge (PROC), conceptual knowledge (CONC), knowledge of cognition (KC), and regulation of cognition (RC) were administered in this study. Meanwhile, in qualitative part, semi-constructed interviews were conducted to 14 students in experimental group so as to have them express their experiences during the implementation of IMPROVE instructional method.

Before the treatment, both groups took pre-tests of procedural knowledge (Pre-PROC), conceptual knowledge (Pre-CONC), knowledge of cognition (Pre-KC), and regulation of cognition (Pre-RC) on topics of composition and inverse functions, infinite sequence and series, and line equations. Subsequently, after nine-week implementation period, both control and experimental group took post-tests of procedural knowledge (Post-PROC), conceptual knowledge (Post-CONC), knowledge of cognition (Post-KC), and regulation of cognition (Post-RC) on the same topics. The data obtained from the pre-test and post-test were analysed using MANCOVA to decide whether the proposed hypotheses were rejected or failed to be rejected.

Before conducting MANCOVA, the covariates were calculated and determined. The assumptions underlying MANCOVA then were evaluated and statistically no violations were found. Subsequently, MANCOVA was conducted with four dependent variables (Post-PROC, Post-CONC, Post-KC, and Post-RC), two independent variables (teaching methods and gender), and four covariates (Pre-PROC, Pre-CONC, Pre-KC, and Pre-RC).

The result of MANCOVA indicated that there are statistically significant mean differences on the collective dependent variables of the Post-PROC, Post-CONC, Post-KC, and Post-RC across groups [$F (4, 55) = 9.34$, Wilks' Lambda = .595, $p =.000$] and gender [$F (4, 55) = 4.51$, Wilks' Lambda = .753, $p =.003$] in moderate effect size. In addition, there is no interaction between teaching methods and gender on collective dependent variables [$F (4,55) = 2.19$, Wilks' Lambda = .863, $p =.082$]

In addition to result of MANCOVA, the follow-up ANCOVA results showed that IMPROVE instructional method could promote students' procedural knowledge ($F (1, 58) = 9.767$, $p < .0125$), conceptual knowledge ($F (1, 58) = 36.357$, $p < .0125$), and regulation of cognition ($F (1, 58) = 8.074$, $p < .0125$) on composition and inverse function, infinite sequence and series, and line equations topics better than traditional instruction did. Although there was little evidence of the effect of the IMPROVE instructional method on procedural and conceptual knowledge in mathematics, the literatures gave broad evidences that the implementation of IMPROVE instructional method became reasonable account of the improvement of students' achievement in mathematics (Anggoro, Bambang Sri Kusumah et al., 2014; Kramarski et al., 2002; Kramarski, 2004; Mevarech & Fridkin, 2006; Mevarech & Kramarski, 1997). The results of the present study provide further empirical support for the studies reported significant results about the effectiveness of IMPROVE instructional method over traditional instruction on students' mathematics achievements. In the literature, for instance, Mevarech and Fridkin (2006) found that pre-college students were exposed to IMPROVE instructional method significantly outperformed their counterparts who were taught with traditional instruction on mathematical knowledge and mathematical reasoning within the course on mathematics functions. Significant result also was found by Kramarski (2008) in which mathematics elementary teachers took benefit from IMPROVE instructional method on the improvement of numerous algebraic procedural and real-life tasks respecting conceptual mathematical explanations. Therefore, in this study the obtained significant mean difference in procedural and conceptual knowledge scores might be associated to this type of instruction.

With reference to Mevarech and Kramarski (1997), the effectiveness of IMPROVE instructional method may be attributed to excellent learning environment that was supported by metacognitive questioning, cooperative setting, and systematic provision of feedback-corrective-enrichment. Within this environment, students were provided opportunities to activate their prior knowledge, connect it to the new topics, actively engage in whole learning process by means of proposing and answering metacognitive questions, discuss and share their ideas within group, review the materials, obtain mastery, verify their understanding, and refine and advance their comprehension based on given feedback. Students' procedural knowledge might be developed in mathematics teaching and learning which emphasizes on solving routine problem which leads students to make use of certain strategies through application of specified rules (Rittle-Johnson et al., 2001; Star, 2000). In this instruction, students were given with various types of problems (routine problems and non-routine problems) in various occasions, thus it might imply to development of procedural knowledge. Meanwhile, students' conceptual knowledge might develop in learning environment that encouraged them to reason flexibly and constructed connections between prior knowledge and new knowledge (NCTM, 2000). Prior knowledge plays remarkable role in constructing knowledge, thus it might contribute to meaningful learning (Ausubel, 1969). Procedural knowledge and conceptual knowledge could not be separated each other and they might be developed iteratively (Rittle-Johnson & Koedinger, 2009; Rittle-Johnson et al., 2001). In this instruction, students were not only presented the way how to solve problem, yet they were encouraged to comprehend the meaning of the concept and the reason underlying the procedures. Therefore, rather than presenting mathematics formulas directly through teacher-centred lectures, reasons behind it were considered.

In the wake of comprehending concepts, students were presented with various mathematical problems to be solved with their partners. Within groups, students were encouraged to share, express, and discuss their understanding using their own words to reach agreement with their partner in effort to cope with mathematical problems. Considerable studies revealed the effectiveness and the importance of cooperative learning in promoting students' mathematics achievement (Hossain & Ahmad, 2013; Zakaria, Solfiri, Daud, & Abidin, 2013; Zakaria, 2010). In addition, students in experimental group were provided opportunity to refine and advance their knowledge using feedback. According to Bandura (1997), advancing knowledge or enrichment allowed students to experience greater depth, breadth, as well as context in learning. Meanwhile, refinement or remediation served to the needs of students who were behind the expected level of achievement and incapable of keeping pace with teaching-learning in a normal classroom (Selvarajan & Vasanthagumar, 2012). These enterprises in experimental group might promote students procedural and conceptual knowledge on composition and inverse function, infinite sequence and series, and line equation concepts. On the other hand, most activities found in control group were listening their teacher, doing exercise individually, and taking notes most of the class times.

The above results were also supported by looking at students' obtained scores in procedural and conceptual knowledge tests. With reference to analysis of students answers relative to scores and the items across groups, it could be revealed that majority students in experimental group obtained score 3, score 4, and score 5 in all procedural and conceptual knowledge items more than majority students in control group. Therefore, it means that students in experimental group could provide reasonable and comprehensive arguments with respect to given questions. Then, it could be said that majority students in experimental group possessed better procedural and conceptual knowledge than majority students in control group. The result of this study in line with description of conceptual understanding proposed by National Council of Teachers of Mathematics (NCTM, 2000) in which conceptual understanding reflected students' ability to reason in setting including the precise application of definitions and connection of concepts and their representations. Besides, result from interview of students in experimental group with respect to their procedural and conceptual knowledge, it showed that in order to solve various mathematical problems successfully and provide comprehensive arguments, there were several things that had to be taken into consideration: meaning of mathematical symbols and expressions, calculation, algebraic manipulation, comprehension of certain concept, and understanding of related concepts.

By considering interviews on students' post-test of procedural and conceptual knowledge, students' development of procedural and conceptual knowledge could be illuminated. Procedural knowledge according to Hiebert and Lefevre (1986) consisted of knowledge of mathematical formal language or symbol representation system and knowledge of collection of formulas and algorithms which are applied to tackle mathematical tasks. According to students, in order to solve questions related to procedural knowledge successfully, the requirements were understanding of meaning of mathematical symbols and expressions, frequently working with mathematical formal language, and solving numerous problems. As a matter of fact, IMPROVE instructional method provided these requirements as during instruction students were encouraged to grasp the meaning of each mathematical symbol and expression. The example of this is an example when students learned the topic of line equations, a student posed a question "What is the meaning of $\beta - \alpha$?". This indicated that students made effort in understanding mathematical expression. According to Ploger and Hecht (2009) awareness of meaning of mathematical symbol and expression might lead to improvement in conceptual knowledge. Subsequently, this situation might lead to enhancement in their writing as they became aware of what they wrote in their note books. Therefore, they got used to write formal mathematical language. In this case, mathematics teachers were encouraged to provide opportunity for students to engage in writing activities as it might influences students development in metacognitive skills (Pugalee, 2001). In addition, students' success in coping with problems related to procedural knowledge was assisted with their effort in dealing with various types of mathematical problems in various occasions such as classroom activities, homework, formative assessments, and enrichment and remedial.

Meanwhile, conceptual knowledge consisted of knowledge of general principle and knowledge of principles underlying procedures (Crooks & Alibali, 2014). Each of knowledge was measured by explanation of concepts tasks and evaluation of examples tasks for knowledge of general principle, and application and justification of procedures tasks and valuation of procedures tasks for knowledge of principles underlying procedures. In order to solve these tasks, according to the result of students' interviews, comprehending certain concept and connecting it to other related concepts, successful in performing calculation, ability in algebraic manipulation, understanding about meaning of mathematical symbols and expressions, and focus on procedures from the beginning until the end were required. As a matter of fact, IMPROVE instructional method provided great opportunity for students to build connection among mathematical concepts using metacognitive questions (Mevarech & Kramarski, 1997). This connection also might assist them in dealing with the meaning of mathematical symbol and expressions. Besides, the instruction took students' awareness into consideration. This awareness might help students perform calculation and monitoring their progress when processing algorithm in effort to solve problems. It was in line with what several researchers claimed that metacognitive skills might contribute to students' success in problem solving (Desoete, Roeyers, & De Clercq, 2003; Mevarech & Amrany, 2008; Ozsoy & Ataman, 2009). In addition, development of ability in algebraic manipulation was obviously seen when students were encouraged to solve numerous problem in various occasions. Newton, Star, and Lynch (2010) claimed that familiarity, comprehensibility, and types of the problems might assist students in addressing the problems.

Corresponding to the improvement of students' procedural knowledge and conceptual knowledge, significant mean different also was found in students' regulation of cognition in favour of experimental group ($F(1, 58) = 8.074$, $p < .0125$). This result was in parallel with the result of the study conducted by Mevarech and Amrany (2008) in which they investigated direct and delayed impact of IMPROVE instructional method on regulation of cognition and mathematics achievement. They found that the instruction could enhance students' mathematics achievement and regulation of cognition. One of the three interdependent substantial elements in the instruction that played important role in enhancing students' regulation of cognition was self-directed metacognitive questions. It is important to note that during the implementation, students in experimental group were guided by self-directed metacognitive questions.

In IMPROVE instructional method, there are four types of metacognitive questions, to wit, connection questions, comprehension questions, strategic questions, and reflective questions (Mevarech & Kramarski, 1997). With reference to Breed et al. (2013), students took benefit from self-directed metacognitive both cognitively and meta-cognitively. The questions could lead students to monitor their learning progress, recognize and remedy their deficiencies and examine whether the selected and carried out strategies were appropriate to cope with presented problems. In addition, it directed students to think deeply with respect to procedures, strategies, and methods to address mathematical problems. Therefore, through the medium of those questions, they were guided to plan, monitor, debug, evaluate, and manage information properly while learning and solving mathematical problems. By its nature, questioning is a crucial part in the learning process by which it contributes to meaningful learning, conceptual understanding, and problem solving (Almeida, 2012).

With reference to study of Mevarech and Fridkin (2006), the impact of IMPROVE instructional method were visible on two elements of general metacognitive awareness, to wit, knowledge of cognition (declarative, procedural, and conditional) and regulation of cognition (planning, monitoring, debugging, evaluating, and information managing). In this study, nevertheless, improvement in regulation of cognition was not in line with another element of metacognitive skill, namely knowledge of cognition in which students' knowledge of cognition in both groups were not found significantly different ($F (1, 58) = 1.845, p > .0125$). Interestingly, the study conducted by Mevarech and Amrany (2008) also came up with similar result. They claimed that even though IMPROVE instructional method allowed students to capitalize on various sort of cognitive regulation processes, it didn't serve to improve their knowledge of cognition. They argued that knowledge about cognition had not strong correlation to a high level of regulation. It was supported by Schraw and Dennison (1994) who revealed that there was moderate correlation between the two components. A second reason was that regulation of cognition and knowledge of cognition were not related which meant that there was no guarantee that students who develop knowledge of cognition would regulate their cognition (Mevarech & Amrany, 2008). It was also due to the fact that their study emphasized heavily on regulation of cognition, rather than knowledge about cognition. In this study, during the implementation, questions related to regulation of cognitive were frequently posed by the teacher that questions related to knowledge of cognition. Besides, as Meloth (1990) claimed that the improvement of knowledge of cognition was found moderately over the school year, the implementation of the study which lasted for 9 weeks might be another reason students' knowledge of cognition was not increased significantly. Furthermore, the use of reflective journal as a matter of fact was expected to amplify students' knowledge of cognition (Lew & Schmidt, 2011). Ideally it should be given after each lesson, however due to load work it was given three times during implementation. Therefore, the impact of the instruction on knowledge of cognition didn't seem to appear significantly.

Since issues with respect to differences in education based on gender are often investigated by researchers around the world, thus analysis with respect to it was conducted to examine the impact of gender on the dependent variables. Based on result of the statistical analysis, the effect of gender was proved statistically on students' procedural knowledge ($F(1, 58) = 15.615$, $p < .0125$), conceptual knowledge ($F(1, 58) = 6.786$, $p > .0125$), and regulation of cognition ($F(1, 58) = 7.118$, $p < .0125$). While several studies indicated that there was no significant difference in students' procedural and conceptual knowledge based on gender (Hutkemri & Zakaria, 2012; Hyde, Lindberg, Linn, Ellis, & Williams, 2008; Mosia, 2014), in this study female students were observed to have more procedural knowledge and conceptual knowledge on the three topics than male students. It was at odds with several previous studies such as Else-Quest et al. (2010) and Lindberg et al. (2010) who showed that there was gender difference in mathematics performance and achievement in favour of male students. However, other research suggested that female students had better mathematics achievement than male students (Kenney-Benson, Pomerantz, Ryan, & Patrick, 2006; Mullis et al., 2012; Voyer & Voyer, 2014). In this study, female students outperformed male students in procedural knowledge. Belenky, Clinchy, & Goldberger (1997) described several characteristics that represented women's cognitive development, one of which was procedural knowledge. They claimed that women tended to emphasize heavily on obtaining and applying procedures for acquiring and communicating knowledge. Therefore, according to them, common things in women were procedures, skills, and techniques. What is more, in this study, conceptual knowledge of female students tended to be greater than that of male students. The possible reason is that according to Chouinard and Roy (2008), in some cases, high school female students had more positive attitudes toward mathematics than high school male students. Besides, result of TIMSS assessment in 2011 showed that Indonesian female students were more successful than male students in mathematics achievement (Mullis et al., 2012). In Indonesia, female students tend to be more careful and diligent than male students. In most classrooms, they sit in front line and they take note neatly. This state of affairs might lead female students to have better procedural and conceptual knowledge than male students.

Gender difference was also investigated in students' knowledge of cognition and regulation of cognition. The result of the present study indicated that there was gender difference on regulation cognition ($F(1, 58) = 7.118$, $p < .0125$). However, the result of this study didn't indicate significant difference on knowledge of cognition ($F(1, 58) = 2.262$, $p > .0125$) across gender. In fact, previous studies reported inconsistent findings with respect to the differences in metacognition based on students' gender. Sperling, Howard, Miller, and Murphy (2002) revealed that there were no significant effects of gender on knowledge of cognition and regulation of cognition. Studies conducted by Ciascai and Lavinia (2011) indicated that boys and girls use differently their metacognitive knowledge and skills in the learning process, yet the study didn't confirm evidently whether female or male students possessed higher metacognitive skills. Several studies that suggested that that female students had better metacognitive skills than male students were, for example, Wu (2014) and Bidjerano (2005). Wu (2014) claimed that female students performed better in knowledge of metacognitive strategies, navigation skills and printed reading assessment, whereas Bidjerano (2005) revealed that female students outperformed male students in the use of rehearsal, organization, metacognition, time management skills, elaboration, and effort. According to Kolic-Vehovec and Bajsanki (2006), female students have better metacognition due to their ability in monitoring tasks.

Other purpose of this study was to investigate whether the effectiveness of teaching methods differ across gender. In other words, interaction between gender and teaching methods related to students' procedural knowledge, conceptual knowledge, metacognitive skills was tried to be understood. The results of the statistical analysis indicated that the effectiveness of either IMPROVE instructional method or traditional instruction in improving procedural knowledge ($F(1, 58) = .018$, $p > .0125$), conceptual knowledge ($F(1, 58) = 1.637$, $p > .0125$), knowledge of cognition ($F(1, 58) = 2.539$, $p > .0125$), and regulation of cognition ($F(1, 58) = .001$, $p > .0125$) do not differ across gender statistically. The reason may lay in the fact that during the implementation, the activities carried out in classroom might not favor neither males nor females. In addition, during the implementation, the teacher tried to provide equal opportunities for both male and female students to pose questions, answer questions, share and express their ideas, and to be guided by the teacher. Therefore, male and female students would acquire the advantages of IMPROVE instructional method similarly and participation in the instruction did not promote scores of male or female students compared to male or female students who participated in traditional instruction. This result was in line with study conducted by Grizzle-Martin (2014) in which there was no significant interaction between the IMPROVE group and non-IMPROVE group and gender. She unfolded that it might be due to the low power of the study, thus participation in IMPROVE instructional method did not promote mathematics achievement of both male or female students with mathematical learning difficulties compared to male or female students with mathematical learning difficulties who were not taught with IMPROVE instructional method. However, this result was at odds with what Mevarech and Fridkin (2006) had predicted in their study in which IMPROVE instructional method would have stronger impact of female students' achievement, in as much as the instruction was of highly verbalized instruction. The issue of interaction between gender differences and IMPROVE instructional method in fact is not much reported in literatures.

Semi-structured interview and classroom observation were also conducted in this study to understand students' experiences with the implementation of IMPROVE instructional method. Based on those, by comparing to the previous instruction they claimed that IMPROVE instructional method lead to pose and answer more questions, to reason and make connection, to deal with various type of problems. In this instruction, the teacher and students used metacognitive questions frequently with the aim of activating students' cognitive and metacognitive skills. Metacognitive question is one of the three interdependent elements in IMPROVE instructional method (Mevarech & Kramarski, 1997). Students were requested not only to answer questions but also pose questions. Di Teodoro, Donders, Kemp-Davidson, Robertson, and Schuyler (2011) revealed that focusing on questions might heighten students metacognition and a way to develop better understanding. The questions might lead the students to present reasons behind the formula or procedures and to connect among existing concepts. As mathematics topics are interconnected each other and in order to grasp the new concept, it is required to have a good grip of previous concepts. Then, understanding mathematical concepts might help them deal with various types of mathematical problems such as application of single or multi-concepts, algorithm, proofing the formulas, showing the properties, interpreting properties, and drawing figures. It was supported by Geary (2004) who argued that both conceptual understanding and procedural knowledge were fundamental skills in problem solving.

Students also expressed that within IMPROVE instructional method environment they feel more confident, better in comprehension, better in self-awareness, and enjoy lesson. According to them, as they were trained to express their thinking and cope with various mathematical problems in many occasions, they felt more confident in learning mathematics and dealing with mathematical problems. Bandura (1977) argued that one source of self-efficacy was performance accomplishment in which previous success might raise confidence in coping with subsequent tasks. This confidence also might lead students to enjoy learning mathematics and it might contribute to improvement of their understanding. According to Mac Iver, Stipek, and Daniels (1991) self-confidence has been associated with enjoyment. Enjoyment in learning mathematics as one of component in motivational orientations (Saxe, Gearhart, & Nasir, 2001) which had positive implication on development of students' achievement (Murayama, Pekrun, Lichtenfeld, & vom Hofe, 2013). Besides, the instruction guided the students to be more aware of what they were doing, thus it might play important role in enhancing students' understanding of mathematical concepts. Mulligan and Mitchelmore (2009) supported that awareness was crucial for students as it was correlated with general mathematical understanding. Therefore, according to the result of post-test of metacognitive awareness inventory, students who were exposed to

IMPROVE instructional method had better regulation of cognition than students who were taught with traditional instruction.

They also revealed that IMPROVE instructional method provided great opportunity for them to discuss, visualize the concepts, review, conduct enrichment and remedial, as well as check answer. In order to address difficulties in comprehending mathematical concepts and solving mathematical problems, students were encouraged to draw figure or make visualization along with guidance of the teacher. Arcavi (2003) revealed that visualization was recognized as a key element of reasoning, problem solving, as well as proving. Therefore, thanks to visualization, students might engage with concepts and meaning, and see solution of the problem in whole manner. In this instruction, the teacher tried to present figures related to concepts and problems given encouraged students to make visualization so that they could cope with it. In addition, as this instruction put emphasis on cooperative setting (Mevarech & Kramarski, 1997), students worked in groups which encouraged them to communicate each other to discuss about the problem and they made effort collectively to reach common solution. Subsequently, reviewing after all material learned might help students to strengthen, evaluate, and refine their understanding of main ideas. Based on classroom observation, several students were able to express their understanding using their own words in proper manner. Furthermore, enrichment and remedial were provided in the instruction as ways to refine and enrich students' knowledge. In other instructions, enrichment and remedial were not provided, thus students felt that the availability of enrichment and remedial was important in enhancing their achievement. IMPROVE instructional method also let students to perform reflection by checking answer after they have solved certain mathematics problems. Checking answer in fact as part of problem solving process namely looking back (Polya, 2014). According to Polya (2014), through looking back, students reconsidered and re-examined the result and the strategies carried out, thus they might strengthen their understanding and elaborate their ability to solve problems. The teacher encouraged students to check their answer by presenting metacognitive questions such as, "Can you check the result?" By considering this, students who were exposed to IMPROVE instructional method could outperform their counterparts who were taught with traditional instruction in procedural and conceptual knowledge, and regulation of cognition.

In addition to positive responses, they express the challenges or weakness in IMPROVE instructional method such as high level of mathematical problems, lack of prior knowledge, noise, feeling inconvenient at the beginning of the instruction, and limited time. As stated, one way to improve students' conceptual knowledge and procedural knowledge was to present problems ranged from straightforward to complex problems. Most low achievers who were interviewed encountered difficulties in dealing with various problems. The reason might be that they had problem in comprehending mathematical concepts. According to Johari, Nor Hasniza, and Mahani, (2012), students' inability in solving problems was due to their lack of conceptual understanding. However, the instruction provided great opportunity for them to improve their procedural and conceptual knowledge, and eventually their scores in post-tests were better than their scores in pre-tests. In addition, students' statements with respect to their weakness in prior knowledge were reasonable as mastery of prior knowledge played important role in effort to comprehend the new concepts. In this instruction, the teacher emphasized students to activate their prior knowledge. At the beginning of the lesson, the teacher posed numerous questions so that students could remember their prior knowledge required to grasp new concepts. Besides, as this instruction encouraged students to pose and response questions, in several occasions noise within classroom inevitably could not be eliminated, thus several students felt annoyed. Several students also encountered difficulties at the beginning of the implementation of IMPROVE instructional method, as they dealt with mathematical concepts heavily. As stated, this instruction was different from the previous instruction, thus adaptation to the new instruction took time. However, students claimed that they got used to follow the teaching paces as time passed by. In addition to dealing with procedural knowledge, effort in understanding mathematical concepts in IMPROVE instructional method had a consequence for students in which they had to accomplish given tasks in restricted time. IMPROVE instruction consisted of teaching steps that students had to engage with, thus they were necessary to be more discipline and work effectively. In the previous instruction, students were not requested to work in certain period of time, yet in this instruction there was time limitation. For several students, time limitation became a problem.

By and large, the results of quantitative part were supported with the result of qualitative part. Majority students in experimental expressed that IMPROVE instructional method had many advantages for them. Understanding students' experience with IMPROVE instructional method in fact is not available in literature review. However, study about embedded metacognitive skills instructions such as inquiry-based mathematics teaching (Chin, Lin, Chuang, & Tuan, 2007) was conducted and it unfolded students experiences with it. In that study, students expressed that they feel better on comprehending the problem as the instruction was interesting and helpful to think more deeply. They also found more opportunities to investigate and analyse mathematical problem individually and collectively. In addition, they said that there were more chances to try various approaches to a problem and discuss it with other students which could inspire them with brilliant and novel ideas. They also suggested that presented mathematical problems should not in large number and more time should be allocated. By considering the findings of that study, generally speaking it supported the interview conducted in this study.

5.2. Conclusions

The following are conclusions of this study.

- IMPROVE instructional method was more effective than traditional instruction in increasing students' procedural and conceptual knowledge and regulation of cognition.
- There was a gender related difference in students' procedural and conceptual knowledge and regulation of cognition in favor of female students.
- IMPROVE instructional method was not different from traditional instruction in increasing students' knowledge of cognition and there was no gender difference in students' knowledge of cognition.
- Students who were exposed to IMPROVE instructional method are more active and engage in learning process than students in control group cognitively and meta-cognitively. Questioning, discussion, presentation, and giving-and-taking feedback are dominant in experimental group.

- IMPROVE instructional method provide a classroom environment that lead students to learn meaningfully by activating their prior knowledge and connecting it to the new knowledge.
- Students express their pleasure with metacognitive instructional method as the instruction provides supported environment for them to think deeply.
- The most salient challenges of IMPROVE instructional method are that students in experimental group have difficulty at the beginning of the instruction as they encounter distinct teaching method, and the instruction requires more time than traditional instruction.
- Majority students in experimental group could provide reasonable and comprehensive arguments with respect to procedural and conceptual knowledge items in post-test.
- In order to be able to solve mathematical problems and present reasonable arguments, students should recognize meaning of mathematical symbols and expressions, frequently work with mathematical formal language, solve numerous and various problems, comprehend certain concept and connect it to other related concepts, perform calculation and algebraic manipulation successfully, and focus on procedures from the beginning until the end.

5.3. Implications

The following suggestions can be made as a result of the current study.

- The new developed Indonesian national curriculum literally emphasizes that students should be able to improve their fact knowledge, procedural knowledge, conceptual knowledge, as well as metacognitive skills. Based on the result of this study, IMPROVE instructional method has been proved to be able to improve students' procedural knowledge, conceptual knowledge of composition and inverse function, infinite sequence and series, and line equations topics rather than traditional instruction. Therefore, broad implication of the result of this study could provide insights and contributions for high school mathematics teachers to improve their practice, as well as to inform decision maker and policy developer in mathematics education concerning the advantage of IMPROVE instructional method.

- The developed lesson plans and instruments in this study might be used as instance for high school mathematics teachers, textbook writers, curriculum developers and also other researchers for designing effective mathematics lesson in topics of composition and inverse function, infinite sequence and series, and line equations.
- IMPROVE instructional method also is found more effective than conventional classroom instruction on improving students' regulation of cognition as it could create a classroom environment in which students can be engaged meaningfully with the assistance of metacognitive questioning, cooperative setting, and systematic provision of feedback-corrective-enrichment.
- By considering its effectiveness, mathematics teacher could modify their instruction and apply IMPROVE instructional method. Training for in-service and pre-service teachers could be conducted so that they might gain detail information about this instructional method. Furthermore, guidance from mathematics education experts could be conducted when they are implementing this instruction within real classroom situation.
- IMPROVE instructional method also consider differences in students' ability as students who has achieved intended learning objective could enrich their knowledge. Meanwhile, students who didn't reach learning objectives are provided with remedial.

5.4. Recommendation for Further Research

The following suggestions can be made by taking into account the experiences of the researcher and results of the current study.

- The effect of IMPROVE instructional method on different topics or subject areas. It is interesting to determine whether the effect of the instruction on students' procedural knowledge and conceptual knowledge will differ for various mathematics topics.
- Both students and teachers encountered difficulties in implementing IMPROVE instructional method at the beginning since they were exposed to novel environment and they were not accustomed to being involved in this environment. Thus, it is interesting to learn and observe teacher and students' process of adaptation and internalization to this new instruction.

- The effect of IMPROVE instructional method integrated with technology and that of without technology for secondary school can be investigated.
- Increasing the sample size to recognize more fully the likelihood implications of IMPROVE instructional method on procedural and conceptual knowledge.
- This study was conducted within a 9-week period. Extending the period of the study to the entire school year to confirm whether IMPROVE instructional method has a significant impact on students' procedural and conceptual knowledge and metacognitive skills.
- IMPROVE instructional method can be selected as one of teaching method when implementing design based research to improve instructional practice in mathematics teaching and learning.
- A larger study including data from multiple high schools can be conducted to confirm whether IMPROVE instructional method influences significantly students' procedural knowledge, conceptual knowledge, and metacognitive skills more significantly from one high school to another high school, while controlling for distinct variables.
- Comparison between IMPROVE instructional method to other teaching method involving Indonesian student also can be conducted to determine whether there are other alternatives of social cognitive-based instructions that will generate outcome similar to IMPROVE instructional method.
- Investigation of relation between teacher's gender and students' metacognitive skills based on gender differences also can be conducted to understand whether teacher gender influences directly or indirectly students' metacognitive ability.
- In order to understanding the gender differences in procedural and conceptual knowledge, it is necessary to investigate students' attitudes and motivation and the relation among them.

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APPENDICES

APPENDIX A

TABLE OF SPECIFICATIONS OF PROC AND CONC

No	Topics	Sub-topics	Procedural Knowledge		Conceptual Knowledge				Total
			PROCA	PROCB	CONCA1	CONCA2	CONCB1	CONCB2	
1	Composite and Inverse Function	Algebraic operation on function	1			1	1		3
		Composition of function and its properties		1		1		1	3
		Inverse function	1	1	1	1		1	5
	Total topic 1		2	2	1	3	1	2	11
2	Infinite Sequences and Series	Concept of infinite sequence and series	1			2			3
		Infinite geometric sequence and series		2	2		2	1	7
	Total topic 2		1	2	2	2	2	1	10
3	Straight line equation	Slope of a line		1		1	1		3
		Parallel lines and Perpendicular lines			1		1		2
		Line equations	1			1	2	1	5
	Total topic 3		1	1	1	2	4	1	10
TOTAL 3 TOPICS			**4**	**5**	**4**	**7**	**7**	**4**	**31**

Note:

PROCA	: Knowledge of mathematical formal language or symbol representation system
PROCB	: Knowledge of collection of formulas and algorithms
CONCA	: Knowledge of general principles
CONCA1	: Explanation of concepts tasks
CONCA2	: Evaluation of examples tasks
CONCB	: Knowledge of the principles underlying procedures
CONCB1	: Application and justification of procedures tasks
CONCB2	: Evaluation of procedures tasks

APPENDIX B

PROCEDURAL AND CONCEPTUAL KNOWLEDGE TEST (PROC & CONC)
AND THE ANSWER KEY OF PROC & CONC

Question 1 (PROCA)
Objective: 1. Students are able to describe the concept of function and apply algebraic operation (addition, subtraction, multiplication, and division) on function.

According to formal mathematical language, which of the following statements is false with regard to algebraic operation on function for $f:\mathbb{R}\to\mathbb{R},\ g:\mathbb{R}\to\mathbb{R}$ *?*
Explain why?

a. $f(x)+g(x)=(f+g)(x)$

b. $f(x)-g(x)=(f-g)(x)$

c. $f(x)\cdot g(x)=f(g).(x)$

d. $f(x)\div g(x)=\left(\frac{f}{g}\right)(x),\ g(x)\neq 0$

e. $f(x)\mp g(x)=(f\mp g)(x)$

Answer : C
Reason :
It should be $f(x)\cdot g(x)=(f\cdot g)(x)$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reason does not make sense. 4. Score 3 : The students choose the correct option and the reason makes sense, but there is a little mistake in the procedure. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in writing correct mathematical symbol. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 2 (CONCB1)

Objective: 1. Students are able to describe the concept of function and apply algebraic operation (addition, subtraction, multiplication, and division) on function.

Let $f(x) = 2x$*, and* $g(x) = \sqrt{x-4}$ *. Determine a)* $(f \times g)(x)$ *and b)* $(f \div g)(x)$ *and its domain.*

<table>
<tr><td>Answer for part a and give your reason</td></tr>
<tr><td>

$f(x) = 2x$ and $g(x) = \sqrt{x-4}$

$f(x) \div g(x) = \left(\frac{f}{g}\right)(x)$

$\frac{2x}{\sqrt{x-4}}$

We cannot divide by zero, therefore $\sqrt{x-4} \neq 0$ and on the other hand the inequality $x-4>0$ should be hold.

$x-4>0$, then $x>4$. Thus, the domain is $D_{f \div g} = \{x \mid x>4, x \in \mathbb{R}\}$

</td></tr>
<tr><td>Answer for part b and give your reason</td></tr>
<tr><td>

$f(x) \cdot g(x) = (f \cdot g)(x)$

$2x\sqrt{x-4} = 2x\sqrt{x-4}$

Since the expression inside square root should be equal to or greater than zero. Thus, the domain is $D_{f \cdot g} = \{x \mid x \geq 4, x \in \mathbb{R}\}$

</td></tr>
<tr><td>Rubric :</td></tr>
<tr><td>

1. Score 0 : The student does not answer the question.
2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures.
3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures.
4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely.
5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language.

</td></tr>
</table>

6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 3 (CONCA2)
Objective: 1. Students are able to describe the concept of function and apply algebraic operation (addition, subtraction, multiplication, and division) on function.

Which of the following statements is true with regard to algebraic operation on functions? (Give your reasons)

a. *If* $(f+g)(a)=0$, *then* $f(a)$ *and* $g(a)$ *must be equal.*
b. *If* $(f-g)(a)=0$, *then* $f(a)$ *and* $g(a)$ *must be opposites or additive inverses.*
c. *If* $(f\cdot g)(a)=0$, *then only* $g(a)$ *must be zero.*
d. *If* $\left(\frac{f}{g}\right)(a)=0$, *then* $f(a)$ *must be zero.*
e. *The subtraction of two functions is a commutative operation.*

Answer : D
Reason :
The value $\left(\frac{f}{g}\right)(a)$ will be zero, if $f(a)=0$. If zero is divided by any number or expression, the result will be zero.
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 4 (PROCA)
Objective: 2. Students are able to analyze the concepts and properties of function and perform algebraic manipulation in determining inverse function and inverse of a function.

According to formal mathematical language, which of the following statements is false with regard to inverse function for f is one-to-one function and $f:\mathbb{R}\to\mathbb{R},\ g:\mathbb{R}\to\mathbb{R}$ *? (Explain why)*

a. $\left(f^{-1}\circ f\right)(x)=\left(f\circ f^{-1}\right)(x)$

b. $\left(f^{-1}\right)^{-1}(x)=f(x)$

c. $(f\circ g)^{-1}(x)=g^{-1}\circ f^{-1}(x)$

d. $\left(f^{-1}\circ f\right)(x)=x=I(x)$

e. $\left(f^{-1}\circ g^{-1}\right)(x)=f^{-1}(x)\cdot g^{-1}(x)$

Answer : E
Reason :
It should be $\left(f^{-1}\circ g^{-1}\right)(x)=f^{-1}\left(g^{-1}\right)(x)=(g\circ f)^{-1}(x)$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reason does not make sense. 4. Score 3 : The students choose the correct option and the reason makes sense, but there is a little mistake in the procedure. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in writing correct mathematical symbol. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 5 (CONCA1)

Objective: 2. Students are able to analyze the concepts and properties of function and perform algebraic manipulation in determining inverse function and inverse of a function.

Given the sets $A = \{a, b, c, d\}$ *and* $B = \{1, 2, 3\}$*. Is it possible to define a function f from A to B that has an inverse of function? (Explain why)*

Answer : It is impossible
Reason :
It is impossible that f has an inverse function, since the member of set A and set B is different and then one-to-one correspondence cannot exist. Eventually, this condition does not lead to obtain inverse function.
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student is misunderstood the question or the student's solution is not fully related to the question. 3. Score 2 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts. 4. Score 3 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts. 5. Score 4 : The student understands the complete questions, but does not translate all the questions into inappropriate mathematical concepts. 6. Score 5 : The student understands the complete questions, translates all the questions into appropriate mathematical concept and the student's answer is consistent with the question
The question is taken from : Bayazit, I., & Gray, E. (2004). Understanding inverse functions: the relationship between teaching practice and student learning. In Proceedings of the 28th Conference of the International (Vol. 2, pp. 103-110).

Question 6 (CONCA2)

Objective: 2. Students are able to analyze the concepts and properties of function and perform algebraic manipulation in determining inverse function and inverse of a function.

Which of the following statements is correct in general with regard to the concept of inverse of a function? Give your reason?

a. *If f is bijective function then the domain of a function is the same as the range of its inverse; and the range of a function is the same as the domain of its inverse.*

b. *Given* $f=\{(-1,1),(0,0),(1,1),(2,4),(3.9)\}$ *and f is invertible.*

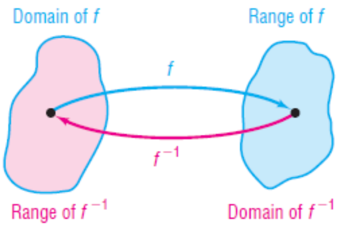
c. *In inverse function,* f^{-1} *means* $\frac{1}{f}$.

d. *Inverse of* $\{(1,3),(2,5)\}$ *is* $\{(-1,-3),(-2,-5)\}$

e. *Not all bijective functions are invertible.*

Answer : A

Reason :



For instance, $f=\{(a,c),(b,d)\}$ and $f^{-1}=\{(c,a),(d,b)\}$

$D_f = R_{f^{-1}} = \{a,b\}$ and $R_f = D_{f^{-1}} = \{c,d\}$

Rubric :

1. Score 0 : The students choose the incorrect option.
2. Score 1 : The students choose the correct option, but without explanation.
3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements.
4. Score 3 : The students choose the correct option and some of the reasons do not make sense.
5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology.
6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.

The question is developed by researcher

Question 7 (PROC2)

Objective: 3. Students are able to describe and analyze properties of a function as a result of operation of two or more other function

Given that $h(x)=(x+1)^5$, $f(x)=x^5$ *and* $g(x)=x+1$. *Which of the following statements is true based on the above condition?*

a. $(g\circ g)(x)=h(x)$
b. $(g\circ f)(x)=h(x)$
c. $(f\circ f)(x)=h(x)$
d. $(f\circ g)(x)=h(x)$
e. $(g\circ f\circ g)(x)=h(x)$

Answer : D
Reason :
$(f\circ g)(x)=f(g(x))=f(x+1)=(x+1)^5$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reason does not make sense. 4. Score 3 : The students choose the correct option and the reason makes sense, but there is a little mistake in the procedure. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in writing correct mathematical symbol. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes
The question is developed by researcher

Question 8 (CONCB2)
Objective: 2. Students are able to analyze the concepts and properties of function and perform algebraic manipulation in determining inverse of a function.

Given that $f(x)=3x+1$ *and* $g(x)=2x+2$. *Based on the properties* $(f\circ g)^{-1}=g^{-1}\circ f^{-1}$, *evaluate the following the procedure why does it make differ?*

<table>
<tr><th>A. $(f\circ g)^{-1}(x)$</th><th>B. $g^{-1}\circ f^{-1}(x)$</th></tr>
<tr><td>
1. Finding the composition of the two functions
a. $y=(f\circ g)(x)=f(g(x)), x\in\mathbb{R}$
b. $f(g(x))=3(2x+2)+1$
c. $f(g(x))=6x+7$
2. Finding the inverse of the composition of the function
a. $y=f(g(x))=6x+7$
b. $x=\frac{y-6}{7}$, $(f\circ g)^{-1}(x)=\frac{x-6}{7}$
</td><td>
1. Finding f^{-1}
a. $y=f(x)=3x+1$
b. $x=\frac{y+1}{3}$, $f^{-1}(x)=\frac{x+1}{3}$
2. Finding g^{-1}
1. $y=g(x)=2x+2$
2. $x=\frac{y-2}{2}$, $g^{-1}(x)=\frac{x-2}{2}$
3. Compose the two functions.
$(g^{-1}\circ f^{-1})(x)=g^{-1}(f^{-1}(x))$
$g^{-1}(f^{-1}(x))=\frac{\frac{x+1}{3}-2}{2}=\frac{x-5}{6}$
</td></tr>
</table>

The correct procedure is : None of them are true
Reason :
Procedure A The mistake is found in 2.b. It should be $(f\circ g)^{-1}(x)=\frac{x-7}{6}$, $x\in\mathbb{R}$ Procedure B The mistake is found in 1.b. To find the inverse, it should be $x=\frac{y-1}{3}$, then $f^{-1}(x)=\frac{x-1}{3}$, $x\in\mathbb{R}$ $(g^{-1}\circ f^{-1})(x)=g^{-1}(f^{-1}(x))=\frac{\frac{x-1}{3}-2}{2}=\frac{x-7}{6}$

Rubric :
1. Score 0 : The student does not answer the question or claims that one of the two procedures is correct. 2. Score 1 : The student claims that the two procedures are incorrect but he/she doesn't provide explanation. 3. Score 2 : The student claims that the two procedures are incorrect but he/she doesn't provides plausible explanation. 4. Score 3 : The student claims that the two procedures are incorrect and provides plausible explanation, but he/she doesn't make correction. 5. Score 4 : The student claims that the two procedures are incorrect, provides plausible explanation, and makes correction, but the correction is wrong. 6. Score 5 : The student claims that the two procedures are incorrect, provides plausible explanation, makes correction, and the correction is true.
The question is developed by researcher

Question 9 (PROCB)
Objective: 3. Students are able to describe and analyze properties of a function as a result of operation of two or more other function.

Given $f(x)=3x+1$ *and* $g(x)=2x-m$ *such that* $(f\circ g)(x)=(g\circ f)(x)$, *find* $g(10)$.

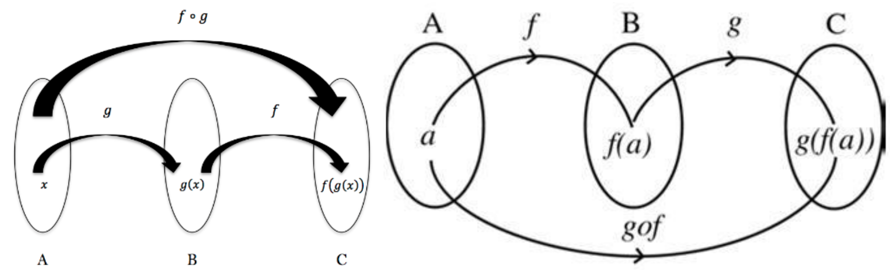
Answer:
$(f\circ g)(x)=f(g(x))=3\cdot g(x)+1=3(2x-m)+1=6x-3m+1,\ x\in\mathbb{R}$ $(g\circ f)(x)=g(f(x))=2\cdot g(x)-m=2(3x+1)-m=6x+2-m,\ x\in\mathbb{R}$ Since $(f\circ g)(x)=(g\circ f)(x)$, $6x-3m+1=6x+2-m$ which gives $m=-\frac{1}{2}$ So we have $g(x)=2x-\left(-\frac{1}{2}\right)=2x+\frac{1}{2},\ x\in\mathbb{R}$. Therefore, $g(10)=2(10)+\frac{1}{2}=20,5$
Rubric
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely.

5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 10 (CONCA2)
Objective: 4. Students are able to describe concepts of composition of function

Which one of the following statements is true concerning $f \circ g$ *?*

a. *If* $f \neq g^{-1}$ *then* $f \circ g \neq g \circ f$.
b. *The* $f \circ g$ *can be defined if there is intersection between range of f and domain of g.*
c. *The function f is applied first and then the function g.*
d. *If* $f(7) = 5$ *and* $g(4) = 7$*, then* $(f \circ g)(4) = 35$
e. *Sign (*$\circ$*) on composition means multiplication*

Answer : A
Reason :
 $f \circ g$ *means that the function g is applied first and then the function f. Meanwhile* $g \circ f$ *means that the function f is applied first and then the function g.*
Rubric
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology.

6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 11 (CONCB2)
Objective: 4. Students are able to describe concepts of composition of function
Given $f(x) = x^2 + 7$ *and* $g(x) = \sqrt{x-3}$. *Find* $f \circ g$ *, and state the domain.*
Solution:

1. $f \circ g = f(g(x))$
2. $f(g(x)) = \sqrt{(x^2+7)-3}$
3. $f(g(x)) = \sqrt{x^2+4}$
4. *Domain* $f \circ g$ = $\{x \mid x \geq 3, x \in \mathbb{R}\}$

This is incorrect. What mistake was made.

Answer:
The mistake is in the second step. It should be $f(g(x)) = (\sqrt{x-3})^2 + 7$ $f(g(x)) = x - 3 + 7$ $f(g(x)) = x + 4$
Rubric :
1. Score 0 : The student does not answer the question or he/she finds the incorrect mistake. 2. Score 1 : The student finds the mistake, but he/she doesn't provide explanation. 3. Score 2 : The student finds the mistake and provides incomplete explanation. 4. Score 3 : The student finds the mistake and provides complete explanation but there is unreasonable statement. 5. Score 4 : The student finds the mistake and provides reasonable explanation but he/she does not make correction. 6. Score 5 : The student finds the mistake, provides reasonable explanation and makes correction.
The question is taken from: Young, Cynthia Y.(2014). Precalculus. Florida, USA: John Wiley & Sons, Inc.

Question 12 (PROCA)
Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

According to formal mathematical language, which of the following statements is false with regard to formulas used in the topic of sequences and series?

a. $S_n = a_1\left(\dfrac{1-r^n}{1-r}\right), r \neq 1$

b. $\{u_n\}_{n=1}^{\infty} = u_1, u_2, u_3, \ldots, u_n$

c. *Formula for infinite geometric series* $S = \dfrac{a}{1-r}, -1 < r < 1$

d. $S_n = u_1, u_2, ..., u_n$

e. $a_n = a_1 \cdot r^{n-1}, n \in \mathbb{N}$

Answer : D
Reason :
It should be $S_n = u_1 + u_2 + ... + u_n$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reason does not make sense. 4. Score 3 : The students choose the correct option and the reason makes sense, but there is a little mistake in the procedure. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in writing correct mathematical symbol. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 13 (CONCA1)
Objective: 2. Students are able to describe the concept of infinite sequence as a function whose domain is natural numbers.

Could all infinite geometric series use the formula $S = \frac{a}{1-r}$ *?*

Answer :
No. The formula could be use if the common ratio of an infinite geometric sequence fall between -1 and 1. The value would converge to a real number. If the common ratio is greater than 1 or less than -1, then the infinite geometric series never converge (or approach) to a real number.
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student is misunderstood the question or the student's solution is not fully related to the question. 3. Score 2 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts. 4. Score 3 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts. 5. Score 4 : The student understands the complete questions, but does not translate all the questions into inappropriate mathematical concepts. 6. Score 5 : The student understands the complete questions, translates all the questions into appropriate mathematical concept and the student's answer is consistent with the question
The question is developed by researcher

Question 14 (PROCB)
Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

Find the sum of an infinite geometric sequence of $18, 6, 2, \frac{2}{3}, ...$

Answer :
1. The ratio of the sequence is $r = \frac{u_2}{u_1} = \frac{6}{18} = \frac{1}{3}$ 2. The ratio falls between -1 and 1, then it is categorized as infinite geometric sequence.

3. By means of $S = \frac{a}{1-r}$, then $S = \frac{18}{1-\frac{1}{3}}$ 4. $S = \frac{18}{\frac{2}{3}} = 18 \cdot \frac{3}{2} = 27$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 15 (PROCB)

Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

Convert the repeating decimal of $0.\overline{36}$ *into simple rational number.*

Answer :
$0.\overline{36} = 0.36363636...$ $0.36363636... = 0.36 + 0.0036 + 0.000036 + ...$ $a = 0.36$ $r = \frac{u_2}{u_1} = \frac{0.0036}{0.36} = 0.01$ $S = \frac{a}{1-r} = \frac{0.36}{1-0.01} = \frac{36}{99} = \frac{4}{11}$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely.

5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language.
6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.

The question is developed by researcher

Question 16 (CONCA1)
Objective: 2. Students are able to describe the concept of infinite sequence as a function whose domain is natural numbers.

Recall the following formula we derived for the sum of the first n-terms of any geometric sequence, $S_n = a_1\left(\frac{1-r^n}{1-r}\right), r \neq 1$. *If the value of r is* $-1 < r < 1$,

a. *What happens to* r^n *as n gets larger and larger?*
b. *So then, what happens to 1-* r^n *as n approaches infinity?*
c. *What happens to the above formula?*

Answer :

a. If r^n as *n* gets larger and larger, the value of r^n close to zero.
b. If *n* approaches infinity, then the value of 1- r^n approach to 1.
c. The formula $S_n = a_1\left(\frac{1-r^n}{1-r}\right), r \neq 1$ will be $S = \frac{a_1}{1-r}$

Rubric :

1. Score 0 : The student does not answer the question.
2. Score 1 : The student is misunderstood the question or the student's solution is not fully related to the question.
3. Score 2 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts.
4. Score 3 : The student understands one portion of the question and translates the question into inappropriate mathematical concepts.
5. Score 4 : The student understands the complete questions, but does not translate all the questions into inappropriate mathematical concepts.
6. Score 5 : The student understands the complete questions, translates all the questions into appropriate mathematical concept and the student's answer is consistent with the question.

The question is developed by researcher

Question 17 (CONCB1)
Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

Given the following infinite geometric series. For what values of x does the following infinite *series* converge?

$2+2(3x)+2(3x)^2+2(3x)^3+...$

Answer :
In order the sum exist, the common ration of the series must be $(-1,1)$. $r=\frac{2(3x)}{2}=3x$ $-1<3x<1$ $-\frac{1}{3}<x<\frac{1}{3}$ Then, the value of x must be an element of the open interval $\left(-\frac{1}{3},\frac{1}{3}\right)$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 18 (CONCA2)

Objective: 2. Students are able to describe the concept of infinite sequence as a function whose domain is natural numbers.

Which of the following statements is true with regard to the concept of sequences?

a. *Infinite sequences is sequences that have unlimited terms.*

b. *Divergent infinite sequences is sequence that has a limit L for n approaches infinite value.*

c. *All arithmetics sequences are convergent sequences.*

d. *Convergent geometric sequences are sequences whose ratios are greater than 1 and less than -1.*

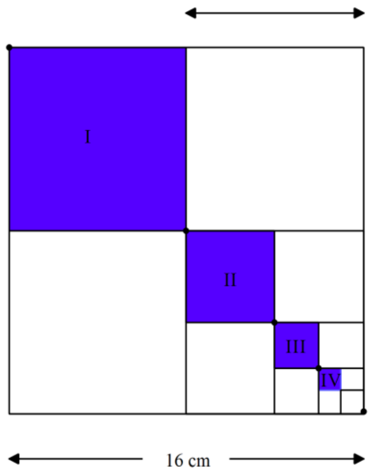
e. $\left\{\frac{n^3}{n^2+n}\right\}_{n=1}^{\infty}$ *is divergent infinite sequence.*

Answer : D
Reason :
As $\lim_{n\to\infty}\frac{n^3}{n^2+1}=\infty$, $\left\{\frac{n^3}{n^2+n}\right\}_{n=1}^{\infty}$ is divergent infinite sequence.
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 19 (CONCB1)

Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

Pay attention to the picture below. A big square with a side length of 16 cm. What is the total square area of the shaded if the shading process is carried out continuously without stopping?



Answer:
The area of the first shaded square is 64 m^2 The area of the second shaded square is 16 m^2 The area of the third shaded square is 4 m^2 The areas of all shaded square form a sequence 16, 8, 4, … So as to find the sum of the areas of all square we are going to use Sum of an Infinite Geometric Series formula, that is, $S = \frac{a}{1-r}$. The first term is 64 and the common ratio is $\frac{1}{4}$. So, $S_\infty = \frac{64}{1-\frac{1}{4}} = \frac{64}{\frac{3}{4}} = 64 \cdot \frac{4}{3}$ So, the sum of the areas of all square is $\frac{256}{3}$ m^2
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely.

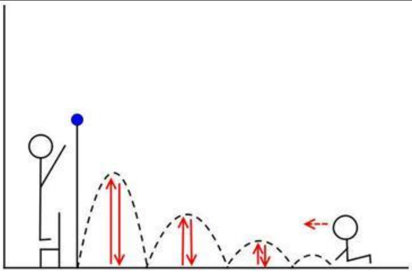
5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is adapted from Aufmann & Nation (2005). *Essentials of Pre-calculus*. Boston New York: Houghton Mifflin Company. (page 524)

Question 20 (CONCB2)

Objective: 1. Students are able to apply the concept of infinite sequence and series in solving problem.

A ball is dropped from a height of 15 m and bounces to 60% of the previous height. Find the total distance traveled by the ball.

Solution

<table>
<tr><th>Procedure A</th><th>Procedure B</th></tr>
<tr><td>1. Finding the ratio
$r = 60\% = \frac{60}{100} = \frac{3}{5}$
2. Applying the formula infinite geometric series.
$S = \frac{a}{1-r}$
$S = \frac{15}{1-\frac{3}{5}}$
$S = \frac{15}{1} \bullet \frac{5}{2} = 37.5$</td><td>1. Finding the ratio
$r = 60\% = \frac{60}{100} = \frac{3}{5}$
2. Applying the formula infinite geometric series.
$S = a_1 + 2 \bullet \frac{a_1}{1-r}$
$S = 15 + 2 \bullet \frac{15}{1-\frac{3}{5}}$
$S = 15 + 75 = 90$</td></tr>
<tr><td colspan="2">The correct procedure is : None of them are correct</td></tr>
<tr><td colspan="2">Reason :</td></tr>
<tr><td></td><td>The formula should be $S_n = a_1 + 2 \bullet \frac{a_2}{1-r}$ (based on the figure)
Therefore, $S = 15 + 2 \bullet \frac{9}{1-\frac{3}{5}}$, and
$S = 15 + 45 = 60$</td></tr>
</table>

Rubric :
1. Score 0 : The student does not answer the question or claims that the two procedures are correct or incorrect or claims that one of the two procedures is correct. 2. Score 1 : The student claims that procedure B is correct but he/she doesn't provide explanation. 3. Score 2 : The student claims that procedure B is correct but he/she doesn't provides plausible explanation. 4. Score 3 : The student claims that procedure B is correct and provides plausible explanation, but he/she doesn't make correction. 5. Score 4 : The student claims that procedure B is correct, provides plausible explanation, and makes correction, but the correction is wrong. 6. Score 5 : The student claims that procedure B is correct, provides plausible explanation, makes correction, and the correction is true.
The question is adapted from http://www.bmlc.ca/Math12/principles%20of%20Math%2012%20%20-%20Geometric%20Series%20Lesson%202.pdf

Question 21 (CONCA2)

Objective: 2. Students are able to describe the concept of infinite sequence as a function whose domain is natural numbers.

Which of the following statements is true with regard to infinite series?

a. *The sum of the infinite series* $U_n = \frac{1}{2}(4)^{n-1}$ *exists.*
b. *A geometric series with common ratio 1 has a sum.*
c. *A convergent infinite series is an infinite series that does not have a finite sum.*
d. *A divergent infinite series is an infinite series with a finite sum.*
e. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$ *is a convergent infinite series.*

Answer : E
Reason :
a. We cannot find the sum of the series $U_n = \frac{1}{2}(4)^{n-1}$ since the ratio falls beyond (-1,1) b. A geometric series with common ratio 1 has not sum since the ratio falls beyond (-1,1) c. A convergent infinite series is an infinite series with a finite sum. d. A divergent infinite series is an infinite series does not have a finite sum.

e. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...$ is a convergent infinite series, since the ratio is between (-1,1).	
Rubric :	
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.	
The question is developed by researcher	

Question 22 (PROCA)

Objective: 1. Students are able to analyze properties of parallel and perpendicular line and apply it in solving problem

According to formal mathematical language, which of the following statements is false with regard to straight line equation?

a. $l_1 \| l_2 \Leftrightarrow m_1 = m_2$

b. $l_1 \perp l_2 \Leftrightarrow m_1 = -\frac{1}{m_2}$

c. $A(x_1, y_1), B(x_2, y_2)$, $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

d. $ax + by + c = 0 \Rightarrow m = -\frac{a}{b}x$

e. $l_1 \perp l_2 \Leftrightarrow m_1 \cdot m_2 = -1$

Answer : D
Reason :
It should be $ax + by + c = 0 \Rightarrow m = -\frac{a}{b}$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reason does not make sense.

4. Score 3 : The students choose the correct option and the reason makes sense, but there is a little mistake in the procedure. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in writing correct mathematical symbol. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 23 (PROCB)

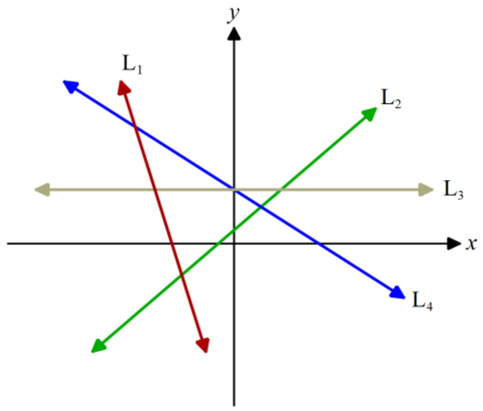
Objective: 1. Students are able to analyze properties of parallel and perpendicular line and apply it in solving problem

The inclination of the line which passes through the points $A(2,-1)$ *and* $B(k,2)$ *is* $45°$. *Determine the value of k.*

Answer:
$m = \tan 45° \Rightarrow m = 1$ $m = \dfrac{2-(-1)}{k-2}$ $1 = \dfrac{3}{k-2}$ $k-2 = 3 \Rightarrow k = 5$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 24 (CONCB1)

Let m_1, m_2, m_3 *and* m_4 *be the slopes of lines* L_1, L_2, L_3 *and* L_4 *respectively. Which of the following statements is true?*



a. $m_4 < m_1 < m_3 < m_2$

b. $m_3 < m_2 < m_4 < m_1$

c. $m_3 < m_4 < m_2 < m_1$

d. $m_1 < m_3 < m_4 < m_2$

e. $m_1 < m_4 < m_3 < m_2$

Answer : E
Reason :
a. $m_1 < 0$, since the graph is decreasing from left to the right. b. $m_2 > 0$, since the graph is increasing from left to the right. c. $m_3 = 0$, since the graph is horizontal. d. $m_4 < 0$, since the graph is decreasing from left to the right. e. Since the angle formed betweend L_4 and positive x-axis is greater than the angle formed between L_1 and positive x-axis, therefore $m_4 > m_1$
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is adapted from Blitzer.(2013).*Intermediate Algebra for College Students*. Upper Saddle River, NJ.:Pearson Education, Inc.

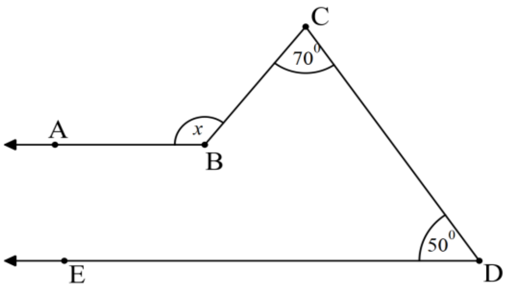
Question 25 (CONCB1)
Objective: 2. Students are able to analyze curves through several points to conclude a straight line, parallel line, or perpendicular line

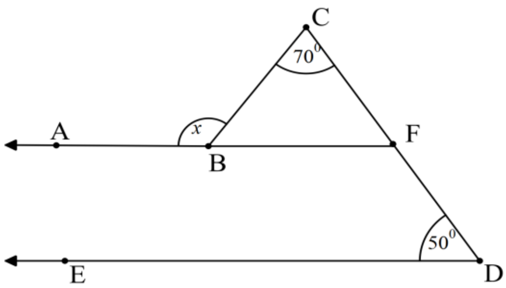
Find the equation of the line which is parallel to the line joining the points $K(-1,-4)$ *and* $L(-5,-10)$, *and which intercepts the x-axis at 5.*

Answer:
The slope of a line which passes through points K and L is $m_{KL} = \frac{-10-(-4)}{-5-(-1)} = \frac{-6}{-4} = \frac{3}{2}$. Since the intended line is parallel to the available line, therefore $m = m_{KL}$. We are going to use the formula $y-b=m(x-a)$ $y-b=\frac{3}{2}(x-a)$ The line intercepts the x-axis at 5 means that the line passes through (5,0) Therefore, $y-0=\frac{3}{2}(x-5)$ $2y=3(x-5)$ $3x-2y-15=0$ or $-3x+2y+15=0$ or $3x-2y=15$ or $-3x+2y=-15$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 26 (CONCB1)
Objective: 2. Students are able to analyze curves through several points to conclude a straight line, parallel line, or perpendicular line

Based on the figure beside, given $BA \parallel DE$, $\angle CDE = 50°$, and $\angle DCB = 70°$. Determine $\angle CBA$



Answer:
$\angle CDE = 50°$, $\angle DCB = 70°$ $\angle BFC = \angle CDE = 50°$ $\angle BCF + \angle FBC + \angle BFC = 180°$ $70° + \angle FBC + 50° = 180°$ $\angle FBC = 60°$ $\angle ABC + \angle FBC = 180°$ $x = 180° - \angle FBC$ $x = 120°$ 
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

Question 27 (CONCA2)

Objective: 1. Students are able to analyze properties of parallel and perpendicular line and apply it in solving problem

Which of the following statements is false?

a. *Lines with positive gradients slope up, from left to right, whereas line with negative gradients slope down, from left to right.*

b. *If a point P is not on a line L, there is only one possibility line which passes through P and parallel to L.*

c. *The slope of vertical lines is 0 (zero), while horizontal lines do not have slope (the slope is undefined).*

d. *Lines that have a slope equal to 1 form an angle* 45° *to the x-axis.*

e. *The slope of a line is the tangent of angle formed between the line with the x-axis.*

Answer : C
Reason :
The slope of vertical lines is undefined, while slope of horizontal lines is zero. Vertical lines mean that $x_1 = x_2$, and $m = \frac{y_2 - y_1}{x_2 - x_1}$, so m is undefined Horizontal lines mean that $y_1 = y_2$, and $m = \frac{y_2 - y_1}{x_2 - x_1}$, so m equals 0 (zero)
Rubric :
1. Score 0 : The students choose the incorrect option. 2. Score 1 : The students choose the correct option, but without explanation. 3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 4. Score 3 : The students choose the correct option and some of the reasons do not make sense. 5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 28 (CONCA1)
Objective: 1. Students are able to analyze properties of parallel and perpendicular line and apply it in solving problem

Based on the figure below, which the following statements is false with respect to the angle formed between two lines?

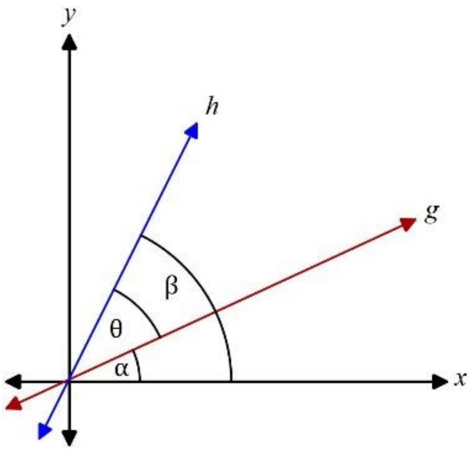
a. $\tan\theta = \tan(\beta - \alpha)$

b. $m_h = \tan\beta$ dan $m_g = \tan\alpha$

c. $\tan\theta = \dfrac{\tan\beta - \tan\alpha}{1 + \tan\beta \cdot \tan\alpha} = \dfrac{m_h - m_g}{1 + m_h \cdot m_g}$

d. *If the angle formed between two angles is* $90°$ *or* $\theta = 90°$, *then* $1 - m_h \cdot m_g = 0$

e. *If the angle formed between two angles is* $0°$ *or* $\theta = 0°$, *then* $\tan\alpha = \tan\beta \Rightarrow m_g = m_h$



Answer : D
Reason :
If the angle formed between two angles is $90°$ *or* $\theta = 90°$, *then* $m_h \cdot m_g = -1$ *or* $m_h \cdot m_g + 1 = 0$
Rubric :
7. Score 0 : The students choose the incorrect option. 8. Score 1 : The students choose the correct option, but without explanation. 9. Score 2 : The students choose the correct option, but the reasons do not relate to the statements. 10. Score 3 : The students choose the correct option and some of the reasons do not make sense. 11. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology. 12. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 29 (CONCA2)

Objective: 2. Students are able to analyze curves through several points to conclude a straight line, parallel line, or perpendicular line.

Which of the following statement is false with regard to $y = mx + c$ *?*

a. $m = \tan\theta$, *where* θ *is the angle between the line and the positive direction of the x-axis.*

b. *When* $c = 0$ *and* $m = 0$ *then the line coincides with x-axis and its equation is of the form* $y = 0$.

c. *When* $c \neq 0$ *and* $m = 0$ *then the line is parallel to x-axis and its equation is of the form* $y = c$.

d. *When* $c = 0$ *and* $m \neq 0$ *then the line passes through the origin and its equation is of the form* $y = mx$.

e. *The line intersect x-axis, then its equation is of the form* $y = c$.

Answer : E
Reason :
It should be "The line intersect x-axis then its equation is of the form $x = -\frac{c}{m}$."   Or "The line intersect y-axis, then its equation is of the form $y = c$."
Rubric
1. Score 0 : The students choose the incorrect option.   2. Score 1 : The students choose the correct option, but without explanation.   3. Score 2 : The students choose the correct option, but the reasons do not relate to the statements.   4. Score 3 : The students choose the correct option and some of the reasons do not make sense.   5. Score 4 : The students choose the correct option and the reason makes sense, but there is a mistake in using appropriate terminology.   6. Score 5 : The students choose the correct option, the reason makes sense, and no mistakes.
The question is developed by researcher

Question 30 (CONCB2)

Objective: 2. Students are able to analyze curves through several points to conclude a straight line, parallel line, or perpendicular line

The lines $ax+2y-4=0$ *and* $2x+by+6=0$ *are perpendicular and intersect each other on the y-axis. Determine the value of a and b.*

1. *Let* $l_1 \equiv ax+2y-4=0$ *and* $l_2 \equiv 2x+by+6=0$
2. $l_1 \perp l_1 \Rightarrow m_1 \bullet m_2 = -1$
3. $\left(-\frac{a}{2}\right)\bullet\left(-\frac{2}{b}\right)=-1$, *so* $\frac{a}{b}=-1$, *so* $a=-b$
4. *Let* $P(k,0)$ *be the intersection point of the lines*

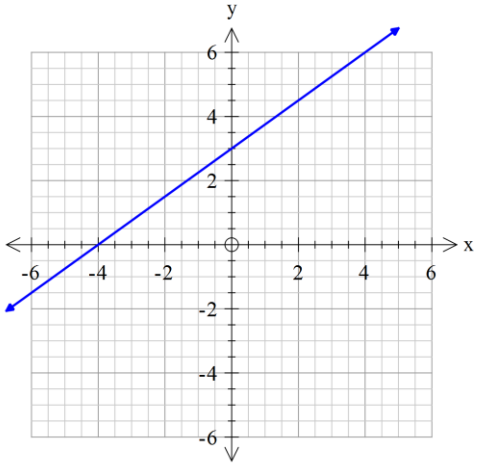
   $P(k,0)$ *satisfies both equations, so*
5. $a\cdot k+2\cdot 0-4=0$ *gives* $k=\frac{4}{a}$ *and* $2\cdot k+b\cdot 0+6=0$ *gives* $k=-3$
6. $-3=\frac{4}{a}$, *so* $a=-\frac{4}{3}$
7. $\therefore\left(-\frac{4}{3},\frac{4}{3}\right)$

Is the procedure correct to determine the value of a and b? The procedure is incorrect
Reason :
The procedure is incorrect when providing the intersection point of the lines. From the 4th step, there is a mistake. The following procedure is correct. Let $P(0,k)$ be the intersection point of the lines. $P(0,k)$ satisfies both equations, so $a\cdot 0+2\cdot k-4=0$ gives $k=2$ and $2\cdot 0+b\cdot k+6=0$ gives $k=-\frac{6}{b}$ $2=-\frac{6}{b}$, so $b=-3$ $\therefore(3,-3)$
Rubric :
1. Score 0 : The student does not answer the question or he/she claims that the procedure is correct. 2. Score 1 : The student finds the mistake, but he/she doesn't provide explanation. 3. Score 2 : The student finds the mistake and provides incomplete explanation.

4. Score 3 : The student finds the mistake and provides complete explanation but there is unreasonable statement. 5. Score 4 : The student finds the mistake and provides reasonable explanation but he/she does not make correction. 6. Score 5 : The student finds the mistake, provides reasonable explanation and makes correction.
The question is developed by researcher

Question 31 (CONCB1)
Objective: 2. Students are able to analyze curves through several points to conclude a straight line, parallel line, or perpendicular line

The graph of a line is shown at the below



Show your work how to determine the equation of a line that is perpendicular to the line shown and goes through the point (3, -1)

Answer:
A line is passing through (0,3) and (4,0), thus the equation of the line is $l_1 \equiv \frac{x-0}{0-4} = \frac{y-3}{0-3}$ $l_1 \equiv 4y - 3x = 12$ The other line, let l_2, is perpendicular to l_1. Thus $m_1 = -\frac{1}{m_2} \Rightarrow m_2 = -\frac{1}{m_1}$, $m_2 = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$ $l_2 \equiv y - b = m_2(x-a)$ $l_2 \equiv y - (-1) = -\frac{4}{3}(x-3)$

$y+1=-\frac{4}{3}(x-3)\Rightarrow 3y+4x-9=0$
Rubric :
1. Score 0 : The student does not answer the question. 2. Score 1 : The student misuses principles or translates the problem into inappropriate procedures. 3. Score 2 : The student uses principles but unable to translate the problem into appropriate procedures. 4. Score 3 : The student uses principles and translate the problem into appropriate procedures, but the student is unable to carry out a procedure completely. 5. Score 4 : The student uses principles, translate the problem into appropriate procedures, carry out a procedure completely, but does not use appropriate mathematical language. 6. Score 5 : The student uses appropriate mathematical terms and strategies. The student uses mathematical principles and language precisely. The student solves and verifies the problem.
The question is developed by researcher

APPENDIX C

METACOGNITIVE AWARENESS INVENTORY ENGLISH VERSION

Gender : ☐ Male ☐ Female
Age :
Grade :
Direction :

1. Please response each statements in this questionnaire as accurately as possible.
2. Put the sign (√) or (X) under the word that most closely describes your overall opinion of each item.

No	Category	Statements	Always False	False	Neutral	True	Always True
1	MO	I ask myself periodically if I am meeting my goals.					
2	MO	I consider several alternatives to a problem before I answer.					
3	PK	I try to use strategies that have worked in the past.					
4	PL	I pace myself while learning in order to have enough time.					
5	DK	I understand my intellectual strengths and weaknesses.					
6	PL	I think about what I really need to learn before I begin a task					
7	EV	I know how well I did once I finish a test.					
8	PL	I set specific goals before I begin a task.					

9	STR	I slow down when I encounter important information.					
10	DK	I know what kind of information is most important to learn.					
11	MO	I ask myself if I have considered all options when solving a problem.					
12	DK	I am good at organizing information.					
13	STR	I consciously focus my attention on important information.					
14	PK	I have a specific purpose for each strategy I use.					
15	CK	I learn best when I know something about the topic.					
16	DK	I know what the teacher expects me to learn.					
17	DK	I am good at remembering information.					
18	CK	I use different learning strategies depending on the situation.					
19	EV	I ask myself if there was an easier way to do things after I finish a task.					
20	DK	I have control over how well I learn.					

21	MO	I periodically review to help me understand important relationships.					
22	PL	I ask myself questions about the material before I begin.					
23	PL	I think of several ways to solve a problem and choose the best one.					
24	EV	I summarize what I've learned after I finish.					
25	DB	I ask others for help when I don't understand something.					
26	CK	I can motivate myself to learn when I need to					
27	PK	I am aware of what strategies I use when I study.					
28	MO	I find myself analysing the usefulness of strategies while I study.					
29	CK	I use my intellectual strengths to compensate for my weaknesses.					
30	STR	I focus on the meaning and significance of new information.					
31	STR	I create my own examples to make information more meaningful.					

32	DK	I am a good judge of how well I understand something.					
33	PK	I find myself using helpful learning strategies automatically.					
34	MO	I find myself pausing regularly to check my comprehension.					
35	CK	I know when each strategy I use will be most effective.					
36	EV	I ask myself how well I accomplish my goals once I'm finished.					
37	STR	I draw pictures or diagrams to help me understand while learning.					
38	EV	I ask myself if I have considered all options after I solve a problem.					
39	STR	I try to translate new information into my own words.					
40	DB	I change strategies when I fail to understand.					
41	STR	I use the organizational structure of the text to help me learn.					
42	PL	I read instructions carefully before I begin a task.					

43	STR	I ask myself if what I'm reading is related to what I already know.					
44	DB	I re-evaluate my assumptions when I get confused.					
45	PL	I organize my time to best accomplish my goals.					
46	DK	I learn more when I am interested in the topic.					
47	STR	I try to break studying down into smaller steps.					
48	STR	I focus on overall meaning rather than specifics.					
49	MO	I ask myself questions about how well I am doing while I am learning something new.					
50	EV	I ask myself if I learned as much as I could have once I finish a task.					
51	DB	I stop and go back over new information that is not clear.					
52	DB	I stop and reread when I get confused.					

Note: DK = Declarative Knowledge, PK = Procedural Knowledge, CK = Conditional Knowledge, PL = Planning, STR = Information Management Strategies, MO = Monitoring, DB = Debugging, EV = Evaluating

APPENDIX D

METACOGNITIVE AWARENESS INVENTOY INDONESIAN VERSION

Kuesioner Kesadaran Metakognitif untuk Pelajaran Matematika
(*Metacognitive Awareness Inventory for Mathematics Lesson***)**

Jenis Kelamin : ☐ Laki-laki ☐ Perempuan

Usia : tahun

Kelas : SMA

Petunjuk Pengisian

1. Seluruh pernyataan dalam kuesioner diisi tanpa ada yang dikosongkan dan dijawab sesuai dengan kondisi yang sebenarnya.
2. Pilihan yang mewakili keadaan sebenarnya diisi dengan menggunakan contreng (√) atau silang (X).

No	Pernyataan	Sangat tidak benar	Tidak benar	Ragu-ragu	Benar	Sangat benar
1	Ketika menjawab soal matematika, saya selalu bertanya pada diri sendiri apakah yang saya lakukan sudah sesuai dengan tujuan yang saya tetapkan.					
2	Sebelum saya mengerjakan soal matematika, saya mempertimbangkan beberapa pilihan cara untuk menjawabnya.					
3	Dalam menjawab soal matematika, saya mencoba menggunakan cara yang berhasil saya gunakan sebelumnya.					
4	Ketika sedang belajar matematika, saya memacu diri sendiri agar memiliki waktu yang cukup.					

5	Saya tahu kelebihan dan kekurangan kemampuan berpikir saya dalam belajar matematika.					
6	Sebelum saya memulai mengerjakan tugas matematika, saya berpikir tentang konsep yang harus saya pelajari dan kuasai.					
7	Saya tahu apakah saya gagal atau berhasil dalam ujian matematika setelah saya menyelesaikannya.					
8	Sebelum saya mulai menjawab soal matematika, saya mengurainya ke dalam beberapa langkah dan menentukan langkah yang terpenting.					
9	Saya memperlambat bacaan ketika saya menemukan informasi penting.					
10	Ketika belajar, saya tahu bagian yang paling penting untuk dipelajari.					
11	Ketika sedang mengerjakan soal matematika, saya bertanya pada diri sendiri apakah saya telah mempertimbangkan semua pilihan cara penyelesaian.					
12	Saya dapat mengorganisasikan informasi atau konsep dengan baik.					
13	Secara sadar saya memfokuskan pada informasi atau konsep yang penting ketika belajar matematika.					
14	Saya memiliki maksud tertentu pada setiap cara yang saya gunakan untuk menyelesaikan soal matematika.					

15	Saya bisa belajar dengan sangat baik ketika saya mengetahui beberapa informasi tentang topik yang sedang dipelajari.					
16	Saya mengetahui hal-hal yang guru harapkan untuk saya pelajari.					
17	Saya dapat mengingat informasi atau konsep dengan baik.					
18	Saya menggunakan strategi belajar yang berbeda tergantung situasi.					
19	Setelah saya mengerjakan soal matematika, saya bertanya kepada diri sendiri apakah ada cara yang lebih mudah untuk menemukan jawaban soal tersebut.					
20	Saya memiliki kontrol terhadap sejauh mana kemampuan belajar saya.					
21	Saya biasanya melakukan pengulangan untuk membantu saya memahami hubungan yang penting di antara konsep atau informasi yang ada.					
22	Saya bertanya pada diri sendiri tentang materi atau konsep sebelum saya memulai mengerjakan soal.					
23	Saya memikirkan beberapa cara untuk menyelesaikan suatu soal dan memilih cara yang terbaik.					
24	Saya membuat kesimpulan tentang apa yang telah saya pelajari setelah saya selesai belajar.					

25	Saya meminta tolong kepada orang lain ketika saya tidak memahami suatu konsep atau soal.					
26	Saya bisa memotivasi diri untuk belajar ketika saya perlu melakukannya.					
27	Ketika saya belajar, saya tahu strategi yang saya pakai.					
28	Ketika saya belajar, saya meneliti kegunaan strategi yang digunakan.					
29	Saya memaksimalkan kemampuan berpikir saya untuk mengatasi kelemahan saya dalam belajar matematika.					
30	Saya memfokuskan diri pada maksud dan kegunaan dari konsep yang baru.					
31	Saya membuat contoh-contoh sendiri untuk membuat konsep lebih mudah dimengerti.					
32	Saya dapat menilai diri sendiri tentang seberapa baik saya memahami suatu konsep.					
33	Saya menggunakan strategi belajar yang tepat secara otomatis.					
34	Saya biasanya berhenti sejenak untuk memeriksa pemahaman saya.					
35	Saya tahu kapan setiap cara yang saya gunakan akan menjadi lebih efektif.					
36	Setelah saya selesai menyelesaikan soal, saya bertanya pada diri sendiri tentang seberapa baik saya mengerjakan soal.					

37	Ketika belajar, saya membuat gambar atau diagram untuk membantu saya memahami suatu konsep.					
38	Setelah saya mengerjakan sebuah soal, saya bertanya pada diri sendiri apakah saya telah mempertimbangkan semua pilihan cara penyelesaian.					
39	Saya mencoba menjelaskan konsep baru dengan menggunakan kata-kata sendiri.					
40	Saya merubah strategi ketika saya gagal untuk memahami suatu soal atau konsep.					
41	Saya menggunakan tulisan yang tersusun rapi untuk membantu saya belajar memahami konsep.					
42	Saya membaca petunjuk dengan teliti sebelum saya memulai mengerjakan soal.					
43	Saya bertanya pada diri sendiri apakah konsep atau soal yang sedang saya baca berhubungan dengan konsep atau soal yang saya ketahui sebelumnya.					
44	Saya mengevaluasi kembali cara yang saya gunakan ketika saya bingung.					
45	Saya mengatur waktu saya untuk bisa menyelesaikan soal dengan sebaik-baiknya.					
46	Saya belajar banyak ketika saya tertarik terhadap suatu topik.					
47	Saya mencoba mengurai proses belajar ke dalam beberapa langkah kecil.					

48	Saya lebih memfokuskan diri pada konsep materi secara keseluruhan daripada hal-hal yang spesifik.					
49	Ketika sedang belajar konsep yang baru, saya bertanya pada diri sendiri tentang seberapa baik usaha yang saya lakukan untuk memahaminya.					
50	Setelah saya menyelesaikan soal, saya bertanya pada diri sendiri apakah saya telah belajar secara maksimal.					
51	Saya berhenti dan melihat kembali konsep atau informasi baru yang belum jelas.					
52	Saya berhenti dan membaca kembali ketika saya bingung.					

APPENDIX G

IMPLEMENTATION OF IMPROVE INSTRUCTIONAL METHOD CHECKLIST

Classroom : ______

Date / Time : _____________ Observer : _____________

No	Activities	Yes	Partially	Never
Introduction				
1	Teacher express the learning objective.			
2	Teacher motivates students to use metacognitive questions to students.			
3	Teachers to take the time to discuss the importance of metacognitive knowledge and regulation.			
3	Teacher reviews the previous topics briefly.			
4	Teacher motivates students to understand the new topics.			
Introducing New Concept				
5	Teacher uses connection questions to introduce the new topic.			
6	Teacher activates students' prior knowledge.			
7	Teacher uses comprehension questions to introduce the new topic.			
8	Teacher uses reflection questions to introduce the new topic.			
9	Teacher provide clear explanation and reason behind process by paying attention students differences in terms of pre-existing knowledge and thinking levels.			
10	Teacher give students chance to write important information in their book in organized manner.			

11	Students answer teachers' metacognitive questions.			
12	Teacher selects appropriate examples by considering students difficulty level.			
13	Teacher asks students about the reason and explanation behind the procedures.			
Practicing				
14	Students work together in pairs.			
15	Students read problems loudly.			
16	Teacher monitors students work by walking around the classroom.			
17	Teacher joins groups and models the use of metacognitive questions.			
18	Students use metacognitive questions loudly to solve the problems.			
19	Teacher listens to how students cope with the problems and provide assistance when needed.			
20	Students express their ideas conveniently.			
21	Students ask their teacher if there is an unclear information or they encounter difficulties.			
Verification				
22	Teacher asks several students to present their work in board.			
23	Students explain their work by means of metacognitive questions.			
24	Teacher verify students answer by asking metacognitive questions.			
25	Teacher gives chance for other students to ask questions.			
26	Teacher gives chance for other students to present their alternative strategies.			
Reviewing				

27	Teacher reviews the main ideas of the lesson with the entire class.			
28	When common difficulties are observed, the teacher provided additional explanations to the whole class.			
29	Teacher asks students to express their ideas about the concept by using their own words.			
30	Teacher provide reflective journal writing.			
Obtaining Mastery				
31	Teacher conducted small quiz after all material related to the certain topics were given			
Enrichment and Remedial				
32	Providing enrichment for students who achieved scores greater than the intended score.			
33	Providing remedial for students who achieved scores less than the intended score.			

APPENDIX H

INTERVIEW QUESTIONS

1. What are the differences between the current instruction and the previous instruction?
2. What are advantages that you take in this new instruction?
3. What are useful activities in this new instruction?
4. What are the challenges or problems which you encounter when you are taught with the new instruction?

APPENDIX I

REFLECTIVE JOURNAL WRITING

Name :

Class : **Date :**

Reflective Journal Writing

1. What were the goals for todays' lesson?

2. What part of the concept or task did you find the most difficult?

3. What strategy did you use to deal with the difficulty?

4. How did you decide which strategy would be most helpful?

5. How would you grade your performance today?

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

APPENDIX J

EXAMPLES OF METACOGNITIVE QUESTIONS

A. Introducing New Concepts

Introducing the whole class to the new material, concepts, problems or procedures by modelling the activation of metacognitive processes.

No	Questions	Note
Connection Questions		
1	What are concepts that become prerequisite of this concept?	
2	What are the similarities/differences between the concepts at hand and concepts we have learned in the past?	
3	What are the similarities or the differences between the two explanations? Why?	
Comprehension Questions		
1	What do we mean by …?	
2	What do you already know about this topic?	
3	What do you think this topic will be about, based on the title? Why do you think so?	
4	In what condition does the formula work?	
5	What is the goal of the lesson?	
6	Why can’t we do this?	
7	What would happen if …?	
8	Can we explain new information in our own words?	
9	What is the mathematical big idea of the lesson?	
10	What things that we have to take into consideration about…?	
Strategic Questions		
1	Please, tell us what formula or strategies we used in the previous topics?	

2	How should we proceed this?	
3	What are strategies that we will use to help us understand the concept?	
4	Are picture and drawing helping us grasp the idea? What, when and why should you use different representations in the solution, different techniques in the lesson?	
5	Why do we have to use this formula for this concept?	
6	What are we going to calculate? How are we going to calculate it?	
7	What is going wrong?	
8	Can we think of different ways to prove the formula?	
Reflection Questions		
1	Do we understand the previous topics very well?	
2	Did we read anything that we found unclear? If so, perhaps we can clarify so that we will understand before we read further.	
3	Can we apply this way of thinking to other problems or situations?	
4	Is there anything we don't understand—any gaps in my knowledge?	
5	What would a summary of this topics be? Include the main idea and three supporting details.	
6	How do we justify our conclusion?	
7	Can we explain the reasons behind the ideas and concepts? Why?	
8	Can we add new information to the existing body of knowledge? What information?	
9	Are we reaching our goals? Why?	
10	Do we need to make changes? Why? What changes?	

B. Presentation

No	Questions	Note
Connection Questions		
1	What are the similarities/differences between the task at hand and the task you have solved before?	
2	What are the similarities or the differences between the two explanations? Why?	
Comprehension Questions		

1	What is the goal of the lesson?	
2	What do you mean by …?	
3	In what condition does the formula work?	
4	Why can't we do this?	
5	What would happen if …? How?	
6	What information is important for you to consider?	
7	Write down your reasons?	
Strategic Questions		
1	How did you determine this to be true?	
2	Why didn't you consider a different route to the problem?	
3	Why do you think this works? Does it always? Why?	
Reflection Questions		
1	Why does that answer make sense to you?	
2	(In response to an answer):…, What if I said that's not true?	
3	Is there any way to show exactly what you mean by that?	
4	What questions did I not answer correctly? Why? How did my answer compare with the suggested correct answer?	
5	What questions did I not answer correctly? Why? What confusions do I have that I still need to clarify?	
6	What would we do differently next time?	
7	Are we on the right track?	
8	Does anyone in this class want to add something to the solution?	
9	How might you convince us that your way is the best way?	
10	How did you determine this to be true?	
11	Are there any other similar answers you can think of with alternative routes?	
12	What strategies worked well for you?	
13	What strategies did not work for you?	

C. Reviewing

No	Questions	Note
Connection Questions		
1	How does this concept relate to the previous concepts?	
2	What are the similarities and differences between the concept at hand and the previous ones?	
3	How does this concept relate daily occurrences?	
Comprehension Questions		
1	What were the goals of the todays learning?	
2	Where are you getting stuck?	
3	What do you know about the topic so far? Can you explain the concept by your own words?	
4	Is there any important information that might help figure out what the concept you are learning?	
Strategic Questions		
1	In what situation, you can use this formula? Why?	
2	How many possibilities strategies are there to solve the problem?	
3	What are the steps to solve the problem? Explain in your own words?	
4	Can the strategy be applied in another situation?	
Reflection Questions		
1	What was today's class session about?	
2	What are your difficulties in this lesson?	
3	What do you think about what was said? How would you agree or disagree with this?	
4	What did you hear today that is in conflict with your prior understanding?	
5	How did the ideas of today's class session relate to previous class sessions?	
6	What do you need to actively go and do now to get your questions answered and your confusions clarified?	
7	What did you find most interesting about class today?	
8	How am I feeling about this activity?	

APPENDIX K

AN EXAMPLE OF LESSON PLAN FOR EXPERIMENTAL GROUP

School Level	: High School
Grade / Semester	: 11 / 1
Subject	: Mathematics-Compulsory
Topics	: Composite and Inverse Function
Subtopics	: Inverse Function
Time	: 2 x 45 minutes
Session	: 1

Basic Competences :

1. Analyzing the concepts and properties of function and perform algebraic manipulation in determining inverse function and inverse of a function.
2. Selecting effective strategy and presenting model of mathematics in solving problem related to inverse function and inverse of function.

Indicators :

4. Describing the concept of inverse of function.
5. Determining the inverse of algebraic function.

Lesson Objectives :

1. Students are able to describe the concept of inverse of function.
2. Students are able to determine inverse of algebraic function.

Instructional Method:

IMPROVE instructional method: Introduction, Metacognitive Questioning, Practicing, Reviewing, Obtaining Mastery, Verification, Enrichment and Remediation.

Sources :

1. Students Mathematics textbook for 11th graders Grafindo Media Pratama.
2. Teacher Mathematics textbook for 11th graders.

Teaching Material :

INTRODUCTION

Opening

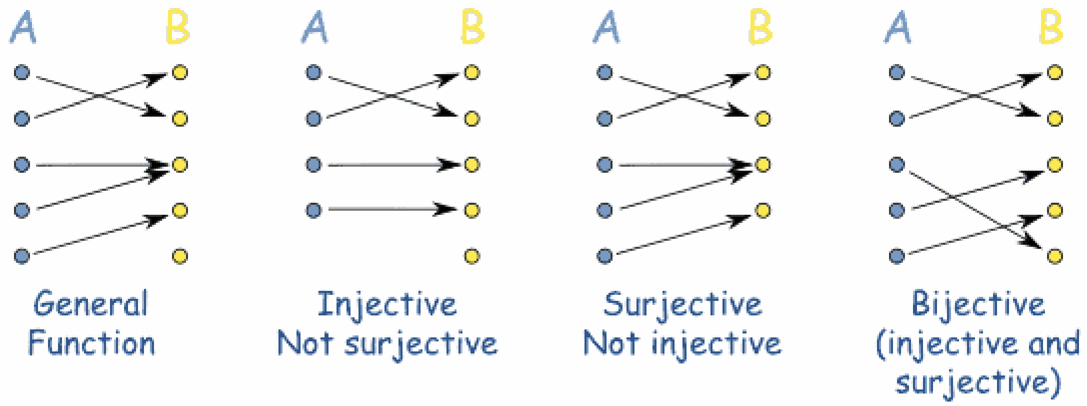
1. The teacher greets students and checks their attendance.
2. The teacher provides information about the topics to be studied today. The teacher writes the title on the board.

3. The teacher asks students
   What do you think this topic will be about, based on the title? Why do you think so?
4. The teacher reveals competences and indicators to be achieved by students.
5. The teacher gives idea of the importance of understanding the inverse function and provides an overview of the application of inverse functions in everyday life, for example in the exchange of money, buying and selling, converting temperature, and so forth.
6. The teacher also gives emphasis to students about the importance of the metacognitive questions in the learning process and solving mathematics problems.

Activating Prior Knowledge

Before getting into the topic of the inverse function, the teacher recalls the concept of bijective functions.

1. The teacher asks the students about the concept of bijective functions.
   i. Do you remember about the concept bijective function?
   ii. What do you already know about this concept?
   iii. Can you explain or describe bijective function?
   - Students are expected to explain the bijective function using their own words.
   - After paying attention to students' answers regarding bijective function, the teacher illuminates explicitly definition of bijective function, namely, the function where each element of the range of a function of the pair have exactly one element in the function domain.
2. The teacher shows a picture below to help students remember bijective function.



3. The teacher asks the students to choose one of the pictures that is categorized as a bijective function and express their reasons. They are also asked to identify the types of the function of other pictures.
   i. Which one of existing images is a bijective function? Why?
   ii. What about the other pictures?

4. The teacher confirms that understanding bijective function is main key to understand the inverse function.

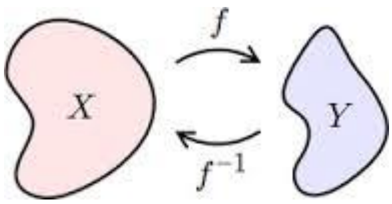
INTRODUCING THE NEW CONCEPT

(At this stage the teacher introduces the new concept and new formula using the metacognitive questions. The teacher introduces new material by asking students in which the questions make students become active in learning process and they can recognize their knowledge. The questions were prepared by the researcher and the teacher to guide students to understand the concept or the material being taught.)

Explanation

1. The teacher guides students to understand the concept of inverse function by activating knowledge that have been discussed previously.
2. The teacher asked the meaning of the inverse.
   a. What is the meaning of inverse?
   (The teacher reminds students about the concept of matrix inverse which had previously been studied)
   (If no one answers, the teacher mentions that the meaning of inverse meaning is opposite. The opposite or inverse of addition is subtraction and the opposite or inverse of multiplication is division)
3. The teacher writes the definition of inverse function on the board.
   Definition:

   Let f be a one-to-one function with domain A and range B and it is expressed in ordered pairs as $f = \{(x, y) | x \in A, y \in B\}$, then the inverse of f, denoted f^{-1}, is the function formed by reversing all the ordered pairs in f, that is, $f^{-1} = \{(y, x) | y \in B, x \in A\}$ or the domain of f^{-1} is B and its range is A.

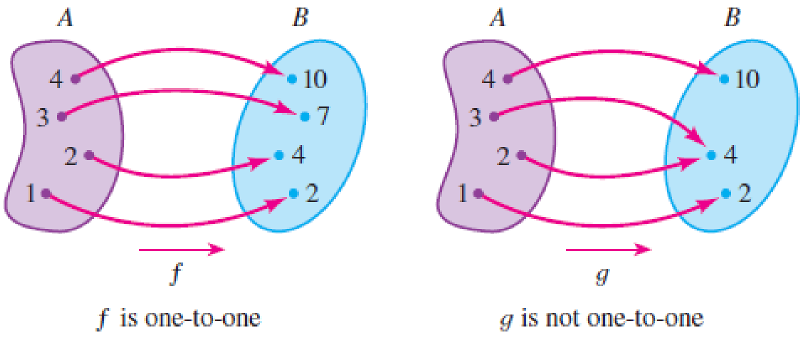


4. The teacher uses the above image to help students understand the definition of inverse function.
5. The teacher asks students:

a. Are notation of the inverse function f^{-1} equal to $\frac{1}{f}$?

(Different, in the inverse function it means inverse. -1 for $\frac{1}{f}$ is used in exponent)

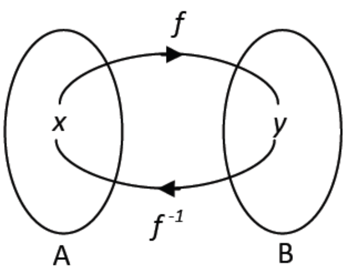
b. Do all inverse of functions as function?
(To help students answer this question, the teacher shows the following figure)



6. Then the teacher asked,
   a. Is the inverse function f a function? Why?
   b. Is the inverse function g a function? Why?
7. The teacher guides students to conclude that the only bijective functions have inverse function.
   So what is function that has inverse function? Bijective function.
8. She wrote the following conclusion,

A function has an inverse function if and only if the function is bijective

9. The teacher shows a picture below to learn the domain and range of function and inverse functions.



10. With reference to the above figure, the teacher asks students some questions as follows:
   a. What is the domain of function f ? ($x \in A$)
   b. What is the range of function f ? ($y \in B$)
   c. What is the domain of function f^{-1}? ($y \in B$)
   d. What is the range of unction f^{-1}? ($x \in A$)
   e. What can you conclude from the above conditions?

(Domain of f equals to range of f^{-1} and range of f equals to domain of f^{-1})

11. The teacher writes the conclusion on the blackboard.
12. The teacher again emphasizes definition of inverse function and ensure that students understand that the domain f should be the same as the range f^{-1} and the range f should be the same as the domain f^{-1}.
13. The teacher reviews students' comprehension by asking questions.
    a. Now can you explain what is the inverse function using your own words?
    b. Why is only one-to-one function that has inverse function?
    c. Are there things that are unclear about the inverse function?
14. The teacher gives chance to the students to write important information in organized manner in their notes.

Examples of Question

1. Questions

For each function below, determine whether the function has an inverse function or not. Indicate your reasons, and specify the inverse function.
a. $f=\{(1,3),(2,5),(3,4),(4,9),(5,0)\}$
b. $g=\{(4,16),(-4,16),(2,4),(-2,4),(5,25)\}$
c. $f(x)=5x+7$

2. By modelling metacognition questions, the teacher solves the problem in the example above.
    a. What does the above example talk about?
    b. What should be considered to complete this examples?
    c. By considering the definition of inverse function.
    d. Teachers can provide an explanation of the answer to the problem by showing a picture.

Solutions

a. The function is a bijective function, for the values of x and y are used once or does not appear twice. The inverse is $f=\{(3,1),(5,2),(4,3),(9,4),(0,5)\}$
Hint: Domain of f equals to range of f^{-1} and range of f equals to domain of f^{-1}

b. The function is not a bijective function, because there is the value of y used twice.
Not a bijective function: there is no inverse function.

3. The teacher asks students if there are things that are unclear about determining one-to-one function, and domain and range of inverse function.
4. The teacher provides opportunities for students to write down important information in their notebook.

Finding the inverse of a function (10 minutes)

1. The teacher says, "After we have studied whether a function has the inverse function or not, then the next step is to learn how to determine the inverse of a function."
2. By providing a simple example, the teacher guides students to learn how to determine the inverse of a function.
3. The teacher asks the students if the function is a one-on-one function or not, and students are expected to answer by providing reasonable arguments.
4. The teacher asks the students about the domain and range of the function.
   $D_f = \{x | x \in \mathbb{R}\}$ and $R_f = \{y | y \in \mathbb{R}\}$
5. The teacher also asks the students about the domain and range of the inverse function.
   $D_{f^{-1}} = \{y | y \in \mathbb{R}\}$ and $R_{f^{-1}} = \{x | x \in \mathbb{R}\}$.
6. Determine the inverse of a function.

1. First we write $y = f(x)$, therefore $y = 5x + 7$
2. Then we will look for the value of x.
   $5x = y - 7$ (Add 7)
   $x = \frac{y-7}{5}$ (Divided by five 5)
3. Replace $f^{-1}(x)$ with x and y with x.
   $f^{-1}(x) = \frac{x-7}{5}$
4. So the inverse of the function is $f^{-1}(x) = \frac{x-7}{5}$

7. When determining inverse of function, the teacher asks the students reasons behind the carried out procedures.
   a. Why the f (x) is equal to y?
   b. What steps have been done is correct? Try to check every step!
8. The teachers' explanations are broken into small steps and if there is unclear information, the teacher reviews the concepts learned.

9. Determining procedures applicable in general to determine the inverse of a function. As students have learned how to determine the inverse of a function, then by means of the teachers' guidance, students are asked to conclude the steps determining the inverse of a function.
    a. Now, could you express the steps to determine the inverse function?
10. Taking into account students' opinions and responses, the teacher writes down steps to determine the inverse of a function on the board.

Steps to determine the inverse of a function. For one-to-one function f defined by an equation $y = f(x)$, find the defining equation of the inverse as follows: Step 1. Replace the symbol $f(x)$ with y Step 2. Solve the equation for x in terms of y Step 3. Replace x with $f^{-1}(x)$ and y with x

11. The teacher parses explanations into simple parts and asks the students if there is unclear information, the teacher reminds the concepts learned.
12. The teacher ensures that the students understand these steps by asking them back read or express it by means of their own words.
13. The teacher gives the students chance to write important information in their notes.

PRACTICING

(Students are requested to apply metacognitive questions)

1. The teacher asks the students to solve problems presented in activity sheets with their partners.
2. The activity sheet consists of five questions that students have to answer and discuss with their partners.
3. The activity sheet is designed for students so that they understand whether a function is a function of one-on-one or not, drawing functions and inverse functions and recognize a relationship between them, as well as the application of the inverse function.
4. The students use metacognitive questions when they discuss with their partners to solve problems in the activity sheet.
5. The students are also given a list of metacognitive questions they have to ask and answer.

REVIEWING

(Teachers and students review the new material using metacognitive questions)

Presentation

1. The teacher asks several students to present the results of their discussion to the class.

2. Students are also asked to explain their work using their own words.
3. The teacher also poses questions to students about their works.
   a. Why do you think so?
   b. Why did not you think of another way to solve the problem?
   c. Try to check every step you performed.
   d. Why is the answer you have found a reasonable answer?
4. Providing the opportunity for other students to ask the students who present their work in front of the class.
   Do any of you who want to ask questions or add something?
5. The teacher ensures that students understand each step.
6. The teacher checks the answer and the steps performed by the students.
   a. How can you convince friends that the strategy you choose is the best?
   b. How can you be sure that it is the right answer?
   c. Are there any other ways that can be taken to find the similar correct answer?
7. The teacher revises mistakes made by the student if necessary.
8. The teacher provides the opportunity for students to ask questions and if there are alternative strategies to solve problem.
9. The teacher also asks students to decide which the best strategy to solve the problem.

Reviewing

1. As in practice, students are confronted with challenging problems. At this stage, the teacher tries to review the mistakes that the students made in effort to understand the concept and solve mathematical problems.
2. The teacher reviews the solution of problems presented in the activity sheet by means of metacognitive questions.
3. The teacher also reviews the main ideas of learning and minimizes difficulties encountered by students by means of metacognitive questions.
   a. What did we learn today?
   b. Can you reveal about what the inverse function?
   c. Do all functions have an inverse function?
   d. What are the domain and range of the inverse function?
   e. Are there similarities between the topics you have learned earlier in this topic? What and why?
4. The teacher evaluates students' progress and provides feedback.
5. If there are any difficulties faced by most students the teacher provides additional explanation.
6. Collect the students' work as part of the assessment.
7. Give homework.

OBTAINING MASTERY

The instruction expects students to possess high and low cognitive process.

1. Environment that is supported with revision and feedback can adjust learning time needed by each student and therefore enable students to obtain mastery of the material and deepen mathematical thinking.
2. After completing learning process, the teacher will provide formative tests for students. The purpose of this test is to understand students' progress in learning process. The results of the test will help the teacher know the students who have obtained mastery and have not.

VERIFICATION

(Based on feedback and remedial, the teacher verifies students' achievement of cognitive and metacognitive skills)
After conducting tests and see the results, teachers identify students by grouping the students who have achieved mastery of the material and that has been based on the limits set value.

ENRICHMENT AND REMEDIAL

1. Formative test will be administered at sixth week, after completing the topic of composition and inverse function.
2. The results of the test will be used as a consideration for the teacher to provide enrichment and remedial.
3. Enrichment will be given to students who gain score more than 75%, while remedial will be given to students whose scores below 75%.

ACTIVITY SHEET

Group	: ____	Class	: _____
Name	: 1________________________	Date	: ____________________
	2________________________		

Direction:

1. Read the questions carefully.
2. Use metacognitive questions to help you solve problems.
3. Express your reason underlying the steps conducted.
4. Solve the problems within 25 minutes.

A. Determine whether the functions below are included in the bijective function or not. Give reasons underlying the answer.

No	Functions	Bijective or not	Reasons
1	$f(x)=\{(-3,3),(-2,2),(0,0),(2,2)\}$		
2	$f(x)=\{(1,1),(2,8),(3,27)\}$		

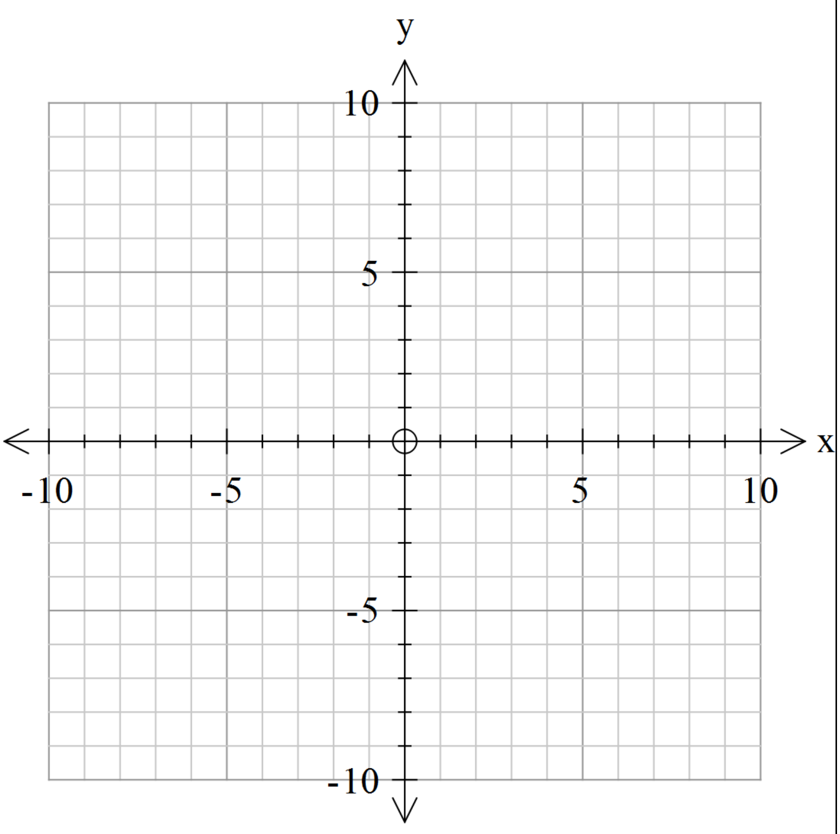
B. Determine the inverse of $f(x)=\{(-1,3),(2,5),(-3,5),(2,0)\}$ and

$$f(x)=\frac{3x+5}{x+2}$$

Solution

1	
2	

C. Determine inverse of the following functions and make graphs of these functions in Cartesians coordinates.

$f(x)=2x+1$	
Determining f^{-1} algebraically	$f(x)=2x+1$
Drawing f and f^{-1}	
Conclusion	Based on the graph you pictures, how is the relationship between the graph f and f^{-1}? (hint: draw $y=x$)

APPENDIX L

Difficulty Level and Discriminant Index

	Pilot Study		**Main Study**	
Items	Difficulty Index	Discriminant Index	Difficulty Index	Discriminant Index
1	0.46	.436**	0.82	.402**
2	0.4	.568**	0.46	.612**
3	0.44	.429**	0.45	.140
4	0.22	.343**	0.64	.569**
5	0.53	.458**	0.63	.436**
6	0.35	.504**	0.38	.099
7	0.73	.334**	0.76	.369**
8	0.56	.329**	0.53	.574**
9	0.64	.288**	0.74	.515**
10	0.35	.524**	0.39	.318**
11	0.44	.489**	0.66	.608**
12	0.38	.589**	0.63	.698**
13	0.36	.385**	0.38	.435**
14	0.17	.516**	0.80	.485**
15	0.43	.469**	0.71	.595**
16	0.51	.324**	0.28	.363**
17	0.19	.429**	0.48	.442**
18	0.22	.404**	0.36	.461**
19	0.29	.288**	0.54	.448**
20	0.35	.512**	0.61	.488**
21	0.33	.363**	0.48	.688**
22	0.32	.234*	0.75	.493**
23	0.3	.205*	0.52	.520**
24	0.45	.123	0.52	.335**
25	0.37	.220*	0.79	.541**
26	0.32	.300**	0.68	.525**

27	0.33	.164	0.52	.472**
28	0.25	.306**	0.45	.526**
29	0.38	.102	0.46	.518**
30	0.22	.174	0.48	.471**
31	0.28	.342**	0.58	.630**

APPENDIX M

Kolmogorov Smirnov Normality Test

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Post-RC	.148	34	.057	.934	34	.041

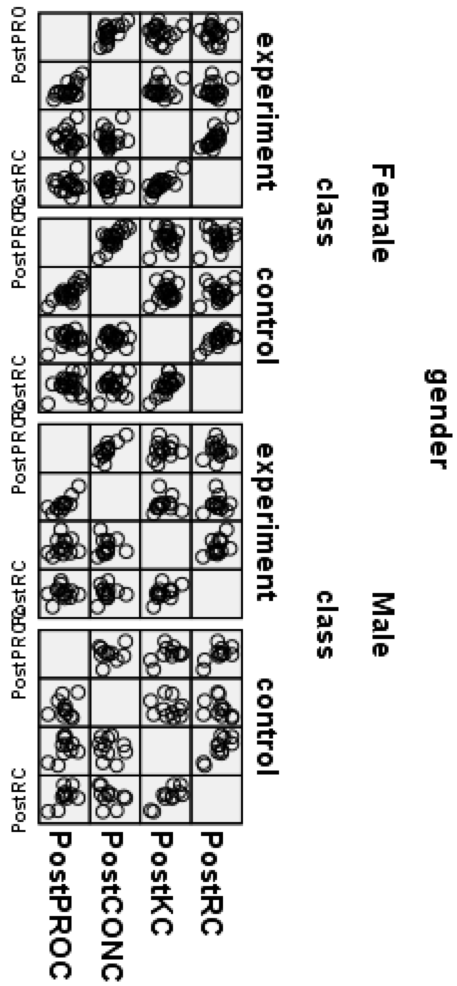
a. Lilliefors Significance Correction

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
PostCONC	.119	41	.150	.985	41	.860

a. Lilliefors Significance Correction

APPENDIX N


gender
Female
Male
class
class
experiment
control
experiment
control
PostPROC
PostCONC
PostKC
PostRC

APPENDIX O

Homogeneity of Regression

For Groups

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
Groups * Pre-PROC	Post-PROC	22.830	1	22.830	.527	.471
	Post-CONC	88.633	1	88.633	1.047	.311
	Post-KC	52.613	1	52.613	1.610	.210
	Post-RC	82.746	1	82.746	.660	.420
Groups * Pre-CONC	Post-PROC	.392	1	.392	.009	.925
	Post-CONC	37.451	1	37.451	.442	.509
	Post-KC	21.465	1	21.465	.657	.421
	Post-RC	64.710	1	64.710	.517	.475
Groups * Pre-KC	Post-PROC	1.045	1	1.045	.024	.877
	Post-CONC	2.326	1	2.326	.027	.869
	Post-KC	.701	1	.701	.021	.884
	Post-RC	1.535	1	1.535	.012	.912
Groups * Pre-RC	Post-PROC	.627	1	.627	.014	.905
	Post-CONC	55.915	1	55.915	.661	.420
	Post-KC	13.363	1	13.363	.409	.525
	Post-RC	275.668	1	275.668	2.200	.144

For Gender

Source	**Dependent Variable**	**Type III Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Gender * Pre-PROC	Post-PROC	41.366	1	41.366	1.129	.293
	Post-CONC	242.626	1	242.626	1.938	.170
	Post-KC	3.544	1	3.544	.096	.758
	Post-RC	136.026	1	136.026	.941	.336
Gender * Pre-CONC	Post-PROC	.392	1	.392	.011	.918
	Post-CONC	.064	1	.064	.001	.982
	Post-KC	16.257	1	16.257	.441	.509
	Post-RC	35.852	1	35.852	.248	.621
Gender * Pre-KC	Post-PROC	.271	1	.271	.007	.932
	Post-CONC	3.450	1	3.450	.028	.869
	Post-KC	.001	1	.001	.000	.995
	Post-RC	1.113	1	1.113	.008	.930
Gender * Pre-RC	Post-PROC	50.796	1	50.796	1.386	.244
	Post-CONC	19.192	1	19.192	.153	.697
	Post-KC	16.838	1	16.838	.457	.502
	Post-RC	6.405	1	6.405	.044	.834

For Gender*Teaching Methods

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.
gender * groups * Pre-PROC	Post-PROC	52.249	2	26.125	.800	.456
	Post-CONC	274.171	2	137.086	1.751	.186
	Post-KC	77.136	2	38.568	1.072	.351
	Post-RC	539.773	2	269.887	1.993	.149
gender * groups * Pre-CONC	Post-PROC	48.941	2	24.470	.749	.479
	Post-CONC	73.720	2	36.860	.471	.628
	Post-KC	101.326	2	50.663	1.409	.255
	Post-RC	313.156	2	156.578	1.156	.324
gender * groups * Pre-KC	Post-PROC	89.167	2	44.583	1.365	.266
	Post-CONC	26.089	2	13.045	.167	.847
	Post-KC	2.402	2	1.201	.033	.967
	Post-RC	77.639	2	38.819	.287	.752
gender * groups * Pre-RC	Post-PROC	45.162	2	22.581	.691	.506
	Post-CONC	58.213	2	29.107	.372	.692
	Post-KC	58.717	2	29.359	.816	.449
	Post-RC	356.612	2	178.306	1.316	.279

APPENDIX P

	N		Skewness	Std. Error of Skewness	Kurtosis	Std. Error of Kurtosis
	Valid	Missing				
item1	169	0	-.153	.159	-.266	.316
item2	169	0	-.821	.159	.398	.316
item11	169	0	-.211	.159	.180	.316
item21	169	0	-.321	.159	-.083	.316
item28	169	0	-.376	.159	.477	.316
item34	169	0	-.504	.159	.375	.316
item49	169	0	-.390	.159	.262	.316
item4	169	0	-.384	.159	-.443	.316
item6	169	0	-.283	.159	-.538	.316
item8	169	0	-.050	.159	-.804	.316
item22	169	0	-.240	.159	-.133	.316
item23	169	0	-.394	.159	-.165	.316
item42	169	0	-.638	.159	.354	.316
item45	169	0	-.583	.159	.115	.316
item9	169	0	-.552	.159	-.370	.316
item13	169	0	-.502	.159	.570	.316
item30	169	0	-.466	.159	.422	.316
item31	169	0	-.151	.159	-.541	.316
item37	169	0	-.203	.159	-.636	.316
item39	169	0	-.451	.159	.107	.316
item41	169	0	-.463	.159	-.171	.316
item43	169	0	-.613	.159	.903	.316
item47	169	0	-.303	.159	.159	.316
item48	169	0	-.135	.159	-.245	.316
item25	169	0	-.777	.159	.795	.316
item40	169	0	-.387	.159	.497	.316
item44	169	0	-.554	.159	.398	.316
item51	169	0	-.269	.159	-.066	.316
item52	169	0	-.737	.159	.590	.316

item7	169	0	-.473	.159	.032	.316
item19	169	0	-.460	.159	-.349	.316
item24	169	0	-.389	.159	-.217	.316
item36	169	0	-.437	.159	-.337	.316
item38	169	0	-.319	.159	-.173	.316
item50	169	0	-.224	.159	-.345	.316
item5	169	0	-.512	.159	-.283	.316
item10	169	0	-.466	.159	.645	.316
item12	169	0	-.152	.159	.034	.316
item16	169	0	-.241	.159	.262	.316
item17	169	0	-.125	.159	.127	.316
item20	169	0	-.361	.159	-.173	.316
item32	169	0	-.662	.159	1.294	.316
item46	169	0	-.602	.159	.105	.316
item3	169	0	-.810	.159	1.988	.316
item14	169	0	-.444	.159	-.110	.316
item27	169	0	-.312	.159	.364	.316
item33	169	0	-.342	.159	.202	.316
item15	169	0	-.676	.159	1.203	.316
item18	169	0	-.792	.159	.816	.316
item26	169	0	-.725	.159	.564	.316
item29	169	0	-.538	.159	.616	.316
item35	169	0	-.320	.159	.093	.316

APPENDIX Q

	item1	item2	item11	item21	item28	item34	item49	item4	item6	item8
item1	1	.397**	.321**	.150*	.217**	.255**	.285**	.173**	.221**	.199**
item2	.397**	1	.230**	.081	.184**	.189**	.172**	.235**	.235**	.168**
item11	.321**	.230**	1	.163*	.163*	.228**	.244**	.243**	.258**	.164*
item21	.150*	.081	.163*	1	.115	.285**	.172**	.243**	.415**	.164*
item28	.217**	.184**	.163*	.115	1	.111	.264**	.126	.252**	.065
item34	.255**	.189**	.228**	.285**	.111	1	.347**	.092	.250**	.120
item49	.285**	.172**	.244**	.172**	.264**	.347**	1	.239**	.182**	.140*
item4	.173**	.235**	.243**	.243**	.126	.092	.239**	1	.247**	.256**
item6	.221**	.235**	.258**	.415**	.252**	.250**	.182**	.247**	1	.177**
item8	.199**	.168**	.164*	.164*	.065	.120	.140*	.256**	.177**	1
item22	.262**	.249**	.252**	.326**	.234**	.184**	.286**	.209**	.281**	.262**
item23	.251**	.266**	.172**	.186**	.298**	.233**	.265**	.219**	.111	.251**
item42	.133*	.094	.143*	.187**	.194**	.225**	.281**	.120	.132*	.005
item45	.274**	.229**	.272**	.189**	.286**	.326**	.356**	.323**	.226**	.274**
item9	.195**	.056	.166*	.190**	.032	.103	.136*	.185**	.175**	.195**
item13	.218**	.193**	.279**	.221**	.191**	.164*	.243**	.199**	.330**	.218**
item30	.159*	.076	.081	.168*	.343**	.242**	.192**	.129*	.144*	.105
item31	.209**	.151*	.139*	.198**	.187**	.294**	.220**	.269**	.188**	.209**
item37	.101	.079	.084	.182**	.320**	.202**	.282**	.115	.155*	.101
item39	.238**	.232**	.142*	.292**	.165*	.253**	.342**	.264**	.148*	.117
item41	.121	.046	.185**	.154*	.126	.343**	.212**	.007	.008	.031
item43	.223**	.136*	.055	.291**	.365**	.228**	.243**	.174**	.303**	.049
item47	.074	.130*	.211**	.226**	.286**	.250**	.327**	.209**	.231**	.121
item48	.044	.117	.222**	.231**	.224**	.265**	.360**	.195**	.264**	.150*
item25	.141*	.054	.015	.260**	.041	.151*	.239**	.248**	.046	.152*
item40	.155*	.154*	.242**	.245**	.200**	.279**	.164*	.181**	.141*	0.58
item44	.110	.116	.087	.209**	.203**	.301**	.319**	.119	.211**	-.045
item51	.187**	.131*	.156*	.280**	.244**	.269**	.409**	.251**	.205**	.191**
item52	.250**	.159*	.176**	.218**	.191**	.233**	.337**	.071	.062	0.050

APPENDIX R

DATE: 6/ 8/2016
TIME: 18:05

L I S R E L 8.80

BY

Karl G. Jöreskog and Dag Sörbom

The following lines were read from file **C:\Users\tian abdul aziz\Dropbox\ANALYSIS\final.SPJ:**

```
Latent Variables  Regulation Knowledge
Relationships
item1 = Regulation
item2 = Regulation
item11 = Regulation
item21 = Regulation
item28 = Regulation
item34 = Regulation
item49 = Regulation
item4 = Regulation
item6 = Regulation
item8 = Regulation
item22 = Regulation
item23 = Regulation
item42 = Regulation
item45 = Regulation
item9 = Regulation
item13 = Regulation
item30 = Regulation
item31 = Regulation
item37 = Regulation
item39 = Regulation
item41 = Regulation
```

```
 item43 = Regulation
 item47 = Regulation
 item48 = Regulation
 item25 = Regulation
 item40 = Regulation
 item44 = Regulation
 item51 = Regulation
 item52 = Regulation
 item7 = Regulation
 item19 = Regulation
 item24 = Regulation
 item36 = Regulation
 item38 = Regulation
 item50 = Regulation
 item5 = Knowledge
 item10 = Knowledge
 item12 = Knowledge
 item16 = Knowledge
 item17 = Knowledge
 item20 = Knowledge
 item32 = Knowledge
 item46 = Knowledge
 item3 = Knowledge
 item14 = Knowledge
 item27 = Knowledge
 item33 = Knowledge
 item15 = Knowledge
 item18 = Knowledge
 item26 = Knowledge
 item29 = Knowledge
 item35 = Knowledge
 Path Diagram
 End of Problem

 Sample Size =   169
```

Covariance Matrix

	item1	item2	item11	item21	item28	item34
item1	0.76					
item2	0.30	0.75				
item11	0.23	0.17	0.71			
item21	0.11	0.06	0.11	0.67		
item28	0.16	0.13	0.11	0.08	0.67	
item34	0.18	0.14	0.16	0.19	0.08	0.69
item49	0.21	0.13	0.18	0.12	0.18	0.25
item4	0.14	0.19	0.19	0.19	0.10	0.07
item6	0.18	0.19	0.20	0.31	0.19	0.19
item8	0.16	0.14	0.13	0.13	0.05	0.09
item22	0.18	0.17	0.17	0.21	0.15	0.12

item23	0.18	0.19	0.12	0.13	0.20	0.16
item42	0.11	0.08	0.11	0.14	0.15	0.17
item45	0.22	0.18	0.21	0.14	0.21	0.25
item9	0.16	0.05	0.14	0.15	0.03	0.08
item13	0.14	0.13	0.18	0.14	0.12	0.10
item30	0.10	0.05	0.05	0.10	0.20	0.14
item31	0.18	0.13	0.11	0.16	0.15	0.24
item37	0.09	0.07	0.08	0.16	0.28	0.18
item39	0.19	0.18	0.11	0.22	0.12	0.19
item41	0.11	0.04	0.16	0.13	0.10	0.29
item43	0.15	0.09	0.04	0.18	0.23	0.15
item47	0.05	0.09	0.15	0.15	0.19	0.17
item48	0.03	0.09	0.17	0.17	0.17	0.20
item25	0.10	0.04	0.01	0.18	0.03	0.11
item40	0.11	0.11	0.17	0.17	0.14	0.19
item44	0.08	0.08	0.06	0.13	0.13	0.20
item51	0.13	0.09	0.11	0.18	0.16	0.18
item52	0.19	0.12	0.13	0.16	0.14	0.17
item7	-0.05	-0.10	0.08	0.06	0.07	0.09
item19	0.20	0.20	0.24	0.17	0.20	0.12
item24	0.18	0.15	0.20	0.33	0.13	0.20
item36	0.24	0.22	0.10	0.17	0.24	0.11
item38	0.10	0.14	0.12	0.13	0.32	0.12
item50	0.26	0.15	0.14	0.22	0.21	0.25
item5	0.10	0.12	0.13	0.06	0.03	0.17
item10	0.07	0.05	0.17	0.03	0.10	0.13
item12	0.00	0.06	0.14	0.10	0.11	0.08
item16	0.07	0.06	0.07	0.13	0.13	0.07
item17	0.03	-0.02	0.13	0.18	0.14	0.05

item20	0.10	0.11	0.15	0.14	0.11	0.15
item32	0.08	0.05	0.05	0.08	0.06	0.20
item46	0.19	0.15	0.12	0.11	0.12	0.18
item3	0.15	0.13	0.08	0.06	0.06	0.07
item14	0.15	0.19	0.21	0.12	0.13	0.12
item27	0.13	0.14	0.22	0.11	0.20	0.16
item33	0.10	0.07	0.18	0.16	0.20	0.17
item15	0.07	0.12	0.06	0.12	0.13	0.04
item18	-0.11	0.01	0.06	0.10	0.11	-0.01
item26	0.18	0.12	0.13	0.25	0.14	0.18
item29	0.17	0.12	0.07	0.13	0.23	0.18
item35	0.12	0.15	0.14	0.07	0.14	0.21

Covariance Matrix

	item49	item4	item6	item8	item22	item23
item49	0.73					
item4	0.19	0.89				
item6	0.14	0.21	0.83			
item8	0.11	0.23	0.15	0.89		
item22	0.20	0.16	0.20	0.19	0.64	
item23	0.19	0.17	0.08	0.15	0.25	0.70
item42	0.22	0.10	0.11	0.00	0.18	0.11
item45	0.28	0.28	0.19	0.09	0.19	0.20
item9	0.11	0.17	0.15	0.14	0.12	0.15
item13	0.16	0.14	0.22	0.20	0.26	0.11
item30	0.12	0.09	0.09	0.07	0.12	0.11
item31	0.18	0.25	0.17	0.20	0.12	0.13
item37	0.26	0.12	0.15	0.20	0.11	0.12
item39	0.27	0.23	0.12	0.10	0.25	0.28
item41	0.18	0.01	0.01	0.03	0.03	0.09
item43	0.16	0.13	0.21	0.04	0.15	0.16

item47	0.23	0.16	0.17	0.09	0.06	0.09
item48	0.28	0.17	0.22	0.13	0.07	0.12
item25	0.17	0.20	0.04	0.12	0.14	0.13
item40	0.12	0.14	0.11	0.04	0.09	0.14
item44	0.21	0.09	0.15	-0.03	0.13	0.23
item51	0.28	0.19	0.15	0.14	0.17	0.21
item52	0.25	0.06	0.05	0.04	0.19	0.19
item7	0.13	0.02	0.02	0.00	0.14	0.06
item19	0.17	0.26	0.15	0.09	0.23	0.25
item24	0.23	0.26	0.16	0.22	0.29	0.15
item36	0.22	0.16	0.11	0.00	0.19	0.25
item38	0.24	0.18	0.20	0.12	0.14	0.19
item50	0.41	0.10	0.15	0.07	0.21	0.19
item5	0.06	0.16	0.15	0.04	0.10	0.10
item10	0.11	0.19	0.19	0.22	0.12	0.06
item12	0.03	0.10	0.21	0.16	0.17	0.07
item16	0.09	0.22	0.24	0.14	0.24	0.19
item17	0.03	0.07	0.18	0.15	0.16	0.00
item20	0.12	0.10	0.12	0.20	0.29	0.13
item32	0.18	0.17	0.08	0.05	0.04	0.11
item46	0.29	0.15	0.17	0.03	0.16	0.23
item3	0.07	0.08	0.10	0.12	0.04	0.10
item14	0.06	0.15	0.13	0.14	0.14	0.15
item27	0.26	0.19	0.18	0.11	0.20	0.13
item33	0.16	0.10	0.17	0.05	0.09	0.09
item15	0.16	0.20	0.11	0.12	0.21	0.15
item18	0.01	0.09	0.00	0.00	0.07	0.07
item26	0.20	0.16	0.12	0.12	0.19	0.17
item29	0.20	0.20	0.10	0.09	0.20	0.22

item35	0.15	0.21	0.12	0.04	0.11	0.11

Covariance Matrix

	item42	item45	item9	item13	item30	item31
item42	0.87					
item45	0.26	0.83				
item9	0.15	0.03	0.94			
item13	0.09	0.12	0.11	0.56		
item30	0.14	0.12	0.02	0.11	0.51	
item31	0.25	0.34	0.08	0.07	0.16	0.96
item37	0.23	0.27	-0.07	0.09	0.13	0.32
item39	0.22	0.25	0.10	0.11	0.18	0.32
item41	0.37	0.25	0.07	0.06	0.19	0.19
item43	0.16	0.30	0.09	0.12	0.17	0.21
item47	0.22	0.23	-0.02	0.01	0.08	0.22
item48	0.10	0.16	-0.03	0.01	0.13	0.18
item25	0.17	0.11	0.18	0.08	0.08	0.11
item40	0.34	0.24	0.05	0.07	0.15	0.25
item44	0.16	0.28	0.03	0.11	0.15	0.18
item51	0.25	0.28	0.10	0.12	0.16	0.23
item52	0.21	0.28	0.21	0.11	0.07	0.19
item7	0.11	0.08	0.14	0.08	0.12	0.11
item19	0.21	0.22	0.12	0.18	0.11	0.22
item24	0.22	0.20	0.20	0.18	0.11	0.21
item36	0.14	0.16	0.01	0.04	0.14	0.15
item38	0.19	0.26	0.04	0.02	0.09	0.19
item50	0.31	0.37	0.17	0.10	0.13	0.25
item5	0.11	0.12	0.11	0.14	0.11	0.09
item10	0.05	0.16	0.09	0.11	0.08	0.13
item12	0.07	0.02	0.01	0.28	0.13	0.05
item16	0.13	0.05	0.04	0.21	0.10	0.14

item17	0.08	0.03	-0.02	0.13	0.07	0.02
item20	0.13	0.16	0.02	0.23	0.14	0.03
item32	0.14	0.23	-0.05	0.08	0.08	0.23
item46	0.20	0.27	0.13	0.16	0.10	0.16
item3	0.08	0.07	0.19	0.07	0.03	0.14
item14	0.09	0.19	0.13	0.14	0.05	0.20
item27	0.21	0.23	0.00	0.17	0.10	0.13
item33	0.18	0.16	0.01	0.06	0.17	0.21
item15	0.01	0.12	0.05	0.24	0.01	0.03
item18	0.13	0.06	0.06	0.08	0.04	0.04
item26	0.25	0.19	0.11	0.13	0.10	0.16
item29	0.13	0.28	0.11	0.15	0.17	0.17
item35	0.16	0.29	0.06	0.06	0.13	0.29

Covariance Matrix

	item37	item39	item41	item43	item47	item48
	--------	--------	--------	--------	--------	--------
item37	1.16					
item39	0.25	0.83				
item41	0.23	0.16	1.02			
item43	0.22	0.29	0.12	0.59		
item47	0.34	0.16	0.26	0.21	0.68	
item48	0.24	0.12	0.10	0.12	0.29	0.82
item25	-0.08	0.11	0.16	0.07	0.03	0.00
item40	0.21	0.24	0.25	0.20	0.23	0.16
item44	0.10	0.32	0.15	0.29	0.14	0.08
item51	0.32	0.30	0.33	0.23	0.21	0.17
item52	0.14	0.18	0.22	0.23	0.16	0.04
item7	0.01	0.10	0.16	0.09	0.10	0.00
item19	0.20	0.17	0.06	0.12	0.13	0.10
item24	0.21	0.26	0.12	0.17	0.20	0.16
item36	0.19	0.22	0.08	0.24	0.16	0.14

item38	0.36	0.22	0.11	0.26	0.24	0.17
item50	0.26	0.29	0.27	0.32	0.26	0.21
item5	0.01	0.11	0.13	0.10	0.06	-0.01
item10	0.08	0.10	0.04	0.08	0.06	0.07
item12	0.10	0.08	0.04	0.08	0.09	0.12
item16	0.09	0.11	0.06	0.15	0.20	0.11
item17	0.12	0.01	0.06	0.06	0.12	0.15
item20	0.07	0.12	0.10	0.08	0.05	0.08
item32	0.04	0.11	0.20	0.11	0.10	0.14
item46	0.08	0.26	0.12	0.26	0.20	0.17
item3	-0.02	0.09	0.11	0.07	0.07	0.01
item14	0.11	0.16	0.04	0.10	0.07	0.05
item27	0.14	0.18	0.16	0.12	0.19	0.07
item33	0.26	0.13	0.26	0.09	0.22	0.29
item15	0.09	0.13	-0.02	0.12	0.07	0.04
item18	0.14	0.10	0.09	0.12	0.17	0.06
item26	0.07	0.21	0.22	0.15	0.14	0.06
item29	0.10	0.25	0.13	0.22	0.07	0.09
item35	0.17	0.22	0.17	0.16	0.13	0.17

Covariance Matrix

	item25	item40	item44	item51	item52	item7
item25	0.73					
item40	0.09	0.68				
item44	0.10	0.23	0.62			
item51	0.15	0.25	0.26	0.64		
item52	0.25	0.19	0.22	0.25	0.77	
item7	0.07	0.17	0.09	0.14	0.12	0.93
item19	0.17	0.20	0.14	0.17	0.24	0.04
item24	0.20	0.23	0.13	0.19	0.19	0.19
item36	0.05	0.10	0.15	0.16	0.16	0.04

item38	-0.03	0.19	0.10	0.19	0.20	0.02
item50	0.12	0.23	0.25	0.26	0.39	0.09
item5	0.03	0.12	0.12	0.13	0.01	0.16
item10	0.03	0.07	0.11	0.14	-0.01	0.16
item12	-0.07	0.07	0.03	0.09	-0.06	0.09
item16	0.05	0.15	0.10	0.09	0.02	0.24
item17	-0.06	0.11	0.00	0.08	0.05	0.14
item20	0.06	0.13	0.13	0.13	0.15	0.19
item32	0.09	0.19	0.20	0.19	0.15	0.14
item46	0.21	0.23	0.29	0.25	0.35	0.11
item3	0.17	0.12	0.10	0.11	0.15	0.08
item14	0.03	0.18	0.11	0.08	0.08	0.00
item27	0.16	0.19	0.18	0.21	0.15	0.05
item33	-0.05	0.13	0.09	0.14	-0.03	0.08
item15	0.14	0.10	0.12	0.16	0.15	0.06
item18	0.11	0.21	0.05	0.12	0.11	0.21
item26	0.17	0.25	0.13	0.23	0.24	0.12
item29	0.18	0.18	0.22	0.23	0.26	0.12
item35	0.00	0.22	0.18	0.14	0.07	0.13

Covariance Matrix

	item19	item24	item36	item38	item50	item5
	--------	--------	--------	--------	--------	--------
item19	0.89					
item24	0.15	0.89				
item36	0.13	0.09	0.75			
item38	0.17	0.09	0.26	0.72		
item50	0.15	0.24	0.32	0.27	0.79	
item5	0.08	0.19	0.00	0.02	0.01	0.70
item10	0.08	0.17	-0.01	0.02	0.10	0.16
item12	0.03	0.17	0.03	0.02	0.07	0.14
item16	0.08	0.17	0.13	0.13	0.12	0.14

item17	0.10	0.16	0.01	0.13	0.03	0.10
item20	0.16	0.22	0.10	0.06	0.13	0.18
item32	0.07	0.10	0.11	0.11	0.20	0.14
item46	0.25	0.16	0.19	0.13	0.29	0.13
item3	0.10	0.07	0.00	-0.01	0.03	0.09
item14	0.16	0.18	0.06	0.14	0.09	0.13
item27	0.20	0.21	0.07	0.10	0.18	0.15
item33	0.04	0.14	0.16	0.09	0.16	0.14
item15	0.14	0.17	0.04	0.11	0.11	0.11
item18	0.23	0.12	0.05	0.11	0.01	0.09
item26	0.09	0.33	0.12	0.13	0.28	0.02
item29	0.27	0.15	0.16	0.15	0.24	0.08
item35	0.18	0.26	0.14	0.13	0.18	0.20

Covariance Matrix

	item10	item12	item16	item17	item20	item32
item10	0.60					
item12	0.18	0.66				
item16	0.12	0.33	0.76			
item17	0.11	0.27	0.20	0.69		
item20	0.12	0.22	0.27	0.15	0.65	
item32	0.13	0.11	0.13	0.14	0.16	0.74
item46	0.06	0.00	0.08	0.01	0.10	0.14
item3	0.07	-0.01	0.03	-0.02	0.04	0.03
item14	0.15	0.13	0.07	0.08	0.11	0.11
item27	0.22	0.13	0.21	0.14	0.26	0.13
item33	0.19	0.18	0.17	0.15	0.18	0.14
item15	0.05	0.14	0.14	0.06	0.14	0.10
item18	0.05	0.12	0.21	0.08	0.11	-0.04
item26	0.11	0.12	0.21	0.13	0.20	0.13
item29	0.14	0.08	0.08	0.07	0.15	0.15

item35	0.12	0.07	0.13	0.10	0.15	0.22

Covariance Matrix

	item46	item3	item14	item27	item33	item15
item46	0.72					
item3	0.13	0.47				
item14	0.11	0.10	0.59			
item27	0.18	0.10	0.17	0.65		
item33	0.04	0.07	0.08	0.15	0.68	
item15	0.19	0.07	0.17	0.13	0.02	0.57
item18	0.12	0.06	0.05	0.12	0.04	0.13
item26	0.22	0.06	0.10	0.32	0.08	0.18
item29	0.23	0.07	0.09	0.10	0.07	0.15
item35	0.10	-0.02	0.20	0.21	0.17	0.06

Covariance Matrix

	item18	item26	item29	item35
item18	0.77			
item26	0.15	0.89		
item29	0.10	0.16	0.61	
item35	0.07	0.11	0.19	0.66

Number of Iterations = 20

LISREL Estimates (Maximum Likelihood)

Measurement Equations

```
 item1 = 0.37*Regulati, Errorvar.= 0.62  , Rý = 0.18
        (0.056)                  (0.059)
         6.50                     10.62

 item2 = 0.31*Regulati, Errorvar.= 0.65  , Rý = 0.13
        (0.057)                  (0.061)
         5.38                     10.69

item11 = 0.33*Regulati, Errorvar.= 0.60  , Rý = 0.15
        (0.055)                  (0.056)
         6.02                     10.65

item21 = 0.38*Regulati, Errorvar.= 0.52  , Rý = 0.22
        (0.052)                  (0.049)
```

```
         7.33                          10.56

 item28 = 0.39*Regulati, Errorvar.= 0.52  , Rý = 0.22
        (0.052)                      (0.049)
         7.42                          10.55

 item34 = 0.41*Regulati, Errorvar.= 0.52  , Rý = 0.24
        (0.053)                      (0.050)
         7.74                          10.52

 item49 = 0.50*Regulati, Errorvar.= 0.48  , Rý = 0.34
        (0.053)                      (0.046)
         9.52                          10.33

  item4 = 0.39*Regulati, Errorvar.= 0.74  , Rý = 0.17
        (0.061)                      (0.069)
         6.37                          10.63

  item6 = 0.38*Regulati, Errorvar.= 0.68  , Rý = 0.18
        (0.059)                      (0.064)
         6.45                          10.62

  item8 = 0.26*Regulati, Errorvar.= 0.82  , Rý = 0.074
        (0.063)                      (0.076)
         4.07                          10.74

 item22 = 0.41*Regulati, Errorvar.= 0.47  , Rý = 0.27
        (0.051)                      (0.045)
         8.19                          10.48

 item23 = 0.41*Regulati, Errorvar.= 0.54  , Rý = 0.23
        (0.054)                      (0.051)
         7.58                          10.54

 item42 = 0.45*Regulati, Errorvar.= 0.67  , Rý = 0.23
        (0.059)                      (0.063)
         7.54                          10.54

 item45 = 0.55*Regulati, Errorvar.= 0.52  , Rý = 0.37
        (0.056)                      (0.051)
         9.93                          10.28

  item9 = 0.22*Regulati, Errorvar.= 0.89  , Rý = 0.051
        (0.065)                      (0.083)
         3.36                          10.77

 item13 = 0.30*Regulati, Errorvar.= 0.47  , Rý = 0.16
        (0.049)                      (0.044)
         6.12                          10.64

 item30 = 0.29*Regulati, Errorvar.= 0.42  , Rý = 0.17
        (0.046)                      (0.039)
         6.36                          10.63

 item31 = 0.47*Regulati, Errorvar.= 0.74  , Rý = 0.23
        (0.063)                      (0.071)
         7.49                          10.54

 item37 = 0.45*Regulati, Errorvar.= 0.96  , Rý = 0.17
```

```
        (0.070)                    (0.090)
         6.45                       10.62

item39 = 0.52*Regulati, Errorvar.= 0.56  , Rý = 0.32
        (0.056)                    (0.054)
         9.18                       10.38

item41 = 0.39*Regulati, Errorvar.= 0.87  , Rý = 0.15
        (0.066)                    (0.082)
         5.87                       10.66

item43 = 0.45*Regulati, Errorvar.= 0.39  , Rý = 0.34
        (0.047)                    (0.038)
         9.49                       10.34

item47 = 0.40*Regulati, Errorvar.= 0.51  , Rý = 0.24
        (0.052)                    (0.049)
         7.73                       10.52

item48 = 0.34*Regulati, Errorvar.= 0.71  , Rý = 0.14
        (0.059)                    (0.066)
         5.64                       10.67

item25 = 0.25*Regulati, Errorvar.= 0.66  , Rý = 0.087
        (0.057)                    (0.062)
         4.43                       10.73

item40 = 0.44*Regulati, Errorvar.= 0.49  , Rý = 0.28
        (0.052)                    (0.047)
         8.47                       10.45

item44 = 0.42*Regulati, Errorvar.= 0.45  , Rý = 0.28
        (0.050)                    (0.043)
         8.37                       10.46

item51 = 0.50*Regulati, Errorvar.= 0.39  , Rý = 0.39
        (0.049)                    (0.038)
         10.29                      10.23

item52 = 0.44*Regulati, Errorvar.= 0.57  , Rý = 0.25
        (0.056)                    (0.055)
         7.96                       10.50

 item7 = 0.22*Regulati, Errorvar.= 0.88  , Rý = 0.052
        (0.065)                    (0.082)
         3.39                       10.77

item19 = 0.41*Regulati, Errorvar.= 0.71  , Rý = 0.19
        (0.061)                    (0.067)
         6.80                       10.60

item24 = 0.47*Regulati, Errorvar.= 0.66  , Rý = 0.25
        (0.060)                    (0.063)
         7.93                       10.51

item36 = 0.38*Regulati, Errorvar.= 0.60  , Rý = 0.20
        (0.056)                    (0.057)
         6.88                       10.59
```

```
item38 = 0.41*Regulati, Errorvar.= 0.55  , Rý = 0.24
        (0.054)                   (0.052)
         7.61                      10.53

item50 = 0.57*Regulati, Errorvar.= 0.47  , Rý = 0.41
        (0.054)                   (0.046)
         10.62                     10.18

 item5 = 0.32*Knowledg, Errorvar.= 0.60  , Rý = 0.14
        (0.057)                   (0.057)
         5.54                      10.53

item10 = 0.32*Knowledg, Errorvar.= 0.50  , Rý = 0.17
        (0.052)                   (0.047)
         6.17                      10.45

item12 = 0.32*Knowledg, Errorvar.= 0.55  , Rý = 0.16
        (0.055)                   (0.053)
         5.83                      10.49

item16 = 0.40*Knowledg, Errorvar.= 0.60  , Rý = 0.21
        (0.058)                   (0.058)
         6.96                      10.34

item17 = 0.27*Knowledg, Errorvar.= 0.62  , Rý = 0.11
        (0.057)                   (0.058)
         4.81                      10.60

item20 = 0.42*Knowledg, Errorvar.= 0.47  , Rý = 0.27
        (0.053)                   (0.047)
         8.02                      10.15

item32 = 0.35*Knowledg, Errorvar.= 0.61  , Rý = 0.17
        (0.058)                   (0.059)
         6.08                      10.46

item46 = 0.41*Knowledg, Errorvar.= 0.55  , Rý = 0.23
        (0.056)                   (0.053)
         7.34                      10.28

 item3 = 0.17*Knowledg, Errorvar.= 0.44  , Rý = 0.063
        (0.048)                   (0.041)
         3.61                      10.70

item14 = 0.33*Knowledg, Errorvar.= 0.48  , Rý = 0.18
        (0.051)                   (0.046)
         6.42                      10.42

item27 = 0.48*Knowledg, Errorvar.= 0.42  , Rý = 0.36
        (0.051)                   (0.042)
         9.41                      9.83

item33 = 0.34*Knowledg, Errorvar.= 0.57  , Rý = 0.17
        (0.055)                   (0.054)
         6.12                      10.46

item15 = 0.32*Knowledg, Errorvar.= 0.47  , Rý = 0.18
        (0.051)                   (0.045)
         6.25                      10.44
```

```
 item18 = 0.24*Knowledg, Errorvar.= 0.71  , Rý = 0.077
        (0.061)                    (0.067)
         4.03                       10.67

 item26 = 0.45*Knowledg, Errorvar.= 0.68  , Rý = 0.23
        (0.062)                    (0.066)
         7.31                       10.28

 item29 = 0.40*Knowledg, Errorvar.= 0.46  , Rý = 0.26
        (0.051)                    (0.045)
         7.74                       10.20

 item35 = 0.40*Knowledg, Errorvar.= 0.50  , Rý = 0.24
        (0.054)                    (0.049)
         7.50                       10.25


        Correlation Matrix of Independent Variables

           Regulati   Knowledg
           --------   --------
Regulati       1.00
Knowledg       0.86       1.00
             (0.03)
              28.40

                    Goodness of Fit Statistics

                  Degrees of Freedom = 1273
        Minimum Fit Function Chi-Square = 2729.84 (P = 0.0)
Normal Theory Weighted Least Squares Chi-Square = 2994.35 (P = 0.0)
         Estimated Non-centrality Parameter (NCP) = 1721.35
    90 Percent Confidence Interval for NCP = (1565.38 ; 1884.96)

              Minimum Fit Function Value = 11.67
        Population Discrepancy Function Value (F0) = 7.36
      90 Percent Confidence Interval for F0 = (6.69 ; 8.06)
     Root Mean Square Error of Approximation (RMSEA) = 0.056
    90 Percent Confidence Interval for RMSEA = (0.072 ; 0.080)
        P-Value for Test of Close Fit (RMSEA < 0.05) = 0.00

          Expected Cross-Validation Index (ECVI) = 13.69
     90 Percent Confidence Interval for ECVI = (13.03 ; 14.39)
                ECVI for Saturated Model = 11.78
              ECVI for Independence Model = 60.24

 Chi-Square for Independence Model with 1326 Degrees of Freedom =
                           13993.18
                  Independence AIC = 14097.18
                      Model AIC = 3204.35
                    Saturated AIC = 2756.00
                  Independence CAIC = 14329.08
                      Model CAIC = 3672.60
                    Saturated CAIC = 8901.31

                 Normed Fit Index (NFI) = 0.90
              Non-Normed Fit Index (NNFI) = 0.88
```

Parsimony Normed Fit Index (PNFI) = 0.87
Comparative Fit Index (CFI) = 0.88
Incremental Fit Index (IFI) = 0.89
Relative Fit Index (RFI) = 0.90

Critical N (CN) = 120.43

Root Mean Square Residual (RMR) = 0.059
Standardized RMR = 0.047
Goodness of Fit Index (GFI) = 0.93
Adjusted Goodness of Fit Index (AGFI) = 0.92
Parsimony Goodness of Fit Index (PGFI) = 0.87

The Modification Indices Suggest to Add the

Path to	from	Decrease in Chi-Square	New Estimate
item22	Knowledg	7.9	0.37
item13	Knowledg	26.4	0.66
item7	Knowledg	10.6	0.57
item50	Knowledg	10.5	-0.43
item12	Regulati	14.1	-0.52
item20	Regulati	8.3	-0.39
item46	Regulati	32.5	0.81
item29	Regulati	21.8	0.61

The Modification Indices Suggest to Add an Error Covariance

Between	and	Decrease in Chi-Square	New Estimate
item2	item1	20.6	0.19
item11	item1	8.4	0.12
item6	item21	18.2	0.17
item13	item6	9.3	0.12
item13	item8	10.0	0.13
item13	item22	21.7	0.15
item30	item28	8.3	0.09
item37	item9	8.1	-0.17
item41	item34	9.0	0.14
item41	item4	8.0	-0.15
item41	item6	8.0	-0.15
item41	item22	10.6	-0.14
item41	item42	16.7	0.21
item43	item11	13.6	-0.12
item47	item22	11.6	-0.11
item47	item13	11.4	-0.11
item47	item37	12.4	0.17
item48	item49	9.1	0.12
item48	item47	15.1	0.16
item25	item37	13.5	-0.19
item40	item49	11.9	-0.11
item40	item22	9.9	-0.10
item40	item42	15.9	0.15
item44	item8	12.9	-0.15
item44	item39	11.7	0.12
item44	item43	14.9	0.11
item51	item41	14.4	0.15
item52	item6	8.9	-0.12
item52	item25	12.6	0.15
item7	item2	11.3	-0.17
item24	item21	14.9	0.15
item38	item28	22.5	0.17
item38	item13	10.4	-0.11

item38	item37	14.3	0.18
item38	item25	12.0	-0.14
item38	item36	8.1	0.11
item50	item49	18.4	0.14
item50	item4	11.1	-0.13
item50	item52	19.5	0.16
item50	item36	9.9	0.11
item5	item50	10.2	-0.12
item10	item8	12.7	0.15
item10	item52	9.2	-0.11
item12	item6	8.9	0.12
item12	item13	32.9	0.20
item12	item25	10.3	-0.13
item12	item52	14.7	-0.15
item16	item22	8.3	0.10
item16	item45	10.6	-0.12
item16	item52	8.2	-0.11
item16	item7	9.4	0.15
item16	item12	30.2	0.22
item17	item21	8.1	0.11
item17	item12	23.4	0.19
item20	item22	22.6	0.15
item20	item13	11.4	0.11
item20	item31	9.5	-0.12
item20	item12	7.9	0.10
item20	item16	9.1	0.11
item32	item22	8.8	-0.11
item46	item44	10.6	0.11
item46	item52	22.3	0.18
item46	item12	15.0	-0.15
item3	item9	14.1	0.15
item3	item25	13.0	0.13
item14	item11	9.4	0.11
item33	item37	9.0	0.15
item33	item41	10.2	0.15
item33	item47	8.3	0.10
item33	item48	21.3	0.20
item33	item25	10.6	-0.13
item33	item52	17.5	-0.16
item33	item46	8.2	-0.11
item15	item22	8.0	0.09
item15	item42	9.1	-0.11
item15	item13	19.2	0.14
item15	item41	8.1	-0.12
item18	item1	16.8	-0.18
item18	item40	9.1	0.12
item18	item7	8.4	0.15
item18	item19	9.7	0.15
item18	item32	9.1	-0.13
item26	item5	10.1	-0.14
item26	item27	9.4	0.12
item29	item47	9.6	-0.10
item29	item52	8.1	0.10
item29	item27	11.8	-0.11
item35	item45	8.7	0.10
item35	item31	10.6	0.13
item35	item3	9.1	-0.10

Time used: 1.232 Seconds

CURRICULUM VITAE

PERSONAL INFORMATION

Surname, Name: Abdul Aziz, Tian

Nationality: Indonesian (ID)

Date and Place of Birth: October 18th 1985, Bandung

Marital Status: Married

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EDUCATION

Degree	Institution	Year of Graduation
Ph.D. on B.S.	METU Secondary Science and Mathematics Education	2016
B.S.	UPI (Universitas Pendidikan Indonesia) Mathematics Education	2008
High School	SMA Negeri 6 Bandung	2004
Middle School	SMP Negeri 15 Bandung	2001
Elementary School	SD Muhammadiyah 4 Bandung	1998

FOREIGN LANGUAGE

English and Turkish