CREDIT DEFAULT SWAP VALUATION: AN APPLICATION VIA STOCHASTIC INTENSITY MODELS

A THESIS SUBMITTED TO
THE GRADUATE SCHOOL OF APPLIED MATHEMATICS
OF
MIDDLE EAST TECHNICAL UNIVERSITY

BY

MERVE NAMUSLU

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR
THE DEGREE OF MASTER OF SCIENCE
IN
FINANCIAL MATHEMATICS

AUGUST 2016

Approval of the thesis:

CREDIT DEFAULT SWAP VALUATION: AN APPLICATION VIA STOCHASTIC INTENSITY MODELS

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ABSTRACT

CREDIT DEFAULT SWAP VALUATION: AN APPLICATION VIA STOCHASTIC INTENSITY MODELS

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August 2016, 81 pages

The objective of this thesis is to study the pricing of a single-name credit default swap (CDS) contract via the discounted cash flow method with reduced-form survival probability functions depending on stochastic intensity. The ability of the model in predicting the market-observed spreads is tested as well by using bond and CDS data from the US market. In credit risk modeling, the CIR (Cox-Ingersoll-Ross) model is used. The main reason for using a reduced-form model in pricing the CDS contracts is the advantages of such models in terms of being more flexible, practical and tractable. In model calibration, each sample firm's bond price is used while determining the optimal set of parameters for the CIR default intensity process. For this purpose, the firm's stochastic default probabilities are estimated within the least squares framework. Data on two of the Dow Jones 30 Index constituents, the Coca-Cola Company and JPMorgan Chase, are used in the analyses. The term structure of daily bond prices that are used to estimate the hazard rate parameters and the daily prices of the firms' CDS contracts that are used to test the success of the model are obtained from Thomson Reuters. The model's success is tested over two distinct time periods. The first period is from July 2008 to September 2008 (pre-crisis) and the second period is from January 2016 to March 2016 (post-crisis). The proxy for the risk-free interest rate is the federal funds rates.

Keywords : Credit default swap, intensity-based model, CIR model, default intensity, survival probability, probability of default

ÖZ

KREDİ TEMERRÜT SWAP PRİMİNİN HESAPLANMASI: BİR STOKASTİK YOĞUNLUK MODELİ UYGULAMASI

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Ağustos 2016, 81 sayfa

Bu çalışmanın amacı indirgenmiş nakit akımı yöntemi kullanılarak kredi temerrüt swap priminin hesaplanmasını stokastik yoğunluğa bağlı olan indirgenmiş formdaki temerrüte düşme olasılığı fonksiyonu ile incelemektir. Ayrıca kullanılan bu modelin piyasa kredi temerrüt swap primlerini tahmin edebilme becerisi bono ve tahvil ve kredi temerrüt swap verileri kullanılarak test edilmiştir. Kredi riski modellemesinde CIR (Cox-Ingersoll-Ross) modeli kullanılmıştır. Bu tezde kredi temerrüt swap priminin indirgenmiş formdaki bir model ile hesaplanmasındaki temel neden bu modellerin daha pratik, daha kolay uygulanabilir ve çözümlenebilir olmasıdır. Kalibrasyon aşamasında herbir firmanın bono ve tahvilleri, CIR temerrüt yoğunluğu hesaplama modelinin optimal parametre grubunu belirlemek için kullanılmıştır. Bu amaçla, firmaların stokastik temerrüt olasılıkları en küçük kareler yöntemi kullanılarak hesaplanmıştır. Analizde Dow Jones 30 endeksinde bulunan firmalardan Coca-Cola Company ve JPMorgan Chase kullanılmıştır. Stokastik yoğunluk fonksiyonlarını hesaplamak için kullanılan günlük bono ve tahvil verileri ve modelin başarısını test etmek için kullanılan kredi temerrüt swap kontratı verileri her firma için Thomson Reuters veri tabanından alınmıştır. Modelin başarısı iki farklı zaman aralığında test edilmiştir. Birinci zaman aralığı Temmuz- Eylül 2008 (kriz öncesi) ve ikinci zaman aralığı Ocak-Mart 2016 (kriz sonrası) şeklindedir. Risksiz faiz oranı olarak New York Merkez Bankası bankalararası para piyasası faiz oranları kullanılmıştır.

Anahtar Kelimeler : Kredi temerrüt swap primi, indirgenmiş formdaki modeller, CIR modeli, yoğunluk değeri, batmama olasılığı, batma olasılığı

To My Family and
My Fiance

ACKNOWLEDGMENTS

I would like to express my very great appreciation to my thesis supervisor Assist. Prof. Dr. Seza Danışoğlu and my thesis co-supervisor Dr. Hande Ayaydın Hacıömeroğlu for his patient guidance, enthusiastic encouragement and valuable advices during the development and preparation of this thesis. Their willingness to give his time and to share their experiences has brightened my path.

I woud like to thank also my thesis defence committe for their encouragements and valuable comments.

I am grateful to all members of the Institute of Applied Mathematics for their understanding, help and supply of effective study programs.

Also, I would like to thank the Scientific and Technological Research Council of Turkey (TÜBİTAK) for providing funds during my master science education and also appreciate the assistance of members working at TÜBİTAK-BİDEB.

Furthermore, I would like to express my pleasure to having such a great family and thank to each of them for their endless love and existence.

Finally, I would like to specially thank my dear fiance Oğuz Yıldız to whom I am grateful beyond words. It would have been impossible for me to achieve any of my educational goals without his sacrifice, encouragement and support.

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CHAPTER 1

INTRODUCTION

One of the fundamental risks that financial and non-financial institutions are exposed to is the credit risk. Credit risk is the uncertainty in a counterparty's ability to meet its financial obligations. In the years leading to the Global Financial Crisis (GFC) of 2008, credit markets grew rapidly and several new financial instruments were designed that allowed the market participants to trade credit risk around the world. As a matter of fact, the fundamental nature of credit risk is not a new idea since it is the first and foremost risk that is addressed by the Basel Committee regulations since the 1980s. In spite of their popularity and very high volumes of trade, most credit-risk-based financial instruments have quite complex cash flow and payment structures that are difficult to understand. In fact, one of the main reasons of the credit crisis that started in the US is argued to be this lack of understanding of the credit and liquidity risk concepts in financial markets. According to the Basel Committee, today banks are still exposed to credit risk as a leading threat to the viability of the banking sectors in the long-run. As a result, it has become even more important for all participants of the financial markets to develop a better understanding of the credit risk [89].

Since its emergence in the 1990's, the credit derivatives market has experienced dramatic growth because it provides financial institutions and investors with adaptable tools to create synthetic risk exposure, customized by focusing on their specific needs. Before the increase in the volume of credit derivatives, risk management techniques were limited to traditional financial analysis. The International Swaps and Derivatives Association (ISDA) introduced credit derivatives in 1992 at their annual meeting. Since the market's inception in 1993, financial institutions have developed many new and exotic credit risk related derivative products as major hedging instruments. Some examples of such instruments are the credit default swap (CDS), collateralized debt obligations (CDOs) and credit linked notes (CLN) [37, 55, 51].

The CDS contracts are one of the most commonly traded types of credit derivatives. According to ISDA, the notional amount of credit default swaps, representing one of the largest and fastest growing financial product markets globally, passed $2 trillion in 2002 [1]. Moreover, the notional amount outstanding of credit default swaps (CDS) grew by 37 percent to $62.2 in the second half of 2007 from $45.5 trillion at mid-year. The CDS market's notional principal growth in 2007 was 81 percent from $34.5 trillion at year-end 2006 [2].

A credit default swap is a contract providing a protection to its buyer against the losses caused by the occurrence of credit events like downgrades or defaults. The underlying asset for a CDS contract is a debt instrument issued by a corporation or a government. In exchange for the protection provided by the CDS contract, the buyer of the CDS makes periodic fixed payments (CDS spreads/premiums) to the protection seller until the contract's maturity or until a credit event occurs, whichever comes first. The CDS contracts can be written on single-name or a basket of reference entities (underlying assets). The contracts are quoted with a CDS spread and a recovery rate on reference entities in the market where bid quotes represent the price paid by the protection buyer and offer quotes represent the price received by the protection seller [88].

The CDS contract has some advantages that made it one of the most popular credit derivative instruments. For example, they are the most liquid hedging tools among credit derivatives. As indicated by ISDA, CDSs strengthen the financial system. By using CDSs to transfer credit risk to other investors, banks can provide more debt to the market. Also, CDSs help reduce credit risk concentration since they distribute the risk throughout the financial market. Moreover, credit default swaps provide information about the credit quality of firms since they are important indicators of market's perception of the credit risk of firms. They have very high trading volume and they respond to underlying asset's price changes rapidly [7].

In fact, the importance of credit risk management has been well understood by financial institutions during the GFC. Starting with the subprime mortgage problems in 2007, GFC created a need for the better understanding of the credit derivative instruments and their associated risks. As part of environment preceding the GFC, some financial institutions took large positions in credit derivatives and loaned an excessive amount of subprime mortgages. As a result of the complications that were created by these positions, some of the famous and major players in the financial markets, such as Bear Stearns, AIG, Lehman Brothers, Freddie Mac. and Fannie Mae needed government assistance to cover their losses so that they could survive in the aftermath of the GFC. Some of these firms even bankrupt as a result of their credit risk exposure [76].

Following the GFC, it was realized that although credit derivatives have a lot of advantages, they may also expose their holders to large amounts of risk. Credit derivatives were blamed for being the main reason for the GFC because of their risks due to their complex structures and large notional amounts arising from their speculative use. Especially after the bankruptcy of the major derivatives dealers during the credit crisis, the counterparty risk estimation and management has become an important issue for the financial system as a whole. Not surprisingly, the modeling of credit risk also gained more interest from financial institutions, regulators and academics [78, 43]. ISDA and Bank of International Settlements (BIS) imposed heavy regulatory measures on CDS trading in order to strengthen the financial system. With the new guidelines, financial institutions had to meet certain obligations such as providing detailed reports about their projected risks to regulatory institutions after determining the mark-to-market value of their portfolios accurately. From an academic perspective, a significant amount of research has been conducted in order to improve the pricing methodology of credit derivatives. Needless to say, developing effective financial regulations needs a better understanding of the pricing practices for the related instruments. For that purpose, after the credit crisis, practitioners as well as academics searched for better and effective models for estimating credit derivative prices and associated risks that might be faced as a result of a position in these instruments [37].

In line with the increased interest in financial markets, this thesis puts forward a methodology that first evaluates the credit risk of the underlying asset and then uses this information to price the credit default swap contract written on that asset.

For the purpose of pricing the CDS spreads, two types of credit risk models have been proposed in the literature since the 1970s: structural (firm value-based) and reduced form (intensity-based) models. In the structural approach, the probability of default is associated with the capital structure of a firm and default time is defined as the time that firm's asset value falls to a sufficiently low level compared to its liabilities. In the reduced-form approach, default is modeled as a jump process with an intensity function and default time is defined as a non-negative random variable with a distribution depending on economic factors. In reduced-form models, at any instant, there is a possibility of default [49]. In this thesis, a reduced-form model is used to price a CDS contract. The main reason for choosing a reduced-form model is its advantage of being more flexible, practical and tractable.

The specific objective of this thesis is to study the pricing of a single-name CDS contract via the discounted cash flow methodology. First, the survival probability for the underlying asset is estimated by using a reduced-form approach that models the default intensity within a stochastic framework. The widely accepted Cox-Ingersoll-Ross (CIR) model is used during this step of the study. Second, the estimated survival probabilities are used to price the CDS contracts. Third, the ability of the pricing model is tested by using market data.

This thesis makes several empirical contributions to the existing literature in exploring the information content of CDS prices for credit risk. Differently from other studies on pricing CDS contracts via the discounted cash flow methodology, this thesis utilizes a stochastic calibration process to determine the optimal set of parameters for estimating the survival probabilities for the issuer of the underlying asset of a CDS contract. In addition,the ability of the model in predicting CDS prices is tested by using market data that allows to perform the tests in two different industries (financial versus non-financial) and two different time periods (pre-GFC and post-GFC). The remainder of the thesis is structured as follows: Chapter 2 provides a detailed literature review of the pricing methodologies of credit default swaps. Chapter 3 discusses the characteristics of CDSs and provides background information on the credit derivatives market. This chapter also discusses the role of the CDS contracts in the GFC. Chapter 4 introduces a pricing methodology for valuation of CDS contracts, discusses the pros and cons of different pricing models and justifies the selection of the model used in the thesis. Chapter 5 provides information regarding the calibration methodology and presents the data used in the empirical part of the study. Chapter 6 presents the results of model estimation as well as performance tests regarding the ability of the model in predicting CDS spreads. Finally, Chapter 7 concludes the thesis and provides recommendations to further improve the selected model.

CHAPTER 2

LITERATURE REVIEW

There is a large literature on credit risk modeling and the number of studies conducted on the subject increased especially after the Global Financial Crisis of 2008 [42]. Since credit risk is the risk of default on a debt due to the borrower's failure to make payments, a credit risk model generates an estimate of the default risk of the debtor. The spread paid on a CDS contract is paid as a compensation for bearing this credit risk and its estimation has become a popular topic in the literature along with the expansion of the credit risk studies. The rapid growth in the CDS market in the 2000s brought about the development of several methods for pricing the CDS spreads. This chapter of the thesis provides a detailed review of the existing literature on the methods of determining the CDS spreads based on the default probability of the reference obligor.

When the literature on CDS contract pricing is examined, it is seen that there are two major lines of research on the subject. The first line of research uses structural models and the second one utilizes reduced form models in the process of CDS pricing. In structural models, the idea is to describe the default as an event based on the internal structure of the firm, so that default is driven by the value of firm's assets and debts [48]. In other words, structural default models associate creditworthiness of a firm with its economic and financial status. These models depend on the assumption that asset prices follow a geometric Brownian motion which is the simplest continuous stochastic process. Further, these models assume that the default occurs endogenously when the market value of the firm falls below either some predetermined default boundary or the value of its debt [98]. These types of models are based on the option pricing model of Black and Scholes(1973) and Merton (1973) [10, 83]. The literature of firm-value based models on credit risk starts with the 1974 study of Merton in which he applies the option pricing theory to the modeling of the debt of a firm [84, 44, 34].

Observing that both equity and debt can be seen as options on the value of a firm's assets, Merton (1974) proposes a simple firm-value-based model to price risky corporate debt. His model is typically accepted as the first modern model of default. The intuition behind the Merton model is that default occurs at the time of debt maturity if the value of a firm's assets is lower than its outstanding debt maturing at a promised time. In Merton's model, the capital structure of the firm is assumed to be composed of equity and debt which is represented by a zero-coupon bond with maturity T and face value D. Following this assumption, the equity of the firm can be regarded as a European call option written on the asset value with maturity T and strike price D.

A default occurs at time T if the option is not exercised, since it means that the face value of the bond is larger than the value of the firm at time T. This approach indicates that default can only happen at the maturity of the zero-coupon bond. Following this financial invention, Merton derived an explicit formula for the value of the risky bonds which can be used to estimate both the credit spread between a risky and a default-free bond and the default probability of a firm [34, 13, 56, 76].

Besides being simple and intuitive, Merton's model is a major breakthrough in the field of pricing defaultable debt [94, 7]. However, it contains some limitations because of the assumptions made in order to set up the model and this makes the model unrealistic, therefore, not applicable in practice. As Mason and Rosenfeld (1984) and Jarrow and Van Deventer (1999) have shown, as a result of such limitations, Merton's model fails to predict the default probability properly, and when it is used as an input in CDS pricing, it tends to underestimate the CDS spreads [65, 63, 45].The first limitation of the Merton model is the assumption that the default or credit event can only happen at maturity. However, in practice, default can take place at any time during the life of a bond. Second, the assumption regarding the capital structure of the firm is not realistic since most firms do not have debt that consists of only zero-coupon bonds. In practice, most firms have more complex debt instruments [94]. Third, Merton assumes that firms cannot issue additional debt until maturity. Therefore, the marginal default probabilities decrease in the Merton model. In practice, as a result of firms issuing additional debt while they have other debt already outstanding, the marginal default probabilities may increase. Moreover, some restrictions arise from the constant interest rate assumption and the omission of the correlation between credit risk and interest rates [7]. An additional important problem about Merton's model is the unobservability of the firm's asset value process which leads to some valuation errors in the pricing process [37]. Over the years several structural models that extend and implement Merton's model have been proposed in the literature in order to overcome these and some other shortcomings of the Merton model [94, 45].

A pioneering first passage model was proposed by Black and Cox (1976) as an extention of the Merton model [9]. In their model, Black and Cox relax the assumption that default can only happen at maturity [37]. This study is the first to introduce the so called first-passage models in which a firm goes into default when its asset value falls for the first time below a certain threshold [34]. On the one hand, similar to the Merton model, Black and Cox also describe the asset value process as a geometric Brownian motion [49]. On the other hand, unlike the Merton model, Black and Cox's model adjusts for more advanced debt structures and defines the default time as the first time that a firm's asset value hits an exogenously fixed, time-dependent exponential default barrier. These default barriers represent safety covenants included in the debt indenture. When the asset value of the firm hits the fixed covenant level, debtholders have the right to force the firm into default and obtain control over the firm's assets, while the shareholders receive nothing. That is to say, the equityholders have a barrier option on the firm's assets getting knocked out if the firm's asset value hits a fixed barrier. In such a setting, it is possible to estimate the risk-neutral survival probabilities until the time that the value of the firm's assets stays above the fixed barrier. Since in the first-passage models default is possible at any time, default probability, and as a result, credit spreads are larger in magnitude compared to those obtained by the Merton model. This result may imply a better approximation of market spreads by the Black and Cox model [94, 7, 9].

Recall that in Merton model, the debt of the firm consists of a single zero coupon bond. Geske (1977) extended the Merton model by adding more complex debt structures like coupons and payment limitations [40]. He modeled the firm's debt structure as a collection of several coupon bonds and by taking the equity of the firm as a compound option, Geske obtained a formula to price the firm's coupon-paying debt [7, 43]. On each payment date, the equityholders have the option to continue controlling the firm until the next coupon date by paying the exercise price that is equal to the coupon payment. The last option held by the equityholders becomes an option on the value of the firm with an exercise price equal to the last coupon payment plus the bond's face value. The shareholders finance the coupon payments by issuing new equity. If the equityholders are unable to refinance -in other words, when the value of the equity is less than the coupon payment- the shareholders decide not to pay the coupon and the firm defaults. In such case, the bondholders take control of the firm. Even though the Geske model introduced some improvements over the Merton model, the assumption that default can occur only on the debt payment dates makes his model unrealistic as well [94, 7, 40].

There are additional models in the literature that, similar to Black and Cox, relax the assumption that default can only occur at maturity, including the models proposed by Kim, Ramaswamy and Sundaresan (1993), Hull and White (1995), Longstaff and Schwartz (1995) and Bryis and de Varenne (1997) [67, 53, 80, 12, 76].One of the common shortcomings of these models is the unrealistic assumption that the interest rate is constant throughout the life of the bond. In order to relax the constant interest rate assumption, stochastic interest rate processes are introduced and used in several structural models [43]. For this purpose, Longstaff and Schwartz (1995) build on the Black and Cox model in their study and use the model of Vasicek (1977) for the risk-free interest rate process [37, 80, 96]. Longstaff and Schwartz model a stochastic interest rate, assume the interest rate and the credit risk to be correlated, but the default barrier in their model is constant. Bryis and de Varenne (1997) also use a generalized Vasicek process for modelling the interest rate and adjust the Black and Cox model with the aim of obtaining a stochastic default level equal to the principal debt payment discounted at the risk-free interest rate [12]. Another model with a stochastic threshold is proposed by Kim et al. (1993) [67]. In this model, Kim et al. assume the default barrier to be a function of the coupon rate and model the risk-free interest rate process by using the Cox-Ingersoll-Ross Model (CIR Model) described in Cox et al. (1985) [18]. By replacing the Vasicek model with the CIR model, Kim et al. are able to avoid generating negative interest rates as a result of parameter selection [7, 67].

The traditional structural models that are extensions of the Merton model use a diffusion process where the firm's asset value is assumed to follow a continuous path within a stochastic process [49]. With the increased popularity of the CDS contracts in the early 2000s, it became apparent that such structural models did not generate prices that were consistent with the unusually high spreads that were observed on short-term contracts [78]. In the structural models, the default of a firm is not an unexpected event because under this process, a sudden drop in the firm value is impossible. However, in reality, the default probability of firms that are not in financial distress is not equal to zero and downward jumps in the asset values may result in unexpected default events. Zhou's (1997) study is the first one to address such problems in stochastic models of asset value and he proposed that it would be more reasonable to include random jumps in the asset value process. In his model, the dynamics of the firm asset value consist of two random components: a continuous diffusion component as in the classical structural models accounting for the marginal changes in the firm's asset value and a discontinuous jump component accounting for the unexpected shocks in the firm's value process [94, 7, 99][1]. In Zhou's model, default may occur from either the diffusion or the jump-diffusion process such that for the first, the asset value of the firm can hit the default barrier for only one time whereas in the second one, it is possible for the asset value to be below the default barrier [7]. The continuous changes in the value of the firm assets are modeled as a geometric Brownian motion while the jumps are modeled as Poisson-distributed events. Zhou (2001) argued that adding jumps into the diffusion models is more convenient when modeling the default risk of a firm and he used a geometric jump-diffusion process to explain the level of a credit spread [49, 8, 100].

Leland (1994) proposed to use endogenous default barriers in his study due to the tendency of the stockholders to choose the default barrier level maximizing the firm value [4, 8]. The extensions to Leland's model were proposed by Hilberink and Rogers (2002) who introduced a Lévy process with no positive jumps and by Lipton (2002) who considered log-exponentially distributed jumps [50, 74, 7, 19, 77, 49, 13].

Traditionally, credit risk models have focused on the role of downward jumps for explaining negative surprises in credit quality. However, by allowing both upward and downward exponentially distributed jumps, Kou (2002) modeled the asset value of the firm as a double exponential jump diffusion (DEJD) process. Two-sided exponential jumps allow for both the under- and overreaction of the market to the surprise arrival of exogenous news or information. Kou's model is a good representation of the leptokurtic and highly skewed returns that often exist in financial markets. In fact, the double exponential jump diffusion model initiated by Kou is a particular case of the Lévy processes. The double exponential distribution has two empirical features. First, it has the leptokurtic feature of the jump size that makes an important contribution to the literature by providing more peaked and more fat-tailed return distributions compared to those provided by the Merton model. Second, it has the memoryless feature that provides an easier calculation of expected mean and variance terms and makes it easier to solve the problem of overshoots due to the fact that the overshoot of a jump through a strike is also exponential [68, 8, 19].

Some of the other studies on double exponential jump diffusion models of CDS spread pricing are carried out by Ramezani and Zeng (1998) who assumed that good and bad news are produced by two independent Poisson processes in their Pareto-Beta Jump-Diffusion model in which jump magnitudes are drawn from the pareto and beta distributions; Asmussen et al. (2004) who obtained an explicit formula for the first-passage time problem; Kou and Wang (2004) who extended the analytical tractability of the Black-Scholes-Merton model for path-dependent options with jump risk via Laplace

[1] Jumps are rare events that are associated with the arrival of new information that has the potential of affecting firm value, such as unexpected financial results, macroeconomic or company-specific announcements etc.

transformation; Asmussen et al. (2008) computed the finite-time survival probabilities exactly; Madan and Schoutens (2008) who applied a fast double Laplace transformation inversion in order to calculate survival probabilities; and, Chen and Kou (2009) who studied credit spreads, jump risk, optimal capital structure and the implied volatility of equity options via a two-sided jump model extending the Leland–Toft endogenous default model [91, 5, 69, 17, 49, 82, 81]. Ramezani and Zeng (2007) stated one advantage of DEJD as the linearity of the process with independent increments and an explicit transition density due to which they developed the econometric approach to the DEJD process parameters estimation [92, 8].

The literature presents other reasonable alternatives for CDS pricing. For instance, a pure- jump process with infinite activity can reflect both frequent small moves and rare large moves. Carr et al. (2002) stated with some evidence from market prices of equity and options that risk-neutral processes for equity prices appear to be pure-jump processes of infinite activity and finite variation [15]. Hao, Li and Shimizu (2013) addressed this important issue by studying the first passage time model over a fixed default barrier for a pure-jump subordinator with a negative drift. They considered an asset value process of infinite activity but finite variation in the model of Madan and Schoutens (2008) [49].

Although the structural models have made important progress over the years in pricing CDS spreads by making a strong reference to the financial characteristics of the firm, in other words, by using a firm's balance sheet and stock market information, they still have some limitations which led to the introduction of reduced-form models with the aim of overcoming such limitations [37, 76, 31]. In the traditional structural models, the value of the firm follows a diffusion process so that default can be predicted just before it happens since there is no sudden drop in the firm value; therefore, firms never default by surprise [31, 100]. Reduced-form models do not relate the default with any observable characteristic of the firm, and, in such models, the default is defined as a stochastic variable i.e. an unexpected event whose likelihood is evaluated by a default intensity process [76, 44]. Since in practice the default of a firm is often unexpected, reduced-form models have a superior ability to capture such events, and, thus, outperform the traditional structural models [37]. Even though Zhou (1997) came up with the idea to include random jumps in the asset value process in structural models, the empirical performance of this improved version of the structural-form models is still relatively poor [31, 48, 99].

The biggest challenge with the structural models is that they require very detailed internal company information, such as the firm's asset value, volatility of the asset values, outstanding debt and its maturity, that may not be commonly observable in the market if the firm is not a publicly held firm [58, 36]. Therefore, such models may be hard to calibrate. Reduced-form models require less detailed information that is typically available to the market participants so these models are easier to implement in practice, allowing for the valuation of other securities and derivatives [76, 94]. In fact, in reduced-form approaches, the probability of default can be obtained from market prices [44]. Also, while structural approaches cannot incorporate credit rating changes occuring quite frequently for risky corporate debt, reduced-form models are able to catch such changes [31].

The other disadvantage of structural models is that they cannot easily fit a given term structure of spreads, specifically, they cannot easily match short term yield spreads while reduced form models have the flexibility to refit the spreads of several credit instruments, also short term yields spreads, of different maturities [47]. In fact, different parameters may be obtained when calibrating a reduced-form model to CDS market quotes and this provides a good fit to the market [31]. Therefore, comparing with the structural approach, the reduced-form approach is flexible and tractable [48]. Moreover, reduced-form models are computationally faster. In this thesis, a reduced-form model is used due to the abovementioned advantages of such models.

In reduced-form models, default is described via an exogenous jump process. The default time is the first jump time of a Poisson process with a stochastic or deterministic arrival intensity (hazard rate); in other words, it is a random stopping time of a given hazard rate process [94, 48]. A Poisson process is a stochastic process and usually used to model rare or discretely countable events. Since defaults are rare and discretely countable, it fits for modeling defaults [94]. In such models, a firm's default which does not depend on the value of the firm's assets may occur with a positive probability at each instance of time because firm defaults whenever the exogeneous random variable shifts which is an unexpected event [90]. Reduced-form or intensity based models aim to determine the statistical characteristics of the default time instead of looking for the reasons of the default [45].

Jarrow and Turnbull (1995) are the first to suggest using the Poisson process for pricing the derivative securities involving credit risk. They introduced the reduced-form models in 1992 and then these models have been widely mentioned by later studies [62, 90]. In their approach, they directly model the probability of default itself in order to predict the default time instead of modeling the firm's equity as in the structural models. They achieved this by using a security pricing model when estimating the probability of default [45]. Jarrow and Turnbull assumed that the stochastic process of default-free term structure and the default process are independent; therefore, term structure and default issues can be handled separately in their model. In reduced-form models, the goal is to model the intensity, so that the probability of default which depends on the intensity (hazard rate) process can be calculated. Jarrow and Turnbull model has two case, namely homogeneous and inhomogeneous cases. As seen in Jarrow and Turnbull (1995), the simplest version of the reduced-form models is the homogeneous Poisson process case where the intensity is taken as constant. However, with constant intensity, the default time is exponentially distributed and only constant CDS spreads can be obtained across maturities which is unrealistic since the CDS spreads in reality are upwards and downwards sloping for a given maturity [94]. Therefore, homogeneous intensity model fails when calibrating to the market data. The solution for this problem is to define the intensity as a deterministic function of time which is to get an inhomogeneous case for Jarrow and Turnbull's model. By an inhomogeneous Poisson process, a more realistic CDS spread curve can be obtained in the calibration [94].

Jarrow, Lando and Turnbull (1997) proceeded with the studies on reduced-form models [61]. They extended the work of Litterman and Iben (1991) by studying the term structure of credit risk spreads in a model in which the default process follows a discrete state space Markov chain in credit ratings [79]. They are the first to explicitly incorporate credit rating information into the valuation method in their model and this approach is practical when estimating the probability of being in a given credit class for a certain time period starting from a specific credit class. They assumed that the default-free term structure and the default process are statistically independent so that it would be adequate to determine a distribution for the default time on the purpose of uniquely determining the progress of the risky debt's term structure with the martingale probabilities. Therefore, they made contribution to the literature of reduced-form models in the sense that they explicitly model this distribution as the first hitting time of a Markov chain where the credit ratings and default are the relevant aspects. They exemplified the fact that the default probability increases as the credit rating decreases by showing that default probability for first class is lower than the second class and default probability for second class is lower than the third class among three different ratings [90].

Although inhomogeneous intensity model can reproduce the CDS term structure outstandingly when calibrated to the market data, it misses out an important point. That is, in a model with deterministic intensity, the obtained survival probabilities are also deterministic and the only information related to default risk arriving over time is the only fact of survival to date. However in reality, there may be new information associated with the credit quality of the firm, beyond the survival, arriving as time passes. Arrival of new information would change the intensity randomly. Therefore, it can be assumed that the intensity varies with an underlying state variable such as credit ratings, equity price of an issuer or distance to default. In this sense, modeling intensity as a random process is reasonable. It was Lando (1998) who proposed the stochastic intensity model [72]. Following Lando, default is modeled as the first jump of a doubly stochastic Poisson process i.e. as a Cox process. This model is not different from that of Jarrow and Turnbull, in fact, the only difference is that default follows a Cox process instead of a Poisson process. A popular choice of this approach is modeling intensity with CIR process named after Cox, Ingersoll and Ross (1985). In fact, they suggested a single factor model for the term structure of interest rate; however, it can be easily applied for intensity process by changing the interest rate for the intensity [18, 94, 29, 73].

Similar approach was used by Duffie and Singleton (1999) as they assumed that the default is an unexpected event given by a hazard-rate process as in the other reduced-form models. Their framework was different from the other reduced-form approaches in the sense of parametrization of losses at default in terms of the fractional reduction in market value with respect to risk neutral probability measure [27, 57, 90]. They focused on applying the techniques which is originally developed for default-free term structure modeling to the term structure of defaultable interest rates [51, 90]. They indicated that when estimating default hazard rates, it is more tractable to use loss-of-market value assumption rather than a loss-of-face value assumption. They showed that the present value of the promised payoff which is discounted by the default adjusted short rate would give the price of relevant defaultable claim. An important characteristic of their valuation method is that the mean-loss rate is given exogenously meaning that default rates and fractional recovery rates does not depend on the value of the contingent claim and so the securities with default risk can be priced as in the standard valuation models by using a default adjusted short rate instead of default-free rate [90].

In their article, Madan and Unal (1998) presented one of the first intensity-based credit risk models by assigning the intensity to be a function of the excess return on the firm's equity which implies that their model combines characteristics of both structural and reduced-form models so that it is called a hyrid credit risk modeling [82]. When the intensity is assumed to change with time and to be directly linked to the properties of the loan, default intensities can be treated as they are dependent on some observable variables affecting probability of default such as market equity price, GDP index, duration of loan. In fact, Carling et al. (2007) showed that accounting variables and macroeconomic variables are most powerful to explain the credit risk [14, 90].

Duffee (1999) used the framework of Duffie and Singleton (1999) with the aim of pricing corporate bonds which are not callable. Duffee assumed that the instantaneous default probability of a firm follows a translated single-factor square-root diffusion process and default process is correlated with the default-free term structures that means default is an unexpected event represented by a jump in a Poisson process and correlated with the term structure. With the specifications in Duffee's model, closed-form solutions to risky zero-coupon bond prices can be obtained [23, 90].

One of the most famous reduced-form models of CDS valuation belongs to Hull and White (2000). Their model starts out by calculating the risk-neutral probability of default at different future times from the yields on bonds issued by the reference entity. To do this, the model assumes the default probability of the risky bond's issuer as the only explanation of a price differential between a riskless and risky bond so that this differential equals to the present value of the cost of reference entity's default. The model calculates the probability of default of the firm at different future times by using expected recovery rate and bonds with different maturities [55]. Hull and White (2001) extended their valuation method by incorporating the default correlations between different entities so that their new model reflects counterparty default risk in CDS valuations. As the default correlation between CDS buyer and seller increases, the impact of the default correlation on the counterparty default risk also increases [54, 56].

Some other researches on CDS contract valuation with reduced-form models are also remarkable for the literature. Nielsen and Ronn (1998) utilized non-linear least squares on cross-sectional data to calculate a log-normal spread model [87]. Duffee (1998) and Keswani (2000) applied maximum likelihood method with Kalman filtering to get parameter estimates of CIR process from time series data where Dülmann and Windfuhr (2000) and Geyer, Kossmeier, and Pichler (2001) applied it to obtain parameter estimates of CIR and Vasicek models for instantaneous credit spread [22, 66, 30, 41]. Janosi, Jarrow and Yildirim (2002) and Frühwirth and Sögner (2001) calculated the hazard rate parameters from cross-sectional data on a day-to-day basis by using non-linear squares [60, 39]. Duffie, Pedersen and Singleton (2003) estimated a multi factor model with Vasicek and CIR processes by using an approximate maximum likelihood method [51, 26]. Jarrow and Yu (2001) applied the primary-secondary framework in a way that default process of secondary firms are dependent on macrovariables such as interest rates and also on the default processes of primary firms whose default process is merely dependent on macrovariables [64, 31]. In a simple jump-diffusion setting for correlated default intensities, Duffie and Garleanu (2001) showed the impact of correlation for the market valuation by following a pre-intensity model in which default time of each obligor has some pre-intensity process [25, 36]. Duffie and Singleton (2003) showed that the relation between the intensity and the survival probabilities can be seen as the relation between a zero coupon bond and the short rate [28, 94]. Leung and Kwok (2005) introduced the concept of interdependent default correlations between the protection buyer, the protection seller and the reference entity [75, 31]. One attempt to improve the reduced-form models by linking default intensities to the firm's fundamentals is Bakshi-Madan-Zhang (2006) (BMZ). They related default intensities to several measures of default risk and found that the leverage gives the best model in that sense. BMZ also showed that yield spreads of corporate bond are susceptible of short term interest rate [6, 93]. By relaxing default intensity specifications and allowing heterogeneous default probabilities, Mortensen (2006) extended the model of Duffie and Garleanu (2001) and presented a semi-analytical valuation methodology in a multi-variate intensity-based model in which default intensities are modeled as correlated affine jump-diffusions [86, 36]. Dunbar (2007) introduced a three-factor reduced-form model incorporating liquidity proxy of market conditions [32].

Literature on modeling CDS spreads is still growing. Some of the abovementioned CDS pricing models will be investigated in detail in the following sections. In this thesis, Cox model will be applied to model the default time, in other words, the intensity function will be considered as a doubly stochastic Poisson process which provides flexibility by letting the intensity not only depend on time but also allowing it to be a stochastic process. In order to model the intensity function, CIR process will be used.

CHAPTER 3

CDS CONTRACTS AND THEIR ROLE IN THE GLOBAL FINANCIAL CRISIS OF 2008

3.1 Credit Default Swap Contracts

Credit risk is the probability assigned to the risk of a loss due to the inadequacy of a counterparty in a financial contract to meet its obligations [7]. This risk can be isolated and traded by credit derivatives through a partial or total risk transfer [21]. A credit derivative for which the payoff depends on the occurrence of the credit event is a financial instrument used to transfer credit risk from the investor exposed to the risk to another investor willing to take that risk [70]. In fact, credit default swaps are the simplest type of credit derivatives.

A credit default swap is a contract that provides the holder of an underlying asset protection against the losses caused by the occurrence of a credit event that is defined very clearly on the contract [51]. Being an agreement between the protection buyer and the protection seller, the CDS contract acts as a form of insurance and offers investors the opportunity to either buy or sell default protection on a reference entity [35]. The main idea is to transfer the financial risk of a reference entity from the protection buyer (risk seller) to the protection seller (risk buyer). The underlying asset to the credit default swap is often a bond or a loan. A CDS written on the bond of a single firm is called a single-name CDS whereas a CDS written on a portfolio of bonds is called a multi-name or basket CDS [45]. The contracts are quoted on reference entities in the market with a CDS spread and a recovery rate where bid quotes indicate the price to be paid by the protection buyer and offer quotes indicate the price to be paid to the protection seller [7]. Since CDS contracts are traded in the over-the-counter market, the maturity of the CDS contract is negotiated between the counterparties; therefore, CDS maturities vary. The most frequently observed maturity is five years [45].

The CDS transaction mechanism includes three parties: a credit protection buyer, a reference entity (a company) and a credit protection seller [98]. The CDS contract is entered between two parties, the buyer of protection and the seller of protection that agree to a contract terminating at either maturity of the underlying asset or credit event, whichever occurs earlier. In order to acquire insurance against a credit event by a third party (i.e. the reference entity who could be a corporation or sovereign issuer), the protection buyer makes a regular stream of payments (premium payments) to the protection seller until the earlier of maturity or a pre-specified credit event. These payments are expressed in basis points per notional amount of the contract. In return, the protection buyer receives a contingent payment from the protection seller in the case of a credit event. In other words, if a credit event occurs before maturity, the payments come to an end and the protection seller gets a physical or cash settlement on the exposure in order to cover the losses by paying the par value to the protection buyer [36, 76]. In short, the protection buyer takes a long position in a CDS to obtain a protection in case of a credit event of the reference entity and pays a premium to the seller for that purpose whereas the protection seller takes a short position in this contract and agrees to compensate for the loss caused by the credit event in exchange for the premium payments [7]. In this manner, a CDS is similar to an insurance contract [24]. The basic structure of a CDS contract is depicted in Figure 3.1.

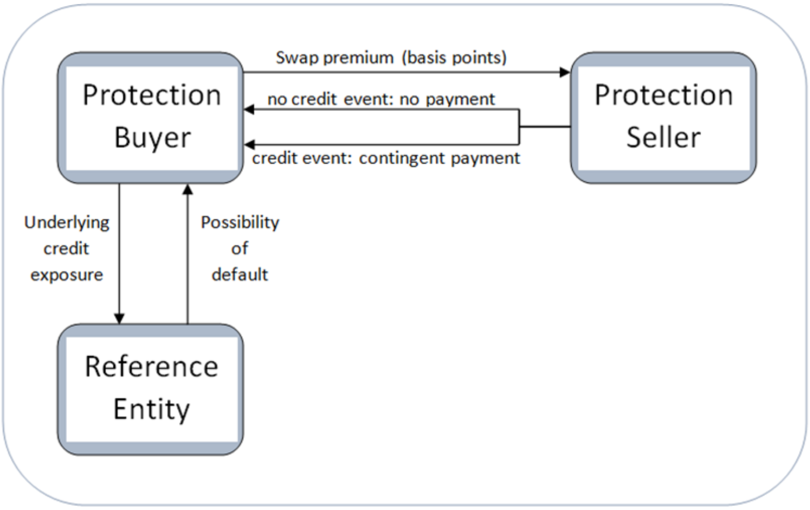

Figure 3.1: CDS Transaction Structure

The CDS contract is a form of an over the counter (OTC) credit derivative and is used by investors for speculation, arbitrage and hedging purposes. If an investor thinks that the CDS spread is higher or lower than its fair price, it may be possible to obtain arbitrage profits by taking advantage of such a mispricing in the market. The accurate pricing of CDS contracts is an important issue for identifying such profit opportunities in the market [76].

3.2 Elements of CDS

CDS contracts are usually constructed in accordance with the standards of ISDA and include information on several terms related to the execution of the contract. These issues are described below.

Reference obligation (Reference asset): This is the underlying asset for the CDS contract. The underlying asset can be any debt instrument issued by a company. In the case of a credit event, the insurance buyers acquire the right to sell the underlying debt instrument at its par value to the insurance seller [55].

Reference entity: The reference entity is the issuer of a reference obligation upon which protection is bought and sold via a CDS contract. In other words, CDS contracts provide insurance against losses caused by the occurrence of a credit event to the reference entity [7]. The reference entity is generally a bank, a corporation or sovereign issuer.

Credit event: A credit event is any unexpected and tangible negative change in the credit quality of a borrower. The CDS contract is initiated to provide the buyer of the contract an insurance against a credit event related to the reference entity [7]. If a pre-determined credit event occurs, then the protection buyer receives a contingent payment from the protection seller in return [45]. A credit event often refers to the default of an obligor. ISDA has made an important contribution to the growth of the CDS market by enhancing the efficiency and safety through standardization of the definition of a credit event. The standard definition of a credit event given by ISDA (2003) includes the following events, one or more of which can be involved in the settlement of a credit default swap: [94].

(i) failure of reference entity to meet any payments,

(ii) obligation acceleration,

(iii) obligation default (e.g. violations of bond covenants) ,

(iv) repudiation/moratorium (the situation where the reference entity disclaims or challenges the validity of the relevant obligation),

(v) bankruptcy, or

(vi) restructuring that makes any creditor worse-off (e.g. coupon reduction or maturity extension) [44, 7, 45].

Notional principal: The CDS notional principal is the face value of the reference obligation that is put under the protection by the CDS contract [44]. The face value of the bond (the reference obligation) is usually the amount that the issuer promises to pay at the maturity date of the bond [21].

CDS spread (premium): The CDS spread measured in basis points is the price paid for default protection. In other words, it is a specified percentage of the notional amount paid regularly for the insurance purchased against the credit event. Periodic premium payment is equal to the multiplication of the CDS spread by the notional value. CDS spreads are expressed in annual terms and the payments are adjusted for the actual number of days included in the payment period (3 months, 6 months, etc.). The CDS premium represents the credit risk of the issuer of the reference asset and is generally quoted as a premium over a swap rate or reference rate such as LIBOR. The period between successive spread payments is stated in the contract and the first payment is made at the end of the first period. If any pre-determined credit event occurs between two payment dates, the protection buyer is required to make a final accrual payment to the protection seller for the protection between the last payment date and the default date [7, 16].

The starting date of the CDS: The starting date of the CDS refers to the date when the default protection starts.

The maturity date: The maturity date of the CDS refers to the end date of the contract provided that no credit event occurs.

Default payment: Default payment is the payment that would be paid to the protection buyer by the protection seller in the case of a credit event in order to buy the defaulted bond. The contingent payment can be settled either physically or in cash. In the case of physical settlement, the protection buyer delivers the reference security to the protection seller and receives cash payment in amount of the par value (notional value) [21]. Alternatively, in a cash settlement procedure, the protection buyer keeps the underlying asset, but receives a payment equal to the difference between the notional value and post-default market value of the reference asset [16]. Some pre-specified number of days after the credit event, or default, the mid-market price or the recovery rate (R) of the underlying asset is determined by a dealer poll or from price quote services [76]. The cash settlement is then equal to 100 minus R percent of the notional principal [37].

CDS contracts are generally settled physically. This is because with physical settlement, protection sellers take the advantage of any rebound in prices caused by the rush of purchasing deliverable bonds by protection buyers after the credit event becomes public information [16]. An example may help to illustrate the difference between physical and cash settlement. Assume that two parties enter into a five-year credit default swap on March 1, 2005 with a notional principal of $100 million and the buyer agrees to pay 90 basis points annually for insurance against a credit event or default by the reference entity. If the reference entity does not default , the buyer of the protection receives no payoff and pays $900,000 on March 1 of each of the years 2006, 2007, 2008, 2009, and 2010. Assume a credit event occurs on September 1, 2008 (half way through the fourth year). If the contract specifies physical settlement, the buyer has the right to sell $100 million par value of the reference asset for $100 million. On the other hand, if the contract specifies cash settlement, the calculation agent gathers poll dealers to determine the mid-market value of the reference obligation on a pre-specified date after the credit event. If the value of the reference obligation is $40 per $100 of par value on this date, then the cash payoff would be $60 million. In both settlement procedures, the buyer is required to pay to the seller the accrual amount for the days between March 1, 2008 and September 1, 2008 (approximately $450,000), but no further payments would be required [55].

3.3 The Role of CDS Contracts in the Global Financial Crisis of 2008

In the run-up to financial crisis of 2008, the rapid innovation and excess usage of financial instruments are observed in financial markets. As stated before, the CDS contracts were one of these popular instruments. It is now widely agreed that the unregulated multi-trillion dollar over-the-counter CDS market played a significant role in the housing price boom and the financial crisis. This section aims to explain how the CDS market played a role in the financial crisis.

In the days following September 11, 2011, the Federal Reserve dramatically lowered interest rates to historically low levels, from about 6.5% to just 1% in order to keep the economy strong. 1% is a very low return for investors who are looking for profitable investment opprtunities. These low rates also made it very easy for banks to extend loans since they were able to borrow at very low rates from the Fed. The mortgage interest rates are good example of such a decline. As published by Freddie Mac., the rate on 30-year fixed-rate mortgages fell from 8% to about 5.5% between 2000 and 2003. With the decrease in mortgage rates that reduced the cost of borrowing, housing prices increased as a result of increased demand. It can be said that the low interest-rate policies of the Fed unintentionally caused speculation in the housing market [46].

As the house prices were increasing, investors who were looking for good investments wanted to profit from this raise. Mortgage loans taken out by homeowners were bundled together by the originating banks and sold on Wall Street as CDO (collateralized debt obligation) slices to the investors looking for a return higher than the 1% offered by the Federal Reserve. As investors earned much higher returns compared to the 1% on Treasury bills, they asked the investment banks for more CDO slices. At the same time, since the house prices were in an increasing trend, people viewed on owning a house as a good investment; therefore, everyone who was qualified and willing to take out a mortgage loan already had taken out a loan. As a result, the lending standard became lax since the originating banks thought that even if the homeowners default on their mortgages, the lenders would get the houses whose prices were steadily increasing. Conventionally, mortgage loans were only issued to good credit borrowers and large down payments and a documentation of income were required. However, in the 2000s, loans were given to higher credit risk borrowers i.e. low-income families, called subprime borrowers, with little or no down payments and often no documentation of income was required [59].

Subprime mortgages, however, have significant default risk. Therefore, some investors who held the top slice of CDOs that were triple A rated investments, wanted to insure their investments via CDS contracts. Selling a CDS is analogous to selling insurance on a debt issue. While insurance companies are required by regulations to hold a certain amount of capital to operate in the industry, the similar cash reserve requirement for CDS sellers is much smaller. In addition, during the housing bubble, the issuers of CDS contracts had the idealistic view that housing prices would continue to go up. Under this view, they sold CDS contracts without posting sufficient collateral or equity capital. This inadequate posting of collateral and insufficient equity capital was partly caused by the misrating of the credit risk of the financial institutions such as Lehman Brothers, Merrill Lynch that held large positions in subprime mortgages. Credit rating agencies used poor models to evaluate mortgage default risk and they underestimated the default risk of the financial institutions who sold large amounts of CDS contracts [59].

As the market experienced this expansion process, the notional amounts of outstanding CDS contracts increased exponentially during period leading up to the financial crisis. In fact, the overall notional amount for outstanding CDS contracts was larger than the sum of the face values of the underlying assets of the contracts. Being a part of the largely unregulated over-the-counter (OTC) derivatives markets, the CDS market was gradually changing from something of an insurance product to a gambling market [85]. Financial institutions continued to sell CDS contracts because of the lack of transparency in the OTC market and their high credit ratings. In fact, the amount of CDS contracts a bank buys and sells is not reported on their balance sheet and this makes it difficult for the counterparties to assess the riskiness of such positions. As a result, financial institutions such as investment banks could never figure out the true leverage amount in the market for CDS contracts written on mortgage loans.

Low interest rates and the lax lending standards created the excess demand for residential home ownership. After a while, and not surprisingly, the subprime homeowners started to default on their mortgages. At first, this was not a problem for the investment banks who owned the mortgages since the houses that served as collateral for the defaulting loans were put up for sale. However, when the number of the defaults increased, there were so many houses for sale on the market. This created an excess supply of houses compared to demand and house prices started to decline which further means that the CDO slices sold to investors lost value. Then the financial institutions insuring the CDO slices via CDS contracts faced high collateral requirements and began experiencing a significant deterioration in their financal positions. This caused the rating agencies to lower the credit ratings of these institutions. The loss in aggregate wealth and the correlated failures of financial institutions froze financial markets with acute negative outcomes to the real economy, ultimately causing unemployment and a deep recession [59].

In 2009, policymakers began to address the systemic vulnerabilities regarding the CDS market. U.S. and European regulators developed ways to regulate the CDS contracts. For example, in the United States, the Treasury proposed regulations for the CDS market that would shift trading of standardized credit derivatives to exchanges and away from the OTC market. The speculative use of CDS contracts is still regarded as a factor that increases the risk in financial markets. In the paper "Has the CDS market influenced the borrowing cost of European countries during the sovereign crisis?" published in the *Journal of International Money and Finance (April 2012)*, Delatte, Gex and López-Villavicencio argue that credit default swaps have actually made the European debt crisis worse, driving up interest rates for unsteady sovereign borrowers such as Greece and Italy [20]. The speculative use of credit derivatives is in fact a controversial issue. Most practitioners as well as academics agree that there is a need for additional regulations to correct the debt misratings issued by the credit rating agencies and to guarantee the execution of the CDS contracts by requiring better capitalized participants.

CHAPTER 4

CDS VALUATION

4.1 Risk Neutral Valuation and Assumptions

In this section, a single-name, cash-settled CDS contract is valued via the discounted cash flow methodology. The discounted cash flow method uses the risk-neutral probability of default when determining the expected cash flows of the CDS and calculating its value [7]. In fact, the fundamental theorem of asset pricing implies that in a complete market, the price of a derivative is given by the discounted expected value of the future payoffs under the unique risk-neutral measure that exists if and only if the market is arbitrage-free. The risk-neutral valuation concept was first presented by Black and Scholes (1973) and Merton (1973). Later, Harrison and Pliska (1981) showed that, under some conditions with the absence of arbitrage opportunities, the real probability measure can be replaced with an equivalent martingale measure $(\mathbb{P})$ under which the discounted price of an asset is a martingale. The discount factor is the risk-free rate, the martingale probability measure $(\mathbb{P})$ is referred to as the risk-neutral probability measure and the present value of the CDS is equal to the expected value of its future cash flows discounted at the risk-free rate [94, 7].

Before discussing the firm's survival probability that determines the expected cash flows and then presenting the formulas to price a CDS contract, the assumptions for CDS pricing can be listed as follows. These assumptions are made in order to value single-name, cash-settled CDS contracts:

Assumption 1. Financial markets are arbitrage-free.

Assumption 2. There is no correlation between the occurrence of the default, the risk-free interest rate and the recovery rate.

Assumption 3. The CDS spread specified in the contract is constant and upfront spread payments are not allowed.

Assumption 4. The recovery rate is the recovery rate on which the CDS is quoted.

Assumption 5. Only default of the reference entity is considered and the counterparty credit risk is ignored. [7, 98].

4.2 Survival Probability

The risk-neutral survival probability is defined as the probability under the risk-neutral measure of the survival of an entity through time T, conditioned on its survival at time t and all other market information at time t. This probability can be denoted as follows:

$$\mathbb{P}(\tau > T \mid \tau > t) = P(t, T) \tag{4.1}$$

where τ is the time of default [94].

This survival probability can be used to define the following indicator function:

$$\mathbb{1}_{\{\tau > T\}} = \begin{cases} 1 & \text{if } \tau > T \\ 0 & \text{if } \tau \leqslant T \end{cases} \tag{4.2}$$

The indicator function has a value of one with probability $P(t, T)$ conditional on no default event up to time T. If a default event occurs at time τ , the indicator function has a value of zero.

Using 4.1 and 4.2, the following expression is obtained.

$$\mathbb{E}_t^{\mathbb{P}}[\mathbb{1}_{\{\tau > T\}}] = P(t, T) \cdot 1 + (1 - P(t, T)) \cdot 0 = P(t, T) \tag{4.3}$$

4.3 Price of a CDS Contract

The protection seller of the contract receives periodic payments from the protection buyer until maturity or credit event, whichever comes first; in other words, as long as the reference entity survives. These payments stop if a default event occurs and then the protection seller makes a default payment to the protection buyer. The value of a CDS contract is the present value of the expected cash flows. Therefore, in order to construct the pricing formula, the present value of the cash flows from the protection buyer (the premium leg) and the seller (protection leg) are need to be determined. Consider a CDS initiated at time t and maturing at time t_n with payment dates $t_1, t_2, ..., t_n$. First, some notations that will be used from this point on are given below:

N	: notional amount
S^{CDS}	: spread of a single-name CDS
$D(t,t_i)$	: discount factor for time interval $[t,t_i]$
$P(t,t_i)$	: survival probability at time t_i
Δt_i	$= t_i - t_{i-1}$
τ	: time of default
M	: number of discrete time points each year at which default can happen
R	: constant recovery rate

4.3.1 Premium Leg

The premium leg is the series of periodic payments of the CDS until maturity or default whichever occurs first. The premium leg also includes the payment of premium accrued from the previous premium payment date until the time of the default.

- If a credit event occurs before time t_i, then the i^{th} and following payments are not paid. Therefore, the premium payment at time t_i is:
$$NS^{CDS}\Delta t_i \mathbb{1}_{\{\tau>t_i\}}$$
- The accrual payment is paid by protection buyer when a default event occurs between two payment dates. One approach to simplify the calculations is to assume that if a default occurs between two payment dates, it will on average take place midway between them (O'Kane & Turnbull). Under this assumption, the accrual payment would be calculated as follows:
$$NS^{CDS}\frac{\Delta t_i}{2}\mathbb{1}_{\{t_{i-1}<\tau<t_i\}}$$

By using these two expressions, the expected present value of all possible premium payments, also called the expected present value of the premium leg can be expressed as [52]:

$$\mathbb{E}_t^{\mathbb{P}}\Bigg[\sum_{i=1}^{n}(N\,S^{CDS}\,\Delta t_i\,D(t,t_i)\,\mathbb{1}_{\{\tau>t_i\}}+N\,S^{CDS}\,\frac{\Delta t_i}{2}\,D(t,\Delta t_i)\,\mathbb{1}_{\{t_{i-1}<\tau<t_i\}})\Bigg]$$
$$=\sum_{i=1}^{n}\Bigg(N\,S^{CDS}\,\Delta t_i\,D(t,t_i)\,\mathbb{E}_t^{\mathbb{P}}[\mathbb{1}_{\{\tau>t_i\}}]+N\,S^{CDS}\,\frac{\Delta t_i}{2}\,D(t,\Delta t_i)\,\mathbb{E}_t^{\mathbb{P}}[\mathbb{1}_{\{t_{i-1}<\tau<t_i\}}]\Bigg) \tag{4.4}$$

The probability of defaulting between t_{i-1} and t_i is equal to the probability of defaulting before t_i minus the probability of defaulting before t_{i-1}. By using 4.1 and 4.3, this probability can be written as:

$$\begin{aligned}\mathbb{E}_t^{\mathbb{P}}[\mathbb{1}_{\{t_{i-1}<\tau<t_i\}}] &= \mathbb{P}(t_{i-1} < \tau < t_i) \\ &= \mathbb{P}(\tau < t_i \mid \tau > t) - \mathbb{P}(\tau < t_{i-1} \mid \tau > t) \\ &= 1 - P(t,t_i) - (1 - P(t,t_{i-1})) \\ &= P(t,t_{i-1}) - P(t,t_i)\end{aligned} \tag{4.5}$$

Therefore, by combining 4.4 and 4.5, an expression for the present value of the premium leg can be given as:

$$N\,S^{CDS} \sum_{i=1}^{n} \Delta t_i \left(D(t,t_i)\,P(t,t_i) + \frac{1}{2}\,D(t,\Delta t_i)\,[P(t,t_{i-1}) - P(t,t_i)] \right) \tag{4.6}$$

A similar expression will be derived for the protection leg in the next subsection.

4.3.2 Protection Leg

The protection leg is the contingent payment that the protection seller makes to the protection buyer following the default at time τ. The method of settlement of the CDS contract in the case of default for the protection payment is specified when the two parties enter the contract. In this thesis, the pricing formula is given for a cash-settled CDS contract. Remember that R is the constant recovery rate. The protection payment can be denoted as $(1 - R)$ percent of the notional amount insured. Assuming that the default can only happen on a finite number of discrete time points each year (M), the present value of the expected premium payments is [94, 88]:

$$\mathbb{E}_t^{\mathbb{P}}\left[\sum_{m=1}^{M\times(t_n-t)} \big(N\,(1-R)\,D(t,t+\frac{m}{M})\,\mathbb{1}_{\{t+\frac{m-1}{M}<\tau<t+\frac{m}{M}\}}\big) \right] \tag{4.7}$$

Similar to the premium leg, the expression for the protection leg can be written as follows:

$$N\,(1-R) \sum_{m=1}^{M\times(t_n-t)} \left(D(t,t+\frac{m}{M})\,\big[P(t,t+\frac{m-1}{M}) - P(t,t+\frac{m}{M})\big] \right) \tag{4.8}$$

4.3.3 The Value of a Credit Default Swap

The market value of a long position in a CDS is the present value of the protection leg minus the present value of the premium leg. Let $V(t,t_n,S^{CDS})$ denote the value of a

CDS to the protection buyer at time t, maturing at time $t_n$with an annual spread S^{CDS}. Then, an expression of its value can be found by subtracting 4.6 from 4.8:

$$V(t,t_n,S^{CDS}) = N\,(1-R)\sum_{m=1}^{M\times(t_n-t)}\left(D(t,t+\frac{m}{M})\left[P(t,t+\frac{m-1}{M}) - P(t,t+\frac{m}{M})\right]\right) \\ - N\,S^{CDS}\sum_{i=1}^{n}\Delta t_i\left(D(t,t_i)\,P(t,t_i) + \frac{1}{2}\,D(t,\Delta t_i)\left[P(t,t_{i-1}) - P(t,t_i)\right]\right) \tag{4.9}$$

In an arbitrage-free market, a CDS contract is constructed such that the value of the contract is zero at initiation. This is done by equating the present value of the protection and default payments. Therefore, setting Equation 4.9 equal to zero and solving for the credit spread gives the fair spread, meaning the spread guaranteeing that the contract is fairly priced [88]:

$$S^{CDS} = \frac{(1-R)\sum_{m=1}^{M\times(t_n-t)}\left(D(t,t+\frac{m}{M})\left[P(t,t+\frac{m-1}{M}) - P(t,t+\frac{m}{M})\right]\right)}{\sum_{i=1}^{n}\Delta t_i\left(D(t,t_i)\,P(t,t_i) + \frac{1}{2}\,D(t,\Delta t_i)\left[P(t,t_{i-1}) - P(t,t_i)\right]\right)} \tag{4.10}$$

As seen in the Equation 4.10, in order to value a CDS contract, two processes should be modelled: the discount factor and the survival probability. Since the focus of this thesis is on valuing the CDS contracts with a stochastic intensity model, a deterministic risk-free rate will be used so that the intensity models can be employed easily. In fact, if the short-term interest rate process is denoted by $r = \{r_t,\ 0 \leqslant t \leqslant T\}$, then discount factor $D(t,T)$ is given by:

$$D(t,T) = \mathbb{E}\left\{\exp(-\int_t^T r_s\,ds)\right\} \tag{4.11}$$

If r is deterministic, then

$$D(t,T) = \exp(-\int_t^T r_s\,ds) \tag{4.12}$$

Moreover, when r is assumed to be constant, the discount factor becomes:

$$D(t,T) = \exp(-r\,(T-t)) \tag{4.13}$$

As the recovery rate is constant, only the survival probability needs to be modeled. In the following subsections, the modeling of survival probabilities with the structural and reduced form models is addressed.

4.4 Structural Approach and the Merton Model

In the structural framework, a firm's total value is modeled in order to develop an estimate for the probability of default. Let's review some basic features of Merton's model. In 1974, Merton presented an application of the Black-Scholes-Merton option pricing model to the pricing of risky corporate debt. In order to do so, Merton modeled firm's equity as a European call option written on the firm's assets so that the stock-holders can be considered as European call option holders. The intuition behind the Merton model is that default occurs at the time of debt maturity if the value of a firm's assets is lower than that of its outstanding debt. Figure 4.1 illustrates the default concept in the Merton model. In this setting, total debt (D) is constant over time and the value of equity fluctuates with the value of the firm's assets. Default occurs when the firm value drops below the default barrier at maturity.

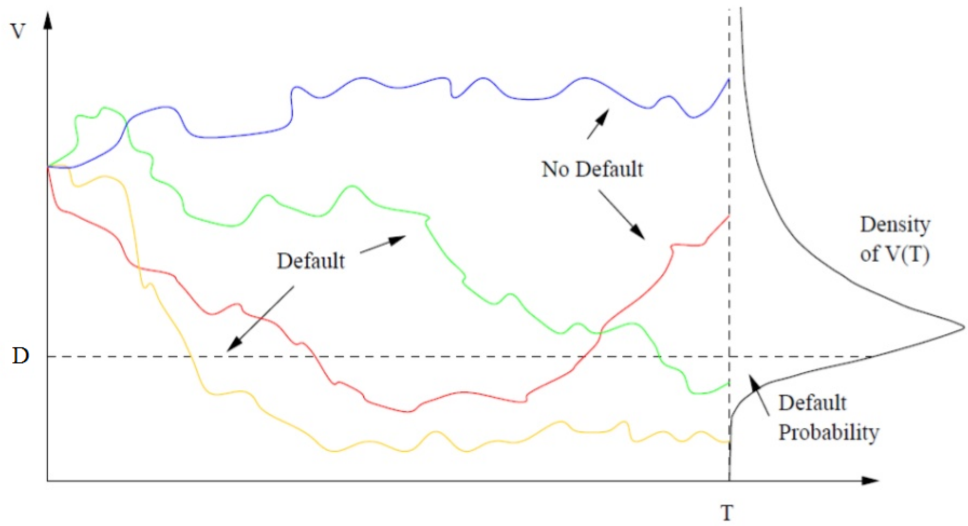

Figure 4.1: Structural approach: default in the classical Merton model (1974).

Retrieved from http://www.slideshare.net/ujjmishra1/credit-risk-models.

Assuming that the company's debt is entirely represented by a zero-coupon bond, if the market value of the firm at maturity is greater than the face value of the bond, then the bondholders get back the face value of the bond and the residual value flows to the equityholders. On the other hand, if the market value of the firm is less than the face value of bond, the bondholders take over the firm hence getting the market value of the firm with the equityholders getting nothing. Therefore, the payoff at maturity to the bondholders is equal to the face value of the bond minus the value of a put option on the value of the firm, with a strike price equal to the face value of the bond and a maturity equal to the maturity of the bond. In other words, the defaultable bond is equivalent to holding a risk free bond with a short put position on the assets of the firm from the bondholders' point of view. Following this basic intuition, Merton derived an explicit formula for the value of risky bonds. This formula can be used to estimate both the probability of default of a firm and the yield differential between a risky bond and a default-free bond [37, 31].

Using the following notation, Table 4.1 is obtained:

V_t : firm value at time t for $t \in [0, T]$
D : face value of the debt at maturity T
Z_t : value of a single zero coupon bond at time t with maturity T and face value D
E_t : value of equity at time t

Table 4.1: Payoffs to bondholders and equityholders.

	Equity Holders receive	Bond Holders receive
If $V_T > D$	$V_T - D$	D
If $V_T < D$	0	V_T
Net position	Max$(0, V_T - D)$	Min(V_T, D)

With this notation, the total asset value of the firm is equal to the following:

$$V_t = Z_t + E_t \tag{4.14}$$

The payoff at maturity of bonds and firm's equity can be written as follows:

$$\begin{aligned} &Z_T = min(V_T, D) = -max(-V_T, -D) = D - max(D - V_T, 0) = D - Put(V_T, D) \\ &E_T = max(V_T - D, 0) = Call(V_T, D) \end{aligned} \tag{4.15}$$

This means that by going long on bonds, the bondholders are long the face value of the bond and short a put option on the assets of the firm with the strike price equal to the face value of the bonds whereas shareholders have a long position in the call option with a strike price equal to the face value of the bonds [37].

Now, applying Black-Scholes-Merton option pricing formula, the value of the firm's debt and equity at time t $(0 \leqslant t \leqslant T)$ are determined as [37]:

$$\begin{aligned} Z_t &= V_t\, \Phi(-d_1) + D\, \mathrm{e}^{-r\,(T-t)}\, \Phi(d_2) \\ E_t &= V_t\, \Phi(d_1) - D\, \mathrm{e}^{-r\,(T-t)}\, \Phi(d_2) \end{aligned} \tag{4.16}$$

where $\Phi(.)$ is the cumulative standard normal distribution function and d_1 and d_2 are given by:

$$d_1 = \frac{\log(V_t/D) + (r + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sqrt{T-t} \tag{4.17}$$

where σ is the volatility of firm's assets and r is risk-free interest rate.

In the Merton framework, the risk- neutral probability $Q(t, T)$ of default at time T can be calculated as:

$$Q(t, T) = \Phi(-d_2) \tag{4.18}$$

[for the proof see Appendix A.]

That is to say

$$P(t, T) = 1 - Q(t - T) = \Phi(d_2) \tag{4.19}$$

Therefore, putting 4.19 into 4.10 will give the price of the CDS spread [84].

4.5 Reduced Form Approach

Unlike the structural approach, reduced form models do not connect the event of default to the capital structure of the firm, i.e. value of a firm's assets or its debt. Instead, the time of default is modeled as a jump process that is not directly linked to any balance sheet information of the firm [11, 94]. The most widely used reduced form approach for calculating default probability is based on the work by Jarrow and Turnbull (1995). They characterized default as the first jump of a Poisson counting process [45]. When the time of default is modeled as the first jump of a Poisson process, market data can be used to determine the parameters associated with the default intensity.

Definition 4.1. *A* Poisson process N_t *with intensity* $\lambda > 0$ *is a non-decreasing, integer valued process satisfying the following conditions:[94, 44, 95].*

- *initial value* $N_0 = 0$,
- *the process has independent and stationary increments*,
- *the density function of process has form* $\mathbb{P}(N_t = n) = \frac{(\lambda\, t)^n}{n!} \exp(-\lambda\, t)$,
- *when* $0 < s < t$*, then the random increment* $N_t - N_s$ *has Poisson distribution with parameter* $\lambda(t - s)$ *and*

$$\mathbb{P}(N_t - N_s = n) = \frac{\lambda^n\, (t - s)^n}{n!} \exp(-\lambda\, (t - s)) \tag{4.20}$$

The next subsections introduce the framework of the most common specifications of the intensity of the Poisson process, namely homogeneous and inhomogeneus Poisson process cases of the Jarrow-Turnbull model and the Cox process.

4.5.1 Jarrow-Turnbull Model

Homogeneous Case:

The simplest specification for reduced-form models is to let the intensity to be constant. The jump process with constant intensity is referred to as a homogeneous Poisson process. As the default is the first jump of the jump process, the survival probability within the time interval $[t, T]$ conditional on surviving until time t, is equal to the probability that the process does not jump in this time period. From the Equation 4.20, the survival probability be written as follows:

$$\begin{aligned} P(t,T) &= \mathbb{P}(N_T - N_t = 0) \\ &= \frac{\lambda^0\,(T-t)^0}{0!}\exp(-\lambda\,(T-t)) \\ &= \exp(-\lambda\,(T-t)) \end{aligned} \tag{4.21}$$

Inhomogeneous Case:

In this case, Jarrow and Turnbull modeled λ as a process of deterministic function of time $\lambda = \{\lambda_t, 0 \leqslant t \leqslant T\}$.

Definition 4.2. *Let τ be the time of default which is the first jump of a Poisson process. Then the intensity of default $\lambda = \{\lambda_t, 0 \leqslant t \leqslant T\}$ is defined as:*

$$\lambda_t = \lim_{\Delta t \to 0} \frac{\mathbb{P}(t < \tau < t + \Delta t \mid \tau > t)}{\Delta t}$$

or equivalently

$$\mathbb{P}[\tau \leqslant t + \Delta t \mid \tau > t] = \lambda_t\,\Delta t \tag{4.22}$$

Definition 4.2 can be interpreted as that the probability of a default within the time interval $[t, t+\Delta t]$ conditional on surviving until time t, is proportional to time dependent intensity function λ_t (hazard rate) and the length of the time interval Δt.

Definition 4.3. *Let $F(t)$ denote the cumulative distribution function of default time τ. Then $F(t) := \mathbb{P}(\tau \leqslant t)$ where $t \geqslant 0$ and $F(0) = 0$ and if f represents the density function of τ, then*

$$F(t) = \int_{-\infty}^{t} f(u)\,du \tag{4.23}$$

By definition 4.3, the survival probability that is the probability that default does not occur before time t can be written as follows:

$$P(t,T) = \mathbb{P}(\tau \geqslant t) = \int_t^\infty f(u)\,du = 1 - F(t) \tag{4.24}$$

Therefore, $P(t+\Delta t, T) = \mathbb{P}(\tau \geqslant t+\Delta t) = 1 - F(t+\Delta t)$

Now, using Equations 4.21 and 4.23 ,

$$\begin{aligned}
\lambda_t\,\Delta t &= \mathbb{P}(t < \tau \leqslant t+\Delta t \mid \tau > t) \\
&= \frac{\mathbb{P}(t < \tau \leqslant t+\Delta t)}{\mathbb{P}(\tau > t)} \\
&= \frac{\mathbb{P}(\tau \leqslant t+\Delta t) - \mathbb{P}(\tau \leqslant t)}{\mathbb{P}(\tau > t)} \\
&= \frac{1 - P(t+\Delta t, T) - (1 - P(t,T))}{P(t,T)} \\
&= \frac{P(t,T) - P(t+\Delta t, T)}{P(t,T)}
\end{aligned} \tag{4.25}$$

Therefore,

$$-\,\lambda_t\,\Delta t = \frac{P(t+\Delta t, T) - P(t,T)}{P(t,T)} \tag{4.26}$$

Then leaving λ_t alone on the left side and taking the limit as $\Delta t \to 0$,

$$-\,\lambda_t = \lim_{\Delta t \to 0} \frac{P(t+\Delta t, T) - P(t,T)}{P(t,T)\,\Delta t} \tag{4.27}$$

Since $\lim\limits_{\Delta t \to 0} \dfrac{P(t+\Delta t, T) - P(t,T)}{\Delta t} = P'(t,T)$, the equation becomes

$$-\,\lambda_t = \frac{P'(t,T)}{P(t,T)} \tag{4.28}$$

Solving for P(t,T) by integrating both sides from t to T gives:

$$P(t,T) = \exp\left(- \int_t^T \lambda_s\,ds \right) \tag{4.29}$$

[44, 45]

Here, if the intensity is taken as a constant, then

$$P(t,T) = \exp\left(-\int_t^T \lambda\, ds\right) = \exp(-\lambda(T-t)) \tag{4.30}$$

as it is obtained in 4.21.

Jarrow and Turnbull assumed that the intensity is a piecewise flat function of maturity time and extending this model to multiple time periods, they modeled default using a binomial tree as shown in the Figure 4.2.

As an example, define λ_t as a step function where $T_{i-1} \leqslant t \leqslant T_i, i = 1,2,3,4$, then

$$P(t,T) = \begin{cases} \exp(\lambda_{0,T_1} t) & if\ 0 \leqslant t < T_1 \\ \exp(\lambda_{0,T_1} T_1 - \lambda_{T_1,T_2}(t-T_1)) & if\ T_1 \leqslant t < T_2 \\ \exp(\lambda_{0,T_1} T_1 - \lambda_{T_1,T_2}(T_2-T_1) - \lambda_{T_2,T_3}(t-T_2)) & if\ T_2 \leqslant t < T_3 \\ \exp(\lambda_{0,T_1} T_1 - \lambda_{T_1,T_2}(T_2-T_1) - \lambda_{T_2,T_3}(T_3-T_2) & \\ -\lambda_{T_3,T_4}(t-T_3)) & if\ T_3 \leqslant t < T_4 \end{cases}$$

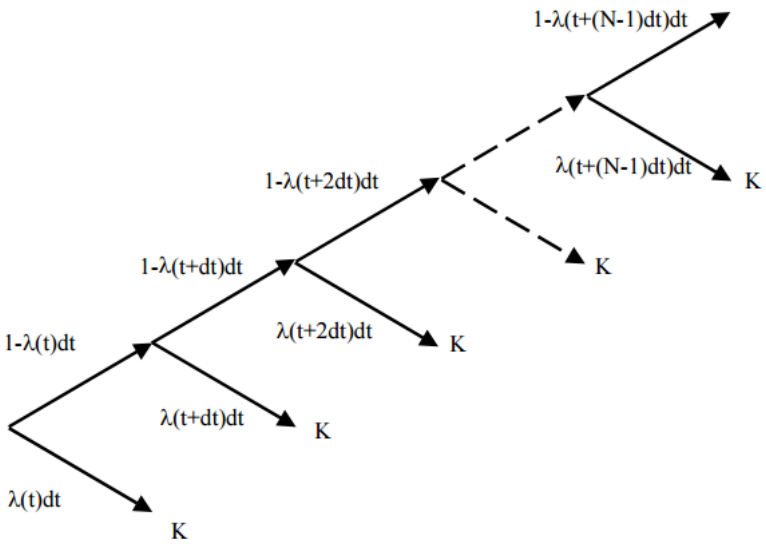

Figure 4.2: Default modelling by using a binomial tree that terminates and makes a payment K at default.

O'Kane D. and Turnbull S.,Valuation of Credit Default Swaps, *pp.1-19, Lehman Brothers International Fixed Income Qantitative Credit Research, 2003.*

This model can be calibrated to market data and replicate the CDS term perfectly. However, since the intensity is deterministic, the survival probabilities also would have to be deterministic. This would mean that together with the constant risk-free interest and recovery rates, the forward CDS spread would be deterministic as well but this would not be a realistic model. Therefore, the models allowing stochastic intensity should be used when pricing the CDS contracts so that the intensity can change randomly as new additional information such as credit ratings or equity price arrive into the market.

4.5.2 Cox Model

Different from the previous two processes, the Cox process assumes a time varying and stochastic intensity. Following Lando(1998), default is modeled as the first jump of a doubly stochastic Poisson process, or as a Cox process.

Definition 4.4. *A Poisson process with stochastic intensity λ_t is called* a Cox process. *The process is doubly stochastic due to the stochastic nature of the jump component and the stochasticity in the probability of jumping (i.e. the intensity) [44].*

Stochastic intensity λ_t is assumed to be positive, right continuous and at least $\mathcal{F}_t$ adapted; in other words, given the default free market information $\mathcal{F}_t$ up to time t, λ is known to time t.

The cumulated intensity is defined as:

$$\Lambda(t) := \int_0^t \lambda_s \, ds$$

Then, the survival probability is given as: [44, 71].

$$\mathbb{P}(\tau \geqslant t) = \mathbb{P}\{\Lambda(\tau) \geqslant \Lambda(t)\} = \mathbb{P}\{\Lambda(\tau) \geqslant \int_0^t \lambda_s \, ds\} = \mathbb{E}\Big[\mathbb{P}\{\Lambda(\tau) \geqslant \int_0^t \lambda_s \, ds\} \;\Big|\; \mathcal{F}_t\Big] \tag{4.31}$$

Since the cumulated intensity at the first jump time, i.e. default time $\Lambda(\tau) = \xi$ is an exponential random variable which is independent of the default free information $\mathcal{F}_t$ where $\tau = \Lambda^{-1}(\xi)$,

$$\mathbb{E}\Big[\mathbb{P}\{\Lambda(\tau) \geqslant \int_0^t \lambda_s \, ds\} \;\Big|\; \mathcal{F}_t\Big] = \mathbb{E}\Big[\mathbb{P}\{\xi \geqslant \int_0^t \lambda_s \, ds\}\Big] = \mathbb{E}\Big\{ \exp\big(-\int_0^t \lambda_s \, ds\big)\Big\} \tag{4.32}$$

Moreover, given all current available information, the conditional survival probability between time t and maturity T is given by the following expression: [44].

$$P(t,T) = \mathbb{E}\Big\{ \exp\big(- \int_t^T \lambda_s \, ds\big) \Big\} \tag{4.33}$$

In fact, letting the intensity be constant or deterministic generates the same expression for the survival probability as in the previous subsection. In order to obtain closed form expressions for survival probabilities, a suitable term structure of interest rate model can be applied to the stochastic intensities. In this thesis, the CIR process is chosen in order to model the intensity of a single-name CDS. This process is named after Cox, Ingersoll and Ross (1985) who propose a single-factor model for the term structure of interest rates. If the interest rate is replaced with the intensity, the dynamics of the default probability process is given as follows:

$$d\lambda_t = \kappa\,(\overline{\lambda} - \lambda_t)\,dt + \sigma\sqrt{\lambda_t}\,W_t \tag{4.34}$$

where the long term mean $\overline{\lambda}$, the mean reversion rate κ and the volatility of intensity σ is positive and where W_t is a standard Brownian motion. It can be seen from the first term of 4.34 that if the intensity at any point λ_t is below $\overline{\lambda}$, the drift will be positive and push the intensity towards its long term value. In the opposite case the drift will push the intensity down. The parameter κ determines the speed of the adjustment towards the long term value [94]. Positivity of intensity which is required for Cox process is guaranteed by the Feller condition: $2\kappa\overline{\lambda} \geqslant \sigma^2$ [38]. Then, under a CIR process, the expression for the survival probability in 4.34 has a closed form:

$$\mathbb{P}(\tau > t|\mathcal{F}_t) = \mathbb{E}\Big[e^{-\int_t^T \lambda_s\, ds} \Big] = A(t,T)e^{-B(t,T)\lambda_t} \tag{4.35}$$

where

$$\begin{aligned} A(t,T) &= \left(\frac{2\gamma\, e^{(\kappa+\gamma)(T-t)\frac{1}{2}}}{(\kappa+\gamma)(e^{\gamma(T-t)}-1)+2\gamma} \right)^{\frac{2\kappa\overline{\lambda}}{\sigma^2}} \\ B(t,T) &= \frac{2(e^{\gamma(T-t)}-1)}{(\kappa+\gamma)(e^{\gamma(T-t)}-1)+2\gamma} \\ \gamma &= \sqrt{\kappa^2 + 2\sigma^2} \end{aligned} \tag{4.36}$$

as given in Cox et al. (1985).

4.6 Bond Pricing

Each firm's bond prices are used to estimate the firm's default probabilities by non-linear least squares and then these probabilities are used obtain CDS prices. In order to do so, first the formula for the price of a bond needs to be obtained.

As stated before, the value $D(t,T)$ at time t of a default-free zero coupon bond with maturity T and face value 1 can be expressed as follows:

$$D(t,T) = \mathbb{E}\Big\{ \exp(-\int_t^T r_s \, ds) \Big\} \tag{4.37}$$

The price $Z(t,T)$ at time $t < T$ of a defaultable zero coupon bond with maturity T and face value 1 and generating a recovery payment of a fraction R of the face value in case of default is given as:

$$Z(t,T) = \mathbb{E}[D(t,T)\,\mathbb{1}_{\{\tau>T\}}] + \mathbb{E}[D(t,\tau)\,R] \tag{4.38}$$

Since default-free interest rate and default time are independent, Equation 4.38 can be expressed by:

$$\begin{aligned} Z(t,T) &= D(t,T)\,\mathbb{E}[\mathbb{1}_{\{\tau>T\}}] + R\;\mathbb{E}[D(t,\tau)] \\ &= D(t,T)\,P(t,T) + R\int_t^T D(t,T)\,f(s)ds \end{aligned} \tag{4.39}$$

where $f(t)$ is the probability density function associated with the intensity process λ_t:

$$f(t) = \lambda_t \, \exp\left(-\int_0^t \lambda_s ds \right)$$

Then the price $Z(t,t_n,c)$ of a defaultable coupon bond with face value F, coupon payment (expressed as a percentage of the face value) C and payment dates $t_1, ..., t_n$ is the sum of the expected discounted value of its coupons and face value, and a potential recovery payment if default occurs. The i^{th} coupon payment is made only if the bond issuer has not yet gone bankrupt at time t_i and the face value is paid only if the bond is still alive at time t_n. The recovery payment of $R\,F$ is made if the bond defaults at time τ before maturity. Therefore,

$$Z(t,t_n,c) = \sum_{i=1}^{n} D(t,t_i)\,\mathbb{E}[\mathbb{1}_{\{\tau>t_i\}}]\,C\,F + D(t,t_n)\,\mathbb{E}[\mathbb{1}_{\{\tau>t_n\}}]\,F + \mathbb{E}[D(t,\tau)\,\mathbb{1}_{\{\tau\leqslant t_n\}}]\,R\,F \tag{4.40}$$

Equivalently,

$$Z(t,t_n,c) = \sum_{i=1}^{n} D(t,t_i)\,P(t,t_i)\,C\,F + D(t,t_n)P(t,t_n)\,F + R\,F\int_{t}^{t_n} D(t,s)\,f(s)ds \tag{4.41}$$

As Houweling and Vorst (2004), we use the following approximation:

$$\int_{t}^{t_n} D(t,s)\,f(s)ds \approx \sum_{j=1}^{m} D(h,h_j)(P(t,h_{j-1}) - P(t,h_j)) \tag{4.42}$$

where $(h_0, ..., h_m)$ is a monthly grid of maturities such that $h_0 = t_0$ and $h_m = t_n$ [33, 52]

CHAPTER 5

METHODOLOGY AND DATA

In this thesis, a methodology based on the technique proposed by Cox et al. (1985) and given in Equations 4.35 and 4.36 is employed in order to price a single-name credit default swap (CDS) contract. The methodology uses a discounted cash flow approach with reduced-form survival probability functions depending on stochastic intensity for two time periods: pre-crisis and post-crisis. The CIR model is used for modeling the default risk. For model calibration, each firm's bond prices are used to determine the optimal set of parameters of the CIR default intensity process. The calibration step generates the firm's stochastic default probabilities and then the parameters obtained in the calibration step are used to obtain the CDS contract prices. In this chapter, the calibration methodology and the data set are explained in detail.

5.1 Calibration

As the model for bond pricing is set up, the parameters $(\kappa, \overline{\lambda}, \sigma, \lambda_0)$ of the CIR process need to be calibrated. The availability of an analytic formula makes the calibration of the parameters to the market-observed bond prices straightforward. The survival probability given in the Equations 4.35 and 4.36 is plugged into the Equation 4.41 in order to get the analytical formula for bond pricing within the CIR process. Next, the model can be directly calibrated to bond market prices by minimizing the Sum of Squared Errors (SSE):

$$SSE = \sum_{q=1}^{K} (Z_q^{Market}(t) - Z_q^{Model}(t, t_n, c))^2 \tag{5.1}$$

where $Z^{Market}(t)$ denotes the bond price observed in the market at time t and $Z^{Model}(t, t_n, c)$ denotes the bond price calculated by the model as given in Equation 4.41. K is the number of bond prices retrieved from the market.

The parameters of the CIR process for a firm are obtained for each day during the sample period. These parameters also allow the determination of the survival probabilities of that firm for each day. Once the survival probabilities are obtained, they are plugged into the Equation 4.10 in order to calculate the CDS premium for the firm for a given day. This procedure is repeated for each day during the sample period.

5.2 Market Data

In order to estimate the hazard rate (intensity) parameters, the analyses are carried out using daily bond and CDS data for two of the Dow Jones 30 Index constituents, namely The Coca-Cola Co. (non-financial) and JPMorgan Chase and Co. (financial). These firms are selected in order to test the model in two different industries which are expected to be affected by the Global Financial Crisis in different ways. On the one hand, since JP Morgan Chase is a financial institution, its assets and liabilities are very sensitive to the changing conditions in markets. This type of sensitivity also is expected to affect the default probability of a financial firm. On the other hand, Coca-Cola is a soft drink producer whose main products have low price and income elasticities even when the market conditions change in an extreme manner. As such, in comparison to a financial institution, the Coca-Cola Company's default probability is expected to be affected with much less severity during a financial crisis. By using data from these firms, it will be possible to demonstrate some of the potential differences in the market's pricing of the default risk in two different industries. Also, the model is tested for two seperate 3-month periods. The first of these periods is between July and September 2008 and accounts for a pre-crisis market environment. The second is between January and March 2016 and accounts for a post-crisis market environment. The reason for testing over two separate periods is to find out whether the model is equally successful in estimating CDS prices after the CDS market has gone through major regulatory changes following the Global Financial Crisis of 2008.

The bond data are collected for fixed-coupon, non-convertible and non-callable bonds denominated in US dollars with an outstanding amount higher than $300 million and with semiannual coupon payments. Bond prices represent mid quotes and are obtained from Thomson Reuters database.

The model also requires the use of a risk-free rate. Among the available proxies for default-free rates, daily federal funds rates that are downloaded from the website of the New York Fed are used.

The recovery rate is assumed to be constant at 40%. This is a standard assumption for pricing credit derivatives. Moreover, Houweling and Vorst (2005) show that the pricing of credit default swaps is relatively insensitive to the assumed recovery rate [52].

The CDS data also are obtained from Thomson Reuters and contain daily mid quote prices. All CDS contracts have a notional amount of 10 million USD. Each quote is given by a quarterly premium expressed in basis points on an annual basis. For each reference entity whose reference obligation is the reference entity's senior debt, CDS quotes with maturities of 6 months and 1, 2, 3, 4, 5, 7, 10 years are obtained for each day. Table 5.1 gives the beginning and ending dates for the final data set.

Table 5.1: Final dataset given for both bond prices and CDS quotes for both firms.

	3-month period	Number of days
Year 2008	01.07.2008 - 30.09.2008	92
Year 2016	01.01.2016 - 31.03.2016	92

Average CDS premiums by firm and maturity for years 2008 and 2016 respectively are given in Table 5.2. As can be seen from the table, for both the pre- and post-crisis periods, the obtained average market quotes increase monotonically with maturity for both firms. This is expected because as time passes, credit quality deterioration is more likely than credit quality improvements [97]. The average CDS premium of Coca-Cola is smaller than that of JPMorgan Chase for both periods. Since the CDS quotes are indicators of credit risk, the perceived credit quality of Coca-Cola seems to be higher compared to JPMorgan Chase. This is an expected situation since JPMorgan Chase is a financial institution, it bears more credit risk, especially in times of crisis and therefore, its CDS quotes are higher for both periods. In addition, for both companies, most of the CDS spreads seem to decline from 2008 to 2016. This is to be expected in retrospect since the first period includes approximately the first two weeks of the Global Financial Crisis. The exceptions are the 7- and 10-year maturities for Coca-Cola and the 10-year maturity for JPMorgan Chase. The observation that not all CDS spreads decline between 2008 and 2010 imply that there has been a slight increase in the slope of the term structure of CDS spreads.

Table 5.2: Average CDS premiums in basis point by firm and maturity for years 2008 and 2016

	The Coca-Cola Co.		**JPMorgan Chase and Co.**	
Maturity	**Year 2008**	**Year 2016**	**Year 2008**	**Year 2016**
6 months	24.8479	5.0659	77.7739	25.7925
1 year	25.8348	6.7145	80.1349	40.6424
2 years	27.3045	9.8043	90.0979	51.3246
3 years	30.3237	14.3509	98.2831	57.3334
4 years	33.1199	18.9338	108.1253	67.1541
5 years	37.1312	26.0743	115.4675	81.5528
7 years	40.5536	41.2815	116.4017	100.6767
10 years	48.2632	51.3760	117.8630	120.0797

Figures 5.1 to 5.4 present the CDS spreads for Coca Cola and JPMorgan Chase for the two periods under analysis. When the 5-year CDS spreads of Coca-Cola in Figures 5.1 and 5.2 are examined, it is seen that in 2008, the quotes remain within a range between 30 to 50 basis points and experience the biggest increase around the time of the Lehman Brothers bankruptcy (September 15, 2008). For the post-crisis period, the CDS spreads exhibit a flat pattern for Coca-Cola. Compared to the premiums of Coca Cola, those of JPMorgan Chase are larger for both periods. In fact, although these two companies have very similar bond ratings as can be seen in Table B.1 in Appendix B, their CDS spreads are still different from each other by magnitude. When

the 5-year CDS spreads of JPMorgan Chase are examined in Figures 5.3 and 5.4, it is seen that in 2008, the quotes remain within the range between 50 to 150 basis points until September and there is a dramatic spike reaching levels of 200 basis points in mid-September, again around the time of the Lehman Brothers bankruptcy. In late September, the CDS spreads start to decline down to pre-September 15 levels. In 2016, the JPMorgan Chase quotes fluctuate within a range of 20 and 150 basis points with the lower boundary of the range being much smaller than the lower boundary in 2008. An additional observation regarding the figures is that the JPMorgan Chase quotes always fluctuate more compared to the Coca-Cola quotes. This is not surprising since, being a financial institution, JPMorgan Chase has assets and liabilities that are a lot more sensitive to changes in macroeconomic conditions. Also, in February 2016, a spike is observed in the JP Morgan quotes. Examining the news related to the company around that time reveals an announcement that JP Morgan is to set aside funds to cover potential losses in the oil and gas sectors that are much larger than previously anticipated. In fact, the price of JPMorgan shares fell 4.3% after the announcement [3].

Additional descriptive statistics regarding the data are given in Appendix B.

In the next chapter, the emprical results are presented and the performance of the CIR model in pricing the CDS premiums of chosen firms is tested.

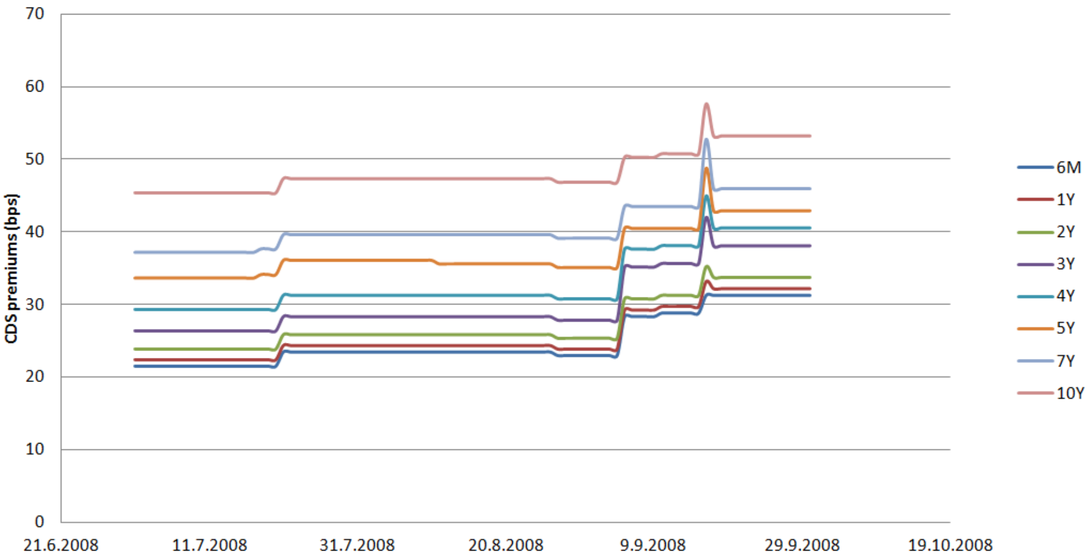

Figure 5.1: CDS premiums of The Coca-Cola Co. for 3-months period in 2008 in basis points by maturity.

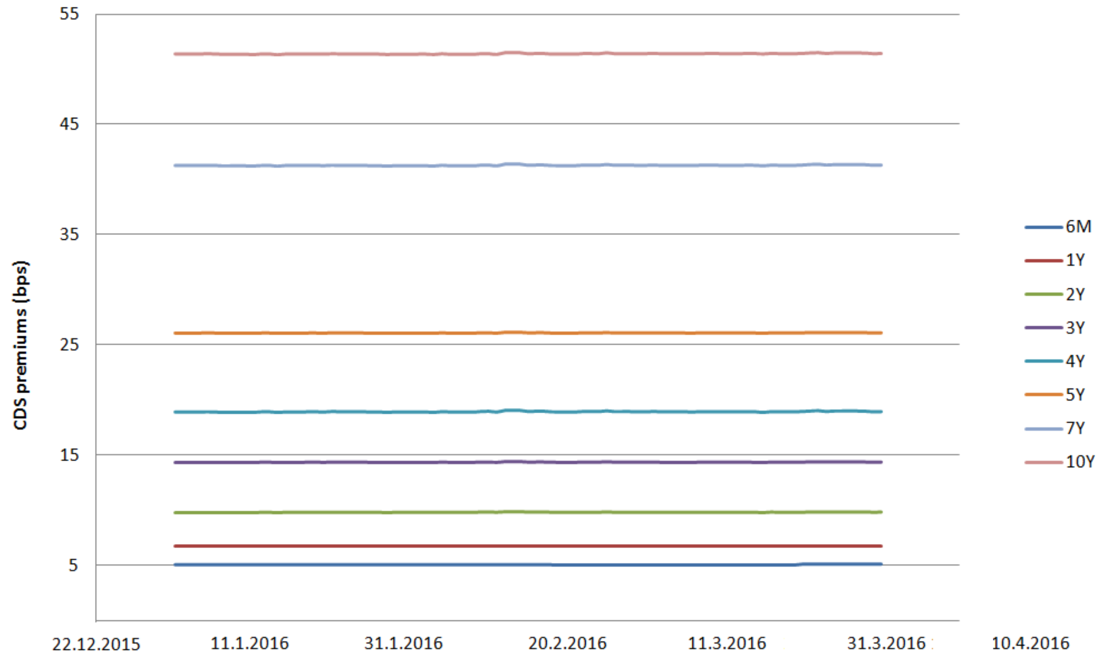

Figure 5.2: CDS premiums of The Coca-Cola Co. for 3-months period in 2016 in basis points by maturity.

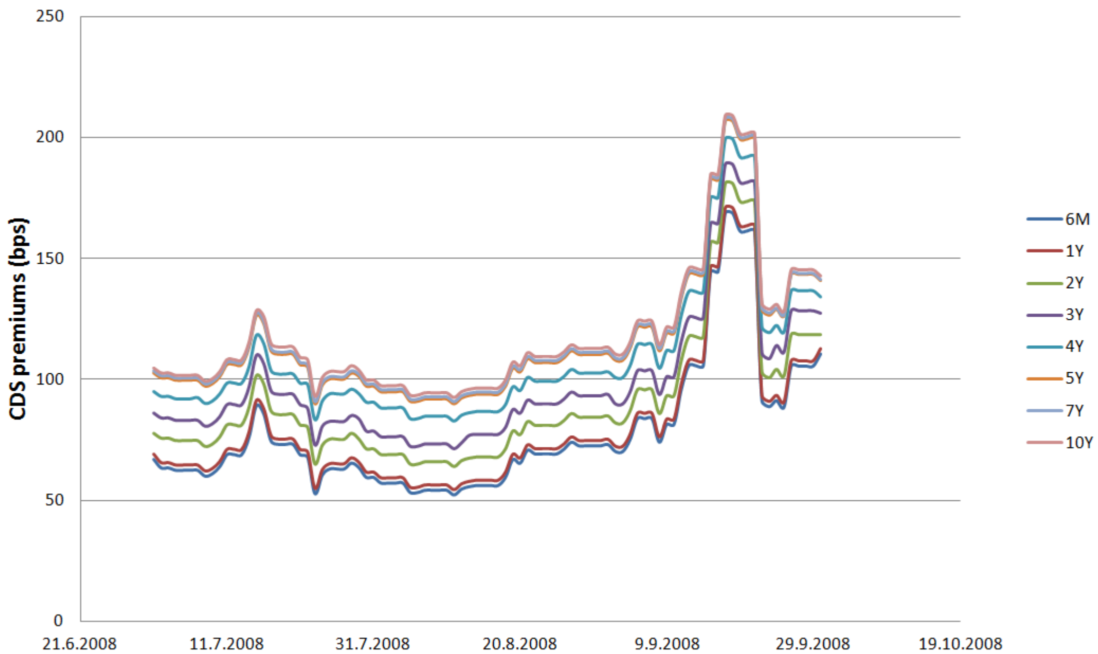

Figure 5.3: CDS premiums of JPMorgan Chase and Co. for 3-months period in 2008 in basis points by maturity.

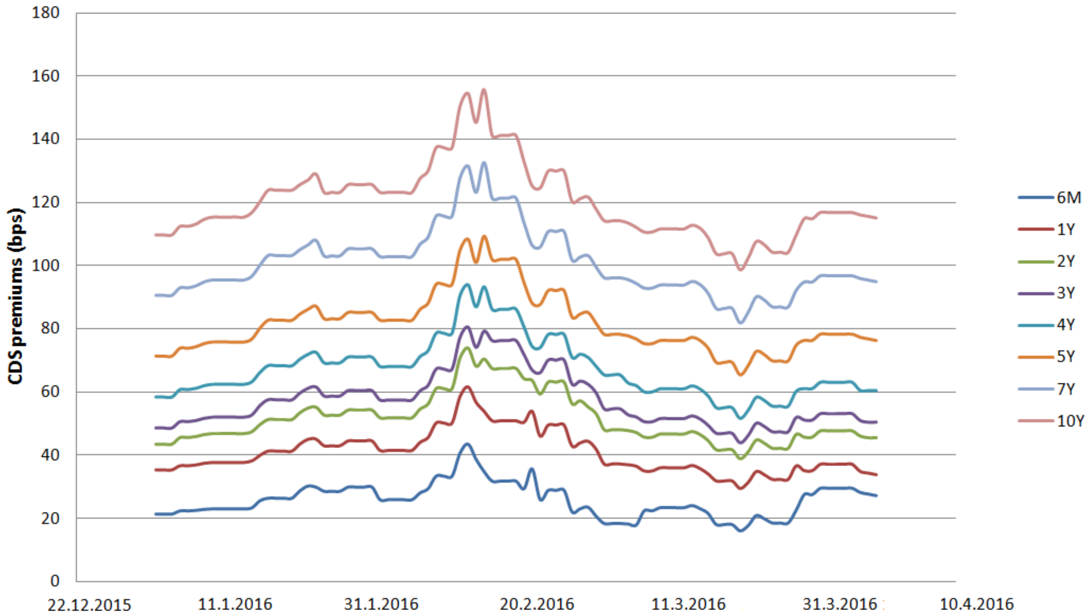

Figure 5.4: CDS premiums of JPMorgan Chase and Co. for 3-months period in 2016 in basis points by maturity

CHAPTER 6

RESULTS

In this chapter, the results of model calibration and performance tests are presented. For each day and firm, hazard rate parameters are obtained via the CIR stochastic intensity model by minimizing the sum of the squared bond pricing errors between the market and model spreads. In order to achieve this, the MATLAB code is run 10,000 times iteratively for each day and firm. These estimates are used in computing the CDS spreads.

6.1 Comparing Model and Market Bond Prices

The ability of the model to fit bond prices is tested by computing the root mean squared error (RMSE) of the deviations between the model and market prices of bonds. Table 6.1 displays RMSEs by firm and year. For both firms and periods, the RMSEs are much smaller than 1 percent of the average bond prices. These RMSE estimates imply that the model is successful in calibrating the parameters. The output of this step of the calculations is the estimated survival probabilities for each firm and day in the sample period.

Table 6.1: RMSE (%) by firm and year

	Coca-Cola Co.		**JPMorgan Chase**	
	Year 2008	**Year 2016**	**Year 2008**	**Year 2016**
RMSE	0.8095	0.4239	0.8058	0.8829
RMSE/Avr. Bond Price	0.0067	0.0032	0.0083	0.0087

The next step is to test the success of the CDS spread estimates obtained by using these survival probabilities.

6.2 Comparing Model and Market CDS Spreads

In order to compare the model and market CDS spreads, Mean Absolute Pricing Errors (MAPEs) are calculated as a representation of the pricing errors. The MAPEs are expressed in basis points and calculated by using the following formula:

$$MAPE = \frac{1}{92}\sum_{t=1}^{92}\sum_{T=1}^{8}\frac{\left|CDS^{Market}(t,T) - CDS^{Model}(t,T)\right|}{8} \tag{6.1}$$

where $CDS^{Market}(t, T)$ denotes the CDS spread observed in the market and $CDS^{Model}(t, T)$ denotes the CDS spread calculated by the model. For each firm, absolute pricing errors are calculated for each of the 92 days and the 8 different maturities. The MAPE measure is the equally-weighted average of these pricing errors over days and maturities.

Table 6.2 presents MAPEs by firm and year, considering all CDS maturities. When each firm's MAPE figures are compared between 2008 and 2016, it is seen that the pricing errors are much smaller for the 2016 sample. This result implies that the model is more successful in predicting the CDS spreads in relatively more stable market environments. This implication is verified by looking at Figures 5.1 through 5.4 above and Tables B.2 through B.5 in Appendix B. The standard deviations of the CDS spreads are much higher in 2008 compared to those in 2016. In addition, the pricing errors for the financial firm (JPMorgan Chase) is larger than those of the non-financial firm (Coca-Cola). Once again, this is not surprising since financial institutions like JPMorgan Chase suffered a lot more from the effects of the Global Financial Crisis compared to some of the manufacturing industries.

Also, MAPEs for a naive model that uses the last observed CDS spread as a predictor of the next day's spread are given in Table 6.2. For the naive model, the absolute pricing error is equal to the absolute difference between the market prices of two consecutive days where the previous day's price is the predictor for the next day's price. As can be seen in Table 6.2, the MAPEs for both firms and periods are smaller for the naive model when compared to the MAPEs for the CIR model. This is not a surprising result for two reasons. First, the raw CDS spread data for both firms include consecutive days where the change in the spreads is equal to zero. The constant spreads over several days imply that either the perceived default probabilities of the sample firms are not changing or the CDS contracts are infrequently traded. The spreads staying constant for several days at a time leads to small pricing errors in the naive model. Second, the CIR model uses no historical CDS spread data in pricing the contracts. As a matter of fact, it is possible to price a new CDS contract by using bond prices and the CIR model. Obviously, such pricing would not be possible with the naive model.

Table 6.2: MAPE in basis point by firm and year considering all CDS maturities.

	Year 2008	**Year 2016**
CIR MAPE for Coca-Cola Co.	9.6832	3.9530
CIR MAPE for JPMorgan Chase	15.0644	4.8591
Naive MAPE for Coca-Cola Co.	0.1958	0.0102
Naive MAPE for JPMorgan Chase	3.9047	1.4910

Table 6.3 presents MAPEs by CDS maturity and firm in years 2008 and 2016. In addition, Figures 6.1 through 6.4 present the 5-year CDS spreads obtained from the

market and the model for the two firms and periods.[1] There are no obvious patterns in the table or the figures that would suggest a relationship between maturities and pricing errors. Generally, within each period, the pricing errors for JPMorgan Chase are larger than those of Coca-Cola. This is an observation in line with the previous results.

Table 6.3: MAPE in basis point by maturity and firm.

	Coca-Cola 2008	**Coca-Cola 2016**	**JPMorgan 2008**	**JPMorgan 2016**
6-months	8.2724	0.7839	8.3254	2.0927
1-year	5.1577	1.2701	8.5365	5.9462
2-years	1.3939	4.9689	9.4036	1.5828
3-years	5.4225	6.7681	11.8975	6.0388
4-years	9.2346	8.0382	17.4626	7.8895
5-years	11.1496	6.2263	22.5897	3.2802
7-years	17.4666	1.6687	21.1074	1.7360
10-years	19.3683	1.9000	21.1927	10.3069

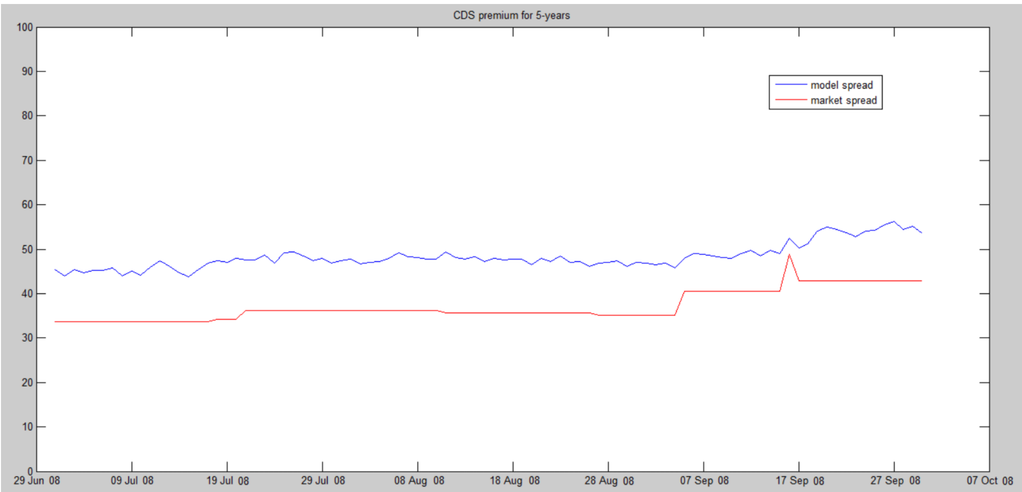

Figure 6.1: Model and market CDS premiums of The Coca-Cola Co. for 5-years maturity in 2008.

[1] Similar graphs for other maturities are provided in Appendix C.

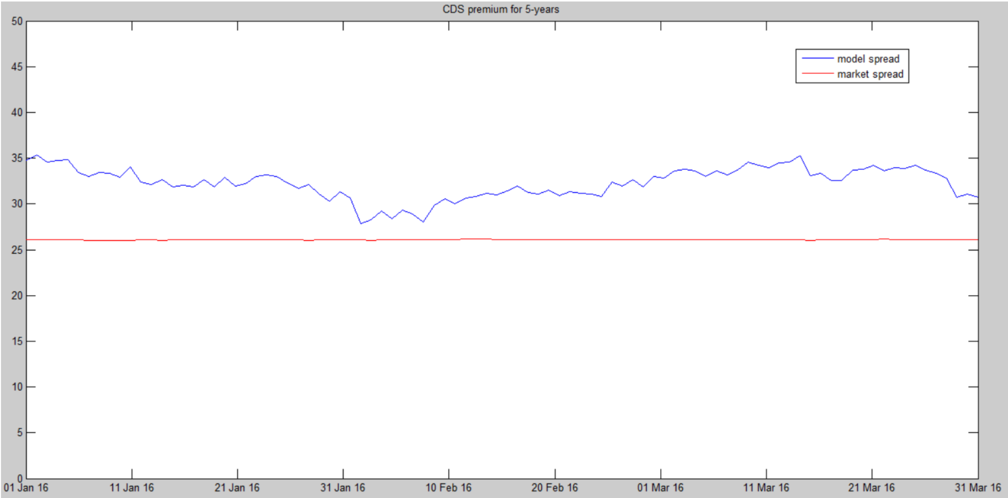

Figure 6.2: Model and market CDS premiums of The Coca-Cola Co. for 5-years maturity in 2016.

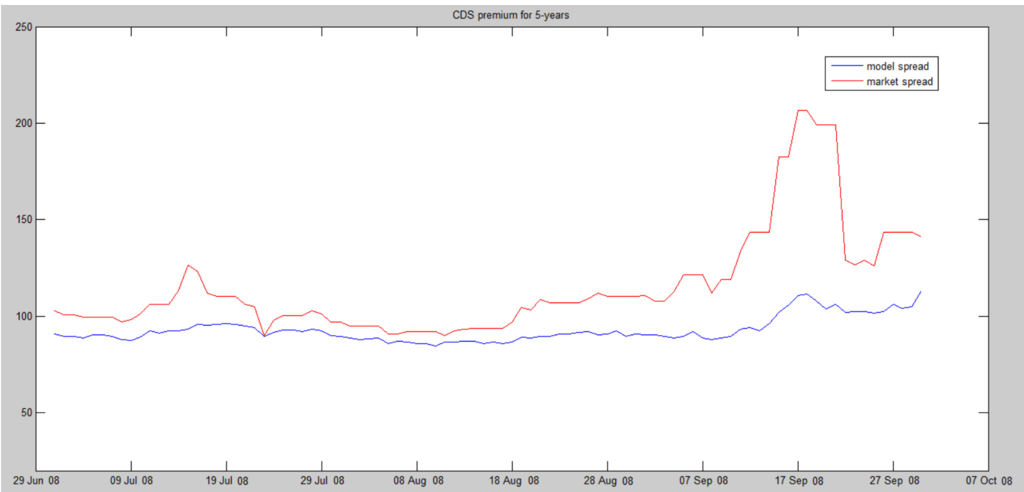

Figure 6.3: Model and market CDS premiums of JPMorgan Chaseand Co. for 5-years maturity in 2008.

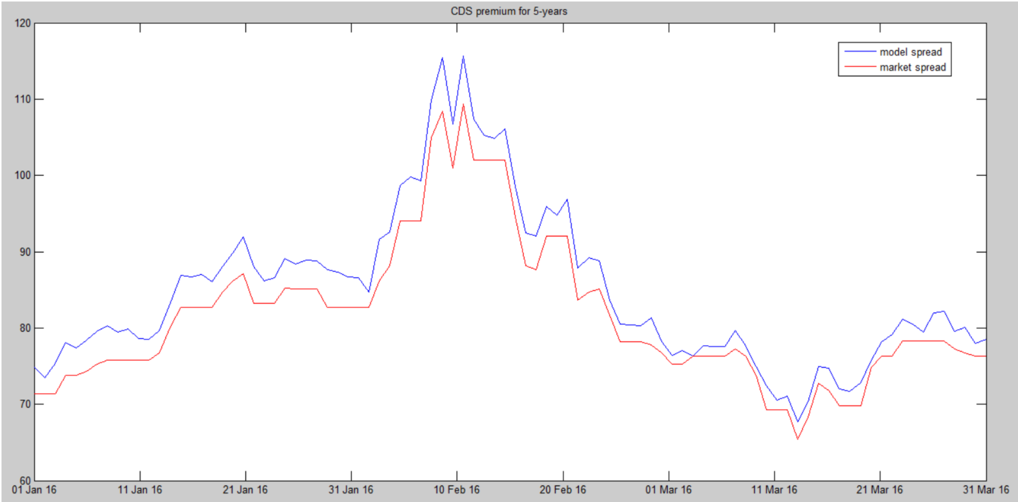

Figure 6.4: Model and market CDS premiums of JPMorgan Chase and Co. for 5-years maturity in 2016.

Examining Figures 6.1 through 6.4 it is seen that for both firms and periods, model CDS spreads are more volatile than the market-observed spreads. This is because the model generates new survival probabilities stochastically via a least square method each day. Except for mid-Semtember in 2008, the model seems to estimate the 5-year CDS premiums of both firms with relatively small pricing errors. For Coca-Cola in both periods and for JPMorgan in 2016, the model seems to overstimate the 5-year premiums whereas it underestimates the JPMorgan premiums in 2008 with small errors. Unfortunately the model does not estimate the CDS premiums with the same precision during the mid-September 2008 period. The reason might be the asymmetric change between the bond prices and CDS premiums. For this period, the decrease in the bond prices is smaller than the increase in the CDS premiums. This asymmetric change may mean that the model cannot catch the premiums at their peak points by using the survival probabilities obtained from the bond prices.

6.3 Regression Analysis

In order to understand the effect of different maturities and sample periods on the magnitude of the absolute value of the pricing errors, MAPEs of each firm are regressed on dummy variables that represent these potential effects. The model below is estimated within a least squares framework:

$$\begin{aligned} MAPE_t = c_0 + c_1(CC)_t + c_2(YR08)_t + c_3(MAT1Y)_t + c_4(MAT2Y)_t + c_5(MAT3Y)_t \\ + c_6(MAT4Y)_t + c_7(MAT5Y)_t + c_8(MAT7Y)_t + c_9(MAT10Y)_t + \epsilon_t \end{aligned} \tag{6.2}$$

where dummy variables $(CC)_t$ representing The Coca-Cola Co., $(YR08)_t$ representing year 2008 and $(MATiY)_t$ representing the i-year(s) maturity for $i = 1, 2, 3, 4, 5, 7$ and

10.

Regression results are presented in the Table 6.4. In general, the results confirm earlier observations. Maturity effects, with the exception of the 1-year and 2-year horizons are statistically different from zero. Since the 6-month maturity is represented by the intercept of the model, these significantly positive estimates imply that longer maturities have larger pricing errors. Since the model parameters are calibrated based on the bond data and used to estimate the CDS spreads, if the bond and CDS prices do not move together in the market, this mismatch may be influencing the estimates at a larger scale for longer maturities. When the parameter estimates for the year dummies are examined, it is seen that pricing errors in 2008 are larger than those in 2016. This is a result in line with the observations in Table 6.2 and Figures 6.1 through 6.4. Finally, the negative and significant coefficient for the Coca-Cola dummy indicates that the pricing errors are smaller for Coca-Cola compared to JPMorgan Chase.

In order to observe the explanatory power of the maturities, firms and years seperately in explaining the MAPEs, MAPEs are regressed on all dummy variables in separate models. The results provided in Appendix D. The findings from Table 6.4 are confirmed with the separate regression model estimations.

Table 6.4: Regression Analysis.

The table documents the results of regressing MAPEs on the dummy variables for firm, CDS maturity and year. The model given in the table contains dummy variables $(CC)_t$ representing The Coca-Cola Co., $(YR08)_t$ representing year 2008 and $(MATiY)_t$ representing the i-year(s) maturity for $i = 1, 2, 3, 4, 5, 7$ and 10. Numbers in parantheses are t-statistics.

Model:

$$\text{MAPE}_t = c_0 + c_1(CC)_t + c_2(YR08)_t + c_3(MAT1Y)_t + c_4(MAT2Y)_t + c_5(MAT3Y)_t$$

$$+ c_6(MAT4Y)_t + c_7(MAT5Y)_t + c_8(MAT7Y)_t + c_9(MAT10Y)_t + \epsilon_t$$

c_0	c_1	c_2	c_3	c_4	c_5
2.4566	-3.1437	7.9677	.3590	-.5313	2.6631
(5.75)	(-7.91)	(20.04)	(0.61)	(-0.81)	(3.94)
c_6	c_7	c_8	c_9	R^2	observation
5.7876	5.9429	5.6260	8.3234	0.1951	2944
(8.19)	(7.91)	(7.09)	(10.40)		

CHAPTER 7

CONCLUSION

In the months leading to the Global Financial Crisis of 2008, credit markets grew rapidly and financial institutions developed many new derivative products to hedge against credit risk. The most common type of such credit derivatives is the CDS contracts, quoted with a CDS spread and a recovery rate on reference entities in the market. Being an agreement between the protection buyer and the protection seller, the CDS acts as a form of insurance and offers investors the opportunity to either buy or sell default protection on a reference entity . These contracts transfer the financial risk of a reference entity from the protection buyer to the protection seller in exchange for periodic fixed payments, called the CDS spread or premium, paid until the contract maturity or a default event, whichever comes first.

As these contracts became more popular, their speculative usage also increased and around the time of the Global Financial Crisis, the notional amount of outstanding CDS contracts exceeded the sum of the face values of their reference assets. Soon, it was realized that in addition to their great advantages such as improving market depth and reducing the information asymmetry of financial markets, credit derivatives such as CDSs also carry high levels of risk due to their complex structures.

In such an enviroment, credit risk modeling gained more interest from financial institutions, regulators and academics and a large body of literature developed on the subject. Since credit risk is the risk of default on a debt due to a failure of the borrower to make payments, a credit risk model generates an estimate of the default risk of the debtor. When it comes to the CDS contracts, the most critical element to be studied is the spread that is paid by the buyer of the contract.

In this thesis, a reduced-form CDS pricing model with stochastic intensity is used. Such a model has practical advantages over the structural-form models. Reduced-form models do not relate the default event with any observable characteristics of the firm and require less detailed information, making the calibration process easy. In fact, with reduced-form approaches, the probability of default can be obtained directly from the market-observed bond prices. Therefore, reduced-form models are more flexible, practical and tractable. Also, while structural approaches cannot incorporate credit rating changes occuring quite frequently for risky corporate debt, reduced-form models are able to integrate such changes. Finally, in reduced-form models, the default is defined as an unexpected event whose likelihood is evaluated by a default intensity process and this feature of the reduced-form model makes it appropriate for modeling defaults as the default of a firm is often an unexpected event.

When using reduced-form models, the intensity needs to be modeled, so that the probability of default which depends on the intensity process can be calculated. Modeling the intensity as a constant, the default time is exponentially distributed and only constant CDS spreads can be obtained but such spreads would be unrealistic in light of the volatility observed in market data. As a result, homogeneous intensity models that use constant default intensities fail when calibrating to the market data. In order to overcome this problem, the intensity can be defined as a deterministic function of time. However, in a model with deterministic intensity, the obtained survival probabilities also are deterministic and the only information related to default risk arriving over time is the firm's survival up to that point in time. However, in reality, in addition to a firm's survival, there may be new information associated with the credit quality of the firm, arriving as time passes. Arrival of new information would change the intensity randomly. Therefore, it can be assumed that the intensity varies with an underlying state variable such as credit ratings, the equity price of an issuer or the firm's distance to default. In this sense, modeling the intensity as a random process is reasonable. A popular choice is modeling intensity as a CIR process named after Cox, Ingersoll and Ross (1985) [18].

In this thesis, a methodology to evaluate the price of a single-name credit default swap (CDS) via the discounted cash flow method is studied. In this approach, the survival probabilities depend on a stochastic intensity process and are estimated in a reduced-form framework. This research exercise is chosen with the purpose of making a contribution to the better understanding of credit risk since CDS contracts are very popular and the CDS premiums are important indicators for the credit risk of an obligor. The model used in this thesis uses the information obtained from the firm's bond prices for estimating the survival probabilities. Each firm's bond price is used in calibration to determine the optimal set of parameters for the CIR default intensity process by a least squares method. Once the parameters are obtained, they are used to obtain CDS prices. Data for two of the Dow Jones 30 Index constituents, namely the Coca Cola Company (non-financial) and JPMorgan Chase (financial), are used for carrying out the analyses. After obtaining the model CDS spreads, the model's ability in predicting the market spreads is tested. The model is tested for two separate 3-month periods. The first of these periods is between July and September 2008 and accounts for a pre-crisis market environment. The second is between January and March 2016 and accounts for a post-crisis (stable) market environment.

In order to evaluate the performance of the model, first, the ability of the model to fit bond prices is tested by looking at the Root Mean Squared Error (RMSE) of the deviations between the model and market prices. The results show that the model is quite successful in estimating the survival probabilities.

Second, the Mean Absolute Pricing Errors (MAPEs) are calculated in order to estimate the pricing errors for the CDS premiums. Results show that the CDS premiums of both firms are better priced for the 2016 period compared to the 2008 period. Such a difference in the success of the model is attributed to the effect of the Global Financial

Crisis that created a highly volatile market environment where the CDS prices also fluctuated in large amounts, especially for the CDS contracts whose underlying assets were mostly issued by financial institutions.

Finally, in order to understand the effect of different maturities and sample periods on the magnitude of the absolute value of the pricing errors, MAPEs of each firm are regressed on dummy variables that represent these potential effects.The parameter estimates for the year dummies show that pricing errors in 2008 are higher than those in 2016 due to the deviations in bond and CDS price changes during the crisis period. Also, the parameter estimates for the firm dummies show that the Coca-Cola Co. CDS contracts are priced with a smaller error compared to those of JPMorgan Chase.

Another observation from the comparison of market and model spreads is that the model CDS spreads are more volatile compared to the market-observed spreads. This is due to the scarcity of traded bonds that are used to obtain the parameters of the survival probabilities. As stated by Elizalde (2005), small variations in the estimated parameters affect the survival probabilities, and therefore, the model CDS spreads fluctuate significantly [34].

For further research, in an attempt to improve the performance of the model, the assumptions used when pricing the CDS contracts can be simplified. First, the protection seller default risk and the default correlation can be taken into the consideration as explained by Hull and White (2001) [54]. Second, factors other than credit risk that affect the spread between risk-free and corporate debt, such as liquidity risk, can be taken into account. Finally, instead of a constant recovery rate, a stochastic recovery rate model can be used in order to include the negative correlation between default probabilities and recovery rates as shown in Altman et al. (2005) [4].

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APPENDIX A

Merton's Model

A.4.1: In the Merton framework, the risk- neutral probability $Q(t,T)$ of default at time T can be calculated as:

$$Q(t,T) = \Phi(-d_2)$$

Proof: Under Merton model, the asset values of a firm can be described by diffusion type stochastic process following a geometric Brownian motion with stochastic differential equation

$$\mathrm{d}V_t = \mu\, V_t\, \mathrm{d}t + \sigma\, V_t\, \mathrm{d}W_t$$

where μ is the mean rate of return on the assets and σ is the asset volatility.
The solution of this stochastic equation is

$$V_t = V_0\, \exp\left\{ (\mu - \frac{1}{2}\sigma^2)t + \sigma\, W_t \right\}$$

Also,

$$V_T = V_t\, \exp((r - \frac{1}{2}\sigma^2)(T-t) + \sigma(W_T - W_t))$$

In order to find the survival probability, $P_t(V_T \geq D)$ should be calculated since $V > D$ implies that there is no default.

$$
\begin{aligned}
P_t(V_T \geq D) &= P_t\big(V_t \, \exp((r - \frac{1}{2}\sigma^2)\,(T-t) + \sigma(W_T - W_t)) \geq D\big) \\
&= P_t\big(x \, \exp((r - \frac{1}{2}\sigma^2)(T-t) + \sigma \, W_{T-t}) \geq D\big) \\
&= P_t\big(x \, \exp((r - \frac{1}{2}\sigma^2)(T-t) + \sigma \, y \, \sqrt{T-t}) \geq D\big) \\
&= P_t\big(\log(x) + (r - \frac{1}{2}\sigma^2)(T-t) + \sigma \, y \, \sqrt{T-t}) \geq \log(D)\big) \\
&= P_t\bigg(y \geq \frac{\log(\frac{D}{x}) - (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\bigg) \\
&= P_t\bigg(-y < \frac{\log(\frac{x}{D}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\bigg) \\
&= P_t\bigg(z < \frac{\log(\frac{x}{D}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}\bigg)
\end{aligned}
$$

where $V_t = x$, $y = \frac{W_{T-t}}{\sqrt{T-t}}$ and $z = -y$

Therefore, the survival probability can be written as:

$$P(t,T) = P_t(V_T \geq D) = \Phi(d_2)$$

since

$$d_2 = \frac{\log(\frac{x}{D}) + (r - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

Therefore, $Q(t,T) = 1 - P(t,T) = \Phi(-d_2)$ [84]

APPENDIX B

Firms' Characteristics and Descriptive Statistics

Table B.1 describes the sector and rating of the firms considered.

Table B.1: Firms' characteristics

Firm	Sector	Year	Rating Moody's/S&P
The Coca-Cola Co.	Consumer Goods	2008	/A+
The Coca-Cola Co.	Consumer Goods	2016	/AA-
JPMorgan Chase and Co.	Financial	2008	Aa2/AA-
JPMorgan Chase and Co.	Financial	2016	A3/A-

The following Tables from B.2 to B.5give the descriptive statictics for the CDS spreads of the firms depending on the CDS maturity. Table B.2 and table B.3 describe the descriptive statictics for the CDS spreads of The Coca-Cola Co. for 2008 and 2016 respectively. Table B.4 and table B.5 describe the descriptive statictics for the CDS spreads of JPMorgan Chase and Co. for 2008 and 2016 respectively.

Table B.2: Descriptive statistics for The Coca-Cola Co. for the time period July-November 2008

Maturity	Mean	Std. Error	Median	Mode	Std. Dev.	Sample Var.	Kurtosis	Skewness	Range	Minimum	Maximum	Sum
6months	24.848	0.363	23.431	23.431	3.478	12.098	-0.688	0.952	9.763	21.479	31.242	2286.007
1-year	25.835	0.365	24.408	24.408	3.499	12.246	-0.658	0.962	10.739	22.455	33.194	2376.802
2-years	27.305	0.366	25.872	25.872	3.511	12.327	-0.630	0.970	11.228	23.919	35.147	2512.020
3-years	30.324	0.462	28.313	28.313	4.426	19.591	-0.582	1.021	15.621	26.360	41.981	2789.777
4-years	33.120	0.441	31.242	31.242	4.228	17.874	-0.460	1.039	15.621	29.289	44.910	3047.0323
5-years	37.131	0.356	36.123	36.123	3.411	11.636	0.287	1.095	15.133	33.682	48.815	3416.0737
7-years	40.554	0.333	39.540	39.540	3.189	10.172	1.067	1.157	15.621	37.099	52.720	3730.931
10-years	48.263	0.289	47.351	47.351	2.774	7.695	0.407	1.115	12.204	45.398	57.602	4440.212

Table B.3: Descriptive statistics for The Coca-Cola Co. for the time period January-March 2016

Maturity	Mean	Std. Error	Median	Mode	Std. Dev.	Sample Var.	Kurtosis	Skewness	Range	Minimum	Maximum	Sum
6months	5.066	0.003	5.07	5.07	0.026	0.000653	0.973	1.233	0.099	5.03	5.13	466.07
1-year	6.715	0.00235	6.71	6.72	0.0225	0.000506	0.863	1.126	0.090	6.68	6.77	617.74
2-years	9.804	0.00133	9.8	9.8	0.0128	0.000163	0.708	0.423	0.060	9.78	9.84	901.999
3-years	14.351	0.00177	14.35	14.34	0.017	0.000289	0.892	0.998	0.080	14.32	14.4	1320.28
4-years	18.934	0.00235	18.93	18.93	0.0226	0.00051	1.257	1.030	0.100	18.9	19	1741.91
5-years	26.074	0.00281	26.07	26.07	0.0269	0.000726	0.920	1.0193	0.120	26.03	26.15	2398.84
7-years	41.282	0.00348	41.27	41.27	0.033	0.00112	0.763	0.913	0.150	41.22	41.37	3797.9
10-years	51.376	0.0044	51.375	51.36	0.042	0.00179	-0.0099	0.581	0.190	51.29	51.48	4726.59

Table B.4: Descriptive statistics for JPMorgan Chase and Co. for the time period July-November 2008

Maturity	Mean	Std. Error	Median	Mode	Std. Dev.	Sample Var.	Kurtosis	Skewness	Range	Minimum	Maximum	Sum
6months	77.774	2.886	68.998	62.193	27.684	766.413	3.585	1.979	116.720	51.999	168.718	7155.198
1-year	80.135	2.886	71.359	64.554	27.684	766.413	3.585	1.979	116.720	54.360	171.079	7372.4102
2-years	90.098	2.881	81.359	118.553	27.638	763.873	3.578	1.974	116.720	64.165	180.885	8289.007
3-years	98.283	2.887	89.942	89.942	27.689	766.675	3.427	1.935	117.220	71.443	188.662	9042.0462
4-years	108.125	2.888	99.192	88.109	27.698	767.176	3.665	2.004	116.720	82.693	199.412	9947.528
5-years	115.468	2.884	106.942	99.637	27.665	765.367	3.667	1.997	116.720	89.942	206.662	10623.009
7-years	116.402	2.881	107.914	100.609	27.634	763.614	3.705	2.004	116.720	90.915	207.634	10708.952
10-years	117.863	2.882	109.386	101.581	27.643	764.147	3.699	2.003	116.720	92.387	209.106	10843.394

Table B.5: Descriptive statistics for JPMorgan Chase and Co. for the time period January-March 2016

Maturity	Mean	Std. Error	Median	Mode	Std. Dev.	Sample Var.	Kurtosis	Skewness	Range	Minimum	Maximum	Sum
6months	25.793	0.557	25.88	25.88	5.340	28.512	0.661	0.602	27.37	16.06	43.43	2372.91
1-year	40.642	0.698	37.74	37.74	6.695	44.819	0.336	0.895	31.88	29.59	61.47	3739.1
2-years	51.325	0.831	47.845	46.74	7.967	63.479	0.260	1.009	35.17	38.76	73.93	4721.86
3-years	57.333	0.904	54.025	57.57	8.673	75.214	0.272	1.028	36.48	44.09	80.57	5274.67
4-years	67.154	0.962	64.36	68.08	9.229	85.178	0.672	1.014	42.06	51.75	93.81	6178.18
5-years	81.553	0.988	78.27	82.7	9.475	89.770	0.844	1.064	43.99	65.37	109.36	7502.860
7-years	100.677	1.097	96.83	95.48	10.525	110.777	0.933	1.021	50.68	81.94	132.62	9262.26
10-years	120.080	1.210	116.8	123.17	11.610	134.782	0.957	0.943	56.99	98.67	155.66	11047.33

Descriptive statistics are in agreement with Table 5.2. Mean of market quotes and also maximum and minimum values of the market quotes grow monotonically with the maturity for both firms. The average CDS premiums of The Coca-Cola Co. is less than JPMorgan Chase and Co. for both periods. This is expected as JPMorgan Chase and Co. is a financial institution, it bears more credit risk. If we compare the years 2008 and 2016, we conclude that in 2008, all of the CDS quotes on different maturities are higher than the one with the same maturity in 2016, except the CDS quotes for The Coca Cola Co. with a maturity of 7 and 10 years and for JPMorgan Chase and Co. with a maturity of 10 years. This is because the stock market in U.S. is experiencing Great Financial Crisis during 2008. Standard deviations and the sample variances are both higher in 2008 for both firms. Especially for JPMorgan Chase and Co. in 2008, both statictics are far greater than the others. This can be explained by the fact that being a financial institution, JPMorgan Chase and Co. is affected by the cirisis more than The Coca-Cola Co. and, therefore, its CDS premiums are more volatile.

APPENDIX C

CDS Premium Graphs for Different Maturities

C.1 Coca-Cola Co. 2008

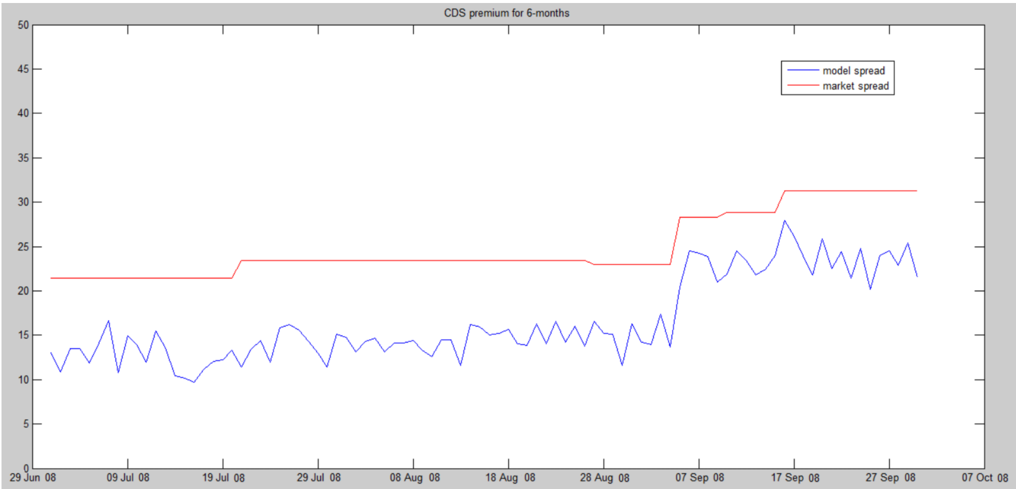

Figure C.1: CDS premiums of The Coca-Cola Co. for 6-months maturity.

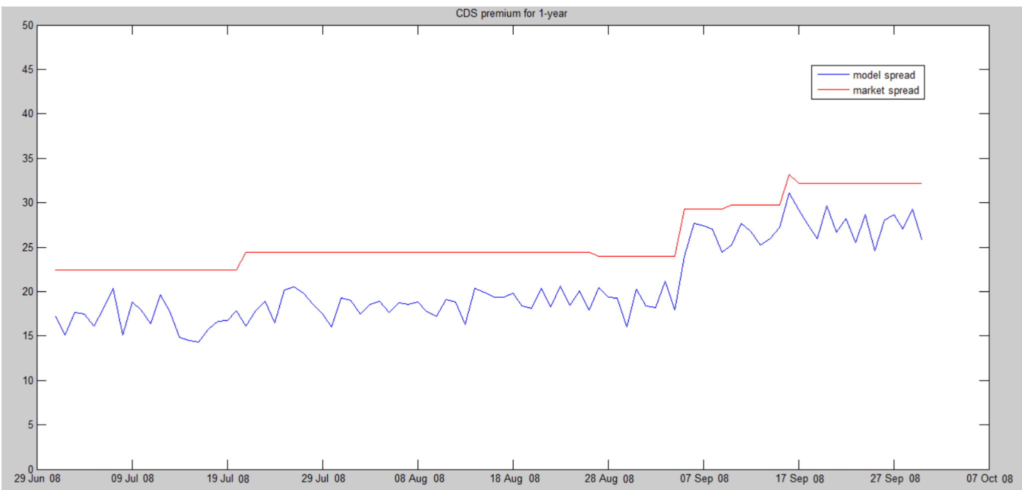

Figure C.2: CDS premiums of The Coca-Cola Co. for 1-year maturity.

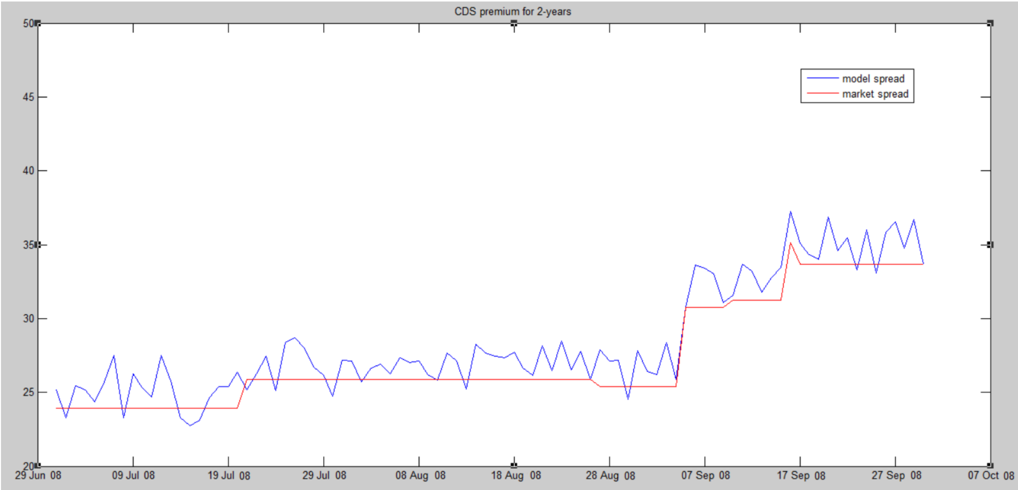

Figure C.3: CDS premiums of The Coca-Cola Co. for 2-years maturity.

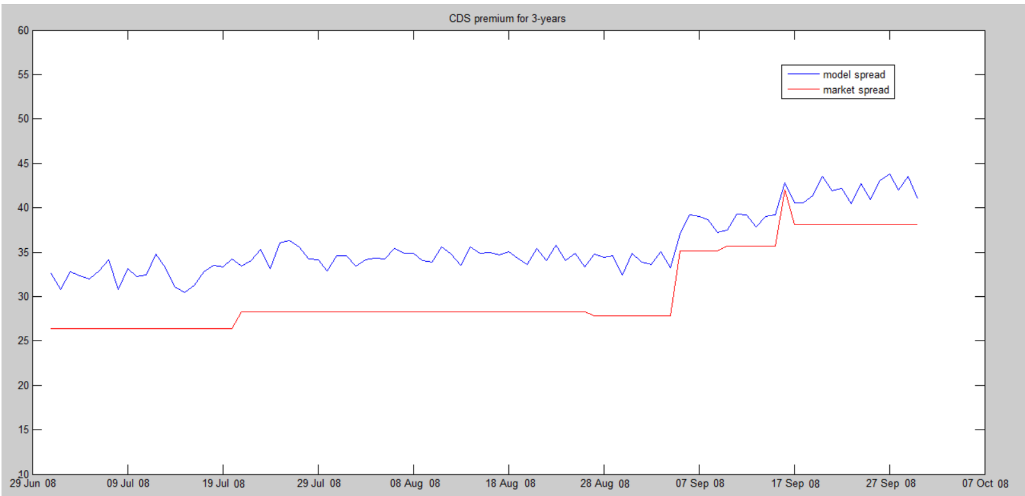

Figure C.4: CDS premiums of The Coca-Cola Co. for 3-years maturity.

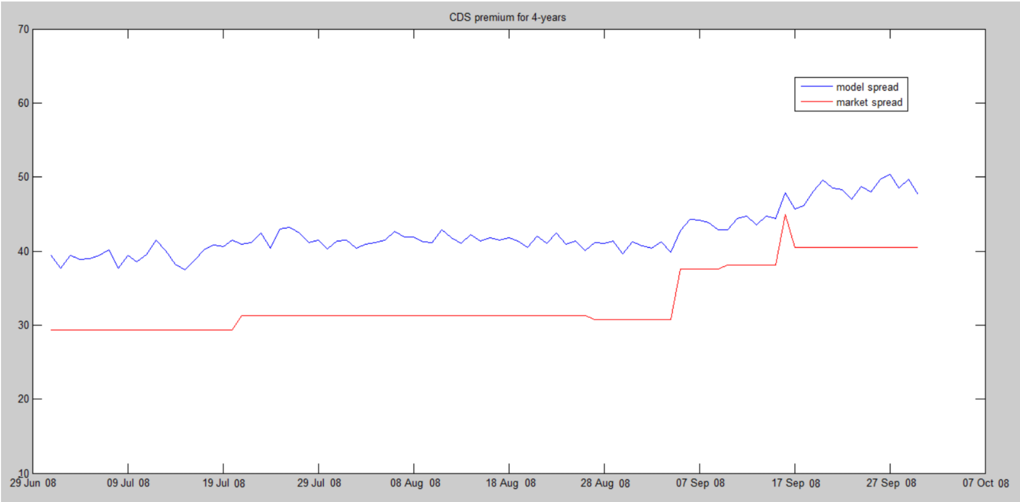

Figure C.5: CDS premiums of The Coca-Cola Co. for 4-years maturity.

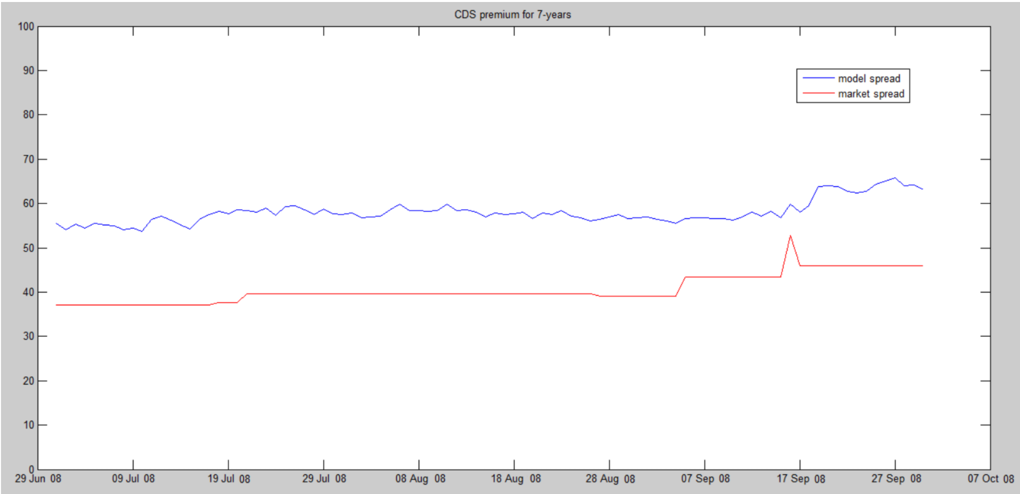

Figure C.6: CDS premiums of The Coca-Cola Co. for 7-years maturity.

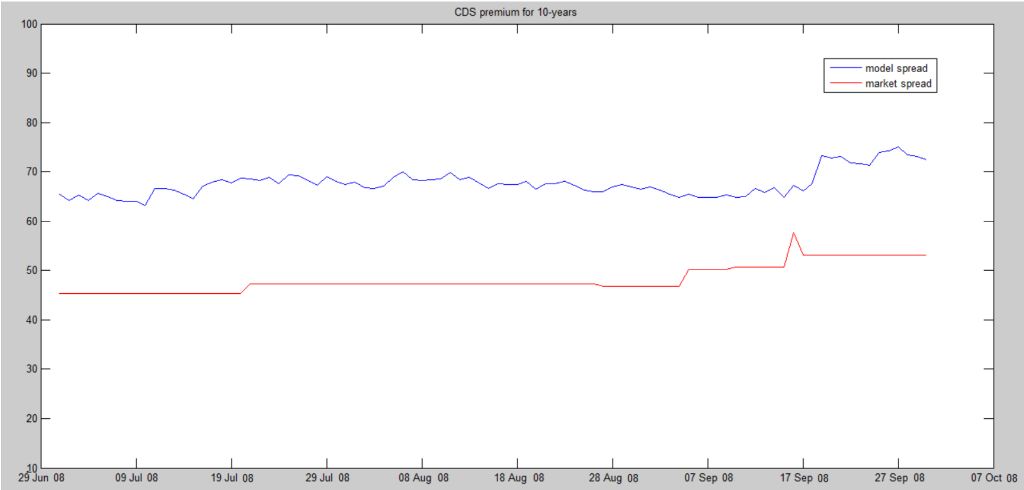

Figure C.7: CDS premiums of The Coca-Cola Co. for 10-years maturity.

C.2 Coca-Cola Co. 2016

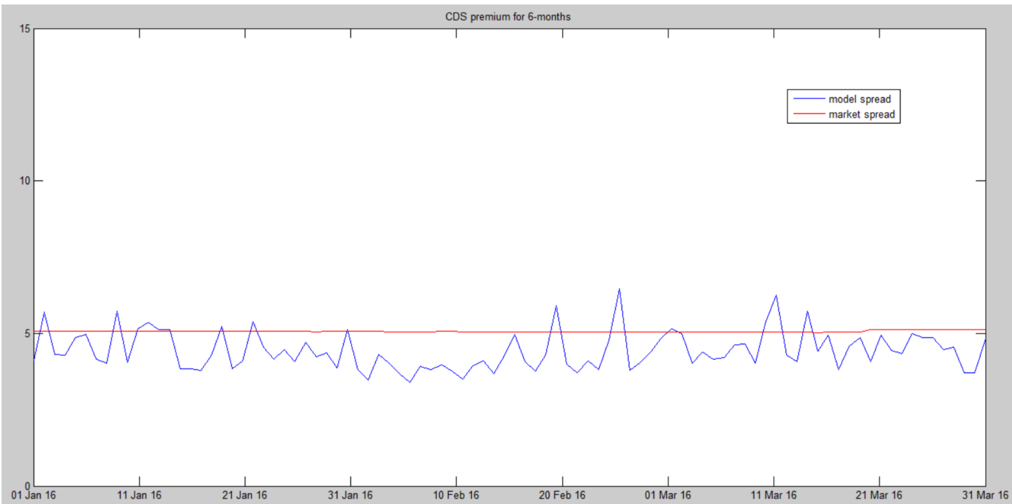

Figure C.8: CDS premiums of The Coca-Cola Co. for 6-months maturity.

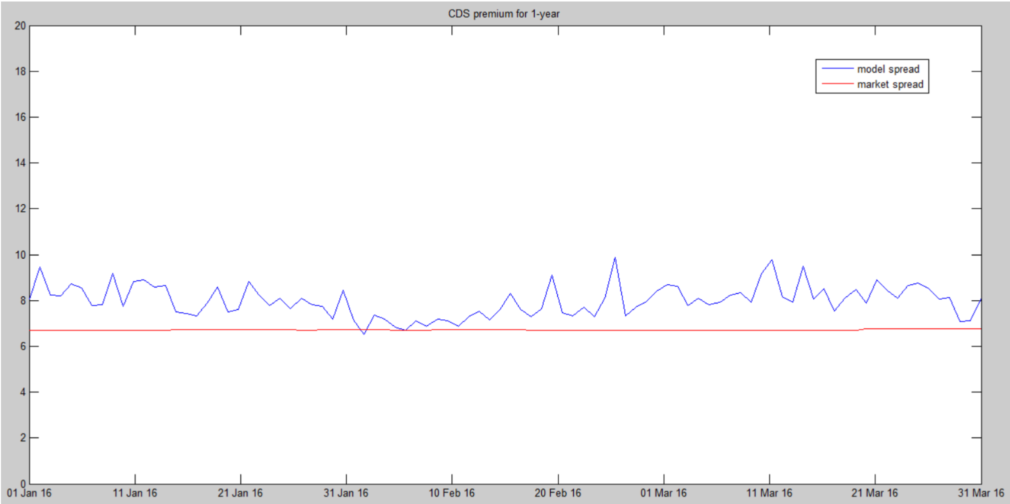

Figure C.9: CDS premiums of The Coca-Cola Co. for 1-year maturity.

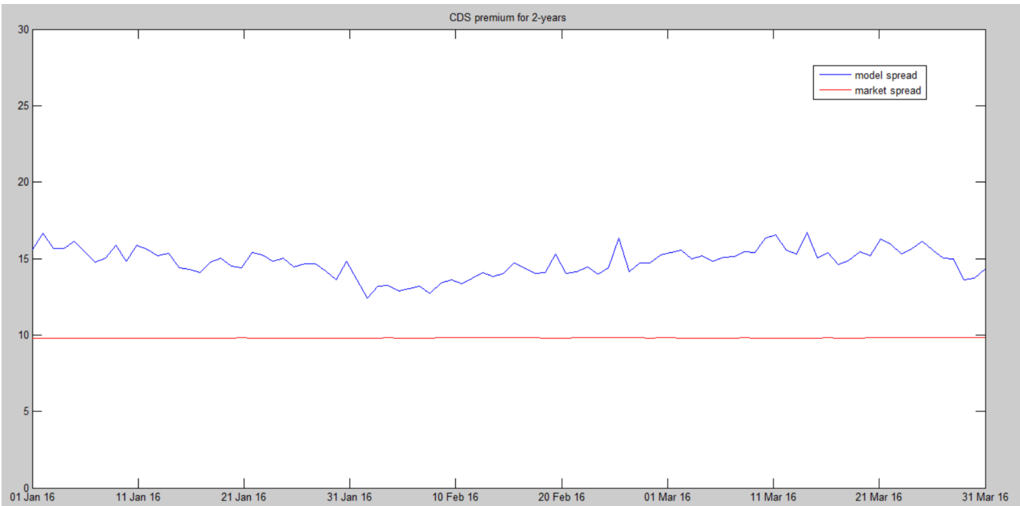

Figure C.10: CDS premiums of The Coca-Cola Co. for 2-years maturity.

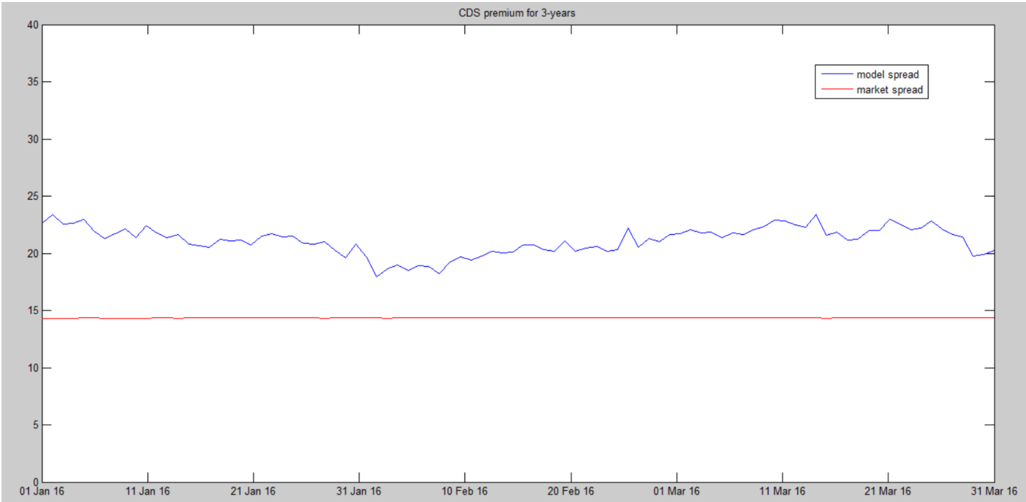

Figure C.11: CDS premiums of The Coca-Cola Co. for 3-years maturity.

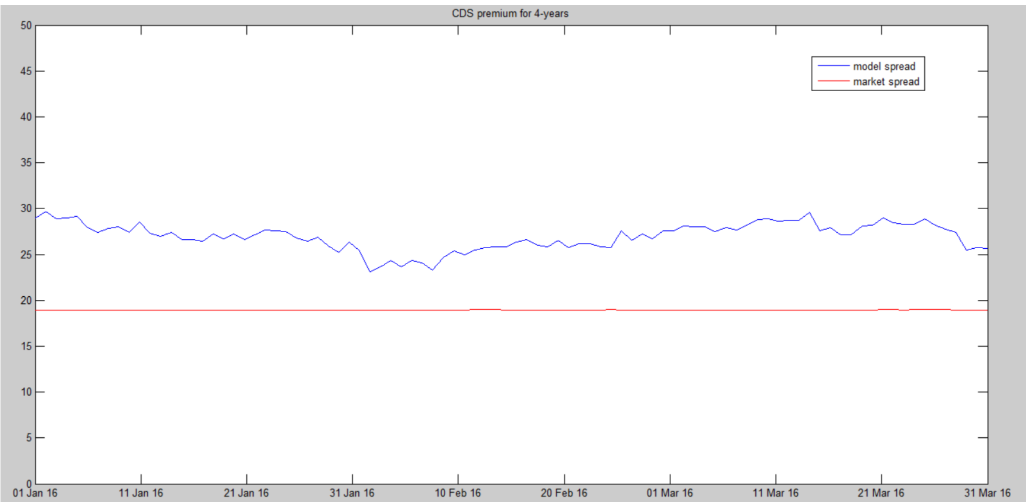

Figure C.12: CDS premiums of The Coca-Cola Co. for 4-years maturity.

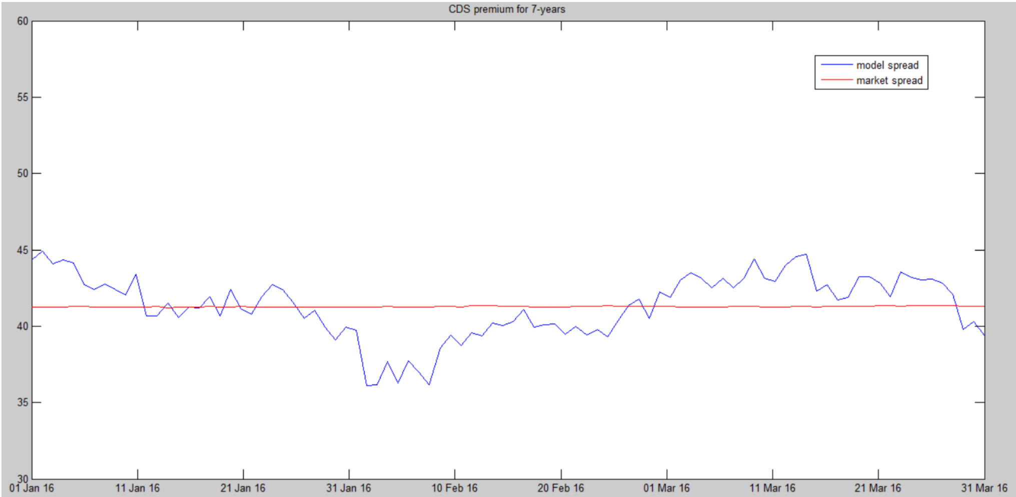

Figure C.13: CDS premiums of The Coca-Cola Co. for 7-years maturity.

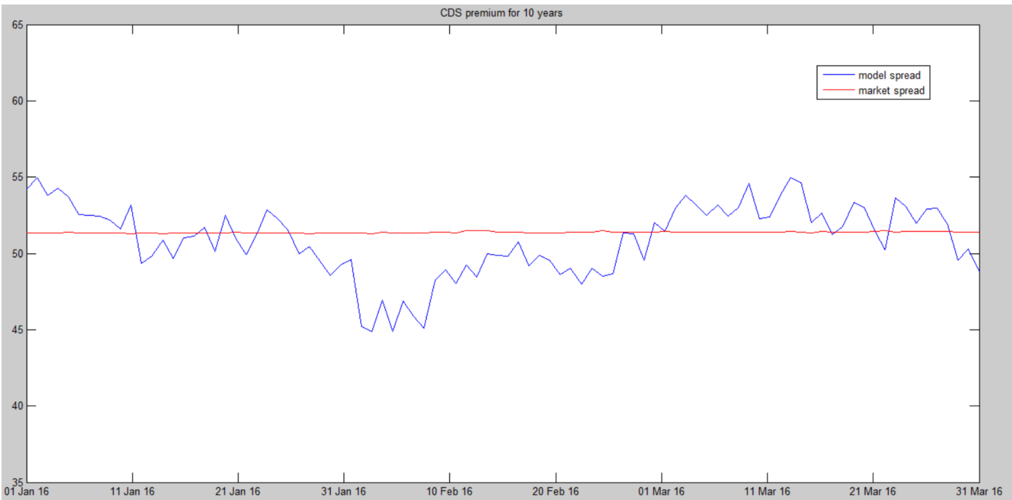

Figure C.14: CDS premiums of The Coca-Cola Co. for 10-years maturity.

C.3 JPMorgan Chase and Co. 2008

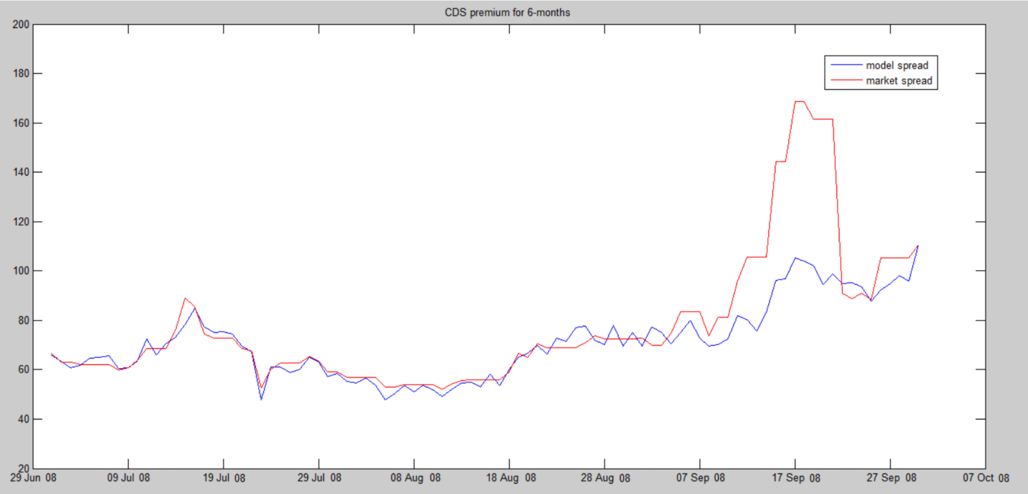

Figure C.15: CDS premiums of JPMorgan Chase and Co. for 6-months by maturity.

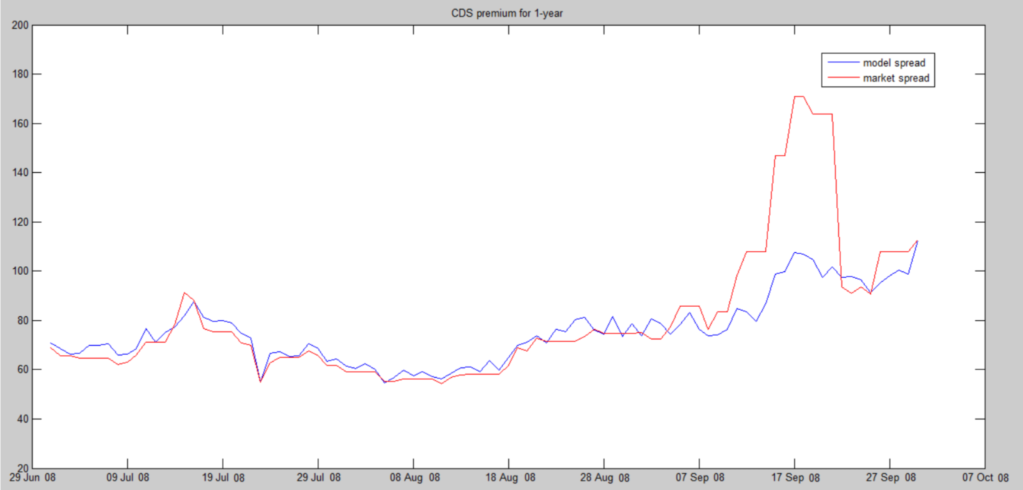

Figure C.16: CDS premiums of JPMorgan Chase and Co. for 1-year maturity.

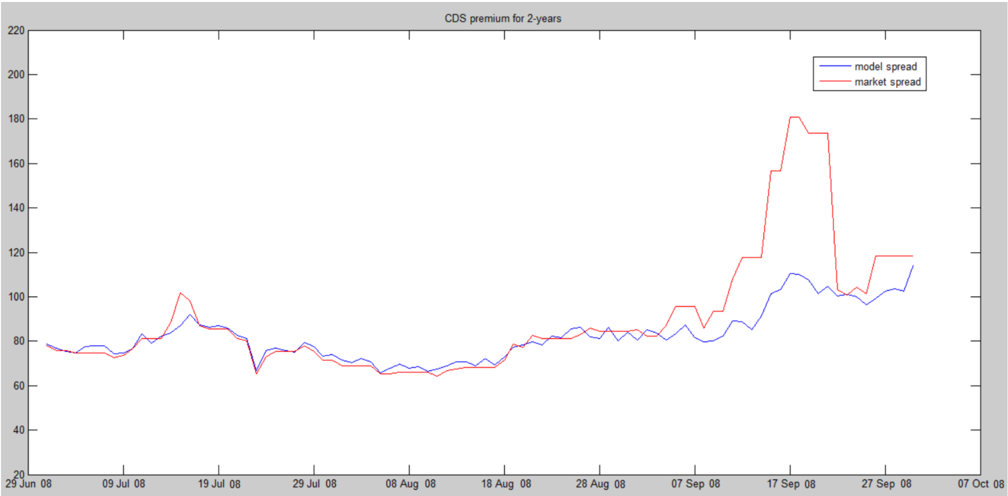

Figure C.17: CDS premiums of JPMorgan Chase and Co. for 2-years maturity.

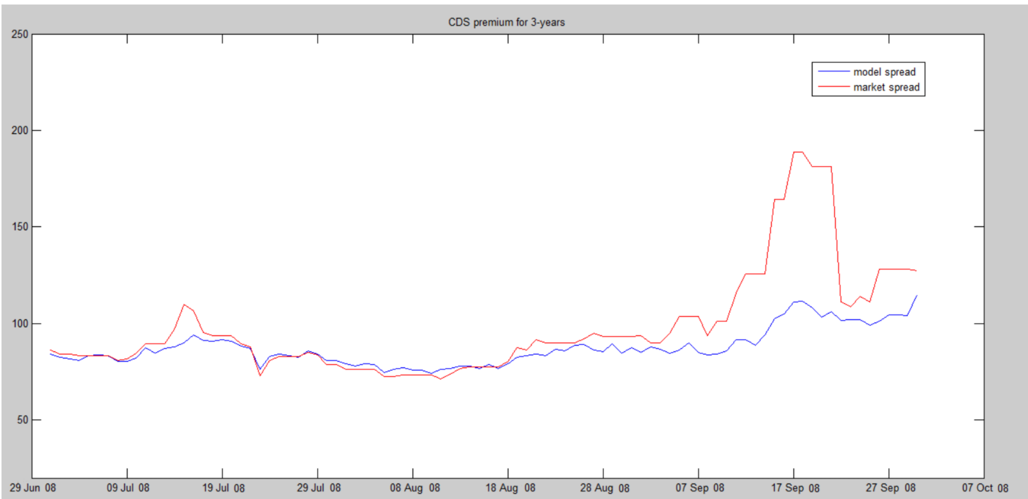

Figure C.18: CDS premiums of JPMorgan Chase and Co. for 3-years maturity.

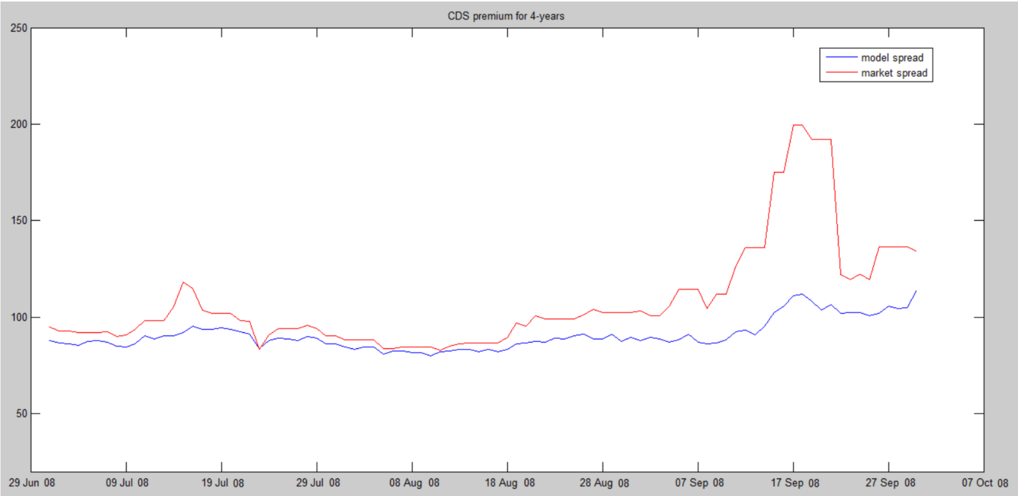

Figure C.19: CDS premiums of JPMorgan Chase and Co. for 4-years maturity.

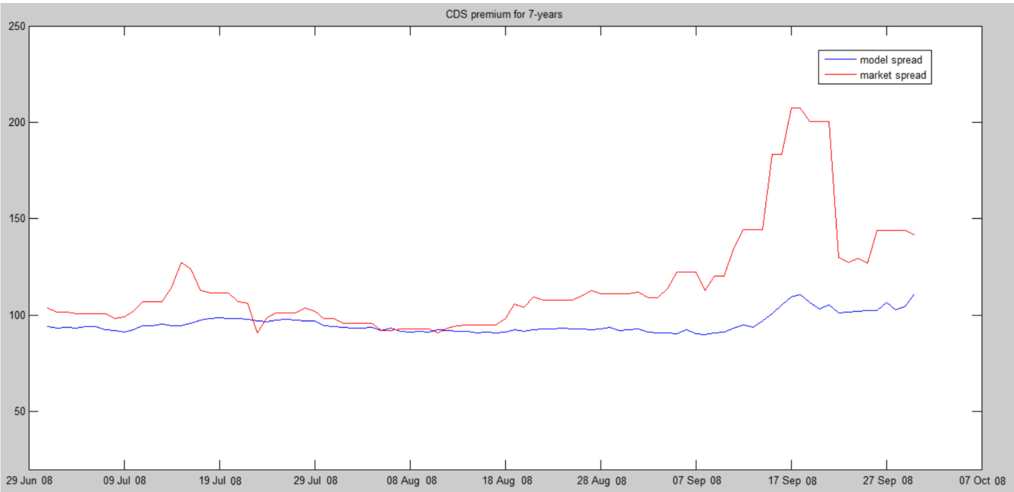

Figure C.20: CDS premiums of JPMorgan Chase and Co. for 7-years maturity.

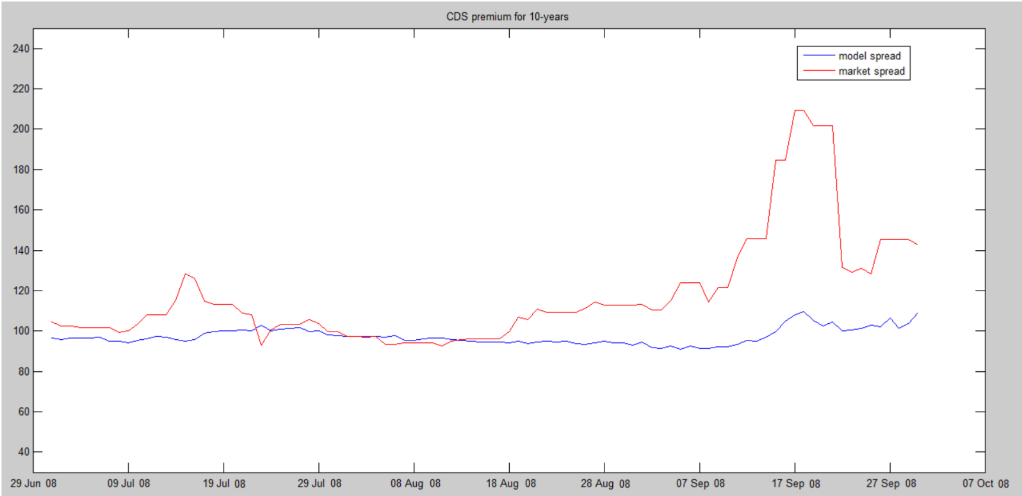

Figure C.21: CDS premiums of JPMorgan Chase and Co. for 10-years maturity.

C.4 JPMorgan Chase and Co. 2016

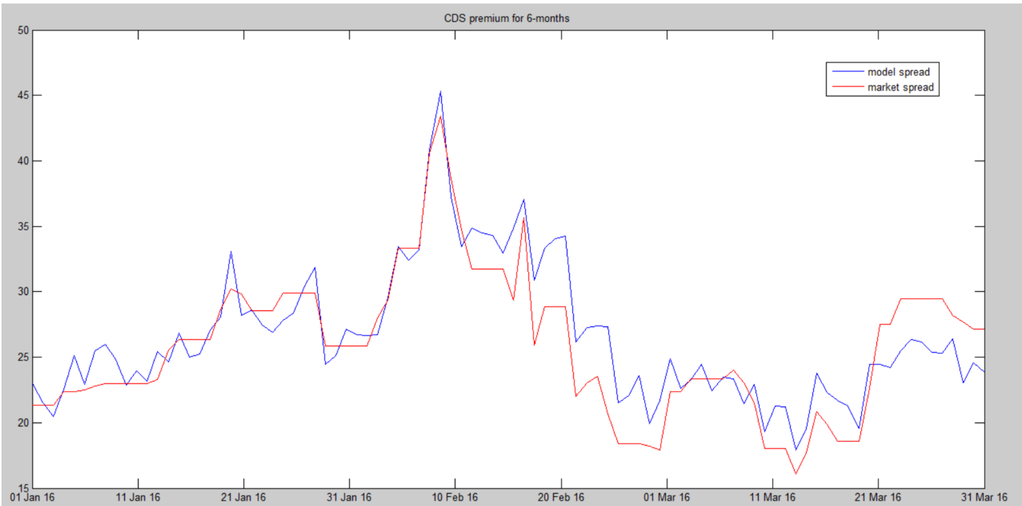

Figure C.22: CDS premiums of JPMorgan Chase and Co. for 6-months by maturity.

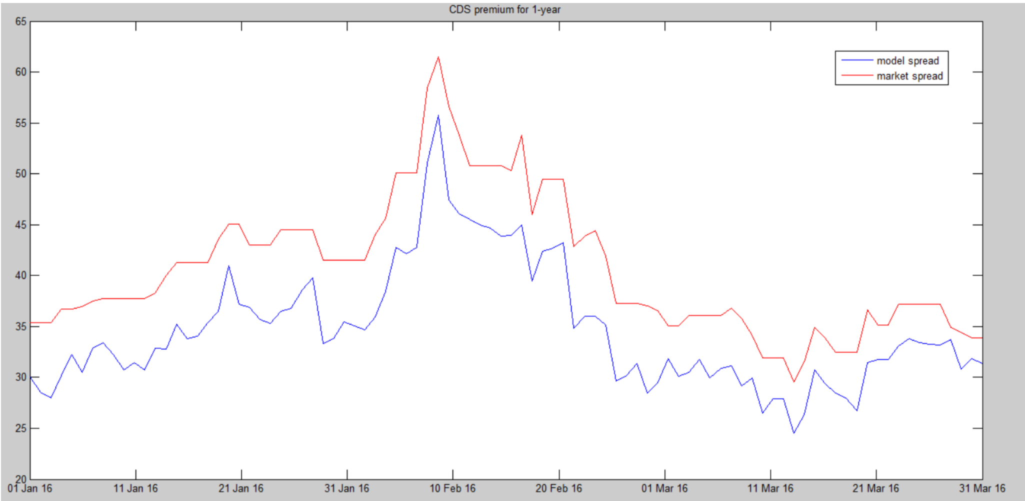

Figure C.23: CDS premiums of JPMorgan Chase and Co. for 1-year maturity.

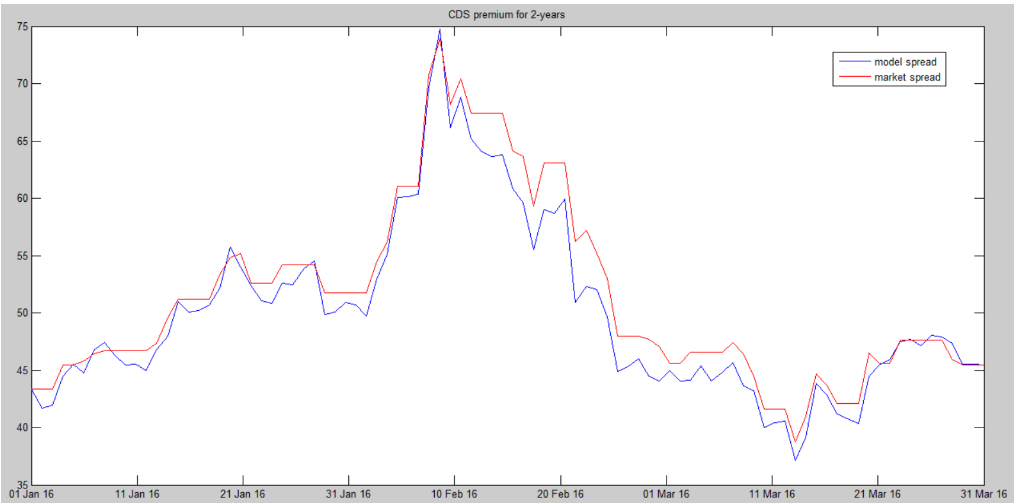

Figure C.24: CDS premiums of JPMorgan Chase and Co. for 2-years maturity.

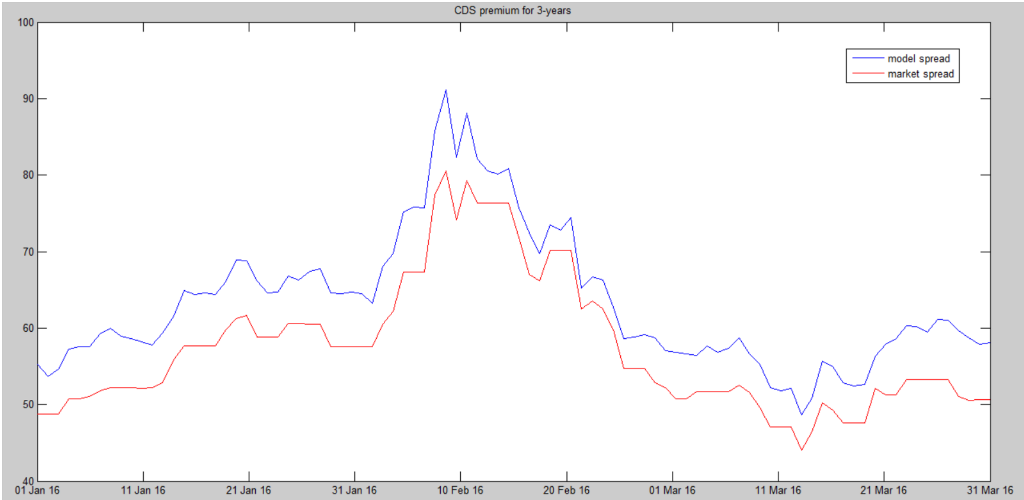

Figure C.25: CDS premiums of JPMorgan Chase and Co. for 3-years maturity.

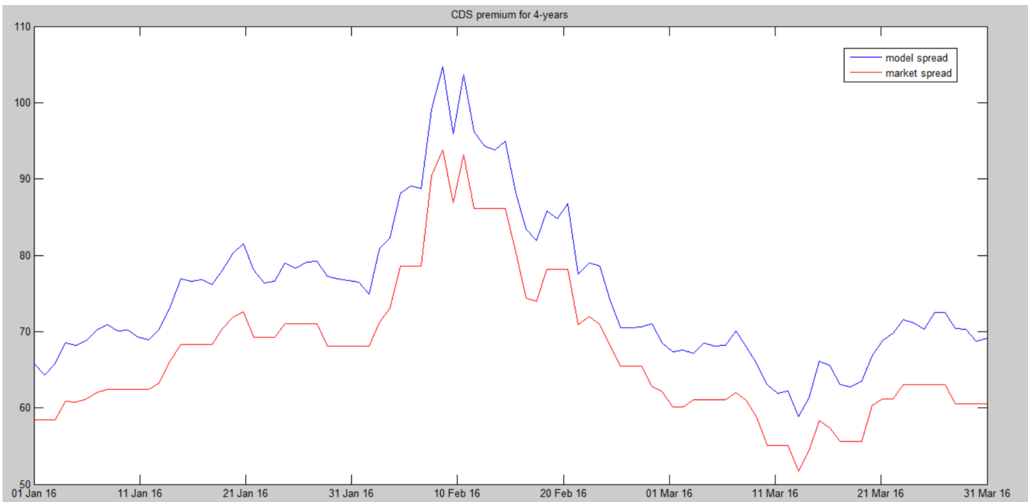

Figure C.26: CDS premiums of JPMorgan Chase and Co. for 4-years maturity.

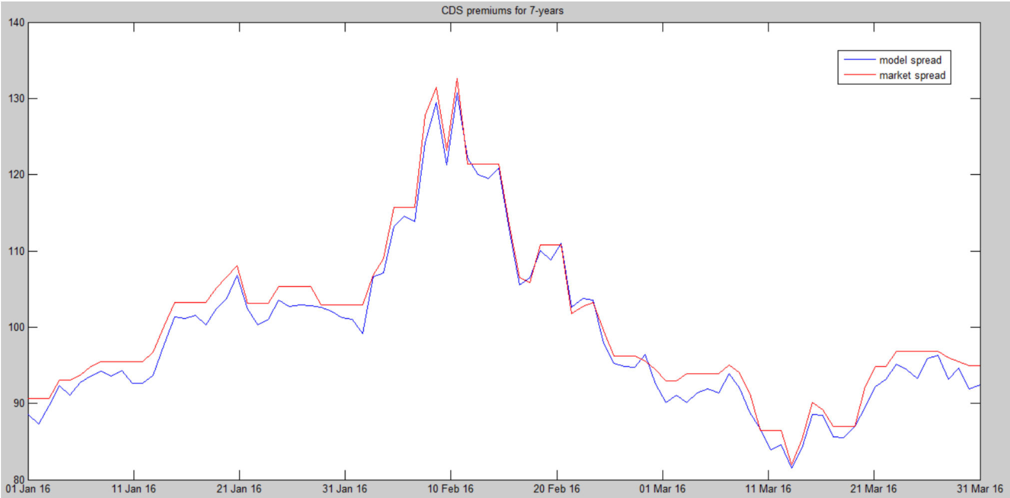

Figure C.27: CDS premiums of JPMorgan Chase and Co. for 7-years maturity.

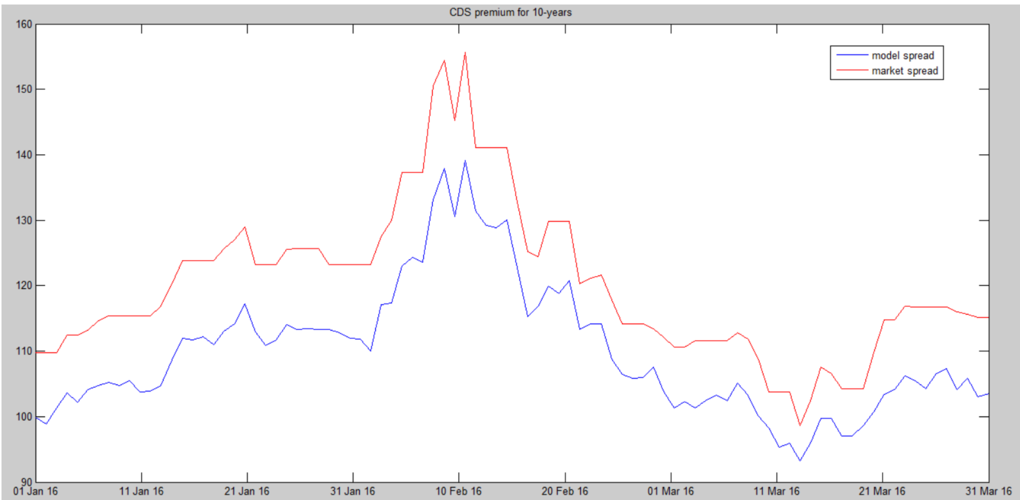

Figure C.28: CDS premiums of JPMorgan Chase and Co. for 10-years maturity.

APPENDIX D

Regression Analysis

Table D.1: Regression analysis considering only maturity as dummy variable.

The table documents the results of regressing MAPEs on the dummy variable for CDS maturity. The model given in the table contains dummy variable $(MATiY)_t$ representing the i-year(s) maturity for $i = 1, 2, 3, 4, 5, 7$ and 10. Numbers in parantheses are t-statistics.

Model:

$$\text{MAPE}_t = c_0 + c_1(MAT1Y)_t + c_2(MAT2Y)_t + c_3(MAT3Y)_t + c_4(MAT4Y)_t$$

$$+ c_5(MAT5Y)_t + c_6(MAT7Y)_t + c_7(MAT10Y)_t + \epsilon_t$$

c_0	c_1	c_2	c_3	c_4
4.8686	.3590	-.5313	2.6631	5.7876
(10.87)	(0.58)	(-0.81)	(3.88)	(7.78)
c_5	c_6	c_7	R^2	observation
5.9429	5.6261	8.3234	0.0677	2944
(7.15)	(6.27)	(9.35)		

Table D.2: Regression analysis considering only firms as dummy variable.

The table documents the results of regressing MAPEs on the dummy variable for firm. The model given in the table contains dummy variable $(CC)_t$ representing The Coca-Cola Co.. Numbers in parantheses are t-statistics.

Model:			
$\text{MAPE}_t = c_0 + c_1(CC)_t + \epsilon_t$			
c_0	c_1	R^2	observation
9.9618	-3.1437	0.0172	2944
(24.06)	(-7.17)		

Table D.3: Regression analysis considering only years as dummy variable.

The table documents the results of regressing MAPEs on the dummy variable for year. The model given in the table contains dummy variable $(YR08)_t$ representing year 2008. Numbers in parantheses are t-statistics.

Model:			
$\text{MAPE}_t = c_0 + c_1(YR08)_t + \epsilon_t$			
c_0	c_1	R^2	observation
4.4068	7.9677	0.1102	2944
(53.03)	(19.09)		